Mid Semester Examination 2021

Name of the Program: B.Tech
Name of the Course: Engineering Mathematics I
Time: $1\frac{1}{2}$ Hour

Semester: I
Course Code: TMA-101
Maximum Marks: 50

Note: (i) Answer **all the questions** by choosing **any one of the sub questions.**

(ii) Each question carries 10 marks.

Q1	(10 marks)	
(a)	Find the Eigen values and Eigen vectors of the matrix $A=\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.	CO1
	OR	
(b)	For what value of m and n the system of linear equations $x+y+z=6, x+2y+5z=10$ and $2x+3y+mz=n$ has (i) Infinite number of solution (ii) No solution	
Q2	(10 marks)	
(a)	Write short notes on the following: (i) Cayley-Hamilton Theorem (ii) Algebraic and Geometric multiplicity (iii) Rank of matrix (iv) Unitary matrices (v) Orthogonal vectors	CO1
	OR	
(b)	(i) Show that the set of vectors $X=[1,2,-3,4], Y=[3,-1,2,1], Z=[1,-5,8,-7]$ is linearly dependent. (ii) Consider matrix $\begin{bmatrix} 4 & 0 \\ 2 & -4 \end{bmatrix}$. Is the matrix diagonalizable? If yes, find P such that $P^{-1}AP = diag[4,-4]$.	
Q3	(10 marks)	
(a)	(i) Show that the function $f(x,y)=\frac{xy^3}{x^2+y^6}, x\neq 0, y\neq 0$ and $f(0,0)=0$ is not continuous at (0, 0) in (x, y). (ii) Find the first six terms of the expansion of the function $e^x \sin y$ in a Taylor's series about (0, 0).	CO3
	OR	

(b)	(i) If $u=\left(x^2+y^2+z^2\right)^{-1/2}$, prove that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0$. (ii) If $u=\tan^{-1}\left(\frac{x^3+y^3}{x+y}\right)$, then show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u$.	
Q4	(10 marks)	
(a)	(i) If A is non-singular matrix show that $rank\left(AA^T\right)=rank(A)$. (ii) Find non-singular matrices P and Q such that PAQ is normal form where $A=\begin{bmatrix}1&1&2\\1&2&3\\0&1&1\end{bmatrix}$.	CO1
	OR	
(b)	Expresses the Hermitian matrix $A=\begin{bmatrix}1&-i&1+i\\i&0&2-3i\\1-i&2+3i&2\end{bmatrix}$ as $P+iQ$ where P is real symmetric and Q is real skew symmetric matrix.	
Q5	(10 marks)	
(a)	If $y=(x^2-1)^n$, use Leibnitz's theorem to show $(1-x^2)y_{n+2}-2xy_{n+1}+n(n+1)y_n=0$	CO3
	OR	
(b)	(i) Show that the sequence $\langle S_n\rangle=\frac{4n}{n+7n^{1/2}}$ has the limit 4. (i) Define Cauchy's root test, and test for convergence of the series whose nth terms is $\frac{n^{n^2}}{(n+1)^{n^2}}$.	