

# Lecture 3 – Spatial Filtering (空间滤波)

This lecture will cover:

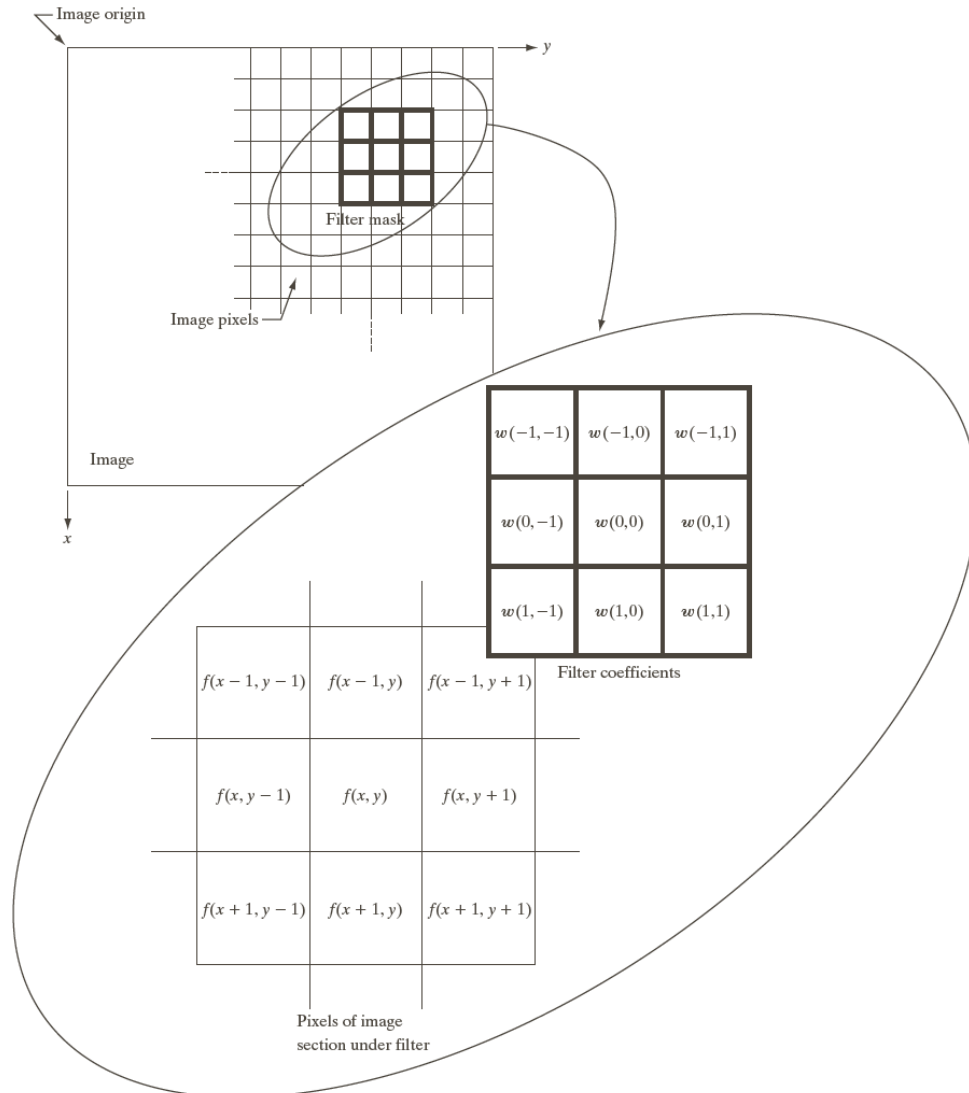
- Spatial domain (空间域)
- Intensity Transformation (灰度变换)
- Histogram (直方图)
- **Spatial Filtering (空间滤波器)**
  - ✓ **Smoothing (平滑)**
  - ✓ **Sharpening (锐化)**

# Spatial Filtering

## A Spatial filter

- is directly applied on the image
- is also called spatial masks (掩模)、kernels (核)、templates (模板)、windows (窗口)
- consists of
  - 1) neighborhood
  - 2) a predefined operation
- can be linear and nonlinear
  - Linear spatial filter corresponds to spectral filter in frequency domain
  - Nonlinear spatial filter cannot be accomplished in frequency domain

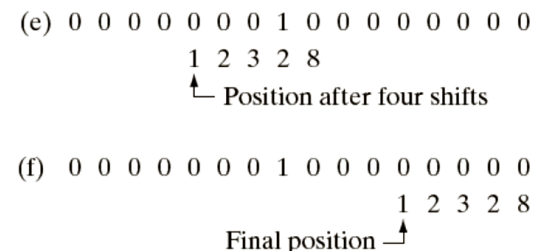
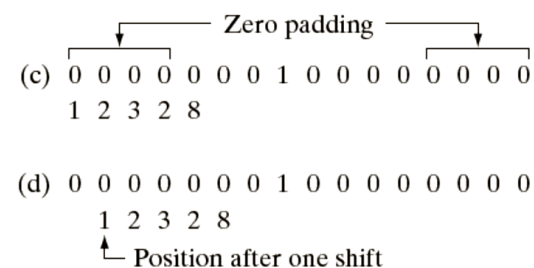
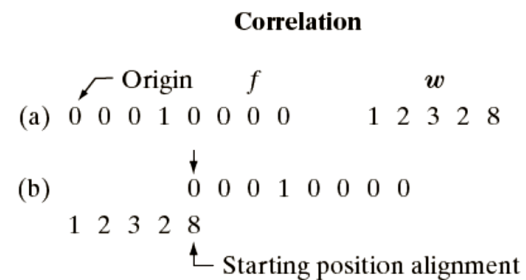
# Spatial Filter



$$g(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x + s, y + t)$$

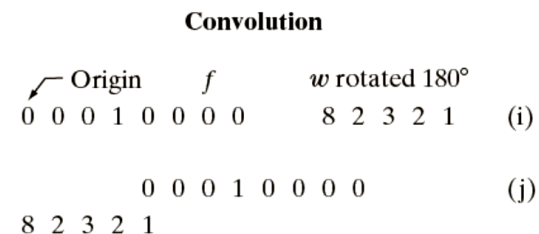
- $f(x, y)$ : input image
- $g(x, y)$ : output filtered image
- $w(s, t)$ :  $m \times n$  spatial filter, where  $m = 2a + 1, n = 2b + 1$

# Correlation(相关) and Convolution(卷积) (1D)



(g) **Full correlation result**  
 0 0 0 8 2 3 2 1 0 0 0 0

(h) **Cropped correlation result**  
 0 8 2 3 2 1 0 0



(k) 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0  
 8 2 3 2 1

(l) 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0  
 8 2 3 2 1

(m) 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0  
 8 2 3 2 1

(n) 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0  
 8 2 3 2 1

(o) **Full convolution result**  
 0 0 0 1 2 3 2 8 0 0 0 0

(p) **Cropped convolution result**  
 0 1 2 3 2 8 0 0

# Correlation and Convolution (2D)

Figure 1 illustrates the correlation and convolution process. The figure is divided into eight sub-figures (a) through (h).

- (a) **Origin  $f(x, y)$** : A 5x5 grid of values. The kernel  $w(x, y)$  is a 3x3 grid of values:  $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ .
- (b) **Padded  $f$** : The 5x5 grid from (a) with a padding of 1. The padded grid is 7x7.
- (c) **Initial position for  $w$** : The 3x3 kernel  $w$  is positioned at the top-left corner of the 7x7 padded grid.
- (d) **Full correlation result**: A 7x7 grid showing the result of the correlation operation. The value 9 is at the center (4,4).
- (e) **Cropped correlation result**: A 5x5 grid showing the cropped result of the correlation operation. The value 9 is at the center (3,3).
- (f) **Rotated  $w$** : The 3x3 kernel  $w$  is rotated 90 degrees clockwise. The rotated kernel is  $\begin{bmatrix} 3 & 2 & 1 \\ 6 & 5 & 4 \\ 9 & 8 & 7 \end{bmatrix}$ .
- (g) **Full convolution result**: A 7x7 grid showing the result of the convolution operation. The value 9 is at the center (4,4).
- (h) **Cropped convolution result**: A 5x5 grid showing the cropped result of the convolution operation. The value 9 is at the center (3,3).

# Equations

## Correlation

$$w(s, t) \star f(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x + s, y + t)$$

## Convolution

$$w(s, t) \star f(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x - s, y - t)$$

## Vector Operation

$$R = w_1 z_1 + w_2 z_2 + \cdots + w_{mn} z_{mn} = \sum_{k=1}^{mn} w_k z_k = w^T z$$

$w_1$	$w_2$	$w_3$
$w_4$	$w_5$	$w_6$
$w_7$	$w_8$	$w_9$

# Spatial Filter Masks

## ➤ Linear Spatial Filter (线性滤波器)

- $R = \frac{1}{9} \sum_{k=1}^9 Z_k$
- $h(x, y) = e^{-\frac{x^2+y^2}{2\sigma^2}}$

## ➤ Nonlinear Spatial Filter (非线性滤波器)

- Max filter (最大值滤波)
- Median filter (中值滤波)

# Smooth Filters (平滑滤波器)

- **Blurring – for preprocessing tasks**
- **Noise deduction**
  - **Linear filter : average filtering – lowpass filter in frequency domain**
  - **Nonlinear filter**





# Smooth Filters (平滑滤波器)

$$g(x, y) = \frac{\sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x + s, y + t)}{\sum_{s=-a}^a \sum_{t=-b}^b w(s, t)}$$

 $\frac{1}{9} \times$ 

1	1	1
1	1	1
1	1	1

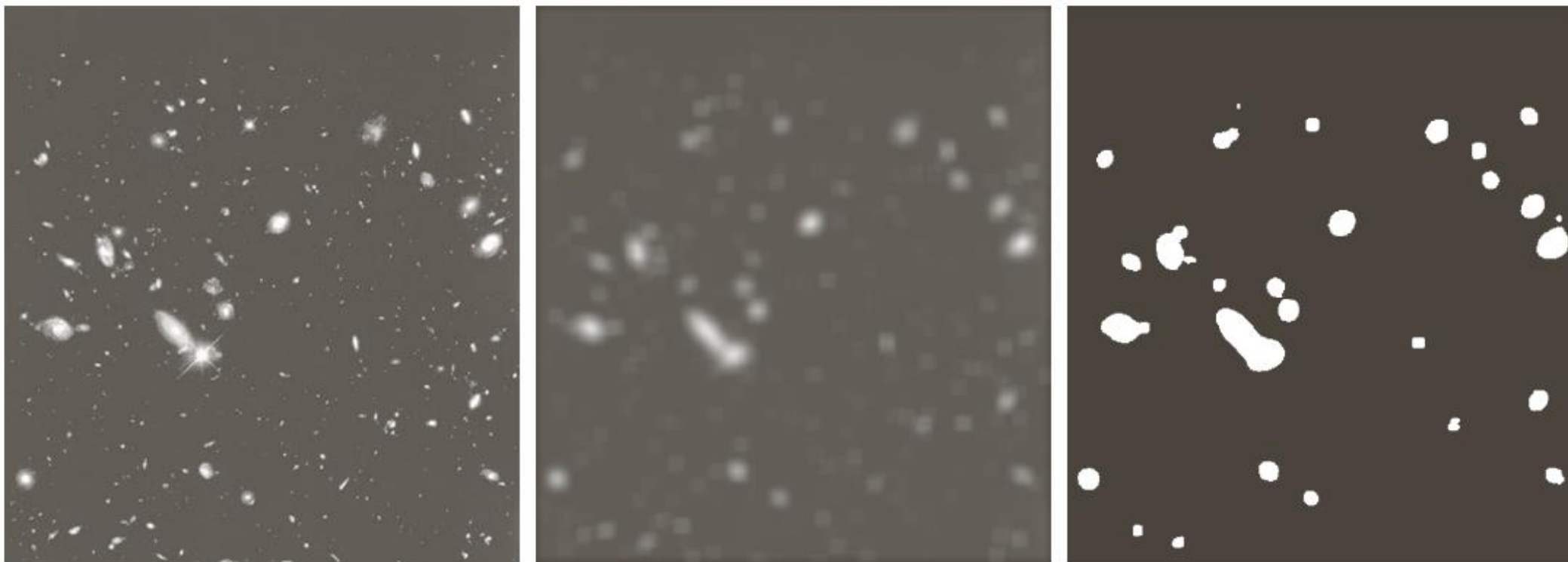
 $\frac{1}{16} \times$ 

1	2	1
2	4	2
1	2	1

# Filter size



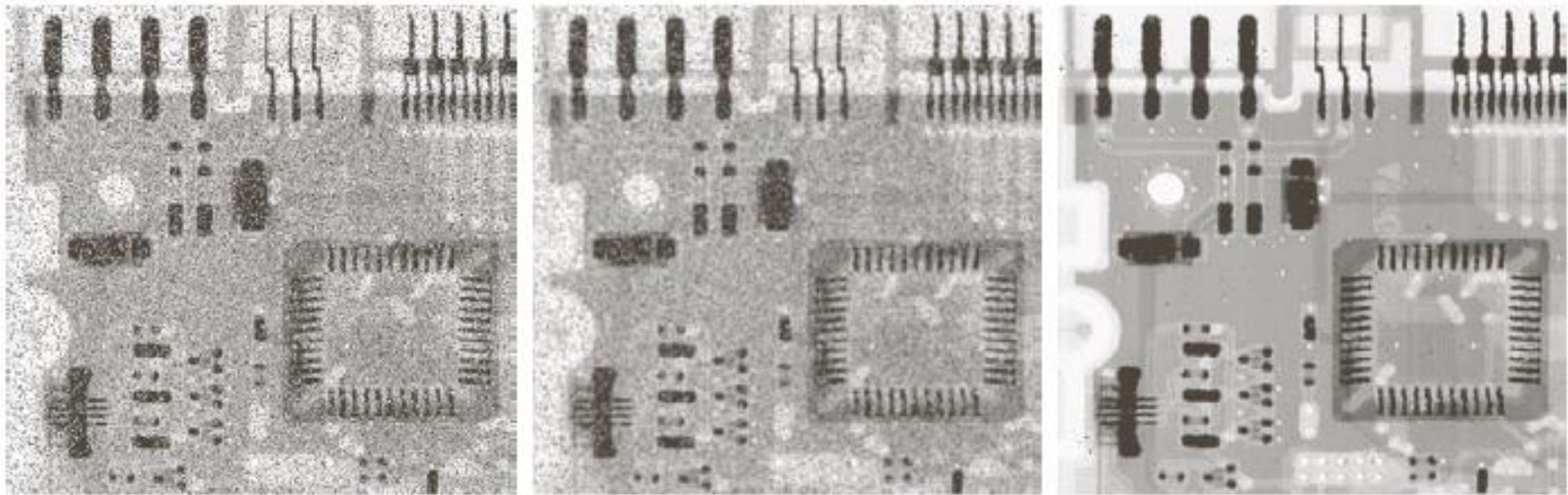
# Smooth Filter and Thresholding(阈值处理)



# Nonlinear Smooth Filters

➤ Order-statistic filter (统计排序滤波器)

$$R = H\{z_k | k = 1, 2, \dots, mn\}$$



# Sharpening Filter

- Spatial differentiation (空间微分)
- Sharpening filter
  - Laplacian filtering (拉普拉斯算子)
  - Unsharp Masking (非锐化掩蔽)
  - Gradient filtering (梯度算子)

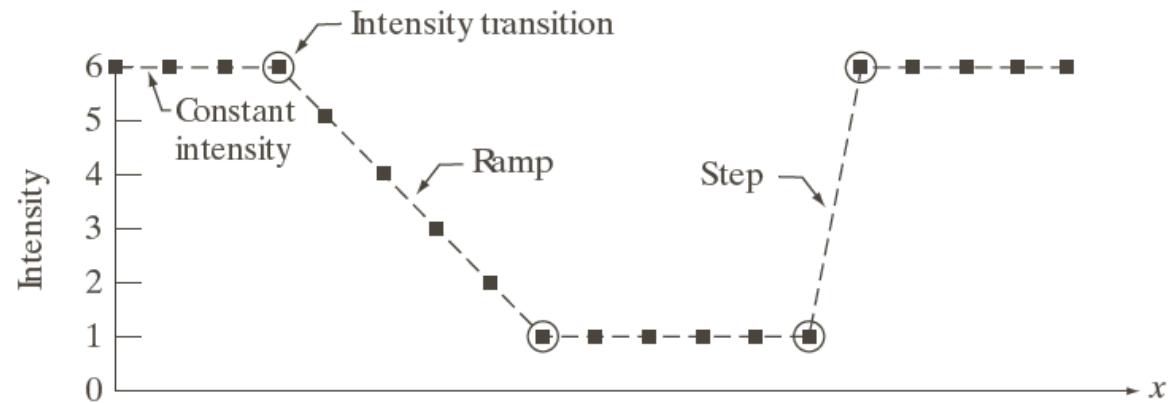
# Sharpening Filter

- To highlight transitions in intensity
- Accomplished by spatial differentiation
  - First-order derivative:  $\frac{\partial f}{\partial x} = f(x + 1) - f(x)$
  - Second-order derivative:  $\frac{\partial^2 f}{\partial x^2} = f(x + 1) + f(x - 1) - 2f(x)$

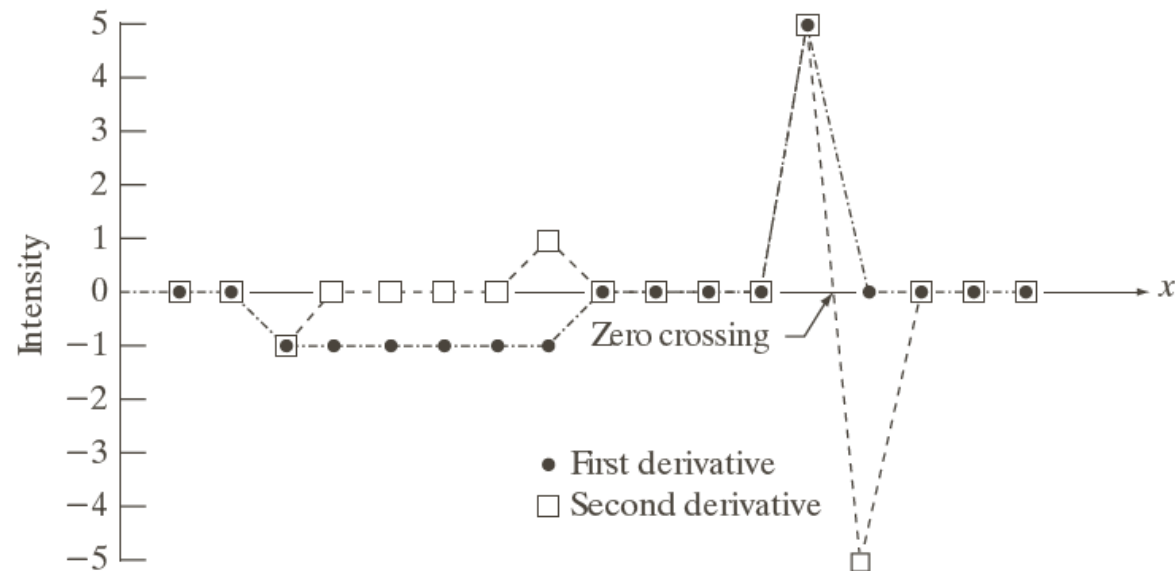
# Sharpening Filter

1. Zero in area of constant intensity
2. Nonzero at the onset of intensity step or ramp
3. (1) Nonzero along intensity ramp – 1<sup>st</sup> order derivative  
(2) Zero along intensity ramp with constant slope – 2<sup>nd</sup> order derivative

# Derivative



Scan line	6	6	6	6	5	4	3	2	1	1	1	1	1	1	6	6	6	6	6	$x$
1st derivative	0	0	-1	-1	-1	-1	-1	0	0	0	0	0	0	5	0	0	0	0	0	
2nd derivative	0	0	-1	0	0	0	0	1	0	0	0	0	0	5	-5	0	0	0	0	





# Laplacian(拉普拉斯算子)

For an image function  $f(x, y)$ ,

$$\text{X direction: } \frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\text{Y direction: } \frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

$$\begin{aligned}\nabla^2 f(x, y) &= \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \\ &= f(x, y+1) + f(x, y-1) + f(x+1, y) + f(x-1, y) - 4f(x, y)\end{aligned}$$

# Laplacian Filter Masks

0	1	0	1	1	1	0	-1	0	-1	-1	-1
1	-4	1	1	-8	1	-1	4	-1	-1	8	-1
0	1	0	1	1	1	0	-1	0	-1	-1	-1

# Laplacian Filter Masks

$$g(x, y) = f(x, y) + c\nabla^2 f(x, y), \quad \text{where } c = \pm 1$$

	1	

−

	1	
1	−4	1
	1	

=

	−1	
−1	5	−1
	−1	

	1	

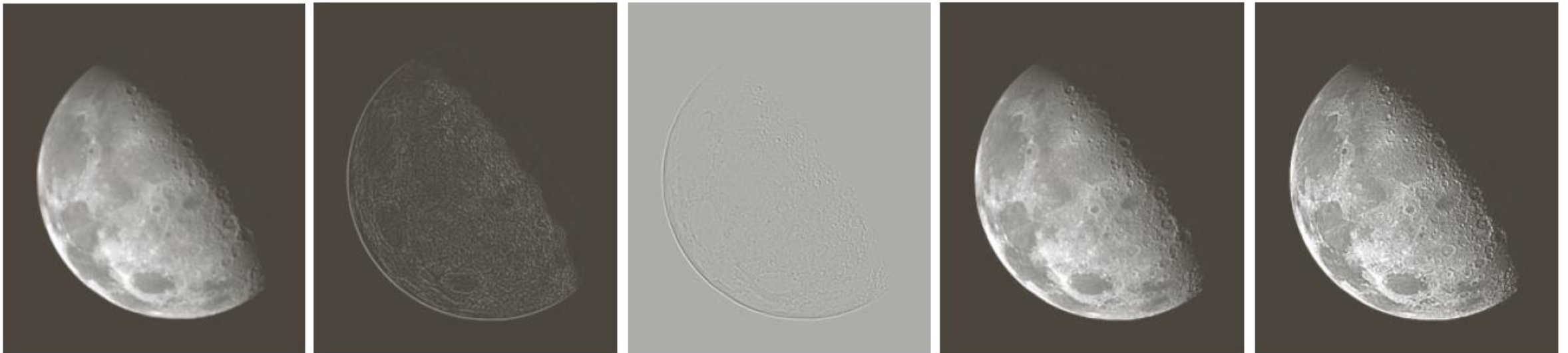
+

	−1	
−1	4	−1
	−1	

=

	−1	
−1	5	−1
	−1	

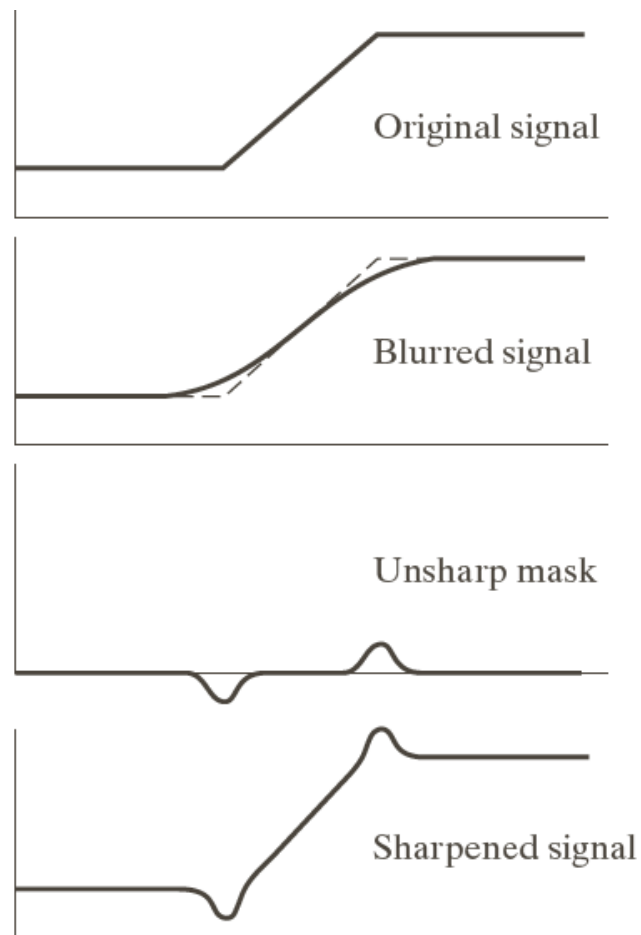
# Image Sharpening with Laplacian



# Unsharpen Mask(非锐化掩蔽)

$$g_{\text{mask}}(x, y) = f(x, y) - \overline{f(x, y)}$$

$$g(x, y) = f(x, y) + k * g_{\text{mask}}(x, y)$$



DIP-XE

DIP-XE

DIP-XE

DIP-XE

DIP-XE

# Gradient(梯度)

The first-order derivative of  $f(x, y)$ :  $\nabla f \equiv \mathbf{grad}(f) \equiv \begin{cases} g_x \\ g_y \end{cases} = \begin{cases} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{cases}$

The amplitude:  $M(x, y) = \mathbf{mag}(\nabla f) = \sqrt{g_x^2 + g_y^2}$

$$M(x, y) \approx |g_x| + |g_y|$$

# Gradient(梯度)

- Roberts cross-gradient operator (罗伯特交叉梯度算子)

$$M(x, y) = |z_9 - z_5| + |z_8 - z_6|$$

- Sobel operator (Sobel算子)

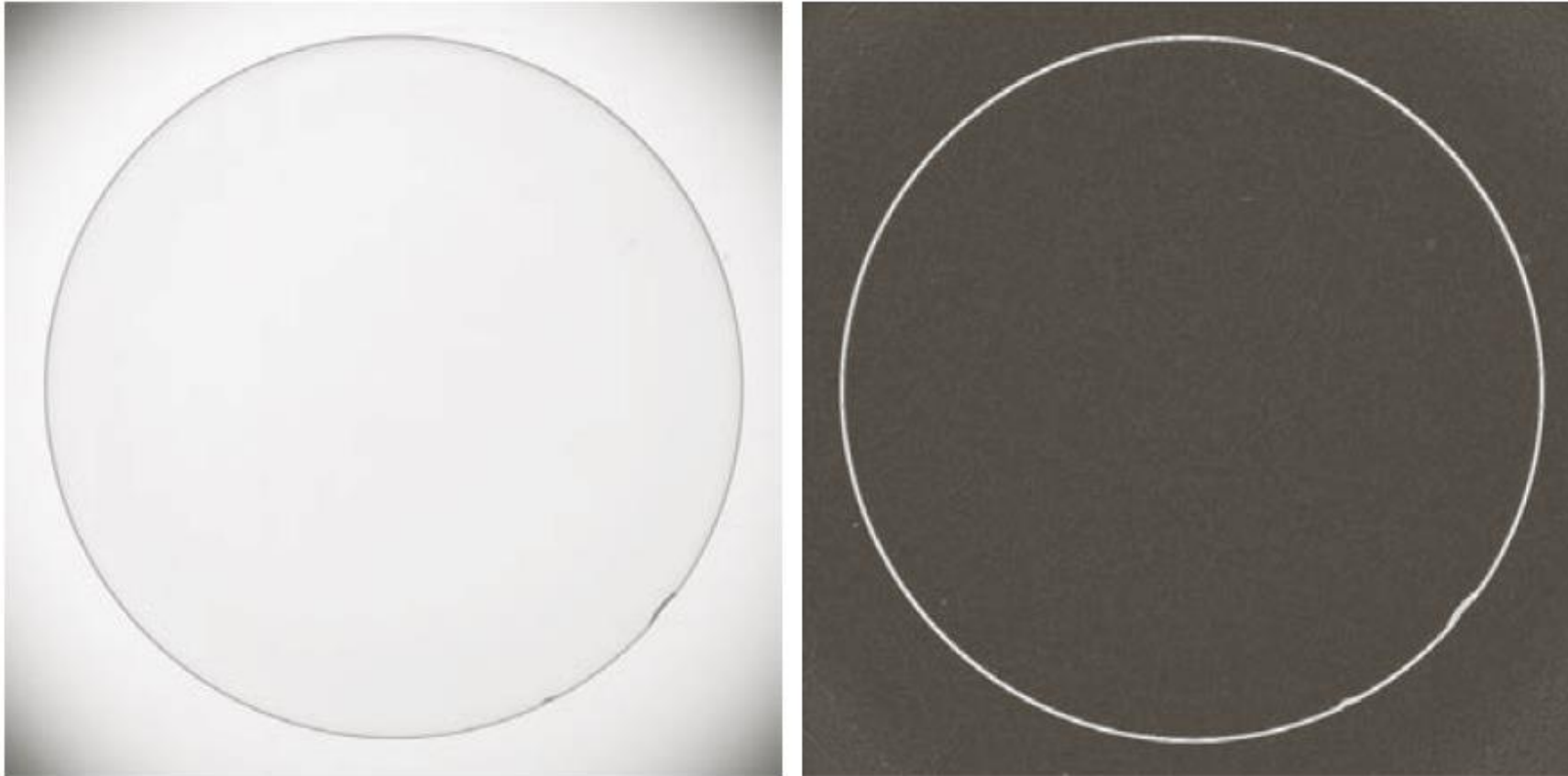
$$M(x, y) = |(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)| \\ + |(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)|$$

$z_1$	$z_2$	$z_3$
$z_4$	$z_5$	$z_6$
$z_7$	$z_8$	$z_9$

-1	0	0	-1
0	1	1	0

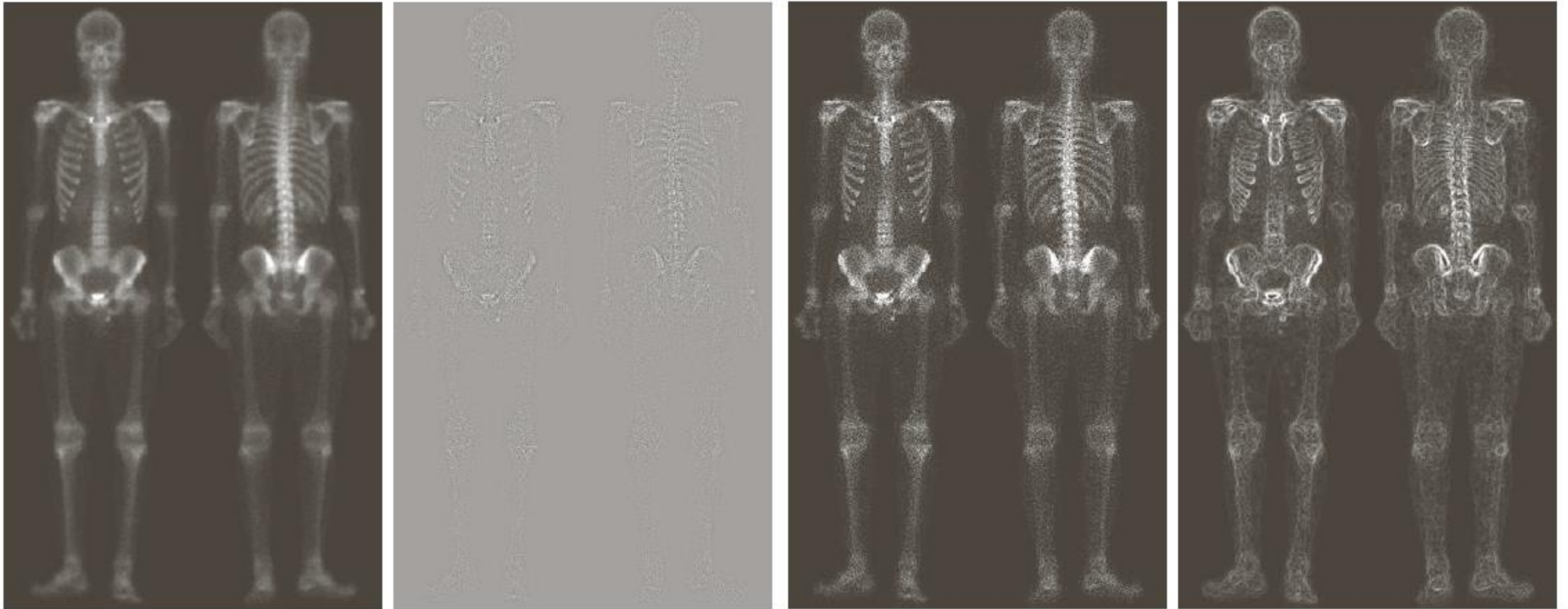
-1	-2	-1	-1	0	1
0	0	0	-2	0	2
1	2	1	-1	0	1

# Edge Enhancement





# Combined Enhancement Methods



# Combined Enhancement Methods

