Quantifying the Effects of Raising

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Abstract

Noting from practical experience that the raising process can both thicken and extend the workpiece, this article analyses the processes involved and presents mathematical, approximate and computational methods for quantifying and visualising these effects; albeit under conditions that are unlikely to be realized in the workshop.

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Introduction

Raising is a process in which a flat disc is deformed into, typically, a solid of revolution. As the vessel is formed the metal has to be compressed: for example, the outside radius of the disc is generally smaller than the radius of the rim of the vessel. Practical experience tells us that the wall thickness can increase (or indeed, disappear) and that the vessel can also gain height.

This article considers two quite unlikely possibilities. One is that the deformation only ever increases wall thickness, never height. The item would be raised in such a fashion that if circles were engraved on the disc at, say, 5mm intervals the raised piece would preserve the spacing of these circles.

The other, perhaps more achieveable modus operandi, would be to maintain a constant thickness when raising which would require the item to gain height.

A theoretical quantification of these modes of working may give us insight into what actually happens in the workshop.

Effect of Raising

Figure 1 below shows a vessel sitting on it's initial disc. An annulus in the disc is shown in grey and we suppose this is transformed into the grey annulus in the vessel.

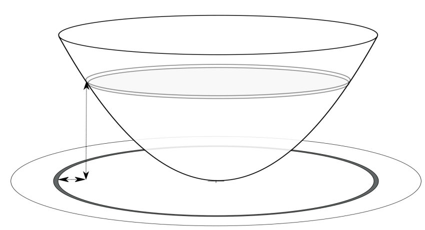

Figure 1: The disc and raised vessel. It is apparent that the radius of disc's annulus is larger than that of the vessel.

Raising Whilst Preserving Arc Length

We imagine a ruler engraved on the disc, and that when raised the vessel's ruler is still a true measure, see Figure 2.

Suppose our disc has thickness T (see figure 3) and we consider an annulus of width δs at a distance s from the centre. Since arc length is preserved, the disc's

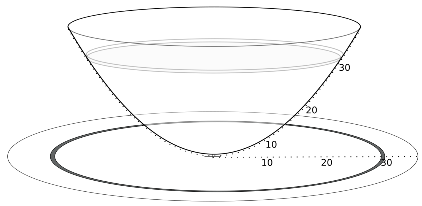

Figure 2: The vessel is raised in such a way that the dots on the disc correspond to those on the vessel

annulus is deformed into an annulus of width δs at arc length s along the vessel and of unknown thickness t.

The disc's annulus's volume is

$(\Pi(s+\delta s)^2 - \Pi s^2)T$

ie $T\Pi(s^2 + 2s\delta s + \delta s^2 - s^2)$

which reduces to $2s\delta sT\Pi$ as $\delta s \to 0$

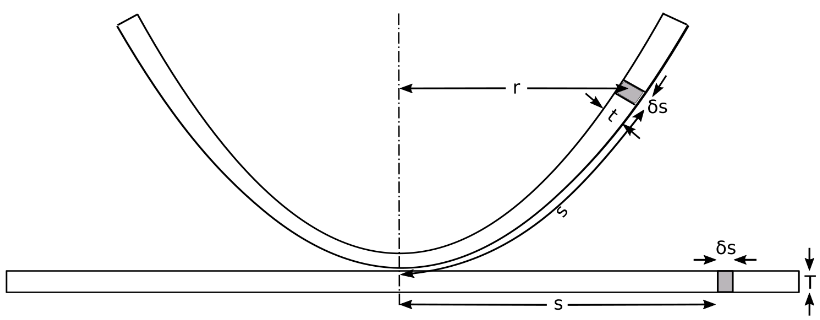

Figure 3: Computing volumes: annulus at distance s from the centre, having width δs. It's thickness is known to be T in the disc and has thickness t in the vessel. r is the effective (hand wave) radius of the vessel's annulus.

The volume of the vessel's annulus is approximately the swept volume of a rectangle δs x t, that is

$2\Pi r\delta st$

where r is known and may be expressed as a function $\rho(s)$ of arc length.

equating the two volumes gives

$2s\delta sT\Pi = 2\Pi r\delta st$

thus $sT = rt$

so $t = \frac{sT}{r}$

in order to solve this we need to express r as a function of arc length, s. In other words

$$t = \frac{sT}{\rho(s)} \text{ where } r = \rho(s) \tag{1}$$

Example - a cylinder

Suppose our initial disc is 300mm diameter and 1mm thick. We wish to form it into a can 100mm diameter and 100mm high and are proposing to keep the arc length constant as described above.

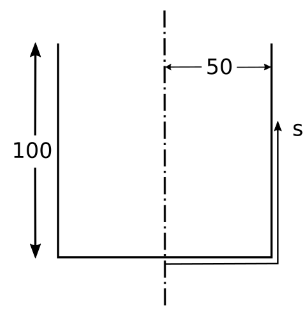

Figure 4: A cylinder 100mm high, 100mm diameter. Arc length s is measured as shown.

$r = \rho(s)$ is such that

$$\rho(s) = \begin{cases} s & : 0 \leq s \leq 50 \\ 50 & : 50 < s \end{cases} \tag{2}$$

Substituting (2) into (1) with $T = 1$ we have

$$t = \begin{cases} \frac{s1}{s} & : 0 \leq s \leq 50 \\ \frac{s1}{50} & : 50 < s \end{cases}$$

giving us

$$t = \begin{cases} 1 & : 0 \le s \le 50 \\ \frac{s}{50} & : 50 < s \end{cases} \tag{3}$$

The wall thickness, t, is 1mm for values of s from zero to 50mm, as we would expect: no work needs to be done to the base. For values of s greater than 50 $t = s/50$, so at the bottom of the side the $t = 51/50 = 1.02$mm and at the top, where $s = 150$mm we would have $t = 150/50 = 3$mm

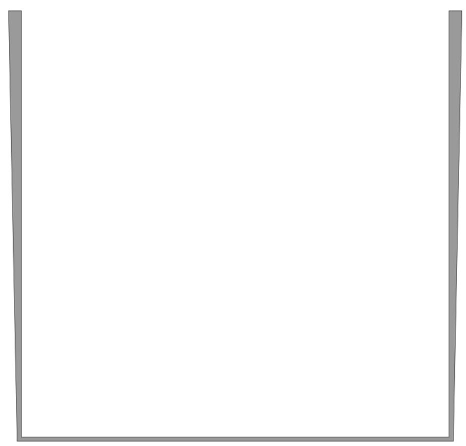

Figure 5: Predicted wall thicknesses. The cylinder is 100mm high, 100mm diameter internally. Wall thickness is 1mm on the base, the sides increase linearly as $s/50$ achieving 3mm at the rim.

As a check, let us estimate the total volumes of disc and raising.

For the disc we have

volume $= \Pi r^2 T = \Pi$ x 150 x 150 x 1 $\approx$ 70685mm^3.

For the raising we have

total volume = volume of base + swept volume of side.

volume of base is $\Pi r^2 T = \Pi$ x 50 x 50 x 1 $\approx$ 7850mm^3.

swept volume of side is (area of side x circumference)

The cross section of the side wall is a rectangle circa 3mm x 100mm less a triangle with base 100mm and height 2mm.

area of rectangle is 3 x 100 = 300mm^2

area of triangle is 1/2 x base x height = (1/2 x 100 x 2) = 100mm^2

cross sectional area = rectangle less triangle = 200mm^2.

circumference of cylinder $2\Pi r = 2\Pi$ x 50 ≈ 315mm

swept volume of the side is 200mm x $315\text{mm}^2 = 63000\text{mm}^3$.

Volume of raising is therefore $63000 + 7850 = 70850\text{mm}^3$ which is more or less in agreement with with initial disc's volume.

This example omits the relation between arc length and radius. We were able to visualise the end result because we 'just know' the radius. In addition the visualisation works because the value of t is measured normal to the surface and we know which direction those normals go in this case.

Computation of Wall Thickness for Arbitrary Profiles

Unless you're making a parabolic refelector, it's unlikely that the profile of the raising will be reducible to a nice function as in the example above. We can get round this by capturing the profile, which gives us radius and height data, and computing arc length at each point. Given knowledge of arc length and radius at a point we can use equation (1) to compute the wall thickness at that point. Having done that, we can visualise the profile.

Capturing the Profile

A two colour image of a profile, for example Figure 6, can be processed by computer (details yet to be written up) to capture radius and height at the level of individual pixels.

Further processing gives us arc length and tangent at each point, enabling calculation of wall thickness and the normal to the surface, finally leading to the visualisation in Figure 7.

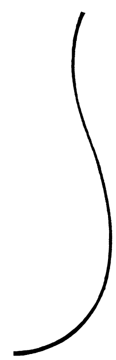

Figure 6: Half profile of a vessel. Note that this is a two colour image and that the base of black outline meets the left hand side of the image. The left hand side is the centre line of the vessel. Bigger images are better images, the one illustrated has been reduced.

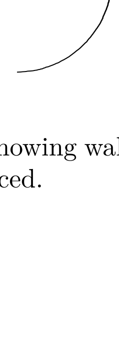

Figure 7: Half profile of a vessel showing wall thicknesses generated from Figure 6. This image has also been reduced.

Raising Whilst Preserving Wall Thickness

In progress...

This is a rather more practical proposition, being more or less what happens when spinning an item. If the piece was being raised this would be achieved by forging out thickened areas. The deformation effectively enlarges the disc, increasing the height and / or width of the vessel.