

# Lecture 5 – Image Segmentation (图像分割)

This lecture will cover:

- **Morphological Image Processing (形态学图像处理)**
  - Morphological operation
  - Morphological algorithms
- Image Segmentation (图像分割)
  - Point, Line and Edge Detection (点、线和边缘检测)
  - Thresholding (阈值处理)
  - Segmentation using Morphological Watersheds (形态学分水岭分割)

# Mathematical Morphology

- **Language: Set theory (集合论)**
- **Advantages comparing to other spatial or frequency domain methods**
  - Keep more information from image;
  - Parallel processing
  - Insensitive to noise
  - Smooth edge
  - Continuous skeletons
- **Key operations**
  - HMT (Hit or Miss Transformation)(击中与否变换)
  - Dilation (膨胀) and Erosion (腐蚀)

# Steps

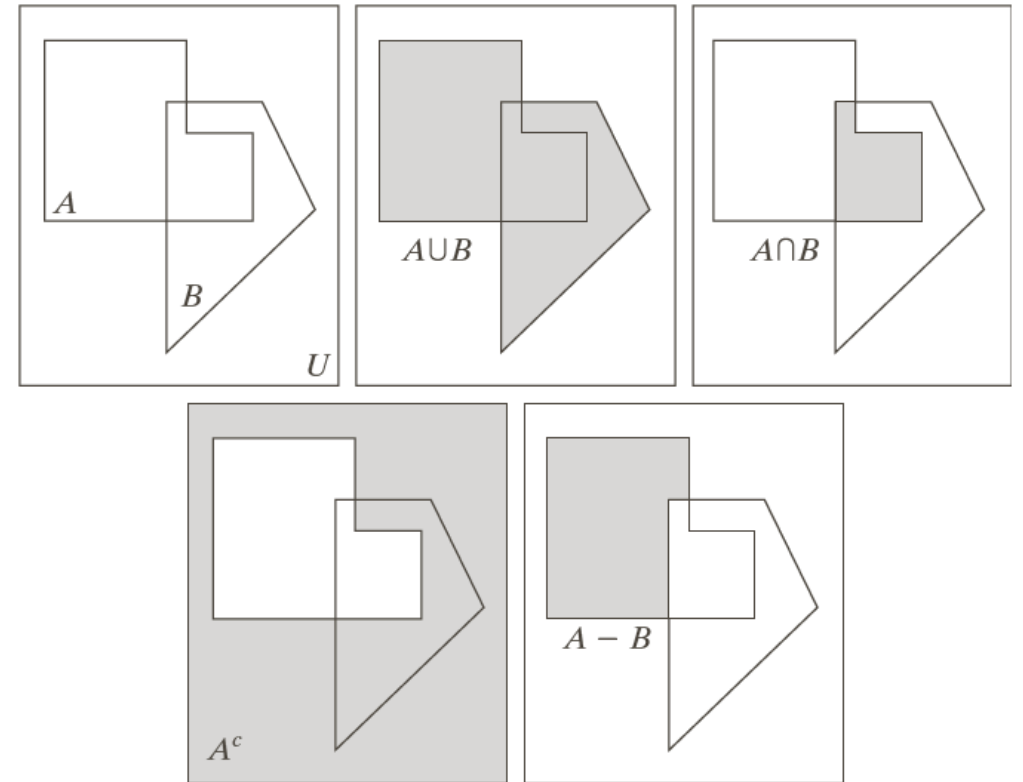
**To analyze image with mathematical morphological methods,**

- ① Specify the geometrical structural pattern of the object;
- ② Choose structuring element (SE) based on the pattern
- ③ Proceed HMT with the selected SE to acquire characteristic image of the object
- ④ Emphasize the desired information to extract attributes

# Preliminaries --- Set Operation

An digital image  $f(x, y)$  can be considered as a set  $A$ , if  $w = (x, y)$  in 2D integer space  $\mathbf{Z}^2$ , Then

- $w \in A$ :  $w$  is an element of  $A$
- $w \notin A$ :  $w$  is not an element of  $A$
- $B = \{w \mid \text{condition}\}$ : all elements which meet the specified condition
  - $A \cup B = \{w \mid w \in A \text{ or } w \in B\}$ : union (并集)
  - $A \cap B = \{w \mid w \in A \text{ and } w \in B\}$ : intersection (交集)
  - $A^c = \{w \mid w \notin A\}$ : complement (补集)
  - $A - B = \{w \mid w \in A, w \notin B\}$ : difference (差集)



# Reflection and Translation

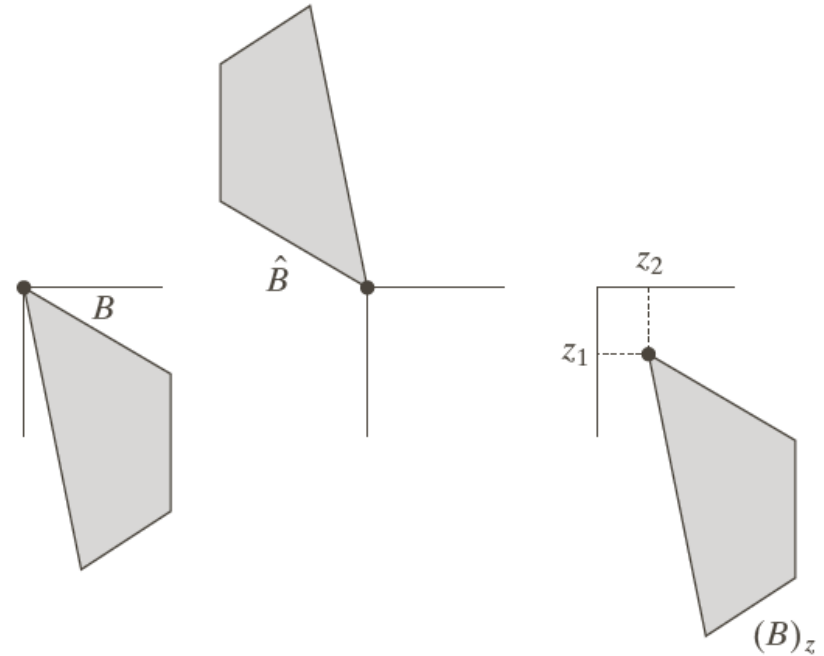
## ➤ Reflection (反射)

$$\hat{B} = \{w \mid w = -b, b \in B\}$$

## ➤ Translation (平移)

$$(B)_z = \{w \mid w = b + z, b \in B\},$$

where  $z = (z_1, z_2)$



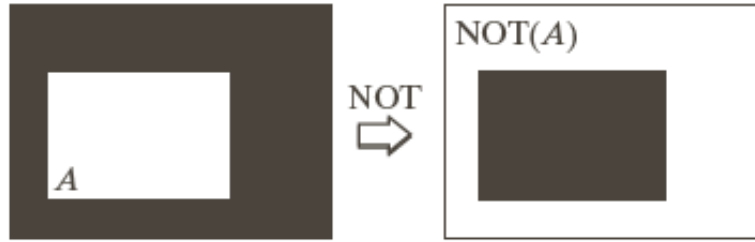
\* *Foreground (前景) and Background(背景):*

*the sets of pixels in an image defined to be objects and non-objects, respectively*

# Preliminaries --- Logical Operation

## Matlab Function:

- $\sim A$  / `not(A)`
- $A \& B$  / `and(A,B)`
- $A | B$  / `or(A,B)`
- $A \oplus B$  / `xor(A,B)`



# Structuring Element (结构元)

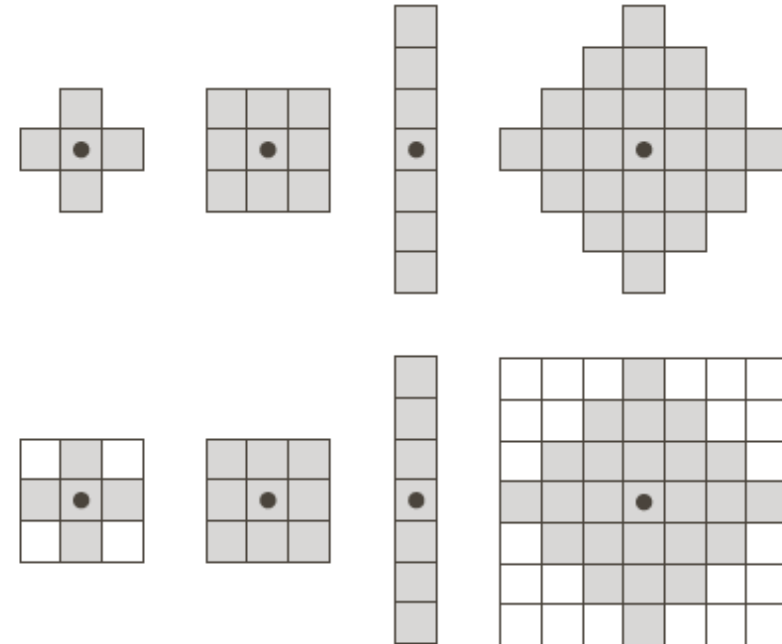
- **Structuring Element (SE)** --- small sets or subimages used to probe an image under study for properties of interest.

- **SE Selection**

- Simpler than the image
- With boundary
- Convex

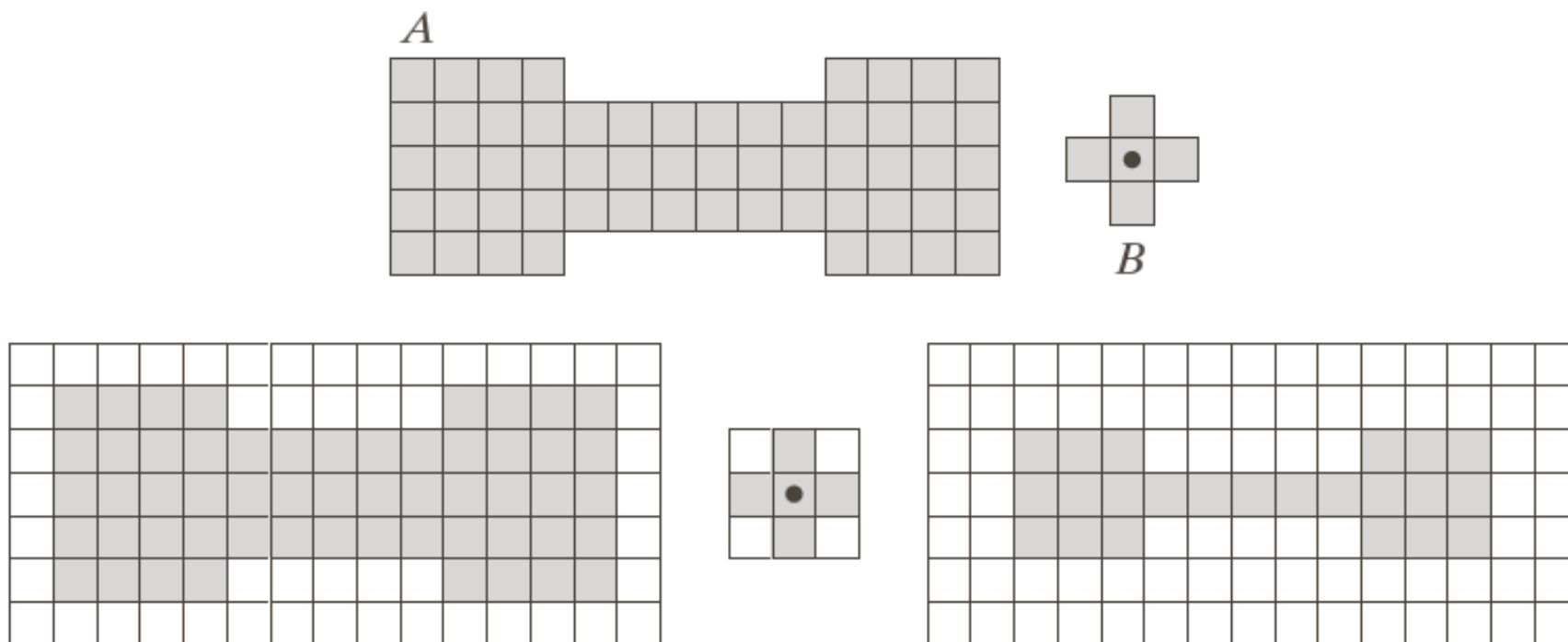
- **Structures**

- Origin
- Rectangular



# Structuring Element (结构元)

➤ **Matlab Function:** `se = strel(shape, parameters)`





# Erosion (腐蚀)

➤ Definition:

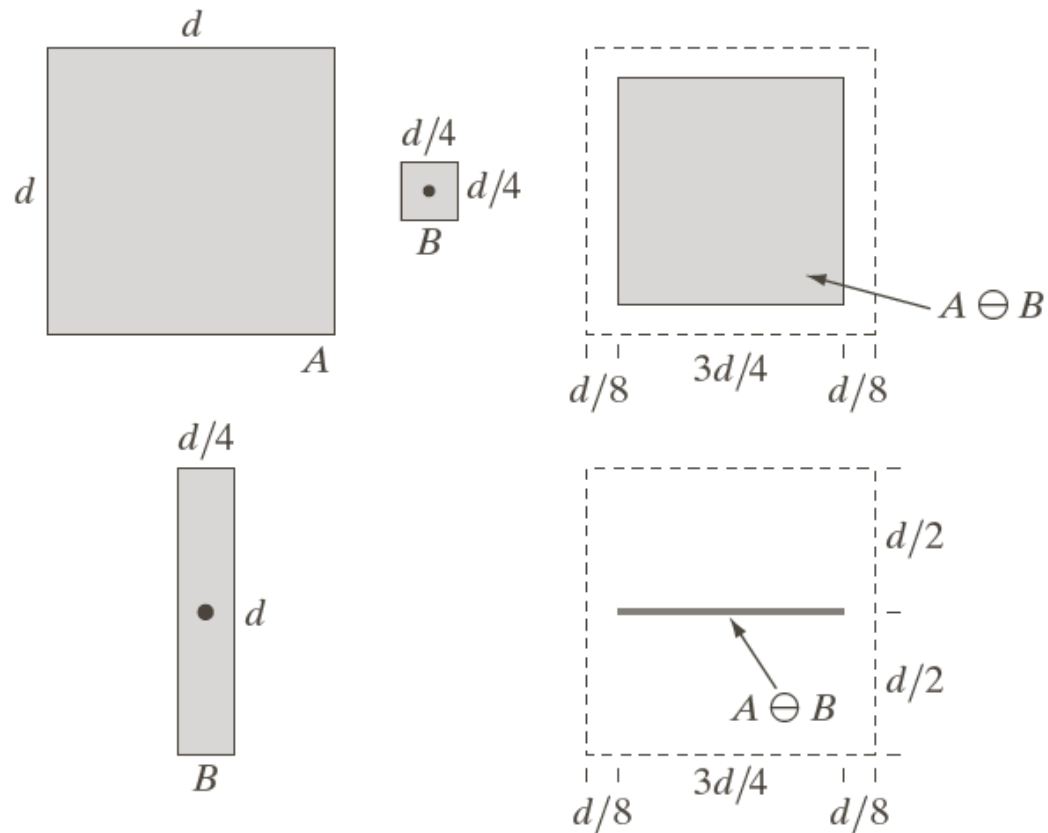
$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$

or

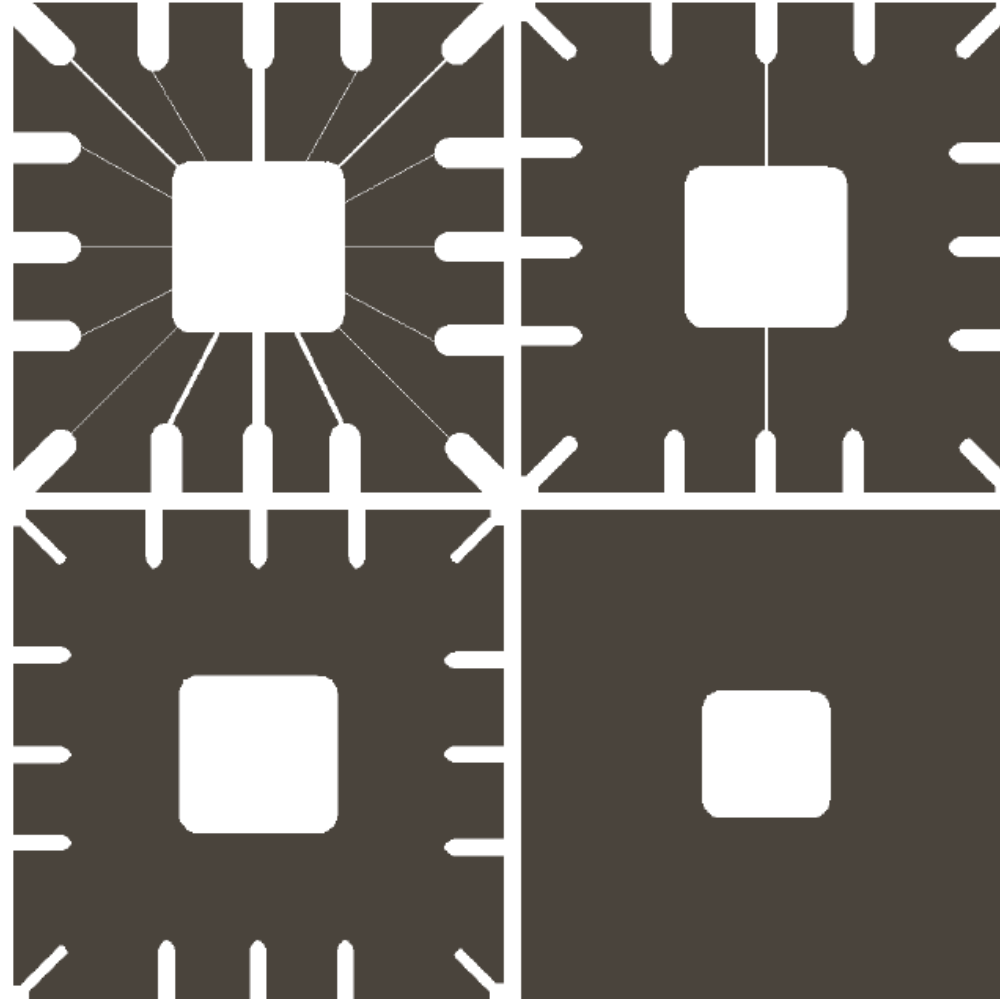
$$A \ominus B = \{z \mid (B)_z \cap A^c = \emptyset\}$$

➤ Matlab Function:

$$J = \text{imerode}(I, SE)$$



# Erosion (腐蚀)



# Dilation (膨胀)

➤ **Definition:**

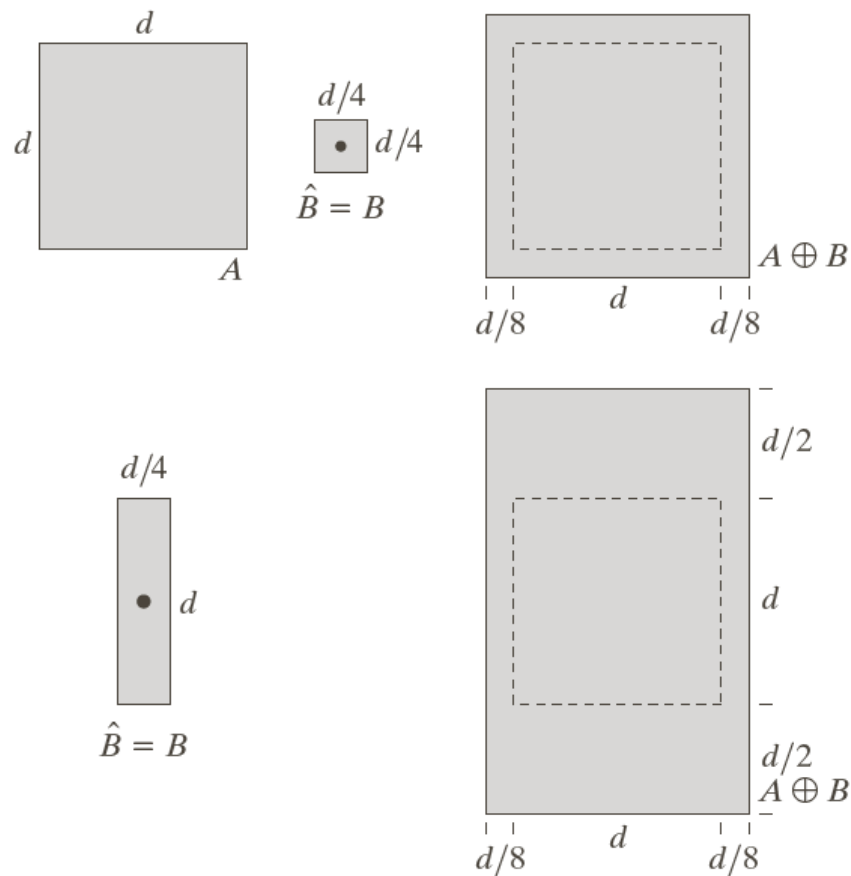
$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \emptyset\}$$

or

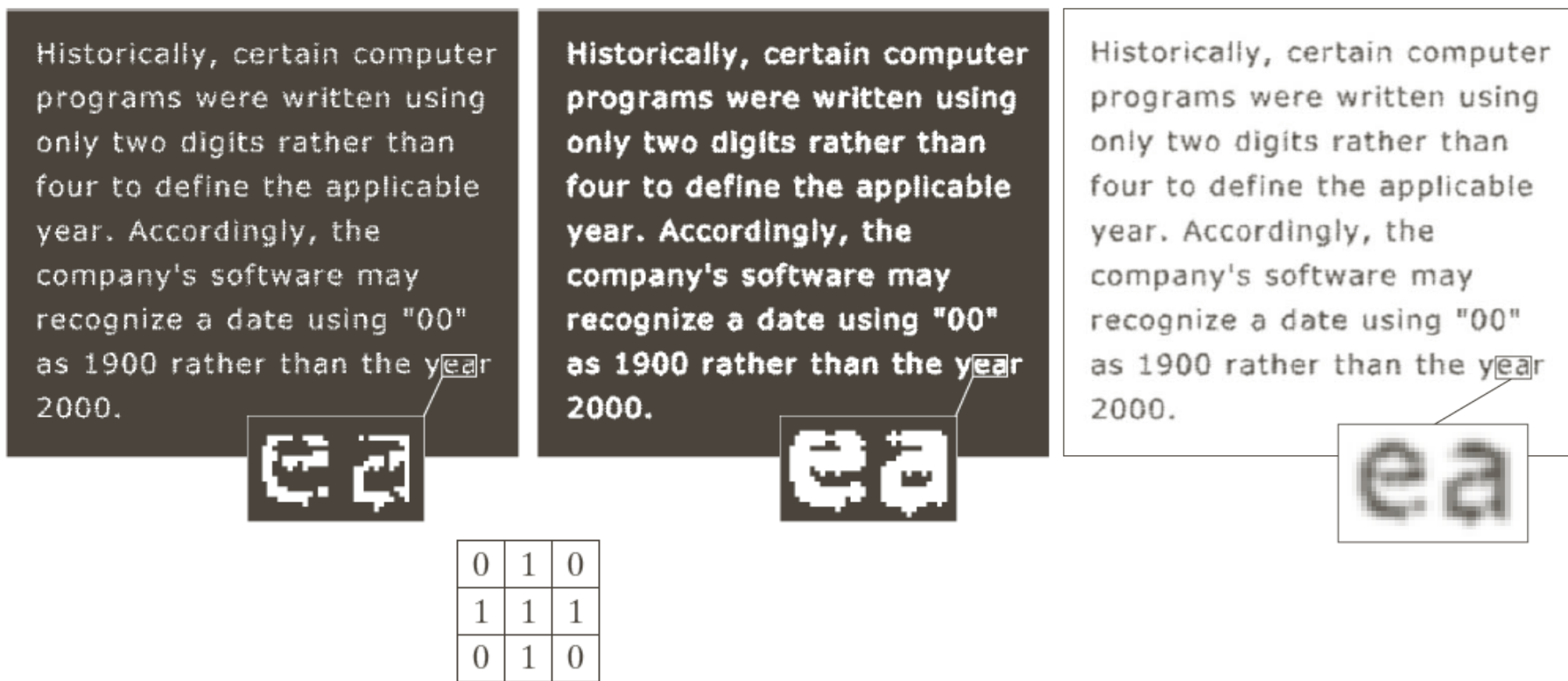
$$A \oplus B = \{z \mid [(\hat{B})_z \cap A] \subseteq A\}$$

➤ **Matlab Function:**

$$J = \text{imdilate}(I, \text{SE})$$



# Dilation (膨胀)



# Properties

## ➤ Duality (对偶性)

- $(A \ominus B)^c = (A)^c \oplus \hat{B}$

- $(A \oplus B)^c = (A)^c \ominus \hat{B}$

## ➤ Associativity (结合律)

$$(A \oplus B) \oplus C = A \oplus (B \oplus C)$$

or

$$(A \oplus B) = A \oplus (B_1 \oplus B_2) = (A \oplus B_1) \oplus B_2$$

# Opening & Closing (开操作和闭操作)

## ➤ Opening (开操作)

- Smooth the contour of an object
- Break narrow isthmuses
- Eliminate thin protrusions

## ➤ Closing (闭操作)

- Smooth the contour of an object
- Fuse narrow breaks and long thin gulfs
- Eliminate small holes
- Fill gaps in the contour



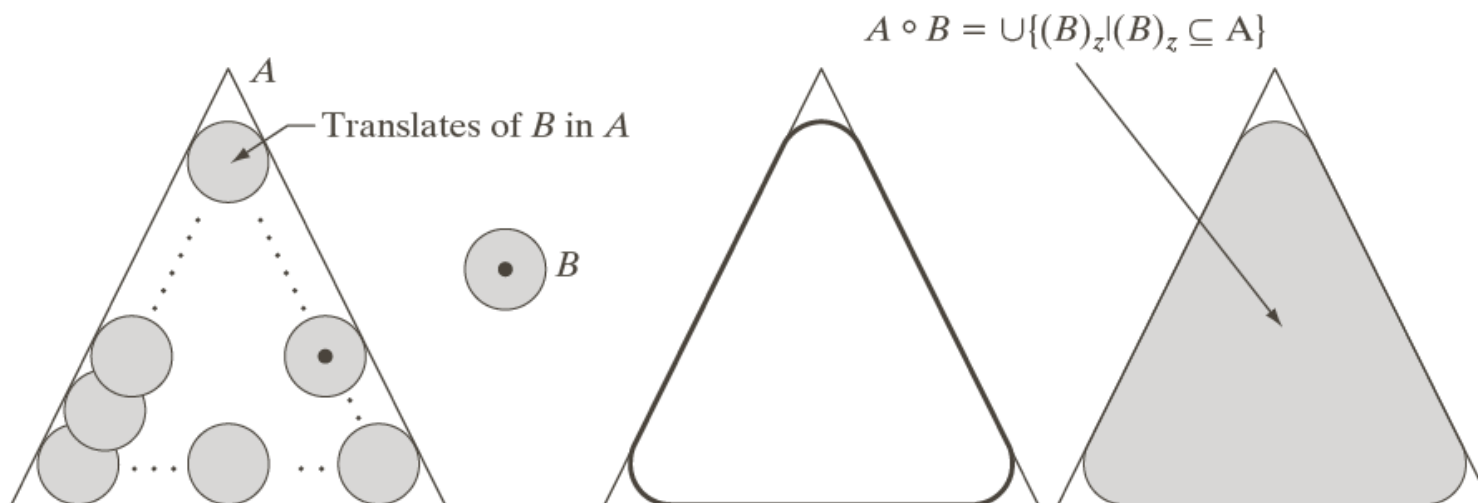
# Opening (开操作)

➤ **Definition:**  $A \circ B = (A \ominus B) \oplus B$  or  $A \circ B = \bigcup \{ (B)_z \mid (B)_z \subseteq A \}$

➤ **Matlab Function:**  $J = \text{imopen}(I, SE)$

➤ **Properties:**

- ①  $A \circ B$  is a subset (subimage) of  $A$
- ② If  $C$  is a subset of  $D$ , the  $C \circ B$  is a subset of  $D \circ B$
- ③  $(A \circ B) \circ B = A \circ B$



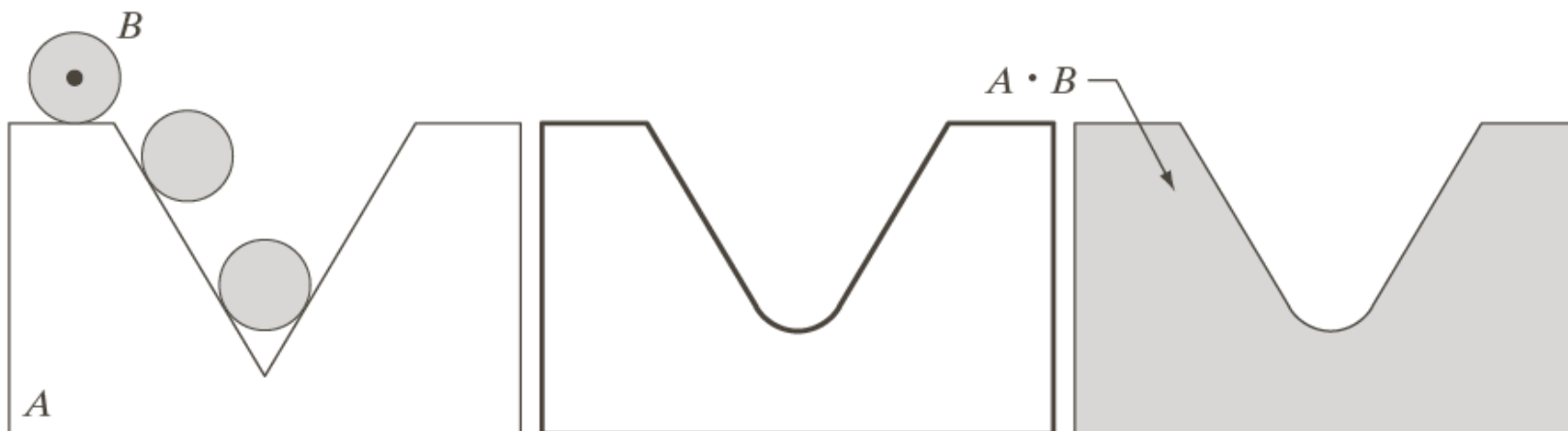
# Closing (闭操作)

➤ **Definition:**  $A \bullet B = (A \oplus B) \ominus B$

➤ **Matlab Function:**  $J = \text{imclose}(I, SE)$

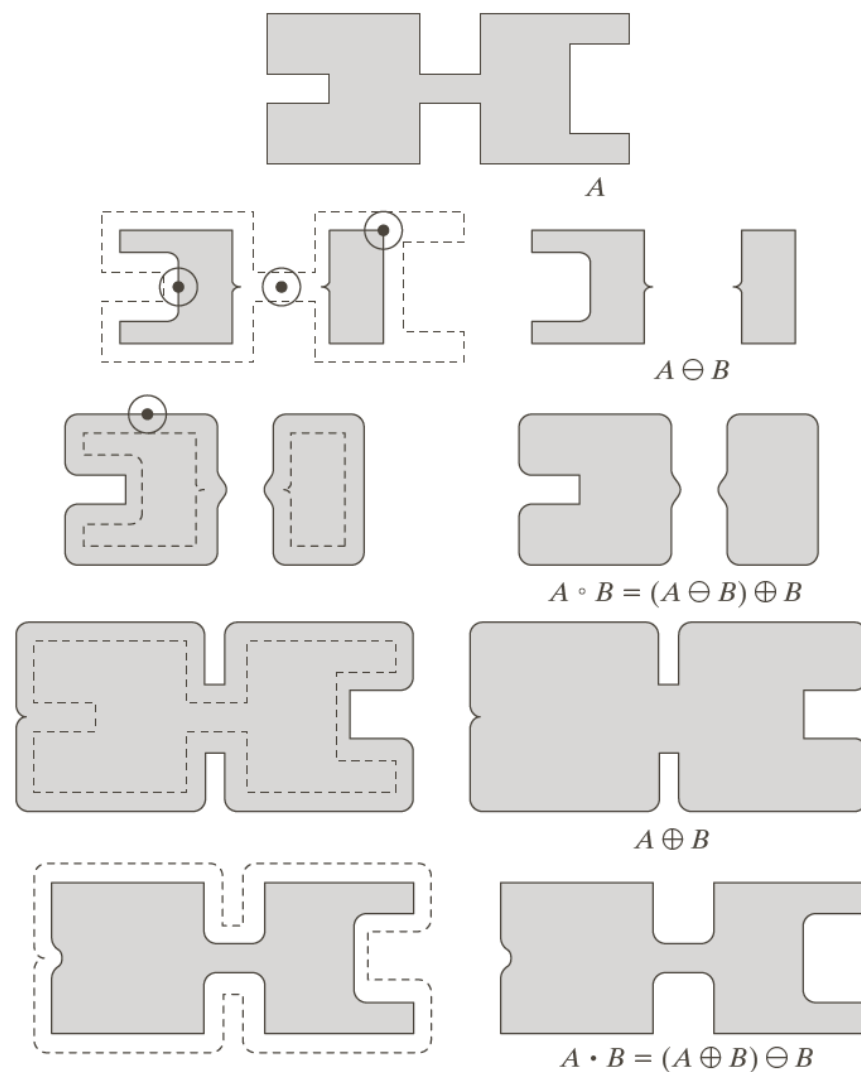
➤ **Properties:**

- ①  $A$  is a subset (subimage) of  $A \bullet B$
- ② If  $C$  is a subset of  $D$ , the  $C \bullet B$  is a subset of  $D \bullet B$
- ③  $(A \bullet B) \bullet B = A \bullet B$





# Opening & Closing (开操作和闭操作)



➤ Duality (对偶性)

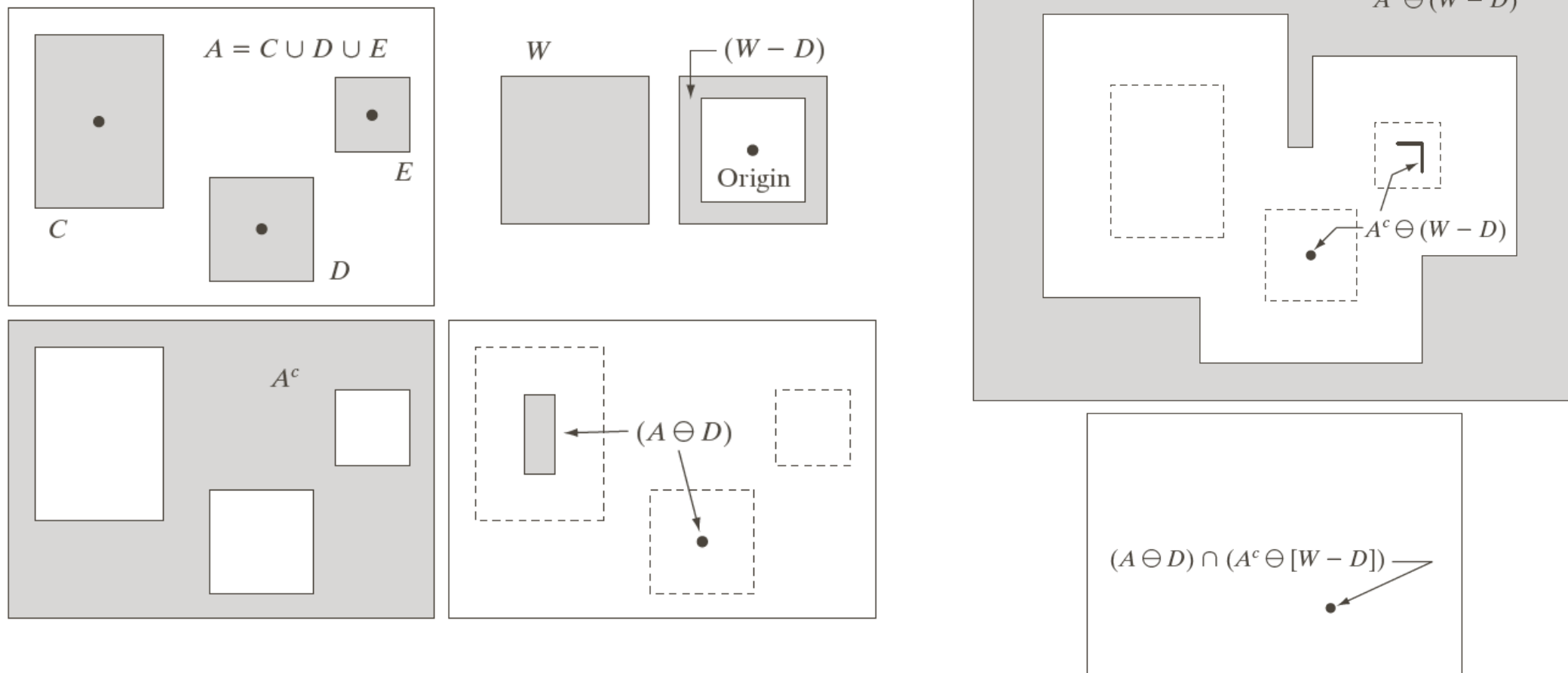
$$(A \bullet B)^c = (A)^c \circ \hat{B}$$

$$(A \circ B)^c = (A)^c \bullet \hat{B}$$

# Morphological Filtering (形态学滤波)



# The Hit-or-Miss Transformation (击中或击不中变换)



# The Hit-or-Miss Transformation (击中或击中不中变换)

➤  $A \circledast B = (A \ominus D) \cap [A^c \ominus (W - D)]$

➤  $A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$

where  $B = (B_1, B_2)$ ,

$B_1$  is associated with the object;

$B_2$  is associated with the corresponding background.

➤  $A \circledast B = (A \ominus B_1) - (A \oplus \hat{B}_2)$

where  $(A \oplus \hat{B}_2)^c = (A^c \ominus B_2)$  and  $M - N = M \cap N^c$

➤ **Matlab Function:**  $J = \text{bwhitmiss}(BW, SE1, SE2)$