CPEN 432: Homework Assignment 2
Deadline: 11:59 PM, 21 February, 2022

1 Implicit Deadlines [15 points]

In a periodic task system with implicit deadlines, rate monotonic (RM) scheduling may do as well as earliest deadline first (EDF) scheduling if there is a specific relationship among the task periods. Specifically, n tasks are to be scheduled on a uniprocessor, and their periods are T, kT, k^2T, $\ldots$, $k^{n-1}T$, respectively (where k is an integer greater than 1), then the utilization bound is 1 and not $n(2^{1/n} - 1)$. For example, any four tasks with periods 4, 8, 16, 32, have a utilization bound of 1. In fact, when a set of tasks have periods that follow such a pattern (called a *harmonic sequence*), then that set of tasks can be replaced by one virtual task for the purpose of analysis.

Using the aforementioned idea, develop a schedulability analysis using utilization bounds giving some attention to the possibility of a harmonic chain existing. **(i)** Provide a general method first, and then **(ii)** use it to test the schedulability of task sets:

$$\tau = \{(C_1 = 1.5, T_1 = 6), (2, 12), (2, 18), (1, 24), (4, 48), (6, 54), (4, 96)\}, \text{ and}$$

$$\tau' = \left\{ \begin{matrix} (C_1=1, T_1=5), (1,10), (1,15), (2,20), (1,25), \\ (3,30), (2,40), (5,75), (2,80), (5,225) \end{matrix} \right\}.$$

2 Constrained Deadlines [35 points]

2.1 Yet Another Exact Schedulability Test [20 points]

Consider n constrained-deadline periodic tasks $\tau_1, \ldots, \tau_n$, where task τ_i is characterized by a period T_i, a relative deadline $D_i \leq T_i$, and a WCET $C_i > 0$. All tasks arrive at time 0. Suppose that the tasks are scheduled on one processor using the deadline monotonic (DM) scheduling policy. Assume for simplicity that the tasks are ordered by decreasing priorities, *i.e.*, T_i has a higher priority than T_j *iff* $i < j$.

Let $\Delta_i(t) = \delta_i(t)/t$, where $\delta_i(t)$ is defined as follows.

$$\delta_i(t) = \sum_{j=1}^{i-1} \left\lceil \frac{t}{T_j} \right\rceil C_j + C_i \qquad (0 < t \leqslant T_i).$$

1. Show that τ_i is schedulable if and only if $\min_{(0,D_i]} \Delta_i(t) \leq 1$. Conclude that the entire task set is *feasible* if and only if $\max_{i \in \{1,\ldots,n\}} \min_{(0,D_i]} \Delta_i(t) \leq 1$.

2. Give an economical interpretation of the test in the previous part (think of *supply* and *demand*).

3. Show that in order to deem task τ_i schedulable, it is sufficient that $\Delta_i(t) \leq 1$ for *some* point in the testing set

$$S_i = \Big\{ T_j : j \in \{1,\ldots,i-1\}, k \in \big\{1,\ldots,\lfloor \min(T_i, D_i)/T_j \rfloor \big\} \Big\}.$$

4. From the previous part, derive the asymptotic running time of the schedulability test for the entire task set. Is this a polynomial-time test? Justify your answer.

2.2 EDF Schedulability Analysis [15 points]

You need to implement a real-time application with three periodic tasks.

- $\tau_1 = (T_1 = 100\,ms,\ D_1 = 80\,ms,\ C_1 = 10\,ms)$

- $\tau_1 = (T_1 = 75\,ms,\ D_1 = 50\,ms,\ C_1 = 15\,ms)$

- $\tau_1 = (T_1 = 150\,ms,\ D_1 = 100\,ms,\ C_1 = 30\,ms)$

Luckily for you, the operating system supports EDF scheduling and hence you can get better processor utilization. On the other hand, you would like to conserve power and run the processor at the lowest possible speed. If the timing information was obtained using a 500 MHz processor, what is the lowest processor speed that would ensure that all jobs meet their deadlines? Assume that execution times scale linearly with speed.

Note that EDF utilization for implicit deadline tasks does not apply because τ_1, τ_2, and τ_3 have constrained deadlines.