

$dW = \vec{F} \cdot d\vec{s} = m \frac{d\vec{v}}{dt} \cdot d\vec{s} = m \vec{v} \cdot d\vec{v}$   
 $\int_A^B dW = \int_A^B m \vec{v} \cdot d\vec{v} = \frac{1}{2} m v_B^2 - \frac{1}{2} m v_A^2 = \Delta E_k$

**Lavoro forza peso:**  $L = \int_A^B \vec{mg} \cdot d\vec{s} = m\vec{g} \cdot \int_A^B d\vec{s} = m\vec{g} \cdot \vec{AB} = -mg(z_B - z_A) = -\Delta E_p$   
 $L = -mg(z_B - z_A) = E_{pA} - E_{pB} = -\Delta E_p$

**Lavoro forza elastica:**  $L = \int_A^B -kx dx = -\frac{kx^2}{2} \Big|_A^B = -\Delta E_p$ ,  $E_p = \frac{1}{2} kx^2$

**Lavoro forza attrito:**  $L = \int_A^B \vec{f} \cdot d\vec{s} = \int_A^B \mu_d N \hat{u}_v \cdot d\vec{s} = \mu_d N \int_A^B ds = \mu_d N \Delta s$   
 $N$ : forza normale,  $\Delta s$ : percorso

$dW = \vec{F} \cdot d\vec{s} = F_x dx + F_y dy + F_z dz = -dE_p$  se  $\vec{F} = -\nabla E_p$   
 $\oint dW = 0 \Rightarrow \exists f(x, y, z)$  t.c.  $F_x = -\frac{\partial f}{\partial x}$ ,  $F_y = -\frac{\partial f}{\partial y}$ ,  $F_z = -\frac{\partial f}{\partial z}$   $f = E_p$   
 $\vec{F} = -\nabla E_p \rightarrow \vec{F} = m\vec{g} \Rightarrow \frac{dE_p}{dz} = F_z = mg$   $E_p = \int mg dz = mgz$   
 $\vec{F} = -kx \Rightarrow \frac{dE_p}{dx} = -kx$   $E_p = \int -kx dx = -\frac{kx^2}{2}$

**Conservazione del momento angolare:**  
 $\vec{L}_O = \vec{r}_O \times \vec{p} = \vec{r}_O \times m\vec{v}$   
 $\vec{r}_O = \vec{OO'} + \vec{r}_{O'}$   $\Rightarrow \vec{L}_O = \vec{OO'} \times \vec{p} + \vec{r}_{O'} \times \vec{p} = \vec{L}_O' + \vec{L}_O$   
 $\vec{L}_O = \vec{r}_O \times m(\vec{v}_r + \vec{v}_O) = \vec{r}_O \times m\vec{v}_O + \vec{r}_O \times m\vec{v}_r$   
 $\vec{M}_O = \vec{r}_O \times \vec{F}$   $\vec{M}_{O'} = \vec{M}_O + \vec{OO'} \times \vec{F}$   $\vec{M} = \sum \vec{r}_i \times \vec{F}_i = \vec{r} \times \sum \vec{F}_i = \vec{r} \times \vec{R}$   
 $\frac{d\vec{L}}{dt} = \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt} = \vec{v} \times m\vec{v} + \vec{r} \times \vec{F} = \vec{M}$

**Velocità in coordinate polari:**  
 $\vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt} \hat{u}_r + r \frac{d\hat{u}_r}{dt} = v \hat{u}_r + r \frac{d\theta}{dt} \hat{u}_\theta$   
 $\vec{L} = \vec{r} \times m \frac{d\theta}{dt} \hat{u}_\theta$

**Se  $\vec{r} \perp \hat{u}_\theta \Rightarrow L = mrv^2 \frac{d\theta}{dt}$**   
**Forza centrale  $\Rightarrow L = \text{cost.} \Rightarrow \frac{1}{2} r^2 \frac{d\theta}{dt} = \text{cost.}$**   
 $dA = \frac{1}{2} r^2 d\theta$   $\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} = \frac{L}{2m}$   
 $A = \frac{L}{2m} T$  periodo:  $T = \frac{2m A}{L}$

**Lavoro forze centrali:**  $L = \int_A^B \vec{F} \cdot d\vec{s} = \int_A^B F(r) \hat{u}_r \cdot d\vec{s}$   
 $ds \cos \theta = dr \Rightarrow \hat{u}_r \cdot d\vec{s} = dr$   
 $d\vec{r} = r d\theta \hat{u}_\theta$   $|d\vec{r}| = r d\theta = dr$   
 $\Rightarrow L = \int_A^B F(r) dr = f(r_A) - f(r_B)$

**Velocità in coordinate cartesiane:**  
 $\vec{r} = \vec{r}_O + \vec{r}'$   
 $\vec{v} = \frac{d\vec{r}}{dt} = \frac{d\vec{r}_O}{dt} + \frac{d\vec{r}'}{dt} = \vec{v}_O + \vec{v}' + \vec{\omega} \times (\vec{x}' \hat{x} + \vec{y}' \hat{y} + \vec{z}' \hat{z}) = \vec{v}_O + \vec{v}' + \vec{\omega} \times \vec{r}'$