On the Efficiency of Competitive Equilibria with Pandemics

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Introduction

- Most pandemics create *local* and not *global* externalities
- Viruses typically do not travel great distances
  - Epi literature suggests droplets can travel at most 27 feet
- Policy intervention literature treats virus as global externality
  - Equilibria are inefficient
  - Lockdowns are very valuable

Introduction

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- Viruses typically do not travel great distances
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- Policy intervention literature treats virus as global externality
  - Equilibria are inefficient
  - Lockdowns are very valuable
- This paper: model pandemics as generating local externalities

Modeling pandemics

- Typical economic approach:
  - Prob of infection depends on aggregate economic activity
  - Implies that pandemics create global externalities
- Epidemiological approach:
  - Transmission occurs in meetings
  - But have little to say about meetings and economic outcomes
- Our approach:
  - Model relationship between meetings and economic activity
  - Implies pandemics create local externalities

Framework

- Embed SIR framework in search/matching/wage posting model
- Allow establishments to offer contracts based on infection status
  - Refer to establishments that offer the same contracts as “islands”
- Allow the virus to travel within but not across islands
  - Externalities are local

Main findings

- Our approach: modeling pandemics as local externalities
  - CE are efficient if contracts can be contingent on infection status
  - High social value of recovered/vaccinated agents
    - Recovered agents receive a premium over their marginal product
    - Susceptible agents willing to pay to pool with these agents.

Main findings

- Our approach: modeling pandemics as local externalities
  - CE are efficient if contracts can be contingent on infection status
  - High social value of recovered/vaccinated agents
    - Recovered agents receive a premium over their marginal product
    - Susceptible agents willing to pay to pool with these agents.
- Typical approach: modeling pandemics as global externalities
  - While competitive equilibria are obviously inefficient
  - Policy implications different from those commonly asserted
    - Susceptible agents can work too little relative to optimal outcome
  - Targeted policies very valuable

Literature

- Epidemiological literature
  - Kermack, McKendrick and Walker (1927), Bourouiba et al. (2014), Morawska et al, (2020), Somsen et al. (2020)
- Local public/club goods
  - Tiebout (1956), Stiglitz (1982), Cole and Prescott (1997), Ellickson et al. (1999).
- Econ-epi literature
  - Atkeson (2020), Stock (2020), Barro et al. (2020), Fernandez–Villaverde and Jones (2020), Alvarez et al. (2020), Acemoglu et al. (2020), Chari et al. (2020), Glover et al. (2020), Jones et al. (2020)
  - Eichenbaum et al. (2020), Bethune and Korinek (2020), Melosi and Rottner (2020), Toxvaerd and Rowthorn (2020)

Outline of talk

- Our approach: pandemics generate local externalities
- Typical approach: pandemics generate global externalities
- Generalizations

Pandemics generate local externalities

Model

- Discrete time model $t = 0, 1, ..., T$
- Continuum of workers/agents
  - Endowed with one unit of time
  - Can be in one of four health states denoted by η

$$\eta \in \left\{ \underbrace{U_S}_{\text{Susceptible}}, \underbrace{U_I}_{\text{Asymptomatic Infected}}, \underbrace{I}_{\text{Symptomatic Infected}}, \underbrace{R}_{\text{Recovered}} \right\}$$

  - Masses $\mu_{\eta t}$

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  - Masses $\mu_{\eta t}$
  - U_S and U_I types indistinguishable, refer to as "unknown" type U
  - I, R types publicly observable
- Continuum of *islands* $j \in \mathcal{J}$
  - $j > 0$: *work* islands in which production takes place
  - $j = 0$: *home* island in which no production takes place

Islands

Each island is associated with

- A production technology
  - For $j > 0$ one unit of labor generates A units of cons good
  - For $j = 0$ no production technology exists

- A triple of wage rates $\{w_{jUt}, w_{jIt}, w_{jRt}\}$

$$w_{\underbrace{j}_{\text{island}}\underbrace{\eta}_{\text{type}}\underbrace{t}_{\text{time}}}$$

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Firms choose which island to operate in

- If they operate on j have to pay wages $w_{j\eta t}$ to type η

Agents

- Endowed with one unit of time
- Preferences over the final consumption good are given by

$$U(c) = \sum_{t=0}^{T} \beta^t u(c_t)$$

- Infected agents suffer utility cost κ per period

Insurance firms

- Large number of insurance firms
- Offer contracts as a function of infection history $h_t = \eta^t$

$$z = (c, l) = \left\{ \underbrace{c_t(h_t)}_{\text{consumption}}, \underbrace{l_{jt}(h_t)}_{\text{labor supply on island } j} \right\}$$

Transmission of the virus

- Susceptible agents become infected in the process of production
  - Production requires meetings between agents
  - No infections take place on the home island

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- Let $\pi_t(h_t)$ denote the mass of history h_t on some island j

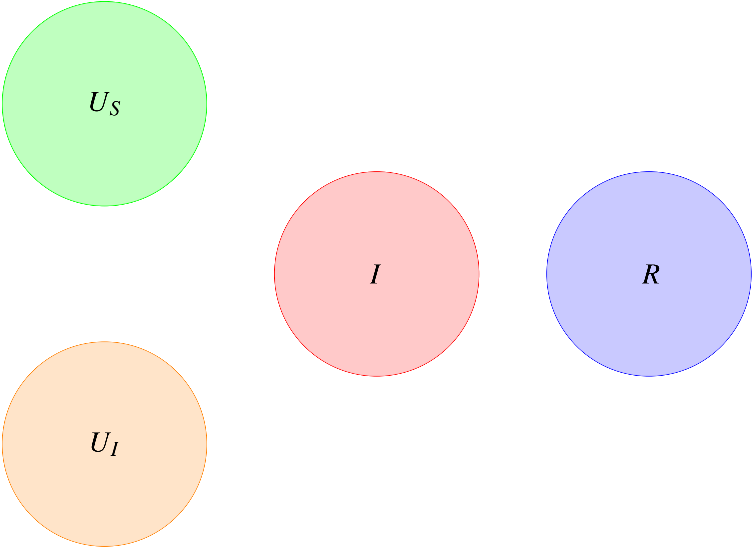
- Infection probability that island

$$\psi\left(\lambda_{jIt}\right) = \chi\lambda_{jIt}$$

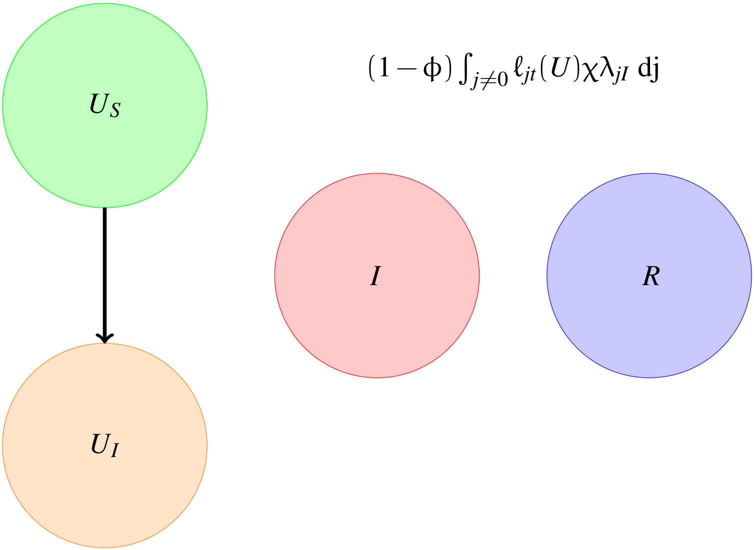
where

$$\lambda_{jIt} = \frac{\overbrace{\sum_{h_{t-1}} \left[\pi_t\left(h_{t-1}, U_I\right) l_{jt}\left(h_{t-1}, U\right) + \pi_t\left(h_{t-1}, I\right) l_{jt}\left(h_{t-1}, I\right)\right]}^{\text{labor supply of infected agents}}}{\underbrace{\sum_{h_{t-1}} \sum_{\eta} \left[\pi_t\left(h_{t-1}, \eta\right) l_{jt}\left(h_{t-1}, \eta\right)\right]}_{\text{total labor supply } L_{jt}}}$$

State transitions



State transitions



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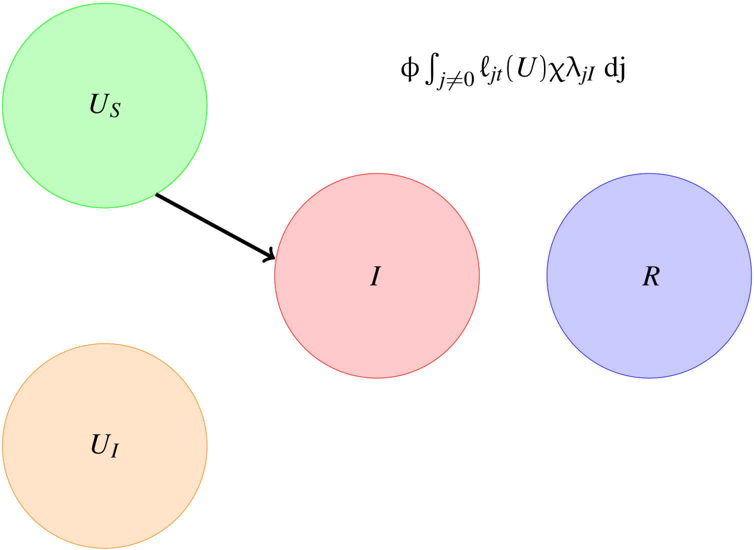

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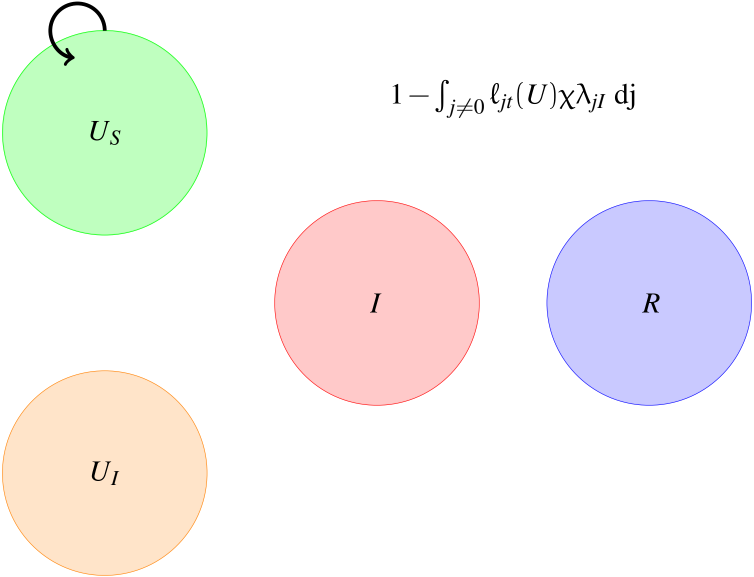

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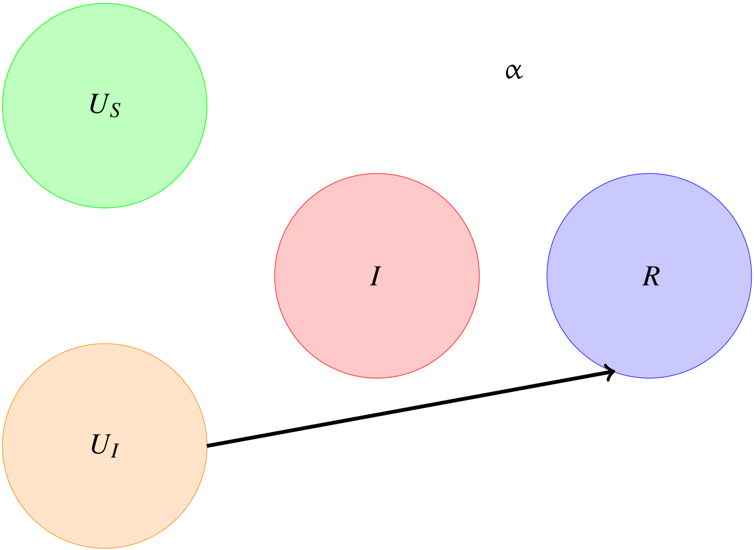

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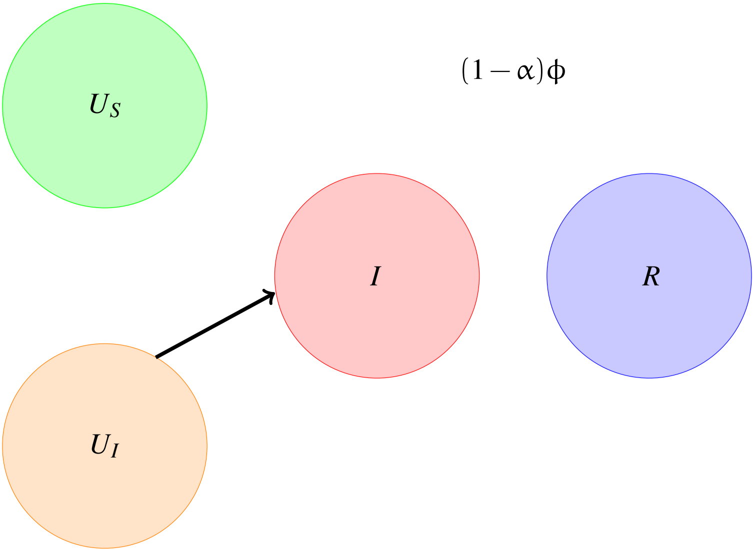

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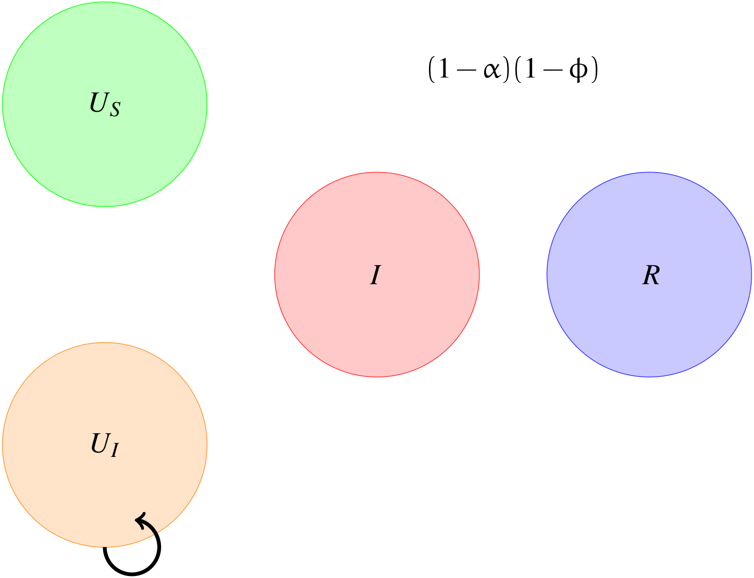

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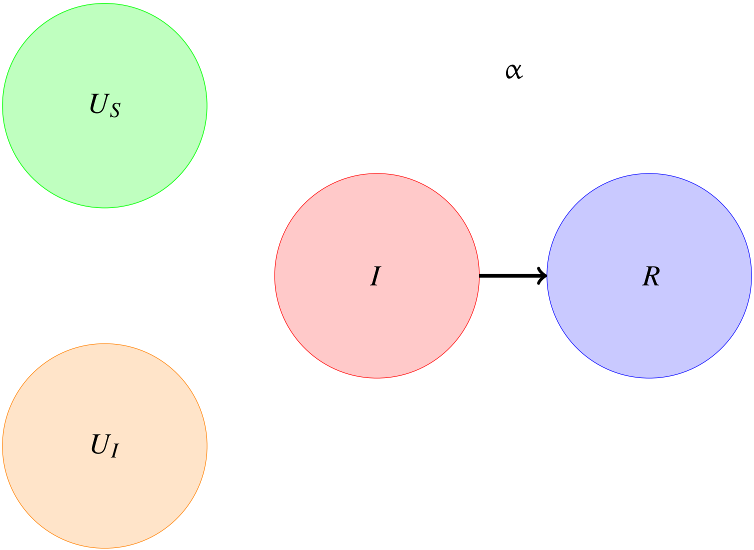

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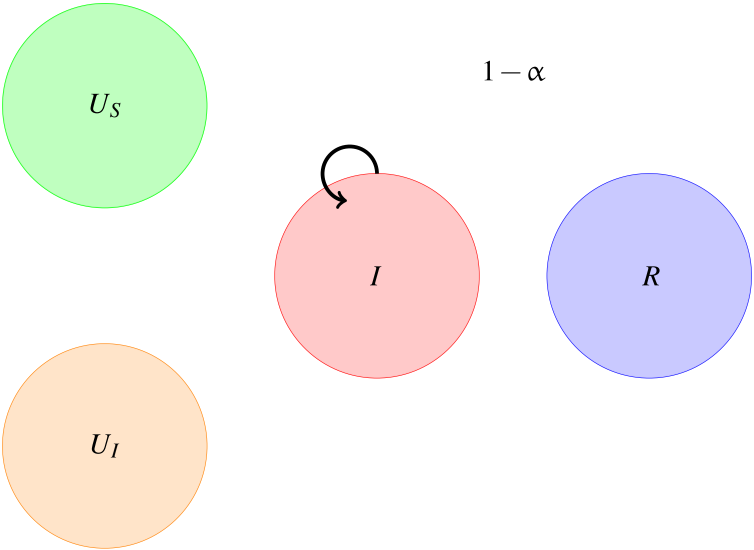

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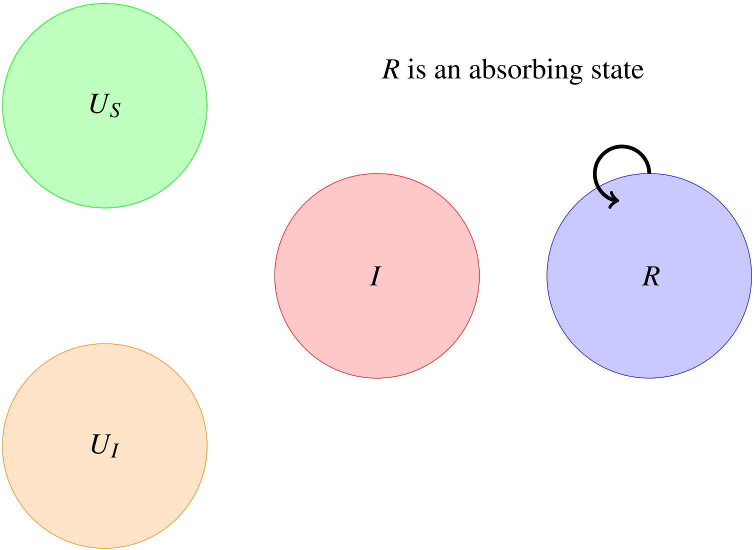

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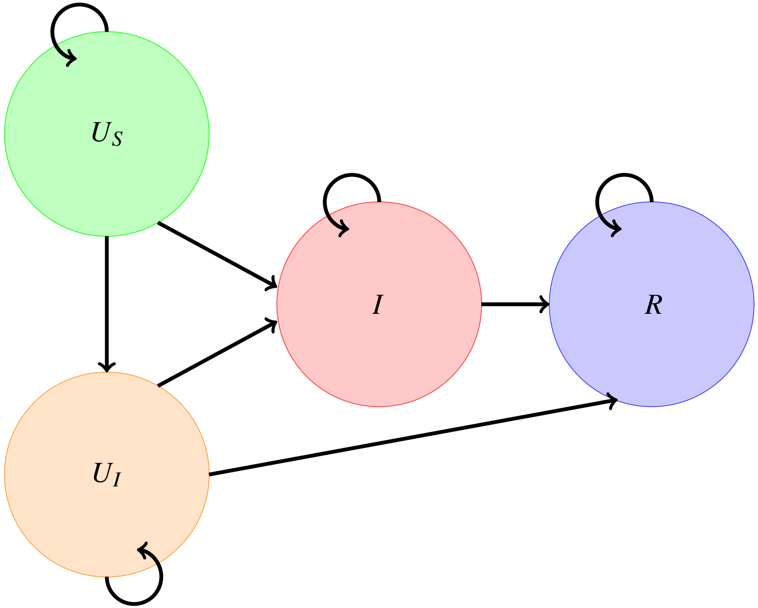

Infection probability

- L^*_{jt}: Mass of workers allocated to island j by all other firms
- Insurance firm takes L^*_{jt} as given

Infection probability

- L_{jt}^{*}: Mass of workers allocated to island j by all other firms
- Insurance firm takes L_{jt}^{*} as given
- If $L_{jt}^{*} > 0$ firm take infection probability $\chi\lambda_{jIt}^{*}$ as given

Infection probability

- L^*_{jt}: Mass of workers allocated to island j by all other firms
- Insurance firm takes L^*_{jt} as given
- If $L^*_{jt} > 0$ firm take infection probability $\chi\lambda^*_{jIt}$ as given
- If $L^*_{jt} = 0$ firm internalizes effect of choices on infection prob

$$\chi\lambda_{jIt} = \chi \frac{\sum_{h_{t-1}} \left[\tilde{\pi}_t\left(h_{t-1}, U_I\right) l_{jt}\left(h_{t-1}, U\right) + \tilde{\pi}_t\left(h_{t-1}, I\right) l_{jt}\left(h_{t-1}, I\right)\right]}{\sum_{h_{t-1}} \sum_{\eta} \left[\tilde{\pi}_t\left(h_{t-1}, \eta\right) l_{jt}\left(h_{t-1}, \eta\right)\right]}$$

$\tilde{\pi}$: mass of agents signed to the particular insurance firm

Matching technology

- Competitive production firms choose which island to locate in
  - Let γ_{jt} be that mass of firms on island j
- Workers and firms on j matched according to $M\left(L_{jt}, \gamma_{jt}\right)$
  - Market tightness $\theta_{jt} \equiv \gamma_{jt}/L_{jt}$.
  - $m_w\left(\theta_{jt}\right)$: probability that a worker is matched with a firm
  - $m_f\left(\theta_{jt}\right)$: probability that a firm is matched with a worker
- Matched firm/worker produce A units of goods per unit of time.
- Unmatched workers do not produce.
- Agents on a work island can get infected even if not matched
  - Besides the home island

Insurance firm's problem

$$\max_{z,\tilde{\pi}_0(\eta_0)} \left(\sum_{t\geqslant 0} Q_t \sum_{\eta_0} \sum_{h_t} \tilde{\pi}\left(h_t \mid \eta_0\right) \left[\int_{j\neq 0} m_w\left(\theta_{jt}\right) w_{j\eta t} l_{jt}\left(h_t\right) - c_t\left(h_t\right) \right] \right)$$

subject to

$$\sum_{t,h_t} \beta^t \frac{\tilde{\pi}_t\left(h_t \mid \eta_0\right)}{\tilde{\pi}_0\left(\eta_0\right)} \left[u\left(c_t\left(h_t\right)\right) - \int_{j\neq 0} l_{jt}\left(h_t\right) \mathbf{1}_{\{\eta_t = U_S\}} \psi\left(\lambda_{jIt}\right) \kappa \mathrm{d}j - \mathbf{1}_{\{\eta_t = U_I, I\}} \kappa \right]$$

$\geqslant \underline{V}\left(\eta_0\right)$ (market price for η_0), $\quad \forall \eta_0$,

and the state transition equations

Competitive equilibrium

Define the set of active islands

$$\Gamma_t = \left\{ j \in \mathcal{J} : l_{jt}\left(h_t\right) > 0 \;\text{ for some } h_t \right\}$$

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Allocation z, prices Q_t, market tightness Θ, and market utilities $\underline{V}(\eta_0)$

1. Insurance firm optimality
2. $m_f\left(\theta_{jt}\right) \sum_\eta \lambda_{j\eta t}\left[A - w_{j\eta t}\right] \leqslant 0$ for all j, with equality if $j \in \Gamma_t$
3. Resource constraint

$$\sum_{h_t} \pi_t\left(h_t\right) c_t\left(h_t\right) = \sum_{h_t} \pi_t\left(h_t\right) \int_{j \neq 0} m_w\left(\theta_{jt}\right) A l_{jt}\left(h_t\right) \mathrm{d}j$$

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4. For $j \notin \Gamma_t$ if $A - w_{j\eta t} > 0$ for all η then $m_w\left(\theta_{jt}\right) = 1$

Equilibrium characterization

In any competitive equilbrium there is

- No mixing of U and I types
  - Infection prob higher with mixing than if U types on their own
  - Separating them makes U types strictly better off
- Mixing of U and R types
  - Infection prob lower when mixing with R agents
  - Allowing for mixing makes U types strictly better off
- No involuntary unemployment
- I and R agents always work

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We now characterize the equilibrium in more detail

A Pareto problem

Consider the following Pareto problem

- Initial symptomatic infected consume A in all periods
  - All symptomatic infected assigned to island 1
- Initial U types get utility V_U
- Trace out the frontier by maximizing welfare of initial recovered

A Pareto problem

$$V_R(V_U) = \max \sum_{t \geqslant 0} \beta^t \sum_{h_t} \pi_t(h_t \mid R)\, u(c_t(h_t \mid R))$$

subject to

$$\sum_{\eta_0} \sum_{h_t} \pi_t(h_t \mid \eta_0) \left[A \int_{j \neq 0} l_{jt}(h_t \mid \eta_0)\, \mathrm{d}j - c_t(h_t \mid \eta_0) \right] \geqslant 0$$

$$c_t(h_t \mid I) = A, \quad l_{1t}(h_{t-1}, I \mid I) = 1$$

$$\sum_{t, h_t} \beta^t \frac{\pi_t(h_t \mid U)}{\pi_0(U)} \left[u(c_t(h_t \mid U)) - \int_{j > 1} l_{jt}(h_t \mid \eta_0)\, \mathbf{1}_{\{\eta_t = U_S\}} \psi\left(\lambda_{jIt}\right) \kappa - \mathbf{1}_{\{\eta_t = U_I, I\}} \kappa \right]$$
$$\geqslant V_U$$

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Proposition

There exists a CE that is efficient and solves the Pareto problem for values V_U^* and $V_R(V_U^*)$ with

$$V_U^* > V^a(U), \quad V_R(V_U^*) > V^a(R).$$

This CE has cross-subsidization from initial U to initial R agents.

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This CE has cross-subsidization from initial U to initial R agents.

- Initial R agents receive consumption $>$ marginal product
- Initial U agents receive consumption $<$ marginal product
- R valuable to initial U agents since lower infection prob
- U agents willing to give up consumption to pool with them

Robustness to private information

- Suppose R types are public but other types are private
- Competitive equilibrium coincides with the earlier one
  - R types get paid more than marginal product
  - U types get paid less than marginal product
  - I types get paid marginal product
  - No type has incentive to mimic any other type

Role of “insurance” firms

- Solve *coordination problem* that arises with local public goods
- Similar to “clubs” in the club good literature
- Of course they also provide insurance

Efficiency of competitive equilibria

Two key assumptions drive the efficiency results

- Contracts can be a function of worker types
- Pandemics are local externalities

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We now relax the last assumption

Pandemics as an (almost) global externality

Single work island

- Suppose there is a single work island $j = 1$ and home island $j = 0$
  - Assume that $w_{1\eta t} = A$ for all η, t
- Contract $z = (c, l) = \{c_t(h_t), l_t(h_t)\}$
  - $l_t(h_t)$: probability of allocation to work island

Infection probability

- Let z^* denote the equilibrium contract
- Insurance firms take infection probability as *given*

$$\psi\left(\lambda_{It}^*\right) = \chi \frac{\sum_{h_{t-1}} \left[\pi_t\left(h_{t-1}, I\right) l_t^*\left(h_{t-1}, I\right) + \pi_t\left(h_{t-1}, U_I\right) l_t^*\left(h_{t-1}, U\right)\right]}{\sum_{h_{t-1}} \sum_{\eta} \pi_t\left(h_{t-1}, \eta\right) l_t^*\left(h_{t-1}, \eta\right)}$$

- Key to inefficiency result in this environment

Characterization of equilibrium

In any equilibrium

- There is no cross-subsidization
  - Otherwise insurance firms will cream skim
- There is mixing between U and I types
  - Symptomatic infected agents always work
  - Unknown type agents will have to mix in order to consume

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The competitive equilibrium is inefficient

- Firms don’t internalize effect of labor supply on infection prob.
- We show that there are *positive* congestion externalities

Parametric Assumption

Pandemics are costly so that not efficient for U agents to always work

▸ condition

Efficient allocation

Assume for simplicity that there are no asymptomatic agents

Characterize PO allocations that leave agents as well off as CE

Proposition

Suppose pandemics are costly. Then, there exists some $\mu^* > 0$ *such that if* $\mu_{I0} < \mu^*$*, then in the solution to the Pareto problem*

- *Infected agents never work in any period*
- *Unknown (i.e. susceptible) and recovered agents work in all periods*

Targeted lockdowns are optimal

- If mass of infected agents small efficient to never work ($l_t = 0$)
  - In contrast to CE where they always work ($l_t = 1$)
- Tax U type agents to finance their consumption
- U types work more than in the CE

Simple Pigouvian taxes

- Suppose government can only levy untargeted Pigouvian taxes
- Agents have to be at least as well off as the CE
- Consider dual of insurance firms problem
  - Maximize welfare of initial unknown type

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Consider first the static problem

Problem for insurance firm

$$\max_{l_0(U)\in[0,1]} u\left(\left(1-\tau\right)Al_0\left(U\right)+T\right)-\xi_0 l_0\left(U\right)\chi\lambda_I^*\kappa-\left(1-\xi_0\right)\kappa$$

where $\xi_0 = \frac{\mu_{U_S 0}}{\mu_{U0}}$ and

$$\lambda_I^* = \frac{\mu_{I0}+\mu_{U_I 0} l_0^*\left(U\right)}{\mu_{U0} l_0^*\left(U\right)+\mu_{I0}+\mu_{R0}}$$

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I and R agents work one unit independent of the tax

Optimal Pigouvian tax

Implements solution to the following problem:

$$\max_{l}\ u(Al) - \xi_0 l \chi \lambda_I \kappa - (1 - \xi_0)\kappa$$

where

$$\lambda_I = \frac{\mu_{I0} + \mu_{U_I 0} l}{\mu_{U0} l + \mu_{I0} + \mu_{R0}}$$

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Evaluating the foc at the CE allocation

$$-\xi_0 \chi \frac{\partial \lambda_I}{\partial l} \kappa$$

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Evaluating the foc at the CE allocation

$$-\xi_0 \chi \frac{\partial \lambda_I}{\partial l} \kappa$$

Infection probability is *decreasing* in l if $\mu_{U_I 0}$ is small enough

Positive congestion externality in static model

- Aggregate economic activity can be too *low*
- Conventional wisdom
  - People left on their own will work too much
  - Aggregate economic activity will be too *high*
  - Lockdowns are a good idea
- If U agent works more
  - Other U_S agents less likely to meet I agents
  - But more likely to meet U_I agents
- If most U agents are susceptible optimal policy is labor *subsidy*

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- If most U agents are susceptible optimal policy is labor *subsidy*
- In dynamic model additional externality details
  - Increasing labor supply of U increases flow of newly infected
  - Increases probability of future infection
  - Race between the static and dynamic externalities

Labor supply in dynamic model

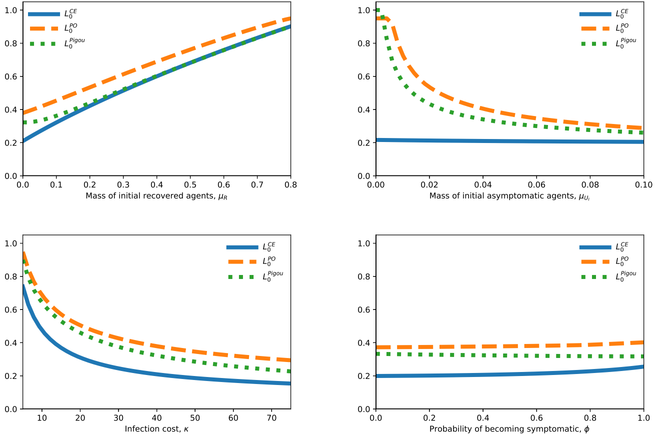


Figure: Employment in the first period as a function of parameters.

Alternate infection technologies

Overview

- Consider a generalization of the infection technology
- Efficiency results in multi-island world unchanged
- Results in single island world technology dependent

Infection technology

- Generalization based on Acemoglu et. al (2020)

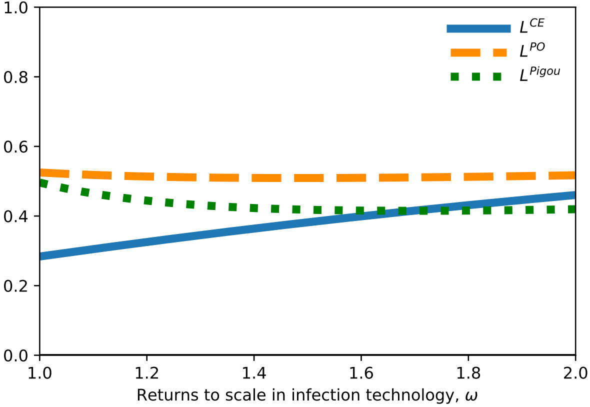
$$\psi(\mu, l) = \chi \frac{\mu_{U_I} l_j(U) + \mu_I l_j(I)}{\left[\mu_U l_j(U) + \mu_I l_j(I) + \mu_R l_j(R)\right]^{2-\omega}}$$

- $\omega \in [1, 2]$ governs returns to scale
- $\omega = 1$ is our baseline and $\omega = 2$ is a quadratic technology

Multi-island World

- If an equilibrium exists it is efficient
- Insurance firms can still solve coordination problem
- Existence issue when technology has increasing returns
  - Incentive to split U types across an increasing number of islands

Single island model (static)



Positive congestion externality present for ω not too large

In paper

- Limited commitment
  - Welfare theorems can fail due to a pecuniary externality
- Positive vacancy cost so that there is unemployment
  - Results unchanged

Evolution of histories

$$\pi_{t+1}(h_t, S) = \pi_t(h_{t-1}, S)\left(1 - l_t(h_{t-1}, S)\chi\lambda_{It}\right)$$

$$\pi_{t+1}(h_{t-1}, S, I) = \pi_t(h_{t-1}, S)\, l_{jt}(h_{t-1}, S)\chi\lambda_{It}$$

$$\pi_{t+1}(h_{t-1}, I, I) = (1 - \alpha)\pi_t(h_{t-1}, I)$$

$$\pi_{t+1}(h_{t-1}, I, R) = \alpha\pi_t(h_{t-1}, I)$$

$$\pi_{t+1}(h_{t-1}, R, R) = \pi_t(h_{t-1}, R)$$

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Parametric Assumption

$$u'(A)\frac{A}{\mu_{U0}} - \mu_{U_S0}\chi\kappa < 0.$$

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Dynamic model

Insurance firm chooses (c_η, l_η) to maximize

$$
\begin{aligned}
V_t(\boldsymbol{\mu}_t, \lambda^*_{It}) = \max_{c_\eta, l_\eta} \sum \mu_{\eta t} \left[u(c_\eta) - l_S \psi(\lambda^*_t) \kappa \mathbf{1}_{\eta=S} - \kappa \mathbf{1}_{\eta=I} \right] \\
+ \beta V_{t+1}\left(\boldsymbol{\mu}_{t+1}, \lambda^*_{It+1}\right)
\end{aligned}
$$

subject to

$$
\sum_\eta \mu_\eta c_\eta \leqslant \sum \mu_\eta l_\eta A
$$

and the laws of motion for $\boldsymbol{\mu}_t$

Don't internalize effect of l_U on current *and* future infection prob

Effect of a small increase in labor supply of S

Total derivative of objective wrt l_S evaluated at equilibrium allocation

$$\underbrace{-\mu^*_{Ut} l^*_{Ut} \chi \frac{\partial \lambda^*_{It}}{\partial l^*_U} \kappa + \beta \left[\frac{\partial V_{t+1}}{\partial \mu_{It+1}} - \frac{\partial V_{t+1}}{\partial \mu_{St+1}} \right] \mu^*_{Ut} l^*_{Ut} \chi \frac{\partial \lambda^*_{It}}{\partial l^*_U}}_{\text{externality from current infection}}$$

$$+ \quad \underbrace{\beta \frac{\partial V_{t+1}}{\partial \lambda^*_{It+1}} \frac{\partial \lambda^*_{It+1}}{\partial l^*_U}}_{\text{externality from future infection}}$$

Two externalities in dynamic model

Externality from current infection.

- Static component identical to static model
  - Always positive
- Dynamic component due to change in future masses of types
  - Typically positive

Externality from future infection

- Increasing l_U increases the *flow* of newly infected agents $\mu_{Ut} l_U \lambda^*_{It}$
- This increases infection probability in the future
- Negative externality

Overall effect on welfare ambiguous Back

Type transitions with asymptomatic agents

$$\pi_{t+1}\left(h_{t-1},U_S,U_I\right)=\left(1-\phi\right)\pi_t\left(h_{t-1},U_S\right)\int_{j\neq 0} l_{jt}\left(h_{t-1},U\right)\chi\lambda_{Ijt}\mathrm{d}j$$

$$\pi_{t+1}\left(h_{t-1},U_S,I\right)=\phi\pi_t\left(h_{t-1},U_S\right)\int_{j\neq 0} l_{jt}\left(h_{t-1},U\right)\chi\lambda_{Ijt}\mathrm{d}j$$

$$\pi_{t+1}\left(h_{t-1},U_S,U_S\right)=\pi_t\left(h_{t-1},U_S\right)\left[1-\int_{j\neq 0} l_{jt}\left(h_{t-1},U\right)\chi\lambda_{Ijt}\mathrm{d}j\right]$$

$$\pi_{t+1}\left(h_{t-1},U_I,U_I\right)=\left(1-\phi\right)\left(1-\alpha\right)\pi_t\left(h_{t-1},U_I\right)$$

$$\pi_{t+1}\left(h_{t-1},U_I,I\right)=\phi\left(1-\alpha\right)\pi_t\left(h_{t-1},U_I\right)$$

$$\pi_{t+1}\left(h_{t-1},U_I,R\right)=\alpha\pi_t\left(h_{t-1},U_I\right)$$

$$\pi_{t+1}\left(h_{t-1},R,R\right)=\pi_t\left(h_{t-1},R\right)$$

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Autarky values

$$V^a(U) = \max \sum_{t,h_t} \beta^t \pi(h_t \mid U)\left[u(c_t(h_t)) - l_t(h_t)\mathbf{1}_{\{\eta_t = U_S\}}\psi(\lambda_{It})\kappa - \mathbf{1}_{\{\eta_t = U_I\}}\kappa\right]$$

subject to

$$\sum_{h_t} \pi(h_t \mid U)(c_t(h_t \mid U) - l_t(h_t \mid U)A) \leqslant 0, \quad \forall t,$$

and

$$\lambda_{It} = \frac{\sum_{h_{t-1}}[\pi_t(h_{t-1}, U_I)\, l_t(h_{t-1}, U)]}{\sum_{h_{t-1}}\sum_{\eta=\{U,R\}}[\pi_t(h_{t-1}, \eta)\, l_t(h_{t-1}, \eta)]}$$

$$V^a(R) = \sum_{t=0}^{T} \beta^t u(A)$$

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