Solutions to Quiz-2 (Group-A) IEC102

(Q1) Consider the circuit shown in Fig. Q1. The switch has been in position-1 for a very long time and is moved to position-2 at time $t=0$. Find $i(t)$ for $t>0$.
(Assume that the circuit is in steady state at $t=0-$)

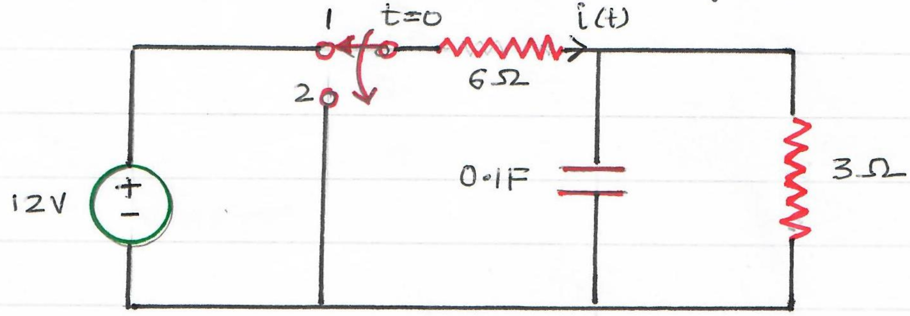
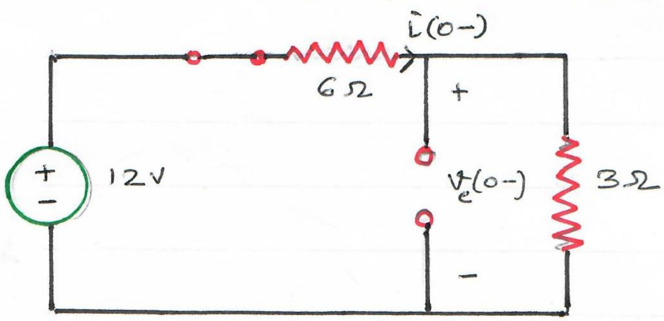


Fig. Q1

Sol.

Circuit at ~~stear~~ $t=0-$ (circuit is in steady state)

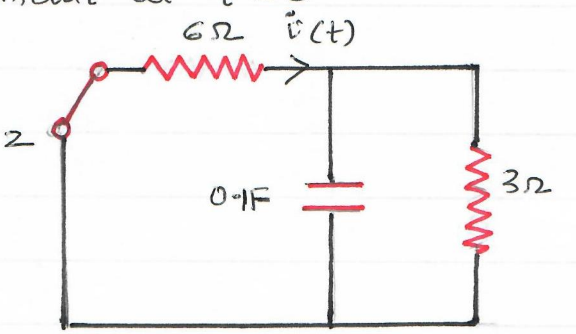


$$v_c(0-) = \frac{12 \times 3}{6+3} = \frac{12\times 3}{9} = 4V$$

$$= v_c(0) = v_c(0+)$$

$$i(0-) = \frac{12}{6+3} = \frac{4}{3} A$$

Circuit at $t=0$



$$v_c(t) = A e^{-t/\tau}$$

where $A = v_c(0) = 4V$

$$R_{eq} = \frac{6\times 3}{6+3} = 2\Omega$$

$$\tau = R_{eq}C$$

$$C = 0.1F$$

$$\tau = 2\times 0.1 = 0.2s$$

$$\therefore v_c(t) = Ae^{-t/\tau} = 4e^{-5t}$$

$$-i(t) + C\frac{dv_c}{dt} + \frac{v_c}{3} = 0 \Rightarrow i(t) = C\frac{dv_c}{dt} + \frac{v_c}{3}$$

$$= 0.1 \times -20e^{-5t} + \frac{4}{3}e^{-5t}$$

$$= -2e^{-5t} + \frac{4}{3}e^{-5t} = -\frac{2}{3}e^{-5t}$$

Free Lined Graph Paper from http://incompetech.com/graphpaper/lined

(Q2) If the switch in the network shown in Fig. Q2 closes at $t = 0$, find $v_o(t)$ for $t > 0$. Assume that the circuit is in steady state at $t = 0-$

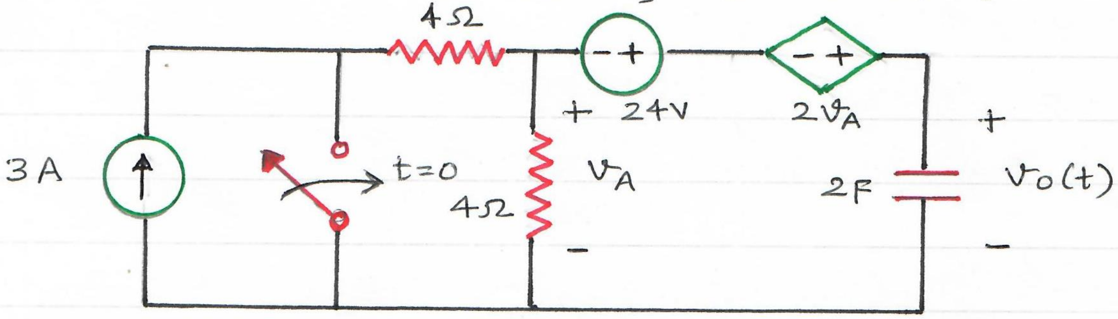
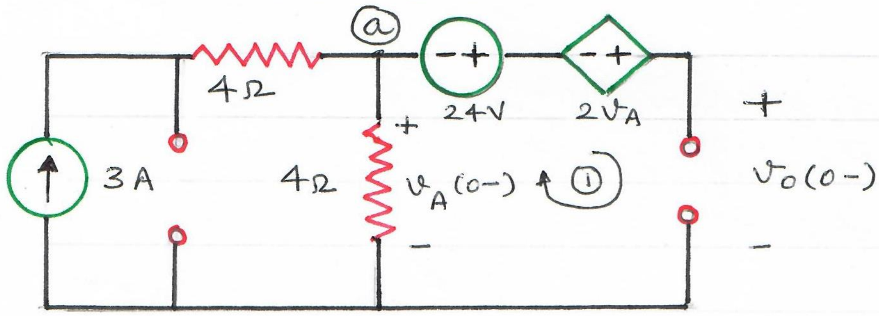


Fig. Q2

Sol.

The circuit at $t = 0-$



Applying KCL at node (a)

$$-3 + \frac{v_A(0-)}{4} = 0 \quad \Rightarrow \quad v_A(0-) = 12V$$

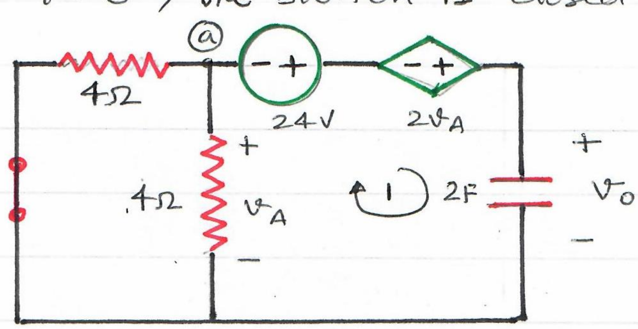
Applying KVL around loop (1)

$$-v_A(0-) - 24 - 2v_A(0-) + v_o(0-) = 0$$

$$\Rightarrow \quad v_o(0-) = 3v_A(0-) + 24 = 3 \times 12 + 24 = 36 + 24 = 60\ V$$

$v_o(0-) = v_o(0) = v_o(0+) = 60V$ ($\because$ since it is the voltage across capacitor)

At $t = 0$, the switch is closed and the circuit is



Applying KCL at node 'a'

$$\frac{v_A}{4} + \frac{v_A}{4} + 2\frac{dv_o}{dt} = 0$$

$$\Rightarrow \frac{v_A}{2} = -2\frac{dv_o}{dt}$$

$$\boxed{v_A = -4\frac{dv_o}{dt}}$$

Applying KVL around loop 1

$$-v_A - 24 - 2v_A + v_o = 0$$

$$\Rightarrow v_o = 3v_A + 24$$

$$\Rightarrow v_o = 3 \times -4\frac{dv_o}{dt} + 24$$

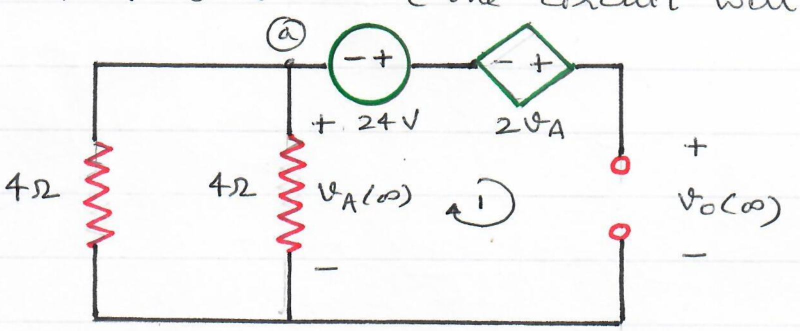
$$\Rightarrow 12\frac{dv_o}{dt} + v_o = 24$$

$$\Rightarrow \boxed{\frac{dv_o}{dt} + \frac{1}{12}v_o = 2}$$

$$\therefore v_o(t) = K + Ae^{-\frac{t}{12}}$$

$v_o(\infty) = K$

Circuit at $t = \infty$ (the circuit will be in steady state)



Applying KCL at node a

$$\frac{v_A(\infty)}{4} + \frac{v_A(\infty)}{4} = 0$$

$$\Rightarrow v_A(\infty) = 0$$

Applying KVL around loop 1

$$-v_A(\infty) - 24 - 2v_A + v_o(\infty) = 0$$

$$\Rightarrow v_o(\infty) = 24\text{ V}$$

$$v_o(\infty) = K = 24$$

$$\therefore v_o(t) = 24 + Ae^{-\frac{t}{12}}$$

$$v_o(0) = 60 = 24 + A \Rightarrow A = 36$$

$$\therefore \boxed{v_o(t) = 24 + 36e^{-\frac{t}{12}}}$$

(Q3) Find $V_c(t)$ for $t \geq 0$ in the circuit shown in Fig. Q3. Given that the circuit is in steady state at $t = 0-$

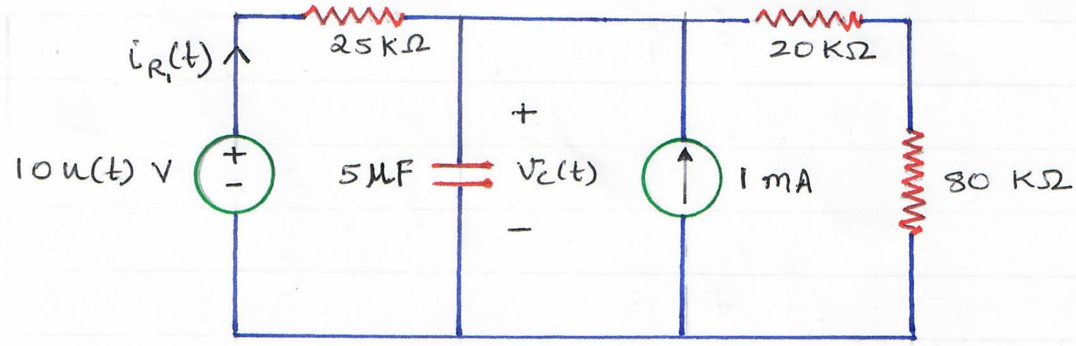


Fig. Q3

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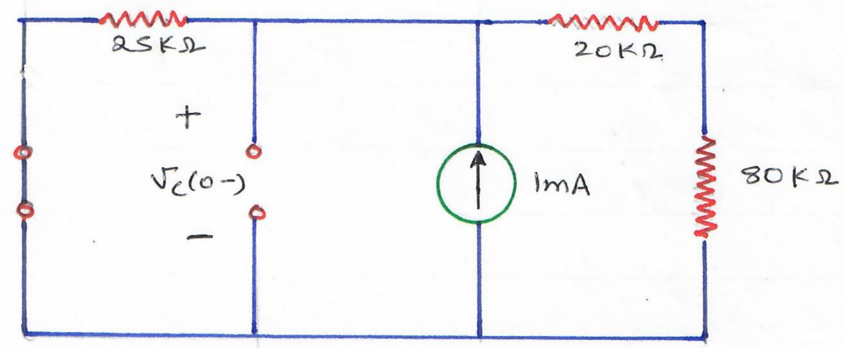
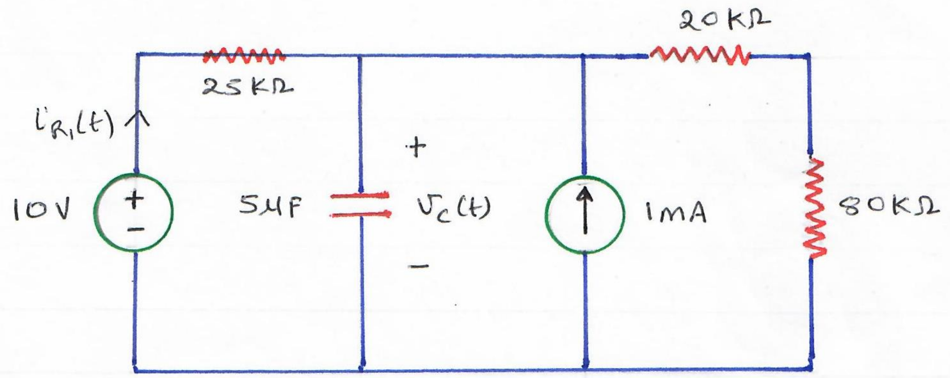
Circuit at $t = 0-$

Since the circuit is in steady state, capacitor acts as open circuit. and the voltage source $10u(t) = 0$



$$V_c(0-) = \frac{100K}{125K} \times 10^{-3} \times 25K = 20V$$

Circuit for $t > 0$



$$V_c(t) = V_c(\infty) + (V_c(0) - V_c(\infty))\, e^{-t/\tau}$$

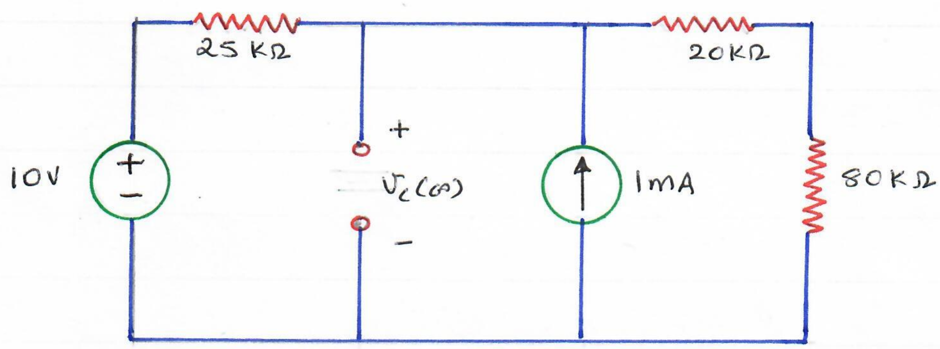
where $\tau = R_{eq} C$

$$R_{eq} = 25\,K\Omega \parallel (20+80)K\Omega$$
$$= 25\,K\Omega \parallel 100\,K\Omega$$
$$= 20\,K\Omega$$

$$\therefore \tau = R_{eq}C = 20\times10^{3}\times5\times10^{-6}$$
$$= 100\times10^{-3} = 0.1\ \text{sec}$$

$$\therefore v_c(t) = v_c(\infty) + (v_c(0) - v_c(\infty)\, e^{-t/0.1}$$
$$= v_c(\infty) + (v_c(0) - v_c(\infty))\, e^{-10t}$$

circuit at $t=\infty$ (the circuit will be in steady state)



$$v_c(\infty) = \left(10 - \frac{10}{125K}\times25K\right) + \frac{100K}{125K}\times10^{-3}\times25K$$
$$= 8 + 20$$
$$= 28\,V$$

$$\therefore v_c(t) = 28 + (20-28)\,e^{-10t}\ V$$
$$= 28 - 8e^{-10t}\ V \qquad \text{for } t \geq 0$$

$$i_{R_1}(t) = \frac{10 - v_c(t)}{25K} = \frac{10-(28-8e^{-10t})}{25}\ mA$$
$$= \frac{-18+8e^{-10t}}{25}$$
$$= \frac{2}{25}(-9+4e^{-10t})\ mA \qquad \text{for } t\geq 0$$