End Semester

Science I (Classical and Quantum Mechanics)

Total Marks:75 , Time: 3 hrs

Q1. The time dependent Lagrangian of a particle moving in one dimension is given by

$$L=e^{\lambda t}\left(\frac{1}{2}m\dot{x}^2+\frac{1}{2}m\omega^2x^2\right)$$

(1) Write down the Lagrange equation of motion

(2) Obtain the expression for generalized momentum and the Hamiltonian H(p,x).

(3) Write the Hamilton's equation of motion. 4+4+4

Q2. The matrix representations of two quantum operators are given by

$a1=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$ and $a2=\begin{pmatrix}1 & i\\ -i & 1\end{pmatrix}$

(1) Calculate the commutation relation [a1,a2].

(2) Evaluate eigenvalues and normalized eigenstates of a1 and a2

(3) What are the measured values of a1? Calculate their probabilities in one of the eigenstates of a2.

(4) Calculate the uncertainty of measuring a1 in the above case, i.e. Δa1.

4+4+4+3

Q3. The normalized eigenstate of the Hamiltonian of an electron in harmonic oscillator at ground state is given by

$$\phi_0(x)=\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\exp\left(\frac{-m\omega}{2\hbar}x^2\right)$$

(1) Calculate $\langle x\rangle$ and $\langle x^2\rangle$ for position operator

(2) Calculate the uncertainty in measuring X, i.e. Δx and $\langle p\rangle$ for momentum operator

(4) Calculate the probability of momentum value p 3×4

Q4. An electron is moving inside a infinite well potential

$$v(x)=\infty \; at \; x<0, x>a$$
$$v(x)=0 \; at \; x>0, x<a$$

The wave-function for the electron is given by $|\psi\rangle=\sqrt{\frac{3}{5}}|\phi_1\rangle+\sqrt{\frac{2}{5}}|\phi_2\rangle$

$|\phi_1\rangle$ and $|\phi_2\rangle$ are two lowest energy eigenstates.

(1) What are the energy values their probabilities you will get if energy is measured.

(2) Calculate the average energy $\langle E \rangle$ and energy uncertainty ΔE.

(3) What is the probability of the particle to remain within 0 to a/2.

(4) How will the probability change at time t, Calculate the minimum probability value achieved at some time t.

3+4+4+4

Q5. An electron of charge e is moving under a central potential $\frac{-e^2}{r}$

(1) Write down the Lagrangian and Hamiltonian of the electron if angular momentum is L in polar coordinate.

(2) Write the classical Hamilton's equation of motion

(3) Calculate the maximum radius r_{max} in the classical bound state, if total energy is E<0 and L=0

(4) If the electron is at the ground state energy eigenstate, evaluate the probability that the electron will be found within r_{max} .

4×4

The ground state eigenfunction, $\phi_0(r) = \frac{1}{\sqrt{\pi} a_0^{3/2}} \exp\left(\frac{-r}{a_0}\right)$ and energy E= $\frac{-e^2}{2a_0}$

Q6. In a region of space, a particle with mass m and with zero energy has a energy eigenfunction where A and L are constants.

$$\psi(x) = a \exp\left(\frac{-x^2}{L^2}\right)$$

(1) Determine the potential energy U(x) of the particle using Schrodinger equation

(2) Determine the ground state energy eigenvalue.

3+2

Useful Integrals

$$\int_0^{\frac{a}{2}} \sin^2\left(\frac{\pi x}{a}\right) dx = \frac{a}{4} \qquad \int_0^{\frac{a}{2}} \sin^2\left(\frac{2\pi x}{a}\right) dx = \frac{a}{4} \qquad \int_0^{\frac{a}{2}} \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{2\pi x}{a}\right) dx = \frac{a}{4}$$

$$\int_0^{\frac{a}{2}} \cos^2\left(\frac{\pi x}{a}\right) dx = \frac{a}{4} \qquad \int_0^{\frac{a}{2}} \cos^2\left(\frac{2\pi x}{a}\right) dx = \frac{a}{4} \qquad \int_0^{\frac{a}{2}} \cos\left(\frac{\pi x}{a}\right) \cos\left(\frac{2\pi x}{a}\right) dx = \frac{a}{3\pi}$$

$$\int x^2 \exp(-x/a)\, dx = -a \exp(-x/a)(2a^2 + 2ax + x^2)$$