

1. Consider a three-level single particle system with six microstates with energies  $0, \varepsilon, \varepsilon, 2\varepsilon, 2\varepsilon, 2\varepsilon$ . What is the mean energy of the system if it is in equilibrium with a bath at temperature  $T$ ? In the region where  $\beta\varepsilon \rightarrow 0$ , what will the graph of heat capacity of the system as a function of  $\varepsilon$  look like at a constant temperature?

**Ans.** 
$$U = \frac{\sum_j E_j e^{-\beta E_j}}{\sum_j e^{-\beta E_j}} = \frac{\varepsilon \cdot 2e^{-\beta\varepsilon} + 2\varepsilon \cdot 3e^{-2\beta\varepsilon}}{1 + 2e^{-\beta\varepsilon} + 3e^{-2\beta\varepsilon}} = \varepsilon \cdot \frac{2e^{-\beta\varepsilon} + 6e^{-2\beta\varepsilon}}{1 + 2e^{-\beta\varepsilon} + 3e^{-2\beta\varepsilon}}$$

heat capacity,  $C_V = \frac{\partial U}{\partial T} = -\frac{\beta}{T} \frac{\partial U}{\partial \beta} = -\frac{\beta\varepsilon}{T} \cdot \left[ -\varepsilon \cdot \frac{2e^{-\beta\varepsilon} + 12e^{-2\beta\varepsilon}}{1 + 2e^{-\beta\varepsilon} + 3e^{-2\beta\varepsilon}} + \varepsilon \cdot \frac{(2e^{-\beta\varepsilon} + 6e^{-2\beta\varepsilon})(2e^{-\beta\varepsilon} + 6e^{-2\beta\varepsilon})}{(1 + 2e^{-\beta\varepsilon} + 3e^{-2\beta\varepsilon})^2} \right] = \text{const.} \cdot \varepsilon^2$

$\therefore$  the graph will look like a parabola.

2. **State** briefly the **difference** in **assumptions** made by Einstein and Debye in developing the theory for heat capacity of solid crystals.

**Ans.** Einstein : all atoms in different lattice points oscillate with the same frequency

Debye : There is a continuous distribution of the frequency of oscillations of the normal modes of the lattice vibrations starting from 0 to a cutoff frequency.

3. Explain qualitatively why the pressure of an ideal Fermi gas is different from that of the classical ideal gas. Mention also if it is lower or higher.

**Ans.** For the classical ideal gas, the molecules occupy continuous energy states and there is no restriction on how many molecules may be in a certain energy state. For the Fermi gas, there is a restriction that only one molecule may be in a certain energy state. This results in a 'quantum' repulsive interaction that increases the pressure of the gas.

4. How will the density of states of an ideal gas like system change if its volume is doubled?

**Ans.** density of states  $\omega(\varepsilon) \propto V^N$

$$\therefore \frac{\omega_{2V}(\varepsilon)}{\omega_V(\varepsilon)} = \frac{(2V)^N}{V^N} = 2^N$$

5. Obtain the value for:  $\frac{\Theta_{x,H_2}}{\Theta_{x,HD}}$ , for x=r(rotational) at high temperatures, without using the Tables.

**Ans.** The equilibrium bond distance and the force constant is determined by electronic effects, so it will be the same for both  $H_2$  and  $D_2$ . But the reduced masses will change. The symmetry number for both is 2.

$$\frac{\Theta_{x,H_2}}{\Theta_{x,HD}} = \frac{I_{HD}}{I_{H_2}} = \frac{\mu_{HD}}{\mu_{H_2}} = \frac{\frac{m_H \cdot m_D}{m_H + m_D}}{\frac{m_H}{2}} = \frac{\frac{2}{3}}{\frac{1}{2}} = \frac{4}{3}$$