2.2.1. ALTERNATIVE CURRENT CIRCUITS

2.2.1.1. PART I: SERIAL RL CIRCUIT

Figure 2.7: $\boldsymbol{RL}$-$\boldsymbol{RC}$ Experimental Setup

MAIN PRINCIPLE

1) Investigation of an alternating current circuit consisting of a series resistor and a coil.
2) Determination of the self-induction coefficient of the coil.
3) Investigation of an alternating current circuit consisting of a series resistor and capacitor
4) Determination of capacitance of capacitor

EQUIPMENT

AC Voltage Source, Voltmeter, Ammeter, Coil, Rheostat, Capacitor, Ruler, Compass, Angle Meter, Connection Cable

THEORY

1. SELF-INDUCTION COEFFICIENT

A magnetic field is produced around a coil through which current flows Figure 2.8. The intensity B (magnetic flux density) of this field is proportional to the current flowing through the coil.

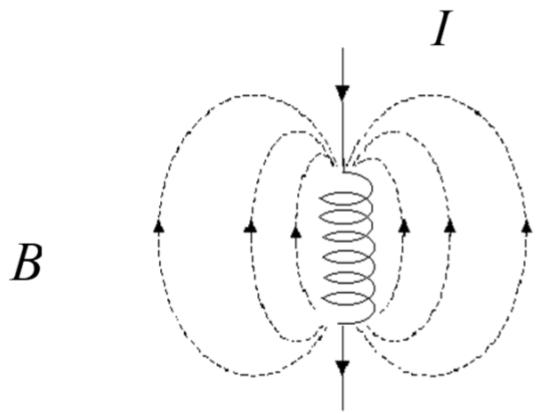

Figure 2.8:

$$B \sim I \tag{2.16}$$

or $B = KI$. Here, K (geometric factor) is a constant that depends on the given circuit. On the other hand, at the ends of a coil in a time-varying field B, an induction electromotive force (iemf) is generated, proportional to this change, given by the relation,

$$\varepsilon_i \approx \frac{dB}{dt} \tag{2.17}$$

This force causes a current I to flow through the coil. As shown in Figure 2.9, according to **Ampere's law**, the coil through which I current flows generates the magnetic field B_L, which is opposite to the B field that forms it. In this case, the coil remains under the influence of its own B_L magnetic field and if the current is variable, a self-induction electromotive force (siemf) appears at the ends of the coil.

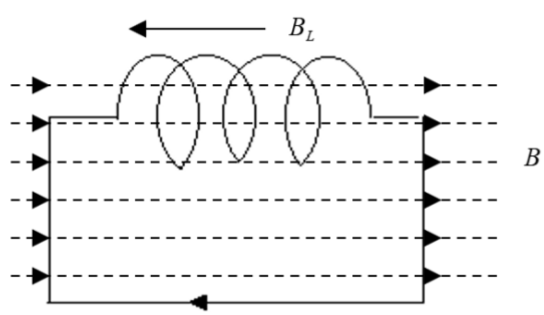

Figure 2.9:

The self-induction electromotive force ε_L is proportional to the time variation of the current flowing through the coil, according to relations (**2.16**) and (**2.17**). According to **Lenz's law**, the siemk is opposite to the self-inducing cause, taking into account that it is opposite to the direction of change of the current,

$$\varepsilon_L \approx -\frac{di}{dt} \text{ or } \varepsilon_L \approx -L\frac{di}{dt} \tag{2.18}$$

relation can be written. The coefficient of proportionality L is called the self-induction coefficient and L,

$$L \equiv -\frac{\varepsilon}{\frac{di}{dt}} \tag{2.19}$$

is given by the relation Here, $[\varepsilon_L] = Volt$, $\left[\frac{di}{dt}\right] = \frac{A}{s}$ and $[L] = \frac{Volt \cdot s}{A} = Henry\ (H)$ is in units.

2. ALTERNATIVE CURRENT

A voltage source whose potential difference between its terminals changes periodically with time is called an **alternative voltage source**. This change is usually sinusoidal. The change of current intensity in the circuit connected to an alternating voltage source is also sine-shaped depending on time. Such currents are called **alternative currents**. **From now on, we will use lower case letters for time-varying quantities.** The time dependence of alternative current intensity and voltage is given by the relation below.

$$i = I_M \sin \omega t \tag{2.20}$$

$$v = V_M \sin(\omega t + \phi) \tag{2.21}$$

Where I_M and V_M are the maximum values (amplitudes) of current and voltage, ω is the frequency of change and is called **angular frequency**. The angular frequency depends on the frequency f and period τ with the relation $\omega = 2\pi f = {2\pi}/{\tau}$. The angle f indicates the “phase difference” between voltage and current intensity.

If $\phi > 0$, the voltage is in advance of the current,

If $\phi < 0$, the voltage is behind the current,

If $\phi = 0$, the voltage is in the same phase with the current.

Here, $[V] = Volt$, $[I] = Ampere$, $[\tau] = s$ and $[f] = [\omega] = s^{-1}$ $(Hertz)$ is in units.

The impedance Z of an alternating current circuit with voltage V at its terminals and current I flowing through it is defined as the ratio of the amplitudes of the voltage and current magnitudes:

$$Z = \frac{V_M}{I_M} \tag{2.22}$$

As can be seen, this relation is the equivalent of Ohm's law in an alternating current circuit and impedance is the resistance of the circuit to alternative current. If the circuit element is only a resistor of value R, the impedance is

$$Z = R \tag{2.23}$$

and the phase difference is

$$\phi = 0 \tag{2.24}$$

If the circuit element is a coil with a self-induction coefficient L, the impedance is

$$Z = \omega L = X_L \tag{2.25}$$

and phase difference is,

$$\phi = \frac{\pi}{2} \tag{2.26}$$

The impedance of the coil is also called "inductance" or "inductive reactance" and is represented by X_L. Here it is $[Z] = [L] = [R] = Ohm\ (\Omega)$ in units.

3. SERIAL R-L CIRCUITS

In the circuit in Figure 2.10, which is formed by connecting a resistor R and a coil L in series, the total voltage V is given by $v = V_m \sin(\omega t + \phi)$ according to relation (**2.21**). The voltage at the resistance terminals can be written using equation (**2.24**),

$$v_R = V_{RM} \sin \omega t \tag{2.27}$$

is obtained. The voltage at the coil terminals is according to relation (**2.26**),

$$v_L = V_{LM} \sin\left(\omega t + \frac{\pi}{2}\right) \tag{2.28}$$

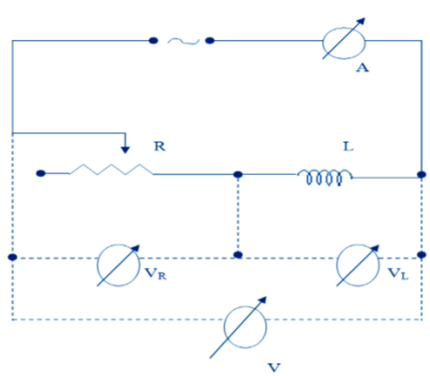

Figure 2.10:

When the quantities in alternating current circuits are vectorial, $\vec{v} = \overrightarrow{v_R} + \overrightarrow{v_L}$ and these three quantities can be represented by the vector diagram in Figure 2.11-Left. If both sides of the equations (**2**.**20**), (**2**.**27**), (**2**.**28**) are divided by I, the vector diagram in Figure 2.11-Left is reduced to the vector diagram in Figure 2.11-Right.

Figure 2.11: Vector diagram (Left), reduced vector diagram (Right)

The impedance of the circuit given by the relation (**2**.**22**) in these diagrams can be written more precisely in the form

$$Z_L = \sqrt{R^2 + \omega^2 L} \tag{2.29}$$

The phase angle of the circuit is given by relation,

$$\tan\phi \equiv \frac{V_{LM}}{V_{RM}} \equiv \frac{\omega L}{R} \Rightarrow \phi \equiv \tan^{-1}\frac{V_{LM}}{V_{RM}} \equiv \tan^{-1}\frac{\omega L}{R} v_L \tag{2.30}$$

Since the resistance of the winding wires of the coil cannot be ignored, the coil itself can be treated as a series $R - L$ circuit as shown in Figure 2.6.

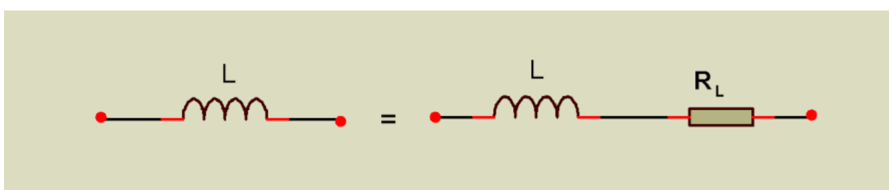

Figure 2.12

If the internal resistance of the coil is represented by R_L, if an R resistor is connected in serial to such a coil, the vector diagrams in Figure 2.11-Left and Figure-11-Right would be transformed into the diagrams in Figure 2.13-Left and Figure 2.13-Right.

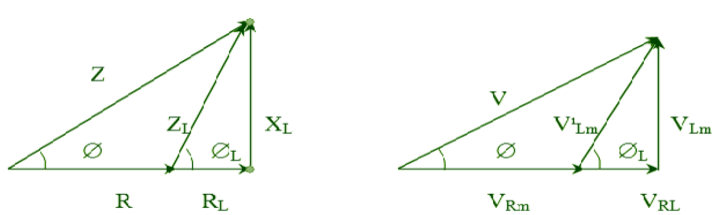

Figure 2.13: Coil (Left), Coil with internal resistance (Right)

Here, the total impedance of the coil is the maximum value of the voltage measured at the terminals of the coil. In this case, relations (**2.29**), (**2.30**) become,

$$Z_L = \sqrt{(R^2 + R_L) + {X_L}^2} \tag{2.31}$$

$$\phi \equiv \tan^{-1} \frac{v_{LM}}{v_{RM} + v_{R_L M}} \tag{2.32}$$

The phase difference ϕ_L between the voltage on the coil and the current flowing through the circuit is obtained as from Figure 2.13-Left and Figure 2.13-Right as,

$$\tan \phi_L = \frac{v_{LM}}{V_{R_L M}} = \frac{\omega L}{R_L} \Rightarrow \phi_L = \tan^{-1} \frac{v_{LM}}{V_{R_L M}} = \tan^{-1} \frac{\omega L}{R_L} \tag{2.33}$$

4. EFFECTIVE VALUES

In addition to the instantaneous value and maximum value in alternating currents, there are also effective values or effective values for both potential difference and current intensity. Like direct current, when alternating current passes through a resistance, it causes that resistance to heat up. The amount of heat generated by an alternating current passing through a certain resistance in a certain time interval is called the effective intensity and effective potential difference of that alternating current. Suppose that an alternating current whose change with time is $i = I_M \sin \omega t$ passes through a resistance such as R. The amount of heat released during a period will be

$$Q = \int_0^T i^2 R \, dt = I_M^2 R \int_0^T \sin^2 \omega t \, dt \tag{2.34}$$

$$Q = \frac{1}{2} I_M^2 \ (Joule) \tag{2.35}$$

The amount of heat that a direct current of intensity i will generate in the same resistance at the same time,

$$Q = i^2 R T \ (Joule) \tag{2.36}$$

According to the definition, since these two amounts of heat will be equal, the effective intensity of the alternative current,

$$i = \frac{I_M}{\sqrt{2}} \tag{2.37}$$

is found. Similarly, the effective potential difference is

$$v = \frac{V_M}{\sqrt{2}} \tag{2.38}$$

is obtained. Such as, the effective value of the alternative voltage in the city electricity network is $220\ volts$. For the maximum value, $220\ x\ =\ 311\ volts$ is obtained.

Accordingly, effective values can be written instead of the maxima of voltages and currents in equations (**2.22**), (**2.30**), (**2.33**). Therefore, the vector diagrams in Figure 2.11 and Figure 2.13 can also be drawn with effective values.

SETUP AND PROCEDURE

1. Set up the circuit in Figure 2.10 and let current flow through the circuit. Set the current strength to a suitable value with the help of variable resistor R (rheostat) and record this value.

$i =$.....................

2. Measure v_R across the resistor, v_L'across the coil and the total voltage v. Disconnect the current and from these three values draw the vector diagram in Figure-14 to scale (this diagram is called the three-voltmeter method).

$v_R =$..................... $v_L' =$..................... $v =$.....................

3. A vector diagram is completed to include v_{RL} and V_L'. Measure the values of v_{R_L} and v_L on the diagram.

$v_{R_L} =$..................... $v_L =$.....................

4. Taking $f = 50\ s^{-1}$, calculate the angular frequency ω from the relation $\omega = 2\pi f = {}^{2\pi}/_{\tau}$.

$\omega =$.....................

5. Determine the self-induction coefficient L of the coil.

$v_L = i\omega L \Rightarrow L = \frac{v_L}{I\omega} =$.....................

6. Calculate the total impedance Z of the circuit from the relation (**2**.**25**).

$Z =$.....................

7. Calculate the internal resistance R_L of the coil.

$R_L = \frac{v_{R_L}}{i} =$.....................

8. Calculate the value of the resistor R.

$R = \frac{v_R}{i} =$.....................

9. Recalculate the self-induction coefficient L of the coil by using the relation (**2**.**31**).

$L = \frac{1}{\omega}\sqrt{Z^2 - (R + R_L)^2} =$.....................

10. Calculate the phase angle ϕ using the relation (**2**.**32**).

$\phi =$.....................

11. Calculate the phase angle ϕ_L using the relation (**2.33**).

$$\phi_L = \tan^{-1}\frac{\omega L}{R_L} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

12. Record the quantities calculated and found from the figure in Table-II.

13. Interpret the results of the experiment. Briefly explain the information you gained from the experiment.

Table 2.2:

Physical Quantities	Calculated	Figure
$\boldsymbol{i\ (A)}$		
$\boldsymbol{v_R\ (V)}$		
$\boldsymbol{v'_L\ (V)}$		
$\boldsymbol{v\ (V)}$		
$\boldsymbol{v_{R_L}\ (V)}$		
$\boldsymbol{v_L\ (V)}$		
$\boldsymbol{L\ (H)}$		
$\boldsymbol{Z\ (\Omega)}$		
$\boldsymbol{R_L\ (\Omega)}$		
$\boldsymbol{R\ (\Omega)}$		
$\boldsymbol{\omega\ (s^{-1})}$		
$\boldsymbol{\phi\ (deg)}$		
$\boldsymbol{\phi_L\ (deg)}$		

2.2.1.2. PART II: SERIAL RC CIRCUIT

MAIN PRINCIBLE

Consider an alternative current generator (generator) connected to a series circuit containing R and C elements. If the voltage amplitude and frequency of the generator are given together with the values of R and C, the current flowing through the circuit can be found in terms of amplitude and phase constant. To simplify our work when analyzing more complex circuits involving two or more elements, we will use graphical structures called phasor diagrams. In these structures, alternating quantities such as current and voltage are represented by phasors. The projection of the phasor on the vertical axis shows the instantaneous value of the quantity in question, and its length shows the amplitude (maximum value) of the quantity. The phasor rotates counterclockwise. As we will see later, the method of summing various currents and voltages of different sinusoidally varying phases is considerably simplified by this procedure.

Capacitors are widely used in various electrical circuits. For example, they are used to adjust the frequency of radio receivers, as filters in power supplies, to eliminate sparks in automobile ignition systems, and as an energy storage device in electronic flash units. A capacitor is essentially an insulator placed between two conductors.

EQUIPMENT

AC Voltage Source, Voltmeter, Ammeter, Coil, Rheostat, Capacitor, Ruler, Compass, Angle Meter, Connection Cable

THEORY

1. DEFINITION OF CAPACITANCE:

Take two conductors with a potential difference of V between them and assume that they have equal and opposite charges. This can be achieved by connecting two uncharged conductors to the poles of a battery. Such a combination of two conductors is called a capacitor. The magnitude of the charge on the capacitor is found to be proportional to the potential difference V. The capacitance C of a capacitor is defined as the ratio of the magnitude of the charge on one of the conductors to the magnitude of the potential difference between these conductors.

$$C = \frac{Q}{V} \qquad (2.39)$$

Note that, according to the definition, the capacitance will always be a positive quantity. Moreover, since the potential difference increases as the accumulated charge increases, the Q/V ratio for a given capacitor is constant. It is easy to see from the equation that in SI units the capacitance is coulombs per volt. The name of the capacitance in **SI** units is farad. In short $[Capacitance\,] = 1F = C/V$. Farad is a very large unit of capacitance. In practice, the capacitance of many devices ranges from milli-farads ($1\ mF = 10^{-3}\ F$) to pico-farads ($1pF = 10^{-12}\ F$).

2. SERIAL RC CIRCUIT:

In the previous sections we have investigated the effects of placing capacitors and resistors separately between the terminals of a voltage source. Now we will examine what happens when these elements are used together. As before, we will assume that the applied voltage varies sinusoidally with time.

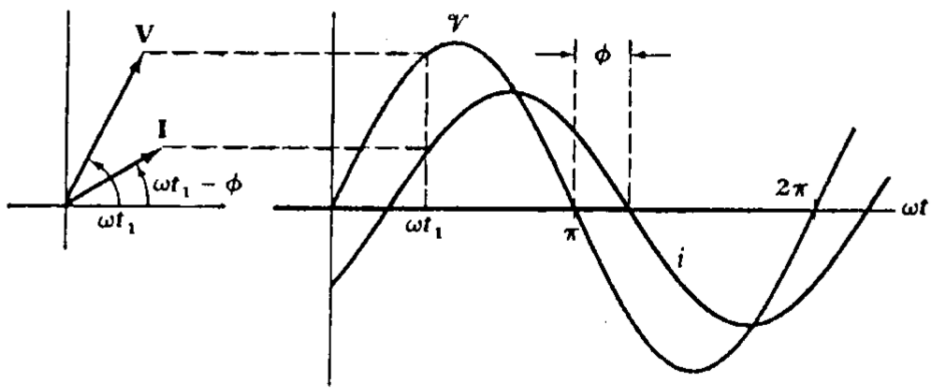

Figure 2.14: a) Left b) Right

It is reasonable to think of the voltage as

$$v = V_M \sin \omega t \tag{2.40}$$

and the current as

$$i = I_M \sin(\omega t - \phi) \tag{2.41}$$

The ϕ quantity is called the phase angle between the current and the applied voltage. Our goal is to determine ϕ and I_M. Recall that since the elements are in series, the current everywhere in the circuit must be the same at any given moment. That is, the alternating current at each point of an alternating current series circuit has the same amplitude and phase. Therefore, as we found in the previous section, the voltage between the terminals of each element will have different amplitude and different phases. In particular, the voltage between the terminals of the resistor is in phase with the current Figure 2.15a. The voltage between the terminals of the capacitor is 90° behind the current Figure 2.15b.

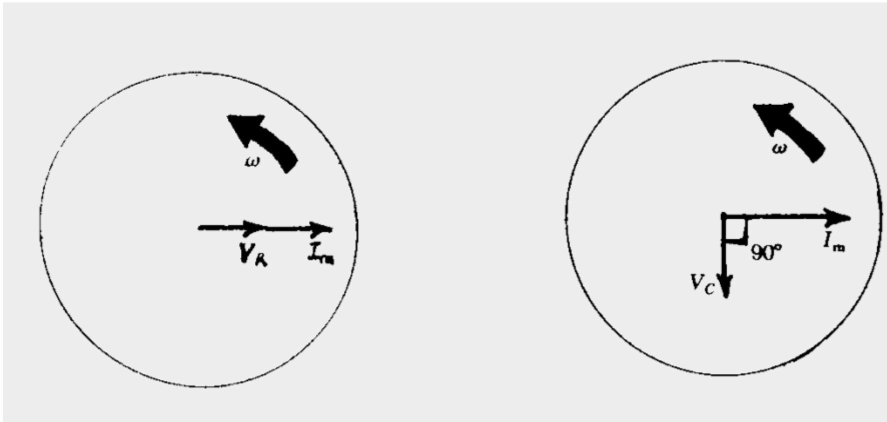

Figure 2.15: Phase of the current with the potential difference between the terminals of a) the resistor and b) the capacitor

Using these phase relationships, we can express the voltage drops between the terminals of both elements as follows:

$$v_R = I_M \sin \omega t = V_{RM} \sin \omega t \tag{2.42}$$

$$v_C = I_M X_C \sin\left(\omega t - \frac{\pi}{2}\right) = -V_{CM} \cos \omega t \tag{2.43}$$

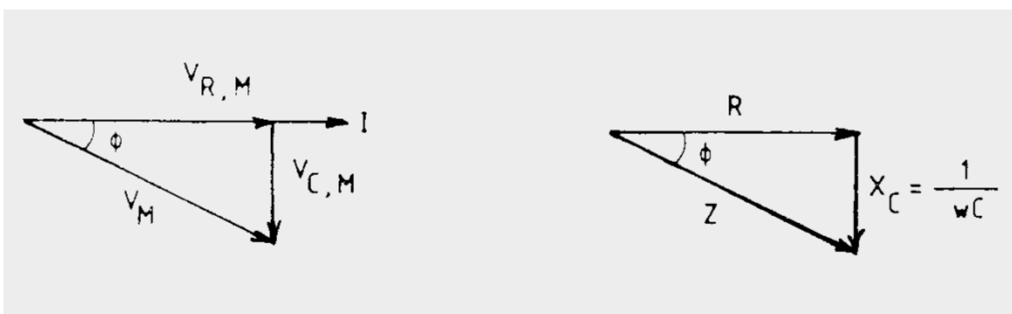

Figure 2.16

Since quantities in alternative current circuits can be expressed in phasors,

$$\vec{v} = \overrightarrow{v_R} + \overrightarrow{v_C} \tag{2.44}$$

and these three quantities can be represented by a phasor diagram. From the diagram,

$$V = \sqrt{V_{RM}^2 + V_{CM}^2} \tag{2.45}$$

relation can be written. The impedance of the circuit can be written in the form,

$$Z = \sqrt{R^2 + \frac{1}{\omega^2 C^2}} \tag{2.46}$$

The phase angle of the circuit is,

$$\tan\phi = -\frac{V_{CM}}{V_{RM}} = -\frac{1/_{\omega C}}{R} \tag{2.47}$$

$$\phi = \tan^{-1} - \frac{V_{CM}}{V_{RM}} = \tan^{-1} - \frac{1/_{\omega C}}{R} \tag{2.48}$$

SETUP AND PROCEDURE

1. The circuit in the Figure 2.17 is established. Measure v_R across the resistor, v_C across the capacitor and the total voltage v.

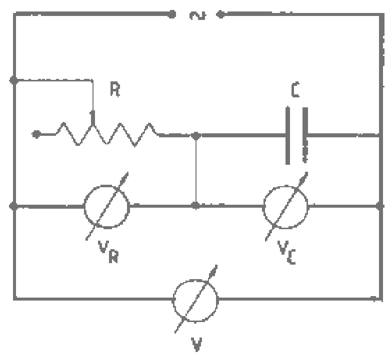

Figure 2.17

$v_R =$..................... $v_C =$..................... $v =$.....................

2. Using the voltages v, v_R and v_C,

$v = \sqrt{V_R^2 + V_C^2} =$.....................

is calculated.

3. Phase difference ϕ,

$\phi = \tan^{-1} - \frac{V_C}{R} =$.....................

4. The intensity of the current i in the circuit, if the resistance R is known,

$i = \frac{v_R}{R} =$.....................

5. The angular frequency ω is calculated by taking the frequency $f = 50\ s^{-1}$.

$\omega = 2\pi f =$.....................

6. The capacitance of the capacitor is

$C = \frac{1}{\omega}\frac{i}{v_C} =$.....................

7. A vector diagram is drawn to scale with the measured v_R, v_C, v values. From this drawn diagram, the phase angle ϕ is found with an angle meter.

Table 2.3:

Physical Quantities	Calculated	Figure
v_R (V)		
v_C (V)		
v (V)		
ϕ (deg)		
R (Ω)		
i (A)		
ω (s^{-1})		
C (F)		