Computer Animation & Physical Simulation

Lecture 8: Soft-Body Simulation – Hair I

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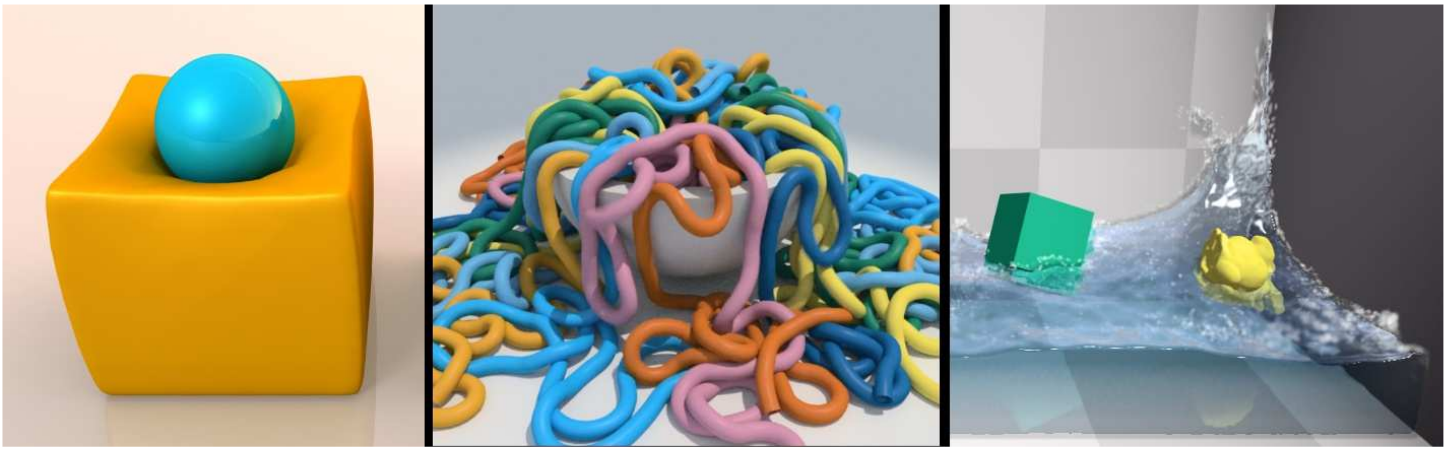
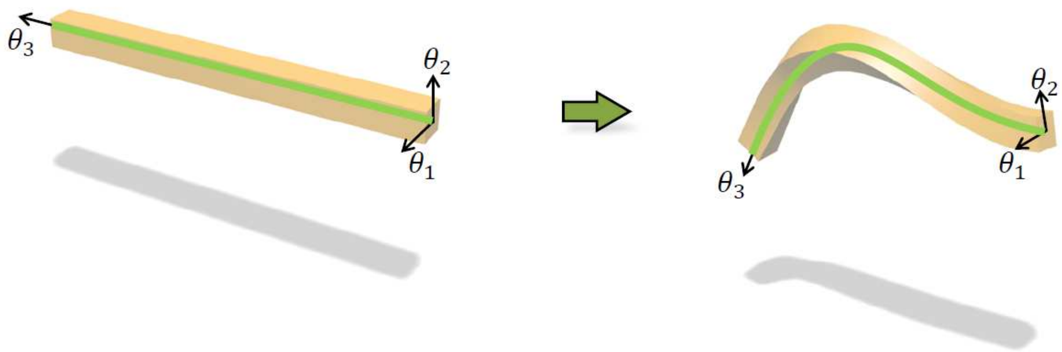
What is a soft body?

- **Unlike rigid body**
  - Shape of soft bodies can change visually
  - The relative distance of two points on the object is not fixed



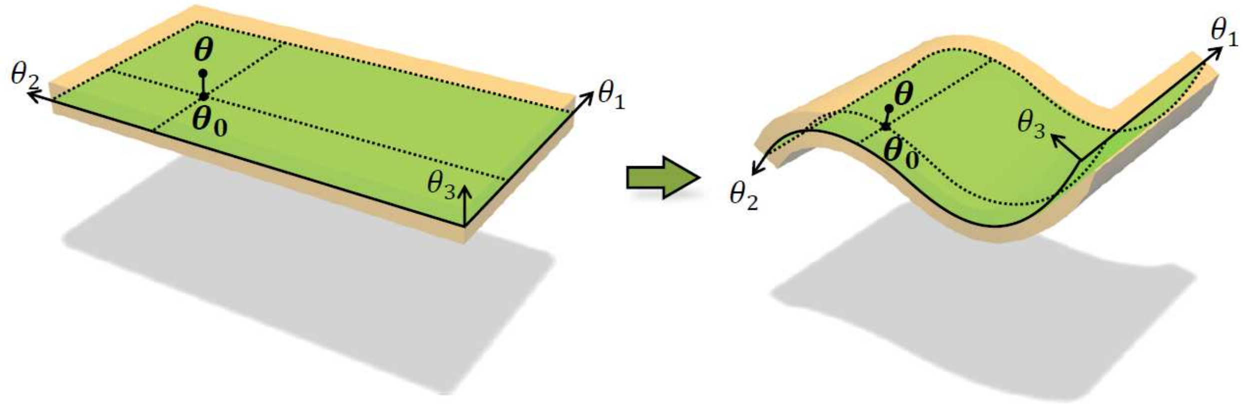
Type of Soft Body

- **1D soft body – rod**
  - A volumetric curve-like solid
  - Extent along tangent direction is much greater than along normal directions



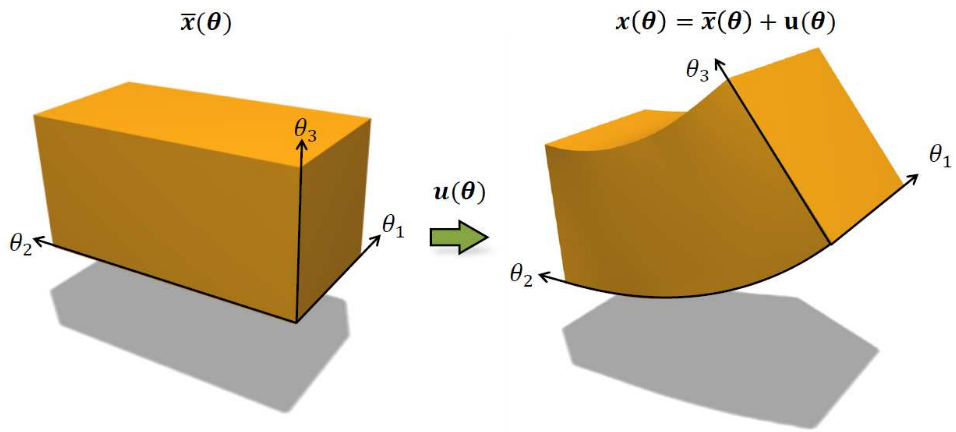
Type of Soft Body

- **2D soft body – (thin) shell**
  - A volumetric surface-like solid
  - Extent along tangent plan directions is much greater than along normal direction



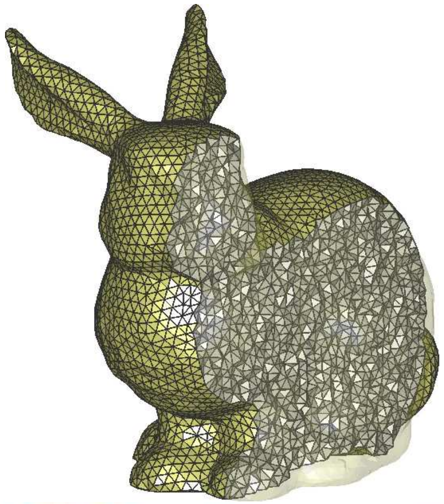
Type of Soft Body

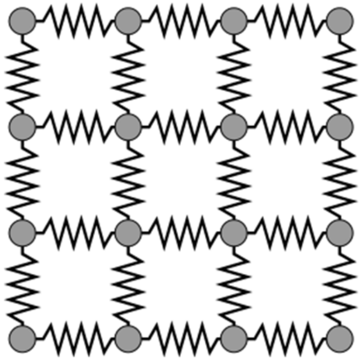
- **3D soft body – (volumetric) solid**
  - A volumetric body
  - No dominance on extension along specific directions

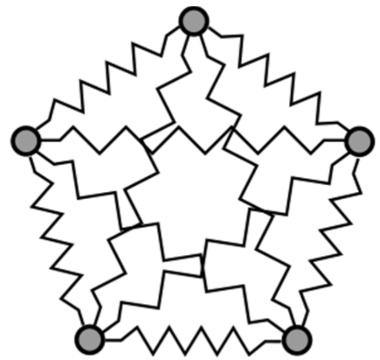


How to simulate soft body motion?

- **Mass-spring system**
  - Represent the solid with meshes
  - Apply spring dynamics to each mesh element

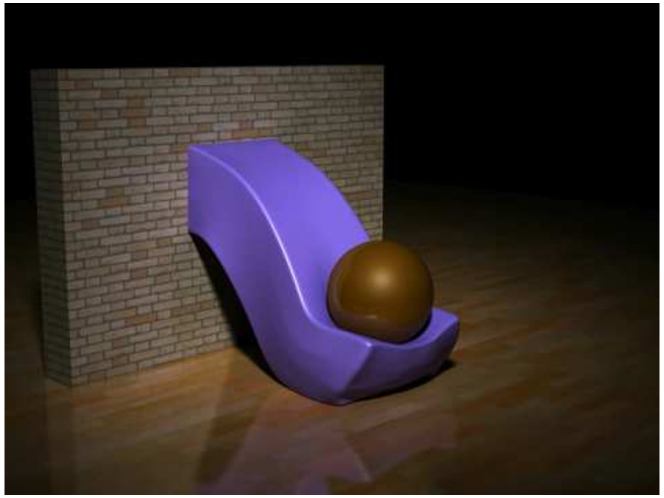


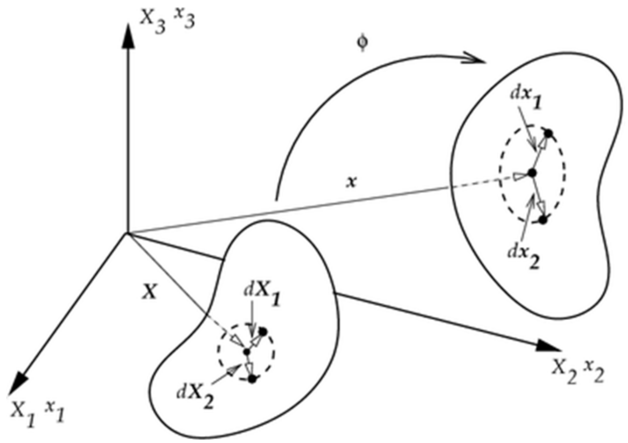




How to simulate soft body motion?

- **Continuum mechanics**
  - The substance is assumed to be continuously distributed
  - Strain and stress relation through constitutive laws

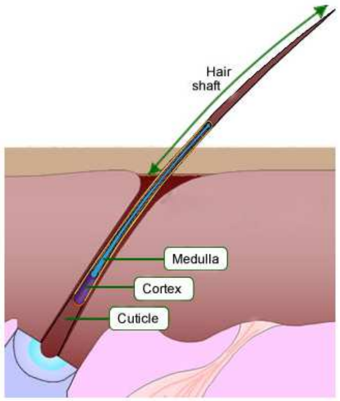


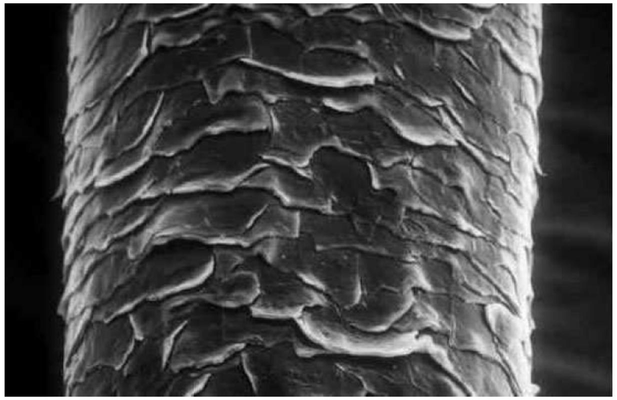


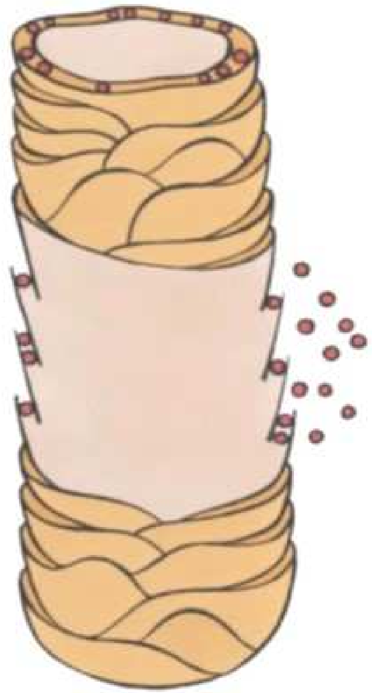
I. Hair Basics

Structure of Hair

- **Close look of hair strand**
  - Hair fiber is a thin structure (0.1mm in diameter)
  - Circular or oval cross section
  - Different frictions along different directions





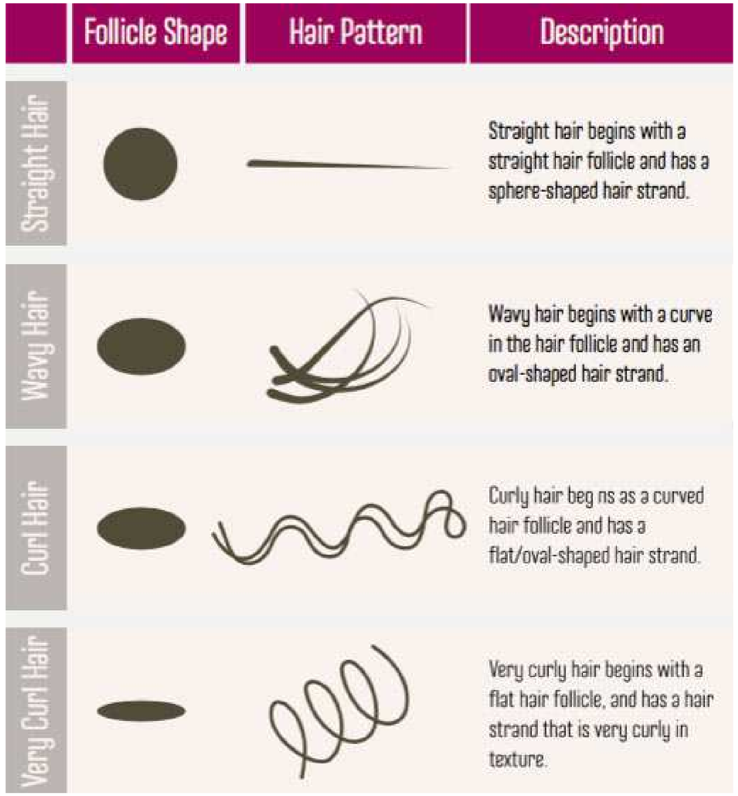


Physical Property of Hair

- **Nonlinear dynamics**
  - Bending and twisting instabilities

Physical Property of Hair

- **Different hair style**
    - Circular cross section: straight hair
    - Oval cross section: curly hair



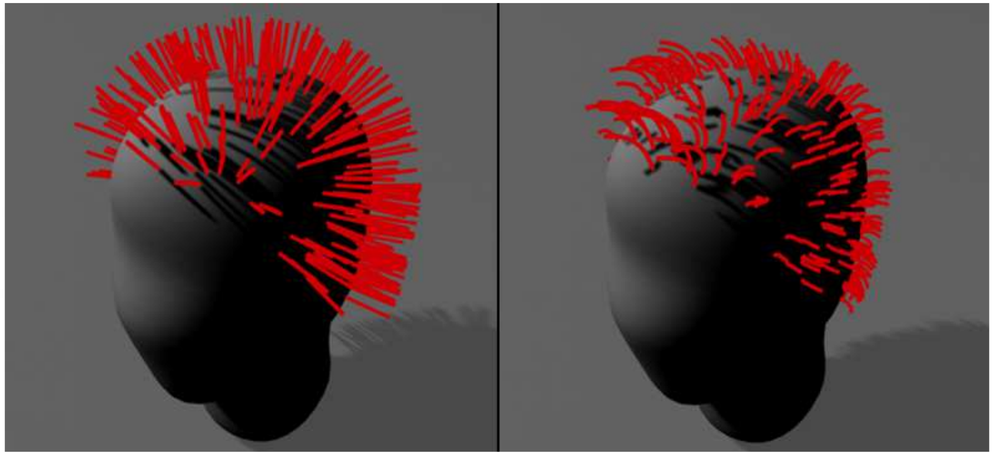
	Follicle Shape	Hair Pattern	Description
Straight Hair			Straight hair begins with a straight hair follicle and has a sphere-shaped hair strand.
Wavy Hair			Wavy hair begins with a curve in the hair follicle and has an oval-shaped hair strand.
Curl Hair			Curly hair begins as a curved hair follicle and has a flat/oval-shaped hair strand.
Very Curl Hair			Very curly hair begins with a flat hair follicle, and has a hair strand that is very curly in texture.

Hair Representation

- **Full representation**
  - Each hair strand is represented explicitly by curves

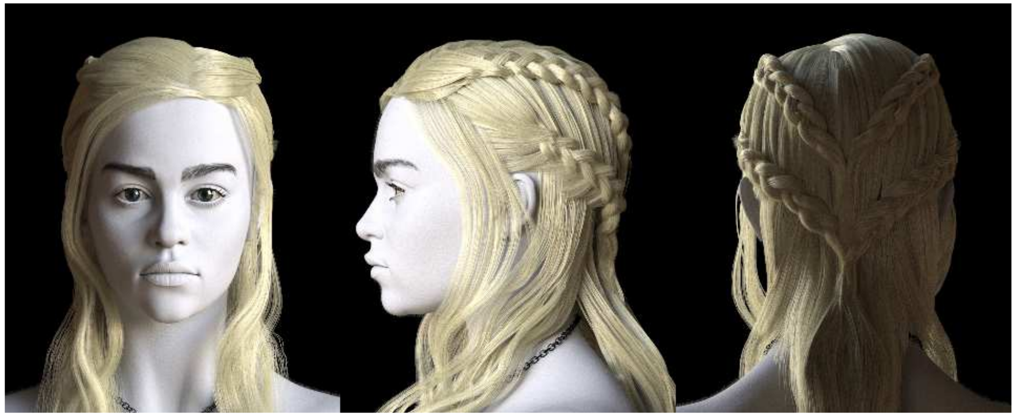
Hair Representation

- **Clustered representation**
  - The hair is represented by guide strands
    - Coherence between guide strands



Hair Modeling

- **Geometric modeling**
  - Representing the hair geometry
  - Hair styling (NURBS)



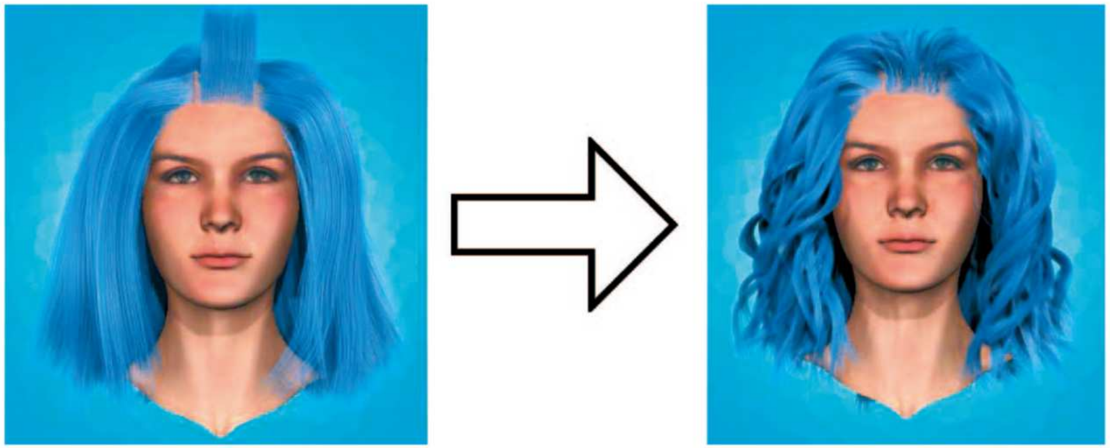
- **Physical modeling**
  - No mature physical model
  - Empirical construction
    - With physical arguments

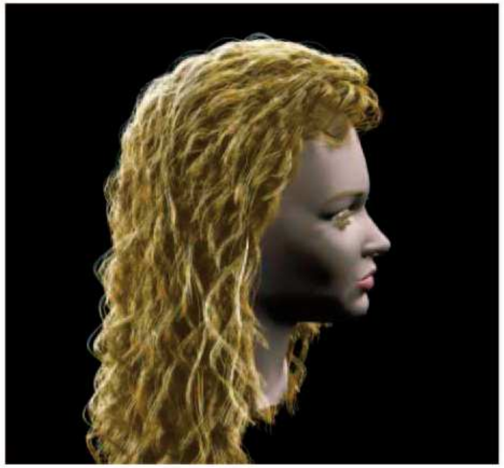
$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}_{iQ}}\right) - \frac{\partial T}{\partial q_{iQ}} + \frac{\partial U}{\partial q_{iQ}} + \frac{\partial D}{\partial \dot{q}_{iQ}} = \int_0^L \mathbf{J}_{iQ}(s, \mathbf{q}, t) \cdot \mathbf{F}(s, t)\, \mathrm{d}s$$

Super-Helix hair model

Hair Styling

- **Creating a desired hairstyle**
  - Purely geometric and physically-based



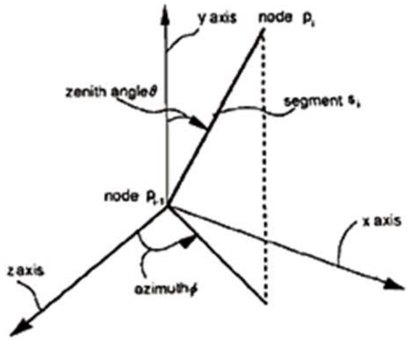


Hair Simulation

- **Simulate the motion of hair**
  - Given initial hair geometry (from modeling)
  - Based on a chosen physical model
  - Possibly interact with hair itself, other solids and fluids

Early Simulation Methods

- **One-dimensional projective equations**
  - A chain of rigid sticks
  - Each stick is assimilated as a direction
    - Parameterized by its polar angles

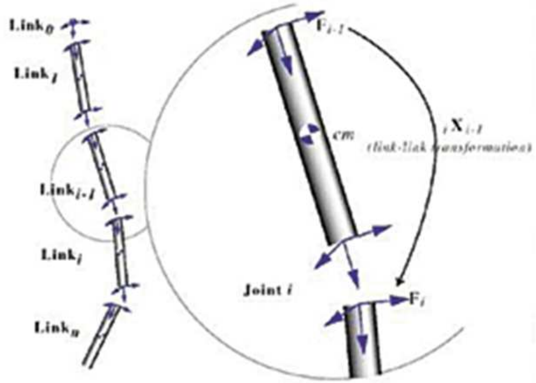


(a)

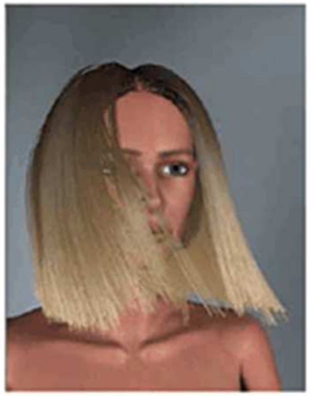
(b)

Early Simulation Methods

- **Rigid multi-body serial chain**
  - Represented as a serial, rigid, multibody open chain
  - Limited to straight hair, curly hair is not modeled



(a)



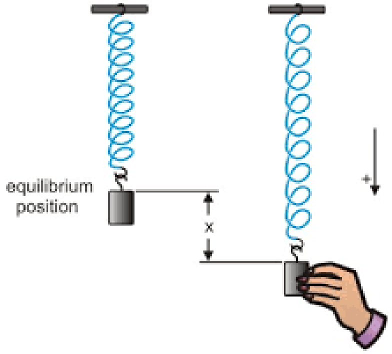
(b)

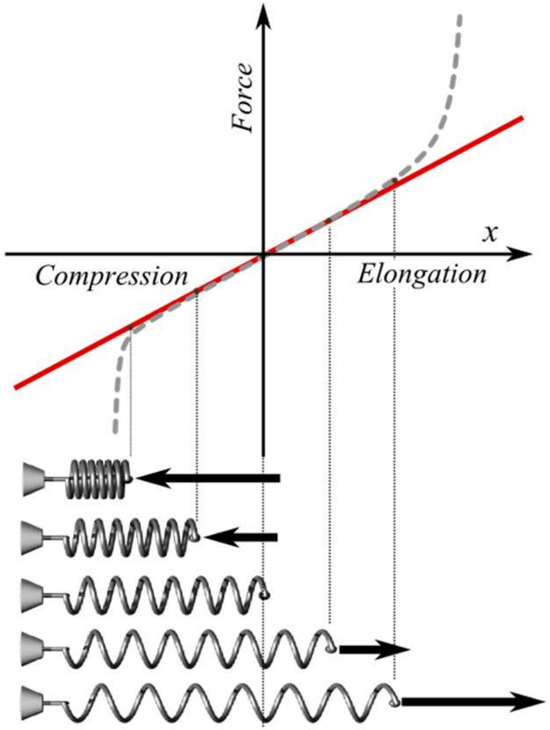
II. Mass-Spring System for Hair Simulation

Mass-Spring Model

- **Linear elastic model**
  - The spring is connected to a non-negligible mass m
  - Hook's law

$$\vec{F} = -k\vec{x}$$





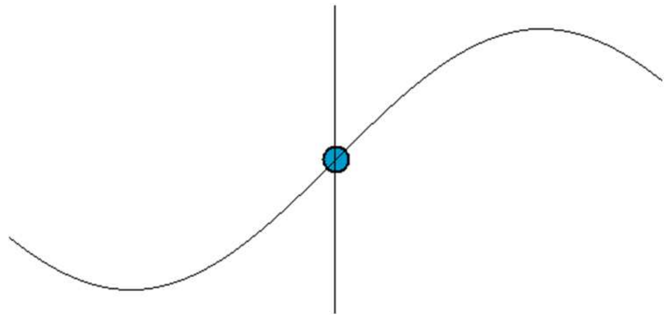
Mass-Spring Model

- **Harmonic oscillator**
  - Without damping

$$F = ma = m\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = m\ddot{x} = -kx$$

    - Analytical solution

$$x(t) = A\cos(\omega t + \phi)$$



Mass-Spring Model

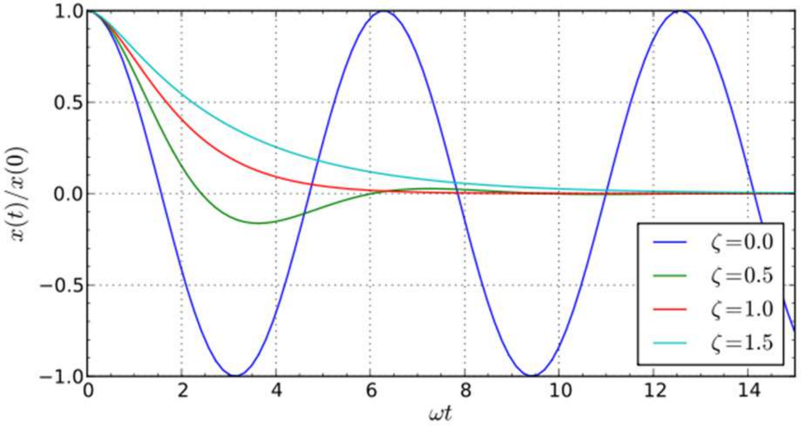
- **Harmonic oscillator**
  - With damping

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2\zeta\omega_0\frac{\mathrm{d}x}{\mathrm{d}t} + \omega_0^2 x = 0,$$

where

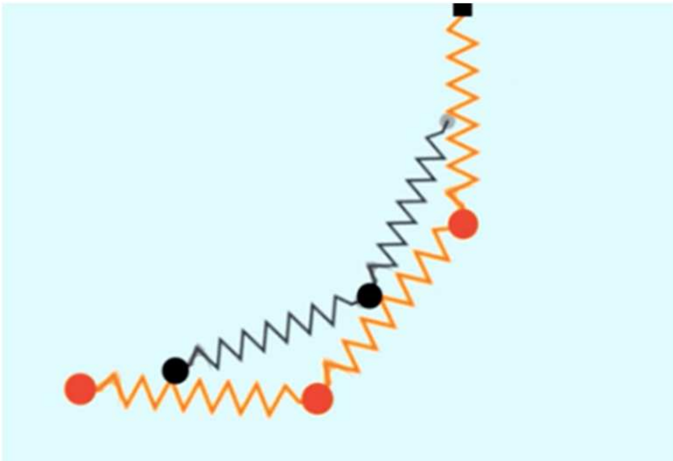
$\omega_0 = \sqrt{\frac{k}{m}}$ is called the 'undamped angular frequency of the oscillator' and

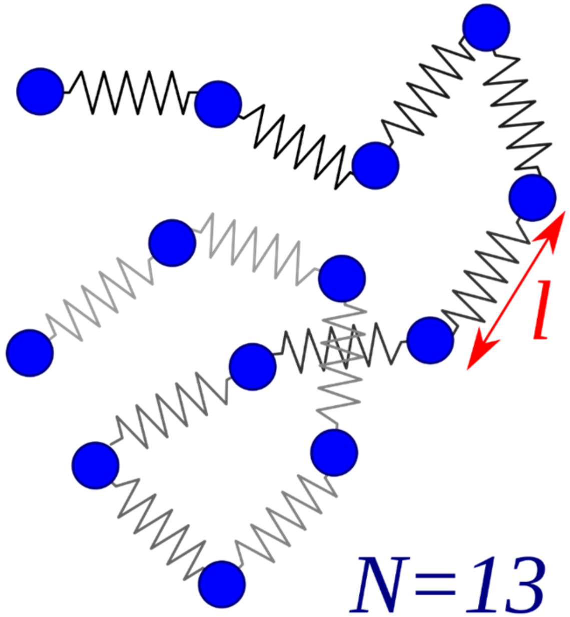
$\zeta = \frac{c}{2\sqrt{mk}}$ is called the 'damping ratio'.



Mass-Spring System

- **A system of mass objects connected by springs**
  - An object is modeled as point masses
  - Objects are connected by one or more springs





Mass-Spring System

- **Governing dynamic equation**
  - Using linear spring model
  - With linear damping

$$m_i\ddot{\mathbf{x}}_i = -\gamma_i\dot{\mathbf{x}}_i + \sum_j \mathbf{g}_{ij} + \mathbf{f}_i$$

$\mathbf{g}_{ij}$: forces exerted on mass i by spring between masses i and j

Mass-Spring System

- **Governing dynamic equation**
  - Writing the equation for entire system

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f}$$

M: mass matrix, diagonal
C: damping matrix, diagonal
K: stiffness matrix, encodes spring forces from nearby connected springs

Re-expression in first-order system

$$\begin{aligned} \dot{\mathbf{v}} &= \mathbf{M}^{-1}\left(-\mathbf{C}\mathbf{v} - \mathbf{K}\mathbf{x} + \mathbf{f}\right) \\ \dot{\mathbf{x}} &= \mathbf{v} \end{aligned}$$

velocity of mass point

Mass-Spring System

- **Numerical solution**
  - Explicit Euler time discretization

$$\frac{d}{dt}\begin{pmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{pmatrix} = \frac{d}{dt}\begin{pmatrix} \mathbf{x} \\ \mathbf{v} \end{pmatrix} = \begin{pmatrix} \mathbf{v} \\ \mathbf{M}^{-1}\mathbf{f}(\mathbf{x}, \mathbf{v}) \end{pmatrix}$$

$\mathbf{x}_0 = \mathbf{x}(t_0)$ and $\mathbf{v}_0 = \mathbf{v}(t_0)$

$\Delta\mathbf{x} = \mathbf{x}(t_0 + h) - \mathbf{x}(t_0)$ and $\Delta\mathbf{v} = \mathbf{v}(t_0 + h) - \mathbf{v}(t_0)$

$$\begin{pmatrix} \Delta\mathbf{x} \\ \Delta\mathbf{v} \end{pmatrix} = h\begin{pmatrix} \mathbf{v}_0 \\ \mathbf{M}^{-1}\mathbf{f}_0 \end{pmatrix}$$

Mass-Spring System

- **Numerical solution**
  - Implicit Euler discretization
  - Linearize

$$\frac{d}{dt}\begin{pmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{pmatrix} = \frac{d}{dt}\begin{pmatrix} \mathbf{x} \\ \mathbf{v} \end{pmatrix} = \begin{pmatrix} \mathbf{v} \\ \mathbf{M}^{-1}\mathbf{f}(\mathbf{x}, \mathbf{v}) \end{pmatrix}$$

$$\Downarrow$$

$$\begin{pmatrix} \Delta\mathbf{x} \\ \Delta\mathbf{v} \end{pmatrix} = h\begin{pmatrix} \mathbf{v}_0 + \Delta\mathbf{v} \\ \mathbf{M}^{-1}\mathbf{f}(\mathbf{x}_0 + \Delta\mathbf{x}, \mathbf{v}_0 + \Delta\mathbf{v}) \end{pmatrix}$$

$$\Downarrow \quad \mathbf{f}(\mathbf{x}_0 + \Delta\mathbf{x}, \mathbf{v}_0 + \Delta\mathbf{v}) = \mathbf{f}_0 + \frac{\partial\mathbf{f}}{\partial\mathbf{x}}\Delta\mathbf{x} + \frac{\partial\mathbf{f}}{\partial\mathbf{v}}\Delta\mathbf{v}$$

$$\begin{pmatrix} \Delta\mathbf{x} \\ \Delta\mathbf{v} \end{pmatrix} = h\begin{pmatrix} \mathbf{v}_0 + \Delta\mathbf{v} \\ \mathbf{M}^{-1}\left(\mathbf{f}_0 + \frac{\partial\mathbf{f}}{\partial\mathbf{x}}\Delta\mathbf{x} + \frac{\partial\mathbf{f}}{\partial\mathbf{v}}\Delta\mathbf{v}\right) \end{pmatrix}$$

$$\Downarrow$$

$$\Delta\mathbf{v} = h\mathbf{M}^{-1}\left(\mathbf{f}_0 + \frac{\partial\mathbf{f}}{\partial\mathbf{x}}h(\mathbf{v}_0 + \Delta\mathbf{v}) + \frac{\partial\mathbf{f}}{\partial\mathbf{v}}\Delta\mathbf{v}\right)$$

Mass-Spring System

- **Numerical solution**
  - Implicit Euler discretization
  - Regrouping

$$\Delta \mathbf{v} = h\mathbf{M}^{-1}\left(\mathbf{f}_0 + \frac{\partial \mathbf{f}}{\partial \mathbf{x}} h(\mathbf{v}_0 + \Delta \mathbf{v}) + \frac{\partial \mathbf{f}}{\partial \mathbf{v}} \Delta \mathbf{v}\right)$$

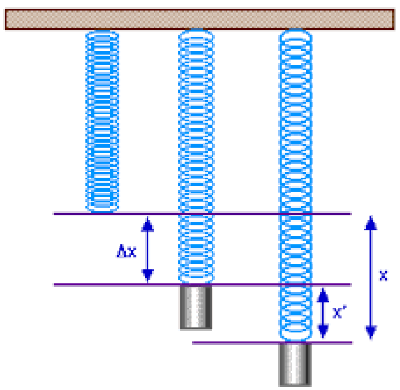
$$\Downarrow$$

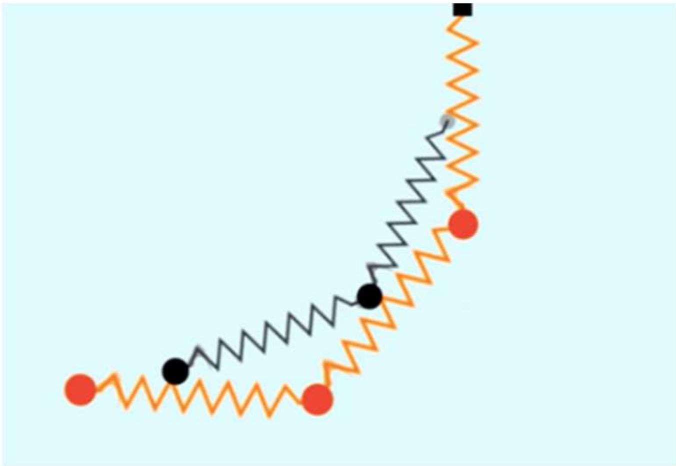
$$\left(\mathbf{I} - h\mathbf{M}^{-1}\frac{\partial \mathbf{f}}{\partial \mathbf{v}} - h^2\mathbf{M}^{-1}\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right)\Delta \mathbf{v} = h\mathbf{M}^{-1}\left(\mathbf{f}_0 + h\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\mathbf{v}_0\right)$$

- We then solve for $\Delta \mathbf{v}$ (sparse linear system, conjugate gradient)
- Given $\Delta \mathbf{v}$, we then compute $\Delta \mathbf{x} = h(\mathbf{v}_0 + \Delta \mathbf{v})$

Simple Modeling for Hair Simulation

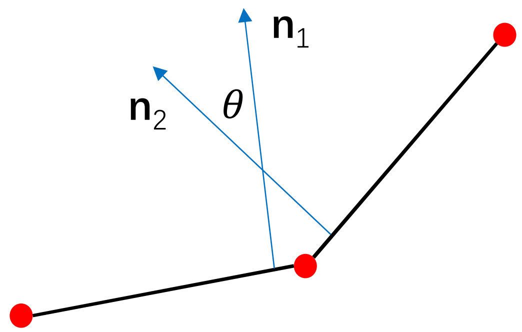
- **Stretching force**
  - Based on the length difference w.r.t rest shape

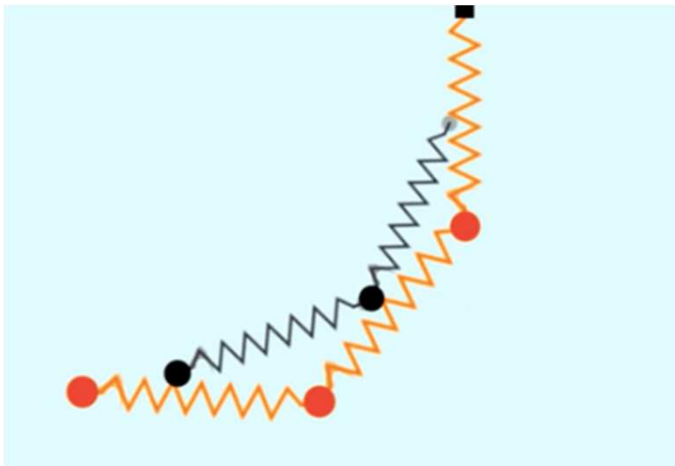




Simple Modeling for Hair Simulation

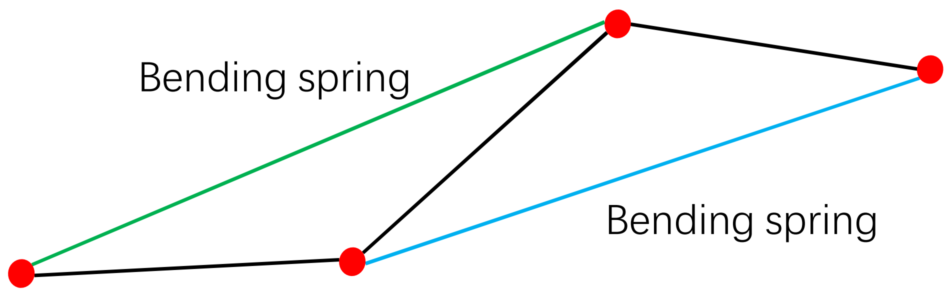
- **Bending force**
  - Based on the angle between adjacent normals of segments





Simple Modeling for Hair Simulation

- **Bending force**
  - Place bending strings across a mass point

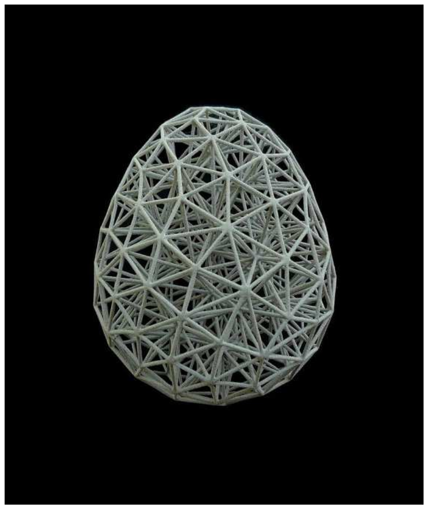


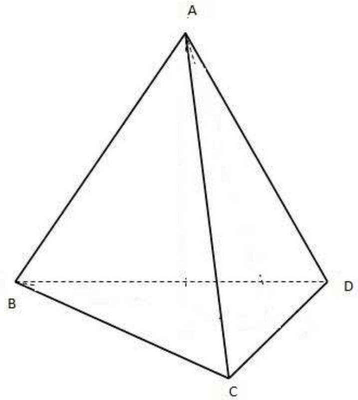
Simple Modeling for Hair Simulation

- **Problem**
  - Some nonlinear behaviors cannot be handled
  - Twisting

Advanced Mass-Spring Model

- **Mass-spring system applied to solids**
  - Tetrahedral mesh

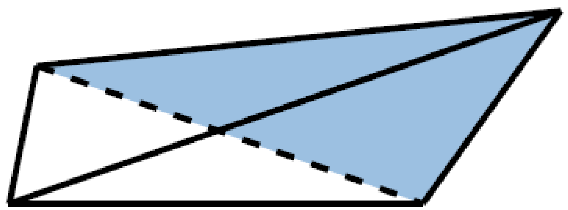




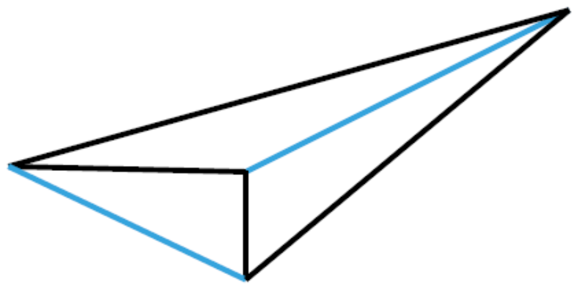
Advanced Mass-Spring Model

- **Problem with volumetric simulations**
  - Tetrahedron may collapse to zero volume
  - For both mass-spring and finite-element simulations

(coplanar)

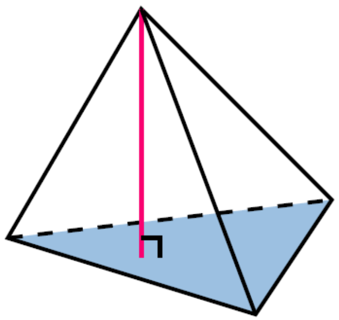


(coplanar)

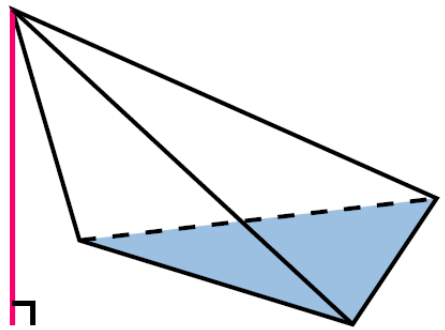


Advanced Mass-Spring Model

- **Point-face altitude spring**
  - Between each particle of the tetrahedron and a virtual node projected onto the plane of the opposite face



(a) Spring has all non-negative barycentric weights

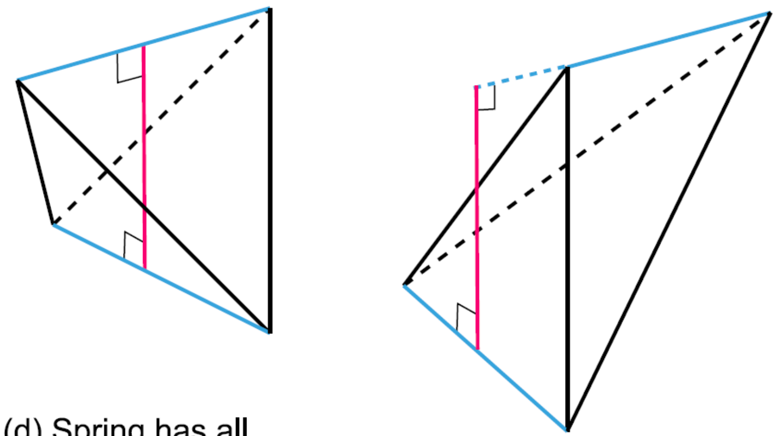


(b) Spring has negative barycentric weights

- Equal and opposite forces are applied to both the particle and the virtual node;
- The virtual node distributes its force barycentrically to the particles of the face

Advanced Mass-Spring Model

- **Edge-edge altitude spring**
  - Mutually orthogonal to the two lines containing the edges



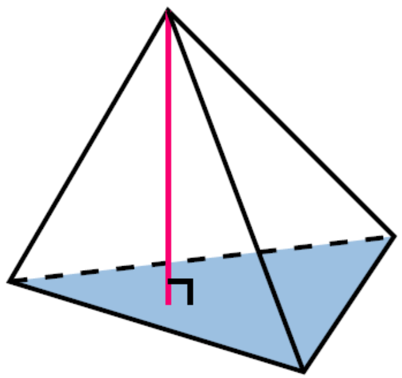
(d) Spring has all non-negative barycentric weights

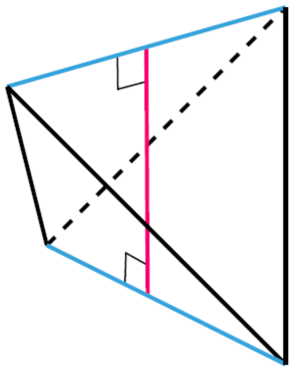
(e) Spring has negative barycentric weights

- The barycentric weights can also be negative if the virtual nodes are not on the segment

Advanced Mass-Spring Model

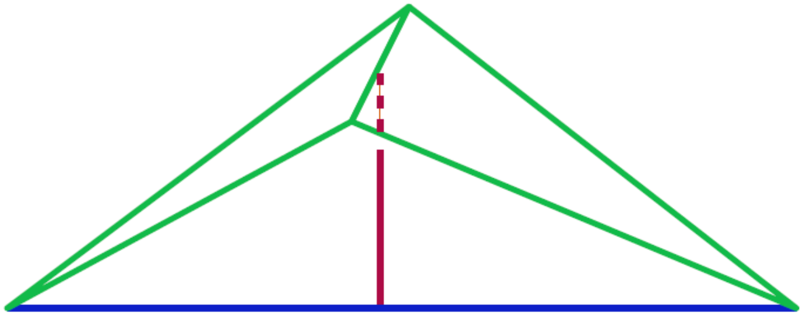
- **Fortunately, for any tetrahedron**
  - There is at least one point/face or edge/edge altitude spring that has nonnegative barycentric weights
    - The edge/edge or point/face spring that currently has the least length is guaranteed to have all non-negative barycentric weights





Advanced Mass-Spring Model

- **Bending string**
  - A pair of triangles sharing an edge can have its bending modeled by two springs

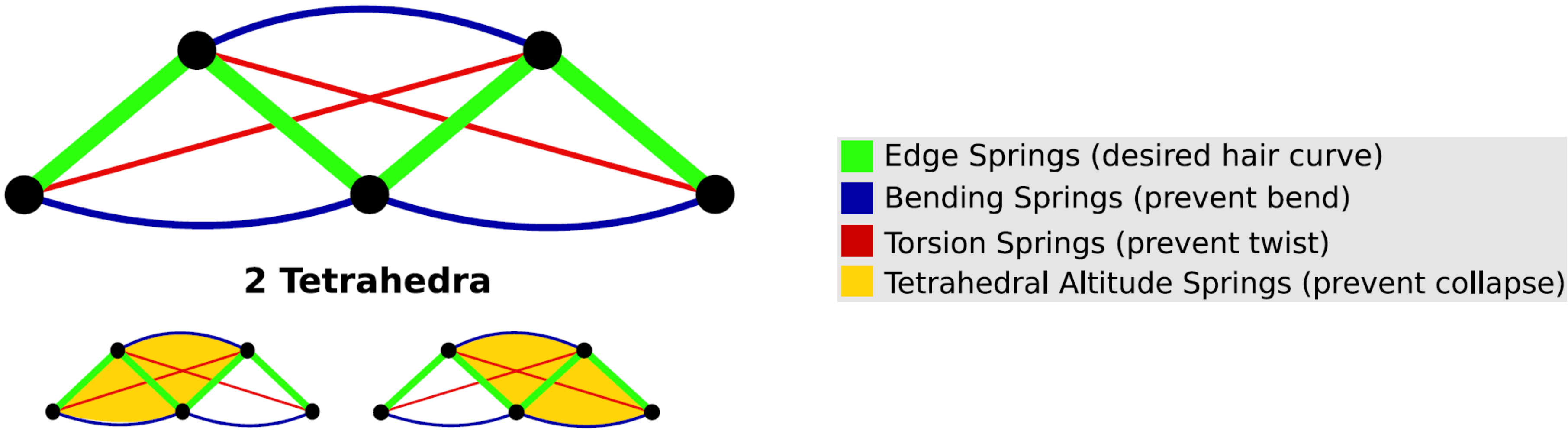


Advanced Mass-Spring Model

- **An advanced altitude spring hair model**
  - If we model a hair as a series of connected line segments
    - Stretching: edge springs between every consecutive particle
    - Bending: bending springs between every other particle
    - Twist: attaching torsion springs that connect each particle to a particle three particles away from it
    - Orientation of hair: edge springs and bending springs together form triangles that implicitly represent the orientation

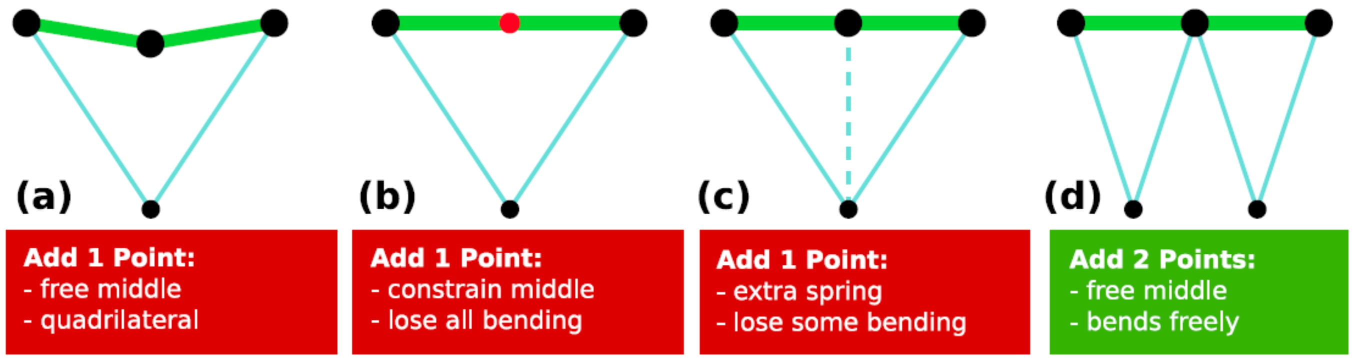
Advanced Mass-Spring Model

- **An advanced altitude spring hair model**
  - Curly hair springs



Advanced Mass-Spring Model

- **An advanced altitude spring hair model**
  - Straight hair springs
    - All the particles may be collinear: zero area tetrahedra
    - Introduce additional particles perturbed from the main hair axis (more rigid spring)



Advanced Mass-Spring Model

- **An advanced altitude spring hair model**
  - Straight hair springs

Advanced Mass-Spring Model

- **Strain limiting**
  - Complex head motions can cause severe stretching
  - Apply momentum conserving velocity impulses to particles attached by springs that exceed 10% deformation
- **Self-repulsion**
  - Using only the edge/edge repulsion

Advanced Mass-Spring Model

- **Animation results**

Advanced Mass-Spring Model

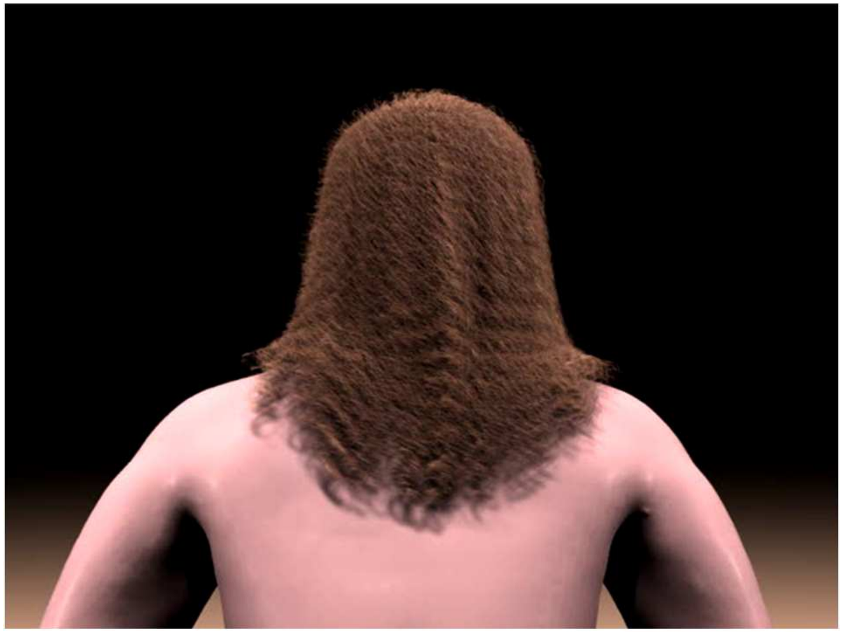
- **Animation results**

Advanced Mass-Spring Model

- **Animation results**

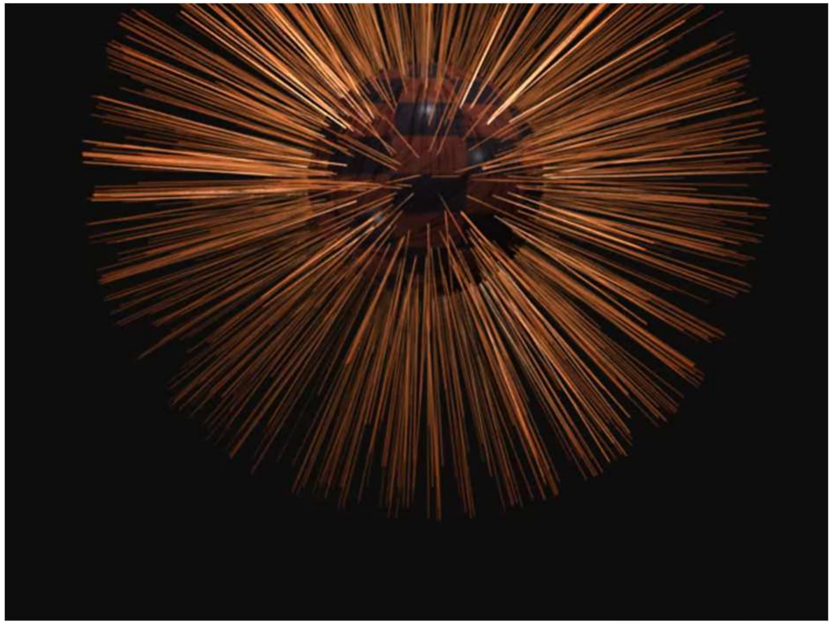
Advanced Mass-Spring Model

- **Animation results**



Advanced Mass-Spring Model

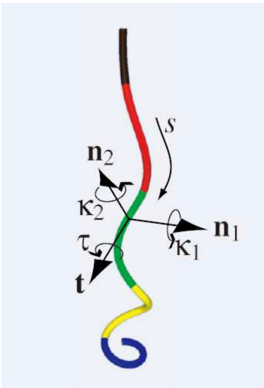
- **Animation results**

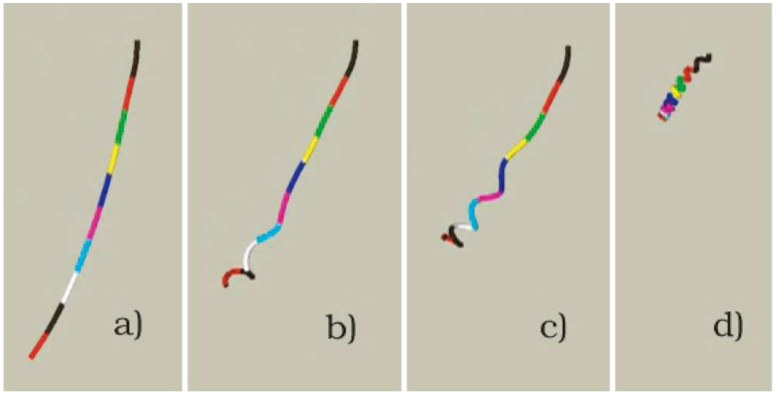


III. Physical Modeling for Hair Simulation

Super-Helix Hair Model

- **Kinematics**
  - Built upon the Cosserat and Kirchhoff theories of rods
  - Centerline: $\mathbf{r}(s,t) \qquad s \in [0,L]$
  - Material (moving) frame: $\mathbf{n}_i(s,t) \qquad \mathbf{r}'(s,t) = \mathbf{n}_0(s,t)$





Super-Helix Hair Model

- **Kinematics**
  - Kirchhoff model for elastic rod: *inextensibility* and *unshearability*
  - The frame $\mathbf{n}_i(s,t)$ is *orthonormal* for all *s*

  - Darboux vector

$$\mathbf{n}_i'(s,t) = \boxed{\boldsymbol{\Omega}(s,t)} \times \mathbf{n}_i(s,t) \quad \text{for } i = 0,1,2.$$

  - Boundary condition

$$\begin{cases} \mathbf{r}(0,t) = \mathbf{r}_{\mathrm{c}}(t) \\ \mathbf{n}_i(0,t) = \mathbf{n}_{i,\mathrm{c}}(t) \quad \text{for } i = 0,1,2, \end{cases}$$

Super-Helix Hair Model

- **Rod's material curvatures & twist** $\mathbf{n}_i'(s,t) = \mathbf{\Omega}(s,t) \times \mathbf{n}_i(s,t) \quad \text{for } i = 0,1,2.$
  - Curvature along two directions: $(\kappa_\alpha(s,t))_{\alpha=1,2}$
  - Twist: $\tau(s,t)$
  - The coordinates of the vector $\mathbf{\Omega}(s,t)$ in the local material frame

$$\mathbf{\Omega}(s,t) = \tau(s,t)\,\mathbf{n}_0(s,t) + \kappa_1(s,t)\,\mathbf{n}_1(s,t) + \kappa_2(s,t)\,\mathbf{n}_2(s,t)$$

  - Introducing a redundant notation for the twist $\kappa_0 = \tau$
  - We can refer to these parameters collectively $(\kappa_i(s,t))_{i=0,1,2}$

The degrees of freedom of a Kirchhoff rod

Super-Helix Hair Model

- **Spatial discretization**
  - Divide the strand $s \in [O, L]$ into N segments S_Q $\quad 1 \leq Q \leq N$
  - Define the material curvatures and twist with piecewise constant functions over these segments

$$q_{i,Q}(t) \longrightarrow \text{Constant functions of } (\kappa_i(s,t))_{i=0,1,2}$$

  - Explicit formula for the material curvatures and twist

$$\kappa_i(s,t) = \sum_{Q=1}^{N} q_{i,Q}(t)\, \chi_Q(s) \longleftarrow 1 \text{ if } s \in S_Q \text{ and } 0 \text{ otherwise}$$

Super-Helix Hair Model

- **Generalized coordinates and reconstruction**
  - Gather all $q_{i,Q}(t)$ $\qquad q_{i,Q}(t) \longrightarrow \mathbf{q}(t)$
  - Can be used to reconstruct the rod shape at any given time

$$\kappa_i(s,t) = \sum_{Q=1}^{N} q_{i,Q}(t)\,\chi_Q(s)$$

$$\Downarrow$$

$$\boldsymbol{\Omega}(s,t) = \tau(s,t)\,\mathbf{n}_0(s,t) + \kappa_1(s,t)\,\mathbf{n}_1(s,t) + \kappa_2(s,t)\,\mathbf{n}_2(s,t)$$

$$\Downarrow$$

$$\mathbf{n}_i'(s,t) = \boldsymbol{\Omega}(s,t) \times \mathbf{n}_i(s,t) \quad \text{for } i = 0,1,2. \qquad \begin{cases} \mathbf{r}(0,t) = \mathbf{r}_c(t) \\ \mathbf{n}_i(0,t) = \mathbf{n}_{i,c}(t) \quad \text{for } i = 0,1,2, \end{cases}$$

$$\Downarrow$$

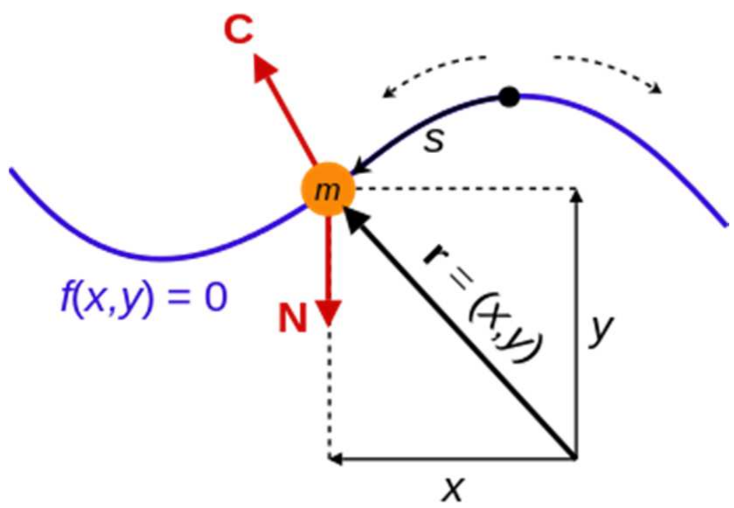
$$\mathbf{r}'(s,t) = \mathbf{n}_0(s,t)$$

Can be integrated to obtain positions

Super-Helix Hair Model

- **Dynamic equations for a Super-Helix**
  - Newton dynamic equations

$$\mathbf{v}_1 = \frac{d\mathbf{r}_1}{dt}, \mathbf{v}_2 = \frac{d\mathbf{r}_2}{dt}, \ldots, \mathbf{v}_N = \frac{d\mathbf{r}_N}{dt} \qquad \sum \mathbf{F} = m\frac{d^2\mathbf{r}}{dt^2}$$



$$\mathbf{r}_1 = (x_1, y_1, z_1), \mathbf{r}_2 = (x_2, y_2, z_2) \quad \ldots$$

Require impulse forces to maintain constraints

Super-Helix Hair Model

- **Dynamic equations for a Super-Helix**
  - Lagrangian dynamic equations
    - Use the energies in the system
    - Lagrangian: a function which summarizes the dynamics of the entire system

$$L = T - V$$

$$T = \frac{1}{2} \sum_{k=1}^{N} m_k v_k^2$$

Total kinetic energy

Potential energy

Lagrange's equations *(First kind)*

$$\frac{\partial L}{\partial \mathbf{r}_k} - \frac{\mathrm{d}}{\mathrm{d}t} \frac{\partial L}{\partial \dot{\mathbf{r}}_k} + \sum_{i=1}^{C} \lambda_i \frac{\partial f_i}{\partial \mathbf{r}_k} = 0$$

Constraints

Super-Helix Hair Model

- **Dynamic equations for a Super-Helix**
  - Given deformable body whose configuration depends on generalized coordinates $\mathbf{q}(t)$
  - Lagrangian mechanics provides a systematic method for deriving its equation of motion

$$\ddot{\mathbf{q}} = \mathbf{a}(\mathbf{q}, \dot{\mathbf{q}}, t)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}_{iQ}}\right) - \frac{\partial T}{\partial q_{iQ}} + \frac{\partial U}{\partial q_{iQ}} + \frac{\partial D}{\partial \dot{q}_{iQ}} = \int_0^L \mathbf{J}_{iQ}(s, \mathbf{q}, t) \cdot \mathbf{F}(s, t)\, \mathrm{d}s$$

Work due to external forces

Dynamic Super-Helix Hair Model

- **Dynamic equations for a super-helix**
  - Lagrangian mechanics formulation

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}_{iQ}}\right) - \frac{\partial T}{\partial q_{iQ}} + \frac{\partial U}{\partial q_{iQ}} + \frac{\partial D}{\partial \dot{q}_{iQ}} = \int_0^L \mathbf{J}_{iQ}(s,\mathbf{q},t)\cdot\mathbf{F}(s,t)\,\mathrm{d}s$$

- Kinetic energy: $T(\mathbf{q},\dot{\mathbf{q}},t) = \frac{1}{2}\int_0^L \rho\, S\left(\dot{\mathbf{r}}^{\mathrm{SH}}(s,\mathbf{q})\right)^2 \mathrm{d}s$
- Internal energy: $U(\mathbf{q},t) = \frac{1}{2}\int_0^L \sum_{i=0}^{2}(EI)_i\,(\kappa_i^{\mathrm{SH}}(s,\mathbf{q}) - \kappa_i^{\mathrm{n}}(s))^2\,\mathrm{d}s$
- Dissipation potential: $D(\mathbf{q},\dot{\mathbf{q}},t) = \frac{1}{2}\int_0^L \gamma\sum_{i=0}^{2}\left(\dot{\kappa}_i^{\mathrm{SH}}(s,\mathbf{q})\right)^2 \mathrm{d}s$

Dynamic Super-Helix Hair Model

- **Dynamic equations for a super-helix**
  - Lagrangian mechanics formulation

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}_{iQ}}\right) - \frac{\partial T}{\partial q_{iQ}} + \frac{\partial U}{\partial q_{iQ}} + \frac{\partial D}{\partial \dot{q}_{iQ}} = \int_0^L \mathbf{J}_{iQ}(s,\mathbf{q},t)\cdot\mathbf{F}(s,t)\,\mathrm{d}s$$

  - Jacobian: $\mathbf{J}_{iQ} = \partial \mathbf{r}^{\mathrm{SH}}(s,\mathbf{q})/\partial q_{iQ}$
  - Force contributions: hair weight, viscous drag, interaction forces with surrounding strands and body

$$\mathbf{F}(s,t) = \rho S \mathbf{g} - \nu \dot{\mathbf{r}}^{\mathrm{SH}}(s,\mathbf{q}) + \mathbf{F}^{\mathrm{i}}(s,t)$$

Dynamic Super-Helix Hair Model

- **Dynamic equations for a super-helix**
  - Lagrangian mechanics formulation

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}_{iQ}}\right) - \frac{\partial T}{\partial q_{iQ}} + \frac{\partial U}{\partial q_{iQ}} + \frac{\partial D}{\partial \dot{q}_{iQ}} = \int_0^L \mathbf{J}_{iQ}(s,\mathbf{q},t)\cdot\mathbf{F}(s,t)\,\mathrm{d}s$$

$$\Downarrow$$

$$\mathbb{M}[s,\mathbf{q}]\cdot\ddot{\mathbf{q}} + \mathbb{K}\cdot(\mathbf{q}-\mathbf{q}^{\mathrm{n}}) = \mathbf{A}[t,\mathbf{q},\dot{\mathbf{q}}] + \int_0^L \mathbf{J}_{iQ}[s,\mathbf{q},t]\cdot\mathbf{F}^{\mathrm{i}}(s,t)\,\mathrm{d}s$$

$\mathbf{q}^{\mathrm{n}}$ defines the rest position in generalized coordinates

Dynamic Super-Helix Hair Model

- **Time discretization**
  - Classical Newton semi-implicit scheme with fixed time step

$$\mathbb{M}[s,\mathbf{q}]\cdot\ddot{\mathbf{q}}+\mathbb{K}\cdot(\mathbf{q}-\mathbf{q}^{\mathrm{n}})=\mathbf{A}[t,\mathbf{q},\dot{\mathbf{q}}]+\int_0^L \mathbf{J}_{iQ}[s,\mathbf{q},t]\cdot\mathbf{F}^{\mathrm{i}}(s,t)\,\mathrm{d}s$$

Implicit

Implicit

Conjugate gradient for the solution

Dynamic Super-Helix Hair Model

- **Hair strand interpolation**
  - Different interpolation styles
  - Interpolate smoothly at the root, but clustered at the tip

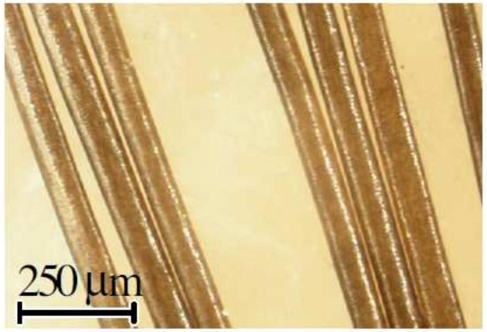
Dynamic Super-Helix Hair Model

- **Determine parameters**
  - Density ρ=1.3 $g \cdot cm$^(−3)
  - Natural curliness:

$$\kappa_1^n = 1/r_h \qquad \kappa_2^n = 0 \qquad \tau^n = \frac{\Delta_h}{2\pi r_h^2}$$

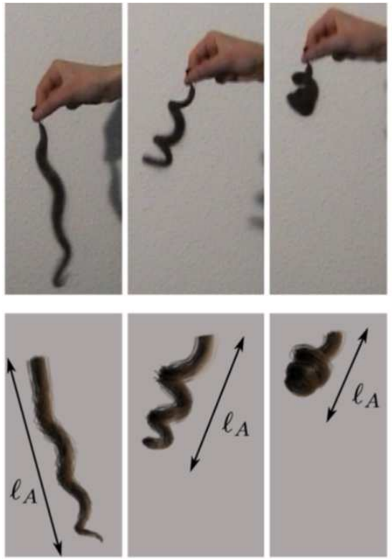
Parameter values for natural hair

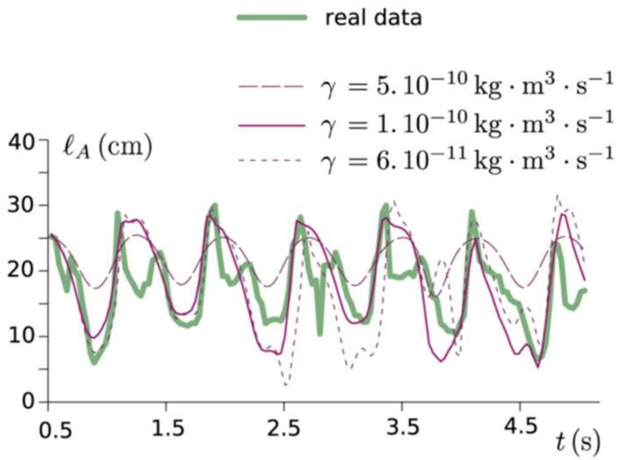
	Asian (smooth)	Caucasian 1 (wavy)	Caucasian 2 (curly)	African (fuzzy)
Radius (μm)	50	35	50	50
Ellipticity	1	1.1	1.1	1.2
Helix radius (cm)	0	1	0.6	0.1
Helix step (cm)	0	0.5	0.5	1
Young's mod. (GPa)	1	2	1.5	0.5
Poisson's ratio	0.48	0.48	0.48	0.48



Dynamic Super-Helix Hair Model

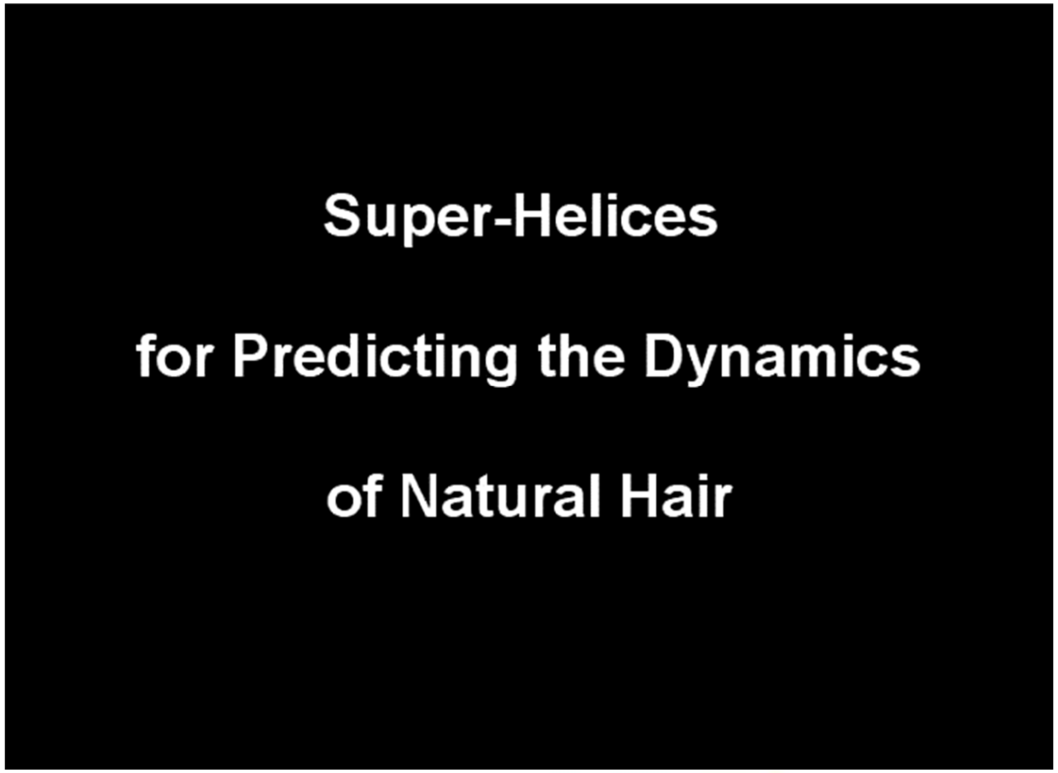
- **Fitting data experimentally**
  - Friction (in collision) by fitting hair lengths





Dynamic Super-Helix Hair Model

- **Animation**



IV. Data-Driven Approach

A Reduced Model for Interactive Hairs

- **High-quality hair simulation is expensive**
  - Building upon precomputed simulation data
  - Constructs a reduced model
    - Optimally represent hair motion with a small number of guide hairs and the corresponding interpolation relationships

A Reduced Model for Interactive Hairs

Next Lecture: Soft-Body Simulation – Hair II