POPL Monsoon 2018 Final Exam

POPL Instructors

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1 IF+/ language (60 points)

Refer to the IF+/ language specified at the end of this question paper.

1.1 Type safety (15 points)

State the type safety theorems (preservation and progress) for the the IF+/ language.

1.2 Language extension (45 points)

Extend the IF+/ language with a < (denoting the less than operator on numbers) construct. Show how the following get extended: semantic domains, syntax, rewrite rules, reduction rules, argument type mismatch judgements and the type system.

2 Determinacy (60 points)

State and prove that the reduction rules of IF+/ form a deterministic transition system: i.e., for any expression e, e reduces to atmost one expression. [Hint: show $e \to e'$ and $e \to e''$ implies $e' = e''$.] State clearly the induction scheme employed. Also, clearly state the reason for each step in the proof. To save time, you may restrict your proof to rules that deal only with the numbers, addition, and the `if` construct in the syntax.

3 CPS Transformations (30 points)

Transform the following direct style programs into continuation passing style.

3.1 Hemachandra-Fibonacci function

```
(define (hem n)
  (cond
    [(= n 0) 0]
    [(= n 1) 1]
    [else (+ (hem (- n 1)) (hem (- n 2)))]))
```

3.2 Reduce

```
(define (reduce x0 f us)
  (if (null? us) x0
      (f (reduce x0 f (rest us))
         (first us))))
```

4 AST evaluation (30 points)

Assume the top-level continuation is the identity function. Assume, further, that +, sub1 and * are part of the language's syntax, i.e., keywords. Assume that the top-level environment contains closures corresponding to the following two functions:

```
(define (f n)
  (+ 4 (g (sub1 n))))

(define (g m)
  (* m 5))
```

Draw an AST annotation diagram illustrating the evaluation of the expresion below, assuming the evaluation is done in continuation passing style:

```
(+ 3 (f 2))
```

Recap of the IF+/ language

Semantic domains and Syntax

$v ::=$
- n
- b

$n ::=$

$b ::=$
- true
- false

$+ : [NUM, NUM] \to NUM$		Addition function
$/ : [NUM, NUM \setminus \{0\}] \to NUM$		Division function
	VAL	Value
	NUM_VAL	Number Value
	BOOL_VAL	Boolean Value
	NUM.	Number
	BOOL	Boolean
	TRUE_VAL	True value
	FALSE_VAL	False value

$e \in EXP$

$e ::=$ *EXP*		IF+/ Expression
$\overline{n}$	*NUM_LIT*	Number Literal
$\overline{b}$	*BOOL_LIT*	Boolean literal
$e \boxed{+} e$	*PLUSEXP*	Plus Expression
$e \boxed{/} e$	*DIVEXP*	Div Expression
if e e e	*IFEXP*	If Expression

Values and Rewrite rules

$e \ \overline{VAL}$

$$\frac{}{\overline{n} \ \overline{VAL}} \ NUM \qquad \frac{}{\overline{b} \ \overline{VAL}} \ BOOL$$

$\hookrightarrow \subseteq EXP \times EXP$

$$\overline{n_1} \boxed{+} \overline{n_2} \hookrightarrow \overline{n_1 + n_2} \ \ PLUS$$

$$\overline{n_1} \boxed{/} \overline{n_2} \hookrightarrow \overline{n_1 / n_2} \quad n_2 \neq 0 \ \ DIV$$

$$\text{if } \overline{\text{true}} \ e \ e' \hookrightarrow e \ \ IF\text{-}TRUE$$

$$\text{if } \overline{\text{false}} \ e \ e' \hookrightarrow e' \ \ IF\text{-}FALSE$$

antics: Reduction and Simplification

$\boxed{\rightarrow \subseteq EXP \times EXP}$

$$\frac{e \hookrightarrow e'}{e \rightarrow e'} \; REWRITE$$

$$\frac{e_1 \rightarrow e_1'}{e_1 \boxed{+} e_2 \rightarrow e_1' \boxed{+} e_2} \; P\_LEFT$$

$$\frac{e_1 \;\; \overline{VAL} \qquad e_2 \rightarrow e_2'}{e_1 \boxed{+} e_2 \rightarrow e_1 \boxed{+} e_2'} \; P\_RIGHT$$

$$\frac{e_1 \rightarrow e_1'}{e_1 \boxed{/} e_2 \rightarrow e_1' \boxed{/} e_2} \; D\_LEFT$$

$$\frac{e_1 \;\; \overline{VAL} \qquad e_2 \rightarrow e_2'}{e_1 \boxed{/} e_2 \rightarrow e_1 \boxed{/} e_2'} \; D\_RIGHT$$

$$\frac{e_1 \rightarrow e_1'}{\text{if } e_1 \; e_2 \; e_3 \rightarrow \text{if } e_1' \; e_2 \; e_3} \; IF\_TEST$$

$\boxed{\overset{*}{\rightarrow} \subseteq EXP \times EXP}$

$$\frac{}{e \overset{*}{\rightarrow} e} \; REF$$

$$\frac{e_1 \rightarrow e_2}{e_1 \overset{*}{\rightarrow} e_2} \; RED$$

$$\frac{e_1 \rightarrow e_2 \qquad e_2 \overset{*}{\rightarrow} e_3}{e_1 \overset{*}{\rightarrow} e_3} \; TRANS$$

xpressions stuck due to DIV/0

$\boxed{e \; DIV/0}$

$$\frac{}{\overline{n_1} \boxed{/} \overline{0} \;\; DIV/0} \; ZERO$$

$$\frac{e_1 \;\; DIV/0}{e_1 \boxed{+} e_2 \;\; DIV/0} \; PLUS\text{-}LEFT$$

$$\frac{e_1 \;\; \overline{VAL} \qquad e_2 \;\; DIV/0}{e_1 \boxed{+} e_2 \;\; DIV/0} \; PLUS\text{-}RIGHT$$

$$\frac{e_1 \;\; DIV/0}{e_1 \boxed{/} e_2 \;\; DIV/0} \; DIV\text{-}LEFT$$

$$\frac{e_1 \;\; \overline{VAL} \qquad e_2 \;\; DIV/0}{e_1 \boxed{/} e_2 \;\; DIV/0} \; DIV\text{-}RIGHT$$

$$\frac{e_1 \;\; DIV/0}{\text{if } e_1 \; e_2 \; e_3 \;\; DIV/0} \; IF$$

Expressions stuck due to Argument Type mismatch

$\boxed{e \quad ATM}$

$$\frac{}{\overline{b_1} \boxed{+} \overline{n_2} \quad ATM} \; BOOL{+}NUM$$

$$\frac{}{\overline{b_1} \boxed{+} \overline{b_2} \quad ATM} \; BOOL{+}BOOL$$

$$\frac{}{\overline{n_1} \boxed{+} \overline{b_2} \quad ATM} \; NUM{+}BOOL$$

$$\frac{}{\overline{b_1} \boxed{/} \overline{n_2} \quad ATM} \; BOOL/NUM$$

$$\frac{}{\overline{b_1} \boxed{/} \overline{b_2} \quad ATM} \; BOOL/BOOL$$

$$\frac{}{\overline{n_1} \boxed{/} \overline{b_2} \quad ATM} \; NUM/BOOL$$

$$\frac{e_1 \quad ATM}{e_1 \boxed{+} e_2 \quad ATM} \; PLUS\text{-}LEFT$$

$$\frac{e_1 \quad \overline{VAL} \qquad e_2 \quad ATM}{e_1 \boxed{+} e_2 \quad ATM} \; PLUS\text{-}RIGHT$$

$$\frac{e_1 \quad ATM}{e_1 \boxed{/} e_2 \quad ATM} \; DIV\text{-}LEFT$$

$$\frac{e_1 \quad \overline{VAL} \qquad e_2 \quad ATM}{e_1 \boxed{/} e_2 \quad ATM} \; DIV\text{-}RIGHT$$

$$\frac{}{\text{if } \overline{n_1}\ e_2\ e_3 \quad ATM} \; IF\text{-}NUM$$

$$\frac{e_1 \quad ATM}{\text{if } e_1\ e_2\ e_3 \quad ATM} \; IF\text{-}ATM$$

Types: syntax and static semantics

$\boxed{\tau \in TYPE}$

$\tau ::=$		$TYPE$	Type
	`num`	$NUMTYPE$	Number Type
	`bool`	$BOOLTYPE$	Boolean Type

$\boxed{e : \tau \subseteq EXP \times TYPE}$

$$\frac{}{\overline{n} : \texttt{num}} \; NUM$$

$$\frac{}{\overline{b} : \texttt{bool}} \; BOOL$$

$$\frac{e_1 : \texttt{num} \quad e_2 : \texttt{num}}{e1 \boxed{-} e_2 \quad \texttt{num}} \; PLUS$$

$$\frac{e_1 : \texttt{num} \quad e_2 : \texttt{num}}{e1 \boxed{/} e_2 : \texttt{num}} \; DIV$$

$$\frac{e_1 : \texttt{bool} \quad e_2 : \tau \quad e_3 : \tau}{\text{if } e_1\ e_2\ e_3 : \tau} \; IF$$