Lecture 5 – Image Restoration （图像复原）

This lecture will cover:

– Model of Image Degradation Process（图像退化过程模型）

– Noise Reduction（噪声消除）

- Noise Models（噪声模型）
- Spatial Filtering （空间域滤波方法）
- Frequency Domain Filtering（频率域滤波方法）

– Image Restoration（图像复原）

- Degradation Function（退化函数）
- Inverse Filtering (逆滤波)
- Wiener Filtering（维纳滤波）
- Constrained Least Squares Filtering（约束最小二乘方滤波）
- Geometric Mean Filtering（几何均值滤波）

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Model of Image Degradation （图像退化模型）

In Spatial domain:

$$g(x,y) = h\,(x,y) \star f(x,y) + \eta(x,y)$$

Where

$g(x,y)$: a degraded image $f(x,y)$: input image

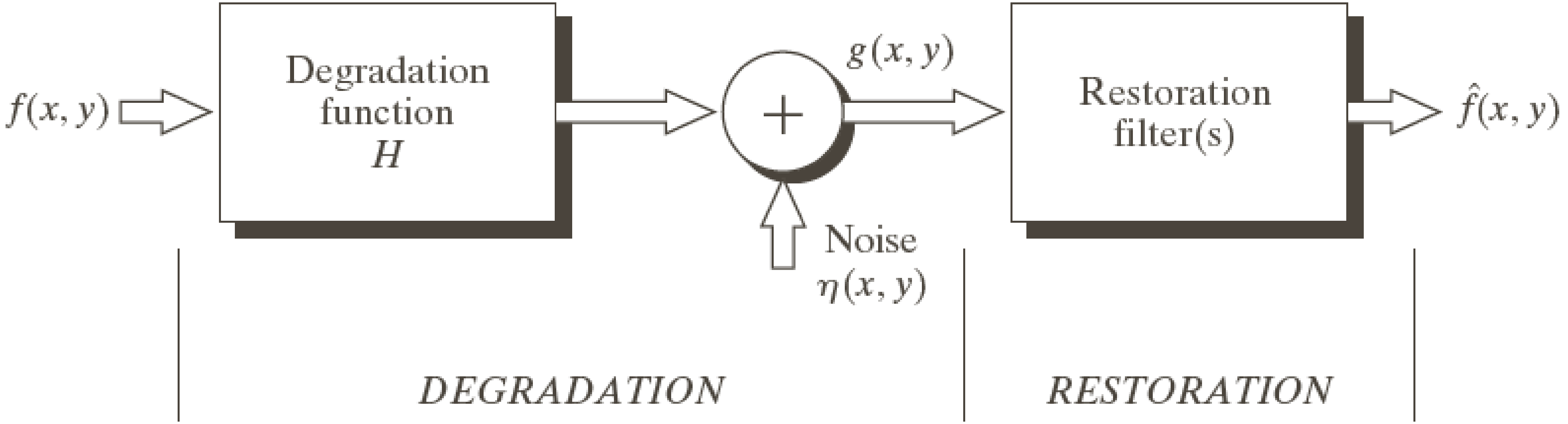
$h(x,y)$: degradation function $\eta(x,y)$: additive noise term

In Frequency domain:

$$G(u,v) = H\,(u,v)F(u,v) + N(u,v)$$

$\boldsymbol{H}$ is a linear, position-invariant process

Model of Image Degradation （图像退化模型）



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Noise Models（噪声模型）

- Properties of Noise
- Noise Probability Density Function (PDF)(概率密度函数)
- Periodic Noise
- Estimation of Noise Parameter（噪声参数）

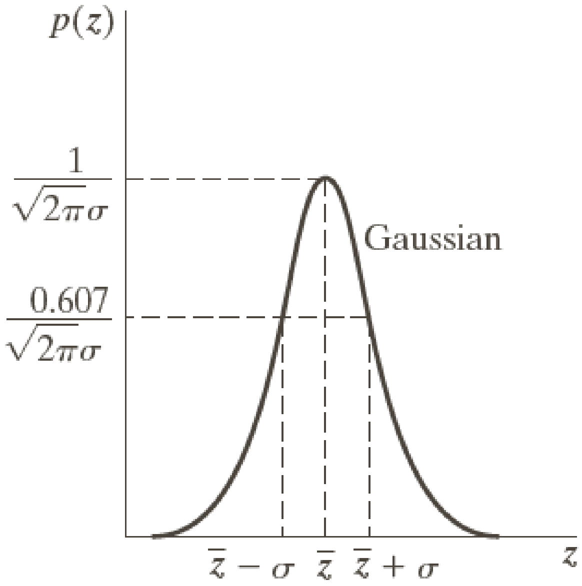
Properties of Noise

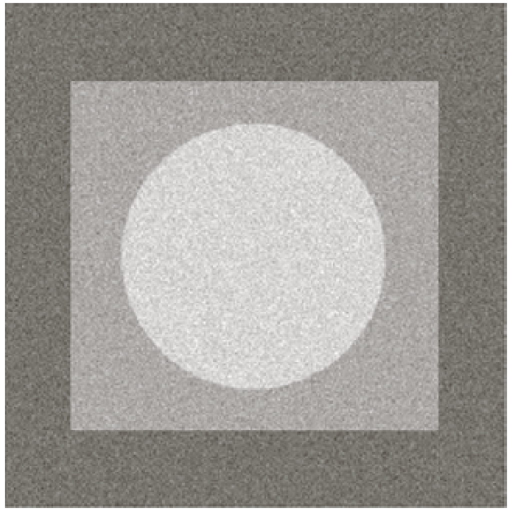
- **Spatial properties -** parameters that define spatial characteristics of noise
- **Frequency properties –** frequency content of noise
  - White noise
- **Independent of spatial coordinates**
- **Uncorrelated with respect to the image itself**

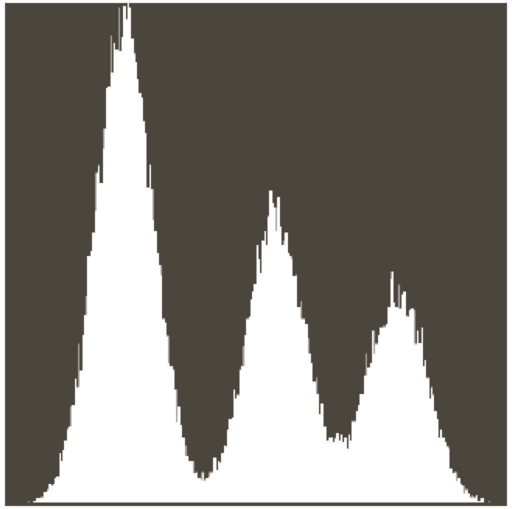
Gaussian Noise (高斯噪声)

Gaussian Noise(高斯噪声): $p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z-\bar{z})^2}{2\sigma^2}}$

$\bar{z}$: mean (average) σ: standard deviation σ^2: variance



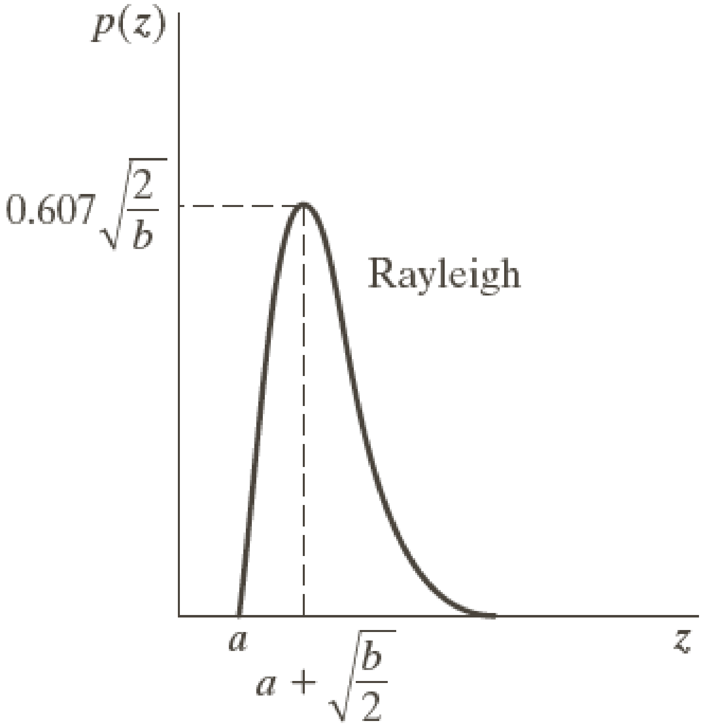


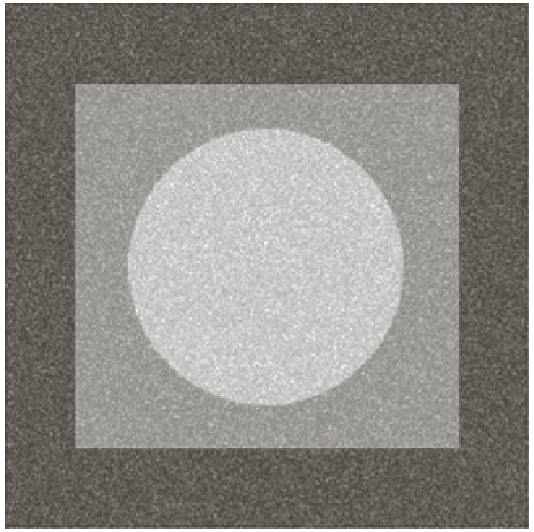


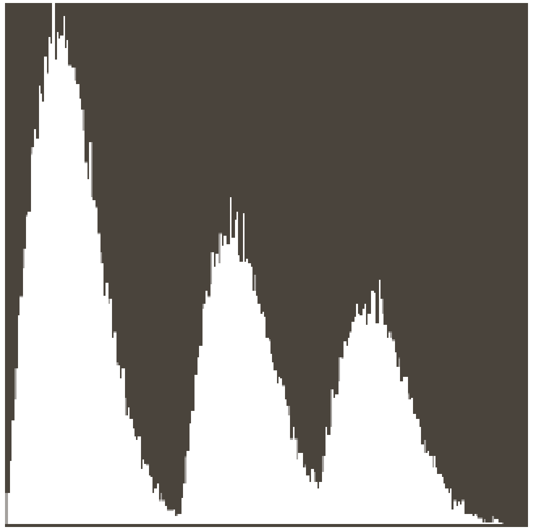
Rayleigh Noise（瑞利噪声）

Rayleigh Noise（瑞利噪声）： $p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-\frac{(z-a)^2}{b}}, & \text{for } z \geq a \\ 0, & \text{for } z < a \end{cases}$

$$\bar{z} = a + \sqrt{\pi b/4} \qquad \sigma^2 = \frac{b(4-\pi)}{4}$$



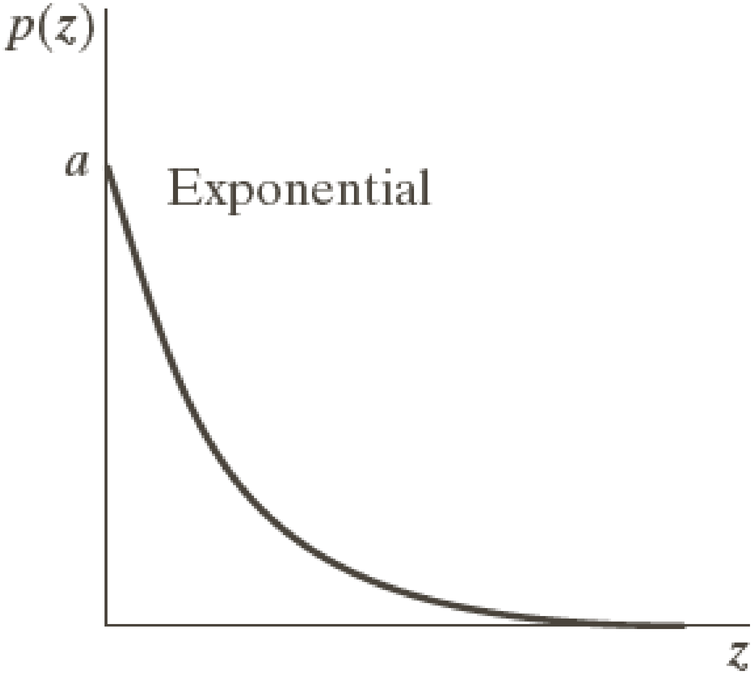


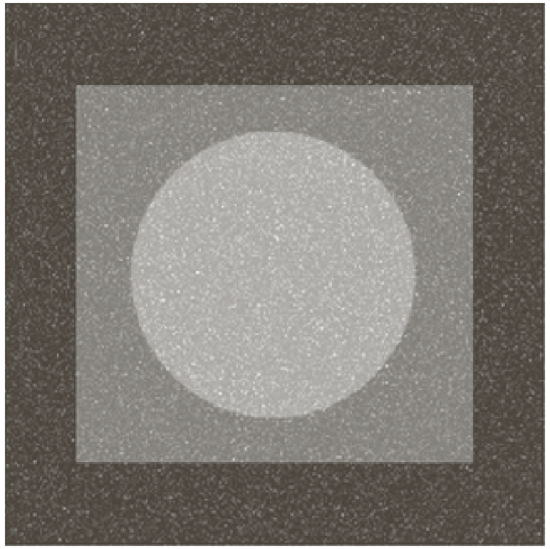


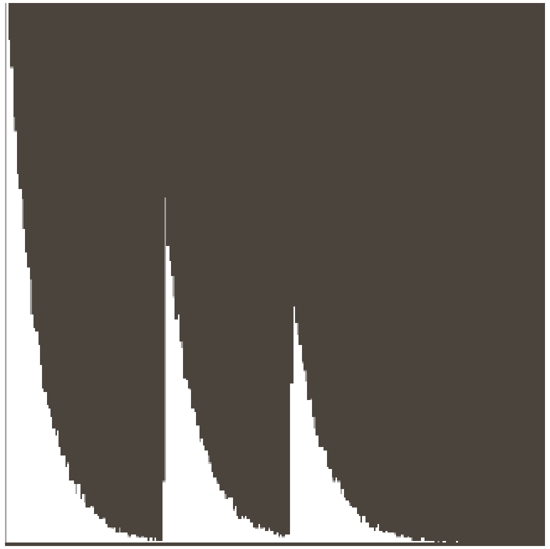
Exponential Noise（指数噪声）

Exponential Noise（指数噪声）： $p(z) = \begin{cases} ae^{-az}, \text{for } z \geq 0 \\ 0, \qquad \text{for } z < 0 \end{cases}$

$$\bar{z} = \frac{1}{a} \qquad \sigma^2 = \frac{1}{a^2}$$



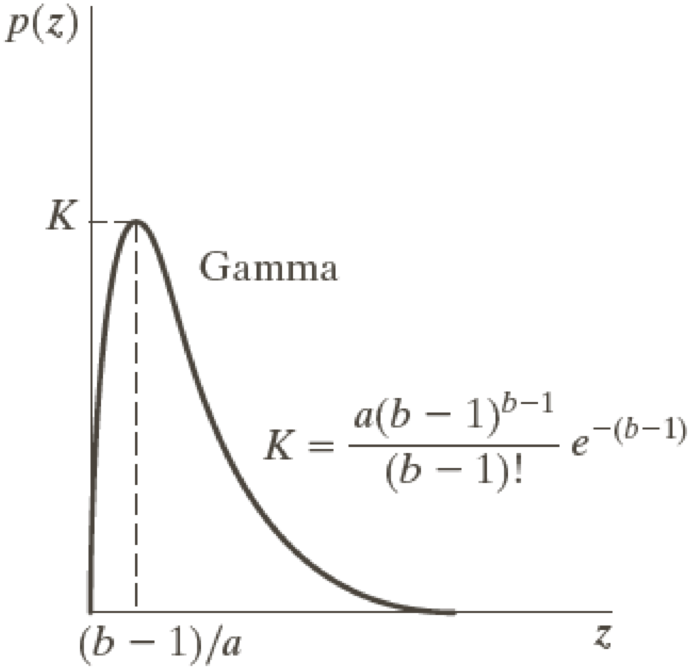


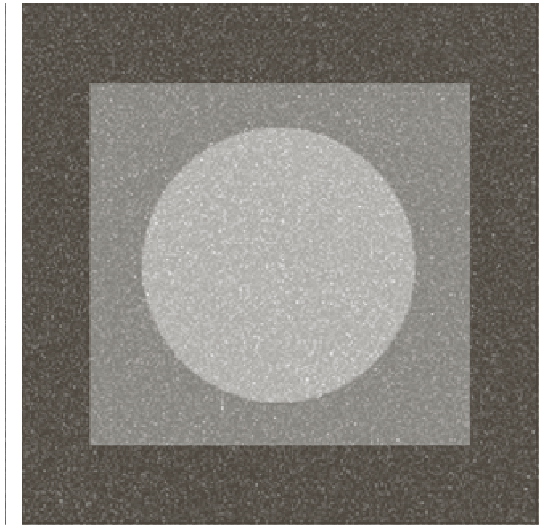


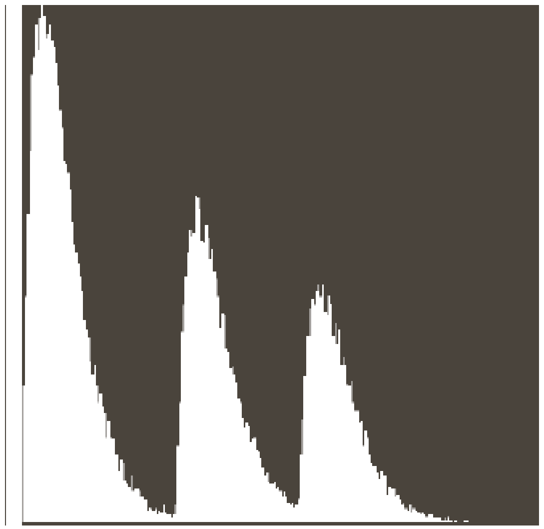
Erlang (gamma) Noise（爱尔兰/伽马噪声）

Erlang (gamma) Noise（爱尔兰/伽马噪声）： $p(z) = \begin{cases} \dfrac{a^b z^{b-1}}{(b-1)!} e^{-az}, & \text{for } z \geq 0 \\ 0, & \text{for } z < 0 \end{cases}$

$$\bar{z} = \frac{b}{a} \qquad \sigma^2 = \frac{b}{a^2}$$



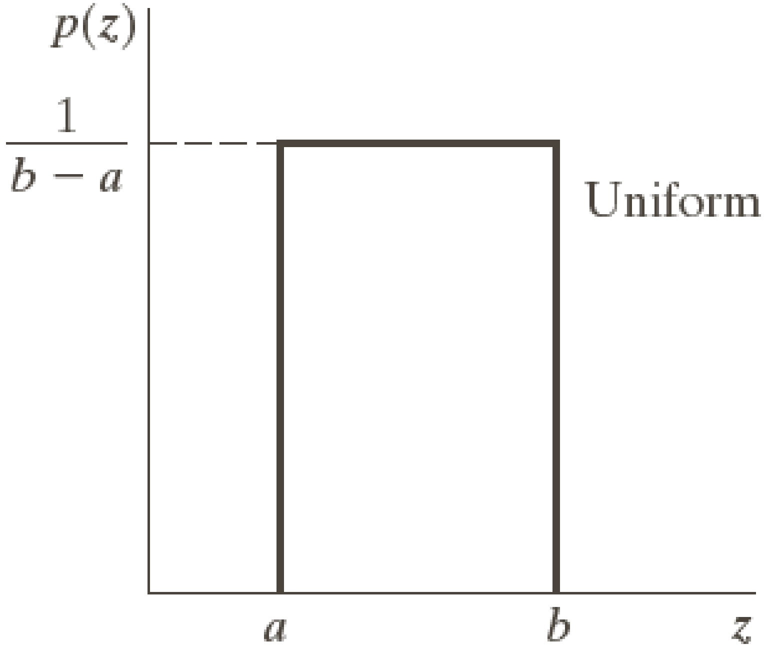


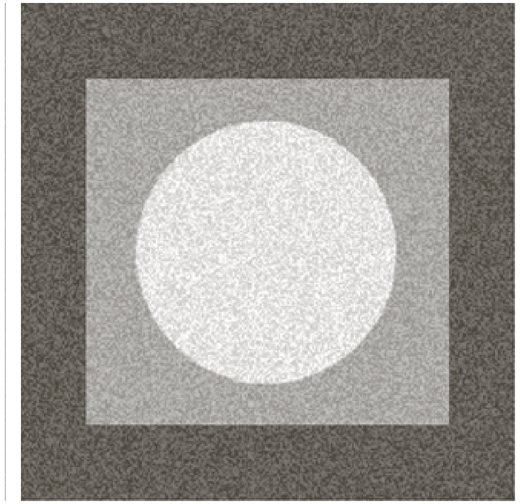


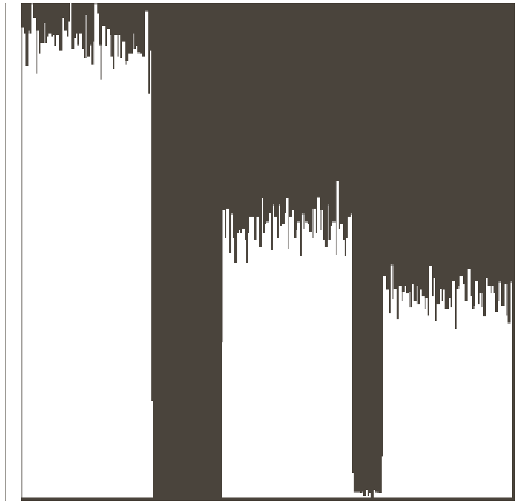
Uniform Noise（均匀噪声）

Uniform Noise（均匀噪声）:　　$p(z) = \begin{cases} \frac{1}{b-a}, & \text{for } a \le z \le b \\ 0, & \text{otherwise} \end{cases}$

$\bar{z} = \frac{a+b}{2}$　　$\sigma^2 = \frac{(b-a)^2}{12}$



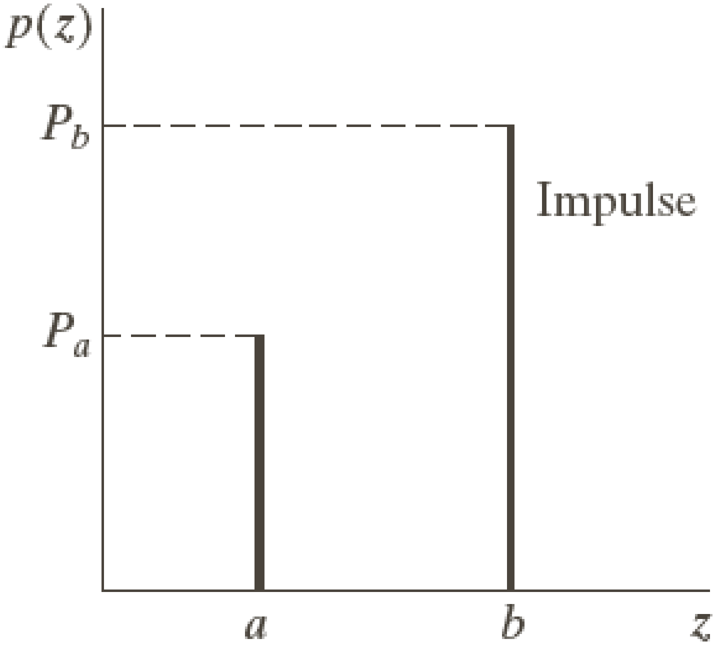


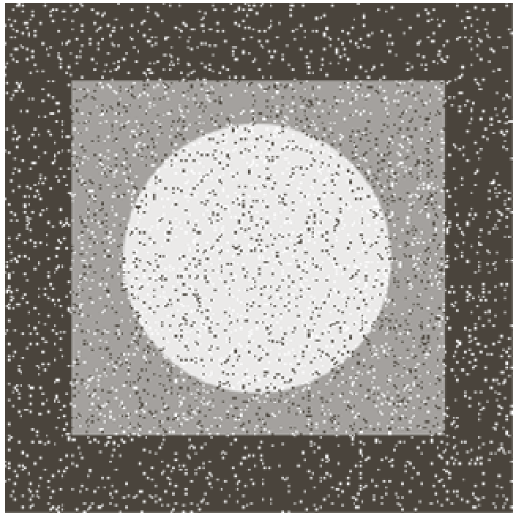


Impulse (salt-and-pepper) Noise （脉冲/椒盐噪声）

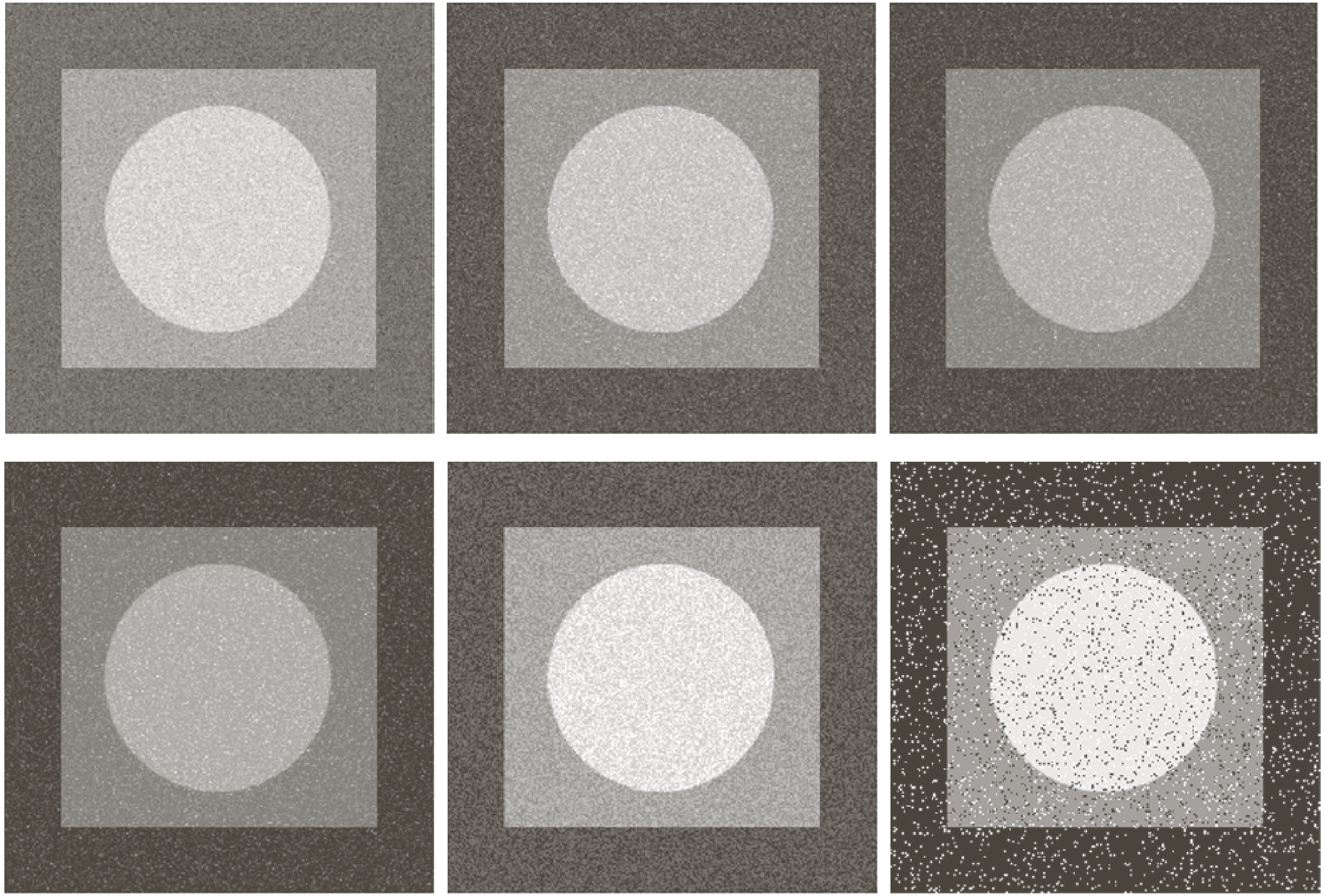
Impulse (salt-and-pepper) Noise（脉冲/椒盐噪声）：

$$p(z) = \begin{cases} P_a, & \text{for } z = a \\ P_b, & \text{for } z = b \\ 1 - P_a - P_b, & \text{otherwise} \end{cases}$$

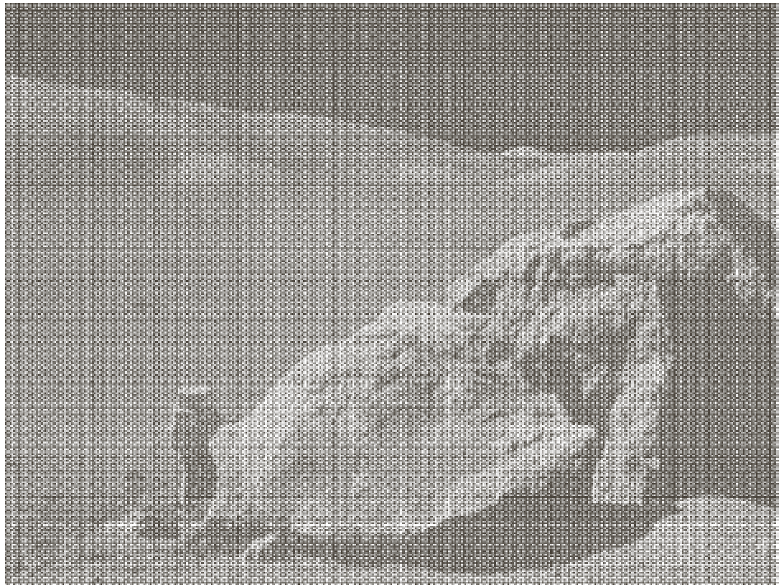


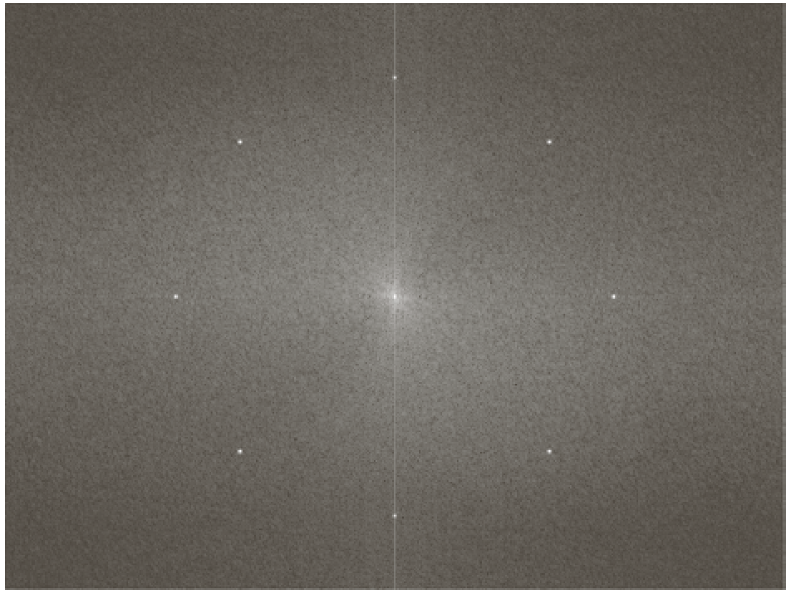


Noise



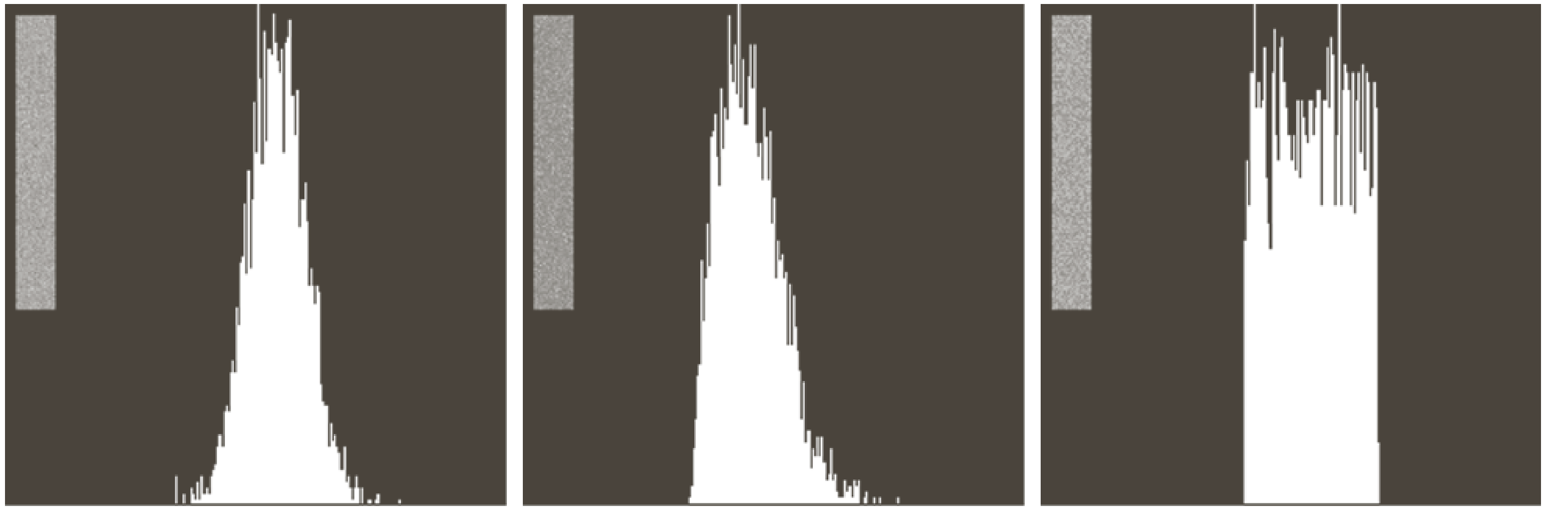
Periodic Noise （周期噪声）





Estimation of Noise Parameter（噪声参数估计）

$$\bar{z} = \sum_{i=0}^{L-1} z_i P_s(z_i) \qquad \sigma^2 = \sum_{i=0}^{L-1} (\bar{z} - z_i)^2 P_s(z_i)$$



Spatial Filtering（空间滤波）

Mean Filters (均值滤波器)

- Arithmetic mean filter (算术均值滤波器)
- Geometric mean filter (几何均值滤波器)
- Harmonic mean filter (谐波均值滤波器)
- Contraharmonic mean filter (逆谐波均值滤波器)

Order-statistic Filters (统计排序滤波器)

- Median filter (中值滤波器)
- Max and Min filter (最大值和最小值滤波器)
- Midpoint filter (中点滤波器)
- Alpha-trimmed mean filter (修正的阿尔法均值滤波器)

Adaptive Filters (自适应滤波器)

- Adaptive local noise reduction filter (自适应局部降噪滤波器)
- Adaptive median filter (自适应中值滤波器)

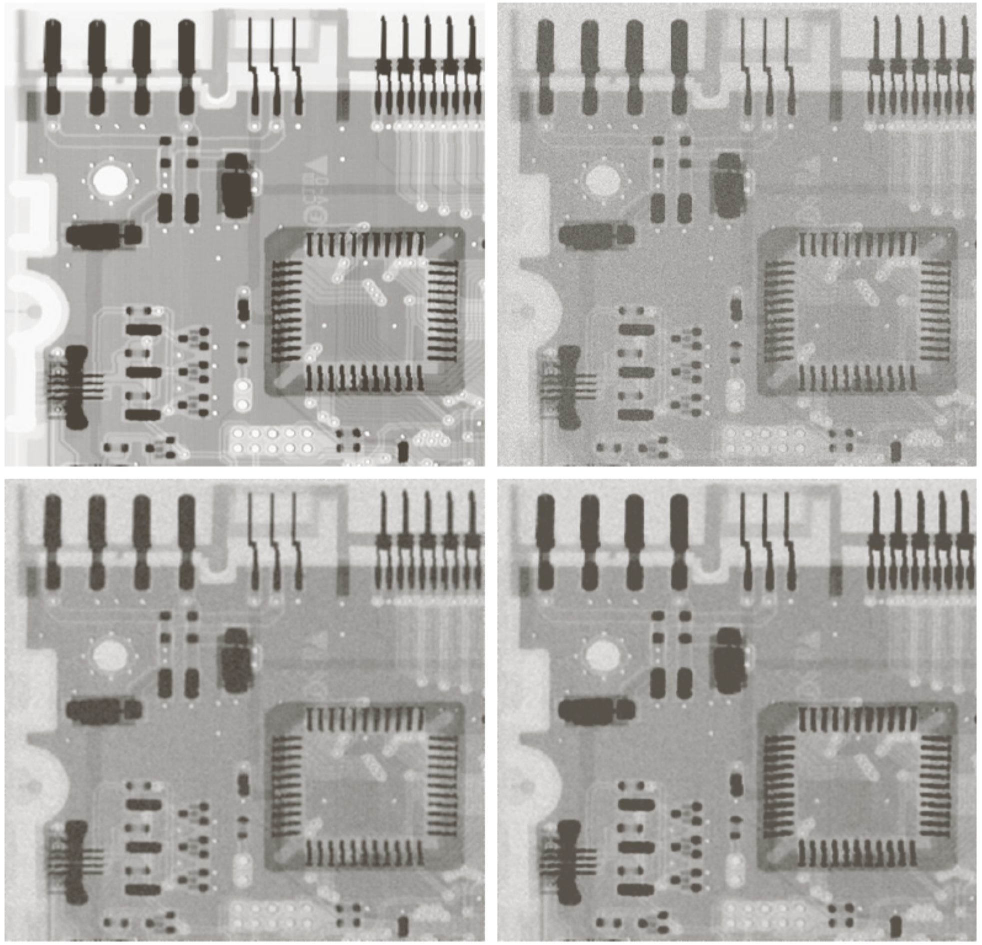
Mean Filters (均值滤波器)

- **Arithmetic mean filter (算术均值滤波器):**

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

- **Geometric mean filter (几何均值滤波器):**

$$\hat{f}(x,y) = \left(\prod_{(s,t)\in S_{xy}} g(s,t) \right)^{\frac{1}{mn}}$$



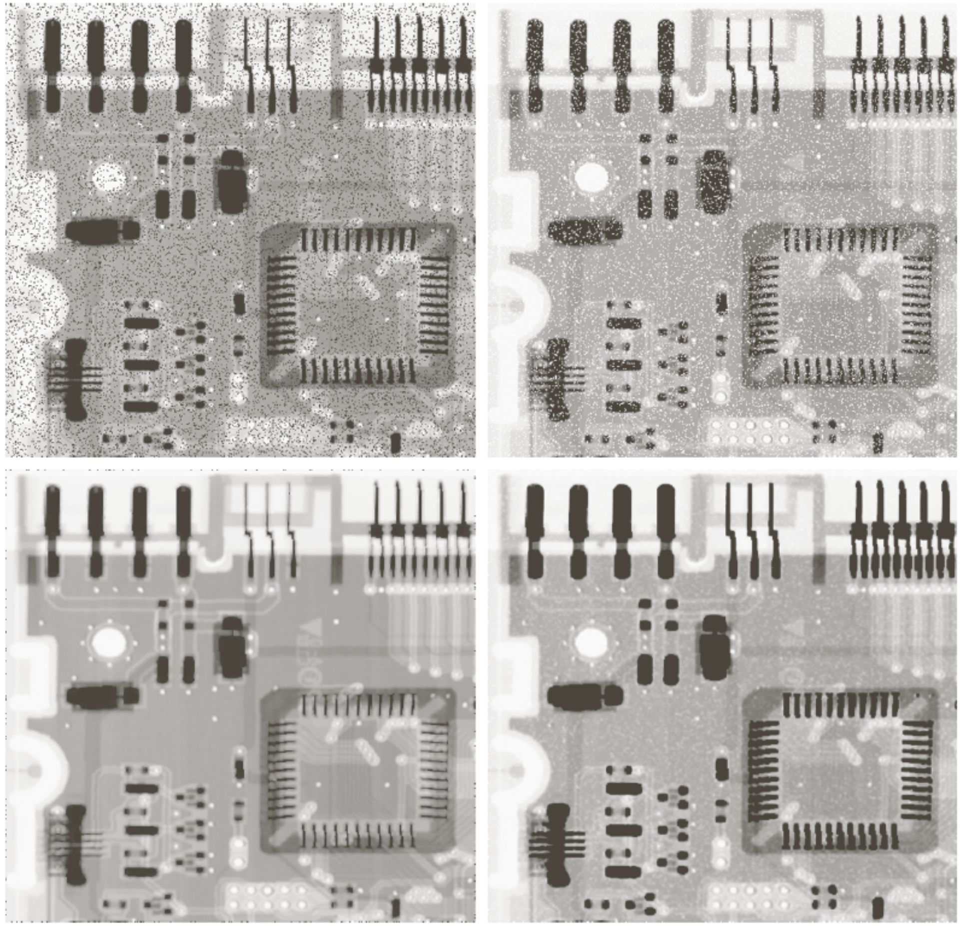
Mean Filters (均值滤波器)

- **Harmonic mean filter (谐波均值滤波器):**

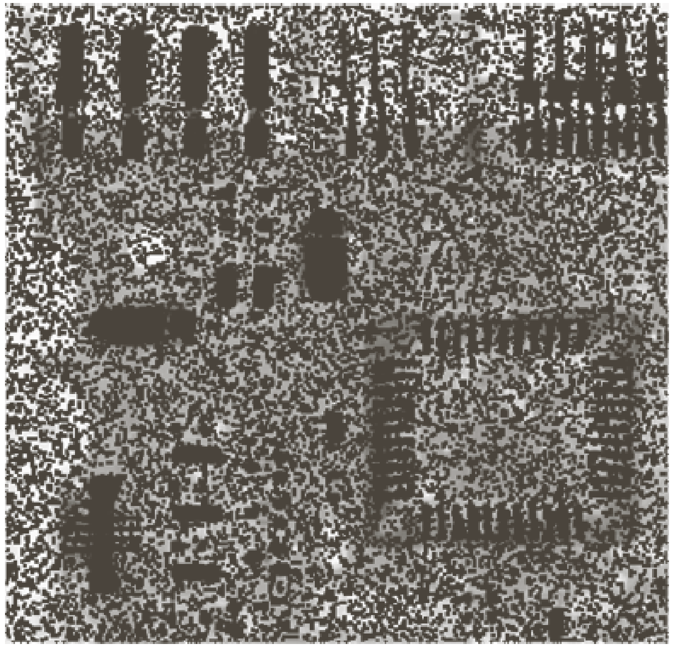
$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

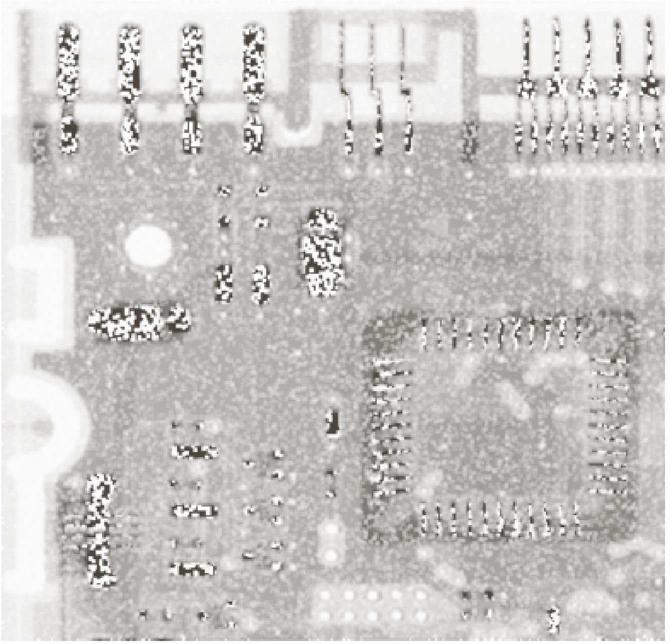
- **Contraharmonic mean filter (逆谐波均值滤波器):**

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$



Contraharmonic mean filter

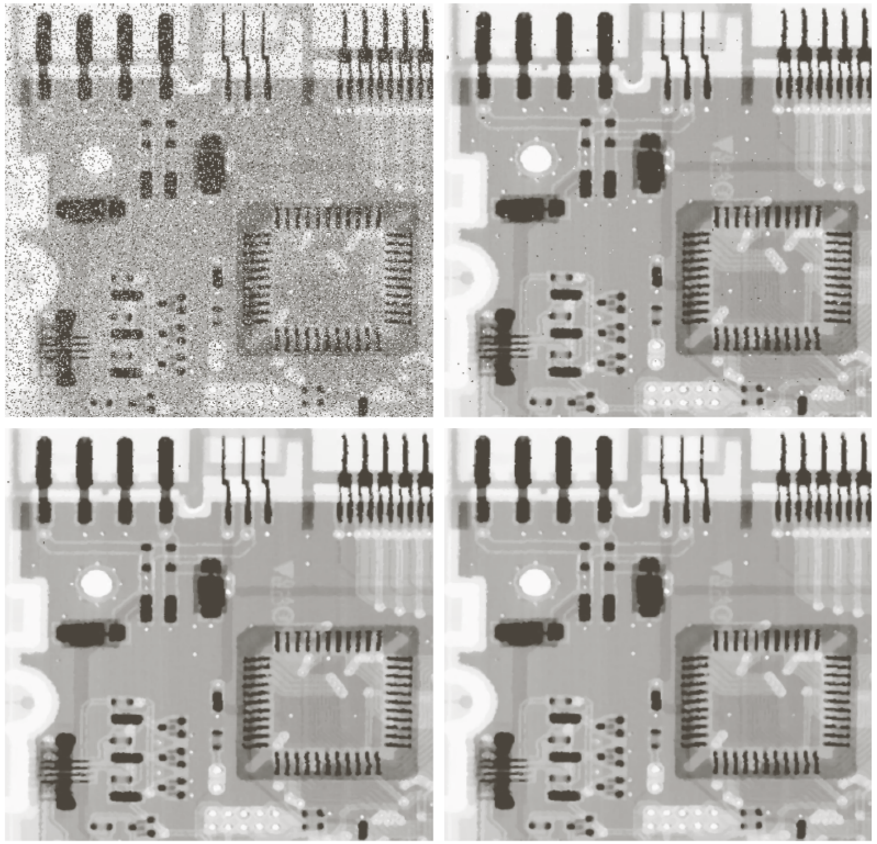




Order-statistic Filters (统计排序滤波器)

- **Median filter (中值滤波器):**

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{\mathrm{median}}\{g(s,t)\}$$



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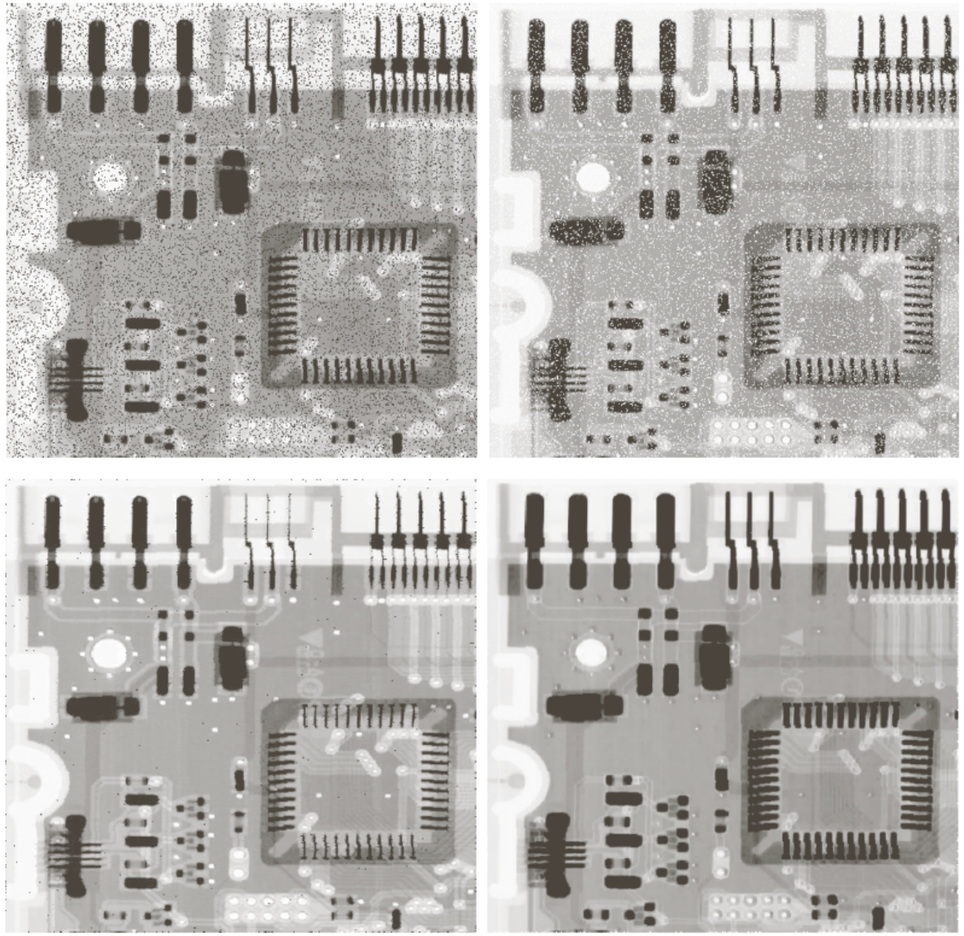
Order-statistic Filters (统计排序滤波器)

- **Max filter**
  (最大值滤波器)

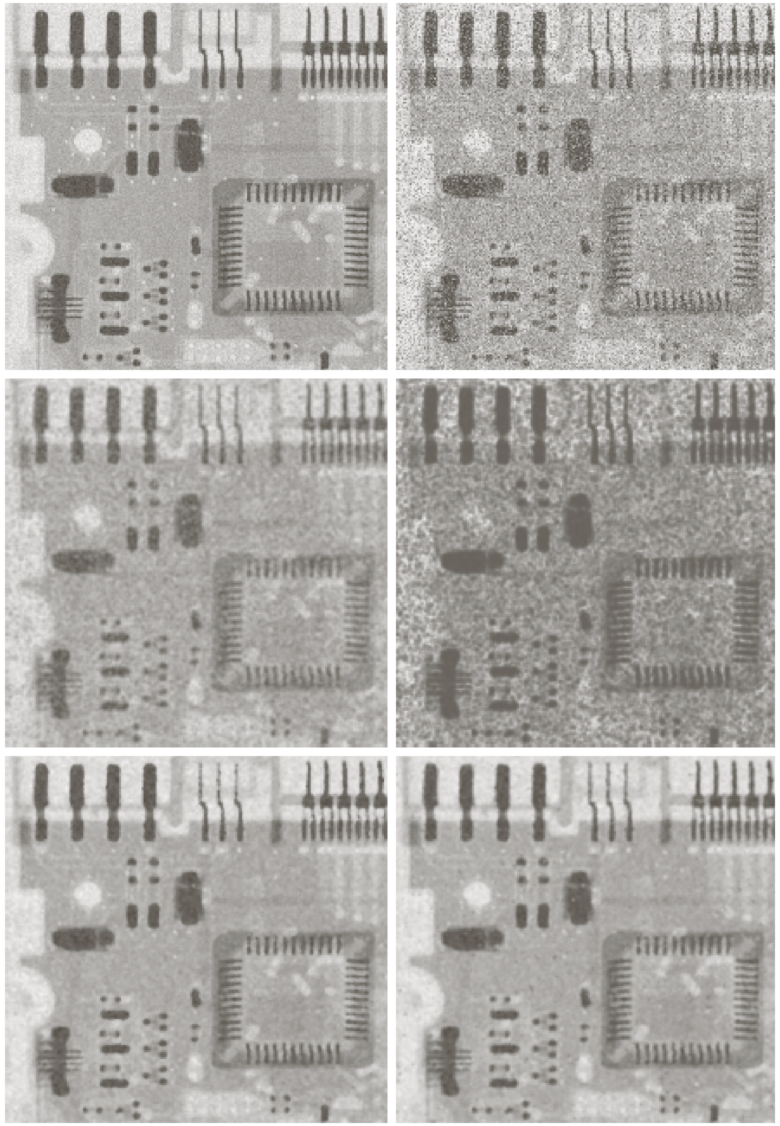
$$\hat{f}(x,y) = \max_{(s,t)\in S_{xy}} \{g(s,t)\}$$

- **Min filter**
  (最小值滤波器)

$$\hat{f}(x,y) = \min_{(s,t)\in S_{xy}} \{g(s,t)\}$$



Order-statistic Filters (统计排序滤波器)



- **Midpoint filter (中点滤波器)**

$$\hat{f}(x,y) = \frac{1}{2}\left[\max_{(s,t)\in S_{xy}}\{g(s,t)\} + \min_{(s,t)\in S_{xy}}\{g(s,t)\}\right]$$

- **Alpha-trimmed mean filter (修正的阿尔法均值滤波器)**

$$\hat{f}(x,y) = \frac{1}{mn-d}\sum_{(s,t)\in S_{xy}} g_r(s,t)$$

Adaptive Filters (自适应滤波器)

- **Adaptive local noise reduction filter (自适应局部降噪滤波器):**

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}[g(x,y) - m_L]$$

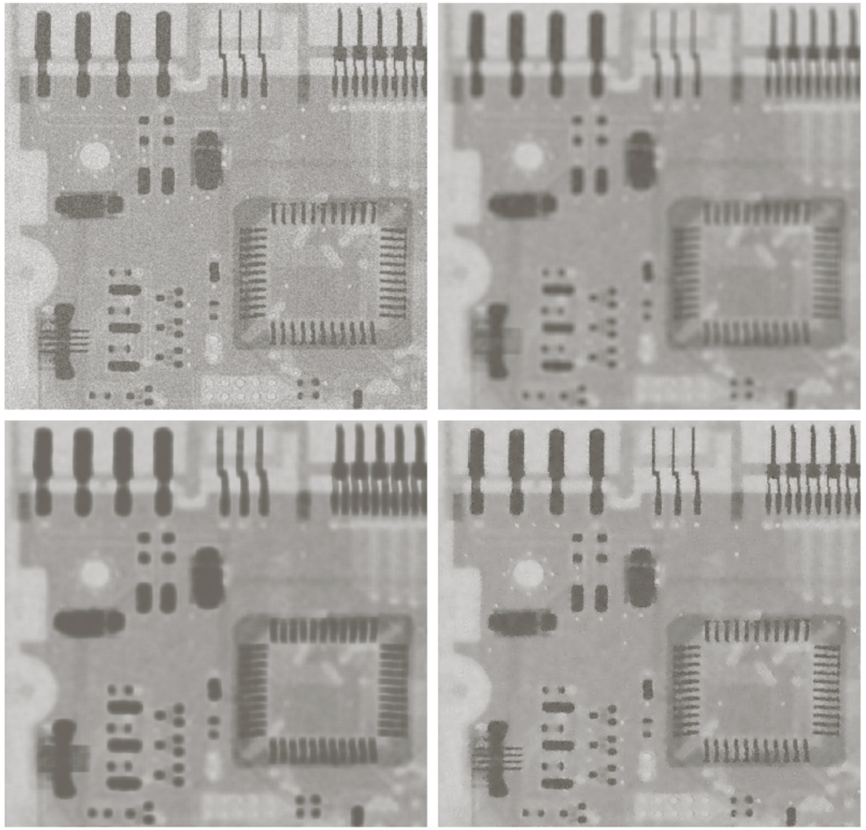
Where σ_L^2

$g(x,y)$: noisy image σ_η^2 : the variance of noise

m_L: the local mean in S_{xy} σ_L^2 : the local variance in S_{xy}

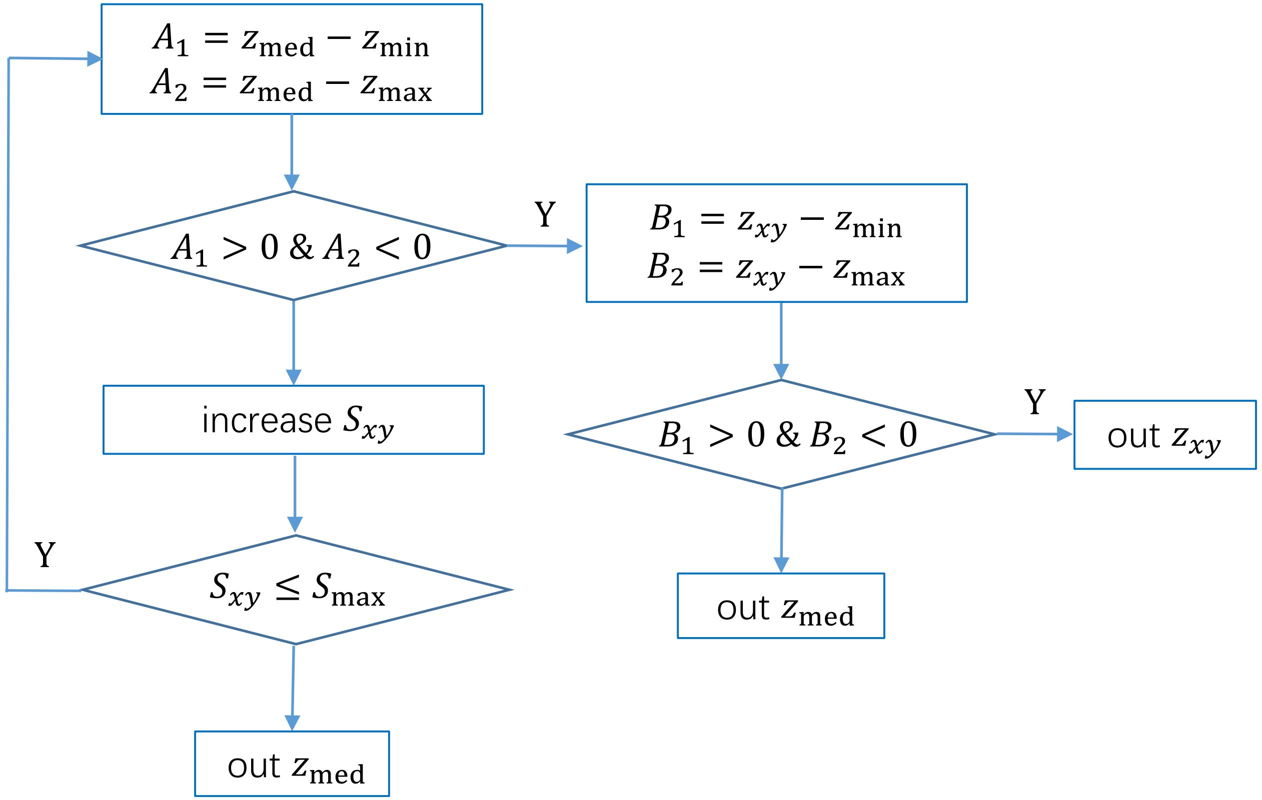
and

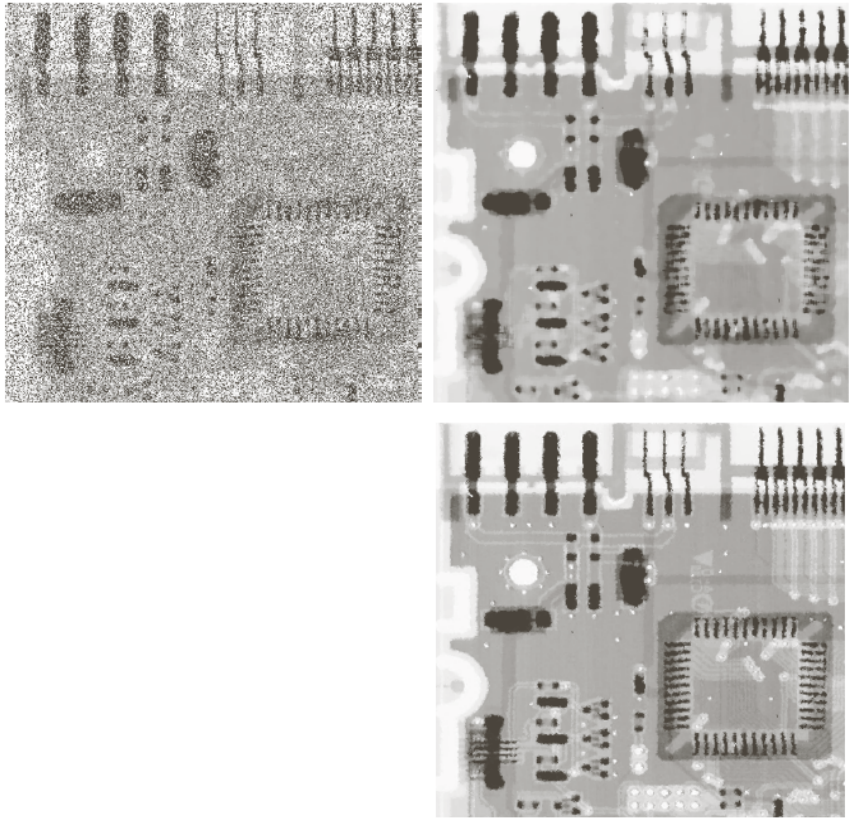
$$g(x,y) = f(x,y) + \eta(x,y)$$



Adaptive Filters (自适应滤波器)

➢ **Adaptive Median filter (自适应中值滤波器):**





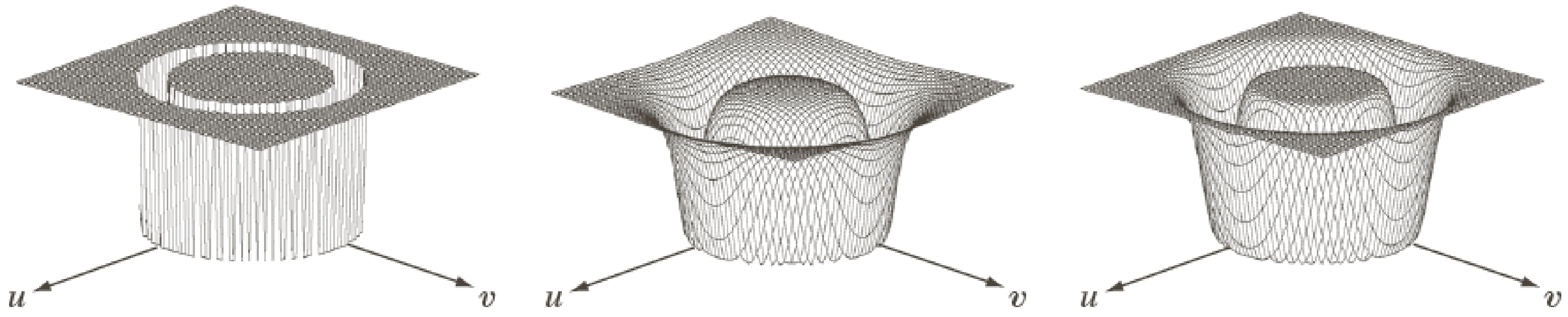
Frequency Domain Filtering（频率域滤波）

Mainly for periodic noise

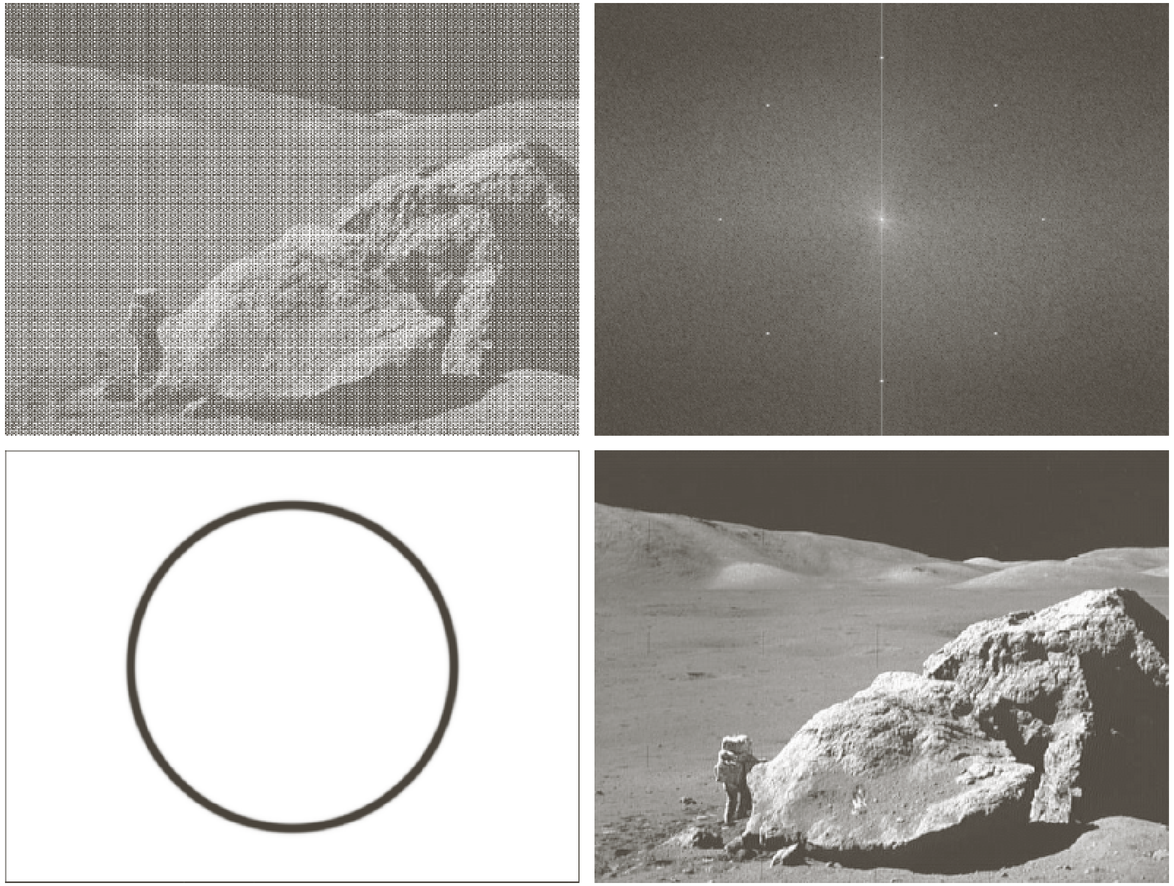
- Bandreject Filters (带阻滤波器)
- Bandpass Filters (带通滤波器)
- Notch Filters (陷波滤波器)
- Optimum Notch Filters (最佳陷波滤波器)

Bandreject Filters (带阻滤波器)

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 0 & \text{if } D_0 - \frac{W}{2} \le D \le D_0 + \frac{W}{2} \\ 1 & \text{otherwise} \end{cases}$	$H(u,v) = \dfrac{1}{1 + \left[\dfrac{DW}{D^2 - D_0^2}\right]^{2n}}$	$H(u,v) = 1 - e^{-\left[\frac{D^2 - D_0^2}{DW}\right]^2}$

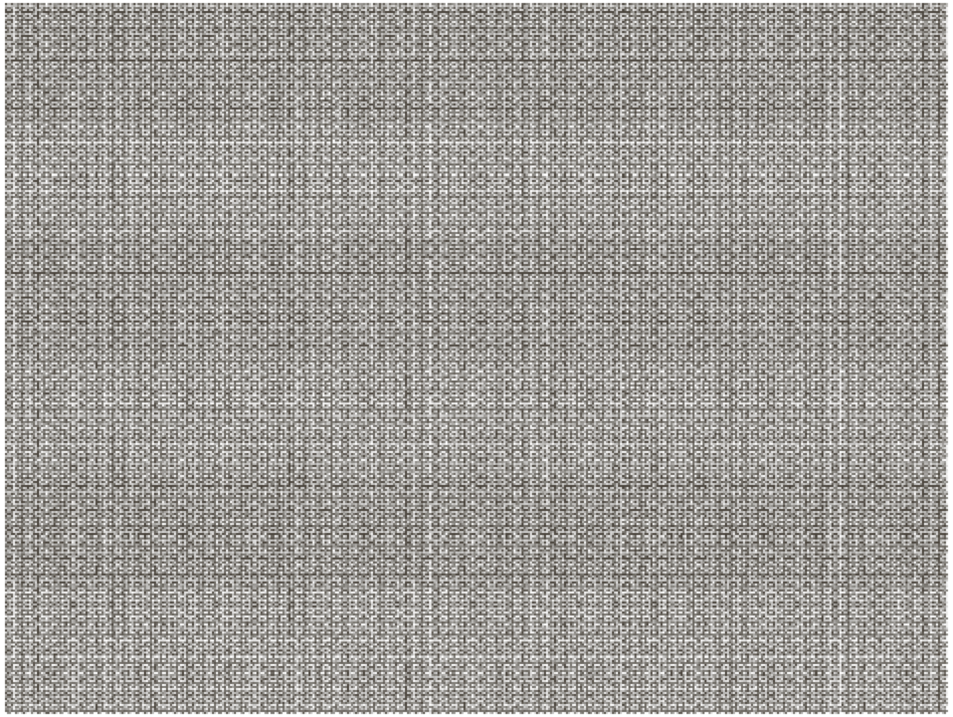


Bandreject Filters (带阻滤波器)



Bandpass Filters (带通滤波器)

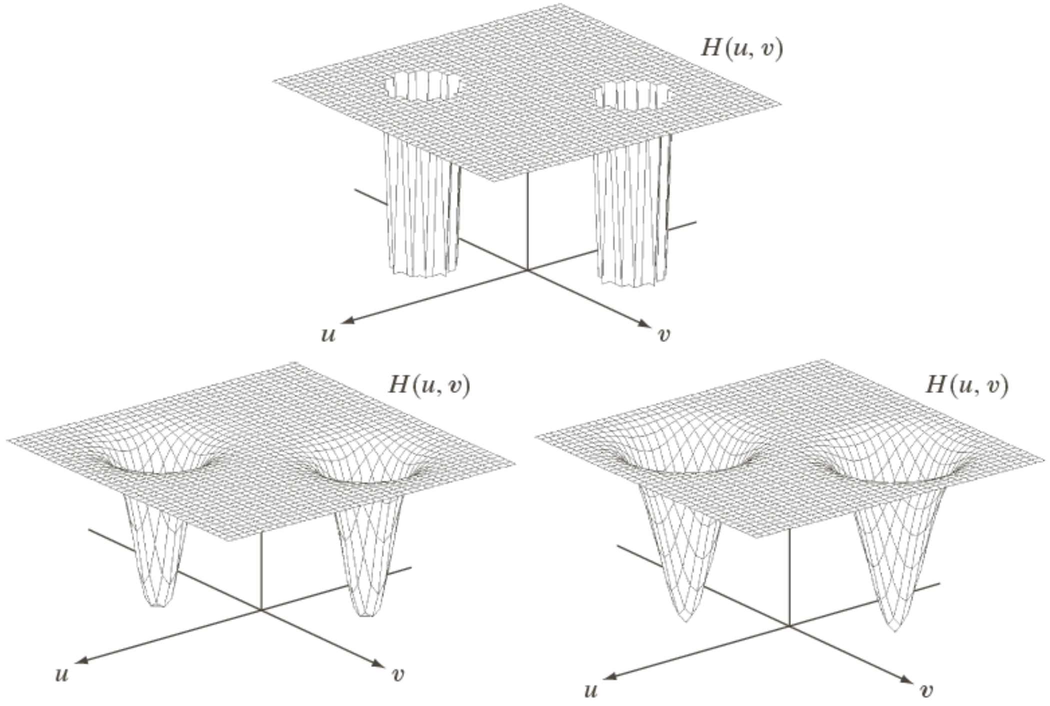
$$H_{\mathrm{BP}}(u,v) = 1 - H_{\mathrm{BR}}(u,v)$$



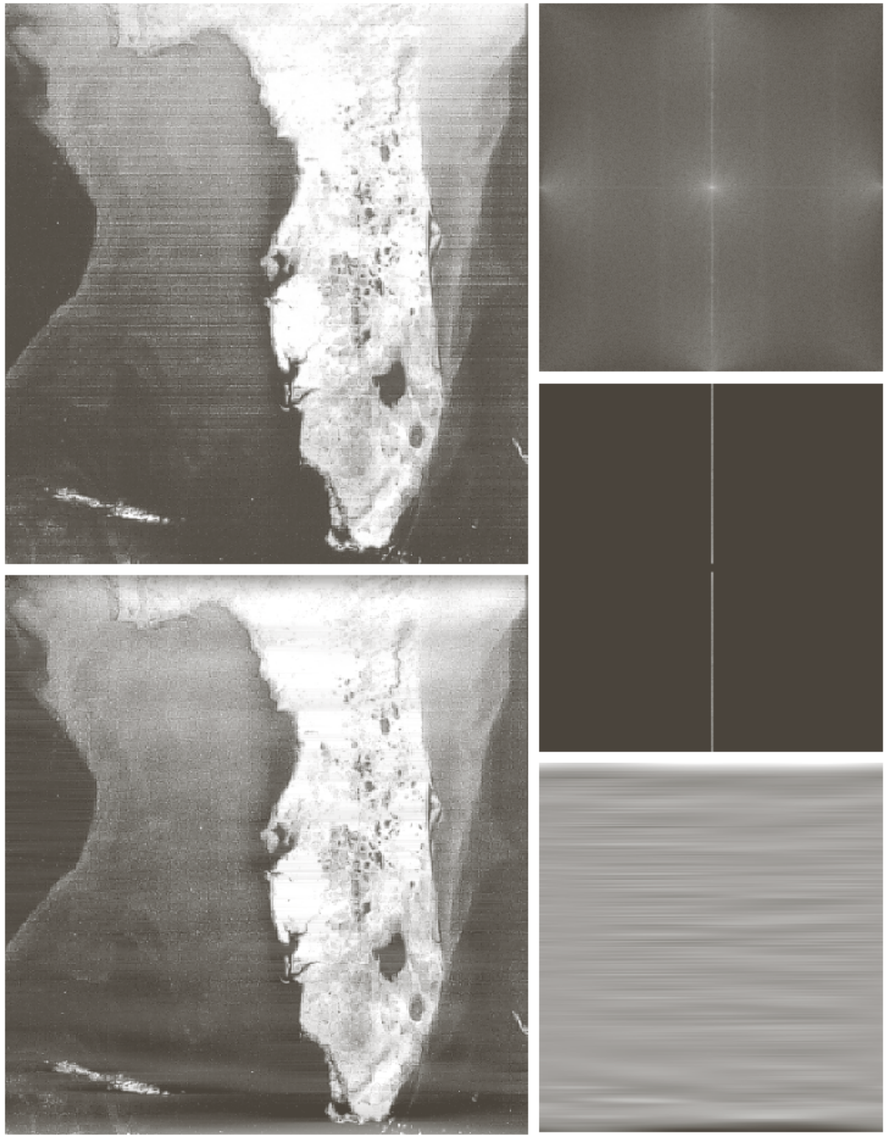
Notch Filters (陷波滤波器)

$$H_{NR}(u,v) = \prod_{k=1}^{Q} H_k(u,v) H_{-k}(u,v)$$

$$H_{\text{NP}}(u,v) = 1 - H_{\text{NR}}(u,v)$$



Notch Filters (陷波滤波器)



Optimum Notch Filters (最佳陷波滤波器)

- **Noise pattern in spatial domain**

$$\eta(x,y) = \mathcal{F}^{-1}\{H_{\mathrm{NP}}(u,v)G(u,v)\}$$

- **Obtain estimate of Noise pattern $\boldsymbol{f(x,y)}$**

$$\hat{f}(x,y) = g(x,y) - w(x,y)\eta(x,y)$$

- **Estimate variance of $\boldsymbol{\hat{f}(x,y)}$**

$$\sigma^2(x,y) = \frac{1}{(2a+1)(2b+1)}\sum_{s=-a}^{a}\sum_{t=-b}^{b}\left[\hat{f}(x+s,y+t) - \bar{\hat{f}}(x,y)\right]^2$$

- **Minimize $\sigma^2(x,y)$, and solve $w(x,y)$**

$$\frac{\partial\sigma^2(x,y)}{\partial w(x,y)} = 0 \quad \Rightarrow \quad w(x,y) = \frac{\overline{g(x,y)\eta(x,y)} - \bar{g}(x,y)\bar{\eta}(x,y)}{\overline{\eta^2}(x,y) - \bar{\eta}^2(x,y)}$$

Optimum Notch Filters (最佳陷波滤波器)