KAN Kolmogorov Arnold Network Note

研究任務

1. 閱讀 KAN Kolmogorov-Arnold Network 論文
2. 設計 KAN 網路系統階層式架構 IDEF0
3. 設計 KAN 網路系統每個功能模組離散事件建模 Grafcet
4. 以 MIAT 方法論合成每個 Grafcet 控制器電路
5. 以 ChatGPT 合成每個 Grafcet Datapath 電路
6. FPGA 整合驗證

> **Attention**
>
> Kolmogorov-Arnold Network 之研究任務須於六月底完成，作為暑期實習前置條件。

> **Seealso**
>
> 「20240527」目標預定之所有（第一到第六點）研究任務，當前初版皆已完成，接下來主要目標為模型量化和優化相關電路設計及輸出結果。
>
> - 「20240610」Integer Quantization 部分已完成優化改寫。
> - 「20240617」KAN Pipeline Grafcet 完成初版架構設計。

開放原始碼參考

- Open Source Code: https://github.com/KindXiaoming/pykan
- Research Paper Reference: https://arxiv.org/abs/2404.19756

論文內容統整

> **Summary**
>
> KAN（Kolmogorov-Arnold Network）論文基本概念。

1. Research Background
    - Kolmogorov-Arnold Representation Theorem（KART）: 此定理說明任何多變量的連續函數都可以表示為單變量連續函數和加法操作的有限組合。
    - Limitations of Multi-Layer Perceptron（MLP）: 傳統由 MLP 形成的神經網路雖然具有強大的表達能力，但在某些應用中存在固定的 Activation Function，使得其解釋性較差且參數效率低下。
2. Kolmogorov-Arnold Network（KAN）
    - Network Structure: 與 MLP 不同，KAN 在 Edge（即權重）上使用可學習的 Activation Function，而不是在 Node（即神經元）上使用固定的 Activation Function。
    - Activation Function: KAN 中的每個權重參數被替換為一個參數化為樣條函數的單變量函數。節點只進行簡單的信號相加操作，不應用任何非線性操作。
3. Advantages of KAN
    - Higher Precision: KAN 在數據擬合和偏微分方程求解方面比 MLP 更準確。例如，在偏微分方程求解中，一個兩層且寬度為十的 KAN 比一個四層且寬度為一百的 MLP 準確度高百倍。
    - Explainability: KAN 可以直觀地可視化，並能與人類用戶進行互動，有助於科學家重新發現數學和物理定律。

> **Summary**
>
> KAN（Kolmogorov-Arnold Network）和 MLP（Multi-Layer Perceptron）之間的主要差異與比較。

1. 激活函數的位置和特性
    - MLP: 激活函數固定，位於節點（神經元）上；且激活函數一般是非線性函數，例如 ReLU、Sigmoid 等。
    - KAN: 激活函數是可學習的，位於邊（權重）上；且每個權重參數被替換為一個參數化為樣條函數的單變量函數。
2. 網路結構和權重表示
    - MLP: 使用線性權重矩陣進行計算，然後應用固定的非線性激活函數；節點進行非線性變換。
    - 結構公式:

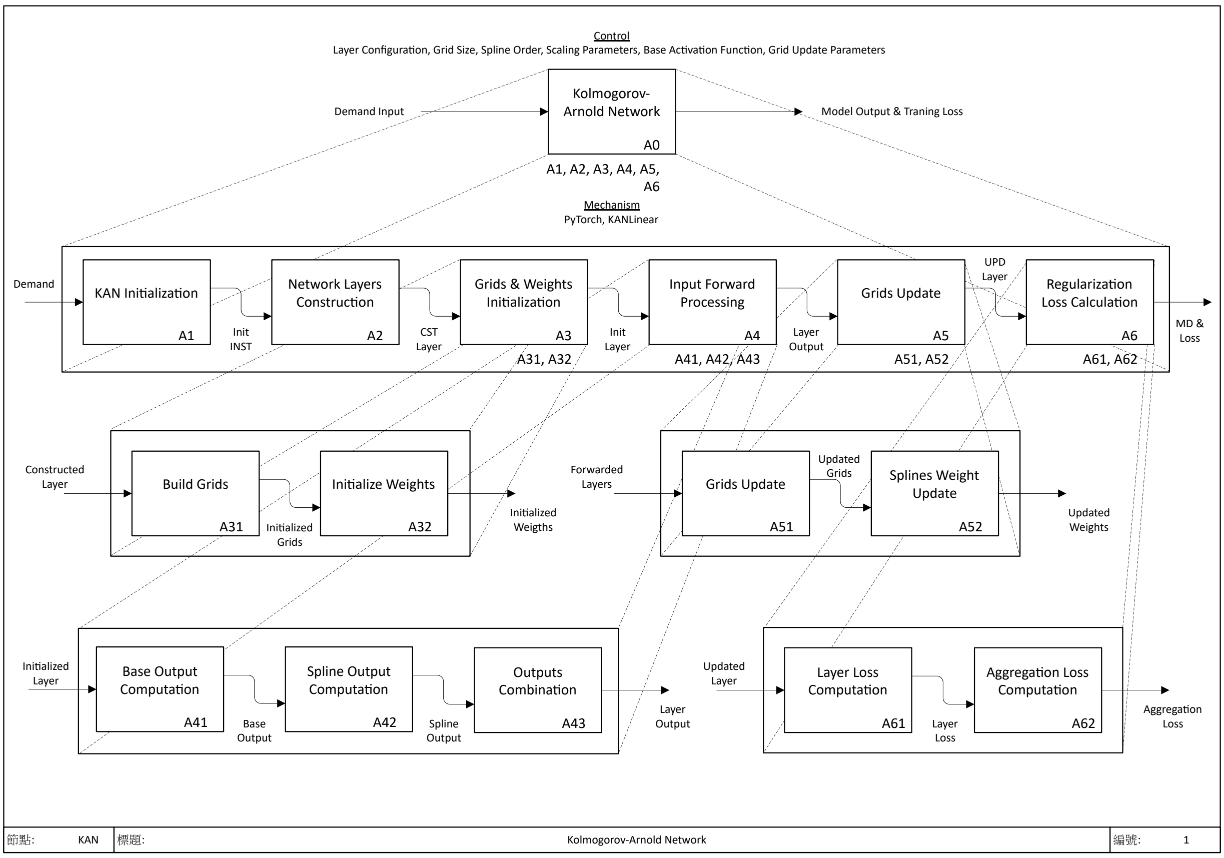
$$MLP(x) = ((W3 \circ \sigma 2 \circ W2 \circ \sigma 1 \circ W1)(x))$$

    - KAN: 沒有線性權重矩陣，所有權重都被樣條函數替代；節點僅進行簡單的信號相加操作，不進行非線性變換。
    - 結構公式:

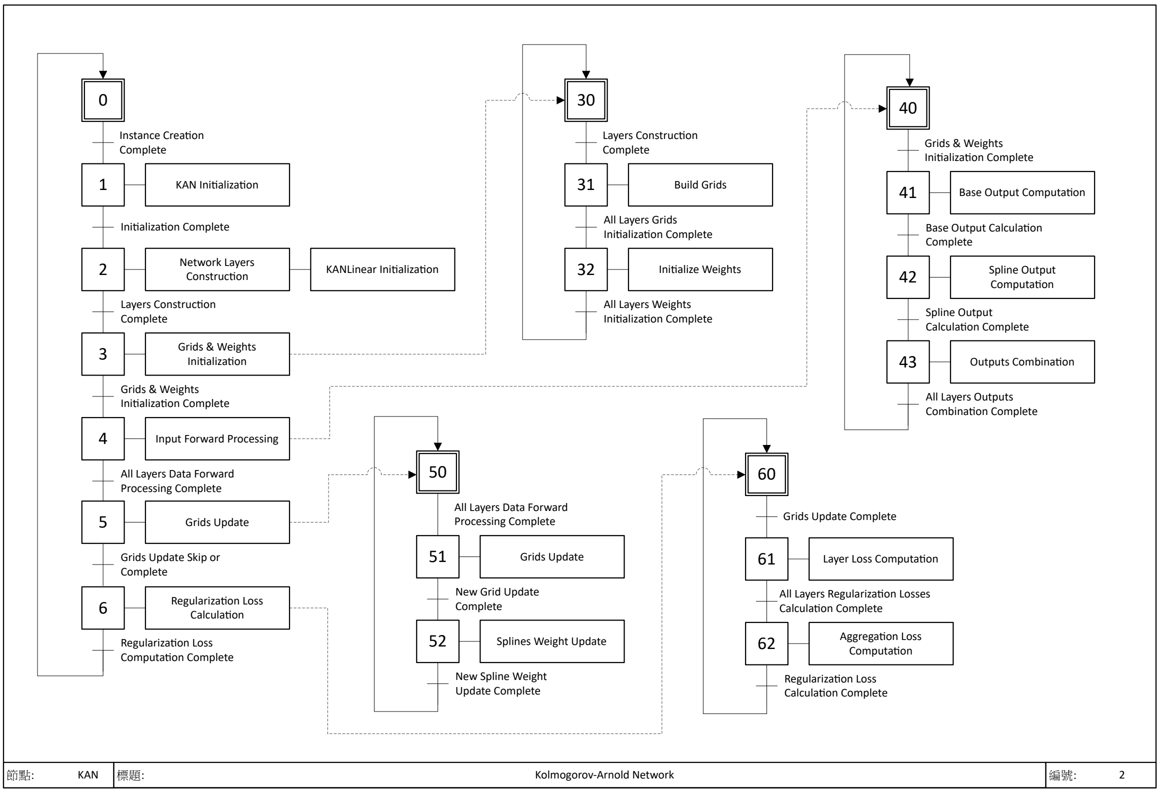
$$KAN(x) = ((\Phi 3 \circ \Phi 2 \circ \Phi 1)(x))$$

3. 訓練方法和參數優化
    - MLP: 權重矩陣通過梯度下降法進行訓練；訓練過程需要調整大量的線性權重參數。
    - KAN: 核心在於樣條函數的參數化和學習；樣條函數通過調整其參數進行優化。
4. 表達能力和適用範圍
    - MLP: 基於普適近似定理，MLP 能夠逼近任意連續函數；常用於各種回歸和分類問題，但在高維數據下可能效率低下。
    - KAN: 基於 Kolmogorov-Arnold 表示定理，能夠表達高維數據的組合結構和單變量函數；對於需要高準確度和解釋性的應用，如數學模型和物理模型，有顯著優勢。
5. 可解釋性和直觀性
    - MLP: 由於固定的激活函數和複雜的權重矩陣，MLP 在解釋性方面較為薄弱；解釋模型需要額外的工具和方法，如 SHAP、LIME 等。
    - KAN: 由於激活函數是可學習的單變量函數，KAN 的結構更易於直觀理解；KAN 的節點僅進行信號相加，使得整體網路更易於可視化和解釋。
6. 計算和資源需求
    - MLP: 訓練和推理過程中，計算資源需求較大，特別是在高維數據和大模型情況下。
    - KAN: 由於樣條函數的引入，KAN 在同樣準確度下所需的參數和計算資源相對較少；能夠在較小的計算圖上達到與大型 MLP 相同甚至更好的準確度。

設計階層式架構 IDEF0



設計功能模組離散事件建模 Grafcet



Python 模擬驗證

- 基於 IDEF0 和 Grafcet 重構後之 Kolmogorov-Arnold Network (PyTorch)

```
import math

import torch
import torch.nn.functional as F


class KANLinear(torch.nn.Module):
    def __init__(
            self,
            in_features,
            out_features,
            grid_size=5,
            spline_order=3,
            scale_base=1.0,
            scale_spline=1.0,
            enable_standalone_scale_spline=True,
            base_activation=torch.nn.SiLU,
            grid_eps=0.02,
            grid_range=[-1, 1],
    ):
        super(KANLinear, self).__init__()
        self.in_features = in_features
        self.out_features = out_features
        self.grid_size = grid_size
        self.spline_order = spline_order

        # 構建網格點
        self.grid = self.build_grid(grid_range, grid_size, spline_order)

        # 初始化基礎權重和樣條權重
        self.base_weight, self.spline_weight, self.spline_scaler = self.initialize_weights(
```

```
            out_features, in_features, grid_size, spline_order, scale_base, scale_spline,
enable_standalone_scale_spline
        )

        self.scale_base = scale_base
        self.scale_spline = scale_spline
        self.enable_standalone_scale_spline = enable_standalone_scale_spline
        self.base_activation = base_activation()
        self.grid_eps = grid_eps

    def build_grid(self, grid_range, grid_size, spline_order):
        h = (grid_range[1] - grid_range[0]) / grid_size
        grid = (
            (
                    torch.arange(-spline_order, grid_size + spline_order + 1) * h
                    + grid_range[0]
            )
            .expand(self.in_features, -1)
            .contiguous()
        )
        return grid

    def initialize_weights(self, out_features, in_features, grid_size, spline_order, scale_base, scale_spline,
                           enable_standalone_scale_spline):
        base_weight = torch.nn.Parameter(torch.Tensor(out_features, in_features))
        spline_weight = torch.nn.Parameter(
            torch.Tensor(out_features, in_features, grid_size + spline_order)
        )
        if enable_standalone_scale_spline:
            spline_scaler = torch.nn.Parameter(
                torch.Tensor(out_features, in_features)
            )
        else:
            spline_scaler = None
        torch.nn.init.kaiming_uniform_(base_weight, a=math.sqrt(5) * scale_base)
        torch.nn.init.kaiming_uniform_(spline_weight, a=math.sqrt(5) * scale_spline)
        if enable_standalone_scale_spline:
            torch.nn.init.kaiming_uniform_(spline_scaler, a=math.sqrt(5) * scale_spline)
        return base_weight, spline_weight, spline_scaler

    def b_splines(self, x: torch.Tensor):
        bases = self.calculate_b_spline_bases(x)
        return bases.contiguous()

    def calculate_b_spline_bases(self, x: torch.Tensor):
        grid: torch.Tensor = (
            self.grid
        )  # (in_features, grid_size + 2 * spline_order + 1)
        x = x.unsqueeze(-1)
        bases = ((x >= grid[:, :-1]) & (x < grid[:, 1:])).to(x.dtype)
        for k in range(1, self.spline_order + 1):
            bases = (
                            (x - grid[:, : -(k + 1)])
                            / (grid[:, k:-1] - grid[:, : -(k + 1)])
                            * bases[:, :, :-1]
                    ) + (
                            (grid[:, k + 1:] - x)
                            / (grid[:, k + 1:] - grid[:, 1:(-k)])
                            * bases[:, :, 1:]
                    )
        return bases

    def curve2coeff(self, x: torch.Tensor, y: torch.Tensor):
        A = self.b_splines(x).transpose(
            0, 1
        )  # (in_features, batch_size, grid_size + spline_order)
        B = y.transpose(0, 1)  # (in_features, batch_size, out_features)
        solution = torch.linalg.lstsq(
            A, B
        ).solution  # (in_features, grid_size + spline_order, out_features)
        result = solution.permute(
```

```python
            2, 0, 1
        )  # (out_features, in_features, grid_size + spline_order)
        return result.contiguous()

    @property
    def scaled_spline_weight(self):
        if self.enable_standalone_scale_spline:
            return self.spline_weight * self.spline_scaler.unsqueeze(-1)
        else:
            return self.spline_weight

    def forward(self, x: torch.Tensor):
        base_output = self.compute_base_output(x)
        spline_output = self.compute_spline_output(x)
        return base_output + spline_output

    def compute_base_output(self, x: torch.Tensor):
        return F.linear(self.base_activation(x), self.base_weight)

    def compute_spline_output(self, x: torch.Tensor):
        return F.linear(
            self.b_splines(x).view(x.size(0), -1),
            self.scaled_spline_weight.view(self.out_features, -1),
        )

    @torch.no_grad()
    def update_grid(self, x: torch.Tensor, margin=0.01):
        batch = x.size(0)

        splines = self.b_splines(x)  # (batch, in, coeff)
        splines = splines.permute(1, 0, 2)  # (in, batch, coeff)
        orig_coeff = self.scaled_spline_weight  # (out, in, coeff)
        orig_coeff = orig_coeff.permute(1, 2, 0)  # (in, coeff, out)
        unreduced_spline_output = torch.bmm(splines, orig_coeff)  # (in, batch, out)
        unreduced_spline_output = unreduced_spline_output.permute(
            1, 0, 2
        )  # (batch, in, out)

        x_sorted = torch.sort(x, dim=0)[0]
        grid_adaptive = x_sorted[
            torch.linspace(
                0, batch - 1, self.grid_size + 1, dtype=torch.int64, device=x.device
            )
        ]

        uniform_step = (x_sorted[-1] - x_sorted[0] + 2 * margin) / self.grid_size
        grid_uniform = (
                torch.arange(
                    self.grid_size + 1, dtype=torch.float32, device=x.device
                ).unsqueeze(1)
                * uniform_step
                + x_sorted[0]
                - margin
        )

        grid = self.grid_eps * grid_uniform + (1 - self.grid_eps) * grid_adaptive
        grid = torch.concatenate(
            [
                grid[:1]
                - uniform_step
                * torch.arange(self.spline_order, 0, -1, device=x.device).unsqueeze(1),
                grid,
                grid[-1:]
                + uniform_step
                * torch.arange(1, self.spline_order + 1, device=x.device).unsqueeze(1),
            ],
            dim=0,
        )

        self.grid.copy_(grid.T)
        self.spline_weight.data.copy_(self.curve2coeff(x, unreduced_spline_output))
```

```
    def regularization_loss(self, regularize_activation=1.0, regularize_entropy=1.0):
        l1_fake = self.spline_weight.abs().mean(-1)
        regularization_loss_activation = l1_fake.sum()
        p = l1_fake / regularization_loss_activation
        regularization_loss_entropy = -torch.sum(p * p.log())
        return (
                regularize_activation * regularization_loss_activation
                + regularize_entropy * regularization_loss_entropy
        )


class KAN(torch.nn.Module):
    def __init__(
            self,
            layers_hidden,
            grid_size=5,
            spline_order=3,
            scale_base=1.0,
            scale_spline=1.0,
            base_activation=torch.nn.SiLU,
            grid_eps=0.02,
            grid_range=[-1, 1],
    ):
        super(KAN, self).__init__()
        self.grid_size = grid_size
        self.spline_order = spline_order

        # 構建 KAN 的層
        self.layers = self.build_layers(
            layers_hidden, grid_size, spline_order, scale_base, scale_spline, base_activation, grid_eps,
grid_range
        )

    def build_layers(self, layers_hidden, grid_size, spline_order, scale_base, scale_spline, base_activation,
grid_eps,
                     grid_range):
        layers = torch.nn.ModuleList()
        for in_features, out_features in zip(layers_hidden, layers_hidden[1:]):
            layers.append(
                KANLinear(
                    in_features,
                    out_features,
                    grid_size=grid_size,
                    spline_order=spline_order,
                    scale_base=scale_base,
                    scale_spline=scale_spline,
                    base_activation=base_activation,
                    grid_eps=grid_eps,
                    grid_range=grid_range,
                )
            )
        return layers

    def forward(self, x: torch.Tensor, update_grid=False):
        for layer in self.layers:
            if update_grid:
                layer.update_grid(x)
            x = layer(x)
        return x

    def regularization_loss(self, regularize_activation=1.0, regularize_entropy=1.0):
        return sum(
            layer.regularization_loss(regularize_activation, regularize_entropy)
            for layer in self.layers
        )
```

- 創建 Kolmogorov-Arnold Network 進行測試，資料集使用 MNIST 手寫數字辨識

```python
from EfficientKAN import KAN

# Train on MNIST
import torch
import torch.nn as nn
import torch.optim as optim
import torchvision
import torchvision.transforms as transforms
from torch.utils.data import DataLoader
from tqdm import tqdm

# Load MNIST
transform = transforms.Compose(
    [transforms.ToTensor(), transforms.Normalize((0.5,), (0.5,))]
)
trainset = torchvision.datasets.MNIST(
    root="./data", train=True, download=True, transform=transform
)
valset = torchvision.datasets.MNIST(
    root="./data", train=False, download=True, transform=transform
)
trainloader = DataLoader(trainset, batch_size=64, shuffle=True)
valloader = DataLoader(valset, batch_size=64, shuffle=False)

# Define model
model = KAN([28 * 28, 64, 10])
device = torch.device("cuda" if torch.cuda.is_available() else "cpu")
model.to(device)
# Define optimizer
optimizer = optim.AdamW(model.parameters(), lr=1e-3, weight_decay=1e-4)
# Define learning rate scheduler
scheduler = optim.lr_scheduler.ExponentialLR(optimizer, gamma=0.8)

# Define loss
criterion = nn.CrossEntropyLoss()
for epoch in range(10):
    # Train
    model.train()
    with tqdm(trainloader) as pbar:
        for i, (images, labels) in enumerate(pbar):
            images = images.view(-1, 28 * 28).to(device)
            optimizer.zero_grad()
            output = model(images)
            loss = criterion(output, labels.to(device))
            loss.backward()
            optimizer.step()
            accuracy = (output.argmax(dim=1) == labels.to(device)).float().mean()
            pbar.set_postfix(loss=loss.item(), accuracy=accuracy.item(), lr=optimizer.param_groups[0]['lr'])

    # Validation
    model.eval()
    val_loss = 0
    val_accuracy = 0
    with torch.no_grad():
        for images, labels in valloader:
            images = images.view(-1, 28 * 28).to(device)
            output = model(images)
            val_loss += criterion(output, labels.to(device)).item()
            val_accuracy += (
                (output.argmax(dim=1) == labels.to(device)).float().mean().item()
            )
    val_loss /= len(valloader)
    val_accuracy /= len(valloader)

    # Update learning rate
    scheduler.step()

    print(
        f"Epoch {epoch + 1}, Val Loss: {val_loss}, Val Accuracy: {val_accuracy}"
    )
```

```python
# Print model weights
print("Trained Model Weights:")

for i, layer in enumerate(model.layers):
    print(f"Layer {i + 1}:")
    print("Spline Weights:")
    print(layer.spline_weight)
    print("Base Weights:")
    print(layer.base_weight)
    print()

# Save model weights (need to create KAN instance then "torch.load")
torch.save(model.state_dict(), "kan_mnist_weights.pth")
# Save the entire model (just get with "torch.load")
torch.save(model, "kan_mnist_model.pth")
```

- Model 於 MNIST 之訓練結果

```
100%|██████████| 938/938 [00:40<00:00, 23.07it/s, accuracy=1, loss=0.031, lr=0.000134]
Epoch 10, Val Loss: 0.08577567627701677, Val Accuracy: 0.9750199044585988
```

- 創建 Kolmogorov-Arnold Network 進行測試，函數擬合簡易乘法 a x b

```python
import torch
import torch.nn as nn
from tqdm import tqdm

from EfficientKAN import KAN


def test_mul():
    kan = KAN([2, 3, 3, 1], base_activation=nn.Identity)
    optimizer = torch.optim.LBFGS(kan.parameters(), lr=0.001)

    with tqdm(range(200)) as pbar:
        for i in pbar:
            loss, reg_loss = None, None

            def closure():
                optimizer.zero_grad()
                x = torch.rand(1024, 2)
                y = kan(x, update_grid=(i % 20 == 0))
                assert y.shape == (1024, 1)
                nonlocal loss, reg_loss
                u = x[:, 0]
                v = x[:, 1]
                loss = nn.functional.mse_loss(y.squeeze(-1), u * v)
                reg_loss = kan.regularization_loss(1, 0)
                (loss + 1e-5 * reg_loss).backward()
                return loss + reg_loss

            optimizer.step(closure)
            pbar.set_postfix(mse_loss=loss.item(), reg_loss=reg_loss.item())

    for layer in kan.layers:
        print(layer.spline_weight)

    torch.save(kan, 'model/kan_multiple_model.pth')
    torch.save(kan.state_dict(), "model/kan_multiple_weights.pth")

    # Test the trained model
    test_model(kan)


def test_model(model):
    model.eval()
    with torch.no_grad():
        test_x = torch.rand(1024, 2)
```

```
        test_y = model(test_x)
        u = test_x[:, 0]
        v = test_x[:, 1]
        expected_y = u * v
        test_loss = nn.functional.mse_loss(test_y.squeeze(-1), expected_y)
        print(f"Test Loss: {test_loss.item():.4f}")


test_mul()
```

- Model 於函數擬合簡易乘法之訓練結果

```
100%|██████████| 200/200 [01:03<00:00,  3.13it/s, mse_loss=0.0938, reg_loss=9.12e-7]
```

訓練模型 Quantization 處理

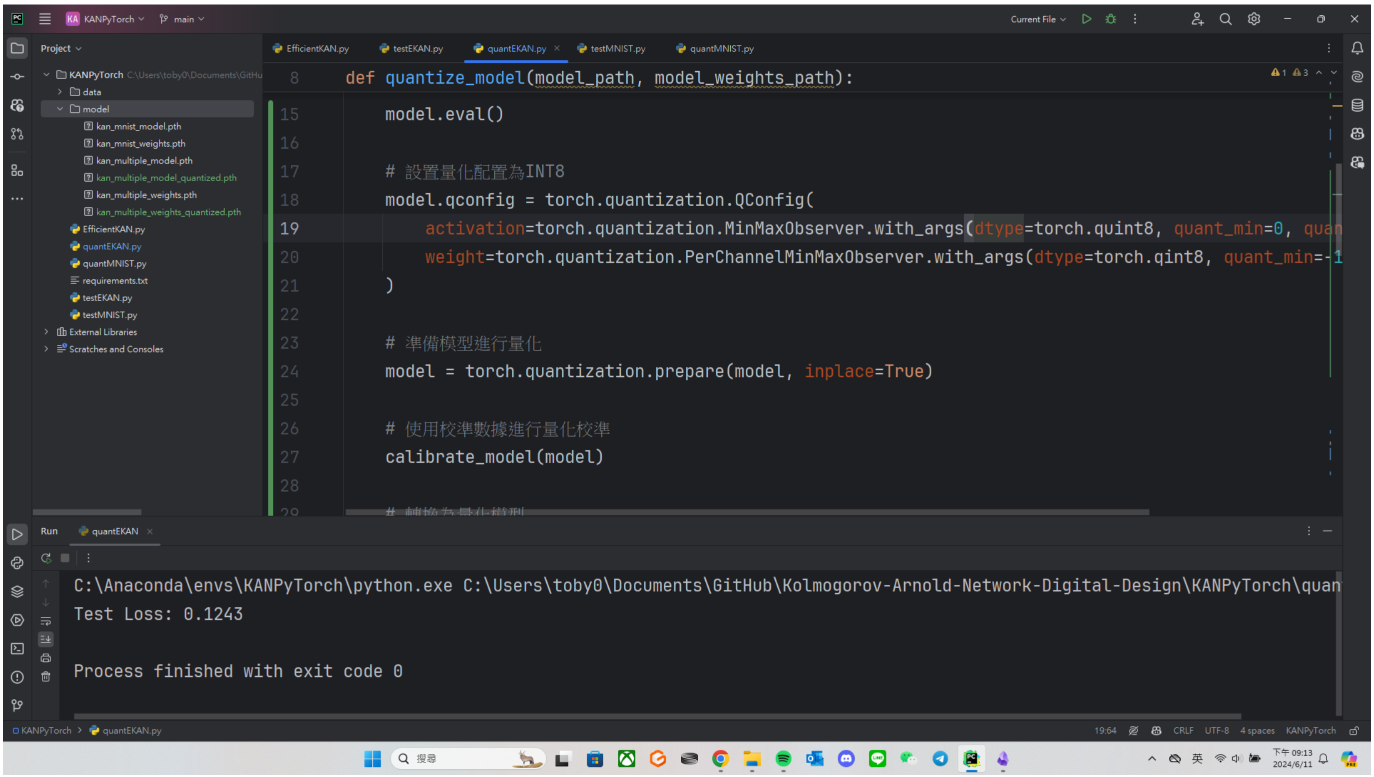
Info

最基本的 Quantization 實現，使用 PyTorch 庫進行 INT8 Quantization 操作並保存模型。

- PyTorch INT8 Quantization 代碼實現

```
import torch
import torch.nn as nn
import torch.quantization

from EfficientKAN import KAN


def quantize_model(model_path, model_weights_path):
    # 加載模型架構
    model = torch.load(model_path)

    # 加載模型權重
    model.load_state_dict(torch.load(model_weights_path))

    model.eval()

    # 設置量化配置為INT8
    model.qconfig = torch.quantization.QConfig(
        activation=torch.quantization.MinMaxObserver.with_args(dtype=torch.quint8, quant_min=0, quant_max=255),
        weight=torch.quantization.PerChannelMinMaxObserver.with_args(dtype=torch.qint8, quant_min=-128,
quant_max=127)
    )

    # 準備模型進行量化
    model = torch.quantization.prepare(model, inplace=True)

    # 使用校準數據進行量化校準
    calibrate_model(model)

    # 轉換為量化模型
    model = torch.quantization.convert(model, inplace=True)

    # 保存量化後的模型
    torch.save(model, 'model/kan_multiple_model_quantized.pth')
    torch.save(model.state_dict(), "model/kan_multiple_weights_quantized.pth")

    # 測試量化後的模型
    test_model(model)


def calibrate_model(model):
    # 使用隨機數據進行校準
    with torch.no_grad():
        for _ in range(100):
```

```python
            test_x = torch.rand(1024, 2)
            model(test_x)


def test_model(model):
    model.eval()
    with torch.no_grad():
        test_x = torch.rand(1024, 2)
        test_y = model(test_x)
        u = test_x[:, 0]
        v = test_x[:, 1]
        expected_y = u * v
        test_loss = nn.functional.mse_loss(test_y.squeeze(-1), expected_y)
        print(f"Test Loss: {test_loss.item():.4f}")


# 使用訓練好的模型路徑和權重文件路徑
model_path = 'model/kan_multiple_model.pth'
model_weights_path = 'model/kan_multiple_weights.pth'

quantize_model(model_path, model_weights_path)
```



Info

進一步將 Quantization 後的 Weights 導出成 CSV，提供給後續推論調用進行使用。

Warning

發現導出的 CSV 結果依舊是浮點數，需要再次確認為何是 CSV 是吃到原始的 Weights 而非 Quantization 後的 Weights。

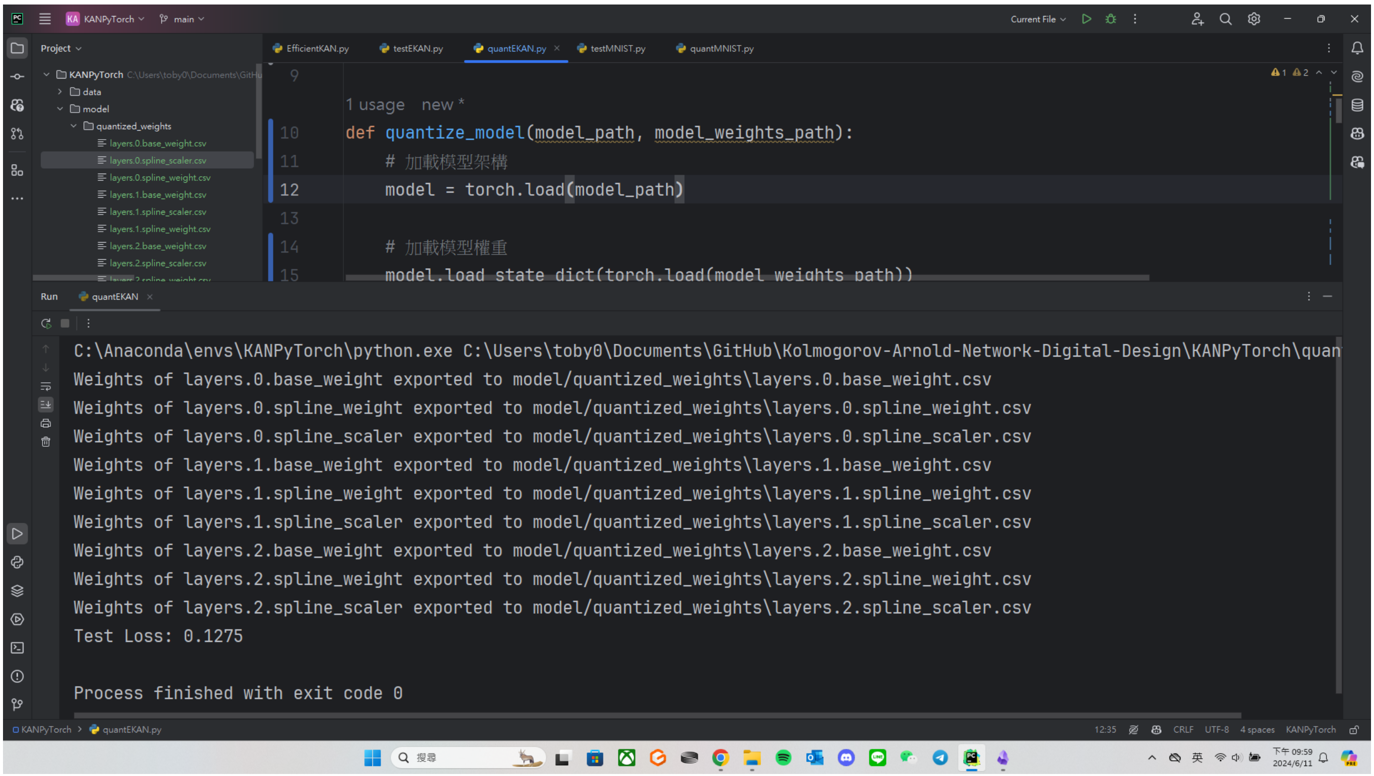
Check

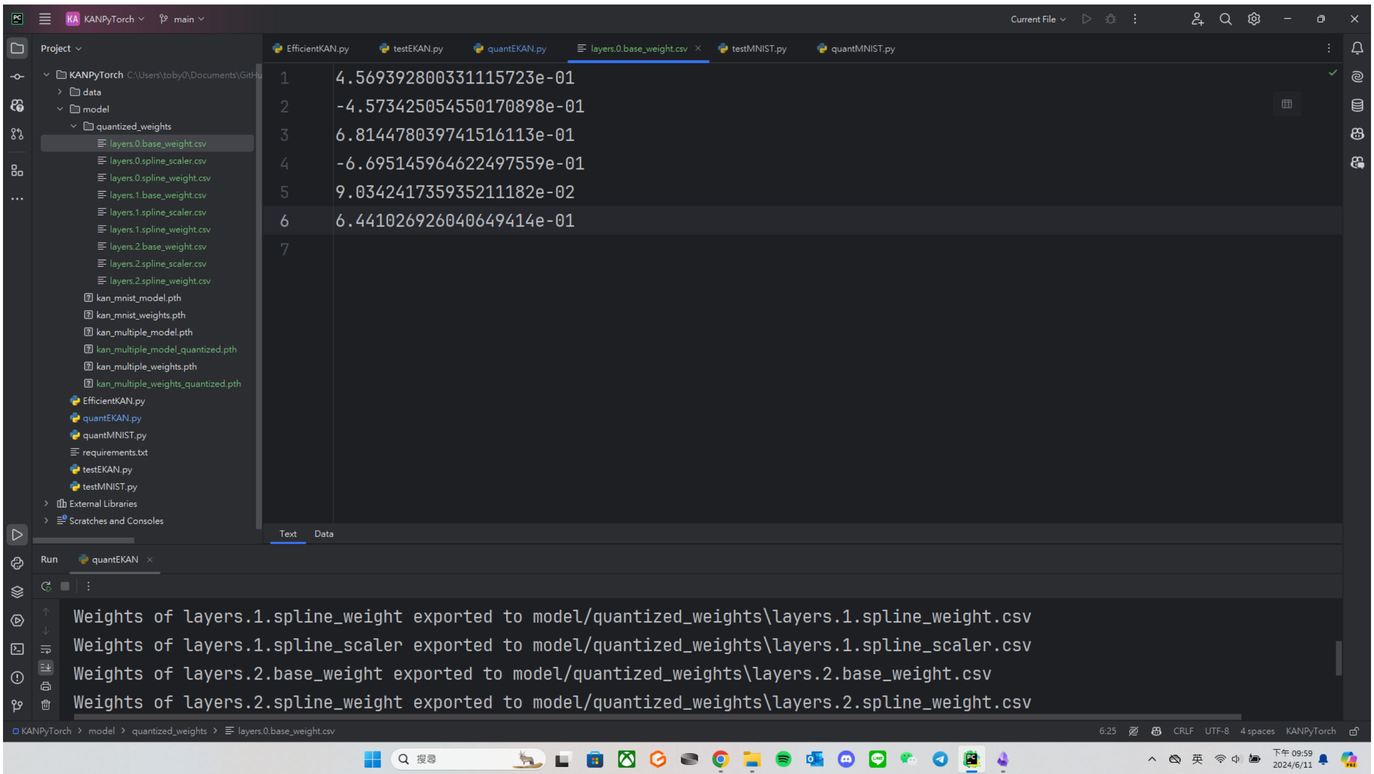
在 PyTorch 中進行量化時，模型的權重並不會直接在 `state_dict` 中顯示為 INT8。這是因為 PyTorch 量化模型的權重會被存儲為浮點數，但在推理時會被視為 INT8。具體來說，量化過程會引入 `FakeQuantize` 模塊來模擬 INT8 行為，但權重仍然以浮點數形式存在。

- PyTorch INT8 Quantization 含 CSV 權重導出

```python
import os

import numpy as np
import torch
import torch.nn as nn
import torch.quantization


def quantize_model(model_path, model_weights_path):
    # 加載模型架構
    model = torch.load(model_path)

    # 加載模型權重
    model.load_state_dict(torch.load(model_weights_path))

    model.eval()

    # 設置量化配置為INT8
    model.qconfig = torch.quantization.default_qconfig

    # 準備模型進行量化
    torch.quantization.prepare(model, inplace=False)

    # 使用校準數據進行量化校準
    calibrate_model(model)

    # 轉換為量化模型
    torch.quantization.convert(model, inplace=False)

    # 保存量化後的模型
    torch.save(model.state_dict(), "model/kan_multiple_weights_quantized.pth")
    torch.save(model, 'model/kan_multiple_model_quantized.pth')

    # 導出量化後的權重
    export_weights_to_csv(model, "model/quantized_weights")

    # print(model.state_dict())

    # 測試量化後的模型
    test_model(model)


def calibrate_model(model):
    # 使用隨機數據進行校準
    with torch.no_grad():
        for _ in range(100):
            test_x = torch.rand(1024, 2)
            model(test_x)


def test_model(model):
    model.eval()
    with torch.no_grad():
        test_x = torch.rand(1024, 2)
        test_y = model(test_x)
        u = test_x[:, 0]
        v = test_x[:, 1]
        expected_y = u * v
        test_loss = nn.functional.mse_loss(test_y.squeeze(-1), expected_y)
        print(f"Test Loss: {test_loss.item():.4f}")


def export_weights_to_csv(model, folder_path):
    if not os.path.exists(folder_path):
        os.makedirs(folder_path)

    # 遍歷模型中的每一層，並將權重存儲在單獨的 CSV 文件中
    for name, param in model.named_parameters():
        if param.requires_grad:
            weight_array = param.detach().cpu().numpy()
```

```
            file_path = os.path.join(folder_path, f"{name}.csv")
            np.savetxt(file_path, weight_array.flatten(), delimiter=",")
            print(f"Weights of {name} exported to {file_path}")


# 使用訓練好的模型路徑和權重文件路徑
model_path = 'model/kan_multiple_model.pth'
model_weights_path = 'model/kan_multiple_weights.pth'

quantize_model(model_path, model_weights_path)
```





Info

由於 PyTorch Quantization 的性質，改為使用手動實現 Quantization 工具，方法採用 Symmetric Quantization。

Summary

`Symmetric Quantization` 是指將數值範圍對稱地映射到量化級別。具體來說，就是把模型參數或激活值的浮點數範圍對稱地映射到固定的整數範圍，使得量化後的數值在正負兩個方向上具有相同的動態範圍。

1. 對稱映射：浮點數的正負最大值絕對值相等，映射到整數範圍時也是對稱的。例如，如果浮點數的範圍是 `[-a, a]`，那麼在 `INT8` 量化後的範圍就是 `[-127, 127]`（因為 `INT8` 的範圍是 `-128` 到 `127`，但通常會保留一個值作為零點）。
2. 縮放因子（`scaling factor`）：這個對稱的範圍由一個縮放因子來確定，縮放因子是浮點數最大絕對值與整數最大值之間的比例。公式如下：

$$textScalingFactor = \frac{\text{Max Absolute Float Value}}{\text{Max Absolute Integer Value}}$$

3. 零點（`zero point`）：對稱量化的零點通常是 `0`，這樣可以確保零值在量化和反量化後不會發生偏移。
4. 簡化運算：由於對稱量化的特性，運算過程中可以避免某些復雜的偏移和對齊操作，從而提升計算效率。

Example

假設有一組浮點數值範圍 `[-6, 6]`，我們希望將它量化到 `INT8` 範圍 `[-127, 127]`。這時的縮放因子就是：

$$textScalingFactor = \frac{6}{127} \approx 0.0472$$

任何一個浮點數 `x` 經過量化後的整數值就是：

$$textQuantizedValue = \text{round}(x/\text{Scaling Factor})$$

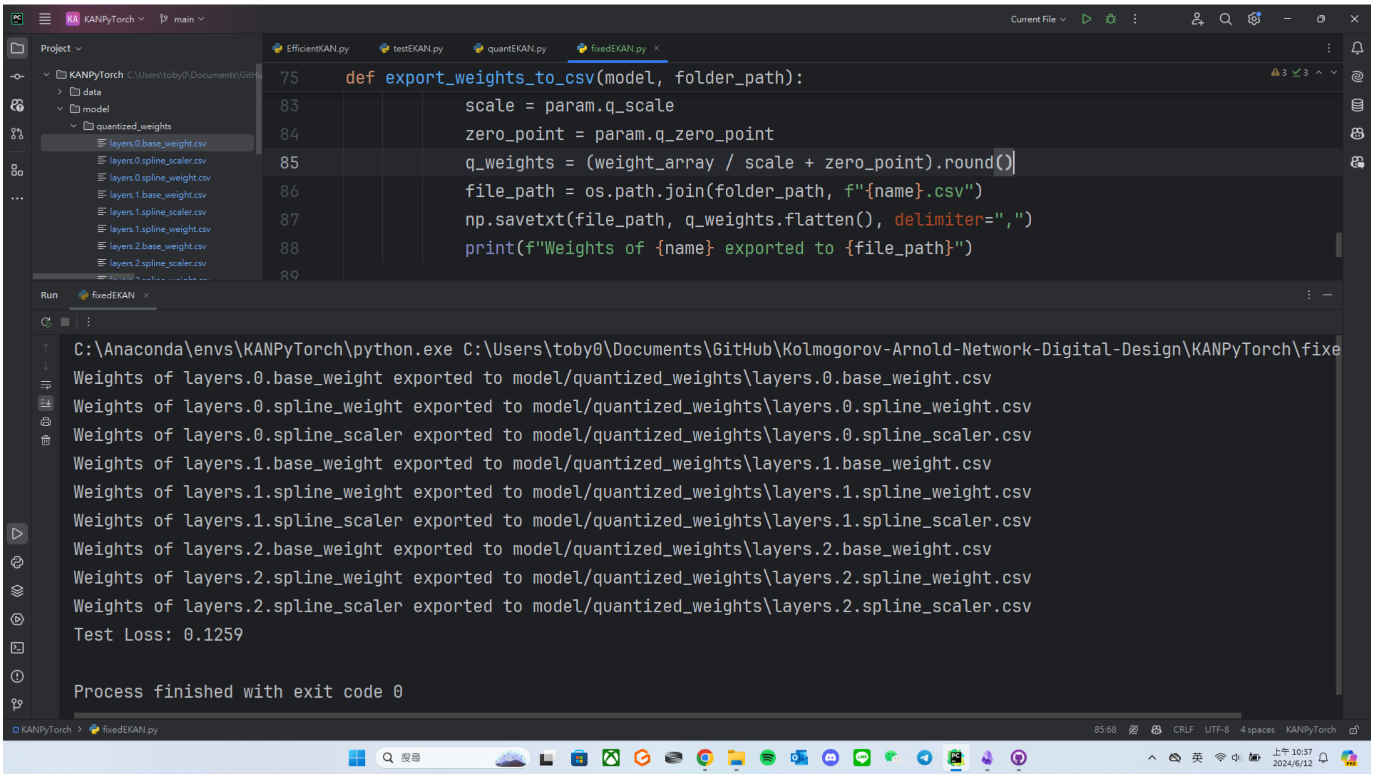
例如，浮點數 `3` 經過量化後的整數值是：

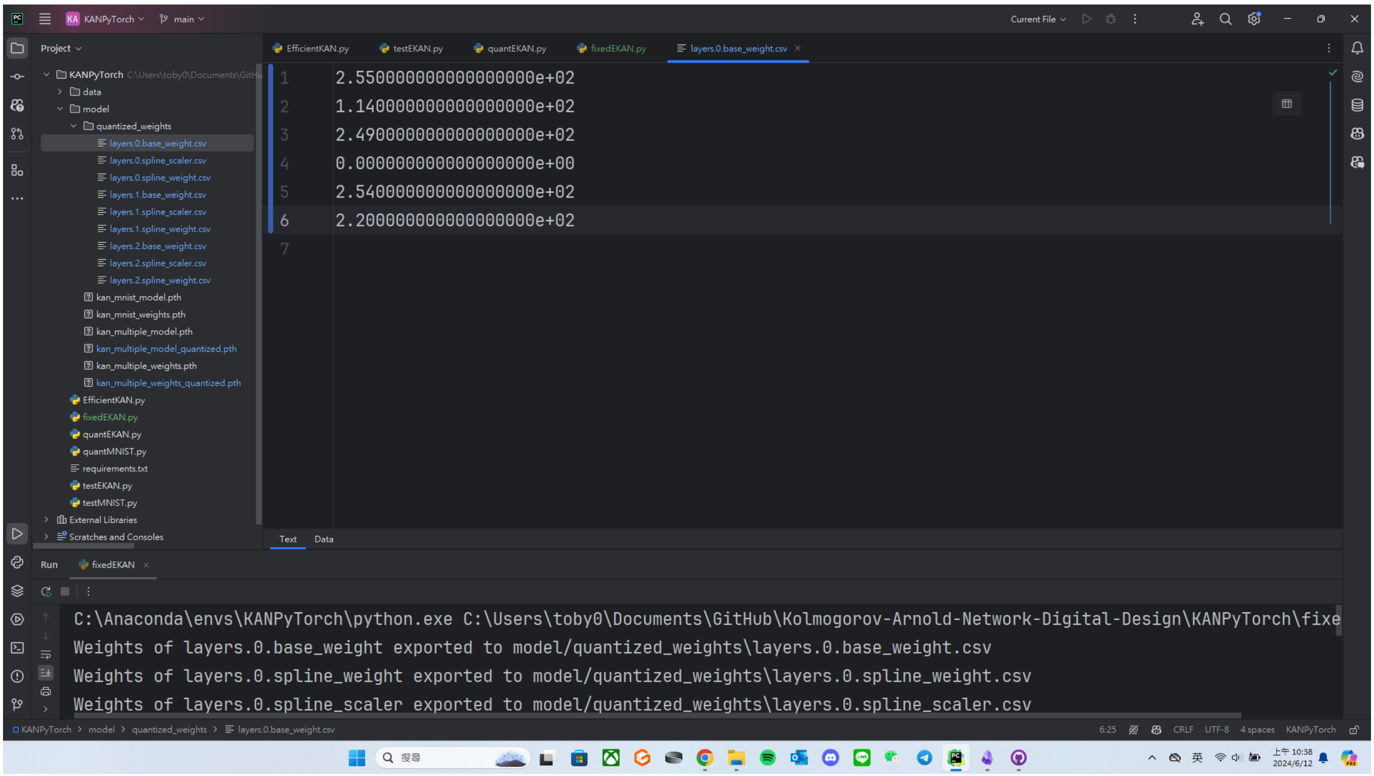
$$textQuantizedValue = \text{round}(3/0.0472) \approx 64$$

- 手動實現 `Quantization`

```python
import os

import numpy as np
import torch
import torch.nn as nn
import torch.quantization


def quantize_tensor(tensor, num_bits=8):
    qmin = 0.
    qmax = 2. ** num_bits - 1.

    min_val, max_val = tensor.min(), tensor.max()
    scale = (max_val - min_val) / (qmax - qmin)
    zero_point = qmin - min_val / scale

    zero_point = int(zero_point)
    q_tensor = (tensor / scale + zero_point).round().clamp(qmin, qmax)

    return q_tensor, scale, zero_point


def dequantize_tensor(q_tensor, scale, zero_point):
    return scale * (q_tensor - zero_point)


def quantize_model(model_path, model_weights_path):
    # 加載模型架構
    model = torch.load(model_path)

    # 加載模型權重
    model.load_state_dict(torch.load(model_weights_path))

    model.eval()

    # 手動量化模型中的權重
```

```python
    for name, param in model.named_parameters():
        if param.requires_grad:
            q_param, scale, zero_point = quantize_tensor(param.data)
            param.data = dequantize_tensor(q_param, scale, zero_point)
            param.q_scale = scale
            param.q_zero_point = zero_point

    # 保存量化後的模型
    torch.save(model.state_dict(), "model/kan_multiple_weights_quantized.pth")
    torch.save(model, 'model/kan_multiple_model_quantized.pth')

    # 導出量化後的權重
    export_weights_to_csv(model, "model/quantized_weights")

    # 測試量化後的模型
    test_model(model)


def calibrate_model(model):
    # 使用隨機數據進行校準
    with torch.no_grad():
        for _ in range(100):
            test_x = torch.rand(1024, 2)
            model(test_x)


def test_model(model):
    model.eval()
    with torch.no_grad():
        test_x = torch.rand(1024, 2)
        test_y = model(test_x)
        u = test_x[:, 0]
        v = test_x[:, 1]
        expected_y = u * v
        test_loss = nn.functional.mse_loss(test_y.squeeze(-1), expected_y)
        print(f"Test Loss: {test_loss.item():.4f}")


def export_weights_to_csv(model, folder_path):
    if not os.path.exists(folder_path):
        os.makedirs(folder_path)

    # 遍歷模型中的每一層，並將權重存儲在單獨的 CSV 文件中
    for name, param in model.named_parameters():
        if param.requires_grad:
            weight_array = param.detach().cpu().numpy()
            scale = param.q_scale
            zero_point = param.q_zero_point
            q_weights = (weight_array / scale + zero_point).round()
            file_path = os.path.join(folder_path, f"{name}.csv")
            np.savetxt(file_path, q_weights.flatten(), delimiter=",")
            print(f"Weights of {name} exported to {file_path}")


# 使用訓練好的模型路徑和權重文件路徑
model_path = 'model/kan_multiple_model.pth'
model_weights_path = 'model/kan_multiple_weights.pth'

quantize_model(model_path, model_weights_path)
```





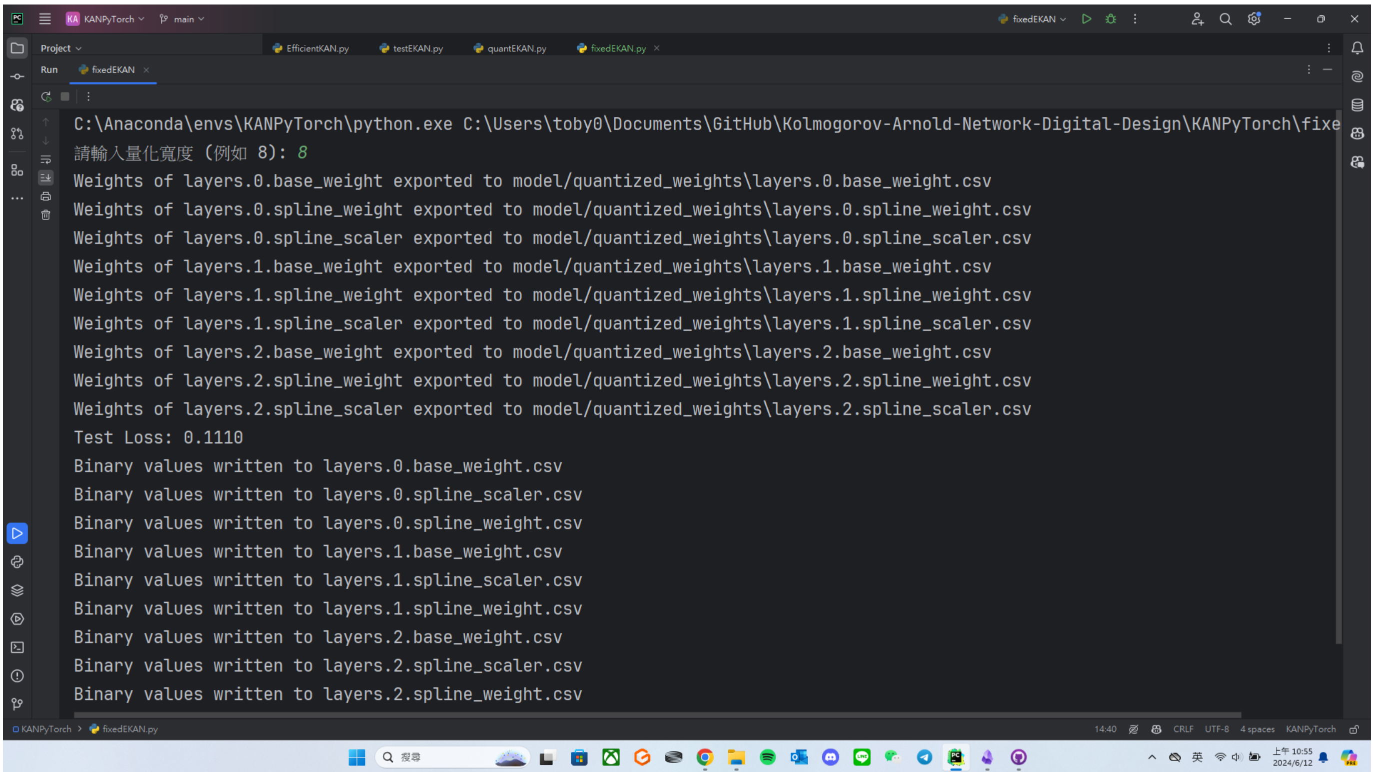
> **Info**
>
> 將 Quantization 後的 Weights 進一步處理為二進制輸出（同時自定義 Quantization 寬度）

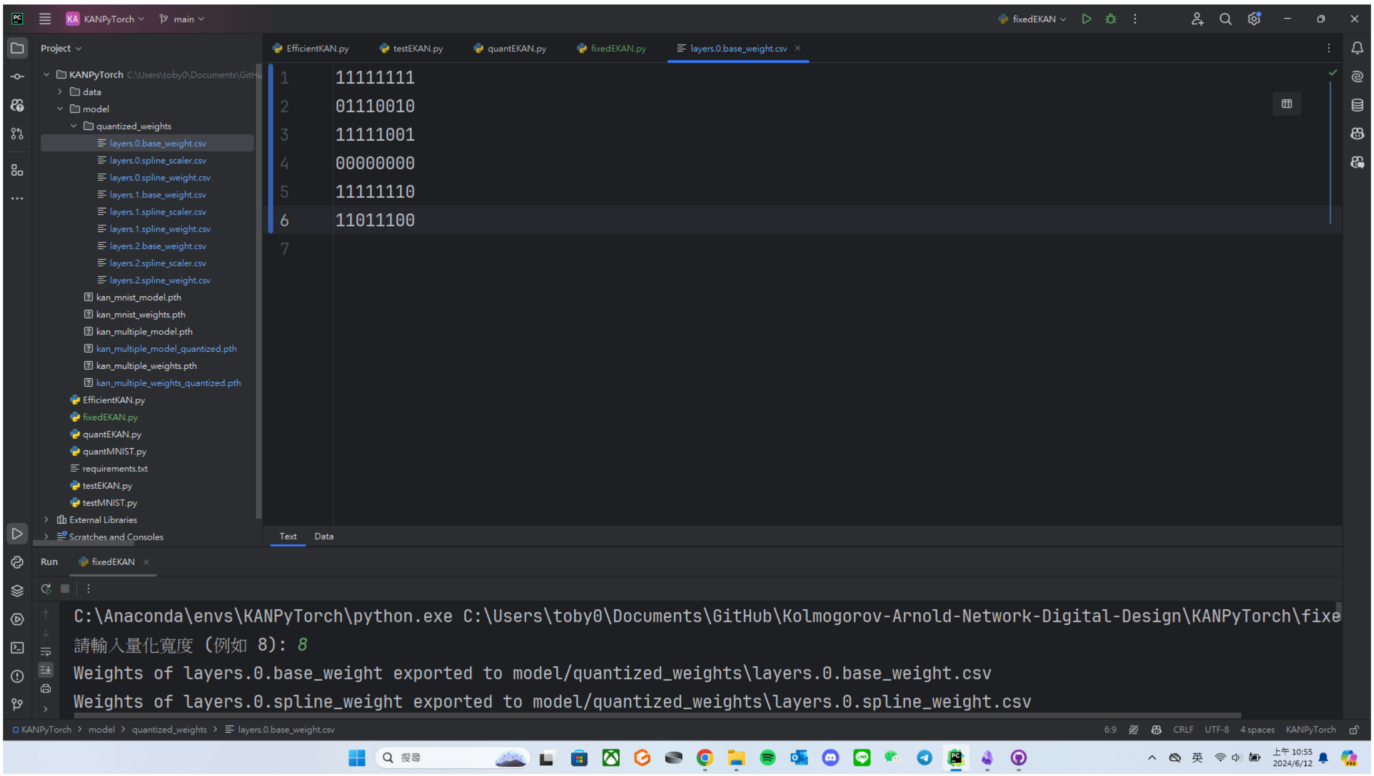
- 二進制輸出及自定義量化寬度

```python
import os
import numpy as np
import torch
import torch.nn as nn
import torch.quantization


def quantize_tensor(tensor, num_bits):
    qmin = 0.
```

```python
    qmax = 2. ** num_bits - 1.

    min_val, max_val = tensor.min(), tensor.max()
    scale = (max_val - min_val) / (qmax - qmin)
    zero_point = qmin - min_val / scale

    zero_point = int(zero_point)
    q_tensor = (tensor / scale + zero_point).round().clamp(qmin, qmax)

    return q_tensor, scale, zero_point


def dequantize_tensor(q_tensor, scale, zero_point):
    return scale * (q_tensor - zero_point)


def quantize_model(model_path, model_weights_path, num_bits):
    # 加載模型架構
    model = torch.load(model_path)

    # 加載模型權重
    model.load_state_dict(torch.load(model_weights_path))

    model.eval()

    # 手動量化模型中的權重
    for name, param in model.named_parameters():
        if param.requires_grad:
            q_param, scale, zero_point = quantize_tensor(param.data, num_bits)
            param.data = dequantize_tensor(q_param, scale, zero_point)
            param.q_scale = scale
            param.q_zero_point = zero_point

    # 保存量化後的模型
    torch.save(model.state_dict(), "model/kan_multiple_weights_quantized.pth")
    torch.save(model, 'model/kan_multiple_model_quantized.pth')

    # 導出量化後的權重
    export_weights_to_csv(model, "model/quantized_weights", num_bits)

    # 測試量化後的模型
    test_model(model)


def calibrate_model(model):
    # 使用隨機數據進行校準
    with torch.no_grad():
        for _ in range(100):
            test_x = torch.rand(1024, 2)
            model(test_x)


def test_model(model):
    model.eval()
    with torch.no_grad():
        test_x = torch.rand(1024, 2)
        test_y = model(test_x)
        u = test_x[:, 0]
        v = test_x[:, 1]
        expected_y = u * v
        test_loss = nn.functional.mse_loss(test_y.squeeze(-1), expected_y)
        print(f"Test Loss: {test_loss.item():.4f}")


def export_weights_to_csv(model, folder_path, num_bits):
    if not os.path.exists(folder_path):
        os.makedirs(folder_path)

    # 遍歷模型中的每一層，並將權重存儲在單獨的 CSV 文件中
    for name, param in model.named_parameters():
        if param.requires_grad:
```

```python
            weight_array = param.detach().cpu().numpy()
            scale = param.q_scale
            zero_point = param.q_zero_point
            q_weights = (weight_array / scale + zero_point).round()
            file_path = os.path.join(folder_path, f"{name}.csv")
            np.savetxt(file_path, q_weights.flatten(), delimiter=",")
            print(f"Weights of {name} exported to {file_path}")


def read_convert_and_write_binary(folder_path, num_bits):
    # 遍歷文件夾中的所有CSV文件
    for file_name in os.listdir(folder_path):
        if file_name.endswith(".csv"):
            file_path = os.path.join(folder_path, file_name)
            # 讀取CSV文件
            data = np.loadtxt(file_path, delimiter=",")
            # 將數據轉換為二進制格式
            binary_data = np.vectorize(np.binary_repr)(data.astype(int), width=num_bits)
            # 將二進制數據寫回CSV文件
            with open(file_path, 'w') as f:
                for binary_value in binary_data:
                    f.write(f"{binary_value}\n")
            print(f"Binary values written to {file_name}")


if __name__ == "__main__":
    # 從終端獲取量化寬度
    num_bits = int(input("請輸入量化寬度 (例如 8): "))

    # 使用訓練好的模型路徑和權重文件路徑
    model_path = 'model/kan_multiple_model.pth'
    model_weights_path = 'model/kan_multiple_weights.pth'

    quantize_model(model_path, model_weights_path, num_bits)

    # 調用函數，讀取並轉換文件夾中的CSV文件
    folder_path = "model/quantized_weights"
    read_convert_and_write_binary(folder_path, num_bits)
```





方法論合成 Grafcet 控制器電路

> ✅ Success
>
> 更換測試資料及 Network 設計以求可以將電路設計容納進去（學習函數擬合簡易乘法，KAN 的架構為 [2, 3, 3, 1]）。

> 💧 Tip
>
> Verilog 僅供模型推論使用。

- KANLayer 實現（KANLayer.v）

```
module KANLinear #(parameter IN_FEATURES = 2, OUT_FEATURES = 3, integer LAYER_NUM = 0) (
    input clk,
    input reset,
    input [15:0] data_in [IN_FEATURES-1:0],
    output reg [15:0] data_out [OUT_FEATURES-1:0]
);

    reg [15:0] base_weight [OUT_FEATURES*IN_FEATURES-1:0];
    reg [15:0] spline_weight [OUT_FEATURES*IN_FEATURES-1:0];
    reg [15:0] base_weight_2d [OUT_FEATURES-1:0][IN_FEATURES-1:0];
    reg [15:0] spline_weight_2d [OUT_FEATURES-1:0][IN_FEATURES-1:0];

    integer i, j;

    initial begin
        if (LAYER_NUM == 0) begin
            $readmemh("C:/intelFPGA_lite/18.1/KAN2/base_weight_layer_0.txt", base_weight);
            $readmemh("C:/intelFPGA_lite/18.1/KAN2/spline_weight_layer_0.txt", spline_weight);
        end else if (LAYER_NUM == 1) begin
            $readmemh("C:/intelFPGA_lite/18.1/KAN2/base_weight_layer_1.txt", base_weight);
            $readmemh("C:/intelFPGA_lite/18.1/KAN2/spline_weight_layer_1.txt", spline_weight);
        end else if (LAYER_NUM == 2) begin
            $readmemh("C:/intelFPGA_lite/18.1/KAN2/base_weight_layer_2.txt", base_weight);
            $readmemh("C:/intelFPGA_lite/18.1/KAN2/spline_weight_layer_2.txt", spline_weight);
        end
```

```
        for (i = 0; i < OUT_FEATURES; i = i + 1) begin
            for (j = 0; j < IN_FEATURES; j = j + 1) begin
                base_weight_2d[i][j] = base_weight[i * IN_FEATURES + j];
                spline_weight_2d[i][j] = spline_weight[i * IN_FEATURES + j];
            end
        end
    end

    always @(posedge clk or posedge reset) begin
        if (reset) begin
            for (i = 0; i < OUT_FEATURES; i = i + 1) begin
                data_out[i] <= 16'd0;
            end
        end else begin
            for (i = 0; i < OUT_FEATURES; i = i + 1) begin
                data_out[i] <= 16'd0;
                for (j = 0; j < IN_FEATURES; j = j + 1) begin
                    data_out[i] <= data_out[i] + base_weight_2d[i][j] * data_in[j];
                end
                if (data_out[i] < 16'd0) begin
                    data_out[i] <= 16'd0;
                end
            end
        end
    end

endmodule
```

> ⚠ Caution
>
> MNIST 版本之 Reference

- KANLayer 實現 (KANLayer.v)

```
module KANLayer #(
    parameter IN_FEATURES = 784,
    parameter OUT_FEATURES = 64,
    parameter SCALE = 256,  // Quantization scale factor
    parameter BASE_WEIGHT_FILE = "C:/intelFPGA_lite/18.1/KAN/base_weight_layer_0.txt",
    parameter SPLINE_WEIGHT_FILE = "C:/intelFPGA_lite/18.1/KAN/spline_weight_layer_0.txt"
)(
    input wire clk,
    input wire reset,
    input wire [7:0] in_data [0:IN_FEATURES-1],  // Input data
    output reg [7:0] out_data [0:OUT_FEATURES-1] // Output data
);
    // Weights stored in on-chip memory (BRAM)
    reg signed [15:0] base_weights [0:OUT_FEATURES*IN_FEATURES-1];
    reg signed [15:0] spline_weights [0:OUT_FEATURES*IN_FEATURES-1];

    // Load weights from memory (initialization)
    initial begin
        $readmemh(BASE_WEIGHT_FILE, base_weights);
        $readmemh(SPLINE_WEIGHT_FILE, spline_weights);
    end

    // Output registers
    reg signed [31:0] base_output [0:OUT_FEATURES-1];
    reg signed [31:0] spline_output [0:OUT_FEATURES-1];
    reg signed [31:0] total_output [0:OUT_FEATURES-1];

    integer i, j;

    // Forward pass
    always @(posedge clk or posedge reset) begin
        if (reset) begin
            for (i = 0; i < OUT_FEATURES; i = i + 1) begin
                base_output[i] <= 0;
                spline_output[i] <= 0;
```

```
                total_output[i] <= 0;
            end
        end else begin
            for (i = 0; i < OUT_FEATURES; i = i + 1) begin
                base_output[i] <= 0;
                spline_output[i] <= 0;
                for (j = 0; j < IN_FEATURES; j = j + 1) begin
                    base_output[i] <= base_output[i] + in_data[j] * base_weights[i*IN_FEATURES + j];
                    spline_output[i] <= spline_output[i] + in_data[j] * spline_weights[i*IN_FEATURES + j];
                end
                total_output[i] <= (base_output[i] + spline_output[i]) / SCALE;  // Combine and scale the outputs
                out_data[i] <= total_output[i][15:8];  // Convert to 8-bit output
            end
        end
    end
endmodule
```

ChatGPT 合成 Grafcet Datapath 電路

Success

更換測試資料及 Network 設計以求可以將電路設計容納進去（學習函數擬合簡易乘法，KAN 的架構為 [2, 3, 3, 1]）。

Tip

Verilog 僅供模型推論使用。

- 完整 Network 實現（KAN.v）

```
module KAN #(parameter IN_FEATURES = 2, L1_FEATURES = 3, L2_FEATURES = 3, OUT_FEATURES = 1) (
    input clk,
    input reset,
    input [15:0] input_data [IN_FEATURES-1:0],
    output [15:0] output_data
);

    wire [15:0] layer1_out [L1_FEATURES-1:0];
    wire [15:0] layer2_out [L2_FEATURES-1:0];
    wire [15:0] layer3_out [OUT_FEATURES-1:0];

    KANLinear #(.IN_FEATURES(IN_FEATURES), .OUT_FEATURES(L1_FEATURES), .LAYER_NUM(0)) layer1 (
        .clk(clk),
        .reset(reset),
        .data_in(input_data),
        .data_out(layer1_out)
    );

    KANLinear #(.IN_FEATURES(L1_FEATURES), .OUT_FEATURES(L2_FEATURES), .LAYER_NUM(1)) layer2 (
        .clk(clk),
        .reset(reset),
        .data_in(layer1_out),
        .data_out(layer2_out)
    );

    KANLinear #(.IN_FEATURES(L2_FEATURES), .OUT_FEATURES(OUT_FEATURES), .LAYER_NUM(2)) layer3 (
        .clk(clk),
        .reset(reset),
        .data_in(layer2_out),
        .data_out(layer3_out)
    );

    assign output_data = layer3_out[0];

endmodule
```

> ⚠ Caution
>
> MNIST 版本之 Reference

- 完整 Network 實現（KAN.v）

```
module KAN (
    input wire clk,
    input wire reset,
    input wire [7:0] in_data [0:783],  // 28x28 = 784 pixels, 8-bit each
    output wire [7:0] out_data [0:9]   // 10 classes, 8-bit each
);
    // Internal signals for each layer
    wire [7:0] layer1_out [0:63];
    wire [7:0] layer2_out [0:9];

    // Instantiate layers
    KANLayer #(
        .IN_FEATURES(784),
        .OUT_FEATURES(64),
        .BASE_WEIGHT_FILE("C:/intelFPGA_lite/18.1/KAN/base_weight_layer_0.txt"),
        .SPLINE_WEIGHT_FILE("C:/intelFPGA_lite/18.1/KAN/spline_weight_layer_0.txt")
    ) layer1 (
        .clk(clk),
        .reset(reset),
        .in_data(in_data),
        .out_data(layer1_out)
    );

    KANLayer #(
        .IN_FEATURES(64),
        .OUT_FEATURES(10),
        .BASE_WEIGHT_FILE("C:/intelFPGA_lite/18.1/KAN/base_weight_layer_1.txt"),
        .SPLINE_WEIGHT_FILE("C:/intelFPGA_lite/18.1/KAN/spline_weight_layer_1.txt")
    ) layer2 (
        .clk(clk),
        .reset(reset),
        .in_data(layer1_out),
        .out_data(layer2_out)
    );

    // Connect the final output
    assign out_data = layer2_out;
endmodule
```

FPGA 整合驗證

> ✔ Success
>
> 更換測試資料及 Network 設計以求可以將電路設計容納進去（學習函數擬合簡易乘法，KAN 的架構為 [2, 3, 3, 1]）。

> Tip
>
> Verilog 僅供模型推論使用。

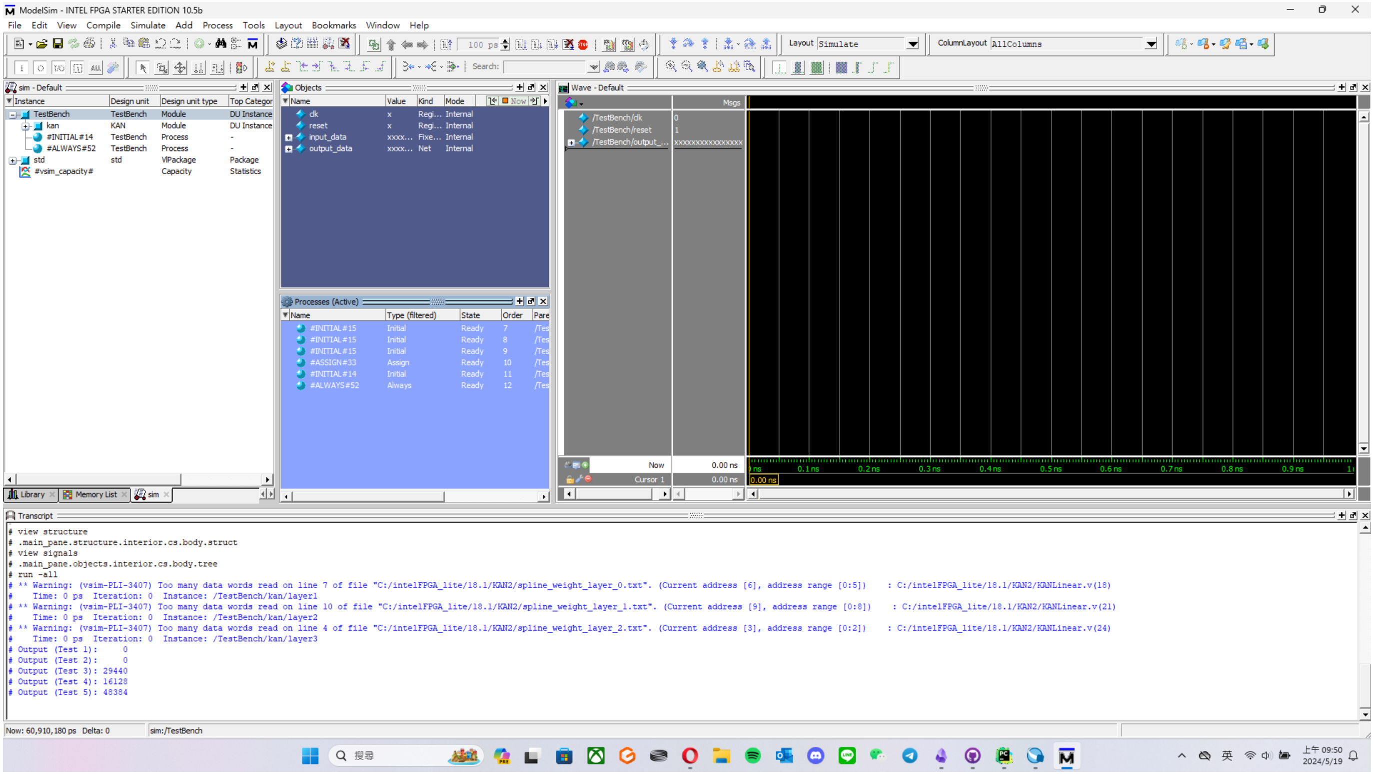
- Testbench 實現（Testbench.v）

```
module TestBench;
    reg clk;
    reg reset;
    reg [15:0] input_data [0:1];
    wire [15:0] output_data;

    KAN #(.IN_FEATURES(2), .L1_FEATURES(3), .L2_FEATURES(3), .OUT_FEATURES(1)) kan (
```

```
        .clk(clk),
        .reset(reset),
        .input_data(input_data),
        .output_data(output_data)
    );

    initial begin
        clk = 0;
        reset = 1;
        input_data[0] = 16'd0;
        input_data[1] = 16'd0;
        #10 reset = 0;

        // test data 1
        input_data[0] = 16'd50;
        input_data[1] = 16'd30;
        #10;
        $display("Output (Test 1): %d", output_data);

        // test data 2
        input_data[0] = 16'd100;
        input_data[1] = 16'd200;
        #10;
        $display("Output (Test 2): %d", output_data);

        // test data 3
        input_data[0] = 16'd150;
        input_data[1] = 16'd250;
        #10;
        $display("Output (Test 3): %d", output_data);

        // test data 4
        input_data[0] = 16'd75;
        input_data[1] = 16'd125;
        #10;
        $display("Output (Test 4): %d", output_data);

        // test data 5
        input_data[0] = 16'd175;
        input_data[1] = 16'd225;
        #10;
        $display("Output (Test 5): %d", output_data);
    end

    always #5 clk = ~clk;

endmodule
```

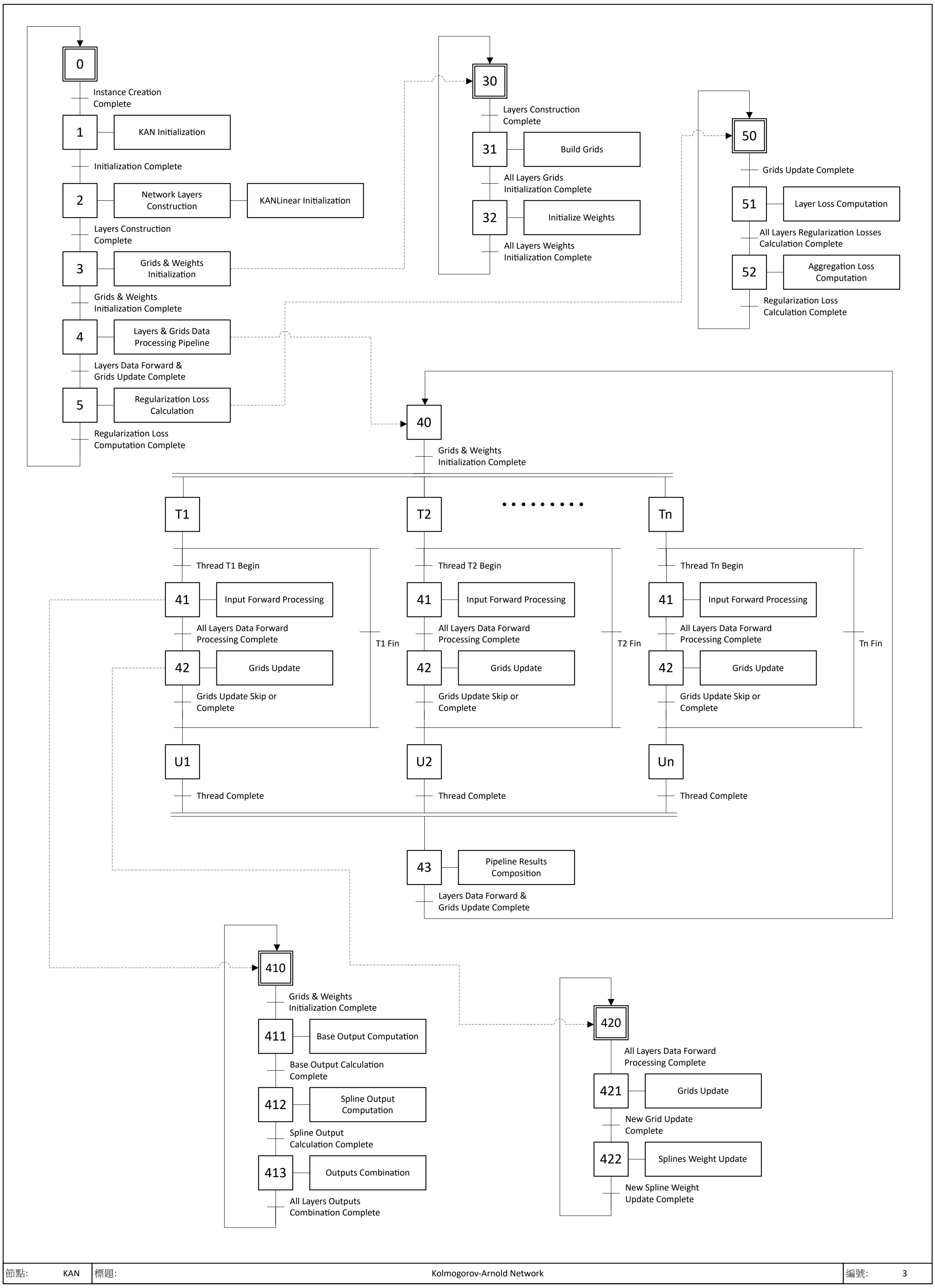
> ⚠ Warning
>
> 硬體設計完仿真的結果如下，誤差老實說非常大，基本上是 `Quantization` 後的 `Weight` 發生狀況。

- `ModelSim` 仿真結果



Pipeline 架構處理

- 功能模組離散事件建模 Grafcet（Pipeline 架構調整）



✅ Success

更換測試資料及 Network 設計以求可以將電路設計容納進去（學習函數擬合簡易乘法，KAN 的架構為 [2, 3, 3, 1]）。

💧 Tip

Verilog 僅供模型推論使用。