Schedulability with resource sharing

Priority inheritance protocol Priority ceiling protocol Stack resource policy

Lecture overview

- We have discussed the occurrence of unbounded priority inversion
- We know about blocking and blocking times

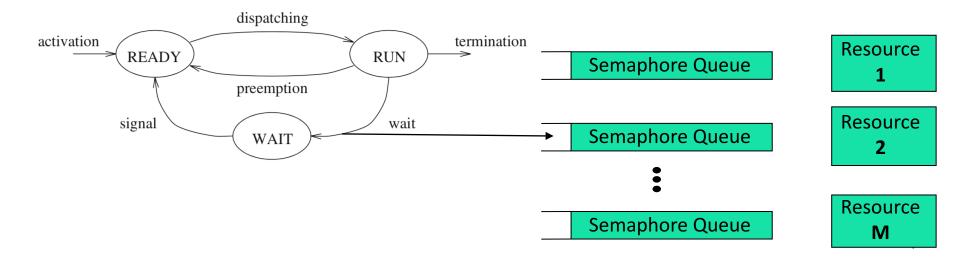
- Now: Evaluating schedulability in combination with protocols for avoiding unbounded priority inversion
- Priority ceiling protocol to prevent deadlocks
- Stack-based resource policy
 - Improves on other policies
 - Extends to EDF

Blocking

- Tasks have synchronization constraints
 - Use semaphores to protect critical sections
- Blocking can cause a *higher priority* task to wait for a *lower priority* task to unlock a resource
 - We always assumed that higher priority tasks can preempt lower priority tasks
 - To make rules consistent, we discussed the priority inheritance approach

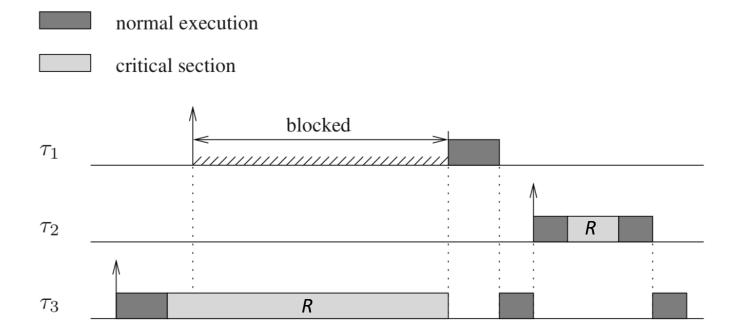
General Model and Assumptions

- Assumption: Each resource has one instance only (binary semaphores)
- Assumption: Resource requests are properly nested
- **Assumption:** We have perfect knowledge of all task resource requirements
- Except for SRP, all protocols are designed for static-priority scheduling
- Each resource has a semaphore queue



Approach #1: Non-Preemptive Protocol (NPP)

- Whenever a task requests a resource, make it the highest priority task for the duration of its critical section
- The good: Easy to Implement
- The bad: unnecessarily blocks higher priority tasks that do not request resources



Non-Preemptive Protocol: Blocking time computation

- A task T_i can be blocked only by a **lower priority** task that has requested a resource before T_i 's arrival
- Whenever a task is in a critical section, it cannot be preempted
- Only one resource can be locked at time t
- The highest priority task available will acquire the processor as soon as the lower priority task releases the resource, and it will not be blocked again
- Conclusion: Worst case blocking time is the duration of the longest critical section of any lower priority task

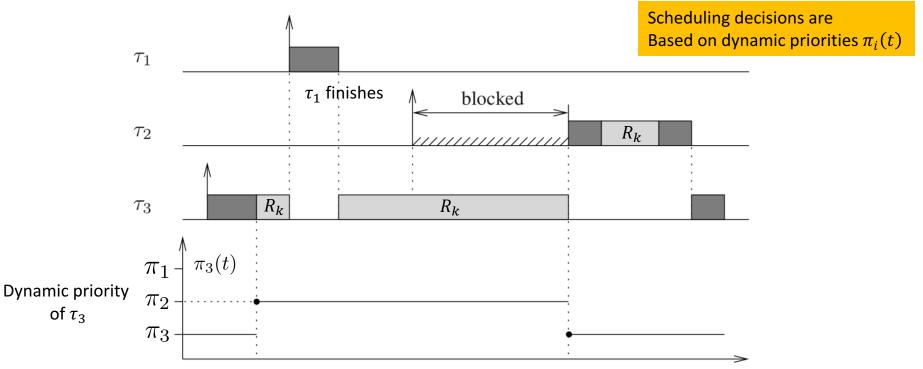
 $\delta_{j,k}$: duration of *longest* critical section of task j using resource R_k

$$B_i = \max\{\delta_{j,k}: \pi_j < \pi_i, R_k \text{ a resource}\}$$

Note: If no lower priority task uses any resources, then $B_i = 0$; i.e., $\max \emptyset = 0$

Approach #2: Highest Locker Priority (HLP)

- When a task requests resource R_k , elevate its priority to the priority of the highest priority task that ever shares (uses) resource R_k
- Avoids the unnecessary blocking of higher priority tasks that do not need resources (present in NPP)



Highest Locker Priority: Blocking time computation

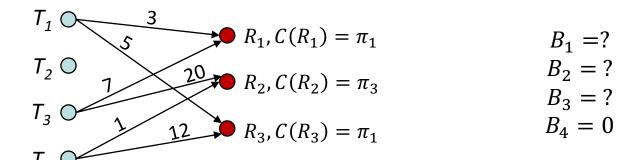
- When a task requests resource R_k , elevate its priority to the priority of the highest priority task that ever shares resource R_k
- Observation: Task T_i can be blocked only by lower priority tasks that uses a resource that is used by a task with priority greater than or equal to T_i
- Claim: A task T_i can be blocked for at most the duration of a single critical section of at most one lower priority task that uses a resource that is used by a task with priority $\geq \pi_i$
- Ceiling of resource R_k is the priority of the highest priority task that uses R_k
 - $C(R_k) = \max_{i \in [n]} \{\pi_i : T_i \text{ uses } R_k\}$
- Claim recast: A task T_i can be blocked for at most the duration of a single critical section of at most one lower priority task that ever uses a resource R_k with $C(R_k) \ge \pi_i$

Highest Locker Priority: Blocking time computation

• A task T_i can be blocked for at most the duration of a **single** critical section of at most one lower priority task that ever uses a resource R_k with $C(R_k) \ge \pi_i$

$$B_i = \max\{\delta_{j,k}: \pi_j < \pi_i, T_j \text{ uses } R_k, C(R_k) \ge \pi_i\}$$

A useful tool: The *resource graph*

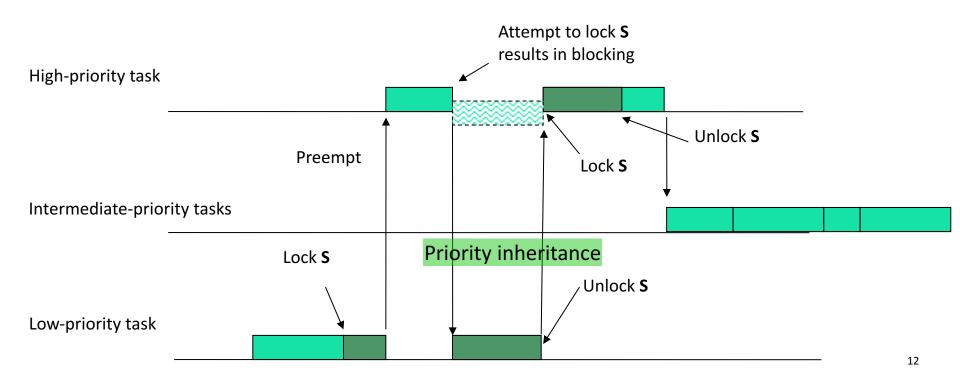


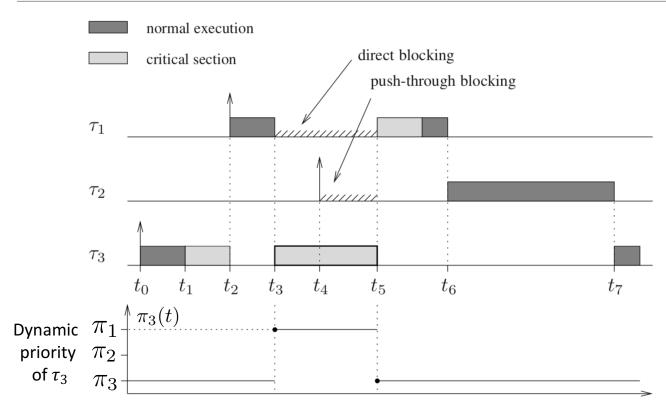
- List jobs in priority order and resources in any order, creating a node for each
- Create an edge between task T_j and resource R_k if T_j uses R_k
- Label arc (T_i, R_k) with the length of the <u>longest</u> critical section of T_i that uses R_k , $\delta_{i,k}$ (even if critical sections nests are nested)
- Label each resource node R_k by its ceiling $C(R_k)$

Highest Locker Priority: Problems

- HLP causes unnecessary blocking
 - A higher priority task is blocked on its *arrival*, not when it attempts to request the resource
 - Solution: delay blocking until the task attempts to request shared resource → PIP!

- Allow a task to inherit the priority of the highest priority task that it is blocking
- When a hp task is blocked as it attempts to acquire a resource that is held by a lower priority task, it **transfers** its priority to that lower priority task

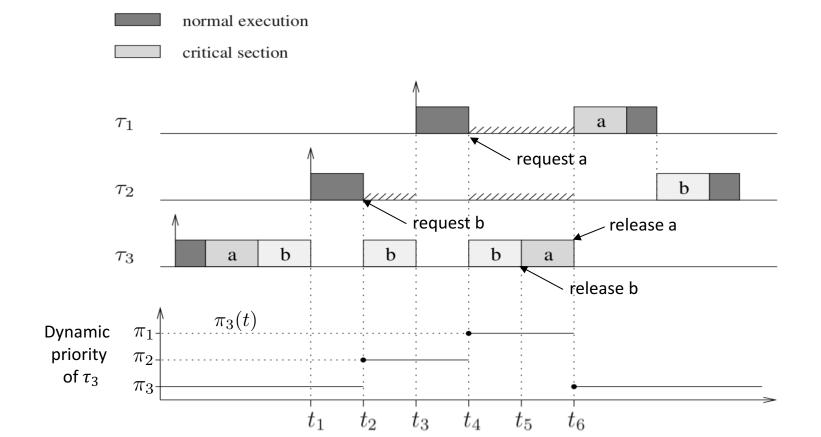




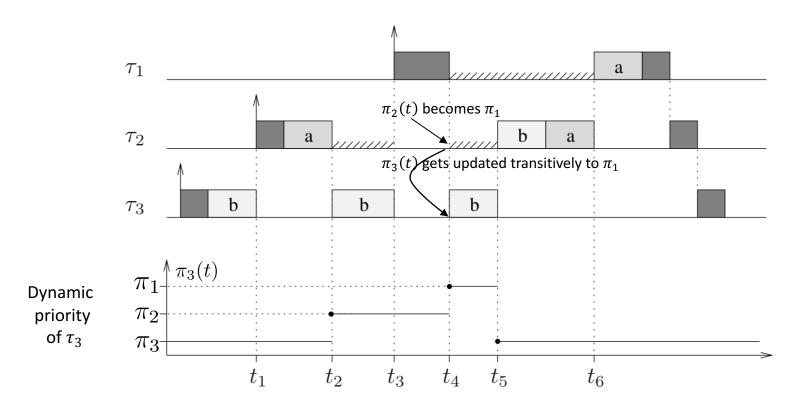
If I am a task, priority inversion occurs when

- (a) Lower priority task holds a resource I need (direct blocking)
- (b) Lower priority task inherits a higher priority than me because it holds a resource the higher-priority task needs (push-through blocking)

- When a task releases a resource, its dynamic priority $\pi(t)$ is set to the highest priority of the tasks blocked by it
- Q: When a task exits a critical section, does it always resume the priority it had when it entered?



- Priority inheritance is transitive
 - However, transitive priority inheritance can occur only in the presence of **nested critical sections** (proof in book Lemma 7.2)



Maximum blocking time

- Claim1: If there are ℓ_i lower-priority tasks that can block task τ_i , then τ_i can be blocked for at most the duration of ℓ_i critical sections (one for each of the ℓ_i lower-priority tasks), regardless of the number of semaphores used by τ_i
 - A critical section $z_{j,k}$ of a lower priority task T_j can block T_i if it causes either direct or pushthru blocking to T_i
- Claim2: If there are s_i distinct semaphores that can block task τ_i , then τ_i can be blocked for at most the duration of s_i critical sections, one for each of the s_i semaphores, regardless of the number of critical sections used by τ_i

- ullet Then, if all critical sections are of equal length, b_i
- Blocking time $B_i = b_i \times \min(\ell_i, s_i)$
 - What if the critical sections are of differing lengths?

General approach to computing blocking times

- What if the critical sections are of differing lengths?
- Will consider a safe approximation to blocking time.
- Assumption: no nested critical sections
- For a high-priority task
 - Examine all tasks with lower priority
 - Determine the worst-case blocking that it may offer (consider the highest priority that it can inherit)
 - Examine all semaphores/resources
 - Determine the worst-case blocking due to that resource
 - Consider lower-priority tasks that may inherit a higher priority when they hold the semaphore

Maximum blocking time

This is just a safe approximation (upper bound on exact blocking time)

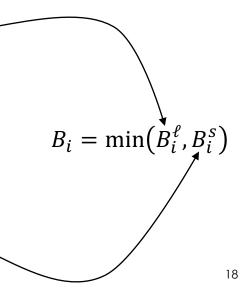
Exact blocking time computation is intractable

- What if the critical sections are of differing lengths?
- $\delta_{j,k}$: length of *longest* critical section among all those of T_j guarded by sempahore S_k
- Let $z_{j,k}$ denote the critical section (CS) whose length is $\delta_{j,k}$
- (1) Blocking due to lower priority tasks that $can block T_i$ (claim1)
 - A task T_j can block T_i if it has lower priority than T_i and uses some resource R_k that is also used by a task with priority greater than or equal to T_i

$$B_i^{\ell} = \sum_{j=i+1}^n \max_k \{\delta_{j,k} : z_{j,k} \text{ is max. length CS that can block } T_i \}$$

- (2) Blocking due to semaphores that can block T_i (claim 2)
 - A resource *can block* T_i if it is used by a lower priority task
 - Assuming we have m semaphores (Resources)

$$B_i^s = \sum_{k=1}^m \max_{j>i} \{\delta_{j,k}: z_{j,k} \text{ is max. length CS that can block } T_i\}$$



Simplifying matters

- Use resource ceilings (very useful device)
- Recall: $C(R_k) = \max_{i \in [n]} \{\pi_i : T_i \text{ uses } R_k\}$
- Claim: In the absence of nested critical sections, a critical section $z_{j,k}$ of τ_j guarded by semaphore S_k can block τ_i only if $P_i < P_i \leq C(S_k)$
 - Proof in text; Lemma 7.5

$$B_i^{\ell} = \sum_{j=i+1}^n \max_k \{\delta_{j,k} : z_{j,k} \text{ is max. length CS that can block } T_i \}$$

$$B_i^{s} = \sum_{k=1}^m \max_{j>i} \{\delta_{j,k} : z_{j,k} \text{ is max. length CS that can block } T_i \}$$

$$B_i^{\ell} = \sum_{j=i+1}^{n} \max_{k} \{ \delta_{j,k} : C(R_k) \ge \pi_i \}$$

$$B_i^{s} = \sum_{k=1}^{m} \max_{j>i} \{ \delta_{j,k} : C(R_k) \ge \pi_i \}$$

Schedulability tests

- For the fixed-priority scheduling case
 - We can use the Liu & Layland bound with some modifications
- For task T_k : we need to consider the blocking by lower priority tasks

$$\frac{e_k + B_k}{P_k} + \sum_{i=1}^{k-1} \frac{e_i}{P_i} \le k(2^{1/k} - 1)$$

Each instance of a task may experience blocking (worst case); equivalent to increasing the execution time of the task by the blocking time.

For task T_k , we need to consider:

- (a) preemption by higher priority tasks
- (b) blocking from lower priority tasks bound for T_k involves only k tasks-

Why do we test each task separately? Why can we not have one utilization bound test like we did earlier?

- Consider the following set of tasks, which share resources R_1 , R_2 and R_3
 - Relative deadline are equal to periods; tasks scheduled using RM policy
 - T_1 : P_1 =20, e_1 =3, uses R_1 and R_2 separately for 1 time unit each
 - T_2 : P_2 =30, e_2 =6, uses R_2 and R_3 simultaneously for 2 time units
 - T_3 : P_3 =50, e_2 =10, uses R_1 and R_3 separately for 3 and 4 time units respectively
 - T_4 : P_4 =80, e_2 =8, uses R_2 for 5 time units

Is there a difference?

Without resource constraints

$$U = \frac{3}{20} + \frac{6}{30} + \frac{10}{50} + \frac{8}{80} = 0.65 < 0.69$$

The task set satisfies the Liu and Layland bound; easily schedulable by RM

- Consider the following set of tasks, which uses resources R_1 , R_2 and R_3
 - Relative deadline are equal to periods; tasks scheduled using RM policy
 - T_1 : P_1 =20, e_1 =3, uses R_1 and R_2 separately for 1 time unit each
 - T_2 : P_2 =30, e_2 =6, uses R_2 and R_3 simultaneously for 2 time units
 - T_3 : P_3 =50, e_2 =10, uses R_1 and R_3 separately for 3 and 4 time units respectively
 - T_4 : P_4 =80, e_2 =8, uses R_2 for 5 time units

With resource constraints

 T_1 can potentially be blocked by T_2 , T_3 and T_4

It can be blocked by T_2 on resource R_2 for upto 6 time units (because it may wait for T_3)

It can be blocked by T_3 on resource R_1 for upto 3 time units

It can be blocked by T_4 on resource R_2 for upto 5 time units

Then maximum wait on lower priority tasks is $B_1^{\ell} = 6 + 3 + 5 \neq 14$

The worst-case wait for R_1 is 3 units (only T_3 can block T_1)

The worst-case wait for R_2 is 6 units (T_2 can block T_1 for 6 units or T_4 can block T_1 for 5 units)

Then maximum wait for resources is $B_1^s = 3 + 6 = 9$

Then
$$B_1 = \min(14/9) = 9$$

$$\frac{9}{20} + \frac{3}{20} < 1$$

T₁ is schedulable

- Consider the following set of tasks, which uses resources R_1 , R_2 and R_3
 - Relative deadline are equal to periods; tasks scheduled using RM policy
 - T_1 : P_1 =20, e_1 =3, uses R_1 and R_2 separately for 1 time unit each
 - T_2 : P_2 =30, e_2 =6, uses R_2 and R_3 simultaneously for 2 time units
 - T_3 : P_3 =50, e_2 =10, uses R_1 and R_3 separately for 3 and 4 time units respectively
 - T4: P4=80, e2=8, uses R2 for 5 time units

With resource constraints

 T_2 can be blocked by T_3 and T_4

 T_3 can block T_2 in two ways:

directly on R_3 (upto 4 units)

by obtaining priority of T_1 when using R_1 (upto 3 units) (push-through)

 T_4 can block T_2 in two ways:

directly when using R_2 (upto 5 units)

by obtaining priority of T_1 when using R_2 (upto 5 units) (push-through)

The worst-case blocking by T_3 is 4 time units

The worst-case blocking by T₄ is 5 time units

Maximum wait for resources is $B_2 = 5 + 4 = 9 = B_2^{\ell}$ (check for yourself that $B_2^s = 13$)

$$\left(\frac{9}{30} + \left(\frac{3}{20} + \frac{6}{30}\right) = 0.65 < 0.82\right)$$

T₂ is schedulable

once.

$$\frac{B_k}{P_k} + \sum_{i=1}^k \frac{e_i}{P_i} \le k(2^{1/k} - 1)$$

A low priority task can block a high

priority task at most once. With

priority inheritance, it will get a

higher priority and continue till it

releases the lock. Therefore, it can

block a high priority task at most

- Consider the following set of tasks, which uses resources R_1 , R_2 and R_3
 - Relative deadline are equal to periods; tasks scheduled using RM policy
 - T_1 : P_1 =20, e_1 =3, uses R_1 and R_2 separately for 1 time unit each
 - T_2 : P_2 =30, e_2 =6, uses R_2 and R_3 simultaneously for 2 time units
 - T_3 : P_3 =50, e_2 =10, uses R_1 and R_3 separately for 3 and 4 time units respectively
 - T₄: P₄=80, e₂=8, uses R₂ for 5 time units

With resource constraints

 T_3 can be blocked by T_4

even when it shares no resource with T_4 (lower priority task)

Notice that T_4 may execute with priority of T_1 (priority inheritance)

 T_4 may execute with the priority of T_1 for at most 5 time units

Classic case of push-through blocking

Maximum blocking due to T_4 is 5 time units; $B_3 = 5$

$$\frac{5}{50} + \left(\frac{3}{20} + \frac{6}{30} + \frac{10}{50}\right) = 0.65$$

T₃ is schedulable

 $\frac{B_k}{P_k} + \sum_{i=1}^{\kappa} \frac{e_i}{P_i} \le k(2^{1/k} - 1)$

- Consider the following set of tasks, which uses resources R_1 , R_2 and R_3
 - Relative deadline are equal to periods; tasks scheduled using RM policy
 - T_1 : P_1 =20, e_1 =3, uses R_1 and R_2 separately for 1 time unit each
 - T_2 : P_2 =30, e_2 =6, uses R_2 and R_3 simultaneously for 2 time units
 - T_3 : P_3 =50, e_2 =10, uses R_1 and R_3 separately for 3 and 4 time units respectively
 - T₄: P₄=80, e₂=8, uses R₂ for 5 time units

$$\frac{B_k}{P_k} + \sum_{i=1}^k \frac{e_i}{P_i} \le k(2^{1/k} - 1)$$

With resource constraints

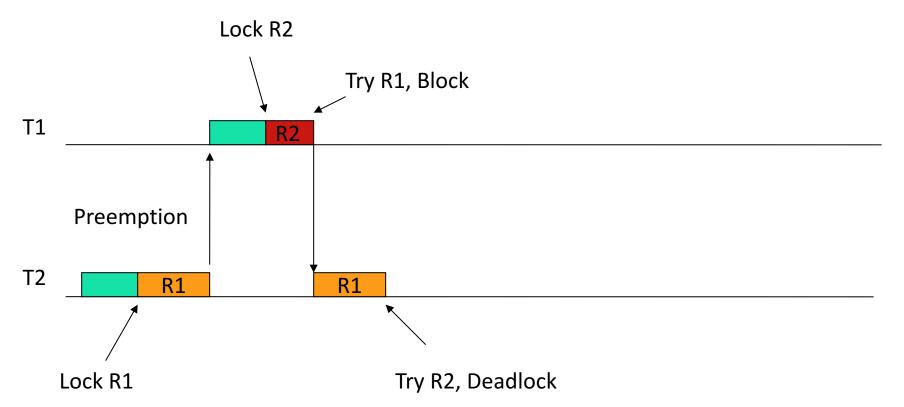
 T_4 can never be blocked because it is the lowest priority task Maximum wait for resources is $B_4 = 0$

$$\left(\frac{3}{20} + \frac{6}{30} + \frac{10}{50} + \frac{8}{80}\right) = 0.65$$

T₄ is schedulable

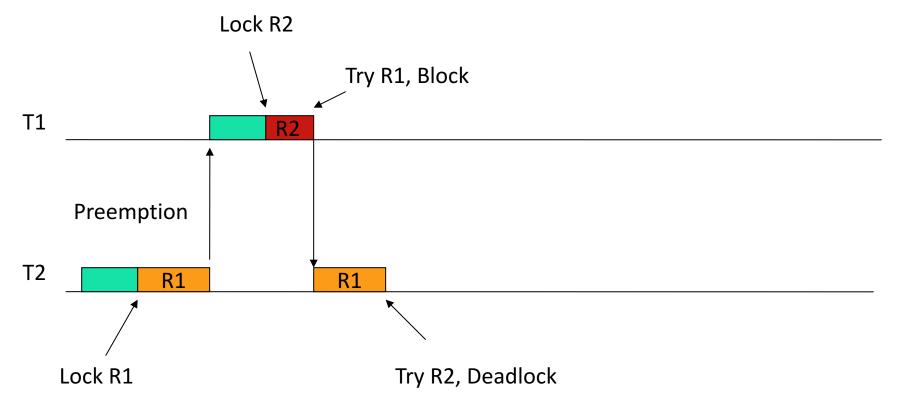
Does priority inheritance solve all problems?

- Actually, not all problems
- We can still have a deadlock if resources are locked in opposing orders



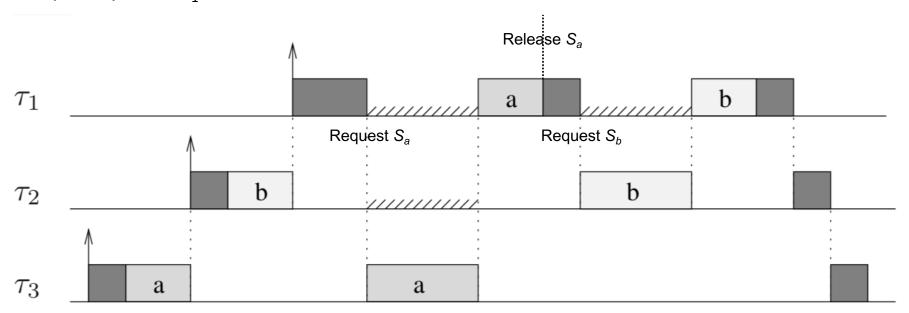
Deadlocks

- Can attribute it to sloppy programming
- But can we solve the problem in a different way
- Avoid deadlocks by designing a suitable protocol



Another problem with PIP: Chained blocking

- When τ_1 attempts to use its resources, it is blocked for the duration of **2** critical sections:
 - once to wait for τ_3 to release S_a
 - and then to wait for τ_2 to release S_b
- In the worst case, if τ_1 accesses n distinct semaphores that have been locked by n lower-priority tasks, τ_1 will be blocked for the duration of n critical sections.



Priority ceiling protocol

- **Definition**: the **priority ceiling** of a semaphore is the highest priority among all tasks that can lock the semaphore
- A task that requests lock R_k is denied if its priority is not strictly higher than the highest priority ceiling of all *currently* locked semaphores (let us say this belongs to semaphore R_h ; there may be more than one)
 - The task is said to be blocked by the task holding semaphore R_h
- A task inherits the priority of the top higher-priority task it is blocking

Priority ceiling protocol

- Recall: Priority Ceiling of resource R_k : $C(R_k) = \max_{i \in [n]} \{\pi_i : T_i \text{ uses } R_k\}$
- Suppose task τ_i requests a resource R_k at time t
- Let $R_h = \operatorname{argmax}_{j} \{C(R_j) : \operatorname{resource} R_j \text{ is locked at time } t \}$
 - This might be a set but assume one resource for simplicity
- Define System (current) Ceiling $C(t) = C(R_h)$
 - System ceiling updated whenever a resource is acquired/released
- If $\pi_i \leq C(t)$, then τ_i is denied access to the resource
 - Exception: If $\pi_i = C(t)$ and T_i is the task locking R_h , then grant T_i access to R_k (o/w T_i will block itself!)
 - τ_i is said to be blocked by the task holding semaphore R_h
 - τ_i then trasfers its priority to task holding R_h

Priority ceiling protocol

- To avoid multiple blocking, this rule does not allow a task to enter a critical section if there are locked semaphores that could block it.
- This means that once a task enters its first critical section, it can **never** be blocked by lower-priority tasks until its completion

Similarity to PIP

Priority Inheritance rule

Fundamental difference from PIP

PIP is *greedy*, PCP is not! In what sense?

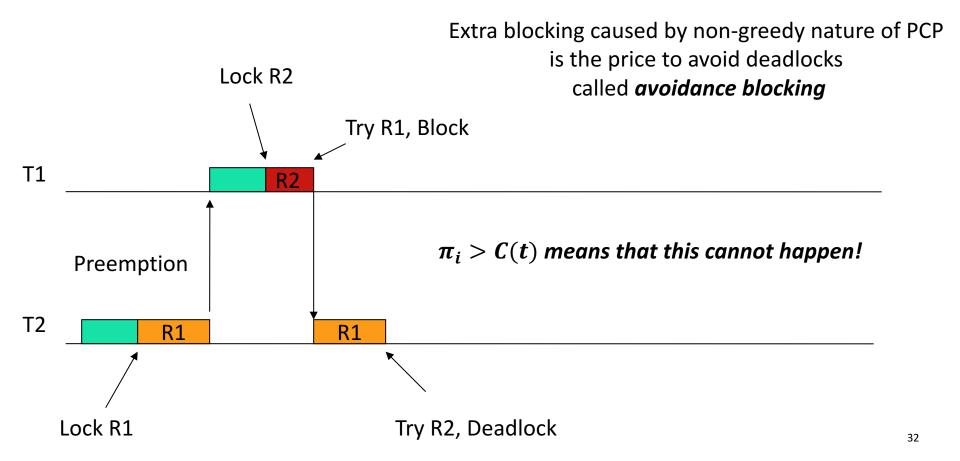
A task can be blocked on a *free* resource in PCP *Impossible in PIP*



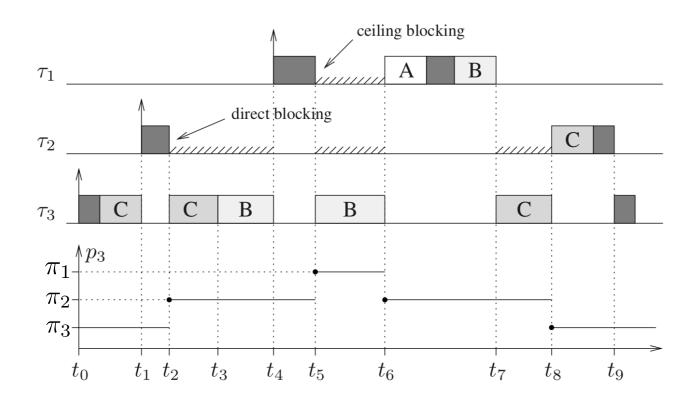
Extra blocking caused by non-greediness of PCP is the price to avoid deadlocks & chained blocking called *avoidance blocking* or *ceiling blocking*

Deadlocks?

• A deadlock can occur if two tasks locked semaphores in opposite order. Can it occur with the priority ceiling protocol?

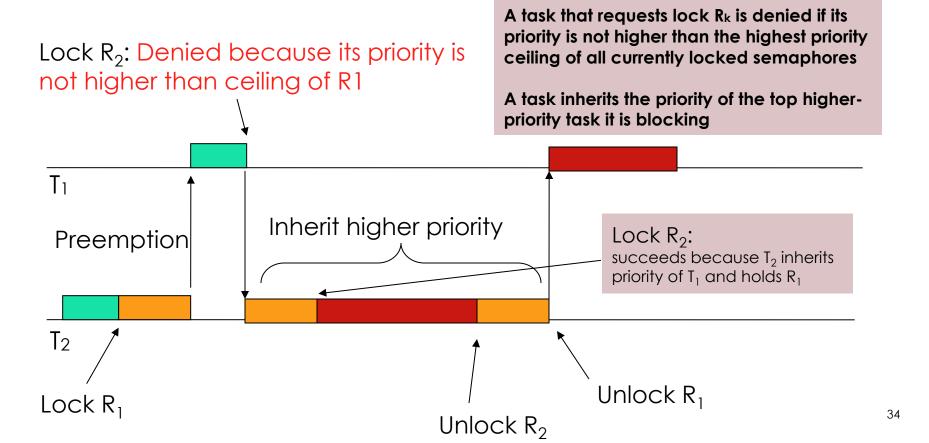


PCP Example



OS kernel locked resources (Ceiling: 1), T2 Current system
R1 (ceiling: 1), T2 Ceiling: 14
R3 (ceiling: 4), T5-

• T_1 and T_2 use R_1 and R_2 : the priority ceiling of a resource is the priority of the highest priority task that uses it, therefore the priority ceilings of R_1 and R_2 are the same: the priority of T_1

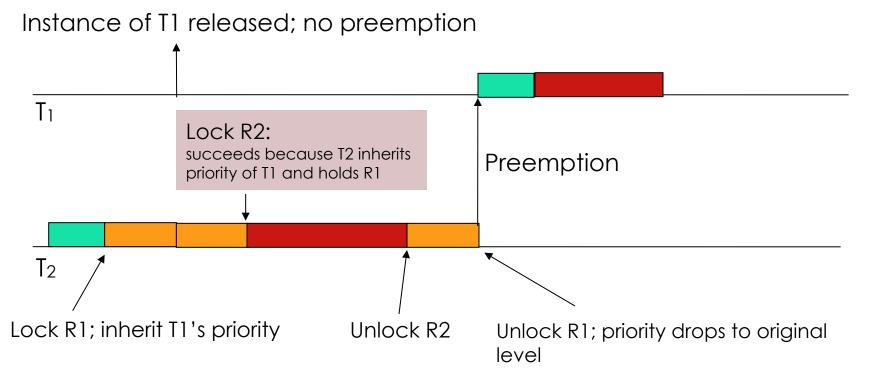


PCP blocking time computation

- A task can be blocked by the duration of at most one critical section of at most one lower priority task
- Much simpler to compute than PIP
- Should consider the three types of blocking and the max of them
- Resource graph to our rescue!

Immediate inheritance

 Priority ceiling protocol with slight difference: when a semaphore is locked, the locking task raises its priority to the ceiling priority of the semaphore (immediate inheritance). When the semaphore is unlocked the task's priority is restored.



Schedulability test for priority ceiling protocol

- The test is the same as with the priority inheritance protocol
 - · Worst-case blocking time may change when compared to PIP

$$\frac{B_k}{P_k} + \sum_{i=1}^k \frac{e_i}{P_i} \le k(2^{1/k} - 1)$$

For task T_k

- Let us attempt to simplify PCP
- There is a class of dynamic-priority systems called fixed preemption-level systems
 - In such systems, the potentials of resource contentions do not change with time, just as in fixed-priority systems, and hence can be analyzed *statically*
- Fact: A job J_h has a higher priority than another job J_l and they both require some resource does not imply that J_l can directly block J_h . This blocking can occur **only when it is possible for J_h to** preempt J_l
- Consequence of fact: when determining whether a free resource can be granted to a job, it is not
 necessary to be concerned with the resource requirements of all higher-priority jobs; only those
 that can preempt the job

- Since for resource contention purposes we only care about the jobs that a job can possibly preempt, let us think of a quantity that encodes a job's ability to preempt other jobs
- (*) Formally, we want to associate job J_i with quantity ψ_i such that if $\psi_i \ge \psi_k$, then it is not possible for J_k to preempt J_i
- J_k cannot preempt $J_i \Leftrightarrow \text{either } r_k \leq r_i \text{ or } \pi_k \leq \pi_i$
- Then (*) translates to:

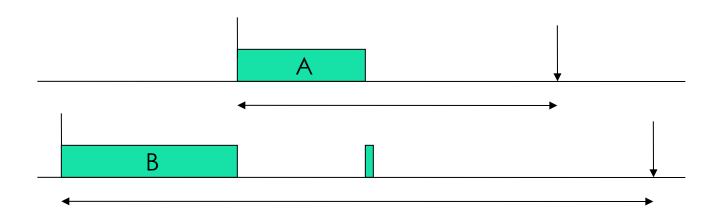
(**) if
$$r_k > r_i$$
 and $\pi_k > \pi_i$, then $\psi_k > \psi_i$ (J_k can potentially preempt J_i)

- A ψ_i satisfying (**) is called the **preemption level** of job J_i
- **Q:** How does ψ_i look like for EDF?

- The preemption level ψ_i of J_i is any quantity satisfying the statement: if $r_k > r_i$ and $\pi_k > \pi_i$, then $\psi_k > \psi_i$ (J_k can potentially preempt J_i)
- **Q:** How does ψ_i look like for EDF?
- EDF:
 - $\pi_k > \pi_i$ iff $r_k + D_k < r_i + D_i$
 - So $r_i < r_k$ implies $r_i + D_k < r_i + D_i \Rightarrow D_k < D_i$
 - $\psi_k > \psi_i \Leftrightarrow D_k < D_i$
 - For EDF, this quantity is for the entire task, not only a job!
- EDF is one such fixed preemption-level system
- The possibility that a task preempts other tasks remains constant throughout all its invocations
 - Task's Preemption level can be computed offline (static) once and for all

Stack-based resource policy with EDF

- Priority is inversely proportional to the absolute deadline
- Preemption level is inversely proportional to the relative deadline
- Observe that:
 - If A arrives after B and Priority(A) > Priority(B) then PreemptionLevel(A)
 > PreemptionLevel(B)



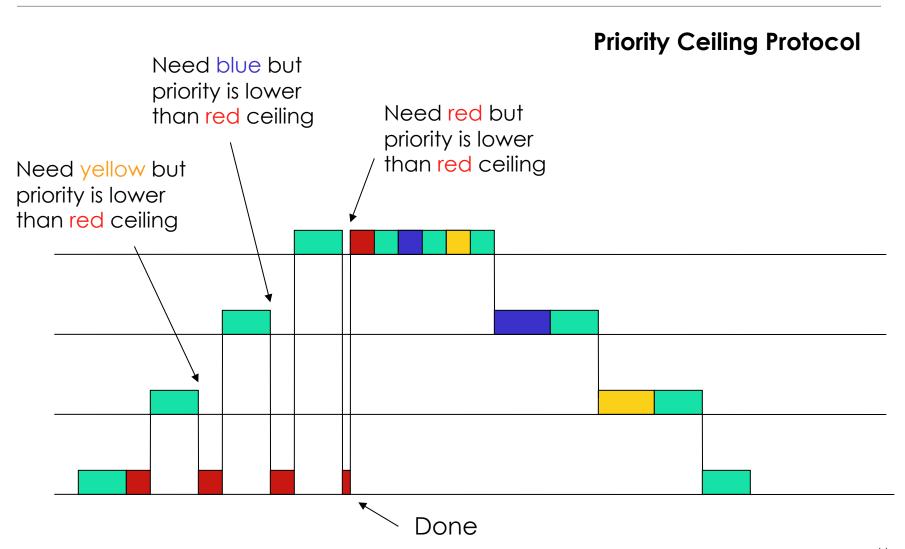
- Priority inheritance protocol and priority ceiling protocol are easy to analyze in a fixed-priority setting
- What about dynamic priority scheduling?
- Stack-based resource policy [SRP]
 - Preemption level: Any fixed value that satisfies the statement "if A arrives after B and priority(A) > priority(B), then PreemptionLevel(A) > PreemptionLevel(B)."
 - Resource ceiling for resource R: Highest preemption level of all tasks that may access the resource R
 - System ceiling: Highest resource ceiling among all currently locked resources
 - A task can preempt another task if
 - · it has the highest priority and
 - its preemption level is higher than the system ceiling

- A task can preempt another task if
 - it has the highest priority and
 - its preemption level is higher than the system ceiling

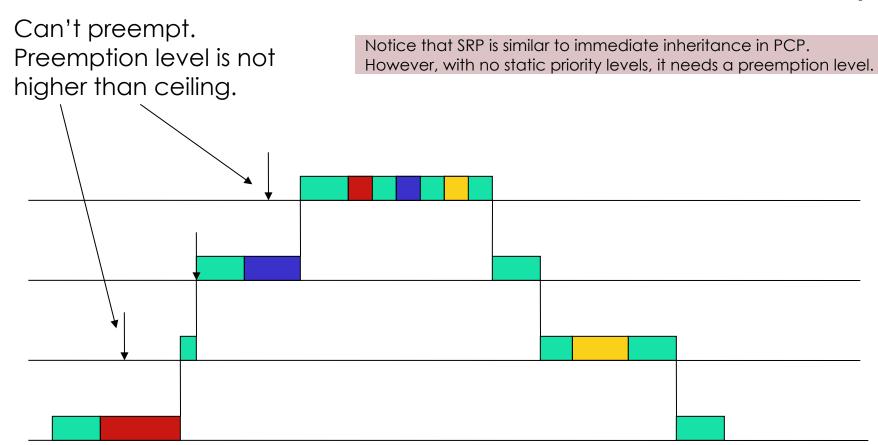
SRP Preemption Test:

- A task is not permitted to preempt until its priority is the highest among those of all the tasks ready to run, and its preemption level is higher than the system ceiling
- Whenever a task arrives or C(t) decreases (a resource is released), perform preemption test on highest priority task (top of ready queue) T_h
- If $\psi_h > C(T)$ then allow T_h to preempt, otherwise block it

Priority ceiling vs. stack-based resource policy



Priority ceiling vs. stack-based resource policy



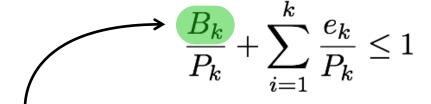
- Q: What does it mean when a task passes the preemption test?
- A: the resources that are currently available are sufficient to satisfy the maximum requirement of task T_h and the maximum requirement of every task that could preempt T_h .
 - This means that once T_h starts executing, it will never be blocked for resource contention.

Remarks

- SRP avoids deadlocks. Why?
- Resources are only allocated when a task requests them, not when it preempts
- A task can be blocked by the preemption test even though it does not require any resource. This is needed to avoid unbounded priority inversion.
- The preemption test has the effect of imposing priority inheritance
 - an executing task that holds a resource modifies the system ceiling and resists preemption
 as though it inherits the priority of any tasks that might need that resource
 46

Analysis with EDF and SRP

As simple as other protocols



For task T_k

Maximum blocking due to task with lower preemption level; in the case of EDF: with period P_j such that $P_k < P_j$.

Tasks are sorted such that the task with shortest period is T_1 and so on.

What is the "stack" in Stack-based Resource Sharing Protocol?

• Two things:

- 1. Can be implemented using a stack. How?
- 2. Allows tasks to share the run-time stack

Highlights

- Schedulability analysis needs to account for blocking due to low priority tasks
- Priority inheritance protocol (PIP) may not prevent deadlocks
- Deadlocks can be prevented with the priority ceiling protocol (PCP)
- To deal with dynamic priority policies (such as EDF), we need a different policy: the stack-based resource policy (SRP)
- SRP (and the immediate inheritance version of the PCP) have efficient implementations
 - Reduce the number of context switches
 - SRP also prevents deadlocks (note the similarities between PCP and SRP)