Soft Real-Time Systems

CPEN 432 – Real-Time Systems Design

Elastic scheduling of real-time tasks

Ref: Elastic task model for adaptive rate control. Buttazzo, Lipari and Abeni. IEEE Real-Time Systems Symposium, 1998.

Many models for soft real-time systems

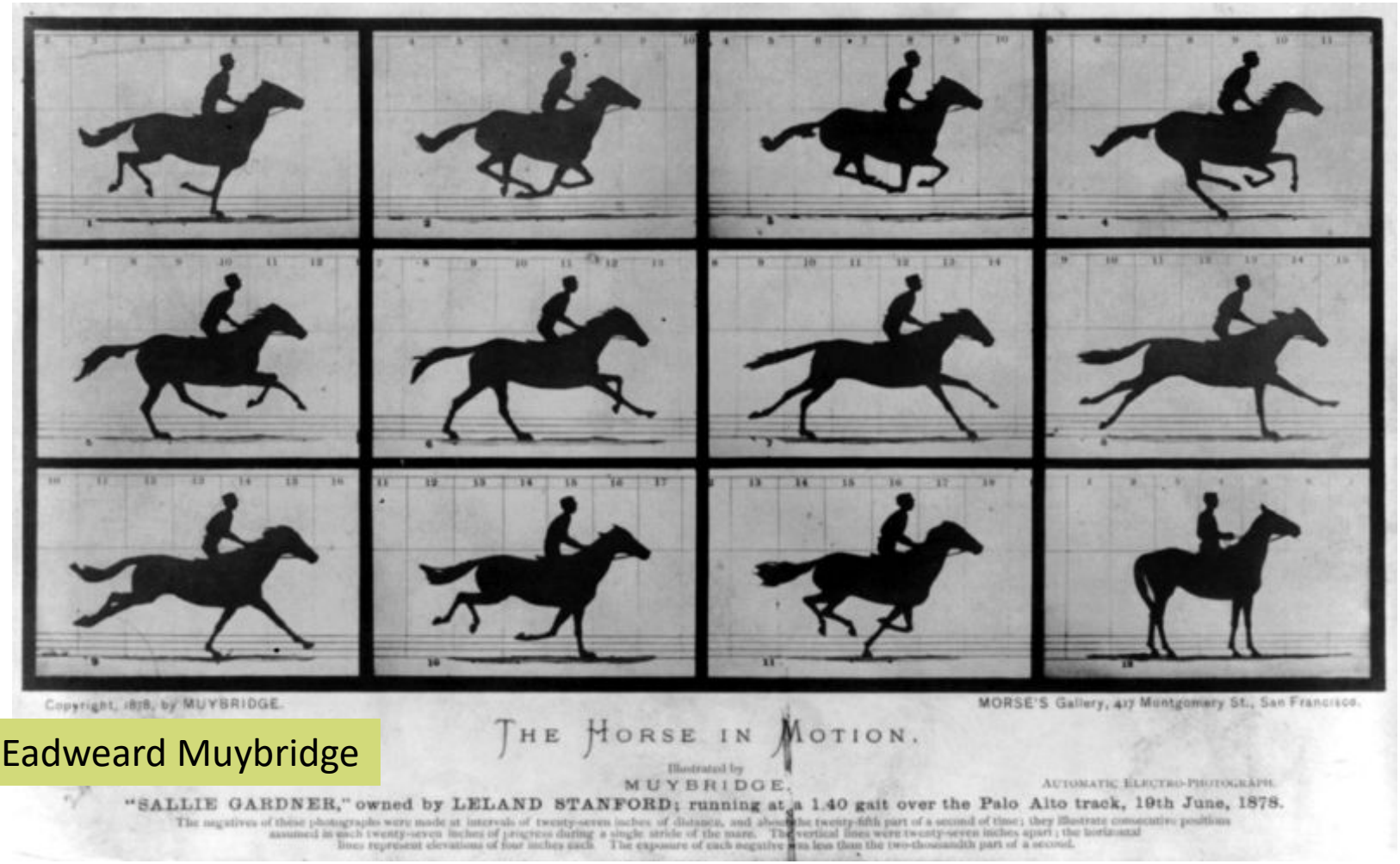
- Why?

- There are a variety of soft real-time systems.

- And there are a variety of ways in which their behaviour can be altered.

  - In a multimedia system…
    - Frames can be dropped, i.e., jobs can be skipped.
    - Alternatively, it is possible to reduce the frame rate.
      - Move from 60 fps to 30 fps or to 24 fps.
      - We are changing the periodicity of a task.

The elastic task model provides a natural abstraction for such tasks.



Eadweard Muybridge

Elastic task scheduling

- What if we need all jobs to meet their deadlines (cannot drop jobs or portions of them) ... Even if this entails allowing tasks to execute at lower frequencies (higher periods)?

- Then we need to understand
  - When should we increase task periods?
  - And increase the periods by what extent?
  - And for which tasks?

Task model for elastic scheduling

- For each task T_i :
  - There is an operating period range $[P_{i,min}, P_{i,max}]$
    - $P_{i,min}$ is the best possible period
  - There is also a value $P_{i,0}$ that represents a nominal period
    - This nominal period is preferred when we can not run the task at the best possible period

- For a hard real-time task:
  - $P_{i,min} = P_{i,max} = P_{i,0}$
- If $P_{i,0} = P_{i,max}$ and $P_{i,min} << P_{i,max}$, the task is very flexible (highly elastic)
- Only periods can be varied; execution times are constant

Task model for elastic scheduling

- For each task T_i :
  - There is also an elasticity factor k_i that represents the flexibility of the task [how much it prefers to stay close to nominal]
    - **k_i = 0:** T_i would always stay at nominal period (hard real-time)
    - **k_i large relative to other tasks:** T_i relatively flexible in being stretched away from nominal period

Additional parameters

Task	e_i	$P_{i,0}$	$P_{i,min}$	$P_{i,max}$	k_i
T_1	10	20	20	25	1
T_2	10	40	40	50	1
T_3	15	70	35	80	1

Schedulability analysis

- We will assume utilization bounds are used for testing schedulability.
  - Depending on the scheduling policy, we know the utilization bound U_b
  - The goal of the elasticity is to ensure that the utilization of the set of tasks does not exceed U_b
  - Of course, we need to scale down task utilizations if the processor utilization threatens to exceed U_b (for instance, when we add a new task)
  - How do we scale down task utilizations?

An example

Task	e_i	$P_{i,0}$	$P_{i,min}$	$P_{i,max}$	k_i
T_1	10	20	20	25	1
T_2	10	40	40	50	1
T_3	15	70	35	80	1

- Is this task set schedulable using EDF?
  - If each task were to use its nominal period then:
    - U = 10/20 + 10/40 + 15/70 = 0.964 < 1.
  - To improve the QoS for T_3, we would like to use a period of 50 (between 35 and 80):
    - U = 10/20 + 10/40 + 15/50 = 1.05 > 1.
  - We could adjust the periods of T_1 (set to 22) and T_2 (set to 45):
    - U = 10/22 + 10/45 + 15/50 = 0.977.

The adjustment seems *ad hoc*. Can we systematically adjust task periods?

Period adjustment (rate adaptation): **Compression**

- At time instant t, suppose we have an overload condition so that the system utilization is > U_b
  - Need to *compress* task utilizations (increase periods) to bring system utilization down to U_b
- At time instant t, let us suppose that task T_i is operating with period P_i
  - Some of the tasks could be operating at the highest possible period $P_i = P_{i,\max}$ (slowest possible rate)
  - We cannot increase the periods of these tasks
  - Denote this *set of tasks* by M

$$U_M = \sum_{T_i \in M} \frac{e_i}{P_{i,max}}$$

If the utilization bound is U_b, then the remaining tasks – those not in M – cannot have a combined utilization greater than U_b-U_M.

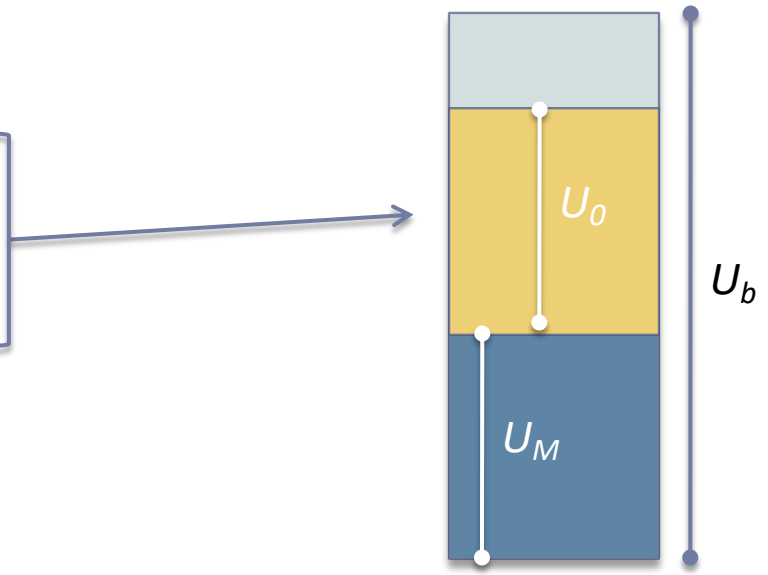
Period adjustment (rate adaptation): **Compression**

- Let the set of tasks with variable/adjustable periods be V.
- U_0 is the combined nominal utilization of tasks in set V.

If the utilization bound is U_b, then the remaining tasks – those not in M – cannot have a combined utilization greater than U_b-U_M.

$$U_0 = \sum_{T_i \in V} \frac{e_i}{P_{i,0}}$$

- If $U_0+U_M=U_b$, we set the periods of all tasks in V to their nominal periods.
- If $U_0 + U_M < U_b$, we set the periods of all tasks in V to their nominal period, and then we use the excess U_b-(U_M-U_0) to improve the QoS for some tasks
- If $U_0+U_M > U_b$, we need to reduce the QoS for some tasks in V below their nominal rate



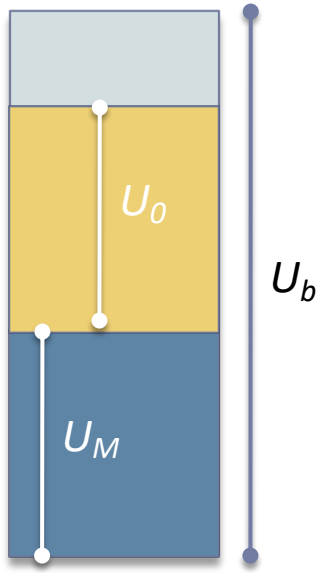
Period adjustment: Compression with excess

- If $U_0 + U_M < U_b$, we can improve the QoS for some tasks beyond their nominal rates [but most likely the net result is that some of these tasks will still suffer QoS degradation compared to their current operating frequencies]

$$U_0 = \sum_{T_i \in V} \frac{e_i}{P_{i,0}}$$

- How do we adjust task utilizations?
  - Use the elasticity coefficients, k_i.

$$U_M = \sum_{T_i \in M} \frac{e_i}{P_{i,max}}$$



- The excess capacity is $U_b - (U_M + U_0) > 0$. For simplicity we distribute this among tasks in V.

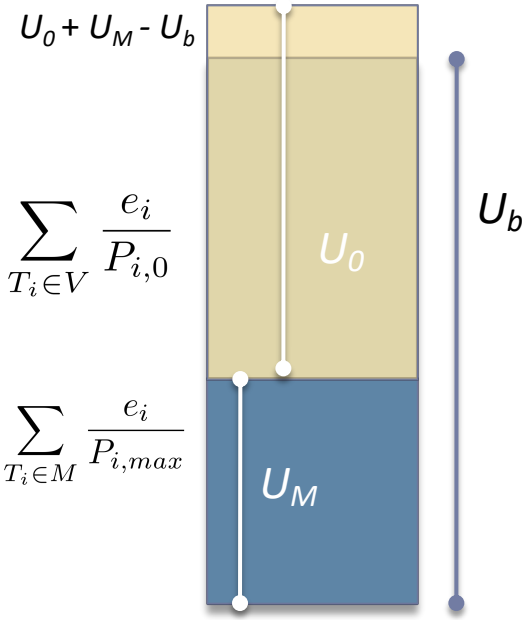
$$\forall T_i \in V : U_i = U_{i,0} + (U_b - U_M - U_0)\frac{k_i}{\sum_{T_j \in V} k_j}$$

Period adjustment (rate adaptation): **Compression**

- What if $U_0 + U_M > U_b$? (cannot even give U_0 to variable tasks)
- We need to reduce the QoS for some tasks in V below their nominal rate after initially giving them U_0
- Note: $U_b - U_M - U_0 < 0$
- We have to *remove* a total utilization of $(U_0 + U_M - U_b)$ from the tasks in V when each operates at its nominal period
  - proportionately to their relative elasticity

$$U_0 = \sum_{T_i \in V} \frac{e_i}{P_{i,0}}$$

$$U_M = \sum_{T_i \in M} \frac{e_i}{P_{i,max}}$$



$$\forall T_i \in V : U_i = U_{i,0} + (U_b - U_M - U_0)\frac{k_i}{\sum_{T_j \in V} k_j}$$

Period adjustment: Compression

- What do we do if the adjustment causes a task T_i to have a period greater then $P_{i,max}$?

  - We have to set the period of that task to $P_{i,max}$,
  - Remove the task from set *V*,
  - Add this task to set *M*,
  - And try to adjust the periods once more for the task in *V*.

Detailed algorithm

Elastic task model for adaptive rate control. Buttazzo, Lipari and Abeni. RTSS 1998.

Algorithm Task_compress(Γ, U_d) {

$U_0 = \sum_{i=1}^{n} C_i/T_{i_0}$;
$U_{min} = \sum_{i=1}^{n} C_i/T_{i_{max}}$;
if ($U_d < U_{min}$) return INFEASIBLE;

do {

$U_f = E_v = 0$;
for (*each* τ_i) {
if (($e_i == 0$) or ($T_i == T_{i_{max}}$))
$U_f = U_f + U_i$;
else $E_v = E_v + e_i$;
}

$ok = 1$;
for (*each* $\tau_i \in \Gamma_v$) {
if (($e_i > 0$) and ($T_i < T_{i_{max}}$)) {
$U_i = U_{i_0} - (U_0 - U_d + U_f)e_i/E_v$;
$T_i = C_i/U_i$;
if ($T_i > T_{i_{max}}$) {
$T_i = T_{i_{max}}$;
$ok = 0$;
}
}
}

} **while** ($ok == 0$);
return FEASIBLE;
}

Period adjustment: **Decompression**

- At time instant *t*, suppose overload condition improves so that the system utilization is < U_b
  - Need to *decompress* task utilizations (decrease periods) to bring system utilization up to U_b
- At time instant t, let us suppose that task T_i is operating with period $P_i = P_i(t)$
  - Let *C* be the set of tasks that have their utilizations compressed
    - $C = \{ T_i: P_i > P_{i,0} \}$
  - We wish to decompress tasks in *C*
  - Let the set of remaining tasks be *A*
    - *A*: "uncompressed" tasks; $A = \{ T_i: P_i <= P_{i,0} \}$

$$U_C = \sum_{T_i \in C} \frac{e_i}{P_i}$$

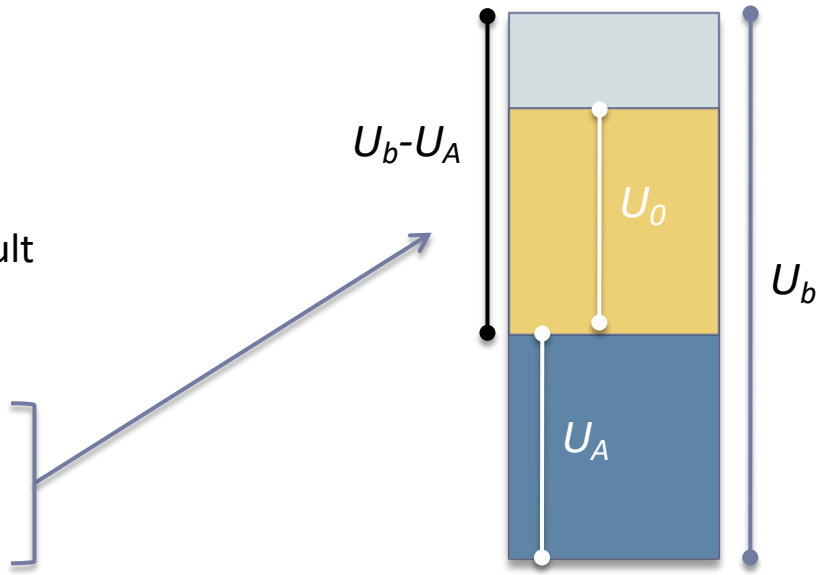
Period adjustment: **Decompression**

- U_0 is the combined nominal utilization of tasks in C.
- When decompressing tasks in C, we do not want to degrade the QoS of the tasks in A → *We keep* U_A *fixed*

$$U_0 = \sum_{T_i \in C} \frac{e_i}{P_{i,0}}$$

If the utilization bound is U_b, then the tasks in C cannot have a combined utilization greater than U_b-U_A.

- If $U_0+U_A=U_b$, we set the periods of all tasks in C to their nominal periods.
- If $U_0+U_A > U_b$, we cannot set all periods in C to nominal [will set to below nominal, but still net result might be QoS improvement compared to current operating frequencies]
- If $U_0 + U_A < U_b$, we set the periods of all tasks in C to nominal, and then we use the excess (U_b-U_A-U_0) to further improve QoS for some tasks



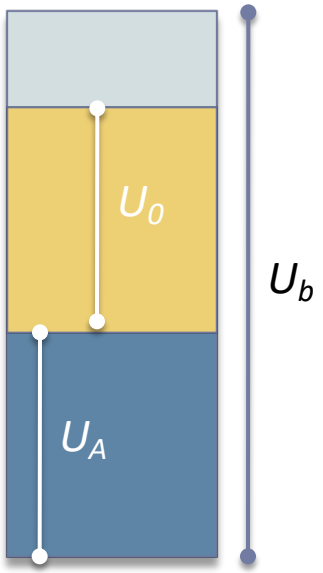
Period adjustment: **Decompression**

- If $U_0 + U_A < U_b$, we can improve the QoS for some tasks beyond their nominal rates

$$U_0 = \sum_{T_i \in C} \frac{e_i}{P_{i,0}}$$

- How do we adjust task utilizations?
  - Use the elasticity coefficients, k_i.

$$U_A = \sum_{T_i \in A} \frac{e_i}{P_i}$$



- The excess capacity is $U_b - (U_A + U_0) > 0$. For simplicity we distribute this among tasks in C.

$$\forall T_i \in C \ : U_i = U_{i,0} + (U_b - U_A - U_0)\frac{k_i}{\sum_{T_j \in C} k_j}$$

The elasticity analogy

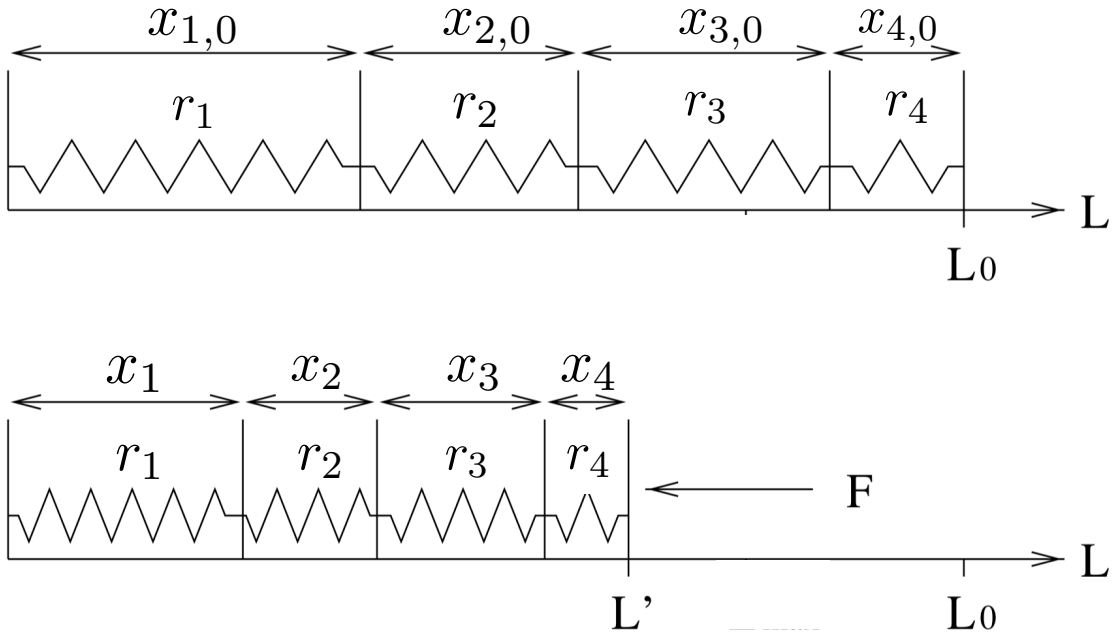
Do not worry about the terminology here.

k_i (task elasticity) $\longleftrightarrow r_i = 1/k_i$ (spring rigidity)

$$U_i \longleftrightarrow x_i$$

$$U = \sum_{i=1}^{n} U_i \longleftrightarrow L = \sum_{i=1}^{n} x_i$$

$$U_0 = \sum_{i=1}^{n} U_{i,0} \longleftrightarrow L_0 = \sum_{i=1}^{n} x_{i,0}$$



For equilibrium: $F = (x_{i,0} - x_i) r_i \ \ \forall i$

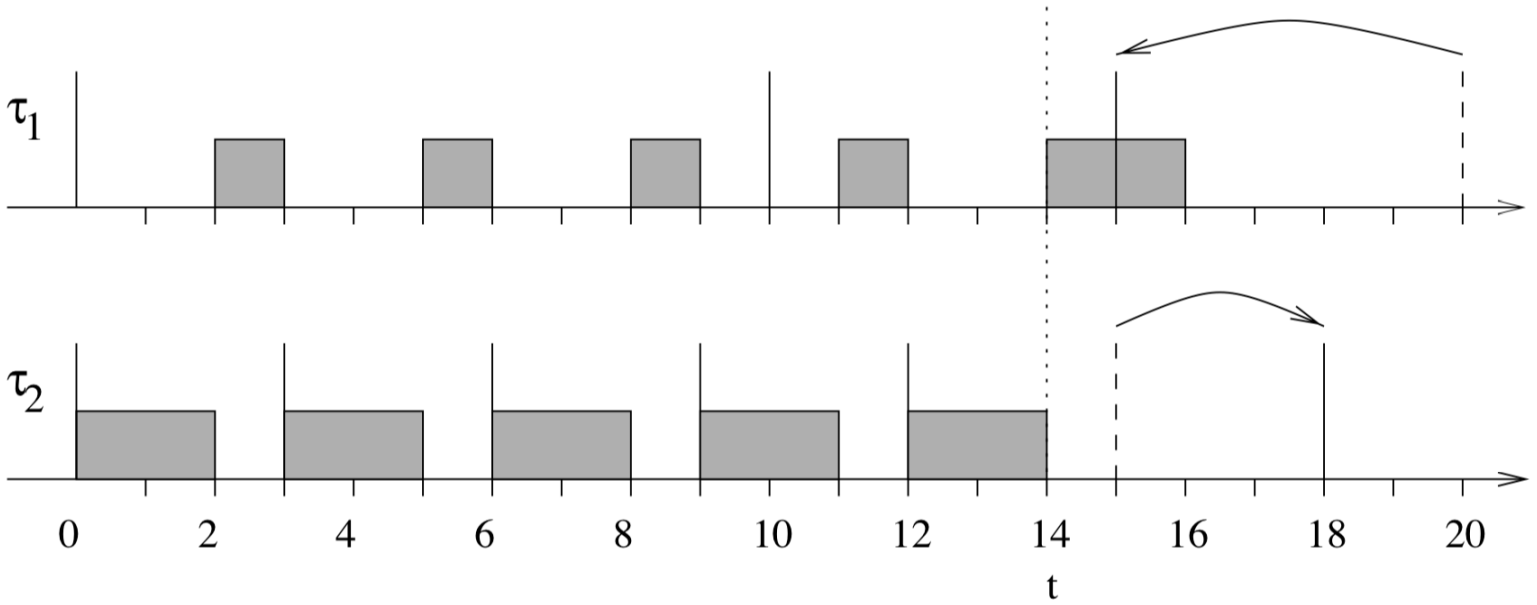
$$\forall i \ \ x_i = x_{i,0} - (L_0 - L') \frac{\left(\frac{1}{\sum_{j=1}^{n} \frac{1}{r_j}} \right)}{r_i}$$

When do we compress/decompress?

- Can we increase or decrease the period of a task at any time instant?
  - Periods can be increased at any time (immediately).
  - Periods can be decreased (utilization increased) only at the next release time of the task.
  - [If you want the detailed proof, consult the reference article.]

Why decompress at specific instants?

- Originally, U = 3/10 + 2/3 = 0.9666 < 1.
- After changing *both* periods at t=14: U = 3/5 + 2/6 = 0.9666 < 1.
- If the period of τ_1 is changed at once (at t=14), τ_1 misses a deadline.



$$P_1 = 10 \rightarrow 5$$

$$P_2 = 3 \rightarrow 6$$

Highlights

- The elastic task model
  - Allows period (rate) adaptation in a real-time system.
  - Analogous to physical spring systems.
  - Like skip-based scheduling (job dropping), elastic scheduling is suitable for multimedia applications.
  - Also useful in manufacturing applications.
    - Silicon wafers processed in a semiconductor plant.
    - We can reduce the rate of processing but we cannot skip a wafer.

What you should know

- How do we adjust the periods of tasks?
- When can we adjust the periods of tasks?
- Period changes are performed, typically, when a new task is added to the system (may need to compress tasks) or when a task is removed (can decompress remaining tasks).
- The choice of which soft real-time model to adopt depends on the application and the expected behaviour.

Tasks with variable execution times

Ref: Algorithms for scheduling imprecise computations. Liu, et al. IEEE Computer, vol. 25, no. 5, May 1991.

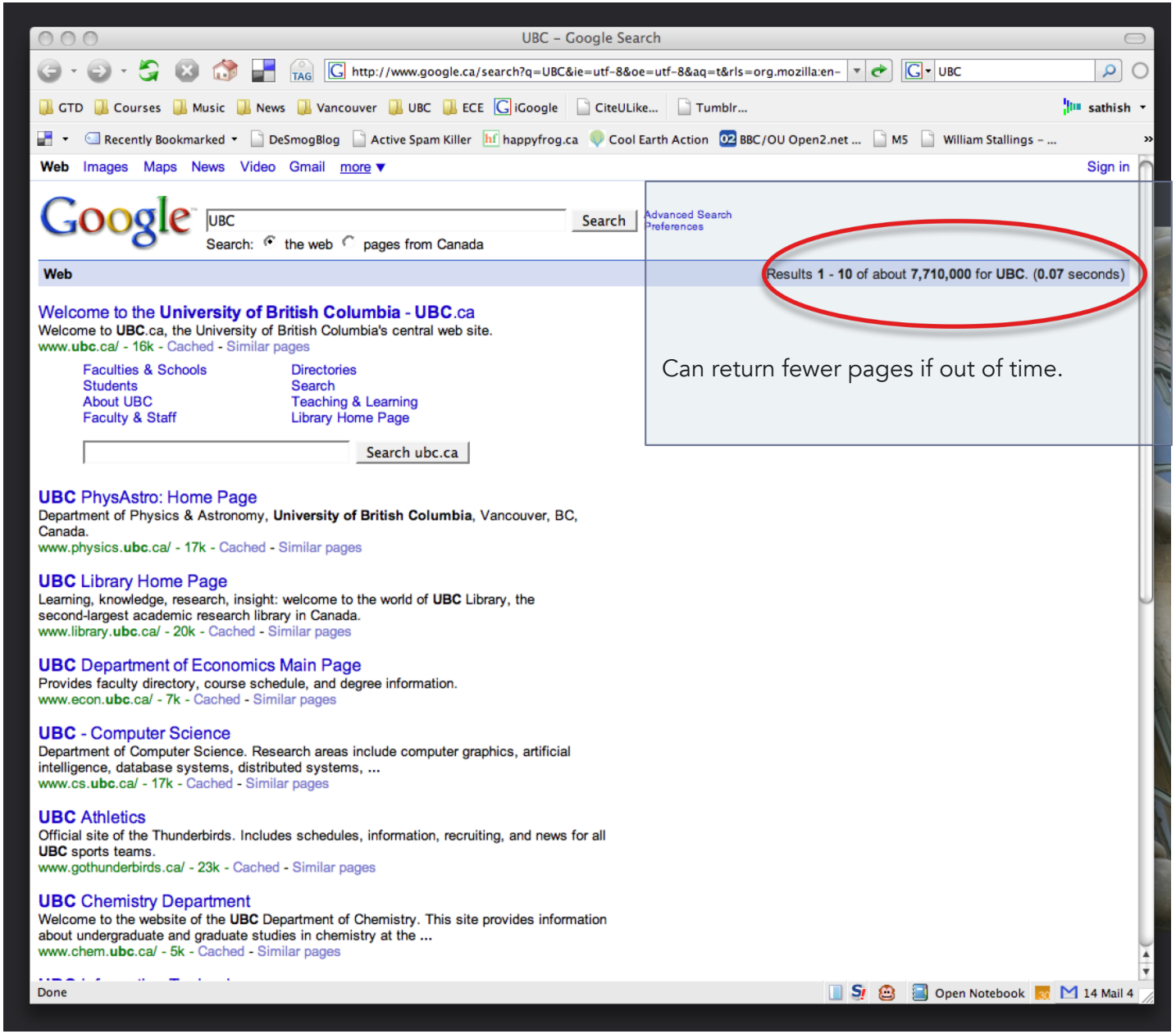
Lecture overview

- Elastic scheduling allows us to adjust task periods at times of overload

- In this lecture, we will examine a second approach
  - Imprecise computation, which trades accuracy of computation for schedulability
  - Assumes that the accuracy of computation is related to the execution time allotted to the task

Why is the imprecise computation model useful?

- The case for imprecise computations
- For specific applications, approximate results may suffice
  - Image processing (fuzzy frames)
  - Object tracking (location *estimates* rather than accurate location)
  - Artificial intelligence algorithms typically perform a search (shorter search time results in a lower quality result)
    - Google's search is not a bad example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How do we achieve the imprecise computation model?

- We assume that tasks are iterative
  - The number of iterations suggests quality (fewer iterations imply lower quality)
  - Terminate the task after a few iterations with acceptable quality
  - Tasks might have a mandatory part
    - Any computation beyond this mandatory portion improves the quality but the task is meaningless without the mandatory portion

More examples

- Radar tracking: Get estimated target locations in a timely fashion rather than accurate information that is too late to be of use
- Multimedia systems: Transmit a low quality image in time rather than missing the deadline, e.g., to meet the 24 fps requirement
- Control systems: Produce an approximate result by a control law as long as the controlled system, e.g., cruise control system, remains stable

Scheduling imprecise tasks

- Tasks are periodic with known period (P_i) and an execution time range [$e_{i,min}$, $e_{i,max}$]
- $e_{i,min}$ represents the mandatory execution required by each task
- The accuracy of a task is highest when each job executes for $e_{i,max}$ time units

- First step
  - Ensure that the mandatory portions of all tasks are schedulable
  - If tasks are scheduled with rate monotonic priorities, we can use the Liu & Layland bound

$$\sum_{i=1}^{n} \frac{e_{i,min}}{P_i} \leq n(2^{1/n} - 1)$$

Example task set

Task	e_{min}	e_{max}	P
T1	2	7	10
T2	4	8	25
T3	6	10	30

$$\frac{2}{10} + \frac{4}{25} + \frac{6}{30} = 0.56 < 3(2^{1/3} - 1)$$

The mandatory portion of this task set is schedulable.

Scheduling imprecise tasks

- Once we have ensured that the mandatory portions are schedulable, we have many options
- The loss in accuracy for each task can be specified by some error function *F*
  - Let e_i be the execution time of T_i; then the error is some function of e_i and $e_{i,max}$
- We could decide to minimize the (weighted) sum of errors
  - How do we find the values for $\{e_i\}$ that minimize the error?

Objective function

Subject to constraints

Mathematical program formulation

$$\min \sum_{i=1}^{n} w_i F(e_i, e_{i,max})$$

Relative importance of the tasks

$$e_i \geq e_{i,min}, \forall i$$

$$e_i \leq e_{i,max}, \forall i$$

Constraints on execution time

$$\sum_{i=1}^{n} \frac{e_i}{P_i} \leq n(2^{1/n} - 1)$$

Schedulability constraint

A simple error function

- $F(e_{i,\max}, e_i) = e_{i,\max} - e_i$ is a simple error function

- Let us also assume that the weight of each task is 1.

$$\min \sum_{i=1}^{n} (e_{i,\max} - e_i)$$
$$\text{subject to:}$$
$$e_{i,\min} \leq e_i \leq e_{i,\max} \; \forall i$$
$$\sum_{i=1}^{n} \frac{e_i}{P_i} \leq n(2^{1/n} - 1)$$

A simple linear program

Solving the linear program

$$\min \sum_{i=1}^{n} (e_{i,\max} - e_i)$$

$$\text{subject to:}$$

$$e_{i,\min} \leq e_i \leq e_{i,\max} \; \forall i$$

$$\sum_{i=1}^{n} \frac{e_i}{P_i} \leq n(2^{1/n} - 1)$$

Task	e_{min}	e_{max}	P
T1	2	7	10
T2	4	8	25
T3	6	10	30

Greedy algorithm

- Intuition: increasing the execution time of a task by *x* decreases error by *x*.
- The task with the largest period has the least utilization penalty.
  - Determine $1/P_i$ for each task. This is the utilization penalty for increasing the execution time of T_i by 1 time unit.
  - **Simplifying assumption**: execution times are integers (or can be represented as integers).
- Start off by setting the execution times of all tasks to their minimum
- Increase the execution time of the task with the smallest utilization penalty to the maximum extent possible.
- Then move to the task with the next smallest utilization penalty.
- Repeat until the utilization bound is reached or no further progress is possible.

Solving the linear program

Task	e_{min}	e_{max}	P
T1	2	7	10
T2	4	8	25
T3	6	10	30

- The mandatory utilization is 0.56. The Liu & Layland bound for 3 tasks is 0.7797.
- The utilization penalty for T3 is 1/30=0.0333, for T2 is 1/25=0.04, for T1 is 1/10=0.1.

- We can increase the execution time of T3 by 4 units at a cost of $4 \times 0.0333 = 0.1333$
  - The total utilization now becomes 0.6933.
- We can increase the execution time of T2 by 2 units at a cost of $2 \times 0.04 = 0.08$
  - The total utilization now becomes 0.7733.
- We cannot make any further increases without violating the utilization bound.
  - Thus we stop.
- *An optimal solution is e_1=2, e_2=6, e_3=10.*

Highlights

- We examined another task model for providing good QoS for soft real-time systems.
- The imprecise computation model trades off execution time for accuracy.
- Different applications need different approaches to obtaining good *quality of service.*
- For each task parameter, we have studied some mechanism by which they can be controlled and the entire system behaves in a "predictable" manner.

What you should know

- What is the imprecise computation model?
- Why, and where, is it useful?
- How do you decide execution times under this model?
  - Solve for the simple case of linear error function.

- Ponder: Can we use elastic scheduling when tasks have variable execution times? What should the parameters for such task sets be?