During the rendering process with ray tracing, we need to generate a ray from the camera, which intersects objects in the scene, resulting in an intersection point. Based on the intersection point, outgoing radiance can be calculated based on a lighting model.

1. As shown in the figure below, we have a pin-hole camera located at $\mathbf{C} = (0, 0, 0)$ with view direction $\vec{\mathbf{V}} = (0, 0, -1)$. Its focal length is 1.0. The vertical field of view (FoV) of the camera is 2θ given that $\tan(\theta) = 0.5$. Assuming that the resolution of the imaging plane is 800 (horizontal) by 600 (vertical), we generate a ray which passes through the center of the pixel $(0, 0)$ located at the bottom-left of the imaging plane.

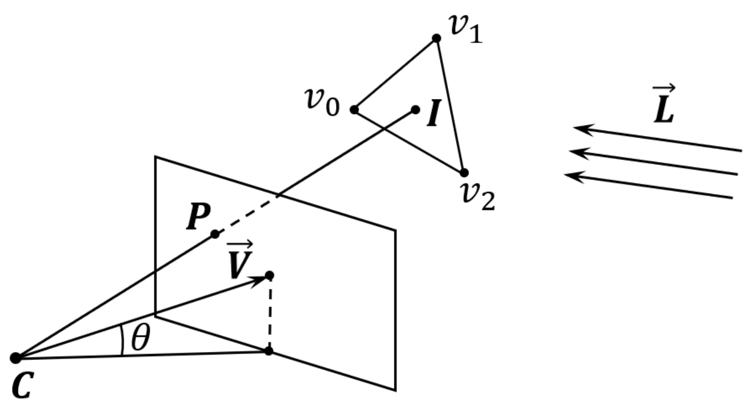
   Write the parametric form of the ray equation for the above ray. You should keep the numerically accurate form of your result.

2. Consider another ray with direction $(0, 0, -1)$, which intersects a triangle in the scene at point $\boldsymbol{I}$, where the three vertices of the triangle are v_0, v_1 and v_2, whose positions are $(-3, -1, -4)$, $(1, -1, -4)$ and $(-3, 7, -6)$, respectively.

   Calculate the barycentric coordinate of the intersected point $\boldsymbol{I}$ with respect to the three vertices of the triangle.

3. Suppose that the surface normal at an intersection point is $(0, 0, 1)$, and there is only one parallel light source in the scene with direction $\vec{\mathbf{L}} = (\sqrt{2}/2, 0, -\sqrt{2}/2)$ and its radiance is 1.0 (we use a scalar to represent radiance for simplicity). Assuming that we use *Phong* model to compute the lighting at the intersection point $\boldsymbol{I}$, the specular reflectance is set to 1.0, and the exponent used in the specular component is set to 2.

   Only considering the specular part of the *Phong* model, calculate the outgoing radiance (a scalar) from the intersection point $\boldsymbol{I}$.



Answer:

1. Using the information of focal length and FoV, we can calculate the height of the imaging plane

$$h = 2f\tan\theta = 1.$$

The side length of a pixel is then $1 \div 600 = 1/600$. Obviously, the center of the imaging plane is at $(0, 0, -1)$, and the position of the center of pixel $(0, 0)$ can be calculated by offsetting the center of the imaging plane by $(-399.5, -299.5)$ pixels, which is

$$\left(-\frac{799}{1200}, -\frac{599}{1200}, -1\right).$$

To obtain the parametric form of the ray equation, we should normalize the direction vector. The norm of the vector is $\sqrt{2437202}/1200$, so the answer is

$$\mathbf{p} = (0, 0, 0) + t\left(-\frac{799}{\sqrt{2437202}}, -\frac{599}{\sqrt{2437202}}, -\frac{1200}{\sqrt{2437202}}\right).$$

2. Given $\mathbf{e_1} = \mathbf{v_1} - \mathbf{v_0}$ and $\mathbf{e_2} = \mathbf{v_2} - \mathbf{v_0}$, the ray-triangle intersection equation using parametric ray equation is

$$\mathbf{v_0} + b_1\mathbf{e_1} + b_2\mathbf{e_2} = \mathbf{o} + t\mathbf{d},$$

which could be written into a linear system:

$$\begin{bmatrix}\mathbf{e_1} & \mathbf{e_2} & -\mathbf{d}\end{bmatrix}\begin{bmatrix}b_1 \\ b_2 \\ t\end{bmatrix} = \mathbf{o} - \mathbf{v_0}.$$

Solving the linear system yields $b_1 = 3/4$, $b_2 = 1/8$ and $t = 17/4$. Using the property that three components of the barycentric coordinates sum to 1, the answer is

$$b_0 = \frac{1}{8},\ b_1 = \frac{3}{4} \text{ and } b_2 = \frac{1}{8}.$$

3. The specular part of the *Phong* lighting model writes (with only one light source)

$$k(\mathbf{R} \cdot \mathbf{V})^{\alpha} i,$$

where k is specular reflectance and i is light radiance. The normal at the intersection point is $(0, 0, 1)$, which means the reflection direction could be easily calculated by multiplying -1 to the z component of the light direction

$$\mathbf{R} = \left(\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}\right).$$

Using the formula yields the answer

$$\text{ans} = \frac{1}{2}.$$