Χειμερινό Εξάμηνο 2009-2010

Πανεπιστήμιο Κρήτης
Τμήμα Επιστήμης Υπολογιστών
ΗΥ-110 Απειροστικός Ι
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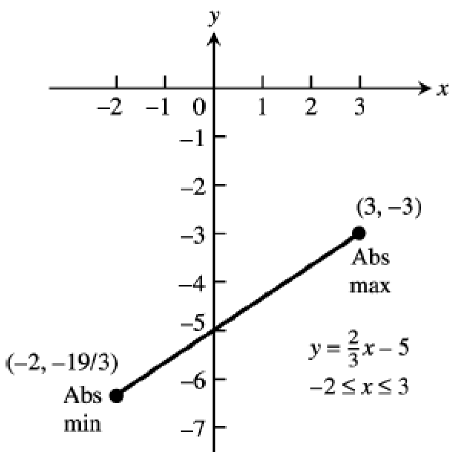
<u>Άσκηση 1η</u>

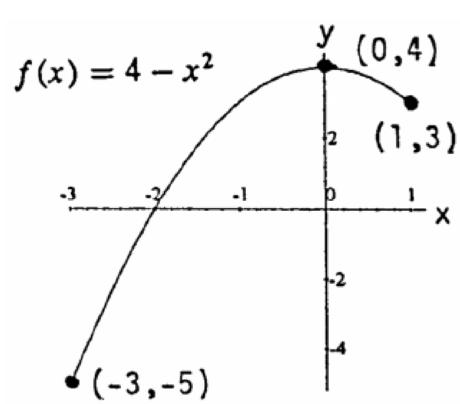
(α)

$f(x) = \frac{2}{3}x - 5 \Rightarrow f'(x) = \frac{2}{3} \Rightarrow$ no critical points; $f(-2) = -\frac{19}{3}$, $f(3) = -3 \Rightarrow$ the absolute maximum is -3 at $x = 3$ and the absolute minimum is $-\frac{19}{3}$ at $x = -2$



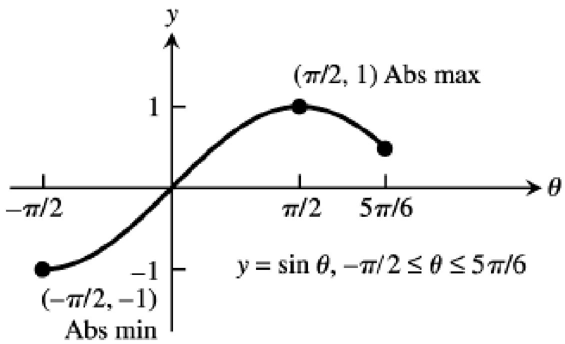
(β)

$f(x) = 4 - x^2 \Rightarrow f'(x) = -2x \Rightarrow$ a critical point at $x = 0$; $f(-3) = -5$, $f(0) = 4$, $f(1) = 3 \Rightarrow$ the absolute maximum is 4 at $x = 0$ and the absolute minimum is -5 at $x = -3$



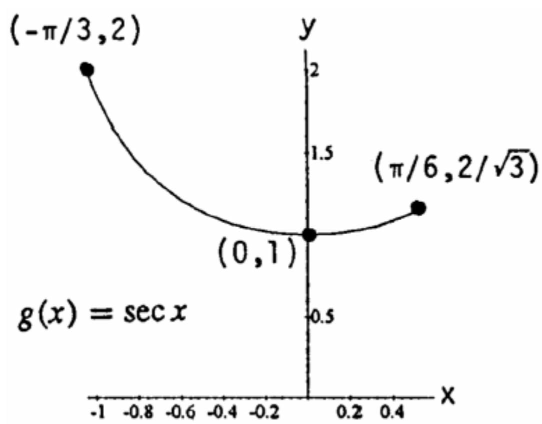
(γ)

$f(\theta) = \sin\theta \Rightarrow f'(\theta) = \cos\theta \Rightarrow \theta = \frac{\pi}{2}$ is a critical point, but $\theta = \frac{-\pi}{2}$ is not a critical point because $\frac{-\pi}{2}$ is not interior to the domain; $f\left(\frac{-\pi}{2}\right) = -1$, $f\left(\frac{\pi}{2}\right) = 1$, $f\left(\frac{5\pi}{6}\right) = \frac{1}{2}$ $\Rightarrow$ the absolute maximum is 1 at $\theta = \frac{\pi}{2}$ and the absolute minimum is -1 at $\theta = \frac{-\pi}{2}$



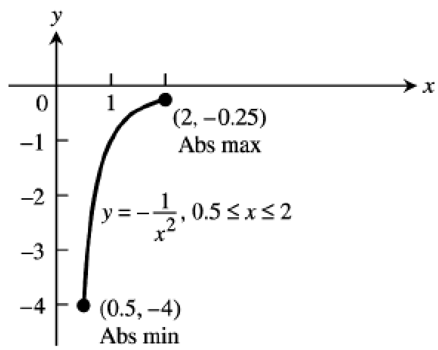
(δ)

$g(x) = \sec x \Rightarrow g'(x) = (\sec x)(\tan x) \Rightarrow$ a critical point at $x = 0$; $g\left(-\frac{\pi}{3}\right) = 2$, $g(0) = 1$, $g\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}} \Rightarrow$ the absolute maximum is 2 at $x = -\frac{\pi}{3}$ and the absolute minimum is 1 at $x = 0$



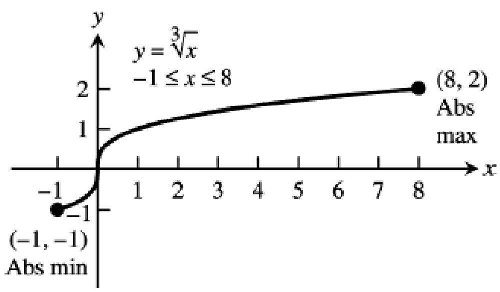
(ε)

$F(x) = -\frac{1}{x^2} = -x^{-2} \Rightarrow F'(x) = 2x^{-3} = \frac{2}{x^3}$, however $x = 0$ is not a critical point since 0 is not in the domain; $F(0.5) = -4$, $F(2) = -0.25 \Rightarrow$ the absolute maximum is -0.25 at $x = 2$ and the absolute minimum is -4 at $x = 0.5$



(στ)

$h(x) = \sqrt[3]{x} = x^{1/3} \Rightarrow h'(x) = \frac{1}{3}x^{-2/3} \Rightarrow$ a critical point at $x = 0$; $h(-1) = -1$, $h(0) = 0$, $h(8) = 2 \Rightarrow$ the absolute maximum is 2 at $x = 8$ and the absolute minimum is -1 at $x = -1$



Άσκηση 2

$s = -\frac{1}{2}gt^2 + v_0t + s_0 \Rightarrow \frac{ds}{dt} = -gt + v_0 = 0 \Rightarrow t = \frac{v_0}{g}$. Now $s(t) = s_0 \Leftrightarrow t\left(-\frac{gt}{2} + v_0\right) = 0 \Leftrightarrow t = 0$ or $t = \frac{2v_0}{g}$.
Thus $s\left(\frac{v_0}{g}\right) = -\frac{1}{2}g\left(\frac{v_0}{g}\right)^2 + v_0\left(\frac{v_0}{g}\right) + s_0 = \frac{v_0^2}{2g} + s_0 > s_0$ is the <u>maximum</u> height over the interval $0 \le t \le \frac{2v_0}{g}$.

Άσκηση 3

(α) Does not; $f(x)$ is not differentiable at $x = 0$ in $(-1, 8)$.

(β) Does; $f(x)$ is continuous for every point of $[0, 1]$ and differentiable for every point in $(0, 1)$.

(γ) Does; $f(x)$ is continuous for every point of $[0, 1]$ and differentiable for every point in $(0, 1)$.

(δ) Does not; $f(x)$ is not continuous at $x = 0$ because $\lim_{x \to 0^-} f(x) = 1 \neq 0 = f(0)$.

Άσκηση 4

Yes. By Corollary 2 we have $f(x) = g(x) + c$ since $f'(x) = g'(x)$. If the graphs start at the same point $x = a$, then $f(a) = g(a) \Rightarrow c = 0 \Rightarrow f(x) = g(x)$.

Άσκηση 5

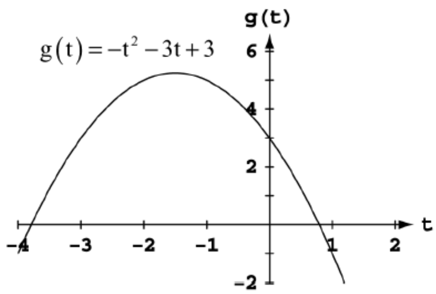
I)

(a) $g(t) = -t^2 - 3t + 3 \Rightarrow g'(t) = -2t - 3 \Rightarrow$ a critical point at $t = -\frac{3}{2}$; $g' = \underset{-3/2}{+++ \mid ---}$, increasing on $\left(-\infty, -\frac{3}{2}\right)$, decreasing on $\left(-\frac{3}{2}, \infty\right)$

(b) local maximum value of $g\left(-\frac{3}{2}\right) = \frac{21}{4}$ at $t = -\frac{3}{2}$

(c) absolute maximum is $\frac{21}{4}$ at $t = -\frac{3}{2}$

(d)



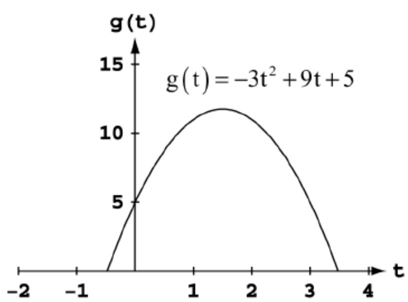
II)

(a) $g(t) = -3t^2 + 9t + 5 \Rightarrow g'(t) = -6t + 9 \Rightarrow$ a critical point at $t = \frac{3}{2}$; $g' = \underset{3/2}{+++ \mid ---}$, increasing on $\left(-\infty, \frac{3}{2}\right)$, decreasing on $\left(\frac{3}{2}, \infty\right)$

(b) local maximum value of $g\left(\frac{3}{2}\right) = \frac{47}{4}$ at $t = \frac{3}{2}$

(c) absolute maximum is $\frac{47}{4}$ at $t = \frac{3}{2}$

(d)



III)

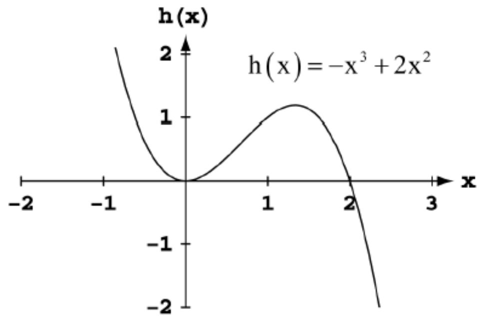
(a) $h(x) = -x^3 + 2x^2 \Rightarrow h'(x) = -3x^2 + 4x = x(4 - 3x) \Rightarrow$ critical points at $x = 0, \frac{4}{3}$

$\Rightarrow h' = \underset{0}{--- \mid} \underset{4/3}{+++ \mid} ---$, increasing on $\left(0, \frac{4}{3}\right)$, decreasing on $(-\infty, 0)$ and $\left(\frac{4}{3}, \infty\right)$

(b) local maximum value of $h\left(\frac{4}{3}\right) = \frac{32}{27}$ at $x = \frac{4}{3}$; local minimum value of $h(0) = 0$ at $x = 0$

(c) no absolute extrema

(d)



IV)

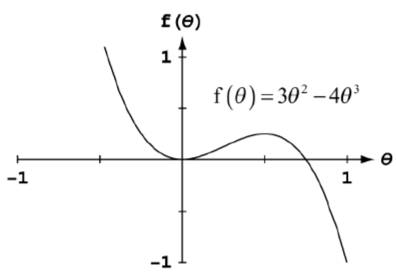
(a) $f(\theta) = 3\theta^2 - 4\theta^3 \Rightarrow f'(\theta) = 6\theta - 12\theta^2 = 6\theta(1 - 2\theta) \Rightarrow$ critical points at $\theta = 0, \frac{1}{2} \Rightarrow f' = \underset{0}{---|}\underset{1/2}{+++|}---$,

increasing on $\left(0, \frac{1}{2}\right)$, decreasing on $(-\infty, 0)$ and $\left(\frac{1}{2}, \infty\right)$

(b) a local maximum is $f\left(\frac{1}{2}\right) = \frac{1}{4}$ at $\theta = \frac{1}{2}$, a local minimum is $f(0) = 0$ at $\theta = 0$

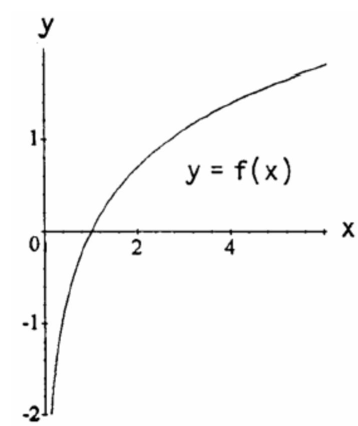
(c) no absolute extrema

(d)



<u>Άσκηση 6</u>

The graph must be concave down for $x > 0$ because $f''(x) = -\frac{1}{x^2} < 0$.



Άσκηση 7

(a) $T = 2\pi\left(\frac{L}{g}\right)^{1/2} \Rightarrow dT = 2\pi\sqrt{L}\left(-\frac{1}{2}g^{-3/2}\right)dg = -\pi\sqrt{L}\,g^{-3/2}\,dg$

(b) If g increases, then $dg > 0 \Rightarrow dT < 0$. The period T decreases and the clock ticks more frequently. Both the pendulum speed and clock speed increase.

(c) $0.001 = -\pi\sqrt{100}\left(980^{-3/2}\right)dg \Rightarrow dg \approx -0.977$ cm/sec^2 $\Rightarrow$ the new $g \approx 979$ cm/sec^2