

# Periodic task scheduling

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Optimality of rate monotonic scheduling (among static priority policies)

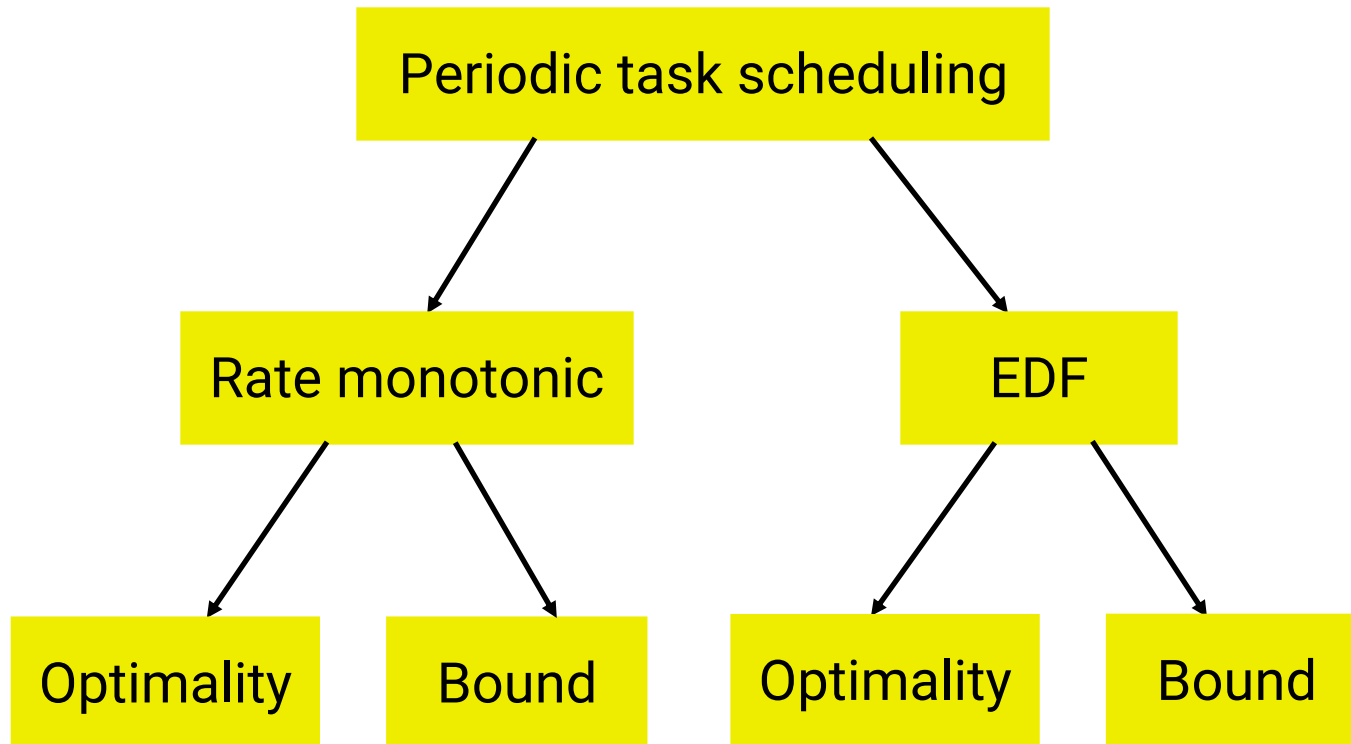
Utilization bound for EDF

Optimality of EDF (among dynamic priority policies)

Tick-driven scheduling (OS issues)

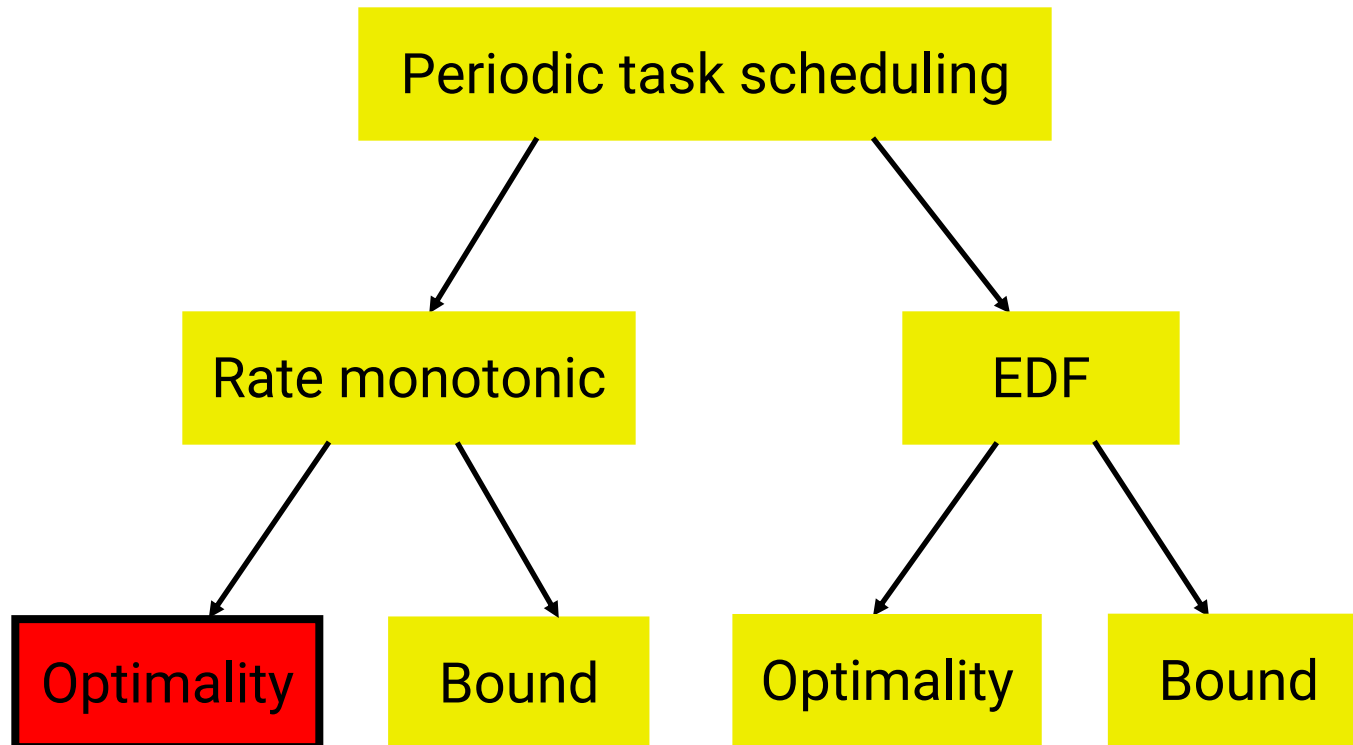
# Lecture outline

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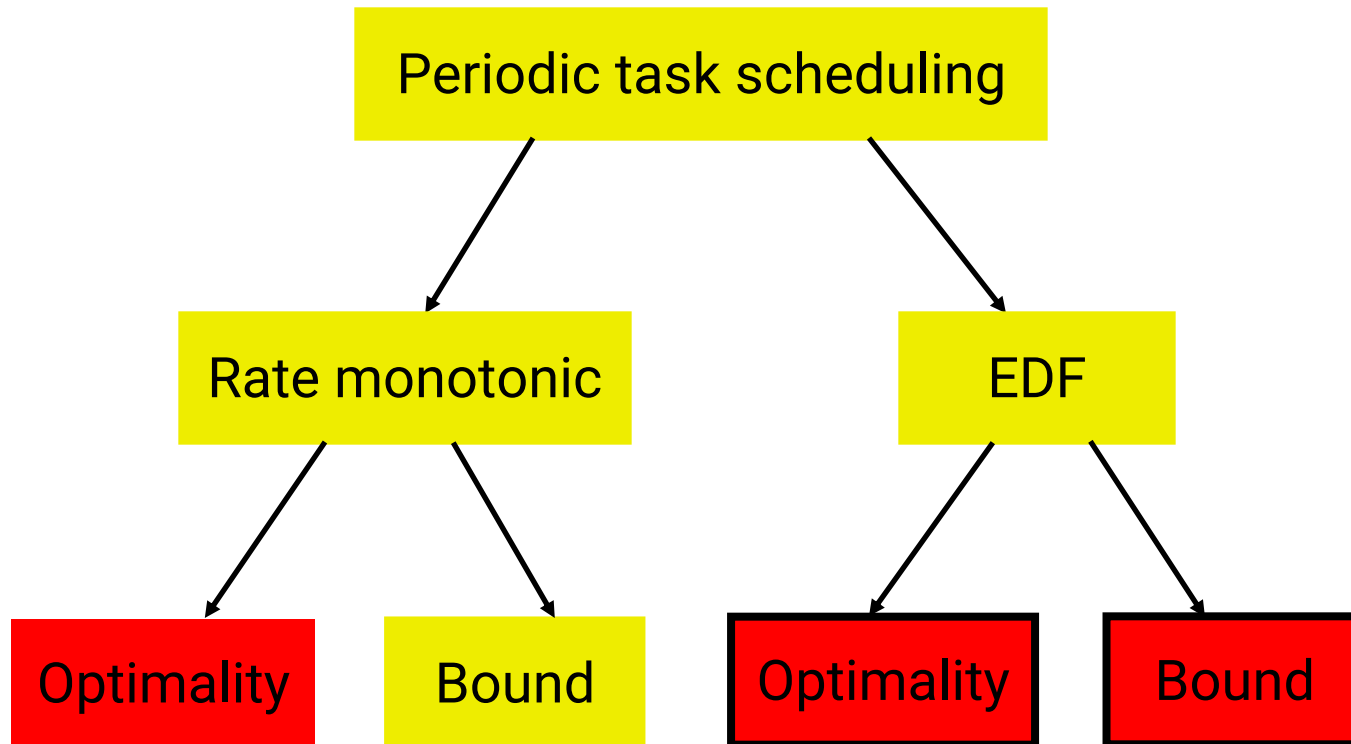
# Lecture outline

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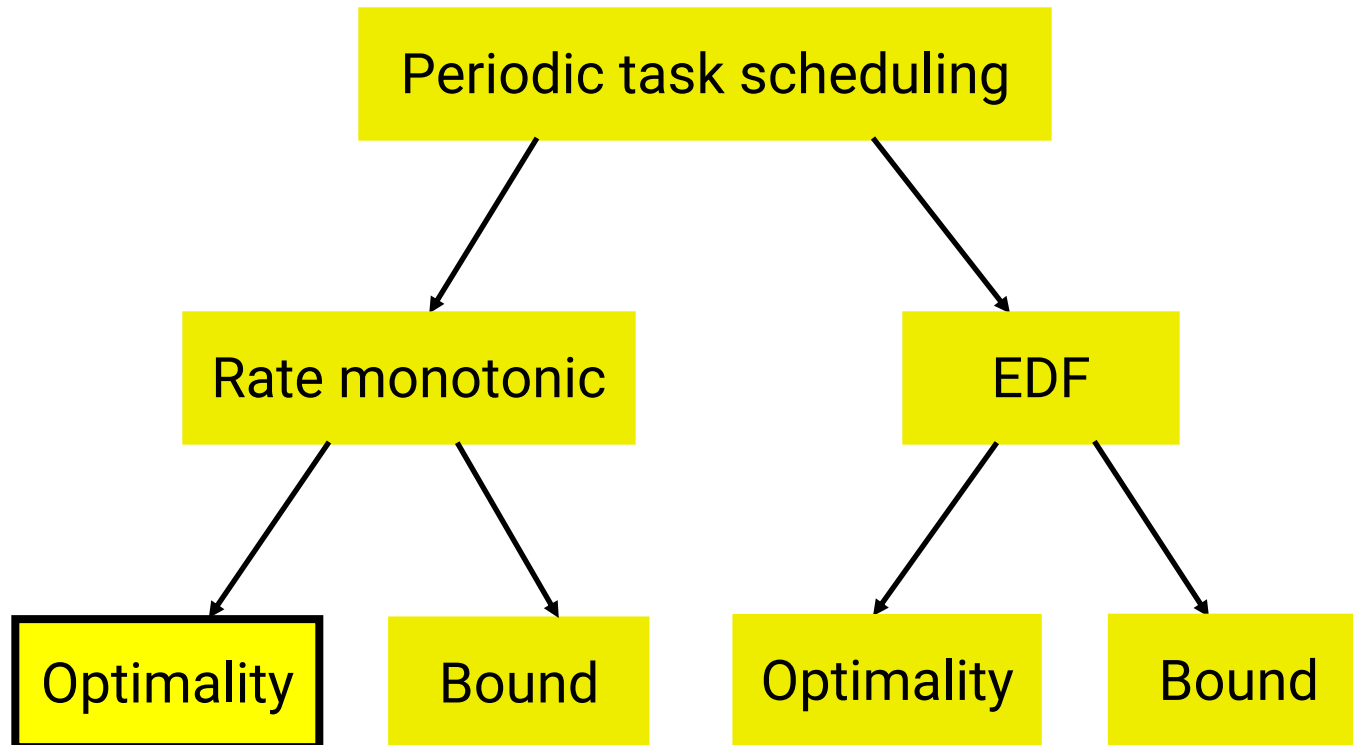
# Lecture outline

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# Next

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# Rate monotonic scheduling

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- Rate monotonic scheduling is an optimal fixed-priority (or static-priority) scheduling policy for periodic tasks.
  - Optimality (Trial #1):

# Rate monotonic scheduling

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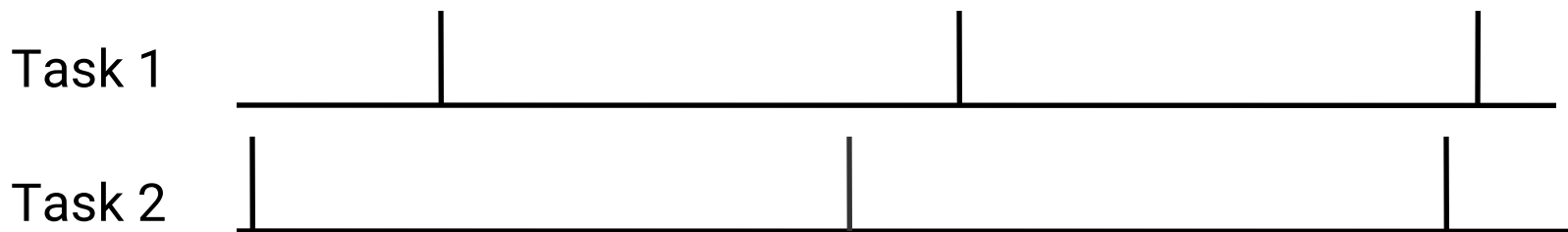
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# Rate monotonic scheduling

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$$P_1 < P_2$$



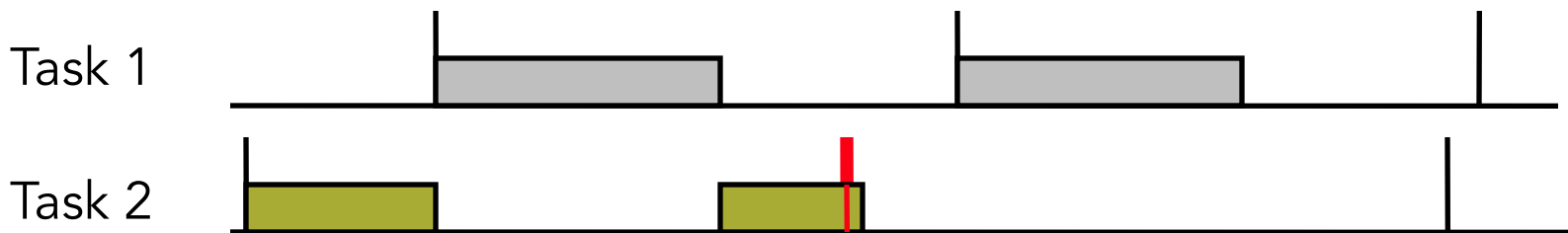


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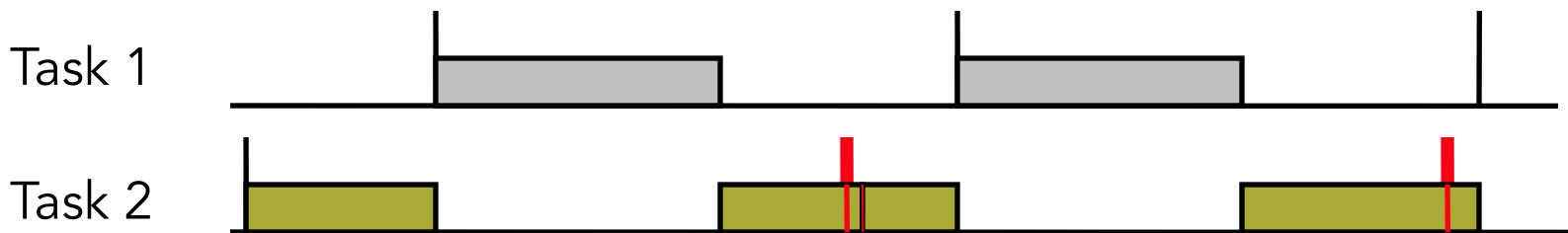


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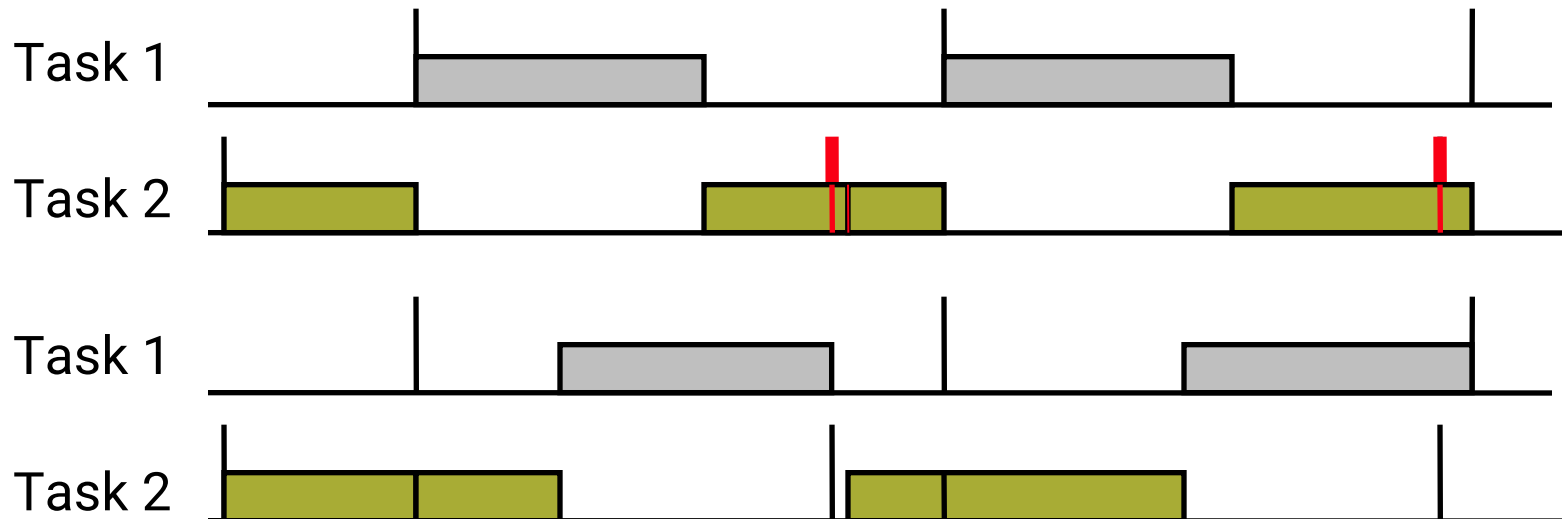
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# Rate monotonic scheduling

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# Rate monotonic scheduling

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- Rate monotonic scheduling is an optimal fixed-priority (or static-priority) scheduling policy for periodic tasks.
  - Optimality (Trial #2): If any other fixed-priority scheduling policy can meet deadlines **in the worst-case scenario**, so can RM.
- How do we prove it?

# Rate monotonic scheduling

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- Rate monotonic scheduling is the optimal fixed-priority (or static-priority) scheduling policy for periodic tasks.
  - Optimality (Trial #2): If any other fixed-priority scheduling policy can meet deadlines **in the worst-case scenario**, so can RM.
- How do we prove it?
  - Consider the worst-case scenario
  - Show that if someone else can schedule then RM can

# The worst-case scenario

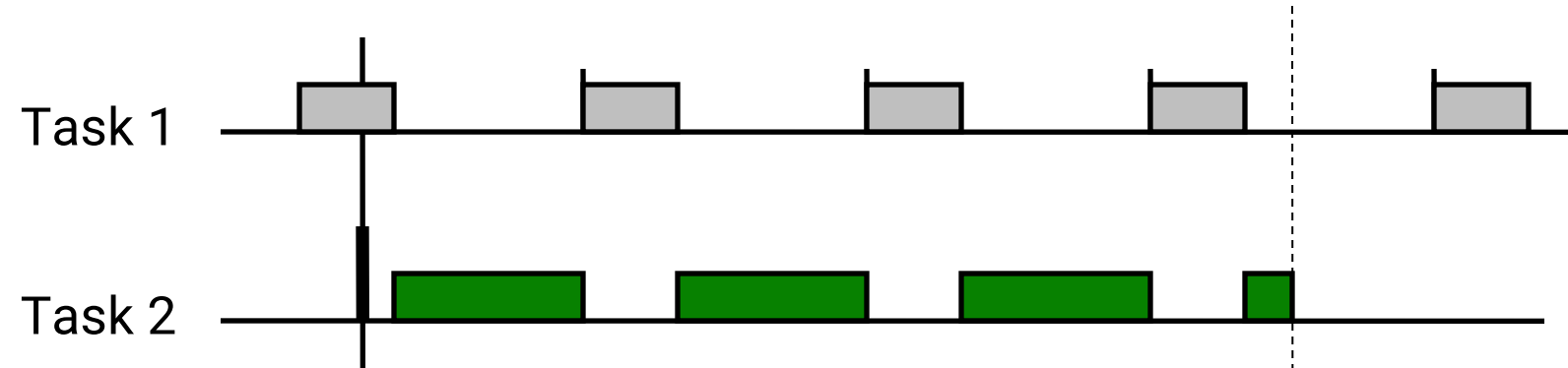
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- **Q:** When does a periodic task,  $T$ , experience the maximum delay?
  - Which arrival time produces the largest response time for  $T$ ?
- **A:** When it arrives together with all the higher-priority tasks (critical instant)
  - Liu and Layland
- Idea for the proof
  - If some higher-priority task does not arrive together with  $T$ , aligning the arrival times can only increase the completion time of  $T$ .

**Critical instant theorem**

## Critical Instant: Proof (Case 1)

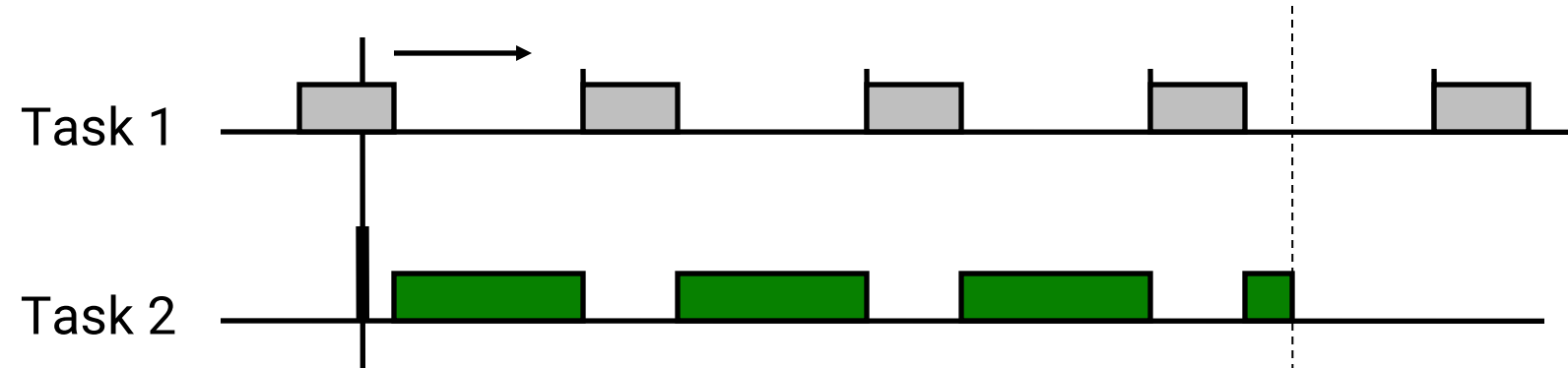
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Case 1: Higher priority task 1 is running when task 2 arrives.

# Critical Instant: Proof

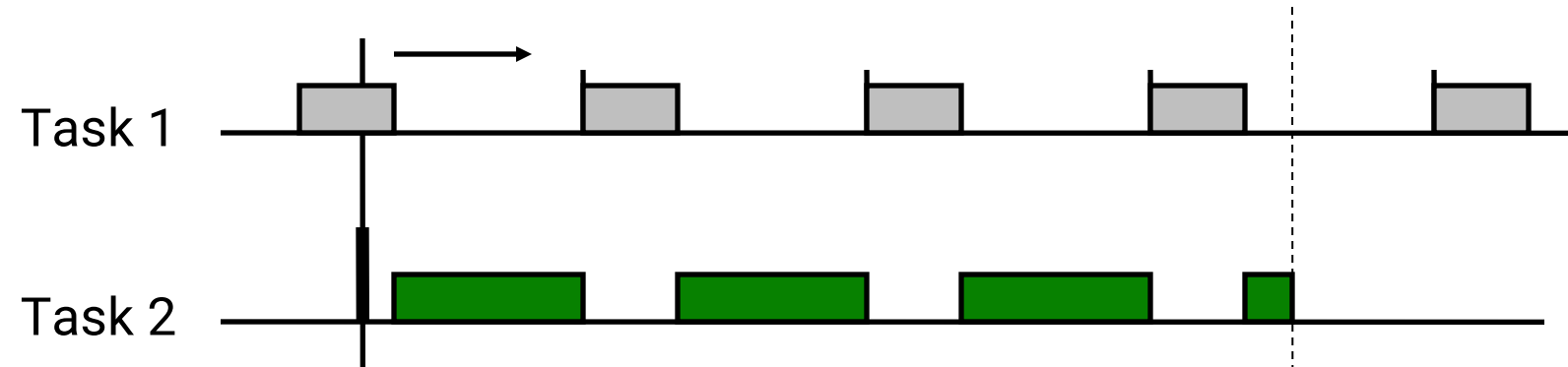
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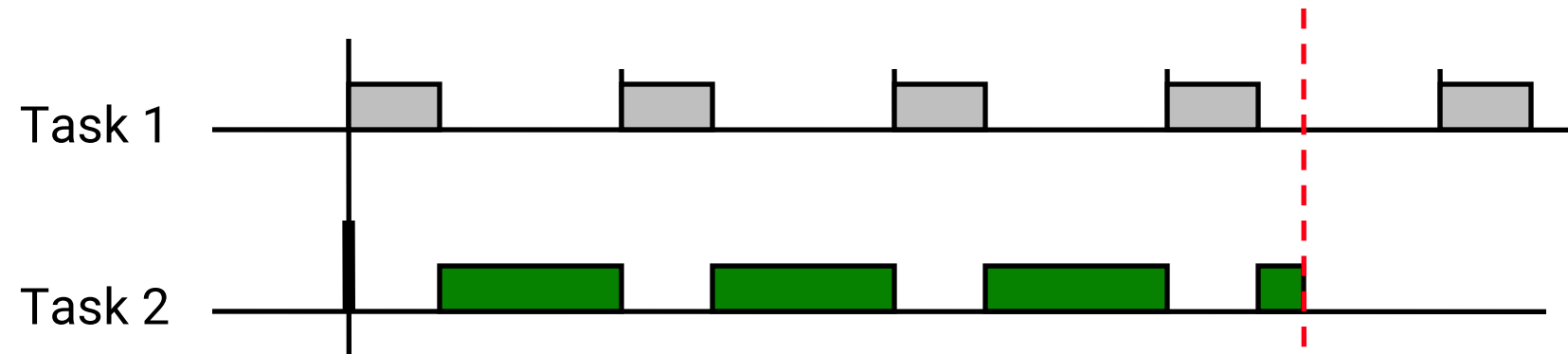
Case 1: higher priority task 1 is running when task 2 arrives  
→ shifting task 1 right will increase completion time of 2



# Critical Instant: Proof

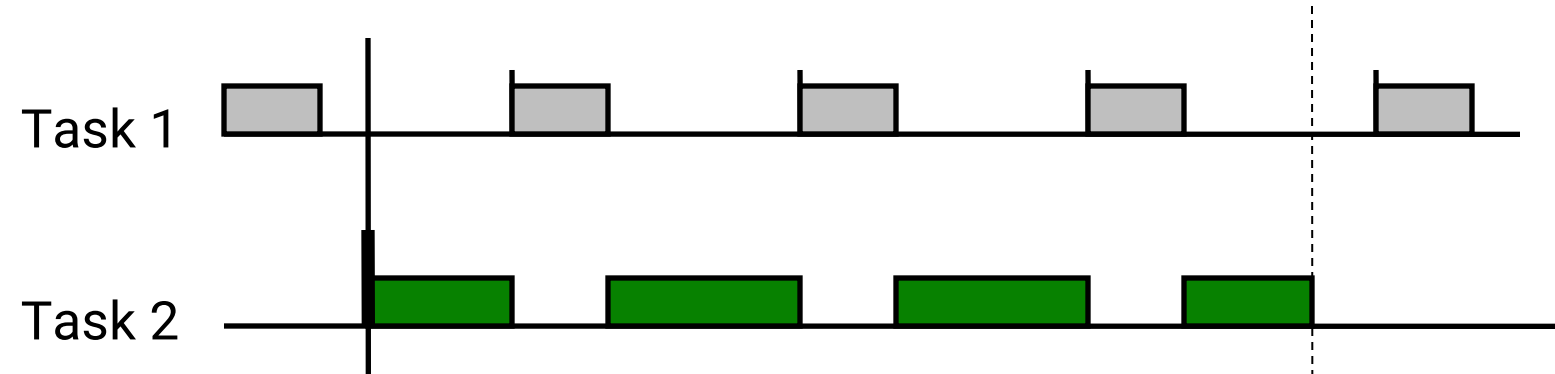


Case 1: higher priority task 1 is running when task 2 arrives  
shifting task 1 right will increase completion time of 2



## Critical Instant: Proof (Case 2)

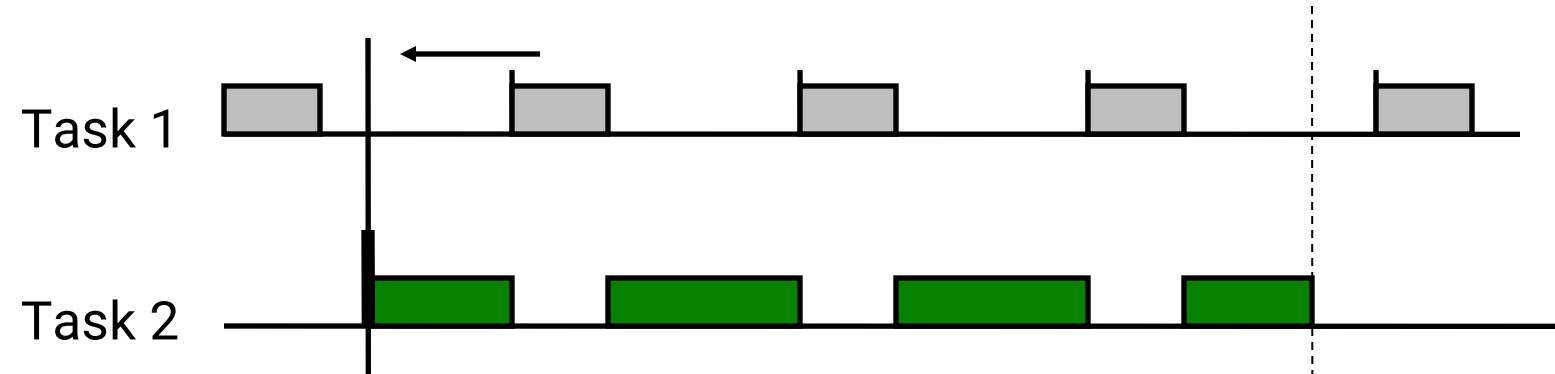
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Case 2: processor is idle when task 2 arrives

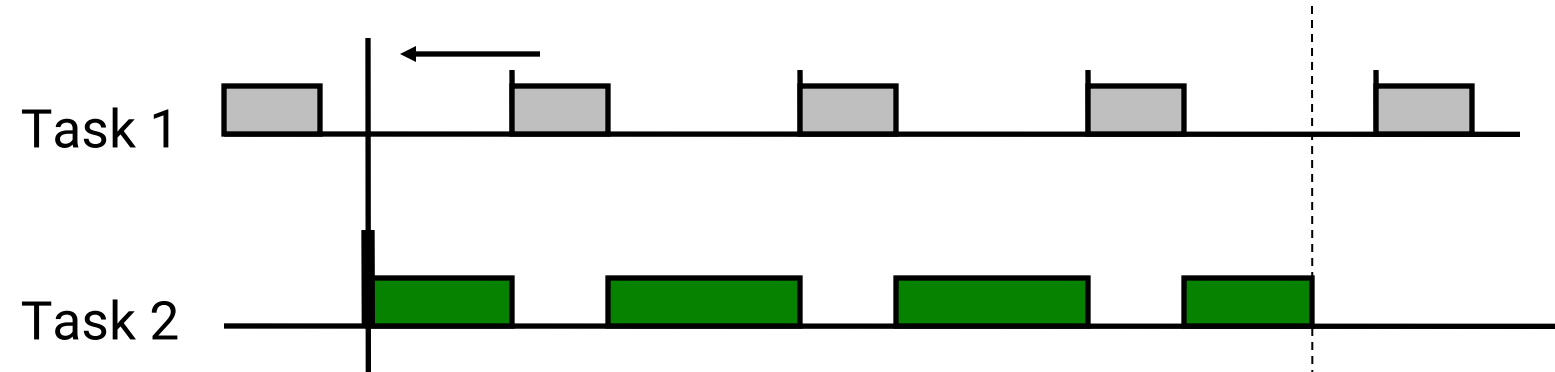
## Critical Instant: Proof (Case 2)

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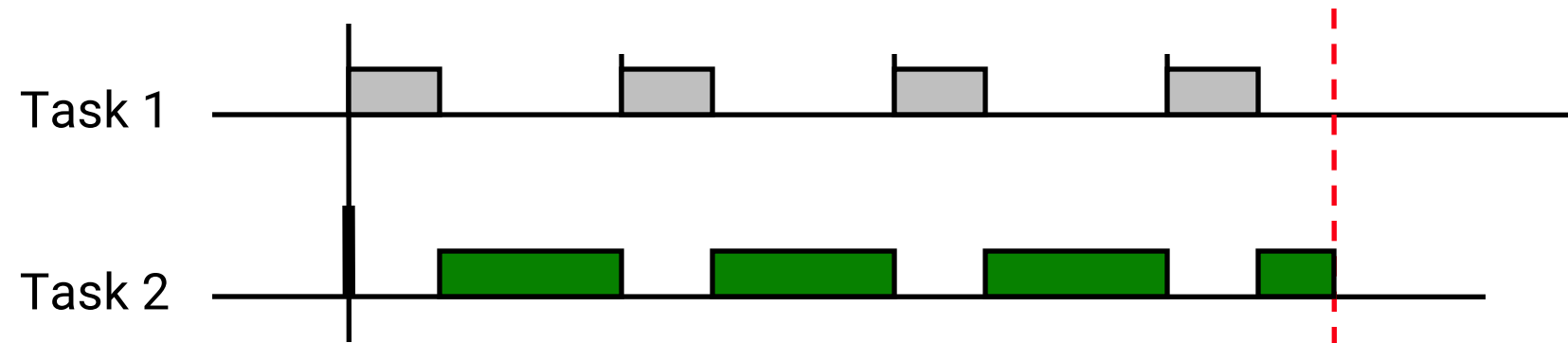


Case 2: processor is idle when task 2 arrives  
shifting Task 1 left cannot decrease completion time of 2

## Critical Instant: Proof (Case 2)



Case 2: processor is idle when task 2 arrives  
shifting task 1 left cannot decrease completion time of task 2



# Critical Instant: Remarks

- **All analyses hereafter will assume the critical instant theorem in effect**
- Why is it important to identify the critical instant?
  - Characterizes the **worst-case** scenario when a task experiences the max delay (remember the pitfall in trial #1 in proving RM optimality earlier?)
  - For task schedulability, need to reason only about the feasibility of the job arriving at the critical instant only until its deadline

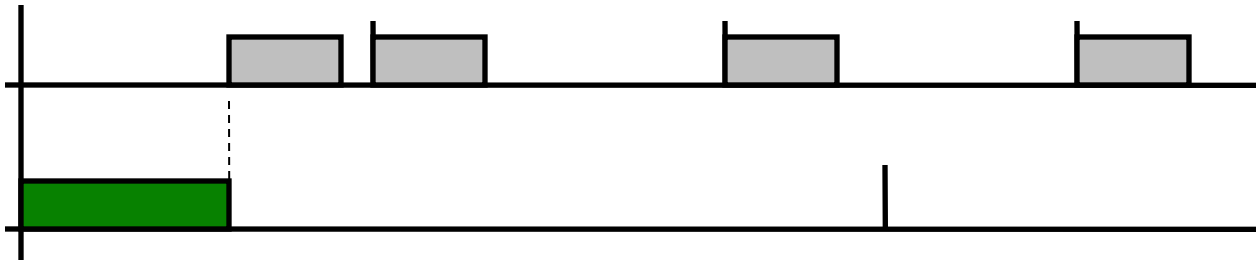
# Assumptions

- All scheduling is preemptive
- A simultaneous release exists even if tasks have non-zero phases
  - Thus critical instant theorem assumed
- Implicit deadlines (deadline = period)
- A task does not suspend itself (on I/O, for instance)
- All tasks in a task set are independent (there are no precedence relations and no resource constraints.)
- All overheads in the kernel are assumed to be zero (context switching and others)

# Optimality of the RM policy

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- If any other fixed-priority policy can meet deadlines so can RM

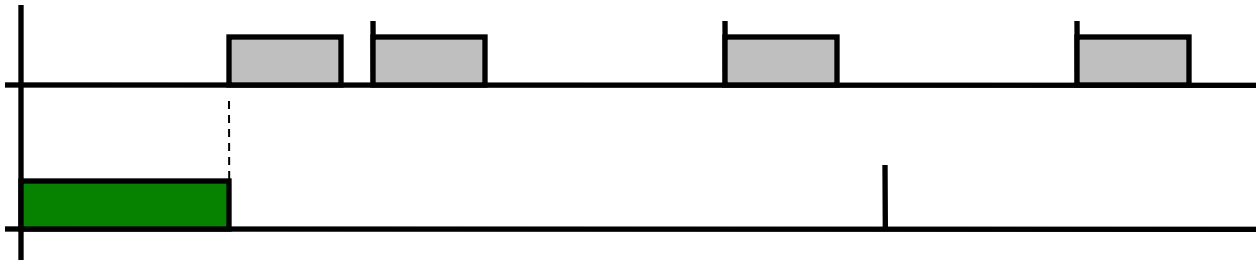


Policy X meets deadlines?

# Optimality of the RM policy

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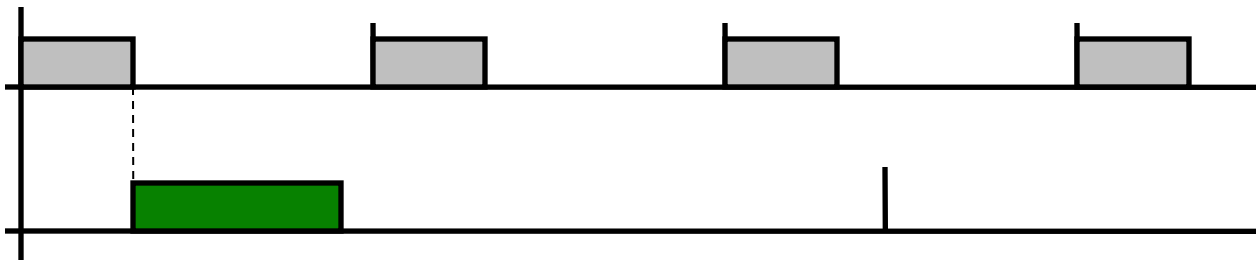
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Policy X meets deadlines?

**YES**

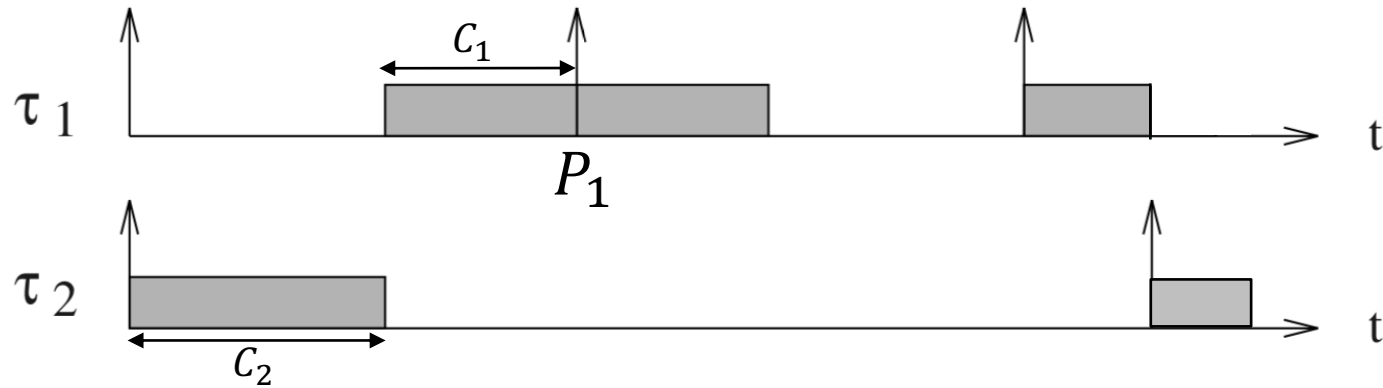
→ RM meets deadlines





# Optimality of the RM policy: Proof

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Two tasks scheduled **not** according to RM

For feasibility in a non-RM policy, we need  $c_1 + c_2 \leq P_1$  to hold at critical instant

Why?

# Optimality of the RM policy: Proof

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For feasibility in a non-RM policy, we need  $C_1 + C_2 \leq P_1$  to hold at critical instant

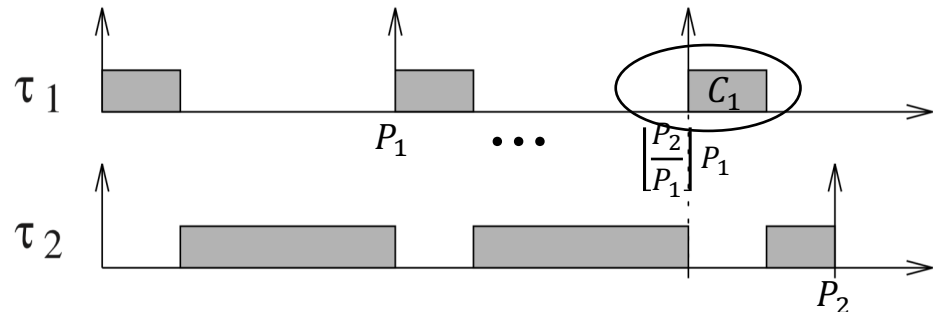
- Now **exchange priorities** of tasks to make it into an RM assignment
- Plan:
  - identify all possible cases
  - In each case derive feasibility condition
  - Show that if  $C_1 + C_2 \leq P_1$  then derived feasibility condition in RM holds

# Optimality of the RM policy: Case 1

For feasibility in a non-RM policy, we need  $C_1 + C_2 \leq P_1$  to hold at critical instant

- **Case 1:** computation time of  $\tau_1$  is short enough that all its *requests* are completed before the second request of  $\tau_2$ 
  - Number of periods  $P_1$  **entirely contained** in  $P_2$  is  $\left\lfloor \frac{P_2}{P_1} \right\rfloor \rightarrow$  Let  $F = \left\lfloor \frac{P_2}{P_1} \right\rfloor$
  - **Case 1** translates to  $C_1 + FP_1 \leq P_2$
  - **Feasibility:** All computation *requested* by  $\tau_1$  during  $P_2$ , in addition to  $C_2$ , should be completed by  $P_2$ 

$$(F + 1)C_1 + C_2 \leq P_2 \quad (*)$$
  - Need to show that  $C_1 + C_2 \leq P_1$  implies (\*)



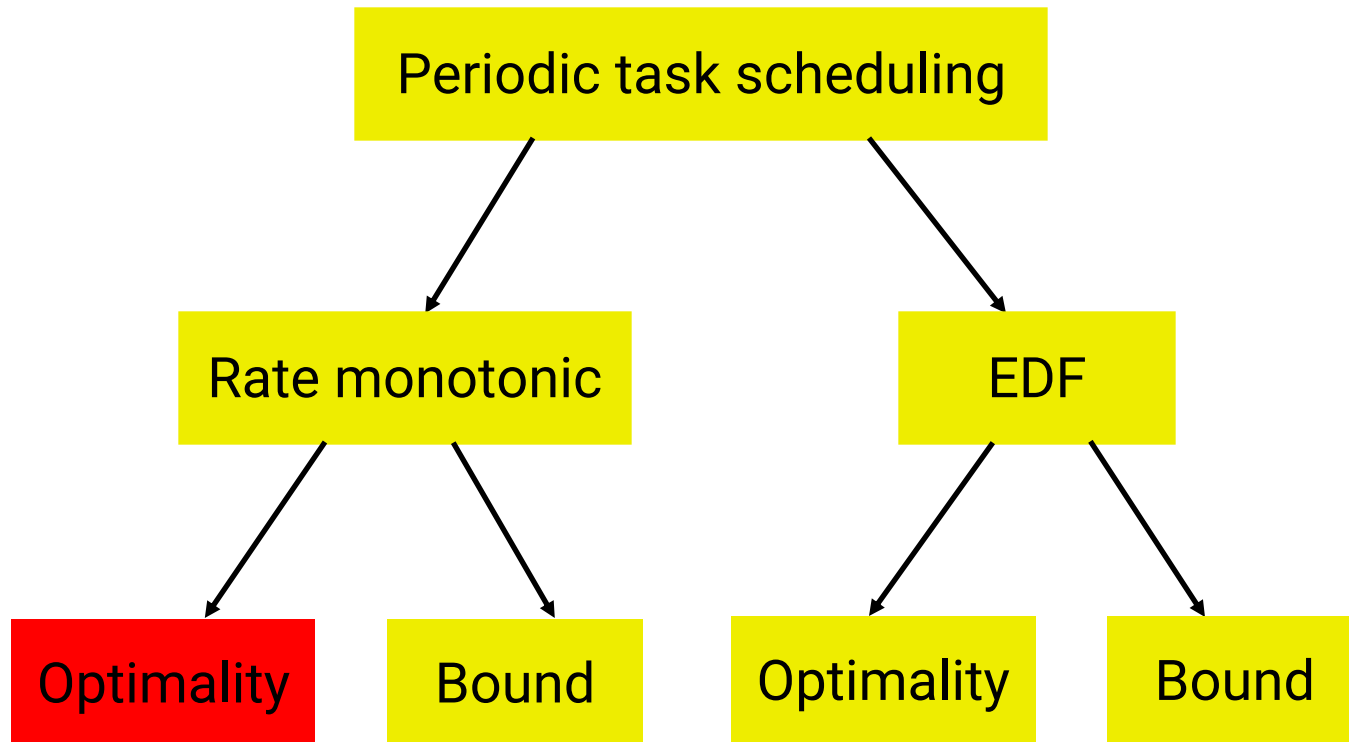
# Optimality of the RM policy: Case 2

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- homework

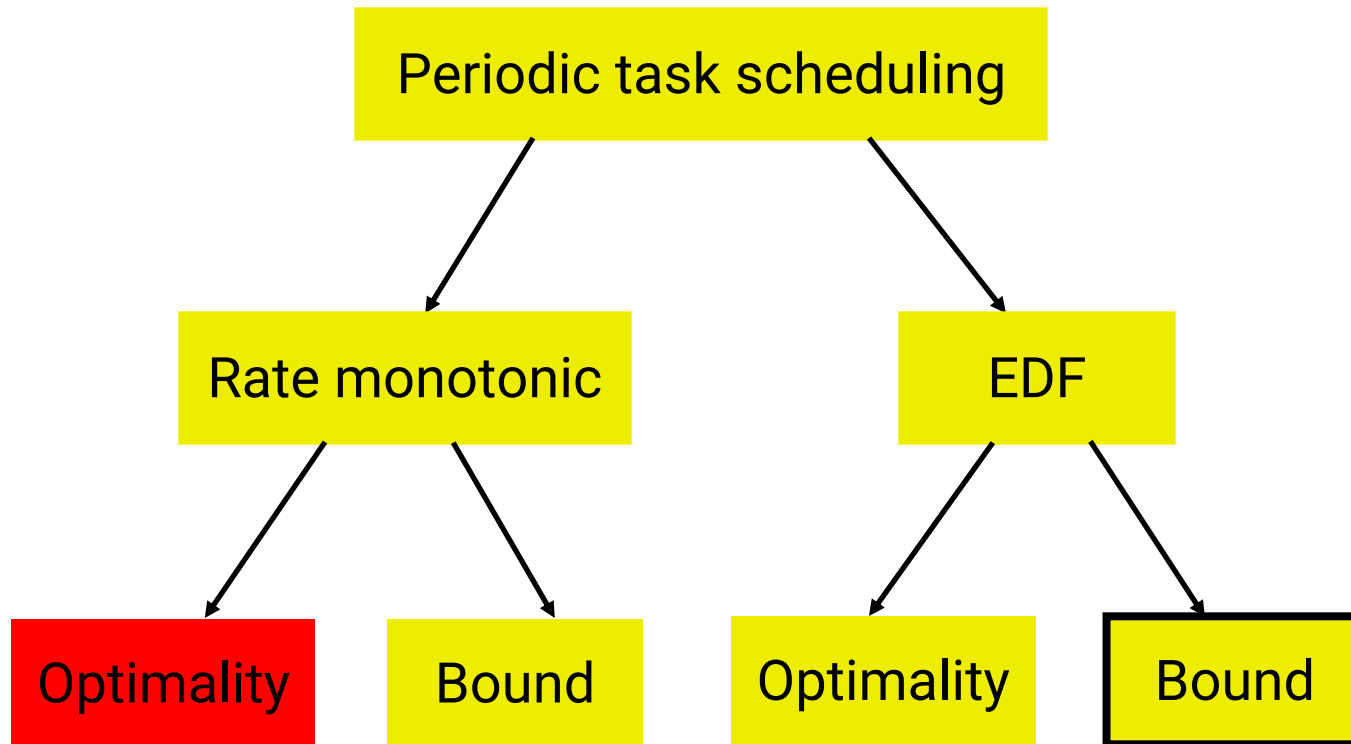
# What have we achieved?

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# Next

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# Recall: Utilization bounds for schedulability

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- $U_S$  is called a **utilization bound** for a given scheduling policy  $S$  if

All task sets with utilization factor  $\leq U_S$  can be scheduled using policy  $S$

- $U_S$  is **tight** if, in addition, for a given scheduling policy  $S$  the following holds:

*For every  $\epsilon > 0$ , there exists at least one task set with utilization  $(U_S + \epsilon)$  that **cannot** be scheduled using policy  $S$*

- A tight bound is the best possible utilization bound: If  $U_S$  is tight, then no other  $U > U_S$  can be a utilization bound for scheduling policy  $S$
- Of course, the maximum value that  $U_S$  can attain for any  $S$  is 1. Why? In class
- $U_S$  is also called the **schedulable utilization** of algorithm  $S$

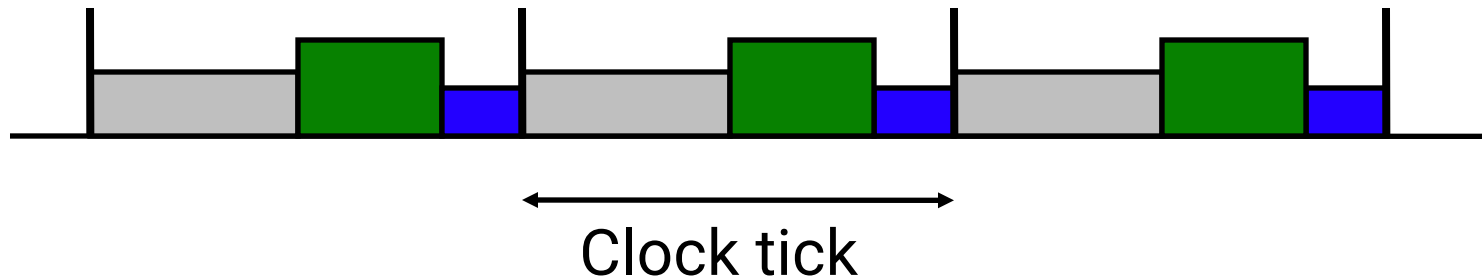
# Utilization bound for EDF

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- Why is it 100%?
- Consider a task set where:

$$\sum_i \frac{C_i}{P_i} = 1$$

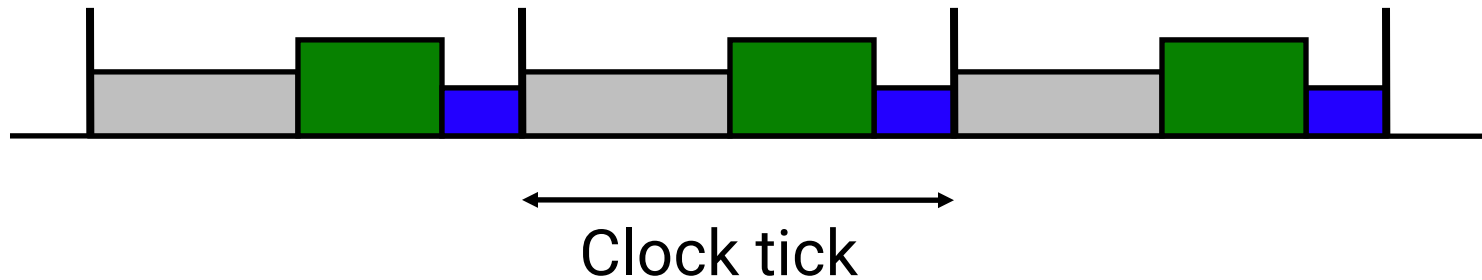
- **Plan:** construct an EDF schedule for this taskset
- Imagine a policy that reserves for each task  $i$  a fraction  $u_i$  of each clock tick, where  $u_i = C_i/P_i$





# Utilization bound for EDF

- Imagine a policy that reserves for each  $T_i$  a fraction  $u_i$  of each time unit, where  $u_i = C_i / P_i$
- Assume wlog that the periods are integers (perhaps after proper scaling)
- Divide time into, say, integer  $L$ -length ticks (e.g.,  $L = \text{GCD}(P_1, \dots, P_n)$ )

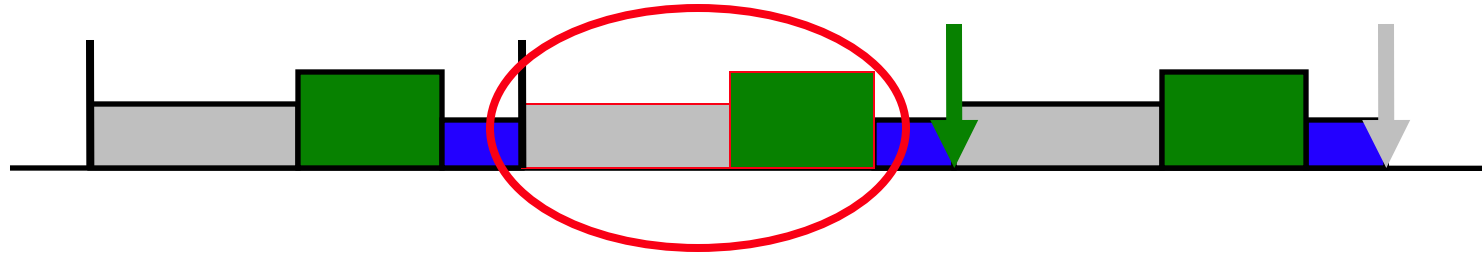


- This policy meets all deadlines, because:
  - Time given to  $T_i$  in its period =  $u_i \times \text{tick length} [\text{time}/\text{tick}] \times (\# \text{ ticks}/\text{period})$   
 $= u_i L (P_i/L) = (C_i/P_i) P_i = C_i [\text{time}/\text{period}]$   
(i.e., enough to finish)

# Utilization bound for EDF

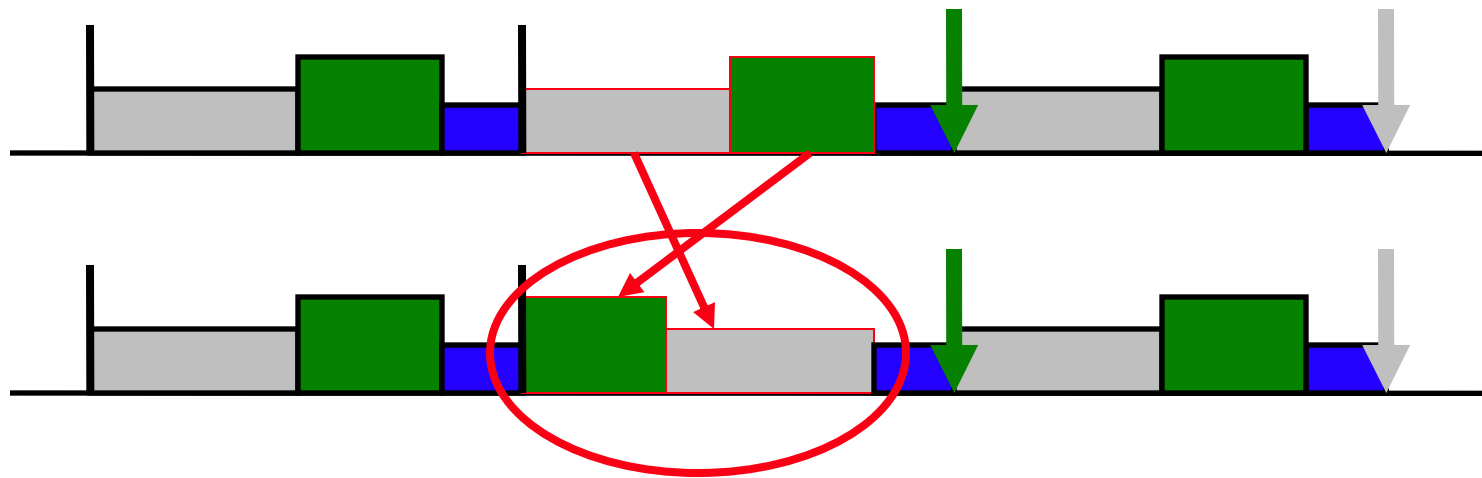
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- Pick any two execution chunks that are not in EDF order and swap them



## Utilization bound for EDF

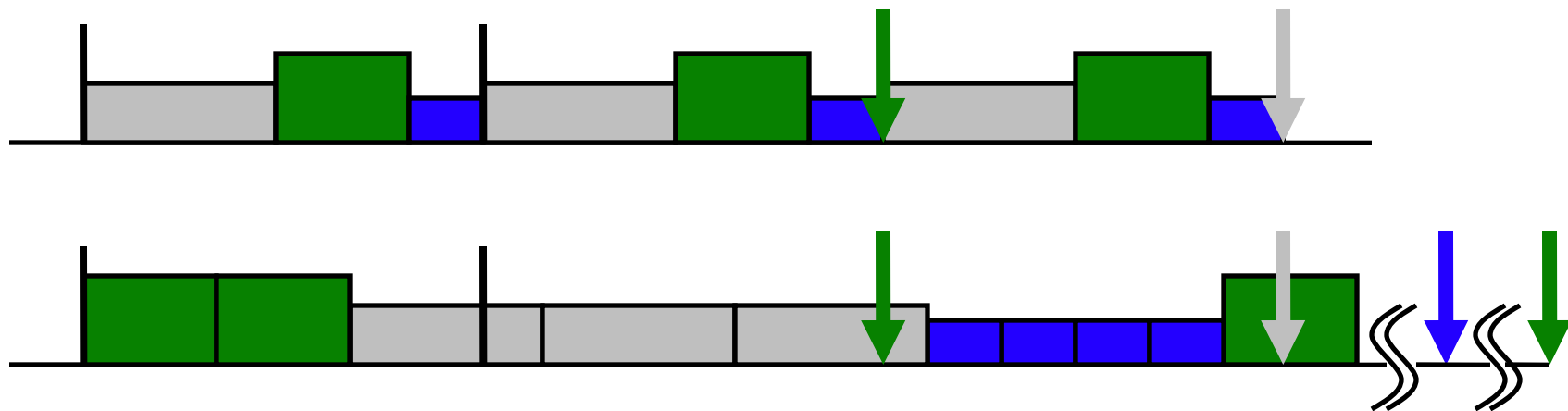
- Pick any two execution chunks that are not in EDF order and swap them



- Still meets deadlines! Why?

# Utilization bound for EDF

- Pick any two execution chunks that are not in EDF order and swap them



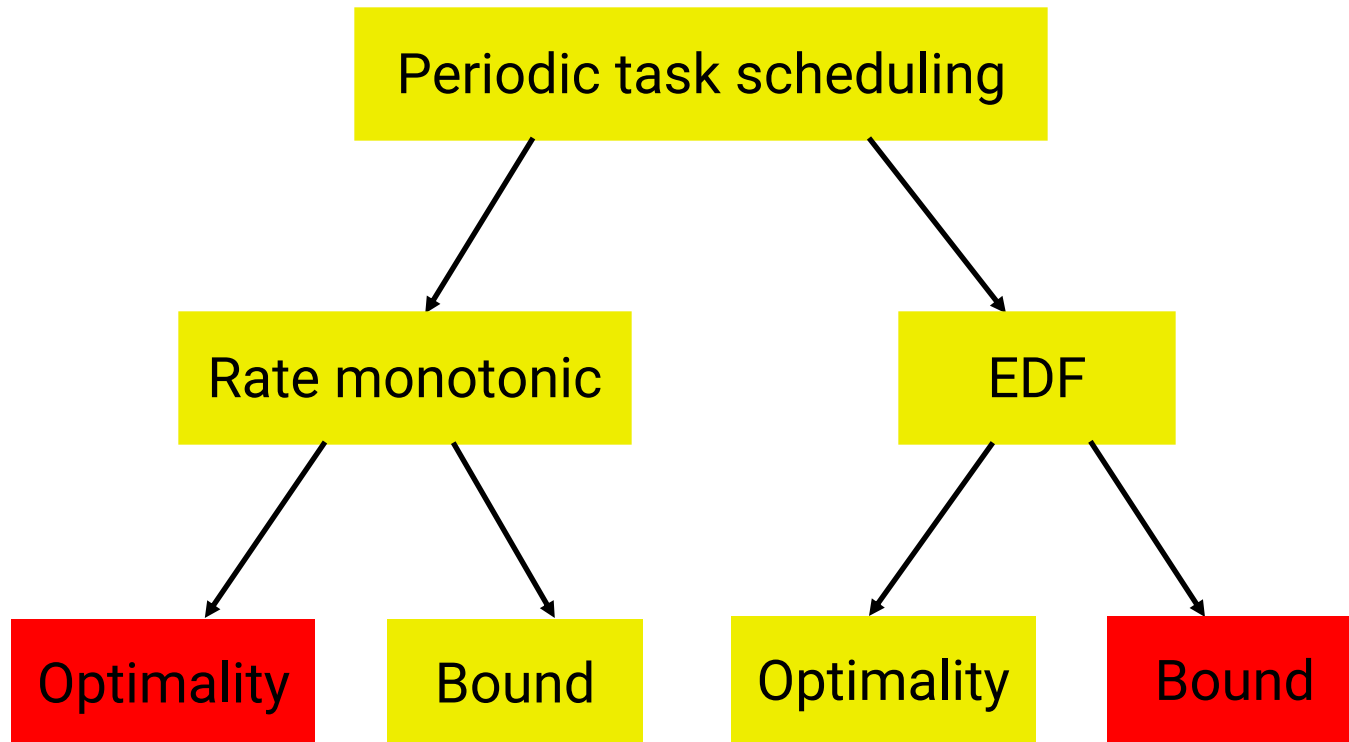
- Still meets deadlines!
- Repeat swap until all in EDF order
  - → EDF meets deadlines

# Utilization bound for EDF

- Why does this prove that the utilization bound of EDF is 1?
  - We showed that every taskset with  $U = 1$  is feasible under EDF
  - Must also show that every taskset with  $U < 1$  is also feasible under EDF
  - This is not needed! Previous argument follows for any  $U \leq 1$
  - Consequences:
    - EDF is optimal!
    - EDF is able to schedule every task set who utilization is 1 or less

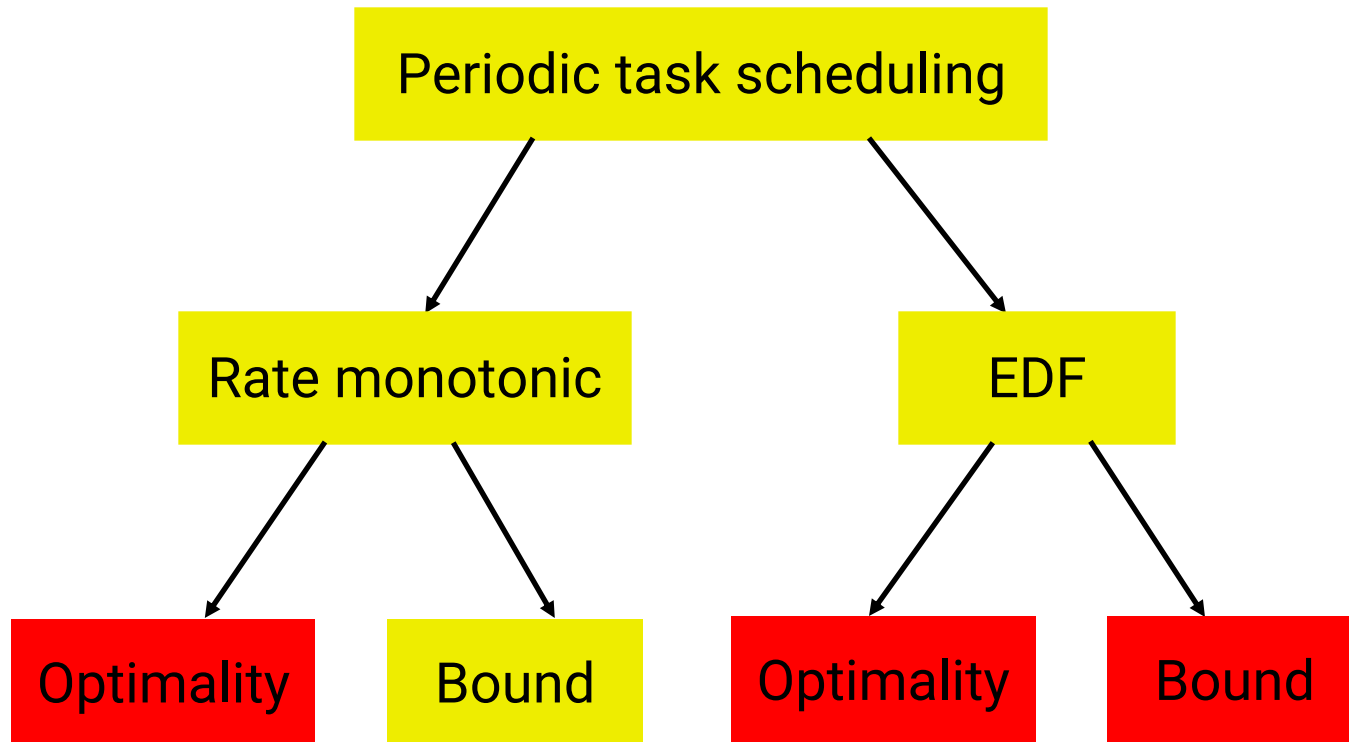
# Next

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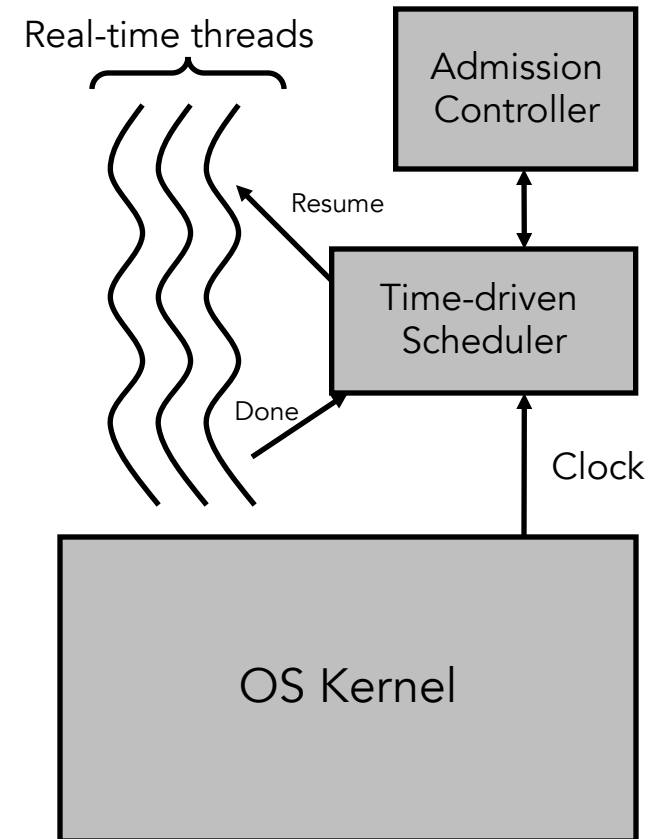
# Next

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# Tick-based scheduling within an OS

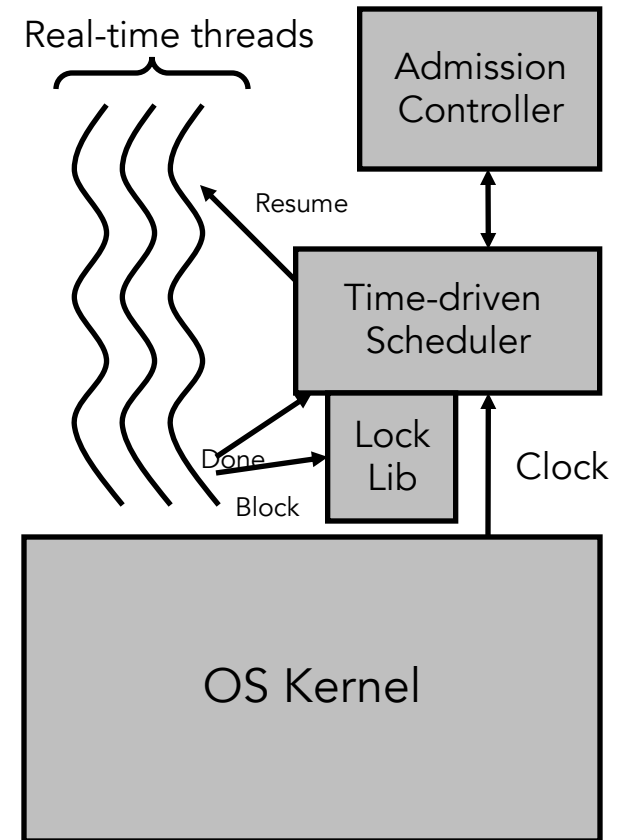
- A real-time library for periodic tasks on Linux or Windows
  - There is need to provide approximate real-time guarantees on common operating systems (as opposed to specialized real-time OSes)
  - A high-priority “real-time” thread pool is created and maintained
  - A higher-priority scheduler is invoked periodically by timer-ticks to check for periodic invocation times of real-time threads. The scheduler resumes threads whose arrival times have come.
  - Resumed threads execute one invocation then block.
  - Scheduling is preemptive
  - The scheduler can implement arbitrary scheduling policies including EDF, RM, etc.
  - An admission controller is responsible for spawning new periodic threads if the new task set can meet its deadlines.





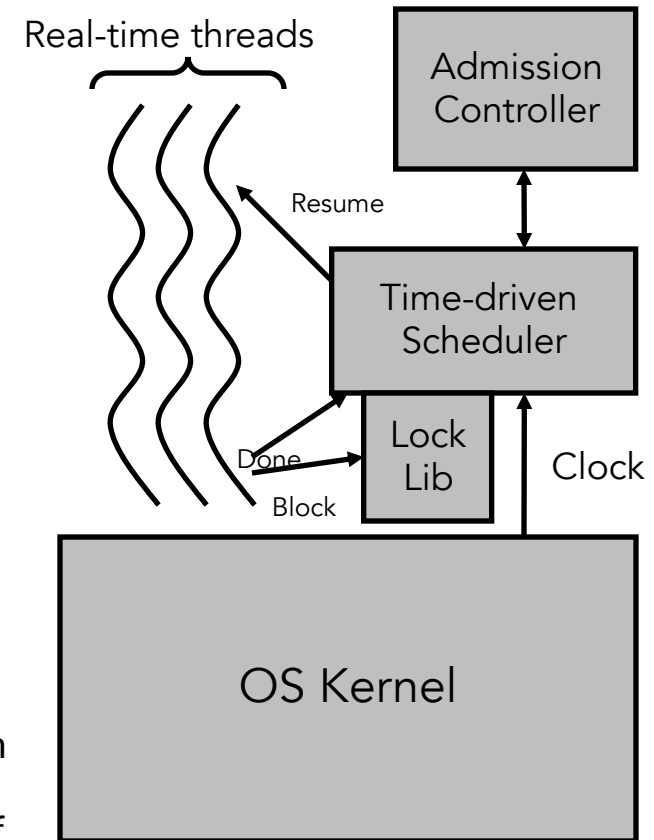
# Tick-based scheduling within an OS

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  - The scheduler can implement arbitrary scheduling policies including EDF, RM, etc.
  - An admission controller is responsible for spawning new periodic threads if the new task set can meet its deadlines.
  - Scheduler implements wrappers for blocking primitives



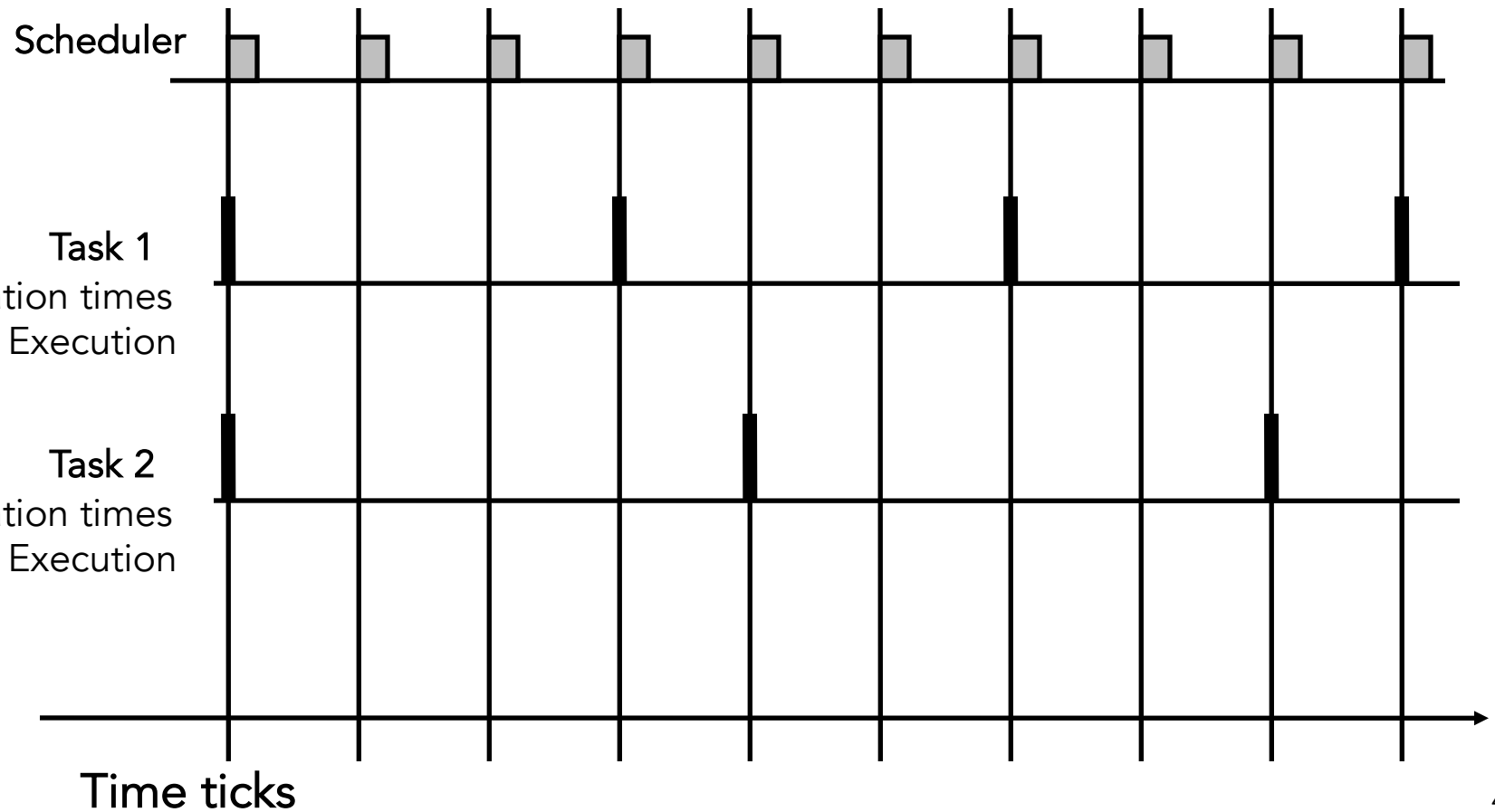
# The time-driven scheduler

- /\* N is the number of periodic tasks \*/
- For i=1 to N
- if (current\_time = next\_arrival\_time of task i)
- put task i in ready\_queue
- /\* ready\_queue is a priority queue that implements
- the desired scheduling policy. \*/
- Inspect top task from ready queue, call it j
- If (a task is running and its priority is higher than priority of j) return
- Else resume task j (and put the running task into the ready queue if applicable); return



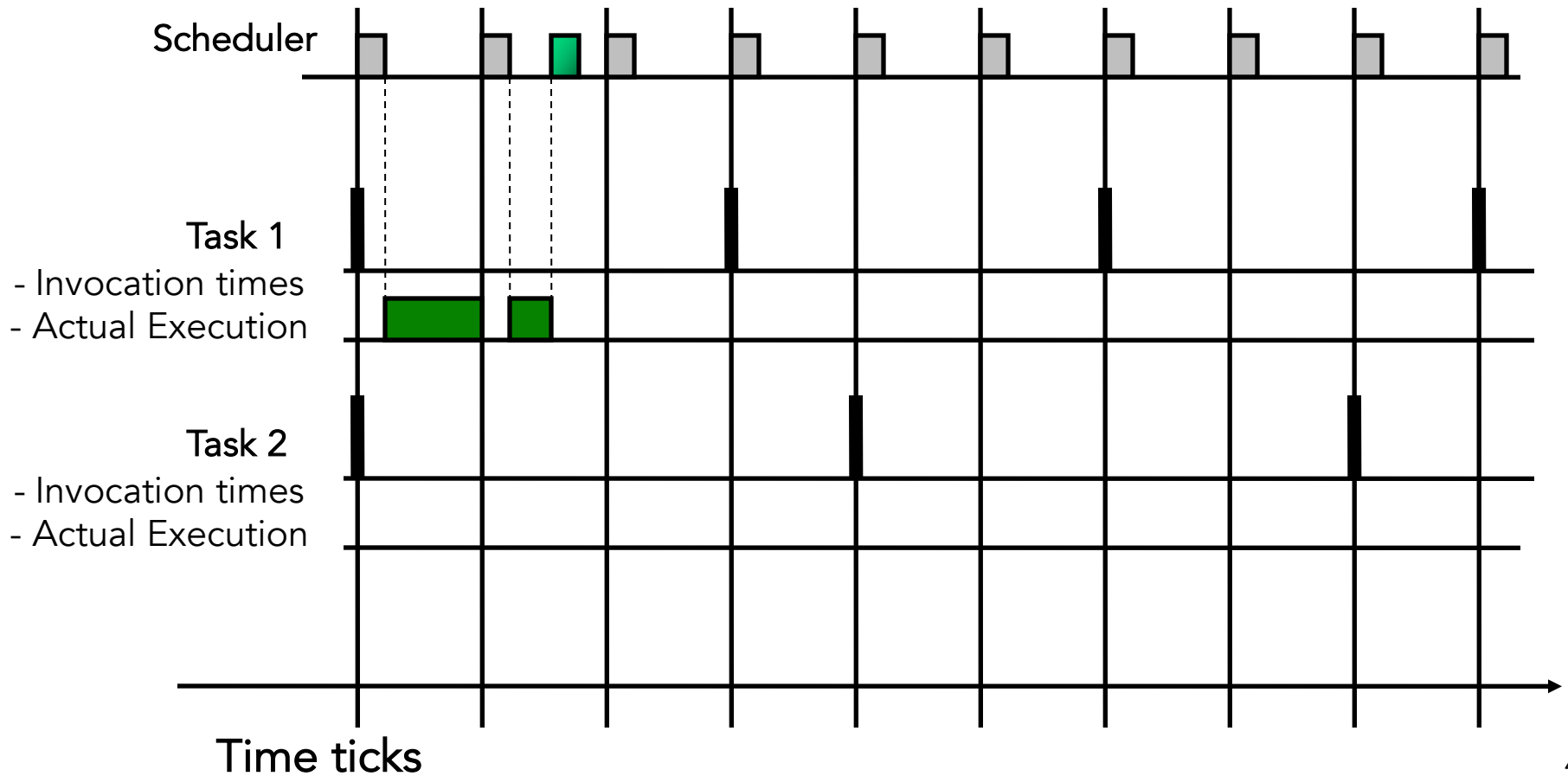
# An example schedule

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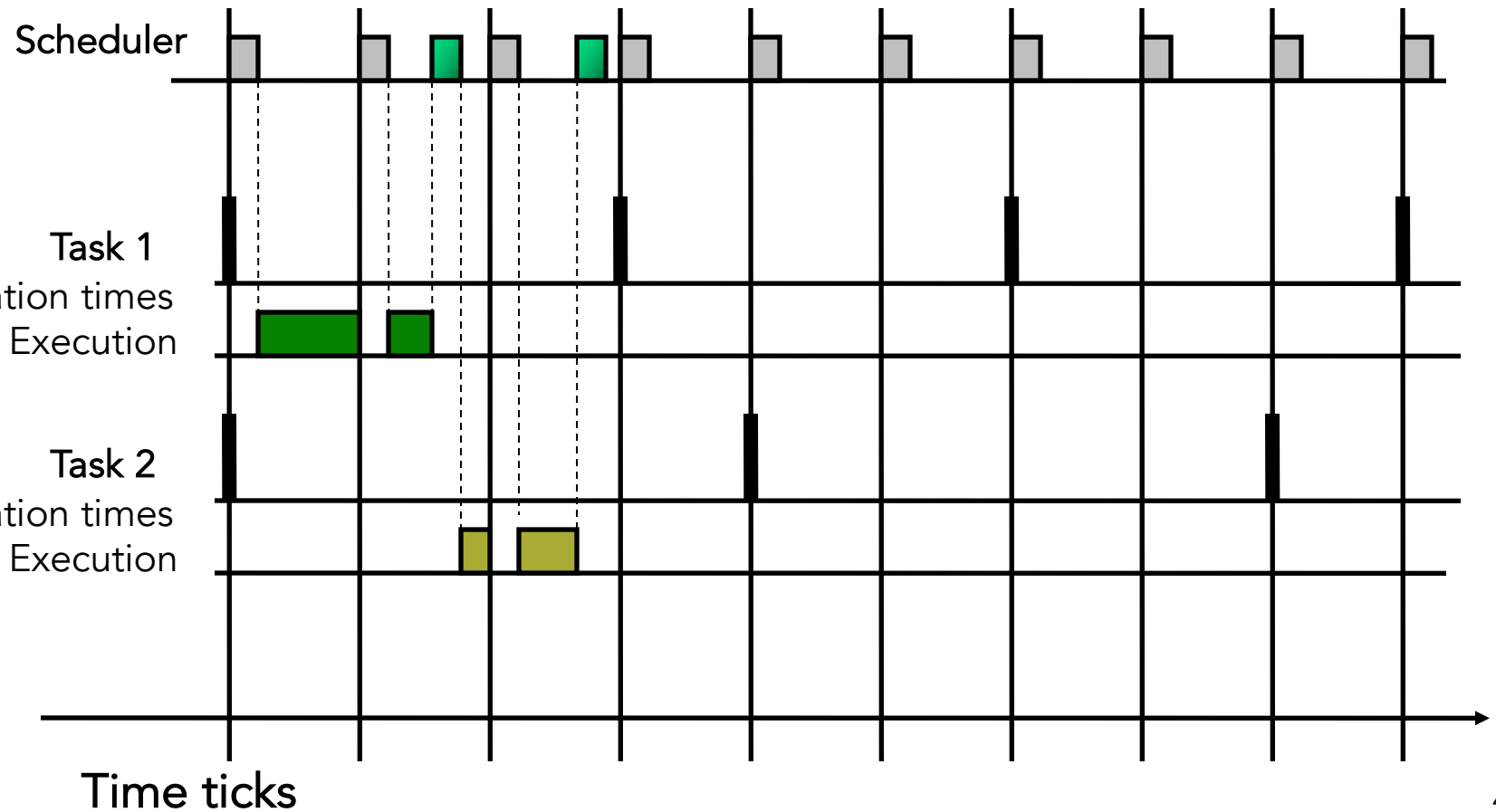


# An example schedule

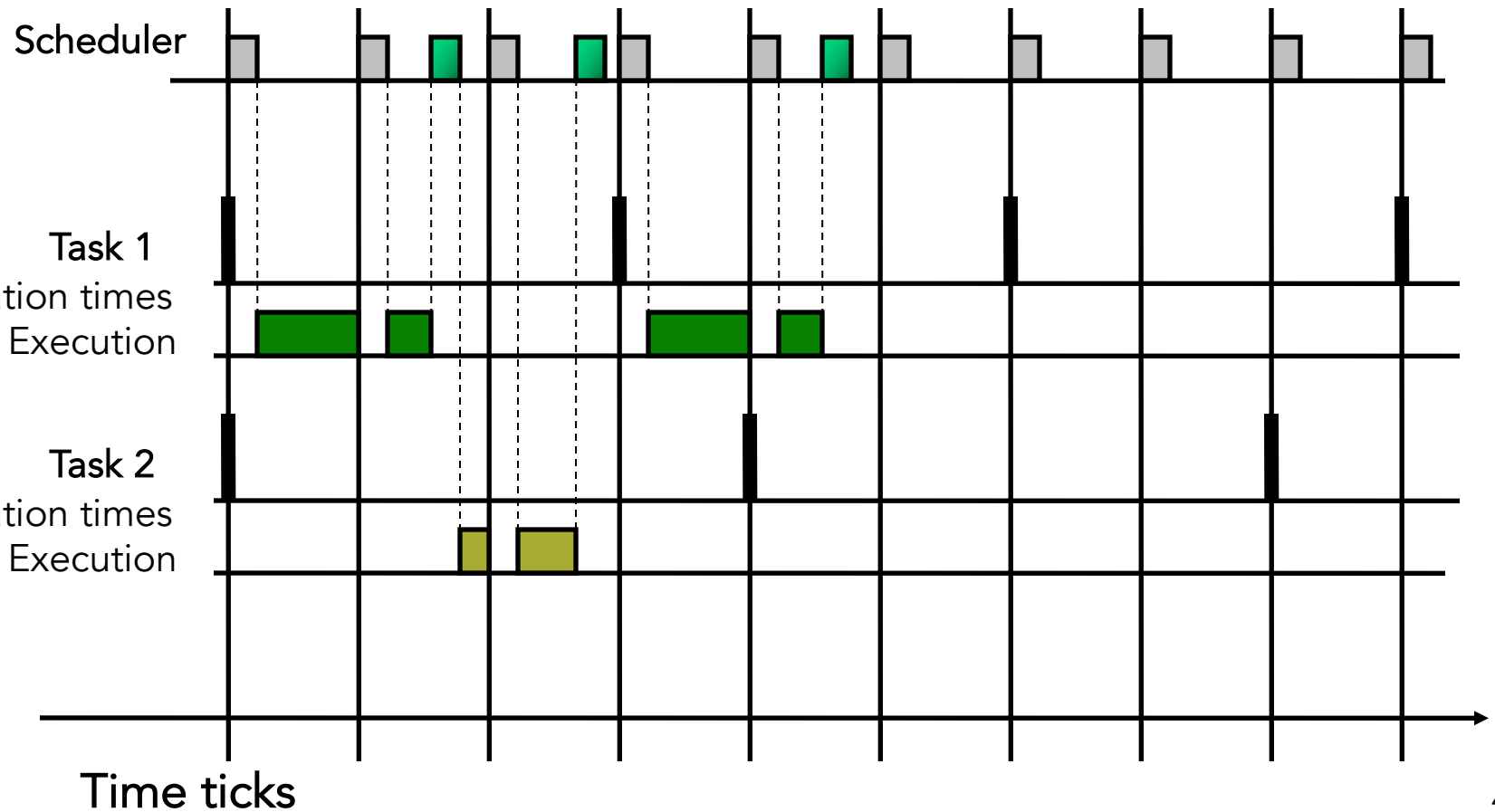
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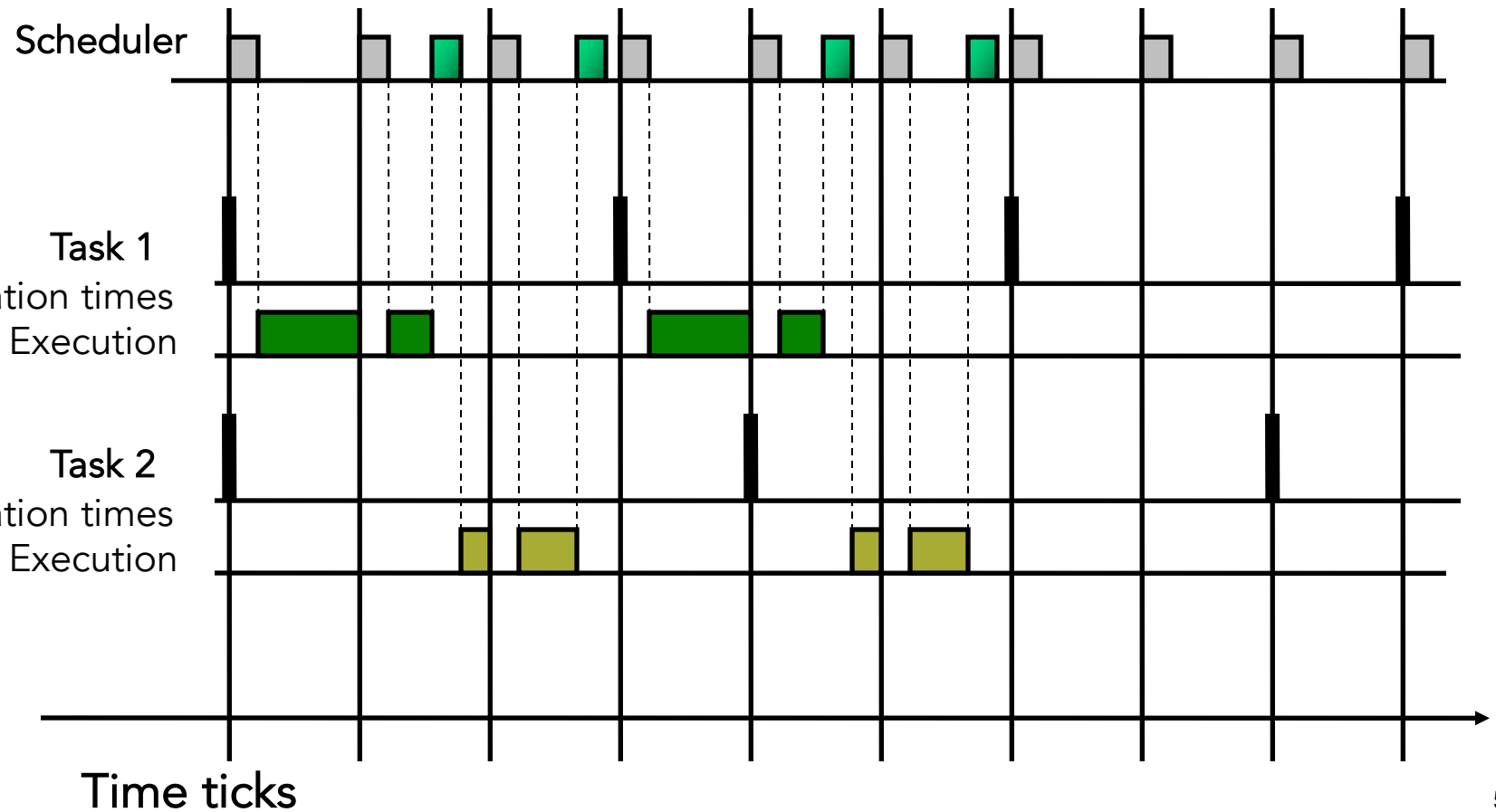
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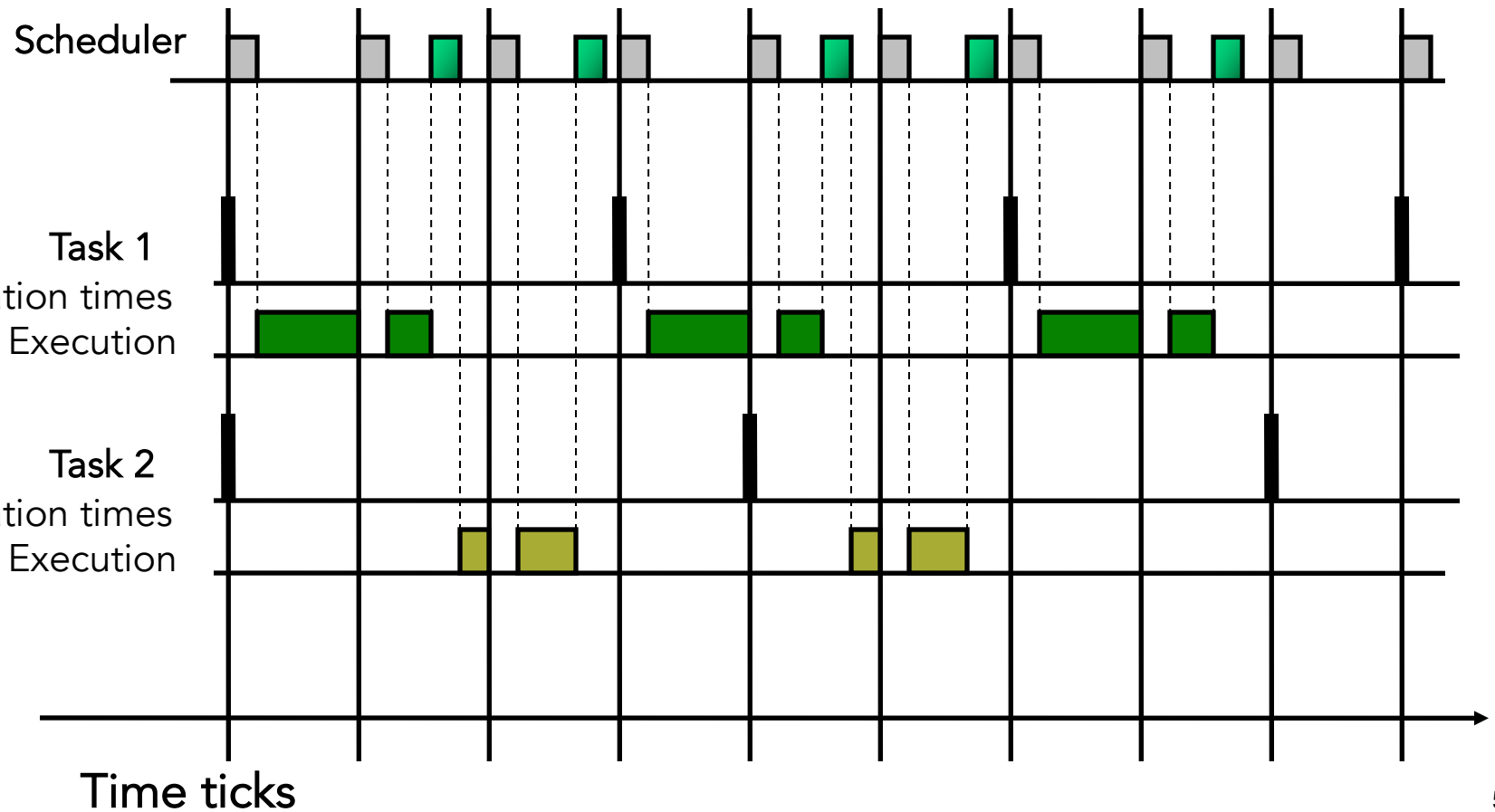
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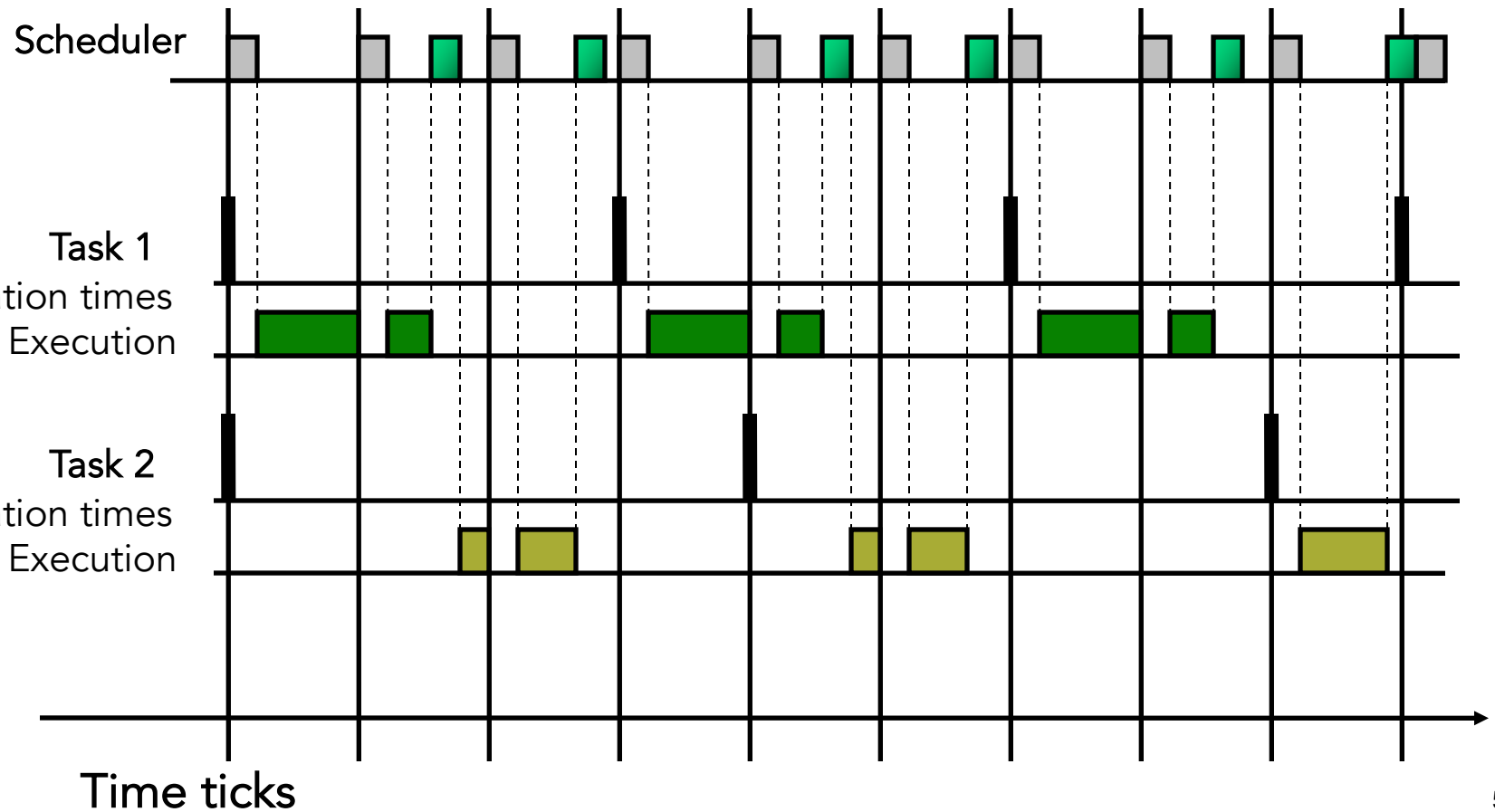


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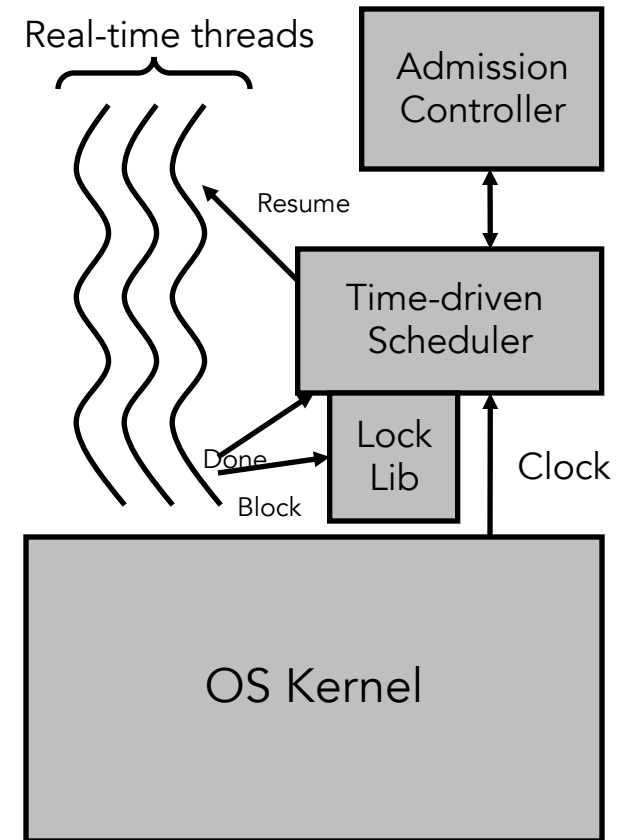


# An example schedule



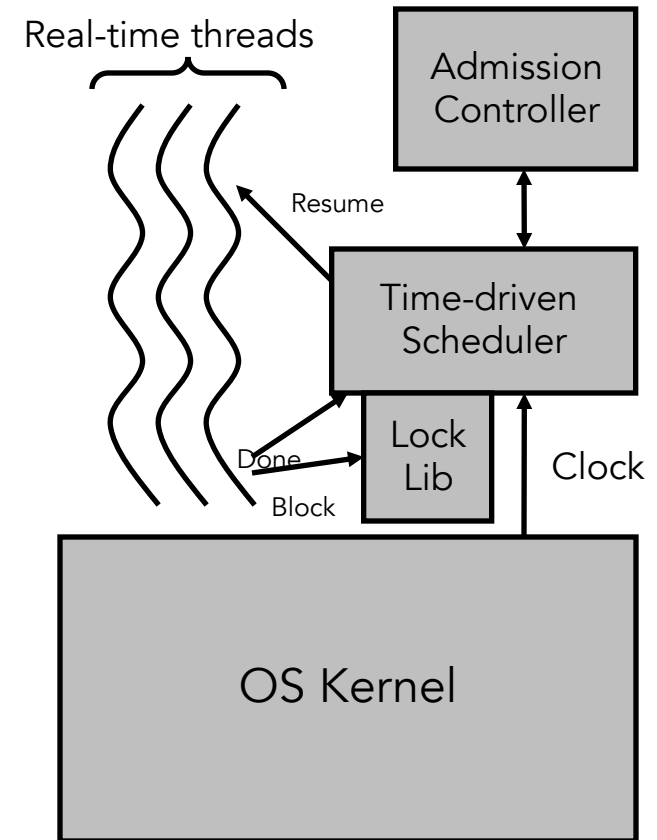
# Admission controller

- Implements schedulability analysis
  - If  $U + C_{new}/P_{new} < U_{bound}$  admit task
  - Must account for various practical overheads. How?
  - Examples of overhead:
    - How to account for the overhead of running the time-driven scheduler on every time-tick?
    - How to account for the overhead of running the scheduler after task termination?
- If new task admitted
  - $U = U + C_{new}/P_{new}$
  - Create a new thread
  - Register it with the scheduler



# Library with lock primitives

- Lock (S) {
  - Check if semaphore S = locked
  - If locked
    - enqueue running tasks in semaphore queue
  - Else
    - let semaphore = locked
- }
- Unlock (S) {
  - If semaphore queue empty then
    - semaphore = unlocked
  - Else
    - Resume highest-priority waiting task
- }



Problem: some threads may execute blocking OS calls (e.g., disk or network read/write and block without calling your lock/unlock!)