Periodic task scheduling

Static priorities

- ★Better utilization bounds
- ★Deadlines less than periods
- ★Exact test for schedulability

Quick review

- Why is rate monotonic scheduling optimal (among static priority policies)?
    - **Critical instant theorem:** The worst-case execution time of a job when tasks are scheduled with fixed priorities occurs when jobs belonging to all tasks release at the same instant
    - It is sufficient, then, to verify that the job that is released at the critical instant meets its deadline
    - In this worst case, rate monotonic scheduling is optimal (easy to see; if tasks are feasibly scheduled in any other order, swap based on deadlines)
- Utilization bound and optimality of EDF
    - The utilization bound is 1 (or 100%)
    - EDF is optimal because no policy can do better (may do as well but not better)

Exercise

Know Your Worst Case Scenario

- Consider a periodic system of two tasks
- Let $U_i = C_i/P_i$ (for $i = 1, 2$)
- What is the maximum value of $\prod_i (1 + U_i)$ for a schedulable system?
- **Motivation:** There may be other functions of a task set rather then just utilization that also indicate schedulability.

Hyperbolic bound for RM

worst case conditions
for schedulability
of **2** tasks under RM

Critically schedulable

$$\begin{cases} C_1 = P_2 - P_1 \\ C_2 = P_1 - C_1 = 2P_1 - P_2 \\ U_1 + 1 = \dfrac{C_1}{P_1} + 1 = \dfrac{C_1 + P_1}{P_1} = \dfrac{P_2}{P_1} \\ U_2 + 1 = \dfrac{C_2}{P_2} + 1 = \dfrac{C_2 + P_2}{P_2} = 2\dfrac{P_1}{P_2} \\ \prod_i (U_i + 1) = 2 \end{cases}$$

Schedulable

$$\prod_i (U_i + 1) \leq 2$$

Hyperbolic bound

Solutions

Critically schedulable

$$\begin{cases} C_1 = P_2 - P_1 \\ C_2 = P_1 - C_1 = 2P_1 - P_2 \\ U_1 + 1 = \dfrac{C_1}{P_1} + 1 = \dfrac{C_1 + P_1}{P_1} = \dfrac{P_2}{P_1} \\ U_2 + 1 = \dfrac{C_2}{P_2} + 1 = \dfrac{C_2 + P_2}{P_2} = 2\dfrac{P_1}{P_2} \\ \prod_i (U_i + 1) = 2 \end{cases}$$

Generalizes to task sets with *n* tasks

Schedulable $\prod_i (U_i + 1) \leq 2$

Hyperbolic bound

Hyperbolic bound for **rate monotonic** scheduling

- A set of periodic tasks is schedulable if

$$\prod_i (U_i + 1) \leq 2$$

Hyperbolic bound for rate monotonic scheduling

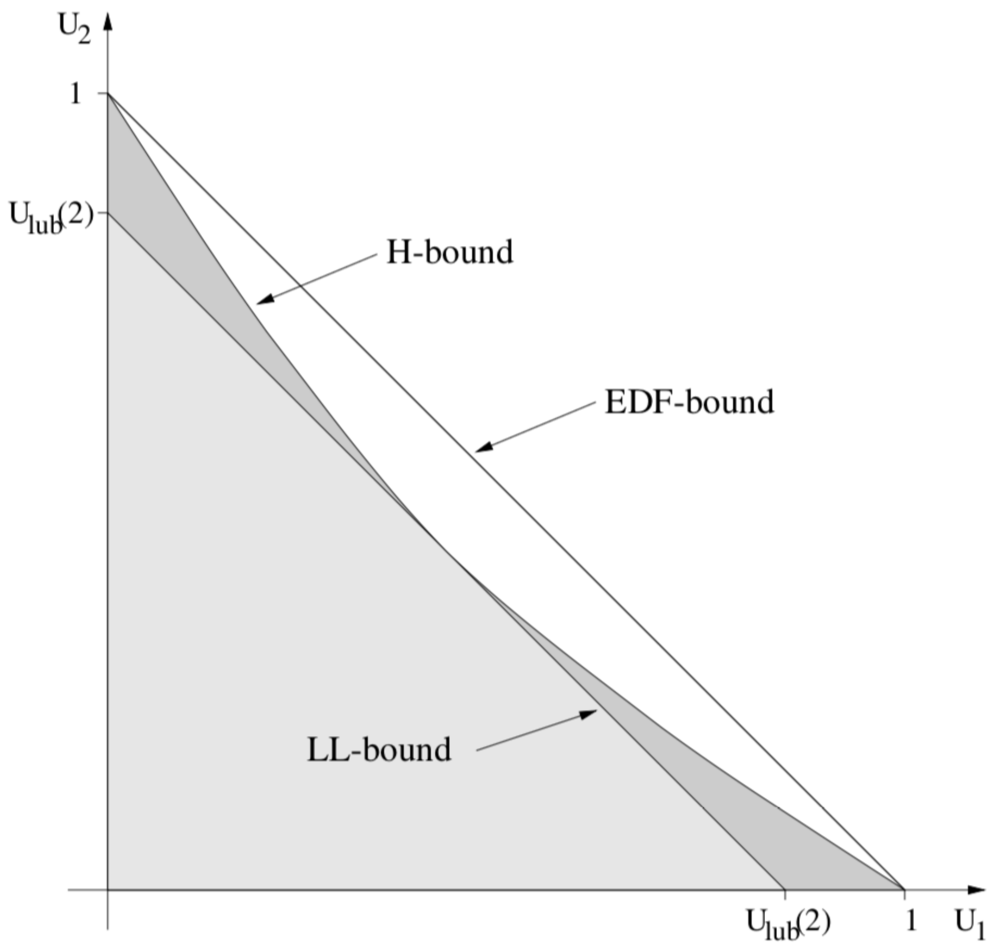
A set of periodic tasks is schedulable if

$$\prod_i (U_i + 1) \leq 2$$

- It is a better bound than the Liu and Layland bound $U \leq n(2^{1/n} - 1)$
- **Example:** consider a system with two tasks such that $U_1 = 0.8$ and $U_2 = 0.1$
  - $U = 0.9 > 0.83$ (**unschedulable** according to the Liu and Layland bound)
  - $(1 + U_1)(1 + U_2) = (1.8)(1.1) = 1.98 < 2$ (**schedulable** according to the hyperbolic bound)

- **Question:** What happens to the hyperbolic bound if task utilizations are equal?

Feasibility regions in U-space

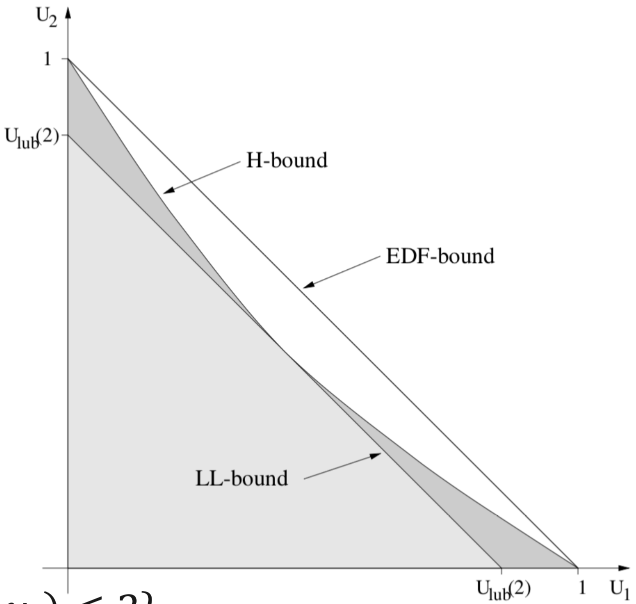


Hyperbolic bound is *tight*

- It is the best possible bound with only knowledge of utilization factors and the number of jobs
- A utilization-based condition $C(u_1, \dots, u_n)$ for a scheduling algorithm is ***tight*** if for every utilization set $(u_1, \dots, u_n)$ with $0 \le u_i \le 1$ for which $C(u_1, \dots, u_n)$ does ***not*** hold, there exists a task set $T_1, \dots, T_n$ with utilizations $u_1, \dots, u_n$ that is ***not*** schedulable by the scheduling algorithm
    - We can construct a task set with the prescribed utilizations (which violate the schedulability condition) that is ***infeasible*** under the given algorithm
- Tightness was proved for H-bound → With the algorithm being RM and $C(u_1, \dots, u_n) \equiv \prod_{i=1}^{n}(1 + u_i) \le 2$
- **Q:** Is the LL bound tight?

How much "better" is Hyperbolic bound relative LL?

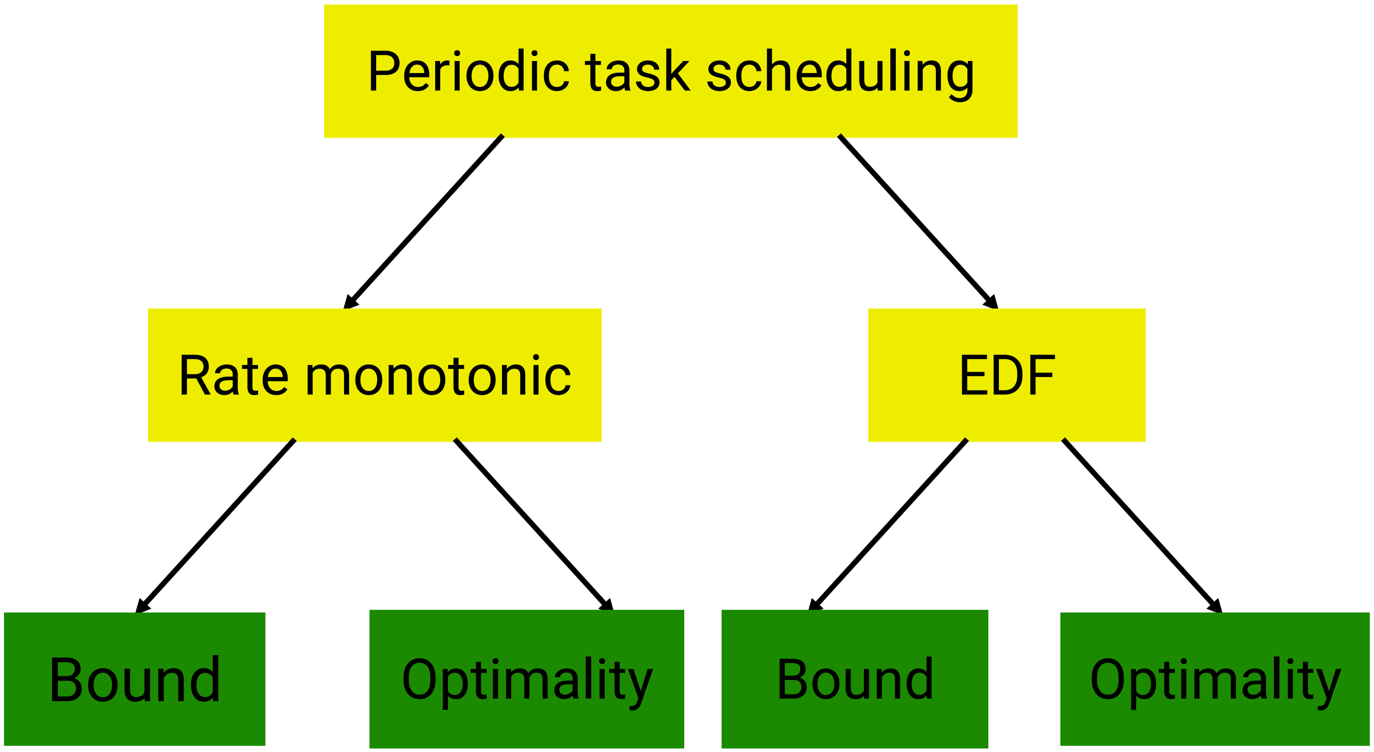
- **How do we measure "better" or the "gain" of H-bound over LL-bound?**



- Consider the utilization space
  - **U-space:** Subset of n-dimensional Euclidean space consisting of vectors $(u_1, \ldots, u_n) \in [0,1]^n$
- Fix number of jobs n
- Volume vol$^n(A)$: n-dimensional Lebesgue measure of (measurable) set $A \subset \mathbb{R}^n$
- Take volume of H-bound region
  - Here need to find vol$^n(H)$, $H = \{u \in \mathbb{R}^n\colon\ u_i \in [0,1], \prod_{i=1}^n (1+u_i) \leq 2\}$
- Take volume of LL-bound region
  - need to find vol$^n(LL)$, $LL = \{u \in \mathbb{R}^n : u_i \in [0,1], \sum_{i=1}^n u_i \leq n\left(2^{1/n} - 1\right)\}$

Asymptotic Gain: $\rho_n = \frac{\text{vol}^n(H)}{\text{vol}^n(LL)} = \sqrt{2} + O(n^{-1})$

Scheduling taxonomy



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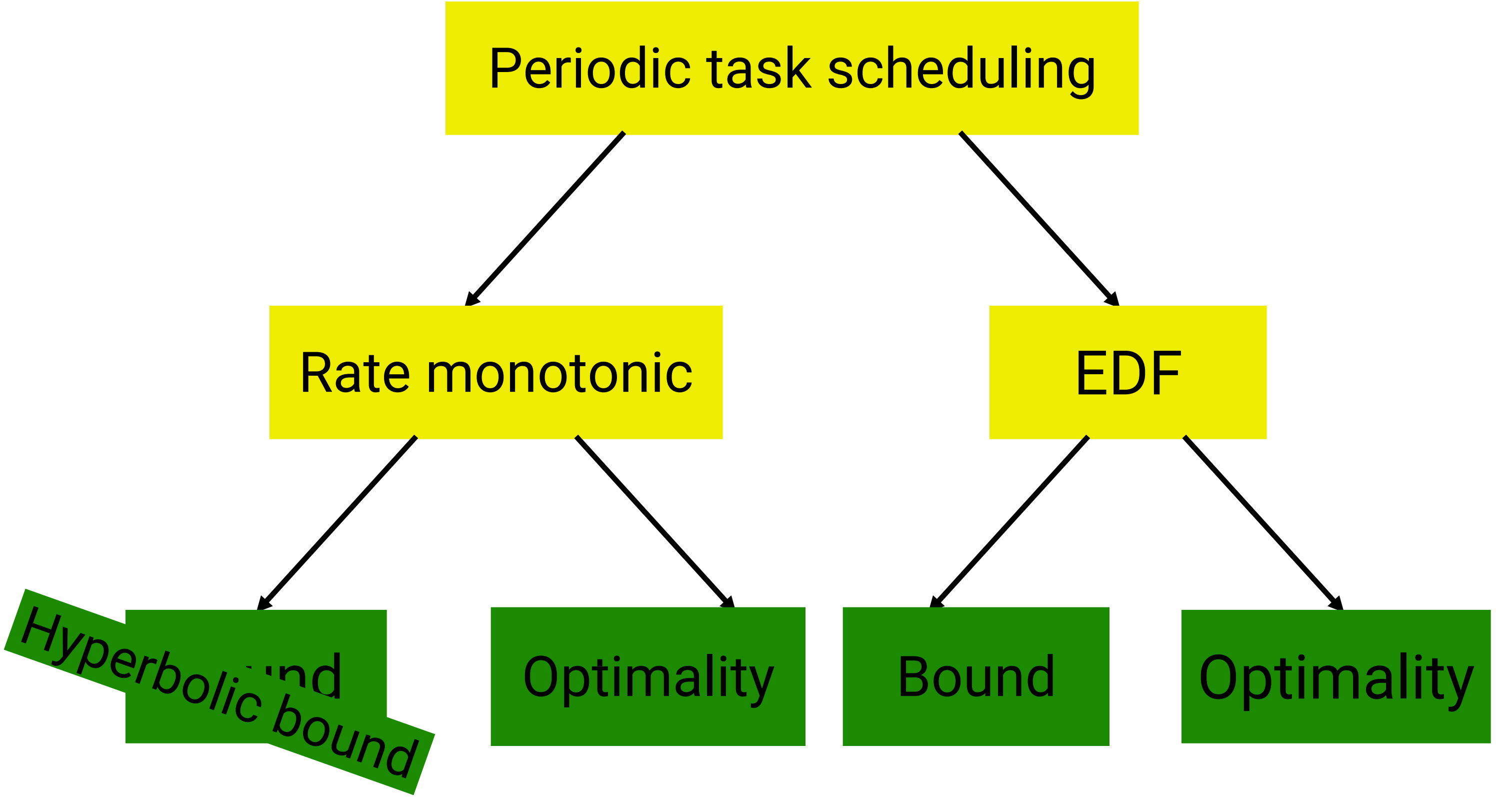

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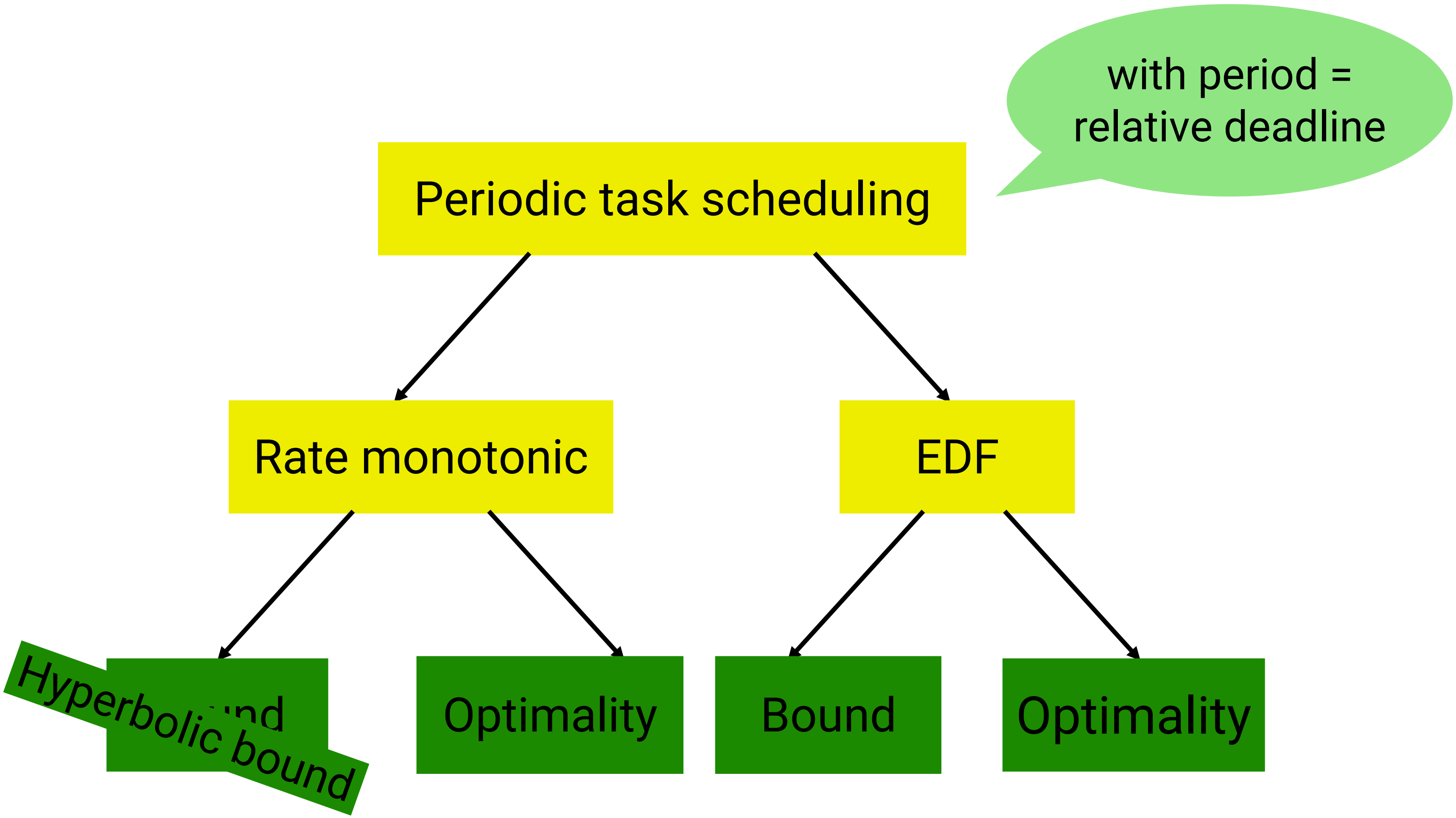

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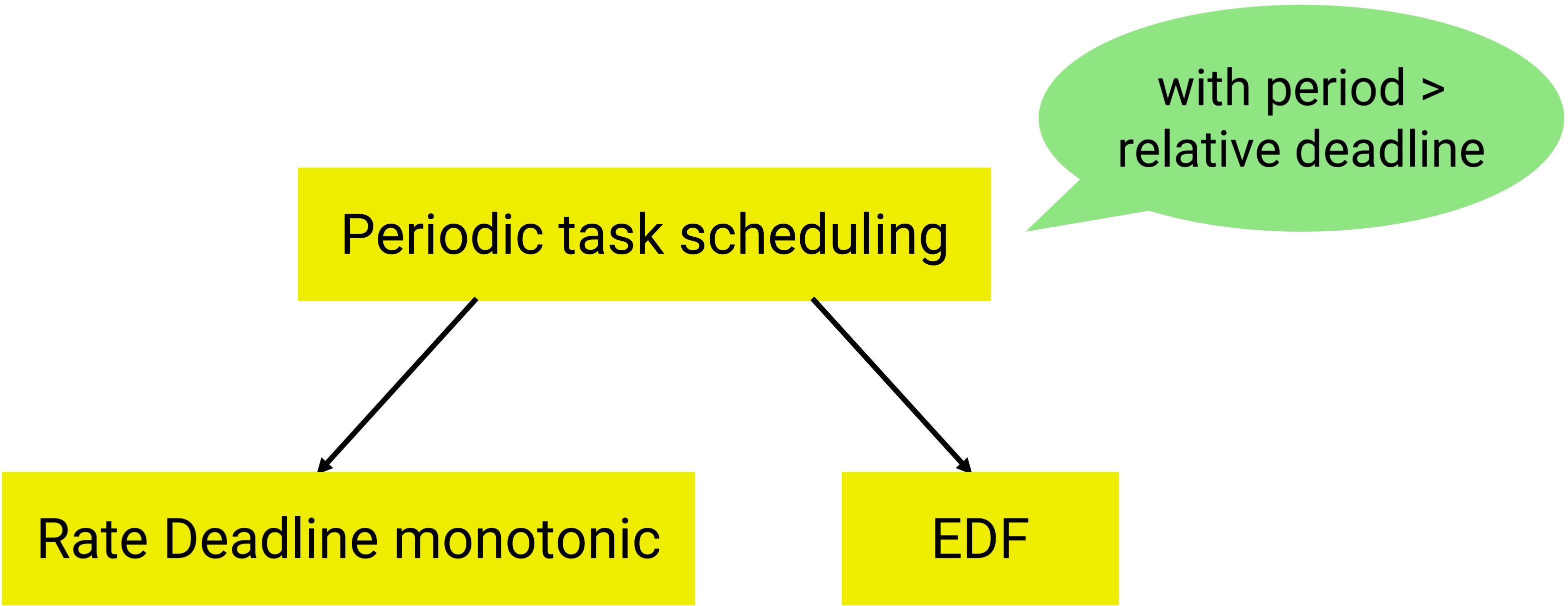
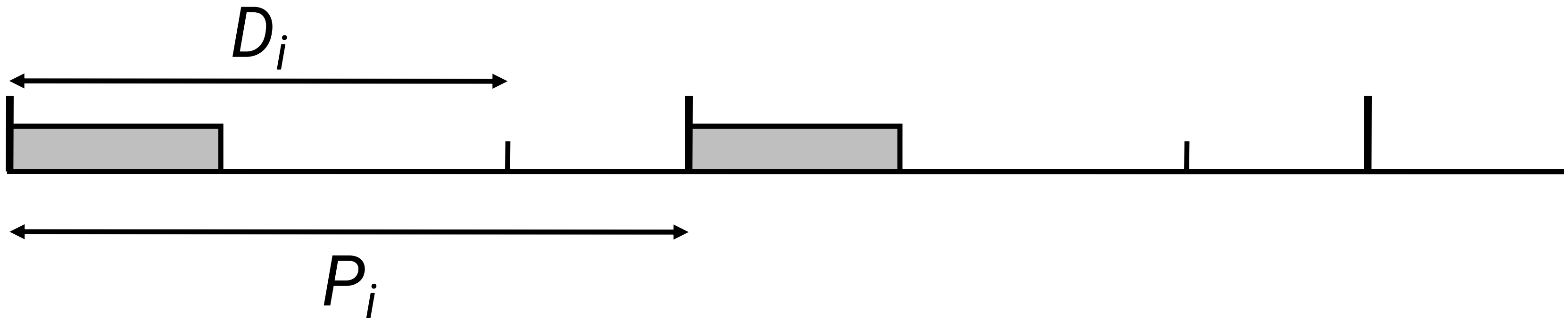


Deadline monotonic scheduling

- Consider a set of periodic tasks where each task, *i*, has a computation time, C_i, a period, P_i, and a relative deadline $D_i < P_i$.



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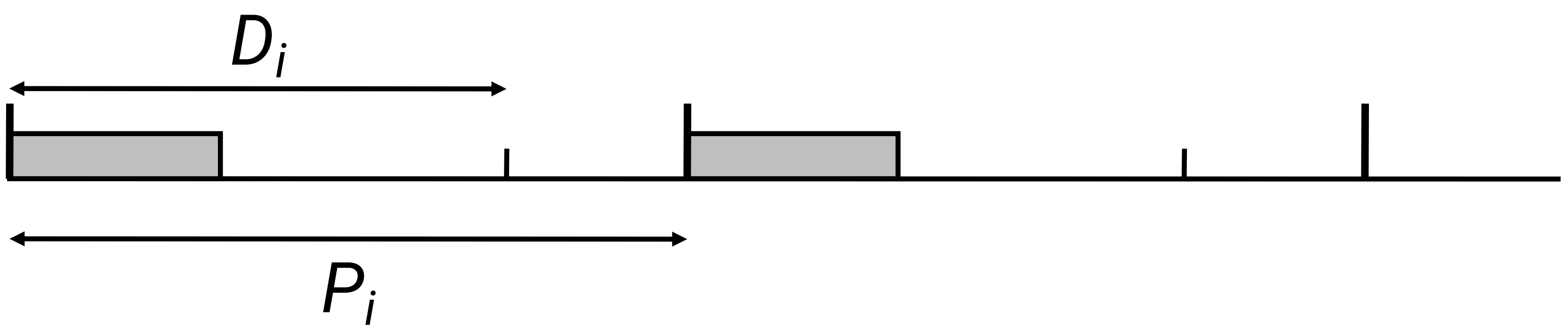

- What is the schedulability condition?

- Can not be worse than when the period of each task is reduced to D_i.

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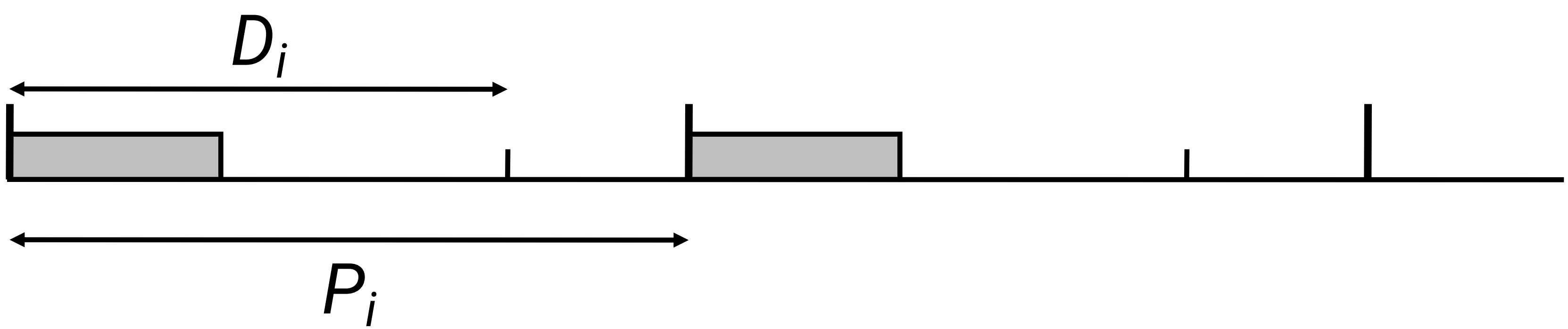

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$$\sum_i \frac{C_i}{D_i} \leq n(2^{1/n} - 1)$$

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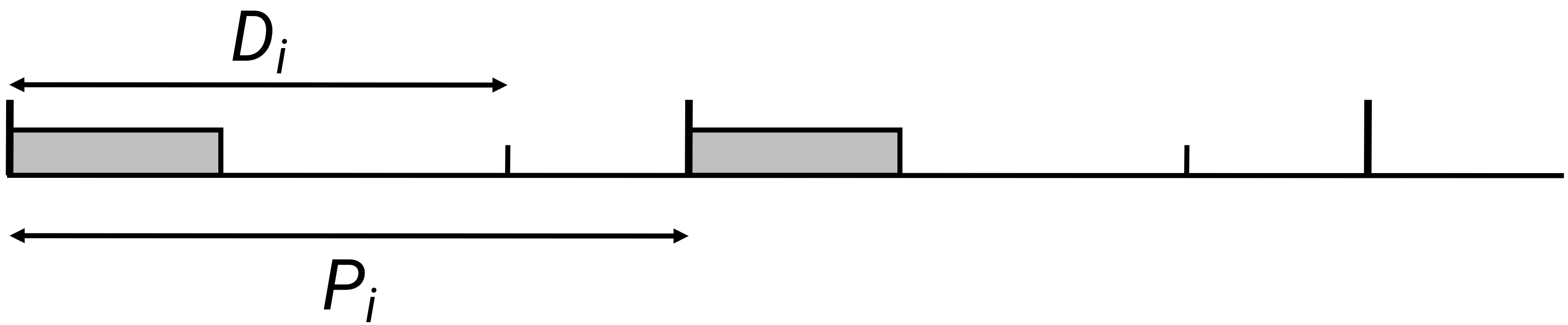

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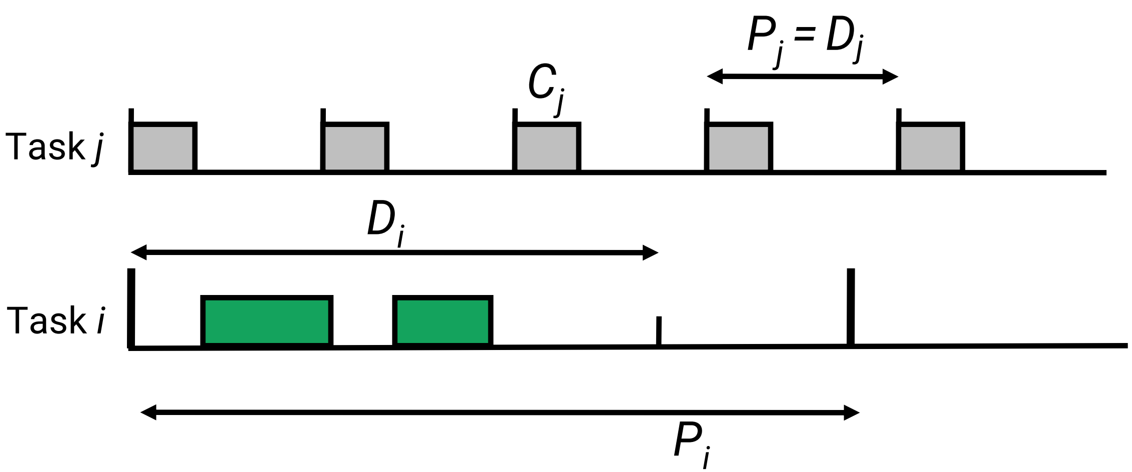
What is the problem?

A better condition for schedulability

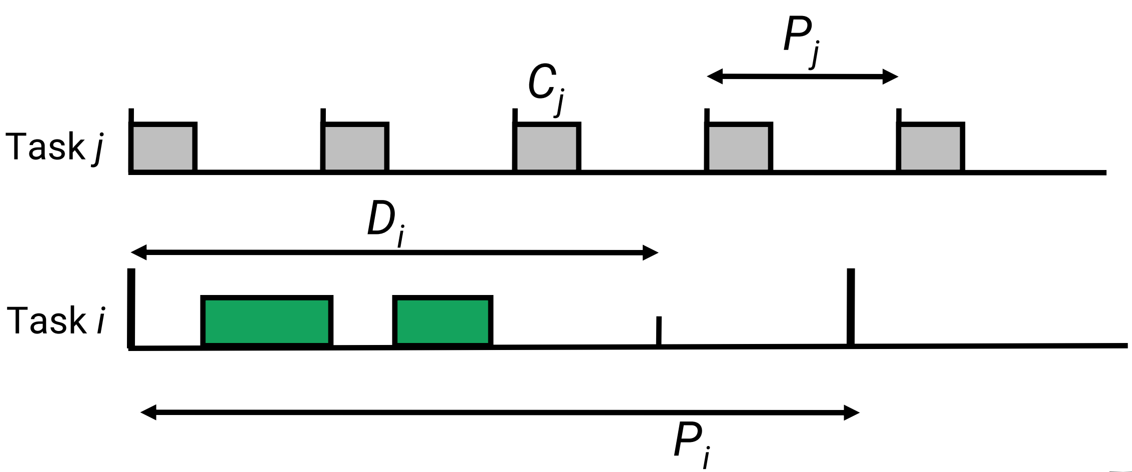
- Worst case interference from a higher priority task, *j*?



A better condition for schedulability

- Worst case <u>interference</u> from a higher priority task, *j*?

Time required by a higher priority task in an interval of length that corresponds to the relative deadline of task *i*.



$$I_i(j) = \left\lceil \frac{D_i}{P_j} \right\rceil C_j$$

Worst case Interference task *j* exercises on task *i*
→ upper bound on total workload *requested* by task *j* during D_i at the critical instant

Number of execution *requests of task j in duration of length D_i assuming critical instant*

A better condition for schedulability

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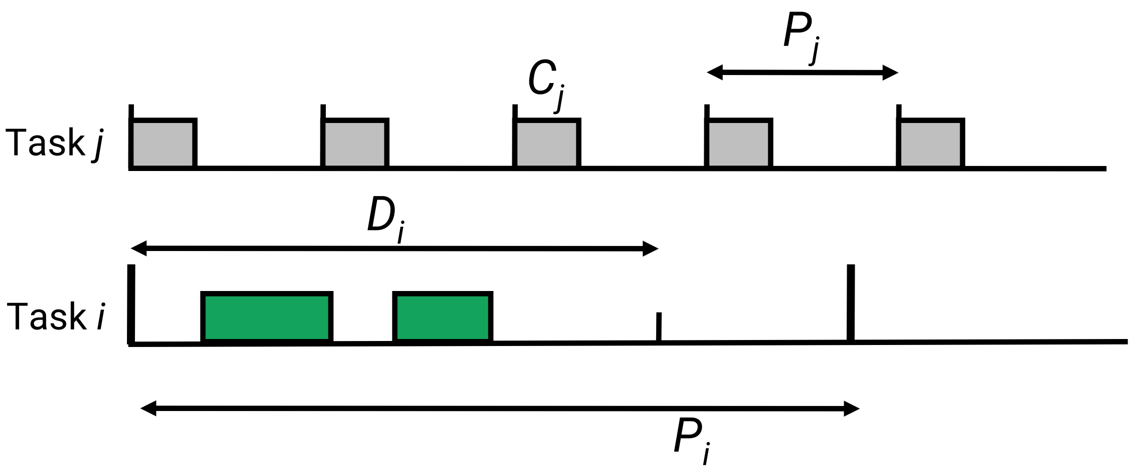

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$$\sum_{j \in \mathrm{hp}(i)} \left\lceil \frac{D_i}{P_j} \right\rceil C_j + C_i \leqslant D_i$$

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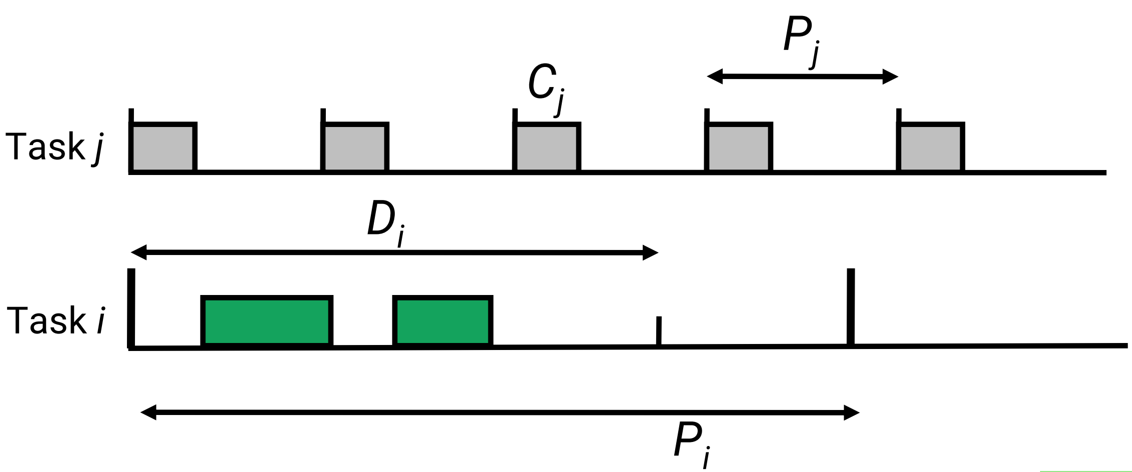

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Interference from higher priority tasks →

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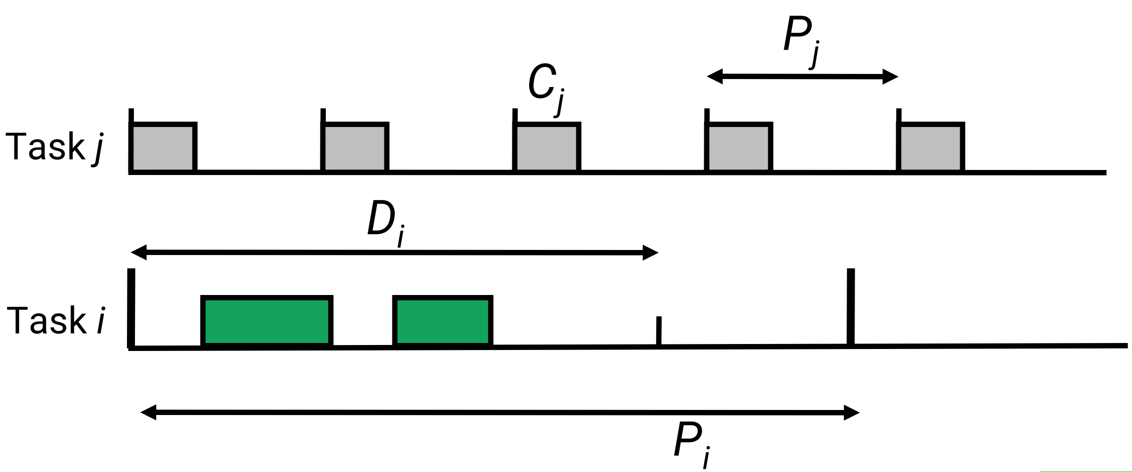
Execution time of T_i

A better condition for schedulability

- Worst case interference from a higher priority task, *j*?

There still is a problem!



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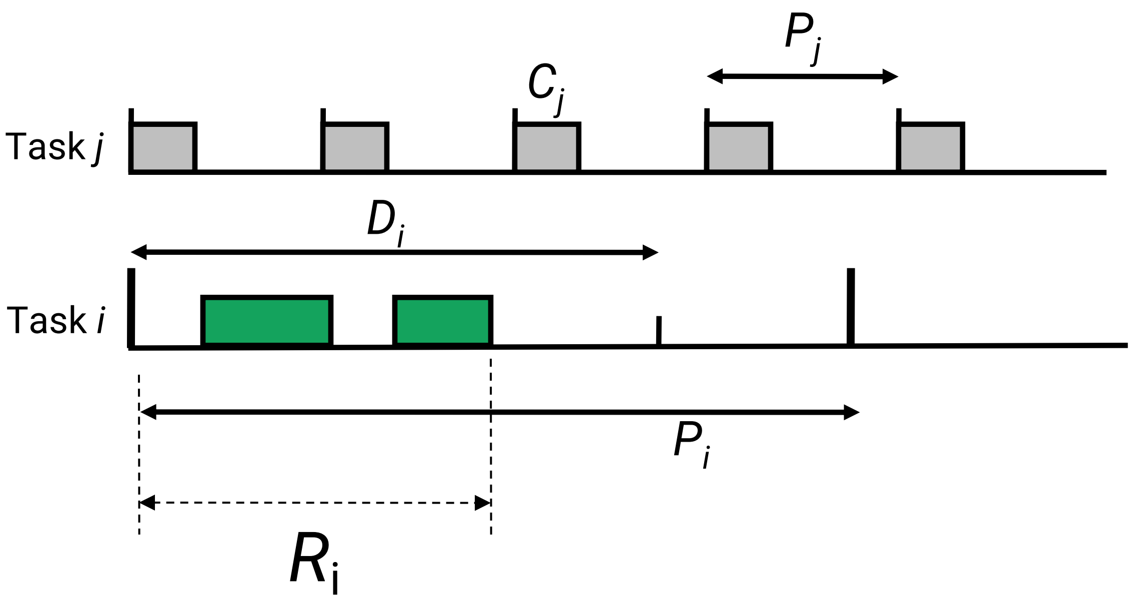
Interference from higher priority tasks →

$$\sum_{j \in \mathrm{hp}(i)} \left\lceil \frac{D_i}{P_j} \right\rceil C_j + C_i \leqslant D_i$$

Execution time of T_i

An exact condition for schedulability

- Interference exists only till a job completes execution, i.e., up to the response time R_i
- Not necessarily up to the relative deadline D_i



$$I_i(j) = \left\lceil \frac{\overset{R_i}{\cancel{D_i}}}{P_j} \right\rceil C_j$$

An exact condition for schedulability

(1) $\left\lceil \frac{R_i}{P_j} \right\rceil C_j$: the *exact* workload interfering with task i in an interval of length R_i starting at the latest critical instant

But $\left\lceil \frac{R_i}{P_j} \right\rceil C_j$ is the workload *requested* by higher priority task τ_j in an interval of length R_i starting at the latest critical instant

And (1) is saying that this workload indeed *completes* by R_i. Why?

Because task *j* has higher priority than task *i* so all instances of task *j* that arrive in interval of length R_i finish before task *i*

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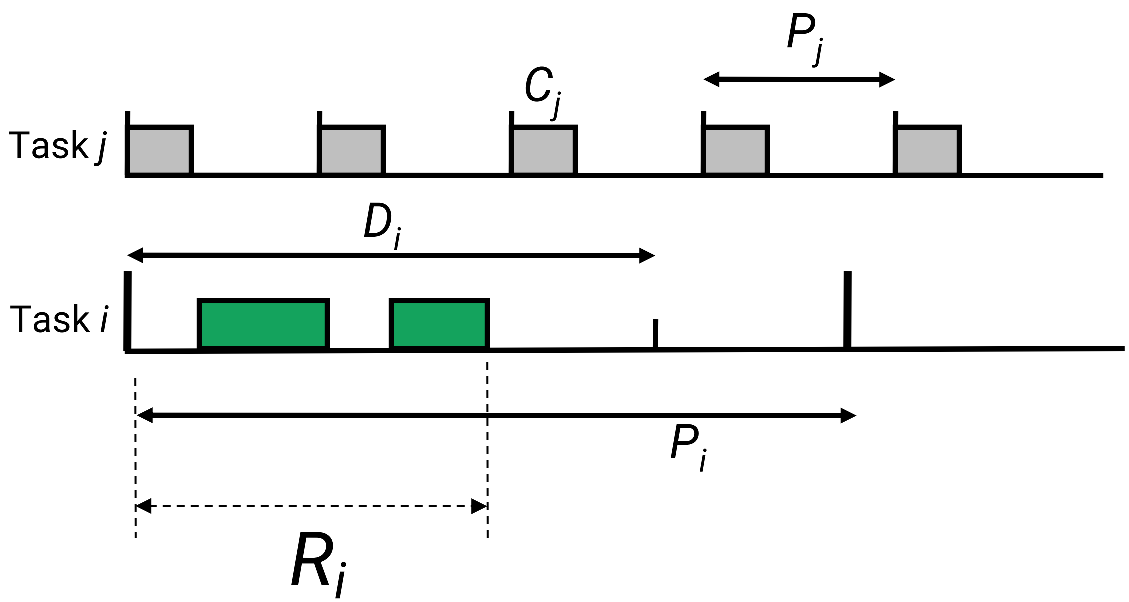
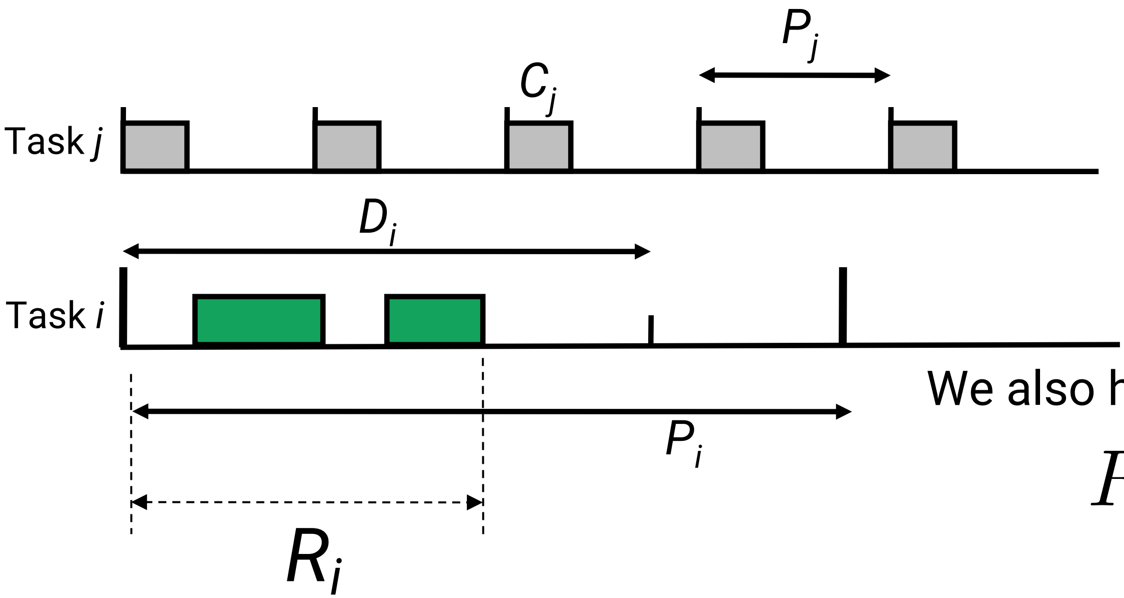


$$I_i = \sum_{j \in \mathrm{hp}(i)} I_i(j) = \sum_{j \in \mathrm{hp}(i)} \left\lceil \frac{R_i}{P_j} \right\rceil C_j$$

Interference on task *i* from all higher priority tasks

An exact condition for schedulability

- Interference exists only till a job completes execution, i.e., up to the response time R_i
- Not necessarily up to the relative deadline D_i



We also have the following relation:

$$R_i = I_i + C_i$$

$$I_i = \sum_{j \in \mathrm{hp}(i)} \left\lceil \frac{R_i}{P_j} \right\rceil C_j$$

An exact condition for schedulability

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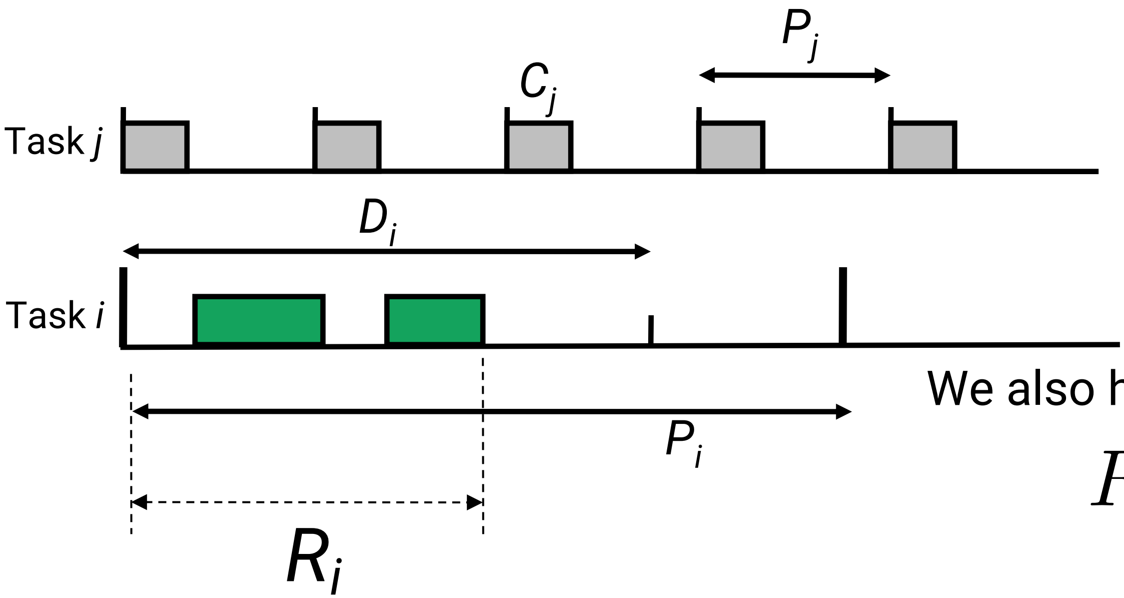


We also have the following relation:

$$R_i = I_i + C_i$$

Solve iteratively for the smallest R_i to satisfy both relations

$$I_i = \sum_{j \in \mathrm{hp}(i)} \left\lceil \frac{R_i}{P_j} \right\rceil C_j$$

Computing Response Time

variable

$$R_i = \underbrace{\sum_{j \in \mathrm{hp}(i)} \left\lceil \frac{R_i}{P_j} \right\rceil C_j + C_i}_{f(R_i)}$$

- What is a solution to this recurrence?
- Need to find a **fixed point**

Computing Response Time

variable

$$R_i = \underbrace{\sum_{j \in \mathrm{hp}(i)} \left\lceil \frac{R_i}{P_j} \right\rceil C_j + C_i}_{f(R_i)}$$

- **What is a solution to this recurrence?**
  - The smallest $t > 0$ such that $t = \sum_{j \in \mathrm{hp}(i)} \left\lceil \frac{t}{P_j} \right\rceil C_j + C_i$ → called *fixed-point*
- Does a solution always exist?
- If so, how can a solution be computed in a finite number of steps? (convergence)

Finding Response Times in a Real-Time System M. Joseph P. Pandya.
***The Computer Journal*, Volume 29, Issue 5, 1 January 1986, Pages 390–395.**

Computing Response Time

$$\text{Recurrence:}\ R_i^{(n+1)} = \underbrace{\sum_{j \in \mathrm{hp}(i)} \left\lceil \frac{R_i^{(n)}}{P_j} \right\rceil C_j}_{I_i^{(n)}} + C_i$$

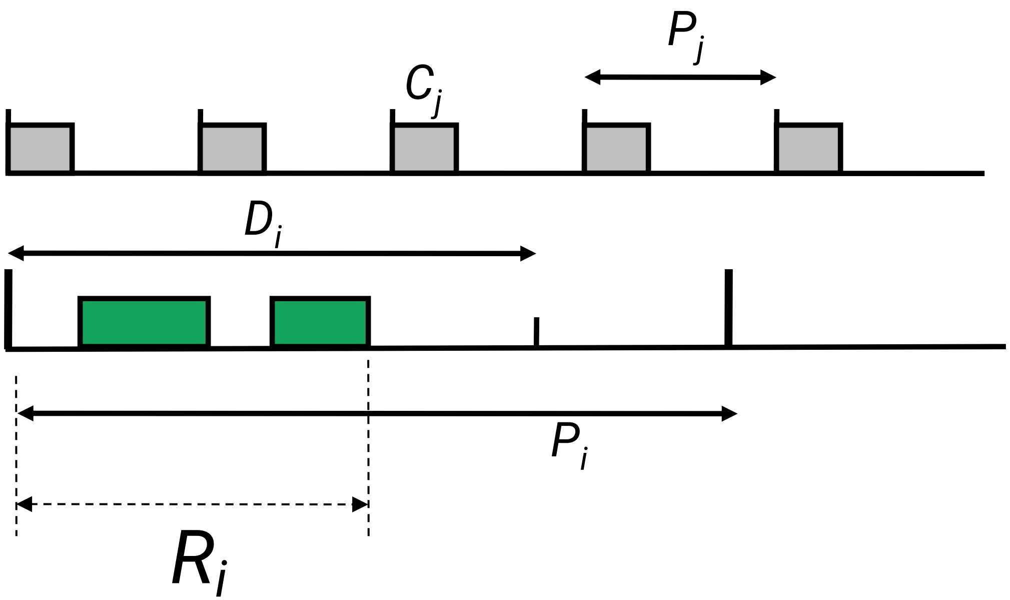
- What is a proper initial value $R_i^{(0)}$?
  - Any *lower* bound on the response time → take $R_i^{(0)} = C_i \text{ or } R_i^{(0)} = \sum_{j \in \mathrm{hp}(i)} C_j$
  - Affects the rate of convergence

Exact Response Time: Solution Existence

- It was shown that recurrence converges if $\sum_{j \in \text{hp}(i)} u_j \leq 1$
- Easy to see that $R_i^{(n+1)} \geq R_i^{(n)}$ →
  - Induction on n + reason about $\left(R_i^{n+1} - R_i^{n}\right)$ + use fact that $x \mapsto \lceil x \rceil$ is increasing
- Stop at first n for which $R_i^{(n+1)} = R_i^{(n)}$
- Recurrence might not converge if $\sum_{j \in \text{hp}(i)} u_j > 1$
  - If only want to know whether or not taskset is schedulable → Terminate as soon as $R_i^{(n)} > D_i$ or $R_i^{(n)} > P_i$

Example

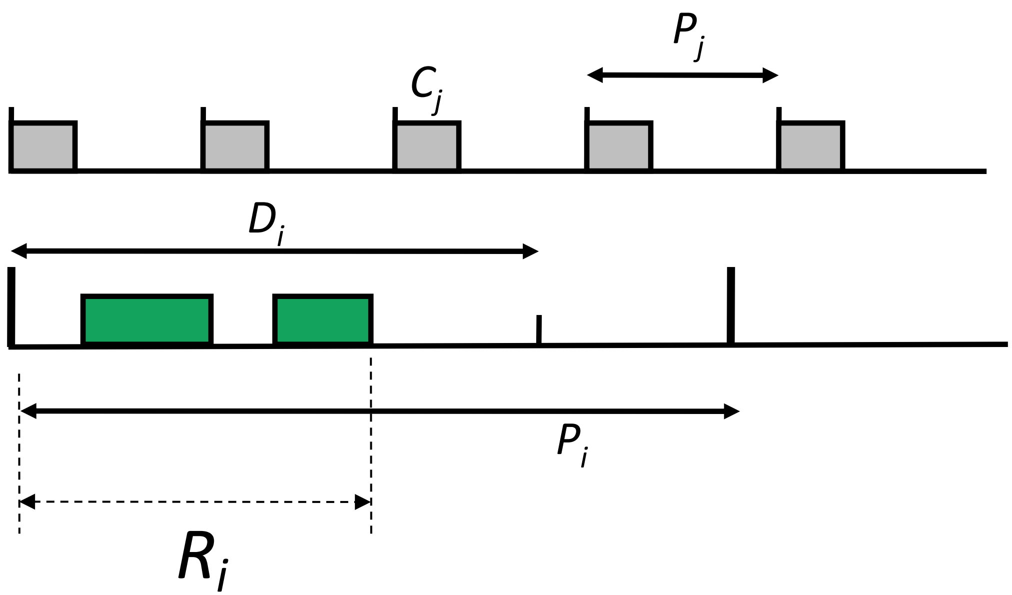
$$R_i^{(n+1)} = \underbrace{\sum_{j\in \mathrm{hp}(i)} \left\lceil \frac{R_i^{(n)}}{P_j} \right\rceil C_j}_{I_i^{(n)}} + C_i$$



Consider a system of two tasks:

Task 1: P_1 = 1.7, D_1 = 0.5, C_1 = 0.5
Task 2: P_2 = 8, D_2 = 3.2, C_2 = 2

Example



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$$I^{(0)} = \lceil 2/1.7 \rceil (0.5) = 1$$

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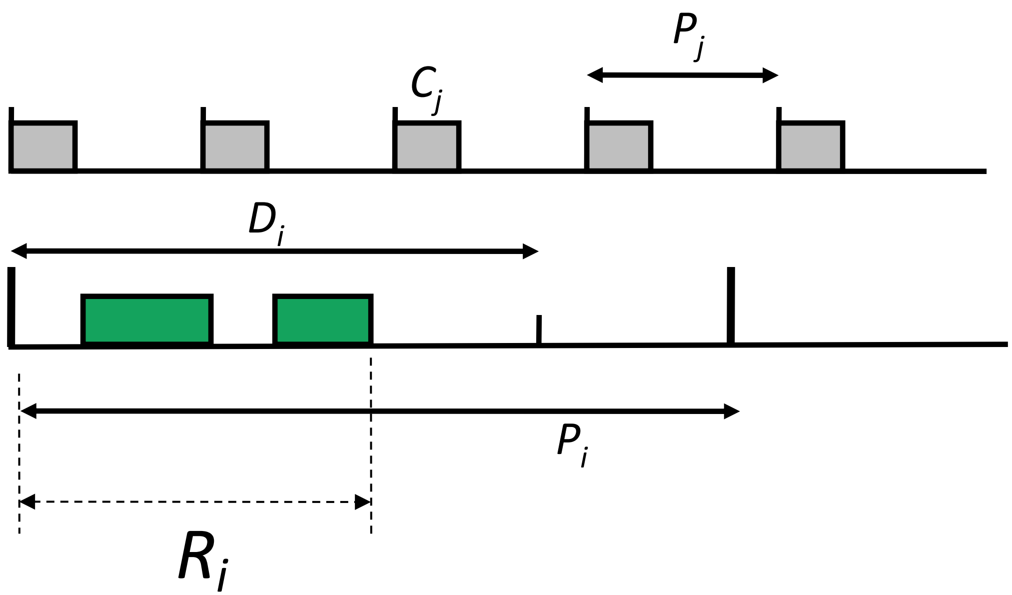

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$$R_2^{(1)} = I_2^{(0)} + C_2 = 3$$
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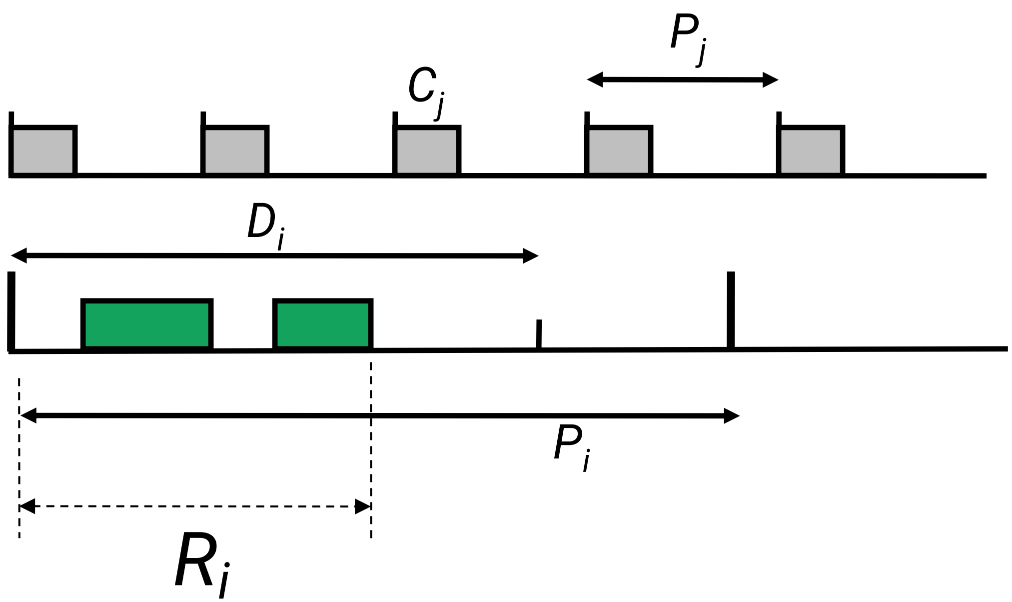

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$$R_2^{(2)} = I^{(1)} + C_2 = 3$$
$$R_2^{(2)} = R_2^{(1)}$$

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3 < 3.2; Task 2 is schedulable.

RTA Algorithm

DM_guarantee (Γ) $\{$

  for (each $\tau_i \in \Gamma$) $\{$

    $I_i = \sum_{k=1}^{i-1} C_k$;

    do $\{$

      $R_i = I_i + C_i$;

      if $(R_i > D_i)$ return(UNSCHEDULABLE);

      $I_i = \sum_{k=1}^{i-1} \left\lceil \frac{R_i}{T_k} \right\rceil C_k$;

    $\}$ **while** $(I_i + C_i > R_i)$;

  $\}$

  return(SCHEDULABLE);

$\}$

Response Time Analysis: Time Complexity

- Is this test efficient?
- Assuming all instance parameters are integers:
  - Inner loop adds at most 1 to interference until deadline is reached
- Runs in time $O(nP_{\max})$, where $P_{\max}$ is the largest period
  - test runs in **pseudo-polynomial** time, not efficient as periods become larger
  - Not suitable for online admission control

Lecture summary

- There are better utilization bounds than the Liu & Layland utilization bound: the hyperbolic bound
- When the relative deadline of a task is less than its period, we can apply utilization bounds
  - But such tests are even more pessimistic than normal
- We can apply exact tests for schedulability when deadlines are less than or equal to periods
  - Such tests require more computation
  - Iterative process