ECO 3302 – Intermediate Macroeconomics

Lecture 6: A Primer on Economic Growth

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February 17–19, 2025

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Introduction

Introduction

- Today we start the study of Economic Growth
- Not only a big block in this course...
- ...but, more importantly, a fundamental part of economics
- Listen to Robert Lucas Jr.:

> "Is there some action a government of India could take that would lead the Indian economy to grow like Indonesia's or Egypt's? If so, *what* exactly? If not, what is it about the 'nature of India' that makes it so? The consequences for human welfare involved in questions like these are simply staggering: once one starts to think about them, it is impossible to think about anything else"

Before we can get in depth into the study of economic growth, we must understand some basics:

- Which variables are best to make cross-country income comparisons? Why?
- Any caveats with these variables that we should be aware of?
- Why is economic growth so important?
- Are there any empirical regularities we should be aware of?

Understanding the Basics

How to make cross-country income comparisons?

- Economists use **real GDP per capita in PPPs** to make cross-country income **comparisons at specific points in time**
  - Why GDP? It highly correlates with measures of economic development
  - Why real? It adjusts for inflation, highlighting actual increases in production
  - Why per capita? It adjusts for population size to make meaningful comparisons
  - Why PPPs (purchasing power parities)? It adjusts currencies to eliminate differences in price levels b/w countries, equalizing purchasing powers See

- Economists use **growth rates of real GDP or real GDP per capita in PPPs** to make cross-country **comparisons in terms of growth potential**
  - Slightly larger growth rates can have tremendous welfare implications
  - Why? The power of compounding! (Back to Bob Lucas!)

Cross–country income differences

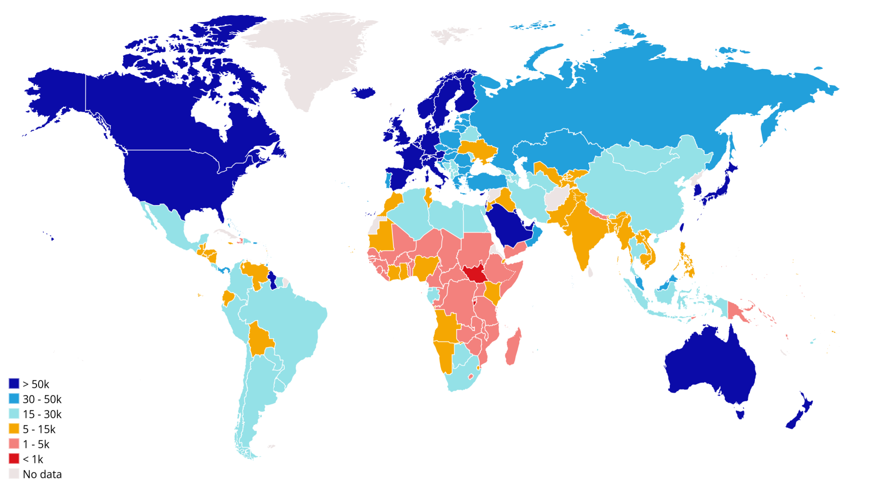


Figure 1: GDP per capita in 2023 (in 2020 PPPs and international dollars)

Caveats with GDP per capita

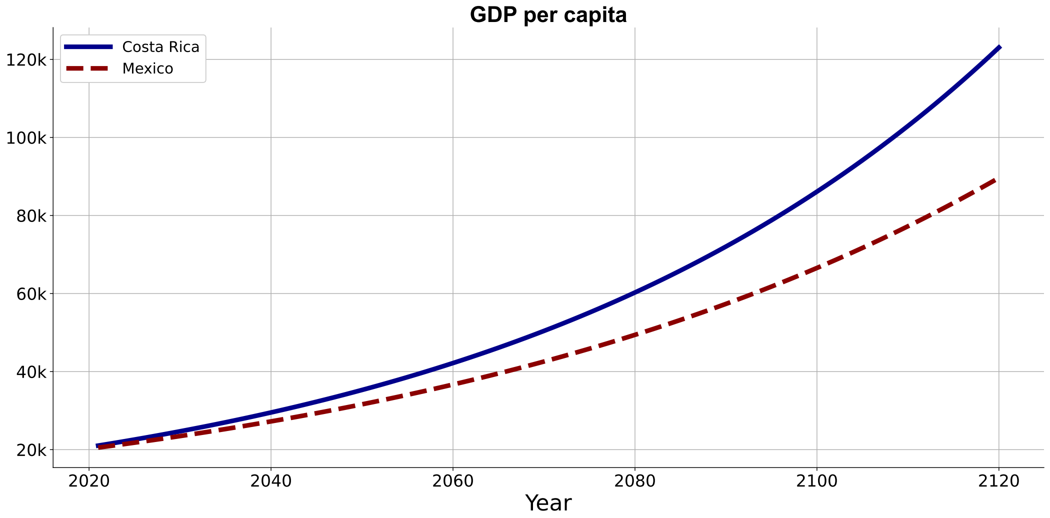
- As any other measure, **GDP presents important drawbacks:**
  - It does not measure non-market activity
    (eg, illegal activities, home production, ...)
  - It counts "goods" and "bads"
    (eg, when hurricane Maria destroyed Puerto Rico in Sept 2017, rebuilding counted as GDP)
  - It doesn't capture many important things
    (ie, two countries with identical GDPs may have very different: work hours per capita, pollution levels, income distributions, levels of security, ...)
- More fundamental and somewhat philosophical question: **Does GDP really measure what we ultimately want to measure (ie, welfare)?**
  - Well-being/happiness vs. income?
- Despite not being a perfect measure, GDP highly correlates with many measures we associate with welfare and is easy to quantify

Why is economic growth so important?

Small differences in growth rates have tremendous welfare implications over the medium/long-run

- **Two countries with similar GDP per capita** in 2021 (in PPPs and intl. dollars):
  - Costa Rica: \$20,666
  - Mexico: \$20,226
- **Suppose one grows** (Costa Rica) **on average at 1.8% per year for next 100 years, and the other** (Mexico) **at 1.5% per year**
- **How much richer would Costa Rica be in 10 years in comparison to Mexico? And in 25, 50, 100?** 5%, 10%, 19%, and 37%, respectively

Our thought experiment



Is there a better way to look at the data?

- Now that we understand the importance of growth, let me ask you:

  Is there a better way to look at the data when looking at growth trajectories?

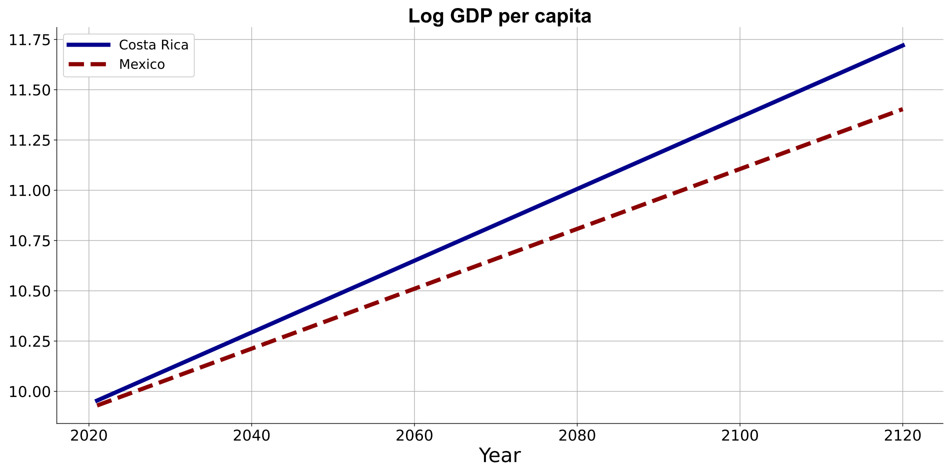
- **Yes!** Economists usually look at growth trajectories **using natural logs**. Why?
  (Natural logs, denoted $\ln$, are in base e; ie, $\ln(x) = \log_e(x)$. We use $\log$ as $\log(x) = \log_{10}(x)$)

  - Because when Y_t grows at a constant rate, $\ln Y_t$ grows linearly

$\Longrightarrow$ If Y_t and Z_t both grow at rate g, then $Y_t - Z_t$ also grows if $Y_0 \neq Z_0$ while $\ln Y_t - \ln Z_t$ will remain constant.

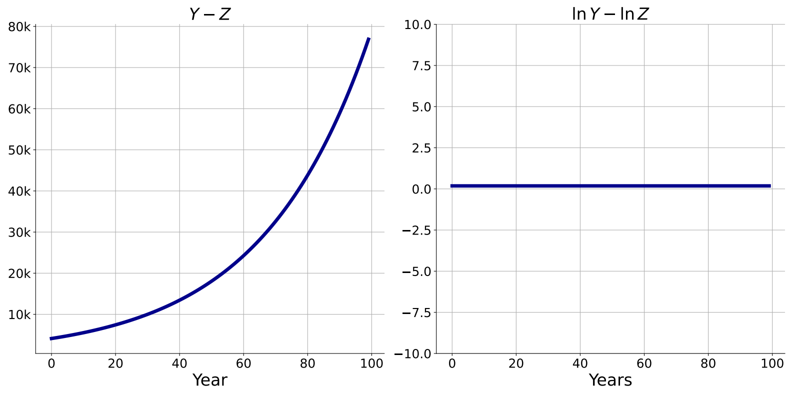
Logs: Linear vs. exponential growth

When Y_t grows at a constant rate, $\ln Y_t$ grows linearly



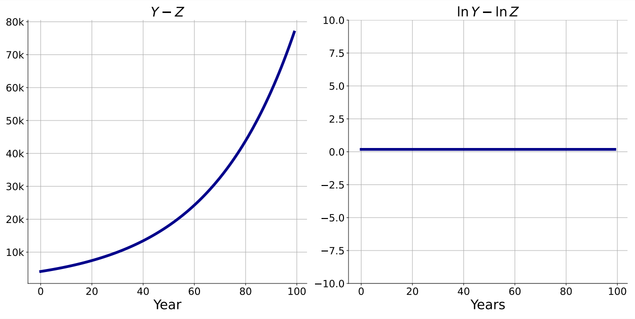
Logs: Do differences come from growth rates or levels?

- If Y_t and Z_t both grow at rate g, $Y_t - Z_t$ will also grow if $Y_0 \neq Z_0$, while $\ln Y_t - \ln Z_t$ will remain constant
- **Thought experiment II**: $Y_0 = 24\text{k}$ and $Z_0 = 20\text{k}$ both grow at 3% per year



Logs: Do differences come from growth rates or levels?

- **Thought experiment II**: $Y_0 = 24\text{k}$ and $Z_0 = 20\text{k}$ both grow at 3% per year



- **Left panel**: Tells differences between Y and Z grow larger over time
- **Right panel**: Tells Y and Z grow at the same rate
- **Left + right panel**: increasing differences between Y and Z come from levels not growth rates

Some Empirical Regularities

Growth is a modern phenomenon

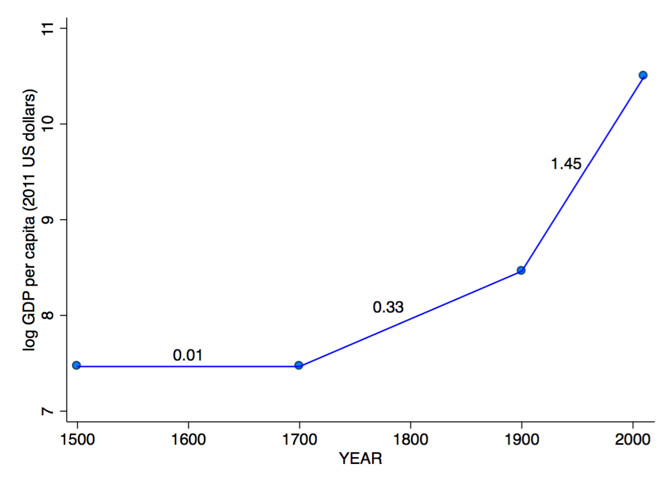
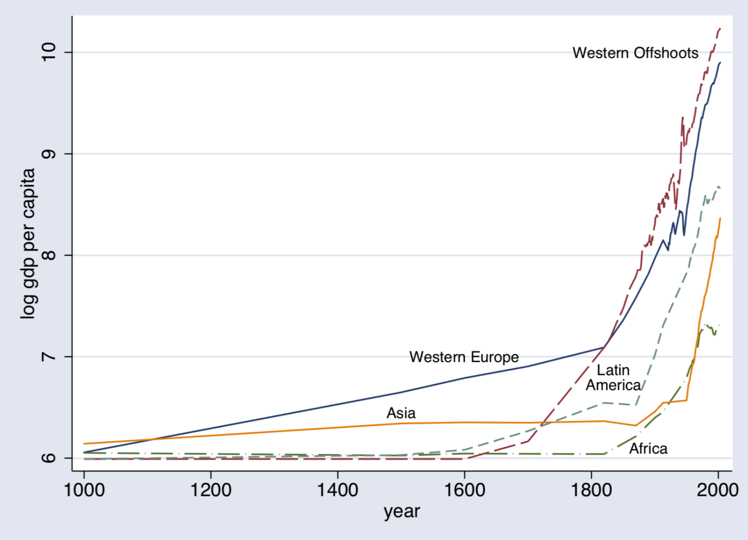


Figure 2: The world's GDP per capita and its growth rates, 1500–2016

Growth is a modern phenomenon: True for all regions



Economic standing is not immutable

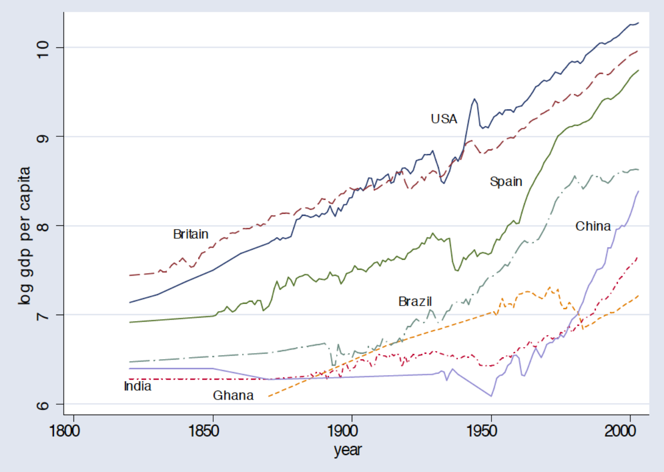


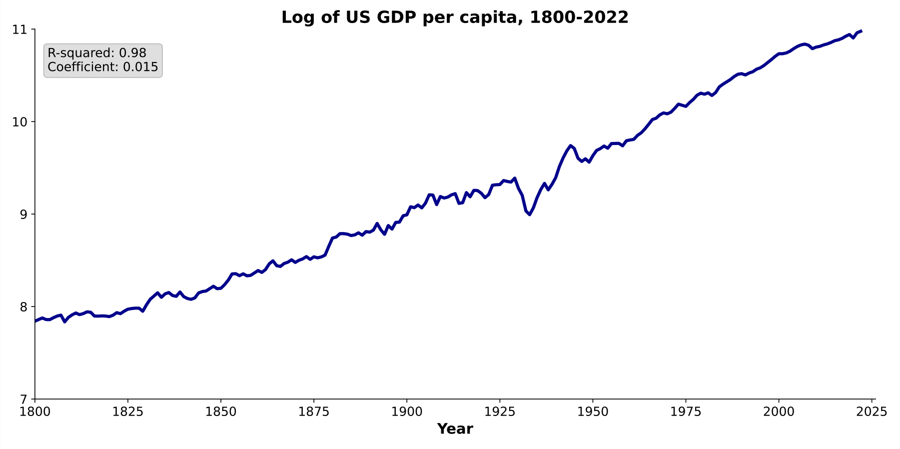
Figure 3: Growth over the last 200 years for selected countries

The Kaldor Facts

- The Kaldor (1961) (long-run economic growth) facts:
  1. Growth of labor productivity (Y/L) is constant over time
  2. Growth of capital per worker (K/L) is constant over time
  3. The real interest rate (r) is constant over time
  4. The capital-output ratio (K/Y) is constant over time
  5. Factor income shares $(1-\alpha)$ are constant over time
  6. Substantial variation in growth rates (in the order of 2–5 p.p.) among the world's fastest growing countries
- Economists believe successful growth models consistent with Kaldor facts
  - Recently, one of these facts has been challenged. Any guess?

Fact 1: Constancy of per-capita growth

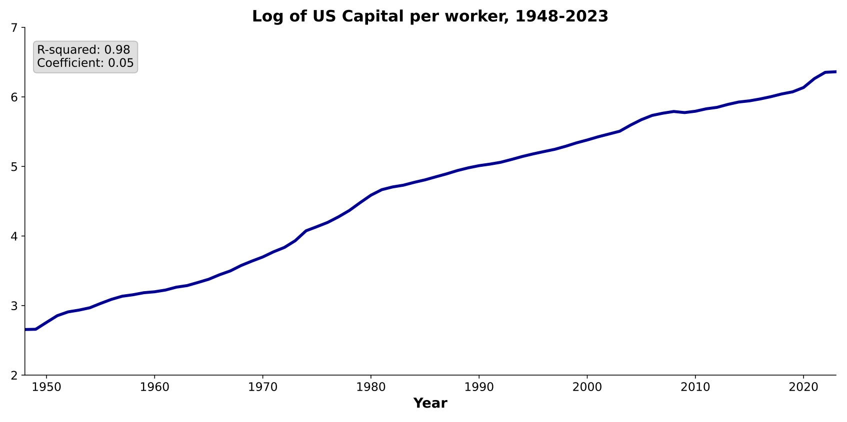
Linear fit suggests US series well approximated by 1.5% annual growth rate



Source: Maddison Project Database 2023.

Fact 2: Constancy of capital-per-worker growth

Linear fit suggests US series well approximated by 5% annual growth rate

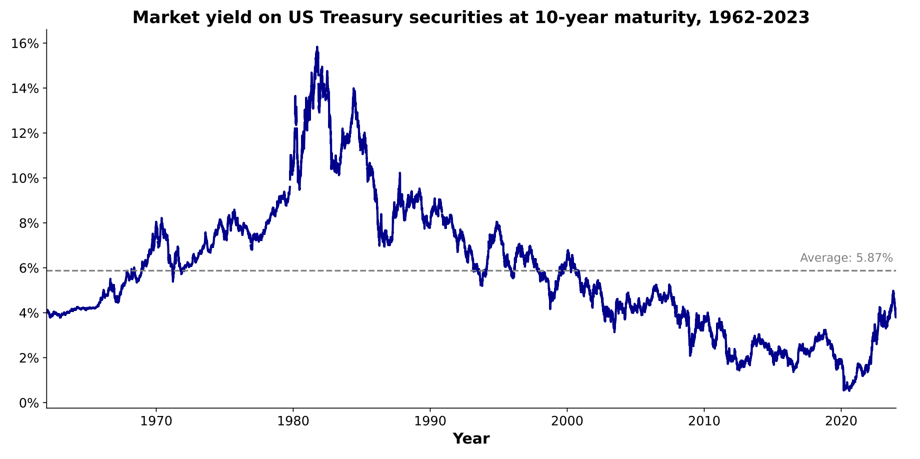


Source: FRED. Numerator: Current-cost net stock of fixed assets and consumer durable goods (K1WTOTL1ES000).

Denominator: Civilian labor force (CLF16OV).

Fact 3: Constancy of real interest rate

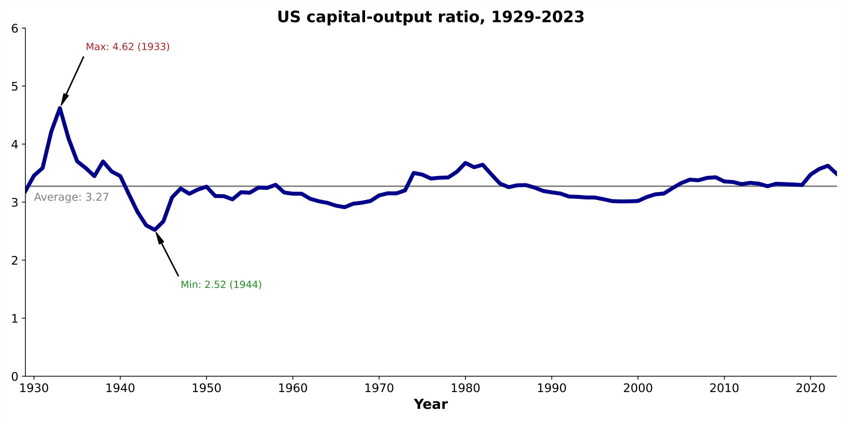
US real interest roughly constant at 6%



Source: FRED. Market yield on U.S. Treasury securities at 10-year constant maturity (DGS10).

Fact 4: Constancy of capital-output ratio

US capital-output ratio roughly constant around 3–3.5



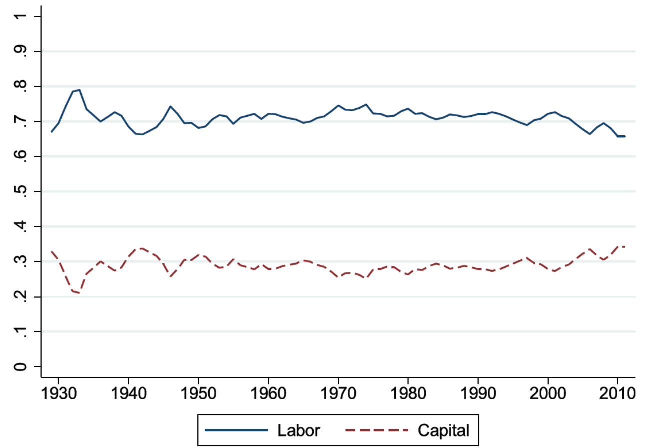
Source: FRED. Numerator: Current-cost net stock of fixed assets and consumer durable goods (K1WTOTL1ES000).

Denominator: Nominal GDP (GDPA).

Fact 5: Constancy of factor shares

US factor shares roughly constant over time? Fact now being challenged!

Remember our discussion on income shares?



Fact 6: Variation of 2–5 p.p. in growth rates among fastest growing countries

In 2023, there was a 2.1% percentage point difference in the annual growth rate of GDP per capita among 5 fastest growing countries

Country	Growth rate in 2023 GDP per capita
Armenia	8.8%
Ukraine	8.0%
Fiji	7.3%
Mauritius	7.1%
India	6.7%

Some Useful Math for Growth Analysis

Useful properties for growth analysis

- **Annual growth rate of Y in any year t** is annual percentage change in Y from previous year:

$$g_{Y,t} \equiv \hat{Y}_t = \frac{Y_t - Y_{t-1}}{Y_{t-1}} \tag{1}$$

- **Growth rates are compounding over time** (starting from $t = 0$):

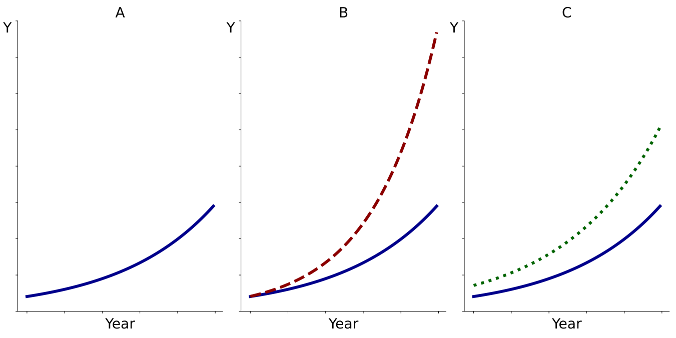
$$\begin{aligned} Y_1 &= (1 + g_{Y,1})Y_0, \\ Y_2 &= (1 + g_{Y,2})Y_1, \\ &= (1 + g_{Y,1})(1 + g_{Y,2})Y_0, \\ &\vdots \\ \implies \qquad Y_t &= (1 + g_{Y,1})(1 + g_{Y,2}) \cdots (1 + g_{Y,t})Y_0 \end{aligned}$$

Useful properties for growth analysis

- **With constant growth rates** (ie, $g_{Yt} = g_Y$ for all t):

$$\begin{aligned}
Y_1 &= (1 + g_Y)Y_0, \\
Y_2 &= (1 + g_Y)Y_1, \\
&= (1 + g_Y)(1 + g_Y)Y_0, \\
&= (1 + g_Y)^2 Y_0, \\
&\vdots \\
Y_t &= (1 + g_Y)^t Y_0 \qquad (2)
\end{aligned}$$

Useful properties for growth analysis



A. $Y_t = (1 + g_Y)^t Y_0$ for $g_Y > 0$
(Our thought experiment, in levels, for a given country)

B. $Y_t = (1 + g_Y)^t Y_0$ vs. $Y_t = (1 + \tilde{g}_Y)^t Y_0$, where $\tilde{g}_Y > g_Y > 0$
(Our thought experiment, in levels)

C. $Y_{1,t} = (1 + g_Y)^t Y_{1,0}$ vs. $Y_{2,t} = (1 + g_Y)^t Y_{2,0}$ for $g_Y > 0$
(What we would see if we plotted Y_t and Z_t of slides 11–12)

Useful properties for growth analysis

- **How to calculate the average annual growth for a given country between years t and $t+j$?**

$$Y_{t+j} = (1+g_Y)^j Y_t \qquad \implies \qquad g_Y = \left(\frac{Y_{t+j}}{Y_t}\right)^{\frac{1}{j}} - 1$$

- Doing this calculation for Chile in period 2000–2020: $g_Y \approx 0.0315$

- Number should be interpreted as a 3.15% average annual growth rate for the Chilean economy from 2000 to 2020

- **Useful approximation for any small number x:**

$$\ln(1+x) \approx x \tag{3}$$

Useful properties for growth analysis

- **Why is $\ln(1+x) \approx x$ a useful approximation?**
- Suppose we observe Y_t and Y_{t+j} and want to calculate the average annual growth rate g_Y
- We know:

$$Y_{t+j} = (1+g_Y)^j Y_t$$

- Taking logs: See properties of logs

$$\ln Y_{t+j} = j \cdot \ln(1+g_Y) + \ln Y_t$$

- Rearranging:

$$\ln(1+g_Y) = \frac{\ln Y_{t+j} - \ln Y_t}{j}$$

- **With our approximation:** $g_Y \approx \frac{\ln Y_{t+j} - \ln Y_t}{j}$

Useful properties for growth analysis

How long will take for a country to double its standards of living?

- Mathematically, we are asking: in what year t will GDP per capita for country c be twice that of year 0, where year 0 is the base year?
- To find the answer we can again exploit the relationship between Y_t, Y_0, and a *predicted* constant growth rate g_Y: $Y_t = (1 + g_Y)^t Y_0$
- We know:

$$Y_t = 2Y_0$$

- Substituting for Y_t:

$$(1 + g_Y)^t Y_0 = 2Y_0 \quad \iff \quad (1 + g_Y)^t = 2.$$

- **Taking logs and solving for t:** $t = \frac{\ln 2}{\ln(1+g_Y)} \approx \frac{0.7}{g_Y}$

Useful properties for growth analysis

How long will it take for a country to double its standards of living?

$$t = \frac{\ln 2}{\ln(1 + g_Y)} \approx \frac{0.7}{g_Y} \qquad (4)$$

- Let's put the math into practice
- **How long will it take for Puerto Rico to double its standards of living if it were to grow at an annual constant rate of:**
  - 1%? 70 years
  - 2%? 35 years
  - 3%? 23 years
  - 5%? 14 years
- What's the difference in years that it takes to double GDP per capita for two countries with annual growth rates of 1% and 1.1%, respect.?

Growth Accounting

Growth accounting in the Solow model

- **Solow's growth model** (due to Solow 1956, Swan 1956) **was a game changer!**
  - Simple framework to think about *proximate causes* of economic growth (technology, capital, labor), and how these drive cross-country income differences

- **At the center of Solow's growth model is the aggregate, neoclassical production function.** In its most common parametrization:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}, \qquad \alpha \in (0,1)$$

Y: output, A: technology parameter, K: capital, L: labor, α: capital share

Growth accounting in the Solow model

- **Multiple ways to look at the data through the Solow model:**
  1. Growth accounting exercises: Solow's (1957) contribution
  2. Regression-based approaches
  3. Calibration exercises
- **Solow's (1957) growth accounting exercise starts with**

$$Y(t) = A(t)K(t)^{\alpha}L(t)^{1-\alpha}$$

- Notice the difference in notation between:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \qquad \text{(discrete time)}$$

$$Y(t) = A(t)K(t)^{\alpha}L(t)^{1-\alpha} \qquad \text{(continuous time)}$$

- Sometimes it is more convenient to work in continuous time

Growth accounting in the Solow model

- Consider
$$Y(t) = A(t)K(t)^{\alpha}L(t)^{1-\alpha}$$
- Taking logs:
$$\ln Y(t) = \ln A(t) + \alpha \ln K(t) + (1-\alpha)\ln L(t)$$
- Differentiating both sides with respect to time:
$$\begin{aligned} \frac{\mathrm{d}\ln Y(t)}{\mathrm{d}t} &= \frac{\mathrm{d}\ln A(t)}{\mathrm{d}t} + \alpha\frac{\mathrm{d}\ln K(t)}{\mathrm{d}t} + (1-\alpha)\frac{\mathrm{d}\ln L(t)}{\mathrm{d}t} \\ \iff \qquad g_Y(t) &= g_A(t) + \alpha g_K(t) + (1-\alpha)g_L(t) \qquad (5) \end{aligned}$$
(In continuous time, time derivatives of logs are equal to growth rates)

Missing steps

- **Can decompose growth rate of Y into A and K, L contributions**

Growth accounting in the Solow model

- **Key equation:**

$$g_Y(t) = g_A(t) + \alpha g_K(t) + (1-\alpha)g_L(t)$$

  says the growth rate of GDP can be decomposed into the contributions of technology g_A, capital αg_K, and labor $(1-\alpha)g_L$

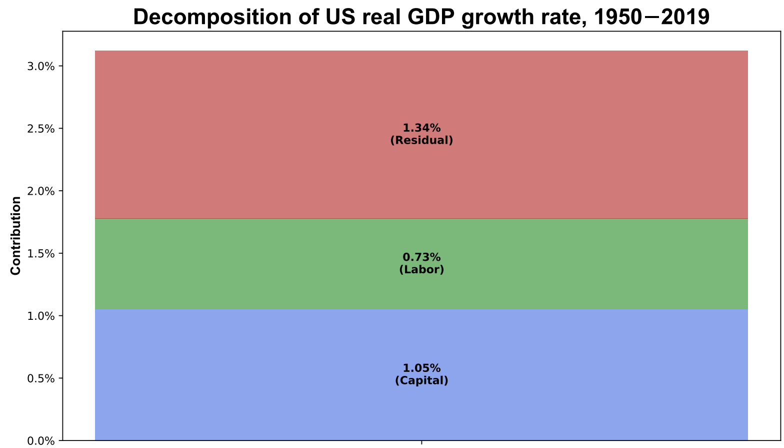
- **How to take key equation to data?**
  - Compute $(1-\alpha) = WL/\text{GDP}$ in data
  - Compute g_Y, g_K, g_L using data on GDP, capital stock, and labor force
  - **What about g_A?** It is not observed, but **it can be backed out as the residual**:

$$g_A(t) = g_Y(t) - \alpha g_K(t) - (1-\alpha)g_L(t)$$

  - **Residual captures much more than just technological change: it is a measure our ignorance!** (captures everything else that affects GDP growth and is not in model)

Growth accounting through the lens of the Solow model

Solow's accounting exercise revealed that **the residual largely explains growth in many countries**. One such example is the United States:



Taking Stock

Taking stock

Hopefully, you are now as fascinated about economic growth as I first was when I encountered this topic!

- Hope you now understand:
  - How to make **cross-country income comparisons** (both at particular points in time and over time)
  - Why **GDP, although not perfect, is a good metric to measure growth**
  - The **welfare implications of designing good economic policies**
  - **GDP per capita rankings are not immutable**: The relative position of a country in the world income distribution can change relatively quickly
  - The **properties of logs and how useful these are**

Taking stock

- We also learnt:
  - **The Kaldor facts** (long-run economic growth facts)
  - **How to do growth accounting**

- Next in our agenda:
  - Understand more mechanically the process of growth (Solow model in depth)
  - Play with the Solow model and add some tweaks to it!
  - Proximate vs. fundamental causes of growth

Questions?

Thank You!

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What could go wrong if we don't use PPPs?

- **Suppose we adjust GDP using market exchange rates (MERs) rather than PPPs to make cross-country comparisons in 2022 between US and Spain**
- Consider two (of many) exchange rates:
  - USD/EUR $= 0.88$ (Jan 2022) vs. USD/EUR $= 1.03$ (Sept 2022)
- **Which exchange rate is the "right" one to take for comparisons?**
  - Taking different exchange rates and same GDPs in domestic currencies: **US is 18% richer than Spain in September than in January**
- **Using PPPs instead of MERs or similar measures avoids these problems**
- While PPPs capture differences in costs of given bundle of goods & services between countries, MERs balance demand and supply for intl currencies and, as such, can be extremely volatile Back

Useful properties of logs

From your calculus classes you must know: ▸ Back

$$\ln(x \cdot y) = \ln x + \ln y \qquad \text{(logarithm of product)}$$

$$\ln\left(\frac{x}{y}\right) = \ln x - \ln y \qquad \text{(logarithm of quotient)}$$

$$\ln(x^a) = a \cdot \ln x \qquad \text{(logarithm of power)}$$

$$\ln(e^x) = x \quad \text{and} \quad e^{\ln x} = x \qquad \text{(log \& exp conversions)}$$

$$\ln\left(\frac{Y_t}{Y_{t-1}}\right) \approx \frac{Y_t - Y_{t-1}}{Y_{t-1}} \qquad \text{(log differences approx. growth rates)}$$

$$\frac{\mathrm{d}\ln x}{\mathrm{d}x} = \frac{1}{x} \qquad \text{(derivative of log)}$$

$$\int \ln x \,\mathrm{d}x = x \cdot \ln x - x + C \qquad \text{(integral of log)}$$

$$\ln 0 = -\infty, \qquad \ln 1 = 0, \qquad \ln 2 \approx 0.7, \qquad \ln 3 \approx 1.1, \qquad \ln 4 \approx 1.4$$

Missing Steps

- Notice that

$$\begin{aligned}\frac{\mathrm{d}\ln Y(t)}{\mathrm{d}t} &= \frac{\mathrm{d}\ln Y(t)}{\mathrm{d}Y(t)}\frac{\mathrm{d}Y(t)}{\mathrm{d}t} \\ &= \frac{1}{Y(t)}\dot{Y}(t) \\ &= g_Y(t)\end{aligned}$$

- Hence,

$$\begin{aligned} & \frac{\mathrm{d}\ln Y(t)}{\mathrm{d}t} = \frac{\mathrm{d}\ln A(t)}{\mathrm{d}t} + \alpha\frac{\mathrm{d}\ln K(t)}{\mathrm{d}t} + (1-\alpha)\frac{\mathrm{d}\ln L(t)}{\mathrm{d}t} \\ \iff \quad & g_Y(t) = g_A(t) + \alpha g_K(t) + (1-\alpha)g_L(t)\end{aligned}$$

Back