H Roll No. 2299037

TMA–316

B. TECH. (THIRD SEMESTER)
END SEMESTER
EXAMINATION, Dec., 2023

DISCRETE MATHEMATICAL STRUCTURES AND COMBINATORICS

Time : Three Hours

Maximum Marks : 100

Note : (i) All questions are compulsory.

(ii) Answer any *two* sub-questions among (a), (b) and (c) in each main question.

(iii) Total marks in each main question are **twenty**.

(iv) Each sub-question carries 10 marks.

1. (a) Define transitive closure of relation. Let $A=\{1,2,3\}$ and $R=\{(1,2),(2,3),(3,1)\}$. Find the transitive closure of R using Warshall's algorithm. (CO1)

(b) Define a Lattice. Give an example of an infinite lattice with : (CO1)

(i) neither a least nor a greatest element.

(ii) a least but not a greatest element.

(iii) a greatest but not a least element.

(iv) both a least and a greatest element.

(c) Let $f: R \to R$ and $g: R \to R$, where R is the set of real numbers. Find f o g and g o f, where $f(x) = x^2$ and $g(x) = x + 4$. State whether functions are injective, surjective of bijective. (CO1)

2. (a) In a bolt factory, three machines M_1, M_2, and M_3 manufacture 2000, 2500, and 4000 bolts every day. Of their output 3%, 4%, and 2.5% are defective bolts. One of the bolts is drawn very randomly from a day's production and is found to be defective. What is the probability that it was produced by machine M_2? (CO2)

(b) A die is thrown 6 times. If "getting an odd number" is a "success". What is the probability of : (CO2)

(i) 5 successes

(ii) at least 5 successes

(iii) at most 5 successes

(c) A random variable has the following probability distribution : (CO2)

x	P (x)
4	0.1
5	0.3
6	0.4
8	0.2

Find the expectation and Standard Deviation of random variable.

3. (a) Define a valid argument. Check the validity of the following argument : If I take breakfast, then I go to school. I do not take breakfast. Therefore, I do not go to school. (CO3)

(b) Explain quantifiers. Symbolize the following sentence using (i) universe of discourse and (ii) without universe of discourse : (CO3)

(1) All states in India are not highly populated.

(2) Some students are tall but not healthy.

(c) Using mathematical induction prove that $6^{n+2} + 7^{2n+1}$ is divisible by 43 for each positive integer n. (CO3)

4. (a) Consider the set Q of rational numbers, and let * be the operation on Q defined by $a * b = ab/2$. Check whether $(Q, *)$ is an abelian group ? (CO4)

(b) Find the solution of the recurrence relation : (CO4)

$$a_n + 2a_{n-1} - 3a_{n-2} = 2^n$$

(c) Define a cyclic group. If G is a cyclic group, then prove that G has exactly two generators. (CO4)

5. (a) Define a connected graph. A disconnected simple graph G (without self-loops and parallel edges) with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges. (CO5)

(b) Prove that the minimum height of a full binary tree with n vertices is $\lceil \log_2(n+1)-1 \rceil$, where $\lceil x \rceil$ is the smallest integer greater than or equal to x. (CO5)

(c) Define Euler and Hamiltonian Graphs and give the example of graph : (CO5)

(i) Euler but not Hamiltonian

(ii) Hamiltonian but not Euler

(iii) Both Euler and Hamiltonian