Scaling laws for lens systems

Adolf W. Lohmann

Situations exist in the area of optical information processing where one may choose between either a single large lens or many small lenslets side by side. The choice will be influenced by many parameters, among others by the space–bandwidth product SW. The SW is an upper limit for the number of data channels which can be handled in parallel. Hence, we investigate the scaling behavior of the space–bandwidth product. Professional lens designers sometimes have viewpoints different from those of designers of systems for digital optics. That fact may justify this investigation, performed by a nonlens designer.

I. Origin of the Problem

In the context of optical data processing one faces the choice between a few large lens systems [Fig. 1(A)] or many small lenslets side by side [Fig. 1(B)]. For some operations a combination of big and small lenses [Fig. 1(C)] may be appropriate.[1] The decision will depend primarily on the data processing job to be performed. For example, in the context of spatial filtering, the underlying systems matrix obeys the Toeplitz constraint if macrolenses are used [Fig. 1(A)]. The Toeplitz constraint for discrete signals is equivalent to space invariance for continuous signals.

In this study we are concerned with the influence of scaling on the number of data channels which can be handled in parallel. The space–bandwidth product SW is proportional to the number of data channels. Hence, the SW is an appropriate measure of the system quality. The SW can be interpreted also as the number of degrees of freedom of the wavefield at the exit of the lens system. Yet another interpretation: the SW is the number of resolvable points in the image plane.

Before discussing the SW in Sec. III, we briefly mention in Sec. II the scaling behavior of some basic parameters such as size and aberrations. In Sec. IV we take note of an empirical law, which connects focal length f and stop number F. We represent our conclusions in Sec. V.

The author is with University of Erlangen-Nuremburg, Physics Institute, D-8520 Erlangen, Federal Republic of Germany.

Received 6 February 1989.
0003-6935/89/234996-03$02.00/0.

II. Some Basic Scaling Laws

For a lens designer scaling is trivial, at least in principle. If the focal length f, the lens diameter B, and other length parameters are enlarged by a factor M, the lateral aberrations ξ will be enlarged in proportion to M. However, all angles and curvatures remain the same (Fig. 2).

Obvious consequences are: the travel time Δt of the light scales with M, the size of the image field $\Delta x\ \Delta y$ with M^2, and the volume and the weight with M^3.

The resolution δx due to diffraction does not change with M since $\delta x = \lambda F$ depends only on the wavelength λ and on the aperture angle, which is represented by the stop number F (Fig. 2). Also the depth of focus $\delta z = 4\lambda F^2$ remains unchanged, so far as it is caused by diffraction.

The wave aberration W, which describes the deviation of the actual wavefront from a perfect spherical wavefront, scales with M; however, wavelength λ remains unchanged. Hence, the Strehl definition (in the Marechal approximation) scales like:

$$I \approx 1 - M^2K^2\overline{W_1^2} \qquad (K = 2\pi/\lambda). \tag{1}$$

The index (1) refers to $M = 1$, which is an arbitrary convention, for example, associated with the focal length $f = 1$ mm.

The lateral aberration ξ scales with M, as can be seen in Fig. 2. Hence the Gaussian moment $\overline{\xi^2}$ scales with M^2.

The area of a pixel A_p in the image plane consists of a purely diffraction-dependent term and a geometric term[2,3]:

$$A_p = (\delta x)^2 + (\delta\xi)^2 = \lambda^2F^2 + \overline{\xi^2}. \tag{2}$$

The scaling behavior of the pixel area A_p is expressed by

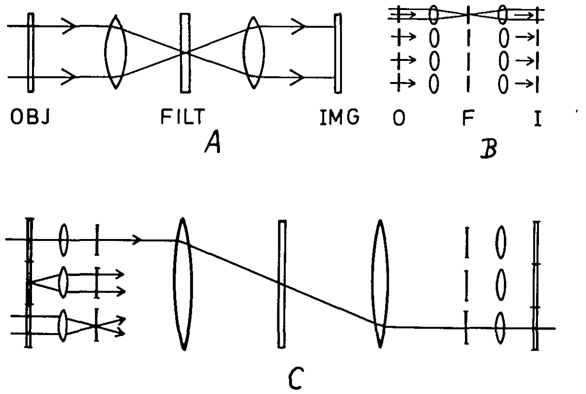

Fig. 1. Lens systems for spatial filtering: (A) macro; (B) micro; (C) hybrid.

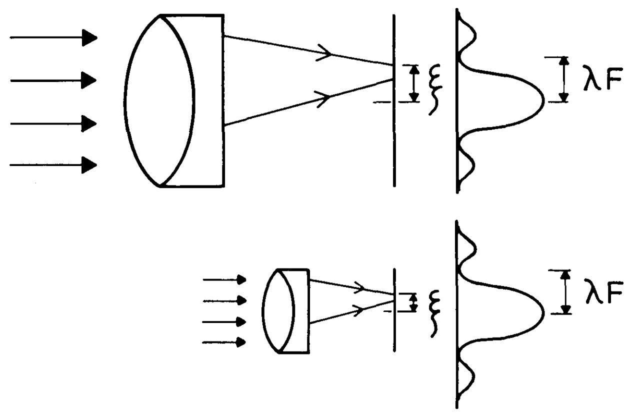

Fig. 2. Scaling behavior. The lateral aberration ξ varies with scale change. Curvatures, angles, and diffraction blur remain constant.

$$A_p(M) = \lambda^2F^2 + M^2\overline{\xi_1^2}. \quad (3)$$

In the context of nonlinear optics one wants this area to be as small as possible in order to concentrate the available light power as much as possible.[4]

III. Scaling of the Space–Bandwidth Product SW

The SW is defined as the number of resolvable pixels within the image field of size A_F. The field size scales as M^2:

$$A_F(M) = \Delta x \Delta y = M^2 \Delta x_1 \Delta y_1. \quad (4)$$

The SW $= A_F/A_p$ is now

$$\mathrm{SW}(M) = \frac{M^2\Delta x_1 \Delta y_1}{\lambda^2F^2 + M^2\overline{\xi_1^2}}. \quad (5)$$

Two special cases are of interest. Without aberration ($\xi = 0$) the SW_D is proportional to M^2:

$$\mathrm{SW}_D = M^2\,\mathrm{SW}_{D1}. \quad (6)$$

If the diffraction blur λF is negligible in comparison with the lateral aberration, the SW is insensitive to scaling:

$$\mathrm{SW}_A = \Delta x_1 \Delta y_1/\overline{\xi_1^2} = \mathrm{SW}_{A1}. \quad (7)$$

The true SW(M) approaches SW_A as M increases (Fig. 3). The 50% level is achieved if the effects of diffraction blur and aberration blur are the same. We call the associated scaling factor M_C:

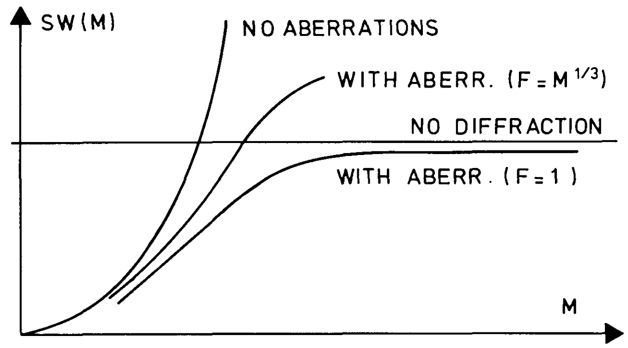

Fig. 3. Space–bandwidth product SW, for (from the top) no aberrations; with aberration (variable stop number); no diffraction; with aberration (fixed stop). M is the scaling factor, with $M = 1$ referring to a focal length of 1 mm.

$$M_C^2 = \lambda^2F^2/\overline{\xi_1^2}. \quad (8)$$

Any $M > M_C$ does not improve the SW(M) by very much. Yet, long focal length lens systems are needed for telephotography and in aerial survey cameras.

IV. Influence of the Stop Number

Aberrations are usually worst at the outer parts of the pupil. Hence, one reduce the aberration blur by reducing the aperture, which means an increase of the stop number F. The price one pays is an increase of the diffraction blur λF. In practice, one often chooses a compromise which can be described by an empirical law:

$$f\,(\mathrm{mm}) = F^3. \quad (9)$$

Table I corroborates this law. There are many exceptions; nevertheless, this law may serve as a guide to the availability of a lens system for a reasonable price.

Next we corroborate this rule by discussing a simple design example. Suppose the aberration ξ consists only of spherical aberration (first order) and defocusing. Expressed as a function of the pupil coordinate x and scaling factor M, the transverse aberration is

$$\xi(x,M) = M[a(x/M) - b(x/M)^3]. \quad (10)$$

The parameter a describes the amount of defocusing and b the amount of spherical aberration. The pupil coordinate x is limited by the pupil diameter B:

$$|x| \le B/2. \quad (11)$$

The pupil diameter $B = f/F$ is connected with the focal length $f = f_1M$ and the stop number F which now

Table I. Tabular Representation of Eq. (9)

Lens	F	f [mm]
Micro	1	1
Wide angle	3	27
Tele	5	125
Aerial	10	1000

scales in accordance with the empirical law [Eq. (9)]. F and B vary as

$$F = (f)^{1/3} = M^{1/3}; \quad f_1 = 1\ \text{mm},$$
$$B = M^{2/3}. \tag{12}$$

Hence, the contribution of the diffraction blur to the pixel area is

$$(\delta x)^2 = \lambda^2 F^2 = \lambda^2 M^{2/3}. \tag{13}$$

To compute the contribution of the aberration to the pixel area we optimize the Gaussian moment $\overline{\xi^2}(a,b,M)$ which is a measure of the pixel area from the geometrical optics point of view. We do this by picking the most suitable amount a of defocusing. A simple minimax calculation yields a as a function of b and M. The optimal amount of defocusing varies as a function of the scaling parameter M. However, the aberration blur remains constant in this example,

$$\overline{\xi^2} = b^2/2800. \tag{14}$$

By combining the results of Eqs. (13) and (14) we obtain for the pixel area

$$A_p = \lambda^2 M^{2/3} + b^2/2800. \tag{15}$$

The scaling behavior of the space–bandwidth product SW is described by Eq. (16):

$$\mathrm{SW} = \frac{M^2 \Delta x_1 \Delta y_1}{\lambda^2 M^{2/3} + b^2/2800}. \tag{16}$$

V. Conclusions

Figure 3 represents our conclusions. Without aberrations the SW would increase quadratically with the scaling factor M. That statement, based on wave optics, is quite different from the purely geometric result which claims SW to be independent of scaling. A straightforward combination of both results leads to the curve which approaches the geometric result from below asymptotically. That result cannot be realistic, otherwise, very long focal length lens systems would be fairly useless. In practice, the apertures of these long systems are reduced, more or less according to the empirical rule in Sec. IV. The corresponding space–bandwidth product SW(M) is indicated in Fig. 3 by the empirical rule $F = M^{1/3}$. Systems for lithography are notable exceptions to this rule. However, they obey the general trend: for a larger space–bandwidth product one needs a larger lens system.

References

1. H. Bartelt and F. Sauer, "Space-Variant Filtering with Holographic Multifacet Elements," Opt. Commun. **53**, 296–301 (1985).
2. N. Bareket, "Second Moment of the Diffraction Point Spread Function as an Image Quality criterion," J. Opt. Soc. Am. **69**, 1311–1312 (1979).
3. V. Gerbig and A. W. Lohmann, "Is lens design legal?"; submitted to Appl. Opt.
4. J. L. Jewell, S. L. McCall, Y. H. Lee, A. Scherer, A. C. Gossard, and J. H. English, J. Phys. **49**, C2–39 (1988).

K. Hildebrand (Wild-Heerbrugg) gave some valuable advice in the early stages of this project.