Lecture 3 – Spatial Filtering（空间滤波）

This lecture will cover:

- Spatial domain（空间域）
- Intensity Transformation （灰度变换）
- Histogram（直方图）
- **Spatial Filtering（空间滤波器）**
  - ✓ **Smoothing（平滑）**
  - ✓ **Sharpening（锐化）**

Spatial Filtering

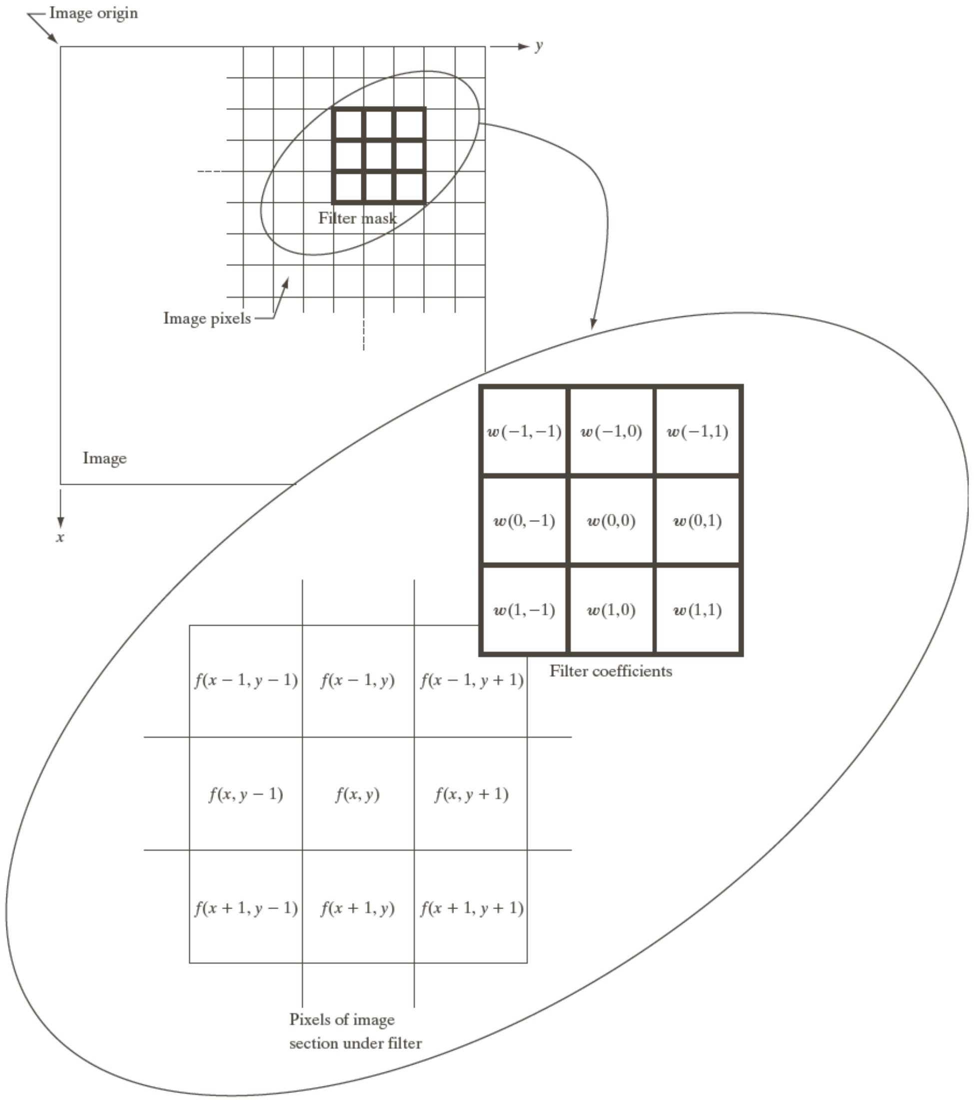
A Spatial filter

- **is directly applied on the image**
- **is also called spatial masks（掩模）、kernels（核）、templates（模板）、windows（窗口）**
- **consists of**

  1) neighborhood 2) a predefined operation

- **can be linear and nonlinear**
  - Linear spatial filter corresponds to spectral filter in frequency domain
  - Nonlinear spatial filter cannot be accomplished in frequency domain

Spatial Filter



$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

- $f(x,y)$: input image
- $g(x,y)$: output filtered image
- $w(s,t)$: $m \times n$ spatial filter, where $m = 2a+1, n = 2b+1$

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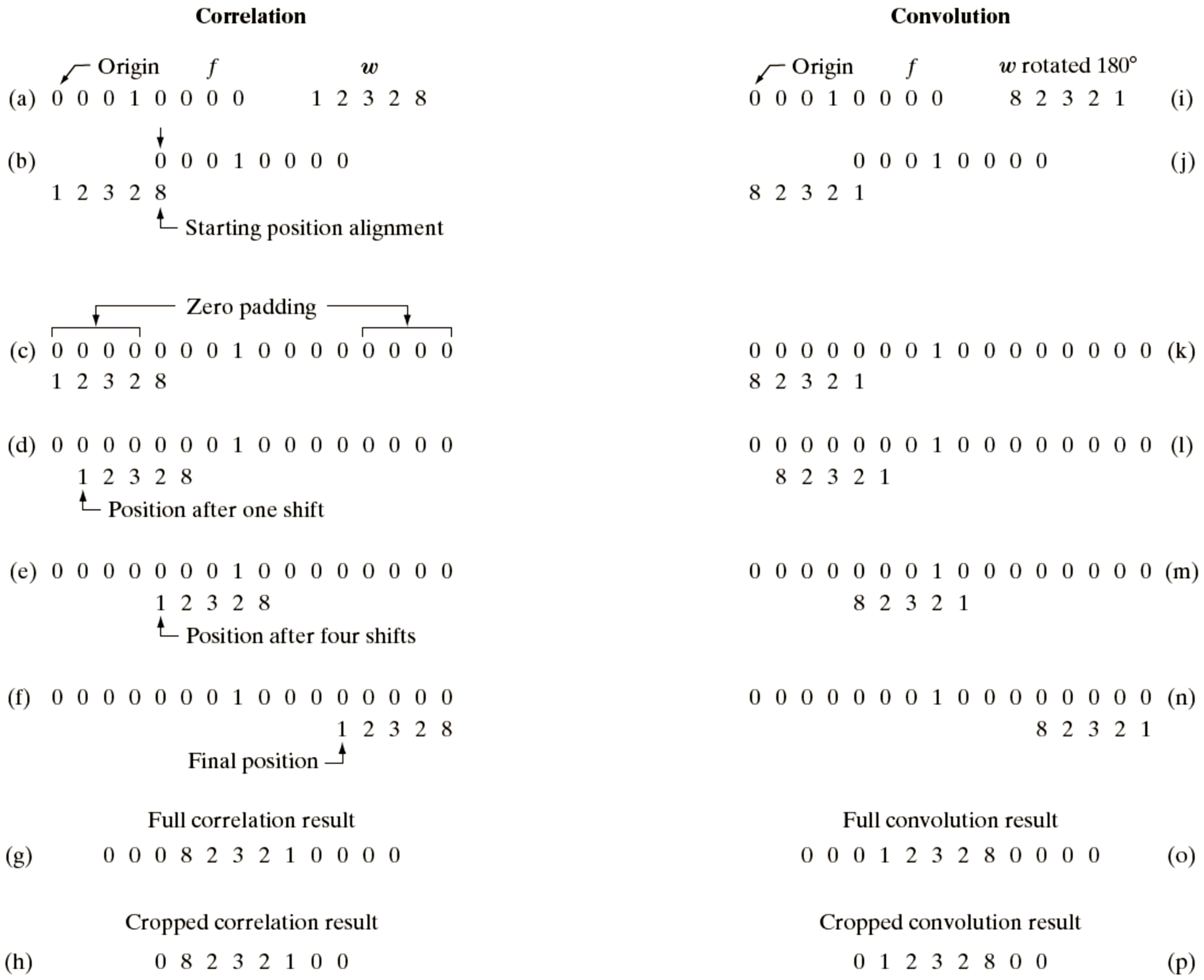
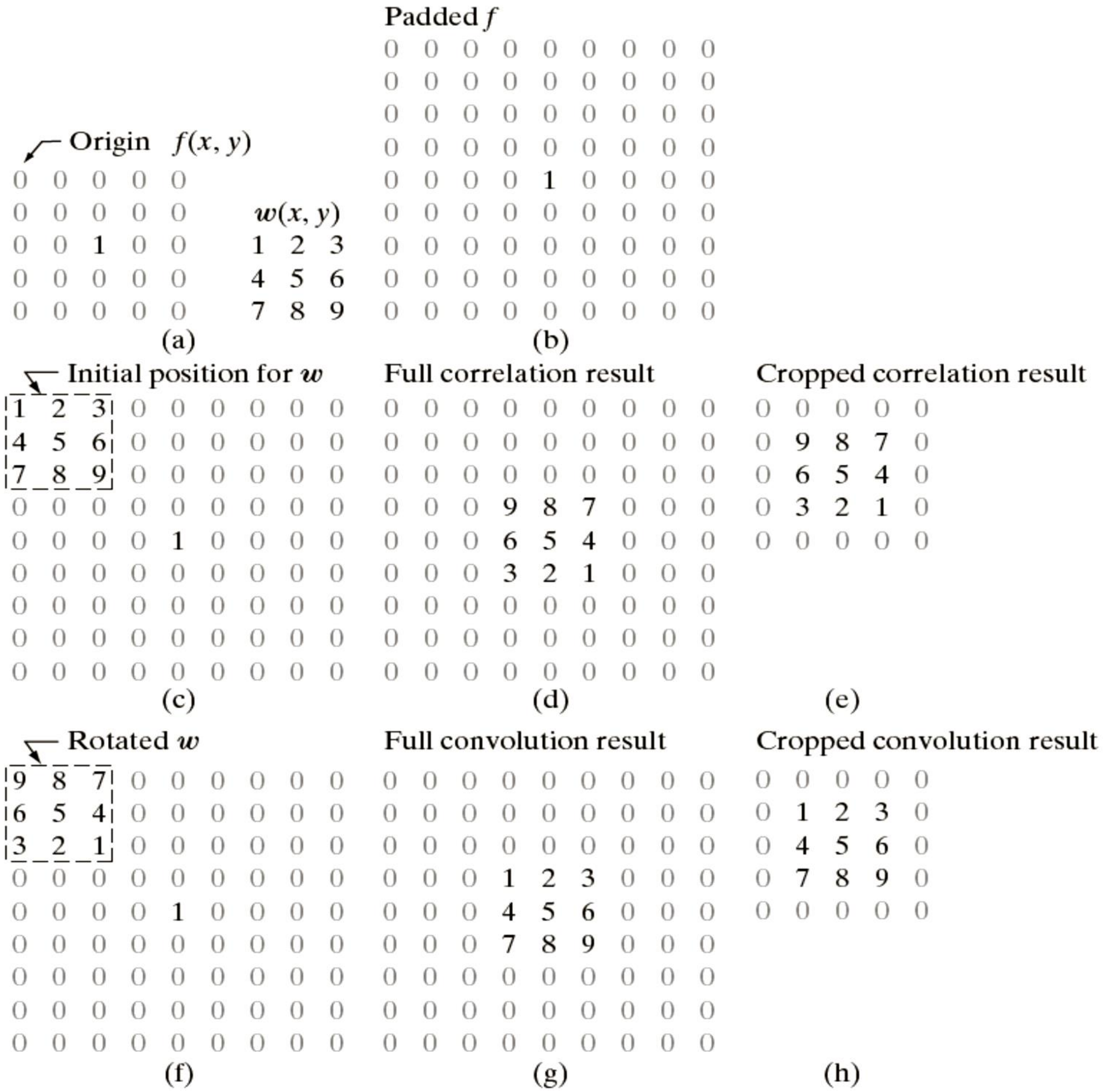
Correlation(相关) and Convolution(卷积) (1D)



Correlation and Convolution (2D)



Equations

Correlation

$$w(s,t) \star f(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

Convolution

$$w(s,t) \bigstar f(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x-s, y-t)$$

Vector Operation

$$R = w_1 z_k + w_2 z_2 + \cdots + w_{mn} z_{mn} = \sum_{k=1}^{mn} w_k z_k = w^T z$$

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

Spatial Filter Masks

➢ Linear Spatial Filter (线性滤波器)

- $R = \frac{1}{9}\sum_{k=1}^{9} z_k$

- $h(x, y) = e^{-\frac{x^2+y^2}{2\sigma^2}}$

➢ Nonlinear Spatial Filter（非线性滤波器）

- Max filter (最大值滤波)
- Median filter （中值滤波）

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Smooth Filters (平滑滤波器)

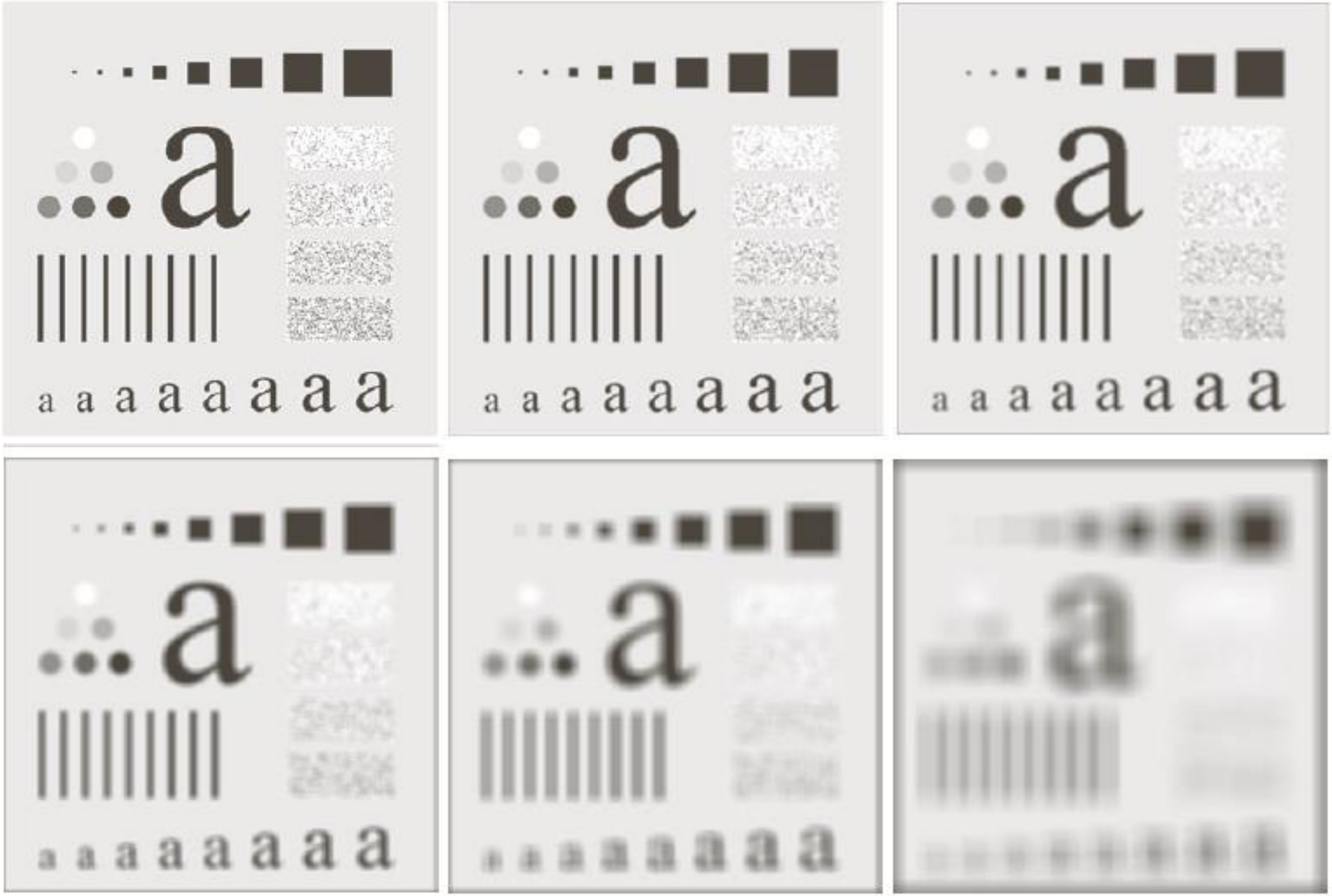
- Blurring – for preprocessing tasks
- Noise deduction
  - Linear filter : average filtering – lowpass filter in frequency domain
  - Nonlinear filter

Smooth Filters (平滑滤波器)

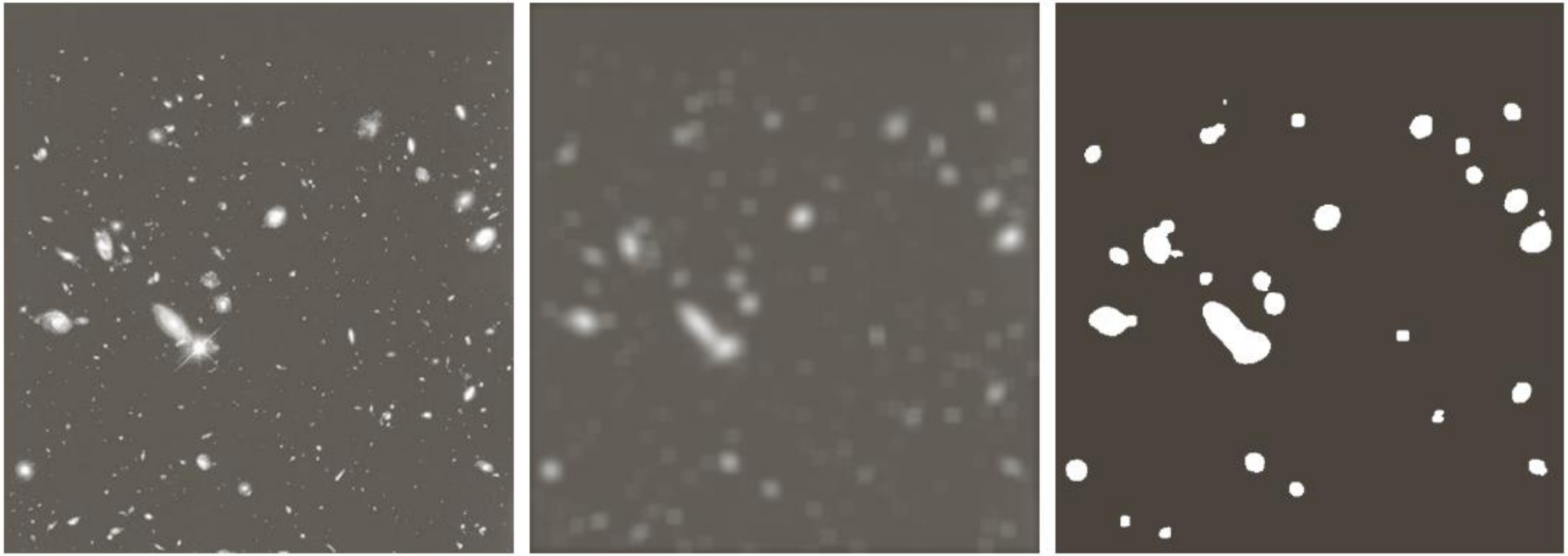
$$g(x,y)=\frac{\sum_{s=-a}^{a}\sum_{t=-b}^{b}w(s,t)f(x+s,y+t)}{\sum_{s=-a}^{a}\sum_{t=-b}^{b}w(s,t)}$$

$$\frac{1}{9}\times\begin{array}{|c|c|c|}\hline 1 & 1 & 1\\\hline 1 & 1 & 1\\\hline 1 & 1 & 1\\\hline\end{array}\qquad\frac{1}{16}\times\begin{array}{|c|c|c|}\hline 1 & 2 & 1\\\hline 2 & 4 & 2\\\hline 1 & 2 & 1\\\hline\end{array}$$

Filter size



Smooth Filter and Thresholding(阈值处理)



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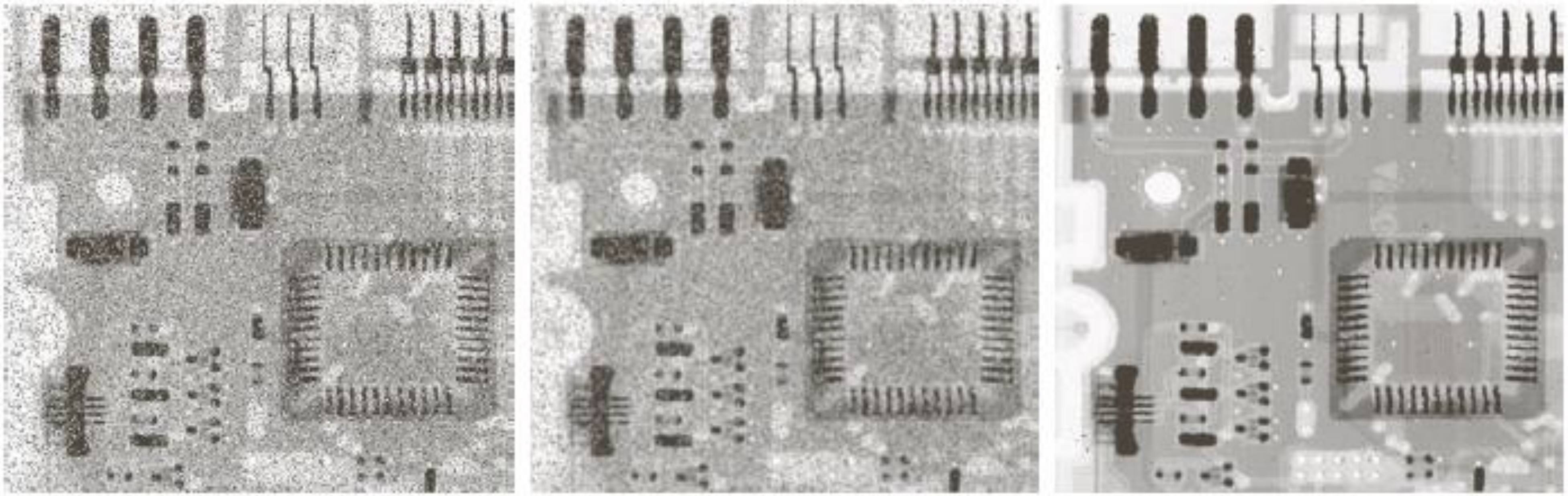
Nonlinear Smooth Filters

- **Order-statistic filter (统计排序滤波器)**

$$R = H\{z_k | k = 1,2,\cdots mn\}$$



Sharpening Filter

- Spatial differentiation（空间微分）

- Sharpening filter

  - Laplacian filtering（拉普拉斯算子）
  - Unsharp Masking（非锐化掩蔽）
  - Gradient filtering（梯度算子）

Sharpening Filter

- To highlight transitions in intensity

- Accomplished by spatial differentiation

  - First-order derivative: $\frac{\partial f}{\partial x} = f(x+1) - f(x)$

  - Second-order derivative: $\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$

Sharpening Filter

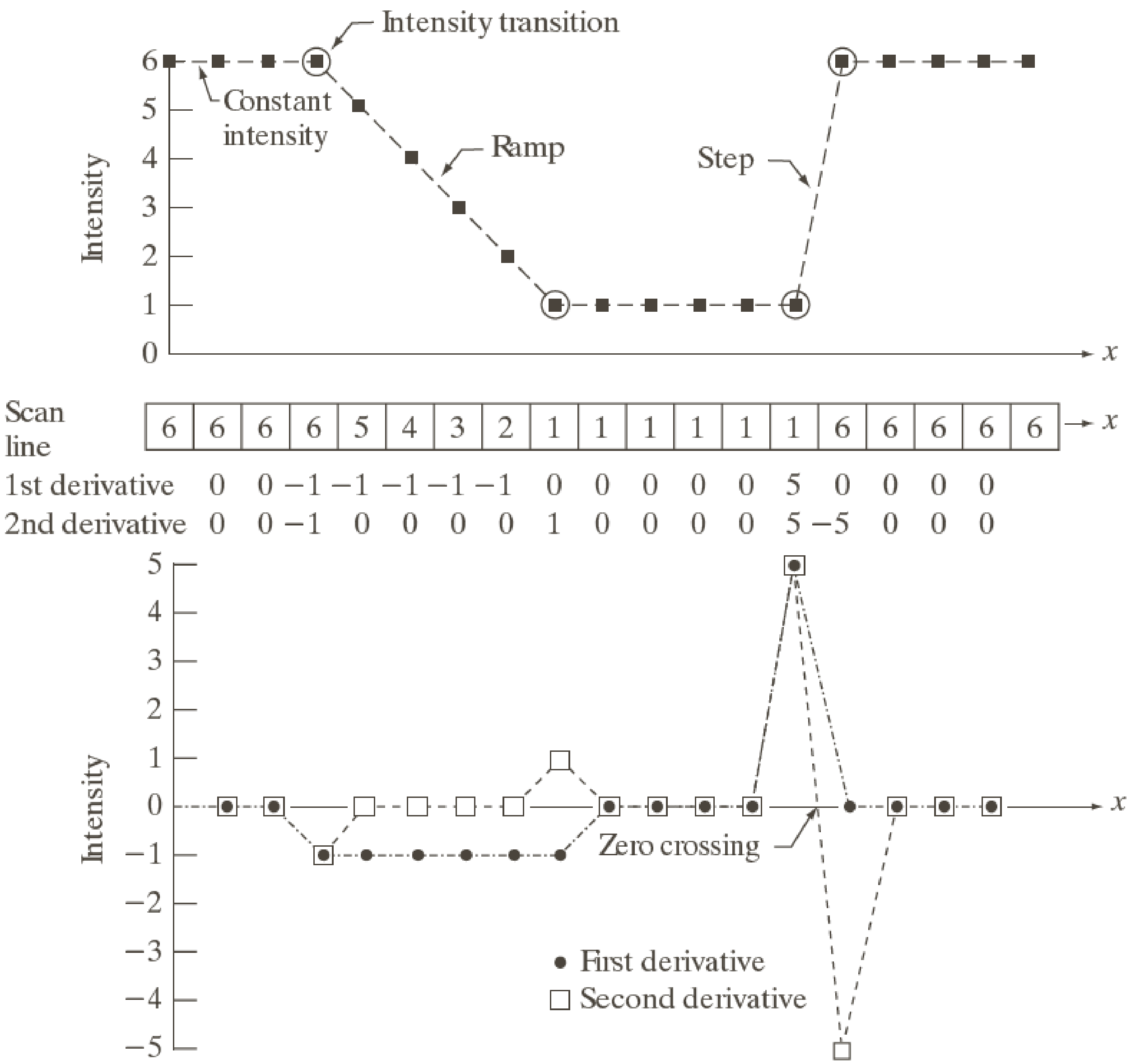
1. Zero in area of constant intensity

2. Nonzero at the onset of intensity step or ramp

3. (1) Nonzero along intensity ramp – 1st order derivative

   (2) Zero along intensity ramp with constant slope – 2nd order derivative

Derivative


Intensity transition
Constant intensity
Ramp
Step
Intensity
x
Scan line
6 6 6 6 5 4 3 2 1 1 1 1 1 1 6 6 6 6 6
1st derivative 0 0 −1 −1 −1 −1 −1 0 0 0 0 0 5 0 0 0 0
2nd derivative 0 0 −1 0 0 0 0 1 0 0 0 0 5 −5 0 0 0
Zero crossing
First derivative
Second derivative

Laplacian(拉普拉斯算子)

For an image function $\boldsymbol{f(x,y)}$,

X direction： $\frac{\partial^2 f}{\partial x^2} = f(x+1,y) + f(x-1,y) - 2f(x,y)$

Y direction： $\frac{\partial^2 f}{\partial y^2} = f(x,y+1) + f(x,y-1) - 2f(x,y)$

$$\nabla^2 f(x,y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$= f(x,y+1) + f(x,y-1) + f(x+1,y) + f(x-1,y) - 4f(x,y)$$

Laplacian Filter Masks

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

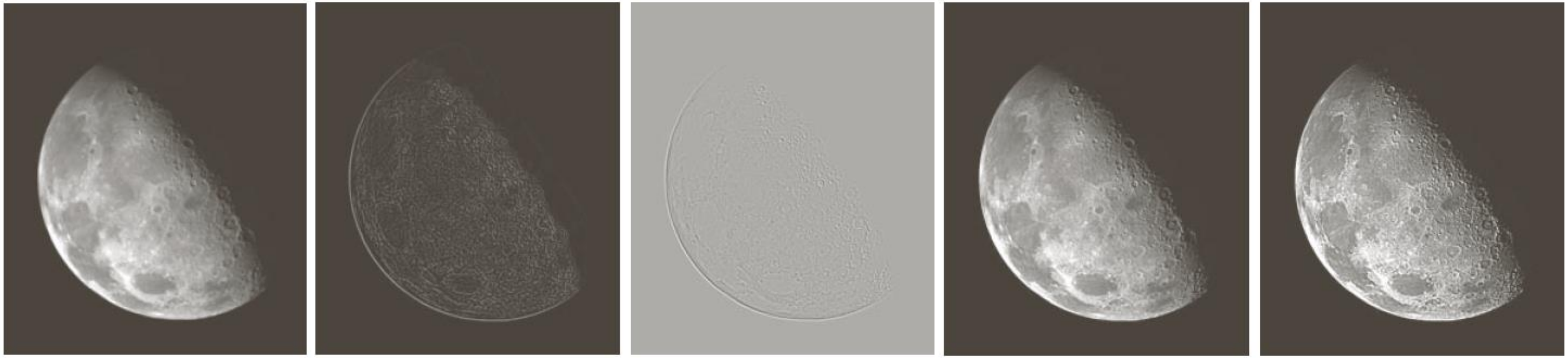
Laplacian Filter Masks

$$\boldsymbol{g(x,y) = f(x,y) + c\nabla^2 f(x,y),} \qquad \textbf{where } \boldsymbol{c = \pm 1}$$

$$\begin{array}{|c|c|c|}\hline & & \\ \hline & 1 & \\ \hline & & \\ \hline \end{array} - \begin{array}{|c|c|c|}\hline & 1 & \\ \hline 1 & -4 & 1 \\ \hline & 1 & \\ \hline \end{array} = \begin{array}{|c|c|c|}\hline & -1 & \\ \hline -1 & 5 & -1 \\ \hline & -1 & \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|}\hline & & \\ \hline & 1 & \\ \hline & & \\ \hline \end{array} + \begin{array}{|c|c|c|}\hline & -1 & \\ \hline -1 & 4 & -1 \\ \hline & -1 & \\ \hline \end{array} = \begin{array}{|c|c|c|}\hline & -1 & \\ \hline -1 & 5 & -1 \\ \hline & -1 & \\ \hline \end{array}$$

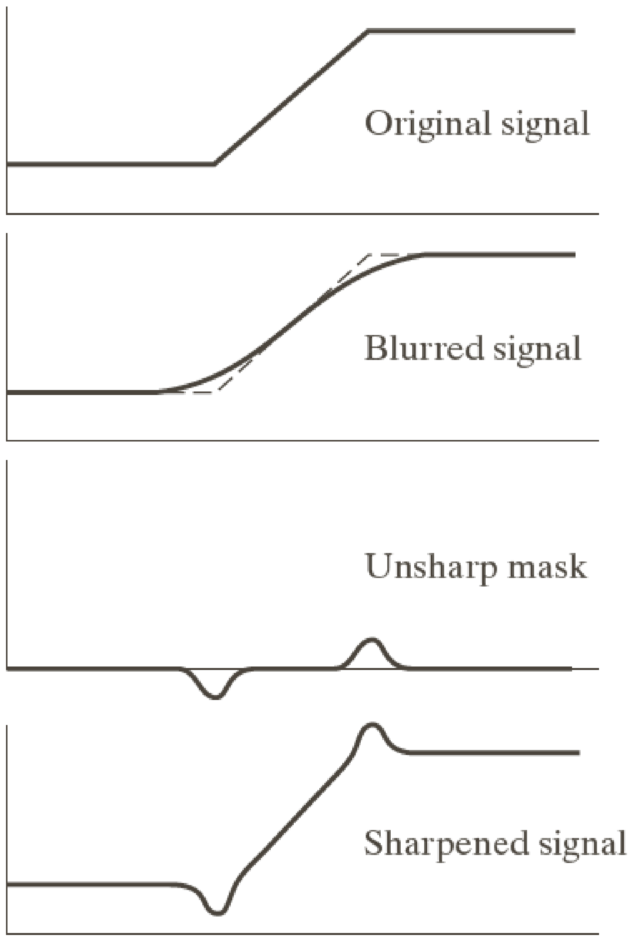
Image Sharpening with Laplacian



Unsharpen Mask(非锐化掩蔽)

$$g_{\text{mask}}(x,y) = f(x,y) - \overline{f(x,y)}$$

$$g(x,y) = f(x,y) + k * g_{\text{mask}}(x,y)$$



Gradient(梯度)

The first-order derivative of $f(x,y)$: $\boldsymbol{\nabla f} \equiv \mathbf{grad}(\boldsymbol{f}) \equiv \begin{cases} \boldsymbol{g_x} \\ \boldsymbol{g_y} \end{cases} = \begin{cases} \dfrac{\boldsymbol{\partial f}}{\boldsymbol{\partial x}} \\ \dfrac{\boldsymbol{\partial f}}{\boldsymbol{\partial y}} \end{cases}$

The amplitude： $\boldsymbol{M(x,y)} = \mathbf{mag}(\boldsymbol{\nabla f}) = \sqrt{\boldsymbol{g_x^2} + \boldsymbol{g_y^2}}$

$$\boldsymbol{M(x,y)} \approx |\boldsymbol{g_x}| + |\boldsymbol{g_y}|$$

Gradient(梯度)

➢ Roberts cross-gradient operator（罗伯特交叉梯度算子）

$$M(x,y) = |z_9 - z_5| + |z_8 - z_6|$$

➢ Sobel operator （Sobel算子）

$$M(x,y) = |(z_7+2z_8 + z_9) - (z_1+2z_2+z_3)|$$

$$+|(z_3+2z_6 + z_9) - (z_1+2z_4+z_7)|$$

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

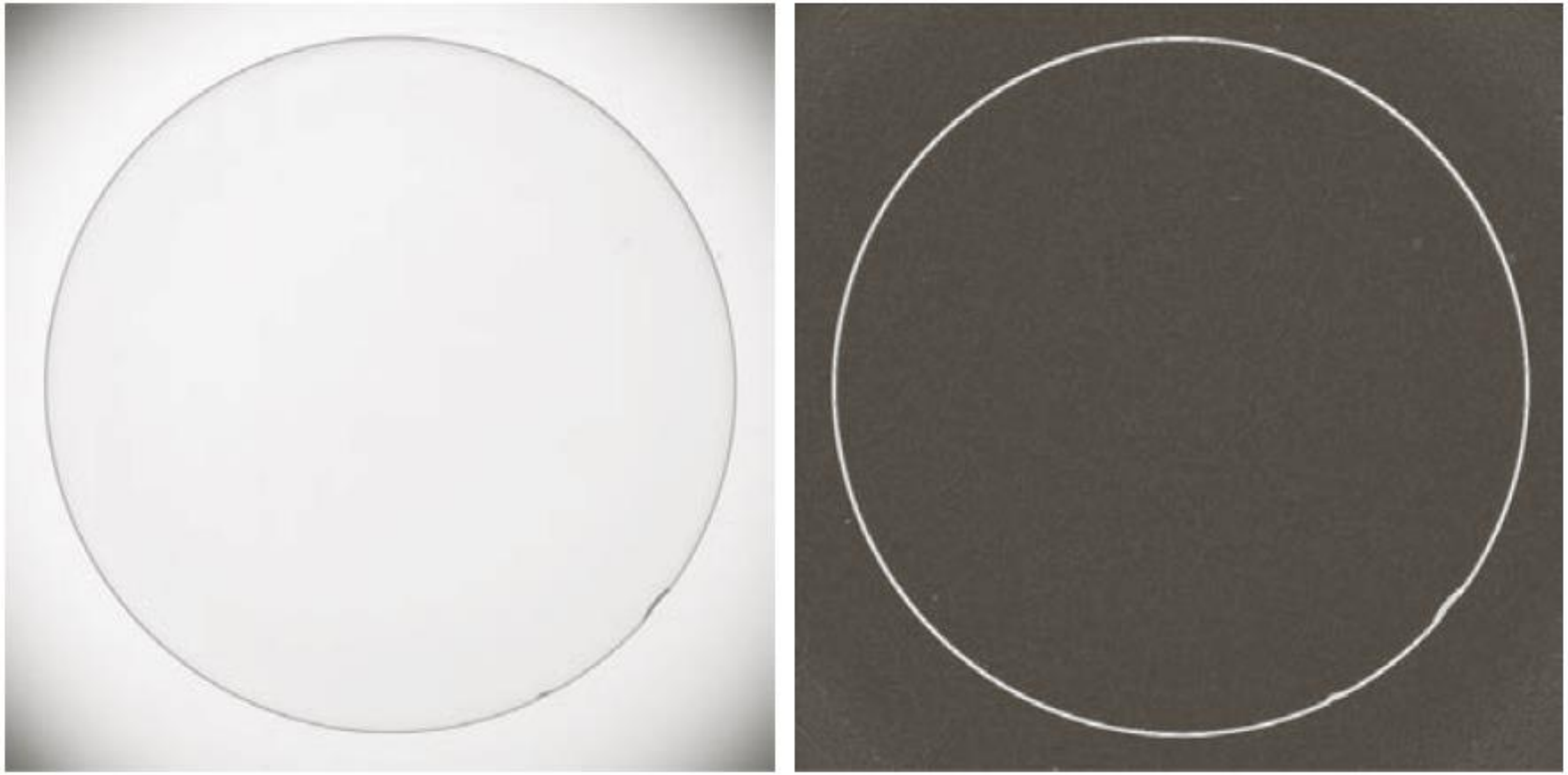
−1	0
0	1

0	−1
1	0

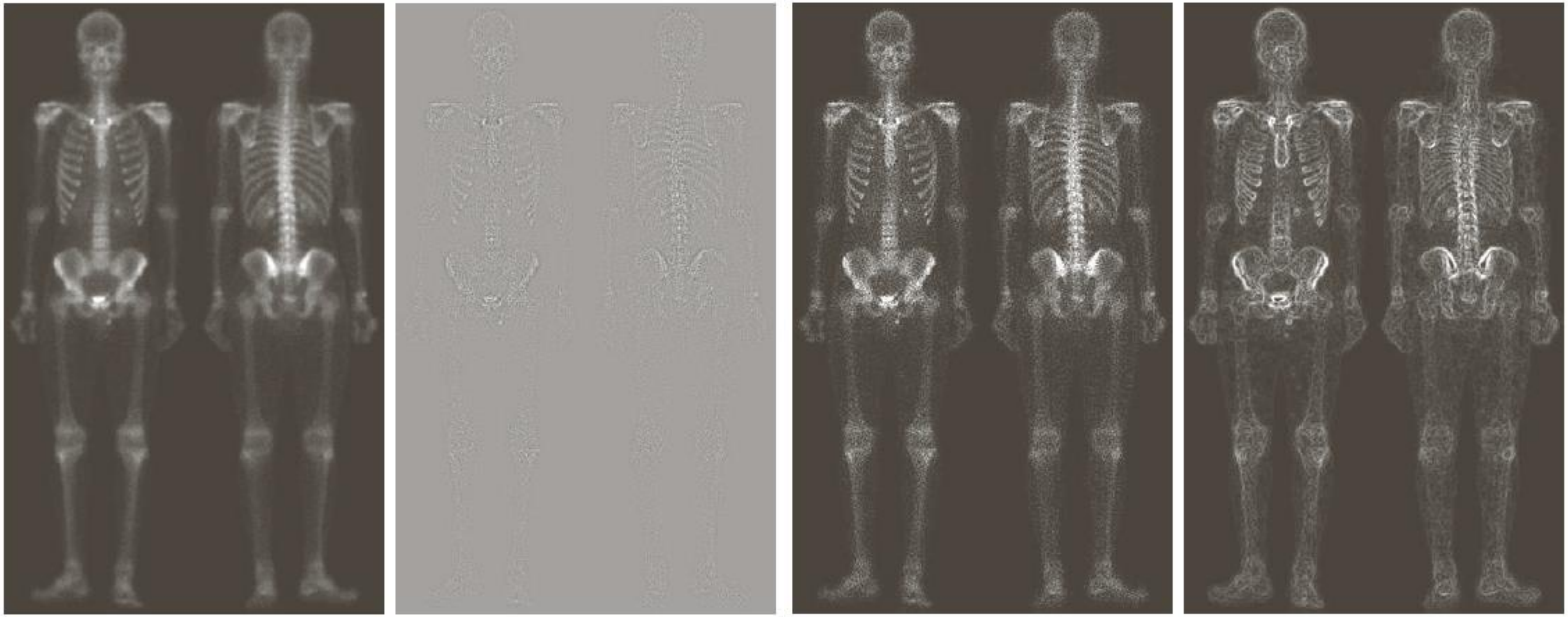
−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

Edge Enhancement



Combined Enhancement Methods



Combined Enhancement Methods