1-1

Monday, August 30, 2021 6:12 AM

Chapter 1 Text Coverage:

1.0 Introduction

In this chapter we provide some basic concepts and definitions. We begin with a brief discussion of what probability is. Then we review some mathematical foundations that are needed for developing probability theory. Next we discuss the concept of random experiments and the axioms of probability. We then introduce discrete and continuous probability models. Finally, we discuss conditional probability.

1.1.0 Introduction: What Is Probability?

odds of a certain event occurring.
outcome / total possible outcomes.

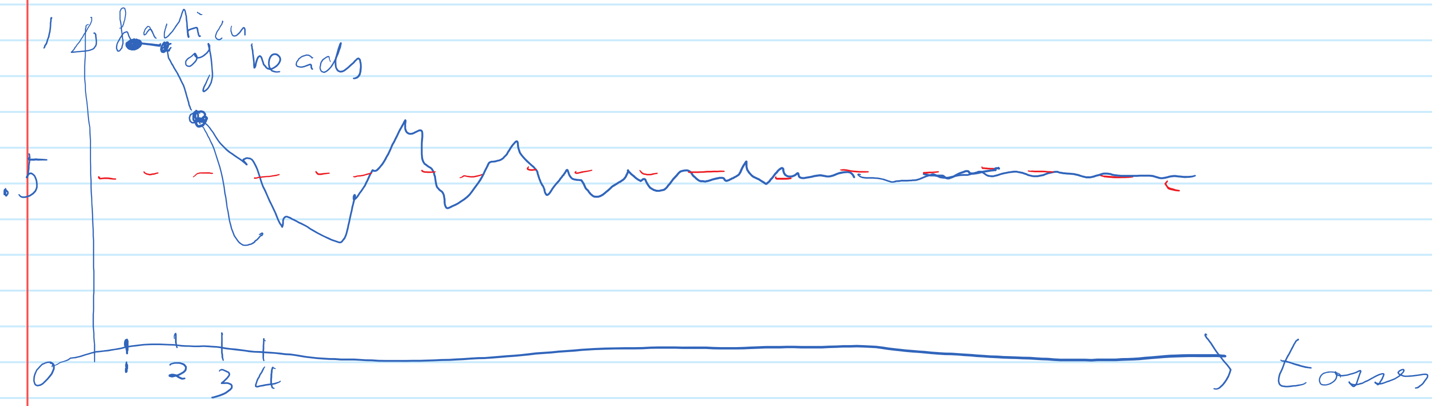
Discussion: Terms: "randomness" and "probability"

What is meant by randomness?

Something unpredictable, possibly inconsistent (different results in repetition)
equal chances for all outcomes (called "simple random sampling")

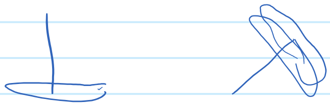
Think of tossing a fair coin repeatedly, don't know the next outcome. But we know that
in repeated samples, the fraction of heads will be roughly 0.5

The more tosses, the closer the fraction of heads will be to 0.5. The **law of large numbers**!
(Jakob Bernoulli)
Tossing a coin: A **binary experiment** . 2 outcomes: Head or Tail.



Two Interpretations of Probability:

Think of tossing a fair coin.
How about a thumbtack?



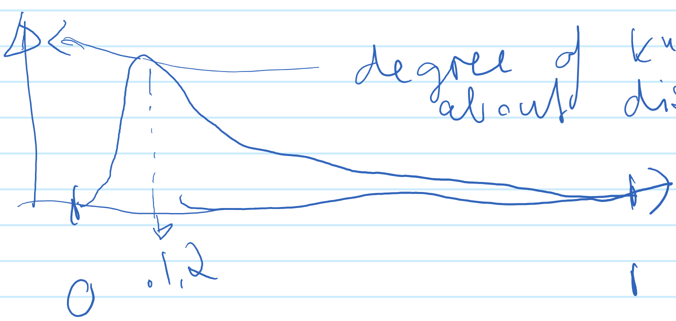
lands on base   on side

chance of landing on its base ?

1) Frequentist **interpretation** of probability: Long run relative frequency ("fraction of times") of an event.

How about COVID? Chance of ***a person*** having COVID (or measles, or .....). Need to define clearly the "universe" of persons being studied : the "sample space".
The fraction of persons having COVID keeps changing! Frequentist interpretation may not be feasible.

2) "Subjective" knowledge about the fraction of COVID infected (ie the rate),. Degree of knowledge about the unknown quantities.



degree of knowledge about disease rate ⟺ degree of plausibility

prob of a disease
chance

uses Bayes' Rule

The beauty of probability theory is that it is applicable regardless of the interpretation of probability that we use (i.e., in terms of long-run frequency or degree of belief). Probability theory provides a solid *framework to study random phenomena. It starts by assuming **axioms of probability**, and then building the entire theory using mathematical arguments.*

1.1.1 Example: Communication Systems

Discuss - read it!

1.2 Review of Set Theory

Why?

Random Experiments, for which we assign probabilities, need to have a "set" structure, so that prob's are well defined and give consistent results.

A **set** is a collection of things (elements).

A **set** is a collection of some items (elements). We often use capital letters to denote a set. To define a set we can simply list all the elements in curly brackets, for example to define a set A that consists of the two elements ♣ and ♢, we write $A = \{♣, ♢\}$. To say that ♢ belongs to A, we write $♢ \in A$, where "$\in$" is pronounced "belongs to." To say that an element does not belong to a set, we use $\notin$. For example, we may write $♡ \notin A$.

Example 1.1: Numerical Sets

Read in the text:

$\mathbb{R}$: real numbers

$\mathbb{Z}$: integers ... $-3, -2, -1, 0, 1, 2$ . . . . .

$\mathbb{N}$: nat. #s

$\mathbb{C}$: complex #s

Notation: (Restricted) Sets

A={x|x satisfies some property} or A={x: x satisfies some property}

$A = \{x \mid x \in \mathbb{Z} \ \& \ -5 \le x \le 3\}$ → intervals

etc

$U = \{x \mid x \in \mathbb{R} \ \& \ -\infty < x < -3.5\}$

$$U = \{x \mid x \in \mathbb{R} \,\&\, -\infty < x < -3.5\}$$
or
$$-\infty < x \leq -3.5$$

Alternative Terminology: outcome ; events versus elements ; sets

Subset ; Superset ; Null-Set (or Empty Set); Universal Set (or Sample Space)

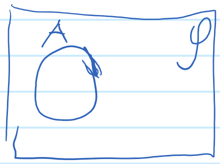
events (sets) $\underline{A \subset B} \iff$ ϕ $\mathcal{S}$: all outcomes

sample spaces all elements

Superset $A \supset B$

(visual aid)

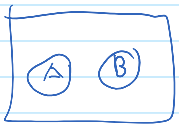
1.2.1 Venn Diagrams



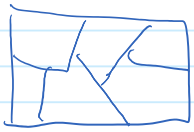
1.2.2 Set Operations

Union, Intersection, Complement, Difference,

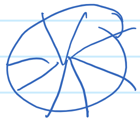
Mutially Exclusuive (or Disjoint)



Partition



Pizza



Theorem 1.1: De Morgan's law
For any sets $A_1, A_2, \cdots, A_n$, we have

- $(A_1 \cup A_2 \cup A_3 \cup \cdots A_n)^c = A_1^c \cap A_2^c \cap A_3^c \cdots \cap A_n^c$;
- $(A_1 \cap A_2 \cap A_3 \cap \cdots A_n)^c = A_1^c \cup A_2^c \cup A_3^c \cdots \cup A_n^c$.

Theorem 1.2: Distributive law
For any sets A, B, and C we have

- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$;
- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Cartesian Product, Multiplication Principle

Def: A **Cartesian product** of two sets A and B, written as $A \times B$, is the set containing **ordered** pairs from A and B.

That is, if $C = A \times B$, then each element of C is of the form (x,y), where $x \in A$ and $y \in B$:

Note that here the pairs are ordered, so for example, $(1,H) \neq (H,1)$. Thus $A \times B$ is **not** the same as $B \times A$.