Math is hard

And you should stop trying to do it

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MAT-101

Consider the following system of equations:

$$x + y = 0$$
$$x - y = 0$$

Can you solve it?

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Can you solve it?

1. Solve the first equation for a single variable.
2. Plug this solution into the second equation to eliminate one variable.
3. Repeat with the following equations until all variables are eliminated.

Example

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Takeaway 1: This process is easy to describe, and always[1]works.

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[1]Almost always, but let's move on.

Consider the following harder problem:

$$\pi x + 2y - 7z + \frac{2}{9}w = -1$$

$$e^5 x - y + z + \frac{1}{9}w = 1$$

$$-9x + y + 4z - \frac{1}{9}w = 0$$

$$\sqrt{3}x + 10y - 3z - \frac{2}{9}w = 2$$

Our previous method works flawlessly. (But you won't!)

In an instant, my computer gives:

$$w = \frac{9(-73e^5 - 53\pi + 10\sqrt{3} + 603)}{4(-10\pi + 5\sqrt{3} + 153 + 13e^5)}, \quad x = \frac{33}{-10\pi + 5\sqrt{3} + 153 + 13e^5},$$

$$y = \frac{3(-3e^5 - 7\pi - 2\sqrt{3} + 117)}{4(-10\pi + 5\sqrt{3} + 153 + 13e^5)}, \quad z = \frac{-4e^5 - 2\pi + \sqrt{3} + 90}{-10\pi + 5\sqrt{3} + 153 + 13e^5}$$

Takeaway 2: Humans are only suited to solve trivial problems. They are much better at devising procedures to solve problems, then letting computers handle it.

(This is a hot take.)

Another mathematically hot take

"Without computers we will be stuck only proving theorems that have short proofs."

— Kenneth Appel, co-prover of the four color theorem

Computer algebra system

Software system to solve mathematical problems, particularly those related to algebra.

Computer algebra systems are *tools*. Some tools are better than others.

We should find "the best."

Ergonomics

The study of human relationship with work.

Our tools should be simple and convenient. This includes minimizing *mental workload*.

Extreme example: Studies in aviation accidents suggest that as much as 18% of pilot errors are due to confusing instruments[2].

My real thesis: Some computer algebra systems are more confusing than others.

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[2] *Handbook of Human Factors and Ergonomics*, 4th edition, pg. 244

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Idea:

1. Look at programs written in two computer algebra systems[3] that solve the same problem.
2. Compute the Halstead metrics on both programs and compare the results.
3. Make pretty pictures.

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[3] Python's *SymPy* library and *Maple*, specifically. The former is free, and the latter is available on campus!

Pretty pictures

Log-effort histogram
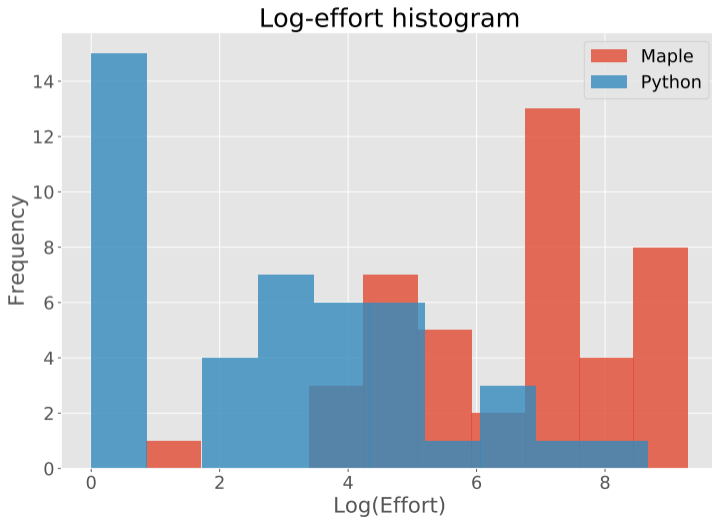
Conclusions

The metrics quantify our gut feelings: Some languages are "simpler" than others when solving the same tasks.

If we want to use computers more effectively, we should notice this.

Mathematicians are bad programmers, and probably don't care. Maybe the next generation will.

Questions?