

$dW = F_T ds = m \frac{dv}{dt} ds = m v dv$

$L_{gravitazionale}: L = \int_A^B \vec{F}_{grav} \cdot d\vec{s} = -mg \int_A^B dz = -mg(z_B - z_A)$

$L = mg(z_A - z_B) = (r_A - r_B) = -\Delta r$

$L_{forza elastica}: L = \int_A^B -Kx dx = -\frac{Kx^2}{2} \Big|_A^B = -\frac{K}{2}(x_B^2 - x_A^2)$

$L_{forza statica}: L = \int_A^B \vec{f} \cdot d\vec{s} = \int_A^B \mu_s N \hat{u}_v \cdot d\vec{s} = \mu_s N \int_A^B ds = \mu_s N \Delta s$

$dW = \vec{F} \cdot d\vec{s} = F_x dx + F_y dy + F_z dz = -d\epsilon_p$

$\oint dW = 0 \Rightarrow \exists f(x,y,z) \text{ t.c. } F_x = -\frac{\partial f}{\partial x}, F_y = -\frac{\partial f}{\partial y}, F_z = -\frac{\partial f}{\partial z}$

$F = -\vec{\nabla} \epsilon_p$

$\rightarrow F = mg \Rightarrow \frac{d\epsilon_p}{dz} = F \Rightarrow \epsilon_p = mgy$

$\rightarrow F = -Kx \Rightarrow \frac{d\epsilon_p}{dx} = -Kx \Rightarrow \epsilon_p = -\frac{Kx^2}{2}$

$\vec{L}_O = \vec{r}_O \times \vec{p} = \vec{r}_O \times m\vec{v}$

$\vec{r}_O = \vec{OO'} + \vec{r}_{O'}$

$\Rightarrow \vec{L}_O = \vec{OO'} \times \vec{p} + \vec{r}_{O'} \times \vec{p}$

$\vec{L}_{O'} = \vec{r}_{O'} \times \vec{p} + \vec{L}_{O'}$

$\vec{L} = \vec{r} \times m(\vec{v}_r + \vec{v}_e) = \vec{r} \times m\vec{v}_e$

$\vec{M}_O = \vec{r}_O \times \vec{F}$

$\vec{M}_{O'} = \vec{M}_O + \vec{OO'} \times \vec{F}$

$\vec{M} = \sum \vec{r}_i \times \vec{F}_i = \vec{r} \times \sum \vec{F}_i = \vec{r} \times \vec{F}$

$\frac{d\vec{L}}{dt} = \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt} = \vec{v} \times m\vec{v} + \vec{r} \times \vec{F} = \vec{M}$

$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt} \hat{u}_r + r \frac{d\hat{u}_r}{dt} = v \hat{u}_r + r \frac{d\theta}{dt} \hat{u}_\theta$

$\Rightarrow \vec{L} = \vec{r} \times m r \frac{d\theta}{dt} \hat{u}_\theta$

Se  $O$  sta nel piano del moto,  $\vec{r} \perp \hat{u}_\theta \Rightarrow L = m r^2 \frac{d\theta}{dt}$

$F$  centrale  $\Rightarrow L = \text{cost.} \Rightarrow \frac{1}{2} r^2 \frac{d\theta}{dt} = \text{cost.}$

$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt}$

$dA = \frac{1}{2} r^2 d\theta$

$\frac{dA}{dt} = \omega \frac{L}{2m}$

$\frac{dA}{dt} = \frac{L}{2m}$

$\int_{A_0}^A dA = \int_0^t \frac{L}{2m} dt$

$A = \frac{L}{2m} T$  periodo:  $T = \frac{2m A}{L}$

Lavoro forze centrali:  $L = \int_A^B \vec{F} \cdot d\vec{s} = \int_A^B F(r) \hat{u}_r \cdot d\vec{s}$

$d\vec{s} = dr \hat{u}_r \Rightarrow \hat{u}_r \cdot d\vec{s} = dr$

$\Rightarrow L = \int_A^B F(r) dr = f(r_B) - f(r_A)$

$d\vec{r} = r d\theta \hat{u}_\theta$

$|d\vec{r}| = r d\theta = dr$

$\vec{r} = \vec{r}_O + \vec{r}'$

$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d\vec{r}_O}{dt} + \frac{d\vec{r}'}{dt} = \vec{v}_O + \vec{v}' + \vec{\omega} \times (\vec{r}' \cdot \hat{x} + y \cdot \hat{y} + z \cdot \hat{z}) = \vec{v}_O + \vec{v}' + \vec{\omega} \times \vec{r}'$

$\vec{v} = \vec{v}_O + \vec{v}' + \vec{\omega} \times \vec{r}'$