ECO 3302 – Intermediate Macroeconomics

Lecture 4: National Income—How It Is Earned

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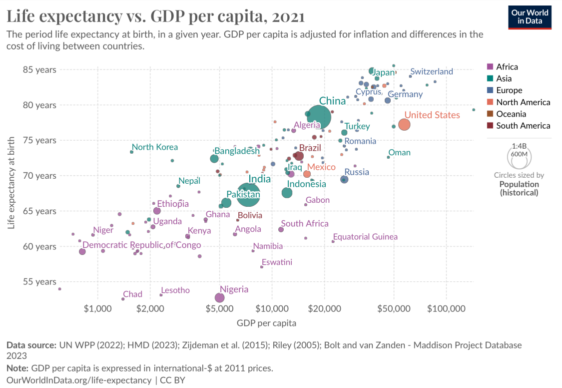
Introduction

- We said that **GDP is the most important macro variable**
- To see why, it suffices to look at the correlation of GDP with other measures of economic development (eg, life expectancy, mortality, literacy, HDI, ...)



GDP (highly) positively correlates with life expectancy

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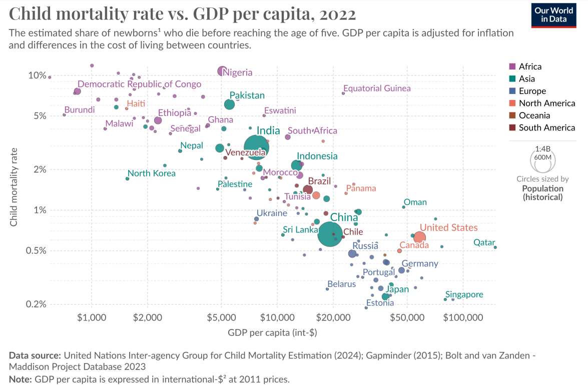

GDP (highly) negatively correlates with child mortality

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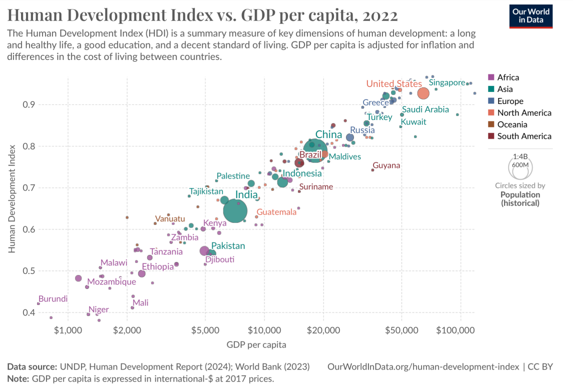

GDP (highly) positively correlates with human development

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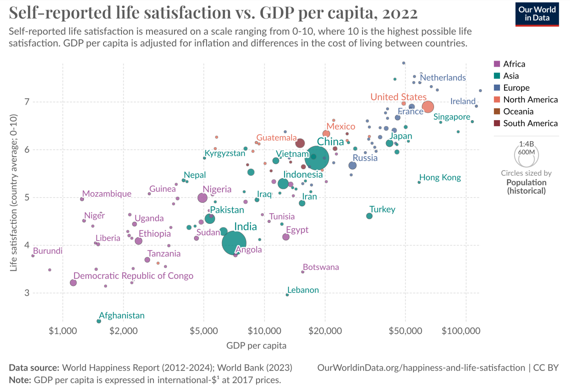


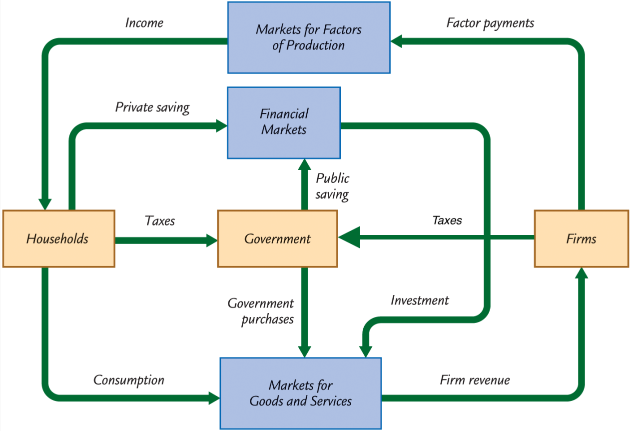
GDP (highly) positively correlates with human welfare

Introduction

- We said that **GDP is the most important macro variable**
- To see why, it suffices to look at the correlation of GDP with other measures of economic development (eg, life expectancy, mortality, literacy, HDI, ...)
- GDP per capita highly correlates with measures of economic development:
  - Life expectancy
  - Mortality
  - Human development index
  - Self-reported life satisfaction
  - ...and many more!

Introduction

- Today, we focus on understanding what determines a nation's income and who receives it. A good place to start is the (more realistic) circular flow chart



Production Functions

Production functions

- The output produced in an economy depends on available technologies and quantities of production factors
  - Technologies: reflect ability to turn inputs into outputs
  - Production factors: inputs used in production
    - Capital: buildings, machines, etc.
    - Labor: hours of work
- Technologies often represented with production functions that relate inputs to outputs
  - Eg, it takes one professor (me), many students (you), computers, and time to produce SMU intermediate macroeconomists

Types of production functions

- **Macro vs. Micro production functions:**
  - Macro production functions (aka *aggregate* production functions) relate factors of production in an economy (capital K and labor L) to real GDP (Y):
$$Y = F(K, L)$$
  - Micro production functions relate factors of production of individual producers (eg, establishments, firms, sectors) to their outputs. Eg,
$$y_i = f(k_i, \ell_i)$$
- **Gross-output vs. value-added production functions:**
  - Gross-output production functions relate all production inputs—that is, factors and material inputs—to output: $y_i = f(\{x_{ij}\}_j, k_i, \ell_i)$
  - Value-added production functions relate *factor* inputs to VA: $y_i = f(k_i, \ell_i)$

Our production function

In this course, we focus on macro, value-added production functions

The *aggregate* production function

The *aggregate* production function F relates factors of production (K, L) to real GDP (Y):

$$Y = F(K, L)$$

Remark:

- Modern macro analysis relies more and more on micro production functions
- And then aggregates to arrive at macro aggregates like GDP See
- This approach permits tackling new interesting questions
  (eg, macro effects of micro shocks, misallocation of production factors, ...)

Properties of production functions $Y = F(K, L)$

- **Twice-continuously differentiable**: F is continuous with respect to K and L and is also differentiable (ie, first and second derivatives exist)
  - Technical assumption made for convenience

- **Positive marginal products**: level of output increases with amount of inputs

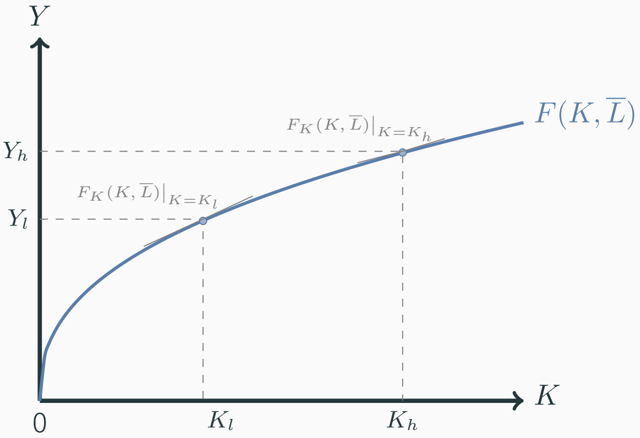
$$F_K(K, L) \equiv \frac{\partial F(K, L)}{\partial K} > 0, \qquad F_L(K, L) \equiv \frac{\partial F(K, L)}{\partial L} > 0$$

- **Diminishing marginal products**: more of an input, keeping all else constant, increases output by less and less

$$F_{KK}(K, L) \equiv \frac{\partial^2 F(K, L)}{\partial K^2} < 0, \qquad F_{LL}(K, L) \equiv \frac{\partial^2 F(K, L)}{\partial L^2} < 0$$

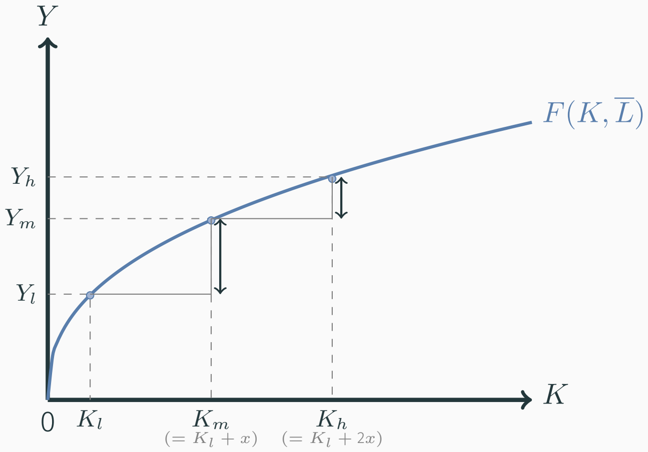
Positive marginal products

The higher the level of an input, the higher the level of output (all else equal)



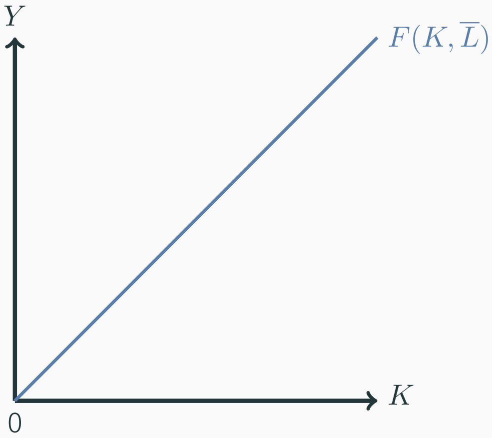
Diminishing marginal products

More of an input increases output by less and less (all else equal)



Question

Does plotted production function have increasing MPK? And diminishing MPK?



Properties of production functions $Y = F(K, L)$

- **Homogeneity of degree k**: function F is homogeneous of degree k in K, L if

$$F(\lambda K, \lambda L) = \lambda^k F(K, L), \qquad \forall \lambda > 0.$$

Eg, if you scale all inputs by common factor (λ), output scales by a predictable power of that factor (λ^k)

  - $k = 0$: output doesn't change when inputs are scaled
  - $k = 1$: function is linearly homogeneous (doubling all inputs doubles output)
  - $k = 2$: scaling inputs by λ would multiply output by λ^2

    $\vdots$

Properties of production functions $Y = F(K, L)$

- Returns to scale:
  - **Constant returns to scale (CRS)**: function F exhibits constant returns to scale if

    $$F(\lambda K, \lambda L) = \lambda F(K, L), \qquad \forall \lambda > 0$$

    A function exhibits CRS if it is homogeneous of degree 1

    Eg, If we double all inputs, we double output

  - **Decreasing returns to scale (DRS)**: F exhibits decreasing returns to scale if

    $$F(\lambda K, \lambda L) < \lambda F(K, L), \qquad \forall \lambda > 1$$

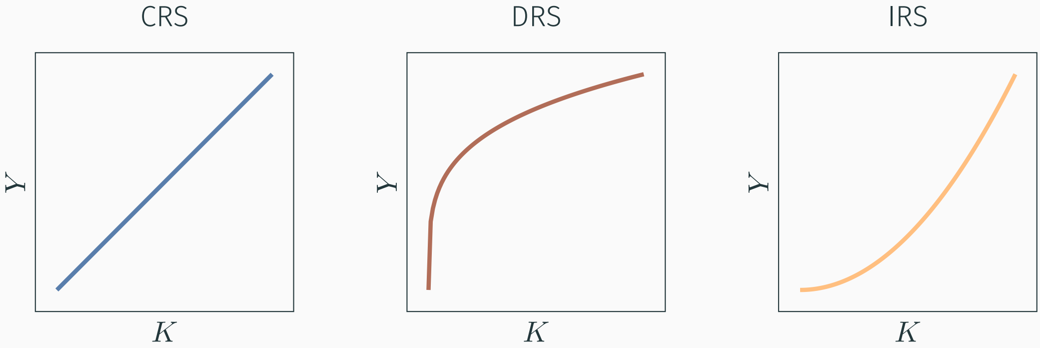
    Eg, If we double all inputs, we less than double output

  - **Increasing returns to scale (CRS)**: F exhibits constant returns to scale if

    $$F(\lambda K, \lambda L) > \lambda F(K, L), \qquad \forall \lambda > 1$$

    Eg, If we double all inputs, we more than double output

Examples: Returns to scale



Properties of production functions $Y = F(K, L)$

- **Inada conditions**: F satisfies

$$F(K,0) = 0 \quad \text{and} \quad F(0,L) = 0 \qquad \text{(Essential inputs)}$$

$$\lim_{K \to 0} F_K(K,L) = +\infty \quad \text{and} \quad \lim_{K \to +\infty} F_K(K,L) = 0$$

$$\lim_{L \to 0} F_L(K,L) = +\infty \quad \text{and} \quad \lim_{L \to +\infty} F_K(K,L) = 0$$

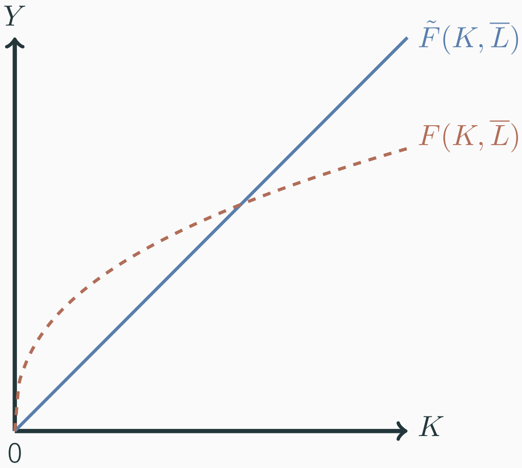
Last two lines: the effect of the first input unit is the largest, and the effect of one additional input unit when the use of that input is approaching infinite is zero

Neoclassical production function

A production function F is **neoclassical** if it is twice-continuously differentiable, has positive and diminishing marginal products, CRS, and meets Inada conditions

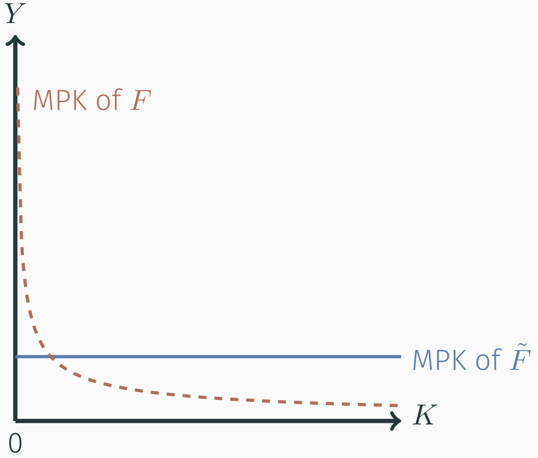
Example: Inada Conditions

Production function F satisfies Inada conditions but $\tilde{F}$ does not. *Why?*



Example: Inada Conditions

MP of $\tilde{F}$ is constant so marginal gain from adding more capital always the same



Properties of production functions

Direction of technological change

Consider production function $\tilde{F}(K, L; A)$, where A is technological progress. Production function $\tilde{F}$ admits different types of technological progress:

1. **Hicks-Neutral**: $\tilde{F}(K, L; A) = AF(K, L)$

   (Ie, technology increases productivity of both factors proportionally)

2. **Solow-Neutral or capital-augmenting**: $\tilde{F}(K, L; A) = F(AK, L)$

   (Ie, technology increases productivity of capital)

3. **Harrod-Neutral or labor-augmenting**: $\tilde{F}(K, L; A) = F(K, AL)$

   (Ie, technology increases productivity of labor)

In practice, technological change is a mixture of three types above, so realistic production functions are of type $\tilde{F}(K, L, \mathbf{A}) = AF(A_K K, A_L L)$

Popular production functions

1. Cobb–Douglas

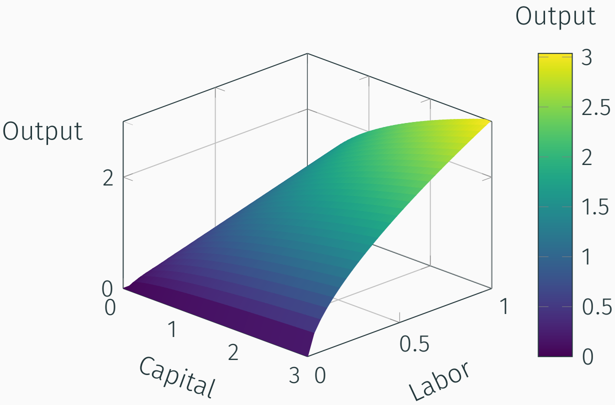
$$Y = AK^{\alpha}L^{\beta}, \qquad \alpha, \beta \in \mathbb{R}_{+}$$

- A: TFP, technological parameter governing the level of output
- α, β: scale parameters determining the returns to scale

- Key properties of Cobb-Douglas production function:
  1. Hicks-neutral tech. progress (other types easily squeezed in)
  2. Constant Elasticity of Substitution (CES) between K and L, equal to 1
  3. Returns to scale $= \alpha + \beta$
  4. Positive and diminishing marginal products for both K and L
  5. K and L are q-complements
  6. Log-linear form amenable to regression analysis: $\ln Y = \ln A + \alpha \ln K + \beta \ln L$

Visual representation: Cobb–Douglas production function

$$\text{GDP} = 2 \times \text{Capital}^{0.38} \times \text{Labor}^{0.62}$$



Cobb–Douglas: Elasticity of substitution between factors

2. Constant Elasticity of Substitution (CES) between K and L, equal to 1

- Elasticity of substitution between K and L defined as percentage change in capital-labor ratio divided by percentage change in MRTS_{KL}:

$$\sigma := \frac{\mathrm{d}(K/L)/(K/L)}{\mathrm{dMRTS}_{LK}/\text{MRTS}_{LK}} = \frac{\mathrm{d}(K/L)}{\mathrm{dMRTS}_{LK}} \times \frac{\text{MRTS}_{LK}}{K/L}$$

Intuitively, it tells us how easily a firm can adjust the mix of capital and labor in response to changes in relative prices of inputs (e.g., wages for labor and the cost of capital)

- By definition, $\text{MRTS}_{LK} = \text{MPL}/\text{MPK}$. Hence,

$$\sigma = \underbrace{\frac{\mathrm{d}(K/L)}{\mathrm{dMRTS}_{LK}}}_{=\frac{\alpha}{\beta}} \times \underbrace{\frac{\text{MPL}}{\text{MPK}}}_{=\frac{\beta}{\alpha}\cdot\frac{K}{L}} \times \frac{L}{K} = 1$$

$\sigma = 1$ means K and L are neither perfect substitutes nor perfect complements. (They can be substituted for each other at a constant rate, but NOT one for one; Ie, if relative price changes by $x\%$, the capital labor ratio will adjust by $x\%$)

Cobb–Douglas: K and L are q-complements

q-complements

Given production function $F(K,L)$, factors K and L are said to be **q-complements** if the cross-partial derivatives of the production function are positive; ie,

$$\frac{\partial^2 F(K,L)}{\partial K \partial L} > 0 \quad \text{and} \quad \frac{\partial^2 F(K,L)}{\partial L \partial K} > 0$$

Intuitively, two factors are q-complements if an increase in the quantity of one input (e.g., labor) raises the marginal product of the other input (e.g., capital)

5. **K and L are q-complements**. With Cobb–Douglas:

$$\frac{\partial \text{MPK}}{\partial L} = \frac{\partial \text{MPL}}{\partial K} = \alpha\beta \frac{Y}{KL} > 0$$

2. Leontief (aka Fixed Proportions)

$$Y = \min\left(\frac{K}{a}, \frac{L}{b}\right), \qquad a, b \in \mathbb{R}_{++}$$

- a, b: constants, representing fixed proportions required to produce

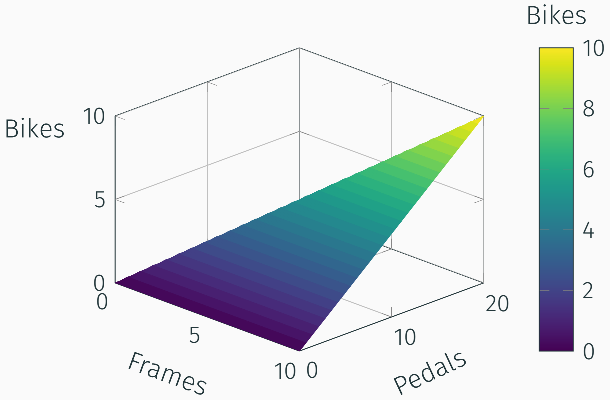
Useful for industries with rigid production requirements

Eg, Building 1 bike requires 1 frame and 2 pedals (10 frames and 30 pedals $\rightarrow$ 10 bikes)

- Key properties of Leontief production function:
  1. No substitutability between K and L
  2. Output determined by limiting input—the one in short supply
  3. CRS
  4. NO positive and diminishing marginal products for both K and L

Visual representation: Leontief production function

$$\text{Bikes} = \min\left\{\text{Frames}, \frac{\text{Pedals}}{2}\right\}$$



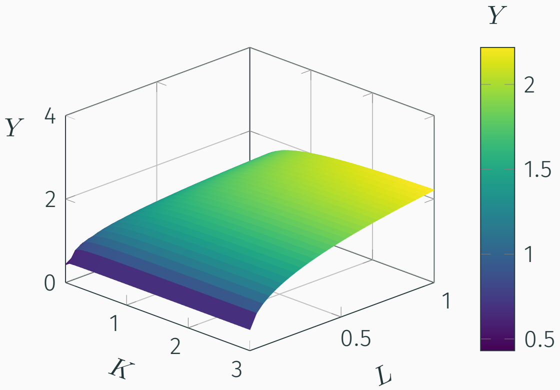
3. Constant Elasticity of Substitution (CES)

$$Y = A\left[\gamma(A_K K)^{\frac{\sigma-1}{\sigma}} + (1-\gamma)(A_L L)^{\frac{\sigma-1}{\sigma}}\right]^{\nu\cdot\frac{\sigma}{\sigma-1}}, \qquad \gamma \in (0,1), \quad \sigma \in [0,\infty)$$

- A: TFP, technological parameter governing the level of output
- $A_i,\ i \in \{K, L\}$: factor-augmenting technological change
- γ: share parameter determining the importance of factors
- σ: elasticity of substitution between factors
  - $\sigma = 0$: no substitution (Leontieff)
  - $\sigma < 1$: gross complements
  - $\sigma = 1$: neither perfect substitutes nor perfect complements (Cobb–Douglas)
  - $\sigma > 1$: gross substitutes
  - $\sigma = \infty$: perfect substitutes (linear production)
- ν: parameter governing returns to scale

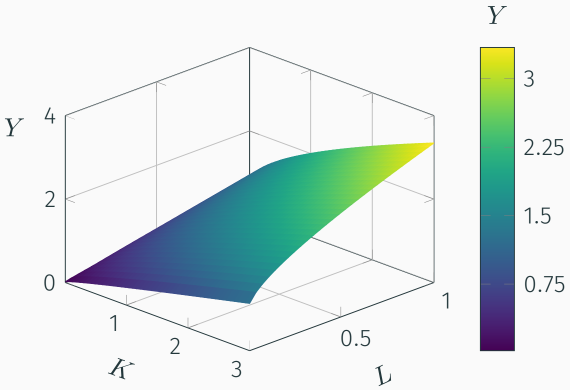
Visual representation: CES production function I (gross complements)

$$Y = 2 \times \left[\gamma K^{\frac{\sigma-1}{\sigma}} + (1-\gamma) L^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \qquad A_K = A_L = \nu = 1,\ \gamma = 0.4,\ \sigma = 0.5$$



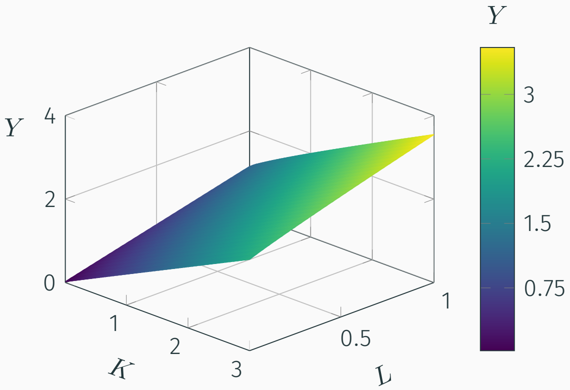
Visual representation: CES production function II (gross substitutes)

$$Y = 2 \times \left[\gamma K^{\frac{\sigma-1}{\sigma}} + (1-\gamma) L^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \qquad A_K = A_L = \nu = 1,\ \gamma = 0.4,\ \sigma = 2$$



Visual representation: CES production function III ($\to$ perfect substitutes)

$$Y = 2 \times \left[\gamma K^{\frac{\sigma-1}{\sigma}} + (1-\gamma) L^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \qquad A_K = A_L = \nu = 1,\ \gamma = 0.4,\ \sigma = 10$$



CES production function

- Key properties of CES production function:
  1. Constant Elasticity of Substitution (CES) between K and L, equal to σ
  2. Returns to scale $= \nu$ (often assumed to be CRS; ie, $\nu = 1$)
  3. Positive and diminishing marginal products for both K and L
  4. K and L may be q-complements ($\sigma \leq 1$) or q-substitutes ($\sigma > 1$)
- **CES more general than Cobb–Douglas**:
  - Elasticity of substitution: σ vs. 1
  - Factor shares: change with input levels vs. constant (will see later)
  - Factor-augmenting technological change: biased growth vs. not
- CES very useful to understand inequality b/w high- and low-skilled workers

CES: Factor-augmenting technological change

$$Y = A\left[\gamma(A_K K)^{\frac{\sigma-1}{\sigma}} + (1-\gamma)(A_L L)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

- Relative marginal product of two factors:

$$\frac{\text{MPK}}{\text{MPL}} = \underbrace{\left(\frac{\gamma}{1-\gamma}\right)}_{\text{relative importance of capital}} \underbrace{\left(\frac{A_K}{A_L}\right)^{\frac{\sigma-1}{\sigma}}}_{\text{technology bias}} \underbrace{\left(\frac{K}{L}\right)^{-\frac{1}{\sigma}}}_{\text{substitution bias}}$$

  - **Substitution bias**: relative MP decreasing in relative factor abundance for $\sigma > 0$
  - **Technology bias**:
    - $\sigma > 1$: increase in A_K (relative to A_L) increases relative MP of K **(capital bias)**
    - $\sigma < 1$: increase in A_K (relative to A_L) reduces relative MP of K **(labor bias)**
    - $\sigma = 1$: neither a change in A_K or A_L is biased towards any factor

Popular production functions

4. Stone–Geary

$$Y = \begin{cases} A(K - \overline{K})^{\alpha}(L - \overline{L})^{\beta}, & \text{if } K > \overline{K} \text{ and } L > \overline{L} \\ 0, & \text{otherwise} \end{cases}$$

- A: TFP, technological parameter governing the level of output
- $\alpha, \beta \in \mathbb{R}_+$: scale parameters determining the returns to scale
- $\overline{K}, \overline{L} \in \mathbb{R}_+$: minimum input requirements

Useful for industries with fixed costs

Eg, Need 2 grids and 1 worker to start producing electricity

- Stone–Geary production function has similar properties as Cobb–Douglas once minimum input requirements met

The Decision-Making of Firms

Firm decision-making

- Economy is populated by many firms
- **Firms make production and pricing decisions**
  - **Production decisions** (eg, how much to produce?) **determine GDP**
  - Pricing decisions (eg, at what price to sell output? How much to pay production factors?) tell us about the competitive environment of the economy
- To simplify analysis, macroeconomists often assume a **representative firm**
  - Many small, identical firms (ie, same goals, same tech, same constraints)
- We assume **firm makes decisions to either***:
  - **Maximize profits** subject to technology constraint + pricing rule(s)
  - **Minimize costs** subject to minimal output constraint + pricing rule(s)

*Under certain conditions, profit max and cost min equivalent

Firm decision-making

- Examples of pricing rules:
  - Firm takes all prices (P, W, L) as given
  - Firm sets output price equal to a markup over marginal cost:

$$P = \mu \times \text{MC}, \qquad \mu \geq 1.$$

  - Firm sets factor prices equal to markdown over marginal product:

$$W = \nu_L \times \text{MPL} \qquad \text{and} \qquad R = \nu_K \times \text{MPK},$$

  where $\nu_L, \nu_K \in (0, 1]$

- Firms have market power when able to set prices away from marginal cost or marginal products
  - **Monopoly**: firm can set output price away from marginal cost ($\mu \neq 1$)
  - **Monopsony**: firm can set factor prices away from marginal products ($\nu \neq 1$)

Firm decision-making

- For simplicity, we **assume *all* markets are competitive**:
  - Firms price at marginal cost ($\mu = 1$)
  - Factors earn their marginal products ($\nu_L, \nu_K = 1$)
- **Pricing behavior of firms and nature of technology both crucial to determine distribution of income**:
  - All markets competitive + CRS $\Rightarrow$ No profits
  - All markets competitive + DRS $\Rightarrow$ Positive profits/rents of fixed factor
  - Some market is not competitive + CRS $\Rightarrow$ Positive profits

In general, whether there are profits or not depends on ratio of returns to scale to markup (see Hasenzagl Perez 2023)

Profit maximization with competitive markets

- Assuming *all* markets competitive
- Profit maximization:

$$\max_{K,L\geq 0} \quad \Pi := PY - WL - RK$$
$$\text{s.t.} \qquad Y = F(K,L)$$

- FOCs:

$$W = P \times \text{MPL} \equiv P \times \frac{\partial F(K,L)}{\partial L}$$
$$R = P \times \text{MPK} \equiv P \times \frac{\partial F(K,L)}{\partial K}$$

Profit maximization with competitive markets (Cobb–Douglas + CRS)

$$\max_{K,L \geq 0} \quad \Pi := PAK^{\alpha}L^{1-\alpha} - WL - RK$$

- FOCs:

$$W = P \times \text{MPL} \equiv P \times (1-\alpha)\frac{Y}{L}$$
$$R = P \times \text{MPK} \equiv P \times \alpha\frac{Y}{K}$$

Rental rate of factor decreasing in quantity available of factor

Profit maximization with competitive markets (CES + CRS)

$$\max_{K,L\geq 0} \quad \Pi := PA\left[\gamma(A_K K)^{\frac{\sigma-1}{\sigma}} + (1-\gamma)(A_L L)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} - WL - RK$$

- FOCs:

$$W = P \times \mathsf{MPL} \equiv P \times (1-\gamma)\Omega(K,L)YA_L^{\frac{\sigma-1}{\sigma}}L^{-\frac{1}{\sigma}}$$
$$R = P \times \mathsf{MPK} \equiv P \times \gamma\Omega(K,L)YA_K^{\frac{\sigma-1}{\sigma}}K^{-\frac{1}{\sigma}}$$

Rental rate of factor decreasing in quantity available of factor, increasing in its importance, ambiguous direction with respect to technology (depends on σ)

The National Distribution of Income

The National Distribution of Income

- Accounting identity states economy's total output equals total income:

$$\underbrace{PY}_{\text{Nominal GDP}} = \underbrace{WL}_{\text{labor income}} + \underbrace{RK}_{\text{capital income}} + \underbrace{\Pi}_{\text{profits}}$$

- Dividing both sides by nominal GDP:

$$1 = \underbrace{\frac{WL}{PY}}_{\substack{\equiv \Lambda_L \\ \text{(labor share)}}} + \underbrace{\frac{RK}{PY}}_{\substack{\equiv \Lambda_K \\ \text{(capital share)}}} + \underbrace{\frac{\Pi}{PY}}_{\substack{\equiv \Lambda_\Pi \\ \text{(profit share)}}}$$

- GDP exhausted compensating factors of production and firm owners

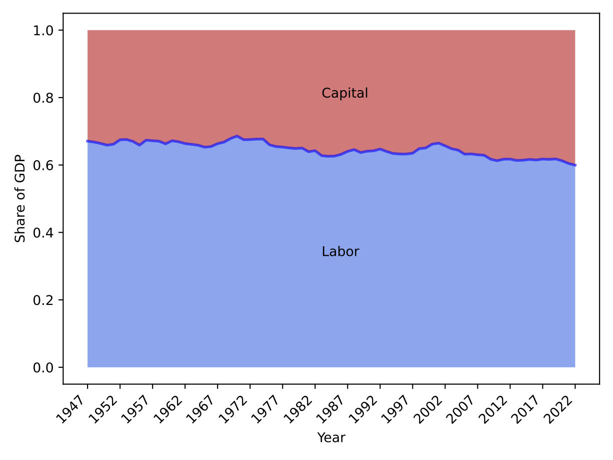
Calculating income shares: Data

$$\underbrace{\frac{WL}{PY}}_{\substack{\equiv\Lambda_L \\ \text{(labor share)}}} + \underbrace{\frac{RK}{PY}}_{\substack{\equiv\Lambda_K \\ \text{(capital share)}}} + \underbrace{\frac{\Pi}{PY}}_{\substack{\equiv\Lambda_\Pi \\ \text{(profit share)}}} = 1$$

- **Can calculate income shares with data on WL, RK or Π and PY**
  - Challenge: Neither R nor Π observed
  - Standard approaches:
    1. Get WL and PY from National Accounts, compute Λ_L, and assume competitive economy (ie, $\Lambda_\Pi = 0$) so that $\Lambda_K = 1 - \Lambda_L$
    2. Get WL, PY and K from National Accounts, impute/estimate R, compute Λ_L and Λ_K, back out $\Lambda_\Pi = 1 - \Lambda_L - \Lambda_K$

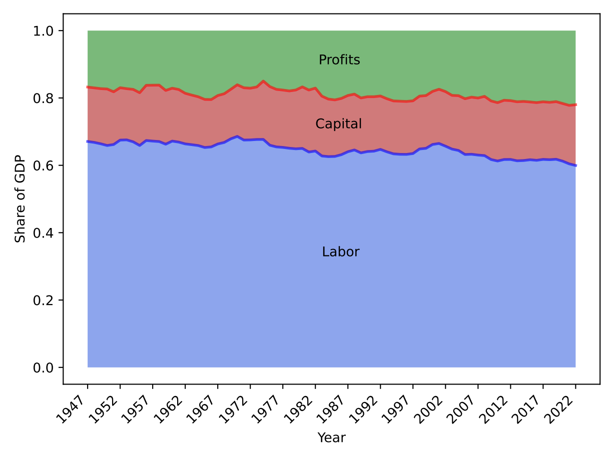
Calculating income shares: US Data

- **Approach 1**: Get WL and PY from National Accounts, compute Λ_L, and assume competitive economy (ie, $\Lambda_\Pi = 0$) so that $\Lambda_K = 1 - \Lambda_L$



Calculating income shares: US Data

- **Approach 2**: Get WL, PY and K from National Accounts, impute $R = 0.12$, compute Λ_L and Λ_K, back out $\Lambda_\Pi = 1 - \Lambda_L - \Lambda_K$



Calculating income shares: Data

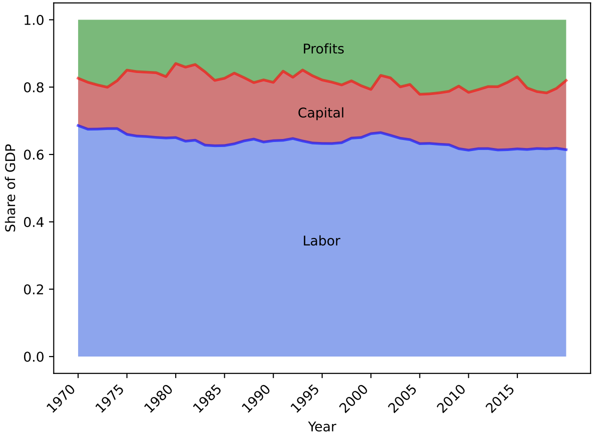
$$\underbrace{\frac{WL}{PY}}_{\equiv \Lambda_L \atop \text{(labor share)}} + \underbrace{\frac{RK}{PY}}_{\equiv \Lambda_K \atop \text{(capital share)}} + \underbrace{\frac{\Pi}{PY}}_{\equiv \Lambda_\Pi \atop \text{(profit share)}} = 1$$

- **Can calculate factor shares with data on WL, RK or Π and PY**
  - Challenge: Neither R nor Π observed
  - Modern approach: Use econ theory and micro data to estimate income shares
    (See Hasenzagl and Perez 2023, "*The Micro-Aggregated Profit Share*")
    - Micro production functions + constrained cost minimization + aggregation theory
    - Profit share: function of returns to scale and market power indicators

$$\Lambda_\Pi = \frac{\text{Sales}}{\text{GDP}} \left(1 - \frac{\overline{\text{RS}}}{\overline{\mu}_{\text{hsw}}} - \text{Cov}_\omega \left[RS, \frac{1}{\mu} \right] \right)$$

Calculating income shares: US Data

- **Approach 3**: Econ theory + macro (NIPA) and micro (Compustat) data
  (Results from Hasenzagl and Perez 2023) ▸ See micro-aggregated profit share and market power indicators



Calculating income shares: US Data

Takeaways:

- **Several ways to compute income shares:**
  - Standard (macro) approaches: Use data from NIPA to compute labor share, and then back out capital and profit shares with assumptions/imputations
  - Modern (micro) approaches: Use economic theory with macro and micro data to estimate income shares
- **Income shares in US data:**
  - Labor share declined 8 p.p. from 1947 to 2023 (broad agreement)
  - Profit share constant at 18% of GDP (controversial)

Calculating income shares: Models

- As with US data, we **can compute income shares in economic models**
- **Income shares in model depend on model details**:
  - Technology: CRS vs. DRS vs. IRS
  - Market power in output markets: perfect competition vs. monopoly
  - Market power in factor markets: perfect competition vs. monopsony
- Market power and technology both crucial to determine income shares
- We calculate income shares under diff. assumptions to illustrate this point

Income shares with Cobb–Douglas (CRS + perfect competition)

- Recall profit maximization yields:

$$W = P \times (1-\alpha)\frac{Y}{L}$$
$$R = P \times \alpha\frac{Y}{K}$$

- Rearranging:

$$\frac{WL}{PY} = 1-\alpha$$
$$\frac{RK}{PY} = \alpha$$

- With CD production technology, CRS, and perfect competition, factor shares constant and given by exponents of production function (profit share is 0)

Income shares with CES (CRS + perfect competition)

- Recall profit maximization yields:

$$W = P \times (1-\gamma)\Omega(K,L)YA_L^{\frac{\sigma-1}{\sigma}}L^{-\frac{1}{\sigma}} \implies \frac{WL}{PY} = (1-\gamma)\Omega(K,L)\left(A_L L\right)^{\frac{\sigma-1}{\sigma}}$$

$$R = P \times \gamma\Omega(K,L)YA_K^{\frac{\sigma-1}{\sigma}}K^{-\frac{1}{\sigma}} \implies \frac{RK}{PY} = \gamma\Omega(K,L)\left(A_K K\right)^{\frac{\sigma-1}{\sigma}}$$

- Profit share:

$$\Lambda_\Pi = 1 - \Lambda_K - \Lambda_L = 0$$

(Verify to earn BP)

- With CES production technology, CRS, and perfect competition, factor shares change with input levels and profit share is 0

Income shares and Euler's Theorem

- The fact fact that profit share is 0 with profit maximization, CRS technology, and perfect competition is a consequence of Euler's Theorem

Euler's Theorem

If $F : \mathbb{R}^n_+ \to \mathbb{R}_+$ is homogeneous of degree k, then

$$k \cdot F(X_1, \dots, X_n) = \sum_{i=1}^{n} \frac{\partial F}{\partial X_i} X_i \tag{1}$$

- By Euler's Theorem, if $F(K, L)$ has CRS (ie, it is homogeneous of degree 1):

$$\begin{aligned} & Y = \text{MPL} \times L + \text{MPK} \times K \\ \iff \quad & Y = WL + RK \quad \text{(using profit max + competitive factor markets)} \end{aligned}$$

All output is exhausted compensating factors of production

Income shares with Cobb–Douglas (DRS + perfect competition)

- Profit maximization ($\alpha + \beta < 1$):

$$W = P \times \beta \frac{Y}{L} \implies \frac{WL}{PY} = \beta$$
$$R = P \times \alpha \frac{Y}{K} \implies \frac{RK}{PY} = \alpha$$

- Profit share:

$$\Lambda_\Pi = 1 - \Lambda_K - \Lambda_L = 1 - (\alpha + \beta) > 0$$

- With CD production technology, DRS, and perfect competition, factor shares constant and profit share is positive

Income shares with CES (DRS + perfect competition)

- Suppose $\nu \in (0,1)$

- Profit maximization yields:

$$W = P \times \nu(1-\gamma)\Omega(K,L)YA_L^{\frac{\sigma-1}{\sigma}}L^{-\frac{1}{\sigma}} \implies \frac{WL}{PY} = \nu(1-\gamma)\Omega(K,L)\left(A_L L\right)^{\frac{\sigma-1}{\sigma}}$$

$$R = P \times \nu\gamma\Omega(K,L)YA_K^{\frac{\sigma-1}{\sigma}}K^{-\frac{1}{\sigma}} \implies \frac{RK}{PY} = \nu\gamma\Omega(K,L)\left(A_K K\right)^{\frac{\sigma-1}{\sigma}}$$

- Profit share:

$$\Lambda_\Pi = 1 - \Lambda_K - \Lambda_L = 1 - \nu > 0 \qquad \text{(Verify to earn BP)}$$

- With CES production technology, DRS, and perfect competition, factor shares change with input levels and profit share is positive

Income shares with CRS + monopoly ($\mu > 1$)

- Take as given Hasenzagl–Perez result:

$$\Lambda_\Pi = 1 - \frac{\text{RS}}{\mu}$$

- By CRS assumption:

$$\Lambda_\Pi = 1 - \frac{1}{\mu} > 0 \qquad (\text{since } \mu > 1)$$

- With CRS and market power in output markets, profit share is positive

Income shares with CD (CRS + monopsony: $\nu_L < 1$ and $\nu_K = 1$)

- Take as given Hasenzagl–Perez result:

$$\begin{aligned}
\Lambda_\Pi &= 1 - \alpha\nu_K - (1-\alpha)\nu_L \\
&= (1-\alpha)(1-\nu_L) && \text{(since } \nu_K = 1\text{)} \\
&> 0 && \text{(since } \alpha, \nu_L \in (0,1)\text{)}
\end{aligned}$$

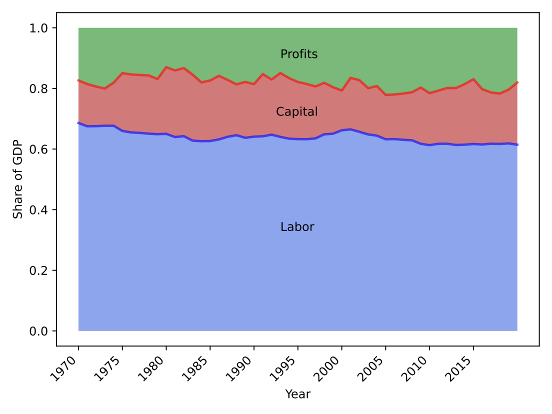
- With CD production technology, CRS, and market power in labor market, profit share is positive

Inequality

Inequality

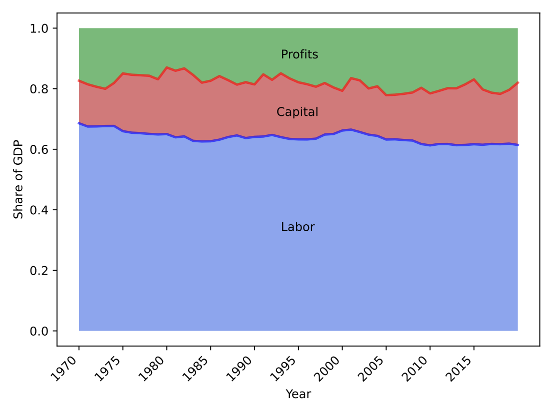
- **Many types of economic inequality**: income, consumption, wealth, ...
- Income shares related to income inequality
- Income shares allow us to talk about income inequality *under certain assumptions on ownership*:
  - Representative agent owns production factors and firms $\rightarrow$ no inequality
  - Capitalists vs. Workers $\rightarrow$ inequality
  - High-skilled vs. low-skilled workers $\rightarrow$ inequality
- Let's take a look ...!

US Income inequality: Representative agent



- Assuming we are all equal (work the same, invest the same, have same ownership of firms), there is no inequality: our income comes from different sources!

US Income inequality: Capitalists vs. Workers



- Assuming two types of workers (capitalists own capital and firms, workers supply labor), workers make WL, capitalists get $RK + \Pi$. There is inequality if

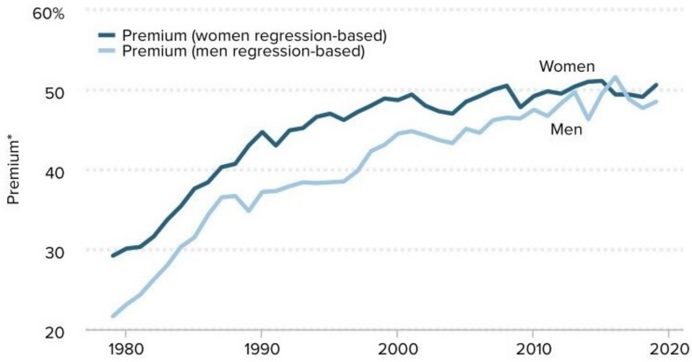
$$\frac{WL}{N_{\text{workers}}} \neq \frac{RK + \Pi}{N_{\text{capitalists}}}$$

US Income inequality: High- vs. low-skilled workers

- Aggregate income shares, although speak to inequality, still mask important sources of inequality due to heterogeneity:
  - Skills of workers
  - Returns on capital investments
  - Ownership of firms
- We now try to understand labor income inequality (ie, how WL is split) in terms of differences in skills (eg, college vs. non-college workers)

US college wage premium

College-educated workers in US earn increasingly more than high-school grads

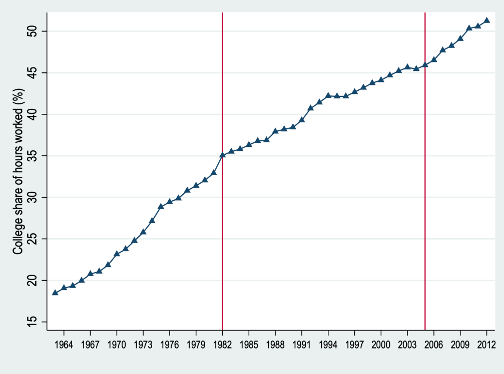


*Percent by which wages of college graduates exceed those of otherwise-equivalent high school graduates, regression-adjusted.

Source: Authors' analysis of State of Working America Data library: **College wage premium**. See Gould (2020).

US college wage premium

College-educated workers in US earn increasingly more than high-school grads yet college-educated workers have become relatively more abundant



How can this be? CES production function can help rationalize these patterns!

CES: Inequality between high- and low-skilled workers

$$Y = A\left[\gamma(A_H H)^{\frac{\sigma-1}{\sigma}} + (1-\gamma)(A_L L)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

H: high-skilled workers L: low-skilled workers

- With profit-maximizing firms and competitive markets:

$$\underbrace{\frac{W_H}{W_L}}_{\text{skill premium}} = \underbrace{\left(\frac{\gamma}{1-\gamma}\right)}_{\substack{\text{relative importance}\\ \text{of skilled workers}}} \underbrace{\left(\frac{A_H}{A_L}\right)^{\frac{\sigma-1}{\sigma}}}_{\text{technology bias}} \underbrace{\left(\frac{H}{L}\right)^{-\frac{1}{\sigma}}}_{\text{substitution bias}}$$

- Taking logs:

$$\ln\left(\frac{W_H}{W_L}\right) = \tilde{\gamma}_H + \left(\frac{\sigma-1}{\sigma}\right)\ln\left(\frac{A_H}{A_L}\right) - \frac{1}{\sigma}\ln\left(\frac{H}{L}\right)$$

CES: Inequality between high- and low-skilled workers

- **Estimating equation**:

$$\ln\left(\frac{W_H}{W_L}\right) = \tilde{\gamma}_H + \left(\frac{\sigma - 1}{\sigma}\right)\ln\left(\frac{A_H}{A_L}\right) - \frac{1}{\sigma}\ln\left(\frac{H}{L}\right)$$

- Can use data $\{W_H, W_L, A_H, A_L, H, L\}$ to estimate key parameter σ
  - Data on hourly wages $\{W_H, W_L\}$ and hours worked $\{H, L\}$ available in CPS
  - Challenge: No data on efficiency of workers $\{A_H, A_L\}$.
    Solution: proxy relative efficiency of workers as (linear) function of time

$$\ln\left(\frac{A_H}{A_L}\right) = \gamma_0 + \gamma_1 \times t$$

- **New estimating equation**:

$$\ln\left(\frac{W_H}{W_L}\right) = \text{constant} + \left(\frac{\sigma - 1}{\sigma}\right) \times t - \frac{1}{\sigma}\ln\left(\frac{H}{L}\right)$$

CES: Inequality between high- and low-skilled workers

$$\ln\left(\frac{W_H}{W_L}\right) = \text{constant} + \left(\frac{\sigma - 1}{\sigma}\right) \times t - \frac{1}{\sigma}\ln\left(\frac{H}{L}\right)$$

- Autor, Katz, and co-authors estimated equations of this type for US workers and found elasticity of substitution $\sigma \in [1.4, 2]$:
  - $\Rightarrow$ **College and non-college workers are (gross) substitutes**
  - $\Rightarrow$ **Technical change biased towards skilled workers**

- **Understanding US college wage premium in light of $\sigma > 1$:**
  - $+$ As technology becomes more biased toward skill workers, wage premium rises
  - $-$ As skilled workers become relatively more abundant, wage premium falls
  - $\Rightarrow$ **US college wage premium increasing due to strong skill bias of tech. since relative supply of skilled workers increasing over time**

Taking Stock

Taking stock

- **GDP per capita highly positively correlates with economic development** (life expectancy, HDI, life satisfaction, ...)
- **Output produced depends on technology** (ability to turn inputs into outputs) **and production factors** (inputs in production process)
- **Technologies represented via production functions**
- Can distinguish **production functions by type**:
  - Micro vs. macro
  - Gross output vs. value added
- Our focus is on **aggregate production function** (macro + value added type)

Taking stock

- **Properties of production functions**: marginal products, returns to scale, homogeneity, Inada conditions, direction of technical change, …
- **Neoclassical production function** is twice-continuously differentiable, has positive and diminishing marginal products, CRS, and meets Inada conditions
- **Popular production functions**: Cobb–Douglas, CES, Leontief, Stone–Geary, …
- Studied **firm decision making** under different assumptions
- **National accounting** ($PY = WL + RK + \Pi$) **implies GDP exhausted compensating production factors and firm owners** ($\Lambda_L + \Lambda_K + \Lambda_\Pi = 1$)

Taking stock

- **Income shares can be obtained in both data and models**
  - US labor share declined by 8pp from 67% in 1947 to 59% in 2023 (broadly accepted)
  - My work suggests profit share constant throughout at 18% of GDP (more controversial; very hard to discern pure profits from capital rents)
- **Income shares speak to income inequality** under ownership assumptions
- **Income inequality in the US between high- and low-skilled labor**:
  - US college-educated workers earn increasingly more than high-school workers, yet college-educated workers increasingly more abundant
  - CES model of production rationalizes this with strong skill bias of technology

Questions?

Thank You!

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GDP aggregation: From Micro to Macro

- Let economy be populated by N producers

- Each producer $i \in \mathcal{N} = \{1, \dots, N\}$ produces output according to:

$$y_i = F_i\left(\{x_{ij}\}_{j \in \mathcal{N}}, \ell_i, k_i\right)$$

x_{ij}: intermediate demand of input j $\quad$ ℓ: labor $\quad$ k: capital

- Nominal GDP is total value of *final* goods produced in domestic economy:

$$\text{GDP} = \sum_{i \in \mathcal{N}} p_i q_i = \sum_{i \in \mathcal{N}} p_i \left(y_i - \sum_{j \in \mathcal{N}} x_{ji} \right)$$

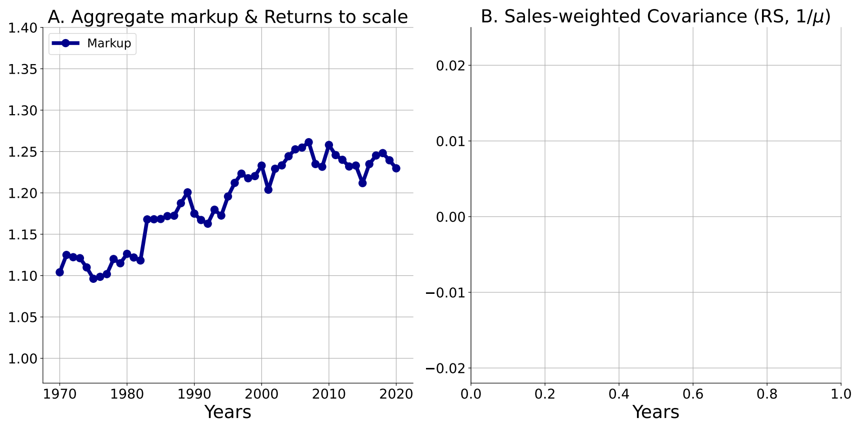
q: value added $\quad$ y: gross output $\quad$ x_{ij}: demand of good i by producer j

* Real GDP can be obtained using Divisia indices to convert nominal GDP (outside scope of this course)

Back

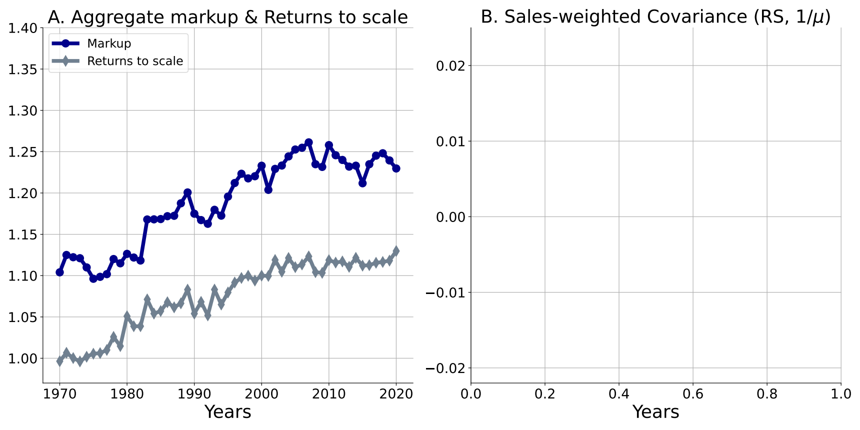
Hasenzagl and Perez (2023): $\Lambda_{\Pi} = \chi\left(1 - \frac{\overline{RS}}{\overline{\mu}_{hsw}} - Cov_{\omega}\left[RS, \frac{1}{\mu}\right]\right)$

- Aggregate markup increasing since 1970, from 10% to 23%



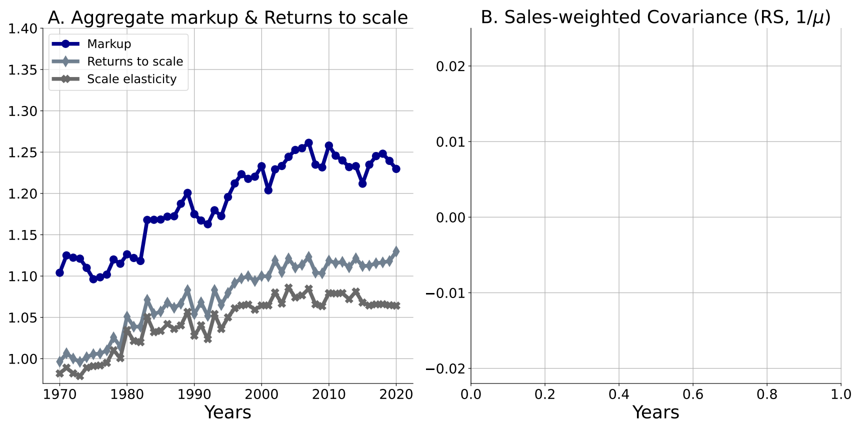
Hasenzagl and Perez (2023): $\Lambda_\Pi = \chi\left(1 - \frac{\overline{RS}}{\overline{\mu}_{hsw}} - Cov_\omega\left[RS, \frac{1}{\mu}\right]\right)$

- Returns to scale increased from 1.00 to 1.13.



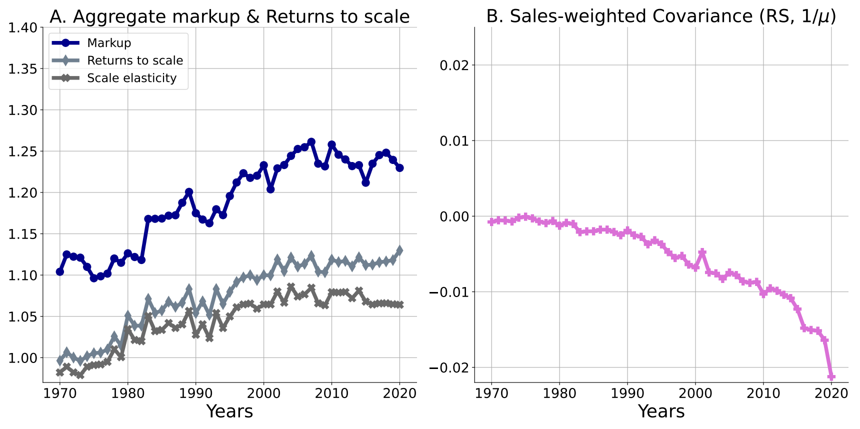
Hasenzagl and Perez (2023): $\Lambda_\Pi = \chi\left(1 - \frac{\overline{RS}}{\overline{\mu}_{hsw}} - Cov_\omega\left[RS, \frac{1}{\mu}\right]\right)$

- Scale elasticity increased from around 0.98 to 1.06. (RS $=$ SE $\times$ FC adj. factor)



Hasenzagl and Perez (2023): $\Lambda_\Pi = \chi \left(1 - \frac{\overline{RS}}{\overline{\mu}_{hsw}} - Cov_\omega\left[RS, \frac{1}{\mu}\right]\right)$

- Small negative correlation between returns to scale and inverse markups.



Hasenzagl and Perez (2023): $\Lambda_\Pi = \chi\left(1 - \frac{\overline{RS}}{\overline{\mu}_{hsw}} - Cov_\omega\left[RS, \frac{1}{\mu}\right]\right)$

Profit share in the US has been roughly constant at around 18% ▸ Back

(consistent with average profit rate of 10% because of double marginalization)