Assignment II

Logistic regression

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Exercise 2:1

This report analyses the data on periodontitis from a group of adult patients at a large dental clinic. The goal is to understand the influence of the parameter estimates between logistic regression models of the probability for periodontitis in the population as a function of dental floss use and the probability for using dental floss as a function of periodontitis status.

Table 1: Regular Use of Dental Floss and Periodontitis

	Periodontitis	No Periodontitis	Total
Used Dental Floss	22	75	148
Not Used Dental Floss	265	97	413
Total	170	340	510

Question 1: Logistic regression model of the probability for periodontitis

The logistic regression model of the probability for periodontitis in the population as a function of dental floss use is defined as:

$\text{logit}(p_x) = \log\left(\frac{p_x}{1-p_x}\right) = \beta_0 + \beta_1 x$

where:

- $x = 1$ if dental floss is regularly used, $x = 0$ otherwise.
- $p_x = P(\text{periodontitis} \mid x)$, the probability of periodontitis given x.
- β_0: The log-odds of periodontitis when $x = 0$ (no floss use).
- β_1: The change in log-odds of periodontitis when x changes from 0 to 1 (effect of using floss).

Approach:

Fit the logistic regression model to the data and interpret the parameter estimates.

Estimate Parameters:

1. Fit the logistic regression model:

The summary of the logistic regression model coefficients is presented below:

Table 2: Summary of Logistic Regression Model Coefficients

	Term	Estimate	Std. Error	z value	P-value
(Intercept)	(Intercept)	-0.5825176	0.1026175	-5.676593	0.0000000
floss	floss	-0.6439281	0.2632835	-2.445759	0.0144548

2. Expected Estimates of the parameters:

2.1. Intercept (β_0):

- β_0 represents the log-odds of periodontitis when $x = 0$ (no dental floss use): $\beta_0 = \log\left(\frac{p_0}{1-p_0}\right)$ where: $p_0 = \frac{\text{Periodontitis (No Floss)}}{\text{Total (No Floss)}} = \frac{148}{413} \approx 0.3585$
- Hence: $\beta_0 \approx \log\left(\frac{0.3585}{1-0.3585}\right) = \log(0.558) \approx -0.583$

2.2. Slope (β_1):

- β_1 represents the change in log-odds when $x = 1$ (dental floss is used): $\beta_1 = \log\left(\frac{p_1}{1-p_1}\right) - \log\left(\frac{p_0}{1-p_0}\right)$ where: $p_1 = \frac{\text{Periodontitis (Floss)}}{\text{Total (Floss)}} = \frac{22}{97} \approx 0.2268$
- Hence: $\beta_1 \approx \log\left(\frac{0.2268}{1-0.2268}\right) - \log\left(\frac{0.3585}{1-0.3585}\right) \approx \log(0.293) - \log(0.558) = -1.229 + 0.583 = -0.646$

The final logistic regression equation: $\text{logit}(p_x) = -0.583 - 0.646x$

Interpret Parameters:

- 1. β_0 = -0.583:
  - The log-odds of periodontitis for individuals who do not use dental floss is approximately -0.583.
  - The corresponding probability of periodontitis is: $p_0 = \frac{e^{-0.583}}{1+e^{-0.583}} \approx 0.358$
- 2. β_1 = -0.646:
  - The log-odds of periodontitis decreases by 0.646 when individuals use dental floss regularly.
  - This corresponds to an odds ratio of: $\text{Odds Ratio} = e^{\beta_1} = e^{-0.646} \approx 0.524$

Individuals who use dental floss regularly have approximately 52.4% lower odds of developing periodontitis compared to those who do not.

Conclusion:

- The logistic regression model provides interpretable estimates of the effect of dental floss use on periodontitis.
- Regular dental floss use is associated with a significant reduction in the odds of periodontitis.
- The logistic regression model is appropriate because the data is binary (periodontitis: yes/no) and the explanatory variable (dental floss use) is categorical.

Question 2: Logistic regression model of the probability for using dental floss

Approach :

R Output :

Observation :

Question 3:

Approach :

R Output :

Observation :

Exercise 2:2

Approach :

The risk of developing a tumor is calculated as:

$Risk = \frac{Number of Tumor}{Total Observations}$

The odds of developing a tumor is calculated as:

$Odds = \frac{Risk}{1-Risk}$

The log-odds of developing a tumor is calculated as:

$\log(Odds) = \log(\frac{Risk}{1-Risk})$

We are working with the following table:

Code :

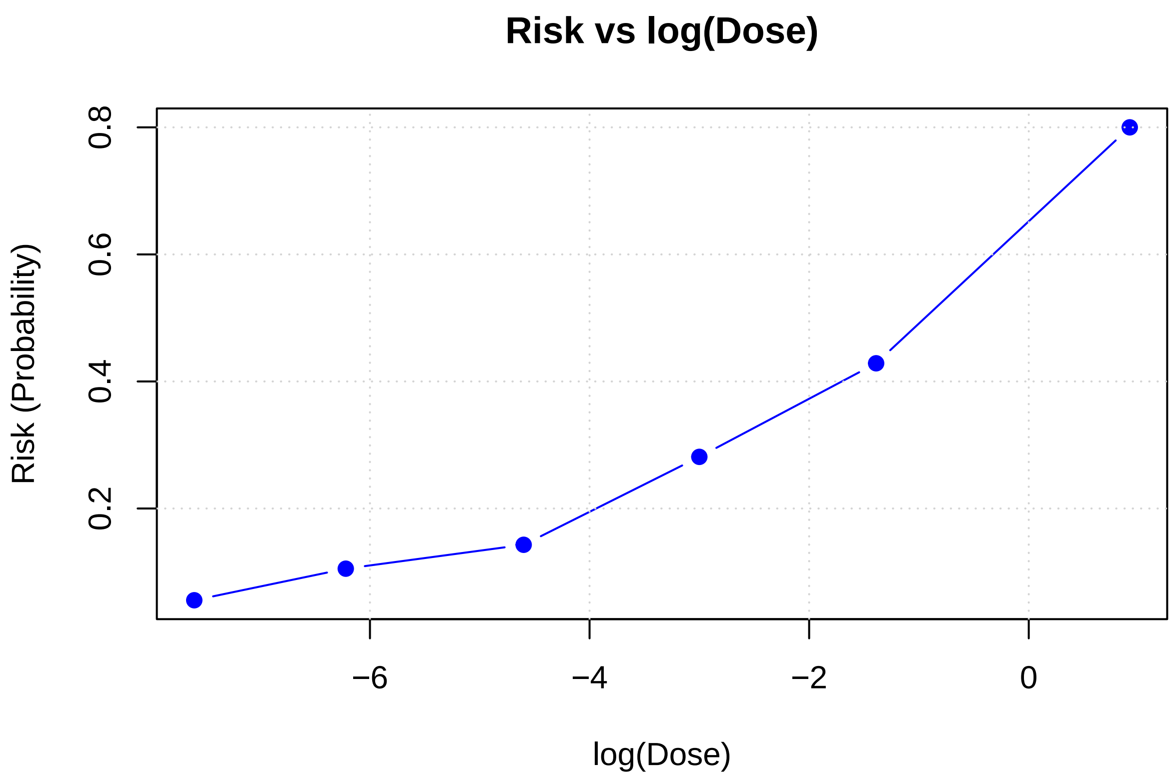
Table 3: Original Data Table

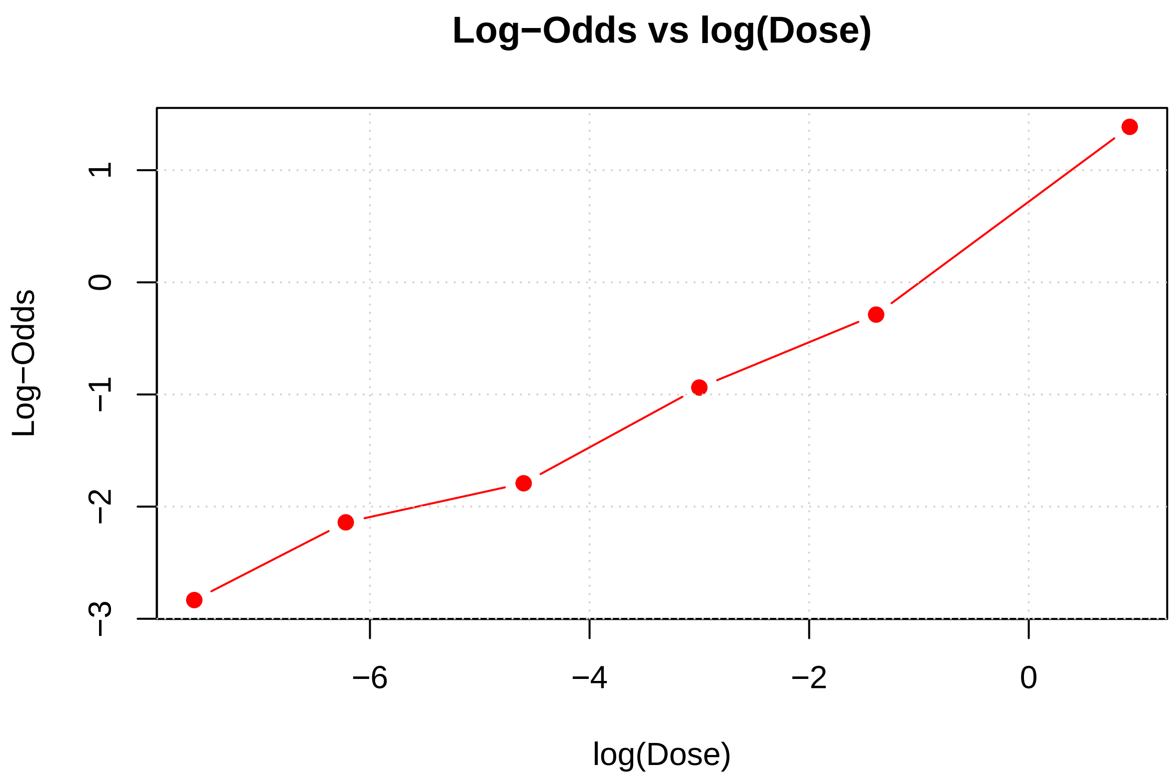
log.dose.	-7.6	-6.22	-4.6	-3	-1.39	0.92
Tumor	1.0	2.00	4.0	9	12.00	32.00
No.tumor	17.0	17.00	24.0	23	16.00	8.00
Total	18.0	19.00	28.0	32	28.00	40.00

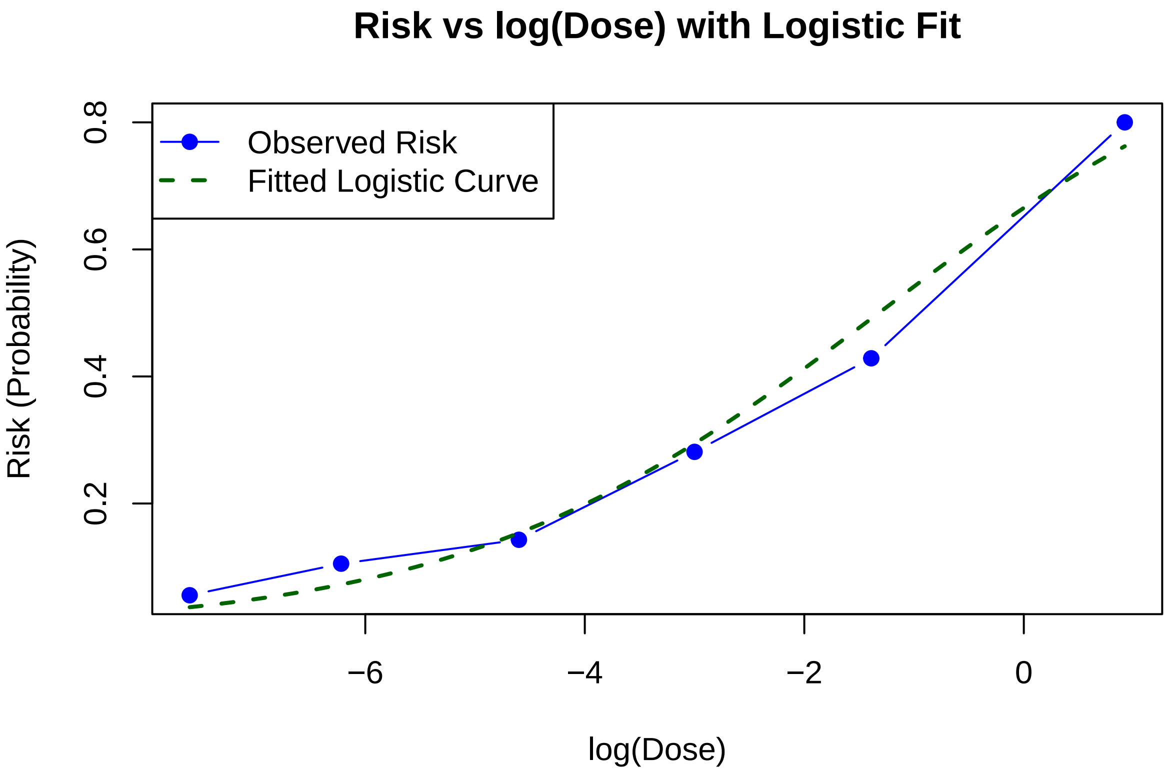
Table 4: Original Data Table

log.dose.	-7.6000	-6.2200	-4.6000	-3.0000	-1.3900	0.9200
Tumor	1.0000	2.0000	4.0000	9.0000	12.0000	32.0000
No.tumor	17.0000	17.0000	24.0000	23.0000	16.0000	8.0000
Total	18.0000	19.0000	28.0000	32.0000	28.0000	40.0000
Risk	0.0556	0.1053	0.1429	0.2812	0.4286	0.8000
Odds	0.0588	0.1176	0.1667	0.3913	0.7500	4.0000
Log.Odds	-2.8332	-2.1401	-1.7918	-0.9383	-0.2877	1.3863

As observed in the updated table, the risk, odds, and log-odds of developing a tumor are calculated for each dose level.


Risk vs log(Dose)
Risk (Probability)
0.2
0.4
0.6
0.8
−6
−4
−2
0
log(Dose)


Log−Odds vs log(Dose)
Log−Odds
−3
−2
−1
0
1
−6
−4
−2
0
log(Dose)



Output :

Observation :