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MODULE *Zarb*

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The specification of the *Zarb* consensus algorithm based on Practical *Byzantine* Fault Tolerant. For more information check here: <https://zarb.network/learn/consensus/consensus-mechanism.html>

EXTENDS *Integers*, *Sequences*, *FiniteSets*, *TLC*

CONSTANT

The total number of faulty nodes

*NumFaulty*,

The maximum number of round per height.

this is to restrict the allowed behaviours that *TLC* scans through.

*MaxRound*

ASSUME

$\wedge \text{NumFaulty} \geq 1$

VARIABLES

*log*,

*states*

Total number of replicas that is  $3f + 1$  where  $f$  is number of faulty nodes.

*Replicas*  $\triangleq (3 * \text{NumFaulty}) + 1$

2/3 of total replicas that is  $2f + 1$

*QuorumCnt*  $\triangleq (2 * \text{NumFaulty}) + 1$

1/3 of total replicas that is  $f + 1$

*OneThird*  $\triangleq \text{NumFaulty} + 1$

A tuple with all variables in the spec (for ease of use in temporal conditions)

*vars*  $\triangleq \langle \text{states}, \text{log} \rangle$

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Helper functions

Fetch a subset of messages in the network based on the *params* filter.

*SubsetOfMsgs(params)*  $\triangleq$

$\{msg \in log : \forall field \in \text{DOMAIN } params : msg[field] = params[field]\}$

*IsProposer* checks if the replica is the proposer for this round

*IsProposer(index)*  $\triangleq$

$(\text{states}[index].\text{round} + \text{states}[index].\text{proposerIndex}) \% \text{Replicas} = index$

*HasPrepareQuorum* checks if there is a quorum of the *PREPARE* votes in each round.

*HasPrepareQuorum(index)*  $\triangleq$

$\text{Cardinality}(\text{SubsetOfMsgs}([$   
 $\text{type} \mapsto \text{"PREPARE"},$   
 $\text{height} \mapsto \text{states}[index].\text{height},$   
 $\text{round} \mapsto \text{states}[index].\text{round}])) \geq \text{QuorumCnt}$

*HasPrecommitQuorum* checks if there is a quorum of the *PRECOMMIT* votes in each round.

$$\begin{aligned} \text{HasPrecommitQuorum}(index) &\triangleq \\ &\text{Cardinality}(\text{SubsetOfMsgs}([ \\ &\quad type \mapsto \text{"PRECOMMIT"}, \\ &\quad height \mapsto \text{states}[index].height, \\ &\quad round \mapsto \text{states}[index].round])) \geq \text{QuorumCnt} \end{aligned}$$

*HasChangeProposerQuorum* checks if there is a quorum of the *CHANGE-PROPOSER* votes in each round.

$$\begin{aligned} \text{HasChangeProposerQuorum}(index) &\triangleq \\ &\text{Cardinality}(\text{SubsetOfMsgs}([ \\ &\quad type \mapsto \text{"CHANGE-PROPOSER"}, \\ &\quad height \mapsto \text{states}[index].height, \\ &\quad round \mapsto \text{states}[index].round])) \geq \text{QuorumCnt} \end{aligned}$$

$$\begin{aligned} \text{HasOneThirdOfChangeProposer}(index) &\triangleq \\ &\text{Cardinality}(\text{SubsetOfMsgs}([ \\ &\quad type \mapsto \text{"CHANGE-PROPOSER"}, \\ &\quad height \mapsto \text{states}[index].height, \\ &\quad round \mapsto \text{states}[index].round])) \geq \text{OneThird} \end{aligned}$$

$$\begin{aligned} \text{GetProposal}(height, round) &\triangleq \\ &\text{SubsetOfMsgs}([type \mapsto \text{"PROPOSAL"}, height \mapsto height, round \mapsto round]) \end{aligned}$$

$$\begin{aligned} \text{HasProposal}(height, round) &\triangleq \\ &\text{Cardinality}(\text{GetProposal}(height, round)) > 0 \end{aligned}$$

$$\begin{aligned} \text{IsCommitted}(height) &\triangleq \\ &\text{Cardinality}(\text{SubsetOfMsgs}([type \mapsto \text{"BLOCK-ANNOUNCE"}, height \mapsto height])) > 0 \end{aligned}$$


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#### Network functions

*SendMsg* broadcasts the message iff the current height is not committed yet.

$$\begin{aligned} \text{SendMsg}(msg) &\triangleq \\ &\text{IF } \neg \text{IsCommitted}(msg.height) \\ &\quad \text{THEN } log' = log \cup \{msg\} \\ &\quad \text{ELSE } log' = log \end{aligned}$$

*SendProposal* is used to broadcast the *PROPOSAL* into the network.

$$\begin{aligned} \text{SendProposal}(index) &\triangleq \\ &\text{SendMsg}([ \\ &\quad type \mapsto \text{"PROPOSAL"}, \\ &\quad height \mapsto \text{states}[index].height, \\ &\quad round \mapsto \text{states}[index].round, \\ &\quad index \mapsto index]) \end{aligned}$$

*SendPrepareVote* is used to broadcast *PREPARE* votes into the network.

$SendPrepareVote(index) \triangleq$   
 $SendMsg([$   
 $\quad type \mapsto \text{"PREPARE"},$   
 $\quad height \mapsto states[index].height,$   
 $\quad round \mapsto states[index].round,$   
 $\quad index \mapsto index])$

*SendPrecommitVote* is used to broadcast *PRECOMMIT* votes into the network.

$SendPrecommitVote(index) \triangleq$   
 $SendMsg([$   
 $\quad type \mapsto \text{"PRECOMMIT"},$   
 $\quad height \mapsto states[index].height,$   
 $\quad round \mapsto states[index].round,$   
 $\quad index \mapsto index])$

*SendChangeProposerRequest* is used to broadcast *CHANGE-PROPOSER* votes into the network.

$SendChangeProposerRequest(index) \triangleq$   
 $SendMsg([$   
 $\quad type \mapsto \text{"CHANGE-PROPOSER"},$   
 $\quad height \mapsto states[index].height,$   
 $\quad round \mapsto states[index].round,$   
 $\quad index \mapsto index])$

*AnnounceBlock* announces the block for the current height and clears the logs.

$AnnounceBlock(index) \triangleq$   
 $log' = \{msg \in log : (msg.type = \text{"BLOCK-ANNOUNCE"}) \vee msg.height > states[index].height\} \cup \{[$   
 $\quad type \mapsto \text{"BLOCK-ANNOUNCE"},$   
 $\quad height \mapsto states[index].height,$   
 $\quad round \mapsto states[index].round,$   
 $\quad index \mapsto -1]\}$

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#### States functions

*NewHeight* state

$NewHeight(index) \triangleq$   
 $\wedge states[index].name = \text{"new-height"}$   
 $\wedge states' = [states \text{ EXCEPT}$   
 $\quad ![index].name = \text{"propose"},$   
 $\quad ![index].height = states[index].height + 1,$   
 $\quad ![index].round = 0]$   
 $\wedge \text{UNCHANGED } \langle log \rangle$

*Propose* state

$Propose(index) \triangleq$   
 $\wedge states[index].name = \text{"propose"}$

$\wedge$  IF  $IsProposer(index)$   
     THEN  $SendProposal(index)$   
     ELSE  $log' = log$   
 $\wedge$   $states' = [states \text{ EXCEPT } ![index].name = \text{"prepare"}]$

**Prepare state**  
 $Prepare(index) \triangleq$   
 $\wedge$   $states[index].name = \text{"prepare"}$   
 $\wedge$  IF  $\wedge HasProposal(states[index].height, states[index].round)$   
      $\wedge \neg HasOneThirdOfChangeProposer(index)$   
      $\vee states[index].round \geq MaxRound$   
 THEN  $\wedge SendPrepareVote(index)$   
      $\wedge$  IF  $HasPrepareQuorum(index)$   
         THEN  $states' = [states \text{ EXCEPT } ![index].name = \text{"precommit"}]$   
         ELSE  $states' = states$   
 ELSE  $\wedge SendChangeProposerRequest(index)$   
      $\wedge states' = [states \text{ EXCEPT } ![index].name = \text{"change-proposer"}]$

**Precommit state**  
 $Precommit(index) \triangleq$   
 $\wedge states[index].name = \text{"precommit"}$   
 $\wedge SendPrecommitVote(index)$   
 $\wedge$  IF  $HasPrecommitQuorum(index) \wedge \neg HasOneThirdOfChangeProposer(index)$   
     THEN  $states' = [states \text{ EXCEPT } ![index].name = \text{"commit"}]$   
     ELSE  $states' = states$

**Commit state**  
 $Commit(index) \triangleq$   
 $\wedge states[index].name = \text{"commit"}$   
 $\wedge AnnounceBlock(index)$   
 $\wedge states' = [states \text{ EXCEPT }$   
      $![index].name = \text{"new-height"},$   
      $![index].proposerIndex = (states[index].round + 1) \% Replicas]$

**ChangeProposer state**  
 $ChangeProposer(index) \triangleq$   
 $\wedge states[index].name = \text{"change-proposer"}$   
 $\wedge$  IF  $HasChangeProposerQuorum(index)$   
     THEN  $states' = [states \text{ EXCEPT }$   
          $![index].name = \text{"propose"},$   
          $![index].round = states[index].round + 1]$   
     ELSE  $states' = states$   
 $\wedge$  UNCHANGED  $\langle log \rangle$

*Sync* checks the *log* for the committed blocks at the current height.

If such a block exists, it commits and moves to the next height.

$$\begin{aligned}
 \text{Sync}(index) &\triangleq \\
 &\text{LET} \\
 &\quad \text{blocks} \triangleq \text{SubsetOfMsgs}([type \mapsto \text{"BLOCK-ANNOUNCE"}, height \mapsto \text{states}[index].height]) \\
 &\text{IN} \\
 &\quad \wedge \text{Cardinality}(\text{blocks}) > 0 \\
 &\quad \wedge \text{states}' = [\text{states} \text{ EXCEPT} \\
 &\quad \quad ![\text{index}].name = \text{"propose"}, \\
 &\quad \quad ![\text{index}].height = \text{states}[index].height + 1, \\
 &\quad \quad ![\text{index}].round = 0, \\
 &\quad \quad ![\text{index}].proposerIndex = ((\text{CHOOSE } b \in \text{blocks} : \text{TRUE}).round + 1) \% \text{Replicas}] \\
 &\quad \wedge \text{log}' = \text{log}
 \end{aligned}$$


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$$\begin{aligned}
 \text{Init} &\triangleq \\
 &\quad \wedge \text{log} = \{\} \\
 &\quad \wedge \text{states} = [index \in 0 \dots \text{Replicas} - 1 \mapsto [ \\
 &\quad \quad \text{name} \mapsto \text{"new-height"}, \\
 &\quad \quad \text{height} \mapsto 0, \\
 &\quad \quad \text{round} \mapsto 0, \\
 &\quad \quad \text{proposerIndex} \mapsto 0]]
 \end{aligned}$$

$$\begin{aligned}
 \text{Next} &\triangleq \\
 &\quad \exists index \in 0 \dots \text{Replicas} - 1 : \\
 &\quad \vee \text{Sync}(index) \\
 &\quad \vee \text{NewHeight}(index) \\
 &\quad \vee \text{Propose}(index) \\
 &\quad \vee \text{Prepare}(index) \\
 &\quad \vee \text{Precommit}(index) \\
 &\quad \vee \text{Commit}(index) \\
 &\quad \vee \text{ChangeProposer}(index)
 \end{aligned}$$

$$\begin{aligned}
 \text{Spec} &\triangleq \\
 &\quad \text{Init} \wedge \Box[\text{Next}]_{\text{vars}}
 \end{aligned}$$

*TypeOK* is the type-correctness invariant.

$$\begin{aligned}
 \text{TypeOK} &\triangleq \\
 &\quad \wedge \quad \forall index \in 0 \dots \text{Replicas} - 1 : \\
 &\quad \quad \wedge \text{states}[index].name \in \{\text{"new-height"}, \text{"propose"}, \text{"prepare"}, \\
 &\quad \quad \quad \text{"precommit"}, \text{"commit"}, \text{"change-proposer"}\} \\
 &\quad \quad \wedge \neg \text{IsCommitted}(\text{states}[index].height) \Rightarrow \\
 &\quad \quad \quad \wedge \text{states}[index].name = \text{"new-height"} \wedge \text{states}[index].height > 1 \Rightarrow \\
 &\quad \quad \quad \quad \text{IsCommitted}(\text{states}[index].height - 1) \\
 &\quad \quad \quad \wedge \text{states}[index].name = \text{"propose"} \Rightarrow
 \end{aligned}$$

$$\begin{aligned}
& \text{Cardinality}(\text{SubsetOfMsgs}([index \mapsto index, height \mapsto \text{states}[index].height, round \mapsto \text{states}[index].round] \\
& \wedge \text{states}[index].name = \text{"precommit"} \Rightarrow \\
& \quad \text{HasPrepareQuorum}(index) \\
& \wedge \text{states}[index].name = \text{"commit"} \Rightarrow \\
& \quad \text{HasPrecommitQuorum}(index) \\
& \wedge \forall round \in 0 \dots \text{states}[index].round : \\
& \quad \wedge \text{Cardinality}(\text{GetProposal}(\text{states}[index].height, round)) \leq 1 \quad \text{not more than two proposals per round} \\
& \quad \wedge round > 0 \Rightarrow \text{Cardinality}(\text{SubsetOfMsgs}([type \mapsto \text{"CHANGE-PROPOSER"}, round \mapsto round]
\end{aligned}$$


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