Chapter 4 part 1

Chapter 4 Expected Values

§4.1 The expected value of a r.v. $E(X)$,

$E(X)$ parallels the notion of a weighted average. The possible values of r.v. are weighted by their probabilities.

Definition: X is a discrete r.v.

X	x_1	x_2	$\cdots$	x_n	$\cdots$
$p(x)$	$p(x_1)$	$p(x_2)$	$\cdots$	$p(x_n)$	$\cdots$

i.e. $p(x_n) = P(X = x_n)$,

$$E(X) = x_1 \cdot p(x_1) + x_2 \cdot p(x_2) + \cdots + x_n \cdot p(x_n) + \cdots$$

$$= \sum_{i=1}^{\infty} x_i \cdot p(x_i) \qquad \text{if} \quad \sum_i |x_i|\, p(x_i) < \infty.$$

if $\sum_i |x_i|\, p(x_i) = \infty$, the $E(X)$ is undefined.

notation: $\mu = E(X)$, or $\mu_X = E(X)$. the mean of X.

the center of mass of the frequency function of r.v. X.

Expl. Let X be the number of die, when a die is tossed. Find $E(X)$.

Ans.

X	1	2	3	4	5	6
$p(x)$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$

$$E(X) = 1 \cdot 1/6 + 2 \cdot 1/6 + 3 \cdot 1/6 + 4 \cdot 1/6 + 5 \cdot 1/6 + 6 \cdot 1/6 = 21/6 = 3.5.$$

Expl. Let X be a r.v, with following p.m.f. find $E(X)$.

X	-1	0	1
$p(x)$	0.2	0.5	0.3

Ans: $E(X) = \sum_i x_i \cdot p(x_i) = (-1)(0.2) + 0 \cdot (0.5) + 1 \cdot (0.3) = 0.1$.

Expl A: (Roulette). A roulette wheel has the number 1 through 36, as well as 0 and 00. If you bet \$1 that an odd number comes up, you win or lose \$1 according to whether that event occurs. If X denotes your net gain, $X = 1$ with probability $18/38$ and $X = -1$ with prob. $20/38$.

i.e.

X	1	-1
$p(x)$	$18/38$	$20/38$

The $E(X) = 1 \cdot 18/38 + (-1) \cdot 20/38 = -1/19 \approx -0.05$.
Thus, if you play a long sequence of independent games, you expect average loss \$0.05 per game.

Expl. B. X is a Geometric r.v. $E(X) = ?$
Suppose that items produced in a plant are independently defective with prob. p. Items are inspected one by one until a defective item is found. On the average, how many items must be inspected?
Ans: Let X be the number of items inspected until a defective item is found.
then, $P(X=1) = p$. $P(X=2) = (1-p) \cdot p$. $P(X=3) = (1-p)^2 \cdot p$, $\cdots$
or $P(X=k) = (1-p)^{k-1} \cdot p = q^{k-1} \cdot p$, $q = 1-p$, $k = 1, 2, \cdots$ X is a geometric r.v.

$$E(X) = \sum_i x_i \cdot P(x_i) = \sum_{k=1}^{\infty} k \cdot P(X=k) = \sum_{k=1}^{\infty} k \cdot q^{k-1} \cdot p = p \cdot \sum_{k=1}^{\infty} k \cdot q^{k-1}$$

$$= p \cdot \left(\sum_{k=1}^{\infty} q^k \right)'_q \qquad \left(\text{using } (q^k)'_q = k \cdot q^{k-1} \text{ and interchange } \Sigma \text{ and } ' \right)$$

$$= p \cdot \left(\frac{q}{1-q} \right)'_q \qquad \left(q + q^2 + q^3 + \cdots = q \cdot (1 + q + q^2 + \cdots) = q \cdot \frac{1}{1-q} = \frac{q}{1-q} \right)$$

$$= p \cdot \frac{1 \cdot (1-q) - q(-1)}{(1-q)^2} = p \cdot \frac{1-q+q}{(1-q)^2}$$

$$= p \cdot \frac{1}{p^2} = \frac{1}{p}. \quad (q = 1-p)$$

i.e. $X \sim$ Geometric r.v. with p. $E(X) = 1/p$.
e.g. if 10% of items are defective. ($p = 10\% = 0.1$). an average of 10 items must be examined to find one that is defective.

Expl. C. Poisson dist. $X \sim P(\lambda)$, $P(X=k) = \frac{\lambda^k e^{-\lambda}}{k!}$, $k = 0, 1, 2, \cdots$
$E(X) = ?$
Ans: $$E(X) = \sum_k k \cdot P(X=k) = \sum_{k=0}^{\infty} k \cdot \frac{\lambda^k e^{-\lambda}}{k!} = \sum_{k=1}^{\infty} k \cdot \frac{\lambda^k e^{-\lambda}}{k \cdot (k-1)!}$$

$$= \sum_{k=1}^{\infty} \frac{\lambda \cdot \lambda^{k-1} \cdot e^{-\lambda}}{(k-1)!} = \lambda \cdot e^{-\lambda} \cdot \sum_{k=1}^{\infty} \frac{\lambda^{k-1}}{(k-1)!} = \lambda \cdot e^{-\lambda} \cdot \sum_{j=0}^{\infty} \frac{\lambda^j}{j!} \quad \left[e^{\lambda} = \sum_{j=0}^{\infty} \frac{\lambda^j}{j!} = 1 + \lambda + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \cdots \right]$$

$$= \lambda \cdot e^{-\lambda} \cdot e^{\lambda} = \lambda.$$

i.e. $X \sim P(\lambda)$, $E(X) = \lambda$.

The parameter λ of the Poisson dist. as the average count.

Expl. The following gambling game, known as the wheel of fortune, is quite popular at many carnivals and gambling casinos: A player bets on one of numbers 1 through 6. Three dice are then rolled, and if the number bet by the player appear i times, $i = 1, 2, 3$. then the player wins i units; on the other hand, if the number bet by the player doesn't appear on any of the dice, then the player loses 1 unit. Is this game fair to the player?

Ans: Let X be the number of units player wins.

$$P(X=1) = \binom{3}{1}\left(\frac{1}{6}\right)^1\left(1-\frac{1}{6}\right)^2 = \frac{75}{216} \qquad P(X=-1) = \binom{3}{0}\left(\frac{1}{6}\right)^0\left(1-\frac{1}{6}\right)^3 = 125/216$$

$$P(X=2) = \binom{3}{2}\left(\frac{1}{6}\right)^2\left(1-\frac{1}{6}\right)^1 = 15/216$$

$$P(X=3) = \binom{3}{3}\left(\frac{1}{6}\right)^3\left(1-\frac{1}{6}\right)^0 = 1/216.$$

X	-1	1	2	3
$p(x)$	$125/216$	$75/216$	$15/216$	$1/216$

$$E(X) = (-1) \cdot 125/216 + 1 \cdot \frac{75}{216} + 2 \cdot \frac{15}{216} + 3 \cdot \frac{1}{216} = -17/216. \quad \text{unfair!}$$

Expl. D. St. Petersburg Paradox.

A gambler has the following strategy for playing a sequence of games. He starts off betting \$1; if he loses, he doubles his bet; and he continues to double his bet until he finally wins. Suppose the game is fair.

At trial 0, he bets \$1. if he loses, he bets \$2 at trial 1; and if he has not won by the k-th trial, he bets $\$2^k$. When he finally wins, he will be \$1 ahead.

This seems like a foolproof way to win \$1. what could be wrong with it?

Ans.

Let X denote the amount of money bet on the very last game (the game he wins). then X has following dist.

	trial 0,	trial 1,	2,	3	$\cdots$	trial k	
X	1	2	2^2	2^3	$\cdots$	2^k	$\cdots$
$p(x)$	$\frac{1}{2}$	$\frac{1}{2}\cdot\frac{1}{2}$	$\frac{1}{2^3}$	$\frac{1}{2^4}$	$\cdots$	$1/2^{k+1}$	$\cdots$

$$E(X) = \sum_i x_i \cdot p(x_i) = \sum_{k=0}^{\infty} 2^k \cdot \frac{1}{2^{k+1}} = \sum_{k=0}^{\infty} \frac{1}{2} = \infty.$$

Formally, $E(X)$ is not defined. One doesn't take into account the enormous amount of capital required.

Expectation for continous r.v. $X \sim f(x)$, $E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$.

provided that $\int |x| \cdot f(x)\,dx < \infty$.

Expl E Gamma Density

$X \sim \text{Gamma}(\alpha, \lambda)$, $X \sim f(x) = \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}$, $x > 0$.

$$E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx = \int_0^\infty x \cdot \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}\,dx = \frac{\lambda^\alpha}{\Gamma(\alpha)} \int_0^\infty x^\alpha e^{-\lambda x}\,dx$$

$$= \frac{\lambda^\alpha}{\Gamma(\alpha)} \int_0^\infty x^{(\alpha+1)-1} e^{-\lambda x}\,dx$$

(Consider Gamma density $(\alpha+1, \lambda)$

$$\int_0^\infty \frac{\lambda^{\alpha+1}}{\Gamma(\alpha+1)} x^{(\alpha+1)-1} e^{-\lambda x}\,dx = 1$$)

and $\Gamma(1+\alpha) = \alpha \cdot \Gamma(\alpha)$

$$= \frac{\lambda^\alpha}{\Gamma(\alpha)} \cdot \frac{\Gamma(\alpha+1)}{\lambda^{\alpha+1}} = \frac{\alpha \cdot \Gamma(\alpha)}{\Gamma(\alpha) \cdot \lambda}$$

$$= \frac{\alpha}{\lambda}.$$

i.e., $X \sim \text{Gamma}(\alpha, \lambda)$, $E(X) = \frac{\alpha}{\lambda}$.

Expl. F. Normal dist. $X \sim N(\mu, \sigma^2)$, $E(X) = \mu$.

$X \sim N(\mu, \sigma^2)$, $f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$.

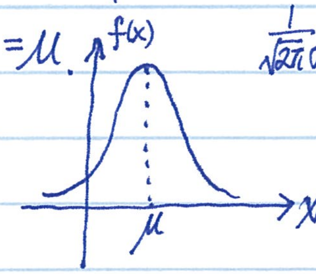
$x - \mu = z$,

$$E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx = \int_{-\infty}^{\infty} x \cdot \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\,dx = \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{\infty} (z+\mu) e^{-\frac{z^2}{2\sigma^2}}\,dz$$

$$= \frac{1}{\sqrt{2\pi}\,\sigma}\int_{-\infty}^{\infty} z\, e^{-\frac{z^2}{2\sigma^2}}\, dz + \frac{\mu}{\sqrt{2\pi}\,\sigma}\int_{-\infty}^{\infty} e^{-\frac{z^2}{2\sigma^2}}\, dz$$

(2nd integrand is pdf $N(0,\sigma^2)$. $\frac{1}{\sqrt{2\pi}\,\sigma}e^{-\frac{x^2}{2\sigma^2}}$)

$$= 0 + \mu \cdot \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{z^2}{2\sigma^2}}\, dz = 0 + \mu \cdot 1 = \mu.$$



i.e. $X \sim N(\mu, \sigma^2)$, $E(X) = \mu$.

Expl G. Cauchy density. $X \sim f(x) = \frac{1}{\pi}\frac{1}{1+x^2}$. $-\infty < x < \infty$.

It appears $E(X) = 0$, since symmetry. But $\int_{-\infty}^{\infty} |x| \cdot f(x)\,dx = \int_{-\infty}^{\infty} \frac{|x|}{\pi(1+x^2)}\,dx = \infty$, $E(X)$ does not exist!

§4.1.1. Expectations of functions of r.v.s.

X is a r.v. $Y = g(X)$. find $E[Y] = E[g(X)] = ?$

A. If X is a discrete r.v.

X	x_1	x_2	......	x_n	....
$p(x)$	$p(x_1)$	$p(x_2)$	.....	$p(x_n)$	....

then $E(Y) = E[g(X)] = g(x_1)\cdot p(x_1) + g(x_2)p(x_2) + \cdots + g(x_n)p(x_n) + \cdots$

$$= \sum_{i=1}^{\infty} g(x_i)\,p(x_i). \quad \text{if } \sum_{i=1}^{\infty} |g(x_i)|\,p(x_i) < \infty$$

B. If X is a continuous r.v.

$X \sim f(x)$. then $E[g(X)] = \int_{-\infty}^{\infty} g(x) f(x)\,dx$, if $\int_{-\infty}^{\infty} |g(x)| f(x)\,dx < \infty$.

Expl.

X	1	2
$p(x)$	1/2	1/2.

$E(X) = 1\cdot \frac{1}{2} + 2 \cdot \frac{1}{2} = \frac{3}{2} = 1.5.$

$E(\frac{1}{X}) = \frac{1}{1}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2} = \frac{3}{4} = 0.75.$

$E(X^2) = 1^2 \cdot \frac{1}{2} + 2^2 \cdot \frac{1}{2} = \frac{1}{2} + \frac{4}{2} = 2.5.$

note: $E[g(X)] \neq g(E(X))$. e.g. $g(x) = x^2$. $Eg(X) = 2.5 \neq 1.5^2 = g(EX)$

$g(x) = 1/x$. $Eg(X) = 0.75 \neq 1/1.5 = 1/E(X) = g(E(X))$.

Expl A. Find average kinetic energy of a gas molecule.

the magnitude of the velocity of a gas molecule is a r.v. X,

$X \sim f(x) = \frac{\sqrt{2/\pi}}{\sigma^3} x^2 e^{-\frac{1}{2}\frac{x^2}{\sigma^2}}$, (Maxwell's dist. σ depends on the temperature of the gas.)

average kinetic energy $Y = \frac{1}{2} m X^2$.

$$E(Y) = E\left(\tfrac{1}{2} m X^2\right) = \int_0^\infty \frac{1}{2} \cdot m \cdot x^2 \cdot \frac{\sqrt{2/\pi}}{\sigma^3} x^2 e^{-\frac{1}{2}\frac{x^2}{\sigma^2}} dx$$

$$= \frac{m}{2} \cdot \frac{\sqrt{2/\pi}}{\sigma^3} \int_0^\infty x^4 e^{-\frac{x^2}{2\sigma^2}} dx.$$

let $u = \frac{x^2}{2\sigma^2}$, $x^2 = 2\sigma^2 \cdot u$

$x = \sqrt{2}\sigma \cdot \sqrt{u}$, $dx = \sqrt{2} \cdot \sigma \cdot \frac{du}{2\sqrt{u}}$

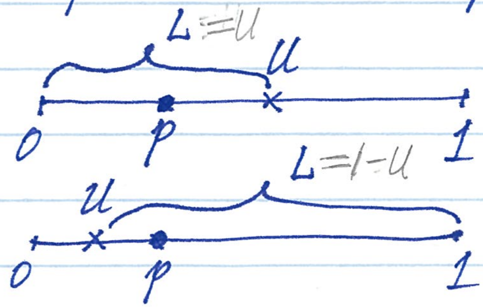
$$= \frac{m}{2} \cdot \frac{\sqrt{2/\pi}}{\sigma^3} \int_0^\infty 4 \sigma^4 \cdot u^2 \cdot e^{-u} \cdot \sqrt{2} \cdot \sigma \cdot \frac{du}{2\sqrt{u}}$$

$$= \frac{2m\sigma^2}{\sqrt{\pi}} \int_0^\infty u^{2-\frac{1}{2}} e^{-u} du = \frac{2m\sigma^2}{\sqrt{\pi}} \int_0^\infty u^{\frac{5}{2}-1} e^{-u} du = \frac{2m\sigma^2}{\sqrt{\pi}} \Gamma\left(\tfrac{5}{2}\right) = \frac{2m \cdot \sigma^2}{\sqrt{\pi}} \cdot \frac{3}{4}\sqrt{\pi}$$

$$= \frac{3}{2} m \cdot \sigma^2.$$

using $\Gamma\left(\frac{5}{2}\right) = \Gamma\left(1+\frac{3}{2}\right) = \frac{3}{2}\Gamma\left(\frac{3}{2}\right) = \frac{3}{2}\Gamma\left(1+\frac{1}{2}\right) = \frac{3}{2} \cdot \frac{1}{2} \Gamma\left(\frac{1}{2}\right) = \frac{3}{4} \cdot \sqrt{\pi}$

Expl. A stick of length 1 is split at a point U that is a uniformly distributed over $(0,1)$. Determine the expected length of the piece that contains the point p, $0<p<1$.



Ans. $U \sim f(u) = \begin{cases} 1, & 0 \le u < 1 \\ 0, & \text{else.} \end{cases}$

$L = \begin{cases} U, & \text{if } U \ge p \\ 1-U, & \text{if } U < p. \end{cases}$

$$E(L) = E[g(U)] = \int g(u) f(u) du = \int_0^p g(u) f(u) du + \int_p^1 g(u) f(u) du$$

$$= \int_0^p (1-u) \cdot 1 \cdot du + \int_p^1 u \cdot 1 \cdot du = \left(u - \frac{u^2}{2}\right)\Big|_0^p + \frac{u^2}{2}\Big|_p^1$$

$$= p - \frac{p^2}{2} + \frac{1}{2} - \frac{p^2}{2} = \frac{1}{2} + p - p^2.$$

e.g. $p=\frac{1}{2}$, $E(L_1)=\frac{1}{2}+\frac{1}{2}-(\frac{1}{2})^2=3/4$.

$(X_1, X_2) \sim \begin{matrix} p(x_1,x_2) \\ f(x_1,x_2) \end{matrix}$, $\quad Y=g(X_1,X_2)$. $\quad E(Y)=?$

A. $(X_1, X_2)\sim p(x_{1i}, x_{2j})$, $\quad i,j=1,2,\cdots$

$$E(g(X_1,X_2)) = \sum_{x_{1i},x_{2j}} g(x_{1i},x_{2j})\cdot p(x_{1i},x_{2j}).$$

B. $(X_1, X_2)\sim f(x_1,x_2)$. $\quad E[g(X_1,X_2)] = \iint g(x_1,x_2)\, f(x_1,x_2)\,dx_1 dx_2$.

Expl B A stick of unit length is broken randomly in two places. What is the average length of the middle piece?

Ans: Assume that the locations of the two break points are U_1, U_2.

Then U_1 and U_2 are two indep. r.v.s. following $U[0,1]$.

The length of the middle piece is $|U_1-U_2|$. We compute $E|U_1-U_2|$.

i.e. $E[g(U_1,U_2)] = E[|U_1-U_2|]$, where $U_1 \perp\!\!\!\perp U_2$, $\quad \begin{matrix} U_1\sim f(u_1)=1, & 0\le u_1\le 1 \\ U_2\sim f(u_2)=1, & 0\le u_2\le 1. \end{matrix}$

$$E[|U_1-U_2|] = \iint |u_1-u_2|\cdot f(u_1,u_2)\,du_1 du_2$$

$$=\int_0^1\int_0^1 |u_1-u_2|\,du_1 du_2 = \int_0^1\left[\int_0^1 |u_1-u_2|\,du_2\right]du_1$$

$$=\int_0^1\left[\int_0^{u_1}|u_1-u_2|\,du_2+\int_{u_1}^1|u_1-u_2|\,du_2\right]du_1$$

$$=\int_0^1\left[\int_0^{u_1}(u_1-u_2)\,du_2+\int_{u_1}^1(u_2-u_1)\,du_2\right]du_1$$

$$=\int_0^1\left[\left(u_1u_2-\frac{1}{2}u_2^2\right)\Big|_0^{u_1}+\left(\frac{1}{2}u_2^2-u_1u_2\right)\Big|_{u_1}^1\right]du_1$$

$$=\int_0^1\left[\left(u_1^2-\frac{1}{2}u_1^2\right)-0+\left(\frac{1}{2}-u_1\right)-\left(\frac{1}{2}u_1^2-u_1^2\right)\right]du_1$$

$$=\int_0^1\left[u_1^2-u_1+\frac{1}{2}\right]du_1 = 1/3.$$

Corollary A. if r.v.s $X \perp\!\!\!\perp Y$, g and h are fixed functions.

then $E[g(X)\cdot h(Y)] = E[g(X)]\cdot E[h(Y)]$.

proof. $LHS = \iint g(x)\, h(y)\cdot f(x,y)\,dx\,dy = \iint g(x)\cdot h(y)\cdot f_X(x)\cdot f_Y(y)\,dx\,dy$

$= \int g(x) f_X(x)\,dx \int h(y) f_Y(y)\,dy = E[g(X)]\cdot E[h(Y)]$.

§ 4.1.2. Expectations of linear combinations of r.v's.

Result. Let $Y = aX + b$, then $E(Y) = E[aX+b] = a\cdot E(X) + b$.

proof. $E[Y] = E[aX+b] = \int (ax+b) f_X(x)\,dx = a\int x f_X(x)\,dx + b\int f_X(x)\,dx$

(i.e. $Y = g(X)$) $\qquad = a\cdot E(X) + b$.

Theorem A. $Y = a + b_1X_1 + b_2X_2 + \cdots + b_nX_n$.

then $E(Y) = a + b_1E(X_1) + b_2E(X_2) + \cdots + b_nE(X_n)$.

Expl. A. $Y \sim B(n,p)$, $E(Y) = ?$

Ans: $P(Y=k) = \binom{n}{k} p^k (1-p)^{n-k}$, $k = 0, 1, 2, \cdots, n$.

$$E(Y) = \sum_{k=0}^{n} k\cdot P(Y=k) = \sum_{k=0}^{n} k\cdot \binom{n}{k} p^k (1-p)^{n-k} = \sum_{k=1}^{n} k\cdot \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$$

$$= \sum_{k=1}^{n} \frac{n\cdot(n-1)!}{(k-1)!\,(n-k)!} p\cdot p^{k-1}\cdot(1-p)^{n-k} = n\cdot p\cdot \sum_{k=1}^{n} \frac{(n-1)!}{(k-1)!(n-k)!} p^{k-1}\cdot(1-p)^{n-k}$$

$$\overset{k-1=l}{=\!=} n\cdot p\cdot \sum_{l=0}^{n-1} \frac{(n-1)!}{l!\,(n-1-l)!} p^{l}\cdot(1-p)^{n-1-l}$$ [note $n-k = n-1-(k-1) = n-1-l$]

$= np.$ [note $\frac{(n-1)!}{l!\,(n-1-l)!} p^l (1-p)^{n-1-l} = P(W=l)$, where $W \sim B(n-1, p)$

thus $\sum_{l=0}^{n-1} P(W=l) = 1$]

i.e. $Y \sim B(n,p)$, $E(Y) = n\cdot p$.

Another way: $Y \sim B(n,p)$, $Y = \sum_{i=1}^{n} X_i$, $X_i = 1$ with prob. p. $X_i = 0$, with prob $1-p$.

X_i is a Bernoulli r.v.

X_i	1	0
$p(x_i)$	p	$1-p$

thus $E(X_i) = 1 \cdot p + 0 \cdot (1-p) = p$.

thus $E(Y) = E(X_1) + E(X_2) + \cdots + E(X_n) = p + p + \cdots + p = np$.

Expl B (Coupon Collection)

Suppose that you collect coupons, that there are n distinct types of coupons, and that on each trial you are equally likely to get a coupon of any of the types. How many trials would you expect to go through until you had a complete set of coupons? (This might be a model for collecting baseball cards or for certain grocery promotions).

Ans. representing the number of trials as a sum.

X_1 = the number of trials up to and including the trial on which the 1st coupon is collected. then $X_1 = 1$.

X_2 = the number of trials from that point up to and including the trial on which the next coupon different from the first is obtained.

X_3 = ... — the third distinct coupon is collected; and so on, up to X_n.

Then, the total number of trials, X, is the sum of the X_i, i.e. $X = X_1 + X_2 + \cdots + X_n$.

$E(X) = \sum_{r=1}^{n} E(X_r)$. $X_1 = 1$, $E(X_1) = 1$.

X_2	1	2	3	$\cdots$
$p(x)$	p	$(1-p)p$	$(1-p)^2 p$	$\cdots$

$p = \frac{n-1}{n}$. $1-p = 1/n$

thus $E(X_2) = 1/p = \frac{n}{n-1}$ from Geometric dist.

similarly.

X_3	1	2	3	$\cdots$
$p(x)$	p_1	$(1-p_1)p_1$	$(1-p_1)^2 p_1$	$\cdots$

$p_1 = \frac{n-2}{n}$.

$E(X_3) = 1/p_1 = \frac{n}{n-2}$. $\cdots$ $E(X_n) = n/(n-(n-1)) = n/1$.

thus $E(X) = 1 + \frac{n}{n-1} + \frac{n}{n-2} + \cdots + n/1 = n\left(\frac{1}{n} + \frac{1}{n-1} + \cdots + \frac{1}{1}\right)$

$= n \cdot \sum_{r=1}^{n} \frac{1}{r}$. if $n = 10$, $E(X) = 29.3$.