Dynamical processes in complex networks-Mid-Sem

Question 1 [3+4=7 Marks]

The equations of motion of two coupled phase oscillators are described by

$$\dot{\theta}_1 = \omega_1 + k_1 \sin(\theta_2 - \theta_1) \tag{1}$$

$$\dot{\theta}_2 = \omega_2 + k_2 \sin(\theta_1 - \theta_2) \tag{2}$$

Where $\theta_{1,2}$, $\omega_{1,2}$, and $k_{1,2}$ are the phase, internal frequency, and coupling strength of oscillator 1 and 2, respectively.

(a) Calculate the critical coupling strength at which both oscillators will follow the common frequency.

(b) Calculate the common frequency. Also, explain if one can obtain a common (and stable) frequency at $k_1 = k_2 = k = 2$ and $\omega_1 = 2, \omega_2 = 10$.

Question 2 [3+4=7 Marks]

Assume that motion of a particle is captured by

$$\dot{z} = (\mu + i\omega - |z|^2) z \tag{3}$$

where $z = x + iy$.

(a) Transform this equation into (r, θ) plane.

(b) Find the condition in which the system will reveal stable limit cycle. Explain it with proper bifurcation analysis.

Question 3 [3.5+3.5=7 Marks]

(a) A system is described by a differential equation:

$$\dot{x} = rx - x^2 \tag{4}$$

where r is the parameter of the system.

Describe the role of r. What type of bifurcation do you expect?

(b) Another system is described by

$$\dot{x} = 1 + rx + x^2 \tag{5}$$

Describe the bifurcations obtained from this system by varying system parameter r.

Question ~~4~~ [4+1.5+1.5=7 Marks]

An epidemic model (Susceptible-infected-susceptible model) can be described by the following differential equation:

$$\frac{dI}{dt} = \beta SI - \gamma I \tag{6}$$

With the condition $S + I = 1$. Here S and I represent the density susceptible and infected populations, and β and γ are the rates of infection and recovery, respectively.

(a) Solve this model and find out the solution I as a function of time, β and γ.

(b) What will be the endemic equilibrium (I^* at $t \to \infty$)?

(c) Explain in which condition the disease will not spread.

Question 5 [3.5+3.5=7 Marks]

Let the Susceptible-infected-susceptible model in network be described by

$$\dot{i}_k = -\gamma i_k + \beta(1 - i_k) \sum_{j=1}^{N} A_{kj} x_j \tag{7}$$

Where i_k is the density of infected population in k^{th} node. β and γ are rates of infection and recovery. A is the adjacency matrix, and N is the number of nodes.

(a) Explain how the network affects the infection at the initial stage ($t \approx 0$). (b) Also, explain in which case the disease will not spread.