Quantifying the Effects of Raising

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03-03-2018

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Abstract

Noting from practical experience that the raising process can both thicken and extend the workpiece, this article analyses the processes involved and presents mathematical, approximate and computational methods for quantifying and visualising these effects; albeit under conditions that are unlikely to be realized in the workshop.

Introduction

Raising is a process in which a flat disc is deformed into, typically, a solid of revolution. The most obvious change is that the workpiece gains height, as if the radius of the initial disc had increased. This can amount to as much as a 50% increase. Less obviously the metal can also thicken when worked: this can be quite dramatic, I've seen an increase from 1mm to 5mm in thickness occur.

So what's going on? Whatever else is happening, volume is conserved: nothing is added, nothing taken away. Nor is the metal being appreciably compressed but rather displaced or deformed. The thickening effect is most noticeable where the vessel has been brought in the most: the insight here is that the metal is being forced in on itself and, for whatever reason is gaining thickness rather than height. Any gain in height presumably comes at the expense of wall thickness, whether by thinning the original disc or by redistributing metal gained by the thickening process described above.

These effects are local, there may be thickening at one point and a gain in height in another part of the item. Indeed both effects could be happening at once. This situation is pretty much impossible to analyse. In the hope that even a limited, preferably quantitative, analysis may give some insight into what's happening I consider two unlikely possibilities.

One is that the deformation only ever increases wall thickness, never height. The item would be raised in such a fashion that if circles were engraved on the disc at, say, 5mm intervals the raised piece would preserve the spacing of these circles. This is equivalent to saying that the arc length, that is the distance measured along the cross section, of the raising is the same as the radius of the initial disc.

The other, perhaps more practical, modus operandi would be to maintain a constant thickness when raising, which would require the item to gain height. This transformation increases the arc length of the finished item relative to the initial disc, the increase in height being gained by eliminating any thickening that may occur.

In both cases I assume that no thinning takes place, that is the wall thickness of the finished item is at least the same as the initial disc.

Effects of Raising

Figure 1 below shows a vessel sitting on it's initial disc. An annulus in the disc is shown in grey and we suppose this is transformed into the grey annulus in the vessel. If we maintain at least the initial disc's thickness then the annulus in the disc must be transformed (eventually) into a smaller diameter annulus in the item. For simplicity the following analysis assumes that either

- the transformation only increases thickness, or

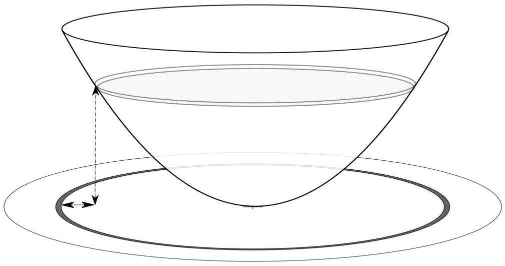

Figure 1: The disc and raised vessel. It is apparent that the radius of disc's annulus is larger than that of the vessel.

- the transformation only increases arc length

These scenarios are taken in turn.

Raising Whilst Preserving Arc Length

We imagine a ruler engraved on the disc, and that when raised the vessel's ruler is still a true measure, see Figure 2.

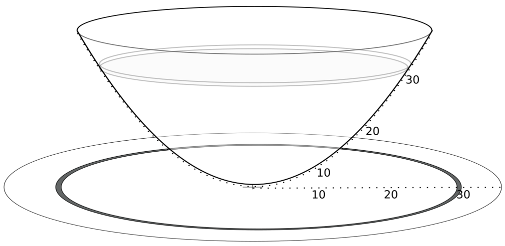

Figure 2: We suppose the vessel is raised in such a way that the distance between the dots on the disc is the same as the distance between those on the vessel.

Suppose our disc has thickness T (see figure 3) and we consider an annulus of width δs at a distance s from the centre. Since arc length is preserved, the disc's annulus is deformed into an annulus of width δs at arc length s along the vessel and of unknown thickness t.

Volume of disc's annulus is area of annulus times T

$(\Pi(s + \delta s)^2 - \Pi s^2)T$

ie $T\Pi(s^2 + 2s\delta s + \delta s^2 - s^2)$

which reduces to $2T\Pi s\delta s$ as $\delta s \to 0$

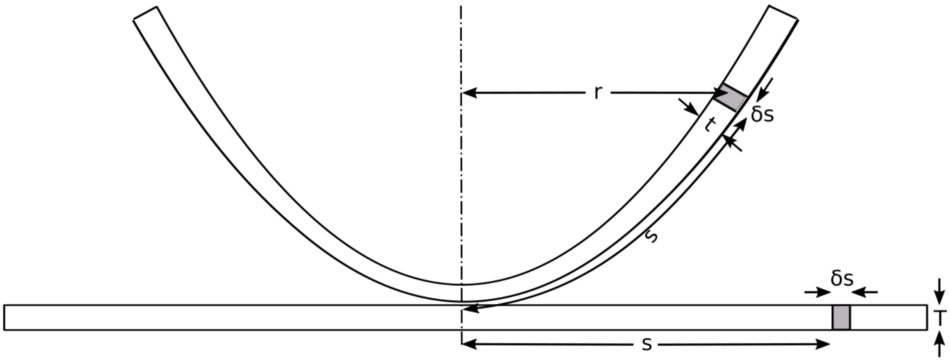

Figure 3: Computing volumes: annulus at distance s from the centre, having width δs. It's thickness is known to be T in the disc and has thickness t in the vessel. r is the effective (hand wave) radius of the vessel's annulus.

The volume of the vessel's annulus is approximately [1] the swept volume of a rectangle δs x t, that is

$2t\Pi r\delta s$

where r is known and may be expressed as a function $\rho(s)$ of arc length.

Equating volumes

$2T\Pi s\delta s = 2t\Pi r\delta s$

thus $sT = rt$

so $t = \frac{sT}{r}$

in order to solve this we need to express r as a function of arc length, s. In other words

$$t = \frac{sT}{\rho(s)} \text{ where } r = \rho(s) \tag{1}$$

Example - a cylinder

Suppose our initial disc is 300mm diameter and 1mm thick. We wish to form it into a can 100mm diameter and 100mm high and are proposing to keep the arc length constant as described above.

$r = \rho(s)$ is such that

$$\rho(s) = \begin{cases} s & : 0 \leq s \leq 50 \\ 50 & : 50 < s \end{cases} \tag{2}$$

[1]Pappus' Theorem states "the area of the region times the distance travelled by the centroid of the region", so r is a bit short.

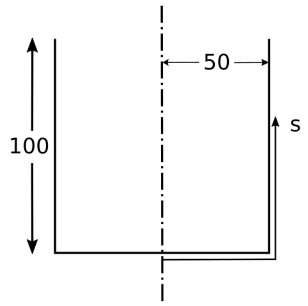

Figure 4: A cylinder 100mm high, 100mm diameter.
Arc length s is measured as shown.

Substituting (2) into (1) with $T = 1$ we have

$$t = \begin{cases} \frac{s1}{s} & : 0 \leq s \leq 50 \\ \frac{s1}{50} & : 50 < s \end{cases}$$

giving us

$$t = \begin{cases} 1 & : 0 \leq s \leq 50 \\ \frac{s}{50} & : 50 < s \end{cases} \tag{3}$$

The wall thickness, t, is 1mm for values of s from zero to 50mm, as we would expect: no work needs to be done to the base. For values of s greater than 50 $t = s/50$, so at the bottom of the side the $t = 51/50 = 1.02$mm and at the top, where $s = 150$mm we would have $t = 150/50 = 3$mm

As a check, let us estimate the total volumes of disc and raising.

For the disc we have

volume $= \Pi r^2 T = \Pi$ x 150 x 150 x 1 $\approx 70685\text{mm}^3$.

For the raising we have

total volume = volume of base + swept volume of side.

volume of base is $\Pi r^2 T = \Pi$ x 50 x 50 x 1 $\approx 7850\text{mm}^3$.

swept volume of side is (area of side x circumference)

The cross section of the side wall is a rectangle circa 3mm x 100mm less a triangle with base 100mm and height 2mm.

area of rectangle is 3 x 100 = 300mm^2

area of triangle is 1/2 x base x height = (1/2 x 100 x 2) = 100mm^2

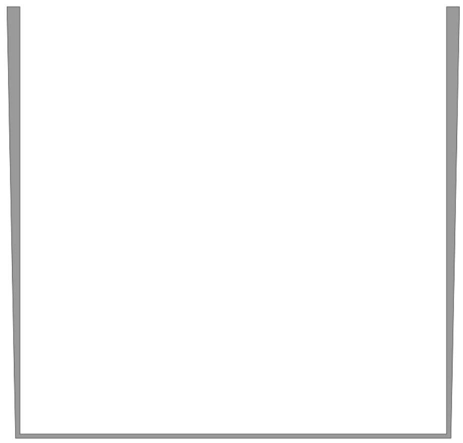

Figure 5: Predicted wall thicknesses. The cylinder is 100mm high, 100mm diameter internally. Wall thickness is 1mm on the base, the sides increase linearly as $s/50$ achieving 3mm at the rim.

cross sectional area = rectangle less triangle = 200mm^2.

circumference of cylinder $2\Pi r = 2\Pi$ x 50 $\approx$ 315mm

swept volume of the side is 200mm x 315mm^2 = 63000mm^3.

Volume of raising is therefore 63000 + 7850 = 70850mm^3 which is more or less in agreement with with initial disc's volume.

This example omits the relation between arc length and radius. We were able to visualise the end result because we 'just know' the radius. In addition the visualisation works because the value of t is measured normal to the surface and we know which direction those normals go in this case.

Computation of Wall Thickness for Arbitrary Profiles

Unless you're making a parabolic reflector, it's unlikely that the profile of the raising will be reducible to a nice function as in the example above. We can get round this by capturing the profile, which gives us radius and height data, and computing arc length at each point. Given knowledge of arc length and radius at a point we can use equation (1) to compute the wall thickness at that point. Having done that, we can visualise the profile.

Capturing the Profile

I have written a program called pbmGetProfile (zip file, documentation) which, given a two colour image of a profile, for example Figure 7, will capture a series of (radius, height) coordinates in a csv file.

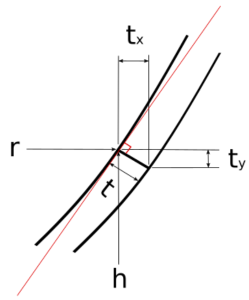

Figure 6: Calculating exterior position. Given an interior point of the profile, radius r at height h and the slope of the tangent (in red), we work out thickness t using our formula. In order to draw the outside of the vessel, we have to find t_x and t_y, which we do via Pythagoras: $t^2 = t_x^2 + t_y^2$ and knowing that t is normal to the tangent.

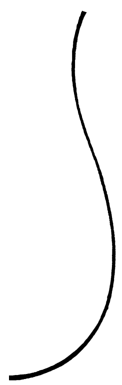

Figure 7: Half profile of a vessel. Note that this is a two colour image and that the base of black outline meets the left hand side of the image. The left hand side is the centre line of the vessel. Bigger images are better images, the one illustrated has been reduced.

This becomes input into a program called rpal (zip file, documentation) which calculates arc length and tangent at each point and from there wall thickness and the normal to the surface. From that the program produces an svg file, which is the basis of the visualisation in Figure 8.

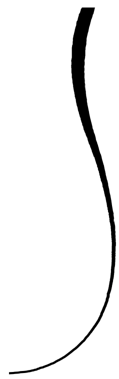

Figure 8: Half profile of a vessel showing wall thicknesses generated from Figure 7. This image has also been reduced.

Raising Whilst Preserving Wall Thickness

In progress...

This is a rather more practical proposition, being more or less what happens when spinning an item. If the piece was being raised this would be achieved by forging out thickened areas. The deformation effectively enlarges the disc, increasing the height and / or width of the vessel.

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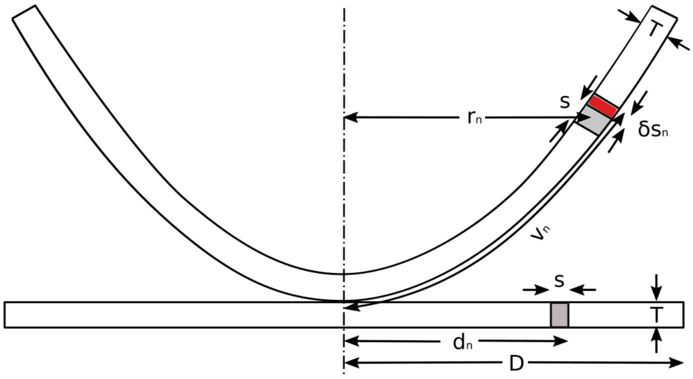

Figure 9: Computing volumes: a disc of radius D has an annulus at a distance d_n from the centre, having width s and thickness T.
In the raising the corresponding annulus is at a distance v_n along the arc (corresponding to a radius of r_n) and has a cross section $(s + \delta s_n)$ x T.

Volume of disc's annulus is area of annulus times T

$T\Pi(d_n^2 - (d_n - s)^2)$

$T\Pi(d_n^2 - (d_n^2 - 2d_n s + s^2)$

$T\Pi(2d_n s - s^2)$

Volume of raising's annulus is swept volume of area ($(s + \delta s_n)$ times T)

$2T\Pi r_n(s + \delta s_n)$

Defining $\delta v_n = s + \delta s_n$ and equating volumes

$T\Pi(2d_n s - s^2) = 2T\Pi r_n(\delta v_n)$

$2d_n s - s^2 = 2r_n \delta v_n$

Let the radius of original disc be divided into N equal parts.
s is now equal to $\frac{D}{N}$ and $d_n = \frac{nD}{N}$ and so $d_n = ns$ and we have

$2ns^2 - s^2 = 2r_n \delta v_n$

$s^2(2n - 1) = 2r_n \delta v_n$

$\delta v_n = \frac{s^2(2n-1)}{2r_n}$

$$\delta v_n = \frac{s^2(2n - 1)}{2r_n} \tag{4}$$

The arc length v_n is the sum of δv_n it contains:

$v_n = \sum_{i=1}^{n} \delta v_i$

$v_n = \sum_{i=1}^{n} \frac{s^2(2i-1)}{2r_i}$

$$v_n = \frac{ns^2}{2} \sum_{i=1}^{n} \frac{(2i-1)}{r_i} \tag{5}$$

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In the limit as $\delta v_n \to 0$

$2d_n s - s^2 = 2r_n \delta v_n$ is

$d_n s = r_n \delta v_n$

$\delta v_n = \frac{d_n s}{r_n}$

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