1. Consider a plane with normal $\mathbf{n} = (0, 1, 0)$ pointing upwards along the $+y$ axis and a *Lambertian diffuse* (perfect diffuse reflection) surface whose material reflectance is π (a non-physics scalar for now), with an *infinitely large uniform area light* illuminating the plane. Thus the incident radiance $L_i(\mathbf{p}, \omega) = 1$ is over all directions and the BRDF of the diffuse surface is a constant $f = 1$.

   Calculate the outgoing radiance (a scalar) $L_o(\mathbf{p}, \omega)$ along $+y$ axis at position $(0, 0, 0)$. You should provide a clear calculation step to get the full score.

   Answer: Consider $\mathbf{p} = (0, 0, 0)$ and $\mathbf{n} = (0, 1, 0)$,

$$
\begin{aligned}
L_o(\mathbf{p}, \mathbf{n}) &= \int_\Omega f \cdot L_i(\mathbf{p}, \omega) \cos\theta \, \mathrm{d}\omega \\
&= \int_0^{2\pi} \left(\int_0^{\pi/2} 1 \cdot \cos\theta \sin\theta \cdot \mathrm{d}\theta \right) \mathrm{d}\phi \\
&= 2\pi \cdot \int_0^{\pi/2} \frac{\sin 2\theta}{2} \, \mathrm{d}\theta \\
&= \pi \cdot \frac{\cos(0) - \cos(\pi)}{2} \\
&= \pi
\end{aligned}
$$

2. During the rendering process, interpolation within triangles is commonly performed for position, color, normal, as well as texture coordinates, etc., to obtain per-pixel information needed. The interpolation is usually calculated with barycentric coordinates.

   Now consider a triangle with the positions and texture coordinates of the associated three vertices as follows:

$$\mathbf{v_0} = (0, 0, 0),\ \mathbf{v_1} = (5, 5, 0),\ \mathbf{v_2} = (0, 5, 10),$$

$$\mathbf{t_0} = (0, 0),\ \mathbf{t_1} = (1, 0),\ \mathbf{t_2} = (0, 1),$$

   and a point inside the triangle (you can prove it by computing barycentric coordinates):

$$\mathbf{p} = (1, 3, 4).$$

   Calculate the interpolated texture coordinates t at point p using barycentric coordinates and show your calculation steps in detail.

   Answer: Given a point inside a triangle, its barycentric coordinate corresponding to a vertex can be given by the ratio of the area of the triangle formed by this point and the other two vertices of the triangle to the area of the whole triangle. The area of a triangle could be calculated as

$$A = \frac{1}{2}||(\mathbf{v_1} - \mathbf{v_0}) \times (\mathbf{v_2} - \mathbf{v_0})||,$$

   where the cross-product can be calculated in determinant form

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = (a_2b_3 - a_3b_2)\mathbf{i} - (a_1b_3 - a_3b_1)\mathbf{j} + (a_1b_2 - a_2b_1)\mathbf{k}.$$

   Thus we can get the area of the whole triangle to be 75/2, the area of the triangle formed by $\mathbf{p}$, $\mathbf{v_1}$, $\mathbf{v_2}$ to be 15, and the triangle formed by $\mathbf{p}$, $\mathbf{v_0}$, $\mathbf{v_2}$ to be 15/2.

   Using the property that three components of the barycentric coordinates sum to 1, the barycentric coordinates of $\mathbf{p}$ is

$$\left(\frac{2}{5}, \frac{1}{5}, \frac{2}{5}\right).$$

   We can easily get the texture coordinates to be

$$\mathbf{t} = \left(\frac{1}{5}, \frac{2}{5}\right).$$