

$dW = F_T ds = m \frac{dv}{dt} ds = m v dv$

$L_{gravitazionale} L = \int_A^B \vec{F}_g \cdot d\vec{s} = -mg \int_A^B dz = -mg(z_B - z_A)$

$L = mg(z_A - z_B) = \Delta E_{pot} = -\Delta U$

$L_{elastica} L = \int_A^B -kx dx = -\frac{k}{2} x^2 \Big|_A^B = -\frac{k}{2} (x_B^2 - x_A^2)$

$L_{forza\ generica} L = \int_A^B \vec{f} \cdot d\vec{s} = \int_A^B \mu_s N \hat{u}_v \cdot d\vec{s} = \mu_s N \int_A^B ds = \mu_s N (s_B - s_A)$

$dW = \vec{F} \cdot d\vec{s} = F_x dx + F_y dy + F_z dz = -dE_p$

$\oint dW = 0 \Rightarrow \exists f(x,y,z) \text{ t.c. } F_x = -\frac{\partial f}{\partial x}, F_y = -\frac{\partial f}{\partial y}, F_z = -\frac{\partial f}{\partial z}$

$\vec{F} = -\nabla E_p \Rightarrow \vec{F} = -mg \Rightarrow \frac{dE_p}{dz} = -mg \Rightarrow E_p = -mgz + C$

$\vec{F} = -kx \Rightarrow \frac{dE_p}{dx} = -kx \Rightarrow E_p = -\frac{1}{2} kx^2 + C$

$\vec{L}_O = \vec{r}_O \times \vec{p} = \vec{r}_O \times m\vec{v}$

$\vec{r}_O = r_O \hat{e}_r + z_O \hat{e}_z \Rightarrow \vec{L}_O = r_O \hat{e}_r \times p \hat{e}_\theta + z_O \hat{e}_z \times p \hat{e}_\theta$

$\vec{L}_O = r_O \hat{e}_r \times p \hat{e}_\theta + L_{Oz} \hat{e}_z$

$\vec{L} = \vec{r} \times m(\vec{v}_r + \vec{v}_\theta) = \vec{r} \times m\vec{v}_\theta$

$\vec{M}_O = \vec{r}_O \times \vec{F}$

$\vec{M}_O = \vec{M}_O + r_O \hat{e}_r \times \vec{F}$

$\vec{M} = \sum \vec{r}_i \times \vec{F}_i = \vec{r} \times \sum \vec{F}_i = \vec{r} \times \vec{F}$

$\frac{d\vec{L}}{dt} = \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt} = \vec{v} \times m\vec{v} + \vec{r} \times \vec{F} = \vec{M}$

$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt} \hat{u}_r + r \frac{d\hat{u}_r}{dt} = v \hat{u}_r + r \frac{d\theta}{dt} \hat{u}_\theta$

$\vec{L} = \vec{r} \times m r \frac{d\theta}{dt} \hat{u}_\theta$

Se $\vec{r} \perp \hat{u}_\theta \Rightarrow L = m r^2 \frac{d\theta}{dt}$

$F \text{ centrale} \Rightarrow \vec{L} = \text{cost.} \Rightarrow \frac{1}{2} r^2 \frac{d\theta}{dt} = \text{cost.}$

$dA = \frac{1}{2} r^2 d\theta \Rightarrow \frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} = \frac{L}{2m}$

$A = \frac{L}{2m} T \text{ periodo: } T = \frac{2\pi A}{L}$

Lavoro forze centriche: $L = \int_A^B \vec{F} \cdot d\vec{s} = \int_A^B F(r) \hat{u}_r \cdot d\vec{s}$

$d\vec{s} = dr \hat{u}_r \Rightarrow \hat{u}_r \cdot d\vec{s} = dr \Rightarrow L = \int_A^B F(r) dr = f(r_B) - f(r_A)$

$d\vec{r} = r d\theta \hat{u}_\theta \quad |d\vec{r}| = r d\theta = dr$

$\vec{r} = r_O \hat{e}_r + r' \hat{e}_r$

$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dr_O}{dt} \hat{e}_r + \frac{dr'}{dt} \hat{e}_r + r' \frac{d\hat{e}_r}{dt} + r' \frac{d\hat{e}_\theta}{dt}$

$= \vec{v}_O + \vec{v}' + \vec{\omega} \times (x' \hat{x} + y' \hat{y} + z' \hat{z}) = \vec{v}_O + \vec{v}' + \vec{\omega} \times \vec{r}'$