1. You are given the following data set (x,y) (where $y=f(x)+\mathcal{E}$): (x1, y1), (x2, y2), (x3, y3), (x4, y4), (x5, y5), (x6, y6) corresponding to values (1,2), (2,4), (3,8), (4,15), (5,26), (6,36). You split the data into two folds and performed 2-fold cross validation to obtain 2 different realizations of the model, say MR1 and MR2. You obtained the following model fits for the points {x1,x2,x3,x4,x5,x6} using MR1 and MR2: {y1+1,y2-2,y3+1,y4-2,y5+1,y6-2} using MR1 and {y1+2,y2-1,y3+2,y4-1,y5+2,y6-1} using MR2. Please answer the questions below along with all the steps and computations involved.

[If there is anything unclear, please make suitable interpretation (in lines of assignment 2 of the course), present the interpretation made and reason for it so we know it is reasonable and solve the question. Please assume noise $\mathcal{E}$ has mean 0 and variance σ^2. Please show the final computation values to reasonable accuracy.]

a. Please write down the formula for bias ? Briefly explain what the formula represents ? **[1+1 points]**

Formula for Bias is $E[\hat{f}(\theta) - f(\theta)]$ where $\hat{f}$ is the function values predicted by the model and f is the original function.

2

The formula represents, the expected or average of deviation of the predicted value $\hat{f}(\theta)$ from the original value $f(\theta)$ from the original value for all θ

basically expectation of deviation from the original

b. Please compute bias using the information provided for the models MR1 and MR2. **[3 points]**

MR1 :- $E[\hat{f}(\theta) - f(\theta)]$ $\{y_1+1, y_2-2, y_3+1, y_4-2, y_5+1, y_6-2\}$

$E(\hat{f}(\theta) - f(\theta))$

original	
y_1	1
y_2	-2
y_3	1
y_4	-2
y_5	1
y_6	-2

$\Sigma \hat{f}(\theta) - f(\theta) = -3$

$E(\hat{f}(\theta) - f(\theta)) = \frac{-3}{6} = -0.5$

we need not substitute point because it is the difference

MR 2:- $E(\hat{f}(\theta) - f(\theta)]$ $\{y_1+2, y_2-1, y_3+2, y_4-1, y_5+2, y_6-1\}$

original	$\hat{f}(\theta)-f(\theta)$
y_1	2
y_2	-1
y_3	2
y_4	-1
y_5	2
y_6	-1

3

$\Sigma \hat{f}(\theta) - f(\theta) = 3$

$E(\hat{f}(\theta) - f(\theta)) = \frac{3}{6} = 0.5$

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c. Please write down formula for variance ? Briefly explain what the formula represents ? [1+1 points]

$$Var(\hat{f}(0)) = E[(\hat{f}(0) - \mu)^2]$$

$Var(x) = E(x^2) - (E(x))^2$

$E((x-\mu)^2)$

The formula indicates the expectation (average) of deviation of each data point (here predicted) from the mean. The deviation is a squared deviation so as to indicate distance from mean

Essentially average of distance from mean.

2

d. Please compute variance using the information for the models MR1 and MR2. [3 points]

MR1: $\{y1+1, y2-2, y3+1, y4-2, y5+1, y6-2\}$

for $x = \{1,2,3,4,5,6\}$

$y = \{2,4,8,15,26,36\}$ $\quad E[y] = mean = \frac{2+4+8+15+26+36}{6} = \frac{91}{6}$

15.16

$\hat{y}_1 = \{3,2,9,13,27,34\}$

$E(\hat{y}-\mu)^2 = (15.16-3)^2 + (15.16-2)^2 + (15.16-9)^2 + (15.16-13)^2 + (15.16-27)^2 + (15.16-34)^2$

$E[(\hat{y}-\mu)^2] = \frac{(\frac{91}{6}-3)^2 + (\frac{91}{6})^2 + (\frac{91}{6}-4)^2 + (\frac{91}{6}-2)^2 + (\frac{91}{6}-6)^2 + (\frac{91}{6}-6)^2}{6}$

$= \frac{920.968}{6} = 153.49$

$= \frac{858.792}{6}$ 143.132 143.13

142.8

3

MR2:- $\{y1+2, y2-1, y3+2, y4-1, y5+2, y6-1\}$

$x = \{1,2,3,4,5,6\}$

$y = \{2,4,8,15,26,36\} \rightarrow E[y] = mean = \frac{91}{6} = 15.16$

$\hat{y} = \{4,3,10,14,28,35\}$

$E((y-\mu)^2) = (15.6-4)^2 + (15.6-3)^2 + (15.6-10)^2 + (15.6-14)^2 + (15.6-28)^2 + (15.6-35)^2$ / 6

$= \frac{858.8736}{6} = \frac{89466}{625} = 143.15$

142.8

(625) 89466 (143.14
625
2696
2500
1966
1875
9160
625
2950
2500
4500

e. Please compute the MSE (Mean Square Error) for MR1 and MR2. Please write down the formula being used before presenting the computations. [3+1 points]

1

$MSE(\hat{f}(0)) = Bias(\hat{f}(0))^2 + Var(\hat{f}(0)) + \sigma^2$

for $f(x)$ $\sigma^2 =$ $E((x-\mu)^2)$ $= \mu(f(x)) = \frac{91}{6}$

Should use $E[(f(x) - \hat{f}(x))]^2$ for calc.

$$\frac{(2-\frac{91}{6})^2 + (4-\frac{91}{6})^2 + (8-\frac{91}{6})^2 + (15-\frac{91}{6})^2 + (26-\frac{91}{6})^2 + (36-\frac{91}{6})^2}{6}$$

$\sigma^2 = 150.14 = \frac{900.8336}{6}$ 150.13

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$MSE(MR1) = (0.5)^2 + (143.13)^2 + (150.14)^2$

$= \cancel{43028.10} \quad 293.62$

$MSE(MR2) = (0.5)^2 + (143.15)^2 + (150.14)^2$

$= 4\cancel{3034.1921} \quad 293.64$

150.14
143.11
0.25
293.62

f. Based on the results above, what would your final model or suggestion be ? Please explain the choice made in detail. [3 points]

We see that bias of MR1 and Bias of MR2 are equal in magnitude.

The irreducible error is common.

What we find the diff. is in the variance

Var(MR2) > var(MR1) by a slight margin

By the bias-variance tradeoff it is better for a model to have low bias and low variance so hence

MR1 realization is slightly better off compared to MR2.

1

same MSE any model

2. Please answer the following questions regarding POMDPs along with all the steps and computations involved. With the IPL season ongoing, popular site Cricinfo provides the winning chance of the teams (T1, T2) at any point as (z%, 100-z%). This represents the belief (z/100, 1-(z/100)) of winning the match for each team from the current point.

a. You construct your belief vector using Cricinfo and take part in a bet where you bet that T1 would win. Both winning and losing the bet would be same value of 100 rupees (i.e., +100 or -100). Please mention whether the following statement is True or False and present the calculations to support: Using the belief vector, you would expect to receive 60 rupees if z is 60%. [3 points]

Ans :- If z = 60%. my belief vector would be $\begin{array}{cc} T_1 & T_2 \\ (0.6, & 0.4) \end{array}$

Expected utility = $P_{outcome} \times U_{outcome}$ utility

0

$= 0.6 \times (+100) + (0.4) \times (-100)$

$= ₹ 20$

so is Yes (True) I would expect

₹0 if z = 60%.

b. If you are a risk neutral agent, you would play the bet in (a). Please mention True or False and present a suitable utility function to make the case for your answer along with calculations involved. [3 points]

A risk neutral agent's characteristic is to play the bet independent of the probability of losing as long he gets so utility which is ~~positive~~, not zero

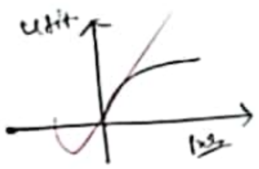
3

so If I were a risk neutral agent :-

we can keep the same utility function
£100 & −100
and I would still PLAY the bet in (a)

because $E[U_1] = 0.6 \times 100 = 60$
$E[U_2] = 0.4(-100) = 40$ [But independent of prob. of loss I play the bet]

c. If you are a risk averse agent, you would definitely not play the bet in (a). Please mention True or False and present a suitable utility function to make the case for your answer along with calculations involved. [3 points]



A risk averse agent's characteristic is to not just look the positive utility he is getting but also weigh the negative utility.

2

despite $E(U|_{win}) > E(U_{loss})$ a risk Averse agent would NOT want to play the bet because

say x is the initial money he has

$$U(x+c) - U(x) < U(x) - U(x-c)$$

$$U(x) > \frac{U(x+c) + U(x-c)}{2}$$

New utility Here $U(win) = 100$ $U(loss) = -160$

so $EU[win] = 100 \times 0.6 = 60$
$E(U|loss) = -160 \times 0.4 = 96$

so the loss has more effect than the gain

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d. If the current status from Cricinfo is (z%, 100-z%) and if action Watch will help you understand the game better before taking a bet, please compute the new belief b' if post Watch action you realize that T2 has increased its chances of winning by 15%. [3 points]

3. Consider a robot that is moving in an environment. The goal of the robot is to move from an initial point to a destination point as fast as possible. However, the robot has the limitation that if it moves fast, its engine can overheat and stop the robot from moving. The robot can move with two different speeds: Slow and Fast. If the robot moves Fast, it gets a (immediate) reward of 10 and if it moves Slow, it gets a (immediate) reward of 4. We can model this problem as an MDP by having three states: Cool, Warm, and Off. The transitions are shown as below. Assume that the discount factor is 0.9 and also assume that when the robot reaches the (terminal) state Off, it will remain there without getting any reward. 2 actions

s	A	s'	P(s'|s,a)
Cool	Slow	Cool	1
Cool	Fast	Cool	1/4
Cool	Fast	Warm	3/4
Warm	Slow	Cool	1/2
Warm	Slow	Warm	1/2
Warm	Fast	Warm	1/4
Warm	Fast	Off	3/4

a) Consider the conservative policy J when the robot always moves Slow. Assume that the robot starts at state Cool. What is the value of J(cool) i.e., expected discounted sum of rewards for state Cool. Please show steps of computation. [4 points]

Since it is an MDP the general formula for utility of a state

States - {Cool, warm, off}

$U_{t+1}(I) = R(I,A) + \max_a \sum \gamma P(J \mid I,A)\, U(J)$

we initally assume all utility to be zero.

we start at cool and given a deterministic policy that we would only take slow action when we start from cool

+1

So $U_{t+1}(\text{cool})^{t+1} = 4 + \gamma(1 \times \cancel{U_t(\text{cool})})$ → reward for action slow → 0 → assumption

$U_{t+1}(\text{cool}) = \underline{4}$

and since it is a deterministic policy and with $P_{ab} = 1$ we are here

we ... therefore

$\therefore U_{t+1}(\text{cool}) = \underline{4}$

+5

b) What is the optimal policy for the robot at each state ? Please explain in detail or show computations for how you arrived at the solution. Answer without clear supporting details will receive 0 points. -- [5 points]

assume $U_t(I) = 0$ for all states and now we are calc.

State ① cool

we calc. above

$U_{t+1}(\text{cool}) = 4$

policy = slow (deterministic)

State 2 cool (fast = action)

$U_{t+1}(\text{cool}) = 10 +$

for state = cool $\quad U_{t+1}(I) = \max_A (R(I,A) + \sum_{s'} P(s'|s,A) \times U_t(s'))$ initially $U_t(\text{cool}) = 0$

$U_{t+1}(\text{cool}) = \max\left(4 + 0, \; 10 + \gamma(\right.$

$\boxed{\text{written at the last.}}$ after question - 9

0

c) Is it possible to change the discount factor to get a different optimal policy? If yes, please provide the discount factor and the policy it gives and if not, please justify your answer with detailed reasoning. -- [3 points, mentioning Yes/No without relevant explanation will not receive points]

No. Decreasing would.

Yes it is possible to change the policy by the discount factor, if the value of discount factor is increased, then the "focus" of the agent on future

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reward is increased because it is essentially weighted probability

so say $\gamma = 0.99$ would change the optimal policy

d) Is it possible to change the immediate reward function so that J(Cool) [not the policy J but the value provided by J at state Cool] changes but the optimal policy would remain unchanged? If yes, please give such a change and if no justify your answer in a couple of sentences. [3 points, mentioning Yes/No without relevant explanation will not carry points]

x3

It depends on the change of the value, if the change of value is not that high, we might end up getting the same optimal policy, but if the change in reward is huge we might have a greater chance of getting a diff. policy

because policy = argmax(max(Reward + (")

so policy is a function of reward.

e) If you plan to solve the above MDP using the Linear Programming approach, please present the complete A matrix for this problem. [5 points]

0

max $(r^T x)$ → reward at each state for action a. $r_{x_i a}$

to maximize → no. of times I take the action 'a' at state(i)

Constraint $Ax = L$

4. Assume that there are 100 students in a class you are part of. Each of the 100 students in the class is provided with a fair coin i.e. P(Heads) = P(Tails) = 0.5. All of you toss your coin simultaneously in each round. Please answer the following questions and present all the relevant reasoning:

a) If the class tossed once, what is the probability that you and your friend both tossed a heads in that round? **[1 points]**

Me	Friend
H	H
H	T
T	H
T	T

are the possibilites so Prob(me = Heads, my friend = heads)

$= \frac{1}{4} = 0.25$

1

b) If the class plans to toss twice, what is the probability that you would toss a heads in second round conditioned on the fact that you tossed a heads in the first round ? **[2 points]**

me	me 2
H	H
H	T
T	H
T	T

$P(2^{nd} = Head \mid 1^{st} = heads) = \frac{P(2^{nd} = Heads \wedge 1^{st} = Heads)}{P(1^{st} = head)}$

$= \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2} = 0.5$

2

c) If the class tossed twice, what is the probability that you obtained heads twice in the two rounds while your friend obtained a tails both times ? **[2 points]**

me 1 Me 2 My frnd 1 my frnd 2

Me and my friend tossing coins are independent activity

~~total poss~~

so $P(me = 2 heads$ and $friend = 2 tails)$

$= P(me = 2 heads) \times P(friend = 2 tails)$

$\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

2

d) If the class tossed once, what is the probability that you are the only one that tossed a heads while everyone else in the class tossed a tails ? **[3 points]**

total possible combinations $= 2^{100}$ of heads and tails.

me getting heads and rest all getting tails is just one possibility so $P = \frac{1}{2^{100}} = \frac{1}{2^{100}}$

3

Or we can write as product of independent events

as 100 of us are independt

$$P = P(me = H) \times \prod_{i=1}^{99} P(\text{others} = T)$$

$$= \frac{1}{2} \times \frac{1}{2^{99}} = \frac{1}{2^{100}}$$

e) If the class tossed once, what is the probability that both you and your friend tossed a Heads given the information that you are risk averse but your friend is risk seeking? You can define suitable utility functions that model risk averse and risk seeking behavior but please present the inputs modeled. **[3 points]**

0

5. Prove the following using truth table method: $(A \rightarrow (B \rightarrow C)) \Leftrightarrow ((A \wedge B) \rightarrow C)$ **[5 points]**

A	B	C
T	T	T
T	T	F
F	T	T
F	F	T

P.T.O

Roll No:

Q.

A	B	C	B→C	A→(B→C)	A∧B	(A∧B)→C
F	F	F	T	T	F	T
F	F	T	T	T	F	T
F	T	F	F	T	F	T
F	T	T	T	T	F	T
T	F	F	T	T	F	T
T	F	T	T	T	F	T
T	T	F	F	F	T	F
T	T	T	T	T	T	T

Appearance of for A⇒B

if A=T then B=T

if A=F then B=F

5

Last 2 columns at the last

6. What are all the *infrequent* candidates (after pruning) that would be generated by Apriori if the frequent itemsets are: F = {A, B, C, D, AB, AC, BC, AD, ABC} [Available items are A, B, C, D] [10 points]

Ans :- applying Apriori on the F′

~~F = A, B, C, D, ABC~~ AC

for F = A, B, C, D we generate more frequent elements and then prune the non-frequent ones

we get {AB, AC, AD, BC, BD, CD}

~~but~~ Now we prune all the infrequent ones and add them to the Ans List = {BD, CD} (not in the frequent list)

for F = AB, AC, BC, AD

we get {ABC, ABD, ACD} we generated here element no we prune the infrequent ones based on our frequent list and append to ans list

Ans List = {BD, CD, ABD, ACD}

+10

for F = ABC

there is nothing we can generate because it is a single element of length = 3

Ans = {BD, CD, ABD, ACD} are the infrequent ones that are generated

7. The probability of cancer in a population is 1%. A test (T1) for cancer identifies cancer patients with a probability of 30% and identifies non-cancer patients with a probability of 99%. [15 points]

(a) Given that a patient has tested positive, what is the probability that he actually has cancer? ______

		Actual Yes	Actual No
Predicted	Yes	0.3	0.01
	No	0.7	0.99

Confusion matrix for test T1

Given that he is tested positive, prob he

actualy has $= \frac{0.3}{0.3+0.7} = 0.3$

But this does not end here he also has to be a part of the population to actually have cancer

so $P = 0.01 \times 0.3 = \underline{3 \times 10^{-3}}$ 1

(b) What is the entropy of the population? 0.08

The entropy of the population is fairly independent of the test so we need not worry about confusion matrix

So Population has $\rightarrow$ cancer patients with $P_1 = 0.01$

$\rightarrow$ non-canc " with $P_2 = 0.99$

$$\text{entropy} = -P_1 \log_2 P_1 - P_2 \log_2 P_2$$

$$= -0.01 \log_2 0.01 - 0.99 \log_2 0.99 = \underline{0.080} = \underline{0.08}$$

5

(c) What is the information gain of T1? 0.80

To calc. the information gain of T1 we calc. it's entropy first.

T1 $\rightarrow$ 0.3 prob predicts correctly regarding cancer 3

$\rightarrow$ 0.99 " " " that it is not cancer

So initial entropy would be of not predicting anything right

$\Rightarrow$ $E(\text{init}) = -0.01 \log_2 0.01 - 0.99 \log_2 0.99 = 0.0807$

The final entropy would be after predicting cancer

$E(\text{final}) = -0.7 \log_2 0.7 - 0.3 \log_2 0.3 = 0.8812$

The Information gain $= E(\text{final}) - E(\text{init}) = 0.8812 - 0.0807 = 0.8005 = 0.80$

8. The spam class of a spam dataset has the following probabilities:

P(A|spam) = 0.2, P(B|spam) = 0.3, P(C|spam) = 0.4, P(AB|spam) = 0.15

$P(Y|X) = \frac{P(X|Y) \times P(Y)}{P(X)}$

Given an email with the keywords, {A, B, C}, what will naive bayes compute as the probability that the email is spam? ________ [5 points]

Ans:- $P(\text{spam} | ABC)$

$\Rightarrow P\left(\frac{\text{spam}}{ABC}\right) = \frac{P(\text{spam} \cap A \cap B \cap C)}{P(A \cap B \cap C)}$?

0.5

$\Rightarrow P\left(\frac{AB}{\text{spam}}\right) = 0.15$

9. Data: {(Ram,65,60),(Shyam,60,60),(Gita,60,70),(Mohan,70,70)}. Given that Ram, Shyam and Mohan are in one cluster and Gita is in the other cluster, determine: [5x3 points]

(i) Single-link distance between the two clusters: ______

Cluster 1 { Ram(65,60), Shyam(60,60) Mohan(70,70) }

Cluster 2 { Gita (60,70) }

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

G↔R = $\sqrt{25+100}$ = $\sqrt{125}$

G→S = $\sqrt{100}$ = 10

G→M = $\sqrt{10}$ = 10

single link dist = min(C_1, C_2) → min. distance across all points of cluster

= 10

(ii) Complete-link distance between the two clusters: ______

Similarly for the distances already calc.

15

G→R = $\sqrt{125}$

G→S = 10

G→M = 10

Complete Link : $\sqrt{125}$ = $5\sqrt{5}$

⇓

farthest dist across diff. all point of cluster = $5\sqrt{5}$

(iii) Average-link distance between the two clusters: ______

Average Link = ~~sum of dist~~ Average of distance over all points across different clusters

G→R = $5\sqrt{5}$ $(\sqrt{25+100})$