3/12/19

Roll No. ....

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TMA–101

B. TECH. (FIRST SEMESTER)

END SEMESTER EXAMINATION, 2019

(ALL BRANCHES)

ENGINEERING MATHEMATICS—I

Time : Three Hours

Maximum Marks : 100

Note : (i) This question paper contains five questions with alternative choice.

(ii) All questions are compulsory.

(iii) Instructions on how to attempt a question are mentioned against it.

(iv) Total marks assigned to each question are **twenty.**

1. Attempt any *two* parts of choice from (a), (b) and (c). (10×2=20 Marks)

(a) For what values of k, the equations $x + y + z = 1$, $2x + y + 4z = k$, $4x + y + 10z = k^2$ has a solution.

(b) If a matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$, find the matrix A^{32}, using Cayley-Hamilton Theorem.

(c) Let $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$. Find matrix P such that $P^{-1}AP$ is diagonal matrix.

2. Attempt any *two* parts of choice from (a), (b) and (c). (10×2=20 Marks)

(a) If $y = \cos^{-1}\left(\dfrac{x - \frac{1}{x}}{x + \frac{1}{x}}\right)$, prove that $y_n = 2(-1)^{n-1}(n-1)\,!\sin^n\theta \sin n\theta$,

where $\theta = \tan^{-1}\left(\dfrac{1}{x}\right)$.

(b) If $y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$, prove that :

$$(x^2 - 1)\,y_{n+2} + (2n+1)\,xy_{n+1} + (n^2 - m^2)\,y_n = 0.$$

(c) If z be a homogeneous function of degree n, show that :

(i) $x\dfrac{\partial^2 z}{\partial x^2} + y\dfrac{\partial^2 z}{\partial x \partial y} = (n-1)\dfrac{\partial z}{\partial x}$

(ii) $x\dfrac{\partial^2 z}{\partial x \partial y} + y\dfrac{\partial^2 z}{\partial y^2} = (n-1)\dfrac{\partial z}{\partial y}$

(iii) $x^2\dfrac{\partial^2 z}{\partial x^2} + 2xy\dfrac{\partial^2 z}{\partial x \partial y} + y^2\dfrac{\partial^2 z}{\partial y^2} = n(n-1)\,z$

3. Attempt any *two* parts of choice from (a), (b) and (c). (10×2=20 Marks)

(a) In a plane triangle ABC, find the maximum value of cos A cos B cos C.

(b) Use the method of the Lagrange's multipliers to find the volume of the largest rectangular parallelopiped that can be inscribed in the ellipsoid $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$.

(c) If u, v, w are the roots of the equation $(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$ in λ, find $\dfrac{\partial(u, v, w)}{\partial(x, y, z)}$.

4. Attempt any *two* parts of choice from (a), (b) and (c). (10×2=20 Marks)

(a) Show that $\nabla^2\left(\dfrac{x}{r^3}\right) = 0$, where r is the magnitude of the position vector $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

(b) Evaluate $\iint \vec{A}\,.\,\hat{n}\, dS$, where $\vec{A} = 18z\hat{i} - 12\hat{j} + 3y\hat{k}$ and S is the part of the plane $2x + 3y + 6z = 12$ included in the first octant.

(c) Apply Stokes' theorem to calculate $\int 4y\,dx + 2z\,dy + 6y\,dz$ over the curve of interaction of $x^2 + y^2 + z^2 = 6z$ and $z = x + 3$.

5. Attempt any *two* parts of choice from (a), (b) and (c). (10×2=20 Marks)

(a) Evaluate :

$$\int_0^{\log 2}\int_0^x\int_0^{x+y} e^{x+y+z}\, dx\, dy\, dz.$$

(b) Change the order of integration and evaluate :

$$\int_0^a\int_0^x \frac{\sin y\, dy\, dx}{\sqrt{[(a-x)(x-y)](4 - 5\cos y)^2}}.$$

(c) Find by double integration the area enclosed by the curve $9xy = 4$ and the line $2x + y = 2$.