Advanced Model Predictive Control — Fall 2022
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Programming Exercise 4

Recovery Initialization and Indirect Feedback SMPC

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1. Implementation of stochastic MPC methods.

   a. **(Graded.)** Choose $p = 0.9$ and compute ellipsoidal PRS set $\mathcal{F}_\infty^p$ and the respective controller in the `compute_PRS_min` method in the provided `PRS_SMPC.m` file using an SDP for some p. Use the synthesised controller to compute $\mathcal{F}_i^p$. Compute the constraint tightenings for different choices of the probability p and observe how the tightenings and $\mathcal{F}_i^p$ change.
   Hint: You can use $\text{tr}(\cdot)$ as a volume upper bound and implement the Lyapunov equation seen in Recitation 7 as an inequality (recall Recitation 3 and 4).

   b. **(Graded.)** Implement the recovery initialization SMPC

$$
\begin{aligned}
\min_{v_i} \ & \|\bar{x}_N\|_P^2 + \sum_{i=0}^{N-1} \|\bar{x}_i\|_Q^2 + \|\bar{u}_i\|_R^2 \\
\text{s.t. } & \bar{x}_{i+1} = A\bar{x}_i + B\bar{u}_i \\
& \bar{x}_i \in \mathcal{X} \ominus \mathcal{F}_{i+k}^p \\
& \bar{u}_i \in \mathcal{U} \ominus K\mathcal{F}_{i+k}^p \\
& \bar{x}_N = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\
& \bar{x}_0 = \begin{cases} x(k), \text{ if feasible} \\ \bar{x}_{1|k-1}, \text{ otherwise} \end{cases}
\end{aligned}
$$

   in the `PRS_SMPC.m` and `main_PE4.m` files.

   c. **(Graded.)** Implement the indirect feedback SMPC problem

$$
\begin{aligned}
\min_{v_i} \ & \|\bar{x}_N\|_P^2 + \sum_{i=0}^{N-1} \|\bar{x}_i\|_Q^2 + \|\bar{u}_i\|_R^2 \\
\text{s.t } & z_{i+1} = Az_i + Bv_i \\
& \bar{x}_{i+1} = A\bar{x}_i + B\bar{u}_i \\
& \bar{e}_i = \bar{x}_i - z_i \\
& \bar{u}_i = K\bar{e}_i + v_i \\
& z_i \in \mathcal{X} \ominus \mathcal{F}_{k+i}^p, \quad v_i \in \mathcal{U} \ominus K\mathcal{F}_{k+i}^p \\
& z_N = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\
& \bar{x}_0 = x(k), z_0 = z_{1|k-1}
\end{aligned}
$$

   in the provided `Indirect_Feedback_SMPC.m` file.