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# TMA–101

## B. Tech. (First Semester)
## Mid Semester EXAMINATION, 2016

**(All Branches)**

ENGINEERING MATHEMATICS—I

*Time : Two Hours* ]          [ *Maximum Marks : 60*

**Note :** (i) This question paper contains *three* questions with alternative choice.

(ii) All questions are compulsory.

(iii) Each question carries four Parts (a), (b), (c) and (d). Attempt either Parts (a) and (b) or (c) and (d) of each question.

(iv) Each Part carries **ten** marks. Total marks assigned to each question are **twenty**.

1. (a) If $y = a\cos(\log x) + b\sin(\log x)$, show that :

(i) $x^2 y_2 + xy_1 + y = 0$

(ii) $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$

(b) If $u, v$ are function of $x$ and $y$ defined by $x = u + e^{-v} \sin u$ and $y = v + e^{-v} \cos u$, prove that $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$.

*Or*

(c) If $y = x^2 e^x$, show that :

$$y_n = \frac{1}{2} n(n-1) y_2 - n(n-2) y_1 + \frac{1}{2}(n-2) y$$

(d) If $u(u^2 + 3x) + 3y = 0$, prove that :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{2u(x-1)}{(u^2 + x^3)}$$

2. (a) Expand $\sin xy$ in powers of $(x - 1)$ and $\left(y - \frac{\pi}{2}\right)$ up to second degree terms.

(b) Verify Euler's theorem for the functions :

$$u = \log \frac{x^2 + y^2}{xy}$$

*Or*

(c) Test the function $f(x, y) = x^2 + y^2 + \frac{2}{x} + \frac{2}{y}$ for maxima or minima.

(d) If $y_1 = \frac{x_2 x_3}{x_1}, y_2 = \frac{x_1 x_3}{x_2}, y_3 = \frac{x_1 x_2}{x_3}$; then show that $\frac{\partial (y_1, y_2, y_3)}{\partial (x_1, x_2, x_3)} = 4$.





3. (a) Discuss the maximum and minimum values of $x^2 + y^2 + 6x + 12$.

(b) If $u, v, w$ are the roots of the equation in $\lambda$ and $\frac{x}{a+\lambda} + \frac{y}{b+\lambda} + \frac{z}{c+\lambda} = 1$, then find $\frac{\partial (u, v, w)}{\partial (x, y, z)}$.

*Or*

(c) If $u = e^{xyz}$, prove that :

$$\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2 y^2 z^2) e^{xyz}$$

(d) Using Leibnitz theorem, prove that the $n$th differential coefficient of $\frac{x^n}{x+1}$ is $\frac{n!}{(x+1)^{n+1}}$.