ISMONTIC – Tangier

M102 - Algorithms - Problem Solving

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Instructions

For each exercise:

1. Analyze the given mathematical formula
2. Identify the required loops (FOR)
3. Identify any necessary conditions (IF)
4. Write the corresponding pseudocode
5. Test mentally with small values

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1. Basic Exercises (Simple FOR Loops)

Exercise 1: Sum of integers

Transform into pseudocode:

$$S = \sum_{i=1}^{n} i = 1 + 2 + 3 + \cdots + n$$

Example:
n = 5 ⇒ S = 1 + 2 + 3 + 4 + 5 = 15

Exercise 2: Sum of squares

Transform into pseudocode:

$$S = \sum_{i=1}^{n} i^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2$$

Example:
n = 4 ⇒ S = 1 + 4 + 9 + 16 = 30

Exercise 3: Product (Factorial)

Transform into pseudocode:

$$P = \prod_{i=1}^{n} \quad i = 1 \times 2 \times 3 \times \cdots \times n = n!$$

Example:
n = 5 ⇒ P = 120

Exercise 4: Sum of cubes

Transform into pseudocode:

$$S = \sum_{i=1}^{n} \quad i^3 = 1^3 + 2^3 + 3^3 + \cdots$$

Example:
n = 3 ⇒ S = 1 + 8 + 27 = 36

Exercise 5: Sum with coefficient

Transform into pseudocode:

$$S = \sum_{i=1}^{n} \quad 2i = 2 + 4 + 6 + \cdots + 2n$$

Example:
n = 5 ⇒ S = 30

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2. Exercises with Conditions

Exercise 6: Sum of even numbers

$$S = \sum_{i=1}^{n,\ i \text{ even}} \quad i$$

Example: n = 10 ⇒ S = 30

Exercise 7: Sum of odd numbers

$$S = \sum_{i=1}^{n,\ i \text{ odd}} \quad i$$

Example: n = 9 ⇒ S = 25

Exercise 8: Conditional sum

$$S = \sum_{i=1}^{n} \begin{cases} i^2 & \text{if i is even} \\ i & \text{if i is odd} \end{cases}$$

Example: n = 5 ⇒ 29

Exercise 9: Counting multiples of 3

$$C = |\ \{i \in [1,n] : i \equiv 0 (\text{mod} 3)\}\ |$$

Example: n = 10 ⇒ C = 3

Exercise 10: Sum with threshold

$$S = \sum_{i=1}^{n} \begin{cases} i & i \leq 50 \\ 0 & i > 50 \end{cases}$$

Example: n = 52 → sum only from 1 to 50

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3. Nested Loops

Exercise 11: Double sum

$$S = \sum_{i=1}^{n} \sum_{j=1}^{i} j$$

Example: n = 3 ⇒ S = 10

Exercise 12: Product of sums

$$P = \prod_{i=1}^{n} \left(\sum_{j=1}^{i} j \right)$$

Example: n = 3 ⇒ P = 18

Exercise 13: Multiplication table sum

$$S = \sum_{i=1}^{n} \sum_{j=1}^{n} (i \times j)$$

Example: n = 2 ⇒ S = 9

Exercise 14: Triangular conditional sum

$$S = \sum_{i=1}^{n} \sum_{j=1}^{i} \begin{cases} j & \text{if j is even} \\ 0 & \text{otherwise} \end{cases}$$

Example: n = 4 ⇒ S = 10

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4. Advanced Exercises

Exercise 15: Partial harmonic series

$$H = \sum_{i=1}^{n} \frac{1}{i}$$

Example: n = 4 → 2.083

Exercise 16: Alternating series

$$S = \sum_{i=1}^{n} (-1)^{i+1} i$$

Example: n = 5 ⇒ 3

Exercise 17: Powers of 2

$$S = \sum_{i=0}^{n} 2^i$$

Example: n = 4 ⇒ 31

Exercise 18: Conditional maximum

$$M = \max \begin{cases} i^2 & \text{if i is prime} \\ -\infty & \text{otherwise} \end{cases}$$

Example: n = 7 ⇒ M = 49

Exercise 19: Fraction sum with condition

$$S = \sum_{i=1}^{n} \begin{cases} \frac{1}{i^2} & i \text{ odd} \\ \frac{1}{i} & i \text{ even} \end{cases}$$

Example: n = 4 ⇒ ≈ 1.861

Exercise 20: Product with variable bounds

$$P = \prod_{i=2}^{n} (1 + \frac{1}{i})$$

Example: n = 4 ⇒ 2.5

Exercise 21: Double condition sum

$$S = \sum_{i=1}^{n} \begin{cases} i^2 & i \equiv 0 \bmod 2 \text{ AND } i \equiv 0 \bmod 3 \\ i & i \equiv 0 \bmod 2 \text{ OR } i \equiv 0 \bmod 3 \\ 0 & \text{otherwise} \end{cases}$$

Example: n = 12 ⇒ 216

Exercise 22: Sum until condition

Add numbers 1+2+3+… until the total exceeds *n*.

Example: n = 20 → S = 15

Exercise 23: Conditional geometric sum

$$S = \sum_{i=1}^{n} r^i \text{where } r = \begin{cases} 2 & i \text{ even} \\ 3 & i \text{ odd} \end{cases}$$

Example: n = 4 ⇒ S = 50

Exercise 24: GCD of multiple numbers

$$G = \text{GCD}(1,2,3,\dots,n)$$

Example: n = 6 → G = 1

Bonus Exercises

Exercise 25: Fibonacci sum

$$S = \sum_{i=1}^{n} F_i$$

Example: n = 6 → 20

Exercise 26: Moving average

$$M_i = \frac{i + (i+1) + (i+2)}{3}$$

Example: n = 5 → $M_1 = 2$, $M_2 = 3$, $M_3 = 4$

Exercise 27: Nested conditional product

$$P = \prod_{i=1}^{n} \begin{cases} \prod_{j=1}^{i} j & i \text{ even} \\ i & i \text{ odd} \end{cases}$$

Example: n = 4 → P = 144