# REINFORCE, Baselines and Actor-Critic

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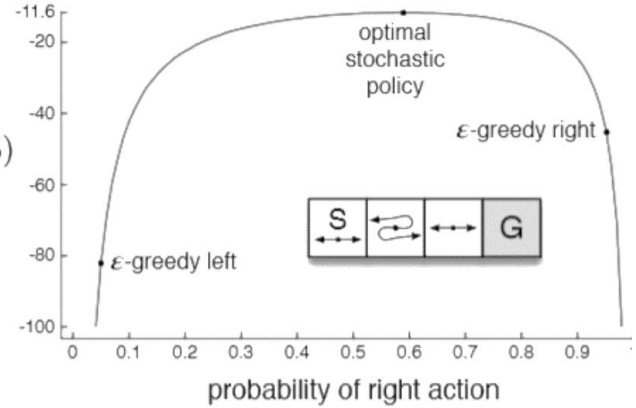
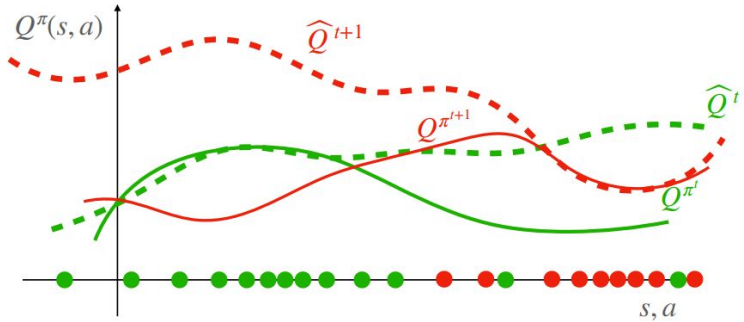
# Recap

# Policy from Value: Problems

So far, we derived a policy out of a tabular or parameterized state-action value function


$J(\theta) = v_{\pi_\theta}(S)$




Green dots: $(s, a)$ from $\pi^t$

Red dots: $(s, a)$ from $\pi^{t+1}$

# Parameterized Policy

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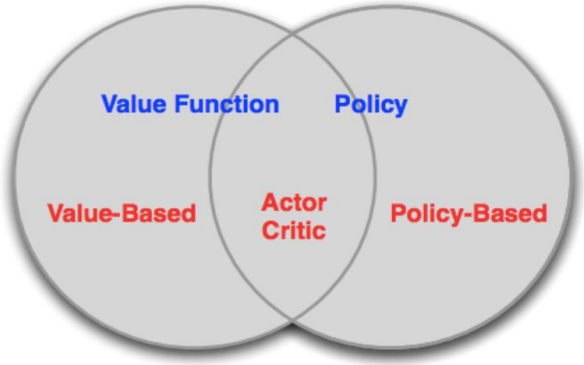
Can we directly represent and learn a parameterized policy?

$$a \sim \pi_{\theta}(s) \text{ or } p(.|s,\theta)$$


Credits: David Silver

# Finding a Parameterized Policy

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How do we search for a parameterized policy $\pi_\theta(s)$?

We want to find the parameters $\theta$ such that $\pi_\theta$ is the best policy

# Quality of a Parameterized Policy

How do we measure the quality for a policy?

In other words, what is our objective function to maximize through the policy parameters?

- For episodic tasks:
  - Value at start state (undiscounted)
- For continuing tasks:
  - Value at start state (discounted)
  - Average value

    see [Sutton&Barto 10.4: Deprecating the Discounted Setting]

  - Average reward per timestep

    see [Sutton&Barto 10.4: Deprecating the Discounted Setting]

$$J(\pi) := \mathbb{E}\left[\sum_{h=0}^{H-1} r(s_h, a_h) \,\middle|\, s_0 \sim \mu, s_{h+1} \sim P_{s_h,a_h}, a_h \sim \pi(\cdot \mid s_h)\right]$$

$$J(\pi) := \mathbb{E}\left[\sum_{h=0}^{\infty} \gamma^h r(s_h, a_h) \,\middle|\, s_0 \sim \mu, s_{h+1} \sim P_{s_h,a_h}, a_h \sim \pi(\cdot \mid s_h)\right]$$

# Policy Optimization

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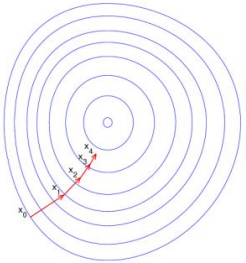
Can use gradient free optimization

- Hill climbing Simplex / Nelder Mead Genetic algorithms
- Cross-Entropy method (CEM)
- Covariance Matrix Adaptation (CMA)

Evolution strategies can rival the performance of standard RL techniques on modern RL benchmarks (e.g. Atari/MuJoCo): https://openai.com/blog/evolution-strategies/

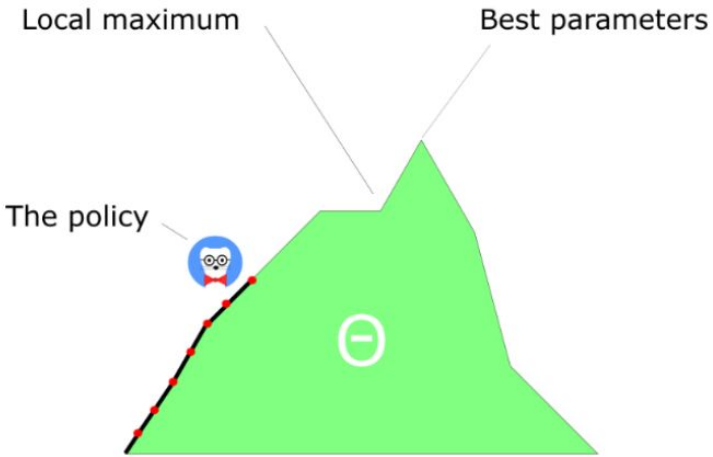
Can work with any policy parameterizations, including non-differentiable ones, but can be sample inefficient and requires higher outcome variability

# Policy Gradient



An efficient and widely used technique to find a parameterized policy is the policy gradient

Guaranteed to converge to local maximum or global maximum, *but it often converges only to a local maximum*



# Policy Gradient

$$\pi_\theta(a \mid s) = \pi(a \mid s; \theta) \qquad J(\pi_\theta) = \mathbb{E}_{\pi_\theta}\left[\sum_{h=0}^{\infty} \gamma^h r_h\right]$$

$$\theta_{t+1} = \theta_t + \eta \nabla_\theta J(\pi_\theta)\big|_{\theta=\theta_t}$$

HOW?

# Finite Differences

$$\pi_\theta(a \mid s) = \pi(a \mid s; \theta) \qquad J(\pi_\theta) = \mathbb{E}_{\pi_\theta}\left[\sum_{h=0}^{\infty} \gamma^h r_h\right]$$

$$\theta_{t+1} = \theta_t + \eta \nabla_\theta J(\pi_\theta)\big|_{\theta=\theta_t}$$

Simple approach: compute $\nabla_\theta J(\pi_\theta)$ using finite differences

IDEA: perturb $\theta$ by small amount in $k$-th dimension

$$\frac{\partial V(s_0, \theta)}{\partial \theta_k} \approx \frac{V(s_0, \theta + \epsilon u_k) - V(s_0, \theta)}{\epsilon}$$

$$\begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \leftarrow k\text{-th entry}$$

Uses $n$ evaluations to compute policy gradient in $n$ dimensions

Works for arbitrary policies, even if policy is not differentiable. Noise can dominate, but can be reduced by averaging over many samples or by reducing randomness if possible

# Policy Gradient

$$\pi_\theta(a \mid s) = \pi(a \mid s; \theta) \qquad J(\pi_\theta) = \mathbb{E}_{\pi_\theta}\left[\sum_{h=0}^{\infty} \gamma^h r_h\right]$$

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Can we compute the gradient analytically?

# Differentiable Policy Classes

Softmax Linear Policy

$$\pi_\theta(a \mid s) = \frac{\exp(\theta^\top \phi(s,a))}{\sum_{a'} \exp(\theta^\top \phi(s,a'))}$$

Softmax Policy

$$\pi_\theta(a \mid s) = \frac{\exp(f_\theta(s,a))}{\sum_{a'} \exp(f_\theta(s,a'))}$$

Gaussian Policy

$$\pi_\theta(a \mid s) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(a - f_\theta(s))^2}{2\sigma^2}\right)$$

f can be a neural network

# Likelihood Ratio Policy Gradient

---

Suppose we have a trajectory $\tau = \{s_0, a_0, s_1, a_1, \ldots\}$

it's probability distribution, depending on $\theta$ is

$$\rho_\theta(\tau) = \mu(s_0)\pi_\theta(a_0 \mid s_0)P(s_1 \mid s_0, a_0)\pi_\theta(a_1 \mid s_1)\ldots$$

$$J(\pi_\theta) = \mathbb{E}_{\tau\sim\rho_\theta(\tau)} \underbrace{\left[\sum_{h=0}^{\infty} \gamma^h r(s_h, a_h)\right]}_{R(\tau)} = \sum_\tau P(\tau;\theta) R(\tau)$$

probability of each trajectory from the distribution

# Likelihood Ratio Policy Gradient

---

$$\nabla_\theta J(\theta) \approx \hat{g} = (1/m) \sum_{i=1}^{m} R(\tau^{(i)}) \nabla_\theta \log P(\tau^{(i)}; \theta)$$

$$= (1/m) \sum_{i=1}^{m} \boxed{R(\tau^{(i)})} \sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t^{(i)} | s_t^{(i)})$$

Measure how good the sample trajectory is

# Policy Gradient Theorem (Finite Setting)

---

The policy gradient theorem generalizes the likelihood ratio approach

$$\nabla_\theta J(\pi_\theta) = \sum_{h=0}^{H-1} \mathbb{E}_{s_h, a_h \sim \mathbb{P}_h^{\pi_\theta}} \left[ \nabla \ln \pi_\theta(a_h \mid s_h) \cdot Q_h^{\pi_\theta}(s_h, a_h) \right]$$

# End Recap

# Policy Gradient Theorem (Infinite Setting)

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$$= \frac{1}{1-\gamma} \mathbb{E}_{s,a\sim d_\mu^{\pi_\theta}} \left[ \nabla_\theta \ln \pi_\theta(a \mid s) \cdot Q^{\pi_\theta}(s,a) \right]$$

HOW?

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This is a familiar problem!

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**Policy Evaluation!**

# Policy Gradient Theorem (Infinite Setting)

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We can use the return G as an unbiased estimate of Q (MC)

# REINFORCE

Initialize policy parameters $\theta$ arbitrarily
**for** each episode $\{s_1, a_1, r_2, \cdots, s_{T-1}, a_{T-1}, r_T\} \sim \pi_\theta$ **do**
  **for** $t = 1$ to $T - 1$ **do**
    $\theta \leftarrow \theta + \alpha \nabla_\theta \log \pi_\theta(s_t, a_t) G_t$
  **endfor**
**endfor**
**return** $\theta$

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Unbiased, but remember the issue?

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VARIANCE!

# Baseline

---

To reduce the variance we can introduce baselines (function of state)

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$$\nabla_\theta J(\pi_\theta) = \frac{1}{1-\gamma}\mathbb{E}_{s,a\sim d_\mu^{\pi_\theta}}\left[\nabla_\theta \ln \pi_\theta(a\,|\,s)\cdot\left(Q^{\pi_\theta}(s,a) - b(s)\right)\right]$$

# Baseline

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$$\nabla_\theta J(\pi_\theta) = \frac{1}{1-\gamma} \mathbb{E}_{s,a\sim d_\mu^{\pi_\theta}} \left[ \nabla_\theta \ln \pi_\theta(a \mid s) \cdot \left( Q^{\pi_\theta}(s,a) - b(s) \right) \right]$$

Is this term introducing a bias?

# Baseline

---

Baselines do not introduce bias

$$\mathbb{E}_{a\sim\pi_\theta(\cdot|s)}\nabla_\theta \ln \pi_\theta(a\,|\,s)b(s)$$

Let’s expand this term

# Baseline

---

Baselines do not introduce bias

$$\mathbb{E}_{a\sim\pi_\theta(\cdot|s)} \nabla_\theta \ln \pi_\theta(a \mid s) b(s)$$

$$= \sum_a \pi_\theta(a \mid s) \boxed{\frac{\nabla \pi_\theta(a \mid s)}{\pi_\theta(a \mid s)}} b(s)$$

And fully write down the gradient of ln

# Baseline

---

Baselines do not introduce bias

$$\mathbb{E}_{a\sim\pi_\theta(\cdot|s)} \nabla_\theta \ln \pi_\theta(a\,|\,s) b(s)$$

$$= \sum_a \pi_\theta(a\,|\,s) \frac{\nabla \pi_\theta(a\,|\,s)}{\pi_\theta(a\,|\,s)} b(s)$$

And fully write down the gradient of ln as well as the expectation

# Baseline

---

Baselines do not introduce bias

$$\mathbb{E}_{a\sim\pi_\theta(\cdot|s)} \nabla_\theta \ln \pi_\theta(a\,|\,s) b(s)$$

$$= \sum_a \pi_\theta(a\,|\,s) \frac{\nabla \pi_\theta(a\,|\,s)}{\pi_\theta(a\,|\,s)} b(s) \quad = b(s) \sum_a \nabla \pi_\theta(a\,|\,s)$$

We can get simplify and highlight b, as it does not depend on a (the sum)

# Baseline

Baselines do not introduce bias

$$\mathbb{E}_{a\sim\pi_\theta(\cdot|s)} \nabla_\theta \ln \pi_\theta(a\,|\,s) b(s)$$

$$= \sum_a \pi_\theta(a\,|\,s) \frac{\nabla \pi_\theta(a\,|\,s)}{\pi_\theta(a\,|\,s)} b(s) = b(s) \sum_a \nabla \pi_\theta(a\,|\,s) = b(s) \nabla \left[ \sum_a \pi_\theta(a\,|\,s) \right] = b(s) \nabla 1 = 0$$

This holds for any baseline as long as it is action-independent

# Value Function as Baseline

---

As baselines have to be action-independent, a common choice for a baseline is

$$b(s) = V^{\pi_\theta}(s)$$

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Called Advantage Function

$$\nabla_\theta J(\theta_t) = \frac{1}{1-\gamma} \mathbb{E}_{s,a \sim d^{\pi_{\theta_t}}} \left[ \nabla_\theta \ln \pi_{\theta_t}(a \mid s) \left( A^{\pi_{\theta_t}}(s,a) \right) \right]$$

# Advantage Function

---

**Intuition:** the advantage function tells us how good an action is compared to the average value of the state

Value of an action in the state

$$Q^{\pi_\theta}(s,a) - V^{\pi_\theta}(s)$$

Average value of the state

# Advantage Function

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**Intuition:** the advantage function tells us how good an action is compared to the average value of the state

$$Q^{\pi_\theta}(s, a) - V^{\pi_\theta}(s)$$

Remember in REINFORCE we estimate our Q by the return

# REINFORCE with Baseline

Initialize policy parameter $\theta$, baseline $b$
**for** iteration=$1, 2, \cdots$ **do**
  Collect a set of trajectories by executing the current policy
  At each timestep $t$ in each trajectory $\tau^i$, compute
    *Return* $G_t^i = \sum_{t'=t}^{T-1} r_{t'}^i$, and
    *Advantage estimate* $\hat{A}_t^i = G_t^i - b(s_t)$.
  Re-fit the baseline, by minimizing $\sum_i \sum_t ||b(s_t) - G_t^i||^2$,
  Update the policy, using a policy gradient estimate $\hat{g}$,
    Which is a sum of terms $\nabla_\theta \log \pi(a_t|s_t, \theta)\hat{A}_t$.
    (Plug $\hat{g}$ into SGD or ADAM)
**endfor**

Credits: Emma Brunskill

# REINFORCE with Baseline

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    (Plug $\hat{g}$ into SGD or ADAM)
**endfor**

We're still using the return and collecting MC samples

Credits: Emma Brunskill

# Advantage Function

$$Q^{\pi_\theta}(s,a) - V^{\pi_\theta}(s)$$

If we can access the true value function, the TD error is an unbiased estimate of the advantage function

$$\begin{aligned}\mathbb{E}_{\pi_\theta}\left[\delta^{\pi_\theta}|s,a\right] &= \mathbb{E}_{\pi_\theta}\left[r + \gamma V^{\pi_\theta}(s')|s,a\right] - V^{\pi_\theta}(s) \\ &= Q^{\pi_\theta}(s,a) - V^{\pi_\theta}(s) \\ &= A^{\pi_\theta}(s,a)\end{aligned}$$

$$\delta^{\pi_\theta} = r + \gamma V^{\pi_\theta}(s') - V^{\pi_\theta}(s)$$

Credits: David Silver

# Advantage Function

$$Q^{\pi_\theta}(s,a) - V^{\pi_\theta}(s)$$

So we can use the TD error to compute the policy gradient

$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(s,a)\ \delta^{\pi_\theta}\right]$$

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Can be approximated!

# Advantage Function

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**Introduces bias**, but it's fine as we know from TD

# Advantage Function

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So we can use the TD error to compute the policy gradient

$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(s,a)\, \delta^{\pi_\theta}\right]$$

This leads us to the notion of critic

# Reducing Variance with Critic

---

**Motivation:** Monte-Carlo policy gradient still has high variance!

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We can estimate V/Q by using a *critic*

Such critic is also parameterized

$$Q_w(s,a) \approx Q^{\pi_\theta}(s,a)$$

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$$\nabla_\theta J(\theta) \approx \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(s,a)\ Q_w(s,a)\right]$$

$$\Delta\theta = \alpha \nabla_\theta \log \pi_\theta(s,a)\ Q_w(s,a)$$

We can select any blend between TD and MC estimators for $Q_w$

# Reducing Variance with Critic

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Such critic is also parameterized

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$$\Delta\theta = \alpha \nabla_\theta \log \pi_\theta(s,a)\ Q_w(s,a)$$

Actor-Critic Algorithms use 1-step TD

# MC vs TD Policy Gradient

---

- In MC policy gradient, the target is the return G

$$\Delta\theta = \alpha(G_t - V_v(s_t))\nabla_\theta \log \pi_\theta(s_t, a_t)$$

- In Actor-Critic the target is a TD target and relies on bootstrapping
  - Multiple timescales are possible (not only 1-step)
  - Also TD-lambda with forward/backward view

$$\Delta\theta = \alpha(r + \gamma V_v(s_{t+1}) - V_v(s_t))\nabla_\theta \log \pi_\theta(s_t, a_t)$$

Credits: David Silver

# Actor-Critic VS Baseline

**REINFORCE with Baseline (episodic), for estimating $\pi_\theta \approx \pi_*$**

Input: a differentiable policy parameterization $\pi(a|s,\boldsymbol{\theta})$
Input: a differentiable state-value function parameterization $\hat{v}(s,\mathbf{w})$
Algorithm parameters: step sizes $\alpha^{\boldsymbol{\theta}} > 0$, $\alpha^{\mathbf{w}} > 0$
Initialize policy parameter $\boldsymbol{\theta} \in \mathbb{R}^{d'}$ and state-value weights $\mathbf{w} \in \mathbb{R}^d$ (e.g., to $\mathbf{0}$)

Loop forever (for each episode):
  Generate an episode $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T$, following $\pi(\cdot|\cdot,\boldsymbol{\theta})$
  Loop for each step of the episode $t = 0, 1, \ldots, T-1$:
    $G \leftarrow \sum_{k=t+1}^{T} \gamma^{k-t-1} R_k$ $(G_t)$
    $\delta \leftarrow G - \hat{v}(S_t,\mathbf{w})$
    $\mathbf{w} \leftarrow \mathbf{w} + \alpha^{\mathbf{w}} \delta \nabla \hat{v}(S_t,\mathbf{w})$
    $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha^{\boldsymbol{\theta}} \gamma^t \delta \nabla \ln \pi(A_t|S_t,\boldsymbol{\theta})$

**One-step Actor–Critic (episodic), for estimating $\pi_\theta \approx \pi_*$**

Input: a differentiable policy parameterization $\pi(a|s,\boldsymbol{\theta})$
Input: a differentiable state-value function parameterization $\hat{v}(s,\mathbf{w})$
Parameters: step sizes $\alpha^{\boldsymbol{\theta}} > 0$, $\alpha^{\mathbf{w}} > 0$
Initialize policy parameter $\boldsymbol{\theta} \in \mathbb{R}^{d'}$ and state-value weights $\mathbf{w} \in \mathbb{R}^d$ (e.g., to $\mathbf{0}$)
Loop forever (for each episode):
  Initialize $S$ (first state of episode)
  $I \leftarrow 1$
  Loop while $S$ is not terminal (for each time step):
    $A \sim \pi(\cdot|S,\boldsymbol{\theta})$
    Take action $A$, observe $S', R$
    $\delta \leftarrow R + \gamma \hat{v}(S',\mathbf{w}) - \hat{v}(S,\mathbf{w})$ (if $S'$ is terminal, then $\hat{v}(S',\mathbf{w}) \doteq 0$)
    $\mathbf{w} \leftarrow \mathbf{w} + \alpha^{\mathbf{w}} \delta \nabla \hat{v}(S,\mathbf{w})$
    $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha^{\boldsymbol{\theta}} I \delta \nabla \ln \pi(A|S,\boldsymbol{\theta})$
    $I \leftarrow \gamma I$
    $S \leftarrow S'$

# Actor-Critic with LFA

---

Critic $Q_w(s,a) = \phi(s,a)^T w$ updates weights w by linear TD(0)
Actor updates weights by policy gradient

**function** QAC
  Initialise $s$, $\theta$
  Sample $a \sim \pi_\theta$
  **for** each step **do**
    Sample reward $r = \mathcal{R}^a_s$; sample transition $s' \sim \mathcal{P}^a_{s,\cdot}$
    Sample action $a' \sim \pi_\theta(s', a')$
    $\delta = r + \gamma Q_w(s', a') - Q_w(s, a)$
    $\theta = \theta + \alpha \nabla_\theta \log \pi_\theta(s, a) Q_w(s, a)$
    $w \leftarrow w + \beta \delta \phi(s, a)$
    $a \leftarrow a', s \leftarrow s'$
  **end for**
**end function**

Credits: David Silver

# Eligibility Traces in Policy Gradient

---

Integrating eligibility traces in policy gradient (actor has multiple timescales)

$$\delta = r_{t+1} + \gamma V_v(s_{t+1}) - V_v(s_t)$$
$$e_{t+1} = \lambda e_t + \nabla_\theta \log \pi_\theta(s, a)$$
$$\Delta\theta = \alpha \delta e_t$$

# Policy Gradient Summary

$$
\begin{aligned}
\nabla_\theta J(\theta) &= \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(s,a) G_t\right] && \text{REINFORCE} \\
&= \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(s,a)\ Q^w(s,a)\right] && \text{Q Actor-Critic} \\
&= \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(s,a)\ A^w(s,a)\right] && \text{Advantage Actor-Critic} \\
&= \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(s,a)\ \delta\right] && \text{TD Actor-Critic} \\
&= \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(s,a)\ \delta e\right] && \text{TD}(\lambda)\text{ Actor-Critic}
\end{aligned}
$$

Critic does policy evaluation to estimate Q, V or A using bootstrapping (*if it uses MC we do not call it a critic*)

Credits: David Silver