(CS1.301) Algorithm Analysis and Design (Monsoon 2022) Date: 19.10.2022

Deep Quiz 2

Alloted time: 45 minutes Total marks: 15

Instructions:

- There are a total of 3 questions.
- Discussions amongst the students are not allowed. No electronic devices nor notes/books of any kind are allowed.
- Any dishonesty shall be penalized heavily.
- Place your identity cards on the table for verification.
- Be clear in your arguments. Partial marking is available but vague arguments shall not be given any credit.

Question 1 [3 marks]

Let $G = (V, E)$ be a directed graph and for every edge $u \to v \in E$, a capacity $c(u \to v)$ is defined. Let $f(u \to v)$ (for all $u \to v \in E$) be a given feasible flow. Compute the residual capacities and the residual graph, with respect to f.

Question 2 [5 marks]

Let n be an integer which is much larger than 5. Formulate a recursive function to compute the number of ways of providing change for n rupees only using (sufficient supply of) 1 rupee and 2 rupee coins.

Question 3 [7 marks]

Suppose you are given three strings, S_1, S_2, and S_3, where $|S_1| = n$, $|S_2| = m$, and $|S_3| = m+n$. We say that S_3 is an interleaf of S_1 and S_2 if and only if S_3 can be formed by interleaving sequences of characters from S_1 and S_2 in a way that maintains the left-to-right ordering of S_1 and S_2. For example, "split" is an interleaving of "spit" and "l", but "splti" is not, and "cchocohilaptes" is an interleaf of "chocolate" and "chips".

Give an efficient dynamic programing algorithm[1] that takes S_1, S_2, and S_3 as parameters and determines whether S_3 is an in interleaf of S_1 and S_2.

[1] Hint: Memoization matrix could take True or False values in each entry where True in entry $M_{i,j}$ could represent if the first $i + j$ letters of S_3 are formed by interleaving of first i letters of S_1 and first j letters of S_2.