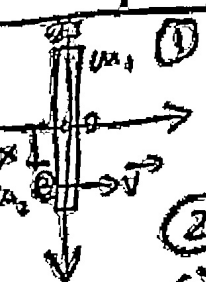


Conservation of angular momentum

$$L_0 = I\omega_0 = \frac{m_1 r^2}{2} \omega_0 = L_1 = \frac{m_1 r^2}{2} \omega + m_2 r^2 \omega$$

$$\frac{m_1 r^2 \omega_0}{2} = \frac{r^2}{2} \omega (m_1 + m_2) \quad \omega = \frac{m_1 \omega_0}{m_1 + 2m_2} = \frac{9}{12} \frac{\text{rad}}{\text{s}} = 4.5 \frac{\text{rad}}{\text{s}}$$



① $m_2 v = (m_1 + m_2) v_{cm} \quad v_{cm} = \frac{m_2 v}{m_1 + m_2} \quad r_{cm} = \frac{x m_1 l}{m_1 + m_2}$

$$(x - r_{cm}) m_2 v = I \omega = \left(\frac{m_1 l^2}{12} + r_{cm}^2 m_1 + m_2 (x - r_{cm})^2 \right) \omega \quad \omega = \frac{(x - r_{cm}) m_2 v}{I}$$

② $m_1 = m_2 = m \quad x = \frac{l}{2} \quad r_{cm} = \frac{l}{4} \quad r_{cm} v = I \omega = \left(\frac{m l^2}{12} + m r^2 \right) \omega$

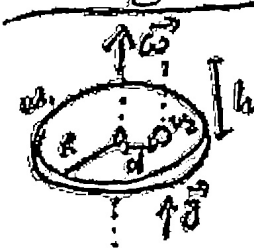
$$\omega = \frac{r v}{\frac{l^2}{12} + r^2} \quad J = \Delta p = m \omega r - m v$$



$$J_x = \Delta p_x = -m_2 v$$

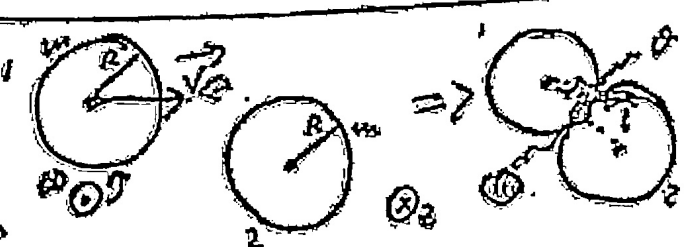
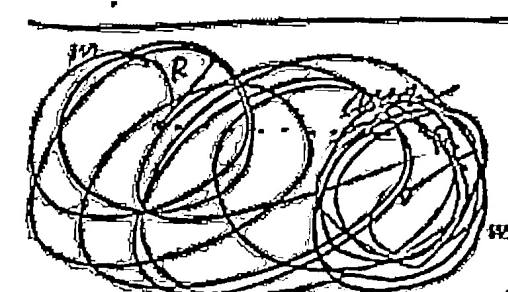
$$I \omega = I' \omega' \quad \frac{m_1 l^2}{12} \omega = \left(\frac{m_1 l^2}{12} + \frac{m_2 l^2}{4} \right) \omega' \quad \omega' = \frac{m_1 \omega}{m_1 + 3m_2}$$

$$J_y = \Delta p_y = m_2 \omega' \frac{l}{2} \quad J = \sqrt{J_x^2 + J_y^2} \quad \theta = \arcsin \frac{J_y}{J}$$



$$L \omega = L' \omega' \quad \frac{1}{2} m_1 R^2 \omega = \left(\frac{1}{2} m_1 R^2 + m_2 d^2 \right) \omega' \quad \omega' = \frac{m_1 R^2 \omega}{m_1 R^2 + 2m_2 d^2}$$

$$J_y = m_2 \sqrt{g h} \quad \text{Impulse angular} = J \omega$$



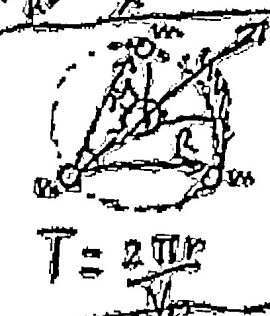
- 1: $m, R, v \neq 0, \omega = 0$
- 2: $m, R, v = 0, \omega \neq 0$
- 3: $m \neq 0, v \neq 0, \omega \neq 0$

$L_f = 0 \Rightarrow \frac{m v}{R} = \frac{R \omega}{2} \Rightarrow \omega = \frac{2v \sin \theta}{R}$

Also $P_{cm} = 0 \quad L_i = R m v \sin \theta - I \omega = \frac{1}{2} m R^2 \omega$

$$\omega = \frac{v \sin \theta}{R} \quad \theta = \frac{\pi}{6}$$

$$\frac{G M m}{R^2} = \frac{v^2}{R} \quad T = \frac{2\pi R}{v} \quad T^2 = \frac{4\pi^2 R^3}{v^2} = \frac{4\pi^2}{G M} R^3$$



$P = m v_{cm} = 0 \quad L = 3 R m v$

$$\frac{m v^2}{R} = \frac{G M m}{R^2} + \frac{2 G m^2}{(2R \sin \frac{\theta}{2})^2} = \frac{G M m}{R^2} + \frac{2 G m^2}{3 R^2}$$

$$v = \sqrt{\frac{R}{3} \left(\frac{M}{R} + \frac{2m}{3} \right)}$$



$$T = \frac{2\pi R}{v}$$

$$E_p = \frac{1}{2} m v^2 = \frac{1}{2} m \left(\frac{R}{3} \left(\frac{M}{R} + \frac{2m}{3} \right) \right)^2 \quad m = \frac{M m}{m + 3M}$$

$$E_p = 3 \frac{G M m}{R} \quad \text{or} \quad \frac{6 m^2}{(M + 3M)^2}$$