

Homework #2 Solution (for blue numbers, refer to appendix B)

3.1

8. c.

$$B(x) = \{-8, -6, -4, -2, 0, 2, 4, 6, 8\}$$

18. c.

$\forall s$ in D, if C(s) then $\sim E(s)$

24. b.

\exists a question x such that x is easy.

$\exists x$ such that x is a question and x is easy.

3.2

12.

Correct negation: There is an irrational number and a rational number such that its product is rational.

40.

If an (integer) number is divisible by 8 then it is divisible by 4.

46.

\exists a function that is polynomial but it does not have a real root.

3.3

12. c.

negation: $\exists x$ in D such that $\forall y$ in E, $xy < y$.

The original statement is true.

41.c.

False, there is no real number that is always one greater than every real number. $\because 10+1 = 11, 11+1 = 12, \dots$

d.

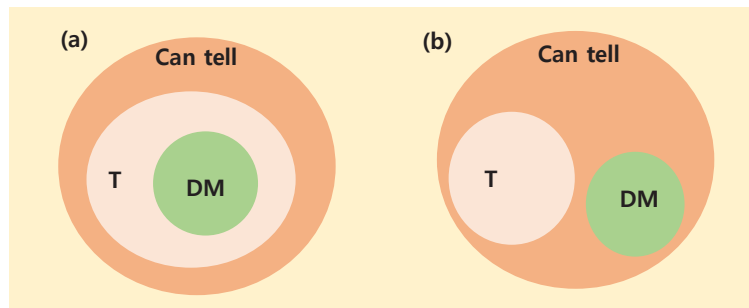
True, for any positive real number x, we can choose y as $1/x$ (i.e., reciprocal) which make $xy = 1$.

56.

(a) Two statements don't necessarily have the same truth value. In the 1st statement, some x should satisfy both P(x) and Q(x) at the same time. In the 2nd statement, however, the choice of x for P(x) (being true) doesn't necessarily make Q(x) true.

3.4

22.



Let "Can tell" denotes "can tell a valid argument from an invalid one", and T = thoughtful people, DM = discrete mathematics students, the diagram (a) is the same as the given argument. However, the diagram (b) is also possible, or T can be inside of DM, or T and DM can have intersection. Therefore, the original argument is invalid.

34.

1. All writers who understand human nature are clever.

If a writer can understand human nature, then he is clever. (if-then)

2. No one is a true poet unless he can stir the human heart.

If a poet cannot stir the human heart, he is not a true poet. (if-then)

If one is a true poet, then he can stir the human heart. (contrapositive)

3. Shakespeare wrote *Hamlet*.

4. No writer who does not understand human nature can stir the human heart.

If a writer does not understand human nature, then he cannot stir the human heart. (if-then)

If a write can stir the human heart, then he understands human nature. (contrapositive)

5. None but a true poet could have written *Hamlet*.

If one wrote *Hamlet*, then he was a true poet. (if-then)

Reordering

3. Shakespeare wrote *Hamlet*.

5. If one wrote *Hamlet*, then he was a true poet. (if-then)

2. If one is a true poet, then he can stir the human heart. (contrapositive)

4. If a write can stir the human heart, then he understands human nature. (contrapositive)

1. If a writer can understand human nature, then he is clever. (if-then)

∴ Shakespeare was clever. (**conclusion**)