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TMA–101

B. TECH. (NON-CS)
(FIRST SEMESTER)
MID SEMESTER EXAMINATION, 2018
ENGINEERING MATHEMATICS—I

Time : 1:30 Hours

Maximum Marks : 50

Note : (i) This question paper contains two Sections.

(ii) Both Sections are compulsory.

Section—A

1. State True/False/One-line answer :

(1×5=5 Marks)

(a) Maximum rank of matrix is equal to maximum no. of rows of matrix. (True/False)

(b) Every square matrix satisfies its own characteristic equation. (True/False)

(c) The sum of Eigen values is equal to the trace of matrix. (True/False)

(d) State Leibnitz theorem.

(e) State Euler's theorem.

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2. Attempt any *five* parts : (3×5=15 Marks)
   (a) State Leibnitz test with example.
   (b) Test the convergence of $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{-n^2}$.
   (c) Define null matrix and show the matrix of $O_{5\times 3}$.
   (d) Find the nth order of derivative of $\log(ax+b)$.
   (e) Using row elementary operation, find the inverse of the matrix A, where
   $$A=\begin{bmatrix}3 & -3 & 4\\ 2 & -3 & 4\\ 0 & -1 & 1\end{bmatrix}.$$
   (f) Define Skew-Hermitian matrix with example.

Section—B

3. Attempt any *two* parts of choice from (a), (b) and (c). (5×2=10 Marks)
   (a) Find the nth order derivative of $e^{cx}\sin(ax+b)$.
   (b) If $y = x\log\left(\frac{x-1}{x+1}\right)$, show that :
   $$y_n = (-1)^{n-2}.(n-2)!\left[\frac{x-n}{(x-1)^n}-\frac{x+n}{(x+1)^n}\right]$$

   (c) If
   $$y = \sin(m\sin^{-1}x)$$
   prove that :
   $$(1-x^2)y_{n+2}-(2n-1)x.y_{n+1}+(m^2-n^2)y_n = 0$$

4. Attempt any *two* parts of choice from (a), (b) and (c). (5×2=10 Marks)
   (a) Reduce the matrix A to canonical form and hence find the rank, where
   $$A=\begin{bmatrix}1 & 2 & 1 & 0\\ -2 & 4 & 3 & 0\\ 1 & 0 & 2 & -8\end{bmatrix}.$$
   (b) Verify the Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$.
   (c) Find Eigen value of the matrix
   $$A=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}.$$

5. Attempt any *two* parts of choice from (a), (b) and (c). (5×2=10 Marks)

(a) Using row elementary operation, find the inverse of the matrix A, where

$$A = \begin{bmatrix} 0 & 2 & 1 & 3 \\ 1 & 1 & -1 & -2 \\ 1 & 2 & 0 & 1 \\ -1 & 1 & 2 & 6 \end{bmatrix}.$$

(b) Reduce the matrix A to echelon form and hence find the rank, where

$$A = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 1 & 3 & 2 & 2 \\ 2 & 4 & 3 & 4 \\ 3 & 7 & 4 & 6 \end{bmatrix}.$$

(c) If u is homogeneous function in the variables x and y of degree n, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu.$