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TMA–302

B. Tech. (CE) (Third Semester)
Mid Semester EXAMINATION, 2017
ENGINEERING MATHEMATICS—III

Time : 1:30 Hours] [*Maximum Marks : 50*

Note : (i) This question paper contains two Sections.

(ii) Both Sections are compulsory.

Section—A

1. Fill in the blanks/True-False : (1×5=5 Marks)

(a) The transform of $f(x)$ defined by :

$$\int_{x_1}^{x_2} K(s, x)\, f(x)\, dx$$

is called ..... .

(b) The Cartesian form of the Cauchy-Riemann equation is ..... .

(c) If Z_1 and Z_2 are two complex variation, then $|Z_1 - Z_2| \geq |Z_1| - |Z_2|$. (True/False)

(d) A bilinear transformation maps straight line into straight line. (True/False)

(e) The Fourier transformation is a linear operation. (True/False)

2. Attempt any *five* parts : (3×5=15 Marks)
(Define/Short Numerical\Short Programming/ Draw)

(a) Find the Fourier cosine transformation of :

$$f(x)=\begin{cases} x & \text{for } 0<x<1 \\ 2-x & \text{for } 1<x<2 \\ 0 & \text{for } x>2 \end{cases}$$

(b) Find the Fourier sine transform of $\frac{e^{-ax}}{x}$.

(c) Find harmonic conjugate of the analytic function whose real part is :

$$x^3-3xy^2+3x^2-3y^2$$

(d) Define the Bilinear transformation.

(e) Show that the function $u=4xy-3x+2$ is harmonic.

(f) Using Fourier integral show that :

$$\int_0^\infty \frac{\cos\lambda x}{1+\lambda^2}d\lambda=\frac{\pi}{2}e^{-x},\ x>0$$

Section—B

3. Attempt any *two* parts of choice from (a), (b) and (c). (5×2=10 Marks)

(a) Find the Fourier transform of :

$$f(z)=\begin{cases} 0-x^2, & |x|\le 1 \\ 0, & |x|>1 \end{cases}$$

Hence evaluate :

$$\int_0^\infty \left(\frac{x\cos x-\sin x}{x^3}\right)\cos\left(\frac{x}{2}\right)dx$$

(b) Find the Fourier sine and cosine transform of $x^{n-1}, n>0$.

(c) Find the Fourier cosine transform of e^{-x^2}.

4. Attempt any *two* parts of choice from (a), (b) and (c). (5×2=10 Marks)

(a) Derive the Cauchy-Riemann equation in Polar form.

(b) Prove that $u=x^2-y^2-2xy-2y+3y$ is harmonic, find analytic function $f(z)=u+iv$ in term of z.

(c) Show that the function $f(z)$ defined by :

$$f(z)=\begin{cases} \frac{x^2y^5(x+iy)}{x^4+y^{10}}, & z\neq 0 \\ 0, & z=0 \end{cases}$$

is not analytic at the origin even though it satisfies Cauchy-Riemann equation at the origin.

5. Attempt any *two* parts of choice from (a), (b) and (c). (5×2=10 Marks)

(a) If $u-v=e^x(\cos y-\sin y)$ and $f(z)=u+iv$ is analytic function of $z=x+iy$, find $f(z)$ in term of z.

(b) Find the Fourier sine transform of :

$$\frac{1}{x(x^2+a^2)}$$

(c) Find the bilinear transformation which maps $z_1=1, z_2=i, z_3=-1$ of z-plane to $w_1=i$, $w_2=0$, $w_3=-i$ of w-plane.