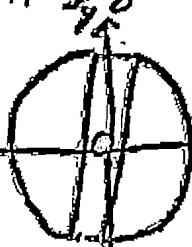


Conservation of angular momentum

$L_0 = I\omega_0 = \frac{m_1 r^2}{2} \omega_0 = L_1 = \frac{m_1 r^2}{2} \omega + m_2 r^2 \omega$

$\frac{m_1 r^2 \omega_0}{2} = \frac{r^2}{2} \omega (m_1 + m_2) \quad \omega = \frac{m_1 \omega_0}{m_1 + 2m_2} = \frac{9}{12} \frac{\text{rad}}{\text{s}} = 4.5 \frac{\text{rad}}{\text{s}}$

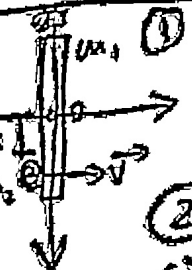


① $m_2 v = (m_1 + m_2) v_{cm} \quad v_{cm} = \frac{m_2 v}{m_1 + m_2} \quad r_{cm} = \frac{x m_1}{m_1 + m_2}$

$(x - r_{cm}) m_2 v = I \omega = \left(\frac{m_1 \ell^2}{12} + r_{cm}^2 m_1 + m_2 (x - r_{cm})^2 \right) \omega \quad \omega = \frac{(x - r_{cm}) m_2 v}{I}$

② $m_1 = m_2 = m \quad x = \ell \quad r_{cm} = \frac{\ell}{2} \quad r_{cm} v = I \omega = \left(\frac{m \ell^2}{12} + m r^2 \right) \omega$


$\omega = \frac{r v}{\frac{\ell^2}{12} + r^2} \quad J = \Delta p = m \omega r - m v$



$J_x = \Delta p_x = -m_2 v$

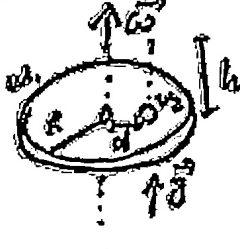
$I \omega = I' \omega' \quad \frac{m_1 \ell^2}{12} \omega = \left(\frac{m_1 \ell^2}{12} + \frac{m_2 \ell^2}{4} \right) \omega' \quad \omega' = \frac{m_1 \omega}{m_1 + 3m_2}$

$J_y = \Delta p_y = m_2 \omega' \frac{\ell}{2} \quad J = \sqrt{J_x^2 + J_y^2} \quad \theta = \arcsin \frac{J_y}{J}$



$L \omega = L' \omega' \quad \frac{1}{2} m_1 R^2 \omega = \left(\frac{1}{2} m_1 R^2 + m_2 d^2 \right) \omega' \quad \omega' = \frac{m_1 R^2 \omega}{m_1 R^2 + 2m_2 d^2}$

$J_y = m_2 \sqrt{g h} \quad \text{Impulse angular} = J \ell$



1: $m, R, v \neq 0, \omega = 0$
 2: $m, R, v = 0, \omega \neq 0$
 3: $m, R, v \neq 0, \omega \neq 0$

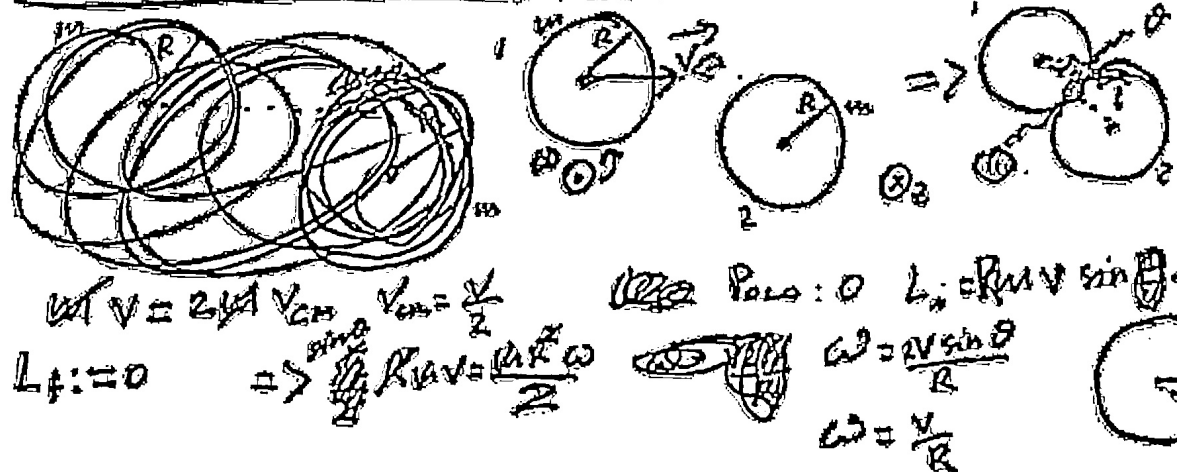
$m v = 2 M v_{cm} \quad v_{cm} = \frac{v}{2}$

$L_f = 0 \Rightarrow \frac{m R v \sin \theta}{2} = \frac{R^2 \omega}{2} \quad \omega = \frac{2 v \sin \theta}{R}$

$\omega = \frac{v}{R}$

$L_i = R M v \sin \theta - I \omega = \frac{3}{2} R M v \sin \theta - \frac{1}{2} m R^2 \omega$

$2 R \sin \theta = \frac{v}{R} \quad \sin \theta = \frac{1}{2} \quad \theta = \frac{\pi}{6}$



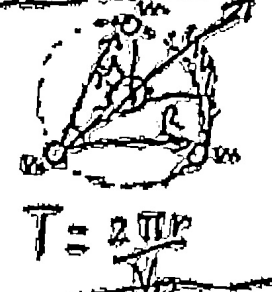
$\frac{G M m}{R^2} = \frac{v^3}{R} \quad T = \frac{2\pi R}{v} \quad T^2 = \frac{4\pi^2 R^3}{v^2} = \frac{4\pi^2}{G M} R^3$

$P = m v_{cm} = 0 \quad L = 3 R m v_{cm}$

$\frac{m v^2}{R} = \frac{G M m}{R^2} + \frac{2 G m^2}{(2 R \sin \frac{\pi}{3})^2} = \frac{G M m}{R^2} + \frac{2 G m^2}{3 R^2}$

$v = \sqrt{\frac{G}{R} (M + \frac{2}{3} m)}$

$T = \frac{2\pi R}{v}$



$E_p = \frac{1}{2} m v^2 = \frac{1}{2} m \left(\frac{G M}{R} + \frac{2 G m}{3 R} \right) = \frac{G M m}{2 R} + \frac{G m^2}{3 R}$