**H** Roll No. ..............................

# TMA–101

## B. TECH. (FIRST SEMESTER)
## MID SEMESTER EXAMINATION, 2019
## (ALL BRANCHES)

ENGINEERING MATHEMATICS—I

**Time : 1 : 30 Hours**

**Maximum Marks : 50**

**Note :** (i) This question paper contains two Sections.

(ii) Both Sections are compulsory.

### Section—A

1. Indicate True *or* False for the following

(1×5=5 Marks)

(a) The non-zero rows of a matrix in echelon form are linearly dependent. (True/False)

(b) Every matrix A is a root of its characteristic polynomial. (True/False)

(c) For a real matrix A, the eigen values and corresponding eigen vectors can be complex. (True/False)

(d) If $y = (2x + 3)^5$, then $\frac{d^5y}{dx^5} = 5!$

(True/False)

(e) If $f(x, y)$ is a homogeneous function of degree $n$ in $x$ and $y$ and has continuous first and second order partial derivatives, then :

$$x^2 \frac{\partial^2 f}{\partial x^2} + 2xy \frac{\partial^2 f}{\partial x \partial y} + y^2 \frac{\partial^2 f}{\partial y^2} = n(n-1) f.$$

(True/False)

2. Attempt any *five* parts : (3×5=15 Marks)

(a) Examine whether A is similar to B, where $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

(b) Prove that the matrix is $A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ unitary.

(c) Evaluate $\lim\limits_{\substack{x \to 0 \\ y \to 0}} \frac{y^2 - x^2}{y^2 + x^2}$.

(d) If $y = \left[x + \sqrt{1+x^2}\right]^m$, prove that :

$$(1+x^2) y_{n+2} + (2n+1) x y_{n+1} + (n^2 - m^2) y_n = 0$$





(e) If $u = f\left(\frac{y}{z}, \frac{x}{y}, \frac{z}{x}\right)$, then show that :

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$$

(f) State Taylor's theorem for two variables.

**Section—B**

3. Attempt any *two* parts of choice from (a), (b) and (c). (5×2=10 Marks)

(a) Find the rank of the matrix A, by reducing it to Normal form :

$$A = \begin{bmatrix} 1 & 3 & 4 & 2 \\ 2 & -1 & 3 & 2 \\ 3 & -5 & 2 & 2 \\ 6 & -3 & 8 & 6 \end{bmatrix}$$

(b) Find for what value of $\lambda$ and $\mu$ the system of linear equations :

$$x + y + z = 6$$
$$x + 2y + 5z = 10$$
$$2x + 3y + \lambda z = \mu$$

has (i) a unique solution (ii) no solution (iii) infinite solutions. Also find the solution for $\lambda = 2$ and $\mu = 8$.

(c) Using Cayley-Hamilton theorem, find the inverse of the matrix :

$$A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$$

4. Attempt any *two* parts of choice from (a), (b) and (c). (5×2=10 Marks)

(a) Find the Eigen values and Eigen vectors of the matrix :

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

(b) Show that :

$$A = \begin{bmatrix} -i & 3+2i & -2-i \\ -3+2i & 0 & 3-4i \\ 2-i & -3-4i & -2i \end{bmatrix}$$

is Skew-Hemitian matrix.

(c) If $y = (\sin^{-1} x)^2$, prove that :

$y_n(0) = 0$, for $n$ odd and

$y_n(0) = 2.2^2.4^2.6^2 \ldots . (n-2)^2$ , $n \neq 2$

for $n$ even





5. Attempt any *two* parts of choice from (a), (b) and (c). (5×2=10 Marks)

(a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that :

$$\left(\frac{\partial}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$$

(b) If $u = \tan^{-1}\left(\frac{x^3 + y^2}{x - y}\right)$, prove that :

(i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin u$

(ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$

$= 2\cos 3u \sin u$

(c) If $f(x, y) = \tan^{-1}(xy)$, find an approximate value of $f(0.1, 0.8)$ using the Taylor's series quadratic approximation.