H Roll No. ....

TMA–102

B. TECH. (CS) (FIRST SEMESTER) END SEMESTER EXAMINATION, 2018

CALCULUS AND LINEAR ALGEBRA

Time : Three Hours

Maximum Marks : 100

Note : (i) This question paper contains five questions.

(ii) All questions are compulsory.

(iii) Each question carries three Parts (a), (b), and (c). Attempt any *two* question of choice from (a), (b) and (c).

(iv) Total marks assigned to each question are **twenty.**

1. Attempt any *two* questions of choice from (a), (b) and (c). (2×10=20 Marks)

(a) Define singular and non-singular matrix with an example and find A^{-1} with the help of Gauss-Jordan method, where :

$$A = \begin{pmatrix} 2 & 1 & 3 \\ 0 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}.$$

F. No. : c-41 ***P. T. O.***

(b) Solve the following system of equations :

$$x-3y+2z-w=2,$$

$$z+2w=8,$$

$$2x-6y+5z=12,$$

$$3x-9y+8z+4w=31.$$

(c) Show that row vectors of the matrix $\begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$ are linearly independent.

2. Attempt any *two* questions of choice from (a), (b) and (c). (2×10=20 Marks)

(a) Define vector space, and Let V be the set of all pairs (x, y) of real numbers and let F be the field of real numbers, define :

$$(x, y)+(x_1, y_1)=(3y+3y_1, -x-x_1)$$

$$c(x, y)=(3cy, -cx)$$

Verify that V with these operations is not a vector space over the field of real numbers.

(b) Define Basis of vector space and show that the vectors (1, 2, 1) (2, 1, 0), (1, –1, 2) form a basis of R^3.

(c) Find the range, rank, kernel and nullity of the linear transformation :

$$T: R^3 \rightarrow R^3 \text{ given by}$$

$$T(x_1, x_2, x_3)=(3x_1, -x_1-x_2, 2x_1+x_2+x_3).$$

3. Attempt any *two* questions of choice from (a), (b) and (c). (2×10=20 Marks)

(a) Let $T: R^3 \rightarrow R^3$ be a linear operator where :

$$T(e_1)=5e_1-6e_2-6e_3,$$

$$T(e_2)=-e_1+4e_2+2e_3,$$

$$T(e_3)=3e_1-6e_2-4e_3$$

Find the characteristic values of T and compute the corresponding characteristic vectors.

(b) A square matrix A is defined by $A=\begin{pmatrix} 1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{pmatrix}$. Find the Eigen bases and the resulting diagonal matrix D of A.

(c) Define inner product space, if :

$$\alpha=(a_1, a_2),\ \beta=(b_1, b_2) \in V_2(R)$$

Define :

$$(\alpha, \beta) = a_1 b_1 - a_2 b_1 - a_1 b_2 + 4\, a_2 b_2$$

Verify that this is inner product space or not.

4. Attempt any *two* questions of choice from (a), (b) and (c). (2×10=20 Marks)

   (a) Find the area bounded by the lines :

   $$y = x + 2,\ y = -x + 2,\ x = 5.$$

   (b) Define Beta and Gamma function with examples and prove that :

   $$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}.$$

   (c) To prove that :

   $$\int_0^{\frac{\pi}{2}} \sin^p \theta \cos^q \theta \, d\theta = \frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}{2\,\Gamma\left(\frac{p+q+2}{2}\right)}$$

5. Attempt any *two* questions of choice from (a), (b) and (c). (2×10=20 Marks)

   (a) Examine :

   $$f(x, y) = x^3 + y^3 - 3\,axy$$

   for maximum and minimum values.

   (b) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, prove that :

   $$x^2 \frac{\partial^2 u}{\partial x^2} + 2\,xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 2 \cos 3u \sin u.$$

   (c) Define mean value theorem with an example and solve :

   $$\lim_{x \to 0} \frac{e^x + e^{-x} - 2\cos x}{x \sin x}$$