一. 1. $P(A-B)=P(A-AB)=P(A)-P(AB)=0.5-P(AB)=0.3 \Rightarrow P(AB)=0.2$

$\therefore P(A\cup B)=P(A)+P(B)-P(AB)=0.5+0.4-0.2=0.7$

2. $DX=np(1-p)=10(p-p^2)=\frac{5}{2} \Rightarrow 4p^2-4p+1=0 \Rightarrow (2p-1)^2=0 \Rightarrow p=\frac{1}{2}$

3. $X\sim N(2,0.2)$, $Y\sim N(2,0.2)$, 且X,Y独立, $\therefore$ $X-2Y$也服从正态分布,

$E(X-2Y)=EX-2EY=2-2\times2=-2$, $D(X-2Y)=DX+D(2Y)=DX+4DY=0.2+4\times0.2=1$

$\therefore Z-1=X-2Y\sim N(-2,1) \Rightarrow U=\frac{Z-1-(-2)}{1}=Z+1\sim N(0,1)$

$U=Z+1 \Rightarrow Z=U-1$

解一: 设Z的分布函数为$F(z)$, 密度函数$f(z)$, 则

$F(z)=P(Z\le z)=P(U-1\le z)=P(U\le z+1)=\int_{-\infty}^{z+1}\frac{1}{\sqrt{2\pi}}e^{-\frac{u^2}{2}}du$

$\therefore f(z)=F'(z)=\frac{1}{\sqrt{2\pi}}e^{-\frac{(z+1)^2}{2}}$

解二: 已知结论: 若$Y=aX+b$, $f_Y(y)=\frac{1}{|a|}f_X\left(\frac{y-b}{a}\right)$

$Z=U-1 \Rightarrow a=1, b=-1$ $\therefore f(z)=\varphi\left(\frac{z-(-1)}{1}\right)=\varphi(z+1)=\frac{1}{\sqrt{2\pi}}e^{-\frac{(z+1)^2}{2}}$

4. $F(10,5)$

5. $p=P(X\le\frac{1}{2})=\int_0^{\frac{1}{2}}2x\,dx=x^2\Big|_0^{\frac{1}{2}}=\frac{1}{4}$

$P(Y=2)=b(2;3,p)=C_3^2p^2(1-p)=3\times\frac{1}{16}\times\frac{3}{4}=\frac{9}{64}$

6. $X\sim N(0,4)\Rightarrow \bar{X}\sim N(0,\frac{2}{5})$, $Y\sim N(0,9)\Rightarrow \bar{Y}\sim N(0,\frac{3}{5})$, 又$\because X,Y$独立,

$\therefore \bar{X}-\bar{Y}$也服从正态分布, $E(\bar{X}-\bar{Y})=E\bar{X}-E\bar{Y}=0-0=0$, $D(\bar{X}-\bar{Y})=D\bar{X}+D\bar{Y}=\frac{2}{5}+\frac{3}{5}=1$

$\therefore \bar{X}-\bar{Y}\sim N(0,1)$, 令$\bar{X}-\bar{Y}=T$, 则

$E|\bar{X}-\bar{Y}|=E|T|=\int_{-\infty}^{+\infty}|t|\frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{2}}dt=\frac{2}{\sqrt{2\pi}}\int_0^{+\infty}te^{-\frac{t^2}{2}}dt=-\frac{2}{\sqrt{2\pi}}e^{-\frac{t^2}{2}}\Big|_0^{+\infty}=\sqrt{\frac{2}{\pi}}$

二. 1. (A) A,B,C两两独立时, A,B,C相互独立 $\Leftrightarrow P(ABC)=P(A)P(B)P(C)$

若A与BC独立, 则 $P(A(BC))=P(A)P(BC)\Rightarrow P(ABC)=P(A)P(B)P(C)$

反之: $P(ABC)=P(A(BC))=P(A)P(B)P(C)=P(A)(P(B)P(C))=P(A)P(BC)$

$\therefore A$与BC独立.

2. $P(\bar{A}B)=P(\bar{A})P(B|\bar{A})=(1-P(A))P(B|\bar{A})=\frac{3}{5}\cdot\frac{5}{6}=\frac{1}{2}$

$\therefore P(\bar{A}|B)=\frac{P(\bar{A}B)}{P(B)}=\frac{\frac{1}{2}}{\frac{4}{5}}=\frac{5}{8}$ $\therefore$ 选 C

3. (D)

4.(A) 设取出的黑球数为X，则所求概率为：

$$P(X\geq 1)=1-P(X=0)=1-\frac{C_4^3}{C_6^3}=\frac{4}{5}$$

5.(A) $P(|X-\mu_1|<1)>P(|Y-\mu_2|<1)\Rightarrow P\left(\left|\frac{X-\mu_1}{\sigma_1}\right|<\frac{1}{\sigma_1}\right)>P\left(\left|\frac{Y-\mu_2}{\sigma_2}\right|<\frac{1}{\sigma_2}\right)$

$X\sim N(\mu_1,\sigma_1^2)\Rightarrow \frac{X-\mu_1}{\sigma_1}\sim N(0,1)$，$Y\sim N(\mu_2,\sigma_2^2)\Rightarrow \frac{Y-\mu_2}{\sigma_2}\sim N(0,1)$

$\therefore \frac{1}{\sigma_1}>\frac{1}{\sigma_2}\Rightarrow \sigma_1<\sigma_2$

6.(D) $E\left(\sum_{i=1}^{9}X_i\right)=\sum_{i=1}^{9}EX_i=9$，$D\left(\sum_{i=1}^{9}X_i\right)=\sum_{i=1}^{9}DX_i=9$

由切比雪夫不等式得 $P\left(\left|\sum_{i=1}^{9}X_i-E\left(\sum_{i=1}^{9}X_i\right)\right|\geq\varepsilon\right)\leq\frac{D\left(\sum_{i=1}^{9}X_i\right)}{\varepsilon^2}$，即 $P\left(\left|\sum_{i=1}^{9}X_i-9\right|\geq\varepsilon\right)\leq\frac{9}{\varepsilon^2}$

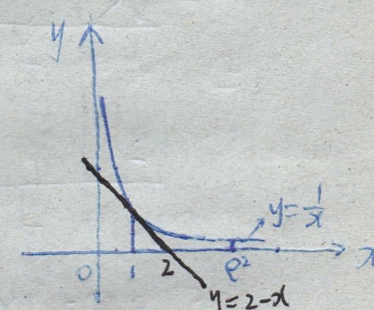
$\Rightarrow 1-P\left(\left|\sum_{i=1}^{9}X_i-9\right|<\varepsilon\right)\leq\frac{9}{\varepsilon^2}\Rightarrow P\left(\left|\sum_{i=1}^{9}X_i-9\right|<\varepsilon\right)\geq 1-9\varepsilon^{-2}$

三. 记 A_i={乙在第i次投篮中投中}，B_i={甲在第i次投篮中投中}，$i=1,2$

(1) $P(A_1)=P(B_1)P(A_1|B_1)+P(\bar{B}_1)P(A_1|\bar{B}_1)=0.7\times0.5+0.3\times0.6=0.53$

(2) $P(B_2)=P(A_1)P(B_2|A_1)+P(\bar{A}_1)P(B_2|\bar{A}_1)=0.53\times0.4+0.47\times0.7=0.541$

四. 记所围区域为D，则 $S_D=\int_1^{e^2}\frac{1}{x}dx=\ln x\Big|_1^{e^2}=2$，



故(X,Y)的联合概率密度函数为：

$$f(x,y)=\begin{cases}\frac{1}{2}, & (x,y)\in D\\ 0, & (x,y)\notin D\end{cases}$$

(1) 当 $x\in[1,e^2]$，$f_X(x)=\int_0^{\frac{1}{x}}\frac{1}{2}dy=\frac{1}{2x}$

$x\notin[1,e^2]$，$f_X(x)=0$

当 $y\in[0,e^{-2}]$，$f_Y(y)=\int_1^{e^2}\frac{1}{2}dx=\frac{1}{2}(e^2-1)$

$y\in[e^{-2},1]$，$f_Y(y)=\int_1^{\frac{1}{y}}\frac{1}{2}dx=\frac{1}{2y}-\frac{1}{2}$

$y\notin[0,1]$，$f_Y(y)=0$

显然 $f(x,y)\neq f_X(x)f_Y(y)$，∴ X与Y不独立.

(2) $P(X+Y<2)=\iint\limits_{x+y<2}f(x,y)\,dx\,dy=\int_1^2dx\int_0^{2-x}\frac{1}{2}dy=\frac{1}{2}\int_1^2(2-x)dx=\frac{1}{2}\left(2x-\frac{x^2}{2}\right)\Big|_1^2$

$=\frac{1}{2}\left(4-2-2+\frac{1}{2}\right)=\frac{1}{4}$

$\therefore P(X+Y\geq 2)=1-P(X+Y<2)=\frac{3}{4}$

五. 显然 $U=1$ 或 2, $V=1$ 或 2.

$(U=1)=(X=1,Y=1)$

$(U=2)=(X=1,Y=2)\cup(X=2,Y=1)\cup(X=2,Y=2)$

$(V=1)=(X=1,Y=1)\cup(X=1,Y=2)\cup(X=2,Y=1)$

$(V=2)=(X=2,Y=2)$

$\therefore P(U=1,V=1)=P(X=1,Y=1)=\frac{2}{3}\cdot\frac{2}{3}=\frac{4}{9}$

$P(U=1,V=2)=P(\phi)=0$

$P(U=2,V=1)=P((X=1,Y=2)\cup(X=2,Y=1))=P(X=1,Y=2)+P(X=2,Y=1)$

$=P(X=1)P(Y=2)+P(X=2)P(Y=1)=\frac{2}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{3}=\frac{4}{9}$

$P(U=2,V=2)=P(X=2,Y=2)=P(X=2)P(Y=2)=\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$

$\therefore$ (U,V) 的联合概率分布如下:

$V \backslash U$	1	2	$P(V=i)$
1	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{8}{9}$
2	0	$\frac{1}{9}$	$\frac{1}{9}$
$P(U=i)$	$\frac{4}{9}$	$\frac{5}{9}$	

$\therefore EU=1\times\frac{4}{9}+2\times\frac{5}{9}=\frac{14}{9}$ $\quad EV=1\times\frac{8}{9}+2\times\frac{1}{9}=\frac{10}{9}$

$UV=1,2,4$, $P(UV=1)=P(U=1,V=1)=\frac{4}{9}$.

$P(UV=2)=P((U=1,V=2)\cup(U=2,V=1))=P(U=1,V=2)+P(U=2,V=1)=0+\frac{4}{9}=\frac{4}{9}$

$P(UV=4)=P(U=2,V=2)=\frac{1}{9}$ $\therefore E(UV)=1\times\frac{4}{9}+2\times\frac{4}{9}+4\times\frac{1}{9}=\frac{16}{9}$

$\therefore Cov(U,V)=E(UV)-EU\cdot EV=\frac{16}{9}-\frac{14}{9}\times\frac{10}{9}=\frac{4}{81}$

六. 将1万个鸡蛋编号为1, 2, …10000, 令 $X_i=\begin{cases}1 & \text{第}i\text{号鸡蛋育成种鸡}\\ 0 & \text{否则}\end{cases}$, $i=1,2\cdots10000$

则 X_i 是相互独立的服从0-1分布的随机变量, $P(X_i=1)=0.84\times0.9=0.756$, $P(X_i=0)=0.244$

且 $\sum_{i=1}^{10000}X_i\sim B(10000,0.756)$, 所求概率为 $P(\sum_{i=1}^{10000}X_i\geqslant7500)$

$n=10000$ 较大, 故可据棣莫弗-拉普拉斯定理作近似计算, 由于 $np=10000\times0.756=7560$

$\sqrt{np(1-p)}=\sqrt{7560\times0.244}\approx42.95$

$\therefore P(\sum_{i=1}^{10000}X_i\geqslant7500)=1-P(\sum_{i=1}^{10000}X_i<7500)=1-P\left(\frac{\sum_{i=1}^{10000}X_i-7560}{42.95}<\frac{7500-7560}{42.95}\right)$

$=1-\Phi(-1.40)=\Phi(1.40)=0.92$

七.(1) $f(x;\beta)=F'(x;\beta)=\begin{cases}\dfrac{\beta}{x^{\beta+1}} & x>1\\ 0 & x\le 1\end{cases}$

$\therefore E[X]=\int_1^{+\infty} x\cdot\dfrac{\beta}{x^{\beta+1}}dx=\dfrac{\beta}{1-\beta}x^{1-\beta}\Big|_1^{+\infty}=\dfrac{\beta}{1-\beta}\Rightarrow \beta=\dfrac{E[X]}{E[X]+1}$

$\therefore$ β的矩法估计量 $\hat\beta=\dfrac{\overline{X}}{\overline{X}+1}$

(2) 构造似然函数 $L(x_1,\cdots x_n;\beta)=\beta^n\cdot\prod_{i=1}^n x_i^{-(\beta+1)}$

取对数：$\ln L=n\ln\beta-(\beta+1)\sum_{i=1}^n\ln x_i$

求导得，$\dfrac{d\ln L}{d\beta}=\dfrac{n}{\beta}-\sum_{i=1}^n\ln x_i=0\Rightarrow\beta=\dfrac{n}{\sum_{i=1}^n\ln x_i}$

$\therefore$ β的极大似然估计量为 $\hat\beta=\dfrac{n}{\sum_{i=1}^n\ln X_i}$

八：(1) σ^2已知，μ的置信度为$1-\alpha$的置信区间为 $[\overline{X}-u_{1-\frac{\alpha}{2}}\frac{\sigma}{\sqrt n},\ \overline{X}+u_{1-\frac{\alpha}{2}}\frac{\sigma}{\sqrt n}]$

已知 $\overline{x}=125$，$\sigma=2.71$，$n=7$，$1-\alpha=0.9\Rightarrow\alpha=0.1\Rightarrow 1-\frac{\alpha}{2}=0.95$，$u_{0.95}=1.65$

$\therefore u_{1-\frac{\alpha}{2}}\frac{\sigma}{\sqrt n}=1.65\cdot\dfrac{2.71}{\sqrt7}\approx1.69$

$\therefore$ 所求置信区间为：$[125-1.69,125+1.69]=[123.31,126.69]$

(2) 已知 $\sigma_1^2=84^2$，$\sigma_2^2=96^2$，假设 $H_0:\mu_1-\mu_2=0$

取统计量 $U=\dfrac{\overline{X}-\overline{Y}-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{m}+\frac{\sigma_2^2}{n}}}\overset{\wedge}{\sim}N(0,1)$，其中$m$.$n$分别为两个样本的样本容量.

对$\alpha=0.05$，临界值 $u_{1-\frac{\alpha}{2}}=u_{0.975}=1.96$

观察值 $U=\dfrac{1295-1230-0}{\sqrt{\frac{84^2}{60}+\frac{96^2}{60}}}=3.95$ $\because U=3.95>1.96$ $\therefore$ 拒绝H_0.

即两厂生产的灯泡寿命有显著差异.

草稿纸

学生签名______