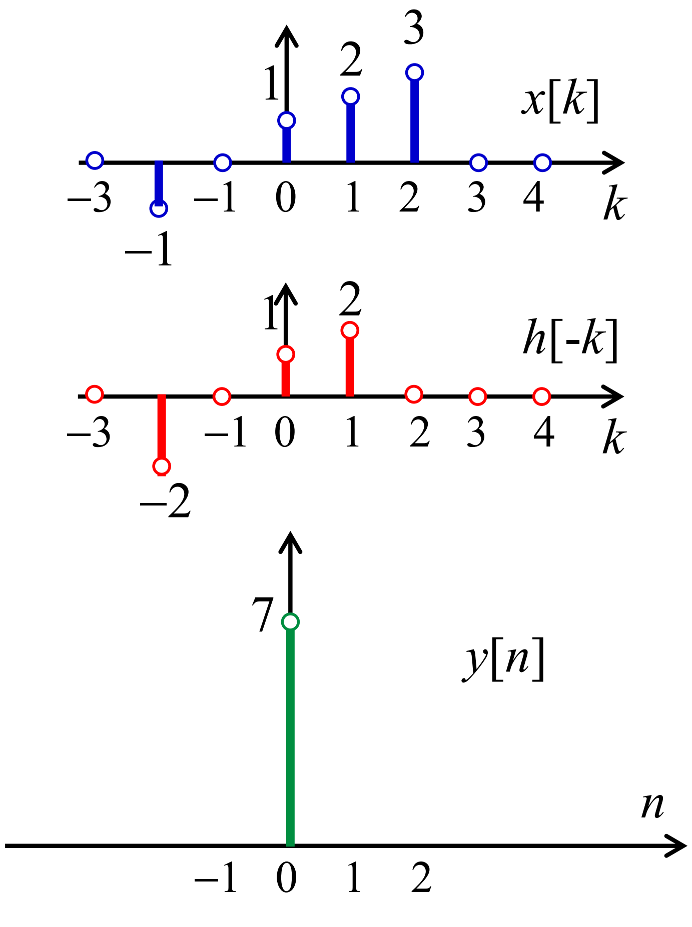
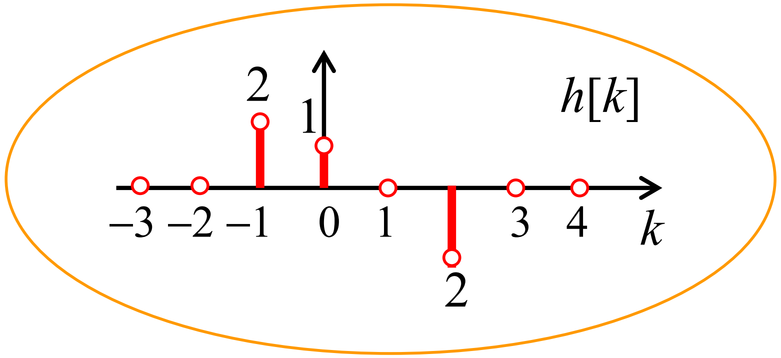
Discrete Convolution: An Example





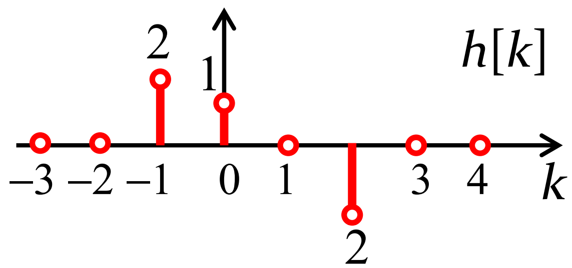
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

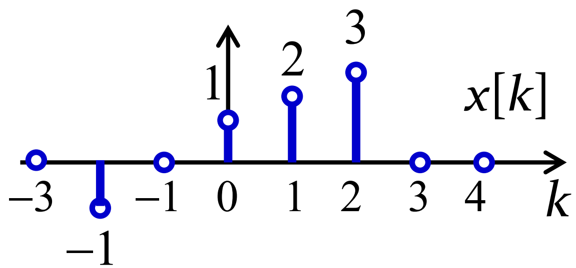
$$y[0] = \sum_{k=-\infty}^{\infty} x[k]h[-k]$$

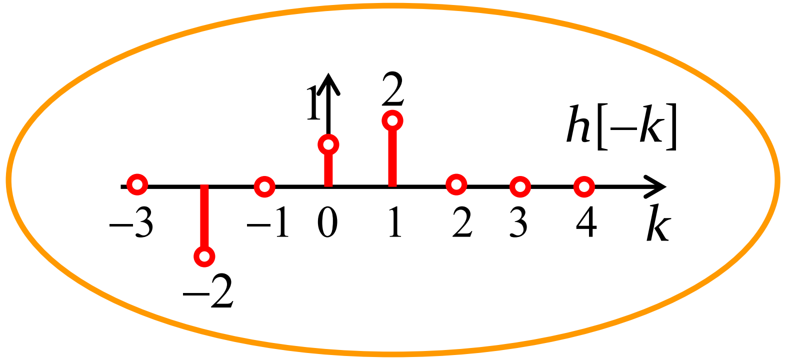
$$= x[-2]h[2] + x[-1]h[1] + x[0]h[0]$$

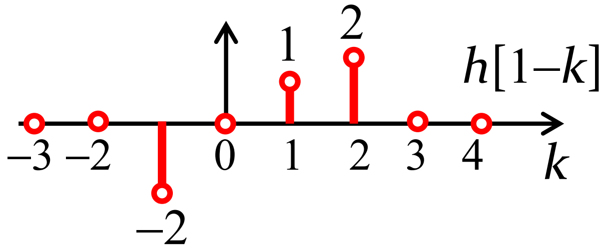
$$+x[1]h[-1] + x[2]h[-2]$$

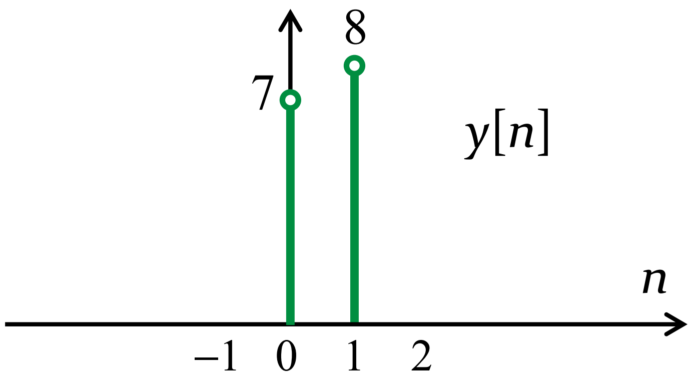
$$= -1\times(-2) + 1\times1 + 2\times2 = 7$$











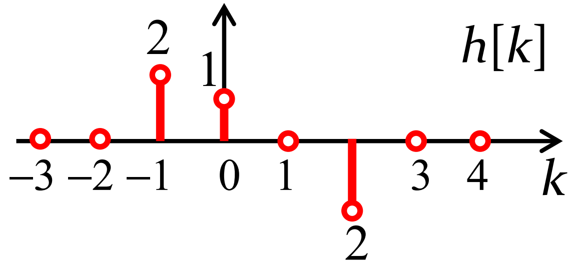
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

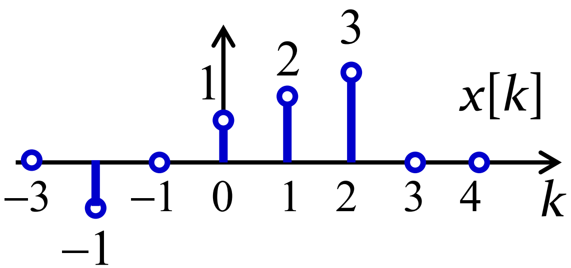
$$y[1] = \sum_{k=-\infty}^{\infty} x[k]h[1-k]$$

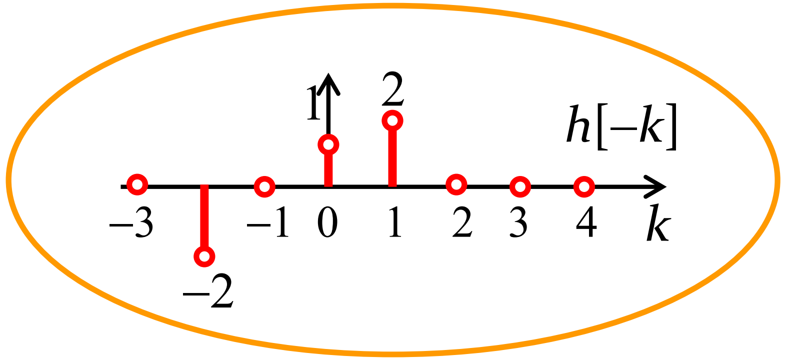
$$= x[-2]h[3] + x[-1]h[2] + x[0]h[1]$$

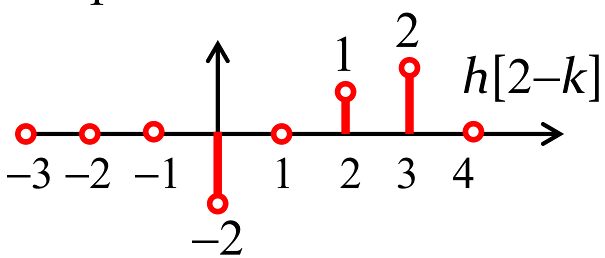
$$+x[1]h[0] + x[2]h[-1]$$

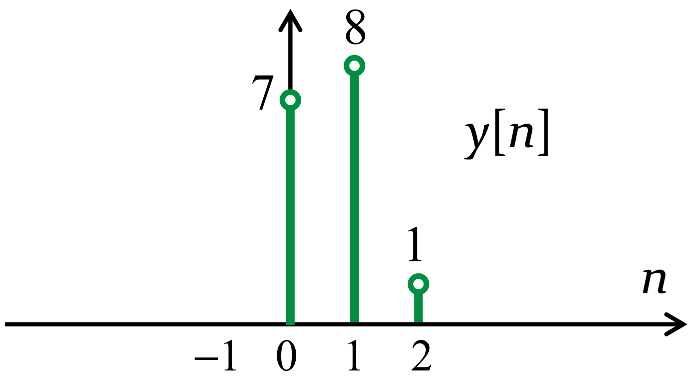
$$= 2\times 1 + 3\times 2 = 8$$







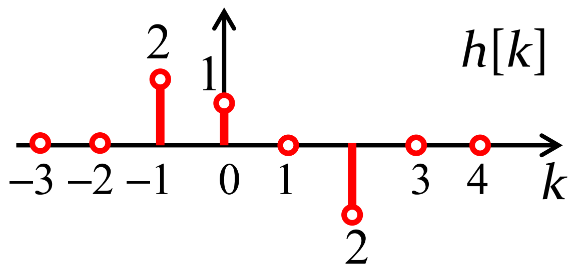


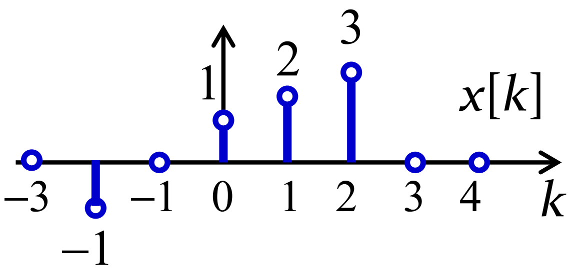


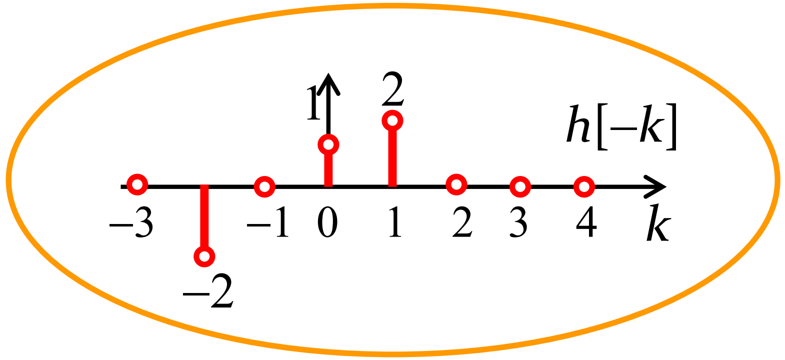
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

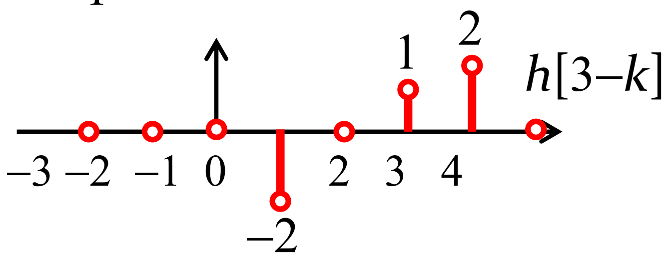
$$y[2] = \sum_{k=-\infty}^{\infty} x[k]h[2-k]$$

$$= x[-2]h[4] + x[-1]h[3] + x[0]h[2]$$
$$+x[1]h[1] + x[2]h[0]$$
$$= 1\times(-2)+3\times1=1$$

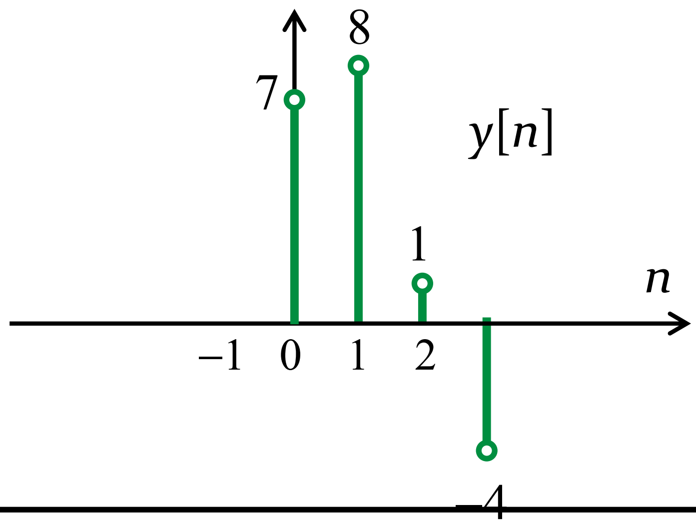






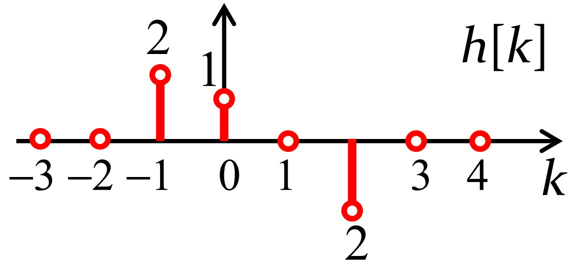


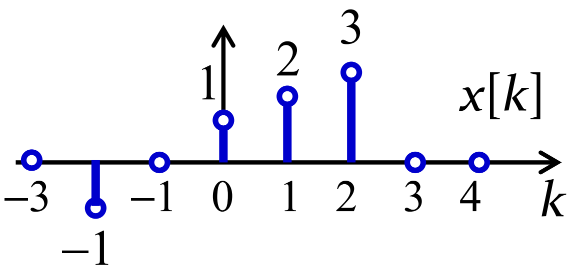
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

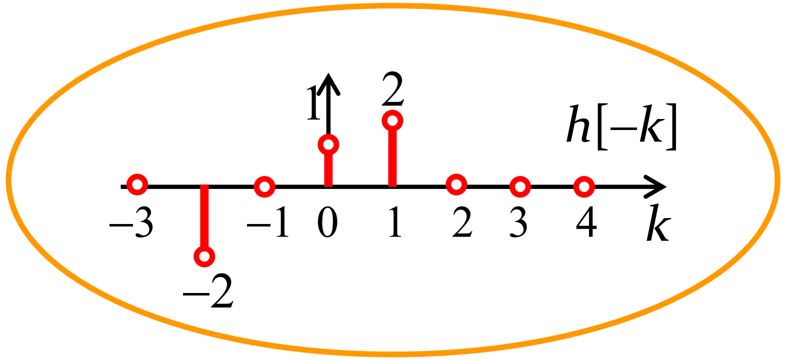


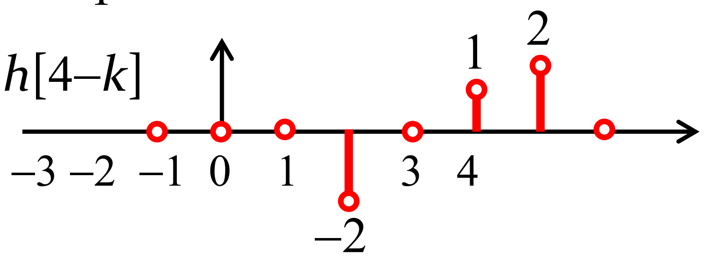
$$y[3] = \sum_{k=-\infty}^{\infty} x[k]h[3-k]$$

$$= x[-2]h[5] + x[-1]h[4] + x[0]h[3]$$
$$+x[1]h[2] + x[2]h[1]$$
$$= 2\times(-2) = -4$$

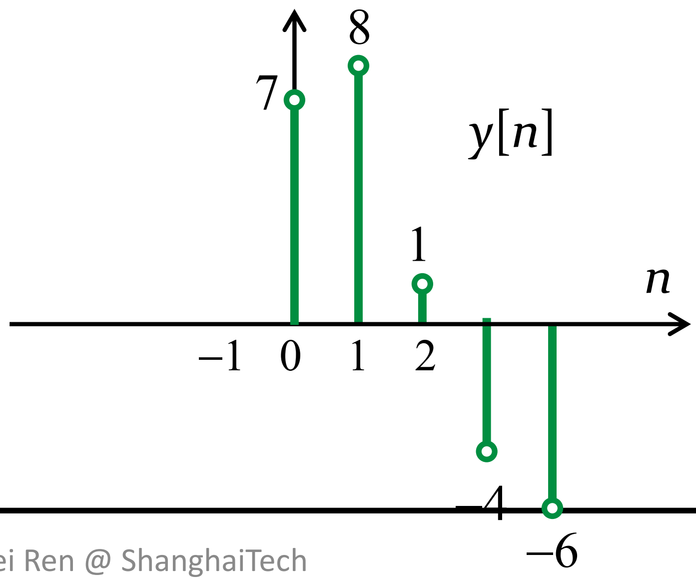






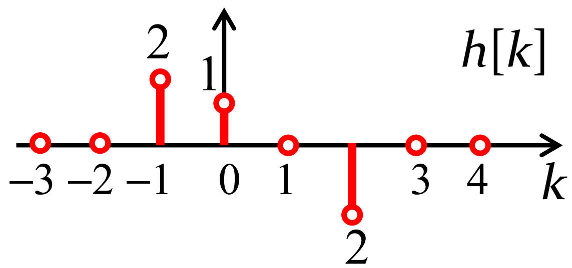


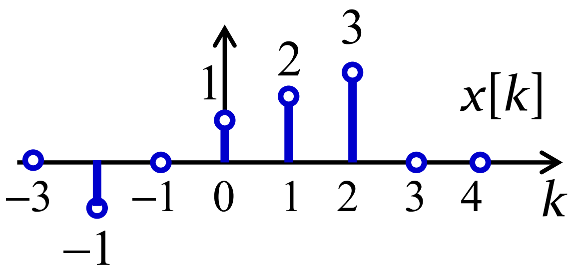
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

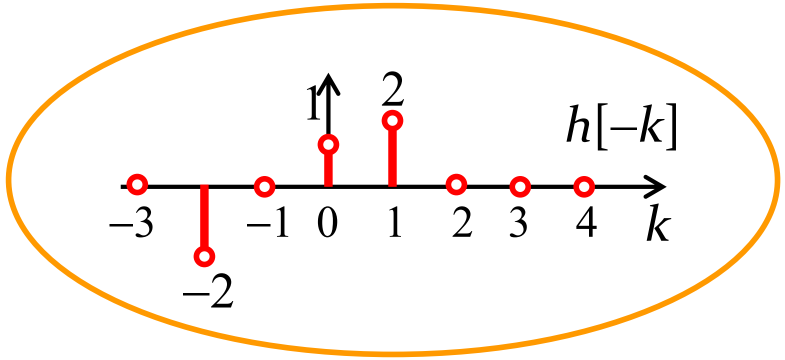


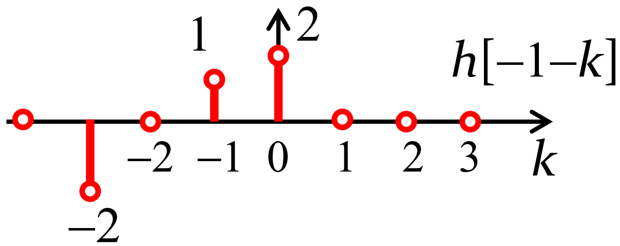
$$y[4] = \sum_{k=-\infty}^{\infty} x[k]h[4-k]$$

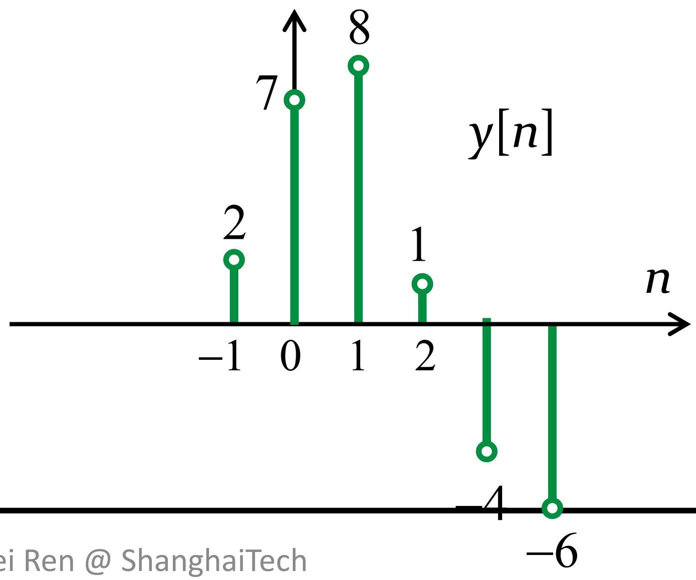
$$= x[-2]h[6] + x[-1]h[5] + x[0]h[4]$$
$$+x[1]h[3] + x[2]h[2]$$
$$= 3\times(-2) = -6$$











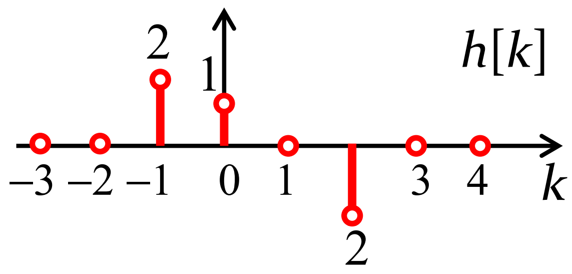
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

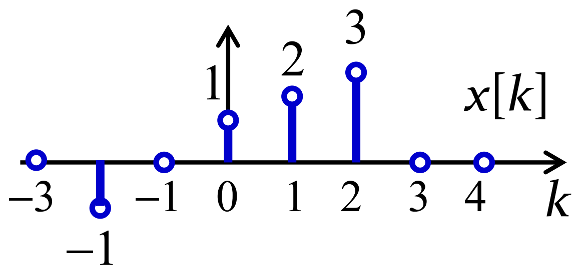
$$y[-1] = \sum_{k=-\infty}^{\infty} x[k]h[-1-k]$$

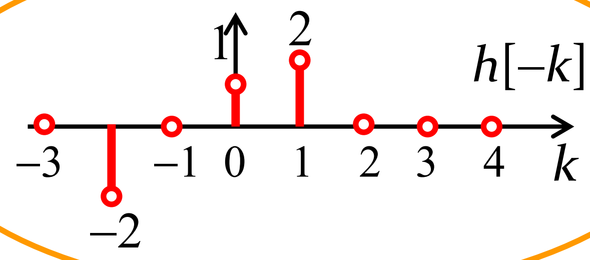
$$= x[-2]h[1] + x[-1]h[0] + x[0]h[-1]$$

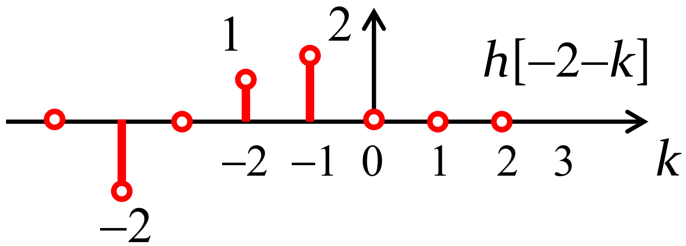
$$+x[1]h[-2] + x[2]h[-3]$$

$$= 1\times 2 = 2$$

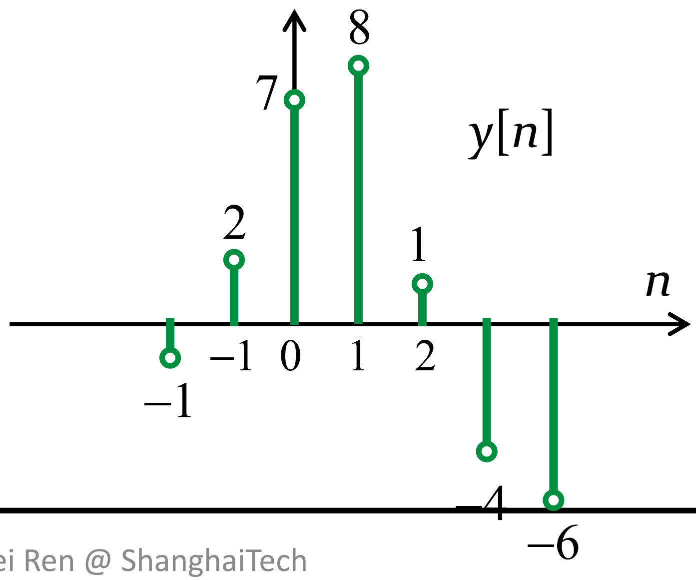








$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

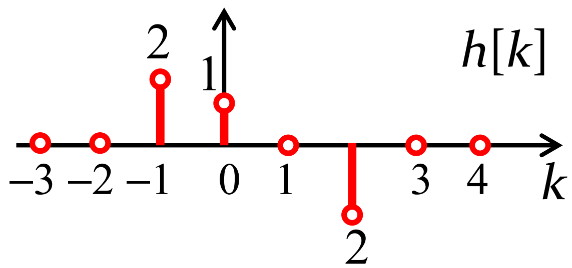


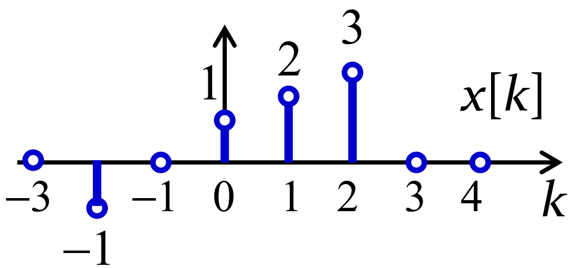
$$y[-2] = \sum_{k=-\infty}^{\infty} x[k]h[-2-k]$$

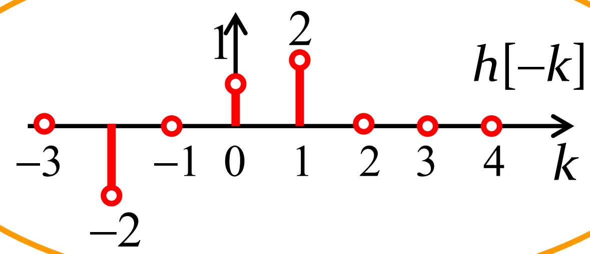
$$= x[-2]h[0] + x[-1]h[-1] + x[0]h[-2]$$

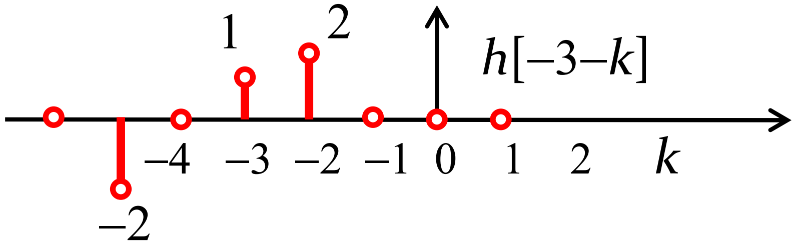
$$+x[1]h[-3] + x[2]h[-4]$$

$$= -1\times 1 = -1$$

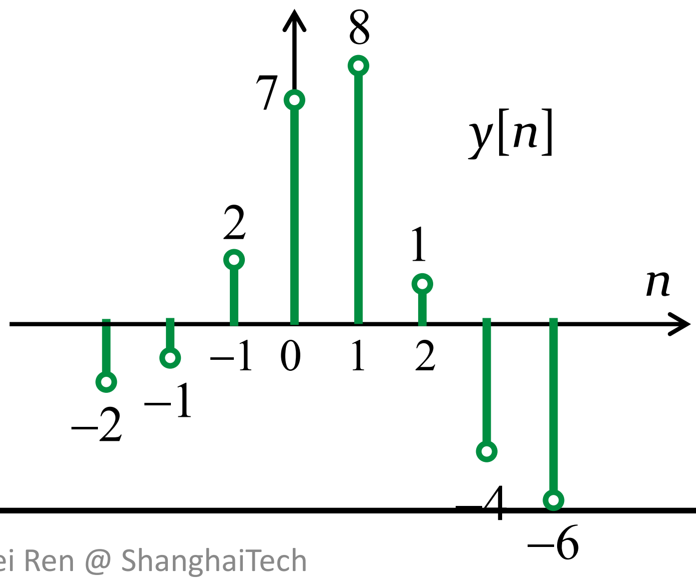








$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$



$$y[-3] = \sum_{k=-\infty}^{\infty} x[k]h[-3-k]$$

$$= x[-2]h[-1] + x[-1]h[-2] + x[0]h[-3]$$
$$+x[1]h[-4] + x[2]h[-5]$$
$$= -1\times 2 = -2$$

Computation of Discrete Convolution

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

- Fold $h[k]$ with respect to the origin to obtain $h[-k]$
- Shift to right (if $n \geq 0$) or to the left (if $n<0$) by $|n|$ samples
- Compute the products of the corresponding samples of sequences $h[n-k]$ and $x[k]$
- Sum the all products to obtain $y[n]$
- Fold, shift, product, and sum

Computation of Convolution

- **Note:** The sum of indices of each sample product inside the convolution sum is equal to the index of the sample being computed

$$y[-3] = \sum_{k=-\infty}^{\infty} x[k]h[-3-k]$$

$$= x[-2]h[-1] + x[-1]h[-2] + x[0]h[-3] + x[1]h[-4] + x[2]h[-5]$$

- If the lengths of the two sequences are M and N, the result of the convolution is of length $M+N-1$