INTERNATIONAL INSTITUTE OF INFORMATION TECHNOLOGY
HYDERABAD

Monsoon Semester (Aug-Nov), 2023
Discrete Structures (DS, Section A)

Quiz I
30.08.2023

1. **(Sets)** [4]

    1. Let $D_i = \{x \in \mathbb{R} \mid -i \leq x \leq i\} = [-i, i]$ for all non negative integers i. Are $D_0, D_1, \ldots,$ **mutually disjoint**? Explain.
    2. Let $A_1 = \{1, 2, 3\}, A_2 = \{u, v\}$, and $A_3 = \{m, n\}$. Find $A_1 \times (A_2 \times A_3)$.
    3. For all sets A, B, C prove that $(A - C) \cap (B - C) \cap (A - B) = \phi$.
    4. Prove or disprove that $X - (Y \cap Z) = (X - Y) \cup (X - Z)$.

2. **(Induction Proofs)** [4]

    Prove the following using induction.

    1. Show that
    $$\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{9}\right)\cdots\left(1 - \frac{1}{n^2}\right) = \left(\frac{n+1}{2n}\right).$$
    2. Suppose that $f_0, f_1, \ldots,$ is a sequence defined as follows
    $$f_0 = 5,\ f_1 = 16, \quad f_k = 7f_{k-1} - 10f_{k-2}, \quad \forall k \geq 2.$$
    Prove that $f_n = 3 \cdot 2^n + 2 \cdot 5^n$ for all integers $n \geq 0$.

3. **(Pigeon hole principle)** [3]

    The **pigeon-hole principle** states that:

    If we put $N + 1$ pigeons in N pigeon-holes, then there will be at-least one pigeon hole with at least two pigeons. Prove this statement using contrapositive proof.

    A **general pigeon-hole principle** is stated as follows:

    If we must put $Nk + 1$ or more pigeons into N pigeon holes, then some pigeon-hole must contain at least $k + 1$ pigeons. Prove this using contrapositive proof.

    Prove the following using pigeon-hole principle.

    1. Given 12 integers, show that two of them can be chosen so that their difference is divisible by 11.

End of Tutorial

Student's name: