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PHYSICS LABORATORY II EXPERIMENT BOOK

ELECTRICITY AND MAGNETISM

2024

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1. CHAPTER

1.1. ERROR CALCULATION

PHYSICS MEASUREMENTS AND ERRORS

Every measurement includes errors in the limits you accept that are not in your significant figures. You must obey and calculate your errors and their limits to achieve reliable measurements. Otherwise, your results do not show the correct values you obtained. Numerical results from experiments are meaningless unless measurement errors are identified. That is, for each measured result, the reliability limits of that result, i.e. the error limits, must be specified. The main purpose of laboratory studies is not the measurement of physical constants or the comprehensive statistical analysis of data. However, it is useful to determine the limits within which the measurement results are reliable. For this purpose, some practical information on error detection is presented below.

There are two types of errors: **Systematic** and **Statistical** errors;

i) Systematic Errors

As the name implies, this type of error is a constant error inherent in the system itself and always affects the result in the same direction. For example, if weights are measured with a weight that weighs more than 1 kg, the measurement result will be smaller by the same amount. In the presence of such errors, the errors are unidirectional; they are either consistently larger or consistently smaller. Systematic errors can be eliminated by the following methods;

1. Necessary corrections are made from measurement results.
2. By correcting the error in the measurement system.
3. By changing the measurement method.

ii) Statistical Errors

In physics, these are non-significant, usually small, bidirectional errors caused by the inherent limitation of measurement precision or by instability in the object being measured or in the measurement system. The presence of such errors can be seen by repeating the same measurement many times. The measured results differ from each other and are distributed around a certain value. These errors cannot be eliminated from the measurement results, but it is possible to approximate the margin of error and the limits within which the measured quantity is reliable. The influence of such errors on the measurement results can be reduced by multiple repetitions of the same measurement and statistical evaluation of the results.

When a physical quantity x is measured N times, let the measurement results by x_1 $x_2, x_3, \dots . x_N,$. In this case the mean value of x is given below.

$$\bar{x} = \frac{(x_1 + x_2 + x_3 + \cdots + x_N)}{N} \tag{1.1}$$

The value of $\bar{x}$ is the most approximated value of x. Hence, if a quantity is measured N times, we can take its average value as the measurement result. Although the reliability of the measurement result increases in proportion to the number of measurements N, we must be satisfied with a practical number of repetitions in experiments.

What is the error in the mean value of x? To determine this, we can make use of the distribution table we call a “histogram” as shown in the table below. For example, let's repeat an experiment in which we measure time 18 times and let the measurement results be as follows in Table 1.1;

Table 1.1: A time measurement as an example

Number of measurements	Time (s)
1	86.2
2	86.5
3	86.4
4	86.5
5	86.7
6	86.6
7	86.6
8	85.6
9	86.4

Number of measurements	Time (s)
10	86.3
11	86.5
12	86.5
13	86.4
14	86.6
15	86.3
16	86.7
17	86.4
18	86.6

When these results are analyzed, it can be seen that 86.7 is measured 2 times, 86.6 5 times, 86.5 is 4 times, 86.4 is 4 times, 86.3 is 2 times, and 86.2 is 1 time. If we show on a graph how many times this value is measured against the measured value, we get a histogram or frequency distribution curve as shown in the Figure 1.1,

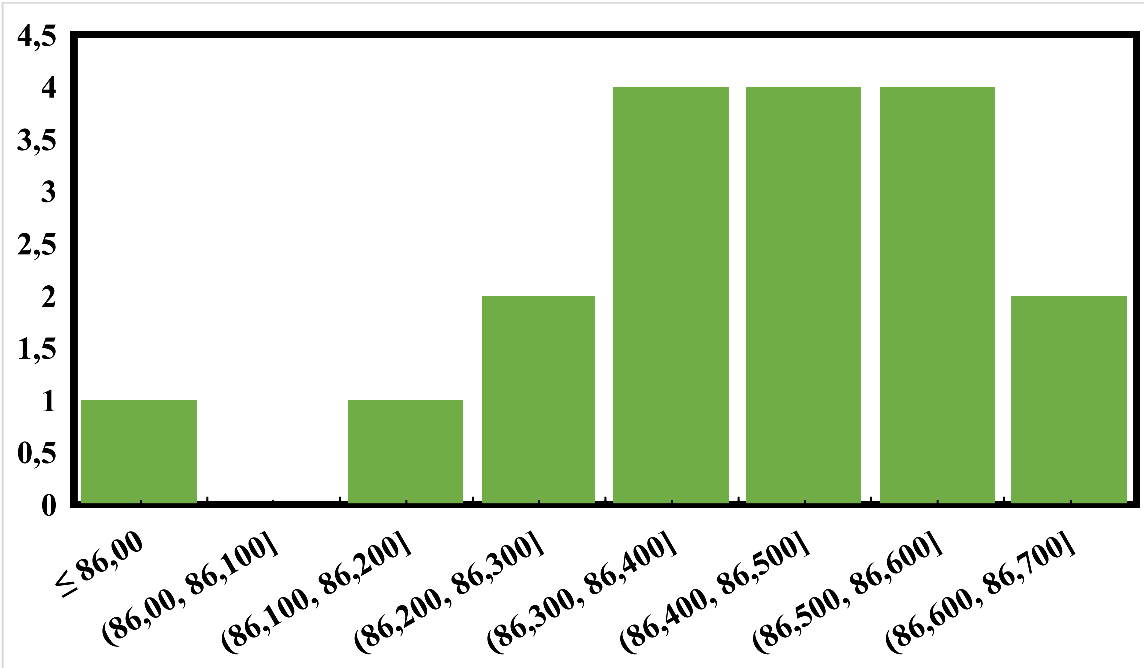


Figure 1.1: Histogram graph of measured time datas

It is necessary to know how reliable the $\bar{x}$ value obtained with the (1.1) equation is. For this example, it is $\bar{x} \approx 86.5$. (The "Standard Deviation" method, which will be explained later, can also be used instead of the equation (1.1) to obtain $\bar{x}$). A common method for determining errors is to determine the mean deviation. For example, the deviation in measurement x_i;

$$d_i = x_i - \bar{x}_i \tag{1.2}$$

and mean deviation;

$$\bar{d} = \frac{(d_1 + d_2 + d_3 + \cdots + d_N)}{N} \tag{1.3}$$

is calculated. This relation gives the mean deviation from $\bar{x}$ and can be taken as *the mean statistical error*. Returning to the previous experiment with 18 measurements, we find $\bar{d} \approx 0.1$ s using relation (1.3). The measurement result of x is $x = \bar{x} \mp \bar{d}$, for our example $x = 86.5 \mp 0.1$ s. In some cases, errors are given as a percentage of errors. Since the error percentage in this case would be $\frac{\bar{d}}{\bar{x}} x100$, the error percentage for the given example is $\frac{0.1}{86.5} x100$ and hence the measurement result is $x = 86.5 \mp \%0.1$ s. As can be seen from these examples, the deviation from the mean value for N times measurement can be a criterion for determining the precision of the measured value. However, this deviation is not the true error. It should only be considered as an approach for determining statistical error. In laboratory work, the student may take $\bar{d}$ as an error even though it is a deviation from the mean value. If $\bar{d}$ is small in a series of measurements, it indicates that $\bar{x}$ has been measured accurately, if it is large, it indicates that $\bar{d}$ has been measured less accurately. Therefore, standard deviation is only one criterion for determining the magnitude of statistical error. Since the significance of the mean value and the deviation will be proportional to the size of N, in student laboratory work, a practical approach should be used to determine the number of measurements, N. Another method frequently used for determining statistical errors is the 'Standard Deviation' method. The standard deviation is σ.

$$\sigma = \sqrt{\frac{({d_1}^2 + {d_2}^2 + {d_3}^2 + \cdots + {d_N}^2)}{N^2}} \tag{1.4}$$

In analyzing student experiment results, either $\bar{d}$ or σ can be used. The choice of σ indicates that greater importance is given to larger deviations. Standard deviation is a simple approach for determining within what limits repeated measurement results may vary. If the distribution follows a Gaussian curve, the results will, with 85% probability, fall within the range of the mean value $\mp$. When it is not possible to repeat measurements many times, when there is suspicion of systematic error, or when imprecise measuring instruments are used, the most appropriate way to determine measurement errors is to take the maximum possible error value. For example, for a length x measured with a meter with the smallest division of 1 mm, the maximum possible error would be $\Delta x = 1\ mm$ mm. In this case, the true value of a measured length x will vary between $x - \Delta x$ and $x + \Delta x$. Measurements are often not directly possible. Instead, other values are measured, and the physical quantity of interest is calculated. Then, it is necessary to determine the combined effect of the error contributions from measuring different quantities on the final result. In these cases, let's briefly review the methods used for calculating errors. Let's assume that the physical quantity r given by the relation $r = f(x, y, z)$ will be calculated from the measurements of the $x, y\ and\ z$ quantities. If the maximum possible errors in the measurements of $x, y\ and\ z$ are $\Delta x, \Delta y\ and\ \Delta z$ respectively, then the effect of these values on the variation of r will be as follows.

$$\begin{aligned}\Delta r = |f(x + \Delta x, y, z) - f(x, y, z)| + |f(x, y + \Delta y, z) - f(x, y, z)| \\ + |f(x, y, z + \Delta z) - f(x, y, z)|\end{aligned} \tag{1.5}$$

This impractical expression can actually be written in terms of partial derivatives;

$$\Delta r = \left|\frac{\partial f}{\partial x}\Delta x\right| + \left|\frac{\partial f}{\partial y}\Delta y\right| + \left|\frac{\partial f}{\partial z}\Delta z\right| \tag{1.6}$$

Let's examine some examples related to the application of the above expression.

a) Rule 1: Addition Operation

If $r = x + y$, Δr is found as $\Delta r = |\Delta x| + |\Delta y| = \Delta x + \Delta y$. The error in the addition is equal to the sum of the errors.

b) Rule 2: Subtraction Operation

If $r = x - y$, Δr is found as $\Delta r = |\Delta x| + |-\Delta y| = \Delta x + \Delta y$. The error in the subtraction is equal to the sum of the errors.

c) Rule 3: Multiplication Operation

If $r = x.y$, Δr is found as $\Delta r = |y\Delta x| + |x\Delta y| = y\Delta x + x\Delta y$. When both sides of the equation are divided by $r = x.y$, ${}^{\Delta r}/_{r} = {}^{\Delta x}/_{x} + {}^{\Delta y}/_{y}$ is obtained. The error in r is equal to the sum of the error ratios in x and y.

d) Rule 4: Division Operation

If $r = x/y$, Δr is found $\Delta r = |\Delta x/y| + |\Delta y/x| = \Delta x/y + \Delta y/x$. When both sides of the equation are divided by $r = x/y$, ${}^{\Delta r}/_{r} = {}^{\Delta x}/_{x} + {}^{\Delta y}/_{y}$ is obtained. The error in r is equal to the sum of the error ratios in x and y.

e) Rule 5: Exponential Functions

If $r = x^n$ where n is any natural number, Δr is found $\Delta r = nx^{n-1}\Delta x$. When both sides of the equation are divided by $r = x^n$, ${}^{\Delta r}/_{r} = n({}^{\Delta x}/_{x})$ is obtained. The error in r is equal to the sum of the error ratios in x and y. For the nth power of x, the error ratio is n times the error ratio of x.

f) Rule 6: Trigonometric functions

If $r = \sin x$, Δr is found $\Delta r = |\Delta x \cos x| = \Delta x \cos x$. In error calculation for trigonometric functions, perhaps the easiest method is to use a trigonometric table. For example,

If $x = 30° \mp 1°$, from $\Delta r = |\sin(x + \Delta x) - \sin(x)|$ relation, $\Delta r = \sin 31 - \sin 30 = 0.515 - 0.5 = 0.015 \approx 0.2$, and $r = 0.5 \mp 0.2$.

SIGNIFICANT FIGURES

Numerical results obtained from experiments must be consistent with the measurement precision. With a ruler that has divisions of 1 mm, measurements can only be made with a precision of up to 1 mm. For example, a measurement result like 32.2 cm can only vary between 32.15 cm and 32.25 cm. That is, the measurement precision is 1 mm. Reporting a measurement as 32.222 cm with the same ruler would be incorrect because such precision cannot be achieved with a ruler with 1 mm divisions.

In some cases, multiple quantities are measured with different precisions, and the experimental results are calculated accordingly. For example, consider the calculation of the area of a rectangle with sides A and B measured. Suppose side A is measured with a vernier caliper with a precision of 0.001 cm, and side B is measured with a ruler that has divisions of 1 mm. The values of A and B are found as follows.

$A = 5.34 \mp 0.001$ (includes 3 significant figures)

$B = 124.2 \mp 0.1$ cm (includes 4 significant figures)

The area of this rectangle cannot be taken as $AxB = 5.34\ cm\ x124.2\ cm = 663.228\ cm^2$. This result would be incorrect because a size measurement result with significant figures of 3 gives an area with significant figures of 6. A general rule is that results cannot be determined with more significant figures than the least significant figures of measurements. For this example, side A is at most 3 reliable numbers and should be taken for multiplication as follows.

$s = AxB = 663\ cm^2$

The error in the product AxB is easily determined by looking at the measurement errors in A and B. The measurement error of A will be about 0.2% and the measurement error of B will be about 0.2%. $\Delta S = 663x0.4\% \approx 2.6\%$ and the result is $\Delta S = 663 \mp 2.6\ cm^2$.

What is important in experiments is to determine the error limits in the results. Significant figures can be easily determined from the results given above. For a measurement with 1% precision, the results must be 2 or 3 significant figures. Similarly, a measurement with an accuracy of 0.001% should be 5 or 6 significant figures.

1.2. GRAPHICS

DRAWING AND UTILIZING GRAPHS

Graphical representation of experimental results is widely used in almost every branch of science because it is practical and easy to understand. Graphs should give all kinds of information in a way that is understandable to everyone.

The following rules should be followed when drawing a graph:

1. The name and date of the graph should be written.

2. It should indicate which quantities the axes correspond to and what the units are.

3. All kinds of text and numbers should be placed in an easily readable way.

4. The unit lengths in the graph should be chosen in such a way that the graph should cover the whole paper.

5. After the data are marked as points on the graph, they should be circled, and an error line should be drawn in proportion to the measurement errors. If the error for a value of y is Δy, the error line should be $2\Delta y$.

In this section, we will only give an overview of linear graphs.

ANALYSIS OF LINEAR GRAPHS:

After all data and error magnitudes have been marked, the most approximate line through them is drawn. Criteria for drawing the most approximate line;

1. There should be as equal a number of data points as possible at the top and bottom of the line,

2. All data points must fall on the line,

3. The drawn line should intersect the error lines.

It is useful to draw with a transparent ruler for all these operations. An example graph is given in Figure 1.2.

The line is the graph of an expression of the form $y = mx + b$ expressed by two constants m and b. Once you have plotted your data (x_i, y_i) $(i = 1, 2, 3, \dots N)$, where N is the number of measurements, the next task is to calculate the constants m and b. Similarly, you need to know the errors Δm and Δb.

As you know, m is the slope of the line and b is the distance where the y-axis crosses the origin. The calculation of m is done by choosing two points on the drawn line that are far apart from each other, as shown in the example, and finding $m = (4.46 - 6.97)/(7.8 - 1.8) = -2.61/6.0 = -0.435\ {}^{m}/_{s^2}$ from the coordinates (1.8 , 6.97) and (7.8 , 4.36) as follows.

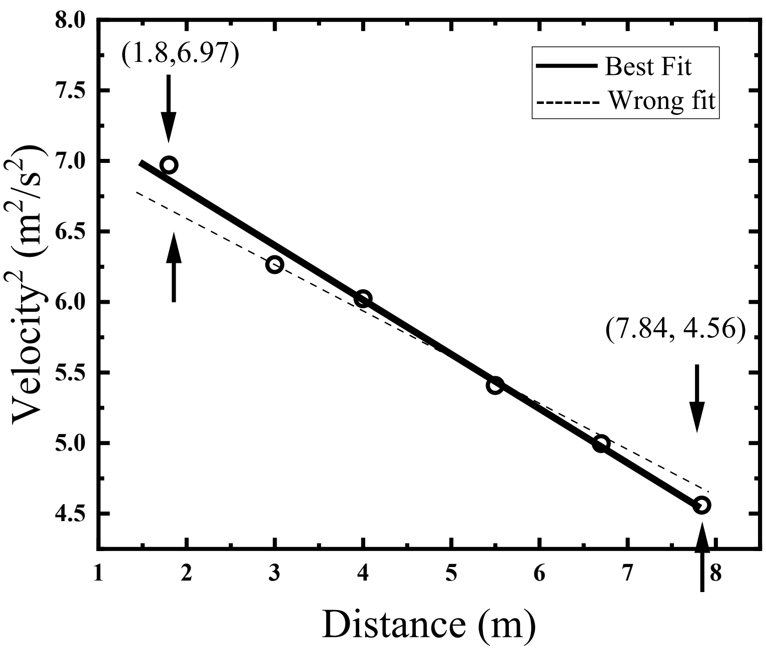


Figure 1.2: Variation of the square of the velocity with distance

The calculation of b from the origin may be slightly different as the horizontal axis does not always pass through zero. As can be seen in Figure 1.2, the horizontal axis should be offset from the origin by 1.8 m and since we know the slope, we have $b = 6.97 - (1.8x0.435) = 7.75\ {}^{m^2}/_{s^2}$.

The error rates in these two parameters can be calculated from the m' and b' values of the worst line that can be drawn. For the inaccurate line in the graph, we took as an example,

$$m' = \frac{4.56-6.73}{7.8-1.8} = -0.362\ {}^{m}/_{s^2}$$

$$b' = 6.73 + (1.8x0.362) = 7.38\ {}^{m^2}/_{s^2}$$

is found. And $\Delta m = |m - m'| = |0.435 - 0.362| = 0.07\ {}^{m}/_{s^2}$ and $\Delta b = |b - b'| = |7.75 - 7.38| = 0.4\ {}^{m^2}/_{s^2}$ is obtained. Constants $m = (-0.44 \mp 0.7)\ {}^{m}/_{s^2}$ and $b = (7.8 \mp 0.4)\ {}^{m^2}/_{s^2}$ are found as a result of graphical analysis. It should be noted that these errors are not real errors, but the maximum possible error value. In experiments, the resulting formulas are not always in the form of the typical line equation. For example, it can also be in the form of equations below.

$$v^2 = v_0^2 - 2ax$$

or

$$I = kv^n$$

In this case, the line equation $y = mx + b$ can be obtained by transferring to the new variables. Taking the first expression above as $v^2 = y$, $v_0^2 = b$, $2a = b$, or taking the logarithm of the second expression,

$$\log I = \log k + n \log v$$

$\log I = y$, $\log k = b$, $n = m$ and $\log v = x$linear graphs can be drawn.

2. CHAPTER

2.1. GENERAL INFORMATION ABOUT DIRECT CURRENT

CURRENT

Current is a charge moving from one region to another. In ordinary metals such as copper or aluminum, some electrons move freely in the conducting material. The movement of these free electrons is random in all directions and their speed is very high ($10^6\ m/s$). If there is a constant and uniform $\vec{E}$ electric field inside the conductor, the electric field does work on the moving charges. The resulting kinetic energy is transferred to the ions vibrating around the equilibrium position as a result of collisions with the ions, and the vibration energy of the ions and therefore the temperature of the conductor increases. In different current-carrying materials, the charges in motion can be positive or negative. In metals, the moving charges are always (negative) electrons. In semiconductors such as germanium and silicon, the conduction is partly electrons and partly vacancies, which act as positive charges due to the lack of electrons. Figure 2.1 (a,b) shows particles of matter carrying two different currents.

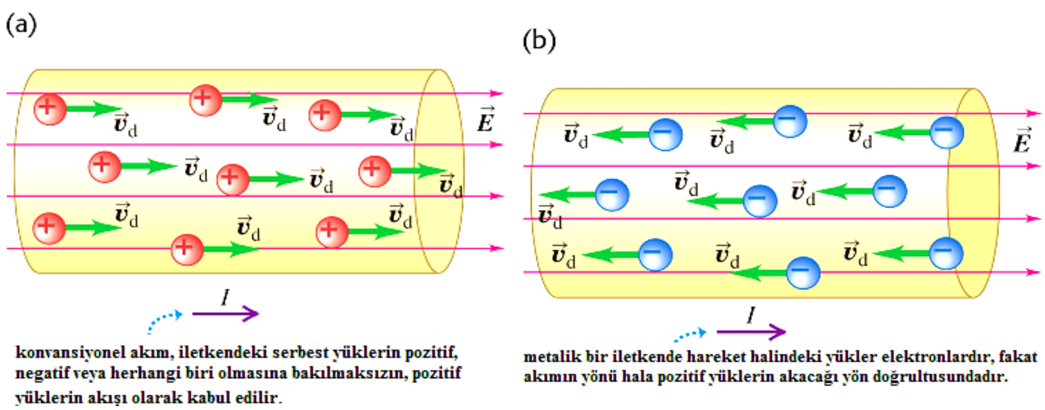


Figure 2.1: As shown in (a) and (b), in both cases there is a positive flow from left to right, and positive charges are located to the right of negative charges. The current, denoted by I, is defined in the direction of positive charge flow. Even in cases where we know that the actual current is due to electrons, their current is characterized as a flow of positive charge only. Current is defined as the net charge passing through a unit surface per unit time.

$$I = \frac{Q}{t} \tag{2.1}$$

Even though the direction of the current is mentioned, the current is not a vector quantity. The SI unit of current is the ampere; an ampere is defined as $coulomb/sec$. milliampere ($1\,mA = 10^{-3}\,amps$), microampere ($1\,\mu A = 10^{-6}\,amps$), nanoampere ($1\,nA = 10^{-9}\,amps$), picoampere ($1\,pA = 10^{-12}\,amps$. When n number of charged particles moving in a unit volume move with the same drift velocity of magnitude v_d, each particle travels a distance $v_d dt$ in time interval dt. At time dt, the particles coming out of the right end of a cylindrical conductor of length $v_d dt$ are the particles that were inside the cylinder at the beginning of the time interval dt.

The volume of a cylinder with a cross-sectional area A is $Av_d dt$ and the number of particles in this volume is expressed as $nAv_d dt$. If each particle carries a charge q, then the charge dQ flowing out of the cylinder at the time dt is $dQ = q(nAv_d dt)$ and the current is expressed as,

$$I = \frac{dQ}{t} = n|q|v_d A \tag{2.2}$$

The current per unit cross-sectional area is called current density (J).

$$J = \frac{I}{A} = n|q|v_d \tag{2.3}$$

The unit of current density is $A/_{m^2}$. The vectorial current density including the direction of the drift velocity is expressed by $\vec{J} = nq\overrightarrow{v_d}$. If q has a positive sign, the quantity v_d is in the same direction as the electric field $\vec{E}$; if q has a negative sign, v_d and the electric field $\vec{E}$ are in opposite directions. In both cases, $\vec{J}$ and $\vec{E}$ are in the same direction.

RESISTIVITY

The current density $\vec{J}$ in a conductor depends on the electric field $\vec{E}$ and the properties of the conducting material. This dependence, which can be complex, means that in some materials, especially metals, at a given temperature $\vec{J}$ is almost completely proportional to $\vec{E}$, while the ratio of the magnitudes E and J is constant. This relationship, known as **Ohm's Law**, was discovered in 1826 by the German physicist **Georg Simon Ohm**.

Table 2.1: Resistivity of some materials at room temperature

	Materials	**Resistivity $\boldsymbol{\rho(\Omega m)x10^{-8}}$**
Metals	Silver	1.47
	Copper	1.72
	Gold	2.44
	Aluminum	2.75
	Tungsten	5.25
	Steel	20
	Lead	22
	Mercury	95
Alloys	Manganite	44
	Constantan	49
	Nichrome	100

	Materials	**Resistivity $\boldsymbol{\rho(\Omega m)}$**
Semiconductors	Graphite	3.5
	Germanium	0.6
	Silicon	2300
Insulators	Amber	5
	Glass	$10^{10} - 10^{14}$
	Lucite	$> 10^{13}$
	Isinglass	$10^{11} - 10^{15}$
	Quartz	$75x10^{16}$
	Teflon	$> 10^{13}$
	Wood	$10^{8} - 10^{11}$

The resistivity is defined as the ratio of the magnitude of the electric field to the magnitude of the current density, symbol ρ.

$$\rho = \frac{E}{J} \tag{2.4}$$

The unit of resistivity is called ${(V/m)}/{(A/m^2)} = {V.m}/{A} = \Omega m$ ($\Omega = V/A$). As can be seen from Table 2.1, metals and alloys have the lowest resistivity. The inverse of resistivity is conductivity and its unit is $(\Omega m)^{-1}$. Many materials do not obey Ohm's law, they are called non-ohmic or non-linear materials.

The resistivity of metallic conductors is directly proportional to increasing temperatures as shown in Figure 2.2. As the temperature increases, the ions of the conductor vibrate at higher amplitude and this vibration increases the probability of collision of mobile electrons with ions. In this case, the friction of electrons in the conductor becomes more difficult, ultimately reducing the current.

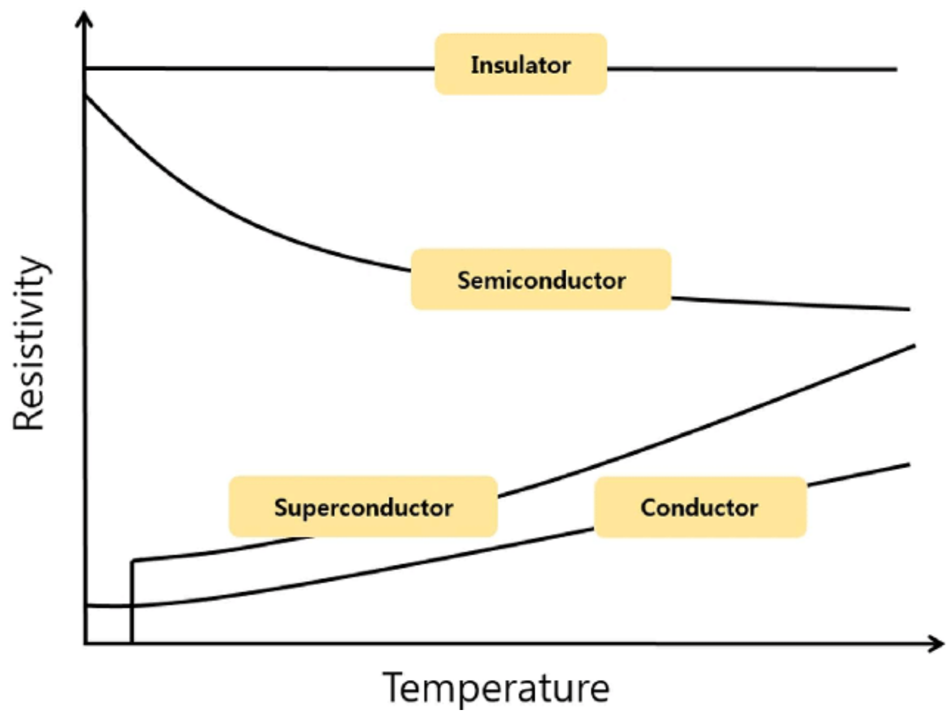


Figure 2.2: Variation of resistivity of metals, semiconductors, insulators, and superconductors with temperature

The resistivity of a metal is expressed by $\rho(T) = \rho_0[1 + \alpha(T - T_0)]$ within a low-temperature window. Here ρ_0 is the resistivity at $T_0(20°)$. $\rho(T)$ represents the resistivity at temperature T. The factor α is called the temperature coefficient of resistivity.

RESISTANCE

Since it is much easier to measure current and potential differences than to measure $\vec{J}$ and $\vec{E}$, if $E = {}^{V}/_{L}, J = {}^{I}/_{A}$ are substituted for $\vec{J}$ and $\vec{E}$, the expression $R = \frac{V}{I}$ is obtained considering that ρ is $\rho = R\frac{A}{L}$.

The SI unit of resistance is Ohm (Ω) ($1\ \Omega\ =\ 1\,{}^{V}/_{A}$), Kilo-ohm ($1\ k\Omega\ =\ 10^3\,{}^{V}/_{A}$), Mega-ohm ($1\ M\Omega\ =\ 10^6\,{}^{V}/_{A}$).

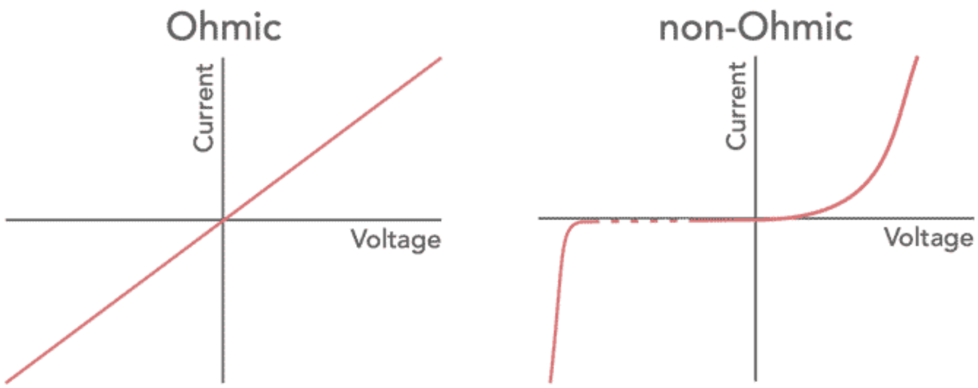


Figure 2.3: Ohmic and non-ohmic behavior.

ELECTRO-MOTOR FORCE (EMF)

For a conductor to have a uniform current, it must be a closed loop or closed circuit. If an electric field $\overrightarrow{E_1}$ is applied to a conductor of resistivity ρ, which is not part of an isolated closed circuit, a current of current density J is generated. As a result, the net positive charge rapidly accumulated at one side of the conductor, while the net negative charge is accumulated at the other side of the conductor.

These charges generate their own internal electric field $\overrightarrow{E_2}$ and the direction of this electric field is opposite to $\overrightarrow{E_1}$. There is such an accumulation of charges at the sides of the conductor that the total electric field inside the conductor becomes $\overrightarrow{E_1} + \overrightarrow{E_2} = 0$. Then $J =\ 0$ and the current in the circuit stops completely. If a charge q circulates in a circuit and comes back to the point where it started, the potential energy at the end of the circuit must be the same as the potential energy at the beginning. When charges move through ordinary matter with resistance, there is always a decrease in potential energy. Therefore, the potential energy must increase in one part of the circuit (Figure 2.4).

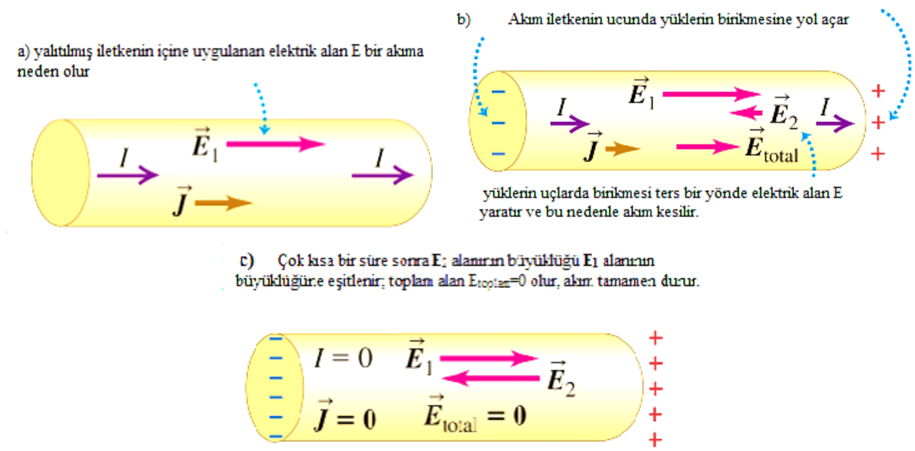


Figure 2.4: A schematic drawing showing the behavior of charges in a conductor with an external electric field ($\overrightarrow{\boldsymbol{E_1}}$)

This is like a fountain pool that circulates its own water. Water flows through the holes above in the direction of decreasing gravitational potential and collects below. Through a pump, the water is brought up again, that is, the potential energy rises. Without a pump, the water would remain where it is collected. In electrical circuits, there must be a device in the pool that acts as a pump (Figure 2.5).

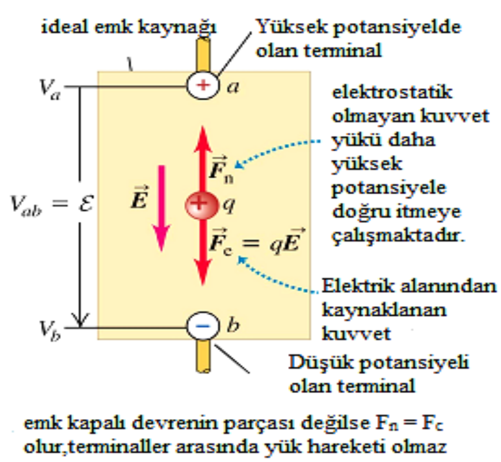


Figure 2.5: Generating of EMF

In this device, even if the electrostatic force tries to push the charge from high potential to low, the charge moves "uphill", that is, from low potential energy to high potential energy. Unlike an ordinary conductor, the direction of current in this device is from low potential energy to high potential energy. This effect, which causes the current to flow from low potential energy to high potential energy, is called the electromotive force, or emf. Emf is not a force, like potential, it is the amount of energy per unit charge. The SI unit of emf, which has the same symbol as potential, is the volt ($1\ V = 1\ {}^{J}/_{C}$). Every smooth-current closed circuit must contain a device that provides an emitter. Such devices are called sources of emission. Batteries, electric generators, solar cells, and fuel cells are some examples of sources of emission. An ideal EMF source maintains a constant potential difference between the terminals regardless of the current flowing through the circuit.

An ideal EMF source, schematically shown in Figure 2.4, maintains the potential difference between conductors a and b (these are the terminals of the device). Terminal a, denoted by +, is at a higher potential than terminal b, denoted by -. This potential difference results in an electric field $\vec{E}$ around the terminals inside and outside the EMF source. The charge q inside the source is subjected to the electric force $\vec{F}_e = q\vec{E}$. However, the source creates another non-electrostatic force: $\vec{F}_n$. This force inside the source pushes the charges from b and an in the "uphill" direction, opposite to the electric force $\vec{F}_e$. Thus $\vec{F}_n$ maintains the potential difference between the terminals. If there were no $\vec{F}_n$, the charge would flow between the terminals until the potential difference was zero. This additional force $\vec{F}_n$ depends on the type of source. In batteries or fuel cells, it is linked to the diffusion process and variable electrolyte densities due to chemical reactions.

The $\vec{F}_n$ force does positive $W_n = q\varepsilon$ work on the charge. This displacement is in the opposite direction of $\vec{F}_e$. Therefore, the potential energy associated with the charge increases by qV_{AB}. $V_{AB} = V_A - V_B$. $\vec{F}_e$ and $\vec{F}_n$ are equal in magnitude but opposite in direction, so the total work done on q is zero. The potential energy of the charge q increases while its kinetic energy does not change. The increase in potential energy is equal to the non-electrostatic work W_n, that is, $q\varepsilon = qV_{AB}$, or $W_{AB} = \varepsilon$ (in an ideal EMF source).

If we make a closed circuit by connecting the terminals of a source with resistance R, the potential difference between terminals a and b creates an electric field inside the wire; this creates a current in the circuit from high potential to low potential, from terminal a to terminal b (Figure 2.6). The potential difference at the sides of the wire is expressed by $\varepsilon = V_{AB} = IR$.

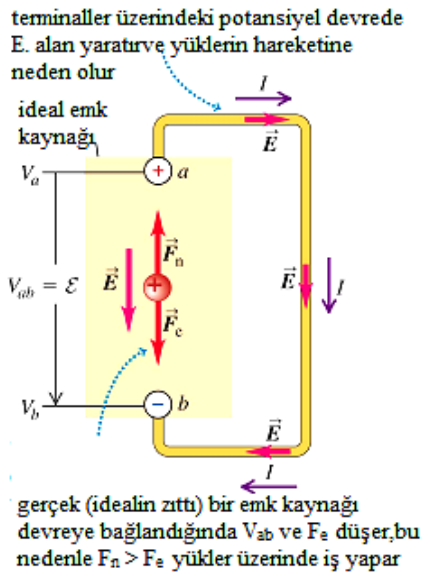


Figure 2.6: EMF in a non-ideal wire

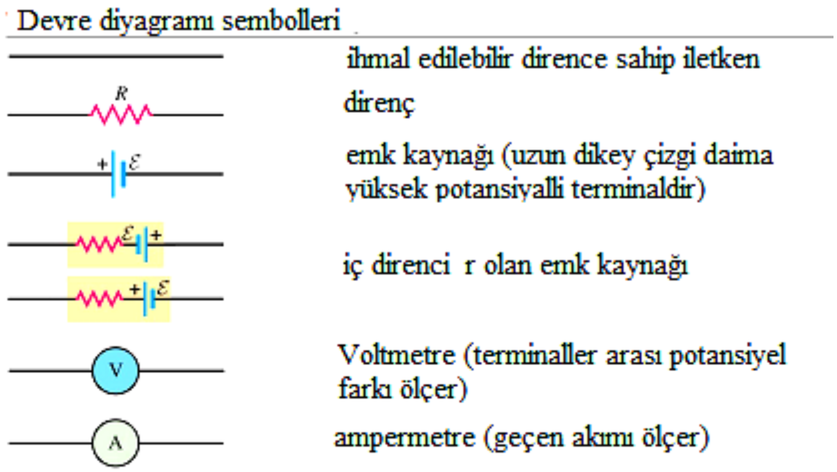
INTERNAL RESISTANCE

The potential difference across the actual EMF source in a circuit is not equal to the EMF expressed by $\varepsilon = V_{AB} = IR$. This is because the charge moving through the actual source material encounters a certain resistance. This resistance is called the internal resistance of the source and is denoted by r. As the current flows through r, there is a drop in potential by Ir. Therefore, the potential difference between the terminals is given by $V_{AB} = \varepsilon - IR$.

CIRCUIT DIAGRAM SYMBOLS

One of the most important ways to analyze an electrical circuit is to draw a circuit diagram schematically. For this reason, the figures in Table 2.2 are used.

Table 2.2: Representation of some electronic circuit components in circuit drawings



Devre diyagramı sembolleri	
	ihmal edilebilir dirence sahip iletken
R	direnç
$+ \ \mathcal{E}$	emk kaynağı (uzun dikey çizgi daima yüksek potansiyalli terminaldir)
$\mathcal{E} \ +$ $+ \ \mathcal{E}$	iç direnci r olan emk kaynağı
V	Voltmetre (terminaller arası potansiyel farkı ölçer)
A	ampermetre (geçen akımı ölçer)

POTENTIAL CHANGES IN A CLOSED CIRCUIT

For a charge q making a full revolution in a closed circuit, the net change in potential energy must be zero. Then the net change in the potential difference in the circuit must also be zero. Figure 2.7 shows how the potential changes in a closed circuit. If we take the potential at the negative terminal of the battery to be zero, then we see an increase in ε and a decrease in Ir at the battery, plus a decrease in IR at the external resistance. When a full revolution is made around the ring, the potential has returned to the level at which it started.

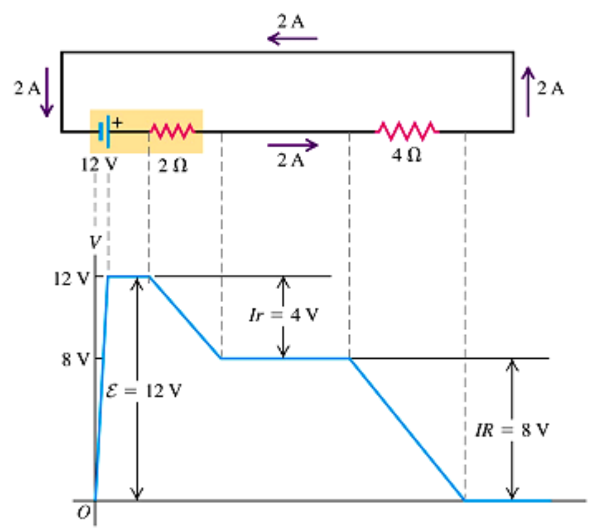


Figure 2.7: Schematic representation of the potential drop in a closed circuit

ENERGY AND POWER IN ELECTRICAL CIRCUITS

In electrical circuits, our interest is usually in the amount of energy given to or received from the circuit element. If the current flowing through the circuit element is III, the amount of charge passing in the time interval dt is $dQ = Idt$. The potential energy change for this amount of charge is $V_{AB}dQ = V_{AB}Idt$. When we divide this expression by dt, we obtain the amount of energy given to or taken from the circuit element. The amount of energy transfer per unit of time is called power and is denoted by P and expressed by $P = V_{AB}I$. The SI unit of power is represented by W and is called Watt.

$$1W = Volt\ x\ Amp = \left(1\frac{J}{C}\right)\left(1\frac{C}{s}\right) = 1\ {}^{J}/_{S} = 1\ Watt$$

The power given to the resistor is expressed by.

$$P = V_{AB}I = I^2R = \frac{V_{AB}}{R}$$

What happens to this energy given to the circuit? The moving charges collide with the atoms in the resistor and transfer some of their energy to these atoms, thus increasing the internal energy of the material. In this case, the temperature of the resistor rises, or heat flows out, or both.

KIRCHHOFF'S RULES

Many resistance patterns cannot be reduced to simple series-parallel arrangements. To calculate the currents in such complex circuits, two rules developed by the German physicist **Gustav Robert Kirchhoff** are used.

The points where three or more conductors meet in a circuit are called junctions or branch points. Closed conductor paths are called loops.

Kirchhoff's junction rule: The algebraic sum of the currents entering a junction is always zero.

$$\sum I = 0 \qquad (2.5)$$

Kirchhoff's loop rule: The algebraic sum of potential differences over a loop must always be zero. This sum includes both emf terms and resistance elements.

$$\sum V = 0 \qquad (2.6)$$

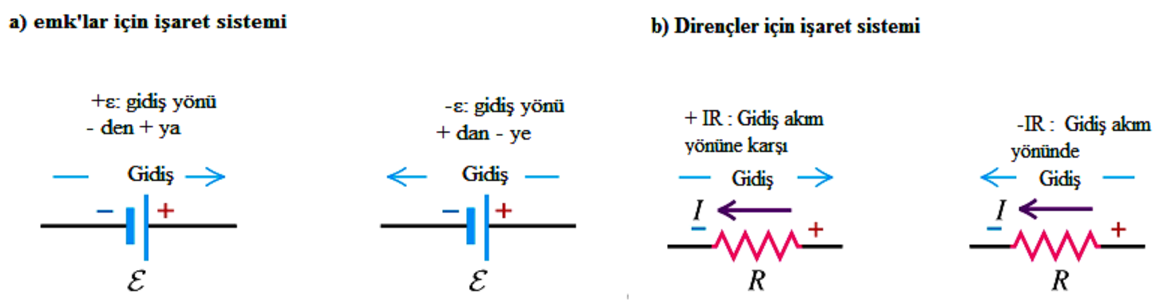


Figure 2.8: Kirchhoff's junction rule

When applying the loop rule, we first select a direction for the current in each branch of the circuit and mark it on the circuit diagram. Then, starting from any point on the circuit, we assume that we are traveling in a loop. Along the way, we add up the EMF and IR values we encounter (Figure 2.9 (a) and (b)).

The emf of a source is positive when we go from the $-$ side to the $+$ side and negative when we go from the $+$ side to the $-$ side. Figure 2.8(a)

Since the current flows in the direction of decreasing potential, the IR term in the resistors is negative when we go in the same direction as the assumed current; the IR term in the resistors is negative when we go in the same direction as the assumed current; the IR term in the resistors is positive when we go in the opposite direction to the assumed current because the potential is increasing. Figure 2.8(b).

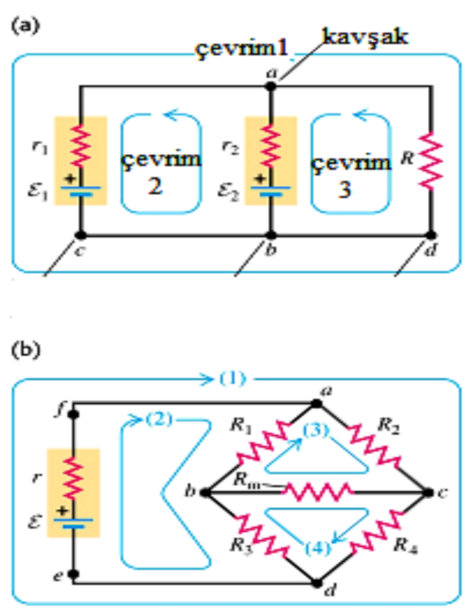


Figure 2.9: Kirchhoff's loop rule.

2.2. EXPERIMENTS

2.2.1. OHM's LAW and THERMISTOR

2.2.1.1. PART I: OHM's LAW

MAIN PRINCIPLE:

I. Ohm's law is used to find the values of unknown resistors using the current flowing through known resistors. Testing the concept of equivalent resistance in series-parallel circuits.

II. Determination of internal resistance and EMF.

EQUIPMENT

Ammeter, Voltage Source, Resistor, Connection Cables.

Figure 2.10: Experimental setup

THEORY

Electric Measuring Instruments: The most basic measuring instruments used in electrical circuits are an ammeter, voltmeter, and potentiometer.

Ammeter: An instrument that measures current is called an ammeter. The ammeter is connected in series with the circuit. In order to make the measurement precisely, the internal resistance R_A of the ammeter must be very small compared to the external circuit resistance.

$$R_A \ll r + R_1 + R_2 \tag{2.7}$$

The ideal ammeter has zero internal resistance. Only in such an ammeter is the power loss zero.

Voltmeter: An instrument that measures the potential difference is called a Voltmeter. The voltmeter is connected parallel to the circuit. For the measurement to be accurate, the internal resistance R_V of the voltmeter must be very large compared to the resistance of the circuit part whose potential difference is to be measured.

The internal resistance of an ideal voltmeter should be infinite so that the power loss in the voltmeter is zero.

$$R_V \gg R_1 \tag{2.8}$$

Galvanometers are the most sensitive instruments that can also act as voltmeters and ammeters.

Potentiometer: ε_x for a potentiometer that measures the electro-motor force of a standard ε_s of known value compared to the electro-motor force of an $\varepsilon_x$of unknown value,

$$\varepsilon_x = \varepsilon_s \left(\frac{R_x}{R_s}\right) \tag{2.9}$$

A. OHM's LAW

Ohm's Law: The ratio of the potential difference between the two terminals of a conductor to the current flowing through it is constant. This constant is called the resistance of the conductor and

$$R = \frac{V}{I} \tag{2.10}$$

is given with. Here, $[V] = Volt$, $[I] = Ampere$, $[R] = {}^{Volt}/_{Ampere} = Ohm(\Omega)$ units. Powers of the Ohm unit are often used, which are $1\ K\Omega = 10^3\ \Omega, 1\ M\Omega = 10^6\ \Omega$.

B. RESISTORS ARE GENERALLY USED IN ELECTRICAL CIRCUITS BY CONNECTING IN TWO WAYS

1. Serial Connection

R_s the equivalent resistance is obtained by connecting resistors R_1, R_2 as seen in Figure 2.11. R_s is given by,

$$R_s = R_1 + R_2 + R_3 \tag{2.11}$$

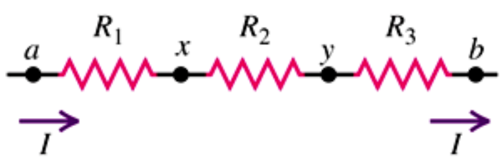


Figure 2.11: Series resistance connection

2. Parallel Connection

The equivalent resistance R_p obtained by connecting resistors R_1, R_2 as shown in Figure 2.12 is calculated by the following relation.

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \tag{2.12}$$

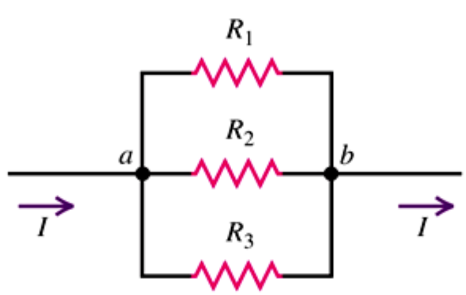


Figure 2.12: Parallel resistance connection

C. ELECTRO-MOTOR FORCE (EMF)

The potential difference is measured between the terminals of a voltage source from which no current is supplied.

D. KIRCHHOFF'S LAW

Two rules that are often used in analyzing electrical circuits and solving problems.

I. Kirchhoff's Law

In a multi-ring circuit, the algebraic sum of the currents at any intersection is zero. According to Figure-3, It is given by the following relation.

$$I_1 - I_2 + I_3 - I_4 = 0 \tag{2.13}$$

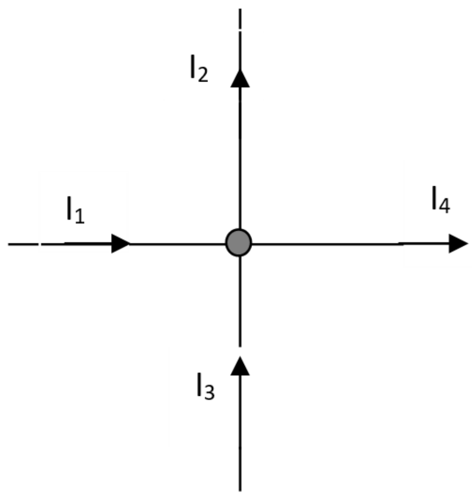


Figure 2.13: Schematic representation of currents passing through a knot point

II. Kirchhoff's Law

The electro-motor force ε in a closed electric circuit is equal to the sum of the potential drops across all resistances in the circuit. Let R_i represent the internal resistance of the voltage source and R the external equivalent resistance of the circuit,

$$\varepsilon = IR_i + IR \tag{2.14}$$

the relation can be written. If the potential drop across R is represented by V, the relation (2.14),

$$\varepsilon = IR_i + V \tag{2.15}$$

or

$$V = \varepsilon - IR_i \tag{2.16}$$

can also be written as.

SETUP AND PROCEDURE

1- The circuit in Figure 2.14 is built. R in the figure is replaced by 10 different resistors selected from the resistor box and *I* currents passing through the milli ammeter are read and recorded in Table 2.3.

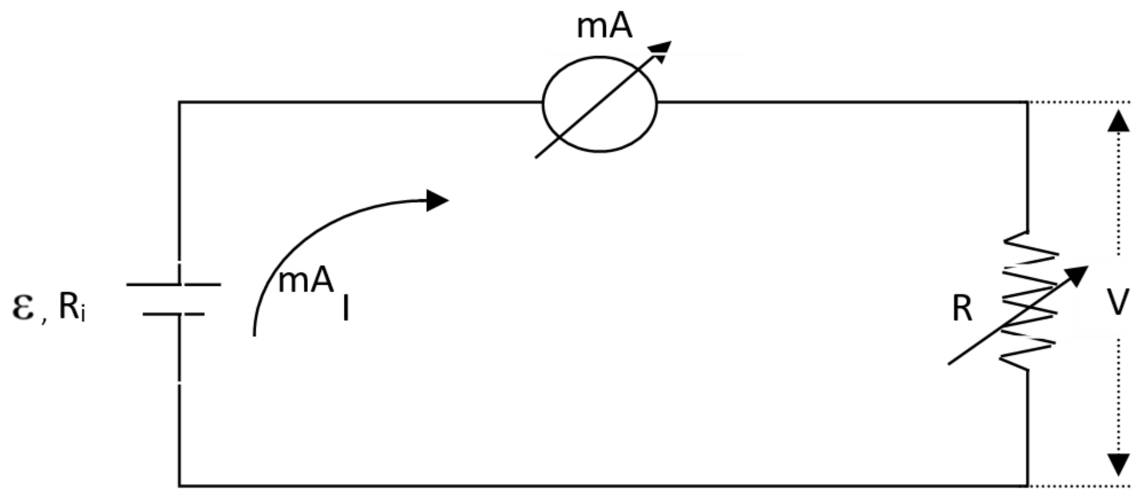


Figure 2.14: Circuit

Table 2.3

R (kΩ)	I (mA)	V (mV)
0		
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

2- The graph of $I = f(R)$ showing the dependence of current intensity on resistance values is drawn (Figure 2.15).

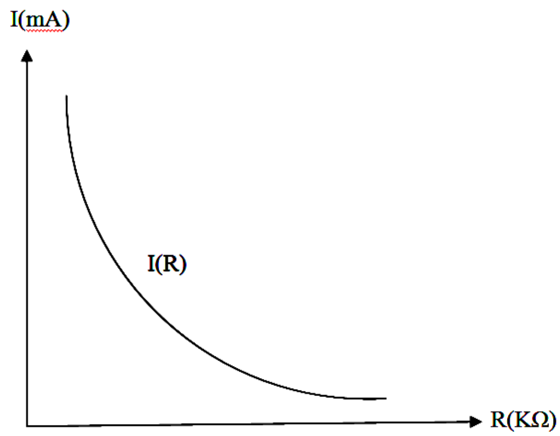


Figure 2.15: Relation of current intensity vs resistance

3- This time unknown resistors R_1 and R_2 are connected instead of resistance R in Figure 2.14.

4- R_1 and R_2 are connected once in series and once in parallel. The currents passing through the circuit are recorded in Table 2.4.

5- The resistance values found by plotting the currents corresponding to the unknown resistors on the graph $I = f\ (R)$ are recorded in Table 2.4.

Table 2.4

R	I (mA)	R (Ω) - Graph	R (Ω) - Formula
R_1			
R_2			
R_s			
R_p			

6- Using R_1 and R_2 values found by the graphical method, R_s series, and R_p parallel equivalent resistances are calculated from relations (2.11) and (2.12). They are entered in Table 2.4. R_s and R_p values found by graphical method and calculation are compared with each other.

$R_s = R_1 + R_2 =$ ……………………=……………… Ω

$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} =$……………………=……………… Ω

7- The potential drops on the resistors R in Table 2.3 selected from the resistor box are calculated by the relation (2.10) and recorded in Table 2.3.

8- $V = f(I)$ graph showing the dependence of potential drops on current intensities is drawn (Figure 2.16). This should be a line according to the relation (2.16). From this graph, the EMF of the voltage source is found from $V = \varepsilon$ for $I = 0$.

$\varepsilon =$……………… Volts

9- A selected I-V pair is found in the graph $V = f(I)$ and the internal resistance R_i of the voltage source is found in the relation (2.16).

$I =$ ……………… mA = ……………… A

$V =$ ……………… Volts

$R_i = \frac{\varepsilon - V}{I} =$ ……………… = ………………Ω

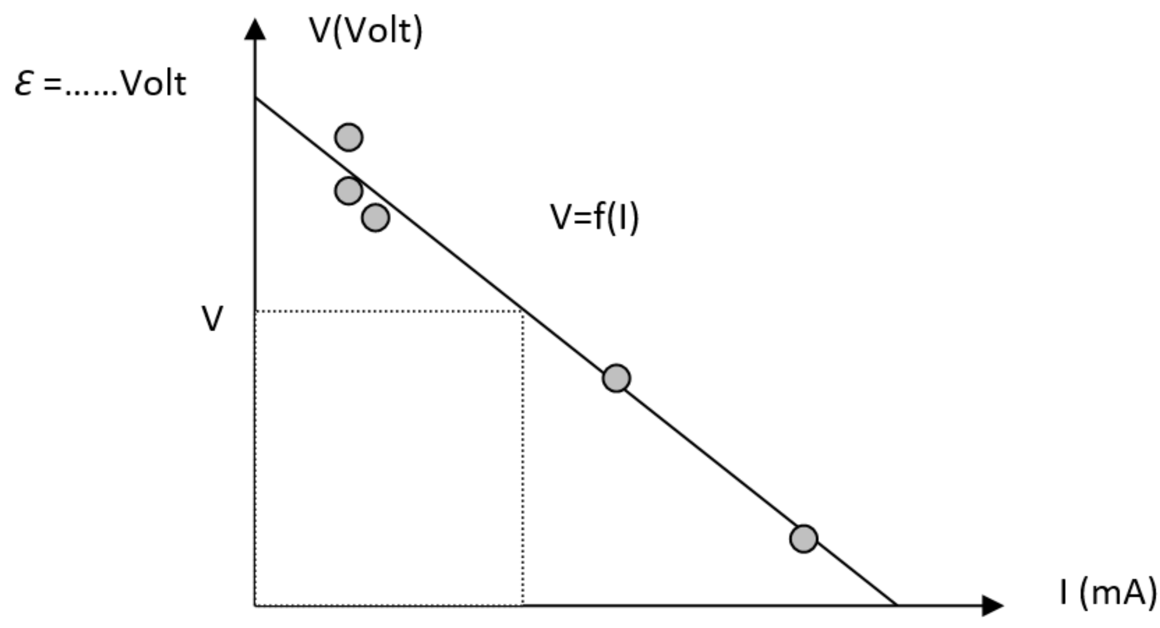


Figure 2.16

2.2.1.2. PART 2: TEMPERATURE MEASUREMENT WITH THERMISTOR

MAIN PRINCIBLE

The structure of a thermistor, observation of the relationship between temperature and resistance of the thermistor, and determination of a method to be used to determine the temperature of any substance by using this relationship.

EQUIPMENT

Heater, Beaker, Glass tube, Thermistor (470 Ω), Milli Ammeter (DC), Power supply, Thermometer, Connection cables.

THEORY

Today, thermal sensors of various structures and types are widely used in many fields of technology. One of the most widely used of these is the thermistor, due to its low cost, robust structure, and ease of manufacture.

After thermometers, which were the first detectors used to observe temperature changes, mankind made a real start in its search for detectors with the use of metal resistance elements with thin plate structures and small heat capacities, which we can call detectors in the real sense. The need to meet the needs accelerated the research on the elements on which the conductivity properties of matter depend.

The most basic properties that a good detector element should have are a heat capacity as small as possible, the ability to respond as quickly as possible to changes in temperature, and the ability to operate in a wide radiation region (frequency range). The need to obtain such a sensing element led to the idea of continuing the studies on semiconductors, which have a greater advantage in terms of the thermal properties of the above-mentioned material. As a result of studies on semiconductors, thermistors were first produced in Germany during World War II. Since they have 100 times smaller heat capacity compared to previously used thin-plate metal resistors, they have gradually replaced them and continue to be used in many fields to this day.

The thermistors used today are circuit elements made of oxides of transition elements Cr, Mn, Fe, Co, and Ni. Depending on the need, there are thermistor types used at very low temperatures such as 3 - 4 K in the context of the relationship between heat capacity and temperature, both at room temperature and in the case of more precise measurements. It is possible to obtain them in various geometric shapes according to the need and service brought by technology. The resistance of a thermistor is determined by its value at 20 °C. If we give an example; From the expression “a 400 Ω thermistor” we understand that the resistance of this thermistor is 400 Ω at 20 °C. The relationship between the temperature of a substance and its resistance is simply given as $R = R_0(1 + \alpha\Delta T)$. Here α is a constant that determines the change in resistance with temperature and is related to the structure of the material, R_0 is the initial value of resistance, R is the final value of resistance, and ΔT is the change in temperature of resistance. This relation given above appears as a result of the $\alpha = \frac{1}{R}\frac{\partial R}{\partial T}$ derivative equation. As can be seen, it shows the response of the α coefficient (resistance temperature coefficient) to the change in temperature.

a. If the coefficient takes a negative value, the thermistor is called an NTC (Negative Temperature Coefficient) type thermistor.

b. If the coefficient takes a positive value, then the thermistor is called a PTC (Positive Temperature Coefficient) type thermistor.

While the resistance of NTC-type thermistors shows an inversely proportional change with temperature, in PTC-type thermistors the resistance-temperature relationship is directly proportional.

Note: For more information about thermistors, you can use the resources related to Thermal Detectors.

SETUP AND PROCEDURE

1- The circuit in Figure 2.17 is established. Before the heater is switched on, the temperature of the water and therefore the temperature of the thermistor is measured with the help of a thermometer and recorded on the table.

2- The heater is switched on and the current flowing through the circuit at 5 °C intervals is measured with the help of an ammeter and recorded in Table 2.5. The measurement process continues until approximately 80 °C.

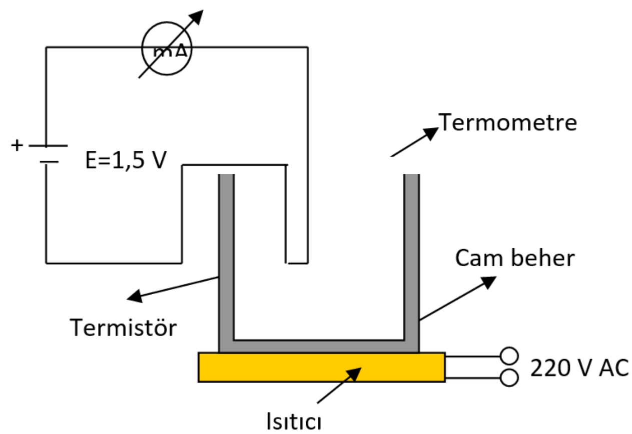


Figure 2.17: Experimental setup

Table 2.5

T (°C)	I (mA)	R (Ω)

3- Since the emf of the voltage source in the circuit is 1.5 $volts$ and the internal resistance is negligibly small, the voltage at the thermistor terminals is $V = 1.5\ volts$. Using this value of the voltage and the current magnitudes in the table and using Ohm's law ($V = I.R$), the resistance values corresponding to each temperature value of the thermistor are calculated and recorded in Table 2.5.

Note: It is possible to determine the temperature of any object using these two graphs. These graphs are a ruler of measurement for us.

4- $I = f(t)$ and $R = f(t)$ graphs are shown in Figure 2.18 and Figure 2.19 are drawn from the table.

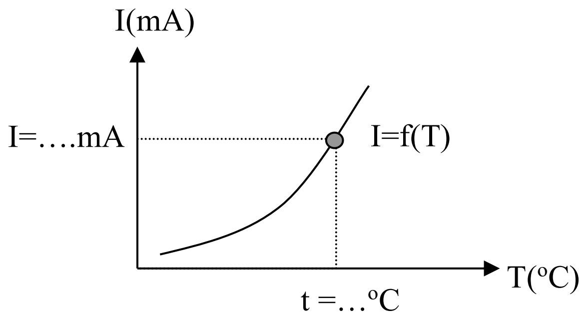


Figure 2.18

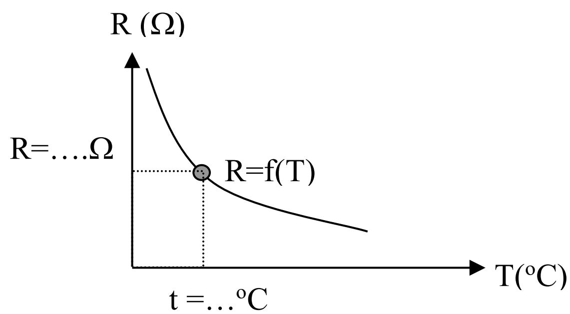


Figure 2.19

QUESTIONS

1. What is a thermistor?

2. What is the type of thermistor you used in the experiment?

3. What are the characteristics of a quality thermistor?

4. What are the possible uses of thermistors?

5. What is the resistance temperature coefficient (α), and what its function is in determining the type of thermistor?

6. What is the basic structure of a thermistor?

7. What do we need to know about the transition elements that play a role in the conductivity of a thermistor and why do we use transition elements?

2.2.2. DETERMINATION OF RESISTANCE BY WHEATSTONE BRIDGE METHOD

MAIN PRINCIPLE

I. Measurement of a resistance of unknown value,

II. Determination of the resistivity of the wire.

EQUIPMENT

Galvanometer, Voltage Source, Resistance Wire, Resistors of Various Values, Resistance Junction Box, Connection Cable, Micrometer, and Ruler.

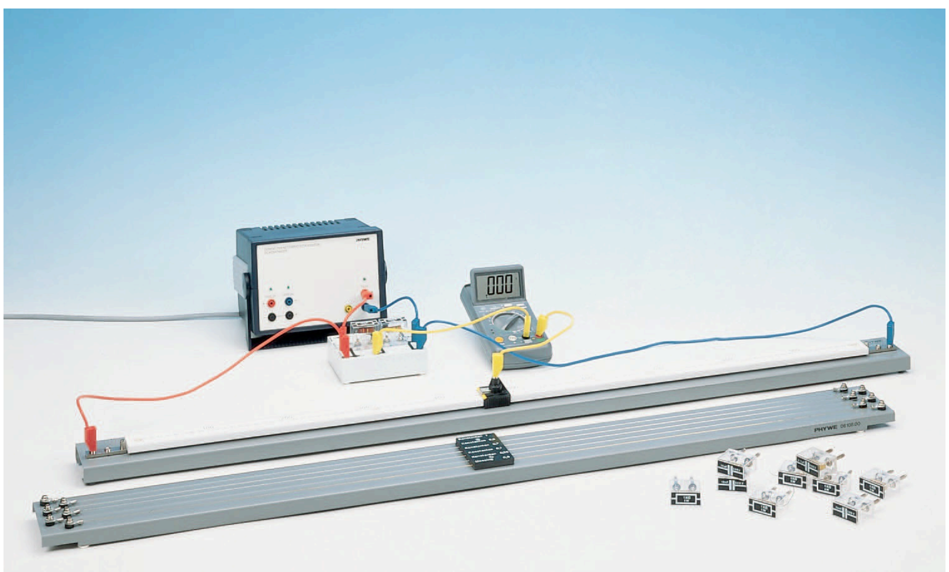


Figure 2.20: Wheatstone Bridge Experimental Setup

THEORY

OHM's LAW:

We know that when an electric potential difference is applied to the terminals of a conductor, current flows through the conductor. Now let us change the electric potential difference applied to the terminals of the same type of conductor.

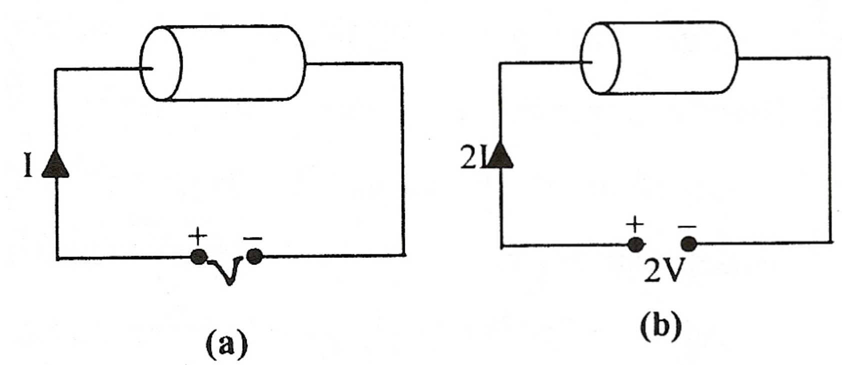


Figure 2.21: Circuits with different voltage sources a) V b) 2V

When a potential difference V is applied to the conductor in Figure 2.21-a, a current of I passes through the conductor. As seen in Figure 2.21-b, when the potential difference is made $2V$, the current intensity becomes 2I. Current changes proportionally with a potential difference. Therefore, the V/I the ratio is constant. This constant is called the resistance of the conductor and is represented by the symbol R. Therefore, the ratio between the potential difference between the terminals of a conductor and the current intensity is constant and this constant ratio is called the resistance of the conductor.

$$R = \frac{V}{I} \tag{2.17}$$

R is given by the expression. In this expression, V is $Volts$, A is $ampere$ and R is $volt/ampere$.

RESISTIVITY

The resistance of a wire conductor is directly proportional to its length and inversely proportional to its cross-section.

$$R \sim \frac{L}{I} \Rightarrow R = \rho \frac{L}{A} \tag{2.18}$$

In this expression, L is the length of the wire and A is the cross-sectional area of the wire. The coefficient of proportionality ρ depends on the material the wire is made of and is called "resistivity". In other words, resistivity refers to the resistance of a material of unit length and unit cross-section.

Here $[R] = \Omega$, $[L] = m$, $[A] = m^2$ and $[\rho] = \Omega m$ units.

WHEATSTONE BRIDGE

The circuit is given in Figure 2.23, consisting of a voltage source, four resistors, and a galvanometer, is called a “Wheatstone Bridge”. By changing the values of the resistors, it can be ensured that current does not pass through the arm where the galvanometer is located. In this case, the bridge is said to be in equilibrium. The equilibrium condition of the bridge is shown in Figure 2.22. As can be seen from the result of Figure 2.22, the equilibrium condition is suitable for calculating a resistance only with the help of other resistances.

$$\frac{R_1}{R_4} = \frac{R_2}{R_3} \tag{2.19}$$

For this purpose, the bridge is formed as in Figure 2.23.

Resistance R1 of unknown resistance is found between points A - C. In the circuit, a resistance wire placed on a 1 m ruler is used as the unknown resistance. Thus, a desired length of the wire can be selected.

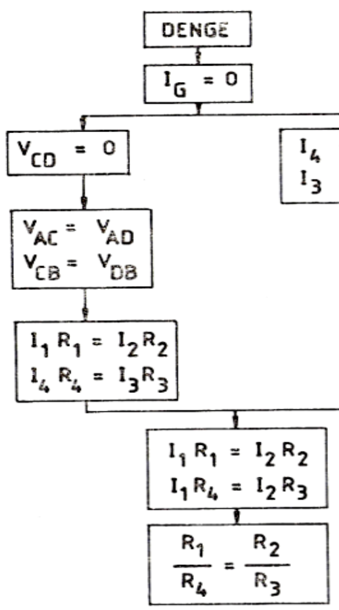


Figure 2.22

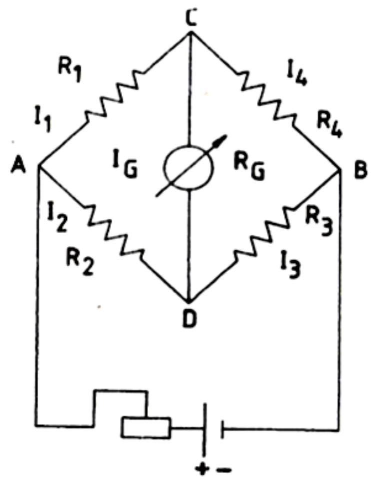


Figure 2.23

Between points A and B there is a resistance wire stretched on the ruler. By sliding the slider D on the wire, the lengths L_2 and L_3 of resistors R_2 and R_3 can be changed. The relation between these lengths and resistances is found in relation (2.18) as,

$$R_2 = \rho \frac{L_2}{A}, R_3 = \rho \frac{L_3}{A} \tag{2.20}$$

Since,

$$\frac{R_2}{R_3} = \frac{L_2}{L_3} \tag{2.21}$$

there is no need to calculate the resistances separately. It is sufficient to measure the lengths. Between points $C - B$ there is a resistor box whose value can be changed gradually. Therefore, R_4 can be read directly. From relations (2.19) and (2.21),

$$\frac{R_1}{R_4} = \frac{R_2}{R_3} = \frac{L_2}{L_3} \Rightarrow R_1 = R_4 \frac{L_2}{L_3} \tag{2.22}$$

unknown resistance R_1 is calculated with the help of lengths L_2, L_3 and resistance R_4.

SETUP AND PROCEDURE

1. Take R_1, R_4 as $(82\ k\Omega, 100\ k\Omega)$; $(4.7\ k\Omega, 10\ k\Omega)$; $(330\ \Omega, 150\ \Omega)$ and find the values of L_2, L_3 at the point where the current is zero with the slider D and write them to Table 2.6.

Table 2.6

$R_1\ (k\Omega)$	$R_4\ (k\Omega)$	$L_2\ (cm)$	$L_3\ (cm)$	$\frac{R_1}{R_4}$	$\frac{L_2}{L_3}$

2. The circuit in Figure 2.24 is established. Set up the circuit with CuNi (constantan) resistance wires with diameters $d_1 = 1\ mm$, $d_2 = 0.7\ mm$, $d_3 = 0.5\ mm$, $d_4 = 0.35\ mm$ respectively as R_1 wire in the circuit diagram.

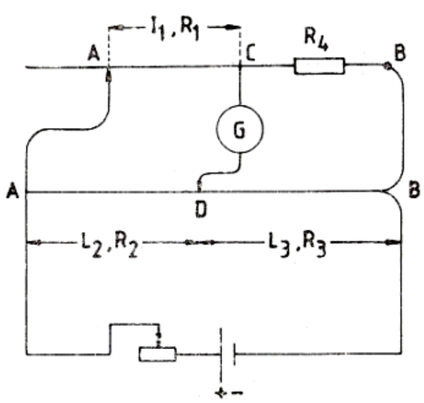


Figure 2.24

3. With the help of the D slider, the ammeter is adjusted until zero current passes through it. This condition is called a system in equilibrium. After equilibrium is achieved for various diameters, the lengths L_2, L_3 are read and processed in Table 2.6.

4. Draw the graph of $R(d)$ with the help of Table 2.6. Take a point of the line you found on the graph.

$R =$ .... Ω , $d =$ .... mm

Using the R, d values you found, find the resistivity ρ from the formula below.

$\rho = \frac{RA}{L} = \frac{R\pi d^2}{4L} =$................. Ωcm

Table 2.7

$d\ (mm)$	$L_1\ (cm)$	$L_2\ (cm)$	$R_3\ (\Omega)$	$R_3\ (\Omega)$
1				
0.7				
0.5				
0.35				

For $d_1 \rightarrow R_1 = R_4 \frac{L_2}{L_3} =$ .... = ... Ω

For $d_2 \rightarrow R_1 = R_4 \frac{L_2}{L_3} =$ .... = ... Ω

For $d_3 \rightarrow R_1 = R_4 \frac{L_2}{L_3} =$ .... = ... Ω

For $d_4 \rightarrow R_1 = R_4 \frac{L_2}{L_3} =$ .... = ... Ω

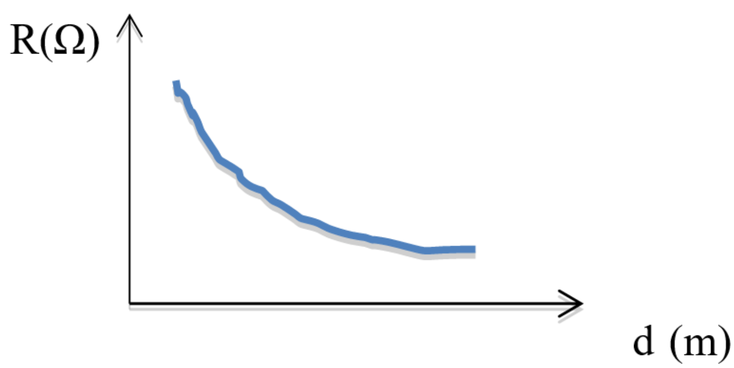


Figure 2.25

5. Take a brass wire with a diameter of $d = 0.5\ mm$ as resistance R_1 and use the slider D in the circuit to make the current zero. Calculate the resistance and resistivity of the brass wire.

$R =$................. Ω, ρ................. Ωcm

2.2.3. CHARGING AND DISCHARGING OF THE CAPACITOR AND THE CHANGE OF CURRENT FLOWING THROUGH THE COIL

2.2.3.1. PART I: INVESTIGATION OF CAPACITOR CHARGING AND DISCHARGING

MAIN PRINCIPLE

I- Investigation of charging and discharging of a capacitor,

II- Calculation of the capacity of the capacitor.

EQUIPMENT

Power Supply, Capacitor Circuit, Voltmeter, Stopwatch, Millimeter paper

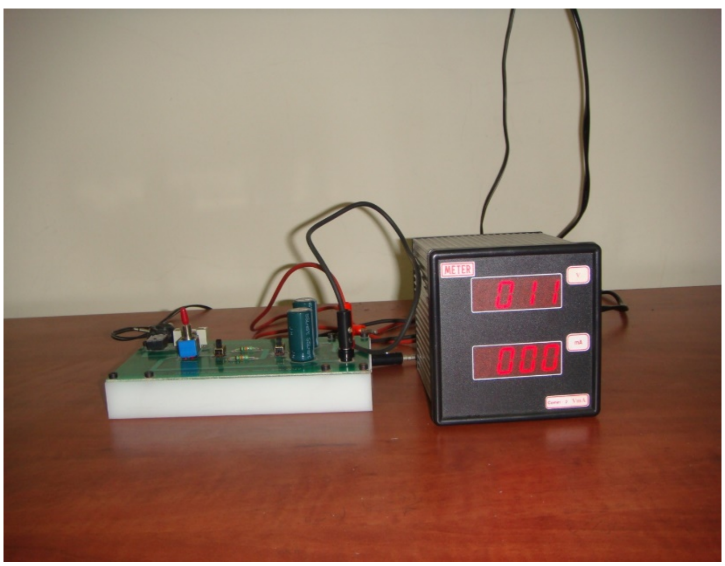


Figure 2.26: Capacitor Experiment Setup

THEORY

CHARGING OF CAPACITOR

Figure 2.27 shows a simple circuit used to charge a capacitor. Such circuits with a series-connected resistor and capacitor are called $R - C$ circuits. Let's assume that the capacitor is empty at the beginning; we take the first time ($t = 0$) as the moment when we close the switch and complete the circuit, allowing current to flow and the capacitor to start charging.

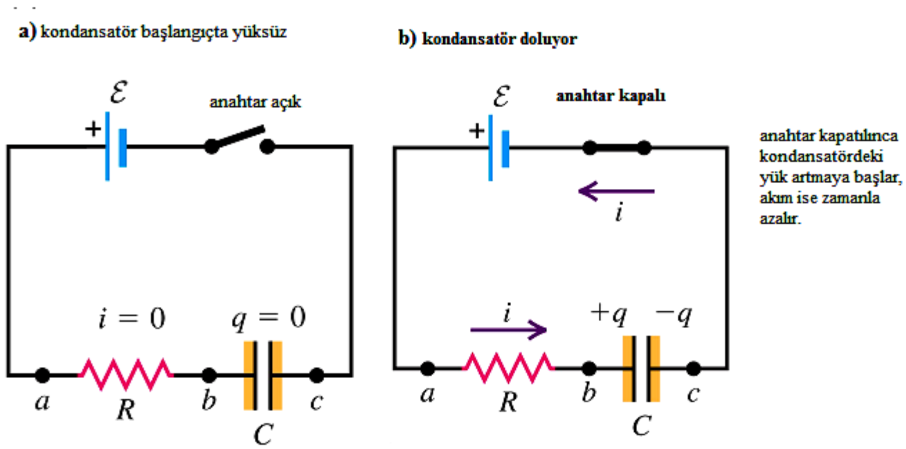


Figure 2.27

Since the capacitor is initially empty, the potential difference V_{bc} across it at $t = 0$ is zero. From this moment, according to **Kirchhoff's loop rule**, the voltage V_{ab}across the resistor R is equal to the source's emf ε. The current I_0 passing through the resistor at $t = 0$ is $I_0 = V_{ab}/R = \varepsilon/R$ according to Ohm's law. As the capacitor fills, the voltage V_{bc} across it increases and the voltage V_{ab} across the resistor and the current decreases. The sum of these two voltages is constant and equal to the emf ε. After a very long time, when the capacitor is completely full, the current and the voltage across the resistor V_{ab} go to zero. All the emf ε of the source appears on the capacitor and $V_{bc} = \varepsilon$.

If we take the charge q and current i in the capacitor at an instant t after the switch is turned off, we assume that the positive direction of the current is the direction in which the positive charges flow to the left plate of the capacitor as shown in Figure 2.27. Instantaneous values of potential differences V_{ab} and V_{bc} are $V_{ab} = iR$, $V_{bc} = q/C$. If we apply the **Kirchhoff loop rule**;

$$\varepsilon - iR - \frac{q}{C} = 0 \Rightarrow i = \frac{\varepsilon}{R} - \frac{q}{RC} \tag{2.23}$$

The potential decreases by iR from a to b and by q/C from b to c. When the switch is first closed at $t = 0$, the charge in the capacitor is $q = 0$. If we substitute $q = 0$ in the above equation, we see that the initial current I_0 is $I_0 = \varepsilon/R$. As the charge q in the capacitor increases, the q/RC term grows, and the charge of the capacitor approaches a final value such as Q_s. When $i = 0$, the above equation becomes $\varepsilon/R = Q_s/RC$, which simplifies to $Q_s = \varepsilon C$. It is seen that the charge Q_s does not depend on the resistor R.

Current and capacitor charge as a function of time is shown in Figure 2.28. At the moment $t = 0$ when the switch is closed, the current suddenly jumps from zero to its initial value $I_0 = \varepsilon/R$, then approaches zero. The charge on the capacitor starts from zero and gradually increases to reach $Q_s = \varepsilon C$.

We can derive general expressions that give the charge q current i as a function of time. When the positive direction of the current is taken as in Figure 2.27, i is equal to the speed at which the positive charges reach the left plate of the capacitor. Therefore, if we substitute $i = dq/dt$ in equation (2.23) is,

$$\frac{dq}{dt} = \frac{\varepsilon}{R} - \frac{q}{RC} = -\frac{1}{RC}(q - \varepsilon C) \tag{2.24}$$

and

$$\frac{dq}{q - \varepsilon C} = -\frac{dt}{RC} \tag{2.25}$$

written in the form. We integrate both sides. If we use and as the integration variables and q' and t' as the upper limits of the integrals,

$$\int_0^q \frac{dq'}{q' - \varepsilon C} = -\int_0^t \frac{dt'}{RC} \tag{2.26}$$

and if integrated,

$$\ln\left(\frac{q - \varepsilon C}{-\varepsilon C}\right) = -\frac{t}{RC} \tag{2.27}$$

is found. If the inverse logarithm is taken on both sides and solved for q,

$$\frac{q-\varepsilon C}{-\varepsilon C} = e^{-\frac{t}{RC}} \; and \; q = \varepsilon C\left(1 - e^{-\frac{t}{RC}}\right) = Q_s(1 - e^{-\frac{t}{RC}}) \tag{2.28}$$

is found. To find the instantaneous current i, the derivative of this expression with respect to time is taken.

$$i = \frac{dq}{dt} = \frac{\varepsilon}{R}e^{-\frac{t}{RC}} = I_0 e^{-\frac{t}{RC}} \tag{2.29}$$

If both sides of equation (2.28) are divided by C to express the charging of the capacitor in terms of voltage,

$$V_C = V_0\left(1 - e^{-\frac{t}{RC}}\right) \tag{2.30}$$

is found. As can be seen, both charge and current are exponential functions of time.

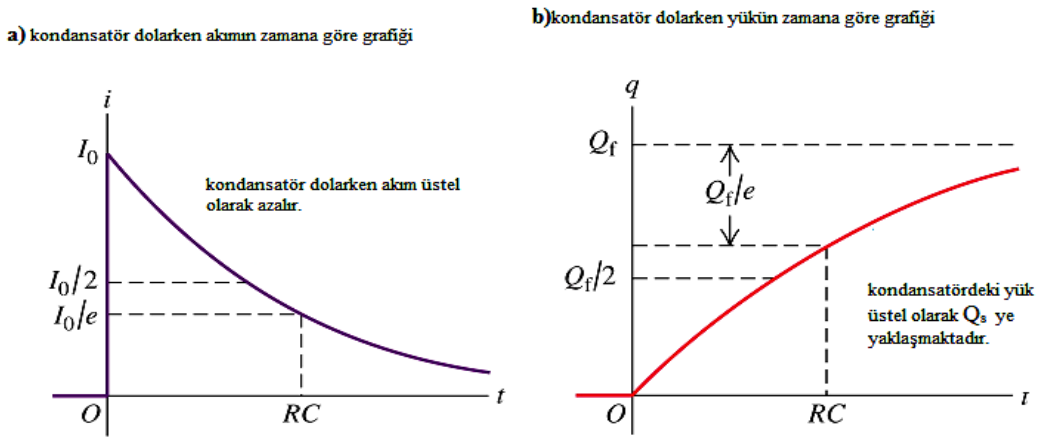


Figure 2.28

The current in the $R - C$ circuit drops to $1/e$ of its initial value ($about$ 0.368 $times$) in RC time. At this time, the charge in the capacitor reaches its final value of $(1 - 1/e) = 0.632$ $times$ of $Q_s = \varepsilon C$. The RC constant is a measure of how long it takes the capacitor to charge. RC is called the time constant of the circuit (relaxation time). $\tau = RC$, if τ is small the circuit charges quickly, if it is large it takes a long time to charge. If the resistance is small, the current passes easily and the capacitor fills up faster. A capacitor or coil is charged and discharged in 4τ time. Because after 4τ the effect is negligibly small.

In Figure 2.28-a, the horizontal axis is the asymptote of the curve. In fact, i is never zero; the longer we wait, the closer it gets to zero. It goes down to 0.000045 times its initial value in about $10RC$ time. Similarly, the curve in Figure 2.28-b asymptotically approaches its final value indicated by Q_s. In $10RC$ time, the difference between q and Q_s decreases to 0.000045 times of Q_s.

DISCHARGING OF CAPACITOR

Let's assume that at a moment when the capacitor in Figure 2.27 carries a charge Q_0, we remove the power supply in the RC circuit and connect points a and c to an open switch. We close the switch and determine the moment $t = 0$ at which we start measuring time. At this moment the charge in the capacitor is $q = Q_0$. As the capacitor discharges through the resistor, the charge decreases and approaches zero over time.

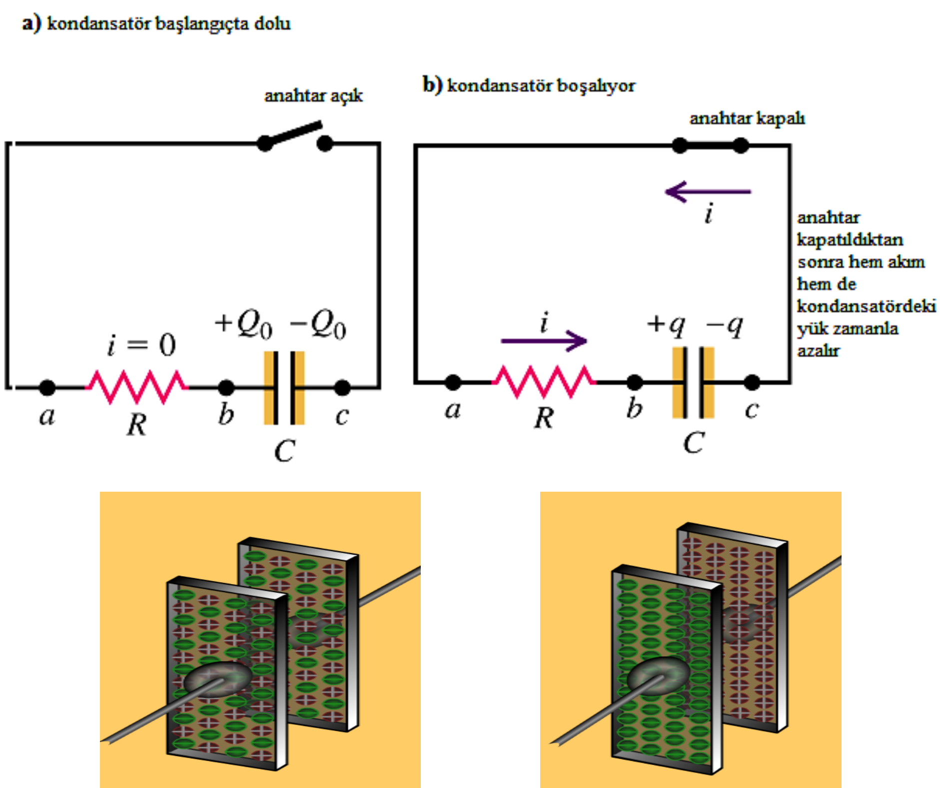


Figure 2.29: The change of charges between the plates of the capacitor is seen.

In Figure 2.29-b, let us use the same direction of current as in Figure 2.27. If the Kirchhoff loop rule is applied, since $\varepsilon = 0$,

$$-iR - \frac{q}{C} = 0 \Rightarrow i = -\frac{q}{RC} \tag{2.31}$$

is found. The fact that the current i is now negative because the positive charge q is leaving the left plate of the capacitor. In other words, the current direction is reversed from the direction we have chosen. At time $t = 0$, when the charge $q = Q_0$, the initial current is $I_0 = -Q_0/RC$.

To find q as a function of time, we integrate the expression (2.31) again taking q' and t'. Now the variable q is between the boundaries Q_0 and q.

$$\int_{Q_0}^{q} \frac{dq'}{q'} = -\frac{1}{RC}\int_{0}^{t} dt' \tag{2.32}$$

and if integrated.

$$\ln\left(\frac{q}{Q_0}\right) = -\frac{t}{RC} \tag{2.33}$$

$$q = Q_0 e^{-\frac{t}{RC}} \tag{2.34}$$

is found. This expression shows that a capacitor charged with a charge Q_0 has a charge q at instant t. The instantaneous current is found by taking the derivative of this expression with respect to time.

$$i = \frac{dq}{dt} = -\frac{Q_0}{RC} e^{-\frac{t}{RC}} = I_0 e^{-\frac{t}{RC}} \tag{2.35}$$

If expression (2.34) is divided by C, the expression that gives the voltage when the capacitor discharges is found.

$$V_C = V_0 e^{-\frac{t}{RC}} \tag{2.36}$$

The variations of current and charge with time are shown in Figure 2.30. Both approach zero exponentially with time. If we compare the i and q expressions, we found for the charging of the capacitor with those we found for the discharging of the capacitor, we see that the current expressions (except the sign of I_0) are the same.

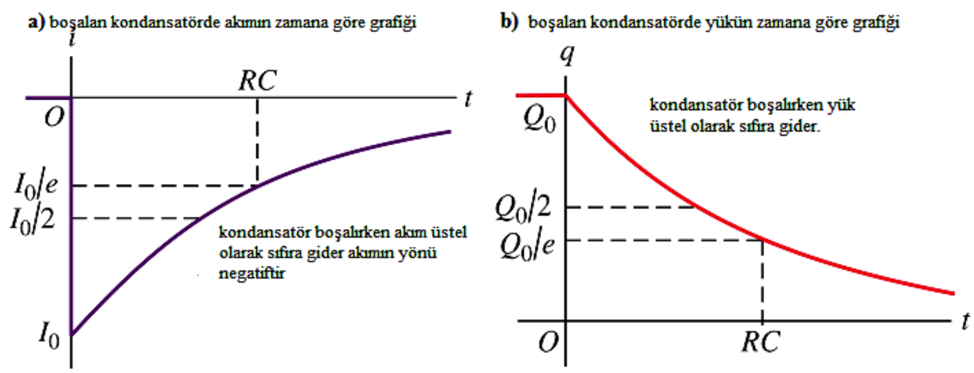


Figure 2.30

Energy relations give us a different view of the behavior of the RC circuit. When the capacitor is charging, the instantaneous power supplied by the source to the circuit is $P = \varepsilon i$ and the instantaneous power dissipated in the resistor is I^2R and the amount of energy stored in the capacitor is $iV_{bc} = iq/C$. If we multiply $\varepsilon - iR - \frac{q}{C} = 0$ by i,

$$\varepsilon i = i^2 R + \frac{iq}{C} \tag{2.37}$$

is found. This equation means that I^2R of the power εi from the source is dissipated in the resistor (converted into heat) and the remaining iq/C is stored in the capacitor.

SETUP AND PROCEDURE

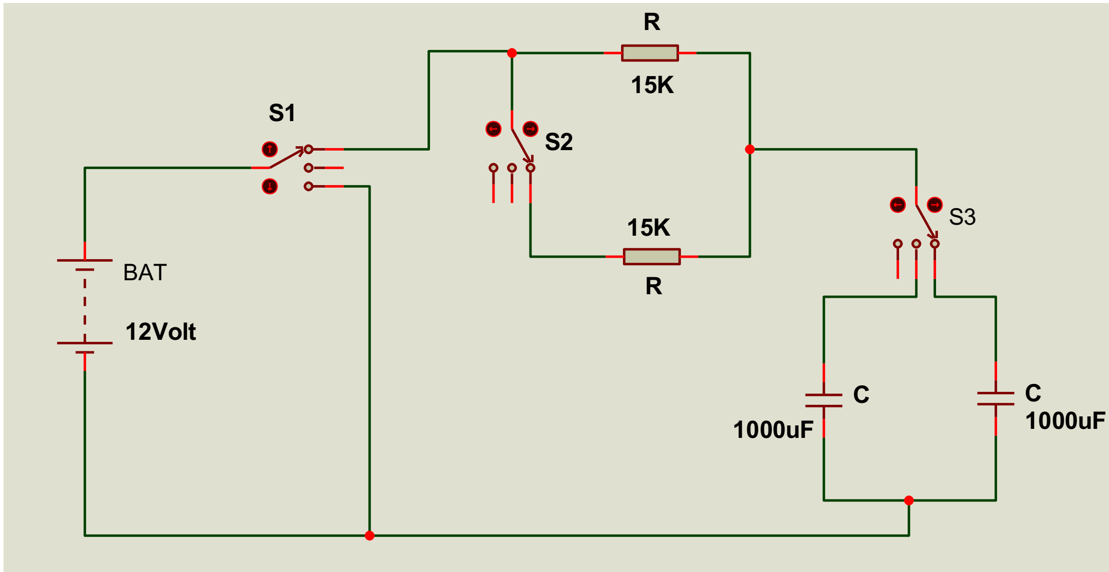


Figure 2.31

In the circuit used in the experiment, after switch S1 is switched to the charging state and the circuit is energized, the voltage on the capacitor is recorded every 5 seconds until the voltage stabilizes. Then switch S2 is switched to the discharge state and the voltage value is measured every 5 seconds. This process is done for all combinations of S2 and S3 switches (4 times in total).

1- Turn the switch to the charge position and charge the $1000\ \mu F$ capacitor through a $15\ k\Omega$ resistor. Record the charging voltages on the capacitor up to the supply voltage of the capacitor at 5 s intervals from the start in Table 2.8. Then discharge the capacitor by turning the S1 switch to the discharge position and repeat the above procedure.

2- Repeat the same process in Step 1 by placing the capacitor in parallel and making its value $2000\ \mu F$.

3- Repeat the same process by placing the resistors in parallel and setting the capacitor to $1000\ \mu F$.

4- With the values you obtained in Step 1, draw the graph between time and $\ln V_c$ for charging and discharging.

5- In step 1, draw the time variation graphs of the capacitor's charging and discharging voltages on the same axes. and explain the overlap in the graphs with the help of the τ parameter.

6- Find the time corresponding to the value where the voltage drops to $1/e$ in a discharge curve of your choice. This value is your τ value. Assuming that the voltage across the capacitor drops to zero for $t = 4\tau$,

$$\ln V_c = \ln V_0 - \frac{t}{\tau} \tag{2.38}$$

draw the line for values of τ from RC to $4RC$.

Table 2.8

	$C = 1000\mu F$ $R = 15\ k\Omega$				$C = 2000\mu F$ $R = 15\ k\Omega$	
Time (s)	**Charging** $V_c\ (V)$	**Discharging** $V_c\ (V)$	**Charging** $\ln V_c$	**Discharging** $\ln V_c$	**Charging** $V_c\ (V)$	**Discharging** $V_c\ (V)$
5						
10						
15						
20						
25						
30						
35						
40						
45						
50						
55						
60						
65						
70						
75						

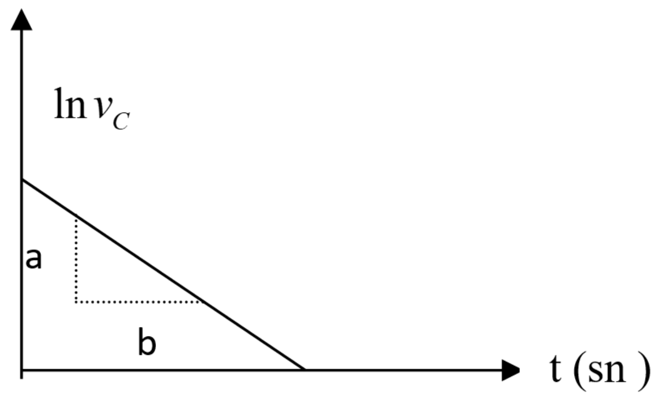


Figure 2.32

As seen in Figure 2.32, with the help of a and b, since it is,

$\tan(180-\alpha)=\frac{a}{b} \Rightarrow \tan(a)=-\frac{a}{b}=-\frac{1}{RC}$

calculate the value of the capacity from the formula below using the value of R on which the capacity discharges.

$C=\frac{b}{a}\frac{1}{R}=$..................... F

Table 2.9

	$C = 1000\mu F$ $R = 7.5\ k\Omega$		$C = 2000\mu F$ $R = 7.5\ k\Omega$	
Time (s)	**Charging** $V_c\ (V)$	**Discharging** $V_c\ (V)$	**Charging** $\ln V_c$	**Discharging** $\ln V_c$
5				
10				
15				
20				
25				
30				
35				
40				
45				
50				
55				
60				
65				
70				
75				

QUESTIONS

1- Which of direct current or alternating current is used in this experiment?

2- What are the initial conditions for the charging and discharging of a capacitor in the equation (**2.27**)?

3- Draw the equivalent circuit for the discharge of a capacitor.

2.2.3.2. PART II: INCREASE AND DECREASE OF CURRENT IN R-L CIRCUIT

MAIN PRINCIPLE

To investigate the change of current over time in the R-L circuit.

EQUIPMENT

Power Supply, Coil Circuit, Ammeter, Stopwatch, Millimeter paper.

THEORY

INCREASE OF CURRENT WITH TIME IN THE R-L CIRCUIT

A circuit containing a resistor and a coil is called an R-L circuit, as shown in Figure 2.33. The coil prevents a rapid change in current: a coil can be very useful if a constant current is desired in the circuit while the emf in the external circuit fluctuates. The resistance R can be an independent element in the circuit, or it can be the resistance of the coil windings. By switching off switch S1, we connect the R-L assembly to a constant source of ε EMF.

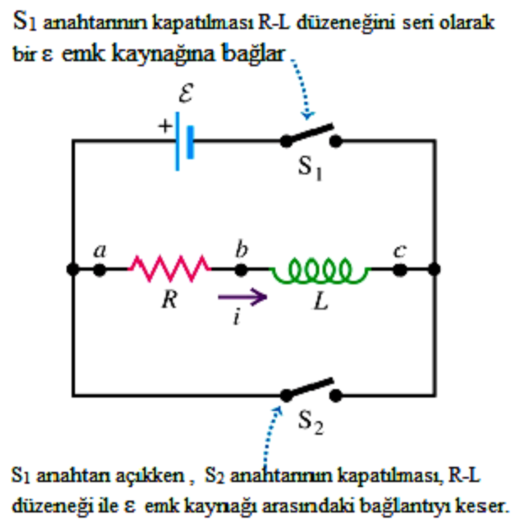


Figure 2.33

Let's assume that both switches are on at the beginning. Let's close the switch S1 at $t = 0$. The current will not rise rapidly from zero to its final value because ${}^{di}/_{dt}$ and the induction EMF in the coil are both infinite. Instead, the current will only start to increase depending on the value of L in the circuit.

Let i be current t seconds after switch S1 is closed and di/dt be the rate of change at that moment. The potential difference between the two terminals of the resistor at that moment is given by V_{ab}, $V_{ab} = iR$ and the potential difference between the terminals of the coil is given by V_{bc}, $V_{bc} =$

$L\,{}^{di}\!/_{dt}$. If the direction of the current flowing through the circuit is as in Figure 2.33 and increasing (note that V_a and V_a are both positive), point a is at a higher potential than point b and therefore at a higher potential than point c. If we apply Kirchhoff's loop rule in the counterclockwise direction on a closed circuit starting from the negative terminal,

$$\varepsilon - iR - L\frac{di}{dt} = 0 \tag{2.39}$$

and so,

$$\frac{di}{dt} = \frac{\varepsilon - iR}{L} = \frac{\varepsilon}{L} - \frac{R}{L}i \tag{2.40}$$

is found. When switch S1 is closed, $i = 0$ and the potential drop across the resistance R is zero. The rate of change of the current at the initial moment,

$$\left(\frac{di}{dt}\right)_{initial} = \frac{\varepsilon}{L} \tag{2.41}$$

As expected, the larger L is, the slower the rate of increase in current. As the current increases, the R/L term increases and the rate of increase in current becomes progressively smaller. This means that the current is now approaching a constant final value I. When the current reaches this value, the rate of increase becomes zero. Hence our equation,

$$\left(\frac{di}{dt}\right)_{final} = 0 = \frac{\varepsilon}{L} - \frac{R}{L}I \Rightarrow I = \frac{\varepsilon}{R} \tag{2.42}$$

The final current I no longer depends on L; I now has the same value as when only the resistor R is connected to the EMF source.

Figure 2.34 shows the variation of the current in time. If the numerator and denominator of the right part of the equation (2.40) are divided by R and rearranged,

$$\frac{di}{i - {}^{\varepsilon}\!/_{R}} = -\frac{R}{L}dt \tag{2.43}$$

is found. If we rename the variables to i' and t', i and t are the upper limits of the integrals. The lower limit values are zero. $t = 0$ corresponds to zero current at the initial instant.

$$\int_0^i \frac{di'}{i' - {}^{\varepsilon}\!/_{R}} = -\int_0^t \frac{R}{L}d\,t' \Rightarrow \ln\left(\frac{i - {}^{\varepsilon}\!/_{R}}{-\varepsilon R}\right) = -\frac{R}{L}t \Rightarrow i = \frac{\varepsilon}{R}\left(1 - e^{-({}^{R}\!/_{L})t}\right) \tag{2.44}$$

is obtained. If the derivative of this expression is taken with respect to time,

$$\frac{di}{dt} = \frac{\varepsilon}{R}e^{-({}^{R}\!/_{L})t} \tag{2.45}$$

is found. At the initial instant $t = 0$, $i = 0$ and $di/dt = \varepsilon/L$, while at the instant $t \to \infty$, $i \to \varepsilon/R$ and $di/dt \to 0$. These results are in agreement with our investigation above. As seen in Figure 2.34, the instantaneous current i first increases rapidly, then increases slowly and asymptotically reaches the final value $I = \varepsilon/R$. At time $t = L/R$, the current increases to $(1 - 1/e)$ or 63% of its final value.

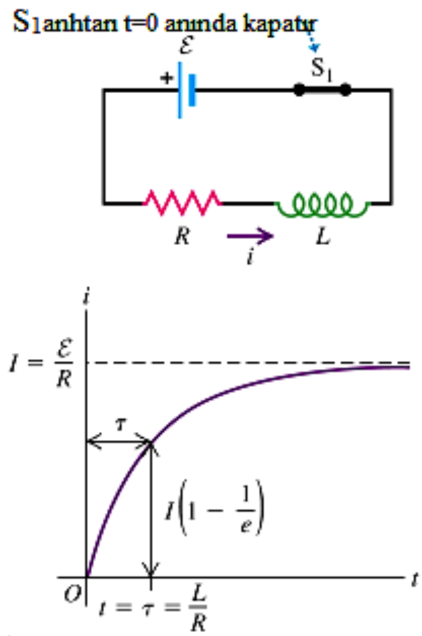


Figure 2.34

Therefore, the L/R value shows how fast the current increases towards the final value. This quantity is called the time constant of the circuit and is known as the induction time constant $\tau_L = L/R$.

At time 2τ the current increases to 86% of its final value; at the time 5τ the current reaches 99.3% of its final value. At the 10τ time, the current reaches 99.995% of its final value. The graph of the current i with respect to the time variable t gives the same shape for all values of L. For a given value of R, the time constant τ is large for large values of L. When L is small, the current rises quickly to its final value; when L is large, the rise is slower.

The law of conservation of energy gives us additional information about the behavior of R-L circuits. The rate of energy transfer from the source to the circuit at any instant is $P = i\varepsilon$. The instantaneous rate of dissipation of energy across the resistor is i^2R and the rate of energy stored in the coil is $iV_{bc} = li\,{}^{di}/_{dt}$. The i^2R part of the $i\varepsilon$ power transferred to the circuit by the power supply is converted into heat on the resistor and the $li\,{}^{di}/_{dt}$ part is stored in the magnetic field in the coil.

ATTENUATION OF CURRENT IN R-L CIRCUIT

Assume that the S1 switch of the circuit in Figure 2.33 is closed for a certain period of time and the current reaches I. Let's remove the emf from the circuit by closing the S2 switch at a moment we set as $t = 0$. (At the same time, the S1 switch is kept open to protect the battery) The current flowing through R and L does not suddenly go to zero. It weakens smoothly as seen in Figure 2.35.

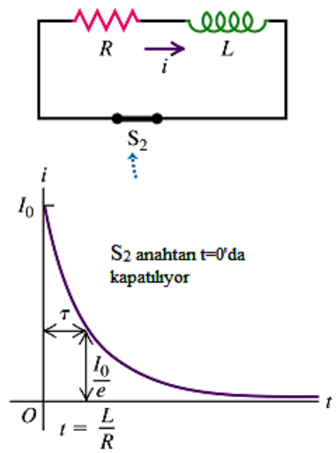


Figure 2.35

If term ε is neglected and Kirchhoff's loop rule is applied,

$$iR = -L\frac{di}{dt} \Rightarrow \frac{R}{L}dt = -\frac{dt}{i} \tag{2.46}$$

is obtained. If this equation is considered that I_0 current passes through the circuit at $t = 0$, if i' and $t'$are taken as variables,

$$\int_{I_0}^{i} \frac{di'}{i'} = -\int_{0}^{t} \frac{R}{L}d\,t' \Rightarrow \,-\ln\left(\frac{i}{I_0}\right) = -\frac{R}{L}t \Rightarrow i = I_0 e^{-(R/L)t} \tag{2.47}$$

is found. The time constant $\tau = L/R$ is the time at which the current drops to $(1/e)$ or 37% of its actual value. It drops to 13.5% in the 2τ time interval and to 0.0045% in the 5τ time. During this attenuation of the current, the energy required to sustain the current is provided by the magnetic energy stored in the magnetic field of the coil.

3. CHAPTER

3.1. BASIC INFORMATION ABOUT MAGNETIC FIELD

MAGNETIC FIELD

MOBILE CHARGES AS A SOURCE OF MAGNETIC FIELD

An electric charge q moving with constant velocity v produces a magnetic field. The electric and magnetic fields produced by the same charge show similarities and interesting differences. Experiments show that the magnetic field $\vec{B}$ is proportional to ${|q|}/{r^2}$ just like the electric field. However, the direction of $\vec{B}$ is not on the line passing from the source to the field point: the magnetic field $\vec{B}$ is perpendicular to the plane formed by this line and the velocity vector v of the particle (Figure 3.1).

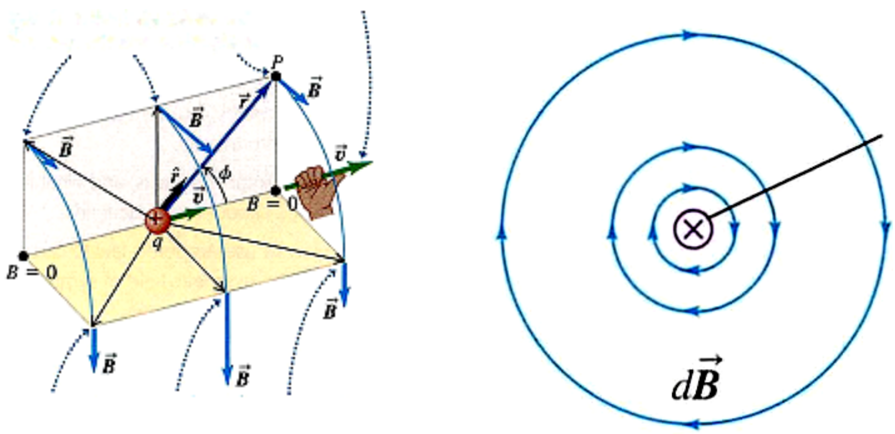


Figure 3.1

Furthermore, the magnetic field magnitude B is proportional to the velocity v of the particle and the sine of the angle φ. Let us define the position of the moving charged particle at any instant as the source point and the field point, we want to calculate as P. Thus, the magnitude of the magnetic field at a point P, given as,

$$B = \frac{\mu_0}{4\pi}\frac{|q| v \sin\varphi}{r^2} \tag{3.1}$$

Here the ratio $\mu_0/4\pi$ is constant, μ_0 is the magnetic permeability of the vacuum. We can express the magnitude and direction of $\vec{B}$ in a single vector equation using vector multiplication. Instead of repeating the expression "direction from the source q to the field point P" each time, let us use the unit vector $\hat{\vec{r}}$ from the source to the field point. This unit vector is obtained by dividing the vector $\vec{r}$ connecting the source to the field point P by the magnitude r. $\hat{\vec{r}} = \vec{r}/_r$

In this case the magnetic field of the moving point charge, $\vec{B}$ can be expressed by,

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q(\vec{v}x\hat{\vec{r}})}{r^2} \tag{3.2}$$

In Figure 3.1-a, the magnetic field is zero everywhere along the line passing through the charged particle in the direction of velocity $\vec{v}$, because $\sin\varphi = 0$ at these points. At a certain distance r from q, $\vec{B}$ has a maximum value at points in a plane perpendicular to the velocity vector $\vec{v}$.

MAGNETIC FIELD OF CURRENT

As in the electric field, magnetic fields have the principle of superposition: The total magnetic field created by more than one moving particle is the vector sum of the fields created by each particle.

Let us calculate the area created by a small current-carrying dl element is shown in Figure 3.1a. The volume of this element is Adl, where A is the cross-sectional area of the conductor. If there are n particles per unit volume and the charge of each particle is q, the total amount of charge moving through the element is;

$$dQ = nqAdl \tag{3.3}$$

The charges moving in this element are equivalent to a single dQ charge moving with a drift velocity V_d. According to Equation (3.1), the magnitude of the magnetic field $d\vec{B}$ generated at any field point P;

$$dB = \frac{\mu_0}{4\pi}\frac{|dQ|V_d\sin\varphi}{r^2} = \frac{\mu_0}{4\pi}\frac{n|q|V_d Adl\sin\varphi}{r^2} \tag{3.4}$$

Since the I current is equal to nAv_d,

$$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\varphi}{r^2} \tag{3.5}$$

is found as. In vector form, the magnetic field of the current element,

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}x\hat{\vec{r}}}{r^3} \tag{3.6}$$

is obtained as. This equation is known as **Biot-Savart**. It is used to find the total magnetic field $\vec{B}$ created by the circuit at any point in space in the current flowing through a closed circuit. For this, we integrate all the elements in the wire over dl.

$$d\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I d\vec{l} x \vec{\hat{r}}}{r^3} \qquad (3.7)$$

Here $\vec{dl}$ is a vector of length dl and in the same direction as the current I in the conductor.

MAGNETIC FIELD CREATED BY A STRAIGHT CONDUCTOR WIRE CARRYING CURRENT

Find the magnetic field $d\vec{B}$ at a point P at a distance x from the center of a conductor of length $2a$ and carrying current I in Figure 3.2 using the **Biot-Savart law**.

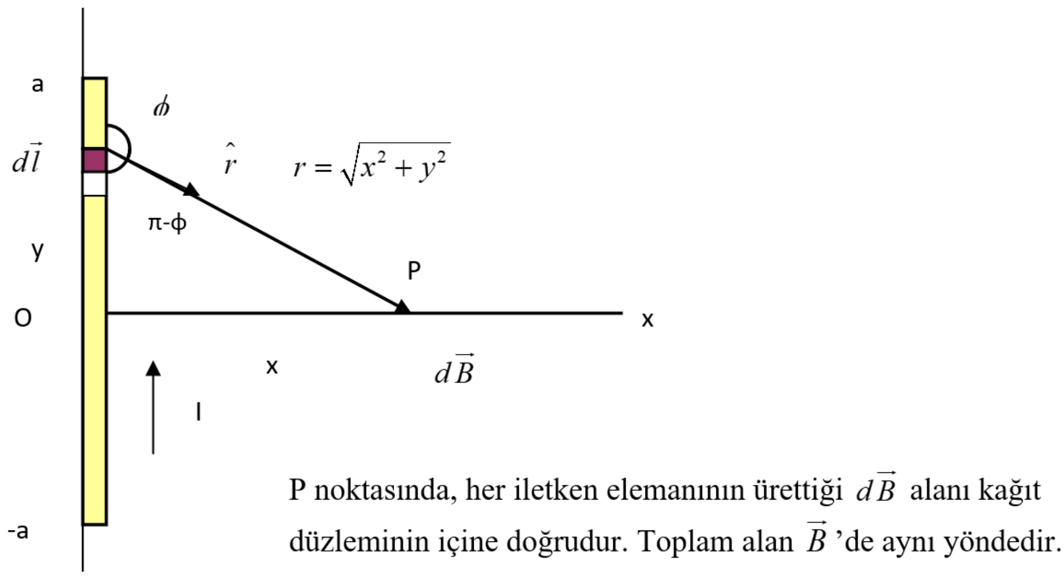


Figure 3.2: Magnetic field of a straight wire carrying a current of length $\mathbf{2a}$

Let us find the magnetic field $d\vec{B}$ created by the conductor element with $dl = dx$. It can be seen from the figure that $r = \sqrt{x^2 + y^2}$ and $\sin\varphi = \sin(\pi - \varphi) = {}^{x}\!/\!\sqrt{x^2 + y^2}$. If the right-hand rule is applied to the vector product $d\vec{l} x \vec{\hat{r}}$, it is seen that the direction of $d\vec{B}$ is towards and perpendicular to the plane of the figure. In addition, the magnetic field produced by all elements of the conductor is in the same direction. Therefore, taking the integral of dB is the same as adding the magnitudes of all dB magnetic fields.

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{+a} \frac{x dy}{(x^2 + y^2)^{3/2}} \qquad (3.8)$$

integral using the relevant variable transformations,

$$B = \frac{\mu_0 I}{4\pi} \frac{2a}{x(x^2 + y^2)} \qquad (3.9)$$

is found. When the length $2a$ of the conductor is much larger than x, the distance of the point P from the wire, $x(x^2+a^2)$ is approximately equal to a; then in the limit $a \to \infty$, equation is reduced to,

$$B = \frac{\mu_0 I}{2\pi x} \tag{3.10}$$

In this case, the magnetic field source has axial symmetry with respect to the y axis. Nothing changes if the wire is rotated about the y axis. Then $\vec{B}$ has the same magnitude at every point along the circle in the plane perpendicular to the conductor wire, with its center on the conductor wire, and the direction of $\vec{B}$ is tangent to this circle at every point.

In the electric field caused by an infinitely long linear charge, the magnitude of the field is proportional to $1/r$. But the shape of the $\vec{B}$ magnetic field lines are completely different from the $\vec{E}$ electric field lines. While electric field lines move away from the positive linear charge distribution towards infinity, magnetic field lines form circles around the current-carrying wire that close in on themselves. Magnetic field lines do not have a starting or ending point. This behavior of magnetic field lines is independent of the shape of the wire generating the magnetic field. This is explained by **Gauss's law** for the magnetic field.

$$B = \oint \vec{B} d\vec{A} = 0 \tag{3.11}$$

this expression indicates that there are no isolated magnetic charges or magnetic monopoles.

MAGNETIC FIELD CREATED BY A CURRENT-CARRYING CIRCULAR RING

Apply the Biot-Savart law to a point P on the axis of a closed circular circuit of radius a and carrying current I by making the shape of the closed current circuit through which q charges flow circular (Figure 3.3).

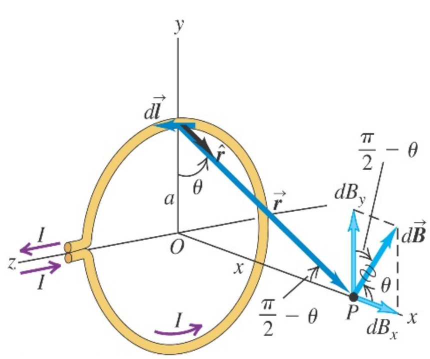


Figure 3.3

The distance of the point P from the center of the circle is x. Since $dl \perp dr$, the magnetic field $\vec{B}$ produced by the element $d\vec{l}$ is in the xy plane. Since $r^2 = x^2 + a^2$, the magnetic field produced by the element $d\vec{l}$,

$$d\vec{B} = \frac{\mu_0}{4\pi}\int \frac{Id\vec{l}x\vec{\hat{r}}}{r^3} = 0 \tag{3.12}$$

$$dB_x = dB\cos\theta = \frac{\mu_0 I}{4\pi}\frac{dl}{(x^2+a^2)}\frac{a}{(x^2+a^2)^{1/2}} \tag{3.13}$$

$$dB_y = dB\sin\theta = \frac{\mu_0 I}{4\pi}\frac{dl}{(x^2+a^2)}\frac{x}{(x^2+a^2)^{1/2}} \tag{3.14}$$

is found. Since the setup has rotational symmetry about the x axis, the total field $\vec{B}$ cannot have a component perpendicular to this axis. This is because each $d\vec{l}$ differential element and its corresponding $d\vec{l}$ element on the other side of the ring are in opposite directions. These two elements contribute equally to the x component of $d\vec{l}$ given by equation (3.14), but since they are opposite to the x axis, they cancel each other out in the sum. Only components parallel to the x axis remain. To get the x component of the total field $\vec{B}$, we integrate equation (3.13) across the circuit.

In this integral, all quantities except dl are constant and are excluded from the integral; in this case

$$B_x = \int \frac{\mu_0 I}{4\pi}\frac{adl}{(x^2+a^2)^{3/2}} = \frac{\mu_0 I}{4\pi}\frac{a}{(x^2+a^2)^{3/2}}\int_0^{2\pi a} dl \tag{3.15}$$

$$B_x = \frac{\mu_0 I a^2}{2(x^2+a^2)^{3/2}} \tag{3.16}$$

MAGNETIC FIELD ALONG THE AXIS OF THE COIL

Suppose that instead of a single turn in Figure 3.3, we have a coil consisting of N turns, all with the same radius. If we keep the turns tight, the distance of the plane of each turn from the field point P is essentially x. In this case, each turn contributes equally to the magnetic field and the total field is the single ring multiplied by N turns.

$$B_x = \frac{N\mu_0 I a^2}{2(x^2 + a^2)^{3/2}} \quad (3.17)$$

The reason for the coefficient N in the equation is that the total field is generated by N concentric circular circuits. If a high magnetic field were to be generated by a single ring, a very strong current would be needed; the wire may not withstand such a high current.

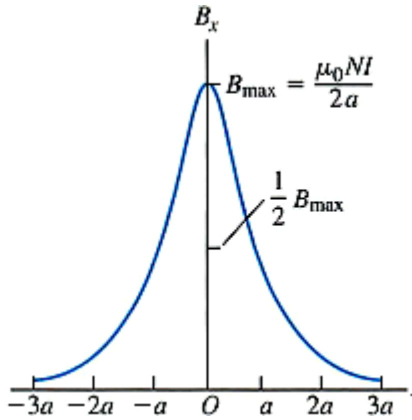


Figure 3.4

Figure 3.4 shows B_x as a function of x. The field strength is maximum at $x = 0$, at the center of the ring or coil.

$$B_x = \frac{N\mu_0 I}{2a} \ (N \ at \ the \ center \ of \ the \ turns) \quad (3.18)$$

AMPERE'S LAW

To find the magnetic field, up to now we have been trying to find the very small $\vec{B}$ magnetic fields of the current element and then add all the $\vec{B}$ to get the total field. For highly symmetric charge distributions in electric field problems, we have been using Gauss's law. Similarly, there is a similar law that allows us to calculate the magnetic field generated by highly symmetric current distributions. This law, known as **Ampere's law** is expressed by,

$$\oint \vec{B}.d\vec{l} = \mu_0 I \quad (3.19)$$

To calculate this integral, we divide the curve into infinitesimal $d\vec{l}$ pieces, calculate the $\vec{B}.d\vec{l}$ scalar product on each piece and add the products. Since in general the value of $\vec{B}$ changes from one point to another, at each $d\vec{l}$ position we must use the value of $\vec{B}$ at that point. An alternative notation can be given as follows.

$$\oint \overrightarrow{B_{\parallel}}.d\vec{l} = \mu_0 I \quad (3.20)$$

It is the component of $\vec{B}$ parallel to $d\vec{l}$. The circle symbol on the integral indicates that this integral is taken over a closed curve with the same starting and ending points.

MAGNETIC FIELD OF THE SOLENOID

A solenoid consists of a wire wound in a helix on a cylinder. Its cross-sectional area is usually circular. Since there are hundreds or thousands of tightly wound windings, each winding is treated as a circular ring. The same I current flows through all the windings of the solenoid in Figure 3.5 and the total $\vec{B}$ magnetic field at each point is the vectorial sum of the magnetic fields produced by all the windings.

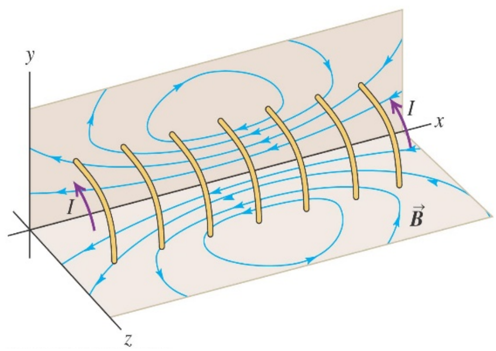


Figure 3.5

Figure 3.5 shows the field lines in the xy and xz planes. We draw the set of field lines equidistant from each other in the center of the solenoid. According to precise calculations, in a long tightly wound solenoid, half of these field lines come out of the ends of the solenoid, and the other half leak out through the windings somewhere between the center and the end.

Near the center of the coil, the field lines are approximately parallel to each other; $\vec{B}$ indicates that the magnetic field is uniform in this region. Outside the solenoid, the field lines diverge and spread over a wide area and the magnetic field weakens. If the length of the solenoid is much greater than the diameter of the cross-sectional area and the solenoid is tightly wound, the magnetic field is almost uniform and parallel to the axis in the cross-sectional area near the center inside the solenoid; the external magnetic field near this center point is too weak.

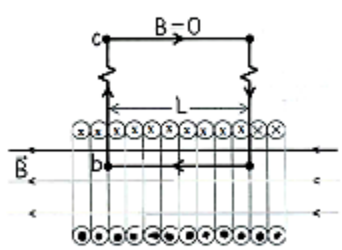


Figure 3.6

Using Ampere's law, a magnetic field can be found at or near the center of such a long solenoid. There is a high symmetry in the solenoid. Inside the solenoid $\vec{B}$ is uniformly distributed and outside the magnetic field is approximately zero. For the magnetic field inside the solenoid, we therefore use a suitable integral curve from Ampere's law. Figure 3.6 shows the integral curve consisting of the magnetic field and the rectangle $abcd$; the edge ab of length L is parallel to the axis of the solenoid. The edges bc and da are assumed to be very long in order to show the edge cd very far away.

Due to symmetry, the $\vec{B}$ field is parallel to this edge along edge ab and its magnitude is constant. When applying Ampere's law integral we move along edge ab in the direction of $\vec{B}$, where $B_{\parallel} = +B$, so

$$\int_a^b \vec{B}.d\vec{l} = BL \tag{3.21}$$

along the edge of bc and da, $B_{\parallel}$ is zero. Because $\vec{B}$ is perpendicular to this edge and $B_{\parallel}$ is zero along the cd edge. Because $\vec{B} = 0$. In this case, the integral $\int \vec{B}.d\vec{l}$ is equal to BL along the entire self-enclosed path.

The number of turns along the length L is equal to nL. Each turn passes through the rectangle $abcd$ once and carries a current I, where I is the current in the turns. The total current passing through the rectangle is $I_{internal} = nLI$. According to Ampere's law, since $\int \vec{B}.d\vec{l}$ is positive, $I_{internal} = nLI$ must also be positive. Then the direction of the current passing through the rectangle is shown in Figure 3.6. Ampere's law also gives the magnitude of $\vec{B}$:

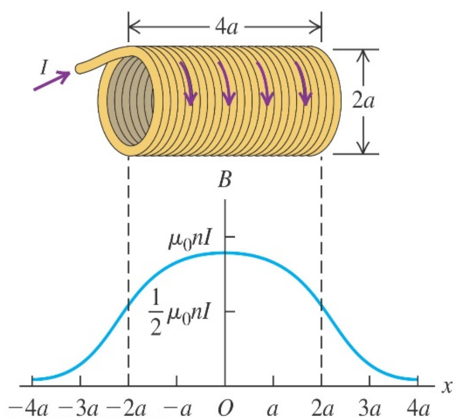


Figure 3.7

$$BL = \mu_0 nLI \tag{3.22}$$

$$B = \mu_0 nI \tag{3.23}$$

The edge ab need not be on the axis of the solenoid. This calculation also shows that the magnetic field inside the solenoid is uniform across the entire cross-section. Figure 3.7 shows the variation of the magnetic field at points on the axis of a solenoid of length $4a$ and radius a.

INDUCTION EXPERIMENTS

In the 1830s, **Michael Faraday** conducted extraordinary experiments at the Smithsonian Institute on the emf produced by magnetic attraction. Figure 3.8 shows one of these experiments. In Figure 3.8-a, a coil of wire is connected to a galvanometer. The galvanometer shows no current when the nearby magnet is stationary. But when the magnet is moved closer to (moved away from) the coil, we see current flowing through the galvanometer (Figure 3.8-b). Although no power supply is connected to the circuit, current flows through the circuit.

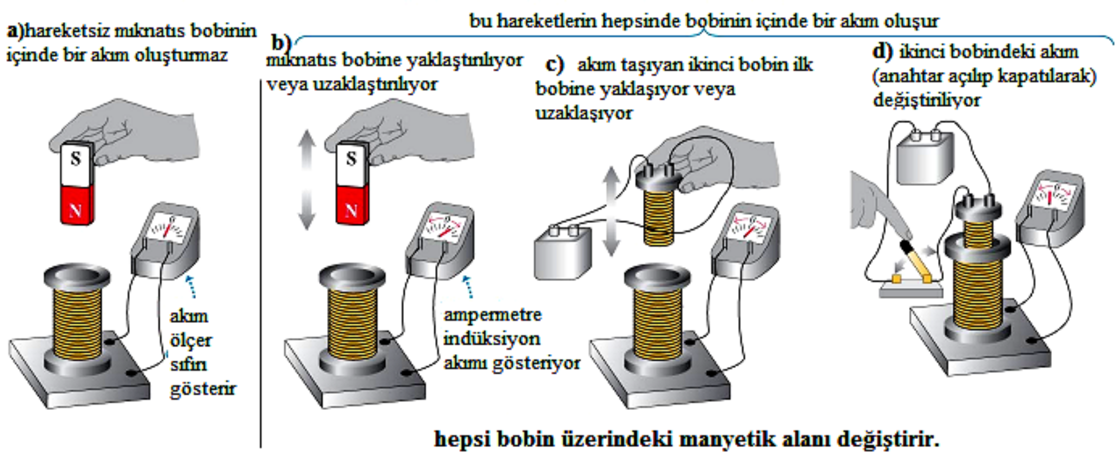


Figure 3.8: Experimental proof of the induction current phenomenon

In Figure 3.8-c, when a second coil carrying current instead of a magnet is moved closer or further away from the coil connected to the galvanometer, the same effect is observed, i.e. current passes through the galvanometer. The events in Figure 3.8-b,c are the events in which current passes through the galvanometer only when the magnet or the coil is moved. In Figure 3.8-d, if we place the second coil inside the coil connected to the galvanometer and leave it still, it is seen that no current passes through the galvanometer. If the current to the first coil is cut off with the help of a switch, it is seen that the current in the galvanometer suddenly increases and reaches zero again, and when the switch is turned on again, it is seen that the current in the galvanometer suddenly changes in the opposite direction to its previous movement and reaches zero again. The common feature of all these experiments is the changing Φ_B magnetic flux in the coil connected to the galvanometer. In each case, the magnetic flux in the coil changes as the magnetic field changes with time or as the coil is moved in a non-uniform magnetic field.

FARADAY's LAW

Considering these experiments, Faraday concluded that "**the variation of the OOB magnetic flux passing through the coil with time produces induction electromotive force (IEMF) in the coil**". For a very small slice of surface $d\vec{A}$ in the $\vec{B}$ magnetic field, the magnetic flux $d\Phi_B$ passing through this surface is: $d\Phi_B = \vec{B}.d\vec{A} = B_\perp dA = BdA\cos\varphi$.

Here $B_\perp$ is the component of $\vec{B}$ perpendicular to the surface element and φ is the angle between $\vec{B}$ and $d\vec{A}$. The total magnetic flux Φ_B through the finite surface is equal to the integral of this expression over the surface.

$$\Phi_B = \int \vec{B}.d\vec{A} = \int BdA\cos\varphi \tag{3.24}$$

In the final form of Faraday's law of induction, "**the induction emf in a closed circuit is equal to the negative of the rate of change with time of the magnetic flux passing through this circuit.**"

$$\varepsilon = -\frac{d\Phi_B}{dt} \; (Faraday's\ induction\ law) \tag{3.25}$$

LENZ'S LAW

H.F.E. Lenz, a Russian scientist, independently repeated many of the phenomena discovered by **Faraday** and **Henry**. He presented a method that makes it easy to find the direction of the induction current or emf. Accordingly, he expressed Lenz's law as "**the direction of any magnetic induction event is opposite to that of the effect which produced it**". This is the meaning of the minus sign in Faraday's law.

RECIPROCAL INDUCTION

We know that due to the current in one of two wires carrying a constant current, a magnetic field is generated around the wire. If one of the circuits has a time-varying current, an additional induction occurs between the two circuits. Consider two coils with adjacent conducting wire turns as in Figure 3.9. The current flowing through coil 1 produces a $\vec{B}$ magnetic field around it and hence an Φ_B magnetic flux flowing through coil 2. According to **Faraday's law**, this generates an induction EMF in coil 2.

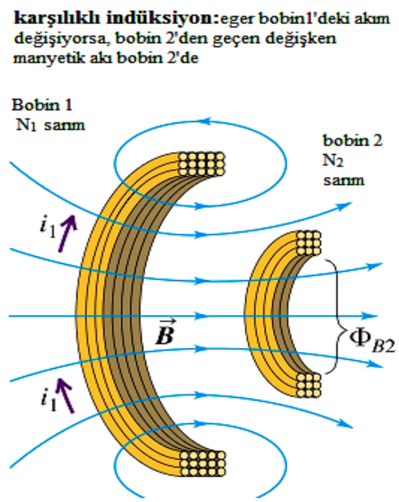


Figure 3.9

Thus, current flowing through one of the circuits causes an induction current in the other circuit (**lowercase letters are used to describe the time-varying physical quantities, and lower indices are used to describe the circuit**). The current i_1 in coil 1 with turns N_1 generates a magnetic field around it and some of these field lines pass through coil 2 with turns N_2. Let us define the magnetic flux arising from the current i_1 in coil 1 and passing through each wire of coil 2 by Φ_{B_2}. Since the $\vec{B}$ magnetic field in the region of the second coil is proportional to i_1, the magnetic flux Φ_{B_2} passing through coil 2 is also proportional to the current i_1. When the current i_1 changes, the magnetic flux Φ_{B_2} also changes; this changing flux causes an induction emission ε_2 in coil 2, defined by the following equation.

$$\varepsilon_2 = -N_2 \frac{d\Phi_{B_2}}{dt} \tag{3.26}$$

Let us define the ratio of Φ_{B_2} to i_1 as $\Phi_{B_2} = (constant)\ x\ i_1$. However, it is more convenient to use the number of windings N_2 in the relation instead. For the mutual induction of the two coils we can write the following equation using the proportionality constant M_{21}.

$$N_2\Phi_B = M_{21}i_1 \tag{3.27}$$

If we take the derivative of both sides with respect to time to make it similar to the equation (3.26),

$$N_2 \frac{d\Phi_{B_2}}{dt} = M_{21}\frac{di_1}{dt} \tag{3.28}$$

If this expression is substituted in the equation (3.26),

$$\varepsilon_2 = -M_{21}\frac{di_1}{dt} \tag{3.29}$$

is found. That is, the variation of the current i_1 through coil 1 with time affects the nearby coil 2 in proportion to the variation of the current i_1 with respect to time (di_2/dt) and produces an induction EMF of one-tenth.

Equation (3.25) can also be written as follows.

$$M_{21} = \frac{N\Phi_{B_2}}{i_1} \tag{3.30}$$

If the coils are in free space, the magnetic flux Φ_{B_2} through each turn of coil 2 is directly proportional to the current i_1. Therefore, the mutual induction constant M_{21} depends only on the geometry of the coils. If there is a magnetic material in the environment, M_{21} also depends on the magnetic property of the material.

If we do the opposite, that is, a time-varying current i_2 in coil 2 results in a time-varying magnetic flux Φ_{B_1} and an induction emf ε_1 in coil 1. In this case, the mutual induction constant M_{12} will be different from M_{21}. This is because the two coils are not identical and the magnetic flux passing through them cannot be the same. However, even if the coils are not symmetrical, M_{12} and M_{21} are always equal to each other. We simply call this common value mutual induction and denote it by M.

$$\varepsilon_1 = -M\frac{di_2}{dt}, \varepsilon_2 = -M\frac{di_1}{dt} \Rightarrow M = \frac{N_2\Phi_{B_2}}{i_1} = \frac{N_1\Phi_{B_1}}{i_2} \tag{3.31}$$

The SI unit of the reciprocal induction constant is **henry** (1H) in honor of **Joseph Henry**, one of the scientists who discovered electromagnetic induction.

$$1H = 1\,{}^{Wb}/_{A} = 1\,{}^{Vs}/_{A} = 1\Omega s = 1\,{}^{J}/_{A^2}$$

SELF-INDUCTION AND INDUCTANCE

Current flowing through a circuit generates a magnetic field that leads to a magnetic flux flowing through the same circuit; when the current changes, this flux also changes (Figure 3.10).

Therefore, in any circuit carrying a time-varying current, an induction emission occurs due to the change in the circuit's own magnetic field. Such an induction EMF is called **self-induction EMF**. Self-induction EMF can occur in any circuit because there is always a magnetic flux, even if small, passing through a closed current-carrying circuit.

In Figure 3.10, a coil of N turns wires increases the self-induction power. As a result of the passing current i, the average value of magnetic flux passing through each winding of the coil will be Φ_B. We define the self-induction L as follows.

$$L = \frac{N\Phi_B}{i} \tag{3.32}$$

The SI unit for self-induction is **henry**. If we consider the above expression that the current changes with time and the Φ_B magnetic flux also changes with time,

$$N\frac{d\Phi_B}{dt} = L\frac{di}{dt} \tag{3.33}$$

is obtained. If substituted in the $\varepsilon = -N\, {}^{d\Phi_B}\!/_{dt}$ equation in Faraday's law,

$$\varepsilon = -L\frac{di}{dt} \tag{3.34}$$

is found.

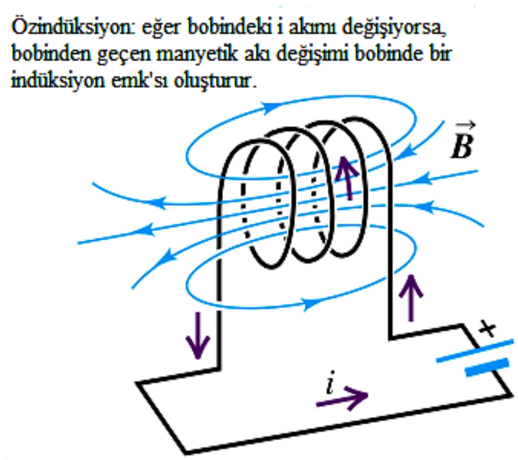


Figure 3.10

MAXWELL'S EQUATIONS FOR ELECTROMAGNETISM

Maxwell's equations, which give the relations between electric and magnetic fields and their sources in a single package, consist of four equations.

Two of Maxwell's equations involve the integration of $\vec{E}$ and $\vec{B}$ over a closed surface. The first is simply Gauss's law for the electric field, which states that "**the integral of $E_\perp$ over any closed surface is equal to $(1/\varepsilon)$ multiplied by the total charge $Q_{internal}$ inside the surface**".

$$\oint \vec{E}.d\vec{A} = \frac{Q_{internal}}{\varepsilon_0}, (Gauss\ law\ for\ \vec{E}) \tag{3.35}$$

The second, similar relation for magnetic fields, states that the surface integral of $E_\perp$ over a closed surface is always zero. This means that there cannot be magnetic monopoles (magnetic charges) acting as sources of magnetic fields.

$$\oint \vec{B}.d\vec{A} = 0, (Gauss\ law\ for\ \vec{B}) \tag{3.36}$$

The third expression is Ampere's law, whose Maxwell's equation includes the displacement current. The conduction current i_c and the displacement current $i_D = \varepsilon_0 \, {}^{d\Phi_E}/_{dt}$ act as a source of magnetic field.

$$\oint \vec{B}.d\vec{l} = \mu_0 \left(i_c + \varepsilon_0 \frac{d\Phi_E}{dt} \right), (Ampere's\ law) \tag{3.37}$$

The fourth equation is Faraday's law and says that a changing magnetic field or flux will cause an induction emission.

$$\oint \vec{E}.d\vec{l} = -\frac{d\Phi_E}{dt}, (Faraday's\ law) \tag{3.38}$$

3.1.1. INVESTIGATION OF THE MAGNETIC FIELD OF A SOLENOID

3.1.1.1. PART I: MAGNETIC FIELD OUTSIDE THE SOLENOID

MAIN PRINCIPLE

Calculation of the magnetic field generated by a solenoid at a point on its axis using direct current.

EQUIPMENT

Multi-turn Coil, Power Supply, Connection Cable, Compass

Figure 3.11: Experimental setup

THEORY

An electric charge in motion generates a magnetic field in the surrounding space. Within the magnetic field, a second charge in motion is influenced by a force. The unit of magnetic field is Tesla in the SI unit system. It is defined as,

$$1\,Tesla = 1\,{}^{Newton}/_{Ampere.Meter}$$

A device through which current i flows and which is tightly wound along a helix is called a solenoid. The length of the helix is taken to be quite large compared to its radius. A winding close to a turn in the solenoid

The observer at the point does not realize that the single turn is along a curve. To this observer, the wire looks like a long wire in terms of its magnetic properties. Therefore, the magnetic field lines generated by a single turn are approximately like circles with the same center. The total magnetic field generated in the solenoid is equal to the sum of the magnetic fields generated by the individual turns.

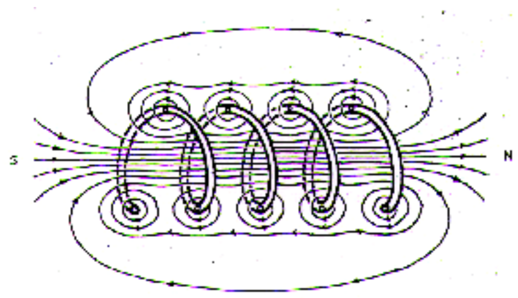


Figure 3.12: Turns and magnetic field lines of a solenoid

A detailed look at Figure 3.12 shows that the magnetic field between the windings is mutually destructive. At points far enough away from the windings, the magnetic field $\vec{B}$ is parallel to the axis of the solenoid.

A very densely wound solenoid looks like a cylindrical sheet through which current flows. The field at point P in Figure 3.12 is generated by the upper windings of the solenoid. In this region, the field is directed to the left. The field created by the lower windings of the solenoid is directed to the right and is such that they destroy each other. If the number of windings increases and the windings become denser, it approaches the ideal solenoid, which is characterized by a cylindrical layer through which current flows. The magnetic field $\vec{B}$ at the outer points of such a long cylindrical current system is approximately zero. By taking the length of the solenoid longer than its diameter, it is correct to assume that the magnetic field at the outer points is zero. The places where the field is zero are outside the solenoid, away from the ends, and close to the midpoint. If the length of the solenoid is not greater than its radius, the magnetic field lines are as shown in Figure 3.13. The frequency of magnetic lines in the center of the solenoid indicates that the magnetic field in the center is more intense than the magnetic field at the outer points. The direction of the magnetic field generated by a solenoid in the direction of its axis is also in the direction of the axis of the solenoid. Its direction is determined by the right-hand rule depending on the direction of the current flowing through the solenoid.

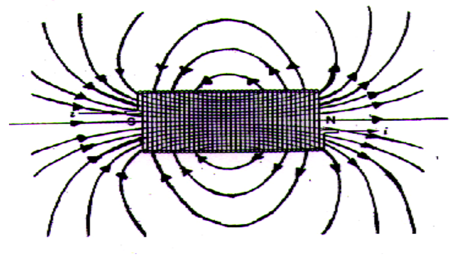


Figure 3.13

Figure 3.13 shows the magnetic field $\vec{B}$ generated by the solenoid at point O in the direction of its axis. However, if the B_y the horizontal component of the earth's magnetic field is also considered as in Figure 3.14, the magnetic field at point O will be B_r. At point O, when there is no magnetic field generated by the solenoid, a compass needle at this point will point in the north-south direction. If current is passed through the solenoid and a magnetic field $\vec{B}$ is generated at this point, the compass needle will deviate from its initial position by angle α and the total magnetic field will take the direction B_r.

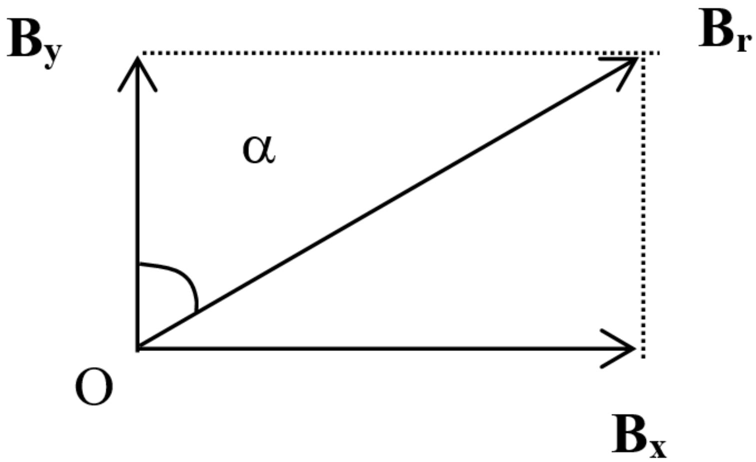


Figure 3.14

From Figure 3.14; $\tan\alpha = B/B_y$ and $B = B_y \tan\alpha$ can be written. Since it is known that the horizontal component of the Earth's magnetic field is $B_y = 26\,\mu T$, the intensity of the magnetic field B to be generated by the coil at the point in question at the angle of deviation α of the compass can be calculated.

SETUP AND PROCEDURE

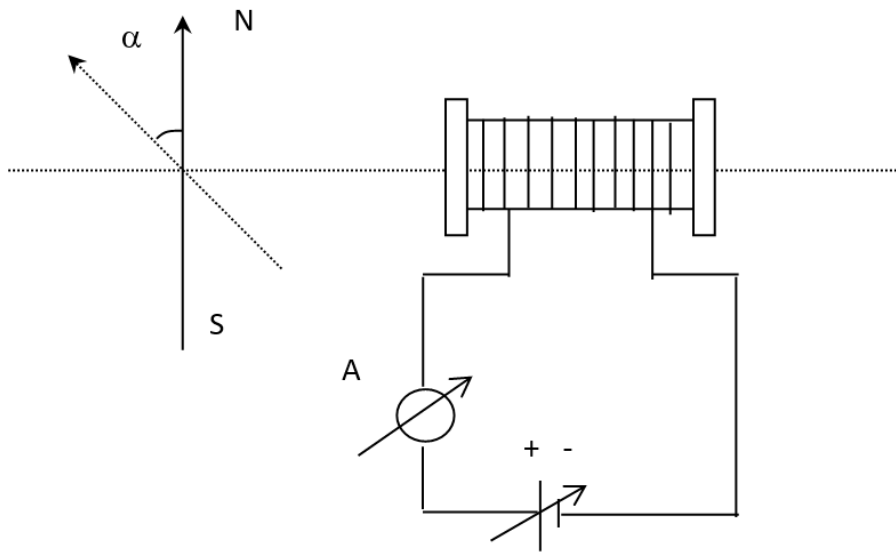


Figure 3.15

The experimental setup consists of a solenoid, an ammeter, a variable direct current source, and a compass needle as shown in Figure 3.15.

1- The compass needle is placed in the north-south direction pointing to zero degrees and the coil is placed in the west-east direction with its axis pointing in the compass direction.

2- Maximum current i ($i \cong 1.5\ A$) is passed through the circuit and the compass-coil distance is adjusted so that the needle of the compass needle deviates about 70° and the current is reset.

3- The current values shown in the table are passed through the current source and solenoid and the deviation angles α on the compass needle are measured and recorded in the Table 3.1. From here $I = f(\alpha)$ graph is drawn as shown in Figure 3.16.

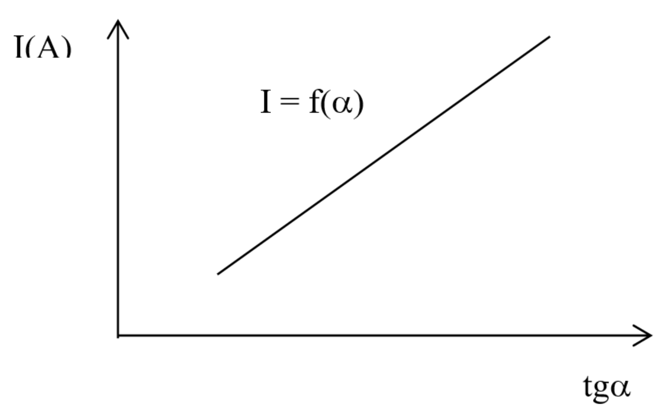


Figure 3.16

4- Taking $B_y = 26\ \mu T$ from the expression $B = B_y \tan\alpha$, the intensity of the magnetic field B created by the solenoid at the point in question corresponding to each current value is determined and recorded in the Table 3.1.

Table 3.1

I (A)	α	$\tan\alpha$	B (μT)
0			
0.2			
0.4			
0.6			
0.8			
1			
1.2			
1.4			
1.6			
1.8			
2.0			

3.1.1.2. PART II: MAGNETIC FIELD INSIDE THE SOLENOID

MAIN PRINCIPLE

1. Measurement of the magnetic flux density in the center of different rings with a Hall bar and determination of its dependence on radius and number of turns.

2. Finding the magnetic field constant.

3. Measurement of magnetic field intensities along the axis of long coils and comparison with theoretical values.

EQUIPMENT

Coils of various sizes and turns, Power Supply, Ruler, Tesla meter, Connection Cable, Ammeter, Voltmeter

Figure 3.17: Biot-Savart experiment setup

THEORY

Magnetic flux in a vacuum according to Maxwell's equations:

$$\oint \vec{B}.d\vec{l} = \mu_0 I \tag{3.39}$$

where l is the length of the closed ring, B and μ_0 are the magnetic flux density and the permittivity of the vacuum respectively. Theoretically, the permittivity of the cavity is $\mu_0 = 1.2566x10^{-6}\ {}^{H}/_{m}$. According to the notation in Figure 3.18 using the Biot-Savart law,

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{l}x\vec{r}}{r^3} \tag{3.40}$$

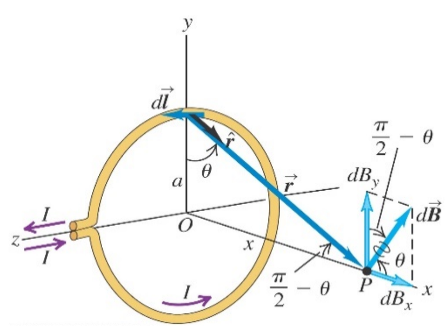


Figure 3.18: Magnetic field vectors at a point on the wire ring axis

The result of the calculation is dB as follows.

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{dl}{a^2 + x^2} \tag{3.41}$$

dB can be solved for the angular dB_y and vertical axis dB_x components.

The dB_x components of each turn are in the same direction as the length unit element dl and can therefore be summed, but the angular components dB_y cancel each other out.

Hence,

$$B_y(x) = 0 \tag{3.42}$$

$$B(x) = B_x(x) = \frac{\mu_0 I}{2} \frac{a^2}{(a^2 + x^2)^{3/2}} \tag{3.43}$$

If a small number of closely similar-sized rings are used, the magnetic flux density can be found by multiplying n by the number of turns.

1- At the center of the rings ($x = 0$)

$$B(0) = \frac{\mu_0 n I}{2a} \tag{3.44}$$

2- Ampere's law is applied to calculate the magnetic flux density of coils of length l and the number of turns n consisting of homogeneously wound windings.

$$B(x = 0) = \mu_0 n I, \qquad (at\ the\ center) \tag{3.45}$$

and

$$B = \frac{\mu_0 n I}{2}, \qquad (at\ the\ ends\ of\ the\ solenoid\ ) \tag{3.46}$$

SETUP AND PROCEDURE

1- The experimental setup is prepared as shown in Figure 3.17. With the help of the power supply, the desired constant current of 0.5 $Amperes$ is applied to the circuit.

2- For multi-wound coils, the magnetic flux densities are measured at various distances by moving the axis bar along the x-axis. ($at\ I = 1\ ampere$)

3- Draw the $B(x)$ curves between the magnetic flux densities B and x for coils with different turns for fixed diameter $2a$ and length l ($l = 160\ mm, 2a = 26\ mm, n = 75,150,300$)

4- Draw the $B(x)$ graphs of the coils with a fixed number of turns n and the same length and different radius a. ($l = 160\ mm, n = 300, 2a = 26, 33, 41\ mm$)

5- Using the table, you created in item 3, determine the constant μ_0 by plotting $B(n)$ for different turns for a fixed distance x and using the slope of the line you found.

Table 3.2

	n: 75 turns 2a: 26 mm	**n: 150 turns 2a: 26 mm**	**n: 300 turns 2a: 26 mm**	**n: 300 turns 2a: 33 mm**	**n: 300 turns 2a: 41 mm**
$x\ (mm)$	$B_x\ (mT)$	$B_x\ (mT)$	$B_x\ (mT)$	$B_x\ (mT)$	$B_x\ (mT)$
0					
10					
20					
30					
40					
50					
60					
70					
80					
90					
100					
110					
120					
130					

140					
150					
160					
170					
180					
190					
200					
210					
220					

Questions

1) Why do we use the Biot-Savart law?

2) Does electric current create a magnetic field?

3) Does a magnetic field create an electric field?

4) Write three units of the magnetic field.

5) Explain the constant μ_0.

3.1.2. MAGNETIC INDUCTION

MAIN PRINCIPLE

The induction EMF of a time-varying magnetic field in another coil inside a long coil,

1- Magnetic field as a function of the intensity of $\vec{B}$,

2- Function of the frequency of the magnetic field,

3- Function the number of turns in the coil

4- To study as a function of the diameter of the induction coil.

EQUIPMENT

Power supply, Function Generator, Ammeter, Voltmeter, Coils, Connection cable

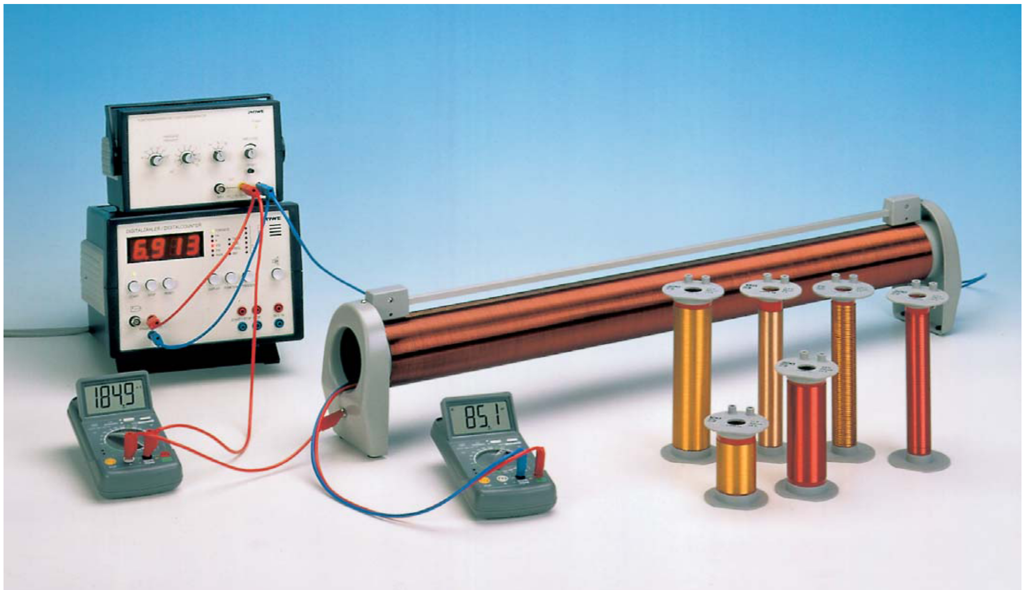


Figure 3.19: Induction Experimental Setup

THEORY

Induction experiments

In the 1830s, Michael Faraday conducted extraordinary experiments at the Smithsonian Institute on the emf produced by magnetic attraction. Figure 3.20 shows one of these experiments. In Figure 3.20-a, a coil of wire is connected to a galvanometer. The galvanometer shows no current when the nearby magnet is stationary. But when the magnet is moved towards or away from the coil, we see current flowing through the galvanometer (Figure 3.20-b). Although no power supply is connected to the circuit, current flows through the circuit.

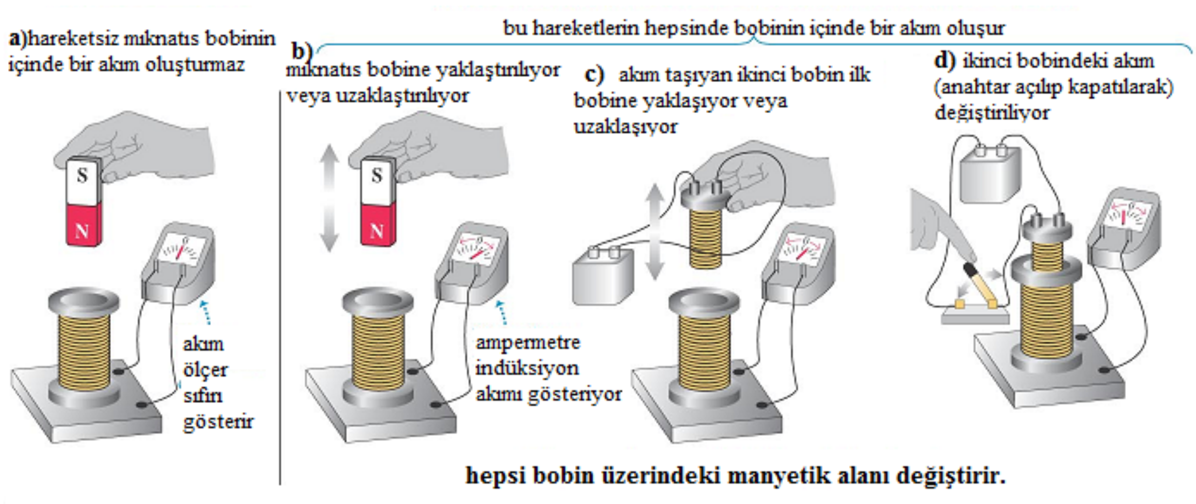


Figure 3.20: Experimentally proving the induction current phenomenon

In Figure 3.20-c, when a second coil carrying current instead of a magnet is moved closer or further away from the coil connected to the galvanometer, the same effect is observed, that is, current passes through the galvanometer. The events in Figure 3.20-b,c are the events in which current passes through the galvanometer only when the magnet or the coil is moved. In Figure 3.20-d, if we place the second coil into the coil connected to the galvanometer and leave it motionless, it is seen that no current passes through the galvanometer. If the current to the first coil is cut off with the help of a switch, it is seen that the current in the galvanometer suddenly increases and reaches zero again, and when the switch is turned on again, this time the current in the galvanometer suddenly changes in the opposite direction to the previous movement and reaches zero again.

The common feature of all these experiments is the changing Φ_B magnetic flux inside the coil connected to the galvanometer. In each case, the magnetic flux in the coil changes as the magnetic field changes with time or as the coil is moved in a non-uniform magnetic field.

Faraday's law

Considering these experiments, Faraday concluded that "the variation of the Φ_B magnetic flux passing through the coil with time releases induction electromotive force (IEMF) in the coil".

For a very small slice of surface dA in a magnetic field $\vec{B}$, the magnetic flux $d\Phi_B$ passing through this surface;

$$d\Phi_B = \vec{B}.d\vec{A} = B_\perp dA = BdA\cos\varphi \tag{3.47}$$

Where $B_\perp$ is the component of $\vec{B}$ perpendicular to the surface element and φ is the angle between $\vec{B}$ and $d\vec{A}$. The total magnetic flux Φ_B through the finite surface is equal to the integral of this expression over the surface.

$$\Phi_B = \int \vec{B}.d\vec{A} = \int BdA\cos\varphi \tag{3.48}$$

In the final form of Faraday's law of induction, "**the induction emf in a closed circuit is equal to the negative of the rate of change with time of the magnetic flux passing through this circuit.**"

$$\varepsilon = -\frac{d\Phi_B}{dt}, \qquad (Faraday's\ induction\ rule) \tag{3.49}$$

Lenz's law

H.F.E. Lenz, a Russian scientist, independently repeated many of the phenomena discovered by Faraday and Henry. He presented a method that makes it easy to find the direction of the induction current or emf. Accordingly, he expressed Lenz's law as "**the direction of any magnetic induction event is opposite to that of the effect which produced it**". This is the meaning of the minus sign in Faraday's law.

A long, thin cross-sectional area A and a conductor with a number of n turns are wound in a ring. A current I in the solenoid turns causes a magnetic field $\vec{B}$ along the solenoid axis. The magnitude of this field was calculated in the theoretical information as $B = \mu_0 nI$. Let us have a second ring with n_1 turns that we will put inside this ring. If we choose the surface vector $\vec{A}$ in the direction of $\vec{B}$, the Φ_B flux through the ring,

$$\Phi_B = BA = \mu_0 nIA \tag{3.50}$$

is found. When the flux I of the solenoid changes with time, the Φ_B magnetic flux also changes and according to Faraday's law, the induction emf in the second loop is expressed as follows.

$$\varepsilon = -n_1\frac{d\Phi_B}{dt} = -\mu_0 nA\frac{dl}{dt} \tag{3.51}$$

Since the current I flowing through the solenoid varies with time if the expression $I = I_0\sin\omega t$ is substituted equation above,

$$\varepsilon = -\mu_0 nn_1 A\omega I_0\cos\omega t \tag{3.52}$$

is found. This expression gives the induction voltage of the small coil inside the coil.

SETUP AND PROCEDURE

PART I:

Determination of the induction EMF of a time-varying magnetic field in another coil inside a long coil as a function of the magnetic field B.

1- Set up the experimental setup in Figure 3.19. Measure the primary coil's alternative current and induction voltage with a multimeter.

2- Insert the induction coil with $n = 300$ turns and $d = 41\ mm$ diameter into the first long coil as a secondary coil.

3- Fix the current frequency of the primary coil at 10 kHz.

4- Measure the induction voltage of the secondary coil for the magnitudes of the current values of the primary coil in the table below and record them in Table 3.3.

Table 3.3: $\boldsymbol{f = 10KHz}$, $\boldsymbol{n = 300\ turns}$ **and** $\boldsymbol{d = 41\ mm}$

$\boldsymbol{I_0\ (mA)}$	$\boldsymbol{\varepsilon\ (mV)}$
2.0	
5.0	
10.0	
15.0	
20.0	
25.0	

5- Draw the graph of the change in $\varepsilon(I_0)$ using $\log \varepsilon$ and $\log I_0$. Explain what the slope of the line you found means to you.

$$\varepsilon = -\mu_0 n n_1 A \omega I_0 \cos \omega t \tag{3.53}$$

$$\varepsilon = k{I_0}^m \Rightarrow \log \varepsilon = \log k + m \log I_0 \tag{3.54}$$

PART II:

Determination of the induction voltage as a function of the number of turns of the induction coil.

1- Set up the experimental setup in Figure 3.19. Keep the current passing through the primary coil as $I_0 = 25\,mA$, the frequency is fixed at $10\,kHz$ and measure the induction voltages using secondary coils with a radius of $41\,mm$ and turns $n = 300, 200, 100$ and record them in Table 3.4 below.

Table 3.4: $\boldsymbol{I_0 = 25\,mA, f = 10\,kHz}$ **and** $\boldsymbol{d = 41\,mm}$

$\boldsymbol{n}$	$\boldsymbol{\varepsilon\,(mV)}$
300	
200	
100	

2- Draw the graph expressing the change in $\varepsilon(n)$ using $\log\varepsilon$ and $\log n$ values. Explain what the slope of the line means to you.

$$\varepsilon = -\mu_0 n n_1 A \omega I_0 \cos\omega t \tag{3.55}$$

$$\varepsilon = k n^m \Rightarrow \log\varepsilon = \log k + m\log n \tag{3.56}$$

PART III:

Determination of the induction voltage as a function of the diameter of the induction coil.

1- Set up the experimental setup in Figure 3.19. Keep the current passing through the primary coil constant at I0=25mA and the frequency fixed at 10 kHz and measure the induction voltages using secondary coils of different diameters, each with 300 turns, and record them in Table-III below.

Table 3.5: $\boldsymbol{I_0 = 25\,mA, f = 10\,kHz}$ **and** $\boldsymbol{n = 300}$

$\boldsymbol{n}$	$\boldsymbol{\varepsilon\,(mV)}$
300	
200	
100	

2- Using $\log \varepsilon$ and $\log d$ values, draw the graph expressing the change in $\varepsilon(d)$ using the data in Table 3.5. Explain what the slope of the line means to you.

$$A = \frac{\pi d^2}{4} \tag{3.57}$$

$$\varepsilon = -\mu_0 n n_1 A \omega I_0 \cos \omega t \tag{3.58}$$

$$\varepsilon = k d^m \Rightarrow \log \varepsilon = \log k + m \log d \tag{3.59}$$

Note: In all three experiments, the graphs we want will not be linear. Therefore, read the explanations below carefully.

In experiments, the resulting formulas are not always in the form of the typical line equation. For example, It can be of the form

$v^2 = v_0^2 - 2ax$

or

$I = kv^n$

In this case, the correct equation can be obtained by transferring to new variables. Taking $v^2 = y$, $v_0^2 = b$ and $2a = m$ in the first expression above, or taking the logarithm of the second expression,

$\log I = \log k + n \log v$

$\log I = y,\ \log k = b\ ,\ m = n,\ \log v = x$

linear graphs can be drawn.