OpenMP

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A running example: Sparse Matrix Vector Multiply (SpMV)

- sparse matrix : a matrix whose elements are mostly zeros
- i.e. the number of non-zero elements (nnz) $\ll$ the number of all elements ($M \times N$)
  - M : the number of rows
  - N : the number of columns

Sparse matrices appear everywhere

- meshes in scientific simulation
  - $A_{i,j}$ = a weight connecting nodes i and j in the mesh
- graphs, which in turn appear in many applications
  - $A_{i,j}$ = the weight of the edge $i \to j$ (or $j \to i$)
  - Web, social network, road/traffic networks, citation networks, metabolic pathways, etc.
- many problems can be solved using SpMV
  - eigenvalues (including PageRank, graph partitioning, etc.)
  - partial differential equation
  - ...

What makes "sparse" matrix different from ordinary (dense) matrix?

- the number of non-zero elements are so small that representing it as $M \times N$ array is too wasteful (or just impossible)
- $\rightarrow$ use a data structure that takes memory/computation only (or mostly) for non-zero elements (coordinate list, compressed sparse row, etc.)

Coordinate list (COO)

- represent a matrix as a list of $(i, j, A_{i,j})$'s
- data format:

```
1  struct coo {
2    int n_rows, n_cols, nnz;
3    /* nnz elements */
4    struct { i, j, Aij } * elems;
5  };
```

- SpMV $(y = Ax)$

```
1  for (k = 0; k < A.nnz; k++) {
2    i,j,Aij = A.elems[k];
3    y[i] += Aij * x[j];
4  }
```

Compressed sparse row (CSR)

- puts elements of a single row in a contiguous range
- an index (number) specifies where a particular row begins in the `elems` array
- $\rightarrow$ *no need to have i for every single element*
- data format:

```
1  struct coo {
2    int n_rows, n_cols, nnz;
3    struct { j, Aij } * elems; // nnz elements
4    int * row_start; // n_rows elements
5  };
```

`elems[row_start[`i`]]` $\cdots$ `elems[row_start[`$i+1$`]]` are the elements in the ith row

- SpMV ($y = Ax$)

```
1  for (i = 0; i < A.n_rows; i++) {
2    for (k = A.row_start[i]; k < A.row_start[i+1]; k++) {
3      j,Aij = A.elems[k];
4      y[i] += Aij * x[j];
5    } }
```

OpenMP

- *de fact* standard model for programming shared memory machines
- C/C++/Fortran + directives + APIs
  - by `#pragma` in C/C++
  - by comments in Fortran
- many free/vendor compilers, including GCC, LLVM, NVIDIA HPC SDK

OpenMP reference

- official home page: `http://openmp.org/`
- specification: `http://openmp.org/wp/openmp-specifications/`
- latest version is 5.0 (`https://www.openmp.org/spec-html/5.0/openmp.html`)
- section numbers below refer to those in OpenMP spec 5.0

Compiling OpenMP programs for multicores

- GCC and LLVM (`clang/clang++`) : compile with `-fopenmp`

```
$ clang -Wall -fopenmp program.c
$ gcc -Wall -fopenmp program.c
```

- NVIDIA HPC SDK (`nvc/nvc++`) : compile with `-mp`

```
$ nvc -Wall -mp program.c
```

- In this lecture, we use LLVM and NVIDIA HPC SDK, as they support OpenMP for multicore, GPU offloading, and CUDA

Running OpenMP programs

- run the executable specifying the number of threads with `OMP_NUM_THREADS` environment variable

```
1 $ OMP_NUM_THREADS=1 ./a.out  # use 1 thread
2 $ OMP_NUM_THREADS=4 ./a.out  # use 4 threads
```

- if `OMP_NUM_THREADS` is unspecified, it uses the number of available processors visible to OS, including hyperthreading
- see 2.6.1 "Determining the Number of Threads for a parallel Region" for more details and other ways to control the number of threads

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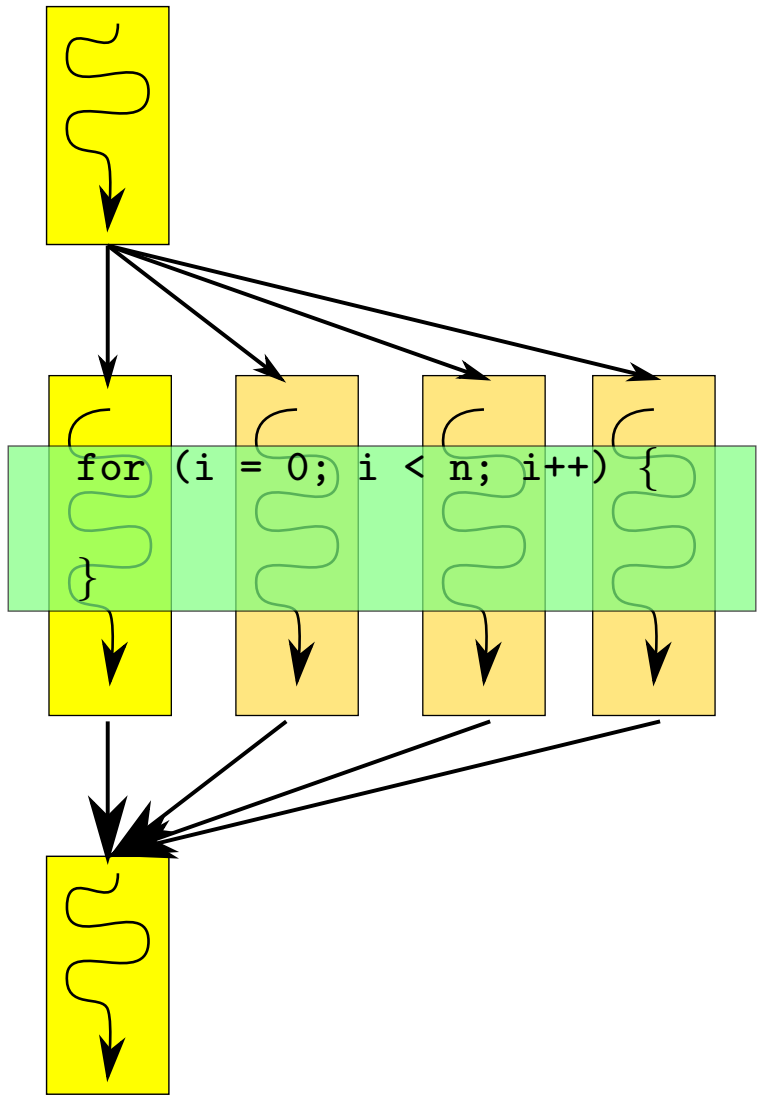
Two pragmas you must know first

- `#pragma omp parallel` to launch a team of threads (2.6)
- then `#pragma omp for` to distribute iterations to threads (2.9.2)

Note: all OpenMP pragmas have the common format: `#pragma omp ...`

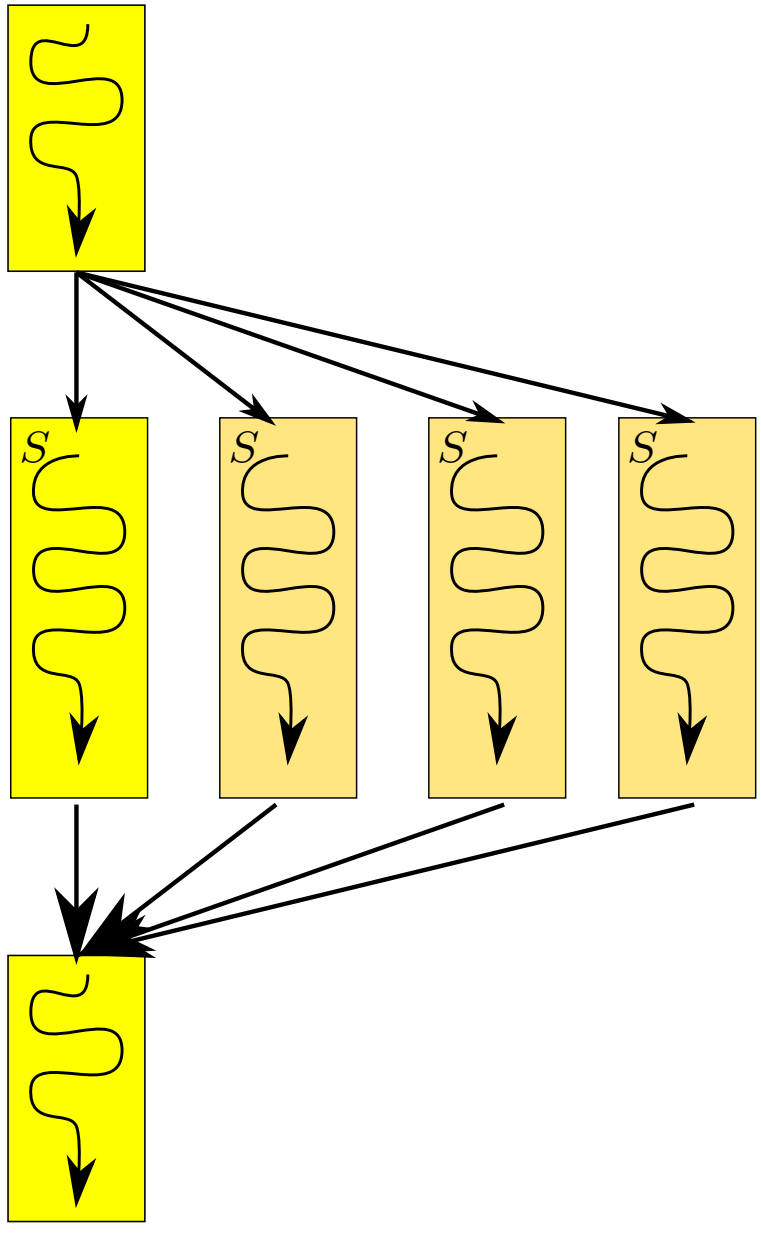


#pragma omp parallel

- basic syntax:

```
1   ...
2   #pragma omp parallel
3     S
4   ...
```

- basic semantics:
  - create a team of `OMP_NUM_THREADS` threads
  - the current thread becomes the *master* of the team
  - *S will be executed by each member of the team*
  - the master thread waits for all to finish S and continue



`parallel` pragma example

```
1  #include <stdio.h>
2  int main() {
3    printf("hello\n");
4  #pragma omp parallel
5    printf("world\n");
6    printf("bye\n");
7    return 0;
8  }
```

```
1  $ OMP_NUM_THREADS=1 ./a.out
2  hello
3  world
4  $ OMP_NUM_THREADS=4 ./a.out
5  hello
6  world
7  world
8  world
9  world
10 bye
```

Remarks : what does `parallel` do?

- you may assume an OpenMP thread ≈ OS-supported thread (e.g., Pthread)
- that is, if you write this program

```
1 int main() {
2 #pragma omp parallel
3   worker();
4 }
```

and run it as follows,

```
1 $ OMP_NUM_THREADS=50 ./a.out
```

you will get 50 OS-level threads, each doing `worker()`

- ⇒ specifying a number (> the hardware core count) causes more harm than good on CPUs

How to distribute work among threads?

- `#pragma omp parallel` creates threads, *all executing the same statement*
- it's not a means to parallelize work, *per se*, but just a means to create a number of similar threads
  - Single Program Multiple Data (SPMD) model
- so how to distribute (or partition) work among them?
  1. do it yourself
  2. use *work sharing* constructs

Do it yourself: functions to get the number/id of threads

- `omp_get_num_threads()` (3.2.2) : the number of threads *in the current team*
- `omp_get_thread_num()` (3.2.4) : the current thread's id (0, 1, ...) in the team
- they are primitives with which you may partition work yourself by whichever ways you prefer
- e.g.,

```
1 #pragma omp parallel
2 {
3   int t  = omp_get_thread_num();
4   int nt = omp_get_num_threads();
5   /* divide n iterations evenly amont nt threads */
6   for (i = t * n / nt; i < (t + 1) * n / nt; i++) {
7     ...
8   }
9 }
```

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Work sharing constructs

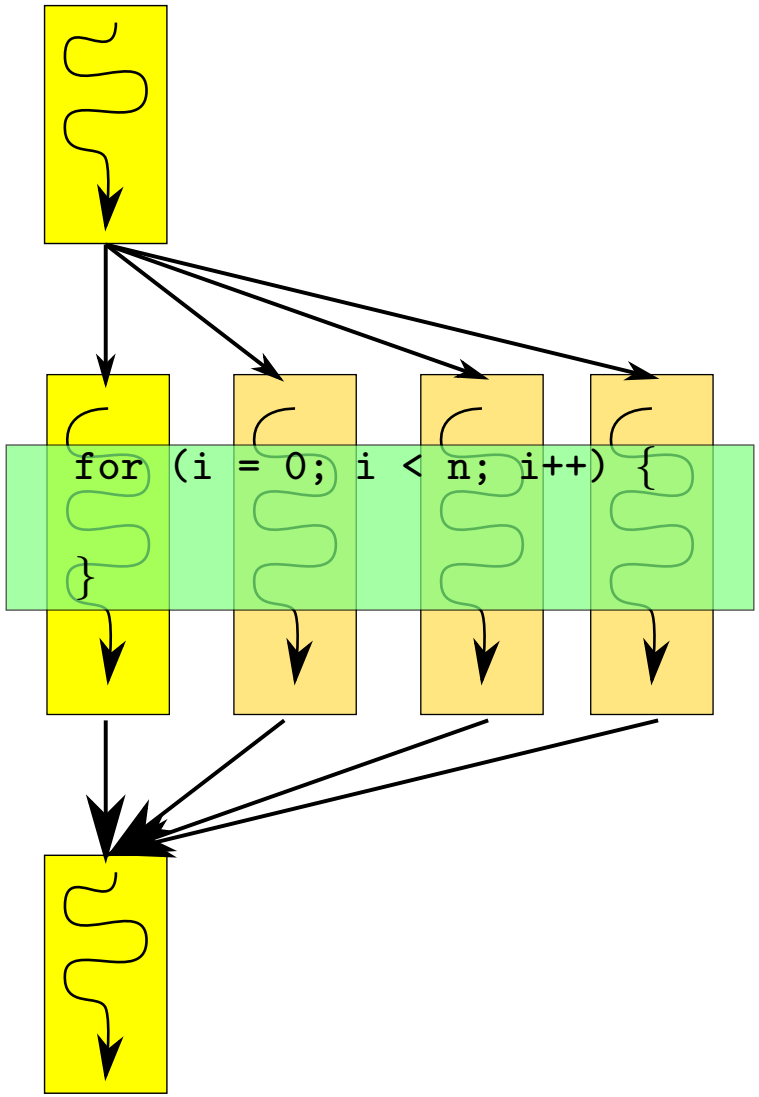
- in theory, `parallel` construct is all you need to do things in parallel
- but it's too inconvenient
- OpenMP defines ways to *partition* work among threads *(work sharing constructs)*
  - for
  - task

`#pragma omp for` (work-sharing for)

- basic syntax (2.9.2):

```
1  #pragma omp for
2  for(i=...; i...; i+=...){
3    S
4  }
```

- basic semantics:
  the threads in the team divde the iterations among them
- but how? ⇒ scheduling



`#pragma omp for` restrictions

- iterations are executed in any order may interleave
  - the program must not rely on the order in which they are executed
- strong syntactic restrictions apply (2.9.1); basically, *the iteration space must be easily identifiable at the beginning* of the loop
  - roughly, it must be of the form:

```
1  #pragma omp for
2  for(i = init; i < limit; i += incr)
3    S
```

    except `<` and `+=` may be other similar operators
  - *init*, *limit*, and *incr* must be loop invariant

Parallelizing loop nests by `collapse`

- `collapse(`l`)` can be used to work-share nested loops. e.g.,

```
1  #pragma omp for collapse(2)
2  for (i = 0; i < n; i++)
3    for (j = 0; j < n; j++)
4      S
```

  will partition n^2 iterations of the doubly-nested loop
- `schedule` clause applies to nested loops as if the nested loop is an equivalent flat loop
- restriction: the loop must be *"perfectly nested"* (the iteration space must be a rectangular and no intervening statement between different levels of the nest)

Parallel SpMV for CSR using `#pragma omp for`

- it only takes to work-share the outer for loop

```
1  // assume inside #pragma omp parallel
2     ...
3  #pragma omp for
4  for (i = 0; i < A.n_rows; i++) {
5    for (k = A.row_start[i]; k < A.row_start[i+1]; k++) {
6      j,Aij = A.elems[k];
7      y[i] += Aij * x[j];
8    }
9  }
```

- note: the inner loop (k) is executed sequentially

Parallel SpMV COO using `#pragma omp for`?

- the following code does *not* work (why?)

```
1  // assume inside #pragma omp parallel
2     ...
3  #pragma omp for
4  for (k = 0; k < A.nnz; k++) {
5    i,j,Aij = A.elems[k];
6    y[i] += Aij * x[j];
7  }
```

- for a similar reason, we cannot apply `collapse` to the CSR loop
- a possible remedy will be described later

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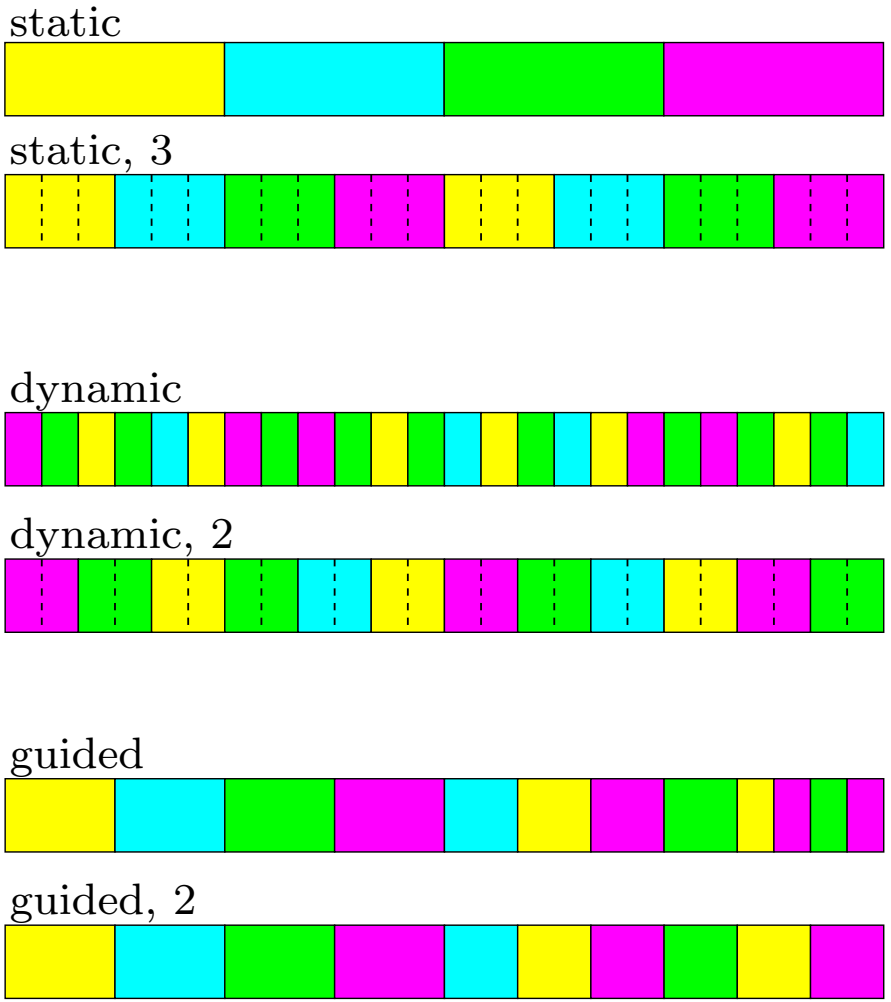
Scheduling (2.9.2)

- `schedule` clause in work-sharing for loop determines how iterations are divided among threads
- There are three alternatives (`static`, `dynamic`, and `guided`)

static, dynamic, and guided

- `schedule(static[,`*chunk*`])`: predictable round-robin
- `schedule(dynamic[,`*chunk*`])`: each thread repeats fetching *chunk* iterations
- `schedule(guided[,`*chunk*`])`: threads grab many iterations in early stages; gradually reduce iterations to fetch at a time
- *chunk* specifies the minimum granularity (iteration counts)



Other scheduling options and notes

- `schedule(runtime)` determines the schedule by `OMP_SCHEDULE` environment variable. e.g.,

```
1 $ OMP_SCHEDULE=dynamic,2 ./a.out
```

- `schedule(auto)` or no `schedule` clause choose an implementation dependent default

Visualizing schedulers

See demo

Scheduling for SpMV on CSR

```
1  // assume inside #pragma omp parallel
2     ...
3  #pragma omp for schedule(???)
4  for (i = 0; i < A.n_rows; i++) {
5    for (k = A.row_start[i]; k < A.row_start[i+1]; k++) {
6      j,Aij = A.elems[k];
7      y[i] += Aij * x[j];
8    }
9  }
```

- static? depending on the number of elements in rows, load imbalance may be significant
- dynamic/guided? load balancing will be better, but extremely dense rows may still be an issue
- the more robust strategy is to partition non-zeros, not rows

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Task parallelism in OpenMP

- OpenMP's initial focus was simple parallel loops
- since 3.0, it supports task parallelism
- but why it's necessary?
- aren't `parallel` and `for` all we need?

Limitation of `parallel for`

- what if you have a parallel loop inside another

```
1 for ( ... ) {
2   ...
3   for ( ...) ...
4 }
```

- perhaps in a function?

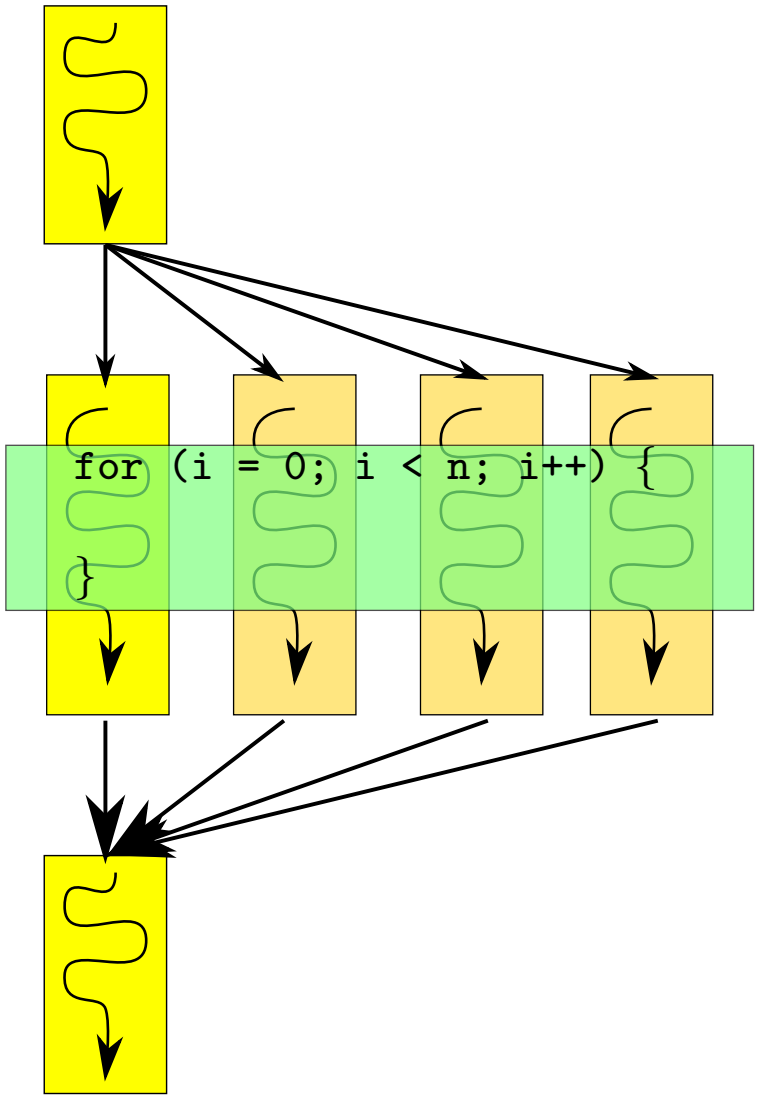
```
1 main() {
2   for ( ... ) {
3     ...
4     g();
5   }
6 }
7 g() {
8   for (...) ...
9 }
```

- what about parallel recursions?

```
1 qs() {
2   if (...) { ... }
3   else {
4     qs();
5     qs();
6   }
7 }
```

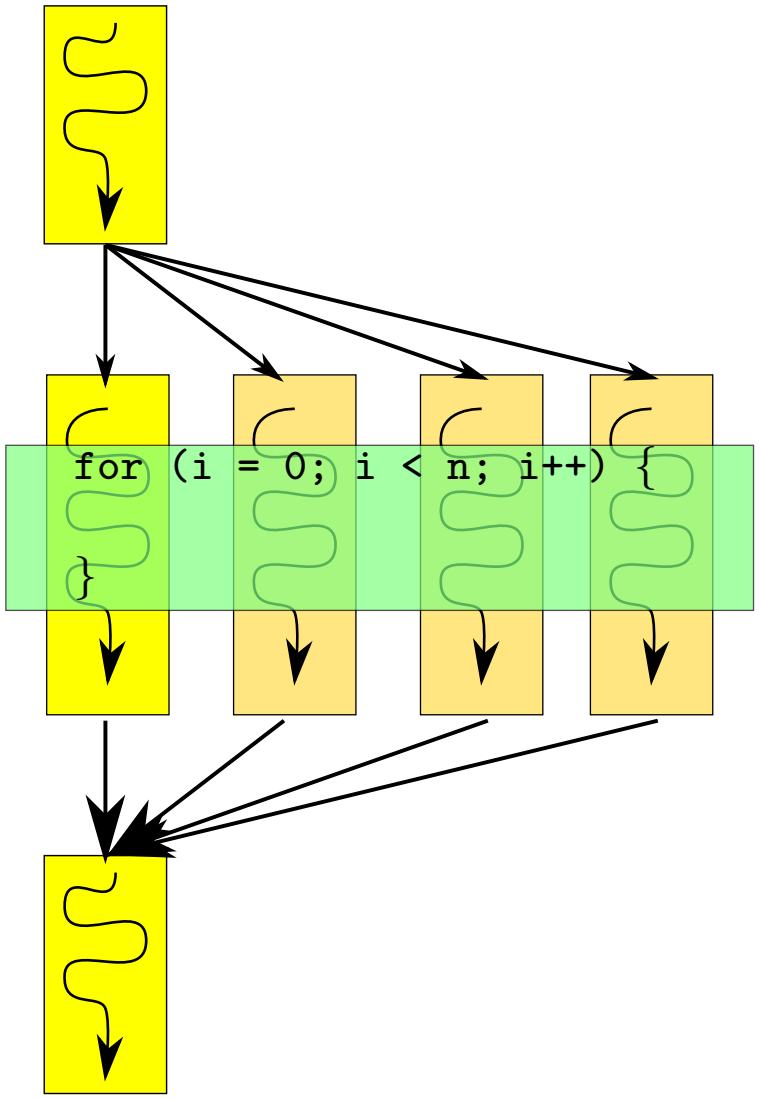
`parallel for` can't handle nested parallelism

- OpenMP generally ignores nested `parallel` pragma when enough threads have been created by the outer `parallel` pragma, for good reasons



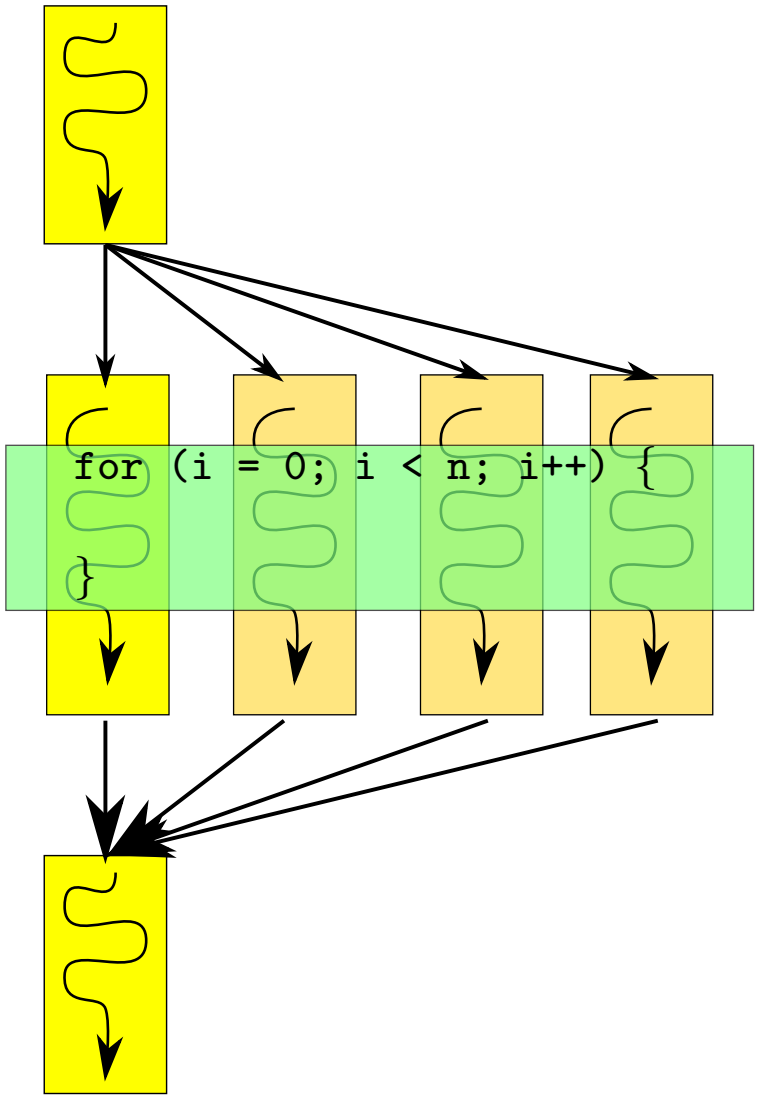
`parallel for` can't handle nested parallelism

- OpenMP generally ignores nested `parallel` pragma when enough threads have been created by the outer `parallel` pragma, for good reasons
- the fundamental limitation is its simplistic work-sharing mechanism



`parallel for` can't handle nested parallelism

- OpenMP generally ignores nested `parallel` pragma when enough threads have been created by the outer `parallel` pragma, for good reasons
- the fundamental limitation is its simplistic work-sharing mechanism
- *tasks* address these issues, by allowing tasks to be created at arbitrary points of execution (and a mechanism to distribute them across cores)



Task parallelism in OpenMP

- syntax:
  - `task` creates a task executing S (2.10.1)

```
1 #pragma omp task
2   S
```

  - `taskwait` waits for child tasks to finish (2.17.5)

```
1 #pragma omp taskwait
```

OpenMP task parallelism template

- don't forget to create a `parallel` region
- don't also forget to enter a `master` region, which says only the master executes the following statement and others "stand-by"

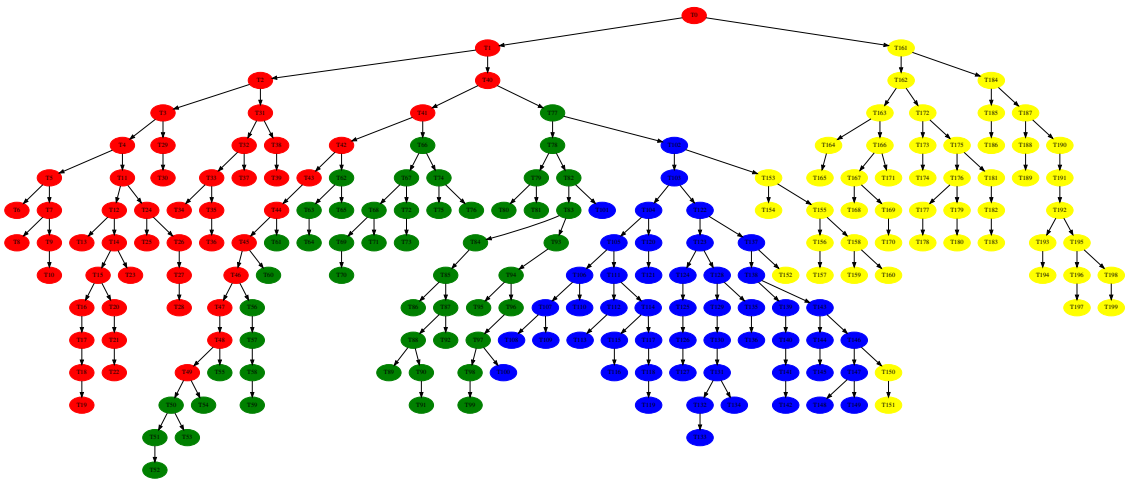
```
1  int main() {
2  #pragma omp parallel
3  #pragma omp master
4  // or #pragma omp single
5    ms(a, a + n, t, 0);
6  }
```

- and create tasks in the master region

```
1  void ms(a, a_end, t, dest) {
2    if (n == 1) {
3      ...
4    } else {
5      ...
6  #pragma omp task
7      ms(a, c,     t,      1 - dest);
8  #pragma omp task
9      ms(c, a_end, t + nh, 1 - dest);
10 #pragma omp taskwait
11     ...
12 }
```

What are tasks good for?

- the strength of tasks as opposed to for loop is its flexibility
  - create tasks at any point during the computation
  - they get distributed to cores
- especially good for “nested parallelism” and “parallel recursions (divide and conquer)”



- even for loops, you may consider reformulating them into divide-and-conquer as an alternative dynamic load-balancing strategy

Pros/cons of various approaches

- `static:`
  - partitioning iterations is simple and does not require communication
  - mapping between work $\leftrightarrow$ thread is deterministic and predictable (why it's important?)
  - may cause load imbalance (leave some threads idle, even when other threads have many work to do)
- `dynamic:`
  - less prone to load imbalance, if chunks are sufficiently small
  - partitioning iterations needs communication (no two threads execute the same iteration) and may become a bottleneck
  - mapping between iterations and threads is non-deterministic
  - OpenMP's dynamic scheduler is inflexible in partitioning nested loops

Pros/cons of schedulers

- divide and conquer + tasks :
  - less prone to load imbalance, as in dynamic
  - distributing tasks needs communication, but efficient implementation techniques are known
  - mapping between work and thread is non-deterministic, as in dynamic
  - you can flexibly partition loop nests in various ways (e.g., keep the space to square-like)
  - need some coding efforts (easily circumvented by additional libraries; e.g., TBB's `blocked_range2d` and `parallel_for`)

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Data sharing

- `parallel`, `for`, `task` pragma accept clauses specifying which variables should be shared among threads or between the parent/child tasks (or otherwise privatized/replicated to each thread)
- 2.19 "Data Environments"
  - `private`
  - `firstprivate`
  - `shared`
  - `reduction` (only for `parallel` and `for`)
  - `copyin`

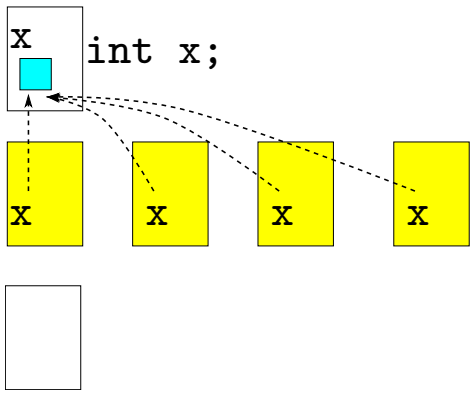
Data sharing/privatizing example

```
1  int main() {
2    int S;  /* shared */
3    int P;  /* made private below */
4  #pragma omp parallel private(P) shared(S)
5    {
6      int L; /* automatically private */
7      printf("S at %p, P at %p, L at %p\n",
8             &S, &P, &L);
9    }
10   return 0;
11 }
```

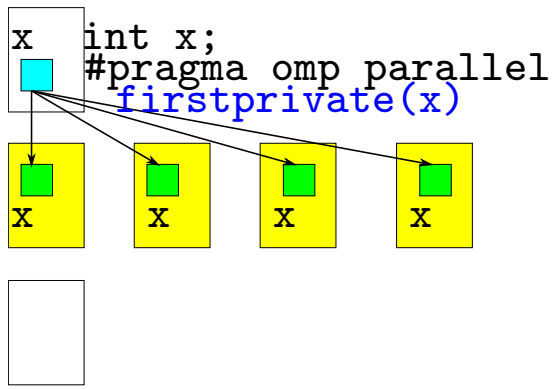
```
1  $ OMP_NUM_THREADS=2 ./a.out
2  S at 0x..777f494, P at 0x..80d0e28, L at 0x..80d0e2c
3  S at 0x..777f494, P at 0x..777f468, L at 0x..777f46c
```

Data sharing behavior

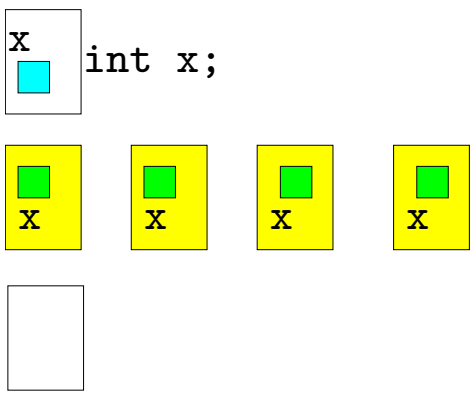
shared



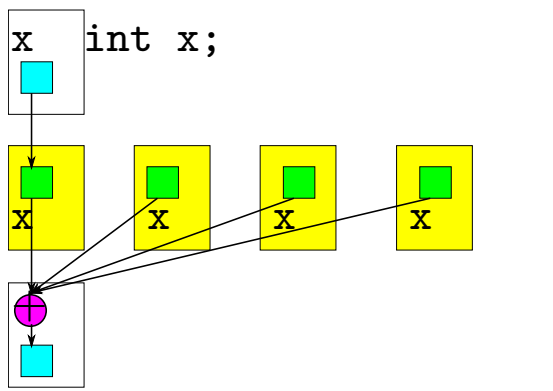
firstprivate



private



reduction



Race condition

- definition: there is *a race condition* when concurrent threads access the same location and one of which writes to it
- *a race condition* almost always implies your program won't work
- even something as simple as this (some accumulations may be lost)

```
1  x = 123;
2  #pragma omp parallel // assume we have 5 threads
3  {
4    ...
5    x++;
6    ..
7  }
8  printf("x = %d\n", x)
```

Race condition

thread 1	thread 2
x (123) $\rightarrow$ t	
x $\leftarrow$ 124	

Race condition

thread 1	thread 2
x (123) $\rightarrow$ t	
	x (123) $\rightarrow$ t
x $\leftarrow$ 124	
	x $\leftarrow$ 124

Race condition

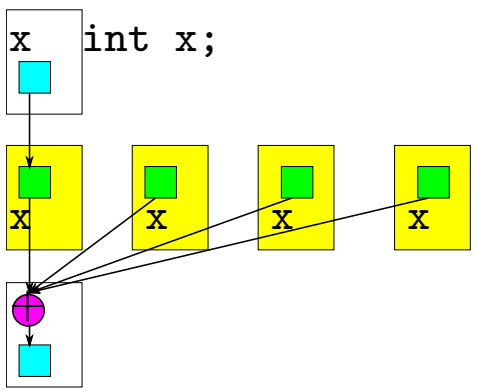
thread 1	thread 2
x (123) $\rightarrow$ t	
	x (123) $\rightarrow$ t
x $\leftarrow$ 124	
	x $\leftarrow$ 124

- The increment by a thread is “lost”

Two basic tools to resolve race conditions

- *"make it atomic"* `#pragma omp atomic` and `#pragma omp critical` : gaurantee the specified operation to be done *atomically*
- *"all you need may be a reduction"* `reduction` clause performs efficient *reduction* operations on behalf of you

thread 1	thread 2
x (123) → t x ← 124	
	x (124) → t x ← 125



#pragma omp critical

- syntax:

```
1 #pragma omp critical
2   statement
```

- effect: the execution of *statement* will not overlap with other executions of *statement* (or any other statement labeled `#pragma omp critical`, for that matter)
- note: most general, but likely to be slow

#pragma omp atomic

- syntax:

```
1 #pragma omp atomic
2    var = var op exp
```

  op is a predefined operation such as `+`, `-`, `*`, ...

- effect: guarantee the read-update is done atomically (is not lost); that is, *var* is not updated by someone else between the read and update
- note: semantically, it is like

```
1   e = exp;
2 #pragma omp critical
3   var = var op e
```

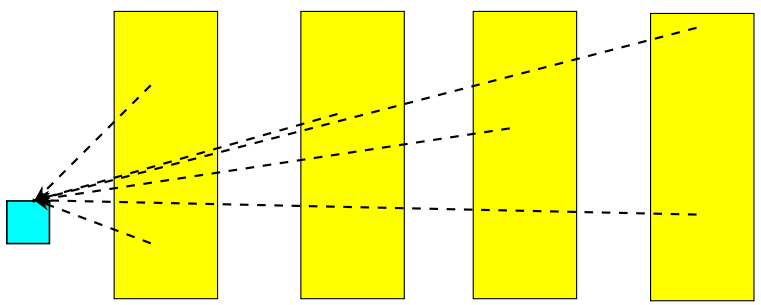
  but typical implementations take advantage of atomic instructions supported by CPU, such as fetch-and-add or compare-and-swap

Reduction

- in general, "reduction" refers to an operation to combine many values into a single value. e.g.,
  - $v = v_1 + \cdots + v_n$
  - $v = \max(v_1, \cdots, v_n)$
  - ...
- simply sharing the variable (v) does not work (race condition)
- one way to fix is to make updates atomic, but it will be slow

```
1  v = 0.0;
2  for (i = 0; i < n; i++) {
3    v += f(a + i * dt) * dt;
4  }
```

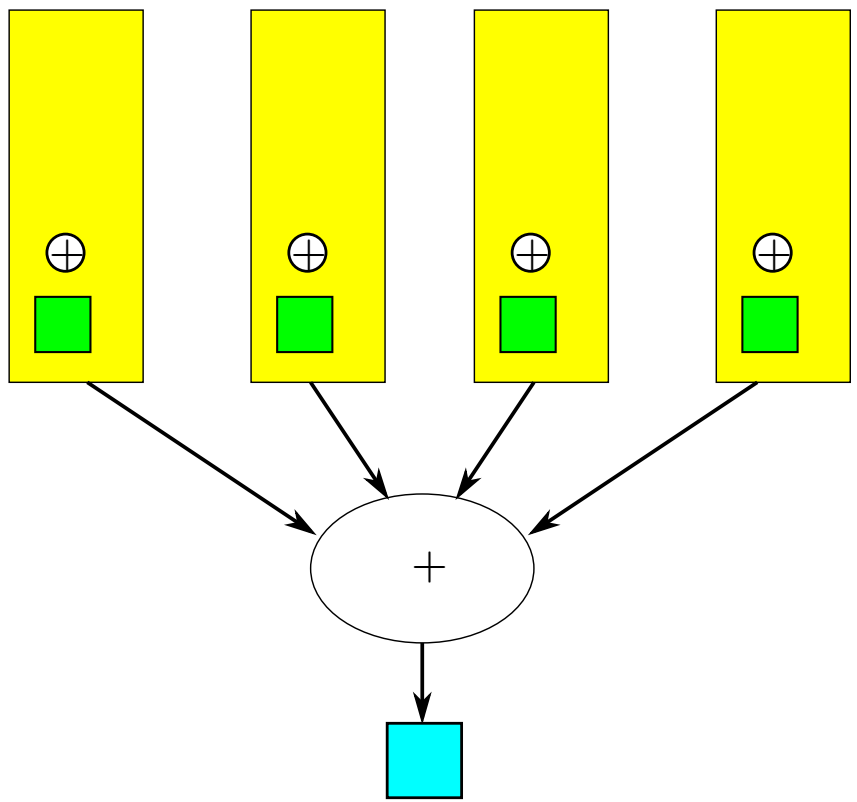
```
1  v = 0.0;
2  #pragma omp parallel for
3  for (i = 0; i < n; i++) {
4  #pragma omp atomic
5    v += f(a + i * dt) * dt;
6  }
```



Reduction clause in OpenMP

- a more efficient strategy:
  - let each thread work (reduce) on its private variable, and
  - when threads finish, combine their partial results into one
- `reduction` clause in OpenMP does just that (2.19.5)

```
1  v = 0.0;
2  #pragma omp parallel for
                reduction(+:v)
3  for (i = 0; i < n; i++) {
4    v += f(a + i * dt) * dt;
5  }
```



Builtin reduction and user-defined reduction (2.9.2)

- reduction syntax:

```
1 #pragma omp parallel reduction(op:var,var,...)
2     S
```

  - builtin reductions
    - *op* is one of `+`, `*`, `-`, `&`, `^`, `|`, `&&`, and `||`
    - (Since 3.1) `min` or `max`
  - builtin reductions are *limited to simple types and common operations* → *user-defined reductions* (since 4.0)

Why do you want user-defined reductions?

- consider how to do reduction on 3-element vector
- e.g., how to parallelize this loop safely

```
1  typedef struct {
2    double a[3];
3  } vec_t;
4
5  int main() {
6    vec_t y;
7    vec_init(&y);                    /* y = {0,0,0} */
8  #pragma omp parallel
9  #pragma omp for
10   for (long i = 0; i < 10000; i++) {
11     y.a[..] += 1;
12   }
13 }
```

- you cannot say `reduction(+:y.a[0], y.a[1], y.a[2])` (what if you have 100 elements?)
- we define a reduction operation on `vec_t` type instead

User-defined reduction

- syntax: (2.19.5.7)

```
1 #pragma omp declare reduction (name : type : combine_statement)
```

or

```
1 #pragma omp declare reduction (name : type : combine_statement) initializer
          (init_statement)
```

- effect:
  - you can specify `reduction(`*name* `:` *var*`)` for a variable of type *type*
  - *init_statement* is executed by each thread before entering the loop, typically to initialize its private copy of *var*
  - *combine_statement* is executed to merge a partial result to another variable

User-defined reduction: a simple example

- introduce reduction

```
1 #pragma omp declare reduction \
2   (vp : vec_t : vec_add(&omp_out,&omp_in)) \
3   initializer(vec_init(&omp_priv))
```

`vec_add` must be defined somewhere and not shown

- add `reduction(vp : y)` to the for loop

```
1 int main() {
2   vec_t y;
3   vec_init(&y);                    /* y={0,0,0} */
4 #pragma omp parallel
5 #pragma omp for reduction(vp : y)
6   for (long i = 0; i < 10000; i++) {
7     y.a[..] += 1;
8   }
9 }
```

User-defined reduction : how it works

with

```
1 #pragma omp declare reduction \
2   (vp : vec_t : vec_add(&omp_out,&omp_in)) \
3   initializer(vec_init(&omp_priv))
```

```
1 #pragma omp for reduction(vp :  y)
2   for (long i = 0; i < 10000; i++) {
3     y.a[..] += 1;
4   }
```

≈

```
1   vec_t y_priv; // thread−local copy of y
2   vec_init(&y_priv); // initializer
3 #pragma omp for
4   for (long i = 0; i < 10000; i++) {
5     y_priv.a[..] += 1;
6   }
7   // merge the partial result into the shared variable
8   // actual implementation may be (is likely to be) different
9   vec_add(&y, &y_priv); // y += y_priv
```

User-defined reduction : limitations

- *combine-statement* can reference only two local variables (`omp_in` and `omp_out`)
  - it should reduce (merge) `omp_in` into `omp_out` (e.g., `omp_out += omp_in`)
- *init-statement* can reference only two local variables (`omp_priv` and `omp_orig`)
  - `omp_priv` : the private copy *init-statement* should initialize
  - `omp_orig` : the original shared variable
- $\Rightarrow$ local contexts necessary for initialization and reduction must be encapsulated in the variables subject to reduction

An exercise : reduction on variable-length vectors

- a variable-length version of the previous example

```
1  typedef struct {
2    long n;      // number of elements (variable)
3    double * a; // n elements
4  } vec_t;
```

- and a reduction for it

```
1    vec_t y;
2    long n = 100;
3    vec_init(&y, n); // n is a local context
4  #pragma omp parallel
5  #pragma omp for     // how to do a proper reduction for y?
6    for (long j = 0; j < 1000000; j++) {
7      y.a[j % n] += 1;
8    }
```

- the point is you cannot reference n in the initializer

```
1  (!) #pragma omp declare reduction \
2    (vp : vec_t : vec_add(&omp_out,&omp_in)) \
3    initializer(vec_init(&omp_priv, n))
```

An exercise : reduction on variable-length vectors

- initializer can reference `omp_orig` to obtain the context (i.e. vector length in this example)
- $\Rightarrow$ define a function, `vec_init_from`, which takes the shared `y` and initialize the private copy of `y`

```
1  int vec_init_from(vec_t * v, vec_t * orig) {
2    long n = orig->n;
3    double * a = (double *)malloc(sizeof(double) * n);
4    for (long i = 0; i < n; i++) {
5      a[i] = 0;
6    }
7    v->n = n;
8    v->a = a;
9    return 0;
10 }
```

- and say

```
1  #pragma omp declare reduction \
2    (vp : vec_t : vec_add(&omp_out,&omp_in)) \
3    initializer(vec_init_from(&omp_priv, &omp_orig))
```

Contents

SIMD constructs

- `simd` pragma (2.9.3)
  - allows an explicit vectorization of for loops
  - syntax restrictions similar to `omp for` pragma apply
- `declare simd` pragma (2.9.3.3)
  - instructs the compiler to generate vectorized versions of a function
  - with it, loops with function calls can be vectorized

`simd` pragma

- basic syntax (similar to `omp for`):

```
1 #pragma omp simd clauses
2 for (i = ...; i < ...; i += ...)
3     S
```

- clauses
  - `aligned(`*var,var,...*`:`*align*`)`
  - `uniform(`*var,var,...*`)` says variables are loop invariant
  - `linear(`*var,var,...*`:`*stride*`)` says variables have the specified stride between consecutive iterations

`declare simd` pragma

- basic syntax (similar to `omp for`):

```
1 #pragma omp declare simd clauses
2 function definition
```

- clauses
  - those for `simd` pragma
  - `notinbranch`
  - `inbranch`

SIMD pragmas, rationales

- most automatic vectorizers give up vectorization in many cases
  1. conditionals (lanes may branch differently)
  2. inner loops (lanes may have different trip counts)
  3. function calls (function bodies are not vectorized)
  4. iterations may not be independent
- `simd` and `declare simd` directives should eliminate obstacles 3 and 4 and significantly enhance vectorization opportunities

A note on GCC OpenMP SIMD implementation

- GCC `simd` and `declare simd` $\approx$ existing auto vectorizer − dependence analysis
- `declare simd` functions are first converted into a loop over all vector elements and then passed to the loop vectorizer

```
1 #pragma omp declare simd
2 float f(float x, float y) {
3   return x + y;
4 }
```

$\longrightarrow$

```
1 float8 f(float8 vx, float8 vy) {
2   float8 r;
3   for (i = 0; i < 8; i++) {
4     float x = vx[i], y = vy[i]
5     r[i] = x + y;
6   }
7   return r;
8 }
```

- the range of vectorizable loops in a recent version I investigated (7.3.0) seems very limited
  - innermost loop with no conditionals
  - doubly nested loop with a very simple inner loop

Strategies for SpMV

- parallelize only across different rows (a single row is processed sequentially)
  - especially natural for CSR
  - extremely long rows may limit speedup
- parallelize all non-zeros, with careful handling of `y[i] +=`
  - atomic accumulation (`#pragma omp atomic`)
  - reduction (`#pragma omp reduction`). you must have user-defined reduction
- divide rows until the number of non-zeros becomes small (e.g., ≤ 5000)
  - further divide a single row if a row contains many zeros
  - can be done naturally with tasks