Solutions to Mid Exam-1 Paper

(Q1) A linear time invariant system yields output $y(t)$ for an input $x(t)$ as shown in Fig. Q1a). What will be the system's output for an input $x_1(t)$ shown in Fig. Q1b)?

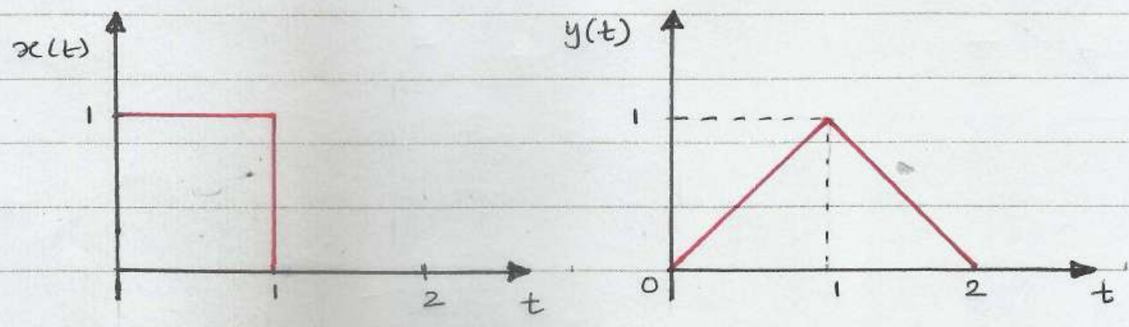


Fig. Q1a

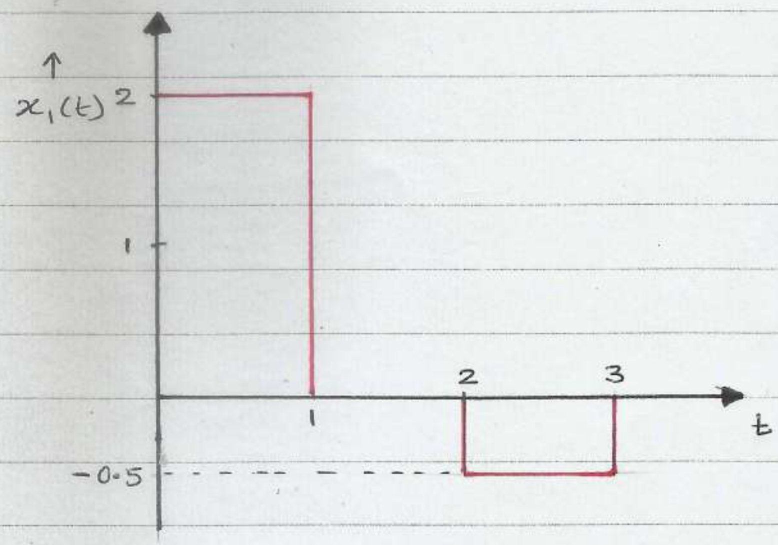
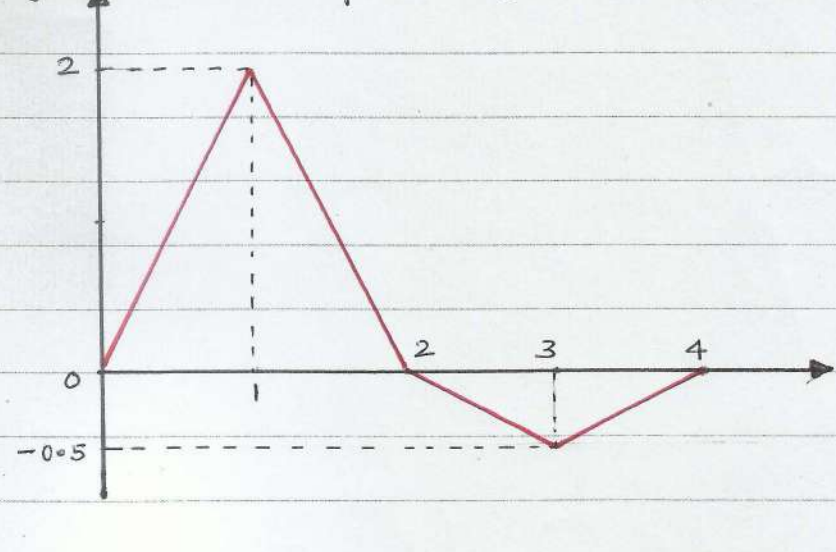


Fig. Q1b

Sol. The output $y(t)$ due to input $x_1(t)$ is shown below.



Q2 Derive the expression for v_o interms of v_1 and v_2 for the circuit shown in Fig. Q2. Assume that the op-amps are ideal and operate in linear region.

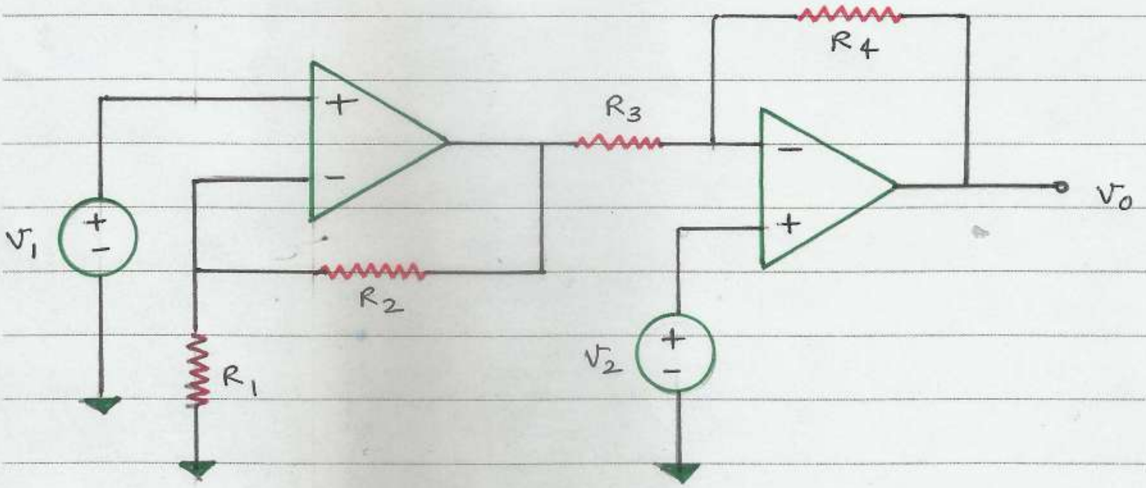


Fig. Q2

How can this amplifier be used as a subtractor?

Sol.

Output voltage of 1st op-amp is

$$v' = \left(1 + \frac{R_2}{R_1}\right) v_1$$

$$v_o = \left(-\frac{R_4}{R_3}\right) v' + \left(1 + \frac{R_4}{R_3}\right) v_2$$

$$= -\left(\frac{R_4}{R_3}\right)\left(1 + \frac{R_2}{R_1}\right) v_1 + \left(1 + \frac{R_4}{R_3}\right) v_2$$

$$\Rightarrow v_o = \left(1 + \frac{R_4}{R_3}\right) v_2 - \left(\frac{R_4}{R_3} + \frac{R_4}{R_3}\frac{R_2}{R_1}\right) v_1$$

If $R_1 = R_4$ and $R_2 = R_3$ then

$$v_o = \left(1 + \frac{R_4}{R_3}\right) v_2 - \left(1 + \frac{R_4}{R_3}\right) v_1$$

$$\Rightarrow v_o = \left(1 + \frac{R_4}{R_3}\right)(v_2 - v_1)$$

(Q3) Specify the values of R_1 and R_2 in Fig. Q3 that are required to cause v_3 to be related to v_1 and v_2 by the equation $v_3 = 6v_1 - 0.8v_2$.
Assume that all the op-amps are ideal and operate in linear region.

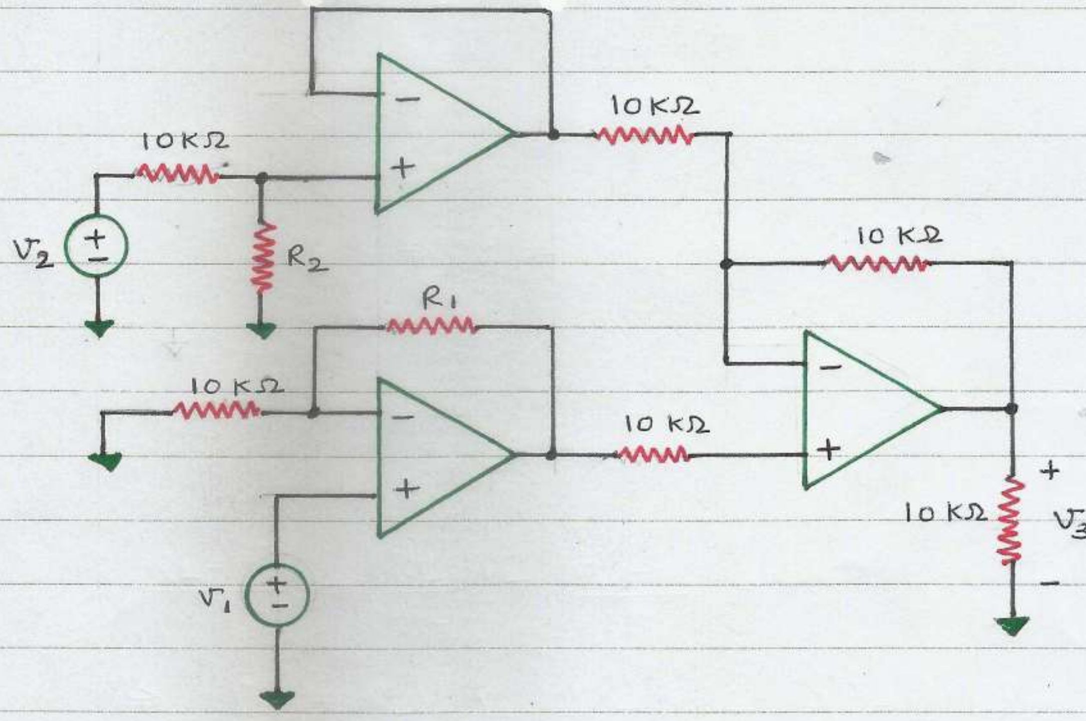
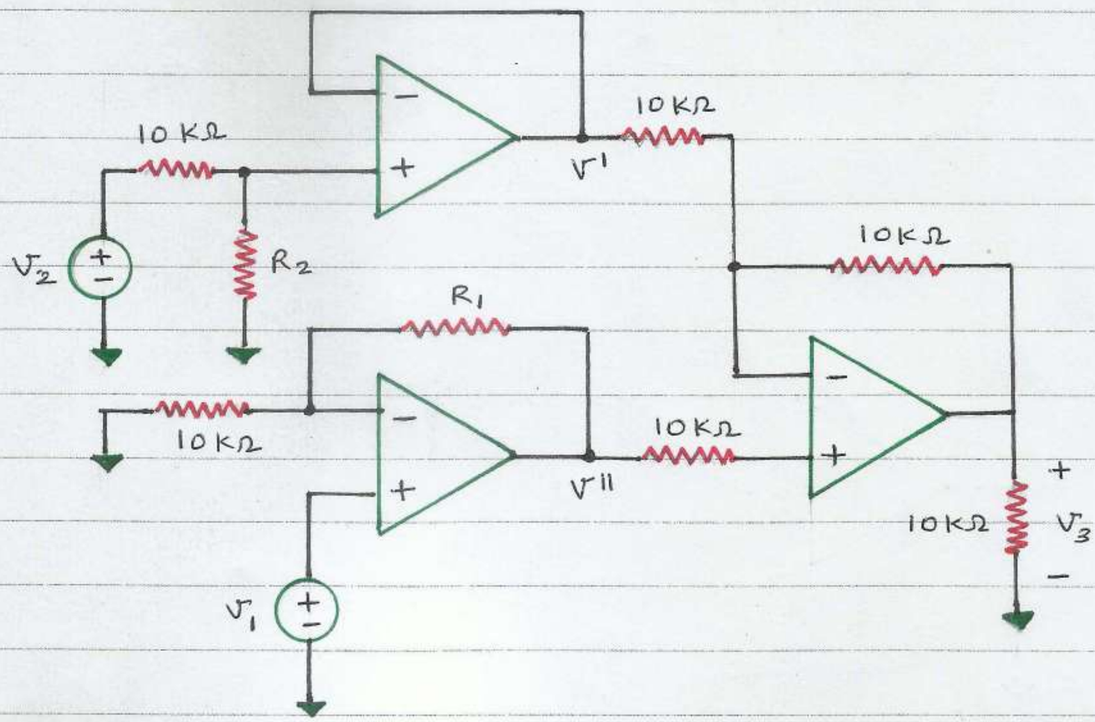


Fig. Q3

Sol.



Let R_1 and R_2 be in $k\Omega$

$$v' = \left(\frac{R_2}{R_2+10}\right) \times v_2$$

$$v'' = \left(1 + \frac{R_1}{10}\right) \times v_1$$

$$v_3 = \left(-\frac{10}{10}\right) \times v' + \left(1 + \frac{10}{10}\right) \times v''$$

$$= -v' + 2v''$$

$$= -\left(\frac{R_2}{R_2+10}\right) \times v_2 + 2\left(1 + \frac{R_1}{10}\right) v_1$$

$$= 2\left(1 + \frac{R_1}{10}\right) v_1 - \left(\frac{R_2}{R_2+10}\right) v_2$$

$2 \times \left(1 + \frac{R_1}{10}\right) = 6$ and $\frac{-R_2}{R_2+10} = -0.8$

$\Rightarrow 1 + \frac{R_1}{10} = 3$

$\Rightarrow \frac{R_1}{10} = 2$

$\Rightarrow R_1 = 20\ k\Omega$

$\Rightarrow \frac{R_2+10}{R_2} = \frac{5}{4}$

$\Rightarrow 1 + \frac{10}{R_2} = \frac{5}{4}$

$\Rightarrow \frac{10}{R_2} = \frac{1}{4}$

$\Rightarrow R_2 = 40\ k\Omega$

$\therefore R_1 = 20\ k\Omega$ and $R_2 = 40\ k\Omega$ for v_3 to be equal to $6v_1 - 0.8v_2$

Q4 For the amplifier circuit shown in Fig. Q4, find the expression for i_L in terms of v_S, if $R_2 = R_4$ and $R_1 = R_3$. What type of amplifier is it ?
Assume that the op-amp is ideal and operate in linear region.

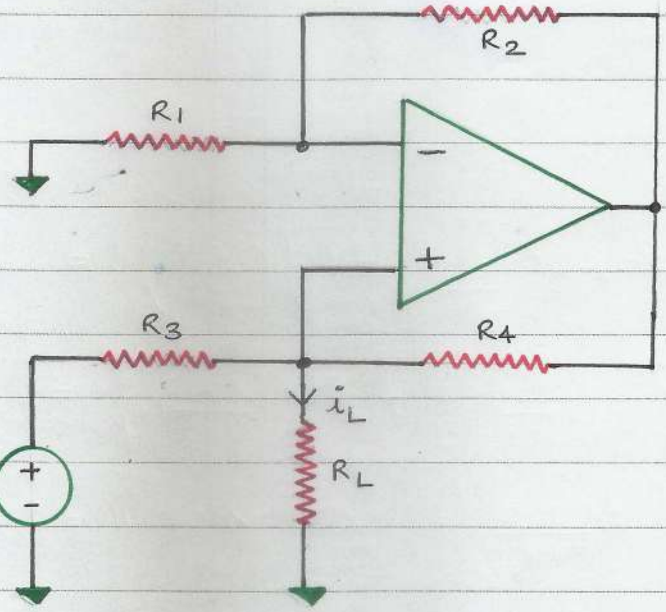
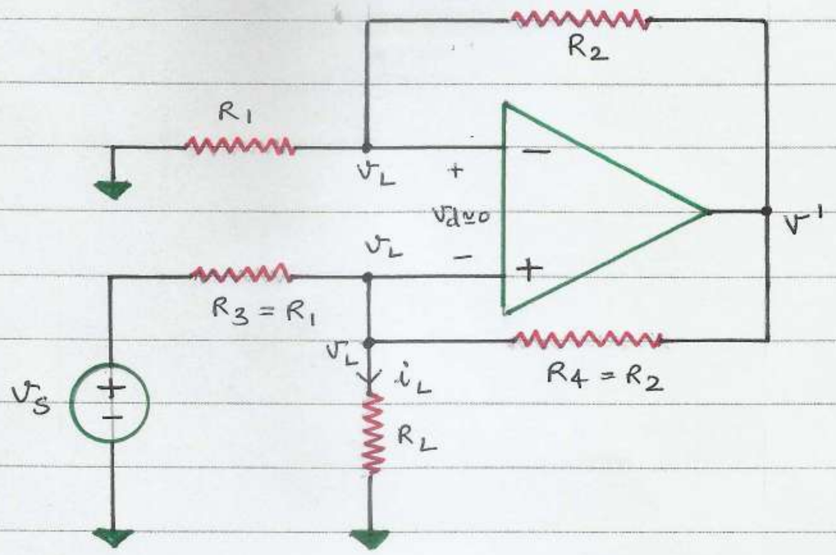


Fig. Q4

Sol.



Applying KCL at non-inverting input of op-amp

$$\frac{v_L - v_s}{R_1} + \frac{v_L}{R_L} + \frac{v_L - v'}{R_2} = 0 \quad \cdots (A)$$

Applying KCL at inverting input of op-amp

$$\frac{v_L}{R_1} + \frac{v_L - v'}{R_2} = 0$$

$$\Rightarrow v_L\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{v'}{R_2}$$

$$\Rightarrow v' = R_2 \times \frac{v_L(R_1+R_2)}{R_1 R_2}$$

$$\Rightarrow v' = \frac{(R_1+R_2)v_L}{R_1} \quad \ldots \quad (B)$$

Substituting the expression for v' in (B) in eq. (A)

Eq (A)

$$\frac{v_L - v_S}{R_1} + \frac{v_L}{R_L} + \frac{v_L - v'}{R_2} = 0$$

$$\Rightarrow v_L\left(\frac{1}{R_1} + \frac{1}{R_L} + \frac{1}{R_2}\right) - \frac{v'}{R_2} = \frac{v_S}{R_1}$$

$$\Rightarrow v_L\left(\frac{1}{R_1} + \frac{1}{R_L} + \frac{1}{R_2}\right) - \frac{(R_1+R_2)v_L}{R_1 R_2} = \frac{v_S}{R_1}$$

$$\Rightarrow v_L\left(\frac{1}{R_1} + \frac{1}{R_L} + \frac{1}{R_2}\right) - \left(\frac{1}{R_1} + \frac{1}{R_2}\right)v_L = \frac{v_S}{R_1}$$

$$\Rightarrow \frac{v_L}{R_L} = \frac{v_S}{R_1}$$

$$\Rightarrow \frac{i_L R_L}{R_L} = \frac{v_S}{R_1}$$

$$\Rightarrow \boxed{i_L = \frac{v_S}{R_1}}$$

∴ Load current depends on source (input) voltage irrespective of load resistance (R_L). It is a transconductance amplifier.

(Q5) Find the output voltage v_o of the amplifier circuit shown in Fig. Q5. Assume that all op-amps are ideal and none of the op-amps saturate.

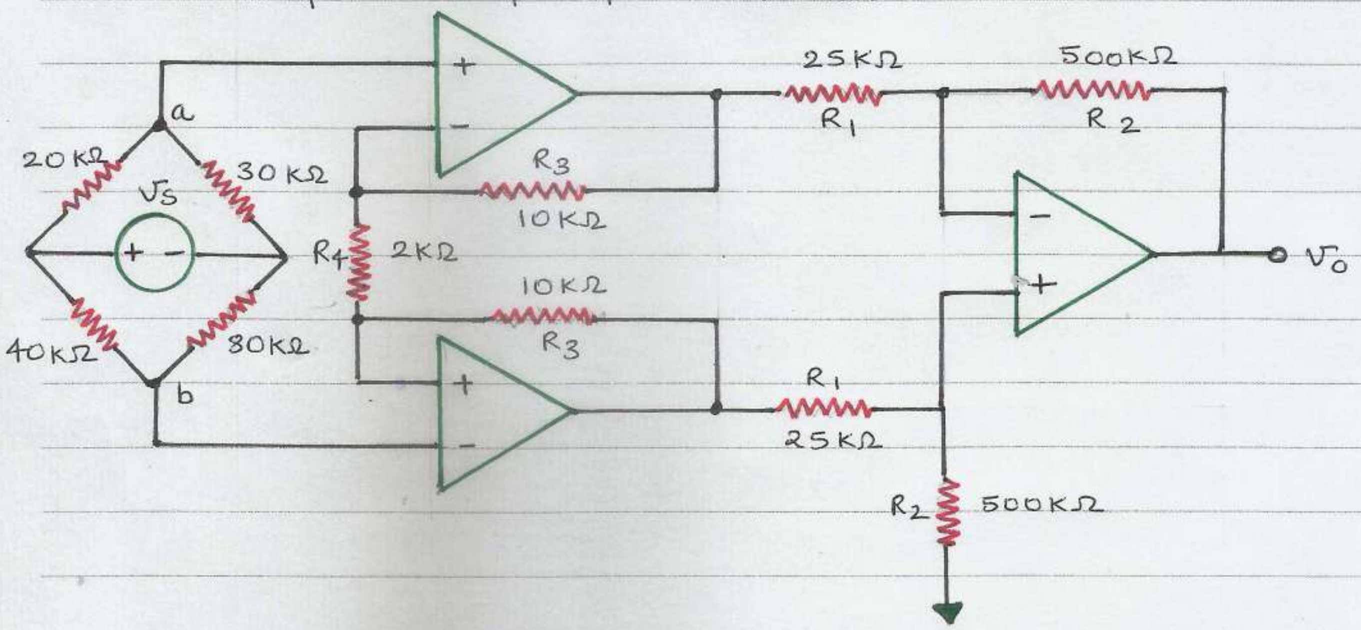
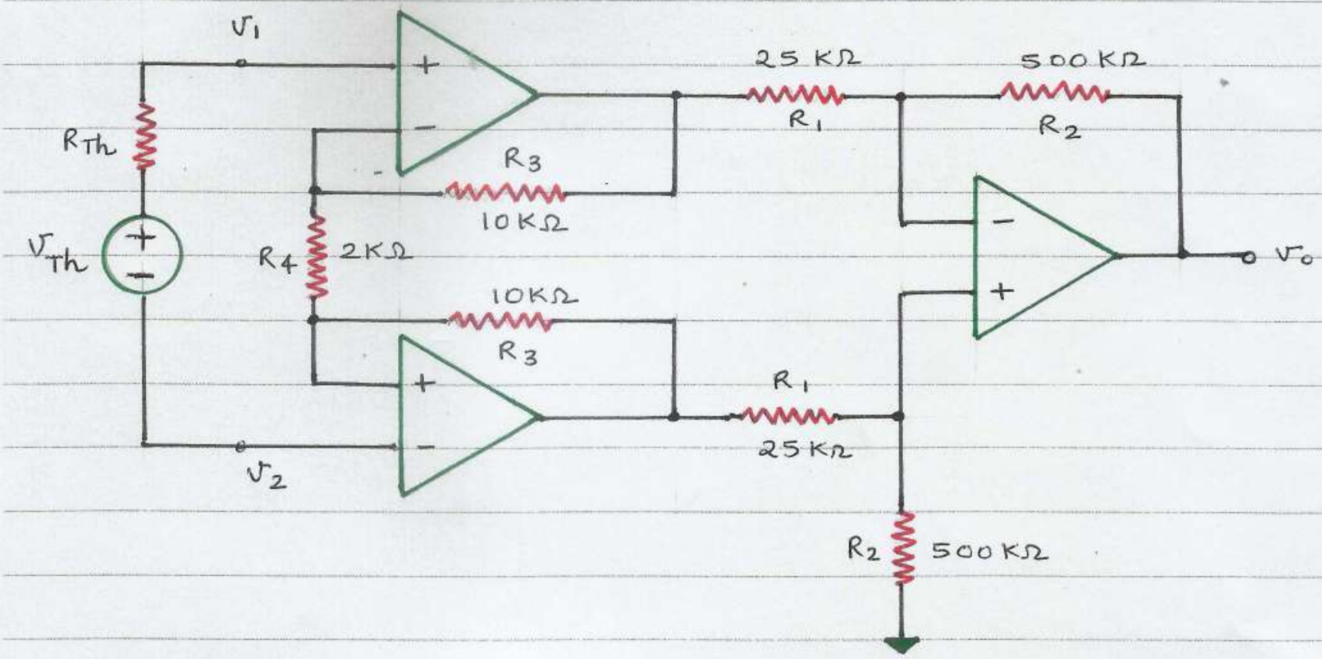


Fig. Q5

Sol. Thevenizing bridge circuit across a-b.



$$R_{Th} = 20\,K\Omega \parallel 30\,K\Omega + 40\,K\Omega \parallel 80\,K\Omega$$

$$= \frac{20 \times 30}{50} + \frac{40 \times 80}{120}\ K\Omega$$

$$= \frac{60}{5} + \frac{80}{3}\ K\Omega = \frac{116}{3}\ K\Omega$$

(B)

$$v_{th} = v_1 - v_2 = \frac{30}{30+20} v_s - \frac{80}{80+40} v_s$$

$$= \frac{30}{50} v_s - \frac{80}{120} v_s$$

$$= \frac{3}{5} v_s - \frac{8}{12} v_s = \frac{3}{5} v_s - \frac{2}{3} v_s$$

$$= -\frac{1}{15} v_s = v_1 - v_2$$

$$v_o = \left(\frac{R_2}{R_1}\right)\left(1 + \frac{2R_3}{R_4}\right)(v_2 - v_1)$$

$$= \left(\frac{500k}{25k}\right)\left(1 + \frac{2 \times 10k}{2k}\right) \times \frac{v_s}{15} \qquad \because v_2 - v_1 = \frac{v_s}{15}$$

$$= 20 \times 11 \times \frac{v_s}{15} = \frac{220}{15} v_s = \frac{44}{3} v_s = 14.667\, v_s$$