Advanced Model Predictive Control — Fall 2022
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Exercise

Programming Exercise 5

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Coding Exercises [graded]

1. In the provided `SM.m` file, implement the set-membership update in `update_estimate` by computing the non-falsified parameter set Δ_i and intersecting it with the previous set estimate Ω_{i-1}.

2. Remove redundant halfspaces by completing the `remove_redundant_halfspace` method in `SM.m`. Use either the method presented in the lecture/recitation or suitable MPT3 functionalities.
   Hint: If you use the method presented in the lecture/recitation: Make sure to exclude halfspace constraints that where already marked as redundant when solving the linear program. Additionally, check if the linear program attains a solution in each iteration (the objective can grow unbounded).

3. Implement the `compute_robust_tightening` method in `SMMPC.m`, which computes the disturbance reachable sets (DRS)

$$\mathcal{F}_i^k = \bigoplus_{j=0}^{i-1} (A_{\bar{\theta}} + BK)^j \bar{\mathcal{W}}^k$$

   where K, and $\bar{\mathcal{W}}^k$ are arguments passed to the method and $\bar{\theta}$ is the initial parameter estimate. Subsequently, the method computes the tightened state and input constraints given the DRS, i.e., $\mathcal{X} \ominus \mathcal{F}_i^k$ and $\mathcal{U} \ominus K\mathcal{F}_i^k$.
   Hint: $A_{\bar{\theta}}$ is provided by the `LinearAffineSystem.m` class and does not need to be computed.

4. Implement the following SM-MPC with DRS tubes in the `SMMPC.m` file

$$\begin{aligned}
\min_{v_i, z_i, \hat{x}_i} \quad & \|\hat{x}_N\|_P^2 + \sum_{i=0}^{N-1} \|\hat{x}_i\|_Q^2 + \|\hat{v}_i\|_R^2 \\
\text{s.t.} \quad & z_0 = x(k), \quad \hat{x}_0 = x(k), \\
& \hat{x}_{i+1} = A_{\hat{\theta}_k} \hat{x}_i + B\hat{u}_i, && i = 0, \dots, N-1, \\
& \hat{u}_i = K(\hat{x}_i - z_i) + v_i, && i = 0, \dots, N-1, \\
& z_{i+1} = A_{\bar{\theta}} z_i + B v_i, && i = 0, \dots, N-1, \\
& z_i \in \mathcal{X} \ominus \mathcal{F}_i^k, && i = 0, \dots, N-1, \\
& v_i \in \mathcal{U} \ominus K\mathcal{F}_i^k, && i = 0, \dots, N-1, \\
& z_N \in \mathcal{X}_f \ominus \mathcal{F}_N^k,
\end{aligned}$$

   where $\bar{\theta}$ is the initial parameter estimate, $\hat{\theta}_k$ is the parameter estimate given by the set membership estimator at time step k, $\mathcal{F}_i^k = \bigoplus_{j=0}^{i-1} (A_{\bar{\theta}} + BK)^j \bar{\mathcal{W}}^k$ are DRS tubes with $\bar{\mathcal{W}}^k = \mathcal{W} \oplus \mathcal{W}_{\theta}^k$, $A_{\hat{\theta}_k} = A + A_1\hat{\theta}_{k,1} + A_2\hat{\theta}_{k,2}$, $A_{\bar{\theta}} = A + A_1\bar{\theta}_1 + A_2\bar{\theta}_2$, and $\mathcal{W}_{\theta}^k = \{w \mid w = \begin{bmatrix} A_1 x & A_2 x \end{bmatrix} (\theta - \bar{\theta}), x \in \mathcal{X}, \theta \in \Omega_k\}$. The tightened terminal set is provided to you, please see the template for more detailed instructions on how to include it in the constraints.

Optional Pen & Paper Exercise [not graded]

5. Consider the system matrices

$$A_\theta = \begin{bmatrix} 1 & \theta_1 \\ 0 & 1 \end{bmatrix} \text{ and } B_\theta = \begin{bmatrix} 0 \\ \theta_2 \end{bmatrix}$$

and a disturbance $w \in \mathbb{R}^2$ with

$$\|w\|_\infty \leq 0.5.$$

It holds that $\theta \in \Omega_0 = \{\theta \in \mathbb{R}^{n_\theta} \mid H_{\theta_0}\theta \leq h_{\theta_0}\}$, where

$$\|\theta_1\| \leq 2 \text{ and } \|\theta_2\| \leq 1.$$

Let the initial state be $x_0 = \begin{bmatrix} 3 & -1 \end{bmatrix}^\top$, the input $u_0 = -1$ and measured state at the next time step $y_1 = \begin{bmatrix} 1.3 & -1.9 \end{bmatrix}^\top$. Perform the set-membership update with the given data.