(b) Prove that the following prepositions are tautology : (CO5)

(i) $p \vee \sim p$

(ii) $\sim (p \wedge q) \vee q$

(iii) $p \rightarrow (p \vee q)$

(c) What is proposition ? Define different laws of the algebra of propositions. (CO5)

H Roll No. .....

TBC–103

B. C. A. (FIRST SEMESTER) END SEMESTER EXAMINATION, Jan., 2023

MATHEMATICAL FOUNDATION OF COMPUTER SCIENCE

Time : Three Hours

Maximum Marks : 100

Note : (i) All questions are compulsory.

(ii) Answer any *two* sub-questions among (a), (b) and (c) in each main question.

(iii) Total marks in each main question are **twenty.**

(iv) Each sub-question carries 10 marks.

1. (a) What is Equivalence relation ? Find the equivalence class of 0, 1, 2, 3 for congruence modulo 4. (CO1)

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(b) Define partial order relation. Show that the relation '≥' is partial order relation on the set of integers Z. (CO1)

(c) Draw the Hasse diagram representing the partial ordering $\{(a.b) \mid a \text{ divided } b\}$ on $\{1, 2, 3, 4, 5, 8, 12\}$. (CO1)

2. (a) Show that the mapping $f : R \to R$ be defined by $f(x) = ax + b$ is inertible. Define its inverse. (CO2)

(b) Show that the function $(x, y) = x + y$ is a primitive recursive function. Also find the value of $f(2, 4)$. (CO2)

(c) Define the following with suitable example : (CO2)

(i) One-to-one function

(ii) Onto function

(iii) Into function

(iv) Bijective function

3. (a) Prove that the sum of first n natural number is given by $\frac{n(n+1)}{2}$ by mathematical induction. (CO3)

(b) Find the generating function of the sequence $\{a_k\}$ is $a_k = 2 + 3k$. (CO3)

(c) Solve the recurrence relation : (CO3)

$$y_{n+2} + y_{n+1} + 2y_n = n^2.$$

4. (a) Prove that the fourth root of unity $\{1, -1, i, -i\}$ form an abelian multiplicative group. (CO4)

(b) What is cyclic group ? Show that the multiplicative group $\{1, \omega, \omega^2\}$ is a cyclic group. (CO4)

(c) Define group with suitable examples. (CO4)

5. (a) Construct a truth table for each compound proposition : (CO5)

(i) $P \wedge (\sim q \vee q)$

(ii) $\sim (p \vee q) \vee (\sim p \wedge \sim q)$