Compositional Performance Analysis

CPEN 432 Real-Time System Design

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So far …

- Arrival model
  - Aperiodic
  - Periodic (with and without offsets)
  - Sporadic

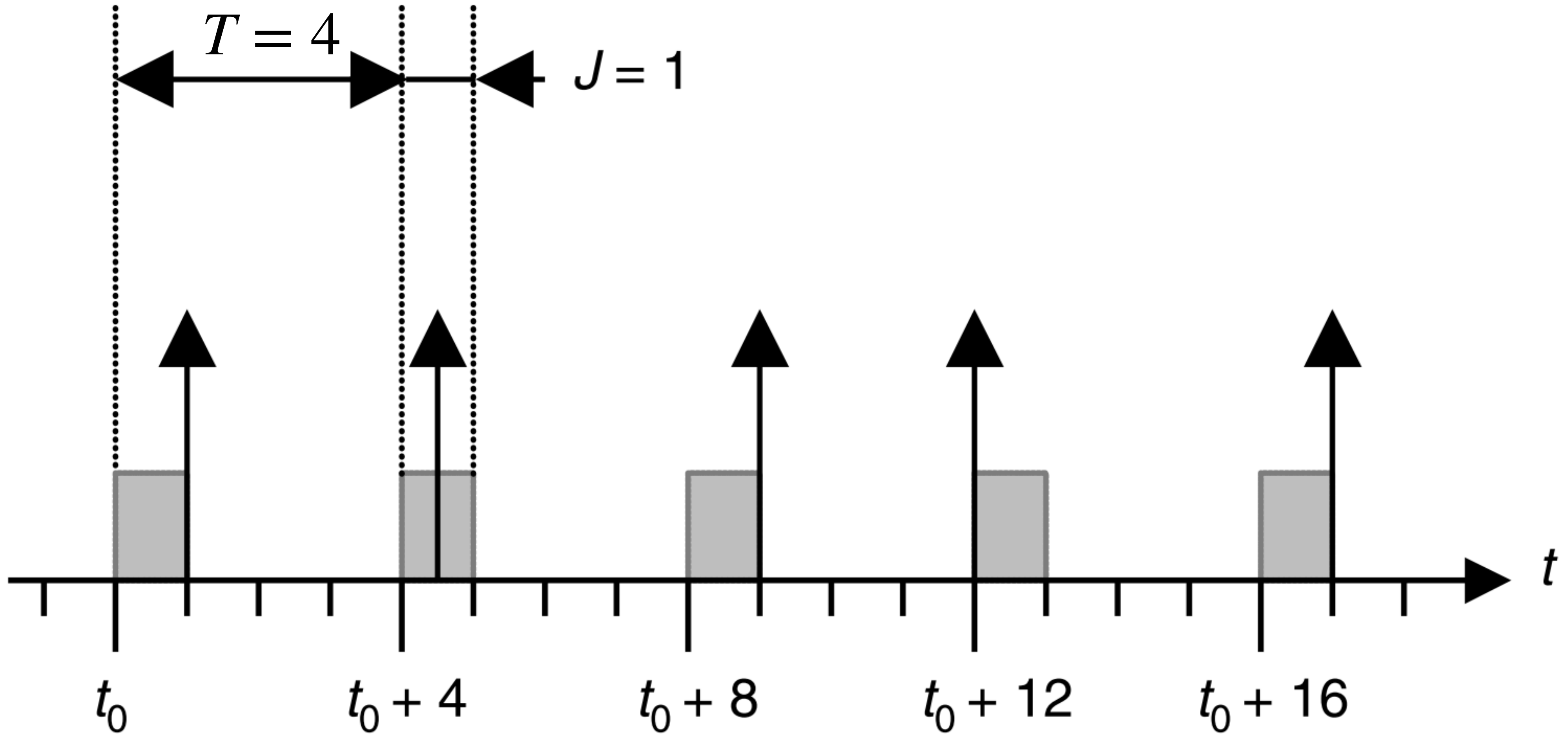
- Response time analyses
  - Uniprocessor: $R_k = C_k + \sum_{\tau_i \in hp(k)} \left(\left\lceil \frac{R_k}{T_i} \right\rceil \cdot C_i \right)$
  - Symmetric multiprocessors: $R_k = C_k + \frac{1}{m} \sum_{\tau_i \in hp(k)} \left(\left\lceil \frac{R_k}{T_i} \right\rceil \cdot C_i + C_i \right)$
  - …

- In a distributed systems, complex embedded applications, MPSoCs, etc.
  - Periodic task model too rigid, as it does not account for variations

“Periodic with Jitter” Event Model

- Two parameters: period T and jitter J
  - Each event **generally occurs periodically** with period T
  - But it **can vary a bit** around its exact position within a jitter interval J

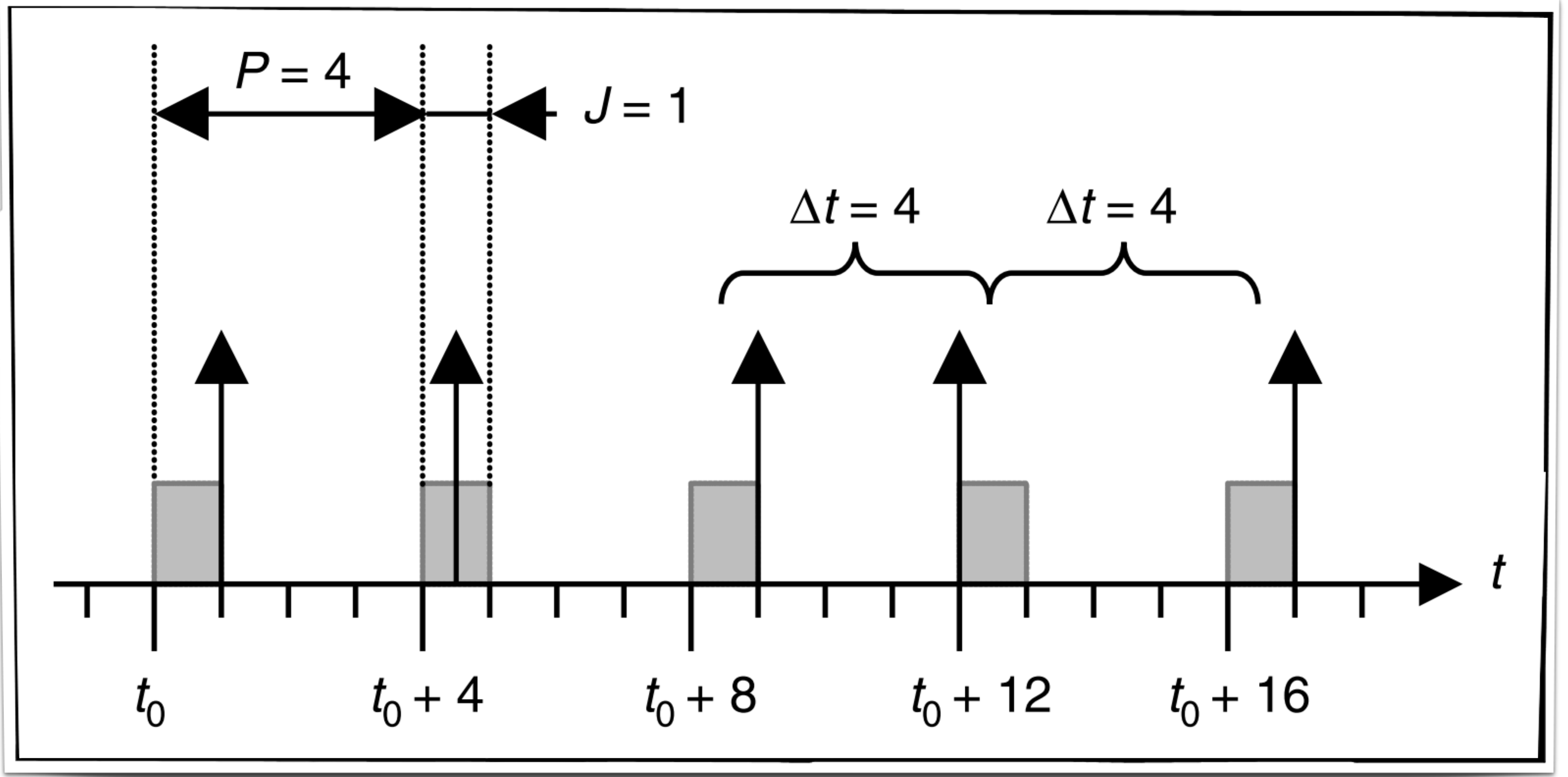
- Example event stream satisfying $(T, J) = (4,1)$

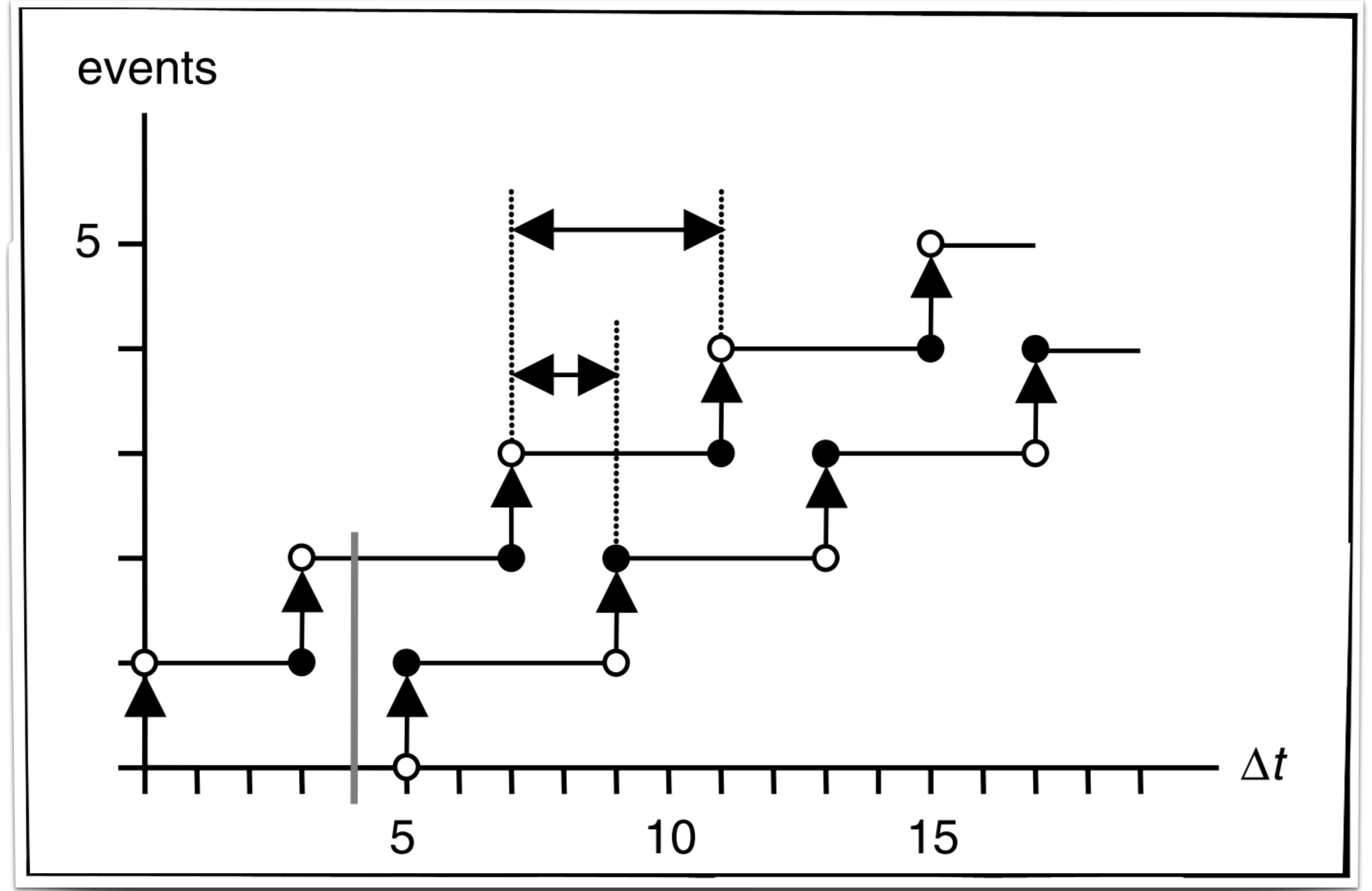


Event Functions

Definition 1 (Upper event function): The upper event function $\eta^u(\Delta t)$ specifies the maximum number of events that can occur during any time interval of length Δt.

Definition 2 (Lower event function): The lower event function $\eta^l(\Delta t)$ specifies the minimum number of events that have to occur during any time interval of length Δt.





$$\eta^u_{T+J}(\Delta t) = \left\lceil \frac{\Delta t + J}{T} \right\rceil$$

$$\eta^l_{T+J}(\Delta t) = \max\left(0, \left\lceil \frac{\Delta t - J}{T} \right\rceil\right)$$

Distance Functions

Definition 3 (Minimum distance function): The minimum distance function $\delta^{min}(N \geq 2)$ specifies the minimum distance between $N \geq 2$ consecutive events in an event stream.

Definition 4 (Maximum distance function): The maximum distance function $\delta^{max}(N \geq 2)$ specifies the maximum distance between $N \geq 2$ consecutive events in an event stream.

$$\delta^{min}(N \geq 2) = \max\{0, (N-1) * T - J\}$$

$$\delta^{max}(N \geq 2) = (N-1) * T + J$$

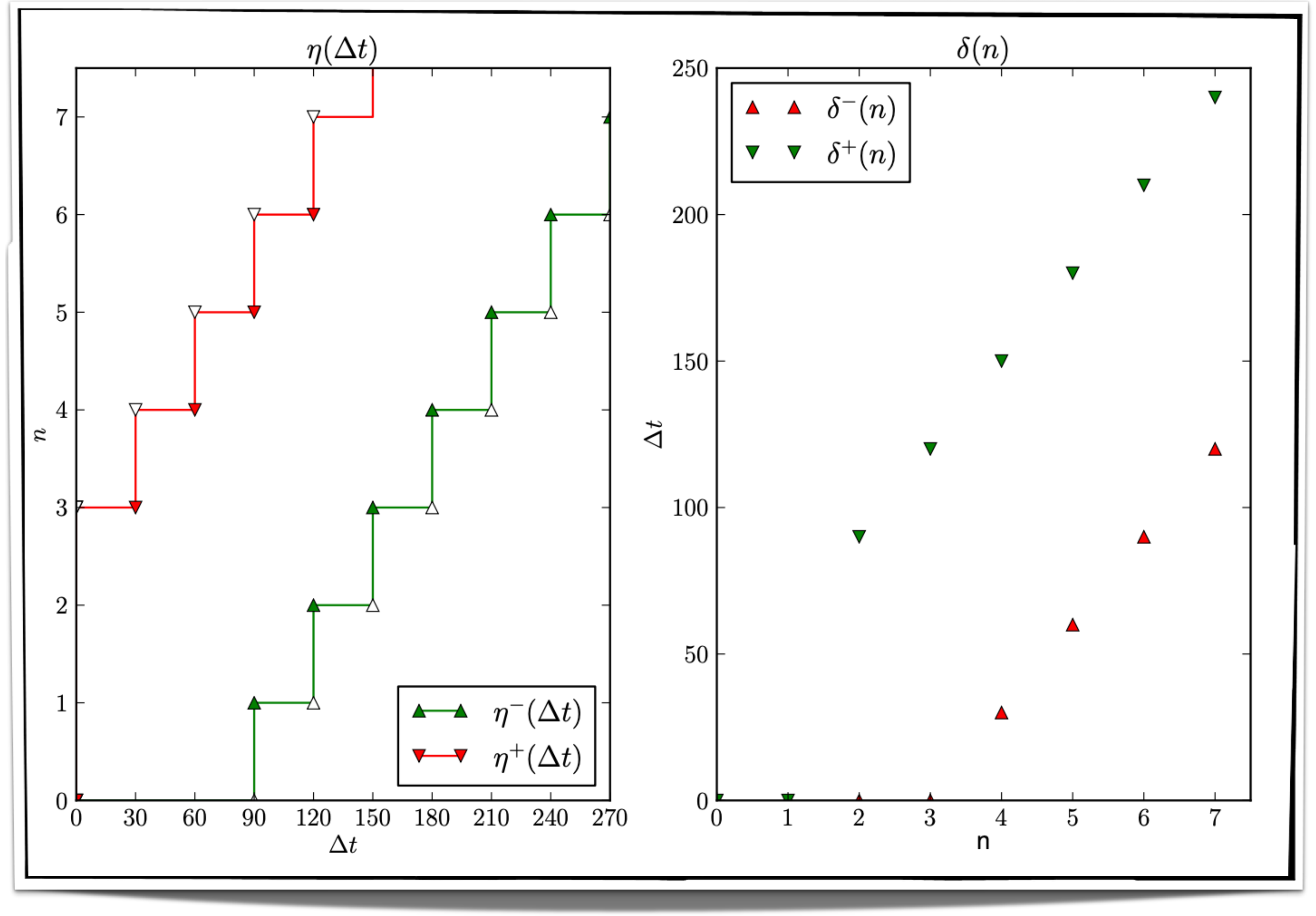
- Questions
  - How can you define $\eta^u(\Delta t)$ and $\eta^l(\Delta t)$ in terms of $\delta^{min}(N)$ and $\delta^{max}(N)$?
    - $\eta^u(\Delta t) = \max_{n \geq 1,\, n \in \mathbb{N}} \{n \mid \delta^{min}(n) < \Delta t\}$
    - $\eta^l(\Delta t) = \min_{n \geq 1,\, n \in \mathbb{N}} \{n \mid \delta^{max}(n+2) > \Delta t\}$

Questions

- For sporadic events with a minimum inter-arrival time T
  - Is the jitter parameter J meaningful?
  - How is the lower event function $n^l(\Delta t)$ defined?
  - How is the maximum distance function $\delta^{max}(N \geq 2)$ defined?

Questions

- What happens if $J > T$ in a “periodic with jitter” event model?
  - Two or more events can occur at the same time, leading to **bursts**
  - New parameter d_{min} that captures the minimum distance between events in a burst
  - Can you draw event and distance functions for $(T, P) = (30,60)$?



Response-Time Analysis (Local)

- Compute the maximum q-event busy window $B_i^+(q)$
  - An upper bound on the time a resource requires to service q activations of task τ_i
  - Assumption: all q activations arrive "sufficiently early"
    - i.e., q^{th} event arrives prior to the completion of its preceding event (the $(q-1)$-event busy window)

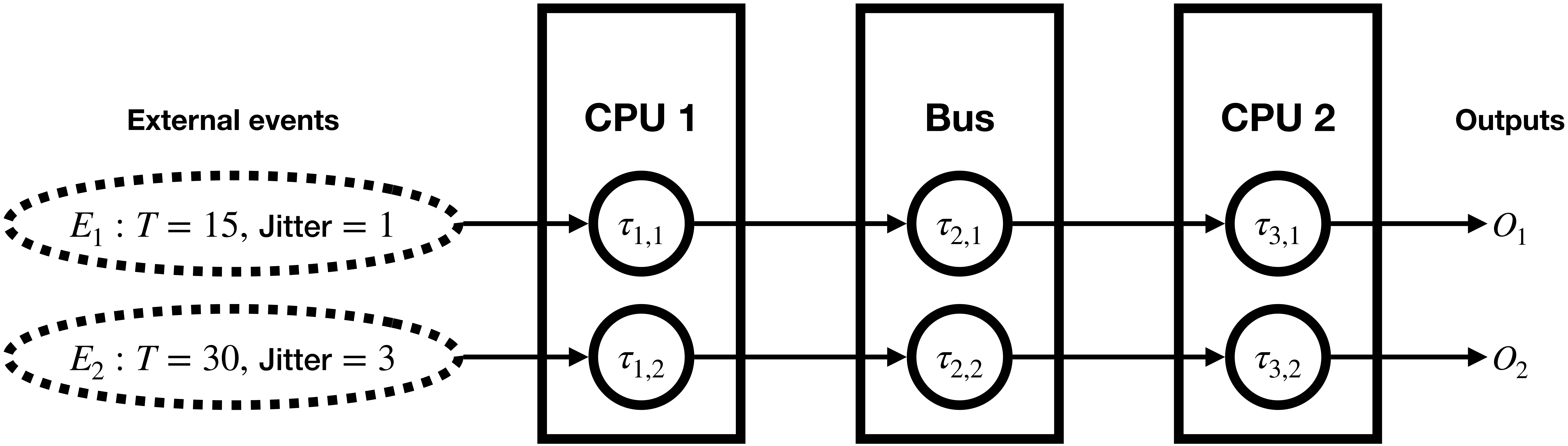
- For fixed-priority preemptive scheduling,
  - Starting with $B_k^u(q) = q \cdot C_k$, solve for $B_k^u(q) = q \cdot C_k + \sum_{i \in hp(k)} \eta_i^u(B_k^u(q)) \cdot C_i$
  - Stopping condition
    - Consider only the first q_k^u activations, where $q_k^u = \min\{q \in \mathbb{N}^+ \mid \delta_k^{min}(q+1) \leq B_k^u(q)\}$
  - $R_k^u = \max_{q \in \mathbb{N}^+ | q \leq q_k^u} \left(B_k^u(q) - \delta_i^{min}(q)\right)$

Output Event Function of a Task

- R_k^u from response-time analysis, $R_k^l = C_k$
  - Thus, the scheduling policy adds an additional jitter of $R_k^u - R_k^l$
  - That is, the output jitter is $J_{k,out} = J_k + \left(R_k^u - R_k^l\right)$
    - Often, J_k is denoted as $J_{k,in}$or $J_{k,act}$

- The output event model period obviously equals the activation period
  - That is, $P_{k,out} = P_{k,in}$

Example 1



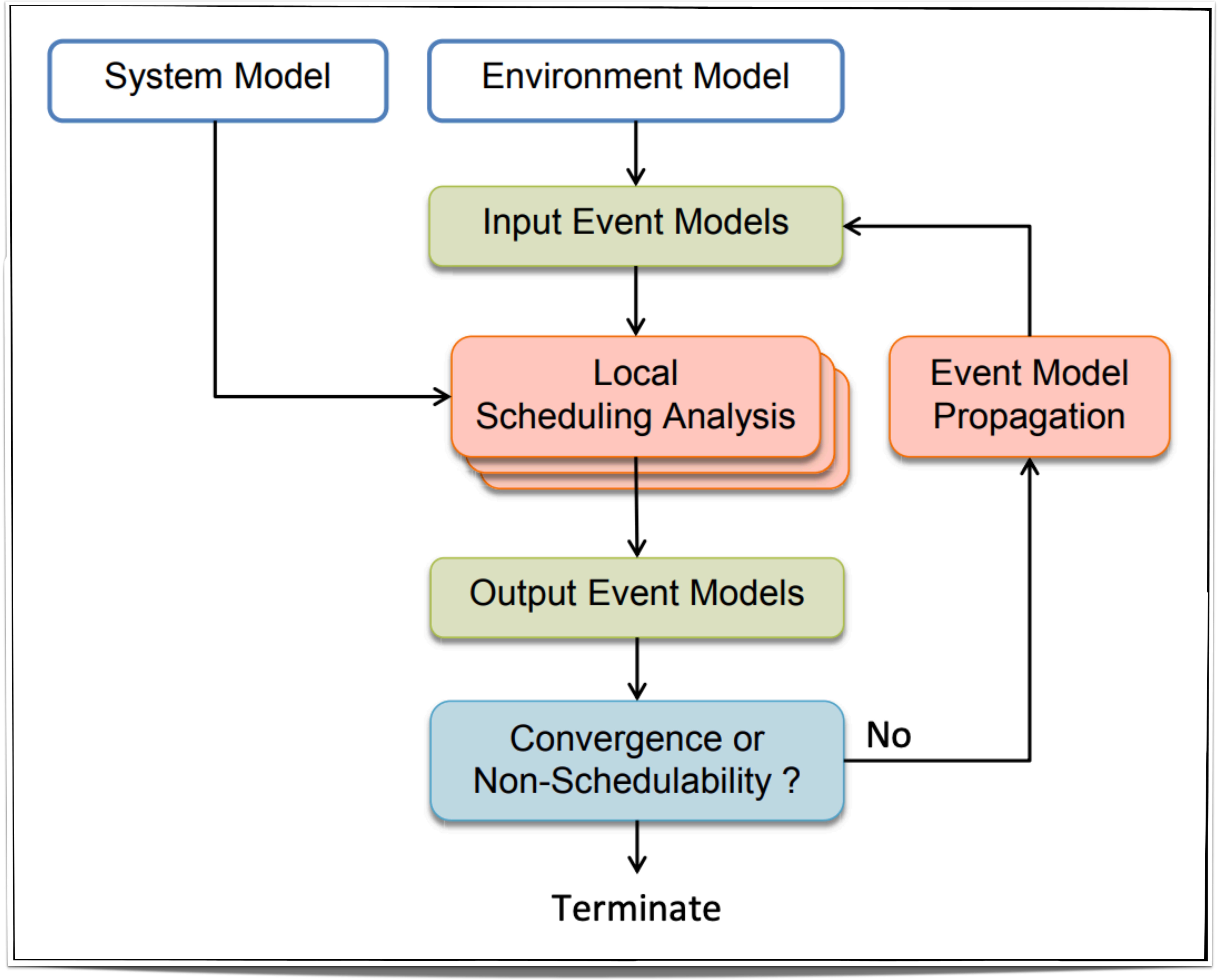
What is the end-to-end path latency of $E_1 \rightarrow O_1$?
What is the end-to-end path latency of $E_2 \rightarrow O_2$?

Example 2

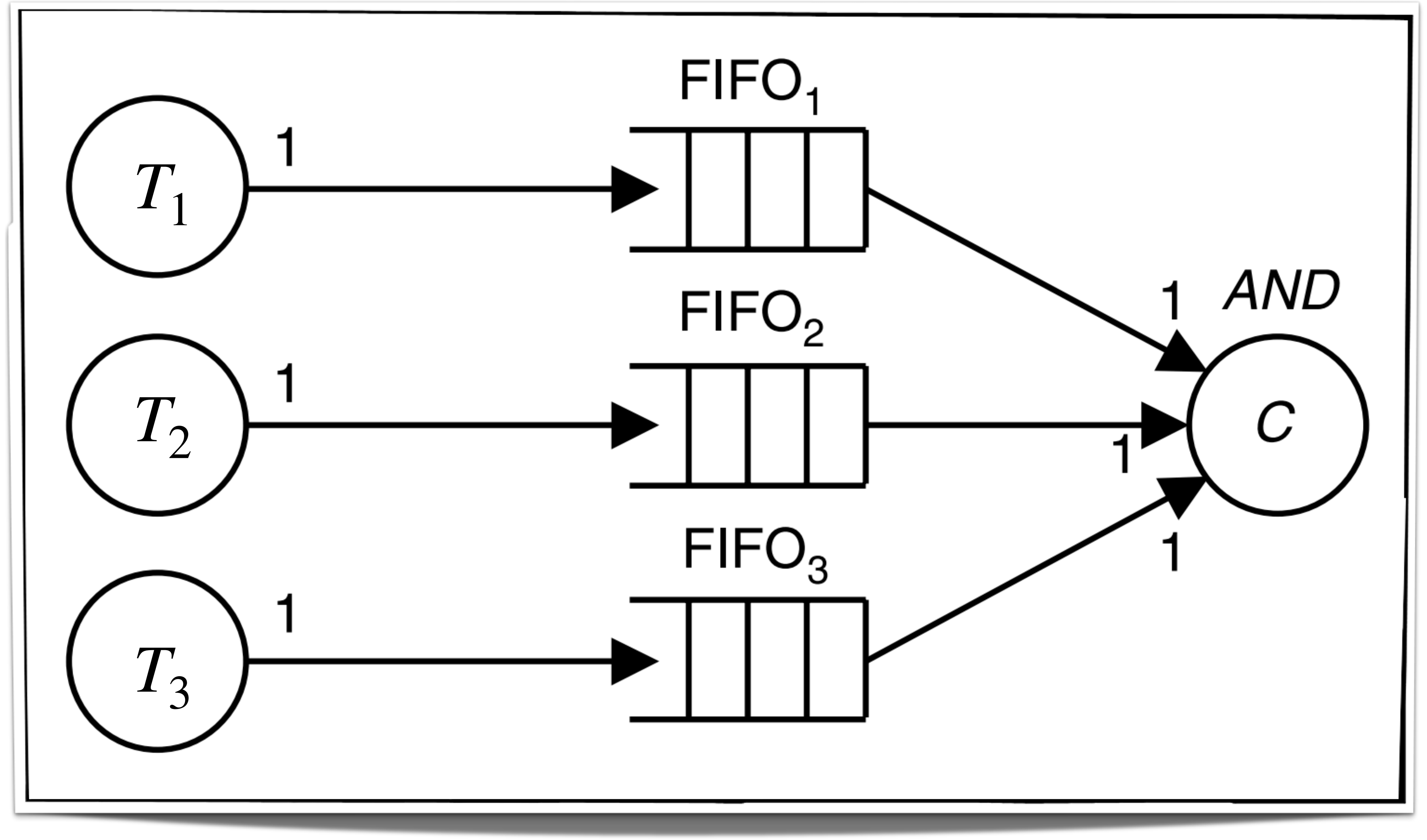
What is the end-to-end path latency of $E_1 \rightarrow O_1$?

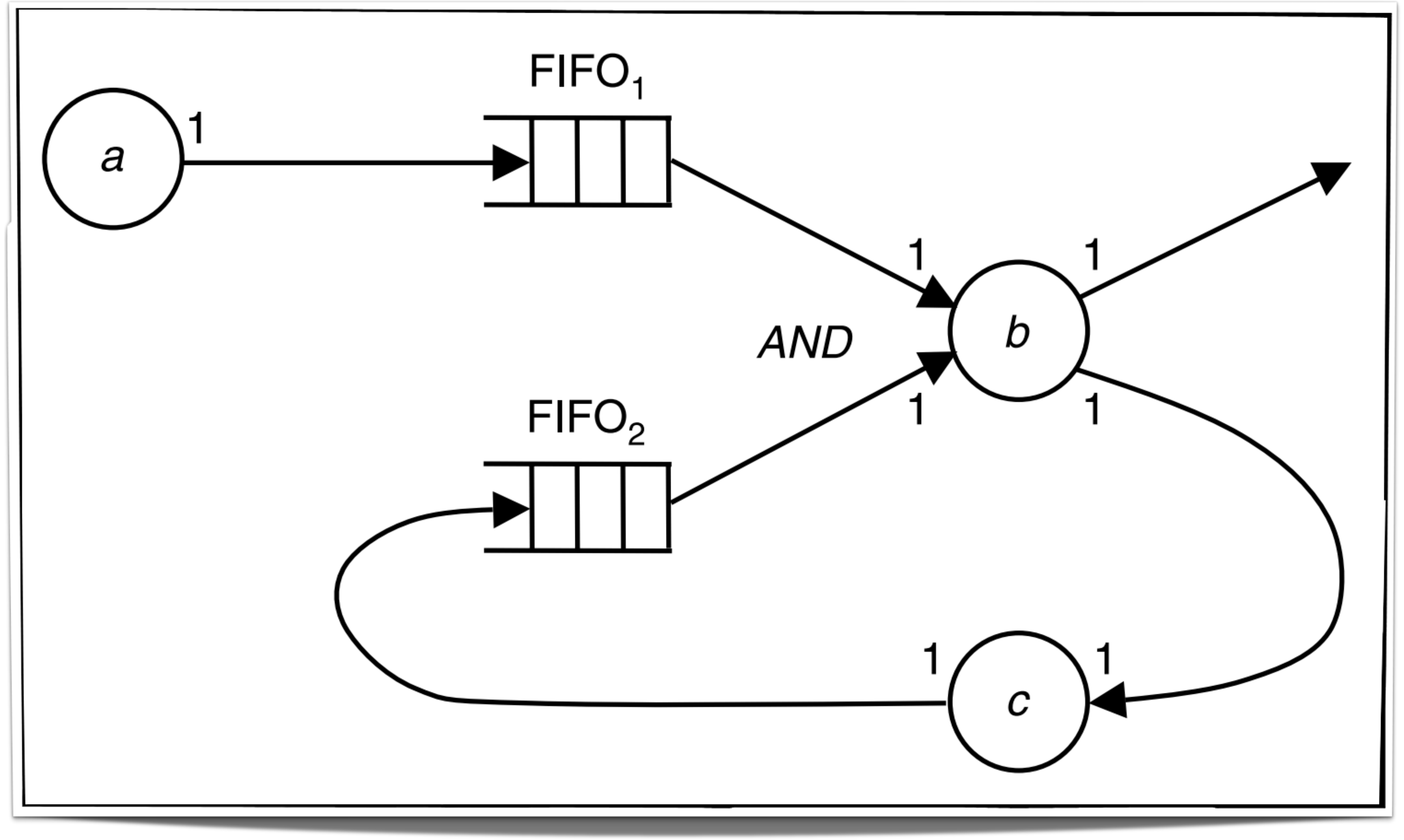
What is the end-to-end path latency of $E_2 \rightarrow O_2$?

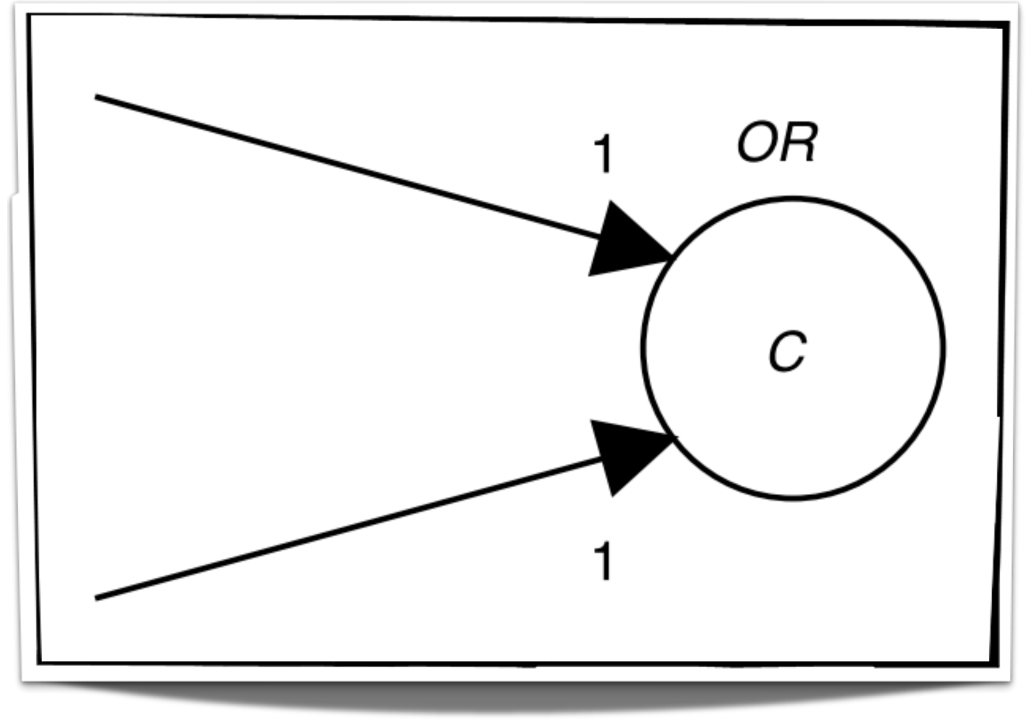
Compositional Performance Analysis



Complex Embedded Applications

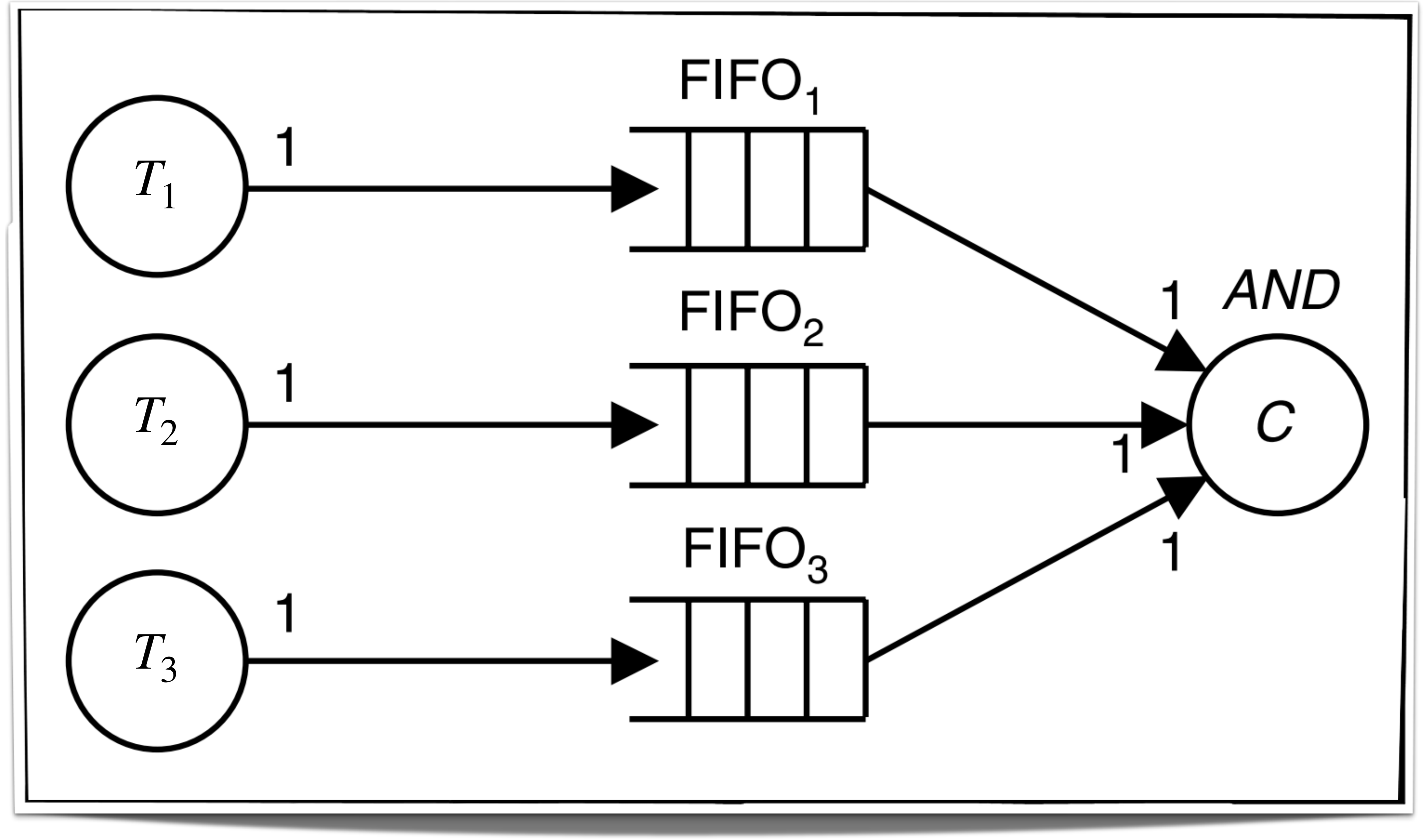


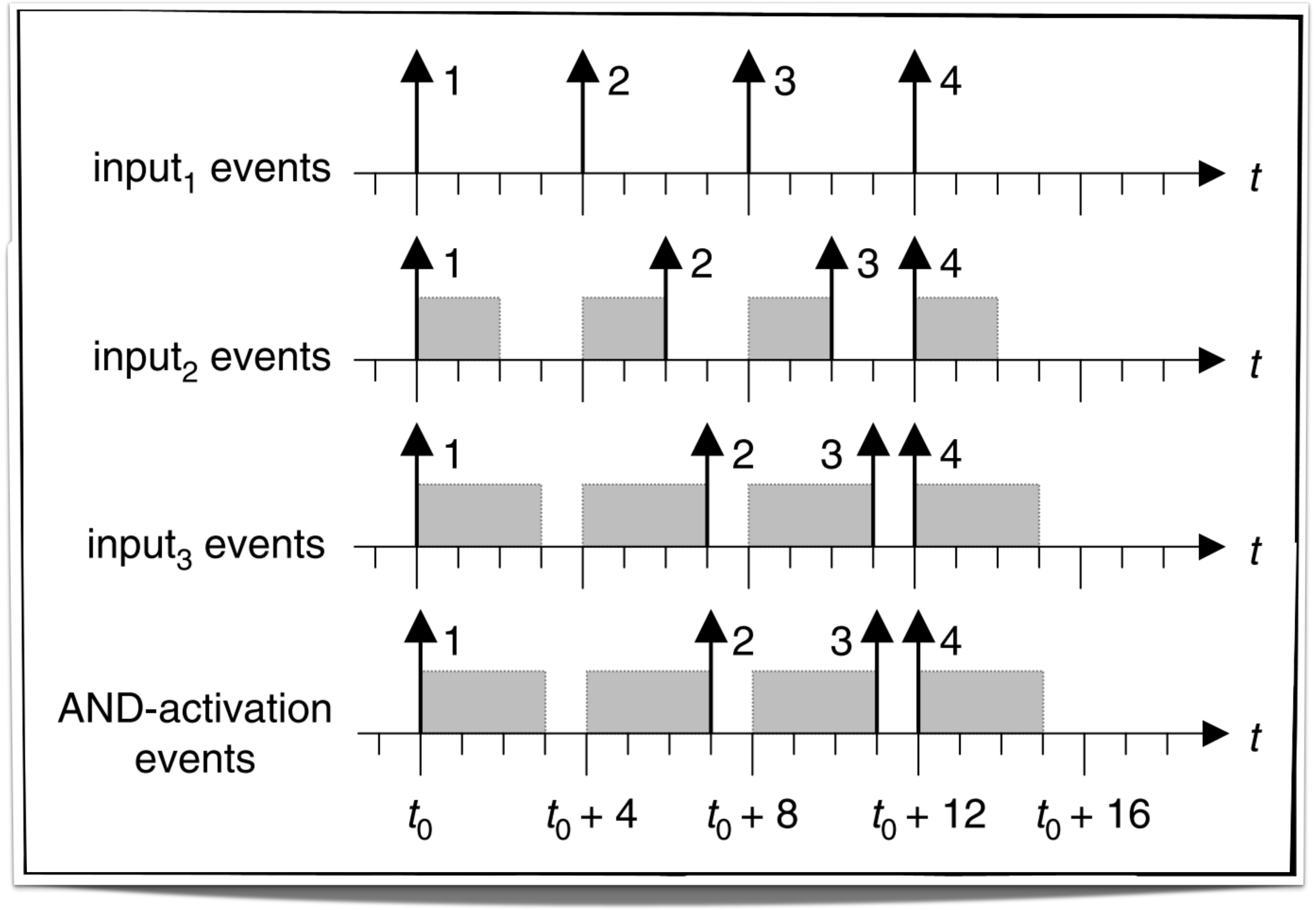




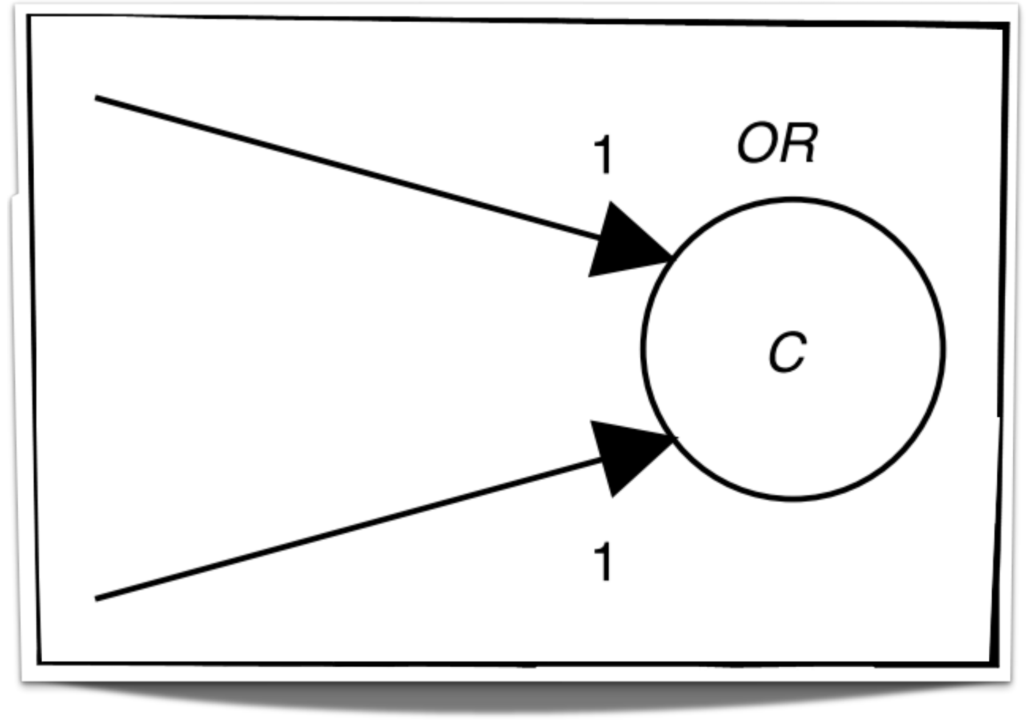
AND-activation

- FIFO channels?
  - Input data buffering: Data may have to wait at some inputs until all other inputs have the necessary data
- Tokens?
  - Amount of data required per input event
- AND-activation period?
  - *"To ensure bounded AND-buffer sizes the period of all input event models must be the same."*
  - Example: $T_{AND} = T_1 = T_2 = T_3$
- AND-activation jitter?
  - Example: $J_{AND} = \max\{J_1, J_2, J_3\}$





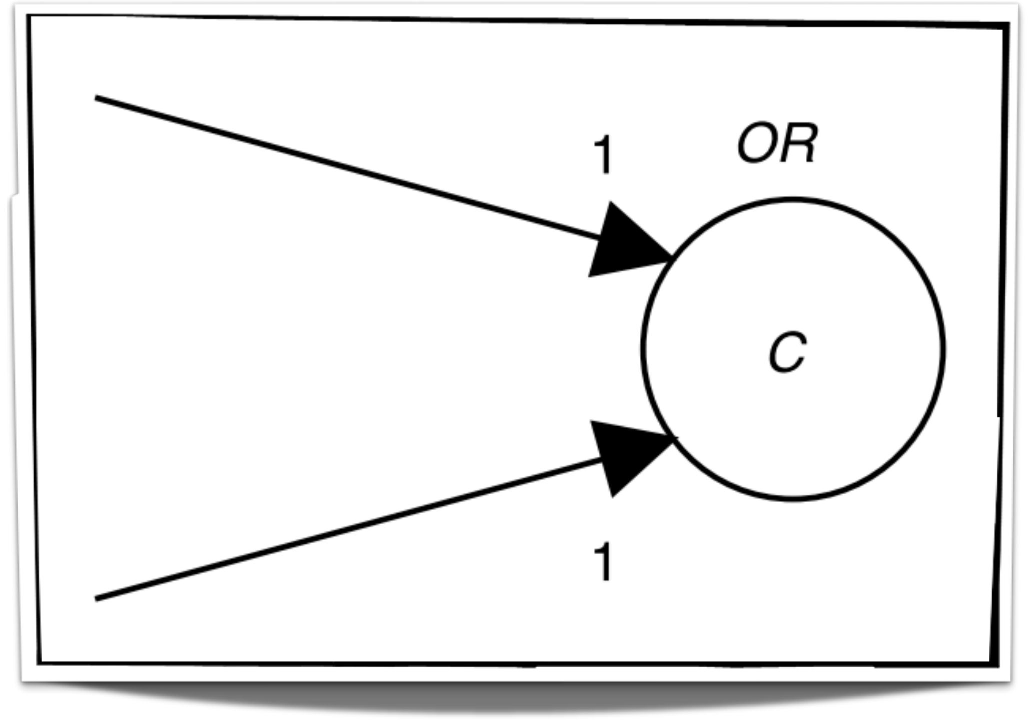
OR-activation



- No FIFO channels. Why?
  - Data at one input never has to wait for data to arrive at a different input for activation

- OR-activation period?
  - Example input event models
    - Event stream $1 : T_1 = 4, J_1 = 2$
    - Event stream $2 : T_2 = 3, J_2 = 2$

OR-activation



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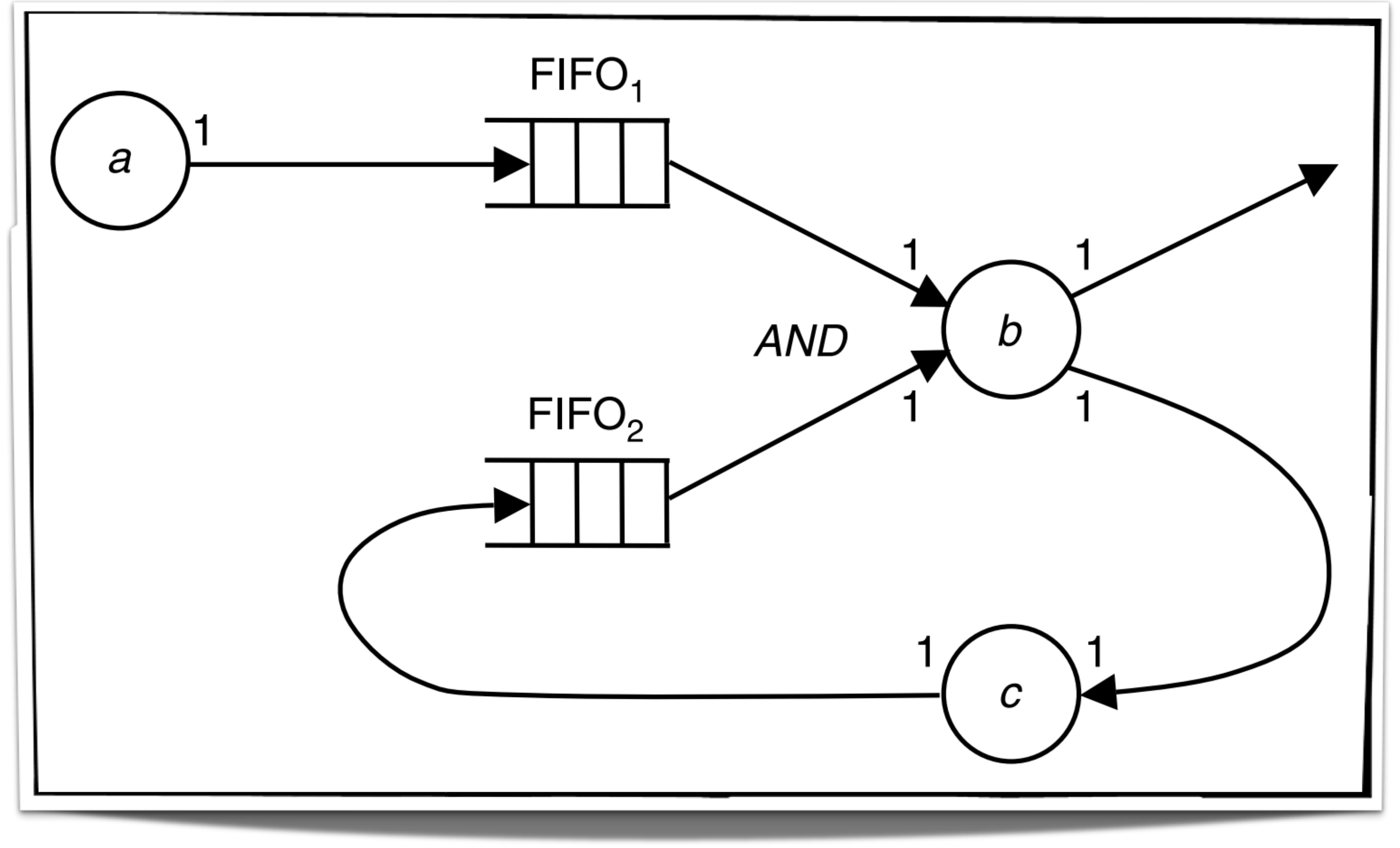
- OR-activation period?
  - Example input event models
    - Event stream $1 : T_1 = 4, J_1 = 2$
    - Event stream $2 : T_2 = 3, J_2 = 2$
  - Consider the *macro* period — the least common multiple of all input event model periods
    - $T_{macro} = LCM(T_i)$
  - Consider the total events across all input streams arriving in the macro period (assuming zero jitter)
    - $N_{total} = \sum_{i=1}^{n} \frac{LCM(T_i)}{T_i}$
  - The OR-activation period is thus the average period, given by
    - $T_{OR} = \frac{T_{macro}}{N_{total}} = \frac{1}{\sum_{i=1}^{n} \frac{1}{T_i}}$

Cyclic Task Dependencies



- Idea
  - Recall that AND-activation jitter is the maximum of input jitters
  - Start with zero jitter for the cyclic input
  - Update, if there are changes
  - This approach may not work. Why?

- Problem
  - Timing of cycle-external and cycle-internal inputs is correlated
    - But the AND-activation jitter (maximum of input jitter) ignores this

- Solution
  - Consider possible phases between inputs arriving at cycle-external and cycle-internal inputs

Open-Source Tool

https://github.com/IDA-TUBS/pycpa | https://pycpa.readthedocs.io/

Welcome

pyCPA is a pragmatic Python implementation of Compositional Performance Analysis (aka the SymTA/S approach provided by Symtavision (now: Luxoft)) used for research in worst-case timing analysis. Unlike the commercial SymTA/S tool, pyCPA is not intended for commercial-grade use and does not guarantee correctness of the implementation.

Thank You!