Noesis : Physics Documentation

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1 Introduction

This document holds all physics related principles used in Noesis. A general description of the phenomenon, the mathematical equations used, and further results will be included.

2 Pendulum

2.1 Introduction

A simple pendulum follows periodic, sinusoidal motion. The equation of motion can be derived from the principle of least action.

2.2 Derivation Of The Equations Of Motion

From a Lagrangian mechanics point of view, the simple pendulum has only one degree of freedom, and therefore needs one dimension, which is the angle described by the mass (θ).
We can write the Lagrangian (in terms of displacement from the mean position x) as the difference between the kinetic and potential energy, which are given here.

$$L = T - V$$
$$L = \frac{1}{2}m\dot{x}^2 - mgh$$

where h is the vertical height of the mass from the mean position.
We can rewrite the Lagrangian in terms of θ using $x = l \cdot \theta$, where l is the length of the pendulum. We also change h to $mgl(1 - cos\,\theta)$ from trigonometric analysis.

$$L = \frac{1}{2} ml\dot{\theta}^2 - mgl(1 - cos\,\theta) \tag{1}$$

We also need to calculate the damping, and add the functions to the generalized forces (Θ).

$$\begin{align} \Theta &= \sum_i F_i \frac{dx_i}{d\theta} \tag{2} \\ &= F_x \frac{dx}{d\theta} + F_y \frac{dy}{d\theta} \tag{3} \end{align}$$

The damping force is assumed to be $f(\dot{\theta}) = b\dot{\theta}$, where b is the damping coefficient. Therefore the F_x and F_y components are the resolved vectors of the same function.

$$\Theta = b\dot{\theta} cos\,\theta \frac{dx}{d\theta} - b\dot{\theta} sin\,\theta \frac{dy}{d\theta} \tag{4}$$

To convert to standard rectangular coordinates, the transforms are $x = lsin\,\theta$ and $y = lcos\,\theta$. Therefore, Θ becomes

$$\begin{align} \Theta &= b\dot{\theta} cos\,\theta (lcos\,\theta) + b\dot{\theta} sin\,\theta (l \sin\,\theta) \tag{5} \\ &= b\dot{\theta} lcos^2\,\theta + b\dot{\theta} lsin^2\,\theta \tag{6} \\ &= bl\dot{\theta} \tag{7} \end{align}$$

Plugging the Lagrangian into the Euler-Lagrange equation,

$$\begin{align} \frac{\partial L}{\partial \theta} - \frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}} &= \Theta \tag{8} \\ -ml^2 \ddot{\theta} - mglsin\,\theta &= bl\dot{\theta} \tag{9} \end{align}$$

This is a difficult equation to solve, therefore, we assume θ to be a small angle, and approximate $\sin\theta \approx \theta$. The equation reduces to

$$\begin{align} ml^2 \ddot{\theta} - bl\dot{\theta} + mgl\theta &= 0 \tag{10} \\ \ddot{\theta} - \frac{b}{ml} \dot{\theta} + \frac{g}{l} \theta &= 0 \tag{11} \end{align}$$

which is our desired equation of motion.

2.3 Application Of Equation Of Motion

To apply the equation of motion to Noesis, the function $\theta(t)$ must be found by solving the derived differential equation, which can be solved using the Laplace transform.
The first and second derivatives of $\theta(t)$ transform under the Laplace transform as

$$\mathcal{L}(\dot{\theta}) = s\mathcal{L}\{\theta(t)\} - \theta(0) \tag{12}$$

$$\mathcal{L}(\ddot{\theta}) = s^2\mathcal{L}\{\theta(t)\} - s\theta(0) - \dot{\theta}(0) \tag{13}$$

For the boundary conditions, We know that the angular velocity ($\dot{\theta}(0) = \omega$) is maximum at the mean position ($\theta(0) = 0$). Hence, the transforms convert to

$$\mathcal{L}(\dot{\theta}) = s\mathcal{L}\{\theta(t)\} \tag{14}$$

$$\mathcal{L}(\ddot{\theta}) = s^2\mathcal{L}\{\theta(t)\} - \omega \tag{15}$$

Applying the Laplace transform to equation (11), we get

$$\mathcal{L}\{\ddot{\theta}(t)\} - \frac{b}{ml}\mathcal{L}\{\dot{\theta}(t)\} + \frac{g}{l}\mathcal{L}\{\theta(t)\} = 0 \tag{16}$$

Substituting the transforms (14) and (15) into the previous equation,

$$s^2\mathcal{L}\{\theta(t)\} - \omega - \frac{b}{ml}s\mathcal{L}\{\theta(t)\} + \frac{g}{l}\mathcal{L}\{\theta(t)\} = 0 \tag{17}$$

Solving for $\mathcal{L}\{\theta(t)\}$,

$$\mathcal{L}\{\theta(t)\} = \frac{\omega}{s^2 - \frac{b}{ml}s + \frac{g}{l}} \tag{18}$$

Taking Inverse Laplace transform to obtain $\theta(t)$,

$$\theta(t) = \frac{2ml\omega}{\sqrt{4m^2lg - b^2}} \sin\left(\frac{\sqrt{4m^2lg - b^2}}{2ml}t\right) \tag{19}$$

For the sake of simplicity, we denote the constant term $A = \frac{2ml\omega}{\sqrt{4m^2lg-b^2}}$, such that the function $\theta(t)$ becomes

$$\theta(t) = A\sin\frac{\omega}{A}t \tag{20}$$