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TMA–316

B. TECH. (C.S.E.) (THIRD SEMESTER) END SEMESTER EXAMINATION, Jan., 2023

DISCRETE STRUCTURE AND COMBINATORICS

Time : Three Hours

Maximum Marks : 100

Note : (i) All questions are compulsory.

(ii) Answer any *two* sub-questions among (a), (b) and (c) in each main question.

(iii) Total marks in each main question are **twenty**.

(iv) Each sub-question carries 10 marks.

1. (a) Let $f : \mathbf{R} \to \mathbf{R}$ is a mapping defined by $f(x) = \sin x$. Find : (CO1)

(i) The image set of **R** under f and hence conclude whether f is a surjective.

(ii) $\left\{x : x \in \mathbf{R}, f(x) = \frac{1}{2}\right\}$ and hence conclude whether f is an injective.

(b) Consider the set of ordered pair of natural number $\mathbf{N} \times \mathbf{N}$ defined by $(a, b) \mathrm{R} (c, d) \Leftrightarrow a + d = b + c$. Prove that R is an equivalence relation. (CO1)

(c) In a survey of 600 television viewers given the following information : 385 watch cricket matches, 295 watch hockey matches, 215 watch football matches, 145 watch cricket and football matches, 170 watch cricket and hockey matches, 150 watch hockey and football matches and 150 do not watch any of the three kinds of matches. (CO1)

(i) How many people in the survey watch all three kinds of matches ?

(ii) How many people watch exactly one of the sports ?

2. (a) Let X have the density function

$$f(x) = \begin{cases} 0.75\left(1 - x^2\right), & \text{if } x \in [-1, 1] \\ 0, & \text{otherwise} \end{cases}.$$

Find the distribution function. Find the probabilities P $\left(\frac{-1}{2} \le x \le \frac{1}{2}\right)$ and $P\left(\frac{1}{4} \le x \le 2\right)$. Find x such that P (X, $\le x$). (CO2)

(b) Find mean, variance and moment generating function of Exponential distribution. Also prove the lack of memory property of the exponential distribution. (CO2)

(c) A die is thrown 6 times. If getting an even number is success, find the probabilities of (i) at least one success, (ii) $\le$ 3 success and (iii) 4 successes. (CO2)

3. (a) Express the statement

$$\left(\neg(P \vee Q) \vee (\neg P) \wedge Q\right)$$

in simplest possible form. (CO3)

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(b) Using truth table, verify the following equivalencies : (CO3)

(i) $\neg(P \leftrightarrow Q) \equiv (P \wedge \neg Q) \vee (\neg P \wedge Q)$

(ii) $P \rightarrow (Q \vee R) \equiv (P \rightarrow Q) \vee (P \rightarrow R)$

(c) Prove the statement by contradiction : There does not exist an integer k such that $4k + 3$ is a perfect square. (CO3)

4. (a) The questions paper of discrete contains two questions divided into two groups of five questions each. In how many ways can an examinee answer six questions taking at least two from each group ? (CO5/4)

(b) State and prove Lagrange's theorem. (CO5/4)

(c) If for every element a of a group G, $a^2 = e$; then prove that G is an abelian group. (CO5/4)

5. (a) For any simple graph of n vertices, prove that the number of edges of G is less than or equal to $\frac{n(n-1)}{2}$. (CO6)

(b) Define cycle graph, bipartite graph, path graph and star graph. Give one example of each of the graph. (CO6)

(c) Prove that, if G is a tree with n vertices then it has exactly n-1 edges. (CO6)

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