H Roll No. ....

TMA–201

B. TECH. (SECOND SEMESTER)

END SEMESTER

EXAMINATION, June, 2023

ENGINEERING MATHEMATICS–II

Time : Three Hours

Maximum Marks : 100

Note : (i) All questions are compulsory.

(ii) Answer any *two* sub-questions among (a), (b) and (c) in each main question.

(iii) Total marks in each main question are **twenty**.

(iv) Each sub-question carries 10 marks.

1. (a) Solve the differential equation : (CO1)

$$(2xy^4e^y + 2xy^3 + y)dx + (x^2y^4e^y - x^2y^2 - 3x)dy = 0$$

P. T. O.

(b) Solve the differential equation : (CO1)

$$x^2 \frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = x + \sin x.$$

(c) Using Method of Variation of Parameters solve the differential equation : (CO1)

$$\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + y = xe^x \log x.$$

2. (a) A periodic square-wave function $f(t)$, in terms of unit step function is written as : (CO2)

$$f(t) = k[u_0(t) - 2u_a(t) + 2u_{2a}(t) - 3u_{3a}(t) + \ldots].$$

Show that the Laplace Transform of $f(t)$ is given by $L\{f(t)\} = \frac{k}{s} \tan h\left(\frac{as}{2}\right)$.

(b) Evaluate the Integral $\int_0^\infty \frac{e^{-t} - e^{-3t}}{t} dt$. (CO2)

(c) Apply Convolution Theorem, solve the initial value problem : (CO2)

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 2y = 5\sin t,$$

$$y(0) = \left(\frac{dy}{dt}\right)_{t=0} = 0.$$

3. (a) Obtain the Fourier series for the function :

$$f(x)=\begin{cases}-\pi, & -\pi<x<0,\\ x, & 0<x<\pi.\end{cases}$$

Deduce that : (CO3)

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\ldots\ldots=\frac{\pi^2}{8}.$$

(b) Expand the function $f(x)=x\sin x$, as a Fourier Series in the interval $-\pi\le x\le\pi$. Hence deduce that : (CO3)

$$\frac{1}{1.3}-\frac{1}{3.5}-\frac{1}{5.7}-\frac{1}{7.9}+\ldots\ldots=\frac{\pi-2}{4}.$$

(c) Let

$$f(x)=\begin{cases}wx & , 0<x<l/2,\\ w(l-x), & l/2<x<l.\end{cases}$$

Show that :

$$f(x)=\frac{4wl}{\pi^2}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^2}\sin\frac{(2n+1)\pi x}{l}.$$

Hence obtain the sum of the series :

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\ldots..$$

4. (a) Solve : (CO3)

$$r-s-2t=(2x^2+xy-y^2)\sin xy-\cos xy.$$

P. T. O.

(b) Show how the wave equation : (CO4)

$$c^2 \frac{\partial^2 y}{\partial y^2} = \frac{\partial^2 y}{\partial t^2}$$

can be solved by the method of separation of variables. If the initial displacement and velocity of a string stretched between $x = 0$ and $x = l$ are given by $y = f(x)$ and $\frac{\partial y}{\partial t}$, determine the constants in the series solution. (CO4)

(c) A bar with insulated sides is initially at temperature 0°C thoroughly. The end $x = 0$ is kept at 0°C, and heat is suddenly applied at the end $x = l$ so that $\frac{\partial u}{\partial x} = A$ for $x = l$, where A is constant. Find the temperature function $u(x, t)$. (CO4)

5. (a) Solve in series the differential equation

$$x^2 \frac{d^2 y}{dx^2} + 5x \frac{dy}{dx} + x^2 y = 0.$$ (CO5)

(b) Show that : (CO5)

$$\int_{-1}^{1} P_m(x) P_n(x)\, dx = \begin{cases} 0 & , \text{if } m \neq n, \\ \dfrac{2}{2n+1}, & \text{if } m = n. \end{cases}$$

(c) Show that : (CO5)

$$xJ_n' = -nJ_n + xJ_{n-1}$$

and

$$J_{3/2}(x) = \sqrt{\frac{2}{\pi x}} \left(\frac{\sin x}{x} - \cos x \right).$$