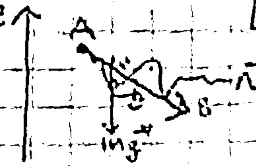
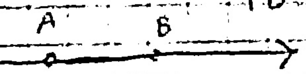
$dW = F_T\, ds = m\frac{dv}{dt} ds = m v\, dv$ $\quad \int_A dW = \int_{v_A} m v\, dv = \frac{\dots}{2}\Big|_{v_A} = \dots$



L forza peso: $L = \int_A^B m\vec{g}\cdot d\vec{s} = m\vec{g}\cdot\int_A^B d\vec{s} = m\vec{g}\cdot\vec{AB} = -mg(z_B - z_A)$

$L = mg(z_A - z_B) = E_{pA} - E_{pB} = -\Delta E_p$

L forza elastica 1D: $L = \int_A^B -kx\,dx = -\frac{kx^2}{2}\Big|_A^B = -\Delta E_p, \quad E_p = \frac{1}{2}kx^2$



L forza attrito: $L = \int_A^B \vec{f}\cdot d\vec{s} = \int_A^B \mu_d N \hat{u}_v \cdot \underbrace{d\vec{s}}_{=ds} = \mu_d N \int_A^B ds = \mu_d N \Delta s_{\text{percorso}}$

$dW = \vec{F}\cdot d\vec{s} = F_x dx + F_y dy + F_z dz = -dE_p$ se F è cons.

$\oint dW = 0 \Rightarrow \exists\, f(x,y,z)$ t.c. $F_x = -\frac{\partial f}{\partial x}, F_y = -\frac{\partial f}{\partial y}, F_z = -\frac{\partial f}{\partial z}$ $\quad f = E_p$

$\vec{F} = -\vec{\nabla} E_p \longrightarrow F = mg \Rightarrow \frac{dE_p}{dz} = F = mg \quad E_p = \int mg\,dz = mgz$

$\quad \longrightarrow F = -kx \Rightarrow \frac{dE_p}{dx} = -kx \quad E_p = \int -kx\,dx = -k\frac{x^2}{2}$

$\vec{L}_o = \vec{r}_o \times \vec{p} = \vec{r}_o \times m\vec{v} \qquad \vec{r}_o = \vec{OO'} + \vec{r}_{o'} \Rightarrow \vec{L}_{o'} = \vec{OO'}\times\vec{p} + \vec{r}_{o'}\times\vec{p}$

$\vec{L}_{o'} = \vec{OO'}\times\vec{p} + \vec{L}_o$

$\vec{L} = \vec{r}\times m(\vec{v}_r + \vec{v}_\theta) = \vec{r}\times m\vec{v}_\theta$

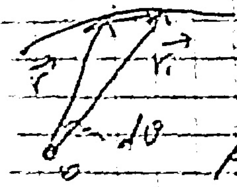
$\vec{M}_o = \vec{r}_o \times \vec{F} \qquad \vec{M}_{o'} = \vec{M}_o + \vec{O'O}\times\vec{F} \qquad \vec{M} = \sum \vec{r}_i \times \vec{F}_i = \vec{r}\times\sum\vec{F}_i = \vec{r}\times\vec{R}$

$\frac{d\vec{L}}{dt} = \frac{d\vec{r}}{dt}\times\vec{p} + \vec{r}\times\frac{d\vec{p}}{dt} = \cancel{\vec{v}\times m\vec{v}} + \vec{r}\times\vec{F} = \vec{M}$

$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{u}_r + r\frac{d\hat{u}_r}{dt} = v\hat{u}_r + r\frac{d\theta}{dt}\hat{u}_\theta \qquad \vec{L} = \vec{r}\times m r\frac{d\theta}{dt}\hat{u}_\theta$

Se O sta nel piano del moto, $\vec{r}\perp\hat{u}_\theta \Rightarrow L = mr^2\frac{d\theta}{dt}$

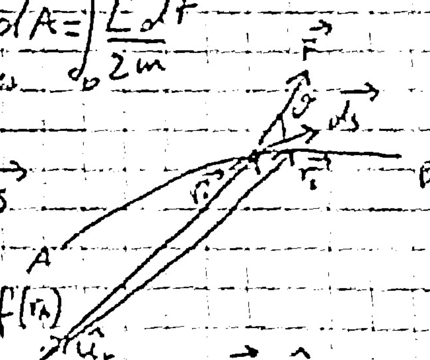
F centrale $\Rightarrow \vec{L} = \text{cost.} \Rightarrow \frac{1}{2}r^2\frac{d\theta}{dt} = \text{cost.} \qquad \frac{dA}{dt} = \frac{1}{2}r^2\frac{d\theta}{dt}$



$dA = \frac{1}{2}r\,r\,d\theta \qquad \frac{dA}{dt} = \text{cost.} \qquad \frac{dA}{dt} = \frac{L}{2m} \qquad \int_{A_{tot}} dA = \int_0^T \frac{L}{2m}dt$

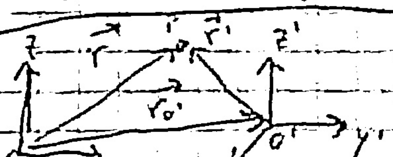
$A = \frac{L}{2m}T$ $\quad$ periodo: $T = \frac{2mA}{L}$

Lavoro forze centrali: $L = \int_A^B \vec{F}\cdot d\vec{s} = \int_A^B F(r)\,\hat{u}_r\cdot d\vec{s}$



$ds\cos\theta = dr \Rightarrow \hat{u}_r\cdot d\vec{s} = dr \Rightarrow L = \int_A^B F(r)\,dr = f(r_B) - f(r_A)$

$d\vec{r} = r\,d\theta\,\hat{u}_\theta \qquad |d\vec{r}| = r\,d\theta = dr$



$\vec{r} = \vec{r}_{o'} + \vec{r}'$

$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d\vec{r}_{o'}}{dt} + \frac{dx'}{dt}\hat{x}' + \frac{dy'}{dt}\hat{y}' + \frac{dz'}{dt}\hat{z}' + \underbrace{x'\frac{d\hat{x}'}{dt}}_{\vec{\omega}\times\hat{x}'} + y'\frac{d\hat{y}'}{dt} + z'\frac{d\hat{z}'}{dt}$

$= \vec{v}_{o'} + \vec{v}' + \vec{\omega}\times(x'\hat{x}' + y'\hat{y}' + z'\hat{z}') = \vec{v}_{o'} + \vec{v}' + \vec{\omega}\times\vec{r}'$