The Physics of Inertial Confinement Fusion at the National Ignition Facility

Ronak Desai

Candidacy Oral Exam

March 27th 2023

DEPARTMENT OF PHYSICS

Advisor

Dr. Christopher Orban

THE OHIO STATE UNIVERSITY

COLLEGE OF ARTS AND SCIENCES

Committee

Dr. Alexandra Landsman

Dr. Douglass Schumacher

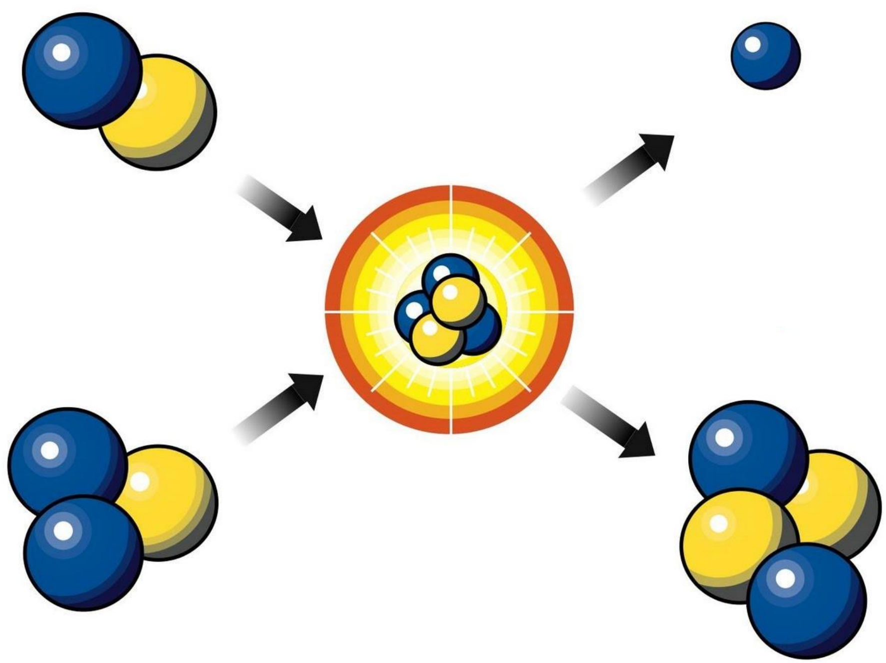
Dr. Brian Skinner

Academic Background

- Undergraduate: Rowan University
  - B.S. Physics
  - B.A. Mathematics
- Gap Year: Brookhaven National Laboratory
  - SULI Internship Program
- 3rd Year Grad Student: OSU
  - Plasma Physics
  - Particle In Cell Simulations
  - Machine Learning

What is Nuclear Fusion?

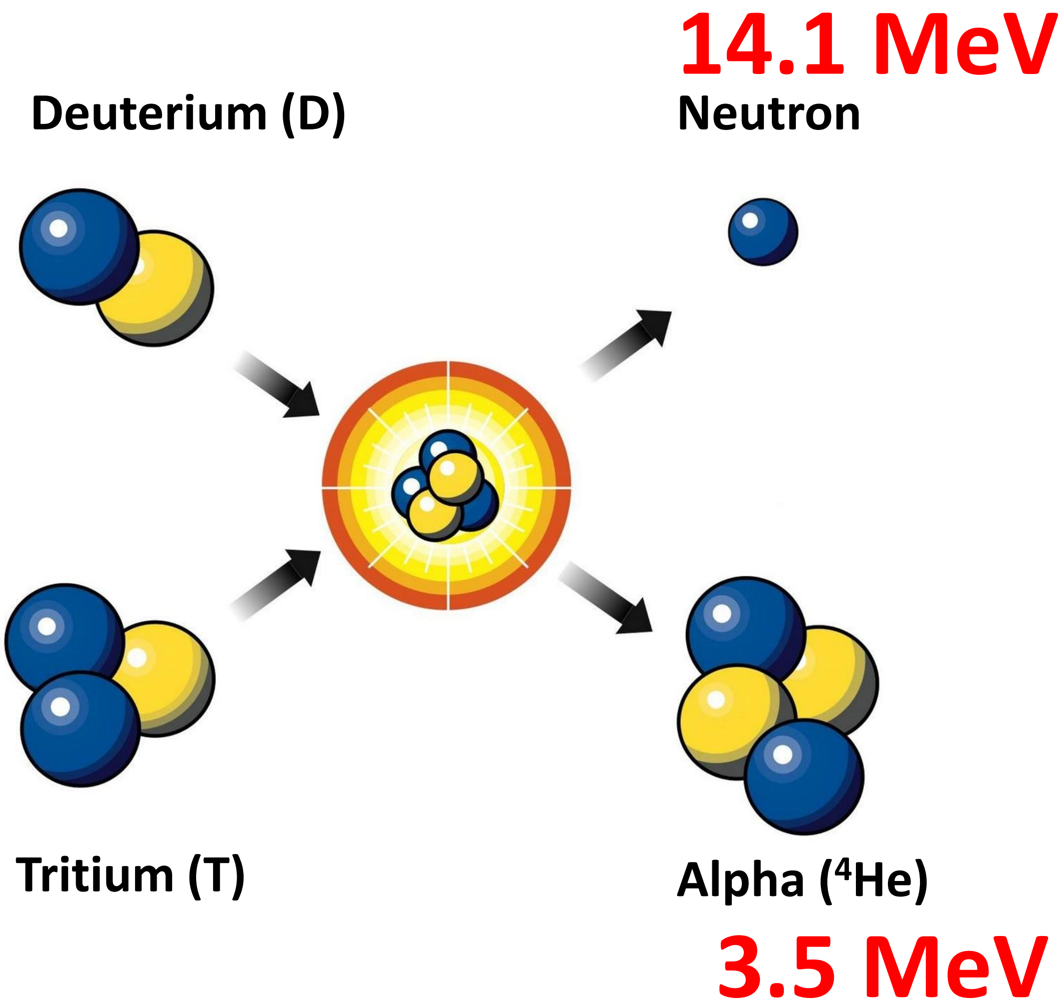
- Two lighter nuclei combine to form heavier nucleus
- $Q = (\Delta m)c^2$
  - $\boldsymbol{Q_{DT}}$ =**17.6 MeV**



https://interestingenergyfacts.blogspot.com/2010/04/nuclear-fusion-facts.html

What is Nuclear Fusion?

- Two lighter nuclei combine to form heavier nucleus
- $Q = (\Delta m)c^2$
  - $\boldsymbol{Q_{DT}}$ =**17.6 MeV**
- Compare DT and Coal:
  - DT: 300 GJ/g
  - Coal: 30 kJ/g
  - Factor of 10 Million!



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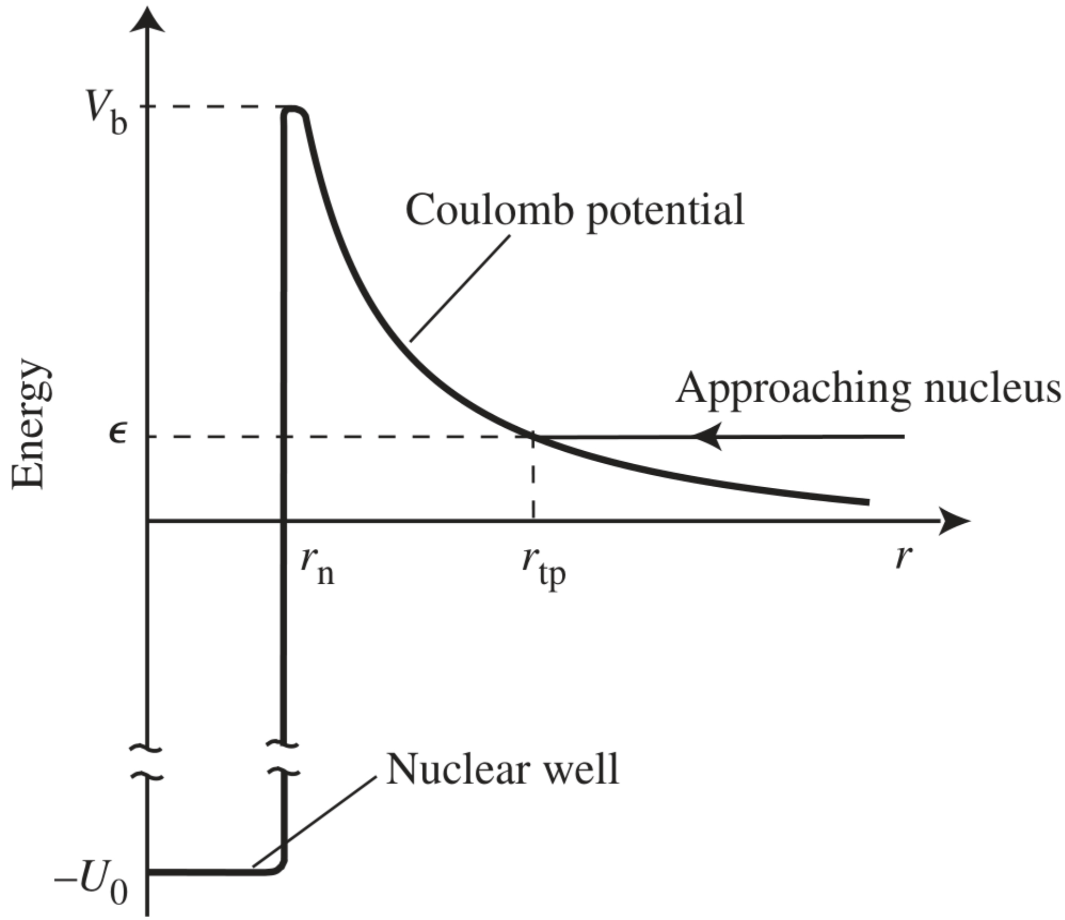
How to Fuse Nuclei

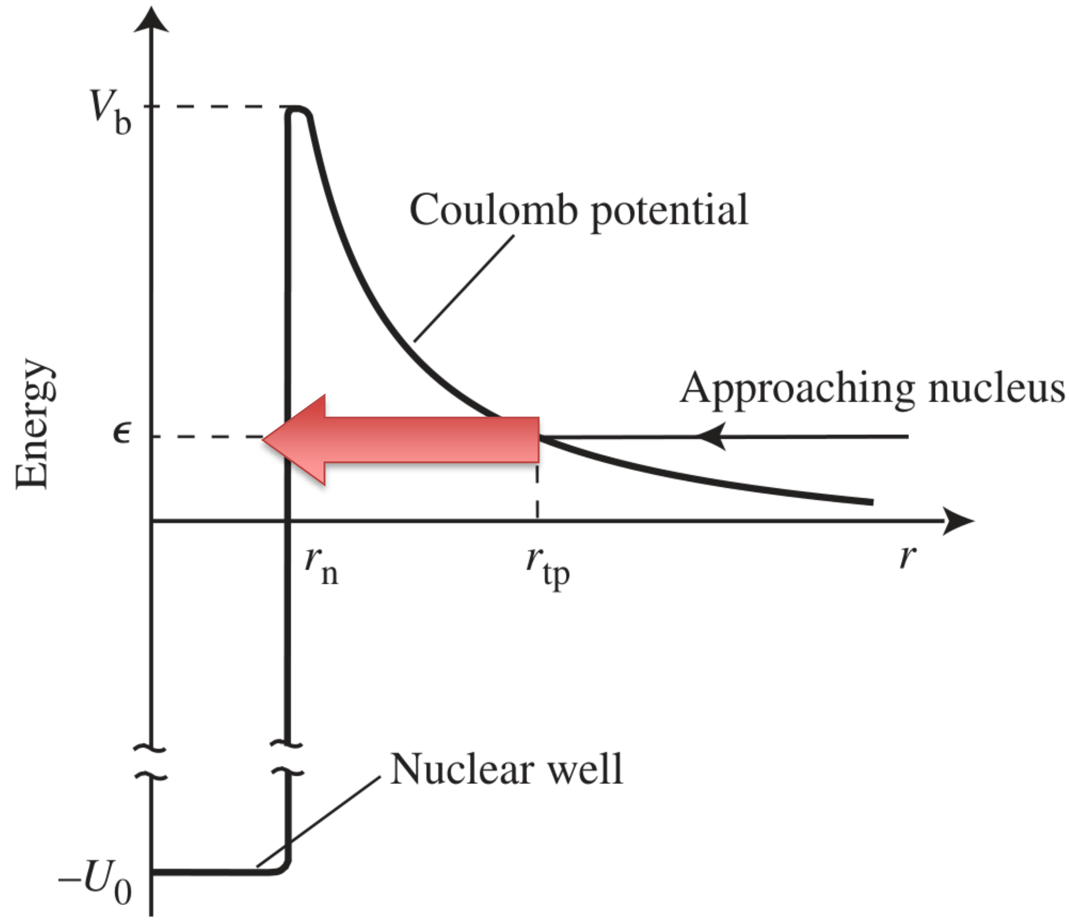
- Need to overcome repulsive coulomb barrier
- $V_b \approx 1\ MeV$
  - 500x hotter than solar core



S. Atzeni, *The Physics of Inertial Fusion: Beam Plasma Interaction, Hydrodynamics, Hot Dense Matter* (2004)

How to Fuse Nuclei

- Need to overcome repulsive coulomb barrier
- $V_b \approx 1\ MeV$
  - 500x hotter than solar core
- Can Tunnel through barrier Quantum Mechanically



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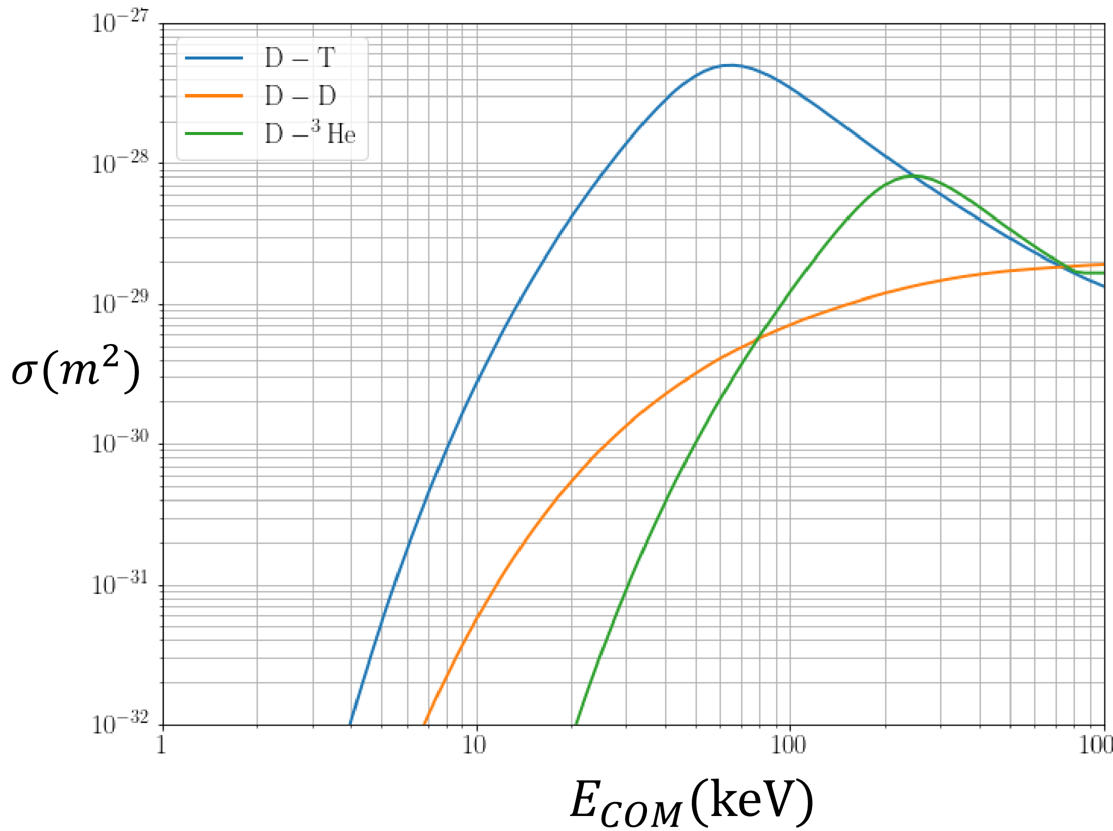
Why DT Fusion?

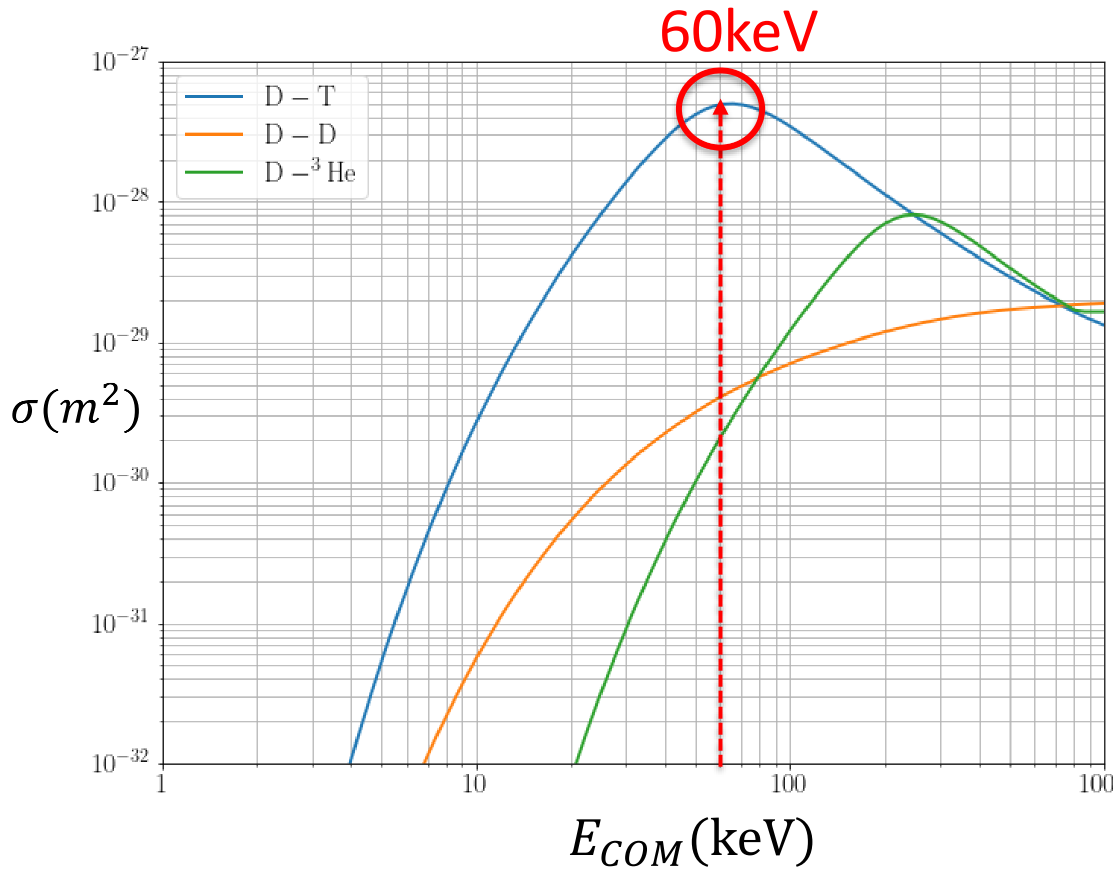
- High Fusion Cross Section (FCS): σ
  - Dependent on Geometric Cross Section and Tunneling Probability



https://scipython.com/blog/plotting-nuclear-fusion-cross-sections/

Why DT Fusion?

- High Fusion Cross Section (FCS): σ
  - Dependent on Geometric Cross Section and Tunneling Probability
- Low Temperature
  - $E_{COM} \sim 60keV$
  - T~10 keV
  - 1 keV ~ 11 Million K



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Lawson Criterion

- Conditions for sustained fusion?
  - n: Number density high enough for frequent collisions
  - τ: Long confinement time for fuel to fully burn

Lawson Criterion

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  - n: Number density high enough for frequent collisions
  - τ: Long confinement time for fuel to fully burn

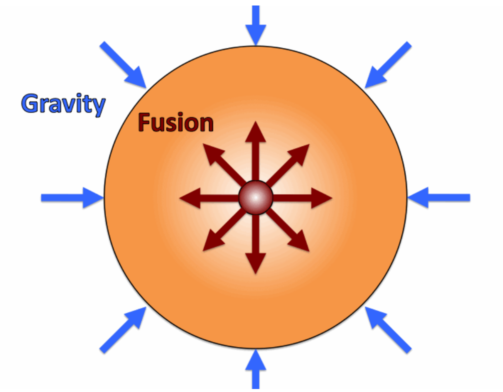
$$\boldsymbol{n\tau > 10^{15} s/cm^3}$$

- for DT fusion

$$\boldsymbol{n\tau > \frac{12 k_B T}{\langle \sigma v \rangle Q}}$$

- T = Temperature
- Q = Energy Released
- $\langle \sigma v \rangle$ = Fusion Cross Section integrated over Maxwell-Boltzmann Distribution

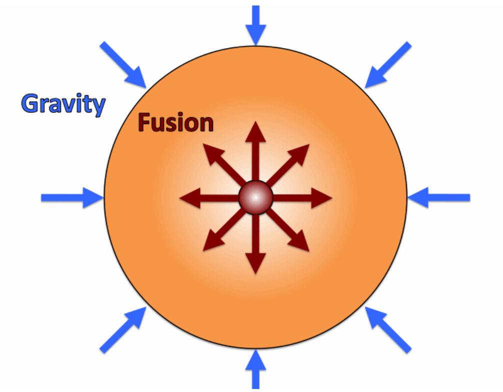
Types of Confinement



http://large.stanford.edu/courses/2011/ph241/olson1/

Gravitational

Types of Confinement



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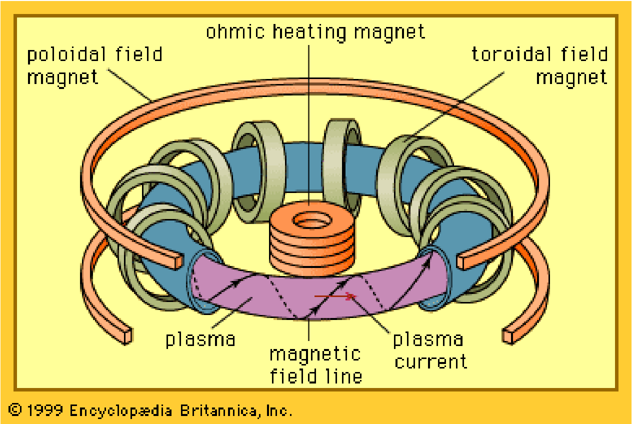
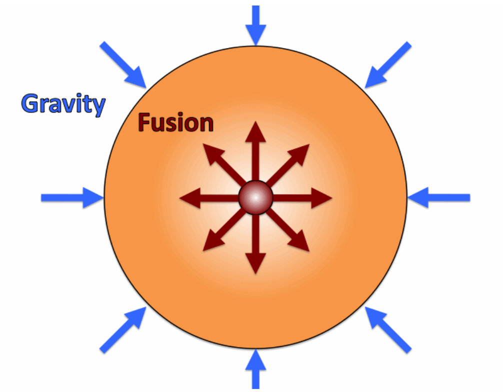


https://www.britannica.com/technology/fusion-reactor/Principles-of-magnetic-confinement

Magnetic

Types of Confinement



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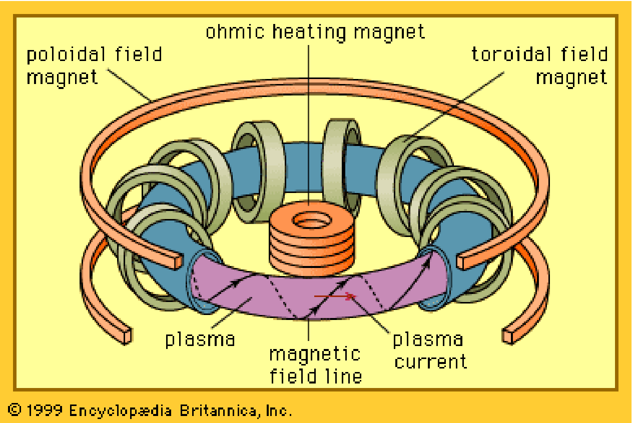

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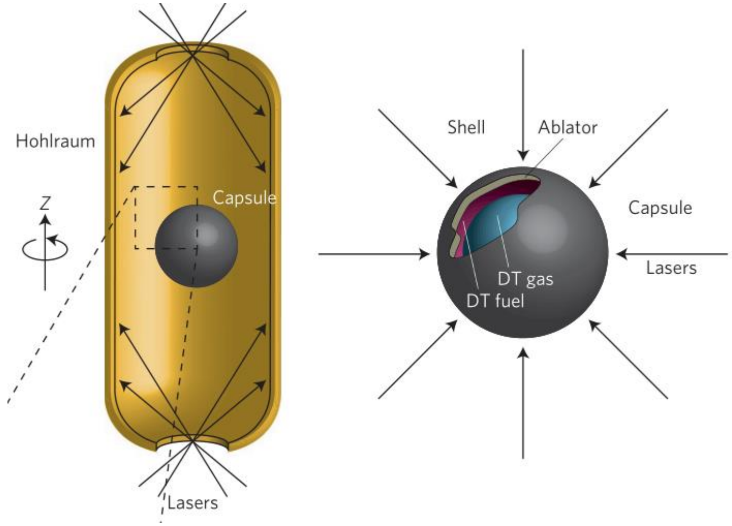
Magnetic

Inertial (ICF)



Nature Phys **12**, 435–448 (2016)

Indirect Drive

Direct Drive

ICF Stages

1. Laser Heating

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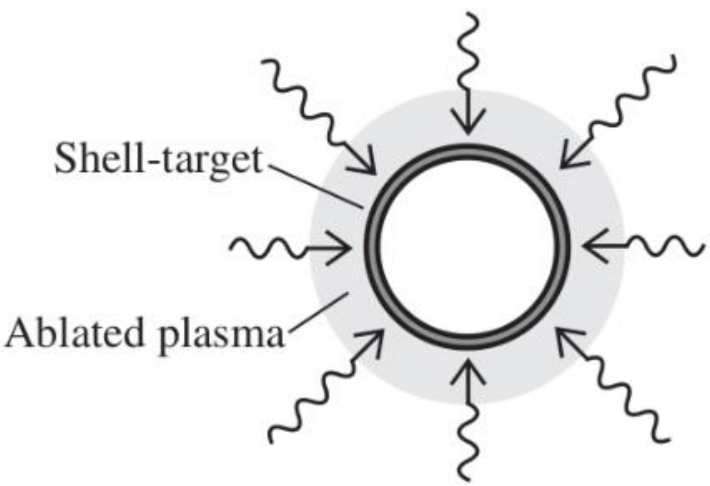

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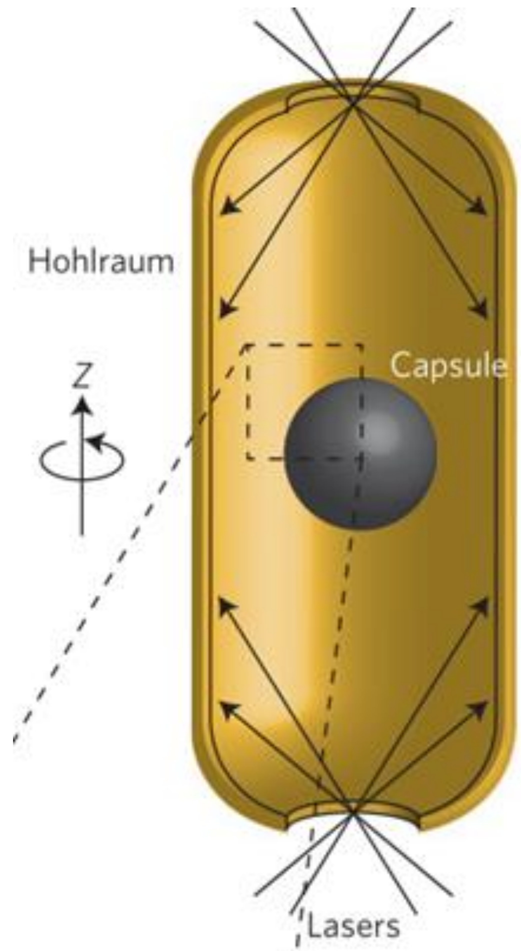

ICF Stages

1. Laser Heating
2. Capsule Expansion (Ablation)

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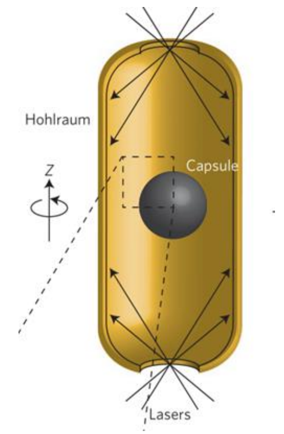

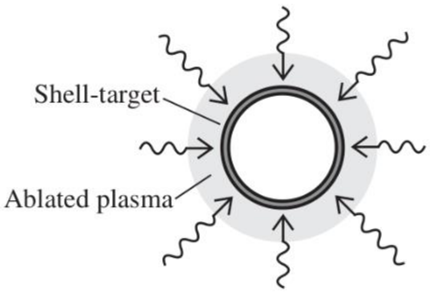

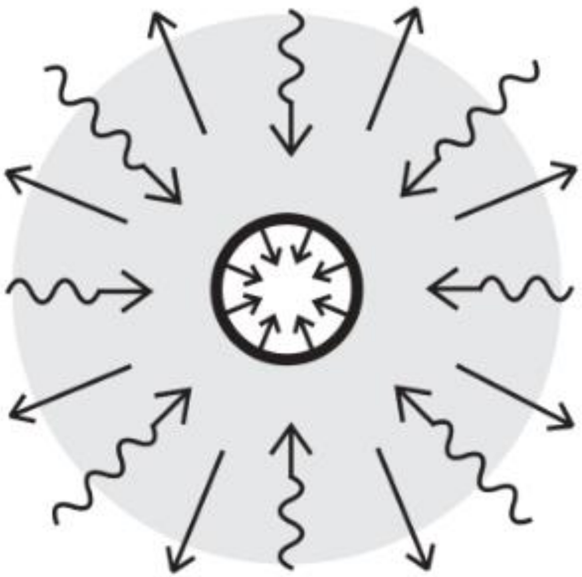

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ICF Stages

1. **Laser Heating**
2. **Capsule Expansion (Ablation)**
3. **Compression and Ignition**

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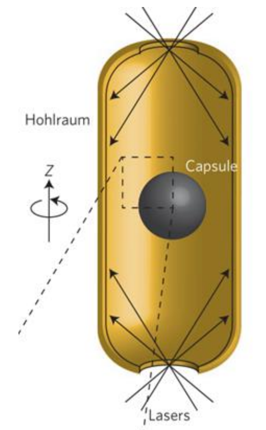

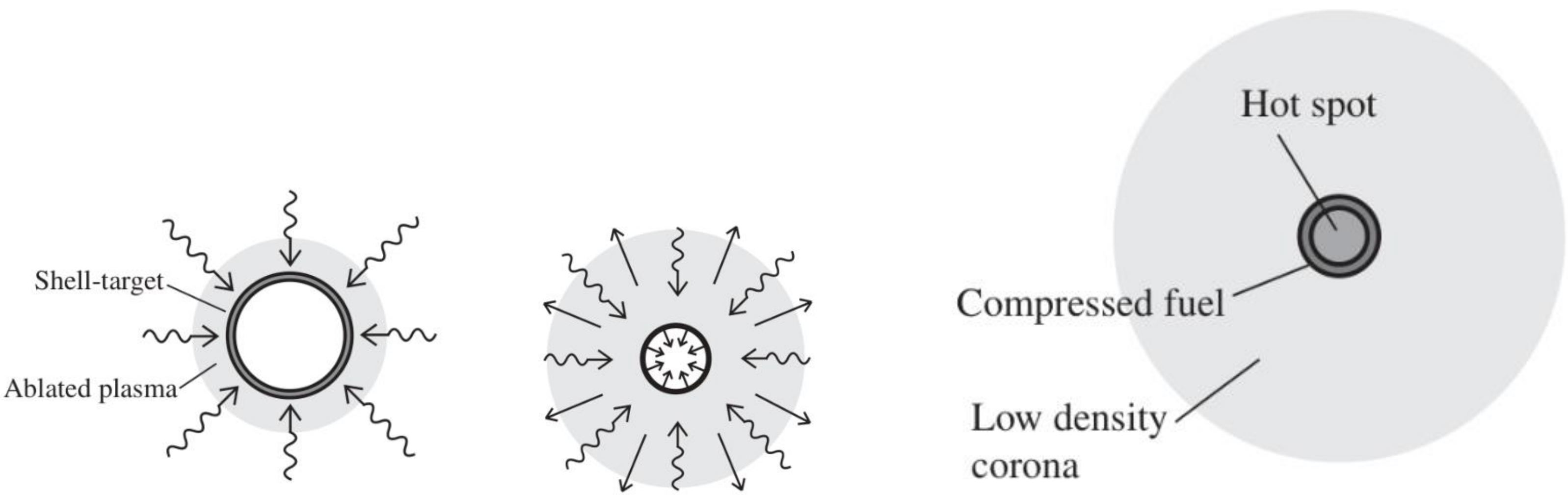

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ICF Stages

1. Laser Heating
2. Capsule Expansion (Ablation)
3. Compression and Ignition
4. Fuel Burn

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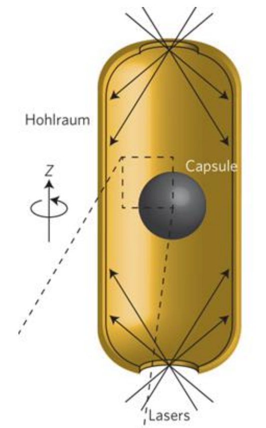

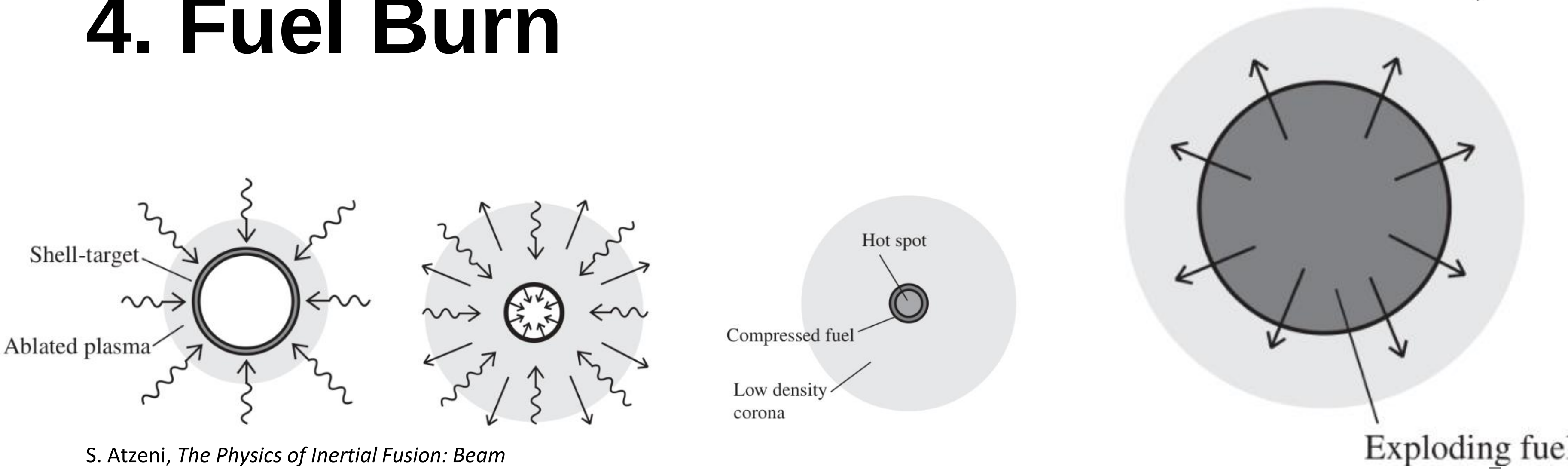

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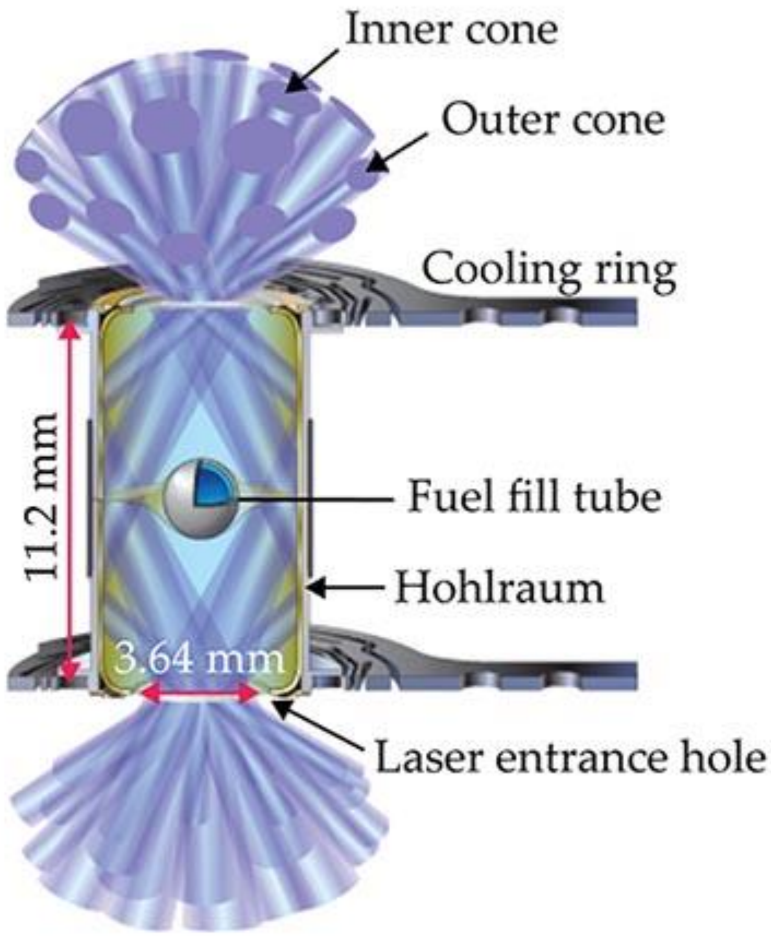
Hohlraum Temperature

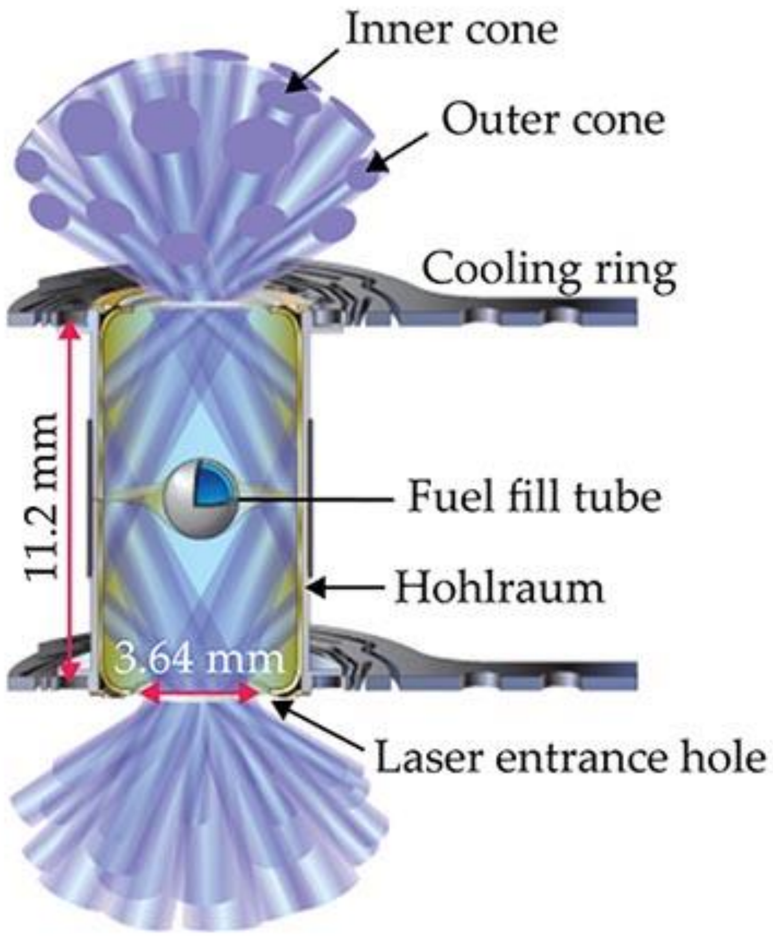
- Laser enters through Laser Entrance Hole (LEH)
- Heats inner surface of cylinder



Nature **601**, 542–548 (2022)

Hohlraum Temperature

- Laser enters through Laser Entrance Hole (LEH)
- Heats inner surface of cylinder
- $I \sim \sigma_{SB} T^4$
  - $I = \frac{P \sim 500\ TW}{A_H \sim 1 cm^2} \sim 10^{15} W/cm^2$
  - $T_r \sim 250\ eV$



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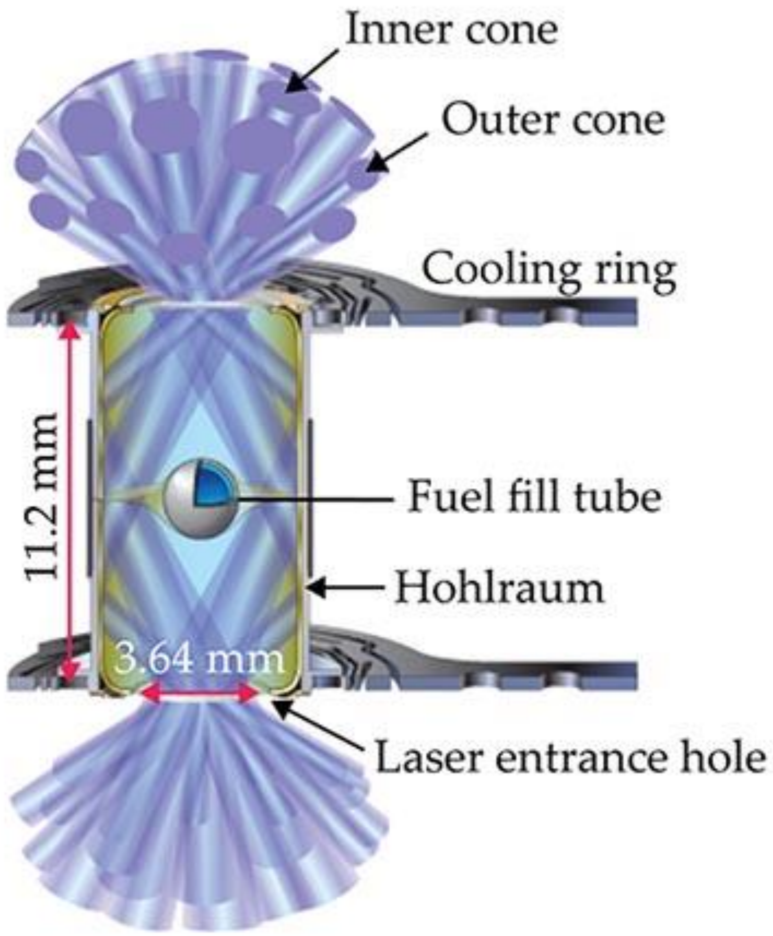
  - $I = \frac{P \sim 500\ TW}{A_H \sim 1cm^2} \sim 10^{15} W/cm^2$
  - $T_r \sim 250\ eV$

- Want High Temperature
  - Small Hohlraum
  - Small LEH



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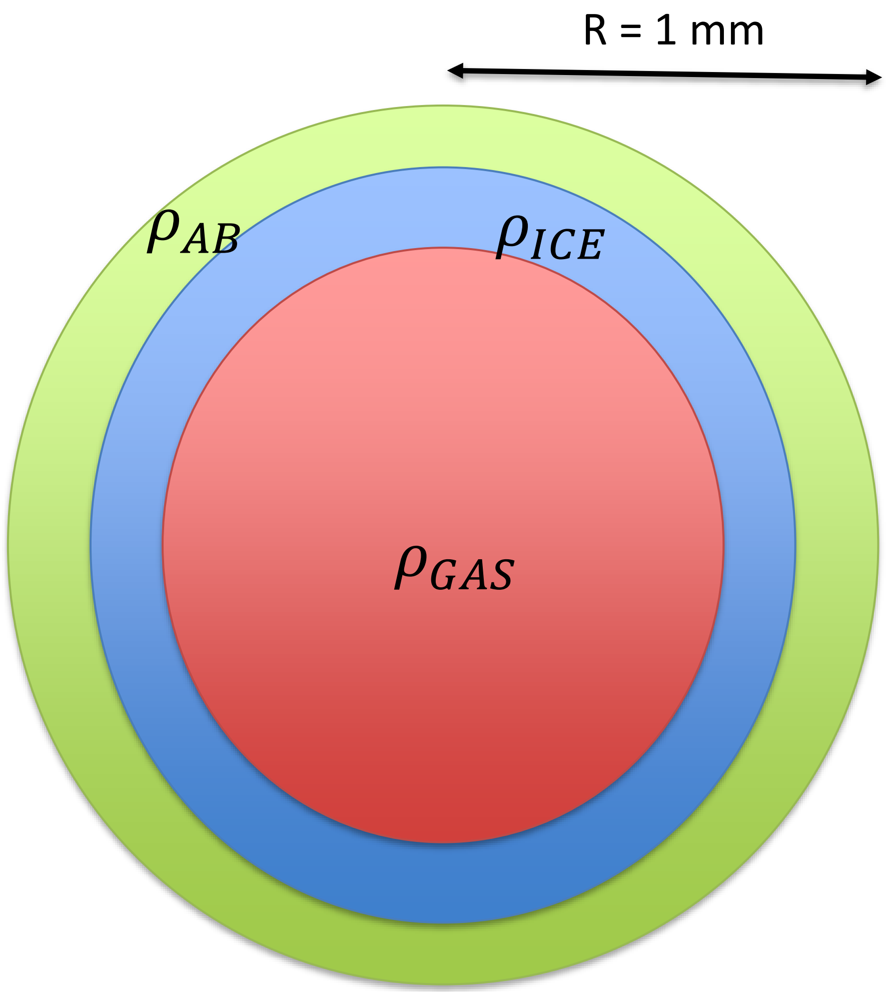
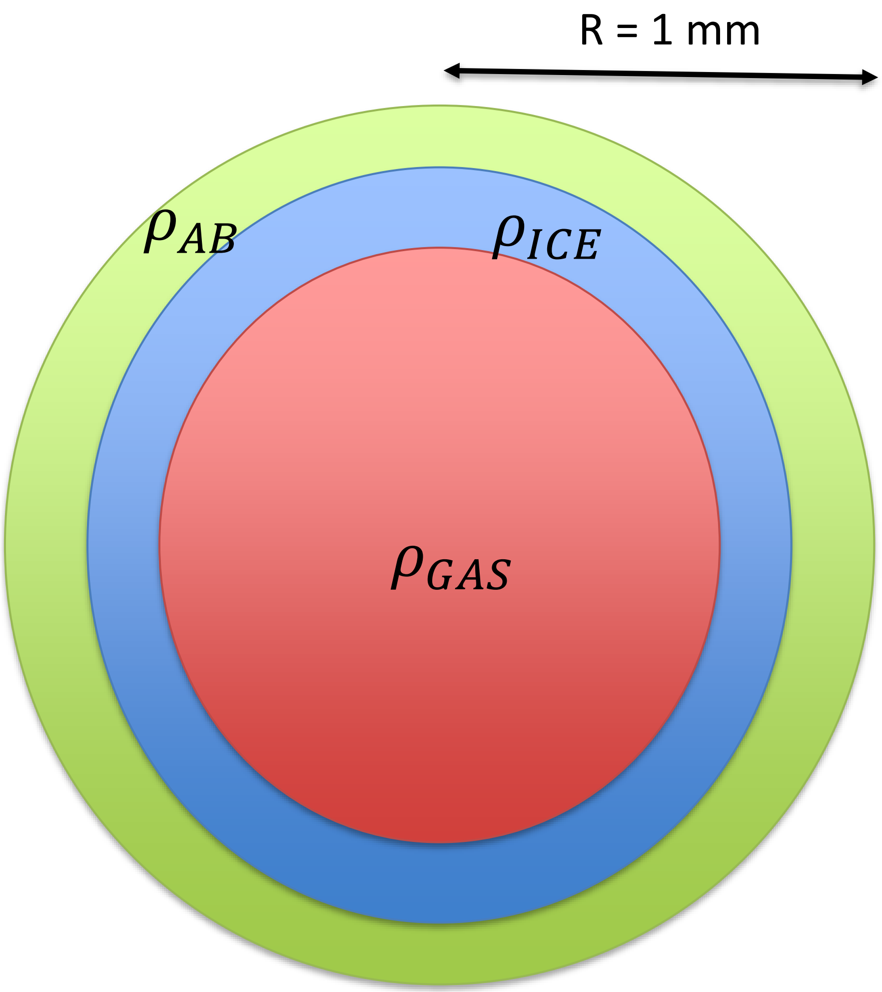
Capsule

- Outer Shell: Ablator
  - Vaporized by Laser



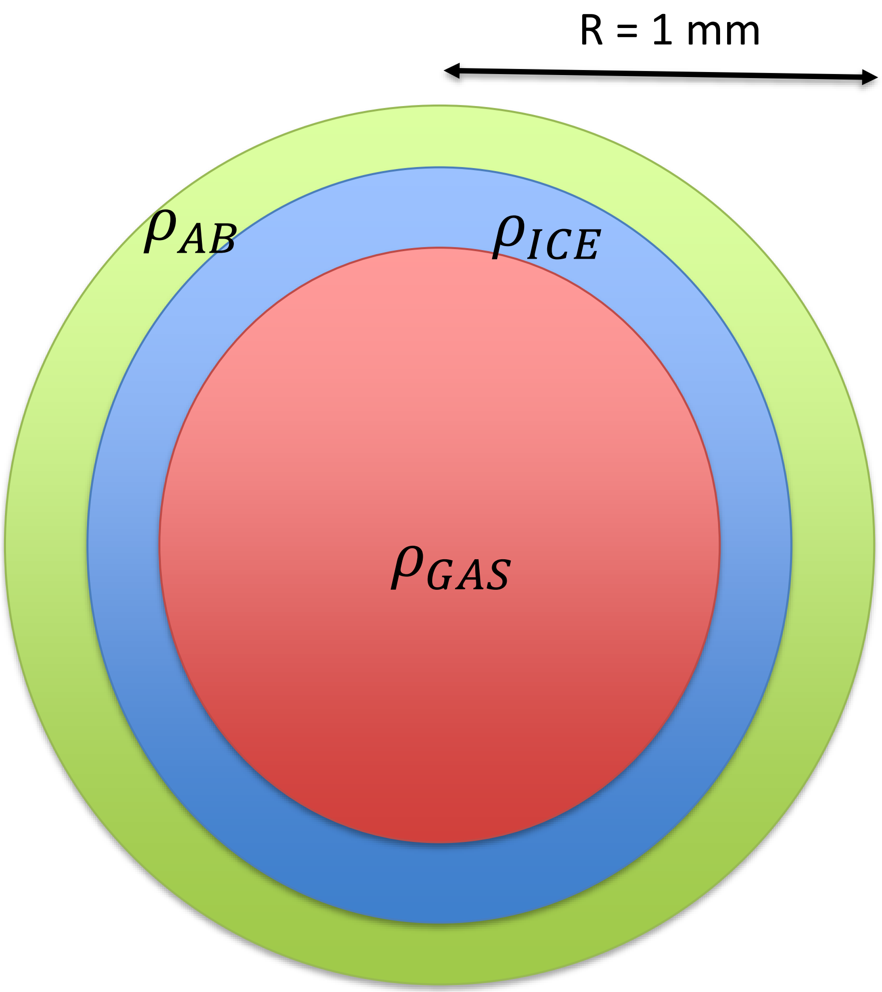
Capsule

- Outer Shell: Ablator
  - Vaporized by Laser
- Inner Shell: DT Ice
  - T~18K
  - Most of Fuel Mass



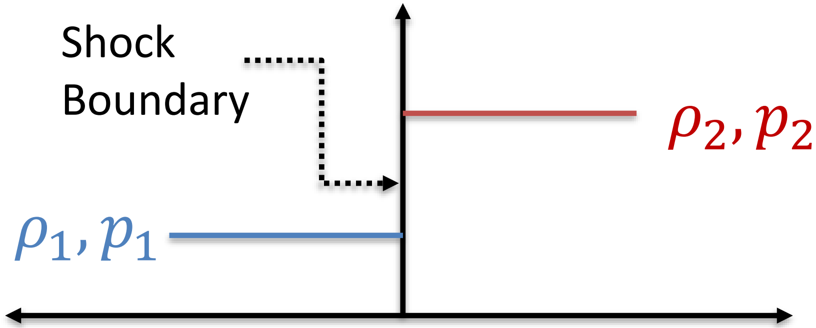
Capsule

- Outer Shell: Ablator
  - Vaporized by Laser
- Inner Shell: DT Ice
  - T~18K
  - Most of Fuel Mass
- Inner Core: DT gas
  - Low Density
  - Reaches Highest Temperature



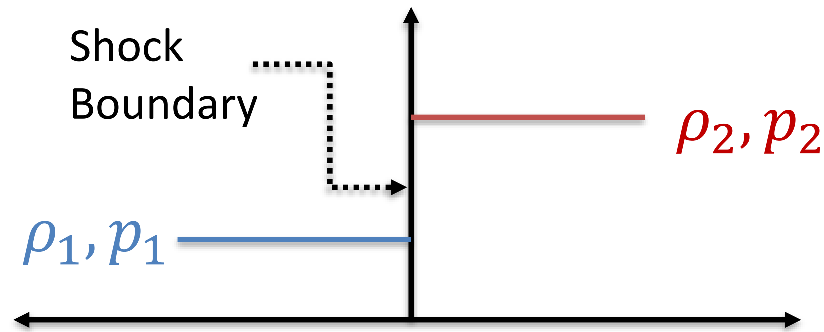
1D Hydrodynamics

- Shock Waves Drive Compression
  - Sharp pressure changes from short pulse laser



1D Hydrodynamics

- Shock Waves Drive Compression
  - Sharp pressure changes from short pulse laser
- Euler Equations of Hydrodynamics
  - mass, momentum, energy

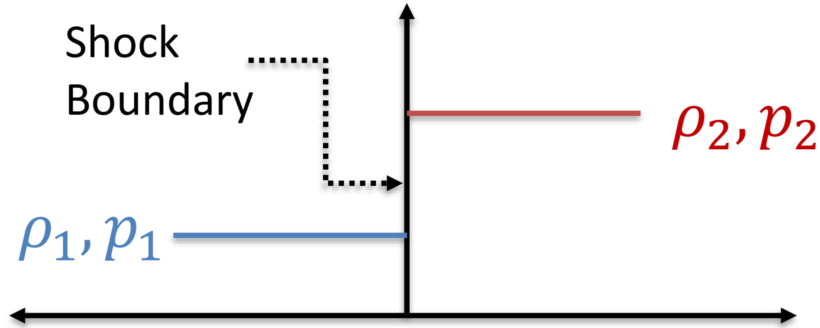


Specific Volume $V \equiv \frac{1}{\rho}$

Shock Compression $$\frac{p_2}{p_1} = \frac{4 - V_2/V_1}{4V_2/V_1 - 1}$$

Isentropic Compression $$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^{5/3}$$

1D Hydrodynamics



- Shock Waves Drive Compression
  - Sharp pressure changes from short pulse laser
- Euler Equations of Hydrodynamics
  - mass, momentum, energy
- Ex) Want $\frac{V_2}{V_1} = \frac{1}{4}$:
  - $\frac{p_2}{p_1} \to \infty$ for 1 shock!
  - $\frac{p_2}{p_1} \approx 10$ isentropically

Specific Volume $V \equiv \frac{1}{\rho}$

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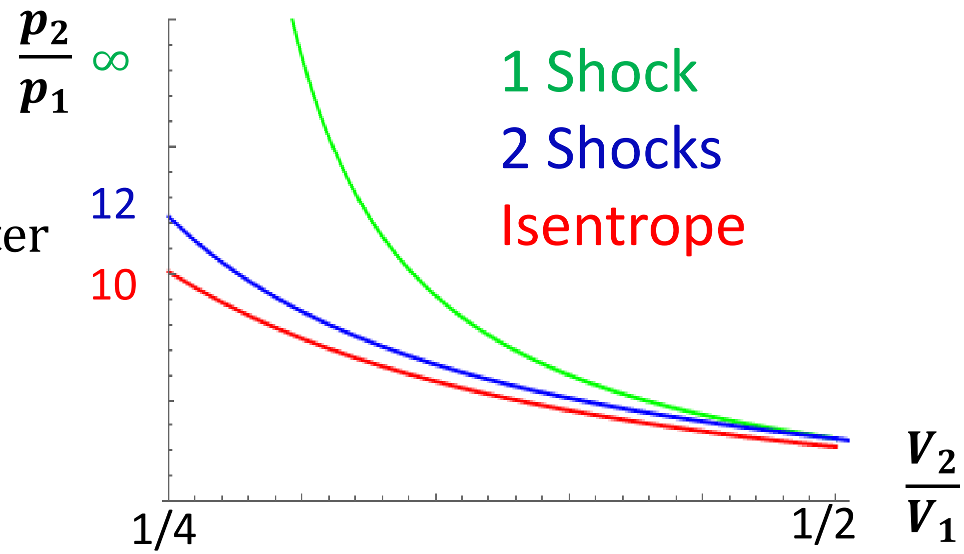
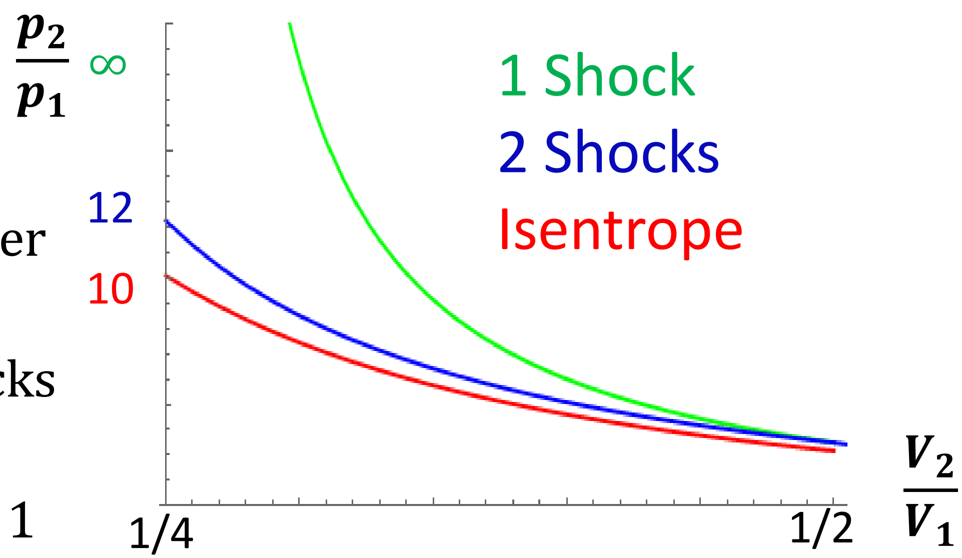
Shocks

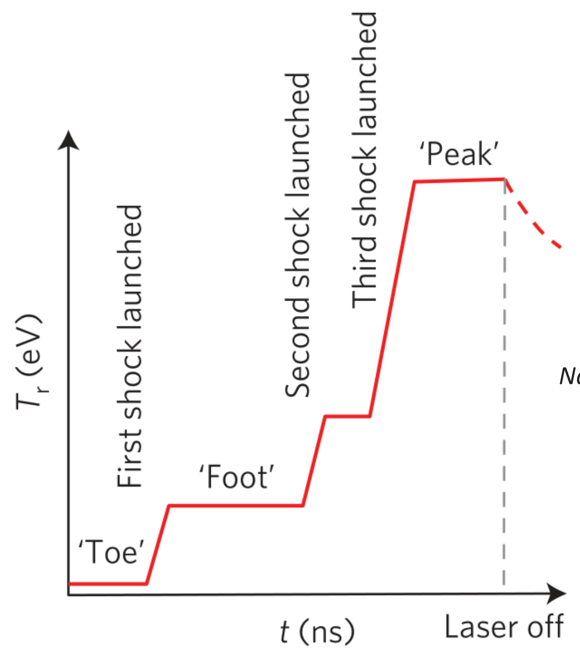
- Several Weak Shocks better than One Strong Shock



Shocks

- Several Weak Shocks better than One Strong Shock
- Laser pulse tuned so shocks converge at center
- Isentrope Parameter $\alpha > 1$
  - Strong Shocks Increase
  - Want to minimize





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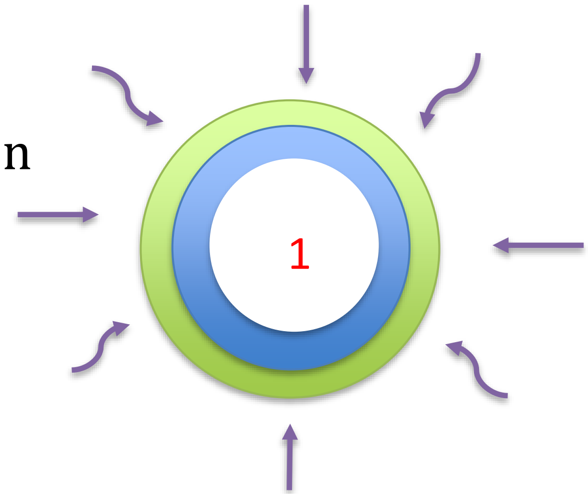
Shell Ablation

- Momentum Conservation
- (1) X-rays heat Ablator
  - Shell gets vaporized



Shell Ablation

- Momentum Conservation
- (1) X-rays heat Ablator
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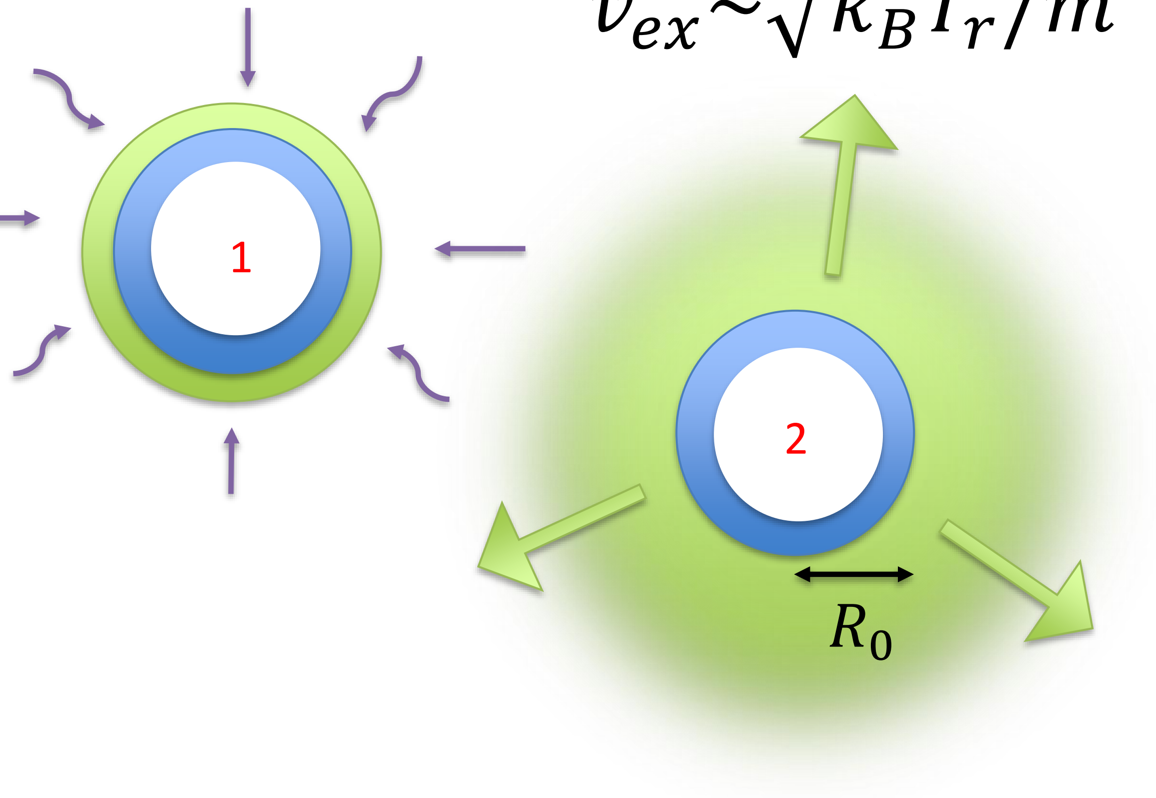
- (2) Material ejected
  - outward
  - speed: v_{ex}

$$v_{ex} \sim \sqrt{k_B T_r / m}$$



Shell Ablation

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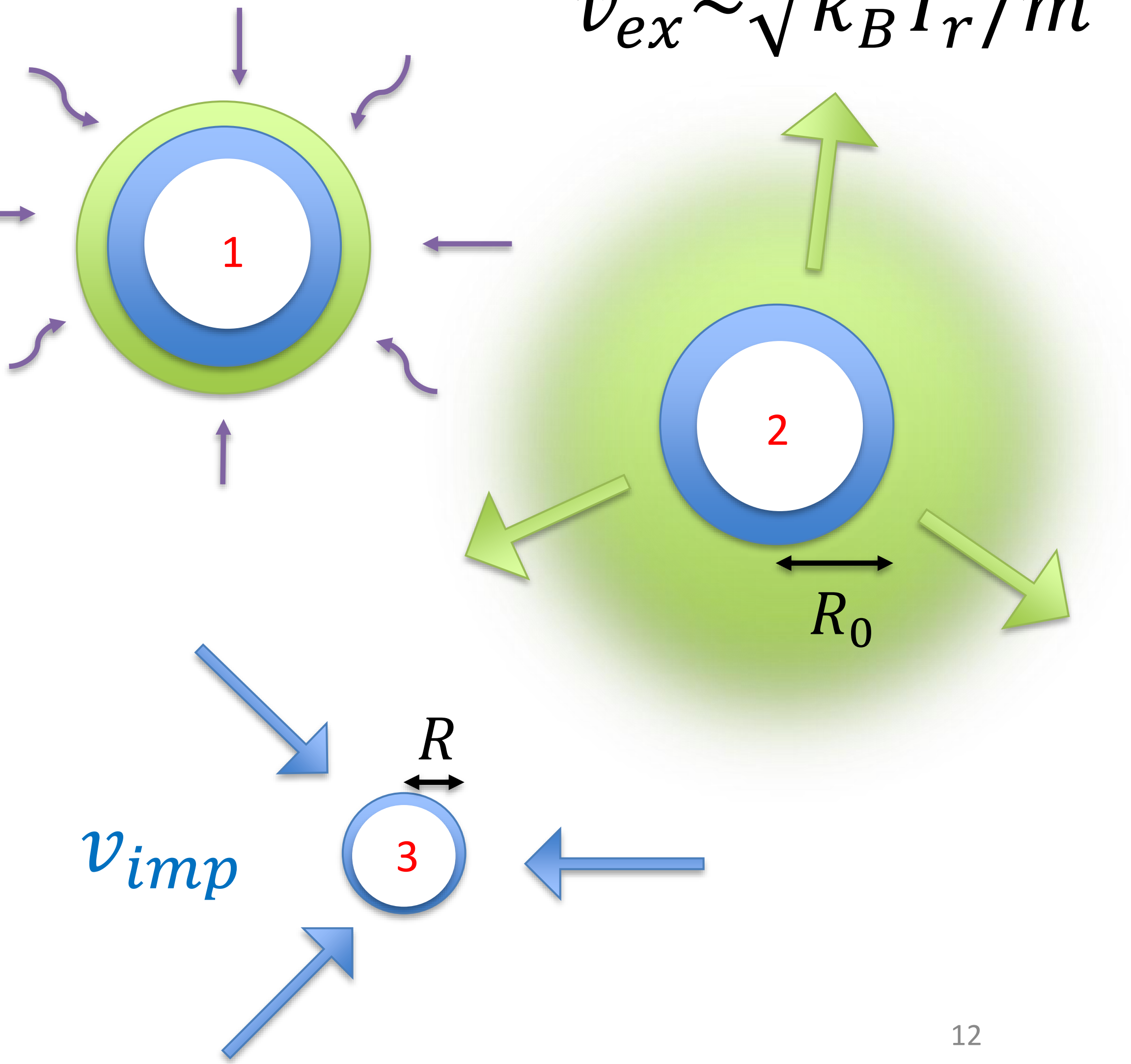
- Momentum Conservation
- (1) X-rays heat Ablator
  - Shell gets vaporized
- (2) Material ejected
  - outward
  - speed: v_{ex}
- (3) Implosion of shell
  - inward
  - speed: v_{imp}



Spherical Rocket

- 1D Rocket Model
  - $M \frac{dv_{imp}}{dt} = v_{ex} \frac{dM}{dt} \rightarrow v_{imp} = v_{ex} \ln\left(\frac{M_0}{M}\right)$ Standard Rocket Equation
  - Radius of shell changes as fuel implodes inwards

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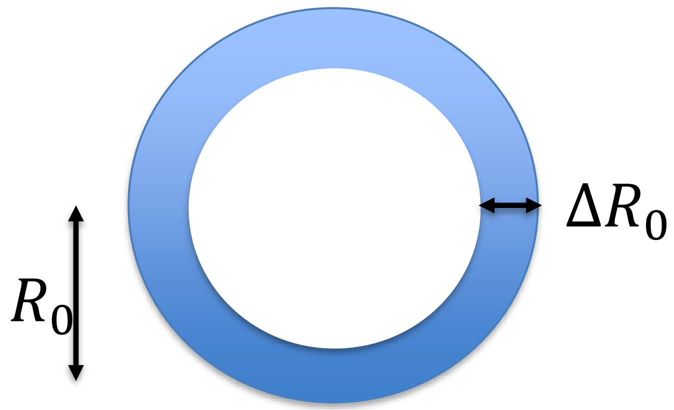
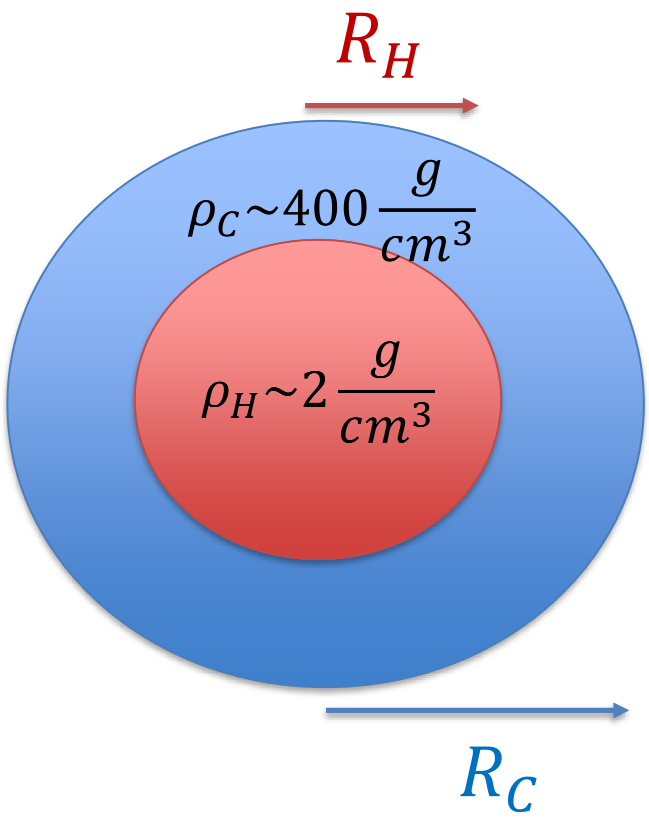
  - Radius of shell changes as fuel implodes inwards
- Implosion Velocity Scaling
  - $v_{imp} \sim v_a A$
  - Aspect Ratio: $A \approx \frac{R_0}{\Delta R_0}$
    - Thin shell drives faster implosions
  - Ablation Velocity: v_a
    - speed at which shell recedes
    - related to hohlraum temperature



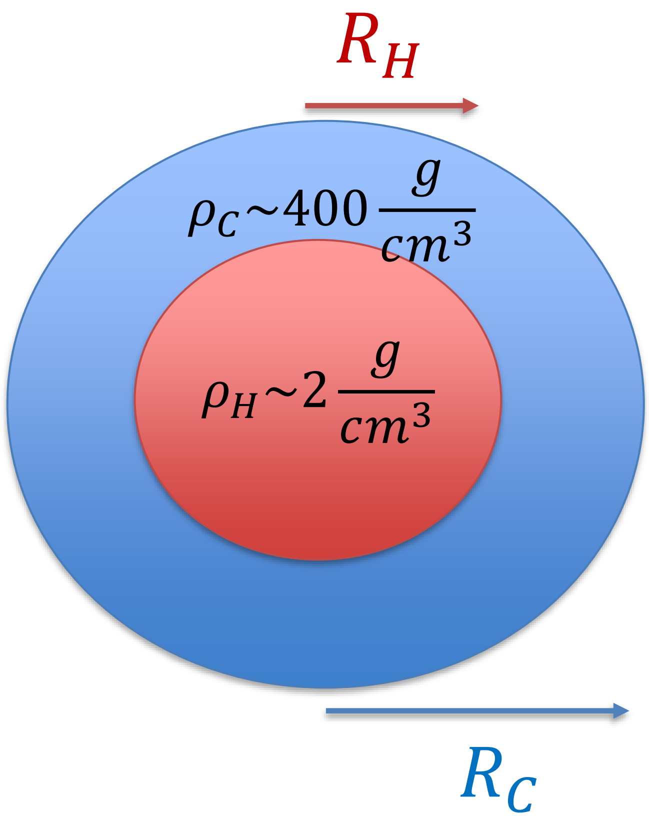
Ignition and Burn

- Kinetic Energy of Imploding Shell goes to Internal Energy of DT Fuel.
  - $\mathrm{KE} = \frac{1}{2} M v_{imp}^2$



Ignition and Burn

- Kinetic Energy of Imploding Shell goes to Internal Energy of DT Fuel.
  - $\mathrm{KE} = \frac{1}{2} M v_{imp}^2$
- Ignition Condition: $n\tau > \frac{10^{15} s}{cm^3}$
  - $n\tau = \frac{\rho}{m} \frac{R}{v_{th}} \sim \rho R$: in $\frac{g}{cm^2}$
- How to Quantify how much is burned?: Φ
  - Burn Efficiency: $\Phi \equiv \frac{\rho R}{\rho R + H_B}$
  - H_B is the burn parameter



Gain and Yield

- Given ρR and T_H, we know fraction of fuel that gets burned Φ
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  - $\mathrm{Q} = 3.5$ MeV per DT pair or 67 MJ/mg
- Multiply by total fuel mass M_f

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- Alpha Particles cause heating:
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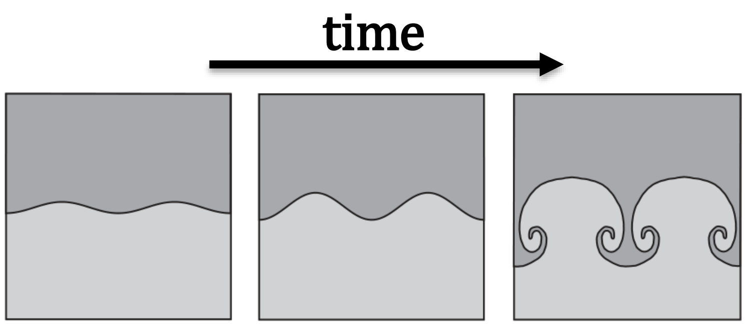
- Multiply by total fuel mass M_f
- Fusion Energy Yield in MJ is "Y"
  - $Y = M_f Q \Phi$
- Gain: $G \equiv \frac{Y}{E_L}$
  - E_L is the laser energy on target

Instabilities and Symmetry



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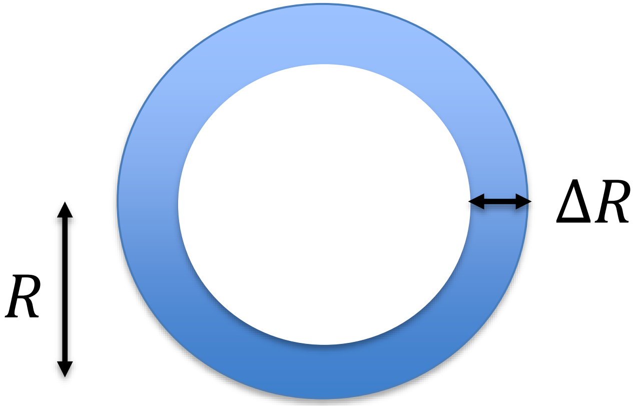
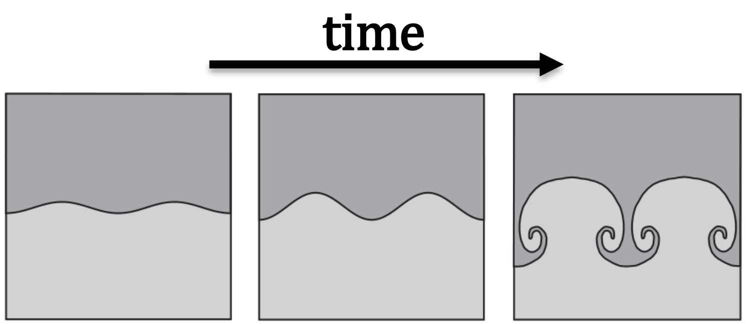
Rayleigh-Taylor Instability



Aspect Ratio: A $\equiv \boldsymbol{R/\Delta R}$

Instabilities and Symmetry



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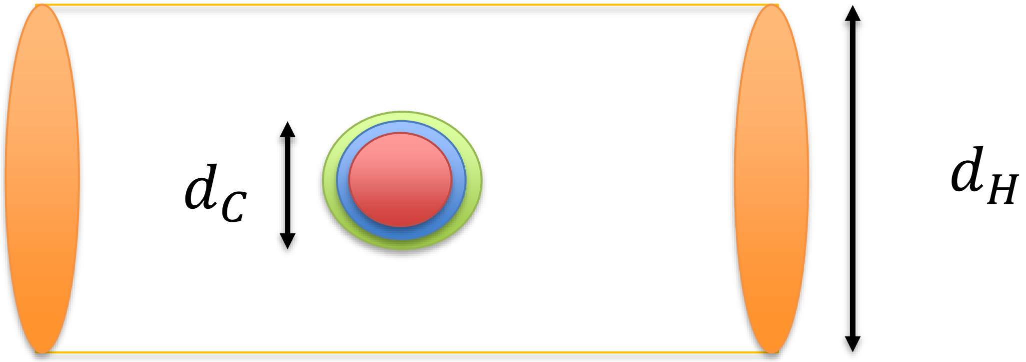
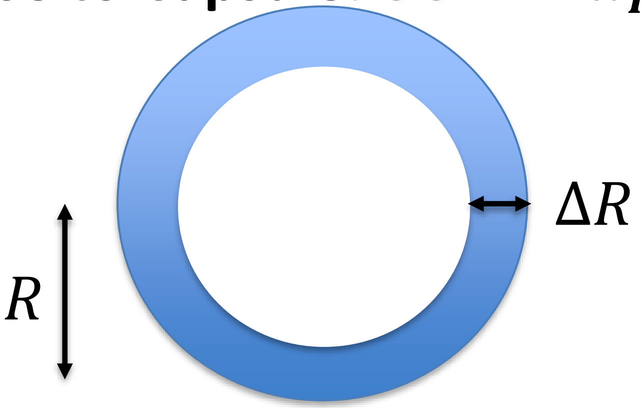


Case to Capsule: $\boldsymbol{CCR = d_H/d_C}$



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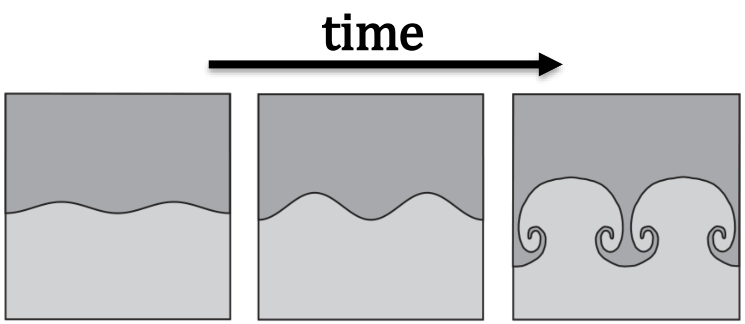

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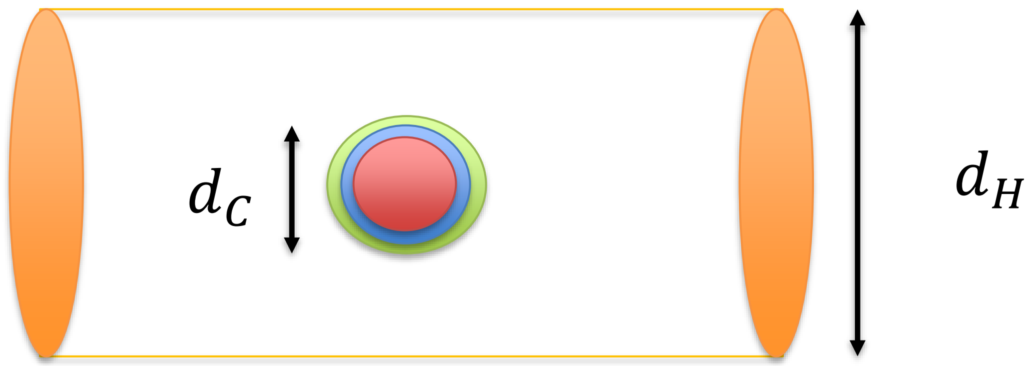
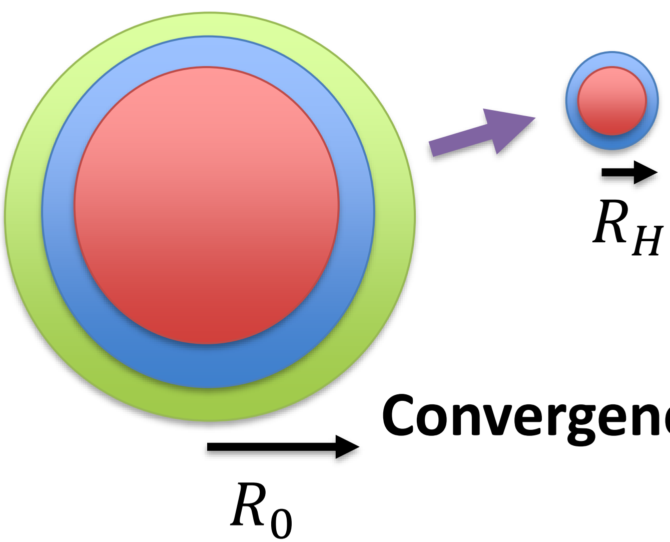
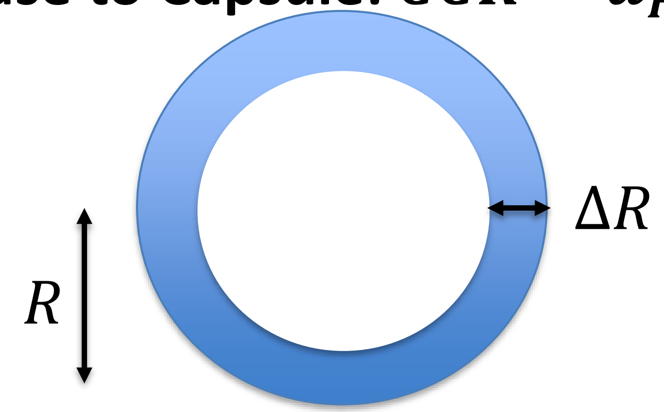
Rayleigh-Taylor Instability



Case to Capsule: $CCR = d_H/d_C$



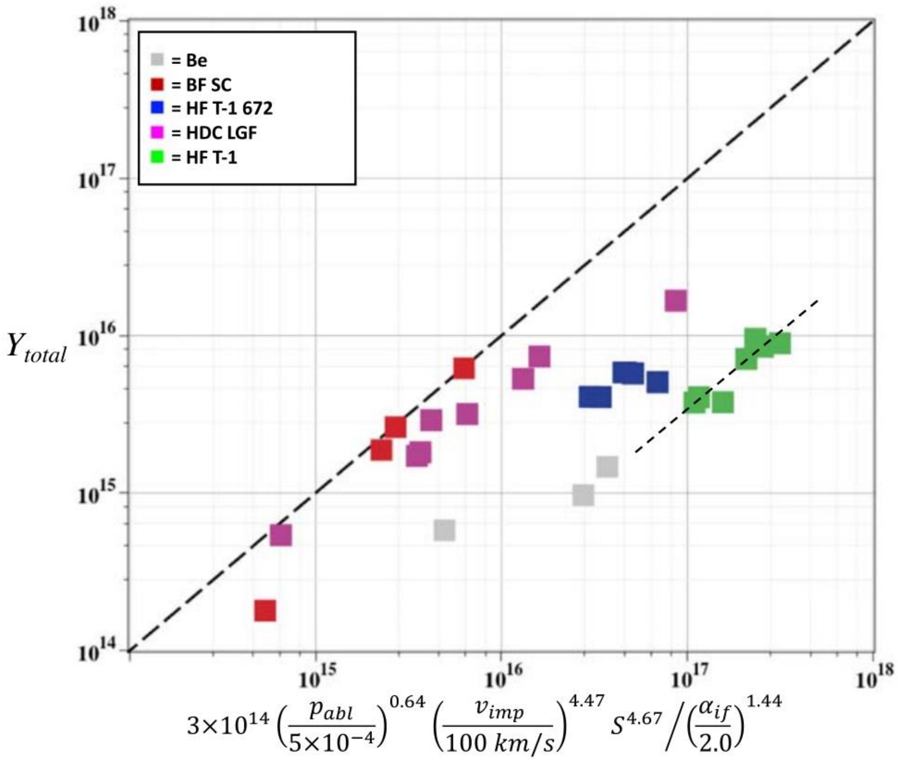
Convergence: $C \equiv R_0/R_H$



Aspect Ratio: $\mathrm{A} \equiv R/\Delta R$

Neutron Yield Scaling

$$Y \sim P_{abl}^{0.64} \frac{v_{imp}^{4.47}}{\alpha^{1.44}} S^{4.67}$$



Plasma Phys. Control. Fusion 61 (2019)

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- P_{abl}: Ablation Pressure
  - Laser Energy
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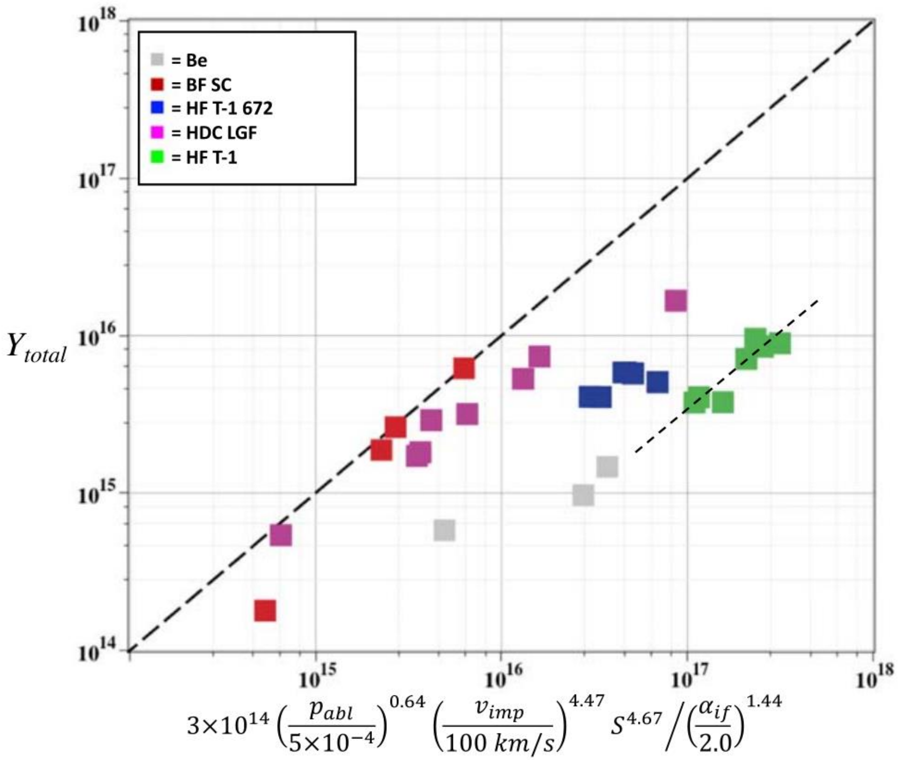
  - Aspect Ratio



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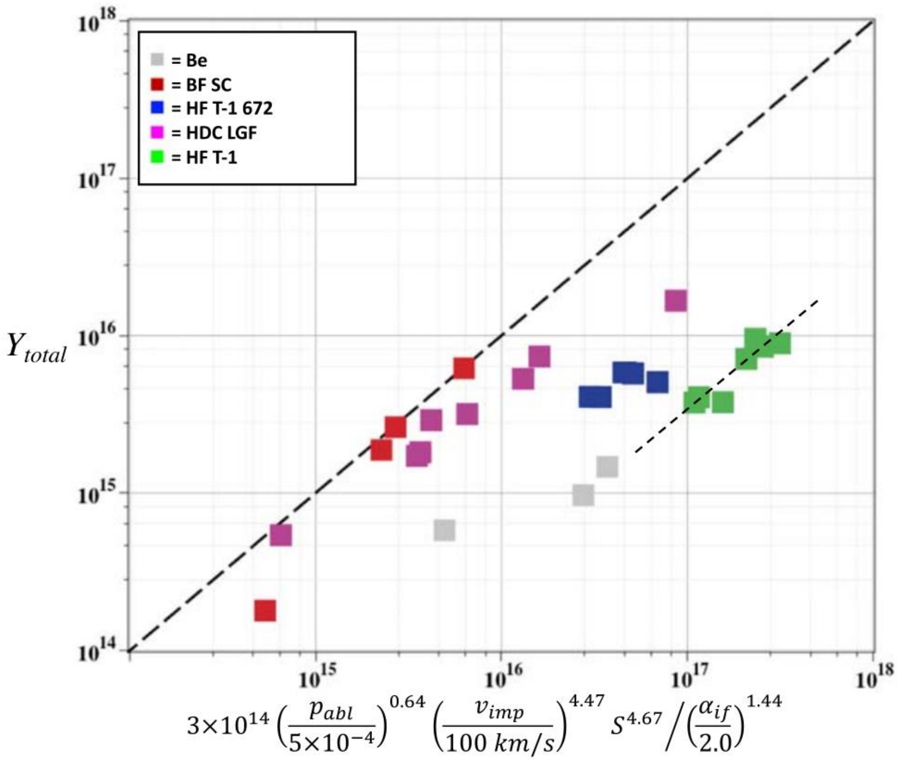
- P_{abl}: Ablation Pressure
  - Laser Energy
- v_{imp}: Implosion Velocity
  - Aspect Ratio
- S : Scale
  - Mass and Radius
- α : Isentrope Parameter
  - Laser Pulse Profile

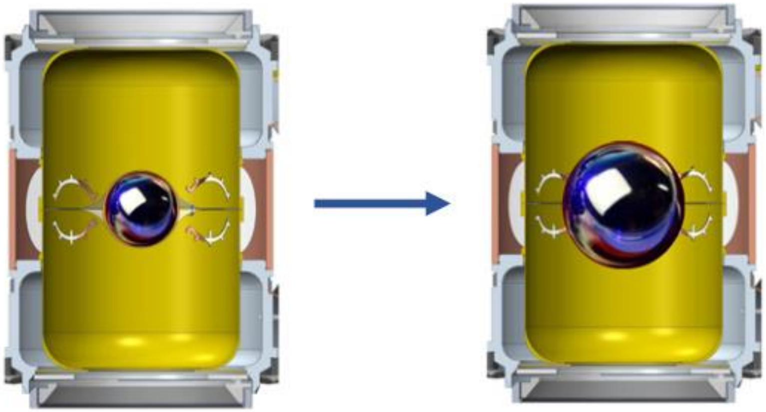


Plasma Phys. Control. Fusion 61 (2019)

Hybrid-E Campaign

- High Yield Big Radius Implosion Design
  - Increased Scale of NIF capsules ~15%
  - Kept Hohlraum Size Same
  - Differences in P_{abl}, v_{imp}, α negligible

Phys. Plasmas 26, 052704 (2019)



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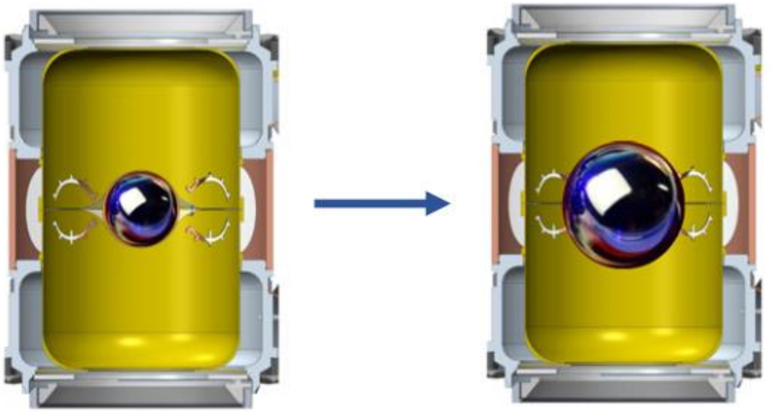
- N170827 (HDC Campaign)
  - $R \approx 910\ \mu m$
  - $Y = 0.053$ MJ
- N210207 (HYBRID-E Campaign)
  - $R \approx 1050\ \mu m$
  - $Y = 0.174$ MJ

Phys. Plasmas 26, 052704 (2019)



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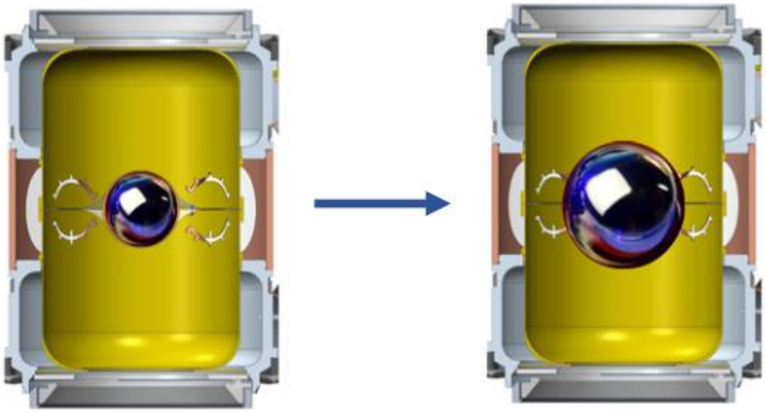
- N210207 (HYBRID-E Campaign)
  - $R \approx 1050\ \mu m$
  - $Y = 0.174$ MJ
- Over 3x increase in yield, scaling predicts 2x increase
  - Scaling works best within same campaign

Phys. Plasmas 26, 052704 (2019)



$$\frac{Y_{21}}{Y_{17}} \sim \left(\frac{R_{21}}{R_{17}}\right)^{4.67}$$
$$= (1.15)^{4.67} \approx 2$$

Conclusion

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- N210207->N210808 Shot
  - 8x gain increase: same capsule size
  - mainly due to engineering advances
- Future of NIF
  - N210808->N221204 had G = 0.72 -> 1.5
  - N221204->N23???? has G=1.5 -> ???
    - 8% thicker ablator, 8% increase in laser energy
    - Symmetry Improvements

References

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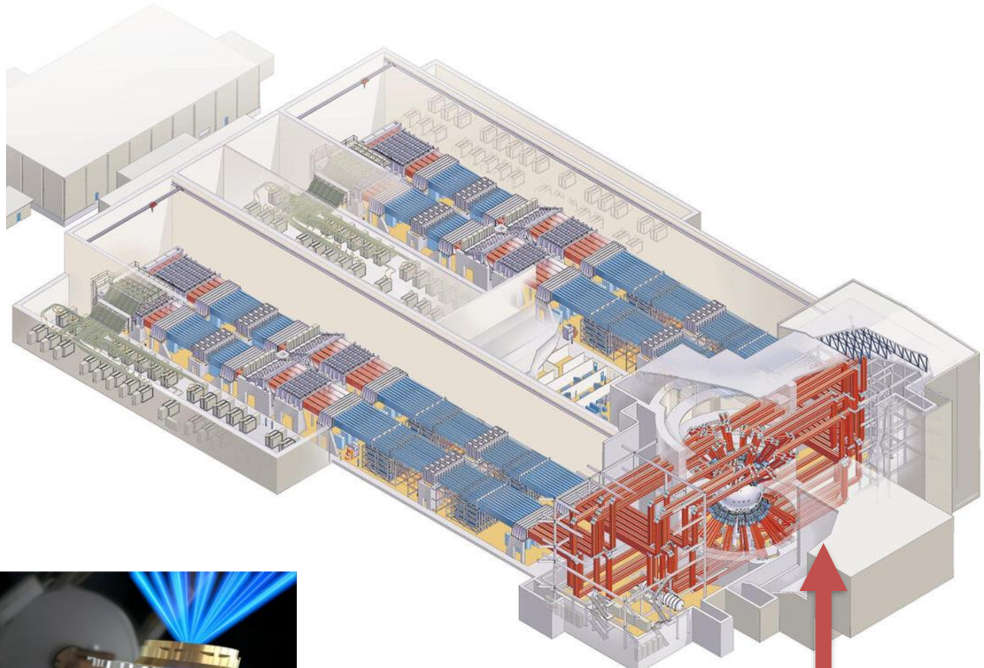
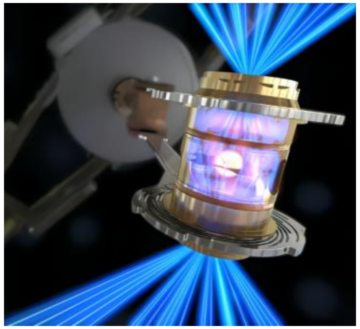
8. S. Le Pape et al., Fusion energy output greater than the kinetic energy of an imploding shell at the national ignition facility, Phys. Rev. Lett. 120, 245003 (2018)
9. O. Hurricane et al., Approaching a burning plasma on the nif, Physics of Plasmas 26, 052704 (2019)
10. A. L. Kritcher et al., Design of an inertial fusion experiment exceeding the lawson criterion for ignition, Phys. Rev. E 106, 025201 (2022)
11. IAEA Webinar Explores NIF's Ignition and Energy Gain Breakthroughs (llnl.gov)

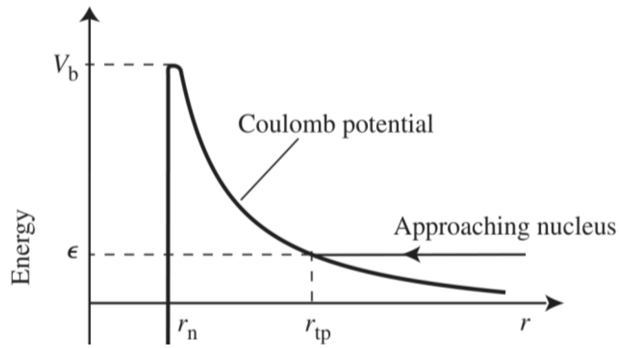
National Ignition Facility

- Size of a Sports Stadium
- 192 laser beams
  - angles: 23, 30, 40, 50
  - ~2 MJ peak energy
  - ~500 TW Peak Power
  - Nd-glass laser
  - $351\mu m$ (frequency tripled)
  - 1 shot every 8 hours





Tunneling Coulomb Barrier

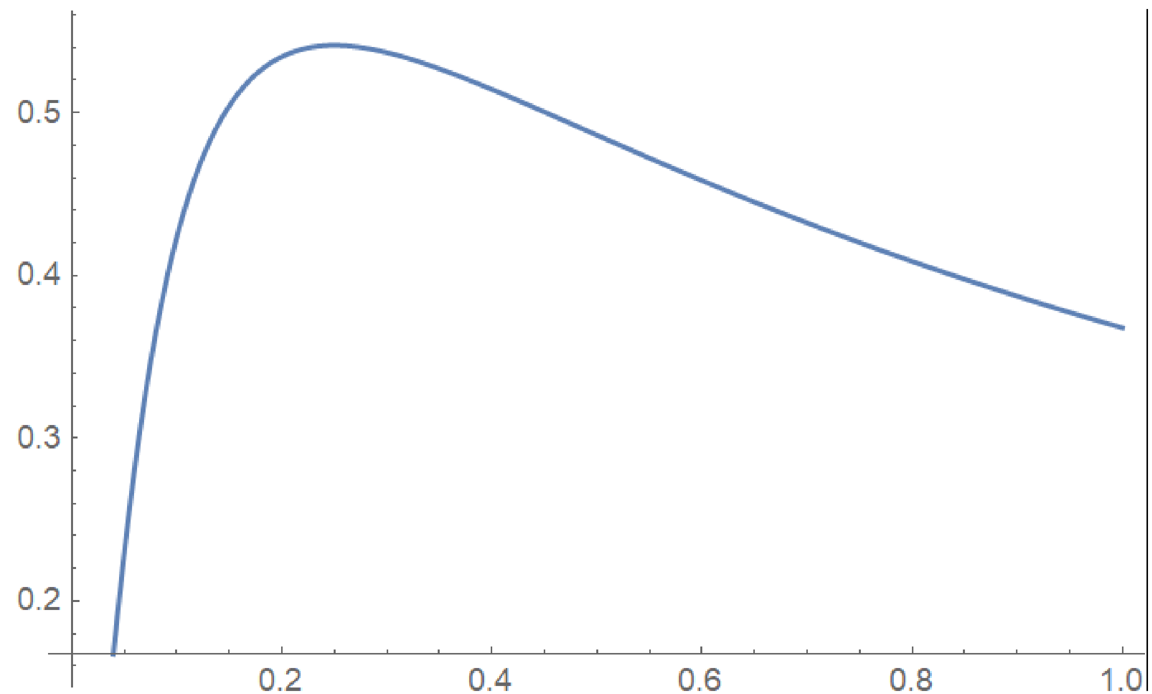


- $\Psi'' = \frac{2m\,(V-E)}{\hbar^2}\Psi$ — (1) Schrodinger Equation
- $\Psi = e^{-\phi(x)}$ — (2) Assume form of Ψ
- $-\phi''(x) + \phi'(x)^2 = \frac{2m\,(V-E)}{\hbar^2}$ — (3) $\phi''(x) = 0$ (slowly varies)
- $\phi(x) \approx \int_{x_0}^{x} \sqrt{\frac{2m\,(V-E)}{\hbar^2}}\,\mathrm{dx}'$ — (4) WKB (w/ eq. (2))
- $V(r) = \frac{e^2}{4\,\pi\epsilon_0 r}, E = \frac{e^2}{4\,\pi\epsilon_0 r_{tp}}$ — (5) For 1D Z=1 Barrier
- $\phi(r_{tp}) \sim \sqrt{r_{tp}} \sim {}^{1}/_{\sqrt{E}}$ — (6) Apply eq. (4) to eq. (5)
- $\Psi(r_{tp}) \sim e^{{}^{1}/_{\sqrt{E}}}$ — (7) Apply eq. (2) to eq. (6)

Fusion Cross Section

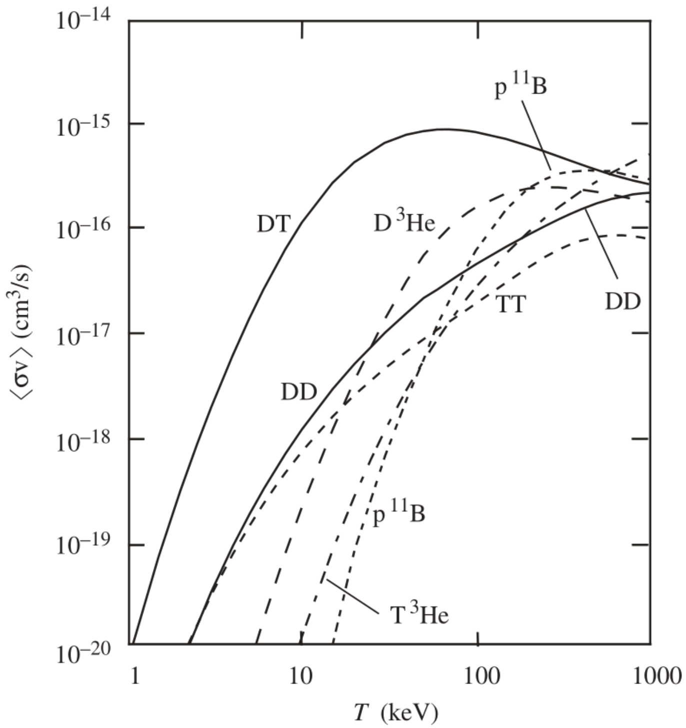
- $\sigma_{geo} \sim \lambda^2 \sim p^{-2} \sim \frac{1}{\epsilon}$
- $P_{tun} \sim e^{-\sqrt{\frac{1}{\epsilon}}}$

```
Plot[Exp[-1/√ϵ]/ϵ, {ϵ, 0, 1}]
```



Reactivity

$$\langle \sigma v \rangle_{DT} = \begin{cases} 4.2 \times 10^{-20} (T_{keV})^4 cm^3 s^{-1} & \text{if } 3 < T_{keV} < 6 \\ 1.1 \times 10^{-18} (T_{keV})^2 cm^3 s^{-1} & \text{if } 8 < T_{keV} < 25 \end{cases}$$



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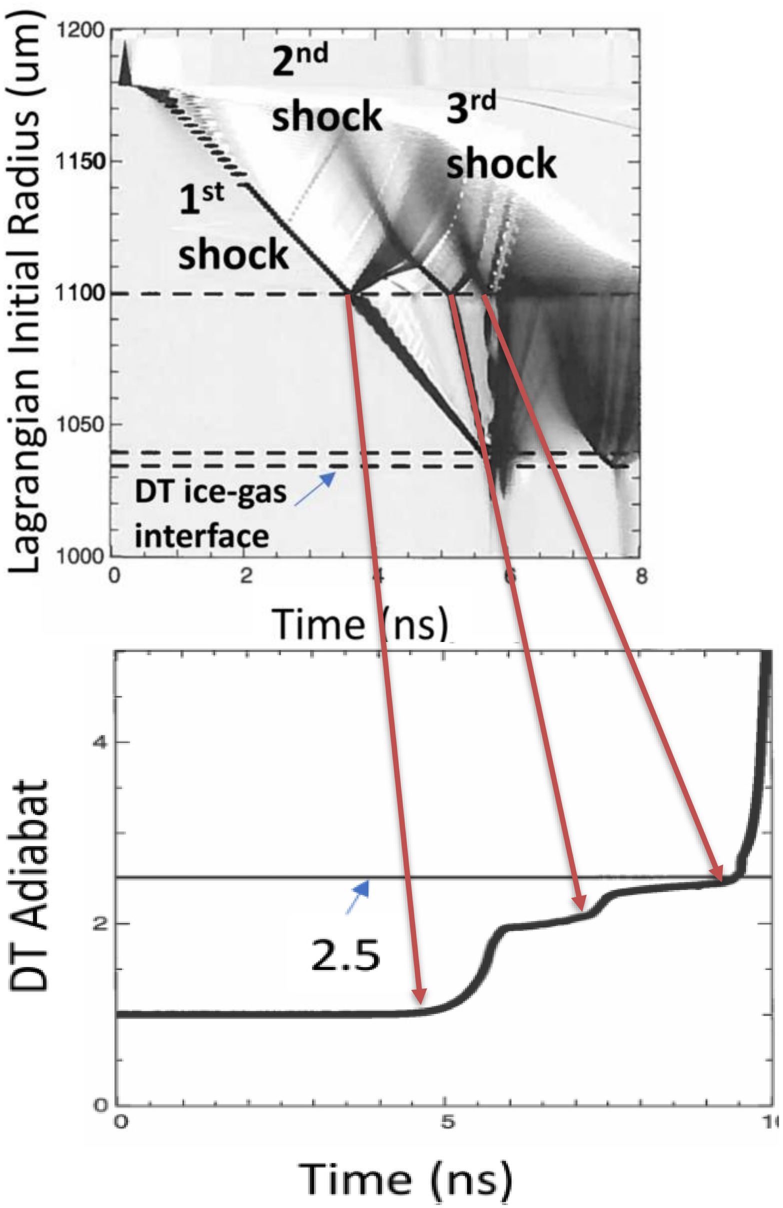
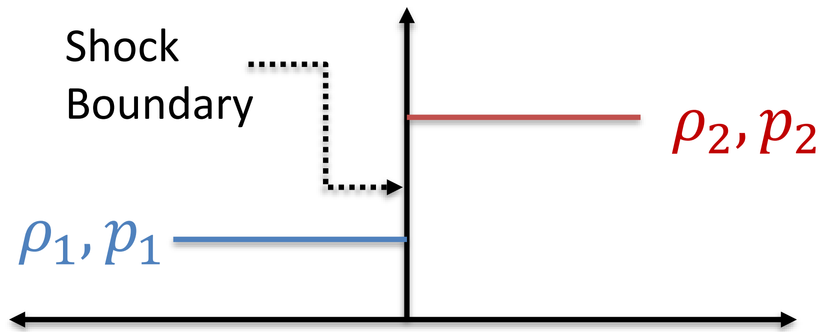
$$\langle \sigma v \rangle = \frac{4\pi}{(2\pi m_r)^{1/2}} \frac{1}{(k_B T)^{3/2}} \int_0^\infty \sigma(\epsilon)\, \epsilon\, \exp(-\epsilon/k_B T) d\epsilon.$$



Shocks Increasing DT Fuel Adiabat (Isentrope Parameter)

Phys. Plasmas 28, 072706 (2021)

1D Hydrodynamics



- Laser pulse is short and intense: Can't compress adiabatically!
  - Shock Waves Drive Compression
- Euler Equations of Hydrodynamics across shock interface
- Ex) Want 4x compression:
  - $\frac{p_2}{p_1} \approx 10$ isentropically
  - $\frac{p_2}{p_1} \rightarrow \infty$ for 1 shock!

Mass: $$\rho_1 v_1 = \rho_2 v_2$$

Momentum: $$p_1 + \rho_1 v_1^2 = p_2 + \rho_2 v_2^2$$

Energy: $$e_1 + p_1/\rho_1 + v_1^2/2 = e_2 + p_2/\rho_2 + v_2^2/2$$

$$V \equiv 1/\rho \qquad \mathrm{e} = \frac{3}{2} pV$$

$$\frac{p_2}{p_1} = \frac{4 - V_2/V_1}{4V_2/V_1 - 1}$$

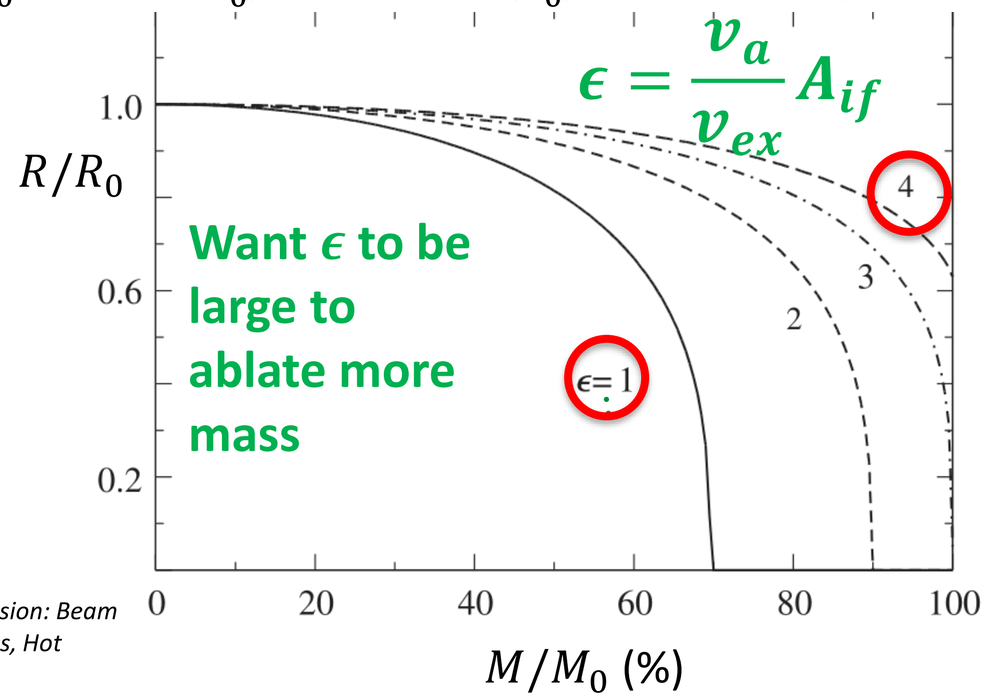
$$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^{5/3}$$

Spherical Rocket

- 1D Rocket Model
  - $M\frac{dv_{imp}}{dt} = v_{ex}\frac{dM}{dt} \rightarrow v_{imp} = v_{ex}\ln\left(\frac{M_0}{M}\right)$ Standard Rocket Equation
  - $\frac{dM}{dt} = -4\pi R^2 \dot{m}_a \rightarrow 1 - \frac{M}{M_0}\left(1 - \ln\frac{M}{M_0}\right) = \frac{\epsilon}{3}(1 - \left(\frac{R}{R_0}\right)^3)$

$$\epsilon = \frac{v_a}{v_{ex}} A_{if}$$

- $v_{imp} \sim v_a A_{if}$
  - Aspect Ratio: $A_{if} \approx \frac{R_0}{\Delta R_0}$
  - Ablation Velocity: v_a
    - speed at which shell recedes



Want ϵ to be large to ablate more mass

S. Atzeni, *The Physics of Inertial Fusion: Beam Plasma Interaction, Hydrodynamics, Hot Dense Matter* (2004)

Example: Yield/Gain Calculation

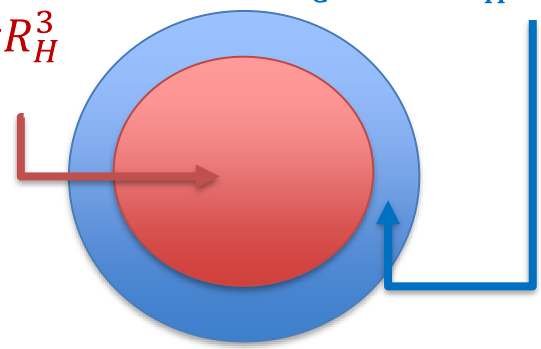
EX) $\rho R = 2\frac{g}{cm^2}, H_B = 6\frac{g}{cm^2},\ \mathrm{M_f} = 0.2\ \mathrm{mg}, \mathrm{E_L} = 1.9\ \mathrm{MJ}$

- $\Phi = \frac{2}{2+6} = 0.25$
- $Y = 0.2 \times 67 \times 0.25 = 3.35\ MJ$
- G = $\frac{3.35}{1.9} = 1.76$
    - More energy in than out!
- But: 400 MJ Laser -> 1.9MJ on target
- Need larger size/mass target for practical fusion power plant

Simplified Scaling Estimate

$$V_C = 4\pi R_H^2 \Delta R$$

$$V_H = \frac{4}{3}\pi R_H^3$$



1. Energy Balance (Assume $p_H = p_S$)

$$\frac{3}{2}pV_H + \frac{3}{2}pV_S = \frac{1}{2}M_C v_{imp}^2$$

$$p = \rho \frac{\Delta R}{R_H} v_{imp}^2$$

2. Partially Fermi Degenerate Shell

$$p \propto \alpha\, \rho_C^{5/3}$$

$$\rho_C \propto \left(\frac{\Delta R}{R_H}\right)^{3/2} \frac{1}{\alpha^{3/2}} v_{imp}^3$$

$$\frac{p}{p_D} \equiv \alpha, \quad p_D \equiv \frac{(3\pi^2)^{\frac{2}{3}}\hbar^2}{5m_e}(\rho)^{5/3}$$

3. Areal Density

$$\rho_C R_H \sim \frac{v_{imp}^3}{\alpha^{3/2}} S$$

$$M_C \sim R_0^3 \sim S^3$$

4. Yield

$$\boxed{Y \sim \Phi M_C \sim \rho_C R_H M_C \sim \frac{v_{imp}^3}{\alpha^{1.5}} S^4}$$

Compare to

$$Y \sim P_{abl}^{0.64} \frac{v_{imp}^{4.47}}{\alpha^{1.44}} S^{4.67}$$