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Chapter 1 HW
Math855 - Fall 2021

1. $S = \{1,2,...,10\}$, $A = \{1,2,3\}$, $B = \{2,3,4,5,6,7\}$, $C = \{7,8,9,10\}$
    a. $\{1,2,3,4,5,6,7\}$
    b. $\{1,8,9,10\}$
    c. $\{2,3,4,5,6,7,8,9,10\}$
    d. A, B, and C do not form a partition because there is overlap- some members of the universal set are represented more than once.
2. The universal set is all real numbers
    a. $[6,8] \cup [2,7) = [2,8]$
    b. $[6,8] \cap [2,7) = [6,7)$
    c. [0,1]^c = (-inf,0) $\cup$ (1,inf)
    d. $[6,8] - (2,7) = [7,8]$
3. Write set for Venn Diagram
    a. $A \cup B - A \cap B$
    b. $B - C$
    c. $(A \cap C) \cup (A \cap B)$
    d. $((A \cup B) - C) \cup (C - (A \cup B))$
4. List set of coin results
    a. A = {(H,H),(H,T)}
    b. B = {(H,T),(T,H),(T,T)}
    c. C = {(H,T),(T,H)}
5. Divisibility sets
    a.

```python
import numpy as np

base_set = np.arange(1,101)

def make_set(input_set=base_set,k=1):
    out_list = [x for x in  input_set if x%k==0]
    return set(out_list)

for i in range(2,6):
    print(f"set A({i}):",make_set(k=i),"\n")
```

    b.

```python
union = make_set(k=2).union(make_set(k=3).union(make_set(k=5)))
print("Length of the union of A, B, and C:",len(union))
```

```
Length of set A(2): 50

Length of set A(3): 33

Length of set A(4): 25

Length of set A(5): 20

Length of the union of A, B, and C: 74
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6. By definition, a partition would have no overlapping members. If B intersect with all three sets in those ways, those are unique matches. Therefore, B's minimum size is 45.

7. Countable or no
    a. Impractical, but countable
    b. Could be said to be multiple of two mappings of natural numbers, and therefore is countable
    c. Too many places within roots, uncountable
8. A = {[0,1)}
    a. The limit approaches 1, but all values are unique, leaving an uncountable span from 0 to 1.
9. A = { }
    a. The limit approaches zero, however, since each value is unique, the intersection is an empty set.
10. *
    a. All binary sequences can represent the summation of orders of magnitude of two. For example, 1010 = 2^3*1 +2^2*0 + 2^1*1 + 2^0*0 = 8 + 0 + 2 + 0 = 10. All natural numbers have a binary representation, and those are unique to each number. Therefore, they correspond in a one-to-one fashion.
    b. Additionally, all rational numbers between [0,1) can be represented using binary in a similar to the above assertion if represented as ratios.
11. Show that the set [0,1) is uncountable
    a. With many series, the limit keeps the function within the bounds of [0,1) while still being infinitely large, such as f(n) = n/(n+1). This set can never be completely listed and therefore is uncountable.
12. f: {H,T}^3
    a. Domain = {H,T} because those are the possible inputs to the function. The codomain is the set of natural numbers, because there are infinitely many coins to flip.
    b. The range of f is equal to the number of coins flipped, that being the maximum of the range of values that could result from f.
    c. If f(x) = 2, we can say that 2 heads were flipped. Statistically, it also implies that something like 2 tails were also flipped, although the odds are not too high on that one.
13. Soccer Match
    a. 1 - 0.5 - 0.25 = 0.25 = P({b})
    b. P({b | d}) = 0.25 + 0.5 = 0.75
14. P(A) = 0.4, P(B) = 0.7, P(A U B) = 0.9
    a. P(A ∩ B) = 0.2
        i. P(A U B) = P(A) + P(B) - P(A ∩ B)
        ii. 0.9 = 1.1 - P(A ∩ B)
    b. P(A^C - B) = (1- P(A)) - (P(B) - P(A ∩ B)) = 0.6 - 0.5 = 0.1
    c. P(A - B) = P(A) - P(A ∩ B) = 0.4 - 0.2 = 0.2
    d. P(A^C - B) = P(A^C) - (P(B) - P(A ∩ B)) = 0.6 - (0.7 - 0.2) = 0.1
    e. P(A^C U B) = P(A^C) + P(A ∩ B) = 0.6 + 0.2 = 0.8
    f. P(A ∩ (B U A^C)) = P(A ∩ B) = 0.2
15. Two Fair Dice

a. X2 = 4 = $1/6$
b. Sum = 7, 12/36 = $1/3$
c. X1 not 2 = $5/6$, X2 >= 4 = $1/2$, X1 and X2 = X1*X2 = 5/12

16. Random X, find *c*
    a. The function defines a probability distribution, meaning that the sum of the infinite series would have to be 1, and 0 < c < 3. C = 1 would not approach 1, but c=2 would.
    b. P({2,4,6}) = (2/9) + (2/81) + (2/729) = 180/729 = 20/81
    c. P({3,4,5,...}) = 1 - P({1,2}) = 1 - ($2/3$) - (2/9) = (1/9)
17. Teams A,B,C,D in tournament
    a. P(A) = P(B)
    b. P(C) = 2P(D)
    c. P(A | C) = 0.6
    d. P(A,B,C,D) = 1
        i. = P(A) + P(B) + P(C) + P(D)
        ii. = 2P(A) + 3P(C)/2
        iii. P(A) + P(C) = 0.6
        iv. 1 = 2(0.6 - P(C)) + 1.5P(C)
        v. 1 = 1.2 - 2P(C) + 1.5P(C)
        vi. 0.5P(C) = 0.2
        vii. P(C) = 0.4
        viii. P(D) = 0.2
        ix. P(A) = P(B) = 0.3
18. Time for job in factory
    a. P(T <= 1) = $\int_0^1 \frac{t^2}{16} dt = \frac{t^3}{48} \Big|_0^1 = 1/48$
    b. P(T > 2) = 1 - P(T <= 2) = $1 - \int_0^2 \frac{t^2}{16} dt = \frac{t^3}{48} \Big|_0^2 = 1 - 1/6 = 5/6$
    c. $\int_1^3 \frac{t^2}{16} dt = \frac{t^3}{48} \Big|_1^3 = 27/48 - 1/48 = 13/24$