TMA -201

Back End Semester Examination

B.Tech – Second Semester ENGINEERING MATHEMATICS - II

Time: Three Hours

MM: 100

Note:

(i) This question paper contains five questions with alternative choice.
(ii) All questions are compulsory.
(iii) Instructions on how to attempt a question are mentioned against it.
(iv) Total marks assigned to each question are twenty.

Q.1 (2X10=20 Marks)

a) Solve the differential equation $(D^4 + 2D^2 + 1)y = x^2 \sin x$

b) Solve the Partial differential equation $(D^2 - DD' - 2D'^2 + 2D + 2D')z = x^2 y$

c) Solve the differential equation $(4x+y)^2 \frac{dx}{dy} = 1$

Q.2 (2X10=20 Marks)

a) Using convolution theorem, evaluate $L^{-1}\left\{\frac{1}{(s+1)(s^2+1)^2}\right\}$

b) Using Laplace transform, solve the differential equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 4e^{2x}, \text{ where } y(0) = -3, y'(0) = 5, \text{ at } x = 0.$$

c) Find Laplace Transform of $\frac{1}{t}\sinh 2t$.

Q.3 (2X10=20 Marks)

a) Prove that $J_{\frac{5}{2}}(x) = \sqrt{\frac{2}{\pi x}}\left(\frac{3-x^2}{x^2}\sin x - \frac{3\cos x}{x}\right)$

b) Prove that $\int_{-1}^{+1} x\, P_n(x)\, P_{n-1}(x)\, dx = \frac{2n}{4n^2-1}$

c) Show that $J_n(x)$ is the coefficient of h^n in the expansion of $e^{\frac{x}{2}\left(h-\frac{1}{h}\right)}$.

Q.4 (2X10=20 Marks)

a) Find half range sine series for $f(x) = x^2 \quad 0 < x < \pi$.

b) Find the Fourier series to represent $f(x) = \left(\frac{\pi - x}{2}\right)^2$, When $0 \le x \le 2\pi$.

c) Develop $\sin\left(\frac{\pi x}{l}\right)$ in half – range cosine series in the range $0 < x < l$.

Q.5

(2X10=20 Marks)

a) Use the method of separation of variables, solve $\dfrac{\partial^2 u}{\partial x \partial t} = e^{-t} \cos x$.

b) Use the method of separation of variables, solve $u_{xy} - 4\,x\,y\,u = 0$.

c) Using the method of separation of variables, solve

$$\frac{\partial^2 u}{\partial x^2} = 2\frac{\partial u}{\partial t} \quad \text{Where} \quad u(x, 0) = x(4 - x).$$