Lecture 6: X-ray Source and Production

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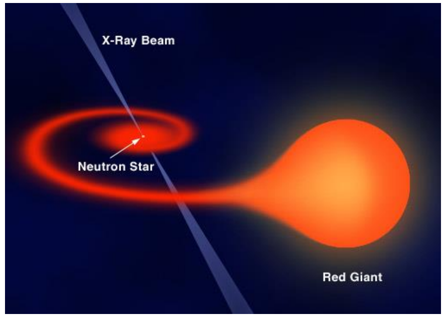
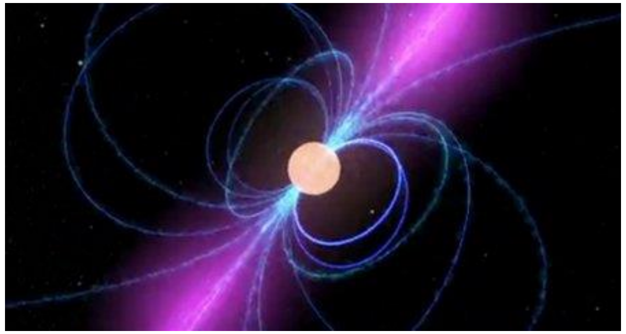
Lecture 6: X-ray Source and Production

- Introduction to X-ray Source
  - Types of X-ray Source
  - History of X-ray Source
  - Bremsstrahlung
- X-ray Tube
  - Cathode/Electron source
  - E-beam focusing
  - Anode
  - Envelope/Window/Filter
  - Cooling
- Bremsstrahlung Physics
  - Bremsstrahlung Production
  - Kramers' Law
  - Bremsstrahlung Spectrum & Characteristic X-rays
  - Focal Spot Size
  - X-ray Exposure
  - Clarification on some terminologies

Sources of X-ray Radiations

- Natural
  - Radioactive isotopes
  - Radon gas (a decay from radium, which is a decay from uranium in earth's crust)
  - Planetary x-ray sources: x-ray stars, x-ray pulsar
  - Wilson cloud chamber - https://www.youtube.com/watch?v=_2NGybNq53U



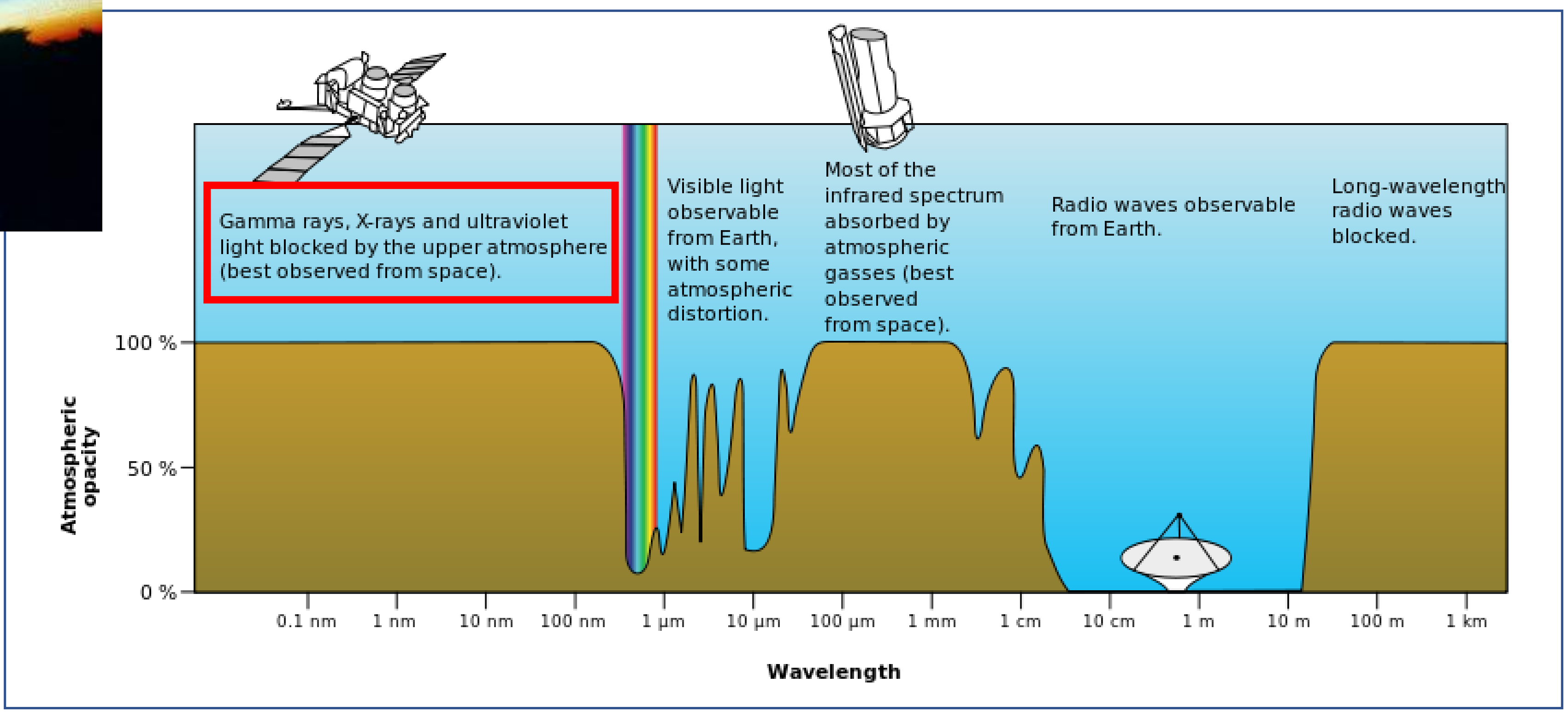


- Man-made
  - X-ray tube
  - Synchrotron
  - Free electron laser

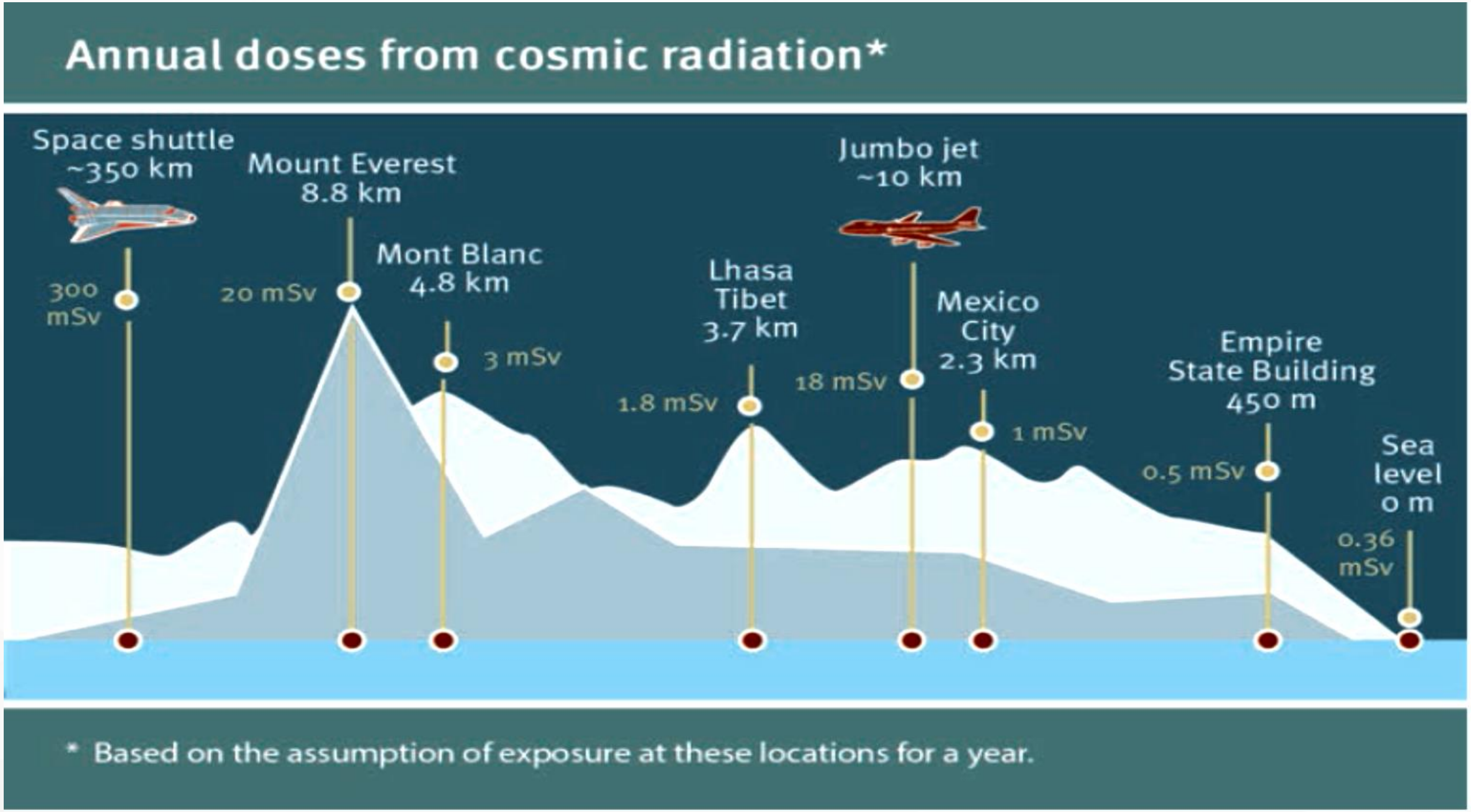
How are we shielded from cosmological radiations?

← biosphere

What about air & space travel?

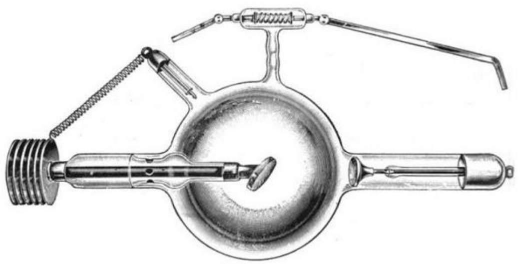


Cosmic radiation at different altitude

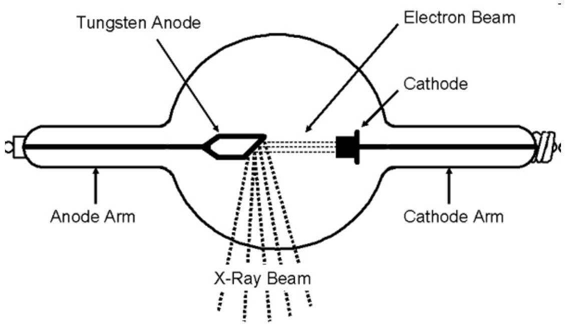


History of X-ray tubes

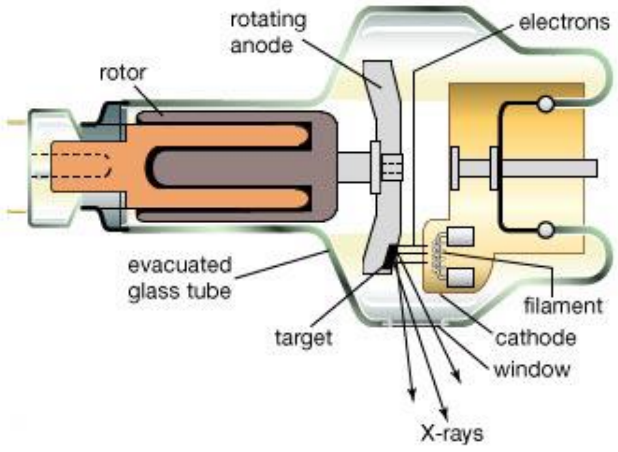
Crookes tube
(~1870)



Coolidge tube
(1913)

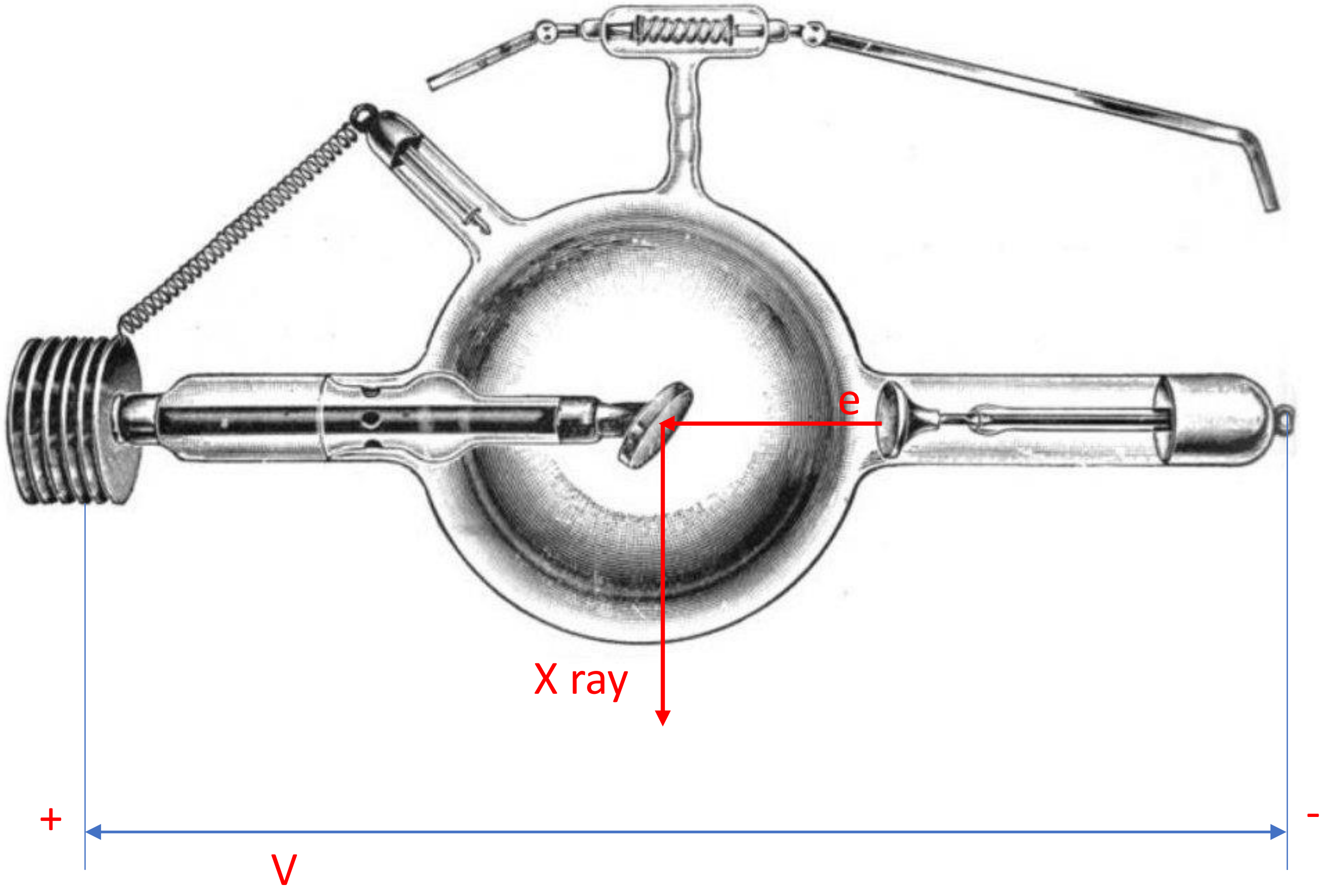


Rotating-anode
tube (~1970)



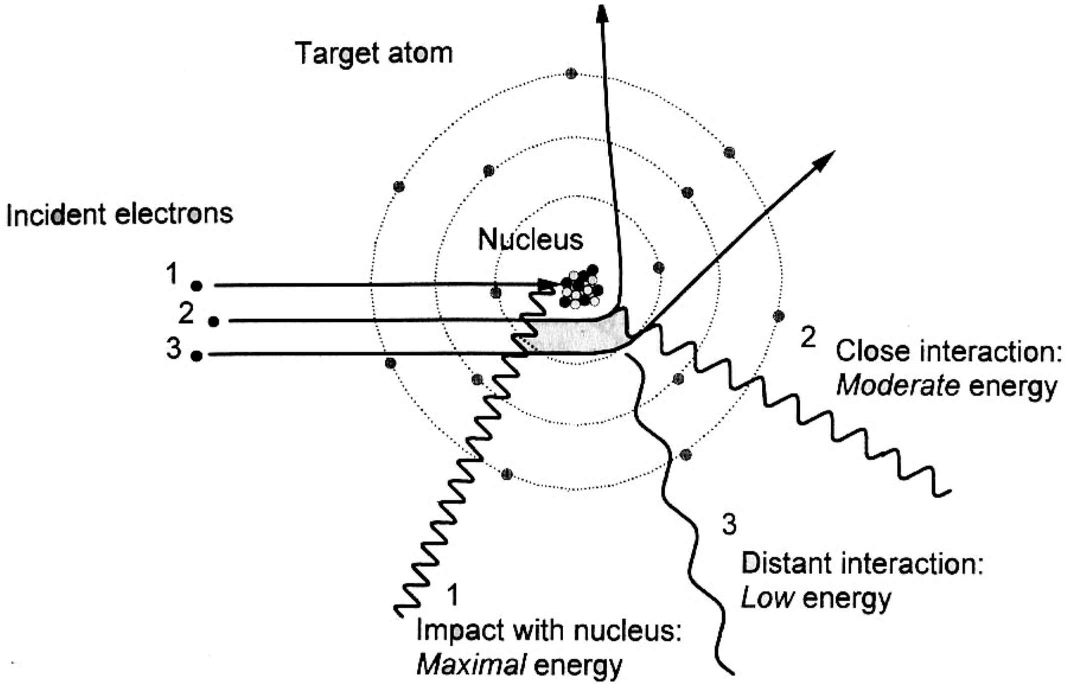
The First X-ray Tube

Crookes X ray tube in early 1900s.



Bremsstrahlung

- EM radiations produced by the deceleration (i.e. braking) of electrons
- In x-ray tube, Bremsstrahlung is responsible for the continuous x-rays
- X-rays from x-ray tube is called “braking” radiations.



Kramers' Law

- For Bremsstrahlung radiation, Kramers' law governs the spectral distribution of x-ray photons vs. the wavelength.

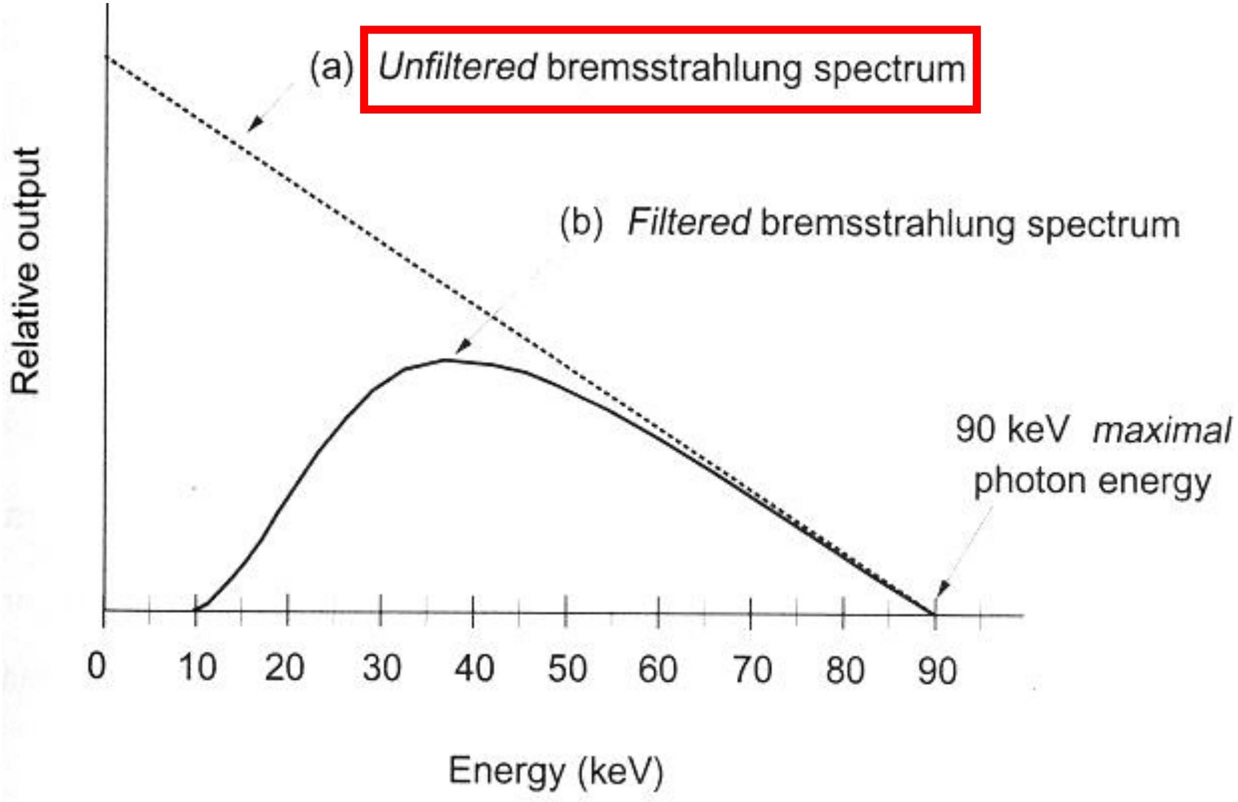
- $I(\lambda)d\lambda = K\left(\frac{\lambda}{\lambda_{min}} - 1\right)\frac{1}{\lambda^2}d\lambda$ (#of photons vs wavelength)
- Angular frequency $\nu = 2\pi c/\lambda$ → $d\nu = -2\pi c/\lambda^2 d\lambda$
- Use $\tilde{I}(\nu)d\nu = -I(\lambda)d\lambda$
- So $\tilde{I}(\nu)d\nu = \frac{K}{2\pi c}\left(\frac{\nu_{max}}{\nu} - 1\right)$ dν, where $\nu_{max} = 2\pi c/\lambda_{min}$

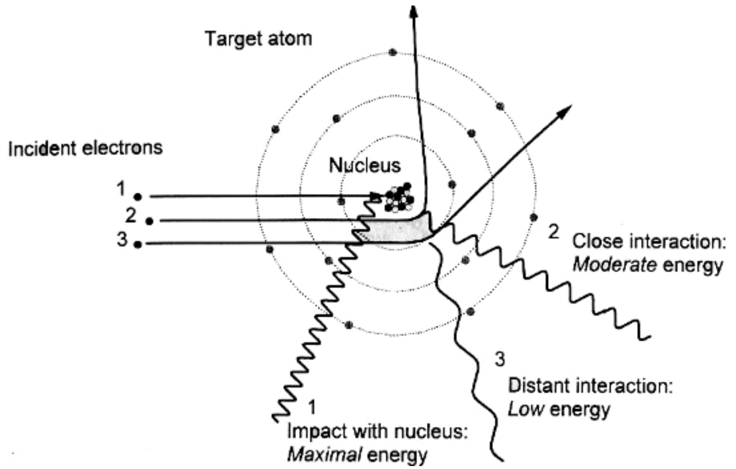
Kramers' Law (cont.)

- $\tilde{I}(\nu)d\nu = \frac{K}{2\pi c}\left(\frac{\nu_{max}}{\nu} - 1\right) \mathrm{d}\nu$
- Multiply $\hbar\nu$ (remember $E = hf = \hbar\nu$) to convert to energy flux
- $\psi(\nu) = \hbar\nu\tilde{I}(\nu) = \frac{K}{2\pi c}(\hbar\nu_{max} - \hbar\nu)$, where $\hbar\nu_{max}$ is the maximum photon energy, that is,
- $\boxed{\psi(E) = \frac{K}{2\pi c}(E_{max} - E)}$, where $E = \hbar\nu$

→energy flux is a linear function of x-ray energy and goes to zero at the maximum x-ray energy E_{max}.

The Bremsstrahlung Spectrum





The distribution of energies in an x-ray spectrum produced by the interaction of 90keV electrons with: (a) unfiltered Bremsstrahlung spectrum; (b) With inherent and added x-ray tube filtration.

Bremsstrahlung output vs tube voltage

Bremsstrahlung output energy flux follows $\psi(\nu) = \frac{K}{2\pi c}(E_{max} - E)$, where $E = \hbar\nu$

Total Bremsstrahlung output is $\int \psi(\nu)\mathrm{d}\nu$

$$\int \psi(\nu)\mathrm{d}\nu = \int \psi(E)\mathrm{d}E$$

$$= \int_0^{E_{max}} \frac{K}{2\pi\hbar c}(E_{max} - E)dE = \frac{K}{4\pi\hbar c}{E_{max}}^2$$

Since E_{max} is entirely dependent on tube voltage kV, the total Bremsstrahlung output ∝ square of tube voltage

That is, neglecting the characteristic x-ray output, we have

Bremsstrahlung output ∝ V^2

Bremsstrahlung Production

The total intensity of the Bremsstrahlung (note: excluding the characteristic x-rays) irradiation, ***I***

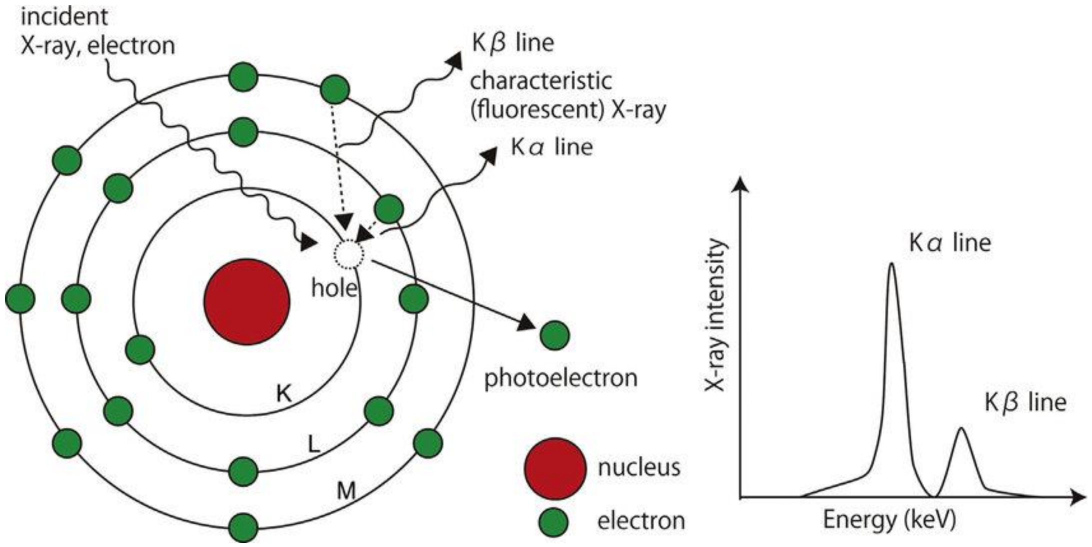
$$I \propto \frac{Z^2 z^2 e^6}{m^2}$$

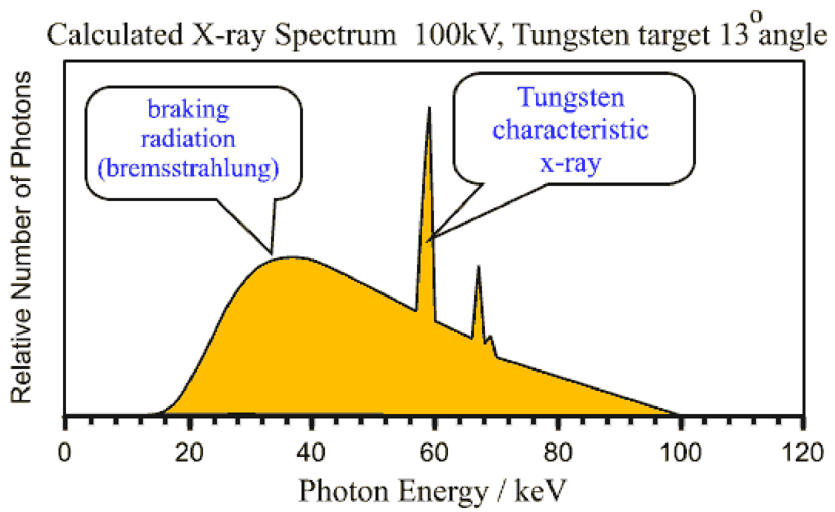
where *Z=target atomic number (with charge Ze), ze and m is the charge and mass of the incoming charged particles, respectively.*

Therefore:

1. electrons are efficient particles in generating x-ray bremsstrahlung because of their small mass;
2. total intensity of x-ray bremsstrahlung is proportional to the square of the target atomic number.

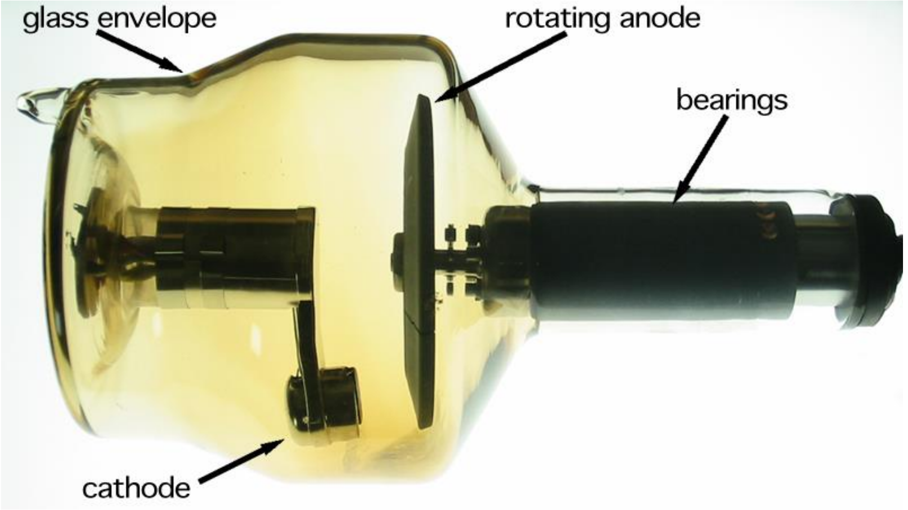
Characteristic x-rays

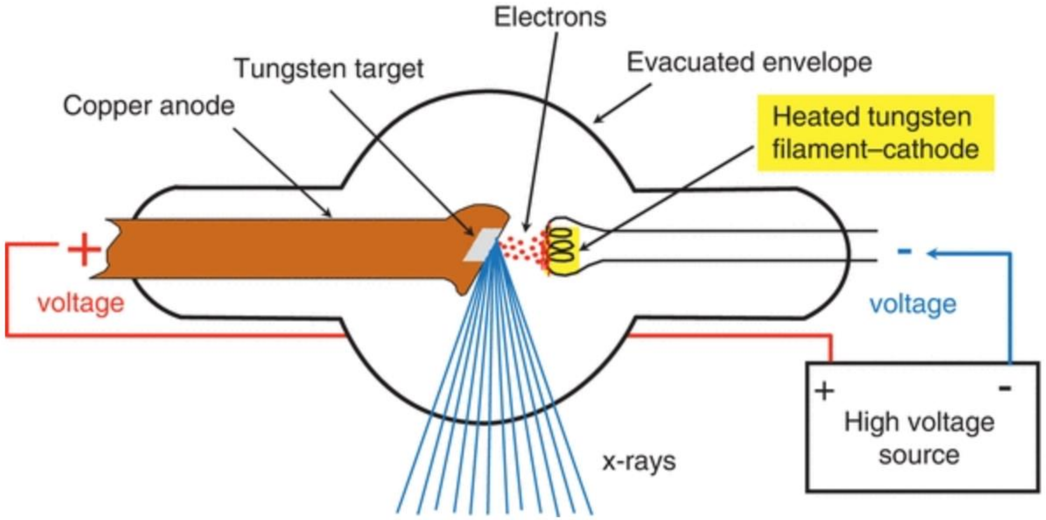




The entire x-ray production=bremsstrahlung + characteristic x-rays.

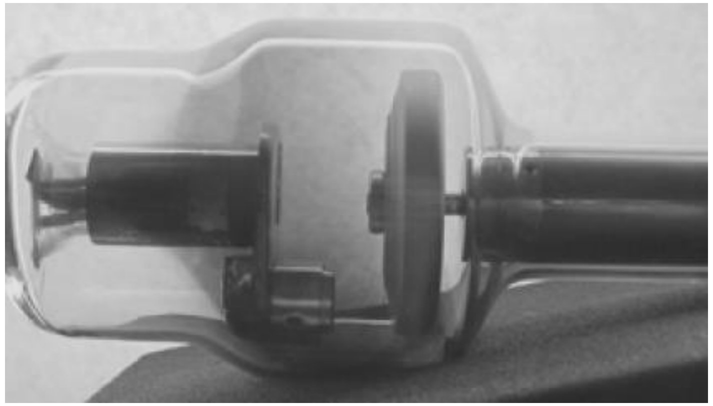
Bare X ray tube

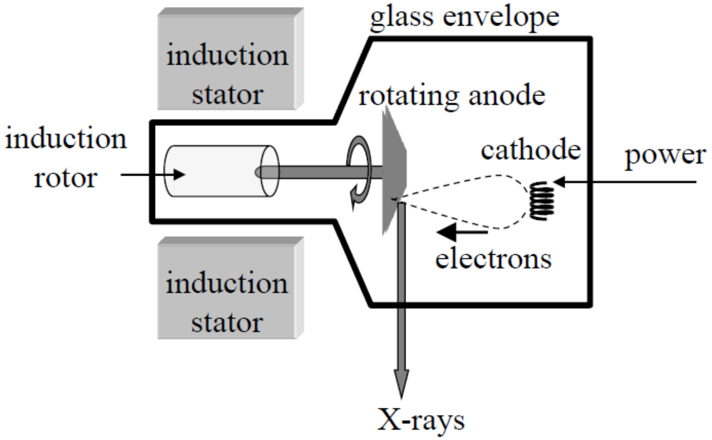




Modern X-ray Tube: Major Components

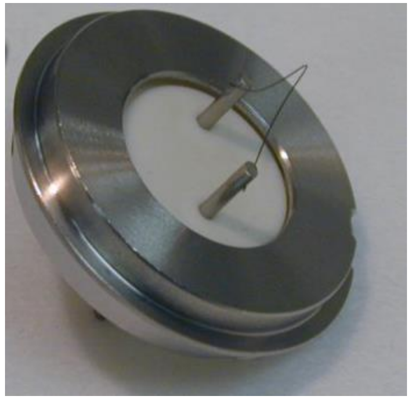
1. Cathode/Electron source
2. E-beam focusing
3. Anode
4. Envelope
5. Window
6. Filter
7. Cooling





Cathode: Electron source in x-ray tubes

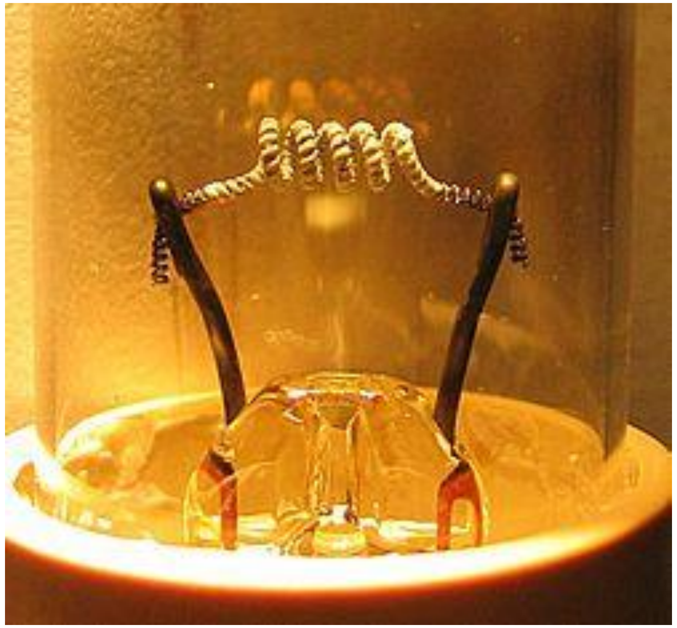
Filament cathode



Dispenser cathode

Material: Tungsten, Barium Aluminate

Thermionic emission electron sources



$$J = A_G T^2 \exp(-W/kT)$$

A_G – constant

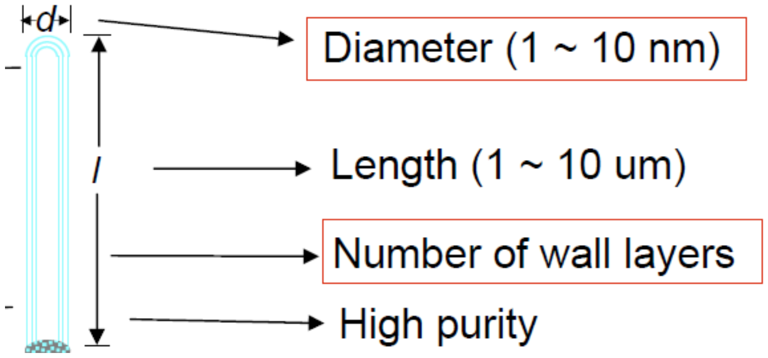
J – current density

T – temperature

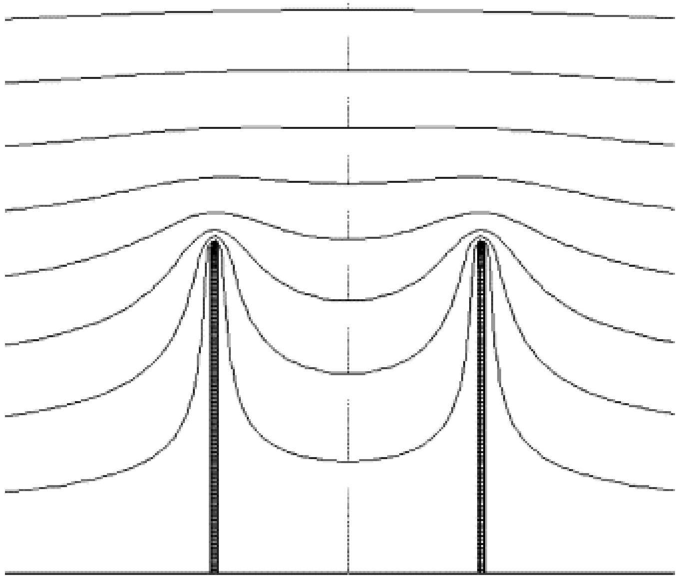
W – work function

k – Boltzmann constant

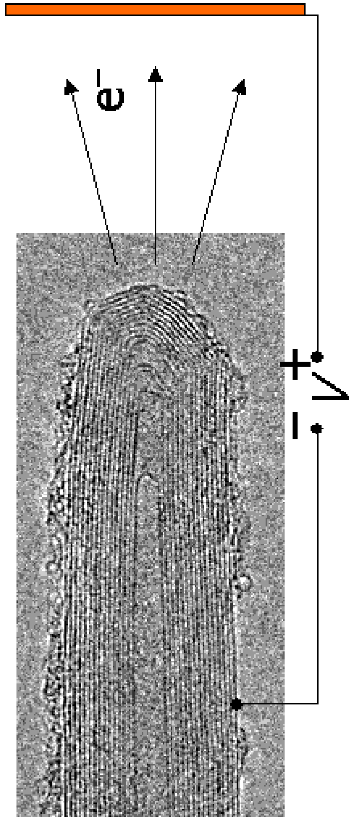
Carbon Nanotube as Field Emitter



Aspect ratio = 10^3-10^4



Electron sources: Field emission sources



Carbon Nanotube

$$J = aV^2 \mathrm{exp}(-b\phi^{3/2}/\beta \mathrm{V})$$

a, b – constants

J – current density

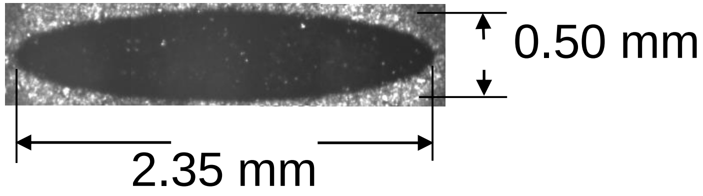
V – applied voltage

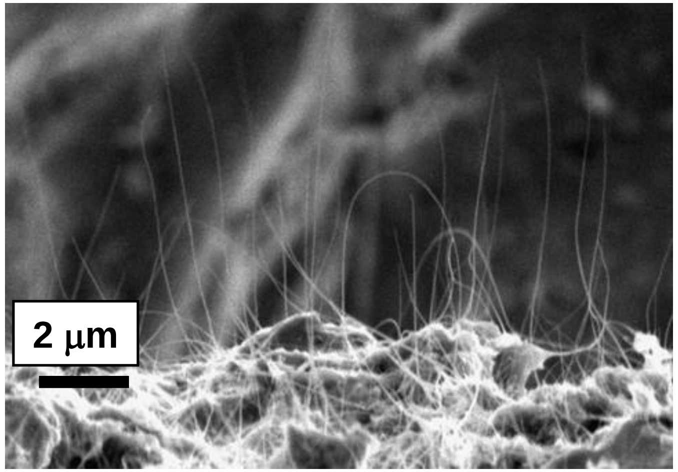
ϕ – work function

β – field enhancement factor

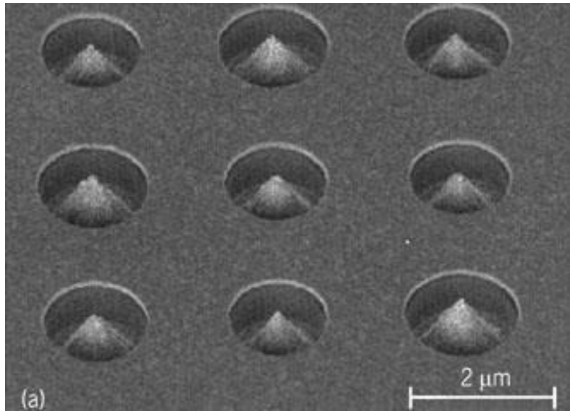
Field emission materials

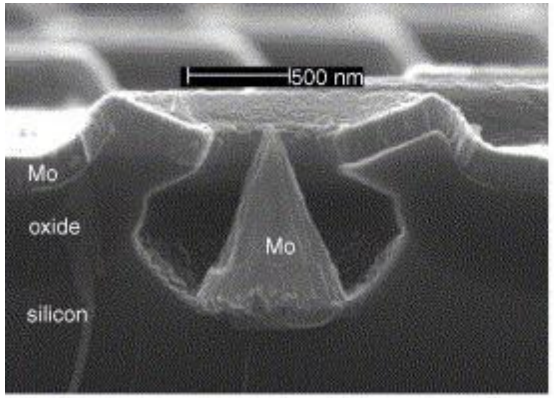
Carbon Nanotube

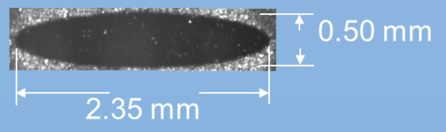
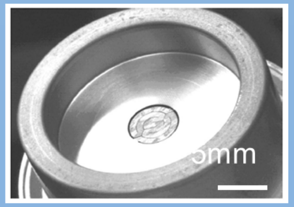




Spindt Tips





	Field-Emission X-ray	*Thermionic-Emission X-ray*
Cathode Material	carbon nanotubes 	tungsten 
Cathode Temperature	room temperature	>1000°C
Switching Speed	<1 μs	>20μs
Electron-Emission Mechanism	$J = aV^2 \exp(-b\phi^{3/2}/\beta V)$ *a*, *b* – constants *J* – current density *V* – applied voltage ϕ – work function β – field enhancement factor	$J = A_G T^2 \exp(-W/kT)$ A_G – constant *J* – current density *T* – temperature *W* – work function

Grid based X-ray tube

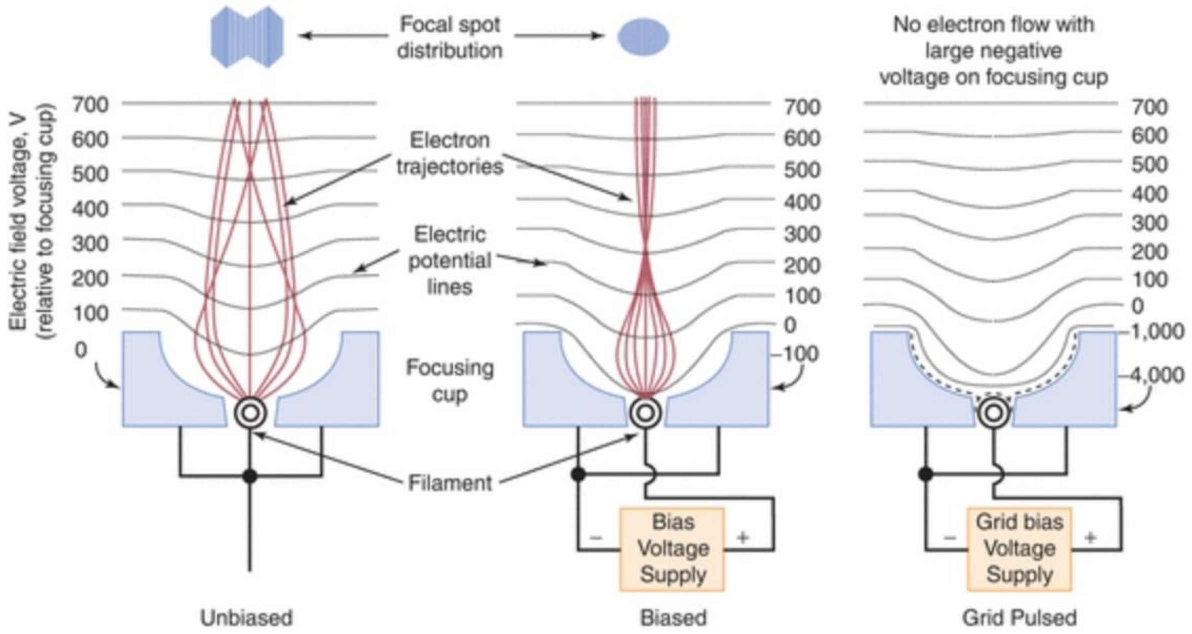


FIGURE 6-10 The focusing cup shapes the electron distribution when it is at the same voltage ("unbiased") as the filament (left). Electrical isolation of the focusing cup from the filament and application of a negative "biased" voltage (~ -100 V) reduces the distribution of emerging electrons by increasing the repelling electric fields surrounding the filament (note the 0 V electric field potential line) and modifying the electron trajectories (middle). At the top are typical electron distributions incident on the target anode (the focal spot) for the unbiased and biased focusing cups. Application of -4,000 V on an electrically isolated focusing cup completely stops electron flow, even with high voltage applied on the tube; this is known as a grid biased or "grid pulsed" tube (right).

E-beam focusing: focusing cup

- Electrons emitted from the cathode can have a distribution of initial energies (i.e. energy spread) and directions.

- Electrons have same negative

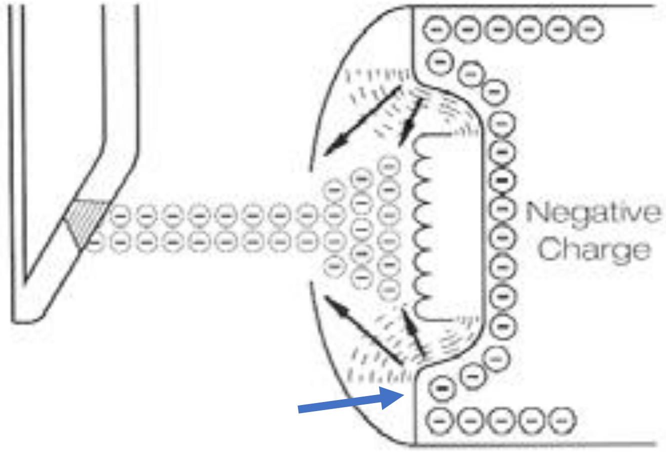
charge and repel each other

(Coulomb force!)

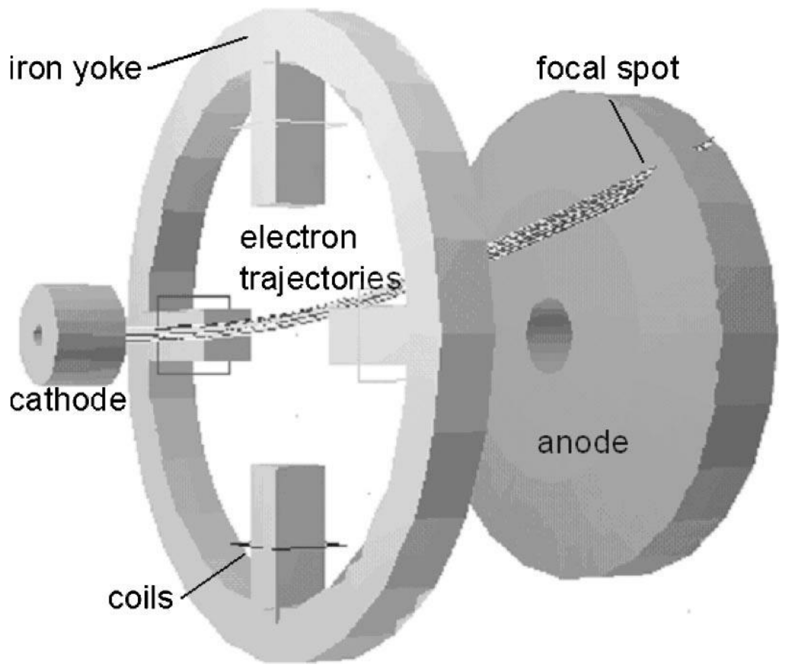
- The goal of e-beam focusing is

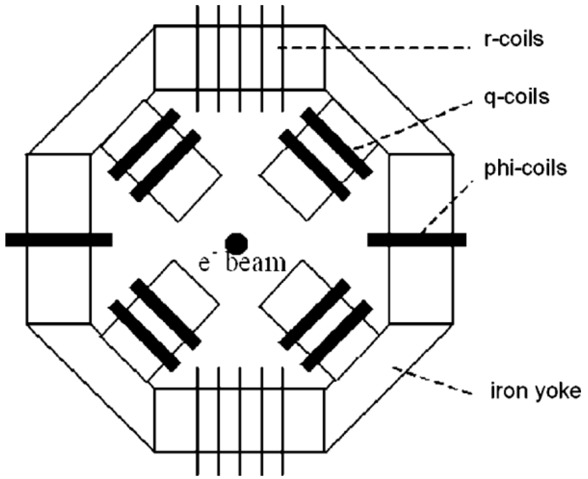
to force electrons to form a small

stream as they move toward the anode.

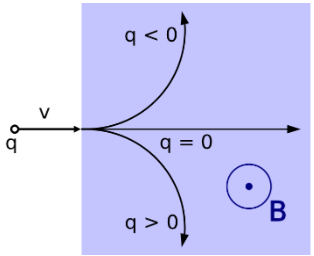


Electron beam focusing: magnetic focusing

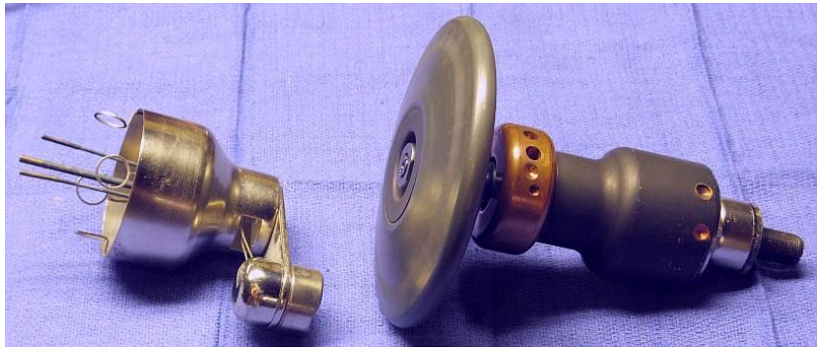




Remember Lorentz force: $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$



Anode



- Anode is the solid target where the electrons are accelerated to, due to kV potential difference between the anode and the cathode.

- 100 kev electrons move at 55% of *c* (**relativistic speed**).

- When electron strike the anode, 99% percent of this K.E. is converted to heat in the anode.

X-ray Conversion Efficiency at Anode

- The electric power at anode

$$P_e = VI$$

- The power of Bremsstrahlung radiation

$$P_r = 0.9 \times 10^{-9} Z V^2 I$$

- X-ray conversion efficiency at anode

$$\psi = \frac{P_r}{P_e} = 0.9 \times 10^{-9} Z V$$

For Tungsten anode (z=74), V=100 kV, ψ= 0.67%

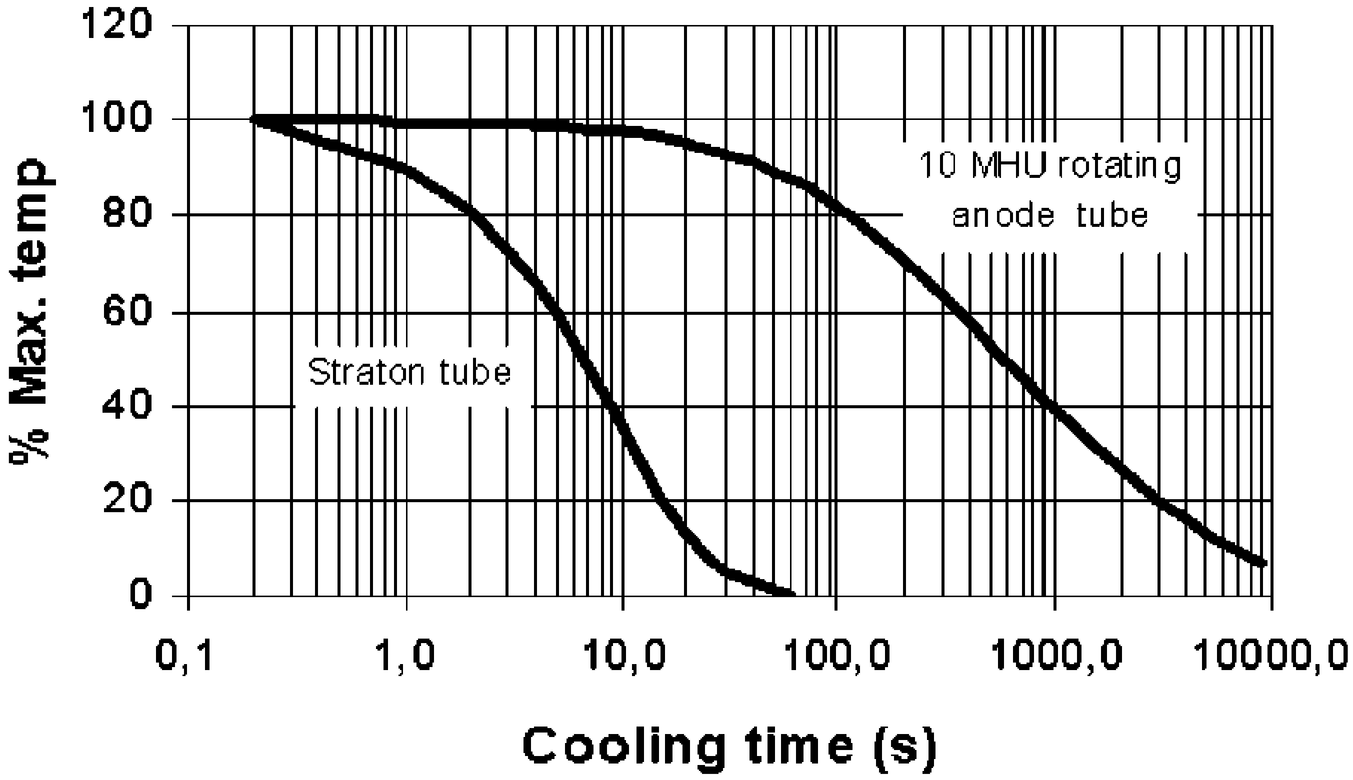
Anode heating

- Global heating
  - Determine how fast the tube can be used for another scan (i.e. affect throughput)
  - Cooling mechanism (radiation cooling, convective cooling)
  - material is tungsten (m.p.=3422 °C, and heat capacity=24.27 J·mol^{-1}·K^{-1})
- Local heating
  - Determine how much tube power for one scan
  - Cooling through thermal conduction in the anode solid

Electron beam welding

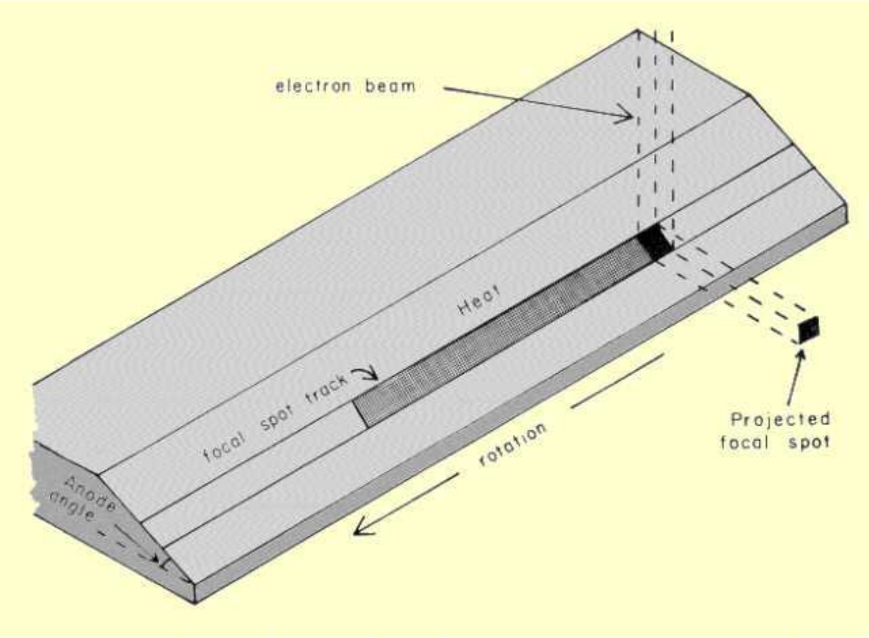
http://www.youtube.com/watch?v=HvYcEEt4K0A

Anode Cooling



Decide how long we need to wait once the tube has reached its max temperature, that is, throughput.

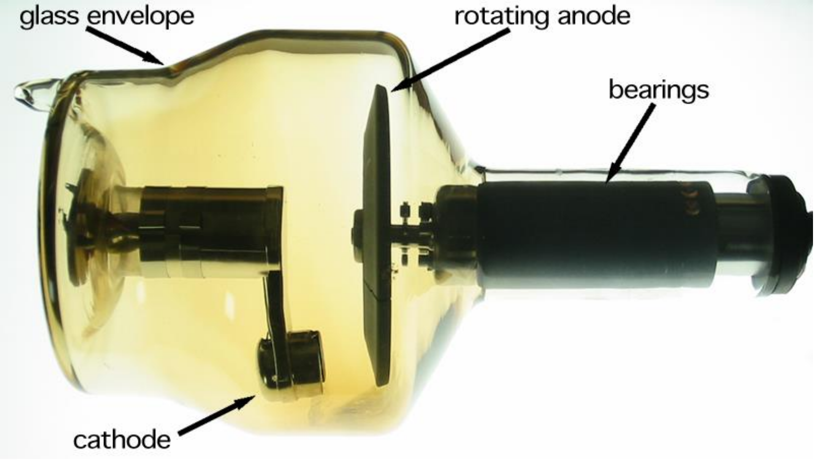
Fixed vs Rotating anode



- For fixed anode, electrons strikes at the same spot, and heat is accumulated at the spot.
- For rotating anode, electrons strikes a rotating anode, thus leaving a **focal spot track** on the anode surface.
- As a result, anode heat during x-ray exposure for image formation is distributed across the track, which has ~10X larger area.
- Therefore, the peak tube power is increased by ~10X.

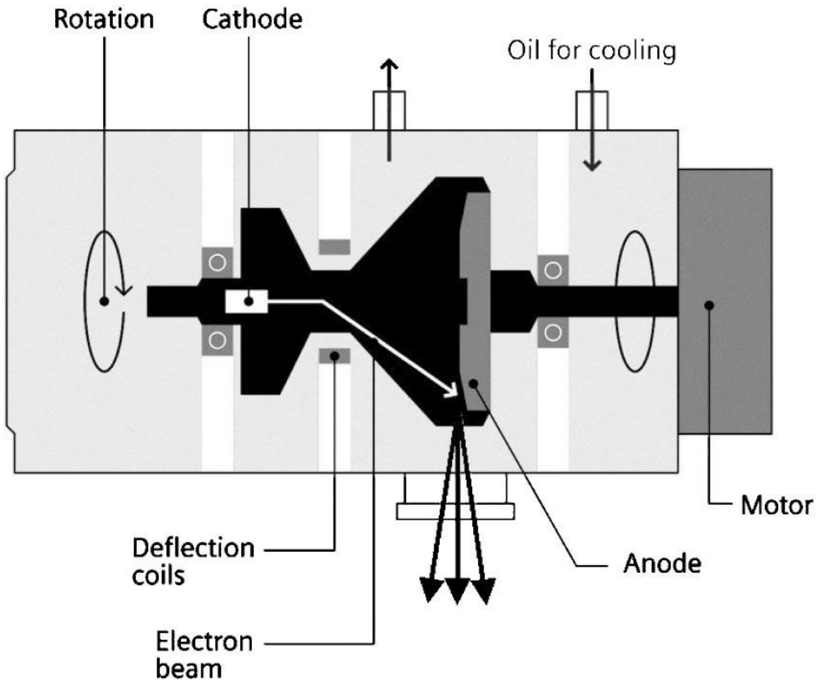
How to rotate an anode (relative to the cathode)?

Rotating anode



Radiation cooling

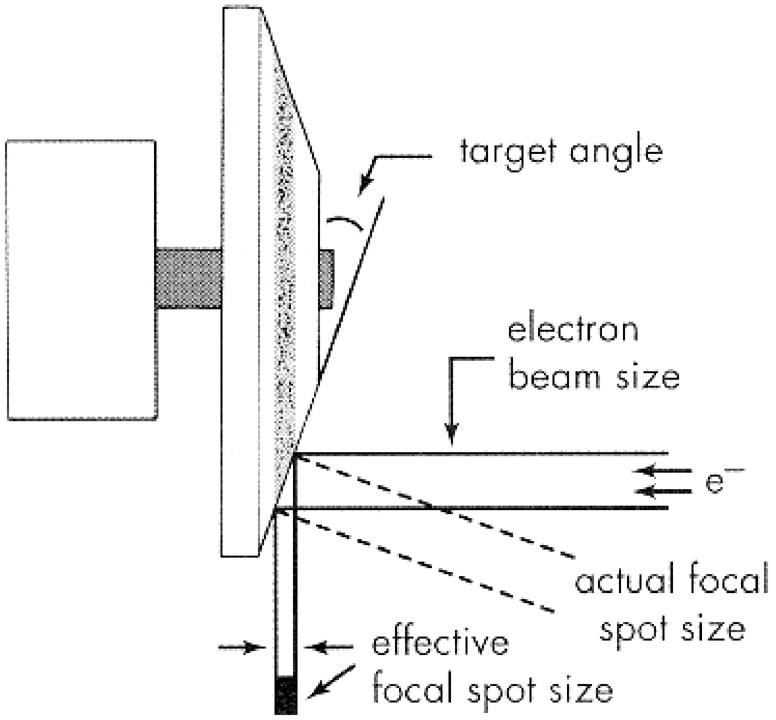
Rotating envelope



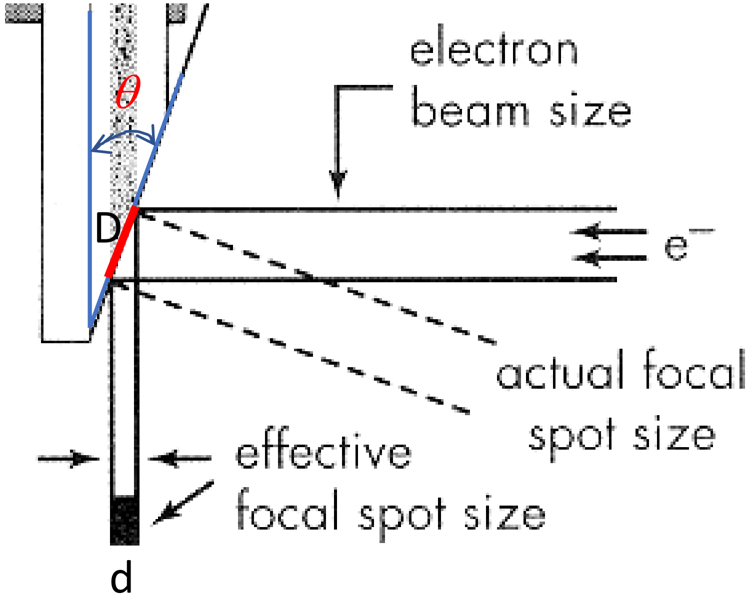
Convective cooling

Target angle (or, anode takeoff angle)

- Actual focal spot on the anode - the area of the target material being bombarded by electrons.
- ***Effective*** focal spot - the *imaginary* geometric line that can be drawn based on the actual focal spot size vs. the anode takeoff angle.
- This is called the Line-Focus principle!

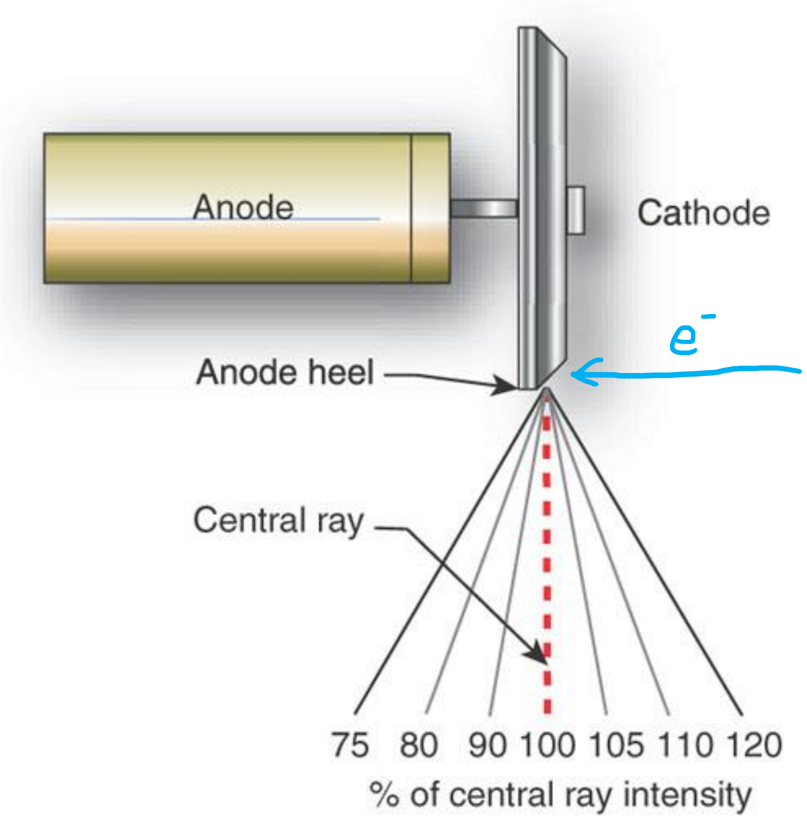


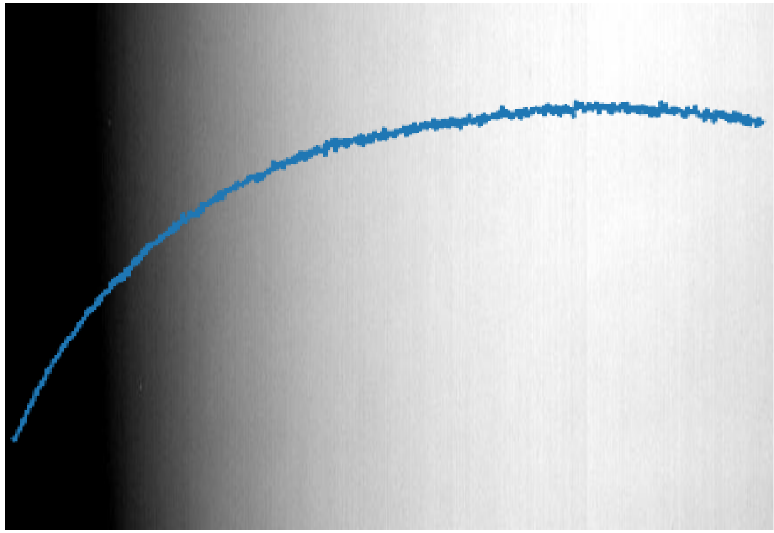
Focal spot area *vs.* effective focal spot



- Effective focal spot size
  - d x d
- Actual focal spot area on anode
  - d x D
- $d = D\ sin\theta$
- Therefore
  - effective focal spot size $= sin\theta *$ the focal spot area on anode.
- A typical target angle is 12 deg, sin 12º = 0.208, that is, effective focal spot size ~1/5 actual focal spot size on anode

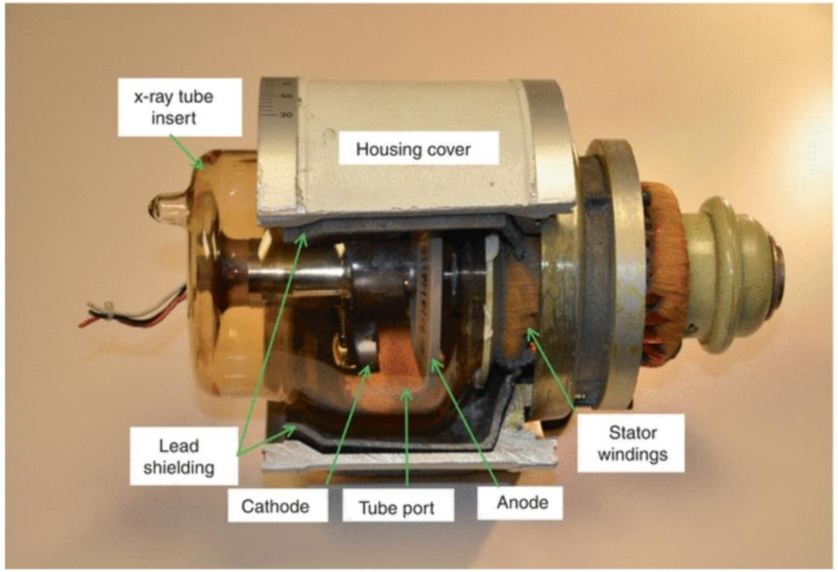
Anode Heel Effect

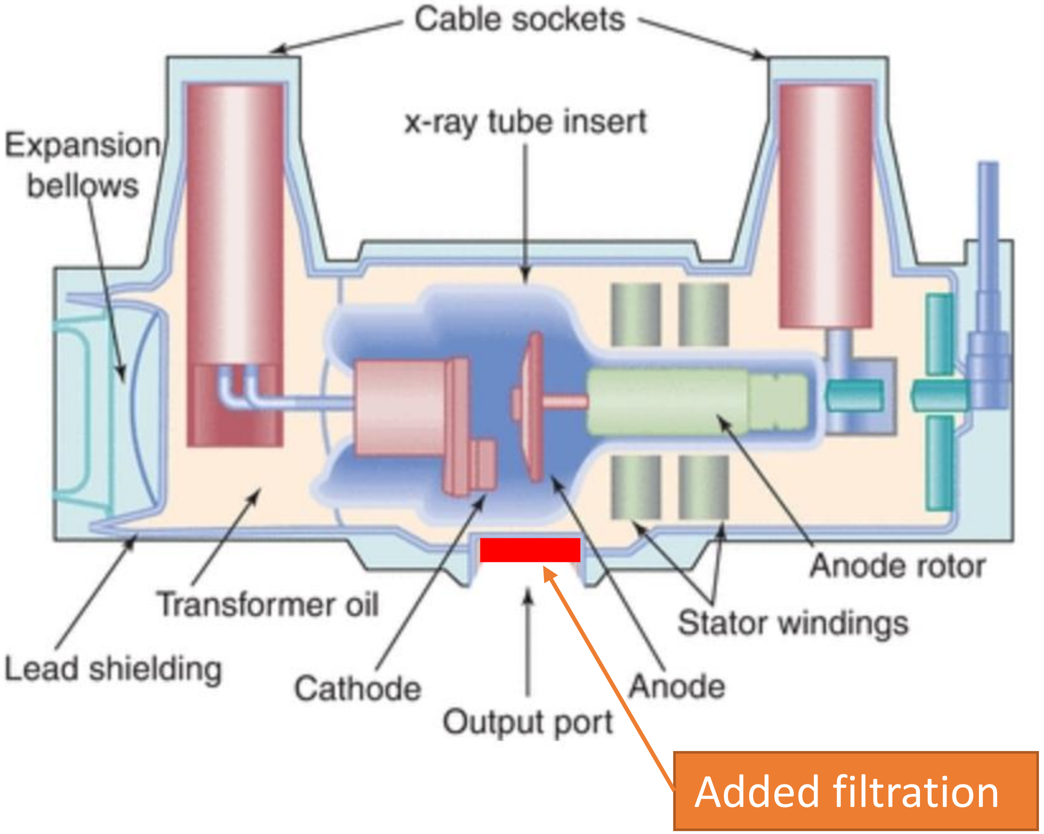




More x-ray attenuation at the anode side!

X-ray Tube Envelope/Housing/Filter





X-ray Tube Spectral Filtration with Added Filter