Roll No. | | | | | | | | **H**

TMA–101

B. Tech. (First Semester)

Mid Semester EXAMINATION, 2017

(All Branch)

ENGINEERING MATHEMATICS—I

Time : 1:30 Hours] [*Maximum Marks : 50*

Note : (i) This question paper contains two Sections.

(ii) Both Sections are compulsory.

Section—A

1. Fill in the blanks/True-False : (1×5=5 Marks)

(a) If $u = y^x$, then $\frac{\partial u}{\partial x} = xy^{x-1}$. (True/False)

(b) The derivative of $\sec(x^\circ + 30^\circ) =$ ....

(c) If $u = x^2$, $v = y^2$, then $\frac{\partial(u, v)}{\partial(x, y)}$ is ....

(d) If $y = \cos x$, then its nth derivative is $\cos\left(x + \frac{n\pi}{2}\right)$. (True/False)

(e) If $p = q = 0$, $rt - s^2 > 0$, $r < 0$, then $f(x, y)$ is minimum. (True/False)

2. Attempt any *five* parts : (3×5=15 Marks)

(a) If :

$$y = a\cos(\log x) + b\sin(\log x)$$

show that :

$$x^2 y_2 + xy_1 + y = 0$$

(b) If :

$$u = \log(x^3 + y^3 + z^3 - 3xyz)$$

show that :

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{(x+y+z)}$$

(c) If :

$$u = \log\frac{x^4 + y^4}{x+y}$$

show that :

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3.$$

(d) Expand $e^x \sin y$ in powers of x and y as far as terms of second degree.

(e) If :

$$x = r\cos\theta$$
$$y = r\sin\theta$$

find $\frac{\partial(x, y)}{\partial(r, \theta)}$.

(f) Find the minimum value of function $x^2 + y^2 + 6x + 12$.

Section—B

3. Attempt any *two* parts of choice from (a), (b) and (c). (5×2=10 Marks)

(a) If :

$$y = \tan^{-1} x$$

show that :

$$(1+x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$$

(b) If $u = e^{xyz}$, prove that :

$$\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2y^2z^2)e^{xyz}$$

(c) If :

$$\frac{x^2}{a^2+u} + \frac{y^2}{b^2+u} + \frac{z^2}{c^2+u} = 1,$$

prove that :

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = 2\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}\right)$$

4. Attempt any *two* parts of choice from (a), (b) and (c). (5×2=10 Marks)

(a) Show that :

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 2\tan u$$

where $u = \sin^{-1}\left(\frac{x^3 + y^3 + z^3}{ax + by + cz}\right)$.

(b) Expand $x^2y+3y-2$ in powers of $(x-1)$ and $(y+2)$ using Taylor's theorem.

(c) Find the extreme values of function x^3+y^3-3axy.

5. Attempt any *two* parts of choice from (a), (b) and (c). (5×2=10 Marks)

(a) If :

$$u=\sin^{-1}\left(\frac{x^{1/3}+y^{1/3}}{x^{1/2}+y^{1/2}}\right)^{\frac{1}{2}},$$

show that :

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}$$

$$=\frac{1}{144}\tan u(\sec^2 u+12)$$

(b) If $y_1=\frac{x_2x_3}{x_1}$, $y_2=\frac{x_1x_3}{x_2}$, $y_3=\frac{x_1x_2}{x_3}$; then show that :

$$\frac{\partial(y_1,y_2,y_3)}{\partial(x_1,x_2,x_3)}=4$$

(c) If $y=\frac{1}{1-5x+6x^2}$, find y_n.