

Problem Formulation

$$\begin{aligned}
& \text{/* Cost Function */} \\
\min_{x(\cdot), u(\cdot), z(\cdot), s(\cdot), s^e} & \int_0^T l(x(\tau), u(\tau), z(\tau), p) + \frac{1}{2} \begin{bmatrix} s_l(\tau) \\ s_u(\tau) \\ 1 \end{bmatrix}^\top \begin{bmatrix} Z_l & 0 & z_l \\ 0 & Z_u & z_u \\ z_l^\top & z_u^\top & 0 \end{bmatrix} \begin{bmatrix} s_l(\tau) \\ s_u(\tau) \\ 1 \end{bmatrix} d\tau + \\
& m(x(T), z(T), p) + \frac{1}{2} \begin{bmatrix} s_l^e \\ s_u^e \\ 1 \end{bmatrix}^\top \begin{bmatrix} Z_l^e & 0 & z_l^e \\ 0 & Z_u^e & z_u^e \\ z_l^{e\top} & z_u^{e\top} & 0 \end{bmatrix} \begin{bmatrix} s_l^e \\ s_u^e \\ 1 \end{bmatrix} \\
& \text{/* Initial value */} \\
\text{s.t.} \quad & \underline{x}_0 \leq J_{\text{bx},0}x(0) \leq \bar{x}_0, \\
& \text{/* Dynamics */} \\
& f_{\text{impl}}(x(t), \dot{x}(t), u(t), z(t), p) = 0, \quad t \in [0, T], \\
& \text{/* Path Constraints with lower bound */} \\
& \underline{h} \leq h(x(t), u(t), p) + J_{\text{sh}}s_{l,h}(t), \quad t \in [0, T], \\
& \underline{x} \leq J_{\text{bx}}x(t) + J_{\text{sbx}}s_{l,\text{bx}}(t), \quad t \in (0, T], \\
& \underline{u} \leq J_{\text{bu}}u(t) + J_{\text{sbu}}s_{l,\text{bu}}(t), \quad t \in [0, T], \\
& \underline{g} \leq Cx(t) + Du(t) + J_{\text{sg}}s_{l,g}(t), \quad t \in [0, T], \\
& s_{l,h}(t), s_{l,\text{bx}}(t), s_{l,\text{bu}}(t), s_{l,g}(t) \geq 0, \quad t \in [0, T], \\
& \text{/* Path Constraints with upper bound */} \\
& h(x(t), u(t), p) - J_{\text{sh}}s_{u,h}(t) \leq \bar{h}, \quad t \in [0, T], \\
& J_{\text{bx}}x(t) - J_{\text{sbx}}s_{u,\text{bx}}(t) \leq \bar{x}, \quad t \in (0, T], \\
& J_{\text{bu}}u(t) - J_{\text{sbu}}s_{u,\text{bu}}(t) \leq \bar{u}, \quad t \in [0, T], \\
& Cx(t) + Du(t) - J_{\text{sg}}s_{u,g}(t) \leq \bar{g}, \quad t \in [0, T], \\
& s_{u,h}(t), s_{u,\text{bx}}(t), s_{u,\text{bu}}(t), s_{u,g}(t) \geq 0, \quad t \in [0, T], \\
& \text{/* Terminal Constraints with lower bound */} \\
& \underline{h}^e \leq h^e(x(T), p) + J_{\text{sh}}^e s_{l,h}^e, \\
& \underline{x}^e \leq J_{\text{bx}}^e x(T) + J_{\text{sbx}}^e s_{l,\text{bx}}^e, \\
& \underline{g}^e \leq C^e x(T) + J_{\text{sg}}^e s_{l,g}^e \leq \bar{g}^e, \\
& s_{l,h}^e, s_{l,\text{bx}}^e, s_{l,\text{bu}}^e, s_{l,g}^e \geq 0, \\
& \text{/* Terminal Constraints with upper bound */} \\
& h^e(x(T), p) - J_{\text{sh}}^e s_{u,h}^e \leq \bar{h}^e, \\
& J_{\text{bx}}^e x(T) - J_{\text{sbx}}^e s_{u,\text{bx}}^e \leq \bar{x}^e, \\
& C^e x(T) - J_{\text{sg}}^e s_{u,g}^e \leq \bar{g}^e, \\
& s_{u,h}^e, s_{u,\text{bx}}^e, s_{u,\text{bu}}^e, s_{u,g}^e \geq 0,
\end{aligned}$$

with

- state vector $x : \mathbb{R} \rightarrow \mathbb{R}^{n_x}$
- control vector $u : \mathbb{R} \rightarrow \mathbb{R}^{n_u}$
- algebraic state vector $z : \mathbb{R} \rightarrow \mathbb{R}^{n_z}$
- model parameters $p \in \mathbb{R}^{n_p}$
- slacks for path constraints $s_l(t) = (s_{l,\text{bu}}, s_{l,\text{bx}}, s_{l,g}, s_{l,h}) \in \mathbb{R}^{n_s}$ and $s_u(t) = (s_{u,\text{bu}}, s_{u,\text{bx}}, s_{u,g}, s_{u,h}) \in \mathbb{R}^{n_s}$
- slacks for terminal constraints $s_l^e(t) = (s_{l,\text{bx}}^e, s_{l,g}^e, s_{l,h}^e) \in \mathbb{R}^{n_s^e}$ and $s_u^e(t) = (s_{u,\text{bx}}^e, s_{u,g}^e, s_{u,h}^e) \in \mathbb{R}^{n_s^e}$

Some of the following restrictions may apply to matrices in the formulation:

DIAG

Diagonal

SPUM

Horizontal slice of a permuted unit matrix

SPUME

Like **SPUM**, but with empty rows intertwined

1 Dynamics

The function $f_{\text{impl}} : \mathbb{R}^{n_x} \times \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \times \mathbb{R}^{n_z} \times \mathbb{R}^{n_p} \rightarrow \mathbb{R}^{n_x+n_z}$ describes the dynamics as a fully implicit DAE.

We offer to discretize F with a classic implicit Runge-Kutta (**irk**) or a structure exploiting implicit Runge-Kutta method (**irk_gnsf**).

Additionally, we offer an explicit Runge-Kutta integrator (**erk**), which can be used with explicit ODE models, i.e. models of the form

$$f_{\text{expl}}(x, u, p) = \dot{x}$$

Mathematical Expression	string identifier	data type	required
f_{impl} respectively f_{expl}	dyn_expr_f	CasADi expression	yes
-	dyn_type	string (explicit or implicit)	yes

2 Cost

There are different **acados** modules to model the cost functions.

- $l : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \times \mathbb{R}^{n_z} \rightarrow \mathbb{R}$ is the Lagrange objective term.
- $m : \mathbb{R}^{n_x} \times \mathbb{R}^{n_z} \rightarrow \mathbb{R}$ is the Mayer objective term.

to decide which one is used set **cost_type** for l , **cost_type_e** for m .

Setting the slack penalties is done in the same way for all cost modules, namely:

Mathematical Expression	string identifier	data type	required
Z_l	cost_Zl	double, DIAG	no
Z_u	cost_Zu	double, DIAG	no
z_l	cost_zl	double	no
z_u	cost_zu	double	no
Z_l^e	cost_Zl_e	double, DIAG	no
Z_u^e	cost_Zu_e	double, DIAG	no
z_l^e	cost_zl_e	double	no
z_u^e	cost_zu_e	double	no

Moreover, you can specify **cost_Z**, to set Z_l, Z_u to the same values, i.e. use a symmetric L2 slack penalty. Similarly, **cost_z**, **cost_Z_e**, **cost_z_e** can be used to set symmetric slack L1 penalties, respectively penalties for the terminal slack variables.

Cost module: auto

Set `cost_type` to `auto` (default). In this case we detect if the cost function specified is a linear least squares term and transcribe it in the corresponding form. Otherwise, it is formulated using the external cost module. Note: slack penalties are optional and we plan to detect them from the expressions in future versions.

Mathematical Expression	string identifier	data type	required
l	<code>cost_expr_ext_cost</code>	CasADi expression	yes
m	<code>cost_expr_ext_cost_e</code>	CasADi expression	yes

Cost module: external

Set `cost_type` to `ext_cost`.

Mathematical Expression	string identifier	data type	required
l	<code>cost_expr_ext_cost</code>	CasADi expression	yes
m	<code>cost_expr_ext_cost_e</code>	CasADi expression	yes

Cost module: linear least squares

Set `cost_type` to `linear_ls`.

The Lagrange cost term has the form

$$l(x, u, z) = \frac{1}{2} \left\| \underbrace{V_x x + V_u u + V_z z}_y - y_{\text{ref}} \right\|_W^2$$

with matrices V_x, V_u, V_z, W of appropriate dimensions.

Similarly, the Mayer cost term has the form

$$m(x, u, z) = \frac{1}{2} \left\| \underbrace{V_x^e x}_y - y_{\text{ref}}^e \right\|_{W^e}^2$$

with matrices V_x^e, W^e of appropriate dimensions.

Mathematical Expression	string identifier	data type	required
V_x	<code>cost_Vx</code>	double	yes
V_u	<code>cost_Vu</code>	double	yes
V_z	<code>cost_Vz</code>	double	yes
W	<code>cost_W</code>	double	yes
y_{ref}	<code>cost_y_ref</code>	double	yes
V_x^e	<code>cost_Vx_e</code>	double	yes
W^e	<code>cost_W_e</code>	double	yes
y_{ref}^e	<code>cost_y_ref_e</code>	double	yes

Cost module: nonlinear least squares

Set `cost_type` to `nonlinear_ls`.

The cost function has the same form as in the linear least squares module.

The only difference is that y , respectively y^e are defined as `CasADi` expressions, instead of the matrices V_x, V_u, V_z , respectively V_x^e

Mathematical Expression	string identifier	data type	required
y	<code>cost_expr_y</code>	<code>CasADi</code> expression	yes
W	<code>cost_W</code>	double	yes
y_{ref}	<code>cost_y_ref</code>	double	yes
y^e	<code>cost_expr_y_e</code>	<code>CasADi</code> expression	yes
y_{ref}^e	<code>cost_y_ref_e</code>	double	yes

3 Constraints

3.1 Initial State

Note: An initial state is not required. For example for MHE problems it should not be set.

Simple syntax for initial constraint $x(0) = \bar{x}_0$:

Mathematical Expression	string identifier	data type	required
\bar{x}_0	<code>constr_x0</code>	double	no

Extended syntax:

Mathematical Expression	string identifier	data type	required
\underline{x}_0	<code>constr_lbx_0</code>	double	no
\bar{x}_0	<code>constr_ubx_0</code>	double	no
$J_{\text{bx},0}$	<code>constr_Jbx_0</code>	double	no

3.2 Path Constraints

Mathematical Expression	string identifier	data type	required
J_{bx}	constr_Jbx	double, PUM	no
\underline{x}	constr_lbx	double	no
\bar{x}	constr_ubx	double	no
J_{bu}	constr_Jbu	double, PUM	no
\underline{u}	constr_lbu	double	no
\bar{u}	constr_ubu	double	no
C	constr_C	double	no
D	constr_D	double	no
\underline{g}	constr_lg	double	no
\bar{g}	constr_ug	double	no
h	constr_expr_h	CasADi expression	no
\underline{h}	constr_lh	double	no
\bar{h}	constr_uh	double	no
J_{sbx}	constr_Jsbx	double, SPUME	no
J_{sbu}	constr_Jsbu	double, SPUME	no
J_{sg}	constr_Jsg	double, SPUME	no
J_{sbx}	constr_Jsh	double, SPUME	no

3.3 Terminal Constraints

Mathematical Expression	string identifier	data type	required
J_{bx}	constr_Jbx_e	double, SPUM	no
\underline{x}^e	constr_lbx_e	double	no
\bar{x}^e	constr_ubx_e	double	no
C^e	constr_C_e	double	no
\underline{g}^e	constr_lg	double	no
\bar{g}^e	constr_ug	double	no
h^e	constr_expr_h_e	CasADi expression	no
\underline{h}^e	constr_lh_e	double	no
\bar{h}^e	constr_uh_e	double	no
J_{sbx}	constr_Jsbx	double, SPUME	no
J_{sg}^e	constr_Jsg_e	double, SPUME	no
J_{sbx}^e	constr_Jsh_e	double, SPUME	no

3.4 External links

<https://docs.google.com/spreadsheets/d/1rVRycLnCyaWJLwnV47u30Vokp7vRu68og30hlDbSjDU/edit?usp=sharing>