Quiz: Probability and Statistics (30 Marks)

[Instruction: Please state reasons wherever applicable.]

5 marks

1. Verify that $P(C|A) = P(C|BA)P(B|A) + P(C|B^cA)P(B^c|A)$ where A, B, C are events in Ω. (AB means $A \cap B$)

2. Find the stationary distribution π for Markov Chains with the following trnasition probability matrix.(3 marks) State if π is unique in each case. (1 mark) Also which of the two chains are irreducible? Give reasons (1mark) .
$P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $Q = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

3. Suppose X is an exponential random variable with parameter λ and CDF denoted by $F_X(\cdot)$. U is a uniform random variable over the interval $[0, 1]$. Now consider another random variable $Y = F_X^{-1}(U)$. Then derive the expression for the CDF $F_Y(y)$.

4. Consider a sequence of random variables $\{X_n\}$ where $X_n \sim Exponential(n)$. Show that X_n converges to X in probability where $X = 0$ with probability 1. Also show that X_n converges to X in distribution (without using the fact that convergence in probability implies convergence in distribution).

10 marks

1. Suppose $u_1 = 0.23, u_2 = .78$ and $u_3 = 0.5$ are 3 samples drawn from a uniform random variable. Convert these samples into samples from
(1) a fair dice
(2) a biased coin with 0.7 as probability of head
(3) An exponential random variable with parameter $\lambda = 1$.
(4) A uniform random variable taking values in the interval $[5, 10]$.