Periodic task scheduling

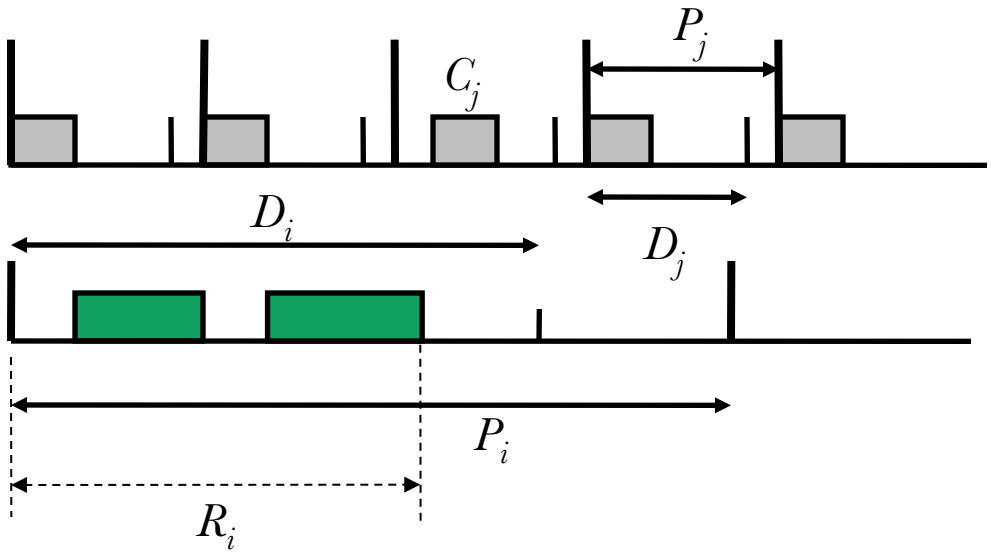
An exact test for EDF
Relative deadlines less than periods
Processor demand criterion

The earliest deadline first policy

- Optimal scheduling policy when relative deadlines are equal to task periods
- Is a task-dynamic priority policy (but not *job*-dynamic)
- How does it behave ***when relative deadlines are less than periods***?
- Exact analysis for EDF

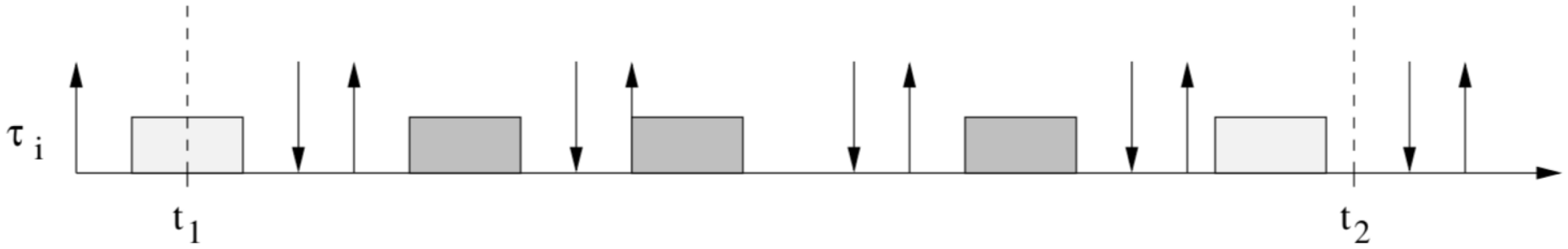
Why not analyze task interference as we did with DM?

- Interference is due to only those jobs with **earlier absolute deadlines**
- Task j exercises interference on task i, and task i exercises interference on task j
- Interference relations are complex due to dynamic priorities → *will abandon this view*



EDF and processor demand

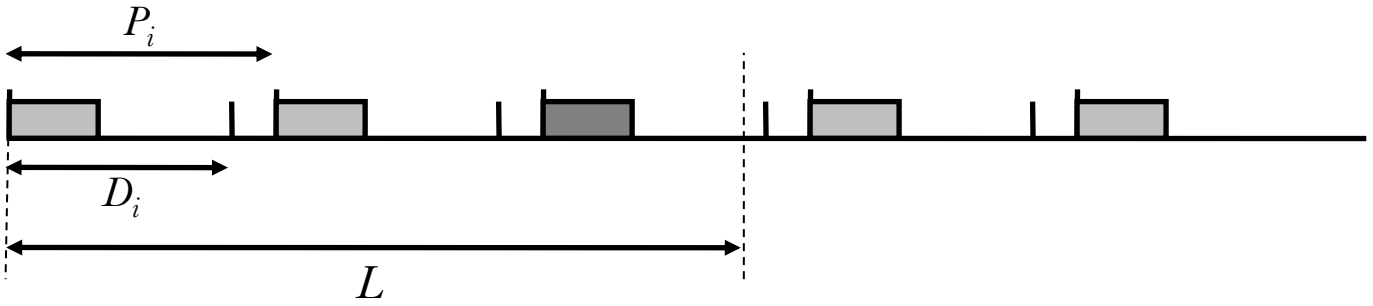
- Task i **requests** service (work) C_i at every arrival $r_{i,k}$
- A request becomes ***demand*** only at the deadline $d_{i,k}$
  - Before the deadline, we are not obliged to complete the request from a feasibility perspective



$$\text{demand}_i(t_1, t_2) = \sum_{\substack{k \geqslant 0 : r_{i,k} \geqslant t_1, \\ d_{i,k} \leqslant t_2}} C_i$$

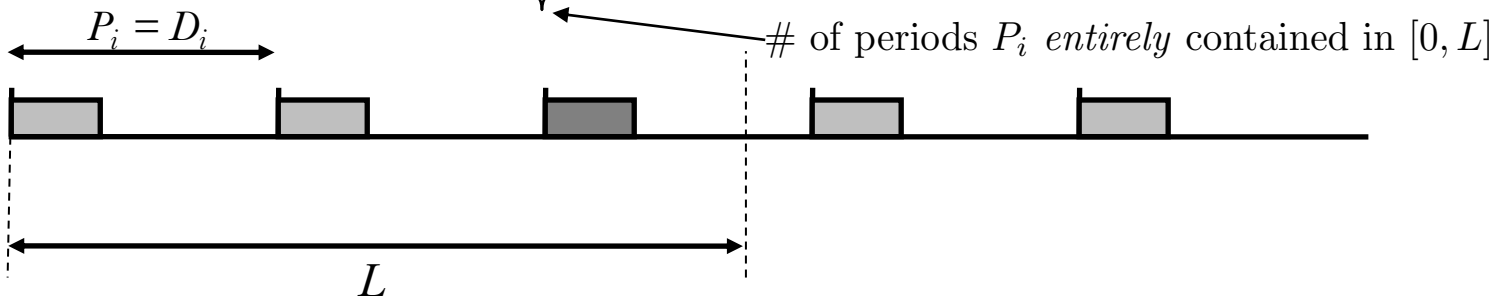
EDF and processor demand

- Task i **requests** service (work) C_i at every arrival $r_{i,k}$
- What is the demand in $[0,\ L]$ for $L > 0$? $(t_1 = 0 \text{ and } t_2 = L)$



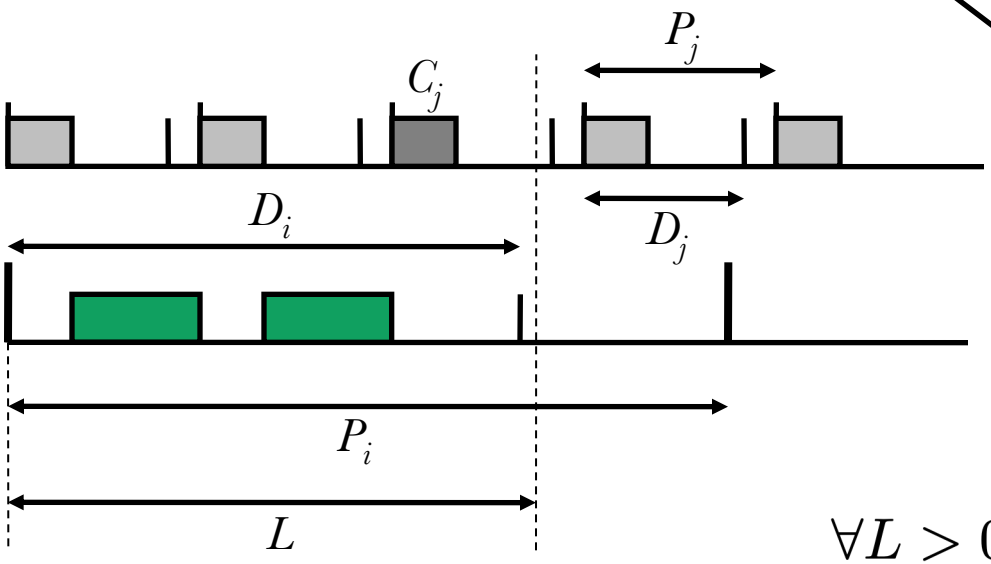
$$\text{demand}_i(0, L) = \sum_{k \geqslant 0 : d_{i,k} \leqslant L} C_i = \left\lfloor \frac{L + P_i - D_i}{P_i} \right\rfloor C_i$$

If $P_i = D_i \Rightarrow \text{demand}_i(0, L) = \underbrace{\left\lfloor \frac{L}{P_i} \right\rfloor}_{\#\text{ of periods } P_i \textit{ entirely} \text{ contained in } [0, L]} C_i$



EDF and processor demand

- Consider demand on the processor due to tasks whose deadlines have passed
- Within any time interval, L, the demand must be less than L.

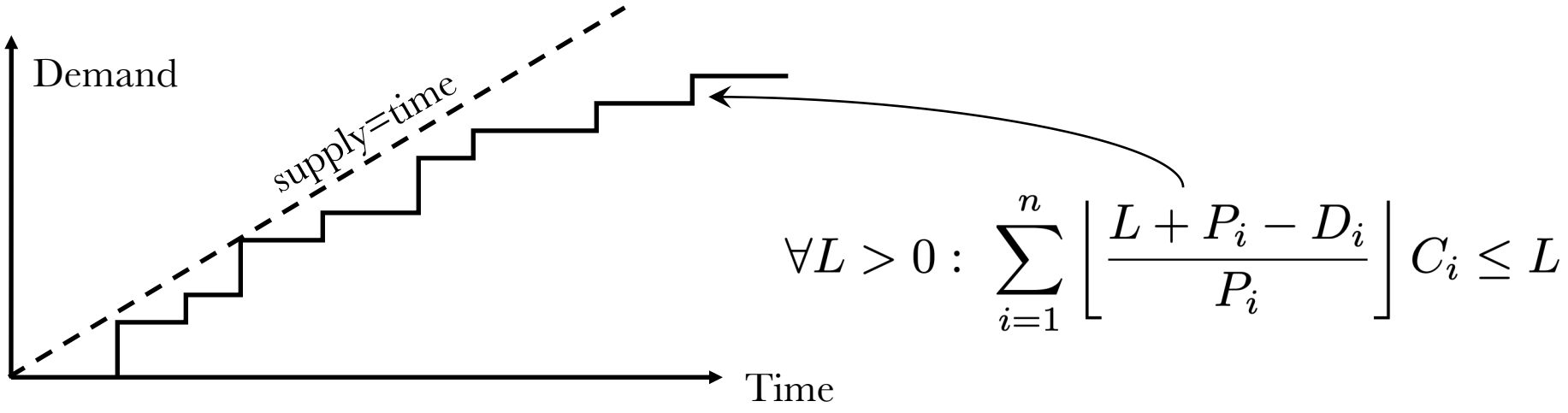


The execution time demanded by jobs with deadline less than L over an interval of length L cannot be greater than L

$$\forall L > 0 : \sum_{i=1}^{n} \left\lfloor \frac{L + P_i - D_i}{P_i} \right\rfloor C_i \leq L$$

Processor demand

- Checking the schedulability for all time instants is not possible
  - Overwhelming complexity
- **Observation 1:** Sufficient to check up to the hyperperiod (schedule repeats itself)
- **Observation 2:** Check only at absolute deadlines. Why?



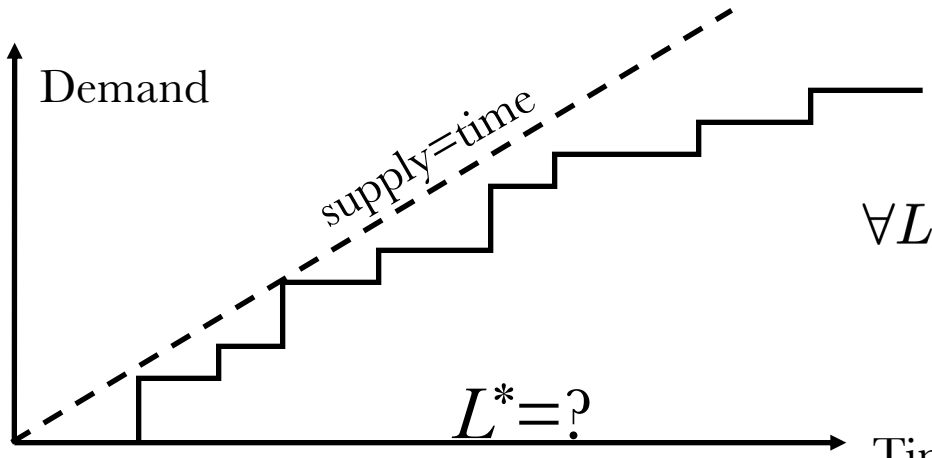
$$\forall L > 0 : \sum_{i=1}^{n} \left\lfloor \frac{L + P_i - D_i}{P_i} \right\rfloor C_i \leq L$$

The map $t \mapsto \left\lfloor \frac{t+P_i-D_i}{P_i} \right\rfloor$ is a *staircase* function with jumps (discontinuities) at absolute deadlines $d_{i,k} = (k-1)P_i + D_i$

Processor demand

- Checking the schedulability for at all time instants is not possible
  - Overwhelming complexity
- **Observation 1:** Sufficient to check up to the hyperperiod (schedule repeats itself)
- **Observation 2:** Check only at absolute deadlines
- Can we do better?

Find a point L^* such that if demand$(0, L^*) \leq L$ for all $L \leq L^*$, then demand$(0, L) \leq L$ for all $L \geq L^* \rightarrow$ sufficient to test only up to L^*



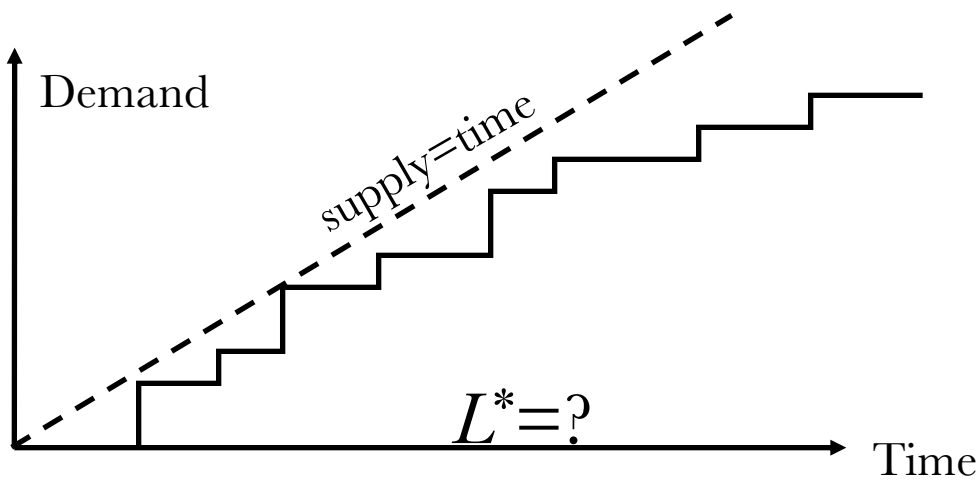
$$\forall L > 0 : \sum_{i=1}^{n} \left\lfloor \frac{L + P_i - D_i}{P_i} \right\rfloor C_i \leq L$$

Processor demand

- Deriving L^*

$$\sum_{i=1}^{n} \left\lfloor \frac{L + P_i - D_i}{P_i} \right\rfloor C_i \leq L$$

$$\sum_{i=1}^{n} \left\lfloor \frac{t + P_i - D_i}{P_i} \right\rfloor C_i \leq \sum_{i=1}^{n} \frac{t - D_i + P_i}{P_i} C_i = tU + \sum_{i=1}^{n} (P_i - D_i) U_i$$

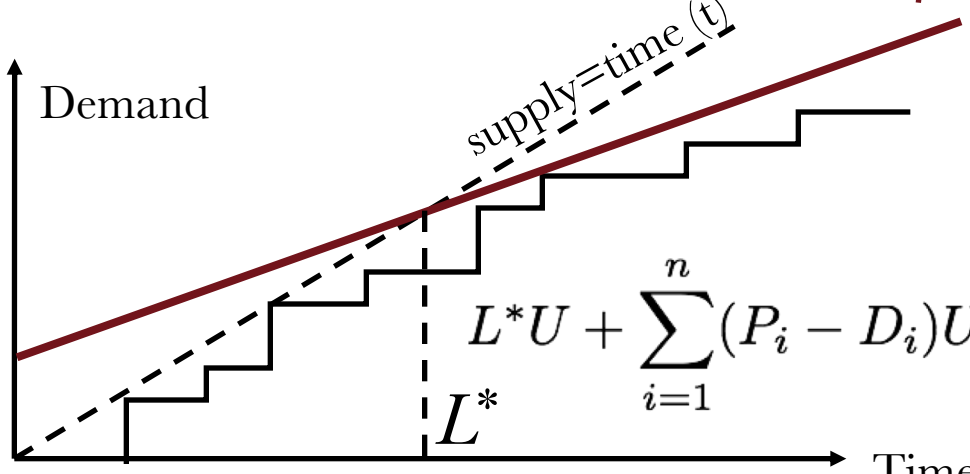


Processor demand

- Deriving L^*

$$\sum_{i=1}^{n} \left\lceil \frac{L + P_i - D_i}{P_i} \right\rceil C_i \leq L$$

$$\sum_{i=1}^{n} \left\lceil \frac{t + P_i - D_i}{P_i} \right\rceil C_i \leq \sum_{i=1}^{n} \frac{t - D_i + P_i}{P_i} C_i = tU + \sum_{i=1}^{n} (P_i - D_i) U_i$$



$$L^*U + \sum_{i=1}^{n} (P_i - D_i) U_i = L^* \Rightarrow L^* = \frac{\sum_{i=1}^{n} (P_i - D_i) U_i}{1 - U}$$

Processor demand criterion

- Check if $$\sum_{i=1}^{n} \left\lfloor \frac{L + P_i - D_i}{P_i} \right\rfloor C_i \leq L$$
  - for all L that are absolute deadlines in the interval $[0,\ L^*]$
  - where $$L^* = \frac{\sum_{i=1}^{n} (P_i - D_i) U_i}{1 - U}$$

- This is an **exact** test for EDF when *relative deadlines are **less** than task periods*
- *What is the complexity of this test? Is it polynomial-time?*
- **Complexity:** Testing constrained-deadine EDF-schedulability is **coNP**-hard. (Eisenbrand & Rothvoß - SODA'10)

Example using processor demand

- Consider the following task set
  - T_1: (C_1=1, P_1=3, D_1=2)
  - T_2: (C_2=2, P_2=7, D_2=5.5)
  - T_3: (C_3=2, P_3=10, D_3=6)
- The task set has a hyperperiod of 210
- However, we only need to test deadlines up to $L^* = \frac{\sum_{i=1}^{n}(P_i - D_i)U_i}{1-U}$
- U = 86/105 = 0.8190; L^* = 8.63
- At $L = 2$:

$$\lfloor \frac{L + P_1 - D_1}{P_1} \rfloor C_1 = \lfloor \frac{2+3-2}{3} \rfloor 1 = 1$$

Example using processor demand

- Consider the following task set
  - T_1: (C_1=1, P_1=3, D_1=2)
  - T_2: (C_2=2, P_2=7, D_2=5.5)
  - T_3: (C_3=2, P_3=10, D_3=6)
- The task set has a hyperperiod of 210
- However, we only need to test deadlines up to $L^* = \frac{\sum_{i=1}^{n}(P_i - D_i)U_i}{1-U}$
- U = 86/105 = 0.8190; L^* = 8.63
- At *L=5.5*: (also need to check at *L=5*; not shown here)

$$\lfloor \frac{L+P_1-D_1}{P_1} \rfloor C_1 + \lfloor \frac{L+P_2-D_2}{P_2} \rfloor C_2 = \lfloor \frac{5.5+3-2}{3} \rfloor 1 + \lfloor \frac{5.5+7-5.5}{7} \rfloor 2 = 2+2 = 4$$

Example using processor demand

- Consider the following task set
  - T_1: (C_1=1, P_1=3, D_1=2)
  - T_2: (C_2=2, P_2=7, D_2=5.5)
  - T_3: (C_3=2, P_3=10, D_3=6)
- The task set has a hyperperiod of 210
- However, we only need to test deadlines up to $L^* = \frac{\sum_{i=1}^{n}(P_i - D_i)U_i}{1-U}$
- U = 86/105 = 0.8190; L^* = 8.63
- At L=6:

$$\lfloor \frac{6+3-2}{3} \rfloor 1 + \lfloor \frac{6+7-5.5}{7} \rfloor 2 + \lfloor \frac{6+10-6}{10} \rfloor 2 = 6$$

Example using processor demand

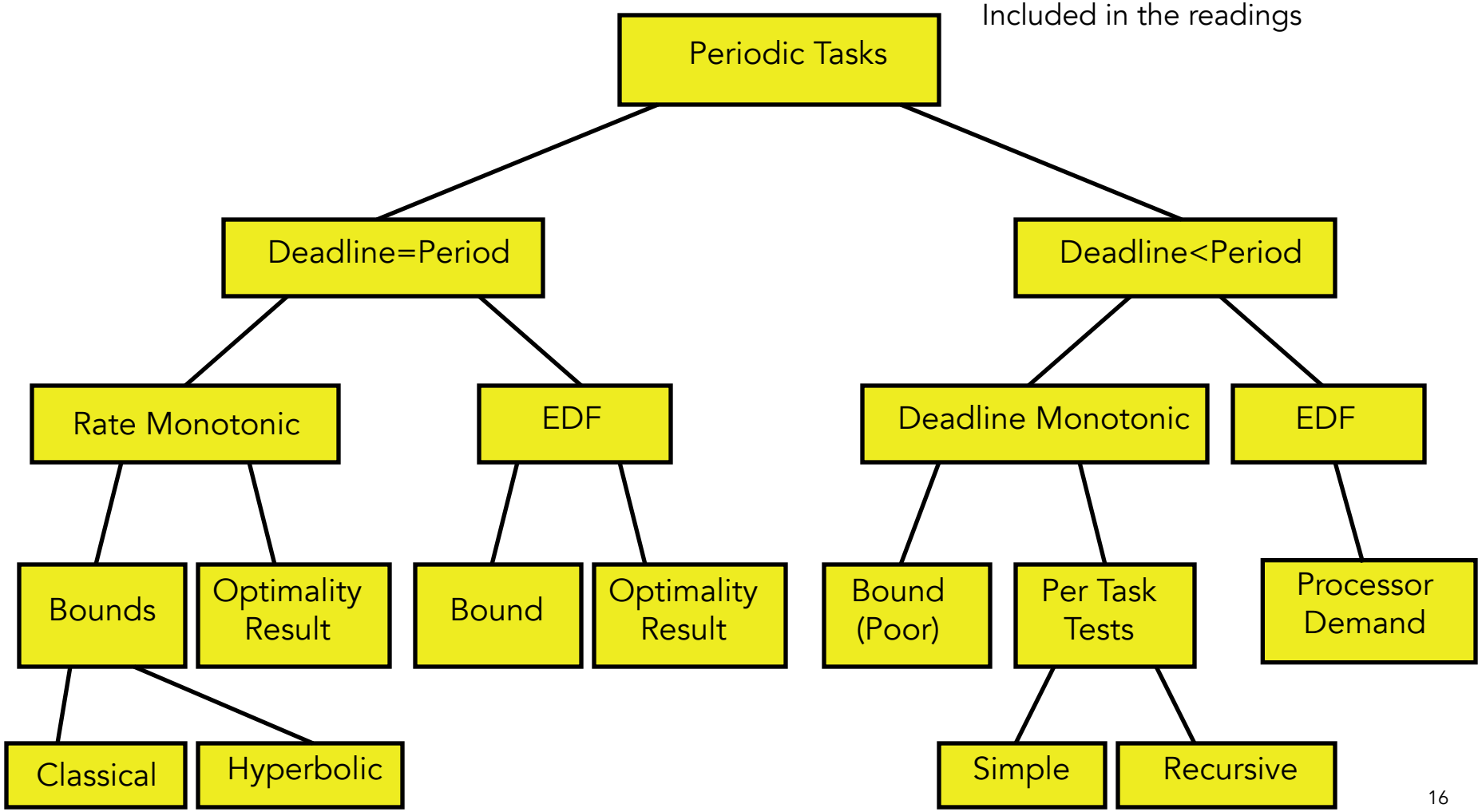
- Consider the following task set
  - T_1: (C_1=1, P_1=3, D_1=2)
  - T_2: (C_2=2, P_2=7, D_2=5.5)
  - T_3: (C_3=2, P_3=10, D_3=6)
- The task set has a hyperperiod of 210
- However, we only need to test deadlines up to $L^* = \frac{\sum_{i=1}^{n}(P_i - D_i)U_i}{1-U}$
- U = 86/105 = 0.8190; L^* = 8.63
- At L=8:

$$\lfloor \frac{8+3-2}{3} \rfloor 1 + \lfloor \frac{8+7-5.5}{7} \rfloor 2 + \lfloor \frac{8+10-6}{10} \rfloor 2 = 3 + 2 + 2 = 7$$

No more absolute deadlines < 8.63. We are done!

Topics covered so far

Chapter 4 of Buttazzo's text
Included in the readings



RM vs. EDF: Judgement Day

- Which policy incurs more preemptions?
- Which has the edge in implementation simplicity and running time complexity?
- How does each behave under *transient* and *permanent* overload conditions?
  - Is one more predictable than the other when overload happens? In what sense?
- Release jitter

Common misconceptions

- EDF incurs more preemptions than RM
- RM is easier to analyze than EDF
- RM introduces less runtime overhead
- EDF is not predictable in transient and permanent overloads
- RM is more predictable in overload conditions, and causes less jitter in task execution

All these statements are either wrong or not precise!
We need to look more carefully into the situation

Debunking the Myths

Implementation Cost

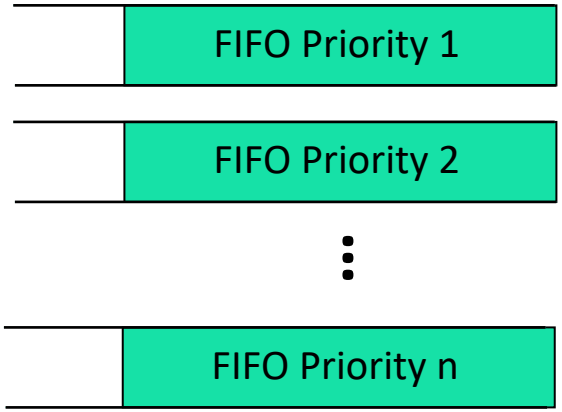
- For a precise comparison, must distinguish two situations
  - **Case 1:** Scheduler is developed on top of a generic priority-based operating system
  - **Case 2:** Scheduler is developed from scratch

Implementation Cost

- **Case 1:** Scheduler is developed on top of a generic priority-based operating system (limited number of priority levels available)
- **RM wins!**
  - Most commercial kernels contain a *fixed* number or priority levels
- Consider the case in which two deadlines d_a and d_b are mapped into two adjacent priority levels
- A new periodic instance is released with absolute deadline d_c s.t. $d_a < d_c < d_b$.
- There is no priority level that can be selected to map d_c, even when the number of active tasks is less than the number of priority levels in the kernel.
- **Solution:** remap d_a and d_b into two new priority levels that are not consecutive → in worst case, all current deadlines may need to be remapped (high cost)

Implementation Cost

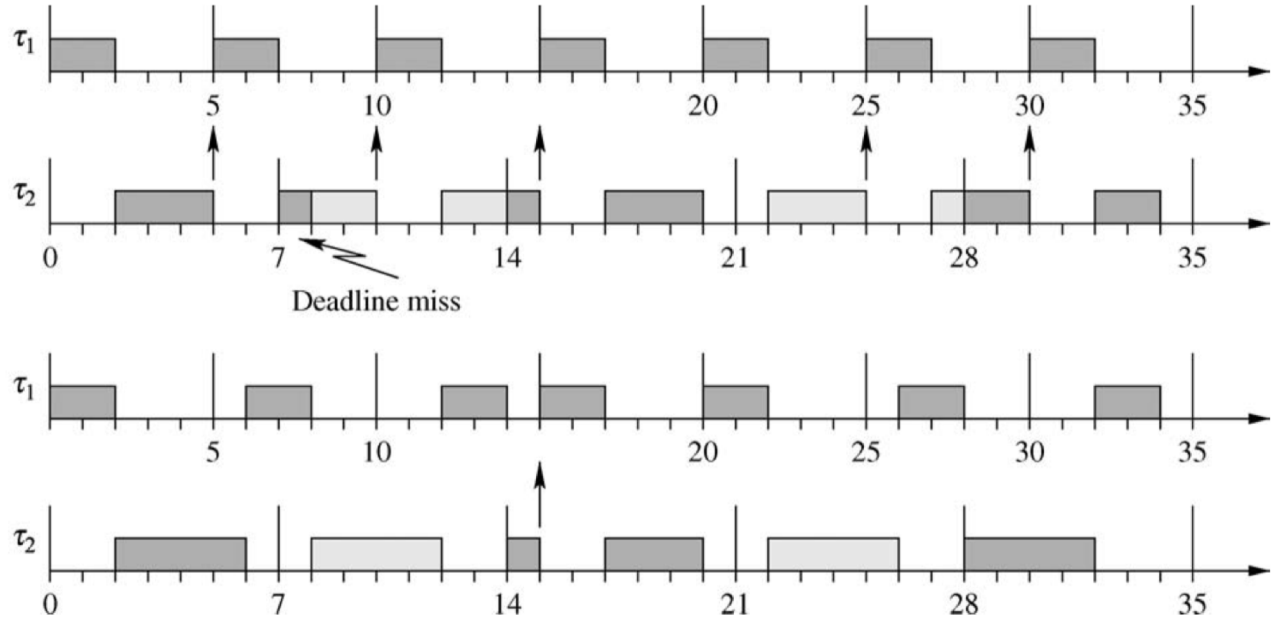
- **Case 2:** Scheduler is developed from scratch
- **A tie!**
- **RM has a tiny edge**: If # of priority levels is low, can split ready queue into separate FIFO queues, one for each priority level → Insertion takes $O(1)$



- This is not possible for EDF → number of queues would be too large (e.g., equal to 2^{32} if system **time** is represented by four byte variables).

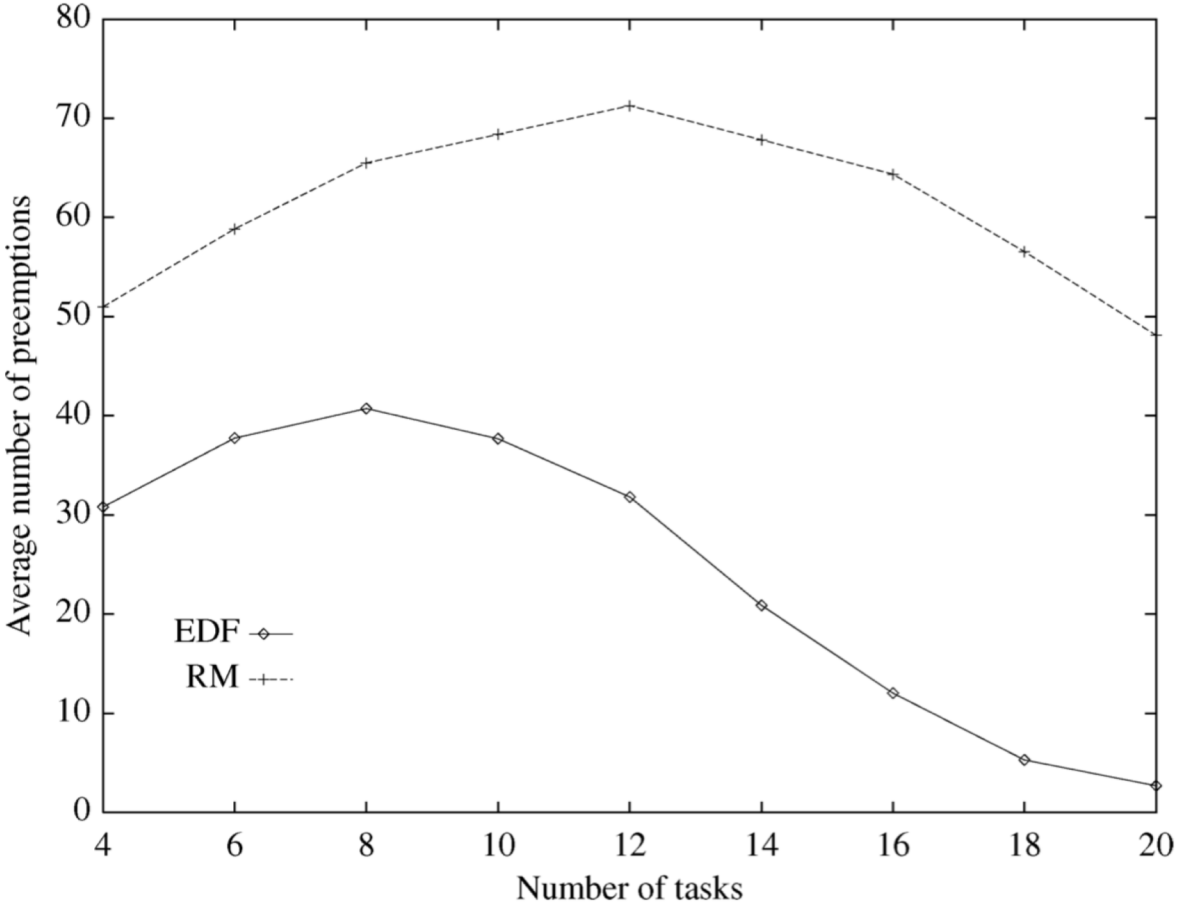
Runtime Cost

- EDF incurs ***less*** context switches than RM!



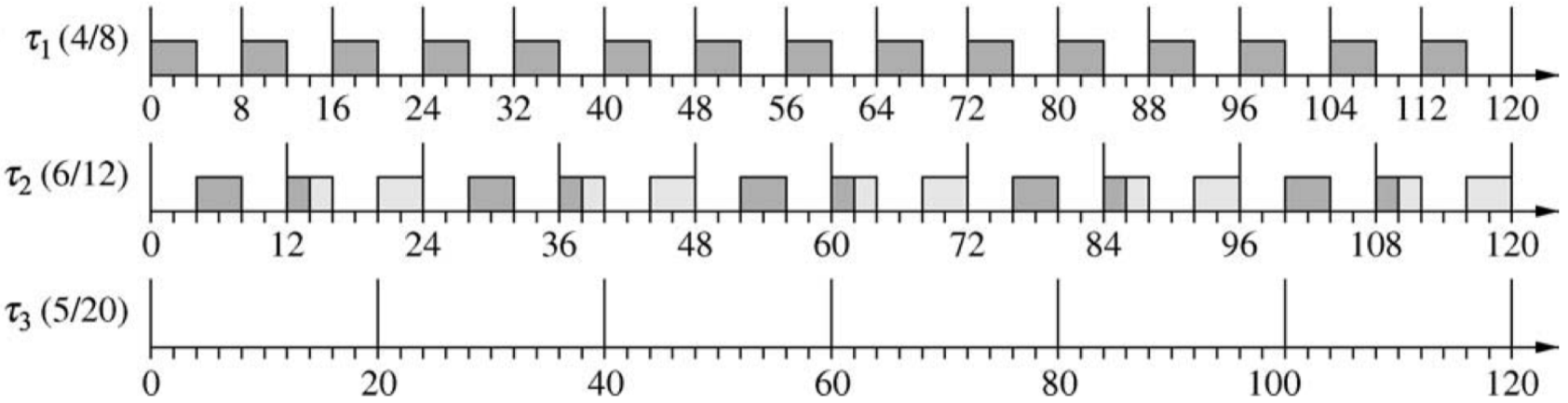
Context Switching Cost: Interesting Phenomenon

- Utilization kept at 0.9 as number of tasks increases



Predictability and Stability: *Permanent* Overload

- *Permanent* Overload → $U > 1$
- Behavior of both RM and EDF *is predictable* in some sense
- **RM:** Might cause complete blocking of *lower* priority tasks



U = 1.25

RM

Predictability and Stability: *Permanent* Overload

- *Permanent* Overload → $U > 1$
- **EDF:** Interesting phenomenon happens; *Period rescaling*
  - *In the long run, EDF will behave such that it appears that tasks were released at a lower rate* → $\overline{P}_i = UP_i$

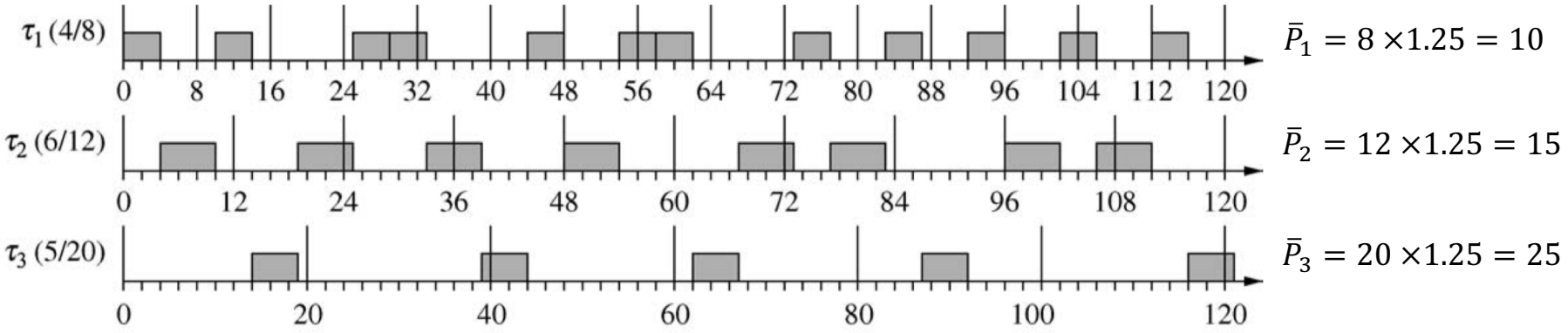
Theorem 1 [Cervin]. *Assume a set of n periodic tasks, where each task is described by a fixed period T_i, a fixed execution time C_i, a relative deadline D_i, and a release offset Φ_i. If $U > 1$ and tasks are scheduled by EDF, then, in stationarity, the average period $\bar{T}_i$ of each task τ_i is given by $\bar{T}_i = T_i U$.*

$$\overline{T}_i := \lim_{t \to \infty} \frac{t}{k_i(t)} = T_i U$$

$k_i(t)$: # of instances of task τ_i that *completed* up to time $t > 0$

Predictability and Stability: *Permanent* Overload

- *Permanent* Overload → $U > 1$
- **EDF** *Period rescaling*



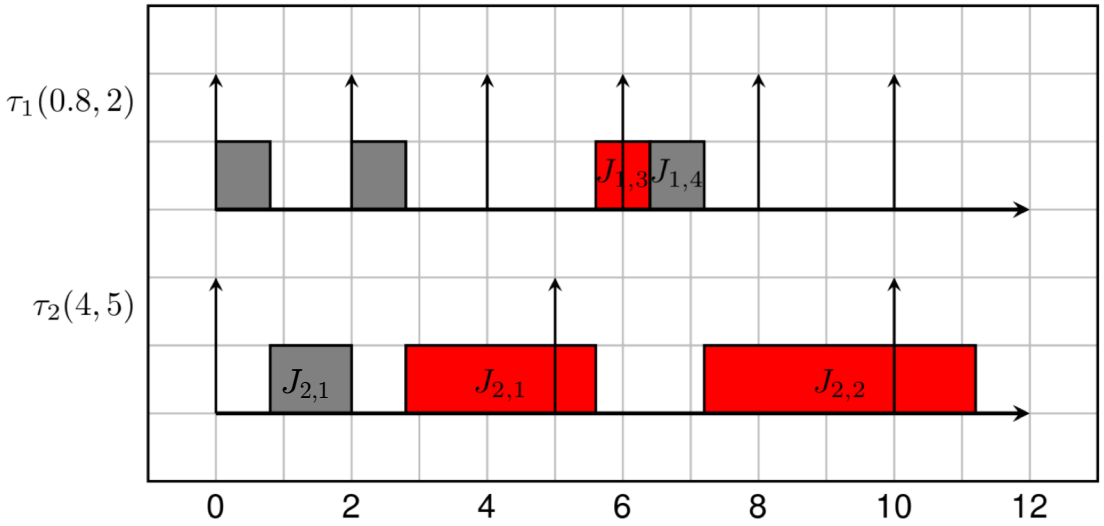
U = 1.25

EDF

$$\overline{U} = \frac{c_1}{\bar{P}_1} + \frac{c_2}{\bar{P}_2} + \frac{c_3}{\bar{P}_3} = \frac{4}{10} + \frac{6}{15} + \frac{5}{25} = 1$$

A deadline miss in EDF *can* be dangerous

- If job misses deadline it executes at a very high priority level (higher than all tasks that have not missed their deadlines)
- If late job allowed to continue, will cause other jobs to miss their deadlines!



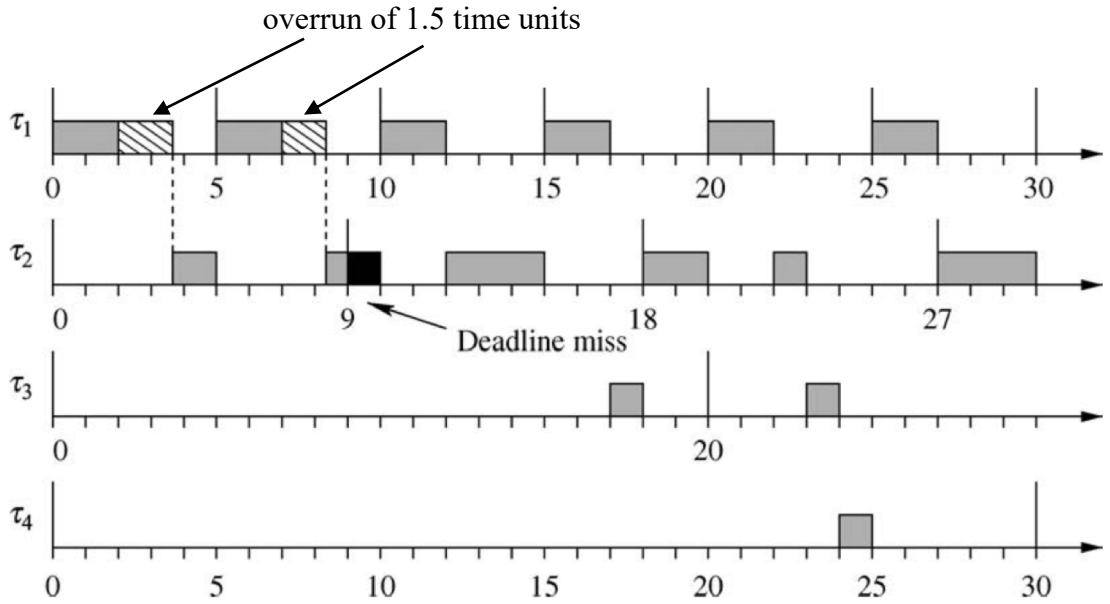
$U = 1.2$

After $J_{1,4}$, *every* job in *both* tasks will be late!

- Need overrun management strategy (application dependent)
  - E.g., schedule each late job at a lower priority than jobs that are not late
  - Drop late job altogether

Predictability and Stability: *Transient* Overload

- **Misconception:** In RM, in the presence of transient overload conditions, deadlines are missed predictably, that is, the ***first*** *tasks that fail are those with the* ***longest period*** (lowest priority tasks fail first)



If a task τ_i overruns, only the tasks with *higher priority than* τ_i are protected under **RM**, but nothing can be ensured for the other tasks.

RM schedule**:** It's **not** τ_4 that misses its deadline, but the higher priority τ_2!

- **EDF:** situation is *not* better than RM!

Handling aperiodic work

- EDF has the edge since it has higher utilization
- **Slack-stealing:** If U_p is utilization of periodic work, then the remaining $1 - U_p$ utilization can be used for aperiodic tasks
  - Increased utilization given to aperiodic servers (discussed later)

Lecture summary

- Exact test for EDF scheduling (relative deadlines < periods)
- Processor demand criterion
- RM vs. EDF

Giorgio C. Buttazo. **Rate monotonic vs. EDF: Judgment day**.
Real-Time Systems, 29:5–26. 2005