2.2.4. RLC SERIAL-PARALLEL CIRCUITS

2.2.4.1. PART I: SERIAL RLC CIRCUITS

MAIN PRINCIBLE

Investigation of frequency performance of a series tuned circuit for the following cases:

1) voltage resonance without damping resistor
2) current resonance without damping resistor
3) current resonance with damping resistor

EQUIPMENT

Resistance, Capacitor, Coil, Computer, Combo3 Software Program, AC Power Source, Connecting cords

Figure 2.26: Experimental setup

THEORY

1. SERIAL RLC CIRCUIT

If an alternating voltage source with a voltage of

$$U(t) = U_0 \cos \omega t \tag{2.61}$$

or its equivalent

$$U(t) = U_0 e^{j\omega t} \tag{2.62}$$

is connected to a series circuit consisting of a coil with inductance L, a capacitor with capacitance C and a resistance R, (see Figure 2.27).

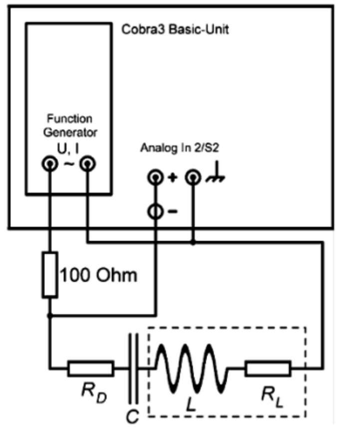

Figure 2.27:

According to the Kirchhoff loop rule:

$$U(t) = I(t)R + L\frac{dI(t)}{dt} + \frac{Q}{C} \tag{2.63}$$

Here; I is the current and Q is the charge on the capacitor. We can write the current as:

$$I(t) = \frac{dQ}{dt} \tag{2.64}$$

If we take the derivative of equation (**2.63**) with respect to time:

$$\frac{dU(t)}{dt} = R\frac{dl(t)}{dt} + L\frac{d^2I(t)}{dt^2} + \frac{I(t)}{C} \tag{2.65}$$

In alternating current circuits we expressed the current as $I(t) = I_0 \cos \omega t = I_0 e^{j\omega t}$. We know that there is a phase difference of ϕ between the current and voltage in RL and RC circuits operating with AC. Taking this phase difference into account, if the current expression above is replaced by

$$I(t) = I_0 e^{j\omega t} e^{-j\phi} \tag{2.66}$$

and placed in equation (**2.65**) together with equation (**2.61**):

$$j\omega U_0 = I_0 e^{-j\phi}\left(\frac{1}{C} + j\omega R - \omega^2 L\right) \tag{2.67}$$

If we write impedance as $Z = {U_0}/{I_0}$ equation (**2.67**) becomes:

$$j\omega Z = e^{-j\phi}\left(\frac{1}{C} + j\omega R - \omega^2 L\right) \tag{2.68}$$

The magnitude of the equation (**2.68**) whose right and left sides are complex numbers is found as:

$$|Z| = \sqrt{R^2 + \left(\frac{1}{C} + j\omega R - \omega^2 L\right)^2} \tag{2.69}$$

Equation (**2.69**) expresses that "at low frequencies the impedance of the circuit will be infinite, and the capacitor will be active in the circuit". A ω_0 value can be found in Equation (**2.69**) that will minimize the impedance (Z) of the circuit:

$$\left(\omega_0 L - \frac{1}{\omega_0 C}\right) \tag{2.70}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{2.71}$$

Such a value of ω_0 is called the angular resonance frequency, and f_0 is called the resonance frequency. The frequency f_0 gives the value at which the current is maximum. In this case, the voltage across the resistor is equal to the voltage of the source. The circuit behaves as if only the resistor were present.

At values below the resonant frequency, capacitive reactance is dominant in the circuit, the voltage lags behind the current and the phase angle ϕ is between zero and $-90°$.

Above the resonance frequency, inductive reactance dominates; voltage leads current and the phase angle ϕ is between zero and $+180°$.

On the other hand, we can insert the identity $e^{-j\phi} = \cos\phi - j\sin\phi$ into equation (**2.68**) and write the real and imaginary parts separately. The imaginary part can be found as:

$$j\left(\omega L - \frac{1}{\omega C}\right)\cos\phi - jR\sin\phi = 0 \tag{2.72}$$

Here we can obtain:

$$\tan\phi = \frac{\omega L - \frac{1}{\omega C}}{R} \tag{2.73}$$

Expression (**2.73**) tells us that at low frequencies the current leads the voltage, while at high frequencies the current lags the voltage.

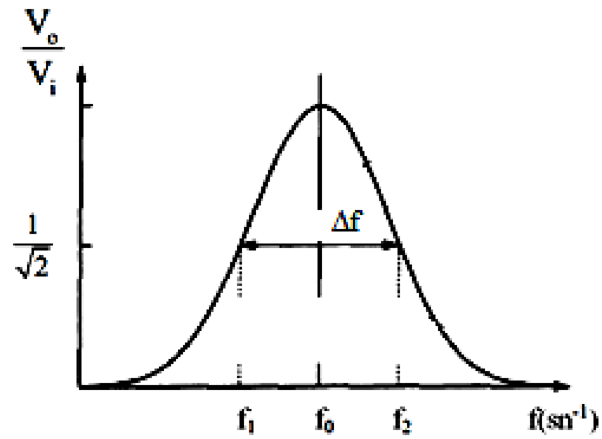

Figure 2.28:

f_1 and f_2 are called the half power frequencies of the circuit. In series RLC circuits the quality factor Q_s is defined:

$$Q_s = \frac{1}{R}\sqrt{\frac{L}{C}} \tag{2.74}$$

The quality factor defines the bandwidth of the circuit.

$$Q_s = \frac{f_0}{\Delta f} = \frac{f_0}{f_2 - f_1} = \sqrt{2}R \tag{2.75}$$

is also the geometric mean of the resonance frequencies ω_1 and ω_2.

$$\omega_0 = \sqrt{\omega_1 \omega_2} = \frac{1}{\sqrt{LC}} \tag{2.76}$$

The bandwidth of a circuit is the region between the frequency values where the voltage taken from the output of the circuit falls below $1/\sqrt{2}$ times its maximum value (Figure 2.28). These frequencies are called half-power frequencies.

SETUP AND PROCEDURE

1) Set up the circuit as in Figure 2.27.
2) Connect the COBRA3 Basic Unit to the computer port **COM1**, **COM2** or to **USB** port.
3) Start the **“measure”** program and select **“Gauge”** → **“Cobra3 PowerGraph”**.
4) Click the **“Analog In2/S2”** and select the **“Module /Sensor”** → **“Burst measurement”** with the parameters seen in Figure 2.29.
5) Click on the **“Function Generator”** symbol and create the parameters in Figure 2.30.
6) Add a **“Virtual device”** by clicking the white triangle in the upper left of the **“PowerGraph”** window or by right-clicking the **“Cobra3 Basic-Unit”** symbol. Turn off all channels but the first and configure this one as seen in Figure 2.31.

7) The **“Settings”** chart of **“PowerGraph”** should look like Figure 2.32.
8) Configure a diagram to be seen during the measurement on the **“Displays”** chart of **“PowerGraph”** as in Figure 2.33 and turn on some Displays for the frequency, the voltages and the current.

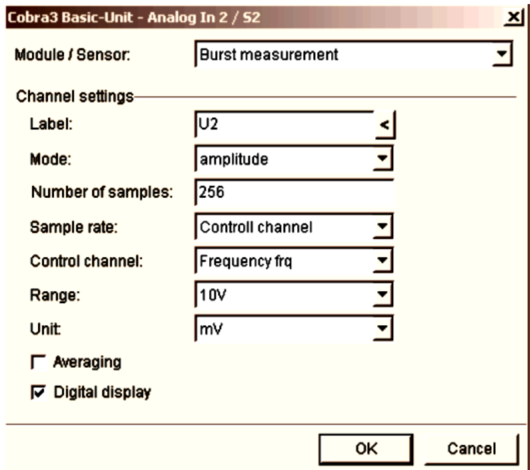

Figure 2.29:

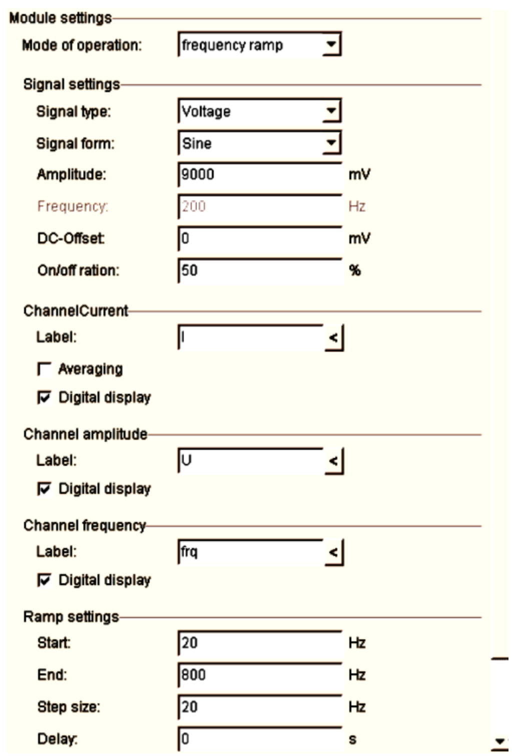

Figure 2.30:

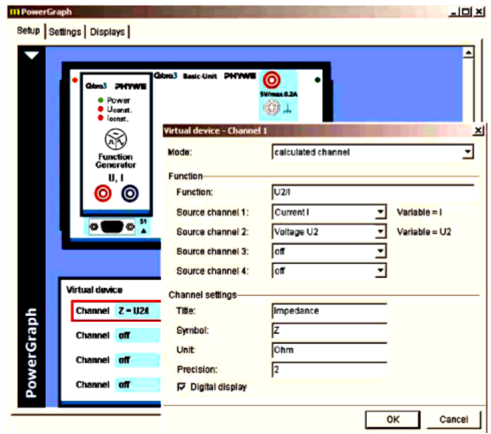

Figure 2.31:

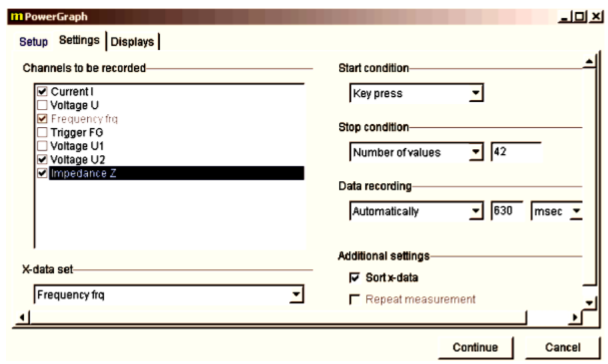

Figure 2.32:

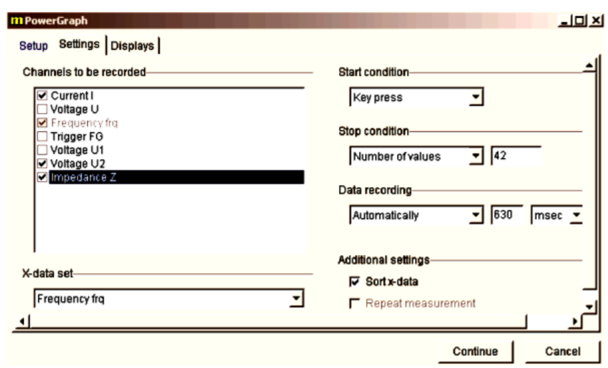

Figure 2.33:

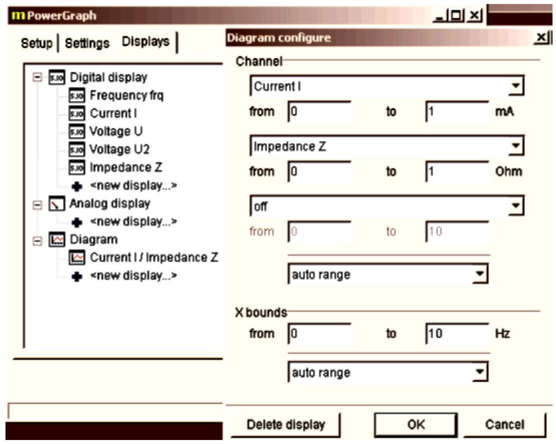

Figure 2.34:

9) Set up a series tuned circuit as seen in Figure 2.27. Start a measurement with the **"Continue"** button. After the measurement has stopped, the recorded curves are visible in the **"measure"** program main menu.

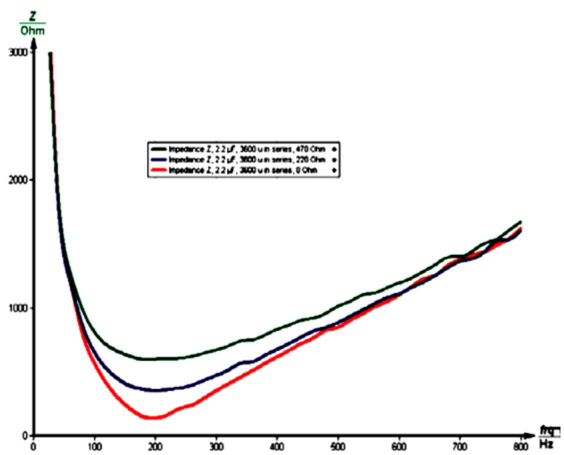

Figure 2.35:

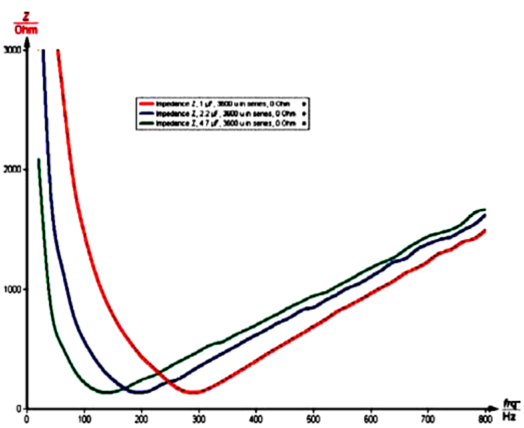

Figure 2.36

10) Record curves for $R_D = 0\ \Omega, 220\ \Omega$ and $470\ \Omega$ with the $2.2\ \mu F$ capacitor.
11) Record curves for $R_D = 0\ \Omega$ with the $1\ \mu F$ capacitor and the $4.7\ \mu F$ capacitor.
12) Use **"Measurement"** → **"Assume channel..."** and **"Measurement"** → **"Channel manager..."** to display the three impedance curves with the damping resistor values $R_D = 0\ \Omega, 220\ \Omega$ and $470\ \Omega$ for the $2.2\ \mu F$ capacitor in a single plot. Scale the impedance curves to the same value either using the **"Scale curves"** tool with the option **"set to values"** or using **"Measurement"** → **"Display options..."** filling appropriate values into the field **"Displayed area"** on the **"Channels"** chart. The result may look like Figure 2.35.
13) In a similar way produce a plot of the impedance over the frequency for the series tuned circuit with no additional damping resistor and the three capacitance values $C = 1\ \mu F, 2.2\ \mu F$ and $4.7\ \mu F$. Figure 2.36 shows a possible result.

2.2.4.2. PART II: PARALLEL RLC CIRCUITS

THEORY

1. PARALLEL RLC CIRCUITS

When making calculations in the circuit in Figure 2.37, the process is more complicated because the R_L internal resistance, which is in series with the coil, and the R_D resistance are parallel to each other. The equivalent resistance is:

$$R = \frac{R_D R_L}{R_D + R_L} \tag{2.77}$$

The total impedance of the circuit is:

$$\frac{1}{Z} = \frac{1}{j\omega L} + j\omega C + \frac{1}{R} \tag{2.78}$$

$$\frac{1}{Z} = j\left(\omega C - \frac{1}{\omega L}\right) + \frac{1}{R} \tag{2.79}$$

The resonance equation for the parallel RLC circuit is:

$$\left(\omega C - \frac{1}{\omega L}\right) = 0 \tag{2.80}$$

For small values of Q_P the resonant frequency is:

$$Q_P = R_D \sqrt{\frac{C}{L}} \tag{2.81}$$

Again, the following equations are also valid in parallel circuits:

$$Q_P = \frac{f_0}{\Delta f} = \frac{f_0}{f_2 - f_1} \tag{2.82}$$

$$\omega_0 = \sqrt{\omega_1 \omega_2} = \frac{1}{\sqrt{LC}} \tag{2.83}$$

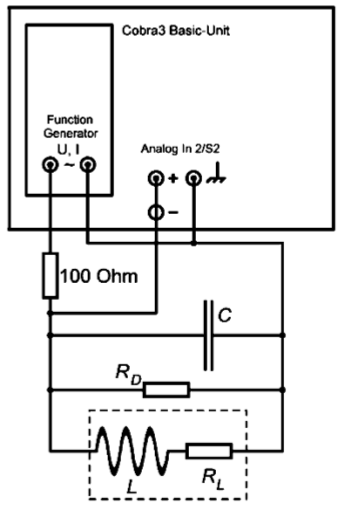

Figure 2.37:

SETUP AND PROCEDURE

1) Set up a parallel tuned circuit as in Figure 2.37.
2) Record curves with the 2.2 μF capacitor and different damping resistors $R_D = \infty\ \Omega, 220\ \Omega$ and $470\ \Omega$.
3) Record curves with the damping resistor $R_D = \infty\ \Omega$ (i.e. no resistor) and $C = 1\ \mu F$ and 4.7 μF. Plot the impedance in dependence on the frequency for $C = 2.2\ \mu F$ and $R_D = \infty\ \Omega, 220\ \Omega$ and $470\ \Omega$ (Figure 2.38).
4) Plot the impedance in dependence on the frequency for $C = 1\ \mu F$, $2.2\ \mu F$ and $4.7\ \mu F$ and $R_D = \infty\ \Omega$ (Figure 2.39).
5) Set up a series tuned circuit with $R_D = \infty\ \Omega$ and $C = 2.2\ \mu F$.
6) Select **"Gauge"** → **"Cobra3 Universal Writer"** and select the parameters as seen in Figure 2.40.
7) Record current and voltage curves in dependence on time for different frequencies between $80\ Hz$ and $360\ Hz$. For frequencies over $200\ Hz$ it is necessary to switch the frequency range under **"Configure FG module"** to **"High frequencies"**.

8) Note which of the curves, current or voltage, was ahead of the other.
9) Use **“Analysis” → “Smooth...”** with the options **“left axis”** and **“add new”** on both current and voltage curves. The curve that was clicked on before will be processed.
10) Use **“Measurement” → “Channel manager...”** to select the **“Current FG' ”** values as x-axis and the **“Analog in 2' ”** voltage values as y-axis (Figure 2.41). The Lissajous-figure to be produced now is no function but a relation so selects in the **“Convert relation to function”** window the option **“Keep measurement in relation mode”**.

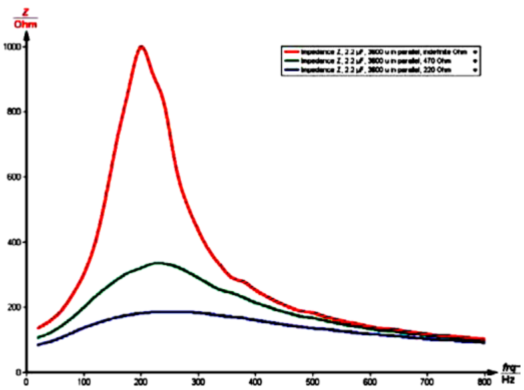

Figure 2.38:

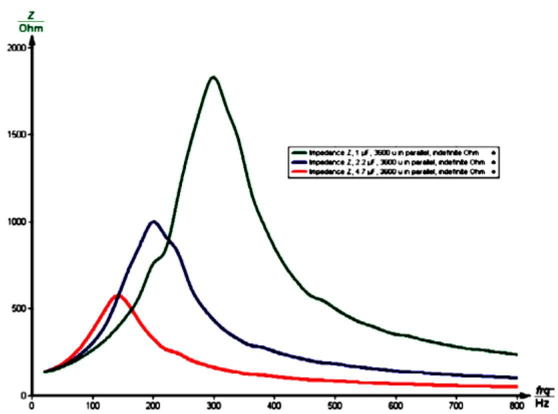

Figure 2.39:

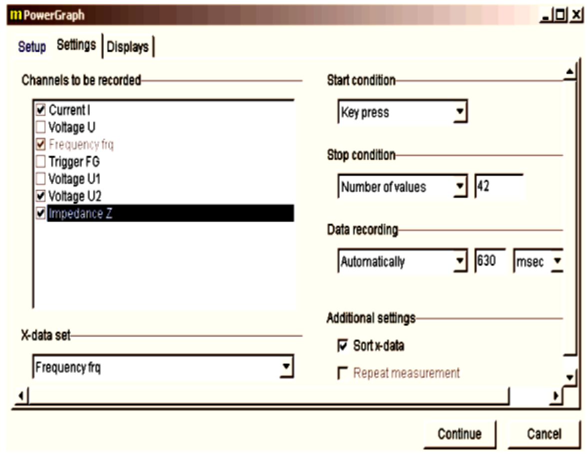

Figure 2.40:

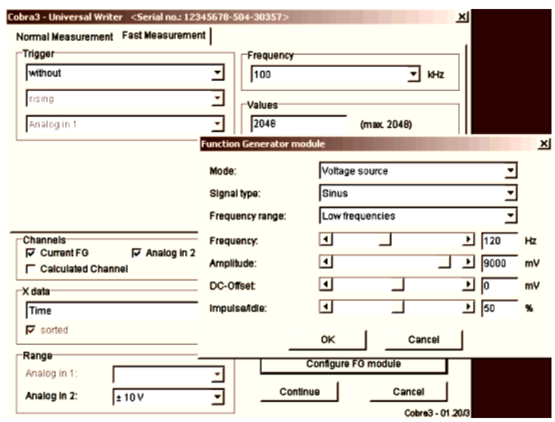

Figure 2.41:

11) Use the **"Survey"** tool to determine the maximal extension of the Lissajous-figure in x-direction ΔI_{max} (Figure 2.42) and the extension of the figure on the $y = 0$ line (ΔI_0) (Figure 2.43).

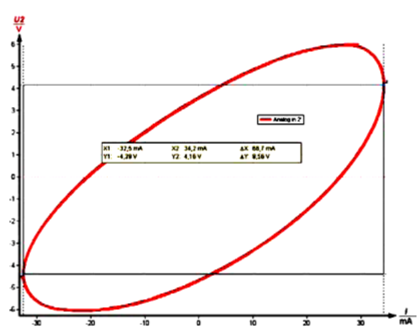

Figure 2.42:

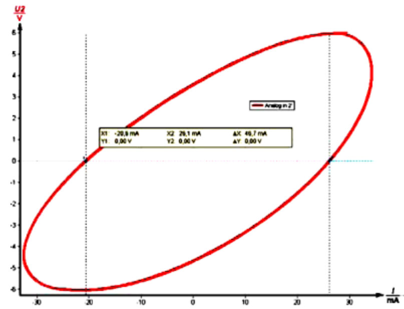

Figure 2.43:

12) The ratio ${}^{\Delta I_0}/_{\Delta I_{max}}$ equals the sine of the phase shift angle $\sin\omega$ between current and voltage.
13) Calculate ω and $\tan\omega$ for the used frequencies and plot them over the frequency using **"Measurement" → "Enter data manually..."** (Figure 2.44).
14) You can use **"Measurement" → "Function generator..."** to compare calculated theoretical values with the measured values. Figure 2.44 shows the equation for coil with $L = 0.3\ mH$ and d.c. resistance $R_L = 150\ \Omega$ in series with a $2.2\ \mu F$ capacitor with no additional damping resistor.

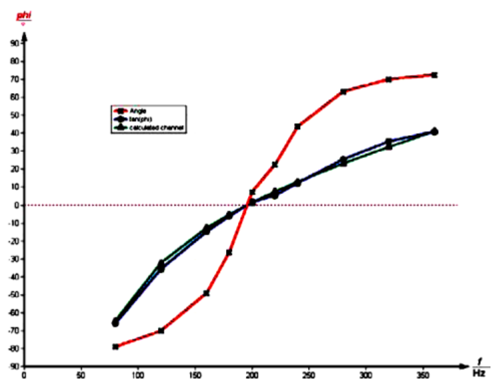

Figure 2.44