H Roll No. 2394081

TMA–101

B. TECH. (FIRST SEMESTER)

MID SEMESTER

EXAMINATION, Oct., 2023

ENGINEERING MATHEMATICS—I

Time : 1½ Hours

Maximum Marks : 50

Note : (i) Answer all the questions by choosing any ***one*** of the sub-questions.

(ii) Each sub-question carries 10 marks.

1. (a) Find the inverse of the following matrix employing elementary transformations :

(CO1)

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}.$$

P. T. O.

OR

(b) Define linear dependence and independence of vectors. Examine for linear dependence [1, 0, 2, 1], [3, 1, 2, 1], [4, 6, 2, –4], [–6, 0, –3, –4] and find the relation between them, if possible. (CO1)

2. (a) Find the rank of the matrix A by normal form method where : (CO1)

$$A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 4 & 1 & 2 & 1 \\ 3 & -1 & 1 & 2 \\ 1 & 2 & 0 & 1 \end{bmatrix}.$$

OR

(b) Discuss the consistency of the following system of equations :

$$2x + 3y + 4z = 11,$$

$$x + 5y + 7z = 15,$$

$$3x + 11y + 13z = 25.$$

If found consistent, solve it. (CO2)

3. (a) Find the eigen values and eigen vectors of the following matrix : (CO1)

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}.$$

OR

(b) State and verify the Cayley-Hamilton theorem for the matrix : (CO1)

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$$

4. (a) If $f(x) = x^n$, prove that : (CO3)

$$f(1) + \frac{f'(1)}{1!} + \frac{f''(1)}{2!} + \frac{f'''(1)}{3!} + \dots + \frac{f^n(1)}{n!} = 2^n$$

OR

(b) If $y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$, prove that : (CO3)

$$(x^2 - 1)y_{n+2} + (2n+1)x.y_{n+1} + (n^2 - m^2)y_n = 0.$$

5. (a) If $\dfrac{x^2}{a^2+u}+\dfrac{y^2}{b^2+u}+\dfrac{z^2}{c^2+u}=1$, show that : (CO3)

$$\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2=2\left(x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}\right)$$

OR

(b) Expand x^y in powers of $(x-1)$ and $(y-1)$ up to the third-degree terms.

(CO3)

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