Constraint Satisfaction Problem (CSP) - Definitions	start time:

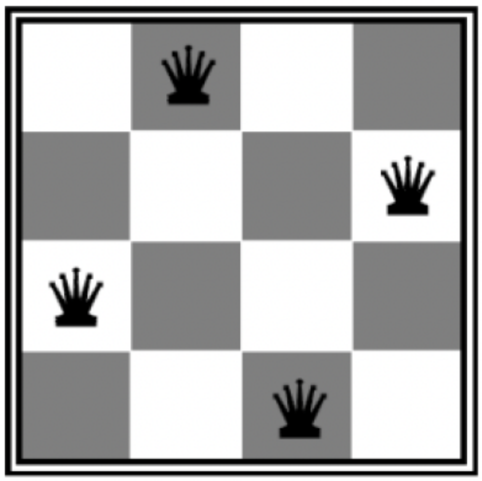
Before you start, share this document with your team member(s) and then complete the form below to assign the role of speaker.

Team Role	Team Member
Speaker: shares your team's ideas with the class.	Addy, Lauren, Arogya

Part A. Components of CSP	start time:

CSPs are defined by three factors:

1. Variables - CSPs possess a set of N variables X1, ...,XN that can each take on a single value from some defined set of values.
2. Domain - A set {x1, ..., xd} representing all possible values that a CSP variable can take on.
3. Constraints - Constraints define restrictions on the values of variables, potentially with regard to other variables.

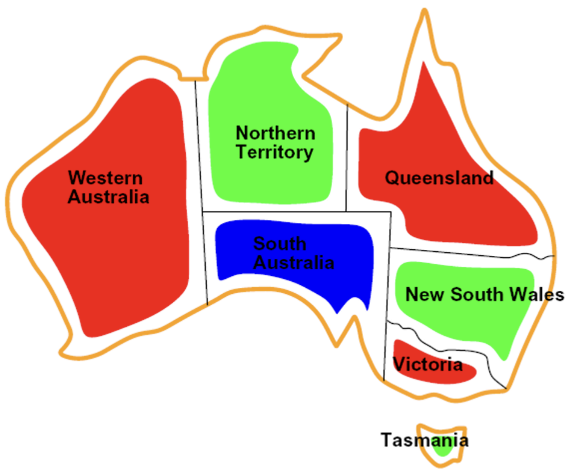


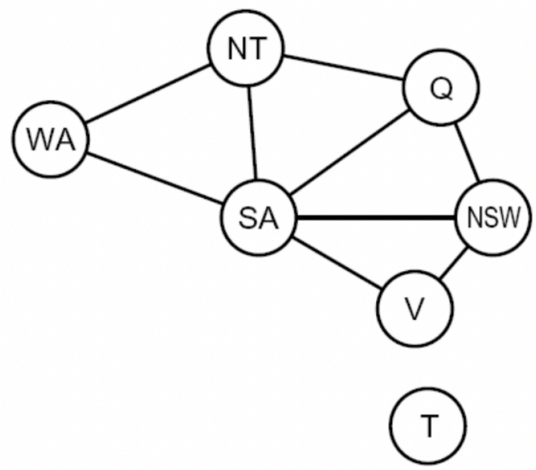
Q1. In the 4-queen problem shown above on a 4 by 4 grid, how many variables are there?
Coordinates (row, column)
4 position of the queen

Q2. What is the domain of each variable?
4 position of the queen ((0,0),(3,3))

Q3. Please list out all constraints of variables and relationships between variables.

Constraints:
No queen can share a row or column or diagonal (Sum of diagonal equals 1)





Consider map coloring the map of Australia using red, green, and blue. The constraints in this problem are simply that no two adjacent states can be the same color.

Q4. How many variables are there in this map coloring problem?
7 states/nodes/variables

Q5. What is the domain of each variable?
3 colors (map colors to numbers aka blue = 1, red = 2, green = 3)

Q6. Please list out all constraints of variables and relationships between variables.

Constraint:
Adjacent nodes cannot share the same color

Relationship between variables:
Adjacent nodes cannot share the same color

Air Traffic Control

We have five planes: A, B, C, D, and E and two runways: international and domestic. We would like to schedule a time slot and runway for each aircraft to either land or take off. We have four time slots: {1, 2, 3, 4} for each runway, during which we can schedule a landing or take off of a plane.

Constraints:

- Plane B has lost an engine and must land in time slot 1.
- Plane D can only arrive at the airport to land during or after time slot 3.
- Plane A is running low on fuel but can last until at most time slot 2.
- Plane D must land before plane C takes off, because some passengers must transfer from D to C.
- Planes A, B, and C cater to international flights and can only use the international runway.
- Planes D and E cater to domestic flights and can only use the domestic runway.
- No two aircrafts can reserve the same time slot for the same runway.

Q7. Complete the formulation of this problem as a CSP in terms of variables, domains, and constraints. Constraints should be expressed implicitly using mathematical or logical notation rather than with words.

Variable:

Planes A,B,C,D,E

Domains:

Tuple (Timeslot, Runway)
Time slots {1,2,3,4}
International versus domestic(1 for international, 2 for domestic)

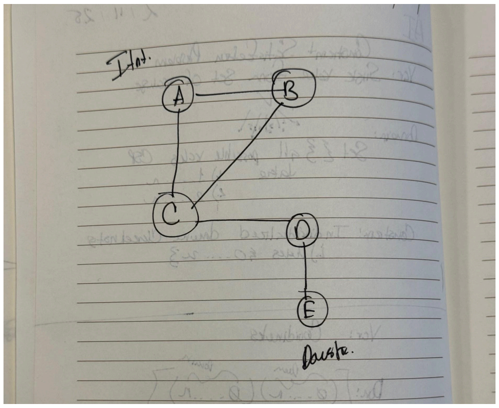
Constraints:

No two aircraft can share runway = y
No two aircraft can share timeslot = x

A - (Ax, Ay) = (1/2, 1) | B - (Bx, By) = (1, 1) | C - (Cx, Cy) = (4, 1) | D - (Dx, Dy) = (3/4, 2) | E - (Ex, Ey) = (1/2/3/4, 2)

Bx = 1
Dx >= 3
Ax <= 2
Dx < Cx
Ay = By = Cy = international
Dy = Ey = domestic
Ax != Bx != Cx
Dx != Ex

Q8. Draw the constraint map below:

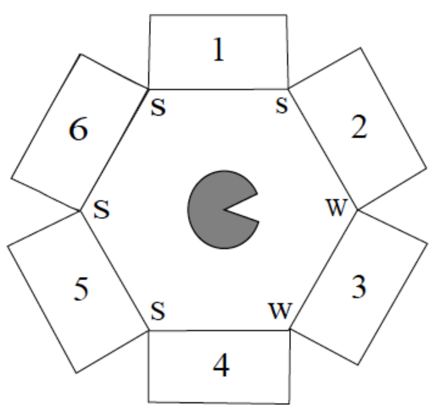


Trapped Pacman

Pacman is trapped! He is surrounded by mysterious corridors, each of which leads to either a pit (P), a ghost (G), or an exit (E). In order to escape, he needs to figure out which corridors, if any, lead to an exit and freedom, rather than the certain doom of a pit or a ghost.

The one sign of what lies behind the corridors is the wind: a pit produces a strong breeze (S) and an exit produces a weak breeze (W), while a ghost doesn't produce any breeze at all. Unfortunately, Pacman cannot measure the strength of the breeze at a specific corridor. Instead, he can stand between two adjacent corridors and feel the max of the two breezes. For example, if he stands between a pit and an exit he will sense a strong(S) breeze, while if he stands between an exit and a ghost, he will sense a weak (W) breeze. The measurements for all intersections are shown in the figure below.

Also, while the total number of exits might be zero, one, or more, Pacman knows that two neighboring squares will not both be exits.



Pacman models this problem using variables Xi for each corridor i and domains P, G, and E.

Q9. State the constraints for this CSP.

Ask Jingsai we suck at formulating math

Xi && Xi+1 != E
Xi && Xi-1 != E

For W:
    Xi == E && (Xi-1 = G || Xi+1 = G)

For S :
    Xi != E && (Xi+1 = P || Xi-1 = P)

P -> S
E -> W
G -> null

Part B. Type of constraints	start time:

CSPs are often represented as constraint graphs, where nodes represent variables and edges represent constraints between them.

• Unary Constraints - Unary constraints involve a single variable in the CSP. They are not represented in constraint graphs, instead simply being used to prune the domain of the variable they constrain when necessary.
• Binary Constraints - Binary constraints involve two variables. They're represented in constraint graphs as traditional graph edges.
• Higher-order Constraint - Constraints involving three or more variables can also be represented with edges in a CSP graph.

Q10. Please classify constraints in N-queen into unary, binary, or higher-order constraints.

Higher Order Constraints: Queens cannot share rows/columns

Q11. Please classify constraints in map coloring into unary, binary, or higher-order constraints.

Binary: Color according to adjacent

Q12. Please classify constraints in air traffic control into unary, binary, or higher-order constraints.

Unary: Runway the plane lands on
Binary:
Higher-order: (Aircraft, Priority of Aircraft, (runway, time))

Q13. Please classify constraints in trapped pacman into unary, binary, or higher-order constraints.

Binary - involves the wind speed from two doors(location, wind_speed)

Part C. Constraint Propagation: Consistency	start time:

Using constraints to reduce the number of legal values for a variable. The idea is that this will leave fewer choices to consider when we make next choice for assigning a value of a variable.

Node consistency - A single variable is node consistency if all values in its domain satisfy the variable's unary constraints.

Arc consistency - Xi is arc-consistent with respect to Xj if for every value in Di, there are some value in Dj that satisfies the binary constraint between Xi and Xj.

FOR HIGHER ORDER CONSTRAINTS:

Path consistency - Xi and Xj is path consistency with respect to Xm if for every consistent pair of values between Xi and Xj, there is a value in Xm that satisfies the binary constraints of Xi,Xm and Xm,Xj.

K-consistency - enhanced version with k X from path consistency.

Q14. Use consistency to eliminate values of other states if coloring WA as red in map coloring problem.

STATE: (WA, RED)
Arc consistency: Binary Constraints -> Adjacent nodes cannot be the same (state,color)
INVALID_ACTIONS: {(NT,RED),(SA,RED),(NSW,RED)}

Q15. Use consistency to eliminate values of other states if coloring WA as red and Q as green in map coloring problem.

WA: RED
Q: GREEN

NT: NOT RED NOT GREEN
SA: NOT RED NOT GREEN
NSW: NOT GREEN

Part D. Definition of CSP	start time:

Q16: What is the difference between CSP and searching problem in a state space covered in previous lectures?

- CSP specifies states based on variables with assigned values and a set of constraints that these values must satisfy
- Searching problem lacks constraints

Q17: What algorithms we have learned in 306 could be used to solve CSP?

Backtracking - algorithm to solve CSP
Greedy, Decrease & Conquer