Some Helper Function:

In [10]:
```
from google.colab import drive
drive.mount('/content/drive')
```

```
Mounted at /content/drive
```

Softmax Function:

In [11]:
```
import numpy as np

def softmax(z):
    """
    Compute the softmax probabilities for a given input matrix.

    Parameters:
    z (numpy.ndarray): Logits (raw scores) of shape (m, n), where
                       - m is the number of samples.
                       - n is the number of classes.

    Returns:
    numpy.ndarray: Softmax probability matrix of shape (m, n), where
                   each row sums to 1 and represents the probability
                   distribution over classes.

    Notes:
    - The input to softmax is typically computed as: z = XW + b.
    - Uses numerical stabilization by subtracting the max value per row.
    """

    z_exp = np.exp(z - np.max(z, axis=1, keepdims=True))  # Numerical sta
    return z_exp / np.sum(z_exp, axis=1, keepdims=True)
```

Softmax Test Case:

This test case checks that each row in the resulting softmax probabilities sums to 1, which is the fundamental property of softmax.

In [12]:
```
# Example test case
z_test = np.array([[2.0, 1.0, 0.1], [1.0, 1.0, 1.0]])
softmax_output = softmax(z_test)

# Verify if the sum of probabilities for each row is 1 using assert
row_sums = np.sum(softmax_output, axis=1)

# Assert that the sum of each row is 1
assert np.allclose(row_sums, 1), f"Test failed: Row sums are {row_sums}"

print("Softmax function passed the test case!")
```

```
Softmax function passed the test case!
```

Prediction Function:

In [13]:
```python
def predict_softmax(X, W, b):
    """
    Predict the class labels for a set of samples using the trained softm

    Parameters:
    X (numpy.ndarray): Feature matrix of shape (n, d), where n is the num
    W (numpy.ndarray): Weight matrix of shape (d, c), where c is the numb
    b (numpy.ndarray): Bias vector of shape (c,).

    Returns:
    numpy.ndarray: Predicted class labels of shape (n,), where each value
    """
    logits = np.dot(X, W) + b
    return np.argmax(softmax(logits), axis=1)
```

Test Function for Prediction Function:

The test function ensures that the predicted class labels have the same number of elements as the input samples, verifying that the model produces a valid output shape.

In [14]:
```python
# Define test case
X_test = np.array([[0.2, 0.8], [0.5, 0.5], [0.9, 0.1]])  # Feature matrix
W_test = np.array([[0.4, 0.2, 0.1], [0.3, 0.7, 0.5]])  # Weights (2 featu
b_test = np.array([0.1, 0.2, 0.3])  # Bias (3 classes)

# Expected Output:
# The function should return an array with class labels (0, 1, or 2)

y_pred_test = predict_softmax(X_test, W_test, b_test)

# Validate output shape
assert y_pred_test.shape == (3,), f"Test failed: Expected shape (3,), got

# Print the predicted labels
print("Predicted class labels:", y_pred_test)
```

```
Predicted class labels: [1 1 0]
```

Loss Function:

In [15]:
```python
def loss_softmax(y_pred, y):
    """
    Compute the cross-entropy loss for a single sample.

    Parameters:
    y_pred (numpy.ndarray): Predicted probabilities of shape (c,) for a s
                            where c is the number of classes.
    y (numpy.ndarray): True labels (one-hot encoded) of shape (c,), where

    Returns:
    float: Cross-entropy loss for the given sample.
    """

    return -np.sum(y * np.log(y_pred + 1e-8)) / y.shape[0]  # Avoid log(0
```

Test case for Loss Function:

This test case Compares loss for correct vs. incorrect predictions.

- Expects low loss for correct predictions.
- Expects high loss for incorrect predictions.

In [16]:
```
import numpy as np

# Define correct predictions (low loss scenario)
y_true_correct = np.array([[1, 0, 0], [0, 1, 0], [0, 0, 1]])  # True one-
y_pred_correct = np.array([[0.9, 0.05, 0.05],
                           [0.1, 0.85, 0.05],
                           [0.05, 0.1, 0.85]])  # High confidence in the

# Define incorrect predictions (high loss scenario)
y_pred_incorrect = np.array([[0.05, 0.05, 0.9],  # Highly confident in th
                             [0.1, 0.05, 0.85],
                             [0.85, 0.1, 0.05]])

# Compute loss for both cases
loss_correct = loss_softmax(y_pred_correct, y_true_correct)
loss_incorrect = loss_softmax(y_pred_incorrect, y_true_correct)

# Validate that incorrect predictions lead to a higher loss
assert loss_correct < loss_incorrect, f"Test failed: Expected loss_correc

# Print results
print(f"Cross-Entropy Loss (Correct Predictions): {loss_correct:.4f}")
print(f"Cross-Entropy Loss (Incorrect Predictions): {loss_incorrect:.4f}"
```

```
Cross-Entropy Loss (Correct Predictions): 0.1435
Cross-Entropy Loss (Incorrect Predictions): 2.9957
```

Cost Function:

In [17]:
```
def cost_softmax(X, y, W, b):
    """
    Compute the average softmax regression cost (cross-entropy loss) over

    Parameters:
    X (numpy.ndarray): Feature matrix of shape (n, d), where n is the num
    y (numpy.ndarray): True labels (one-hot encoded) of shape (n, c), whe
    W (numpy.ndarray): Weight matrix of shape (d, c).
    b (numpy.ndarray): Bias vector of shape (c,).

    Returns:
    float: Average softmax cost (cross-entropy loss) over all samples.
    """

    logits = np.dot(X, W) + b
    y_pred = softmax(logits)
    return loss_softmax(y_pred, y)
```

Test Case for Cost Function:

The test case assures that the cost for the incorrect prediction should be higher than for the correct prediction, confirming that the cost function behaves as expected.

In [18]:
```
import numpy as np

# Example 1: Correct Prediction (Closer predictions)
X_correct = np.array([[1.0, 0.0], [0.0, 1.0]])  # Feature matrix for corr
y_correct = np.array([[1, 0], [0, 1]])  # True labels (one-hot encoded, m
W_correct = np.array([[5.0, -2.0], [-3.0, 5.0]])  # Weights for correct p
b_correct = np.array([0.1, 0.1])  # Bias for correct prediction

# Example 2: Incorrect Prediction (Far off predictions)
X_incorrect = np.array([[0.1, 0.9], [0.8, 0.2]])  # Feature matrix for in
y_incorrect = np.array([[1, 0], [0, 1]])  # True labels (one-hot encoded,
W_incorrect = np.array([[0.1, 2.0], [1.5, 0.3]])  # Weights for incorrect
b_incorrect = np.array([0.5, 0.6])  # Bias for incorrect prediction

# Compute cost for correct predictions
cost_correct = cost_softmax(X_correct, y_correct, W_correct, b_correct)

# Compute cost for incorrect predictions
cost_incorrect = cost_softmax(X_incorrect, y_incorrect, W_incorrect, b_in

# Check if the cost for incorrect predictions is greater than for correct
assert cost_incorrect > cost_correct, f"Test failed: Incorrect cost {cost

# Print the costs for verification
print("Cost for correct prediction:", cost_correct)
print("Cost for incorrect prediction:", cost_incorrect)

print("Test passed!")
```

```
Cost for correct prediction: 0.0006234264070982092
Cost for incorrect prediction: 0.2993086000673444
Test passed!
```

```
/usr/lib/python3.11/tokenize.py:529: RuntimeWarning: coroutine 'main' was
never awaited
  pseudomatch = _compile(PseudoToken).match(line, pos)
RuntimeWarning: Enable tracemalloc to get the object allocation traceback
```

Computing Gradients:

In [19]:
```
def compute_gradient_softmax(X, y, W, b):
    """
    Compute the gradients of the cost function with respect to weights an

    Parameters:
    X (numpy.ndarray): Feature matrix of shape (n, d).
    y (numpy.ndarray): True labels (one-hot encoded) of shape (n, c).
    W (numpy.ndarray): Weight matrix of shape (d, c).
    b (numpy.ndarray): Bias vector of shape (c,).

    Returns:
    tuple: Gradients with respect to weights (d, c) and biases (c,).
    """


    m = X.shape[0]
```

```
    logits = np.dot(X, W) + b
    y_pred = softmax(logits)
    grad_W = np.dot(X.T, (y_pred - y)) / m
    grad_b = np.sum(y_pred - y, axis=0) / m
    return grad_W, grad_b
```

Test case for compute_gradient function:

The test checks if the gradients from the function are close enough to the manually computed gradients using np.allclose, which accounts for potential floating-point discrepancies.

In [20]:
```
import numpy as np

# Define a simple feature matrix and true labels
X_test = np.array([[0.2, 0.8], [0.5, 0.5], [0.9, 0.1]])  # Feature matrix
y_test = np.array([[1, 0, 0], [0, 1, 0], [0, 0, 1]])  # True labels (one-

# Define weight matrix and bias vector
W_test = np.array([[0.4, 0.2, 0.1], [0.3, 0.7, 0.5]])  # Weights (2 featu
b_test = np.array([0.1, 0.2, 0.3])  # Bias (3 classes)

# Compute the gradients using the function
grad_W, grad_b = compute_gradient_softmax(X_test, y_test, W_test, b_test)

# Manually compute the predicted probabilities (using softmax function)
z_test = np.dot(X_test, W_test) + b_test
y_pred_test = softmax(z_test)

# Compute the manually computed gradients
grad_W_manual = np.dot(X_test.T, (y_pred_test - y_test)) / X_test.shape[0
grad_b_manual = np.sum(y_pred_test - y_test, axis=0) / X_test.shape[0]

# Assert that the gradients computed by the function match the manually c
assert np.allclose(grad_W, grad_W_manual), f"Test failed: Gradients w.r.t
assert np.allclose(grad_b, grad_b_manual), f"Test failed: Gradients w.r.t

# Print the gradients for verification
print("Gradient w.r.t. W:", grad_W)
print("Gradient w.r.t. b:", grad_b)

print("Test passed!")
```

```
Gradient w.r.t. W: [[ 0.1031051   0.01805685 -0.12116196]
 [-0.13600547  0.00679023  0.12921524]]
Gradient w.r.t. b: [-0.03290036  0.02484708  0.00805328]
Test passed!
```

Implementing Gradient Descent:

In [21]:
```
def gradient_descent_softmax(X, y, W, b, alpha, n_iter, show_cost=False):
    """
    Perform gradient descent to optimize the weights and biases.

    Parameters:
    X (numpy.ndarray): Feature matrix of shape (n, d).
    y (numpy.ndarray): True labels (one-hot encoded) of shape (n, c).
```

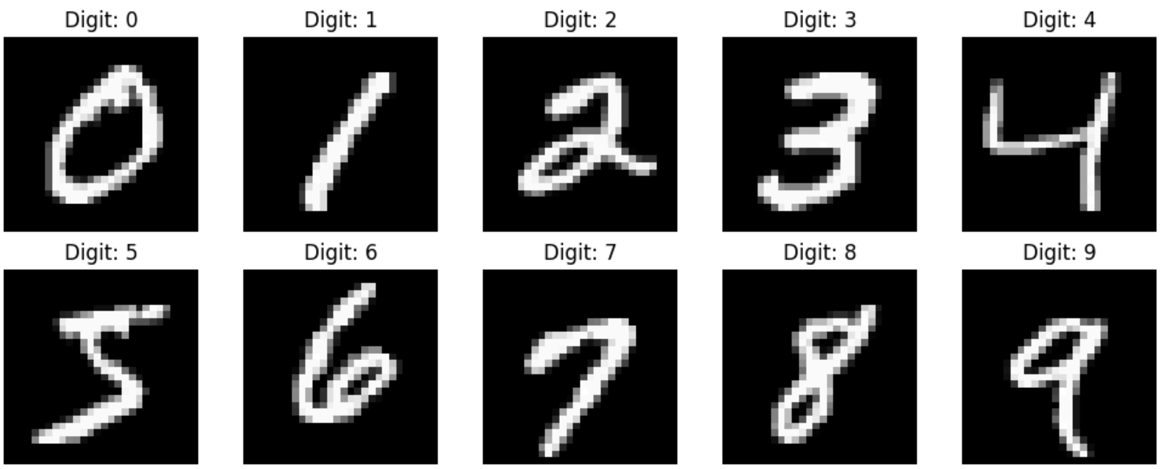
```
    W (numpy.ndarray): Weight matrix of shape (d, c).
    b (numpy.ndarray): Bias vector of shape (c,).
    alpha (float): Learning rate.
    n_iter (int): Number of iterations.
    show_cost (bool): Whether to display the cost at intervals.

    Returns:
    tuple: Optimized weights, biases, and cost history.
    """
    cost_history = []
    for i in range(n_iter):
        grad_W, grad_b = compute_gradient_softmax(X, y, W, b)
        W -= alpha * grad_W
        b -= alpha * grad_b
        cost = cost_softmax(X, y, W, b)
        cost_history.append(cost)
        if show_cost and i % 100 == 0:
            print(f"Iteration {i}, Cost: {cost:.4f}")
    return W, b, cost_history
```

Preparing Dataset:

In [22]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from sklearn.model_selection import train_test_split

def load_and_prepare_mnist(csv_file, test_size=0.2, random_state=42):
    """
    Reads the MNIST CSV file, splits data into train/test sets, and plots

    Arguments:
    csv_file (str)       : Path to the CSV file containing MNIST data.
    test_size (float)    : Proportion of the data to use as the test set
    random_state (int)   : Random seed for reproducibility (default: 42).

    Returns:
    X_train, X_test, y_train, y_test : Split dataset.
    """

    # Load dataset
    df = pd.read_csv(csv_file)

    # Separate labels and features
    y = df.iloc[:, 0].values  # First column is the label
    X = df.iloc[:, 1:].values  # Remaining columns are pixel values

    # Normalize pixel values (optional but recommended)
    X = X / 255.0  # Scale values between 0 and 1

    # Split data into train and test sets
    X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=t

    # Plot one sample image per class
    plot_sample_images(X, y)

    return X_train, X_test, y_train, y_test
```

```python
def plot_sample_images(X, y):
    """
    Plots one sample image for each digit class (0-9).

    Arguments:
    X (np.ndarray): Feature matrix containing pixel values.
    y (np.ndarray): Labels corresponding to images.
    """

    plt.figure(figsize=(10, 4))
    unique_classes = np.unique(y)  # Get unique class labels

    for i, digit in enumerate(unique_classes):
        index = np.where(y == digit)[0][0]  # Find first occurrence of th
        image = X[index].reshape(28, 28)  # Reshape 1D array to 28x28

        plt.subplot(2, 5, i + 1)
        plt.imshow(image, cmap='gray')
        plt.title(f"Digit: {digit}")
        plt.axis('off')

    plt.tight_layout()
    plt.show()
```

In [23]:
```python
csv_file_path = "/content/drive/MyDrive/AI and ML/Week2/mnist_dataset.csv
X_train, X_test, y_train, y_test = load_and_prepare_mnist(csv_file_path)
```



A Quick debugging Step:

In [24]:
```python
# Assert that X and y have matching lengths
assert len(X_train) == len(y_train), f"Error: X and y have different leng
print("Move forward: Dimension of Feture Matrix X and label vector y matc
```

```
Move forward: Dimension of Feture Matrix X and label vector y matched.
```

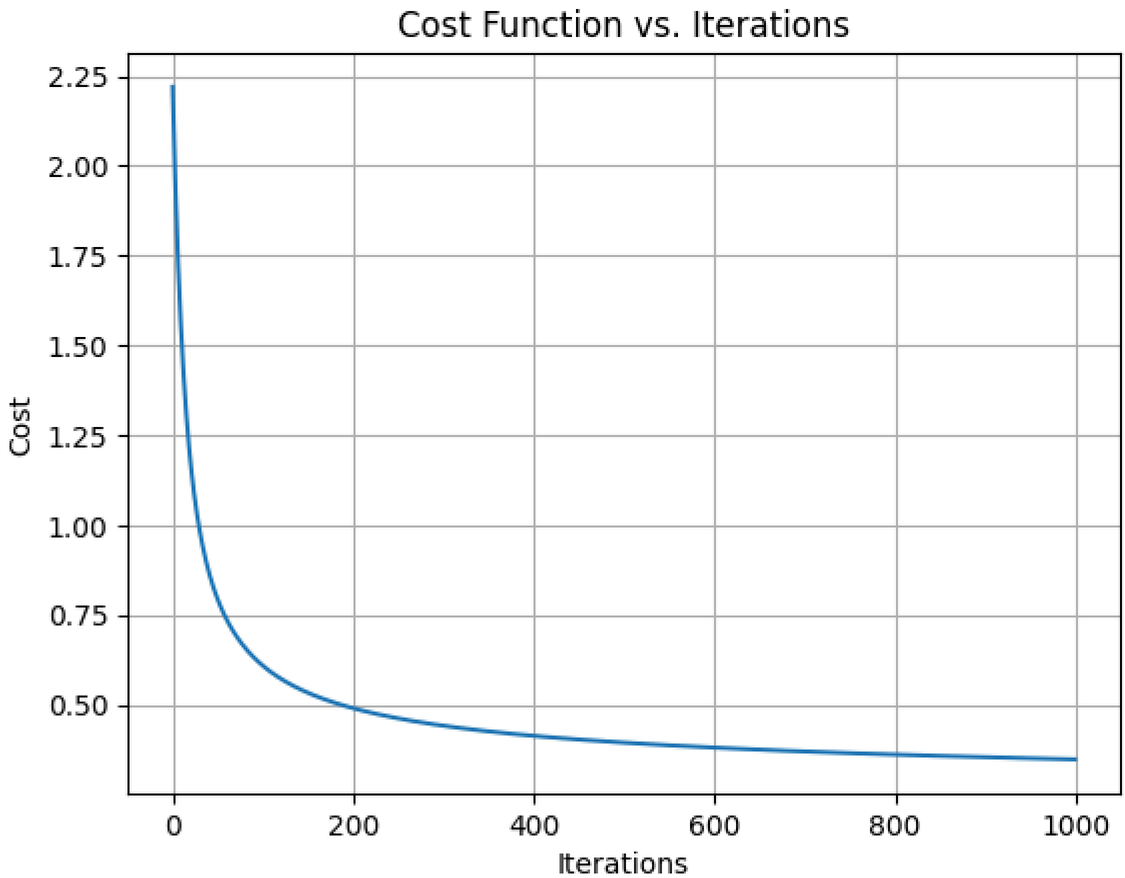
Train the Model:

In [25]:
```python
print(f"Training data shape: {X_train.shape}")
print(f"Test data shape: {X_test.shape}")
```

```
Training data shape: (48000, 784)
Test data shape: (12000, 784)
```

In [26]:
```
from sklearn.preprocessing import OneHotEncoder

# Check if y_train is one-hot encoded
if len(y_train.shape) == 1:
    encoder = OneHotEncoder(sparse_output=False)  # Use sparse_output=Fal
    y_train = encoder.fit_transform(y_train.reshape(-1, 1))  # One-hot en
    y_test = encoder.transform(y_test.reshape(-1, 1))  # One-hot encode t

# Now y_train is one-hot encoded, and we can proceed to use it
d = X_train.shape[1]  # Number of features (columns in X_train)
c = y_train.shape[1]  # Number of classes (columns in y_train after one-h

# Initialize weights with small random values and biases with zeros
W = np.random.randn(d, c) * 0.01  # Small random weights initialized
b = np.zeros(c)  # Bias initialized to 0

# Set hyperparameters for gradient descent
alpha = 0.1  # Learning rate
n_iter = 1000  # Number of iterations to run gradient descent

# Train the model using gradient descent
W_opt, b_opt, cost_history = gradient_descent_softmax(X_train, y_train, W

# Plot the cost history to visualize the convergence
plt.plot(cost_history)
plt.title('Cost Function vs. Iterations')
plt.xlabel('Iterations')
plt.ylabel('Cost')
plt.grid(True)
plt.show()
```

```
Iteration 0, Cost: 2.2202
Iteration 100, Cost: 0.6087
Iteration 200, Cost: 0.4903
Iteration 300, Cost: 0.4415
Iteration 400, Cost: 0.4133
Iteration 500, Cost: 0.3944
Iteration 600, Cost: 0.3805
Iteration 700, Cost: 0.3697
Iteration 800, Cost: 0.3611
Iteration 900, Cost: 0.3539
```



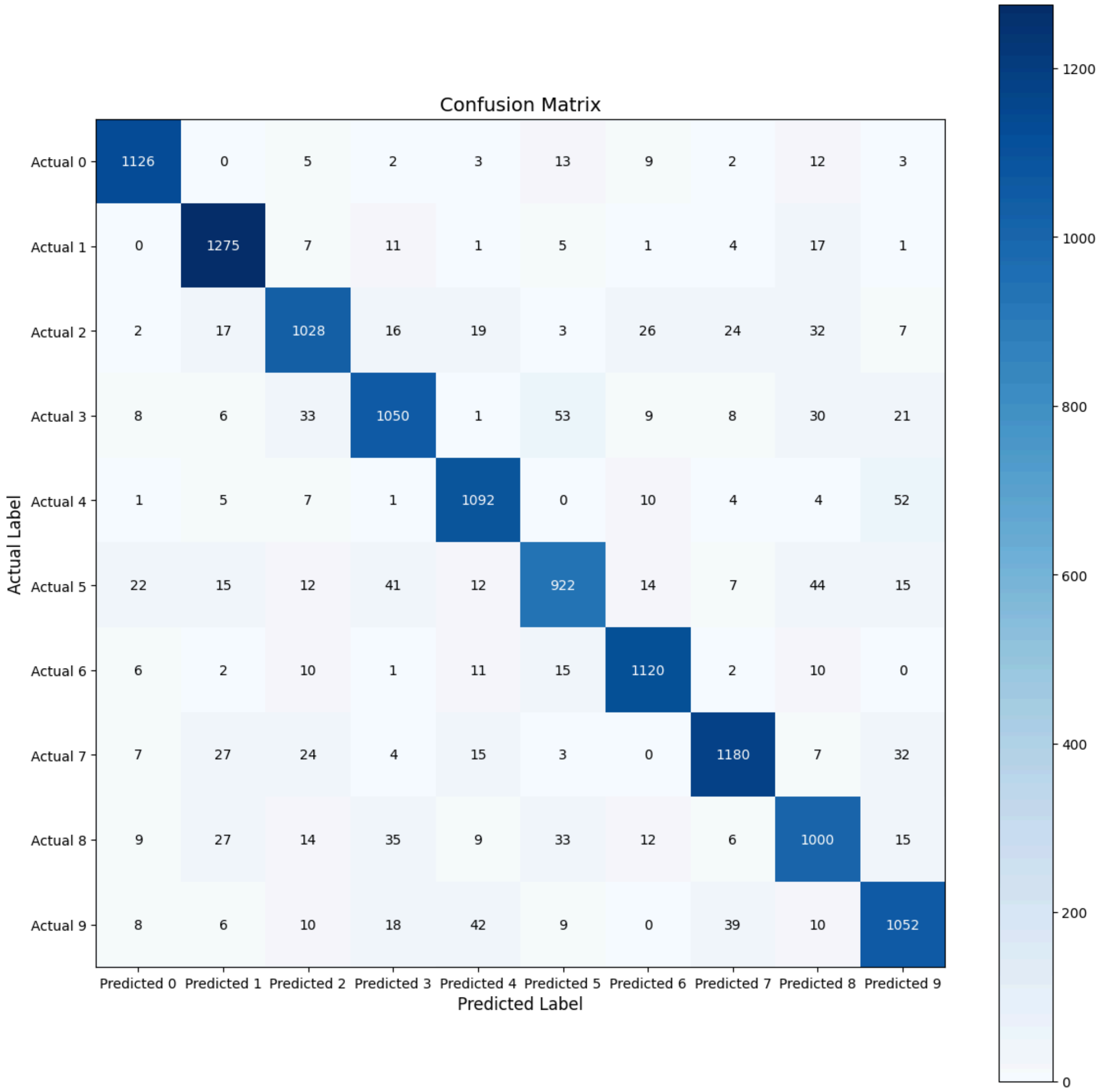
Evaluating the Model:

In [27]:
```
import numpy as np
import matplotlib.pyplot as plt
from sklearn.metrics import confusion_matrix, precision_score, recall_sco

# Evaluation Function
def evaluate_classification(y_true, y_pred):
    """
    Evaluate classification performance using confusion matrix, precision

    Parameters:
    y_true (numpy.ndarray): True labels
    y_pred (numpy.ndarray): Predicted labels

    Returns:
    tuple: Confusion matrix, precision, recall, F1 score
    """
    # Compute confusion matrix
    cm = confusion_matrix(y_true, y_pred)

    # Compute precision, recall, and F1-score
    precision = precision_score(y_true, y_pred, average='weighted')
    recall = recall_score(y_true, y_pred, average='weighted')
    f1 = f1_score(y_true, y_pred, average='weighted')

    return cm, precision, recall, f1
```

In [28]:
```
# Predict on the test set
y_pred_test = predict_softmax(X_test, W_opt, b_opt)
```

```
# Evaluate accuracy
y_test_labels = np.argmax(y_test, axis=1)  # True labels in numeric form

# Evaluate the model
cm, precision, recall, f1 = evaluate_classification(y_test_labels, y_pred

# Print the evaluation metrics
print("\nConfusion Matrix:")
print(cm)
print(f"Precision: {precision:.2f}")
print(f"Recall: {recall:.2f}")
print(f"F1-Score: {f1:.2f}")

# Visualizing the Confusion Matrix
fig, ax = plt.subplots(figsize=(12, 12))
cax = ax.imshow(cm, cmap='Blues')  # Use a color map for better visualiza

# Dynamic number of classes
num_classes = cm.shape[0]
ax.set_xticks(range(num_classes))
ax.set_yticks(range(num_classes))
ax.set_xticklabels([f'Predicted {i}' for i in range(num_classes)])
ax.set_yticklabels([f'Actual {i}' for i in range(num_classes)])

# Add labels to each cell in the confusion matrix
for i in range(cm.shape[0]):
    for j in range(cm.shape[1]):
        ax.text(j, i, cm[i, j], ha='center', va='center', color='white' i

# Add grid lines and axis labels
ax.grid(False)
plt.title('Confusion Matrix', fontsize=14)
plt.xlabel('Predicted Label', fontsize=12)
plt.ylabel('Actual Label', fontsize=12)

# Adjust layout
plt.tight_layout()
plt.colorbar(cax)
plt.show()
```

```
Confusion Matrix:
[[1126    0    5    2    3   13    9    2   12    3]
 [   0 1275    7   11    1    5    1    4   17    1]
 [   2   17 1028   16   19    3   26   24   32    7]
 [   8    6   33 1050    1   53    9    8   30   21]
 [   1    5    7    1 1092    0   10    4    4   52]
 [  22   15   12   41   12  922   14    7   44   15]
 [   6    2   10    1   11   15 1120    2   10    0]
 [   7   27   24    4   15    3    0 1180    7   32]
 [   9   27   14   35    9   33   12    6 1000   15]
 [   8    6   10   18   42    9    0   39   10 1052]]
Precision: 0.90
Recall: 0.90
F1-Score: 0.90
```



Linear Seperability and Logistic Regression:

In [30]:
```
import numpy as np
import matplotlib.pyplot as plt
from sklearn.datasets import make_classification, make_circles
from sklearn.model_selection import train_test_split
from sklearn.linear_model import LogisticRegression

# Set random seed for reproducibility
np.random.seed(42)
# Generate linearly separable dataset
X_linear_separable, y_linear_separable = make_classification(n_samples=20

# Split the data into training and testing sets
X_train_linear, X_test_linear, y_train_linear, y_test_linear = train_test

# Train logistic regression model on linearly separable data
logistic_model_linear_separable = LogisticRegression()
logistic_model_linear_separable.fit(X_train_linear, y_train_linear)

# Generate non-linearly separable dataset (circles)
```

```
X_non_linear_separable, y_non_linear_separable = make_circles(n_samples=2

# Split the data into training and testing sets
X_train_non_linear, X_test_non_linear, y_train_non_linear, y_test_non_lin

# Train logistic regression model on non-linearly separable data
logistic_model_non_linear_separable = LogisticRegression()
logistic_model_non_linear_separable.fit(X_train_non_linear, y_train_non_l
```

Out[30]:

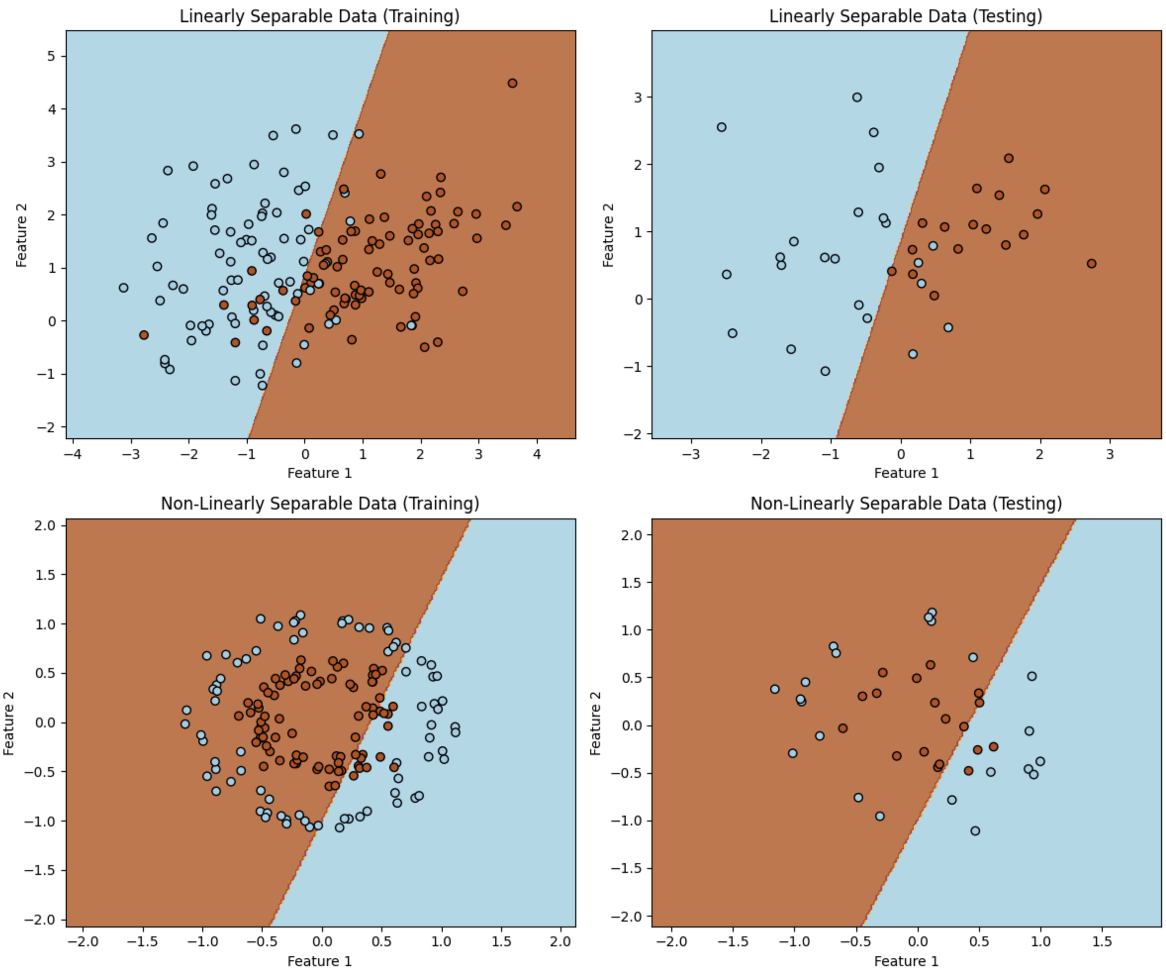
```
▾ LogisticRegression

LogisticRegression()
```

In [31]:

```
# Plot decision boundaries for linearly and non-linearly separable data
def plot_decision_boundary(ax, model, X, y, title):
  h = .02 # step size in the mesh
  x_min, x_max = X[:, 0].min() - 1, X[:, 0].max() + 1
  y_min, y_max = X[:, 1].min() - 1, X[:, 1].max() + 1
  xx, yy = np.meshgrid(np.arange(x_min, x_max, h), np.arange(y_min, y_max
  Z = model.predict(np.c_[xx.ravel(), yy.ravel()])
  Z = Z.reshape(xx.shape)
  ax.contourf(xx, yy, Z, alpha=0.8, cmap=plt.cm.Paired)
  ax.scatter(X[:, 0], X[:, 1], c=y, edgecolors='k', cmap=plt.cm.Paired)
  ax.set_title(title)
  ax.set_xlabel('Feature 1')
  ax.set_ylabel('Feature 2')
```

In [32]:

```
# Create subplots
fig, axes = plt.subplots(2, 2, figsize=(12, 10))
# Plot decision boundary for linearly separable data (Training)
plot_decision_boundary(axes[0, 0], logistic_model_linear_separable, X_tra
# Plot decision boundary for linearly separable data (Testing)
plot_decision_boundary(axes[0, 1], logistic_model_linear_separable, X_tes
# Plot decision boundary for non-linearly separable data (Training)
plot_decision_boundary(axes[1, 0], logistic_model_non_linear_separable, X
# Plot decision boundary for non-linearly separable data (Testing)
plot_decision_boundary(axes[1, 1], logistic_model_non_linear_separable, X
plt.tight_layout()
# Save the plots as PNG files
plt.savefig('decision_boundaries.png')
plt.show()
```


Linearly Separable Data (Training)
Linearly Separable Data (Testing)
Non-Linearly Separable Data (Training)
Non-Linearly Separable Data (Testing)
Feature 1
Feature 2