H Roll No. 2394081..........

TMA–101

B. TECH. (FIRST SEMESTER) END SEMESTER EXAMINATION, Dec., 2023

ENGINEERING MATHEMATICS—I

Time : Three Hours

Maximum Marks : 100

Note : (i) All questions are compulsory.

(ii) Answer any *two* sub-questions among (a), (b) and (c) in each main question.

(iii) Total marks in each main question are **twenty**.

(iv) Each sub-question carries 10 marks.

1. (a) State the Cayley-Hamilton theorem and verify it for : (CO1)

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$$

(b) Check the consistency of the following linear system and solve them completely if consistent : (CO1)

$$x + y + z = 6$$

$$x + 2y + 5z = 10$$

and $$2x + 3y + 2z = 8.$$

(c) Find the eigen values and eigen vector of matrix : (CO1)

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}.$$

2. (a) If $u = \log\left(\frac{x^4 + y^4}{x + y}\right)$, then find : (CO2)

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$$

and $$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}.$$

(b) State the Leibnitz theorem. If $y = e^{a\sin^{-1}x}$, prove that : (CO2)

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + a^2)y_n = 0.$$

(c) Find the Taylor series expansion of $\tan^{-1}\dfrac{y}{x}$ about (1, 1) upto the inclusion of third-degree terms. Hence compute $f(1.1, 0.9)$ approximately. (CO2)

3. (a) If u, v, w are the roots of the equation $(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$ in λ, find $\dfrac{\partial(u, v, w)}{\partial(x, y, z)}$. (CO3)

(b) Write your statement about the functional dependency of the functions :

$$u = \frac{x + y}{z}, v = \frac{y + z}{x}, w = \frac{y(x + y + z)}{xz}.$$

If dependent find the relation between them. (CO3)

(c) Find the point upon the plane $ax + by + cz = p$ at which the function $f = x^2 + y^2 + z^2$ has a maximum value and find this maximum value. (CO3)

4. (a) Solve by changing the order of integration in $\int_0^1 \int_{x^2}^{2-x} xy\, dx\, dy$. (CO4)

(b) Prove the relation between Beta and Gamma functions. (CO4)

(c) Evaluate the integral $\iiint x^{l-1} y^{m-1} z^{n-1} dx\, dy\, dz$, where x, y, z are all positive but limited to the condition

$$\left(\frac{x}{a}\right)^p + \left(\frac{y}{b}\right)^q + \left(\frac{z}{c}\right)^r \leq 1. \qquad \text{(CO4)}$$

5. (a) Prove that :

$$(y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$$

is both solenoidal and irrotational. (CO5)

(b) State Green's theorem for a plane. Using Green's theorem, find $\int_C x^2 y\, dx + x^2\, dy$, where C is the boundary described counter clockwise of the triangle with vertices (0, 0), (1, 0), (1, 1). (CO5)

(c) Evaluate $\iint_S \vec{A} . \hat{n} dS$, where $\vec{A} = 18z\hat{i} - 12\hat{j} + 3y\hat{k}$ and S is part of the plane $2x + 3y + 6z = 12$ included in the first octant. (CO5)