H Roll No. ....

TMA–101

B. TECH. (FIRST SEMESTER) END SEMESTER EXAMINATION, Jan., 2023

ENGINEERING MATHEMATICS—I

Time : Three Hours

Maximum Marks : 100

Note : (i) All questions are compulsory.

(ii) Answer any *two* sub-questions among (a), (b) and (c) in each main question.

(iii) Total marks in each main question are **twenty**.

(iv) Each sub-question carries 10 marks.

1. (a) Find the rank of the matrix

$$A = \begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$$

by reducing it to normal form. (CO1)

P. T. O.

(b) Are the following vectors linearly dependent ? If so, find the relation between them, $X_1 = (1, 1, 2, 3)$, $X_2 = (1, 2, 3, 4)$ and $X_3 = (2, 3, 4, 9)$. (CO1)

(c) If

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix},$$

then find modal matrix P. (CO1)

2. (a) If

$$y^{\frac{1}{m}} + y^{\frac{1}{m}} = 2x,$$

prove that : (CO2)

$$\left(x^2 - 1\right) y_{n+2} + (2n + 1)\, x y_{n+1} + \left(n^2 - m^2\right) y_n = 0.$$

(b) If : (CO2)

$$u = \log_e\left(x^3 + y^3 + z^3 - 3xyz\right),$$

show that :

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x + y + z)^2}.$$

(c) Discuss the convergence of the series : (CO2)

$$\frac{1^2.2^2}{1!}+\frac{2^2.3^2}{2!}+\frac{3^2.4^2}{3!}+\ldots\ldots$$

3. (a) Find the volume of the tetrahedron bounded by the surfaces $x = 0, y = 0$, $z = 0$ and $x + y + z = a$. (CO3)

(b) The temperature T at any point (x, y, z) in space is $T = 400\ xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$. (CO3)

(c) If : (CO3)

$$u^3 + v^3 + w^3 = x + y + z,$$
$$u^2 + v^2 + w^2 = x^3 + y^3 + z^3,$$
$$u + v + w = x^2 + y^2 + z^2,$$

then find the value of $\dfrac{\partial(u, v, w)}{\partial(x, y, z)}$.

4. (a) Change the order of integration and evaluate : (CO4)

$$\int_0^2 \int_{2-\sqrt{4-y^2}}^{2+\sqrt{4-y^2}} dx\, dy.$$

(b) Find the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. (CO4)

(c) (i) Evaluate : (CO4)

$$\int_0^1 \frac{dx}{\sqrt{-\log x}}$$

(ii) Evaluate :

$$\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}\,dx$$

5. (a) Prove that : (CO5)

$$\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$$

(b) Evaluate

$$\iint_S \left(yz\hat{i} + zx\hat{j} + xy\hat{k}\right).\vec{ds}$$

were S is the surface of the sphere $x^2+y^2+z^2=a^2$ in the first octant.

(CO5)

(c) Apply Stoke's theorem to find the value of

$$\int_C (ydx + zdy + xdz),$$

where C is the curve of intersection of $x^2+y^2+z^2=a^2$ and $x+z=a$. (CO5)

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