Lecture 4: General topics in biomedical imaging

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Lecture 4: General topics in biomedical imaging

- ❑General image quality
  - MSE
  - PSNR
  - SSIM
- ❑System specific image quality
  - MTF
  - Noise, SNR, CNR
  - Artifact
- ❑Task specific image quality
  - Sensitivity and Specificity
  - ROC & AUC
  - Model Observers

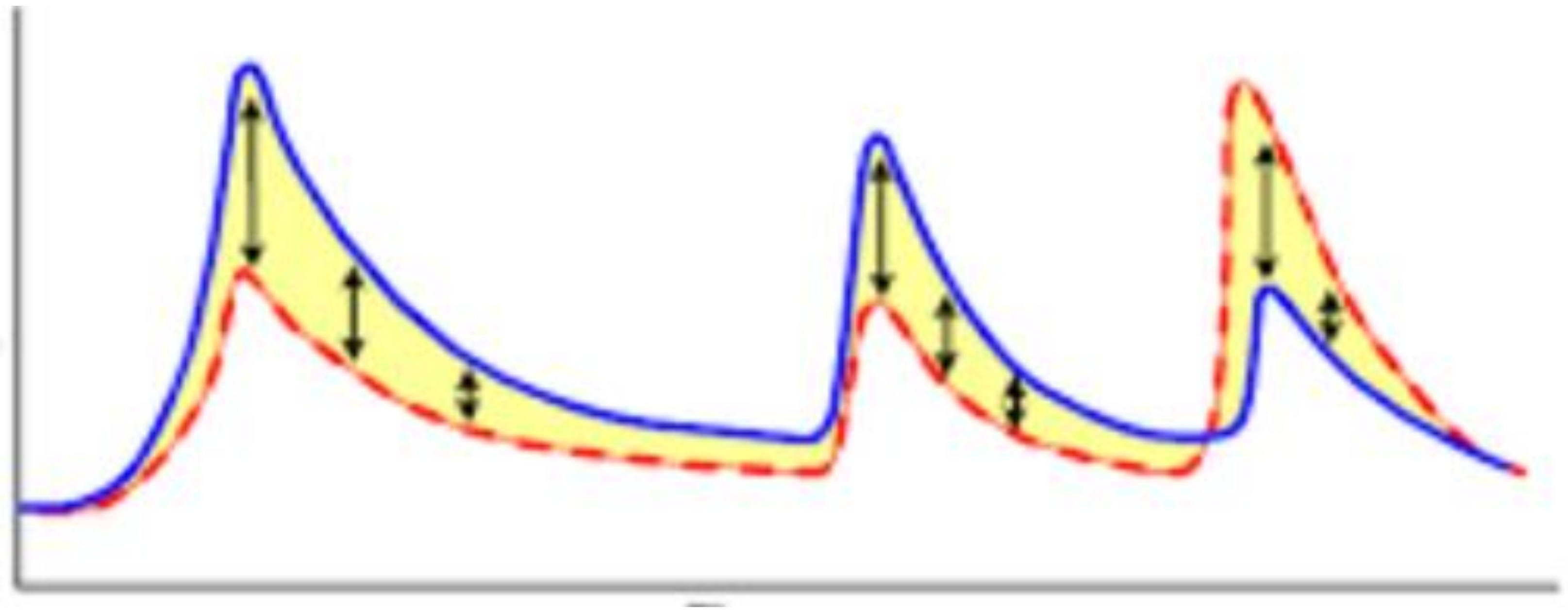
Mean Squared Error

$$\mathrm{MSE} = \frac{1}{n}\sum_{i=1}^{n}(y_i - \tilde{y}_i)^2$$

$$MSE = \frac{\sum_{M,N}[I_1(m,n) - I_2(m,n)]^2}{M * N}$$

M and *N* are the number of rows and columns in the input images

MSE quantifies TOTAL difference between two images!



MSE is an error summation method!

More Variants

Mean squared error	$\text{MSE} = \frac{1}{n}\sum_{t=1}^{n} e_t^2$
Root mean squared error	$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{t=1}^{n} e_t^2}$
Mean absolute error	$\text{MAE} = \frac{1}{n}\sum_{t=1}^{n} \lvert e_t \rvert$
Mean absolute percentage error	$\text{MAPE} = \frac{100\%}{n}\sum_{t=1}^{n} \left\lvert \frac{e_t}{y_t} \right\rvert$

peak signal-to-noise ratio (PSNR)

This ratio is used as a quality measurement between the original and a compressed image. The higher the PSNR, the better the quality of the compressed, or reconstructed image.

$$PSNR = 10 \log_{10}\left(\frac{R^2}{MSE}\right)$$

R is the maximum fluctuation in the input image data type.

For 8-bit image, R=255-0=255

Observation: MSE=225, but obvious different quality!

Philosophy

- HVS (Human Visual System) Extracts Structural Information
- HVS Highly Adapted for Contextual Changes

Classical	“New”
Bottom-up	Top-down
Error Visibility	Structural Distortion

- How to define structural information?
- How to separate structural & nonstructural info?

SSIM Equation

Finally, we combine the three comparisons of (6), (9) and (10) and name the resulting similarity measure the SSIM index between signals $\mathbf{x}$ and $\mathbf{y}$

$$\mathrm{SSIM}(\mathbf{x},\mathbf{y}) = [l(\mathbf{x},\mathbf{y})]^{\alpha} \cdot [c(\mathbf{x},\mathbf{y})]^{\beta} \cdot [s(\mathbf{x},\mathbf{y})]^{\gamma} \tag{12}$$

where $\alpha > 0, \beta > 0$ and $\gamma > 0$ are parameters used to adjust the relative importance of the three components. It is easy to verify that this definition satisfies the three conditions given above. In order to simplify the expression, we set $\alpha = \beta = \gamma = 1$ and $\boxed{C_3 = C_2/2}$ in this paper. This results in a specific form of the SSIM index

$$\boxed{\mathrm{SSIM}(\mathbf{x},\mathbf{y}) = \frac{(2\mu_x\mu_y + C_1)(2\sigma_{xy} + C_2)}{\left(\mu_x^2 + \mu_y^2 + C_1\right)\left(\sigma_x^2 + \sigma_y^2 + C_2\right)}.} \tag{13}$$

The "universal quality index" (UQI) defined in [6] and [7] corresponds to the special case that $C_1 = C_2 = 0$, which produces unstable results when either $(\mu_x^2 + \mu_y^2)$ or $(\sigma_x^2 + \sigma_y^2)$ is very close to zero.

The SSIM Paper

600 IEEE TRANSACTIONS ON IMAGE PROCESSING, VOL. 13, NO. 4, APRIL 2004

Image Quality Assessment: From Error Visibility to Structural Similarity

Zhou Wang, *Member, IEEE*, Alan Conrad Bovik, *Fellow, IEEE*, Hamid Rahim Sheikh, *Student Member, IEEE*, and Eero P. Simoncelli, *Senior Member, IEEE*

***Abstract*—Objective methods for assessing perceptual image quality traditionally attempted to quantify the visibility of errors (differences) between a distorted image and a reference image using a variety of known properties of the human visual system. Under the assumption that human visual perception is highly adapted for extracting structural information from a scene, we introduce an alternative complementary framework for quality assessment based on the degradation of structural information. As a specific example of this concept, we develop a Structural Similarity Index and demonstrate its promise through a set of intuitive examples, as well as comparison to both subjective ratings and state-of-the-art objective methods on a database of images compressed with JPEG and JPEG2000.[1]**

work digital video server can examine the quality of video being transmitted in order to control and allocate streaming resources. Second, it can be used to *optimize* algorithms and parameter settings of image processing systems. For instance, in a visual communication system, a quality metric can assist in the optimal design of prefiltering and bit assignment algorithms at the encoder and of optimal reconstruction, error concealment, and postfiltering algorithms at the decoder. Third, it can be used to *benchmark* image processing systems and algorithms.

Objective image quality metrics can be classified according to the availability of an original (distortion-free) image, with

SSIM Example

SSIM Extensions

- **Color Image Quality Assessment**

  Toet & Lucassen, *Displays, '*03

- **Video Quality Assessment**

  Wang, *et al.*, *Signal Processing: Image Communication,* '04

- **Multi-scale SSIM**

  Wang, *et al.*, Invited Paper, *IEEE Asilomar Conf.* '03

- **Complex Wavelet SSIM**

  Wang & Simoncelli, *ICASSP* '05

MSE = 210, but different SSIM

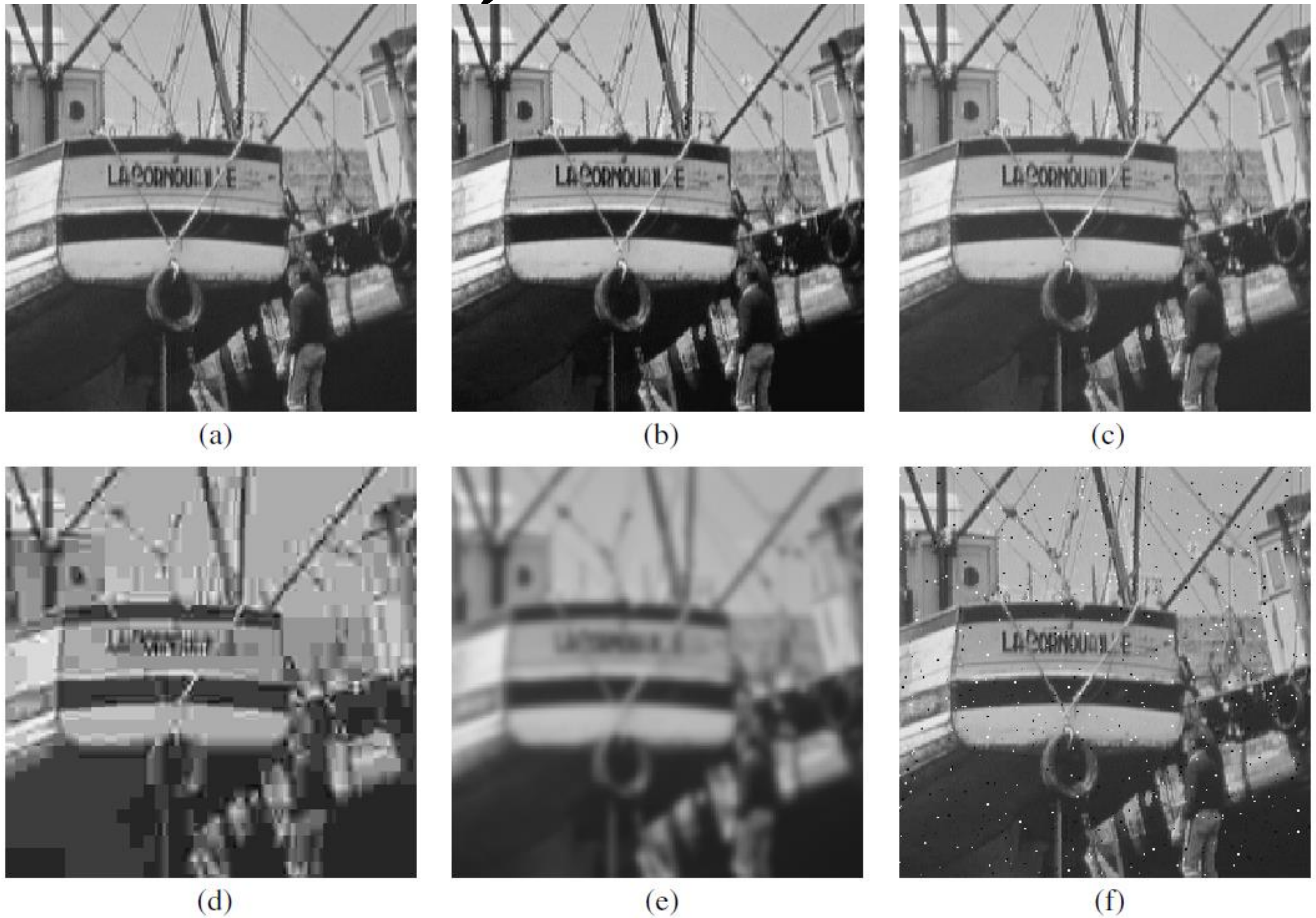


Fig. 2. Comparison of "Boat" images with different types of distortions, all with $\mathrm{MSE} = 210$. (a) Original image (8 bits/pixel; cropped from 512×512 to 256×256 for visibility). (b) Contrast-stretched image, $\mathrm{MSSIM} = 0.9168$. (c) Mean-shifted image, $\mathrm{MSSIM} = 0.9900$. (d) JPEG compressed image, $\mathrm{MSSIM} = 0.6949$. (e) Blurred image, $\mathrm{MSSIM} = 0.7052$. (f) Salt-pepper impulsive noise contaminated image, $\mathrm{MSSIM} = 0.7748$.

mathematical relationship between PSNR & SSIM

For 8-bit images,

$$PSNR = 10\log_{10}\left[\frac{2\sigma_{fg}(l(f,g) - SSIM)}{255^2 SSIM} + \left(\frac{\mu_f - \mu_g}{255}\right)^2\right]$$

2010 International Conference on Pattern Recognition

Image quality metrics: PSNR vs. SSIM

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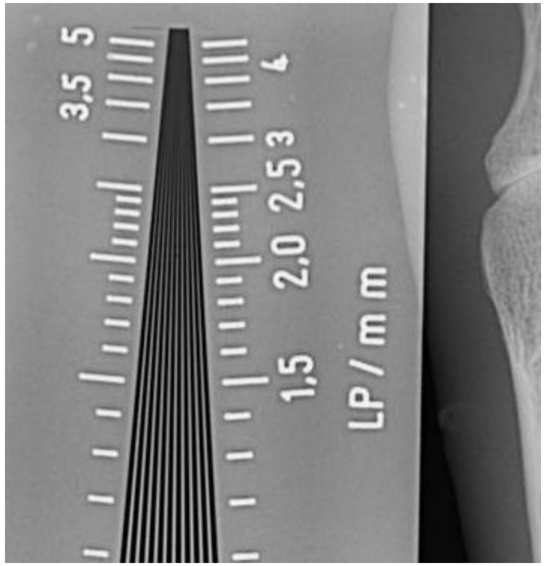
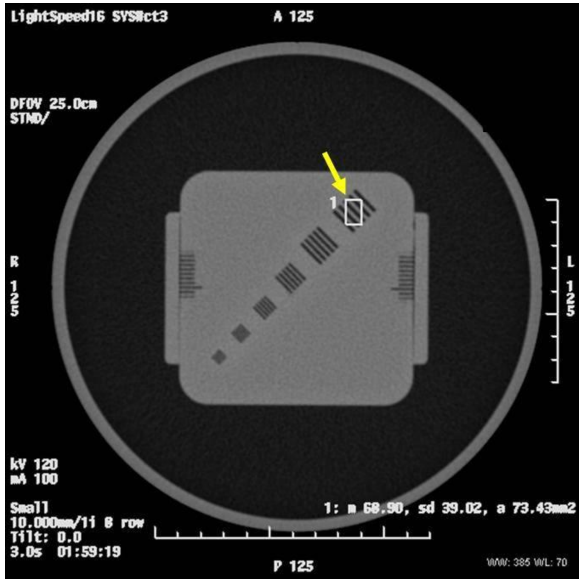
System Specific Image Quality

- Accuracy of diagnostic tests of an imaging system depend on the system’s image quality.
- Quantitative image quality measures:
    1. Spatial resolution (MTF)
    2. SNR (signal-to-noise ratio)
    3. CNR (contrast-to-noise ratio)
- Qualitative image quality
    - Artifact
- Trade-off exists among those image quality measures.

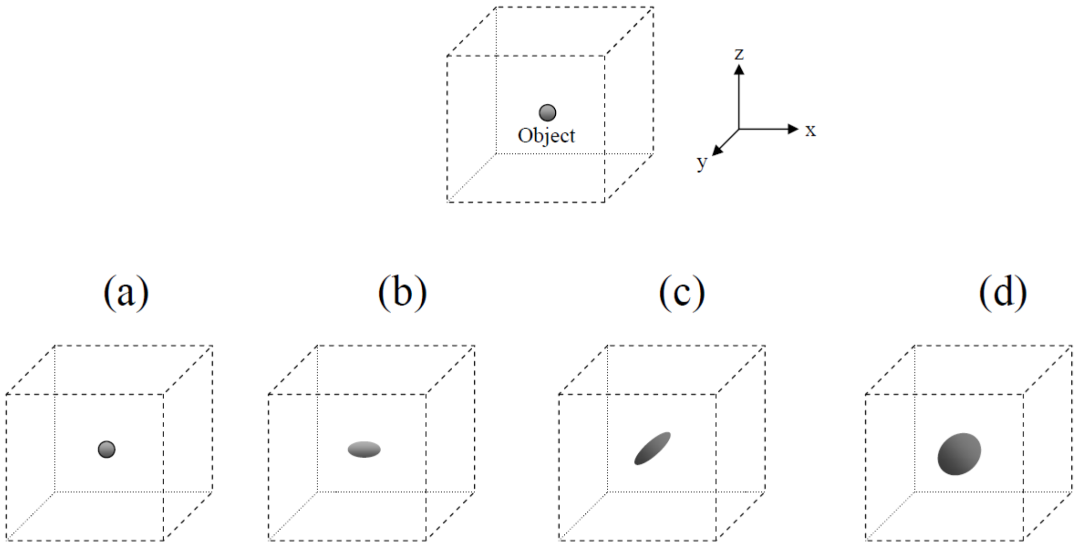
Spatial Resolution

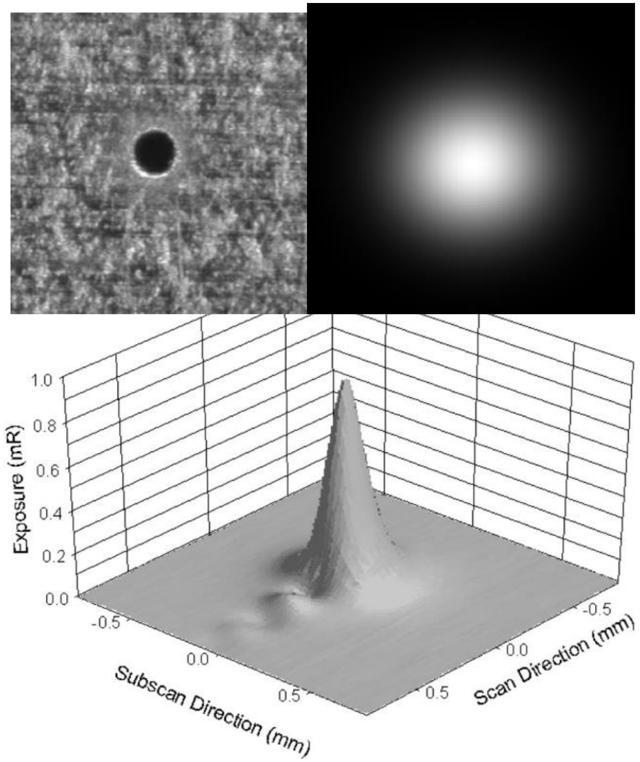
- Spatial resolution
  - Refers to the *smallest* feature that can be visualized by an imaging system, or the *smallest* distance between two features such that the features can be individually resolved.
  - represents by PSF, LSF, or MTF.





Point Spread Function (PSF)



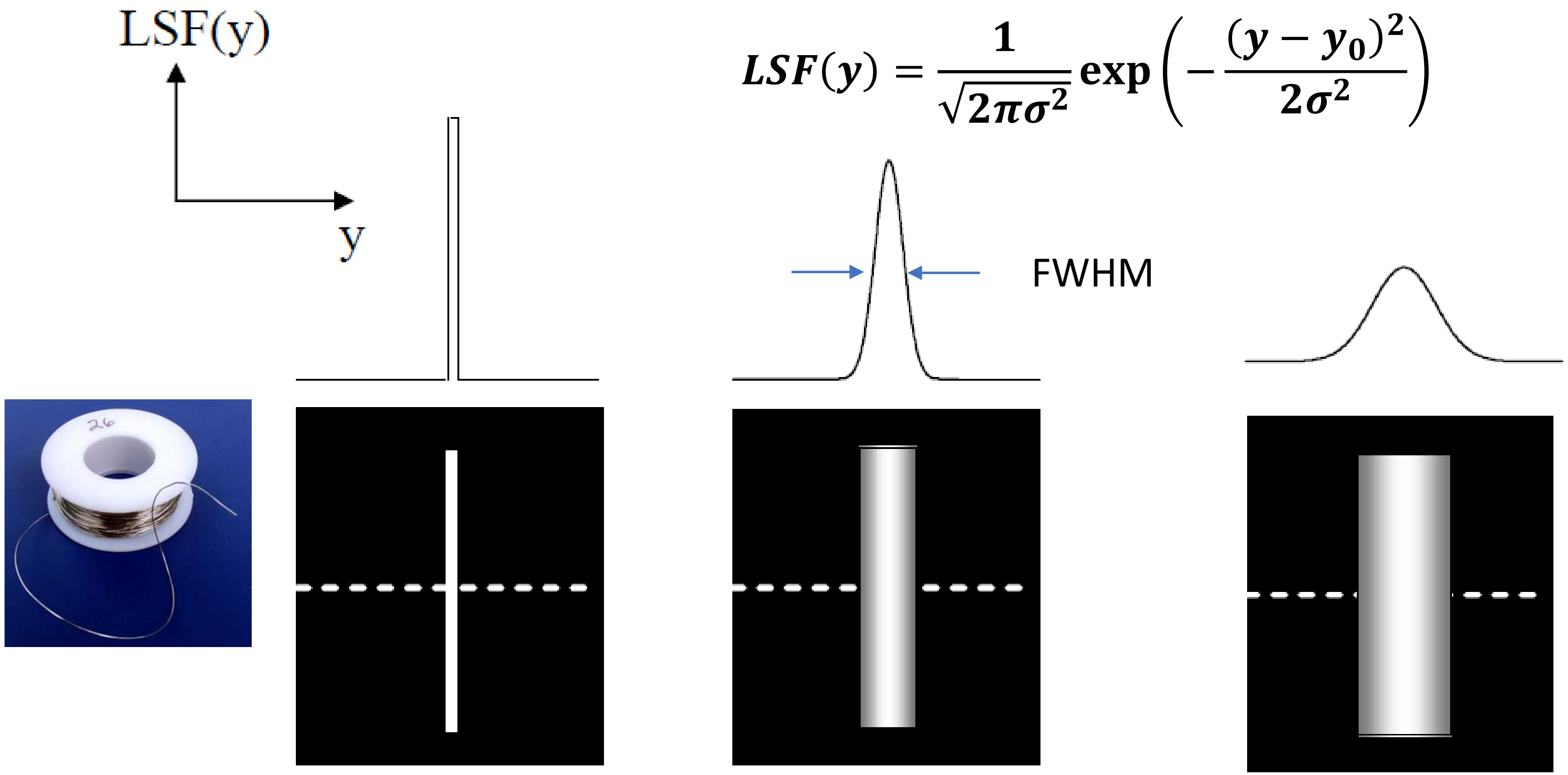


$PSF(x,y,z)$ → $I(x,y,z) = O(x,y,z) * PSF(x,y,z)$

point spread function — image — object

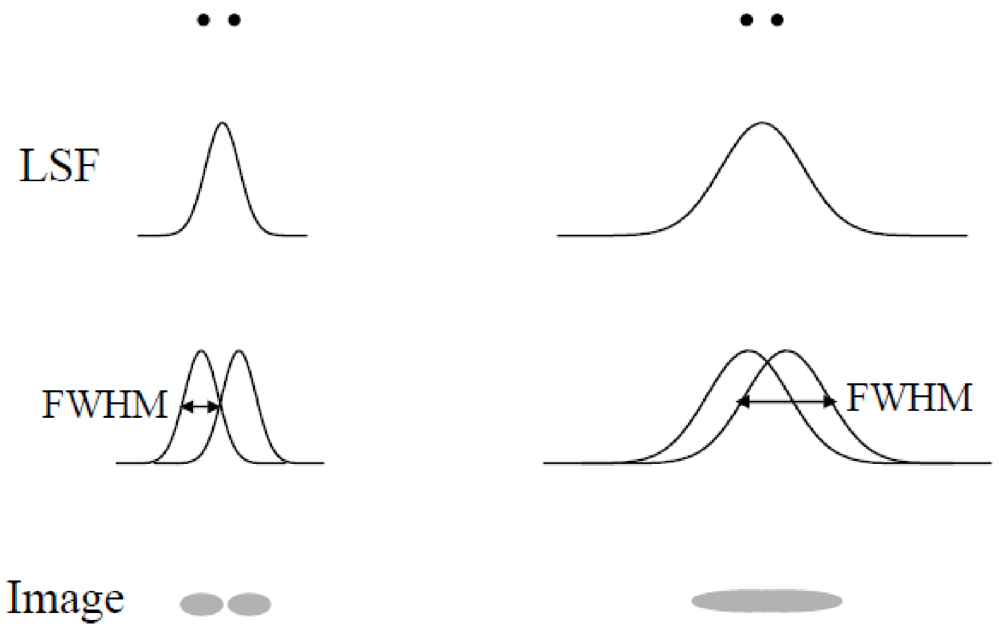
Line Spread Function (LSF)

$$LSF(y) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y-y_0)^2}{2\sigma^2}\right)$$



Spatial resolution

Refers to the *smallest* feature that can be visualized by an imaging system, or the *smallest* distance between two features such that the features can be individually resolved.

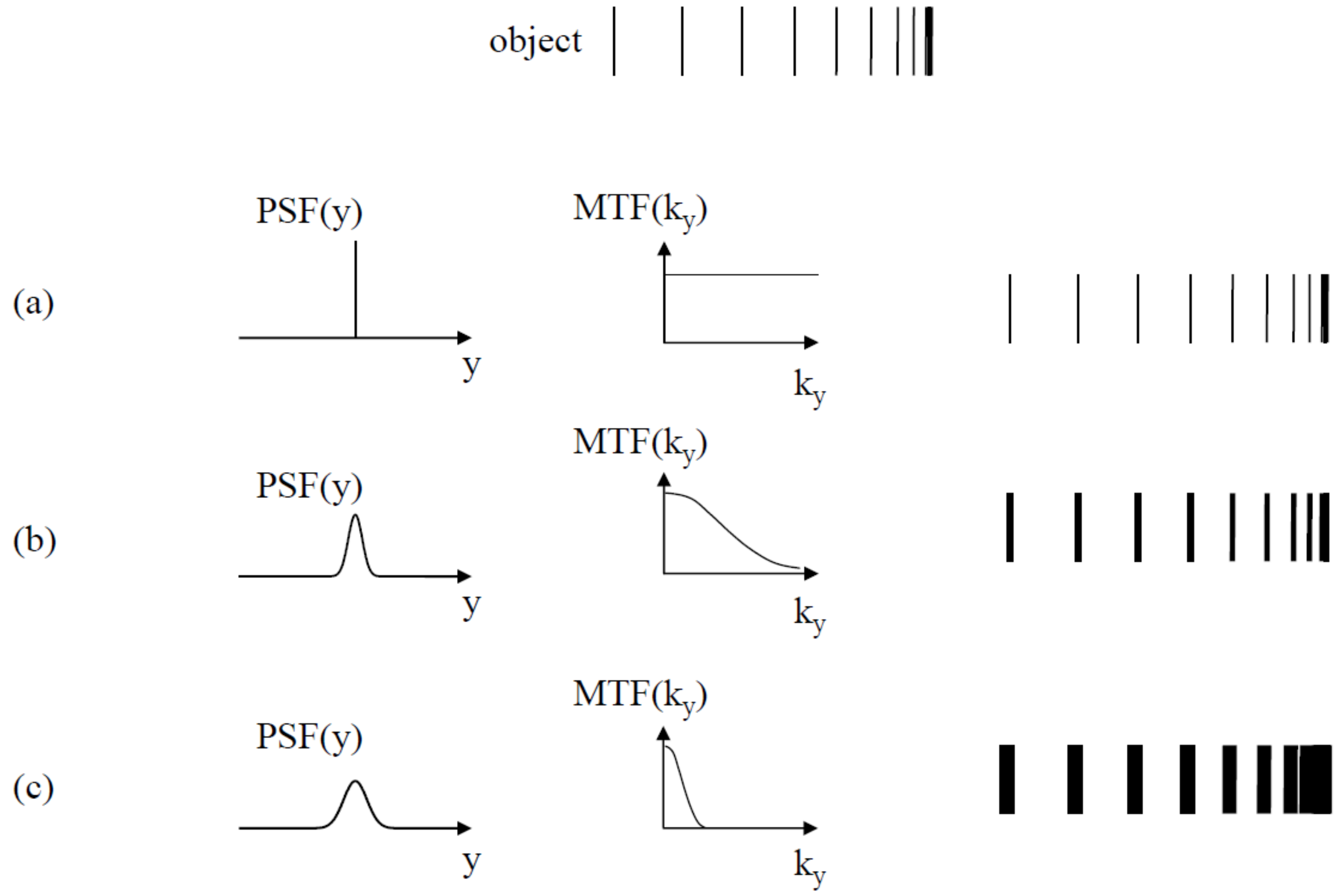


Spatial frequency

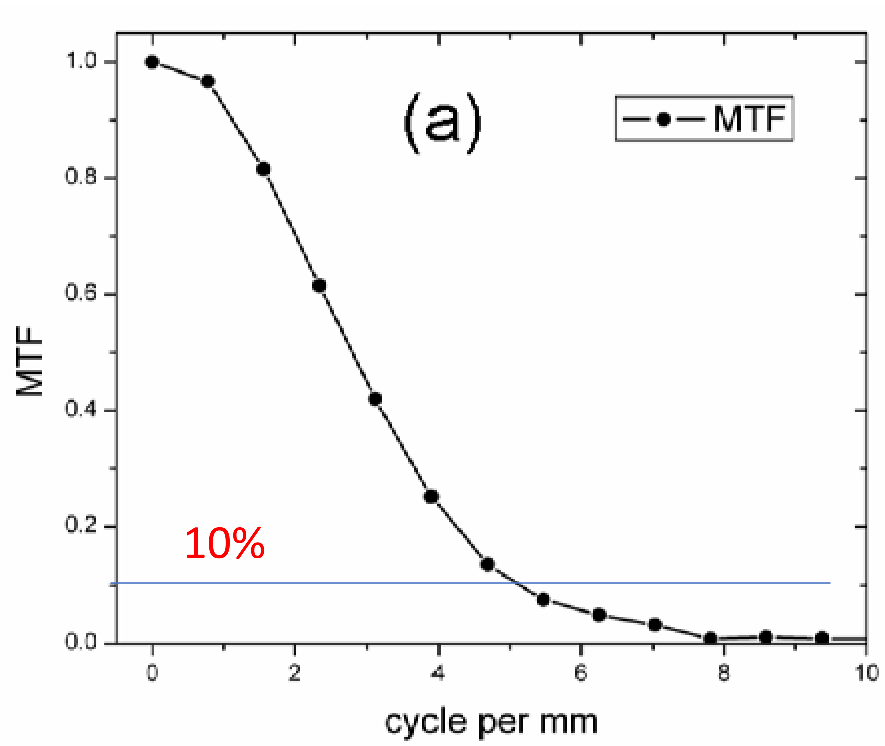
- Is a measure about how a *spatially* periodic structure repeats in space
- Unit: cycles per mm (or cm or m) or line pair per mm (LP/mm)

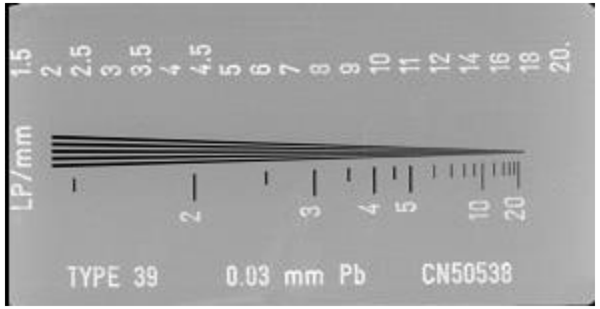
MTF – Modulation Transfer Function

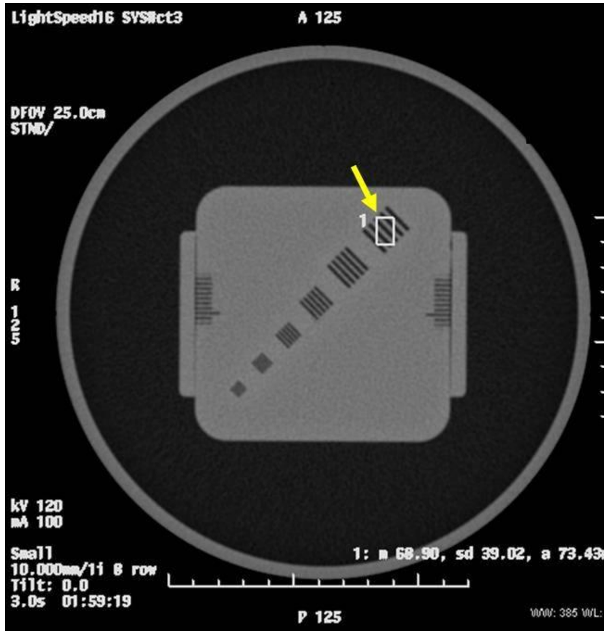
$$MTF(k_x, k_y, k_z) = F\{PSF(x, y, z)\},$$ ***F – Fourier Transform***



MTF Example


(a)
MTF
10%
cycle per mm
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10


LP/mm
TYPE 39
0.03 mm Pb
CN50538


LightSpeed16
A 125
DFOV 25.0cm
STND/
kV 120
mA 100
Small
Tilt: 0.0
P 125

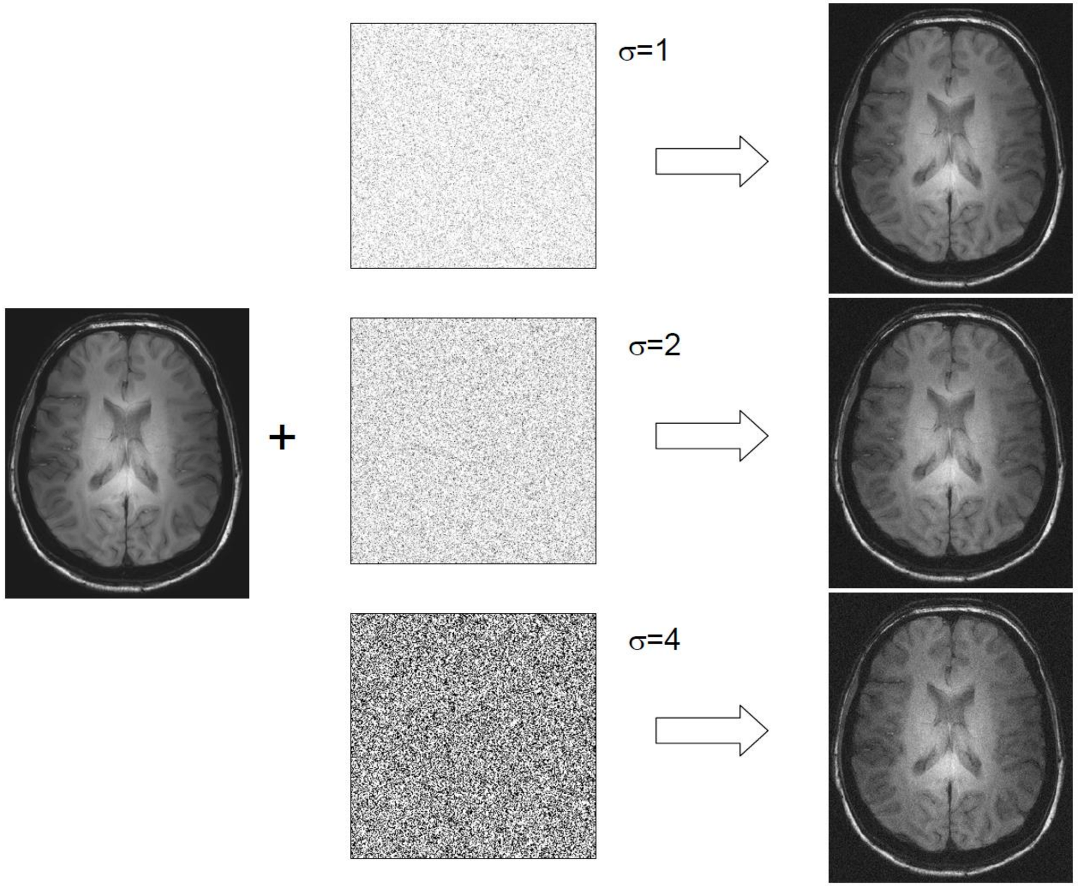
Signal-to-Noise Ratio (SNR)

- Signal
  - ➢The amplitude of the physical quantity that is measured.
  - ➢This is the *REAL* signal.

- Noise
  - ➢any signal that is measured but is not related to the actual physical quantity that one is trying to measure.
  - ➢<u>Noise is NOT systematic error (artifact)</u>!
  - ➢Noise is random with mean value at zero.

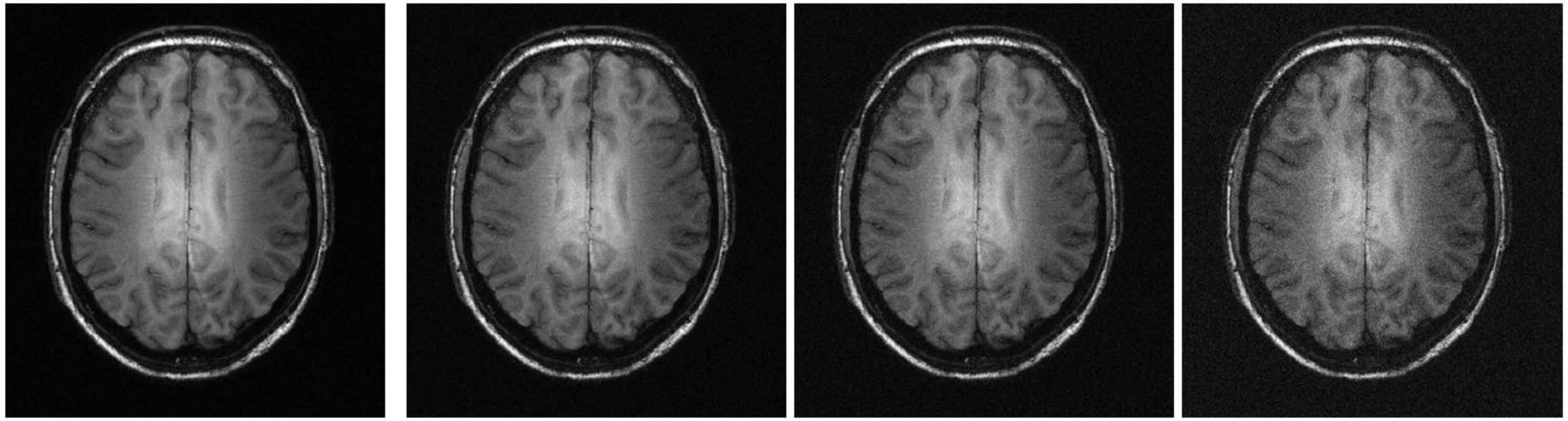
- SNR = Signal/Noise

Effects of noise on image quality



Increase SNR by averaging N scans

$$\mathrm{SNR} \propto \sqrt{N}$$



N= 6 4 2 1

Contrast-to-Noise Ratio (CNR)

✓ Contrast refers to signal difference between two tissues in the same image

$$C_{AB} = |S_A - S_B|$$

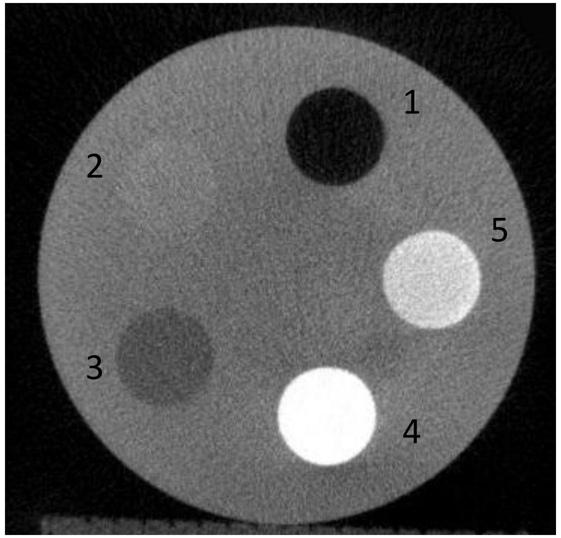
CNR is defined as

$$CNR = \frac{C_{AB}}{\sigma} = \frac{|S_A - S_B|}{\sigma} = |SNR_A - SNR_B|$$

✓ An image can have high SNR but low CNR.

✓ For an image to be diagnostically useful, it needs high CNR.

CNR Example



(1)air,
(2)water,
(3)fat mimic,
(4)contrast agent (30 mgI/mL), and
(5)bone simulator.

	μ	σ
Air	-1000	88
Water	0	97
Fat mimic	-378	90
Contrast agent	1382	113
Bone simulator	935	110

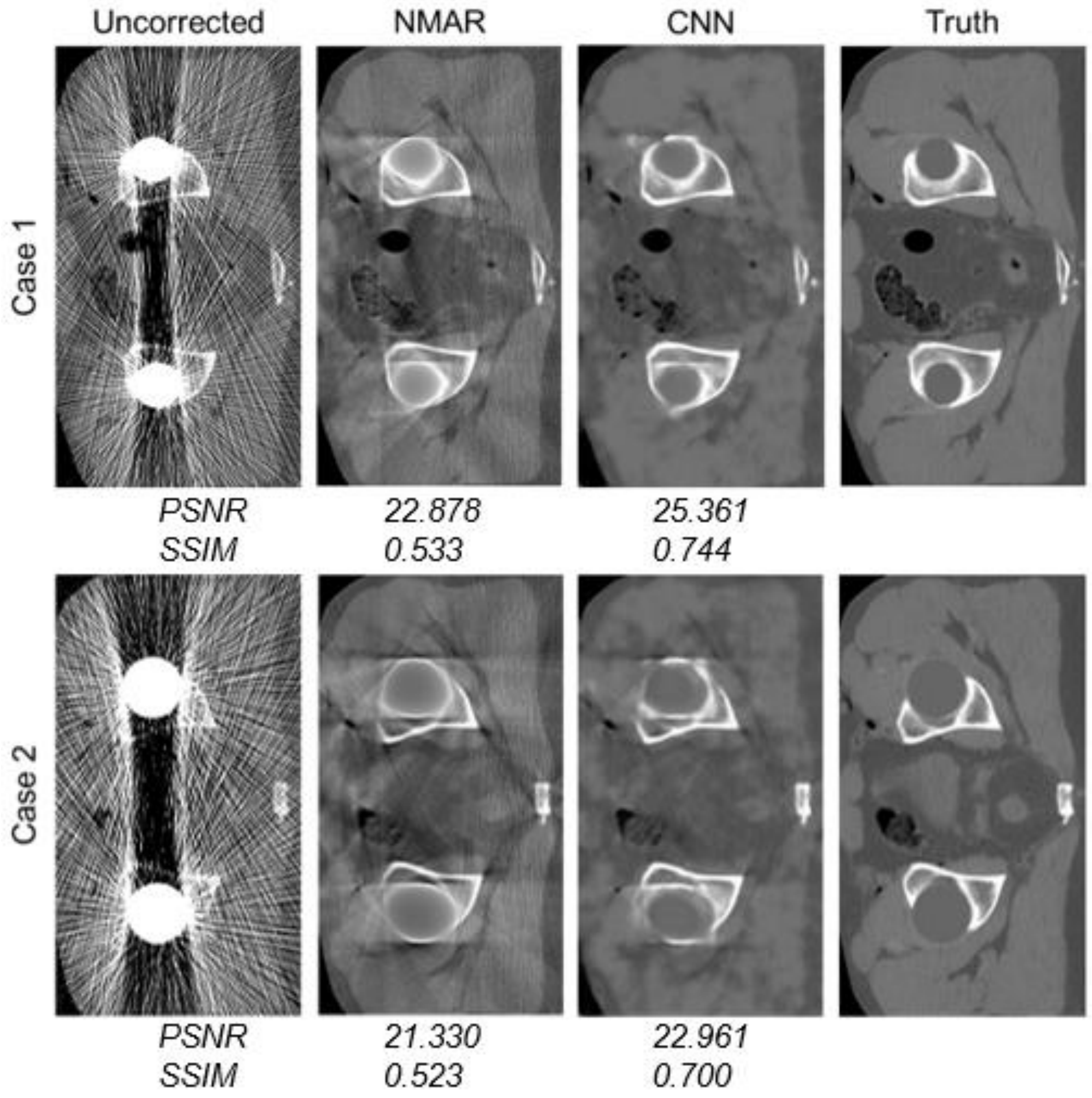
Another definition of CNR:

$$CNR = \frac{\mu_2 - \mu_1}{\sqrt{\sigma_2^2 + \sigma_1^2}}$$

CNR water-to-air = 10.8;

CNR water-to-fat = 4.0

Metal Artifacts in CT



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Need for Task-specific Measures

Four Cases (Two Error Types)

Diagnosis	Actual situation	
	Multiple sclerosis	*Healthy*
Multiple sclerosis	True positive	False positive
Healthy	False negative	True negative

Test Result ↓	Diseased	Healthy ← Actual Situation
Test Positive	**True Positive**	**False Positive**
Test Negative	**False Negative**	**True Negative**

Sensitivity & Specificity

Sensitivity=TP/(TP+FN)

Specificity =TN/(TN+FP)

Likelihood of a positive case

How sure we say YES

Likelihood of a negative case

How sure we say NOPE

Sensitivity – ability of a test to make correct diagnosis of a disease.
= number of true positive / the sum of true positives and false negatives)
= **True Positive Fraction (TPF)**
TPF=1-FPF (False Positive Fraction)

Specificity – ability of a test to correctly rule out a disease.
= number of true negative / the sum of true negatives and false positives
= **True Negative Fraction (TNF)**
TNF=1-FNF (False Negative fraction)

Accuracy – ability of a test to make correct diagnosis out of total number of tests.
=number of correct diagnosis / total number of diagnoses

PPV & NPV

– **Positive Predictive Value (PPV):** fraction of patients whose medical imaging calls abnormal actually have disease

$$PPV = \frac{True\ Positives}{True\ Positives + False\ Positives} = \frac{True\ Positives}{All\ persons\ called\ abnormal}$$

– **Negative Predictive Value (NPV):** fraction of patients whose medical imaging calls Normal and do NOT have disease.

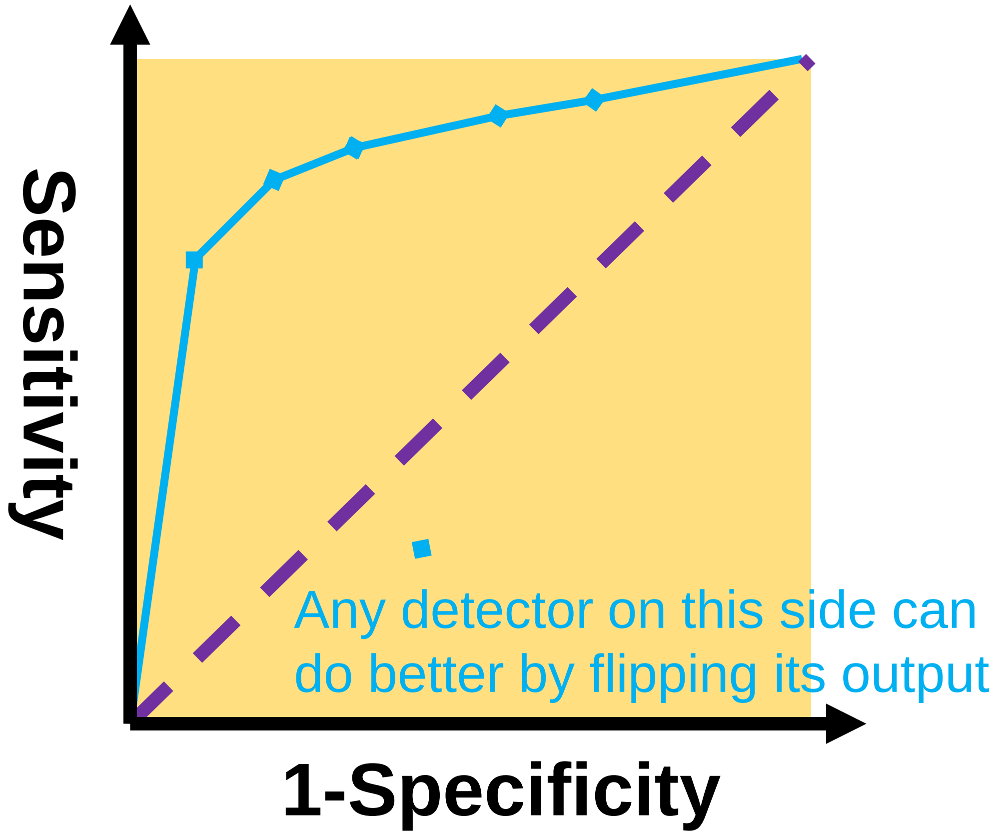
$$NPV = \frac{True\ Negatives}{True\ Negatives + False\ Negatives} = \frac{True\ Negatives}{All\ persons\ called\ normal}$$

Example

Diagnosis	Tuberculosis		
X-Ray	Yes	No	Total
Positive	22	51	73
Negative	8	1739	1747
Total	30	1790	1820

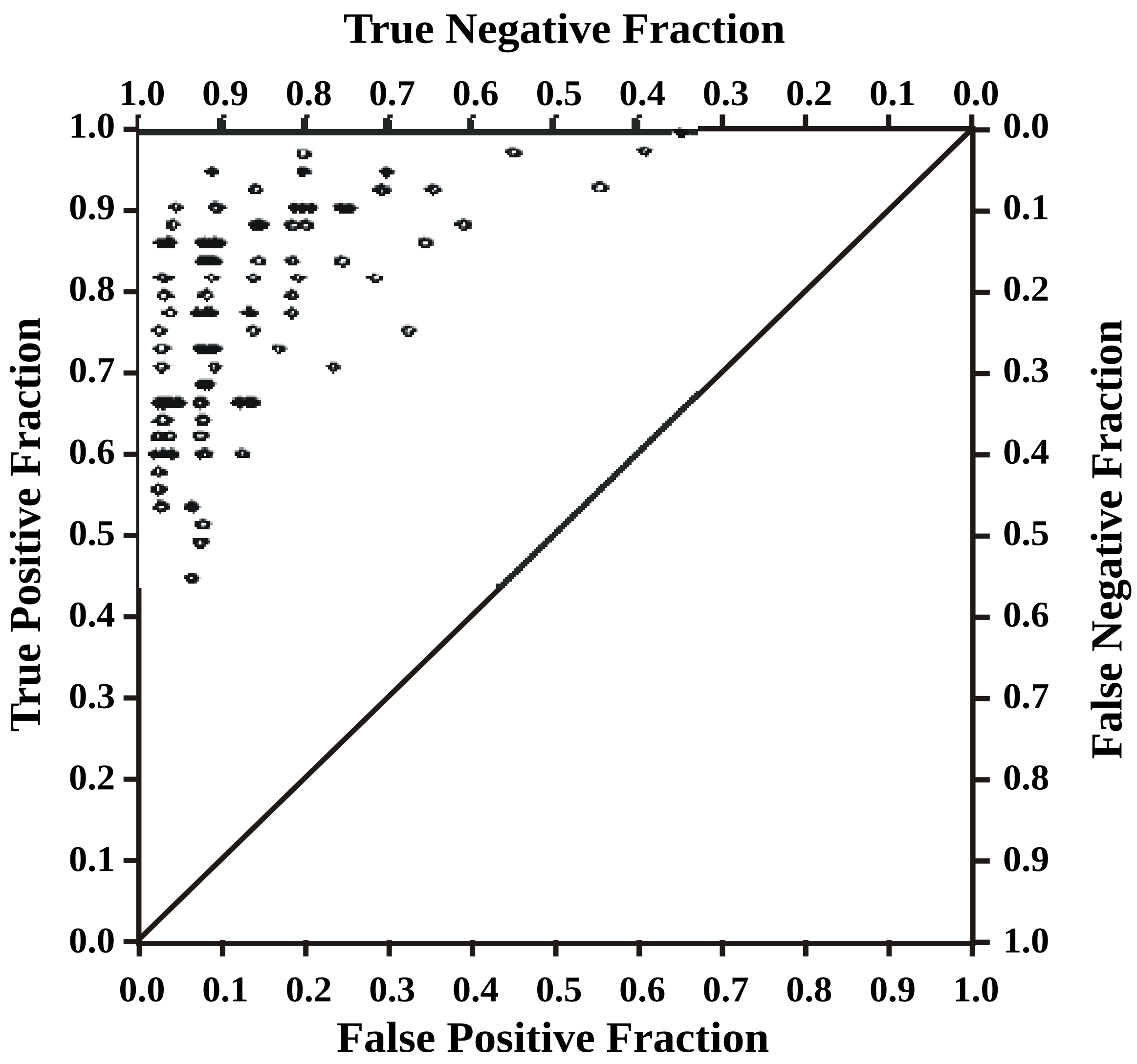
Sensitivity = 22 / 30 = 73.3%
Specificity = 1739 / 1790 = 97.2%
Positive Predictive Value (PPV) = 22 / 73 = 30.1%
Negative Predictive Value (NPV) = 1739 / 1747 = 99.5%
Diagnostic Accuracy (DA) = (22 + 1739) / 1820 = 96.8%
Prevalence (PR) = 30 / 1820 = 1.6%

Receiver Operating Characteristic



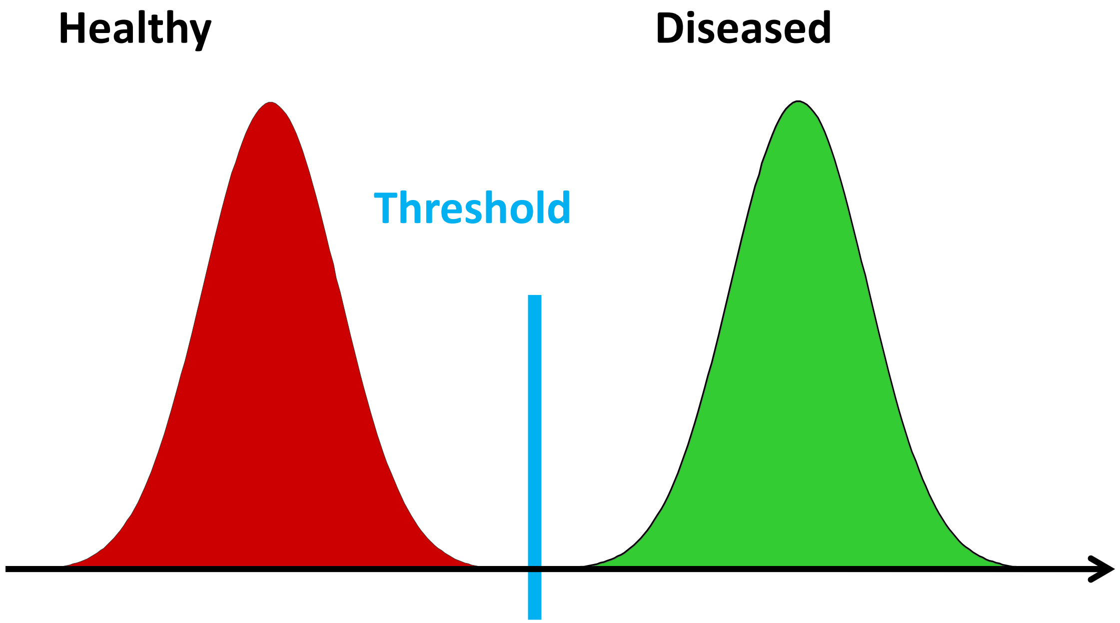
- **Report sensitivity & specificity**
- **Give an ROC curve**
- **Average over many data**

ROC Example

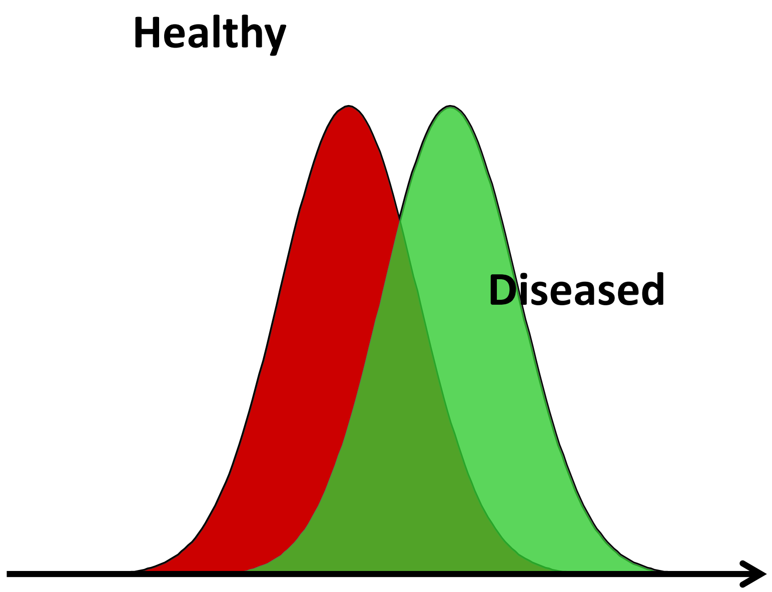


TPF *vs* FPF for 108 US radiologists in study by Beam et al.

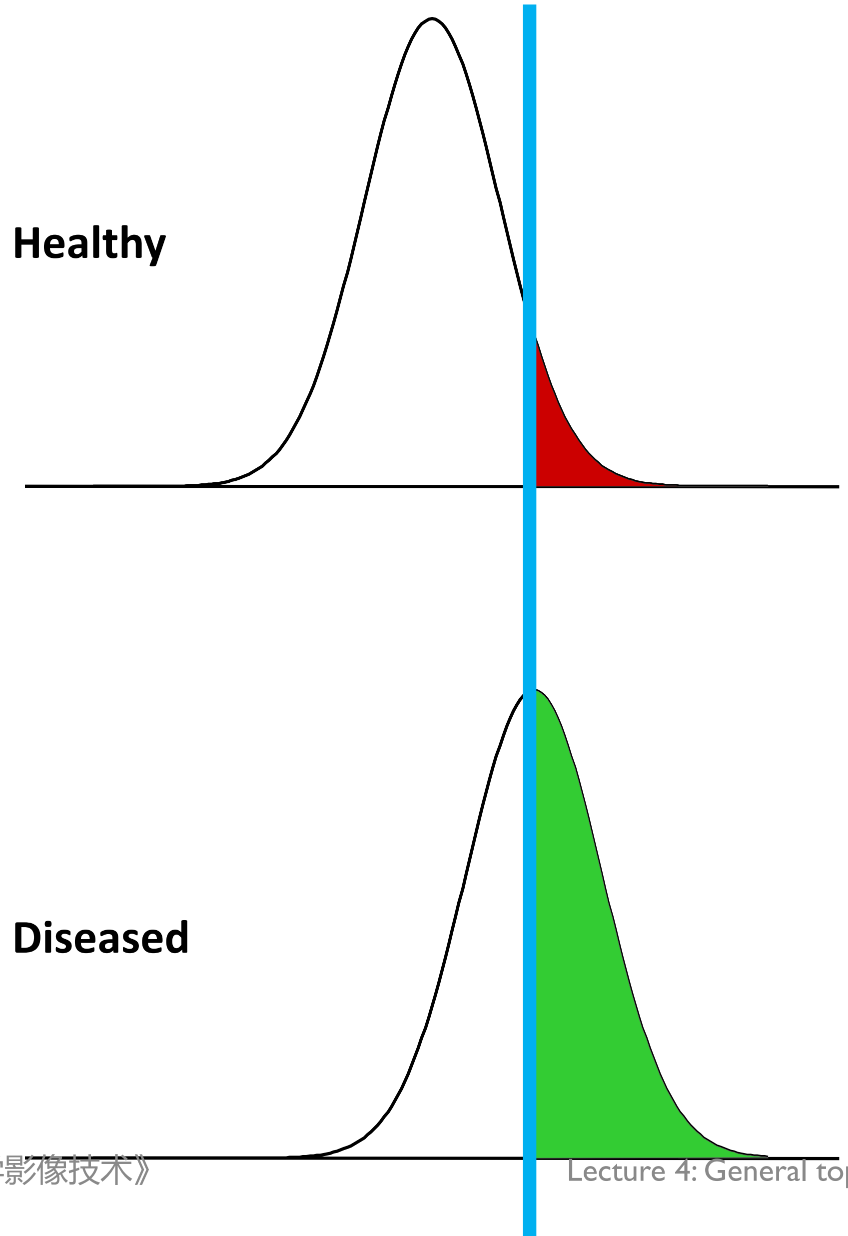
Ideal Case

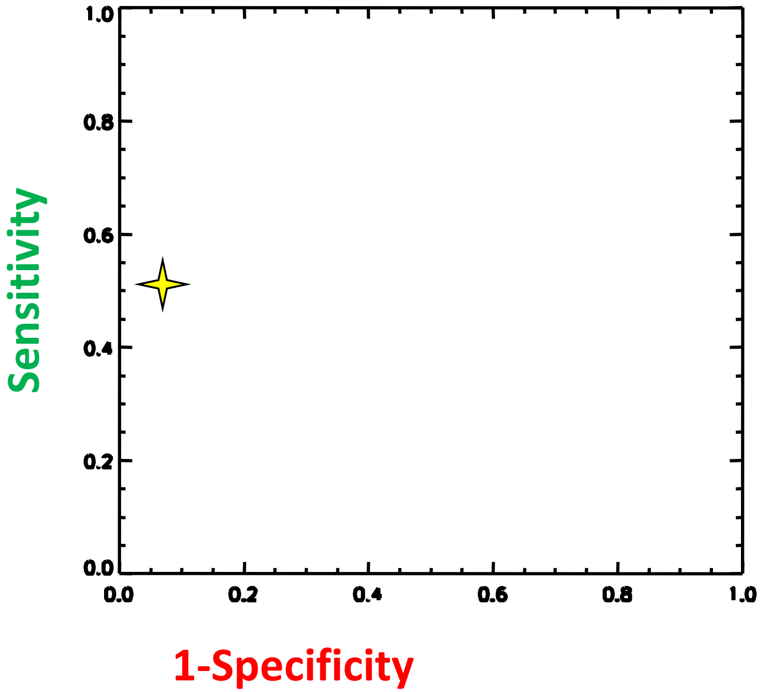


More Realistic Case

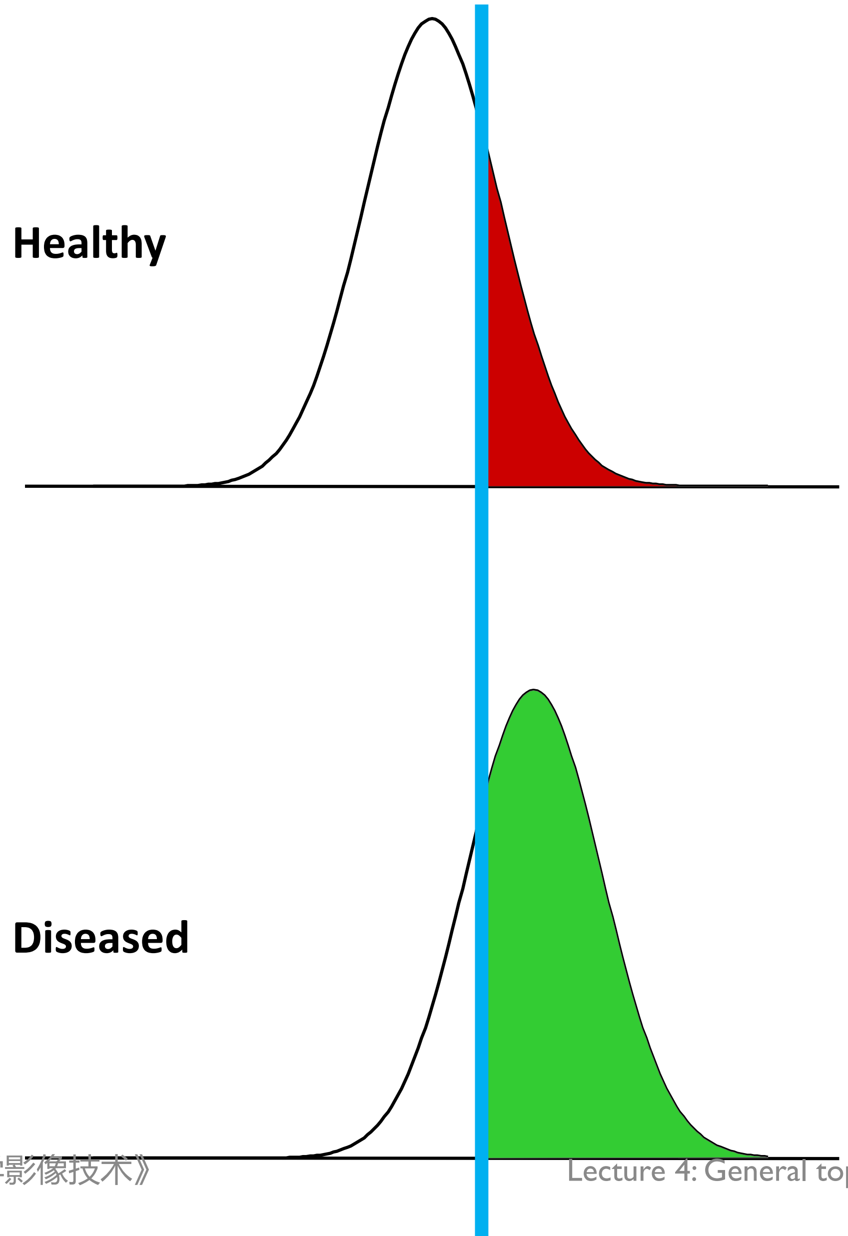


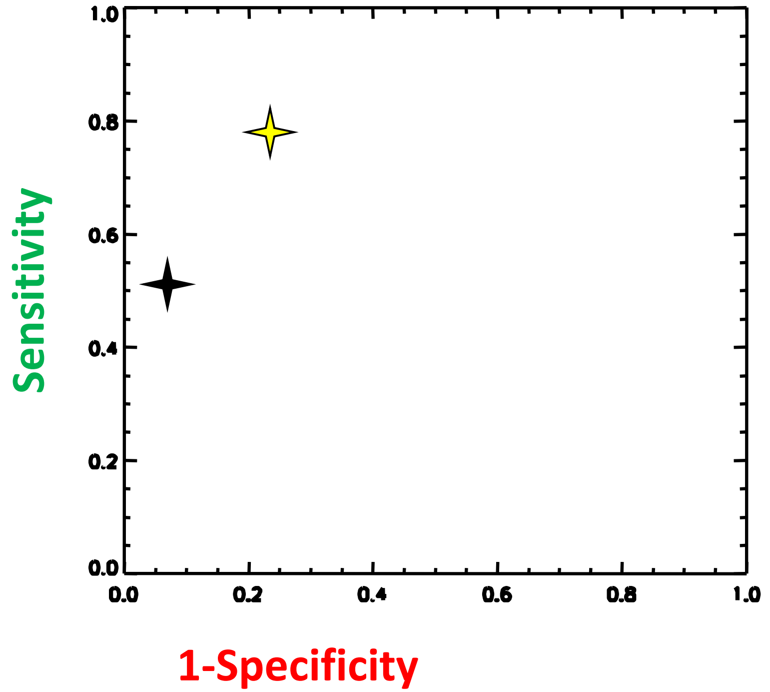
ROC: Least Aggressive



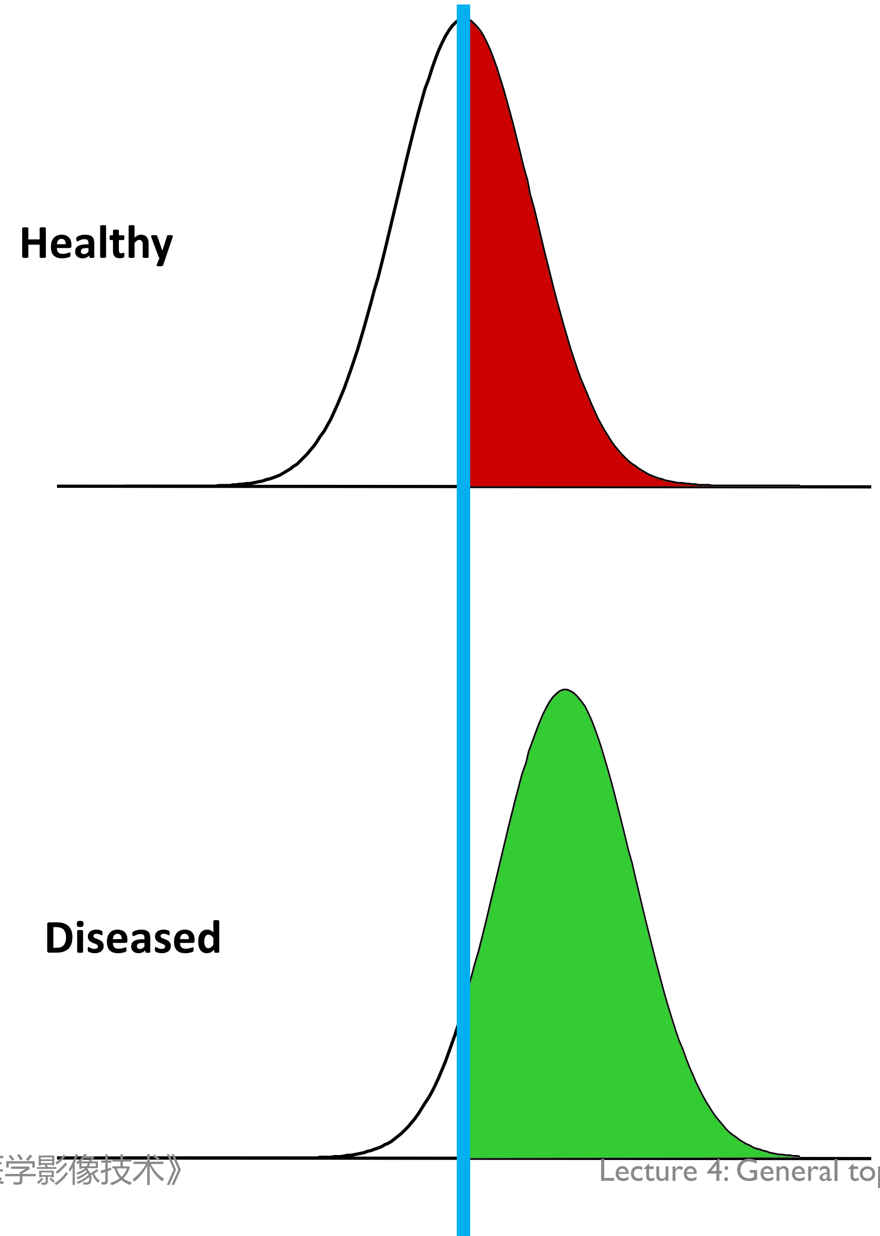


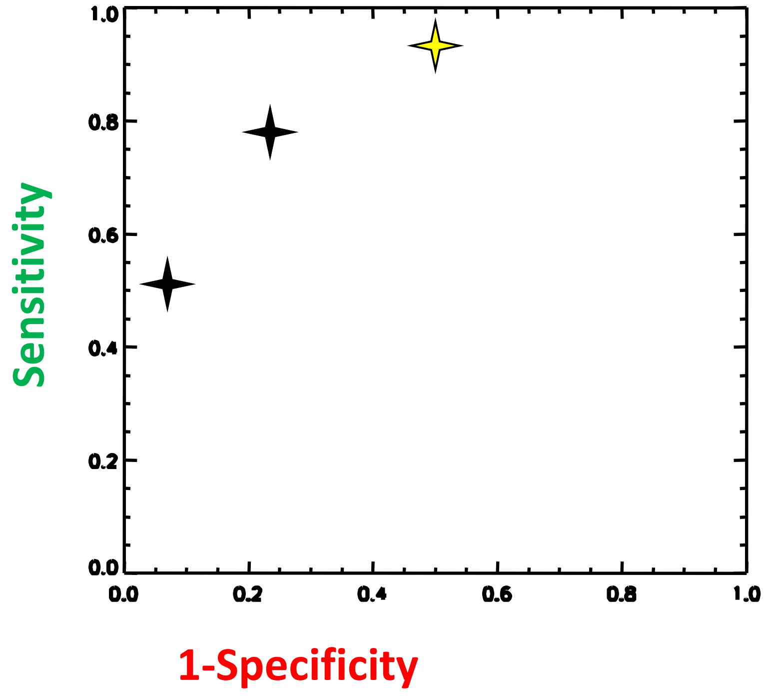
ROC: Moderate



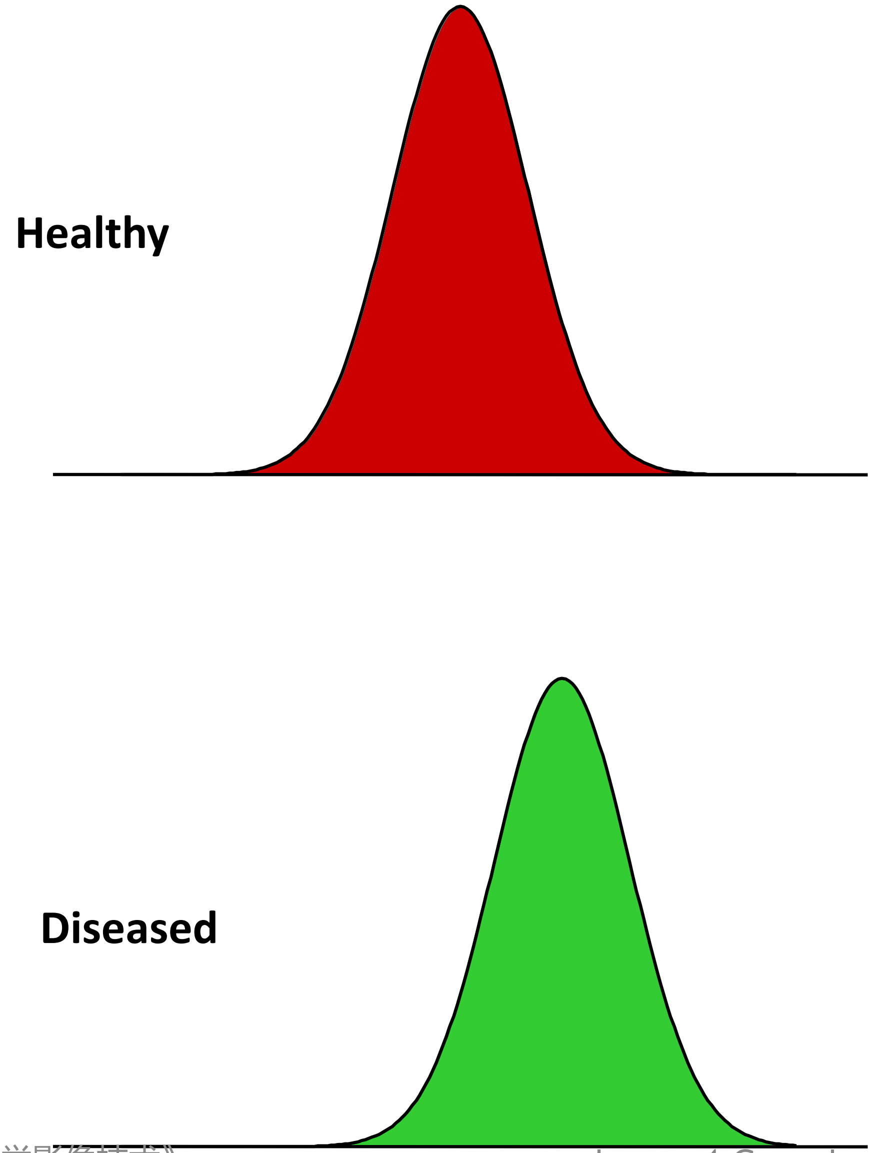


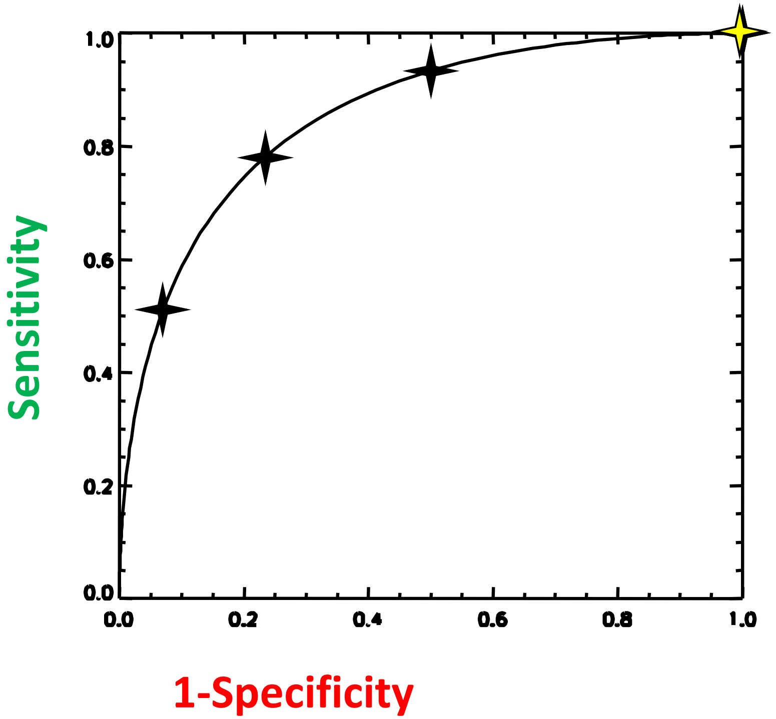
ROC: More Aggressive



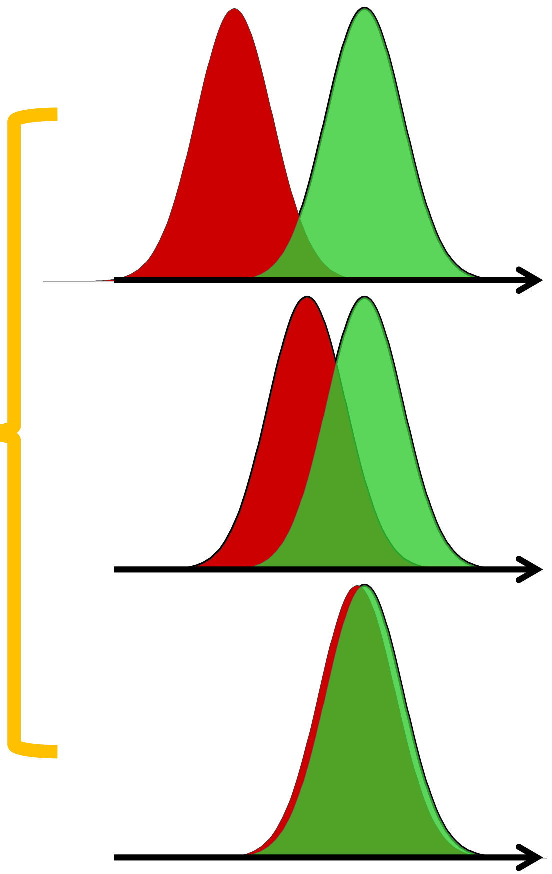


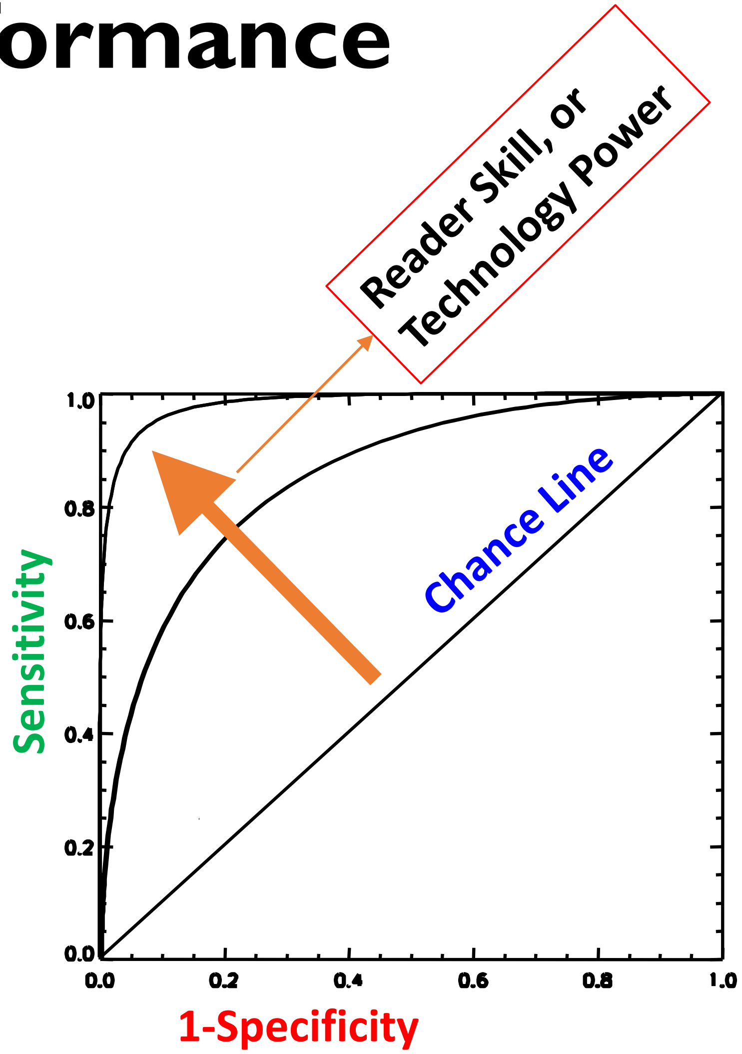
ROC Curve


Healthy
Diseased


Sensitivity
1-Specificity
0.0
0.2
0.4
0.6
0.8
1.0

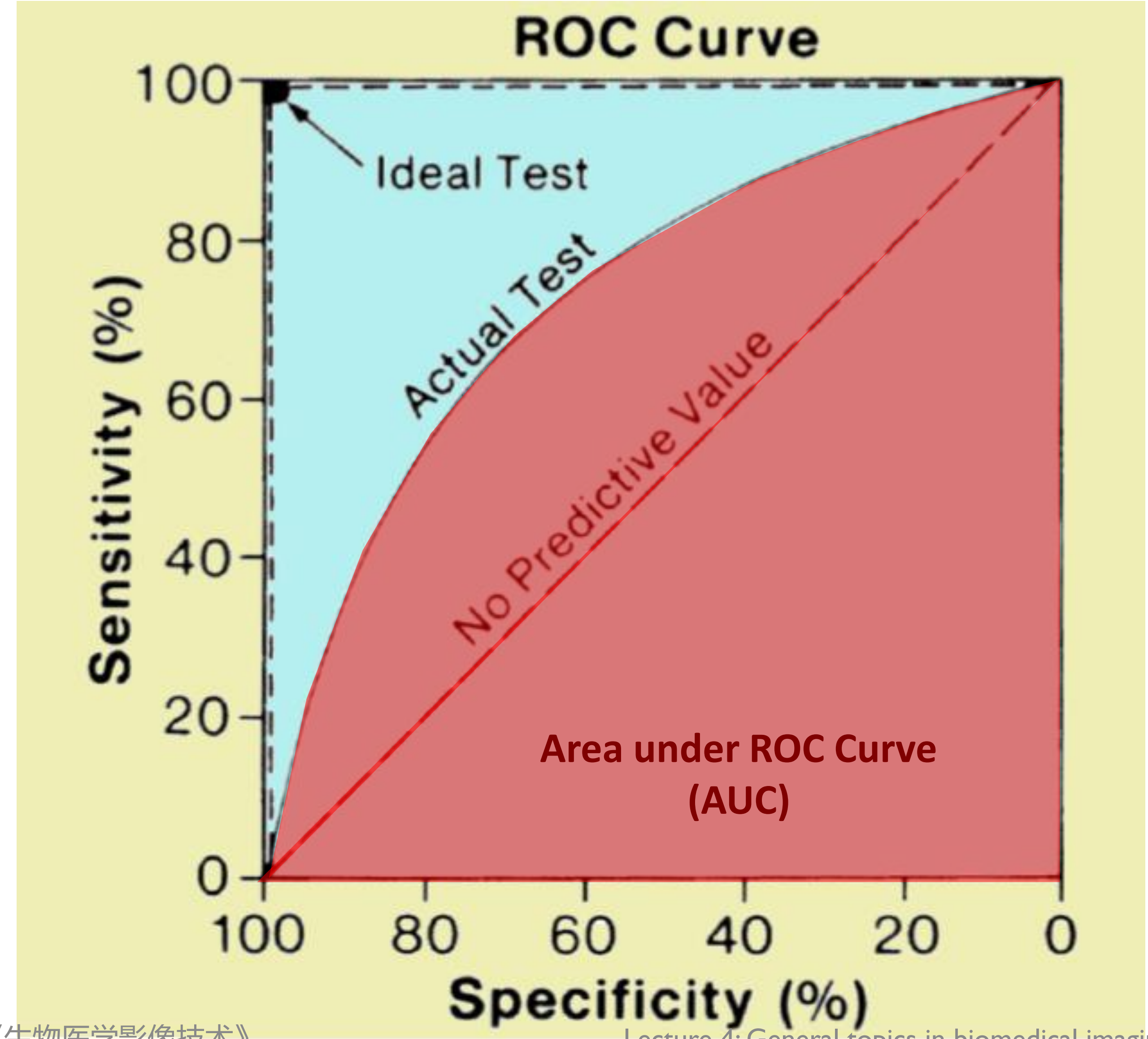
Diagnostic Performance





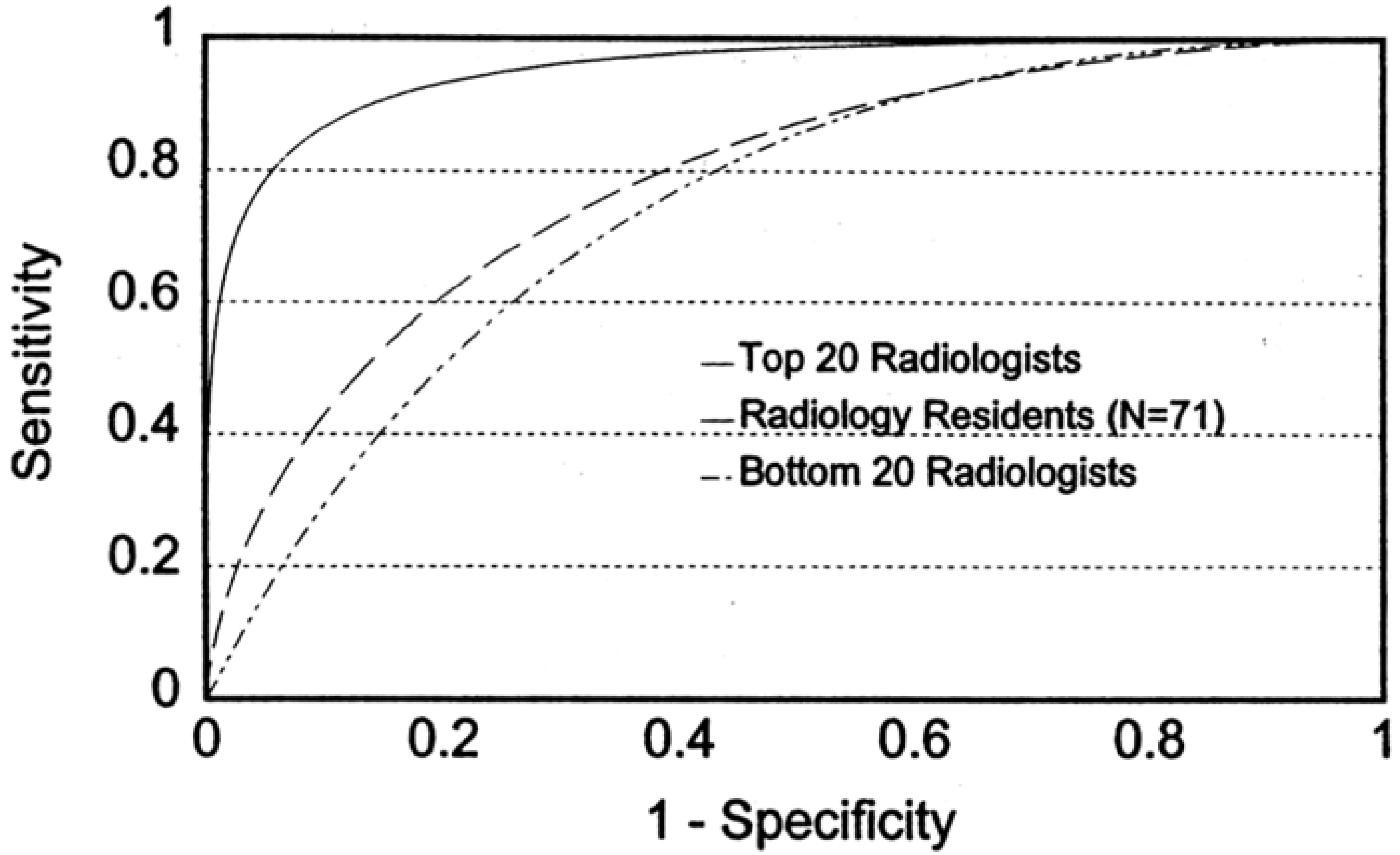
Reader Skill: Same Thing But Viewed Differently

Area Under Curve



- ROC curves are used to compare diagnostic tests.
- The area under the curve gives a quantitative measure of quality of the diagnostic test.

Example



Chest film study by F. James Potchen, M.D., 1999

Imaging Model

In imaging, the process of data acquisition is represented mathematically as (3)

$$\mathbf{g} = \mathrm{H}\mathbf{f} + \mathbf{n} \qquad \ldots(1)$$

where $\mathbf{f}$ is the object being imaged, H denotes the imaging operator which represents the imaging system, $\mathbf{n}$ is the noise generated during the measurement, and $\mathbf{g}$ is the image vector. In nuclear-medicine imaging, for example, the object $\mathbf{f}$ may be the continuous three-dimensional radioactivity distribution in a patient. The imaging system H describes the mapping from the continuous object $\mathbf{f}$ to the discrete

Model Observers

Theranostics 2013, Vol. 3, Issue 10 774

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2013; 3(10):774-786. doi: 10.7150/thno.5138

Review

Model Observers in Medical Imaging Research

Xin He and Subok Park

US Food and Drug Administration.

Corresponding author: xin.he@fda.hhs.gov

Received: 2012.08.31; Accepted: 2013.04.15; Published: 2013.10.04

Abstract

Model observers play an important role in the optimization and assessment of imaging devices. In this review paper, we first discuss the basic concepts of model observers, which include the mathematical foundations and psychophysical considerations in designing both optimal observers for optimizing imaging systems and anthropomorphic observers for modeling human observers. Second, we survey a few state-of-the-art computational techniques for estimating model observers and the principles of implementing these techniques. Finally, we review a few applications of model observers in medical imaging research.

Key words: model observer, medical imaging research

The World of Medical Imaging