

SC1.110 : Quiz : solns.

1. Estimate

- (a) the number of water molecules in your body (Assume that you are nearly all water).

Ans. Average mass of an adult body $\approx 60\text{kg}$. $\implies \frac{60}{\frac{18}{1000}} = 3.3 \times 10^3$ moles $= 3.3 \times 10^3 \times 6.023 \times 10^{23} = 2 \times 10^{27}$ molecules ($\frac{1}{2}$ mark for correct order of magnitude : $(1 - 3) \times 10^{27}$)

- (b) the average rise in sea level due to thermal expansion of sea water for a temperature rise of 1.0°C (volume of the Earth's oceans $= 1.37 \times 10^9 \text{ km}^3$ and their surface area $= 361 \times 10^6 \text{ km}^2$ and the coefficient of thermal expansion of water $= 2.1 \times 10^{-4} \text{ K}^{-1}$).

Ans. $\alpha = \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_p \approx \frac{1}{V} \cdot \frac{\Delta V}{\Delta T} \implies \Delta V \approx \alpha V \Delta T = 2.1 \times 10^{-4} \times 1.37 \times 10^9 = 2.9 \times 10^5 \text{ km}^3$.

$\therefore \Delta h = \frac{\Delta V}{A} = \frac{2.9 \times 10^5}{361 \times 10^6} \text{ km} = 0.8 \text{ m}$ ($\frac{1}{2}$ mark for correct order of magnitude : $0.5 - 1.5 \text{ m}$; $\frac{1}{2}$ mark for the method)

If anyone assumed coefficient of expansion to be linear, then $\alpha = 3 \times 2.1 \times 10^{-4} \text{ K}^{-1}$ and the final result will be $3 \times 0.8 = 2.4 \text{ m}$. In this case, $\frac{1}{2}$ mark will be given for order of magnitude $1.5 - 4.5 \text{ m}$. and $\frac{1}{2}$ mark for the method.

2. Consider two paths for n moles of a perfect gas undergoing a change in pressure and volume, $\{p, V\}$; path

I : $(1, 1) \rightarrow (2, 2)$ on a straight line and path II : $(1, 1) \rightarrow (2, 1) \rightarrow (2, 2)$? What values will the integrals

$\int p dV$ and $\int V dp$ have for the two separate paths? Will the sum of the two integrals depend on the path?

Explain.

Ans. : path I : $\int_{(1,1)}^{(2,2)} p dV = \int_{(1,1)}^{(2,2)} V dp = \int_1^2 p dV = \left. \frac{V^2}{2} \right|_0^1 = \frac{3}{2}$ ($\frac{1}{2}$ mark)

path II : $\int_{(1,1)}^{(2,1)} p dV + \int_{(2,1)}^{(2,2)} p dV = 0 + 2 = 2$ ($\frac{1}{2}$ mark) $\int_{(1,1)}^{(2,1)} V dp + \int_{(2,1)}^{(2,2)} V dp = 1 + 0 = 1$ ($\frac{1}{2}$ mark) ($\frac{1}{2}$ mark)

$\int p dV + \int V dp = \int_{1 \times 1}^{2 \times 2} d(pV) = 3$: pV is a state function (product of two state functions and hence $d(pV)$

is an exact differential \implies as long as the initial and final state are the same, the value of the integral will be path independent ($\frac{1}{2}$ mark).

3. What is the thermodynamic definition of a perfect gas?

Ans. Molecules of a perfect gas have no interactions. \therefore no work is done on expansion. With no energy coming from anywhere

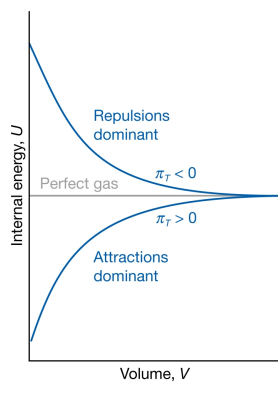
else, $(\frac{\partial U}{\partial V})_T = 0$ (1 mark)

(b) What is meant by thermodynamic internal pressure of a real gas?

Ans. The quantity $\pi_T = (\frac{\partial U}{\partial V})_T$ is called thermodynamic internal pressure of a real gas. It is generally non-zero and represents the interactions between molecules. ($\frac{1}{2}$ mark)

The following is not required, but given here for the sake of completion :

If $\Delta U > 0$ as $\Delta V > 0$ isothermally when there are attractive forces between the particles then a plot of U against V slopes upwards and $\pi_T > 0$

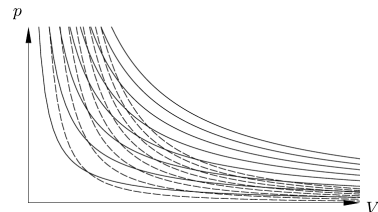


4. A gas obeying the equation of state $p(V - nb) = nRT$ is subjected to a Joule–Thomson expansion. Will the temperature increase, decrease, or remain the same? Why?.

Ans. the temperature will increase (1 mark)

The equation of state implies that only repulsive interactions are taken into account. For such a gas, energy will be released on expansion and hence the temperature will increase. (1 mark)

5. In the picture shown, some isotherms and adiabats are plotted? Which ones (dashed or solid) are isotherms and which are adiabats? Explain. (b) what must be zero for an adiabatic process?



Ans. dashed lines are adiabats and solid lines are isotherms. (1 mark)

The isotherms follow $pV = \text{const} \implies \left| \frac{dp}{dV} \right| = \frac{p}{V}$ and the adiabats follow $pV^\gamma = \text{const} \implies \left| \frac{dp}{dV} \right| = \gamma \frac{p}{V}$

$\because \gamma > 1$, the adiabats have higher slopes; \therefore dashed lines are the adiabats. (1 mark)

The heat flow into or out of a system from or to the surroundings is zero for an adiabatic process. ($\frac{1}{2}$ mark)

6. Consider a perfect gas, inside a cylinder fitted with a piston, undergoing a change of state from initial state be T, V_i to the final state T, V_f . There are two possible paths, Path 1: free expansion against zero external pressure; Path 2 : reversible, isothermal expansion. Calculate w , q , and ΔU for each process.

Ans. $\because \Delta U = 0$ for both paths and $\Delta U = q + w$, \therefore in each case $q = -w$ ($\frac{1}{2}$ mark).

path 1 : for free expansion, $w = 0$ and $\therefore q = 0$ too. ($\frac{1}{2}$ mark)

Path 2 : $w = - \int p dV = -nRT \ln \frac{V_f}{V_i} \implies q = nRT \ln \frac{V_f}{V_i}$ ($\frac{1}{2}$ mark).

7. A block of lead of heat capacity 1 kJ K^{-1} is cooled from 200 K to 100 K in two ways. (a) It is plunged into a large liquid bath at 100 K. (b) The block is first cooled to 150 K in one liquid bath and then to 100 K in another bath. Calculate the total entropy changes (system+surroundings) in the two cases. What will be the total entropy change in the limit of an infinite number of intermediate baths?

Ans. The energy lost by the block of lead flows out as heat to the surroundings at the lower temperature.

To find the entropy change for the system (lead block), we must construct a reversible path and evaluate

$$\Delta S_{\text{sys}} = \int_{T_i}^{T_f} C_V \cdot \frac{dT}{T} = C_V \ln \frac{T_f}{T_i}; \quad T_f < T_i \implies \Delta S_{\text{sys}} < 0$$

The surroundings absorb heat (flowing out of the system) at a constant temperature, T_f

$$\therefore \Delta S_{\text{surr}} = \frac{1}{T_f} \int_{T_i}^{T_f} C_V \cdot dT$$

$$(a) \Delta S_{\text{sys}} = - \int_{100}^{200} C_V \cdot \frac{dT}{T} = -C_V \ln 2; \quad \Delta S_{\text{surr}} = \frac{1}{100} \int_{100}^{200} C_V \cdot dT = C_V;$$

$$\Delta S_{\text{tot}} = C_V(1 - \ln 2) = 0.307 \text{ JK}^{-1} (\frac{1}{2} \text{ mark})$$

$$(b) \Delta S_{\text{tot}} = \Delta S_{\text{I}} + \Delta S_{\text{II}} \text{ and } \Delta S_i = \Delta S_{i,\text{sys}} + \Delta S_{i,\text{surr}} \quad \forall i = \text{I, II}$$

$$\Delta S_{\text{I}} = - \int_{150}^{200} C_V \cdot \frac{dT}{T} + \frac{1}{150} \int_{150}^{200} C_V \cdot dT; \quad \Delta S_{\text{II}} = - \int_{100}^{150} C_V \cdot \frac{dT}{T} + \frac{1}{100} \int_{100}^{150} C_V \cdot dT$$

$$\therefore \Delta S_{\text{tot}} = -C_V \ln 2 + \frac{C_V}{3} + \frac{C_V}{2} = 0.14 \text{ JK}^{-1} (\frac{1}{2} \text{ mark})$$

With infinite number of intermediate baths, the process becomes reversible, so $\Delta S_{\text{tot}} = 0 (\frac{1}{2} \text{ mark})$.

$$\text{Not needed : Algebraically, } \Delta S_{\text{tot}} = - \int_{100}^{200} C_V \cdot \frac{dT}{T} + \left(\int_{200-\delta}^{200} C_V \cdot \frac{dT}{200-\delta} + \int_{200-2\delta}^{200-\delta} C_V \cdot \frac{dT}{200-2\delta} + \dots \right) = 0$$

8. What is the maximum possible efficiency of an engine operating between two thermal reservoirs, one at 100° C and the other at 0° C ?

$$\text{Ans. } \eta = 1 - \frac{273}{373} = 0.27 \text{ (1 mark)}$$

9. In a comic song by Flanders and Swann about the laws of thermodynamics, they summarize the first law by the statement:

Heat is work and work is heat - Is that a good summary?

Ans. No. ($\frac{1}{2}$ mark) Work can be converted to heat. Heat can be partially converted into work. They are not the same thing. Heat is related to disordered motion of molecules in matter. Work involves ordered motion of molecules in matter. ($\frac{1}{2}$ mark)