Quizz 2 MA3.101: Linear Algebra
Spring 2022

Indranil Chakrabarty

May 2022

Answer all questions: (Time - 45 mins) (Full Marks- 30)

1. Let Q be an orthogonal matrix , then show that
(i) Q^{-1} is orthogonal.
(ii) $det(Q) = \pm 1$.
(iii) If λ is an eigenvalue of Q, then $|\lambda| = 1$. (6)

2. Prove that an orthogonal 2×2 matrix must have the form,
$\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ or $\begin{pmatrix} a & b \\ b & -a \end{pmatrix}$ where $\begin{pmatrix} a \\ b \end{pmatrix}$ is a unit vector. (4)

3. Let A be a nilpotent matrix (that is b $A^m = O$ for some m). Show that $\lambda = 0$ is the only eigen value of A. (2)

4. Let A be an idempotent matrix(that is $A^2 = A$). Show that $\lambda = 0$ and $\lambda = 1$ are the only eigen value of A. (2)

5. Let v is an eigen vector of A, with corresponding eigen value λ and c is scalar. Show that v is an eigen vector of $A - cI$ with corresponding eigen value $\lambda - c$. (2)

6. Compute the (a) characteristic polynomial, (b) the eigen values, (c) basis for each eigen space, (d) algebric and geometric multiplicity of each eigen values, for the following matrix,

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 3 & 0 \\ -2 & 1 & 2 & -1 \end{pmatrix}$$ (4)

7. Apply Gram Schmidt process to find an orthogonal basis for the column spaces of the matrix

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ -1 & 1 & 0 \\ 1 & 5 & 1 \end{pmatrix}$$ (4)

8. Suppose that u, v and w are vectors in inner product space such that, $< u, v >= 1$, $< u, w >= 5$, $< v, w >= 0$, $||u|| = 1$, $||v|| = \sqrt{3}$, $||w|| = 2$, then evaluate the expressions,
(i) $< u + w, v - w >$
(ii) $< 2v - w, 3u + 2w >$
(iii) $||u + v||$ (6)