10A - DIFFRACTION GRATING

OBJECTIVES

- Using a spectrometer, determine the grating constant of a given diffraction grating.
- Use this to calculate the wavelength of one of the double lines of sodium.

EQUIPMENTS

- **Sodium lamp.**
- **Spectrometer.**
- **Diffraction grating**.

GENERAL INFORMATION

Imagine placing an opaque object in front of a light source and a screen behind it. You would expect the area on the screen behind the object to be dark while the rest is illuminated. However, a pattern of illumination has been observed in the area that should be dark. This pattern is known as diffraction.

Diffraction is classified in two ways: Fraunhofer and Fresnel diffraction. In Fresnel diffraction, the image is formed at infinity. We aim to observe diffraction in a laboratory setting, which is known as Fraunhofer diffraction. By using an appropriate lens, we can make the image form at a finite distance. On the screen, a wide bright region is observed at the center, bordered by darkness. This pattern of illumination continues with secondary maxima.

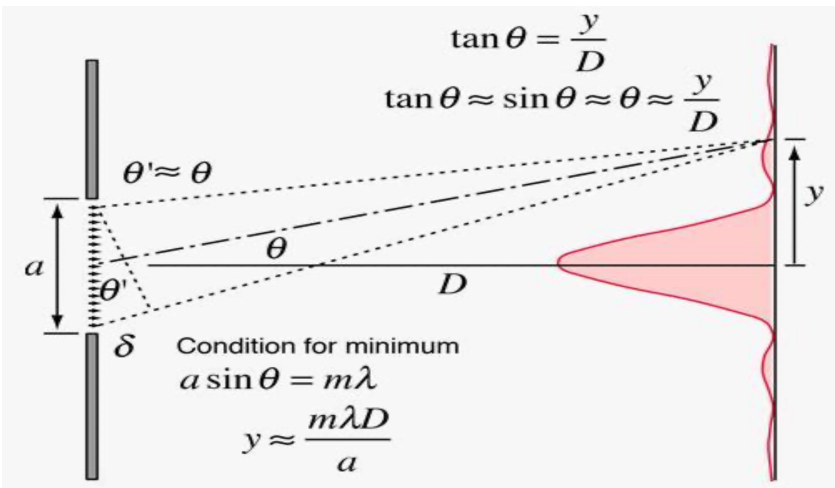

Figure 1: Fraunhofer diffraction.

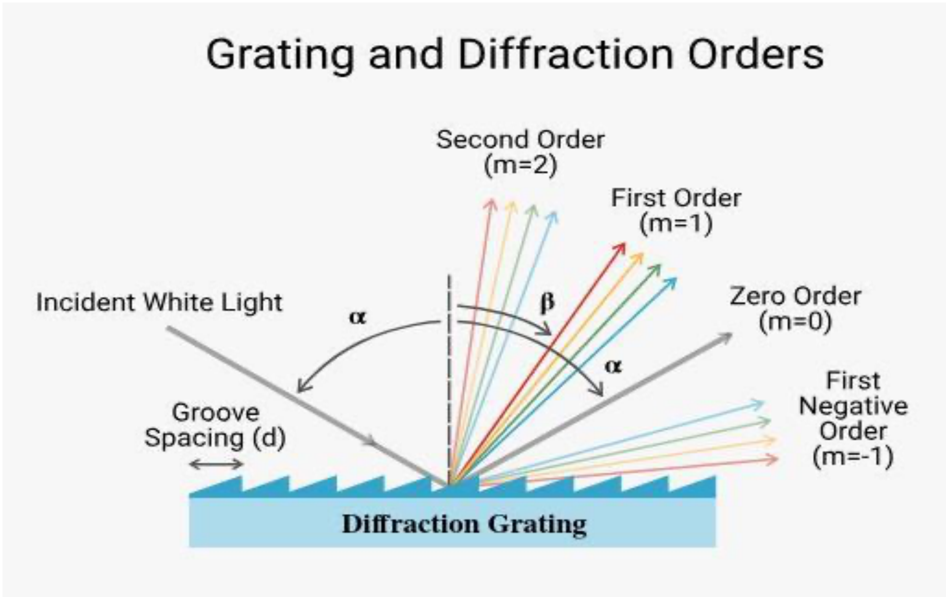

Figure 2: Reflection directions of the diffracted white light.

A diffraction grating, which is a very useful device in the analysis of light sources, consists of a large number of equally spaced slits. The regions between the lines are transparent to light, thus behaving like separate slits. The distance between two slits is defined as the 'grating constant' a. The diffraction grating used in the experiment is a transmission grating. In this case, the light path shown in Figure 3 applies. For inclined incidence on the diffraction grating (Figure 3), the following expression is valid for bright spots.

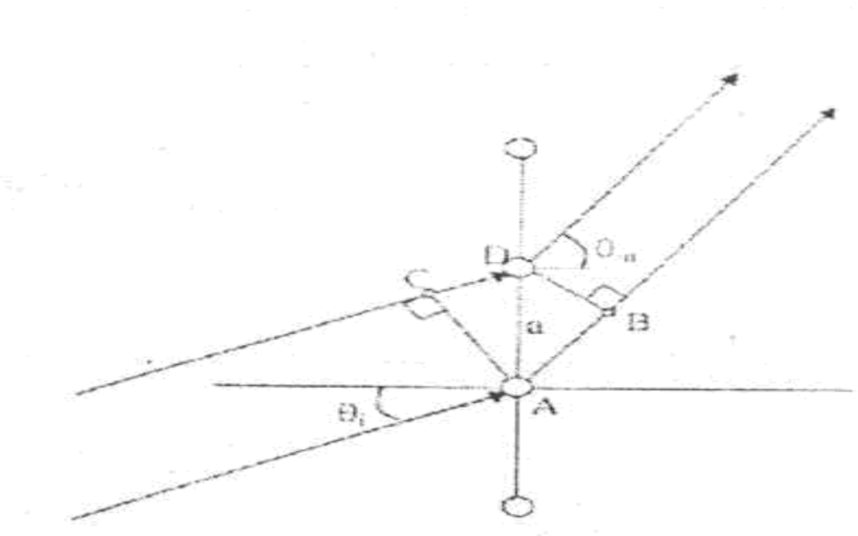

Figure 3: Incidence at an angle on a transmission grating.

$$AB - CD = a(\sin\theta_m - \sin\theta_i) \qquad (1)$$

$$m\lambda = a(\sin\theta_m - \sin\theta_i)$$

Here, $\underline{AB} - \underline{CD}$ is the optical path difference, m is the order of the diffraction grating, λ is the wavelength of the incoming light, and a is the grating constant.

In the case where the incident light is perpendicular to the grating:

$$m\lambda = asin\theta_m \qquad (2)$$

Here, θ_m is the diffraction angle. For m = 0, the central bright spot will be formed, and for m = 1, the first-order bright spot will be formed.

In the case where the optical path difference is an exact multiple of $\lambda/2$, the resulting interference is destructive:

$$(2m - 1)\frac{\lambda}{2} = asin\theta_m \qquad (3)$$

In a diffraction grating, the more slits there are, the more destructive interferences occur, making the bright spots narrower and clearer. If two plane waves with very close wavelengths fall on the diffraction grating, each wave will produce diffraction maxima at different angles according to their wavelengths. The maxima of a given order for all wavelengths form a spectrum. Here, the long-wavelength light is the most deflected, meaning that red light is deflected more than violet light. This is the opposite of the dispersion of light in a prism. The dispersion of the diffraction grating, 'D,' is given by the following expression.

$$D = \frac{d\theta}{d\lambda} = \frac{m}{acos\theta m} \qquad (4)$$

The resolving power of a given diffraction grating is given as:

$$R = mN = \frac{\lambda}{\Delta\lambda_{min}} \qquad (5)$$

$$R = \frac{aN(sin\theta_m - sin\theta_i)}{\lambda} \qquad (6)$$

Here, $(\Delta\lambda)_{min}$ is the minimum wavelength range that the spectrometer can resolve, and N is the total number of slits in the diffraction grating. Diffraction grating spectrometers are used to achieve spectral separation of light with multiple wavelengths and to determine each wavelength. The maximum value of R occurs when both θ_i and θ_m are on the same side of the normal. In this case:

$$R = \frac{2aN\sin\theta_i}{\lambda} \quad (7)$$

EXPERIMENTAL PROCEDURES

A spectrometer essentially consists of four main parts: the collimator, the angular scale table, the table on which the diffraction grating is placed, and the telescope. The collimator has an adjustable slit (slit width). The angular scale table consists of two concentric circular parts. The K2 table, marked with angles up to 360°, can easily rotate around a vertical axis with the telescope. The K1 circle has two vernier scales with 180° intervals. The full degree is the value in front of the zero line of the vernier. The vernier division that aligns with the lower degree divisions gives the minutes to be added to the full degree.

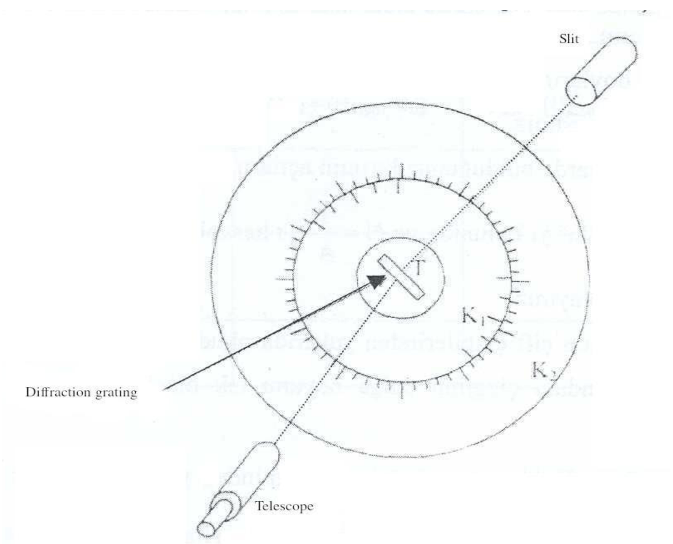

Figure 4: Spectrometer.

1. Turn on the sodium lamp.

2. Adjust the table so that the light is incident perpendicularly onto the diffraction grating.

3. Look through the telescope to see the zero-order bright line and read the angle in this case.

4. Turn the telescope to the left and read the θ_m values for the possible values of m.

5. Considering the zero-order angle you read in step-3, calculate the diffraction angles θ_m and record them into Table-1.

Note: From the first order onwards, you will see that the sodium lines are split into two. These are two bright lines with wavelengths of 589.592 nm and 588.995 nm. When measuring the angles, be careful to read the same line each time.

6. For each value of m, calculate the average of the diffraction angles found on the left and right sides.

7. For each value, determine the diffraction grating constant a and calculate $N = 1/a$.

8. Calculate the average value of a (a_{average}).

9. Using this a value, calculate the wavelength of the line outside the sodium doublet with a wavelength of 589.592 nm for a single m value.

10. Plot the graph of $\sin\theta_m = f(m)$. Use the slope of the line you obtain to find a value for a and compare it with the previously found a_{average}.

11. Calculate the expected dispersion value for sodium with a wavelength of $\lambda = 589.592$ nm at the 1st and 2nd orders.

Question 1: What is the expected resolving power of this diffraction grating at the 1st and 2nd orders?
Question 2: What is the minimum wavelength range that can be resolved at the 1st and 2nd orders?
Question 3: Why does the distance between lines, or the resolving power, increase as the order increases? Provide an explanation.

Table 1: Spectral values obtained for the sodium line with a wavelength of $\lambda = 589.592$ nm (bright yellow line)

m	θ_k (left)	θ_k (right)	θ_{left} ($\theta_{k,left} - \theta_0$)	θ_{right} ($\theta_{k,right} - \theta_0$)	$\theta_{average}$	$\sin\theta_{average}$	a (mm)	N
0								
1								
2								

Table 2: Spectrometer measurements obtained for the Sodium line other than the yellow line.

$a_{average}$	m	$\theta_{k,left}$	$\theta_{k,right}$	θ_{left}	θ_{right}	$\theta_{average}$	$\sin\theta_{average}$	λ (nm)

Table 3: Other spectral information (for the line with a wavelength of λ = 589.592 nm).

m	D (dispersion)	R (resolving power)	Smallest resolvable wavelength range
1			
2			