1. The intensity function

$$f(x,y) = i(x,y)r(x,y) = K \cdot 10^{-(x^2+y^2)^{1/2}} \cdot \frac{1}{x^2+y^2}$$

When $x = 6, y = 8$,

$$f(x,y) = K \cdot 10^{-(6^2+8^2)^{\frac{1}{2}}} \cdot \frac{1}{6^2+8^2} = 1$$

$$\Longrightarrow \quad K = 10^{12}$$

2. (1) Megabyte = 1024*1024 byte

$$\frac{1000 * 1024 * 768 * 8}{1024 * 1024 * 8} = 750 \quad \text{Mb}$$

(2)

$$\frac{1000 * 1024 * 768 * (1 + 8 + 1)}{9600} = 819200 \quad \text{sec}$$

3. The length of image area in real world will fulfill

$$\frac{l}{14} = \frac{500}{35}$$

$$\Longrightarrow \quad l = 200 \quad \text{mm}$$

The resolution

$$\Delta l = \frac{l}{2048} = \frac{200}{2048} = 0.1 \quad \text{mm}$$

4 4. $D_4=7$

5	2	2	1	(Q)2
2	3	6	3	6
1	0	3	2	5
(P)3	2	4	5	2
1	5	3	4	0

$D_8=4$

5	2	2	1	(Q)2
2	3	6	3	6
1	0	3	2	5
(P)3	2	4	5	2
1	5	3	4	0

$D_m=6$

5	2	2	1	(Q)2
2	3	6	3	6
1	0	3	2	5
(P)3	2	4	5	2
1	5	3	4	0

5. $x_0 = r\cos\alpha \qquad y_0 = r\sin\alpha$

$$x_1 = r\cos(\alpha+\beta) = r(\cos\alpha\cos\beta - \sin\alpha\sin\beta) = r\cos\alpha\cos\beta - r\sin\alpha\sin\beta = x_0\cos\beta - y_0\sin\beta$$

$$y_1 = r\sin(\alpha+\beta) = r(\sin\alpha\cos\beta + \cos\alpha\sin\beta) = r\cos\alpha\sin\beta + r\sin\alpha\cos\beta = x_0\sin\beta + y_0\cos\beta$$

$$\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ 1 \end{bmatrix}$$

6

$$T=\begin{bmatrix}1&0&2\\0&1&2\\0&0&1\end{bmatrix}\begin{bmatrix}\cos 30^\circ&-\sin 30^\circ&0\\\sin 30^\circ&\cos 30^\circ&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&-2\\0&1&-2\\0&0&1\end{bmatrix}=\begin{bmatrix}\frac{\sqrt{3}}{2}&-\frac{1}{2}&3-\sqrt{3}\\\frac{1}{2}&\frac{\sqrt{3}}{2}&1-\sqrt{3}\\0&0&1\end{bmatrix}$$

$$T^c=T^{-1}=\begin{bmatrix}\frac{\sqrt{3}}{2}&\frac{1}{2}&1-\sqrt{3}\\-\frac{1}{2}&\frac{\sqrt{3}}{2}&3-\sqrt{3}\\0&0&1\end{bmatrix}$$

or $$T^c=\begin{bmatrix}1&0&2\\0&1&2\\0&0&1\end{bmatrix}\begin{bmatrix}\cos(-30^\circ)&-\sin(-30^\circ)&0\\\sin(-30^\circ)&\cos(-30^\circ)&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&-2\\0&1&-2\\0&0&1\end{bmatrix}=\begin{bmatrix}\frac{\sqrt{3}}{2}&\frac{1}{2}&1-\sqrt{3}\\-\frac{1}{2}&\frac{\sqrt{3}}{2}&3-\sqrt{3}\\0&0&1\end{bmatrix}$$

7. Solution 1:

$$f(a,y_0)=f_{00}+\frac{a-x_0}{x_1-x_0}(f_{10}-f_{00})=f_{00}+(a-x_0)(f_{10}-f_{00})=(f_{10}-f_{00})a-x_0f_{10}+f_{00}(1+x_0)$$
$$=(f_{10}-f_{00})a-x_0f_{10}+x_1f_{00}$$

$$f(a,y_1)=f_{01}+\frac{a-x_0}{x_1-x_0}(f_{11}-f_{01})=f_{01}+(a-x_0)(f_{11}-f_{01})=(f_{11}-f_{01})a-x_0f_{11}+f_{01}(1+x_0)$$
$$=(f_{11}-f_{01})a-x_0f_{11}+x_1f_{01}$$

$$f(a,b)=f(a,y_0)+\frac{(b-y_0)}{y_1-y_0}\left[f(a,y_1)-f(a,y_0)\right]=f(a,y_0)+(b-y_0)\left[f(a,y_1)-f(a,y_0)\right]$$

$$=(f_{10}-f_{00})a-x_0f_{10}+x_1f_{00}+(b-y_0)\left[(f_{11}-f_{01})a-x_0f_{11}+x_1f_{01}-(f_{10}-f_{00})a+x_0f_{10}-x_1f_{00}\right]$$

$$=(f_{10}-f_{00})a-x_0f_{10}+x_1f_{00}+(b-y_0)\left[(f_{11}-f_{10}-f_{01}+f_{00})a-x_0(f_{11}-f_{10})+x_1(f_{01}-f_{00})\right]$$

$$=(f_{11}-f_{10}-f_{01}+f_{00})ab+\left[f_{10}-f_{00}-y_0(f_{11}-f_{10}-f_{01}+f_{00})\right]a+\left[-x_0(f_{11}-f_{10})+x_1(f_{01}-f_{00})\right]b$$
$$+\left[-x_0f_{10}+x_1f_{00}+x_0y_0(f_{11}-f_{10})-x_1y_0(f_{01}-f_{00})\right]$$

$$=(f_{11}-f_{10}-f_{01}+f_{00})ab+(-y_0f_{11}+y_1f_{10}+y_0f_{01}-y_1f_{00})a+(-x_0f_{11}+x_0f_{10}+x_1f_{01}-x_1f_{00})b$$
$$+(x_0y_0f_{11}-x_0y_1f_{10}-x_1y_0f_{01}+x_1y_1f_{00}).$$

Solution 2:

let $f(a,b) = P_1 ab + P_2 a + P_3 b + P_4$.

Then
$$f_{00} = P_1 x_0 y_0 + P_2 x_0 + P_3 y_0 + P_4$$
$$f_{01} = P_1 x_0 y_1 + P_2 x_0 + P_3 y_1 + P_4$$
$$f_{10} = P_1 x_1 y_0 + P_2 x_1 + P_3 y_0 + P_4$$
$$f_{11} = P_1 x_1 y_1 + P_2 x_1 + P_3 y_1 + P_4$$

solve
$$P_1 = f_{11} - f_{10} - f_{01} + f_{00}$$
$$P_2 = -y_0 f_{11} + y_1 f_{10} + y_0 f_{01} - y_1 f_{00}$$
$$P_3 = -x_0 f_{11} + x_0 f_{10} + x_1 f_{01} - x_1 f_{00}$$
$$P_4 = x_0 y_0 f_{11} - x_0 y_1 f_{10} - x_1 y_0 f_{01} + x_1 y_1 f_{00}$$

8.
$$\begin{bmatrix} x_f \\ y_f \\ 1 \end{bmatrix} = T^c \begin{bmatrix} x_g \\ y_g \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 1-\sqrt{3} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 3-\sqrt{3} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{5+\sqrt{3}}{2} \\ \frac{3+\sqrt{3}}{2} \\ 1 \end{bmatrix} \approx \begin{bmatrix} 3.4 \\ 2.4 \\ 1 \end{bmatrix}$$

Therefore $x_0 = 3$ $x_1 = 4$ $y_0 = 2$ $y_1 = 3$

$f_{00} = 2$ $f_{01} = 3$ $f_{10} = 6$ $f_{11} = 9$

substitute to solution of Problem 7

$P_1 = 2$ $P_2 = 0$ $P_3 = -5$ $P_4 = 0$

$$g(3,3) = f(x_f, y_f) = P_1 x_f y_f + P_2 x_f + P_3 y_f + P_4$$
$$= 2 \cdot \frac{5+\sqrt{3}}{2} \cdot \frac{3+\sqrt{3}}{2} - 5 \cdot \frac{3+\sqrt{3}}{2}$$
$$= 4.1$$
$$\approx 4$$