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TMA–101

B. TECH. (NON-CS)
(FIRST SEMESTER)
END SEMESTER EXAMINATION, 2018
ENGINEERING MATHEMATICS

Time : Three Hours

Maximum Marks : 100

Note : (i) This question paper contains five questions with alternative choice.

(ii) All questions are compulsory.

(iii) Each question carries three Parts (a), (b), and (c). Attempt any *two* question of each Part.

(iv) Each Part carries **ten** marks. Total marks assigned to each question are **twenty**.

1. Attempt any *two* questions of choice from (a), (b) and (c). (2×10=20 Marks)

(a) Define Eigen value and Eigen vector of the matrix. Find the Eigen value of the matrix :

$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}.$$

(b) Find for what values of λ and μ the system of linear equations $x+y+z=6$, $x+2y+5z=10$ and $2x+3y+\lambda z=\mu$ has

(i) a unique solution

(ii) no solution

(iii) infinite no solution

(c) A square matrix A is defined by $A=\begin{pmatrix} 1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{pmatrix}$. Find the modal matrix P and the resulting diagonal matrix D of A.

2. Attempt any *two* questions of choice from (a), (b) and (c). (2×10=20 Marks)

(a) Define Leibnitz's theorem, and if $y=(x^2-1)^n$, prove that :

$$(x^2-1)\,y_{n+2}+2\,xy_{n+1}-n\,(n+1)y_n=0.$$

(b) Define Cauchy's root test, and test for convergence of the series whose nth term is $\dfrac{n^{n^2}}{(n+1)^{n^2}}$.

(c) If $z=f(x,y)$ is a homogeneous function of x and y of degree n, then :

$$x^2\frac{\partial^2 z}{\partial x^2}+2\,xy\frac{\partial^2 z}{\partial x\,\partial y}+y^2\frac{\partial^2 z}{\partial y^2}=n\,(n-1)\,z$$

3. Attempt any *two* questions of choice from (a), (b) and (c). (2×10=20 Marks)

(a) If $u^3+v^3=x+y$, $u^2+v^2=x^3+y^3$, then prove that : $\dfrac{\partial(u,v)}{\partial(x,y)}=\dfrac{y^2-x^2}{2\,uv\,(u-v)}$.

(b) Divide 120 into three parts so that the sum of their products taken two at a time shall be maximum.

(c) Use the method of the Lagrange's multipliers to find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}+\dfrac{z^2}{c^2}=1$.

4. Attempt any *two* questions of choice from (a), (b) and (c). (2×10=20 Marks)

(a) Evaluate $\oint \overline{F}.dr$ where $\overline{F}=x^2\hat{i}+xy\hat{j}$ and C is the boundary of the square in the plane $z=0$ and bounded by the lines $x=0$, $y=0$, $x=a$ and $y=a$.

(b) Evaluate $\oint \overline{F} . dr$ by Stokes' Theorem, where $\overline{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ and C is the boundary of the rectangle $x = \pm a$, $y = 0$ and $y = b$.

(c) Show that $\text{div}(\text{grad}\, r^n) = n(n+1)r^{n-2}$, where $r^2 = x^2 + y^2 + z^2$.

5. Attempt any *two* questions of choice from (a), (b) and (c). (2×10=20 Marks)

(a) Find by double integration the whole area of the curve $a^2x^2 = y^3(2a - y)$.

(b) To prove that :

$$\Gamma m \, \Gamma\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}} \Gamma 2m$$

where m is positive.

(c) Change the order of integration of :

$$\int_0^a \int_{\sqrt{a^2 - y^2}}^{y+a} f(x, y)\, dxdy .$$

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