MA2.101: Linear Algebra (Spring 2022)

Quiz 1

April 18, 2022

Attention

1. Quiz is total of 100 points (75 points for in-class and 25 points for take-home).
2. Answer at least 3 questions in class ($75 = 25 \times 3$ points), where *-marked questions are mandatory. That is, the last two questions are to be answered in-class. Maximum time for in-class quiz is 45 minutes.
3. Remaining unanswered questions are going to be take-home (25 points), to be submitted before Monday (i.e., by 24 April 2022).

Question 1

Show that the set of all real 2×2 matrices of the form

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$$

where $a, b \in \mathbb{R}$, with the usual matrix operations, form a field.

Question 2

Find all solutions to the following system of equations:

$$\begin{array}{llrr} x_1 + 2x_2 + x_3 & +x_4 = & 7 & (1) \\ 2x_1 + 4x_2 + 4x_3 & -2x_4 = & 24 & (2) \\ 3x_1 + 6x_2 & +9x_4 = & 6. & (3) \end{array}$$

Notice that the system of linear equations is of the form $AX = Y$, where A and Y are known and one needs to solve for X. Use elementary row operations to derive row-reduced echelon form for A in order to solve for X.

Question 3

Let V be a vector space defined over the field of real numbers $\mathbb{R}$. Consider that $x, y, z \in V$. Show that the set $\{x, y, z\}$ of vectors is linearly independent if and only if the set $\{x + y, y + z, z + x\}$ of vectors is linearly independent.

Question 4*

Consider the vector space $\mathbb{F}^4$ defined over the field $\mathbb{F}$. Determine for each of the following subsets of $\mathbb{F}^4$ if they are subspace of $\mathbb{F}^4$ or not. Answers should include reasonable justification.

1. The set with its only element being the zero vector $(0, 0, 0, 0)$.
2. The set of vectors $(x_1, x_2, x_3, x_4) \in \mathbb{F}^4$ such that $x_1 + 3x_2 + 4x_3 + 5x_4 = 0$.
3. The set of vectors $(x_1, x_2, x_3, 0) \in \mathbb{F}^4$ such that $x_1 + 3x_2 + 4x_3 = 1$.
4. The set of vectors $(x_1, x_2, x_3, x_4) \in \mathbb{F}^4$ such that $x_1 = x_4$.

Question 5*

Derive the values of $a \in \mathbb{R}$ for which the following set of vectors spans the vector space $\mathbb{R}^3$ defined over $\mathbb{R}$:

$$\{(1, 0, a), (1, 2, -3), (a, 1, 0)\}. \tag{4}$$