H Roll No. 2294038

TMA–316

B. TECH. (THIRD SEMESTER)

MID SEMESTER

EXAMINATION, Oct., 2023

DISCRETE MATHEMATICAL STRUCTURES AND COMBINATORICS

Time : 1½ Hours

Maximum Marks : 50

Note : (i) Answer all the questions by choosing any ***one*** of the sub-questions.

(ii) Each sub-question carries 10 marks.

1. (a) Prove that the set $(P(S), \subseteq)$ for any set S is a lattice. Also show that if $(L, \leq)$ is a lattice, then $(L, \geq)$ is also a lattice. (CO1)

P. T. O.

OR

(b) Find the cardinality of a set of integers, defined as :

$X = \{n \mid 1 \leq n \leq 123, n$ is divisible by 2 or 3$\}$.

Also evaluate $| P(P(P(P(P(X))))) |$.

(CO1)

2. (a) Prove the following : (CO1)

(i) $A - B = A \Leftrightarrow A \cap B = \phi$,

(ii) $A - B = \phi \Leftrightarrow A \subseteq B$,

(iii) $A - (A - B) = A \cap B$,

(iv) $(A - C) \cap (B - C) = (A \cap B) - C$.

OR

(b) Define the following with suitable examples :

(i) Reflexive Relation,

(ii) Irreflexive Relation,

(iii) Antisymmetric Relation,

(iv) Asymmetric Relation

If the cardinality of a set is n, determine :

(i) least number of elements,

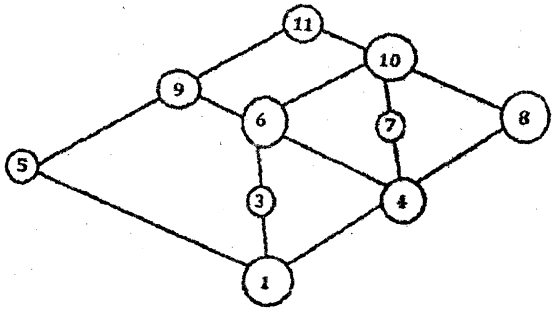
(ii) most number of elements, and

(iii) possible numbers of relations for each case. (CO1)

3. (a) Show that the relation '$\subseteq$' defined on the power set P(A) of a set A is a partial order relation. (CO1)

OR

(b) Let A = {1, 2, 3, 4, ..., 11}, be the poset whose Hasse diagram is shown in the figure :



Find the LUB and GLB of B = {6, 7, 10}, if they exist. (CO2)

4. (a) A bag contains 8 red and 5 white balls. Three balls are drawn at random. Find the probability that : (CO2)

(i) all three balls are white,

(ii) all three balls are red,

(iii) one ball is red and two balls are white

OR

(b) Two dices are thrown. Find the probability that the sum of the numbers coming on them is 9, if it is known that the number 5 always occurs on the first dice. (CO2)

5. (a) Write probability distribution when three coins are tossed. (CO2)

OR

(b) If m things are distributed among 'a' men and 'b' women, show that the probability that the number of things received by men is odd, is : (CO2)

$$\frac{1}{2}\frac{\{(b+a)^m-(b-a)^n\}}{(b+a)^m}.$$