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TCS–304/TIT–304

B. Tech. (CS/IT) (Third Semester)
End Semester EXAMINATION, 2017
GRAPH THEORY

Time : Three Hours] [*Maximum Marks : 100*

Note : (i) This question paper contains five questions.

(ii) All questions are compulsory.

(iii) Instructions on how to attempt a question are mentioned against it.

(iv) Total marks assigned to each question are **twenty**.

1. Attempt any *two* parts of choice from (a), (b) and (c). (10×2=20 Marks)

(a) State whether the statements given below are True/False and give proper explanation against your choice : $2\frac{1}{2}$ each

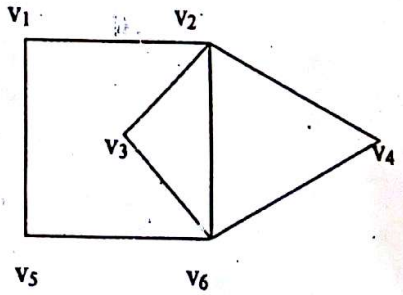
(i) Objective of TSP (Travelling Salesman Problem) is to find out the shortest Hamiltonian cycle in a graph.

(ii) When we say complement of a graph, it is always for simple graph only.

(iii) All trails are path.

(iv) An edge '*e*' is bridge then its obvious that—there exists a cycle which holds '*e*' on it.

(b) Define Semi-Euler, Euler and Hamiltonian graph. For the graph given below mention whether it is—Semi-Euler graph, Euler graph and Hamiltonian graph or not. Give reason behind your answer. 10



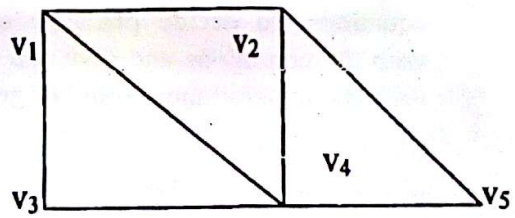
(c) (i) Prove that "summation of degree of all vertices in a graph is twice of the size of the graph". 5

(ii) How many "Hamiltonian-cycle" possible for complete graph of order 5 ? 5

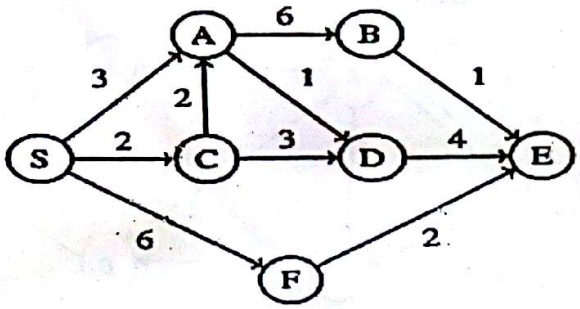
2. Attempt any *two* parts of choice from (a), (b) and (c). (10×2=20 Marks)

(a) (i) What is Spanning Tree ? Mention some important characteristics of Spanning Trees. 4

(ii) Calculate number of spanning trees possible for—Complete graph – K_7, Cyclic graph – C_9 and the graph given below. 6



(b) Find the shortest path from node 'S' to node 'E' in the graph given below (using Dijkstra's algorithm). 10



(c) (i) A complete Binary Tree is having 62 nodes. Then calculate the height of that tree. 5

(ii) Discuss Kruskal's algorithm to find minimal spanning tree formed from a weighted graph with an example. 5

3. Attempt any *two* parts of choice from (a), (b) and (c). (10×2=20 Marks)

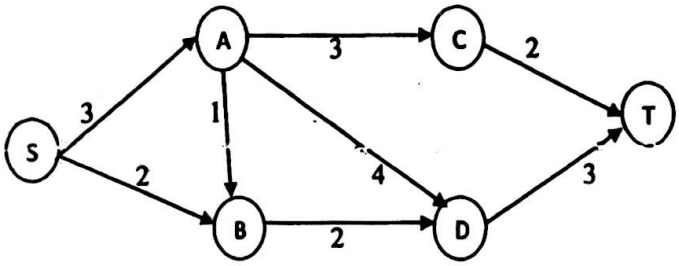
(a) When a graph will be called as a "planar" graph ? Generate the below mentioned equations (to decide planarity of a graph) with the conditions and give *one* example of each (here *e*-size and *n*-order of graph) : 10

(i) $e <= 3n - 6$

(ii) $e <= 2n - 4$

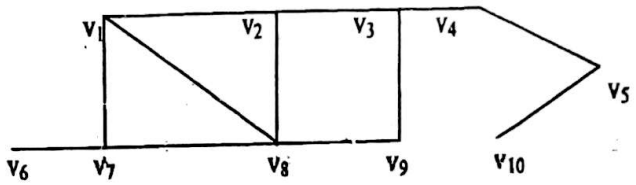
(b) Discuss the concept of network flow. Consider the network given below and solve it to show that—(here 'S' is source and 'T' is destination). 10

Maximal flow = minimal cut.



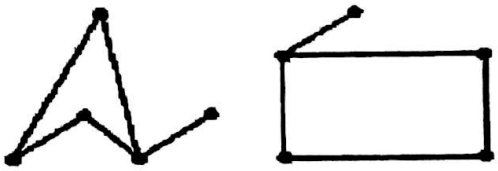
(c) (i) Prove that "For any non-trivial connected graph there will be at least 2 non-cut vertices". 4

(ii) Find out cut-vertices, bridges and blocks in the graph given below : 6



4. Attempt any *two* parts of choice from (a), (b) and (c). (10×2=20 Marks)

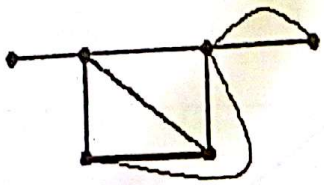
(a) (i) Define isomorphism between two graphs. Verify the following graphs are isomorphic to each other : 5



(ii) Prove that maximum size of a disconnected graph of order '*n*' and component '*k*' could be : 5

$(n - k) * (n - k + 1)/2$

(b) Define cut-set vector and circuit vector of a graph. Find the set of all cut set-vectors and the set of all circuit vectors of the following graph : 10



(c) For a connected graph G = (V, E) and $|V| = n$, define incidence matrix A(G), circuit matrix B(G) and cut-set matrix C(G). Derive the ranks for matrix A, B and C for graph G. 10

5. Attempt any *two* parts of choice from (a), (b) and (c). (10×2=20 Marks)

(a) (i) What is the sum of degree of all the vertices in any wheel graph of order 'n' (W_n) ? 5

(ii) A graph is of order = 20 and its minimum vertex cover number is 8. Then what is the size of maximal independent set possible in that graph ? 5

(b) (i) Define matching. Give an example to explain "Every maximal matching is not perfect matching".

(ii) Find out number of perfect matching for complete graph of order 8. 5

(c) (i) Write down chromatic numbers for the following graphs : 5

Wheel graph—W_n, Complete graph—K_n, Star graph, Cyclic graph—C_n, Bi-partite graph.

(ii) Show that a graph or order 'n' is a tree if and only if its : 5

Chromatic polynomial $= x * (x - 1)^{n-1}$