# Discussion 2

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$X_{t;r}$ , $X_{t+j;m}$

$$V_{rm}(j) = \sqrt{\| g^{(r,m)}(u,v) \|^2} \qquad r,m = 1, \dots, d$$

$$j = 0, \pm 1, \dots$$

$j = 0$

$$\begin{pmatrix} X_{t;1} \\ \vdots \\ X_{t;d} \end{pmatrix} \qquad t = 1, \dots, n$$

$$V^{(2)}(0) = \begin{bmatrix} V^2_{rm}(0) \end{bmatrix} \rightarrow \text{positive definite}$$

$$V(0) = \begin{bmatrix} V_{rm}(0) \end{bmatrix} \qquad V(0) = \sqrt{V^2(0)}$$

↳ square root elementwise

$$R^2_{rm}(j) = \frac{V^2_{rm}(j)}{\{V^2_{rr}(0)\}^{1/2} \{V^2_{mm}(0)\}^{1/2}} \leq 1$$

$$R_{rm}(0) = \frac{V_{rm}(0)}{\sqrt{V_{rr}(0)}\sqrt{V_{mm}(0)}} \rightarrow < 1$$

$$R_x(0) = \left( R_{rm}(0) \right) \qquad r,m=1,\dots,d$$

$$R^{(2)}(0) = \left( R^2_{rm}(0) \right) \longrightarrow \text{o.k}$$

same convention

$$\hat{V}_{rm}(0) = \frac{1}{(n-0)^2} \sum_{t,s=1}^{n-0} A^r_{ts} B^m_{ts}$$

$$\hat{R}_{rm}(0) = \frac{\hat{V}_{rm}(0)}{\sqrt{\hat{V}_{rr}(0)}\sqrt{\hat{V}_{mm}(0)}}$$

$$\hat{V}_{rr}(0) = \frac{1}{(n-0)^2} \sum_{t,s=1}^{n-0} A^r_{ts} B^r_{ts}$$

Given a data set $(X_t)$

<u>Step #1:</u> Calculate all test statistics $\hat{R}_{rm}(j)$,

$j=1,\dots,$ max lag

<u>Step #2:</u> Generate $(W_t)$ i.i.d $N(0,1)$

$$\hat{R}^*_{rm}(j) = \frac{1}{(n-j)^2} \sum_{t,s=1}^{n} W^* h(\dots) W^*$$

Step #3

Calculate $$P_{rm}(j) = \frac{1}{(B+1)} \sum 1\left(\hat{R}^{*}_{rm}(j) > \hat{R}_{rm}(j)\right)$$

$j = 1, \dots, maxlag$

Step 4:

Adjust p-values $\longrightarrow \tilde{P}_{rm}(j)$ ?

Step 5

$$\tilde{P}_{rm}(j) = \frac{\#\left\{\hat{R}^{*}_{rm}(j) \geq c_{rm}(j)\right\}}{B}$$

Step 6 $\quad c = \max_{r,m} c_{rm}(j)$

1.) Computation using matrices, step by step in the case of i.i.d r.v.

2) Consider simulated distribution of $\max_{r,m,j} \hat{R}_{rm}(j)$

(Gumbel ?)