

A Survey on Incomplete Multiview Clustering

Jie Wen^{ID}, Zheng Zhang^{ID}, Senior Member, IEEE, Lunke Fei^{ID}, Member, IEEE,
Bob Zhang^{ID}, Senior Member, IEEE, Yong Xu^{ID}, Senior Member, IEEE,
Zhao Zhang^{ID}, Senior Member, IEEE, and Jinxing Li

Abstract—Conventional multiview clustering seeks to partition data into respective groups based on the assumption that all views are fully observed. However, in practical applications, such as disease diagnosis, multimedia analysis, and recommendation system, it is common to observe that not all views of samples are available in many cases, which leads to the failure of the conventional multiview clustering methods. Clustering on such incomplete multiview data is referred to as incomplete multiview clustering (IMC). In view of the promising application prospects, the research of IMC has noticeable advances in recent years. However, there is no survey to summarize the current progresses and point out the future research directions. To this end, we review the recent studies of IMC. Importantly, we provide some frameworks to unify the corresponding IMC methods and make an in-depth comparative analysis for some representative methods from theoretical and experimental perspectives. Finally, some open problems in the IMC field are offered for researchers. The related codes are released at https://github.com/DarrenZZhang/Survey_IMC.

Index Terms—Data mining, incomplete multiview clustering (IMC), missing views, multiview learning.

I. INTRODUCTION

IN RECENT years, multiview data collected from diverse sources can be seen everywhere [1]–[4]. As shown in Fig. 1, doctors usually combine the information from magnetic resonance imaging (MRI), positron emission tomography (PET), and cerebrospinal fluid (CSF), for diagnosing Alzheimer’s

disease [1]. Features obtained by different feature extractors, such as texture, color, and local binary patterns, can be also regarded as different views of an image [5]. Generally speaking, different views not only contain complementary information and consistent information but also have many redundant information and inconsistent information [6], [7]. This indicates that it is not a good approach to stack different views into a long vector or treat these views individually in practical applications. To address this issue, the research on multiview learning appears, which aims to jointly explore the information of multiple views for different tasks [8], [9].

As an unsupervised multiview learning method, multiview clustering has received a lot of attention in the fields of computer vision, bioinformatics, and natural language processing [10]–[13]. It focuses on partitioning a set of data samples into different groups according to the underlying information of multiple views. In the past decades, plenty of multiview clustering methods have been proposed based on a common assumption that all views are fully observed, where some of these methods are summarized in [10] and [14]–[17]. However, in many practical applications today, the multiview data to be processed are usually incomplete, i.e., some of the samples do not have all views. For instance, as stated in [18], owing to the patient dropout and poor data quality, some images corresponding to the PET and CSF are not available for some individuals, which generates an incomplete multiview data of Alzheimer’s disease. When some views are missing, the natural alignment property of multiple views will be seriously broken, which is harmful to the excavation of the complementary information and consistent information. In addition, the absence of views leads to a serious of information loss and aggravates the information imbalance among different views. These indicate that the incomplete learning problem is challenging. In fields of multiview clustering, clustering on such incomplete multiview data is referred to as incomplete multiview clustering (IMC).

For IMC, two naive approaches are: 1) removing the samples with missing views and then performing clustering on the remaining samples with fully observed views and 2) setting the missing views as 0 or average instance and then processing the data by conventional multiview clustering methods [19], [20]. The first approach goes against the purpose of clustering which aims at partitioning all data points into their respective clusters [19]. For the second approach, the filled missing instances will play a negative role in the clustering process since all missing instances filled in the same vector will be naturally segmented to the same cluster. Besides

Manuscript received 10 December 2021; revised 23 May 2022; accepted 17 July 2022. This work was supported in part by the Shenzhen Science and Technology Program under Grant RCBS20210609103709020, and in part by the Shenzhen Fundamental Research Fund under Grant JCYJ20190806142416685. This article was recommended by Associate Editor G. Nicosia. (Jie Wen and Zheng Zhang contributed equally to this work.) (Corresponding authors: Bob Zhang; Yong Xu.)

Jie Wen, Zheng Zhang, Yong Xu, and Jinxing Li are with the Shenzhen Key Laboratory of Visual Object Detection and Recognition, Harbin Institute of Technology (Shenzhen), Shenzhen 518055, China (e-mail: jiewen_pr@126.com; darrenzz219@gmail.com; laterfall@hit.edu.cn; lijinxing@cuhk.edu.cn).

Lunke Fei is with the School of Computer Science and Technology, Guangdong University of Technology, Guangzhou 510000, China (e-mail: flksxm@126.com).

Bob Zhang is with the Department of Computer and Information Science, University of Macau, Macau, China (e-mail: bobzhang@um.edu.mo).

Zhao Zhang is with the School of Computer Science and the School of Artificial Intelligence, Hefei University of Technology, Hefei 230000, China (e-mail: cszzhang@gmail.com).

This article has supplementary material provided by the authors and color versions of one or more figures available at <https://doi.org/10.1109/TSMC.2022.3192635>.

Digital Object Identifier 10.1109/TSMC.2022.3192635

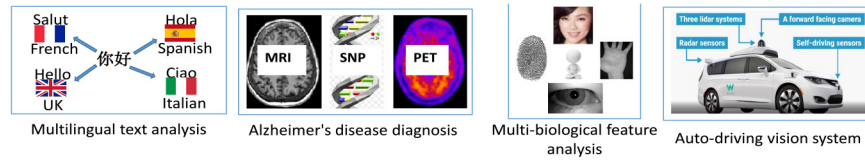


Fig. 1. Some examples of multiview data [10].

this, the experimental results in Section VII also show that the second approach achieves very bad performances, especially for the case with a high missing-view rate. In the past years, many advanced methods, such as cascaded residual autoencoder (CRA) [21] and missing view imputation with generative adversarial networks (VIGANs) [22], are proposed for missing view recovery. CRA stacks many residual autoencoders (AEs) and recovers the missing views by minimizing the residual between the prediction and original data. VIGAN combines the denoising AE and generative adversarial networks (GANs), where denoising AE is used to reconstruct the missing views according to the outputs of GAN. A limitation of VIGAN is that this method is not suitable to process the incomplete data with more than three views. Moreover, recovering the missing views is a promising approach to address the incomplete learning problem but the performance will be highly dependent on the quality of the recovered data.

Motivation and Objectives: In recent years, IMC has received more and more attention and many effective methods have been proposed to address such a challenging problem [23]–[32]. However, there is no survey to systematically summarize the current progresses of IMC. The main purpose of this article is to provide a comprehensive study on IMC and to provide a good start to newcomers who are interested in IMC and its related areas. In this article, almost all of the existing state-of-the-art IMC methods are summarized. Besides this, some representative IMC methods are selected and fairly compared, which lets the readers intuitively observe the clustering performance of these methods. Moreover, some open problems that still have not been solved well are offered for researchers.

Categorization of the Existing Methods: The existing IMC methods can be categorized from different perspectives. For example, from the viewpoint of “missing view recovery,” the existing IMC methods can be categorized into two groups, where the one focuses on addressing the incomplete learning problem by recovering the missing views or missing connections among samples, and the other one does not recover the missing information w.r.t. the missing views but just focuses on the partially aligned information among the available views. From the viewpoint of the main methodologies and learning models exploited in these methods, we can divide the existing IMC methods into four categories, i.e., matrix factorization (MF)-based IMC, kernel learning-based IMC, graph learning-based IMC, and deep learning-based IMC. To summarize more IMC methods and provide an intuitive comparison, we will analyze the existing IMC methods according to the second categorization scheme in this article. Specifically, in our work, we regard all IMC methods that exploit the deep network as

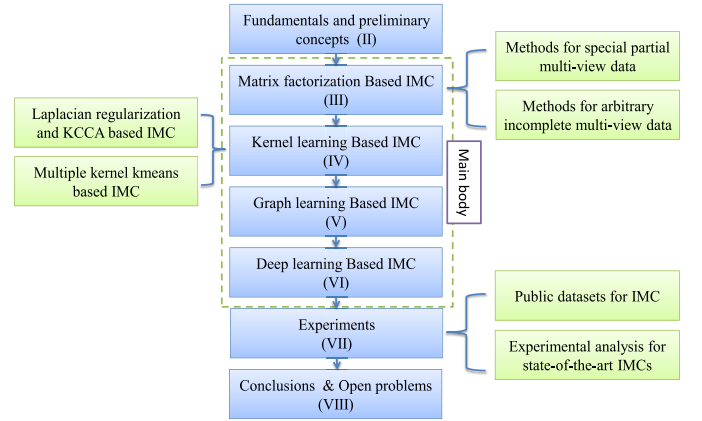


Fig. 2. Structure of this article.

deep learning-based IMC since these methods are very rare in comparison with the other methods. MF-based IMC generally seeks to decompose the multiview data into the consensus representation shared by all views. Kernel learning-based IMC tries to obtain the consensus representation from the incomplete kernel data. Graph learning-based IMC is based on the spectral clustering, which aims to obtain a consensus graph (or several view-specific graphs) from incomplete multiview data or calculate the consensus representation from the incomplete graphs directly.

Organization of This Article: The structure of this article is shown in Fig. 2. We first introduce some preliminary concepts in Section II. In Sections III–VI, we summarize and analyze the four kinds of IMC methods, respectively. In Section VII, we conduct several experiments to compare and analyze the representative IMC methods. In Section VIII, we offer the conclusions and some open problems. More experiments are presented in the supplementary material.

II. FUNDAMENTALS AND PRELIMINARY CONCEPTS

A. Notations

Owing to the space limitation, we move the detailed notations used in this article to Table V of the supplementary file. For a matrix $A \in R^{m \times n}$, its l_1 norm, l_F norm, and $l_{2,1}$ norm are defined as $\|A\|_1 = \sum_{i=1}^m \sum_{j=1}^n |A_{i,j}|$, $\|A\|_F^2 = \sum_{j=1}^n \sum_{i=1}^m A_{i,j}^2$, and $\|A\|_{2,1} = \sum_{i=1}^m (\sum_{j=1}^n A_{i,j}^2)^{1/2}$, respectively. $A \geq 0$ means that all elements of matrix A are non-negative. A^T is the transpose matrix of A . $\text{Tr}(A) = \sum_{i=1}^n A_{i,i}$ is the trace of matrix $A \in R^{n \times n}$. $|Z|$ is the element-wise absolute matrix of Z with all elements as $|Z_{i,j}|$.

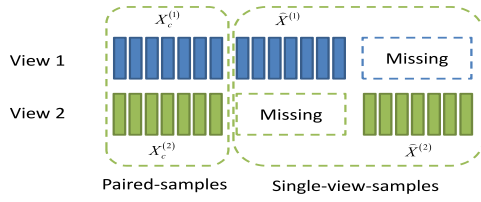


Fig. 3. Incomplete case of data with two views.

B. Basic Background on Single-View/Multiview Clustering

Considering that many IMC methods are derived from the MF-based multiview clustering and multiview spectral clustering, we briefly introduce some related basic knowledge.

MF-Based Multiview Clustering: For multiview data $T^{(v)} \in \mathbb{R}^{m_v \times n}$, a basic assumption is that different views have the same distribution of labels. This is also the so-called semantic consistency of multiple views. A naive MF-based approach is to obtain the consensus representation P as follows [33]:

$$\begin{aligned} \min_{\{U^{(v)}, \bar{P}^{(v)}\}_{v=1}^l, P} & \sum_{v=1}^l \|T^{(v)} - U^{(v)} \bar{P}^{(v)}\|_F^2 + \beta_v \|\bar{P}^{(v)} - P\|_F^2 \\ \text{s.t. } & \{U^{(v)}, \bar{P}^{(v)}, P\} \in \phi \end{aligned} \quad (1)$$

where ϕ and $\varphi(U^{(v)}, \bar{P}^{(v)}, P)$ are the boundary constraint and regularization constraint, respectively.

Multiview Spectral Clustering: Similar to MF-based methods, multiview spectral clustering also aims to obtain the consensus representation of multiview data. Their difference is that multiview spectral clustering seeks to realize such a goal by exploring the information of all graphs jointly, where a naive framework can be formulated as follows [34]:

$$\begin{aligned} \min_{\{\bar{P}^{(v)}\}_{v=1}^l, P} & \sum_{v=1}^l \text{Tr}(\bar{P}^{(v)} L_{Z^{(v)}} \bar{P}^{(v)T}) + \beta_v \varphi(\bar{P}^{(v)}, P) \\ \text{s.t. } & \{\bar{P}^{(v)}, P\} \in \phi \end{aligned} \quad (2)$$

where ϕ is generally defined as the orthogonal constraint. Regularization constraint $\varphi(\bar{P}^{(v)}, P)$ can be chosen as the co-regularizer $\text{Tr}(\bar{P}^{(v)T} P P^T \bar{P}^{(v)})$, which pushes all individual representations toward a consensus representation P by minimizing their disagreements.

C. Categorization of the Incomplete Multiview Data

In this article, we divide the incomplete multiview data into three cases as shown in Figs. 3–5, where Figs. 3 and 4 show the special incomplete multiview data with two views and more than two views, respectively. For these two kinds of incomplete data, samples contain only one view and all views are regarded as the single-view-samples and paired-samples, respectively. Fig. 5 shows the incomplete multiview data with arbitrary missing views.

III. MATRIX FACTORIZATION-BASED IMC

From the conventional MF-based multiview clustering framework (1), we can find that it is impossible to obtain

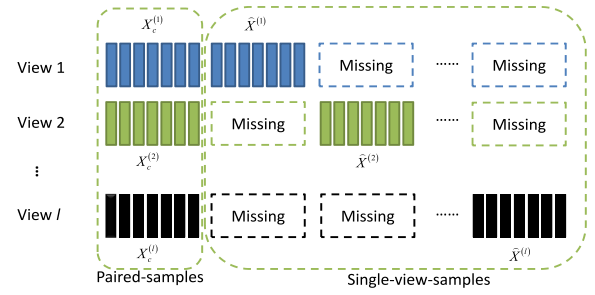


Fig. 4. Special case of incomplete data with more than two views, which only contains paired-samples and single-view-samples.

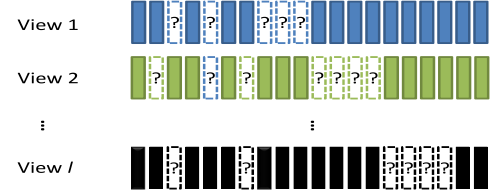


Fig. 5. Incomplete multiview data with arbitrary missing instances.

the consensus representation directly via such a model when some views are missing. For incomplete multiview data, the key problem is how to design an IMC model that can obtain a common cluster indicator matrix or consensus representation from the incomplete multiview data.

In the past years, two approaches are widely considered to design such IMC models based on the MF theory, where one approach aims to explore the consistent information among the partially aligned views, and the other approach focuses on exploring the fully aligned information by recovering the missing views. In this article, we divide the existing MF-based IMC methods into two categories from the viewpoint of application scenario, i.e., MF-based methods for special partial multiview data shown in Figs. 3–4 and weighted MF-based methods for arbitrary incomplete data are shown in Fig. 5.

A. MF-Based Methods for Special Partial Multiview Data

Most of the earliest studies, such as partial multiview clustering (PMVC) [19], incomplete multimodal grouping (IMG) [20], and partial multiview subspace clustering (PMSC) [35], take the incomplete multiview data shown in Fig. 3 as an example to design their models. Among these methods, PMVC is a pioneer work, which seeks to obtain the latent common representation P_c , $P_s^{(1)}$, and $P_s^{(2)}$, for the paired-samples $\{X_c^{(v)}\}_{v=1}^2$ and single-view-samples $\hat{X}^{(1)}$, $\hat{X}^{(2)}$, respectively. However, it ignores the geometric structure of the data, which is important to unsupervised representation learning. IMG [20] and PMSC [35] are two extensions of PMVC, which further introduce the graph embedding technique to capture the geometric structure. In particular, the above three methods can be unified into the following generalized model, referred to as the PMVC framework (PMVCF):

$$\min_{P_c, \{U^{(v)}, P_s^{(v)}\}_{v=1}^l, Z^*} \sum_{v=1}^l \left\| \begin{bmatrix} X_c^{(v)} \\ \hat{X}^{(v)} \end{bmatrix} - U^{(v)} \begin{bmatrix} P_c \\ P_s^{(v)} \end{bmatrix} \right\|_F^2$$

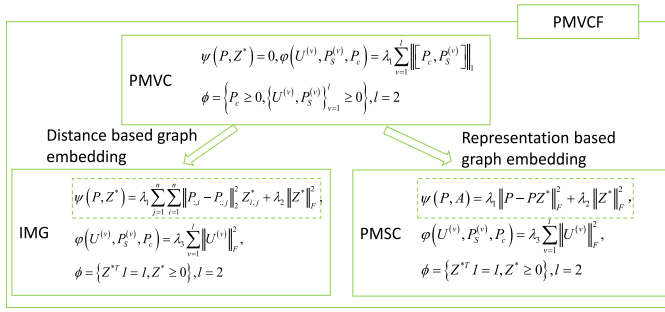


Fig. 6. Connections among PMVCF, PMVC, IMG, and PMSC.

$$+ \psi(P, Z^*) + \varphi(U^{(v)}, P_s^{(v)}, P_c)$$

$$\text{s.t. } \left\{ P_c, \left\{ U^{(v)}, P_s^{(v)} \right\}_{v=1}^l, Z^* \right\} \in \phi \quad (3)$$

where $X_c^{(v)} \in R^{m_v \times n_c}$ denotes the instance set of the paired-samples in the v th view. $\hat{X}^{(v)} \in R^{m_v \times n_{vs}}$ denotes the instance set of samples which only exist in the v th view, n_{vs} denotes the corresponding instance number. $P = [P_c, P_s^{(1)}, \dots, P_s^{(l)}] \in R^{c \times n}$ denotes the consensus representation of all samples, $P_s^{(v)}$ denotes the new representation of $\hat{X}^{(v)}$. $U^{(v)} \in R^{m_v \times c}$ is the basis matrix of the v th view, Z^* denotes the embedded graph of data. $\psi(P, Z^*)$ and $\varphi(U^{(v)}, P_s^{(v)}, P_c)$ are constraints of the corresponding variables. ϕ denotes the boundary constraint of variables.

The connections among PMVCF, PMVC, IMG, and PMSC are shown in Fig. 6, which mainly include the following.

- 1) PMVC, IMG, and PMSC are all based on a partial multiview MF model, i.e., $\min \sum_{v=1}^l \|[X_c^{(v)}, \hat{X}^{(v)}] - U^{(v)}[P_c, P_s^{(v)}]\|_F^2$. Their motivations on addressing the incomplete learning problem are the same, i.e., using the partially aligned information of paired-samples as constraint to obtain the consensus representation shared by all views. Their main differences are the definitions of constraints $\psi(P, Z^*)$, $\varphi(U^{(v)}, P_s^{(v)}, P_c)$, and boundary constraint ϕ .
- 2) Compared with PMVC, IMG and PMSC further explore different kinds of structure information of data by introducing different constraints $\psi(P, Z^*)$.

Generally, IMG and PMSC have the potential to perform better than PMVC by introducing the manifold constraint. However, their computation complexities are much higher than PMVC. Wen *et al.* [23] proposed a graph regularized MF-based method, referred to as IMC_GRMF, which integrates the graph regularization and MF into one item as follows:

$$\min_{\Upsilon} \sum_{v=1}^l \sum_{j=1}^{n_c+n_v} \sum_{i=1}^{n_c+n_v} \|x_i^{(v)} - U^{(v)} P_{:,j}^{(v)}\|_2^2 Z_{ij}^{(v)}$$

$$+ \sum_{v=1}^l \left(\lambda_1 \|P^{(v)} H^{(v)} - P_c\|_F^2 + \lambda_2 \|P^{(v)}\|_1 \right)$$

$$\text{s.t. } U^{(v)T} U^{(v)} = I \quad (4)$$

where $\Upsilon = \{P_c, \{P^{(v)}, U^{(v)}\}_{v=1}^l\}$. $Z^{(v)} \in R^{n_v \times n_v}$ denotes the k -nearest-neighbor graph preconstructed from the available instances of the v th view. $H^{(v)} \in R^{n_v \times n_c}$ is a preconstructed binary matrix according to the view-paired-information, where $H_{i,i}^{(v)} = 1$ ($i = 1, \dots, n_c$) when the first n_c samples are paired samples; otherwise, $H_{i,j}^{(v)} = 0$.

Compared with PMVC, IMG, and PMSC, IMC_GRMF provides a more general model to handle the data with more than two views as shown in Fig. 4. Besides this, IMC_GRMF exploits an indirect approach to obtain the consensus representation from the latent representations of all views, which provides more freedom for representation learning. However, IMC_GRMF is inflexible since it requires the feature dimension $c \leq \min(m_1, \dots, m_l)$ owing to the orthogonal constraint of the basis matrix.

B. MF-Based Methods for Incomplete Data With Arbitrary Missing Views

To handle the incomplete data with arbitrary missing views shown in Fig. 5, many weighted MF-based IMC methods have been proposed, which reduce the negative influence of missing views by imposing some weight matrices preconstructed from the view-missing information of all views on the MF item. The representative works are multiple incomplete views clustering (MIC) [6], online multiview clustering (OMVC) [36], doubly aligned IMC (DAIMC) [37], and one-pass IMC (OPIMC) [36]. Besides these, graph regularized PMVC (GPMVC) [38] proposed by Rai *et al.* can also be regarded as a variant of the weighted MF-based IMC method.

In this article, from the viewpoint of the strategy of achieving the consensus representation, we can unify the existing weighted MF-based IMC methods into the following two frameworks, referred to as WMF_IMCF1 and WMF_IMCF2:

$$\text{WMF_IMCF1} : \min_{\Upsilon} \sum_{v=1}^l \|(Y^{(v)} - U^{(v)} \bar{P}^{(v)}) W^{(v)}\|_F^2$$

$$+ \gamma_v \|(Q^{(v)} \bar{P}^{(v)} - P) W^{(v)}\|_F^2$$

$$+ \varphi(U^{(v)}, \bar{P}^{(v)}, P)$$

$$\text{s.t. } \left\{ P, \left\{ U^{(v)}, \bar{P}^{(v)} \right\}_{v=1}^l \right\} \in \phi \quad (5)$$

$$\text{WMF_IMCF2} : \min_{P, \{U^{(v)}\}_{v=1}^l} \sum_{v=1}^l \|(Y^{(v)} - U^{(v)} P) W^{(v)}\|_F^2$$

$$+ \varphi(U^{(v)}, P)$$

$$\text{s.t. } \left\{ P, \left\{ U^{(v)} \right\}_{v=1}^l \right\} \in \phi \quad (6)$$

where $\Upsilon = \{P, \{U^{(v)}, \bar{P}^{(v)}\}_{v=1}^l\}$. $U^{(v)}$ and $\bar{P}^{(v)} \in R^{c \times n}$ are the basis matrix and latent representation of all instances (i.e., including the available instances and missing instances) of the v th view. $Q^{(v)} \in R^{c \times c}$ is a diagonal weight matrix, which is defined as I in MIC and OMVC, and defined as

$Q_{i,i}^{(v)} = \sum_{j=1}^{m_v} U_{j,i}^{(v)}$ in GPMVC.¹ $W^{(v)}$ is a diagonal weight matrix to reduce the negative influence of the missing views. In the above two frameworks, ϕ denotes the boundary constraint, such as the non-negative constraint in MIC and OMVC. $\phi(U^{(v)}, \bar{P}^{(v)}, P)$ and $\phi(U^{(v)}, P)$ denote the regularization constraint of these variables.

Analysis of the Two Frameworks: MIC, OMVC, and GPMVC are the most representative works of WMF_IMCF1. The most representative methods of WMF_IMCF2 are DAIMC [37] and OPIMC [39]. From models (5) and (6), we can find that WMF_IMCF1 obtains the consensus representation from the latent representations derived from all views, while WMF_IMCF2 directly decomposes the original multiview data into a consensus representation and several basis matrices. Intuitively, compared with WMF_IMCF2, WMF_IMCF1 provides more freedom in learning the consensus representation. However, WMF_IMCF1 introduces at least an extra tunable hyperparameter γ_v , which increases the complexity of the optimal parameter selection.

Connections of the Existing Weighted MF-Based Methods: From Fig. 7, we can observe that these weighted MF methods commonly impose different constraints on the basis matrix or consensus representation for capturing different properties. Among these methods, MIC, OMVC, and OPIMC impose the $l_{2,1}$, l_1 , and l_F norm-based constraints on the latent representations or basis matrices, respectively. These approaches can avoid the trivial solutions, but are not beneficial to improve the discriminability. Compared with these methods, DAIMC and GPMVC attempt to explore more information from the data by imposing the approximate orthogonal constraint or graph constraint, which are beneficial to achieve a better performance. Nevertheless, OMVC and OPIMC have their superiorities in efficiency and memory cost since their objective functions can be optimized in a chunk-by-chunk style. In addition to the above connections, there are many differences among these methods.

- 1) *Filling of the Missing Instance (Definition of $Y^{(v)} \in R^{m_v \times n}$):* Some methods, such as MIC and OMVC, use the average instance to fill in the missing instances. The other methods, such as DAIMC and OPIMC, set all elements of the missing instances as 0.
- 2) *Definition of the Diagonal Weighted Matrices $\{W^{(v)}\}_{v=1}^l$:* Generally, the diagonal element $W_{i,i}^{(v)}$ is set as 1 if the i th sample has the v th view. However, if the v th view is missing for the i th sample, $W_{i,i}^{(v)}$ is set as n_v/n in MIC and 0 in DAIMC. It should be noted that when $W_{i,i}^{(v)}$ is set as n_v/n for the missing instances, the filled virtual instances will affect the consensus representation learning.

¹When 1) the diagonal weighted matrix $W^{(v)}$ is defined as $W_{i,i}^{(v)} = 1$ for the i th sample that has the v th view otherwise $W_{i,i}^{(v)} = 0$; 2) $\phi(U^{(v)}, \bar{P}^{(v)}, P) = \sum_{v=1}^l \beta_v \text{Tr}(\bar{P}^{(v)} G^{(v)} L_{Z^{(v)}} G^{(v)T} \bar{P}^{(v)T})$, where $G^{(v)}$ is defined in Table V in the supplementary file, $L_{Z^{(v)}}$ is the Laplacian matrix constructed from the available instances of the v th view; and 3) $\phi(P, \{U^{(v)}, \bar{P}^{(v)}\}_{v=1}^l) = \{\{U^{(v)}, \bar{P}^{(v)}\}_{v=1}^l \geq 0\}$, then framework (5) is equivalent to the learning model of GPMVC [38].

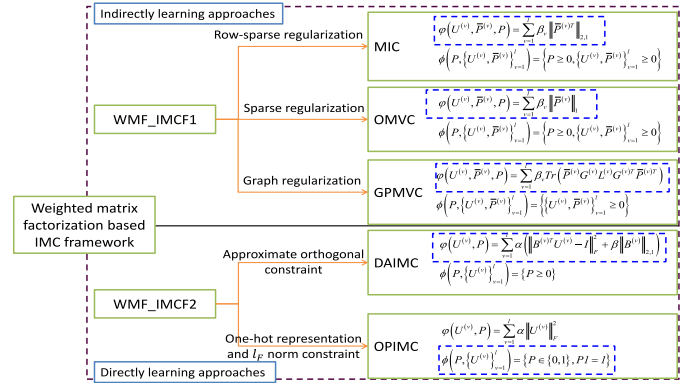


Fig. 7. Frameworks of the weighted MF-based IMC.

Compared with the models of PMVCF, WMF_IMCF1, WMF_IMCF2, and the conventional MF-based multiview clustering model (1), we can find that PMVCF, WMF_IMCF1, and WMF_IMCF2 generally focus on exploring the partially aligned information of the available instances to address the incomplete learning problem, where PMVCF designs the partial MF model and the other two frameworks introduce the weighted constraint. However, these methods suffer from the following problems: 1) they ignore the hidden information of missing views and 2) these methods are all based on the squared Euclidean distance or Frobenius norm, which may be not powerful enough to capture the intrinsic structure of the incomplete multiview data. To address the first issue, two improved methods are proposed from the viewpoint of missing view recovery. For example, Xu *et al.* [40] proposed a very simple missing-view restoration and common-representation joint learning model based on the assumption that all views are generated from a shared subspace. Besides this, Wen *et al.* [26] proposed a missing-view restoration method, called unified embedding alignment framework (UEAF). UEAF can be viewed as an improved version of the method proposed in [40] whose learning model is exactly the first item of UEAF. Compared with the method proposed in [40], UEAF further considers the geometric structure of data and features, and thus has the potential to perform better on missing-view restoration and clustering. However, a shortcoming of UEAF is that it has a relatively high computation complexity. For the second issue, kmeans-based consensus IMC methods proposed in [41] and [42] provide some new ideas, where different distance metrics, such as KL-divergence and Cosine distance, are unified into their proposed frameworks. The two methods can be viewed as the variants of multiview kmeans. Moreover, a common point between the two methods and the previous MF-based frameworks is that they adopt a similar weighted strategy as in WMF_IMCF1 and WMF_IMCF2 to reduce the negative influence of missing views. More specifically, when choosing the square Euclidean distance to measure the matrix decomposition errors, the learning model proposed in [41] can be unified into WFM_IMCF1.

IV. KERNEL LEARNING-BASED IMC

Multiple kernel clustering (MKC) generally seeks to learn a consensus representation or multiple latent representations

corresponding to all views from the multiple kernels preconstructed from all views, followed by kmeans to achieve the clustering results. The conventional MKCs all require that the input kernels are complete. In other words, the existing conventional MKCs cannot handle the kernel clustering tasks where some rows and columns of the preconstructed kernels are absent, caused by the missing views. To solve this problem, many researchers have studied the multiple incomplete kernel clustering (MIKC) in the recent decade.

Most MIKCs solve the incomplete learning problem based on recovering the missing rows and columns of the kernel matrices. In view of the exploited main techniques to solve the incomplete issue, we divide the existing MIKCs into two groups, where the first group is based on the Laplacian regularization and kernel canonical correlation analysis (KCCA), and the second group is based on multiple kernel kmeans.

A. Laplacian Regularization and KCCA-Based IMC

Laplacian regularization and KCCA-based IMC methods generally first perform the kernel matrices completion and then learn the latent representations of all views via KCCA. For example, based on a complete kernel preconstructed from the view without missing instances, Trivedi *et al.* [43] recovered the missing elements of the incomplete kernel by solving the following Laplacian regularized problem:

$$\begin{aligned} \min_{K^{(2)} \geq 0} \quad & \text{Tr}(L^{(1)} K^{(2)}) \\ \text{s.t.} \quad & K_{ij}^{(2)} = k(x_i^{(2)}, x_j^{(2)}) \quad \forall i, j = 1, 2, \dots, n_2 \end{aligned} \quad (7)$$

where $L^{(1)}$ denotes the Laplacian matrix of kernel $K^{(1)} \in R^{n \times n}$ of the 1st view without missing instances. $k(x_i^{(2)}, x_j^{(2)})$ is a value computed by a kernel function $k(*, *)$.

Then, Trivedi *et al.* performed KCCA on the complete kernel $K^{(1)}$ and recovered complete kernel $K^{(2)}$ to obtain the latent representations of two views, followed by kmeans clustering. In this article, we refer to the method proposed by Trivedi *et al.* as MIKC with one complete kernel (MIKC_OCK).

To solve the complete view issue of MIKC_OCK, collective kernel learning (CoKL) is proposed, which interactively recovers the incomplete kernel matrices by optimizing a similar problem as (7) across different views [44]. One of the limitations of CoKL is that it is only suitable to the incomplete data with two views.

In summary, the Laplacian regularization and KCCA-based IMC methods are not suitable to practical applications since these methods can only handle one kind of incomplete cases. Moreover, the two-step-based approach, i.e., perform the kernel matrices completion and latent representations learning separately, cannot guarantee the global optimal kernel matrices and latent representations.

B. Multiple Kernel kmeans-Based IMC

Compared with the Laplacian regularization and KCCA-based IMC methods, multiple kernel kmeans-based IMC methods seek to recover the kernel matrices and learn the consensus representation or cluster index matrix simultaneously

in a joint framework. One of the basic and intuitive multiple incomplete kernel kmeans clustering models proposed in [45] is expressed as follows:

$$\begin{aligned} \min_{\{\alpha^{(v)}, K_{(s_v, s_v)}^{(v)}\}_{v=1}^l, P} \quad & \text{Tr} \left(\sum_{v=1}^l \left(\alpha^{(v)} \right)^2 K^{(v)} (I - PP^T) \right) \\ \text{s.t.} \quad & P^T P = I, \sum_{v=1}^l \alpha^{(v)} = 1, \alpha^{(v)} \\ & \geq 0, K_{(s_v, s_v)}^{(v)} = K_{SV}^{(v)}, K^{(v)} \geq 0 \end{aligned} \quad (8)$$

where $P \in R^{n \times c}$ denotes the consensus representation. s_v ($1 \leq p \leq n_v$) denotes the sample indexes of those instances that are observed in the v th view, $K_{SV}^{(v)}$ denotes the subkernel matrix preconstructed from these available instances. $K^{(v)} \in R^{n \times n}$ is the kernel matrix of the v th view to recover. $K_{(s_v, s_v)}^{(v)}$ denotes the elements corresponding to the observed instances.

For convenience, we refer to the method proposed in [45] as incomplete multiple kernel kmeans clustering (IMKKC). From (8), we can observe that the main idea of IMKKC is to align the fused kernel $\sum_{v=1}^l (\alpha^{(v)})^2 K^{(v)}$ and ideal kernel PP^T . Different from IMKKC, another similar basic model, referred to as consensus kernel kmeans-based IMC (CKK-IMC) proposed in [46] seeks to obtain the consensus P from the latent representations $\{P^{(v)} \in R^{n \times c}\}_{v=1}^l$ by introducing the dissimilarity-based regularization item. The main issues of IMKKC and CKK-IMC are that the two methods ignore the local structure of the data and do not sufficiently consider the complementary information of views. Besides these, IMKKC has a relatively high computational complexity of $O(n^3)$. Based on IMKKC, many improved methods, such as localized IMKKC (LIMKKC) [32] and incomplete multiple kernel kmeans with mutual kernel completion (MKKM-IK-MKC) [28], are proposed. In particular, LIMKKC mainly introduces some neighborhood indication matrices to IMKKC to preserve the local information of the data. In order to recover the missing rows and columns of the kernel matrices better, MKKM-IK-MKC introduces a sparse reconstruction-based constraint to model (8) to capture more complementary information from the kernels.

Generally, the above kernel completion-based methods need to recover $(1/2)(n - n_v)(n + n_v + 1)$ elements for the v th view with n_v observed instances. However, recovering such a large number of elements may trap the model into a local minimum, which affects the clustering performance [29]. Liu *et al.* [29] proposed another multiple kernel-based approach, named efficient and effective IMC (EE-IMVC), to address the IMC problem. Different from the above multiple kernel kmeans-based methods, EE-IMVC does not focus on recovering the kernel matrices. It seeks to jointly compute the latent representations corresponding to the missing instances of each view and obtain the consensus representation $P \in R^{n \times c}$ as follows:

$$\begin{aligned} \max_{\Upsilon} \quad & \text{Tr} \left(P^T \sum_{v=1}^l \alpha^{(v)} \begin{bmatrix} \bar{P}^{(v)} \\ \bar{P}^{(o)} \\ \bar{P}^{(u)} \end{bmatrix} Q^{(v)} \right) \\ \text{s.t.} \quad & P^T P = I, Q^{(v)T} Q^{(v)} = I, \end{aligned}$$

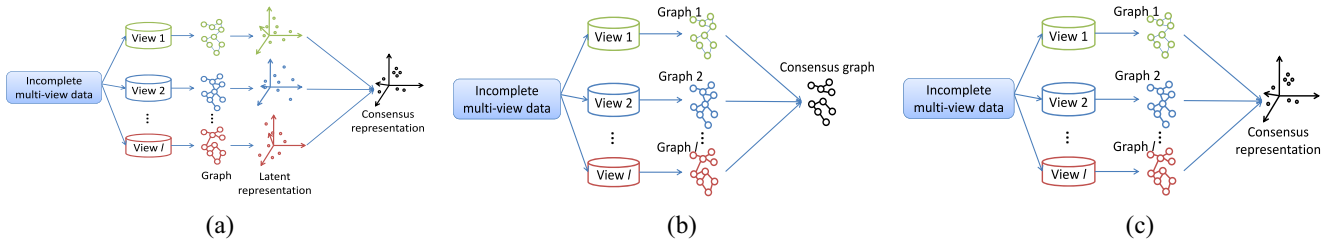


Fig. 8. Three kinds of graph learning-based IMC methods, where (a) and (c) focus on obtaining the consensus representation, and (b) seeks to obtain the consensus graph from the incomplete data.

$$\bar{P}_{(u)}^{(v)T} \bar{P}_{(u)}^{(v)} = I, \sum_{v=1}^l \left(\alpha^{(v)} \right)^2 = 1 \quad (9)$$

where $\Upsilon = \{P, \{Q^{(v)}, \bar{P}_{(u)}^{(v)}, \alpha^{(v)}\}_{v=1}^l\}$. Permutation matrix $Q^{(v)} \in R^{c \times c}$ is used to match $\bar{P}^{(v)} = \begin{bmatrix} \bar{P}_{(o)}^{(v)} \\ \bar{P}_{(u)}^{(v)} \end{bmatrix}$ and P . $\bar{P}_{(o)}^{(v)}$ and $\bar{P}_{(u)}^{(v)}$ are the latent representations of the observed instances and missing instances of the v th view, respectively.

Compared with the other multiple kernel kmeans-based methods, EE-IMVC greatly reduces the computational complexity and memory cost. However, it cannot bring the nearest sample pairs closer and push the other sample pairs far away.

V. GRAPH LEARNING-BASED IMC

Similar to the conventional multiview spectral clustering, the purpose of graph learning-based IMC is to obtain a consensus graph or consensus representation from multiple incomplete graphs constructed from the data, where almost all of the existing methods are based on the preconstructed incomplete graphs and the missing similarity elements are set as 0 or average values. In particular, we can divide the existing methods into three categories as shown in Fig. 8.

As a representative method of Fig. 8(a), spectral clustering-based IMC (SCIMC) exploits a very simple co-training approach to recover the latent representations of missing instances and obtain the consensus representation [47]. Given the similarity matrices $\{Z^{(v)}\}_{v=1}^l$ with a size of $n \times n$ for each view, where missing entries are filled in the average of the column vectors corresponding to the observed instances, SCIMC seeks to obtain the consensus representation $P \in R^{n \times c}$ by optimizing the following four problems one by one:

$$\begin{cases} \max_{\bar{P}^{(v)T} \bar{P}^{(v)} = I} \text{Tr}(\bar{P}^{(v)T} L^{(v)} \bar{P}^{(v)}) \\ \max_{P^T P = I} \sum_{v=1}^l \lambda_v \text{Tr}(P P^T \bar{P}^{(v)} \bar{P}^{(v)T}) \\ \max_{\bar{P}_m^{(v)}} \text{Tr} \left(\begin{bmatrix} \bar{P}_a^{(v)} \\ \bar{P}_m^{(v)} \end{bmatrix} \begin{bmatrix} \bar{P}_a^{(v)} \\ \bar{P}_m^{(v)} \end{bmatrix}^T \begin{bmatrix} P_a \\ P_m \end{bmatrix} \begin{bmatrix} P_a \\ P_m \end{bmatrix}^T \right) \\ \max_{P^T P = I} \sum_{v=1}^l \lambda_v \text{Tr}(P P^T \bar{P}^{(v)} \bar{P}^{(v)T}) \end{cases} \quad (10)$$

where $L^{(v)}$ is a symmetrical normalized matrix calculated as $L^{(v)} = (D^{(v)})^{-1/2} Z^{(v)} (D^{(v)})^{-1/2}$ and diagonal matrix $D_{i,i}^{(v)} = \sum_{j=1}^n Z_{i,j}^{(v)}$. In the third step, P and $\bar{P}^{(v)}$ are reordered as $\begin{bmatrix} P_a \\ P_m \end{bmatrix}$

and $\begin{bmatrix} \bar{P}_a^{(v)} \\ \bar{P}_m^{(v)} \end{bmatrix}$, where $\bar{P}_a^{(v)}$ and $\bar{P}_m^{(v)}$ denote the latent representation of the available instances and missing instances of the v th view, respectively.

From (10) and the conventional multiview spectral clustering model (2), we can find that SCIMC divides the conventional model (2) into several independent steps and seeks to alternately recover the latent representation corresponding to the missing views by borrowing the information of the other views. Then, it combines the latent representations of all views to achieve the consensus representation by minimizing the disagreement of different views. In (10), the first two steps can be viewed as the initialization steps. Although SCIMC can handle the IMC problem, it suffers from three issues: 1) it cannot achieve the optimal consensus representation by optimizing the four problems independently; 2) it is unreasonable to initialize the similarity graphs by setting the missing entries as the average of the columns; and 3) SCIMC is sensitive to the preconstructed similarity graph $\{Z^{(v)}\}_{v=1}^l$.

Wang *et al.* [48] proposed a perturbation-oriented IMC (PIC) method based on the spectral perturbation theory, which is a representative work of Fig. 8(b). For any incomplete data, PIC mainly adopts the following three steps to obtain the consensus graph for spectral clustering.

- 1) *Nearest-Neighbor Graph Construction and Completion*: PIC provides a graph learning method to directly obtain some nearest-neighbor graphs $\{Z^{(v)} \in R^{n \times n}\}_{v=1}^l$, where some rows of $Z^{(v)}$ associated to the missing instances are set as the average of the rows corresponding to the other available views.
- 2) *Fusion Weight Calculation*: PIC establishes a perturbation-oriented model to learn some coefficients for graph fusion.
- 3) *Consensus Graph Learning*: PIC obtains the consensus graph by fusing these multiple graphs with the learned coefficients. PIC can handle all kinds of incomplete multiview data. However, since the graph construction, weight calculation, and consensus graph learning are independent of each other, PIC is also sensitive to the quality of the preconstructed graphs.

Recently, Zhou *et al.* [27] proposed a consensus graph learning approach for IMC (CGL_IMC). Different from PIC, CGL_IMC provides a weighted graph learning approach to construct the similarity graphs $\{Z^{(v)} \in R^{n \times n}\}_{v=1}^l$ of all incomplete views. Then, it obtains the consensus graph from the

following Laplacian regularized graph learning framework:

$$\begin{aligned} \min_{Z^*, \alpha^{(v)}} \quad & \sum_{v=1}^l \alpha^{(v)} \|Z^* - Z^{(v)}\|_F^2 + 2\beta \text{Tr}(F^T L_{Z^*} F) \\ \text{s.t.} \quad & \alpha^{(v)} \geq 0, \sum_{v=1}^l \alpha^{(v)} = 1, F^T F = I, Z^* \geq 0, Z^{*T} I = I \end{aligned} \quad (11)$$

where $Z^* \in R^{n \times n}$ denotes the consensus graph.

By introducing the variant of rank constraint $\text{Tr}(F^T L_{Z^*} F)$, CGL_IMC has the potential to learn the optimal consensus graph that has exactly c blocks for the data with c clusters. However, similar to PIC, CGL_IMC is also sensitive to the quality of the preconstructed graphs.

Wen *et al.* [25] proposed a spectral clustering-based method, named incomplete multiview spectral clustering with adaptive graph learning (IMSC_AGL). As shown in Fig. 8(c), IMSC_AGL tries to directly obtain the consensus representation from multiple graphs adaptively learned from the incomplete data. In particular, different from SCIMC and PIC, IMSC_AGL integrates the adaptive graph construction and spectral-based consensus representation learning into a joint optimization framework, which can naturally address the issues of PIC and SCIMC. The learning model of IMSC_AGL is as follows:

$$\begin{aligned} \min_{Z^{(v)}, E^{(v)}, P} \quad & \sum_{v=1}^l \left(\|Z^{(v)}\|_* + \lambda_2 \|E^{(v)}\|_1 \right) \\ & + \lambda_1 \sum_{v=1}^l \text{Tr}(P G^{(v)} L_{Z^{(v)}} G^{(v)T} P^T) \\ \text{s.t.} \quad & X^{(v)} = X^{(v)} Z^{(v)} + E^{(v)}, Z^{(v)} I = I, \\ & 0 \leq Z^{(v)} \leq 1, Z_{i,i}^{(v)} = 0, P P^T = I \end{aligned} \quad (12)$$

where $P \in R^{c \times n}$ is the consensus representation. $L_{Z^{(v)}}$ is the Laplacian matrix of graph $Z^{(v)}$. $G^{(v)}$ is defined in Table V in the supplementary file.

From the second item of the learning model (12), we can observe that IMSC_AGL does not introduce any uncertain information to guide the consensus representation learning like SCIMC and PIC. This ensures that IMSC_AGL obtains a more reasonable consensus representation. However, IMSC_AGL suffers from the issue of high computational complexity, which is not suitable for large-scale datasets.

All of the above graph learning-based methods commonly separate the model optimization and clustering into two independent steps, which need to implement kmeans as post-processing to partition data into respective groups. Zhuge *et al.* [49] proposed a unified framework, named simultaneous representation learning and clustering (SRLC), which incorporates the graph-based representation learning and label prediction into a joint framework. Compared with the previous works, SRLC has the potential to obtain the global optimal clustering labels. However, its performance is also sensitive to the quality of graphs preconstructed from data.

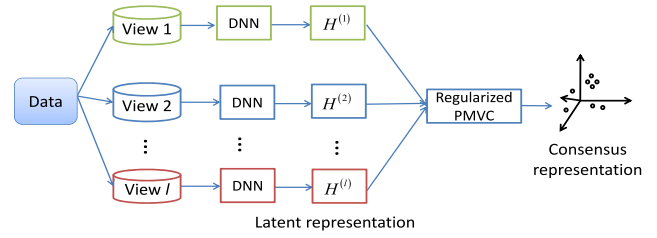


Fig. 9. Framework of IMC_DSM. For simplicity, we refer to the objective function (13) as regularized PMVC.

Generally speaking, compared with MF-based methods, graph learning-based methods can better excavate the geometric information of the data. However, since the graph learning-based methods need to implement some relatively time-consuming operations, such as eigenvalue computation, singular value decomposition, and matrix inverse operation, these methods may not be suitable for large-scale datasets. Thus, it is necessary to develop some efficient graph learning algorithms for IMC. In addition, in view of the fact that there are a lot of information residing in the data, such as the global information and local information, it is better to sufficiently consider these valuable information in IMC models.

VI. DEEP LEARNING-BASED IMC

As presented in the previous section, representation learning plays an important role in most of the existing IMC methods. Learning a more discriminative consensus representation is crucial to achieve a better performance. In recent years, deep learning has been successfully applied in many fields of computer vision and pattern classification owing to its good performance in learning a high-level feature representation. To this end, researchers seek to combine the deep learning and conventional IMC approach to improve the performance, where the most representative works are IMC via deep semantic mapping (IMC_DSM) [50], PMVC via consistent GANs (PMVC_CGAN) [51], and adversarial IMC (AIMC) [52].

IMC_DSM integrates the deep neural networks (DNNs)-based feature extraction, PMVC, and local graph regularization into a framework as shown in Fig. 9. The objective function of IMC_DSM is expressed as follows:

$$\begin{aligned} \min_{P_c, \{U^{(v)}, P_s^{(v)}\}_{v=1}^l} \quad & \sum_{v=1}^l \left\| \begin{bmatrix} A_c^{(v)} \\ A_s^{(v)} \end{bmatrix} - U^{(v)} \begin{bmatrix} P_c \\ P_s^{(v)} \end{bmatrix} \right\|_F^2 \\ & + \lambda_v \text{Tr}(P^{(v)} L_{Z^{(v)}} P^{(v)T}) \\ \text{s.t.} \quad & U^{(v)} \geq 0, P^{(v)} = \begin{bmatrix} P_c \\ P_s^{(v)} \end{bmatrix} \geq 0 \end{aligned} \quad (13)$$

where $A^{(v)} = [A_c^{(v)}, A_s^{(v)}] = f(\vartheta^{(v)} X^{(v)} + b^{(v)})$ denotes the feature representation produced by DNN in the v th view. $L_{Z^{(v)}}$ is the Laplacian matrix of the nearest-neighbor graph preconstructed from the available instances of the v th view. $\vartheta^{(v)}$ and $b^{(v)}$ are the network weights and bias, respectively.

It is easy to observe that: 1) the objective function (13) is a special case of our unified framework (5) and 2) IMC_DSM addresses the incomplete learning problem by exploring the partially aligned information among the available views as the conventional PMVCF introduced in the previous section. In

IMC_DSM, DNN can extract the high-level features from the data and the local graph regularization item is beneficial to obtain a more reasonable structured representation.

PMVC_CGAN provides another clustering approach for incomplete two-view data based on AE and GAN. PMVC_CGAN mainly contains three components: 1) it exploits AE to generate the latent representations of all views; 2) GAN is introduced to generate the missing views; and 3) a “KL-divergence”-based loss function is introduced to guarantee that the learned latent representation is suitable for the clustering task. In particular, PMVC_CGAN provides many interesting points in comparison with the existing works, including IMC_DSM. For example, existing methods need to implement kmeans on the obtained representation for achieving the clustering result, while PMVC_CGAN can directly produce the final clustering result according to the KL-divergence. In addition, almost all of the existing works ignore the information of missing views, while PMVC_CGAN can sufficiently exploit it for model training via GAN. Moreover, the most difference between PMVC_CGAN and the existing works is that PMVC_CGAN focuses on clustering, while the other works focus on learning the consensus representation or graph. To address the issue that PMVC_CGAN cannot handle the incomplete data with more than two views, Xu *et al.* proposed AIMC. AIMC can be viewed as an extension of PMVC_CGAN for the case of more than two views since it exploits a similar framework as PMVC_CGAN. However, AIMC requires that some samples must have complete views.

From the above presentations, we can observe that: 1) exploiting the DNN is beneficial to learn more discriminative representations so as to improve the clustering performance; 2) the deep-based methods can be applied to large-scale datasets owing to the batch-based training style; and 3) the biggest shortcoming of the existing deep-based methods is that these deep-based methods cannot be applied to all kinds of incomplete cases.

In Table VI of the supplementary material, we summarize the main merits and drawbacks of some representative IMC methods mentioned above (refer to Sections III–VI). From the presentation of these methods, we can obtain the following two points: 1) imposing some weighted matrices constructed from the view-missing information on the conventional multiview clustering methods is an effective and flexible approach to address the incomplete learning problem and 2) designing some robust models to recover the missing views or missing elements in the kernels or graphs is a valuable research direction to address the IMC problem. In the second approach, a challenging problem is how to ensure the rationality of missing-view or missing-element recovery theoretically.

VII. EXPERIMENTS

In this section, we mainly conduct several experiments to compare and analyze some representative IMC methods presented in the previous sections. The compared methods include best single view (BSV) [20], Concat [20], multiview non-negative MF (MultiNMF) [33], auto-weighted multiple graph learning (AMGL) [53], multiview clustering

with adaptive neighbors (MLAN) [54], centroid-based co-regularized multiview spectral clustering (CCo-MVSC) [34], seven MF-based methods (i.e., PMVC [19], IMG [20], IMC_GRMF [23], MIC [6], OMVC [36], DAIMC [37], and OPIMC [39]), two kernel learning-based methods (i.e., CoKL [44] and MKKM-IK-MKC [28]), and two graph learning-based methods (i.e., PIC [48] and IMSC_AGL [25]). MultiNMF is a popular non-negative MF multiview clustering method. CCo-MVSC, AMGL, and MLAN are graph-based multiview clustering method, which requires the full observation of views. For CCo-MVSC, the elements corresponding to the missing views in the graph are set as 0. For BSV, Concat, MultiNMF, AMGL, and MLAN, the missing instances are filled in the average instance of the corresponding view. BSV performs kmeans on all views separately and then reports the highest clustering results. Concat stacks all views of a sample into a long vector and then performs kmeans on it to obtain the clustering results. For CoKL and MKKM-IK-MKC, we exploit the “Gaussian kernel” to construct the kernel matrices.

A. Datasets

In this section, we first list some public available multiview datasets and their URLs for researchers. We divide the public datasets into two categories.

- 1) *Multiple-Features*: In this category, some kinds of features are extracted from the same object such as document and image, as different views. Representative public multiple-features-based multiview datasets include: Handwritten digit images,² Corel dataset for image retrieval,³ Caltech101 and NUSWIDE datasets for object recognition,⁴ animal with attributes (AWA) dataset for animal classification,⁵ and some document datasets, such as Cora, CiteSeer, WebKB, and Newsgroup datasets,⁶ and BBCSport dataset.⁷
- 2) *Multiple Modalities*: Data in this category are collected from diverse domains or by different tensors. Some representative multiple modalities-based multiview datasets include: Columbia consumer video (CCV) dataset for consumer video/audio analysis,⁸ BUAA-avisnir face dataset (BUAA),⁹ Berkeley Drosophila Genome Project (BDGP) for gene expression analysis,¹⁰ Reuters dataset for large-scale multilingual text analysis,¹¹ and 3 Sources dataset.¹² Among these datasets, BBCSport and 3 Sources datasets are naturally incomplete.

In our experiments, the following five representative public datasets are selected to compare different IMC methods,

²<https://archive.ics.uci.edu/ml/datasets/Multiple+Features>

³<https://archive.ics.uci.edu/ml/datasets/Corel+Image+Features>

⁴https://drive.google.com/drive/folders/1O_3YmthAZGiq1ZPSdE74R7Nwos2PmnHH

⁵<https://archive.ics.uci.edu/ml/datasets/Corel+Image+Features>

⁶<http://lig-membres.imag.fr/grimal/data.html>

⁷<http://mlg.ucd.ie./datasets/bbc.html>

⁸<http://www.ee.columbia.edu/ln/dvmm/CCV/>

⁹<https://github.com/hdzhao/IMG/tree/master/data>

¹⁰<http://ranger.uta.edu/~heng/Drosophila/data/>

¹¹<https://archive.ics.uci.edu/ml/datasets/Corel+Image+Features>

¹²<http://mlg.ucd.ie./datasets/3sources.html>

where their information is briefly summarized in Table I of the supplementary file.

- 1) *BUAA* [55]: Face images in the BUAA dataset are collected by visual and near-infrared cameras, which can be naturally regarded as two kinds of views of the same person. In our experiments, we select the subset composed of 90 visual and near-infrared images from the first ten classes as that in [20] to evaluate the above representative IMC methods.
- 2) *BDGP* [56], [57]: BDGP is designed for the research of gene expression. In our experiments, a multiview BDGP dataset collected by Cai *et al.* [56] is chosen, where each sample is represented by the texture feature and three kinds of bag-of-words features extracted from the lateral, dorsal, and ventral images, respectively.
- 3) *Caltech101* [58]: Caltech101 is a popular object dataset that contains 9144 images from 102 categories, including a background and 101 objects, such as airplanes, ant, bass, and beaver. In our experiments, four kinds of feature sets, i.e., Cenhist, Hog, Gist, and LBP, extracted by Li *et al.* are chosen as four views [59].
- 4) *BBCSport* [60]: BBCSport is a text dataset collected from the BBC Sport website corresponding to five kinds of sports news articles (i.e., athletics, cricket, football, rugby, and tennis) [60]. We select a subset¹³ which contains 116 samples represented by four views for evaluation [38].
- 5) *NUSWIDE* [61]: NUSWIDE is a real-world Web image dataset collected by the researchers from the National University of Singapore. In our experiments, a multiview subset containing 30000 images and 31 classes is adopted, in which each image is represented by five kinds of low-level features, i.e., color histogram, color correlogram, edge direction histogram, wavelet texture, and block-wise color moments.

B. Experimental Setting

Evaluation Metrics: Four well-known metrics, i.e., clustering accuracy (ACC), normalized mutual information (NMI), purity, and adjusted rand index (ARI), are chosen to assess the above methods [28], [62]–[64].

Incomplete Multiview Data Construction: For BUAA dataset, we randomly select $p\%$ samples as paired samples that have both two views, where p is defined as $\{10, 30, 50\}$. For the remaining $(1-p\%)$ samples, we remove the near-infrared view for half of the samples and remove the visual view for the other half. For the other four datasets, under the condition that all samples at least have one view, we randomly delete $p\%$ instances from every view to construct the incomplete data. Specifically, for each missing rate or paired rate $p\%$, we implement all compared methods on several groups of the randomly constructed incomplete data, then report the average and standard deviations of the clustering results.

C. Experimental Results and Analysis

Experimental results in terms of ACC, NMI, and purity, of different methods on the BUAA and BDGP are listed in Table I. For Caltech101, BBCSport, and NUSWIDE datasets, the experimental results of different methods are listed in Table II in the supplementary material owing to the space limitation. The results in terms of ARI are shown in Fig. 1 in the supplementary material. It should be noted that since four graph learning-based methods and a kernel learning-based method, e.g., AMGL, MLAN, MKKM-IK-MKC, PIC, and IMSC_AGL, require a very large memory and report “out of memory error” on our computer with 64-GB RAM and Win 10 system, we do not report the experimental results of these methods on the large-scale NUSWIDE dataset in Table II and Fig. 1 in the supplementary material. From the experimental results, we have the following observations.

- 1) In most cases, all of the multiview learning-based IMC methods perform better than BSV and Concat on the first four datasets. In addition, the four complete multiview clustering methods, i.e., MultiNMF, CCo-MVSC, AMGL, and MLAN, perform worse than the IMC methods, such as DAIMC, IMC_GRMF, PIC, and IMSC_AGL. These two phenomena demonstrate that: a) all of the multiview learning-based IMC methods have the potential to capture more information from the incomplete multiview data than BSV and Concat; b) simply setting the missing instances or graphs as 0 or average instance is not a good choice to address the incomplete clustering problem; and c) sufficiently exploring the aligned information among the available views is an effective approach to address the incomplete learning problem for the difficult IMC tasks.
- 2) On the BUAA, BBCSport, and Caltech101 datasets, PIC and IMSC_AGL generally perform much better than the MF-based IMC methods in terms of ACC, NMI, and purity. However, on the BDGP dataset, these two graph learning-based methods and the kernel learning-based method, e.g., MKKM-IK-MKC, perform worse than the other methods. Moreover, we can observe that PIC, IMSC_AGL, and MKKM-IK-MKC are not suitable to the clustering task of the NUSWIDE dataset on a computer with 64-GB RAM memory. By analyzing the original features of the data, we observe that BDGP is a very different dataset from the other four datasets since some original instances in this dataset are naturally unavailable and set as a zero vector. On this special dataset with some missing views, it is difficult to obtain the high-quality kernels or graphs by Gaussian kernel or the distance-based graph construction scheme. This is the major reason to the bad performance obtained by the kernel and graph-based IMC methods. According to these phenomena, we can infer that: a) in most cases, the graph learning-based method can obtain a more discriminative representation than the MF-based IMC methods for clustering; b) capturing the correct geometric structure information of data is very important for the unsupervised clustering tasks; and c) compared

¹³<https://github.com/GPMVCDummy/GPMVC/tree/master/partialMV/PVC/recreateResults/data>

TABLE I
EXPERIMENTAL RESULTS W.R.T. ACC (%), NMI (%), AND PURITY (%) OF DIFFERENT METHODS ON THE BUAA AND BDGP DATASETS

Data	Method	ACC			NMI			Purity		
		0.1	0.3	0.5	0.1	0.3	0.5	0.1	0.3	0.5
BUAA	BSV	48.33±3.70	56.96±4.93	64.26±5.19	43.10±3.77	53.03±3.98	61.78±3.85	50.19±3.77	58.66±3.46	65.96±4.05
	Concat	45.62±2.54	46.61±3.43	47.46±3.29	51.22±1.85	51.95±2.78	52.43±2.71	47.99±2.30	49.41±3.17	49.68±3.16
	MultiNMF	39.78±5.69	44.44±6.09	50.44±4.35	40.96±4.86	43.55±5.54	49.03±5.95	42.22±5.09	46.67±5.50	52.67±4.62
	CCo-MVSC	56.74±4.91	67.28±3.69	72.60±3.46	59.75±4.81	69.36±2.94	75.31±2.41	59.29±4.74	69.13±3.61	74.90±3.19
	AMGL	48.78±5.14	55.11±4.92	61.78±4.51	46.93±4.22	53.90±5.22	59.24±2.87	52.00±4.62	58.11±4.48	64.22±2.76
	MLAN	51.78±5.95	51.44±5.78	54.89±5.59	52.49±5.09	48.50±5.58	48.75±6.17	54.56±4.46	53.67±5.11	55.44±5.63
	PMVC	58.00±3.47	65.02±5.99	70.62±2.20	60.40±3.25	68.19±5.45	72.86±2.43	60.44±3.81	67.16±5.46	72.89±1.77
	IMG	54.89±5.89	67.39±5.79	74.00±5.41	53.46±5.55	67.53±5.94	76.26±3.53	56.74±5.07	68.65±3.46	76.15±5.12
	IMC_GRMF	66.89±7.43	76.44±4.40	82.44±2.88	66.84±7.24	76.91±4.37	80.20±2.51	69.11±5.47	78.00±4.67	82.44±2.88
	MIC	56.58±4.48	66.04±0.96	71.02±8.61	59.01±4.08	68.39±0.44	71.66±8.26	59.11±4.53	69.02±0.78	73.02±8.02
	OMVC	58.37±6.77	62.67±5.10	64.74±5.91	59.41±5.40	60.82±3.89	63.89±4.96	60.22±4.43	63.41±5.02	66.89±5.20
	DAIMC	57.56±7.79	65.33±7.30	76.67±4.84	57.65±7.30	69.07±5.95	80.62±2.99	58.89±6.89	69.11±6.40	79.11±3.72
	OPIMC	47.33±7.01	52.67±4.13	54.00±6.01	49.42±6.12	55.26±4.24	55.16±7.62	50.44±5.91	55.33±4.87	56.89±5.85
	CoKL	21.33±0.93	29.56±3.74	38.89±3.96	12.48±1.21	22.48±2.54	34.66±3.79	23.78±0.99	32.44±2.87	41.73±4.49
	MKKM-IK-MKC	69.24±3.41	88.09±6.31	91.11±3.86	70.79±2.62	86.26±5.67	88.98±3.77	70.89±2.70	88.09±6.31	91.11±3.86
BDGP	PIC	64.44±6.82	80.40±3.66	90.44±2.68	66.79±3.08	80.99±1.78	88.91±2.53	67.16±5.93	82.80±1.90	90.44±2.67
	IMSC_AGL	74.89±6.41	81.11±5.21	84.89±5.75	74.13±3.73	78.61±3.99	82.80±4.96	75.89±4.90	81.33±4.80	84.89±5.75
	BSV	51.48±3.96	41.44±3.55	34.74±1.52	35.74±4.01	25.20±2.70	16.39±1.36	53.16±3.94	43.75±2.63	35.92±1.32
	Concat	57.66±4.79	50.04±1.58	40.41±3.52	44.58±4.78	31.81±1.45	19.76±1.78	59.68±4.71	51.14±1.45	41.81±2.79
	MultiNMF	29.14±1.44	30.39±1.33	31.60±0.40	7.97±0.90	10.07±1.10	8.54±0.65	29.15±1.42	30.39±1.33	31.68±0.42
	CCo-MVSC	61.50±5.64	57.63±5.30	52.03±3.40	44.37±2.02	35.78±2.58	27.03±1.94	64.84±3.83	60.03±3.81	53.93±2.36
	AMGL	53.55±6.46	44.99±6.67	47.50±4.44	44.63±7.98	28.61±7.48	28.89±4.87	57.52±6.68	46.62±6.64	47.71±4.28
	MLAN	32.24±4.64	31.53±5.63	27.37±1.62	13.19±4.38	12.46±5.28	7.92±1.53	32.84±4.14	32.24±5.16	27.90±1.49
	MIC	48.31±0.83	40.88±1.18	34.02±1.42	28.52±0.49	23.94±1.21	11.05±0.89	48.54±0.87	43.57±1.05	34.95±1.24
	OMVC	55.23±4.55	46.22±3.15	39.46±1.12	28.78±1.59	19.44±1.20	13.51±1.21	53.23±4.55	46.34±2.89	39.46±1.12
	DAIMC	77.34±2.58	69.30±6.42	52.45±8.57	55.64±2.68	47.87±4.65	28.33±1.38	77.34±2.58	69.34±6.41	52.88±7.73
	OPIMC	79.38±7.69	63.73±7.29	55.17±9.24	61.77±7.41	41.47±3.88	25.94±7.29	79.38±7.69	65.11±6.13	55.54±8.50
	MKKM-IK-MKC	31.80±1.68	29.12±0.29	29.44±1.39	7.21±1.10	5.86±0.56	6.65±1.19	32.97±1.60	30.15±0.64	30.70±1.45
	PIC	32.92±9.14	22.20±1.60	23.52±0.56	10.17±9.43	1.97±1.58	1.94±0.64	33.16±9.28	22.78±1.79	23.95±0.81
	IMSC_AGL	57.64±6.22	52.08±9.20	46.68±4.64	34.77±3.96	27.04±7.77	21.50±3.50	59.37±4.43	53.29±8.28	48.97±4.14

with the MF-based methods, the graph and kernel-based methods need to compute several $n * n$ graph/kernel matrices for the data with n samples, which require a large storage memory.

Moreover, from these observations, we can obtain that no IMC methods can maintain consistently good performance on all kinds of datasets. Therefore, it is significant to choose a suitable algorithm for different datasets.

VIII. CONCLUSION

The incomplete learning problem is challenging in multi-view clustering and its research is significant for practical applications. This article reviewed almost all of the representative IMC methods and divided them into four categories, i.e., MF-based IMC, kernel learning-based IMC, graph learning-based IMC, and deep learning-based IMC. We not only briefly introduced some representative IMC methods but also discussed their connections, differences, advantages, and disadvantages in depth. For the MF-based IMC methods, some unified frameworks were provided to integrate them.

Although many IMC methods have been proposed in the past decades, several challenging problems are still not solved well.

1) *Large-Scale Problem*: In practical clustering tasks, such as recommendation system and financial data analysis, the sample number of the collected data is often very large. However, most IMC methods suffer from a high computational complexity and memory cost, especially for the kernel-based and graph-based methods. As a result, the existing IMC methods are not applicable to large-scale datasets. Although some

methods, such as OMVC and OPIMC, provide the chunk-by-chunk approach to reduce the memory requirement, the performance of these methods is not guaranteed. Therefore, it is significant to design some efficient methods that simultaneously consider the efficiency and performance for large-scale IMC tasks.

2) *Information Imbalance Problem*: Owing to the random absence of views, different views often have different numbers of instances. In addition, different views not only commonly have huge differences in feature dimension and feature amplitude but also describe the object at different levels. These indicate that different views carry different degrees of information for clustering. However, this factor has been consistently ignored by the existing IMC methods, which is not conducive to clustering. Therefore, designing a more robust clustering model by taking into account the information imbalance property is a new research direction for IMC.

3) *Mixed Data Types [10]*: As far as we know, the existing IMC methods are all proposed for the multiview data with vector-based numerical features, but fails with data from other feature types, such as image, symbolic, and ordinal. Although one can preprocess the data to obtain the vector-based features, some underlying information hidden in the data will be lost. For example, extracting the vector-based features from images may miss some structure information. So it is worth to explore more applicable and flexible IMC models for mixed data.

4) *Complex Data With Noises*: From the summarization to the existing IMC methods, it is easy to observe that existing methods all ignore the influence of noises since their models commonly treat all samples (including noisy samples and clean samples) equally important. As a result, the existing methods are not robust to noise. Generally, it is impossible

to collect clean data without noise in practical applications. Thus, it is important to design more robust models that can reduce the negative influence of noises. In the single-view learning fields, self-paced learning has been proved an effective approach to identify the noisy samples and improve the robustness. Therefore, integrating the self-paced learning to design more robust models is a worth research direction.

5) *Missing View Recovery*: As far as we know, there are few works on missing view recovery. Moreover, the existing works suffer from some issues, where UEAF has a relatively high computational cost and the other two methods need sufficient samples whose all views are fully observed for model training. Moreover, the performances of missing-view recovery of these methods are neither guaranteed by experiments nor theory. In fact, missing view recovery not only can naturally solve the incomplete learning problem but also is beneficial to improve the clustering performance. Besides this, missing view recovery has a lot of potential application values, especially in the criminal investigation. Therefore, it is of great significance to study the missing view recovery and design more robust unified missing-view recovery and IMC frameworks.

In addition to the above issues, some other issues existing in the multiview clustering also appear in IMC, such as local minima and multiple solutions pointed out in [10]. Besides this, clustering on the partially view-aligned data [65], [66] is another challenging problem and the researches on both view-missing and view-partially aligned cases are even rarer. In summary, the progress of IMC is still in the theoretical research stage. In the future, researchers need to design more efficient, high performance, and robust IMC methods for practical applications.

REFERENCES

- [1] C. R. Jack, Jr., et al., "The alzheimer's disease neuroimaging initiative (ADNI): MRI methods," *J. Magn. Reson. Imag.*, vol. 27, no. 4, pp. 685–691, 2008.
- [2] K. Yan, X. Fang, Y. Xu, and B. Liu, "Protein fold recognition based on multi-view modeling," *Bioinformatics*, vol. 35, no. 17, pp. 2982–2990, 2019.
- [3] T. Bhadra, S. Mallik, and S. Bandyopadhyay, "Identification of multi-view gene modules using mutual information-based hypograph mining," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 49, no. 6, pp. 1119–1130, Jun. 2019.
- [4] T. Zhou, C. Zhang, X. Peng, H. Bhaskar, and J. Yang, "Dual shared-specific multiview subspace clustering," *IEEE Trans. Cybern.*, vol. 50, no. 8, pp. 3517–3530, Aug. 2020.
- [5] X. Cai, F. Nie, W. Cai, and H. Huang, "Heterogeneous image features integration via multi-modal semi-supervised learning model," in *Proc. ICCV*, 2013, pp. 1737–1744.
- [6] W. Shao, L. He, and P. S. Yu, "Multiple incomplete views clustering via weighted nonnegative matrix factorization with $L_{2,1}$ regularization," in *Proc. ECML PKDD*, 2015, pp. 318–334.
- [7] Y. Guo, D. Tao, W. Liu, and J. Cheng, "Multiview cauchy estimator feature embedding for depth and inertial sensor-based human action recognition," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 47, no. 4, pp. 617–627, Apr. 2017.
- [8] E. Pan and Z. Kang, "Multi-view contrastive graph clustering," in *Proc. NIPS*, vol. 34, 2021, pp. 2148–2159.
- [9] Z. Lin, Z. Kang, L. Zhang, and L. Tian, "Multi-view attributed graph clustering," *IEEE Trans. Knowl. Data Eng.*, early access, Aug. 6, 2021, doi: 10.1109/TKDE.2021.3101227.
- [10] G. Chao, S. Sun, and J. Bi, "A survey on multi-view clustering," 2017, *arXiv:1712.06246*.
- [11] H. Liu, J. Wu, T. Liu, D. Tao, and Y. Fu, "Spectral ensemble clustering via weighted K-means: Theoretical and practical evidence," *IEEE Trans. Knowl. Data Eng.*, vol. 29, no. 5, pp. 1129–1143, May 2017.
- [12] Z. Huang, J. T. Zhou, X. Peng, C. Zhang, H. Zhu, and J. Lv, "Multi-view spectral clustering network," in *Proc. IJCAI*, 2019, pp. 2563–2569.
- [13] Z. Kang et al., "Multi-graph fusion for multi-view spectral clustering," *Knowl. Based Syst.*, vol. 189, Feb. 2020, Art. no. 105102.
- [14] Y. Yang and H. Wang, "Multi-view clustering: A survey," *Big Data Min. Anal.*, vol. 1, no. 2, pp. 83–107, Jun. 2018.
- [15] C. Xu, D. Tao, and C. Xu, "A survey on multi-view learning," 2013, *arXiv:1304.5634*.
- [16] Z. Kang, Z. Lin, X. Zhu, and W. Xu, "Structured graph learning for scalable subspace clustering: From single view to multi-view," *IEEE Trans. Cybern.*, early access, Mar. 17, 2021, doi: 10.1109/TCYB.2021.3061660.
- [17] L. Li and H. He, "Bipartite graph based multi-view clustering," *IEEE Trans. Knowl. Data Eng.*, vol. 34, no. 7, pp. 3111–3125, Jul. 2022.
- [18] M. Liu, J. Zhang, P.-T. Yap, and D. Shen, "View-aligned hypergraph learning for alzheimer's disease diagnosis with incomplete multi-modality data," *Med. Image Anal.*, vol. 36, pp. 123–134, Feb. 2017.
- [19] S.-Y. Li, Y. Jiang, and Z.-H. Zhou, "Partial multi-view clustering," in *Proc. AAAI*, 2014, pp. 1969–1974.
- [20] H. Zhao, H. Liu, and Y. Fu, "Incomplete multi-modal visual data grouping," in *Proc. IJCAI*, 2016, pp. 2392–2398.
- [21] L. Tran, X. Liu, J. Zhou, and R. Jin, "Missing modalities imputation via cascaded residual autoencoder," in *Proc. CVPR*, 2017, pp. 1405–1414.
- [22] C. Shang, A. Palmer, J. Sun, K.-S. Chen, J. Lu, and J. Bi, "VIGAN: Missing view imputation with generative adversarial networks," in *Proc. ICBD*, 2017, pp. 766–775.
- [23] J. Wen, Z. Zhang, Y. Xu, and Z. Zhong, "Incomplete multi-view clustering via graph regularized matrix factorization," in *Proc. ECCV Workshops*, 2019, pp. 593–608.
- [24] Q. Wang, Z. Ding, Z. Tao, Q. Gao, and Y. Fu, "Generative partial multi-view clustering," 2020, *arXiv:2003.13088*.
- [25] J. Wen, Y. Xu, and H. Liu, "Incomplete multiview spectral clustering with adaptive graph learning," *IEEE Trans. Cybern.*, vol. 50, no. 4, pp. 1418–1429, Apr. 2020.
- [26] J. Wen, Z. Zhang, Y. Xu, B. Zhang, L. Fei, and H. Liu, "Unified embedding alignment with missing views inferring for incomplete multi-view clustering," in *Proc. AAAI*, 2019, pp. 5395–5400.
- [27] W. Zhou, H. Wang, and Y. Yang, "Consensus graph learning for incomplete multi-view clustering," in *Proc. PAKDD*, 2019, pp. 529–540.
- [28] X. Liu et al., "Multiple kernel k-means with incomplete kernels," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 42, no. 5, pp. 1191–1204, May 2020.
- [29] X. Liu et al., "Efficient and effective incomplete multi-view clustering," in *Proc. AAAI*, vol. 33, 2019, pp. 4392–4399.
- [30] J. Wen, Z. Zhang, Z. Zhang, L. Fei, and W. Meng, "Generalized incomplete multiview clustering with flexible locality structure diffusion," *IEEE Trans. Cybern.*, vol. 51, no. 1, pp. 101–114, Jan. 2021.
- [31] J. Guo and J. Ye, "Anchors bring ease: An embarrassingly simple approach to partial multi-view clustering," in *Proc. AAAI*, vol. 33, 2019, pp. 118–125.
- [32] X. Zhu et al., "Localized incomplete multiple kernel k-means," in *Proc. IJCAI*, 2018, pp. 3271–3277.
- [33] J. Liu, C. Wang, J. Gao, and J. Han, "Multi-view clustering via joint nonnegative matrix factorization," in *Proc. SIAM ICDM*, 2013, pp. 252–260.
- [34] A. Kumar, P. Rai, and H. Daume, "Co-regularized multi-view spectral clustering," in *Proc. NIPS*, 2011, pp. 1413–1421.
- [35] N. Xu, Y. Guo, X. Zheng, Q. Wang, and X. Luo, "Partial multi-view subspace clustering," in *Proc. ACM MM*, 2018, pp. 1794–1801.
- [36] W. Shao, L. He, C.-T. Lu, and S. Y. Philip, "Online multi-view clustering with incomplete views," in *Proc. ICBD*, 2016, pp. 1012–1017.
- [37] M. Hu and S. Chen, "Doubly aligned incomplete multi-view clustering," in *Proc. IJCAI*, 2019, pp. 2262–2268.
- [38] N. Rai, S. Negi, S. Chaudhury, and O. Deshmukh, "Partial multi-view clustering using graph regularized NMF," in *Proc. ICPR*, 2016, pp. 2192–2197.
- [39] M. Hu and S. Chen, "One-pass incomplete multi-view clustering," in *Proc. AAAI*, 2019, pp. 3838–3845.
- [40] C. Xu, D. Tao, and C. Xu, "Multi-view learning with incomplete views," *IEEE Trans. Image Process.*, vol. 24, no. 12, pp. 5812–5825, Dec. 2015.
- [41] H. Liu and Y. Fu, "Consensus guided multi-view clustering," *ACM Trans. Knowl. Discov. Data*, vol. 12, no. 4, pp. 1–21, 2018.
- [42] J. Wu, H. Liu, H. Xiong, J. Cao, and J. Chen, "K-means-based consensus clustering: A unified view," *IEEE Trans. Knowl. Data Eng.*, vol. 27, no. 1, pp. 155–169, Jan. 2015.
- [43] A. Trivedi, P. Rai, H. Daumé, III, and S. L. DuVall, "Multiview clustering with incomplete views," in *Proc. NIPS Workshop*, vol. 224, 2010, pp. 1–7.
- [44] W. Shao, X. Shi, and S. Y. Philip, "Clustering on multiple incomplete datasets via collective kernel learning," in *Proc. ICDM*, 2013, pp. 1181–1186.
- [45] X. Liu, M. Li, L. Wang, Y. Dou, J. Yin, and E. Zhu, "Multiple kernel k-means with incomplete kernels," in *Proc. AAAI*, 2017, pp. 2259–2265.
- [46] Y. Ye, X. Liu, Q. Liu, and J. Yin, "Consensus kernel k-means clustering for incomplete multiview data," *Comput. Intell. Neurosci.*, vol. 2017, pp. 1–11, Oct. 2017.
- [47] H. Gao, Y. Peng, and S. Jian, "Incomplete multi-view clustering," in *Proc. ICIIP*, 2016, pp. 245–255.

- [48] H. Wang, L. Zong, B. Liu, Y. Yang, and W. Zhou, "Spectral perturbation meets incomplete multi-view data," in *Proc. IJCAI*, 2019, pp. 3677–3683.
- [49] W. Zhuge, C. Hou, X. Liu, H. Tao, and D. Yi, "Simultaneous representation learning and clustering for incomplete multi-view data," in *Proc. IJCAI*, 2019, pp. 4482–4488.
- [50] L. Zhao, Z. Chen, Y. Yang, Z. J. Wang, and V. C. M. Leung, "Incomplete multi-view clustering via deep semantic mapping," *Neurocomputing*, vol. 275, pp. 1053–1062, Jan. 2018.
- [51] Q. Wang, Z. Ding, Z. Tao, Q. Gao, and Y. Fu, "Partial multi-view clustering via consistent GAN," in *Proc. ICDM*, 2018, pp. 1290–1295.
- [52] C. Xu, Z. Guan, W. Zhao, H. Wu, Y. Niu, and B. Ling, "Adversarial incomplete multi-view clustering," in *Proc. IJCAI*, 2019, pp. 3933–3939.
- [53] F. Nie, J. Li, and X. Li, "Parameter-free auto-weighted multiple graph learning: A framework for multiview clustering and semi-supervised classification," in *Proc. IJCAI*, 2016, pp. 1881–1887.
- [54] F. Nie, G. Cai, and X. Li, "Multi-view clustering and semi-supervised classification with adaptive neighbours," in *Proc. AAAI*, 2017, pp. 2408–2414.
- [55] D. Huang, J. Sun, and Y. Wang, "The BUAA-VisNir face dataset instructions," Dept. School Comput. Sci. Eng., Beihang Univ., Beijing, China, Rep. IRIP-TR-12-FR-001, 2012.
- [56] X. Cai, H. Wang, H. Huang, and C. Ding, "Joint stage recognition and anatomical annotation of drosophila gene expression patterns," *Bioinformatics*, vol. 28, no. 12, pp. i16–i24, 2012.
- [57] P. Tomancak *et al.*, "Global analysis of patterns of gene expression during drosophila embryogenesis," *Genome Biol.*, vol. 8, no. 7, p. R145, 2007.
- [58] L. Fei-Fei, R. Fergus, and P. Perona, "Learning generative visual models from few training examples: An incremental Bayesian approach tested on 101 object categories," in *Proc. CVPR Workshop*, 2004, pp. 178–178.
- [59] Y. Li, F. Nie, H. Huang, and J. Huang, "Large-scale multi-view spectral clustering via bipartite graph," in *Proc. AAAI*, 2015, pp. 2750–2756.
- [60] D. Greene and P. Cunningham, "Practical solutions to the problem of diagonal dominance in kernel document clustering," in *Proc. ICML*, 2006, pp. 377–384.
- [61] T.-S. Chua, J. Tang, R. Hong, H. Li, Z. Luo, and Y. Zheng, "NUS-WIDE: A real-world Web image database from national university of Singapore," in *Proc. ACM ICIVR*, 2009, pp. 1–9.
- [62] E. Amigó, J. Gonzalo, J. Artilles, and F. Verdejo, "A comparison of extrinsic clustering evaluation metrics based on formal constraints," *Inf. Retrieval*, vol. 12, no. 4, pp. 461–486, 2009.
- [63] L. Hubert and P. Arabie, "Comparing partitions," *J. Classification*, vol. 2, no. 1, pp. 193–218, 1985.
- [64] C. Zhang, H. Fu, S. Liu, G. Liu, and X. Cao, "Low-rank tensor constrained multiview subspace clustering," in *Proc. ICCV*, 2015, pp. 1582–1590.
- [65] Z. Huang, P. Hu, J. T. Zhou, J. Lv, and X. Peng, "Partially view-aligned clustering," in *Proc. NIPS*, vol. 33, 2020, pp. 1–11.
- [66] X. Zhang, L. Zong, X. Liu, and H. Yu, "Constrained NMF-based multi-view clustering on unmapped data," in *Proc. AAAI*, 2015, pp. 3174–3180.



Jie Wen received the Ph.D. degree in computer science and technology from the Harbin Institute of Technology (Shenzhen), Shenzhen, China, in 2019.

He is currently an Assistant Professor with the School of Computer Science and Technology, Harbin Institute of Technology (Shenzhen). He has authored or coauthored more than 80 technical papers at prestigious international journals and conferences, including the IEEE TRANSACTIONS ON NEURAL NETWORKS AND LEARNING SYSTEMS,

IEEE TRANSACTIONS ON IMAGE PROCESSING, IEEE TRANSACTIONS ON CYBERNETICS, IEEE TRANSACTIONS ON MULTIMEDIA, PATTERN RECOGNITION, ECCV, AAAI, IJCAI, and ACM MM. His research interests include image and video enhancement, pattern recognition, and machine learning.

Dr. Wen serves/served as an Associate Editor for *International Journal of Image and Graphics* and the Area Chair for ACM MM. He served as the Reviewer for more than 30 international journals and conferences, including IEEE TRANSACTIONS ON PATTERN ANALYSIS AND MACHINE INTELLIGENCE, IEEE TRANSACTIONS ON NEURAL NETWORKS AND LEARNING SYSTEMS, IEEE TRANSACTIONS ON IMAGE PROCESSING, IEEE TRANSACTIONS ON MULTIMEDIA, AAAI, IJCAI, CVPR, ICCV, and ACM MM.



Zheng Zhang (Senior Member, IEEE) received the M.S. and Ph.D. degrees in computer science and technology from the Harbin Institute of Technology, Harbin, China, in 2014 and 2018, respectively.

He was a Postdoctoral Research Fellow with The University of Queensland, Brisbane, QLD, Australia. He is currently an Associate Professor with the Harbin Institute of Technology (Shenzhen), Shenzhen, China. He has published over 100 technical papers at prestigious journals and conferences.

His research interests include machine learning, computer vision, and multimedia analytics.

Dr. Zhang is an Editorial Board Member of the *Information Processing and Management*, and also serves/served as the AC/SPC/PC member of several top conferences.



Lunke Fei (Member, IEEE) received the Ph.D. degree in computer science and technology from the Harbin Institute of Technology, Harbin, China, in 2016.

Since 2017, he has been with the School of Computer Science and Technology, Guangdong University of Technology, Guangzhou, China. His research interests include pattern recognition, biometrics, image processing, and machine learning.



Bob Zhang (Senior Member, IEEE) received the B.A. degree in computer science from York University, Toronto, ON, Canada, in 2006, the M.A.Sc. degree in information systems security from Concordia University, Montreal, QC, Canada, in 2007, and the Ph.D. degree in electrical and computer engineering from the University of Waterloo, Waterloo, ON, Canada, in 2011.

Then, he remained with the Center for Pattern Recognition and Machine Intelligence, and later was a Postdoctoral Researcher with the Department of Electrical and Computer Engineering, Carnegie Mellon University, Pittsburgh, PA, USA. He is currently an Associate Professor with the Department of Computer and Information Science, University of Macau, Macau, China. His research interests focus on biometrics, pattern recognition, and image processing.

Dr. Zhang is a Technical Committee Member of the IEEE Systems, Man, and Cybernetics Society, an Associate Editor for the *International Journal of Image and Graphics*, as well as an Editorial Board Member for the *International Journal of Information*.



Yong Xu (Senior Member, IEEE) received the Ph.D. degree in pattern recognition and intelligence system from the Nanjing University of Science and Technology, Nanjing, China, in 2005.

He is currently a Professor with the School of Computer Science and Technology, Harbin Institute of Technology (Shenzhen), Shenzhen, China. His current interests include pattern recognition, biometrics, machine learning, and video analysis. More information please refer to <http://www.yongxu.org/lunwen.html>.



Zhao Zhang (Senior Member, IEEE) received the Ph.D. degree from the Department of Electronic Engineering, City University of Hong Kong, Hong Kong, in 2013.

He is currently a Full Professor with the School of Computer Science, Hefei University of Technology, Hefei, China. His current research interests include data mining and machine learning, image processing, and pattern recognition.

Prof. Zhang is serving/served as an Associate Editor for IEEE ACCESS, *Neurocomputing*, and *IET*

Image Processing. He is also a Senior Member of CCF.



Jinxing Li received the B.Sc. degree from the Department of Automation, Hangzhou Dianzi University, Hangzhou, China, in 2012, the M.Sc. degree from the Department of Automation, Chongqing University, Chongqing, China, in 2015, and the Ph.D. degree from the Department of Computing, The Hong Kong Polytechnic University, Hong Kong, in 2018.

He is currently an Associate Professor with the Harbin Institute of Technology (Shenzhen), Shenzhen, China. His research interests are pattern recognition, deep learning, medical biometrics, and machine learning.