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Kripa Anne
2017115?

IIIT Hyderabad

DIFFERENTIAL EQUATIONS

Spring 2020 – Mathematics Elective IMA303

Instructor: B. S. Lakshmi **Date:** FEBRUARY 26, 2020

Time: 1H 30 Min (4 00 PM- 5 30 PM) **Mid-Semester Examination** Total Marks: 50

Instructions:

- Answer all questions. (√)
- Doubts and Questions will not be entertained during the Exam (×)
- Class notes or books are not permitted. (×)
- Calculators are allowed. (√)

Q 1. (a) Consider the initial value problem

$$y' = \frac{t\,y(4-y)}{3} \qquad y(0) = y_0$$

i. Determine how the behavior of the solution as t increases depends on the initial value y_0.

ii. Suppose that $y_0 = 0.5$. Find the time T at which the solution first reaches the value 3.98.

(b) In the following differential equation verify whether the equation is exact. If not find an integrating factor (if possible) and solve it.

$$y + (2xy - e^{-2y})y' = 0$$

[6+7=13]

Q 2. (a) Suppose that the population $P(t)$ of a country satisfies the differential equation $\frac{dP}{dt} = kP(200 - P)$ with k a constant. Its population in 2000 was 100 million and was then growing at the rate of one million per year. Predict this country's population for the year 2060.

(b) Two large containers A and B of the same size are filled with different fluids. The fluids in containers A and B are maintained at 0° C and 100°C, respectively. A small metal bar, whose initial temperature is 100°C, is lowered into container A. After 1 minute the temperature of the bar is 90°C. After 2 minutes the bar is removed and instantly transferred to the other container. After 1 minute in container B the temperature of the bar rises 10°C. How long, measured from the start of the entire process, will it take the bar to reach 99.9°C?

[6+7=13]

Q 3. (a) Assume that y_1 and y_2 are a fundamental set of solutions of $y'' + p(t)y' + q(t)y = 0$ and let $y_3 = a_1y_1 + a_2y_2$ and $y_4 = b_1y_1 + b_2y_2$, where a_1, a_2, b_1, and b_2 are any constants. Show that the Wronskian of (y_3, y_4)

$$W[y_3, y_4] = (a_1b_2 - a_2b_1)W[y_1, y_2].$$

Are y_3 and y_4 also a fundamental set of solutions? Why or why not?

(b) Suppose that r_1 and r_2 are roots of $ar^2 + br + c = 0$ and that $r_1 \neq r_2$; then write down the solutions the differential equation $ay'' + by' + cy = 0$. Show that

$$\phi(t; r_1, r_2) = \frac{e^{r_2t} - e^{r_1t}}{r_2 - r_1}$$

is also a solution of the equation for $r_1 \neq r_2$. Then think of r_1 as fixed, obtain the second solution in the case of equal roots. (using limits)

[6+6=12]

Q 4. Consider the differential equation

$$ay'' + by' + cy = g(t),$$

where a, b, and c are positive

If $g(t) = d$, a constant, to what value does every solution of the equation approach as $t \to \infty$? What happens if $c = 0$? What if $b = 0$ also?

[12]