

Deep Learning for Computer Vision

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Recap



• Gradient of a scalar valued function $f(\mathbf{x})$: $\mathbf{x} \to \left(\frac{\partial f}{\partial x_1}, \dots, \frac{\partial f}{\partial x_D}\right)^T$

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 ight)^T$
- Gradient of a vector valued function f(x) is called Jacobian:

$$\mathbf{J} = \left[egin{array}{ccc} rac{\partial \mathbf{f}}{\partial x_1} & \cdots & rac{\partial \mathbf{f}}{\partial x_n} \end{array}
ight] = \left[egin{array}{ccc}
abla^{\mathrm{T}} f_1 \ dots \
abla^{\mathrm{T}} f_m \end{array}
ight] = \left[egin{array}{ccc} rac{\partial f_1}{\partial x_1} & \cdots & rac{\partial f_1}{\partial x_n} \ dots & \ddots & dots \
abla^{f_m} rac{\partial f_m}{\partial x_1} & \cdots & rac{\partial f_m}{\partial x_n} \end{array}
ight]$$

Gradient descent on MLP



• Loss is $\mathcal{L}(W, \mathbf{b}) = \sum_n l(f(x_n; W, \mathbf{b}), y_n)$

Gradient descent on MLP



- Loss is $\mathcal{L}(W, \mathbf{b}) = \sum_{n} l(f(x_n; W, \mathbf{b}), y_n)$
- For applying Gradient descent, we need gradient of individual sample loss with respect to all the model parameters

$$l_n = l(f(x_n; W, \mathbf{b}), y_n)$$

$$\frac{\partial l_n}{\partial W_i^{(l)}} \text{ and } \frac{\partial l_n}{\partial \mathbf{b}_i^{(l)}}$$

Forward pass operation



$$x^{(0)} = x \xrightarrow{W^{(1)}, \mathbf{b}^{(1)}} s^{(1)} \xrightarrow{\sigma} x^{(1)} \xrightarrow{W^{(2)}, \mathbf{b}^{(2)}} s^{(2)} \dots x^{(L-1)} \xrightarrow{W^{(L)}, \mathbf{b}^{(L)}} s^{(L)} \xrightarrow{\sigma} x^{(L)} = f(x; W, \mathbf{b})$$

Formally,
$$x^{(0)} = x, f(x; W, \mathbf{b}) = x^{(L)}$$

$$\forall l = 1, \dots, L \quad \begin{cases} s^{(l)} &= W^{(l)} x^{(l-1)} + \mathbf{b}^{(l)} \\ x^{(l)} &= \sigma(s^{(l)}) \end{cases}$$



Core concept of backpropagation



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$$J_{f_N \circ f_{N-1} \circ \ldots f_1(x)} = J_{f_N(f_{N-1}(\ldots f_1(x)))} \cdot J_{f_{N-1}(f_{N-2}(\ldots f_1(x)))} \cdot \ldots \cdot J_{f_2(f_1(x))} \cdot J_{f_1(x)}$$

 $J_{f(x)}$ is Jacobian of f computed at x.

0

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•
$$x^{(l-1)} \xrightarrow{W^{(l)}, \mathbf{b}^{(l)}} s^{(l)} \xrightarrow{\sigma} x^{(l)}$$



- $\bullet \ x_i^{(l)} = \sigma(s_i^{(l)})$



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 \bullet Since $s^{(l)}$ influences loss ${\mathcal L}$ through only $x^{(l)}$,

$$\frac{\partial \ell}{\partial s_i^{(l)}} = \frac{\partial \ell}{\partial x_i^{(l)}} \frac{\partial x_i^{(l)}}{\partial s_i^{(l)}} = \frac{\partial \ell}{\partial x_i^{(l)}} \sigma'(s_i^{(l)})$$



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$$s_i^{(l)} = \Sigma_j W_{i,j}^{(l)} x_j^{(l-1)} + b_i^{(l)}$$



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$$s_i^{(l)} = \sum_j W_{i,j}^{(l)} x_j^{(l-1)} + b_i^{(l)}$$

 \bullet Since $x^{(l-1)}$ influences the loss ${\mathcal L}$ only through $s^{(l)}$,

$$\frac{\partial \ell}{\partial x_{j}^{(l-1)}} = \sum_{i} \frac{\partial \ell}{\partial s_{i}^{(l)}} \frac{\partial s_{i}^{(l)}}{\partial x_{j}^{(l-1)}} = \sum_{i} \frac{\partial \ell}{\partial s_{i}^{(l)}} W_{i,j}^{(l)}$$





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$$\frac{\partial \ell}{\partial b^{(l)}} = \frac{\partial \ell}{\partial s_i^{(l)}} \frac{\partial s_i^{(l)}}{\partial b^{(l)}} = \frac{\partial \ell}{\partial s_i^{(l)}} \tag{2}$$

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Summary of Backprop



- \bullet From the definition of loss, obtain $\frac{\partial l}{\partial x_i^{(l)}}$
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Then wrt the parameters

$$\frac{\partial \ell}{\partial w_{i,j}^{(l)}} = \frac{\partial \ell}{\partial s_i^{(l)}} x_j^{(l-1)} \text{ and } \frac{\partial \ell}{\partial b_i^{(l)}} = \frac{\partial \ell}{\partial s_i^{(l)}}$$

Jocobian in Tensorial form



$$\bullet \ \psi : \mathcal{R}^N \to \mathcal{R}^M \ \text{then} \ \left[\frac{\partial \psi}{\partial x}\right] = \begin{bmatrix} \frac{\partial \psi_1}{\partial x_1} & \cdots & \frac{\partial \psi_1}{\partial x_N} \\ \vdots & \ddots & \vdots \\ \frac{\partial \psi_M}{\partial x_1} & \cdots & \frac{\partial \psi_M}{\partial x_N} \end{bmatrix}$$

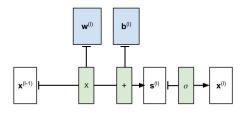
Jocobian in Tensorial form



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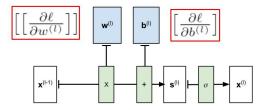
Forward Pass



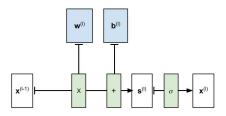


Goal of Backward Pass



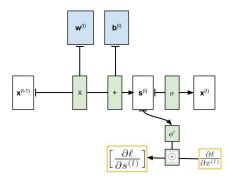




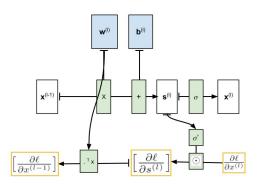




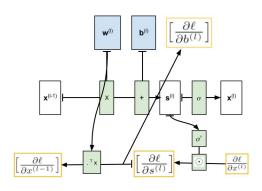




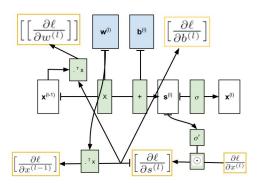












Update the parameters



$$\bullet \ W^{(l)} = W^{(l)} - \eta \left[\left[\frac{\partial \ell}{\partial w^{(l)}} \right] \right] \ \text{and} \ \mathbf{b}^{(l)} = \mathbf{b}^{(l)} - \eta \left[\frac{\partial \ell}{\partial b^{(l)}} \right]$$



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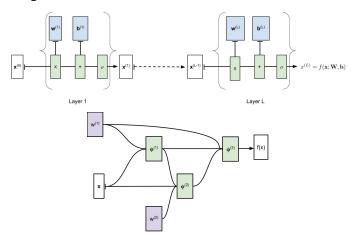


- BP is basically simple: applying chain rule iteratively
- It can be expressed in tensorial form (similar to the forward pass)
- Heavy computations are with the linear operations
- Nonlinearities go into simple element wise operations
- In an untreated situation, BP Needs all the intermediate layer results to be in memory
- Takes twice the computations of forward pass

Beyond MLP

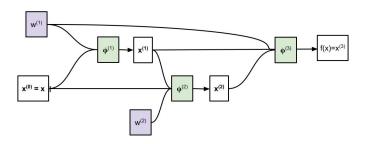


We can generalize MLP



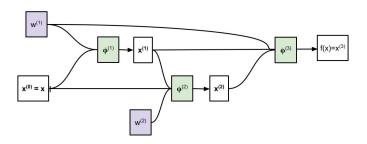
To an arbitrary Directed Acyclic Graph (DAG)





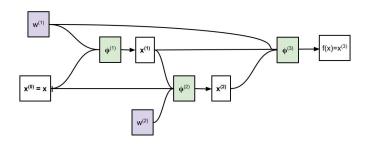
$$x^{(0)} = x$$





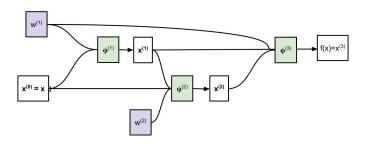
- $x^{(0)} = x$
- $\bullet \ x^{(1)} = \phi^{(1)}(x^{(0)}; w^{(1)})$





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- $x^{(0)} = x$
- $x^{(1)} = \phi^{(1)}(x^{(0)}; w^{(1)})$
- $x^{(2)} = \phi^{(2)}(x^{(0)}, x^{(1)}; w^{(2)})$
- $f(x) = x^{(3)} = \phi^{(3)}(x^{(1)}, x^{(2)}; w^{(1)})$

Notation: Jacobian of a general transformation



if
$$(a_1 \dots a_Q) = \phi(b_1 \dots b_R)$$
 then we use the notation (3)

$$\begin{bmatrix} \frac{\partial a}{\partial b} \end{bmatrix} = J_{\phi}^{T} = \begin{bmatrix} \frac{\partial a_{1}}{\partial b_{1}} & \cdots & \frac{\partial a_{Q}}{\partial b_{1}} \\ \vdots & \ddots & \vdots \\ \frac{\partial a_{1}}{\partial b_{2}} & \cdots & \frac{\partial a_{Q}}{\partial b_{2}} \end{bmatrix}$$
(4)

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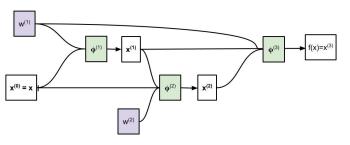
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(4)

if
$$(a_1 \dots a_Q) = \phi(b_1 \dots b_R; c_1 \dots c_S)$$
 then we use the notation (5)

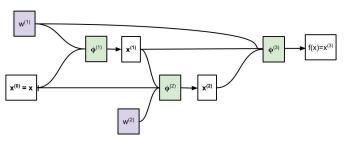
$$\begin{bmatrix} \frac{\partial a}{\partial c} \end{bmatrix} = J_{\phi|c}^T = \begin{bmatrix} \frac{\partial a_1}{\partial c_1} & \cdots & \frac{\partial a_Q}{\partial c_1} \\ \vdots & \ddots & \vdots \\ \frac{\partial a_1}{\partial c_2} & \cdots & \frac{\partial a_Q}{\partial c_2} \end{bmatrix}$$
(6)





 \bullet From the loss equation, we can compute $\left[\frac{\partial \ell}{\partial x^{(3)}}\right]$

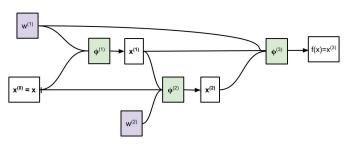




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$$\left[\frac{\partial \ell}{\partial x^{(2)}}\right] = \left[\frac{\partial x^{(3)}}{\partial x^{(2)}}\right] \left[\frac{\partial \ell}{\partial x^{(3)}}\right] = J_{\phi^{(3)}|x^{(2)}}^T \left[\frac{\partial \ell}{\partial x^{(3)}}\right]$$





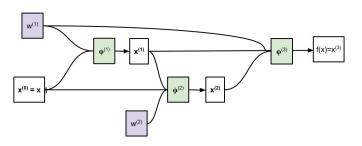
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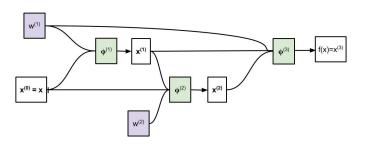
$$\begin{split} \left[\frac{\partial \ell}{\partial x^{(1)}}\right] &= \left[\frac{\partial x^{(3)}}{\partial x^{(1)}}\right] \left[\frac{\partial \ell}{\partial x^{(3)}}\right] + \left[\frac{\partial x^{(2)}}{\partial x^{(1)}}\right] \left[\frac{\partial \ell}{\partial x^{(2)}}\right] \\ &= J_{\phi^{(3)}|x^{(1)}}^T \left[\frac{\partial \ell}{\partial x^{(3)}}\right] + J_{\phi^{(2)}|x^{(1)}}^T \left[\frac{\partial \ell}{\partial x^{(2)}}\right] \end{split}$$





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 - · automatically differentiate them



Autograd

Gradient Computation



 PyTorch automatically constructs on-the-fly graph to compute gradient of any wrt any tensor

Gradient Computation



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Autograd



 Easy to use syntax: only need to define the sequence of forward pass operations

Autograd



- Easy to use syntax: only need to define the sequence of forward pass operations
- Flexible: Computational graph can be dynamic, so is the forward pass

Autograd in PyTorch



A tensor has the Boolean field 'requires_grad'

Autograd in PyTorch



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Autograd in PyTorch



- A tensor has the Boolean field 'requires_grad'
- PyTorch knows if it has to compute gradients wrt this tensor or not
- Default is False
- requires_grad_() function can be used to set to any value

Autograd



 torch.autograd.grad(o/p,i/p) returns gradients of outputs wrt the inputs



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- Tensor.grad field accumulates these gradient
- Standard function used to train the models.
- Since it ACCUMULATES the gradients, one may need to set Tensor.grad to zero before calling it
- Accumulation is helpful (e.g. sum of losses, or sum over different mini-batches, etc.)

torch.no_grad()



Switches the autograd machinery off

torch.no_grad()



- Switches the autograd machinery off
- Useful for operations such as parameter updation

detach()



 Creates a tensor which only shares data but doesn't require gradient computation

detach()



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- Not connected to the current graph

detach()



- Creates a tensor which only shares data but doesn't require gradient computation
- Not connected to the current graph
- Used when gradient should not be propagated beyond a variable, or to update the leaf nodes in the graph



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- Specified with create_graph = True

Demo



► Colab Notebook: Backword()