Simon Kurgan April 20, 2024 1

**Proposition R.231:** Prove that  $A = \{m + n\sqrt{3} \mid m, n \in \mathbb{Z}\}$  is closed under mulitplication. **Proof:** Let  $A = \{m + n\sqrt{3} \mid m, n \in \mathbb{Z}\}$ , and let  $m + n\sqrt{3}$  and  $p + q\sqrt{3}$  be elements of A. Then

$$(m+n\sqrt{3}) (p+q\sqrt{3}) = mp + mq\sqrt{3} + np\sqrt{3} + 3qn$$
  
= 
$$(mp+3qn) + (mq+np)\sqrt{3}.$$

Since  $m, n, p, q \in \mathbb{Z}$ , mp + 3nq and mq + np are both integers. Therefore,

$$(m+n\sqrt{3})(p+q\sqrt{3}) \in A,$$

and A is closed under multiplication.  $\square$