Back End Semester Examination 2017 TMA-101

B.Tech – First Semester

ENGINEERING MATHEMATICS - I

Time: Three Hours

MM: 100

(i) This question paper contains five questions with alternative choice.
(ii) All questions are compulsory.
(iii) Instructions on how to attempt a question are mentioned against it.
(iv) Total marks assigned to each question are twenty.

Q.1.

(2X10=20 Marks)

a) If $u = x\,\phi\left(\frac{y}{x}\right)+\psi\left(\frac{y}{x}\right)$ Show that : $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=x\phi\left(\frac{y}{x}\right)$.

b) If $y=\sin^{-1}x$, find $y_n(0)$.

c) If $y=\frac{x^3}{(x+1)(x+2)}$, find y_n.

Q.2

a) Expand $(1+x+y^2)^{1/2}$ in powers of $(x-1)$ and y. (2X10=20 Marks)

b) Find the extreme values of function $f(x, y) = x^3y^2(1-x-y)$.

c) If $u=\frac{x}{\sqrt{1-r^2}}$, $v=\frac{y}{\sqrt{1-r^2}}$ and $w=\frac{z}{\sqrt{1-r^2}}$,

where $r^2=x^2+y^2+z^2$ then show that $\frac{\partial(u,v,w)}{\partial(x,y,z)}=\left(1-r^2\right)^{\frac{5}{2}}$

Q.3.

(2X10=20 Marks)

a) Using Green's theorem for $\int_c (x^2-2xy)dx+(x^2y+3)dy$ around the boundary c of the region $y^2=8x$ and $x=2$.

b) Find the work done by a force $y\,i + x\,j$ which displaces a particle from origin to a point $(i + j)$.

c) Find the direction in which the directional derivative of $f(x, y) = \frac{(x^2-y^2)}{xy}$ at $(1, 1)$ is zero.

Q.4

(2X10=20 Marks)

a) Show that given equations $3x + 4y + 5z = a$, $4x + 5y + 6z = b$, $5x + 6y + 7z = c$ don't have a solution unless $a + c = 2b$.

b) Examine the system of vectors for linear dependence. If dependence, find the relation between them.

$X_1 = (1, -1, 1)$, $X_2 = (2, 1, 1)$ $X_3 = (3, 0, 2)$

c) Find the Inverse of the matrix, by using E-transformations, If $A = \begin{bmatrix} 1 & -1 & 1 \\ 4 & 1 & 0 \\ 8 & 1 & 1 \end{bmatrix}$

Q. 5

(2X10=20 Marks)

a) Change the order of integration and hence evaluate: $\int_0^1 \int_{x^2}^{x} (x^2 + y^2)^{-1/2} \, dy \, dx$.

b) Evaluate $\int_0^1 (x \log x)^3 \, dx$

c) Evaluate $\int_0^{2a} \int_0^{\sqrt{2ax - x^2}} x^2 \, dy \, dx$