End Semester Examination – Monsoon 2017

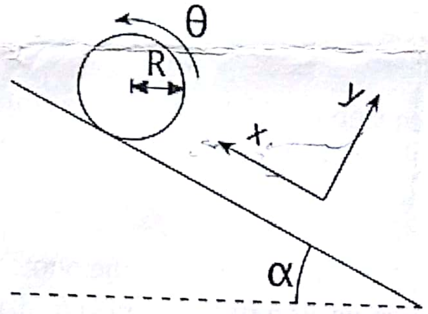
IIIT-Hyderabad

Subject: Science I (ISC201)

Total: 50 marks

Time: 3 hrs

1 A wheel of mass m, radius R, and radius of gyration R_G is released at the top of a hill. Assume that the wheel does not slip as it rolls down the hill (refer Figure). Using Lagrange's equations, derive the equations of motion of this system. (8M)



2 Consider the random walk problem in one dimension and suppose that the probability of a single displacement between s and s+ds is given by

$$w(s)\,ds = \frac{1}{\pi}\frac{b}{s^2+b^2}\,ds$$

Calculate the probability $P(x)\,dx$ that the total displacement after N steps lies between x and $x+dx$. Does $P(x)$ become Gaussian when N becomes large? (4M)

3 a) For a quantum particle of mass m moving on the surface of sphere, express the kinetic energy operator in terms of the spherical polar coordinates.
b) Determine the energy and angular momentum of a quantum particle of mass m travelling on a circular ring. (7M)

4 a) Find the probability that the electron in the ground-state H atom is less than a distance a_0 from the nucleus. The wavefunction of 1s electron is $\psi = \frac{e^{-r/a_o}}{\sqrt{\pi}\,a_o^{3/2}}$.
b) Find the expectation value of $1/r$ for 1s electron. (3M)

5 What are Euler angles? How do you use them to describe the rotational dynamics of a rigid body? (4M)

6 Given the Lagrangian of an isolated system, derive the conservation laws resulting from the (a) homogeneity of time, (b) homogeneity of space, (c) isotropy of space. (6M)

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7. Demonstrate that the uncertainity principle (relating Δx and Δp) is satisfied in the ground- state of a particle in a one-dimensional box. (5M)

8. The dynamics of a quantum particle of mass m moving one-dimensionally in a potential $V(x)$ is governed by the Hamiltonian $H_0 = \frac{p^2}{2m} + V(x)$, where $p = -i\hbar \frac{d}{dx}$ is the momentum operator. Let $E_n^{(0)}$, $n = 1, 2, 3, \ldots$, be the eigenvalues (i.e., energy of the n^{th} state) of H_0. Now consider a new Hamiltonian $H = H_0 + \lambda p/m$, where λ is given parameter. Given m and $E_n^{(0)}$, find the eigenvalues of H. (8M)

$\lambda = \frac{h}{mv}$

9. Using Langevin's equation, calculate the mean square displacement of a solute in a solvent. Discuss the short-time and long-time behavior of the mean square displacement. (3M)

10. Using Bohr theory, find the frequency of the photon emitted by a hydrogen atom due to the transtion of electron from the level $n+1$ to the level n and frequency of revolution of the electron in n^{th} level. Show that at larger values of n, both the frequency of photon and frequency of revolution are approximately same. (2M)

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