Modelling Alternatives for the IHTC 2024 Competition - A Modelling Tutorial

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Key Points

- Tutorial on modelling based on a 2024 competition problem
- Not obvious whether CP is a viable/the best approach
- 32 participating teams with state-of-the-art solutions
- Show different concepts and methods on a live example
- Also use visualisation to understand the issues

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Insight Today

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4
Co-Lead Universities
4 partner institutions

Built on **20** years of research in Data Analytics and AI

1250
Academics, Postdocs, PhDs, RAs

3900+
Scientific conference and journal papers

175+
Funded collaborations with industry partners

350+
Research Awards

16
Spin out companies
107 license agreements

135+
H2020 consortia, 500+ collaborations, 40+ countries

1,137+ school visits, 28,000 students

398
PhDs graduated

Insight

Introduction

Why the Change of Title?

- Planned: Modelling for Scheduling and Rostering Problems with Constraint Programming
- Originally planned more of an overview tutorial
- Then came a message from Gene Freuder on the Constraints mailing list (July 26th):

  Anyone interested in applying CP to this:
  `https://ihtc2024.github.io`
  (I realize the competition itself is over.)

- I soon realised that this problem was perfect for the tutorial
- But: I had to solve the problem first, with only days before the tutorial
- The competition ran over six months...

Why talk about Modelling?

- This is really very simple, two/three concepts:
  - Variables
  - Constraints
  - (Objective)
- Not at all this simple for practical problems
- This only applies if somebody already has made all the difficult choices
- What to include, what to ignore?
- What are hard constraints, what are preferences?
- Why different stakeholders see a different problem?
- Why different solvers produce very different quality solutions?

My Perspective

- Get the job done quickly
  - Are we solving the right problem?
  - Build confidence for ourselves/the customer
- To satisfactory performance
  - Get it working first
  - Then identify bottlenecks
  - Have a baseline for comparison
- So that we can make changes quickly
  - Real-world problems are always evolving
  - We may have new ideas to test and integrate

Dual Role of Modelling

- For the human
  - Which problem are we solving, exactly?
- For the machine
  - What to do?
- We now have tools that can change model representations to make them more suitable for specific solvers
- Concentrate on explaining main concepts we look for, avoid micro-management of large set of primitives
  - *YOU ARE IN A MAZE OF TWISTY LITTLE PASSAGES, ALL ALIKE.*

Problem Description

Short Overview

IHTC 2024 Competition

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Short Overview

IHTC 2024 Competition

Integrated Hospital Capacity Management Problem

- Decide which patients to admit in planning period (2-4 weeks)
- Decide when to admit selected patients
- Surgery performed on day of admission by preassigned surgeon
- Respect capacity constraints of surgeons and theatres
- Assign bed to patient during their stay
- Respect constraints on room capacity and use
- Assign nurses to rooms in each shift to provide required care
- Nurse roster given
- Minimise total, weighted cost

Problem Description

Short Overview

IHTC 2024 Competition

IHTC 2024

- `https://ihtc2024.github.io/`
- Organized by Udine/Italy and KU Leuven/Belgium
- Under EURO umbrella (replaced Roadef 2024)
- Ran from September 24 to March 25
- 32 teams participating
- 70 problem instances
- Solution checker!

IHTC2024 Winners

IHTC 2024 Home Dates Submission Conferences S.I. Instances Validator Teams Results Ranking File formats Organizers Sponsors Contact

Participating Teams

We are pleased to share that 32 teams submitted their solvers to the IHTC-2024 competition. We would like to thank all participating teams for their contributions.

After collecting and validating the results, we are pleased to announce the final ranking of IHTC-2024:

Rank	Team Name	Affiliation	Team members	Solver
1	v777v	independent	Venislav Varbanov	
2	SDU-IMADA	University of Southern Denmark	Ahmad Othman, Marco Chiarandini	github
3	Twente	University of Twente	Daniela Guericke, Rolf van der Hulst, Asal Karimpour, Ieke Schrader, Matthias Walter	
4	ORTEC	independent	Wouter Kool, Lotte Berghman, Martijn van Brink, Charlie Ye, Eva van Rooijen, Judith Mourits, Per Kampman	
5	UGent Mippets	Ghent University	Dries Goossens, David Van Bulck, Samuel Bakker, Karel Devriesere, Lisa Garcia Tercero	

We are also pleased to announce that Team SDU-IMADA is the winner of the open-source software prize.

Is this a good problem for CP?

- There are good and bad points
- Good
  - Not too messy
  - Problem size not super large
  - Decomposes into well-known sub problem types
- Bad
  - Some constraints are totally soft
  - Overall cost is weighted sum of many factors
  - Time resolution limited

Concepts

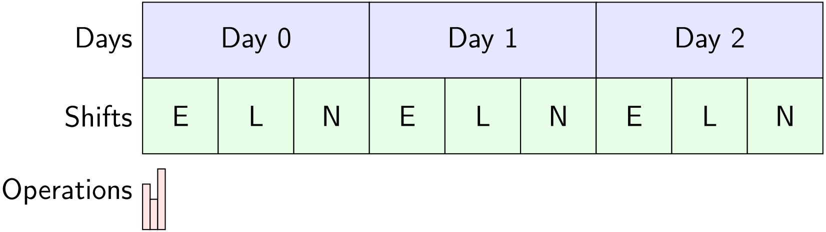
Concepts

- Time
- Patients/Occupants
- Surgeons
- Nurses
- Operating theatres
- Rooms/beds

Data Samples

- We will use two problem instances to show features of problems
- test10, largest of the initial instance set
- m29, largest of the final evaluation set
- Demonstrate scalability of visualisation as well as of solvers

Time Scale (Planning period 2-4 weeks)

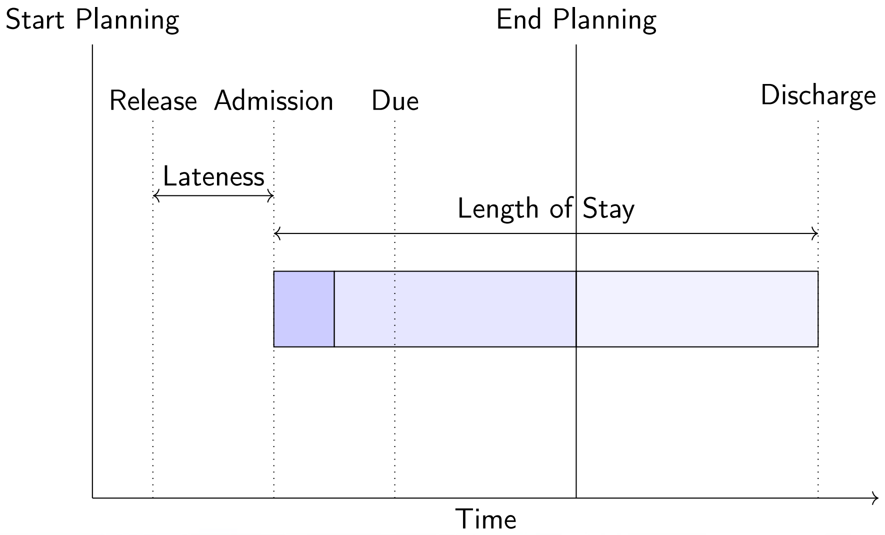


- Three different time scales in problem
  - Days: patient admission, bed/room use, aligned with shifts
  - Shifts: Nurse assignment, workload, skills
  - Procedures: not scheduled, only total load per day

Patients

- Potential inpatients for surgery, assigned to a surgeon
- Mandatory patients must be scheduled
- Optional patients may be scheduled
- Each patient has earliest admission day (release day)
- Mandatory patients have a latest admission day (due date)
- Procedure length and length of stay vary

Patient Events and Intervals



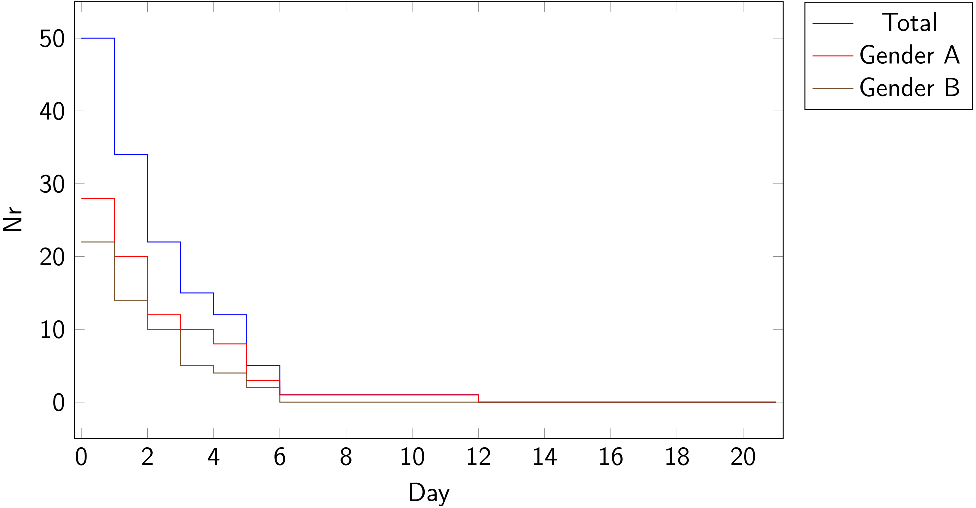
Needed Patient Care on Day of Stay, Top 20 Patients in test10

	0E	0L	0N	1E	1L	1N	2E	2L	2N	3E	3L	3N	4E	4L	4N	5E	5L	5N	6E	6L	6N	7E	7L	7N	8E	8L	8N	9E	9L
p301	3	3	1	2	3	1	2	2	1	2	3	2	3	2	2	2	3	1	2	2	2	3	3	1	3	2	1	3	2
p121	2	2	2	2	3	2	2	3	1	2	3	2	3	3	2	3	2	1	2	2	2	3	2	1	3	2	1	2	2
p227	2	2	1	2	2	1	3	2	1	2	3	1	3	2	2	3	3	2	2	3	1	3	2	1	2	3	1	3	2
p421	2	2	1	3	3	2	2	2	2	2	2	1	2	2	1	2	2	1	3	2	2	3	2	2	3	3	1	3	2
p042	2	2	2	2	2	1	2	2	2	2	3	1	3	2	2	3	3	1	2	2	2	2	3	1	3	2	2	3	2
p062	2	3	1	2	2	1	3	2	1	2	2	2	2	3	1	3	2	1	2	2	2	3	2	2	2	3	2	3	1
p076	3	3	2	2	3	2	3	2	2	2	2	1	2	2	1	2	2	1	2	2	2	3	2	1	2	2	2	3	2
p081	2	2	1	2	2	2	2	2	1	2	2	1	2	2	2	2	2	1	1	3	1	1	1	1	2	2	1	1	2
p181	3	3	2	2	3	1	2	2	2	2	2	1	2	2	1	2	2	2	2	2	1	3	3	1	2	2	1	2	3
p317	2	2	2	3	3	1	2	2	2	2	2	1	3	2	2	2	2	2	3	2	2	2	3	1	3	2	2	3	3
p415	3	3	1	3	2	2	2	3	2	2	2	2	3	2	1	2	2	1	2	2	1	2	2	1	3	1	1	2	1
p149	3	2	2	3	3	2	2	2	2	3	2	1	2	2	2	3	3	2	2	2	1	3	2	2	2	2	2	2	3
p229	3	2	2	3	2	2	2	3	2	2	2	1	2	3	1	3	3	2	2	3	2	2	2	1	3	3	2	3	2
p097	3	2	2	3	2	2	2	2	2	3	2	1	2	2	2	2	2	1	3	2	1	2	3	2	2	3	2		
p104	2	2	1	2	2	1	2	2	2	2	3	1	2	3	1	2	2	2	2	3	2	2	2	2	3	1	1		
p122	2	3	2	2	2	1	3	2	2	3	2	1	3	2	1	2	2	1	2	2	1	2	3	1	2	3	2		
p136	3	2	2	2	2	2	2	3	2	2	3	2	2	2	2	2	2	2	2	2	1	2	2	2	1	2	2		
p190	3	2	2	2	3	2	2	2	1	3	2	1	2	2	1	3	2	2	2	3	1	1	2	1	1	1	2		
p206	2	3	1	3	2	2	2	2	2	2	2	2	2	2	1	2	2	1	2	3	2	3	3	2	3	2	1		
p224	3	3	2	2	2	1	2	3	2	3	3	2	3	2	2	3	2	2	3	1	1	3	1	1	1	1	2		

Occupants

- Patients admitted in previous planning period
- Surgery already done
- Occupy bed from period start until discharge
- Same care needs as patients, create nurse workload
- "Work in progress"

Occupants Beds Assigned - test10



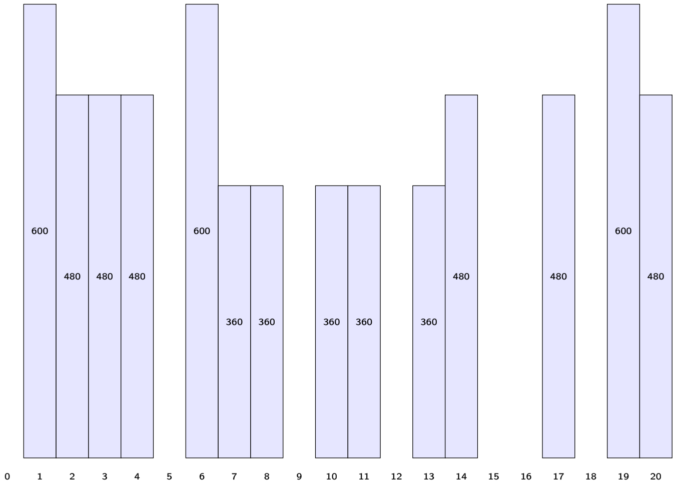
Occupants Room Use - test10

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
r00	1																				
r01	2	2	1	1																	
r02	1	1																			
r03	1	1																			
r04	2	1	1	1	1	1															
r05	2	2	2	2	2	1															
r06	2	2	1																		
r07	2	1	1																		
r08																					
r09	1																				
r10	1																				
r11	1	1	1	1																	
r12																					
r13	1	1																			
r14	2	2	1	1	1																
r15	1	1	1	1	1																
r16	1	1																			
r17	2	2	2	2	2																
r18	1	1																			
r19	1	1																			
r20	1																				
r21	2	1	1																		
r22																					
r23	2																				

Surgeons

- Each surgeon has set list of assigned patients
- In this problem: Only interact with patients by operating
- Surgeon stands for whole team needed to perform procedures
- Hard work limits for surgery only on each day
- May be unavailable due to other duties

Surgeon S5 Availability - test10



Nurses

- Provide care for patients during their stay
- Nurses operate in three non-overlapping shifts
- Roster given
- Nurses are assigned rooms, responsible for patients in those rooms
- Given skill, max workload per shift: soft constraints

Nurse Roster (Part I) - test10

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
n000	L	L	N		E	L	N		E	L	N	N	N	N	N	N	N		E		E
n001	L	L	L	L	L	L		E	L	L	L	L	L	L	L		E	E		E	E
n002	E		E	L	L	L	L	N	N	N		E		E	E	E	L		E	E	E
n003	N	N		E	L	N	N		E	E	N		E	E	L	N		L	L		E
n004	L	L	L	N	N	N	N		L	L	N	N	N	N		L	L	L	L	L	N
n005	L	L	L	N	N		L	N		N		L	L		L	L	L	L	N	N	N
n006	E	E	L	L	L	N	N		E	E	L		E	N		E	E	E	E	E	E
n007	L	L	N		E	E	E	E	E	L	L		E	E	N	N	N	N	N		E
n008	L	L	L	N		E	L	L	L		L	L	L	N	N	N		L		L	L
n009	N	N	N		E	L	L	L	L		E	L	L	L	N	N	N		N		E
n010	L	N		E	L	L	L	N		L	N	N	N		E	E		E	E	E	L
n011	N	N		E	E	E	L	L	L	L	N		E	E	E	E	L	L	L		E
n012	L		E	E	E	E	E	E	N			E	L	L		E	E	L	N	N	N
n013	L	N		E	L	L	L		E	L	L	L	L	L	L		E	N	N		E
n014	L	N	N		L	N		E	E	L	L	L	N	N		E	E	N		E	L
n015	N	N	N		L	L	L	L	N	N		E	L	L	L	L		E	E	L	L
n016	N	N	N		N		E	E	E	L	L	L	L	N	N		E	E	E	L	N
n017	N	N	N	N		E	N		E	L	N	N		N	N	N	N	N	N	N	
n018	L		L	L	N		E	E	N	N	N		L	L	L	L	L	N		L	L
n019	L	L	L	L	L	L	L	L	L	L		E	L	L	L	N	N		E	E	L
n020	L	L	L	N	N		E	E	E	E	E	E	E	L	L	L	L	L		L	L
n021	L	N	N	N		E	E	L	N	N	N	N	N	N		L	N	N	N		E
n022	L	L	L	L	N		E	E	L	N		E	E	E	E	N	N		E	E	E
n023	L	L	L	N	N	N	N		E	N	N	N		E	E	E	E	E	L	N	N
n024	N	N	N	N		E	E	E	E	E	E	E	E	L	L	L	L	L	L	L	L

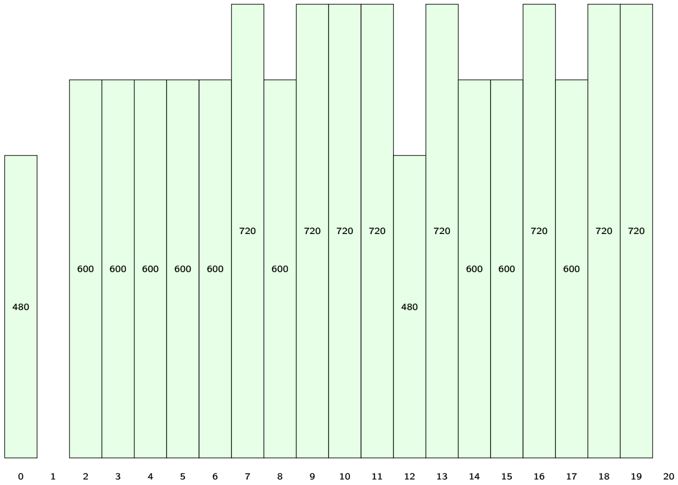
Nurse Max Workload - test10

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
n000	5	5	15		15	15	15		15	5	5	15	15	15	15	15	5		5		15
n001	15	15	15	15	15	15		15	15	5	5	15	15	15	15		15	15		15	15
n002	10		10	10	10	10	10	10	10	10		10		10	10	10	10		10	10	10
n003	15	15		15	5	5	15		15	15	15		15	15	5	15		15	15		15
n004	12	12	12	12	12	12	12		12	12	12	12	12	12		12	12	12	12	12	12
n005	5	15	5	5	15		15	15		5		15	5		15	15	15	15	5	15	15
n006	15	15	5	15	15	15	15		5	15	15		15	15		15	15	15	15	15	15
n007	15	5	5		15	5	15	15	15	15	15		15	15	5	15	15	15	15		15
n008	12	12	12	12		12	12	12	12		12	12	12	12	12	12		12		12	12
n009	15	15	5		15	5	15	15	15		15	15	15	15	5	5	15		15		15
n010	5	15		15	15	15	5	15		15	15	5	15		15	15		15	15	5	15
n011	15	15		15	15	15	15	15	15	5	15		5	15	15	15	15	15	5		5
n012	15		15	15	15	5	15	15	15			15	15	15		5	15	5	15	15	15
n013	10	10		10	10	10	10		10	10	10	10	10	10	10		10	10	10		10
n014	12	12	12		12	12		12	12	12	12	12	12	12		12	12	12		12	12
n015	5	15	15		15	5	5	15	15	15		15	15	15	15	15		15	5	15	5
n016	12	12	12		12		12	12	12	12	12	12	12	12	12		12	12	12	12	12
n017	5	15	15	15		5	15		5	15	15	5		15	5	5	15	15	15	5	
n018	10		10	10	10		10	10	10	10	10		10	10	10	10	10	10		10	10
n019	15	15	15	15	5	15	5	15	15	15		15	15	15	15	15	15		15	15	15
n020	10	10	10	10	10		10	10	10	10	10	10	10	10	10	10	10	10		10	10
n021	12	12	12	12		12	12	12	12	12	12	12	12	12		12	12	12	12		12
n022	12	12	12	12	12		12	12	12	12		12	12	12	12	12	12		12	12	12
n023	15	15	15	15	5	15	15		15	15	5	5		15	15	15	15	15	5	15	5
n024	15	15	15	5		5	15	15	5	15	5	15	15	5	15	15	5	15	15	15	5

Theatres

- Used for procedures by surgeons
- All procedures can be performed in all theatres
- Limited total availability per day
- Cost, when used, per day
- Convenience cost if surgeon assigned to two or more theatres per day

Theatre t06 Availability - test10



Rooms/Beds

- Each patient is assigned a bed in one room during their stay
- Each room has a given bed capacity
- Rooms/beds are assigned by day
- Patients are not moved between rooms
- Not all rooms are compatible with all patients
- On any day, only patients with same gender can be assigned to the same room, hard constraint
- Prefer to have same age-group in the same room, soft constraints

Weight Factors

w_u cost of an unscheduled optional patient, i.e. the patient is not admitted in the planning period

w_l cost of delay per day for admitting a patient after their release date

w_t cost of opening a operating theatre for one day

w_s cost of each additional theatre if a surgeon is allocated to more than one theatre per day

w_w cost of a workload excess for a nurse in one shift per workload unit

w_k cost of skill excess for a nurse in one shift per skill unit

w_a cost of a age group conflict between two patients in the same room per day

w_c cost of continuity of care excess if multiple nurses are responsible for a patient

Decomposition

Some Data Analysis

Decomposition

Some Data Analysis

Instance Data

Name	Day	Skill Level	Shift Type	Age Group	Occ	Pat	Mand Pat	Room	Bed	Thea	Surg	Nurse
test01	21	3	3	3	7	42	11	5	13	2	1	13
test02	14	3	3	3	5	37	12	6	16	3	2	17
test03	14	3	3	3	10	45	12	6	20	2	1	14
test04	14	3	3	3	7	54	32	8	25	3	2	19
test05	14	3	3	3	9	62	19	6	19	2	1	15
test06	14	2	3	2	9	111	50	9	22	3	3	20
test07	21	5	3	5	13	113	55	9	26	3	2	22
test08	21	2	3	4	15	173	39	9	27	3	2	21
test09	21	4	3	2	26	146	88	14	44	2	2	29
test10	21	5	3	3	50	525	349	46	130	11	10	83
m21	14	3	3	3	13	234	32	21	59	11	9	46
m22	14	2	3	4	36	244	119	30	89	9	7	62
m23	14	2	3	3	61	254	71	38	117	14	10	77
m24	21	4	3	3	59	397	264	39	115	11	8	78
m25	21	2	3	2	32	412	25	26	75	6	7	54
m26	28	4	3	5	38	570	247	41	125	6	6	81
m27	14	5	3	5	47	295	87	36	112	16	11	70
m28	28	2	3	2	36	611	420	42	124	11	10	84
m29	28	4	3	5	44	631	110	49	144	5	8	96
m30	21	4	3	4	31	489	267	48	140	8	11	90

Data KPIs (I)

Name	Mandatory Patient Percentage	Gender A Patient Percentage	Implied Room Gender Percentage	Mandatory Theatre Percentage	Max Theatre Percentage	Mandatory Op Percentage	Max Op Percentage
test01	26.19	52.38	100.00	7.89	29.25	36.93	136.93
test02	32.43	48.65	83.33	10.49	28.89	29.35	80.80
test03	26.67	60.00	100.00	11.22	39.17	79.17	276.39
test04	59.26	42.59	75.00	16.63	28.45	56.25	96.25
test05	30.65	45.16	83.33	18.51	71.56	75.78	292.97
test06	45.05	41.44	100.00	22.66	51.64	50.00	113.92
test07	48.67	46.02	88.89	27.39	55.70	82.78	168.33
test08	22.54	52.02	100.00	19.91	73.30	61.26	225.55
test09	60.27	51.37	92.86	60.05	108.29	86.33	155.66
test10	66.48	52.57	80.43	35.25	55.76	72.71	115.04
m21	13.68	43.59	47.62	5.32	43.42	11.30	92.29
m22	48.77	45.90	76.67	25.69	52.16	53.32	108.24
m23	27.95	47.64	92.11	9.62	36.37	23.66	89.48
m24	66.50	51.64	94.87	32.52	49.76	81.46	124.64
m25	6.07	51.21	92.31	4.71	84.50	8.12	145.72
m26	43.33	51.75	73.17	31.22	76.73	57.63	141.65
m27	29.49	45.42	77.78	9.96	34.71	23.74	82.73
m28	68.74	48.28	73.81	30.71	45.98	64.71	96.88
m29	17.43	50.87	69.39	17.13	96.48	23.08	130.01
m30	54.60	52.76	58.33	45.96	84.35	56.31	103.35

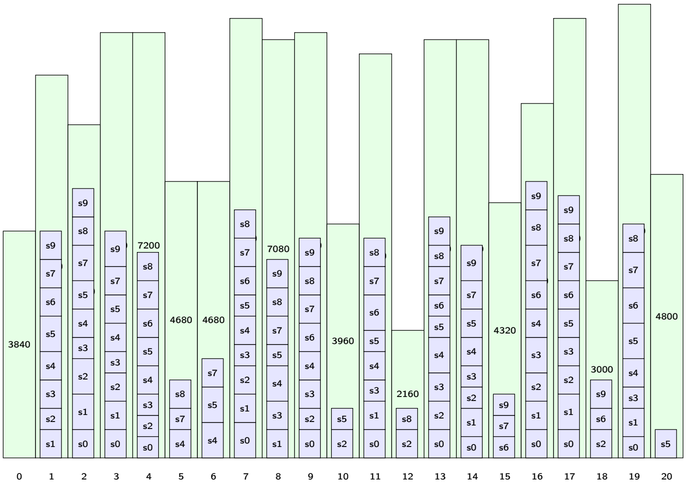
Data KPIs (II)

Name	Worst Mandatory Surgeon Percentage	Worst Max Surgeon Percentage	Required Bed Percentage	Max Bed Percentage	Required Nurse Load Percentage	Max Nurse Load Percentage
test01	36.93	136.93	32.60	92.31	24.65	71.47
test02	29.69	102.70	41.96	96.43	31.97	74.84
test03	79.17	276.39	27.14	80.71	26.37	80.99
test04	58.62	106.90	47.71	79.43	48.98	80.77
test05	75.78	292.97	41.35	122.56	39.69	123.61
test06	59.46	124.32	80.52	170.45	65.10	140.17
test07	84.88	169.77	59.71	117.40	54.32	107.78
test08	63.68	243.42	43.39	153.09	43.78	154.31
test09	81.17	164.17	59.09	92.64	60.32	95.78
test10	82.74	141.92	64.40	94.40	62.71	92.29
m21	21.43	152.78	23.37	137.89	21.76	131.94
m22	64.44	128.91	51.44	96.95	51.06	97.58
m23	41.48	172.73	28.39	80.71	28.99	85.13
m24	87.26	149.50	58.63	84.51	59.74	86.27
m25	19.09	230.50	12.44	132.63	11.75	132.31
m26	66.22	209.50	35.54	77.20	37.71	82.00
m27	38.24	115.63	34.63	96.94	35.58	103.93
m28	79.29	119.29	55.79	81.02	57.22	83.34
m29	27.54	156.64	14.91	73.19	14.82	75.19
m30	65.35	133.54	46.84	83.61	47.55	85.85

Weight Factors per Instance

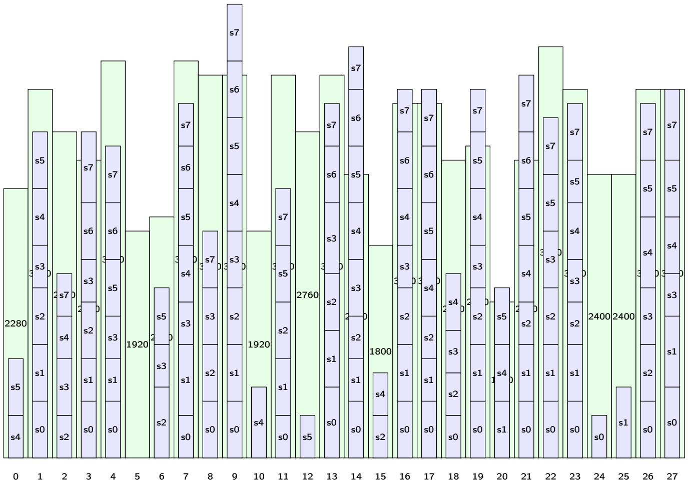
Name	Room Mixed Age	Room Nurse Skill	Continuity of Care	Nurse Excessive Workload	Open Operating Theatre	Surgeon Transfer	Patient Delay	Unscheduled Optional
test01	5	1	5	1	30	1	5	150
test02	5	1	1	1	10	10	5	350
test03	1	1	5	10	10	5	15	350
test04	1	5	1	5	10	1	10	250
test05	5	1	5	1	30	10	5	400
test06	5	1	5	1	30	10	10	500
test07	5	10	5	1	50	10	5	250
test08	1	5	1	1	40	1	10	250
test09	1	5	5	5	20	5	10	350
test10	1	10	5	5	50	10	10	300
m21	1	5	1	5	40	5	15	500
m22	5	10	5	5	20	1	10	400
m23	5	1	5	1	40	10	10	300
m24	5	10	1	1	20	10	15	350
m25	1	1	1	1	20	1	10	200
m26	1	10	1	10	30	10	10	450
m27	5	5	5	10	40	1	15	150
m28	1	5	5	5	40	10	10	350
m29	5	10	1	5	50	10	15	150
m30	5	5	1	1	50	10	10	400

Total Theatre and Surgeon Capacity - test10



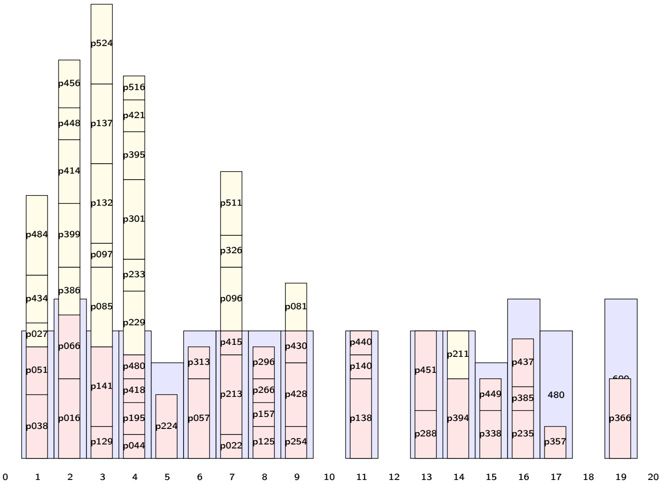
Theatre capacity not limiting

Total Theatre and Surgeon Capacity - m29



Theatre capacity must be considered

Capacity and Demand for Surgeon s7 - test10



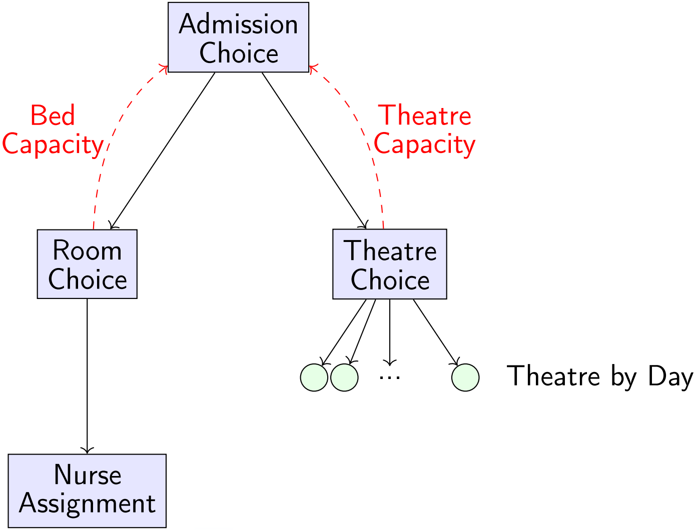
Patients shown on release day

Not much capacity left for optional patients

Results of Analysis

- Finding which patients to admit will be the most important choice.
- To utilise the full surgeon capacity, we will have to delay admission of some patients to use the available time slots.
- We do not have to consider the detailed theatre assignment when making the initial admission selection.
- When admitting a patient, we must have a bed for their full stay available. Estimating full capacity is tough.
- We can initially ignore the nurse workload/skill limits, and age mix constraints as well.

The Resulting Decomposition



Stages

Admission decide which patients to admit when

Room assign a room/bed to each selected

Theatre assign a theatre to each patient

Nurse Assignment assign a nurse to each (non-empty) room

Alternatives

- Combine some/all stages
  - At some point, model gets too large
  - Not enough time to find good solution by complete search
  - Use heuristics to find good solutions quickly
- Resolve part of the assignment (Large Neighborhood Search, LNS)
  - Needs initial solution, second stage refinement
- Integrate stages as Logical Benders Decomposition
  - Probably too time consuming for competition setting

Admission Choice

- Bin Packing Model
- Scheduling Model
- Zero/One Model
- Solution Samples

Summary

- Decide which patients to select
- Choose the day on which they are admitted
- This is the day of the surgery
- Respect capacity limits for each surgeon
- Minimise weighted cost of rejected patients and lateness
- (Anticipate some capacity constraints imposed by later stages)

Admission Choice

Bin Packing Model
Scheduling Model
Zero/One Model
Solution Samples

Idea

- Bin packing with optional variables
- A variable can be present or not
- It is only considered in the constraints if it is present
- Express capacity constraints as bin packing constraint
- Room capacity constraint is a cumulative constraint

Variables

$x_i \in rel_i..due_i$ optional variable, indicating admission day, which is also day of surgery

The domain restriction handles the release and due date limits.

Mandatory Patients must be admitted

We have to admit all mandatory patients, this means that the corresponding variable is not optional, and must be set to true.

$$\forall_{i \in P \textbf{ s.t. } m_i} : \quad \textbf{presence}(x_i) = \textbf{true}$$

Surgeon Capacity

The total resource usage for each surgeon, i.e. the allocated operating time, for every day must be less or equal to their capacity.

$$\begin{aligned}\forall_{s \in S}: \quad \textbf{binPacking}(&[x_i | i \in P \textbf{ s.t. } s_i = s],\\ &[d_i | i \in P \textbf{ s.t. } s_i = s],\\ &[c_{sj} | j \in D])\end{aligned}$$

- Resource needs (item weight) are d_i, the duration of the operation; bin size is capacity of surgeon c_{sj} on day j
- If a patient is not admitted, then their variable is not present, and the surgery does not count in any bin

Anticipated Theatre Capacity

The total operating time scheduled on each day must be limited to the sum of the resource availabilities of all theatres. This is the total over all admitted patients.

$$\textbf{binPacking}([x_i | i \in P],\\ [d_i | i \in P],\\ [\sum_{t \in T} c_{tj} | j \in D])$$

resource needs are d_i, the duration of the operation

Anticipated Bed Capacity

The total number of beds needed on each day for both the admitted patients and the preassigned occupants must be limited by the total bed capacity.

$$
\begin{aligned}
\textbf{cumulative}(&[x_i | i \in P] ++ [0 | o \in O],\\
&[l_i | i \in P] ++ [l_o | o \in O],\\
&[1 | i \in P] ++ [1 | o \in O],\\
&\sum_{r \in R} c_r)
\end{aligned}
$$

- uses ++ for list concatenation
- tasks here have duration l_i, and resource need of 1

Objective

The objective is the minimise the weighted sum of the cost of rejected patients, and the sum of the lateness of the accepted patients.

$$\min \sum_{i \in P} \neg \textbf{presence}(x_i) w_u + \sum_{i \in P} (x_i - rel_i) w_l$$

w_u cost of not admitting a patient in this planning period

w_l cost of admitting a patient after their release day, applies to each day of delay

rel_i release day for patient i

Insight

What is the binPacking constraint?

660 PREDEFINED

5.54 bin_packing_capa

DESCRIPTION LINKS

Origin Derived from bin_packing.

Constraint bin_packing_capa(BINS, ITEMS)

Arguments

BINS : collection(id−int, capa−int)
ITEMS : collection(bin−dvar, weight−int)

Restrictions

$|\mathtt{BINS}| > 0$
required(BINS, [id, capa])
distinct(BINS, id)
$\mathtt{BINS.id} \geq 1$
$\mathtt{BINS.id} \leq |\mathtt{BINS}|$
$\mathtt{BINS.capa} \geq 0$
required(ITEMS, [bin, weight])
in_attr(ITEMS, bin, BINS, id)
$\mathtt{ITEMS.weight} \geq 0$

Purpose Given several items of the collection ITEMS (each of them having a specific weight), and different bins described the the items of collection BINS (each of them having a specific capacity capa), assign each item to a bin so that the total weight of the items in each bin does not exceed the capacity of the bin.

Global Constraint Catalog
On-line Resource on global constraints
https://sofdem.github.io/gccat/

What is the `cumulative` constraint?

5.96 cumulative

DESCRIPTION LINKS GRAPH AUTOMATON

Origin [1]

Constraint `cumulative(TASKS, LIMIT)`

Synonym `cumulative_max`.

Arguments

$$\texttt{TASKS} : \texttt{collection}\begin{pmatrix} \texttt{origin}-\texttt{dvar}, \\ \texttt{duration}-\texttt{dvar}, \\ \texttt{end}-\texttt{dvar}, \\ \texttt{height}-\texttt{dvar} \end{pmatrix}$$

$$\texttt{LIMIT} : \texttt{int}$$

Restrictions

`require_at_least(2, TASKS, [origin, duration, end])`
`required(TASKS, height)`
$\texttt{TASKS.duration} \geq 0$
$\texttt{TASKS.origin} \leq \texttt{TASKS.end}$
$\texttt{TASKS.height} \geq 0$
$\texttt{LIMIT} \geq 0$

Purpose Cumulative scheduling constraint or scheduling under resource constraints. Consider a set $\mathcal{T}$ of tasks described by the `TASKS` collection. The `cumulative` constraint enforces that at each point in time, the cumulated height of the set of tasks that overlap that point, does not exceed a given limit. A task overlaps a point i if and only if (1) its origin is less than or equal to i, and (2) its end is strictly greater than i. It also imposes for each task of $\mathcal{T}$ the constraint $\texttt{origin} + \texttt{duration} = \texttt{end}$.

Caveat

- This version relies on optional variables to model optional patients that may or may not be scheduled
- Not all solvers allow this (MiniZinc as of Version 2.9.3 does not)
- You can model an extra bin to hold unwanted items, use capacity limit wisely
- Or you may want to use a scheduling model instead

Admission Choice

Idea

- Scheduling with optional task variables
- A variable can be present or not
- It is only considered in the constraints if it is present
- Express capacity constraints as cumulative resource limits
- Task duration is one

Variables

$x_i \in rel_i..due_i$ optional task variable with duration 1

This handles the time limits of the task limiting the possible admission days to the values from the release day to the due day.

Mandatory Patients must be admitted

We have to admit all mandatory patients, this means that the corresponding variable is not optional, and must be set to true.

$$\forall_{i \in P \textbf{ s.t. } m_i} : \quad \textbf{presence}(x_i) = \textbf{true}$$

Surgeon Capacity

The total resource use, i.e. the allocated operating time, for every day must be less or equal to the resource capacity.

$$\begin{aligned}\forall_{s \in S}: \quad \textbf{cumulative}(&[x_i | i \in P \textbf{ s.t. } s_i = s],\\ &[1 | i \in P \textbf{ s.t. } s_i = s],\\ &[d_i | i \in P \textbf{ s.t. } s_i = s],\\ &[c_{sj} | j \in D])\end{aligned}$$

resource needs are d_i, the length of the operation

Anticipated Theatre Capacity

The total operating time scheduled on each day must be limited to the sum of the resource availabilities of all theatres.

$$\textbf{cumulative}([x_i | i \in P], \\ [1 | i \in P], \\ [d_i | i \in P], \\ [\sum_{t \in T} c_{tj} | j \in D])$$

resource needs are d_i, the length of the operation

Insight

Anticipated Bed Capacity

The total number of beds needed on each day for both the admitted patients and the preassigned occupants must be limited by the total bed capacity.

$$\begin{aligned}\textbf{cumulative}(&[x_i|i \in P] + +[0|o \in O],\\ &[l_i|i \in P] + +[l_o|o \in O],\\ &[1|i \in P] + +[1|o \in O],\\ &\sum_{r \in R} c_r)\end{aligned}$$

- uses ++ for list concatenation
- tasks here have duration l_i

Objective

$$\min \sum_{i \in P} \neg \textbf{presence}(x_i) w_u + \sum_{i \in P} (x_i - rel_i) w_l$$

w_u cost of not admitting a patient in this planning period

w_l cost of admitting a patient after their release day, applies to each day of delay

rel_i release day for patient i

Admission Choice

Idea

- Decide if patient *i* is admitted on day *j*
- Can pick at most one admission day
- Allows to reject patient by not admitting the patient on any day
- Express capacity restrictions by creating sums of potential resource use
- Index expressions can get complex

Variables

x_{ij} 0/1 variable indicating if surgery for patient i is scheduled on day j

We may decide not to create a variable for days where the patient cannot be scheduled

Schedule Mandatory Patients

All mandatory patients must be scheduled, this means that exactly one of the x_{ij} must be set to one. Optional patients may be scheduled, it is possible that all x_{ij} are set to zero.

$$\forall_{i \in P}: \quad \sum_{j \in D} x_{ij} \begin{cases} = 1 & m_i, \textbf{ patient i is mandatory} \\ \leq 1 & \neg m_i, \textbf{ patient i is optional} \end{cases}$$

Insight

Respect Release Day

No patient can be scheduled before their release day.

$$\forall_{i \in P} \forall_{j \in D \textbf{ s.t. } j < rel_i} x_{ij} = 0$$

Respect Due Day

No mandatory patient can be scheduled after their due date, the due date is not defined for optional patients. Again, in the internal implementation we simply do not create x_{ij} as variable for these dates

$$\forall_{i \in P \, \textbf{s.t.} \, m_i} \forall_{j \in D \, \textbf{s.t.} \, j > due_i} x_{ij} = 0$$

Surgeon Capacity

The total time allocated to each surgeon s on each day j must be less or equal to their operating capacity c_{sj}.

$$\forall_{s \in S} \forall_{j \in D} : \sum_{i \in P \textbf{ s.t. } s_i = s} x_{ij} d_i \leq c_{sj}$$

Anticipated Theatre Capacity

The total surgery time allocated on day j must be less or equal to the total operating time of all theatres on that day. The capacity of theatre t on day j is given by c_{tj}.

$$\forall_{j \in D}: \quad \sum_{i \in P} x_{ij} d_i \leq \sum_{t \in T} c_{tj}$$

Anticipated Bed Capacity

The total number of beds required for admitted patients on each day must be below the available room capacity. Patients occupy a bed if the where admitted at an earlier date so that their length of stay has not yet ended. We must also take any pre-assigned occupants into account when expressing the bed capacity.

$$\forall_{j \in D} : \sum_{i \in P} \sum_{j' \in D \textbf{ s.t. } j - l_i < j' \leq j} x_{ij} \leq \sum_{r \in R} (c_r - o_{rj})$$

Stronger Assumption: Assume Room Gender Pre-assignment

There is a hard constraint that no patients with different gender can be allocated to the same room. One easy way to enforce this is to allocate a designated gender to each room, for example by extending the gender used by the preassigned occupants. The preassignment g_r does not need to be total, i.e. not all rooms need to have a preassigned gender. The maximum bed capacity available for a given gender g then is the total bed capacity minus the capacity preallocated to other genders.

$$\forall_{g \in G} \forall_{j \in D} : \sum_{i \in P \textbf{ s.t. } g_i = g} \; \sum_{j' \in D \textbf{ s.t. } j - l_i < j' \leq j} x_{ij} \leq \sum_{r \in R} (c_r - o_{rj}) - \sum_{r \in R \textbf{ s.t. } r_g \neq g} (c_r - o_{rj})$$

Objective

We want to maximise the number of admitted patients in the planning period, and minimise the total delay over all admitted patients.

$$\min \sum_{i \in P} \sum_{j \in D} x_{ij} \left(w_l (j - rel_i) - w_u \right)$$

w_u cost of not admitting a patient in this planning period

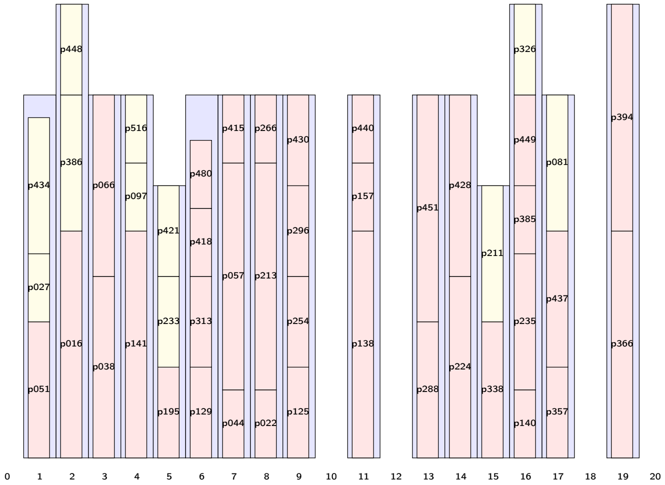
w_l cost of admitting a patient after their release day, applies to each day of delay

rel_i release day for patient i

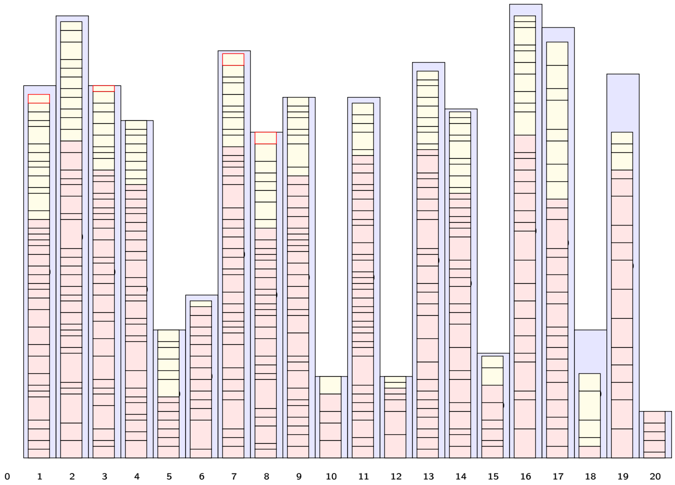
Admission Choice

- Bin Packing Model
- Scheduling Model
- Zero/One Model
- Solution Samples

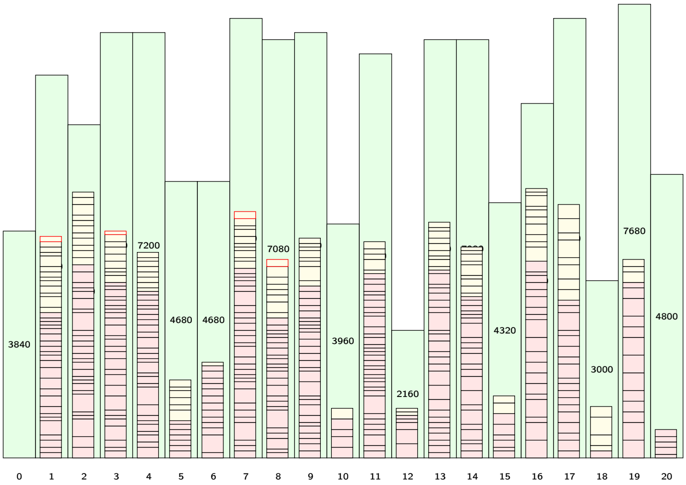
Surgeon s7 and Admitted Patients - test10



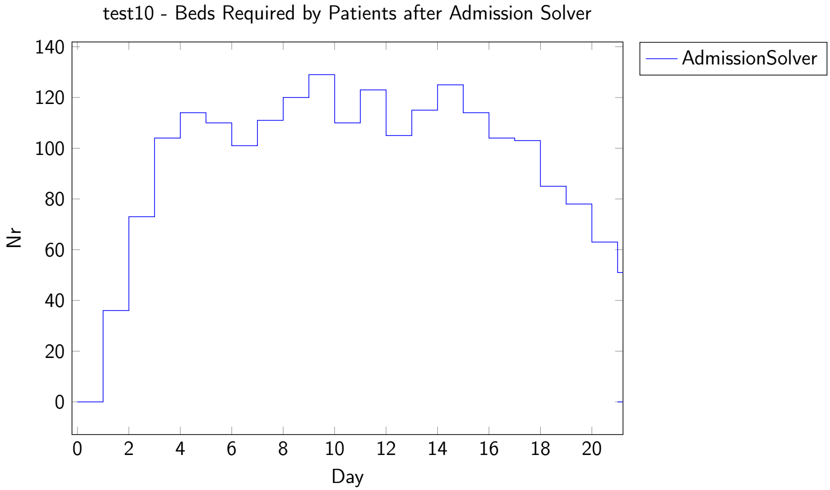
Total Surgeons Capacity and Admitted Patients - test10



Total Theatres Capacity and Admitted Patients - test10

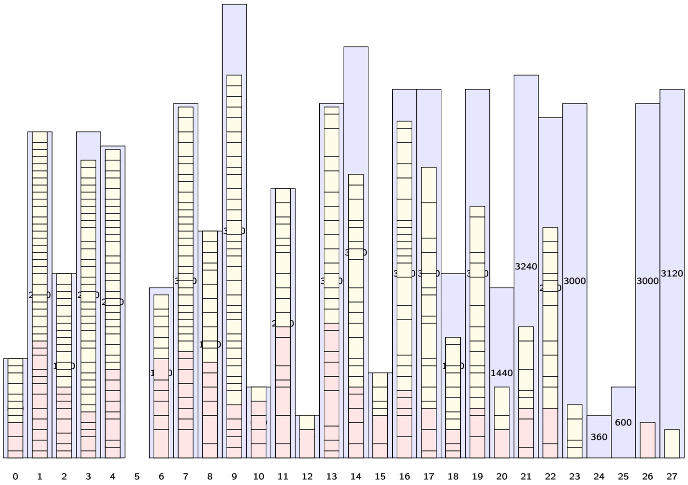


Estimated Bed Requirements - test10



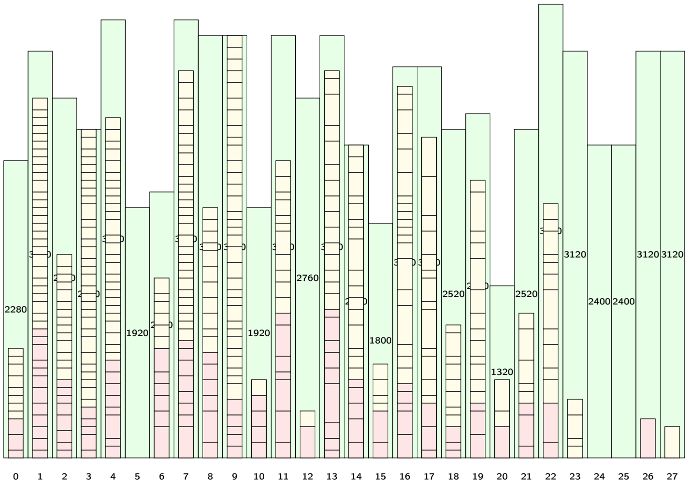
Insight

Total Surgeons Capacity and Admitted Patients - m29



Insight

Total Theatres Capacity and Admitted Patients - m29



Which model is better?

- Depends on the solver used
- Models may be transformed into each other
- Here: optional variables not good for propagation
- Not all solvers support optional variables for the required constraints
- Time resolution very coarse, consider admission was done by the second :-)
- Item height can be scaled to small integer
- Overall problem size moderate (hundreds of tasks)
- Preference to scheduling model

Room Choice

- Finite Domain Model
- Zero/One Model
  - Constraints for Gender Assigned Rooms
- Solution Samples

Room Choice

Finite Domain Model

Zero/One Model

Constraints for Gender Assigned Rooms

Solution Samples

Idea

- Define a finite domain variable which gives the room for a patient
- Express room conflicts with disequality ($\neq$) constraints
- Express room capacity by counting the number of variables assigned to each room for each day
- We still allow to reject a few more patients if that is needed

Information Available

- We can rely on the decisions made in earlier stages
- Set P^a of admitted patients
- Fixed value a_i for the admission day of patients in P^a

Variables

We use a single, optional domain variables for each patient, which ranges over all possible rooms

$x_i \in 1..|R|$ room allocated to patient i

We may have to reject some more patients to find a feasible solution!

Mandatory Patients must be admitted

We have to admit all mandatory patients, this means that the corresponding variable is not optional, and must be set to true.

$$\forall_{i \in P^a \textbf{ s.t. } m_i} : \quad \mathbf{presence}(x_i) = \mathbf{true}$$

Excluded Rooms

A patient i cannot be assigned to an excluded room.

$$\forall_{i \in P_a} \forall_{r \in R \textbf{ s.t. } i_{ir}} : \quad x_i \neq r$$

Room Capacity

For every day, the number of patients occupying a room cannot exceed the room capacity. The following global cardinality (gcc) constraints must hold for each day j.

$$\begin{aligned}\forall_{j \in D}: \quad & gcc([x_i | i \in P^a \textbf{ s.t. } a_i \leq j < a_i + l_i], \\ & [r | r \in 1..|R|], \\ & [c_r - o_{rj} | r \in 1..|R|])\end{aligned}$$

We are counting the number of patients that use each room.

Insight

No Gender Mix Between Patients

If the admission periods for two patients of different genders overlap, then they cannot be assigned to the same room value.

$$\forall_{i_1, i_2 \in P^a \text{ s.t. } g_{i_1} \neq g_{i_2} \wedge \mathbf{overlap}(a_{i_1}, l_{i_1}, a_{i_2}, l_{i_2})} : \quad x_{i_1} \neq x_{i_2}$$

Insight

No Gender Mix between Occupants and Patients

For an occupant o and a patient i of different gender who is admitted before the end of the stay l_o of the occupant, the room r_o used by the occupant o is not available for the patient i.

$$\forall_{o \in O} \forall_{i \in P^a \text{ s.t. } g_o \neq g_i \wedge a_i < l_o} : \quad x_i \neq r_o$$

Objective

$$\min \sum_{i \in P^a} \neg \mathbf{presence}(x_i) w_u$$

w_u cost of rejecting a patient

Global Cardinality (GCC)

5.162 global_cardinality

DESCRIPTION LINKS GRAPH AUTOMATON

Origin CHARME [284]

Constraint global_cardinality(VARIABLES, VALUES)

Synonyms count, distribute, distribution, gcc, card_var_gcc, egcc, extended_global_cardinality.

Arguments

VARIABLES : collection(var−dvar)
VALUES : collection(val−int, noccurrence−dvar)

Restrictions

required(VARIABLES, var)
required(VALUES, [val, noccurrence])
distinct(VALUES, val)
VALUES.noccurrence ≥ 0
VALUES.noccurrence $\leq$ |VARIABLES|

Purpose Each value VALUES[i].val (with $i \in [1, |\texttt{VALUES}|]$) should be taken by exactly VALUES[i].noccurrence variables of the VARIABLES collection.

Example

$$\left(\begin{array}{l}\langle 3,3,8,6\rangle, \\ \left\langle\begin{array}{ll}\texttt{val}-3 & \texttt{noccurrence}-2, \\ \texttt{val}-5 & \texttt{noccurrence}-0, \\ \texttt{val}-6 & \texttt{noccurrence}-1\end{array}\right\rangle\end{array}\right)$$

The global_cardinality constraint holds since values 3, 5 and 6 respectively occur 2, 0 and 1 times within the collection $\langle 3,3,8,6\rangle$ and since no constraint was specified for value 8.

Room Choice

Idea

- Each admitted patient must be assigned to a room
- Some rooms are not compatible with a patient
- Each room has a fixed capacity that holds at all times
- If we do not assign a room, then patient is rejected
- Use special indicator variable to mark rejection of optional patients

Variables

x_{ir} zero/one indicator variable if patient i is assigned to room r

y_{rg} zero/one indicator variable if room r is reserved for patients of gender g

z_i zero/one indicator if selection of optional patient i must be revoked to achieve a feasible solution

Room Assignment

Each patient must be assigned a room, except if we reject the patient in this stage by setting z_i to one. This allows to find a relaxed solution which removes optional patients as required.

$$\forall_{i \in P^a}: \quad z_i + \sum_{r \in R} x_{ir} = 1$$

Excluded Rooms

Some rooms are not compatible with a patient, they are defined in the input data, we use the notation i_{ir} to state whether room r is compatible with patient i.

$$\forall_{i \in P^a} \forall_{r \in R \textbf{ s.t. } i_{ir}} : \quad x_{ir} = 0$$

Respect Room Capacity

We have to respect the bed capacity of each room, that is the number of patient staying in a room on any day must be below the capacity of the room minus the number of preassigned occupants in the room on that day.

$$\forall_{r \in R} \forall_{j \in D} : \sum_{i \in P^a \textbf{ s.t. } a_i - l_i < j \leq a_i} x_{ir} \leq c_r - o_{rj}$$

No Gender Mix between Patients

Two patients of different gender whose stay overlaps in time cannot be assigned to the same room at the same time, i.e. their assignment variables x_{ir} cannot not be both equal to one for the same room.

$$\forall_{i_1,i_2 \in P^a \text{ s.t. } g_{i_1} \neq g_{i_2} \wedge \textbf{overlap}(a_{i_1}, l_{i_1}, a_{i_2}, l_{i_2})} \forall_{r \in R} : \quad x_{i_1 r} + x_{i_2 r} \leq 1$$

We use the function *overlap* to determine if the stay overlaps based on the assigned admission day a_i and length of stay l_i.

$$\textbf{overlap}(a_{i_1}, l_{i_1}, a_{i_2}, l_{i_2}) := a_{i_2} < a_{i_1} + l_{i_1} \wedge a_{i_1} < a_{i_2} + l_{i_2}$$

No Gender Mix between Patients and Occupants

For an occupant o and a patient i of different gender who is admitted before the end of the stay l_o of the occupant, the room r_o used by the occupant o is not available for the patient i.

$$\forall_{o \in O} \forall_{i \in P^a \textbf{ s.t. } g_o \neq g_i \wedge a_i < l_o} : \quad x_{ir_o} = 0$$

Gender Choice for Room

Exactly one gender g must be assigned to each room r, i.e. exactly one of the y_{rg} variables for a room must be set to one.

$$\forall_{r \in R}: \quad \sum_{g \in G} y_{rg} = 1$$

Respect Room Gender Restriction

If a patient i of gender g_i is assigned to a room r, then the gender assignment of that room must be g as well ($y_{rg} = 1$).

$$\forall_{g \in G} \forall_{r \in R} \forall_{i \in P^a \textbf{ s.t. } g_i = g} : \quad x_{ir} \leq y_{rg}$$

Preassigned Gender for some Rooms

Some rooms have predefined gender assignments which must be respected.

$$\forall_{g \in G} \forall_{r \in R \textbf{ s.t. } r_g = g}: \quad y_{rg} = 1$$

Objective

We want to minimise the number of optional patients that are rejected in this solution, ideally no additional patients are removed at this stage.

$$\min \sum_{i \in P^a} z_i w_u$$

w_u cost of not admitting a patient

Room Choice

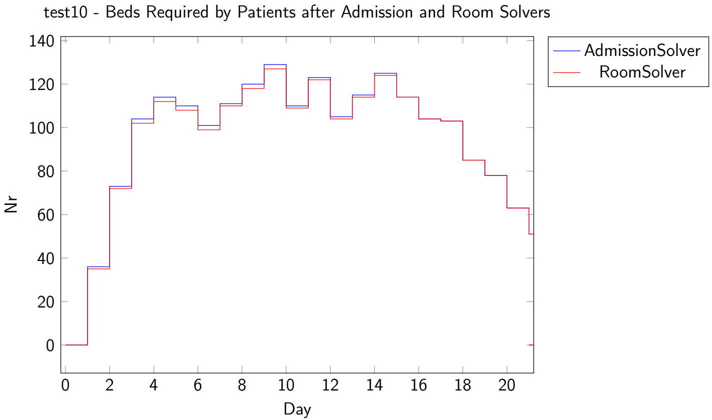
Finite Domain Model

Zero/One Model

Constraints for Gender Assigned Rooms

Solution Samples

Reduction in Admitted Patients in Room Solver - test10



Allocated Beds in Rooms - test10

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
r00	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	2	2	2
r01	2	2	1	2	2	2	1	2	2	2	2	2	1	1	2	1	1	2	2	2	1
r02	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
r03	1	2	2	3	3	3	2	3	3	3	2	3	3	3	3	3	3	3	3	3	2
r04	2	1	2	2	2	2	2	2	2	2	2	2	1	2	2	2	2	1	2	2	2
r05	2	2	2	3	3	3	3	3	3	3	2	3	3	3	3	3	3	3	3	3	2
r06	2	3	4	3	4	4	2	4	3	4	3	2	4	4	4	4	4	3	4	3	4
r07	2	2	2	2	2	2	2	2	2	2	2	2	1	1	2	2	2	2	2	2	2
r08			2	2	2	2	2	2	2	2	2	2	1	2	2	1	2	2	2	2	2
r09	1	3	4	4	4	4	4	4	4	4	3	4	4	4	4	4	4	4	3	4	3
r10	1	2	2	2	2	2	2	1	1	2	1	2	1	2	2	2	2	2	2	2	2
r11	1	2	3	4	4	4	2	4	4	4	4	4	4	4	4	4	4	4	3	4	4
r12		1	1	3	3	3	2	2	3	3	3	3	3	2	3	2	3	3	2	3	3
r13	1	2	2	3	3	3	3	3	3	3	2	3	2	3	3	3	3	3	3	2	2
r14	2	2	2	2	2	1	2	2	2	2	2	2	1	2	2	2	2	2	2	2	2
r15	1	2	2	2	2		1	2	2	2	2	2	1	2	2	2	2	2	2	2	2
r16	1	2	3	3	3	3	3	3	2	3	2	3	3	3	3	3	3	3	1	1	
r17	2	2	4	4	4	3	4	3	4	4	4	4	4	4	4	3	3	4	4	3	2
r18	1	2	2	3	3	3	1	3	3	3	3	3	2	3	3	3	2	2	3	3	3
r19	1	2	2	2	2	2	2	1	1	2	1	2	2	2	2	2	2	2	2		
r20	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1		2	2	2	1
r21	2	2	2	2	2		1	2	2	2	2	2	1		2	2	2	2	1	1	
r22			1	2	2	2	2	2	2	2	2	2	2	1	2	2	2	2	1	2	2
r23	2	1	2	3	3	3	3	3	3	3	3	3	3	3	3	3	2	3	3	3	3
r24		2	3	3	3	3	3	2	3	3	3	3	3	3	3	3	3	3	2	3	2
r25	1	2	3	3	3	3	1	2	3	3	3	3	2	3	3	2	3	3	2	1	1
r26	1	2	3	4	4	4	2	2	4	4	4	4	3	3	4	4	3	4	2	1	1
r27								2	3	4	3	3	3	3	4	4	3	2	2	1	
r28	2	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
r29				1	3	3	2	3	3	3	3	3	3	3	3	3	2	3	3	3	1
r30	1	1	2	2	2	1		1	2	3	3	3	3	3	3	2	2	3	2	3	3
r31	2	2	2	2	2	2	1	2	2	2	2	2	1	2	2	2	2	1		1	1
r32	1	2	3	4	4	4	4	4	4	4	4	4	4	4	4	3	4	4	3	3	2
r33	1	2	3	4	4	4	2	2	4	4	3	4	4	3	4	3	2	2	1		
r34	1			4	4	4	4	3	3	4	3	4	3	4	4	4	4	3			
r35			2	2	2	1	2	2	1	2	1	2	2	2	2	2	1	1	1	1	1
r36	1		1	3	3	3	2	2	3	3	3	3	3	3	3	2	2	2	2	2	
r37	1				2	2	2	1	1	2	1	2	2	2	2	2	1	1			
r38		2	3	4	4	3	3	4	4	4	3	3	1	1	1	1					
r39	3	3	3	3	3	3	3	3	3	3	2	3	3	3	3	3	3	3	2	1	1
r40	1	1	1	3	3	2	3	3	3	3	2	3	2	3	3	3	3	3	2	1	
r41	1	1	2	3	3	3	3	2	3	3	3	3	1	3	3	3	2	1	1		
r42		2	3	3	3	3	3	3	1	1	1	1	1	1	1	1	1				
r43	1	1	1	2	2	2	2	2	2	2	2	2	1	2	1	1	1				
r44	1	1	2	2	4	3	3	4	4	4	2	4	3	3	4	4	1	1	1		
r45	1	1	2	1	1	1	1	1	2	2	2	1	1	1	2	2	2	2	1		

Unused Beds after Room Assignment - test10

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
r00	1																	1			
r01			1				1						1	1		1	1				1
r02	1																				
r03	2	1	1				1				1										1
r04		1											1					1			
r05	1	1	1								1										1
r06	2	1		1			2		1		1	2						1		1	
r07													1	1							
r08	2	2											1			1					
r09	3	1									1								1		1
r10	1							1	1		1		1								
r11	3	2	1				2												1		
r12	3	2	2				1	1						1		1			1		
r13	2	1	1								1		1							1	1
r14						1							1								
r15	1					2	1						1								
r16	2	1							1		1								2	2	3
r17	2	2				1		1								1	1			1	2
r18	2	1	1				2						1				1	1			
r19	1							1	1		1									2	2
r20	1															1	2				1
r21						2	1						1	2					1	1	2
r22	2	2	1											1					1		
r23	1	2	1														1				
r24	3	1						1											1		1
r25	2	1					2	1					1			1			1	2	2
r26	3	2	1				2	2					1	1			1		2	3	3
r27	4	4	4	4	4	4	4	2	1		1	1	1	1			1	2	2	3	4
r28		1																			
r29	3	3	3	2			1										1				2
r30	2	2	1	1	1	2	3	2	1							1	1		1		
r31							1						1					1	2	1	1
r32	3	2	1													1			1	1	2
r33	3	2	1				2	2			1			1		1	2	2	3	4	4
r34	3	4	4					1	1		1		1					1	4	4	4
r35	2	2				1			1		1						1	1	1	1	1
r36	2	3	2				1	1								1	1	1	1	1	3
r37	1	2	2	2				1	1		1						1	1	2	2	2
r38	4	2	1			1	1				1	1	3	3	3	3	4	4	4	4	4
r39											1								1	2	2
r40	2	2	2			1					1		1						1	2	3
r41	2	2	1					1					2				1	2	2	3	3
r42	3	1							2	2	2	2	2	2	2	2	2	3	3	3	3
r43	1	1	1										1		1	1	1	2	2	2	2
r44	3	3	2	2		1	1				2		1	1			3	3	3	4	4
r45	1	1		1	1	1	1	1				1	1	1					1	2	2

Insight

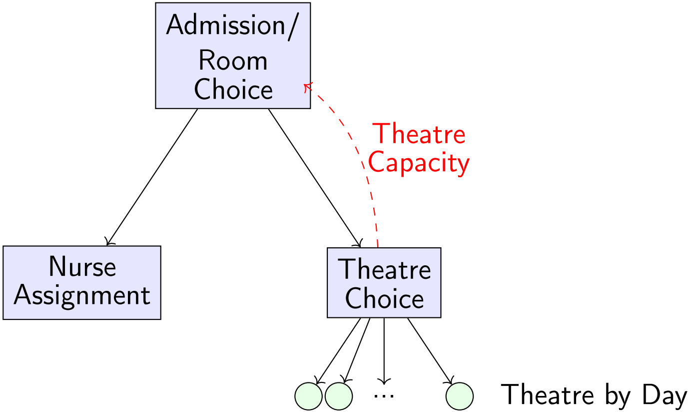
We have a problem

- Why did the room solver have to reject some patients?
- Finding the optimal solution for its sub-problem!
- Bad assumptions in the admission solver
- Overestimated bed capacity in gender pre-allocated rooms
- Problem goes away if we relax gender mix constraint in room assignment
    - But this is a hard constraint
- We need to make a better admission solver

Combined Admission/Room Choice

- Cumulatives Model
- Alternative/Cumulative Model
- Zero/One Model

Combined Admission/Room Choice



Combined Admission/Room Choice

Cumulatives Model

Alternative/Cumulative Model

Zero/One Model

Idea

- Reuses parts of the admission model which are not concerned with room assignment
- Uses "super global constraint" `cumulatives` to handle room capacity
- `cumulatives` does not allow optional variables, use spare values instead

Variables

As the `cumulatives` constraint does not allow optional variables, we have to use special values instead. This makes the model rather messy.

$x_i \in rel_i..due_i \cup \{|D|+1\}$ finite domain variable indicating the admission day of patient i, spare value $|D|+1$ indicates rejected patient

$r_i \in R \cup \{|R|+1, |R|+2\}$ finite domain variable indicating the room that the patient is assigned to, the additional values are used to assign rejected patients for genders A and B

We need two spare values for the room assignment to allow the no gender mix to work with spare values as well.

Mandatory Patients must be admitted

We have to admit all mandatory patients, this means that the corresponding variables cannot take the spare values.

$$\forall_{i \in P \textbf{ s.t. } m_i} : \quad r_i \neq |R| + 1 \wedge r_i \neq |R| + 2 \wedge x_i \neq |D| + 1$$

Surgeon Capacity (as before)

The total resource usage for each surgeon, i.e. the allocated operating time, for every day must be less or equal to their capacity. For the spare value we allow unlimited capacity.

$$\forall_{s \in S}: \quad \textbf{binPacking}([x_i | i \in P \textbf{ s.t. } s_i = s], [d_i | i \in P \textbf{ s.t. } s_i = s], [c_{sj} | j \in D] + +[\infty])$$

- Resource needs (item weight) are d_i, the duration of the operation; bin size is capacity of surgeon c_{sj} on day j
- If a patient is not admitted, then their variable is assigned to the $|D| + 1$ bin, which has unlimited capacity

Anticipated Theatre Capacity (as before)

The total operating time scheduled on each day must be limited to the sum of the resource availabilities of all theatres. WE again use special value $|D|+1$ to indicate rejected patients, that bin has unlimited capacity.

$$\begin{aligned}\textbf{binPacking}(&[x_i | i \in P],\\ &[d_i | i \in P],\\ &[\sum_{t\in T} c_{tj} | j \in D] ++ [\infty])\end{aligned}$$

resource needs are d_i, the duration of the operation

Excluded Rooms

A patient i cannot be assigned to an excluded room.

$$\forall_{i \in P_a} \forall_{r \in R \text{ s.t. } i_{ir}} : \quad r_i \neq r$$

Combined Room Capacity Constraint for all Rooms

The number of beds required by all patients and all occupants assigned to the same room on one day cannot exceed the room capacity.

$$
\begin{aligned}
\textbf{cumulatives}(&[x_i|i \in P] + +[0|o \in O],\\
&[l_i|i \in P] + +[l_o|o \in O],\\
&[1|i \in P] + +[1|o \in O],\\
&[r_i|i \in P] + +[r_o|o \in O],\\
&[c_r|r \in R] + +\{\infty, \infty\}])
\end{aligned}
$$

THere is one cumulative constraint per room, we use assignment variable to decide which room a patient belongs to.

No Gender Mix between Patients

For two patients of different gender, we have to either assign them to different rooms, or one stay has to be before the other.

$$\forall_{i_1, i_2 \in P \text{ s.t. } g_{i_1} \neq g_{i_2}} : \quad \begin{aligned} & r_{i_1} \neq r_{i_2} \vee \\ & x_{i_1} + l_{i_1} \leq x_{i_2} \vee \\ & x_{i_2} + l_{i_2} \leq x_{i_1} \end{aligned}$$

Large set of disjunctions created between all pairs of patients with different gender

No Gender Mix between Occupants and Patients

Patients and occupants of different gender cannot share the same room.

$$\forall_{o \in O} \forall_{i \in P \textbf{ s.t. } g_i \neq g_o} : \quad \begin{aligned} & r_i \neq r_o \vee \\ & x_i \geq l_o \end{aligned}$$

This translates into simpler disjunctions as occupants fixed in time

Objective

$$\min \sum_{i \in P} (x_i == |D| + 1) w_u + \sum_{i \in P} (x_i - rel_i) w_l$$

w_u cost of not admitting a patient in this planning period

w_l cost of admitting a patient after their release day, applies to each day of delay

rel_i release day for patient i

Cumulatives Constraint

5.101 cumulatives

DESCRIPTION LINKS GRAPH

Origin [32]

Constraint `cumulatives(TASKS, MACHINES, CTR)`

Arguments

$$\begin{array}{lll} \texttt{TASKS} & : & \texttt{collection}\begin{pmatrix} \texttt{machine}-\texttt{dvar}, \\ \texttt{origin}-\texttt{dvar}, \\ \texttt{duration}-\texttt{dvar}, \\ \texttt{end}-\texttt{dvar}, \\ \texttt{height}-\texttt{dvar} \end{pmatrix} \\ \texttt{MACHINES} & : & \texttt{collection}(\texttt{id}-\texttt{int}, \texttt{capacity}-\texttt{int}) \\ \texttt{CTR} & : & \texttt{atom} \end{array}$$

Restrictions

required(TASKS, [machine, height])
require_at_least(2, TASKS, [origin, duration, end])
in_attr(TASKS, machine, MACHINES, id)
TASKS.duration ≥ 0
TASKS.origin $\leq$ TASKS.end
|MACHINES| > 0
required(MACHINES, [id, capacity])
distinct(MACHINES, id)
CTR $\in [\leq, \geq]$

Purpose

Consider a set $\mathcal{T}$ of tasks described by the TASKS collection. When CTR is equal to $\leq$ (respectively $\geq$), the cumulatives constraint enforces the following condition for each machine m: At each point in time, where at least one task assigned on machine m is present, the cumulated height of the set of tasks that both overlap that point and are assigned to machine m should be less than or equal to (respectively greater than or equal to) the capacity associated with machine m. A task overlaps a point i if and only if (1) its origin is less than or equal to i, and (2) its end is strictly greater than i. It also imposes for each task of $\mathcal{T}$ the constraint origin + duration = end.

Combined Admission/Room Choice

Cumulatives Model

Alternative/Cumulative Model

Zero/One Model

Idea

- We create a version of the patient variable for each possible assigned room.
- Link it to main decision variable
- Only one of those variables can be selected
- Express room capacity constraints over the optional variables for each room

Variables

$x_i \in rel_i..due_i$ optional task variable to describe admission day of patient i

$y_{ir} \in rel_i..due_i$ optional task variable to describe patient i assigned to room r

Alternative

We link the x and y variables. Only one of the y variables can be present, and, if present, must have the same value as the linked x variable.

$$\forall_{i \in P}: \quad \textbf{alternative}(x_i, [y_{i1}, y_{i2}, ..., y_{i|R|}])$$

Room Capacity Constraint

For every room r we state a cumulative capacity constraint which limits the number of beds used. Most of the y_{ir} will be absent, and therefore do not affect the room capacity. We include the occupants as fixed tasks

$$\begin{aligned}
\forall_{r \in R}: \quad \textbf{cumulative}(&[y_{ir} | i \in P] + +[0 | o \in O \textbf{ s.t. } r_o = r],] \\
&[l_i | i \in P] + +[l_o | o \in O \textbf{ s.t. } r_o = r], \\
&[1 | i \in P] + +[1 | o \in O \textbf{ s.t. } r_o = r], \\
&c_r)
\end{aligned}$$

Excluded rooms

A patient cannot be assigned to a forbidden room.

$$\forall_{i \in P} \forall_{r \in R \ \textbf{s.t.}\ ri_{ir}} : \quad \textbf{presence}(y_{ir}) = \textbf{false}$$

No gender mix between patients

? Still searching for an elegant solution, not introducing new variables

No gender mix between patients and occupants

$$\forall_{o \in O} \forall_{i \in P \text{ s.t. } g_i \neq g_o} : \quad \textbf{presence}(y_{ir_o}) = \textbf{false} \vee$$
$$x_i \geq l_o$$

Objective (as before)

$$\min \sum_{i \in P} \neg \mathbf{presence}(x_i) w_u + \sum_{i \in P} (x_i - rel_i) w_l$$

w_u cost of not admitting a patient in this planning period

w_l cost of admitting a patient after their release day, applies to each day of delay

rel_i release day for patient i

Insight

Combined Admission/Room Choice

- Cumulatives Model
- Alternative/Cumulative Model
- Zero/One Model

Idea

- Keep as much as possible of the original admission model
- Add new set of variables with added dimension (here: assigned room)
- Only one of them can be set

Variables

x_{ij} 0/1 variable indicating if surgery for patient i is scheduled on day j

y_{ijr} 0/1 variable indicating if surgery for patient i is scheduled on day j and patient is allocated to room r

Linking Variables

The x and y variables must be linked, for each patient there is at most a single (for mandatory patients, exactly one) y variable which is set to one.

$$\forall_{i \in P} \forall_{j \in D}: \quad x_{ij} = \sum_{r \in R} y_{ijr}$$

Respect Release Day

No patient can be scheduled before their release day.

$$\forall_{i \in P} \forall_{j \in D \textbf{ s.t. } j < rel_i} x_{ij} = 0$$

Respect Due Day

No mandatory patient can be scheduled after their due date, the due date is not defined for optional patients. Again, in the internal implementation we simply do not create x_{ij} (and the linked y_{ijr} variables) as variable for these dates

$$\forall_{i \in P \,\mathbf{s.t.}\, m_i} \forall_{j \in D \textbf{ s.t. } j > due_i} x_{ij} = 0$$

Schedule Mandatory Patients

All mandatory patients must be scheduled, this means that exactly one of the x_{ij} must be set to one. Optional patients may be scheduled, it is possible that all x_{ij} are set to zero.

$$\forall_{i \in P}: \quad \sum_{j \in D} x_{ij} \begin{cases} = 1 & m_i, \textbf{ patient i is mandatory} \\ \leq 1 & \neg m_i, \textbf{ patient i is optional} \end{cases}$$

Insight

Surgeon Capacity

The total time allocated to each surgeon s on each day j must be less or equal to their operating capacity c_{sj}.

$$\forall_{s \in S} \forall_{j \in D} : \sum_{i \in P \textbf{ s.t. } s_i = s} x_{ij} d_i \leq c_{sj}$$

Anticipated Theatre Capacity

The total surgery time allocated on day j must be less or equal to the total operating time of all theatres on that day. The capacity of theatre t on day j is given by c_{tj}.

$$\forall_{j \in D}: \quad \sum_{i \in P} x_{ij} d_i \leq \sum_{t \in T} c_{tj}$$

Excluded Rooms

Some rooms are forbidden for some patients.

$$\forall_{i \in P} \forall_{r \in R \textbf{ s.t. } i_{ir}} \forall_{j \in D} : y_{ijr} = 0$$

Insight

Respect Room Capacity on Each Day

The total number of patients in a room on each day cannot exceed the bed capacity of the room.

$$\forall_{r \in R} \forall_{j \in D} : \quad \sum_{i \in P} \sum_{k \in D \text{ s.t. } k \leq j < k + l_i} y_{ikr} \leq c_r - O_{rj}$$

No Gender Mix Between Patients

Two patients of different gender whose stay overlaps cannot be assigned to the same room

$$\forall_{i_1, i_2 \in P \text{ s.t. } g_{i_1} \neq g_{i_2}} \forall_{j_1 \in D} \forall_{j_2 \in D \text{ s.t. } \mathbf{overlap}(j_1, l_{i_1}, j_2, l_{i_2})} \forall_{r \in R} : \quad y_{i_1 j_1 r} + y_{i_2 j_2 r} \leq 1$$

This is horrible, number of constraints needed explodes

No Gender Mix between Occupants and Patients

Patients cannot be assigned to a room while an preassigned occupant of a different gender is still present.

$$\forall_{o \in O} \forall_{i \in P \textbf{ s.t. } g_i \neq g_o} \forall_{j \in D \textbf{ s.t. } j < l_o} : \quad y_{ijr_o} = 0$$

Objective

We want to maximise the number of admitted patients in the planning period, and minimise the total delay over all admitted patients. We can therefore express the objective as a minimisation,

$$\min \sum_{i \in P} \sum_{j \in D} x_{ij} \left(w_l (j - rel_i) - w_u \right) \tag{1}$$

w_u cost of not admitting a patient in this planning period

w_l cost of admitting a patient after their release day, applies to each day of delay

rel_i release day for patient i

Checking the number of generated constraints for 0/1 model

Name	Day	Pat	Room	Max Length Stay	Avg Length Stay	Est No Gender Constraints	Actual No Gender Constraints	No Gender Constraints Ratio
test01	21	42	5	12	5.45	2.12e+07	1.18e+05	0.0055
test02	14	37	6	9	5.46	8.79e+06	8.14e+04	0.0093
test03	14	45	6	7	4.31	1.03e+07	9.46e+04	0.0092
test04	14	54	8	12	4.89	2.24e+07	1.71e+05	0.0076
test05	14	62	6	12	4.74	2.14e+07	1.44e+05	0.0067
test06	14	111	9	11	4.50	9.77e+07	7.26e+05	0.0074
test07	21	113	9	12	5.19	2.63e+08	9.36e+05	0.0036
test08	21	173	9	12	4.80	5.70e+08	2.74e+06	0.0048
test09	21	146	14	12	5.15	6.78e+08	0.00e+00	0.0000
test10	21	525	46	13	4.63	2.59e+10	0.00e+00	0.0000
m21	14	234	21	13	4.75	1.07e+09	0.00e+00	0.0000
m22	14	244	30	10	4.58	1.60e+09	0.00e+00	0.0000
m23	14	254	38	13	4.64	2.23e+09	0.00e+00	0.0000
m24	21	397	39	13	4.73	1.28e+10	0.00e+00	0.0000
m25	21	412	26	14	4.87	9.49e+09	0.00e+00	0.0000
m26	28	570	41	12	4.56	4.77e+10	0.00e+00	0.0000
m27	14	295	36	12	4.66	2.86e+09	0.00e+00	0.0000
m28	28	611	42	13	4.44	5.46e+10	0.00e+00	0.0000
m29	28	631	49	12	4.51	6.90e+10	0.00e+00	0.0000
m30	21	489	48	12	4.85	2.46e+10	0.00e+00	0.0000

These work nicely!

These explode!

Back to the Drawing Board

- The combined zero/one model is not feasible for all sizes
- Finite domain models
  - Needs a special solver (Choco?)
  - Or some more modeling
- Zero/one model: Use lazy constraints
- Some form of Logical Benders Decomposition
- Or some LNS on top of decomposed admission/room solvers

Theatre Choice

- Finite Domain Model
- Zero/One Model

Theatre Choice

Finite Domain Model

Zero/One Model

Idea

- Problem decomposes into individual sub-problems per day
- One variable for each patient to give the theatre the patient is assigned to
- Count the number of theatres uses by `nvalue` constraint
- Count the number of theatres used by each surgeon by `nvalue`
- Respect theatre capacity by a `binPacking` constraint

Variables

$x_i \in 1..|T|$ the theatre to which patient i is assigned to

$y_s \in 1..|T|$ the number of theatres in which surgeon s operates

$z \in 1..|T|$ the number of open theatres

Count Number of Theatres Used

We count the number of theatres used by counting the number of values used in the assignment variables of all selected patients.

$$\textbf{nvalue}([x_i | i \in P^j], z)$$

Number of Theatres Used by Surgeon

We count the number of theatres used by a surgeon by counting the number of different values that are assigned to the patients that are handled by the surgeon.

$$\forall_{s \in S}: \quad \textbf{nvalue}([x_i | i \in P^j \textbf{ s.t. } s_i = s], y_s)$$

Theatre Capacity

The capacity constraints for each operating theatre must be respected, i.e. the total surgery time of all patients assigned to that theatre must be less or equal to the capacity of the theatre.

$$\textbf{binPacking}([c_{tj} | t \in T], [x_i | i \in P^j], [d_i | i \in P^j])$$

Objective

$$\min zw_t + \sum_{s \in S} y_s w_s$$

w_t cost of opening a theatre on the day

w_s cost of assigning surgeon to multiple theatres on the day

Nvalue constraint

5.283 nvalue

DESCRIPTION LINKS GRAPH AUTOMATON

Origin [289]

Constraint nvalue(NVAL, VARIABLES)

Synonyms cardinality_on_attributes_values, values.

Arguments
NVAL : dvar
VARIABLES : collection(var−dvar)

Restrictions
required(VARIABLES, var)
NVAL $\geq$ min(1, |VARIABLES|)
NVAL $\leq$ |VARIABLES|
NVAL $\leq$range(VARIABLES.var)

Purpose NVAL is the number of distinct values taken by the variables of the collection VARIABLES.

Example $(4, \langle 3, 1, 7, 1, 6\rangle)$

The nvalue constraint holds since its first argument NVAL $=$ 4 is set to the number of distinct values occurring within the collection $\langle 3, 1, 7, 1, 6\rangle$.

Theatre Choice

Finite Domain Model

Zero/One Model

Idea

- Indicator variable if patient i is assigned to theatre t
- Exactly one of the assignments must hold
- Indicator variables to state if surgeon s operates in theatre t
- Indicator variable if theatre t is open

Variables

x_{it} zero/one indicator if patient i is allocated to theatre t

y_{st} zero/one indicator if surgeon s operates in theatre t

z_t zero/one indicator if threatre t is open

Assign exactly one Theatre to each Patient

In this model, the patients P^j considered on day j have all been admitted. We must therefore select exactly one theatre for each patient.

$$\forall_{i \in P^j}: \quad \sum_{t \in T} x_{it} = 1$$

Force Theatre Open

If one patient i is assigned to a specific theatre t, then the theatre must be open, and $z_t = 1$.

$$\forall_{i \in P^j} \forall_{t \in T} : \quad x_{it} \leq z_t$$

Control theatres used by Surgeon

We use the y_{st} variables to count how many theatres are used by a surgeon in the solution. As soon as one patient for the surgeon is allocated to a theatre t, so that x_{it} is one, we must also have that $y_{st} = 1$.

$$\forall_{s \in S} \forall_{i \in P^j \textbf{ s.t. } s_i = s} \forall_{t \in T}: \quad x_{it} \leq y_{st}$$

Theatre Capacity

We have to respect the capacity of the theatres, and cannot assign more work to them than their capacity on the day.

$$\forall_{t \in T}: \quad \sum_{i \in P^j} x_{it} d_i \leq c_{tj}$$

Objective

$$\min \sum_{t \in T} z_y w_t + \sum_{s \in S} \sum_{t \in T} y_{st} w_s$$

w_t cost of opening a theatre on the day

w_s cost of assigning surgeon to multiple theatres on the day

Nurse Assignment

- 0/1 Model
- Decomposition by Shift

Nurse Assignment

0/1 Model

Decomposition by Shift

Idea

- All of the constraints here are soft
- Find an assignment that overall minimises cost, sounds like MIP
- Local search is also a string candidate
- Finite domain models: not enough to propagate
- Perhaps, reconsider the decomposition?

Variables

x_{rjqn} zero/one indicator, stating if room r on day j shift q is assigned to nurse n; only defined if nurse is working during that shift

y_{jqn} integer, workload assigned to nurse n in shift q of day j

z_{jqn} integer, skill level required by nurse n in shift q of day j

u_{in} zero/one, nurse n is responsible for patient i in at least one shift

Room Cover

One nurse is responsible for any non-empty room in every shift on any day. A room r is non-empty on day j in shift q if $w_{rjq} > 0$. A nurse n can be assigned to a room r on day j shift q is the nurse is working in that shift on the day.

$$\forall_{r \in R} \forall_{j \in D} \forall_{q \in Q \text{ s.t. } w_{rjq} > 0} : \sum_{n \in N \text{ s.t. } cap_{jqn} > 0} x_{rjqn} = 1$$

Nurse Workload Capacity

The total workload assigned to nurse n in shift q of day j should not exceed the available nurse workload capacity cap_{jqn}, but can be exceeded by paying cost for the excess y_{jqn}.

$$\forall_{n \in N} \forall_{j \in D} \forall_{q \in Q \textbf{ s.t. } cap_{jqn} exists} : \sum_{r \in R \textbf{ s.t. } w_{rjqn} > 0} x_{rjqn} w_{rjn} \leq cap_{jqn} + y_{jqn}$$

Respect Nurse Skilllevel

The skill level assigned to a nurse in shift q of day j should not exceed the given skill level of the nurse s_n, but can be exceeded by z_{jqn} units paying the cost in the objective function.

$$\forall_{n \in N} \forall_{j \in D} \forall_{q \in Q} \textbf{ s.t. } cap_{jqn} exists \forall_{r \in R} \textbf{ s.t. } x_{rjqn} exists : \quad x_{rjqn} s_{rjq} \leq s_n + z_{jqn}$$

Nurse Responsibility

A nurse n is responsible for a patient i if during any shift q during the stay of the patient in room r_i the nurse is assigned to that room.

$$\forall_{i \in P^b} \forall_{j \in D \textbf{ s.t. } a_i \leq j < a_i + l_i} \forall_{q \in Q} \forall_{n \in N} : \quad x_{r_i jqn} \leq u_{in}$$

Objective

The objective is to find an assignment of nurses to rooms that minimises the excess workload and skill excess costs, while also providing continuity of care, minimising the number of nurses responsible for each patient.

$$\min \sum_{j \in D} \sum_{q \in Q} \sum_{n \in N} y_{jqn} w_w + \sum_{j \in D} \sum_{q \in Q} \sum_{n \in N} z_{jqn} w_k + \sum_{i \in P} \sum_{n \in N} u_{in} w_c$$

w_w workload excess cost

w_k skill violation cost

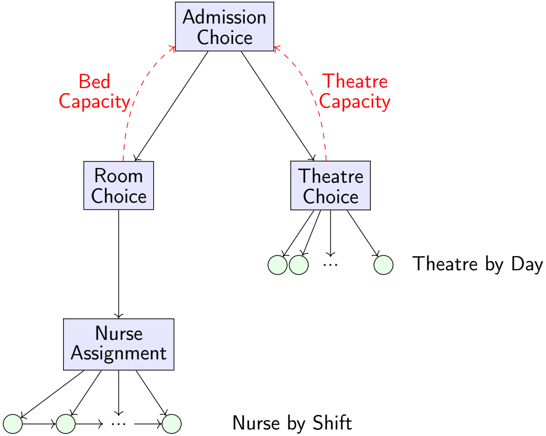
w_c continuity of care cost

Nurse Assignment

0/1 Model

Decomposition by Shift

Nurse Assignment Decomposition



Idea

- Solve the nurse assignment for each day in shift in increasing order
- Remember which nurses have been responsible for which patients in previous steps
- Large reduction if size of model (factor 28*3) for four week planning

Variables

x_{rn} zero/one indicator, stating if room r in the selected shift is assigned to nurse n; only defined if nurse is working during that shift

y_n integer, workload assigned to nurse n in the selected shift

z_n integer, skill level required by nurse n in the selected shift

u_i zero/one, stating if patient i in the selected shift is cared for by a nurse that the patient has not seen before; only defined if patient is in hospital during that shift

Room Cover

One nurse is responsible for any non-empty room in every shift on any day. A room r is non-empty in the selected shift on day j in shift q if $w_{rjq} > 0$. A nurse n can be assigned to a room r during the selected shift if the nurse is working in that shift.

$$\forall_{r \in R \text{ s.t. } w_{rjq} > 0} : \sum_{n \in N \text{ s.t. } cap_{jqn} > 0} x_{rn} = 1 \tag{2}$$

Respect Nurse Workload

The total workload assigned to nurse n in the selected shift should not exceed the available nurse workload capacity cap_{jqn}, but can be exceeded by paying cost for the excess y_n.

$$\forall_{n \in N \textbf{ s.t. } cap_{jqn} exists} : \sum_{r \in R \textbf{ s.t. } w_{rjqn} > 0} x_{rn} w_{rjn} \leq cap_{jqn} + y_n \tag{3}$$

Respect Nurse Skillevel

The skill level assigned to a nurse in the selected shift should not exceed the given skill level of the nurse s_n, but can be exceeded by z_n units paying the cost in the objective function.

$$\forall_{n \in N \textbf{ s.t. } cap_{jqn} exists} \forall_{r \in R \textbf{ s.t. } x_{rn} exists} : \quad x_{rn} s_{rjq} \leq s_n + z_n \tag{4}$$

Patient sees a new nurse in this shift

If in the selected shift a patient sees a new nurse, a cost for continuity of care must be paid.

$$\forall_{i \in P} \forall_{n \in N \textbf{ s.t. } n \notin B_i^{jq}} : \quad x_{r_i n} \leq u_i \tag{5}$$

Insight

Objective

$$\min \sum_{n \in N} y_n w_w + \sum_{n \in N} z_n w_k + \sum_{i \in P} u_i w_c$$

w_w workload excess cost

w_k skill violation cost

w_c continuity of care cost

Stepping Back

Sample Results

Name	Decomp	Total Time	Admission Time	Admission Gap Percent	Admission Solution Status	Theatre Total Time	Theatre Solver Outcome	Room Time	Room Gap Percent	Room Solution Status	Nurse Total Time	Nurse Solver Outcome
test01	AR+Td+N	5.79	1.42	0.00	Optimal	0.05	Opt:11 Sol:0	0.00	0.00	n/a	4.09	Opt:1 Sol:0
test02	AR+Td+N	599.74	2.36	0.06	Optimal	0.14	Opt:9 Sol:0	0.00	0.00	n/a	597.07	Opt:0 Sol:1
test03	AR+Td+N	0.61	0.34	0.08	Optimal	0.02	Opt:5 Sol:0	0.00	0.00	n/a	0.21	Opt:1 Sol:0
test04	AR+Td+N	609.46	7.77	0.20	Optimal	0.11	Opt:10 Sol:0	0.00	0.00	n/a	592.51	Opt:0 Sol:1
test05	AR+Td+N	2.00	1.07	0.12	Optimal	0.04	Opt:9 Sol:0	0.00	0.00	n/a	0.86	Opt:1 Sol:0
test06	AR+Td+N	601.34	301.77	2.03	Solution	0.14	Opt:13 Sol:0	0.00	0.00	n/a	298.10	Opt:0 Sol:1
test07	AR+Td+N	600.41	310.89	0.38	Solution	0.22	Opt:17 Sol:0	0.00	0.00	n/a	289.13	Opt:0 Sol:1
test08	AR+Td+N	600.46	105.78	0.19	Optimal	0.23	Opt:17 Sol:0	0.00	0.00	n/a	494.20	Opt:0 Sol:1
test09	A+R+Td+Nd	48.99	47.56	0.20	Optimal	0.14	Opt:19 Sol:0	0.04	0.00	Optimal	1.16	Opt:63 Sol:0
test10	A+R+Td+Nd	213.16	11.84	0.19	Optimal	83.62	Opt:20 Sol:0	42.22	0.00	Optimal	75.10	Opt:60 Sol:3
m21	A+R+Td+Nd	252.92	1.08	0.13	Optimal	4.91	Opt:14 Sol:0	217.59	0.00	Optimal	29.20	Opt:42 Sol:0
m22	A+R+Td+Nd	10.35	0.45	0.18	Optimal	5.98	Opt:14 Sol:0	1.10	0.00	Optimal	2.69	Opt:42 Sol:0
m23	A+R+Td+Nd	11.75	2.34	0.20	Optimal	7.89	Opt:14 Sol:0	0.14	0.00	Optimal	1.25	Opt:42 Sol:0
m24	A+R+Td+Nd	46.51	2.21	0.17	Optimal	17.30	Opt:21 Sol:0	0.32	0.00	Optimal	26.45	Opt:63 Sol:0
m25	A+R+Td+Nd	24.04	16.45	0.17	Optimal	1.81	Opt:21 Sol:0	3.17	0.00	Optimal	2.46	Opt:63 Sol:0
m26	A+R+Td+Nd	154.92	5.30	0.16	Optimal	3.35	Opt:26 Sol:0	0.34	0.00	Optimal	145.65	Opt:73 Sol:11
m27	A+R+Td+Nd	279.72	0.65	0.18	Optimal	252.98	Opt:14 Sol:0	0.54	0.00	Optimal	25.35	Opt:42 Sol:0
m28	A+R+Td+Nd	352.31	7.10	0.18	Optimal	38.65	Opt:28 Sol:0	300.29	100.00	Solution	5.96	Opt:84 Sol:0
m29	A+R+Td+Nd	88.56	3.05	0.20	Optimal	3.40	Opt:25 Sol:0	0.36	0.00	Optimal	81.45	Opt:77 Sol:7
m30	A+R+Td+Nd	968.59	7.40	0.19	Optimal	575.60	Opt:20 Sol:1	300.27	100.00	Solution	84.70	Opt:52 Sol:11

Cost

Name	Decomp	Total Cost	Admission Cost	Late Cost	Theatre Cost	Surgeon Cost	AgeMix Cost	Continuity Cost	Nurse Overload Cost	Nurse Underskill Cost
test01	AR+Td+N	3,256	1,200	460	330	0	160	1,000	26	80
test02	AR+Td+N	1,664	350	520	140	0	235	217	7	195
test03	AR+Td+N	10,159	9,100	285	50	0	11	685	0	28
test04	AR+Td+N	2,353	250	960	150	0	27	351	35	580
test05	AR+Td+N	15,091	13,600	200	270	0	110	855	21	35
test06	AR+Td+N	16,841	11,000	2,940	570	20	90	2,095	46	80
test07	AR+Td+N	18,326	9,750	445	1,150	0	765	2,640	386	3,190
test08	AR+Td+N	23,958	20,250	1,350	920	0	93	576	134	635
test09	A+R+Td+Nd	20,620	13,300	890	660	0	25	4,400	240	1,105
test10	A+R+Td+Nd	60,594	15,900	6,820	4,550	250	374	16,835	2,165	13,700
m21	A+R+Td+Nd	29,079	10,500	11,685	2,040	55	137	1,492	150	3,020
m22	A+R+Td+Nd	26,861	9,200	3,710	1,040	11	1,150	7,495	625	3,630
m23	A+R+Td+Nd	22,510	5,400	3,920	2,200	90	1,145	8,290	1,029	436
m24	A+R+Td+Nd	33,575	19,950	5,340	1,660	170	1,235	2,914	1,076	1,230
m25	A+R+Td+Nd	34,850	20,000	10,810	1,280	13	34	1,977	263	473
m26	A+R+Td+Nd	74,378	47,250	12,760	2,610	170	670	3,808	40	7,070
m27	A+R+Td+Nd	42,979	2,250	7,695	2,320	19	2,425	10,570	5,060	12,640
m28	A+R+Td+Nd	52,987	11,900	12,480	4,680	200	72	17,875	1,790	3,990
m29	A+R+Td+Nd	53,632	32,250	7,590	3,650	120	2,710	3,437	145	3,730
m30	A+R+Td+Nd	37,134	9,600	10,940	6,050	300	2,690	3,410	1,069	3,075
Total		580,847	263,000	101,800	36,320	1,418	14,158	90,922	14,307	58,922

Cost Percentage

Name	Total Cost	Total Cost Percent	Admission Cost Percent	Late Cost Percent	Theatre Cost Percent	Surgeon Cost Percent	AgeMix Cost Percent	Continuity Cost Percent	Nurse Overload Cost Percent	Nurse Underskill Cost Percent
test01	3,256	100.00	36.86	14.13	10.14	0.00	4.91	30.71	0.80	2.46
test02	1,664	100.00	21.03	31.25	8.41	0.00	14.12	13.04	0.42	11.72
test03	10,159	100.00	89.58	2.81	0.49	0.00	0.11	6.74	0.00	0.28
test04	2,353	100.00	10.62	40.80	6.37	0.00	1.15	14.92	1.49	24.65
test05	15,091	100.00	90.12	1.33	1.79	0.00	0.73	5.67	0.14	0.23
test06	16,841	100.00	65.32	17.46	3.38	0.12	0.53	12.44	0.27	0.48
test07	18,326	100.00	53.20	2.43	6.28	0.00	4.17	14.41	2.11	17.41
test08	23,958	100.00	84.52	5.63	3.84	0.00	0.39	2.40	0.56	2.65
test09	20,620	100.00	64.50	4.32	3.20	0.00	0.12	21.34	1.16	5.36
test10	60,594	100.00	26.24	11.26	7.51	0.41	0.62	27.78	3.57	22.61
m21	29,079	100.00	36.11	40.18	7.02	0.19	0.47	5.13	0.52	10.39
m22	26,861	100.00	34.25	13.81	3.87	0.04	4.28	27.90	2.33	13.51
m23	22,510	100.00	23.99	17.41	9.77	0.40	5.09	36.83	4.57	1.94
m24	33,575	100.00	59.42	15.90	4.94	0.51	3.68	8.68	3.20	3.66
m25	34,850	100.00	57.39	31.02	3.67	0.04	0.10	5.67	0.75	1.36
m26	74,378	100.00	63.53	17.16	3.51	0.23	0.90	5.12	0.05	9.51
m27	42,979	100.00	5.24	17.90	5.40	0.04	5.64	24.59	11.77	29.41
m28	52,987	100.00	22.46	23.55	8.83	0.38	0.14	33.73	3.38	7.53
m29	53,632	100.00	60.13	14.15	6.81	0.22	5.05	6.41	0.27	6.95
m30	37,134	100.00	25.85	29.46	16.29	0.81	7.24	9.18	2.88	8.28
Total	580,847	100.00	45.28	17.53	6.25	0.24	2.44	15.65	2.46	10.14

Compared to Others

- Still a long way to go!
- Encouraging:
  - 4 solutions better than reference solution
  - 1 new best solution (after competition)
- We are not dealing with the soft constraints
- Possible improvement for large admission problems
- We need better time management

Interested in Trying a Competition?

Teasing Challenge EURO/ROADEF 2026-2027

- Organising committee
  - Eric Bourreau
  - Safia Kedad-Sidhoum
  - David Savourey
  - Zacharie Ales
  - Ronan Bocquillon
  - Marc Sevaux
- Industrial Committee
  - Amal Benhamiche
  - Yannick Carliney
  - Morgan Chopin
  - Eric Gourdin
  - Nancy Plerrot

Orange Challenge Euro ROADEF 2026

https://www.euro-online.org/web/pages/1707/roadefeuro-challenge

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