Deep Q-Networks

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Recap

Value function approximators

Instead of using tables, we will represent V and Q with parameterized functions

$$V(s, \mathbf{w}) \approx V^{\pi}(s)$$
$$(\text{or } Q(s, a, \mathbf{w}) \approx Q^{\pi}(s, a))$$

where V(s, **w**) can be a **linear combination of features**, a decision tree, K-nearest neighbor, a neural network.

How do we optimize V?

We iteratively optimize the approx. value function, minimizing the mean squared value error

$$MSVE(\mathbf{w}) = \Sigma_s d(s)[Q(s,a) - Q_\pi(s,a,\mathbf{w})]^2$$

where d: S -> [0,1], such that $\Sigma_s d(s)=1$, is a distribution over the states

We seek for $\mathbf{w}^*$ s.t. $RMSE(\mathbf{w}^*) <= RMSE(\mathbf{w})$ for all w

Cool, but how?

Optimization through Gradient Descent

Gradient Descent optimize a function f(x,**w**) by minimizing an error J(**w**). This is done by iteratively update w in the following way:

$$\mathbf{w}_{t+1} = \mathbf{w}_t - \alpha \nabla_{\mathbf{w}} J(\mathbf{w}_t)$$

Using the mean squared error (MSE):

$$\mathbf{w}_{t+1} = \mathbf{w}_t - \alpha \nabla_{w} [\tfrac{1}{2}(f^* - f(x_t, \mathbf{w}_t))^2] =$$

$$= \mathbf{w}_t - \alpha(f^* - f(x_t, \mathbf{w}_t)) \nabla_{w} f(x_t, \mathbf{w}_t)$$

Optimization through Gradient Descent

Remembering that our function f is Q(s,a,**w**) and that our f^* is Q^π we obtain:

$$\mathbf{w}_{t+1} = \mathbf{w}_t - \alpha \nabla_{\mathbf{w}}[\tfrac{1}{2}(Q^\pi_t - Q(s_t,a_t,\mathbf{w}_t))^2] =$$

$$= \mathbf{w}_t - \alpha(Q^\pi_t - Q(s_t,a_t,\mathbf{w}_t)) \nabla_w Q(s_t,a_t,\mathbf{w}_t)$$

Ok cool again, but what about $Q(s_t,a_t,w_t)$?

Linear Combination State-Action Value Approx.

If we represent our state with a feature vector

$$\mathbf{x}(s,a) = (x_1(s,a), \ldots, x_n(s,a))^T$$

And consider a parametric Q function in the form

$$Q(s,a,\mathbf{w}) = \mathbf{w}^T \mathbf{x}(s,a)$$

Then

$$J(w_t) = \mathbb{E}(Q^\pi(s_t,a_t) - \mathbf{w}^T \mathbf{x}(s_t,a_t))^2$$

$$\mathbf{w}_{t+1} = \mathbf{w}_t + \alpha(Q^\pi{}_t - \mathbf{w}^T{}_t \mathbf{x}(s_t,a_t))\ \mathbf{x}(s_t,a_t)$$

Incremental Prediction Algorithms (forward-view)

What exactly is the target Q^{π}_t?

- For MC, the target is the return G_t
- For TD(0) the TD target is $R_{t+1} + \gamma Q(s_{t+1}, a_{t+1}, \mathbf{w})$
- For forward-view TD(λ), the target is G_t^{λ}
- For backward-view TD(λ) the update is

$$\mathbf{w}_{t+1} = \mathbf{w}_t - \alpha\delta_t e_t$$

$$\delta_t = R_{t+1} + \gamma Q(s_{t+1},a_{t+1},\mathbf{w}) - Q(s_t,a_t,\mathbf{w})$$

$$e_t = \gamma\lambda e_{t-1} + \nabla_w Q(s_t,a_t,\mathbf{w})$$

Least Squares

Given a Q function approx. $Q(s,a,\mathbf{w}) \sim Q^{\pi}(s,a)$

And an experience $D = \{(s_1,a_1,Q^{\pi}_1),(s_2,a_2,Q^{\pi}_2),...,(s_n,a_n,Q^{\pi}_n)\}$

The least squares algorithm finds a parameter vector **w** minimizing the error:

$$J(\mathbf{w}) = \Sigma_i \, (Q^{\pi}_i - Q(s_i,a_i,\mathbf{w}))^2$$

with the Stochastic Gradient Descent or with the closed form solution (linear version)

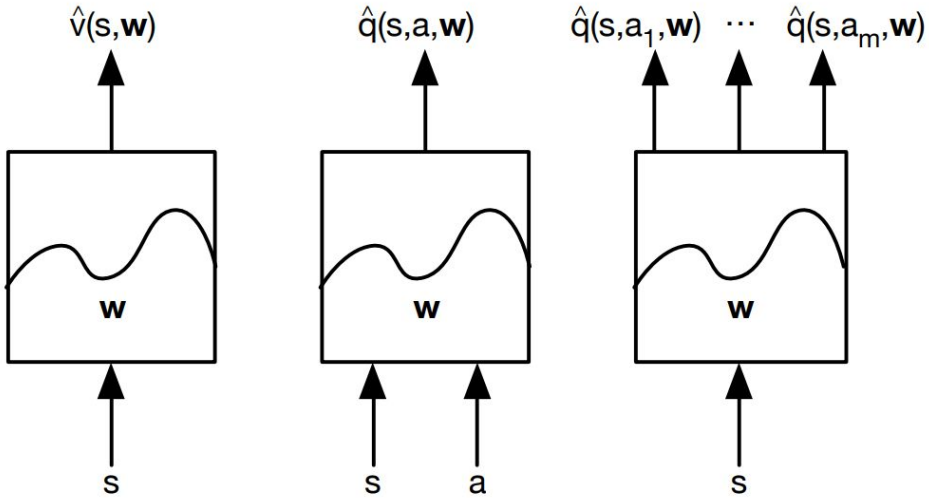
What about the features?

- Feature representation plays an important role
- Most times features should be inspected in combination:

Is the pendulum swinging up or down? The angle and the angular velocity shouldn’t be used independently

End Recap

Types of VFA



Credits: David Silver

Stochastic Gradient Descent with Experience Replay

Given an experience D = $\{(s_1,a_1,Q^{\pi}{}_1),(s_2,a_2,Q^{\pi}{}_2),...,(s_n,a_n,Q^{\pi}{}_n)\}$

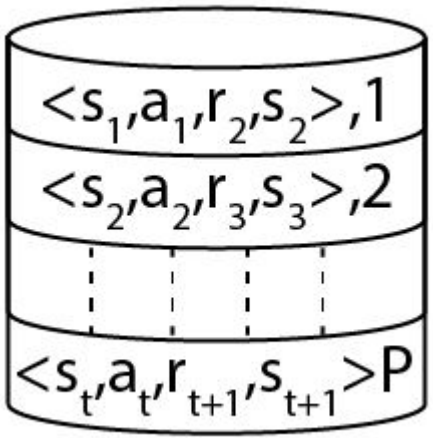
Repeat:

- Sample $(s,a,Q^{\pi}) \sim D$
- Apply the stochastic gradient descent update

$$\boldsymbol{\Delta w}_t = \alpha(Q^{\pi} - Q(s,a,\mathbf{w}))\ \nabla_w Q(s,a,\mathbf{w})$$

Replay Buffer

- Used in most recent RL algorithms
- Stores previous transitions experienced by an agent
- Transitions are used later in time for training

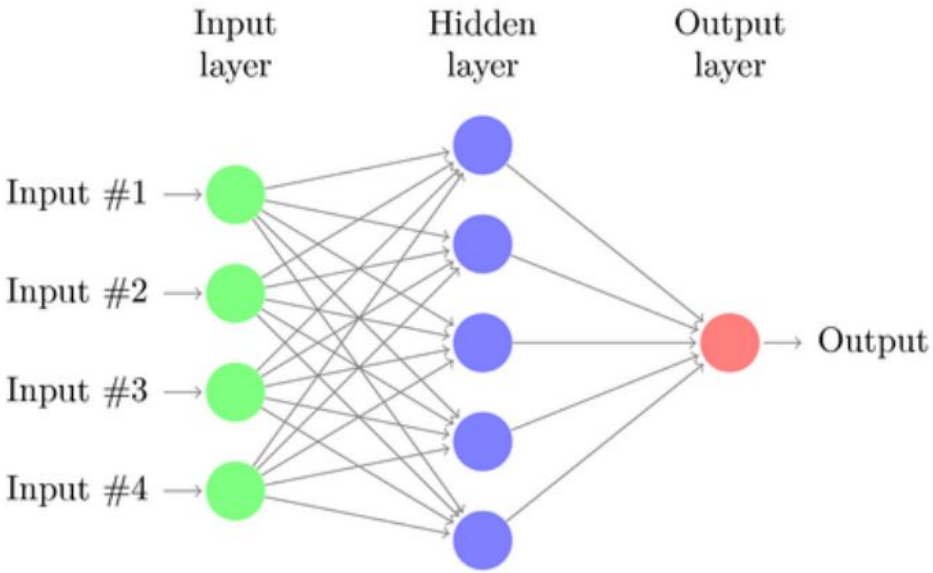


- **Capacity:** total number of transitions stored in the buffer
- **Age of transitions:** number of gradient steps taken by the learner since the transition was generated

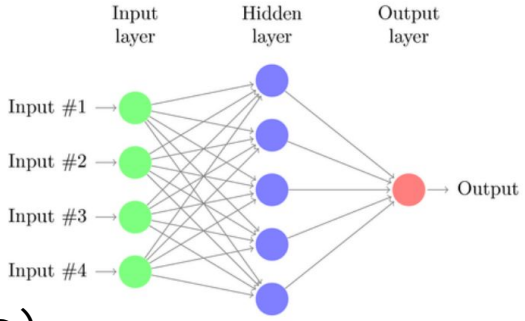
Replay Buffer - Why?

- Efficient use of previous experience
  - Same data used multiple times
  - Collecting real data costs
- Better convergence with function approximators
  - Makes data more i.i.d.
  - More similar to a supervised learning task

Neural Networks



Deep Neural Networks



- Composition of multiple layers (functions)
- To fit the parameters, require a loss function (a measure of error)
- Use chain rule to propagate the error
- Combine linear and non-linear transformations

Deep Neural Networks

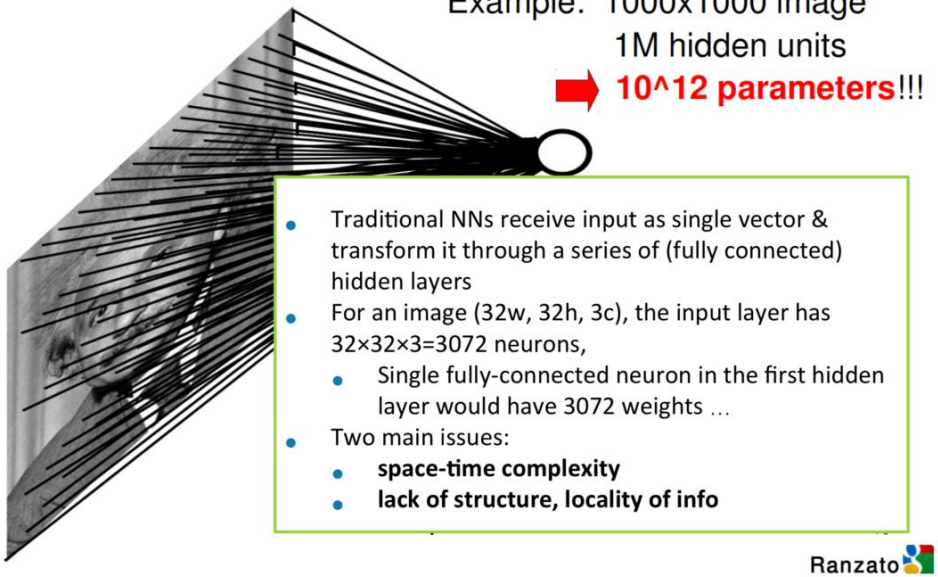
Why DNNs instead of feature engineering + other function approximators?

Deep Neural Networks

Why DNNs instead of feature engineering + other function approximators?

- Use distributed representations instead of local representations (like Kernels in SVMs)
- Universal function approximators
- Can potentially need exponentially less nodes/parameters (compared to a shallow net) to represent the same function
- Can learn the parameters using stochastic gradient descent

Convolutional Neural Networks



Convolutional Neural Networks

But images have structure!

Convolutional Neural Networks

But images have structure!

Consider local structure and common extraction of features

Convolutional Neural Networks

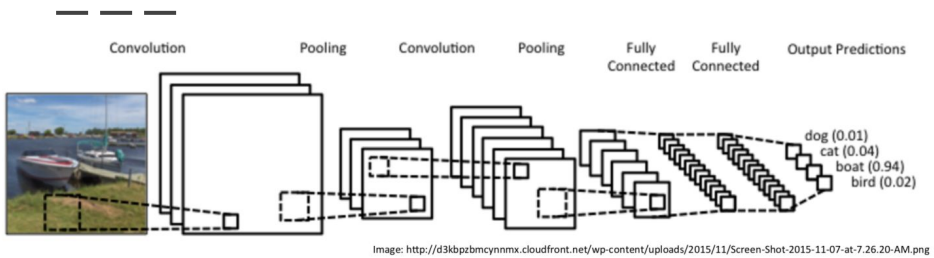
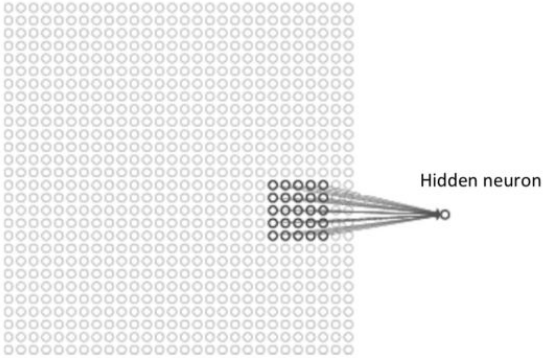
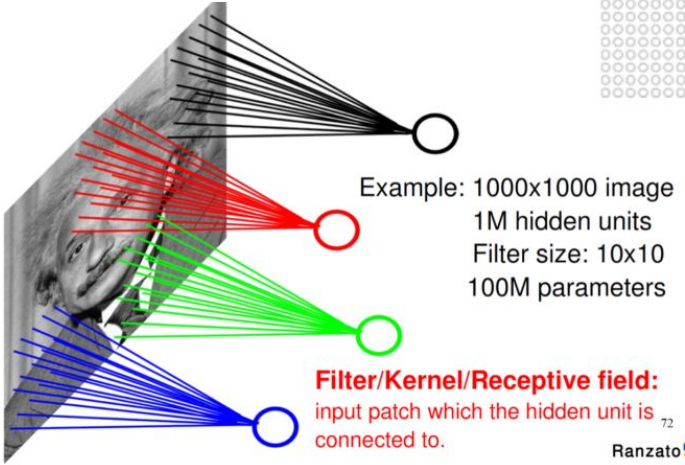


Image: http://d3kbpzbmcynnmx.cloudfront.net/wp-content/uploads/2015/11/Screen-Shot-2015-11-07-at-7.26.20-AM.png





Ranzato

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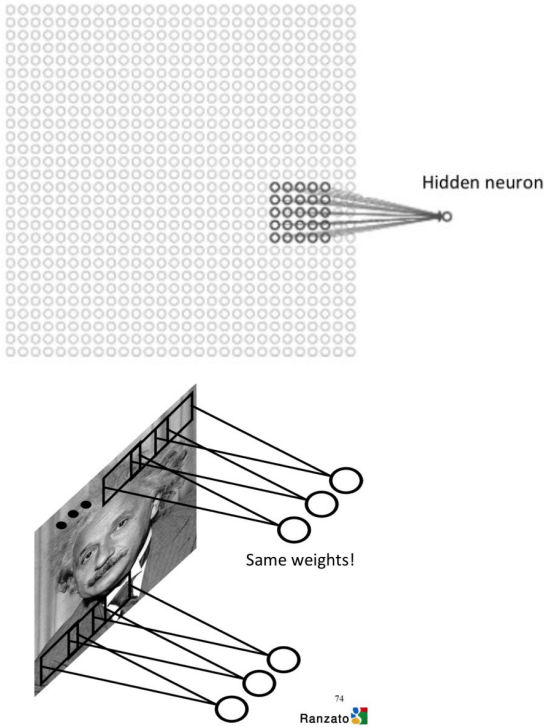
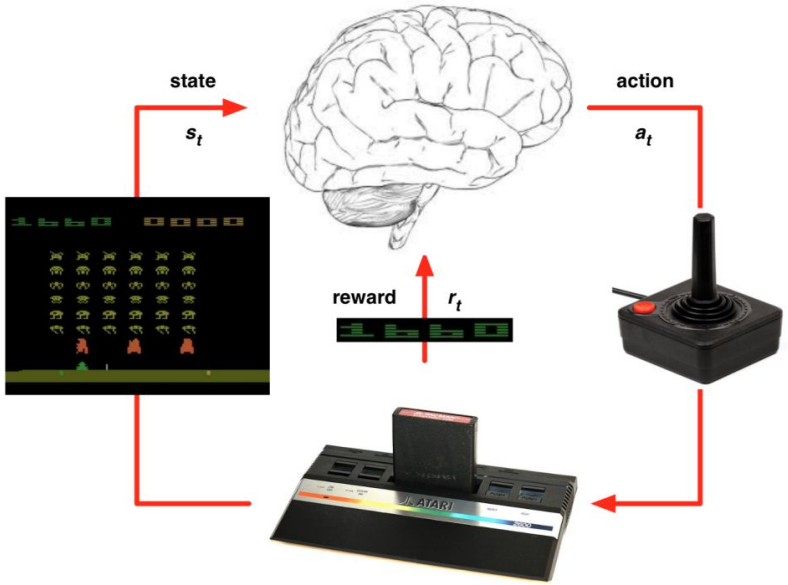
Convolutional Neural Networks

The same weights and bias are used for each of the hidden neurons

All the neurons in the first hidden layer detect exactly the same feature, just at different locations in the input image

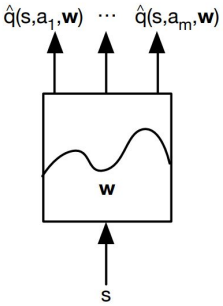


CNNs in Atari

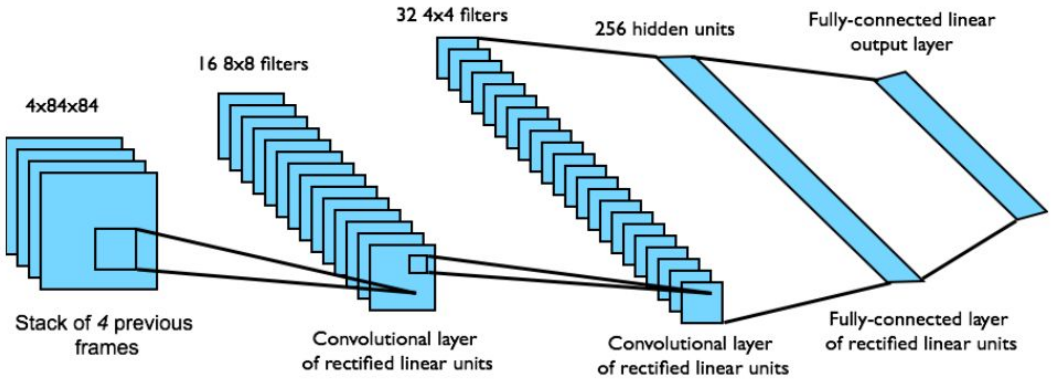


Credits: Emma Brunskill

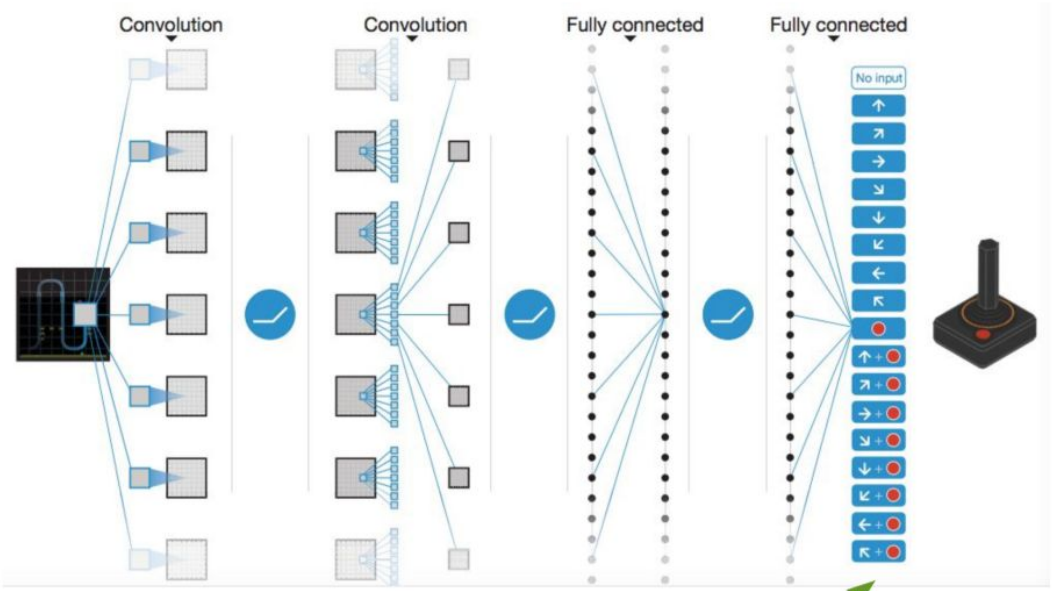
DQN



- End-to-end learning of values Q(s, a) from pixels s
- Input state s is stack of raw pixels from last 4 frames
- Output is Q(s, a) for 18 joystick/button positions
- Reward is change in score for that step



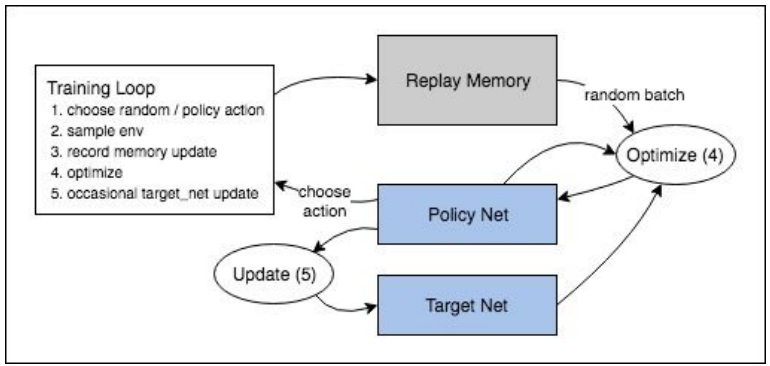
DQN


Convolution
Convolution
Fully connected
Fully connected
No input
1 network, outputs Q value for each action

DQN

DQN uses experience replay:

- The sampled tuple is used to compute the update targets



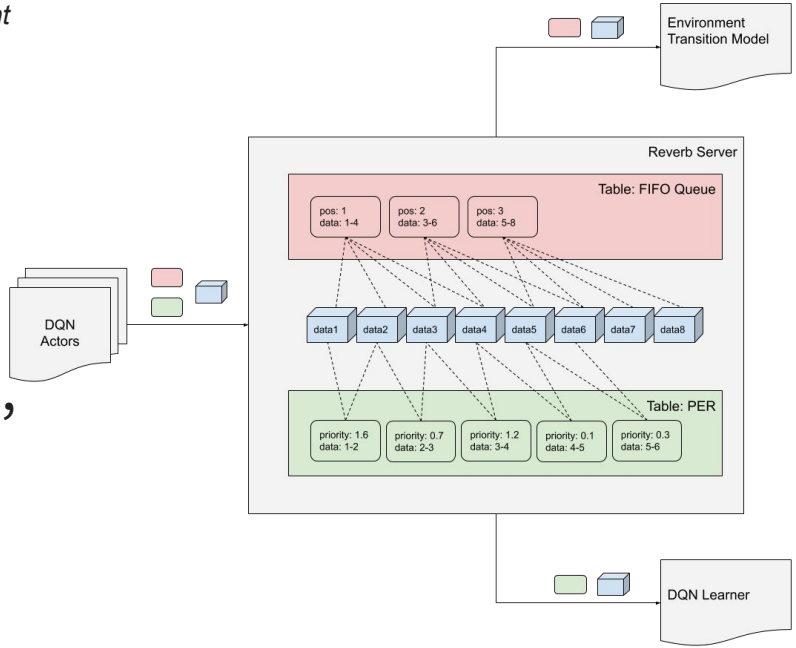
s_1, a_1, r_2, s_2
s_2, a_2, r_3, s_3
s_3, a_3, r_4, s_4
...
$s_t, a_t, r_{t+1}, s_{t+1}$

$\rightarrow \quad s, a, r, s'$

Reverb

Cassirer, Albin, et al. "Reverb: A Framework For Experience Replay." *arXiv preprint arXiv:2102.04736* (2021). - Code: https://github.com/deepmind/reverb

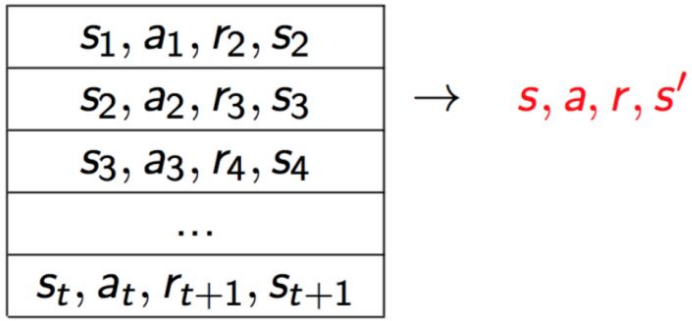
- Efficient and easy-to-use data storage and transport system
- Experience replay system for distributed reinforcement learning
- Other data-structures for FIFO, LIFO and queues



DQN

DQN uses experience replay:

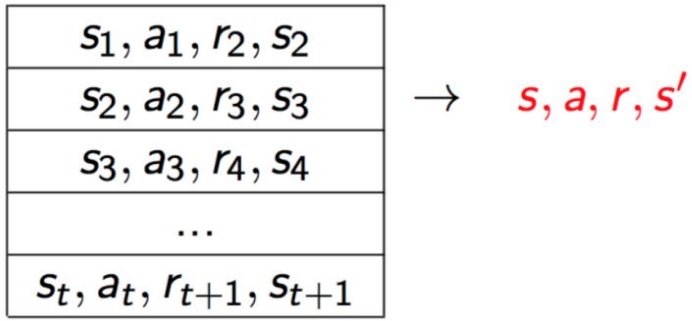
- The sampled tuple is used to compute the update targets
- The target is computed by using the estimation itself
- Since the weights get updated on the next round, the target value also changes



DQN

DQN uses experience replay:

- The sampled tuple is used to compute the update targets
- To improve stability, have a specific **target network** used in the target calculation for multiple updates and fix its weights
- Update such weights every once a while copying the Q network in it

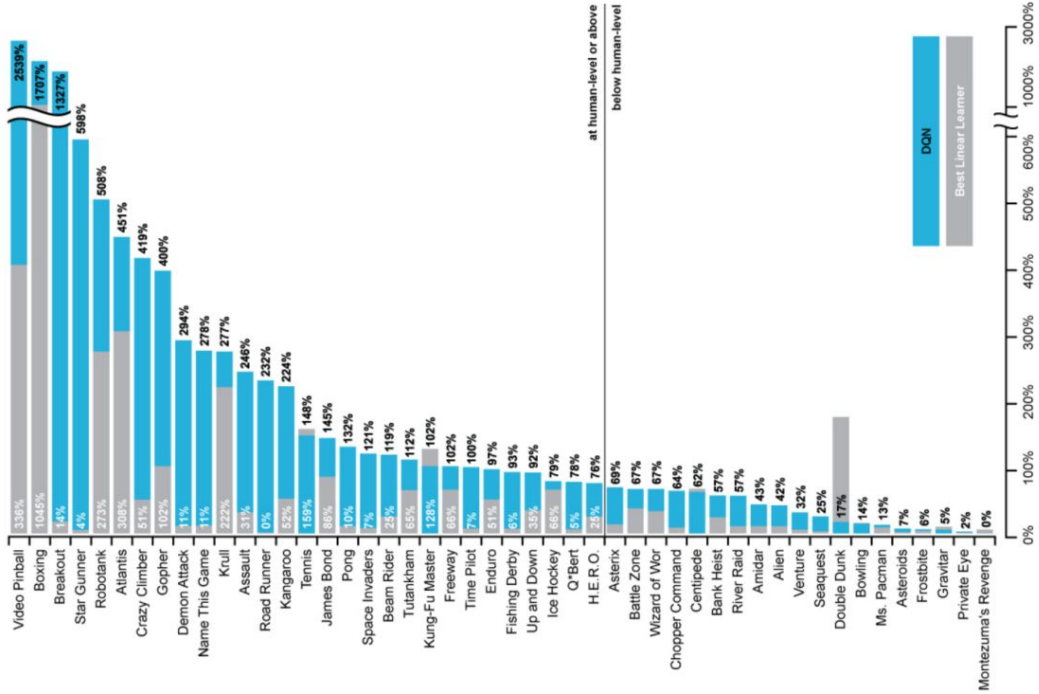


DQN Pseudo-code

1: Input C, α, $D = \{\}$, Initialize $\boldsymbol{w}$, $\boldsymbol{w}^- = \boldsymbol{w}$, $t = 0$
2: Get initial state s_0
3: **loop**
4: Sample action a_t given ϵ-greedy policy for current $\hat{Q}(s_t, a; \boldsymbol{w})$
5: Observe reward r_t and next state s_{t+1}
6: Store transition (s_t, a_t, r_t, s_{t+1}) in replay buffer D
7: Sample random minibatch of tuples (s_i, a_i, r_i, s_{i+1}) from D
8: **for** j in minibatch **do**
9: **if** episode terminated at step $i + 1$ **then**
10: $y_i = r_i$
11: **else**
12: $y_i = r_i + \gamma \max_{a'} \hat{Q}(s_{i+1}, a'; \boldsymbol{w}^-)$
13: **end if**
14: Do gradient descent step on $(y_i - \hat{Q}(s_i, a_i; \boldsymbol{w}))^2$ for parameters $\boldsymbol{w}$: $\Delta \boldsymbol{w} = \alpha(y_i - \hat{Q}(s_i, a_i; \boldsymbol{w}))\nabla_{\boldsymbol{w}} \hat{Q}(s_i, a_i; \boldsymbol{w})$
15: **end for**
16: $t = t + 1$
17: **if** mod(t,C) $==$ 0 **then**
18: $\boldsymbol{w}^- \leftarrow \boldsymbol{w}$
19: **end if**
20: **end loop**

Credits: Emma Brunskill

DQN Results


DQN
Best Linear Learner
at human-level or above
below human-level
0%
100%
200%
300%
400%
500%
600%
1000%
3000%
Video Pinball 2539% 538%
Boxing 1707% 1045%
Breakout 1327% 84%
Star Gunner 598% 4%
Robotank 508% 273%
Atlantis 451% 308%
Crazy Climber 419% 51%
Gopher 400% 102%
Demon Attack 294% 11%
Name This Game 278% 11%
Krull 277% 222%
Assault 246% 31%
Road Runner 232% 0%
Kangaroo 224% 52%
Tennis 148% 159%
James Bond 145% 86%
Pong 132% 10%
Space Invaders 121% 7%
Beam Rider 119% 25%
Tutankham 112% 65%
Kung-Fu Master 102% 128%
Freeway 102% 46%
Time Pilot 100% 7%
Enduro 97% 51%
Fishing Derby 93% 6%
Up and Down 92% 35%
Ice Hockey 79% 86%
Q*Bert 78% 5%
H.E.R.O. 76% 28%
Asterix 69%
Battle Zone 67%
Wizard of Wor 67%
Chopper Command 64%
Centipede 62%
Bank Heist 57%
River Raid 57%
Amidar 43%
Alien 42%
Venture 32%
Seaquest 25%
Double Dunk 17%
Bowling 14%
Ms. Pacman 13%
Asteroids 7%
Frostbite 6%
Gravitar 5%
Private Eye 2%
Montezuma's Revenge 0%

DQN Result Analysis

Game	Linear	Deep Network	DQN w/ fixed Q	DQN w/ replay	DQN w/replay and fixed Q
Breakout	3	3	10	241	317
Enduro	62	29	141	831	1006
River Raid	2345	1453	2868	4102	7447
Seaquest	656	275	1003	823	2894
Space Invaders	301	302	373	826	1089

Credits: Emma Brunskill

Maximization Bias

Greedy and epsilon greedy require a maximization step

This can lead to a positive bias:

- Consider s where many actions have Q(s,a) that are all zero
- Estimated values are uncertain, some above and some below zero
- Maximum of the true values is zero, but max over estimates is positive

Maximization Bias

Problem is related to using same samples both to determine max action and estimate its value

We could learn two independent estimates:

- One could be used to determine maximizing action
- Other could be used to estimate value
- Only one estimate is updated on each episode

Memory requirement is doubled; computation is unchanged

Double Q-Learning

Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$
Initialize $Q_1(s, a)$ and $Q_2(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, such that $Q(terminal, \cdot) = 0$

Loop for each episode:
  Initialize S
  Loop for each step of episode:
    Choose A from S using the policy ε-greedy in $Q_1 + Q_2$
    Take action A, observe R, S'
    With 0.5 probabilility:

$$Q_1(S, A) \leftarrow Q_1(S, A) + \alpha\Big(R + \gamma Q_2\big(S', \operatorname{argmax}_a Q_1(S', a)\big) - Q_1(S, A)\Big)$$

    else:

$$Q_2(S, A) \leftarrow Q_2(S, A) + \alpha\Big(R + \gamma Q_1\big(S', \operatorname{argmax}_a Q_2(S', a)\big) - Q_2(S, A)\Big)$$

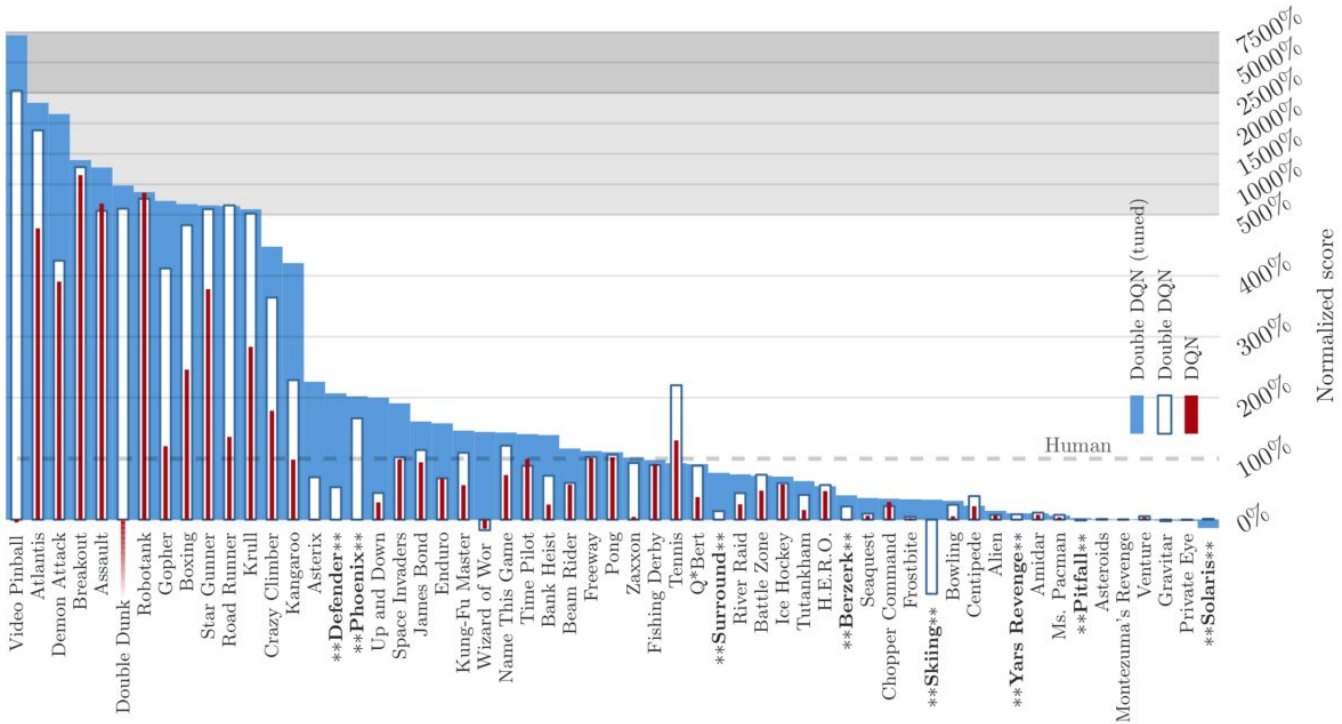
    $S \leftarrow S'$
  until S is terminal

DDQN

- Extend Double Q-Learning to DQN
- Use current Q-network for action selection
- Use older (the target) Q-network (w^-) for action evaluation

$$\Delta \boldsymbol{w} = \alpha(r + \gamma \overbrace{\hat{Q}(\underbrace{\arg\max_{a'} \hat{Q}(s', a'; \boldsymbol{w})}_{\text{Action selection: } \boldsymbol{w}}; \boldsymbol{w}^-)}^{\text{Action evaluation: } \boldsymbol{w}^-} - \hat{Q}(s, a; \boldsymbol{w}))$$

DDQN Results



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Prioritized Experience Replay (PER)

Schaul, Tom, et al. "Prioritized experience replay." *arXiv preprint arXiv:1511.05952* (2015). - Code: https://github.com/google/dopamine/tree/master/dopamine/replay_memory

- **Idea:** *“more frequently replay transitions with high expected learning progress, as measured by the magnitude of their temporal-difference (TD) error”*
- Prioritization can lead to:
  - Loss of diversity:
    - Use stochastic prioritization
  - Bias
    - Use importance sampling

PER - Stochastic VS Greedy Prioritization

- Greedy: Directly store magnitude of TD error in transitions

$$\delta_i = r_t + \gamma \max_{a \in \mathcal{A}} Q_{\theta^-}(s_{t+1}, a) - Q_\theta(s_t, a_t) \qquad (s_t, a_t, r_t, s_{t+1}, |\delta_t|)$$

Problem:

- We would need to keep all of them up-to-date
- ER contains a lot of data

Solution:

- Only update items that are sampled during updates (stochastic)

PER - Sampling Probability

- Compute priorities p:
  - **Rank-based method:** sort items based on TD error magnitude $p_i = 1/\text{rank}(i)$
  - **Proportional method:** (epsilon to ensure non-zero) $p_i = |\delta_i| + \epsilon$
  - Priorities initialized at max value for new samples
- Compute probability of sampling the i-th data-point:

$$P(i) = \frac{p_i^\alpha}{\sum_k p_k^\alpha}$$

  - alpha determines level of prioritization (0 means none)

PER - Bias Correction

Bias arises from the fact that we are changing the same distribution as the one that we are using for our updates

- Changes the distribution & solution

Solution:

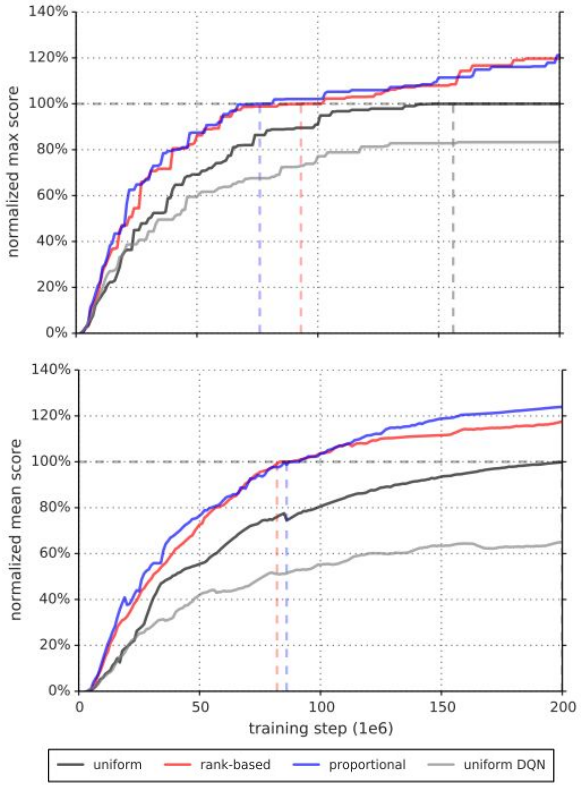
- Use importance sampling for updates
- If P(i) becomes 0, w becomes large
- If P(i) = 1/N, we have uniform sampling and w = 1
- Beta defines amount of correction

$$w_i = \left(\frac{1}{N} \cdot \frac{1}{P(i)}\right)^{\beta}$$

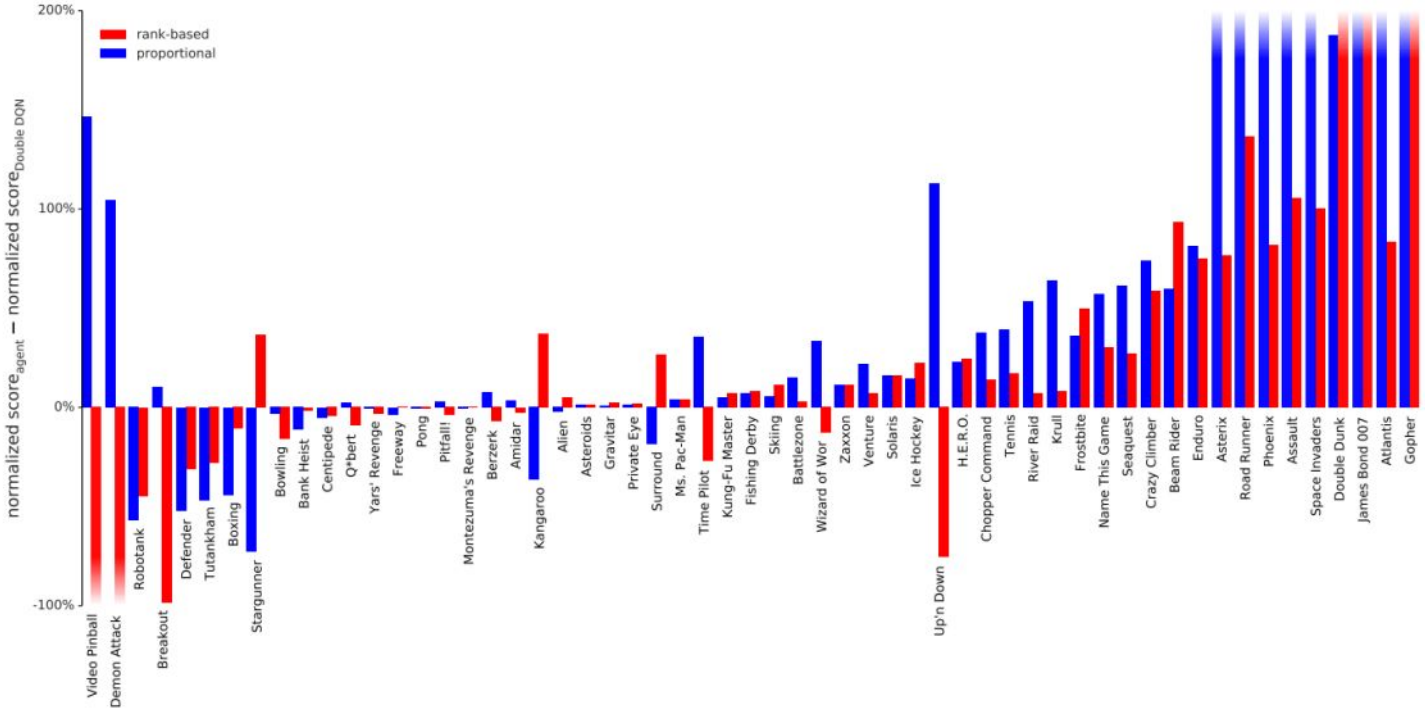
DDQN with PER

Algorithm 1 Double DQN with proportional prioritization

Input: minibatch k, step-size η, replay period K and size N, exponents α and β, budget T.
Initialize replay memory $\mathcal{H} = \emptyset$, $\Delta = 0$, $p_1 = 1$
Observe S_0 and choose $A_0 \sim \pi_\theta(S_0)$
for $t = 1$ **to** T **do**
  Observe S_t, R_t, γ_t
  Store transition $(S_{t-1}, A_{t-1}, R_t, \gamma_t, S_t)$ in $\mathcal{H}$ with maximal priority $p_t = \max_{i<t} p_i$
  if $t \equiv 0 \mod K$ **then**
    for $j = 1$ **to** k **do**
      Sample transition $j \sim P(j) = p_j^\alpha / \sum_i p_i^\alpha$
      Compute importance-sampling weight $w_j = (N \cdot P(j))^{-\beta} / \max_i w_i$
      Compute TD-error $\delta_j = R_j + \gamma_j Q_{\text{target}}(S_j, \arg\max_a Q(S_j, a)) - Q(S_{j-1}, A_{j-1})$
      Update transition priority $p_j \leftarrow |\delta_j|$
      Accumulate weight-change $\Delta \leftarrow \Delta + w_j \cdot \delta_j \cdot \nabla_\theta Q(S_{j-1}, A_{j-1})$
    end for
    Update weights $\theta \leftarrow \theta + \eta \cdot \Delta$, reset $\Delta = 0$
    From time to time copy weights into target network $\theta_{\text{target}} \leftarrow \theta$
  end if
  Choose action $A_t \sim \pi_\theta(S_t)$
end for

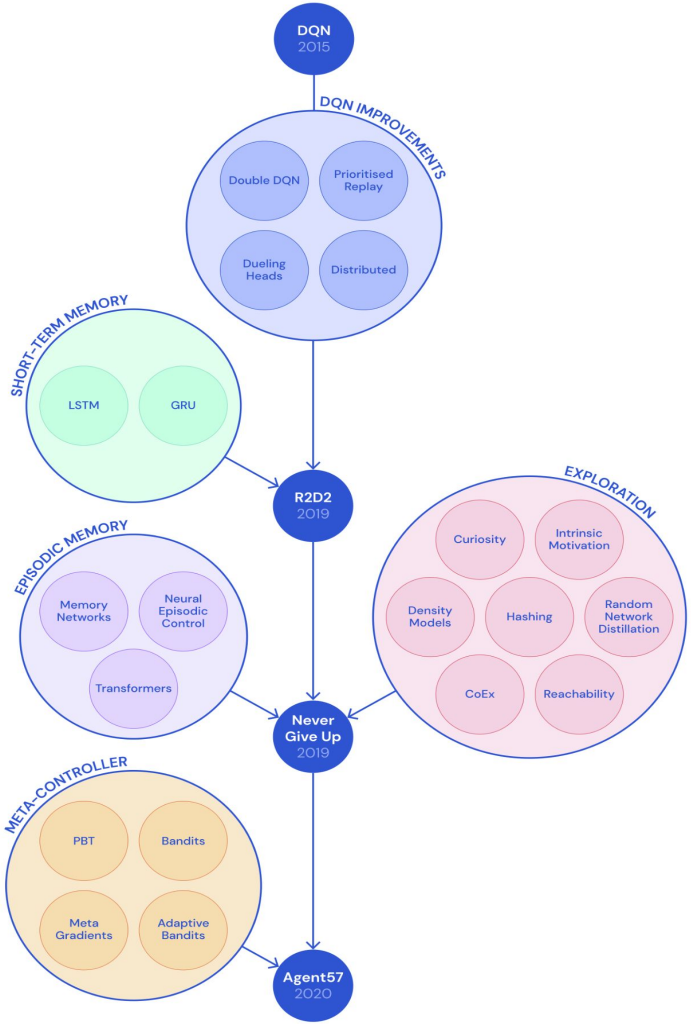


DDQN with PER Results



From DQN to Agent57

- DQN is the first approach beating humans in many Atari games
- Agent57 is the first approach beating humans in all Atari games



More on Off-Policy RL

SAPIENZA
UNIVERSITÀ DI ROMA

On-Policy vs Off-Policy

On-policy: learn by what you do

Off-policy: learn by looking at someone else

- learn from observing other agents or humans
- reuse experience
- learn about optimal policy while following exploratory behaviors
- learn multiple policies while following a single policy

Expectation of Different Distribution

How do you estimate the expectation of a different distribution?

Expectation of Different Distribution

How do you estimate the expectation of a different distribution?

$$\begin{aligned}\mathbb{E}_{X\sim P}[f(X)] &= \sum P(X)f(X) \\ &= \sum Q(X)\frac{P(X)}{Q(X)}f(X) \\ &= \mathbb{E}_{X\sim Q}\left[\frac{P(X)}{Q(X)}f(X)\right]\end{aligned}$$

Importance Sampling

The same idea is applied to RL for off-policy learning

Importance Sampling for MC

The same idea is applied to RL for off-policy learning

Consider the MC setting: we want to use the returns from policy μ to evaluate π

Compute $G^{\pi/\mu}$ by multiplying **importance sampling corrections**

$$G_t^{\pi/\mu} = \frac{\pi(A_t|S_t)}{\mu(A_t|S_t)} \frac{\pi(A_{t+1}|S_{t+1})}{\mu(A_{t+1}|S_{t+1})} \cdots \frac{\pi(A_T|S_T)}{\mu(A_T|S_T)} G_t$$

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if μ is zero and π non-zero this cannot be used

Importance Sampling for TD

The same idea is applied to RL for off-policy learning

Consider the TD setting: we want to weight the TD target

$$\frac{\pi(A_t|S_t)}{\mu(A_t|S_t)}\left(R_{t+1}+\gamma V(S_{t+1})\right)$$

Q-Learning does not need it, why?

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The same idea is applied to RL for off-policy learning

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$$\frac{\pi(A_t|S_t)}{\mu(A_t|S_t)}(R_{t+1} + \gamma V(S_{t+1}))$$

Q-Learning does not need it, why?

We directly use the action from the target policy