2294066

End Semester Back Examination 2023

Name of the course: B.Tech | *Semester: I*
Name of the paper: Engineering Mathematics - I | *Paper Code: TMA - 101*
Time: 3 hours | Maximum Marks: 100

Note:
(i) All questions are compulsory.
(ii) Answer any two sub questions among a, b and c in each main question.
(iii) Total marks in each main question are twenty.
(iv) Each sub part carries 10 marks.

Q.1	$(10\times 2=20$ Marks$)$	
(a)	Using elementary transformations find the inverse of $A=\begin{bmatrix}1 & 1 & 3\\ 1 & 3 & -3\\ -2 & -4 & -4\end{bmatrix}$.	
(b)	Verify Cayley – Hamilton theorem for the matrix, $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$. Hence find A^{-1}.	CO: 1
(c)	Find the eigen values and eigen vectors of the following matrix. $A=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$.	
Q.2	$(10\times 2=20$ Marks$)$	
(a)	If $y=\sin\left(m\sin^{-1}x\right)$, prove that $\left(1-x^2\right)y_{n+2}-(2n+1)x.y_{n+1}+\left(m^2-n^2\right)y_n=0$.	
(b)	If $u=\log_e\left(x^3+y^3+z^3-3xyz\right)$, show that $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u=-\frac{9}{(x+y+z)^2}$.	CO: 2
(c)	Discuss the convergence of the series $\frac{1^2.2^2}{1!}+\frac{2^2.3^2}{2!}+\frac{3^2.4^2}{3!}+\ldots$	
Q.3	$(10\times 2=20$ Marks$)$	
(a)	Find the point upon the plane $ax+by+cz=p$ at which the function $f=x^2+y^2+z^2$ has a maximum value and find this maximum f.	
(b)	The temperature T at any point (x,y,z) in space is $T=400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2+y^2+z^2=1$.	CO: 3
(c)	If $u=xy+yz+zx,\ v=x^2+y^2+z^2$ and $w=x+y+z$, determine whether there is a functional relationship between u, v, w and if so, find it.	

Q.4	(10×2=20 Marks)	
(a)	Change the order of integration $\int_0^1\int_{x^2}^{2-x} xy\,dxdy$ and hence evaluate the same.	CO: 4
(b)	Find the volume of tetrahedron bounded by the planes $x=0,\ y=0,\ z=0$ and $x+y+z=a$.	
(c)	Evaluate (i) $\int_0^\infty \sqrt[4]{x}.e^{-\sqrt{x}}dx$, (ii) $\int_0^1 x^4\left(1-\sqrt{x}\right)^5 dx$	
Q.5	(10×2=20 Marks)	
(a)	Prove that $\nabla^2 f(r)=f''(r)+\frac{2}{r}f'(r)$.	CO: 5
(b)	Evaluate $\iint_S\left(yz\hat{i}+zx\hat{j}+xy\hat{k}\right).\overrightarrow{ds}$ where S is the surface of the sphere $x^2+y^2+z^2=a^2$ in the first octant.	
(c)	Apply Stoke's theorem to find the value of $\int_C(y\,dx+z\,dy+x\,dz)$, where C is the curve of intersection of $x^2+y^2+z^2=a^2$ and $x+z=a$.	