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TMA–201

B. TECH. (SECOND SEMESTER) END SEMESTER EXAMINATION, 2018

(All Branches)

ENGINEERING MATHEMATICS–II

Time : Three Hours

Maximum Marks : 100

Note : (i) This question paper contains five questions with alternative choice.

(ii) All questions are compulsory.

(iii) Instructions on how to attempt a question are mentioned against it.

(iv) Each part carries **ten** marks. Total marks assigned to each question are **twenty**.

1. Attempt any *two* questions of choice from (a), (b) and (c). (2×10=20 Marks)

(a) Solve the differential equation of :

$$(D^4 + D^2)y = 0$$

$$y(0) = y'(0) = y''(0) = 0, y'''(0) = 1.$$

(b) Solve $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$ with the help of variation of parameter.

(c) Solve :

$(D^3 - 3D^2D' - 4DD'^2 + D'^3)z = \sin(y + 2x)$

2. Attempt any *two* questions of choice from (a), (b) and (c). (2×10=20 Marks)

(a) Express $f(t) = \sin 2t, 2\pi < t < 4\pi$ and $f(t) = 0$ otherwise, in terms of unit step function and then find its Laplace transform.

(b) Find the Laplace inverse of $\log\left(\frac{s+1}{s-1}\right)$.

(c) Applying convolution theorem, solve the following initial value problem :

$$y'' + y = \sin 3t$$

given $y(0) = 0, y'(0) = 0$.

3. Attempt any *two* questions of choice from (a), (b) and (c). (2×10=20 Marks)

(a) To prove that :

$$J_{-n}(x) = (-1)^n J_n(x)$$

(b) Define orthogonally of Legendre polynomials and to prove that :

$$\int_{-1}^{1} P_m(x) P_n(x)\, dx = \frac{2}{(2n+1)}$$

if $m = n$.

(c) Prove that :

$$P_n(x) = \frac{1}{n!2^n} \frac{d^n}{dx^n}(x^2 - 1)^n$$

4. Attempt any *two* questions of choice from (a), (b) and (c). (2×10=20 Marks)

(a) Prove that :

$$x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n \frac{\cos nx}{n^2}, -\pi < x < \pi$$

(b) Obtain Fourier series to represent the function $f(x) = |x|$ for $-\pi < x < \pi$ and deduce :

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots..$$

(c) Obtain the half range cosine series for $f(x) = x^2, 0 < x < \pi$.

5. Attempt any *two* questions of choice from (a), (b) and (c). (2×10=20 Marks)

(a) Using the method of separation of variables, solve $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$ and $u = e^{-5y}$ when $x = 0$.

(b) Find the solution of the wave equation :

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

such that $y = P_0 \cos Pt$, (P_0 is a const.) When $x = l$ and $y = 0$ when $x = 0$.

(c) Find the solution of $\frac{\partial^2 u}{\partial x^2} = h^2 \frac{\partial u}{\partial t}$ for which $u(0, t) = u(l, t) = 0$, $u(x, 0) = \sin \frac{\pi x}{l}$ by method of variables separable.

F. No. : a-77