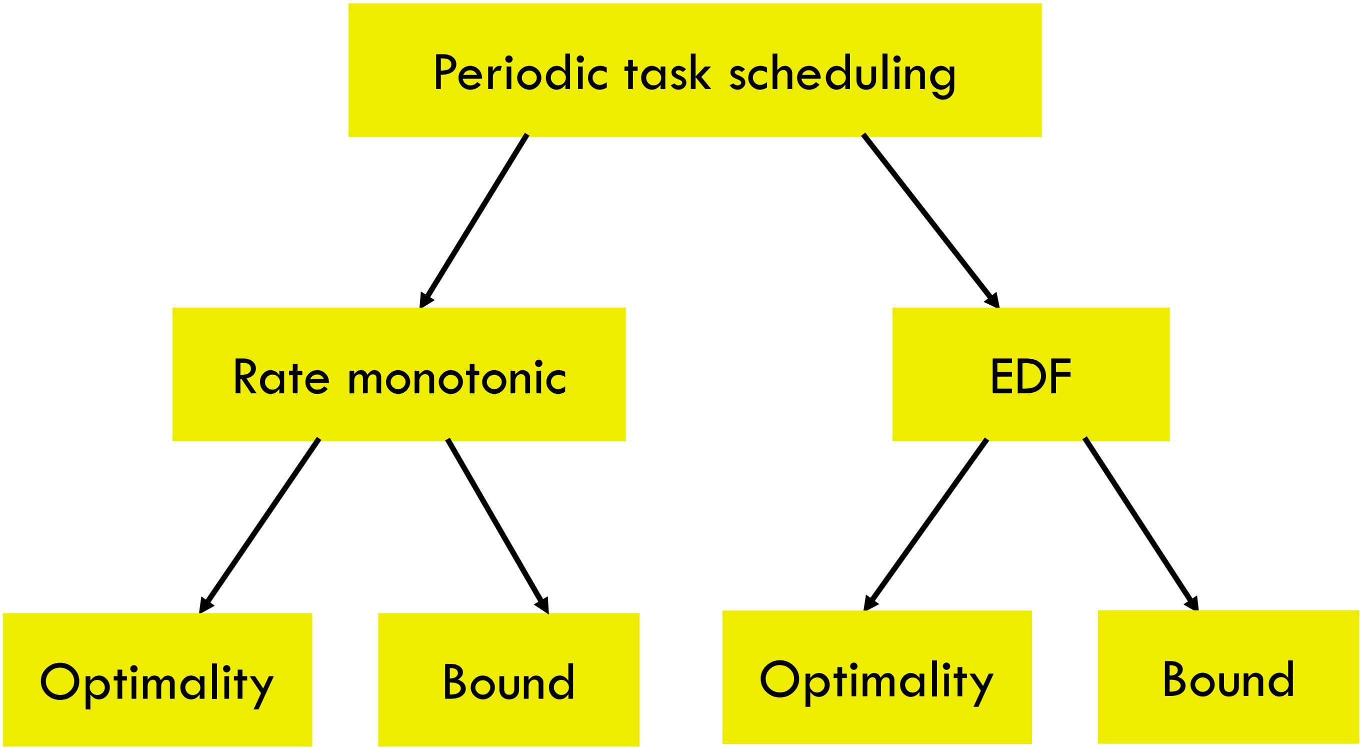
Periodic task scheduling

Optimality of rate monotonic scheduling (among static priority policies)
Utilization bound for EDF
Optimality of EDF (among dynamic priority policies)
Tick-driven scheduling (OS issues)

Lecture outline



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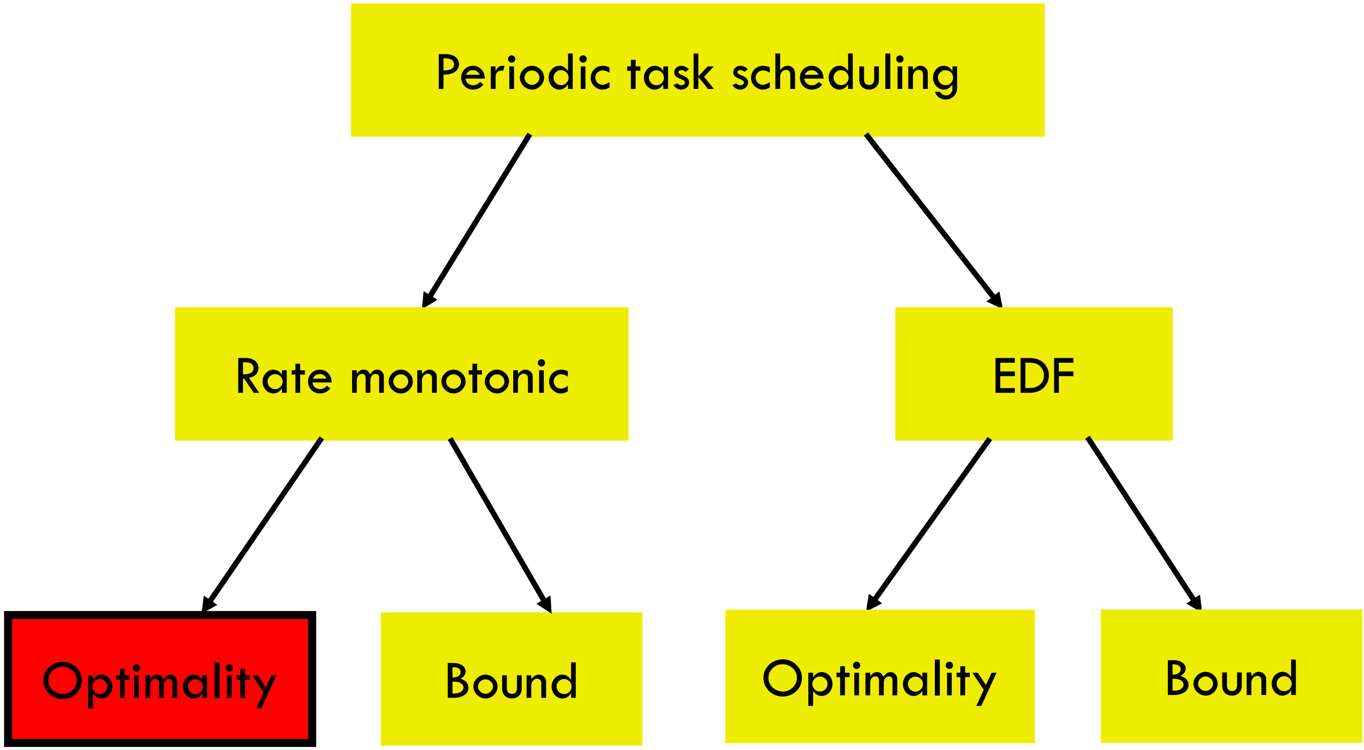

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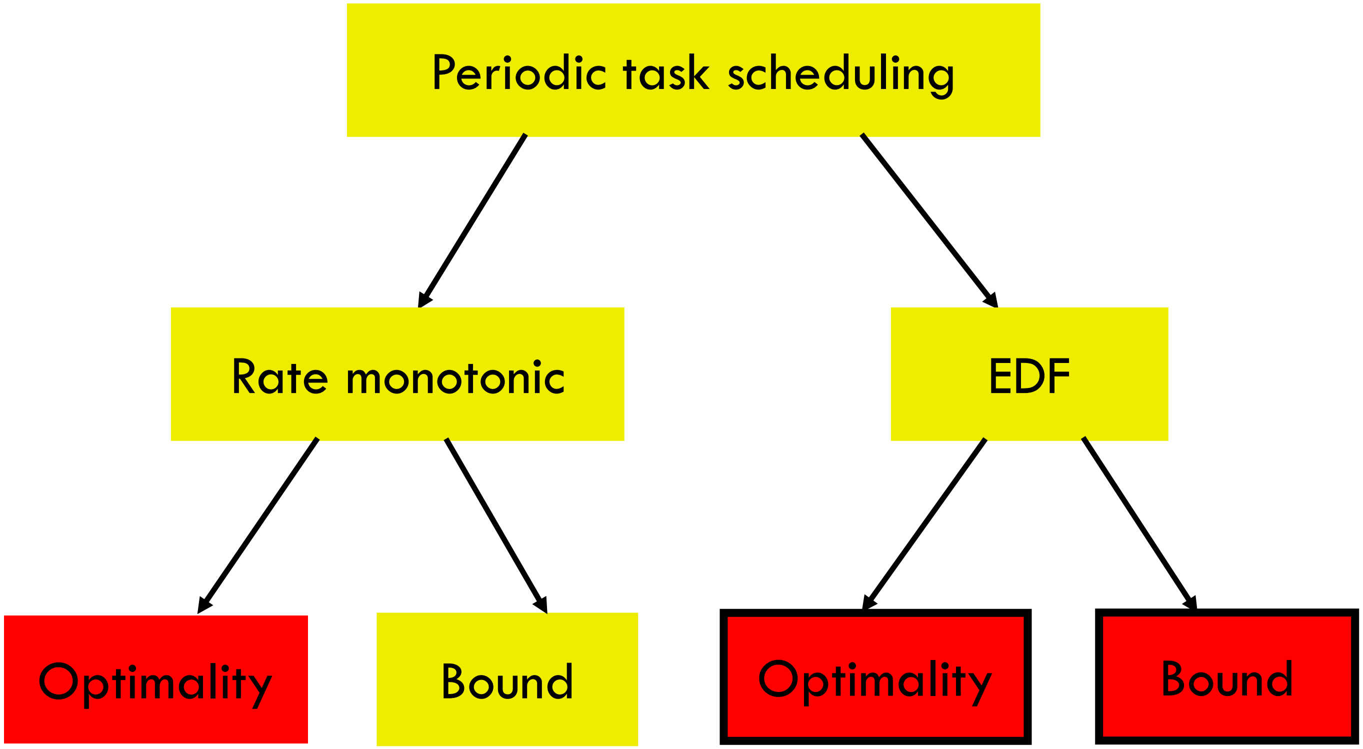

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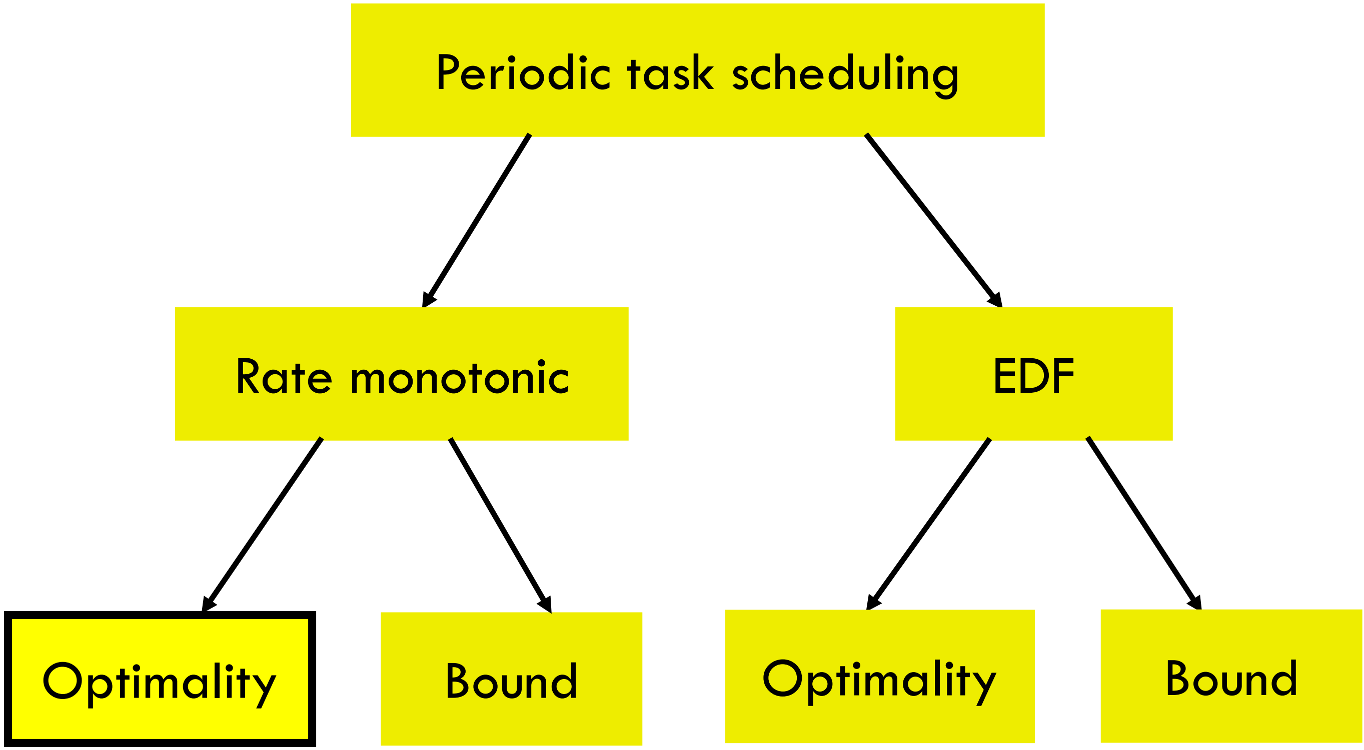

Periodic task scheduling
Rate monotonic
EDF
Optimality
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Rate monotonic scheduling

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  - Optimality (Trial #1):

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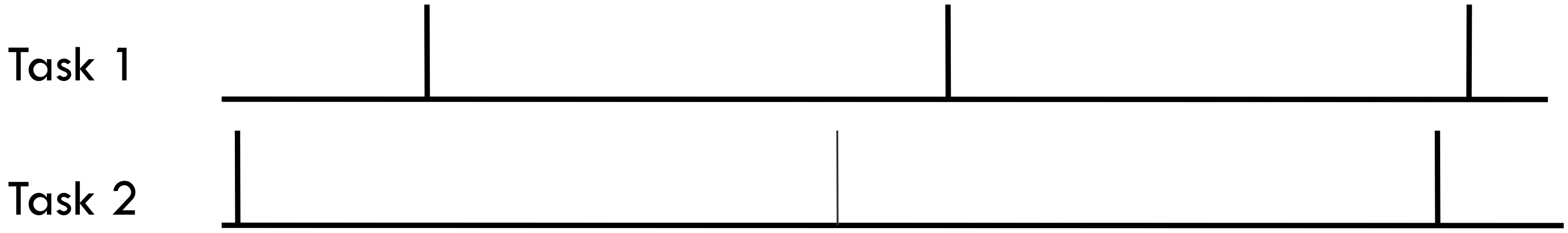
$$P_1 < P_2$$



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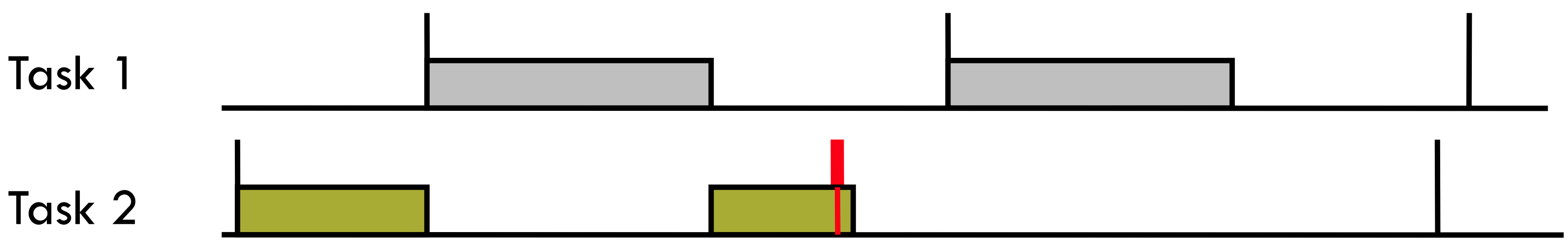

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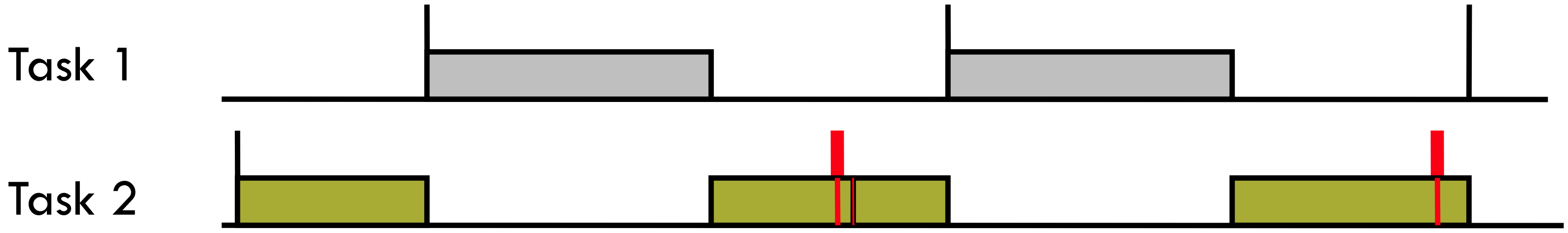

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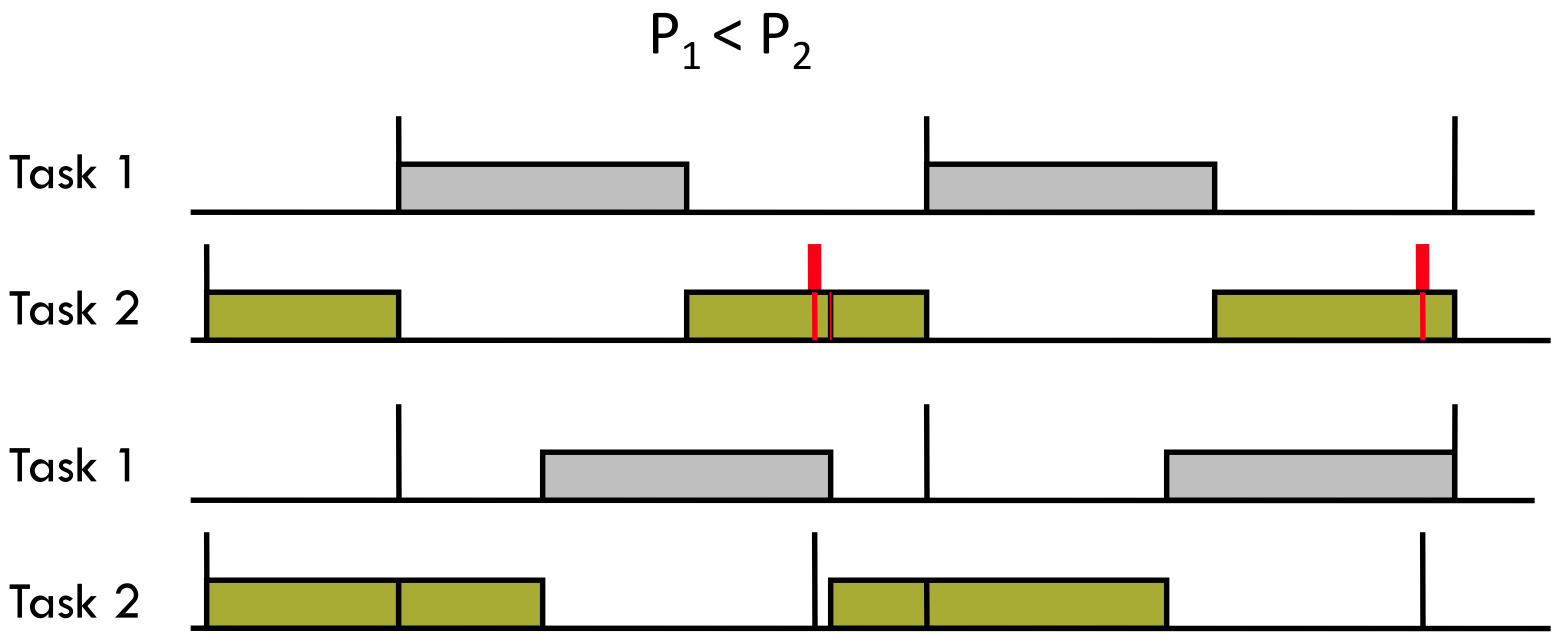


Rate monotonic scheduling

- Rate monotonic scheduling is the optimal fixed-priority (or static-priority) scheduling policy for periodic tasks.
  - Optimality (Trial #2): If any other fixed-priority scheduling policy can meet deadlines in the worst case scenario, so can RM.
- How do we prove it?

Rate monotonic scheduling

- Rate monotonic scheduling is the optimal fixed-priority (or static-priority) scheduling policy for periodic tasks.
  - Optimality (Trial #2): If any other fixed-priority scheduling policy can meet deadlines in the worst case scenario, so can RM.
- How do we prove it?
  - Consider the worst case scenario
  - Show that if someone else can schedule then RM can

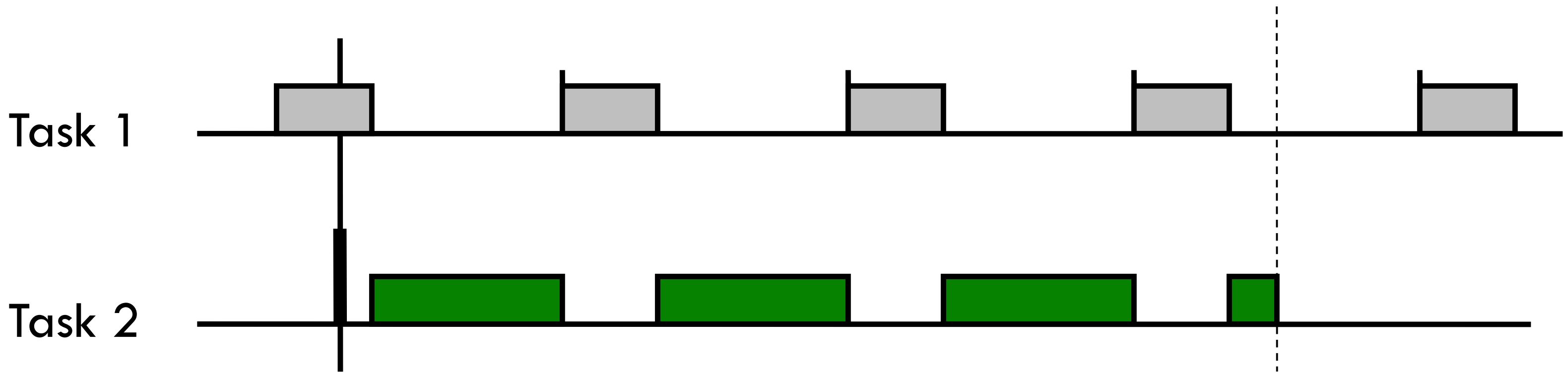
The worst-case scenario

- **Q:** When does a periodic task, *T*, experience the maximum delay?
  - Which arrival time produces the largest *response time* for *T*?
- **A:** When it arrives together with all the higher-priority tasks (critical instant)
  - Liu and Layland

- Idea for the proof
  - If some higher-priority task does not arrive together with *T*, aligning the arrival times can only increase the completion time of *T*.

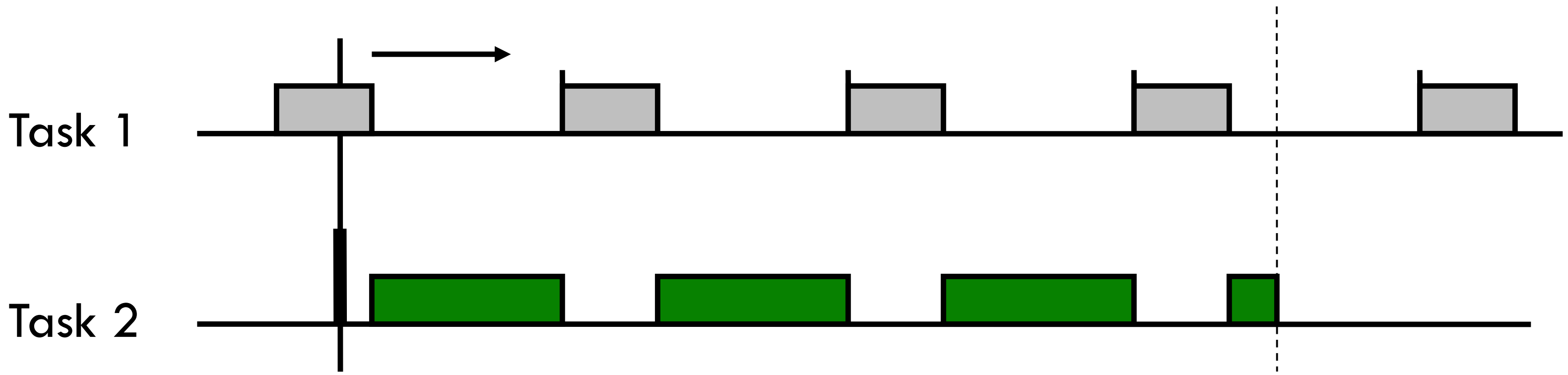
Critical instant theorem

Critical Instant: Proof (Case 1)



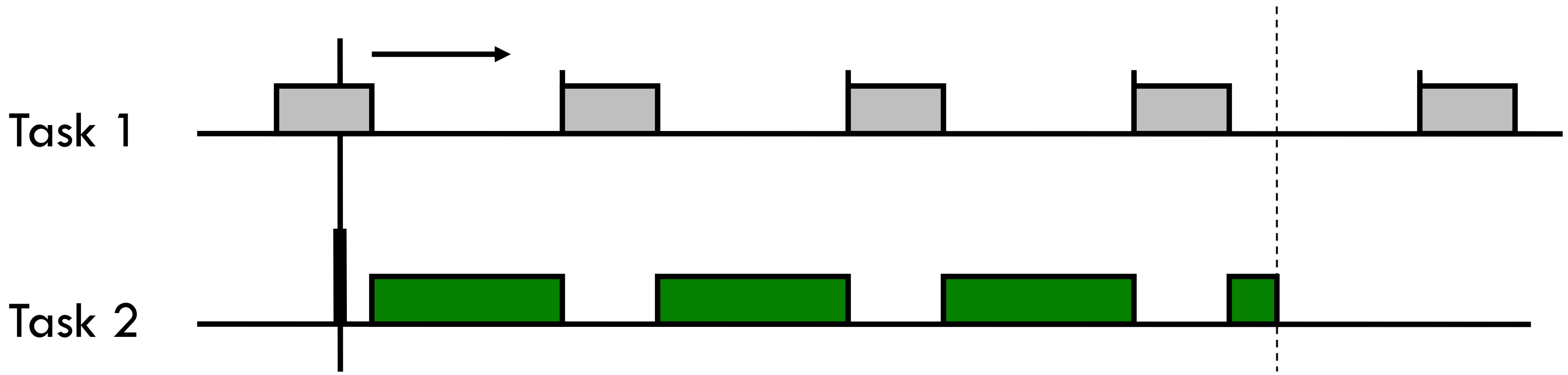
Case 1: Higher priority task 1 is running when task 2 arrives.

Critical Instant: Proof



Case 1: higher priority task 1 is running when task 2 arrives
→ shifting task 1 right will increase completion time of 2

Critical Instant: Proof



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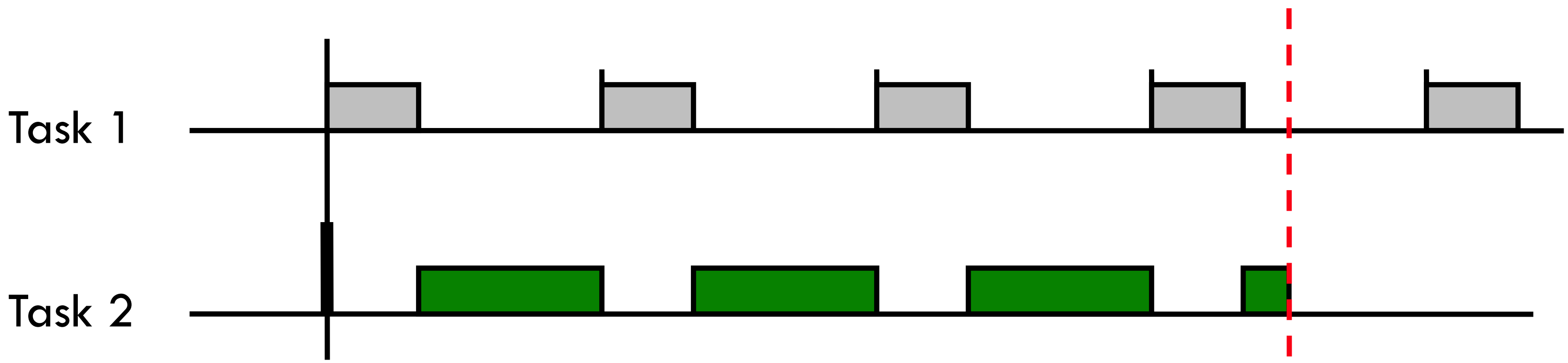
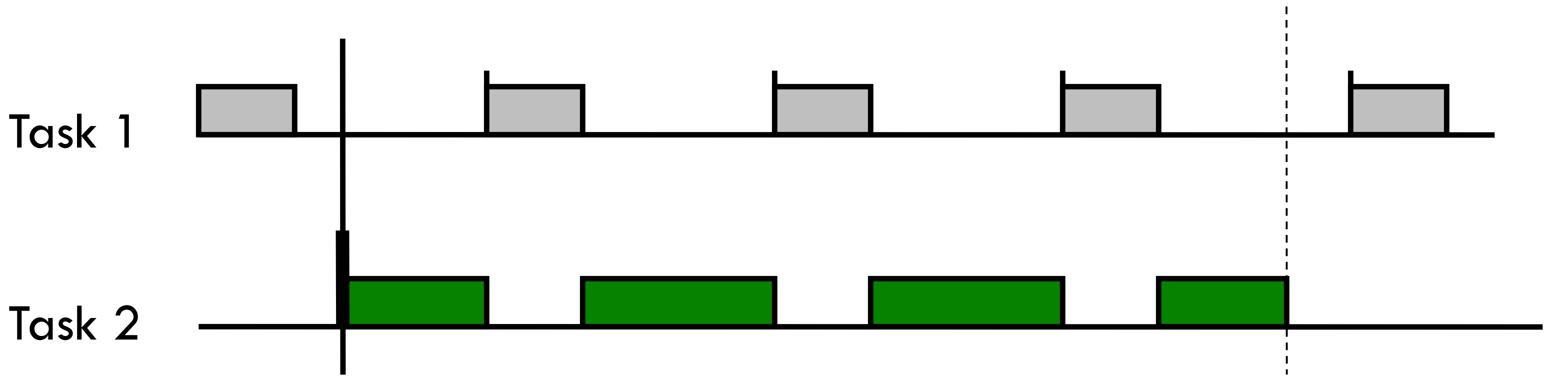
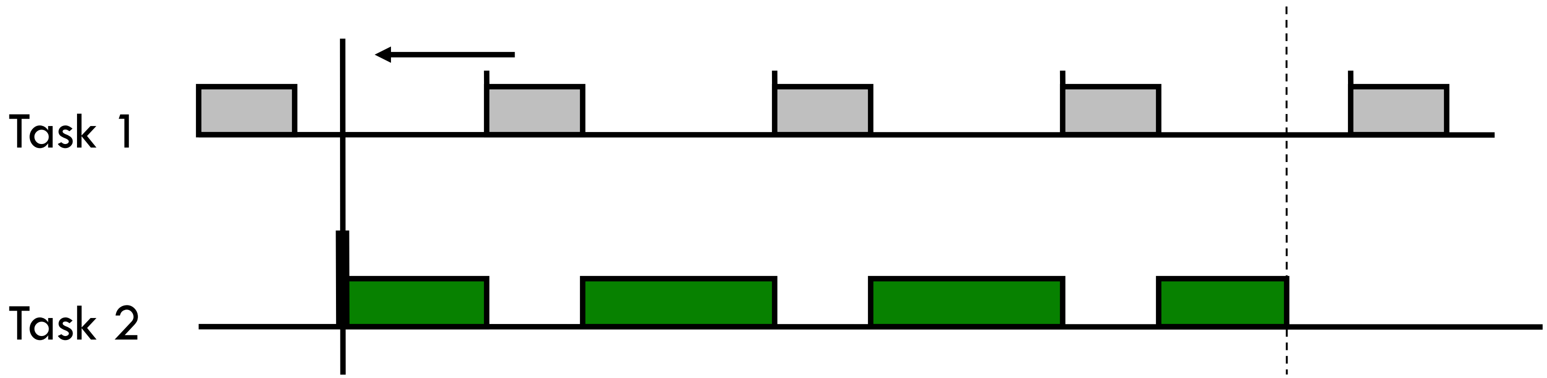


Critical Instant: Proof (Case 2)



Case 2: processor is idle when task 2 arrives

Critical Instant: Proof (Case 2)



Case 2: processor is idle when task 2 arrives
→ shifting task 1 left cannot decrease completion time of 2

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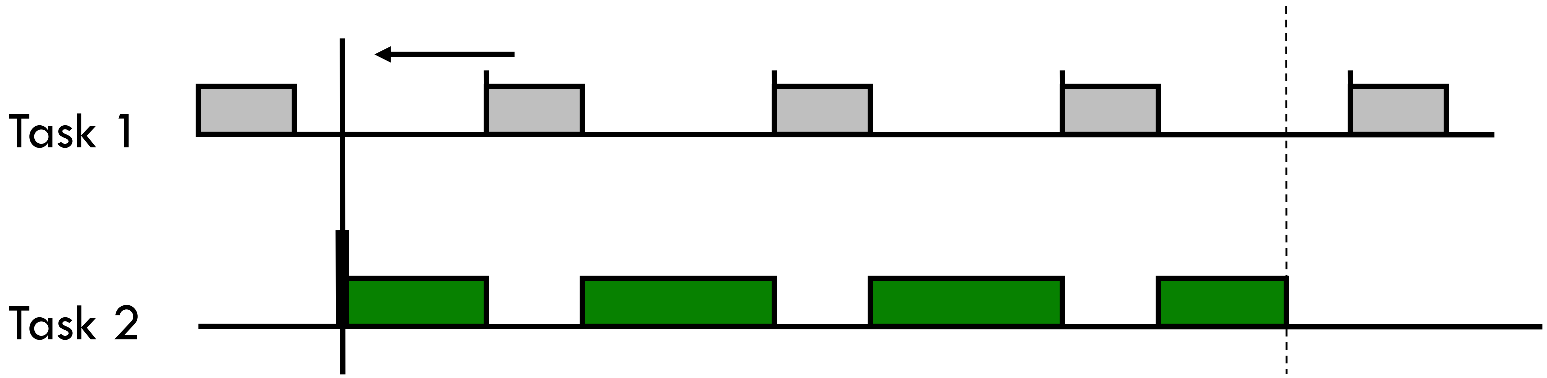

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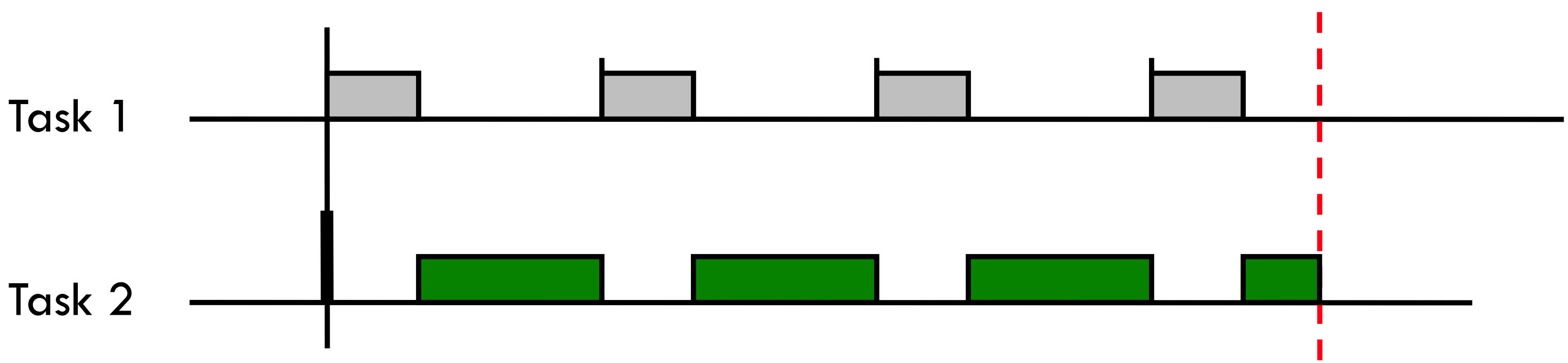


Critical Instant: Remarks

- ***All analyses hereafter will assume the critical instant theorem in effect***
- *Why is it important to identify the critical instant?*
  - *Characterizes the worst case scenario when a task experiences the max delay (remember the pitfall in trial #1 in proving RM optimality earlier?)*
  - *For task schedulability, need to reason only about the feasibility of the job arriving at the critical instant*

Critical Instant: Remarks

- If task **phases** are **not all** 0, does there always exist a point of simultaneous release? Can you come up with a counterexample?
  - It does not necessarily exist in this case and a simple counterexample of 3 tasks exists
- If not, then how easy it is to determine whether a point of simultaneous release exists for non-zero phase task sets?
  - An algorithm exists! (naïve approach takes exponential time, however)

Critical Instant: Remarks

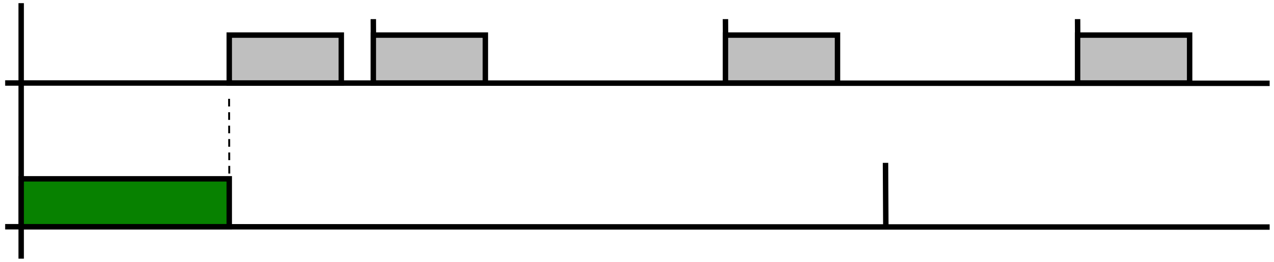
- Is RM optimal if a simultaneous release does not exist?
- What about EDF?
  - **Hint:** study the proof of optimality of EDF later and see whether the critical instant theorem was used.
- Does the critical instant theorem still hold in **non-preemptive** scheduling? (Assuming a simultaneous release exists)

Assumptions

- All scheduling is preemptive
- A simultaneous release exists even if tasks have non-zero phases
    - And thus critical instant theorem assumed
- Implicit deadlines (deadline = period)
- A task does not suspend itself (on I/O, for instance)
- All tasks in a task set are *independent* (there are no precedence relations and no resource constraints.)
- All overheads in the kernel are assumed to be zero (context switching and others)

Optimality of the RM policy

- If any other fixed-priority policy can meet deadlines so can RM



Policy X meets deadlines?

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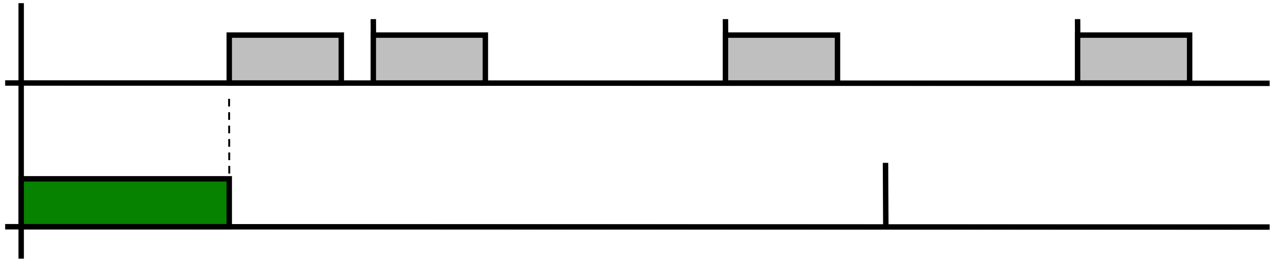
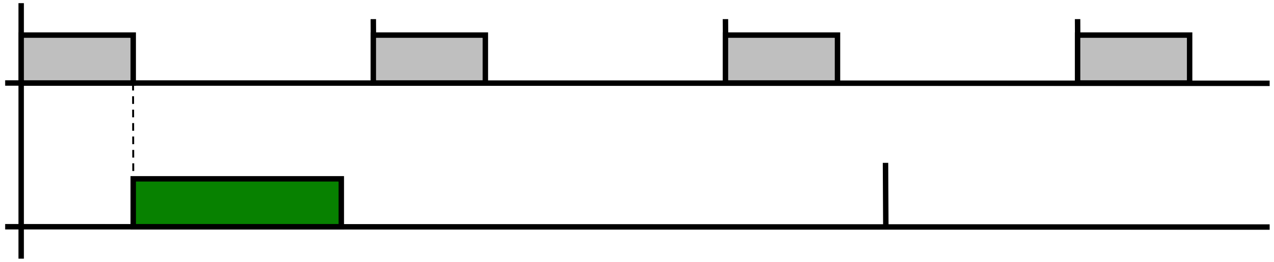


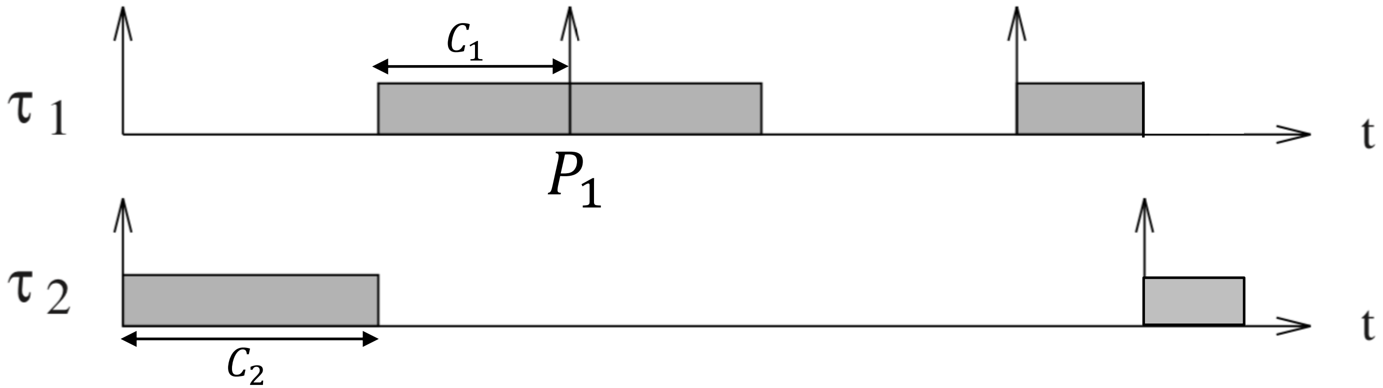
Policy X meets deadlines?

→ RM meets deadlines

YES



Optimality of the RM policy: Proof



Two tasks scheduled not according to RM

For feasibility in a non-RM policy, we need $C_1 + C_2 \leq P_1$ to hold at critical instant

Why?

Optimality of the RM policy: Proof

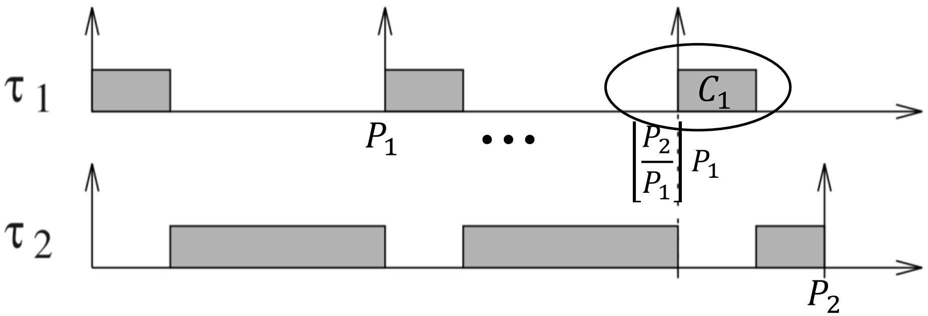
For feasibility in a non-RM policy, we need $C_1 + C_2 \leq P_1$ to hold at critical instant

- Now **exchange priorities** of tasks to make it into an RM assignment
- Plan:
  - identify all possible cases
  - In each case derive feasibility condition
  - Show that if $C_1 + C_2 \leq P_1$ then derived feasibility condition in RM holds

Optimality of the RM policy: Case 1

For feasibility in a non-RM policy, we need $C_1 + C_2 \leq P_1$ to hold at critical instant

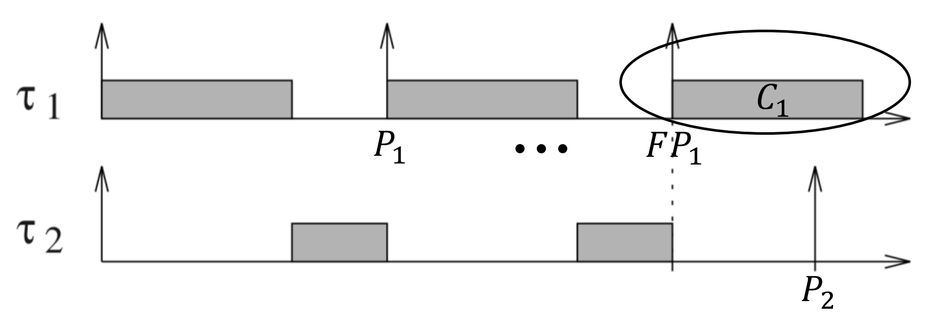
- **Case 1:** computation time of τ_1 is short enough that all its *requests* are completed before the second request of τ_2
  - Number of periods P_1 **entirely contained** in P_2 is $\left\lfloor \frac{P_2}{P_1} \right\rfloor$ → Let $F = \left\lfloor \frac{P_2}{P_1} \right\rfloor$
  - **Case 1** translates to $\boldsymbol{C_1 + FP_1 \leq P_2}$
  - **Feasibility:** All computation *requested* by τ_1 during P_2, in addition to C_2, should be completed by P_2
  - $(F+1)C_1 + C_2 \leq P_2 \qquad (*)$
  - Need to show that $C_1 + C_2 \leq P_1$ implies $(*)$



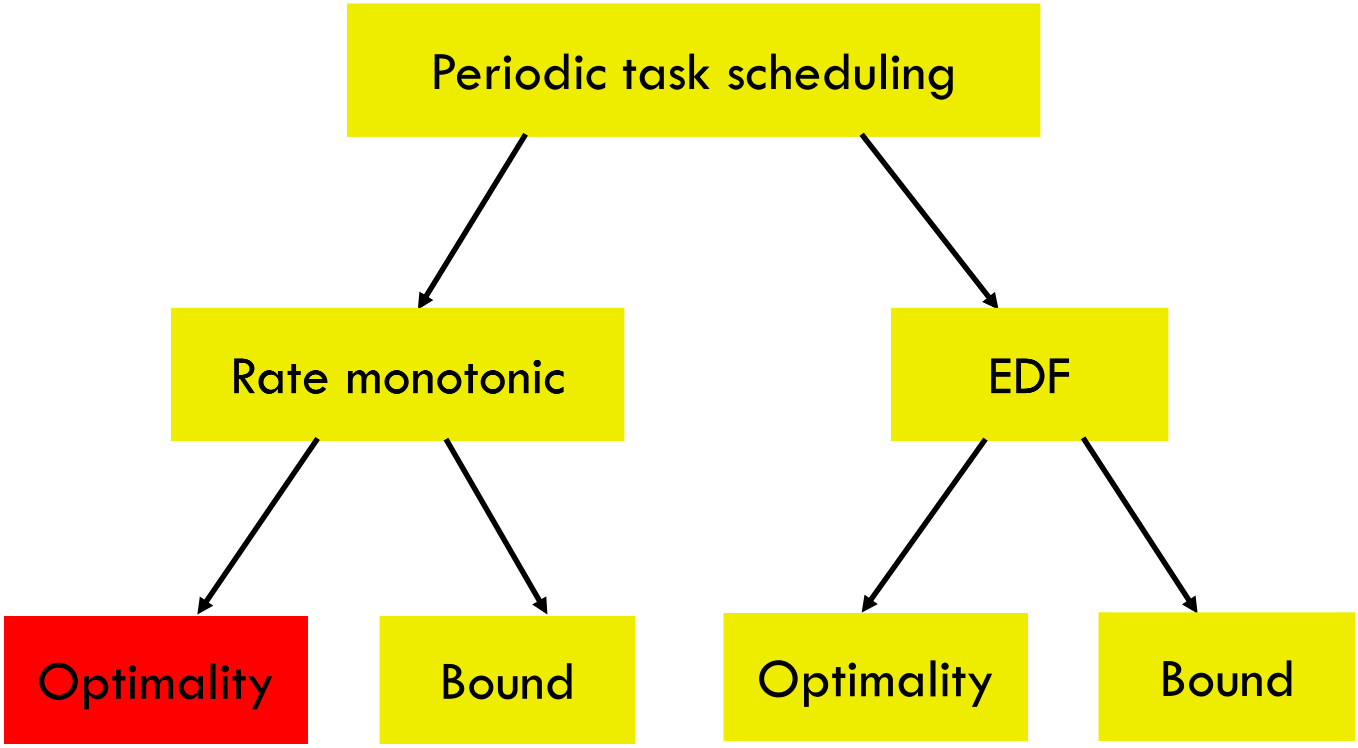
Optimality of the RM policy: Case 2

For feasibility in a non-RM policy, we need $C_1 + C_2 \leq P_1$ to hold at critical instant

- **Case 2:** computation time of τ_1 is long enough to overlap with the second request of τ_2
  - **Case 2 translates to $\boldsymbol{C_1 + FP_1 \geq P_2}$**
  - **Feasibility condition:** $FC_1 + C_2 \leq FP_1$ $(**)$ Why?
    - For feasibility τ_2 must finish before the start of the (FP_1)-th request of τ_1 because τ_1 has higher priority than τ_2 so τ_1 will occupy the processor until P_2 by the condition in **case 2** and thus P_2 cannot execute in $[FP_1, P_2]$
  - Need to show that $C_1 + C_2 \leq P_1$ implies $(**)$



What have we achieved?



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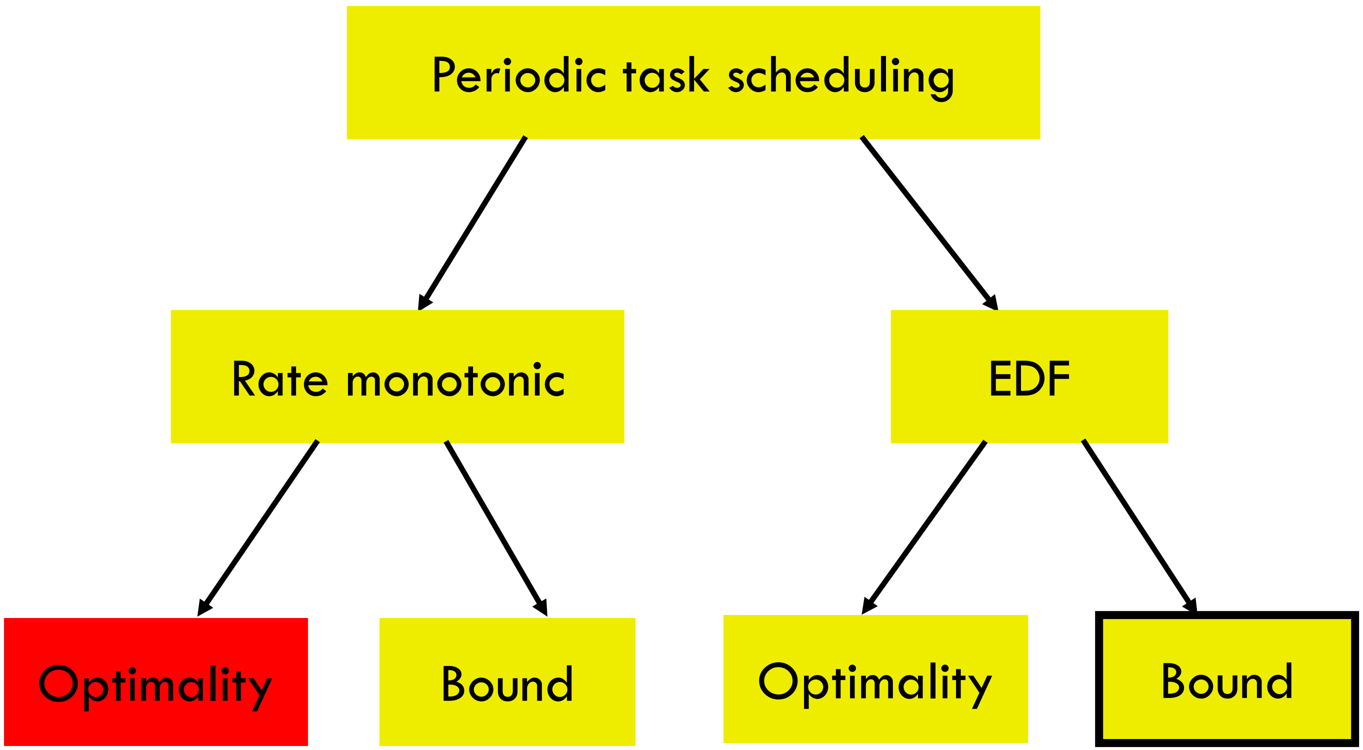
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Recall: Utilization bounds for schedulability

- U_S is called a **utilization bound** for a given scheduling policy S if

  All task sets with utilization factor $\leq U_S$ can be scheduled using policy S

- U_S is **tight** if, in addition, for a given scheduling policy S the following holds:

  For every $\epsilon > 0$ *, there exists at least one* task set with utilization $(U_S + \epsilon)$ that **cannot** be scheduled using policy S

- A tight bound is the best (largest) possible utilization bound: If U_S is tight, then no other $U > U_S$ can be a utilization bound for scheduling policy S

- Of course, the maximum value that U_S can attain for any S is 1. Why? In class

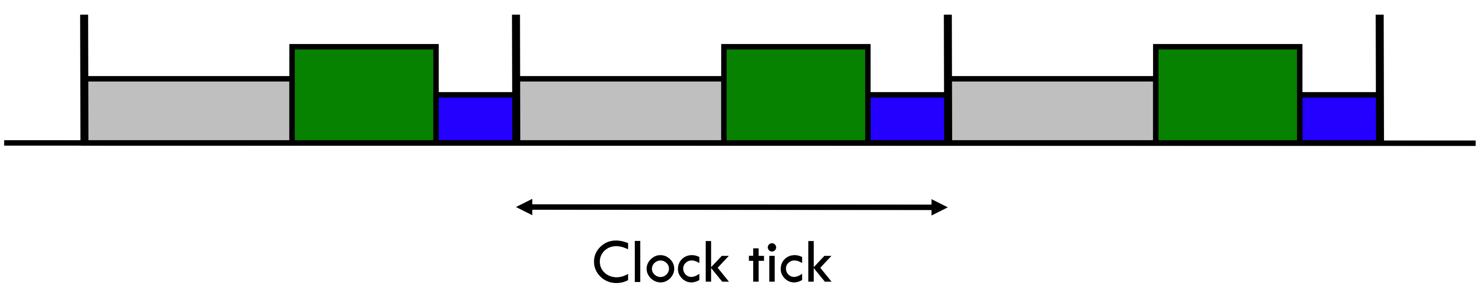
- U_S is also called the ***schedulable utilization*** of algorithm S

Utilization bound for EDF

- Why is it 100%?
- Consider a task set where:

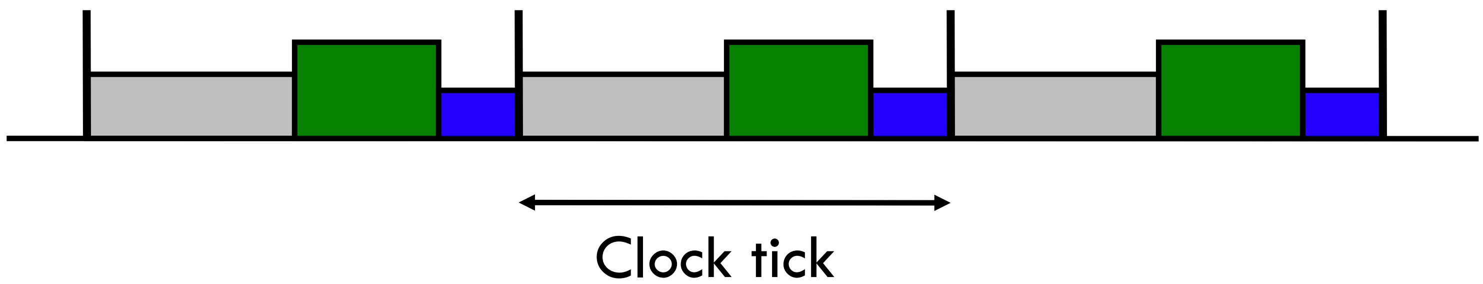
$$\sum_i \frac{c_i}{P_i} = 1$$

- Imagine a policy that reserves for each task i a fraction u_i of each clock tick, where $u_i = C_i / P_i$



Utilization bound for EDF

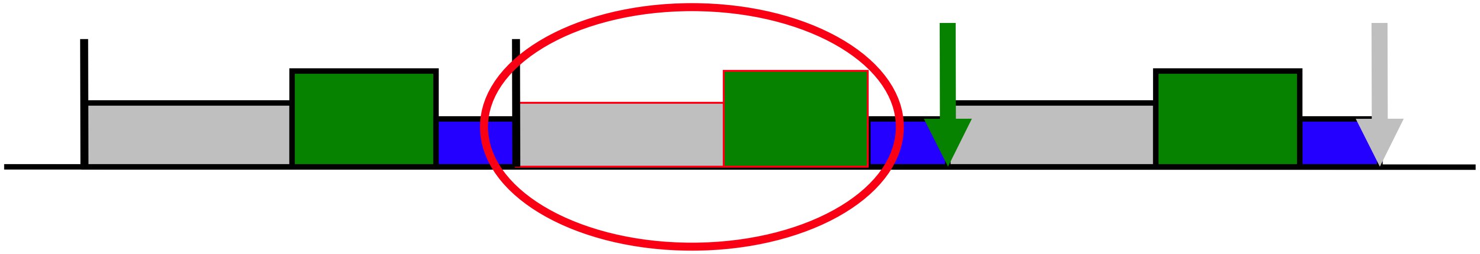
- Imagine a policy that reserves for each T_i a fraction u_i of each time unit, where $u_i = C_i / P_i$
- Divide time into, say, L = GCD(P_1, …, P_n)-length ticks after proper scaling of periods to integers



- This policy meets all deadlines, because:
  - Time given to T_i in its period = $u_i \times$ (# ticks/period) $\times$ tick length (time/tick)
    $= u_i(P_i/L)L = (C_i / P_i)\ P_i = C_i$ time/period
    (i.e., enough to finish)

Utilization bound for EDF

- Pick any two execution chunks that are not in EDF order and swap them



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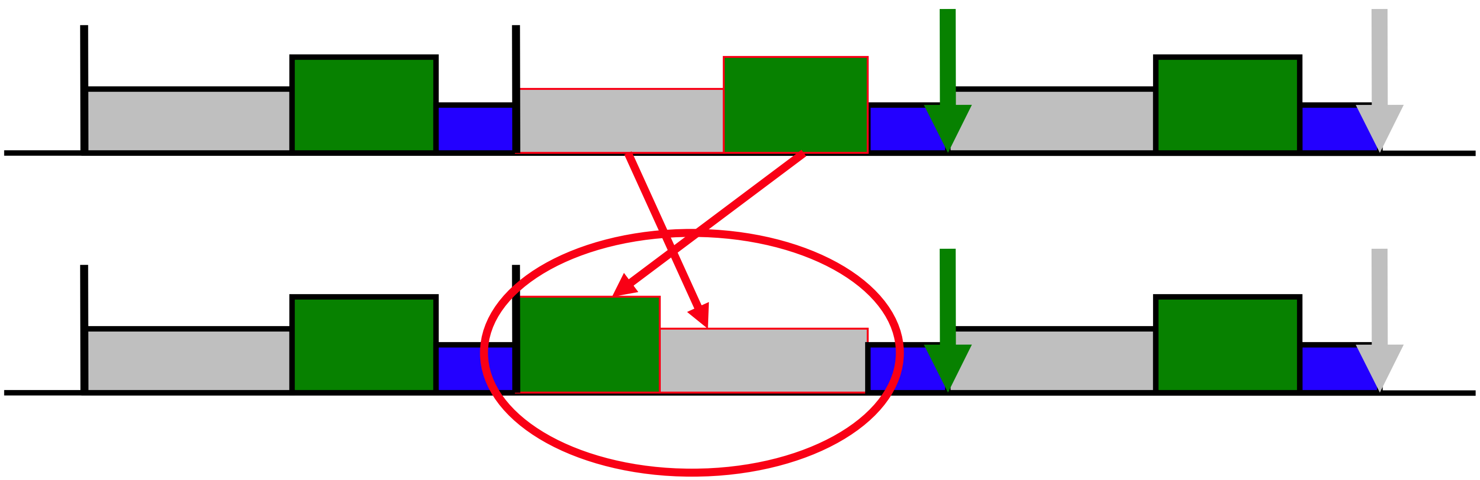

- Still meets deadlines! Why?

Utilization bound for EDF

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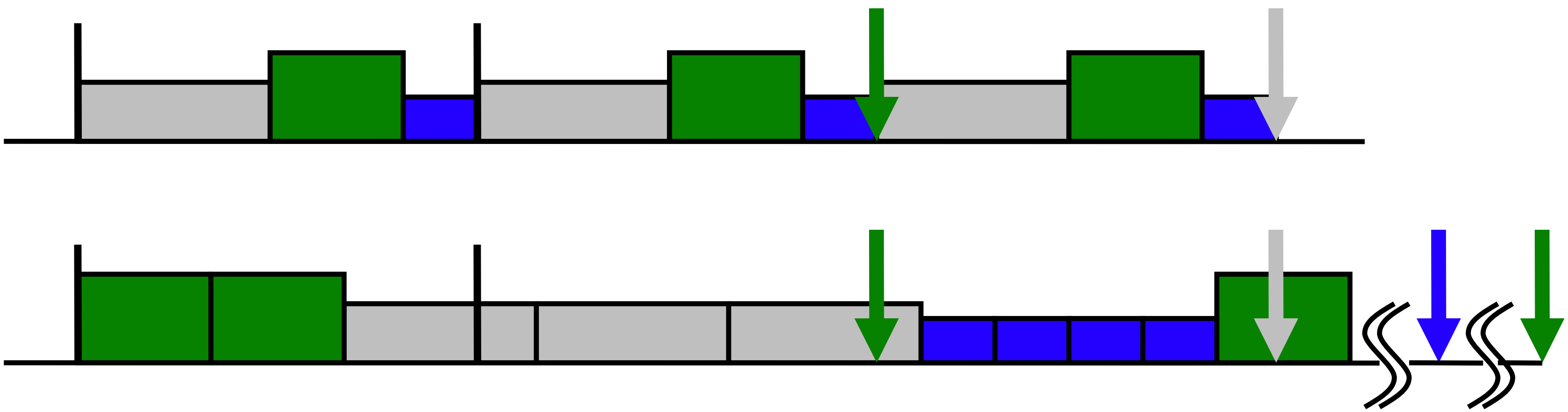
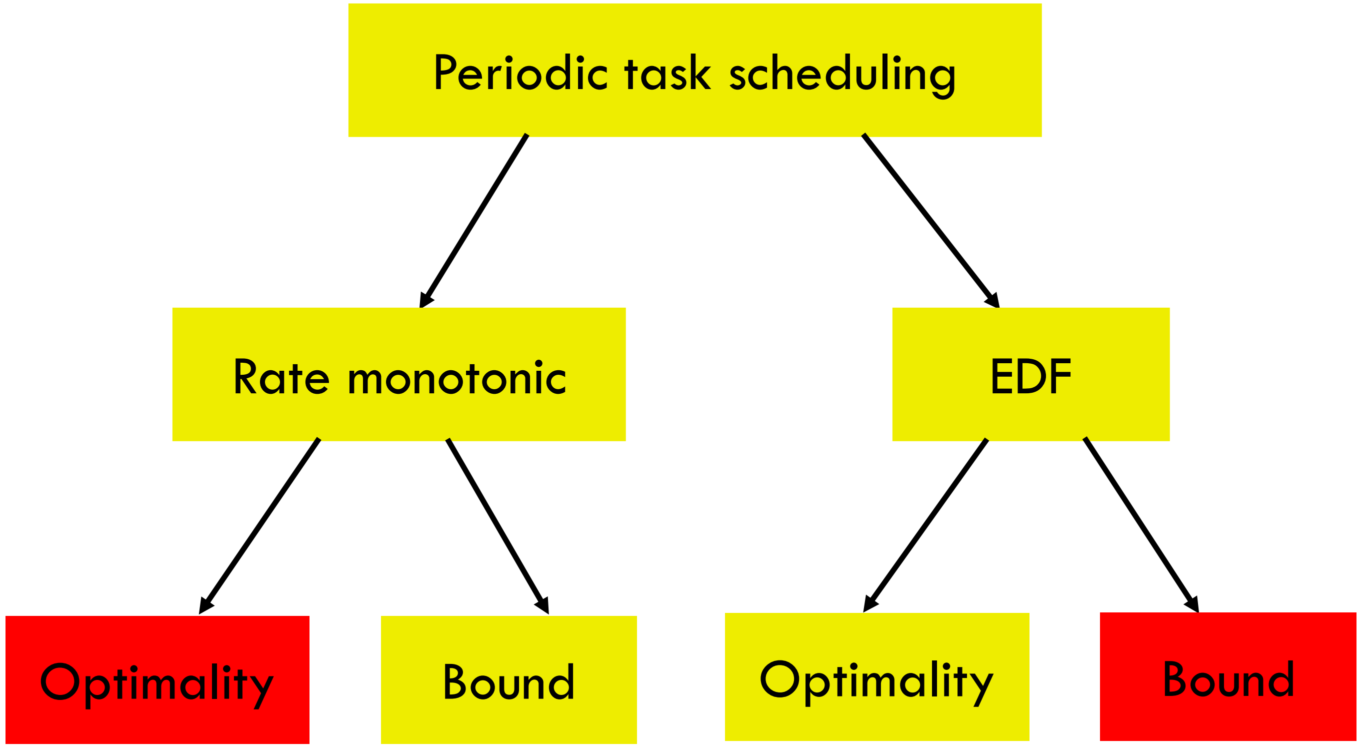


- Still meets deadlines!
- Repeat swap until all in EDF order
  - → EDF meets deadlines

Utilization bound for EDF

- Why does this prove that the utilization bound of EDF is 1?
  - We showed that every taskset with $U = 1$ is feasible under EDF
  - Must also show that every taskset with $U < 1$ is also feasible under EDF
  - This is not needed! Previous argument follows for any $U \leq 1$
  - Consequences:
    - EDF is optimal!
    - EDF is able to schedule every task set who utilization is 1 or less

Next



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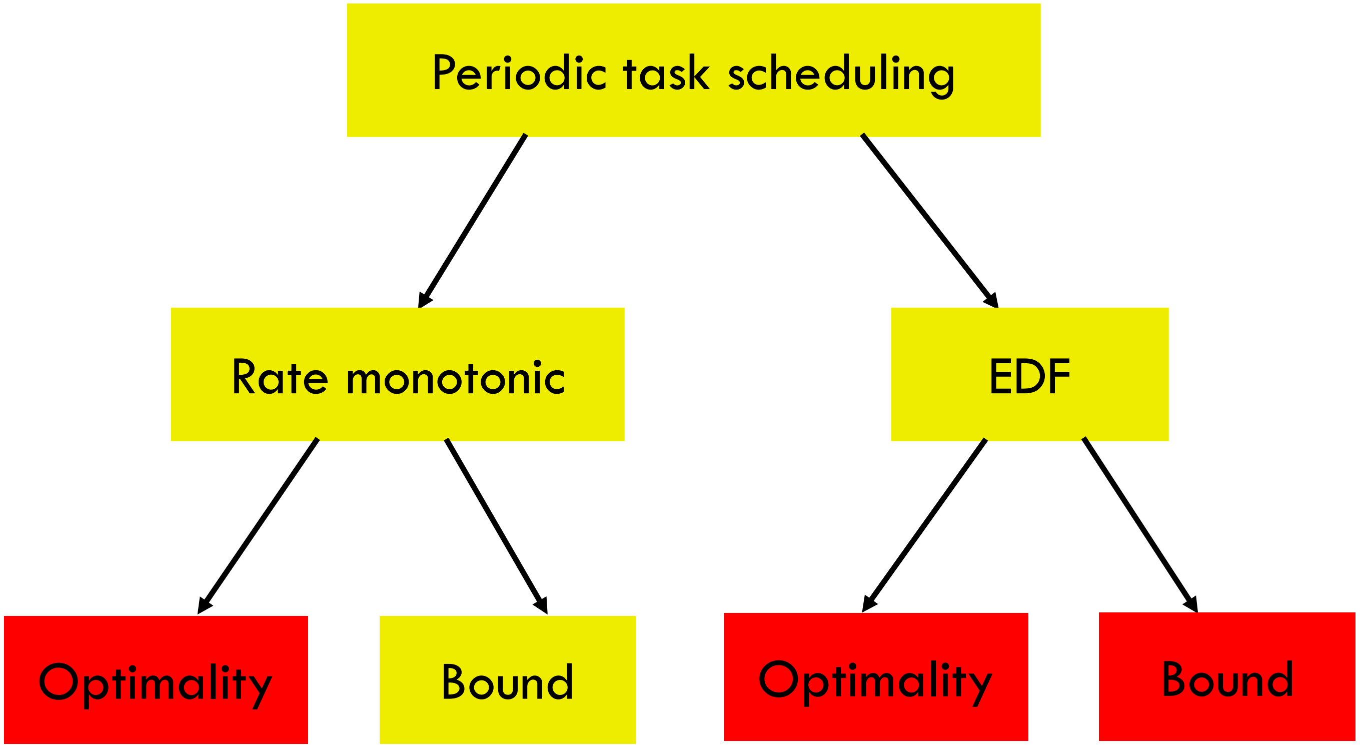

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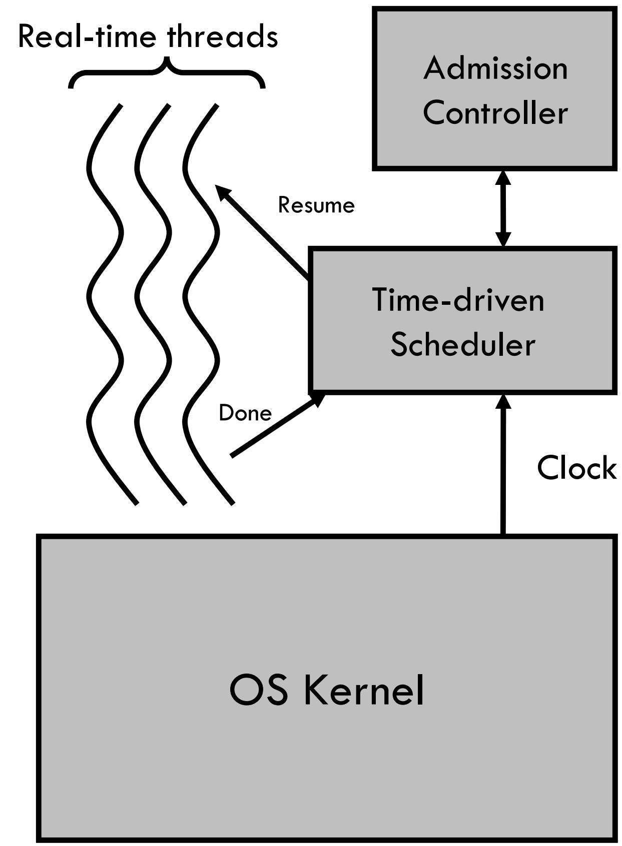
Tick-based scheduling within an OS

- A real-time library for periodic tasks on Linux or Windows
  - There is need to provide approximate real-time guarantees on common operating systems (as opposed to specialized real-time OSes)
  - A high-priority "real-time" thread pool is created and maintained
  - A higher-priority scheduler is invoked periodically by timer-ticks to check for periodic invocation times of real-time threads. The scheduler resumes threads whose arrival times have come.
  - Resumed threads execute one invocation then block.
  - Scheduling is preemptive
  - The scheduler can implement arbitrary scheduling policies including EDF, RM, etc.
  - An admission controller is responsible for spawning new periodic threads if the new task set can meet its deadlines.



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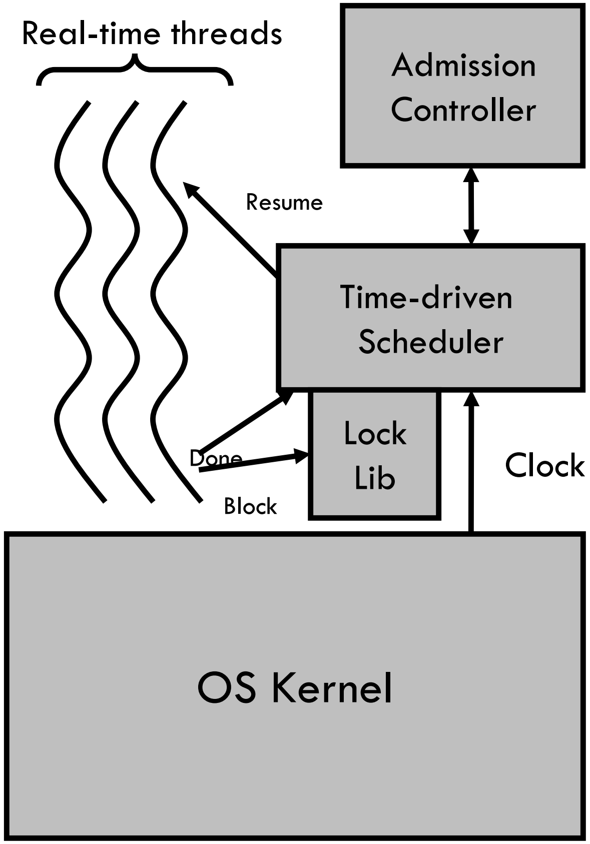
  - Scheduler implements wrappers for blocking primitives

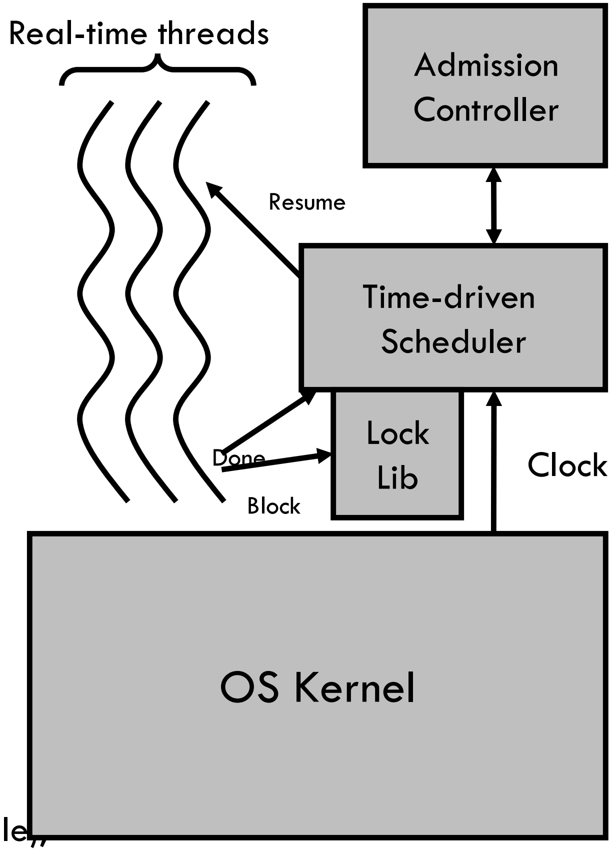


The time-driven scheduler

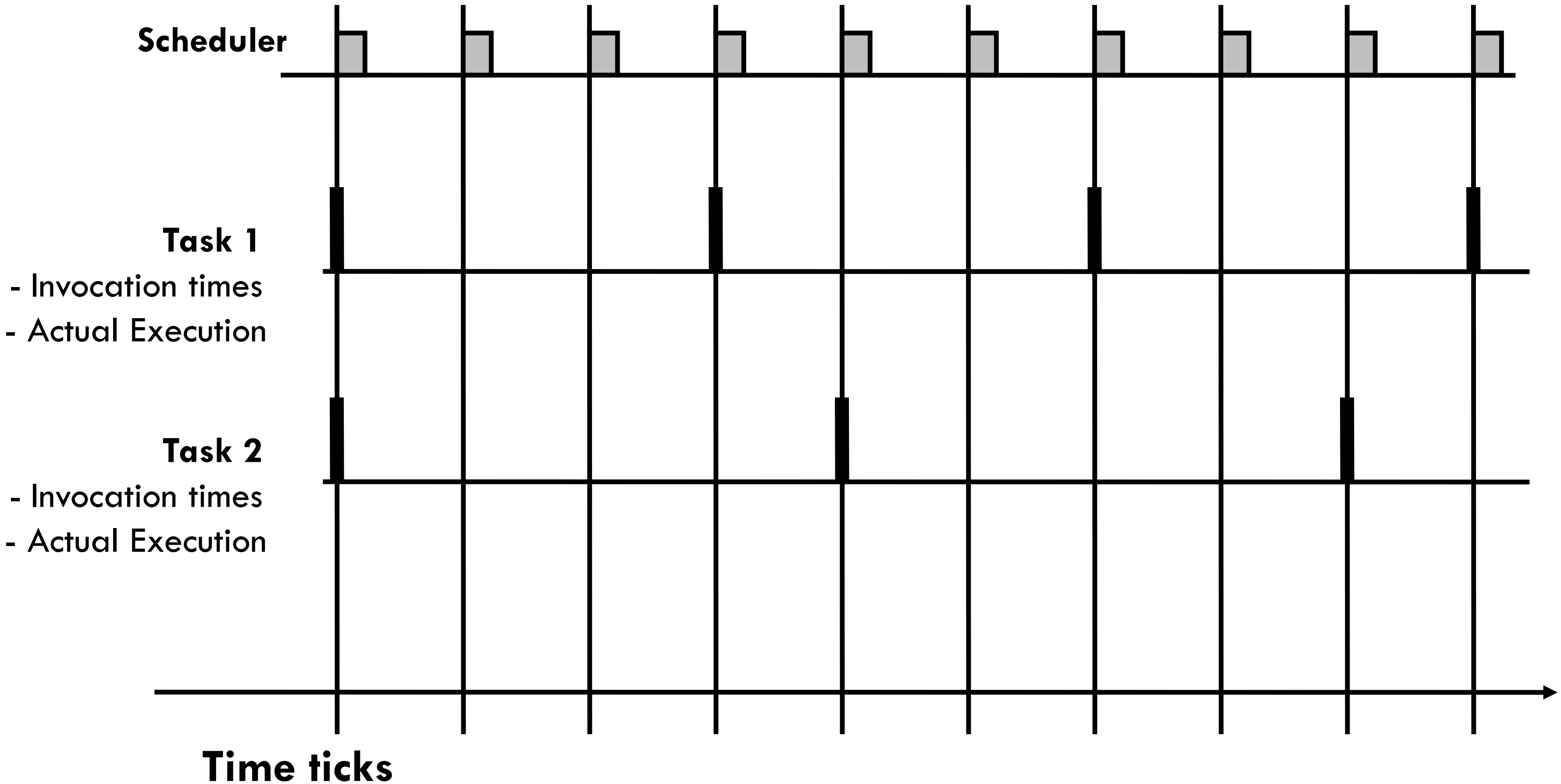
- /* N is the number of periodic tasks */
- For i=1 to N
-    if (current_time = next_arrival_time of task i)
-            put task i in ready_queue

- /* ready_queue is a priority queue that implements
-    the desired scheduling policy. */

- Inspect top task from ready queue, call it j
- If (a task is running and its priority is higher than priority of j) return
- Else resume task j (and put the running task into the ready queue if applicable), return



An example schedule



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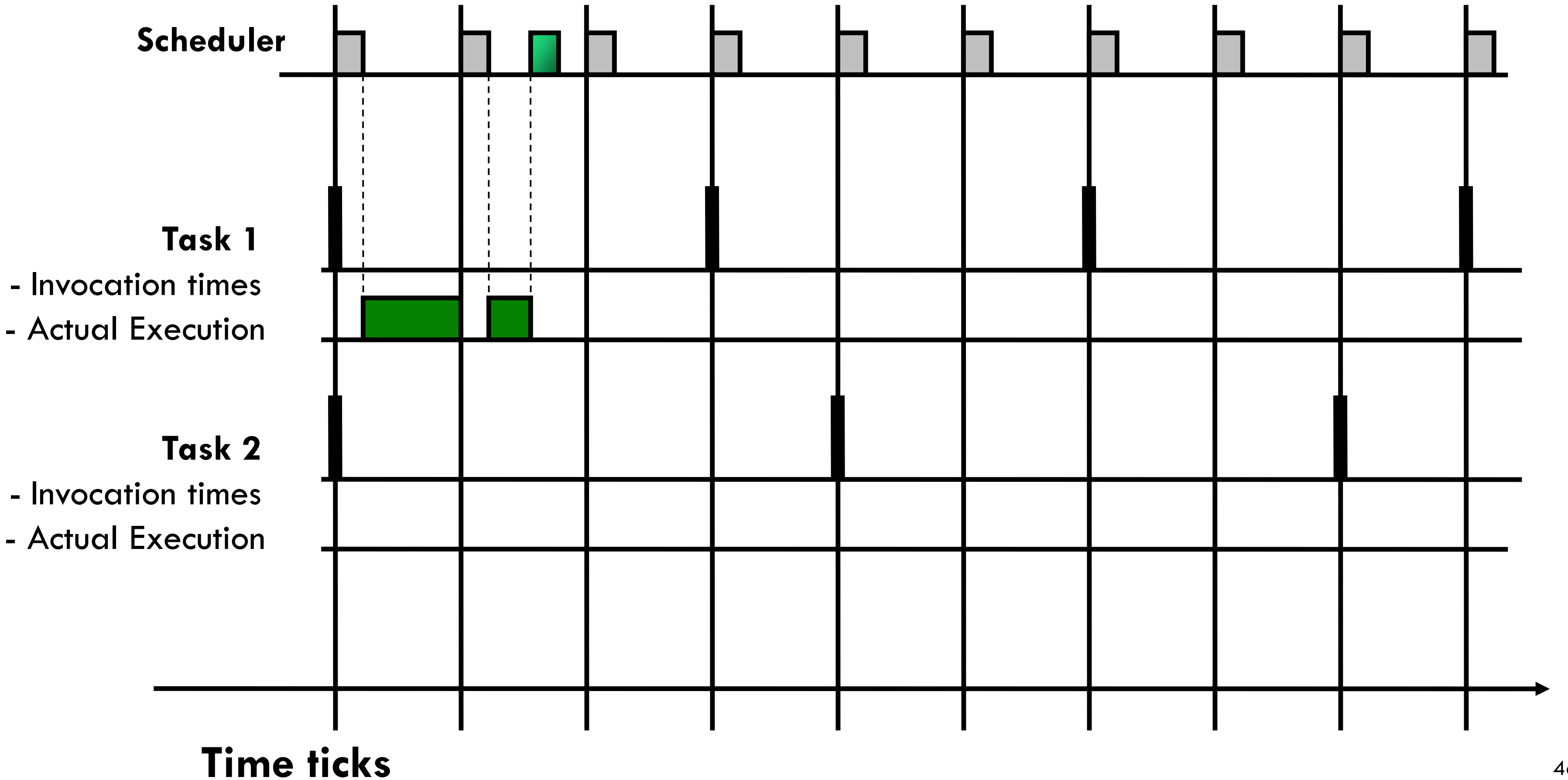

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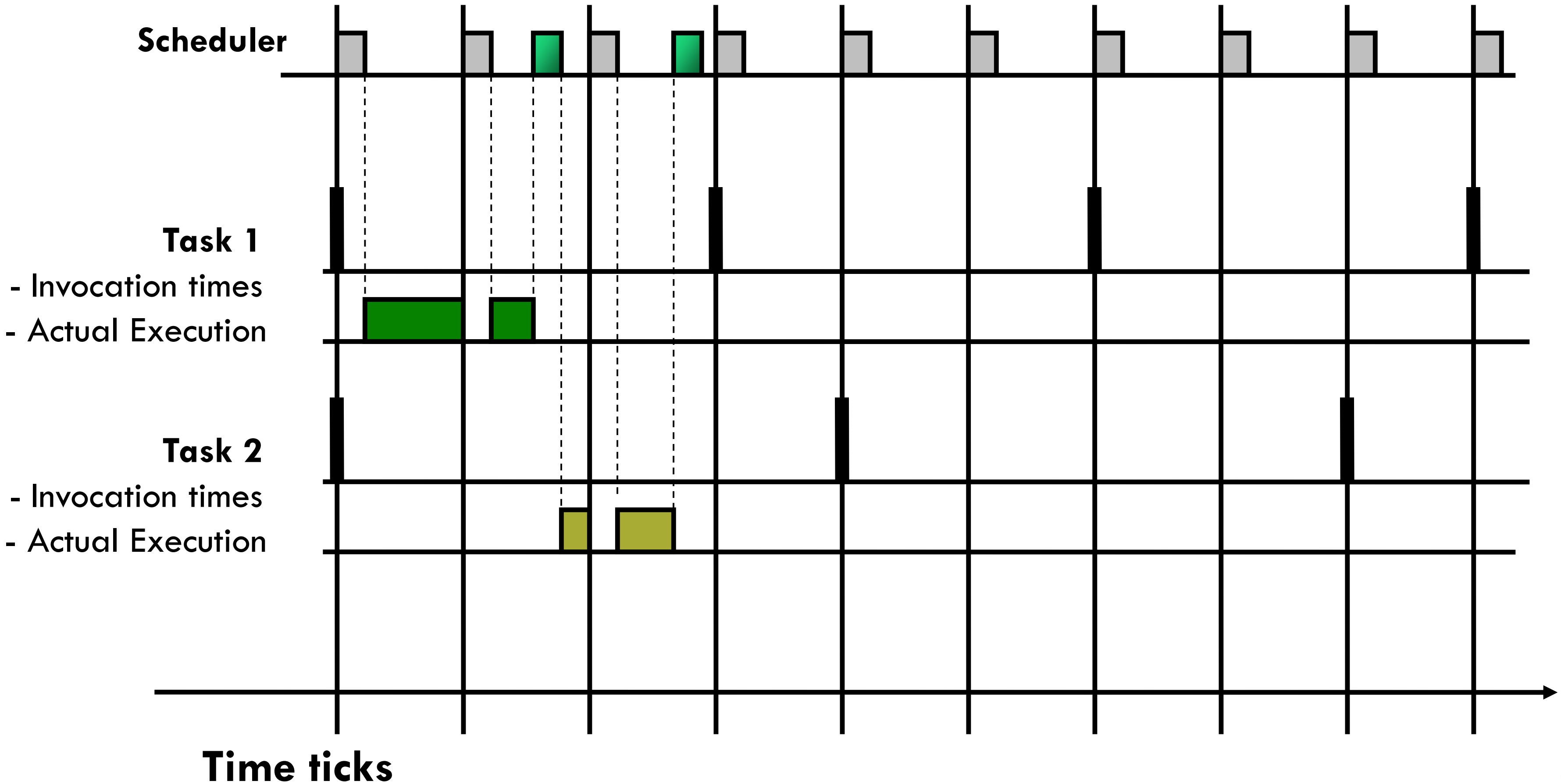

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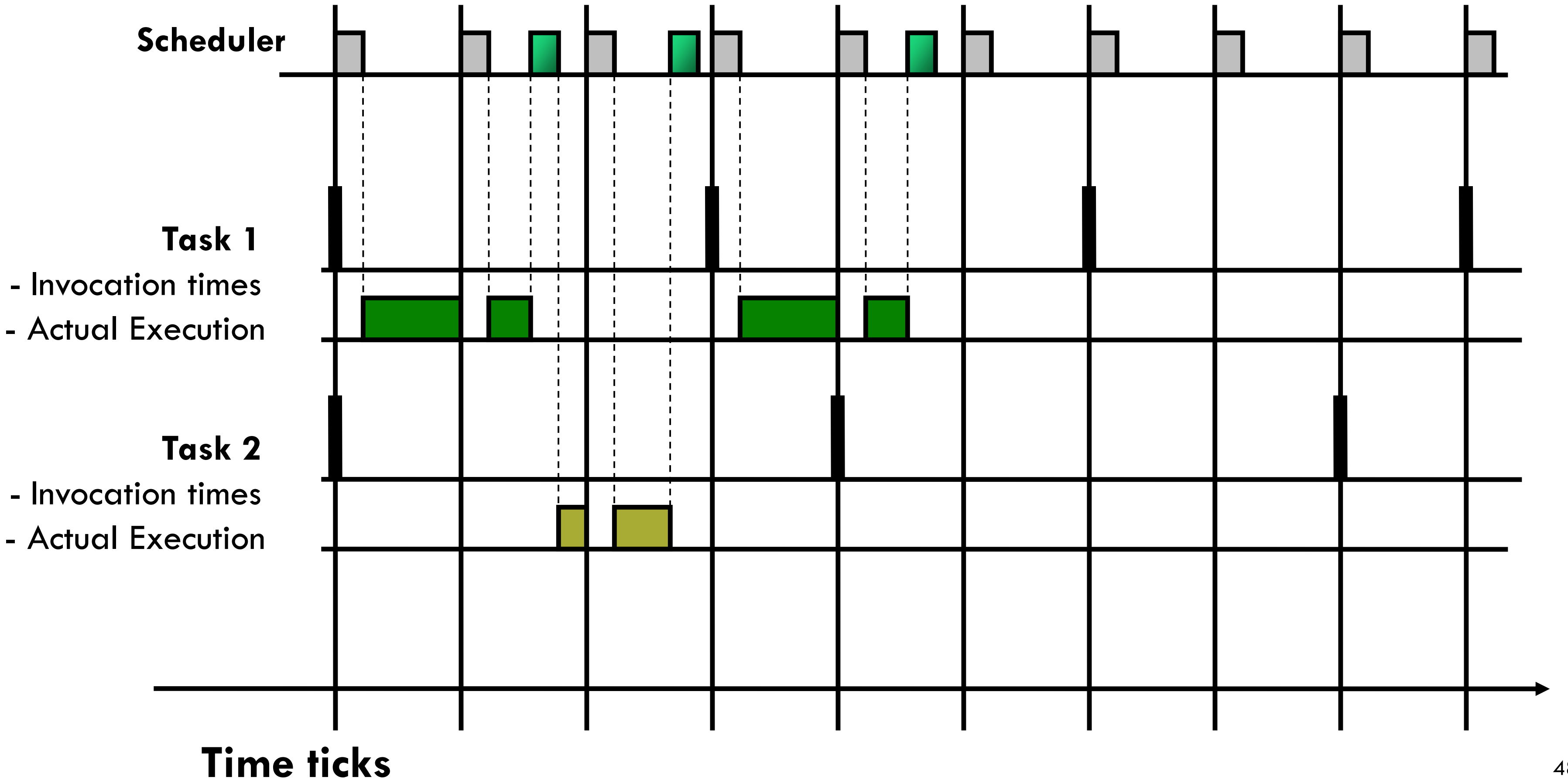

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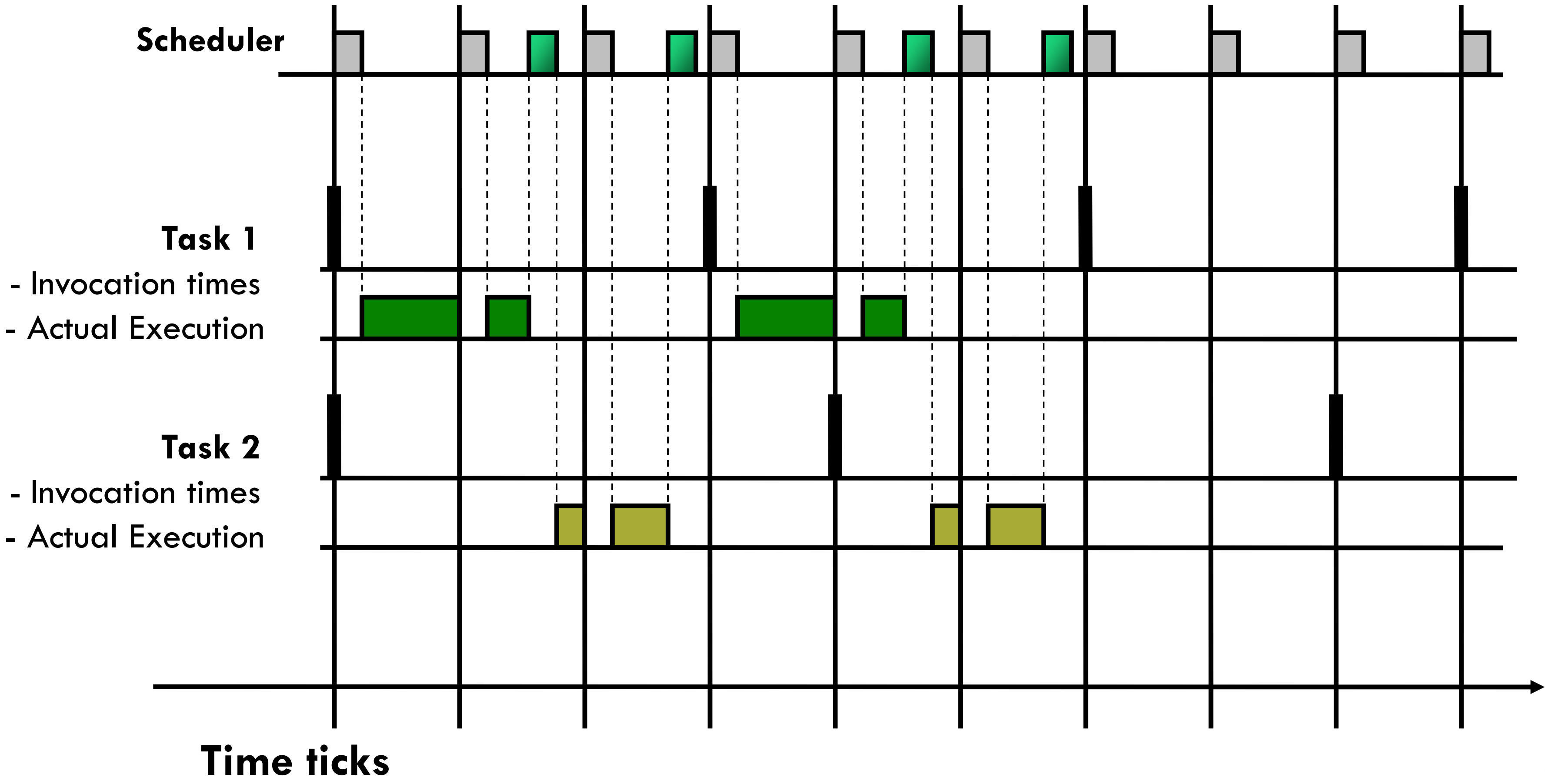

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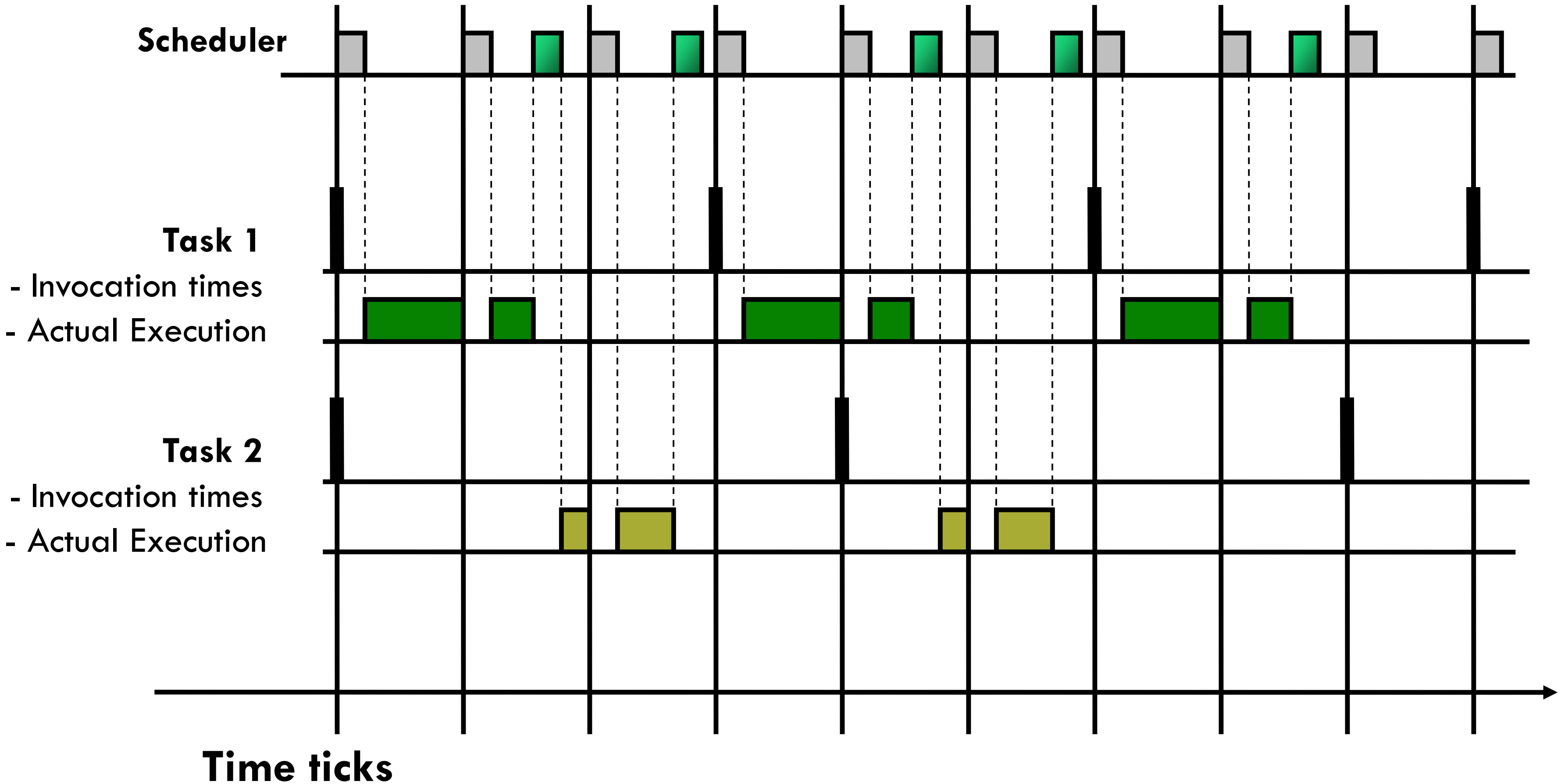

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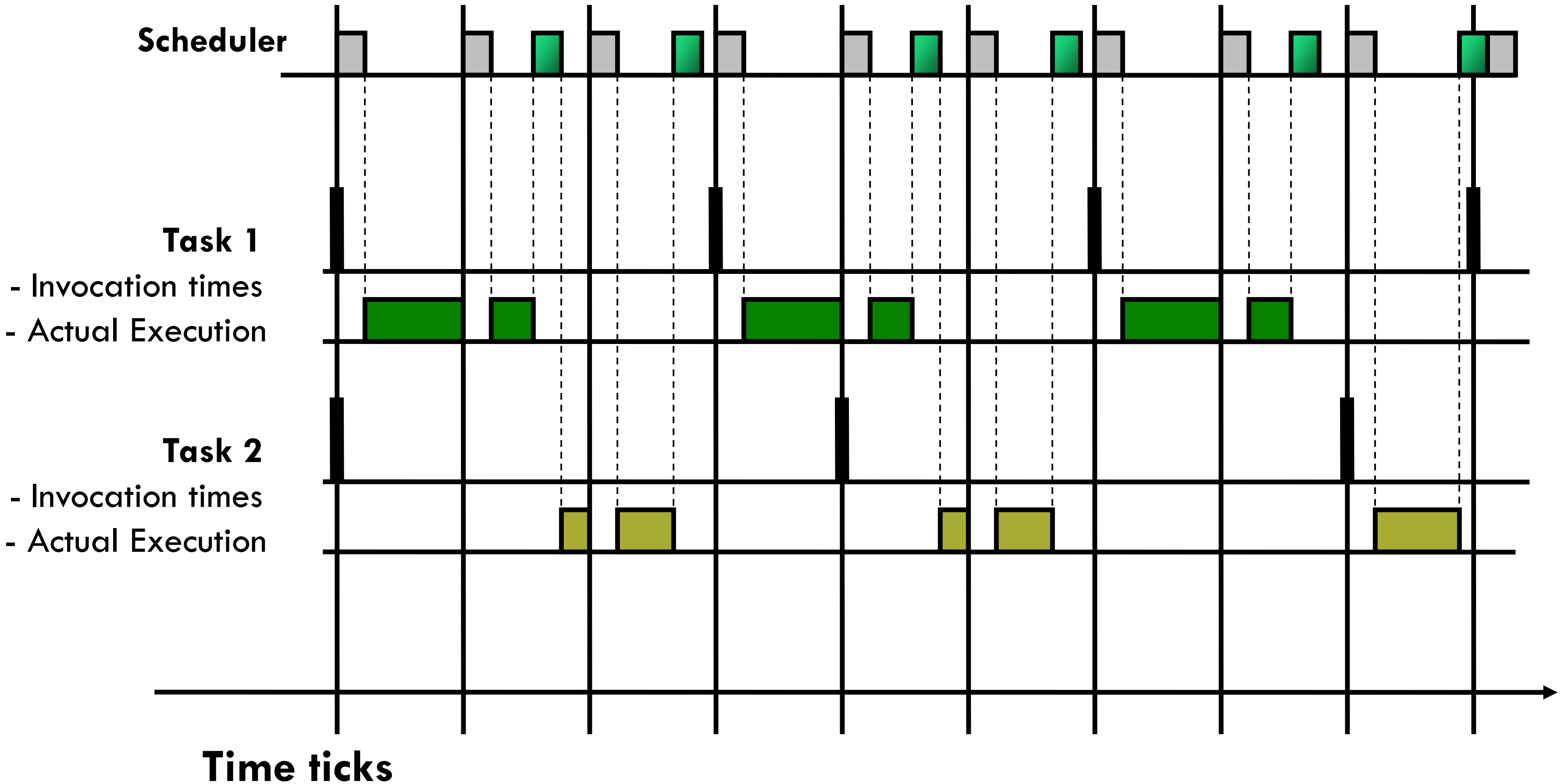
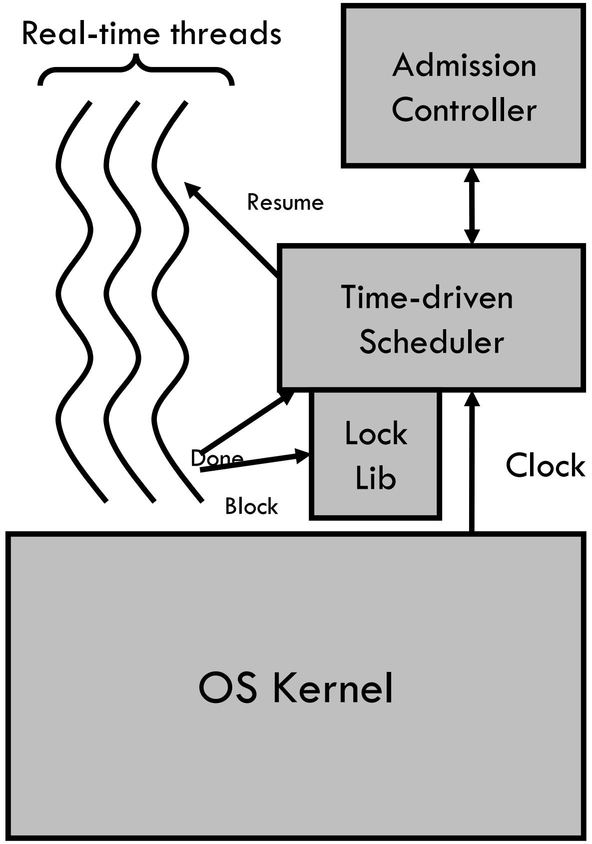


Admission controller

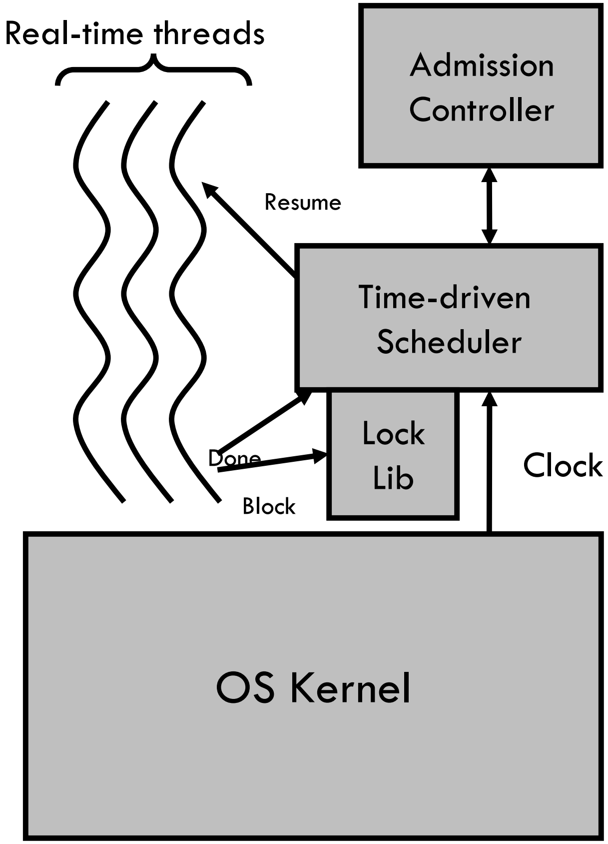
- Implements schedulability analysis
  - If $U + C_{new}/P_{new} < U_{bound}$ admit task
  - Must account for various practical overheads. How?
  - Examples of overhead:
    - How to account for the overhead of running the time-driven scheduler on every time-tick?
    - How to account for the overhead of running the scheduler after task termination?
- If new task admitted
  - $U = U + C_{new}/P_{new}$
  - Create a new thread
  - Register it with the scheduler



Library with lock primitives

- Lock (S) {
- Check if semaphore S = locked
- If locked
- enqueue running tasks in semaphore queue
- Else
- let semaphore = locked
- }

- Unlock (S) {
- If semaphore queue empty then
- semaphore = unlocked
- Else
- Resume highest-priority waiting task
- }



Problem: some threads may execute blocking OS calls (e.g., disk or network read/write and block without calling your lock/unlock!)