2 - FRESNEL EQUATIONS - REFLECTION THEORY

Objectives

- Planar-polarized light is reflected off a glass surface. The rotation of the plane of polarization and the intensity of the reflected light are determined and compared with the Fresnel formula for reflection.

Equipments

- He-Ne Laser(1,0 mW, 230 VAC)
- Polarization Filter
- Prism(60°, $h = 36mm$)
- Prism Holder
- Si-Photodetector Amplifier and Control Unit
- Digital Multimeter
- Connection Cables
- Protractor
- Scaled Jointed Radial Holder

General Information

What are the electromagnetic theory of light, reflection coefficient, reflection factor, Brewster's law, law of refraction, polarization, and level of polarization?

According to electromagnetic theory, light consists of two components: the electric field vector E and the magnetic field vector B. These components oscillate in directions perpendicular to both the direction of propagation of the electromagnetic wave and to each other, and they oscillate in the same phase. The correlation between them can be expressed as:

$$|B| = n|E| \qquad (1)$$

Where n is the refractive index. The Poynting vector that carries the energy in the direction of propagation is determined by the following:

$$S \sim E \times B \quad \text{and} \quad |S| \sim |E|^2 \qquad (2)$$

If light strikes the boundary surface of an isotropic medium with a refractive index of n at an angle of α, part of the incoming light is reflected at the same angle of α, while the remaining part propagates through the isotropic medium at an angle of refraction β.

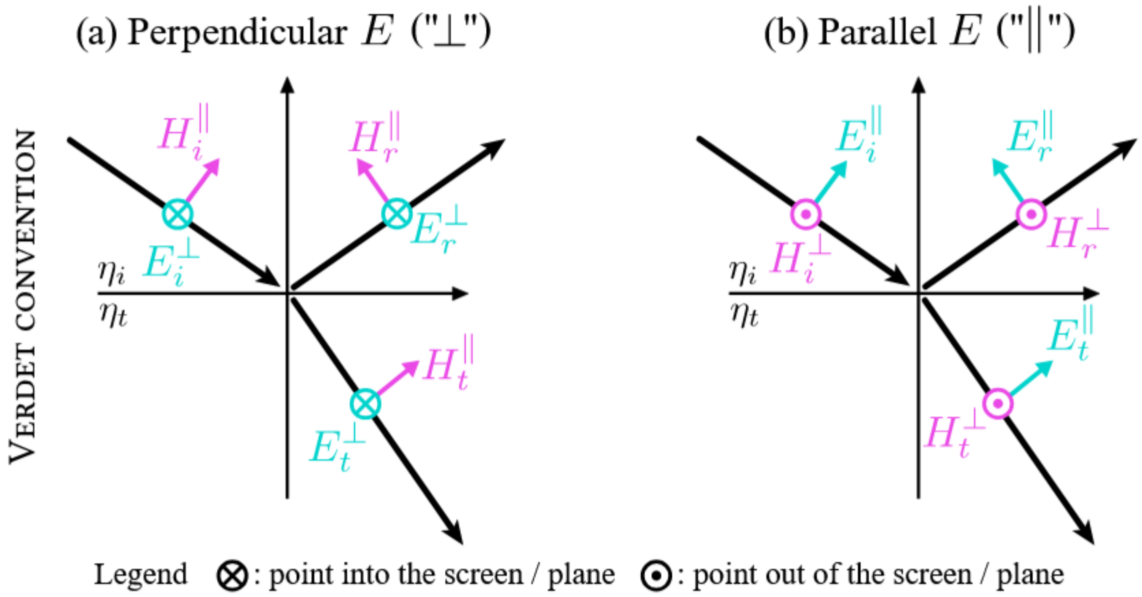

Figure 1: (a) Perpendicular (b) Parallel Polarization

In Figure 1(a), according to expression (2), the electric field vector $E_i^{\perp}$ of the incoming light is polarized perpendicular to the plane of incidence, while the magnetic field vector $H_i^{||}$ is polarized parallel to it. Based on the continuity of the tangential components and considering the direction of the light, the following expressions are obtained:

$$E_i^{\perp} + E_r^{\perp} = E_t^{\perp}; \qquad (H_i^{||} - H_r^{||})cos\alpha = H_t^{||}cos\beta \tag{3}$$

Where α is the angle between the incident ray and the normal and β is the angle between the transmitted ray and the normal.

Using expressions (1) and (3);

$$(E_i^{\perp} - E_r^{\perp})cos\alpha = n(E_i^{\perp} + E_r^{\perp})cos\beta \tag{4}$$

is obtained. Taking reflection law into consideration, the magnitude of electric fields of reflected and incident rays' ratio, which is also known as reflective coefficient, is as follows:

$$\zeta^{\perp} = \frac{E_r^{\perp}}{E_i^{\perp}} = \frac{cos\alpha - n\,cos\beta}{cos\alpha + n\,cos\beta} = -\frac{sin(\alpha-\beta)}{sin(\alpha+\beta)} \tag{5}$$

Figure 1(b) shows an incoming light wave with an electric field vector $E_i^{||}$ oscillating parallel to the plane of incidence. Similar to expression (3):

$$H_i^{\perp} + H_r^{\perp} = H_t^{\perp}; \qquad (E_i^{||} - E_r^{||})cos\alpha = E_t^{||}cos\beta \tag{6}$$

Using expressions (1) and (6):

$$\left(E_i^{||} - E_r^{||}\right)cos\alpha = \frac{1}{n}\left(E_i^{||} - E_r^{||}\right)cos\beta \qquad (7)$$

is obtained. Similar to (5) the magnitude of electric fields of reflected and incident rays ratio is as follows:

$$\zeta^{||} = \frac{E_r^{||}}{E_i^{||}} = \frac{n\,cos\alpha - cos\beta}{n\,cos\alpha + cos\beta} = -\frac{tan(\alpha-\beta)}{tan(\alpha+\beta)} \qquad (8)$$

By using Snell's law of refraction, if the angle of refraction β is eliminated, Fresnel's formulas (5) and (8) can be expressed in a different form.

$$\zeta^{\perp} = \frac{E_r^{\perp}}{E_i^{\perp}} = -\frac{\left(\sqrt{n^2-sin^2\alpha}-cos\alpha\right)^2}{n^2-1} \qquad (9a)$$

$$\zeta^{||} = \frac{E_r^{||}}{E_i^{||}} = \frac{n^2cos\alpha-\sqrt{n^2-sin^2\alpha}}{n^2cos\alpha+\sqrt{n^2-sin^2\alpha}} \qquad (9b)$$

$$tan\alpha_p = n \qquad (\alpha_p = \text{Brewster's angle})$$

For all incident rays between angles 0 and $\pi/2$, $\zeta^{\perp} \geq \zeta^{||}$ is valid.

For special cases:

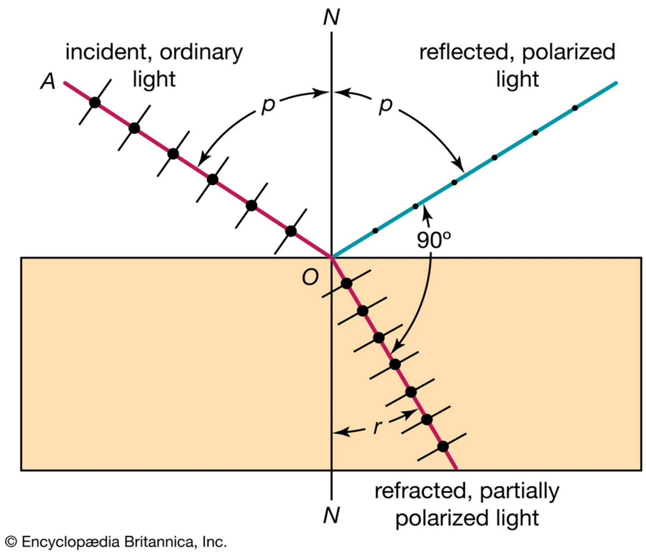

Figure 2: Brewster's Law

1. For a perpendicular incident angle $(\alpha = \beta = 0)$;

$$\zeta^{\perp} = \zeta^{||} = \left|\frac{n-1}{n+1}\right| \qquad (10)$$

2. For incident angle $\alpha = \frac{\pi}{2}$;

$$\zeta^{\perp} = \zeta^{||} = 1 \qquad (11)$$

3. If the reflected and transmitted angles are perpendicular to each other $\alpha + \beta = \frac{\pi}{2}$ (see Figure 2), due to expression (8);

$$\zeta^{||} = 0 \qquad (12)$$

which means the reflected light is completely polarized.In this case, only the electric vector oscillates in the normal plane of incidence. Using Snell's Law:

$$sin\alpha = n\ sin\beta = n\ sin\left(\frac{\pi}{2} - \alpha\right) = n\ cos\alpha \qquad (13)$$

is acquired. In this special case, for the incident angle:

$$tan\ \alpha_p = n$$

is acquired.

Table 1: Current values $i_r^{\perp}$ and $i_r^{||}$ as a function of angle α

α Degr.	i'_µA (235 µA)	ζ" = E'_t / E'_0	i'_µA (230 µA)	ζ" = E'_⊥ / E'_0	√(i' / i'_0)
10.0	11.5	0.221	13.5	0.243	0.243
15.0	11.0	0.216	15.0	0.255	0.255
20.0	10.5	0.211	16.5	0.268	0.268
25.0	10.0	0.206	18.5	0.282	0.282
30.0	9.0	0.2	21.0	0.298	0.298
35.0	8.0	0.187	24.5	0.316	0.316
40.0	7.1	0.187	29.0	0.338	0.338
45.0	6.0	0.16	35.0	0.365	0.365
50.0	5.6	0.149	41.0	0.39	0.39
52.5	5.4	0.144	41.0	0.422	0.422
55.0	5.2	0.138	49.0	0.462	0.462
60.0	4.7	0.123	72.0	0.55	0.55
62.5	4.5	0.117	100.0	0.661	0.661
65.0	4.0	0.105	135.0	0.765	0.765
70.0	3.0	0.08	165.0	0.846	0.846
75.0	2.0	0.058	200.0	0.935	0.935
80.0	1.0	0.032	230.0	1.0	1.0
85.0	0.8	0.027	260.0	1.07	1.07

Tab. 1 contains the i_r current values measured with a photocell for the intensity of light reflected at an angle α. According to expression (2), the current is directly proportional to the intensity of light, which is proportional to the square of the magnitude of the electric (or magnetic) field.

Figure 3 shows the experimentally determined curves for $\zeta_r^{||}$ and $\zeta_r^{\perp}$ as functions of the angle of incidence. The curve for $\zeta^{||}$ shows a distinct minimum at $\alpha_p = 58.5°$. Using this value and the point where the ζ curves intersect via extrapolation, the refractive index $n = 1.63$ is obtained from expressions (13) and (10).

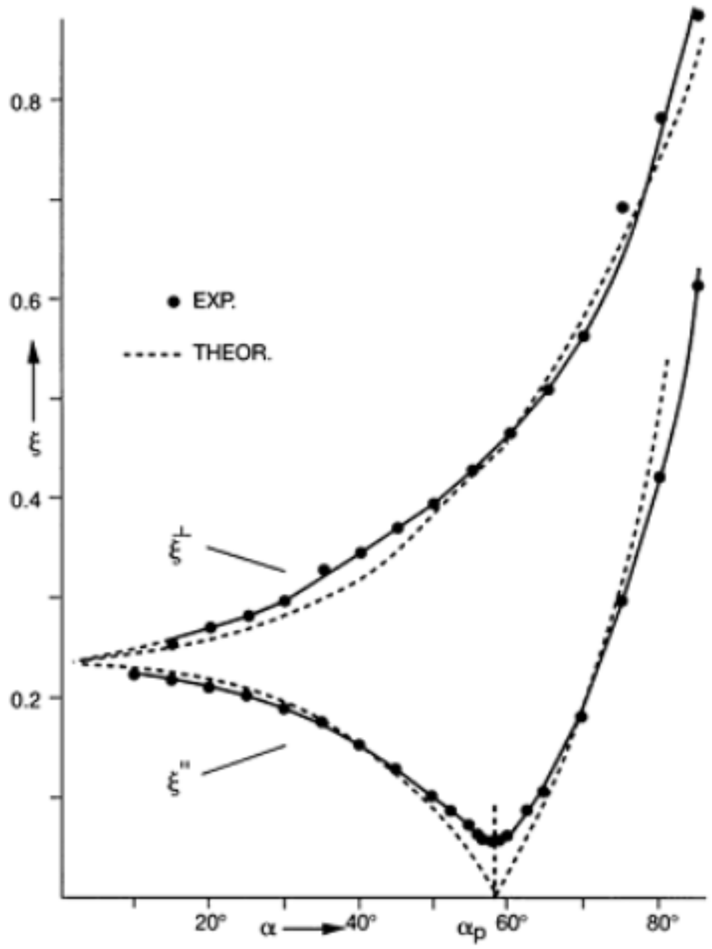

Figure 3: $\zeta_r^{||}$ and $\zeta_r^{\perp}$ as a function of incident angle and experimental values

If reflection components of (9a) and (9b) are squared and summed up reflective factor is:

$$R = \frac{\left(E_r^{\perp}\right)^2 + \left(E_r^{||}\right)^2}{\left(E_0^{\perp}\right)^2 + \left(E_0^{||}\right)^2} = \left(\frac{n-1}{n+1}\right)^2 \qquad (14)$$

In this experiment, the reflection factor(R) is calculated to be $n = 1.63$ for the prism.

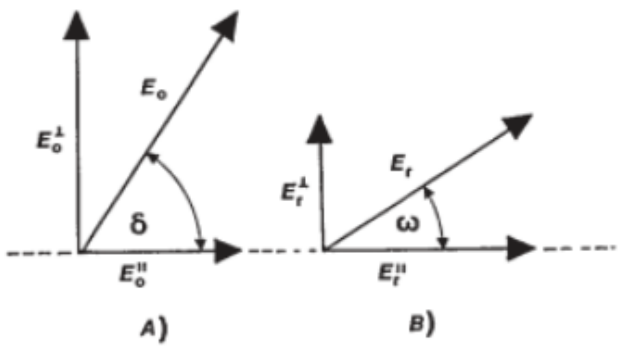

Figure 4: Change in polarization direction by reflection

Another method for verifying Fresnel's formulas is described below:

Plane-polarized light is directed onto glass at a fixed δ angle relative to the plane of incidence. The rotation of the plane of polarization for the reflected light is determined as a function of the angle of incidence. In Figure 4, the paper plane represents the reflective surface.

If the electric field vector oscillates at an angle ω after reflection, a rotation of the plane of polarization occurs as it passes through the angle $\psi = \delta - \omega$. For the parallel and normal components to the plane of incidence:

$$E_r^{||} = E_r cos\ \omega \quad ; \qquad E_r^{\perp} = E_r sin\ \omega \qquad (15)$$

or

$$tan\ \omega = \frac{E_r^{\perp}}{E_r^{||}} = \frac{E_r^{\perp} \times E_0^{||} \times E_0^{\perp}}{E_0^{\perp} \times E_r^{||} \times E_0^{||}} \qquad (16)$$

is acquired. Using expressions (5) and (8), expression (16) is:

$$tan\ \omega = -\frac{sin(\alpha-\beta)}{sin(\alpha+\beta)} \cdot \frac{tan(\alpha-\beta)}{tan(\alpha+\beta)} \cdot tan\ \delta \qquad (17)$$

In the special case of $\delta = \frac{\pi}{4}$:

$$tan\ \psi = tan\left(\frac{\pi}{4} - \omega\right) = \frac{1-tan\ \omega}{1+tan\ \omega} \qquad (18)$$

is acquired. If $tan\ \omega$ from (17) is put in its place in (18) and some adjustments are made the expression becomes:

$$tan\psi = \frac{cos(\alpha+\beta)+cos(\alpha-\beta)}{cos(\alpha+\beta)-cos(\alpha-\beta)} = -\frac{cos\ \alpha\sqrt{1-sin^2\beta}}{sin\alpha \cdot sin\beta} \qquad (19)$$

By using Snell's law of refraction, if the angle of refraction β is eliminated, expression (19) can be expressed as:

$$\psi = arctan\left(\frac{cos\alpha\sqrt{n^2 - sin^2\alpha}}{sin^2\alpha}\right) \qquad (20)$$

If the polarization plane has rotated $\psi = \frac{\pi}{4}$, Brewster's Law is acquired from expression (20):

$$tan\alpha_p = n$$

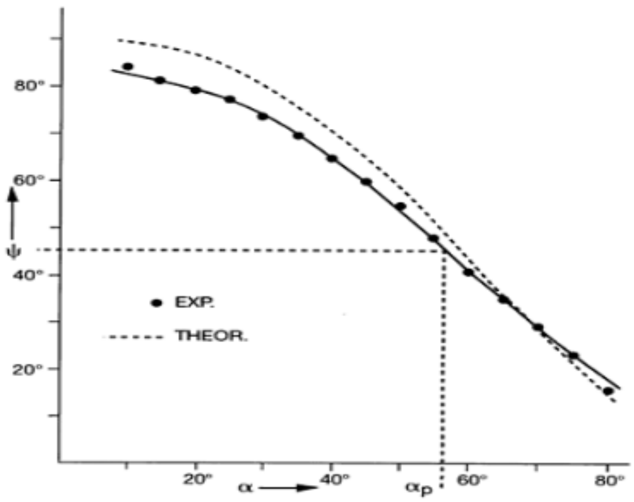

Figure 5: The calculated and measured variations for the rotation of the oscillation direction as a function of the angle of incidence at 45°

Experiment Procedure

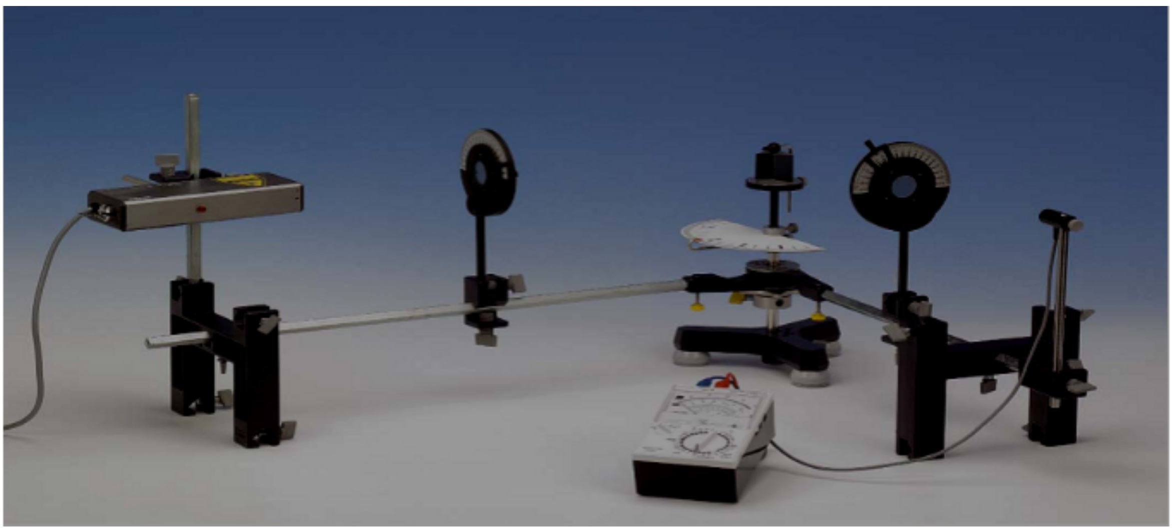

Figure 6: Experiment Setup

1. The experimental setup is shown in Figure 6.

2. The laser should be allowed to heat for approximately 15 minutes before starting the procedures.

3. The laser beam should be positioned over the center of the prism table to find the zero position.

4. The intensity $i_0^{||}$ of the light polarized parallel to the plane of incidence is measured.

5. The protractor scale is set to zero to determine the $\alpha = 0$ angle of incidence.

6. The prism is placed on the table so that it reflects the incoming light back along its path (Figure 7).

7. The angle of incidence α should be changed in 5° increments starting from $\alpha \leq 10°$, and in 1° increments near the Brewster angle region.

8. The photocell is rotated to obtain the maximum current for determining the intensity $i_r^{||}$.

9. First, the intensity $i_0^{\perp}$ is re-measured. Then the angle of incidence is changed in 5° increments, and the intensity corresponding to the reflected light is measured.

10. The laser is readjusted to its normal position to determine the degree of rotation of the plane of polarization due to reflection.

11. The photocell is placed in the path of the beam without the prism.
12. To precisely determine the oscillation plane, a polarization filter is attached in front of the laser, and the filter is rotated until the recorded intensity reaches a minimum.

13. Once the filter is rotated by 45°, the prism is placed in position using a known method.

14. The degree of rotation of the plane of polarization for the reflected beam is found using a second polarization filter placed between the prism and the detector.

15. The angle of incidence is changed in 5° increments.

16. The rotation angle of the plane of polarization is the average of multiple measurements.

17. The reflection coefficients for polarized light perpendicular and parallel to the plane of incidence will be determined as a function of the angle of incidence and represented graphically.

18. The refractive index of the crystal prism will be determined.

19. The reflection coefficients will be calculated using Fresnel's formulas and compared with the measured curves.

20. The reflection factor for the glass prism will be calculated.

21. The rotation of the plane of polarization of plane-polarized light upon reflection will be determined as a function of the angle of incidence and will be presented graphically.

22. Later, the values calculated using Fresnel’s equations should be compared.