

**Real Analysis**  
Mid-Sem 2023  
Time - 1.5 hours  
Full marks 50

- 1.a) Prove that for each  $n \geq 2$ ,  $(n+1)! > 2^n$ .  
b) Prove that for all  $n \in \mathbb{N}$ ,  $(3 + \sqrt{5})^n + (3 - \sqrt{5})^n$  is an even integer. (4+6)
- 2.a) Prove that the set of natural numbers is not bounded from above.  
b) Prove that there is a unique positive real number  $x$ , such that  $x^2 = 2$  (5+5)
- 3.a) Prove that the union and intersection of finite number of open sets in  $\mathbb{R}$  are open sets themselves.**  
b) Show that the set  $\mathbb{N}$  has no limit points. (10+5)
- 4.a) Prove that  $\lim_{n \rightarrow \infty} \frac{S_n}{t_n} = \frac{s}{t}$ , given  $\lim_{n \rightarrow \infty} S_n = s$  and  $\lim_{n \rightarrow \infty} t_n = t$  with  $t_n \neq 0 \forall n \in \mathbb{N}$  and  $t \neq 0$ .  
b) Show whether the following sequence  $(x_n)$  with  $x_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n}$  is convergent or not.  
c) Given  $x \geq 1$ , show that  $\lim_{n \rightarrow \infty} (2x^{1/n} - 1)^n = x^2$  (5+5+5)