

SC3.316: Mathematical Methods in Biology

Final (Spring 2024)

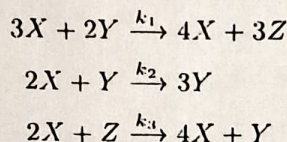
Duration: 3 hrs

Definitions/notations:

- Consider a reaction network $G = (V, E)$. Let $\{\omega_y\}_{y \in C}$ be the standard basis for \mathbb{R}^C . Then
 1. $\Delta_{\rightarrow} = \{\omega_{y'} - \omega_y \in \mathbb{R}^C : y \rightarrow y' \in E\}$.
 2. $\Delta = \{\omega_{y'} - \omega_y \in \mathbb{R}^C : y \sim y' \in E\}$. (Here $y \sim y'$ means that y and y' are in the same linkage class.)
- $\text{supp}(y) = \{i \mid y_i \neq 0\}$.

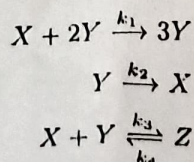
Questions

1. (10 points) Show that complex balanced dynamical systems are quasi-thermodynamic.
2. (5 points) Consider the following dynamical system



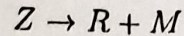
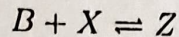
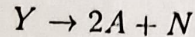
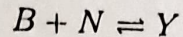
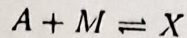
Write the dynamical system generated by the reaction network above in the following form: $\frac{dc}{dt} = Y A_k \Psi$, where Y, A_k, Ψ denote the usual symbols used in the class and $c = (x, y, z)^T$ denotes the vector of concentrations corresponding to the species X, Y, Z .

3. (10 points) Consider a deficiency zero reaction network whose dynamics is given by $\frac{dx}{dt} = Y A_k \Psi$, where Y, A_k, Ψ denote the usual symbols. Then show that $\ker(Y A_k) \subseteq \ker(A_k)$.
4. Show that the following holds:
 - (a) (5 points) $\text{span}(\Delta) = \text{span}(\Delta_{\rightarrow})$.
 - (b) (5 points) For a reaction network having n complexes and ℓ linkage classes, we have $\dim(\text{span}(\Delta)) = \dim(\text{span}(\Delta_{\rightarrow})) = n - \ell$.
5. (5 points) State the Shinar-Feinberg criterion for a dynamical system to exhibit absolute concentration robustness.
6. (10 points) Consider the following reaction network:



- Does the above network satisfy the conditions of the absolute concentration robustness theorem with respect to all three species? (5 marks)
- Is there absolute concentration robustness in any species? Justify. (5 marks)

7. Consider the following reaction network:



- Draw the species-reaction graph corresponding to the reaction network above. (10)
- Does the fully open extension of the network above have the capacity for multiple equilibria? (5)

8. Define the following terms:

- (a) (3 points) Dynamically equivalent systems
- (b) (3 points) Persistence
- (c) (4 points) Permanence

9. State True or False with justification.

- (a) (2 points) Union of siphons is a siphon.
- (b) (2 points) Intersection of siphons is a siphon.
- (c) (3 points) Union of critical sets is critical.
- (d) (3 points) Any subset of a critical set is critical.

10. (15 points) Consider a reaction network. Let S denote the set of all species and let $T \subseteq S$ be a subset of species that is a minimal siphon (note that a set is minimal if it is the smallest set possessing that property. In this case, it means that there is no subset of T which is a siphon). A set A is said to be closed if and only if for every reaction $y \rightarrow y'$, if $\text{supp}(y) \subseteq A$, then $\text{supp}(y') \subseteq A$. For any set A , let $\text{Cl}(A)$ denote the smallest closed set containing A . Then for every species $i \in T$, prove that $\text{Cl}(\{i\} \cup (S - T)) = S$.