$$\overbrace{\qquad\qquad\qquad}^{G_1}\ \overbrace{\qquad\qquad}^{G_2}$$

1.a) $G(f) = \frac{\pi}{20} e^{-\frac{2\pi}{10}|f-20|} + \frac{\pi}{20} e^{-2\frac{\pi}{10}|f+20|}$

Hesapta kolaylık olması açısından G'yi G_1 ve G_2 olarak ayırdım.

$$R(\tau) = \mathcal{F}^{-1}\{G_1(f)\} + \mathcal{F}^{-1}\{G_2(f)\}$$

$$R_1(\tau) = \int_{-\infty}^{\infty} G_1(f) e^{j2\pi f\tau} df \quad u = f-20 \text{ olsun } du = df$$

$$R_1(\tau) = \frac{\pi}{20} e^{j2\pi 20\tau} \int_{-\infty}^{\infty} e^{-\frac{\pi}{5}|u|} e^{j2\pi u\tau}$$

$$R_1(\tau) = \frac{\pi}{20} e^{j2\pi 20\tau} \int_{-\infty}^{0} e^{\frac{\pi}{5}u} e^{j2\pi u\tau} du + \int_{0}^{\infty} e^{-\frac{\pi}{5}u} e^{j2\pi u\tau} du$$

$$R_1(\tau) = \frac{\pi}{20} e^{j2\pi\cdot 20\cdot\tau} \left[\int_{0}^{\infty} e^{-\frac{\pi}{5}u} \underbrace{\left(e^{j2\pi u\tau} + e^{-j2\pi u\tau}\right)}_{2\cos(2\pi u\tau)} du\right]$$

$$R_1(\tau) = \frac{\pi}{20} e^{j2\pi\cdot 20\tau} \left(\int_{0}^{\infty} e^{-\frac{\pi}{5}u} \cdot 2 \cdot \cos(2\pi u\tau) du\right)$$

Laplace dönüşümündeki $\int_0^{\infty} e^{-st} \cos(at) dt = \frac{s}{a^2+s^2}$ ile benzerliğinden

$$\boxed{R_1(\tau) = \frac{\pi}{10} e^{j2\pi\cdot 20\cdot\tau} \cdot \frac{\pi/5}{\left(\frac{\pi}{5}\right)^2 + (2\pi\tau)^2}}$$

$$e^{j2\pi(u-20)\tau}$$

$$R_2(\tau) = \int_{-\infty}^{\infty} G_2(f) e^{j2\pi f\tau} = \int_{-\infty}^{\infty} \frac{\pi}{20} e^{-\frac{\pi}{5}|f+20|} e^{j2\pi f\tau} df$$

$R_1(\tau)$ için yaptığımız gibi $u = f+20$, $du = df$

$$R_2(\tau) = \frac{\pi}{20} e^{-j2\pi\cdot 20\tau} \int_{-\infty}^{\infty} e^{-\frac{\pi}{5}|u|} e^{j2\pi u\tau} du$$

$$= \frac{\pi}{20} e^{-j2\pi\cdot 20\tau} \left(\int_{-\infty}^{0} e^{\frac{\pi}{5}u} e^{j2\pi u\tau} du + \int_{0}^{\infty} e^{-\frac{\pi}{5}u} e^{j2\pi u\tau} du\right)$$

$$= \frac{\pi}{20} e^{-j2\pi\cdot 20\tau} \left[\int_{0}^{\infty} e^{-\frac{\pi}{5}u} \left(e^{j2\pi u\tau} + e^{-j2\pi u\tau}\right)\right]$$

$$= \frac{\pi}{20} e^{-j2\pi 20\tau} \left[\int_{0}^{\infty} e^{-\frac{\pi}{5}u} (2\cos(2\pi u\tau))\right]$$ Tekrar laplace dönüşümü benzerliğinden:

$$\boxed{= \frac{\pi}{10} e^{-j2\pi 20\tau} \left[\frac{\pi/5}{\left(\frac{\pi}{5}\right)^2 + (2\pi\tau)^2}\right] = R_2(\tau)}$$

$$R(\tau) = R_1(\tau) + R_2(\tau)$$

$$R(\tau) = \frac{\pi^2}{50} \cdot \frac{1}{\pi^2\left[\left(\frac{1}{5}\right)^2 + (2\tau)^2\right]} \left[e^{j2\pi \cdot 20\tau} + e^{-j2\pi \cdot 20\tau} \right]$$

$$R(\tau) = \frac{1}{50} \frac{1}{\left(\frac{1}{25}\right) + 4\tau^2} \left[2\cos(40\tau) \right]$$

$$= \frac{1}{25} \cdot \frac{1}{\frac{1}{25} + 4\tau^2} \cdot \cos(40\tau) \Rightarrow \boxed{R(\tau) = \frac{\cos(40\tau)}{1 + 100\tau^2}}$$