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# TMA–301(A)

## B. Tech. (ME) (Third Semester)
## Mid Semester EXAMINATION, 2017

ENGINEERING MATHEMATICS—III

*Time : 1:30 Hours* ] [ *Maximum Marks : 50*

**Note :** (i) This question paper contains two Sections.

(ii) Both Sections are compulsory.

### Section—A

1. Fill in the blanks/True-False : (1×5=5 Marks)

(a) Any function which satisfies the Laplace's equation is known as ..........

(b) The polar form of the Cauchy Riemann equation is ...........

(c) If $Z_1$ and $Z_2$ are two complex variables then $|Z_1 + Z_2| \leq |Z_1| + |Z_2|$. (True/False)

(d) A bilinear transformation maps circle into circle. (True/False)

(e) The value of $\oint_C \frac{dz}{z+2}$ is ........, where C is the circle $|z|=1$.

2. Attempt any *five* parts : (3×5=15 Marks)
(Define/Short Numerical/Short Programming/ Draw)

(a) Using the C-R equation show that $f(z)=z^3$ is the entrie $z$-plane.

(b) Show that the function $u=\cos x \cosh y$ is harmonic.

(c) Find harmonic conjugate of the analytic function whose real part is :

$$x^3-3xy^2+3x^2-3y^2.$$

(d) Evaluate :

$$\int_{(0,0)}^{(1,1)}(3x^2+4xy+3y^2)dx$$
$$+2(x^2+3xy+4y^2)dy$$

along the path $y=x^2$.

(e) Evaluate :

$$\oint_C \frac{z^2+1}{z^2-1}dz$$

where C is the circle $|z|=\frac{3}{2}$.

(f) Define the Bilinear transformation.





Section—B

3. Attempt any *two* parts of choice from (a), (b) and (c). (5×2=10 Marks)

(a) Derive the Cauchy-Riemann equation in Polar form.

(b) Show that the function $f(z)$ defined by :

$$f(z)=\begin{cases}\frac{x^3y(y-ix)}{x^6+y^2}, & z\neq 0\\ 0 & , z=0\end{cases}$$

is not analytic at the origin even though it satisfies Cauchy-Riemann equation at the origin.

(c) Prove that :

$$u=x^2-y^2-2xy-2y+3y$$

is harmonic, find analytic function $f(z)=u+iv$ in terms of $z$.

4. Attempt any *two* parts of choice from (a), (b) and (c). (5×2=10 Marks)

(a) Evaluate :

$$\int_C \frac{3z^2+2}{(z-1)(z^2+9)}dz$$

where C is the circle $|z-2|=2$.

(b) Find the bilinear transformation which maps $z_1 = 1$, $z_2 = i$, $z_3 = -1$ of $z$-plane to $w_1 = i$, $w_2 = 0$, $w_3 = -i$ of $w$-plane.

(c) Using Cauchy integral formula, evaluate :

$$\frac{1}{2\pi}\int_C \frac{ze^z}{(z-a)^3}\,dz$$

where the point $a$ lies within the closed curve C.

5. Attempt any *two* parts of choice from (a), (b) and (c). (5×2=10 Marks)

(a) If :

$$u - v = (x-y)(x^2+4xy+y^2)$$

and $$f(z) = u + iv$$

is analytic function of $z = x + iy$, find $f(z)$ in term of $z$.

(b) Find the fixed point and the normal form of the bilinear transformation $w = \dfrac{3z-4}{z-1}$.

(c) Evaluate the integral :

$$\int_0^{2\pi} \frac{d\theta}{5-3\cos\theta}$$