

## Forest counts! More carbon sequestration & More rational management

As the problem of global warming intensifies, the impact of climate change on life is increasing, and forests play an indispensable role in mitigating climate change. In this paper, we measure the total amount of carbon sequestration in forests, then we evaluate and predict carbon sequestration after considering other factors, to give the most reasonable management decision-making suggestions.

First, we built a **carbon sequestration model**. In this model, taking into account the species and age of trees, we calculated forest carbon sequestration and forest product carbon sequestration separately. By calculating the total carbon sequestration amount through integration, we can find the cutting time of various types of trees. Cutting at or after this time yields more carbon sequestration than if the trees were not cut. Then, we calculated the carbon sequestration of forests in China through the model and then compared the results with the actual data, verifying the validity of the model.

Secondly, taking into account the other values of the forest, we build decision models to inform forest managers of the best use of a forest. In the model, we take into account five indicators: biodiversity, productive capacity, health vitality, carbon sequestration, and economic benefits. The **grey relational analysis** model is used to evaluate the above indicators, and determine the priority of each indicator in decision-making. To minimize the transition time for each of the forest metrics, we use a **mathematical programming model** to find the best management options for each forest. After applying the best management plan, we used the **grey prediction model** GM(1,1) improved by **combined weight method** to predict the carbon sequestration of forests and their products within 100 years.

Furthermore, we apply the decision model to two forests in China, the Great Khingan in the cold temperate zone and the Xishuangbanna rainforest in the tropical rainforest climate. As for Great Khingan, we first used gray correlation analysis to get the priority of each indicator, and the result showed that the priority of fire rate was the highest. Then, using mathematical programming, the most suitable transition point for the forest is 17% of the correlation degree of each index, and the shortest time required to reach the transition point is 19.4 years. Finally, we predict that the total carbon sequestration in the Great Khingan Mountains from 2000 to 2100 is 143735.3297 million tons by GM(1,1). As for Xishuangbanna Tropical Rainforest, we use the same way to calculate. Results show that biomass and carbon sequestration has the highest priority, the transition point is 12% of the correlation degree of each indicator, and the shortest time required to reach the transition point is 9.7 years. The total carbon sequestration from 2000 to 2100 was 129,590.2157 million tons.

Finally, we analyze the sensibility of our model and point out our strengths and weaknesses. Analysis indicates that the Greater Khingan is most sensitive to changes in fire rates and the Xishuangbanna rainforest is most sensitive to changes in tourism.

**Keywords:** Grey relational analysis, Mathematical programming model, GM(1,1), combined weight method

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# 1 Introduction

## 1.1 Problem Background

Nowadays, global warming poses a huge threat to our lives. To reduce carbon dioxide in the air, certain forest products combined with new forest regeneration have the potential to sequester more carbon over time than complete deforestation. Thus, proper logging is beneficial for carbon sequestration, but excessive logging can lead to a reduction in carbon sequestration, so finding a balance between harvesting forest products and allowing trees to continue to grow is crucial. In addition to this, conservation and biodiversity aspects, recreational uses, and cultural considerations also need to be taken into account. Therefore, we decided to construct a carbon sequestration model and xx model to help forest managers use and manage forests better.

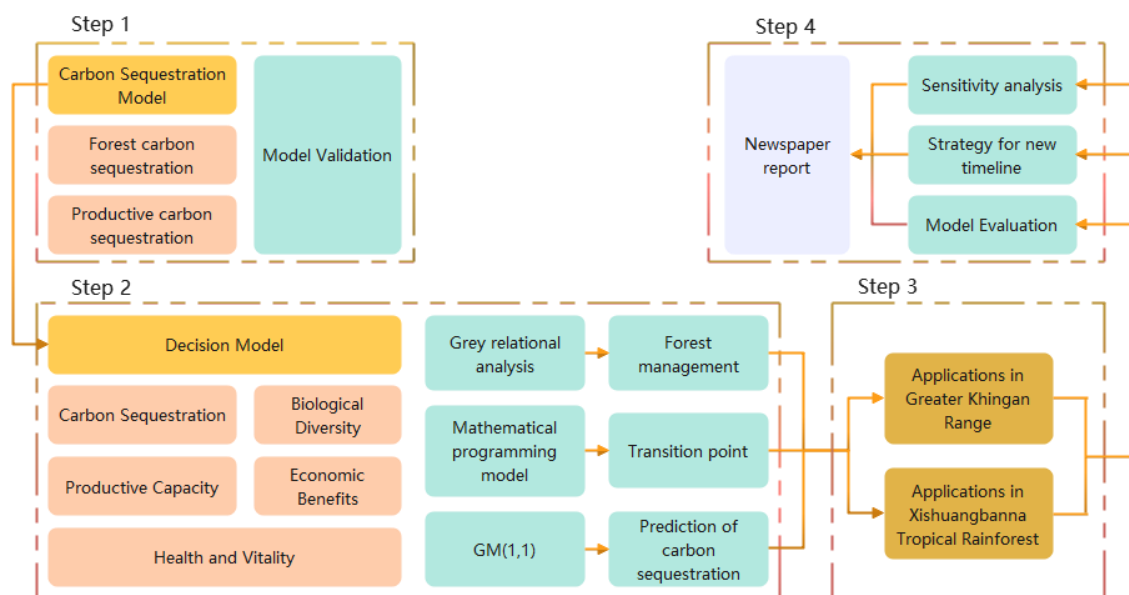
## 1.2 Our work

We developed a carbon sequestration model. We divide the total amount of carbon sequestration into forests and forest products and then calculate them respectively. In the process, we can also obtain the cutting time of various types of trees. Then we selected various types of forests in China as representatives to test the model effectiveness.

While considering other factors, we built a decision model. 17 inferior indicators are classified into 5 superior indicators: biodiversity, productive capacity, health-vitality, carbon sequestration, and economic efficiency. We evaluate forests through grey relational analysis and give corresponding recommendations. Then we use the mathematical programming model to find out the transition time and the best forest management plan. Finally, we use GM(1,1) to how much the forest and its products will sequester carbon dioxide over 100 years.

We apply the model to two forests in China which are Greater Khingan and Xishuangbanna tropical rain forests. Through the evaluation and planning results, we figure out the rational management plan and the suitable transition for the forest.

The entire modeling process is as follows:



**Figure 1: Our Approach**

## 2 Assumptions

To simplify the problem, we make the following basic assumptions, each of which is properly justified.

- **Assumption 1:** Errors due to data corrections or statistical errors by the National Bureau of Statistics are ignored.
- **Assumption 2:** When calculating the value of a forest and its forest products, it is assumed that the number of trees felled is close to that of new trees planted each year.
- **Assumption 3:** Assume that trees are made into forest products shortly after they are felled.
- **Assumption 4:** Ignore the impact of sudden natural disasters on the state of forests and their values.

## 3 Carbon Sequestration Model

Carbon sequestration refers to technologies that capture carbon and store it safely instead of directly releasing CO<sub>2</sub> into the atmosphere. Therefore, to establish carbon sequestration, we use the amount of carbon stored by forests and their products to measure the amount of CO<sub>2</sub> sequestered. The carbon storage in forests is, The carbon storage of forest products is, we calculate them separately.

### 3.1 Model Establishment

#### 3.1.1 Calculate carbon storage in forests

We suppose that trees are felled at age  $T$ , and assume that the total number of trees remains the same. Then, after the following calculation, it is concluded that the number of trees of type in the forest at the age of is  $n_i(t_1)$ .

$$n_i(t_1)|_{t=k} = n_i((t_1 + 1) \% T_i)|_{t=k+1} \quad (1)$$

where

- $T_i$  is the age of the tree of type when it was felled.
- $t_1$  is the age of tree
- $t$  is the current year

For the understanding of the above formula, please refer to the following figure:

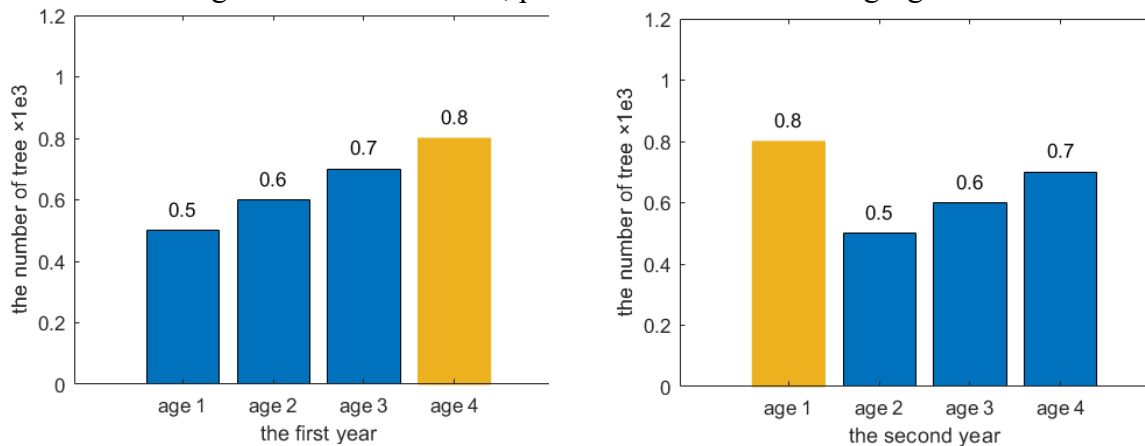


Figure 2: Cutting and Replanting

Explanation: Assume that the tree is cut down at its fifth year. During the first year, the yellow bars represent the number of trees that will be felled. When we cut it down, the same number of trees of the same type are planted. In the second year, the original number of trees in age 4 was moved to age 1, and the number of trees in other ages was moved 1 unit to the right. Therefore, the number of trees is a periodic function of time.

For one tree, its diameter at breast height and tree height gradually increases with age. By collecting the relevant discrete data of different trees at different ages, we obtain the function of DBH  $D(t_1)$  and tree height about  $t_1$  through cubic spline interpolation. Therefore, according to the number of volumes and the number of trees. We calculate the volume over the life cycle of the forest, which in turn calculates the total carbon storage  $A_{forest}$ :

$$V_i(t_1) = \frac{1}{4} \pi \cdot D_i^2(t_1) H_i(t_1) \quad (2)$$

$$A_{forest} = \sum_{i=1}^m \sum_{t_1=1}^{t_{i\max}} V_i(t_1) \cdot Q_i \cdot n_i(t_1) \quad (3)$$

Where

- $m$  is the number of species of trees commonly found in the forest
- $t_{i\max}$  is a tree of type  $i$ 's maximum age
- $n_i(t_1)$  refers to the number of trees in the forest that are  $t_1$  years old for the  $i^{th}$  tree
- $V_i(t_1)$  is the volume of the  $i^{th}$  tree at the age of  $t_1$  ( $m^3$ )
- $Q_i$  is the conversion factor of the  $i^{th}$  tree ( $t/m^3$ )

Conversion factor refers to tons of carbon per cubic meter of wood. Referring to related literature, common conversion factors are shown in the table below:

**Table1: Tons of carbon per cubic meter of wood in different tree species**

Conifer	Poplar	Broad-leaf tree	Oak	Beech	Spruce	Fir	Pine	Larch
0.28	0.28	0.3	0.33	0.34	0.215	0.205	0.245	0.275

### 3.1.2 Calculate carbon storage of forest products

Inspired by carbon stock accounting models in the literature, we believe that the ability of trees to sequester carbon gradually declines after felling, And it is assumed that the carbon sequestration capacity of the trees when they are just felled is the same as the carbon sequestration capacity of the forest products made at this time, so carbon sequestration capacity of the  $i^{th}$  tree when initially made into forest products is  $C_i(0)$ :

$$C_i(0) = n_i(0) \cdot V_i(T_i) \cdot Q_i \quad (4)$$

Where

- $n_i(0)$  is the number of trees made into forest products.
- $V_i(T_i) = \frac{1}{4} \pi D(T_i)^2 H(T_i)$  is the volume of the tree of type  $i$  when it was felled.
- $Q_i$  is the conversion factor of the  $i^{th}$  tree.

Then, with the change of time, the carbon sequestration of forest products decreases year by year until they reached their maximum life  $t_{product\max}$ :

$$C_i(t_2 + 1) = e^{-k} \cdot C_i(t_2) \quad (0 \leq t_2 \leq t_{product\max}) \quad (5)$$

Where

- $t_2 = 0$  is the same time as  $t_1 = T$ , that is, trees turned into products as soon as they were cut down.
- $k(> 0)$  is a first-order decay variable per year.

Calculate the total carbon sequestration over the life cycle of forest products  $A_{product}$ :

$$A_{product} = \sum_{i=1}^m \sum_{t_2=1}^{t_{product\max}} C_i(t_2) \quad (6)$$

$C_i(t_2)$  is carbon sequestration in the  $t_2^{th}$  year after the  $i^{th}$  tree is made into a product.

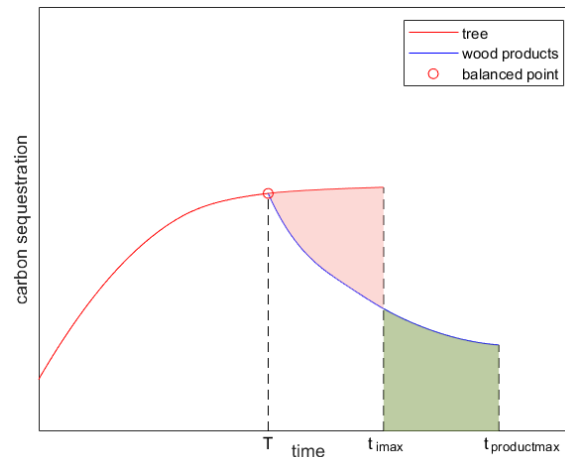
Calculate total  $CO_2$  sequestered by forests and their products over time:

$$A_{sum} = A_{forest} + A_{product} \quad (7)$$

To make our forest management plan most effective at carbon sequestration, we have produced a rough picture of the amount of carbon sequestered overtime when trees are felled to make products and when they are not. To make the carbon sequestration of the tree into a product greater than when it is not made into a product, it is necessary to meet:

$$\int_T^{t_{\max}} (A_{forest} - A_{product}) dt \leq \int_{t_{\max}}^{t_{product\max}} A_{product} dt \quad (8)$$

As shown in the figure below, when the forest product life is long, the carbon sequestration balance between cutting and not cutting can be achieved when the tree life reaches approximately 70% of the total life  $t_{\max}$ . That is, when the area of the pink shaded part in the figure is equal to the area of the green shaded part. Therefore, we consider the location as a balance point where we believe that optimization of carbon sequestration can be achieved when trees are felled at that point and beyond.



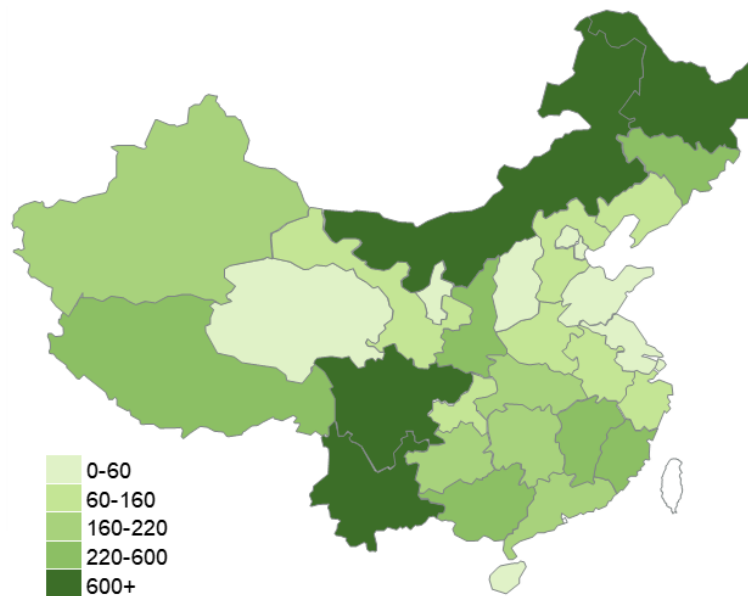
**Figure 3: Carbon Sequestration of Forests and its products**

Therefore, our recommendations for forest management plans are as follows. Firstly, through the calculation results and the query data, we found that the carbon content is related to the age of the tree, which can be generally divided into five types: young age, middle age, near-mature, mature, and over-mature. Cutting down some mature and over-mature trees and planting some new ones can increase the number of middle-aged forests and thus the forest's

carbon sequestration capacity. Secondly, make products that last as long as possible. Some products have a short lifespan, while others may outlive the trees that produce them. The carbon sequestered in certain forest products may allow more carbon sequestration over time than the carbon sequestration benefits of not deforestation at all.

### 3.2 Model Effectiveness

We applied the model to forests across China. By collecting forest data in each province in 2016, we calculated the carbon sequestration (million tons) of forests in each province using the above formula. The intuitive diagram is as follows:



**Figure 4: Carbon Sequestration in China**

By comparing with the relevant data found, we find that the carbon sequestration calculated in this model in Tibet is significantly smaller than the real data. After our analysis, we suppose that's because our model does not take the amount of carbon sequestered in soil and plant photosynthesis carbon sequestration into consideration. However, except for Tibet, the errors between the calculated carbon sequestration and the real data in other provinces are within an acceptable range, which shows that the model is universal and reasonable to a certain extent.

Then we classified different tree species, and the carbon sequestration amount of various trees in China in 2016 is shown in the following table:

**Table 2: The main trees Carbon stock in 2016**

Native Tree	Forest Carbon (million tons)	Introduced tree	Forest Carbon (million tons)
Picea	580.712	Pinus	329.50
Larix	670.45	Betula	550.98
Abies	791.09	Populus	419.23
Eucalyptus	128.74	Quercus	827.84
Remaining	4428.15	Remaining	279.68

## 4 Forest Management Model

The forest management plan corresponding to the carbon sequestration model can obtain the maximum amount of carbon dioxide sequestration, but this is not necessarily the one that is best for society given the other ways that forests are valued. Therefore, for the sustainable development of forests, we have developed a forest management model, which takes into account the five indicators of forest biodiversity, productive capacity, health-vitality, carbon sequestration, and economic benefits.<sup>[1]</sup> In the model, we calculate the scores corresponding to each indicator, and then provide the corresponding forest management methods according to the scores.

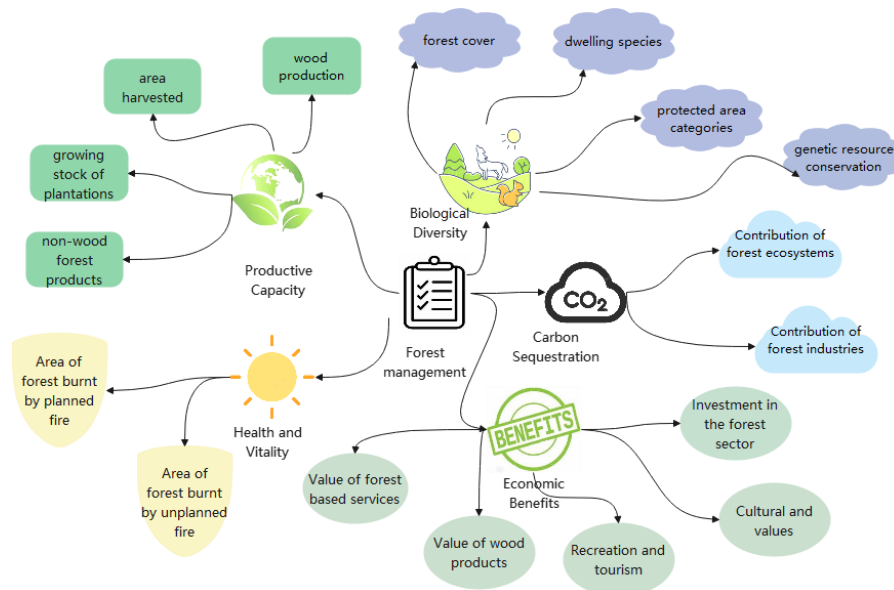


Figure 5: Indicator Frame Diagram

### 4.1 Indicators

#### ● Forest Cover

Biodiversity is one of the most important factors closely related to the sustainable development of society. Forests play an important role in maintaining global, national, and local biodiversity. By comparing the endangered species data with the spatial distribution of existing species globally, it can be found that there is a high correlation between the number of threatened species and forest cover, and forest degradation is the biggest threat to biodiversity.

#### ● Forest Growing Stock

The productivity of forest refers to the number of organics accumulated by green plants per unit area, which represents the amount of carbon entering the vegetation from the air, and the productivity directly determines the value of forest resources.<sup>[2]</sup> The stock of growing trees refers to the total stock of all trees on the land within a certain range, including forest stock, opening forest stock, scattered wood stock, and surrounding trees stock. Timber stock is an important indicator reflecting the productivity of a country or region, and it changes regularly with different tree species and site conditions.

#### ● Fire Rates

The health and vitality of forests face a series of disturbances that affect the composition,



structure, and function of forests, and these disturbances are also largely affected by climate. The increase in the total amount of combustibles caused by climate change, the extension of the fire season, and the extreme weather conditions will lead to higher fire rates. Although some forest ecosystems rely on fire for regeneration, forest fires can cause devastating damage to some sensitive forest ecosystems, as well as loss of property and life. Therefore, when assessing the health and vitality of forests, we focus on the incidence and behavior of forest fires.

- **Carbon Sequestration**

Carbon sequestration refers to a process by which forests absorb and store carbon dioxide in the atmosphere to reduce greenhouse gases. Compared with other terrestrial ecosystems, forests can store more carbon as carbon sinks and can accumulate organic carbon compounds for a long time. Nearly one-third of the world's land is covered by forests, far less than it was before human disturbances and swathes of forests in Europe, Asia, Africa, and the Americas have disappeared from the earth. If we continue to destroy forests at the current rate—about 7,000 square kilometers of forest in the Amazon basin are lost from the earth every year—in three or four decades, we will suffer enormous losses. This will exacerbate global warming, and all efforts to reduce emissions will be in vain.

- **Gross output value of forestry**

Gross forestry output value is the value of products (or operations) produced by the forest industry in a certain period calculated and expressed in monetary units, and usually includes three items: gross industrial output value, industrial commodity output value, and industrial net output value. Among them, there are mainly wood and various wood processing products, economic forest, and forest by-products. The economic forest is a forest whose main purpose is to directly obtain economic benefits from the production of wood or other forest products, as opposed to shelter forest. Therefore, to measure the total output value of forests, it is mainly to measure the economic profits brought by timber and economic forests.

- **Forestry Tourism**

Forest tourism refers to tourism activities such as sightseeing, leisure and vacation carried out by people relying on forest resources and their external environment. The tourism value of forest resources is high, whether it is used for sightseeing, or for developing forestry to satisfy the local people, it has evolved into an economic and social behavior that takes both ecological consumption and ecological production into account. For managers, the development and utilization of forest should be carried out within the capacity of forest resources, to eliminate short-term economic behavior, and to seek sustainable and coordinated development.

## **4.2 Model Establishment**

Based on the above indicators, we collected a relatively small sample of data. Since the system information is partially known, we establish a gray system, and through the existing data, we establish a gray correlation analysis model to evaluate each case. Among them, there are evaluation indicators, and the evaluation object is the data from 2015 to 2020, with a total of  $m$  years of data.

### **4.2.1 Gray Correlation Analysis Model**

#### **Step1 Data processing (normalization)**

Normalize the data of each indicator in each year

- For benefit indicators such as Forest Cover, Growing Stock Volume, Forest Growing Stock, Carbon Sequestration, this method of normalization is used:

$$b_{ij} = \frac{a_{ij} - a_{\min j}}{a_{\max j} - a_{\min j}} \quad (9)$$

- For cost indicators such as fire rates, this method of normalization is used:

$$b_{ij} = \frac{a_{\max j} - a_{ij}}{a_{\max j} - a_{\min j}} \quad (10)$$

- Whether they are benefit indicators or cost indicators, this method of normalization can be used:

$$b_{ij} = a_{ij} / \sqrt{\sum_{i=1}^m a_{ij}^2} \quad (i = 1, \dots, m; j = 1 \dots n) \quad (11)$$

### Step2 Select reference sequence

For the processed data, we select the reference sequence  $x_0 = \{x_0(k) | k = 1, 2, \dots, n\}$ , where  $x_0(k)$  is the optimal value in all data for the  $k^{th}$  indicator. Actually, after normalization, each value of the reference sequence is 1, that is,  $x_0(k) = 1$ . Comparison sequences are  $x_i = \{x_i(k) | k = 1, 2, \dots, n\}$  ( $i = 1, 2, \dots, m$ ), which is  $i^{th}$  year's data.

### Step3 Determine the weight of each indicator

To reduce the influence of subjective factors, we use the combined weight method to determine the corresponding weight of each indicator (combined weight method is detailed below), and get the weight  $w = [w_1, \dots, w_n]$ , where  $w_k$  ( $k = 1, 2, \dots, n$ ) is the  $k^{th}$  indicator's weight.

### Step4 Calculate the grey correlation coefficient

Calculate the correlation coefficient of the  $k^{th}$  indicator between the comparison sequence  $x_i$  and reference sequence  $x_0$ .

$$\xi_i(k) = \frac{\min_s \min_t |x_0(t) - x_s(t)| + \rho \max_s \max_t |x_0(t) - x_s(t)|}{|x_0(k) - x_i(k)| + \rho \max_s \max_t |x_0(t) - x_s(t)|} \quad (12)$$

Where

- $\rho \in (0, 1)$  is the resolution factor, which is taken as 0.5 in our model, the resolution is moderate.
- $\min_s \min_t |x_0(t) - x_s(t)|$  is two-stage minimum difference,  $\max_s \max_t |x_0(t) - x_s(t)|$  two-stage maximum difference.

### Step5 Calculate grey weighted correlation degree

$$r_i = \sum_{k=1}^n w_k \xi_i(k) \quad (13)$$

Where,  $r_i$  is grey weighted correlation degree of the  $i^{th}$  evaluation object to the ideal object, that is, the weighted score of the forest condition in the  $i^{th}$  year.

## 4.2.2 Combined weight method

To combine the impact of subjective and objective evaluation, we use the combination of

the Entropy weight method (EWM) and subjective weight method (AHP) to determine the weight of indicators.

$$w_k = \frac{\sqrt{\alpha_k \beta_k}}{\sum_{k=1}^m \sqrt{\alpha_k \beta_k}} \quad (14)$$

Where

- $\alpha_k$  is the weight calculated by the subjective method.
- $\beta_k$  is the weight calculated by the objective method.

$$p_{ik} = \frac{x_i(k)}{\sum_{i=1}^n x_i(k)} \quad (15)$$

$$E_k = - \frac{\sum_{k=1}^m p_{ik} \ln p_{ik}}{\ln m} \quad (16)$$

$$\beta_k = \frac{1 - E_k}{k - \sum E_k} \quad (17)$$

Where  $x_i(k)$  is the  $k^{th}$  indicator of the  $i^{th}$  year,  $E_k$  is the information entropy of the  $k^{th}$  index.

Through the improved grey correlation analysis based on the combined weights method, we find the degree of correlation between a particular scheme and the optimal scheme, and thus score the forest by criteria formulated by each indicator. Based on the score of the forest, we can give reasonable management suggestions for the forest and propose a reasonable management plan.

### 4.3 Problems Solve

- What is the spectrum of management plans that your decision model may suggest?

In the decision model, we propose a management plan to realize the balanced development of forestry, fully coordinate the relationship between forestry resources, economic, and ecosystems in each region, and promote the harmonious development of forestry. The scope is forest biodiversity, productive capacity, health vitality, carbon sequestration, and economic benefits.

- Are there any conditions that would result in a forest that should be left uncut?

There are no conditions that would leave the forest uncut. The management of forests is to achieve benefits, including ecological benefits and economic benefits.<sup>[3]</sup> In terms of ecological benefits, continuous cutting can form young forests, which can achieve higher carbon sequestration than over-mature forests, promote tree growth and improve land utilization. As for economic benefits, some trees have extremely high economic value, and the rich forest products and forest by-products are used in construction, papermaking, and other industries. Therefore, it is necessary to cut the trees for the sustainable development of the forests.

- Are there transition points between management plans that apply to all forests? How are

characteristics of a specific forest used to determine transition points between management plans?

All forest management plans have transition points. After applying the decision model to the forest to be tested, managers can make a corresponding management plan based on the changes of indicators over time and the changes in the forest score each year. There is a certain transition period between target management plans, related to specific forest species and locations. According to the score obtained by different forests, we use the mathematical programming model to determine the adjustment target of each indicator based on the existing basis, and take the corresponding longest transition time of the indicator as the transition time for the realization of the overall target. The mathematical programming equation is as follows:

$$\begin{aligned} & \min \max \Delta t_i \quad (i = 1, 2, \dots, k) \\ & \begin{cases} \frac{F_i(t + \Delta t_i)}{F_i(t)} = 1 + x\% \quad (i = 1, 2, \dots, k) \\ t \geq t_0 \\ 0 < \Delta t_i \leq E_{\max \Delta t} \end{cases} \end{aligned} \quad (18)$$

Among them,  $F_i$  represents the grey relational degree.  $\Delta t_i$  represents the time it takes for the  $i$  indicator to reach the adjustment target we set.  $x\%$  represents the adjustment scale we set, namely the transition point. The meanings of  $t$  and  $t_0$  are the same as above, representing the current year and the year when the model was applied.  $E_{\max \Delta t}$  is the maximum adjustment time we expect, i.e. we want the transition time not to exceed the maximum value.

#### 4.4 Carbon Sequestration Forecast

Based on the management options determined by the decision model, we can predict the amount of forest carbon sequestration in the future years or even decades. The original data constitutes a gray system, part of the information is known, and the original time series implies a change rule. Since the management plan is time-sensitive, we use the data on the carbon sequestration of forests and their products from 2006 to 2020 to predict the carbon sequestration of forests and their products within 100 years through GM(1,1).

**Step1** Calculate a new sequence based on the original sequence

The original data sequence is  $x^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\}$ . Through the method of

$$x^{(1)}(k) = \sum_{i=1}^k x^{(0)}(i) \quad (1 \leq k \leq n), \text{ we obtain cumulative sequence } x^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n)\}.$$

**Step2** establish grey differential equations

$$x^{(0)}(k) + az^{(1)}(k) = b \quad (k = 2, 3, \dots, n) \quad (19)$$

its corresponding whitening differential equation is

$$\frac{dx^{(t)}}{dt} + ax^{(1)}(t) = b \quad (20)$$

The solution of  $u = [a, b]^T$  is described in step 4.

**Step3** Construct the data matrix  $B$  and data vector  $Y$

$$B = \begin{bmatrix} -\frac{1}{2}(x^{(1)}(1) + x^{(1)}(2)) & 1 \\ -\frac{1}{2}(x^{(1)}(2) + x^{(1)}(3)) & 1 \\ \dots & \dots \\ -\frac{1}{2}(x^{(1)}(n-1) + x^{(1)}(n)) & 1 \end{bmatrix}, \quad Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \dots \\ x^{(0)}(n) \end{bmatrix} \quad (21)$$

**Step4** Calculate  $\hat{u}$

$$\hat{u} = \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = (B^T B)^{-1} B^T Y \quad (22)$$

**Step5** Generate Sequence Predictors

Based on formula(20), we deduce the formula as follow,

$$\hat{x}^{(1)}(k+1) = \left( x^{(0)}(1) - \frac{\hat{b}}{\hat{a}} \right) e^{-ak} + \frac{\hat{b}}{\hat{a}} \quad (23)$$

Based on this formula, we obtained the generated sequence value  $\hat{x}^{(1)} = \{\hat{x}^{(1)}(k+1) | k \geq 0\}$

**Step6** Calculate the model recovery value

$$\hat{x}^{(0)}(k) = \begin{cases} \hat{x}^{(1)}(1) & , k = 0 \\ \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) & , k = 1, 2, \dots \end{cases} \quad (24)$$

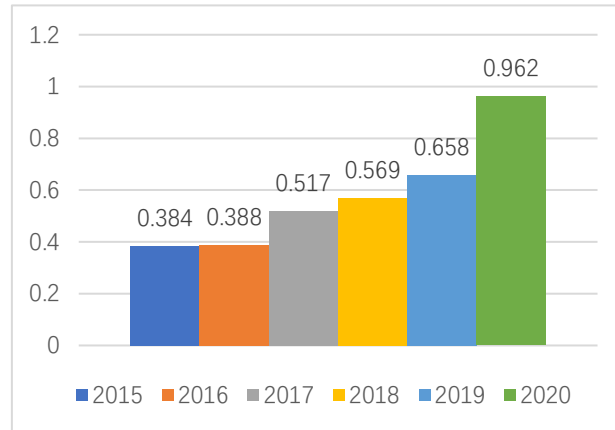
Where  $\hat{x}^{(0)} = \{\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \dots, \hat{x}^{(0)}(n)\}$  is the predictive sequence.

## 5 Case Study: Greater Khingan Range

The Greater Khingan Range is located in Heilongjiang Province, China, with a total length of more than 1,400 kilometers, an average width of about 200 kilometers, an altitude of 1100-1400 meters, and a total area of 327,200 square kilometers. It is cold in winter and warm in summer, with a large temperature difference between day and night. The average and lowest temperature there over the year is -2.8 °C and -52.3 °C respectively, which belongs to the cold temperate continental monsoon climate. It is well-preserved and has the largest square among the primeval forests, and it is one of the important forestry bases in China.

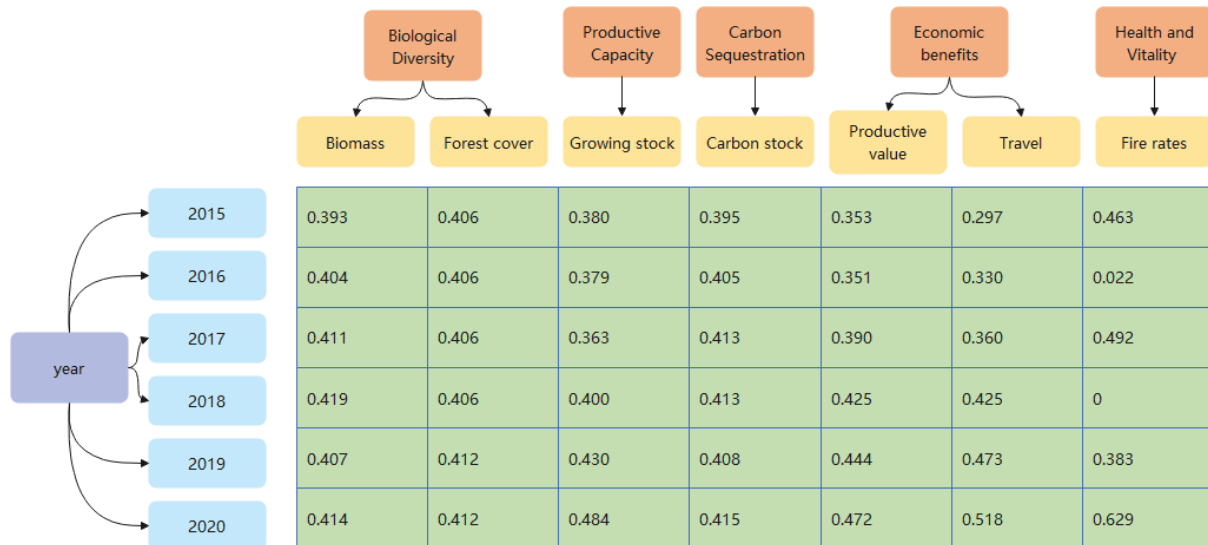
### 5.1 Forest Assessment

Through the grey relational analysis method in the decision-making model, we calculated the grey relational degree of each index in the Greater Khingan Range from 2015 to 2020. We obtain that the gray correlation degree of the Greater Khingan Range shows an upward trend year by year. The reason may be that the forest coverage area of the Greater Khingan Range has steadily increased, and indicators such as gross output value and biodiversity have also risen.



**Figure 6: Grey Relational Degree in Greater Khingan Range**

At the same time, we made a figure to show the degree of change of each indicator in the past six years. We found that the fire frequency in the Greater Khingan Range was random and did not show a stable trend in our model. After searching the information, this may be due to the frequent occurrence of thunder and lightning weather there, and the probability of lightning-caused fire is higher<sup>[4]</sup>. Therefore, we consider that there is a need to increase the priority of fire protection in the management plan for this forest. The remaining indicators are slowly increasing over time, so we can consider the remaining indicators according to the original management plan:



**Figure 7: Detailed Indicators Correlation**

In addition, by solving the mathematical programming equations in the decision model, we calculated the optimal transition point applicable to this forest: when the transition point is set to 17% of the current indicators, the time all the indicators reach the transition point was the shortest, at approximately 19.4 years, with the longest time required for fire protection to reach the transition point. This may be because the growth of fire protection capacity is not stable, and it may take a period of adjustment to gradually adapt:

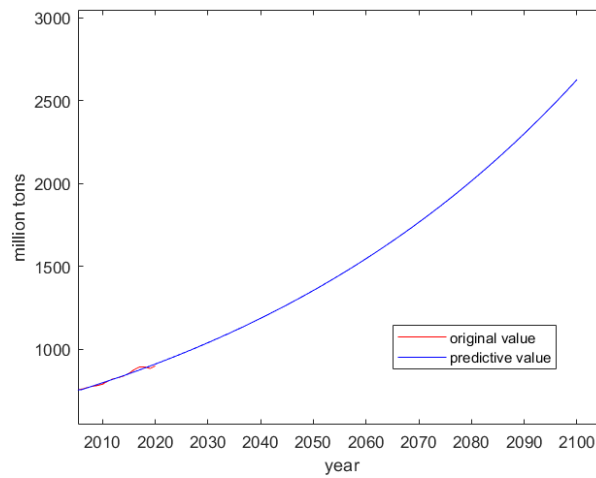
$$\text{the solution is: } x\% = 17\%, \text{ then } \min \max(\Delta t_i) = 19.4 \text{ year} \quad (25)$$

## 5.2 Prediction of carbon sequestration

To predict the total amount of carbon sequestration in the Greater Khingan Range and local products over 100 years, we calculated the carbon sequestration amount in the Greater Khingan Range from 2000 to 2020 under the decision model, and used the GM(1,1) model to predict the carbon sequestration amount from 2000 to 2100. The prediction equation of carbon sequestration from 2000 to 2100 is as follow:

$$\hat{x}^{(1)}(k+1) = \left( \hat{x}^{(0)}(1) - \frac{\hat{b}}{\hat{a}} \right) e^{-\hat{a}k} + \frac{\hat{b}}{\hat{a}} = 56904.93e^{0.01322k} - 56149.9 \quad (26)$$

According to the above equation, we have made a forecast of carbon sequestration in the Greater Khingan Range for 100 years since 2000, as shown in the following figure:



**Figure 8: Carbon Sequestration Forecast in Greater Khingan Range**

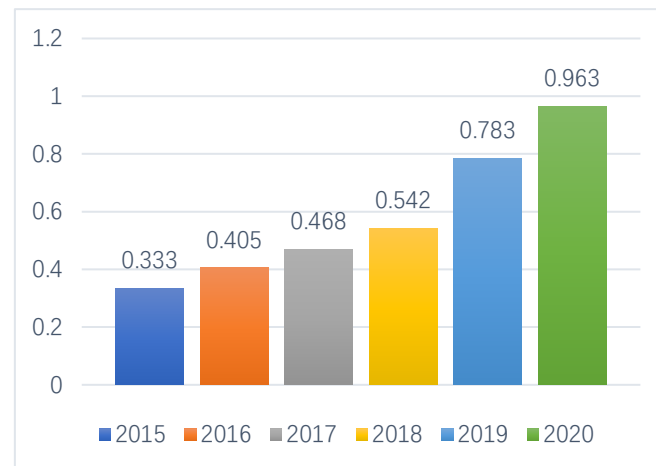
According to the calculation, the total amount of carbon sequestration in the Greater Khingan Mountains in 100 years has reached 143,735.3297 million tons.

## 6 Case Study: Xishuangbanna Tropical Rainforest

The Xishuangbanna Tropical Rainforest is located in the southern part of Yunnan Province, China, with a total area of 2420.2 square kilometers. Its tropical monsoon forest, subtropical evergreen broad-leaved forest, rare animal and plant species, and the entire forest ecology are invaluable, and it is the only preserved forest in the northern edge of the tropics in the world. The intact, contiguous large area of tropical forest has attracted much attention in the world.

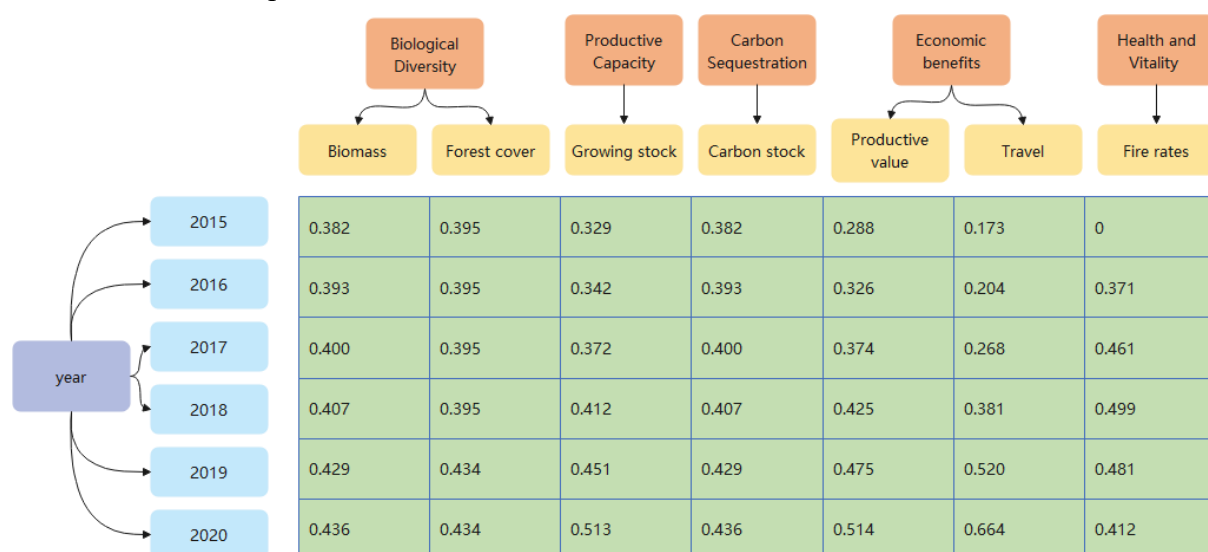
### 6.1 Forest Assessment

In the same way, we made a gray correlation analysis map of Xishuangbanna tropical rainforest from 2015 to 2020. Similar to the Greater Khingan Range, with the steady growth of local biomass and growing stock, the grey correlation degree in Xishuangbanna also increases steadily over time. Based on this analysis, the overall management plan of Xishuangbanna tropical rain forest is reasonable.



**Figure 9: Grey Relational Degree in Xishuangbanna Tropical Rainforest**

Correspondingly, we also made a schematic diagram of the correlation coefficients corresponding to each indicator each year. We found that, compared with Greater Khingan Range, Xishuangbanna's fire protection capacity is stronger, and the degree of change is relatively stable. This may be due to the vigorous development of tourism in the tropical rain forests of Xishuangbanna, so in order to ensure the safety of tourists, the local area pays more attention to safety factors such as fire protection<sup>[5]</sup>. Therefore, in terms of forest management, we can consider reducing its priority slightly, and adjust the priority more to other factors such as biomass and carbon sequestration::



**Figure 10: Carbon Sequestration Forecast in Xishuangbanna Tropical Rainforest**

In addition, by solving the mathematical programming equations in the decision model, we also calculated the optimal transition point applicable to the forest: when the transition point is set to 12% of the current indicators, the time all the indicators reach the transition point is the shortest, about 9.7 years. This may be because Xishuangbanna itself has a large forest area, high forestry output value, and large number of tourists, so it has better adaptability to the new management plan and can quickly reach the transition point we set.



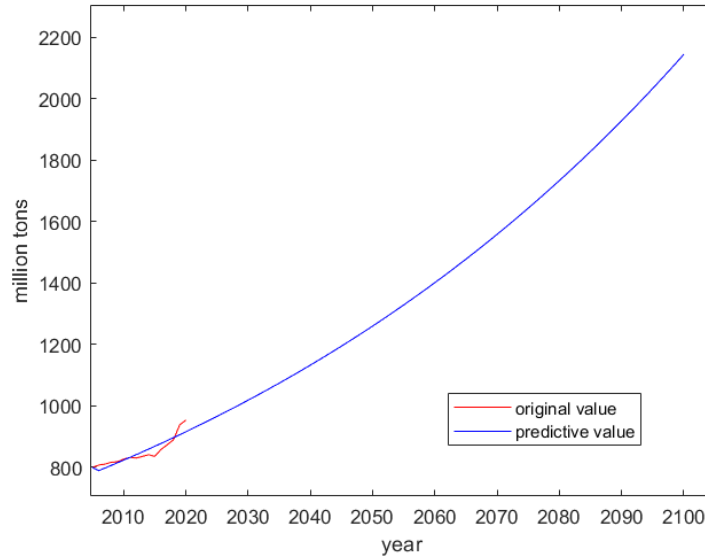
the solution is:  $x\% = 12\%$ , then  $\min \max(\Delta t_i) = 9.7 \text{ year}$  (27)

## 6.2 Prediction of carbon sequestration

Similarly, in order to predict the total carbon sequestration of Xishuangbanna rainforest and local products over 100 years, we calculated the carbon sequestration of Xishuangbanna rainforest from 2000 to 2020 under the decision model, and used GM (1,1) model to predict the carbon sequestration from 2000 to 2100. The prediction equation of carbon sequestration is as follow:

$$\hat{x}^{(1)}(k+1) = \left( \hat{x}^{(0)}(1) - \frac{\hat{b}}{\hat{a}} \right) e^{-\hat{a}k} + \frac{\hat{b}}{\hat{a}} = 73591.87e^{0.01065k} - 72793.9 \quad (28)$$

According to the above equation, we have made a forecast of carbon sequestration in Xishuangbanna tropical rainforest from 2000 to 2100, as shown in the following figure:



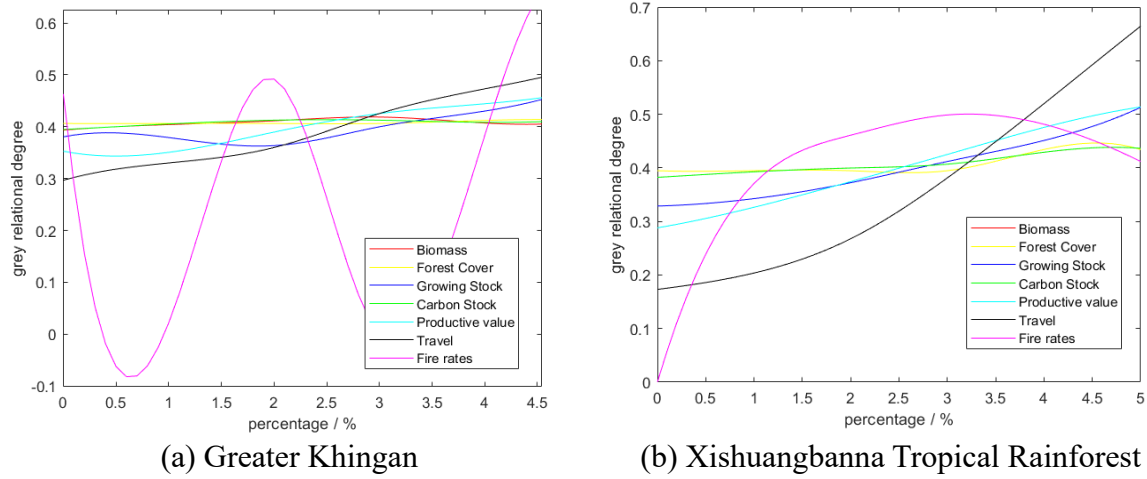
**Figure 11: Carbon Sequestration Forecast in Xishuangbanna Tropical Rainforest**

According to calculations, the total amount of carbon sequestration in Xishuangbanna's tropical rainforest in 100 years has reached 129,590.2157 million tons.

## 7 Discussion

### 7.1 Sensitivity Analysis

By changing the expansion ratio of a certain parameter, while keeping the values of other parameters unchanged, we have drawn the sensitivity analysis diagrams of the indicators of the Greater Khingan and Xishuangbanna Tropical Rainforests as shown below:



**Figure 12: Sensitivity Analysis**

For the Greater Khingan Mountains, with the increase of the expansion ratio, the sensitivity of fire in our model is greater, and even a negative correlation appears. This shows that compared with other indicators, the fire rates are more unstable, while the upward trend of other indicators is obviously more stable.

For the Xishuangbanna tropical rainforest, the increasing trend of tourism with the expansion ratio is more obvious. It shows this factor is more sensitive than other indicators due to the influence of tourism development in the region. The fire factor is more sensitive in the early stage, and gradually tends to be stable in the middle and late stage.

## 7.2 Strategy for the new timeline

After applying the decision model, we consider strategies for transitioning from the existing timeline to the new timeline when the harvest time is 10 years longer than the current practice. Assuming that the current harvest interval for the  $i$  tree is  $x_i$  years and the demand is  $sum_{i \text{ original}}$ , the new timeline corresponds to a harvest interval of  $x_i + 10$  years and the demand is  $sum_{i \text{ original}}$ . Assuming that our model does not account for the influence of market demand, the demand for the new harvest time should satisfy the following equation:

$$\frac{sum_{i \text{ now}}}{sum_{i \text{ original}}} = \frac{x_i + 10}{x_i} \quad (29)$$

Now we consider market demand. Let the market demand factor of the first type of  $i$  trees be  $demand_i$ , which is related to the social demand. Generally speaking, with the increase of interval time and the increase of social demand, the demand factor  $demand_i$  also shows a gradual upward trend. We believe that the demand in the new timeline is proportional to the demand factor, and the demand formula of the new timeline after improvement is:

$$sum_{i \text{ now}} = sum_{i \text{ original}} \cdot \frac{x_i + 10}{x_i} \cdot demand_i \quad (30)$$

We selected some local species in the Greater Khingan, substituted the existing data into the above formula, and also calculated the demand in the new timeline. A comparison chart of

the existing timeline and the new timeline is shown below:

**Table 1: Timeline of Greater Khingan**

Tree Species	Felling Age	Demand(per tree)	Tree Species	Felling Age	Demand(per tree)
Birch	423	1634	Birch	433	2175
Larch	316	3581	Larch	326	3864
Willow	72	2756	Willow	82	4071
pine	538	895	pine	548	763

By comparing the timelines, we found that willows with a shorter growth cycle had the largest increase in demand after a 10-year increase in the harvest interval. This may be because trees with shorter growth cycles are more suitable for felling to make up for the demand of people when the harvest interval is longer. In addition, the growth cycle of willow trees is short, and the local demand for willow trees is relatively high. Therefore, our suggestion for the transition from the existing timeline to the new timeline for the Greater Khingan is to give priority to increasing the harvest of trees with short growth cycles, and then consider trees with large values of demand factors  $demand_i$  on this basis.

Similarly, we substitute the data of various types of trees in Xishuangbanna tropical rain forest into the equation in 5.3, and get the comparison chart between the existing timeline and the new timeline as shown below:

**Table 1: Timeline of Xishuangbanna tropical rain forest**

Tree Species	Felling Age	Demand(per tree)	Tree Species	Felling Age	Demand(per tree)
Birch	423	2859	Birch	433	3169
Larch	316	4756	Larch	326	4893
Willow	72	4279	Willow	82	5078
pine	538	1871	pine	548	1973

In the same way, it can be seen that our strategy for the Greater Khingan Mountains still applies here. Like the willow tree with the shortest growth cycle, we plan to increase its harvest to accommodate the one-time demand for willow trees.

## 8 Model Evaluation

### 8.1 Strengths

- ✧ Both the decision model based on the evaluation criteria and the carbon sequestration prediction model is related to the gray system, which belongs to the same system and has a strong correlation
- ✧ In order to make forest management recommendations, we have created several indicators. For each indicator, we found mass data, so the solution is more reasonable.

- ✧ According to carbon sequestration in various provinces across the country, the forest carbon sequestration calculated by our model is in good agreement with the actual data.
- ✧ In the prediction model, the prediction curve is highly consistent with the original data curve, which proves the validity of the prediction model.
- ✧ Our basic model has been tested by sensitivity analysis, and the error is acceptable, so the model is stable.

## **8.2 Weaknesses**

- ✧ In the prediction model, we did not take the limitation of forest area into account, and carbon sequestration may stop increasing with the limitation of forest area.
- ✧ The impact of cultural factors on forest management plans is ignored. Since cultural factors have more influence on areas of high population density while population densities in forests are lower, we suppose that cultural factors have less influence on forest values.

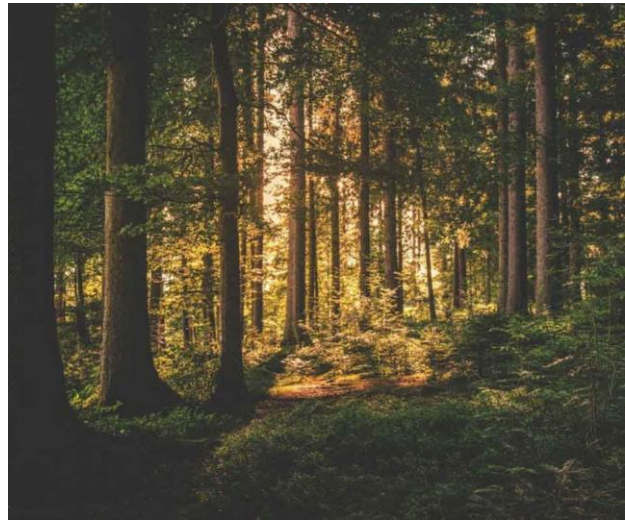
# Leaving forest uncut is to destroy our community

Edited by Team 2208919 on February 17, 2022

When it comes to forests, we all know that they play an extremely significant role in climate change, but sometimes the appropriate deforestation can bring more benefits.

People often get into a mindset that since forest ecosystems are the lungs of the planet, leaving forests untouched is the best way for both humans and the environment. But this idea is very lopsided, and does not take into account the growth habits of trees and the sustainable development of society.

In terms of ecology, continuous cutting and replanting not only can adjust the growth structure of forests but also achieve higher ecological benefits. Humans are mortal. Similarly, trees have their growth cycles. When they reach the end of their lives, the ability of over-mature trees to absorb carbon dioxide is far inferior to that of young trees. However, the space in the forest is limited which will lead to the "aging" of the forest causing worse ecological capacity. But cutting down trees at the right time can help create young thick forests with higher carbon dioxide uptake, which is beneficial to climate change.



Moreover, Products made from forests can replace plastics, be biodegraded, and return to nature again. As for plastics, it will take at least decades or hundreds of years to be degraded into environmentally harmless fragments and then return to the natural cycle.

In terms of social development, deforestation can increase the productivity of forests and provide employment opportunities, which brings more economic benefits for us. Some trees have extremely high economic value and produce abundant forest products or forest by-products, which are widely used in construction, papermaking, and other industries. Moreover, with the in-depth development of society, replanting after cutting can adjust the production pattern of forestry, provide space and support for new industries, resolve the disadvantages of unscientific forestry structure. At the same time, since the COVID-19 outbreak, the employment rate has continued to decline, which has brought huge panic and anxiety to people. However, from production to sales, deforestation has provided a large number of jobs, which relieves people's mental and life stress under the epidemic.

In general, the reason why people hold a negative attitude towards felling is that there are doubts about whether environmental protection and economic development can coexist. But everything needs to be viewed dialectically. As long as an appropriate method can be found, there would be no conflict between the economy and the environment. Of course, how to find the right harvest time and method to cut trees to realize the sustainable development of society still requires our further exploration and efforts.

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