

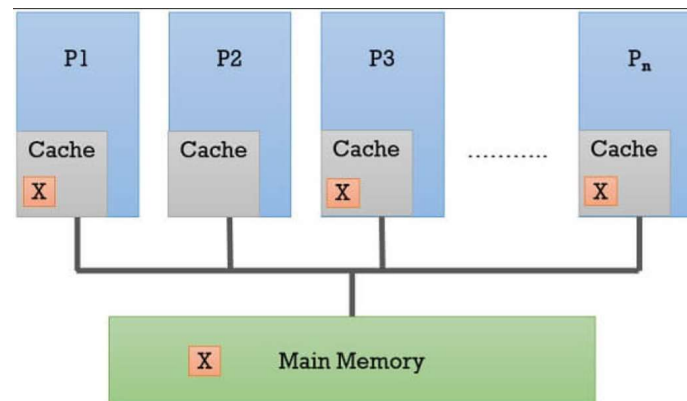


# Communication in Shared and Distributed Memory

CS121 Parallel Computing  
Fall 2023

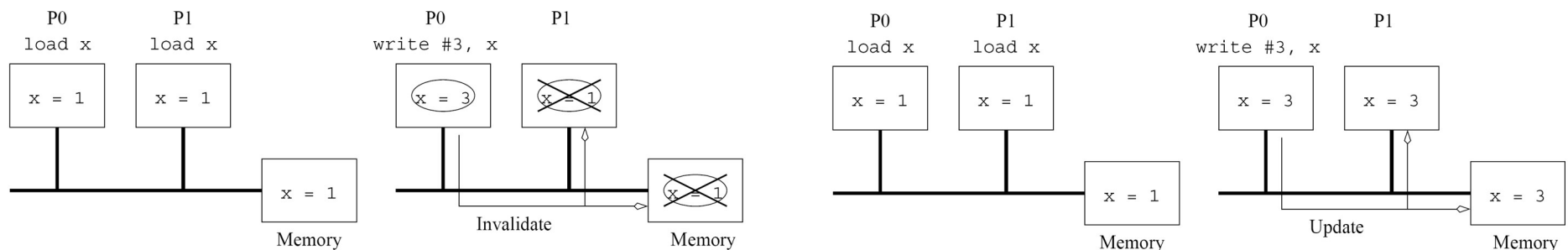
# The cache coherence problem

- In shared memory systems, there is only one logical copy of each variable.
- If several processors want to access the variable, they can each store it in their cache, to avoid latency to access main memory.
- But when one processor modifies the variable, other processors must be aware of the change.
- Since the variable is stored in multiple caches, the caches have to be made coherent through some procedure / protocol.



# Cache coherence protocols

- Two main types of protocols, invalidate and update.
- **Invalidate** When one process modifies variable, all other cached copies and copy in memory are declared invalid.
  - If another process wants to access the variable, first process writes back new value to memory, and second process reads from memory.
- **Update** When one process modifies variable, it writes new value to other caches and memory.
- Invalidate and update trade off communication vs speed.
  - If other processes don't read the variable, update wastes communication.
  - If other processes do read the variable, invalidate causes stall for writeback and read.
- Since bandwidth is limited in parallel systems, most use invalidate.



Source: Introduction to Parallel Computing, Grama et al.



# MSI protocol

- Invalidation based coherency protocol.
- The basis for widely used and higher performance protocols, e.g. MESI and MOESI.
- A variable  $X$  can be in the M (modified), S (shared) or I (invalid) state.
- When a processor performs an action (e.g. read or write), it may cause a state change and coherence messages to be sent to the main memory and all other processors.



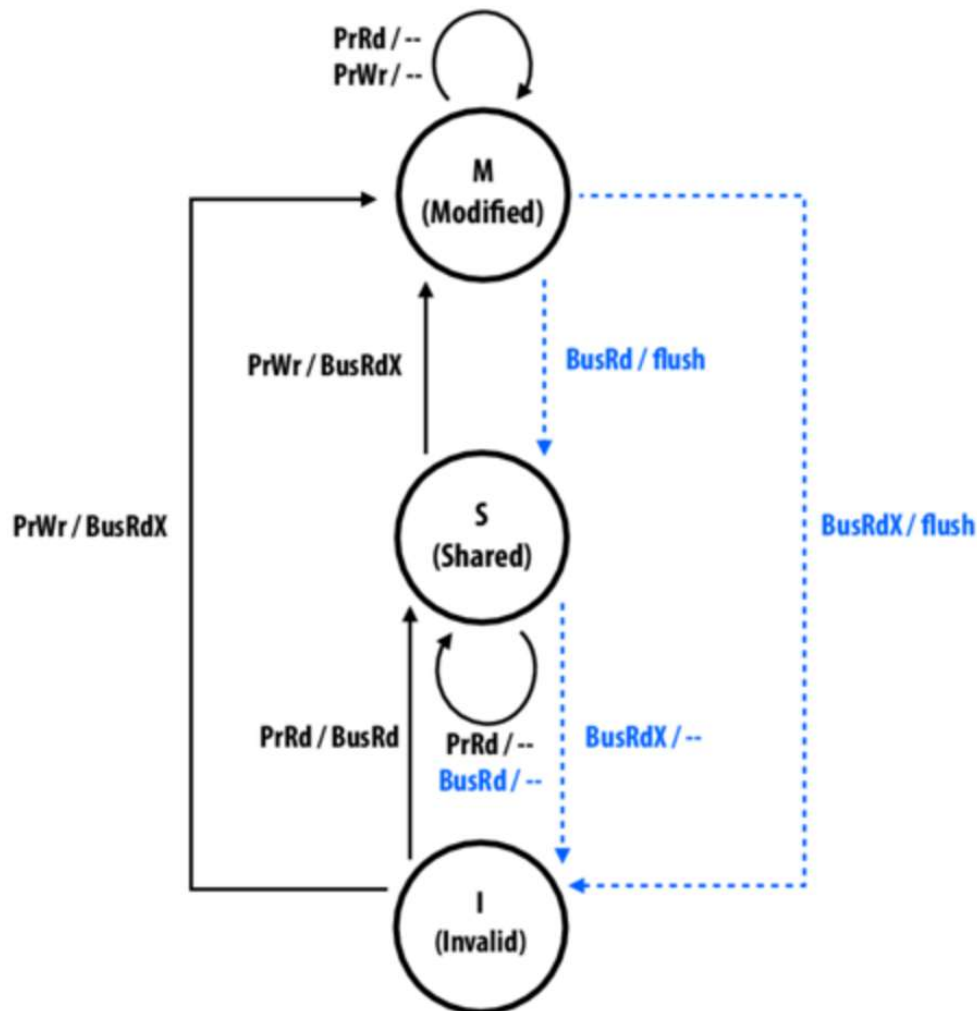
# MSI protocol

- S (shared): X may exist in multiple caches.
  - X's value hasn't changed since last time each processor read X from main memory.
    - All copies of X are up to date.
  - Processors can continue to read X from own caches.
- M (modified / dirty) state
  - A processor p changed its cached value of X.
  - p must inform other processors their cached X value now invalid.
  - p can still read / write to X in its own cache.
  - When p evicts X, must write back (flush) its latest value to main memory.
- I (invalid): Another processor has changed X's value.
  - This processor's value for X is out of date.
  - Must read X from main memory.
  - When reading X, will cause processor holding X in M state to first write its value to main memory.

# MSI example

Time ↓ ▽	Instruction at Processor 0	Instruction at Processor 1	Variables and their states at Processor 0	Variables and their states at Processor 1	Variables and their states in Global mem.
					x = 5, D y = 12, D
	read x	read y	x = 5, S	y = 12, S	x = 5, S y = 12, S
	x = x + 1	y = y + 1	x = 6, D	y = 13, D	x = 5, I y = 12, I
	read y	read x	y = 13, S x = 6, S	y = 13, S x = 6, S	y = 13, S x = 6, S
	x = x + y	y = x + y	x = 19, D y = 13, I	x = 6, I y = 19, D	x = 6, I y = 13, I
	x = x + 1	y = y + 1	x = 20, D	y = 20, D	x = 6, I y = 13, I

# MSI state machine



❑ A / B: Input request A, output action B.

❑ Processor messages in black

❑ Bus messages in blue.

❑ PrRd: processor read request

❑ PrWr: processor write request

❑ flush: processor writes back data to memory

❑ BusRdX: obtain copy with intent to modify

❑ BusRd: obtain copy of var with no intent to modify

## Examples

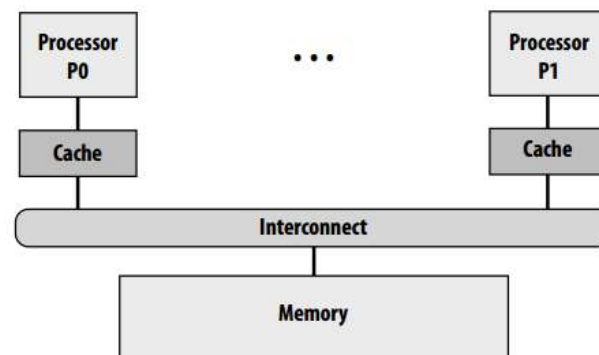
❑ Var currently shared. Proc writes new value (generates PrWr), leading to BusRdX msg. Proc goes to M state. All other procs (currently in S state) go to I state.

❑ Var currently dirty (last updated by a proc in M state). Proc in I state wants to read var (generates PrRd), leading to BusRd msg. Proc in M state flushes current var value to memory. All procs go to S state.

❑ Proc in M state wants to read / write. Generates PrRd / PrWr local message, but no bus messages. Hence no coherence traffic.

# Snoopy cache system

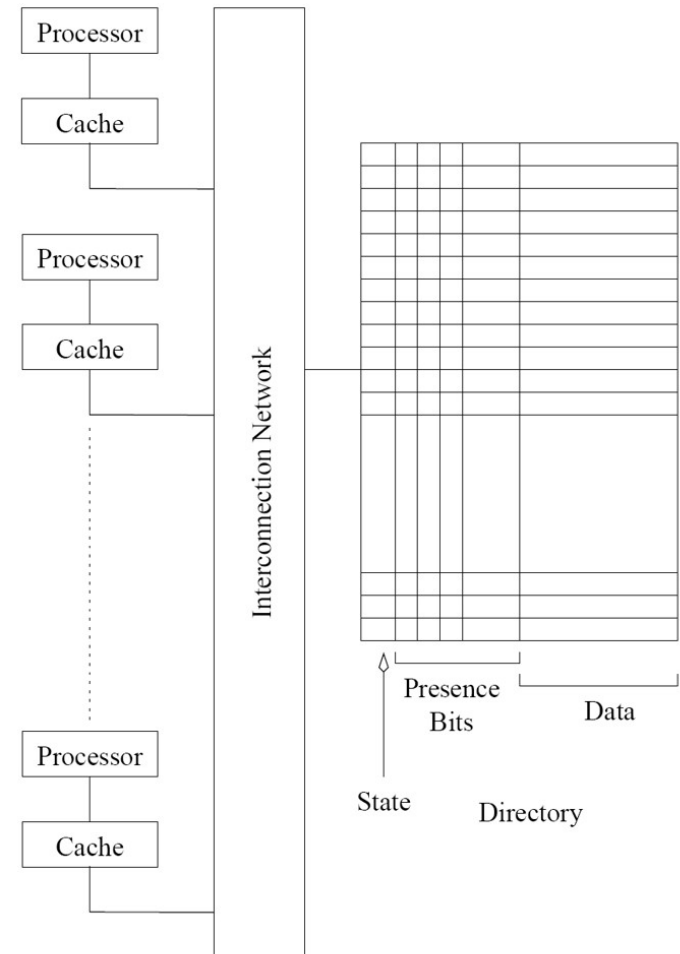
- MSI (and other protocols) can be implemented on different types of hardware.
- A basic (i.e. cheap) setup is a snoopy coherence protocol implemented on a bus or ring.
  - All processes listen on the bus for coherency traffic and respond accordingly.
  - Protocol works correctly because messages are broadcast on bus, so all processes hear all messages.
- If only one processor modifies a variable, all accesses are to its cache, and no coherence traffic  $\Rightarrow$  good performance.
- If multiple processors modify the variable, many flushes and coherency traffic  $\Rightarrow$  bus becomes performance bottleneck.





# Directory cache system

- Based on observation that only processors with shared or dirty copy of variable need to hear its coherence messages.
- Keep a directory data structure in memory, indicating for each memory block which processors hold shared or dirty copies.
  - Given  $p$  procs, for each memory block use  $p$  bits indicating processors relevant for the block.



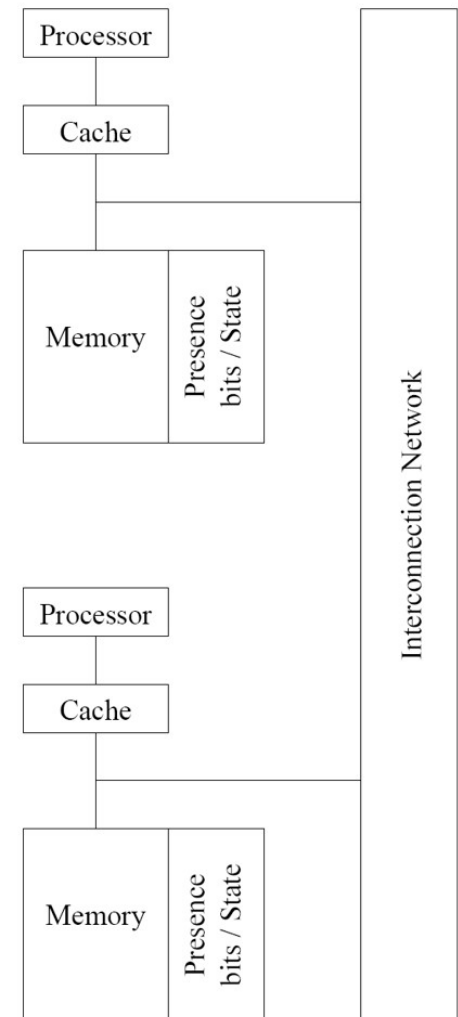


# Example

- Initially several procs hold var X in S state. They're marked in X's presence bits in directory.
- Proc 0 modifies X. Main mem and all present procs besides proc 0 recv invalidate msg.
- All their presence bits for X are reset; only proc 0 still present in directory.
- Later (invalid) proc 1 reads X. Since proc 1 in I state, it reads directory, finds proc 0 in M state, and causes proc 0 to flush X.
- Procs 0 and 1 marked in directory in S state.

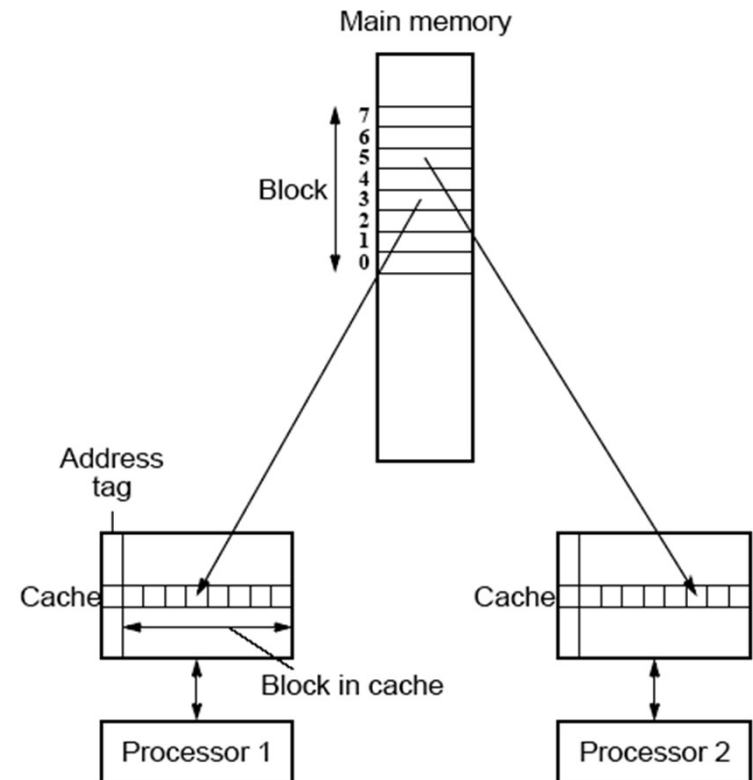
# Distributed directory cache

- Since directory stored in memory, all coherency traffic access memory, which becomes bottleneck.
- Also, directory needs large  $O(mp)$  amount of storage,  $m = \#$  memory blocks,  $p = \#$  processors.
- To improve performance, distribute directory among the processors.
  - Each processor handles a fixed memory range, and stores directory for the range.
  - Processor needing directory info on a var queries processor responsible for the var.



# False sharing

- Data transferred into / from cache in units of cache lines (aka blocks).
  - Typical L1 cache line size is 64-128B.
  - Multiple words fit into one cache line.
- Processors accessing different words can access same cache line.
  - If one processor modifies cache line, other processors with same cache line must be updated or invalidated.
- Padding tries to lay out data so data accessed by different processors are in different lines.



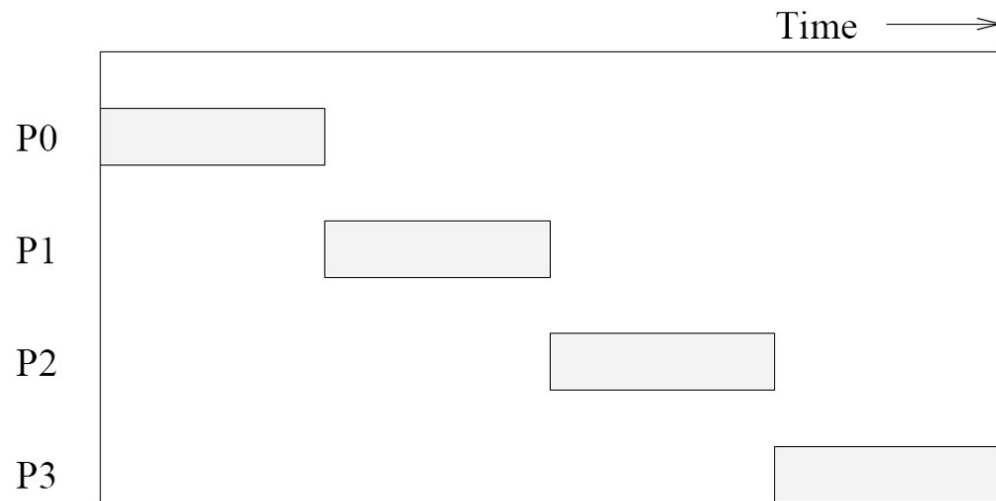


# Communication costs

- First consider cost in message passing systems, then shared memory.
- Time to transfer a message of size  $m$  from one node to another consists of
  - Startup time  $t_s$ : Prepare message header, error correction, running routing protocol, interfacing with router, etc.
    - Once per message.
  - Per hop time  $t_h$ : Used by router to determine next hop.
  - Per word transfer time  $t_w$ : If bandwidth is  $r$ , per word transfer time from source to dest is  $1/r$ .

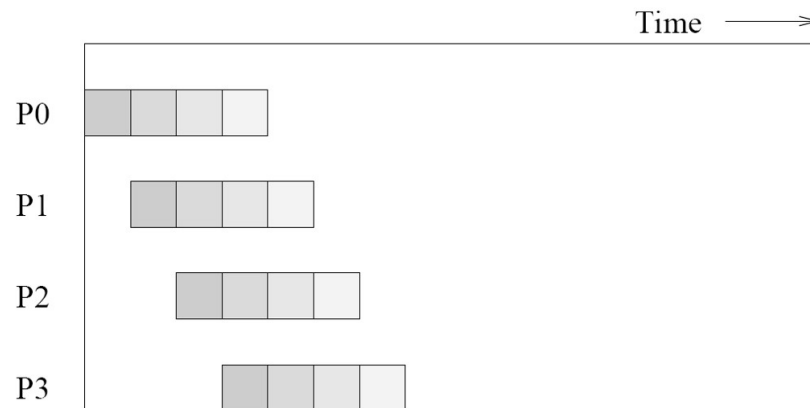
# Store and forward routing

- Each intermediate node waits to receive entire message before forwarding it to next hop on path.
- Takes  $m t_w + t_h$  time per link, so for  $l$  links  $t_s + l(m t_w + t_h)$  total time.
  - $t_h$  small compared to  $m t_w$ , so time is  $\approx t_s + l m t_w$ .



# Packet routing

- Break large message into small packets.
- Intermediate node can receive a packet as soon as it finishes forwarding previous packet it received.
  - Pipeline parallelism.
- Also lets packets take different routes, and easier error correction.
  - Essential for Internet, with unpredictable network state, high error rates.
- But overhead from each packet's routing info, error correction, sequence number etc.
- Total time  $t_s + t_h l + t_w (m+1-1)$ .
  - First packet arrives at time  $t_s + t_h l + t_w l$ .
  - Remaining  $m-1$  packets arrive every  $t_w$  time.





# Cut-through switching

- Similar to packet routing, but optimized for parallel computers.
- Messages take same path. No routing info.
- Small (~ 32B) flits (flow control digits) transmitted at high rate.
  - First flit (header) is allocated path by switches / routers. Remaining (body and tail) flits follow same path.
- Simple, per message (instead of per packet) error correction.
- To reduce latency for high priority flits (e.g. cache lines), use multilane cut-through routing.
- Also called virtual cut-through. A related technique is wormhole routing.





# Overall cost model

- Total communication time depends on data volume  $m$  and number of hops  $l$ .
- However, programmer has little control over  $l$ .
  - Usually can't control process to processor mapping.
  - Many systems use randomized two step routing to minimize congestion: first send message to random node, then send it to destination.
  - Per hop time  $t_h$  usually small compared to  $t_s$  or  $m t_w$ .
- Communication time can be simplified to  $t_s + m t_w$ .
- Assumes uncongested network.
  - If congestion on link is  $c$ , i.e.  $c$  messages are sent over the link, time becomes  $t_s + c m t_w$ .
- **Ex**  $\sqrt{p} \times \sqrt{p}$  mesh where nodes randomly communicate with each other.
  - Across a bisection there are  $O(p)$  communications.
  - But mesh bisection width is  $\sqrt{p}$ .
  - So some link carries  $O(p) / \sqrt{p} = O(\sqrt{p})$  messages.
  - So communication time is  $t_s + O(\sqrt{p} m t_w)$ .



# Comm cost in shared memory

- Shared memory communication costs are more unpredictable.
  - Memory layout determined by system. Hard to estimate amount of local / remote memory accesses.
  - Cache thrashing depends on scheduling and process allocation.
  - Coherency traffic, spatial locality and false sharing also very nondeterministic.
- But to first order, still model communication time as  $t_s + t_w m$ .
  - $t_s$  and  $t_m$  are much smaller in shared memory than distributed memory architectures.

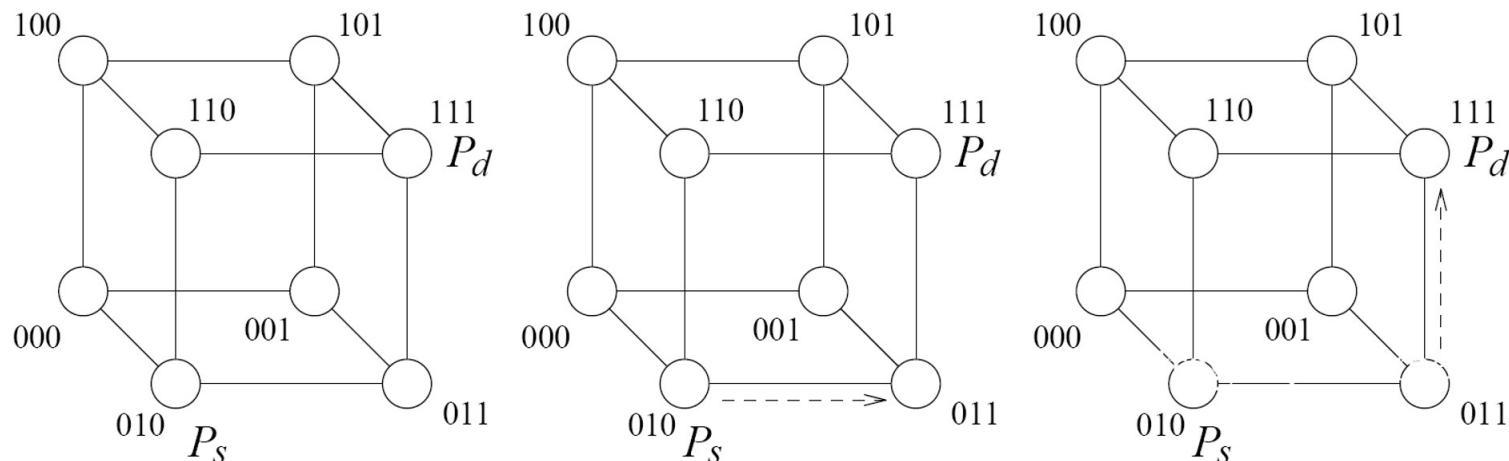


# Routing

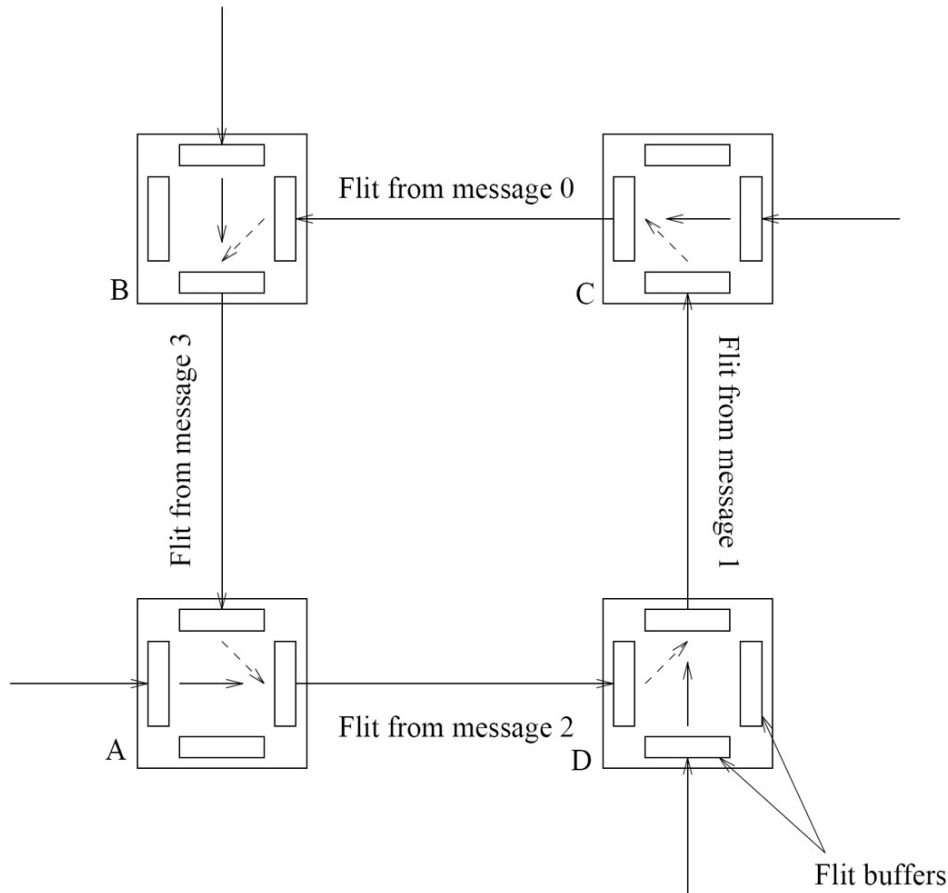
- We saw interconnection networks such as mesh, hypercube, tree, etc. How do processes communicate on these networks?
- Routing algorithm selects a path between two communicating processes.
- Routes are ideally short, uncongested and easy to compute. But tradeoffs exist.
  - **Ex** Short (minimal) routes may be more congested.
  - **Ex** Deterministic, i.e. fixed given source and destination, or adaptive, i.e. route around congestion.

# Dimension and E-cube routing

- Dimension routing for a k-dim mesh orders the dimensions (e.g. XYZ) and routes messages in order of dimension.
- E-cube routing does dimension routing on dimensions of a hypercube.
  - Let  $s, d$  be bit representations of source and destination.
  - Compute  $r = s \text{ XOR } d$ , and from least to most significant digit of  $r$ , route along dimensions with 1-bit.
- Ex Routing from 010 to 111.



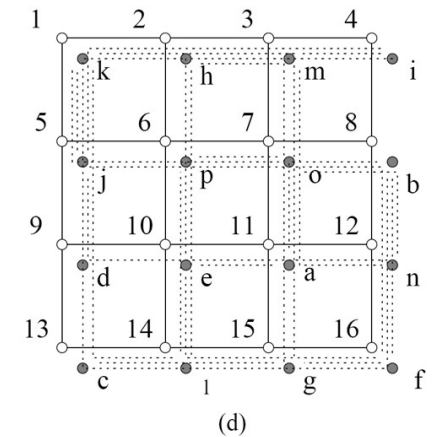
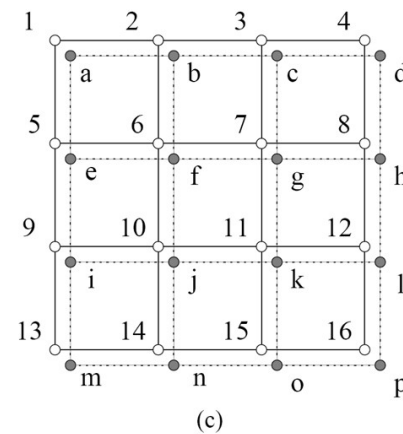
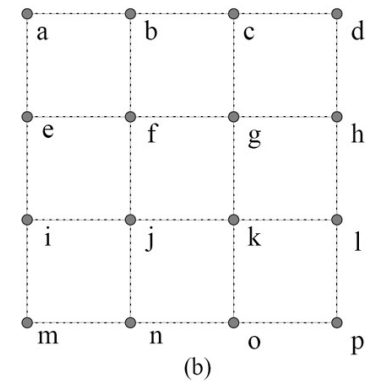
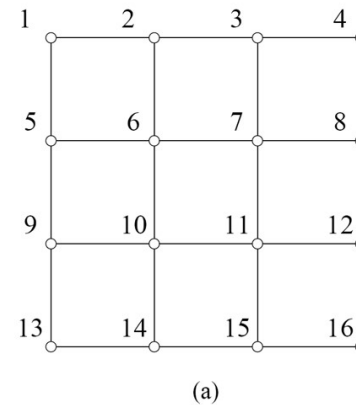
# Deadlocks



- Dimension and E-cube routing prevent deadlocks.
- **Ex** Without dimension routing.
  - $A \rightarrow D \rightarrow C, D \rightarrow C \rightarrow B, C \rightarrow B \rightarrow A, B \rightarrow A \rightarrow D$
  - A's flit can't move from AD to DC, because DC is occupied by D's flit.
  - D's flit can't move from DC to CB, because CB is occupied by C's flit. Etc.
- With XY routing
  - $A \rightarrow D \rightarrow C, D \rightarrow A \rightarrow B, C \rightarrow B \rightarrow A, B \rightarrow C \rightarrow D.$
  - A and C's flits first move along X dim, then B and D's flits.

# Process-processor mappings

- Process communication pattern determined by the program.
- Processor communication pattern determined by the hardware.
- To run a program on a piece of hardware, must map processes to processors.
- Poor mappings can cause communication bottlenecks, reduce performance.
- Ex (a) and (b) show processor and process communication patterns.
  - Mapping (c) causes no bottlenecks.
  - Mapping (d) creates bottlenecks at links 15, 23, etc.



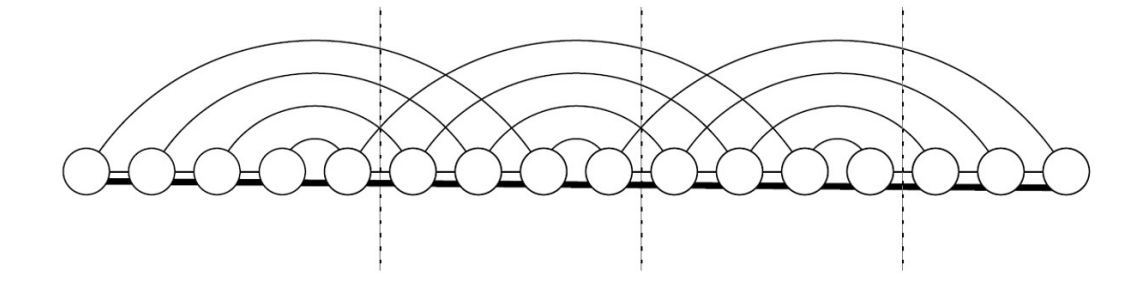
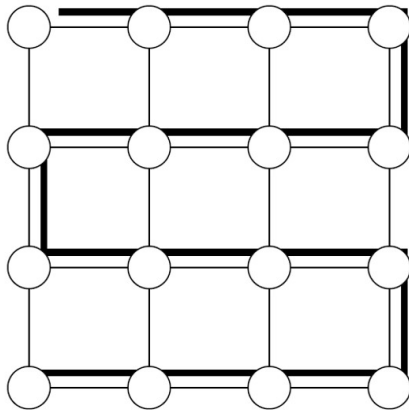


# Embeddings

- Give a map  $f$  from the graph  $P$  of process communication to graph  $R$  of processor communication pattern.
  - Each process of  $P$  maps to a processor in  $R$ .
  - Each edge from  $p_1$  and  $p_2$  in  $P$  maps to a shortest path between  $f(p_1)$  to  $f(p_2)$  in  $R$ .
- Congestion is max number of routing paths that cross an edge in  $R$ .
  - **Ex** On previous example congestion = 5, e.g. on edge (1,5).
- Dilation is max length of route between any two neighbors in  $P$ .
  - **Ex** On previous example dilation = 5, e.g. for edge (b,c).
- Expansion is  $(\# \text{ processes}) / (\# \text{ processors})$ .
  - Assume for simplicity it is 1.

# Line and mesh

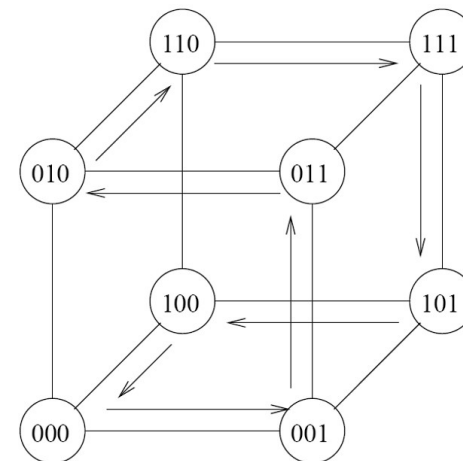
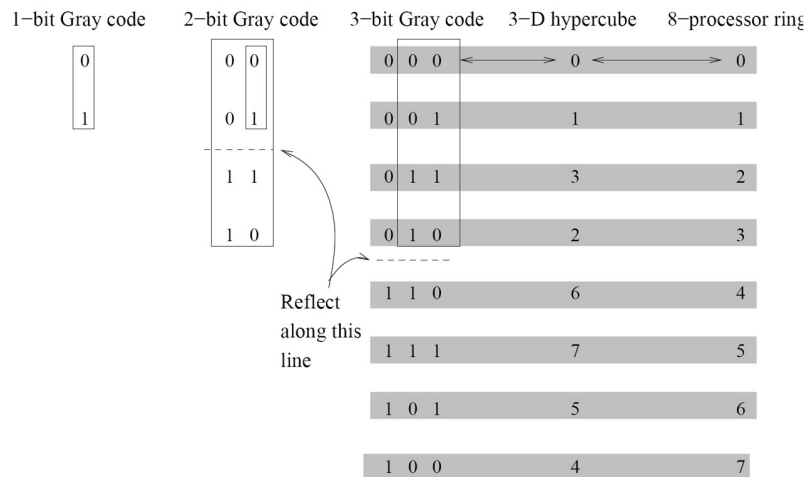
- Line maps into mesh using zig-zag embedding.
  - Congestion and dilation are 1.
- To map mesh to line, invert the mapping.
- For a  $\sqrt{p} \times \sqrt{p}$  mesh, congestion is  $\sqrt{p}$ .
  - Every two consecutive rows must traverse edge connecting them in the line graph.
- Dilation is  $2\sqrt{p} - 1$ , between the first nodes in consecutive rows.





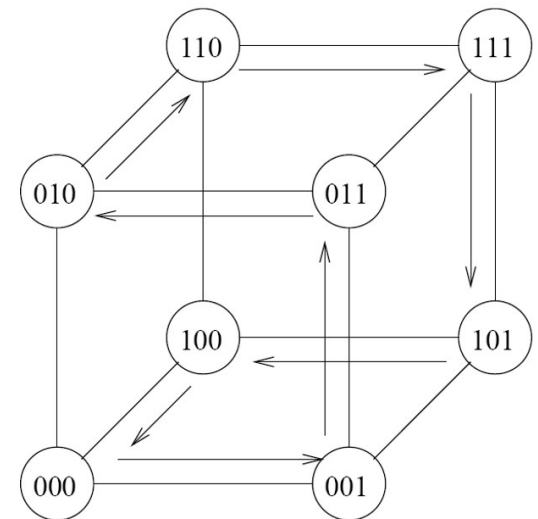
# Line to hypercube

- Map size  $2^k$  line graph into  $k$ -dim hypercube.
- **Gray codes** An ordering of  $k$  digit binary numbers where consecutive values differ in one digit.
  - **Recursive construction** Let  $G_{k-1}$  be a  $k-1$  digit Gray code. Prepend  $G_{k-1}$  with 0. Then take another  $G_{k-1}$ , reverse it and prepend with 1. Concatenate the two copies.
- Map node  $i$  in line graph to node  $G_k(i)$  in hypercube.
- $i$  and  $i+1$  are mapped to neighbors in hypercube.
  - $G_k(i)$  and  $G_k(i+1)$  differ in one bit, so are connected in hypercube.
  - So dilation is 1.
- Each hypercube edge is mapped onto by at most one line graph edge.
  - So congestion is 1.



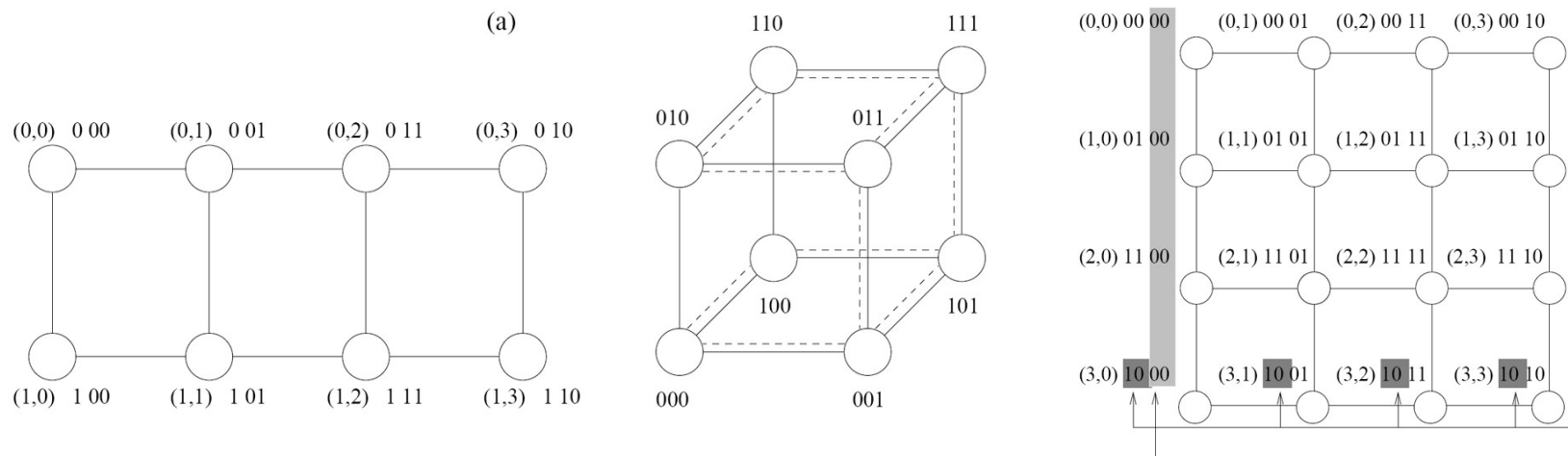
# Hypercube to line

- To map  $k$ -dim hypercube into size  $2^k$  line graph, just use the reverse mapping, i.e. node  $i$  in hypercube maps to  $G_k^{-1}(i)$  in line.
- To compute congestion, recall bisection width of  $k$ -dim hypercube is  $2^{k-1}$ .
  - Bisection width of line is 1.
  - Thus,  $2^{k-1}$  edges of the hypercube need to cross the middle edge of the line graph, and the congestion is  $2^{k-1}$ .
- Dilation is  $\Theta(2^k)$ , e.g. from node 0 to node  $2^{k-1}$ .



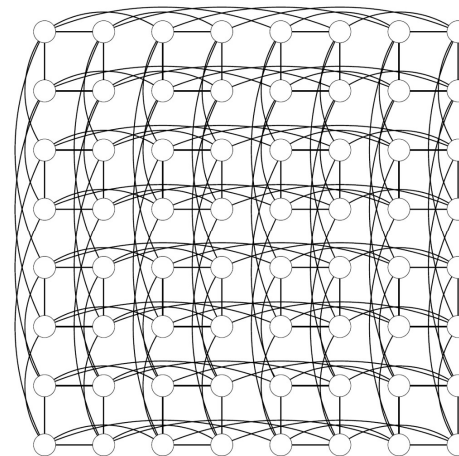
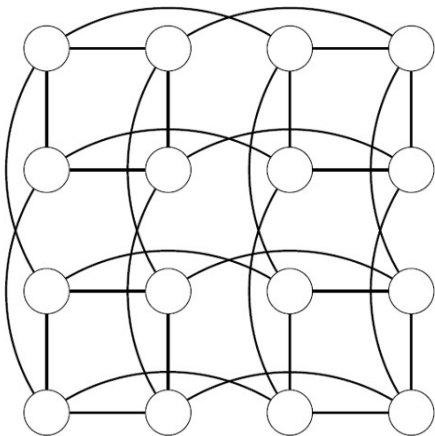
# 2D mesh to hypercube

- Map a  $2^r \times 2^s$  mesh into an  $(r + s)$ -dim hypercube.
- Map node  $(i, j)$  in mesh to node  $G_r(i) \parallel G_s(j)$  in hypercube.
  - Neighbors in the mesh map to nodes that differ in one bit, i.e. neighbors in the hypercube.
  - So dilation and congestion are both 1.
- Note that different rows and columns of the mesh map to distinct sub-hypercubes.



# Hypercube to 2D mesh

- Map  $(\log p)$ -dim hypercube into  $\sqrt{p} \times \sqrt{p}$  mesh.
  - Assume  $\sqrt{p}$  for simplicity is a power of 2, and let  $q = \log \sqrt{p} = (\log p) / 2$ .
- If we fix the first or last  $q$  digits of the nodes in hypercube, we get another  $q$ -dim hypercube.
  - Ex Take 4-dim hypercube, and fix last two digits to 10. Then get another hypercube (0010, 0110, 1110, 1010).
- For each  $q$  digit binary number  $r$ , map the hypercube from fixing the last  $q$  digits in hypercube to row  $r$  of mesh.
  - Use the hypercube to line mapping.
  - Congestion is  $\sqrt{p} / 2$ .
- For each  $q$  digit binary number  $r$ , map the hypercube from fixing the first  $q$  digits in hypercube to column  $r$  of mesh.
- Congestion in rows and column independent. So overall congestion is  $\sqrt{p} / 2$ .





# Cost performance tradeoffs

- Cost of a network can be measured in terms of e.g. number of wires, bisection width (complexity), etc.
- **Wire complexity** Square  $p$  node mesh with  $O(\log p)$  wires per link has same cost as  $p$  node hypercube.
- Average distance in mesh is  $O(\sqrt{p})$ , and in hypercube is  $O(\log p)$ .
- With  $O(\log p)$  wires per link in mesh,  $t_w$  gets reduced by  $O(\log p)$  factor.
  - Avg latency in mesh is  $t_s + O(t_h \sqrt{p}) + O(m t_w / \log p)$ .
  - Avg latency in hypercube is  $t_s + O(t_h \log p) + m t_w$ .
  - For fixed number of processors and large message size,  $m t_w \gg t_h$ , so mesh has lower latency.
- Hypercube better under contention, since it has much greater bisection width.