## Quiz 1

## Automata Theory Monsoon 2021, IIIT Hyderabad

October 27, 2021

Total Points: 20 Time: 45 mins

**General Instructions:** FSM stands for finite state machine. DFA stands for deterministic finite automata. NFA stands for non-deterministic finite automata. PDA stands for Push Down Automata.  $a^*$  is the Kleene Star operation.  $a^+ = a^* \setminus \{\epsilon\}$ , where  $\epsilon$  is the empty string.

- 1. [2 points] Show that any language L containing only finitely many strings is regular.
- 2. [4 points] Let B and C be languages over  $\Sigma = \{0, 1\}$ . Define

 $B \xleftarrow{1} C = \{w \in B \mid \exists \ y \in C, \text{ such that strings } w \text{ and } y \text{ contain equal number of } 1s\}$ 

Prove that the class of regular languages is closed under  $\leftarrow \frac{1}{\cdot}$ .

3. [2 points] Convert the following language into its Chomsky normal form and write the number of production rules.

$$S \to AB$$

$$A \rightarrow B$$

$$A \rightarrow a$$

$$B \to Bb$$

[N.B: Do not have rules that are redundant. For example don't add  $S_0 \to S$  since S itself can be the start symbol.]

4. [2 points] Show that the grammar G whose rules are as follows, is ambiguous.

$$S \rightarrow aS|aSbS|c$$

- 5. [3 points] For a symbol a, define  $a^+ = \{a, aa, \dots\}$ . Is the language  $L = \{wcw^R | w, c \in \{a, b\}^+\}$  regular? If your answer is yes, write the equivalent regular expression and if no, prove that L is not regular using pumping lemma.
- 6. [3 points] Is  $L = \{a^n b^{4n} | n \ge 1\}$  context-free? If so, (i) write the corresponding context free grammar for this language and (ii) draw the PDA that recognizes L.
- 7. [4 points] The language L consisting of all strings having an equal number of 0's and 1's is context-free. State whether the following languages are regular or context-free. Draw the corresponding automaton to support your answer (DFA/PDA).

(i) 
$$L_1 = \{ w | |\#0's - \#1's| \le 1 \}$$

(ii) 
$$L_2 = \{w | |\#0's - \#1's| \le 1, \ \forall \text{ prefixes of } w\}$$

Note: In 7 the string 00001111 will belong to  $L_1$  but will not belong to  $L_2$  since 00001, a prefix of 00001111, does not satisfy  $|\#0's - \#1's| \le 1$ .