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TMA–301

B. TECH. (CSE) (THIRD SEMESTER) END SEMESTER EXAMINATION, 2018

DISCRETE MATHEMATICS

Time : Three Hours

Maximum Marks : 100

Note : (i) This question paper contains five questions with alternative choice.

(ii) All questions are compulsory.

(iii) Instructions on how to attempt a question are mentioned against it.

(iv) Each part carries **ten** marks. Total marks assigned to each question are **twenty.**

1. Attempt any *two* questions of choice from (a), (b) and (c). (2×10=20 Marks)

(a) Explain the following with example :

(i) Tautologies

(ii) Disjunction

(iii) Contradiction

(iv) Validity

(b) If p, q and r are any *three* statements, then construct the truth table :

(i) $(p \to q) \wedge (q \to r) \leftrightarrow (p \leftrightarrow r)$

(ii) $(p \Rightarrow (q \wedge r)) \equiv \Rightarrow \neg (p \vee q)$

(c) Show that :

(i) $p \wedge (q \vee r') \equiv (p \wedge q) \vee (p \wedge r)$

(ii) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

2. Attempt any *two* questions of choice from (a), (b) and (c). (2×10=20 Marks)

(a) In a class of 42 students, each play at least one of the three games : Cricket, Hockey and Football. It is found that 14 play Cricket, 20 play Hockey and 24 play Football, 3 play both Cricket and Football, 2 play both Hockey and Football and none play all the three games. Find the number of students who play Cricket but not Hockey.

(b) Using Principle of Mathematical induction prove that $3^{2n+2} - 8n - 9$ is divisible by 8.

(c) If ${}^nP_r = 3,024$ and ${}^nC_r = 126$, find the value of n and r.

3. Attempt any *two* questions of choice from (a), (b) and (c). (2×10=20 Marks)

(a) Let A be the set of non-zero integer and let R be the relation on A × A defined by $(a,b)R(c,d) \Leftrightarrow a + d = b + c$. Show that R is an equivalence relation.

(b) Define the following :

(i) Hasse diagrams

(ii) Partial order relation

(iii) Composition of function

(iv) Into function

(c) Let $f : R^+ \to R^+$ and $g : R^+ \to R^+$ be function defined by $f(x) = \sqrt{x}$ and $g(x) = 3x + 1$ for all $x \in R^+$, find $f \circ g$ and $g \circ f$.

4. Attempt any *two* questions of choice from (a), (b) and (c). (2×10=20 Marks)

(a) Solve the recurrence relation $a_n - 7a_{n-2} + 6a_{n-3} = 0$ with initial conditions $a_0 = 8, a_1 = 6, a_2 = 22$.

(b) Solve the recurrence relation $a_n - 2a_{n-1} - 3a_{n-2} = 0$, $n \geq 2$ by the generating function method with initial conditions, $a_0 = 3, a_1 = 1$.

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(c) Find the particular solution of the recurrence relation :

$$a_{n+2} - 4a_{n+1} + 4a_n = .2^n .$$

5. Attempt any *two* questions of choice from (a), (b) and (c). (2×10=20 Marks)

(a) Prove that the set Q_+ of positive rational number forms an abelian group with respect to the operation * defined as

$$a * b = \frac{ab}{2}.$$

(b) Define the following :

(i) Group

(ii) Cosets

(iii) Ring

(c) Let L be a complemented distribution lattice. Then prove that De Morgan's laws :

(i) $(a \vee b)' = a' \wedge b'$

(ii) $(a \wedge b)' = a' \vee b'$ hold in L for all $a, b \in L$, where a' denote the complement of a.