Solutions to Quiz-2 (Group-B) IEC102

(Q1) The switch drawn in Fig. Q1 has been open a ponderously long time (i.e., the circuit is in steady state before the switch is closed).

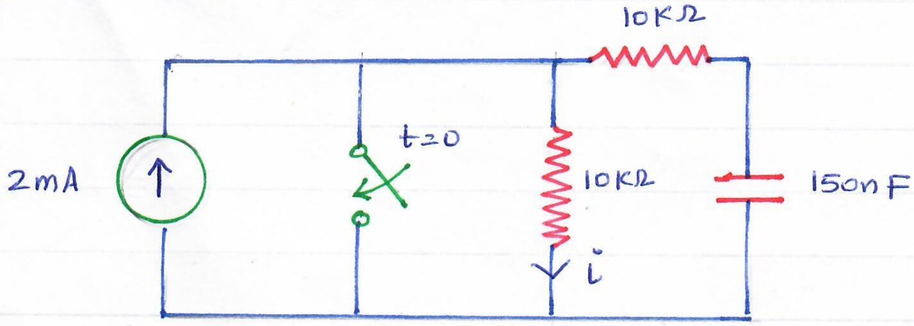


Fig. Q1

a) Determine the value of current labelled 'i' prior to the switch being closed,

b) obtain the value of 'i' just after the switch is closed.

c) Find the expression for $V_c(t)$ for time $t > 0$

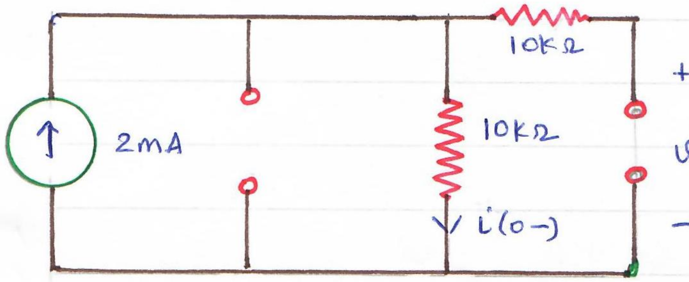
PTO

c) Find the expression for $v_c(t)$ for any time $t>0$.

Sol.

Circuit at $t=0-$ (just before the switch is closed).

The capacitor acts as open circuit the circuit is in steady state at $t=0-$ (given).



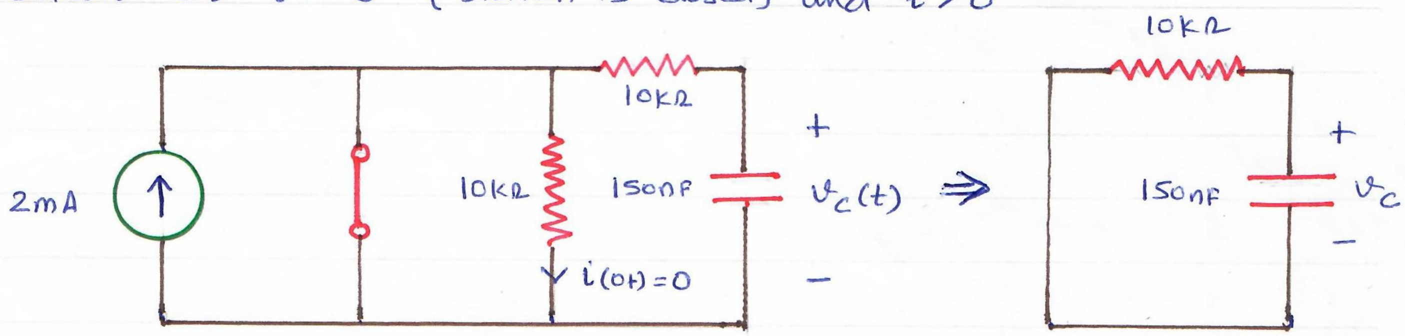
a) $i_L(0-) = 2\,mA$

$$v_c(0-) = 2\times10^{-3}\times10\times10^{3}$$

$$= 20\,V$$

$$v_c(0-) = v_c(0) = v_c(0+)$$

Circuit at $t=0$ (switch is closed) and $t>0$



b)

$i(0+) = 0$ as there is a short circuit in parallel with this resistor as the switch is closed.

c) $$v_c(t) = v_c(0)\, e^{-t/\tau}$$

where $v_c(0) = 20\,V$

$$\tau = R_{eq}C = 10K\times150\times10^{-9} = 10\times10^{3}\times150\times10^{-9}$$

$$= 15\times10^{-4}$$

$$= 1.5\,ms$$

$$\therefore v_c(t) = 20\, e^{\frac{-t}{1.5\times10^{-3}}} = 20\, e^{-\frac{10000t}{15}}$$

$$\boxed{v_c(t) = 20\, e^{-\frac{2000t}{3}}\ V}$$ for $t\geq0$

(Q2) For the circuit shown in Fig. Q2, find $v_c(t)$ for all $t > 0$. Given that the capacitor is initially uncharged i.e., $v_c(0) = 0$.

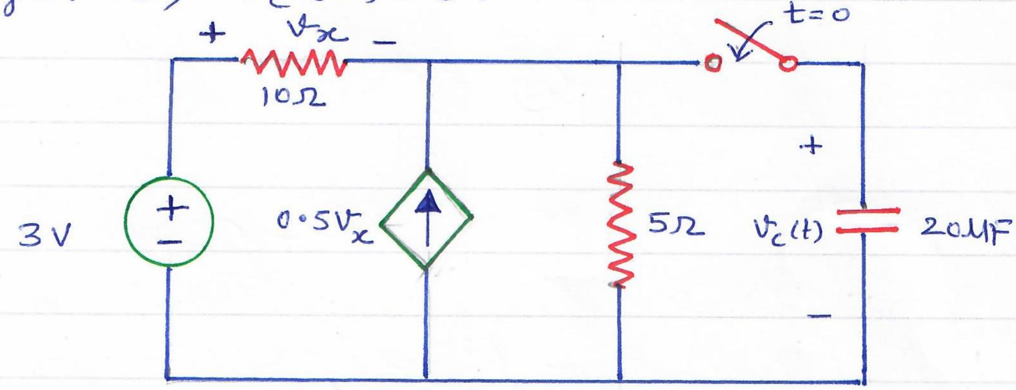
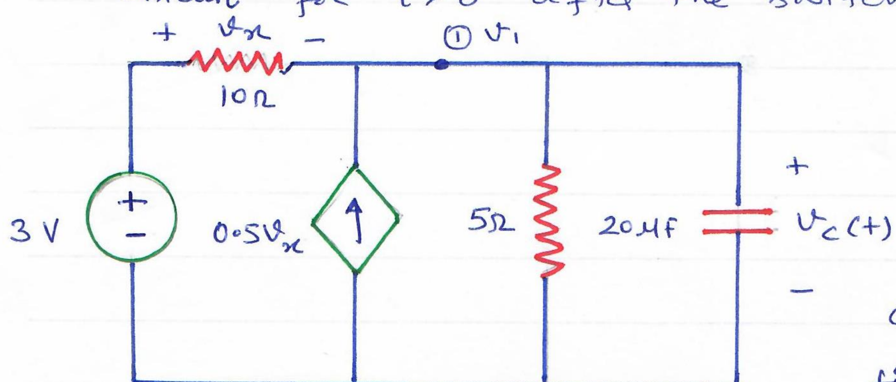


Fig. Q2

Sol.

circuit for $t > 0$ after the switch is closed is



$V_c(0) = 0$ (given)

The voltage across the capacitor can be of the form

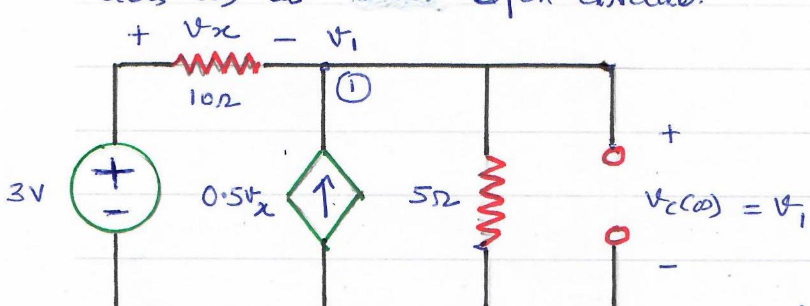
$$v_c(t) = Ae^{-t/\tau} + B$$

where τ is the time constant

$$V_c(0) = 0 = A(1) + B = A + B \quad \ldots$$

$V_c(\infty)$

at $t = \infty$ the circuit will be in steady state and capacitor acts as a open circuit.



Applying KCL at node (1)

$$\frac{v_1 - 3}{10} + \frac{v_1}{5} - 0.5v_x = 0$$

but $v_x = 3 - v_1$

$$\therefore \frac{v_1 - 3}{10} + \frac{v_1}{5} - 0.5(3 - v_1)$$

$$\Rightarrow \frac{v_1 - 3}{10} + \frac{v_1}{5} - 0.5(3 - v_1) = 0$$

$$\Rightarrow (v_1 - 3) + 2v_1 - 5(3 - v_1) = 0$$

$$\Rightarrow 8v_1 = 18$$

$$\Rightarrow v_1 = \frac{18}{8} = \frac{9}{4} = v_c(\infty)$$

$$v_c(t) = Ae^{-t/\tau} + B$$

$$v_c(\infty) = 0 + B = \frac{9}{4} \Rightarrow \boxed{B = \frac{9}{4}}$$

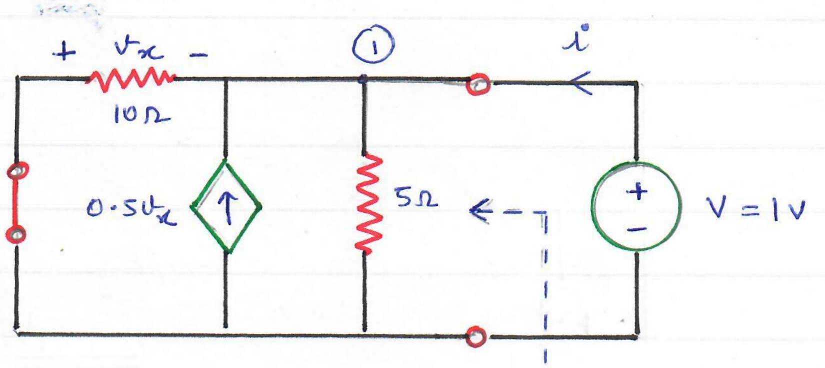
$$v_c(0) = A + B = 0$$

$$\Rightarrow A = -B = -\frac{9}{4} \Rightarrow \boxed{A = -\frac{9}{4}}$$

$$v_c(t) = -\frac{9}{4}e^{-t/\tau} + \frac{9}{4} = \frac{9}{4}\left(1 - e^{-t/\tau}\right)$$

To find τ, we have open the capacitor and find the equivalent resistance of the remaining circuit by turning off all independent sources.

Req



$$Req = \frac{V}{i} = \frac{1}{i}$$

Since there is dependent source connect a known voltage source of say 1V and find the current 'i' pumped by the source

$$Req = \frac{V}{i} = \frac{1}{i}$$

Applying KCL at node 1

$$\frac{v_1}{10} + \frac{v_1}{5} + -0.5v_x - i = 0$$

but $v_x = -v_1$

©

$$\therefore \quad \frac{v_1}{10} + \frac{v_1}{5} + 0.5v_1 - i = 0$$

$$\Rightarrow \quad v_1 + 2v_1 + 5v_1 = 10i$$

$$\Rightarrow 10i = 8v_1$$

but $v_1 = V = 1V$

$$\therefore \boxed{i = 0.8 \ A}$$

$$R_{eq} = \frac{1}{i} = \frac{1}{0.8}\ \Omega$$

$$\therefore \tau = R_{eq}C = \frac{1}{0.8} \times 20\times10^{-6} = \frac{200}{8} \times 10^{-6}$$

$$= 25\times10^{-6}\ s$$

$$\therefore v_c(t) = \frac{9}{4}\left(1 - e^{-\frac{t}{25\times10^{-6}}}\right)$$

(Q3) Find $i(t)$ for $t>0$. Assume that the circuit is in steady state at $t=0-$.

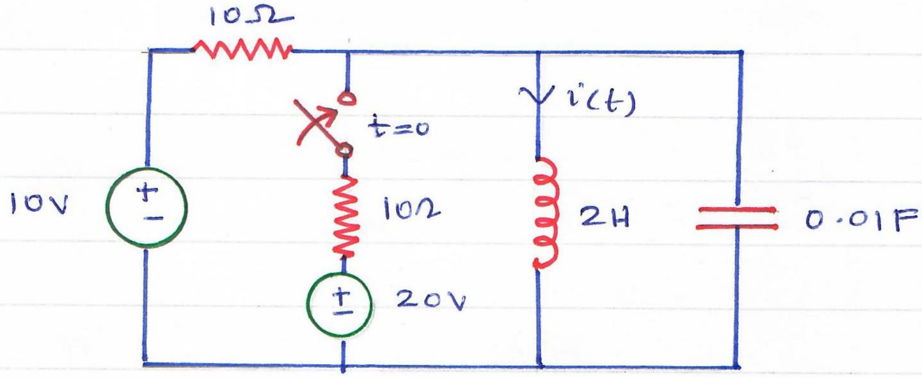
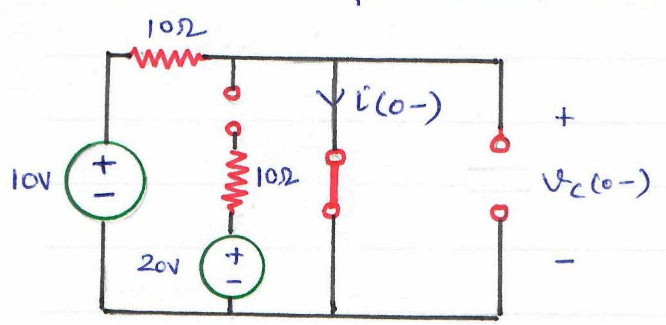


Fig. Q3

Sol.

The circuit at $t=0-$ (since it is in steady state, the capacitor acts as an open circuit and inductor acts as a short circuit)

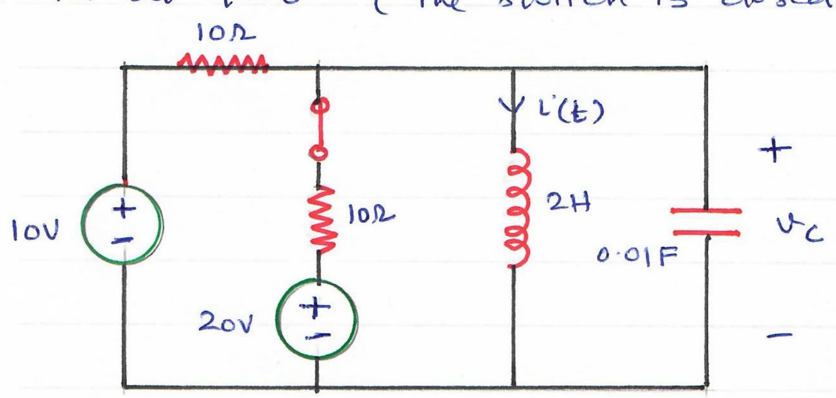


$$i(0-) = \frac{10}{10} = 1A = i(0) = i(0+)$$

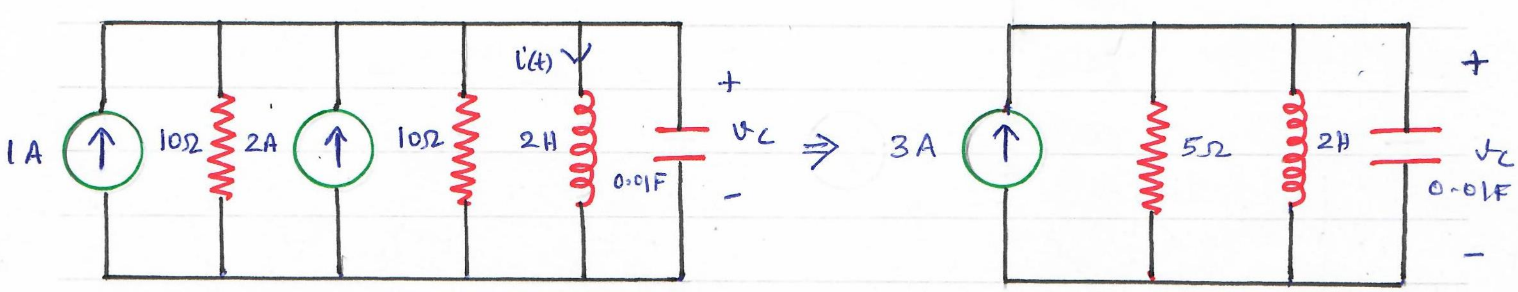
(since i is the current through inductor)

$$v_c(0-) = 0 = v_c(0) = v_c(0+)$$

CKt at $t=0$ (The switch is closed)



Using source transformation



It is a forced parallel RLC circuit

The circuit is parallel RLC circuit with a source.

$$\alpha = \frac{1}{2RC} = \frac{1}{2\times5\times0.01} = \frac{1}{0.1} = 10 \text{ rad/s}$$

$$w_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2\times0.01}} = \frac{1}{\sqrt{2}\times0.1} = \frac{10}{\sqrt{2}} \text{ rad/s}$$

$\alpha > w_0$, $\therefore$ The circuit is overdamped

$$S_1, S_2 = -\alpha \mp \sqrt{\alpha^2 - w_0^2} = -10 \mp \sqrt{50} = -17.07, -2.93$$

The response can be of the form

$$i(t) = K + A_1 e^{s_1 t} + A_2 e^{s_2 t}$$
$$= K + A_1 e^{-17.07t} + A_2 e^{-2.93t}$$

$$i(0-) = i(0) = i(0+) = 1A$$

$$v_c(0-) = 0 = v_c(0) = v_c(0+)$$

$$L\frac{di}{dt} = v_c$$

$$\Rightarrow L\frac{di(0)}{dt} = v_c(0)$$

$$\Rightarrow \frac{di(0)}{dt} = \frac{v_c(0)}{L} = 0$$

$$i(t) = K + A_1 e^{-17.07t} + A_2 e^{-2.93t}$$

$$\frac{di(t)}{dt} = 0 - 17.07 A_1 e^{-17.07t} - 2.93 A_2 e^{-2.93t}$$
$$= -17.07 A_1 e^{-17.07t} - 2.93 A_2 e^{-2.93t}$$

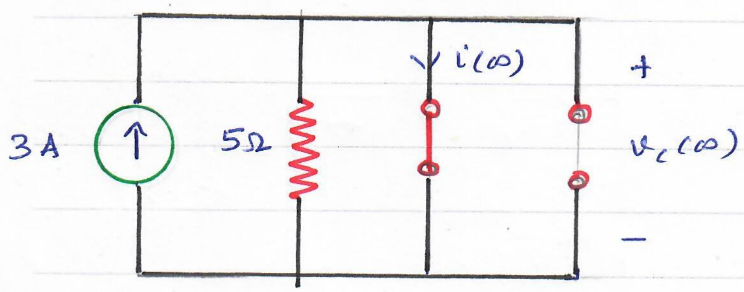
$$i(0) = 1 = K + A_1 + A_2 \Rightarrow K + A_1 + A_2 = 1 \quad \ldots (A)$$

$$\frac{di(0)}{dt} = 0 = -17.07 A_1 - 2.93 A_2 = 0 \quad \ldots (B)$$

We have 3 unknowns and 2 eqns.

The circuit will be in steady state at $t=\infty$
(capacitor acts as open circuit and inductor acts as short circuit)

The circuit at $t=\infty$



$i(\infty) = 3\,A$

$$i(t) = K + A_1 e^{-17.07t} + A_2 e^{-2.93t}$$

$i(\infty) = \boxed{K = 3}$ ..(c)

Substituting the value of K in (A)

$K + A_1 + A_2 = 1$ .. (A)

$\Rightarrow A_1 + A_2 = 1 - 3 = -2$ .. (A)

(B)

$-17.07A_1 - 2.93A_2 = 0$ ..(B)

Solving (A) and (B)

$A_1 = 0.4144$

and $A_2 = -2.4144$

$$\boxed{\therefore i(t) = 3 + 0.4144\,e^{-17.07t} - 2.4144\,e^{-2.93t}}$$