Lecture 3 – Spatial Filtering（空间滤波）

This lecture will cover:

- Spatial domain（空间域）
- Intensity Transformation （灰度变换）
- **Histogram（直方图）**
- Spatial Filtering（空间滤波器）
  - ✓ Smoothing（平滑）
  - ✓ Sharpening（锐化）

Definition

$$h(r_k) = n_k$$

Where r_k : the *k*th intensity value in the level range of [0, L-1]

n_k : the number of pixels in the image with intensity r_k

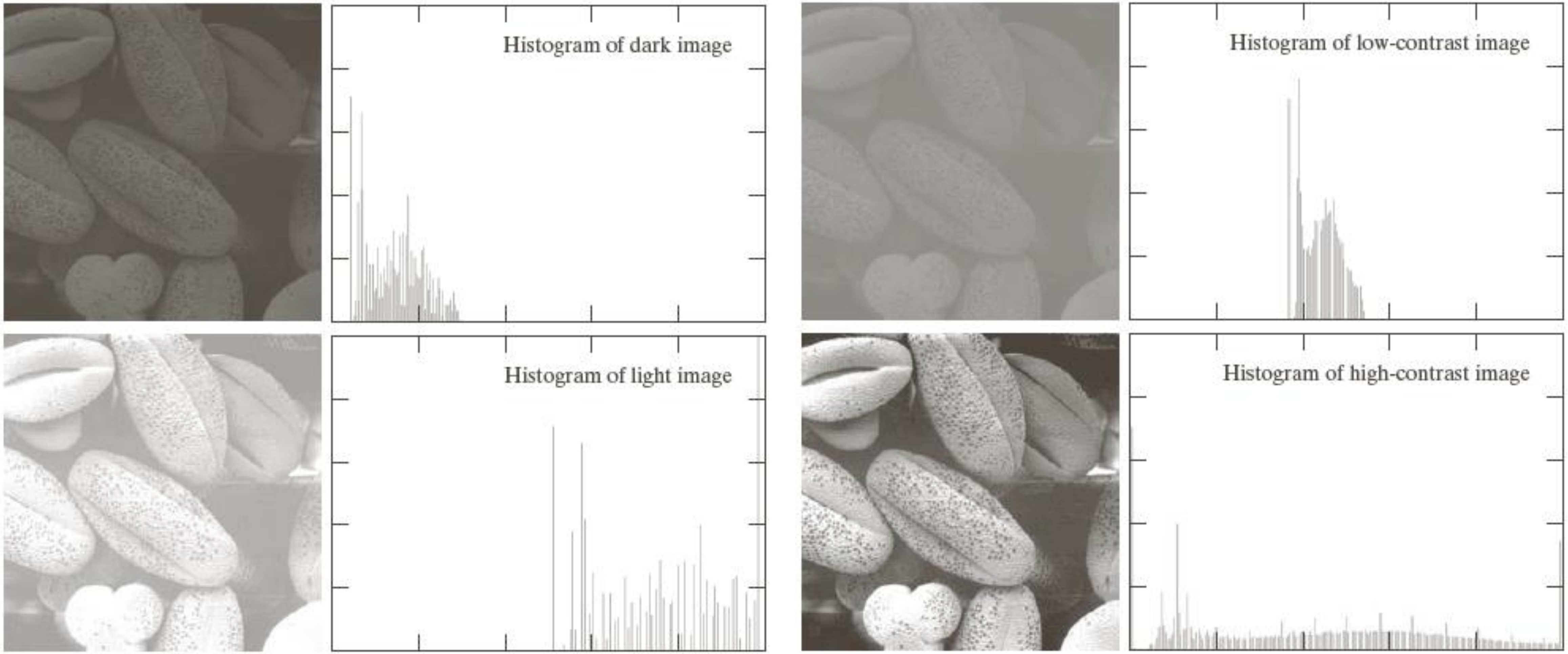
Normalized Histogram（归一化直方图）

$$p(r_k) = \frac{n_k}{MN}$$

Where $p(r_k)$: the probability of occurrence of intensity r_k in an image

M, N: the row and column dimensions of the image

Basic Image Type



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Properties

The histogram of an image

- describe the number or probability of intensity, NO location (spatial) information
- can be same as other images
- $\sum_0^{L-1} n_k = M \cdot N$ or $\sum_0^1 p(r_k) = 1$
- If Region C=A∪B, A and B are disjoint, $H_C = H_A + H_B$

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Basis of Histogram Processing

Given intensity transformation $s = T(r)$, where $T(r)$

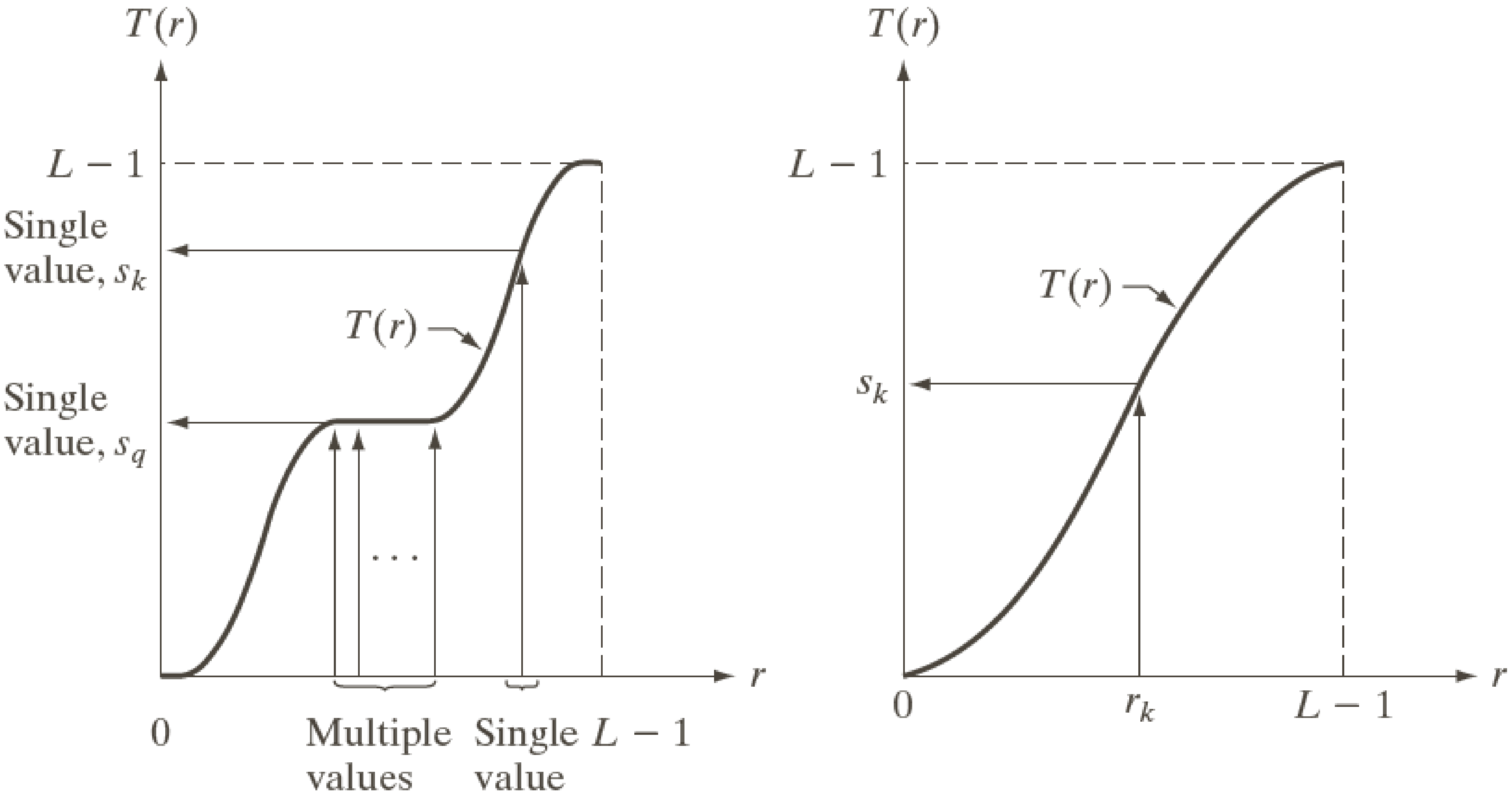
- $T(r)$ is strictly monotonically increasing function （严格单调递增函数, $T(r_2) > T(r_1)$ if $r_2 > r_1$） in the interval $0 \le r \le L-1$
- $0 \le T(r) \le L-1$ for $0 \le r \le L-1$

The inverse transform $r = T^{-1}(s)$

The probability density function (PDF) of $\boldsymbol{s}$ is

$$p_s(s) = p_r(r) \cdot \frac{dr}{ds}$$

Intensity Mapping

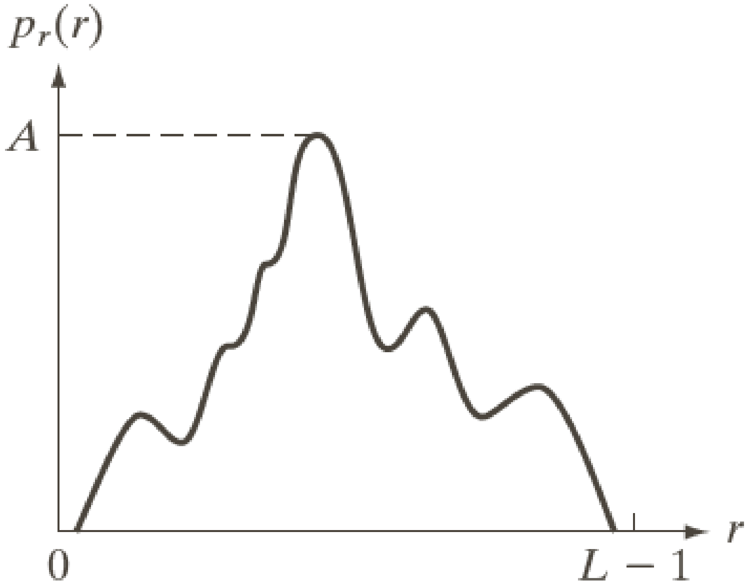


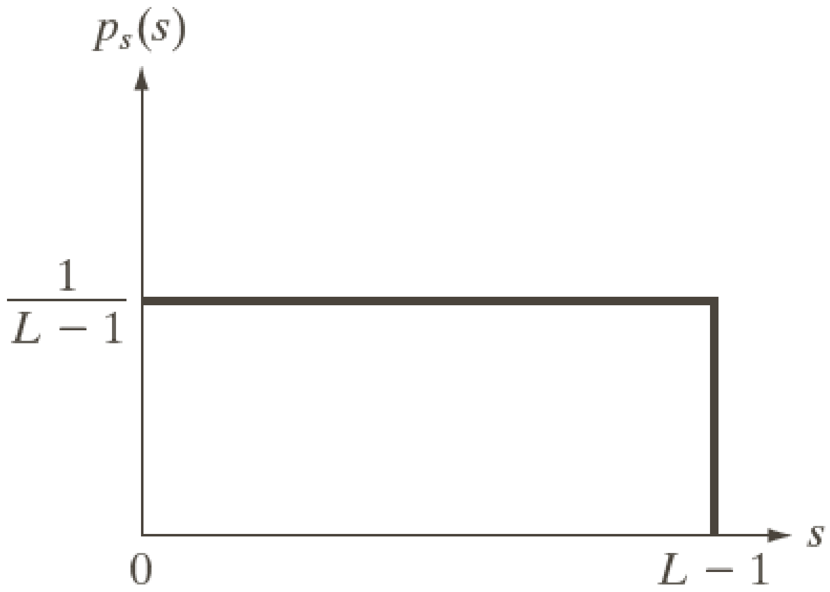
Histogram Processing

- Histogram Equalization （直方图均衡）
- Histogram Matching (Specification)（直方图匹配/规定化）
- Local Histogram Processing（局部处理）
- Histogram Statistics for Image Enhancement （直方图统计）

Histogram Equalization

- **Transformation function :** $s = T(r) = (L-1)\int_0^r p_r(w)dw$
- **Uniform Probability density function :** $p_s(s) = \frac{1}{L-1}$





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Histogram Equalization

$$s = T(r) = (L-1)\sum_{j=0}^{k} p_r(r_j) = (L-1)\sum_{j=0}^{k} \frac{n_j}{MN} = \frac{L-1}{MN}\sum_{j=0}^{k} n_j$$

K	r_k	n_k	$p(r_k)$	s_k		s_k	$p(s_k)$
0	0	790	0.19	1.33	1	0	0
1	1	1023	0.25	3.08	3	1	0.19
2	2	850	0.21	4.55	5	2	0
3	3	656	0.16	5.67	6	3	0.25
4	4	329	0.08	6.23	6	4	0
5	5	245	0.06	6.65	7	5	0.21
6	6	122	0.03	6.86	7	6	0.24
7	7	81	0.02	7.00	7	7	0.11

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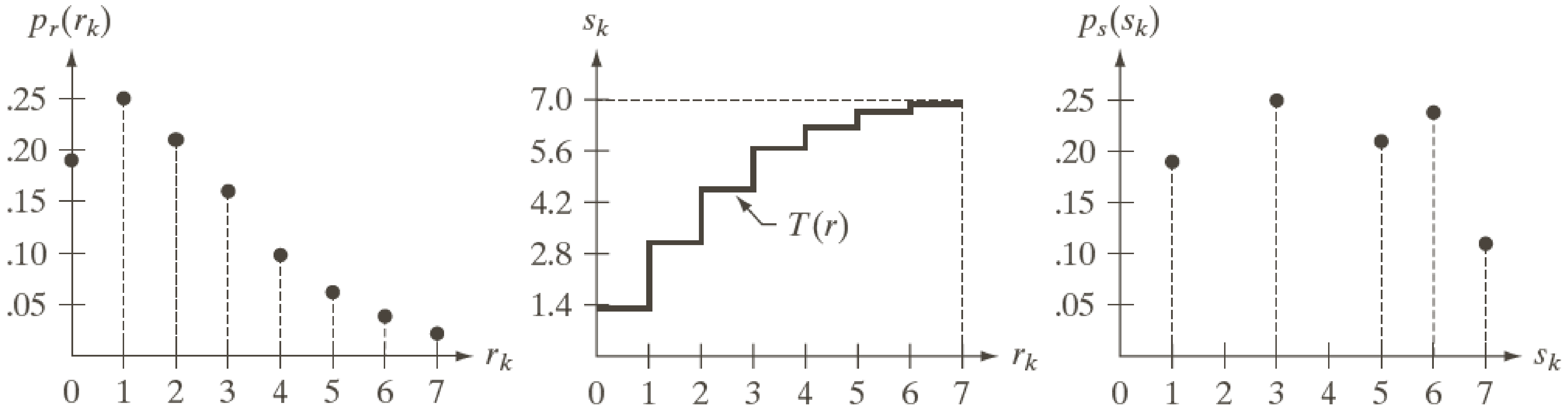
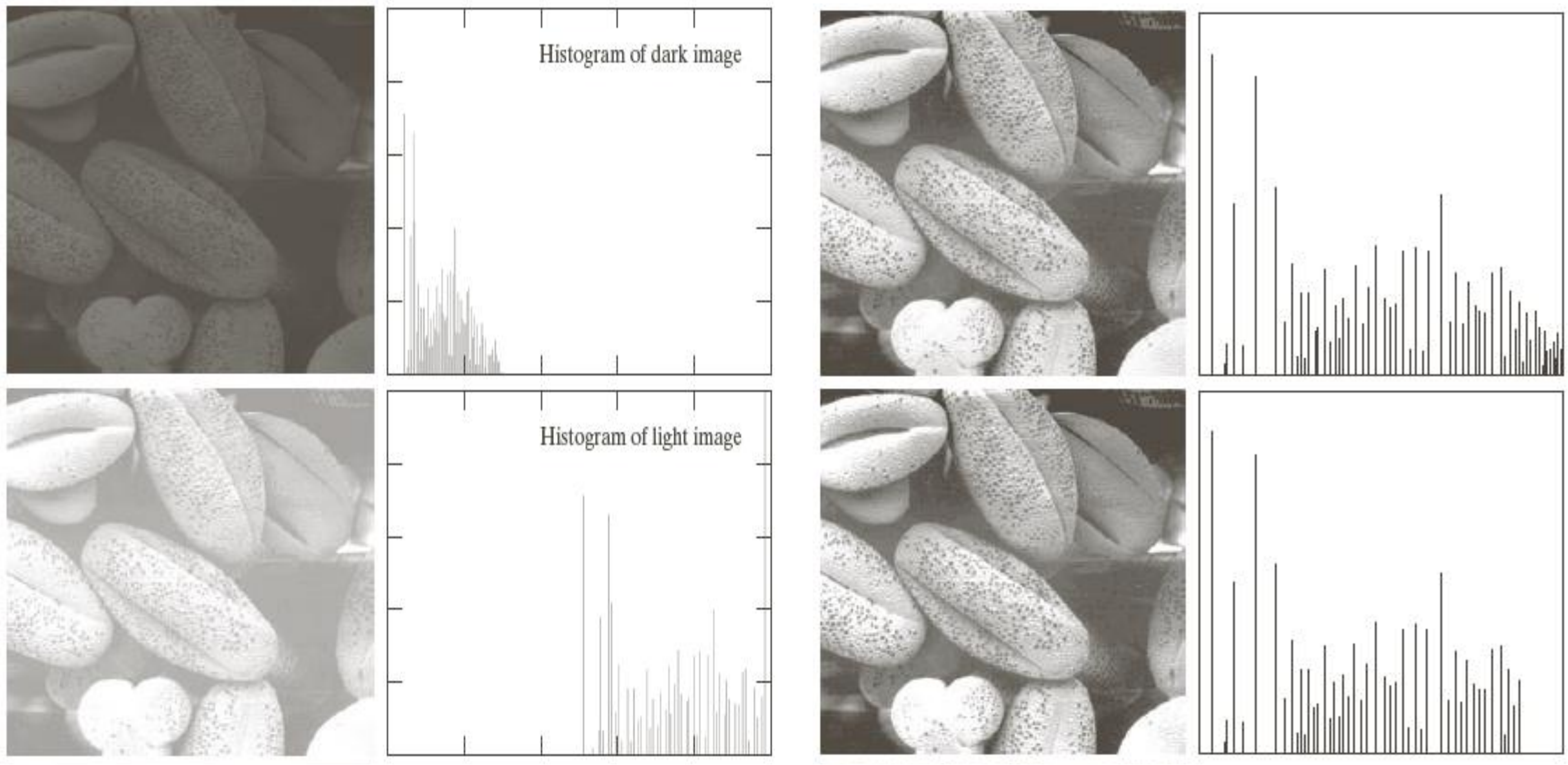
Example

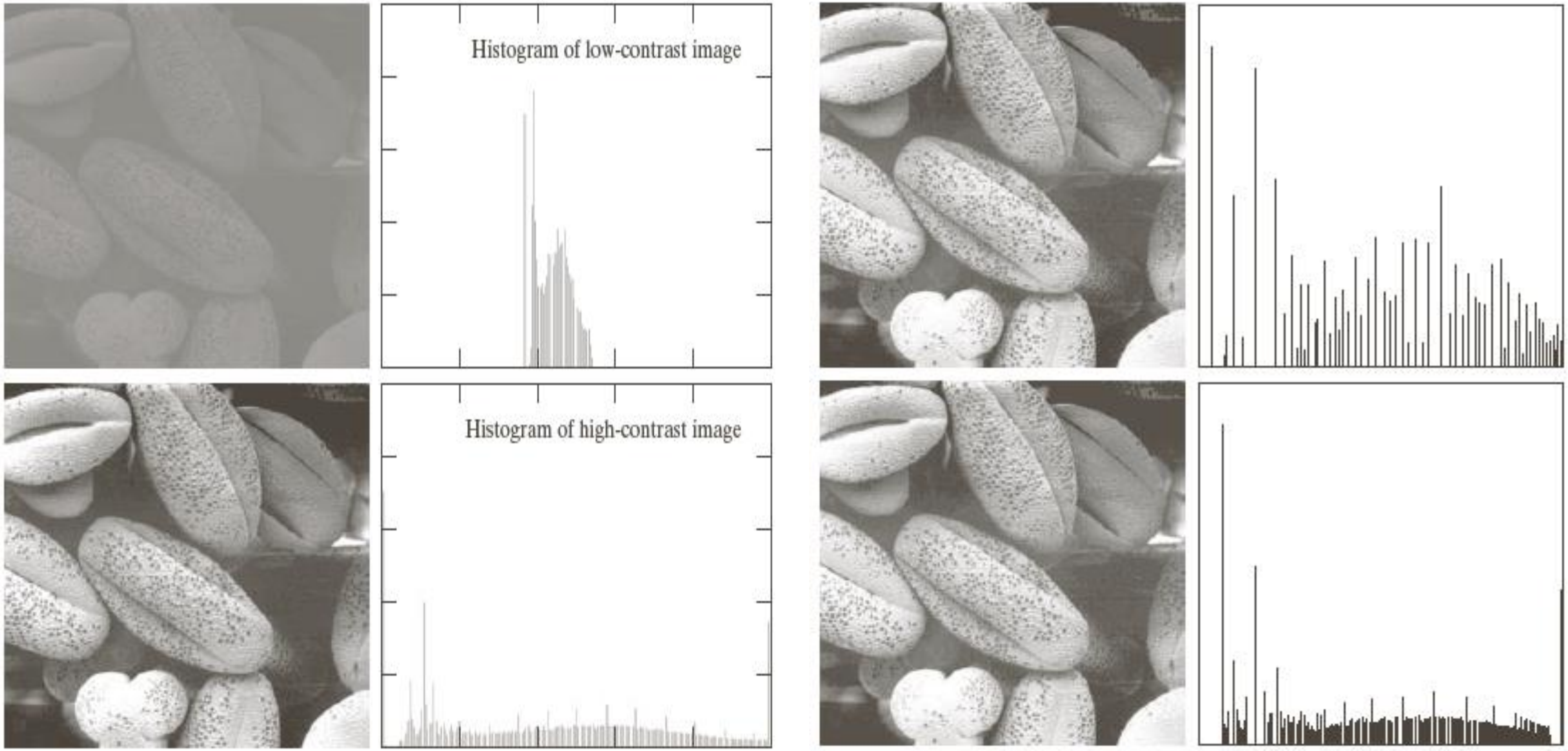


$p_r(r_k)$
.25
.20
.15
.10
.05
0 1 2 3 4 5 6 7
r_k
s_k
7.0
5.6
4.2
2.8
1.4
$T(r)$
0 1 2 3 4 5 6 7
r_k
$p_s(s_k)$
.25
.20
.15
.10
.05
0 1 2 3 4 5 6 7
s_k

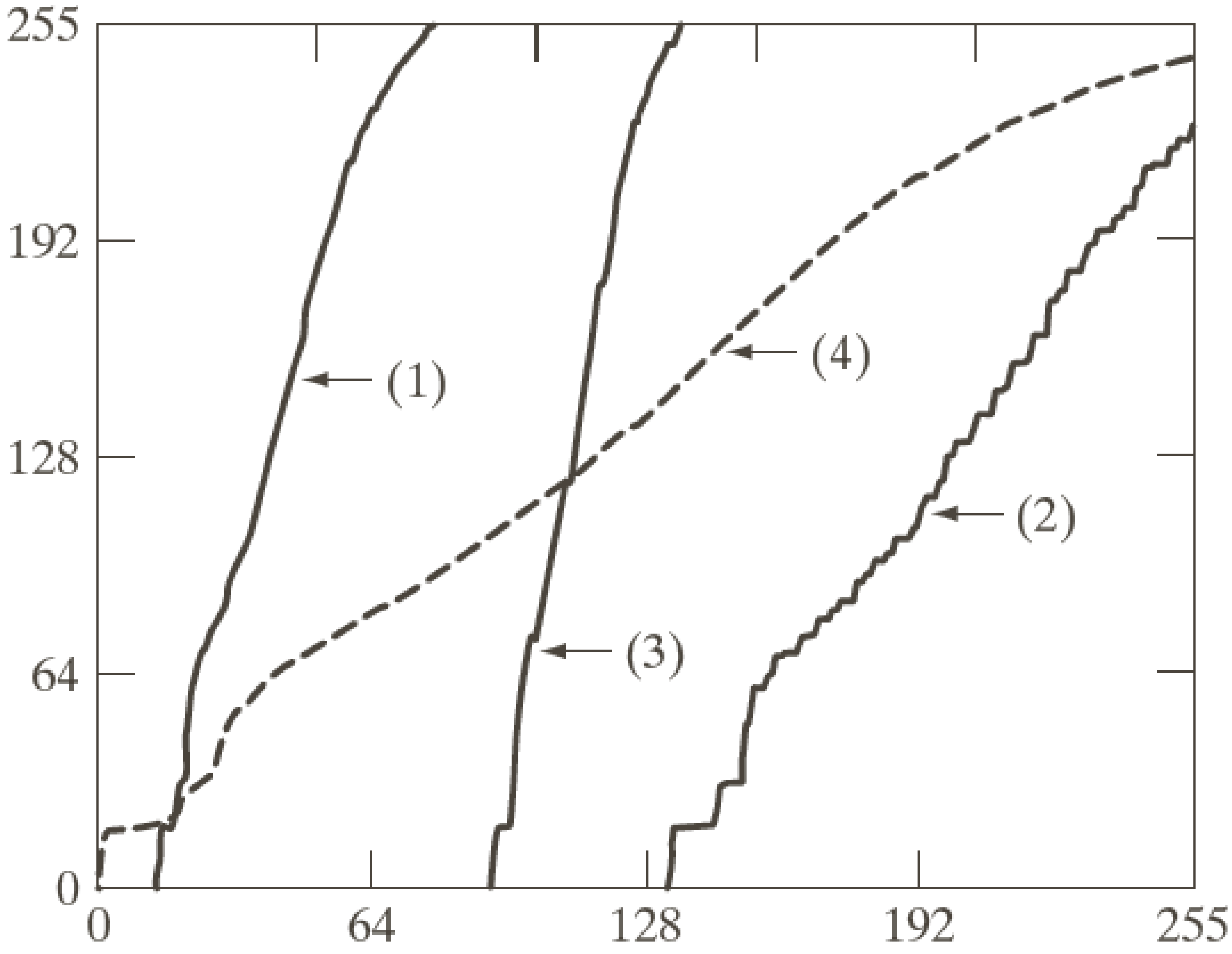
Example



Example


Histogram of low-contrast image
Histogram of high-contrast image

Transformation Function



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Histogram Matching

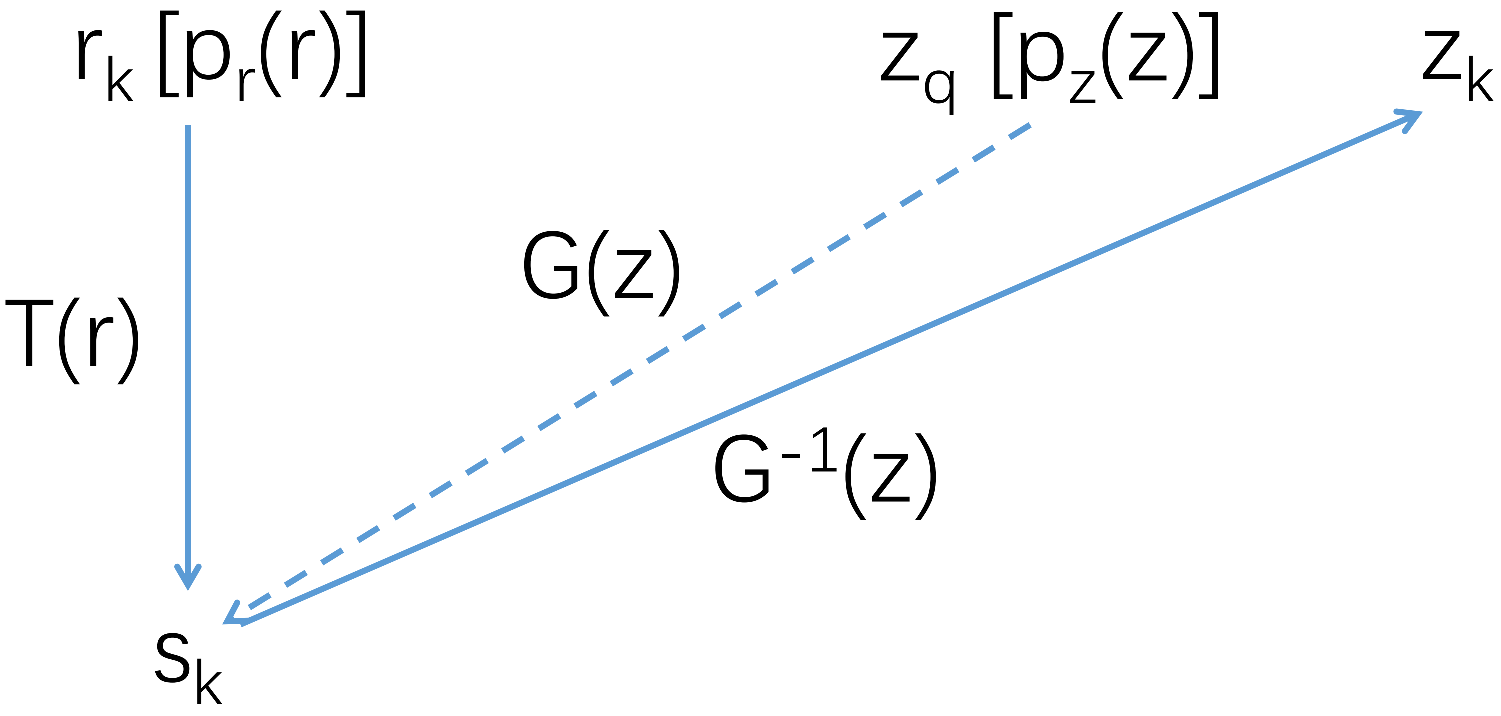
Generate a processed image with a specified histogram

For input : $s = T(r) = (L-1)\int_0^r p_r(w)dw$

For output : $G(z) = (L-1)\int_0^z p_z(t)dt = s$

Therefore $z = G^{-1}(s) = G^{-1}[T(r)]$

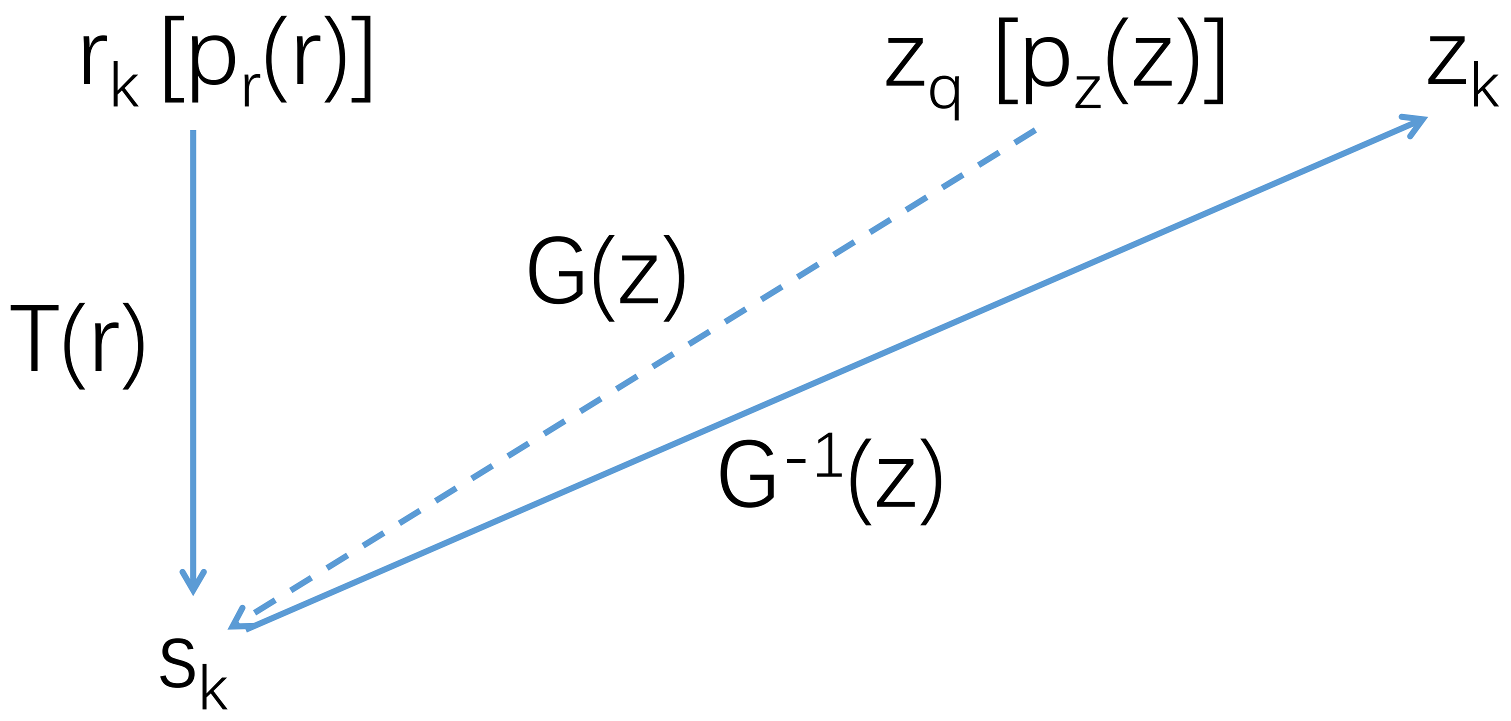
Histogram Matching



Typical Transformation Function

Probability Density Function of Output Image		Transformation Function
Uniform	$P_g(g) = \dfrac{1}{g_{\max} - g_{\min}}, g_{\max} \geq g \geq g_{\min}$	$g = [g_{\max} - g_{\min}]C(f) + g_{\min}$
Exponential	$P_g(g) = \alpha \exp[-\alpha(g - g_{\min})], g \geq g_{\min}$	$g = g_{\min} - \dfrac{1}{\alpha}\ln[1 - C(f)]$
Raleigh	$P_g(g) = \dfrac{g - g_{\min}}{\alpha^2}\exp[-\dfrac{(g - g_{\min})^2}{2\alpha^2}], g \geq g_{\min}$	$g = g_{\min} + \{2\alpha^2 \ln[\dfrac{1}{1 - C(f)}]\}^{1/2}$
Hyperbolic (Cubic)	$P_g(g) = \dfrac{1}{3}[\dfrac{g^{-2/3}}{g^{1/3}{}_{\max} - g^{1/3}{}_{\min}}], g_{\max} \geq g \geq g_{\min}$	$g = \{[g^{1/3}{}_{\max} - g^{1/3}{}_{\min}]C(f) + g^{1/3}_{\min}\}^3$
Hyperbolic (Logarithmic)	$P_g(g) = \dfrac{1}{g[\ln(g_{\max}) - \ln(g_{\min})]}, g_{\max} \leq g \leq g_{\min}$	$g = g_{\min}\left[\dfrac{g_{\max}}{g_{\min}}\right]^{C(f)}$

Histogram Matching



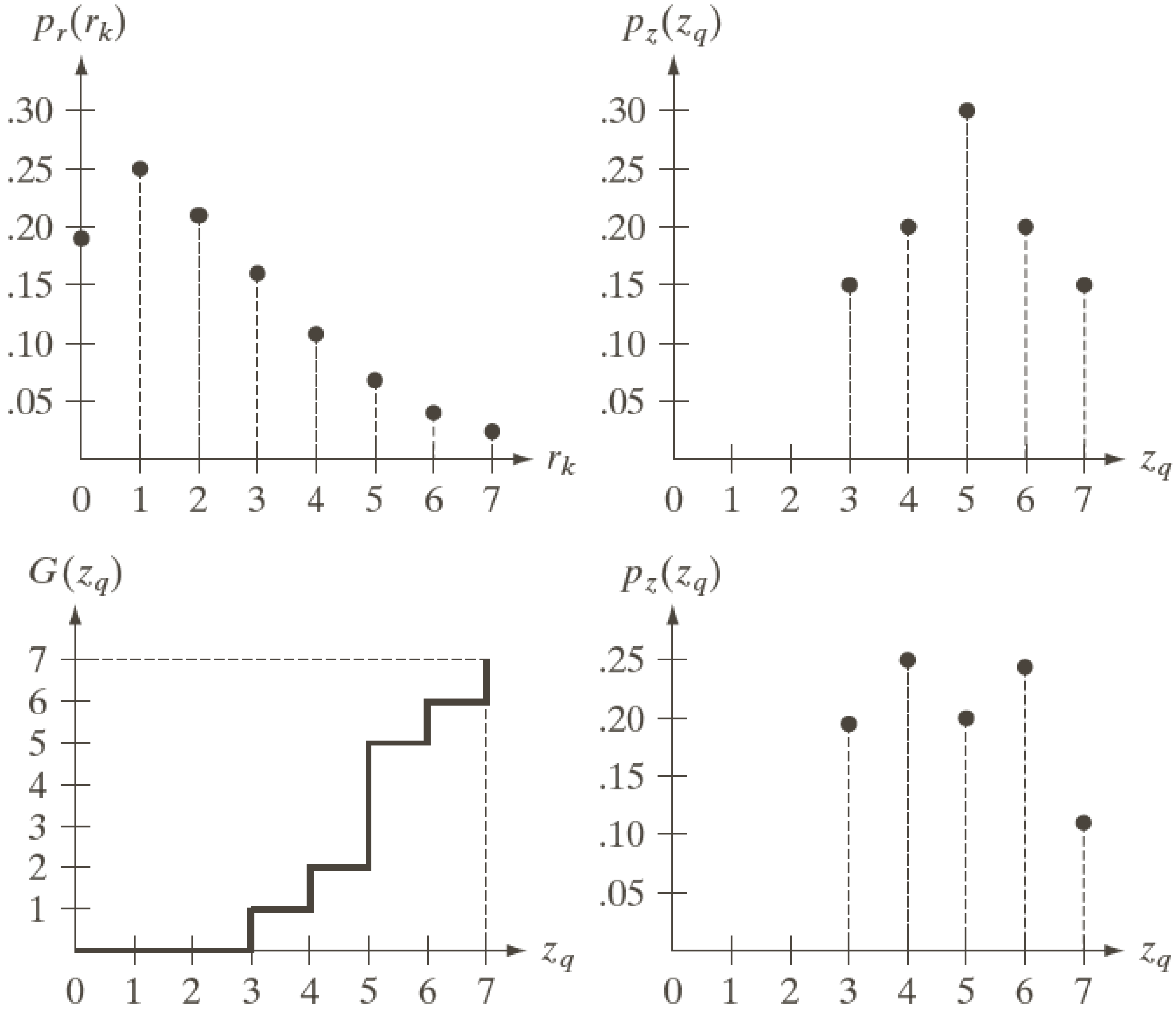
Histogram Matching

r_k	$p(r_k)$	s_k	z_q	$p(z_q)$	$G(z_q)$	$s_k \rightarrow z_k$	$r_k \rightarrow z_k$	z_k	$p(z_k)$
0	0.19	1	0	0	0	3	3	0	0
1	0.25	3	1	0	0	4	4	1	0
2	0.21	5	2	0	0	5	5	2	0
3	0.16	6	3	0.15	1	6	6	3	0.19
4	0.08	6	4	0.20	2	6	6	4	0.25
5	0.06	7	5	0.30	5	7	7	5	0.21
6	0.03	7	6	0.20	6	7	7	6	0.24
7	0.02	7	7	0.15	7	7	7	7	0.11

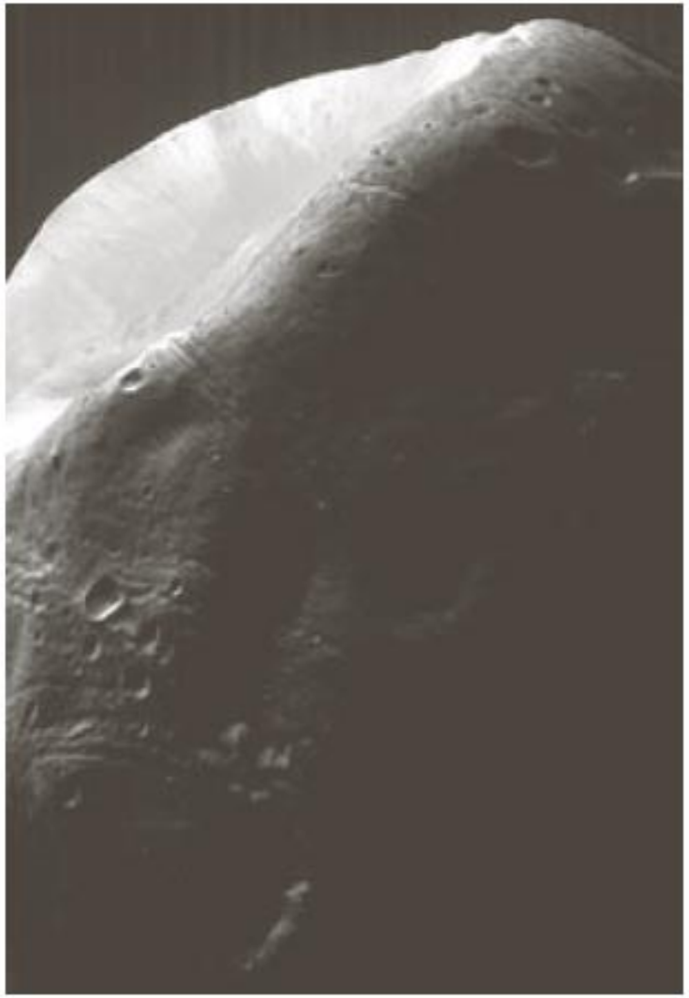
Histogram Matching

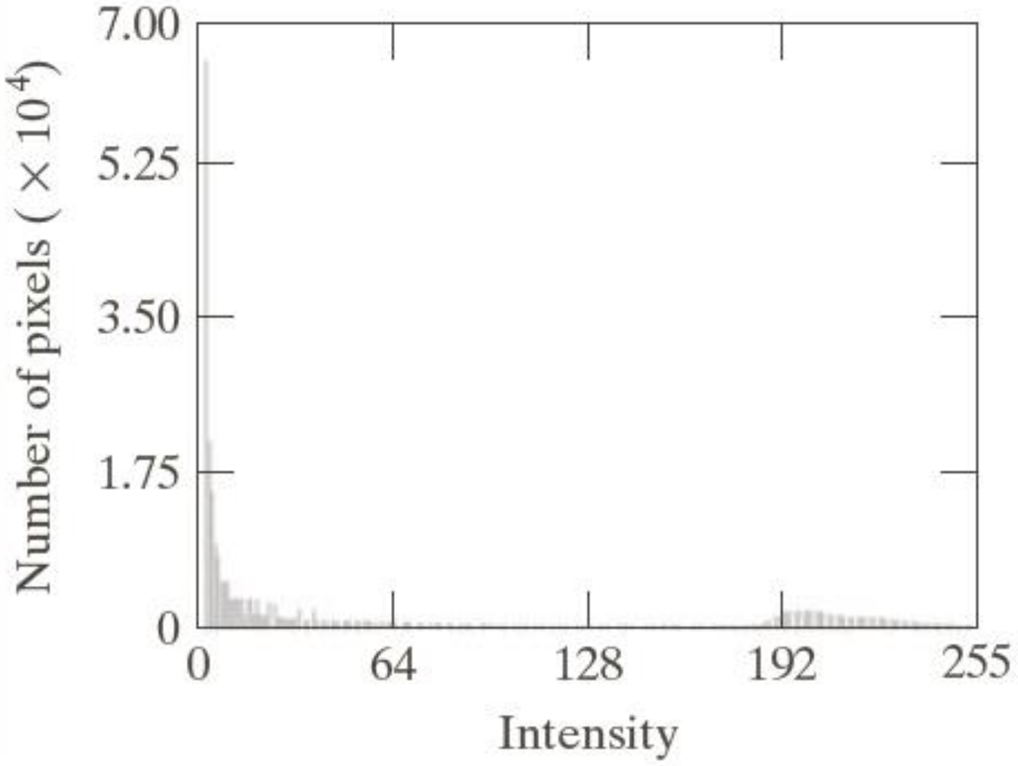
r_k	$p(r_k)$	s_k	z_q	$p(z_q)$	$G(z_q)$	$s_k \rightarrow z_k$	$r_k \rightarrow z_k$	z_k	$p(z_k)$
0	0.19	1	0	0	0	3	3	0	0
1	0.25	3	1	0	0	4	4	1	0
2	0.21	5	2	0	0	5	5	2	0
3	0.16	6	3	0.15	1	6	6	3	0.19
4	0.08	6	4	0.20	2	6	6	4	0.25
5	0.06	7	5	0.30	5	7	7	5	0.21
6	0.03	7	6	0.20	6	7	7	6	0.24
7	0.02	7	7	0.15	7	7	7	7	0.11

Histogram Matching

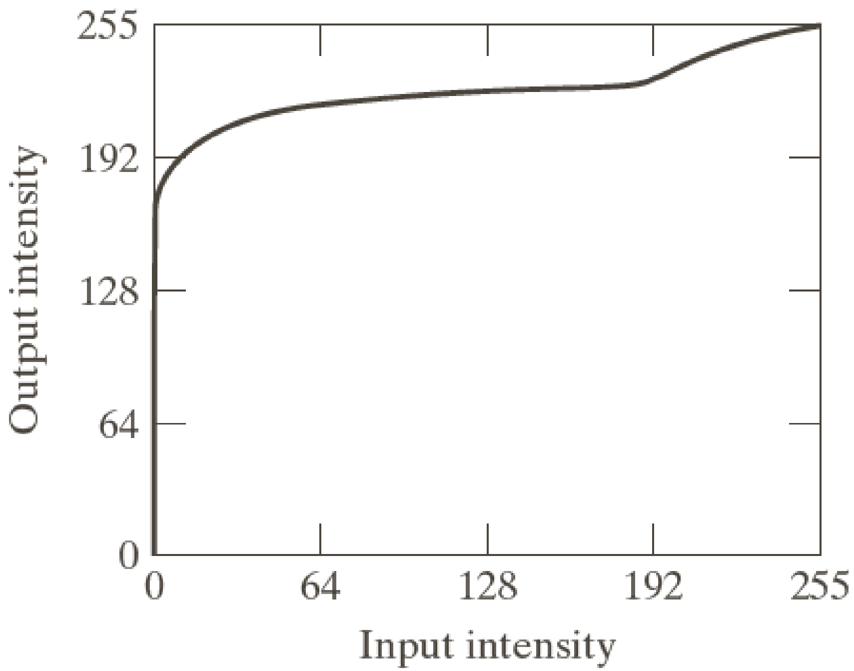


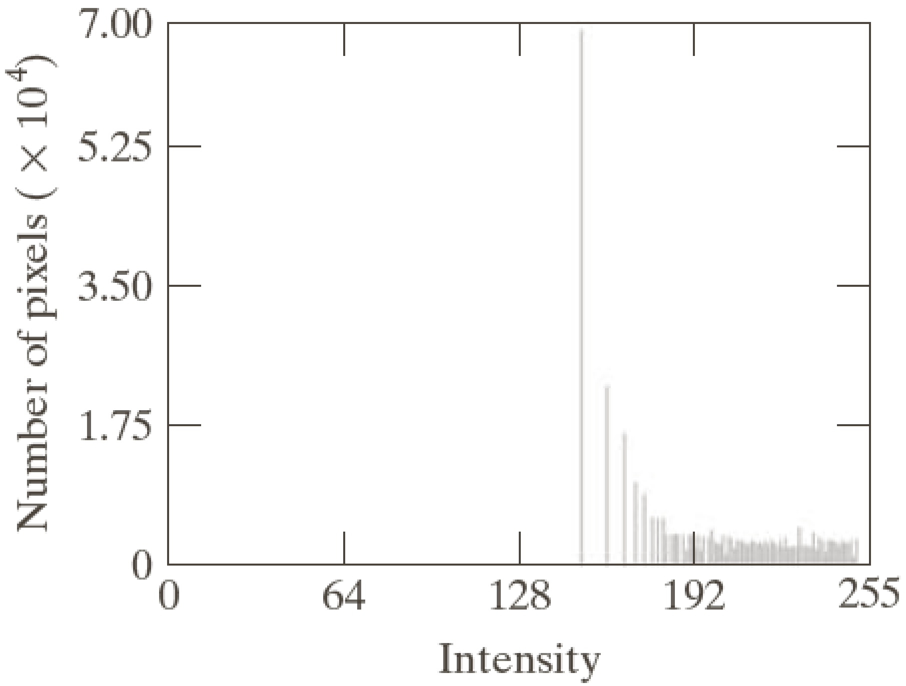
Equalization *vs* Matching



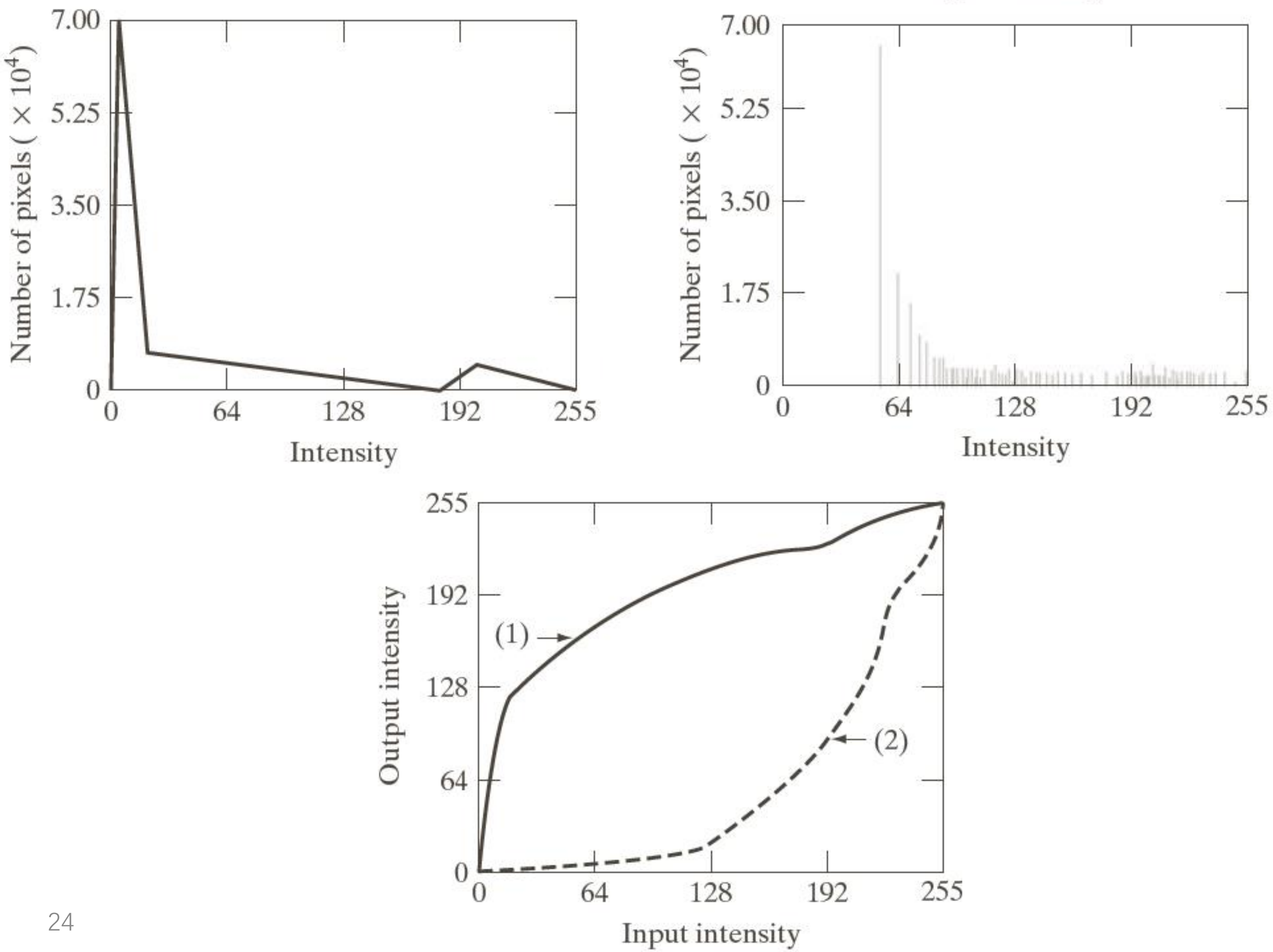


Equalization *vs* Matching

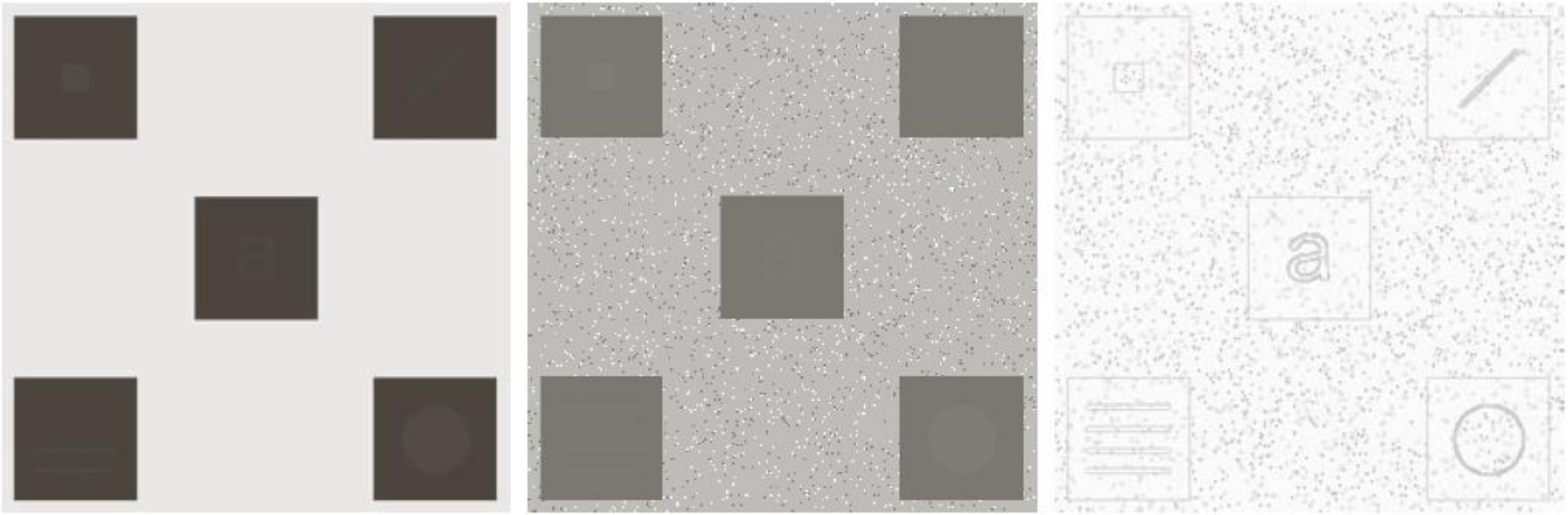




Equalization *vs* Matching



Local Histogram Processing



Histogram Statistics

- **Average Intensity （平均灰度）**

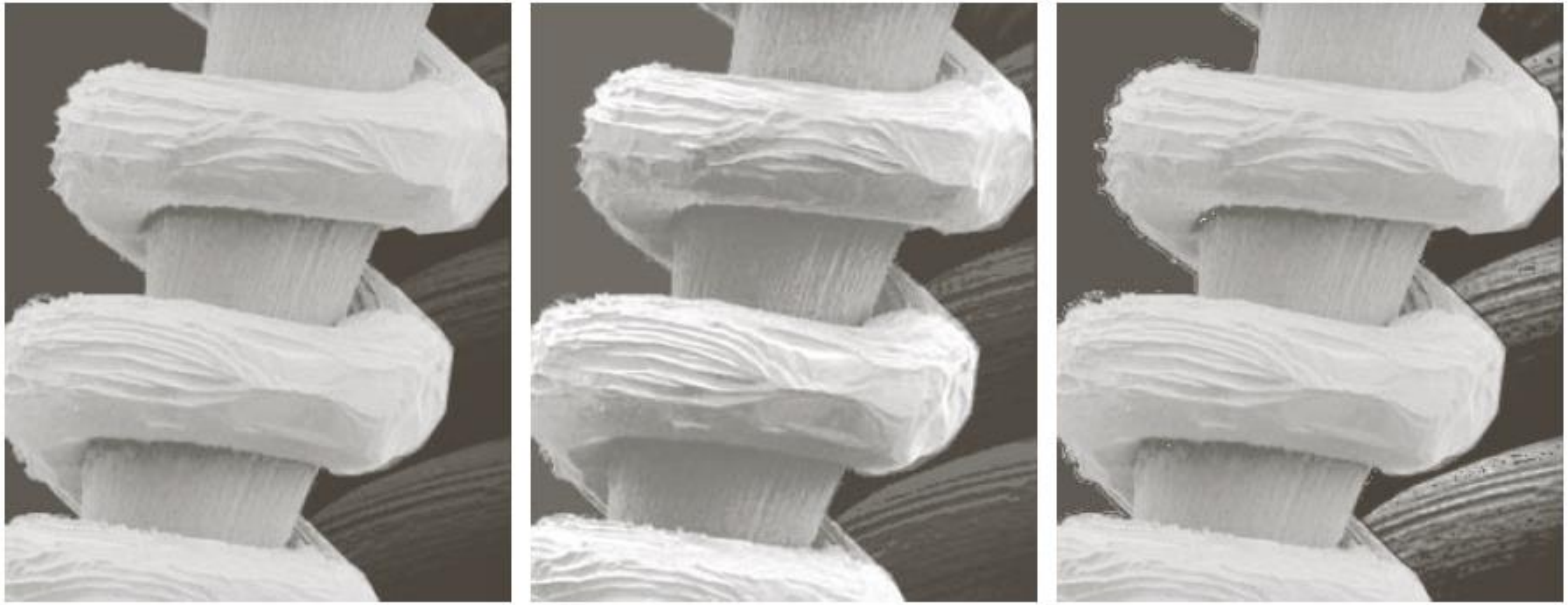
$$m = \sum_{i=0}^{L-1} r_i p(r_i) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y)$$

- **Intensity Variance （灰度方差）**

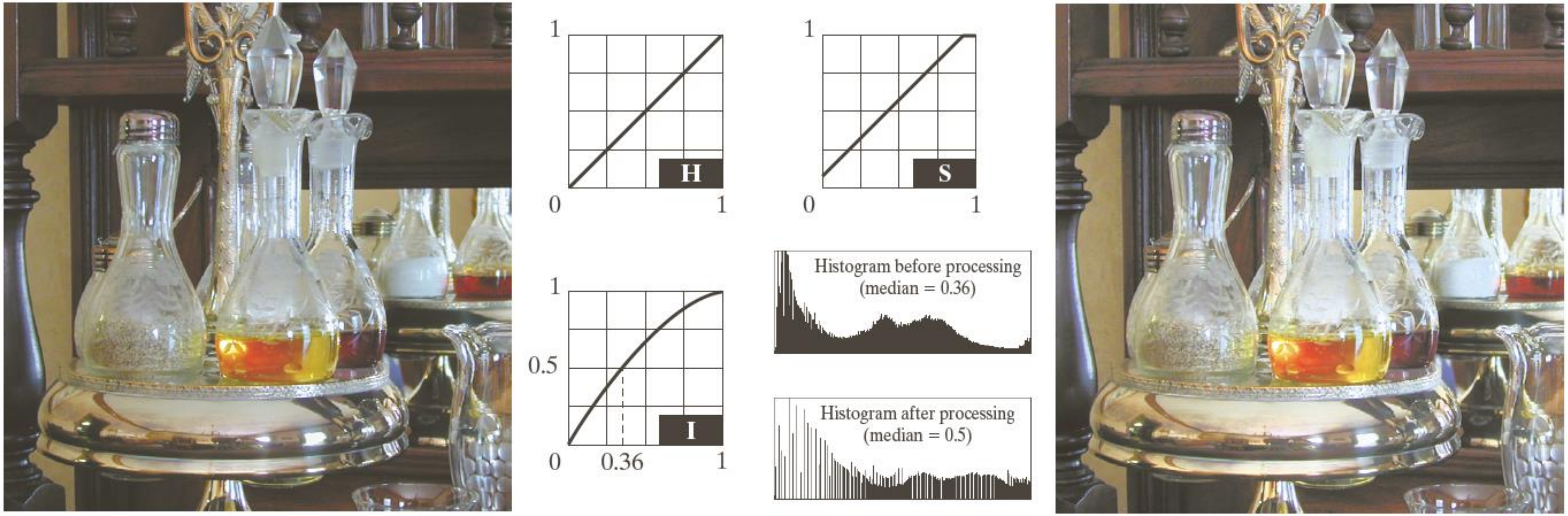
$$\sigma^2 = \sum_{i=0}^{L-1} (r_i - m)^2 p(r_i) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [f(x, y) - m]^2$$

Histogram Statistics

$$g(x,y) = \begin{cases} E \cdot f(x,y), & \text{if } m_{S_{xy}} \le k_0 m_G \text{ AND } k_1\sigma_G \le \sigma_{S_{xy}} \le k_2\sigma_G \\ f(x,y), & \text{otherwise} \end{cases}$$



Color Histogram



Color Histogram