Mid Sem – 1

PART A

Answer ANY SIX. [5X 6=30 marks]

1. A. List any advantages and/or disadvantages to using a k-way merge sort(makes k partitions) instead of a two –way merge sort.
   B. Compare QuickSort and MergeSort. When would you prefer one over the other ?

2. Abhijeet has discovered a remarkable property of the binary search tree. Suppose on searching for a key k in a BST, the search ends up in a leaf. Consider three sets: A, the keys to the left of the search path; B, the keys on the search path and C, the keys to the right of the search path. Abhijeet claims that for any three keys a in A, b in B and c in C must satisfy a<=b <=c. Argue that Abhijeet is right or provide a counter example.

3. How would you convert infix to prefix notation. Determine the complexity and present an example for the algorithm.

4. Derive the maximum and minimum nodes in an AVL tree of height h.

5. What is the maximum height difference possible between two leaf nodes in an AVL tree? Explain.

6. How could you implement three stacks in one array?

7. In a binary search tree with distinct key values, let x be a leaf node and let y be its parent. Show that key[y] is either the smallest key in T larger than key[x] or the largest key in T smaller than key[x]

8. Show how to implement a queue using two stacks. Analyze the running time of the queue operations. Show that for a sequence of n queue operations, the implementation takes a worst case running time of O(n).

PART B

Answer ANY FOUR

[4 X 10 = 40 marks]

1. Let rank of an element be the position of the element in the sorted sequence of the elements(assume there are no duplicates). Say you are building a system where you could do any of three operations on a 23 tree., namely - insertion, deletion, return the rank of an element. What additional information can you store on each node of the 23 tree to answer the rank query and what would be the complexity of a rank query. Can your method be extended to duplicates where rank is the position of the element in a stable sorted sequence. (Stable sort is where two elements R and S with the same value and with R appearing before S in the original list will also have R appear before S in the sorted list.)
Can you obtain a stable sort from an AVL tree? Explain.

2. An inversion in an array A occurs when $A[i] < A[j]$ for $j < i$. The pair (i,j) is called an inversion in A. Determine the number of inversions in any permutation of n numbers in $\Theta(n \log n)$.

3. Consider a 2-AVL tree as a binary search tree where for each given node, the height difference of its children can be at most 2. What is the maximum height of a 2-AVL tree ? Similarly, what can be the maximum height of a c-AVL tree where c is the maximum height difference allowed between two children of each node.

4. Given the sequence of numbers 3,10, 9,2,4,11 how can we determine other permutations that will result in the same binary search tree.

5. Solve ANY TWO of the below recurrences .
Given $T(1)=1$.
a. $T(n) = T(n-1) + \log n$
b. $T(n)=2T(n/2 +17) + n$
c. $T(n) = \sqrt{n}\, T(\sqrt{n}) + n$