International Institute of Information Technology, Hyderabad

(Deemed to be University)

Automata Theory - Monsoon 2022

Final Exam

Maximum Time : 90 Minutes Total Marks : 30

Roll No. _______________ Programme __________ Date __________

Room No. __________ Seat No. __________ Invigilator Sign. _______________

Checked By

Subjective Questions

Question:	1	2	3	4	5	6	7	Total
Points:	3	3	6	5	3	4	6	30
Score:								

General Instructions to the students

1. Place your Permanent / Temporary Student ID card on the desk during the examination for verification by the Invigilator.
2. **Do not answer with pencil. Mark the choice clearly.**
3. **No questions will be answered during the exam. Make necessary and reasonable assumptions.**
4. **No extra sheets will be provided.**
5. Reading material such as books (unless open book exam) are not allowed inside the examination hall.
6. Borrowing writing material or calculators from other students in the examination hall is prohibited.
7. **If any student is found indulging in malpractice or copying in the examination hall, the student will be given 'F' grade for the course and may be debarred from writing other examinations**.

Standard Notation Followed in the Questions

1. FSM stands for finite state machine.
2. DFA stands for deterministic finite automata.
3. NFA stands for non-deterministic finite automata.
4. PDA stands for Push Down Automata.
5. CFG stands for context-free grammar.
6. a^* is the Kleene Star operation.
7. Φ denotes the empty language.
8. ϵ denotes the empty string.

Best of Luck

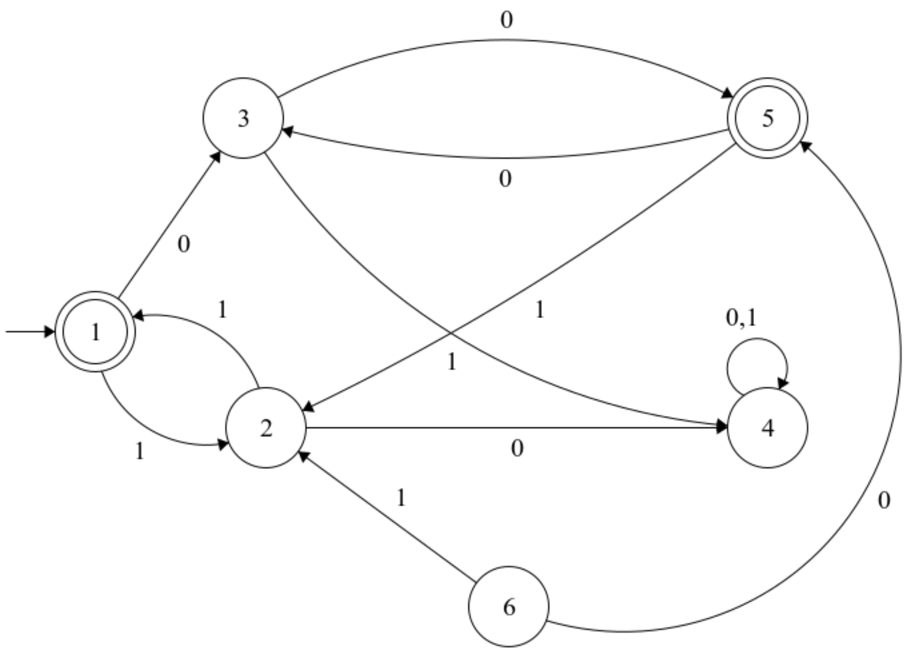
Subjective Questions

Write detailed answers. Adequately explain your assumptions and thought process.

1. [3 points] For a symbol a, prove that the following language is not a Regular Language

$$L = \{a^{2^n} | n \text{ is an integer such that } n \geq 0\}$$

2. [3 points] Find the Unreachable and Dead States in the DFA given below ($\Sigma = \{0, 1\}$) and minimize it. Also find the regular expression of the language accepted by the DFA.



3. [6 points] Let R be a regular language, C be a context free language, RC_1, RC_2 be recursive languages, and RE_1, RE_2 be recursively enumerable languages. State the class the following languages belong to with reasons.

   1. $R \cap C$.
   2. $RE_1 \cup RE_2$.
   3. $RC_1 \cup RE_2$.
   4. $RC_1 \cap RE_2$.

4. [5 points] The language L consisting of all strings having an equal number of 0's and 1's is context-free. State whether the following languages are regular or context-free. Draw the corresponding automaton (DFA/PDA) to support your answer.

   1. $L_1 = \{w | \; |\#0's - \#1's| \leq 1\}$
   2. $L_2 = \{w | \; |\#0's - \#1's| \leq 1, \forall \text{ prefixes of } w\}$

   Note: The string 00001111 will belong to L_1 but not L_2 since 00001, a prefix of 00001111, does not satisfy $|\#0's - \#1's| \leq 1$.

5. [3 points] For a symbol a, define $a^+ = \{a, aa, ...\}$. Is the language $L = \{wcw^R | w, c \in \{a, b\}^+\}$ regular? If your answer is yes, write the equivalent regular expression and if no, prove that L is not regular using pumping lemma.

6. [4 points] Consider a modified push-down automaton where the stack is replaced with a **queue**. A queue is a tape allowing symbols to be written only on the left-hand end and read only at the right-hand end. Each write operation (call it "push") adds a symbol to the left-hand end of the queue and each read operation (call it "pull") reads and removes a symbol at the right-hand end. The input followed by exactly one blank symbol is placed on a separate read-only input tape, and the head on the input tape can move only from **left to right**. We say that the queue automaton accepts its input by entering a special accept state at any time.

   Show that a language can be recognized by a **deterministic** queue automaton iff the language is Turing-recognizable.

7. [6 points] Consider the following languages

$$\text{HALT} = \{\langle M, w\rangle | M(w) \text{ halts}\} \text{ and } \mathrm{A}_{TM} = \{\langle M, w\rangle | M(w) \text{ accepts}\}.$$

In class, we directly proved that A_{TM} is undecidable and obtained that $A_{\text{TM}} \preceq \text{HALT}$ in order to show that HALT is undecidable. It is possible to prove things in the other direction to establish the undecidability of A_{TM}. Namely, to first provide direct proof of the undecidability of HALT and then reduce it to A_{TM}. This is what you are asked to do here.

(a) Prove that HALT is undecidable ***directly*** without reducing it to any known undecidable problem.

(b) Using the undecidability of HALT above, prove that $\text{HALT} \preceq A_{\text{TM}}$.

Rough Work