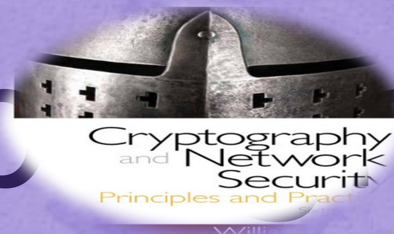


Cryptography and Network Security

Eighth Edition

by William Stallings

Chapter 6

Advanced Encryption Standard

Finite Field Arithmetic

- In the Advanced Encryption Standard (AES) all operations are performed on 8-bit bytes
- The arithmetic operations of addition, multiplication, and division are performed over the finite field $GF(2^8)$
- A field is a set in which we can do addition, subtraction, multiplication, and division without leaving the set
- Division is defined with the following rule:
  - $a / b = a (b^{-1})$
- An example of a finite field (one with a finite number of elements) is the set Z_p consisting of all the integers $\{0, 1, \ldots, p - 1\}$, where *p* is a prime number and in which arithmetic is carried out modulo *p*

Finite Field Arithmetic

If one of the operations used in the algorithm is division, then we need to work in arithmetic defined over a field

- Division requires that each nonzero element have a multiplicative inverse

For convenience and for implementation efficiency we would like to work with integers that fit exactly into a given number of bits with no wasted bit patterns

- Integers in the range 0 through $2^n - 1$, which fit into an *n*-bit word

The set of such integers, Z_{2^n}, using modular arithmetic, is not a field

- For example, the integer 2 has no multiplicative inverse in Z_{2^n}, that is, there is no integer *b*, such that $2b \bmod 2^n = 1$

A finite field containing 2^n elements is referred to as $GF(2^n)$

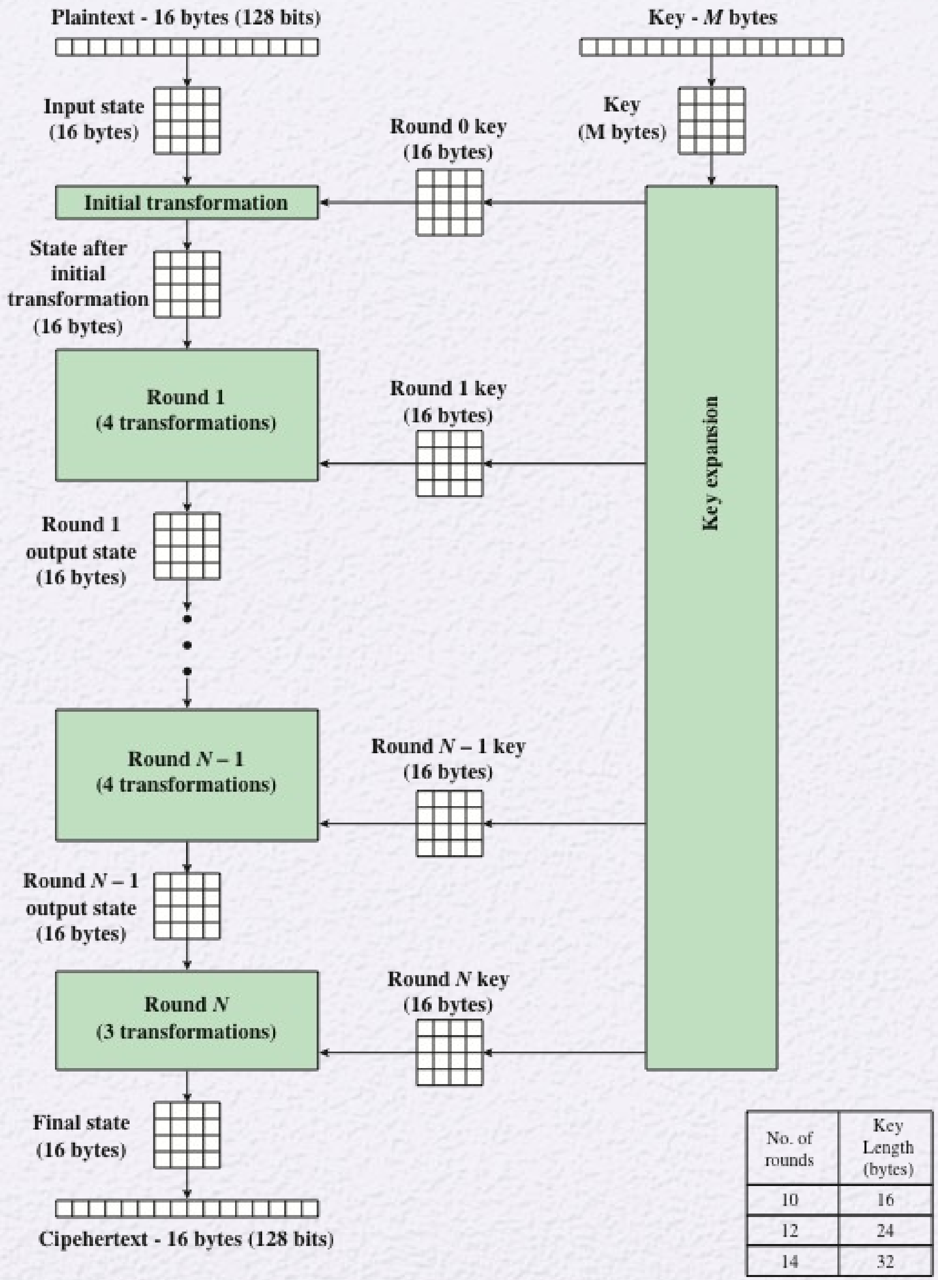
- Every polynomial in $GF(2^n)$ can be represented by an n-bit number

Table 4.5

Average Time Required for Exhaustive Key Search

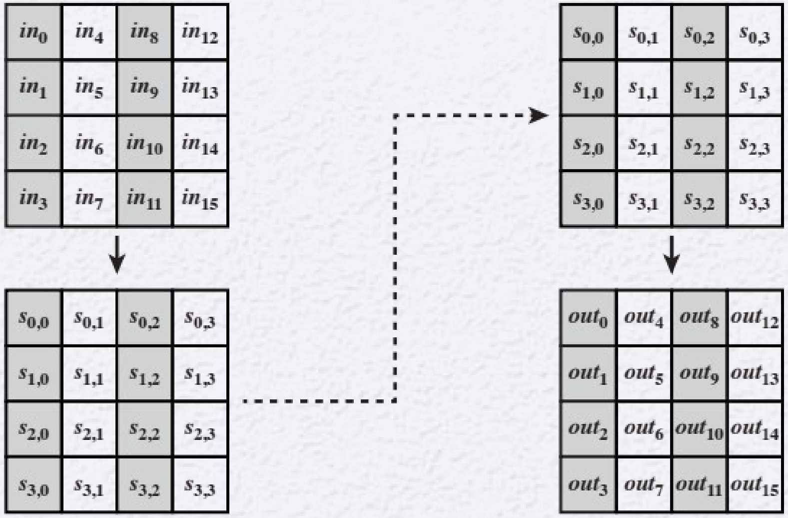
Key size (bits)	Cipher	Number of Alternative Keys	Time Required at 10^9 decryptions/s	Time Required at 10^{13} decryptions/s
56	DES	$2^{56} \approx 7.2 \times 10^{16}$	2^{55} ns = 1.125 years	1 hour
128	AES	$2^{128} \approx 3.4 \times 10^{38}$	2^{127} ns = 5.3×10^{21} years	5.3×10^{17} years
168	Triple DES	$2^{168} \approx 3.7 \times 10^{50}$	2^{167} ns = 5.8×10^{33} years	5.8×10^{29} years
192	AES	$2^{192} \approx 6.3 \times 10^{57}$	2^{191} ns = 9.8×10^{40} years	9.8×10^{36} years
256	AES	$2^{256} \approx 1.2 \times 10^{77}$	2^{255} ns = 1.8×10^{60} years	1.8×10^{56} years
26 characters (permutation)	Monoalphabetic	$26! = 4 \times 10^{26}$	2×10^{26} ns = 6.3×10^{9} years	6.3×10^{6} years

(Table can be found on page 109 in the textbook)

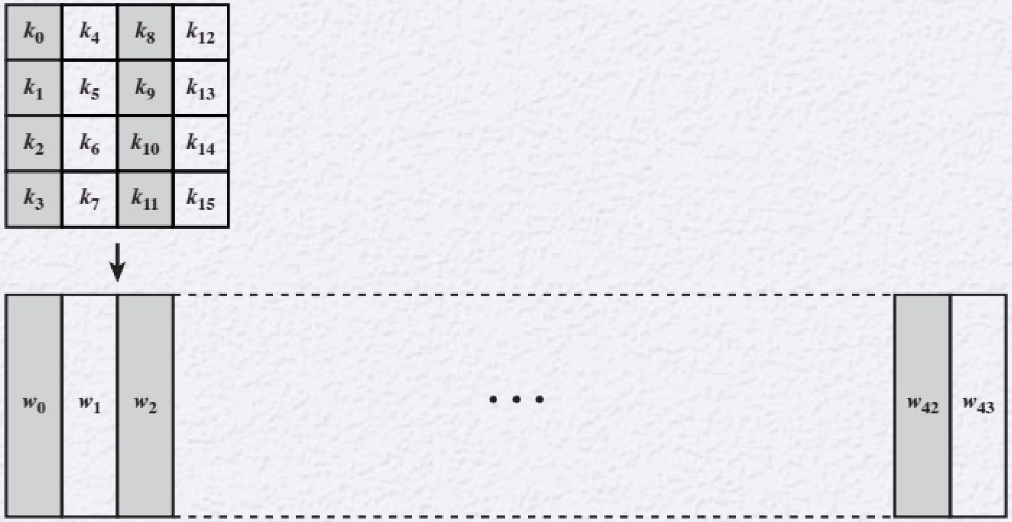


No. of rounds	Key Length (bytes)
10	16
12	24
14	32

Figure 6.1 AES Encryption Process



(a) Input, state array, and output



(b) Key and expanded key

Figure 6.2 AES Data Structures

Table 6.1
AES Parameters

Key Size (words/bytes/bits)	4/16/128	6/24/192	8/32/256
Plaintext Block Size (words/bytes/bits)	4/16/128	4/16/128	4/16/128
Number of Rounds	10	12	14
Round Key Size (words/bytes/bits)	4/16/128	4/16/128	4/16/128
Expanded Key Size (words/bytes)	44/176	52/208	60/240

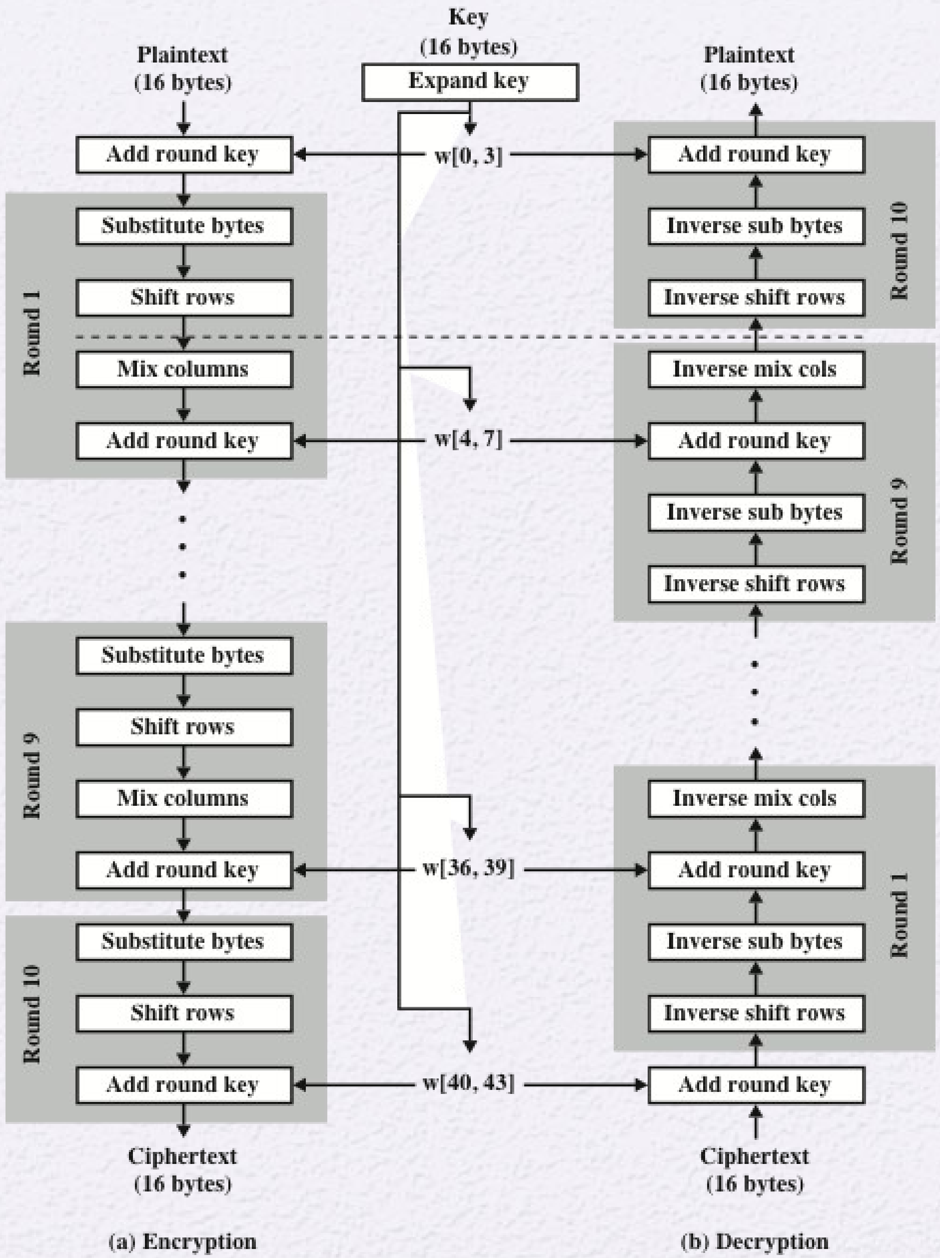


Figure 6.3 AES Encryption and Decryption

Detailed Structure

- Processes the entire data block as a single matrix during each round using substitutions and permutation
- The key that is provided as input is expanded into an array of forty-four 32-bit words, w

Four different stages are used:

- Substitute bytes – uses an S-box to perform a byte-by-byte substitution of the block
- ShiftRows – a simple permutation
- MixColumns – a substitution that makes use of arithmetic over $GF(2^8)$
- AddRoundKey – a simple bitwise XOR of the current block with a portion of the expanded key

- The cipher begins and ends with an AddRoundKey stage
- Can view the cipher as alternating operations of XOR encryption (AddRoundKey) of a block, followed by scrambling of the block (the other three stages), followed by XOR encryption, and so on
- Each stage is easily reversible
- The decryption algorithm makes use of the expanded key in reverse order, however the decryption algorithm is not identical to the encryption algorithm
- State is the same for both encryption and decryption
- Final round of both encryption and decryption consists of only three stages

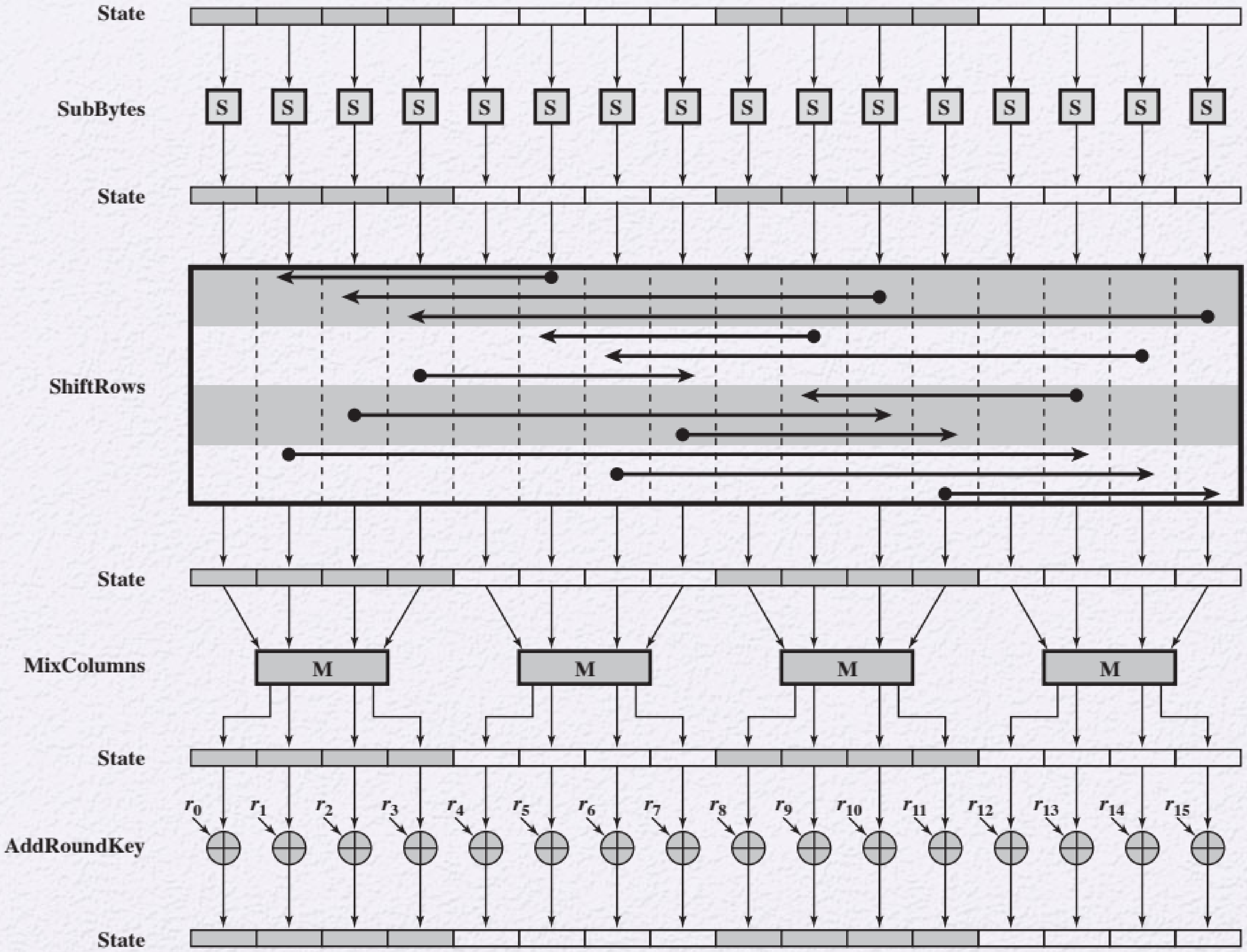


Figure 6.4 AES Encryption Round

AES Implementation

- AES decryption cipher is not identical to the encryption cipher
  - The sequence of transformations differs although the form of the key schedules is the same
  - Has the disadvantage that two separate software or firmware modules are needed for applications that require both encryption and decryption

Two separate changes are needed to bring the decryption structure in line with the encryption structure

The first two stages of the decryption round need to be interchanged

The second two stages of the decryption round need to be interchanged

Implementation Aspects

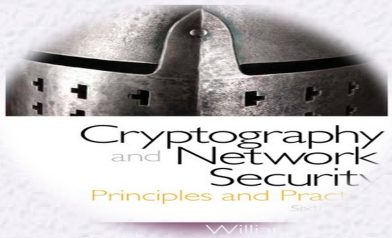
- AES can be implemented very efficiently on an 8-bit processor
- AddRoundKey is a bytewise XOR operation
- ShiftRows is a simple byte-shifting operation
- SubBytes operates at the byte level and only requires a table of 256 bytes
- MixColumns requires matrix multiplication in the field $GF(2^8)$, which means that all operations are carried out on bytes

Implementation Aspects

- Can efficiently implement on a 32-bit processor
  - Redefine steps to use 32-bit words
  - Can precompute 4 tables of 256-words
  - Then each column in each round can be computed using 4 table lookups + 4 XORs
  - At a cost of 4Kb to store tables
- Designers believe this very efficient implementation was a key factor in its selection as the AES cipher

Summary

- Present an overview of the general structure of Advanced Encryption Standard (AES)
- Understand the four transformations used in AES



- Explain the AES key expansion algorithm
- Understand the use of polynomials with coefficients in GF(2^8)