Swaraj

INTERNATIONAL INSTITUTE OF INFORMATION TECHNOLOGY-HYDERABAD

MID I, Monsoon 2018

TIME: 1.5 hrs

Maths-III

Marks: 50

Instructions

- **Answer all Questions**
- **No Formula sheet is allowed**

1. Estimate the bias in a pH meter. Data are collected on the meter by measuring the pH of a neutral substance (pH =7). A sample of size 10 is taken with results given by 7.07, 7.00, 7.10, 6.97, 7.00, 7.03, 7.01, 7.01, 6.98, 7.08. [2]

2. If the probabilities that an automobile mechanic will service 3, 4, 5, 6, 7, or 8 or more cars on any given workday are, respectively, 0.012, 0.19, 0.28, 0.24, 0.10, and 0.07, what is the probability that he will service at least 5 cars on his next day at work? [2]

3. A coin is biased so that a head is twice as likely to occur as a tail. If the coin is tossed 3 times, what is the probability of getting 2 tails and 1 head? [3]

4. In a certain assembly plant, three machines, B1, B2 and B3, make 30%, 45%, and 25%, respectively, of the products. It is known from the past experience that 2%, 3% and 2% of the products made by each machine, respectively, are defective. Now suppose that a finished product is randomly selected. What is the probability that it is defective? [3]

5. The probability density function is given as follows:

   $f(x) = ax^2 0 \le x \le 5, f(x) = 0$, otherwise

   (a) What is the probability that a value selected at random from this distribution will be less than 2? [2]

   (b) What is the probability if the random variable is in between 1 and 3? [1]

   (c) What is the probability that the random variable will be larger than or equal to 4? [1]

   (d) What is the probability that the random variable exceeds 6? [1]

6. Given the joint density function

$$f(x,y) = \begin{cases} \dfrac{x(1+3y^2)}{4} & 0 < x < 2, 0 < y < 1 \\ 0 & Otherwise \end{cases}$$

   (a) Find marginal density function, g(x) [2]

   (b) Find marginal density function, h(y) [2]

$\int_0^1 \frac{x}{4}(1+3y^2) \to \int \frac{x}{4} + \int \frac{x}{4} 3y^3 = \frac{x}{4} + \frac{x}{4} \cdot 3 \cdot \frac{y^2}{2} \Big|_0^1 = \frac{2x}{8} + \frac{3x}{8} = \frac{5x}{8} = \frac{5}{8}\left(\frac{5}{2}\right) = \frac{6}{8}$

$\int_0^2 \frac{x}{4}(1+3y^2)dx = \int_0^2 \frac{x}{4}dx + \int_0^2 \frac{x}{4} 3y^2 dx \to \frac{1}{4} \cdot \frac{x^2}{2}\Big|_0^2 + \frac{3y}{4} \int \left(\frac{x^2}{2}\right)_0^2 = \frac{1}{8}(4) + \frac{3y}{8}(4) = \frac{12y+4}{8} = \frac{12}{8}\left(\frac{1}{2}\right) + \frac{4}{8} \quad \left(\frac{10}{8}\right)$

(c) Find the conditional probability density function, f(x/y) [2]

(d) Find $P(\frac{1}{4} < X < \frac{1}{2} | Y = \frac{1}{3})$ [2]

(e) Test the statistical dependence of random variables X and Y. [2]

(f) Find the expected value of X, E(X) [2]

(g) Find the expected value of Y, E(Y) [2]

(h) Find the expected value of X and Y as E(XY) [2]

7. Suppose that the shelf life, in years, of a certain perishable food product package incardboard containers is a random variable whose probability density function is given by:

$$f(x,y) = \begin{cases} e^{-x} & x > 0 \\ 0 & Otherwise \end{cases}$$

Let X1, X2, and X3 represent the shelf lives for three of these containers selected independently, find P(X1<2, 1<X2 < 3, X3>2). [4]

8. A fair coin is tossed 6 times; consider heads as success. Find

(a) The probability that exactly two heads occur [3]

(b) The probability of getting at least four heads [3]

(c) The probability of getting no heads [2]

9. A random variable X has mean 40 and standard deviation as 5. Find the value of b for which $P(40 - b \leq X \leq 40 + b) \geq 0.95$ [3]

10. A group of 10 individuals is used for a biological case study. The group contains 3 people with blood type O, 4 with blood type A, and 3 with blood type B. What is the probability that a random sample of 5 will contain 1 person with blood type O, 2 people with blood type A, and 2 people with blood type B. [4]

$x > 0$

$E(x) = \int_0^\infty x \cdot f(x) dx \geq \int_a^\infty x \cdot f(x) dx \geq \int_a^\infty a f(x) \cdot dx$

$E(x) \geq a \cdot \int_a^\infty f(x) dx$ $\quad x = x_i \quad g(x_j) = \int f(x,y) dy$

$\sigma^2 = \cancel{E(x^2) - (E(x))^2}$

$\cancel{E(x^2) - (a \int_a f(x) \cdot dx)}$

$E(x - \mu)^2$

$= E(x^2 + \mu^2 - 2\mu x)$

$= E(x^2) + E(\mu^2) - 2\mu E(x)$

$E(x^2) + \mu^2$

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MID II, Monsoon 2018
TIME: 1.5 hrs
Maths-III
Marks: 30

Instructions

- **Answer all Questions**
- **No Formula sheet is allowed**

Part – I (Complex Numbers)

1. Find x, y if $\frac{x}{1+i}+\frac{y}{2-i}=2+4i$ [2.5]
2. Find the roots of $z^2-(1-i)z+7i-4=0$ in the form $a+ib$ [2.5]
3. Show that $\arg(\frac{z_1}{z_2})=\arg(z_1)-\arg(z_2)\pm 2\pi$ [2.5]
4. Find the modulus and argument of z = (2-i)(1+3i) [2.5]

Part – II (Probability and Statistics)

II. Choose the Correct Option

1. In a symmetric random walk of flipping of an unbiased coin, determine the probability of the particle taking the value of 1 in n = 3 trials. [2.5]

(a) $\frac{15}{8}$ (b) $\frac{8}{15}$ (c) $\frac{1}{3}$ (d) 15

2. Two random process X(t) and Y(t) are given as follows:

$$X(t) = A\sin(\omega t+\emptyset_1)Y(t) = B\cos(\omega t+\emptyset_2)$$

The normalized cross-covariance is

(a) 1 (b) 0 (c) C_{XY} (d) ρ_{AB} [2.5]

3. If X and Y are two random variables with PDF as $f(X,Y)=\frac{1}{14}(5-\frac{y}{2}-x)$ with $0\le x\le 2$ and $0\le y\le 2$.

(i) The PDF $g(u,v)$, where $u=x+y$ and $v=\frac{y}{2}$ is [2.5]

(a) $g(u,v)=\frac{1}{7}(5-\frac{y}{2}-x)$

(b) $g(u,v)=\frac{1}{7}(5-v-u)$

3

$\frac{1}{5}+\frac{1}{3}$ z^2-z-4

$3C_1\cdot(\frac{1}{2})^3$ $(\frac{3}{8})$ sina sinb $-2\cos a\cos b$ $\cos(a+b)+\cos(a-b)$

1/8	3/8	1 3/		3/8	1/8	
-3	-2	-1	1	2	3	

1/8 3/8 3/8 1/8
0 1 2 3

$\boxed{21,1}$ HTT THT

(c) $g(u,v)=\frac{1}{7}(5+v-u)$

(d) $g(u,v)=\frac{1}{14}(5-v-u)$

(ii) The limits of u and v are given as [1.0]

(a) $v\le u\le 2+v$ and $0\le v\le 2$
(b) $v\le u\le 2+2v$ and $0\le v\le 2u$
(c) $2v\le u\le 2v+2$ and $2\le v\le u$
(d) $2v\le u\le 2v+2$ and $0\le v\le 1$

4. The probability of getting no mistakes in a page of a book is e^{-4}. The probability that a page of a book contains more than 2 mistakes is [2.5]

(a) $1-e^{-4}$ (b) $1-2e^{-4}$ (c) $1-13e^{-4}$ (d) $13e^{-4}$

5. Fill in the blanks, each carry 1.5 mark [1.5X6 =9]

i) Condition for the first order stationarity........

ii) If X and Y are two random variables with PDF, $f(x,y)$, the transformation to $g(u,v)$, where u and v are functions of x and y is given by ......

iii) If X fallows binomial distribution, with large number of trials as probability is close to zero, the variable can fallow .... distribution with mean as ....

iv) If $\bar{x}$ is the mean of a random sample of size n, from a population with mean μ and variance as σ^2. As $n\to\infty$ the standard normal random variable will follow a form of...........

v) If X is a random variable following exponential distribution, the mean and variance are.....

vi) If x fallows normal distribution with mean μ and standard deviation as σ, then 99.7% of the probability will be in the interval of ..$\mu-3, \mu+3$

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End Sem, Monsoon 2018

TIME: 3 hrs

Maths-III

Marks: 100

Probability and Statistics: Shaik Rehana

Maximum Marks: 60

Note:

1. All questions are compulsory

2. Notations have usual significance

3. Calculators are not allowed

Section A:

Each question carries 5 marks [5X6=30]

1. Briefly explain Bayes theorem

2. Chebshev inequality

3. Central limit theorem

4. Random walk

5. Statistical dependence

6. Hypergeometric distribution

Section B:

1. There are two streams flowing, let A be the event that stream a is polluted, and B is the event that stream b is polluted. In a given day P(A) = 2/5 and P(B) = 3/4. The probability that at least one stream will be polluted in any given day is 4/5. Determine

(a) Probability that stream a is also polluted given that stream b is polluted [2.5]

(b) Probability that stream b is also polluted given that stream a is polluted [2.5]

2. A box contain 25 strain gages, and 4 of them are known to be defective gages. If 6 gages were used in an experiment, what is the probability that there was one defective gages in the experiment? [5]

3. The height of earth dam must allow sufficient freeboard above the maximum reservoir level to prevent waves from washing over the top. The wind tide, in feet, above still-water level is

$$Z = \frac{F}{1400\,d}V^2$$

$P(A|B) = \frac{P(A\cap B)}{P(B)}$

$P(A/B_i) = \frac{P(A\cap B_i)}{P(B_i|A)}$

Where V = wind speed in miles per hour; F = length of water surface over which the wind blows, in feet; d = average depth of lake along the length, in feet. If wind speed fallows exponential distribution with mean speed v_0, then determine the distribution of the tide Z. [10]

4. A fair coin is tossed and if heads come up, a sine wave $x_1(t) = Sin(5\pi t)$ is sent. If tails come up, then $x_2(t) = t$. The resulting random process x(t) is an ensemble of $x_1(t)$. Find the mean and variance of the random process, for t = 0, 0.5, 0.7. [5]

5. A population consists of set, S= {4,7,10} in equiprobable space. Random samples of size 2 are drawn with replacement.

(a) Compute the population mean and standard deviation. [2.5]

(b) Find the sampling distribution for the sample mean. [2.5]

n^k - $\frac{n!}{(n-r)!\,r!}$

Complex Analysis

Indranil Chakrabarty

November 15, 2018

Maximum marks: 40
Note:

1. All questions are compulsory.
2. Notations have usual significance

SECTION A:

Each question carries 5 marks.

1. Solve $\cos(z) = 1/2$ for z
2. For complex numbers $z_i,\ w_i\ (i = 1..n)$ show that $|z_1 w_1 + ...z_n w_n| \leq \sqrt{|z_1|^2 + ... + |z_n|^2}\sqrt{|w_1|^2 + ... + |w_n|^2}$
3. $\int_C \frac{\sin(\pi z^2)+\cos(\pi z^2)}{(z-1)(z-2)}$ where C is the circle : $|z - i| = 3$
4. Find the harmonic conjugates of the following harmonic functions on C (Complex numbers): a) $u(x, y) = x^2 - y^2$ b) $u(x, y) = \sin x \cosh y$
5. Suppose f is an analytic function on a region (an open connected set) A and that $|f(z)|$ is constant on A . Show that f is constant on A.
6. Find the expansion of e^z around the point $i\pi$.

SECTION B:

Each question carries 5 marks.

1. Let $f : A \to C$ and $g : B \to C$ be analytic (A, B are open sets) and let $f(A) \subset B$. Then show that $g \circ f : A \to C$ defined by $(g \circ f)(z) = g(f(z))$ is analytic and $\frac{d}{dz}(g \circ f)(z) = g^{'}(f(z)).f^{'}(z)$. (where C is the set of all Complex numbers)
2. Consider the integration of the function e^{-z^2} around the rectangular contour Γ with vertics $\pm a$, $\pm a + ib$ and oriented positively. By letting $a \to \infty$ while keeping b fixed, show that $\int_{\infty}^{\infty} e^{-x^2} e^{\pm 2ibx} dx = \int_{\infty}^{\infty} e^{-x^2} \cos(2bx) dx = e^{-b^2}\sqrt{\pi}$