INTERNATIONAL INSTITUTE OF INFORMATION TECHNOLOGY-HYDERABAD MID I, Monsoon 2018

TIME: 1.5 hrs Maths-III Marks: 50

Instructions

- Answer all Questions
- No Formula sheet is allowed
- 1. Estimate the bias in a pH meter. Data are collected on the meter by measuring the pH of a neutral substance (pH =7). A sample of size 10 is taken with results given by 7.07, 7.00, 7.10, 6.97, 7.00, 7.03, 7.01, 7.01, 6.98, 7.08. [2]
- 2. If the probabilities that an automobile mechanic will service 3, 4, 5, 6, 7, or 8 or more cars on any given workday are, respectively, 0.012, 0.19, 0.28, 0.24, 0.10, and 0.07, what is the probability that he will service at least 5 cars on his next day at work?
- 3. A coin is biased so that a head is twice as likely to occur as a tail. If the coin is tossed 3 times, what is the probability of getting 2 tails and 1 head?
- In a certain assembly plant, three machines, B1, B2 and B3, make 30%, 45%, and 25%, respectively, of the products. It is known from the past experience that 2%, 3% and 2% of the products made by each machine, respectively, are defective. Now suppose that a finished product is randomly selected. What is the probability that it is defective? [3]
 - 5. The probability density function is given as follows:

$$f(x) = ax^2 0 \le x \le 5$$
, $f(x) = 0$, otherwise

- (a) What is the probability that a value selected at random from this distribution will be less than 2? [2]
- (b) What is the probability if the random variable is in between 1 and 3? [1]
- (e) What is the probability that the random variable will be larger than or equal to 4? [1]
- (d) What is the probability that the random variable exceeds 6? [1]
 - 6. Given the joint density function

$$f(x,y) = \begin{cases} \frac{x(1+3y^2)}{4} & 0 < x < 2, 0 < y < 1\\ 0 & Otherwise \end{cases}$$

- (a) Find marginal density function, g(x)
- [2] (b) Find marginal density function, h(y)

$$\int_{0}^{3} \frac{1}{4} \left(1+39 \right)^{2} = \left(\frac{1}{4} + \left(\frac{1}{4} + \frac{1$$

(e) Find the conditional probability density function, f(x/y)	[2]
---------------------------------------------------------------	-----

(d) Find
$$P(\frac{1}{4} < X < \frac{1}{2} | Y = \frac{1}{3})$$
 [2]

Find the expected value of
$$X$$
, $E(X)$ [2]

7. Suppose that the shelf life, in years, of a certain perishable food product package incardboard containers is a random variable whose probability density function is given by:

$$f(x,y) = \begin{cases} e^{-x} > 0 \\ 0 & Otherwise \end{cases}$$

Let X1, X2, and X3 represent the shelf lives for three of these containers selected independently, find P(X1<2, 1<X2<3, X3>2). [4]

8. A fair coin is tossed 6 times; consider heads as success. Find

- 9. A random variable X has mean 40 and standard deviation as 5. Find the value of b for which $P(40 b \le X \le 40 + b) \ge 0.95$
- 10. A group of 10 individuals is used for a biological case study. The group contains 3 people with blood type O, 4 with blood type A, and 3 with blood type B. What is the probability that a random sample of 5 will contain 1 person with blood type O, 2 people with blood type A, and 2 people with blood type B.

$$E(x) = E \int_{0}^{\infty} x \cdot F(x) dx \geq \int_{0}^{\infty} x \cdot F(x) dx \geq \int_{0}^{\infty} f(x) dx = \int_{0}^{$$

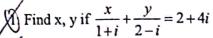
INTERNATIONAL INSTITUTE OF INFORMATION TECHNOLOGY-HYDERABAD MID II, Monsoon 2018

TIME: 1.5 hrs Maths-III Marks: 30

Instructions

- Answer all Questions
- No Formula sheet is allowed

Part - I (Complex Numbers)



[2.5]

Find the roots of $z^2 - (1-i)z + 7i - 4 = 0$ in the form a + ib

[2.5]

3. Show that $\arg(\frac{z_1}{z_2}) = \arg(z_1) - \arg(z_2) \pm 2 \pi$

[2.5]

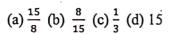
4. Find the modulus and argument of z = (2-i)(1+3i)

[2.5]

Part - II (Probability and Statistics)

II. Choose the Correct Option

1) In a symmetric random walk of flipping of an unbiased coin, determine the probability of the particle taking the value of 1 in n = 3 trials.





2. Two random process X(t) and Y(t) are given as follows:

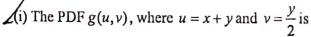
$$X(t) = Asin(\omega t + \emptyset_1)Y(t) = Bcos(\omega t + \emptyset_2)$$

The normalized cross-covariance is

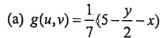
(a) 1 (b) 0 (c)
$$C_{XY}$$
 (d) ρ_{AB}

[2.5]

S. If X and Y are two random variables with PDF as $f(X,Y) = \frac{1}{14}(5 - \frac{y}{2} - x)$ with $0 \le x \le 2$ and

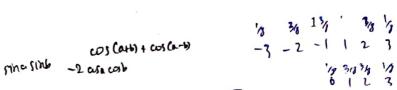


[2.5]



(b)
$$g(u,v) = \frac{1}{7}(5-v-u)$$









(c)
$$g(u,v) = \frac{1}{7}(5+v-u)$$

(d)
$$g(u,v) = \frac{1}{14}(5-v-u)$$

(ii) The limits of u and v are given as

[1.0]

- (a) $v \le u \le 2 + v$ and $0 \le v \le 2$
- (b) $v \le u \le 2 + 2v$ and $0 \le v \le 2u$
- (c) $2v \le u \le 2v + 2$ and $2 \le v \le u$
- (d) $2v \le u \le 2v + 2$ and $0 \le v \le 1$

A. The probability of getting no mistakes in a page of a book is e^{-4} . The probability that a page of a book contains more than 2 mistakes is [2.5]

(a)
$$1-e^{-4}$$
 (b) $1-2e^{-4}$ (c) $1-13e^{-4}$ (d) $13e^{-4}$

5. Fill in the blanks, each carry 1.5 mark

[1.5X6 = 9]

(Condition for the first order stationarity......

- (i) If X and Y are two random variables with PDF, f(x, y), the transformation to g(u, v), where u and v are functions of x and y is given by
- (iv) If \bar{x} is the mean of a random sample of size n, from a population with mean μ and variance as σ^2 . As $n \to \infty$ the standard normal random variable will follow a form of............
- If X is a random variable following exponential distribution, the mean and variance are.....

yi) If x fallows normal distribution with mean μ and standard deviation as σ , then 99.7% of the probability will be in the interval of $\frac{\mu^{-3}}{2}$.

Ewaray

INTERNATIONAL INSTITUTE OF INFORMATION TECHNOLOGY-HYDERABAD

End Sem, Monsoon 2018

TIME: 3 hrs Maths-III Marks: 100





Probability and Statistics: Shaik Rehana

Maximum Marks: 60

Note:

- 1. All questions are compulsory
- 2. Notations have usual significance
- 3. Calculators are not allowed

Section A:

Each question carries 5 marks

[5X6=30]

- 1. Briefly explain Bayes theorem
- 2. Chebshev inequality
- 3. Central limit theorem

4. Random walk

- 5. Statistical dependence
- 6. Hypergeometric distribution

Section B:

- 1. There are two streams flowing, let A be the event that stream a is polluted, and B is the event that stream b is polluted. In a given day P(A) = 2/5 and P(B) = 3/4. The probability that at least one stream will be polluted in any given day is 4/5. Determine
- (a) Probability that stream a is also polluted given that stream b is polluted

[2.5]

(b) Probability that stream b is also polluted given that stream a is polluted

[2.5]

- 2. A box contain 25 strain gages, and 4 of them are known to be defective gages. If 6 gages were used in an experiment, what is the probability that there was one defective gages in the experiment? [5]
- 3. The height of earth dam must allow sufficient freeboard above the maximum reservoir level to prevent waves from washing over the top. The wind tide, in feet, above still-water level is

$$Z = \frac{F}{1400 d} V^2$$



Where V = wind speed in miles per hour; F = length of water surface over which the wind blows, in feet; d = average depth of lake along the length, in feet. If wind speed fallows exponential distribution with mean speed v_0 , then determine the distribution of the tide Z. [10]

- 4. A fair coin is tossed and if heads come up, a sine wave $x_1(t) = Sin(5\pi t)$ is sent. If tails come up, then $x_2(t) = t$. The resulting random process x(t) is an ensemble of $x_1(t)$. Find the mean and variance of the random process, for t = 0, 0.5, 0.7.
- 5. A population consists of set, $S = \{4,7,10\}$ in equiprobable space. Random samples of size 2 are drawn with replacement.

(a) Compute the population mean and standard deviation.

[2.5]

(b) Find the sampling distribution for the sample mean.

[2.5]

Ne Guir.

Complex Analysis

Indranil Chakrabarty

November 15, 2018

Maximum marks: 40

ote:

- 1. All questions are compulsory.
- 2. Notations have usual significance

SECTION A:

Each question carries 5 marks.

- 1. Solve $\cos(z) = 1/2$ for z
- 2. For complex numbers z_i , w_i (i=1..n) show that $|z_1w_1+....z_nw_n| \le \sqrt{|z_1|^2+...+|z_n|^2}\sqrt{|w_1|^2+...+|w_n|^2}$
- 3. $\int_C \frac{\sin(\pi z^2) + \cos(\pi z^2)}{(z-1)(z-2)}$ where C is the circle: |z-i| = 3
- 4. Find the harmonic conjugates of the following harmonic functions on C (Complex numbers): a) $u(x,y)=x^2-y^2$ b) $u(x,y)=\sin x \cosh y$
- 5. Suppose f is an analytic function on a region (an open connected set) A and that |f(z)| is constant on A. Show that f is constant on A.
- 6. Find the expansion of e^z around the point $i\pi$.

SECTION B:

Each question carries 5 marks.

- 1. Let $f:A \to C$ and $g:B \to C$ be analytic (A,B) are open sets) and let $f(A) \subset B$. Then show that $g \circ f:A \to C$ defined by $(g \circ f)(z) = g(f(z))$ is analytic and $\frac{d}{dz}(g \circ f)(z) = g'(f(z)), f'(z)$. (where C is the set of all Complex numbers)
- 2. Consider the integration of the function e^{-z^2} around the rectangular contour Γ with vertics $\pm a$, $\pm a + ib$ and oriented positively. By letting $a \to \infty$ while keeping b fixed, show that $\int_{-c}^{\infty} e^{-x^2} e^{\pm 2ibx} dx = \int_{-c}^{\infty} e^{-x^2} \cos(2bx) dx = \int_{-c}^{c} -b^2 /c$