

Example of stable moving average (inverse of a VAR model):

$$X_t = \sum_{k=0}^q (A)^k \epsilon_{t-k}$$

$$(1 - AL)X_t = \epsilon_t$$

L is backshift operator $LX_t = X_{t-1}$, and H is transfer function:

$$X_t = H(L)\epsilon_t$$

$$X_t = \sum_{k=0}^{\infty} (AL)^k \epsilon_t$$

$$H(L) = (1 - AL)^{-1} = \sum_{k=0}^{\infty} (AL)^k$$

$$X_t = AX_{t-1} + \epsilon_t$$

$$\mathbb{E}[\epsilon_{t-k}\epsilon'_{t-l}] = \delta(k, l)\Sigma$$

$$V(X_t) = \sum_{k,l=0}^q (A)^k \mathbb{E}[\epsilon_{t-k}\epsilon'_{t-l}](A)'^l = \sum_{k,l=0}^q (A)^k \delta(k, l)\Sigma(A)'^l = \sum_{k=0}^q (A)^k \Sigma(A)'^k$$

Autocovariance at lag 0 (variance):

$$V(X_t) = \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^T X_t X_t'$$

Autocovariance in spectral domain (fourrier transform of autocovariance):

$$\Gamma_{\omega} = H_{\omega} \Sigma H_{\omega}^*$$

$$\Gamma_{\omega} = \sum_k \gamma_k \exp(-i\omega k)$$

$$H_{\omega} = H(\exp(-i\omega))$$

With autocovariance at lag k:

$$\gamma_k = \mathbb{E}[X_t X_{t-k}']$$