SC3.316: Mathematical Methods in Biology
Midterm 2
Duration: 1 hr 30 mins

1) (15 points) Find the reduced echelon form of the following matrix:

$$A = \begin{pmatrix} 1 & 2 & 1 & 9 \\ 2 & 4 & 1 & 18 \\ 3 & 5 & 1 & 24 \end{pmatrix}$$

2. (10 points) State the Deficiency Zero Theorem.

3. (15 points) Consider a reaction network with l linkage classes. Let δ_i denote the deficiency of the ith linkage class and let δ denote the deficiency of the whole network. If $\delta = 0$, then show that $\delta_i = 0$ for every linkage class.

4. (15 points) A reaction network is "forest-like" if every direct link connecting two complexes in a linkage class is a cut-link. Show that every forest-like weakly reversible reaction network is reversible.

5. (15 points) A reaction network is consistent if there exists positive real numbers $c_{y \to y'}$ such that

$$\sum_{y \to y'} c_{y \to y'} (y' - y) = 0. \tag{1}$$

Show that a weakly reversible reaction network is consistent.

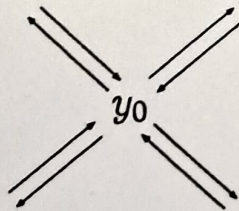6. (15 points) Show that a reversible star-like network (as in Figure 1) is quasi-thermodynamic.

Figure 1: Star-like network.

7. (15 points) Let $A_{n \times n}$ be a square matrix and let $x \in \mathbb{R}^n$. Show that the following are equivalent

1. For every vector $b \in \mathbb{R}^n$, the system $Ax = b$ has at least one solution.
2. For every vector $b \in \mathbb{R}^n$, the system $Ax = b$ has exactly one solution.