Roll No. | | | | | | | |

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TMA–201

B. Tech. (Second Semester)
End Semester EXAMINATION, 2017
(All Branches)
ENGINEERING MATHEMATICS—II

Time : Three Hours] [*Maximum Marks : 100*

Note : (i) This question paper contains *five* questions.

(ii) All questions are compulsory.

(iii) Instructions on how to attempt a question are mention against it.

(iv) Total marks assigned to each question are **twenty.**

1. Attempt any *two* questions of choice from (a), (b) and (c). (2×10=20 Marks)

(a) Solve :

$$\frac{d^2y}{dx^2} + 2h\frac{dy}{dx} + \left(h^2 + p^2\right)y = ke^{-hx}\cos px$$

(b) Solve :

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = e^{-x}\sec^3 x$$

(c) Solve :

$$\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = e^{2x+y}$$

2. Attempt any *two* questions of choice from (a), (b) and (c). (2×10=20 Marks)

(a) Find the Laplace Transform of $t\cos^3 2t$.

(b) Using Laplace transform, solve the differential equation :

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 4e^{2x},$$

where $y(0) = -3, y'(0) = 5$, at $x = 0$.

(c) Using the convolution theorem find inverse Laplace transform $\frac{1}{s^2\left(s^2 - a^2\right)}$.

3. Attempt any *two* questions of choice from (a), (b) and (c). (2×10=20 Marks)

(a) Show that :

$$\left(1 - 2xz + z^2\right)^{\frac{-1}{2}} = \sum_{n=0}^{\infty} z^n P_n(x),$$

where $P_n(x)$ are Legendre's Polynomials of degree n.

(b) Show that :

$$\frac{d}{dx}\left[xJ_n(x)J_{n+1}(x)\right] = x\left[J_n^2(x) - J_{n+1}^2(x)\right]$$

(c) Prove that :

$$\int_{-1}^{1} P_m(x)P_n(x)\,dx = \begin{cases} 0 & \text{if } m \neq n \\ \dfrac{2}{2n+1} & \text{if } m = n \end{cases}$$

4. Attempt any *two* questions of choice from (a), (b) and (c). (2×10=20 Marks)

(a) Find the Fourier series to represent $f(x) = \frac{\left(3x^2 - 6x\pi + 2\pi^2\right)}{12}$, when $0 \le x \le 2\pi$.

(b) Find a Fourier series to represent :

$$f(x) = |\sin x|, \text{ for } -\pi < x < \pi$$

(c) Find a Fourier series to represent :

$$f(x) = \cos(sx), \text{ for } -\pi < x < \pi$$

5. Attempt any *two* questions of choice from (a), (b) and (c). (2×10=20 Marks)

(a) Solve :

$$2x\frac{\partial z}{\partial x} - 3y\frac{\partial z}{\partial y} = 0$$

(b) Solve :

$$16\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \text{ if } u(x,0) = x^2(5 - x)$$

(c) Solve :

$$\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}, \text{ if } u(x,0) = 4x - \frac{1}{2}x^2$$