

Conservative momenta angular

$L_0 = I\omega_0 = \frac{m_1 r^2}{2} \omega_0 = L_1 = \frac{m_1 r^2}{2} \omega + m_2 r^2 \omega$

$\frac{m_1 r^2 \omega_0}{2} = \frac{r^2}{2} \omega (m_1 + m_2) \quad \omega = \frac{m_1 \omega_0}{m_1 + 2m_2} = \frac{\frac{9}{10} \text{ rad}}{\frac{1}{10}} = 9 \text{ rad}$

① $m_2 v = (m_1 + m_2) v_{cm} \quad v_{cm} = \frac{m_2}{m_1 + m_2} v$

$(x - r_{cm}) m_2 v = I \omega = \left(\frac{m_1 \ell^2}{12} + r_{cm}^2 m_1 + m_2 (x - r_{cm})^2 \right) \omega \quad \omega = \frac{(x - r_{cm}) m_2 v}{I}$

② $m_1 = m_2 = m \quad x = \ell \quad r_{cm} = \frac{\ell}{2} \quad v_{cm} = \frac{v}{2}$

$\omega = \frac{r_{cm} v}{\frac{\ell^2}{12} + m r_{cm}^2} = \frac{\frac{\ell}{2} \cdot \frac{v}{2}}{\frac{\ell^2}{12} + m \frac{\ell^2}{4}} = \frac{6v}{\ell(3m + 2m)} = \frac{2v}{\ell}$

$J = \Delta p = m \omega r - m v$

$J_x = \Delta p_x = -m_2 v$

$I \omega = I' \omega' \quad \frac{m_1 \ell^2}{12} \omega = \left(\frac{m_1 \ell^2}{12} + \frac{m_2 \ell^2}{4} \right) \omega' \quad \omega' = \frac{m_1 \omega}{m_1 + 3m_2}$

$J_y = \Delta p_y = m_2 \omega' \frac{\ell}{2} \quad J = \sqrt{J_x^2 + J_y^2} \quad \theta = \arcsin \frac{J_y}{J}$

$I \omega = I' \omega' \quad \frac{1}{2} m_1 R^2 \omega = \left(\frac{1}{2} m_1 R^2 + m_2 d^2 \right) \omega' \quad \omega' = \frac{m_1 R^2 \omega}{m_1 R^2 + 2m_2 d^2}$

$J_y = m_2 \sqrt{2gh} \quad \text{Impulse angular} = J_y d$

1: $m, R, v=v, \omega=0$
 2: $m, R, v=0, \omega=\omega$
 1,2: C.M. $\equiv 0, v=v_{cm}, \omega=\omega$

$L_i = 0 \quad L_f = R m v \sin \theta - I \omega = \frac{1}{2} m R^2 \omega$

$2R \sin \theta = R \quad \sin \theta = \frac{1}{2} \quad \theta = \frac{\pi}{6}$

$\omega = \frac{2v \sin \theta}{R} = \frac{v}{R}$

$\frac{GM}{R^2} = \frac{v^2}{R} \quad T = \frac{2\pi R}{v} \quad T^2 = \frac{4\pi^2 R^2}{v^2} = \frac{4\pi^2}{GM} R^3$

$E = m v_{cm} = 0 \quad L = 3 R m v_m$

$\frac{m v^2}{R} = \frac{GMm}{R^2} + \frac{2GMm}{(2R \sin \frac{\pi}{3})^2} = \frac{GMm}{R^2} + \frac{2GMm}{3R^2} \quad v = \sqrt{\frac{GM}{R} \left(1 + \frac{2}{3} \right)}$

$T = \frac{2\pi R}{v_m}$

$\frac{1}{\mu} = \frac{1}{M} + \frac{3}{m} = \frac{m+3M}{Mm} \quad \mu = \frac{Mm}{m+3M}$

$E_p = -3 \frac{GMm}{R} \quad E_k = 3 \frac{GMm}{R\sqrt{5}}$