Chapter 2: Data Preprocessing

- Preprocess Steps
  - Data cleaning
  - Data integration and transformation
  - Data reduction

Why Data Preprocessing?

- Data in the real world is dirty
  - incomplete: lacking attribute values, lacking certain attributes of interest, or containing only aggregate data
  - noisy: containing errors or outliers
  - inconsistent: containing discrepancies in codes or names
- No quality data, no quality mining results!
  - Quality decisions must be based on quality data
  - Data warehouse needs consistent integration of quality data

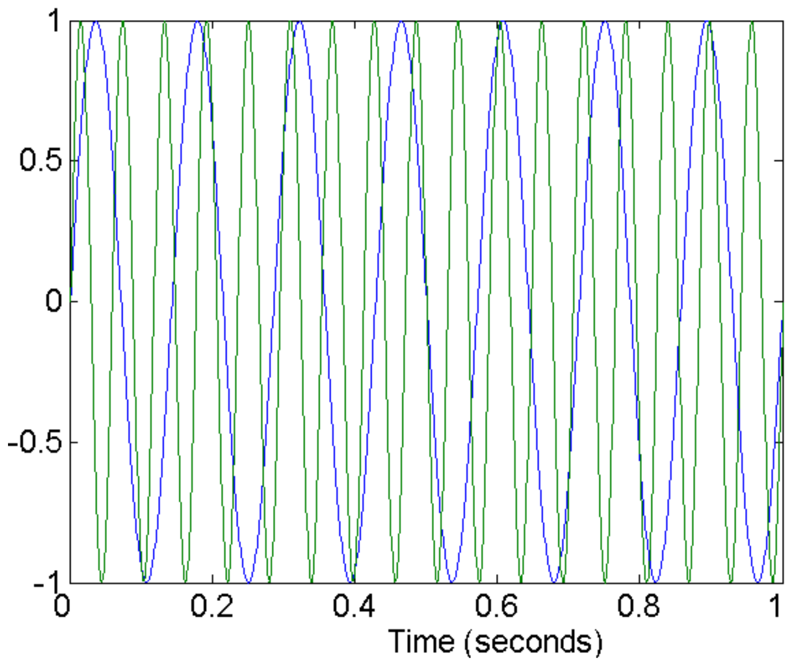
Data Quality

- What kinds of data quality problems?
- How can we detect problems with the data?
- What can we do about these problems?

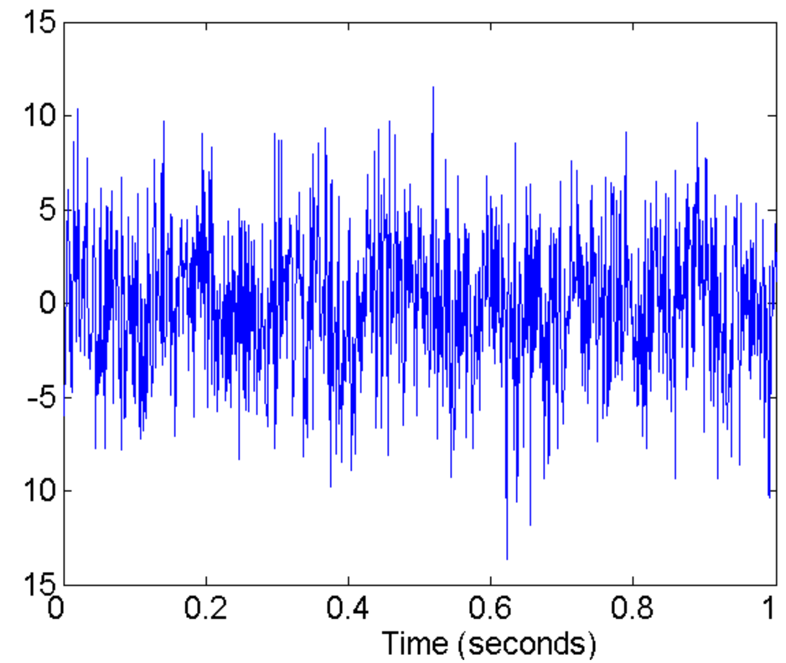
- Examples of data quality problems:
  - Noise and outliers
  - missing values
  - duplicate data

Noise

- Noise refers to modification of original values
  - Examples: distortion of a person's voice when



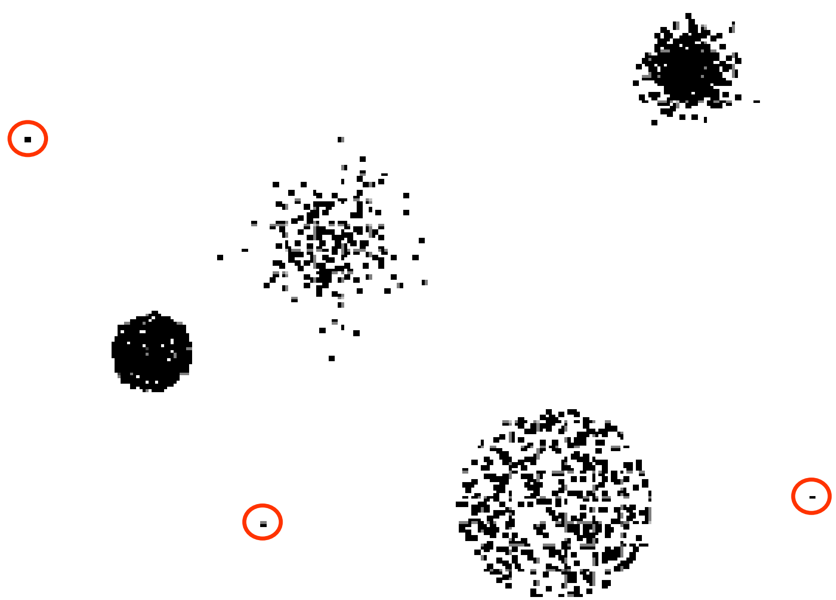
Two Sine Waves



Two Sine Waves + Noise

Outliers

- Outliers are data objects with characteristics that are considerably different than most of the data set



Missing Values

- Reasons for missing values
  - Information is not collected (e.g., people decline to give their age and weight)
  - Attributes may not be applicable to all cases (e.g., annual income is not applicable to children)

- Handling missing values
  - Eliminate Data Objects
  - Estimate Missing Values
  - Ignore the Missing Value During Analysis
  - Replace with all possible values (weighted by their probabilities)

Duplicate Data

- Data set may include data objects that are duplicates, or almost duplicates of one another
  - Major issue when merging data from heterogeous sources
- Examples:
  - Same person with multiple email addresses
- Data cleaning
  - Process of dealing with duplicate data issues

Multi-Dimensional Measure of Data Quality

- A well-accepted multidimensional view:
  - Accuracy
  - Completeness
  - Consistency
  - Timeliness
  - Believability
  - Value added
  - Interpretability
  - Accessibility

Major Tasks in Data Preprocessing

- Data cleaning
  - Fill in missing values, smooth noisy data, identify or remove outliers, and resolve inconsistencies
- Data integration
  - Integration of multiple databases, data cubes, or files
- Data transformation
  - Normalization and aggregation
- Data reduction
  - Obtains reduced representation in volume but produces the same or similar analytical results

Data Cleaning

- Data cleaning tasks
  - Fill in missing values
  - Identify outliers and smooth out noisy data
  - Correct inconsistent data

Missing Data

- Data is not always available
  - E.g., many tuples have no recorded value for several attributes, such as customer income in sales data
- Missing data may be due to
  - equipment malfunction
  - inconsistent with other recorded data and thus deleted
  - data not entered due to misunderstanding
  - certain data may not be considered important at the time of entry
  - not register history or changes of the data
- Missing data may need to be inferred.

How to Handle Missing Data?

- Ignore the tuple: usually done when class label is missing (assuming the tasks in classification—not effective when the percentage of missing values per attribute varies considerably.
- Fill in the missing value manually: tedious + infeasible?
- Use a global constant to fill in the missing value: e.g., “unknown”, a new class?!
- Use the attribute mean to fill in the missing value
- Use the attribute mean for all samples belonging to the same class to fill in the missing value: smarter
- Use the most probable value to fill in the missing value: inference-based such as Bayesian formula or decision tree

Noisy Data

- Noise: random error or variance in a measured variable
- Incorrect attribute values may due to
  - faulty data collection instruments
  - data entry problems
  - data transmission problems
  - technology limitation
  - inconsistency in naming convention
- Other data problems which requires data cleaning
  - duplicate records
  - incomplete data
  - inconsistent data

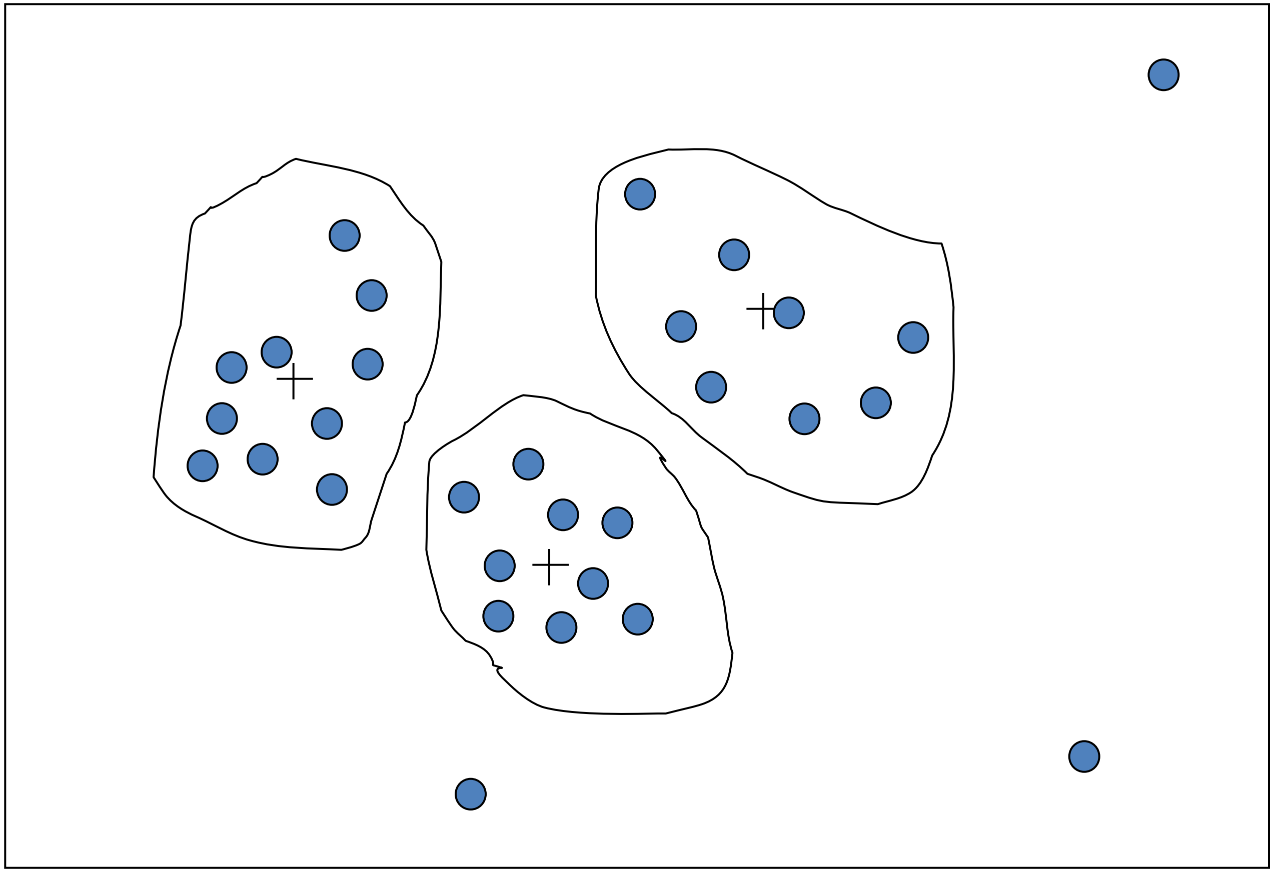
How to Handle Noisy Data?

- Binning method:
  - first sort data and partition into (equi-depth) bins
  - then one can smooth by bin means, smooth by bin median
  - Equal-width (distance) partitioning:
    - It divides the range into *N* intervals of equal size**: uniform grid**
    - if *A* and *B* are the lowest and highest values of the attribute, the width of intervals will be: $W = (B-A)/N$.
  - Equal-depth (frequency) partitioning:
    - It divides the range into *N* intervals, each containing approximately **same number of samples**
    - Managing categorical attributes can be tricky.

- Combined computer and human inspection
  - detect suspicious values and check by human

Cluster Analysis

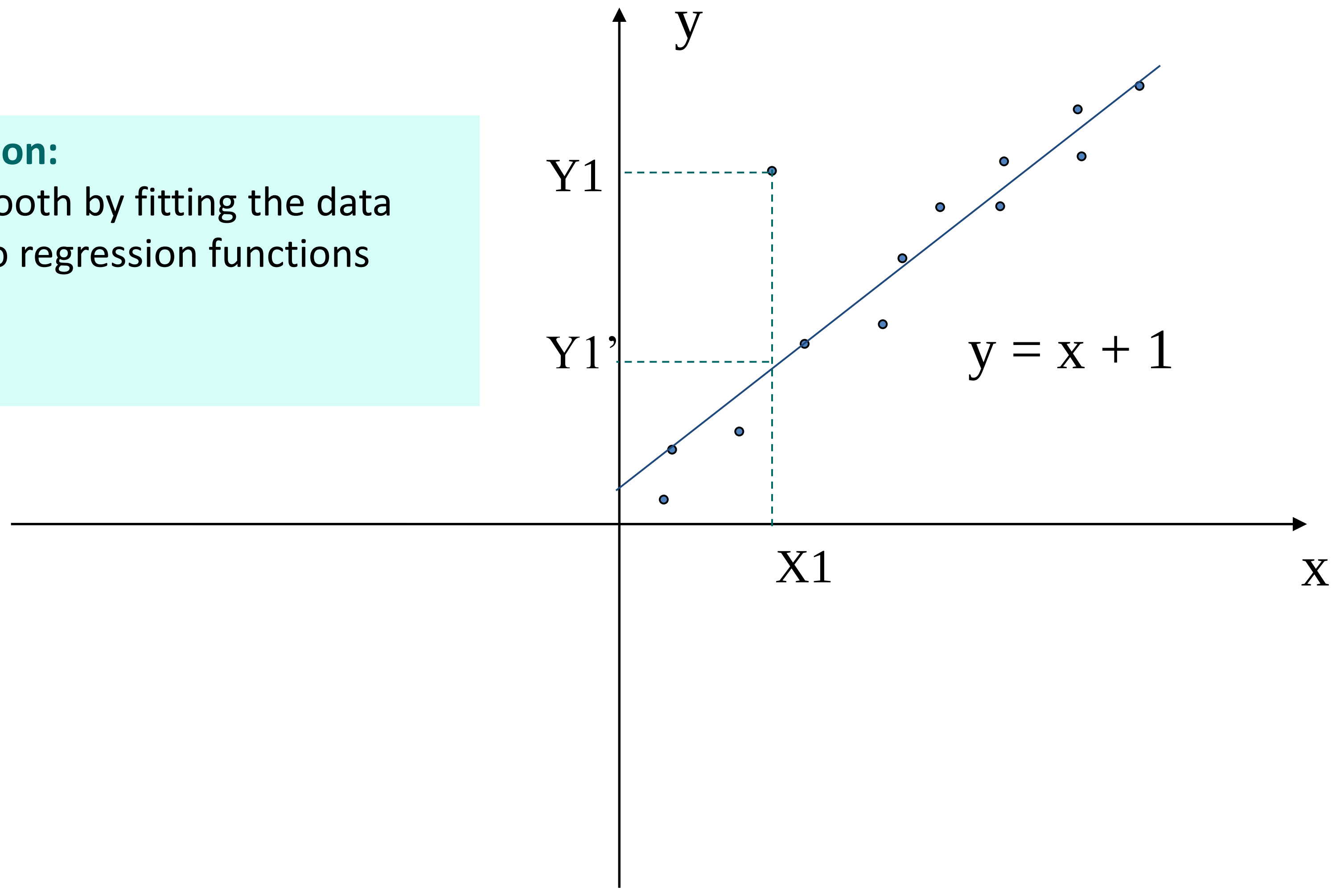
Clustering: detect and remove outliers



Regression

Regression:

smooth by fitting the data into regression functions



Data Integration

- Data integration:
  - combines data from multiple sources.
  - Schema integration
  - integrate metadata from different sources
  - **Entity identification problem: identify real world entities from multiple data sources, e.g., A.cust-id ≡ B.cust-#**
- Detecting and resolving data value conflicts
  - for the same real world entity, attribute values from different sources are different
  - possible reasons: **different representations, different scales, e.g., metric vs. British units**

Handling Redundant Data in Data Integration

- Redundant data occur often when integration of multiple databases
  - **The same attribute may have different names in different databases**
  - **One attribute may be a "derived" attribute in another table.**
  - **Redundant data may be able to be detected by correlational analysis**
- Careful integration of the data from multiple sources may help reduce/avoid redundancies and inconsistencies and improve mining speed and quality

Data Transformation

- Smoothing: remove noise from data
- Aggregation: summarization, data cube construction
- Generalization: concept hierarchy climbing
- Normalization: scaled to fall within a small, specified range
  - min-max normalization
  - z-score normalization
  - normalization by decimal scaling
- Attribute/feature construction
  - New attributes constructed from the given ones

Data Transformation: Normalization

- **min-max normalization**
  - Min-max normalization performs a linear transformation on the original data.
  - Suppose that minA and maxA are the minimum and the maximum values for attribute A. Min-max normalization maps a value v of A to v' in the range [new-minA, new-maxA] by computing:

$$v' = \frac{v - min_A}{max_A - min_A}(new\_max_A - new\_min_A) + new\_min_A$$

  - Ex. Let income range \$12,000 to \$98,000 normalized to [0.0, 1.0]. Then \$73600 is mapped to

$$\frac{73{,}600 - 12{,}000}{98{,}000 - 12{,}000}(1.0 - 0) + 0 = 0.716$$

Data Transformation: Normalization

- **Z-score Normalization:**
  - In z-score normalization, attribute A are normalized based on the mean and standard deviation of A. a value v of A is normalized to v' by computing:

$$v' = \frac{v - \mu_A}{\sigma_A}$$

  - where μ: mean, σ: standard deviation
  - Ex. Let μ = 54,000, σ = 16,000. Then $\frac{73{,}600 - 54{,}000}{16{,}000} = 1.225$
  - This method of normalization is useful when the actual minimum and maximum of attribute A are unknown.

Data Transformation: Normalization

- **Normalization by Decimal Scaling**
  - Normalization by decimal scaling normalizes by moving the decimal point of values of attribute A.
  - The number of decimal points moved depends on the maximum absolute value of A.
  - a value v of A is normalized to v’ by computing: v’ = (v / 10j). Where j is the smallest integer such that Max(|v’|)<1.

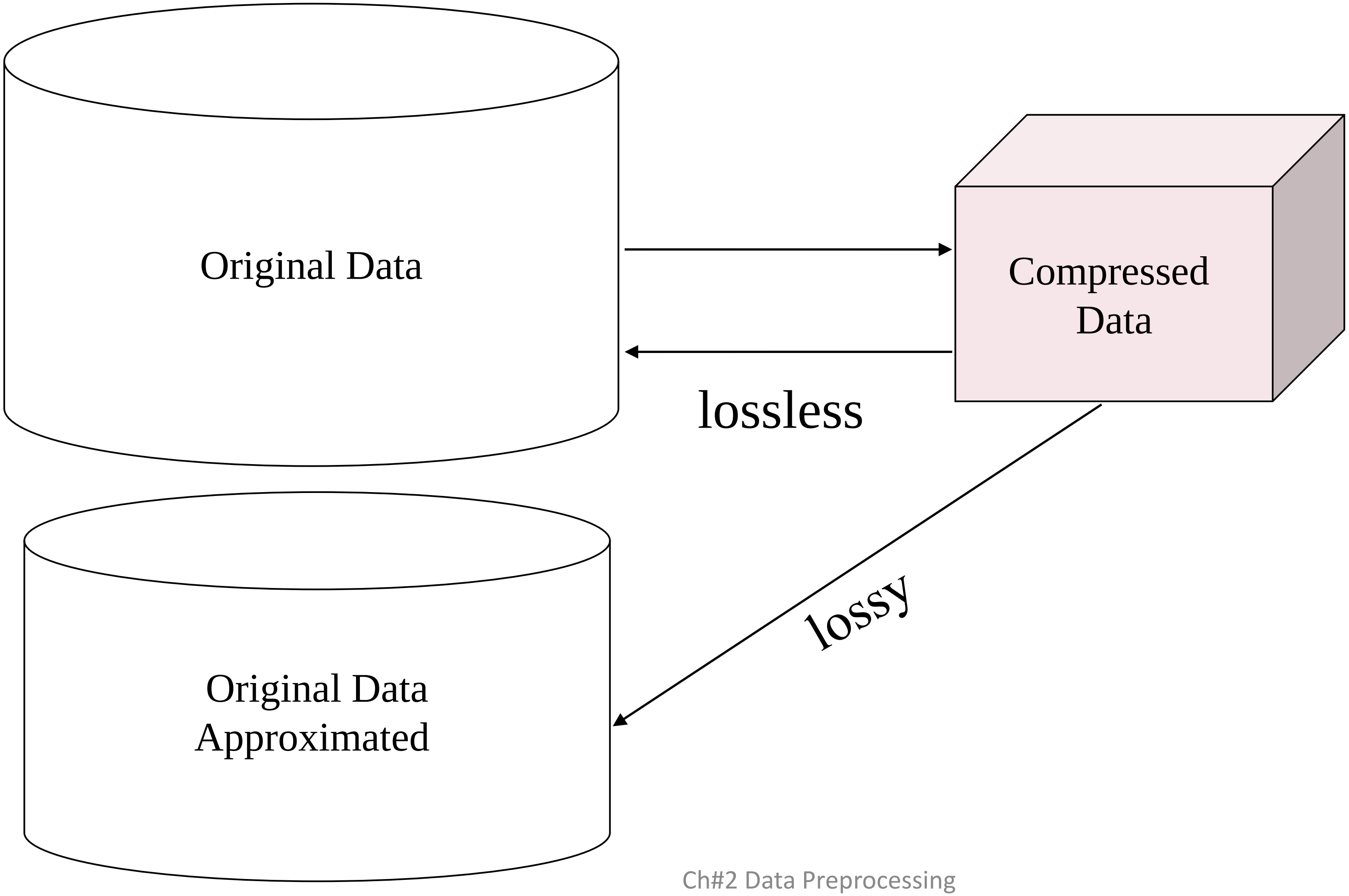
Decimal Scaling Normalization

Suppose that the recorded values of F range from − 986 to 917. The maximum absolute value of F is 986. To normalize by decimal scaling, we therefore divide each value by 1,000 (i.e., j = 3) so that −986 normalizes to −0.986 and 917 normalizes to 0.917.

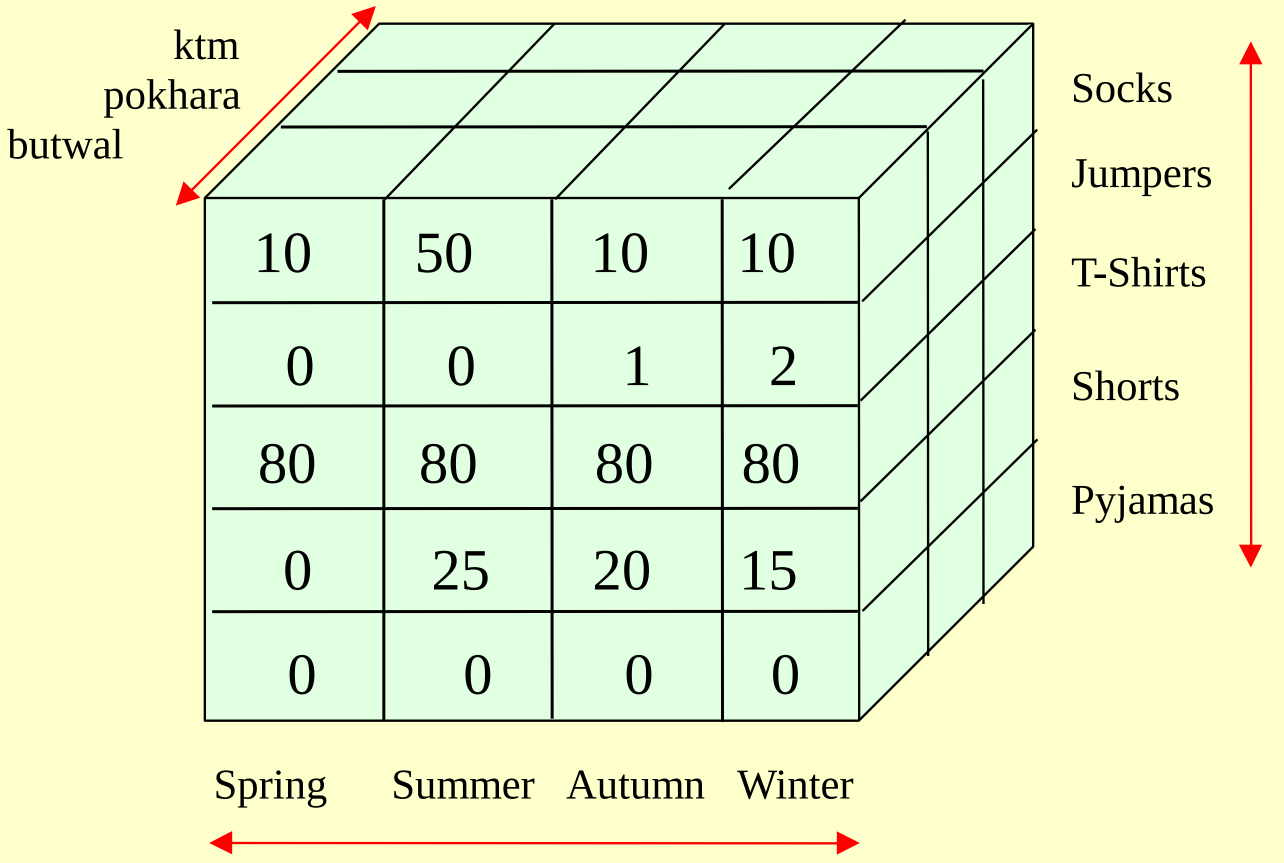
Data Reduction Strategies

- Warehouse may store terabytes of data: Complex data **analysis/mining may take a very long time** to run on the complete data set
- Data reduction
  - Obtains a reduced representation of the data set that is much smaller in volume but yet **produces the same (or almost the same) analytical results**
- Data reduction strategies
  - Sampling: selecting a subset of the data
  - Dimensionality reduction:
  - Data Compression: lossy (audio/video) and lossless (string)

Data Compression



OLAP and MULTIDIMENSIONAL DATA MODEL



Example: Three dimensions – Product, Sales_Area, and Season

- when performed by **dimension reduction**
  - one or more dimensions are removed from the cube
- Ex a sales cube with location and time
  - aggregation of total sales by location
    - rather than by location and by time

	location by country	
	Türkiye	Almanya
PC	50	90
Printer	20	30

Two dimensional cuboid

→

	locat All
PC	140
Printer	50

One dim. cuboid

Key Terms in Data Preprocessing

- Aggregation
- Sampling
- Dimensionality Reduction
- Feature subset selection
- Feature creation
- Discretization and Binarization
- Attribute Transformation

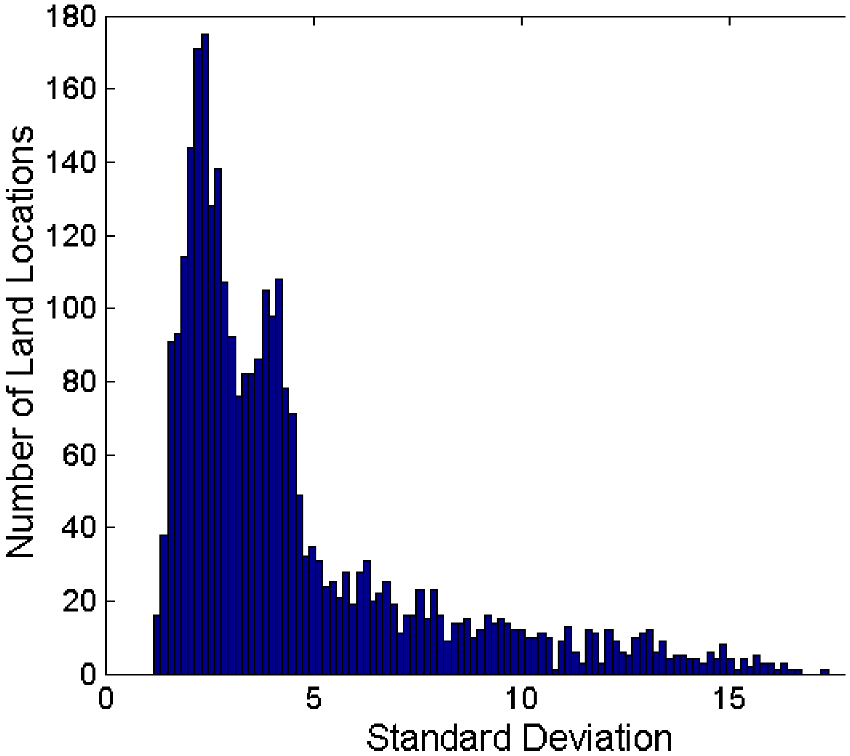
Aggregation

- Combining two or more attributes (or objects) into a single attribute (or object)

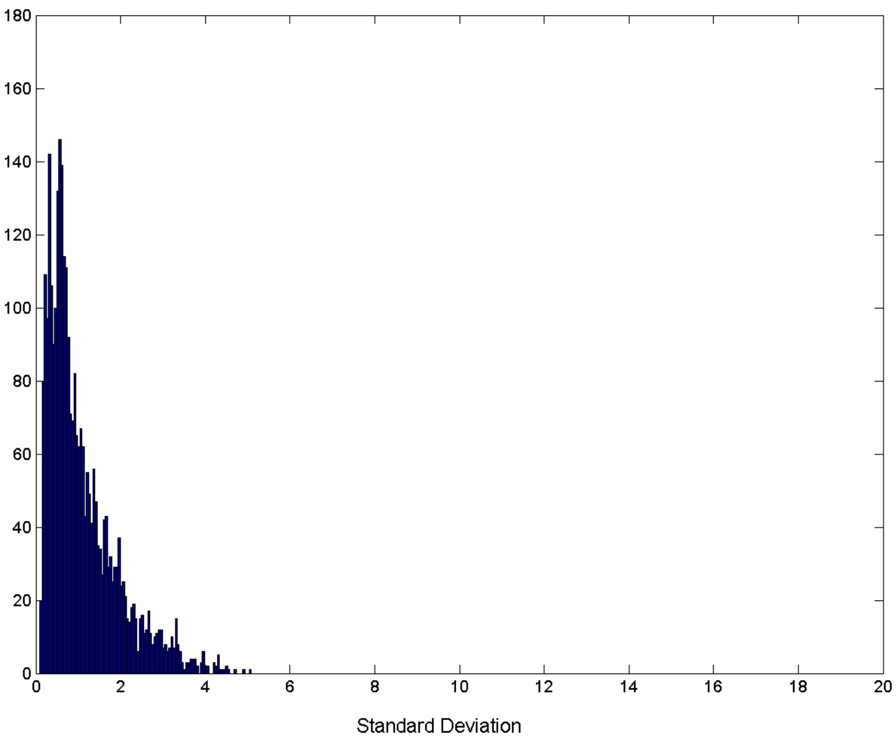
- Purpose
  - Data reduction
    - Reduce the number of attributes or objects
  - Change of scale
    - Cities aggregated into regions, states, countries, etc
  - More “stable” data
    - Aggregated data tends to have less variability

Aggregation

Variation of Precipitation in Australia



Standard Deviation of Average Monthly Precipitation



Standard Deviation of Average Yearly Precipitation

Sampling

- **Sampling is the main technique employed for data selection.**
  - **It is often used for both the preliminary investigation of the data and the final data analysis.**

- **Statisticians sample because obtaining the entire set of data of interest is too expensive or time consuming.**

- **Sampling is used in data mining because processing the entire set of data of interest is too expensive or time consuming.**

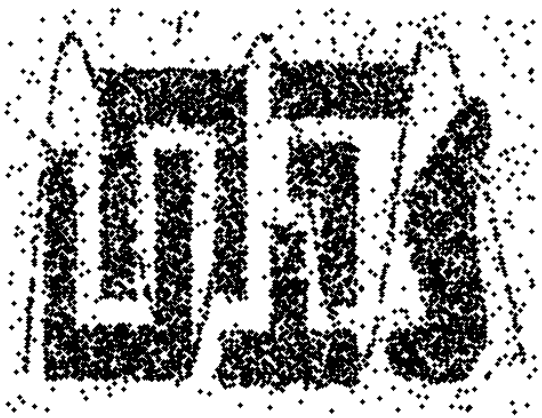
Sampling …

- The key principle for effective sampling is the following:
  - using a sample will work almost as well as using the entire data sets, if the sample is representative
  - A sample is representative if it has approximately the same property (of interest) as the original set of data

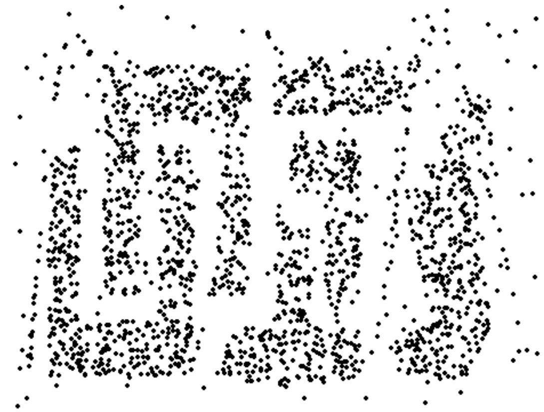
Types of Sampling

- Simple Random Sampling
  - There is an equal probability of selecting any particular item
- Sampling without replacement
  - As each item is selected, it is removed from the population
- Sampling with replacement
  - Objects are not removed from the population as they are selected for the sample.
    - In sampling with replacement, the same object can be picked up more than once
- Stratified sampling
  - Split the data into several partitions; then draw random samples from each partition

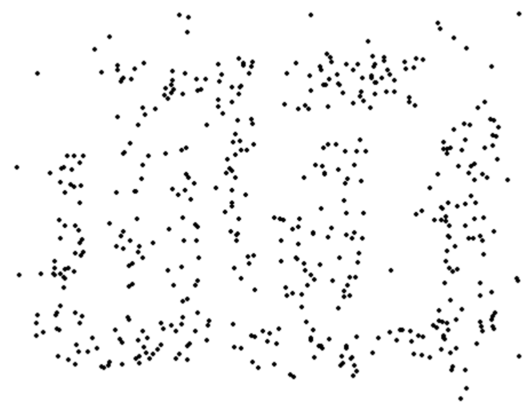
Sample Size



8000 points



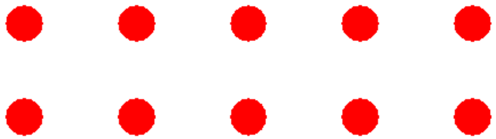
2000 Points

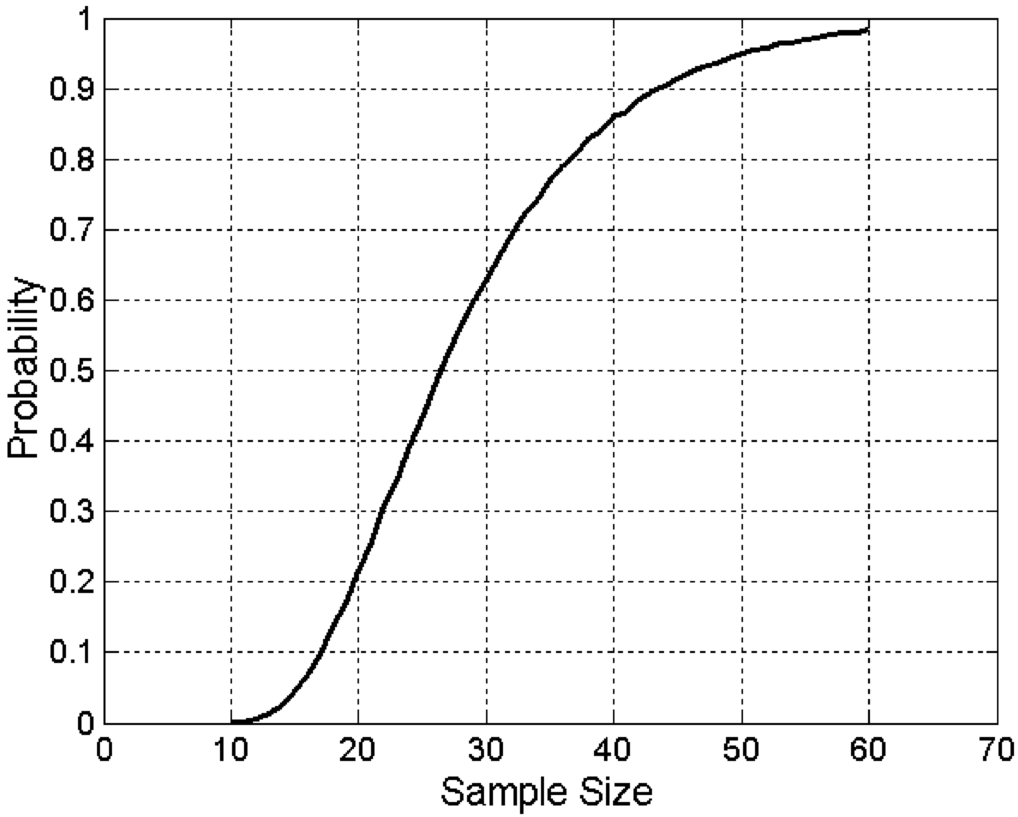


500 Points

Sample Size

- **What sample size is necessary to get at least one object from each of 10 groups.**



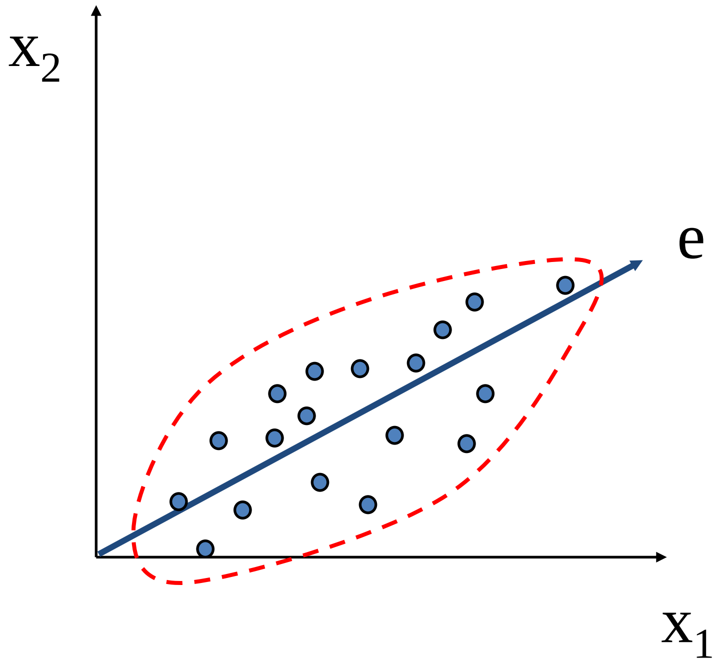


Dimensionality Reduction

- Purpose:
  - Avoid curse of dimensionality
  - Reduce amount of time and memory required by data mining algorithms
  - Allow data to be more easily visualized
  - May help to eliminate irrelevant features or reduce noise
- Techniques
  - Principle Component Analysis
  - Singular Value Decomposition
  - Others: supervised and non-linear techniques

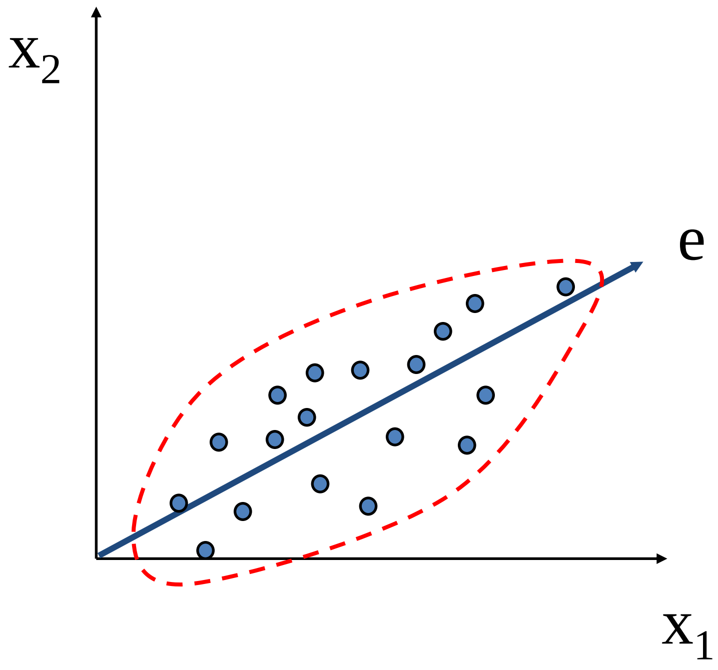
Dimensionality Reduction: PCA

- Goal is to find a projection that captures the largest amount of variation in data



Dimensionality Reduction: PCA

- Find the eigenvectors of the covariance matrix
- The eigenvectors define the new space



Dimensionality Reduction: PCA

Dimensions = 206

Feature Subset Selection

- Another way to reduce dimensionality of data
- Redundant features
  - duplicate much or all of the information contained in one or more other attributes
  - Example: purchase price of a product and the amount of sales tax paid
- Irrelevant features
  - contain no information that is useful for the data mining task at hand
  - Example: students' ID is often irrelevant to the task of predicting students' GPA

Feature Subset Selection

- Techniques:
  - Brute-force approch:
    - Try all possible feature subsets as input to data mining algorithm
  - Embedded approaches:
    - Feature selection occurs naturally as part of the data mining algorithm
  - Filter approaches:
    - Features are selected before data mining algorithm is run
  - Wrapper approaches:
    - Use the data mining algorithm as a black box to find best subset of attributes

Feature Creation

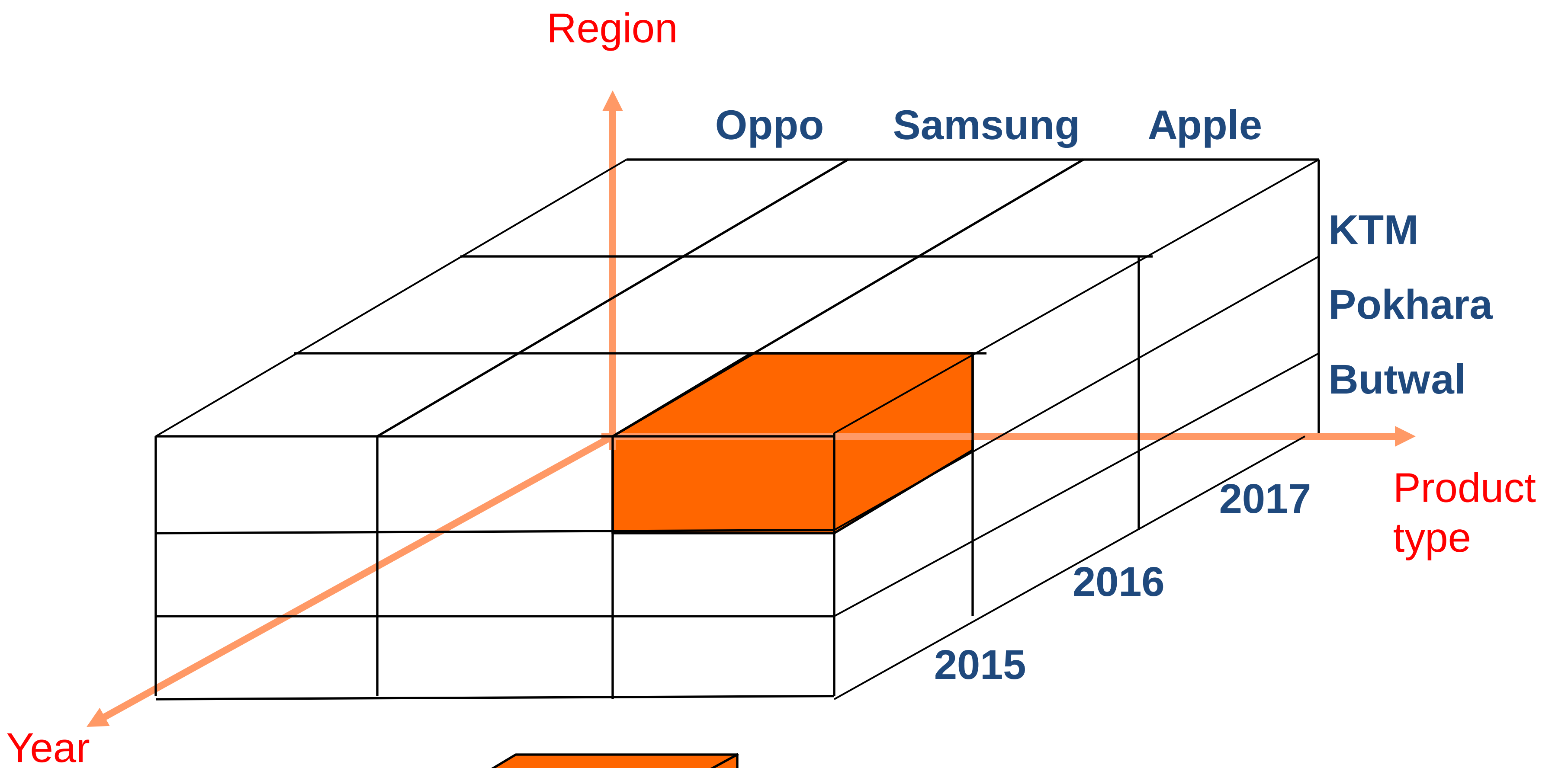
- Create new attributes that can capture the important information in a data set much more efficiently than the original attributes

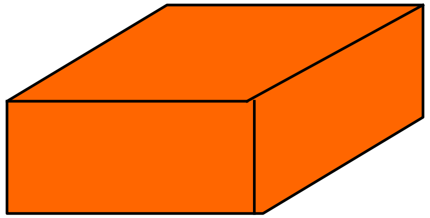
- Three general methodologies:
  - Feature Extraction
    - domain-specific
  - Mapping Data to New Space
  - Feature Construction
    - combining features

Attribute Transformation

- A function that maps the entire set of values of a given attribute to a new set of replacement values such that each old value can be identified with one of the new values
  - Simple functions: x^k, $\log(x)$, e^x, $|x|$
  - Standardization and Normalization

Storage: The Cube





Sales of apple telephones
in 2015 in KTM

OLAP Terminology

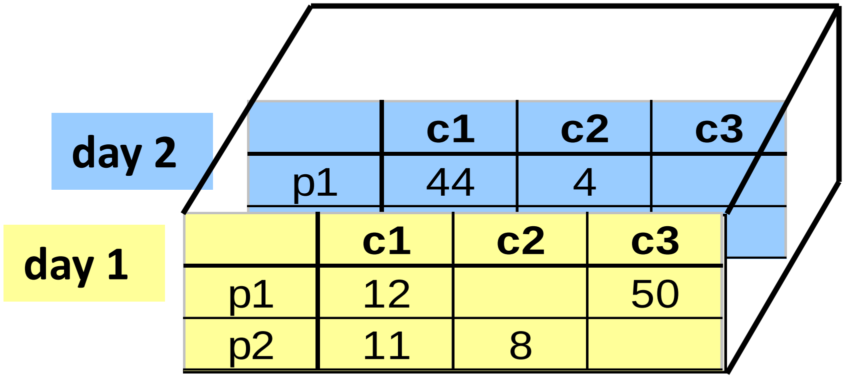
- A **data cube** supports viewing/modeling of a variable (a set of variables) of interest. **Measures** are used to report the values of the particular variable with respect to a given set of dimensions.
- A **fact table** stores measures as well as keys representing relationships to various dimensions.
- **Dimensions** are perspectives with respect to which an organization wants to keep record.
- A **star schema** defines a fact table and its associated dimensions.

3-D Cube

Fact table view:

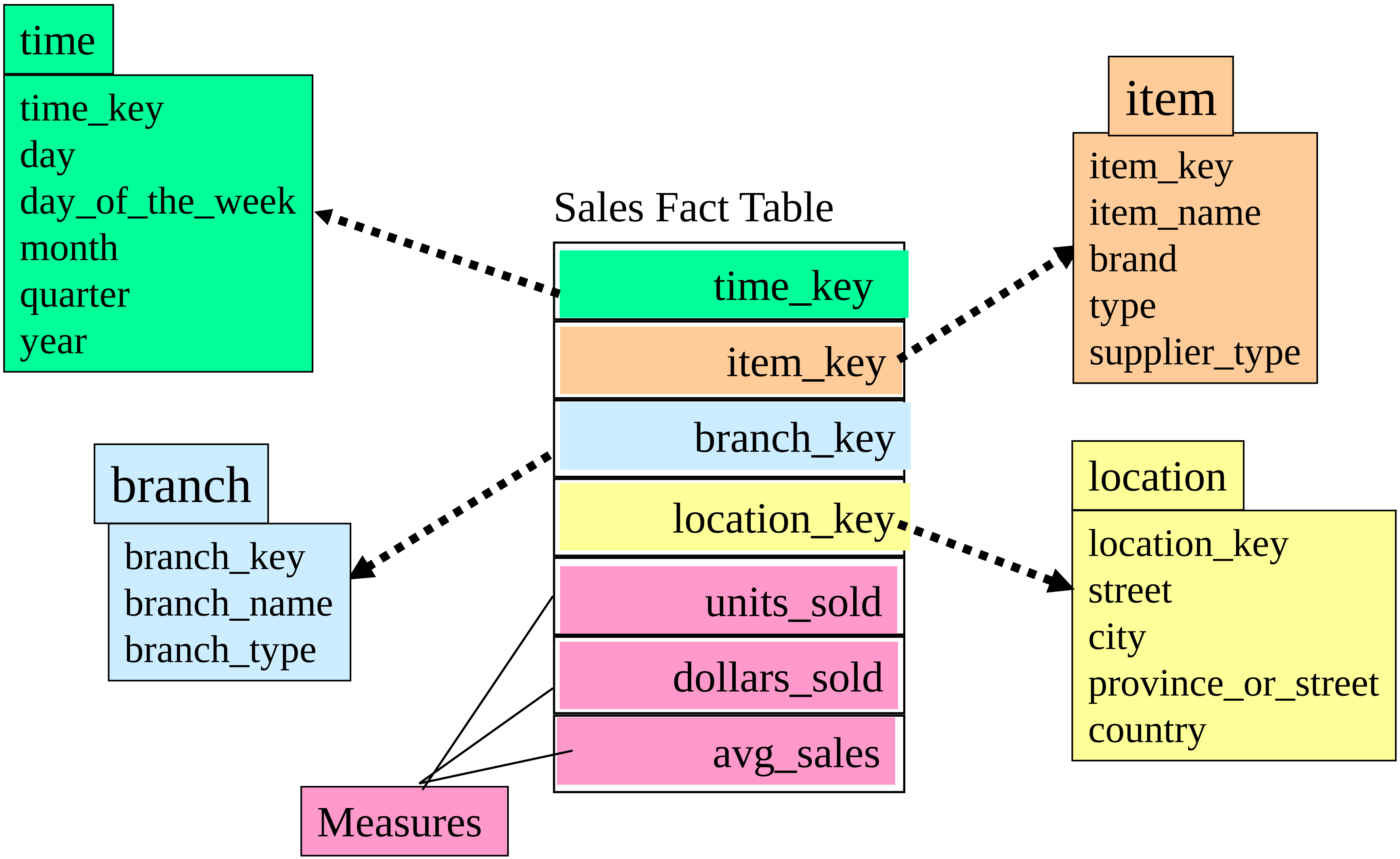
sale	prodId	storeId	date	amt
	p1	c1	1	12
	p2	c1	1	11
	p1	c3	1	50
	p2	c2	1	8
	p1	c1	2	44
	p1	c2	2	4

Multi-dimensional cube:



dimensions = 3

Example of Star Schema



Typical OLAP Operations

- Roll up (drill-up): summarize data
  - *by climbing up hierarchy or by dimension reduction*
- Drill down (roll down): reverse of roll-up
  - *from higher level summary to lower level summary or detailed data, or introducing new dimensions*
- Slice and dice:
  - *project and select*
- Pivot (rotate):
  - *reorient the cube, visualization, 3D to series of 2D planes.*
- Other operations
  - *drill across: involving (across) more than one fact table*

OLAP

TYPICAL OLAP OPERATIONS

Drill Down

Total Sales
Total Sales per city
Total Sales per city per store
Total Sales per city per store per month

Drill Up

Drill Down

Total Sales
Total Sales per city
Total Sales per city by category

Drill Up

Drill Across

By a drill up opperation examine sales By country rather than city level

	location by city			
	Istanbul	Ankara	Berlin	Münih
PC	20	30	50	40
Printer	15	5	10	20

roll up

→

	location y country	
	Türkiyy	Almanya
PC	50	90
Printer	20	30

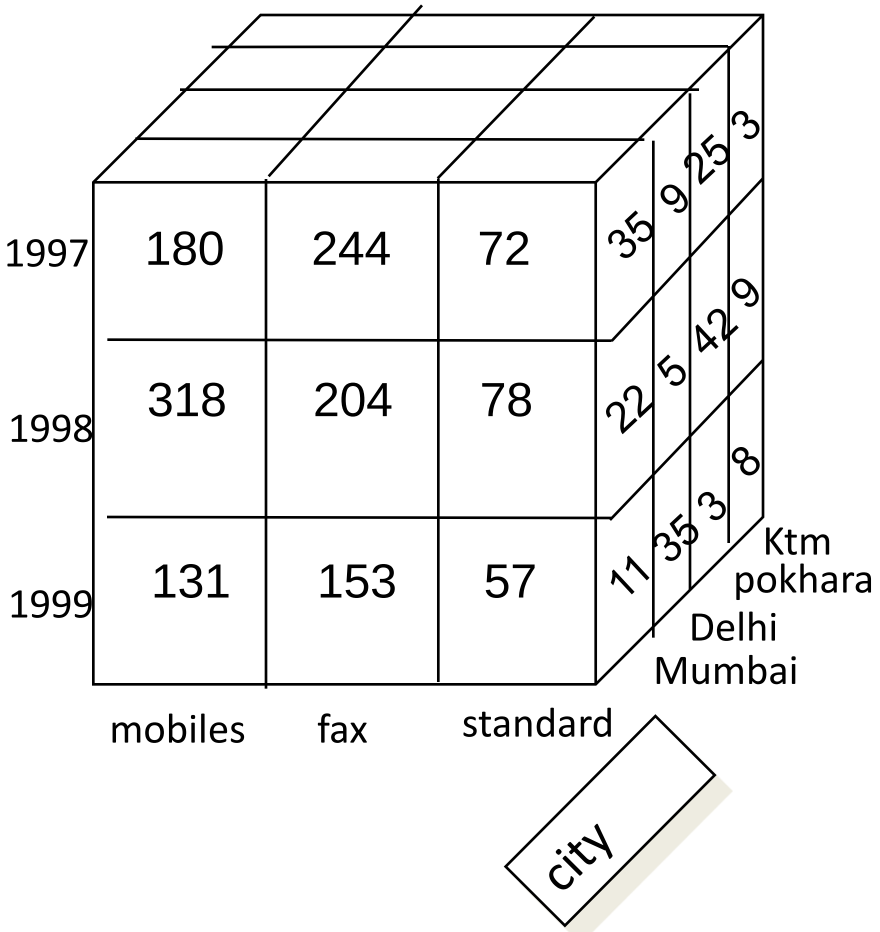
	measure is sales	
	Time 2002	
PC	50	
Printer	23	

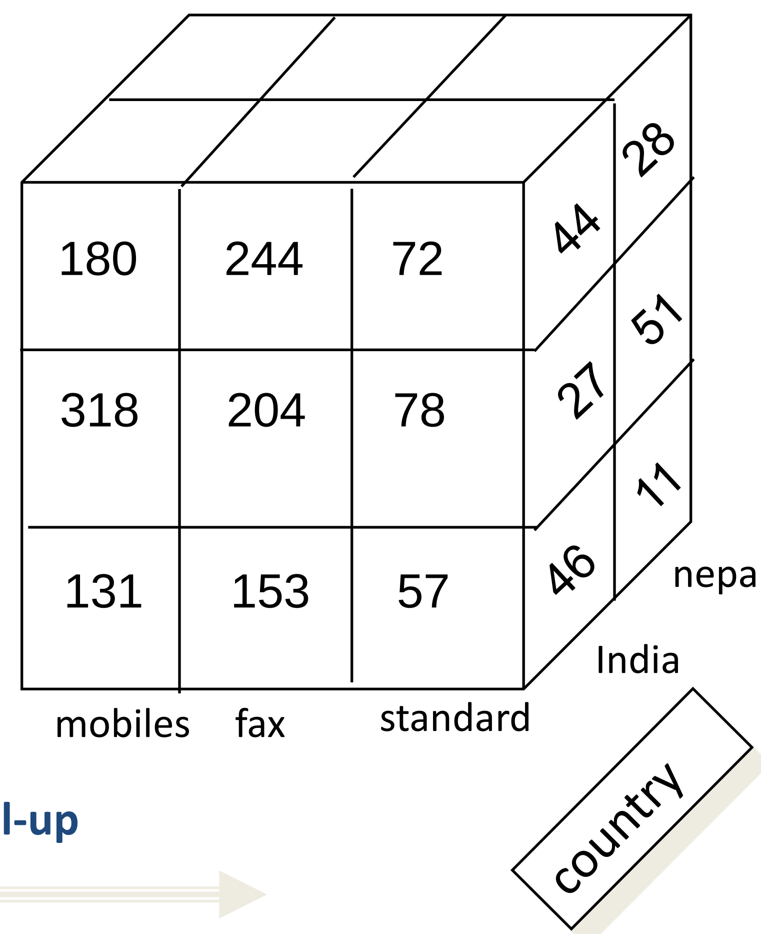
Drill down

→

	2002			
	Q1	Q2	Q3	Q4
PC	10	15	20	5
Printer	5	10	5	3

Roll-up and Drill-down algebraic operators





Roll-up

Less detailed: go up in the granularity hierarchy

Drill-down

More detailed: go down in the granularity hierarchy

Slice and dice

- Slice: a selection on one dimension of the cube resulting in subcube
- Ex: sales data are selected for dimension time using time =spring
- EX: you could slice a cube by using a particular product and view all sales of that product across all dates and customers.
- dice: defines a subcube by performing a selection on two or more dimensions
- EX: look for sales of a particular product on a particular day to a particular customer.
- Ex: a dice opp. Based on
  - location="pokhara" or "ktm" and
  - time =spring or summer and
  - item = "T-shirts" or "Pyjamas"

What is Data?

- Collection of data objects and their attributes
- An attribute is a property or characteristic of an object
  - Examples: eye color of a person, temperature, etc.
  - Attribute is also known as variable, field, characteristic, or feature
- A collection of attributes describe an object
  - Object is also known as record, point, case, sample, entity, or instance

Attributes

Objects

Tid	**Refund**	**Marital Status**	**Taxable Income**	**Cheat**
1	Yes	Single	125K	**No**
2	No	Married	100K	**No**
3	No	Single	70K	**No**
4	Yes	Married	120K	**No**
5	No	Divorced	95K	**Yes**
6	No	Married	60K	**No**
7	Yes	Divorced	220K	**No**
8	No	Single	85K	**Yes**
9	No	Married	75K	**No**
10	No	Single	90K	**Yes**

Attribute Values

- Attribute values are numbers or symbols assigned to an attribute

- Distinction between attributes and attribute values
  - Same attribute can be mapped to different attribute values
    - Example: height can be measured in feet or meters

  - Different attributes can be mapped to the same set of values
    - Example: Attribute values for ID and age are integers
    - But properties of attribute values can be different
      - ID has no limit but age has a maximum and minimum value

Types of Attributes

- There are different types of attributes
  - Nominal
    - Examples: ID numbers, eye color, zip codes
  - Ordinal
    - Examples: rankings (e.g., taste of potato chips on a scale from 1-10), grades, height in {tall, medium, short}
  - Interval
    - Examples: calendar dates, temperatures in Celsius or Fahrenheit.
  - Ratio
    - Examples: temperature in Kelvin, length, time, counts

Properties of Attribute Values

- The type of an attribute depends on which of the following properties it possesses:
  – Distinctness: $=\ \neq$
  – Order: $<\ >$
  – Addition: $+\ -$
  – Multiplication: $*\ /$

  – Nominal attribute: distinctness
  – Ordinal attribute: distinctness & order
  – Interval attribute: distinctness, order & addition
  – Ratio attribute: all 4 properties

Attribute Type	Description	Examples	Operations
Nominal	The values of a nominal attribute are just different names, i.e., nominal attributes provide only enough information to distinguish one object from another. (=, ≠)	zip codes, employee ID numbers, eye color, sex: {*male, female*}	mode, entropy, contingency correlation, χ^2 test
Ordinal	The values of an ordinal attribute provide enough information to order objects. (<, >)	hardness of minerals, {*good, better, best*}, grades, street numbers	median, percentiles, rank correlation, run tests, sign tests
Interval	For interval attributes, the differences between values are meaningful, i.e., a unit of measurement exists. (+, -)	calendar dates, temperature in Celsius or Fahrenheit	mean, standard deviation, Pearson's correlation, *t* and *F* tests
Ratio	For ratio variables, both differences and ratios are meaningful. (*, /)	temperature in Kelvin, monetary quantities, counts, age, mass, length, electrical current	geometric mean, harmonic mean, percent variation

Attribute Level	Transformation	Comments
Nominal	Any permutation of values	If all employee ID numbers were reassigned, would it make any difference?
Ordinal	An order preserving change of values, i.e., *new_value = f(old_value)* where *f* is a monotonic function.	An attribute encompassing the notion of good, better best can be represented equally well by the values {1, 2, 3} or by { 0.5, 1, 10}.
Interval	*new_value =a * old_value + b* where a and b are constants	Thus, the Fahrenheit and Celsius temperature scales differ in terms of where their zero value is and the size of a unit (degree).
Ratio	*new_value = a * old_value*	Length can be measured in meters or feet.

Discrete and Continuous Attributes

- Discrete Attribute
  - Has only a finite or countably infinite set of values
  - Examples: zip codes, counts, or the set of words in a collection of documents
  - Often represented as integer variables.
  - Note: binary attributes are a special case of discrete attributes
- Continuous Attribute
  - Has real numbers as attribute values
  - Examples: temperature, height, or weight.
  - Practically, real values can only be measured and represented using a finite number of digits.
  - Continuous attributes are typically represented as floating-point variables.

Types of data sets

- **Record**
  - **Data Matrix**
  - **Document Data**
  - **Transaction Data**
- **Graph**
  - **World Wide Web**
  - **Molecular Structures**
- **Ordered**
  - **Spatial Data**
  - **Temporal Data**
  - **Sequential Data**
  - **Genetic Sequence Data**

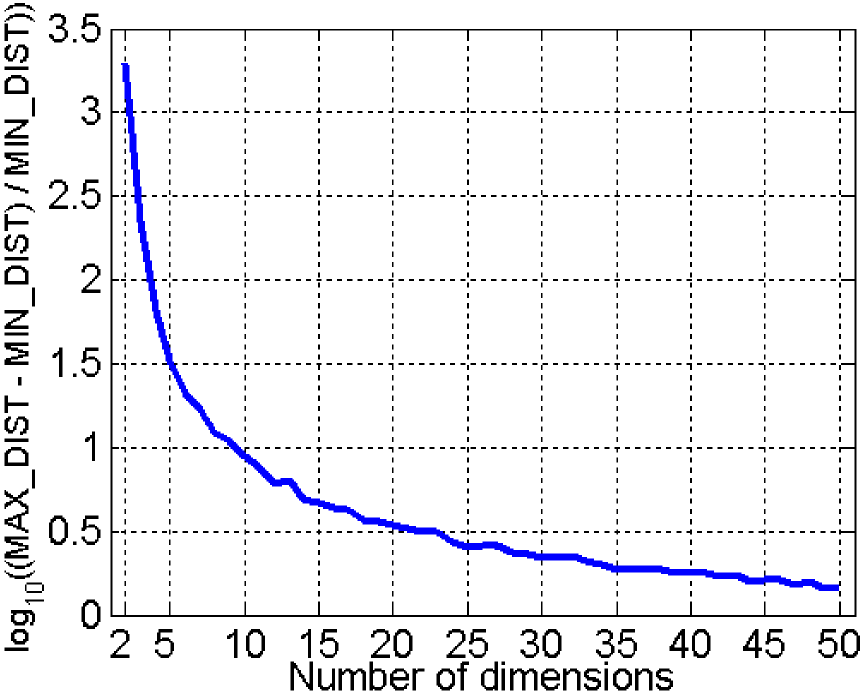
Important Characteristics of Structured Data

- **Dimensionality**
  - **Curse of Dimensionality**
- **Sparsity**
  - **Only presence counts**
- **Resolution**
  - **Patterns depend on the scale**

Curse of Dimensionality

- When dimensionality increases, data becomes increasingly sparse in the space that it occupies

- Definitions of density and distance between points, which is critical for clustering and outlier detection, become less meaningful



- **Randomly generate 500 points**
- **Compute difference between max and min distance between any pair of points**

Record Data

- Data that consists of a collection of records, each of which consists of a fixed set of attributes

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1	Yes	Single	125K	**No**
2	No	Married	100K	**No**
3	No	Single	70K	**No**
4	Yes	Married	120K	**No**
5	No	Divorced	95K	**Yes**
6	No	Married	60K	**No**
7	Yes	Divorced	220K	**No**
8	No	Single	85K	**Yes**
9	No	Married	75K	**No**
10	No	Single	90K	**Yes**

Data Matrix

- If data objects have the same fixed set of numeric attributes, then the data objects can be thought of as points in a multi-dimensional space, where each dimension represents a distinct attribute

- Such data set can be represented by an m by n matrix, where there are m rows, one for each object, and n columns, one for each attribute

Projection of x Load	**Projection of y load**	**Distance**	**Load**	**Thickness**
10.23	5.27	15.22	2.7	1.2
12.65	6.25	16.22	2.2	1.1

Document Data

- Each document becomes a `term' vector,
  - each term is a component (attribute) of the vector,
  - the value of each component is the number of times the corresponding term occurs in the document.

	team	coach	pla y	ball	score	game	wi n	lost	timeout	season
Document 1	3	0	5	0	2	6	0	2	0	2
Document 2	0	7	0	2	1	0	0	3	0	0
Document 3	0	1	0	0	1	2	2	0	3	0

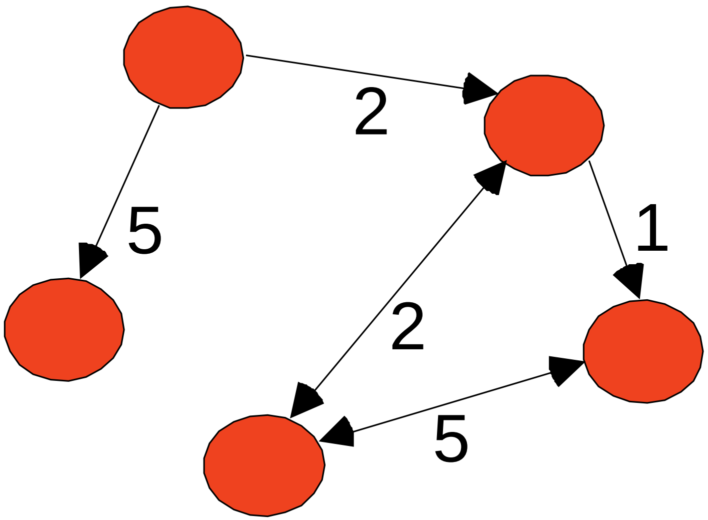
Transaction Data

- A special type of record data, where
  - each record (transaction) involves a set of items.
  - For example, consider a grocery store. The set of products purchased by a customer during one shopping trip constitute a transaction, while the individual products that were purchased are the items.

TID	*Items*
1	**Bread, Coke, Milk**
2	**Beer, Bread**
3	**Beer, Coke, Diaper, Milk**
4	**Beer, Bread, Diaper, Milk**
5	**Coke, Diaper, Milk**

Graph Data

- Examples: Generic graph and HTML Links



Data Mining

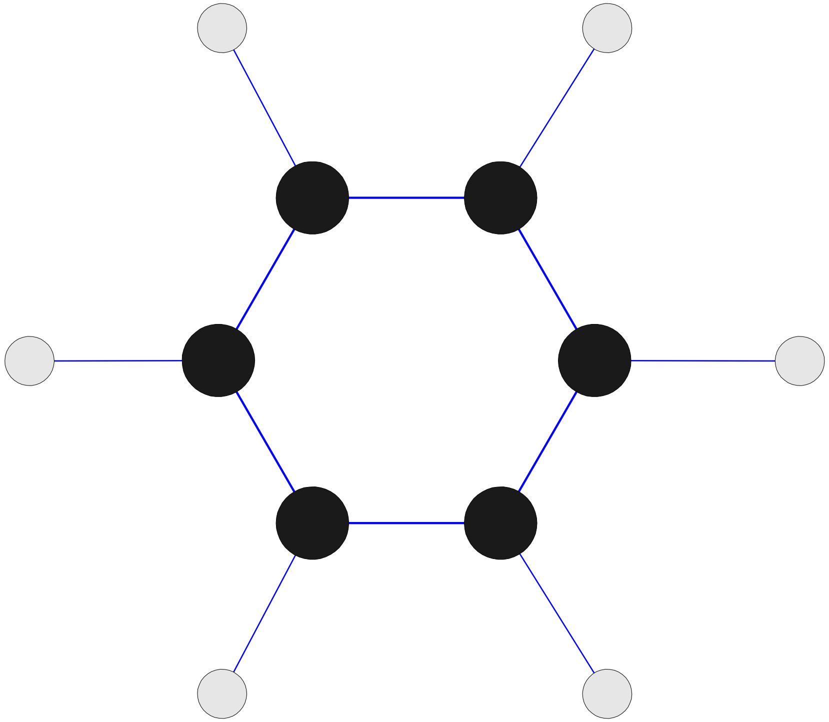
Graph Partitioning

Parallel Solution of Sparse Linear System of Equations

<a href="papers/papers.html#ffff">
N-Body Computation and Dense Linear System Solvers

Chemical Data

- Benzene Molecule: C_6H_6



Ordered Data

- Sequences of transactions

Items/Events

(A B) (D) (C E)
(B D) (C) (E)
(C D) (B) (A E)

An element of the sequence

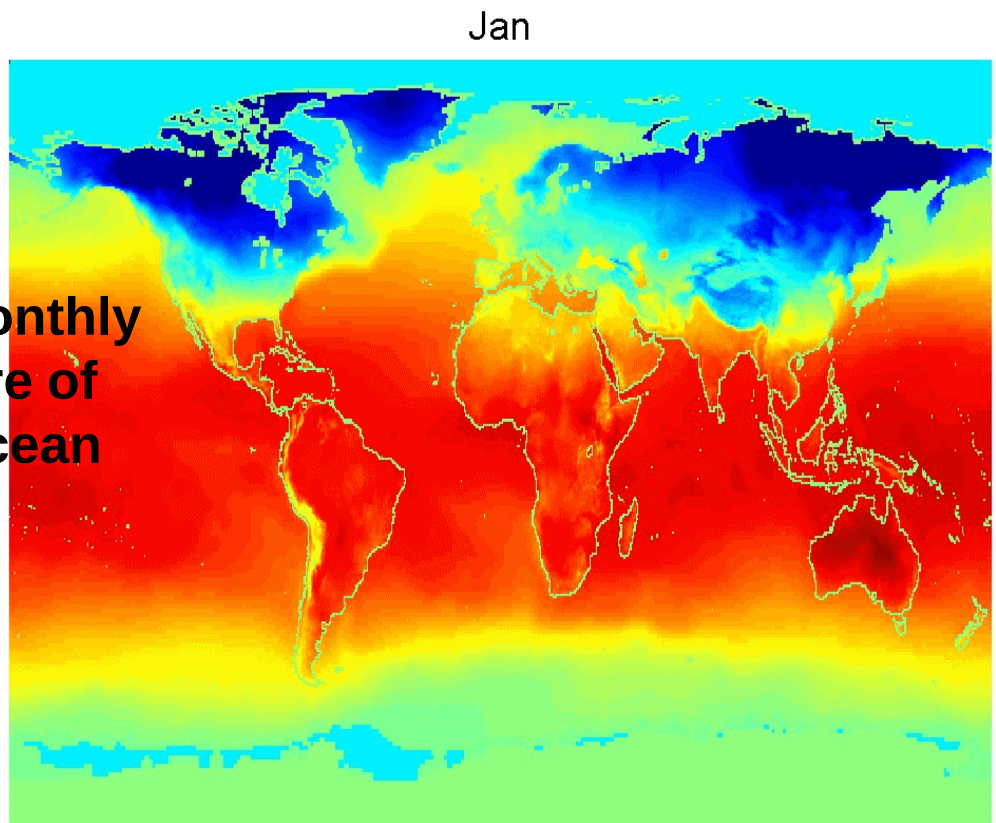
Ordered Data

- Genomic sequence data

GGTTCCGCCTTCAGCCCCGCGCC
CGCAGGGCCCGCCCCGCGCCGTC
GAGAAGGGCCCGCCTGGCGGGCG
GGGGGAGGCGGGGCCGCCCGAGC
CCAACCGAGTCCGACCAGGTGCC
CCCTCTGCTCGGCCTAGACCTGA
GCTCATTAGGCGGCAGCGGACAG
GCCAAGTAGAACACGCGAAGCGC
TGGGCTGCCTGCTGCGACCAGGG

Ordered Data

- Spatio-Temporal Data



Average Monthly Temperature of land and ocean

Similarity and Dissimilarity

- Similarity
  - Numerical measure of how alike two data objects are.
  - Is higher when objects are more alike.
  - Often falls in the range [0,1]
- Dissimilarity
  - Numerical measure of how different are two data objects
  - Lower when objects are more alike
  - Minimum dissimilarity is often 0
  - Upper limit varies

Similarity/Dissimilarity for Simple Attributes

p and *q* are the attribute values for two data objects.

Attribute Type	Dissimilarity	Similarity
Nominal	$d = \begin{cases} 0 & \text{if } p = q \\ 1 & \text{if } p \neq q \end{cases}$	$s = \begin{cases} 1 & \text{if } p = q \\ 0 & \text{if } p \neq q \end{cases}$
Ordinal	$d = \frac{\lvert p-q \rvert}{n-1}$ (values mapped to integers 0 to $n-1$, where n is the number of values)	$s = 1 - \frac{\lvert p-q \rvert}{n-1}$
Interval or Ratio	$d = \lvert p - q \rvert$	$s = -d$, $s = \frac{1}{1+d}$ or $s = 1 - \frac{d - min\_d}{max\_d - min\_d}$

Table 5.1. Similarity and dissimilarity for simple attributes

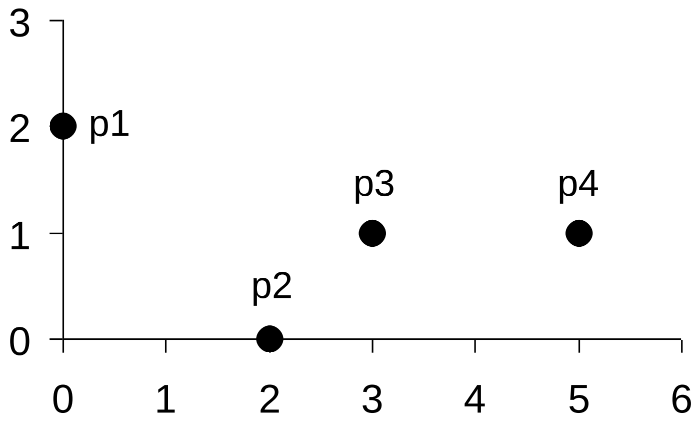
Euclidean Distance

- Euclidean Distance

$$dist = \sqrt{\sum_{k=1}^{n} (p_k - q_k)^2}$$

Where n is the number of dimensions (attributes) and p_k and q_k are, respectively, the k^{th} attributes (components) or data objects p and q.

Euclidean Distance



point	x	y
p1	0	2
p2	2	0
p3	3	1
p4	5	1

	p1	p2	p3	p4
p1	0	2.828	3.162	5.099
p2	2.828	0	1.414	3.162
p3	3.162	1.414	0	2
p4	5.099	3.162	2	0

Distance Matrix

Minkowski Distance

- Minkowski Distance is a generalization of Euclidean Distance

$$dist = \left(\sum_{k=1}^{n} | p_k - q_k |^r \right)^{\frac{1}{r}}$$

Where *r* is a parameter, *n* is the number of dimensions (attributes) and p_k and q_k are, respectively, the kth attributes (components) or data objects *p* and *q*.

Minkowski Distance: Examples

- $r = 1$. City block (Manhattan, taxicab, L_1 norm) distance.
  - A common example of this is the **Hamming distance**, which is just the **number of bits that are different between two binary vectors**
  - Find the distance between the vectors 01101010 and 11011011.
  - 01101010
  - 11011011
  - They differ in four places, so the Hamming distance d(01101010, 11011011) = 4.

- $r = 2$. Euclidean distance

- $r \rightarrow \infty$. "supremum" (L_{max} norm, L_{∞} norm) distance.
  - This is the **maximum difference between any component of the vectors**

Minkowski Distance

point	x	y
p1	0	2
p2	2	0
p3	3	1
p4	5	1

L1	p1	p2	p3	p4
p1	0	4	4	6
p2	4	0	2	4
p3	4	2	0	2
p4	6	4	2	0

L2	p1	p2	p3	p4
p1	0	2.828	3.162	5.099
p2	2.828	0	1.414	3.162
p3	3.162	1.414	0	2
p4	5.099	3.162	2	0

L_∞	p1	p2	p3	p4
p1	0	2	3	5
p2	2	0	1	3
p3	3	1	0	2
p4	5	3	2	0

Distance Matrix

Common Properties of a Distance

- Distances, such as the Euclidean distance, have some well known properties.

1. $d(p, q) \geq 0$ for all p and q and $d(p, q) = 0$ only if $p = q$. (**Positive definiteness)**
2. $d(p, q) = d(q, p)$ for all p and q. (**Symmetry**)
3. $d(p, r) \leq d(p, q) + d(q, r)$ for all points p, q, and r. (**Triangle Inequality**)

where $d(p, q)$ is the distance (dissimilarity) between points (data objects), p and q.

Common Properties of a Similarity

- Similarities, also have some well known properties.

  1. $s(p, q) = 1$ (or maximum similarity) only if $p = q$.

  2. $s(p, q) = s(q, p)$ for all p and q. (Symmetry)

  where $s(p, q)$ is the similarity between points (data objects), p and q.

Similarity Between Binary Vectors

- Common situation is that objects, *p* and *q*, have only binary attributes

- Compute similarities using the following quantities

  M_{01} = the number of attributes where p was 0 and q was 1
  M_{10} = the number of attributes where p was 1 and q was 0
  M_{00} = the number of attributes where p was 0 and q was 0
  M_{11} = the number of attributes where p was 1 and q was 1

- Simple Matching and Jaccard Coefficients

  SMC = number of matches / number of attributes
  $= (M_{11} + M_{00}) / (M_{01} + M_{10} + M_{11} + M_{00})$

  J = number of 11 matches / number of not-both-zero attributes values
  $= (M_{11}) / (M_{01} + M_{10} + M_{11})$

SMC versus Jaccard: Example

$p =$ 1 0 0 0 0 0 0 0 0 0

$q =$ 0 0 0 0 0 0 1 0 0 1

$M_{01} = 2$ (the number of attributes where p was 0 and q was 1)
$M_{10} = 1$ (the number of attributes where p was 1 and q was 0)
$M_{00} = 7$ (the number of attributes where p was 0 and q was 0)
$M_{11} = 0$ (the number of attributes where p was 1 and q was 1)

$SMC = (M_{11} + M_{00})/(M_{01} + M_{10} + M_{11} + M_{00}) = (0+7) / (2+1+0+7) = 0.7$

$J = (M_{11}) / (M_{01} + M_{10} + M_{11}) = 0 / (2 + 1 + 0) = 0$

Cosine Similarity

- If d_1 and d_2 are two document vectors, then

  $\cos(d_1, d_2) = (d_1 \bullet d_2) / ||d_1|| \, ||d_2||$,

  where $\bullet$ indicates vector dot product and $|| d ||$ is the length of vector d.

- Example:

  d_1 = **3 2 0 5 0 0 0 2 0 0**
  d_2 = **1 0 0 0 0 0 0 1 0 2**

$d_1 \bullet d_2$= 3*1 + 2*0 + 0*0 + 5*0 + 0*0 + 0*0 + 0*0 + 2*1 + 0*0 + 0*2 = 5
$||d_1||$ = (3*3+2*2+0*0+5*5+0*0+0*0+0*0+2*2+0*0+0*0)$^{0.5}$ = (42) $^{0.5}$ = 6.481
$||d_2||$ = (1*1+0*0+0*0+0*0+0*0+0*0+0*0+1*1+0*0+2*2) $^{0.5}$ = (6) $^{0.5}$ = 2.245

$\cos(d_1, d_2)$ = .3150

Correlation

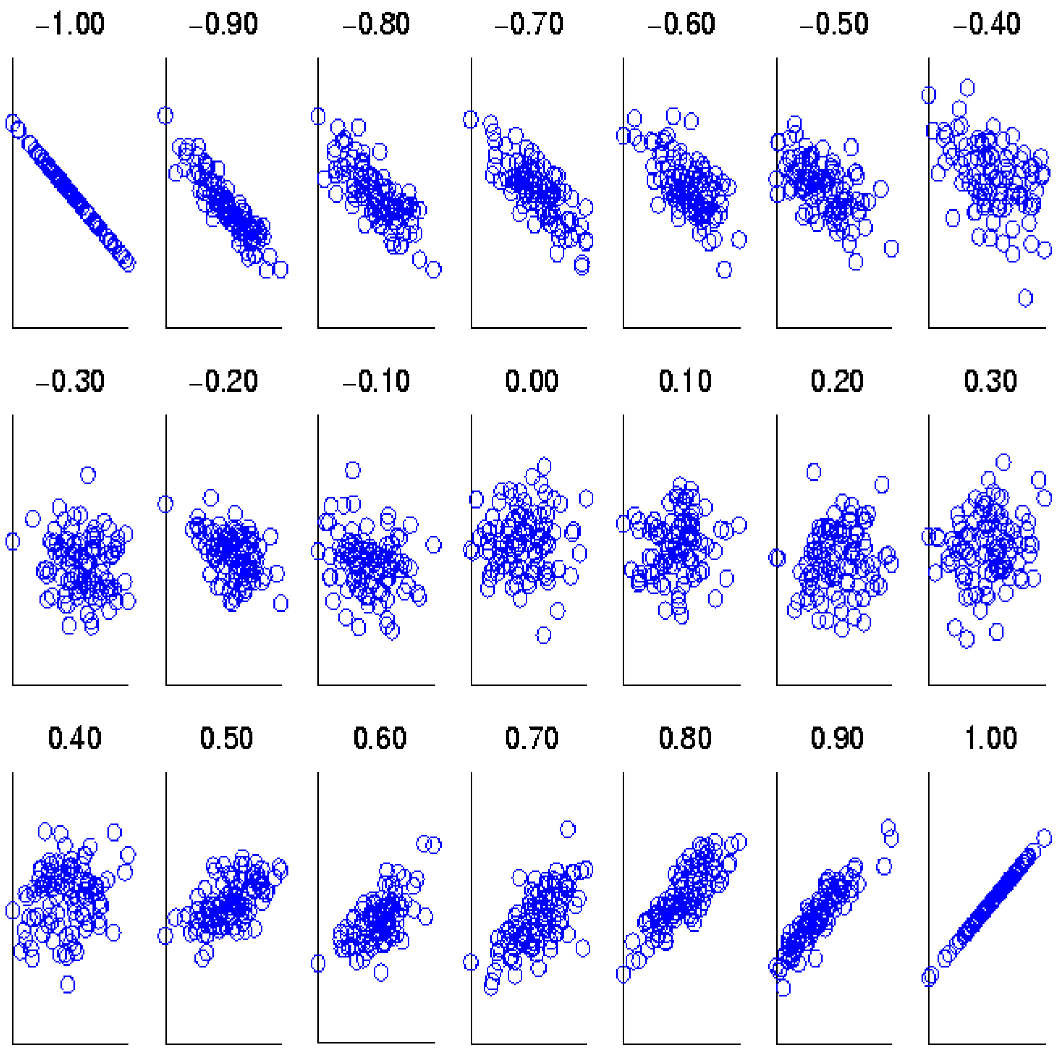
- Correlation measures the linear relationship between objects
- To compute correlation, we standardize data objects, p and q, and then take their dot product

$$p'_k = (p_k - mean(p)) / std(p)$$

$$q'_k = (q_k - mean(q)) / std(q)$$

$$correlation(p, q) = p' \bullet q'$$

Visually Evaluating Correlation



Scatter plots showing the similarity from –1 to 1.