

# Image reconstruction supplementary materials

DIP 2022

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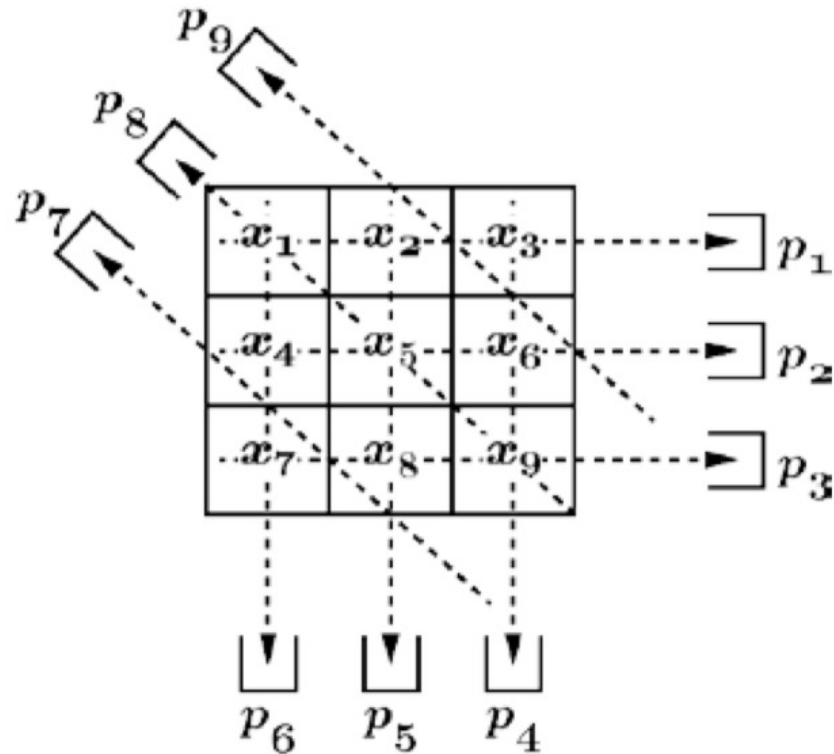
SIST, ShanghaiTech

- CT scanner at high rotational speed
- <https://www.youtube.com/watch?v=4jEtTNKM68I>
- Matlab code to play with CT recon

```
%close all
clear
P = phantom(128);
theta = 0:10:179;
figure(1),imshow(P)
title('Original image')
R = radon(P,theta);
figure(11),imagesc(R);
title('sinogram')
```

```
noise_R = 10*rand(size(R));
RN = R + noise_R; % add some noise
I1 = iradon(RN,theta,'Ram-Lak');
I2 = iradon(RN,theta,'Han');
figure(2)
subplot(1,2,1)
imshow(I1,[])
title('I1')
subplot(1,2,2)
imshow(I2,[])
title('I2')
```

# Algebraic reconstruction



$$x_1 + x_2 + x_3 = p_1$$

$$x_4 + x_5 + x_6 = p_2$$

$$x_7 + x_8 + x_9 = p_3$$

$$x_3 + x_6 + x_9 = p_4$$

$$x_2 + x_5 + x_8 = p_5$$

$$x_1 + x_4 + x_7 = p_6$$

$$2(\sqrt{2} - 1)x_4 + (2 - \sqrt{2})x_5 + 2(\sqrt{2} - 1)x_8 = p_7$$

$$(\sqrt{2})x_1 + (\sqrt{2})x_5 + (\sqrt{2})x_9 = p_8$$

$$2(\sqrt{2} - 1)x_2 + (2 - \sqrt{2})x_3 + 2(\sqrt{2} - 1)x_6 = p_9$$

## Technical Box 5: Algebraic Reconstruction Technique (ART)

The ART possesses an intuitive geometrical interpretation. Each of the linear equations represents a (hyper-) plane in the solution space. For example, for the very simple case of a reconstruction problem with two unknown pixels only, each equation defines a line in the 2D plane. The ART can be illustrated as a successive orthogonal projection of the respective approximate solution onto the (hyper-) planes defined by the individual linear equations.

The orthogonal projection of the approximation  $\mathbf{x}_{i-1}$  onto the (hyper-) plane defined by the  $i$ -th equation yields the approximation  $\mathbf{x}_i$  and is formally given by

$$\mathbf{x}_i = \mathbf{x}_{i-1} + \frac{p_i - \mathbf{A}_i \mathbf{x}_{i-1}}{\mathbf{A}_i \mathbf{A}_i^\top} \mathbf{A}_i^\top,$$

where the row vector  $\mathbf{A}_i$  represents the  $i$ -th matrix row and  $p_i$  denotes the  $i$ -th entry of the right-hand side vector  $\mathbf{p}$ .

- Introduction of CT reconstruction
- <https://www5.cs.fau.de/fileadmin/persons/MaierAndreas/maier/Hornegger16-CRB.pdf>
- PET reconstruction
- <https://www.youtube.com/watch?v=3BC0bnWobLs>
- Iterative reconstruction toolbox
- <http://people.compute.dtu.dk/pcha/AIRtoolsII/>