1. Consider a 3D space using right-handed coordinates. An object is located at $(0, 0, 0)$ but rotated $+90°$ along the $+z$ axis $(0, 0, 1)$ (according to the right-hand rule). A camera is located at $(0, 0, 1)$, facing toward the $-z$ direction $(0, 0, -1)$, and its up direction is the $+y$ direction $(0, 1, 0)$.

   Please give the combined model-view matrix to transform the point from local coordinate to the camera coordinate.

   Answer: According to the rule of 3-dimensional rotation matrix, the rotation of the object can be expressed using a 3 by 3 matrix:

$$\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

   Extending it to 4 by 4 yields the model matrix of the object:

$$\begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

   The view transformation only contains a translation. Its matrix representation is:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

   Multiplying the two matrices in order yields the final combined model-view matrix:

$$\begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2. Consider a *Bézier curve* with degree $n = 2$ (**three control points**). The curve is given by $\mathbf{c}_0 = (1, 0), \mathbf{c}_1 = (3, 3)$ and $\mathbf{c}_2 = (5, 0)$, see Figure 1.

   As you have learned in class, the evaluation of Bézier curve is commonly performed by the *de Casteljau's algorithm*. We perform the iterative calculation $\mathbf{c}_i^{(n+1)} \leftarrow (1-t) \cdot \mathbf{c}_i^{(n)} + t \cdot \mathbf{c}_i^{(n)}$ where $t \in [0, 1]$ is the parameter.

   In fact, we can split a Bézier curve into two continuous Bézier curves containing the same number of control points by viewing the split point on the curve as an extra shared control point between the control points of the two split curves.

   By referring to the iterative evaluation process of the Bézier curve, calculate the three new control points of the two split curves respectively, where we split the curve at $t = 0.5$, see point B in Figure 1.

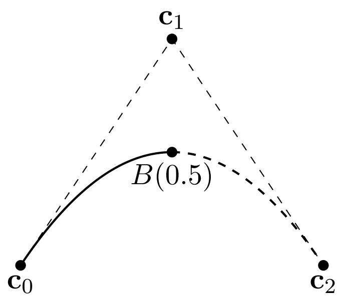

Figure 1: Illustration of the splitting problem for $n = 2$.

Answer: Using the hint from the problem, calculate $\mathbf{c}_0^{(1)} = (2, 1.5)$ and $\mathbf{c}_1^{(1)} = (4, 1.5)$ on the center of line segments $\mathbf{c}_0, \mathbf{c}_1$ and $\mathbf{c}_1, \mathbf{c}_2$ respectively.

By de Casteljau's algorithm, calculate $B(0.5) = (3, 1.5) = 0.5 \cdot \mathbf{c}_0^{(1)} + 0.5 \cdot \mathbf{c}_1^{(1)}$. While the tangent direction of an arbitrary point on the curve is given by the subtraction of two points in the last iteration of de Casteljau's algorithm (in our case, $\mathbf{c}_1^{(1)} - \mathbf{c}_0^{(1)}$), the middle control point of the first split curve should be determined by the intersection of line $\mathbf{c}_0$ to $\mathbf{c}_1$ and $y = 1.5$, which is $\mathbf{c}_0^{(1)} = (2, 1.5)$. The second split curve is determined similarly.

Thus the result is,

(a) The first curve is given by $\mathbf{c}_0 = (1, 0)$, $\mathbf{c}_0^{(1)} = (2, 1.5)$, $B(0.5) = (3, 1.5)$.

(b) The second curve is given by $B(0.5) = (3, 1.5)$, $\mathbf{c}_1^{(1)} = (4, 1.5)$, $\mathbf{c}_2 = (5, 0)$.