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1 Data Structure

1.1 Dynamic Segment Tree

```
const int MAXL = 1000000000;
template <typename T>
struct DynamicSegment {
    struct Node{
        int l, r; // range
        T data;
        Node *left, *right;
        Node():l(1),r(MAXL),data(0),left(nullptr),right(nullptr) {}
    }
    void extend(){
        if(l == r)return;
        if(left == nullptr){ //if leaf node
            left = new Node();
            right = new Node();
            int mid = (l + r) / 2;
            left->l = l;
            left->r = mid;
            right->l = mid + 1;
            right->r = r;
        }
        return;
    }
};
Node *tree;

DynamicSegment() { tree = new Node(); }
void update(Node *cur, int x, T data){
    if(x < cur->l || cur->r < x)return;
    if(cur->l == cur->r)return cur->data = data, (void)0;
    cur->extend();
    update(cur->left, x, data);
    update(cur->right, x, data);
    cur->data = mergeNode(cur->left->data, cur->right->data);
}
void update(int x, T data){ update(tree, x, data); }

T query(Node *cur, int l, int r){
    if(cur->l > cur->r || cur->r < l || cur->l > r)return T(0);
    if(l <= cur->l && cur->r <= r)return cur->data;
    cur->extend();
    return mergeNode(query(cur->left, l, r), query(cur->right, l, r));
}
T query(int l, int r){ return query(tree, l, r); }
T mergeNode(T a, T b){ return a + b; }
};
DynamicSegment<long long> tree;

1.2 Dynamic Segment Tree With Lazy
const int MAXL = 1000000000;
template <typename T>
struct DynamicSegmentLazy {
    struct Node{
        int l, r; // range
        T data, lazy;
        Node *left, *right;
        Node():l(1),r(MAXL),data(0),lazy(0),left(0),right(0) {}
    }
};
```

```
void extend(T lzy=0){
    if(l == r)return;
    if(left == 0){ //if leaf node
        left = new Node();
        right = new Node();
        int m = (l + r) / 2;
        left->l = l;
        left->r = m;
        right->l = m + 1;
        right->r = r;
    }
    left->lazy += lzy;
    right->lazy += lzy;
    return;
}
};
Node *tree;

DynamicSegmentLazy() { tree = new Node(); }
void pushdown(Node *cur){
    if(cur->lazy){
        cur->data += (cur->r - cur->l + 1) * cur->lazy;
        cur->extend(cur->lazy);
        cur->lazy = 0;
    }
}

void update(Node *cur, int l, int r, T data){
    pushdown(cur);
    if(cur->l > cur->r || cur->l > r || l > cur->r)return;
    if(l <= cur->l && cur->r <= r){
        cur->data += (cur->r - cur->l + 1) * data;
        if(cur->l != cur->r)cur->extend(data);
        return;
    }
    cur->extend();
    update(cur->left, l, r, data);
    update(cur->right, l, r, data);
    cur->data = mergeNode(cur->left->data, cur->right->data);
}
void update(int l, int r, T data){ update(tree, l, r, data); }

T query(Node *cur, int l, int r){
    if(cur->l > cur->r || cur->l > r || l > cur->r)return T(0);
    pushdown(cur);
    if(l <= cur->l && cur->r <= r)return cur->data;
    cur->extend();
    return mergeNode(query(cur->left, l, r), query(cur->right, l, r));
}
T query(int l, int r){ return query(tree, l, r); }
T merge(T a, T b) {
    return a + b;
}
};

1.3 Fenwick
template <typename T>
struct Fenwick {
```

```

int N;
vector<T> tree;
Fenwick(int _N):N(_N) { tree.resize(N + 1); }
void update(int idx, T data) {
    for( ; idx <= N; idx += idx & -idx) tree[idx] += data;
}
T query(int idx) {
    T ret = 0;
    for( ; idx; idx -= idx & -idx) ret += tree[idx];
    return ret;
}
T query(int l, int r) {
    return query(r) - query(l - 1);
}
};

```

## 1.4 Hld

```

struct HLD {
    Graph<int> G;
    vector<int> par, top, dep, siz, in, out;
    Segment<int> seg; // Option
    int id;
    HLD(Graph<int> G):G(G) {
        int N = (int)G.size();
        siz = par = top = dep = in = out = vector<int>(N);
        seg = Segment<int>(N); // Option
        id = 0;
    }
    void dfs(int cur=1, int prev=0) {
        siz[cur] = 1;
        par[cur] = prev;
        dep[cur] = dep[prev] + 1;
        for(int &nxt : G[cur]) {
            if(nxt == prev) continue;
            dfs(nxt, cur);
            siz[cur] += siz[nxt];
            if(siz[nxt] > siz[G[cur][0]]) swap(nxt, G[cur][0]);
        }
    }
    void dfs2(int cur=1, int prev=0) {
        in[cur] = ++id;
        if(cur == 1) top[cur] = 1;
        for(int nxt: G[cur]) {
            if(nxt == prev) continue;
            top[nxt] = (nxt == G[cur][0] ? top[cur] : nxt);
            dfs2(nxt, cur);
        }
        out[cur] = id;
    }
    int lca(int a, int b) {
        while(top[a] != top[b]) {
            if(dep[top[a]] < dep[top[b]]) swap(a, b);
            a = par[top[a]];
        }
        if(in[a] > in[b]) swap(a, b);
        return a;
    }
    void update(int, int);
    int query(int, int);
};

```

## 1.5 Kdtree

```

template <typename T>
inline T INF() {
    return numeric_limits<T>::max() / 2;
}
template<typename T> inline T square(T x) { return x * x; }
template<typename T> struct KDTree {
    // axis == 1 ? y : x
    struct Node {
        T x, y;
        int axis;
        T mnx, mxx, mny, mxy;

        Node() {
            mnx = mny = INF<T>();
            mxx = mxy = -INF<T>();
            axis = 0;
        }
        void update(T y, T x) {
            mnx = min(mnx, x); mny = min(mny, y);
            mxx = max(mxx, x); mxy = max(mxy, y);
        }
        T dis(pair<T, T> point) {
            T a = point.first - y, b = point.second - x;
            return square(a) + square(b);
        }
        bool operator==(pair<T, T> point) { return make_pair(y, x) == point; }
        bool operator!=(pair<T, T> point) { return make_pair(y, x) != point; }
        bool operator<(pair<T, T> point) { return make_pair(y, x) < point; }
        bool operator>(pair<T, T> point) { return make_pair(y, x) > point; }
    };

    vector<pair<T, T>> points;
    vector<Node> tree;
    vector<bool> exist;
    T query_answer;
    int siz;
    KDTree(int N = 1 << 17) {
        for(siz = 1; siz < N; siz <= 1);
        tree.resize(siz << 1);
        exist.resize(siz << 1);
    }
    KDTree(const vector<pair<T, T>> &V) : KDTree(V.size()) {
        points = V;
    }
    void build(int l, int r, int pos) {
        Node cur;
        for(int i = l; i <= r; ++i) {
            auto [y, x] = points[i];
            cur.update(y, x);
        }
        tree[pos] = cur;
        exist[pos] = true;
        if(pos == 1) tree[pos].axis = 0;
        else tree[pos].axis = 1 - tree[pos >> 1].axis;
    }
};

```

```

if(tree[pos].axis) sort(points.begin() + 1, points.begin() + r + 1);
else sort(points.begin() + 1, points.begin() + r + 1, [&](const pair<T, T> &a, const pair<T, T> &b) { return a.second != b.second ? a.second < b.second : a.first < b.first; });
int mid = (l + r) / 2;
tree[pos].y = points[mid].first;
tree[pos].x = points[mid].second;
if(l <= mid - 1) build(l, mid - 1, pos << 1);
if(mid + 1 <= r) build(mid + 1, r, pos << 1 | 1);
}
void build() { build(0, (int)points.size() - 1, 1); }
void query(int pos, pair<T, T> point) {
    if(tree[pos] != point) query_answer = min(query_answer, tree[pos].dis(point));
    if(tree[pos].axis) { // y
        if(point.first < tree[pos].y) {
            if(exist[pos << 1]) query(pos << 1, point);
            if(exist[pos << 1 | 1] && square(tree[pos << 1 | 1].mny - point.first) < query_answer) query(pos << 1 | 1, point);
        }
        else {
            if(exist[pos << 1 | 1]) query(pos << 1 | 1, point);
            if(exist[pos << 1] && square(tree[pos << 1].mxy - point.first) < query_answer) query(pos << 1, point);
        }
    }
    else {
        if(point.second < tree[pos].x) {
            if(exist[pos << 1]) query(pos << 1, point);
            if(exist[pos << 1 | 1] && square(tree[pos << 1 | 1].mnx - point.second) < query_answer) query(pos << 1 | 1, point);
        }
        else {
            if(exist[pos << 1 | 1]) query(pos << 1 | 1, point);
            if(exist[pos << 1] && square(tree[pos << 1].mxx - point.second) < query_answer) query(pos << 1, point);
        }
    }
}
T query(pair<T, T> point) {
    query_answer = INF<T>();
    query(1, point);
    return query_answer;
}
};

```

## 1.6 Lca

// 1-index, dist (1l), Need Graph Template

```

struct LCA {
    int N, sz;
    Graph<pair<int, int>> G;
    vector<int> dep;
    vector<ll> dist;
    vector<vector<int>> par;
    LCA(const Graph<pair<int, int>> &_G):G(_G) {

```

```

    for(sz = 1; (1 << sz) < N; ++sz);
    N = G.size();
    dep = vector<int>(N + 1);
    dist = vector<ll>(N + 1);
    par = vector<vector<int>>(sz, vector<int>(N + 1));
    dfs(1, 0);
    for(int j = 1; j < sz; ++j) for(int i = 1; i <= N; ++i)
        par[j][i] = par[j - 1][par[j - 1][i]];
}

void dfs(int cur, int prev) {
    dep[cur] = dep[prev] + 1;
    for(const auto &[nxt, w]: G[cur]) {
        if(nxt == prev) continue;
        par[0][nxt] = cur;
        dist[nxt] = dist[cur] + w;
        dfs(nxt, cur);
    }
}

int lca(int u, int v) {
    if(dep[u] > dep[v]) swap(u, v);
    for(int i = sz - 1; ~i; --i) if(dep[u] <= dep[par[i][v]]) v = par[i][v];
    if(u == v) return u;
    for(int i = sz - 1; ~i; --i) if(par[i][u] != par[i][v]) u = par[i][u], v = par[i][v];
    return par[0][u];
}

ll distance(int u, int v) { return dist[u] + dist[v] - 2 * dist[lca(u, v)]; }

int kth(int u, int v, int k) {
    int l = lca(u, v), dif = dep[u] - dep[l] + 1;
    if(dif < k) k = dep[v] - dep[l] + dif - k, u = v, v = 1;
    else --k;
    for(int i = sz - 1; ~i; --i) if(k & (1 << i)) u = par[i][u];
    return u;
}
};

```

## 1.7 Pbds

```

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
#define ordered_set tree<int, null_type, less_equal<int>,
rb_tree_tag, tree_order_statistics_node_update>
// multiset처럼 less_equal<int>
// set처럼 less<int>
ordered_set pbds;
pbds.insert(x);
pbds.erase(x); // multiset처럼 쓸 때 주의
*pbds.find_by_order(x);
*pbds.find_by_key(x);

```

## 1.8 Pst

```

template <typename T>
struct PST {
    struct Node {
        Node *left, *right;
        T data;
    };
};

```

```

Node(Node *l = nullptr, Node *r = nullptr, T v=0):left(l),
right(r), data(v) { }
Node *push(int l, int r, int x, T _data) {
    if(r < x || x < l) return this;
    if(l == r) return new Node(0, 0, this->data + _data);
    int mid = l + (r - l) / 2;
    Node *L = left->push(l, mid, x, _data);
    Node *R = right->push(mid + 1, r, x, _data);
    return new Node(L, R, L->data + R->data);
}
};

Node *roots[100002];
int siz;

PST() { setting(); }
PST(int N) { setting(N); }
void setting(int N = 2e9 + 10){
    siz = N;
    roots[0] = new Node();
    roots[0]->left = roots[0]->right = roots[0];
}

void expand(int p){ roots[p] = roots[p - 1]; }
void update(int p, int idx, T data, bool _expand=false){
    if(!_expand) expand(p);
    roots[p] = roots[p]->push(1, siz, idx, data);
}

T query(Node *cur, int l, int r, int s, int e){
    if(s <= l && r <= e) return cur->data;
    if(e < l || r < s) return 0;
    int mid = l + (r - l) / 2;
    return query(cur->left, l, mid, s, e) + query(cur->right,
        mid + 1, r, s, e);
}

T query(int s, int e, int p){ return query(roots[p], 1, siz,
s, e); }

T kth(Node *s, Node *e, int l, int r, int k){
    if(l == r) return l;
    int mid = l + (r - l) / 2;
    T data = e->left->data - s->left->data;
    if(data >= k) return kth(s->left, e->left, l, mid, k);
    return kth(s->right, e->right, mid + 1, r, k - data);
}

T kth(int s, int e, int k){ return kth(roots[s], roots[e], 1,
siz, k); }
};

```

## 1.9 Rope

```

#include <ext/rope>
using namespace __gnu_cxx;
string S;
crope rp = S.c_str();
rp.push_back('a');
rp.insert(0, "asdf");
rp.erase(0, 1);
rp.replace(0, 1, "asdf");
rp.substr(0, 2); // idx, cnt
rp.pop_back();

```

```
rp += rp2;
```

## 1.10 Segment Tree

```

template <typename T>
struct Segment {
    vector<T> tree;
    int siz;

    Segment(int N = 1 << 17) {
        for(siz = 1; siz < N; siz <= 1);
        tree = vector<T>(siz << 1);
    }

    void build() {
        for(int i = siz - 1; i > 0; --i) {
            tree[i] = tree[i << 1] + tree[i << 1 | 1];
        }
    }

    void update(int idx, T data) {
        tree[idx += siz] = data;
        while(idx >= 1) tree[idx] = tree[idx << 1] + tree[idx << 1 | 1];
    }

    T query(int l, int r) {
        T ret_L = T(), ret_R = T();
        for(l += siz, r += siz; l <= r; l >= 1, r >= 1) {
            if(l & 1) ret_L = ret_L + tree[l ++];
            if(~r & 1) ret_R = tree[r --] + ret_R;
        }
        return ret_L + ret_R;
    }

    T& operator[](const int &idx) { return tree[idx + siz]; }
};

```

## 1.11 Segment Tree With Lazy

```

template <typename T>
struct SegmentLazy {
    vector<T> tree, lazy;
    int siz;

    SegmentLazy(int N = 1 << 17) {
        for(siz = 1; siz < N; siz <= 1);
        lazy = tree = vector<T>(siz << 1);
    }

    void putItem(int idx, T data) { tree[idx + siz] = data; }
    void build() {
        for(int i = siz - 1; i; --i) tree[i] = merge(tree[i << 1],
            tree[i << 1 | 1]);
    }

    void propagate(int l, int r, int pos) {
        if(!lazy[pos]) return;
        if(l != r) {
            lazy[pos << 1] = merge(lazy[pos << 1], lazy[pos]);
            lazy[pos << 1 | 1] = merge(lazy[pos << 1 | 1],
                lazy[pos]);
        }
        tree[pos] += lazy[pos] * (r - l + 1);
        lazy[pos] = 0;
    }
};

```

```

void update(int l, int r, int s, int e, int pos, T data) {
    if(s <= l && r <= e) {
        lazy[pos] += data;
        propagate(l, r, pos);
        return;
    }
    propagate(l, r, pos);
    if(e < l || r < s) return;
    int mid = (l + r) / 2;
    update(l, mid, s, e, pos << 1, data);
    update(mid + 1, r, s, e, pos << 1 | 1, data);
    tree[pos] = merge(tree[pos << 1], tree[pos << 1 | 1]);
}

void update(int s, int e, T data) { update(0, siz - 1, s, e, 1, data); }

T query(int l, int r, int s, int e, int pos) {
    propagate(l, r, pos);
    if(s <= l && r <= e) return tree[pos];
    if(e < l || r < s) return 0;
    int mid = (l + r) / 2;
    return merge(query(l, mid, s, e, pos << 1), query(mid + 1, r, s, e, pos << 1 | 1));
}

T query(int s, int e) { return query(0, siz - 1, s, e, 1); }

T merge(T a, T b) {
    return a + b;
}
};

```

## 1.12 Union Find Roll Back

```

struct UnionFind {
    vector<int> par, rank;
    stack<tuple<int, int, int>> st;
    UnionFind(int N) {
        par = rank = vector<int>(N + 1);
        iota(par.begin(), par.end(), 0);
    }

    int find(int x) { return par[x] == x ? x : find(par[x]); }
    bool merge(int u, int v) {
        u = find(u); v = find(v);
        if(u == v) return false;
        if(rank[u] < rank[v]) swap(u, v);
        par[v] = u;
        st.emplace(u, v, rank[u] == rank[v]);
        if(rank[u] == rank[v]) ++rank[u];
        return true;
    }

    void revert(int cnt) {
        while(cnt --> 0) {
            auto [u, v, c] = st.top(); st.pop();
            par[v] = v;
            if(c) -- rank[u];
        }
    }

    int conn(int u, int v) { return find(u) == find(v); }
};

```

## 2 Graph

### 2.1 Dinic

```

struct Dinic {
    struct Node {
        int node_idx, cost, flow, rev;
        Node(int _nxt = -1, int _cost = 0, int _rev = -1):node_idx(_nxt),cost(_cost),flow(0),rev(_rev) {}
        int spare() { return cost - flow; }
        void setRev(int _rev) { rev = _rev; }
    };
    vector<Node> nodes;
    vector<vector<int>> G;
    vector<int> level;
    vector<int> work;

    int src, snk, asrc, asnk, N;
    Dinic(int _N) {
        src = _N + 1;
        snk = src + 1;
        asrc = snk + 1;
        asnk = asrc + 1;
        N = asnk;
        G.resize(N + 1);
    }

    bool bfs(int s, int e) {
        level = vector<int>(N + 1, -1);
        level[s] = 0;
        queue<int> Q; Q.push(s);
        while(!Q.empty()) {
            int cur = Q.front(); Q.pop();
            for(const int &x: G[cur]) {
                Node &nxt = nodes[x];
                if(nxt.spare() > 0 && level[nxt.node_idx] == -1) {
                    level[nxt.node_idx] = level[cur] + 1;
                    Q.push(nxt.node_idx);
                }
            }
        }
        return ~level[e];
    }

    int dfs(int s, int e, int f) {
        if(s == e) return f;
        for(int &i = work[s]; i < (int)G[s].size(); ++i) {
            Node &nxt = nodes[G[s][i]];
            if(nxt.spare() > 0 && level[nxt.node_idx] == level[s] + 1) {
                int ret = dfs(nxt.node_idx, e, min(f, nxt.spare()));
                if(ret > 0) {
                    nxt.flow += ret;
                    nodes[nxt.rev].flow -= ret;
                    return ret;
                }
            }
        }
        return 0;
    }

    int flow(int s, int e) {
        int ret = 0;
        while(bfs(s, e)) {

```

```

            work = vector<int>(N + 1, 0);
            while(true) {
                int x = dfs(s, e, numeric_limits<int>::max());
                if(x == 0) break;
                ret += x;
            }
        }
        return ret;
    }

    void addEdge(int u, int v, int cost, bool is_directed = true, bool is_unique = false) {
        if(is_unique) {
            for(const int &x: G[u]) {
                if(nodes[x].node_idx == v) {
                    nodes[x].cost += cost;
                    if(!is_directed) return;
                    break;
                }
            }
        }
        if(!is_directed) {
            for(const int &x: G[v]) {
                if(nodes[x].node_idx == u) {
                    nodes[x].cost += cost;
                    return;
                }
            }
        }
        int a = (int)nodes.size(), b = a + 1;
        Node uv = Node(v, cost, b);
        Node vu = Node(u, is_directed ? 0 : cost, a);
        nodes.push_back(uv); nodes.push_back(vu);
        G[u].push_back(a); G[v].push_back(b);
    }

    void addLREdge(int u, int v, int lower, int upper) {
        if(lower) {
            addEdge(asrc, v, lower);
            addEdge(u, asnk, lower);
        }
        addEdge(u, v, upper - lower);
    }

    int flow() { return flow(src, snk); }
    int lrflow() { return flow(asrc, asnk); }
};

2.2 Mcmf

template <typename T>
struct MinCostMaxFlow {
    struct Edge {
        int edge_id, node_idx, cost, flow, rev;
        T dist;
        Edge(int _edge_id, int _node_idx, int _cost, T _dist, int _rev):edge_id(_edge_id),node_idx(_node_idx),cost(_cost),flow(0),dist(_dist) {}
        int spare() { return cost - flow; }
    };

    vector<Edge> edges;
    vector<vector<int>> G;
    vector<pair<int, int>> par;

```

```

vector<T> dist;

int src, snk, N;
T INF;

MinCostMaxFlow(int _N) {
    src = _N + 1;
    snk = src + 1;
    N = snk;

    INF = numeric_limits<T>::max();

    G.resize(N + 1);
    par.resize(N + 1, make_pair(-1, -1));
}

bool spfa(int s, int e) {
    vector<int> InQ(N + 1);
    dist = vector<T>(N + 1, INF);

    dist[s] = 0;
    deque<int> dq; dq.push_back(s);
    InQ[s] = 1;

    while(!dq.empty()) {
        int cur = dq.front(); dq.pop_front();
        InQ[cur] = 0;
        for(const int &x: G[cur]) {
            Edge &e = edges[x];
            if(e.spare() > 0 && dist[e.node_idx] > dist[cur] +
                e.dist) {
                dist[e.node_idx] = dist[cur] + e.dist;
                par[e.node_idx] = make_pair(cur, e.edge_id);
                if(InQ[e.node_idx] == 0) {
                    dq.push_back(e.node_idx);
                    InQ[e.node_idx] = 1;
                }
            }
        }
    }

    return dist[e] != INF;
}

// min_cost, max_flow
pair<T, int> flow_after_spfa(int s, int e) {
    int mn = numeric_limits<int>::max();
    for(int cur = e; cur != s; cur = par[cur].first) {
        mn = min(mn, edges[par[cur].second].spare());
    }
    if(mn == 0) return make_pair<T, int>(-1, -1);
    T min_cost = 0;
    int max_flow = mn;
    for(int cur = e; cur != s; cur = par[cur].first) {
        min_cost += (T)mn * edges[par[cur].second].dist;
        edges[par[cur].second].flow += mn;
        edges[edges[par[cur].second].rev].flow -= mn;
    }
    return make_pair(min_cost, max_flow);
}

pair<T, int> flow(int s, int e) {

```

```

pair<T, int> ret;
while (spfa(s, e)) {
    pair<T, int> cur = flow_after_spfa(s, e);
    if (cur.first == -1) break;
    ret.first += cur.first;
    ret.second += cur.second;
}
return ret;
}

// addEdge
void addEdge(int u, int v, int cost, T dist) {
    int a = edges.size();
    int b = a + 1;

    Edge uv = Edge(a, v, cost, dist, b);
    Edge vu = Edge(b, u, 0, -dist, a);

    edges.push_back(uv);
    edges.push_back(vu);

    G[u].push_back(a);
    G[v].push_back(b);
}

pair<T, int> flow() { return flow(src, snk); }
};

2.3 2Sat

// 1-indexed, a xor b = (a or b) and (~a or ~b)
int getIdx(int x) { return abs(x) << 1 | (x < 0); }
void addEdge(Graph<int> &G, int u, int v) {
    u = getIdx(u), v = getIdx(v);
    G.addEdge(u ^ 1, v); G.addEdge(v ^ 1, u);
}
bool available(Graph<int> &G) {
    SCC scc(G);
    int N = G.size() - 2 >> 1;
    for(int i = 1; i <= N; ++i) {
        if(scc.scc_id[i << 1] == scc.scc_id[i << 1 | 1]) return
            false;
    }
    return true;
}

```

## 2.4 Scc

```

// 1-indexed, Need Graph template
struct SCC {
    int N, id;
    Graph<int> G;
    vector<int> D, scc_id;
    vector<vector<int>> scc;
    stack<int> st;

    SCC(const Graph<int> &G):G(_G) {
        id = 0;
        N = G.size();
        D.resize(N + 1);
        scc_id.resize(N + 1, -1);
        for(int i = 1; i <= N; ++i) if(!D[i]) dfs(i);
    }

```

```

}

int dfs(int cur) {
    D[cur] = ++id;
    st.push(cur);
    int par = D[cur];
    for(const auto &nxt: G[cur]) {
        if(!D[nxt]) par = min(par, dfs(nxt));
        else if(scc_id[nxt] == -1) par = min(par, D[nxt]);
    }
    if(par == D[cur]) {
        scc.emplace_back();
        while(!st.empty()) {
            int x = st.top(); st.pop();
            scc_id[x] = (int)scc.size() - 1;
            scc.back().push_back(x);
            if(x == cur) break;
        }
    }
    return par;
}

int size() { return scc.size(); }
vector<int> &operator[] (const int idx) { return scc[idx]; }
Graph<int> graph() {
    int K = size();
    Graph<int> sccG(K);
    for(int i = 1; i <= N; ++i) {
        for(const int &nxt: G[i]) {
            if(scc_id[i] == scc_id[nxt]) continue;
            sccG.addEdge(scc_id[i], scc_id[nxt]);
        }
    }
    for(int i = 0; i < K; ++i) {
        sort(sccG[i].begin(), sccG[i].end());
        sccG[i].erase(unique(sccG[i].begin(), sccG[i].end()),
            sccG[i].end());
    }
    return sccG;
}
};

```

## 2.5 Dominator Tree

```

vector<int> DominatorTree(const vector<vector<int>> &G, int
start_node) {
    int N = (int)G.size();
    vector<vector<int>> rG(N);
    for (int cur = 0; cur < N; ++cur) {
        for (int nxt : G[cur]) rG[nxt].push_back(cur);
    }

    vector<int> uf(N), sdom_id(N), idom(N, -1), sdom(N, -1);
    for (int i = 0; i < N; ++i) uf[i] = sdom_id[i] = i;
    function<int(int)> find = [&](int x) -> int {
        if (uf[x] == x) return x;
        int tmp = find(uf[x]);
        if (sdom[sdom_id[x]] > sdom[sdom_id[uf[x]]]) sdom_id[x] =
            sdom_id[uf[x]];
        return uf[x] = tmp;
    };
    vector<int> numbering, par(N);

```

```

function<void(int)> dfs = [&](int cur) -> void {
    sdom[cur] = numbering.size();
    numbering.push_back(cur);
    for (int nxt : G[cur]) {
        if (sdom[nxt] != -1) continue;
        par[nxt] = cur;
        dfs(nxt);
    }
};
dfs(start_node);
int K = (int)numbering.size();
vector<vector<int>> buf(N);
vector<int> final_uf(N);
for (int i = K - 1; i >= 0; --i) {
    int u = numbering[i];
    if (sdom[u] == -1) continue;
    for (int v : rG[u]) {
        if (sdom[v] == -1) continue;
        find(v);
        if (sdom[u] > sdom[sdom_id[v]]) sdom[u] =
            sdom[sdom_id[v]];
    }
    buf[numbering[sdom[u]]].push_back(u);
    for (int nxt : buf[par[u]]) {
        find(nxt);
        final_uf[nxt] = sdom_id[nxt];
    }
    buf[par[u]].clear();
    uf[u] = par[u];
}
idom[start_node] = start_node;
for (const int &x : numbering) {
    if (sdom[x] == sdom[final_uf[x]]) idom[x] = sdom[x];
    else idom[x] = idom[final_uf[x]];
}
for (const int &x : numbering) {
    if (x != start_node) idom[x] = numbering[idom[x]];
}
return idom;
}
}

```

## 2.6 Gomory Hu

```

vector<int> par(N);
int ans = 0;
for (int i = 1; i < N; ++i) {
    Dinic dinic(N);
    for (auto [u, v]: edges) dinic.addEdge(u, v, 1, false);
    int src = i, snk = par[i];
    int flow = dinic.flow(src, snk);
    ans = max(ans, flow);
    for (int j = i + 1; j < N; ++j) {
        if (dinic.level[j] != -1 && par[j] == par[i]) par[j] = i;
    }
}
}

```

## 2.7 Tree Isomorphism

```

// Need Graph Template
struct TreeIsomorphism {
    string tree_str;
    TreeIsomorphism(Graph<int> &G) {

```

```

    int N = G.size();
    function<vector<int>()> get_center = [&]() -> vector<int> {
        vector<int> ind(N), cand;
        for (int i = 0; i < N; ++i) {
            ind[i] = G[i].size();
            if (ind[i] < 2) cand.push_back(i);
        }
        int cnt = N;
        while (cnt > 2) {
            vector<int> tmp;
            for (int x : cand) {
                --cnt;
                for (int y : G[x]) if (--ind[y] == 1)
                    tmp.push_back(y);
            }
            cand = tmp;
        }
        return cand;
    };

    function<string(int, int)> make_string = [&](int cur, int
    prev) -> string {
        vector<string> child;
        for (int nxt : G[cur]) {
            if (nxt == prev) continue;
            child.push_back(make_string(nxt, cur));
        }
        sort(child.begin(), child.end());
        string ret = "";
        for (const string &s : child) ret += s;
        return "(" + ret + ")";
    };

    if (N == 0) { }
    else {
        vector<int> center = get_center();
        if (center.size() == 1) tree_str = make_string(center[0],
        -1);
        else tree_str = min(make_string(center[0], -1),
        make_string(center[1], -1));
    }
}
string get() { return tree_str; }
};

```

## 3 Others

### 3.1 Fastinput

```

// eof 추가해야함.
#define BUFFERMAX 1 << 19
struct IO {
    char buf[BUFFERMAX];
    char _read() {
        static int idx = BUFFERMAX;
        if(idx == BUFFERMAX) fread(buf, 1, BUFFERMAX, stdin), idx =
        0;
        return buf[idx++];
    }
}
char readChar() {
    char ret = _read();

```

```

    while(ret == 10 || ret == 32) ret = _read();
    return ret;
}
string readString() {
    string ret = "";
    char now = _read();
    while(now == 10 || now == 32) now = _read();
    while(true) {
        ret += now;
        now = _read();
        if(now == 10 || now == 32) break;
    }
    return ret;
}
template<typename T> T readInt() {
    T ret = 0;
    bool minus = false;
    char now = _read();
    while(now == 10 || now == 32) now = _read();
    if(now == '-') minus = true, now = _read();
    while(48 <= now && now <= 57) {
        ret = ret * 10 + now - 48;
        now = _read();
    }
    if(minus) ret *= -1;
    return ret;
}
void read(int &x) { x = readInt<int>(); }
void read(long long &x) { x = readInt<long long>(); }
void read(char &x) { x = readChar(); }
void read(string &x) { x = readString(); }
template<typename Type, typename... Types> void read(Type
&arg, Types &...args) { read(arg); read(args...); }
} io;

template<typename T>
IO& operator>> (IO& in, T &x) { in.read(x); return in; }

#define cin io
#define istream IO

```

## 3.2 Main

```

#include <bits/stdc++.h>

using namespace std;

#define all(x) (x).begin(), (x).end()
#define rall(x) (x).rbegin(), (x).rend()
#define sz(x) ((int)(x).size())
#define sortall(x) sort(all(x))
#define Unique(x) (x).erase(unique(all(x)), (x).end())
#define compress(x) sortall(x); Unique(x)

typedef bool i1;
typedef char i8;
typedef short i16;
typedef int i32;
typedef long long i64;

```



```
typedef unsigned char u8;
typedef unsigned short u16;
typedef unsigned int u32;
typedef unsigned long long u64;
```

```
typedef float f16;
typedef double f32;
typedef long double f64;
```

```
template<typename T> using Vec = vector<T>;
template<typename T> using Que = queue<T>;
template<typename T> using Dec = deque<T>;
```

```
template<int fp=0> struct fastio { fastio() {
ios::sync_with_stdio(false); cin.tie(0); if(fp)cout<<fixed<<'
'<<setprecision(fp); } };
```

```
template<typename First, typename Second> inline istream&
operator>>(istream& in, pair<First, Second> &_data) {
in>>_data.first>>_data.second; return in; }
template<typename First, typename Second> inline ostream&
operator<<(ostream& out, pair<First, Second> &_data) {
out<<_data.first<<' '<<_data.second; return out; }
template<typename First, typename Second, typename Third>
inline istream& operator>>(istream& in, tuple<First, Second,
Third> &_data) {
in>>get<0>(_data)>>get<1>(_data)>>get<2>(_data); return in; }
template<typename First, typename Second, typename Third>
inline ostream& operator<<(ostream& out, tuple<First, Second,
Third> &_data) { out<<get<0>(_data)<<' '<<get<1>(_data)<<'
'<<get<2>(_data); return out; }
```

```
template<typename T> auto Vector(const int N, const T& value) {
return vector(N, value); }
template<typename...Ts> auto Vector(const int N, Ts... args) {
return vector(N, Vector(args...)); }
template<typename InputType> void in(InputType& x) { cin>>x; }
template<typename InputType, typename... InputTypes> void
in(InputType& x, InputTypes& ...y) { cin>>x; in(y...); }
template<typename IterableInputType> void vin(IterableInputType
&V, int skip=0) { for(auto &x: V) if(--skip < 0) cin >> x; }
```

```
template<const int p=0, typename OutputType> void
out(OutputType x) { cout<<x<<' '; }
template<const int p=0, typename OutputType, typename...
OutputTypes> void out(OutputType x, OutputTypes ...y) {
cout<<fixed<<setprecision(p)<<x<<' '; out<p>(y...); }
template<const int p=0, typename IterableOutputType> void
vout(const IterableOutputType &V, int skip=0) { for(auto &x: V)
if(--skip<0) out<p>(x); }
```

```
template<i64 modulo=numeric_limits<i64>::max(), typename... T>
i64 Sum(T... x) { return (... + x) % modulo; }
template<i64 modulo=numeric_limits<i64>::max(), typename... T>
i64 Mul(T... x) { return (... * x) % modulo; }
```

```
constexpr int dy[] = {-1,1,0,0,-1,-1,1,1,-2,-1,1,2,2,1,-1,-2};
constexpr int dx[] = {0,0,-1,1,-1,1,-1,1,1,2,2,1,-1,-2,-2,-1};
```

```
int main() {
fastio<>();

return 0;
}
```

### 3.3 Random

```
mt19937 rd = mt19937(0x9402);
uniform_int_distribution<int> ri(0, INT_MAX);
ri(rd);
```

## 4 Math

### 4.1 Euler Phi

```
template <typename T>
struct EulerPhi {
int N;
bool isBig;
vector<T> phi, primes;
EulerPhi(int _N):N(_N) {
if(N <= 5000000) {
isBig = false;
phi.resize(N + 1); iota(phi.begin(), phi.end(), 0);
phi[0] = 0;
for(int i = 2; i <= N; ++i) {
if(phi[i] != i) continue;
for(int j = i; j <= N; j += i) phi[j] = phi[j] / i * (i
- 1);
}
}
else {
isBig = true;
T sq = (T)sqrtl(N);
vector<int> chk(sq + 1);
for(T i = 2; i * i <= N; ++i) {
if(chk[i]) continue;
primes.push_back(i);
for(T j = i + i; j * j <= N; j += i) chk[j] = 1;
}
}
}
T getPhi(T N) {
if(N == 1) return 1;
if(!isBig) return phi[N];
T res = 1;
for(T p: primes) {
T x = 1;
while(N % p == 0) x *= p, N /= p;
res *= x - x / p;
}
if(N != 1) res *= N - 1;
return res;
}
};
```

### 4.2 Fft

```
using ll = long long;
using cpx = complex<double>;
```

```
void FFT(vector<cpx> &a, bool inv = false) {
int N = (int)a.size();
vector<cpx> root(N / 2);
for(int i = 1, j = 0; i < N; ++i) {
int bit = N >> 1;
while(j >= bit) j -= bit, bit >>= 1;
j += bit;
if(i < j) swap(a[i], a[j]);
}
double ang = 2 * acos(-1) / N * (inv ? -1 : 1);
for(int i = 0; i < N / 2; ++i) root[i] = cpx(cos(ang * i),
sin(ang * i));
/*
XOR convolution: set roots[:] = 1.
OR convolution: set roots[:] = 1 and do following
if(!inv) a[j + k] = u + v, a[j + k + i / 2] = u;
else a[j + k] = v, a[j + k + i / 2] = u - v;
*/
for(int i = 2; i <= N; i <= 1) {
int step = N / i;
for(int j = 0; j < N; j += i) {
for(int k = 0; k < i / 2; ++k) {
cpx u = a[j | k], v = a[j | k | i >> 1] * root[step *
k];
a[j | k] = u + v; a[j | k | i >> 1] = u - v;
}
}
}
if(inv) for(int i = 0; i < N; ++i) a[i] /= N;
}
vector<ll> multiply(const vector<ll> &a, const vector<ll> &b) {
{
vector<cpx> a(va.begin(), va.end()), b(vb.begin(), vb.end());
int N = 2;
while(N < a.size() + b.size()) N <= 1;
a.resize(N); b.resize(N);
FFT(a); FFT(b);
for(int i = 0; i < N; ++i) a[i] *= b[i];
FFT(a, true);
vector<ll> res(N);
for(int i = 0; i < N; ++i) res[i] = llround(a[i].real());
ret
}
```

### 4.3 Ntt

```
using ll = long long;
template <typename T>
T power(T a, T b, T mod) {
if(b == 0) return 1;
if(~b & 1) return power(a * a % mod, b >> 1, mod);
return a * power(a, b - 1, mod) % mod;
}
// (MOD) 104,857,601 = 25 * 2^22 + 1, w = 3
// (MOD) 998,244,353 = 119 * 2^23 + 1, w = 3
// (MOD) 2,281,701,377 = 17 * 2^27 + 1, w = 3
// (MOD) 2,483,027,969 = 37 * 2^26 + 1, w = 3
// (MOD) 2,113,929,217 = 63 * 2^25 + 1, w = 5
// (MOD) 1,092,616,193 = 521 * 2^21 + 1, w = 3
template<ll W, ll MOD> void NTT(vector<ll> &V, bool inv=false)
{
```

```

int N = (int)V.size();
vector<ll> root(N >> 1);
for(int i = 1, j = 0; i < N; ++i) {
    int bit = N >> 1;
    while(j >= bit) j -= bit, bit >>= 1;
    j += bit;
    if(i < j) swap(V[i], V[j]);
}
ll ang = power<ll>(W, (MOD - 1) / N, MOD);
if(inv) ang = power<ll>(ang, MOD - 2, MOD);
root[0] = 1;
for(int i = 1; i * 2 < N; ++i) root[i] = root[i - 1] * ang % MOD;
for(int i = 2; i <= N; i <= 1) {
    int step = N / i;
    for(int j = 0; j < N; j += i) {
        for(int k = 0; k * 2 < i; ++k) {
            ll u = V[j | k], v = V[j | k | i >> 1] * root[step * k] % MOD;
            V[j | k] = (u + v) % MOD;
            V[j | k | i >> 1] = ((u - v) % MOD + MOD) % MOD;
        }
    }
}
if(inv) {
    ll t = power<ll>(N, MOD - 2, MOD);
    for(int i = 0; i < N; ++i) V[i] = V[i] * t % MOD;
}
}
template<ll W, ll MOD> vector<ll> multiply(const vector<ll>
&va, const vector<ll> &vb) {
    vector<ll> a(va.begin(), va.end()), b(vb.begin(), vb.end());
    int N = 2;
    while(N < a.size() + b.size()) N <= 1;
    a.resize(N); b.resize(N);
    NTT<W, MOD>(a); NTT<W, MOD>(b);
    for(int i = 0; i < N; ++i) a[i] *= b[i];
    NTT<W, MOD>(a, true);
    return a;
}

```

## 5 String

### 5.1 Aho Corasick

```

struct AhoCorasick {
    struct Trie {
        Trie *nxt[26];
        Trie *fail;
        bool output;

        Trie() {
            for(int i=0;i<26;+i) nxt[i]=nullptr;
            fail=nullptr;
            output=false;
        }
        ~Trie() {
            for(int i=0;i<26;+i) if(nxt[i]) delete nxt[i];
        }
    } *root;
    AhoCorasick() { root = new Trie(); }
}

```

```

void insert(const string &S) {
    Trie *cur = root;
    int N = (int)S.size();
    for(int i = 0; i < N; ++i) {
        int nxt = S[i] - 'a';
        if(cur->nxt[nxt] == nullptr) cur->nxt[nxt] = new Trie();
        cur = cur->nxt[nxt];
    }
    cur->output=true;
}

void build() {
    queue<Trie*> Q;
    root->fail = root;
    Q.push(root);
    while(!Q.empty()) {
        Trie* cur = Q.front(); Q.pop();

        for(int i = 0; i < 26; ++i) {
            Trie *next = cur->nxt[i];
            if(next == nullptr) continue;
            if(cur == root) next->fail = root;
            else {
                Trie *dst = cur->fail;
                while(dst != root && dst->nxt[i] == nullptr) dst = dst->fail;
                if(dst->nxt[i]) dst = dst->nxt[i];
                next->fail = dst;
            }
            if(next->fail->output) next->output = true;
            Q.push(next);
        }
    }
}

bool find(const string &S) {
    Trie *cur = root;
    int N = (int)S.size();
    for(int i = 0; i < N; ++i) {
        int nxt = S[i] - 'a';
        while(cur != root && cur->nxt[nxt] == nullptr) cur = cur->fail;
        if(cur->nxt[nxt]) cur = cur->nxt[nxt];
        if(cur->output) return true;
    }
    return false;
}
}
};

```

### 5.2 Kmp

```

template <typename T>
struct KMP {
    vector<int> fail;
    vector<int> failure(const T& Q) {
        fail.resize((int)Q.size() + 1);
        for(int i = 1, j = 0; i < (int)Q.size(); ++i) {
            while(j > 0 && Q[i] != Q[j]) j = fail[j - 1];
            if(Q[i] == Q[j]) fail[i] = ++j;
        }
        return fail;
    }
}
vector<int> kmp(const T& P, const T& Q) {
}

```

```

if(fail.size() == 0) failure(Q);
vector<int> res;
for(int i = 0, j = 0; i < (int)P.size(); ++i) {
    while(j > 0 && P[i] != Q[j]) j = fail[j - 1];
    if(P[i] == Q[j]) {
        if(j + 1 == (int)Q.size()) res.push_back(i - (int)Q.size() + 1), j = fail[j];
        else ++j;
    }
}
return res;
}
};

```

### 5.3 Manacher

```

struct Manacher {
    vector<int> P;
    Manacher(string S) {
        string T = "$";
        for(char ch: S) T += ch, T += '$';
        int N = (int)T.size();
        P.resize(N);
        for(int i = 0, r = 0, c = 0; i < N; ++i) {
            if(2 * c >= i) P[i] = max(0, min(P[2 * c - i], r - i));
            while(0 <= i - P[i] - 1 && i + P[i] + 1 < N && T[i - P[i] - 1] == T[i + P[i] + 1]) ++ P[i];
            if(r < i + P[i]) r = i + P[i], c = i;
        }
    }
    int& operator[](int idx) { return P[idx]; }
};

```

### 5.4 Z

```

template <typename T>
vector<int> Z(const T &V) {
    int N = (int)V.size();
    vector<int> ret(N); ret[0] = N;
    for(int i = 1, l = 0, r = 0; i < N; ++i) {
        if(i < r) ret[i] = min(r - i - 1, ret[i - l]);
        while(i + ret[i] < N && V[i + ret[i]] == V[ret[i]]) ++ ret[i];
        if(i + ret[i] > r) r = i + ret[i], l = i;
    }
    return ret;
}

```

## 6 Geometry

### 6.1 Ccw

```

T x, y;
Point() : Point(0, 0) {}
Point(T _x, T _y) : x(_x), y(_y) {}
Point operator+(Point p) { return Point(x + p.x, y + p.y); }
Point operator-(Point p) { return Point(x - p.x, y - p.y); }
T operator*(Point p) { return x * p.y - y * p.x; }
bool operator==(Point p) { return x == p.x && y == p.y; }
bool operator<(Point p) { return x == p.x ? y < p.y : x < p.x; }
}

template <typename OT>

```



```

void operator=(Point<OT> p) {
    *this = Point(p.x, p.y); }
    void t() { swap(x, y); }
};

template<typename T> inline istream& operator>>(istream &in,
Point<T> &o) { in >> o.x >> o.y; return in; }
template<typename T> inline ostream& operator<<(ostream &out,
Point<T> &o) { out << o.x << ' ' << o.y; return out; }
// -1: 반시계, 0: 평행, 1: 시계
template<typename T> int ccw(Point<T> a, Point<T> b, Point<T>
c) {
    T x = a * b + b * c + c * a;
    return (x > 0) - (x < 0);
}

template<typename T> T dist(Point<T> a, Point<T> b) {
    return (a.x - b.x) * (a.x - b.x) + (a.y - b.y) * (a.y - b.y);
}

template<typename T> struct Line {
    Point<T> p1, p2;
    Line():Line(0, 0) {}
    Line(T a, T b):Line(Point<T>(0, 0), Point<T>(a, b)) {}
    Line(Point<T> a, Point<T> b):p1(a),p2(b) {
        if(p1.x > p2.x) swap(p1, p2);
        else if(p1.x == p2.x && p1.y > p2.y) swap(p1, p2);
    }
    T dx() { return p1.x - p2.x; }
    T dy() { return p1.y - p2.y; }
    T ccw() { return p1 * p2; }
    void t() { p1.t(); p2.t(); }
};

// 0: 교점 0개, 1: 교점 1개 (끝점 0), 2: 교점 1개 (끝점 X), 3:
// 교점 ∞개
// 4: 평행 교점 1개, 5: 평행 교점 ∞개
template<typename T> int intersect(Line<T> l1, Line<T> l2) {
    int ca = ccw(l1.p1, l1.p2, l2.p1), cb = ccw(l1.p1, l1.p2,
l2.p2);
    int cc = ccw(l2.p1, l2.p2, l1.p1), cd = ccw(l2.p1, l2.p2,
l1.p2);
    if(ca == 0 && cb == 0 && cc == 0 && cd == 0) {
        if(l1.p1.x == l1.p2.x && l2.p1.x == l2.p2.x && l1.p2.x ==
l2.p1.x) l1.t(), l2.t();
        int A = l1.p1.x, B = l1.p2.x, C = l2.p1.x, D = l2.p2.x;
        if(A > D || B < C) return 0;
        if(A == D || B == C) return 4;
        return 5;
    }
    if(ca * cb <= 0 && cc * cd <= 0) return (!ca || !cb || !cc ||
!cd) ? 1 : 2;
    return 0;
}

template<typename T, typename AT> pair<int, Point<AT>>
intersection_point(Line<T> l1, Line<T> l2) {
    int chk = intersect(l1, l2);
    if(chk == 0 || chk == 3) return make_pair(chk, Point<AT>());
    if(chk == 1 || chk == 4) {
        Point<AT> ans;
        if(l1.p1 == l2.p1 || l1.p1 == l2.p2) ans = l1.p1;
        else if(l1.p2 == l2.p1 || l1.p2 == l2.p2) ans = l1.p2;
        else if(ccw(l1.p1, l1.p2, l2.p1) == 0) ans = l2.p1;

```

```

        else if(ccw(l1.p1, l1.p2, l2.p2) == 0) ans = l2.p2;
        else if(ccw(l2.p1, l2.p2, l1.p1) == 0) ans = l1.p1;
        else if(ccw(l2.p1, l2.p2, l1.p2) == 0) ans = l1.p2;
        return make_pair(1, ans);
    }
    T a = l1.ccw() * l2.dx() - l1.dx() * l2.ccw();
    T b = l1.ccw() * l2.dy() - l1.dy() * l2.ccw();
    T d = l1.dx() * l2.dy() - l1.dy() * l2.dx();
    return make_pair(chk, Point<AT>(1. * a / d, 1. * b / d));
}

```

## 6.2 Convex Hull

```

// 더 추가해야함.
template<typename T>
vector<Point<T>> ConvexHull(vector<Point<T>> V) {
    swap(V[0], *min_element(V.begin(), V.end()));
    sort(V.begin() + 1, V.end(), [&](Point<T> a, Point<T> b) {
        int w = ccw(V[0], a, b);
        return w ? w > 0 : dist(V[0], a) < dist(V[0], b);
    });
    int idx = (int)V.size() - 1;
    while(idx > 1 && ccw(V[0], V[idx], V[idx - 1]) == 0) --idx;
    reverse(V.begin() + idx, V.end());
    vector<int> st;
    for(int i = 0; i < (int)V.size(); ++i) {
        // line ok < or <=
        while(st.size() > 1 && ccw(V[st[st.size() - 2]],
V[st.back()], V[i]) < 0) st.pop_back();
        st.push_back(i);
    }
    vector<Point<T>> res;
    for(int x: st) res.push_back(V[x]);
    return res;
}

template<typename T> pair<Point<T>, Point<T>>
get_far_two_point(vector<Point<T>> V) {
    int N = (int)V.size();
    T d = 0;
    pair<Point<T>, Point<T>> res;
    auto upd = [&](Point<T> a, Point<T> b) {
        T cur = dist(a, b);
        if(d < cur) d = cur, res = make_pair(a, b);
    };
    for(int i = 0, r = 0; i < N; ++i) {
        while(r + 1 < N && ccw(Point<T>(), V[(i + 1) % N] - V[i],
V[(r + 1) % N] - V[r]) >= 0) upd(V[i], V[r++]);
        upd(V[i], V[r]);
    }
    return res;
}

```