Advanced Model Predictive Control — Fall 2022
Prof. M. Zeilinger
Dr. A. Carron
Dr. L. Hewing
Dr. J. Köhler

Exercise

Programming Exercise 3

Alexandre Didier and Jérôme Sieber

1 Exercise

Nonlinear Robust MPC

1. **(Graded)** Implementation of nonlinear robust MPC in the `Nonlinear_RMPC.m` file.

   a. Implement the computation of the RPI set Ω and the tightenings, which we derived in the recition in the `compute_tightening` method.

   b. Compute the constraint tightenings for different choices of ρ and observe how the tightenings and Ω change. Choose a ρ for the remainder of the exercise.

   c. Consider the nonlinear robust MPC problem

$$\begin{aligned}
\min_{V,z_0} \quad & \sum_{i=0}^{N-1} z_i^T Q z_i + v_i^T R v_i && && \text{(1a)}\\
\text{s.t.} \quad & \forall i = 0, \dots, N-1, && && \text{(1b)}\\
& z_{i+1} = f(z_i, v_i), && && \text{(1c)}\\
& [A_x]_j z_i \leq [b_x]_j - c_{x,j}\delta, && j \in [1, n_x], && \text{(1d)}\\
& [A_u]_j v_i \leq [b_u]_j - c_{u,j}\delta, && j \in [1, n_u], && \text{(1e)}\\
& z_N = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, && && \text{(1f)}\\
& \|x(k) - z_0\|_P^2 \leq \delta^2, && && \text{(1g)}
\end{aligned}$$

   where $\delta = \frac{\bar{\alpha}_w}{1-\rho} = \max_{w \in \mathcal{W}} \|w\|_P \cdot \frac{1}{1-\rho}$.
   Implement (1) in the provided `Nonlinear_RMPC.m` file.

2. **(Graded)** Implementation of nonlinear robust MPC avoiding state dependent disturbances in the `Nonlinear_RMPC.m` file.

   a. Consider the nonlinear robust MPC problem

$$\min_{V, z_0} \quad \sum_{i=0}^{N-1} z_i^T Q z_i + v_i^T R v_i \tag{2a}$$

$$\text{s.t.} \quad \forall i = 0, \ldots, N-1, \tag{2b}$$

$$z_{i+1} = f(z_i, v_i), \tag{2c}$$

$$[A_x]_j z_i \leq [b_x]_j - c_{x,j}\delta, \qquad j \in [1, n_x], \tag{2d}$$

$$[A_u]_j v_i \leq [b_u]_j - c_{u,j}\delta, \qquad j \in [1, n_u], \tag{2e}$$

$$\|G z_i\|_P + L_w \delta \leq \hat{w} \tag{2f}$$

$$z_N = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \tag{2g}$$

$$\|x(k) - z_0\|_P^2 \leq \delta^2, \tag{2h}$$

   where $\delta = \frac{\hat{w}}{1-\rho}$.
   Implement (2) in the in the provided `Nonlinear_RMPC.m` file by modifying the implementation in 1.c.

   b. Choose `w_hat` as 50% of the previously used bound `w_bar` and simulate the system. Observe what happens when you change `rho` and `w_hat`.