KAN Kolmogorov Arnold Network Note

研究任務

1. 閱讀 KAN Kolmogorov-Arnold Network 論文
2. 設計 KAN 網路系統階層式架構 IDEF0
3. 設計 KAN 網路系統每個功能模組離散事件建模 Grafcet
4. 以 MIAT 方法論合成每個 Grafcet 控制器電路
5. 以 ChatGPT 合成每個 Grafcet Datapath 電路
6. FPGA 整合驗證

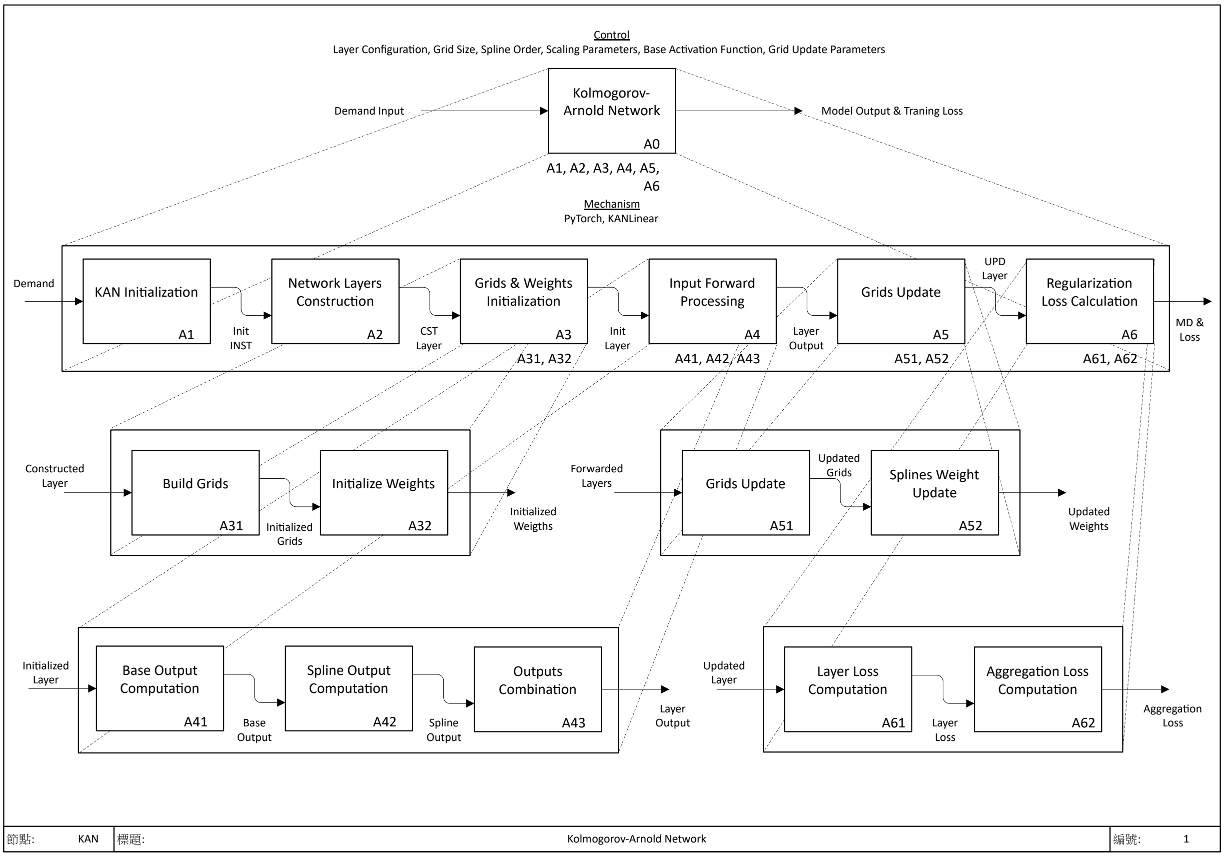
> ⚠ **Attention**
>
> Kolmogorov-Arnold Network 之研究任務須於六月底完成，作為暑期實習前置條件。

> ✎ **Seealso**
>
> 目標預定之所有（第一到第六點）研究任務，當前初版皆已完成，接下來主要目標為優化相關電路設計與輸出結果。

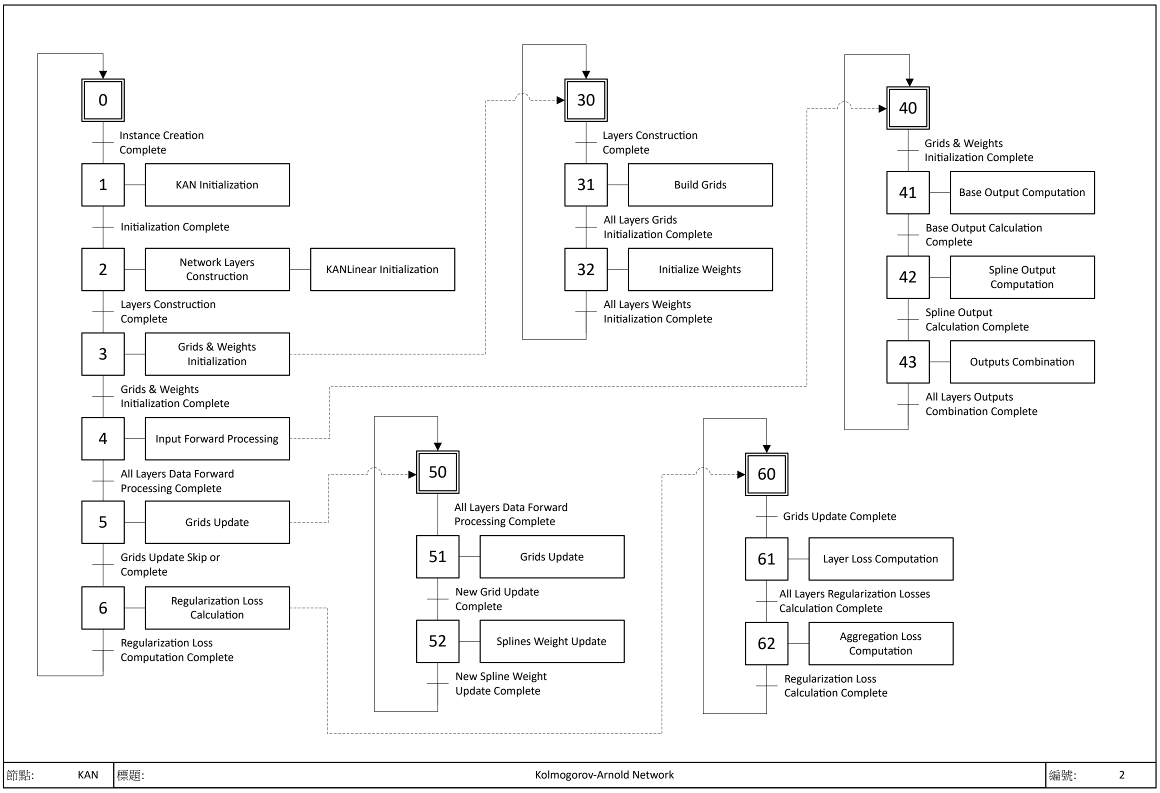
開放原始碼

- Code: https://github.com/KindXiaoming/pykan
- Reference: https://arxiv.org/abs/2404.19756

設計階層式架構 IDEF0



設計功能模組離散事件建模 Grafcet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Instance Creation Complete
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Initialization Complete
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Grids & Weights Initialization Complete
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節點: KAN
標題: Kolmogorov-Arnold Network
編號: 2

Python 模擬驗證

- 基於 IDEF0 和 Grafcet 重構後之 Kolmogorov-Arnold Network (PyTorch)

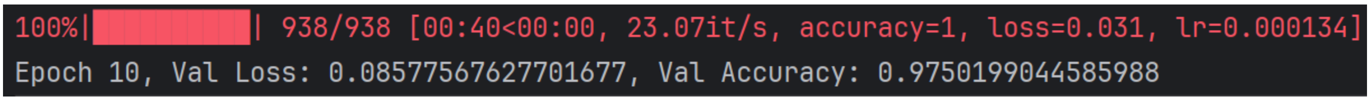
```python
import math

import torch
import torch.nn.functional as F


class KANLinear(torch.nn.Module):
    def __init__(
            self,
            in_features,
            out_features,
            grid_size=5,
            spline_order=3,
            scale_base=1.0,
            scale_spline=1.0,
            enable_standalone_scale_spline=True,
            base_activation=torch.nn.SiLU,
            grid_eps=0.02,
            grid_range=[-1, 1],
    ):
        super(KANLinear, self).__init__()
        self.in_features = in_features
        self.out_features = out_features
        self.grid_size = grid_size
        self.spline_order = spline_order

        # 構建網格點
        self.grid = self.build_grid(grid_range, grid_size, spline_order)

        # 初始化基礎權重和樣條權重
        self.base_weight, self.spline_weight, self.spline_scaler = self.initialize_weights(
            out_features, in_features, grid_size, spline_order, scale_base, scale_spline,
enable_standalone_scale_spline
        )

        self.scale_base = scale_base
        self.scale_spline = scale_spline
        self.enable_standalone_scale_spline = enable_standalone_scale_spline
        self.base_activation = base_activation()
        self.grid_eps = grid_eps

    def build_grid(self, grid_range, grid_size, spline_order):
        h = (grid_range[1] - grid_range[0]) / grid_size
        grid = (
            (
                    torch.arange(-spline_order, grid_size + spline_order + 1) * h
                    + grid_range[0]
            )
            .expand(self.in_features, -1)
            .contiguous()
        )
        return grid

    def initialize_weights(self, out_features, in_features, grid_size, spline_order, scale_base, scale_spline,
                           enable_standalone_scale_spline):
        base_weight = torch.nn.Parameter(torch.Tensor(out_features, in_features))
        spline_weight = torch.nn.Parameter(
            torch.Tensor(out_features, in_features, grid_size + spline_order)
        )
        if enable_standalone_scale_spline:
            spline_scaler = torch.nn.Parameter(
                torch.Tensor(out_features, in_features)
            )
        else:
            spline_scaler = None
        torch.nn.init.kaiming_uniform_(base_weight, a=math.sqrt(5) * scale_base)
        torch.nn.init.kaiming_uniform_(spline_weight, a=math.sqrt(5) * scale_spline)
        if enable_standalone_scale_spline:
            torch.nn.init.kaiming_uniform_(spline_scaler, a=math.sqrt(5) * scale_spline)
        return base_weight, spline_weight, spline_scaler

    def b_splines(self, x: torch.Tensor):
        bases = self.calculate_b_spline_bases(x)
        return bases.contiguous()

    def calculate_b_spline_bases(self, x: torch.Tensor):
        grid: torch.Tensor = (
            self.grid
        )  # (in_features, grid_size + 2 * spline_order + 1)
        x = x.unsqueeze(-1)
        bases = ((x >= grid[:, :-1]) & (x < grid[:, 1:])).to(x.dtype)
        for k in range(1, self.spline_order + 1):
            bases = (
                            (x - grid[:, : -(k + 1)])
                            / (grid[:, k:-1] - grid[:, : -(k + 1)])
                            * bases[:, :, :-1]
                    ) + (
                            (grid[:, k + 1:] - x)
                            / (grid[:, k + 1:] - grid[:, 1:(-k)])
                            * bases[:, :, 1:]
                    )
        return bases

    def curve2coeff(self, x: torch.Tensor, y: torch.Tensor):
        A = self.b_splines(x).transpose(
            0, 1
        )  # (in_features, batch_size, grid_size + spline_order)
        B = y.transpose(0, 1)  # (in_features, batch_size, out_features)
        solution = torch.linalg.lstsq(
            A, B
        ).solution  # (in_features, grid_size + spline_order, out_features)
        result = solution.permute(
            2, 0, 1
        )  # (out_features, in_features, grid_size + spline_order)
        return result.contiguous()

    @property
    def scaled_spline_weight(self):
        if self.enable_standalone_scale_spline:
            return self.spline_weight * self.spline_scaler.unsqueeze(-1)
        else:
            return self.spline_weight

    def forward(self, x: torch.Tensor):
        base_output = self.compute_base_output(x)
        spline_output = self.compute_spline_output(x)
        return base_output + spline_output

    def compute_base_output(self, x: torch.Tensor):
        return F.linear(self.base_activation(x), self.base_weight)

    def compute_spline_output(self, x: torch.Tensor):
        return F.linear(
            self.b_splines(x).view(x.size(0), -1),
            self.scaled_spline_weight.view(self.out_features, -1),
        )

    @torch.no_grad()
    def update_grid(self, x: torch.Tensor, margin=0.01):
        batch = x.size(0)

        splines = self.b_splines(x)  # (batch, in, coeff)
        splines = splines.permute(1, 0, 2)  # (in, batch, coeff)
        orig_coeff = self.scaled_spline_weight  # (out, in, coeff)
        orig_coeff = orig_coeff.permute(1, 2, 0)  # (in, coeff, out)
        unreduced_spline_output = torch.bmm(splines, orig_coeff)  # (in, batch, out)
        unreduced_spline_output = unreduced_spline_output.permute(
            1, 0, 2
        )  # (batch, in, out)

        x_sorted = torch.sort(x, dim=0)[0]
        grid_adaptive = x_sorted[
            torch.linspace(
                0, batch - 1, self.grid_size + 1, dtype=torch.int64, device=x.device
            )
        ]

        uniform_step = (x_sorted[-1] - x_sorted[0] + 2 * margin) / self.grid_size
        grid_uniform = (
                torch.arange(
                    self.grid_size + 1, dtype=torch.float32, device=x.device
                ).unsqueeze(1)
                * uniform_step
                + x_sorted[0]
                - margin
        )

        grid = self.grid_eps * grid_uniform + (1 - self.grid_eps) * grid_adaptive
        grid = torch.concatenate(
            [
                grid[:1]
                - uniform_step
                * torch.arange(self.spline_order, 0, -1, device=x.device).unsqueeze(1),
                grid,
                grid[-1:]
                + uniform_step
                * torch.arange(1, self.spline_order + 1, device=x.device).unsqueeze(1),
            ],
            dim=0,
        )

        self.grid.copy_(grid.T)
        self.spline_weight.data.copy_(self.curve2coeff(x, unreduced_spline_output))
    def regularization_loss(self, regularize_activation=1.0, regularize_entropy=1.0):
        l1_fake = self.spline_weight.abs().mean(-1)
        regularization_loss_activation = l1_fake.sum()
        p = l1_fake / regularization_loss_activation
        regularization_loss_entropy = -torch.sum(p * p.log())
        return (
                regularize_activation * regularization_loss_activation
                + regularize_entropy * regularization_loss_entropy
        )


class KAN(torch.nn.Module):
    def __init__(
            self,
            layers_hidden,
            grid_size=5,
            spline_order=3,
            scale_base=1.0,
            scale_spline=1.0,
            base_activation=torch.nn.SiLU,
            grid_eps=0.02,
            grid_range=[-1, 1],
    ):
        super(KAN, self).__init__()
        self.grid_size = grid_size
        self.spline_order = spline_order

        # 構建 KAN 的層
        self.layers = self.build_layers(
            layers_hidden, grid_size, spline_order, scale_base, scale_spline, base_activation, grid_eps,
grid_range
        )

    def build_layers(self, layers_hidden, grid_size, spline_order, scale_base, scale_spline, base_activation,
grid_eps,
                     grid_range):
        layers = torch.nn.ModuleList()
        for in_features, out_features in zip(layers_hidden, layers_hidden[1:]):
            layers.append(
                KANLinear(
                    in_features,
                    out_features,
                    grid_size=grid_size,
                    spline_order=spline_order,
                    scale_base=scale_base,
                    scale_spline=scale_spline,
                    base_activation=base_activation,
                    grid_eps=grid_eps,
                    grid_range=grid_range,
                )
            )
        return layers

    def forward(self, x: torch.Tensor, update_grid=False):
        for layer in self.layers:
            if update_grid:
                layer.update_grid(x)
            x = layer(x)
        return x

    def regularization_loss(self, regularize_activation=1.0, regularize_entropy=1.0):
        return sum(
            layer.regularization_loss(regularize_activation, regularize_entropy)
            for layer in self.layers
        )
```

- 創建 Kolmogorov-Arnold Network 進行測試，資料集使用 MNIST 手寫數字辨識

```python
from EfficientKAN import KAN

# Train on MNIST
import torch
import torch.nn as nn
import torch.optim as optim
import torchvision
import torchvision.transforms as transforms
from torch.utils.data import DataLoader
from tqdm import tqdm

# Load MNIST
transform = transforms.Compose(
    [transforms.ToTensor(), transforms.Normalize((0.5,), (0.5,))]
)
trainset = torchvision.datasets.MNIST(
    root="./data", train=True, download=True, transform=transform
)
valset = torchvision.datasets.MNIST(
    root="./data", train=False, download=True, transform=transform
)
trainloader = DataLoader(trainset, batch_size=64, shuffle=True)
valloader = DataLoader(valset, batch_size=64, shuffle=False)

# Define model
model = KAN([28 * 28, 64, 10])
device = torch.device("cuda" if torch.cuda.is_available() else "cpu")
model.to(device)
# Define optimizer
optimizer = optim.AdamW(model.parameters(), lr=1e-3, weight_decay=1e-4)
# Define learning rate scheduler
scheduler = optim.lr_scheduler.ExponentialLR(optimizer, gamma=0.8)

# Define loss
criterion = nn.CrossEntropyLoss()
for epoch in range(10):
    # Train
    model.train()
    with tqdm(trainloader) as pbar:
        for i, (images, labels) in enumerate(pbar):
            images = images.view(-1, 28 * 28).to(device)
            optimizer.zero_grad()
            output = model(images)
            loss = criterion(output, labels.to(device))
            loss.backward()
            optimizer.step()
            accuracy = (output.argmax(dim=1) == labels.to(device)).float().mean()
            pbar.set_postfix(loss=loss.item(), accuracy=accuracy.item(), lr=optimizer.param_groups[0]['lr'])

    # Validation
    model.eval()
    val_loss = 0
    val_accuracy = 0
    with torch.no_grad():
        for images, labels in valloader:
            images = images.view(-1, 28 * 28).to(device)
            output = model(images)
            val_loss += criterion(output, labels.to(device)).item()
            val_accuracy += (
                (output.argmax(dim=1) == labels.to(device)).float().mean().item()
            )
    val_loss /= len(valloader)
    val_accuracy /= len(valloader)

    # Update learning rate
    scheduler.step()

    print(
        f"Epoch {epoch + 1}, Val Loss: {val_loss}, Val Accuracy: {val_accuracy}"
    )
```

```python
# Print model weights
print("Trained Model Weights:")

for i, layer in enumerate(model.layers):
    print(f"Layer {i + 1}:")
    print("Spline Weights:")
    print(layer.spline_weight)
    print("Base Weights:")
    print(layer.base_weight)
    print()

# Save model weights (need to create KAN instance then "torch.load")
torch.save(model.state_dict(), "kan_mnist_weights.pth")
# Save the entire model (just get with "torch.load")
torch.save(model, "kan_mnist_model.pth")
```

- Model 於 MNIST 之訓練結果

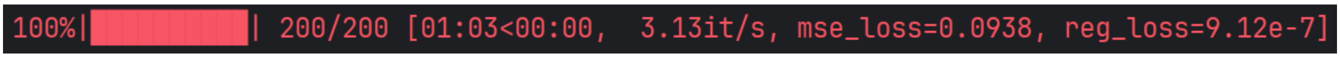


```
100%|██████████| 938/938 [00:40<00:00, 23.07it/s, accuracy=1, loss=0.031, lr=0.000134]
Epoch 10, Val Loss: 0.08577567627701677, Val Accuracy: 0.9750199044585988
```

- 創建 Kolmogorov-Arnold Network 進行測試，函數擬合簡易乘法 a x b

```python
import torch
import torch.nn as nn
from tqdm import tqdm

from EfficientKAN import KAN


def test_mul():
    kan = KAN([2, 3, 3, 1], base_activation=nn.Identity)
    optimizer = torch.optim.LBFGS(kan.parameters(), lr=0.001)

    with tqdm(range(200)) as pbar:
        for i in pbar:
            loss, reg_loss = None, None

            def closure():
                optimizer.zero_grad()
                x = torch.rand(1024, 2)
                y = kan(x, update_grid=(i % 20 == 0))
                assert y.shape == (1024, 1)
                nonlocal loss, reg_loss
                u = x[:, 0]
                v = x[:, 1]
                loss = nn.functional.mse_loss(y.squeeze(-1), u * v)
                reg_loss = kan.regularization_loss(1, 0)
                (loss + 1e-5 * reg_loss).backward()
                return loss + reg_loss

            optimizer.step(closure)
            pbar.set_postfix(mse_loss=loss.item(), reg_loss=reg_loss.item())

    for layer in kan.layers:
        print(layer.spline_weight)

    torch.save(kan, 'model/kan_multiple_model.pth')
    torch.save(kan.state_dict(), "model/kan_multiple_weights.pth")

    # Test the trained model
    test_model(kan)


def test_model(model):
    model.eval()
    with torch.no_grad():
        test_x = torch.rand(1024, 2)
        test_y = model(test_x)
        u = test_x[:, 0]
        v = test_x[:, 1]
        expected_y = u * v
        test_loss = nn.functional.mse_loss(test_y.squeeze(-1), expected_y)
        print(f"Test Loss: {test_loss.item():.4f}")


test_mul()
```

- Model 於函數擬合簡易乘法之訓練結果



```
100%|██████████| 200/200 [01:03<00:00,  3.13it/s, mse_loss=0.0938, reg_loss=9.12e-7]
```

訓練模型 Quantization 處理

Todo

為了解決模型 Floating Point 將會對電路設計產生的影響，需要先針對模型權重的 Quantization 進行實驗，用以尋找合適的 Integer Quantization 模式。

Check

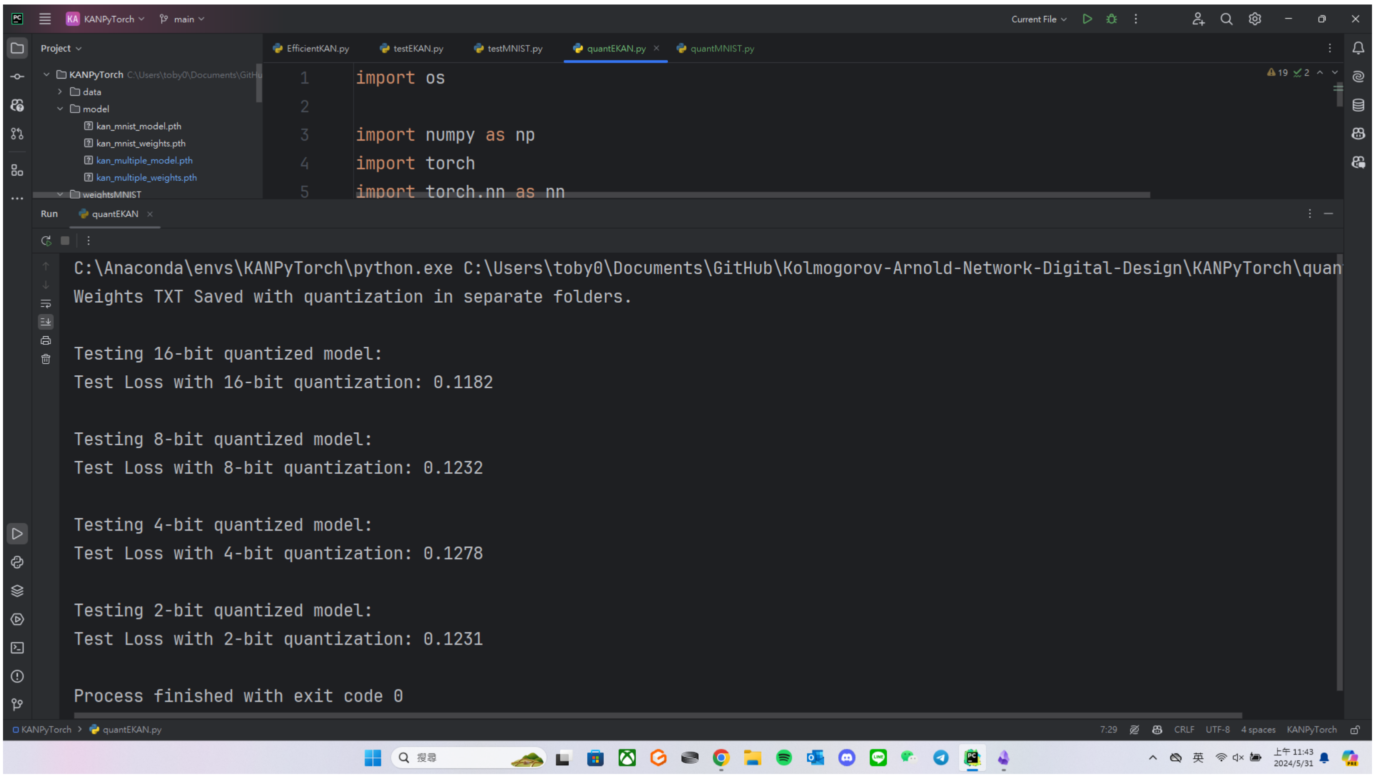
Multiplication 測試模型 Integer Quantization 比較，INT16、INT8、INT4、INT2。

- Quantization 功能設計、新權重保存和新權重評估

```
import os

import numpy as np
import torch
import torch.nn as nn

from EfficientKAN import KAN

# 加載已訓練的模型
model_path = 'model/kan_multiple_weights.pth'
model = KAN([2, 3, 3, 1], base_activation=nn.Identity)
model.load_state_dict(torch.load(model_path))


# 量化工具
def quantize_tensor(tensor, num_bits):
    qmin = 0.
    qmax = 2. ** num_bits - 1.

    min_val, max_val = tensor.min(), tensor.max()
    scale = (max_val - min_val) / (qmax - qmin)
    initial_zero_point = qmin - min_val / scale

    zero_point = 0
    if initial_zero_point < qmin:
        zero_point = qmin
    elif initial_zero_point > qmax:
        zero_point = qmax
    else:
        zero_point = initial_zero_point

    zero_point = int(zero_point)
    quantized_tensor = (tensor / scale + zero_point).round().clamp(qmin, qmax)
    quantized_tensor = quantized_tensor.int()

    return quantized_tensor, scale, zero_point
```

```
# 將權重保存到單獨的 TXT 檔案
def save_layer_weights_to_txt(model, layer_index, base_dir, num_bits):
    layer = model.layers[layer_index]
    folder_name = os.path.join(base_dir, f"{num_bits}bits")
    os.makedirs(folder_name, exist_ok=True)

    base_weight_file = os.path.join(folder_name, f"layer{layer_index}_base_weight.txt")
    spline_weight_file = os.path.join(folder_name, f"layer{layer_index}_spline_weight.txt")
    spline_scaler_file = os.path.join(folder_name, f"layer{layer_index}_spline_scaler.txt")

    with open(base_weight_file, 'w') as f:
        base_weight_data = layer.base_weight.detach().cpu().numpy()
        quantized_base_weight, scale, zero_point = quantize_tensor(torch.tensor(base_weight_data), num_bits)
        f.write(f"scale: {scale}\n")
        f.write(f"zero_point: {zero_point}\n")
        for value in quantized_base_weight.flatten():
            f.write(f'{value}\n')

    with open(spline_weight_file, 'w') as f:
        spline_weight_data = layer.spline_weight.detach().cpu().numpy()
        quantized_spline_weight, scale, zero_point = quantize_tensor(torch.tensor(spline_weight_data), num_bits)
        f.write(f"scale: {scale}\n")
        f.write(f"zero_point: {zero_point}\n")
        for value in quantized_spline_weight.flatten():
            f.write(f'{value}\n')

    with open(spline_scaler_file, 'w') as f:
        spline_scaler_data = layer.spline_scaler.detach().cpu().numpy()
        quantized_spline_scaler, scale, zero_point = quantize_tensor(torch.tensor(spline_scaler_data), num_bits)
        f.write(f"scale: {scale}\n")
        f.write(f"zero_point: {zero_point}\n")
        for value in quantized_spline_scaler.flatten():
            f.write(f'{value}\n')


# 呼叫函數保存每層的權重，使用16位、8位和4位量化和其他位數
bit_levels = [16, 8, 4, 2]
base_dir = 'weightsMultiplication'
for num_bits in bit_levels:
    for i in range(len(model.layers)):
        save_layer_weights_to_txt(model, i, base_dir, num_bits)

print("Weights TXT Saved with quantization in separate folders.")


# 從 TXT 文件加載量化後的權重
def load_quantized_weights_from_txt(model, layer_index, base_dir, num_bits):
    layer = model.layers[layer_index]
    folder_name = os.path.join(base_dir, f"{num_bits}bits")

    base_weight_file = os.path.join(folder_name, f"layer{layer_index}_base_weight.txt")
    spline_weight_file = os.path.join(folder_name, f"layer{layer_index}_spline_weight.txt")
    spline_scaler_file = os.path.join(folder_name, f"layer{layer_index}_spline_scaler.txt")

    def read_quantized_file(file_path):
        with open(file_path, 'r') as f:
            lines = f.readlines()
        scale = float(lines[0].strip().split(": ")[1])
        zero_point = int(lines[1].strip().split(": ")[1])
        quantized_values = np.array([int(v.strip()) for v in lines[2:]])
        return quantized_values, scale, zero_point

    quantized_base_weight, scale, zero_point = read_quantized_file(base_weight_file)
    layer.base_weight.data = torch.tensor((quantized_base_weight - zero_point) * scale,
dtype=torch.float32).view_as(
        layer.base_weight)

    quantized_spline_weight, scale, zero_point = read_quantized_file(spline_weight_file)
    layer.spline_weight.data = torch.tensor((quantized_spline_weight - zero_point) * scale,
                                            dtype=torch.float32).view_as(layer.spline_weight)
    if os.path.exists(spline_scaler_file):
        quantized_spline_scaler, scale, zero_point = read_quantized_file(spline_scaler_file)
        layer.spline_scaler.data = torch.tensor((quantized_spline_scaler - zero_point) * scale,
                                                dtype=torch.float32).view_as(layer.spline_scaler)


# 測試量化後的模型性能
def test_quantized_model(model, base_dir, num_bits):
    for i in range(len(model.layers)):
        load_quantized_weights_from_txt(model, i, base_dir, num_bits)

    model.eval()
    with torch.no_grad():
        test_x = torch.rand(1024, 2)
        test_y = model(test_x)
        u = test_x[:, 0]
        v = test_x[:, 1]
        expected_y = u * v
        test_loss = nn.functional.mse_loss(test_y.squeeze(-1), expected_y)
        print(f"Test Loss with {num_bits}-bit quantization: {test_loss.item():.4f}")


# 測試不同量化位數的模型
for num_bits in bit_levels:
    print(f"\nTesting {num_bits}-bit quantized model:")
    test_quantized_model(model, base_dir, num_bits)
```

- 評估結果 (不同 Quantization 之間差異不大，雖然 Loss 不像 Quantization 之前那樣優異到 10e-7 這樣，但也依舊維持著一個可用的函數擬合水平)



```
import os

import numpy as np
import torch
import torch.nn as nn
```

```
C:\Anaconda\envs\KANPyTorch\python.exe C:\Users\toby0\Documents\GitHub\Kolmogorov-Arnold-Network-Digital-Design\KANPyTorch\quan
Weights TXT Saved with quantization in separate folders.

Testing 16-bit quantized model:
Test Loss with 16-bit quantization: 0.1182

Testing 8-bit quantized model:
Test Loss with 8-bit quantization: 0.1232

Testing 4-bit quantized model:
Test Loss with 4-bit quantization: 0.1278

Testing 2-bit quantized model:
Test Loss with 2-bit quantization: 0.1231

Process finished with exit code 0
```

✅ Check

MNIST 測試模型 Integer Quantization 比較，INT16、INT8、INT4、INT2。

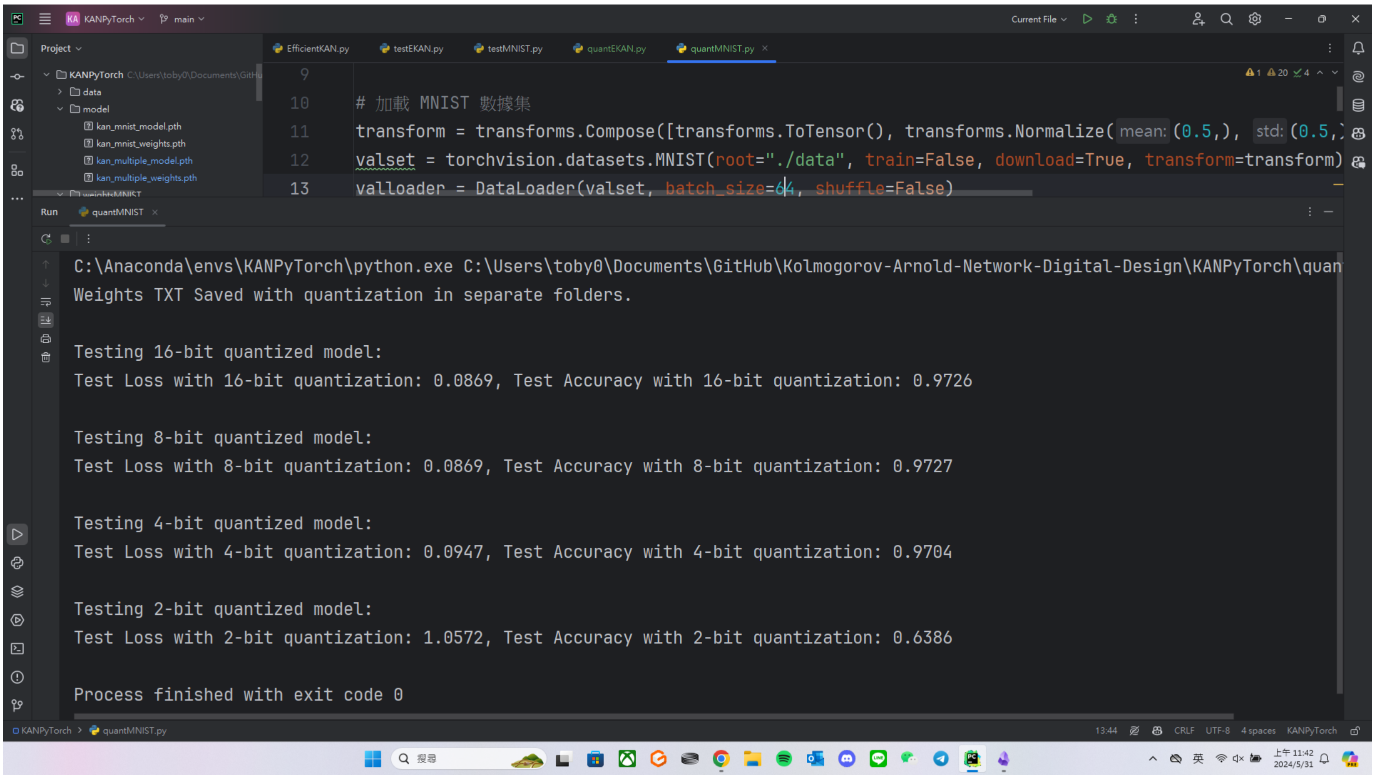
- Quantization 功能設計、新權重保存和新權重評估

```
import os

import numpy as np
import torch
import torch.nn as nn
import torchvision
```

```python
import torchvision.transforms as transforms
from torch.utils.data import DataLoader

# 加載 MNIST 數據集
transform = transforms.Compose([transforms.ToTensor(), transforms.Normalize((0.5,), (0.5,))])
valset = torchvision.datasets.MNIST(root="./data", train=False, download=True, transform=transform)
valloader = DataLoader(valset, batch_size=64, shuffle=False)

# 加載已訓練的模型
model_path = 'model/kan_mnist_model.pth'
model = torch.load(model_path)
device = torch.device("cuda" if torch.cuda.is_available() else "cpu")
model.to(device)

# 定義損失函數
criterion = nn.CrossEntropyLoss()


# 量化工具
def quantize_tensor(tensor, num_bits):
    qmin = 0.
    qmax = 2. ** num_bits - 1.

    min_val, max_val = tensor.min(), tensor.max()
    scale = (max_val - min_val) / (qmax - qmin)
    initial_zero_point = qmin - min_val / scale

    zero_point = 0
    if initial_zero_point < qmin:
        zero_point = qmin
    elif initial_zero_point > qmax:
        zero_point = qmax
    else:
        zero_point = initial_zero_point

    zero_point = int(zero_point)
    quantized_tensor = (tensor / scale + zero_point).round().clamp(qmin, qmax)
    quantized_tensor = quantized_tensor.int()

    return quantized_tensor, scale, zero_point


# 將權重保存到單獨的 TXT 檔案
def save_layer_weights_to_txt(model, layer_index, base_dir, num_bits):
    layer = model.layers[layer_index]
    folder_name = os.path.join(base_dir, f"{num_bits}bits")
    os.makedirs(folder_name, exist_ok=True)

    base_weight_file = os.path.join(folder_name, f"layer{layer_index}_base_weight.txt")
    spline_weight_file = os.path.join(folder_name, f"layer{layer_index}_spline_weight.txt")
    spline_scaler_file = os.path.join(folder_name, f"layer{layer_index}_spline_scaler.txt")

    with open(base_weight_file, 'w') as f:
        base_weight_data = layer.base_weight.detach().cpu().numpy()
        quantized_base_weight, scale, zero_point = quantize_tensor(torch.tensor(base_weight_data), num_bits)
        f.write(f"scale: {scale}\n")
        f.write(f"zero_point: {zero_point}\n")
        for value in quantized_base_weight.flatten():
            f.write(f'{value}\n')

    with open(spline_weight_file, 'w') as f:
        spline_weight_data = layer.spline_weight.detach().cpu().numpy()
        quantized_spline_weight, scale, zero_point = quantize_tensor(torch.tensor(spline_weight_data), num_bits)
        f.write(f"scale: {scale}\n")
        f.write(f"zero_point: {zero_point}\n")
        for value in quantized_spline_weight.flatten():
            f.write(f'{value}\n')

    if layer.spline_scaler is not None:
        with open(spline_scaler_file, 'w') as f:
            spline_scaler_data = layer.spline_scaler.detach().cpu().numpy()
            quantized_spline_scaler, scale, zero_point = quantize_tensor(torch.tensor(spline_scaler_data),
num_bits)
            f.write(f"scale: {scale}\n")
            f.write(f"zero_point: {zero_point}\n")
            for value in quantized_spline_scaler.flatten():
                f.write(f'{value}\n')


# 呼叫函數保存每層的權重，使用16位、8位和4位量化
bit_levels = [16, 8, 4, 2]
base_dir = 'weightsMNIST'
for num_bits in bit_levels:
    for i in range(len(model.layers)):
        save_layer_weights_to_txt(model, i, base_dir, num_bits)

print("Weights TXT Saved with quantization in separate folders.")


# 從 TXT 文件加載量化後的權重
def load_quantized_weights_from_txt(model, layer_index, base_dir, num_bits):
    layer = model.layers[layer_index]
    folder_name = os.path.join(base_dir, f"{num_bits}bits")

    base_weight_file = os.path.join(folder_name, f"layer{layer_index}_base_weight.txt")
    spline_weight_file = os.path.join(folder_name, f"layer{layer_index}_spline_weight.txt")
    spline_scaler_file = os.path.join(folder_name, f"layer{layer_index}_spline_scaler.txt")

    def read_quantized_file(file_path):
        with open(file_path, 'r') as f:
            lines = f.readlines()
        scale = float(lines[0].strip().split(": ")[1])
        zero_point = int(lines[1].strip().split(": ")[1])
        quantized_values = np.array([int(v.strip()) for v in lines[2:]])
        return quantized_values, scale, zero_point

    quantized_base_weight, scale, zero_point = read_quantized_file(base_weight_file)
    layer.base_weight.data = torch.tensor((quantized_base_weight - zero_point) * scale,
dtype=torch.float32).view_as(
        layer.base_weight)

    quantized_spline_weight, scale, zero_point = read_quantized_file(spline_weight_file)
    layer.spline_weight.data = torch.tensor((quantized_spline_weight - zero_point) * scale,
                                            dtype=torch.float32).view_as(layer.spline_weight)

    if os.path.exists(spline_scaler_file):
        quantized_spline_scaler, scale, zero_point = read_quantized_file(spline_scaler_file)
        layer.spline_scaler.data = torch.tensor((quantized_spline_scaler - zero_point) * scale,
                                                dtype=torch.float32).view_as(layer.spline_scaler)


# 測試量化後的模型性能
def test_quantized_model(model, base_dir, num_bits):
    for i in range(len(model.layers)):
        load_quantized_weights_from_txt(model, i, base_dir, num_bits)

    model.eval()
    val_loss = 0
    val_accuracy = 0
    with torch.no_grad():
        for images, labels in valloader:
            images = images.view(-1, 28 * 28).to(device)
            output = model(images)
            val_loss += criterion(output, labels.to(device)).item()
            val_accuracy += ((output.argmax(dim=1) == labels.to(device)).float().mean().item())
    val_loss /= len(valloader)
    val_accuracy /= len(valloader)
    print(
        f"Test Loss with {num_bits}-bit quantization: {val_loss:.4f}, Test Accuracy with {num_bits}-bit
quantization: {val_accuracy:.4f}")
```

```
# 測試不同量化位數的模型
for num_bits in bit_levels:
    print(f"\nTesting {num_bits}-bit quantized model:")
    test_quantized_model(model, base_dir, num_bits)
```

- 評估結果（除 2-bit Quantization 以外，其餘正確率接可維持在 97%，與 Quantization 前的結果相差無幾）



```
# 加載 MNIST 數據集
transform = transforms.Compose([transforms.ToTensor(), transforms.Normalize(mean: (0.5,), std: (0.5,
valset = torchvision.datasets.MNIST(root="./data", train=False, download=True, transform=transform)
valloader = DataLoader(valset, batch_size=64, shuffle=False)
```

```
C:\Anaconda\envs\KANPyTorch\python.exe C:\Users\toby0\Documents\GitHub\Kolmogorov-Arnold-Network-Digital-Design\KANPyTorch\quan
Weights TXT Saved with quantization in separate folders.

Testing 16-bit quantized model:
Test Loss with 16-bit quantization: 0.0869, Test Accuracy with 16-bit quantization: 0.9726

Testing 8-bit quantized model:
Test Loss with 8-bit quantization: 0.0869, Test Accuracy with 8-bit quantization: 0.9727

Testing 4-bit quantized model:
Test Loss with 4-bit quantization: 0.0947, Test Accuracy with 4-bit quantization: 0.9704

Testing 2-bit quantized model:
Test Loss with 2-bit quantization: 1.0572, Test Accuracy with 2-bit quantization: 0.6386

Process finished with exit code 0
```

方法論合成 Grafcet 控制器電路

⚠ Warning

我 Quartus Prime 是選擇用實驗室的板子來進行預設的 FPGA 規格，下列的設計是可以用的（經過其他的驗證程式確認），但因為使用了大量的 PIN 腳，而 MAX10 預設的 PIN 腳不夠用，所以沒有辦法編譯成 MAX10 可用的電路規格，可能會需要優化電路設計，或是更換為比較簡易的測試資料（第一輪採用 MNIST Dataset 進行測試，且 KAN 的架構為 [28 * 28, 64, 10]）。

✖ Missing

MNIST 版本暫時為參考使用

- KANLayer 實現（KANLayer.v）

```
module KANLayer #(
    parameter IN_FEATURES = 784,
    parameter OUT_FEATURES = 64,
    parameter SCALE = 256,  // Quantization scale factor
    parameter BASE_WEIGHT_FILE = "C:/intelFPGA_lite/18.1/KAN/base_weight_layer_0.txt",
    parameter SPLINE_WEIGHT_FILE = "C:/intelFPGA_lite/18.1/KAN/spline_weight_layer_0.txt"
)(
    input wire clk,
    input wire reset,
    input wire [7:0] in_data [0:IN_FEATURES-1],  // Input data
    output reg [7:0] out_data [0:OUT_FEATURES-1] // Output data
);
    // Weights stored in on-chip memory (BRAM)
    reg signed [15:0] base_weights [0:OUT_FEATURES*IN_FEATURES-1];
    reg signed [15:0] spline_weights [0:OUT_FEATURES*IN_FEATURES-1];

    // Load weights from memory (initialization)
    initial begin
        $readmemh(BASE_WEIGHT_FILE, base_weights);
        $readmemh(SPLINE_WEIGHT_FILE, spline_weights);
    end

    // Output registers
    reg signed [31:0] base_output [0:OUT_FEATURES-1];
    reg signed [31:0] spline_output [0:OUT_FEATURES-1];
    reg signed [31:0] total_output [0:OUT_FEATURES-1];

    integer i, j;

    // Forward pass
    always @(posedge clk or posedge reset) begin
        if (reset) begin
            for (i = 0; i < OUT_FEATURES; i = i + 1) begin
                base_output[i] <= 0;
                spline_output[i] <= 0;
                total_output[i] <= 0;
            end
        end else begin
            for (i = 0; i < OUT_FEATURES; i = i + 1) begin
                base_output[i] <= 0;
                spline_output[i] <= 0;
                for (j = 0; j < IN_FEATURES; j = j + 1) begin
                    base_output[i] <= base_output[i] + in_data[j] * base_weights[i*IN_FEATURES + j];
                    spline_output[i] <= spline_output[i] + in_data[j] * spline_weights[i*IN_FEATURES + j];
                end
                total_output[i] <= (base_output[i] + spline_output[i]) / SCALE;  // Combine and scale the outputs
                out_data[i] <= total_output[i][15:8];  // Convert to 8-bit output
            end
        end
    end
endmodule
```

Success

更換測試資料及 Network 設計以求可以將電路設計容納進去（學習函數擬合簡易乘法，KAN 的架構為 [2, 3, 3, 1]）。

- KANLayer 實現（KANLayer.v）

```
module KANLinear #(parameter IN_FEATURES = 2, OUT_FEATURES = 3, integer LAYER_NUM = 0) (
    input clk,
    input reset,
    input [15:0] data_in [IN_FEATURES-1:0],
    output reg [15:0] data_out [OUT_FEATURES-1:0]
);

    reg [15:0] base_weight [OUT_FEATURES*IN_FEATURES-1:0];
    reg [15:0] spline_weight [OUT_FEATURES*IN_FEATURES-1:0];
    reg [15:0] base_weight_2d [OUT_FEATURES-1:0][IN_FEATURES-1:0];
    reg [15:0] spline_weight_2d [OUT_FEATURES-1:0][IN_FEATURES-1:0];

    integer i, j;

    initial begin
        if (LAYER_NUM == 0) begin
            $readmemh("C:/intelFPGA_lite/18.1/KAN2/base_weight_layer_0.txt", base_weight);
            $readmemh("C:/intelFPGA_lite/18.1/KAN2/spline_weight_layer_0.txt", spline_weight);
        end else if (LAYER_NUM == 1) begin
            $readmemh("C:/intelFPGA_lite/18.1/KAN2/base_weight_layer_1.txt", base_weight);
            $readmemh("C:/intelFPGA_lite/18.1/KAN2/spline_weight_layer_1.txt", spline_weight);
        end else if (LAYER_NUM == 2) begin
            $readmemh("C:/intelFPGA_lite/18.1/KAN2/base_weight_layer_2.txt", base_weight);
            $readmemh("C:/intelFPGA_lite/18.1/KAN2/spline_weight_layer_2.txt", spline_weight);
        end

        for (i = 0; i < OUT_FEATURES; i = i + 1) begin
            for (j = 0; j < IN_FEATURES; j = j + 1) begin
```

```
                base_weight_2d[i][j] = base_weight[i * IN_FEATURES + j];
                spline_weight_2d[i][j] = spline_weight[i * IN_FEATURES + j];
            end
        end
    end

    always @(posedge clk or posedge reset) begin
        if (reset) begin
            for (i = 0; i < OUT_FEATURES; i = i + 1) begin
                data_out[i] <= 16'd0;
            end
        end else begin
            for (i = 0; i < OUT_FEATURES; i = i + 1) begin
                data_out[i] <= 16'd0;
                for (j = 0; j < IN_FEATURES; j = j + 1) begin
                    data_out[i] <= data_out[i] + base_weight_2d[i][j] * data_in[j];
                end
                if (data_out[i] < 16'd0) begin
                    data_out[i] <= 16'd0;
                end
            end
        end
    end

endmodule
```

ChatGPT 合成 Grafcet Datapath 電路

⚠Warning

我 Quartus Prime 是選擇用實驗室的板子來進行預設的 FPGA 規格，下列的設計是可以用的（經過其他的驗證程式確認），但因為使用了大量的 PIN 腳，而 MAX10 預設的 PIN 腳不夠用，所以沒有辦法編譯成 MAX10 可用的電路規格，可能會需要優化電路設計，或是更換為比較簡易的測試資料（第一輪採用 MNIST Dataset 進行測試，且 KAN 的架構為 [28 * 28, 64, 10]）。

⊗Missing

MNIST 版本暫時為參考使用

- 完整 Network 實現（KAN.v）

```
module KAN (
    input wire clk,
    input wire reset,
    input wire [7:0] in_data [0:783],   // 28x28 = 784 pixels, 8-bit each
    output wire [7:0] out_data [0:9]    // 10 classes, 8-bit each
);
    // Internal signals for each layer
    wire [7:0] layer1_out [0:63];
    wire [7:0] layer2_out [0:9];

    // Instantiate layers
    KANLayer #(
        .IN_FEATURES(784),
        .OUT_FEATURES(64),
        .BASE_WEIGHT_FILE("C:/intelFPGA_lite/18.1/KAN/base_weight_layer_0.txt"),
        .SPLINE_WEIGHT_FILE("C:/intelFPGA_lite/18.1/KAN/spline_weight_layer_0.txt")
    ) layer1 (
        .clk(clk),
        .reset(reset),
        .in_data(in_data),
        .out_data(layer1_out)
    );

    KANLayer #(
        .IN_FEATURES(64),
        .OUT_FEATURES(10),
        .BASE_WEIGHT_FILE("C:/intelFPGA_lite/18.1/KAN/base_weight_layer_1.txt"),
        .SPLINE_WEIGHT_FILE("C:/intelFPGA_lite/18.1/KAN/spline_weight_layer_1.txt")
    ) layer2 (
        .clk(clk),
        .reset(reset),
        .in_data(layer1_out),
        .out_data(layer2_out)
    );

    // Connect the final output
    assign out_data = layer2_out;
endmodule
```

Success

更換測試資料及 Network 設計以求可以將電路設計容納進去（學習函數擬合簡易乘法，KAN 的架構為 [2, 3, 3, 1]）。

- 完整 Network 實現（KAN.v）

```
module KAN #(parameter IN_FEATURES = 2, L1_FEATURES = 3, L2_FEATURES = 3, OUT_FEATURES = 1) (
    input clk,
    input reset,
    input [15:0] input_data [IN_FEATURES-1:0],
    output [15:0] output_data
);

    wire [15:0] layer1_out [L1_FEATURES-1:0];
    wire [15:0] layer2_out [L2_FEATURES-1:0];
    wire [15:0] layer3_out [OUT_FEATURES-1:0];

    KANLinear #(.IN_FEATURES(IN_FEATURES), .OUT_FEATURES(L1_FEATURES), .LAYER_NUM(0)) layer1 (
        .clk(clk),
        .reset(reset),
        .data_in(input_data),
        .data_out(layer1_out)
    );

    KANLinear #(.IN_FEATURES(L1_FEATURES), .OUT_FEATURES(L2_FEATURES), .LAYER_NUM(1)) layer2 (
        .clk(clk),
        .reset(reset),
        .data_in(layer1_out),
        .data_out(layer2_out)
    );

    KANLinear #(.IN_FEATURES(L2_FEATURES), .OUT_FEATURES(OUT_FEATURES), .LAYER_NUM(2)) layer3 (
        .clk(clk),
        .reset(reset),
        .data_in(layer2_out),
        .data_out(layer3_out)
    );

    assign output_data = layer3_out[0];

endmodule
```

FPGA 整合驗證

Success

更換測試資料及 Network 設計以求可以將電路設計容納進去（學習函數擬合簡易乘法，KAN 的架構為 [2, 3, 3, 1]）。

- Testbench 實現（Testbench.v）

```
module TestBench;
    reg clk;
    reg reset;
    reg [15:0] input_data [0:1];
    wire [15:0] output_data;

    KAN #(.IN_FEATURES(2), .L1_FEATURES(3), .L2_FEATURES(3), .OUT_FEATURES(1)) kan (
        .clk(clk),
        .reset(reset),
        .input_data(input_data),
        .output_data(output_data)
    );

    initial begin
        clk = 0;
        reset = 1;
        input_data[0] = 16'd0;
        input_data[1] = 16'd0;
        #10 reset = 0;

        // test data 1
        input_data[0] = 16'd50;
        input_data[1] = 16'd30;
        #10;
        $display("Output (Test 1): %d", output_data);

        // test data 2
        input_data[0] = 16'd100;
        input_data[1] = 16'd200;
        #10;
        $display("Output (Test 2): %d", output_data);

        // test data 3
        input_data[0] = 16'd150;
        input_data[1] = 16'd250;
        #10;
        $display("Output (Test 3): %d", output_data);

        // test data 4
        input_data[0] = 16'd75;
        input_data[1] = 16'd125;
        #10;
        $display("Output (Test 4): %d", output_data);

        // test data 5
        input_data[0] = 16'd175;
        input_data[1] = 16'd225;
        #10;
        $display("Output (Test 5): %d", output_data);
    end

    always #5 clk = ~clk;

endmodule
```

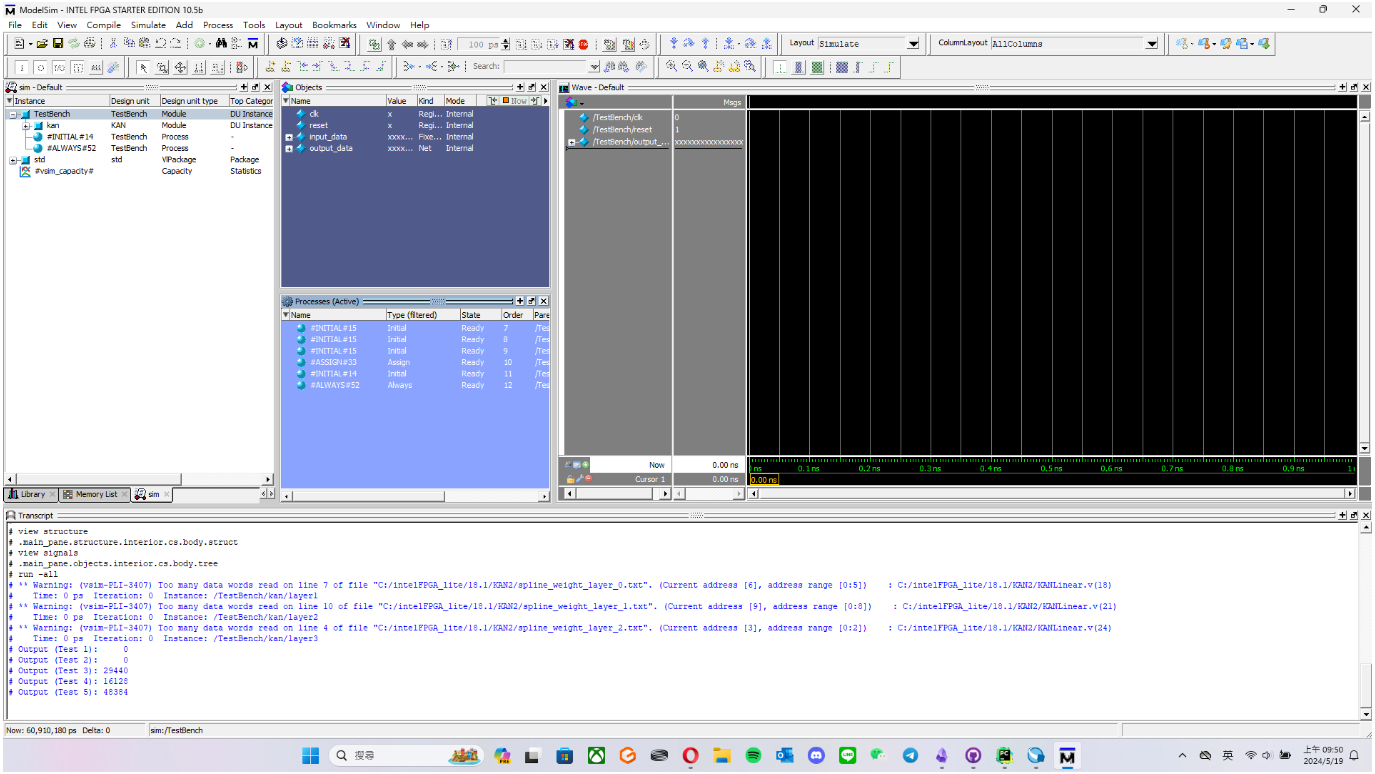
⚠ Warning

硬體設計完仿真的結果如下，誤差老實說非常大，進行排查後發現有可能是 Quantization 後的權重問題，因為 PyTorch 讀取量化後的權重結果一樣糟糕，可能會需要考慮導入特別針對小數進行處理的硬體設計部分或是針對 Quantization 重新進行調整來解決問題。

ℹ Todo

實際怎麼處理還有待研究，因為已經對 Quantization 的部分進行過一次調整，最一開始的問題在於 Spline 部分的 Weights，由於值非常小，在 Quantization 後會全部變成 0；後來改為基於數值動態進行 Scaling 的操作並實際進行仿真，最後的效果還是不太行。目前不確定是 Quantization 的 Function 有寫錯，還是說是應用上有其他問題。

- ModelSim 仿真結果



Instance	Design unit	Design unit type	Top Categor
TestBench	TestBench	Module	DU Instance
kan	KAN	Module	DU Instance
#INITIAL#14	TestBench	Process	-
#ALWAYS#52	TestBench	Process	-
std	std	VlPackage	Package
#vsim_capacity#		Capacity	Statistics

Name	Value	Kind	Mode
clk	x	Regi...	Internal
reset	x	Regi...	Internal
input_data	xxxx...	Fixe...	Internal
output_data	xxxx...	Net	Internal

Name	Type (filtered)	State	Order	Pare
#INITIAL#15	Initial	Ready	7	/Tes
#INITIAL#15	Initial	Ready	8	/Tes
#INITIAL#15	Initial	Ready	9	/Tes
#ASSIGN#33	Assign	Ready	10	/Tes
#INITIAL#14	Initial	Ready	11	/Tes
#ALWAYS#52	Always	Ready	12	/Tes

	Msgs
/TestBench/clk	0
/TestBench/reset	1
/TestBench/output_...	xxxxxxxxxxxxxxxxx

```
# view structure
# .main_pane.structure.interior.cs.body.struct
# view signals
# .main_pane.objects.interior.cs.body.tree
# run -all
# ** Warning: (vsim-PLI-3407) Too many data words read on line 7 of file "C:/intelFPGA_lite/18.1/KAN2/spline_weight_layer_0.txt". (Current address [6], address range [0:5])    : C:/intelFPGA_lite/18.1/KAN2/KANLinear.v(18)
#    Time: 0 ps  Iteration: 0  Instance: /TestBench/kan/layer1
# ** Warning: (vsim-PLI-3407) Too many data words read on line 10 of file "C:/intelFPGA_lite/18.1/KAN2/spline_weight_layer_1.txt". (Current address [9], address range [0:8])    : C:/intelFPGA_lite/18.1/KAN2/KANLinear.v(21)
#    Time: 0 ps  Iteration: 0  Instance: /TestBench/kan/layer2
# ** Warning: (vsim-PLI-3407) Too many data words read on line 4 of file "C:/intelFPGA_lite/18.1/KAN2/spline_weight_layer_2.txt". (Current address [3], address range [0:2])    : C:/intelFPGA_lite/18.1/KAN2/KANLinear.v(24)
#    Time: 0 ps  Iteration: 0  Instance: /TestBench/kan/layer3
# Output (Test 1):     0
# Output (Test 2):     0
# Output (Test 3): 29440
# Output (Test 4): 16128
# Output (Test 5): 48384
```

Now: 60,910,180 ps Delta: 0   sim:/TestBench