# Search 1: Concepts	start time:

This activity introduces some ideas and techniques used in artificial intelligence (AI) to search for possible solutions to problems.

Computer science and most other disciplines are ultimately concerned with **real problems** that can be complicated and involve special cases, boundaries, etc. Thus, we often begin with **toy problems** that are easier to describe, analyze, or solve.

Before you start, share this document with your team member(s) and then complete the form below to assign the role of speaker.

Team Role	Team Member
Speaker: shares your team's ideas with the class.	Makenna, Arogya, Jesse

It's so much easier to suggest solutions
when you don't know too much about the problem.
-- Malcolm Forbes

(8 min) A. 8-Puzzles	start time:

1. (1 min) **8-puzzles** have a 3x3 board with 8 sliding tiles and 1 space (see example below). The goal is to slide one tile at a time until they form a familiar picture or sequence. How many moves are possible if the space is in:

a.	the center?	4
b.	a corner?	2
c.	the middle of a side?	3

2. (2 min) For the initial layout shown below, fill in blank boards to show each layout that is:
   a. **1 move away** from the initial layout.
   b. **2 moves away** from the initial layout (you may not need all of the boards).

Initial Layout

6	8	4
3	1	
2	7	5

1 Move Away

6	8	
3	1	4
2	7	5

6	8	4
3		1
2	7	5

6	8	4
3	1	5
2	7	

2 Moves Away

6		8
3	1	4
2	7	5

6	8	4
3	1	
2	7	5

6		4
3	8	1
2	7	5

6	8	4
	3	1
2	7	5

6	8	4
3	1	
2	7	5

6	8	4
3	7	1
2		5

6	8	4
3	1	
2	7	5

6	8	4
3	1	5
2		7

3. (1 min) Which layout(s) above are repeated, and how many times are they repeated?

At 2 moves away, each set of states contains the initial state.

If you added each layout that is 3 moves away, which layouts would be repeated?

At 3 moves away, the positions from the 1 move away state would be repeated.

4. (2 min) In complete sentences, explain why it might be a good idea to avoid repeated layouts when searching for a solution.

If we allowed repeats in our states then we can have never-ending cycles that would appear in the structure.

5. (2 min) The same idea can be adapted to other size boards. For example, a **15-Puzzle** has 15 pieces on a 4x4 board, and a **3-Puzzle** has 3 pieces on a 2x2 board.

a.	For a 15-Puzzle, what is the minimum number of possible moves?	2
b.	what is the maximum number of possible moves?	4
c.	For a 3-Puzzle, what is the minimum number of possible moves?	2
d.	what is the maximum number of possible moves?	2

(8 min) B. Normal Magic Squares	start time:

1. (2 min) A **normal magic square (NMS) of order N** is a set of distinct integers 1…N^2 laid out in a N-by-N square such that all rows, all columns, and both diagonals sum to the **magic constant M** = $N(N^2+1)/2$.

a.	What is M for a square of order 3?	15
b.	Is square (b) at right a NMS?	No
c.	Is square (c) at right a NMS?	Yes
d.	Is there a NMS of order 1?	Yes
e.	Is there a NMS of order 2?	No

(b)

8	3	4
1	6	9
5	7	2

(c)

2	9	4
7	5	3
6	1	8

(d)

1

(e)

2	3
4	1

2. (2 min) Explain your answers to 1d and 1e:

There is only one cell in d and the order is 1. Therefore if we keep one, then it is the solution. However, in 1e, it doesn't work because there is no two sets of two numbers that add to 5 in 1,2,3,4

3. (2 min) For a **blank square** of **order 4**:

a.	How many cells (positions) does it have?		16
b.	If all cells are empty,	how many could get the value 1?	16
c.	If 1 is in a cell,	how many could get the value 2?	At most 15
d.	If 1 & 2 are in cells,	how many could get the value 3?	At most 14
e.	If 1, 2, 3, & 4 are in cells,	how many could get the value 5?	At most 12

4. (2 min) Answer these questions with an **expression**, not a **value**. If all cells are empty:

a.	How many ways could 1 & 2 be placed?	16*15 = (16P2)
b.	How many ways could 1 2, 3, & 4 be placed?	At most 16*15*14*13 = (16P4)
c.	How many ways could **all numbers** be placed?	At most 16!

(6 min) C. 8-Queens Puzzle	start time:

1. (2 min) A chessboard is an 8x8 grid,
and each position can contain one chess piece.
How many ways are there to place:

a.	1 piece?	64
b.	2 pieces?	64*63
c.	4 pieces?	64*63*62*61

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2. (2 min) The **queen** piece can move any distance in any direction, as shown by the shaded positions at right. The goal of the **8-Queens Puzzle** is to place 8 queens such that none share a row, column, or diagonal.

a.	In diagram at right, do any queens share a row?	No
b.	do any queens share a diagonal?	Yes

3. (1 min) Explain why it is not possible to place 9 queens such that none share a row, column, or diagonal.

Because Queens captures the entire row, column, and diagonal they sit on, you cannot have 9 queens on an 8x8 board.

Science never solves a problem without creating ten more.
-- George Bernard Shaw

(10 min) D. States & Actions	start time:

Many AI **problems** (or approaches to solve them) have similar structures, and consistent terminology can help us to compare problems and approaches. This terminology may be familiar, if you have studied **finite state automata (FSA)**. Each problem involves some set of **actions** (or **moves**) that change its **state**. Each problem starts from an **initial state** and tries to reach a **goal state**, where it passes a **goal test**. Most problems have one initial state (some have more), and zero, one, or more **goal states**. A problem with no goal states can't be solved.

1. (2 min) Describe the **initial state** and a suitable **goal test** for each problem:

Problem	Initial State	Goal Test
8-Puzzle	3x3 grid of cells: 1 blank, the others with random symbols.	Are all symbols in correct order to form a picture or sequence?
8-Queens	8x8 grid of cells; all blank	For all 8 queens, can no one queen capture another queen?
NMS of order N	NxN grid of cells; all blank	Do all the rows, columns, and diagonals add up to the magic number?

2. (2 min) Answer the questions below:

a.	Does a NMS of order 2 have a **goal test**?	Yes
b.	Does a NMS of order 2 have a **goal state**?	No
c.	A **3-Puzzle** has the **initial state** and **goal state** shown below. Is it possible to reach the goal state from the initial state?	No

initial:

	3
1	2

goal:

1	2
3	

In general, each **state** has a set of **applicable actions**, and each action leads to a **successor** state (also called a **next** state). A **transition model** or **transition function** defines how actions move from one state to another.

3. (1 min) An **8-Puzzle** has at most **4 actions**:
slide left, right, up or down.
At right, show states with only 3 and 2 **applicable actions**.

a. 3 actions

x	x	x
x	x	
x	x	x

b. 2 actions

x	x	
x	x	x
x	x	x

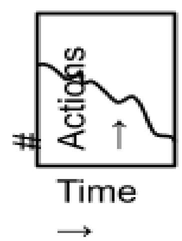
4. (2 min) In an **8-Queens Puzzle**, each action makes other actions **inapplicable**. At right, put a 1 in each position blocked by Q1, and then a 2 in each position blocked by Q2.

Q	1	1 2	1	1 2	1	1 2	1
1	1		2	2	2		
1 2	2	1 2	2	**Q**	2	2	2
1			1 2	2	2		
1		2		1 2		2	
1	2			2	1		2
1 2				2		1	
1				2			1

5. (1 min) The **number of applicable actions** can vary, but there can also be a general trend. Over time, does the number tend to go **up**, **down**, or stay the **same** in:

a.	8-Puzzle	Same
b.	8-Queens	Down
c.	Normal Magic Square (NMS)	Down

goes down



goes up

6. (2 min) In general, will it be easier to search states if the number of applicable actions goes **up** or **down** over time? Explain your answer.

Down because there will be less options to search through, aka the tree will become less dense than if the number of actions is going up

For every complex problem there is an answer that is clear, simple, and wrong.
-- H. L. Mencken

(10 min) E. Paths & State Space	start time:

1. (2 min) A sequence of states connected by applicable actions is a **path**.
In some problems, the challenge is to find a **goal state**; in other problems,
the goal state is known obvious, and the challenge is to find a **goal path**.
Categorize each of the following as a **goal state** or **goal path** problem.

a.	8-Puzzle	Goal state
b.	8-Queens	Goal path
c.	Normal Magic Square	Goal path

2. (3 min) Some paths may be better than others, even if they reach the same goal state;
thus, we can define a **path cost function** to compare paths. For an **8-Puzzle**:

a.	Would most people prefer a path with 20 actions & states, or 200?	20
b.	What is a possible path cost function?	path1 < path2 ? return path1 : return path2

3. (2 min) The initial state, possible actions, and transition model together define
the **set of all possible states** that can be reached. This is called the **state space**.
In complete sentences, describe the state space for **tic-tac-toe**.

The initial state for TTT is an empty board. Possible actions include placing an X or an O in an empty space. The goal state is when the board either has 3 X's or O's in a row, or all spaces on the board are filled.

The set of all possible states can be described as any state where the number of X's and O's are are within 1 of each other, with 3 or less of either X or O.

4. (2 min) Some problems have states that cannot be reached through any sequence of actions,
and thus are not in the state space. Describe 2 **unreachable states** for tic-tac-toe.

- A board filled with just O's.
- A board filled with just X's.
- A board where the difference between the O's and X's is greater than 1

(10 min) F. Searching State Space	start time:

We can think of **state space** as a **map**, where each state is a place on the map. Our goal is to find a set of actions to move from the **initial state** to a **goal state**.

The figure at right is a sample map, with **initial state** IN, **goal states** G1-G3, and up to 4 actions from each state. Each "1" is 1 step away from IN, each "2" is 2 steps away from IN, etc.

							G1		
				3					
			3	2	3				
		3	2	1	2	3			
	3	2	1	IN	1	2	3		
		3	2	1	2	3			
		4	3	2	3	4			
	6			3			6		G3
8								8	
G2									

1. (2 min) On the map above, how many states are:

a.	2 steps away from IN?	8
b.	3 steps away from IN?	12
c.	4 steps away from IN?	16

2. (1 min) Would the number of states within 4 steps increase **faster** or **slower**:

a.	if each state had 2 possible actions?	Slower
b.	if each state had 6 possible actions?	Faster

3. (2 min) A map of state space can help us think about ways to search state space. The number of actions on a path is the **path length**.

a.	Which of the 3 goals has the shortest path length from IN?	G1
b.	Which goal is farthest from IN?	G2

Any search **problem** can be formally defined by:

- **state space** (initial state, actions, transition model)
- **goal test**
- **path cost function**

Artificial intelligence typically considers search problems with large and complex state spaces, in which an algorithm can only explore a small fraction of the states.

4. (3 min) The table below describes general categories of problems.
For each description, decide whether solving the problem will require
one action from each state, **several** actions, or **many** actions (at random).

5. (2 min) Rank the categories in the table below from best (1) to worst (6).
If two categories seem equal, give them an equal ranking.

	Description of Problem Category	Approach	Ranking
a.	You **know** the goal state. You **don't know** which actions lead towards it.	Many	4 | 5
b.	You **know** the goal state. You **have clues** for which actions lead towards it.	Several	3
c.	You **don't know** the goal state. You **have clues** for which actions lead towards it.	Several	5 | 4
d.	You **know** the goal state. You **know** which action leads towards it.	One	1
e.	You **don't know** the goal state. You **don't know** which actions lead towards it.	Many	6
f.	You **don't know** the goal state. You **know** which action leads towards it.	One	2

(8 min) G. Analysis using Factorial	start time:

1. (2 min) The product of the sequence 1*2*...*n is written as **n!** and called **n factorial**.
Answer each question below with an expression that uses factorial(s) -
do not multiply or divide to get a final value.

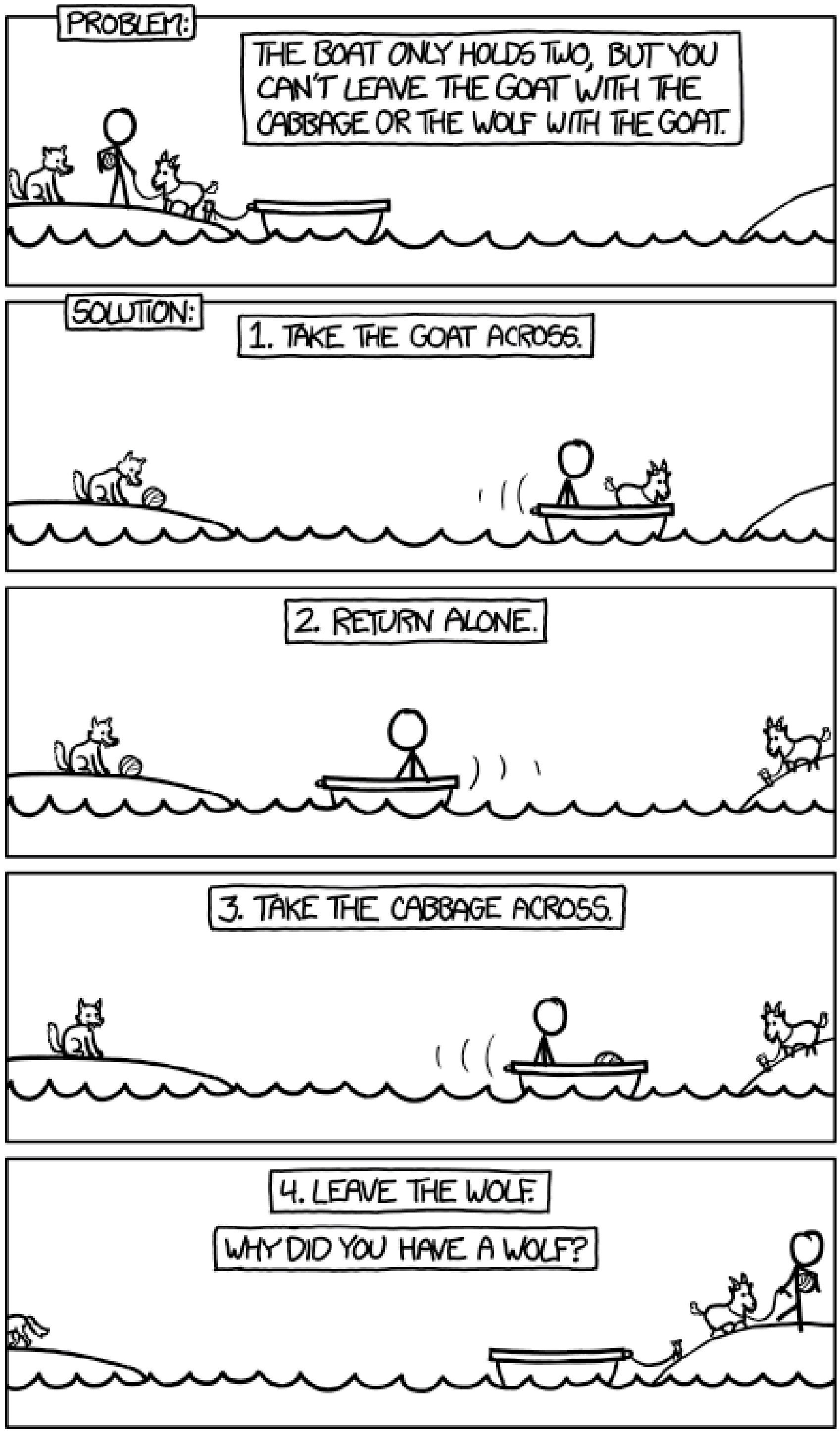
a.	Write out the integers in 5!	5, 4, 3, 2, 1
b.	Write out the integers in 8!	8, 7, 6, 5, 4, 3, 2, 1
c.	Write an expression for 8! that uses 5!	8*7*6*5!
d.	Write an expression for 6*7*8 that uses 5! and 8!	8! / 5!
e.	Write an expression for the number of ways to fill a **blank square** of **order 4**:	4!

2. (2 min) In the **8-Queens Puzzle**:

a.	How many ways could 8 queens be placed?	64*63*...*56
b.	Write the answer to the previous question using factorials	64! / 56!

3. (3 min) In the **general case** of a **blank square** of **order N**.

a.	How many **cells** does it have?	N * N
b.	How many ways could the value 1 be placed?	N!
c.	How many ways could the values 1 & 2 be placed?	N! * (N-1)!
d.	How many ways could all values be placed?	
e.	How many ways could all but the last 2 values be placed?	
f.	How many ways could all but the last 3 values be placed?	
g.	How many ways could all but the last M values be placed?	
h.	How many ways could the first M of N values be placed?	



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