Roll No. | | | | | | | H

TMA–101

B. Tech. (First Semester)

End Semester EXAMINATION, 2017

(All Branches)

ENGINEERING MATHEMATICS—I

Time : Three Hours] [*Maximum Marks : 100*

Note : (i) This question paper contains *five* questions.

(ii) All questions are compulsory.

(iii) Instructions on how to attempt a question are mentioned against it.

(iv) Total marks assigned to each question are **twenty**.

1. Attempt any *two* parts of choice from (a), (b) and (c). (10×2=20 Marks)

(a) If $y = \tan^{-1} x$, find y_n.

(b) If $y = \sin^{-1} x$, find $y_n(0)$.

(c) If $x^x y^y z^z = \lambda$, show that :

$$\left(\frac{\partial^2 z}{\partial x \partial y}\right) = -(x \log ex)^{-1}$$

at $x = y = z$.

2. Attempt any *two* parts of choice from (a), (b) and (c). (10×2=20 Marks)

(a) Find the extreme values of function $f(x, y) = x^3 y^2 (1 - x - y)$.

(b) If $u = x(1-r^2)^{-1/2}$ $v = y(1-r^2)^{-1/2}$, $w = z(1-r^2)^{-1/2}$ where $r^2 = x^2 + y^2 + z^2$, then show that $\frac{\partial(u, v, w)}{\partial(x, y, z)} = (1-r^2)^{-5/2}$.

(c) The pressure P at any point (x, y, z) in space is $P = 400\, x y z^2$. Find the highest pressure at the surface of a unit sphere $x^2 + y^2 + z^2 = 1$.

3. Attempt any *two* parts of choice from (a), (b) and (c). (10×2=20 Marks)

(a) Find the direction in which the directional derivative of $\phi(x, y) = \frac{(x^2 - y^2)}{xy}$ at the point (1, 1) is zero.

(b) If a is constant vector and r is the radius vector, $\vec{r} = xi + yj + zk$, prove that curl

$$(\vec{r} \times \vec{a}) = -2a.$$

(c) Using Green's theorem for :

$$\int_C (x^2 - 2xy)\, dx + (x^2 y + 3)\, dy$$

around the boundary C of the region $y^2 = 8x$ and $x = 2$.

4. Attempt any *two* parts of choice from (a), (b) and (c). (10×2=20 Marks)

(a) Find the inverse of the matrix, by using E-transformations, if $A = \begin{bmatrix} 1 & -1 & 1 \\ 4 & 1 & 0 \\ 8 & 1 & 1 \end{bmatrix}$.

(b) Verify Cayley-Hamilton theorem :

$$A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

(c) Find two non-singular matrix P and Q such that PAQ is normal form $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$.

5. Attempt any *two* parts of choice from (a), (b) and (c). (10×2=20 Marks)

(a) Evaluate :

$$\int_0^\infty x^4 e^{-x^2}\, dx$$

(b) Evaluate :

$$\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dy\, dx}{1 + x^2 + y^2}$$

(c) Evaluate :

$$\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\, dz\, dy\, dx$$