05/12/16

Roll No. | | | | | | | |

H

TMA–101

B. Tech. (First Semester)

End Semester EXAMINATION, 2016

(All Branches)

ENGINEERING MATHEMATICS–I

Time : Three Hours] . [*Maximum Marks : 100*

Note : (i) This question paper contains five questions.

(ii) All questions are compulsory.

(iii) Instructions on how to attempt a question are mentioned against it.

(iv) Total marks assigned to each question are **twenty.**

1. Attempt any *two* questions of choice from (a), (b) and (c). (2×10=20 Marks)

(a) If $y = \sin(m\sin^{-1} x)$, find the value of y_n at $x = 0$.

(b) If $x = r\cos\theta, y = r\sin\theta$, prove that $\frac{\partial r}{\partial x} = \frac{\partial x}{\partial r}$ and $\frac{1\partial x}{r\partial\theta} = r\frac{\partial\theta}{\partial x}$.

(c) If $y = \frac{x^4}{x^2 - 3x + 2}$, find y_n.

2. Attempt any *two* questions of choice from (a), (b) and (c). (2×10=20 Marks)

(a) If $u = x\phi\left(\frac{y}{x}\right) + \psi\left(\frac{y}{x}\right)$, prove that by Euler theorem on homogeneous function that :

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0$$

(b) If $u = \frac{x+y}{z}, v = \frac{y+z}{x}, w = \frac{y(x+y+z)}{xz}$, then show that u, v, w are dependent.

(c) Expand y^x at (1, 1) upto second degree terms.

3. Attempt any *two* questions of choice from (a), (b) and (c). (2×10=20 Marks)

(a) Find the directional derivative of $f(x, y, z) = xyz$ at the point P (1, – 1, – 2) in the direction of the vector $2\hat{i} - 2\hat{j} + 2\hat{k}$.

(b) Verify Green's theorem for

$$\int_C (x^2 - 2xy)\,dx + (x^2 y + 3)\,dy$$

around the boundary c of the region $y^2 = 8x$ and $x = 2$.

(c) Find the total work done by a force :

$$F = (x^2 + y^2)i - 2xyj$$

moving a point at (0, 0) to (a, b) along the rectangle bounded by the lines $x = 0, y = 0, x = a$ and $y = b$.

4. Attempt any *two* questions of choice from (a), (b) and (c). (2×10=20 Marks)

(a) Reduce the matrix into normal form and find of rank of matrix :

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

(b) Find the Eigen values and Eigen vectors of the matrix :

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 3 & 2 & 3 \end{bmatrix}$$

(c) Find the value of k for which the system of equations :

$$3x - y + 4z = 3$$
$$x + 2y - 3z = -2$$
$$6x + 5y + kz = -3$$

has a unique solution. Find the solution for $k = -5$.

5. Attempt any *two* questions of choice from (a), (b) and (c). (2×10=20 Marks)

(a) Evaluate :

$$\int_0^{\pi/2}\int_0^{y} \cos 2y\sqrt{1 - a^2\sin^2 x}\,dx\,dy$$

(b) Evaluate :

$$\int_1^e \int_1^{\log y} \int_1^{e^x} \log z \, dz \, dx \, dy$$

(c) Change the order of integration and hence evaluate :

$$\int_0^1 \int_x^{2-x} \left(\frac{x}{y}\right) dy \, dx$$