Midterm Exam 2

MATH 755/855, November 18, 2020

Name____ Score____________________

1. (15pts) Let X be a continuous random variable with probability density function

$$f(x) = \begin{cases} 2x, & \text{if } 0 \le x \le 1; \\ 0, & \text{otherwise;} \end{cases}$$

(a) (10pts) Find $P\big(X < 0.5\big)$ and $P\big(0.25 \le X \le 0.75\big)$.

(b) (5pts) Find the 90th percentile of the distribution of X, i.e., find the constant $x_{0.9}$ such that $P(X \le x_{0.9}) = 0.9$.

2. (15pts) If X is a Gamma random variable with $\alpha = 2$ and $\lambda = 3$, i.e., X has the following *p.d.f.*

$$X \sim f(x) = \begin{cases} 9xe^{-3x}, & \text{if } x \geq 0; \\ 0, & \text{otherwise}; \end{cases}$$

Let $Y = 2X$. Find the probability density function of random variable Y, what distribution does it follow?

3. (30pts) A fair coin is tossed three times. Let X denote the number of heads on the first toss and Y the total number of heads. The joint frequency function of (X, Y) is derived and given in the following table:

X \Y	0	1	2	3	
0	1/8	1/4	1/8	0	
1	0	1/8	1/4	1/8	

(a) (10pts) Find $P\big(X + Y > 2\big)$ and $P\big(Y - X \geq 1\big)$.

(b) (5pts) Compute the marginal distributions of X and Y.

(c) (5pts) Are X and Y independent? Why?

(d) (5pts) Compute the conditional frequency function of X given $Y = 1$.

(e) (5pts) Find $P_{Y|X}(Y \geq 2|X = 1)$.

4. (25pts) The joint probability density function of X and Y is given by

$$f(x,y) = \begin{cases} x+y, & \text{if } 0 < x < 1,\ 0 < y < 1; \\ 0, & \text{otherwise.} \end{cases}$$

(a) (10pts) Find $P\big(Y \geq X^2\big)$.

(b) (10pts) Find the marginal density functions of X and Y. Are X and Y independent? Why?

(c) (5pts) Find the conditional density $f_{Y|X}(y|0.3)$? i.e., $f_{Y|X}(y|x)$ when $x = 0.3$.

5. (15pts) Let random variable X follow a uniform distribution on $[0, 1]$. Conditional on $X = x$, a random variable Y has a uniform distribution on $[x, 1]$, i.e., $Y|_{X=x} \sim U[x, 1]$.

(a) (5pts) Find the joint density function of (X, Y) and the marginal density function of Y. .

(b) (5pts) Find the conditional density $f_{X|Y}(x|0.6)$, i.e., $f_{X|Y}(x|y)$ when $Y = 0.6$.

(c) (5pts) Find $P(X \geq 0.5)$ and $P(X \geq 0.5|Y = 0.6)$.