Statistical Mechanics

Final Exam

Full marks: 80 , Section A: 8* 5 =40 , Section B: 4* 10 =40 , Time: : 3 Hrs

(For standard temperature and pressure , $T = 293K$ and $P = 1.01 \times 10^5$, $K = 1.38 \times 10^{-23}$.
m of $H_2 = 1.67 \times 10^{-27}$, m of helium = 6.64×10^{-27}, m of oxygen = 25.6×10^{-27} , $h = 6.63 \times 10^{-34}$)

A.

1. A system of N identical bosons is in the ground state of a harmonic oscillator potential with frequency nu. Calculate the average energy and the specific heat at constant volume for this system in the quantum canonical ensemble at temperature T = 200 K.

2. How does the interaction between atoms in a Bose-Einstein condensate affect its behavior and properties?

3. How is the total energy radiated by a blackbody related to its temperature?

4. How do the occupation numbers relate to the distribution of particles among different energy levels in a system?

5. What is the Fermi energy, and how does it relate to the occupation of energy levels in a Fermi gas?

6. What is the Fermi temperature, and how is it related to the Fermi energy?

7. What is the Fermi surface, and how is it related to the distribution of electrons in a metal?

8. What are the properties of Bose-Einstein condensates, and how do they differ from normal gases?

B.

1. A system of N bosons is confined in a 3D box of volume V. What is the maximum energy of a boson in the ground state, and what is the maximum energy of a boson in the first excited state?

2. A particle in a one-dimensional box of length L has energy levels given by $E_n = (n^2 * \pi^2 * h^2)/(2mL^2)$, where n is a positive integer, h is Planck's constant, and m is the mass of the particle. Calculate the average energy and the specific heat at constant volume for this system in the quantum canonical ensemble at temperature T = 100 K, assuming that the particle is a fermion with spin 1/2.

3. Determine the values of the degeneracy discriminant ($n\lambda^3$) for hydrogen, helium, and oxygen at NTP. Make an estimate of the respective temperature ranges where the magnitude of this quantity becomes comparable to unity and hence quantum effects become important.

4. Obtain numerical estimates of the Fermi energy (in eV) and the Fermi temperature (in K) for the following systems:

(a) conduction electrons in silver, lead, and aluminum;

(b) nucleons in a heavy nucleus, such as $_{80}\mathrm{Hg}^{200}$, and

(c) He^3 atoms in liquid helium-3 (atomic volume: 63 Å^3 per atom).