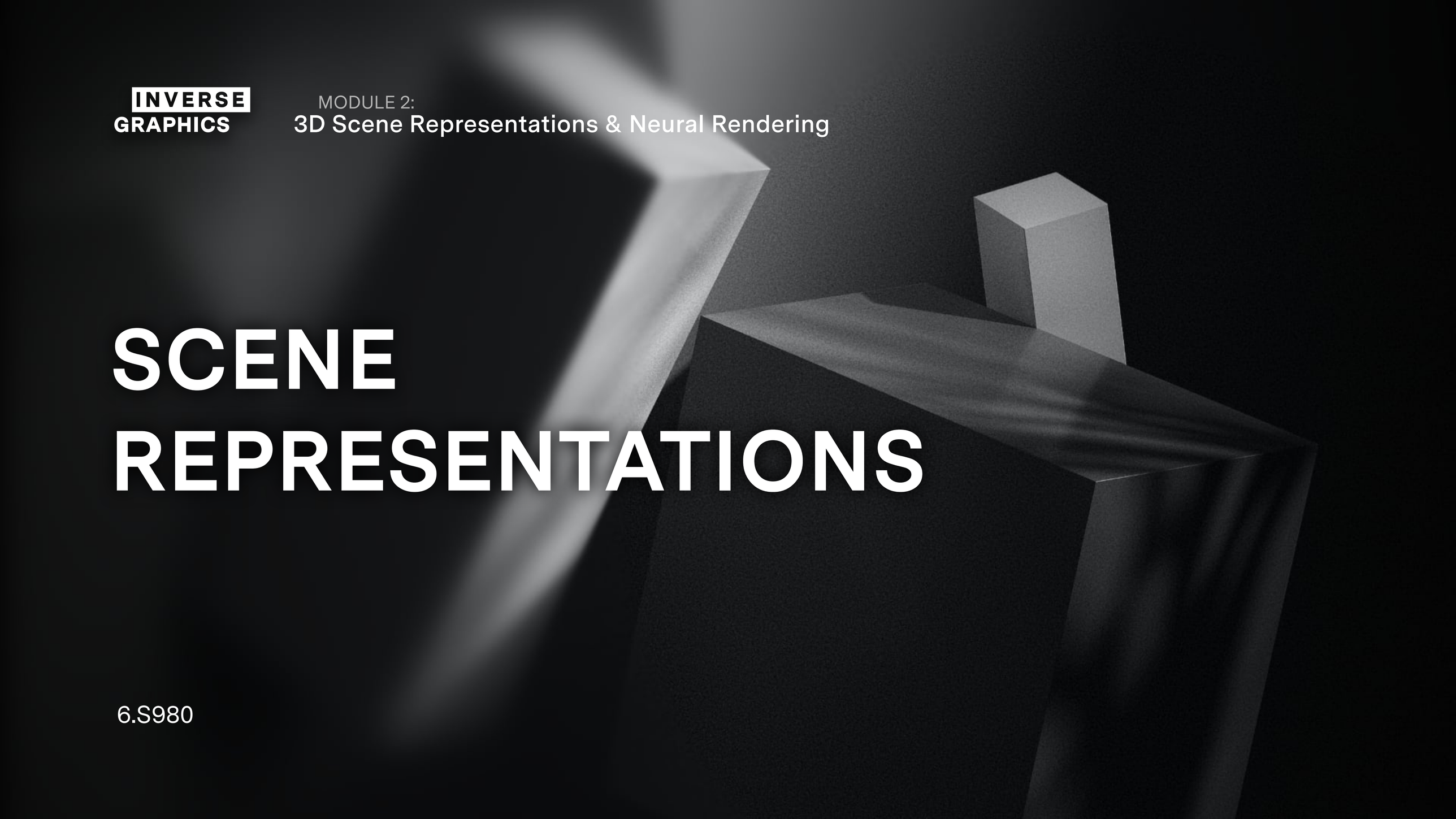
INVERSE GRAPHICS

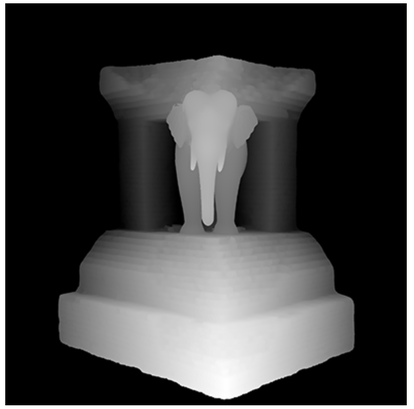
MODULE 2:
3D Scene Representations & Neural Rendering

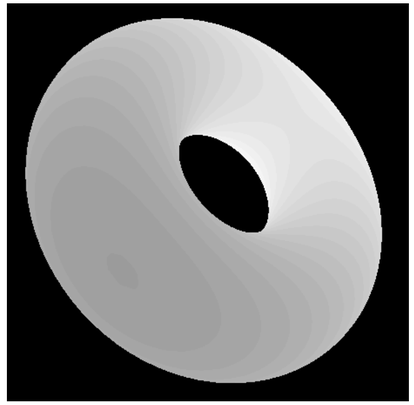


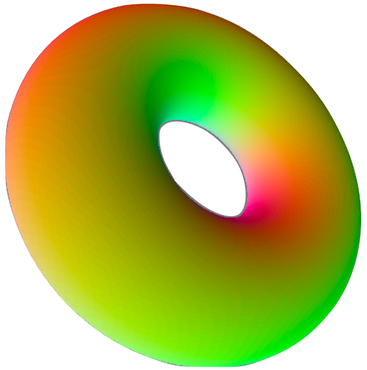
SCENE REPRESENTATIONS

6.S980

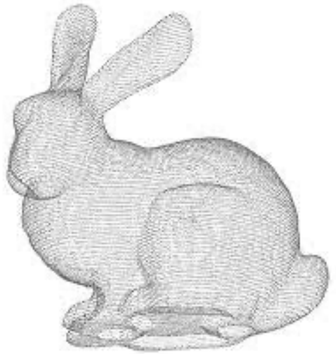
2.5D Representations

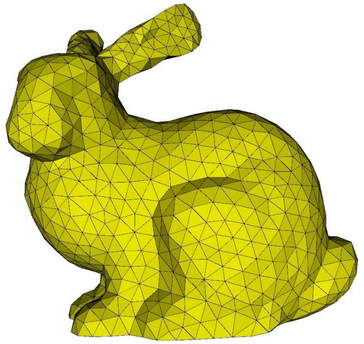




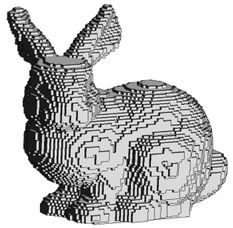


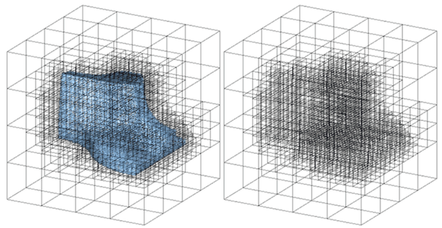
Surface Representations

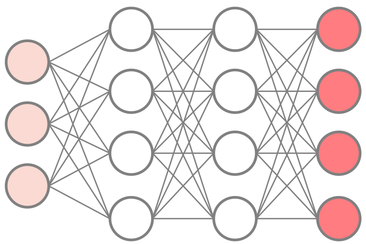


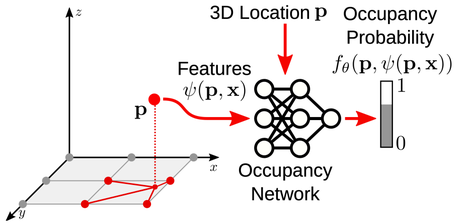


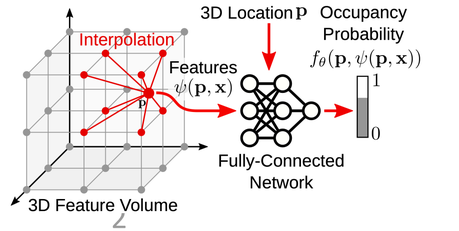
Field Parameterizations



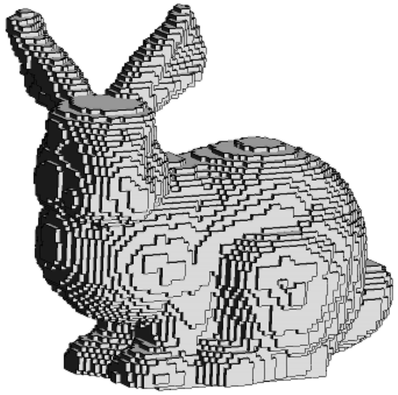


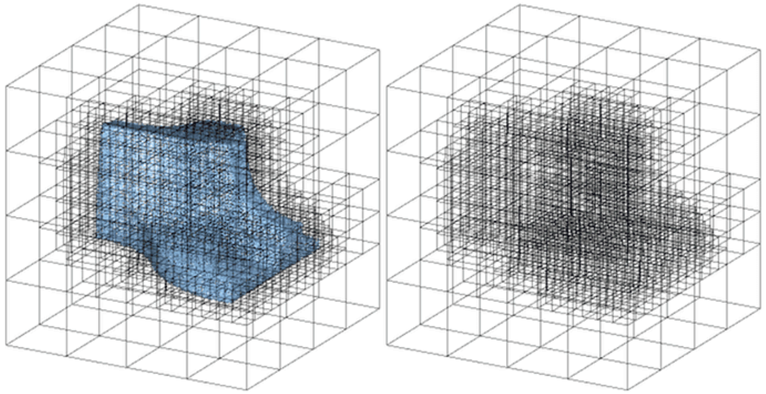




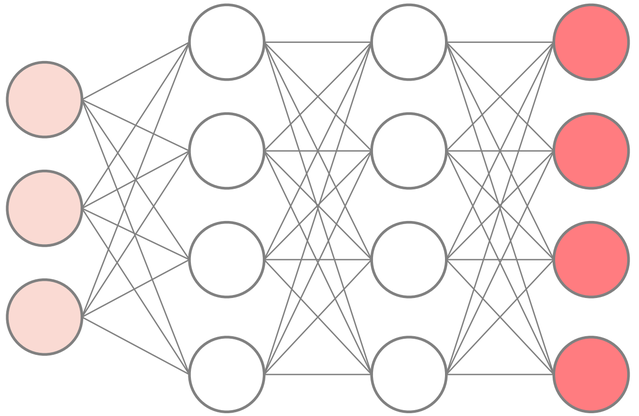


Parameterizing Fields

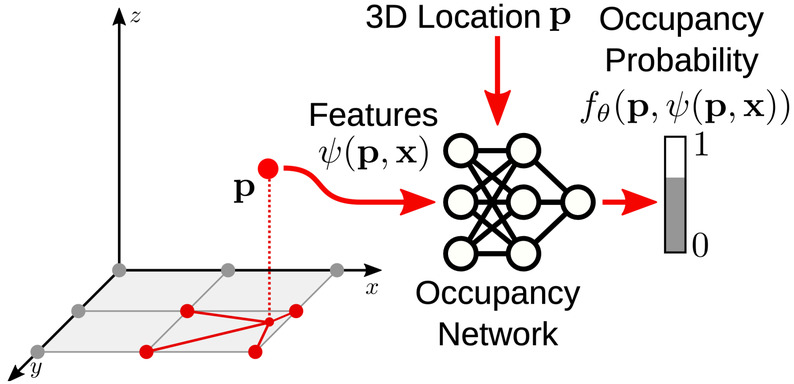


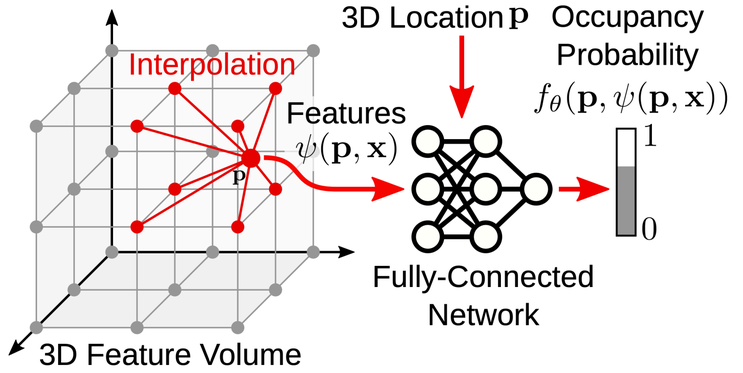


Discrete Parameterizations



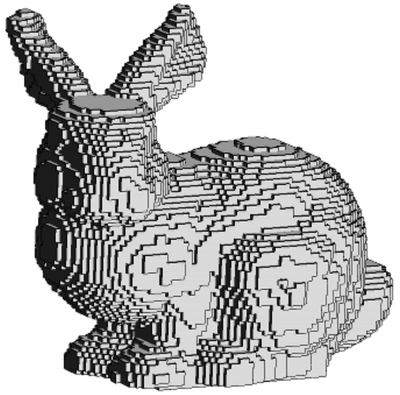
Continuous Parameterizations

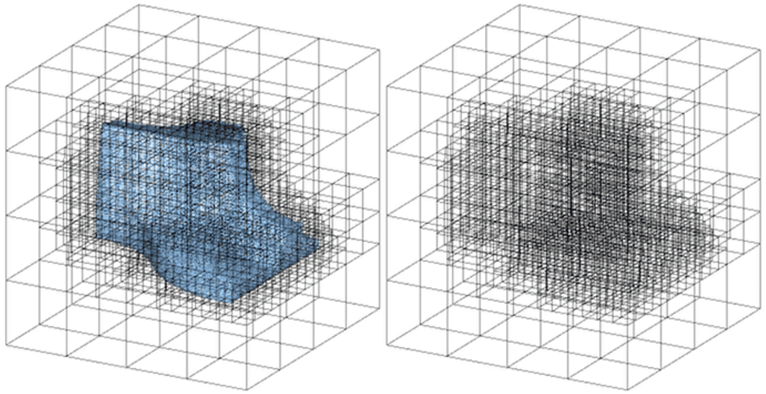




Hybrid Parameterizations

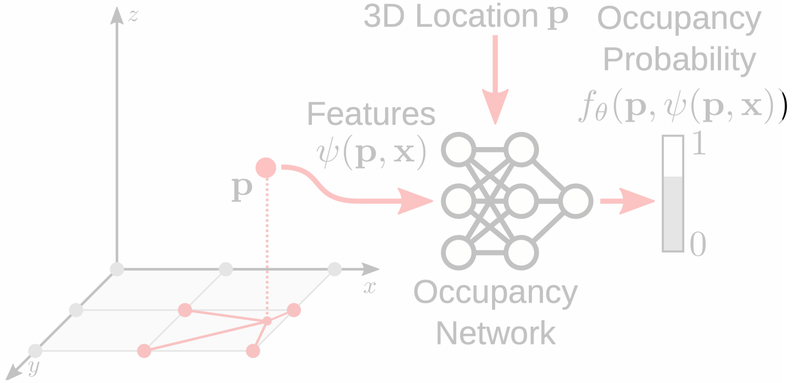
Voxel Grids

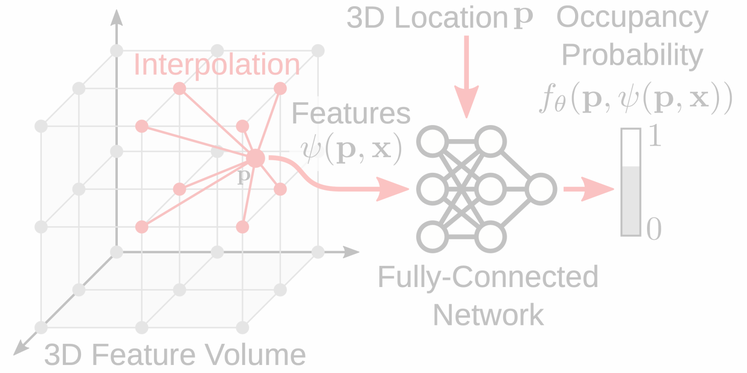




Discrete Parameterizations

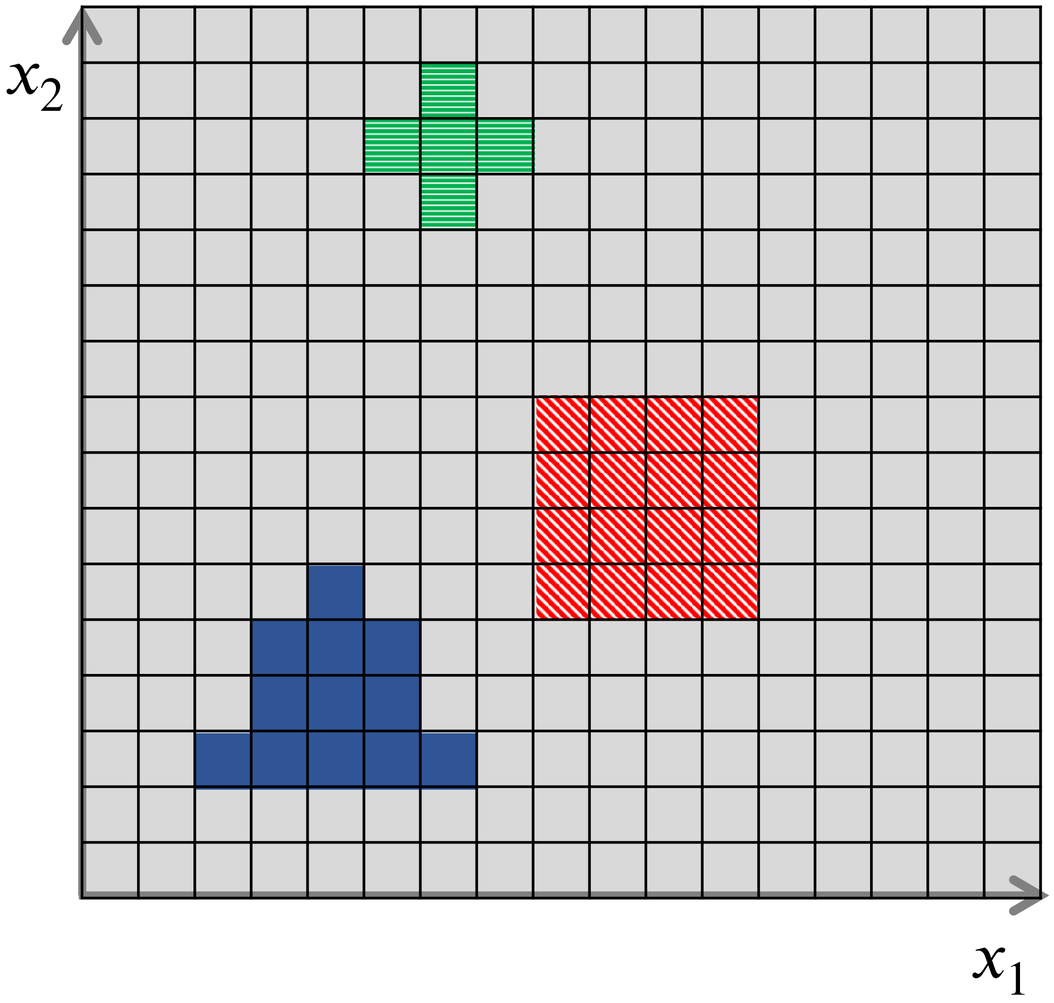
Continuous Parameterizations

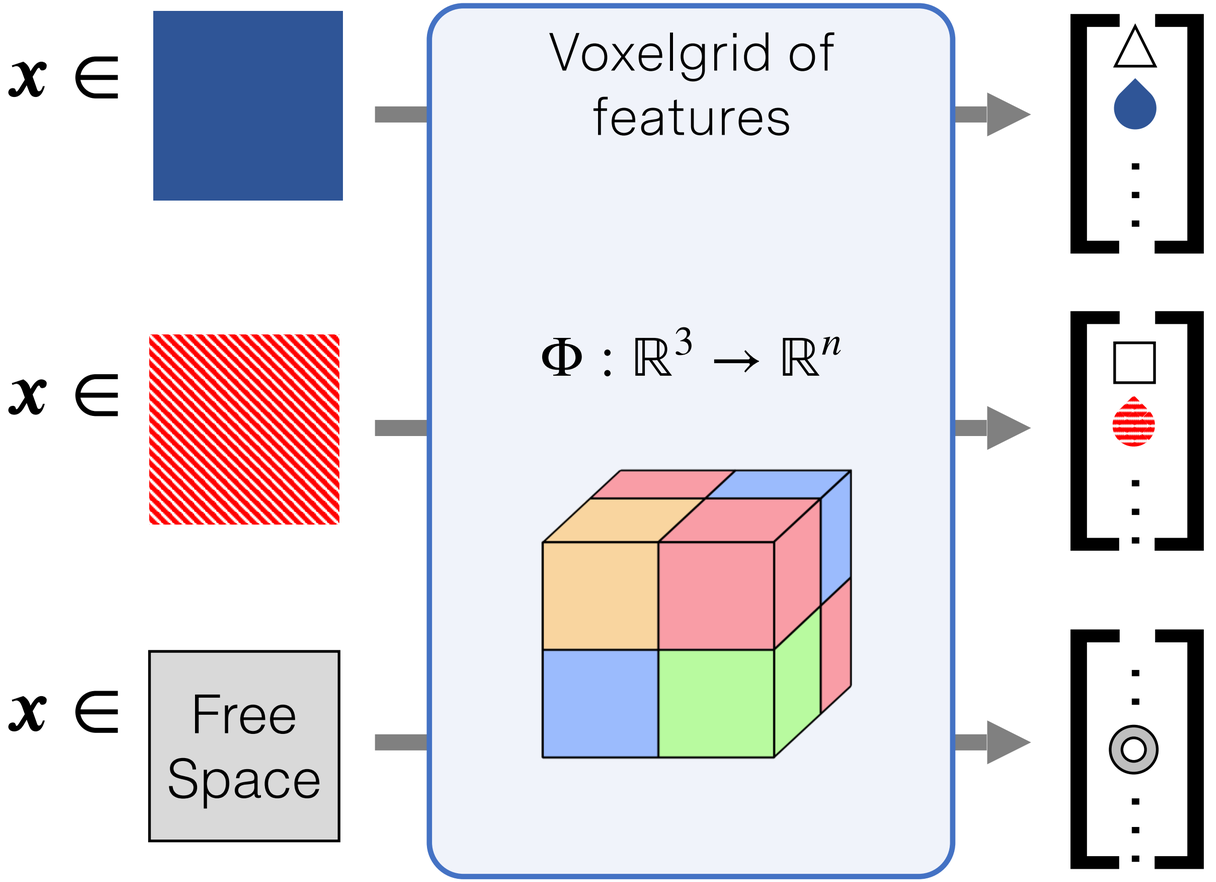




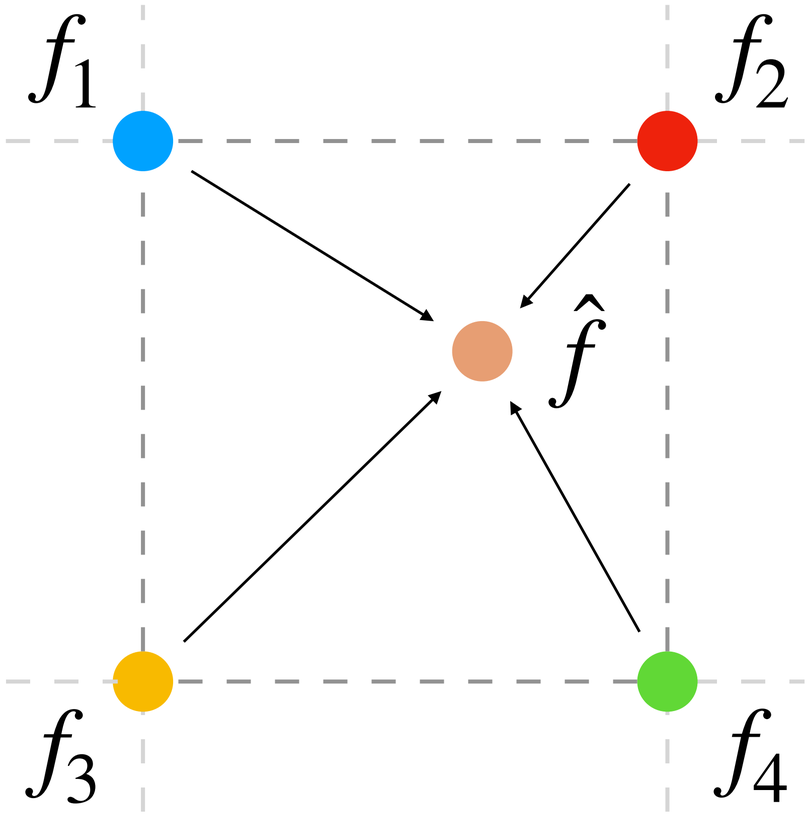
Hybrid Parameterizations

Voxel Grids


x_2
x_1


$x \in$
$x \in$
$x \in$
Free Space
Voxelgrid of features
$\Phi : \mathbb{R}^3 \rightarrow \mathbb{R}^n$

Interpolation: Querying *continuous* values



Interpolation in a 2D voxel grid.

- Only stores values at vertex locations, i.e. corners
- Values at intermediate coordinates are defined via interpolation w/ some kernel k

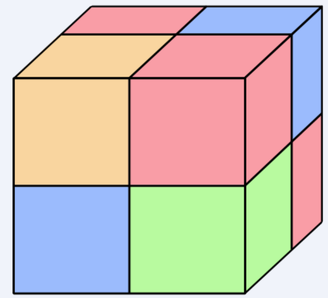
$$\hat{f}(\mathbf{x}) = \sum_{i=1}^{N} f_i k(\mathbf{x}, \mathbf{x}_i)$$

Voxel grids

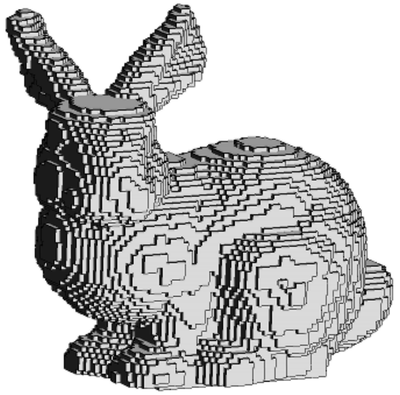
- For d spatial dimensions and resolution n, memory grows $O(n^d)$
- Fast sampling: d-linear interpolation
  = index into array 2^d times & weighted sum.
- Convenient processing as it exposes *locality.* Can easily run convolutions, nearest-neighbor lookups, etc.
- Intractable in higher dimensions :/

Voxelgrid of features

$\Phi : \mathbb{R}^3 \rightarrow \mathbb{R}^n$



Multi-Resolution Representations



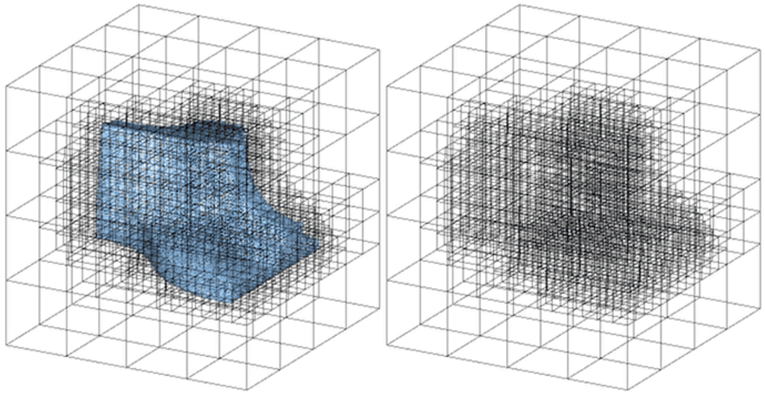
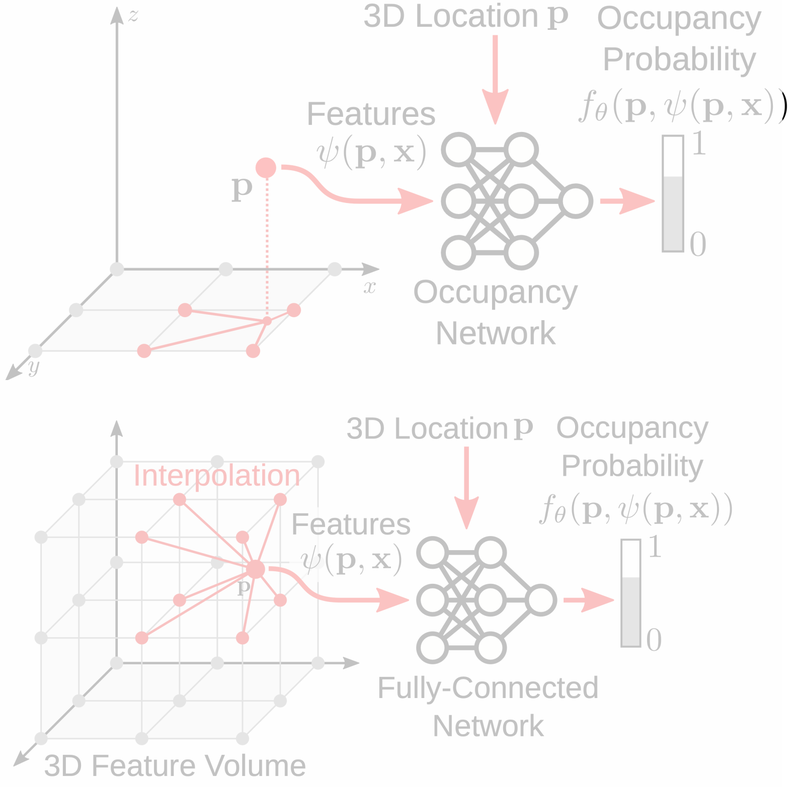


Image Source: https://doc.cgal.org/latest/Orthtree/index.html

Discrete Parameterizations

Continuous Parameterizations



Hybrid Parameterizations

Octrees

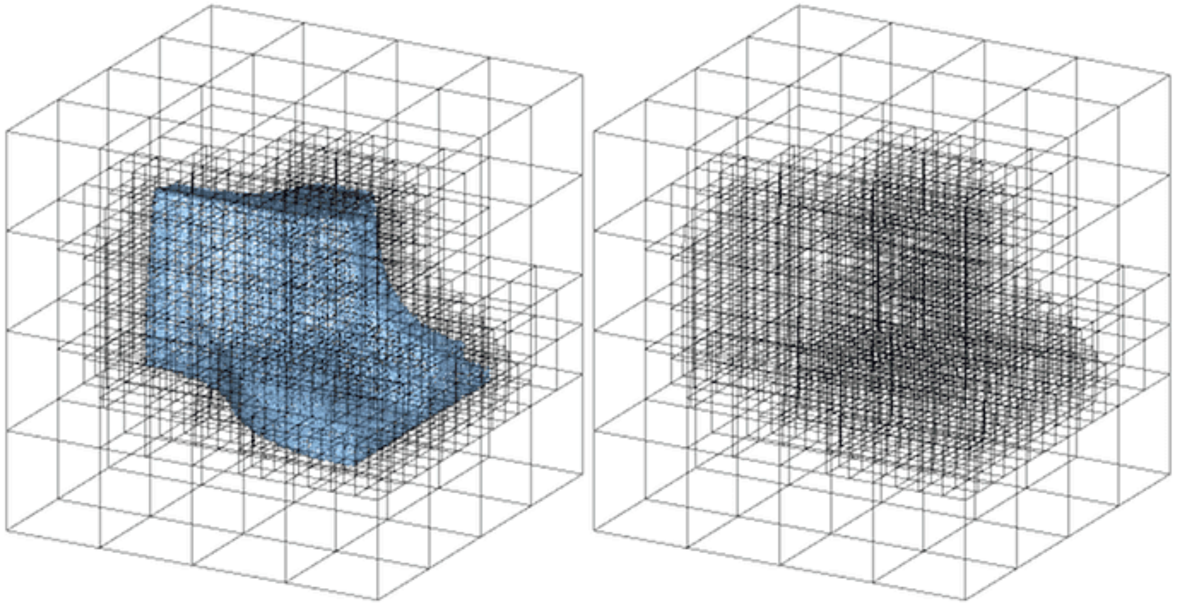


Image Source: https://doc.cgal.org/latest/Orthtree/index.html

Divide volume into 8 voxels

If any box has more than *N* points in it, divide it into 8 voxels

Do not divide voxels with zero points.

Octrees

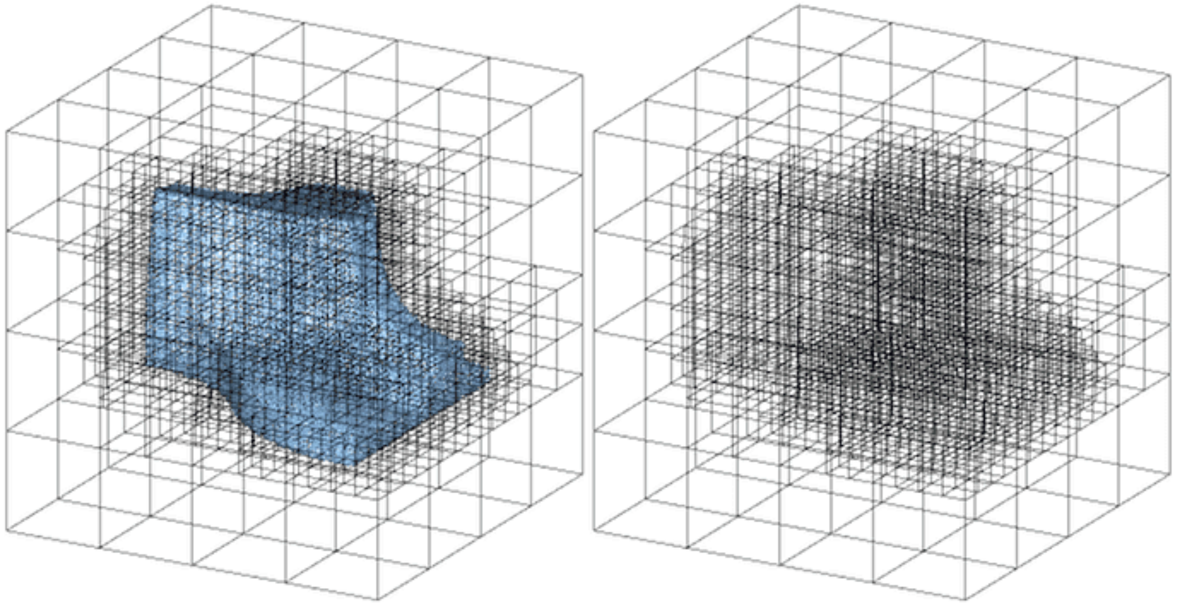


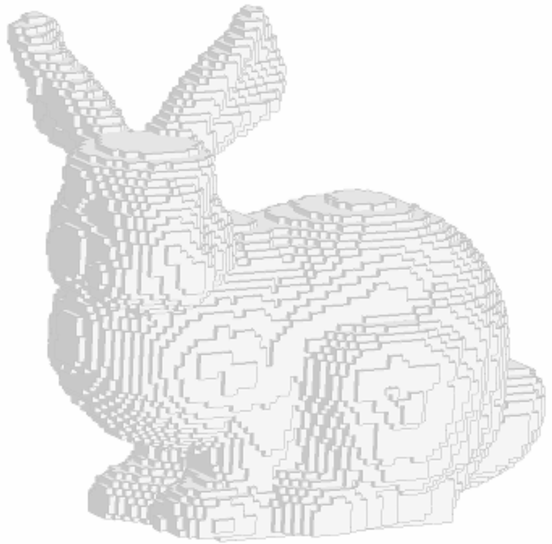
Image Source: https://doc.cgal.org/latest/Orthtree/index.html

Saves memory (less resolution in empty parts)

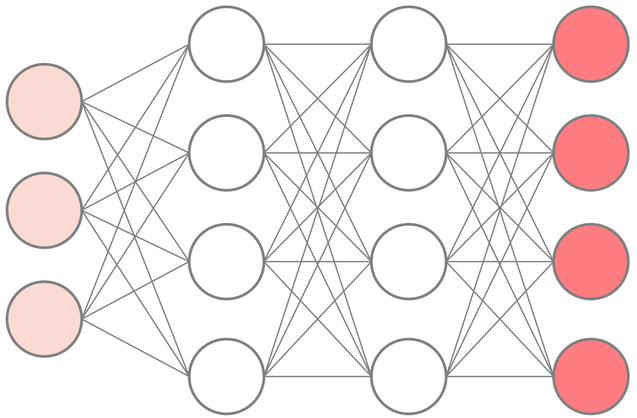
Sub-division is not differentiable: can’t easily build neural network that outputs Octree

To sample point, have to traverse multi-resolution hierarchy. Still fast, though.

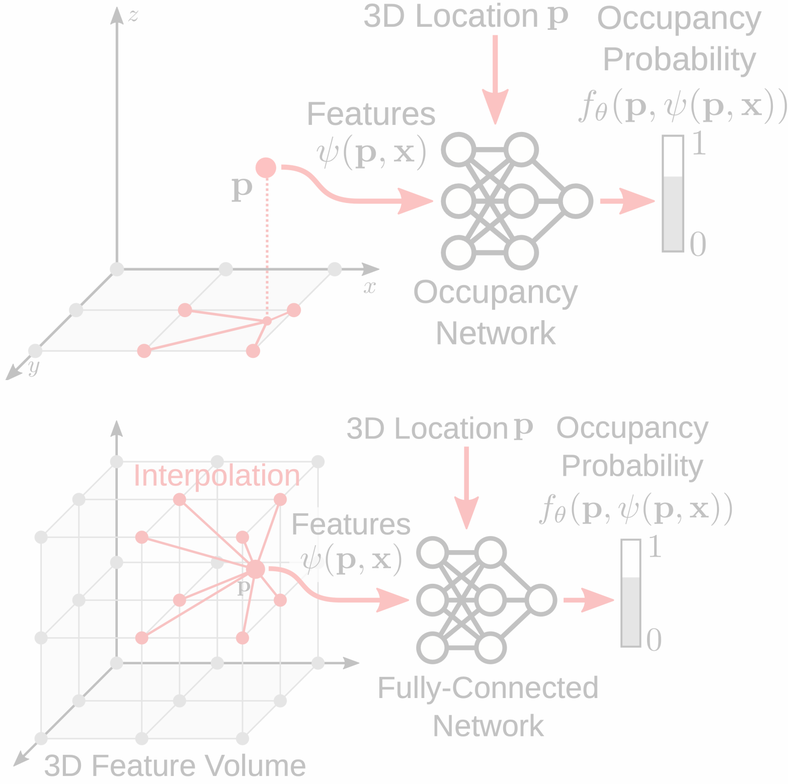
Neural Fields



Discrete Parameterizations

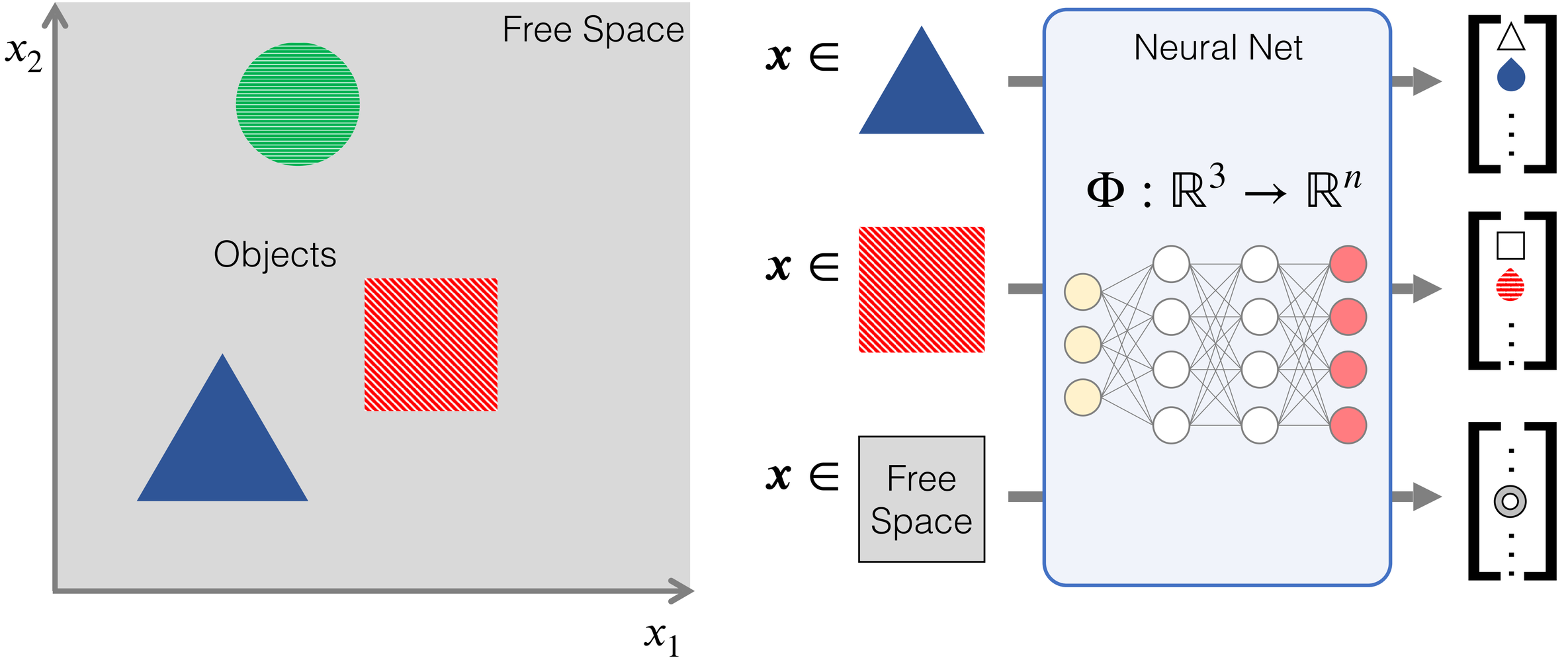


Neural Fields



Hybrid Parameterizations

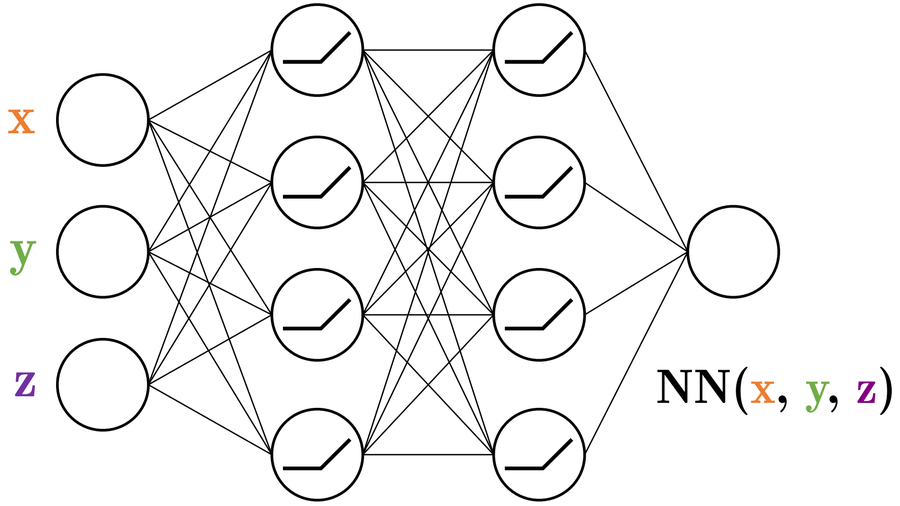
Neural fields: Parameterize Field as Neural Network!



Occupancy Networks: Learning 3D Reconstruction in Function Space, Mescheder et al.
IM-Net: Learning Implicit Fields for Generative Shape Modeling, Chen et al.
DeepSDF: Learning Continuous Signed Distance Functions for Shape Representation, Park et al. 2019
Sitzmann et al: Scene Representation Networks: Continuous 3D-Structure-Aware Neural Scene Representations, NeurIPS 2019.

Shapes

ReLU MLP

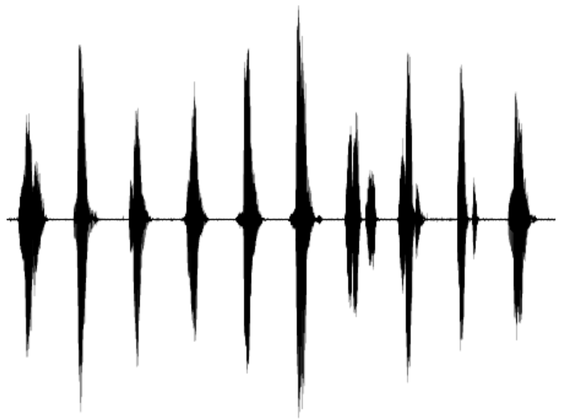


Mescheder et al. 2018
Park et al. 2018
Chen et al. 2018

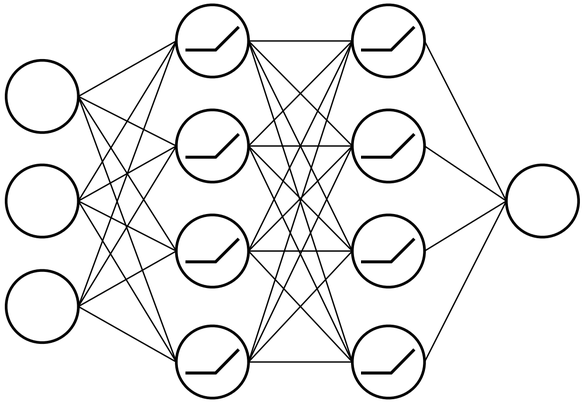
Images

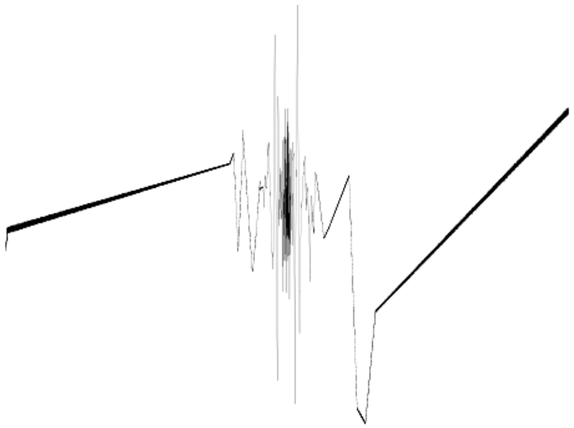
Shapes

Audio



ReLU MLP



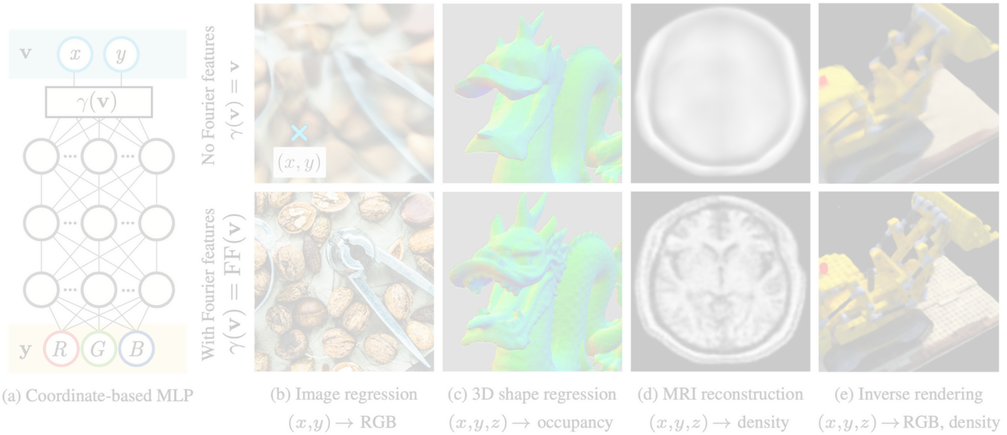


SIREN: Neural Implicit Representations With Periodic Activation Functions

Sitzmann & Martel et al. NeurIPS 2020

Fourier Features Let Networks Learn High Frequency Functions in Low Dimensional Domains

Tancik, Srinivasan, Mildenhall NeurIPS 2020



(a) Coordinate-based MLP

(b) Image regression $(x,y) \rightarrow$ RGB

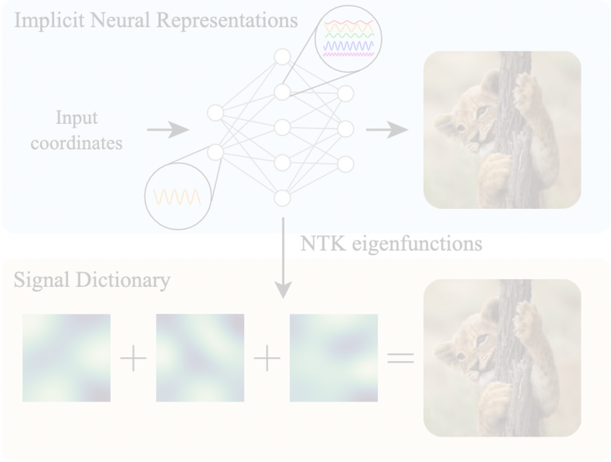
(c) 3D shape regression $(x,y,z) \rightarrow$ occupancy

(d) MRI reconstruction $(x,y,z) \rightarrow$ density

(e) Inverse rendering $(x,y,z) \rightarrow$ RGB, density

A Structured Dictionary Perspective on Implicit Neural Representations

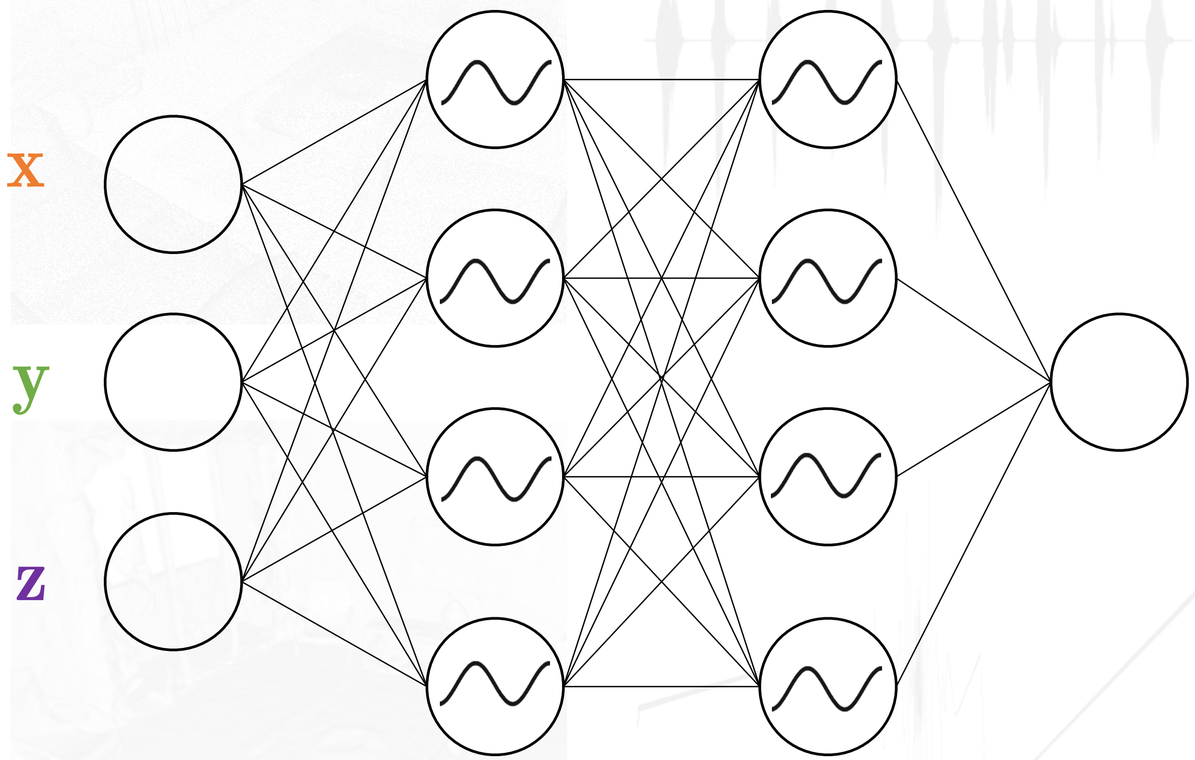
Yüce & Ortiz-Jiménez et al. CVPR 2022



Gaussian Activated Neural Radiance Fields for High Fidelity Reconstruction & Pose Estimation

Chng et al. ECCV 2022

SIREN: Sinusoidal Representation Networks



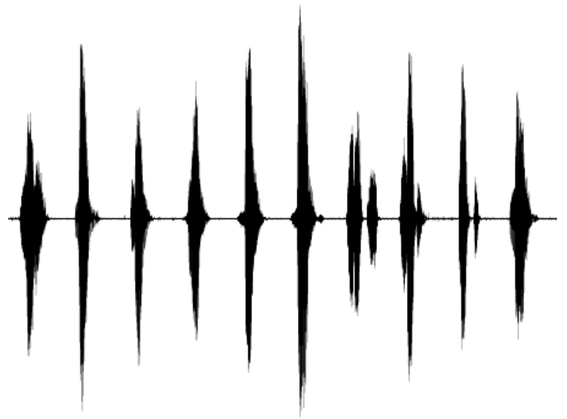
Sitzmann et al.: Implicit neural representations with periodic activation functions, NeurIPS 2020

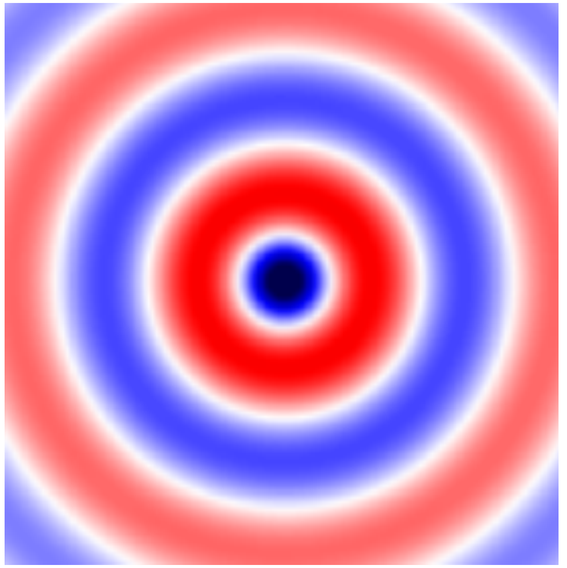
Images

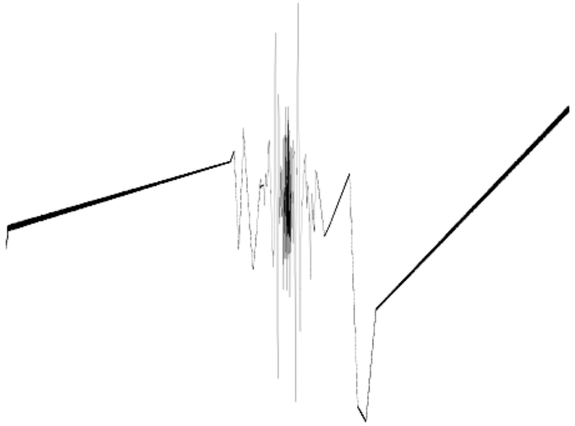
Shapes

Audio

Quantities defined by a differential equation





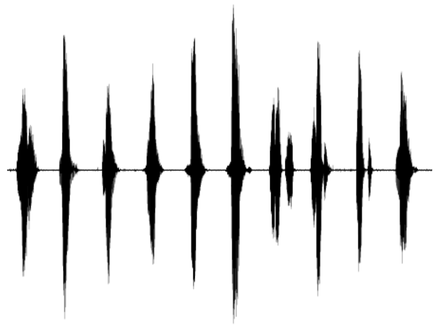


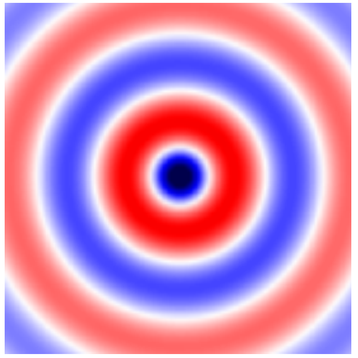
Images

Shapes

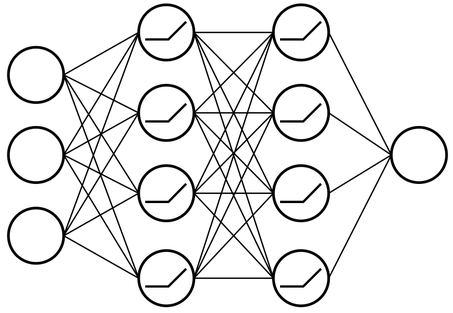
Audio

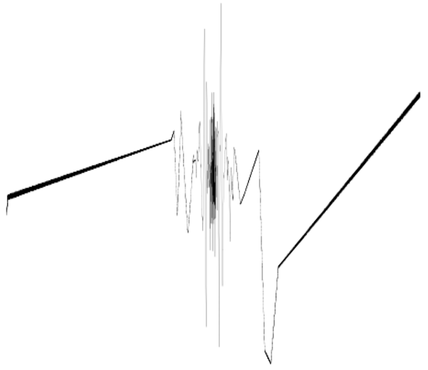
Quantities defined by a differential equation

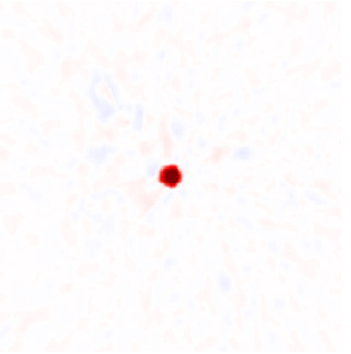




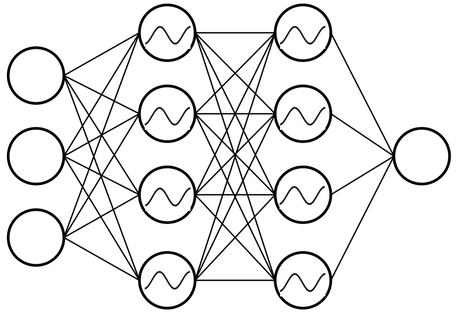
ReLU MLP

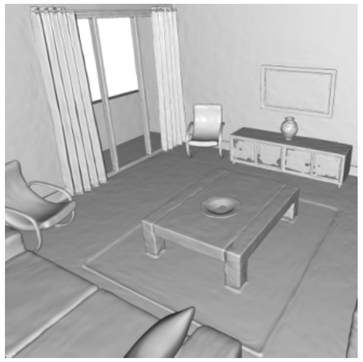


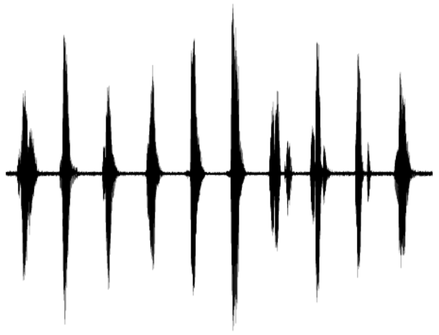


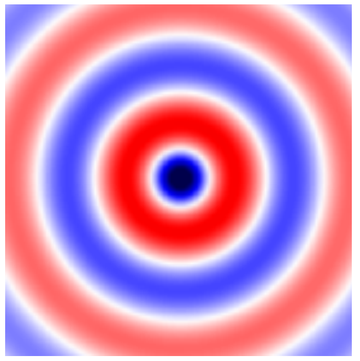


SIREN

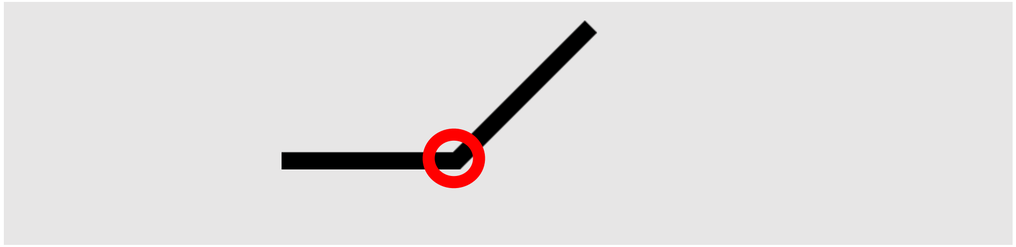


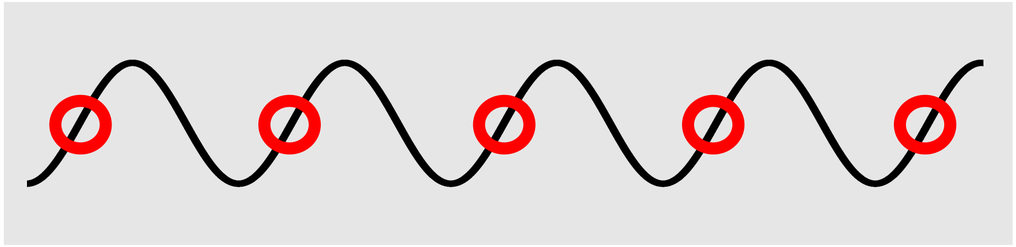






Locality

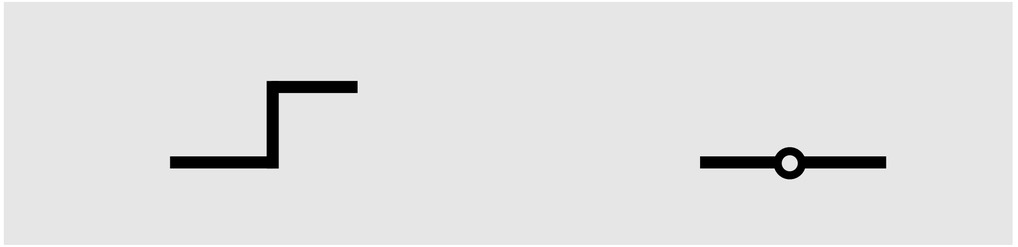


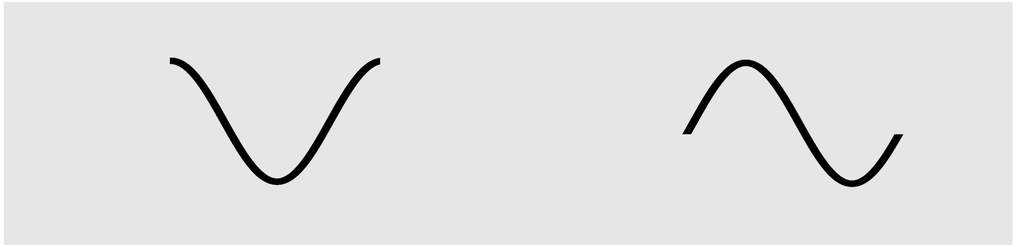


Periodicity allows SIREN to replicate activations across input domain

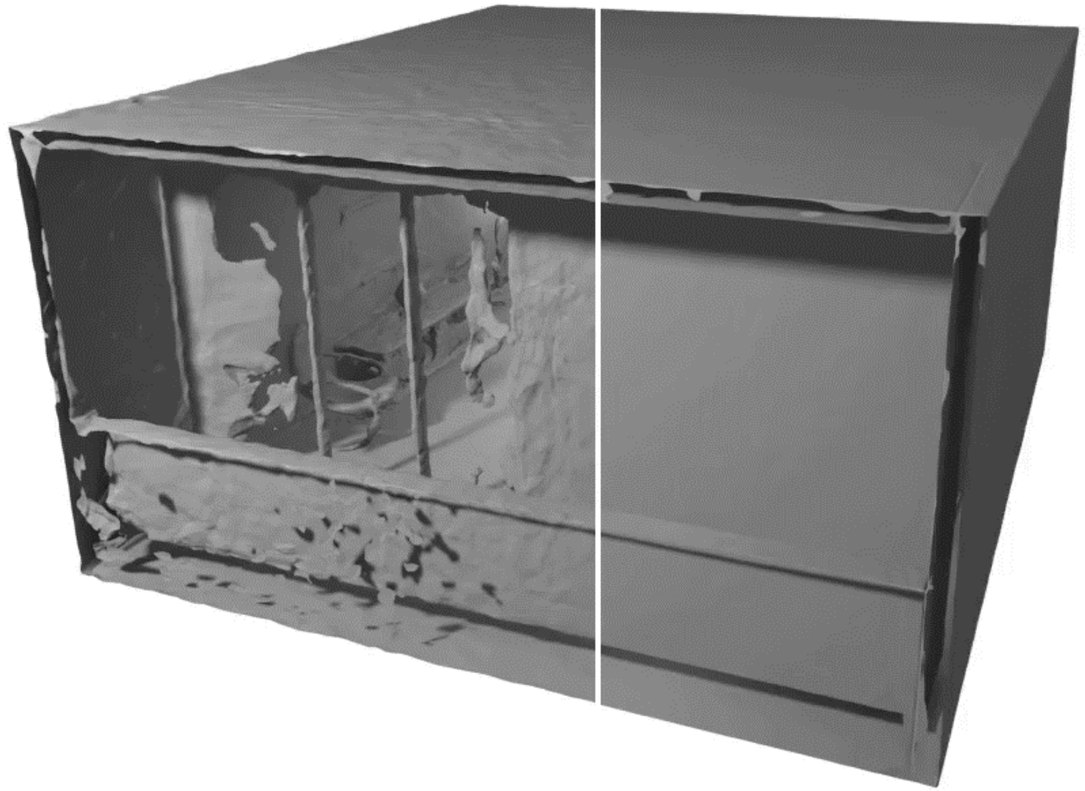
Derivatives

$\delta/\delta x$ $\delta^2/\delta x^2$





All derivatives exist, are nonzero and bounded by 1



~1MB compared to
110MB of full mesh!

ReLU

SIREN

Background: Kernel Regression

Given "training" set $\{(x_i, y_i)\}_i$, a kernel function makes predictions on a point $\mathbf{x}$ by interpolating labels $\mathbf{y}_i$ in the training set according to pairwise weights between $\mathbf{x}_i$ to $\mathbf{x}$ as measured by a kernel function:

$$f(\mathbf{x}) = \sum_{i=1}^{n} (\mathbf{K}^{-1}\mathbf{y})_i k(\mathbf{x}_i, \mathbf{x})$$

Where $\mathbf{K}_{ij} = k(\mathbf{x}_i, \mathbf{x}_j)$, k is kernel function with $k \geq 0$.

The Neural Tangent Kernel

$$f(\mathbf{x}) = \sum_{i=1}^{n} (\mathbf{K}^{-1}\mathbf{y})_i k(\mathbf{x}_i, \mathbf{x})$$

It turns out (Jacot et al. 2018) that in the limit of an infinitely wide MLP $f(x)$ with initialization θ_0, test-time predictions are made according to a kernel:

$$k_{NTK}(\mathbf{x}_1, \mathbf{x}_2) = \langle \nabla_\theta f_{\theta_0}(\mathbf{x}_1), \nabla_\theta f_{\theta_0}(\mathbf{x}_2) \rangle$$

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In other words: “Neural” implicit representations interpolate the training set according to a relatively simple rule. No “intelligence”, no AI, no magic.

The Neural Tangent Kernel

$$f(\mathbf{x}) = \sum_{i=1}^{n} (\mathbf{K}^{-1}\mathbf{y})_i k(\mathbf{x}_i, \mathbf{x})$$

It turns out (Jacot et al. 2018) that in the limit of an infinitely wide MLP $f(x)$ with initialization θ_0, test-time predictions are made according to a kernel:

$$k_{NTK}(\mathbf{x}_1, \mathbf{x}_2) = \langle \nabla_\theta f_{\theta_0}(\mathbf{x}_1), \nabla_\theta f_{\theta_0}(\mathbf{x}_2) \rangle$$

What we can do is design the kernel and pick the space of $(\mathbf{x}, \mathbf{y})$,
the space in which our neural network will interpolate.

The Neural Tangent Kernel

$$f(\mathbf{x}) = \sum_{i=1}^{n} (\mathbf{K}^{-1}\mathbf{y})_i k(\mathbf{x}_i, \mathbf{x})$$

It turns out (Jacot et al. 2018) that in the limit of an infinitely wide MLP $f(x)$ with initialization θ_0, test-time predictions are made according to a kernel:

$$k_{NTK}(\mathbf{x}_1, \mathbf{x}_2) = \langle \nabla_\theta f_{\theta_0}(\mathbf{x}_1), \nabla_\theta f_{\theta_0}(\mathbf{x}_2) \rangle$$

We do **not** know of an equivalent rule for transformers to date - work in progress!

The Neural Tangent Kernel

$$f(\mathbf{x}) = \sum_{i=1}^{n} (\mathbf{K}^{-1}\mathbf{y})_i k(\mathbf{x}_i, \mathbf{x})$$

It turns out (Jacot et al. 2018) that in the limit of an infinitely wide MLP $f(x)$ with initialization θ_0, test-time predictions are made according to a kernel:

$$k_{NTK}(\mathbf{x}_1, \mathbf{x}_2) = \langle \nabla_\theta f_{\theta_0}(\mathbf{x}_1), \nabla_\theta f_{\theta_0}(\mathbf{x}_2) \rangle$$

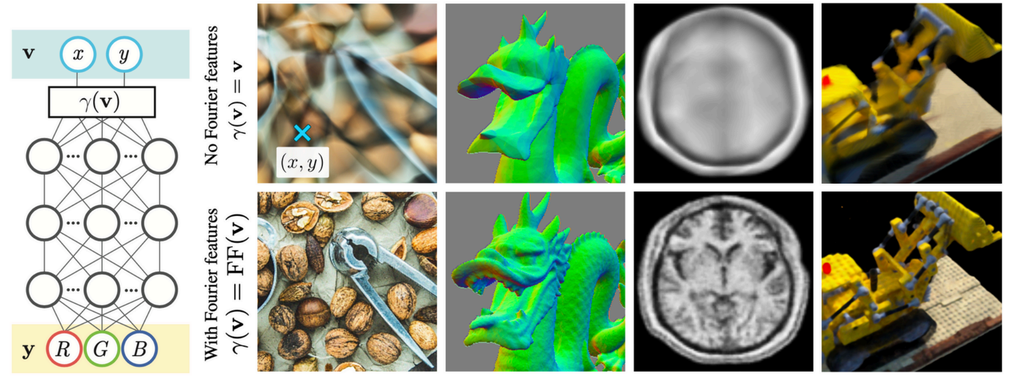
What does this mean for neural fields?

SIREN: Neural Implicit Representations With Periodic Activation Functions

Sitzmann & Martel et al. NeurIPS 2020

Fourier Features Let Networks Learn High Frequency Functions in Low Dimensional Domains

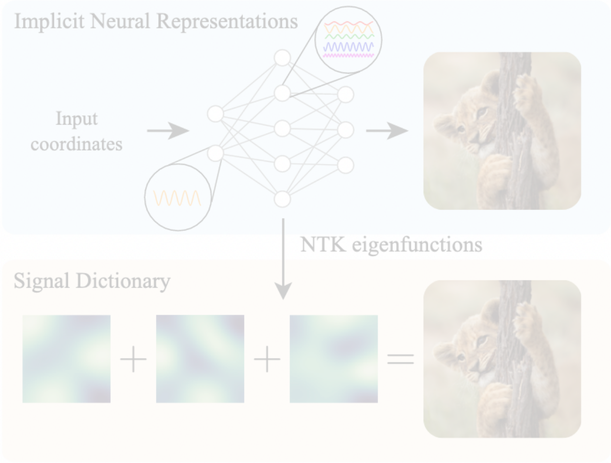
Tancik, Srinivasan, Mildenhall et al, NeurIPS 2020



(a) Coordinate-based MLP | (b) Image regression $(x,y) \rightarrow$ RGB | (c) 3D shape regression $(x,y,z) \rightarrow$ occupancy | (d) MRI reconstruction $(x,y,z) \rightarrow$ density | (e) Inverse rendering $(x,y,z) \rightarrow$ RGB, density

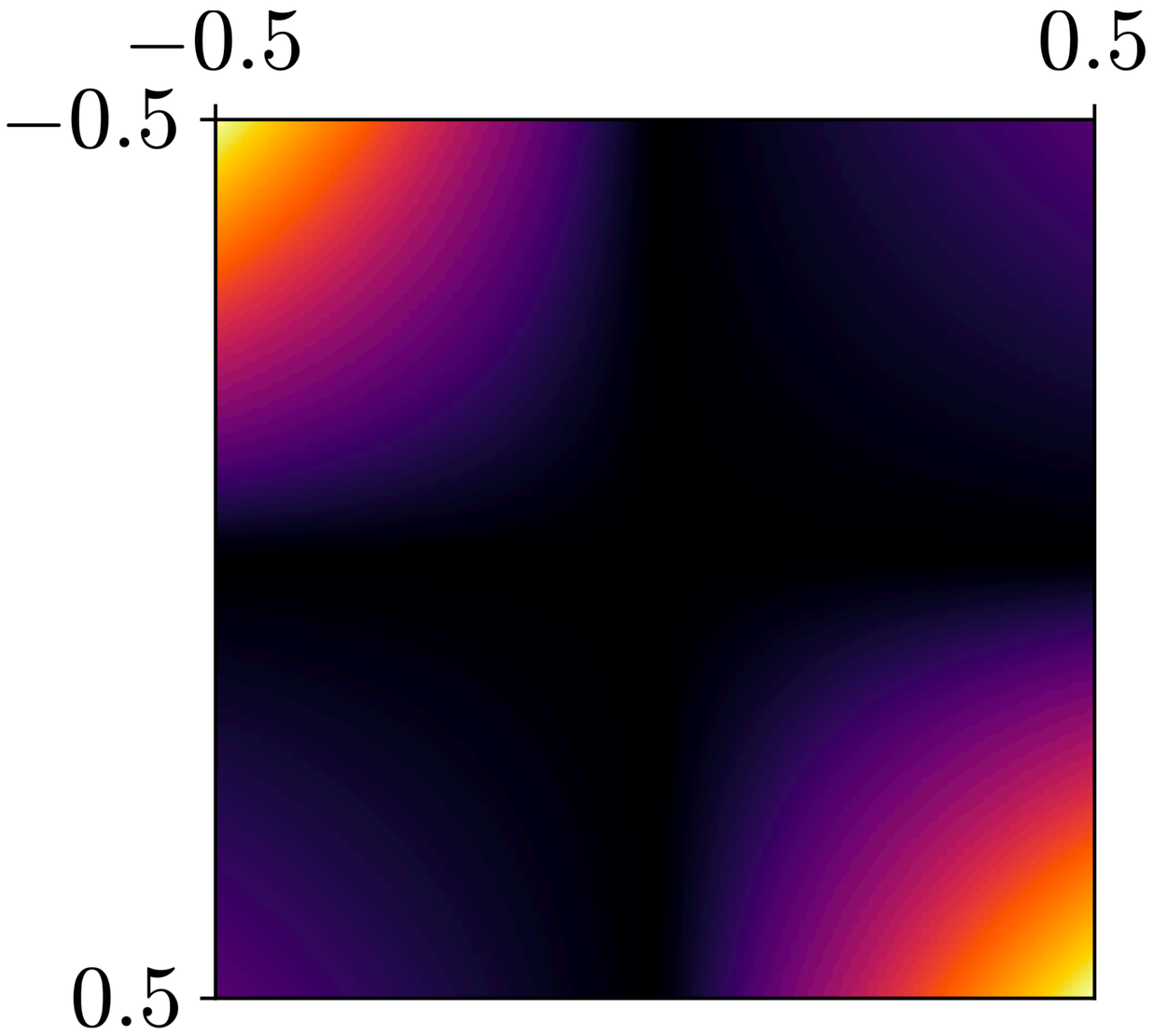
A Structured Dictionary Perspective on Implicit Neural Representations

Yüce & Ortiz-Jiménez et al. CVPR 2022



Gaussian Activated Neural Radiance Fields for High Fidelity Reconstruction & Pose Estimation

Chng et al. ECCV 2022

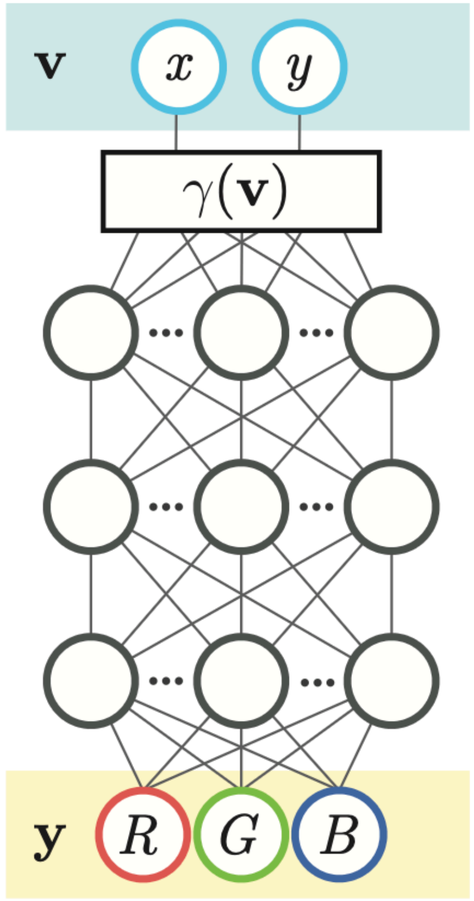


NTK kernel $k_{NTK}(x_i, x_j)$ for a 4-layer ReLU MLP with one scalar input.

From “Fourier Features Let Networks Learn High Frequency Functions in Low Dimensional Domains”, Tancik, Srinivasan, Mildenhall et al.

NTK of ReLU neural net is quite non-local.

Feature Embeddings



$$\gamma(\mathbf{x}) = [\gamma_1(\mathbf{x}), \ldots, \gamma_n(\mathbf{x})]$$

$$\gamma_{2i}(\mathbf{x}) = \sin(2^{i-1}\pi x_i)$$
$$\gamma_{(2i+1)}(\mathbf{x}) = \cos(2^{i-1}\pi x_i)$$

Sinusoidal Embeddings
Zhong et al. ICLR 2020
Mildenhall et al., ECCV 2020

$$\gamma(\mathbf{x}) = \exp(-\frac{\|t - x\|^2}{2\sigma^2})$$

Gaussian Embeddings
Zheng et al., arXiv 2021

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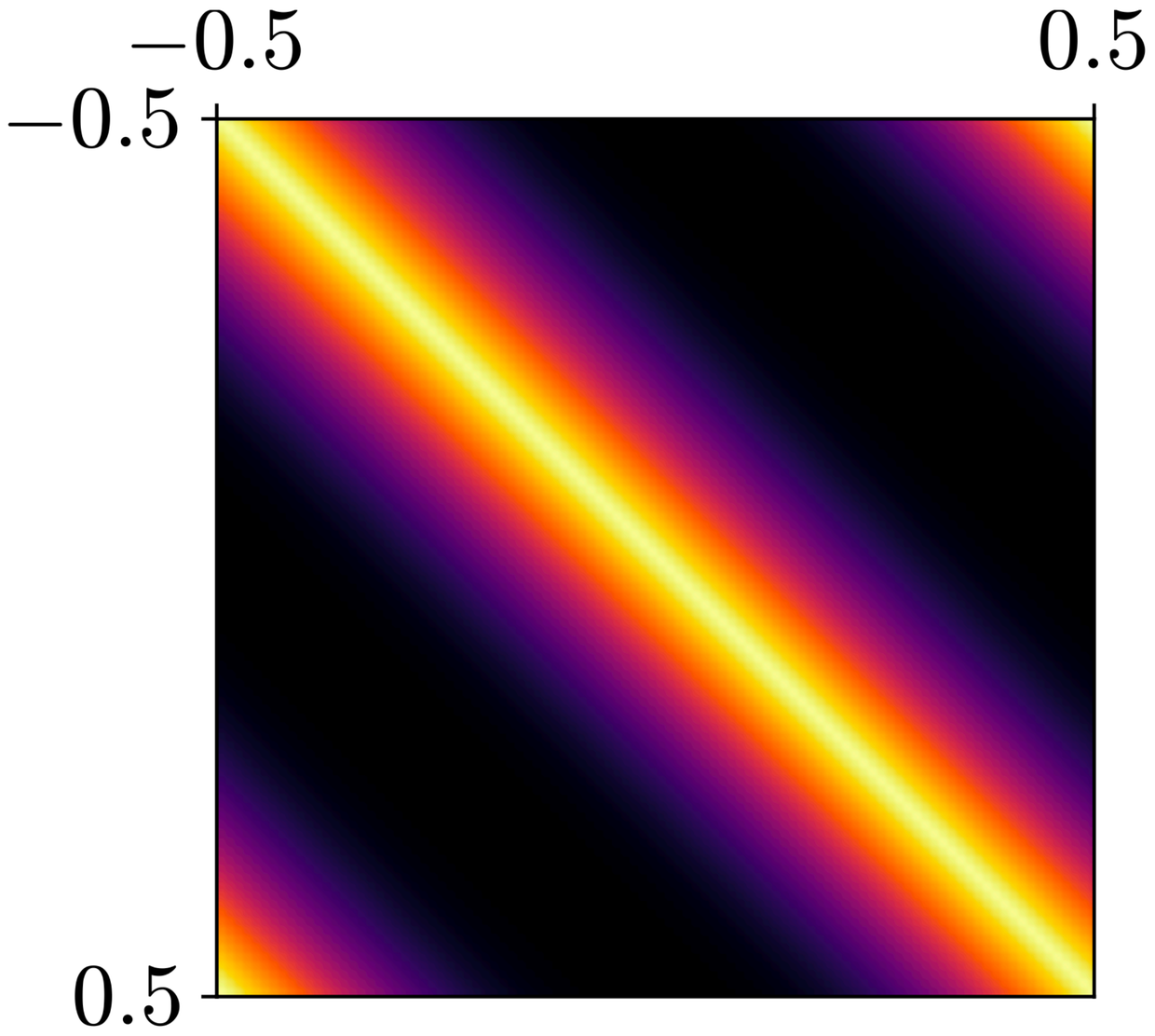
Reconstructing continuous distributions of 3D protein…, Zhong et al.; Neural Radiance Fields, Mildenhall et al.



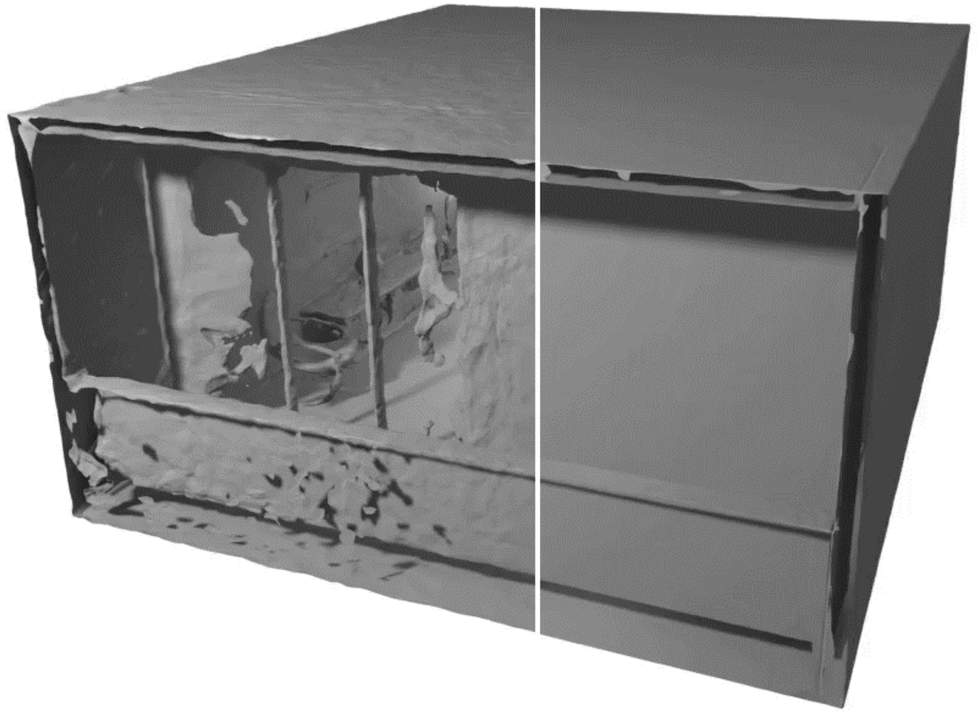
NTK kernel $k_{NTK}(\gamma(x_i), \gamma(x_j))$ for a 4-layer ReLU MLP with one scalar input and **Sinusoidal Feature Mapping**

$$\gamma(x) = [\cos(2\pi x), \sin(2\pi x)].$$

From “Fourier Features Let Networks Learn High Frequency Functions in Low Dimensional Domains”, Tancik, Srinivasan, Mildenhall et al.

NTK is local (note diagonal).

SIREN

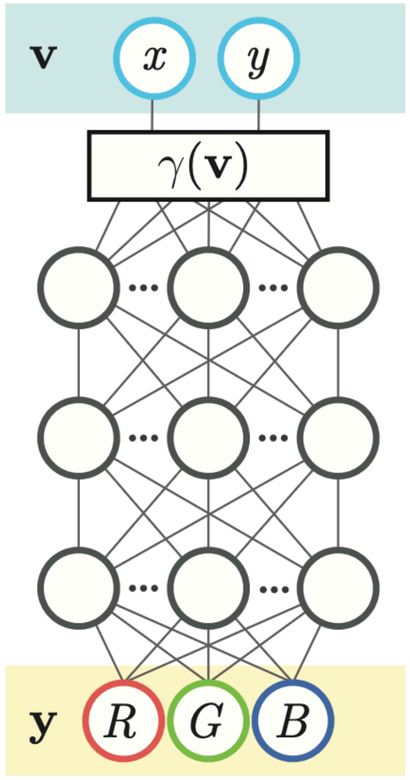


~1MB compared to
110MB of full mesh!

ReLU

SIREN

Fourier Features

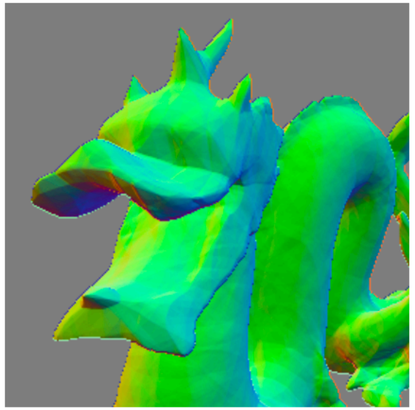


(a) Coordinate-based MLP

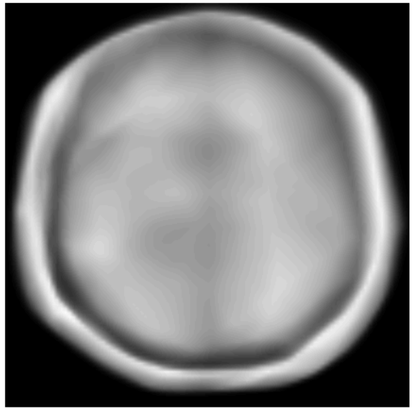
No Fourier features $\gamma(\mathbf{v}) = \mathbf{v}$

With Fourier features $\gamma(\mathbf{v}) = \mathrm{FF}(\mathbf{v})$

(b) Image regression
$(x,y) \rightarrow$ RGB

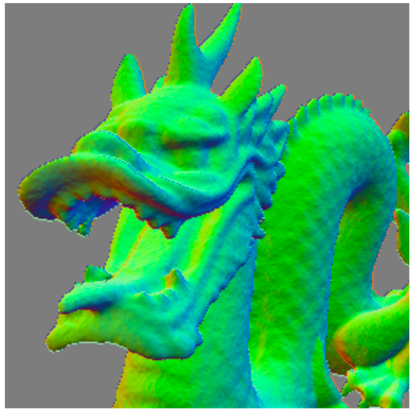


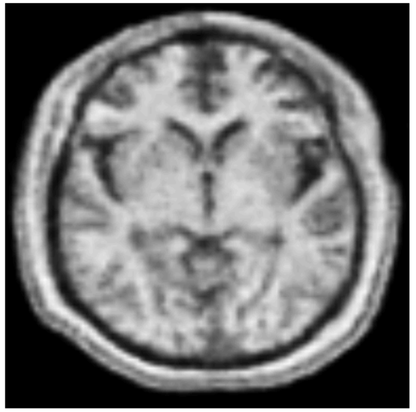
(c) 3D shape regression
$(x,y,z) \rightarrow$ occupancy

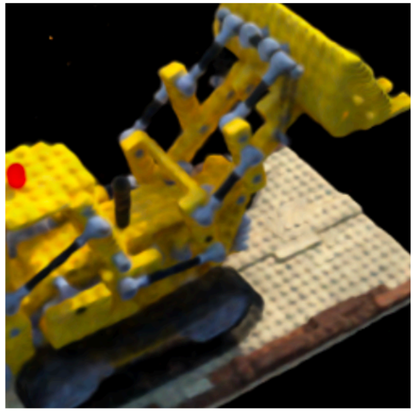


(d) MRI reconstruction
$(x,y,z) \rightarrow$ density

(e) Inverse rendering
$(x,y,z) \rightarrow$ RGB, density





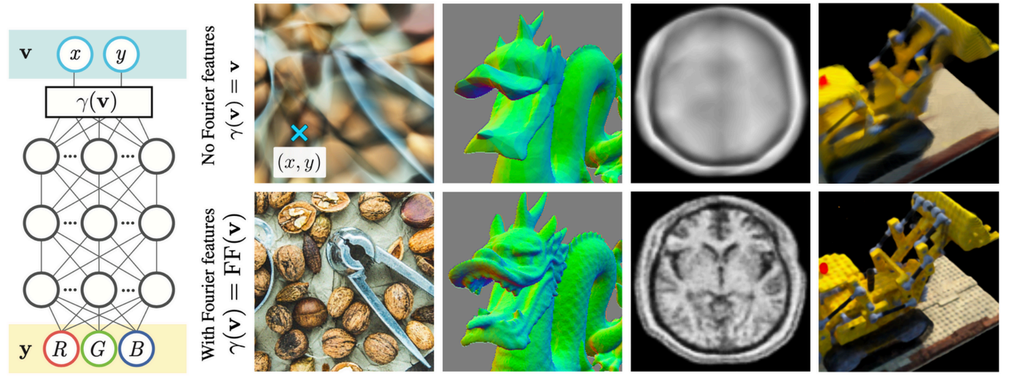


SIREN: Neural Implicit Representations With Periodic Activation Functions

Sitzmann & Martel et al. NeurIPS 2020

Fourier Features Let Networks Learn High Frequency Functions in Low Dimensional Domains

Tancik, Srinivasan, Mildenhall NeurIPS 2020



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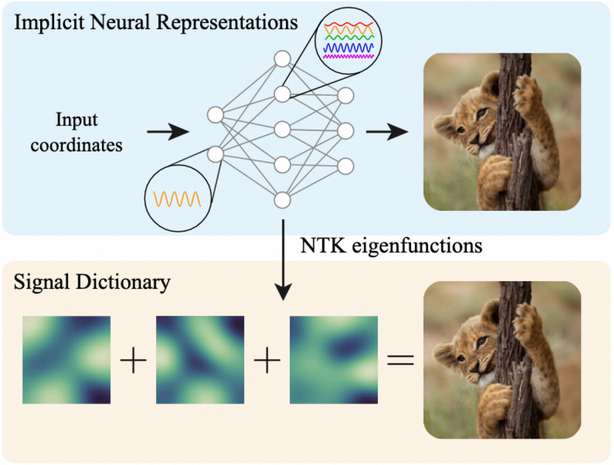
Yüce & Ortiz-Jiménez et al. CVPR 2022

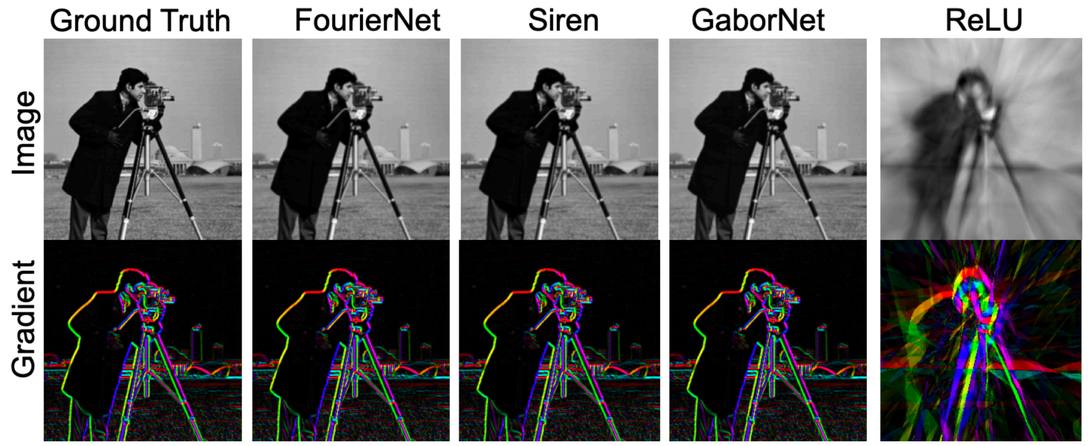


Gaussian Activated Neural Radiance Fields for High Fidelity Reconstruction & Pose Estimation

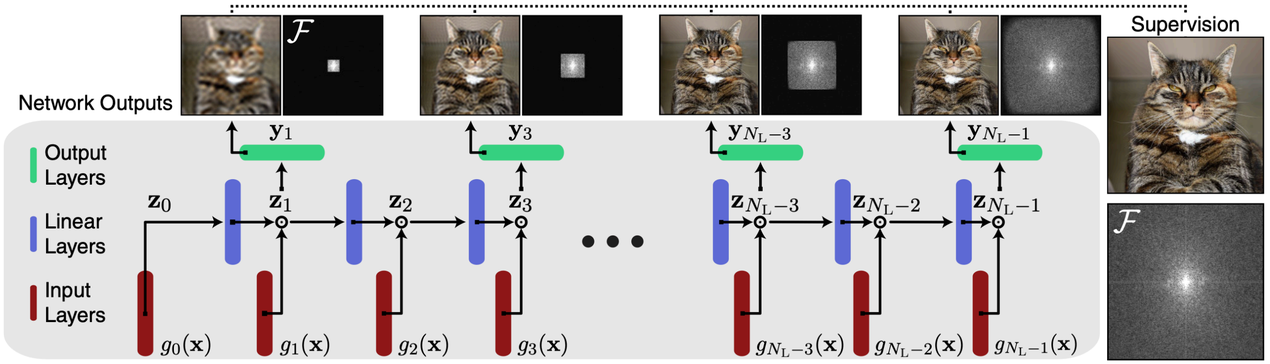
Chng et al. ECCV 2022

Neural (?) Fields with analytical (but not tractably computable) Fourier spectrum!

Multiplicative Filter Networks
Fathony et al. ICLR 2021



BACON: Band-limited coordinate networks for multiscale scene representation
Lindell et al. CVPR 2022



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EUROGRAPHICS 2022
D. Meneveaux and G. Patanè
(Guest Editors)

Volume 41 (2022), Number 2
STAR – State of The Art Report

Neural Fields in Visual Computing and Beyond

Yiheng Xie[1,2] Towaki Takikawa[3,4] Shunsuke Saito[5] Or Litany[4] Shiqin Yan[1] Numair Khan[1] Federico Tombari[6,7]
James Tompkin[1] Vincent Sitzmann[8†] Srinath Sridhar[1†]

[1]Brown University [2]Unity Technologies [3]University of Toronto [4]NVIDIA [5]Meta Reality Labs Research [6]Google [7]Technical University of Munich
[8]Massachusetts Institute of Technology [†]*Equal advising*

https://neuralfields.cs.brown.edu/

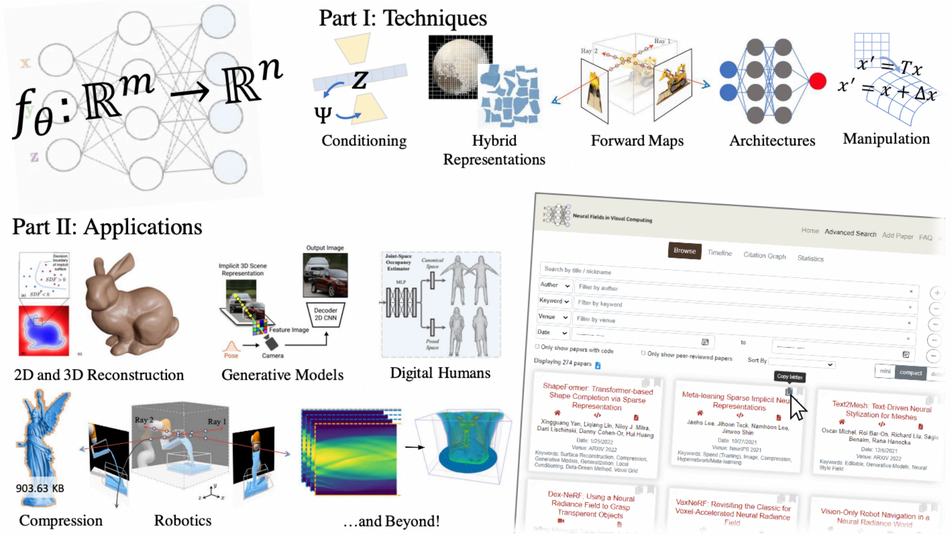
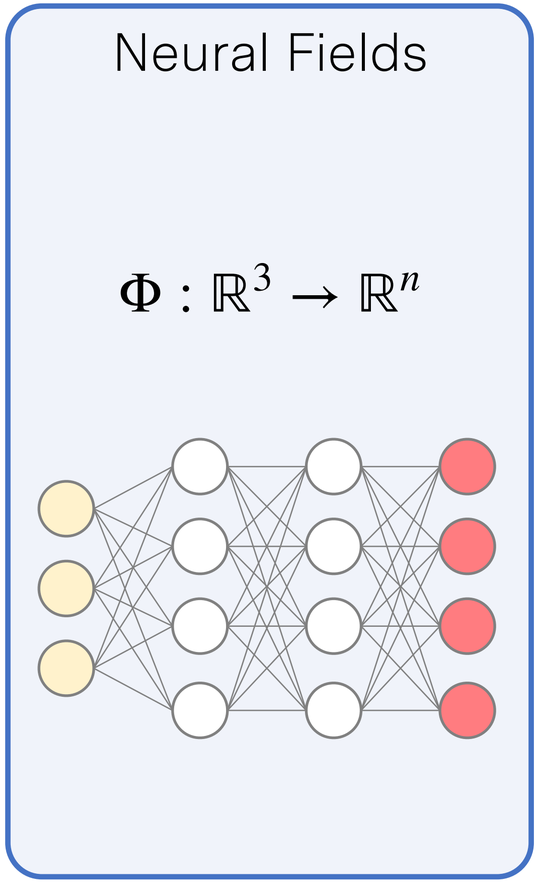


Figure 1: **Contribution of this report.** Following a survey of over 250 papers, we provide a review of (**Part I**) techniques in neural fields such as prior learning and conditioning, representations, forward maps, architectures, and manipulation, and of (**Part II**) applications in visual computing including 2D image processing, 3D scene reconstruction, generative modeling, digital humans, compression, robotics, and beyond. This report is complemented by a community-driven website with search, filtering, bibliographic, and visualization features.

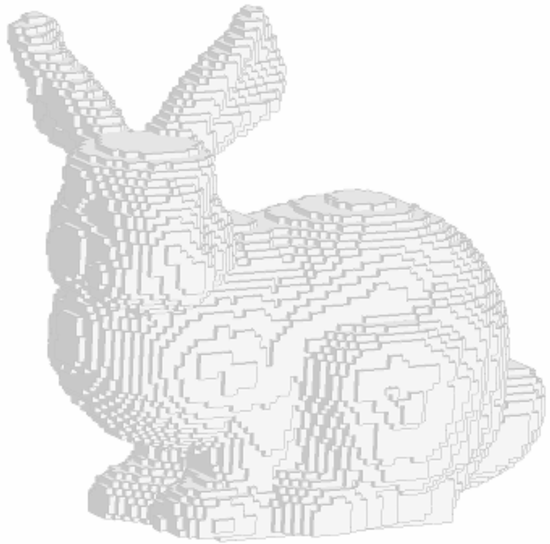
Neural Fields

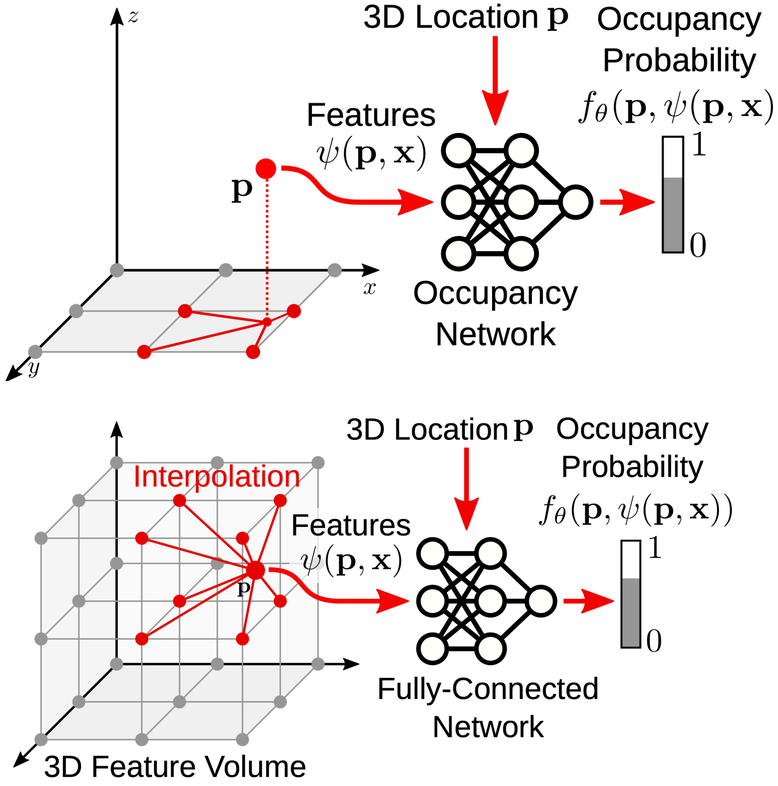
- Storage memory **does not grow with spatial resolution or number of spatial dimensions**.
- **Slow** sampling: Each query takes a forward pass through the neural net. Means GPU-memory intensive forward passes.
- Does **not** expose locality: Can't identify set of parameters / direction in parameter space that encodes particular spatial location.
- **Inconvenient processing**: cannot run convolutions.
- But: automatic adaptive resolution. Over optimization, will converge to assign more compute to higher-frequency areas.



Note: Despite their name, has nothing to do with Machine Learning itself.
It's as "smart" as a voxelgrid.

Hybrid discrete / continuous reps



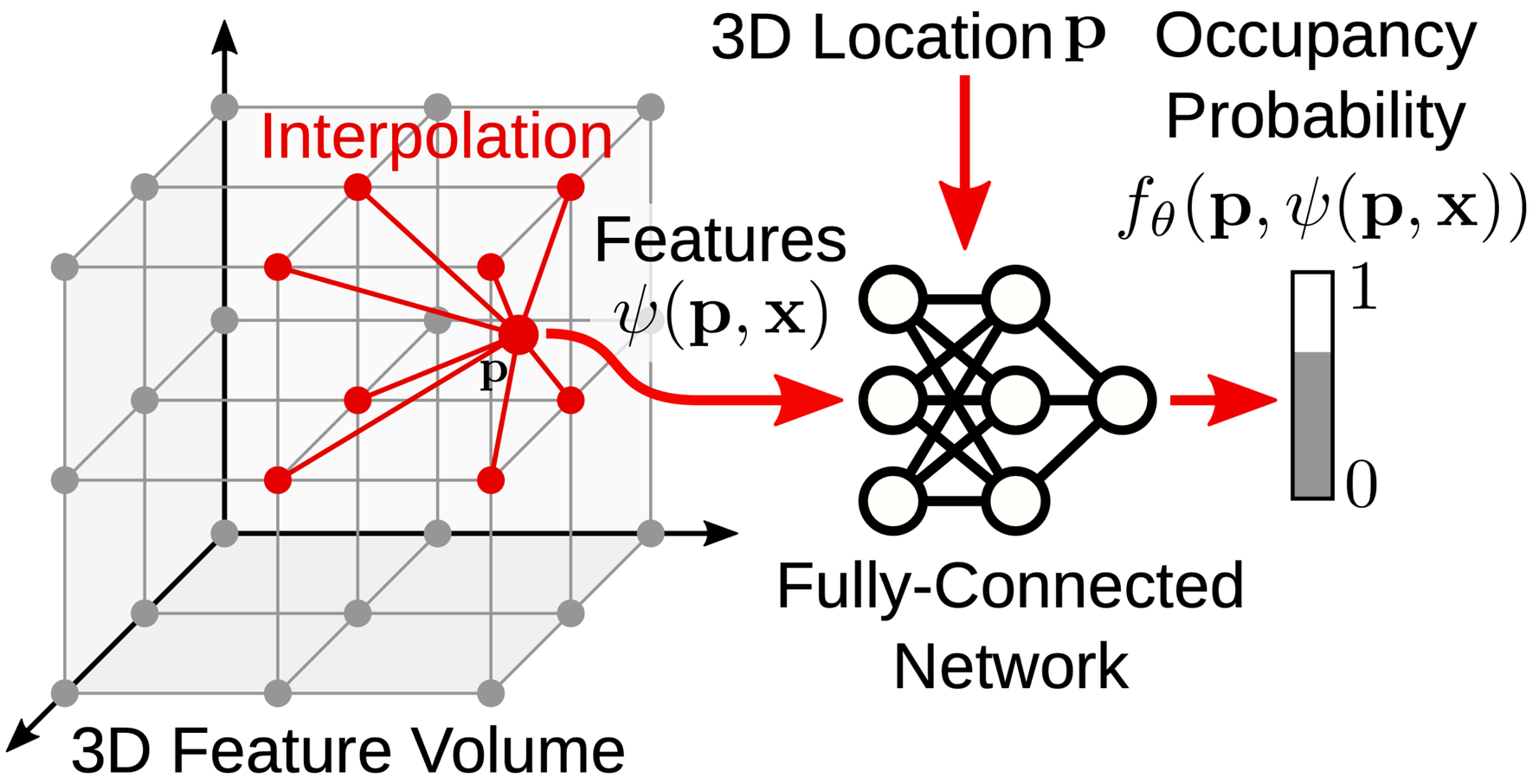


Discrete Parameterizations

Continuous Parameterizations

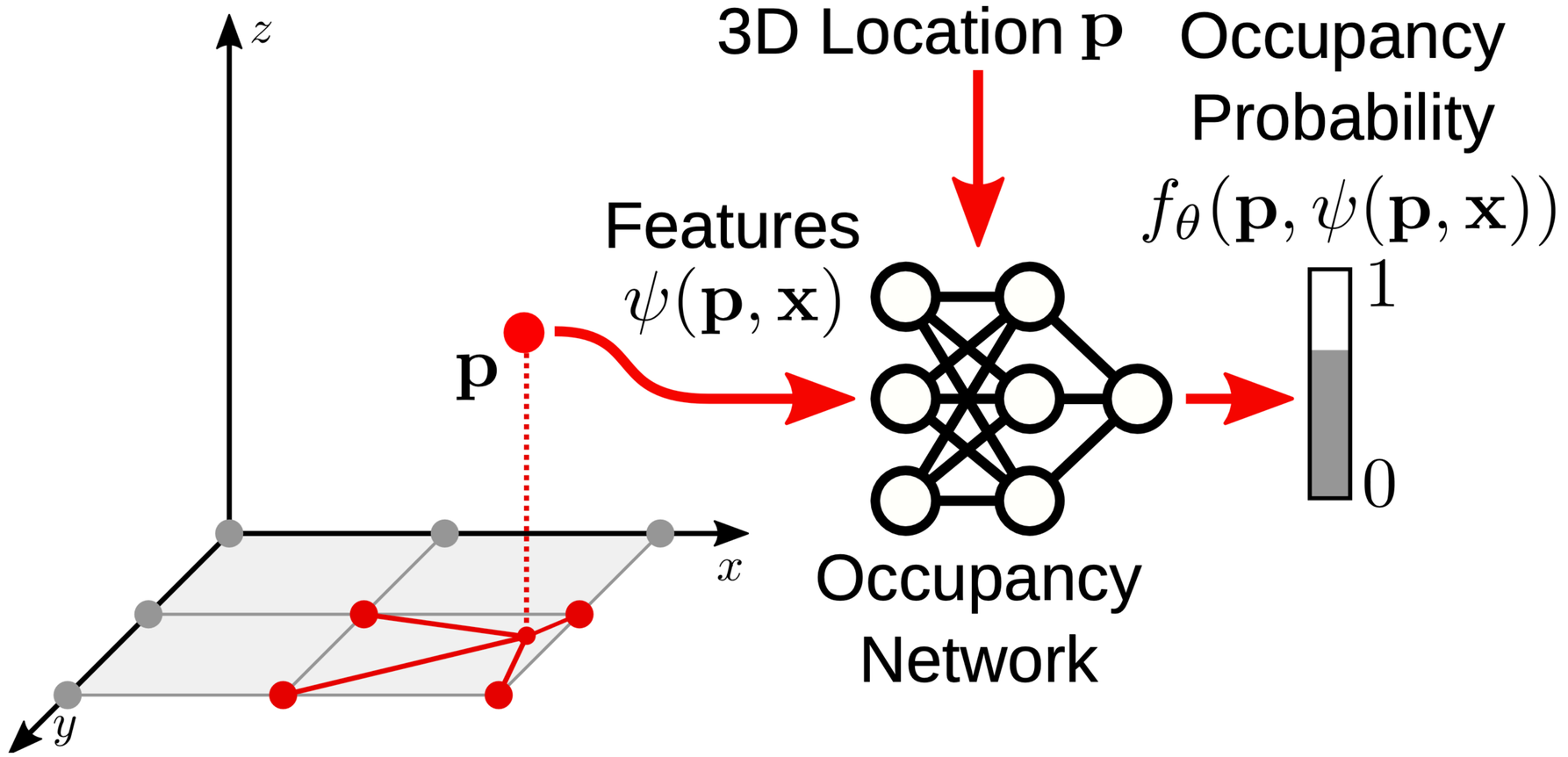
Hybrid Parameterizations

Basic idea: Use Neural Net as Interpolation Kernel



Convolutional Occupancy Networks [Peng et al. 2020]
Local Implicit Grid Representations for 3D Scenes [Jiang et al. 2020]
Implicit Functions in Feature Space for 3D Shape Reconstruction and Completion [Chibane et al. 2020]
Deep Local Shapes: Learning Local SDF Priors for Detailed 3D Reconstruction [Chabra et al. 2020]

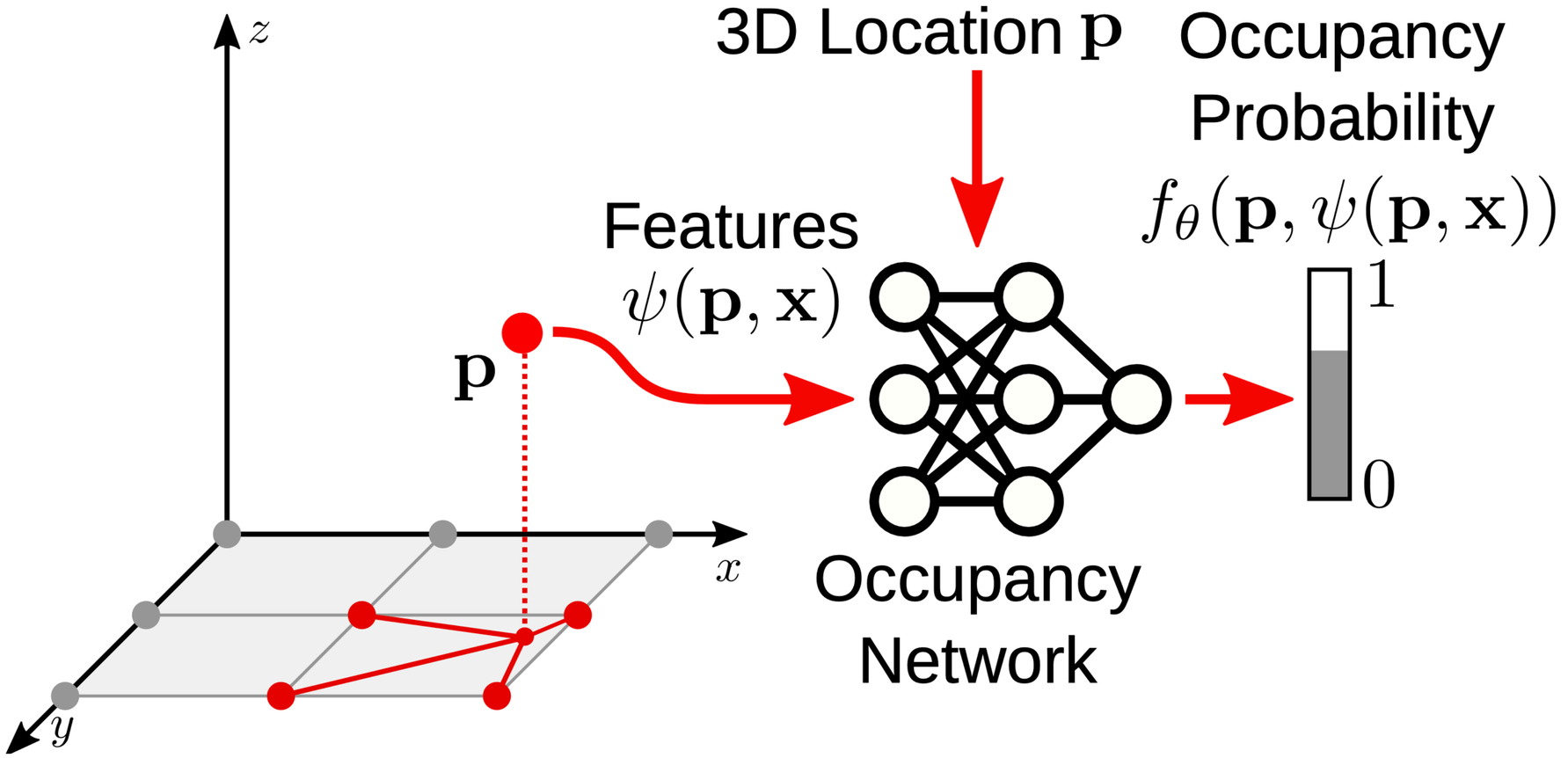
Ground plan & Orthographic Projection



Any assumptions made on scene? Any limitations?

Convolutional Occupancy Networks [Peng et al. 2020]

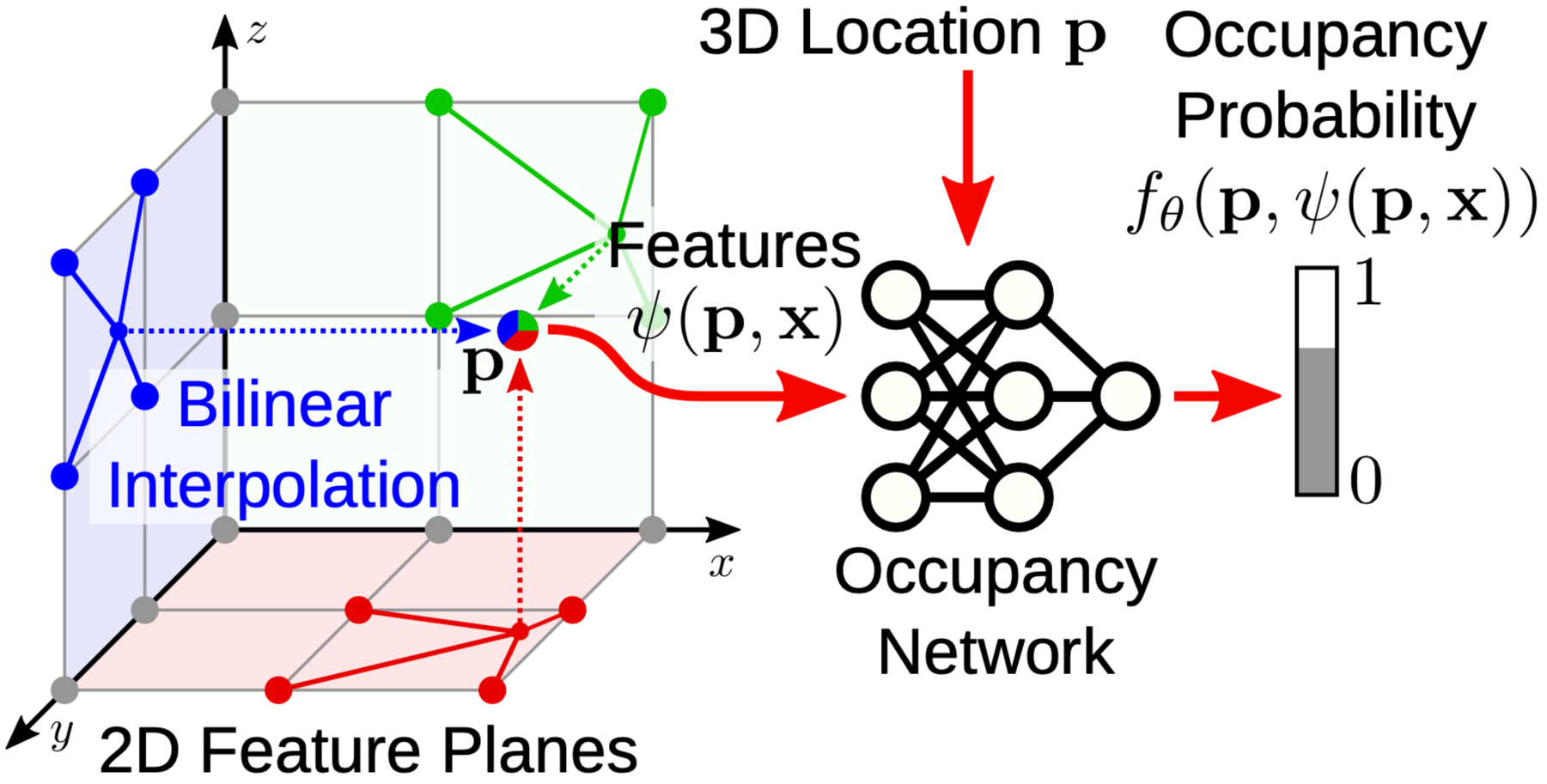
Ground plan & Orthographic Projection



No. You can still represent *any scene* - some info stored in NN, some in grid.

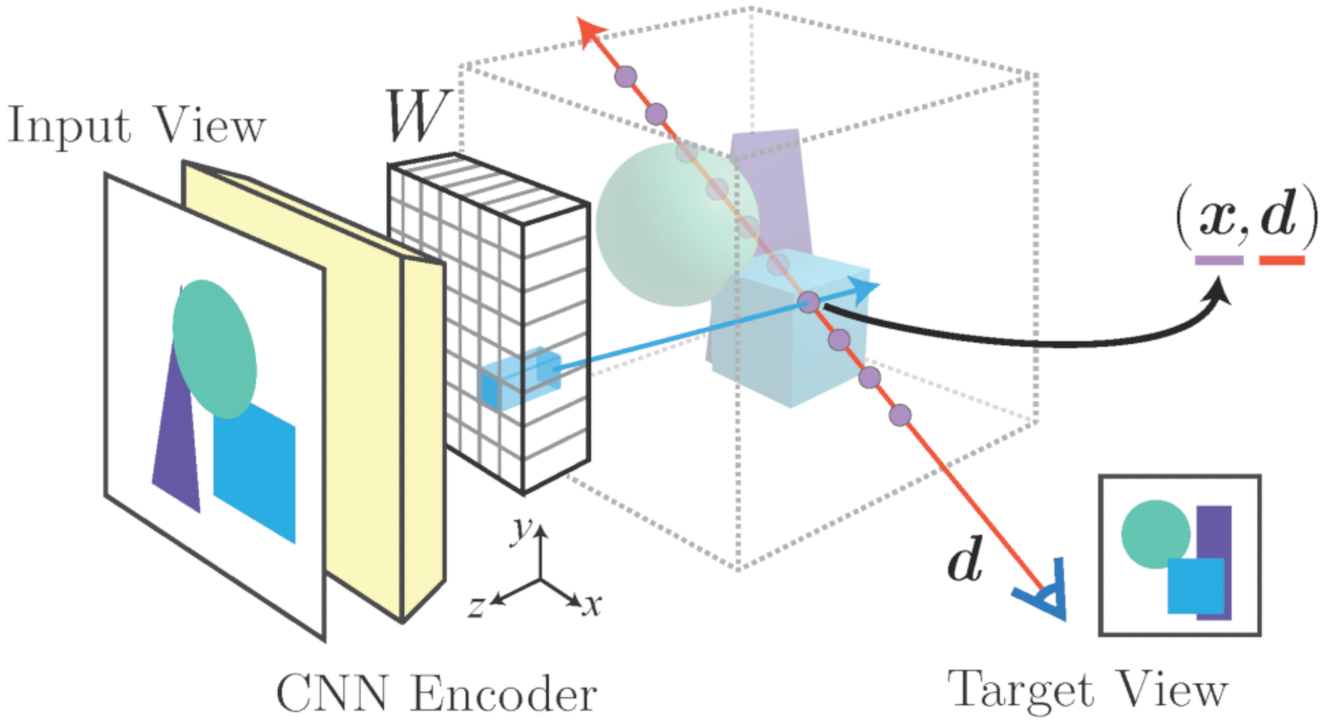
Convolutional Occupancy Networks [Peng et al. 2020]

Tri-Planes and Orthographic Projection



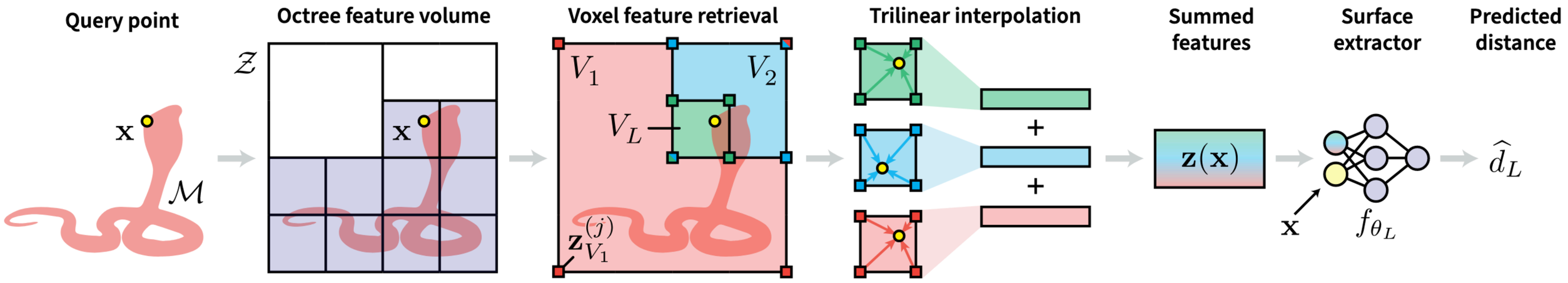
Convolutional Occupancy Networks [Peng et al. 2020]

Image Grids & Perspective Projection



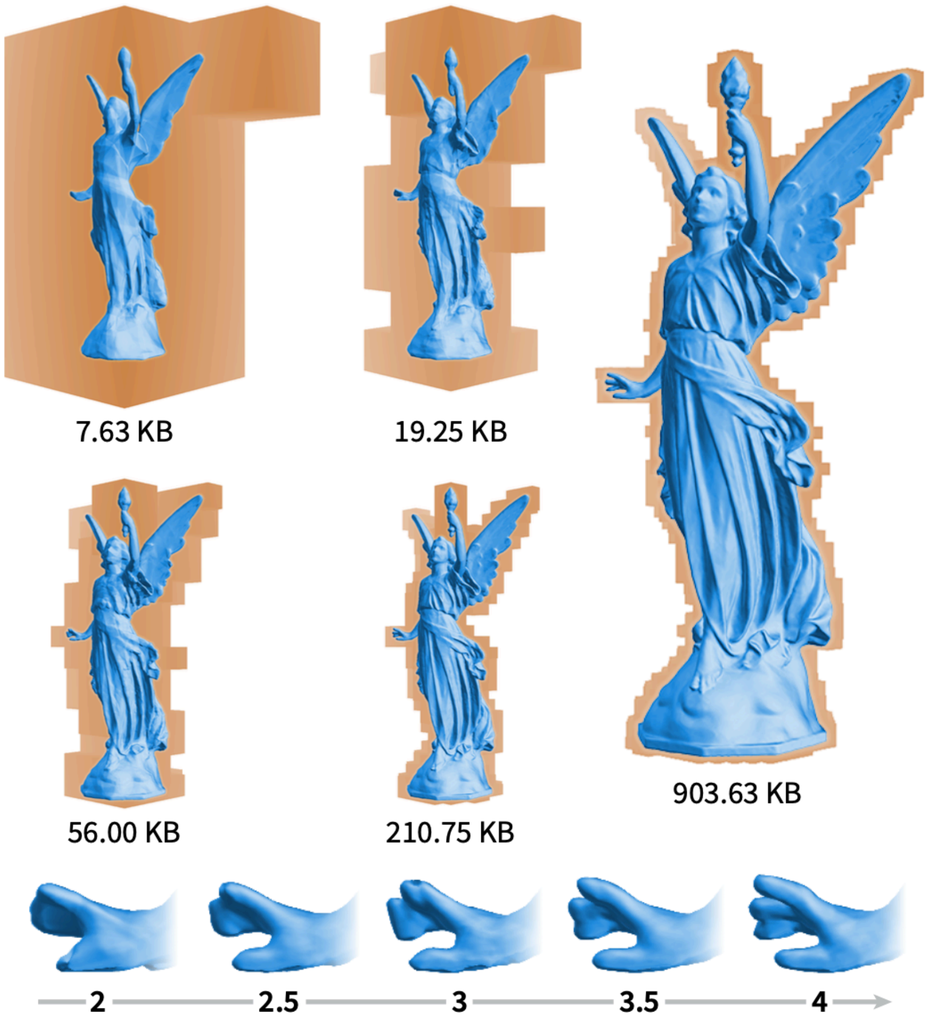
PixelNeRF (Yu et al. 2020)

Multi-scale Voxel Grids



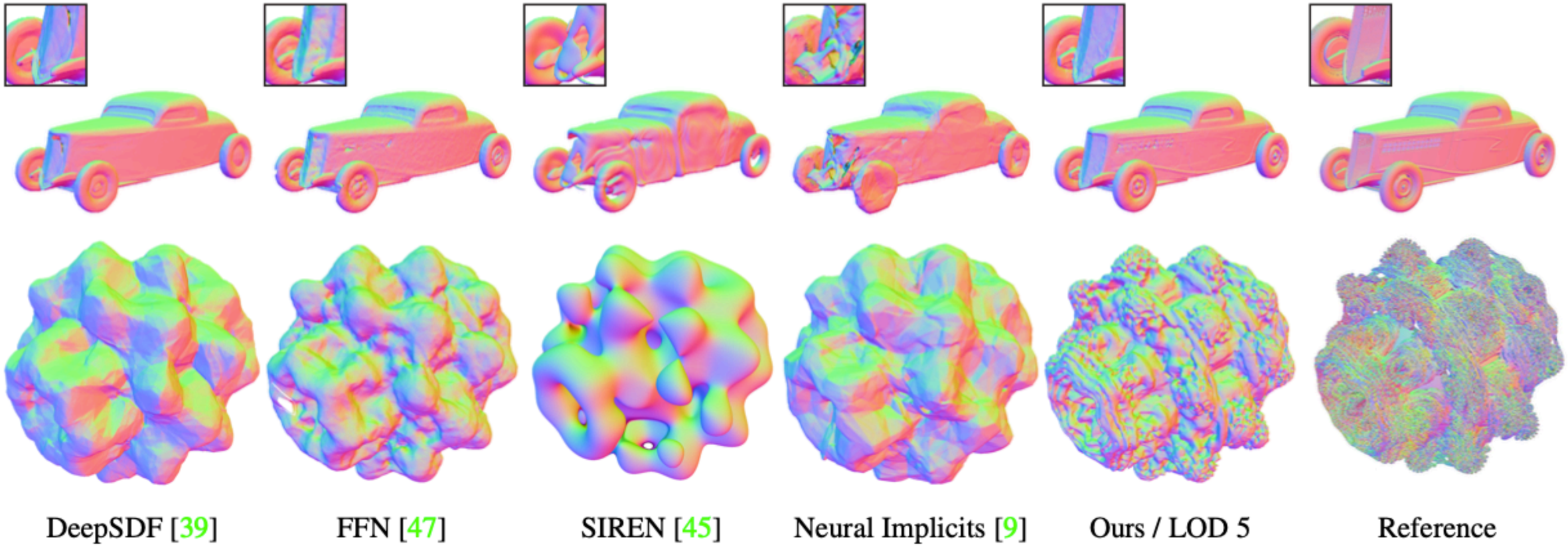
Neural Geometric Level of Detail: Real-time Rendering with Implicit 3D Shapes, Takikawa, Litalien et al. 2021

Multi-scale Voxel Grids



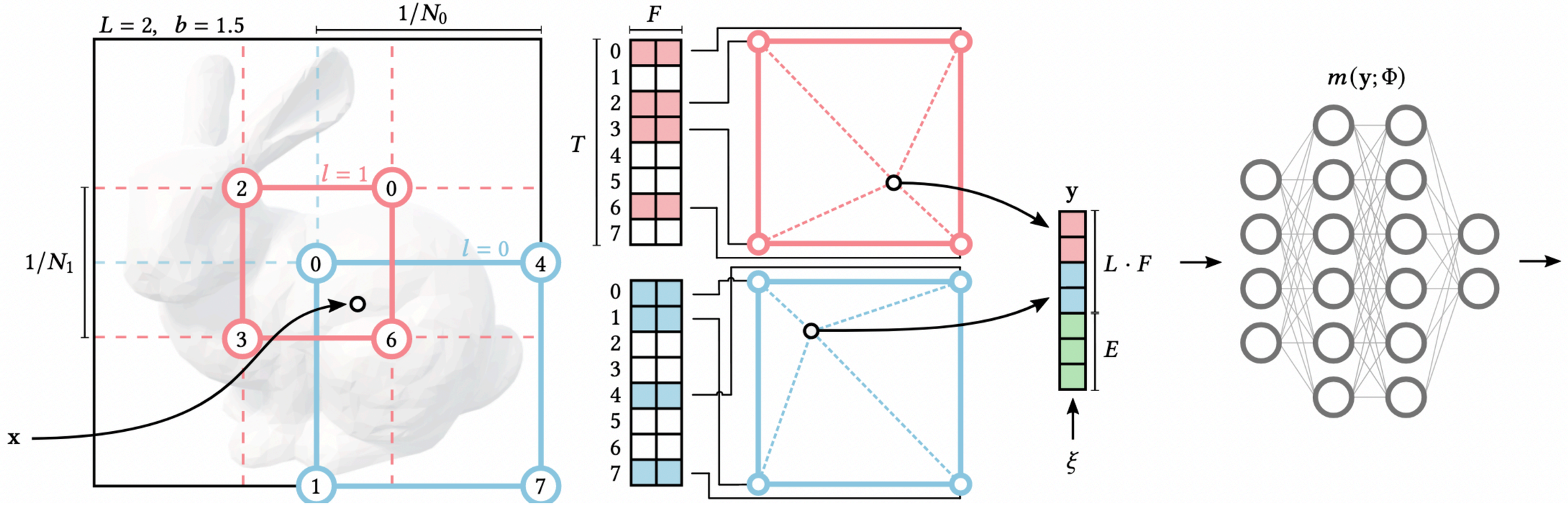
Neural Geometric Level of Detail: Real-time Rendering with Implicit 3D Shapes, Takikawa, Litalien et al. 2021

Multi-scale Voxel Grids



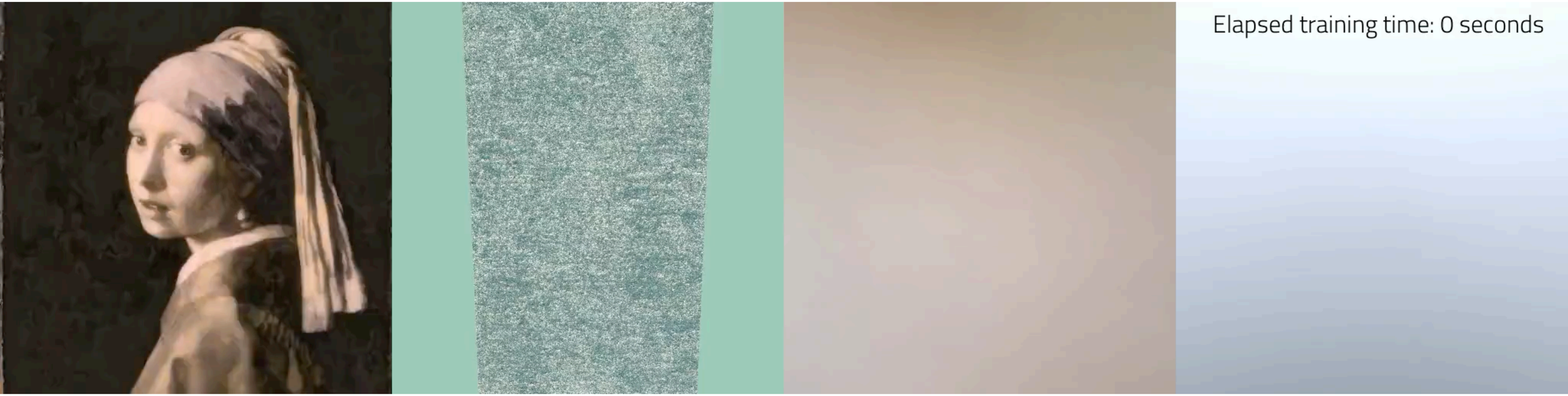
Neural Geometric Level of Detail: Real-time Rendering with Implicit 3D Shapes, Takikawa, Litalien et al. 2021

Multi-scale Voxel Grids + Hash Table



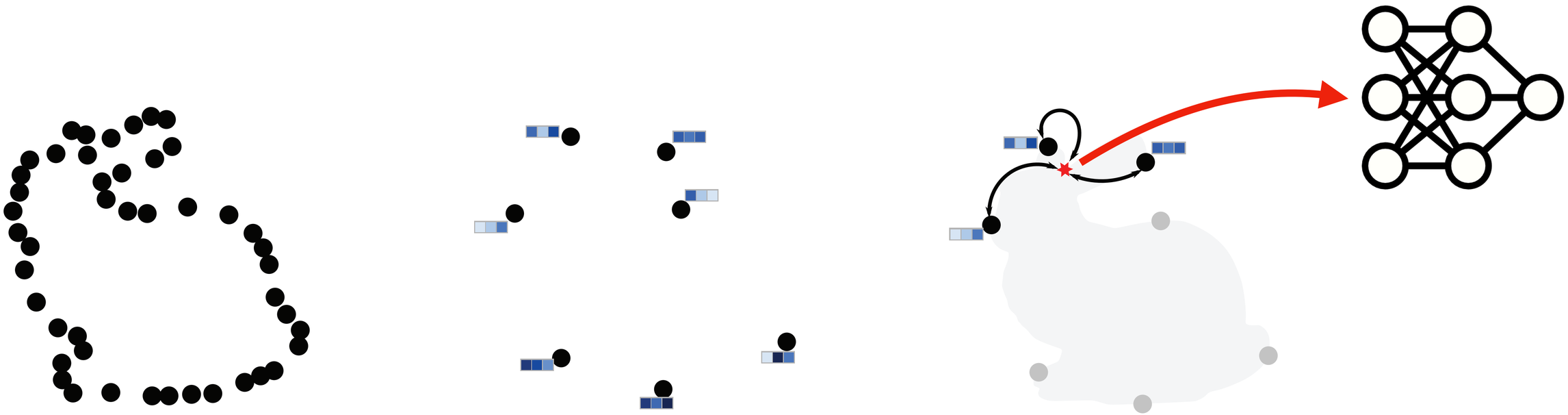
Instant Neural Graphics Primitives with a Multiresolution Hash Encoding, Müller et al. 2022

Multi-scale Voxel Grids + Hash Table



Instant Neural Graphics Primitives with a Multiresolution Hash Encoding, Müller et al. 2022

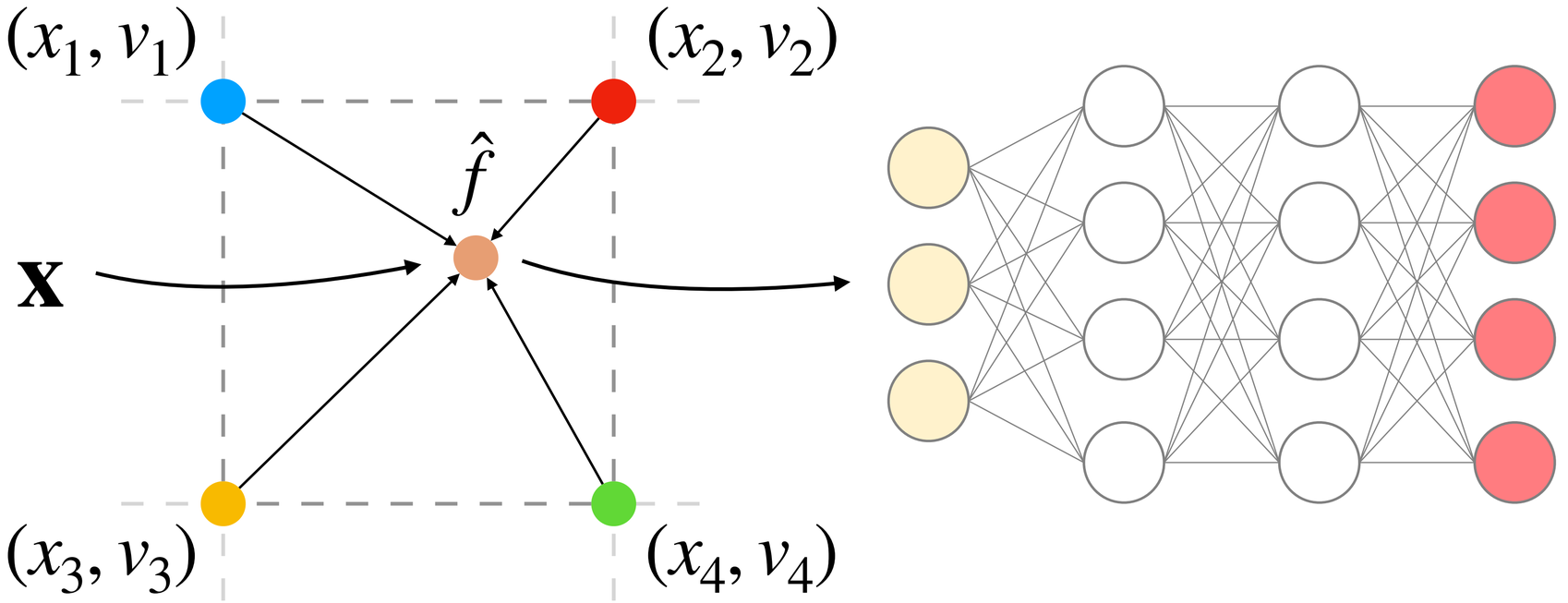
Point Cloud & Nearest Neighbor



Hybrid representations can be used to decode a surface representation into a volume representation!

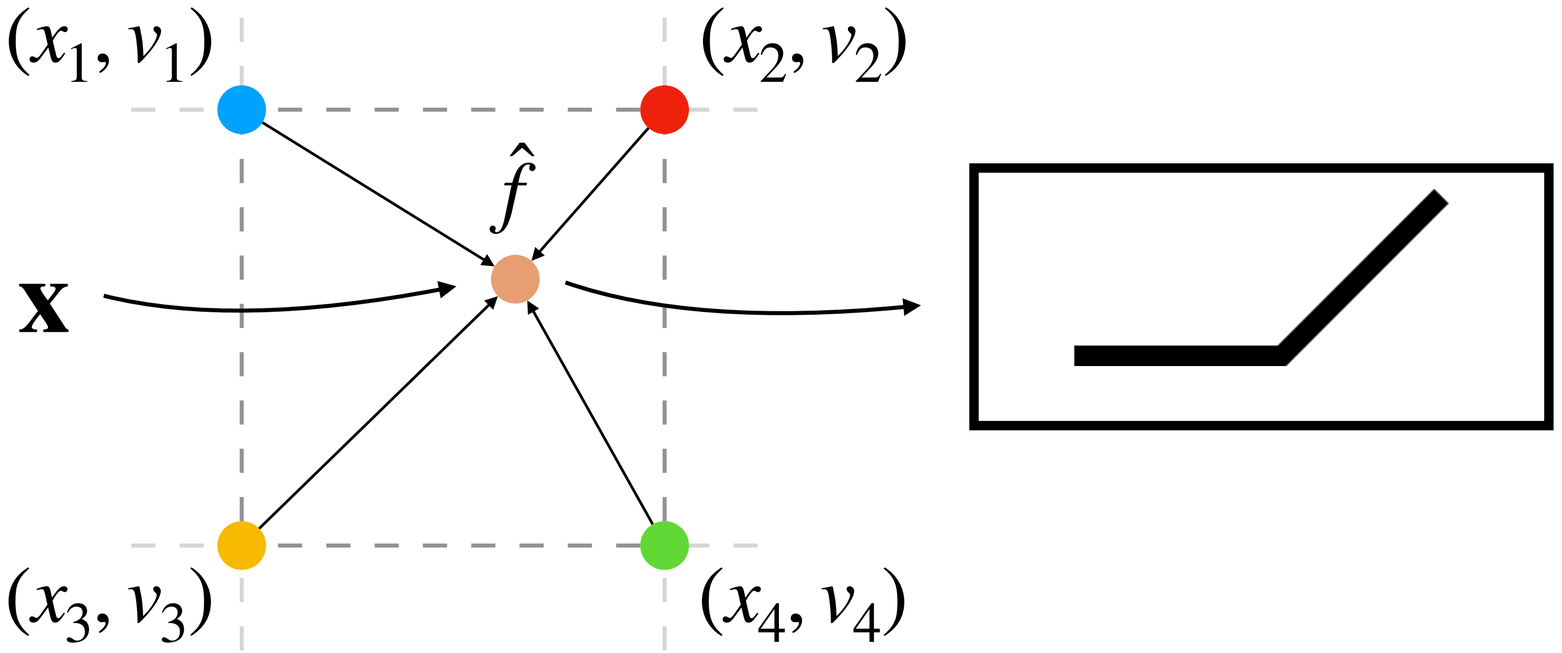
AirNet (Giebenhain et al. 2021)

ReLU Fields: How much MLP do you need?



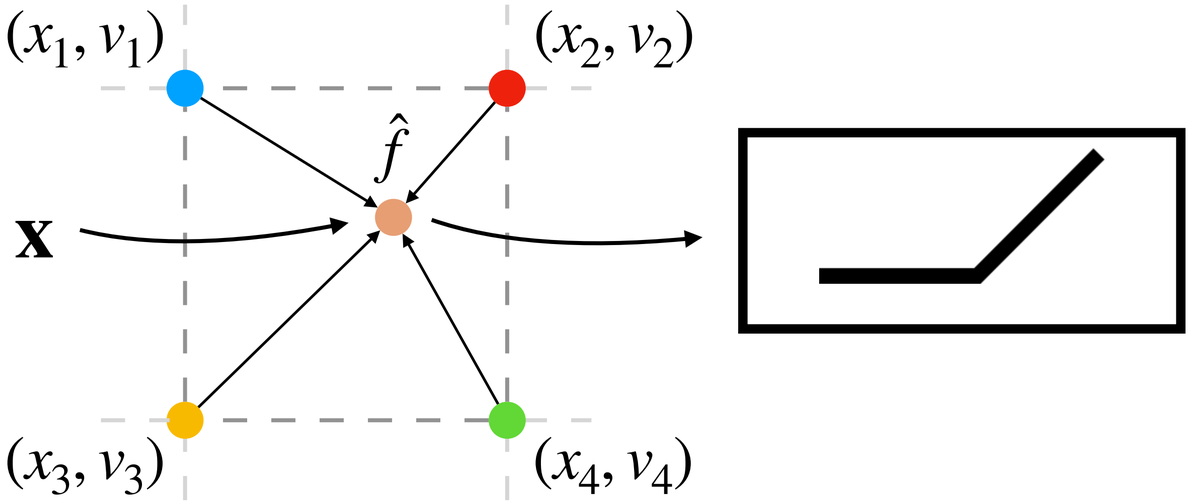
ReLU Fields: The Little Non-linearity That Could (Karnewar et al. 2022)

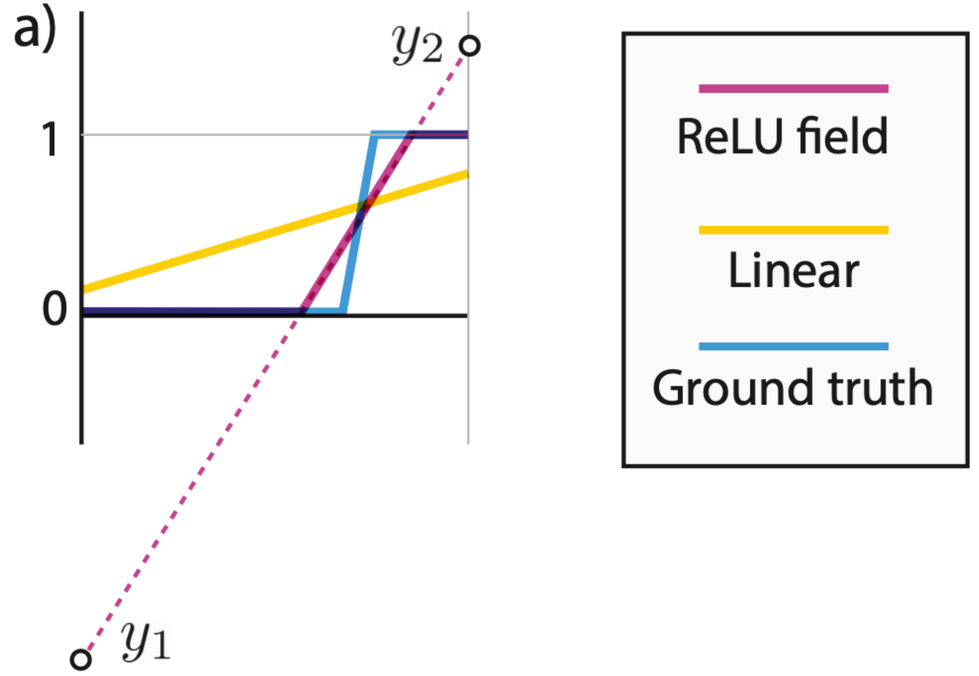
ReLU Fields: How much MLP do you need?



ReLU Fields: The Little Non-linearity That Could (Karnewar et al. 2022)

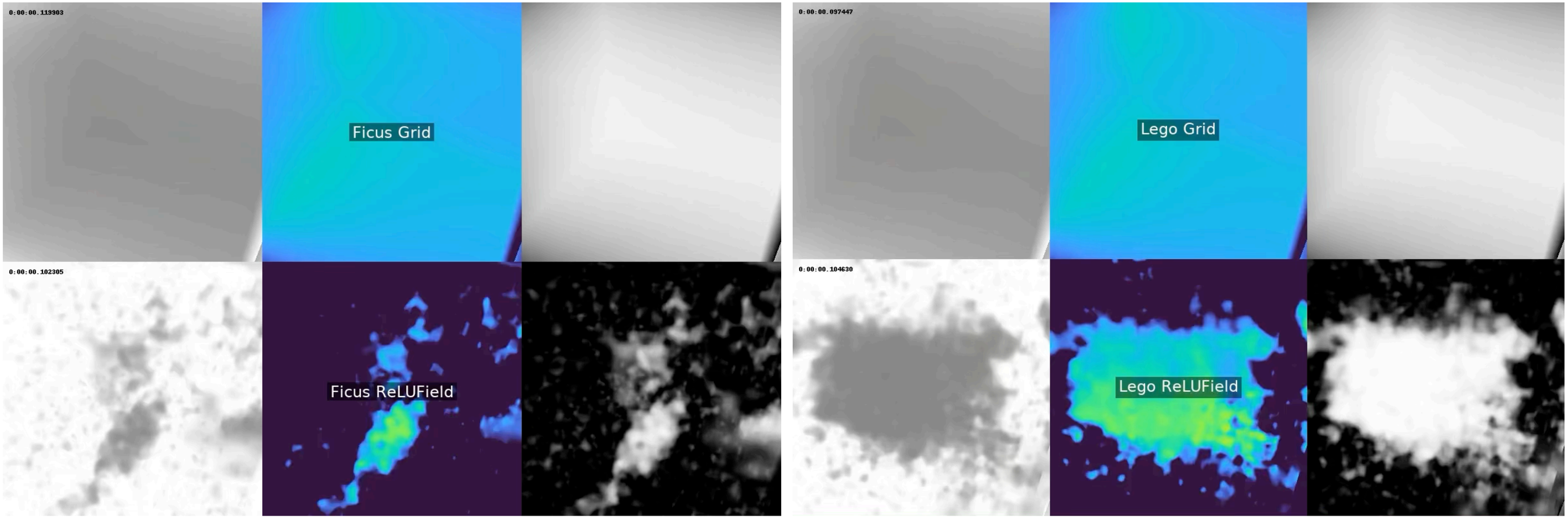
ReLU Fields: How much MLP do you need?





ReLU Fields: The Little Non-linearity That Could (Karnewar et al. 2022)

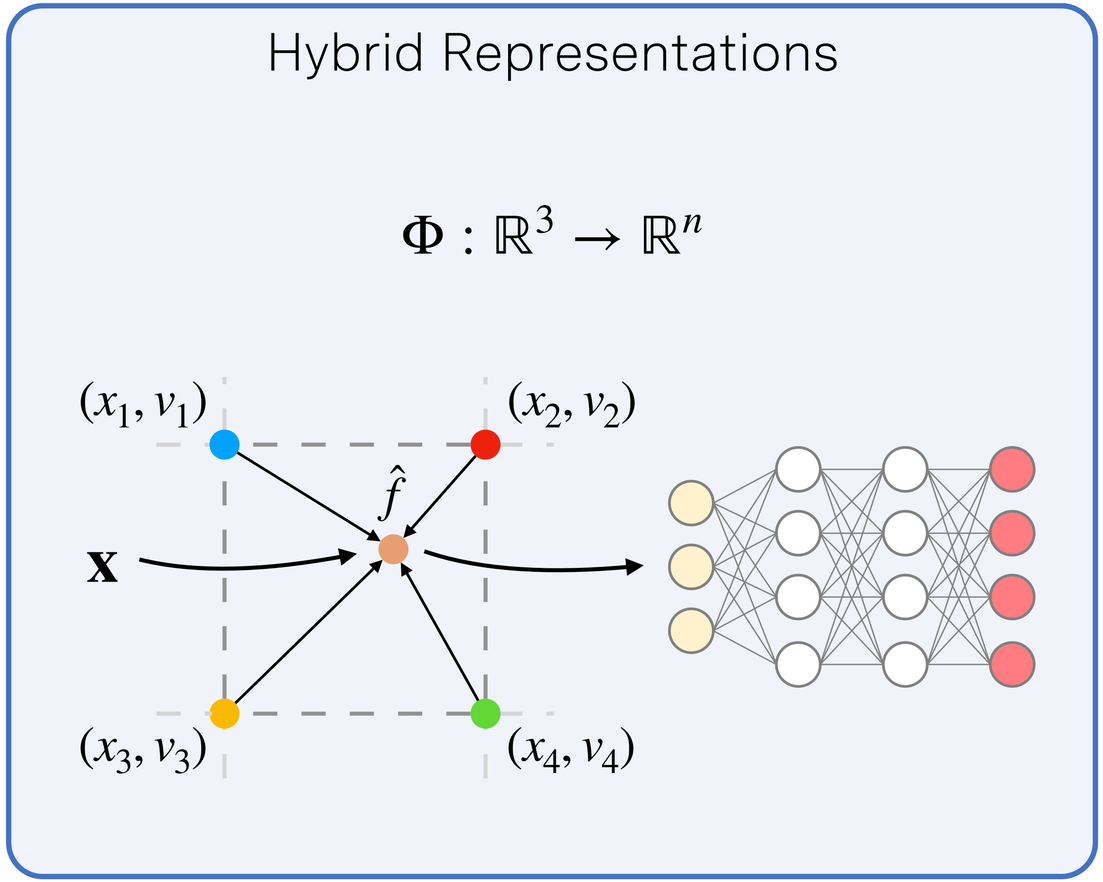
ReLU Fields: How much MLP do you need?



ReLU Fields: The Little Non-linearity That Could (Karnewar et al. 2022)

Hybrid Representations

- Very natural to trade off memory with compute.
- Neural network can locally “super resolve” discrete representation: Basically acts as interpolation kernel.
- Can be the best of both worlds: Fast and “continuous”.



How can we represent unbounded scenes?

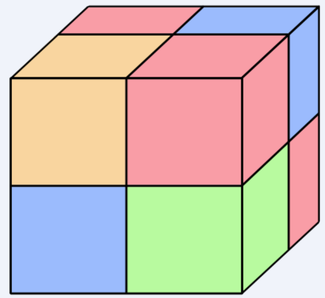
Image Source: Wikipedia

How can we represent unbounded scenes?

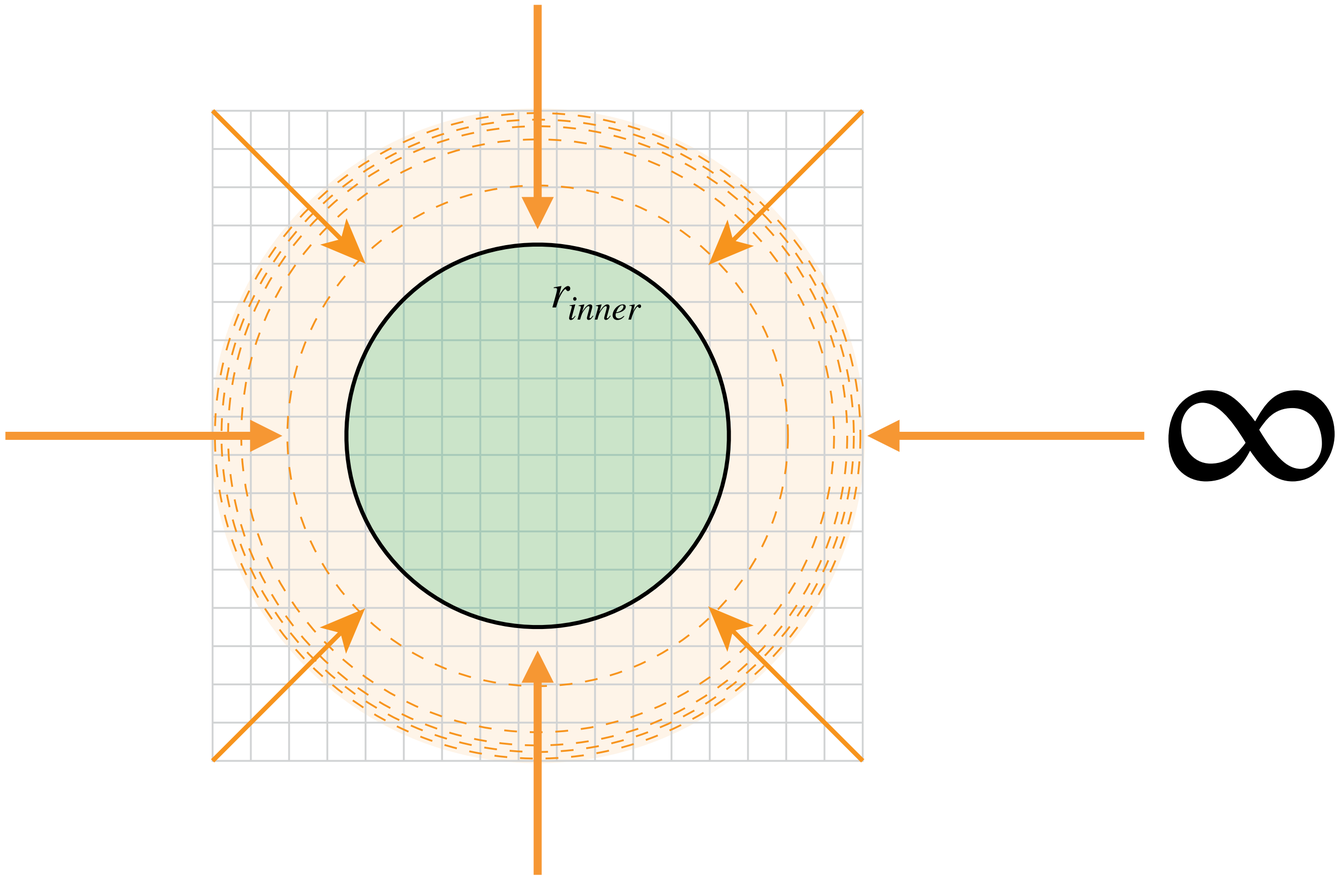
Image Source: Wikipedia

Scene Representation

$$\Phi : \mathbb{R}^3 \rightarrow \mathbb{R}^n$$



Parameterizing Unbounded Scenes: Contraction



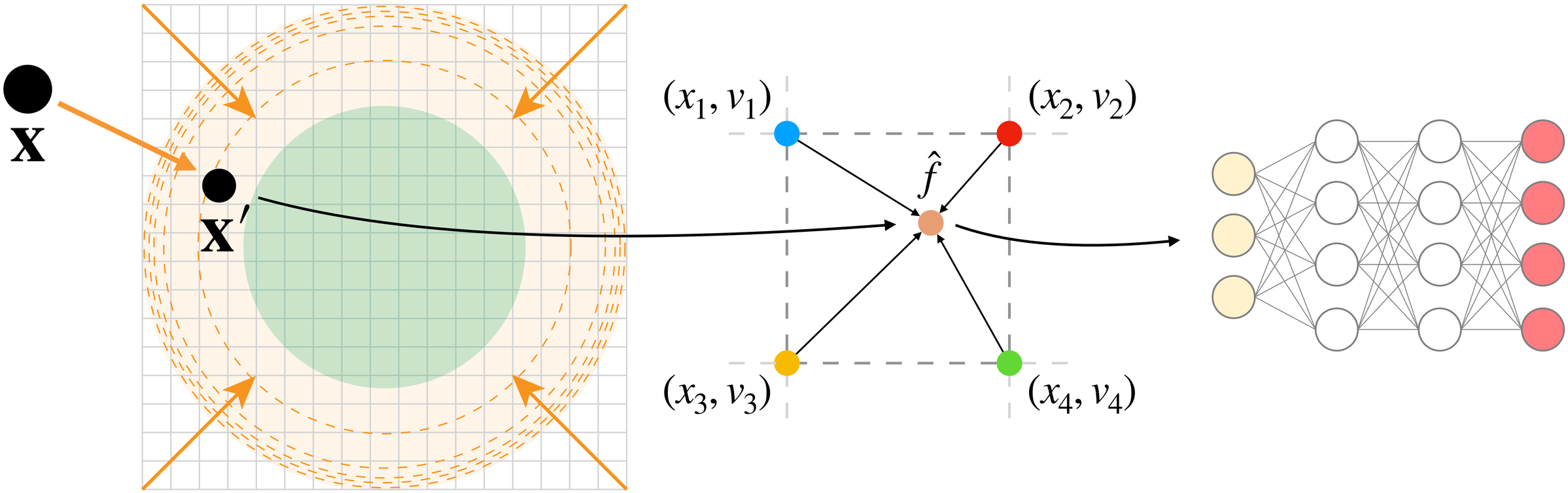
$$\mathbf{x}' = \frac{(1+k) - k/||\mathbf{u}||}{\mathbf{u}/||\mathbf{u}||} r_{inner}$$

$$\mathbf{u} = \frac{\mathbf{x}}{r_{inner}}$$

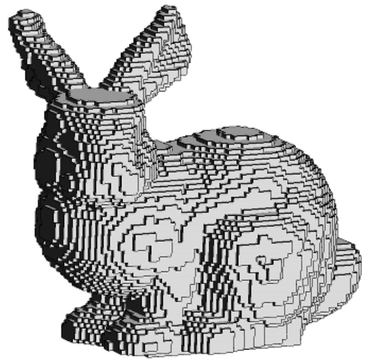
Example Compactification function from "Mip-NeRF 360: Unbounded Anti-Aliased Neural Radiance Fields", Barron et al., CVPR 2022

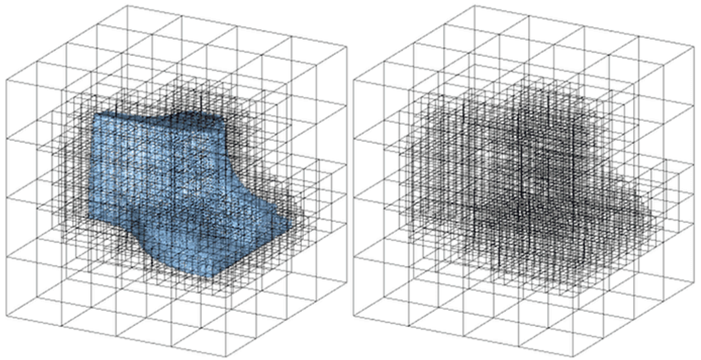
Come up with nonlinear mapping that "squashes" space, such that infinity is mapped to some **finite** value. Aligns with intuition "further away, less resolution".

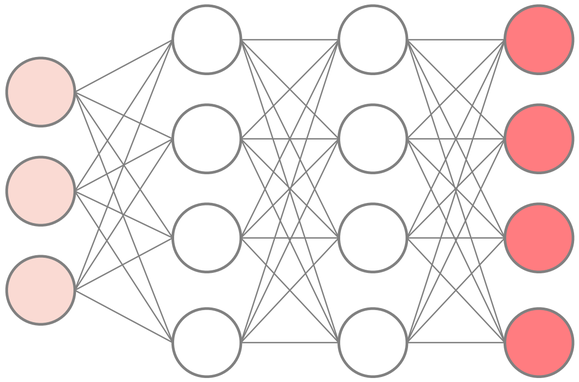
Parameterizing Unbounded Scenes: Contraction

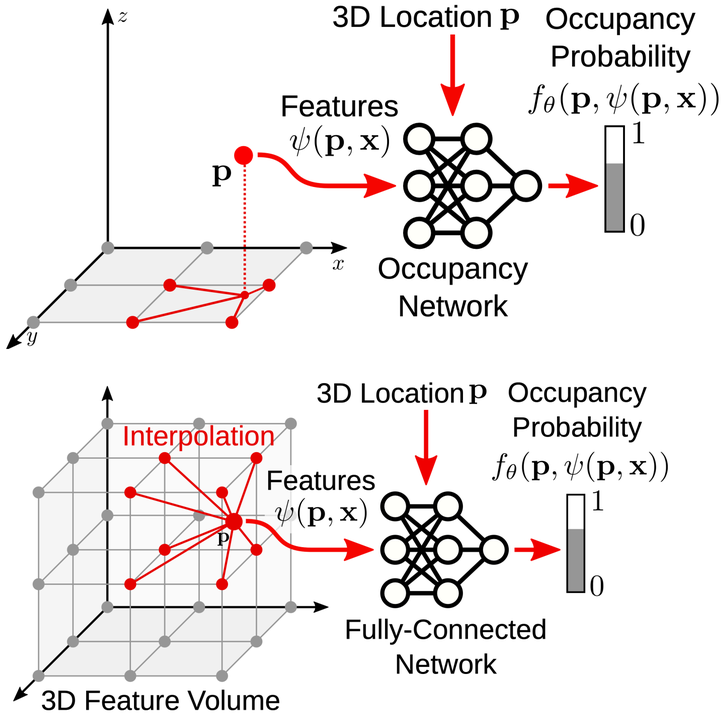


Summary

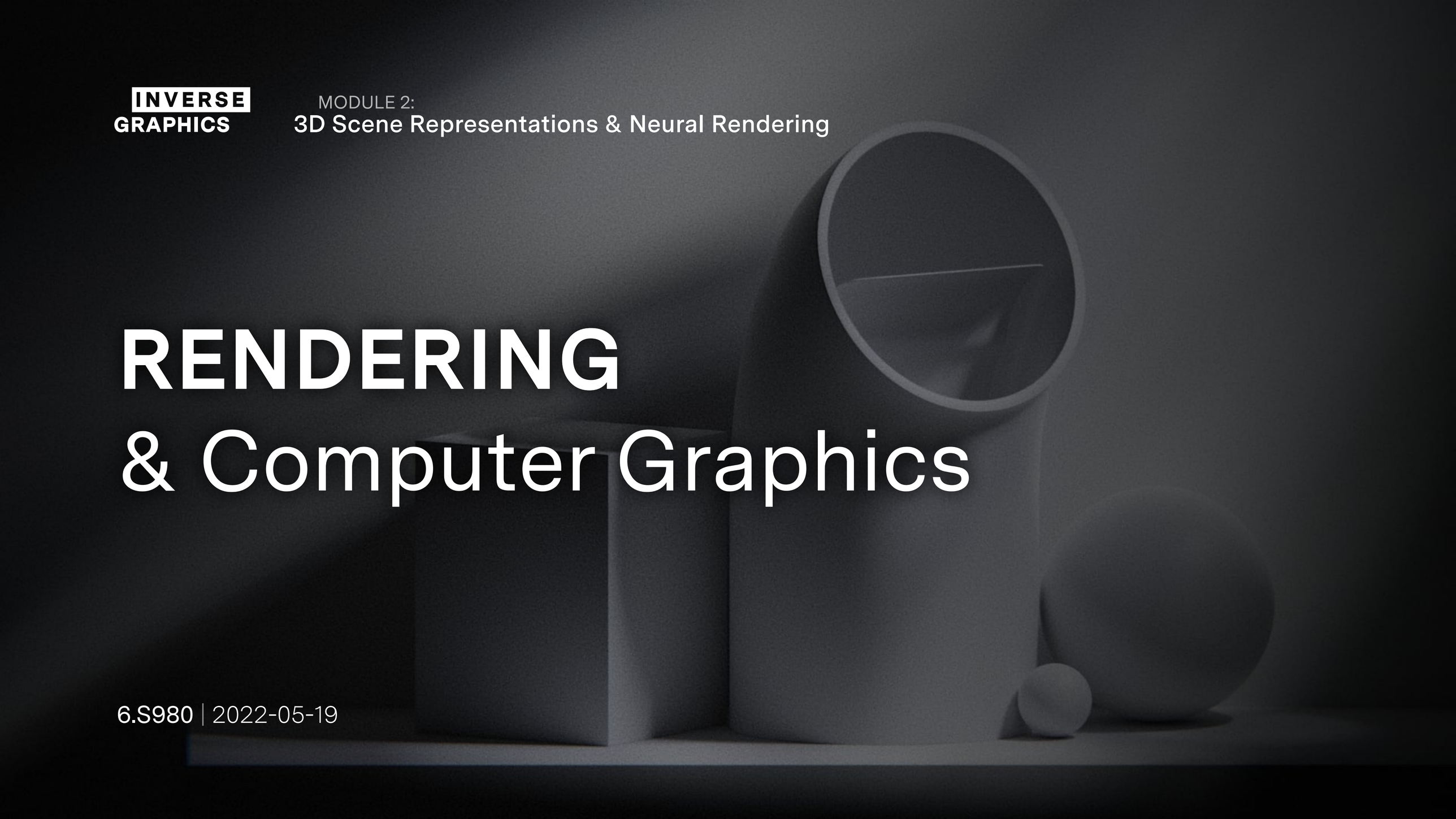








- No “Artificial Intelligence”, “understanding” or “Machine Learning” when fitting a single representation to a single 3D thing, even if there are neural networks in them.
- It becomes *a lot* more interesting when considering *inference,* i.e., we want representation to be output of an encoder! More on that later in “Prior-based reconstruction”.



INVERSE GRAPHICS

MODULE 2:
3D Scene Representations & Neural Rendering

RENDERING & Computer Graphics

6.S980 | 2022-05-19

3D Scene

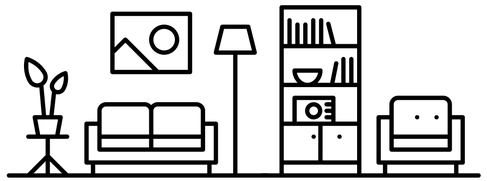


Image Formation

Inference

Neural Scene Representation

Downstream Tasks

Graphics

“inverse graphics”…?

Today: Light Transport & High-level of Physics-based Rendering

3D Scene

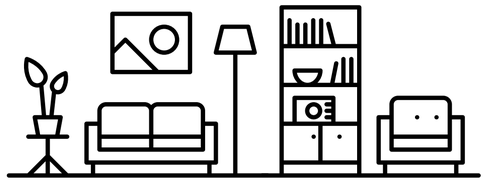


Image Formation

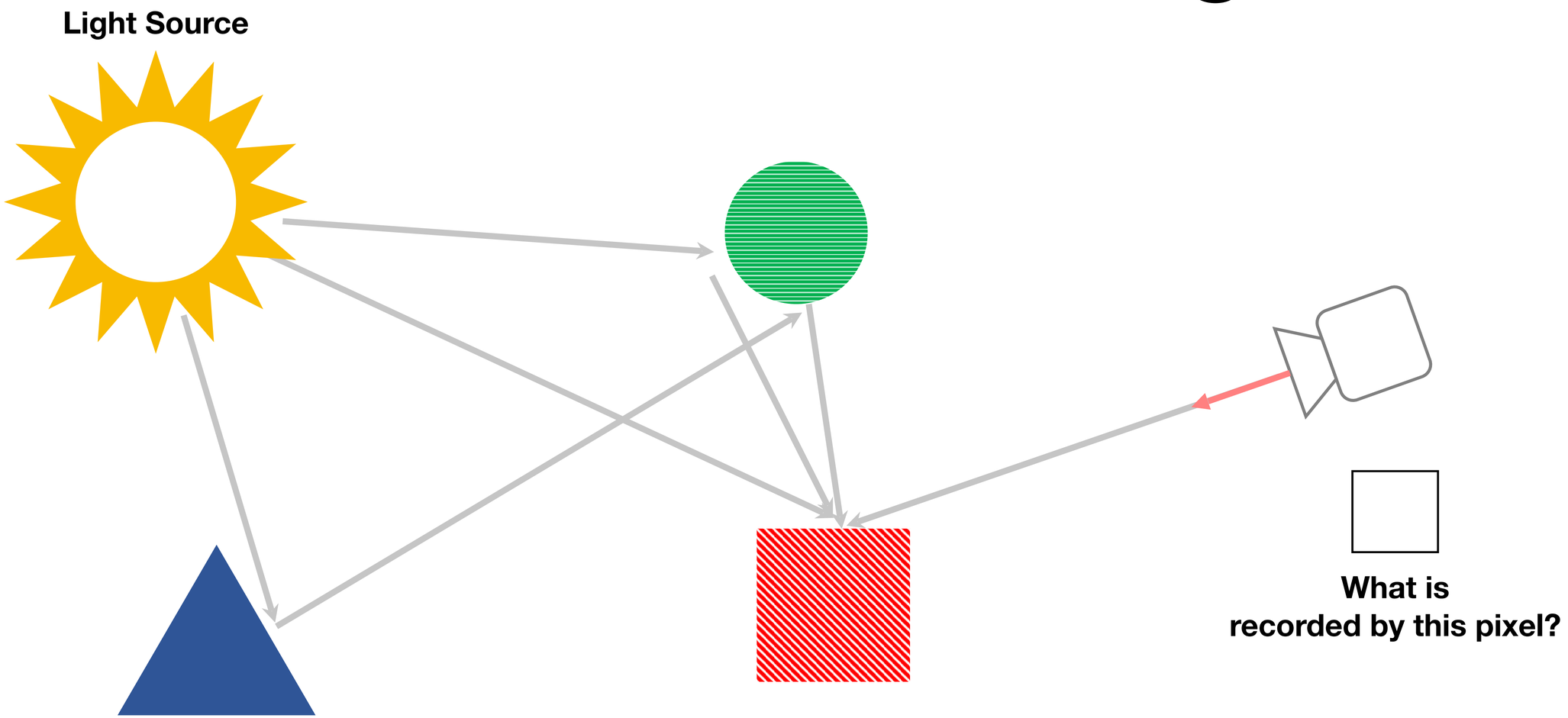
Inference

Neural Scene Representation

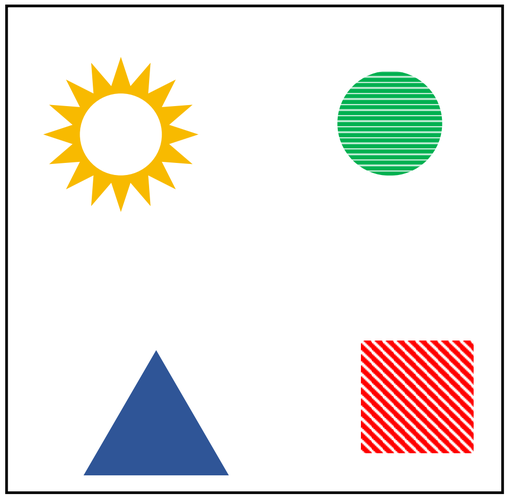
Downstream Tasks

Why?	In order to reason about the 3D world from images, we need to understand how 3D properties such as materials, lighting, etc. relate to the measurements observed by a camera.
What you'll learn.	The rendering equation, simulating light transport via multi-bounce ray-tracing, bidirectional radiance distribution functions, rasterization, rendering.

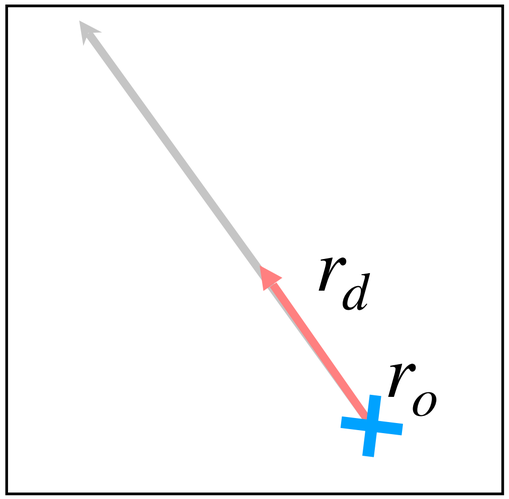
What is Rendering?



Function Signature of Rendering



Scene : Light sources, materials, shapes, …



Camera Ray

Radiance of the ray

Some Slides Adopted From

Stanford CS348I: Computer Graphics in the Era of AI

Profs. C. Karen Liu and Jiajun Wu

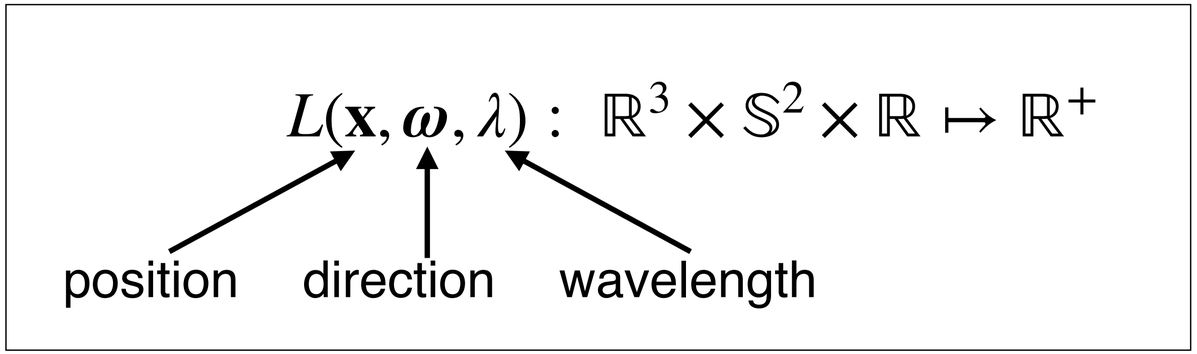
The Rendering equation

The moral of the story:

$$L_o(\mathbf{x}, \boldsymbol{\omega}_o, \lambda) = L_e(\mathbf{x}, \boldsymbol{\omega}_o, \lambda) + \int_\Omega f(\mathbf{x}, \boldsymbol{\omega}_i, \boldsymbol{\omega}_o, \lambda)\, L_i(\mathbf{x}, \boldsymbol{\omega}_i, \lambda)\, (\boldsymbol{\omega}_i \cdot \mathbf{n}) d\boldsymbol{\omega}_i$$

Radiance

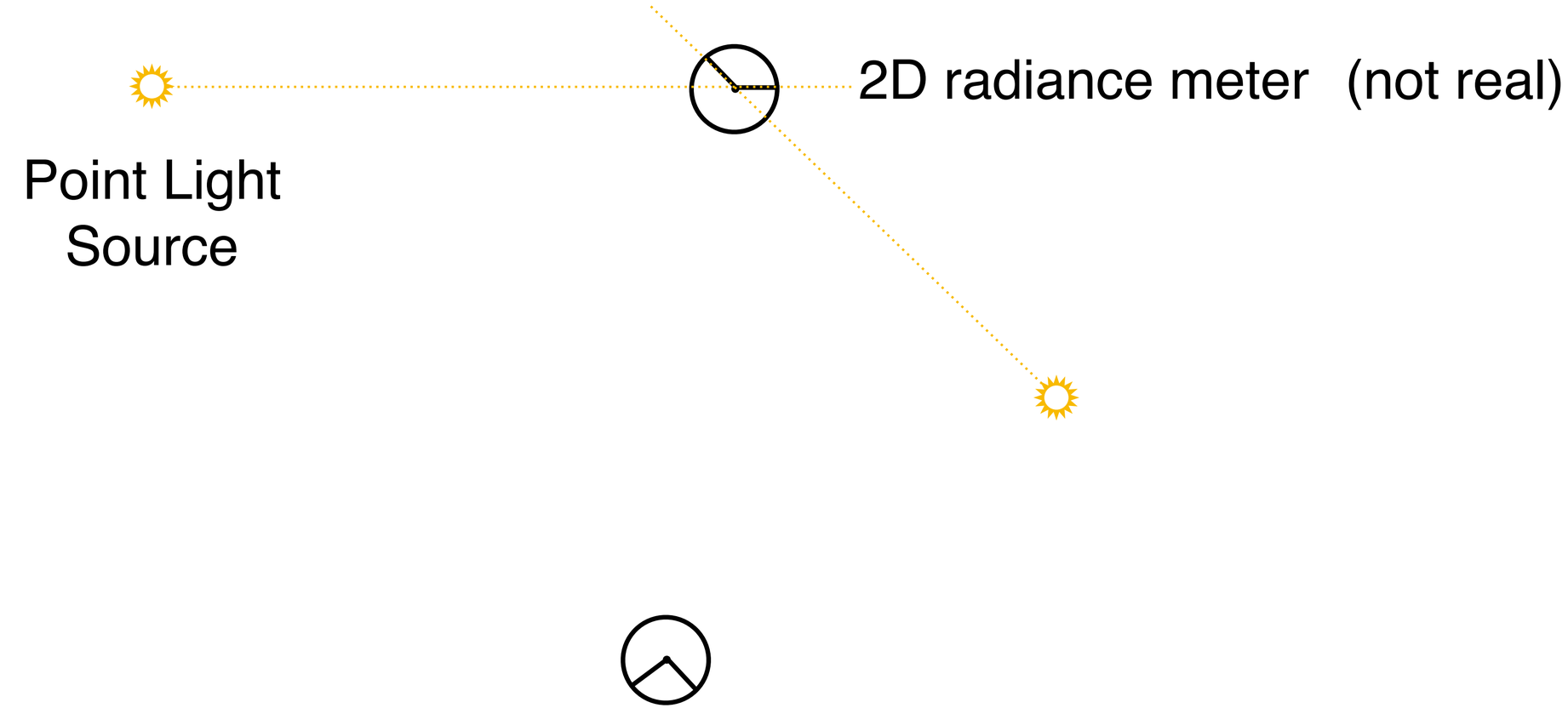
- Radiance is a fundamental quantity of light.
- Radiance is density of photons at a point, traveling in the same direction, at given wavelength.
- Radiance is energy per time, per unit area, per unit solid angle, per wavelength.



The “Light Field”: Radiance for every possible ray

Slide Credit: Stanford CS348I, Computer Graphics in the Age of AI, Stanford, C. Karen Liu & Jiajun Wu

Radiance intuition

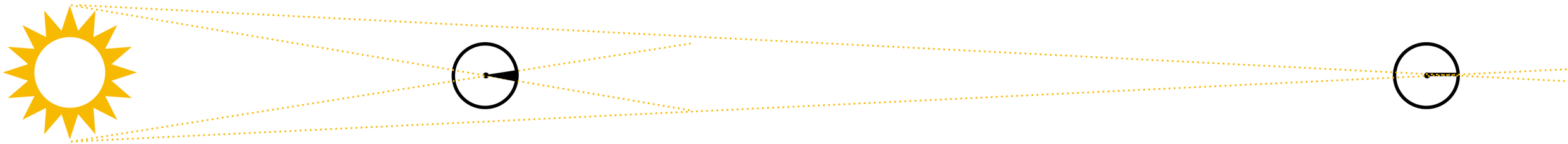


Slide Credit: Stanford CS348I, Computer Graphics in the Age of AI, Stanford, C. Karen Liu & Jiajun Wu

Radiance intuition

Slide Credit: Stanford CS348I, Computer Graphics in the Age of AI, Stanford, C. Karen Liu & Jiajun Wu

Radiance intuition



- Radiance properties
  - Radiance along an unblocked ray is constant.
  - Response of sensors is proportional to radiance of visible surface.
  - Radiance of reflected light is proportional to that of incoming light:

$$L(\mathbf{x}, \boldsymbol{\omega}_o, \lambda) \propto L(\mathbf{x}, \boldsymbol{\omega}_i, \lambda)$$

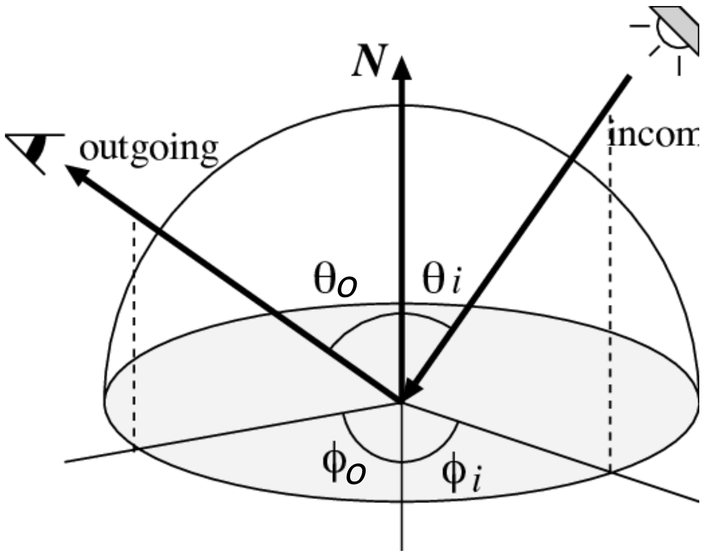
reflected incoming

Slide Credit: Stanford CS348I, Computer Graphics in the Age of AI, Stanford, C. Karen Liu & Jiajun Wu

BRDF

- Constant of proportionality defines Bidirectional Reflectance Distribution Function (BRDF).

$$f(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = \frac{\text{outgoing light in direction } \boldsymbol{\omega}_o}{\text{incoming light in direction } \boldsymbol{\omega}_i}$$



- BRDF describes how a material reflects light.

- Helmholtz reciprocity: $f(\boldsymbol{\omega}_1, \boldsymbol{\omega}_2) = f(\boldsymbol{\omega}_2, \boldsymbol{\omega}_1)$

$$\boldsymbol{\omega} = (\theta, \phi)$$

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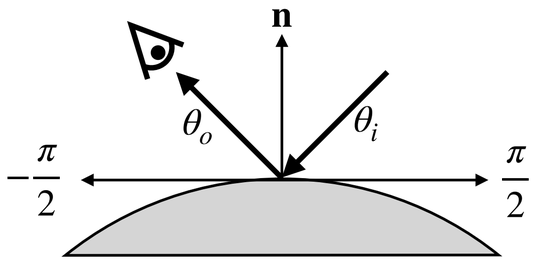
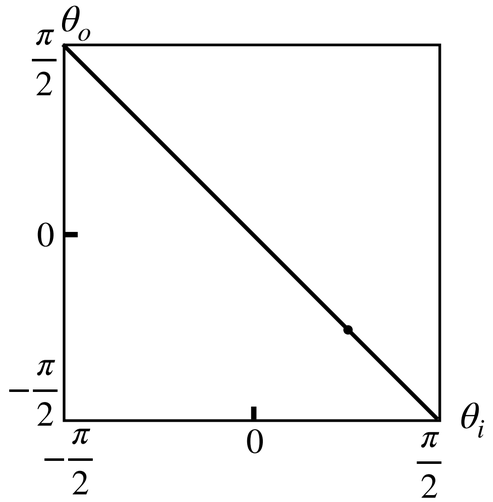
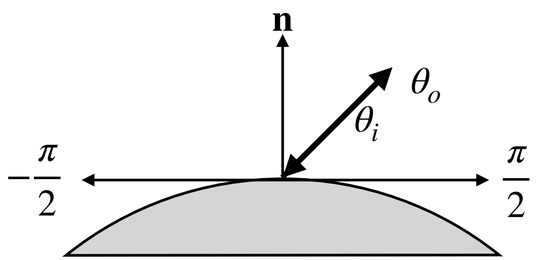
BRDF intuition

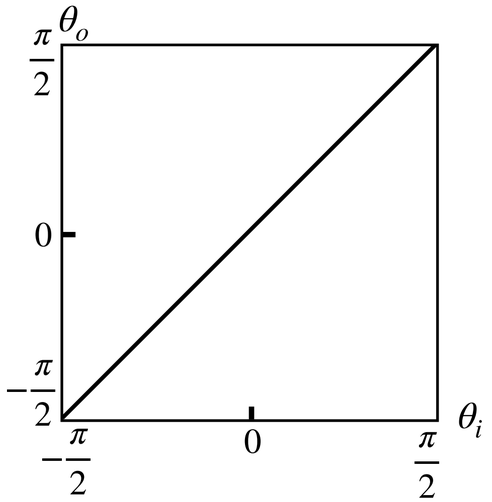
perfect mirror



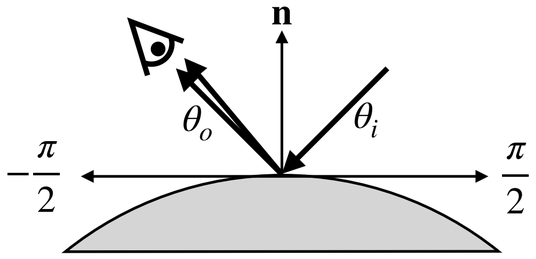


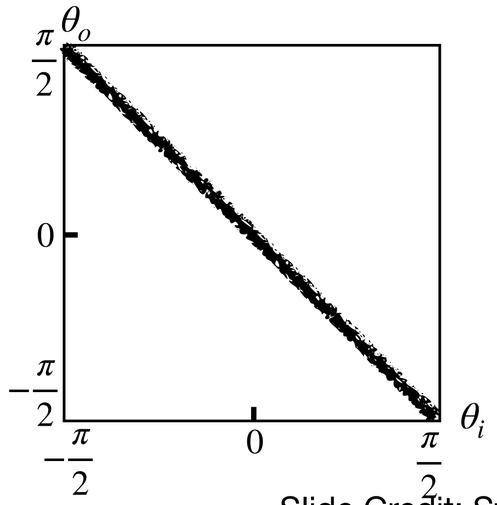
retro-reflector



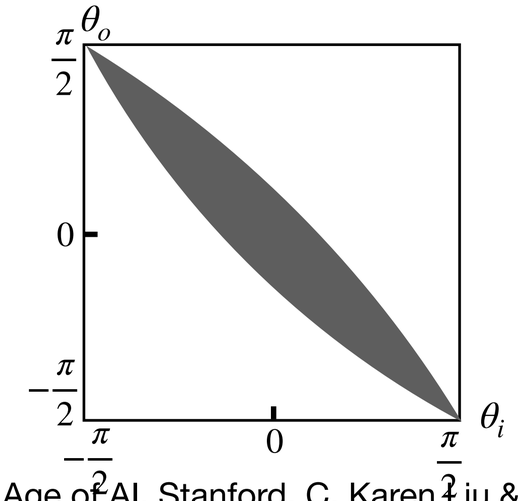


fairly shiny mirror





How does BRDF for paper look like?



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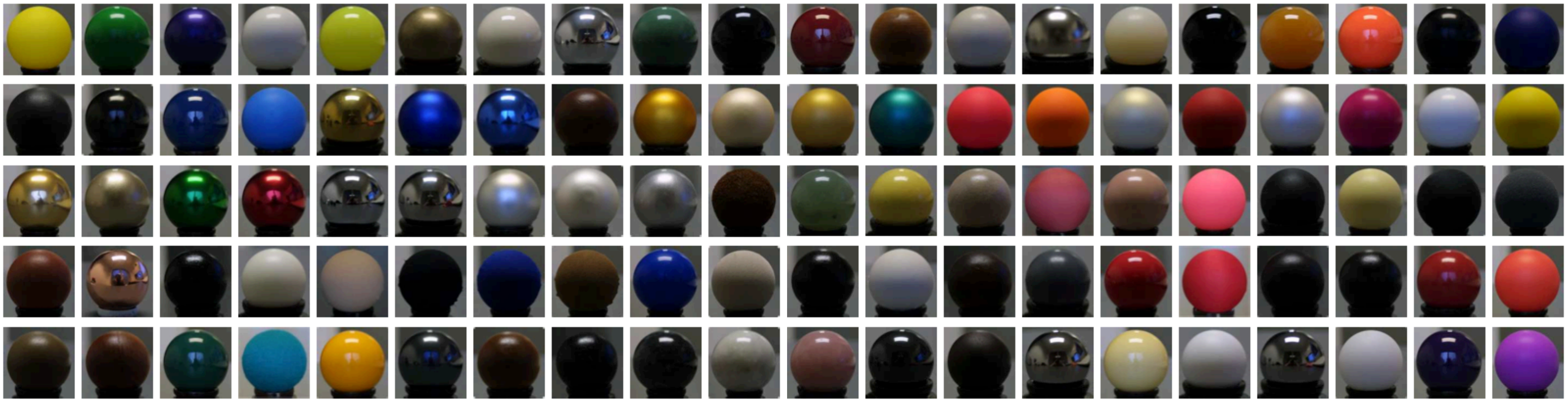
Different materials



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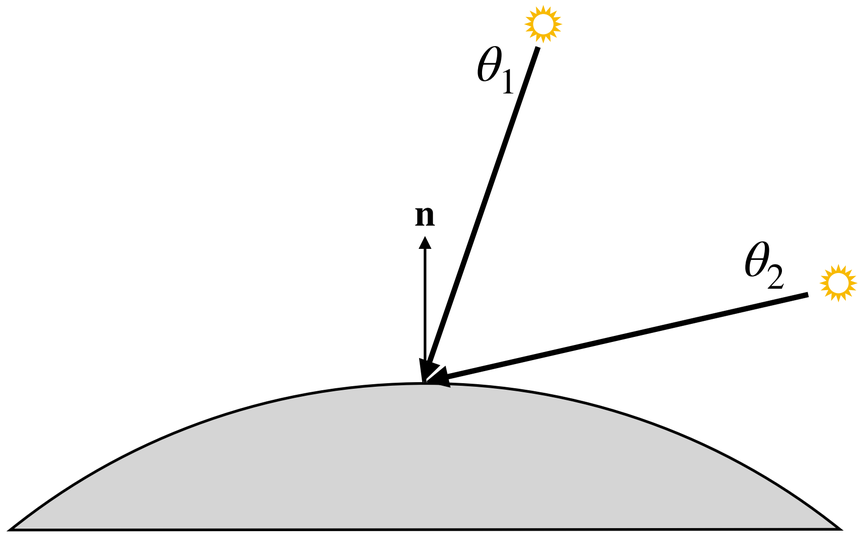
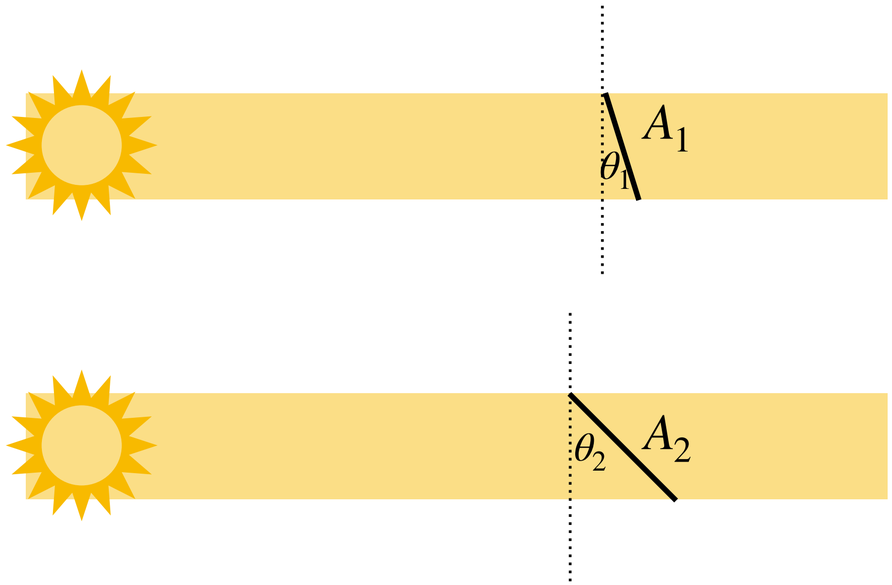
Angle Weight of Photon density

Which incoming light results in more photon density at the surface?





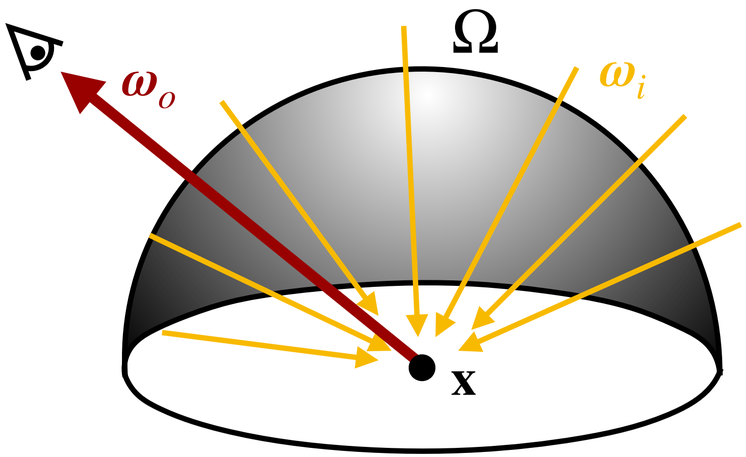
Slide Credit: Stanford CS348I, Computer Graphics in the Age of AI, Stanford, C. Karen Liu & Jiajun Wu

Rendering equation

$$L_o(\mathbf{x}, \boldsymbol{\omega}_o, \lambda) = L_e(\mathbf{x}, \boldsymbol{\omega}_o, \lambda) + \int_{\Omega} f(\mathbf{x}, \boldsymbol{\omega}_i, \boldsymbol{\omega}_o, \lambda)\, L_i(\mathbf{x}, \boldsymbol{\omega}_i, \lambda)\, (\boldsymbol{\omega}_i \cdot \mathbf{n}) d\boldsymbol{\omega}_i$$

- emitted radiance
- outgoing radiance in $\boldsymbol{\omega}_o$
- fraction of incoming light that is reflected into $\boldsymbol{\omega}_o$
- incoming radiance
- angle weight of photon density
- solid angle differential

Conservation of energy: $\int_{\boldsymbol{\omega}_i \in \Omega} f(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o)\, (\boldsymbol{\omega}_i \cdot \mathbf{n})\, d\boldsymbol{\omega}_i \leq 1$



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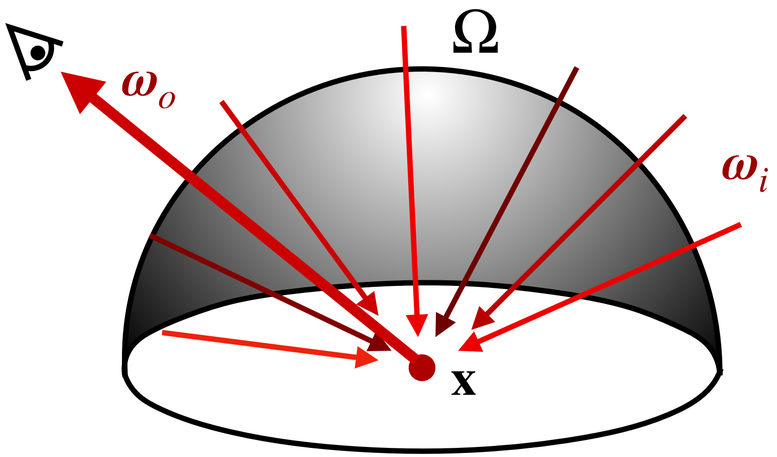
Colors

$$L_o(\mathbf{x}, \boldsymbol{\omega}_o, \lambda) = L_e(\mathbf{x}, \boldsymbol{\omega}_o, \lambda) + \int_\Omega f(\mathbf{x}, \boldsymbol{\omega}_i, \boldsymbol{\omega}_o, \lambda)\, L_i(\mathbf{x}, \boldsymbol{\omega}_i, \lambda)\, (\boldsymbol{\omega}_i \cdot \mathbf{n}) d\boldsymbol{\omega}_i$$

$$L_o(\mathbf{x}, \boldsymbol{\omega}_o, \lambda^R) = L_e(\mathbf{x}, \boldsymbol{\omega}_o, \lambda^R) + \int_\Omega f(\mathbf{x}, \boldsymbol{\omega}_i, \boldsymbol{\omega}_o, \lambda^R)\, L_i(\mathbf{x}, \boldsymbol{\omega}_i, \lambda^R)\, (\boldsymbol{\omega}_i \cdot \mathbf{n}) d\boldsymbol{\omega}_i$$



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Colors

$$L_o(\mathbf{x}, \boldsymbol{\omega}_o, \lambda) = L_e(\mathbf{x}, \boldsymbol{\omega}_o, \lambda) + \int_\Omega f(\mathbf{x}, \boldsymbol{\omega}_i, \boldsymbol{\omega}_o, \lambda)\, L_i(\mathbf{x}, \boldsymbol{\omega}_i, \lambda)\, (\boldsymbol{\omega}_i \cdot \mathbf{n}) d\boldsymbol{\omega}_i$$

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$$L_o(\mathbf{x}, \boldsymbol{\omega}_o, \lambda^G) = L_e(\mathbf{x}, \boldsymbol{\omega}_o, \lambda^G) + \int_\Omega f(\mathbf{x}, \boldsymbol{\omega}_i, \boldsymbol{\omega}_o, \lambda^G)\, L_i(\mathbf{x}, \boldsymbol{\omega}_i, \lambda^G)\, (\boldsymbol{\omega}_i \cdot \mathbf{n}) d\boldsymbol{\omega}_i$$

$$L_o(\mathbf{x}, \boldsymbol{\omega}_o, \lambda^B) = L_e(\mathbf{x}, \boldsymbol{\omega}_o, \lambda^B) + \int_\Omega f(\mathbf{x}, \boldsymbol{\omega}_i, \boldsymbol{\omega}_o, \lambda^B)\, L_i(\mathbf{x}, \boldsymbol{\omega}_i, \lambda^B)\, (\boldsymbol{\omega}_i \cdot \mathbf{n}) d\boldsymbol{\omega}_i$$

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Colors

$$L_o(\mathbf{x}, \boldsymbol{\omega}_o, \lambda) = L_e(\mathbf{x}, \boldsymbol{\omega}_o, \lambda) + \int_\Omega f(\mathbf{x}, \boldsymbol{\omega}_i, \boldsymbol{\omega}_o, \lambda)\, L_i(\mathbf{x}, \boldsymbol{\omega}_i, \lambda)\, (\boldsymbol{\omega}_i \cdot \mathbf{n}) d\boldsymbol{\omega}_i$$

$$L(\mathbf{x}, \boldsymbol{\omega}) : \ \mathbb{R}^3 \times \mathbb{S}^2 \mapsto \mathbb{R}^3$$

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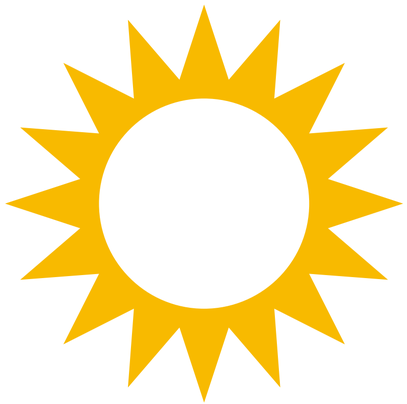
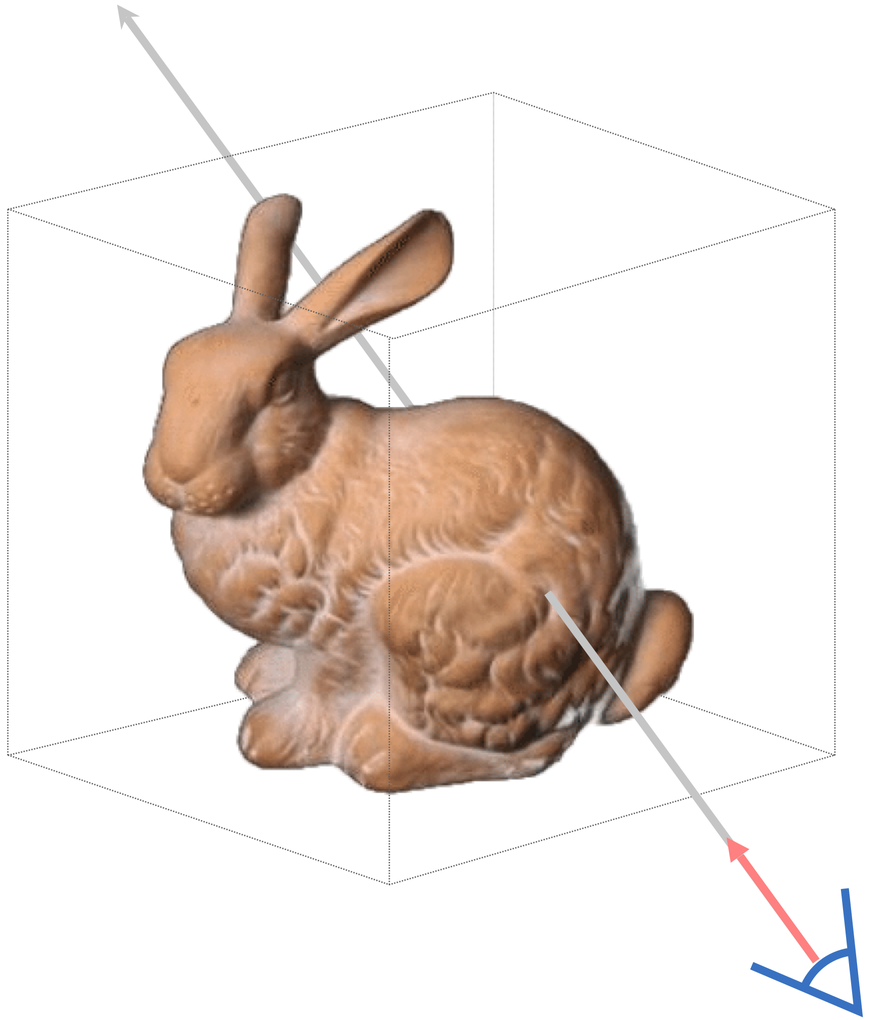
Ok, sure, but how do we actually render our bunny?





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Rendering equation

Rendering equation: $L_o(\mathbf{x}, \boldsymbol{\omega}_o, \lambda) = L_e(\mathbf{x}, \boldsymbol{\omega}_o, \lambda) + \int_{\Omega} f(\mathbf{x}, \boldsymbol{\omega}_i, \boldsymbol{\omega}_o, \lambda)\ L_i(\mathbf{x}, \boldsymbol{\omega}_i, \lambda)\ (\boldsymbol{\omega}_i \cdot \mathbf{n}) d\boldsymbol{\omega}_i$

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Shading

$$L_o(\mathbf{x}, \boldsymbol{\omega}_o, \lambda) = L_e(\mathbf{x}, \boldsymbol{\omega}_o, \lambda) + \int_{\Omega} f(\mathbf{x}, \boldsymbol{\omega}_i, \boldsymbol{\omega}_o, \lambda)\, L_i(\mathbf{x}, \boldsymbol{\omega}_i, \lambda)\, (\boldsymbol{\omega}_i \cdot \mathbf{n}) d\boldsymbol{\omega}_i$$

reflectance property

light source

angle weight

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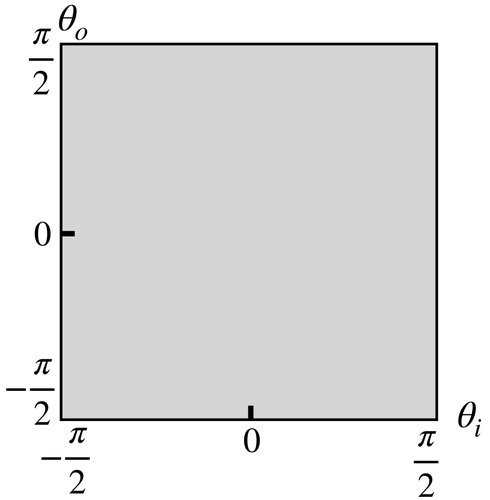
Lambertian surfaces

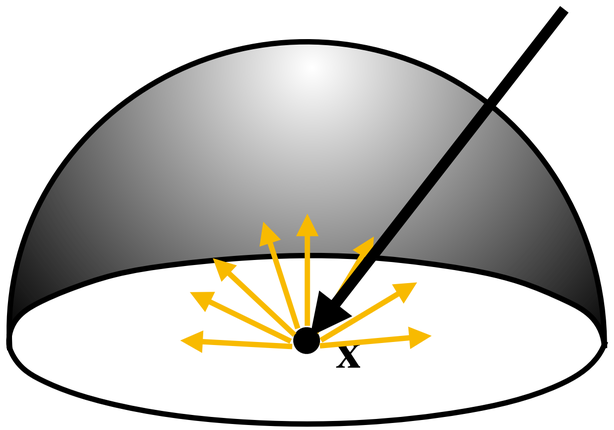
$$L_o(\mathbf{x}, \boldsymbol{\omega}_o, \lambda) = L_e(\mathbf{x}, \boldsymbol{\omega}_o, \lambda) + \int_\Omega f(\mathbf{x}, \boldsymbol{\omega}_i, \boldsymbol{\omega}_o, \lambda)\, L_i(\mathbf{x}, \boldsymbol{\omega}_i, \lambda)\, (\boldsymbol{\omega}_i \cdot \mathbf{n}) d\boldsymbol{\omega}_i$$

reflectance property — light source — angle weight

What does the BRDF look like for Lambertian surfaces?

$$f(\mathbf{x}, \cdot, \cdot, \lambda) = \text{constant} = C_{surf}$$





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Lambertian surfaces

$$L_o(\mathbf{x}, \boldsymbol{\omega}_o, \lambda) = L_e(\mathbf{x}, \boldsymbol{\omega}_o, \lambda) + \int_\Omega f(\mathbf{x}, \boldsymbol{\omega}_i, \boldsymbol{\omega}_o, \lambda)\ L_i(\mathbf{x}, \boldsymbol{\omega}_i, \lambda)\ (\boldsymbol{\omega}_i \cdot \mathbf{n}) d\boldsymbol{\omega}_i$$

reflectance property — light source — angle weight

What does the BRDF look like for

Assumption: Lambertian surfaces

$$f(\mathbf{x}, \cdot, \cdot, \lambda) = \text{constant} = C_{surf}$$

$$L_o(\mathbf{x}, \boldsymbol{\omega}_o, \lambda) = L_e(\mathbf{x}, \boldsymbol{\omega}_o, \lambda) + C_{surf} \int_\Omega L_i(\mathbf{x}, \boldsymbol{\omega}_i, \lambda)\ (\boldsymbol{\omega}_i \cdot \mathbf{n}) d\boldsymbol{\omega}_i$$

Assumption: Direct lights only

$$L_o(\mathbf{x}, \boldsymbol{\omega}_o, \lambda) = L_e(\mathbf{x}, \boldsymbol{\omega}_o, \lambda) + C_{surf} \sum_i L_i(\mathbf{x}, \boldsymbol{\omega}_i, \lambda)\ (\boldsymbol{\omega}_i \cdot \mathbf{n})$$

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Diffuse light

C_{surf} and C_{l_i} are 3x1 vectors representing RGB colors

$\circ$ is elementwise multiplication

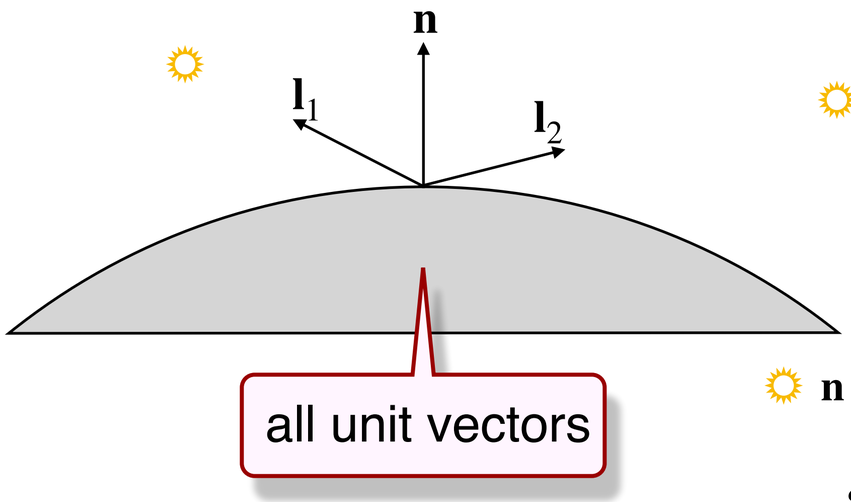
$$C = C_{surf} \circ \sum_{i=1}^{N} C_{l_i} \max(0, \mathbf{n} \cdot \mathbf{l}_i)$$

surface color

light color

Light locations matter,

but eye location doesn't!



$\mathbf{n} \cdot \mathbf{l} < 0$

What's the perceived color if we shine a blue light on a red diffuse surface?

1. purple
2. red
3. black

$$L_o(\mathbf{x}, \boldsymbol{\omega}_o, \lambda) = L_e(\mathbf{x}, \boldsymbol{\omega}_o, \lambda) + C_{surf} \sum_i L_i(\mathbf{x}, \boldsymbol{\omega}_i, \lambda)\ (\boldsymbol{\omega}_i \cdot \mathbf{n})$$

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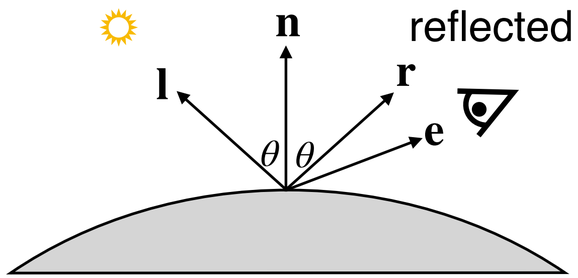
Ambient light and specular light

$$C = C_{surf} \circ C_{ambi} + C_{surf} \circ \sum_{i=1}^{N} C_{l_i} \max(0, \mathbf{n} \cdot \mathbf{l}_i) + \sum_{i=1}^{N} C_{l_i} \max(0, \mathbf{e} \cdot \mathbf{r}_i)^p$$

ambient light

specular light



$(\mathbf{r} \cdot \mathbf{e})^p$

Assumption: Lambertian surfaces

Assumption: Direct lights only

Ambient term: approximates the average color of all surfaces in the scene to make up for indirect lights from every direction.

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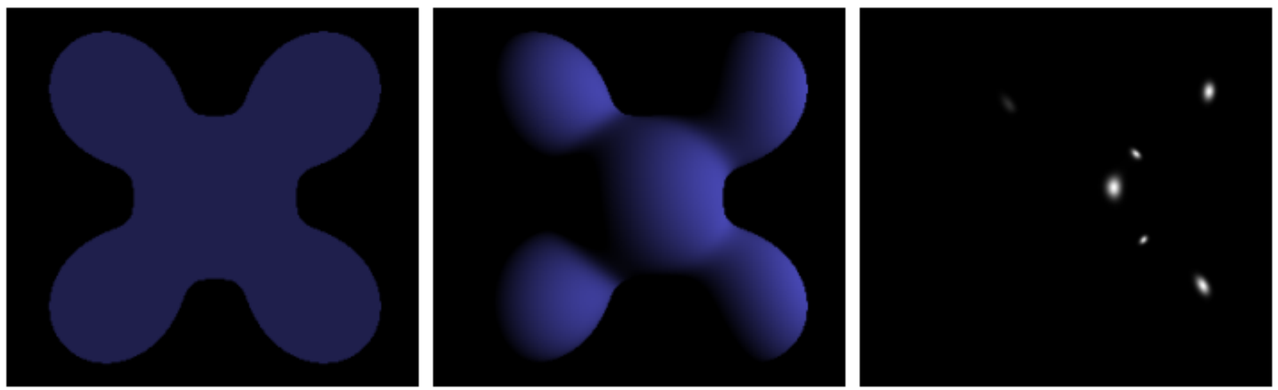
Phong illumination model

$$C = C_{surf} \circ C_{ambi} + C_{surf} \circ \sum_{i=1}^{N} C_{l_i} \max(0, \mathbf{n} \cdot \mathbf{l}_i) + \sum_{i=1}^{N} C_{l_i} \max(0, \mathbf{e} \cdot \mathbf{r}_i)^p$$



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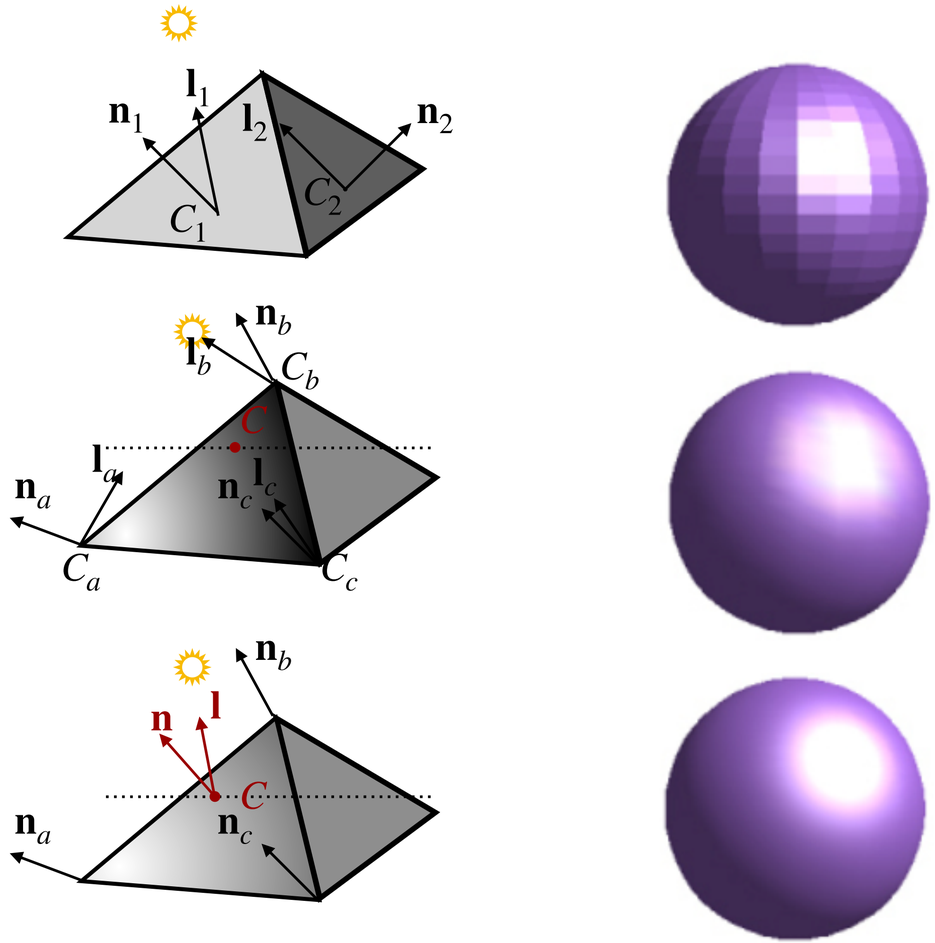
Apply shading

- Where to apply shading equation?
  - Per polygon (Flat Shading): one normal and one light vector per triangle.
  - Per vertex (Gouraud Interpolation): compute color at vertices of meshes and interpolate color during **rasterization**.
  - Per pixel (Phong interpolation): interpolate normals of vertices during rasterization and compute color for each interpolated normal.

The process of computing the mapping from scene geometry to pixels



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Rendering equation

photorealism

Rendering equation: $L_o(\mathbf{x}, \boldsymbol{\omega}_o, \lambda) = L_e(\mathbf{x}, \boldsymbol{\omega}_o, \lambda) + \int_{\Omega} f(\mathbf{x}, \boldsymbol{\omega}_i, \boldsymbol{\omega}_o, \lambda)\ L_i(\mathbf{x}, \boldsymbol{\omega}_i, \lambda)\ (\boldsymbol{\omega}_i \cdot \mathbf{n}) d\boldsymbol{\omega}_i$

Can’t do shadows, refraction, indirect light and inter-reflections…

Phong model with **rasterization**: $C = C_{surf} \circ C_{ambi} + C_{surf} \circ \sum_{i=1}^{N} C_{l_i} \max(0, \mathbf{n} \cdot \mathbf{l}_i) + \sum_{i=1}^{N} C_{l_i} \max(0, \mathbf{e} \cdot \mathbf{r}_i)^p$

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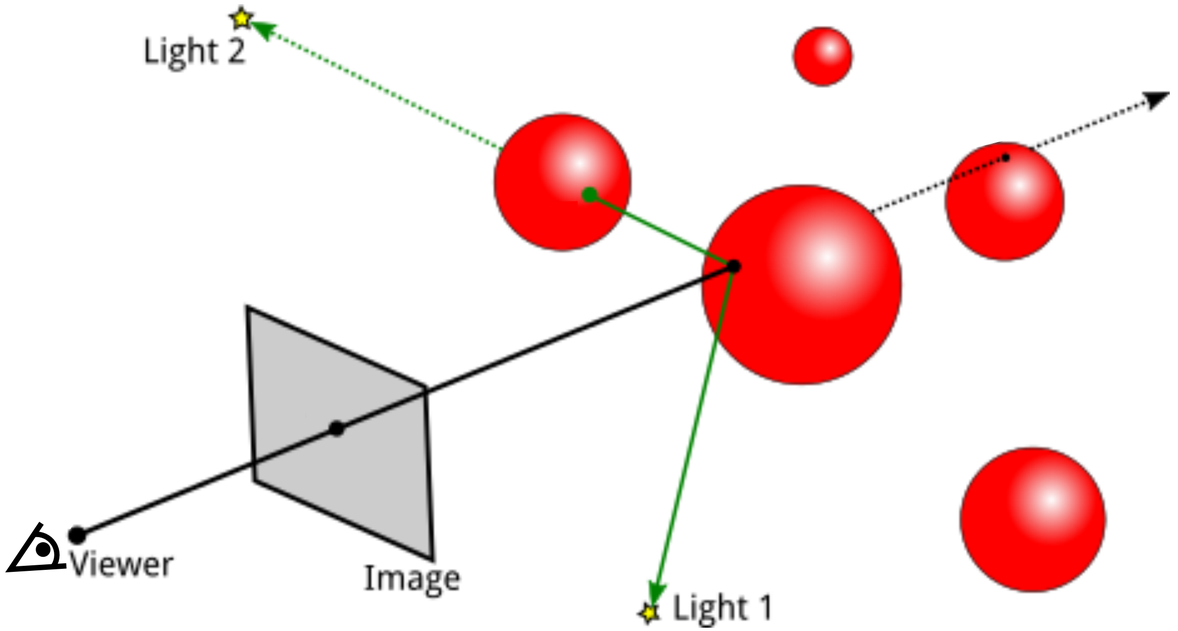
Ray tracing



for each pixel (x, y)

create ray R from eye through (x, y)

for each object O in scene

if R intersects O and is closest so far

record this intersection

shade pixel (x, y) based on nearest intersection

image: David Eck

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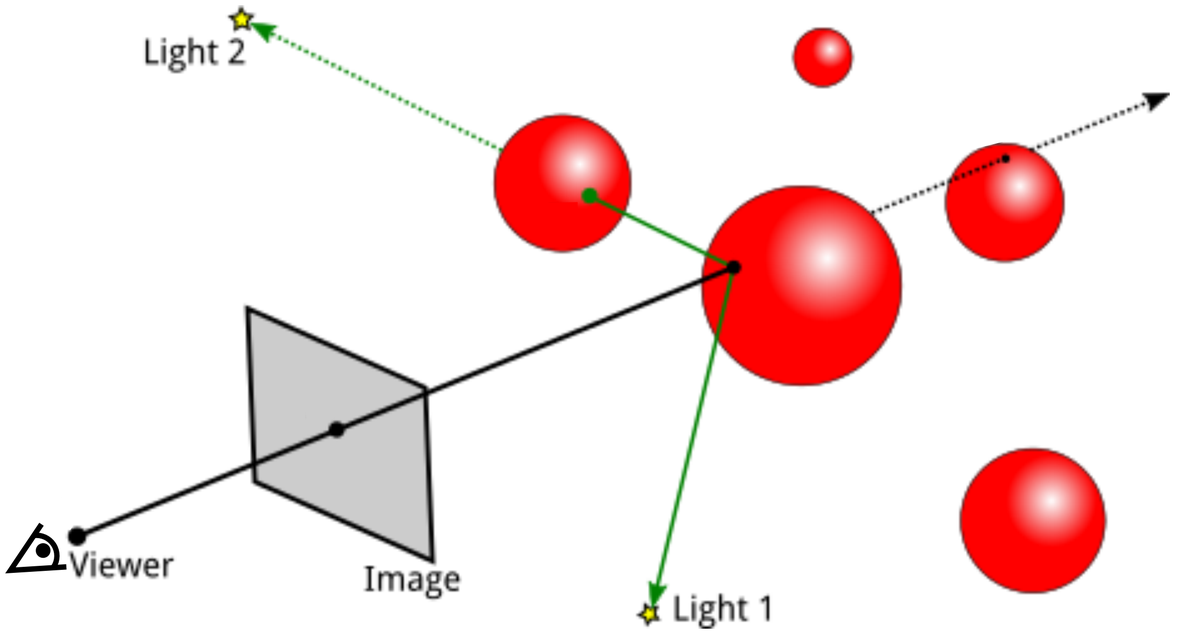
Ray tracing



for each pixel (x, y)

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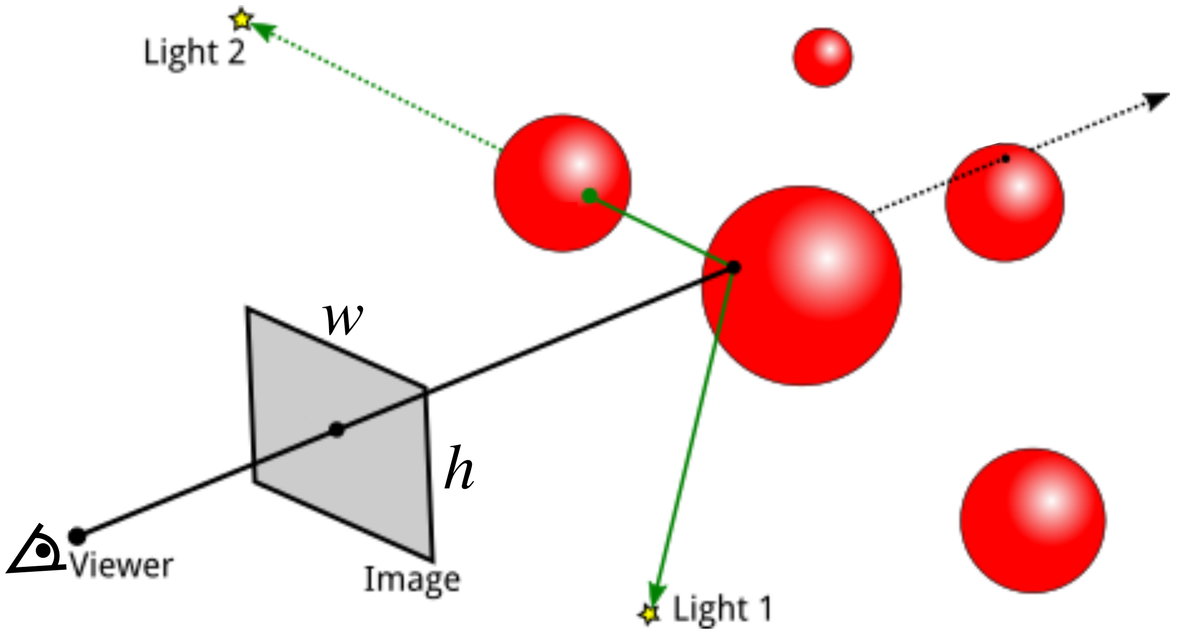
Create an eye ray



$$\mathbf{X}(d) = d * \mathbf{R}_1^{C2W}\mathbf{K}_1^{-1}\tilde{\mathbf{x}}_1 + \mathbf{O}_1$$

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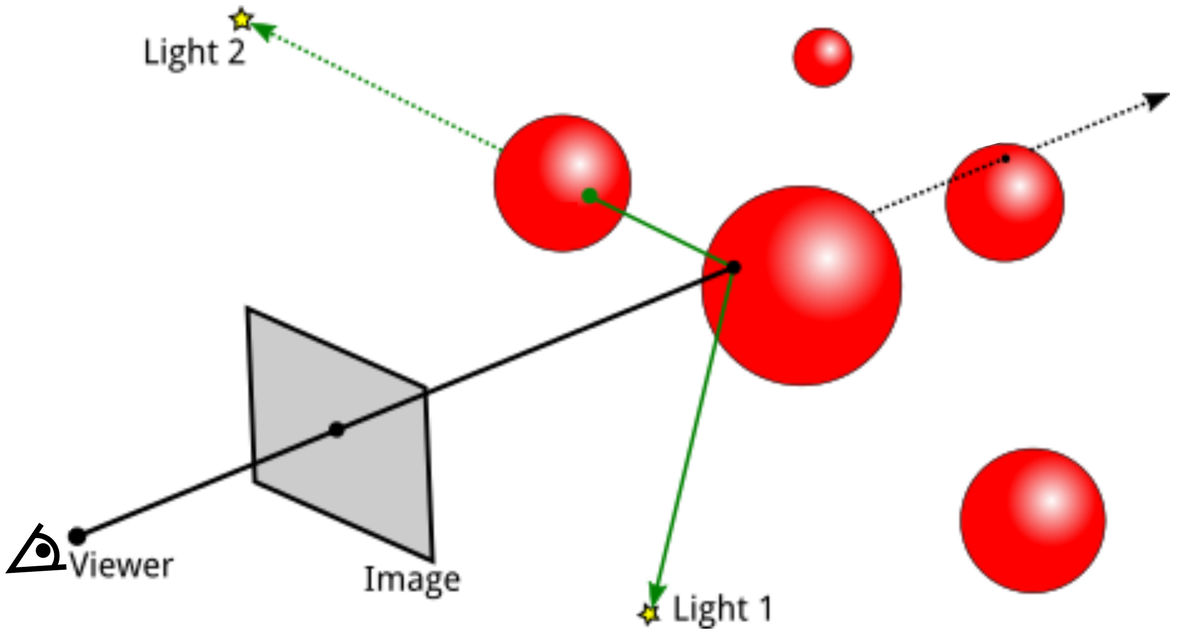
Ray tracing



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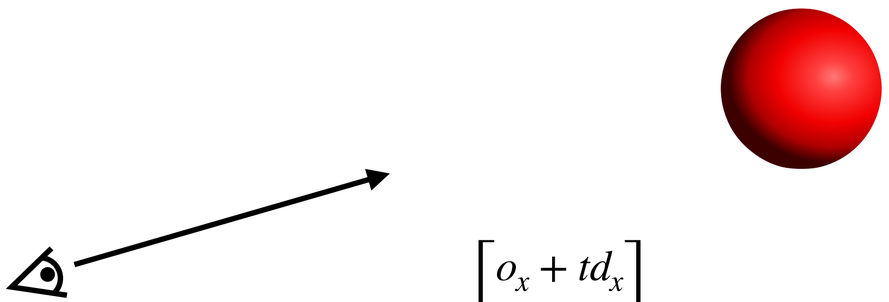
Find intersection

- How to find an intersection of a ray with a sphere?

$$(x - x_c)^2 + (y - y_c)^2 + (z - z_c)^2 - r^2 = 0$$

work out algebra to solve t

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



$$\mathbf{r}(t) = \mathbf{o} + t\mathbf{d} = \begin{bmatrix} o_x + td_x \\ o_y + td_y \\ o_z + td_z \end{bmatrix}$$

- What about a ray with a polygon?

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Find intersection

- Intersect the ray with the plane containing the polygon.
- Find out if the intersection is inside of the polygon.

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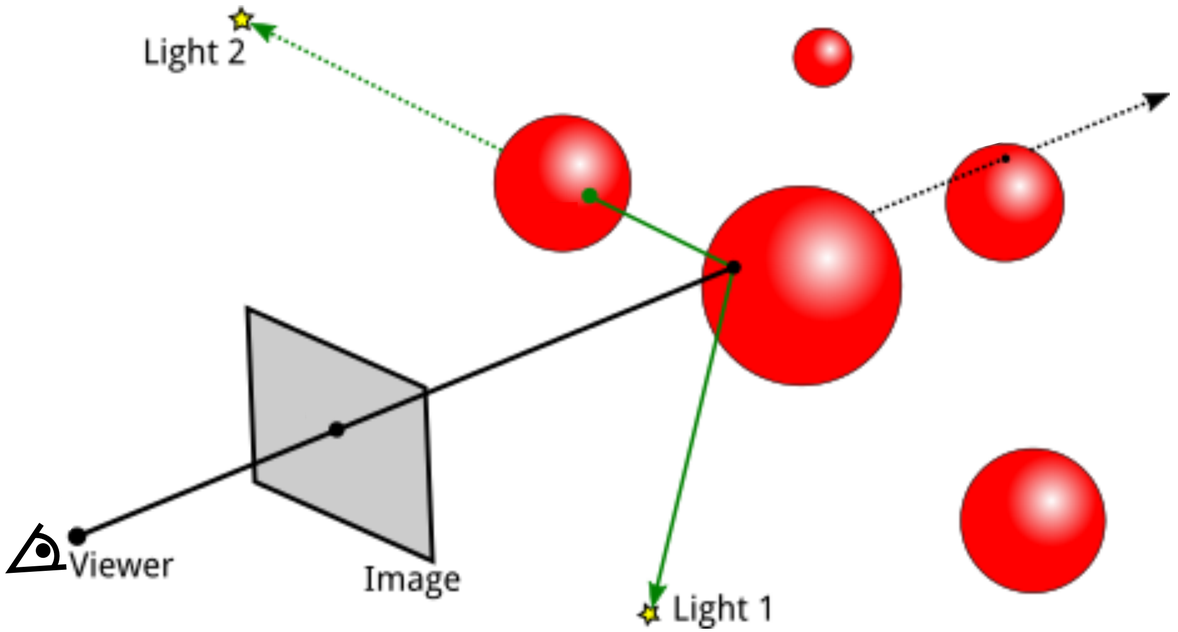
Ray tracing



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for each object O in scene

if R intersects O and is closest so far

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shade pixel (x, y) based on nearest intersection

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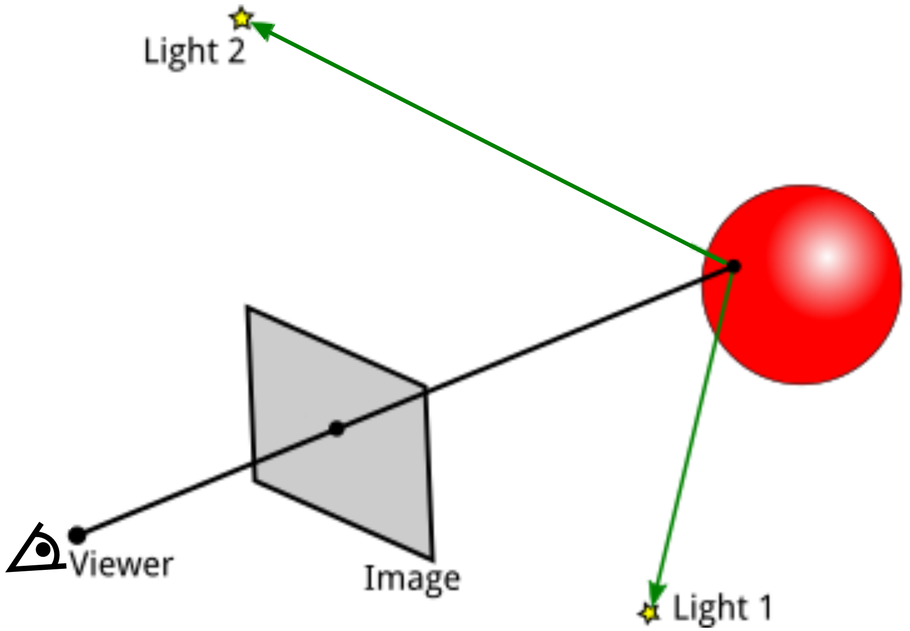
Compute the color of a pixel



$$C = C_{ambi} + \sum_{i=1}^{2} C_{l_i} \circ C_{surf} \, (\mathbf{n} \cdot \mathbf{l}_i) + C_{spec}$$

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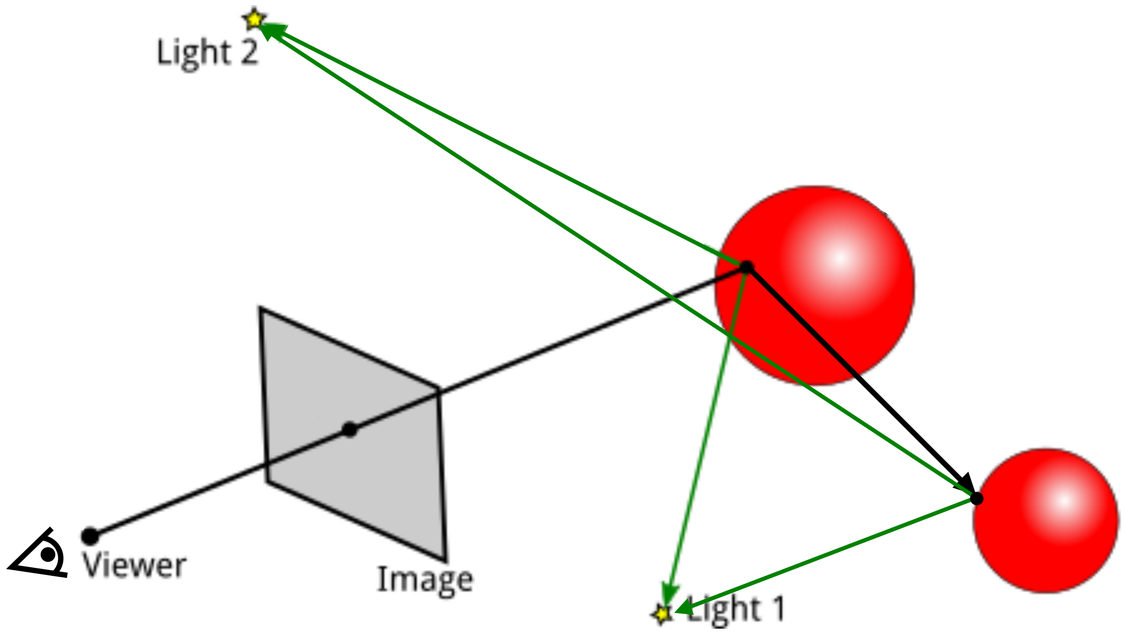
Reflection

Generate single, “most likely” direction



$$C = C_{ambi} + \sum_{i=1}^{2} C_{l_i} \circ C_{surf} \, (\mathbf{n} \cdot \mathbf{l}_i) + C_{spec} + k_{refl}^{(1)} \cdot C_{refl}^{(1)}$$

$$C_{refl}^{(1)} = C_{ambi} + \sum_{i=1}^{2} C_{l_i} \circ C_{surf}^{(1)} \, (\mathbf{n}^{(1)} \cdot \mathbf{l}_i^{(1)}) + C_{spec}^{(1)}$$

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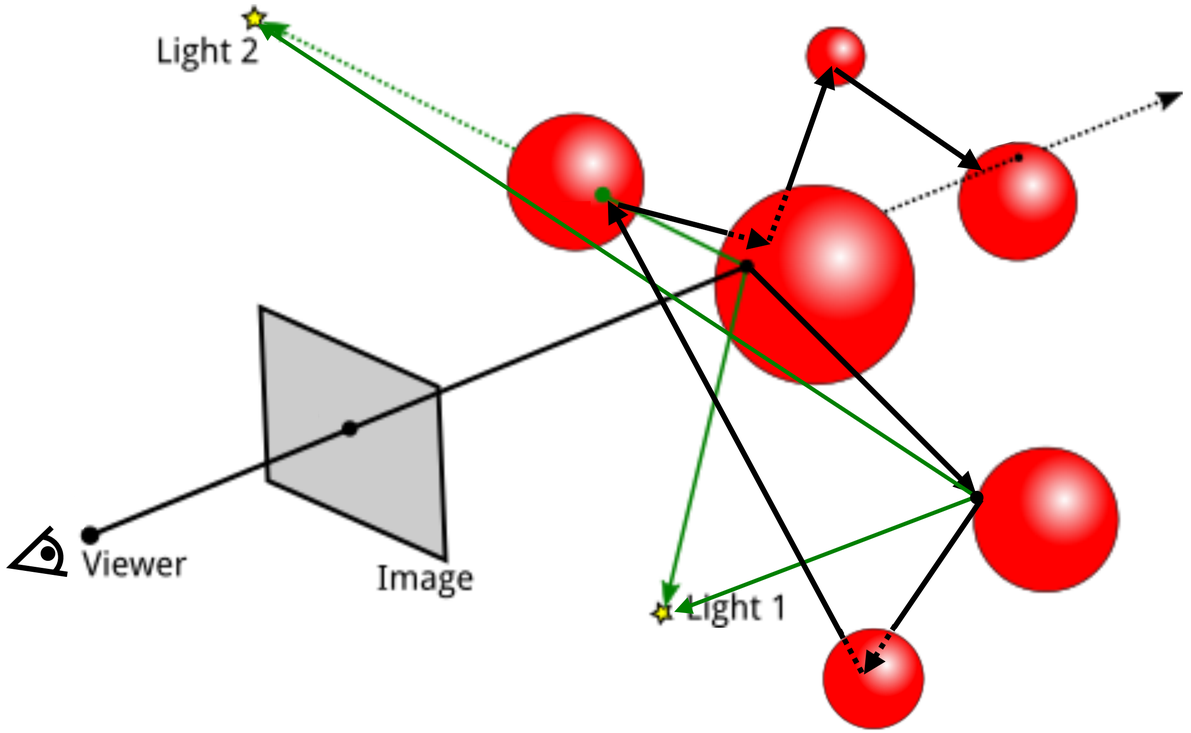
Recursive reflection



$$C = C_{ambi} + \sum_{i=1}^{2} C_{l_i} \circ C_{surf}\,(\mathbf{n} \cdot \mathbf{l}_i) + C_{spec} + k_{refl}^{(1)} \cdot C_{refl}^{(1)}$$

$$C_{refl}^{(1)} = C_{ambi} + \sum_{i=1}^{2} C_{l_i} \circ C_{surf}^{(1)}\,(\mathbf{n}^{(1)} \cdot \mathbf{l}_i^{(1)}) + C_{spec}^{(1)} + k_{refl}^{(2)} \cdot C_{refl}^{(2)}$$

$$C_{refl}^{(2)} = C_{ambi} + \sum_{i=1}^{2} C_{l_i} \circ C_{surf}^{(2)}\,(\mathbf{n}^{(2)} \cdot \mathbf{l}_i^{(2)}) + C_{spec}^{(2)} + k_{refl}^{(3)} \cdot C_{refl}^{(3)}$$

… When to stop recursion?

1. Hit a non-shiny surface.
2. When contribution to final color is small.
3. Max depth is reached.

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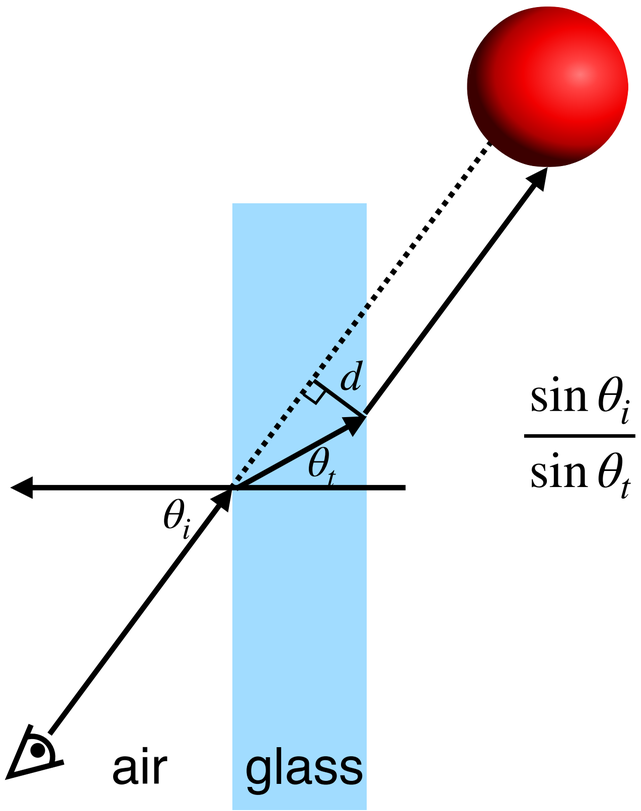
Refraction



$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{\eta_t}{\eta_i}$$

$$\text{index of refraction } (\eta) = \frac{\text{speed of light thru vacuum}}{\text{speed of light thru medium}}$$

materials	η
vacuum	1
air	1.0003
water	1.33
ice	1.309
diamond	2.47

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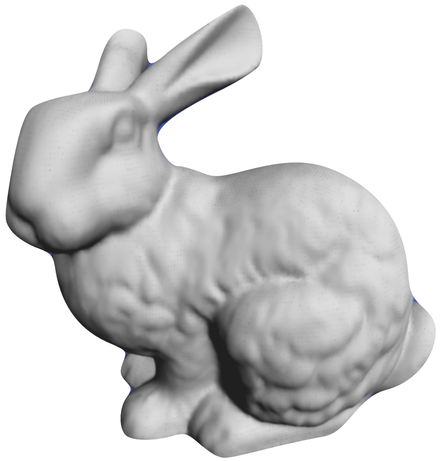
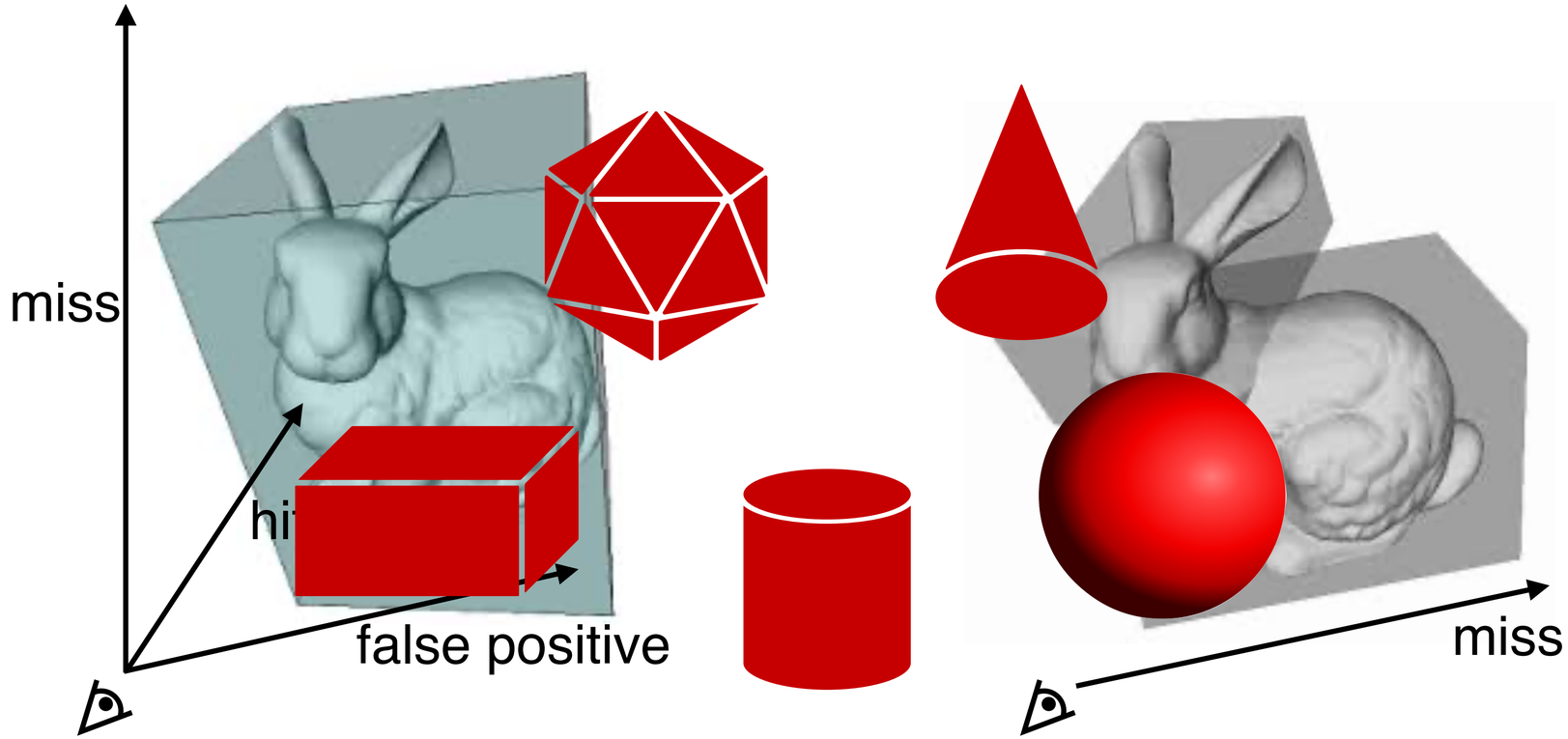
Ray tracing acceleration





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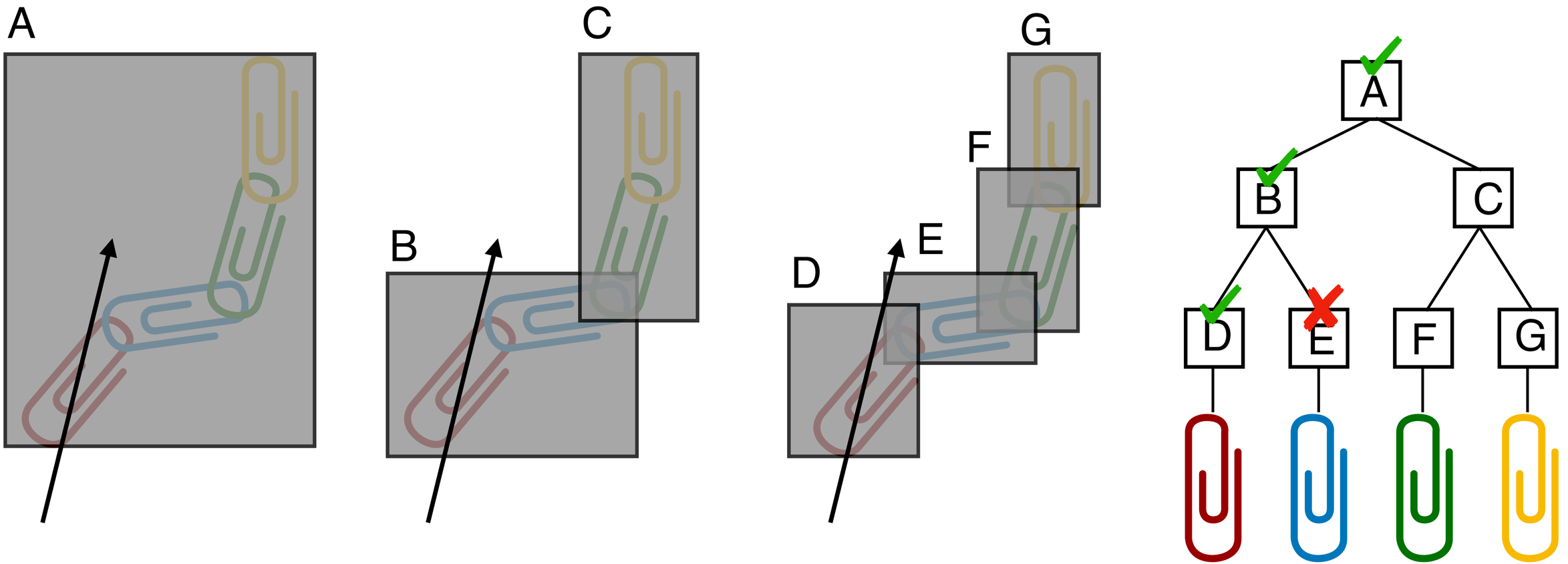
Bounding hierarchy



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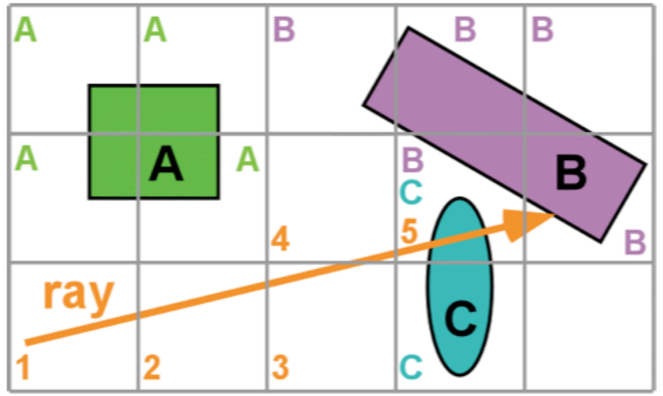
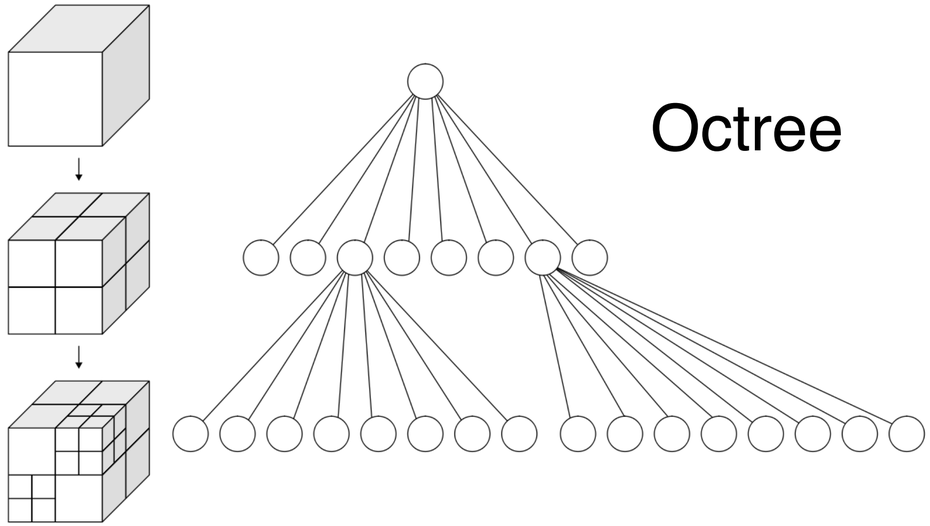
Create bounding hierarchy

- Use a tree-like structure to organize polygons into bounding hierarchy.
- For objects with similar size, **Spatial Partition** (or **Grids**) method works well.
  - Build an object list for each cell.
  - Shoot a ray thru the grid and check the object list of each cell the ray touches.
- For varying sized objects, build an **Octree** to store the objects in a binary hierarchy.

Grids

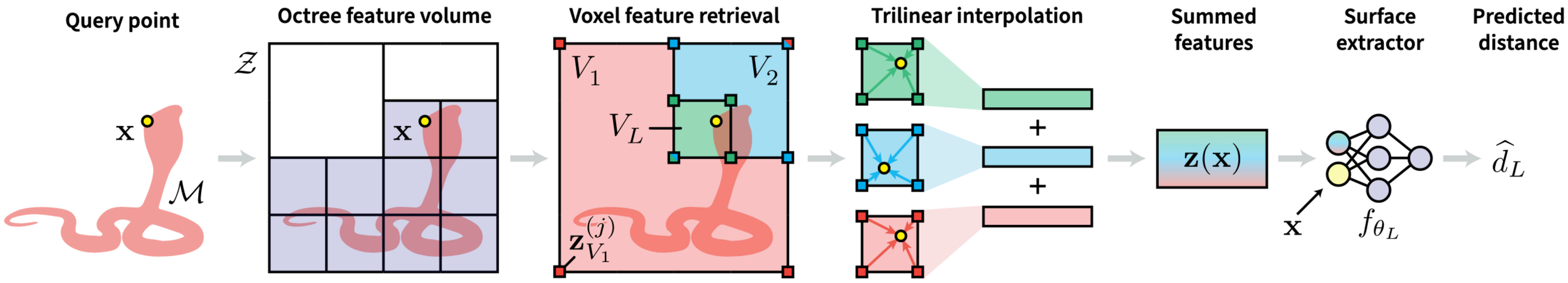


Octree



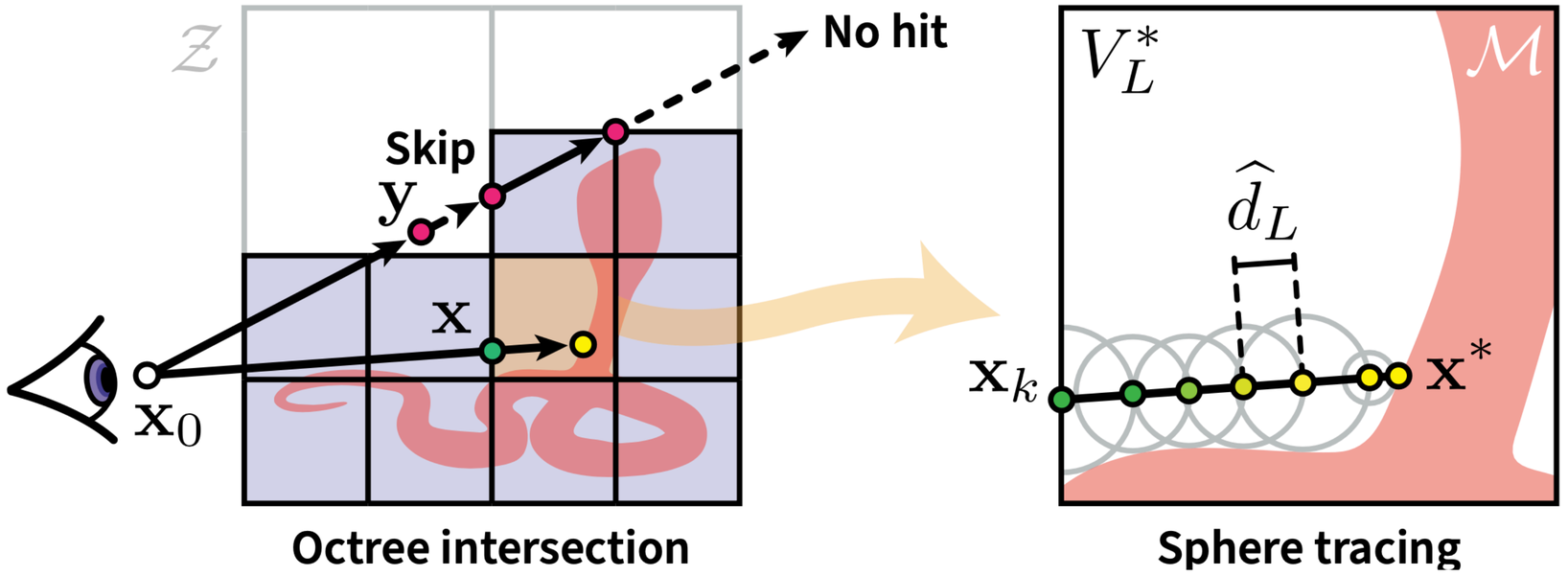
Slide Credit: Stanford CS348I, Computer Graphics in the Age of AI, Stanford, C. Karen Liu & Jiajun Wu

Neural Geometric Level of Detail

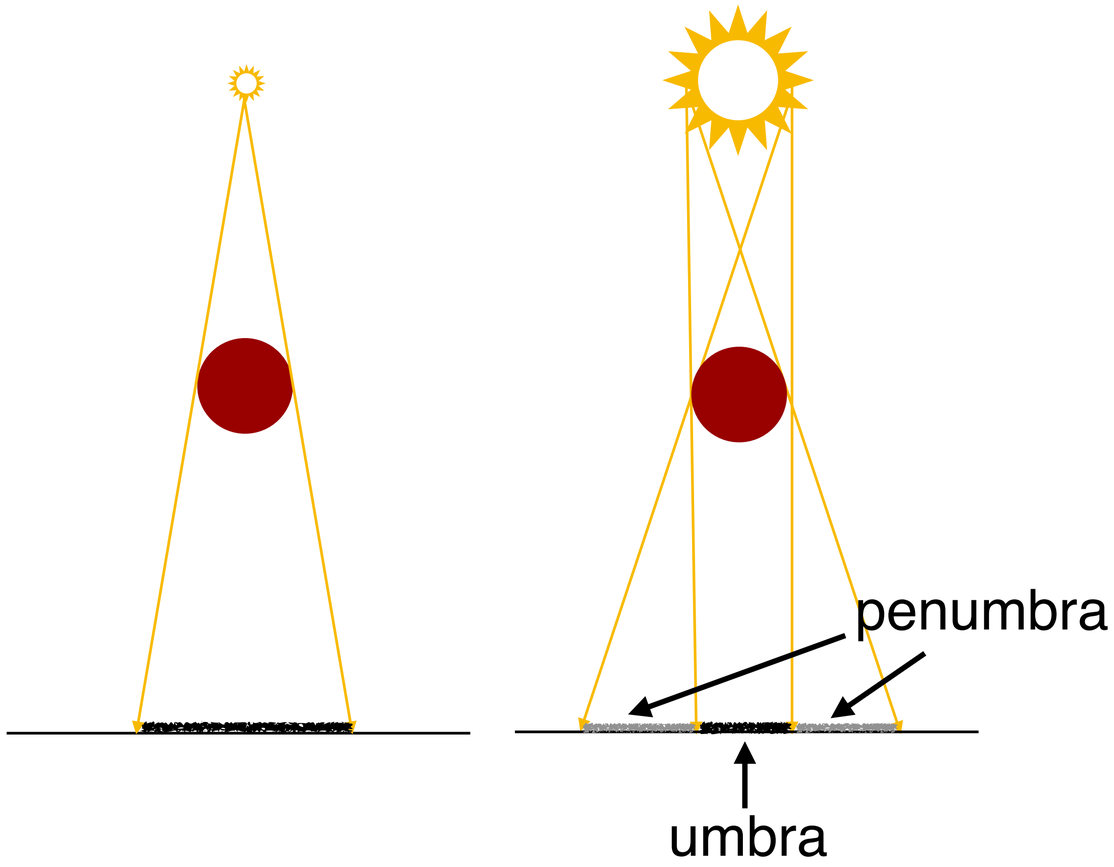


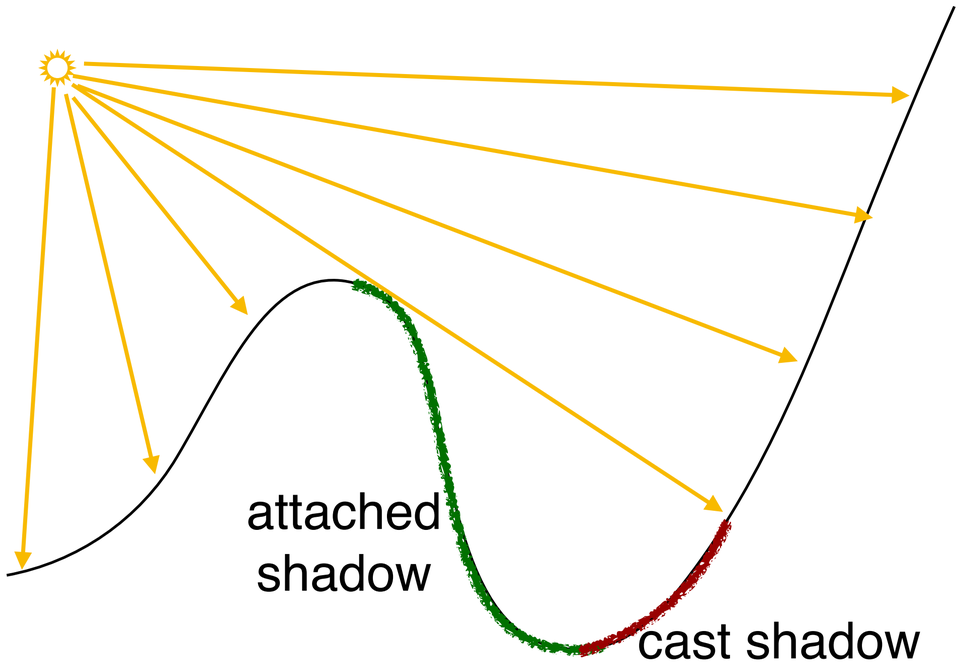
Neural Geometric Level of Detail: Real-time Rendering with Implicit 3D Shapes, Takikawa, Litalien et al. 2021

Neural Geometric Level of Detail



Shadows

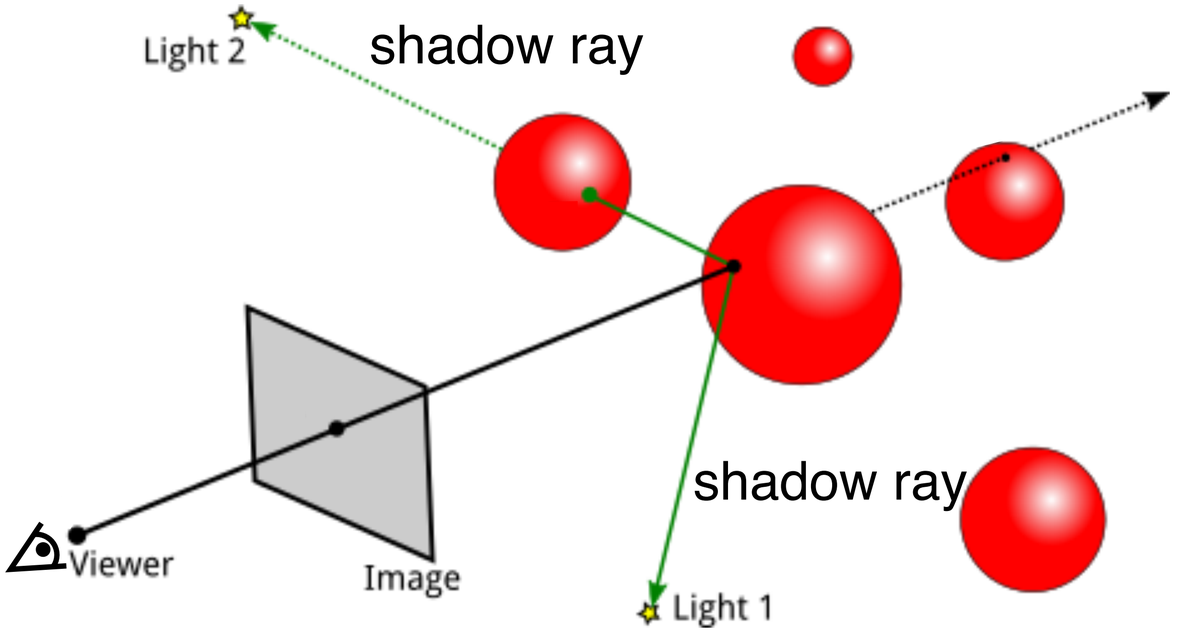




Slide Credit: Stanford CS348I, Computer Graphics in the Age of AI, Stanford, C. Karen Liu & Jiajun Wu

Shadow rays

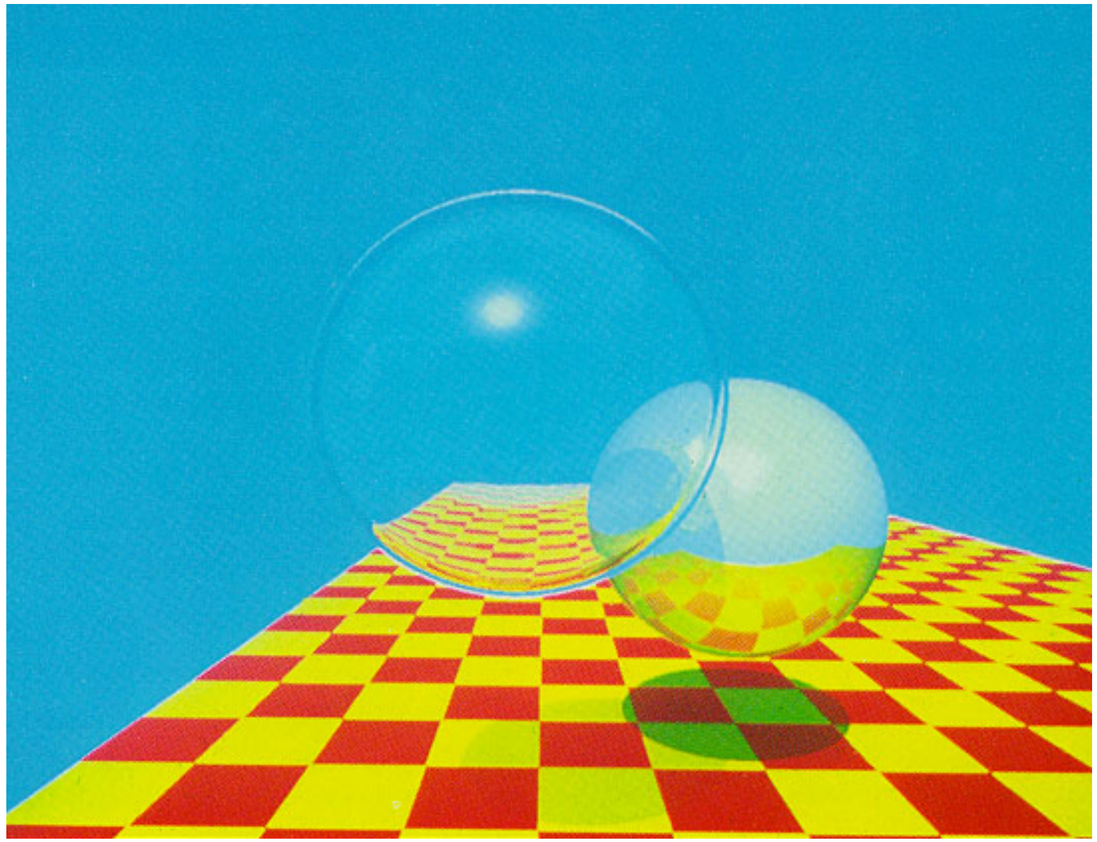
Add shadows in ray tracing

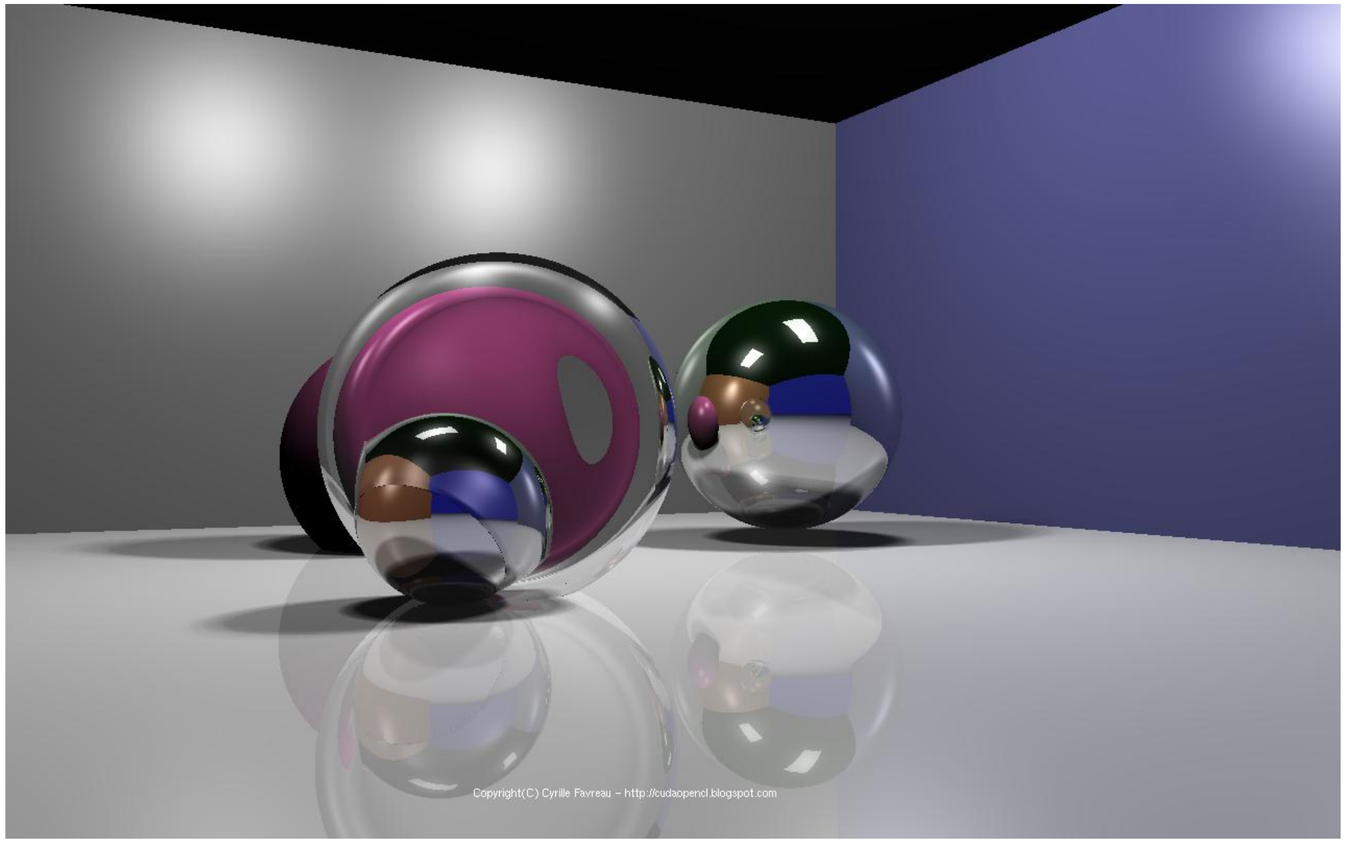


$$C = C_{ambi} + \sum_{i=1}^{2} \text{visible}(\mathbf{l}_i, \mathbf{x}) \cdot C_{l_i} \circ C_{surf} \ \max(\mathbf{n} \cdot \mathbf{l}_i, 0) + C_{spec}$$

Slide Credit: Stanford CS348I, Computer Graphics in the Age of AI, Stanford, C. Karen Liu & Jiajun Wu

Ray tracing examples





Slide Credit: Stanford CS348I, Computer Graphics in the Age of AI, Stanford, C. Karen Liu & Jiajun Wu

Rendering equation

Rendering equation: $L_o(\mathbf{x}, \boldsymbol{\omega}_o, \lambda) = L_e(\mathbf{x}, \boldsymbol{\omega}_o, \lambda) + \int_{\Omega} f(\mathbf{x}, \boldsymbol{\omega}_i, \boldsymbol{\omega}_o, \lambda)\ L_i(\mathbf{x}, \boldsymbol{\omega}_i, \lambda)\ (\boldsymbol{\omega}_i \cdot \mathbf{n}) d\boldsymbol{\omega}_i$

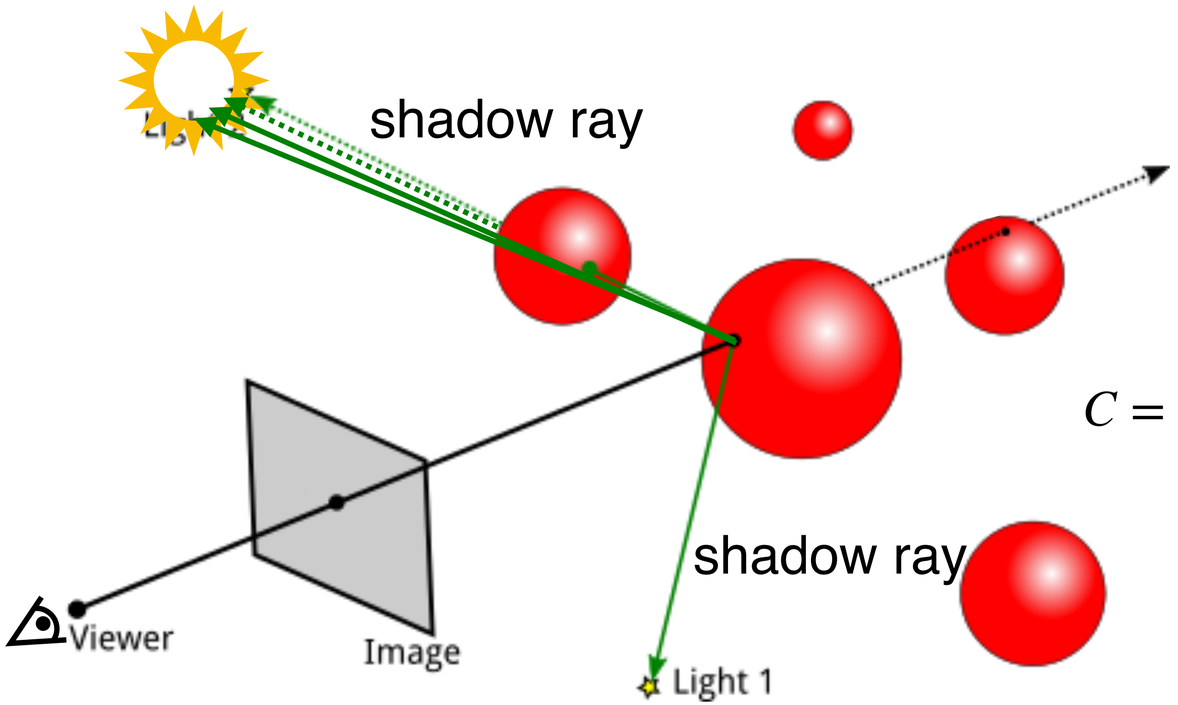
Phong model with **ray tracing**: Add reflection, refraction and hard shadows to basic Phong model.

Phong model with **rasterization**: $C = C_{surf} \circ C_{ambi} + C_{surf} \circ \sum_{i=1}^{N} C_{l_i} \max(0, \mathbf{n} \cdot \mathbf{l}_i) + \sum_{i=1}^{N} C_{l_i} \max(0, \mathbf{e} \cdot \mathbf{r}_i)^p$

Slide Credit: Stanford CS348I, Computer Graphics in the Age of AI, Stanford, C. Karen Liu & Jiajun Wu

Distribution ray tracing

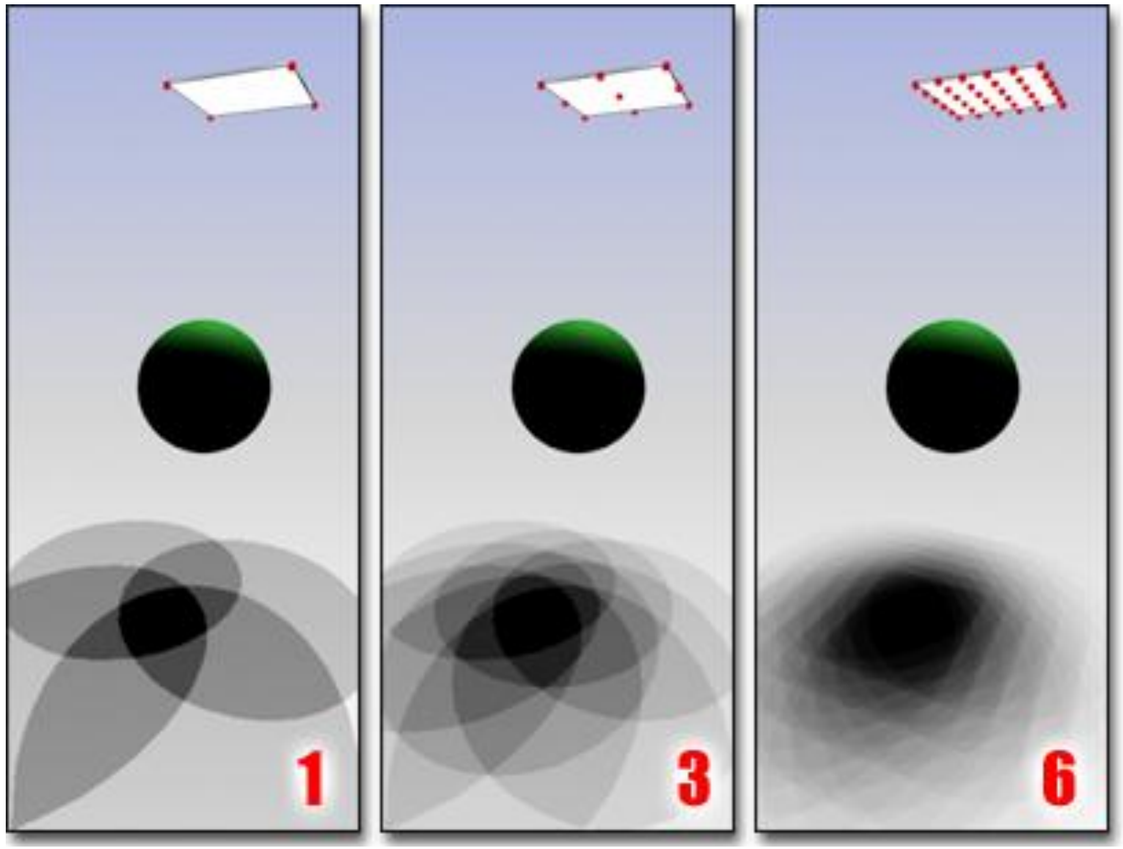
Add soft shadow effects in ray tracing

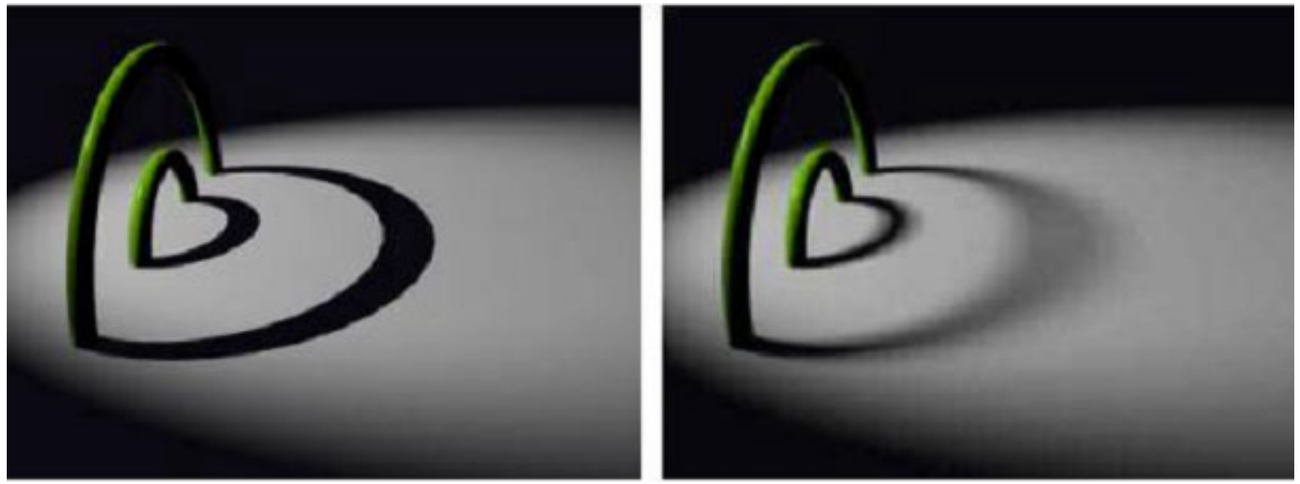


$$C = C_{ambi} + \sum_{i=1}^{2} \left(\frac{1}{M} \sum_{j=1}^{M} \text{visible}(\mathbf{l}_i^{(j)}, \mathbf{x}) \cdot C_{l_i} \circ C_{surf} \ \max(\mathbf{n} \cdot \mathbf{l}_i^{(j)}, 0) \right) + C_{spec}$$

Slide Credit: Stanford CS348I, Computer Graphics in the Age of AI, Stanford, C. Karen Liu & Jiajun Wu

Distribution ray tracing examples

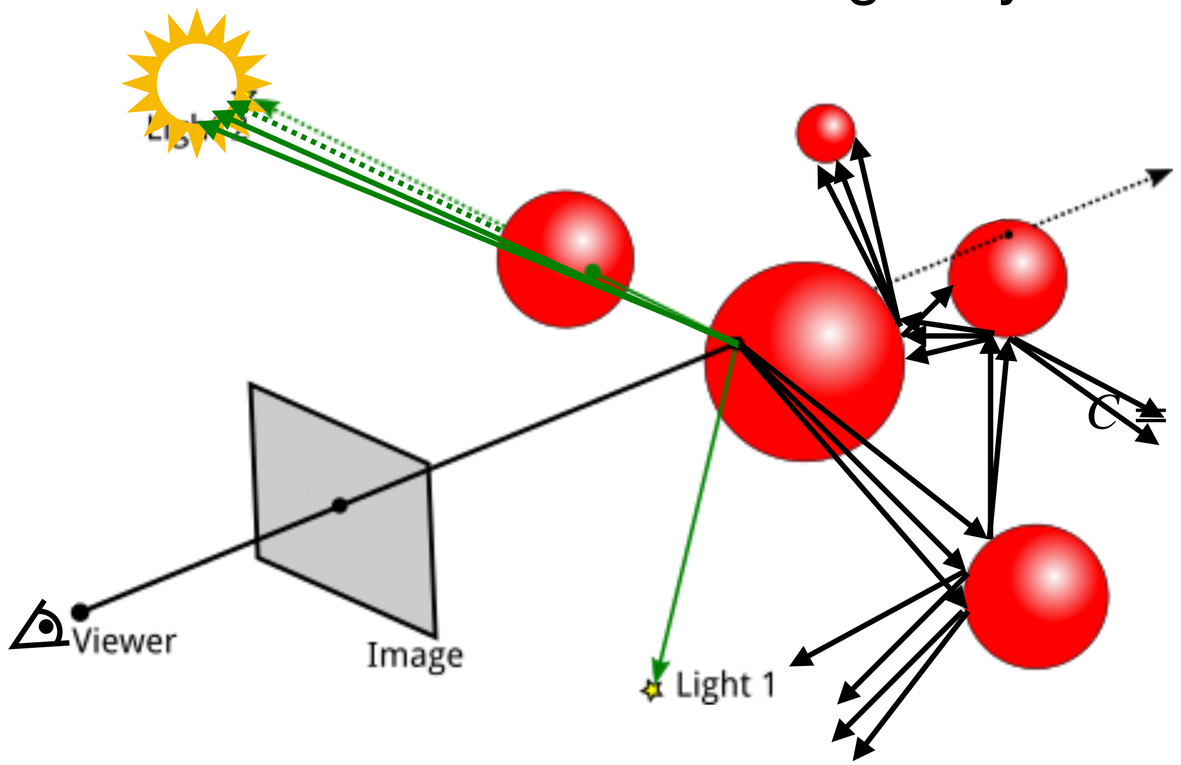




Slide Credit: Stanford CS348I, Computer Graphics in the Age of AI, Stanford, C. Karen Liu & Jiajun Wu

Distribution ray tracing

Add glossy reflection effects in ray tracing



$$C = C_{ambi} + \sum_{i=1}^{2}\left(\frac{1}{M}\sum_{j=1}^{M}\text{visible}(\mathbf{l}_i^{(j)},\mathbf{x})\cdot C_{l_i}\circ C_{surf}\ \max(\mathbf{n}\cdot\mathbf{l}_i^{(j)},0)\right) + C_{spec} + k_{refl}^{(1)}\cdot\sum_{k=1}^{P} C_{refl_p}^{(1)}$$

Exponentially expand the reflection terms.

Slide Credit: Stanford CS348I, Computer Graphics in the Age of AI, Stanford, C. Karen Liu & Jiajun Wu

Rendering equation

Rendering equation: $L_o(\mathbf{x}, \boldsymbol{\omega}_o, \lambda) = L_e(\mathbf{x}, \boldsymbol{\omega}_o, \lambda) + \int_{\Omega} f(\mathbf{x}, \boldsymbol{\omega}_i, \boldsymbol{\omega}_o, \lambda)\ L_i(\mathbf{x}, \boldsymbol{\omega}_i, \lambda)\ (\boldsymbol{\omega}_i \cdot \mathbf{n}) d\boldsymbol{\omega}_i$

Distribution ray tracing: Can approximate soft shadows and glossy (but not perfectly reflective) surfa

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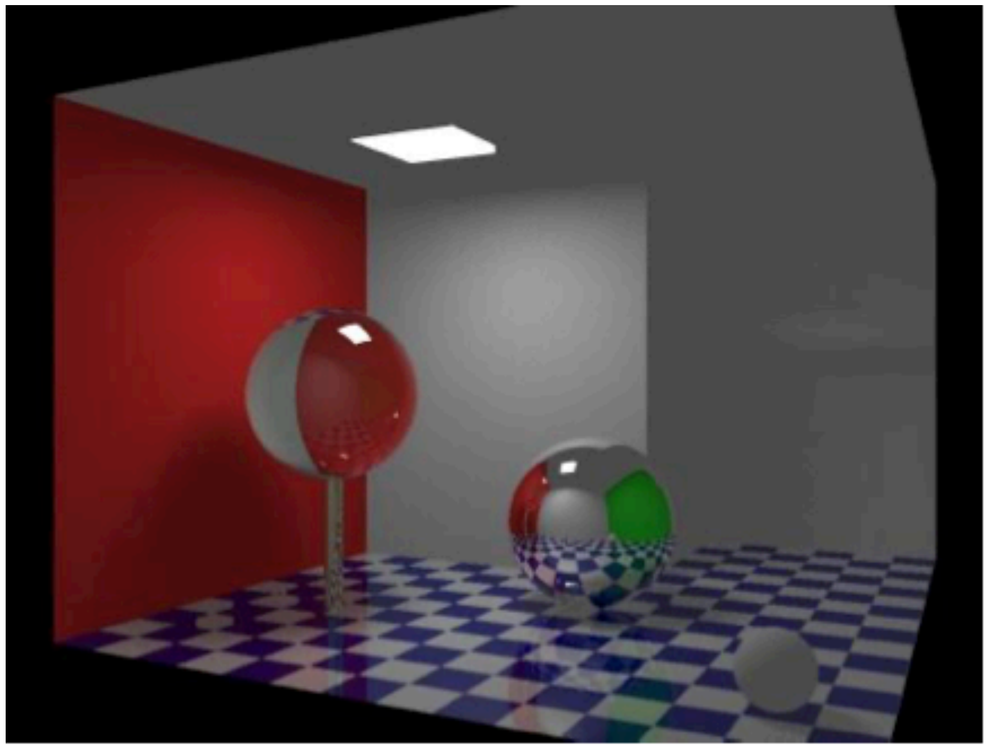
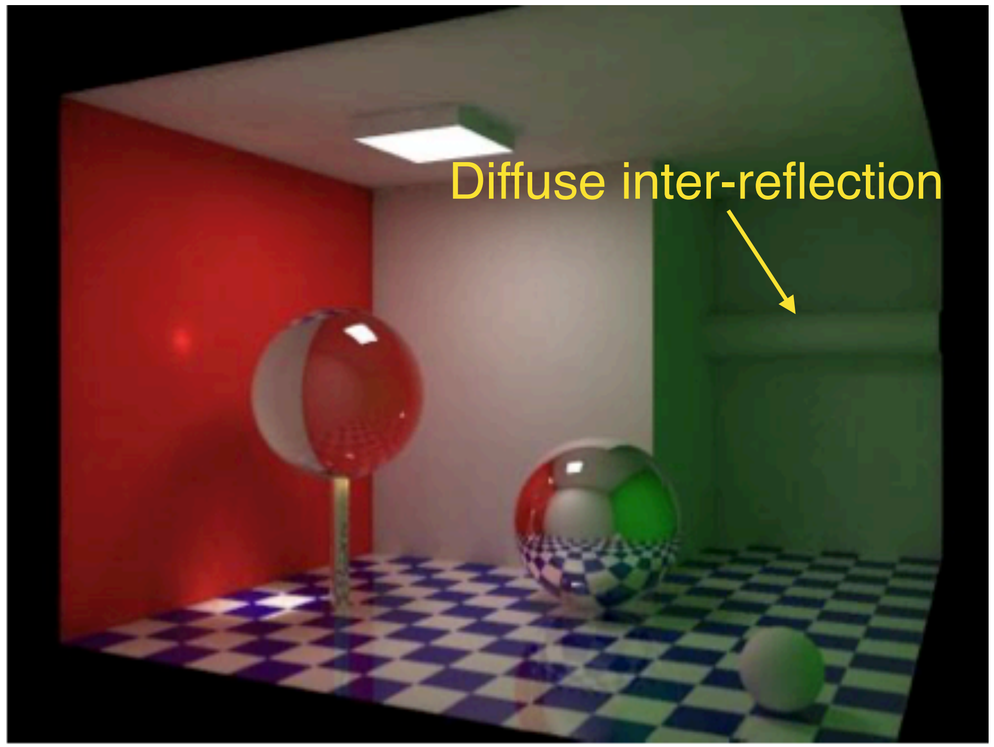
Beyond ray tracing





Slide Credit: Stanford CS348I, Computer Graphics in the Age of AI, Stanford, C. Karen Liu & Jiajun Wu

Rendering equation

photorealism

Rendering equation: $L_o(\mathbf{x}, \boldsymbol{\omega}_o, \lambda) = L_e(\mathbf{x}, \boldsymbol{\omega}_o, \lambda) + \int_{\Omega} f(\mathbf{x}, \boldsymbol{\omega}_i, \boldsymbol{\omega}_o, \lambda)\ L_i(\mathbf{x}, \boldsymbol{\omega}_i, \lambda)\ (\boldsymbol{\omega}_i \cdot \mathbf{n}) d\boldsymbol{\omega}_i$

Path tracing: Close approximation of the solution to the rendering equation.

Distribution ray tracing: Can approximate soft shadows and glossy (but not perfectly reflective) surface

Phong model with ray tracing: Add reflection, refraction and hard shadows to basic Phong model.

Phong model with rasterization: $C = C_{surf} \circ C_{ambi} + C_{surf} \circ \sum_{i=1}^{N} C_{l_i} \max(0, \mathbf{n} \cdot \mathbf{l}_i) + \sum_{i=1}^{N} C_{l_i} \max(0, \mathbf{e} \cdot \mathbf{r}_i)^p$

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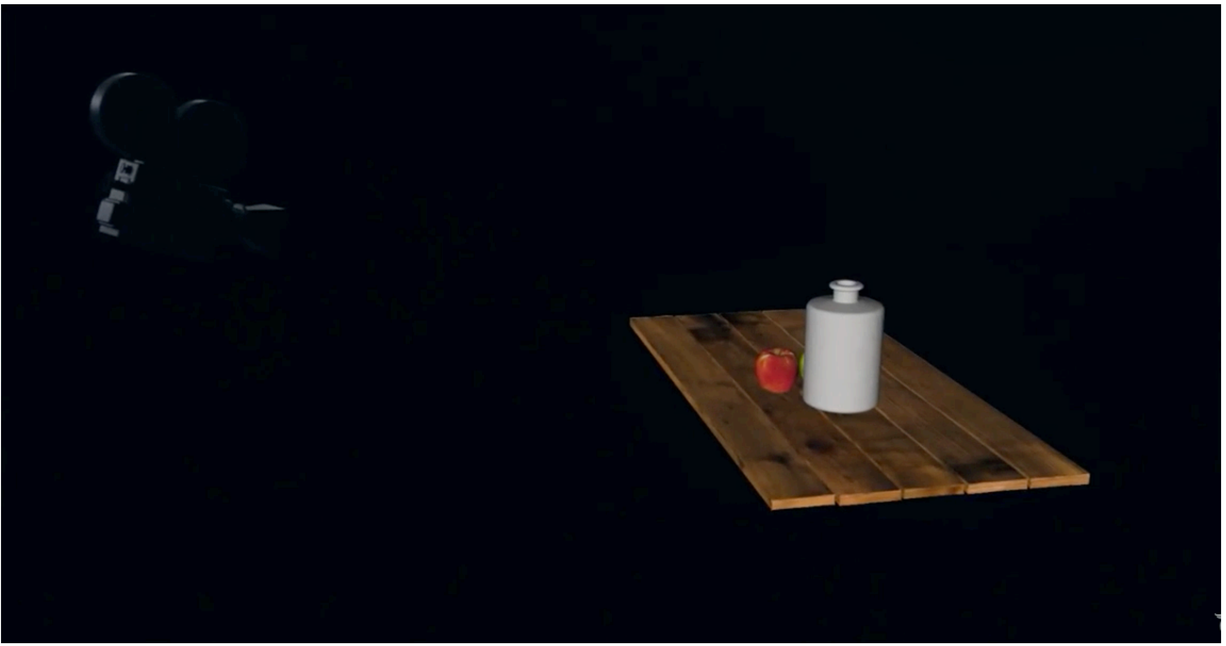
Path tracing



for each pixel *p*

create ray *r* from eye through *p*

for each sample

color += PathTracer(*r*)

render *p* in *color*/numSamples

- Send a ray through a pixel, find the first intersection on the surface of an object, and integrate over all the illuminance arriving to the intersection point.
- Compute illuminance by recursively projecting a ray from the surface to the scene in a bouncing path that terminates when a light source is intersected.
- The light is then sent backwards through the path and to the output pixel.

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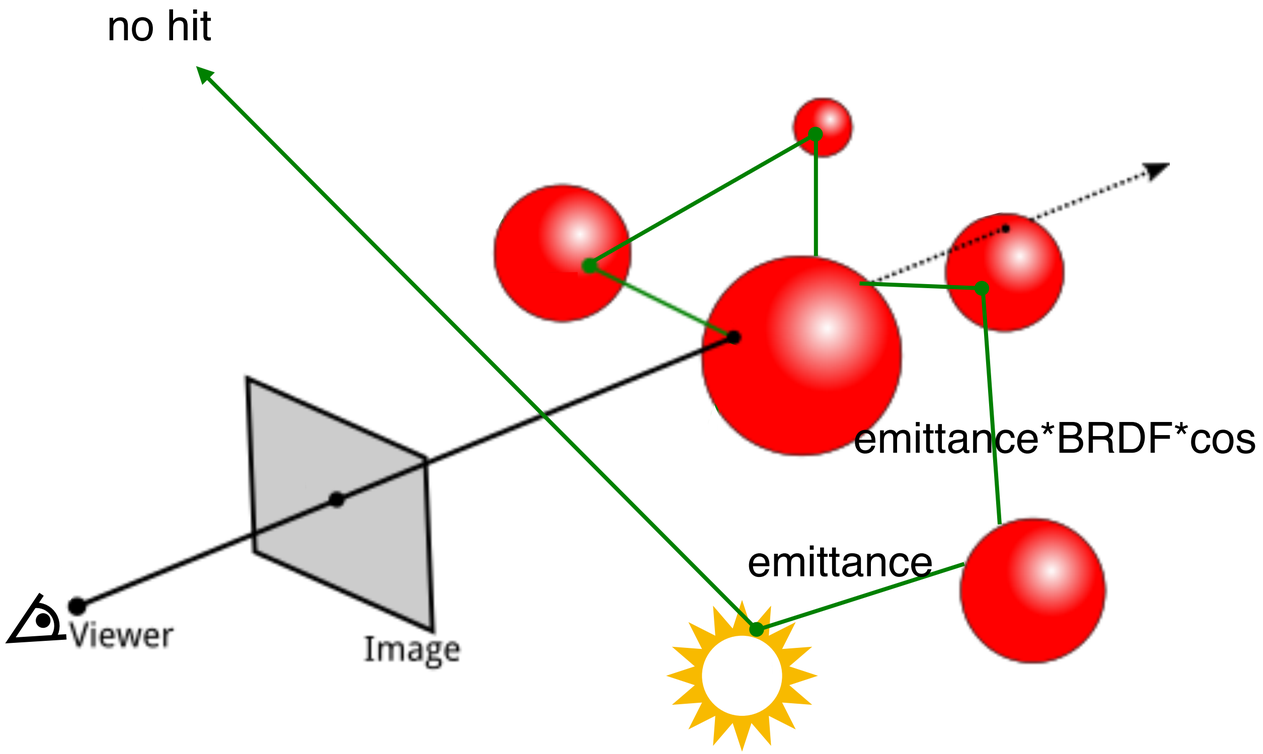
Path tracing




```
PathTracer(r)
    if r.hitNothing
        return Black
    if r.hit.material.emmitance
        return r.hit.material.emmitance
    create a new ray r' from r.hit in random direction
    light = PathTracer(r')
    cos = dot(r'.direction, r.hit.normal)
    BRDF = r.hit.material.reflectance
    return BRDF*light*cos
```

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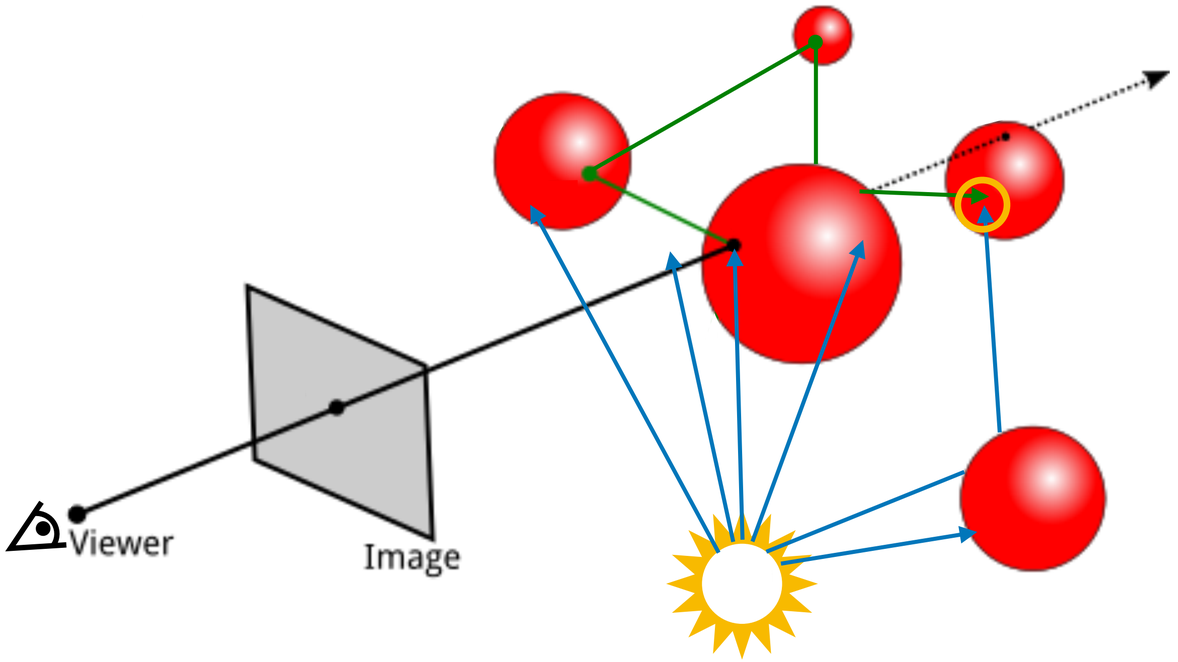
Bidirectional path tracing (BDPT)



- Reflected light for a point can be approximated by two ways:
  - Shooting rays from the light sources into the scene. Billions of paths are created and bounced for a number of steps.
  - Gathering rays from a point on a surface. Bounce rays from the surface to the scene and terminate them at light sources.
- Bidirectional Path Tracing combines both **Shooting** and **Gathering** to obtain faster convergence:
  - Shooting and gathering paths are traced independently.
  - The head of the shooting path is connected to the tail of the gathering path.
  - Attenuate light at every bounce and back out into the pixel.

Slide Credit: Stanford CS348I, Computer Graphics in the Age of AI, Stanford, C. Karen Liu & Jiajun Wu


2spp


2spp denoised


4spp


4spp denoised


8spp


8spp denoised


50spp


50spp denoised


1000spp


50spp denoised
1000spp denoised


12000spp


12000spp denoised


50spp denoised
12000spp

Summary

Rendering equation: $L_o(\mathbf{x}, \boldsymbol{\omega}_o, \lambda) = L_e(\mathbf{x}, \boldsymbol{\omega}_o, \lambda) + \int_{\Omega} f(\mathbf{x}, \boldsymbol{\omega}_i, \boldsymbol{\omega}_o, \lambda)\ L_i(\mathbf{x}, \boldsymbol{\omega}_i, \lambda)\ (\boldsymbol{\omega}_i \cdot \mathbf{n}) d\boldsymbol{\omega}_i$

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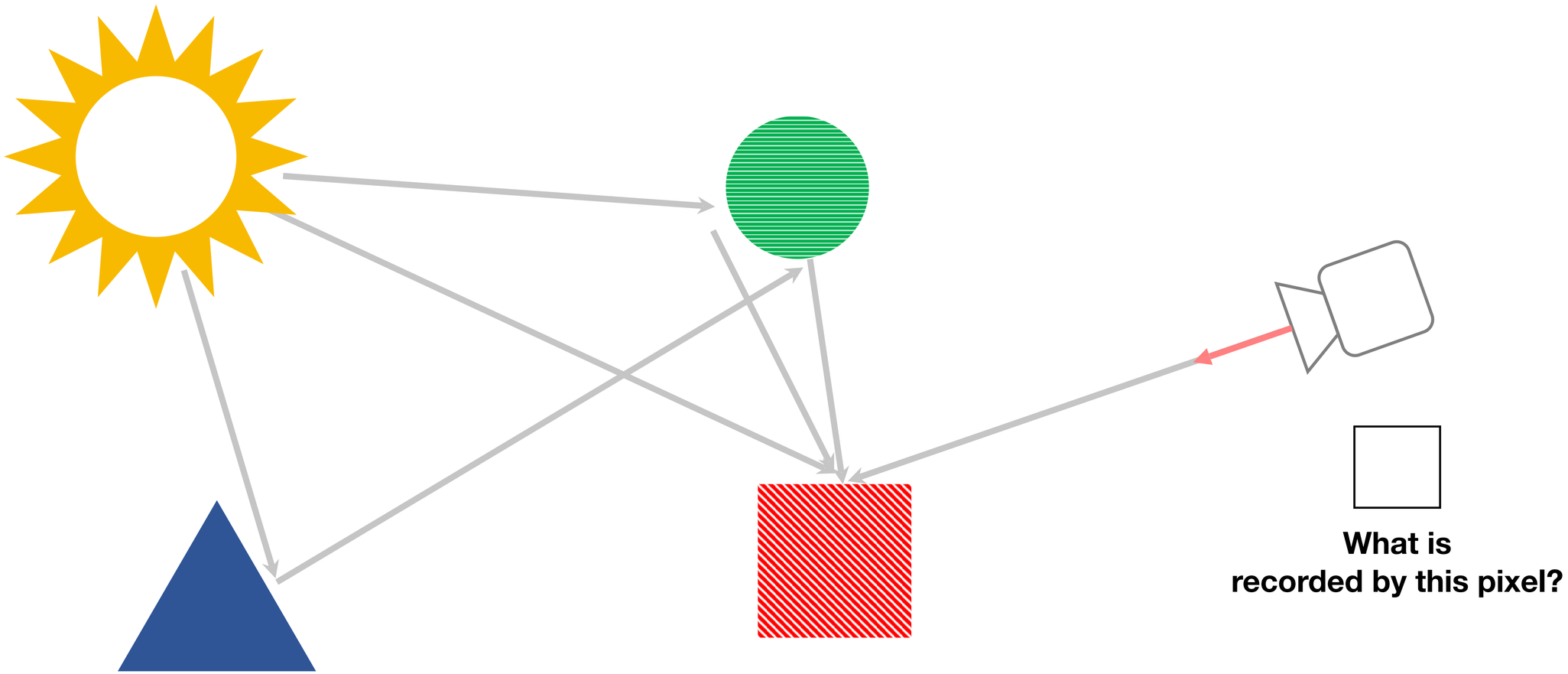
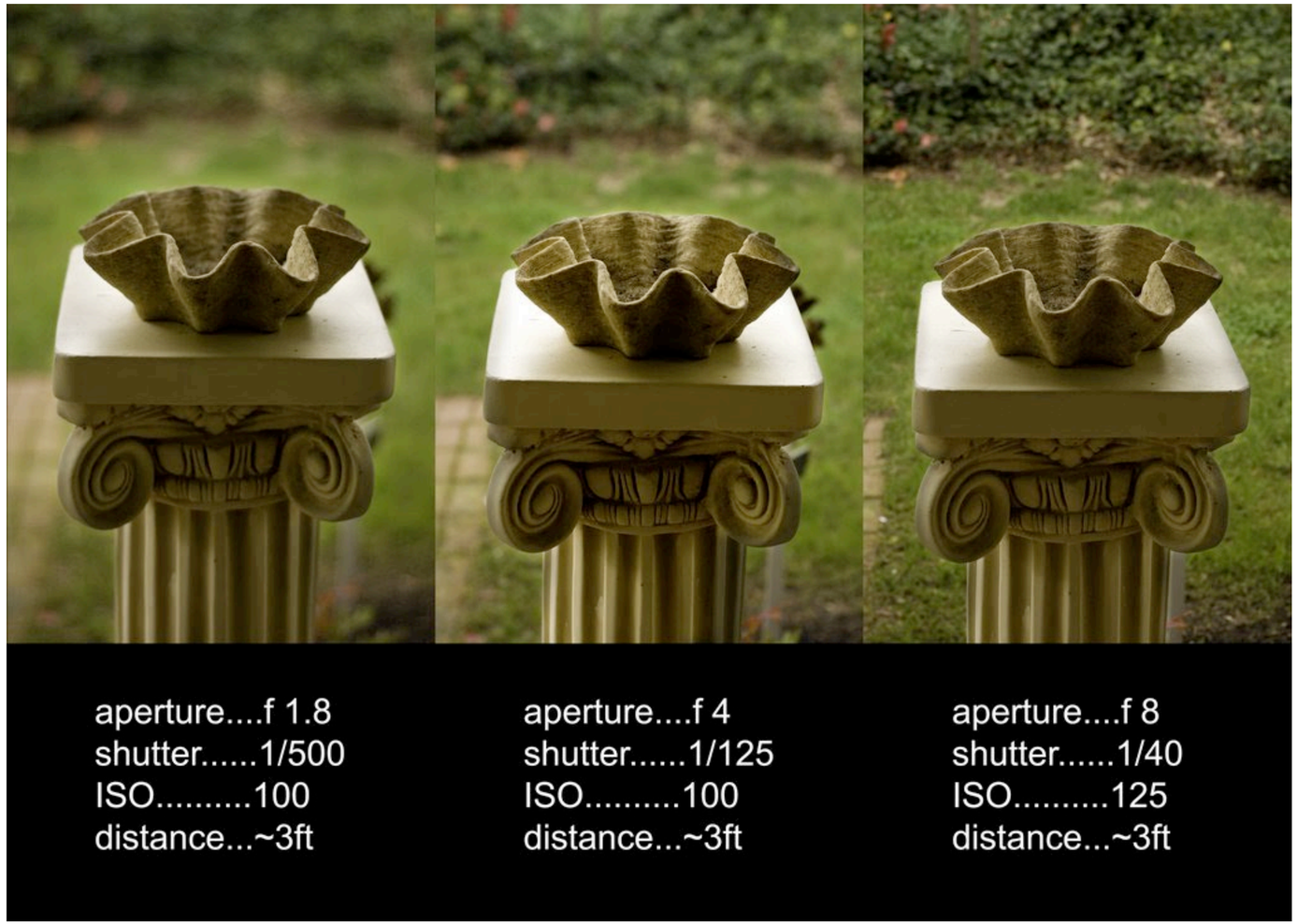
Slide Credit: Stanford CS348I, Computer Graphics in the Age of AI, Stanford, C. Karen Liu & Jiajun Wu

Other effects to be aware of



Depth-of-Field



http://photographywisdom.com

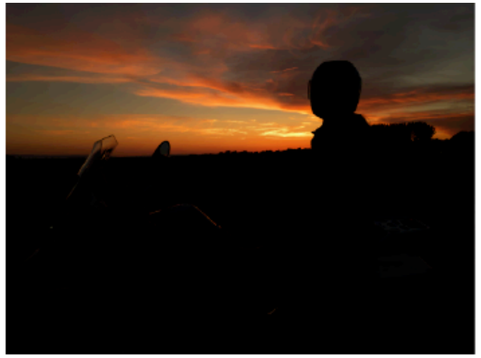
Slide credit: Gordon Wetzstein, Stanford EE367 / CS448I: Computational Imaging

Important Imaging Effects: Dynamic Range

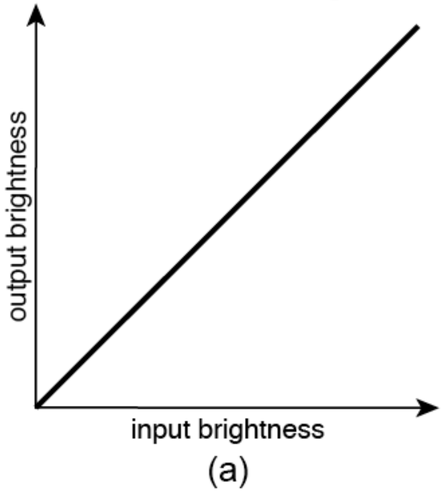
Wikipedia

Slide credit: Gordon Wetzstein, Stanford EE367 / CS448I: Computational Imaging

Important Imaging Effects: Tone Mapping

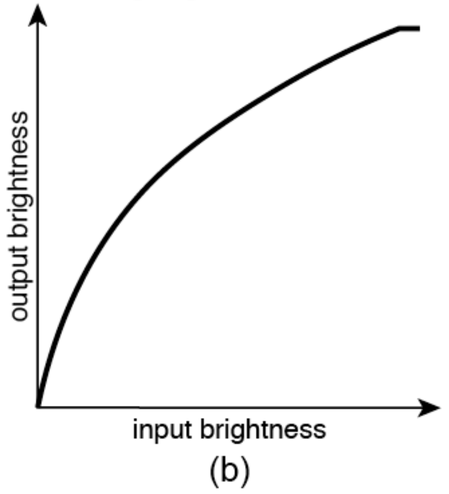


Linear RGB image



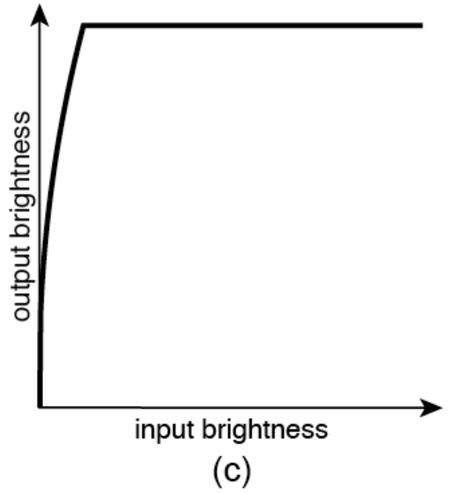
(a)

Highlights visible



(b)

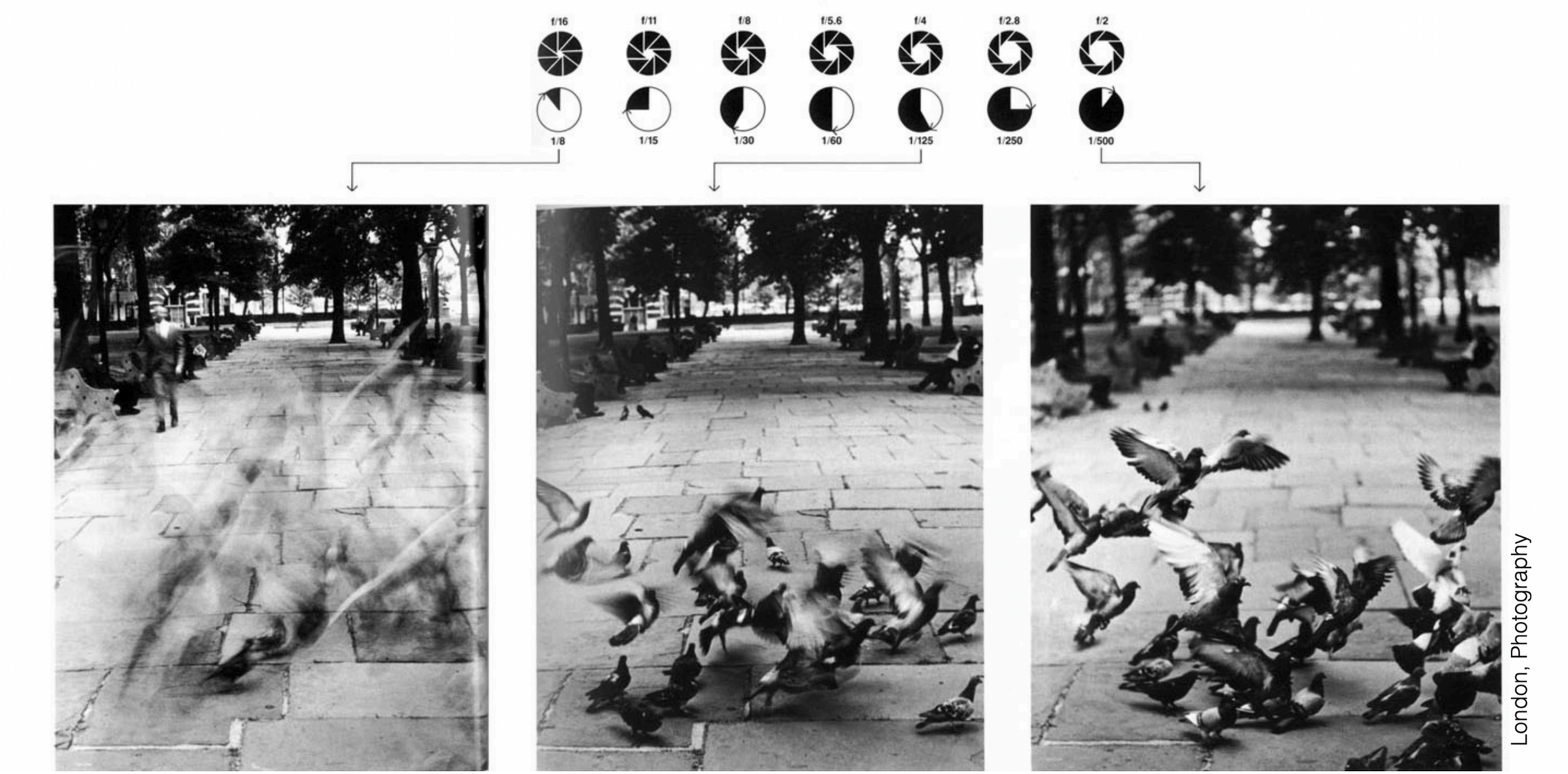
Shadows visible



(c)

https://ai.googleblog.com/2020/08/live-hdr-and-dual-exposure-controls-on.html

Important Imaging Effects: Motion Blur



Slide credit: Gordon Wetzstein, Stanford EE367 / CS448I: Computational Imaging

Today: Light Transport & High-level of Physics-based Rendering

3D Scene

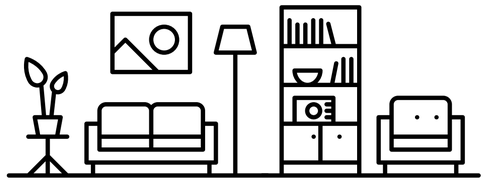


Image Formation

Inference

Neural Scene Representation

Downstream Tasks

Why?	In order to reason about the 3D world from images, we need to understand how 3D properties such as materials, lighting, etc. relate to the measurements observed by a camera.
What you'll learn.	The rendering equation, simulating light transport via multi-bounce ray-tracing, bidirectional radiance distribution functions, rasterization, rendering.