1 Basic Mathematics & Measurements

Section I

Single Correct Option

1, $K = \frac{1}{2}mv^2$

$\therefore \quad \left(\frac{\Delta K}{K}\right)_{max} = \frac{\Delta m}{m} + 2\frac{\Delta v}{v}$

$\therefore$ Maximum error $= 2\% + 2(3\%)$

In the estimate of,

kinetic energy $(K) = 8\%$

Option (c) is correct.

2. $d = \frac{m}{V} = \frac{m}{l^3}$

$\therefore$

$$\left(\frac{\Delta d}{d} \times 100\right)_{max} = \left(\frac{\Delta m}{m} \times 100\right) + 3\left(\frac{\Delta l}{l} \times 100\right)$$

$$= 4\% + 3(3\%)$$

$$= 13\%$$

Option (d) is correct.

3. $p = \frac{F}{A} = \frac{F}{L^2}$

$\therefore$ Permissible error in pressure (p)

$$= 4\% + 2(2\%)$$

$$= 8\%$$

Option (a) is correct.

4. $p_1V_1 = p_2V_2$

$\Rightarrow \quad p_2 = p_1\frac{V_1}{V_2}$

$$= p_1\frac{V_1}{V_1 - 10\% \text{ of } V_1}$$

$$= \frac{p_1}{90\%}$$

$$p_2 = \frac{p_1}{9}$$

$\therefore$ Percentage increase in pressure $= \frac{100}{9}$

$$= 11.1$$

Option (a) is correct.

5. $K = \frac{p^2}{2m}$

$\therefore$ Error in the measurement of kinetic energy (K)

$$= 2 \times 100\%$$

$$= 200\%$$

Option (d) is correct.

6. $3400 = 3.400 \times 10^3$

$\therefore$ Number of significant figures $= 2$

Option (d) is correct.

7. $A = 3.124 \text{ m} \times 3.002 \text{ m}$

$$= 9.378 / 248 \text{ m}^2$$

$$= 9.378 \text{ m}^2$$

Option (a) is correct.

8. $g = \frac{GM}{R^2}$

$$= \frac{\text{constant}}{R^2}$$

$K = \frac{1}{2}I\omega^2$

$$= \frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega$$

$$= \text{constant} \times R^2$$

$\therefore$ Decrease in R (radius) by 2% world increase g by 4% and decrease K (rotational kinetic energy) by 4%.

Option (b) is correct.

9. Heat $(H) = 2^2 Rt$

$\therefore$ Maximum error in measuring heat (H)

$$= 2(2\%) + 1\% + 1\%$$
$$= 6\%$$

Option (b) is correct.

10. $V = lbt$

$$= 12 \times 6 \times 2.45$$
$$= 176.4$$
$$= 1.764 \times 10^2 \text{ cm}^3$$
$$= 2 \times 10^2 \text{ cm}^2$$

Option (b) is correct.

11. $I = \dfrac{P}{4\pi r^2}$

i.e., Ir^2 = constant

i.e., if r is increased by 2% the intensity will decrease by 4%.

Option (d) is correct.

12. Option (b) is correct.

13. $V = \dfrac{4}{3}\pi r^3$

$$\frac{\Delta V}{V} = 3\frac{\Delta r}{r}$$
$$= 3(1\%)$$
$$= 3\%$$

Option (c) is correct.

14. $a^3 = 6a^2$ (given)

$\therefore\ a = 6$

$\Rightarrow V = 6^3 = 216 \text{ m}^3$

Option (b) is correct.

15. $g = 4\pi^2 \dfrac{l}{T^2}$

$$\therefore \left(\frac{\Delta g}{g} \times 100\right)_{\max} = \frac{\Delta l}{l} \times 100 + 2\left(\frac{\Delta T}{T} \times 100\right)$$
$$= \left(\frac{1 \text{ mm}}{100 \text{ m}} \times 100\right) + 2\left(\frac{0.15}{2003} \times 10\right)$$
$$= 0.1\% + 0.1\% = 0.2\%$$

Option (a) is correct.

16. $Q = I\,\alpha\, e^{-\frac{tI}{(\Delta V)\varepsilon_0 \beta}}$ (given)

We know that

$$Q = It$$

$$\therefore \quad t = \alpha\, e^{-\frac{tI}{(\Delta V)\varepsilon_0 \beta}}$$

$$\Rightarrow \quad [\alpha] = [t]$$

and $[\beta] = \left[\dfrac{tI}{(\Delta V)\varepsilon_0}\right]$

$$\Rightarrow \quad \left[\frac{\beta}{\alpha}\right] = \left[\frac{tI}{(\Delta V)\varepsilon_0 t}\right] = \left[\frac{I}{(\Delta V)}\right]\left[\frac{1}{\varepsilon_0}\right]$$
$$= \frac{1}{[\text{Resistance}]}\left[\frac{1}{\varepsilon_0}\right]$$

or $$\left[\frac{\beta}{\alpha}\right] = \frac{1}{[\text{ML}^2\text{T}^{-3}\text{A}^{-2}]}\frac{1}{[\text{M}^{-1}\text{L}^{-3}\text{T}^4\text{A}^2]}$$
$$= \frac{1}{[\text{L}^{-1}\text{T}]} = [\text{velocity}]$$
$$= \frac{1}{[\mu_0 \varepsilon_0]^{1/2}}$$

Option (a) is correct.

17. $a = \dfrac{\sqrt{pq}}{r^2 s^3}$

$$\left(\frac{\Delta a}{a} \times 100\right)_{\max} = \frac{1}{2}\left(\frac{\Delta p}{p} \times 100\right) + \frac{1}{2}\left(\frac{\Delta q}{q} \times 100\right) + 2\left(\frac{\Delta r}{r} \times 100\right) + 3\left(\frac{\Delta s}{s} \times 100\right)$$
$$= \frac{1}{2}(1\%) + \frac{1}{2}(3\%) + 2(0.5\%) + 3(0.33\%)$$
$$= 0.5\% + 1.5\% + 1\% + 1\%$$
$$= 4\%$$

Option (c) is correct.

18. Least count of main scale

$$= \frac{2 \text{ mm}}{4} = 0.5 \text{ mm}$$

$$\text{Least count} = \frac{\text{least count of main scale}}{50}$$
$$= 0.1 \text{ mm}$$

Zero error $= -30 \times 0.01$ mm

$= -0.3$ mm

(–ive sign, zero of circular scale is lying above observed reading of plate thick)

$= 2$ MSR + 20 CSR

$= (2 \times 0.5 \text{ mm}) + (20 \times 0.01 \text{ mm})$

$= 1 \text{ mm} + 0.2 \text{ mm}$

$= 1.2 \text{ mm}.$

Plate thickness (corrected reading)

= observed reading – zero error

$= 1.2 \text{ mm} + 0.3 \text{ mm}$

$= 1.5 \text{ mm}$

Option (d) is correct.

More than One Correct Options

1. Maximum percentage error in x :

$= (\%\text{ error in } a) + 2(\%\text{ error in } b) + 3\ (\%\text{ error in } c)$

$= 15\%$

Assertion and Reason

1. Least count of screw gauge

$$= \frac{\text{Pitch}}{\text{Number of divisions of circular scale}}$$

Less the value of pitch, less will be least count of screw gauge leading to len uncertainty that is more accuracy in the measurement.

Thus, assertion is true.

From the above relation we conclude that least count of screw gauge is inversely proportional to the number of divisions of circular scale.

Thus reason is false.

Option (c) is correct.

Match the Columns

1. (a) $F = \dfrac{GM_1M_2}{r^2}$

$\therefore \quad GM_1M_2 = Fr^2$

$i.e., [GM_1M_2] = [\text{MLT}^{-2}][\text{L}^2]$

$= [\text{ML}^3\text{T}^{-2}]$

$\therefore$ (a) $\rightarrow$ (q)

(b) $\dfrac{3RT}{M} = \dfrac{3pV}{n} = \dfrac{3 \text{ work}}{nM}$

$\therefore \quad \left[\dfrac{3RT}{M}\right] = \left[\dfrac{\text{ML}^2\text{T}^{-2}}{\text{M}}\right] = [\text{L}^2\text{T}^{-2}]$

$\therefore$ (b) $\rightarrow$ (r)

(c) $\dfrac{F^2}{q^2B^2} = \left(\dfrac{F}{qB}\right)^2$

$= \left(\dfrac{qvB\sin\theta}{qB}\right)^2$

$= v^2 \sin^2\theta$

$\therefore \quad \left[\dfrac{F^2}{q^2B^2}\right] = [\text{LT}^{-1}]^2 = [\text{L}^2\text{T}^{-2}]$

$\therefore$ (c) $\rightarrow$ (r)

(d) $g = \dfrac{GM_e}{R_e^2}$

$\dfrac{GM_e}{R_e} = gR_e$

$\Rightarrow \quad \left[\dfrac{GM_e}{R_e}\right] = [\text{LT}^{-2}][\text{L}]$

$= [\text{L}^2\text{T}^{-2}]$

$\therefore$ (d) $\rightarrow$ (r)

2 Units & Dimensions Vectors

Section I

Single Correct Option

1. Pressure $(p) = \dfrac{\text{Force}}{\text{Area}}$

$$\therefore \quad [p] = \frac{[\mathrm{MLT^{-2}}]}{[\mathrm{L^2}]}$$

$$= [\mathrm{ML^{-1}T^{-2}}]$$

Option (d) is correct.

2. $W = I^2Rt$

$$\therefore \quad [R] = \frac{[\mathrm{ML^2T^{-2}}]}{[\mathrm{A^2T}]} = \frac{[\mathrm{ML^2T^{-2}A^{-2}}]}{[\mathrm{T}]} \quad \ldots(\mathrm{i})$$

$$V = L\frac{dI}{dt} \text{ and } W = Vq$$

$$\therefore \quad L = \frac{W}{q}\frac{dt}{dI}$$

$$[L] = \frac{[\mathrm{ML^2T^{-2}}][\mathrm{T}]}{[\mathrm{A^2T}]} = [\mathrm{ML^2T^{-2}A^{-2}}]$$

Using Eq. (i)

$$[R] = \frac{[L]}{[T]}$$

i.e., $$[T] = \left[\frac{L}{R}\right]$$

Option (c) is correct.

3. $F = 6\pi\eta av$

$$\therefore \quad [\eta] = \frac{[F]}{[av]}$$

$$= \frac{[\mathrm{MLT^{-2}}]}{[\mathrm{LLT^{-1}}]}$$

$$= [\mathrm{ML^{-1}T^{-1}}]$$

Option (d) is correct.

4. $\phi = Li$

$$\therefore \quad [\phi] = [L][i]$$

$$= [\mathrm{ML^2T^{-2}A^{-2}}][\mathrm{A}]$$

$$= [\mathrm{ML^2T^{-2}A^{-1}}]$$

Option (a) is correct.

5. Linear impulse $(I) = F \cdot \Delta t$

$$[I] = [\mathrm{MLT^{-2}}][\mathrm{T}]$$

$$= [\mathrm{MLT^{-1}}]$$

Option (c) is correct.

6. $F = G\dfrac{m_1m_2}{d^2}$

$$[G] = \frac{[F][d^2]}{[m_1m_2]} = \frac{[\mathrm{MLT^{-2}}][\mathrm{L^2}]}{[\mathrm{M^2}]}$$

$$= [\mathrm{M^{-1}L^3T^{-2}}]$$

Option (c) is correct.

7. $F = \dfrac{\mu_0}{4\pi}\dfrac{i_1 \cdot i_2}{d}$

$$\therefore \quad [\mu_0] = \frac{[\mathrm{MLT^{-2}}][\mathrm{L}]}{[\mathrm{A^2}]} = [\mathrm{ML^2T^{-2}A^{-2}}]$$

Option (c) is correct.

8. $[k] = \dfrac{[\mathrm{L}]}{[\mathrm{LT^{-1}}]} = [\mathrm{T}]$

Option (c) is correct.

9. $[a] = \dfrac{[\mathrm{MLT^{-2}}]}{[\mathrm{T}]} = [\mathrm{MLT^{-3}}]$

$$[b] = \frac{[\mathrm{MLT^{-2}}]}{[\mathrm{T^2}]} = [\mathrm{MLT^{-4}}]$$

Option (c) is correct.

10. $E = h\nu$

$\therefore \quad [h] = \dfrac{[\mathrm{ML^2\,T^{-2}}]}{[\mathrm{T^{-1}}]} = [\mathrm{ML^2\,T^{-1}}]$

Angular momentum $(J) = \dfrac{nh}{2\pi}$

$[J] = [h] = [\mathrm{ML^2\,T^{-1}}]$

Option (b) is correct.

11. $[\text{Energy}] = [\mathrm{ML^2\,T^{-2}}]$

$= [\mathrm{M}][\mathrm{L\,T^{-1}}]^2$

$\therefore [\text{Mass}] = [Ev^{-2}]$

Option (c) is correct.

12. $\dfrac{1}{2}\varepsilon_0 E^2 = \text{Energy density} = \dfrac{\text{Energy}}{\text{Volume}}$

$\therefore \quad \left[\dfrac{1}{2}\varepsilon_0 E^2\right] = \dfrac{[\mathrm{ML^2\,T^{-2}}]}{[\mathrm{L^3}]}$

$= [\mathrm{ML^{-1}\,T^{-2}}]$

Option (b) is correct.

13. $[a] = [\mathrm{T^2}]$

$[b] = \dfrac{[\mathrm{T^2}]}{[\mathrm{L}][\mathrm{ML^{-1}\,T^{-2}}]}$

$\therefore \quad \left[\dfrac{a}{b}\right] = [\mathrm{M\,T^{-2}}]$

Option (b) is correct.

14. Velocity gradient $= \dfrac{dv}{dx}$

$[\text{Velocity gradient}] = \dfrac{[\mathrm{L\,T^{-1}}]}{[\mathrm{L}]}$

$= [\mathrm{T^{-1}}]$

$= [\mathrm{M^0\,L^0\,T^{-1}}]$

Option (a) is correct.

15. $[\text{Force}] = [\mathrm{ML\,T^{-2}}]$

$\therefore \quad [\text{Mass}] = \dfrac{[\mathrm{F}]}{[\mathrm{L\,T^{-2}}]}$

$= [\mathrm{F\,L^{-1}\,T^2}]$

Option (a) is correct.

16. Coefficient of friction (μ)

$= \dfrac{\text{Limitting frictional force}}{\text{Normal force}}$

$\therefore \quad [\mu] = [\mathrm{M^0\,L^0\,T^0}]$

Option (b) is correct.

17. $q = CV$

and $V = iR$

$\therefore \quad q = iCR$

$it = iCR$

$\Rightarrow \quad [CR] = [t] = [\mathrm{M^0\,L^0\,T\,A^0}]$

Option (a) is correct.

18. $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{r^2}$

$\therefore$ Unit of ε_0 = Newton-metre^2/coulomb^2.

Option (b) is correct.

19. Angular momentum $(J) = \dfrac{nh}{2\pi}$

$I = \Sigma\, mr^2$

$\therefore \quad \dfrac{h}{I} = \dfrac{2\pi J/n}{\Sigma mr^2} = \dfrac{mvr}{\Sigma\, mr^2}$

$\left[\dfrac{h}{I}\right] = \left[\dfrac{\mathrm{L\,T^{-1}}}{\mathrm{L}}\right] = [\mathrm{T^{-1}}]$

= Frequency

Option (a) is correct.

20. $v = at + \dfrac{b}{t+c}$

$[c] = [T]$

$\left[\dfrac{b}{t+c}\right] = [v]$

or $\quad [b] = [\mathrm{L\,T^{-1}}][\mathrm{T}] = [\mathrm{L}]$

$[at] = [v] = [\mathrm{L\,T^{-1}}]$

$\Rightarrow \quad [a] = [\mathrm{L\,T^{-2}}]$

Option (a) is correct.

21. $y = A\sin\left[\dfrac{2\pi}{\lambda}(ct - x)\right]$

$= A\sin\left[\dfrac{2\pi}{\lambda}ct - \dfrac{2\pi x}{\lambda}\right]$

$\dfrac{2\pi x}{\lambda} = \theta$ (angle)

$\therefore \quad [x] = [\lambda] = [\mathrm{L}]$

Further, $y = A\sin\theta$

$\therefore \quad [A] = [y] = [\mathrm{L}]$

Option (a) is correct.

22. $[X] = [\mathrm{M^{-1}\,L^{-3}\,T^3\,A^2}]$

$= \dfrac{[\mathrm{T\,A^2}]}{[\mathrm{ML^2\,T^{-2}}]}$

$$= \frac{[t][i^2]}{[\text{Work}]}$$

$\therefore$ X is resistance. $[\because W = i^2Rt]$

23. $\vec{\mathbf{F}} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$

$\vec{\mathbf{r}} = 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

$$\therefore \quad \vec{\tau} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 3 & 2 & 3 \\ 2 & -3 & 4 \end{vmatrix}$$

or $\vec{\tau} = 17\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - 13\hat{\mathbf{k}}$

24. $\sqrt{(0.5)^2 + (0.8)^2 + (c)^2} = 1$

or $0.25 + 0.64 + c^2 = 1$

or $c^2 = 1 - 0.89$

$c = \sqrt{0.11}$

Option (b) is correct.

25. $|\vec{\mathbf{A}} + \vec{\mathbf{B}}| = |\vec{\mathbf{A}} - \vec{\mathbf{B}}|$

$(\vec{\mathbf{A}} + \vec{\mathbf{B}}) \cdot (\vec{\mathbf{A}} + \vec{\mathbf{B}}) = (\vec{\mathbf{A}} - \vec{\mathbf{B}}) \cdot (\vec{\mathbf{A}} - \vec{\mathbf{B}})$

$A^2 + B^2 + 2\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = A^2 + B^2 - 2\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$

i.e., $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = 0$

$\therefore$ Angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}} = 90°$

26. $(\vec{\mathbf{A}} + \vec{\mathbf{B}}) \cdot (\vec{\mathbf{A}} - \vec{\mathbf{B}}) = 0$

$\vec{\mathbf{A}} \cdot \vec{\mathbf{A}} + \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} - \vec{\mathbf{B}} \cdot \vec{\mathbf{B}} - \vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = 0$

$A^2 - B^2 = 0$

$A = \pm B$

$|\vec{\mathbf{A}}| = |\vec{\mathbf{B}}|$

Option (d) is correct.

27. Work $(= \vec{\mathbf{F}} \cdot \vec{\mathbf{s}})$ is a scalar quantity.

Option (d) is correct.

28. Speed $= |\vec{\mathbf{v}}|$

Option (d) is correct.

29. $|\vec{\mathbf{A}}| = 3$, $|\vec{\mathbf{B}}| = 5$ and angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is 60°.

$\therefore$ $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = |\vec{\mathbf{A}}|\,|\vec{\mathbf{B}}| \cos 60°$

$= (3)(5)\left(\frac{1}{2}\right)$

$= 7.5$

Option (b) is correct.

30. $\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{C}}$

$\therefore$ $(\vec{\mathbf{A}} + \vec{\mathbf{B}}) \cdot (\vec{\mathbf{A}} + \vec{\mathbf{B}}) = \vec{\mathbf{C}} \cdot \vec{\mathbf{C}}$

or $\vec{\mathbf{A}} \cdot \vec{\mathbf{A}} + 2\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} + \vec{\mathbf{B}} \cdot \vec{\mathbf{B}} = \vec{\mathbf{C}} \cdot \vec{\mathbf{C}}$

or $A^2 + 2\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} + B^2 = C^2$

or $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = 0$

or $|\vec{\mathbf{A}}|\,|\vec{\mathbf{B}}| \cos\theta = 0$

or $\cos\theta = 0$

or $\theta = \frac{\pi}{2}$

Option (d) is correct.

31. Magnetic field intensity.

Option (d) is correct.

32. $\vec{\mathbf{P}} + \vec{\mathbf{Q}} = \vec{\mathbf{R}}$

$(\vec{\mathbf{P}} + \vec{\mathbf{Q}}) \cdot (\vec{\mathbf{P}} + \vec{\mathbf{Q}}) = \vec{\mathbf{R}} \cdot \vec{\mathbf{R}}$

$P^2 + Q^2 + 2\vec{\mathbf{P}} \cdot \vec{\mathbf{Q}} = R^2$

$12^2 + 5^2 + 2\vec{\mathbf{P}} \cdot \vec{\mathbf{Q}} = 13^2$

$\vec{\mathbf{P}} \cdot \vec{\mathbf{Q}} = 0$

$\therefore$ Angle between $\vec{\mathbf{P}}$ and $\vec{\mathbf{Q}} = \frac{\pi}{2}$

Option (b) is correct.

33. Option (b) is correct.

34. $\vec{\mathbf{P}} + \vec{\mathbf{Q}} + \vec{\mathbf{R}} = 0$

$\therefore$ $\vec{\mathbf{P}} + \vec{\mathbf{Q}} = -\vec{\mathbf{R}}$

or $(\vec{\mathbf{P}} + \vec{\mathbf{Q}}) \cdot (\vec{\mathbf{P}} + \vec{\mathbf{Q}}) = (-\vec{\mathbf{R}}) \cdot (-\vec{\mathbf{R}})$

or $\vec{\mathbf{P}} \cdot \vec{\mathbf{P}} + \vec{\mathbf{Q}} \cdot \vec{\mathbf{Q}} + 2\vec{\mathbf{P}} \cdot \vec{\mathbf{Q}} = \vec{\mathbf{R}} \cdot \vec{\mathbf{R}}$

or $P^2 + Q^2 + 2\vec{\mathbf{P}} \cdot \vec{\mathbf{Q}} = R^2$ ...(i)

Let $Q^2 = P^2$ and $R = P\sqrt{2}$

Thus, Eq. (i) takes the form

$P^2 + P^2 + 2PQ\cos\theta = 2P^2$

or $2PQ\cos\theta = 0$

or $\cos\theta = 0$

or $\theta = 90°$

$\therefore$ Angle between $\vec{\mathbf{P}}$ and $\vec{\mathbf{Q}}$ is 90°

$$\vec{\mathbf{P}} + \vec{\mathbf{Q}} + \vec{\mathbf{R}} = 0$$

$\therefore$ $\vec{\mathbf{P}} + \vec{\mathbf{R}} = -\vec{\mathbf{Q}}$

or $(\vec{\mathbf{P}} + \vec{\mathbf{R}}) \cdot (\vec{\mathbf{P}} + \vec{\mathbf{R}}) = (-\vec{\mathbf{Q}}) \cdot (-\vec{\mathbf{Q}})$

or $P^2 + R^2 + 2PR\cos\phi = Q^2$

or $2PR\cos\phi = Q^2 - P^2 - R^2$

or $2PR\cos\phi = -R^2$

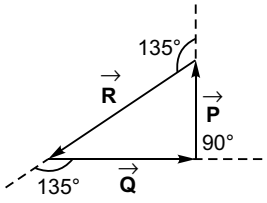
or $2P\cos\phi = -R$

or $2P\cos\phi = -P\sqrt{2}$

or $\cos\phi = -\dfrac{1}{\sqrt{2}}$

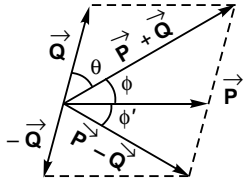
$\therefore$ $\phi = 135°$

$\therefore$ Angle between $\vec{\mathbf{P}}$ and $\vec{\mathbf{R}}$ is 135°.



Option (a) is correct.

35. Angle (ϕ) between $\vec{\mathbf{P}} + \vec{\mathbf{Q}}$ and $\vec{\mathbf{P}} - \vec{\mathbf{Q}}$



$$\tan\phi = \frac{Q\sin\theta}{P + Q\cos\theta}$$

Angle ϕ' between $\vec{\mathbf{P}} - \vec{\mathbf{Q}}$ and $\vec{\mathbf{P}}$

$$\tan\phi' = \frac{Q\sin(\pi + \theta)}{P + Q\cos(\pi + \theta)}$$

$$= \frac{-Q\sin\theta}{P - Q\cos\theta}$$

$$\tan[\phi + (-\phi')] = \frac{\tan\phi - \tan\phi'}{1 + \tan\phi\tan\phi'}$$

$$= \frac{\dfrac{Q\sin\theta}{P + Q\cos\theta} - \dfrac{(-Q\sin\theta)}{P - Q\cos\theta}}{1 - \dfrac{Q\sin\theta}{(P + Q\cos\theta)} \cdot \dfrac{(-Q\sin\theta)}{(P - Q\cos\theta)}}$$

$$= \frac{2PQ\sin\theta}{P^2 + Q^2\cos 2\theta}$$

This implies that angle between $\vec{\mathbf{P}} + \vec{\mathbf{Q}}$ and $\vec{\mathbf{P}} - \vec{\mathbf{Q}}$ will vary from 0 to π.

Option (b) is correct.

36. $R^2 = P^2 + Q^2 + 2PQ\cos\theta$

for $R = P = Q$

$$P^2 = P^2 + P^2 + 2PP\cos\theta$$

or $\cos\theta = -\dfrac{1}{2}$

or $\theta = 120°$

Option (b) is correct.

37. $W = \vec{\mathbf{F}} \cdot \vec{\mathbf{s}}$

$= (3\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}}) \cdot (3\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}})$

$= 25$ J

Option (b) is correct.

38. $\vec{\mathbf{P}} \cdot \vec{\mathbf{Q}} = (a\,\hat{\mathbf{i}} + a\,\hat{\mathbf{j}} + 3\,\hat{\mathbf{k}}) \cdot (a\,\hat{\mathbf{i}} - 2\,\hat{\mathbf{j}} - \hat{\mathbf{k}})$

$= a^2 - 2a - 3$

For $\vec{\mathbf{P}} \perp \vec{\mathbf{Q}}$, $\vec{\mathbf{P}} \cdot \vec{\mathbf{Q}} = 0$

i.e., $a^2 - 2a - 3 = 0$

or $(a - 3)(a + 1) = 0$

$\Rightarrow$ $a = 3$

Other value is – ive.

Option (d) is correct.

39. If a vector makes angles α, β and γ with the co-ordinate axes, then

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

Now, $\left(\dfrac{3}{7}\right)^2 = \dfrac{9}{49}, \left(\dfrac{6}{7}\right)^2 = \dfrac{36}{49}, \left(\dfrac{2}{7}\right)^2 = \dfrac{4}{49}$

and $\dfrac{9}{49} + \dfrac{36}{49} + \dfrac{4}{49} = 1$

$\therefore$ Option (a) is correct.

40. $\vec{\mathbf{A}} = 4\,\hat{\mathbf{i}} - 3\,\mathbf{j}$ and $\vec{\mathbf{B}} = 8\,\hat{\mathbf{i}} + 8\,\hat{\mathbf{j}}$

$\therefore$ $\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{C}} = 12\,\hat{\mathbf{i}} + 5\,\hat{\mathbf{j}}$

$$\hat{\mathbf{C}} = \frac{\vec{\mathbf{C}}}{|C|} = \frac{12\,\hat{\mathbf{i}} + 5\,\hat{\mathbf{j}}}{\sqrt{12^2 + 5^2}}$$

$$= \frac{12}{13}\,\hat{\mathbf{i}} + \frac{5}{13}\,\hat{\mathbf{j}}$$

Option (b) is correct.

41. $\vec{\mathbf{A}} = 2\,\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\,\hat{\mathbf{k}}, \vec{\mathbf{B}} = 5\,\hat{\mathbf{i}} + n\hat{\mathbf{j}} + \hat{\mathbf{k}},$

$\vec{\mathbf{C}} = -\,\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\,\hat{\mathbf{k}}$

$\therefore$ Vectors $\vec{\mathbf{A}}, \vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ will be coplanar if their scalar triple product is zero *i.e.*,

$$(\vec{\mathbf{A}} \times \vec{\mathbf{C}}) \cdot \vec{\mathbf{B}} = 0$$

$$\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 3 & -2 \\ -1 & 2 & 3 \end{vmatrix} \cdot (5\,\hat{\mathbf{i}} + n\,\hat{\mathbf{j}} + \hat{\mathbf{k}}) = 0$$

or $\quad (13\,\hat{\mathbf{i}} - 4\,\hat{\mathbf{j}} + 7\,\hat{\mathbf{k}}) \cdot (5\,\hat{\mathbf{i}} + n\,\hat{\mathbf{j}} + \hat{\mathbf{k}}) = 0$

or $\quad 65 - 4n + 7 = 0$

or $\quad n = 18$

Option (a) is correct.

42. Option (a) is correct.

43. $(\vec{\mathbf{a}} + \vec{\mathbf{b}}) \times (\vec{\mathbf{a}} - \vec{\mathbf{b}})$

$$= \vec{\mathbf{a}} \times \vec{\mathbf{a}} + \vec{\mathbf{b}} \times \vec{\mathbf{a}} - \vec{\mathbf{a}} \times \vec{\mathbf{b}} - \vec{\mathbf{b}} \times \vec{\mathbf{b}}$$

$$= 0 - \vec{\mathbf{a}} \times \vec{\mathbf{b}} - \vec{\mathbf{a}} \times \vec{\mathbf{b}} - 0$$

$$= -\,2(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) = 2(\vec{\mathbf{b}} \times \vec{\mathbf{a}})$$

Option (a) is correct.

44. $\vec{\mathbf{A}} = 3\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}} + 5\,\hat{\mathbf{k}}$

$\vec{\mathbf{B}} = 3\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}} - 5\,\hat{\mathbf{k}}$

$$\cos\theta = \frac{(\vec{\mathbf{A}} \cdot \vec{\mathbf{B}})}{|\vec{\mathbf{A}}|\,|\vec{\mathbf{B}}|}$$

$$= \frac{9 + 16 - 25}{3^2 + 4^2 + 5^2}$$

$$= 0$$

$\Rightarrow \quad \theta = 90°$

Option (c) is correct.

45. $A + B = 7$

$A - B = 3$

$\therefore \quad B = 2\,\text{N}$

Option (c) is correct.

46. Angle between $\vec{\mathbf{A}} = 2\,\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}}$

and $\quad \vec{\mathbf{B}} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$

$$\theta = \frac{\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}}{|\vec{\mathbf{A}}|\,|\vec{\mathbf{B}}|} = \frac{2 + 3}{\sqrt{2^2 + 3^2} \cdot \sqrt{2}}$$

$$= \frac{5}{\sqrt{13} \cdot \sqrt{2}}$$

$$= \frac{5}{\sqrt{26}}$$

Component of $\vec{\mathbf{A}}$ along $\hat{\mathbf{i}} + \hat{\mathbf{j}}$

$$\vec{\mathbf{C}} = \frac{5}{\sqrt{26}}\,(2\,\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}})$$

$$|\vec{\mathbf{C}}| = \frac{5}{\sqrt{2}}$$

Option (a) is correct.

47. $R^2 = (3P)^2 + (2P)^2 + 2 \times 3P \times 2P\cos\theta$

or $\quad R^2 = 13P^2 + 12P^2\cos\theta \quad \ldots$(i)

Further

$(2R)^2 = (6P)^2 + (2P)^2 + 2 \times 6P \times 2P\cos\theta$

or $\quad 4R^2 = 40P^2 + 24P^2\cos\theta \quad \ldots$(ii)

Dividing Eq. (ii) by Eq. (i),

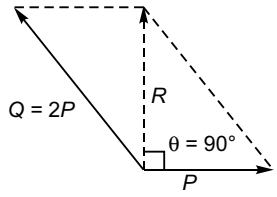
$10P^2 + 6P^2\cos\theta = 13P^2 + 12P^2\cos\theta$

or $\quad 6\cos\theta = -\,P$

or $\quad \theta = 120°$

Option (b) is correct.

48. $$\tan\theta = \frac{Q\sin\alpha}{P + Q\cos\alpha}$$



As $\quad \theta = 90°, \tan\alpha = \infty$

$\therefore \quad P + Q\cos\alpha = 0$

i.e., $\quad \cos\alpha = -\dfrac{P}{Q}$

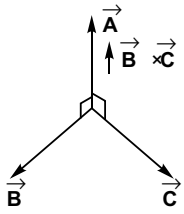
$$= -\frac{P}{2P}$$

$$= -\frac{1}{2}$$

$\therefore \quad \alpha = 120°$

Option (a) is correct.

49. $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = 0$



$\Rightarrow \quad \vec{\mathbf{A}} \perp \vec{\mathbf{B}}$ ...(i)

$\vec{\mathbf{A}} \cdot \vec{\mathbf{C}} = 0$

$\Rightarrow \quad \vec{\mathbf{A}} \perp \vec{\mathbf{C}}$ ...(ii)

From Eq. (i) and Eq. (ii), we conclude that $\vec{\mathbf{A}}$ is perpendicular to the plane containing $\vec{\mathbf{B}}$ and $\vec{\mathbf{C}}$.

This implies that $\vec{\mathbf{A}}$ is perpendicular to $\vec{\mathbf{B}} \cdot \vec{\mathbf{C}}$.

Option (c) is correct.

50. $P^2 + Q^2 + 2PQ\cos\alpha = R^2$

or $\quad P^2 + Q^2 + 2PQ\cos\alpha = 8^2$

or $\quad P^2 + Q^2 + 2PQ + 2PQ\cos\alpha - 2PQ = 64$

or $\quad (P+Q)^2 + 2PQ(\cos\alpha - 1) = 64$

or $\quad (16)^2 + 2PQ(\cos\alpha - 1) = 64$

or $\quad 2PQ(\cos\alpha - 1) = -192$

or $\quad PQ\cos\alpha - PQ = -96$ ...(i)

$$\tan\theta = \frac{Q\sin\alpha}{P + Q\cos\alpha} = \infty \quad \text{(as } \theta = 90°\text{)}$$

$\therefore \quad P + Q\cos\alpha = 0$

$Q\cos\alpha = -P$ ...(ii)

Using Eq. (ii) and Eq. (i),

$P(-P) - PQ = -96$

or $\quad -P(P+Q) = -96$

or $\quad -P \times 16 = -96$

$P = +6\,\text{N}$

$\therefore \quad Q = 10\,\text{N}$

Option (a) is correct.

51. $|\vec{\mathbf{A}} \times \vec{\mathbf{B}}| = \sqrt{3}\,(\vec{\mathbf{A}} \cdot \vec{\mathbf{B}})$

$\Rightarrow \quad |\vec{\mathbf{A}}||\vec{\mathbf{B}}|\sin\theta = \sqrt{3}\,|\vec{\mathbf{A}}|\,|\vec{\mathbf{B}}|\cos\theta$

$\Rightarrow \quad \tan\theta = \sqrt{3}$

$\Rightarrow \quad \theta = 60°$

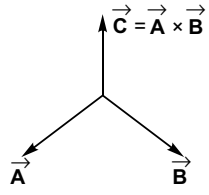
$\therefore |\vec{\mathbf{A}} \times \vec{\mathbf{B}}|^2 = |\vec{\mathbf{A}}|^2 \times |\vec{\mathbf{B}}|^2 + 2|\vec{\mathbf{A}}||\vec{\mathbf{B}}|\cos\theta$

$= A^2 + B^2 + 2AB\cos 60°$

$= A^2 + B^2 + AB$

$|\vec{\mathbf{A}} \times \vec{\mathbf{B}}| = [A^2 + B^2 + AB]^{1/2}$

52. $\vec{\mathbf{C}}$ is perpendicular to both $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$



$\therefore \quad \vec{\mathbf{C}} \cdot \vec{\mathbf{A}} = 0$

or $\quad (\vec{\mathbf{A}} \times \vec{\mathbf{B}}) \cdot \vec{\mathbf{A}} = 0$

Option (d) is correct.

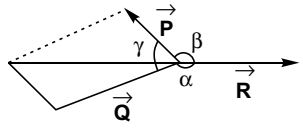
53. $(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 8\hat{\mathbf{k}}) \cdot (-4\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + \alpha\hat{\mathbf{k}}) = 0$

$\Rightarrow \quad -8 + 12 + 8\alpha = 0$

$\therefore \quad \alpha = -\frac{1}{2}$

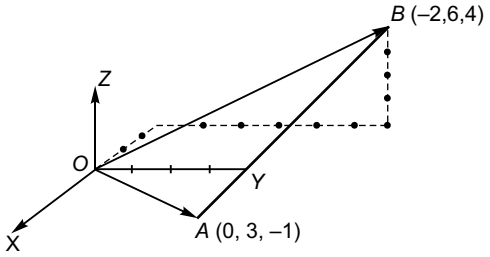
Option (c) is correct.

54. $\vec{\mathbf{P}} + \vec{\mathbf{Q}} + \vec{\mathbf{R}} = \vec{\mathbf{0}}$



If, $\quad \frac{|\vec{\mathbf{P}}|}{\sin\alpha} = \frac{|\vec{\mathbf{Q}}|}{\sin\beta} = \frac{|\vec{\mathbf{R}}|}{\sin\gamma}$

55. $\vec{OA} + \vec{AB} = \vec{OB}$



$\therefore \quad \vec{AB} = \vec{OB} - \vec{OA}$

$= (-2\hat{i} + 6\hat{j} + 4\hat{k}) - (0\hat{i} + 3\hat{j} - \hat{k})$

$= -2\hat{i} + 3\hat{j} + 5\hat{k}$

Option (c) is correct.

56. Using answer to questions no. 35, as angle between $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$ is 90°

$A^2 + B^2 \cos 2\theta = 0$

or $\quad A^2 = -B^2 \cos 2\theta$

or $\quad A^2 = -B^2 \cos 2\left(\frac{\pi}{2}\right) \quad \left(\because \theta = \frac{\pi}{2}\right)$

or $\quad A^2 = -B^2 \cos \pi$

or $\quad A^2 = B^2$

$\Rightarrow \quad A = B$

Option (a) is correct.

Match the Columns

1. (a) $|\vec{A} \times \vec{B}| = |\vec{A} \cdot \vec{B}|$

or $\quad |\vec{A}|\,|\vec{B}| \sin\theta = \pm |\vec{A}|\,|\vec{B}| \cos\theta$

or $\quad \tan\theta = \pm 1$

$\Rightarrow \quad \theta = \frac{\pi}{2}, \frac{3\pi}{4}$

Thus, (a) → (r) (s).

(b) $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$ (given)

or $\quad \vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$

or $\quad |\vec{A}|\,|\vec{B}| \sin\theta = -|\vec{A}||\vec{B}| \sin\theta$

or $\quad \sin\theta = -\sin\theta$

or $\quad 2\sin\theta = 0$

$\Rightarrow \quad \theta = 0$ rad

Thus, (b) → (p).

(c) $|\vec{A} \times \vec{B}| = |\vec{A} \times \vec{B}|$

or $\quad |\vec{A} + \vec{B}|^2 = |\vec{A} - \vec{B}|^2$

$\Rightarrow \quad (\vec{A} + \vec{B}) \cdot (\vec{A} + \vec{B}) = (\vec{A} - \vec{B}) \cdot (\vec{A} - \vec{B})$

or $\vec{A} \cdot \vec{A} + \vec{B} \cdot \vec{A} + \vec{B} \cdot \vec{A} + \vec{B} \cdot \vec{B}$

$= \vec{A} \cdot \vec{A} - \vec{B} \cdot \vec{A} - \vec{A} \cdot \vec{B} + \vec{B} \cdot \vec{B}$

or $\quad 4\vec{A} \cdot \vec{B} = 0$

$\Rightarrow \quad \vec{A} \perp \vec{B}$

Thus, (c) → (q).

(d) $\vec{A} + \vec{B} = \vec{C}$

or $\quad (\vec{A} + \vec{B}) \cdot (\vec{A} + \vec{B}) = \vec{C} \cdot \vec{C}$

or $\quad \vec{A} \cdot \vec{A} + \vec{B} \cdot \vec{A} + \vec{B} \cdot \vec{A} + \vec{B} \cdot \vec{B} = \vec{C} \cdot \vec{C}$

or $\quad A^2 + 2\vec{A} \cdot \vec{B} + B^2 = C^2$

or $\quad 2\vec{A} \cdot \vec{B} = 0 \quad (\because A^2 + B^2 = C^2)$

$\Rightarrow \quad \vec{A} \perp \vec{B}$

Thus, (d) → (q).

Section II

Subjective Questions

1. $2 \times 10^{11}\ \text{N/m}^2 = \frac{(2 \times 10^{11})(10^5\ \text{dyne})}{(10^4\ \text{cm}^2)}$

$= 2 \times 10^{12}\ \text{dyne/cm}^2$

2. $72\ \text{dyne/cm} = \frac{(72)(10^{-5}\text{N})}{(10^{-2}\text{m})} = 0.072\ \text{N/m}$

3. $[a] = [y] = [L]$

$[\omega t] = [M^0L^0T^0] \quad \therefore \quad [\omega] = [T^{-1}]$

$[\theta] = [M^0L^0T^0]$

4. $h = \frac{E}{\nu} = \frac{\text{J}}{\text{per sec}} = \text{J-s}$

$[h] = [ML^2T^{-2}][T] = [ML^2T^{-1}]$

5. $[b] = [x^2] = [L^2]$

$$\left[\frac{b}{at}\right] = [p]$$

$$\therefore\ [a] = \left[\frac{b}{tp}\right] = \left[\frac{L^2}{TML^2T^{-3}}\right] = [M^{-1}T^2]$$

6. $S_t = \left(ut + \frac{1}{2}at^2\right) - \left[u(t-1) + \frac{1}{2}a(t-1)^2\right]$

$= u.1 - \frac{1}{2}a(1)^2 + at(1) = u + at - \frac{1}{2}a$

$= u + \frac{a}{2}(2t - 1)$

Here t in second. Hence the given equation seems to be dimensionally incorrect. But it is correct because 1 is hidden.

7. LHS is dimensionless. While RHS has the dimensions $[L^{-1}]$.

8. LHS is dimensionless. Hence $n = 0$.

9. Just write the dimension of different physical quantities.

10. $E = km^x n^y a^z$.

Here k = a dimensionless constant

$\therefore\quad [E] = [m]^x[n]^y[a]^z$

$\therefore\quad [ML^2T^{-2}] = [M]^x[T^{-1}]^y[L]^z$

$\therefore\quad x = 1,\ y = 2$ and $z = 2$

11. $F = km^x v^y r^z$

($k = a$ dimensionless constant)

$[F] = [m]^x[v]^y[r]^z$

$\therefore\quad [MLT^{-2}] = [M]^x[LT^{-1}]^y[L]^z$

Solving we get,

$x = 1,\ y = 2$ and $z = -1$

$$\therefore\quad F = \frac{kmv^2}{r}$$

12. (a) $[d] = [F]^x[L]^y[T]^z$

$\therefore\ [ML^{-3}] = [MLT^{-2}]^x[L]^y[T]^z$

Equating the powers we get,

$x = 1,\ y = -4,\ z = 2$

$\therefore\quad [d] = [FL^{-4}T^2]$

Similarly other parts can be solved.

13. $\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB} = \frac{6-2+8}{\sqrt{9+1+4}\sqrt{4+4+16}}$

$= \frac{12}{\sqrt{336}} = \frac{3}{\sqrt{21}}$

14. $\vec{A} + \vec{B} = \vec{R}$ (say) $= 3\hat{i} + \hat{j}$

$\vec{A} - \vec{B} = \vec{S}$ (say) $= \hat{i} + 5\hat{j}$

Angle between $\vec{R}$ and $\vec{S}$

$$\cos\theta = \frac{\vec{R}\cdot\vec{S}}{RS} = \frac{3+5}{\sqrt{9+1}\sqrt{1+25}} = \frac{8}{\sqrt{260}} = \frac{4}{\sqrt{65}}$$

$$\therefore\ \theta = \cos^{-1}\left(\frac{4}{\sqrt{65}}\right)$$

15. Their dot product should be zero.

16. Ratio of coefficients of $\hat{i}$, $\hat{j}$ and $\hat{k}$ should be same.

17. No solution is required.

18. Component of $\vec{A}$ along $\vec{B} = A\cos\theta = \frac{\vec{A}\cdot\vec{B}}{B}$

19. $\vec{A} + \vec{B} = \vec{R} = 5\hat{i} - \hat{j} + \hat{k}$

$$\hat{R} = \frac{\vec{R}}{R}$$

20. $\vec{A}\times\vec{B} = \vec{C}$ (say) $= -3\hat{i} + 8\hat{j} + 2\hat{k}$

Now $\vec{A}\cdot\vec{C} = -6 + 8 - 2 = 0$

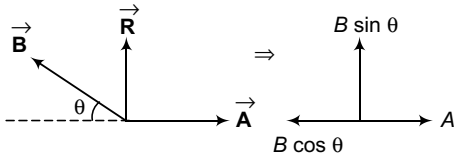
$\therefore\quad \vec{A} \perp \vec{C}$

$\vec{B}\cdot\vec{C} = 0 + 8 - 8 = 0$

$\therefore\quad \vec{B} \perp \vec{C}$

21. Area of parallelogram $= |\vec{A}\times\vec{B}|$

22.



$B\sin\theta = R = \frac{B}{2}$

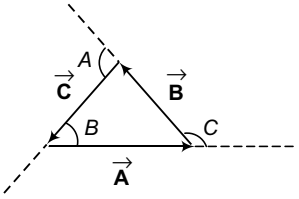
$\therefore\quad \theta = 30°$

$\therefore$ Angle between $\vec{A}$ and $\vec{B} = 180 - \theta = 150°$

23. $\vec{A} + \vec{B} = (4\hat{i} + 6\hat{j}) + \vec{B} = 10\hat{i} + 9\hat{j}$

$\therefore\quad \vec{B} = (6\hat{i} + 3\hat{j})$

24.



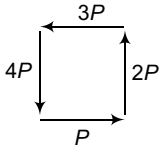
Applying sine law, we have

$$\frac{a}{\sin(180 - A)} = \frac{b}{\sin(180 - B)}$$

$$= \frac{c}{\sin(180 - C)}$$

or $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

25.



$$\vec{\mathbf{R}} = P\hat{\mathbf{i}} + 2P\hat{\mathbf{j}} - 3P\hat{\mathbf{i}} - 4P\hat{\mathbf{j}}$$

$$= (-2P\hat{\mathbf{i}} - 2P\hat{\mathbf{j}})$$

26. $$R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

$$S^2 = P^2 + Q^2 - 2PQ\cos\theta$$

$$\therefore\ R^2 + S^2 = 2(P^2 + Q^2)$$

3 Motion in One Dimension

Introductory Exercise 3.1

1. Suppose a particle is moving with constant velocity v (along the axis of x).

Displacement of particle in time $t_1 = vt_1$

Displacement of particle in time $t_2 = vt_2$

$\therefore$ Displacement of the particle in the time interval $\Delta t (= t_2 - t_1)$

$$= vt_2 - vt_1$$

$$= v(t_2 - t_1)$$

$\therefore$ Average velocity in the time interval

$$\Delta t' = \frac{v(t_2 - t_1)}{(t_2 - t_1)}$$

$$= v$$

Now, as the particle is moving with constant velocity (*i.e.*, with constant speed in a given direction) its velocity and speed at any instant will obviously be v.

Ans. True.

2. As the stone would be free to acceleration under earth's gravity it acceleration will be g.

3. A second hand takes 1 min *i.e.*, 60s to complete one rotation (*i.e.,* rotation by an angle of 2π rad).

$\therefore$ Angular speed of second hand

$$= \frac{2\pi \text{ rad}}{60 \text{ s}}$$

$$= \frac{\pi}{30} \text{ rad s}^{-1}$$

Linear speed of its tip = radius × angular speed

$$= 2.0 \text{ cm} \times \frac{\pi}{30} \text{ rad s}^{-1}$$

$$= \frac{\pi}{15} \text{ cms}^{-1}$$

As the tip would be moving with constant speed.

Average speed $= \frac{\pi}{15} \text{ cms}^{-1}$

In 15 s the second hand would rotate through 90° *i.e.*, the displacement of its tip will be $r\sqrt{2}$.

$\therefore$ Modulus of average velocity of the tip of second hand in 15 s.

$$= \frac{r\sqrt{2}}{15}$$

$$= \frac{2\sqrt{2}}{15} \text{ cms}^{-1}$$

4. (a) Yes. By changing direction of motion, there will be change in velocity and so acceleration.

(b) (i) No. In curved path there will always be acceleration. (As explained in the previous answer no. 3)

(ii) Yes. In projectile motion the path of the particle is a curved one while acceleration of the particle remains constant.

(iii) Yes. In curved path the acceleration will always be there. Even if the path is circular with constant speed the direction of the acceleration of the particle would every time be changing.

5. (a) Time speed $= \dfrac{\text{Circumference}}{\text{Speed}}$

$$= \frac{2\pi \times 4\,\text{cm}}{1\,\text{cm/s}} = 8\pi = 25.13\ \text{s}$$

(b) As particle is moving with constant speed of 1 cm/s, its average speed in any time interval will be 1 cm/s.

$$|\text{Average velocity}| = \frac{r\sqrt{2}}{T/4}$$

$$= \frac{4r\sqrt{2}}{\left(\dfrac{2\pi r}{\text{speed}}\right)} = \frac{2\sqrt{2}}{\pi}\,\text{speed}$$

$$= \frac{2\sqrt{2}}{\pi}\,\text{cms}^{-1}$$

$$= 0.9\,\text{cms}^{-1}$$

$$|\text{Average acceleration}| = \frac{v\sqrt{2}}{T/4}$$

(where v = speed)

$$= \frac{4v\sqrt{2}}{\dfrac{2\pi r}{v}}$$

$$= \frac{2\sqrt{2}}{\pi}\frac{v^2}{r}$$

$$= \frac{2\sqrt{2}}{\pi}\frac{(1)^2}{4}$$

$$= 0.23\ \text{cm s}^{-2}$$

6. Distance = Speed × time

$$D_1 = v_1 t_1$$

$$D_2 = v_2 t_2$$

$$\text{Average speed} = \frac{D_1 + D_2}{t_1 + t_2} = \frac{v_1 t_1 + v_2 t_2}{t_1 + t_2}$$

$$= \frac{(4 \times 2) + (6 \times 3)}{2 + 3}$$

$$= 5.2\,\text{ms}^{-1}$$

Introductory Exercise 3.2

1. Acceleration (due to gravity).

2. $s_t = u + at - \dfrac{1}{2}a$ is physically correct as it gives the displacement of the particle in t^{th} second (or any time unit).

s_t = Displacement in t seconds

– displacement in $(t-1)$ seconds

$$= \left[ut + \frac{1}{2}at^2\right] - \left[u(t-1) + \frac{1}{2}a(t-1)^2\right]$$

$$st = u + at - \frac{1}{2}a$$

Displacement, Velocity, Acceleration

Therefore, the given equation is dimensionally incorrect.

3. Yes. When a particle executing simple harmonic motion returns from maximum amplitude position to its mean position the value of its acceleration decreases while speed increases.

4. $v = t^{3/4}$ (given)

$$\frac{ds}{dt} = t^{3/4} \quad \ldots\text{(i)}$$

$$\therefore \quad s = \int t^{3/4}\,dt = \frac{t^{\frac{3}{4}+1}}{\frac{3}{4}+1} + c$$

or $\quad s = \dfrac{4}{7}t^{7/4} + c$

i.e., $\quad s \propto t^{7/4}$

Differentiating Eq. (i) w.r.t. time t,

$$\frac{d^2s}{dt^2} = \frac{3}{4}t^{\frac{3}{4}-1}$$

$$\Rightarrow \quad a \propto t^{-1/4}$$

5. Displacement (s) of the particle

$$s = (40 \times 6) + \frac{1}{2}(-10)6^2$$

$$= 240 - 180$$

$$= 60\text{m (in the upward direction)}$$

Distance covered (D) by the particle

Time to attain maximum height

$$= \frac{40}{10} = 4 \text{ s} < 6 \text{ s}$$

It implies that particle has come back after attaining maximum height (h) given by

$$h = \frac{u^2}{2g}$$

$$= \frac{(40)^2}{2 \times 10} = 80 \text{ m}$$

$\therefore$ $D = 80 + (80 - 60)$

$= 100$ m

6. $v = 40 - 10t$

$\therefore$ $\frac{dx}{dt} = 40 - 10t$

or $dx = (40 - 10t)\, dt$

or $x = \int (40 - 10t)\, dt$

or $x = 40t - 5t^2 + c$

As at $t = 0$ the value of x is zero.

$c = 0$

$\therefore$ $x = 40t - 5t^2$

For x to be 60 m.

$60 = 40t - 5t^2$

or $t^2 - 8t + 12 = 0$

$\therefore$ $t = 2$ s or 6 s

7. Average velocity $= \frac{\text{Displacement in time } t}{t}$

$$= \frac{ut + \frac{1}{2}at^2}{t}$$

$$= u + \frac{1}{2}at$$

8. $v_2 = v_1 + at$

$\therefore$ $at = v_2 - v_1$

Average velocity $= \frac{\text{Displacement in time } t}{t}$

$$= \frac{v_1 t + \frac{1}{2}at^2}{t}$$

$$= v_1 + \frac{1}{2}at$$

$$= v_1 + \frac{v_2 - v_1}{2}$$

$$= \frac{v_1 + v_2}{2}$$

$\therefore$ **Ans.** True.

9. $125 = 0 \cdot t + \frac{1}{2}gt^2$

$\Rightarrow$ $t = 25$ s

Average velocity $= \frac{125 \text{ m}}{5 \text{ s}}$ (downwards)

$= 25$ m/s (downwards)

10. $v = 10 + 5t - t^2$ ...(i)

$\therefore$ $a = \frac{dv}{dt} = 5 - 2t$

At $t = 2$ s

$a = 5 - 2 \times 2$

$= 1 \text{ m/s}^2$

From Eq. (i),

$\frac{dx}{dt} = 10 + 5t - t^2$

$\therefore$ $x = \int (10 + 5t - t^2)\, dt$

or $x = 10t + \frac{5t^2}{2} - \frac{t^3}{3} + c$

As, at $t = 0$ the value of x is zero

$c = 0$

$\therefore$ $x = 10t + \frac{5}{2}t^2 - \frac{t^3}{3}$

Thus, at $t = 3$ s

$x = (10 \times 3) + \frac{5}{2}(3)^2 - \frac{3^3}{3}$

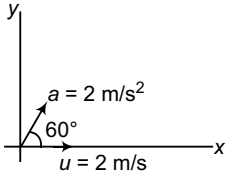
$= 30 + 22.5 - 9$

$= 43.5$ m

11. $\vec{\mathbf{u}} = 2\hat{\mathbf{i}}$ m/s

$\vec{\mathbf{a}} = (2\cos 60^\circ\, \hat{\mathbf{i}} + 2\sin 60^\circ\, \hat{\mathbf{j}})$ m/s^2

$= (1\hat{\mathbf{i}} + \sqrt{3}\,\hat{\mathbf{j}})$ m/s^2



$\vec{\mathbf{v}} = \vec{\mathbf{u}} + \vec{\mathbf{a}}\, t$

$= 2\hat{\mathbf{i}} + (1\hat{\mathbf{i}} + \sqrt{3}\,\hat{\mathbf{j}})2$

$= 4\hat{\mathbf{i}} + 2\sqrt{3}\,\hat{\mathbf{j}}$

$$|\vec{\mathbf{v}}| = \sqrt{4^2 + 12} = 2\sqrt{7} \text{ m/s}$$

$$\vec{\mathbf{s}} = \vec{\mathbf{u}}\,t + \frac{1}{2}\vec{\mathbf{a}}\,t^2$$

$$= (2\,\hat{\mathbf{i}})\,2 + \frac{1}{2}(1\,\hat{\mathbf{i}} + \sqrt{3}\,\hat{\mathbf{j}})\,2^2$$

$$= 4\,\hat{\mathbf{i}} + 2\,\hat{\mathbf{i}} + 2\sqrt{3}\,\hat{\mathbf{j}}$$

$$= 6\,\hat{\mathbf{i}} + 2\sqrt{3}\,\hat{\mathbf{j}}$$

$$\therefore \quad |\vec{\mathbf{s}}| = \sqrt{36 + 12}$$

$$= 4\sqrt{3} \text{ m}$$

12. Part I $\vec{\mathbf{v}} = (2\,\hat{\mathbf{i}} + 2t\,\hat{\mathbf{j}})$m/s ...(i)

$$\frac{\overrightarrow{d\mathbf{v}}}{dt} = 2\,\hat{\mathbf{j}}$$

$$\vec{\mathbf{a}} = 2\,\hat{\mathbf{j}} \text{ m/s}^2$$

From Eq. (i),

$$\frac{d\vec{\mathbf{s}}}{dt} = (2\,\hat{\mathbf{i}} + 2t\,\hat{\mathbf{j}})$$

$$\therefore \quad \vec{\mathbf{s}} = \int (2\,\hat{\mathbf{i}} + 2t\,\hat{\mathbf{j}})\,dt$$

$$\vec{\mathbf{s}} = 2t\,\hat{\mathbf{i}} + t^2\,\hat{\mathbf{j}} + c$$

Taking initial displacement to be zero.

$$\vec{\mathbf{s}} \text{ (at } t = 1 \text{ s)} = (2\,\hat{\mathbf{i}} + \hat{\mathbf{j}}) \text{ m}$$

Part II Yes. As explained below.

$\vec{\mathbf{v}} = 2\,\hat{\mathbf{i}} + 2t\,\hat{\mathbf{j}}$ implies that initial velocity of the particle is $2\,\hat{\mathbf{i}}$ m/s^2 and the acceleration is $2\hat{\mathbf{j}}$ m/s^2

$$\therefore \quad \vec{\mathbf{s}} \text{ (at } t = 1 \text{ s)} = (2\,\hat{\mathbf{i}} \times 1) + \frac{1}{2}(2\,\hat{\mathbf{j}})\,1^2$$

$$= (2\,\hat{\mathbf{i}} + \hat{\mathbf{j}}) \text{ m}$$

13. $x = 2t$ and $y = t^2$

$$\therefore \quad y = \left(\frac{x}{2}\right)^2$$

or, $\quad x^2 = 4y$

(The above is the equation to trajectory)

$x = 2t$

$$\therefore \quad \frac{dx}{dt} = 2 \textit{ i.e., } \vec{\mathbf{v}}_x = 2\,\hat{\mathbf{i}}$$

$y = t^2$

$$\therefore \quad \frac{dy}{dt} = 2t \textit{ i.e., } \vec{\mathbf{v}}_y + 2t\,\hat{\mathbf{j}}$$

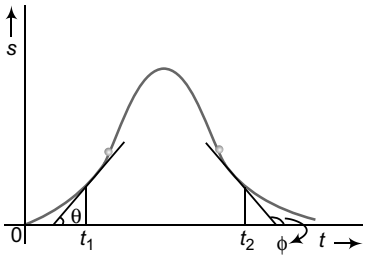
Thus, $\quad \vec{\mathbf{v}} = \vec{\mathbf{v}}_x + \vec{\mathbf{v}}_y$

$$= (2\,\hat{\mathbf{i}} + 2t\,\hat{\mathbf{j}}) \text{ m/s}$$

$$\vec{\mathbf{a}} = \frac{\overrightarrow{d\mathbf{v}}}{dt} = 2\,\hat{\mathbf{j}} \text{ m/s}^2$$

Introductory Exercise 3.3

1. At $t = t_1$



$$v = \tan\theta$$

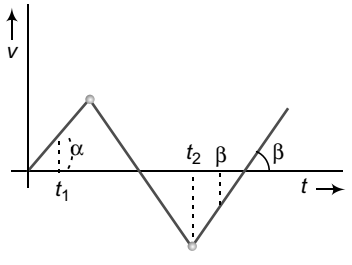
As $\theta < 90°$, v_{t_1} is + ive.

At $t = t_2$

$$v_{t_2} = \tan\phi$$

As $\phi > 90°$, v_{t_2} is – ive.

Corresponding v-t graph will be



Acceleration at $t = t_1$:

$$a_{t_1} = \tan\alpha$$

As $\alpha < 90°$, $a\ t_1$ is + ive constant.

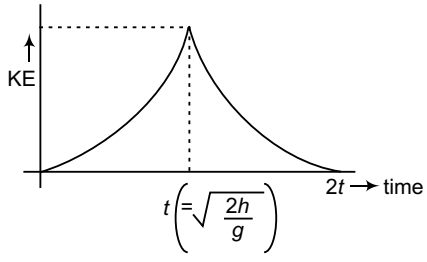
Acceleration at $t = t_2$

$$a_{t_2} = \tan\beta$$

As $\beta < 90°$, a_{t_2} is + ive constant.

2. Let the particle strike ground at time t velocity of particle when it touches ground

would be gt. KE of particle will be $\frac{1}{2}mg^2t^2$ *i.e.*, KE $\propto t^2$. While going up the velocity will get – ive but the KE will remain. KE will reduce to zero at time $2t$ when the particle reaches its initial position.



$$\text{KE} = \frac{1}{2}mg^2t^2 = \frac{1}{2}mg^2\frac{2h}{g}$$

$$= mgh$$

3. Speed of ball (just before making first collision with floor)

$$= \sqrt{2gh} = \sqrt{2 \times 10 \times 80}$$

$$= 40 \text{ m/s}$$

Time taken to reach ground

$$= \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 80}{10}} = 4 \text{ s}$$

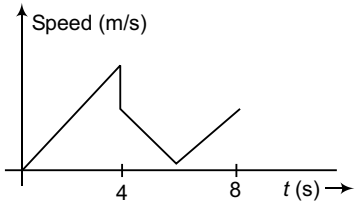
Speed of ball (just after first collision with floor)

$$= \frac{40}{2} = 20 \text{ m/s}$$

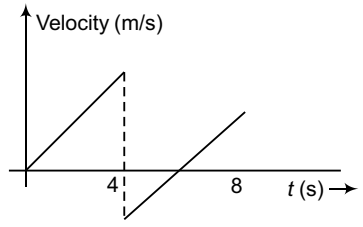
Time to attain maximum height

$$t = \frac{-20}{-10} = 2 \text{ s}$$

$\therefore$ Time for the return journey to floor = 2 s.

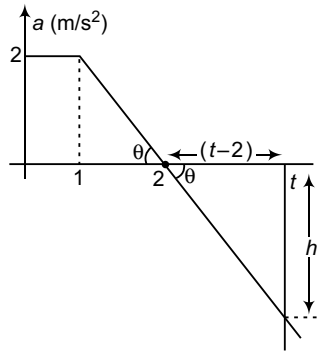


Corresponding velocity-time will be



4. $\frac{h}{(t-2)} = \tan\theta = \frac{2}{(2-1)}$

$\Rightarrow \quad h = 2(t-2)$



Particle will attain its initial velocity *i.e.*, net increase in velocity of the particle will be zero when,

area under a-t graph = 0

$$\frac{(1+2)\times 2}{2} + \frac{(-h)(t-2)}{2} = 0$$

or $\quad 3 - (t-2)^2 = 0$

or $\quad (t-2)^2 = 3$

or $\quad t - 2 = \pm\sqrt{3}$

or $\quad t = 2 \pm \sqrt{3}$

Ans . At time $t = 2 + \sqrt{3}$ s

($t = 2 - \sqrt{3}$ not possible).

Introductory Exercise 3.4

1. Relative acceleration of A w.r.t. B

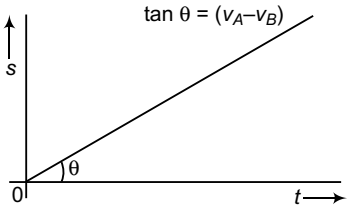
$$a_{AB} = (+g) - (+g) = 0$$

2. Velocity of A w.r.t. $B = v_A - v_B$

$\therefore$ Relative displacement (*i.e.*, distance between A and B) would be

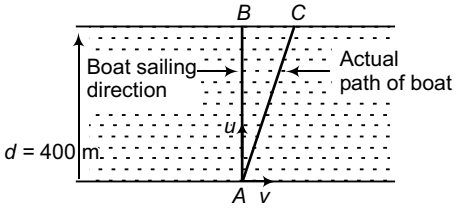
$$s = (v_A - v_B)t + \frac{1}{2}a_{AB}t^2$$

or $\quad s = (v_A - v_B)\,t$



3. In figure, u = speed of boat

v = speed of river flow



Time to cross river

$$= \frac{AB}{u}\left\{ = \frac{BC}{v} = \frac{AC}{\sqrt{u^2+v^2}} \right\}$$

$$= \frac{400\text{ m}}{10\text{ m/s}}$$

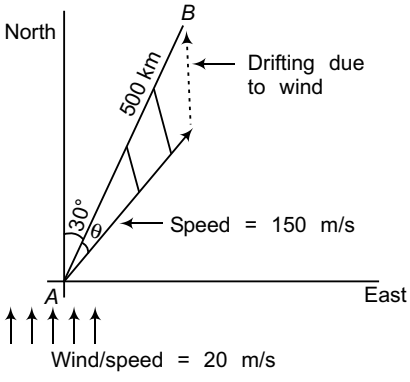
$$= 40\text{ s}$$

$$BC = \frac{v}{u}AB$$

$$= \frac{2\text{ m/s}}{10\text{ m/s}} \times (400\text{ m})$$

$$= 80\text{ m}$$

4. Let C be the point along which pilot should head the plane.



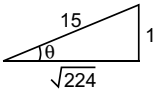
Apply sine formula in $\Delta\, ABC$

$$\frac{\sin 30^\circ}{150\,t} = \frac{\sin\theta}{20t} = \frac{\sin(180^\circ - \overline{30^\circ + \theta})}{500\times 10^3}$$

$$\frac{1}{300} = \frac{\sin\theta}{20}$$

$$\theta = \sin^{-1}\left(\frac{1}{15}\right)$$

Now $\quad \dfrac{1}{300\,t} = \dfrac{\sin(30^\circ+\theta)}{500\times 10^3}$



or $\quad \dfrac{5000}{3\,t} = \sin 30^\circ \cos\theta + \cos 30^\circ \sin\theta$

$$\frac{5000}{3\,t} = \frac{1}{2}\,\frac{\sqrt{224}}{15} + \frac{\sqrt{3}}{2}\cdot\frac{1}{15}$$

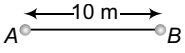
or $\quad \dfrac{5000}{3\,t} = 0.5577$

$\Rightarrow \quad t = \dfrac{5000}{3\times 0.5577}$

$$= 2989\text{ s}$$

$$= 50\text{ min}$$

5.



$a_A = 1\text{ m/s}^2, \qquad a_B = 2\text{ m/s}^2$

$v_A = 3\text{ m/s}, \qquad v_B = 1\text{ m/s}$

Acceleration of A w.r.t. $B = 1 - 2 = -1\text{ m/s}^2$

Velocity of A w.r.t. $B = 3 - 1 = 2$ m/s

Initial displacement of A w.r.t. $B = -10$ m

At time relative displacement of A w.r.t. B

$$s = -10 + 2\,t + \frac{1}{2}(-1)\,t^2$$

or $\quad s = -10 + 2t - 0.5t^2$

For s to be minimum

$$\frac{ds}{dt} = 0$$

or $\quad 2 - (0.5\times 2t) = 0$

i.e., $\quad t = 2$ s

$\therefore \quad s_{\min} = -10 + (2\times 2) - 0.5\times(2)^2$

$$= -10 + 4 - 2$$

$$= -8\text{ m}$$

Minimum distance between A and $B = 8$ m.

AIEEE Corner

Subjective Questions (Level 1)

1. (a) $D = v_1t_1 + v_2t_2$

$$= \left(60\,\frac{\text{km}}{\text{h}} \times 1\text{ h}\right) + \left(80\,\frac{\text{km}}{\text{h}} \times \frac{1}{2}\text{ h}\right)$$

$$= 100\text{ km}$$

(b) Average speed

$$= \frac{100\text{ km}}{1.5\text{ h}} = 66.67\text{ km/h.}$$

2. (a) Displacement in first two seconds

$$= (4)(2) + \frac{1}{2}(6)2^2$$

$$= 20\text{ m}$$

$\therefore$ Average velocity $= \dfrac{20\text{ m}}{2\text{ s}} = 10\text{ m/s}$

(b) Displacement in first four seconds

$$= (4)(4) + \frac{1}{2}(6)\,4^2 = 64\text{ m}$$

$\therefore$ Displacement in the time interval

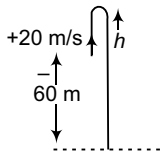
$t = 2$ s to $t = 4$ s

$= 64 - 20$

$= 44$ m

$\therefore$ Average velocity $= \dfrac{44\text{ m}}{2\text{ s}} = 22\text{ m/s.}$

3. Let the particle takes t time to reach ground.



$\therefore \qquad -64 = 20t + \dfrac{1}{2}(-10)\,t^2$

i.e., $\quad 5t^2 - 20t - 64 = 0$

$t = 6.1$ s

If the particle goes h meter above tower before coming down

$$0 = (20)^2 + 2(-10)\,h$$

$\Rightarrow \qquad h = 20$ m

(a) Average speed $= \dfrac{\text{Total distance moved}}{\text{Time taken}}$

$$= \frac{(20 + 20 + 60)}{6.1} = 16.4\text{ m/s}$$

(b) Average velocity $= \dfrac{\text{Net displacement}}{\text{Time taken}}$

$$= \frac{-60\text{ m}}{6.1\text{ s}}$$

$= -9.8$ m/s

$= 9.8$ m/s (downwards)

4. Average velocity $= \dfrac{\Sigma vt}{\Sigma t}$

$$2.5v = \frac{vt_0 + 2vt_0 + 3vT}{t_0 + t_0 + T}$$

$\therefore \qquad 5t_0 + 2.5T = 3t_0 + 3T$

or $\qquad T = 4t_0$

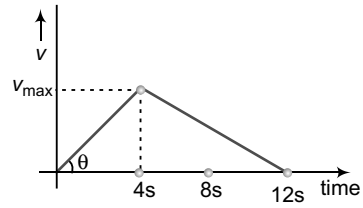
5. (a) Average acceleration

$$= \frac{\text{Final velocity} - \text{initial velocity}}{(4 + 8)\text{ s}}$$

$$= \frac{0 - 0}{12}$$

$= 0\text{ m/s}^2$

(c) $a = \tan\theta = \dfrac{v_{max}}{4}$



$\therefore \qquad 4 = \dfrac{v_{max}}{4} \qquad (\because a = 4\text{ m/s}^2)$

i.e., $\quad v_{max} = 16$ m/s

Displacement of particle in 12 seconds

= Area under v-t graph

$$= 12 \times \frac{v_{max}}{2}$$

$$= \frac{12 \times 16}{2}$$

$= 96$ m

Average velocity

$$= \frac{\text{Displacement (96 m) at 12 s}}{\text{Time (12 s)}}$$

$= +8$ m/s

(b) As the particle did not return back distance travelled in 12 s

$= \text{Displacement at 12 s}$

$\therefore$ Average speed = 8 m/s.

6. (a) Radius (R) of circle $= \dfrac{21}{22}$ m

$\therefore$ Circumference of circle $= 2\pi R$

$$= 2 \times \frac{22}{7} \times \frac{21}{22} \text{ m}$$

$$= 6 \text{ m}$$

Speed (v) of particle = 1 m/s

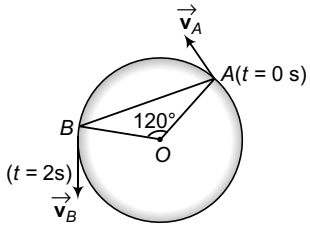
$\therefore$ Distance moved by particle in 2 s = 2 m

Thus, angle through which the particle moved

$$= \frac{2}{6} \times 2\pi = \frac{2\pi}{3} = 120°$$

Magnitude of Average velocity

$$= \frac{\text{Magnitude of displacement}}{\text{Time } (= 2 \text{ s})}$$



$$= \frac{AB}{2} = \frac{2R \sin 60°}{2}$$

$$= R\frac{\sqrt{3}}{2}$$

$$= \frac{21}{22}\frac{\sqrt{3}}{2} = \frac{21\sqrt{3}}{44} \text{ m/s}$$

(b) Magnitude of average acceleration

$$= \left| \frac{\vec{\mathbf{v}}_B - \vec{\mathbf{v}}_A}{2 \text{ s}} \right|$$

$$= \frac{2v \sin \frac{120°}{2}}{2}$$

$[\because \quad |\vec{\mathbf{v}}_A| = |\vec{\mathbf{v}}_B| = v = 1 \text{ m/s})$

$$= v \sin 60°$$

$$= \frac{\sqrt{3}}{2} \text{ m/s}^2$$

7. Position vector at $t = 0$ s

$$\vec{\mathbf{r}}_1 = (1\,\hat{\mathbf{i}} + 2\,\hat{\mathbf{j}}) \text{ m}$$

Position vector at $t = 4$ s

$$\vec{\mathbf{r}}_2 = (6\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}}) \text{ m}$$

(a) Displacement from $t = 0$ s to $t = 4$ s

$$= (\vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1)$$

$$= (6\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}}) - (1\,\hat{\mathbf{i}} + 2\,\hat{\mathbf{j}})$$

$$= (5\,\hat{\mathbf{i}} + 2\,\hat{\mathbf{j}}) \text{ m}$$

$\therefore$ Average velocity $= \dfrac{(5\,\hat{\mathbf{i}} + 2\,\hat{\mathbf{j}}) \text{ m}}{4 \text{ s}}$

$$= (1.25\,\hat{\mathbf{i}} + 0.5\,\hat{\mathbf{j}}) \text{ m/s}$$

(b) Average acceleration

$$= \frac{\text{Final velocity} - \text{Initial velocity}}{4 \text{ s}}$$

$$= \frac{(2\,\hat{\mathbf{i}} + 10\,\hat{\mathbf{j}}) - (4\,\hat{\mathbf{i}} + 6\,\hat{\mathbf{j}})}{4}$$

$$= \frac{-2\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}}}{4}$$

$$= (-0.5\,\hat{\mathbf{i}} + \hat{\mathbf{j}}) \text{ m/s}^2$$

(c) We cannot find the average speed as the actual path followed by the particle is not known.

Uniform acceleration

(a) One dimensional motion

8. If at time t the vertical displacement between A and B is 10 m

$$\frac{1}{2}gt^2 - \frac{1}{2}g(t-1)^2 = 10$$

or $\quad t^2 - (t-1)^2 = 2$

or $\quad t^2 - (t^2 - 2t + 1) = 2$

or $\quad 2t = 3$

$\quad t = 1.5$ s

9. The two bodies will meet if

Displacement of first after attaining highest point = Displacement of second before attaining highest point

$$v_0 t - \frac{1}{2} g t^2 = v_0(t - t_0) - \frac{1}{2} g(t - t_0)^2$$

or $$0 = -v_0 t_0 - \frac{1}{2} g(t_0^2 - 2t_0 t)$$

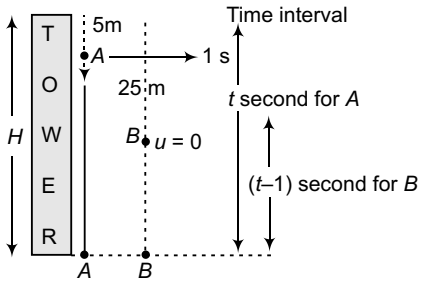
or $$g t_0 t = \frac{1}{2} g t_0^2 + v_0 t_0$$

or $$t = \frac{\frac{1}{2} g t_0 + v_0}{g}$$

$$= \frac{t_0}{2} + \frac{v_0}{g}$$

10. $$5 = \frac{1}{2} g t^2$$

$\Rightarrow$ $$t = 1 \text{ s}$$



For *A*

$$H = 0.t + \frac{1}{2} g t^2$$

or $$H = \frac{1}{2} g t^2 \quad \ldots(i)$$

For *B*

$$H - 25 = \frac{1}{2} g(t-1)^2 \quad \ldots(ii)$$

or $$\frac{1}{2} g t^2 - \frac{1}{2} g(t-1)^2 = 25$$

[Substituting value of H from Eq. (i)]

$$\frac{1}{2} g[t^2 - (t-1)^2] = 25$$

$$t^2 - (t^2 - 2t + 1) = 5$$

$$2t - 1 = 5$$

$\Rightarrow$ $$t = 3 \text{ s}$$

Substituting $t = 3$ s in Eq. (i)

$$H = \frac{1}{2} \times 10 \times 3^2 = 45 \text{ m}$$

11. $s = \frac{1}{2} a t_0^2$ **Forward motion**

$v = a t_0$ **Backward motion**

$$-s = (a t_0)t + \frac{1}{2}(-a)t^2$$

$\therefore$ $$-\frac{1}{2} a t_0^2 = (a t_0)t - \frac{1}{2} a t^2$$

or $$-t_0^2 = 2t_0 t - t^2$$

or $$t^2 - 2t_0 t - t_0^2 = 0$$

$$t = \frac{2t_0 \pm \sqrt{(-2t_0)^2 - 41(-t_0^2)}}{2}$$

$$= \frac{2t_0 \pm 2t_0\sqrt{2}}{2}$$

$= t_0 + t_0\sqrt{2}$ (– ive sign being absurd)

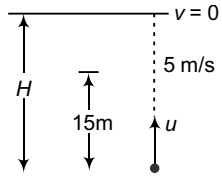
$$= (1 + \sqrt{2})t_0$$

$$= 2.141 t_0$$

From the begining of the motion the point mass will return to the initial position after time $3.141 t_0$.

12. $$5^2 = u^2 + 2(-10)15$$

$\Rightarrow$ $$u^2 = 325$$



(a) **For *H***

$$0^2 = u^2 + 2(-10)H$$

i.e., $$20H = 325$$

or $$H = 16.25 \text{ m}$$

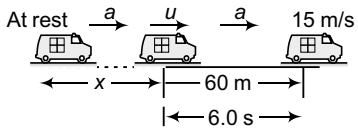
(b) **For *t***

$$0 = \sqrt{325} + (-10)t$$

$\therefore$ $$t = \frac{\sqrt{325}}{10}$$

$$= 1.8 \text{ s}$$

13.



(a) $$15^2 = u^2 + 2a \times 60 \quad \ldots(i)$$

and $$15 = u + a \times 6 \quad \ldots(ii)$$

Substituting the value of $6a$ from Eq. (ii) in Eq. (i)

$$225 = u^2 + 20(15 - u)$$

i.e., $u^2 - 20u + 75 = 0$

$(u - 15)(u - 5) = 0$

$\therefore \quad u = 5$ m/s

(15 m/s being not possible)

(b) Using Eq. (ii)

$$a = \frac{5}{3} \text{ m/s}^2$$

(c) $u^2 = 0^2 + 2ax$

i.e., $x = \dfrac{u^2}{2a} = \dfrac{(5)^2}{2 \times \frac{5}{3}}$

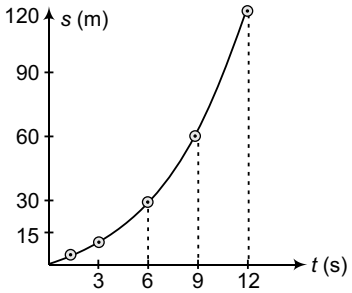
$= 7.5$ m

(d) $s = \dfrac{1}{2} at^2$

$= \dfrac{1}{2} \times \dfrac{5}{3} \times t^2$

$= \dfrac{5}{6} t^2$ ...(iii)

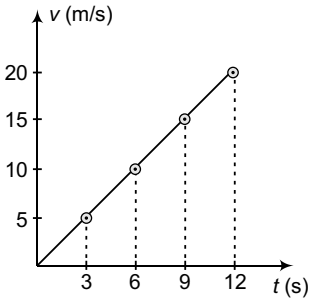
t(s)	0	1	3	6	9	12
v (m/s)	0	5/6	7.5	30	67.5	120



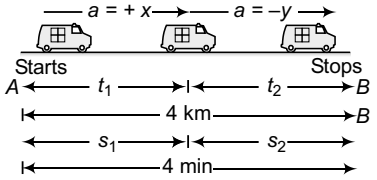
Differentiating Eq. (iii) w.r.t. time t

$$v = \frac{5}{3} t$$

t(s)	0	3	6	9	12
v (m/s)	0	5	10	15	20



14.



Journey *A* to *P*

$v_{max} = 0 + xt_1$ ...(i)

and $v_{max}^2 = 2xs_1$ ...(ii)

$\Rightarrow \quad t_1 = \dfrac{v_{max}}{x}$

Journey *P* to *B*

$0 = v_{max} + (-y)t_2$ ...(iii)

and $v_{max}^2 = 2ys_2$ ...(iv)

$\Rightarrow \quad t_2 = \dfrac{v_{max}}{y}$

$\therefore \quad \dfrac{v_{max}}{x} + \dfrac{v_{max}}{y} = t_1 + t_2 = 4$

or $v_{max}\left[\dfrac{1}{x} + \dfrac{1}{y}\right] = 4$ ...(v)

From Eq. (ii) and Eq. (iv)

$$s_1 + s_2 = \frac{v_{max}^2}{2x} + \frac{v_{max}^2}{2y}$$

or $4 = \dfrac{v_{max}^2}{2}\left[\dfrac{1}{x} + \dfrac{1}{y}\right]$ ...(vi)

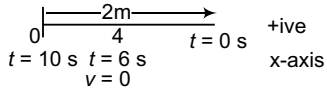
Dividing Eq. (vi) by Eq. (v)

$$v_{max} = 2$$

Substituting the value of v_{max} in Eq. (v)

$\dfrac{1}{x} + \dfrac{1}{y} = 2$ (Proved)

15. Let acceleration of the particle be a using



$v = u + at$

$0 = u + a6$

$\therefore \quad a = -\dfrac{u}{6}$

(a) At $t = 10$ s, $s = -2$ m

$-2 = u \times 10 + \dfrac{1}{2} a \times 10^2$

or $-2 = (-6a)10 + 50a$

or $-10a = -2$

or $a = 0.2$ m/s^2

(b) $v(\text{at } t = 10\text{ s}) = u + a\,10$

$= -6a + 10a$

$= 4a$

$= 0.8$ m/s

(c) Two or three dimensional motion

16. $\vec{\mathbf{a}} = \dfrac{\vec{\mathbf{F}}}{m} = \dfrac{10\text{ N north}}{2\text{ kg}}$

$= 5\text{ m/s}^2$, north

$= 5\,\hat{\mathbf{j}}\text{ m/s}^2$

$\vec{\mathbf{u}} = 10$ m/s, east

$= 10\,\hat{\mathbf{i}}$ m/s

using $\vec{\mathbf{v}} = \vec{\mathbf{u}} + \vec{\mathbf{a}}\,t$

$\vec{\mathbf{v}} = 10\,\hat{\mathbf{i}} + (5\,\hat{\mathbf{j}} \times 2)$

$= 10\,\hat{\mathbf{i}} + 10\,\hat{\mathbf{j}}$

$|\vec{\mathbf{v}}| = 10\sqrt{2}$ m/s

$\vec{\mathbf{v}} = 10\sqrt{2}$, north-east

using $\vec{\mathbf{s}} = \vec{\mathbf{u}}\,t + \dfrac{1}{2}\vec{\mathbf{a}}\,t^2$

$= (10\,\hat{\mathbf{i}} \times 2) + \dfrac{1}{2}(5\,\hat{\mathbf{j}})2^2$

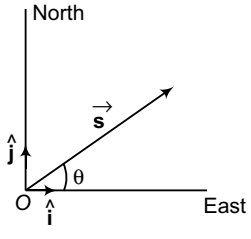
$= (20\,\hat{\mathbf{i}} + 10\,\hat{\mathbf{j}})$ m

$|\vec{\mathbf{s}}| = \sqrt{20^2 + 10^2}$

$= 10\sqrt{5}$ m

$\cot\theta = \dfrac{20}{10} = 2$

$\theta = \cot^{-1} 2$



$\vec{\mathbf{s}} = 10\sqrt{5}$ m at $\cot^{-1}(2)$ from east to north.

17. $\vec{\mathbf{s}}_0 = (2\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}})$ m

$\vec{\mathbf{a}}_1 = 2\,\hat{\mathbf{i}}\text{ m/s}^2$ ($t = 0$ s to $t = 2$ s) $t_1 = 2$ s

$\vec{\mathbf{u}} = \vec{\mathbf{0}}$ m/s

$\vec{\mathbf{a}}_2 = -4\,\hat{\mathbf{j}}\text{ m/s}^2$ ($t = 2$ s to $t = 4$ s) $t_2 = 2$ s

(a) Velocity

$\vec{\mathbf{v}}_1 = \vec{\mathbf{u}} + \vec{\mathbf{a}}_1\,t_1$

$= \vec{\mathbf{0}} + (2\,\hat{\mathbf{i}})\,2$

$= 4\,\hat{\mathbf{i}}$

$\vec{\mathbf{v}}_2 = \vec{\mathbf{v}}_1 + (\vec{\mathbf{a}}_1 + \vec{\mathbf{a}}_2)t_2$

$= 4\,\hat{\mathbf{i}} + (2\,\hat{\mathbf{i}} - 4\,\hat{\mathbf{j}})\,2$

or $\vec{\mathbf{v}}_2 = (8\,\hat{\mathbf{i}} - 8\,\hat{\mathbf{j}})$ m/s

(b) Co-ordinate of particle

$\vec{\mathbf{s}}_1 = \vec{\mathbf{s}}_0 + \vec{\mathbf{u}}\,t_1 + \dfrac{1}{2}\vec{\mathbf{a}}_1\,t_1^2$

$= (2\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}}) + (0)(2) + \dfrac{1}{2}(2\,\hat{\mathbf{i}})\,2^2$

$= 2\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}} + 4\,\hat{\mathbf{i}}$

$= 6\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}}$

$\vec{\mathbf{s}}_2 = \vec{\mathbf{s}}_1 + \vec{\mathbf{v}}_1\,t_2 + \dfrac{1}{2}(a_1 + a_2)t_2^2$

$= 6\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}} + (4\,\hat{\mathbf{i}})\,2 + \dfrac{1}{2}(2\,\hat{\mathbf{i}} - 4\,\hat{\mathbf{j}})\,2^2$

$= 6\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}} + 8\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{i}} - 8\,\hat{\mathbf{j}}$

$= 18\,\hat{\mathbf{i}} - 4\,\hat{\mathbf{j}}$

Co-ordinate of the particle

[18 m, – 4 m]

18. $\vec{\mathbf{u}} = (2\,\hat{\mathbf{i}} - 4\,\hat{\mathbf{j}})$ m/s, $\vec{\mathbf{s}}_0 = \vec{\mathbf{0}}$ m

$\vec{\mathbf{a}} = (4\,\hat{\mathbf{i}} + \hat{\mathbf{j}})\text{ m/s}^2$

(a) Velocity

$\vec{\mathbf{v}} = \vec{\mathbf{u}} + \vec{\mathbf{a}}\,t$

$= (2\,\hat{\mathbf{i}} - 4\,\hat{\mathbf{j}}) + (4\,\hat{\mathbf{i}} + \hat{\mathbf{j}})2$

$= (10\,\hat{\mathbf{i}} - 2\,\hat{\mathbf{j}})$ m/s

(b) Co-ordinates of the particle

$\vec{\mathbf{s}} = \vec{\mathbf{s}}_0 + \vec{\mathbf{u}}\,t + \dfrac{1}{2}\vec{\mathbf{a}}\,t^2$

$= \vec{\mathbf{0}} + (2\,\hat{\mathbf{i}} - 4\,\hat{\mathbf{j}}) + \dfrac{1}{2}(4\,\hat{\mathbf{i}} + \hat{\mathbf{j}})2^2$

$= 10\,\hat{\mathbf{i}} - 2\,\hat{\mathbf{j}}$

$\therefore$ Co-ordinates of particle would be

[10 m, – 2 m]

19. $\vec{\mathbf{u}} = 8\,\hat{\mathbf{j}}$ m/s

$\vec{\mathbf{a}} = (4\,\hat{\mathbf{i}} + 2\,\hat{\mathbf{j}})$ m/s^2

$\vec{\mathbf{s}}_0 = \vec{\mathbf{0}}$

$\vec{\mathbf{s}} = 29\,\hat{\mathbf{i}} + n\,\hat{\mathbf{j}}$

(a) $\vec{\mathbf{s}} = \vec{\mathbf{s}}_0 + \vec{\mathbf{u}}\,t + \frac{1}{2}\vec{\mathbf{a}}\,t^2$

$$29\,\hat{\mathbf{i}} + n\,\hat{\mathbf{j}} = \vec{0} + (8\,\hat{\mathbf{j}})t + \frac{1}{2}(4\,\hat{\mathbf{i}} + 2\,\hat{\mathbf{j}})t^2$$

$$29\,\hat{\mathbf{i}} + n\,\hat{\mathbf{j}} = 8\,\hat{\mathbf{j}}\,t + 2\,\hat{\mathbf{i}}\,t^2 + \hat{\mathbf{j}}\,t^2$$

Comparing the coefficients of $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$

$$29 = 2t^2 \quad \ldots(i)$$

and $$n = 8t + t^2 \quad \ldots(ii)$$

$\Rightarrow$ $$t = \sqrt{\frac{29}{2}} = 3.807 \text{ s}$$

Substituting value of t in Eq. (ii)

$$n = 8 \times 3.807 + (3.807)^2$$
$$= 44.95$$

(b) Speed at $t = \sqrt{\frac{29}{2}}$ s

$$\vec{\mathbf{v}} = \vec{\mathbf{u}} + \vec{\mathbf{a}}\,t$$

$$= 8\,\hat{\mathbf{j}} + (4\,\hat{\mathbf{i}} + 2\,\hat{\mathbf{j}})(3.807)$$

$$= (2 \times 3.807)\,\hat{\mathbf{i}} + \{(4 \times 3.807) + 8\}\,\hat{\mathbf{j}}$$

$$= (7.614\,\hat{\mathbf{i}} + 23.228\,\hat{\mathbf{j}}) \text{ m/s}$$

speed $= |\vec{\mathbf{v}}| = \sqrt{(7.614)^2 + (23.228)^2}$

$= 24.44$ m / s

20. $\vec{\mathbf{s}}_0 = 5\,\hat{\mathbf{i}}$ m

at $t = 0.02$ s, $\vec{\mathbf{s}} = (5.1\,\hat{\mathbf{i}} + 0.4\,\hat{\mathbf{j}})$ m

Average velocity $\vec{\mathbf{v}}_{av} = \dfrac{\vec{\mathbf{s}} - \vec{\mathbf{s}}_0}{0.02}$

$$= \frac{(5.1\,\hat{\mathbf{i}} + 0.4\,\hat{\mathbf{j}}) - (5\,\hat{\mathbf{i}})}{0.02}$$

$$= \frac{0.1\,\hat{\mathbf{i}} + 0.4\,\hat{\mathbf{j}}}{0.02}$$

$$= 5\,\hat{\mathbf{i}} + 20\,\hat{\mathbf{j}} \text{ m/s}$$

$$|\vec{\mathbf{a}}_{av}| = \sqrt{5^2 + 20^2}$$

$$= 20.6 \text{ m/s}$$

$$\tan\theta = \frac{20}{5}$$

i.e., $\theta = \tan^1(4)$

Non uniform acceleration.

21. $x = 2 + t^2 + 2t^3$

(a) **At** $\boldsymbol{t = 0}$, $x = 2$ m

(b) $\dfrac{dx}{dt} = 2t + 6t^2$

$$\left(\frac{dx}{dt}\right)_{t = 0 \text{ s}} = 0 \text{ m/s}$$

(c) $\dfrac{d^2x}{dt^2} = 2 + 12t$

$(a)_{t = 2\text{s}} = 2 + (12 \times 2) = 26$ m/s^2

22. $$a = -v\frac{dv}{dx}$$
$$= -(10) \times 3$$
$$= -30 \text{ m/s}^2$$

23. $s = t^3 - 9t^2 - 15t$

$\therefore$ $$v = \frac{ds}{dt} = 3t^2 - 18t - 15$$

ie, $$a = \frac{dv}{dt} = 6t - 18$$

Acceleration (a) in the interval $0 \le t \le 10$ s will be maximum at $t = 10$ s

$$a(\text{at } t = 10 \text{ s}) = (6 \times 10) - 18$$
$$= 42 \text{ m/s}^2$$

24. $$a = 3 - 2t$$

$$\frac{dv}{dt} = 3 - 2t$$

or $$\int dv = \int (3 - 2t)\,dt$$

or $$v = 3t - t^2 + c$$

or $$v = 3t - t^2 + v_0 \quad [\text{as at } t = 0, v = v_0]$$

or $$\int ds = \int (3t - t^2 + v_0)\,dt$$

or $$s = \frac{3t^2}{2} - \frac{t^3}{3} + v_0 t$$

(a) Displacement at = Displacement at

$t = 0$ s $\qquad t = 5$ s

$$0 = \frac{3.5^2}{2} - \frac{5^3}{3} + v_0 5$$

$\Rightarrow \qquad v_0 = 5/6 = 0.833$ s

(b) $v = 3t - t^2 + v_0$

Velocity at t (= 5.00 s)

$$= 3.5 - 5^2 + 0.833$$
$$= 15 - 25 + 833$$
$$= -9.167 \text{ m/s}$$

25. $v = 3t^2 - 6t$

$\therefore \qquad \int ds = \int (3t^2 - 6t)\, dt$

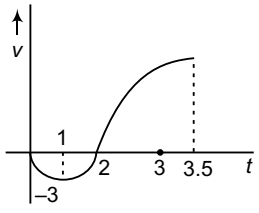
or $\qquad s = t^3 - 3t^2 + c$

or $\qquad s = t^3 - 3t^2 = t^2(t - 3)$

(a) Average velocity $= \dfrac{(3.5)^2(3.5-3)}{3.5}$

$$= 3.5 \times 0.5$$
$$= 1.75 \text{ m/s}$$

(b) Distance covered



$$= \int_0^2 (6t - 3t^2)dt + \int_2^{3.5} (3t^2 - 6r)\, dt$$
$$= [3t^2 - t^3]_0^2 + [t^3 - 3t^2]_2^{3.5}$$
$$= 3(2)^2 - (2)^3 + (3.5)^3 - 3(3.5)^2 - (2)^3 + 3(2)^2$$
$$= 12 - 8 + 42.875 - 36.75 - 8 + 12$$
$$= 14.125 \text{ m}$$

(c) Average speed $= \dfrac{14.125}{3.5}$

$$= 4.036 \text{ m/s}$$

26. $\qquad v = \dfrac{4}{a}$

i.e., $\qquad va = 4 \qquad$ ...(i)

or $\qquad \int v\, dv = \int 4\, dt$

or $\qquad \dfrac{v^2}{2} = 4t + C$

Now, at $t = 2$ s, $v = 6$ m/s

$\therefore \qquad \dfrac{6^2}{2} = (4 \times 2) + C$

Thus, $C = 10$.

i.e., $\qquad \dfrac{v^2}{2} = 4t + 10 \qquad$ ...(ii)

$\therefore$ at $\qquad t = 3$ s,

$$\frac{v^2}{2} = (4 \times 3) + 10$$
$$v^2 = 44$$
$$v = \sqrt{44} = 2\sqrt{11} \text{ m/s}$$

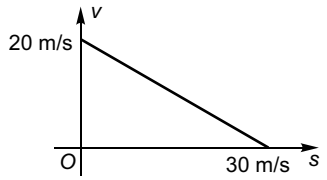
Substituting above found value of v in Eq. (i),

$$2\sqrt{11}\, a = 4$$

i.e., $\qquad a = \dfrac{2}{\sqrt{11}}$

$$= 0.603 \text{ m/s}^2$$

27. According to question, the velocity of the particle varies as shown in figure.



$\therefore \qquad \dfrac{v}{20} + \dfrac{s}{30} = 1$

or $\qquad 3v + 2s = 60$

or $\qquad 3v = 60 - 2s \qquad$ ...(i)

Differentiating above equation w.r.t. time t

$$3 \cdot \frac{dv}{dt} = -2 \cdot \frac{ds}{dt}$$

or $\qquad 3a = -2v$

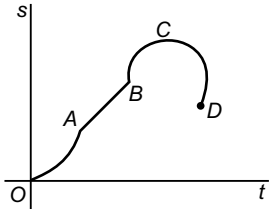
or $\qquad 3a = -2 \cdot \left(\dfrac{60 - 2s}{3}\right) \qquad$ [using Eq. (i)]

or $\qquad a = -\dfrac{2}{9}(60 - 2s)$

$$= -\frac{2}{9}(60 - 2 \times 15) \qquad [\text{at } s = 15 \text{ m}]$$
$$= -\frac{20}{3} \text{ m/s}^2$$

Graphy

28. OA : slope is + ive and increasing.



$\therefore$ velocity is +ive and acceleration is +ive.

AB : slope is +ive and constant

$\therefore$ velocity is +ive and acceleration is zero.

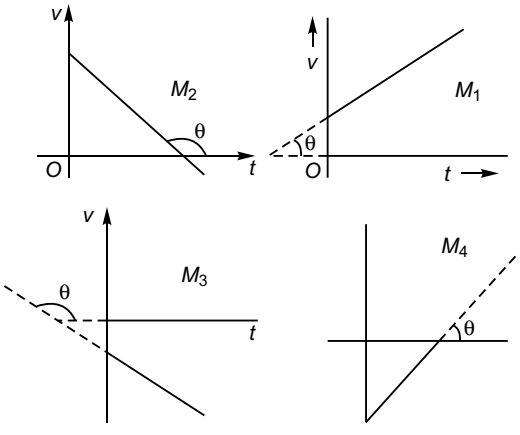
BC : sope is +ive and decreases.

$\therefore$ velocity is +ive and increasing.

CD : slope is –ive and increasing

$\therefore$ velocity is – ive and acceleration is – ive.

29.



In M_1 and M_3 : $0° \le \theta < 90°$

$\therefore$ slope is +ive *i.e.*, acceleration is +ive.

In M_2 and M_4 : $90° < \theta \le 180°$

$\therefore$ slope is – ive *i.e.,* acceleration is – ive.

(a) M_1 : Magnitude of velocity is increasing.

M_2 : Magnitude of velocity is decreasing.

M_3 : Magnitude of velocity is increasing.

M_4 : Magnitude of velocity is decreasing.

Ans : M_1 and M_3.

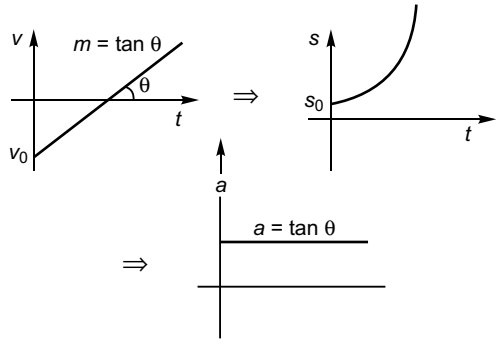
(b) $P \to M_1$

$Q \to M_2$

$R \to M_3$

$S \to M_4$

30. Case I



$$v = mt - v_0 \text{ (}st\text{-line)}$$

$$\therefore \quad \int ds = \int (mt - v_0)\, dt$$

i.e., $\quad s = m \cdot \dfrac{t^2}{2} - v_0 t + s_0 \qquad$ (Parabola)

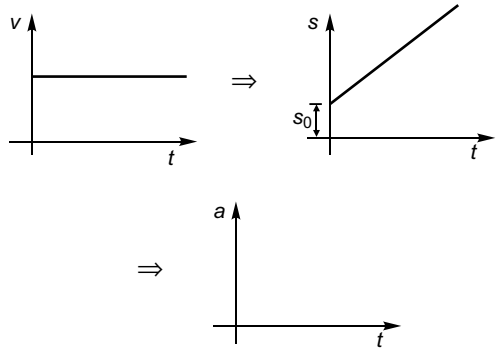
or $\quad s = s_0 - v_0 t + \dfrac{1}{2} a t^2 \qquad (\because m = a)$

Further, $\quad a = \dfrac{dv}{dt} = \tan\theta$

As for $0° \le \theta < 90°$

$\tan\theta$ is + ive, a is + ive.

Case II



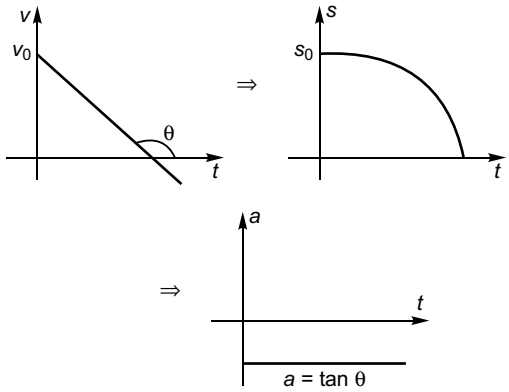
$v = k$ (constant) (st line parallel to time-axis)

$$\Rightarrow \qquad \int ds = \int k\, dt$$

$$s = kt + s_0$$

Further, $\quad a = \dfrac{dv}{dt} = 0$

Case III.



$$v = v_0 + mt$$

$$\therefore \quad \int ds = \int (v_0 + mt)\, dt$$

i.e., $$s = v_0 t + m\frac{t^2}{2} + s_0$$

or $$s = s_0 + v_0 t - \frac{1}{2}at^2 \qquad (\because m = -a)$$

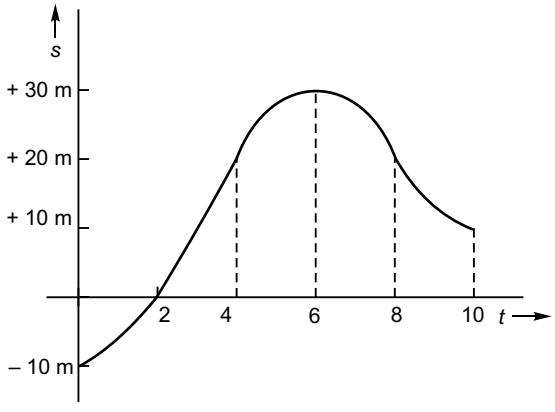
Further, $a = \dfrac{dv}{dt} = \tan\theta$

As for $90° < \theta \le 180°$

$\tan\theta$ is – ive, a is – ive.

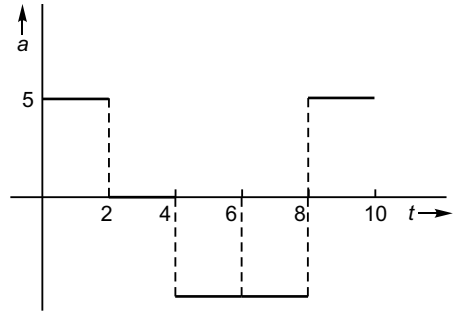
Time-displacement graph

Time	Area under the graph	Initial Displacement	Net displacement
at 0 s	0 m	– 10 m	– 10 m
2 s	10 m	– 10 m	0 m
4 s	30 m	– 10 m	+ 20 m
6 s	40 m	– 10 m	+ 30 m
8 s	30 m	– 10 m	+ 20 m
10 s	20 m	– 10 m	+ 10 m



Time-acceleration graph

time	slope	acceleration
0 - 2 s	5	5 m/s^2
2 - 4 s	zero	0 m/s^2
4 - 6 s	– 5	– 5 m/s^2
6 - 8 s	– 5	– 5 m/s^2
8 - 10 s	+ 5	+ 5 m/s^2



31. a = slope of v-t graph.

S = area under $v - t$ graph.

Corresponding graphs are drawn in the answer sheet.

32. Average velocity $= \dfrac{s}{t} = \dfrac{\text{Net area}}{\text{Time}} = \dfrac{A_1 - A_2}{t}$

Average speed $= \dfrac{d}{t} = \dfrac{A_1 + A_2}{t}$

33. Average acceleration

$$= \frac{v_f - v_i}{\text{time}} = \frac{-10 - 20}{6} = -5\,\text{m/s}^2$$

$v_f - v_i$ = Area under a-t graph.

34. (a) Acceleration = slope of v-t graph.

35. (b) $r_f - r_i = s$ = area under v-t graph.

(c) Equations are written in answer sheet.

Relative Motion

35. (a) Acceleration of 1 w.r.t. 2

$= (-g) - (-g)$

$= 0\ \text{m/s}^2$

(b) Initial velocity of 2 w.r.t. 1

$= (+20) - (-5)$

$= 25$ m/s

(c) Initial velocity of 1 w.r.t. 2

$= (-5) - (+20)$

$= -25$ m/s

$\therefore$ Velocity of 1 w.r.t. 2 at time $t\left(=\frac{1}{2}\text{ s}\right)$

$$= (-25) + (0)\left(\frac{1}{2}\right)$$

$$= -25 \text{ m/s}$$

(d) Initial relative displacement of 2 w.r.t. 1

$$\vec{\mathbf{S}}_0 = -20 \text{ m}$$

Using $\vec{\mathbf{S}} = \vec{\mathbf{S}}_0 + \vec{\mathbf{u}}_{\text{rel}}\, t + \frac{1}{2}\vec{\mathbf{a}}_{\text{rel}}\, t^2$,

as at time t (= time of collision of the particles) the relative displacement of 2 w.r.t. 1 will be zero (*i.e.*, $\vec{\mathbf{S}} = \vec{\mathbf{O}}$)

$$\vec{\mathbf{O}} = (-20) + (-25)\cdot\left(\frac{1}{2}\right)$$

$\Rightarrow$ $t = 0.85$ s

36. Let length of escalator L (= 15 m)

walking speed of man $= \frac{L}{90}$

Speed of escalator $= \frac{L}{60}$

Time taken by man walking on a moving escalator $= \dfrac{L}{\frac{L}{90} + \frac{L}{60}}$

$= 36$ s

Required time has been found without using the actual length of the escalator.

37. $\vec{\mathbf{S}}_0$ = Initial displacement of elevator w.r.t. ball = + 10 m.

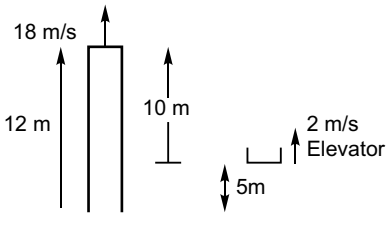
Relative velocity of elevator w.r.t. ball

$= (2) - (18) = -16$ m/s

Accelerator of elevator w.r.t. ball

$= (0) - (-10)$

$= 10 \text{ m/s}^2$



Using $\vec{\mathbf{S}} = \vec{\mathbf{S}}_0 + u_{\text{rel}}\, t + \frac{1}{2} a_{\text{rel}}\, t^2$

$$0 = (+10) + (-16)\, t + \frac{1}{2}(+10)\, t^2$$

or $5t^2 - 16t - 10 = 0$

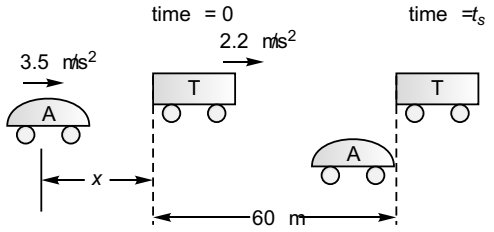
$t = 3.65$ s

Position of elevator when it meets ball

$= 5 + (2 \times 3.65)$

$= 12.30$ m level

38.



(a) $\frac{1}{2}(2.2)\, t^2 = 60$ ...(i)

$\Rightarrow$ $t = 7.39$ s

(b) $\frac{1}{2}(3.5)\, t^2 = 60 + x$ ...(ii)

Dividing Eq. (ii) by Eq. (i),

$$\frac{60 + x}{60} = \frac{3.5}{2.2}$$

or $\frac{x}{60} = \frac{1.3}{2.2}$

$x = 35.5$ m

(c) At the time of overtaking

Speed of automobile $= (3.5)(7.39)$

$= 25.85$ m/s

Speed of truck $= (2.2)(7.39)$

$= 16.25$ m/s

39. Let, acceleration of lift = a (upward)

$\therefore$ acceleration of thrown body w.r.t. lift

$= (-g) - (+a)$

$= -(g + a)$

If time of flight is t, using

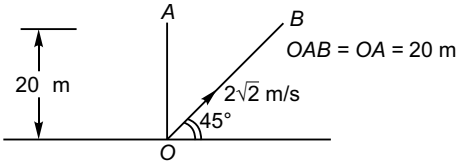
$$v_{\text{rel}} = u_{\text{rel}} + a_{\text{rel}}\, t$$

$$(-v) = (+u) + \{-(a + g)\}\, t$$

$\Rightarrow$ $(a + g)\, t = 2u$

or $a = \frac{2u}{t} - g$

40.



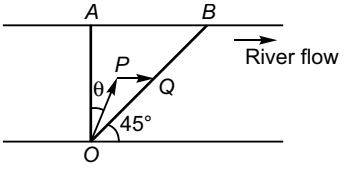
$$OB = \sqrt{(OA)^2 + (AB)^2}$$
$$= \sqrt{(20)^2 + (20)^2}$$
$$= 20\sqrt{2}\text{ m}$$

Speed along OB is $2\sqrt{2}$ m/s

$\therefore$ Time taken to reach $B = \dfrac{20\sqrt{2}\text{ m}}{2\sqrt{2}\text{ ms}^{-1}}$

$$= 10\text{ s}$$

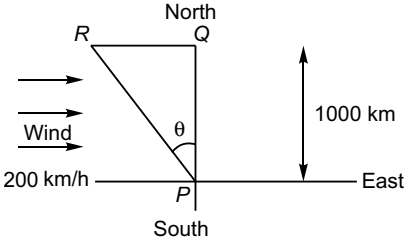
41. In ΔOPQ,



$$OP = |\vec{\mathbf{v}}_{br}|$$
$$OP = |\vec{\mathbf{v}}_{r}|$$
$$\frac{PQ}{\sin\angle POQ} = \frac{OP}{\sin\angle OPQ}$$

$$\frac{2}{\sin(45° - \theta)} = \frac{4}{\sin 45°}$$
$$\theta = \left[\sin^{-1}\left(\frac{1}{2\sqrt{2}}\right) - 45°\right]$$

42. Let pilot heads point R to reach point Q



$$\sin\theta = \frac{RQ}{PR}$$
$$= \frac{200t}{500t}$$

$\Rightarrow$ $\theta = \sin^{-1}(0.4)$

$(PR)^2 = (RQ)^2 + (PQ)^2$

or $(500t)^2 - (200t)^2 = (1000)^2$

or $t = \dfrac{1000}{\sqrt{(500)^2 - (200)^2}}$

$$= \frac{10}{\sqrt{25-4}} = \frac{10}{\sqrt{21}}\text{ h}$$

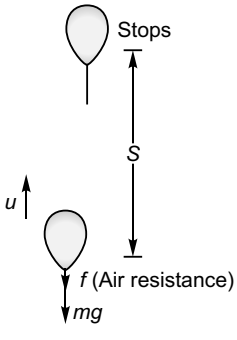
Objective Questions (Level 1)

Single Correct Option

1. As the packet is detached from rising balloon its acceleration will be g in the downward direction.

Option (b) is correct.

2. While going up :



$$0 = u - \left(\frac{mg+f}{m}\right)T_1$$
$$T_1 = \frac{um}{mg+f} \quad \text{...(i)}$$

Using $v^2 = u^2 + 2as$

$$0^2 = u^2 - 2\left(\frac{mg+f}{m}\right)s$$

i.e., $S = \dfrac{mu^2}{2(mg+f)}$

While coming down

$$s = \frac{1}{2}\left(\frac{mg-f}{m}\right)T_2^2$$

or $\dfrac{mu^2}{2(mg+f)} = \dfrac{1}{2}\left(\dfrac{mg-f}{m}\right)T_2^2$

$$\Rightarrow \qquad T_2 = \frac{um}{\sqrt{(mg+f)(mg-f)}} \quad \ldots\text{(ii)}$$

Using Eq. (i) and Eq. (ii)

$$\frac{T_2}{T_1} = \sqrt{\frac{mg+f}{mg-f}}$$

$$\Rightarrow \qquad T_2 > T_1$$

Option (c) is correct.

3. Angular speed (ω) of seconds hand

$$= \frac{2\pi}{60} \text{ rad s}^{-1}$$

Speed of the tip of seconds hand

$v = \frac{\pi}{30} \times 1$ cm/s [$\because$ $v = r\omega$ and $r = 1$ cm]

As in 15s the seconds hand rotates through 90°, the change in velocity of its tip in 15 s will be

$$= v\sqrt{2} = \frac{\pi\sqrt{2}}{30} \text{ cm s}^{-1}$$

Option (d) is correct.

4. Average speed $= \dfrac{5+5}{\frac{5}{30}+\frac{5}{60}} = 40 \text{ ms}^{-1}$

Option (c) is correct.

5. Relative velocity of boat w.r.t. water

$$= (3\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}}) - (-3\,\hat{\mathbf{i}} - 4\,\hat{\mathbf{j}}) = 6\,\hat{\mathbf{i}} + 8\,\hat{\mathbf{j}}$$

Option (b) is correct.

6. $21 = \dfrac{(18 \times 11) + (42 \times v)}{18 + 42}$

$\Rightarrow v = 25.29$ m/s

7. $$x = 32t - \frac{8t^3}{3}$$

$$v = \frac{dx}{dt} = 32 - 8t^2 \quad \ldots\text{(i)}$$

$\therefore$ Particle is at rest when

$$32 - 8t^2 = 0$$

i.e., $\qquad t = 2$ s

Differentiating Eq. (i) w.r.t. time t

$$a = \frac{dv}{dt} = -16t$$

$\therefore$ a at time $t = 2$ s (when particle is at rest)

$$= -(16) \times (2) = -32 \text{ m/s}^2$$

Option (b) is correct.

8. For first one second

$$2 = \frac{1}{2} a \times 1^2$$

$$\Rightarrow \qquad a = 4 \text{ m/s}^2$$

Velocity at the end of next second

$$v = (4) \times (2)$$

$$= 8 \text{ m/s}$$

Option (b) is correct.

9. $x = -3t + t^3$

$\therefore$ displacement at time t (= 1 s)

$$= -3(1) + (1)^3 = -2 \text{ m}$$

and displacement at time t (= 3 s)

$$= -3(3) + (3)^3 = 18 \text{ m}$$

And as such displacement in the time interval ($t = 1$ s to $t = 3$ s)

$$= (18 \text{ m}) - (-2 \text{ m})$$

$$= +20 \text{ m}$$

Option (c) is correct.

10. Acceleration $a = bt$

$$\therefore \qquad \frac{dv}{dt} = bt$$

or $$\int dv = \int bt\, dt$$

or $$v = \frac{1}{2} bt^2 + C$$

Now, at $t = 0, v = v_0$

$$\therefore \qquad C = v_0$$

i.e., $$v = \frac{1}{2} bt^2 + v_0$$

or $$\frac{ds}{dt} = v_0 + \frac{1}{2} bt^2$$

or $$\int ds = \int \left(v_0 + \frac{1}{2} bt^2\right) dt$$

or $$s = v_0 t + \frac{1}{6} bt^3 + k$$

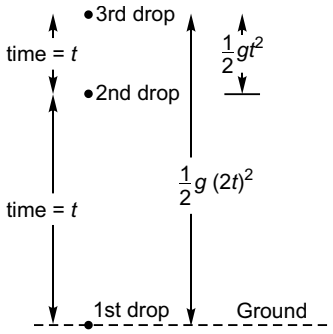
At $t = 0, s = 0,$

$$\therefore \qquad k = 0$$

$$\Rightarrow \qquad s = v_0 t + \frac{1}{6} bt^3$$

Option (a) is correct.

11. $\frac{1}{2}g(2t^2+6)$ m (given)



$\therefore \quad \frac{1}{2}gt^2 = \frac{5}{4}$ m

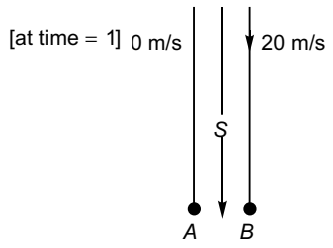
$\therefore$ Height of 2nd drop from ground

$= 5\text{ m} - \frac{5}{4}\text{ m}$

$= 3.75$ m

Option (c) is correct.

12.



[At time = $t-1$]

$$s = \frac{1}{2}\times 10t^2$$

$$= 20(t-1) + \frac{1}{2}\times 10(t-1)^2$$

or $\quad t^2 = 4(t-1) + (t^2 - 2t + 1)$

or $\quad 2t - 3 = 0$

or $\quad t = \frac{3}{2}$ s

$\therefore \quad s = \frac{1}{2}\times 10\times\left(\frac{3}{2}\right)^2$

$= 11.25$ m

Option (c) is correct.

$\therefore \quad v = t\,\hat{\mathbf{i}} + \frac{t^3}{2}\hat{\mathbf{j}}$

Thus, velocity of particle at time t (= 2 s) will be

$$v_2 = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$$

Option (b) is correct.

13. $v_0^2 = 2gh$ and $u^2 = 2g(3h)$

$\therefore \quad \frac{u^2}{v_0^2} = 3$

or $\quad u = v_0\sqrt{3}$

Option (a) is correct.

14. $v_x = 8t - 2$

$\therefore \quad \frac{dx}{dt} = 8t - 2$

or $\quad \int dx = \int (8t-2)\,dt$

or $\quad x = 4t^2 - 2t + k_1$

Now at $t = 2$, $x = 14$

$\therefore \quad 14 = 4\cdot 2^2 - (2)2 + k_1$

i.e., $\quad k_1 = 2$

Thus, $\quad x = 4t^2 - 2t + 2 \quad$ ...(i)

Further, $\quad v_y = 2$

i.e., $\quad \frac{dy}{dt} = 2$

or $\quad \int dy = \int 2\,dt$

or $\quad y = 2t + k_2$

Now, at $t = 2$, $y = 4$

$\therefore \quad k_2 = 0$

Thus, $\quad y = 2t \quad$ ...(ii)

Substituting $t = \frac{y}{2}$ in Eq. (i),

$$x = 4\left(\frac{y}{2}\right)^2 - 2\left(\frac{y}{2}\right) + 2$$

or $\quad x = y^2 - y + 2$

Option (a) is correct.

15. $x = 5t$ and $y = 2t^2 + t$

$\therefore \quad \frac{dx}{dt} = 5$ and $\frac{dy}{dt} = 4t + 1$

Now, $\quad \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

$$\tan 45° = \frac{4t+1}{5}$$

$\Rightarrow \quad t = 1$ s

Option (b) is correct.

16. $y = 8t - 5t^2$ and $x = 6t$

$\therefore \quad \frac{dy}{dt} = 8 - 10t$ and $\frac{dx}{dt} = 6$

At $t = 0$

$$\frac{dy}{dt} = 8 \text{ and } \frac{dx}{dt} = 6$$

$\therefore$ Velocity of projection

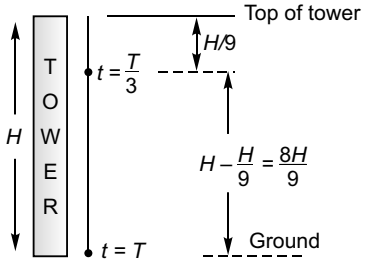
$$= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{6^2 + 8^2}$$

$$= 10\,\text{ms}^{-1}$$

Option (c) is correct.

17. $$T = \sqrt{\frac{2H}{g}}$$

$$\therefore \quad \frac{T}{3} = \sqrt{\frac{2}{g}\left(\frac{H}{9}\right)}$$



Option (c) is correct.

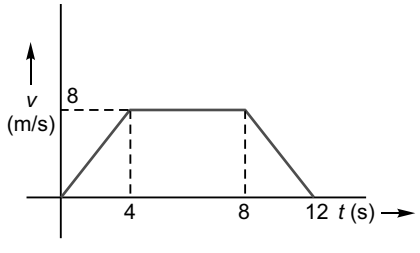
18. Distance of farthest corner from one corner

$$= a + a + a = 3a$$

$\therefore$ Time taken $= \dfrac{3a}{u}$

Option (a) is correct.

19. Time-velocity graph of the given time-acceleration graph will be



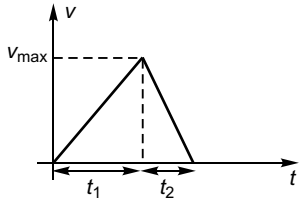
$\therefore$ Height of lift above the starting point when it comes to rest

$$= \text{Area under } t\text{-}v \text{ graph}$$

$$= \frac{4 + 12}{2} \times 8 = 64 \text{ m}$$

Option (b) is correct.

20.



In time interval t_1

$$\text{Acceleration} = \frac{v_{\max}}{t_1} = a$$

In time interval t_2

$$\text{Retardation} = \frac{v_{\max}}{t_2} = 2a$$

$$\therefore \quad \frac{v_{\max}}{t_2} = 2 \cdot \frac{v_{\max}}{t_1}$$

i.e., $\quad t_1 = 2t_2$

Now $\quad t_1 + t_2 = t$

$\therefore \quad 2t_2 + t_2 = t$

or $\quad t_2 = \dfrac{t}{3}$

Depth of shaft = Displacement of lift

$$= \frac{1}{2} v_{\max}(t_1 + t_2)$$

$$= \frac{1}{2} v_{\max}(2t_2 + t_2)$$

$$= \frac{3}{2} v_{\max} t_2$$

$$= \frac{3}{2}(2at_2)\, t_2$$

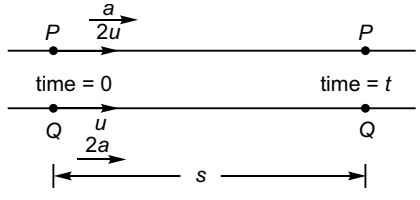
$$= 3at_2^2$$

$$= 3a\left(\frac{t}{3}\right)^2$$

$$= \frac{at^2}{3}$$

Option (b) is correct.

21. $s = 2ut + \dfrac{1}{2}at^2 = ut + \dfrac{1}{2}2at^2$



$$\Rightarrow \quad ut - \frac{1}{2}at^2 = 0$$

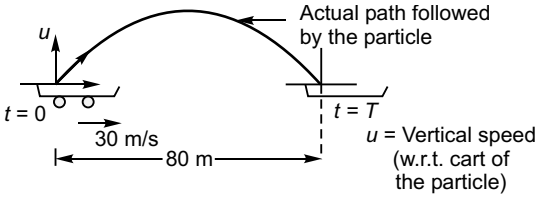
or $$u - \frac{1}{2}at = 0$$

or $$t = \frac{2u}{a}$$

$\therefore$ $$s = 2u\left(\frac{2u}{a}\right) + \frac{1}{2}a\left(\frac{2u}{a}\right)^2$$
$$= \frac{4u^2}{a} + \frac{2u^2}{a}$$
$$= \frac{6u^2}{a}$$

Option (a) is correct.

22. $u =$ vertical speed (w.r.t. cart) of the particle.



For cart $T = \frac{80\text{ m}}{30\text{ m/s}} = \frac{8}{3}$ s

For particle $0 = uT + \frac{1}{2}(-g)T^2$

i.e., $$\frac{1}{2}gT^2 = uT$$
$$T = \frac{2u}{g} = \frac{8}{3}$$

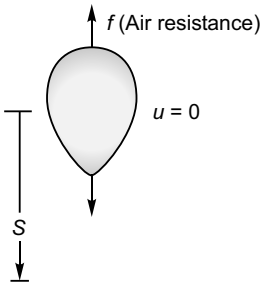
i.e., $$\frac{2u}{10} = \frac{8}{3}$$

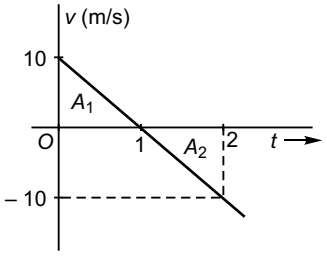
or $$u = \frac{40}{3}\text{ ms}^{-1}$$

Option (c) is correct.

23. $A_1 = \frac{1}{2}(+10)(+1) = +5$ m

$A_2 = \frac{1}{2}(-10)(+1) = -5$ m





$\therefore$ Displacement of particle at time ($t = 2$)
$$= (+5\text{ m}) + (-5\text{ m})$$
$$= 0\text{ m}.$$

i.e., the particle crosses its initial position at $t = 2$ s.

Option (b) is correct.

24. As downward direction is considered to be + ive, velocity of ball at $t = 0$ will be $-v_0$.

Thus, option (a) and (c) are incorrect.

Now, as the ball will have +ive acceleration throughout its motion option (b) is also incorrect.

$\therefore$ Correct option is (d).

25. Let acceleration of lift $= a$ (upwards)

Displacement of ball in time t

= Displacement of lift in time t

$$v_0 t - \frac{1}{2}gt^2 = \frac{1}{2}at^2$$

or $$v_0 - \frac{gt}{2} = \frac{at}{2}$$

$\therefore$ $$a = \frac{2v_0 - gt}{t}$$

Option (a) is correct.

26. $a = -0.2v^2$

$\therefore$ $$\frac{dv}{dt} = -0.2v^2$$

or $$\int v^{-2}dv = -0.2\int dt$$

or $$\frac{v^{-2+1}}{-2+1} = -0.2t + C$$

or $$-\frac{1}{v} = -0.2t + C$$

Now, at $t = 0, v = 10$ m/s (given)

$\therefore$ $$-\frac{1}{10} = 0 + C$$

i.e., $$C = -\frac{1}{10}$$

Thus, $-\frac{1}{v} = -0.2t - \frac{1}{10}$

$\because$ For velocity v at time t (= 2 s)

$$-\frac{1}{v} = -(0.2 \times 2) - \frac{1}{10}$$

$$-\frac{1}{v} = -\frac{4}{10} - \frac{1}{10}$$

$$\frac{1}{v} = \frac{1}{2}$$

$\Rightarrow$ $v = +2$ m/s.

Option (a) is correct.

27. For displacement (S_1) of train 1 before coming to rest

$$0^2 = (10)^2 + 2(-2)S_1$$

i.e., $S_1 = 25$ m

For displacement (S_2) of train 2 before coming to rest

$$0^2 = (20)^2 + 2(-1)S_2$$

i.e., $S_2 = 200$ m

$\therefore$ $S_{min} = S_1 + S_2$

$= 225$ m

Option (b) is correct.

28. Let the balls collide after time t the first ball is shot.

$\therefore$ displacement (S) of ball 1 at time t

= displacement (S) of ball 2 at time $(t-2)$

$$40t + \frac{1}{2}(-g)t^2 = 40(t-2) + \frac{1}{2}(-g)(t-2)^2$$

or $\frac{1}{2}g[t^2 - (t-2)^2] = 80$

or $(t + t - 2)(t - t + 2) = 16$

or $2t - 2 = 8$

or $t = 5$ s

$$S = 40 \times 5 + \frac{1}{2}(-10)5^2$$

$= 200 - 125$

$= 75$ m

Option (b) is correct.

29. $0 = u - gT$

i.e., $u = gT$ ...(i)

$H = uT - \frac{1}{2}gT^2$ ...(ii)

and $h = ut - \frac{1}{2}gt^2$ ...(iii)

Substracting Eq. (ii) from Eq. (iii),

$$h - H = u(t-T) + \frac{1}{2}g(T^2 - t^2)$$

$$= gT(t-T) - \frac{1}{2}g(t-T)(t+T)$$

$$= (t-T)g\left[T - \frac{1}{2}(t+T)\right]$$

$$= (t-T)g\left[\frac{2T - t - T}{2}\right]$$

$$= -\frac{1}{2}g(t-T)^2$$

i.e., $h = H - \frac{1}{2}g(t-T)^2$

Option (d) is correct.

30. $x = \frac{t^2}{2}$

$\therefore$ $v_x = \frac{dx}{dt} = t$

$$y = \frac{x^2}{2} = \frac{(t^2/2)^2}{2} = \frac{t^4}{8}$$

$\therefore$ $v_y = \frac{dy}{dt} = \frac{t^3}{2}$

31. $\sqrt{x} = t + 3$

i.e., $x = t^2 + 6t + 3$

$$\frac{dx}{dt} = 2t + 6$$

i.e., $v \propto t$

Option (b) is correct.

32. Aeroplane's velocity at time t (= 20 s)

= Area under curve (t-a graph)

$$= (20 \times 5) - \left(\frac{1}{2} \times 10 \times 2\right)$$

$= 90$ m/s

Option (c) is correct.

33. $v = 5\sqrt{1+5}$ ...(i)

At $s = 0$ (*i.e.*, initially),

velocity of particle = 5 m/s

Option (b) is correct.

Differentiating equation (i) w.r.t. s

$$\frac{dv}{ds} = 5\frac{1}{2}(1+5)^{-\frac{1}{2}}$$

$$v \cdot \frac{dv}{ds} = \frac{5}{2}(1+5)^{-\frac{1}{2}} \cdot 5(1+5)^{\frac{1}{2}}$$

$= 12.5$ m/s^2

Option (b) is correct.

34. See answer to question no. 2.

$$\frac{\text{Time of ascent}}{\text{Time of descent}} = \sqrt{\frac{mg - f}{mg + f}}$$

$$= \sqrt{\frac{mg - ma}{mg + ma}}$$

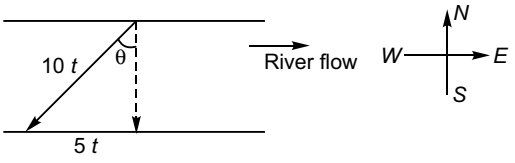
$$= \sqrt{\frac{g - a}{g + a}}$$

$$= \sqrt{\frac{10 - 2}{10 + 2}}$$

$$= \sqrt{\frac{2}{3}}$$

Option (b) is correct.

35.



$$\sin\theta = \frac{5t}{10t} = \frac{1}{2}$$

$\Rightarrow$ $\theta = 30°$

Option (b) is correct.

36. $F_{\text{net}} = 3t^2 - 32$

i.e., $ma_{\text{net}} = 3t^2 - 32$

or $a_{\text{net}} = \dfrac{3t^2 - 32}{10}$

or $a_{\text{net}} = 0.3t^2 - 3.2$

or $\dfrac{dv}{dt} = 0.3t^2 - 3.2$

or $\int dv = \int (0.3t^2 - 3.2)\, dt$

or $v = \dfrac{0.3t^3}{3} - (3.2)\, t + k$

or $v = 0.1t^3 - (3.2)\, t + k$

Now, at $t = 0, v = 10$ m/s

$\therefore$ $10 = k$

Thus, $v = 0.1\, t^3 - (3.2)\, t + 10$

$\therefore$ at $t = 5$ s

$$v = 0.1(5)^3 - (3.2)(5) + 10$$

$$= 12.5 - 16.0 + 10$$

$$= 6.5 \text{ m/s}$$

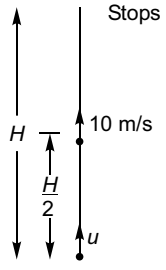
Option (b) is correct.

37. $u^2 = 2gH$

$\Rightarrow$ $H = \dfrac{u^2}{2g}$

$$10^2 = u^2 - 2g\frac{H}{2}$$

$\Rightarrow$ $u^2 = 100 + gH$



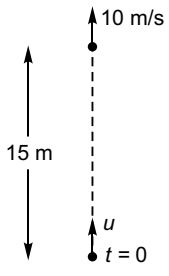
Thus, $H = \dfrac{100 + gH}{2g}$

or $gH = 100$

$\Rightarrow$ $H = 10$ m

Option (b) is correct.

38.



$$10^2 = u^2 - 2g \times 15$$

$$100 = v^2 - 2 \times 10 \times 15$$

i.e., $u = 20$ m/s

Now, $v = u - gt = 20 - 10 \times 3$

$$= 20 - 10 \cdot (3)$$

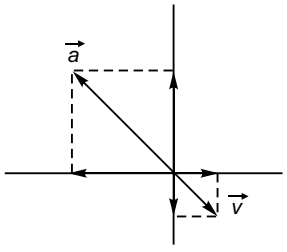
$$= -10 \text{ m/s, downward}$$

Option (d) is correct.

JEE Corner

Assertion and Reason

1. As acceleration ($\vec{\mathbf{a}}$) is just opposite to velocity ($\vec{\mathbf{v}}$), the particle will move along a straight line.



$\therefore$ Assertion is false (motion being one dimensional).

Reason is true as $\vec{\mathbf{v}}$ and $\vec{\mathbf{a}}$ do not depend upon time.

Option (d) is correct.

2. Displacement-time graph is parabolic only when the slope of the straight line velocity-time graph is not zero *i.e.,* acceleration is not zero.

If acceleration is zero.

$$s = ut$$

i.e., the displacement-time graph will be a straight line.

$\therefore$ Assertion is wrong.

Reason : $v = u + at$

i.e., $$\frac{ds}{dt} = u + at$$

or $$\int ds = \int (u + at)\, dt$$

$$s = ut + \frac{1}{2}at^2 + k$$

If at $t = 0$, $s = 0$

The value of k will be zero.

$\therefore$ $$s = ut + \frac{1}{2}at^2$$

Thus, reason is true.

3. Displacement in time t_0

$$= \text{Area under } v\text{-}t \text{ graph}$$

$$= \frac{1}{2}v_0 t_0$$

$\therefore$ average velocity in time interval t_0

$$= \frac{\text{Displacement}}{t_0}$$

$$= \frac{\frac{1}{2}v_0 t_0}{t_0} = \frac{v_0}{2}$$

$\therefore$ Assertion is true.

Reason is also true as proved.

4. Acceleration (a) will be zero only when velocity does not change with time

$$a = v \cdot \frac{dv}{ds}$$

$$= \frac{ds}{dt} \cdot \frac{dv}{ds}$$

$$= \frac{dv}{dt}$$

$\therefore a = 0$ if $\frac{dv}{dt} = 0$

or v is constant with time.

Thus, Assertion is wrong.

Reason is correct as a is equal to $\frac{dv}{dt}$ which is instantaneous acceleration.

5. If acceleration is in the opposite direction to the velocity of the particle the speed will decrease.

$$u = 10 \text{ m/s}$$

$$a = -2t \text{ m/s}^2$$

$$\frac{dv}{dt} = -2t$$

$$\int dv = \int -2t\, dt$$

$$v = -2 \cdot \frac{t^2}{2} + k$$

At $t = 0$ s, velocity $= +10$ m/s

$\therefore$ $k = +10$ m/s

$\therefore$ $v = -t^2 + 10$

Time (t)	v m/s	speed (m/s)
15	9	9
25	6	6
35	1	1

$\therefore$ Assertion is true.

Reason is false as when acceleration is positive the speed will increase.

6. $\frac{da}{dt} = 2\,(\text{ms}^{-3})$ (given)

[This implies that reason is true]

$$\int da = \int 2\,dt$$
$$a = 2t + C$$

If at $t = 0$, $a = 0$

we have $C = 0$

$\therefore$ $a = 2t$

or $\frac{dv}{dt} = 2t$

$\therefore$ Assertion is false.

Option (d) is correct.

7. If initially particle velocity is – ive and acceleration is uniform and + ive, the particle will return to its initial position after a certain time interval. In this time interval the average velocity will be zero as net displacement will be zero.

$\therefore$ Assertion is false.

For average velocity to zero, the particle must return to its initial position (as discussed above) and for this velocity can't remain constant.

$\therefore$ Reason is true.

Option (d) is correct.

8. From O to A velocity of the particle is increasing while from A to B it is decreasing without change in direction as for the velocity to change its direction the slope of s-t graph must be negative.

$\therefore$ Assertion is false.

If the slope of s-t graph is + ive the velocity of the particle will be + ive while if it is – ive the velocity of the particle will also be – ive.

$\therefore$ Reason is true.

9. $S_1 = 2t - 4t^2$

and $S_2 = -2t + 4t^2$

$\Rightarrow$ displacement of particle 2 w.r.t. 1.

$$S_{21} = S_2 - S_1$$
$$= (-2t + 4t^2) - (2t - 4t^2)$$
$$= -4t + 8t^2$$

Time t	Relative Displacement
0 s	0 m
1 s	4 m
2 s	24 m
3 s	60 m

As relative displacement is increasing the relative velocity would also be increasing.

$\therefore$ Assertion is false.

Reason is true.

10. If $v = u + (-a)\,t$ [acceleration being made – ive]

Velocity (v) of the particle will be zero

at $t = \frac{u}{a}$.

Thus, for $t < \frac{u}{a}$, v is + ive

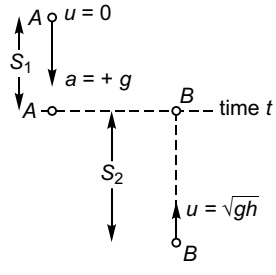
i.e., the acceleration can change its direction without change in direction of velocity.

$\therefore$ Assertion is true.

If ΔV changes sign say from + ive to – ive, the acceleration which equals $\frac{\Delta V}{\Delta t}$ will also change sign from + ive to – ive.

$\therefore$ Reason is true but it is not the correct explanation of the assertion.

11. At time t when the two are at the same height.



$$S_1 + S_2 = h$$
$$\left(\frac{1}{2}gt^2\right) + \left(ut - \frac{1}{2}gt^2\right) = h$$
$$ut = h$$
$$\sqrt{gh} \cdot t = h$$
$$t = \sqrt{\frac{h}{g}}$$

$$S_1 = \frac{1}{2}gt^2$$
$$= \frac{1}{2}g\frac{h}{g}$$
$$= \frac{h}{2}$$

$\therefore$ Assertion is true.

Reason is also true as assertion is based on this.

12. Please see answer to question no. 2 of objective questions (level 1).

$$T_2 = \frac{um}{\sqrt{(mg+f)(mg-f)}}$$

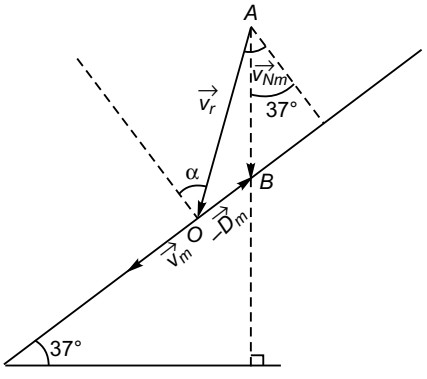
Now, if masses of the bodies are different, the value of T_2 (time to reach earth) will be different if f (air resistance) is same.

Thus, Assertion is false further Reason is correct as explained above.

Objective Questions (Level 2)

Single Correct Option

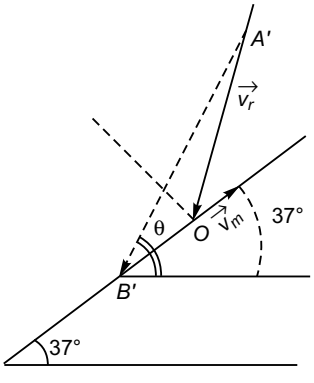
1. Applying sine formula in ΔAOB



$$\frac{AO}{\sin(96° + 37°)} = \frac{OB}{\sin(\alpha - 37°)}$$

$$\frac{v_r}{\cos 37°} = \frac{v_m}{\sin(\alpha - 37°)} \quad \ldots\text{(i)}$$

Applying sine formula in



$\Delta AOB'$

$$\frac{AO}{\sin(\theta - 37°)} = \frac{OB'}{\sin[(\alpha + \theta)]}$$

$$\frac{v_r}{\sin(\theta - 37°)} = \frac{v_m}{\sin[127° - (\alpha + \theta)]} \quad \ldots\text{(ii)}$$

$$\angle B'AO = 180° - (90° + \alpha) - (\theta - 37°)$$
$$= 90° - \alpha - \theta + 37°$$
$$= 127° - (\alpha + Q)$$

Comparing Eqs. (i) and (ii)

$$\frac{\sin(\alpha - 37°)}{\cos 37°} = \frac{\sin[127° - (\alpha + \theta)]}{\sin(\theta - 37°)}$$

$$\frac{\sin\alpha\cos 37° - \cos\alpha\sin 37°}{\cos 37°}$$
$$= \frac{\sin 127°\cos(\alpha + \theta) + \cos 127°\sin(\alpha + \theta)}{\sin\theta\cos 37° - \cos\theta\sin 37°}$$

$$\sin\alpha - \cos\alpha\tan 37°$$
$$= \frac{\frac{4}{5}\cos(\alpha + \theta) + \frac{3}{5}\sin(\alpha + \theta)}{\frac{4}{5}\sin\theta - \frac{3}{5}\cos\theta}$$

$$\text{or } \sin\alpha - \frac{3}{4}\cos\alpha = \frac{4\cos(\alpha + \theta) + 3\sin(\alpha + \theta)}{4\sin\theta - 3\cos\theta}$$

$$\text{or } \sin\alpha - \frac{3}{4}\cos\alpha$$
$$= \frac{4[\cos\alpha\cos\theta - \sin\alpha\sin\theta] + 3[\sin\alpha\cos\theta + \cos\alpha\sin\theta]}{4\sin\theta - 3\cos\theta}$$

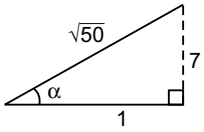
$$\text{or } \sin\alpha - \frac{3}{4}\cos\alpha$$
$$= \frac{4[\cos\alpha - \sin\alpha\tan\theta] + 3[\sin\alpha + \cos\alpha\tan\theta]}{4\tan\theta - 3}$$

$$\text{or } \sin\alpha - \frac{3}{4}\cos\alpha$$
$$= \frac{4[\cos\alpha - \frac{7}{8}\sin\alpha] + 3[\sin\alpha + \frac{7}{8}\cos\alpha]}{4\cdot\frac{7}{8} - 3}$$

$$\text{or } \sin\alpha - \frac{3}{4}\cos\alpha$$
$$= \frac{4[8\cos\alpha - 7\sin\alpha] + 3[8\sin\alpha + 7\cos\alpha]}{4}$$

or $4\sin\alpha - 3\cos\alpha$

$$= 32\cos\alpha - 28\sin\alpha + 24\sin\alpha + 21\cos\alpha$$



or $32\cos\alpha + 21\cos\alpha + 3\cos\alpha$

$$= 28\sin\alpha - 24\sin\alpha + 4\sin\alpha$$

or $$56\cos\alpha = 8\sin\alpha$$

$$\tan\alpha = \frac{7}{1}$$

From Eq. (i)

$$\frac{v_r}{\cos 37^\circ} = \frac{v_m}{\sin(\alpha - 37^\circ)}$$

$$\therefore \quad v_r = \frac{v_m \cos 37^\circ}{\sin\alpha\cos 37^\circ - \cos\alpha\sin 37^\circ}$$

$$= \frac{v_m \cos 37^\circ}{\cos\alpha\cos 37^\circ\,[\tan\alpha - \tan 37^\circ]}$$

$$= \frac{v_m}{\cos\alpha\left[7 - \frac{3}{4}\right]}$$

$$= \frac{4}{25}\cdot\frac{v_m}{\cos\alpha}$$

$$= \frac{4}{25}\times\frac{5}{1}\times\sqrt{50}$$

$$= \sqrt{32}$$

Option (b) is correct.

2. $$\frac{dv}{dt} = -4v + 8$$

$$a = -4v + 8$$

At the time the body acquires terminal speed its acceleration (a) must be zero.

Thus

$$-4v + 8 = 0$$

$$v = 2 \text{ m/s}$$

Terminal speed = 2 m/s.

Option (b) is correct.

3. Particles will collide if

$$\frac{\vec{\mathbf{v}}_1 - \vec{\mathbf{v}}_2}{|\vec{\mathbf{v}}_1 - \vec{\mathbf{v}}_2|} = \frac{\vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1}{|\vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1|}$$

$$\vec{\mathbf{v}}_1 - \vec{\mathbf{v}}_2 = (5\hat{\mathbf{i}} + 10\hat{\mathbf{j}} + 5\hat{\mathbf{k}}) - (10\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 5\hat{\mathbf{k}})$$

$$= -5\hat{\mathbf{i}} + 5\hat{\mathbf{j}}$$

$$|\vec{\mathbf{v}}_1 - \vec{\mathbf{v}}_2| = 5\sqrt{2}$$

$$\therefore \quad \frac{\vec{\mathbf{v}}_1 - \vec{\mathbf{v}}_2}{|\vec{\mathbf{v}}_1 - \vec{\mathbf{v}}_2|} = -\frac{1}{\sqrt{2}}\hat{\mathbf{i}} + \frac{1}{\sqrt{2}}\hat{\mathbf{j}}$$

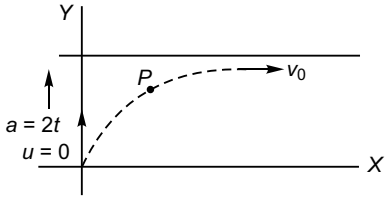
$$\vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1 = \vec{\mathbf{r}}_2 = 30\hat{\mathbf{i}}$$

$$|\vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1| = 30$$

$$\therefore \quad \frac{\vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1}{|\vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1|} = \hat{\mathbf{i}} \neq \frac{\vec{\mathbf{v}}_1 - \vec{\mathbf{v}}_2}{|\vec{\mathbf{v}}_1 - \vec{\mathbf{v}}_2|}$$

$\therefore$ Option (c) is correct.

4. Displacement along y-axis at time t



$$a = 2t$$

$$\frac{dv_y}{dt} = 2t$$

or $$\int dv_y = \int 2t\,dt$$

or $$v_y = t^2 + C_1$$

At $t = 0, v_y = 0$ (given)

$\therefore C_1 = 0$

$$\therefore \quad v_y = t^2$$

$$\Rightarrow \quad \frac{dy}{dt} = t^2$$

or $$\int dy = \int t^2 + C_2$$

or $$y = \frac{t^3}{2} + C_2$$

At $t = 0$, $Y = 0$ (given)

$$\therefore \quad Y = \frac{t^3}{3} \qquad \ldots\text{(i)}$$

Displacement along x-axis at time t :

$$x = v_0 t$$

or $$t = \frac{x}{v_0}$$

Substituting above value of t in Eq. (i)

$$Y = \frac{1}{3}\left(\frac{x}{v_0}\right)^3$$

Option (a) is correct.

5. $t = \alpha x^2 + \beta x$

Differentiating w.r.t. time t

$$1 = \alpha \cdot 2x \frac{dx}{dt} + \beta \cdot \frac{dx}{dt}$$

or $$v(\beta + 2\alpha \cdot x) = 1$$

or $$v = (\beta + 2\alpha \cdot x)^{-1}$$

$\therefore$ $$\frac{dv}{dt} = -\frac{1}{(\beta + 2\alpha\, x)^2} \cdot 2\alpha \cdot \frac{dx}{dt}$$

$$a = -\frac{2\alpha}{(\beta + 2\alpha\, x)^2} \cdot v$$

$$= -\frac{2\alpha}{(\beta + 2\alpha\, x)^2} \cdot v$$

$$= -(2\alpha\, v^2) \cdot v$$

$$= -2\alpha\, v^3$$

$\therefore$ Retardation $= 2\alpha\, v^3$

Option (a) is correct.

6. $f = a - bx$

$\therefore$ $$v \frac{dv}{dx} = a - bx$$

or $$\int v\, dv = \int (a - bx)\, dx$$

or $$\frac{v^2}{2} = a \cdot x - \frac{bx^2}{2} + C$$

As at $x = 0$ car is at rest $C = 0$

$\therefore$ $$\frac{v^2}{2} = ax - \frac{bx^2}{2} \quad \ldots(i)$$

For v to be maximum.

$$\frac{d}{dx}\left(ax - \frac{bx^2}{2}\right) = 0$$

i.e., $$a - bx = 0$$

or $$x = \frac{a}{b}$$

Substituting $x = \frac{a}{b}$ in Eq. (i),

$$\frac{v_{\max}^2}{2} = a \cdot \frac{a}{b} - \frac{b}{2}\left(\frac{a}{b}\right)^2$$

$$= \frac{a^2}{2b}$$

$\therefore$ $$v_{\max} = \frac{a}{\sqrt{b}}$$

Car will come to rest when

$$ax - \frac{bx^2}{2} = 0$$

$$x = \frac{2a}{b}$$

$\therefore$ Distance between two stations $= \frac{2a}{b}$

Option (a) is correct.

7. Force $= -kx^2$

$\therefore$ Acceleration $= -\frac{k^2}{2^m} x^2$

$\Rightarrow$ $$v \frac{dv}{dx} = -\frac{kx^2}{m}$$

or $$\int v\, dv = \int -\frac{kx^2}{m}\, dx$$

or $$\frac{v^2}{2} = -\frac{k}{m}\frac{x^3}{3} + C$$

Now at $x = a, v = 0$

$\therefore$ $$C = \frac{k}{m}\frac{a^3}{3}$$

Thus $$\frac{v^2}{2} = -\frac{kx^2}{m \times 3} + \frac{ka^3}{m \times 3}$$

$\therefore$ Velocity at $x = 0$

$$v = \sqrt{\frac{2\,ka^3}{3m}}$$

Option (d) is correct.

8. $v_x = 4 + 4t$ and $v_y = 4t$

$\therefore \frac{dx}{dt} = 4 + 4t$ and $\frac{dy}{dt} = 4t$

or $$\int dx = \int (4 + 4t)\, dt$$

and $$\int dy = \int 4t\, dt$$

or $$x = 4t + 2t^2 + C_1$$

$\therefore$ $$C_1 = 1$$

Thus $$x = 4t + 2t^2 + 1 \quad \ldots(i)$$

and $$y = 2t^2 + C_2$$

At $$t = 0\; x = 1$$

and at $t = 0$, $y = 2$

$\therefore$ $$C_2 = 0$$

Thus $$y = 2t^2 + 2 \quad \ldots(ii)$$

Substituting value of t in Eq. (i) in terms of y using Eq. (ii) we won't get relationship as mentioned in option (a), (b) or (c).

$\therefore$ Option (d) is correct.

9. $$a = \frac{1}{2} gt^2$$

$$= 4 \times \tan 120^\circ$$

$$= 4 \times [-\tan 60^\circ]$$

$$= -4\sqrt{3}\ \text{m/s}^2$$

$\therefore$ Magnitude of acceleration $= 4\sqrt{3}\ \text{ms}^{-2}$.

Option (a) is correct.

10. $$x_1 = \frac{1}{2}gt^2$$

$$x_1 + x_2 = \frac{1}{2}g(2t)^2$$

$$\therefore \quad x_2 = \frac{1}{2}g \cdot 4t^2 - \frac{1}{2}gt^2$$

$$x_2 = \frac{3}{2}gt^2$$

$$x_2 - x_1 = gt^2$$

$$\Rightarrow \quad t = \sqrt{\frac{x_2 - x_1}{g}}$$

Option (a) is correct.

11. $$y^2 + x^2 = l^2$$

$$\therefore \quad 2y \cdot \frac{dy}{dt} + 2x \cdot \frac{dx}{dt} = 0$$

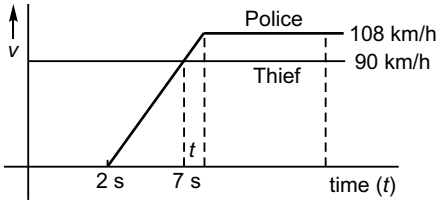
or $$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

$$= -\frac{x}{x\tan 30^\circ}\frac{dx}{dt}$$

$$= -2\sqrt{3} \text{ m/s}$$

Option (c) is correct.

12. Maximum separation (x_{max}) between the police and the thief will be at time t (shown in figure)



$$t - 2 = \frac{v}{a} = \frac{90 \text{ kmh}^{-1}}{5 \text{ ms}^{-2}}$$

$$= \frac{90}{5} \times \frac{1000}{3600} \text{ s}$$

i.e., $$t = 7 \text{ s}$$

$$x_{max} = [90 \text{ kmh}^{-1} \times 7 \text{ s}] - \frac{1}{2} \times 5 \text{ s} \times 90 \text{ kmh}^{-1}$$

$$= \frac{90 \times 1000}{3600} \times \frac{9}{2} \text{ m}$$

$$= 112.5 \text{ m}$$

Option (a) is correct.

13. If meeting time is t

$$\frac{2v}{3}\sin\theta = u\sin 30^\circ$$

i.e., $$\sin\theta = \frac{3}{4}$$

or $$\theta = \sin^{-1}\frac{3}{4}$$

Option (c) is correct.

14. Distance $= \frac{1}{2}\left(\frac{\alpha\beta}{\alpha+\beta}\right)t^2$

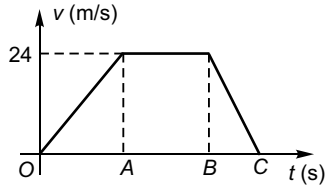
$$\therefore \quad 0 = \frac{1}{2}\left(\frac{1 \times 4}{1+4}\right)t^2$$

$$\Rightarrow \quad t = \sqrt{500} \text{ s}$$

$$= 22.36 \text{ s}$$

Option (a) is correct.

15. Let $BC = t'$



$$\therefore \quad \frac{24}{t'} = 4$$

$$\Rightarrow \quad t' = 6 \text{ s}$$

$$\therefore \quad OB = 50 \text{ s}$$

Let $OA = t$

$$\therefore \quad AB = 56 - t$$

$$1032 = \frac{1}{2}[56 + (56 - t)] \times 24$$

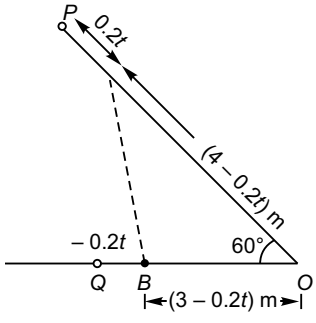
$$\Rightarrow \quad t = 20 \text{ s}$$

$\therefore$ maximum acceleration $= \frac{24 \text{ m/s}}{20 \text{ s}}$

$$= 1.2 \text{ m/s}^2$$

Option (b) is correct.

16. From ΔOAB



$$\cos AOB = \frac{OA^2 + OB^2 - AB^2}{2 \cdot OA \cdot OB}$$

$\therefore \quad OA^2 + OB^2 - AB^2 = OA \cdot OB$

i.e., $\quad AB^2 = OA^2 + OB^2 - OA \cdot OB$

$= (4 - 0.2t)^2 + (3 - 0.2t)^2 + (4 - 0.2t)(3 - 0.2t)$

or $\quad AB^2 = 16 + 0.04t^2 - 1.6t + 9 + 0.04t^2 - 1.2t + 12 - 1.4t + 0.04t^2$

or $\quad AB^2 = 0.12t^2 - 4.2t + 37$

For AB to be minimum

$$0.24t - 4.2 = 0$$

or $\quad t = 17.5$ s

$\therefore \quad (AB)^2_{\min} = 0.12(17.5)^2 - 4.2 \times (17.5) + 37$

$= 36.75 - 73.5 + 37$

$(AB)^2_{\min} = 0.75 \text{ m}^2$

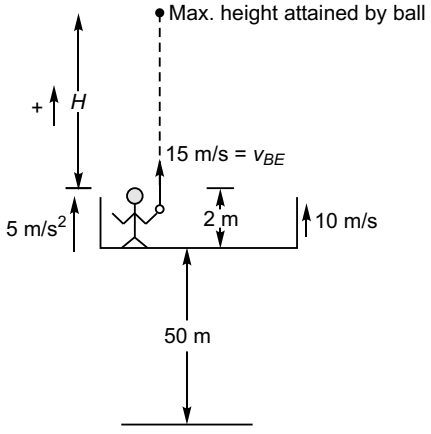
$= 7500 \text{ cm}^2$

$= 50\sqrt{3}$ cm

Option (d) is correct.

17. Velocity of ball w.r.t. elevator = 15 m/s

Acceleration of ball w.r.t. elevator



$= (-10) - (+5) = -15 \text{ m/s}^2$

Final displacement of ball w.r.t. elevator $= -2$ m

$\therefore \quad -2 = 15t + \frac{1}{2}(-15)t^2$

i.e., $\quad 15t^2 - 30t - 4 = 0$

$\therefore \quad t = \frac{30 + \sqrt{900 + 4 \times 4 \times 15}}{2 \times 15}$

$= 2.13$ s

Option (a) is correct.

18. Velocity of ball w.r.t. ground (v_{BE})

$= (15 + 10)$ m/s

$= 25$ m/s

Now $\quad v^2 = u^2 + 2as$

$\therefore \quad 0^2 = (25)^2 + 2(-10)H$

or $\quad H = 31.25$ m

Maximum height by ball as measured from ground $= 31.25 + 2 + 50$

$= 83.25$ m

Option (c) is correct.

19. Displacement of ball w.r.t. ground during its flight $= H = 31.25$ m

Option (d) is correct.

20. Displacement of ball w.r.t. floor of elevator at time t

$$s = 15t + \frac{1}{2}(-15)t^2 + 2$$

s will be maximum, when

$$\frac{ds}{dt} = 0$$

i.e., $\quad 15 - 15t = 0$

i.e., $\quad t = 1$s

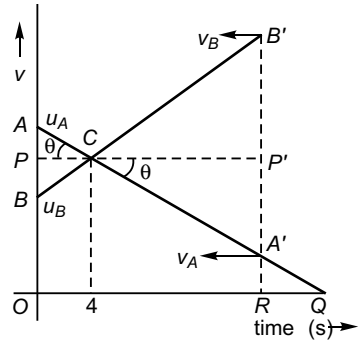
$\therefore s_{\max} = (15 \times 1) + \frac{1}{2}(-10)(1)^2 + 2$

$= 12$ m

Option (a) is correct.

21. Let the particles meet at time t

i.e., displacement of the particles are equal and which is possible when



Area of ΔACB = Area of $\Delta A'B'C$

[Area $OBCA'\theta O$ being common]

or $\quad$ Area of ΔAPC = Area $A'P'C$

$$\frac{1}{2} AP \times PC = \frac{1}{2} A'P' \times P'C$$

or $\quad AP \times PC = A'P' \times P'C$

or $\quad (PC \tan\theta) PC = (P'C \tan\theta) P'C$

or $\quad PC = P'C'$

$\therefore \quad OR = PC + P'C' = 2PC = 8$ s.

Option (c) is correct.

22. Area of $\Delta A'B'O$ = Area of ΔABO

$$\therefore \quad \frac{1}{2}(v_B - v_A)\cdot 4 = \frac{1}{2}(u_A - u_B)\cdot 4$$

i.e., $\quad v_B - v_A = u_A - u_B$

$$= 5 - 15 = -10$$

$$\Rightarrow \quad v_A - v_B = 10 \text{ ms}^{-1}$$

Option (b) is correct.

23. $u_A = 6 \text{ ms}^{-1}, u_B = 12 \text{ ms}^{-1}$

and at $t = 4$ s common velocity = 8 ms^{-1}

To find velocity of A at $t = 10$ s

$$\frac{v_A - 8}{10 - 4} = \frac{u_A - 8}{4}$$

or $$\frac{v_A - 8}{6} = \frac{6 - 8}{4}$$

$$\therefore \quad v_A = 5 \text{ ms}^{-1}$$

Option (d) is correct.

More than One Correct Options

1. $\because \quad a = -\alpha v^{1/2} \quad \ldots$(i)

i.e., $$\frac{dv}{dt} = -\alpha v^{1/2}$$

or $$\int v^{-1/2}\, dv = -\alpha \int dt$$

$$2v^{1/2} = -\alpha t + C_1$$

Now, at $t = 0, v = v_0$

$$\therefore \quad C_1 = 2v_0^{1/2}$$

$$\Rightarrow \quad 2v^{1/2} = -\alpha t + 2v_0^{1/2}$$

i.e., the particle will stop at

$$t = \frac{2v_0^{1/2}}{\alpha}$$

Option (a) is correct.

Option (b) is incorrect.

From Eq. (i),

$$v\frac{dv}{dt} = -\alpha v^{1/2}$$

or $$\int v^{1/2}\, dv = -\alpha \int dx$$

or $$\frac{2}{3}v^{3/2} = -\alpha + C_2$$

As at $x = 0, v = v_0, C_2 = \frac{2}{3}v_0^{3/2}$

$$\therefore \quad \frac{2}{3}v^{3/2} = -\alpha x + \frac{2}{3}v_0^{3/2}$$

i.e., when the particle stops

$$x = \frac{2}{3\alpha}v_0^{3/2}$$

Option (d) is correct.

Option (c) is incorrect.

2. $a = -0.5\,t$ (m/s^2)

$$\frac{dv}{dt} = -\frac{t}{2}$$

or $$\int dv = -\frac{1}{2}\int t\, dt$$

$$v = -\frac{t^2}{4} + C_1$$

At $t = 0, v = 16$ m/s

$$\therefore \quad v = -\frac{t^2}{4} + 16 \quad \ldots\text{(i)}$$

From above relation v is zero at

$$t = 8 \text{ s}$$

Options (a) is correct.

From relation (i),

$$\frac{ds}{dt} = -\frac{t^2}{4} + 16$$

$$\therefore \quad \int ds = \int\left(-\frac{t^2}{4} + 16\right)dt$$

i.e., $$s = -\frac{t^3}{12} + 16t + C_2$$

$$s = -\frac{t^3}{12} + 16t \quad \text{[As at } t = 0, s = 0\text{]}$$

At $t = 4$ s

$$s = 58.67 \text{ m}$$

Option (b) is correct.

The particle returns back at $t = 8$ s

From relation (ii)

$$s_8 = -\frac{8^3}{12} + (16 \times 8) = 85.33 \text{ m}$$

$$s_{10} = -\frac{10^3}{12} + (16 \times 10) = 76.66 \text{ m}$$

Distance travelled in 10 s

$$= s_8 + (s_8 - s_{10})$$

$$= (2 \times 85.33) - 76.66$$

$$= 94 \text{ m}$$

Option (c) is correct.

Velocity of particle at $t = 10$ s

$$v_{10} = -\frac{(10)^2}{4} + 16$$
$$= -25 + 16$$
$$= -9 \text{ m/s}$$

$\therefore$ Speed of particle at $t = 10$ s is 9 m/s.

Option (d) is correct.

3. $|\vec{\mathbf{v}}|$ is scalar.

$\therefore$ Option (a) is incorrect.

$$\frac{d\vec{\mathbf{v}}}{dt} = \vec{\mathbf{a}} \qquad \text{(by definition).}$$

$\therefore$ Option (b) is correct.

v^2 is scalar.

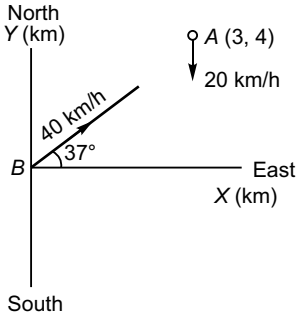
$\therefore$ Option (c) is incorrect.

$$\frac{\vec{\mathbf{v}}}{|\vec{\mathbf{v}}|} \text{ is } \hat{\mathbf{v}} \text{ (unit vector)}$$

$$\therefore \qquad \frac{d\hat{\mathbf{v}}}{dt} = \hat{\mathbf{a}} \neq \vec{\mathbf{a}}$$

$\therefore$ Option (d) is incorrect.

4.



$$\vec{\mathbf{v}}_A = -20\,\hat{\mathbf{j}} \text{ kmh}^{-1}$$
$$\vec{\mathbf{v}}_B = (40\cos 37°)\,\hat{\mathbf{i}} + (40\sin 37°)\,\hat{\mathbf{j}} \text{ kmh}^{-1}$$
$$= (32\,\hat{\mathbf{i}} + 24\,\hat{\mathbf{j}}) \text{ kmh}^{-1}$$
$$\vec{\mathbf{v}}_{AB} = \vec{\mathbf{v}}_A - \vec{\mathbf{v}}_B$$
$$= (-20\,\hat{\mathbf{j}}) - (32\,\hat{\mathbf{i}} + 24\,\hat{\mathbf{j}})$$
$$= -32\,\hat{\mathbf{i}} - 44\,\hat{\mathbf{j}}$$

Option (a) is correct.

Option (c) is incorrect.

At any time

$$\vec{\mathbf{S}}_A = 3\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}} + (-20\,\hat{\mathbf{j}})\,t$$
$$\vec{\mathbf{S}}_B = (32\,\hat{\mathbf{i}} + 24\,\hat{\mathbf{j}})\,t$$

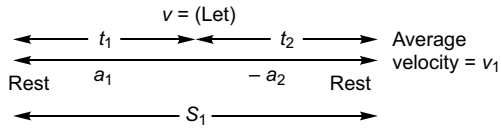
Position of A relative to B :

$$\vec{\mathbf{S}}_{AB} = \vec{\mathbf{S}}_A - \vec{\mathbf{S}}_B$$
$$= (3\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}} - 20t\,\hat{\mathbf{j}}) - (32t\,\hat{\mathbf{i}} + 24t\,\hat{\mathbf{j}})$$
$$= (3 - 32t)\,\hat{\mathbf{i}} + (4 - 44t)\,\hat{\mathbf{j}}$$

Option (b) is correct.

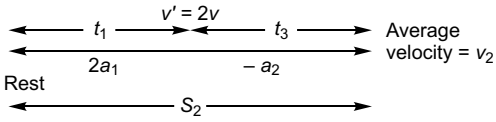
Option (d) is incorrect.

5. Case I.



$$v = a_1 t_1$$
$$v = a_2 t_2$$
$$s_1 = \frac{1}{2} a_1 t_1^2 + \frac{1}{2} a_2 t_2^2$$

Case II.



$$v' = 2a_1 t_1$$
$$= 2v$$
$$2v = a_2 t_3$$
$$2(a_2 t_2) = a_2 t_3$$
$$\Rightarrow \qquad t_3 = 2t_2$$
$$s_2 = \frac{1}{2}(2a_1)\,t_1^2 + \frac{1}{2} a_2 t_3^2$$
$$= a_1 t_1^2 + \frac{1}{2} a_2 (2\,t_2)^2$$
$$= a_1 t_1^2 + 2\,a_2 t_2^2$$
$$2s_2 = a_1 t_1^2 + a_2 t_2^2$$
$$\therefore \qquad s_1 > 2s_1 \qquad \text{...(i)}$$
$$4s_1 = 2a_1 t_1^2 + 2a_2 t_2^2$$
$$\therefore \qquad s_2 < 4s_1 \qquad \text{...(ii)}$$

Combining Eqs. (i) and (ii),

$$2s_1 < s_2 < 4s_1$$

Option (d) is correct.

In Case I. $v_{\text{av}} = \dfrac{s_1}{t_1 + t_2}$

i.e., $$v_1 = \frac{\frac{1}{2} a_1 t_1^2 + \frac{1}{2} a_2 t_2^2}{t_1 + t_2}$$

$$= \frac{(a_1t_1)t_1 + (a_2t_2)t_2}{2(t_1 + t_2)}$$

$$= \frac{vt_1 + vt_2}{2(t_1 + t_2)} = \frac{v}{2}$$

In Case II.

$$v_{av} = \frac{s_2}{t_1 + t_3}$$

i.e., $$v_2 = \frac{a_1t_1^2 + 2a_2t_2^2}{t_1 + t_3}$$

$$= \frac{(a_1t_1)t_1 + a_2t_2(2t_2)}{t_1 + 2t_2}$$

$$= \frac{vt_1 + v(2t_2)}{t_1 + 2t_2}$$

$$= v = 2v_1$$

Option (a) is correct.

Option (b) is incorrect.

6. If the particle's initial velocity is + ive and has some constant – ive acceleration the particle will stop somewhere and then return back to have zero displacement at same time $t\,(>0)$.

Also if particle's initial velocity is – ive and has some constant + ive acceleration the particle will stop somewhere and then return back to have zero displacement at same time $t\,(>0)$.

$\therefore$ Options (b) and (c) are correct.

and options (a) and (d) are incorrect.

7. $F = \alpha t$

$\Rightarrow$ $$ma = \alpha t$$

or $$a = \frac{\alpha}{m} t \quad \ldots(i)$$

$\therefore$ Graph between a (acceleration) and time (t) will be as curve 1.

$\therefore$ Option (a) is correct.

From equation

$$\frac{dv}{dt} = \frac{\alpha}{m} \cdot t$$

$\therefore$ $$\int dv = \frac{\alpha}{m} \int t\, dt$$

i.e., $$v = \frac{\alpha}{m} \cdot \frac{t^2}{2} + c$$

$\therefore$ Graph between velocity (v) and time (t) will be as curve 2.

Option (b) is correct.

8. $$a = -\frac{6}{30} s + 6$$

$$5a = -s + 30$$

$$5 \cdot v \frac{dv}{ds} = -s + 30$$

or $$\int 5v\, dv = \int (-s + 30)\, ds$$

or $$\frac{5v^2}{2} = -\frac{s^2}{2} + 30s + c_1$$

or $$\frac{5}{2} v^2 = -\frac{S^2}{2} + 30s \quad \ldots(i)$$

[$C_1 = 0$ as at $s = 0$ particle is at rest]

Substituting $s = 10$ m in Eq. (i),

$$\frac{5}{2} v^2 = -\frac{(10)^2}{2} + 300$$

i.e., $$v = 10 \text{ m/s}$$

$\therefore$ Option (b) is correct.

From Eq. (i) v to be maximum

$$-s + 30 = 0$$

$$s = 30$$

$\therefore$ $$\frac{5}{2} v_{max}^2 = -\frac{(30)^2}{2} + (30 \times 30)$$

or $$\frac{5}{2} v_{max}^2 = 450$$

or $$v_{max} = \sqrt{180} \text{ m/s}$$

Option (c) is correct.

9. If particle's path is

(i) straight with backward motion

(ii) not straight somewhere.

Distance moved will be greater than the modulus of displacement

$\therefore$ $$|\vec{\mathbf{v}}_{av}| < v_{av}$$

Option (a) is correct.

If particle returns to its initial position, the value of $\vec{\mathbf{v}}_{av}$ will be zero while its average speed (v_{av}) will not be zero.

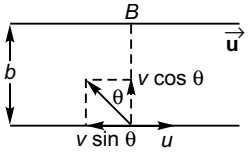
$\therefore$ Option (c) is correct.

10. If $u = 0, v = at$ and $s = \frac{1}{2} at^2$

$\therefore$ v-t graph will be as shown in (a).

s-t graph will be as shown in (d).

11. Swimmer will reach point B



if $\quad v\sin\theta = u$

i.e., $\quad v > u$

Option (c) is correct.

Time to cross the river

$$t = \frac{b}{v\cos\theta} = \frac{b}{\sqrt{v^2 - u^2}} \qquad [\text{as } v\sin\theta = u]$$

Option (b) is correct.

$$t_{\min} = \frac{b}{v(\cos\theta)_{\max}} = \frac{b}{v}$$

Option (a) is correct.

12. At time T, particle's velocity change from – –ive to + ive.

Option (a) is correct.

As slope v-t is same throughout, particle's acceleration is constant.

Option (b) is correct.

As net area under the curve is zero.

$\left[A_{\text{net}} = \left(\frac{vT}{2}\right) + \left(\frac{-vT}{2}\right)\right]$, displacement of particle at time $2T$ is zero.

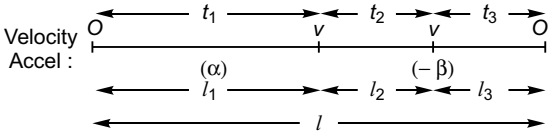
Option (c) is correct.

Initial speed $= |\vec{\mathbf{v}}| = v$

Final speed $= |\vec{\mathbf{v}}| = v$

Option (d) is correct.

13.



$$v = \alpha t_1 \qquad v = \beta t_3$$

$$l_1 = \frac{1}{2}\alpha t_1^2,\ l_2 = vt_2,\quad l_3 = \frac{1}{2}\beta t_3^2$$

$$= \frac{1}{2}vt_1 = \frac{1}{2}vt_3$$

$$t = t_1 + t_2 + t_3$$

$$= \frac{2l_1}{v} + \frac{l_2}{v} + \frac{2l_3}{v}$$

$$= \frac{l_1}{v} + \frac{l_2}{v} + \frac{l_3}{v} + \frac{l_1}{v} + \frac{l_3}{v}$$

$$= \frac{l}{v} + \frac{1}{v}[l_1 + l_2]$$

$$= \frac{l}{v} + \frac{1}{v}\left[\frac{1}{2}\alpha t_1^2 + \frac{1}{2}\beta t_2^2\right]$$

$$= \frac{l}{v} + v\left[\frac{1}{2}\frac{\alpha t_1^2}{v^2} + \frac{1}{2}\frac{\beta t_2^2}{v^2}\right]$$

$$= \frac{l}{v} + v\left[\frac{1}{2}\frac{\alpha t_1^2}{\alpha^2 t_1^2} + \frac{1}{2}\frac{\beta t_2^2}{\beta^2 t_2^2}\right]$$

$$= \frac{l}{v} + \frac{v}{2}\left[\frac{1}{\alpha} + \frac{1}{\beta}\right]$$

Option (b) is correct.

For t to be minimum.

$$\frac{d}{dv}\left[\frac{l}{v} + \frac{v}{2}\left\{\frac{1}{\alpha} + \frac{1}{\beta}\right\}\right] = 0$$

$$-\frac{l}{v_2} + \frac{1}{2}\left\{\frac{1}{\alpha} + \frac{1}{\beta}\right\} = 0$$

or $\quad \dfrac{l}{v^2} = \dfrac{\alpha + \beta}{2\alpha\beta}$

or $\quad v = \sqrt{\dfrac{2l\alpha\beta}{\alpha + \beta}}$

Option (d) is correct.

14. $x = t^2$ and $y = t^3 - 2t$

$$\frac{dx}{dt} = 2t \quad \text{and} \quad \frac{dy}{dt} = 3t^2 - 2$$

At $t = 0$, $\dfrac{dx}{dt} = 0$ but $\dfrac{dy}{dt} = -2$

$\therefore$ at $t = 0$ particle is moving parallel to y-axis.

Option (a) is correct.

Option (d) is incorrect.

At $t = 0$, $\quad \vec{\mathbf{v}} = -2\hat{\mathbf{j}}$

$$\frac{d^2x}{dt^2} = 2$$

and $\quad \dfrac{d^2y}{dt^2} = 6t$

At $t = 0$, $\quad \vec{\mathbf{a}} = 2\hat{\mathbf{i}}$

i.e., $\quad \vec{\mathbf{v}} \perp \vec{\mathbf{a}}$

Option (b) is correct.

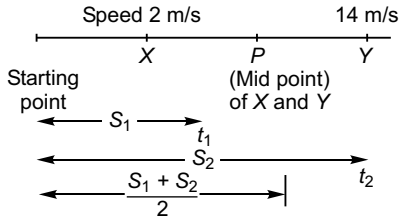
At $t = \sqrt{\frac{2}{3}}$:

$$\frac{dx}{dt} = 2\sqrt{\frac{2}{3}} \text{ but } \frac{dy}{dt} = 3\left(\frac{2}{3}\right) - 2$$

$\therefore$ Particle is moving parallel to x-axis.

Option (c) is correct.

15.



$$at_1 = 2 \text{ and } at_2 = 14$$

$$\therefore \quad t_2 = 7t_1$$

$$S_1 + S_2 = \frac{1}{2}at_1^2 + \frac{1}{2}at_2^2$$

$$= \frac{1}{2}a(t_1^2 + t_2^2)$$

$$= \frac{1}{2}a[t_1^2 + (7t_1)^2]$$

$$= 25a\,t_1^2$$

Speed (v) at $\frac{S_1 + S_2}{2}$:

$$v^2 = 2a\left(\frac{S_1 + S_2}{2}\right)$$

$$= a(S_1 + S_1)$$

$$= a(25a\,t_1^2)$$

$$\therefore \quad v = 5a\,t_1$$

or $\quad v = 10$ m/s

Option (a) is correct.

Time t to reach P from T

$$10 = 2 + at$$

$$\Rightarrow \quad t = \frac{8}{a} \quad \ldots(i)$$

Time t' to reach Y from P

$$14 = 10 + at'$$

$$\therefore \quad t' = \frac{4}{a} \quad \ldots(ii)$$

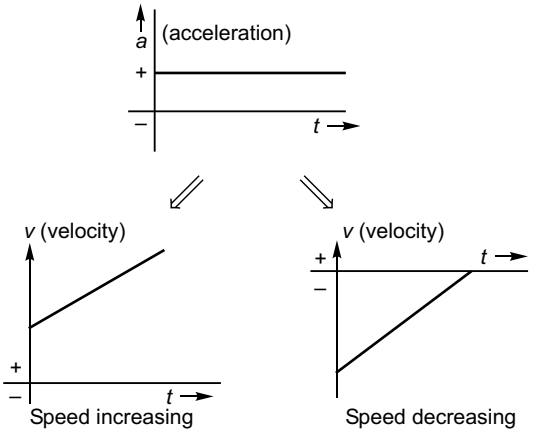
Comparing Eq. (i) and Eq. (ii),

$$t = 2t'$$

Option (c) is correct.

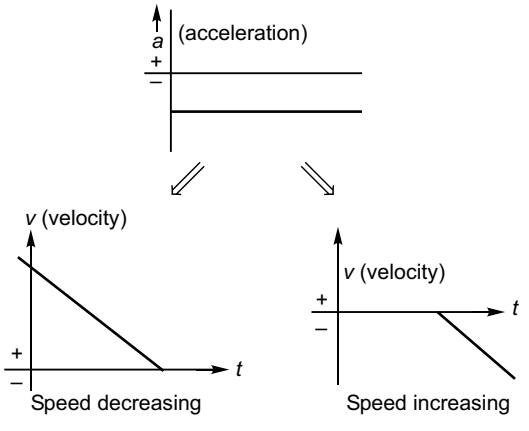
Match the Columns

1. (a)



(a) (a) → (r) (s).

(b)



(b) (b) → (r) (s).

(c) $s = kt^2$

$$v = \frac{ds}{dt} = 2kt$$

$\therefore v$ must be increasing with time.

$\Rightarrow$ speed must also be increasing with time.

(c) $\rightarrow$ (p).

(d) Slope is + ive and decreasing.

$\therefore$ Velocity must be decreasing with time.

$\Rightarrow$ speed must also be decreasing with time.

(d) $\rightarrow$ (q).

2. (a) $\vec{\mathbf{v}} = -2\hat{\mathbf{i}}$

$\therefore$ Initial speed $= |\vec{\mathbf{v}}| = 2$

$$\vec{\mathbf{a}} = -4\hat{\mathbf{j}}$$

Velocity ($\vec{\omega}$) at time t

$$\vec{\omega} = \vec{\mathbf{v}} + \vec{\mathbf{a}}t$$

$$= -2\hat{\mathbf{i}} - 4t\hat{\mathbf{j}}$$

Speed at time $t = |\vec{\omega}| = \sqrt{(-2)^2 + (-4t)^2}$

At $t > 0$

$$|\vec{\omega}| > |\vec{\mathbf{v}}|$$

Speed increasing.

$\therefore$ (a) $\rightarrow$ (p).

(b) $\vec{\mathbf{v}} = 2\hat{\mathbf{i}}$

$$|\vec{\mathbf{v}}| = 2$$

$$\vec{\mathbf{a}} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$$

Using $\vec{\omega} = \vec{\mathbf{v}} + \vec{\mathbf{a}}t$

$\therefore$ $\vec{\omega} = 2\hat{\mathbf{i}} + 2t\hat{\mathbf{i}} + 2t\hat{\mathbf{j}}$

$\therefore$ $|\vec{\omega}| = \sqrt{(2+2t)^2 + (2t)^2}$

As $t > 0$, $|\vec{\omega}| > |\vec{\mathbf{v}}|$

(a) $\rightarrow$ (p).

(c) $\vec{\mathbf{v}} = -2\hat{\mathbf{i}}$ and $\vec{\mathbf{a}} = +2\hat{\mathbf{i}}$

Using $\vec{\omega} = \vec{\mathbf{v}} + \vec{\mathbf{a}}t$

$$\vec{\omega} = -2\hat{\mathbf{i}} + 2t\hat{\mathbf{i}}$$

or $\vec{\omega} = (2t-2)\hat{\mathbf{i}}$

$\therefore$ $|\vec{\omega}| = |2t-2|$

At $t = 0$ s Speed $= |\vec{\omega}| = 2$ m/s

$t = 1$ s $|\vec{\omega}| = 0$ m/s

$t = 2$ s $|\vec{\omega}| = 2$ m/s

$t = 3$ s $|\vec{\omega}| = 4$ m/s

$\therefore$ (c) $\rightarrow$ (s).

(d) $\vec{\mathbf{v}} = 2\hat{\mathbf{i}}$ and $\vec{\mathbf{a}} = -2\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$

$$\vec{\omega} = \vec{\mathbf{v}} + \vec{\mathbf{a}}t$$

$$= 2\hat{\mathbf{i}} - 2t\hat{\mathbf{i}} + 2t\hat{\mathbf{j}}$$

$$= (2-2t)\hat{\mathbf{i}} + 2t\hat{\mathbf{j}}$$

$|\vec{\omega}| = \sqrt{(2-2t)^2 + (2t)^2}$

At $t = 0$ $|\vec{\omega}| = 2$

$t = \frac{1}{2}$ $|\vec{\omega}| = \sqrt{2}$

$t = 1$ $|\vec{\omega}| = 2$

$t = 2$ $|\vec{\omega}| = \sqrt{20}$

$\therefore$ (d) $\rightarrow$ (s).

3. (a) From A to B, v is increasing and area is + ive.

$\therefore$ (a) $\rightarrow$ (p).

(b) From B to C, v is increasing and area is + ive.

$\therefore$ (b) $\rightarrow$ (p).

(c) From C to D, v is decreasing while area is + ive.

$\therefore$ (c) $\rightarrow$ (q).

(d) From D to E, v is – ive and $|\vec{\mathbf{v}}|$ is increasing, area is –ive.

$\therefore$ (d) $\rightarrow$ (r).

4. (a) Displacement = Area

$$= \frac{10 \times 4}{2} = 20 \text{ unit}$$

$\therefore$ Average velocity $= +\frac{20}{4} = +5$ unit

i.e., (a) $\rightarrow$ (r).

(b) Displacement $= +20 - \frac{1 \times 5}{2} = +17.5$ unit

$\therefore$ Average velocity $= \frac{+17.5 \text{ unit}}{3}$

$= 5.83$ unit

$\therefore$ (b) $\rightarrow$ (s).

(c) Displacement $= +20 + \left(\frac{2 \times (-10)}{2}\right)$

$= +10$ unit

$\therefore$ Average velocity $= \frac{+10}{6}$

i.e., (c) $\rightarrow$ (s).

(d) Rate of change of velocity at $t = 4$ s

$$\vec{\mathbf{a}} = \frac{0-10}{4-2} = -5$$

$\therefore$ Rate of change of speed at $t = 4$ s would be 5.

i.e., (d) $\rightarrow$ (r).

5. (a) $x = -20 + 5t^2$

$0 = -20 + 5t^2$

$\Rightarrow$ $t = 2$ s

i.e., (a) $\rightarrow$ (r).

(b) $x = -20 + 5t^2$

$$\frac{dx}{dt} = 10t$$

$$\frac{d^2x}{dt^2} = 10$$

Velocity will be numerically equal to acceleration at $t = 1$ s.

Thus (b) $\rightarrow$ (q).

(c) At $t = 0$ s $x = -20$ m

$t = 1$ s $x = -15$ m

$t = 2$ s $x = 0$ m

$t = 3$ s $x = +25$ m

Particle is always moving along + ive x-axis.

$\therefore$ (c) $\rightarrow$ (s).

(d) $v = \frac{dx}{dt} = 10t$ is zero at $t = 0$ s.

$\therefore$ (d) $\rightarrow$ (p).

6. (a) $x = 1 - 2t + t^2$

$0 = 1 - 2t + t^2$ [for particle to cross y-axis]

i.e., $t = 1$ s

$$Y = 4 - 4t + t^2$$

$$\frac{dy}{dt} = -4 + 2t$$

$$= -4 + (2 \times 1) \quad [\text{at } t = 1 \text{ s}]$$

$$= -2 \text{ unit}$$

$\therefore$ (a) $\rightarrow$ (q).

(b) $Y = 4 - 4t + t^2$

$0 = 4 - 4t + t^2$ [For particle to cross x-axis]

i.e., $t = 2$ s

$$x = 1 - 2t + t^2$$

$$\frac{dy}{dt} = -2 + 2t$$

$$= -2 + (2 \times 2) \quad [\text{at } t = 2 \text{ s}]$$

$$= +2 \text{ unit}$$

$\therefore$ (b) $\rightarrow$ (p).

(c) $\left(\frac{dx}{dt}\right)_{t=0\text{ s}} = -2$ unit

and $\left(\frac{dy}{dt}\right)_{t=0\text{ s}} = -4$ unit

$\therefore$ Initial velocity of particle

$$= \sqrt{(-2)^2 + (-4)^2}$$

$$= 2\sqrt{5} \text{ unit}$$

i.e., (c) $\rightarrow$ (s).

(d) $\frac{d^2x}{dt^2} = 2$

and $\frac{d^2y}{dt^2} = 2$

$\therefore$ Initial acceleration of particle $= 2\sqrt{2}$ unit

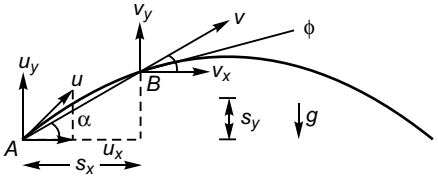
i.e., (d) $\rightarrow$ (s).

4 Projectile Motion

Introductory Exercise 4.1

1. A particle projected at any angle with horizontal will always move in a plane and thus projectile motion is a 2-dimensional motion. The statement is thus **false**.
2. At high speed the projectile may go to a place where acceleration due to gravity has some different value and as such the motion may not be uniform accelerated. The statement is thus **true**.
3. See article 4.1.
4. $u = 40\sqrt{2}$ m/s, $\theta = 45°$

   As horizontal acceleration would be zero.



$$v_x = u_x = u\cos\theta = 40 \text{ m/s}$$
$$s_x = u_x t = (u\cos\theta)\, t = 80 \text{ m}$$

A : position of particle at time = 0.

B : position of particle at time = t.

As vertical acceleration would be $-g$

$$v_y = u_y - gt$$
$$= u\sin\theta = gt$$
$$= 40 - 20$$
$$= 20 \text{ m/s}$$
$$s_y = u_y \cdot t - \frac{1}{2}gt^2$$
$$= 80 - \frac{1}{2}\times 10 \times 2^2 = 60 \text{ m}$$

$\therefore$ $$v = \sqrt{v_x^2 + v_y^2}$$
$$= \sqrt{40^2 + 20^2}$$
$$= 20\sqrt{5} \text{ m/s}$$
$$\tan\phi = \frac{v_y}{v_x} = \frac{20}{40}$$

i.e., $$\phi = \tan^{-1}\left(\frac{1}{2}\right)$$
$$s = \sqrt{s_x^2 + s_y^2}$$
$$= \sqrt{80^2 + 60^2}$$
$$= 100 \text{ m}$$
$$\tan\alpha = \frac{s_y}{s_x}$$
$$= \frac{60}{80}$$

i.e., $$\alpha = \tan^{-1}\left(\frac{3}{4}\right)$$

5. $s_y = u_y t + \frac{1}{2}(-g)\, t^2$

   or $$s_y = (u\sin\theta)t - \frac{1}{2}gt^2$$

   or $$15 = 20t - \frac{1}{2}gt^2$$

   or $$t^2 = 4t + 3 = 0$$

   i.e., $$t = 1 \text{ s and } 3 \text{ s}$$

6. See figure to the answer to question no. 4.

   $u = 40$ m/s, $\theta = 60°$

   $\therefore$ $$u_x = 40\cos 60° = 20 \text{ m/s}$$

Thus, $v_x = u_x = 20$ m/s

As $\phi = 45°$

$$\tan\phi = \frac{v_y}{v_x} = 1$$

$\therefore$ $$y_y = v_x = 20 \text{ m/s}$$

Thus, $$v = \sqrt{v_x^2 + v_y^2} = v_x\sqrt{2}$$
$$= 20\sqrt{2} \text{ m/s}$$

Before reaching highest point

$$v_y = u_y + (-g)t$$

$\therefore$ $$20 = 40\sin 60° - 10t$$

or $$2 = 2\sqrt{3} - t$$

$\Rightarrow$ $$t = 2(\sqrt{3} - 1) \text{ s}$$

After attaining highest point

$$-20 = 40\sqrt{3} - 10t$$

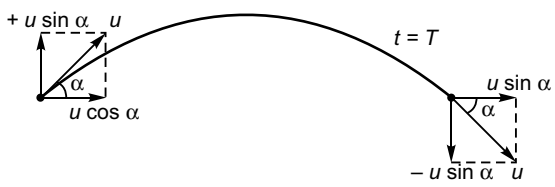
i.e., $$-2 = 2\sqrt{3} - t$$

or $$t = 2(\sqrt{3} + 1) \text{ s}$$

7. Average velocity $= \dfrac{\text{Range}}{\text{Time of flight}}$

$$= \frac{u^2 \sin 2\alpha / g}{2u\sin\alpha / g}$$
$$= \frac{2u^2 \sin\alpha\cos\alpha}{g} \times \frac{g}{2u\sin\alpha}$$
$$= u\cos\alpha$$

8. Change in velocity



$$= (-u\sin\alpha) - (+u\sin\alpha)$$
$$= -2u\sin\alpha$$
$$= 2u\sin\alpha \text{ (downward)}$$

9. Formulae for R, T and H_{max} will be same if the projection point and the point where the particle lands are same and lie on a horizontal line.

10. $\vec{\mathbf{r}} = [3t\,\hat{\mathbf{i}} + (4t - 5t^2)\,\hat{\mathbf{j}}]$ m

y-coordinate will be zero when

$$4t - 5t^2 = 0$$

i.e., $$t = 0 \text{ s}, \frac{4}{5} \text{ s}$$

$t = 0$ belongs to the initial point of projection of the particle.

$\therefore$ $$\vec{\mathbf{r}} = 0\,\hat{\mathbf{i}} \text{ m}$$

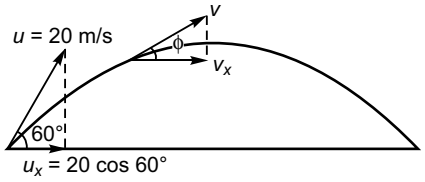
i.e., $$x = 0 \text{ m}$$

At $t = 0.8$ s,

$$\vec{\mathbf{r}} = 2.4\,\hat{\mathbf{i}} \text{ m}$$

i.e., $$x = 2.4 \text{ m}$$

11. $v_x = u_x = 10$ m/s



$$= 10 \text{ m/s}$$
$$v_x = u_x = 10 \text{ m/s}$$
$$v_x = v\cos\phi$$

$\Rightarrow$ $$\cos\phi = \frac{v_x}{v}$$
$$= \frac{10}{v}$$
$$= \frac{10}{10} \qquad \left[\text{as, } v = \frac{20}{2} \text{ (given)}\right]$$
$$= 1$$

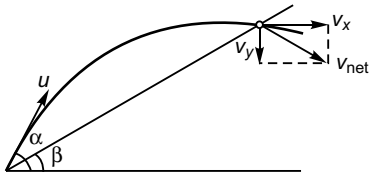
i.e., $$\phi = 0°$$

$\therefore$ Speed will be half of its initial value at the highest point where $\phi = 0°$.

Thus, $$t = \frac{u\sin\theta}{g}$$
$$= \frac{20\sin 60°}{10} = \sqrt{3} \text{ s}$$

Introductory Exercise 4.2

1. Time of flight



$\alpha = 60°$
$\beta = 30°$
$u = 10$ m/s^2
$g = 10$ m/s^2

$$T = \frac{2u \sin(\alpha - \beta)}{g \cos \beta}$$
$$= \frac{(2 \times 10 \sin(60° - 30°)}{10 \cos 30°}$$
$$= \frac{2}{\sqrt{3}} \text{ s}$$

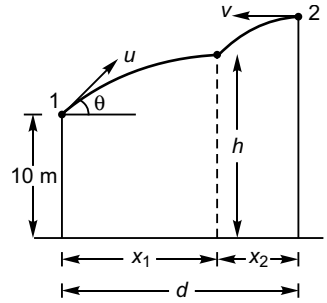
Using, $v = u + at$

$$v_x = u_x = u \cos 60°$$
$$= 10 \times \frac{1}{2} = 5 \text{ m/s}$$
$$v_y = u_y + (-g)\,T$$
$$= u \sin 60° - gT$$
$$= 10 \frac{\sqrt{3}}{2} - 10 \frac{2}{\sqrt{3}}$$
$$= 10\left(\frac{3-4}{2\sqrt{3}}\right)$$
$$= -\frac{5}{\sqrt{3}} \text{ m/s}$$
$$v_{net} = \sqrt{v_x^2 + v_y^2}$$
$$= \sqrt{5^2 + \left(\frac{-5}{\sqrt{3}}\right)^2}$$
$$= 5\sqrt{1 + \frac{1}{3}}$$
$$= \frac{10}{\sqrt{3}} \text{ m/s}$$

2. Component of velocity perpendicular to plane

$$= v_{net} \cos \beta$$
$$= \frac{10}{\sqrt{3}} \times \cos 30°$$
$$= \frac{10}{\sqrt{3}} \times \frac{\sqrt{3}}{2}$$
$$= 5 \text{ m/s}$$

3. Let the particle collide at time t.



$$x_1 = (u \cos \theta)\, t$$

and $$x_2 = v\, t$$

$\therefore$ $$d = x_2 - x_1$$
$$= (v + u \cos \theta)\, t$$
$$= [10 + 10\sqrt{2} \cos 45°]\, t = 20\, t$$

Using equation, $s = ut + \frac{1}{2} at^2$

For vertical motion of particle 1 :

$$h - 10 = (u \sin \theta)\, t + \frac{1}{2}(-g)\, t^2$$

i.e., $$h = 10 + (u \sin \theta)\, t - \frac{1}{2} gt^2 \quad \text{...(i)}$$

or $$h - 10 + 10\, t - \frac{1}{2} gt^2$$

For the vertical motion of particle 2 :

$$20 - h = \frac{1}{2} gt^2$$

i.e., $$h = 20 - \frac{1}{2} gt^2 \quad \text{...(ii)}$$

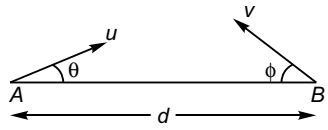
Comparing Eqs. (i) and (ii),

$$10 + 10\, t - \frac{1}{2} g\, t^2 = 20 - \frac{1}{2} g\, t^2$$

$\Rightarrow$ $$t = 1 \text{ s}$$

$\therefore$ $$d = 20 \text{ m}$$

4. $$u = 10 \text{ m/s}$$
$$v = 5\sqrt{2} \text{ m/s}$$



$$\theta = 30°$$
$$\phi = 45°$$
$$d = 15\text{ m}$$

Let the particles meet (or are in the same vertical time t).

$\therefore \quad d = (u\cos\theta)\,t + (v\cos\phi)\,t$

$\Rightarrow \quad 15 = (10\cos 30° + 5\sqrt{2}\cos 45°)\,t$

or $\quad 15 = (5\sqrt{3} + 5)\,t$

or $\quad t = \dfrac{3}{\sqrt{3}+1}\text{ s}$

$$= 1.009\text{ s}$$

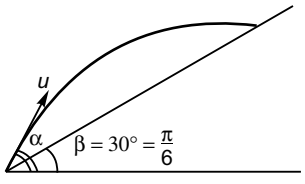
Now, let us find time of flight of A and B

$$T_A = \frac{2u\sin\theta}{g}$$
$$= 1\text{ s}$$

As $T_A < t$, particle A will touch ground before the expected time t of collision.

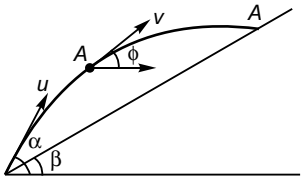
$\therefore$ Ans : NO.

5. For range to be maximum



$$\alpha = \frac{\pi}{4} + \frac{\beta}{2}$$
$$= \frac{\pi}{4} + \frac{\pi/6}{2}$$
$$= \frac{\pi}{3}$$
$$= 60°$$

6. At point A velocity ($\vec{\mathbf{v}}$) of the particle will be parallel to the inclined plane.



$$u = 40\text{ m/s}$$
$$\alpha = 60°$$
$$\beta = 30°$$
$$g = 10\text{ m/s}^2$$

$\therefore \quad \phi = \beta$

$$v_x = u_x = u\cos\alpha$$
$$v_x = v\cos\phi = v\cos\beta$$

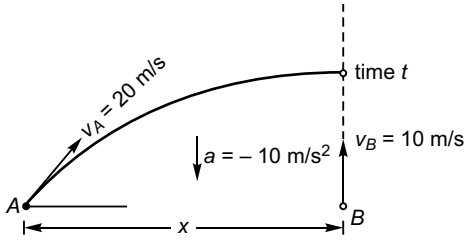
or $\quad u\cos\alpha = v\cos\beta$

$\Rightarrow \quad v = \dfrac{u\cos\alpha}{\cos\beta} = \dfrac{40\cos 60°}{\cos 30°}$

$$= \frac{40\left(\frac{1}{2}\right)}{\frac{\sqrt{3}}{2}}$$
$$= \frac{40}{\sqrt{3}}\text{ m/s}$$

7. (a) At time t, vertical displacement of A

= Vertical displacement of B



$$(v_A\sin\theta)\,t - \frac{1}{2}gt^2 = v_B\,t - \frac{1}{2}gt^2$$

i.e., $\quad v_A\sin\theta = v_B$

$$\sin\theta = \frac{v_B}{v_A}$$
$$= \frac{10}{20} = \frac{1}{2}$$

$\therefore \quad \theta = 30°$

(b) $x = (v_A\cos\theta)\,t$

$$= (20\cos 30°) \times \frac{1}{2}$$
$$= 5\sqrt{3}\text{ m}$$

AIEEE Corner

Subjective Questions (Level 1)

1. (a) $R = \dfrac{u^2 \sin 2\theta}{g} = \dfrac{(20\sqrt{2})^2 \sin 90°}{10}$

$= 80\text{ m}$

$H = \dfrac{u^2 \sin^2 \theta}{2g} = \dfrac{(20\sqrt{2})^2 \sin^2 45°}{10}$

$= 40\text{ m}$

$T = \dfrac{2u \sin\theta}{g} = \dfrac{2(20\sqrt{2}) \sin 45°}{10}$

$= 4\text{ s}$

(b) $\vec{\mathbf{u}} = (20\hat{\mathbf{i}} + 20\hat{\mathbf{j}})$ m/s and $\vec{\mathbf{a}} = -10\hat{\mathbf{j}}$ m/s^2

$-\vec{\mathbf{v}} = \vec{\mathbf{u}} + \vec{\mathbf{a}}\, t$

$= (20\hat{\mathbf{i}} + 20\hat{\mathbf{j}}) + (-10\hat{\mathbf{j}})\, 1 (\text{at } t = 1\text{ s})$

$= (20\hat{\mathbf{i}} - 10\hat{\mathbf{j}})$ m/s

(c) Time of flight

$T = \dfrac{2u \sin\theta}{g}$

$= \dfrac{2 \times 20\sqrt{2} \times \left(\dfrac{1}{\sqrt{2}}\right)}{10}$

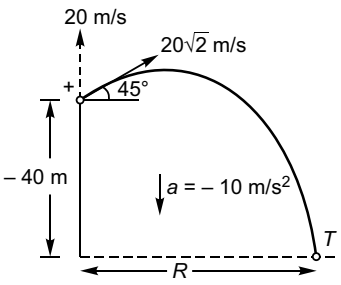
$= 4\text{ s}$

(c) ∴ Velocity of particle at the time of collision with ground.

$= (20\hat{\mathbf{i}} + 20\hat{\mathbf{j}}) + (-10\hat{\mathbf{j}})\, 4$

$= (20\hat{\mathbf{i}} - 20\hat{\mathbf{j}})$ m/s

2. (a) $s = ut + \dfrac{1}{2}at^2$



∴ $(-40) = (+20)\, T + \dfrac{1}{2}(-10)\, T^2$

or $5T^2 - 20T - 40 = 0$

or $T^2 - 4T - 8 = 0$

∴ $T = \dfrac{-(-4) + \sqrt{(-4)^2 - 4(1)(-8)}}{2(1)}$

Leaving – ive sign which is not positive.

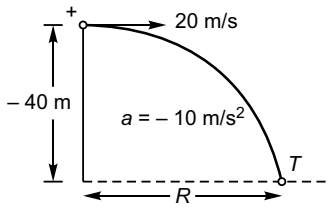
$= \dfrac{4 + \sqrt{48}}{2} = \dfrac{4 + 4\sqrt{3}}{2}$

$= (2 + 2\sqrt{3})\text{ s}$

$= 5.46\text{ s}$

$R = 20 \times 5.46 = 109.2\text{ m}$

(b) $s = ut + \dfrac{1}{2}at^2$



$(-40) = 0 \times T + \dfrac{1}{2}(-10)\, T^2$

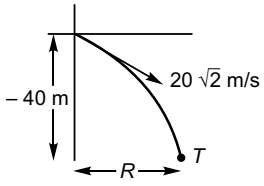
⇒ $T = 2\sqrt{2}\text{ s}$

$= 2.83\text{ s}$

$R = 2.83 \times 20$

$= 56.6\text{ m}$

(c) $s = ut + \dfrac{1}{2}at^2$



$(-40) = (-20)\, T + \dfrac{1}{2}(-10)\, T^2$

or $5T^2 + 20T - 40 = 0$

or $T^2 + 4T - 8 = 0$

∴ $T = \dfrac{-4 + \sqrt{(4)^2 - 4(1)(-8)}}{2(1)}$

$= \dfrac{-4 + 4\sqrt{3}}{2} = -2 + 2\sqrt{3} = 1.46\text{ s}$

$\therefore \qquad R = 20 \times 1.46$

$= 29.2\text{ m}$

3. (a) Change in velocity (v_c)

= Change in vertical velocity (as horizontal velocity does not change).

$= (u \sin\theta - gt) - (u \sin\theta)$

$= -gt$

$= -(10 \times 3)\text{ m/s}$

$= -30\text{ m/s}$

$= 30\text{ m/s (downward)}$

(b) $\vec{\mathbf{u}} = (20\hat{\mathbf{i}} + 20\hat{\mathbf{j}})\text{ m/s}$

$$\vec{\mathbf{a}} = -10\hat{\mathbf{j}}\text{ m/s}^2$$

$\therefore$ Displacement at time t (= 3 s)

$$= \vec{\mathbf{u}}\,t + \frac{1}{2}\vec{\mathbf{a}}\,t^2$$

$$= (20\hat{\mathbf{i}} + 20\hat{\mathbf{j}})3 + \frac{1}{2}(-10\hat{\mathbf{j}})3^2$$

$$= 60\hat{\mathbf{i}} + 60\hat{\mathbf{j}} - 45\hat{\mathbf{j}}$$

$$= 60\hat{\mathbf{i}} + 15\hat{\mathbf{j}}$$

$$\therefore \quad \vec{\mathbf{v}}_{av} = \frac{60\hat{\mathbf{i}} + 15\hat{\mathbf{j}}}{3}$$

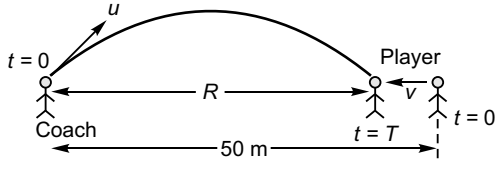
$$= 20\hat{\mathbf{i}} + 5\hat{\mathbf{j}}$$

$$\Rightarrow \quad |\vec{\mathbf{v}}_{av}| = \sqrt{(20)^2 + (5)^2}$$

$$= \sqrt{425}$$

$$= 20.6\text{ m/s}$$

4. $R + vT = 50$



$$\Rightarrow \quad vT = 50 - \frac{u^2 \sin^2\theta}{g}$$

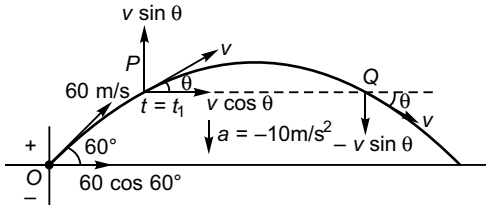
$$v \times \frac{2u\sin\theta}{g} = 50 - \frac{(20)^2}{10}$$

$$v \times \left(\frac{2 \times 20 \times \sin 45°}{10}\right) = 10$$

$$\therefore \quad v = \frac{100}{40 \times \frac{1}{\sqrt{2}}}$$

$$= \frac{5}{\sqrt{2}}\text{ m/s}$$

5. Horizontal compnent of velocity at P = Horizontal component of velocity at O



$$\therefore \quad v\cos\theta = 60\cos 60°$$

$$\Rightarrow \quad v\cos 45° = 60\cos 60°$$

$$\Rightarrow \quad v = \frac{60\cos 60°}{\cos 45°}$$

$$= 30\sqrt{2}\text{ m/s}$$

For point P :

$$v\sin 45° = 60\sin 60° + (-10)\,t_1$$

or $$30 = 60\frac{\sqrt{3}}{2} - 10\,t_1$$

$$t_1 = 3(\sqrt{3} - 1)$$

$$= 2.20\text{ s}$$

For point Q :

$$-v\sin 45° = 60\sin 60° + (-10)\,t_2$$

$$\therefore \quad t_2 = 3(\sqrt{3} + 1)$$

$$= 8.20\text{ s}$$

6. $Y = bx - cx^2$

Differentiating above equation w.r.t. time t

$$\therefore \quad \frac{dy}{dt} = b\frac{dx}{dt} - 2cx\frac{dx}{dt} \qquad \text{...(i)}$$

$$\Rightarrow \quad \frac{dy}{dt} = b\frac{dx}{dt} \qquad (\text{at } x = 0)$$

$$v = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

$$= \sqrt{\left(\frac{dx}{dt}\right)^2 + b^2\left(\frac{dx}{dt}\right)^2}$$

$$= \sqrt{1 + b^2}\,\frac{dx}{dt} \qquad \text{...(ii)}$$

Differentiating Eq. (i) w.r.t. time t

$$\frac{d^2y}{dt^2} = b \cdot \frac{d^2x}{dt^2} - 2c\left(\frac{dx}{dt}\right)^2 - 2cx\frac{d^2x}{dt^2} \quad \text{...(iii)}$$

Acceleration of particle

$$-a = \sqrt{\left(\frac{d^2x}{dt^2}\right)^2 + \left(\frac{d^2x}{dt^2}\right)}$$

$$\therefore \quad \frac{d^2x}{dt^2} = 0$$

Substituting $\frac{d^2x}{dt^2} = 0$ and $\frac{d^2y}{dt^2} = -a$ in Eq. (iii)

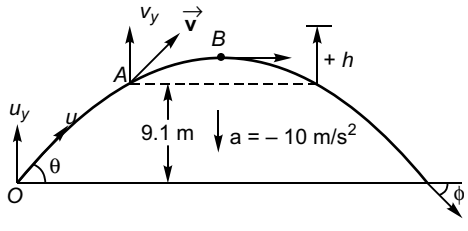
$$-a = -2c\left(\frac{dx}{dt}\right)^2$$

i.e., $$\frac{dx}{dt} = \sqrt{\frac{a}{2c}}$$

Substituting above value of $\frac{dx}{dt}$ in Eq. (ii)

$$v = \sqrt{\frac{a}{2c}(1 + b^2)}$$

7. (a) Velocity at point $A = 7.6\,\hat{\mathbf{i}} + 6.1\,\hat{\mathbf{j}}$



$\therefore$ Initial vertical velocity at $a = 6.1\,\hat{\mathbf{j}}$ m/s.

Final vertical velocity at B (highest point)

$$= 0 \text{ m/s}$$

Using $v^2 = u^2 + 2as$ (Between A and B)

$$0^2 = (6.1)^2 + 2(-10)(+h)$$

$$\Rightarrow \quad h = \frac{(6.1)^2}{20}$$

$$= 1.86 \text{ m}$$

$\therefore$ Maximum height attained by ball

$$= 9.1 + 1.86$$

$$= 10.96 \text{ m}$$

(b) Let magnitude of vertical velocity at O (point of projection) $= u_y$

Using $v^2 = u^2 + 2as$ (Between O and A)

$$(6.1)^2 = u_y^2 + 2(-10)(9.1)$$

$$\Rightarrow \quad u_y = \sqrt{37.21 + 182}$$

$$= 14.8$$

Angle of projection

$$\theta = \tan^{-1}\left(\frac{u_y}{u_x}\right)$$

$$= \tan^{-1}\left(\frac{u_y}{u_x}\right)$$

$$= \tan^{-1}\left(\frac{14.8}{7.6}\right)$$

$$= 62.82°$$

$$\text{Range} = \frac{u^2 \sin 2\theta}{g}$$

$$= \frac{(u_x^2 + u_y^2)\sin 2\theta}{g}$$

$$= \frac{(v_x^2 + u_y^2)\sin 2\theta}{g}$$

$$= \frac{[(7.6)^2 + (14.8)^2]\sin 125.64°}{10}$$

$$= 22.5 \text{ m}$$

(c) Magnitude of velocity just before the ball hits ground

$$= [(-u_y)^2 + (u_x)^2]^{\frac{1}{2}}$$

$$= [(-14.8)^2 + (6.1)^2]^{\frac{1}{2}} \quad [u_x = v_x]$$

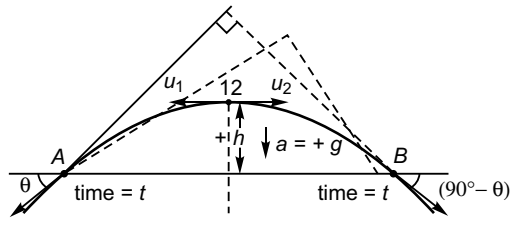
$$= 16 \text{ ms}^{-1}$$

(d) $\phi = \theta = \tan^{-1}\left(\frac{u_y}{u_x}\right)$

$$= \tan^{-1}\left(\frac{14.8}{7.6}\right)$$

$$= \tan^{-1}(1.95)$$

8. As initial vertical velocities of both particles will be zero and both fall under same acceleration (g), at anytime t, the vertical displacement of both will be same



i.e., both will always remain in the same horizontal line as shown in figure.

At time t :

Vertical velocity of A

$$= \text{Vertical velocity of } B$$
$$= 0 + (+g)\,t$$
$$= g\,t$$

At A :

$$\tan\theta = \frac{\text{Vertical velocity of particle 1}}{\text{Horizontal velocity of particle 1}}$$

or $$\tan\theta = \frac{gt}{u_1} \quad \ldots\text{(i)}$$

At B :

$$\tan(90° - \theta) = \frac{\text{Vertical velocity of particle 2}}{\text{Horizontal velocity of particle 2}}$$

or $$\cos\theta = \frac{gt}{u_2} \quad \ldots\text{(ii)}$$

Multiplying Eq. (i) by Eq. (ii),

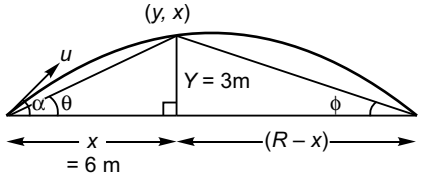
$$1 = \frac{gt}{u_1} \times \frac{gt}{u_2}$$

$\therefore$ $$t = \frac{\sqrt{u_1 u_2}}{g}$$

Distance between A and B

$$= R_1 + R_2$$
$$= u_1 t + u_2 t$$
$$= (u_1 + u_2)\,t$$
$$= (u_1 + u_2)\frac{\sqrt{u_1 u_2}}{g}$$
$$= \frac{7\sqrt{2}}{9.8}$$
$$= 2.47 \text{ m}$$

9. $Y = x\left(1 - \dfrac{x}{R}\right) = \tan\alpha$



$$\frac{YR}{x(R-x)} = \tan\alpha$$

or $$\frac{Y}{x} + \frac{y}{R-x} = \tan\alpha$$

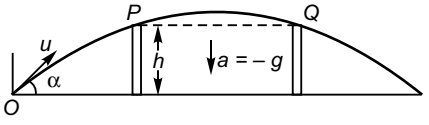
or $$\tan\theta + \tan\phi = \tan\alpha$$

(Students to remember this formula)

$\Rightarrow$ $$\tan\alpha = \frac{3}{6} + \frac{3}{12}$$
$$= \frac{9}{12}$$

$\Rightarrow$ $$\alpha = \tan^{-1}\left(\frac{3}{4}\right)$$

10. On the trajectory there be two points P and Q at height h from ground.



If particle takes t time to reach point A (*i.e.*, vertical displacement of $+h$)

$$(+h) = (u\sin\alpha)\,t + \frac{1}{2}(-g)\,t^2$$

or $$gt^2 - 2(u\sin\alpha)\,t + 2h = 0 \quad \ldots\text{(i)}$$

The above equation is quadratic in t. Two values of t will satisfy Eq. (i). One having lower value will be time ($= t_1$) to reach point P while the higher value will be the time ($= t_2$) to reach point Q.

$\therefore$ Time to reach point Q from point P

$$= t_2 - t_1$$
$$= \sqrt{(t_2 + t_1)^2 - 4t_1 t_2}$$
$$= \sqrt{\left(\frac{2u\sin\alpha}{g}\right)^2 - 4\left(\frac{2h}{g}\right)}$$

[Using Eq. (i)]

$$= \frac{4u^2\sin^2\alpha - 2gh}{g}$$

or $$(t_2 - t_1) = \frac{\sqrt{16gh\sin^2\alpha - 8gh}}{g}$$

$(\because u = 2\sqrt{gh})$

Distance between P and Q :

$$2h = (u\cos\alpha)(t_2 - t_1)$$

$\therefore$ $$4h^2 = u^2\cos^2\alpha\,(t_2 - t_1)^2$$

or $$4h^2 = (4gh)\cos^2\alpha\left(\frac{16gh\sin^2\alpha - 8gh}{g^2}\right)$$

or $\quad 1 = \cos^2 (16 \sin^2 \alpha - 8)$

or $\quad 1 = \cos^2 \alpha\, [16(1 - \cos^2 \alpha) - 8]$

or $\quad 1 = \cos^2 \alpha\, [8 - 16 \cos^2 \alpha]$

or $\quad 16 \cos^4 \alpha - 8 \cos^2 \alpha + 1 = 0$

$\Rightarrow \quad (4 \cos^2 \alpha - 1)^2 = 0$

$\Rightarrow \quad 4 \cos^2 \alpha = \dfrac{1}{4}$

$$\cos \alpha = \frac{1}{2}$$

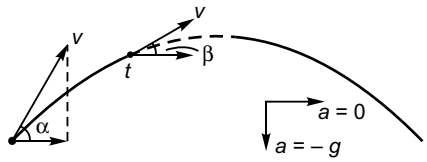
$$\cos \alpha = 60°$$

($\cos \alpha = -\dfrac{1}{2}$ being not possible).

$$\therefore \quad (t_2 - t_1) = \sqrt{\frac{16gh \sin^2 60° - 2gh}{g^2}}$$

$$= 2\sqrt{\frac{h}{g}} \qquad \text{(Proved)}$$

11. Initial vertical velocity $= u \sin \alpha$



At time t vertical velocity $= v \sin \beta$

$$\therefore \quad v \sin \beta = u \sin \alpha = (-g)\, t \qquad \ldots\text{(i)}$$

Now, as horizontal acceleration will be zero.

$$v \cos \beta = u \cos \alpha$$

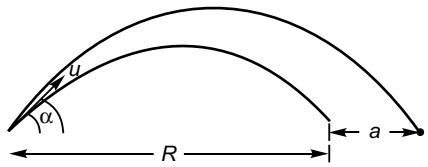
Thus, Eq. (i) becomes

$$\left(\frac{u \cos \alpha}{\cos \beta}\right) \sin \beta = u \sin \alpha - gt$$

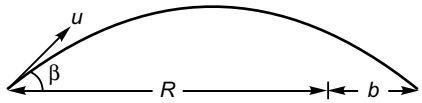
or $\quad u \sin \alpha \cos \beta - u \cos \alpha \sin \beta = gt \cos \beta$

or $\quad u = \dfrac{gt \cos \beta}{\sin (\alpha - \beta)}$

12. $R - a = \dfrac{u^2 \sin 2\alpha}{g}$



i.e., $\quad bR - ab = \dfrac{bu^2 \sin 2\alpha}{g} \qquad \ldots\text{(i)}$



$$R + b = \frac{u^2 \sin 2\beta}{g}$$

i.e., $\quad aR + ab = \dfrac{au^2 \sin 2\beta}{g} \qquad \ldots\text{(ii)}$

Adding Eqs. (i) and (ii), we have

$$bR + aR = u^2 \left(\frac{b \sin 2\alpha + a \sin 2\beta}{g}\right)$$

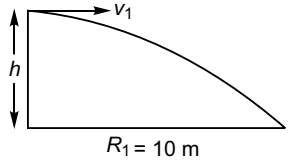
or $\quad R = u^2 \dfrac{(b \sin 2\alpha + a \sin 2\beta)}{(a + b)\, g}$

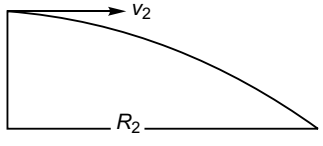
$$\frac{u^2 \sin 2\theta}{g} = u^2 \frac{(b \sin 2\alpha + a \sin 2\beta)}{(a + b)\, g}$$

$$\Rightarrow \quad \theta = \frac{1}{2} \sin^{-1} \left[\frac{b \sin 2\alpha + a \sin 2\beta}{a + b}\right]$$

(Proved.)

13. $R_1 = v_1 \sqrt{\dfrac{2h}{g}}$





i.e., $\quad h = \dfrac{1}{2}\left(\dfrac{R_1 g}{v_1}\right)^2$

$$= \frac{1}{2}\left(\frac{10 \times 9.8}{5}\right)^2$$

$$= 19.6 \text{ m}$$

$$\therefore \quad T = \sqrt{\frac{2h}{g}}$$

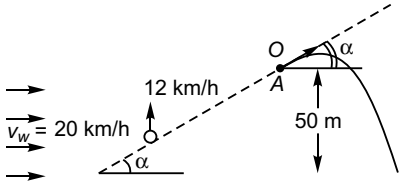
$$= \sqrt{\frac{2 \times 19.6}{9.8}}$$

$$= 2 \text{ s}$$

$$R_2 = v_2 \cdot T = 7.5 \times 2$$

$$= 15 \text{ m}$$

14. Vertical velocity of balloon (+ bag)



$$= 12 \times \frac{5}{18} \text{ m/s}$$

$$= \frac{10}{3} \text{ m/s}$$

Horizontal velocity of balloon (+ bag)

$$= \text{Wind velocity} = 20 \text{ km/h} = 20 \times \frac{5}{18} \text{ m/s}$$

$$= \frac{50}{9} \text{ m/s}$$

$$= 5.55 \text{ m/s}$$

$\therefore \tan\alpha = \frac{12}{20}$. *i.e.*, $\sin\alpha = 0.51$

Bag is released at point A.

Let t be time, the bag takes from A to reach ground.

Using, $s = ut + \frac{1}{2}at^2$

$$(-50) = \left(\frac{10}{3}\sin\alpha\right)t + \frac{1}{2}(-g)\,t^2$$

i.e., $$5t^2 - 1.7t - 50 = 0$$

$$\therefore \quad t = \frac{1.7 + \sqrt{(-1.7)^2 - 4 \times 5 \times (-50)}}{2 \times 5}$$

$$= 3.37 \text{ s}$$

Vertical velocity of bag when it strikes ground

$$v_B = -\frac{10}{3} + (10)(3.37)$$

$$= 37.03 \text{ m/s}$$

$$v_\omega = 5.55 \text{ m/s}$$

$\therefore$ Velocity of bag with which it strikes ground

$$v_{\text{net}} = \sqrt{v_B^2 + v_\omega^2}$$

$$= 37.44 \text{ m/s}$$

15. $$T = \frac{2u\sin(\alpha - \beta)}{g\cos\beta}$$

$$= \frac{2 \times 20\sqrt{2} \times \sin(45° - 30°)}{10\cos 30°}$$

$$= 1.69 \text{ s}$$

$$R = \frac{2u^2 \sin(\alpha - \beta)\cos\alpha}{g\cos^2\beta}$$

$$= \frac{2 \times (20\sqrt{2})^2 \times \sin 15° \times \cos 45°}{10\cos^2 30°}$$

$$= 39 \text{ m}$$

16. $$T = \frac{2u\sin(\alpha + \beta)}{g\cos\beta}$$

$$= \frac{2 \times 20\sqrt{2} \times \sin(45° + 30°)}{10\cos 30°}$$

$$= 6.31 \text{ s}$$

$$R = \frac{u^2}{g\cos^2\beta}[\sin(2\alpha + \beta) + \sin\beta]$$

$$= \frac{(20\sqrt{2})^2}{10\cos^2 30°}[\sin(90° + 30°) + \sin 30°]$$

$$= 145.71 \text{ m}$$

17. $$T = \frac{2u\sin(\alpha + \beta)}{g\cos\beta}$$

$$= \frac{2u\sin\beta}{g\cos\beta} \qquad (\because \alpha = 0°)$$

$$= \frac{2u}{g}\tan\beta$$

$$= \frac{2 \times 20}{10}\tan 30°$$

$$= 2.31 \text{ s}$$

$$R = \frac{u^2}{g\cos^2\beta}[\sin(2\alpha + \beta) + \sin\beta]$$

$$= \frac{u^2(2\sin\beta)}{g\cos^2\beta} \qquad [\text{as } \alpha = 0°]$$

$$= \frac{uT}{\cos\beta} = \frac{20 \times 2.31}{\cos 30°}$$

$$= 53.33 \text{ m}$$

18. $$R = \frac{u^2}{g\cos^2\beta}[\sin(2\alpha + \beta) + \sin\beta]$$

$$= \frac{u^2}{g\cos^2\beta}[\sin\{2(\alpha + \beta) - \beta\} + \sin\beta]$$

$$= \frac{u^2}{g\cos^2\theta}[\sin(\pi - \theta) + \sin\theta]$$

$$[\because (\alpha + \beta) = \frac{\pi}{2}]$$

or $$R = \frac{2u^2}{g}\tan\theta\sec\theta$$

19. (a) Acceleration of particle 1 w.r.t. that of particle 2

$$= (-g) - (-g)$$
$$= 0$$

(b) Initial velocity of 1st particle $= 20\,\hat{\mathbf{j}}$ m/s

Initial velocity of 2nd particle

$$= (20\sqrt{2}\cos 45°\,\hat{\mathbf{i}} + 20\sqrt{2}\sin 45°\,\hat{\mathbf{j}})\text{ m/s}$$

$$= (20\,\hat{\mathbf{i}} + 20\,\hat{\mathbf{j}})\text{ m/s}$$

∴ Initial velocity of 1st particle w.r.t. that of 2nd particle

$$= [(20\,\hat{\mathbf{j}}) - (20\,\hat{\mathbf{i}} + 20\,\hat{\mathbf{j}})]\text{ m/s}$$

$$= -20\,\hat{\mathbf{i}}\text{ m/s}$$

$$= 20\text{ m/s (downward)}$$

(c) Horizontal velocity of 1st particle

$$= 0\text{ m/s}$$

Horizontal velocity of 2nd particle

$$= 20\,\hat{\mathbf{i}}\text{ m/s}$$

∴ Horizontal velocity of 1st particle w.r.t. that of 2nd particle

$$= 0 - (20\,\hat{\mathbf{i}})$$

$$= -20\,\hat{\mathbf{i}}\text{ m/s}$$

Relative displacement of 1st particle w.r.t. 2nd particle at $t = 2$ s

$$= -20\,\hat{\mathbf{i}} \times 2$$

$$= -40\,\hat{\mathbf{i}}\text{ m/s}$$

∴ Distance between the particles at $t = 2$ s

$$= 40\text{ m}$$

20. (a) As observed by passenger

Vertical acceleration of stone

$$= g - 0 = g$$

Horizontal velocity of stone

$$= v - v = 0$$

∴ Path of the stone will be a straight line (downwards).

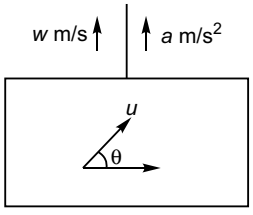
(b) As observed by man standing on ground

Vertical acceleration of stone $= g$

Horizontal velocity of stone $= v$

∴ Path of the stone will be parabolic.

21. (a) $g_{\text{eff}} = g - (-a)$

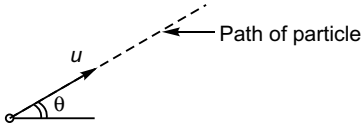


$$= g + a$$
$$= 10 + 1$$
$$= 11\text{ m/s}^2$$
$$T = \frac{2u\sin\theta}{g_{\text{eff}}}$$
$$= \frac{2 \times 2 \times \sin 30°}{11}$$
$$= 0.18\text{ s}$$

(b) Dotted path [(in lift) acceleration upwards]

Full line path [In lift at rest or moving with constant velocity upwards or downwards].

(c) If lift is moving downward with acceleration g.



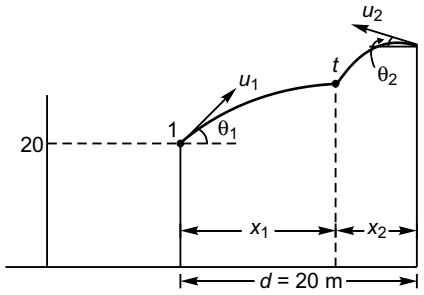
$$g_{\text{eff}} = g - g = 0$$

22. Horizontal motion :

$$x_1 = u_1\cos\theta_1 \text{ and } x_2 = u_2\cos\theta$$

∴ $$u_1\cos\theta_1 + u_2\cos\theta_2 = 20 \quad \text{...(i)}$$

Vertical motion :



$$20 + (u_1 \sin\theta_1)\, t + \frac{1}{2}(-g)\, t^2 = 30 + (u_2 \sin\theta_2)\, t + \frac{1}{2}(-g)\, t^2$$

or $\quad (u_1 \sin\theta_1 - u_2 \sin\theta_2)\, t = 10 \quad$ ...(ii)

Objective Questions (Level 1)

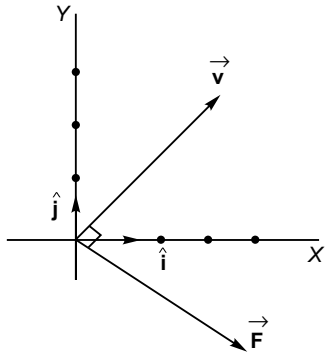
1. $\vec{\mathbf{v}} = 3\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}}$ and $\vec{\mathbf{F}} = 4\,\hat{\mathbf{i}} - 3\,\hat{\mathbf{j}}$

$$\vec{\mathbf{v}} \cdot \vec{\mathbf{F}} = (3\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}}) \cdot (4\,\hat{\mathbf{i}} - 3\,\hat{\mathbf{j}})$$

$$= 12 - 12 = 0$$

$\therefore \quad \vec{\mathbf{F}} \perp \vec{\mathbf{v}}$

Path of the particle is circular.



Option (c) is correct.

2. Projectile motion is uniformly accelerated everywhere even at the highest point.

Option (a) correct.

Option (b) incorrect.

At the highest point acceleration is perpendicular to velocity.

Option (c) incorrect.

3. For range to be maximum

$$\theta = 45°$$

i.e., $\quad \dfrac{u_x}{u} = \cos 45° = \dfrac{1}{\sqrt{2}}$

or $\quad v_x = \dfrac{u}{\sqrt{2}} \quad (\because v_x = u_x)$

$$= \frac{20}{\sqrt{2}}$$

$$= 14.14 \text{ m/s}$$

$$= 14 \text{ m/s (approx)}$$

Option (b) is correct.

4. H (maximum height) $= \dfrac{u^2 \sin^2 \alpha}{2g}$

$$\therefore\ H_1 = \frac{u^2 \sin^2 \theta}{2g} \text{ and } H_2 = \frac{u^2 \sin^2 (90° - \theta)}{g}$$

$$= \frac{u^2 \cos^2 \theta}{g}$$

Thus, $\quad \dfrac{H_1}{H_2} = \dfrac{\sin^2 \theta}{\cos^2 \theta}$

Option (c) is correct.

5. Equation to trajectory is

$$Y = x \tan\theta - \frac{1}{2}\frac{gx^2}{u^2 \cos\theta \sin\theta}$$

$$Y = x \tan\theta - \frac{x^2}{R}$$

$$\text{Area} = \int_0^R Y\, dx$$

$$= \int_0^R \left(x \tan\theta - \frac{x^2}{R} \right) dx$$

$$A = \left[\frac{x^2}{2} \tan\theta - \frac{x^3}{3R} \right]_0^R$$

$$= \frac{R^2}{2} \tan\theta - \frac{R^3}{3}$$

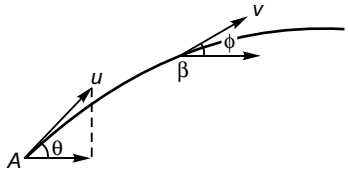
$$= R^2 \left[\frac{\tan\theta}{2} - \frac{1}{3} \right]$$

$$= \frac{4v_0^4 \sin^2\theta\cos^2\theta}{g^2}\left[\frac{\tan\theta}{2} - \frac{1}{3}\right]$$

$$= \frac{4v_0^4}{g^2}\left[\frac{\sin^3\theta\cos\theta}{2} - \frac{\sin^2\theta\cos^2\theta}{3}\right]$$

$$= \frac{2v_0^4}{3g^2}[3\sin^2\theta\cos\theta - 2\sin^2\theta\cos^2\theta]$$

6. $v\cos\phi = u\cos\theta$



or $\quad v = \dfrac{u\cos 60°}{\cos 30°}$

or $\quad v = \dfrac{u}{\sqrt{3}}$

KE at $B = \dfrac{1}{2}mv^2$

$$= \frac{1}{2}m \cdot \frac{u^2}{3}$$

$$= \frac{K}{3} \qquad \left(\because \frac{1}{2}mu^2 = K\right)$$

$\therefore$ Option (b) is correct.

7. $\dfrac{u^2\sin 2\theta}{g} = \dfrac{1}{2}\left(\dfrac{u^2}{g}\right)$

or $\quad \sin 2\theta = \dfrac{1}{2}$

or $\quad 2\theta = 30°$

$\therefore \quad \theta = 15°$

Option (a) is correct.

8. $T_1 = \dfrac{2u\sin\theta}{g}$

$\therefore \quad T_2 = \dfrac{2u\sin(90° - \theta)}{g}$

$$= \frac{2u\cos\theta}{g}$$

Thus, $\quad T_1T_2 = \dfrac{2}{g}\dfrac{2u^2\sin\theta\cos\theta}{g}$

or $\quad T_1T_2 = \dfrac{2R}{g}$

$\Rightarrow \quad R = \dfrac{1}{2}g\,T_1T_2$

Option (d) is correct.

9. $R_{max} = 1.6$ m

$$\frac{u^2}{g} = 1.6$$

$\Rightarrow \quad u = 4$ m/s

$\therefore \quad T = \dfrac{2u\sin 45°}{g}$

$$= \frac{4\sqrt{2}}{10}$$

Number of jumps $= \dfrac{10\sqrt{2}}{T} = \dfrac{10\sqrt{2}}{4\sqrt{2}/10}$

$$= 25$$

$\therefore$ Grass hopper would go

$= 25 \times 1.6$ m *i.e.*, 40 m.

Option (d) is correct.

10. $|\text{Av. velocity}| = \dfrac{|\text{Displacement}|}{\text{time}}$

$$= \frac{1}{T/2}\sqrt{\left(\frac{R}{2}\right)^2 + H^2}$$

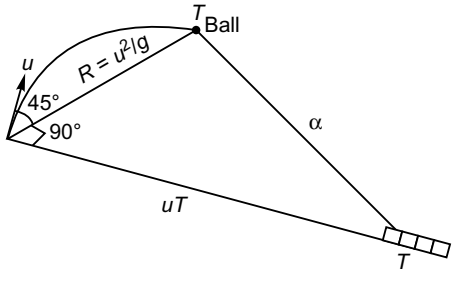
$$= \frac{1}{\frac{u\sin\theta}{g}}\sqrt{\left(\frac{u^2\sin\theta\cos\theta}{g}\right)^2 + \left(\frac{u^2\sin^2\theta}{2g}\right)^2}$$

$$= u\sqrt{\cos^2\theta + \frac{\sin^2\theta}{4}}$$

$$= \frac{u}{2}\sqrt{1 + 3\cos^2\theta}$$

Option (b) is correct.

11. $d^2 = R^2 + u^2T^2$



$$= \left(\frac{u^2}{g}\right)^2 + u^2\left(\frac{2u\sin 45°}{g}\right)^2$$

$$= \frac{u^4}{g^2} + \frac{2u^4}{g^2}$$

$$= \frac{3u^4}{g^2}$$

$\therefore \qquad d = \frac{u^2}{g}\sqrt{3} = \frac{30^2}{10}\sqrt{3} = 90\sqrt{3}$ m

Option (b) is correct.

12. At maximum height

$$\frac{dy}{dt} = 0$$

i.e., $\frac{d}{dt}(10t - t^2) = 0$

or $10 - 2t = 0$

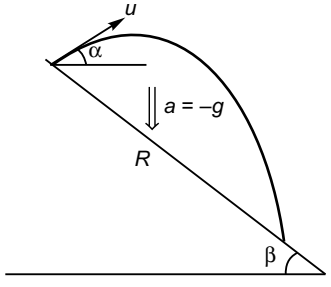
or $t = 5$ s

$\therefore$ Maximum height attained $= 10(5) - 5^2$

$= 25$ m

Option (d) is correct.

13. $R = \frac{u^2}{g\cos^2\beta}[\sin(2\alpha + \beta) + \sin\beta]$



As $\alpha = 0$ (according to question)

$$R = \frac{u^2}{g\cos^2\beta}[2\sin\beta]$$

$$= \frac{(50)^2 \times 2\sin 30°}{10 \times \cos^2 30°}$$

$$= \frac{1000}{3} \text{ m}$$

Option (b) is correct.

14. **First particle** :

$$H_{max} = 102 \text{ m}$$

$\therefore \qquad \frac{u^2 \sin^2\alpha}{2g} = 102$

$\Rightarrow \qquad u^2 = \frac{102 \times 2g}{\sin^2 60°} \quad [\text{As } \alpha = \frac{\pi}{3} = 60°]$

Second particle :

Range of the second particle will be equal to that of particle

if, $\frac{u^2 \sin 2\phi}{g} = \frac{u^2 \sin 2\alpha}{g}$

$\sin 2\phi = \sin 3\alpha$

$2\phi = \pi - 2\alpha$

or $\phi = \frac{\pi}{2} - \alpha$

$= \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6} = 30°$

$\therefore$ Maximum height attained by second particle

$$= \frac{u^2 \sin^2\phi}{2g}$$

$$= \frac{102 \times 2g}{\sin^2 60°} \times \frac{\sin^2 30°}{2g}$$

$$= \frac{102 \times 1/4}{3/4} = 34 \text{ m}$$

Option (d) is correct.

15. $s = ut + \frac{1}{2}at^2$

$\therefore \qquad (-70) = (50\sin 30°)\,t + \frac{1}{2}(-10)\,t^2$

or $5t^2 - 25t - 70 = 0$

or $t^2 - 5t - 14 = 0$

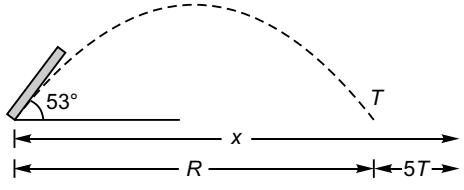
$(t - 7)(t + 2) = 0$

$t = 7$ s

(-2 s not possible)

Option (c) is correct.

16. Initial separation



$x = R + 5T$

$= (u\cos\alpha)\,T + 5T$

$= T[(u\cos\alpha) + 5]$

$= \frac{2u\sin\alpha}{g}[(u\cos\alpha) + 5]$

$= \frac{2 \times 80 \times \frac{4}{5}}{10}\left[\left(80 \times \frac{3}{5}\right) + 5\right] = 256$ m

Option (d) is correct.

JEE Corner

Assertion and Reason

1. Assertion is wrong while the explanation as given in reason is correct.

$\therefore$ Option (d) is correct.

2. Assertion and Reason both are correct. both carry the same meaning.

Option (a) is correct.

3. $H = \dfrac{u^2 \sin^2 \alpha}{2g}$ [H = maximum height]

$$= \frac{g}{8}\left(\frac{2u \sin \alpha}{g}\right)^2 = \frac{g}{8} T^2$$

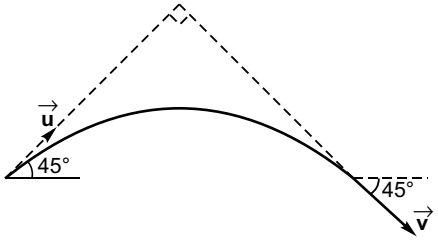
i.e., $H \propto T^2$ (Reason)

$\therefore$ H will become four times if T is made two times.

Thus, assertion is correct and also reason is the correct explanation of the assertion.

Option (a) is correct.

4. Reason is correct as $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = 0$ when $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are perpendicular to each other.

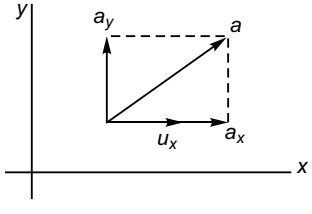


In the case mentioned $\vec{\mathbf{v}}$ is perpendicular to $\vec{\mathbf{u}}$ only at $t = T$ and not at any time t.

Option (b) is correct.

[**Note :** $\vec{\mathbf{v}}$ will never be perpendicular to $\vec{\mathbf{u}}$ if angle of projection is less than 45°.]

5. $x = u_x t + \dfrac{1}{2} a_x t^2$



and $\quad y = \dfrac{1}{2} a_y t^2$

Thus, $\quad x = u_x \sqrt{\dfrac{2y}{a_y}} + \dfrac{1}{2} a_x \dfrac{2y}{a_y}$

or $\quad x = k_1 \sqrt{y} + k_2 y$

or $\quad (x - k_2 y)^2 = k_1^2 y$

or $\quad x^2 + k_2^2 y^2 - 2k_2 y - k_1^2 y = 0$

or $\quad x^2 + k_2^2 y^2 - (2k_2 + k_1^2) y = 0$

6. $\dfrac{\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1}{t_2 - t_1}$

$$= \frac{(\vec{\mathbf{u}} + \vec{\mathbf{a}} t_2) - (\vec{\mathbf{u}} + \vec{\mathbf{a}} t_1)}{t_2 - t_1}$$

$= \vec{\mathbf{a}}$ (which is a constant quantity in projectile motion).

$\therefore$ Assertion is correct and the reason for this also correct, as explained.

Thus, option (a) is correct.

7. Let initial velocity of $A = u_A$

Initial velocity of $B = u_B$

B is projected at an angle α with horizontal.

For time (T_A) taken by A to return to the point of projection :

$$(-u_A) = (+u_A) + (-g) T_A$$

$$\Rightarrow \quad T_A = \frac{2u_A}{g} = 4 \text{ (s)}$$

Time taken by B to reach ground

$$T_B = \frac{2u_B \sin \alpha}{g} = 4 \text{ (s)}$$

($\because T_B = T_A$ given)

$\therefore$ $u_B \sin \alpha = 2g$

Height attained by B

$$H = \frac{u_B^2 \sin^2 \alpha}{2g}$$

$$= \frac{(2g)^2}{2g}$$

$$= 2g$$

$$= 20 \text{ m}$$

∴ Assertion is correct.

Reason for this incomplete as then T_A will not be equal to T_B.

Thus, option (c) is correct.

8. $$H = \frac{u_V^2}{2g}$$

∴ $$H_1 : H_2 = 4H : H$$

i.e., $$\frac{u_{1V}^2}{2g} : \frac{u_{2V}^2}{2g} = 4 : 1$$

or $$u_{1V} : v_{2V} = 2 : 1 \quad \text{...(i)}$$

$$R = \frac{2u_V u_H}{g}$$

For $R_1 = R_2$

$$u_{1V}\, u_{1H} = u_{2V}\, u_{2H}$$

$$\frac{u_{1H}}{u_{2H}} = \frac{u_{2V}}{u_{1H}}$$

or $$\frac{u_{1H}}{u_{2H}} = \frac{1}{2} \quad \text{[Using relation (i)]}$$

∴ Assertion is correct.

Reason is correct as the given relation under heading reason leads to

$$\text{Range} = v_H \times \frac{2v_V}{g}$$

∴ Option (a) is correct.

9. Assertion is incorrect as it is the velocity which would decrease by 10 m/s in the downward direction.

Acceleration is nothing but rate of change of velocity written against reason is correct.

∴ Option (d) is correct.

10. Using law of conservation of mechanical energy

$$\frac{1}{2}mv^2 + mgh = \frac{1}{2}mu^2$$

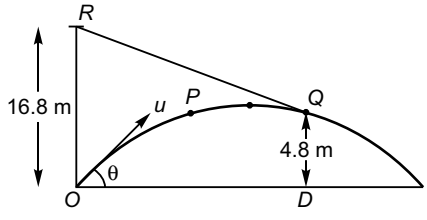
∴ $$v = \sqrt{u^2 - 2gh}$$

Thus, assertion is correct.

If the particle is projected with vertical component of velocity as u_y the vertical component of velocity of the particle at height h would be $\sqrt{u_y^2 - 2gh}$ is correct as written against reason but it is not the correct explanation of the assertion as nothing is given regarding the change in the horizontal component of velocity.

Objective Questions (Level 2)
Single Correct Option

1. $s = ut + \frac{1}{2}at^2$



$$= (u \sin\theta)\, t + \frac{1}{2}(-gt)^2$$

$$= 24 \sin\theta - 7 \times 2$$

For $\theta = 30°$: $s = 4.8$ m

Time for attaining maximum height

$$= \frac{u \sin\theta}{g}$$

$$= 1 \text{ s}$$

∴ Above displacement is for point Q.

For $\theta = 90°$: $s = 16.8$ m

For $\theta = 30°$: $OD = (u \cos\theta)\, t$

$$= (20 \cos 30°)\, 1 \times 2$$

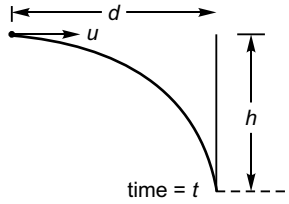
$$= 20.8$$

∴ $$QR = \sqrt{(20.8)^2 + (16.8 - 4.8)^2}$$

$$= 24 \text{ m}$$

Option (c) is correct.

2. $t = \dfrac{d}{u}$ also $h = \dfrac{1}{2}gt^2$



$$\therefore \qquad h = \frac{1}{2}g \cdot \frac{d^2}{u^2}$$

$$\Rightarrow \qquad d^2 = \frac{2u^2h}{g}$$

Option (b) is correct.

3. $R = \dfrac{u^2}{g\cos^2\beta}[\sin(2\alpha + \beta) + \sin\beta]$

Substituting $v = 10$ m/s

$g = 10$ m/s^2

$\alpha + \beta = 90°$

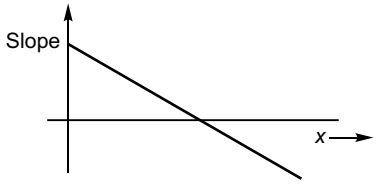
and $\beta = 30°$

$$R = \frac{(10)^2}{10\cos^2 30°}[\sin(90° + 60°) + \sin 30°]$$

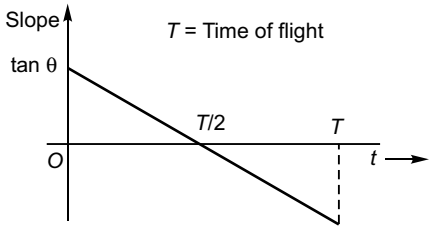
$$= \frac{10}{\left(\frac{3}{4}\right)}\left[\frac{1}{2} + \frac{1}{2}\right] = \frac{40}{3}\text{ m}$$

Option (c) is correct.

4. $y = x\tan\theta - \dfrac{1}{2}g\dfrac{x^2}{u^2\sin\theta\cos\theta}$



$$\frac{dy}{dx} = \tan\theta - \frac{1}{2}\frac{2x}{u^2\sin\theta\cos\theta}$$



Slope of trajectory $= \tan\theta - \dfrac{x}{u^2\sin\theta\cos\theta}$

Substituting $x = (u\cos\theta)\,t$

$$\frac{dy}{dx} = \tan\theta - \frac{(u\cos\theta)\,t}{u^2\sin\theta\cos\theta}$$

$$= \tan\theta - \frac{t}{u\sin\theta}$$

Option (a) is correct.

5. $y = \beta x^2$

$$\therefore \qquad \frac{dy}{dt} = \beta \cdot 2x\frac{dx}{dt}$$

and $$\frac{d^2y}{dt^2} = 2\beta\left[x \cdot \frac{d^2x}{dt^2} + \left(\frac{dx}{dt}\right)^2\right]$$

or $$\alpha = 2\beta\left[x\frac{d^2x}{dt^2} + \left(\frac{dx}{dt}\right)^2\right]$$

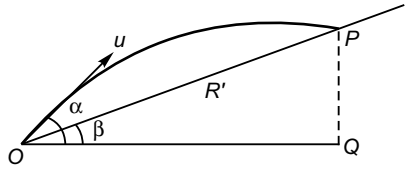
Now, as $\dfrac{d^2x}{dt^2} = 0$

(acceleration being along y-axis only)

$$\frac{dx}{dt} = \sqrt{\frac{\alpha}{2\beta}}$$

Option (d) is correct.

6. As the projectile hits the inclined plane horizontally



$$PQ = H_{\max} = \frac{u^2\sin^2\alpha}{2g}$$

and $$OQ = \frac{u^2\sin\alpha\cos\alpha}{g}$$

$\therefore$ Range on inclined plane

$$R' = \sqrt{(PQ)^2 + (OQ)^2}$$

$$= \sqrt{\left(\frac{u^2\sin^2\alpha}{2g}\right)^2 + \left(\frac{u^2\sin\alpha\cos\alpha}{g}\right)^2}$$

$$= \frac{u^2}{g}\sqrt{\frac{\sin^4\alpha}{4} + \sin^2\alpha\cos^2\alpha}$$

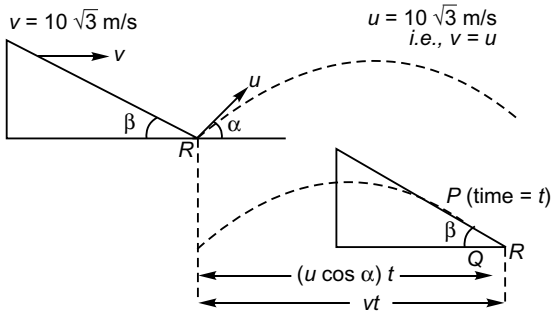
$$= \frac{u^2}{g}\sqrt{\frac{1}{4}\left(\frac{\sqrt{3}}{4}\right)^4 + \left(\frac{\sqrt{3}}{2}\right)^2\left(\frac{1}{2}\right)^2}$$

$$= \frac{u^2}{g}\sqrt{\frac{9}{64}+\frac{3}{16}}$$

$$= \frac{u^2\sqrt{21}}{8g}$$

Option (d) is correct.

7. Let the projectile hits the inclined plane at P at time t



$$PQ = (u\sin\alpha) - \frac{1}{2}gt^2$$

Further, $QR = vt - (u\cos\alpha)\,t$

and $\tan\beta = \dfrac{PQ}{QR}$

i.e., $OQ\tan\beta = PQ$

$$t[v - \cos\alpha]\tan\beta = \left[u\sin\alpha - \frac{1}{2}gt\right]t$$

or $10[1-\cos 60^\circ]\tan 30^\circ$

$$= \left[10\sin 60^\circ - \frac{1}{2}\times 10\times t\right]$$

or $\dfrac{5}{\sqrt{3}} = 5\sqrt{3} - 5t$

or $t = \sqrt{3} - \dfrac{1}{\sqrt{3}}$

$$= \frac{2}{\sqrt{3}}\text{ s}$$

8. $y^2 + 2y + 2 = x$

$\therefore$ $2y\dfrac{dy}{dt} + 2\dfrac{dy}{dt} = \dfrac{dx}{dt}$

or $(2y+2)\dfrac{dy}{dt} = \dfrac{dx}{dt}$

or $\dfrac{dx}{dt} = 10(y+1)$

or $\dfrac{d^2x}{dt^2} = 10\left(\dfrac{dy}{dt}\right)$

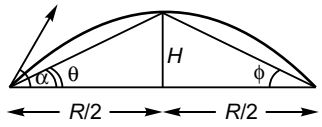
$$= 50\text{ m/s}^2$$

Acceleration of the particle

$$= \frac{d^2x}{dt^2}$$

$$= 50\text{ m/s}^2 \qquad \left[\text{as } \frac{dy}{dt} = \text{constant}\right]$$

9. $\tan\alpha = \tan\theta + \tan\phi$

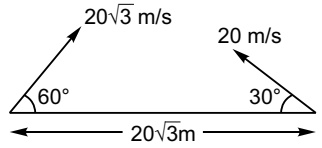


$$= \tan\theta + \tan\theta = 2\tan\theta$$

$\therefore$ $\theta = \tan^{-1}\left(\dfrac{\tan\alpha}{2}\right)$

Option (c) is correct.

10. At any time t,



Horizontal distance between particles

$$x = 20\sqrt{3} - (10\sqrt{3} + 10\sqrt{3})\,t$$

$$= 20\sqrt{3}\,(1-t)$$

Vertical distance between particles at time t

$$y = (30-10)\,t = 20\,t$$

Distance between particles at time t

$$D = \sqrt{x^2 + y^2}$$

or $D^2 = [20\sqrt{3}(1-t)]^2 + [20\,t]^2$

$$= 400[3(1-t)^2] + 400\,t^2$$

For D to minimum

$$\frac{dD}{dt} = 0$$

$$3\times 2(1-t)(-1) + 2\,t = 0$$

or $6t - 6 + 2t = 0$

i.e., $t = \dfrac{3}{4}$ s

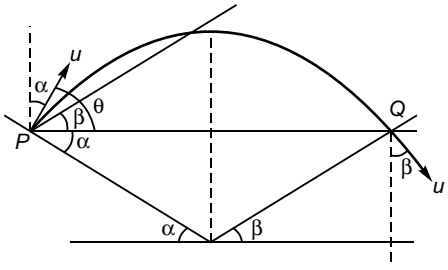
$$\therefore D^2_{\min} = 400\left[3\left(1-\frac{3}{4}\right)^2\right] + 400\left(\frac{3}{4}\right)^2$$

$$= 75 + 75\times 3$$

$\Rightarrow$ $D_{\min} = 10\sqrt{3}$ m

Option (b) is correct.

11. Let time of flight = T



As, horizontal component of the velocity of the particle will not change

$$u\cos(90° - \alpha) = v\cos(90° - \beta)$$

or $$u\sin\alpha = v\sin\beta \quad \ldots(i)$$

Using $v = u + at$ for the horizontal component of velocity

$$(-v\cos\beta) = (+u\cos\alpha) + (-g)\,T$$

$$\Rightarrow \quad gT = u\cos\alpha + v\cos\beta$$

or $$T = \frac{u\cos\alpha + v\cos\beta}{g} \quad \ldots(ii)$$

Now, as $\alpha = \beta\ (= 30°)$, $u = v$ from Eq. (i)

Substituting $v = u$ and $\alpha = \beta = 30°$ in Eq. (ii)

$$T = \frac{u\cos 30° + u\cos 30°}{g} = \frac{2u\cos 30°}{g} = \frac{u\sqrt{3}}{g}$$

Option (b) is correct.

12. $PQ = \dfrac{2u^2\sin\theta\cos\theta}{g}$

or $2a\cos\alpha = \dfrac{2u^2\sin(90° - \alpha)\cos(90° - \alpha)}{g}$

or $$a = \frac{u^2\sin\alpha}{g}$$

or $$u^2 = 2ag \quad (\text{as } \alpha = 30°)$$

or $$u^2 = 2 \times 4.9 \times 9.8$$

or $$u = 9.8 \text{ m/s}$$

Option (a) is correct.

More than One Correct Options

1. Two particles projected at angles α and β with same speed (= μ) will have same range if

$$\alpha + \beta = 90°$$

Option (a) is correct.

$$R = \frac{2u^2\sin\alpha\cos\alpha}{g}$$

h_1 (maximum height attained by first) $= \dfrac{u^2\sin^2\alpha}{2g}$

h_2 (maximum height attained by second) $= \dfrac{u^2\sin^2\beta}{2g}$

$$\therefore \quad \sqrt{h_1h_2} = \frac{u^2\sin\alpha\sin\beta}{2g} = \frac{u^2\sin\alpha\sin(90° - \alpha)}{2g} = \frac{u^2\sin\alpha\cos\alpha}{2g} = \frac{R}{4}$$

Option (b) is correct.

t_1 (time of flight of first) $= \dfrac{2u\sin\alpha}{g}$

t_2 (time of flight of second) $= \dfrac{2u\sin\beta}{g}$

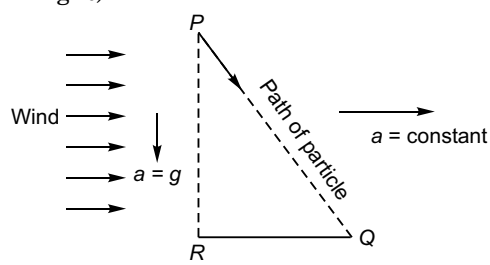
$$\therefore \frac{t_1}{t_2} = \frac{\sin\alpha}{\sin\beta} = \frac{\sin\alpha}{\sin(90° - \alpha)} = \tan\alpha$$

Option (c) is correct.

Also, $$\sqrt{\frac{h_1}{h_2}} = \frac{\sin\alpha}{\sin\beta} = \tan\alpha$$

Option (d) is correct.

2. Horizontal displacement in time t (Time of flight)



$$x = \frac{1}{2}at^2 \quad \ldots(i)$$

Vertical displacement in time t

$$y = \frac{1}{2}gt^2 \qquad \ldots(ii)$$

$$\therefore \qquad \frac{y}{x} = \frac{g}{a}$$

or y = constant x [as a is constant]

Option (a) is correct.

Substituting $y = 49$ m in Eqs. (ii)

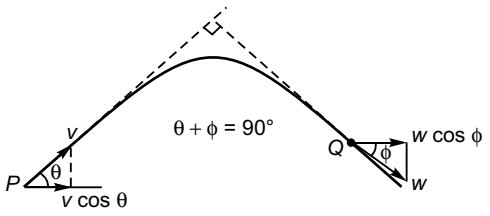
$$49 = \frac{1}{2} \times 9.8 \times t^2$$

$$\Rightarrow \qquad t = \sqrt{10} = 3.16 \text{ s}$$

$PQ > PR$ (= 40 m)

Option (d) is correct.

3. As there will be no change in the horizontal component of the velocity of the particle



$$w \cos \phi = v \cos \theta$$

or $\qquad w \cos (90° - \theta) = v \cos \theta$

or $\qquad w \sin \theta = v \cos \theta$

or $\qquad w = v \cos \theta$

Option (b) is correct.

Option (a) is incorrect.

Vertical velocity of particle at Q

$$= w \sin \phi$$
$$= w \sin (90° - \theta)$$
$$= w \cos \theta$$

Using, $v = u + at$

$$(-w \cos \theta) = (+v \sin \theta) + (-g)\,T$$

[where T = time from P to Q]

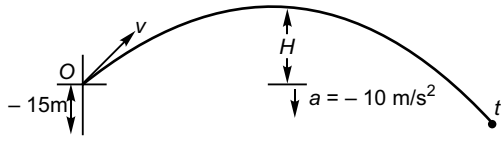
$$\Rightarrow \qquad gT = v \sin \theta + w \cos \theta$$
$$= v \sin \theta + v \cot \theta \cos \theta$$
$$= v \sin \theta + v \cdot \frac{\cos^2 \theta}{\sin \theta}$$
$$= v \operatorname{cosec} \theta$$

$$\therefore \qquad t = \frac{v \operatorname{cosec} \theta}{g}$$

Option (c) is correct.

Option (d) is incorrect.

4. $\vec{\mathbf{v}} = 10\,\hat{\mathbf{i}} + 10\,\hat{\mathbf{j}}$



If θ be the angle of projection

$$\tan \theta = \frac{v_y}{v_x} = \frac{10}{10} = 1$$

i.e., $\qquad \theta = 45°$

Option (a) correct.

Using relation, $s = ut + \frac{1}{2}at^2$

$$(-15) = 10t + \frac{1}{2}(-10)\,t^2$$

i.e., $\qquad t^2 - 2t - 3 = 0$

or $\qquad (t - 3)(t + 1) = 0$

$\therefore \qquad t = 3$ s

Option (b) is incorrect.

Horizontal range of particle

$$= v_x \cdot t$$
$$= 10 \times 3 = 30 \text{ m}$$

Option (c) is incorrect.

Maximum height of projectile from ground

$$= H + 15$$
$$= \frac{u^2 \sin^2 \theta}{2g} + 15$$
$$= \frac{v_y^2}{2g} + 15$$
$$= \frac{(10)^2}{2 \times 10} + 15$$
$$= 20 \text{ m}$$

Option (d) is correct.

5. Average velocity between any two points can't remain constant as in projectile motion velocity changes both in magnitude and direction.

Option (a) is incorrect.

Similarly option (b) is also incorrect.

In projectile motion

$$\vec{\mathbf{a}} = \text{constant}$$

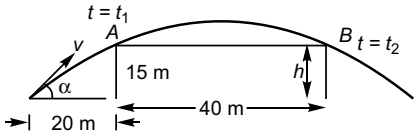
$$\therefore \quad \frac{d\vec{\mathbf{v}}}{dt} = \text{constant}$$

Option (c) is correct.

also $$\frac{d^2\vec{\mathbf{v}}}{dt^2} = 0$$

Option (d) is correct.

6. $(+h) = (+u\sin\alpha)\,t + \frac{1}{2}(-g)\,t^2$



$$\Rightarrow \quad gt^2 - (2u\sin\alpha)\,t + 2h = 0 \qquad \ldots(i)$$

$$t_1 + t_2 = \frac{2u\sin\alpha}{g},\ t_1t_2 = \frac{2h}{g}$$

$$(t_2 - t_1)^2 = (t_2 + t_1)^2 - 4\,t_1t_2$$

$$= \frac{4u^2\sin^2\alpha}{g} = 4\cdot\frac{2h}{g}$$

$$\therefore\ t_{AB} = \sqrt{\frac{8(H-h)}{g}}$$

[where H = Maximum height]

$$\Rightarrow \quad 2 = \sqrt{\frac{8(H-15)}{10}}$$

$$\therefore \quad H = 20\text{ m}$$

Option (b) is correct.

i.e., $$\frac{u^2\sin^2\alpha}{2g} = 20$$

$$u\sin\alpha = \sqrt{40g}$$

$$= 20\text{ m/s}$$

Option (c) is correct.

$$(u\cos\alpha)\,t_{AB} = 40$$

$$\Rightarrow \quad u\cos\alpha = \frac{40}{2}$$

$$= 20\text{ m/s}$$

Option (d) is correct.

Substituting values of g, $u\sin\alpha$ and h in Eq. (i)

$$10t^2 - 40t + 30 = 0$$

i.e., $$t^2 - 4t + 3 = 0$$

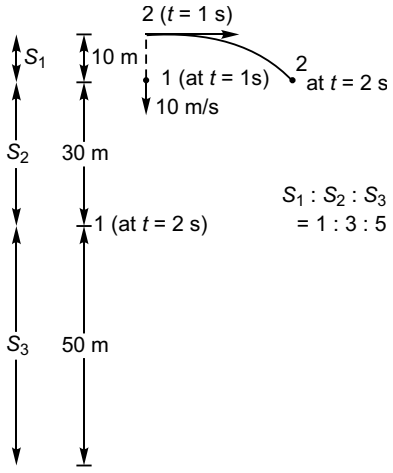
$$\therefore \quad t = 1 \text{ or } 3$$

$t = 1$ s for A and $t = 3$ s for B.

Option (a) is correct.

Match the Columns

1. (a) At $t = 2$ s



horizontal distance between 1 and 2

= horizontal displacement of 2

$= 10 \times 1 = 10$ m

$\therefore$ (a) $\rightarrow$ (p)

(b) Vertical distance between the two at $t = 2$ s $= 30$ m

$\therefore$ (b) $\rightarrow$ (s)

(c) Relative horizontal component of velocity

$$= 10 - 0 = 10\text{ m/s}$$

$\therefore$ (c) $\rightarrow$ (p)

(d) Relative vertical component of velocity (at $t = 2$ s)

= Velocity of 1 at $t = 2$ s

– Vertical velocity of 2 at $t = 2$ s

= Velocity of 1 at $t = 2$ s

– Velocity of 1 at $t = 1$ s

$= (2g) - (1g) = g$

$= 10$ m/s

(d) $\rightarrow$ (p).

2. Given, $H = 20$ m

$$\therefore \quad \frac{u^2 \sin^2 \alpha}{2g} = 20$$

$$u \sin \alpha = 20$$

$\therefore$ (d) $\rightarrow$ (s)

Time of flight $= \dfrac{2u \sin \alpha}{g} = 4$ s

(a) $\rightarrow$ (s)

$$H = \frac{R}{2} \qquad \text{(given)}$$

$$\therefore \quad \frac{u^2 \sin^2 \alpha}{2g} = \frac{1}{2} \frac{2u^2 \sin \alpha \cos \alpha}{g}$$

$$\Rightarrow \quad \frac{u \sin \alpha}{u \cos \alpha} = 2$$

$\therefore$ (b) $\rightarrow$ (q)

$$u \cos \alpha = \frac{u \sin \alpha}{2} = 10 \text{ m/s}$$

$\therefore$ (c) $\rightarrow$ (r)

3. $\dfrac{u^2 \sin (2 \times 15°)}{2g} = 10$

$$\therefore \quad \frac{u^2}{2g} = 20 \text{ m}$$

$$\Rightarrow \quad R_{max} = 20 \text{ m}$$

(a) $\rightarrow$ (q)

(b) Maximum height $= \dfrac{u^2}{4g} = 10$ m

$\therefore$ (b) $\rightarrow$ (p)

(c) Range at $(45° + \theta)$ = Range at $(45° - \theta)$

$\therefore$ Range at 75° = Range at 15°

= 10 m (given)

(c) $\rightarrow$ (p)

(d) Height at 30° $= \dfrac{u^2 \sin^2 30°}{2g}$

$$= \frac{1}{4}\left(\frac{u^2}{2g}\right)$$

$$= 5 \text{ m}$$

$\therefore$ (d) $\rightarrow$ (s)

4. (a) $T = \dfrac{2u \sin \alpha}{g}$

$$= \frac{2u_\perp}{g}$$

$\therefore$ On increasing $u_\perp$ to two times the T (time) of flight will also become two times *i.e.*, (a) $\rightarrow$ (q).

(b) $H = \dfrac{u^2 \sin^2 \alpha}{2g} = \left(\dfrac{u_\perp}{2g}\right)^2$

H_{max} will become 4 times.

$\therefore$ (b) $\rightarrow$ (r)

(c) $R = \dfrac{2(u_\perp)(u_\parallel)}{g}$

R will become two times

(c) $\rightarrow$ (q)

(d) Angle of projection with horizontal

$$\tan \alpha_{new} = \frac{u_{\perp (new)}}{u_\parallel}$$

$$= \frac{2u_\perp}{u_\parallel}$$

$$= 2 \tan \alpha$$

$$\alpha_{new} = \tan^{-1}(2 \tan \alpha) \neq 2\alpha$$

$\therefore$ (d) $\rightarrow$ (s)

5. (a) $\vec{\mathbf{u}}$ (at 0) $= (u \cos \theta)\,\hat{\mathbf{i}} + (u \sin \theta)\,\hat{\mathbf{j}}$

$\vec{\mathbf{v}}$ (at A) $= (u \cos \theta)\,\hat{\mathbf{i}}$

|Change in velocity between A & B|

$$= u \sin \theta$$

$\therefore$ (a) $\rightarrow$ (q)

(b) |Average velocity between 0 and A|

$$= \frac{|\vec{\mathbf{OA}}|}{T/2}$$

$$= \frac{\sqrt{H^2 + \left(\frac{R}{2}\right)^2}}{T/2}$$

$$= \frac{\sqrt{\left[\left(\frac{u^2 \sin^2 \theta}{2g}\right)^2 + \frac{1}{4}\left(\frac{2u \sin \theta \cos \theta}{g}\right)^2\right]}}{\frac{u \sin \theta}{g}}$$

$$= \frac{u}{2}\sqrt{1 + 3\cos^2 \theta}$$

$\therefore$ (b) $\rightarrow$ (s)

(c) Velocity at B

$$\vec{\mathbf{v}}_B = (u\cos\theta)\,\hat{\mathbf{i}} - (u\sin\theta)\,\hat{\mathbf{j}}$$

$\therefore |\vec{\mathbf{v}}_B - \vec{\mathbf{u}}| = 2u\sin\theta$

Thus, (c) $\rightarrow$ (s).

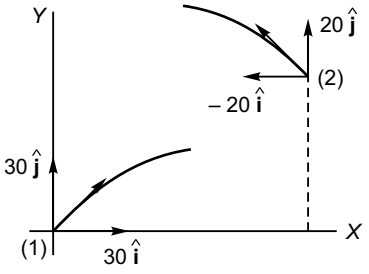
(d) |Average velocity between 0 and B|

$$= \frac{\text{Range}}{\text{Time of flight}}$$

$$= \frac{(u\cos\theta)\,T}{T} = u\cos\theta$$

$\therefore$ (d) $\rightarrow$ (p).

6. (a) Horizontal displacement of (1) at $t = 2$ s



$$\vec{\mathbf{S}}_{1H} = 30\,\hat{\mathbf{i}}\text{ m/s} \times 2\text{ s}$$

$$= +60\,\hat{\mathbf{i}}\text{ m}$$

Horizontal displacement of (2) at $t = 2$ s

$$\vec{\mathbf{S}}_{2H} = 130\,\hat{\mathbf{i}}\text{ m} + (-20\,\hat{\mathbf{i}}\text{ m/s})(1\text{ s})$$

$$= 110\,\hat{\mathbf{i}}\text{ m}$$

Horizontal displacement of (2) w.r.t. (1)

$$= (110\,\hat{\mathbf{i}} - 60\,\hat{\mathbf{i}})\text{ m} = 50\,\hat{\mathbf{i}}\text{ m}$$

$\therefore$ (a) $\rightarrow$ (r)

(b) Vertical displacement of (1) at $t = 2$ s

$$\vec{\mathbf{S}}_{1V} = 30\,\hat{\mathbf{j}}\text{ m/s} \times 2\text{ s} + \frac{1}{2}(-10\,\hat{\mathbf{j}}\text{ m/s}^2)(2\text{s})^2$$

$$= 60\,\hat{\mathbf{j}}\text{ m} - 20\,\hat{\mathbf{j}}\text{ m}$$

$$= 40\,\hat{\mathbf{j}}\text{ m}$$

Vertical displacement of (2) at $t = 2$ s

$$\vec{\mathbf{S}}_{2V} = 75\,\hat{\mathbf{j}}\text{ m} + (20\,\hat{\mathbf{j}}\text{ m/s})(1\text{ s}) + \frac{1}{2}(-10\,\hat{\mathbf{j}}\text{ m/s}^2)(16)^2$$

$$= 90\,\hat{\mathbf{j}}\text{ m}$$

$\therefore$ Vertical displacement of (2) w.r.t. (1),

$$= (90\,\hat{\mathbf{i}} - 40\,\hat{\mathbf{j}})\text{ m}$$

$$= 50\,\hat{\mathbf{j}}\text{ m}$$

$\therefore$ (b) $\rightarrow$ (r)

(c) Horizontal velocity of (1) at $t = 2$ s

$$\vec{\mathbf{v}}_{1H} = 30\,\hat{\mathbf{i}}$$

Horizontal velocity of (2) at $t = 2$ s

$$\vec{\mathbf{v}}_{2H} = -20\,\hat{\mathbf{i}}$$

Relative horizontal component of the velocity of (2) w.r.t. 1

$$\vec{\mathbf{v}}_{RH} = \vec{\mathbf{v}}_{2H} - \vec{\mathbf{v}}_{1H}$$

$$= (-20\,\hat{\mathbf{i}}) - (+30\,\hat{\mathbf{i}})$$

$$= -50\,\hat{\mathbf{i}}\text{ m/s}$$

$\therefore |\vec{\mathbf{v}}_{RH}| = 50$ m/s

Thus, (c) $\rightarrow$ (r)

(d) Vertical velocity of (1) at $t = 2$ s

$$\vec{\mathbf{v}}_{1V} = 30\,\hat{\mathbf{j}} + (-10\,\hat{\mathbf{j}})(2)$$

$$= 10\,\hat{\mathbf{j}}$$

Vertical velocity of (2) at $t = 2$ s

$$\vec{\mathbf{v}}_{1V} = 20\,\hat{\mathbf{j}} + (-10\,\hat{\mathbf{j}})(1)$$

$$= 10\,\hat{\mathbf{j}}$$

Relative vertical component of the velocity of (2) w.r.t. 1

$$\vec{\mathbf{v}}_{RV} = (10\,\hat{\mathbf{j}}) - (10\,\hat{\mathbf{j}}) = 0$$

$\therefore |\vec{\mathbf{v}}_{RV}| = 0$ m/s

Thus (d) $\rightarrow$ (s)

7. $T = \dfrac{2u_y}{g} \quad \therefore \quad T \propto u_y$

$H = \dfrac{u_y^2}{2g} \quad \therefore \quad H \propto u_y^2$

$H_A = H_B = H_C \qquad \therefore \; T_A = T_B = T_C$

Further, $R = u_x T$

T is same. But

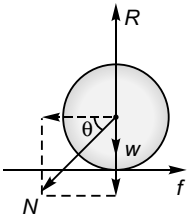
$$R_A < R_B < R_C$$

$\therefore u_{XA} < u_{XB} < u_{XC}$

5 Laws of Motion

Introductory Exercise 5.1

1. N = Normal force on cylinder by plank



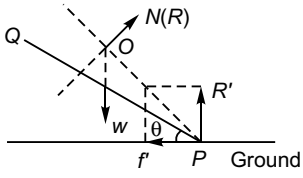
Force acting on cylinder

R = Normal force on cylinder by ground,
f = Force of friction by ground by cylinder,
w = Weight of cylinder.

$$N \cos\theta = f$$

$$w + N \sin\theta = R,$$

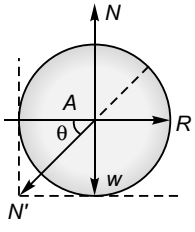
$N(R)$ = Reaction to N,



(Force acting on plank)

i.e.,normal force on plank by cylinder
R' = Normal force on plank by ground,
w = Weight of plank,
f' = frictional force on plank by ground.
Resultant of f' and R', $N(R)$ and w pass through point O.

2.

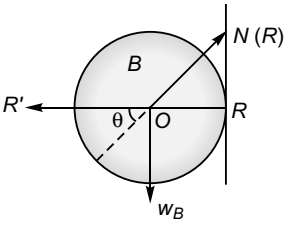


Force on sphere A

R = Normal force on sphere A by left wall,
N = Normal force on sphere A by ground,
N' = Normal force on sphere A by sphere B,
w_A = Weight of sphere A.

$$N' \cos\theta = R$$

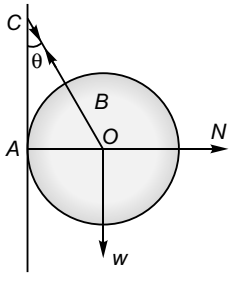
$$N' \sin\theta + w_A = N$$



Force of sphere B

R' = Normal force on sphere B by right wall,
$N(R)$ = Reaction to N *i.e.* normal force on sphere B by sphere A,
w_B = Weight of sphere B,
R', $N(R)$ and w_B pass through point O, the centre sphere B.

3. N = Normal force on sphere by wall,



w = Weight of sphere,
T = Tension in string.

4. Component of $\vec{\mathbf{F}}_1$

along x-axis : $4\cos 30° = 2\sqrt{3}$ N

along y-axis : $4\sin 30° = 2$ N

Component of $\vec{\mathbf{F}}_2$

along x-axis : $4\cos 120° = -2$ N

along y-axis : $4\sin 120° = 2\sqrt{3}$ N

Component of $\vec{\mathbf{F}}_3$

along x-axis : $6\cos 270° = 0$ N

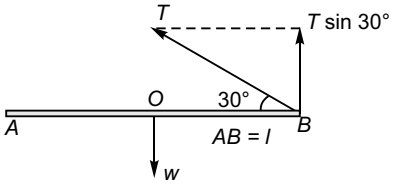
along y-axis : $6\sin 270° = -6$ N

Component of $\vec{\mathbf{F}}_4$

along x-axis : $4\cos 0° = 4$ N

along y-axis : $4\sin 0° = 0$ N

5. Taking moment about point A



$$AB = l$$

$$(T\sin 30°)\, l = w\frac{l}{2}$$

$$\Rightarrow \qquad T = w$$

6. See figure (answer to question no. 3)

$$\sin\theta = \frac{OA}{OB + BC}$$

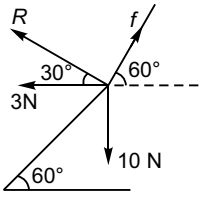
$$= \frac{a}{a+a} = \frac{1}{2}$$

$$T\cos 30° = w$$

or $$T\frac{\sqrt{3}}{2} = w$$

or $$T = \frac{2}{\sqrt{3}}w$$

7. $R\cos 30° + 3 = f\cos 60°$



i.e., $$\frac{R\sqrt{3}}{2} + 3 = \frac{f}{2}$$

or $$R\sqrt{3} + 6 = f \qquad \ldots(i)$$

and $$R\sin 30° + f\sin 60° = 10$$

i.e., $$R\frac{1}{2} + f\frac{\sqrt{3}}{2} = 10$$

or $$R + f\sqrt{3} = 20 \qquad \ldots(ii)$$

Substituting the value of f from Eq. (i) in Eq. (ii)

$$R + (R\sqrt{3} + 6)\sqrt{3} = 20$$

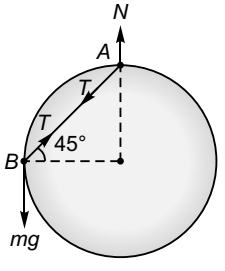
$$4R + 6\sqrt{3} = 20$$

$$\Rightarrow \qquad R = \frac{20 - 6\sqrt{3}}{4} = 2.4 \text{ N}$$

$$\therefore \qquad f = (2.4)\sqrt{3} + 6$$

$$= 10.16 \text{ N}$$

8. At point B (instantaneous vertical acceleration only)



$$\therefore \qquad mg - T\sin 45° = ma \qquad \ldots(i)$$

At point A (instantaneous horizontal acceleration only)

$$\therefore \qquad T\cos 45° = ma \qquad \ldots(ii)$$

Combining Eqs. (i) and (ii)

$$mg - ma = ma$$

$$\Rightarrow \qquad a = \frac{g}{2}$$

Introductory Exercise 5.2

1. Acceleration of system

$$a = \frac{(+120)+(-50)}{1+4+2}$$

$$= 10\,\text{m/s}^2$$

Let normal force between 1 kg block and 4 kg block = F_1

$\therefore$ Net force on 1 kg block = $120 - N$

$\therefore$ $$a = \frac{120 - F_1}{1}$$

or $$10 = 120 - F_1$$

i.e., $$F_1 = 110\,\text{N}$$

Net force on 2 kg block $= 2 \times a$

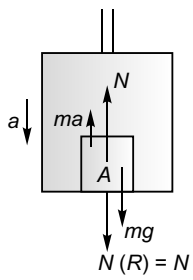
$= 2 \times 10$

$= 20\,\text{N}$

2. As, $4g \sin 30° > 2g \sin 30°$

The normal force between the two blocks will be zero.

3. $N(R) = \dfrac{mg}{4}$



$\therefore$ $$N = \frac{mg}{4}$$

As lift is moving downward with acceleration a, the pseudo force on A will be ma acting in the upward direction.

For the block to be at rest w.r.t. lift.

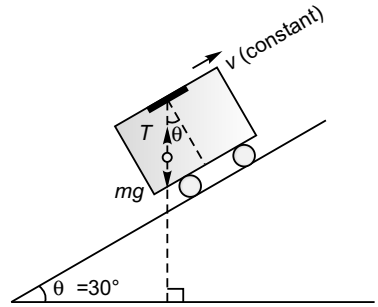
$$N + ma = mg$$

or $$\frac{mg}{4} + ma = mg$$

$\Rightarrow$ $$a = \frac{3}{4}g$$

4. Angle made by the string with the normal to the ceiling $= \theta = 30°$

As the train is moving with constant velocity no pseudo force will act on the plumb-bob.

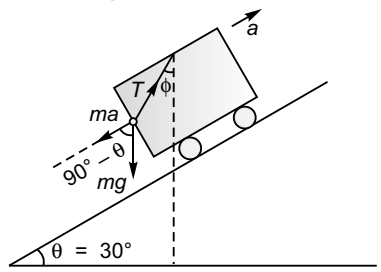


Tension in spring $= mg$

$= 1 \times 10$

$= 10\,\text{N}$

5. Pseudo force ($= ma$) on plumb-bob will be as shown in figure



$$T\cos\phi = mg + ma\cos(90° - \theta)$$

i.e., $$T\cos\phi = mg + ma\sin\theta \quad \ldots\text{(i)}$$

and $$T\sin\phi = ma\cos\theta$$

Squaring and adding Eqs. (i) and (ii),

$$T^2 = m^2g^2 + m^2a^2\sin^2\theta + 2m^2ag\sin\theta + m^2a^2\cos^2\theta \quad \ldots\text{(iii)}$$

$$T^2 = m^2g^2 + m^2a^2 + m^2ag \quad (\because \theta = 30°)$$

$$= m^2g^2 + m^2\frac{g^2}{4} + m^2\frac{g}{2}\cdot g \quad \left(\because a = \frac{g}{2}\right)$$

$$= \frac{7m^2g^2}{4}$$

or $$T = \frac{mg\sqrt{7}}{2}$$

$$= 5\sqrt{7}\,\text{N}$$

Dividing Eq. (i) by Eq. (ii),

$$\tan\phi = \frac{ma\cos\theta}{mg + ma\sin\theta}$$

$$= \frac{a\cos\theta}{g + a\sin\theta}$$

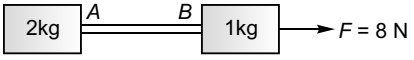
$$= \frac{\cos\theta}{2+\sin\theta}$$

$$= \frac{\cos 30°}{2+\sin 30°}$$

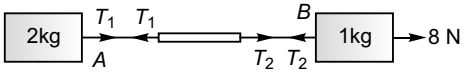
$$= \frac{\sqrt{3}}{5}$$

i.e., $\phi = \tan^{-1}\left[\frac{\sqrt{3}}{5}\right]$

6.



$$a = \frac{8}{2+1+1} = 2\ \text{m/s}^2$$



Net force on 1 kg mass $= 8 - T_2$

$\therefore \quad 8 - T_2 = 1 \times 2$

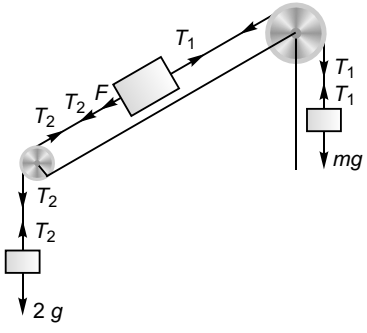
$\Rightarrow \quad T_2 = 6\ \text{N}$

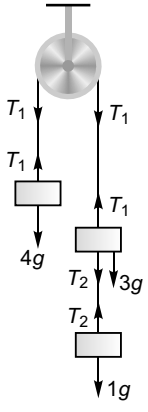
Net force on 1 kg block $= T_1$

$\therefore \quad T_1 = 2a = 2 \times 2 = 4\ \text{N}$

Introductory Exercise 5.3

1. $F = 2g\sin 30° = g$





For the system to remain at rest

$T_2 = 2g$ ...(i)

$T_2 + F = T_1$ ...(ii)

or $T_2 + g = T_1$ ...[ii (a)]

$T_1 = mg$ ...(iii)

Substituting the values of T_1 and T_2 from Eqs. (iii) and (i) in Eq. [ii(a)]

$$2g + g = mg$$

i.e., $m = 3$ kg

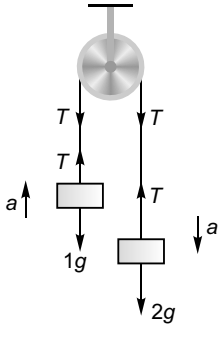
2. As net downward force on the system is zero, the system will be in equilibrium

$\therefore \quad T_1 = 4g$

and $\quad T_2 = 1g$

$\therefore \quad \frac{T_1}{T_2} = 4$

3. $2g - T = 2a$



$T - 1g = 1a$

Adding above two equations

$$1g = 3a$$

$$\therefore \quad a = \frac{g}{3}$$

Velocity of 1kg block 1 section after the system is set in motion

$$v = 0 + at$$
$$= \frac{g}{3} \cdot 1$$
$$= \frac{g}{3} \quad \text{(upward)}$$

On stopping 2 kg, the block of 1kg will go upwards with retardation g. Time (t') taken by the 1 kg block to attain zero velocity will be given by the equation.

$$0 = \left(\frac{g}{3}\right) + (-g)\,t'$$

$$\Rightarrow \quad t' = \frac{1}{3}\text{ s}$$

If the 2 kg block is stopped just for a moment (time being much-much less than $\frac{1}{3}$ s), it will also start falling down when the stopping time ends.

In $t'\left(=\frac{1}{3}\text{s}\right)$ time upward displacement of 1 kg block

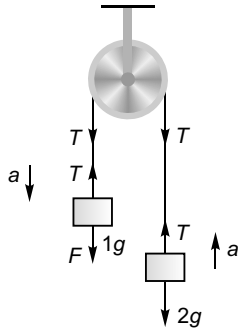
$$= \frac{u^2}{2a} = \frac{(g/3)^2}{2g} = \frac{g}{18}$$

Downward displacement of 2 kg block

$$= \frac{1}{2}at^2 = \frac{1}{2}g \cdot \left(\frac{g}{3}\right)^2 = \frac{g}{18}$$

As the two are just equal, the string will again become taut after time $\frac{1}{3}$ s.

4. $$F + 1g - T = 1a \quad \text{...(i)}$$



and $$T - 2g = 2a \quad \text{...(ii)}$$

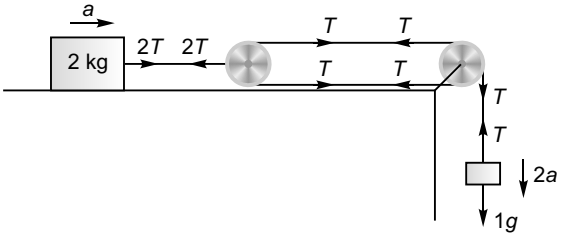
Adding Eqs. (i) and (ii),

$$F - 1g = 3a$$

$$\therefore \quad a = \frac{20 - 10}{3} = \frac{10}{3}\text{ ms}^{-2}$$

Introductory Exercise 5.4

1. $$2T = 2 \times a \quad \text{...(i)}$$



and $$1g - T = 2a \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii),

$$a = \frac{g}{3}$$

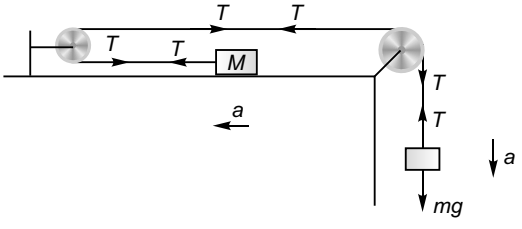
$\therefore$ Acceleration of 1 kg block

$$2a = \frac{2g}{3} = \frac{20}{3}\text{ ms}^{-2}$$

Tension in the string

$$T = \frac{g}{3} = \frac{10}{3}\text{ N}$$

2. $$Mg - T = Ma \quad \text{...(i)}$$



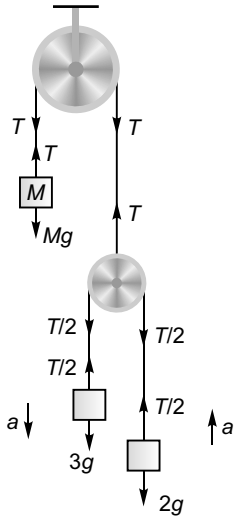
$$T = Ma \qquad \ldots(ii)$$

Solving Eqs. (i) and (ii)

$$a = \frac{g}{2}$$

and $\quad T = Mg$

3. Block of mass M will be at rest if



$$T = Mg \qquad \ldots(i)$$

For the motion of block of mass 3 kg

$$3g - \frac{T}{2} = 3a \qquad \ldots(ii)$$

For the motion of block of mass 2 kg

$$\frac{T}{2} - 2g = 2a \qquad \ldots(iii)$$

Adding Eqs. (ii) and (iii),

$$g = 5a$$

i.e., $\quad a = \frac{g}{5}$

Substituting above value of a in Eq. (iii),

$$\frac{T}{2} = 2(g + a)$$

or $\quad T = 4\left(g + \frac{g}{5}\right)$

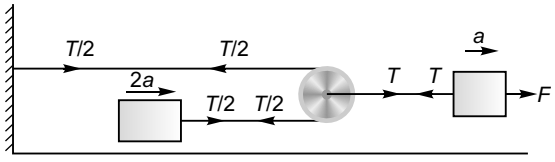
$$= \frac{24\,g}{5}$$

Substituting value of above value of T in Eq. (i),

$$M = \frac{24}{5}$$

$$= 4.8 \text{ kg}$$

4. $\quad \frac{T}{2} = m_1 \cdot 2a$



i.e., $\quad T = 4m_1a \qquad \ldots(i)$

$$F - T = m_2a \qquad \ldots(ii)$$

or $\quad F - 4m_1a = m_2a$

or $\quad a = \frac{F}{4m_1 + m_2}$

$$= \frac{0.40}{(4 \times 0.3) + 0.2}$$

$$= \frac{0.40}{1.4}$$

$$= \frac{2}{7}\text{ ms}^{-2}$$

$\therefore \quad T = 4m_1a$

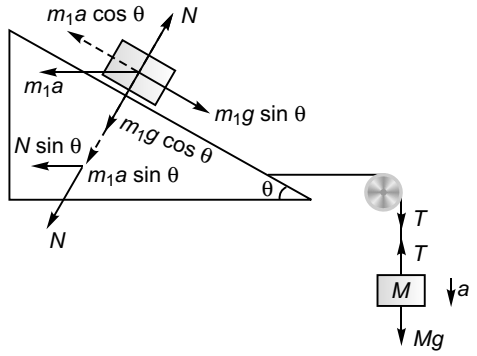
$$= 4 \times 0.3 \times \frac{2}{7}$$

$$= \frac{2.4}{7}$$

$$= \frac{12}{35}\text{ N}$$

Introductory Exercise 5.5

1. Block on triangular block will not slip if



$$m_1a\cos\theta = m_1g\sin\theta$$

i.e., $$a = g\tan\theta \quad \text{...(i)}$$

$$N = m_1g\cos\theta + m_1a\sin\theta \quad \text{...(ii)}$$

For the movement of triangular block

$$T - N\sin\theta = m_2a \quad \text{...(iii)}$$

For the movement of the block of mass M

$$Mg - T = Ma \quad \text{...(iv)}$$

Adding Eqs. (iii) and (iv),

$$Mg - N\sin\theta = (m_2 + M)\,a$$

Substituting the value of N from Eq. (ii) in the above equation

$$Mg - (m_1g\cos\theta + m_1a\sin\theta)\sin\theta = (m_2a + Ma)$$

i.e., $$M(g - a) = m_1g\cos\theta\sin\theta + (m_2 + m_1\sin^2\theta)\,a$$

Substituting value of a from Eq. (i) in the above equation,

$$M(1 - \tan\theta) = m_1\cos\theta\sin\theta + (m_2 + m_1\sin^2\theta)\tan\theta$$

$$\therefore M = \frac{m_1\cos\theta\sin\theta + (m_2 + m_1\sin^2\theta)\tan\theta}{(1 - \tan\theta)}$$

Substituting $\theta = 30°$, $m_1 = 1$ kg and $m_2 = 4$ kg

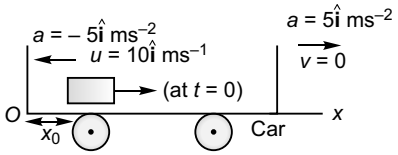
$$M = \frac{(\cos 30°\sin 30° + 4 + \sin^2 30°)\tan 30°}{(1 - \tan 30°)}$$

$$= \frac{0.443 + (4.25)(0.577)}{0.423}$$

$$= 6.82 \text{ kg}$$

2. (a) Using $s = s_0 + ut + \frac{1}{2}at^2$

Displacement of block at time t relative to car would be



$$x = x_0 + 10t + \frac{1}{2}(-5)\,t^2$$

or $$x = x_0 + 10t - 2.5t^2$$

Velocity of block at time t (relative to car) will be

$$v = \frac{dx}{dt} = 10 - 5t$$

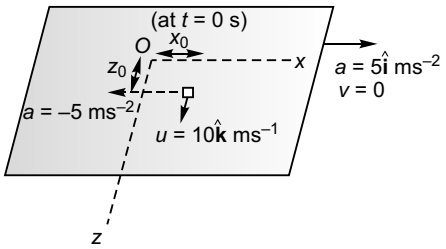
(b) Time (t) for the block to arrive at the original position (*i.e.,* $x = x_0$) relative to car

$$x_0 = x_0 + 10t - 2.5t^2$$

$\Rightarrow$ $$t = 4 \text{ s}$$

3. (a) In car's frame position of object at time t would be given by

In car's frame



$$x = x_0 + 0 \times t + \frac{1}{2}(-5)\,t^2$$

i.e., $$x = x_0 - 2.5t^2 \quad \text{...(i)}$$

and $$z = z + 10t \quad \text{...(ii)}$$

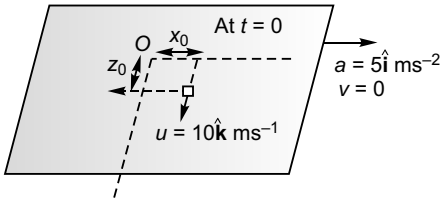
Velocity of the object at time t would be

$$v_x = \frac{dx}{dt} = -5t \quad \text{...(iii)}$$

and $$v_z = \frac{dz}{dt} = 10 \quad \text{...(iv)}$$

(b) In ground frame the position of the object at time t would be given by

In ground frame



$$x = x_0$$

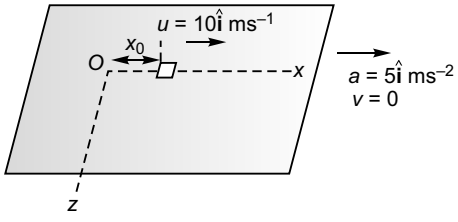
and $$z = z_0 + 10t$$

Velocity of the object at time t would be

$$v_x = \frac{dx}{dt} = 0$$

and $$v_z = \frac{dz}{dt} = 10 \text{ ms}^{-1}$$

4. $m = 2$ kg



Normal force on object $= mg$

Maximum sliding friction $= \mu_s mg$

$$= 0.3 \times 2 \times 10 = 6 \text{ N}$$

Deceleration due to friction $= \frac{6}{2} = 3$ m/s^2

Deceleration due to pseudo force $= 5$ m/s^2

$\therefore$ Net deceleration $= (3 + 5)$ m/s^2

$$= 8 \text{ m/s}^2$$

$\therefore$ Displacement of object at any time t (relative to car)

$$x = x_0 + 10t + \frac{1}{2}(-8)\,t^2$$

or $$x = x_0 + 10t - 4t^2$$

Thus, velocity of object at any time t (relative to car)

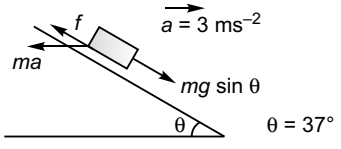
$$v_x = \frac{dx}{dt} = 10 - 8t$$

The object will stop moving relative to car when

$$10 - 8t = 0 \textit{ i.e., } t = 1.25 \text{ s}$$

$\therefore$ $$v_x = 10 - 8t \text{ for } 0 < t < 1.25 \text{ s}$$

For block not to slide the frictional force (f) would be given by



$$f + ma\cos\theta = mg\sin\theta$$

or $$f = mg\sin\theta - ma\cos\theta$$

$$= m \times 10 \times \frac{3}{5} - m \times 3 \times \frac{4}{5}$$

$$= \frac{18m}{5} = \frac{9}{25}mg$$

AIEEE Corner

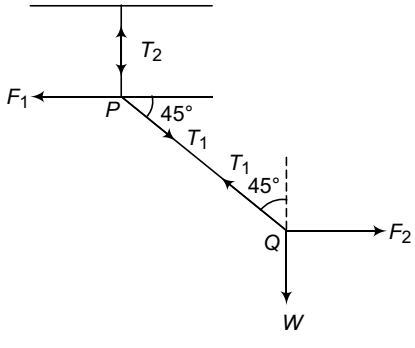
Subjective Questions (Level 1)

1. FBD is given in the answer.

2. FBD is given in the answer.

3. FBD is given in the answer.

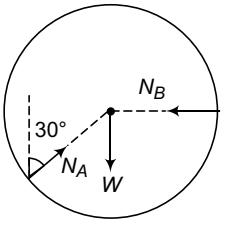
4.



At point *P*, $F_1 = \frac{T_1}{\sqrt{2}}$ and $T_2 = \frac{T_1}{\sqrt{2}}$

At point *Q*, $F_2 = \frac{T_1}{\sqrt{2}}$ and $W = \frac{T_1}{\sqrt{2}}$

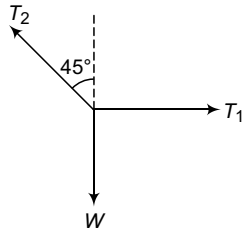
5.



$N_A \sin 30° = N_B$

and $N_A \cos 30° = W$

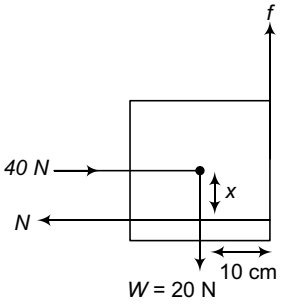
6.



$$\frac{T_2}{\sqrt{2}} = W \text{ and}$$

$$\frac{T_2}{\sqrt{2}} = T_1$$

7.

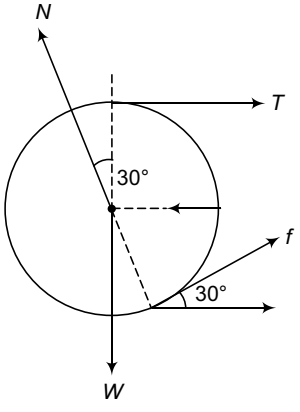


$$N = 40 \quad \ldots(i)$$

$$f = 20 \quad \ldots(ii)$$

and $$f \times 10 = N \times x \quad \ldots(iii)$$

8.

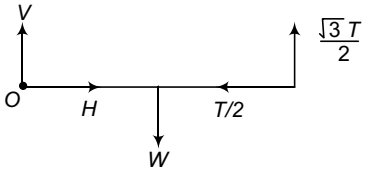


$$T + f\cos 30° = N\sin 30° \quad \ldots(i)$$

$$N\cos 30° + f\sin 30° = W \quad \ldots(ii)$$

$$T \times R = fR \quad \ldots(iii)$$

9.



$$V + \frac{\sqrt{3}T}{2} = W \quad \ldots(i)$$

$$H = \frac{T}{2} \quad \ldots(ii)$$

Net moment about O = zero

$$\therefore \quad W \times \frac{l}{2} = \frac{\sqrt{3}T}{2} \times l \quad \ldots(iii)$$

10. (a) $a = \dfrac{100 - 40}{6 + 4 + 10} = 3 \text{ m/s}^2$

(b) Net force = ma

$\therefore \quad F_6 = 18 \text{ N}, F_4 = 12 \text{ N and } F_{10} = 30 \text{ N}$

(c) $N - 40 = F_{10} = 30$

$\therefore \quad N = 70 \text{ N}.$

11. $a = \dfrac{F}{m_1 + m_2 + m_3} = \dfrac{60}{60} = 1 \text{ m/s}^2$

(a) $T_1 = m_1 a = 10 \text{ N}$

$T_2 - T_1 = m_2 a$

$\therefore \quad T_2 - 10 = 20$

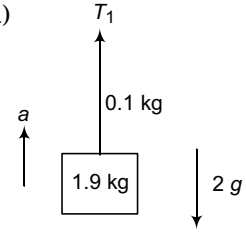
$\therefore \quad T_2 = 30 \text{ N}$

(b) $T_1 = 0$. New acceleration

$$a' = \frac{60}{50} = 1.2 \text{ m/s}^2$$

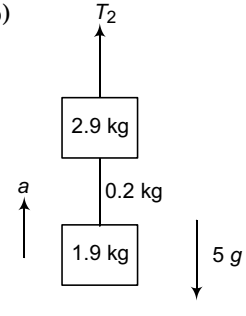
$$T_2 = m_2 a' = 24 \text{ N}$$

12. (a)



$$T_1 - 2g = 2a$$

(b)



$$T_2 - 5g = 5a$$

13. (a) $a = \dfrac{200 - 16g}{16}$

(b) $T_1 - 11g = 11a$

(c) $T_2 - 9g = 9a$

14. If the monkey exerts a force F on the rope upwards, then same force F transfers to bananas also. If monkey releases her hold on rope both monkey and bananas fall freely under gravity.

15. Tension on $B = T$

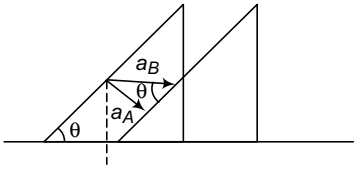
Tension on $A = 3T$

Now in these situations $a \propto \dfrac{1}{T}$

16. $x_A + x_C + 2x_B = \text{constant}$.

Differentiating twice w.r.t. time we get the acceleration relation.

17.



$$\frac{a_A}{a_B} = \sin\theta$$

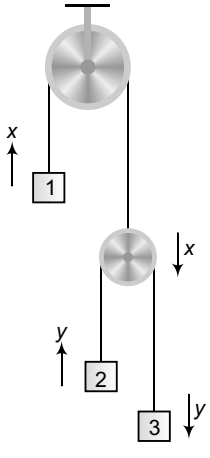
$\therefore \quad a_A = a_B \sin\theta$

18. $x + y = 6 \quad \ldots(i)$

$y - x = 4 \quad \ldots(ii)$

Solving, we get

$x = 1 \text{ m/s}^2$



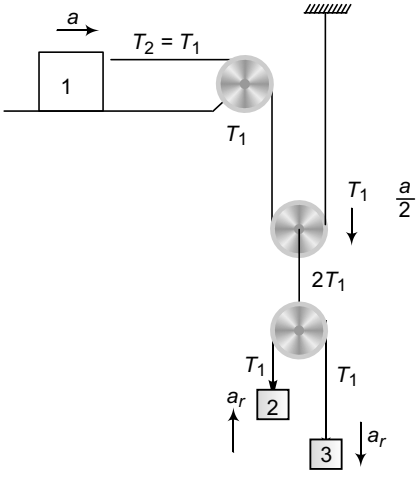
19. $$a = \frac{7g - 3g}{10} = 4 \text{ m/s}^2$$

$40 - T_1 = 4a$

$30 + T_1 - T_2 = 3a$

$T_3 - 10 = 1a$

20.

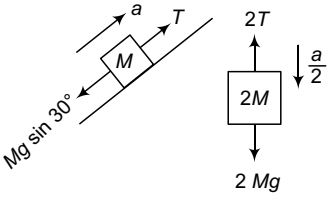


$T_1 = 1a \quad \ldots(i)$

$T_1 - 20 = 2(a_r - a/2) \quad \ldots(ii)$

$30 - T_1 = 3(a_r + a/2) \quad \ldots(iii)$

21.



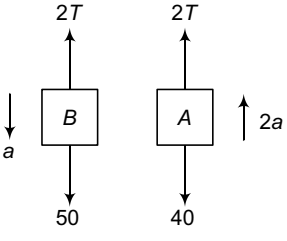
$T - Mg\sin 30° = Ma \quad \ldots(i)$

$2Mg - 2T = 2M \cdot \dfrac{a}{2} \quad \ldots(ii)$

22. $T = 1a \quad \ldots(i)$

$10 - T = 1a \quad \ldots(ii)$

23.



$50 - 2T = 5a \quad \ldots(i)$

$T - 40 = 4(2a) \quad \ldots(ii)$

24. (a) $N = 40 \text{ N}, \mu_s N = 24 \text{ N}$

$F < \mu_s N$

$\therefore f = 20 \text{ N}$ and $a = 0$

(b) $N = 20 \text{ N}, \mu_s N = 12 \text{ N}$

and $\mu_k N = 8 \text{ N}$

$$F > \mu_s N$$

$\therefore \quad f = \mu_k N = 8$ N

and $\quad a = \dfrac{20-8}{2} = 6$ m/s^2

(c) $N = 60 - 20 = 40$ N

$\mu_s N = 8$ N and

$\mu_k N = 4$ N

Since, $F \cos 45° > \mu_s N$

$\therefore \quad f = \mu_k N = 4$ N

and $\quad a = \dfrac{20-4}{6}$

$= \dfrac{8}{3}$ m/s^2

25. $a = \mu g = 3$ m/s^2

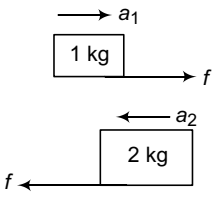
(a) $v = at$

$\therefore \quad 6 = 3t$ or $t = 2$ s

(b) $s = \dfrac{1}{2}at^2$

$\therefore \quad s = \dfrac{1}{2} \times 3 \times 4 = 6$ m.

26. $f = 0.4 \times 1 \times 10 = 4$ N



$a_1 = \dfrac{f}{1} = 4$ m/s^2

$a_2 = \dfrac{f}{2} = 2$ m/s^2

(a) Relative motion will stop when

$$v_1 = v_2$$

or $\quad 2 + 4t = 8 - 2t$

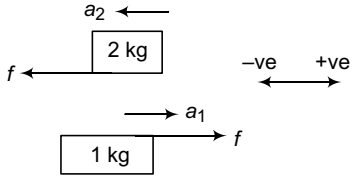
$\therefore \quad t = 1$ s

(b) $v_1 = v_2 = 2 + 4 \times 1 = 6$ m/s

(c) $s_1 = u_1 t + \dfrac{1}{2}a_1 t^2$

$s_2 = u_2 t - \dfrac{1}{2}a_2 t^2$

27. $f = (0.6)(2)(10) = 12$ N



$a_2 = \dfrac{12}{2} = 6$ m/s^2

and $a_1 = \dfrac{12}{1} = 12$ m/s^2

(a) Relative motion will stop when

$$v_1 = v_2$$

or $\quad u_1 + a_1 t = u_2 + a_2 t$

or $\quad 3 - 6t = -18 + 12t$

$\therefore \quad t = \dfrac{7}{6}$ s

(b) Common velocity at this instant is v_1 or v_2.

(c) $s_1 = u_1 t + \dfrac{1}{2}a_1 t^2$ and

$$s_2 = u_2 t + \frac{1}{2}a_2 t^2$$

28. $N = 20$ N

$\mu_s N = 16$ N

and $\mu_k N = 12$ N

Since, $W = 20$ N $> \mu_s N$, friction $\mu_k N$ will act.

$\therefore \quad a = \dfrac{20-12}{2} = 4$ m/s^2

29. $N = 20$ N

$\mu N = 16$ N

Block will start moving when

$$F = \mu N$$

or $\quad 2t = 16$

or $\quad t = 8$ s.

After 8 s

$a = \dfrac{2t-16}{2} = t - 8$

i.e., *a-t* graph is a straight line with positive slope and negative intercept.

30. $N = 60$ N, $\mu_s N = 36$ N, $\mu_k N = 24$ N

Block will start moving when

$$F = \mu_s N$$

or $\quad 4t = 36$

$\therefore \quad t = 9$ s

After 9 s

$$a = \frac{4t - 24}{6} = \frac{2}{3}t - 4$$

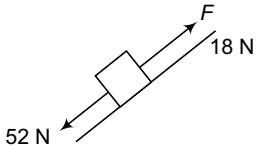
31. $N = mg\cos\theta = 30$ N

$mg\sin 30° = 30\sqrt{3}$ N $\approx$ 52 N.

$\mu_s N = 18$ N and

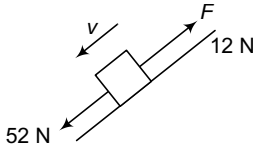
$\mu_k N = 12$ N

(a)



$F = 52 - 18 = 34$ N.

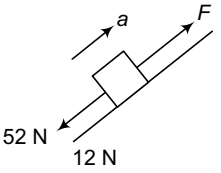
(b)



$F + 12 = 52$

$\therefore \quad F = 40$ N

(c)



$F - 52 - 12 = 6 \times 4$

$\therefore \quad F = 88$ N

Objective Questions (Level-1)

Single Correct Option

1. $a = \dfrac{mg - F}{m} = g - \dfrac{F}{m}$

$m_A > m_B$

$\therefore \quad a_A > a_B$

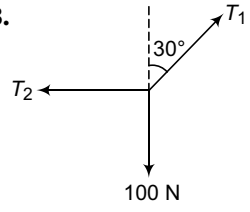
or ball A reaches earlier.

2. $a = \dfrac{4g - 2g}{6} = \dfrac{g}{3}$

Now, $2g - T = 2\dfrac{g}{3}$

$\therefore \quad T = \dfrac{4g}{3} = 13$ N

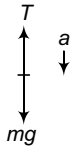
3.



$$\frac{\sqrt{3}T_1}{2} = 100 \quad \text{...(i)}$$

$$\frac{T_1}{2} = T_2 \quad \text{...(ii)}$$

4.



$mg - T = ma$

$\therefore \quad a_{\min} = g - \dfrac{T_{\max}}{m}$

$= g - \dfrac{\frac{2}{3}mg}{m} = \dfrac{g}{3}$

5. $a = \dfrac{10g - 5g}{15} = \dfrac{g}{3}$

$10g - T = 10 \times \dfrac{g}{3}$

$\therefore \quad T = \dfrac{20g}{3} =$ Reading of spring balance.

6. $a = \dfrac{2mg\sin 30° - mg}{3m} = 0$

$\therefore \quad T = mg$

7. $a_1 = g\sin\theta - \mu\, g\cos\theta = g\sin 45° - \mu\, g\cos 45°$

$a_2 = g\sin\theta = g\sin 45°$

Now $t = \sqrt{\dfrac{2s}{a}}$ or $t \propto \dfrac{1}{\sqrt{a}}$

$\therefore \quad \dfrac{t_1}{t_2} = \sqrt{\dfrac{a_2}{a_1}}$

or $\quad 2 = \sqrt{\dfrac{g\sin 45°}{g\sin 45° - \mu\, g\cos 45°}}$

Solving, we get $\mu = \dfrac{3}{4}$

8. $F_1 = mg\sin\theta + \mu mg\cos\theta$

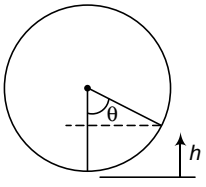
$F_2 = mg\sin\theta - \mu mg\cos\theta$

Given that $F_1 = 2F_2$

9. For equilibrium of block, net force from plane should be equal and opposite of weight.

10. No solution is required.

11. Angle of repose $\theta = \tan^{-1}(\mu) = 30°$



$$\therefore \quad h = R - R\cos\theta = \left(1 - \frac{\sqrt{3}}{2}\right)R$$

12. Net pulling force $= 15g - 5g = 10g = F$

Net retarding force $= (0.2)(5g) = g = f$

$$\therefore \quad a = \frac{F - f}{25} = \frac{9}{25}g$$

$$T_1 - 5g = 5a = \frac{9}{5}g$$

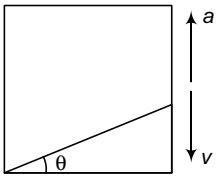
$$\therefore \quad T_1 = \frac{34}{5}g$$

$$15g - T_2 = 15a = \frac{27}{5}g$$

$$\therefore \quad T_2 = \frac{48}{5}g$$

$$\therefore \quad \frac{T_1}{T_2} = \frac{17}{24}$$

13.



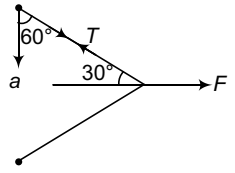
Relative to lift, $a_r = (g + a)\sin\theta$ along the plane.

$$\text{Now, } t = \sqrt{\frac{2s}{a_r}}$$

$$= \sqrt{\frac{2L}{(g + a)\sin\theta}}$$

14. $f = mg\sin\theta$ (if block is at rest)

15. $2T\cos 30° = F$

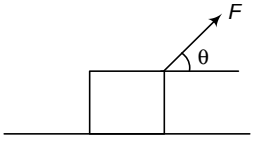


$$\therefore \quad T = \frac{F}{\sqrt{3}}$$

$$a = \frac{T\cos 60°}{m}$$

$$= \frac{F}{2\sqrt{3}\,m}$$

16.



$N = mg - F\sin\theta$

$\mu N = (\tan\phi)\,N = (\tan\phi)(mg - F\sin\theta)$

$F\cos\theta = \mu N$

17. $\mu mg = 0.2 \times 4 \times 10 = 8$ N

At $t = 2$ s, $F = 4$ N

Since $F < \mu mg$

$\therefore$ Force of friction $f = F = 4$ N

18. $a_1 = g\sin\theta$

$a_2 = g\sin\theta - \mu g\cos\theta$

$$t = \sqrt{\frac{2S}{a}} \text{ or } t \propto \frac{1}{\sqrt{a}}$$

$$\frac{t_1}{t_2} = \sqrt{\frac{a_2}{a_1}}$$

$$\frac{1}{2} = \sqrt{\frac{g\sin\theta - \mu g\cos\theta}{g\sin\theta}}$$

19. Net accelaration of man relative to ground $= a + a = 2a$

$$T - mg = m(2a)$$

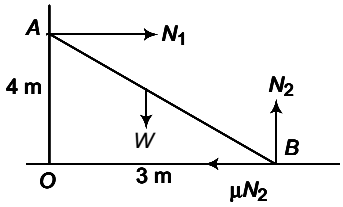
$$\therefore \quad T = m(g + 2a)$$

20. $a = 0$

$3F = (50 + 25)g = 75g$

$\therefore F = 25g = 250$ N

21.

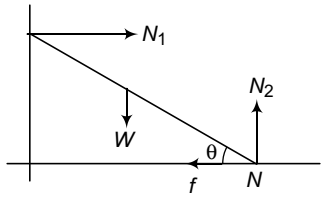


$$N_1 = \mu\, N_2$$
$$N_2 = W$$

Net moment about B should be zero.

$\therefore \quad W \times \frac{3}{2} = N_1 \times 4$

22.



$N_2 = W = 250$ (always)

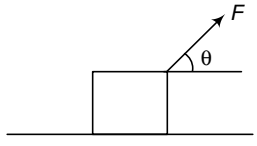
$\therefore \quad f_{max} = \mu N_2 = 75$ N

23. $a = \dfrac{Mg - \frac{1}{2}Mg}{M} = \dfrac{g}{2}$

$$T - \frac{1}{2} \times \frac{M}{2} \times g = \frac{M}{2} \times a = \frac{Mg}{4}$$

$\therefore \quad T = \dfrac{Mg}{2}$

24.

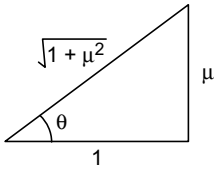


$N = mg - F \sin\theta$

$F \cos\theta = \mu\, N = \mu(mg - F \sin\theta)$

$F = \dfrac{\mu\, mg}{\cos\theta + \mu \sin\theta}$

For F to be minimum, $\dfrac{dF}{d\theta} = 0$



or $\quad \tan\theta = \mu$

$$F_{min} = \frac{\mu mg}{\dfrac{1}{\sqrt{1+\mu^2}} + \dfrac{\mu^2}{\sqrt{1+\mu^2}}}$$
$$= \frac{\mu}{\sqrt{1+\mu^2}}\, mg$$
$$= \frac{1}{2}\, mg$$

25. $\mu mg = 32$ N

Since $\quad \therefore\ f = 30$ N

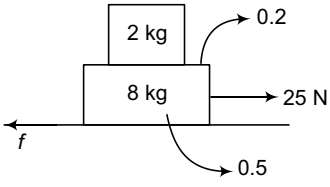
$30\text{ N} > \mu mg$

$\therefore$ Normal reaction $N = mg = 40$ N

$\therefore$ Net contact force $= \sqrt{(30)^2 + (40)^2}$

$= 50$ N

26.



$f_{max} = (0.5)(8 + 2)(10) = 50\text{N} > 25\text{ N}$

$\therefore$ Blocks will not move and therefore force of friction between two blocks = 0.

27. $mg \sin\theta = 10 \times 10 \times \dfrac{3}{5} = 60$ N

This $60\text{ N} > 30\text{ N}$

$\therefore$ Force of friction is upwards.

Net contact force is resultant of friction and normal reaction.

28. In first figure $T = F$

and in second figure, $T = 2F$.

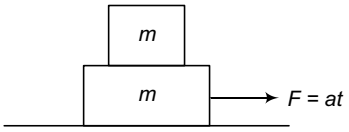
29. $F = 4\left(\dfrac{mg}{2}\right) \rightarrow$ upwards

$W = 2mg \rightarrow$ downwards

$F = W$

$\therefore \quad a = 0$

30.



Maximum acceleration of upper block due to friction

$$a_{max} = \frac{\mu mg}{m} = \mu g$$

$$\frac{F}{2m} = \frac{at}{2m} = \mu g$$

$$\therefore \quad t = \frac{2\mu m g}{a}$$

31. $a_1 = g \sin 30° = g/2$

$a_2 = g \sin 60° = \frac{\sqrt{3}}{2} g$

Angle between $\vec{\mathbf{a}}_1$ and $\vec{\mathbf{a}}_2$ is 30°.

$\therefore$

$$a_r = |\vec{\mathbf{a}}_1 - \vec{\mathbf{a}}_2| = \sqrt{(g/2)^2 + (\sqrt{3}g/2)^2 - 2(g/2)(\sqrt{3}g/2)\cos 30°}$$

$$= g/2$$

JEE Corner

Assertion and Reason

1. Even if net force = 0, rotational motion can take place.

2. No solution is required.

3. $a_1 = g \sin\theta + \mu g \cos\theta$

$a_2 = g \sin\theta - \mu g \cos\theta$

$\frac{a_1}{a_2} = \frac{1+\mu}{1-\mu}$ as $\theta = 45°$ and $\sin\theta = \cos\theta$

Substituting $\mu = \frac{1}{3}$ we have,

$$\frac{a_1}{a_2} = \frac{1+1/3}{1-1/3} = \frac{2}{1}$$

4. There is no force for providing $(-2\hat{\mathbf{i}})$ m/s^2 to the block.

5. If we increase F_1, maximum value of friction will increase. But if we increase F_2 friction acting on the block will increase.

6. No solution is required.

7. $a = \frac{(m_1 - m_2)g}{m_1 + m_2}$ (if $m_1 > m_2$)

$m_1 g - T = m_1 a$

$\therefore \quad T = m_1(g - a)$

$= \frac{2m_1 m_2 g}{(m_1 + m_2)}$

Mathematically, we can prove that

$$m_2 g < T < m_1 g$$

Similarly if $m_2 > m_1$, then we can prove that,

$$m_1 g < T < m_2 g$$

8. If accelerations of both the frames are same then one frame as observed from other frame will be inertial.

Further, a frame moving with constant velocity is inertial.

9. No solution is required.

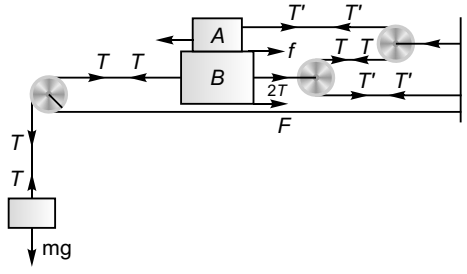
10. No solution is required.

11. Force of friction is in the direction of motion.

Objective Questions (Level 2)

Single Correct Option

1. $T_{max} = F_{max} + f_{max} + 2T'_{max}$



$= \mu(m_A + m_B)g + \mu m_A g + 2\mu m_A g$

$= \mu(4m_A + m_B)g$

But, $T_{max} = M_{max} \cdot g$

$\therefore$ $M_{max} \cdot g = \mu(4m_A + m_B)g$

i.e., $M_{max} = \mu(4m_A + m_B)$

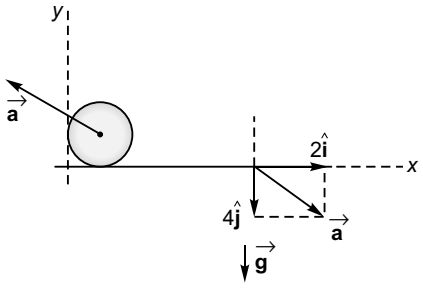
$= 0.3 \times [(4 \times 100) + 70] = 81$ kg

Option (c) is correct.

2. $\vec{\mathbf{v}} = 8t\,\hat{\mathbf{i}} - 2t^2\,\hat{\mathbf{j}}$

$\therefore$ $\vec{\mathbf{a}} = \dfrac{d\vec{\mathbf{v}}}{dt}$

$= 8\,\hat{\mathbf{i}} - 4t\,\hat{\mathbf{j}}$



$\therefore$ Force (pseudo) on sphere

$\vec{\mathbf{F}}_s = 1 \cdot (-8\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}})$

$= -8\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}}$

Gravitational force on sphere

$= -mg\,\hat{\mathbf{j}} = -10\,\hat{\mathbf{j}}$

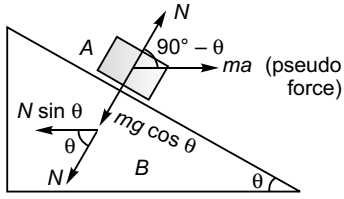
$\therefore$ Net force on sphere

$\vec{\mathbf{F}} = -8\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}} - 10\,\hat{\mathbf{j}} = -8\,\hat{\mathbf{i}} - 6\,\hat{\mathbf{j}}$

$|\vec{\mathbf{F}}| = 10$ N

Option (b) is correct.

3. $N \sin\theta$ will accelerate B towards left.



Let a be the acceleration of B.

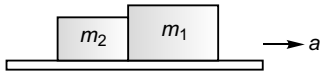
Due to acceleration of B towards left pseudo force equal to ma will act on block, toward right.

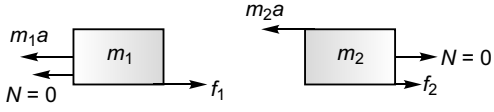
Thus, $N + ma\sin\theta = mg\cos\theta$

$\Rightarrow$ $N = mg\cos\theta - ma\sin\theta$

Option (b) is correct.

4. If $a \le \mu g$





When a is	f_1	N	f_2
Zero	Zero	Zero	Zero
$\frac{\mu g}{2}$	$m_1 \frac{\mu g}{2}$	Zero	$m_2 \frac{\mu g}{2}$
μg	$m_1 \mu g$ (Not yet attained limiting value which is $2\mu m_1 g$)	Zero	$m_2 \mu g$ (Attained limiting value)

Now, if $a > \mu g$, the block of mass m_2 would be just greater like to move towards left than μg as a then

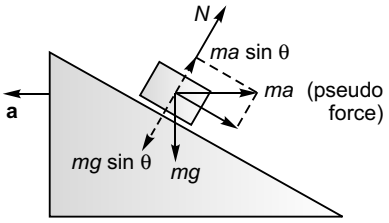
$m_2 a > f_2$ (max)

While the block of mass m_1 will remain at rest as then f_1 has margin to increase.

Thus, m_1 will apply normal force (N) or m_2 and so will do m_1 on m_2 to stop its motion (towards left).

Option (d) is correct.

5. $N + ma\sin\theta = mg\cos\theta$



The block will fall freely if $N = 0$

i.e., $ma\sin\theta = mg\cos\theta$

$\Rightarrow$ $a = g\cos\theta$

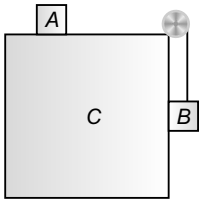
Option (c) is correct.

6. Speed of A w.r.t. $C = v_1$

Speed of C w.r.t. ground $= v_0$

$\therefore$ Speed of A w.r.t. ground $= v_1 - v_0$

Now, speed of B w.r.t. ground = speed of A is w.r.t. ground

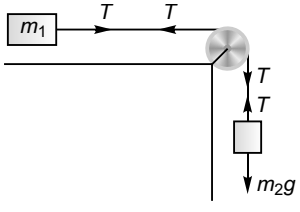


$\therefore$ $v_2 = v_1 - v_0$

$\Rightarrow$ $v_1 - v_2 = v_0$

Option (a) is correct.

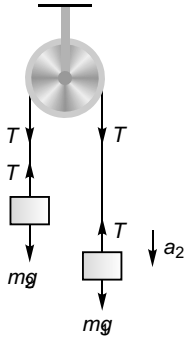
7. **Fig. 1**



$$m_2g - T = m_2a_1$$

and $$a_1 = \frac{m_2}{m_1 + m_2}g$$

Fig. 2

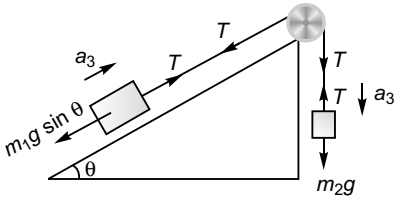


$$m_1g - T = m_2a_2$$

and $$T - m_2g = m_1a_2$$

$\Rightarrow$ $$a_2 = \frac{m_1 - m_2}{m_1 + m_2}g$$

Fig. 3



$$m_2g - T = m_2a_3$$

and $$T - m_1g\sin\theta = m_1a_3$$

$\therefore$ $$a_3 = \frac{m_2 - m_1\sin\theta}{m_1 + m_2}g$$

Substituting $m_1 = 4$ kg, $m_2 = 3$ kg

and $\theta = 30°$

$$a_1 = \frac{3}{4+3}g$$

$$= \frac{3}{7}g$$

$$a_2 = \frac{4-3}{4+3}g$$

$$= \frac{1}{7}g$$

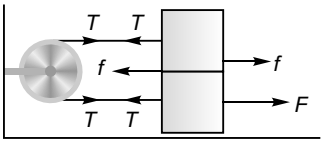
$$a_3 = \frac{3 - 4^2 \cdot \frac{1}{2}}{4+3}g$$

$$= \frac{1}{7}g$$

$\therefore$ $a_1 > a_2 = 3$

Option (b) is correct.

8. $F - f_{max} - T = ma$ (For lower block)



$T - f_{max} = ma$ (For upper block)

$\Rightarrow$ $F - 2f_{max} = 2ma$

or $F - 2\mu mg = 2ma$

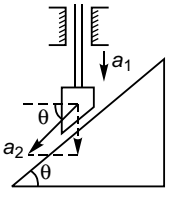
or $a = \frac{F}{2m} - \mu g$

Option (c) is correct.

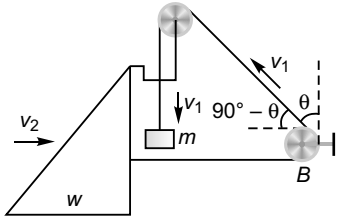
9. $a_2\cos(90° - \theta) = a_1$

i.e., $a_2\sin\theta = a_1$ [$\theta < 0°$]

Option (b) is correct.



10. Had θ been 90°.



For every x displacement of wedge (w) the vertical fall in mass would have been $2x$ as the string passes through pulley B.

i.e., $v_1 = 2v_2$

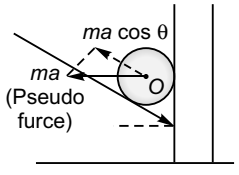
For the situation given the speed of mass would thus be

$$v_1 = 2v_2 \cos(90° - \theta)$$

i.e., $v_1 = 2v_2 \sin\theta$

Option (c) is correct.

11. The cylinder will start rising up the inclined plane if



$$ma\cos\theta > mg\sin\theta$$

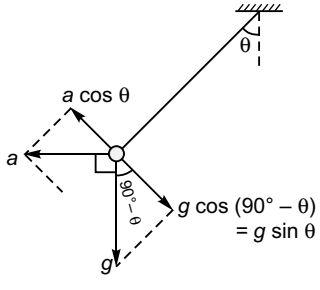
i.e., $a > g\tan\theta$

$\therefore\ a_{min} = g\tan\theta$ [cylinder will be at the point of rising up the inclined plane]

12. At the position of maximum deflection the net acceleration of bob (towards its mean position will be)

$$g\sin\theta - a\cos\theta$$

as explained in figure.



13. $Mg - T = Ma$

$T - mg = ma$

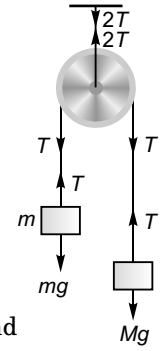
$$\Rightarrow \quad a = \frac{M-m}{M+m}g$$

$$\therefore \quad T = m(g+a)$$

$$= m\left[g + \frac{M-m}{M+m}g\right]$$

$$= mg\left[1 + \frac{M-m}{M+m}\right]$$

$$= mg[1+1]$$



{as $m << M$, $M - m \approx M$ and $M + m \approx M$}

$$= 2mg$$

$\therefore$ Tension in the string suspended from ceiling $= 2T = 4mg$

Option (a) is correct.

14. For $0° < \theta \le \phi$ frictional force $(F) = mg\sin\theta$

For $\theta \ge \phi$ frictional force $(F) = \mu\, mg\cos\theta$

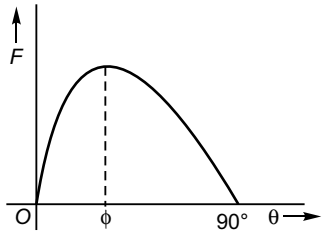
[ϕ = angle of repose]

For example

Let $\mu = \frac{1}{\sqrt{3}}$ $\quad\therefore\quad \tan\phi = \frac{1}{\sqrt{3}}$

$$\phi = 30°$$

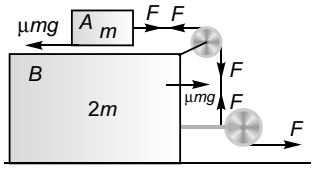
θ	F (force of friction)	Condition
10°	$mg\sin 10° = 0.174\, mg$	
20°	$mg\sin 20° = 0.342\, mg$	Increase but not linearly only
30°	$\mu mg\cos 30° = 0.500\, mg$	
40°	$\mu mg\cos 40° = 0.442\, mg$	Decrease but not linearly only
60°	$\mu mg\cos 60° = 0.287\, mg$	
90°	Zero	



Option (b) is correct.

15. $$a_B = \frac{\mu\, mg}{2m}$$

$$= \frac{\mu g}{2}$$



For no slipping

$$a_A = a_B$$

$$\frac{F - \mu mg}{m} = \frac{\mu g}{2}$$

i.e., $$F = \frac{3}{2}\mu mg$$

$$\mu = \frac{2}{3}\frac{F}{mg}$$

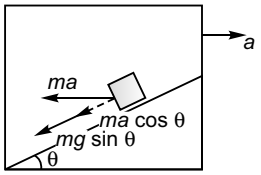
Slipping will obviously be m_2 there if μ is greater than above mentioned value

$$\therefore \quad \mu_{\min} = \frac{2}{3}\frac{F}{mg}$$

For no slipping.

Option (c) is correct.

16. F_{net} (downward) $= mg\sin\theta + ma\cos\theta$



$$= m(g\sin\theta + a\cos\theta)$$

$$\therefore \quad g_{\text{eff}} = g\sin\theta + a\cos\theta$$

Time (T) required to cover $2L$ distance along inclined would be

$$T = \sqrt{\frac{2L}{g_{\text{eff}}}}$$

$$= \sqrt{\frac{2L}{(g\sin\theta + a\cos\theta)}}$$

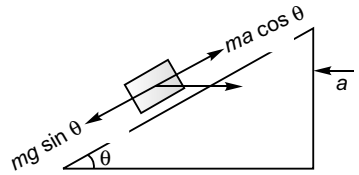
Option (c) is correct.

17. F_{net} on block along incline in the upward direction

$$= ma\cos\theta - mg\sin\theta$$

$$= m(a\cos\theta - g\sin\theta)$$

$$\therefore \quad g_{\text{eff}} = a\cos\theta - g\sin\theta$$



$\therefore$ Time (t) to move s distance would be given by

$$s = \frac{1}{2}g_{\text{eff}}t^2$$

i.e., $$t = \sqrt{\frac{2s}{g_{\text{eff}}}} = \sqrt{\frac{2s}{(a\cos\theta - g\sin\theta)}}$$

Substituting $s = 1$ m,

$$\theta = 30°,$$

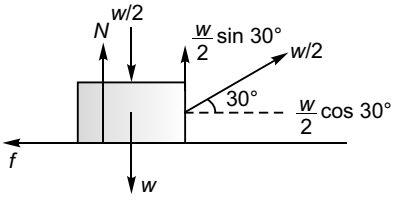
$$a = 10\sqrt{3}\ \text{m/s}^2 \text{ and } g = 10\ \text{m/s}^2$$

$$t = \sqrt{\frac{2\times 1}{\left(10\sqrt{3}\times\frac{\sqrt{3}}{2}\right) - \left(10\times\frac{1}{2}\right)}}$$

$$= \frac{1}{\sqrt{5}}\ \text{s}$$

Option (b) is correct.

18. $N + \frac{w}{2}\sin 30° = w + \frac{w}{2}$



$$f = \text{frictional force}$$

$$N = \frac{5w}{4}$$

$$f_{\max} = \mu N = \mu \cdot \frac{5w}{4}$$

The block will remain stationary if

$$\frac{w}{2}\cos 30° \le \frac{\mu\, 5w}{4}$$

or $$\frac{w}{2}\frac{\sqrt{3}}{2} \le \mu\frac{5w}{4}$$

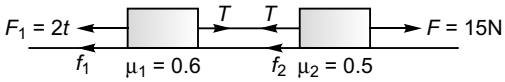
or $$\sqrt{3} \le 5\mu$$

or $$\frac{\sqrt{3}}{5} \le \mu$$

$\therefore$ Block will move if $\mu < \frac{\sqrt{3}}{5}$

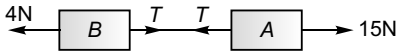
Option (d) is correct.

19.



At $t = 2$ s

$F_1 = 4$ N



f_1 and f_2 are the frictional forces

$$(f_1)_{max} = 0.6 \times 1 \times 10$$
$$= 6\,\text{N}$$
$$(f_2)_{max} = 0.5 \times 2 \times 10$$
$$= 10\,\text{N}$$

At $t = 2$ s

Net external force (F_{net}) on system

$$= 15\,\text{N} - 4\,\text{N}$$
$$= 11\,\text{N}$$

As $F_{net} > (f_1)_{max} + (f_2)_{max}$, the system will remain at rest and the values frictional forces on the blocks will be given

$$T = 4 + f_1 \text{ and } T = 15 - f_2$$
$$4 + f_1 = 15 - f_2 \qquad \text{...(i)}$$
$$f_1 + f_2 = 11\,\text{N}$$

Let direction being + ive for Eq. (i)

Option (a) $f_1 = -4$ N, $f_2 = = 5$ N

$\Rightarrow f_1 + f_2 = 1$ N wrong

Option (b) $f_1 = -2$ N, $f_2 = +5$ N

$\Rightarrow f_1 + f_2 = 3$ N wrong

Option (c) $f_1 = 0$ N, $f_2 = +10$ N

$\Rightarrow f_1 + f_2 = 10$ N wrong

Option (d) $f_1 = +1$ N, $f_2 = +10$ N

$\Rightarrow f_1 + f_2 = 11$ N correct.

OR

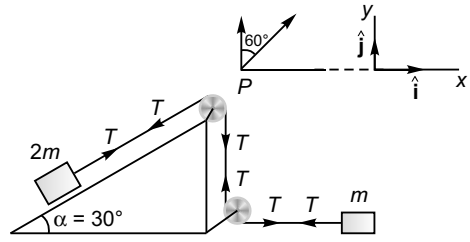
As the likely movement would be towards right f_2 will be at its maximum.

$\therefore \quad f_2 = 10$ N

$\Rightarrow \quad f_1 = 1$ N

Option (d) is correct.

20. $2mg \sin\alpha - T = 2ma$



and $\quad T = ma$

$\therefore \quad 2mg \sin\alpha = 3ma$

$\Rightarrow \quad a = \dfrac{2g \sin 30°}{3} = \dfrac{g}{3}$

$\therefore \quad T = \dfrac{mg}{3}$

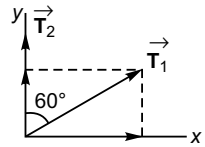
Force (R) applied by clamp on pulley would be

$$|\vec{T}_1| = |\vec{T}_2| = T$$

$$\vec{T}_1 = \frac{mg}{3}(\cos 30°)\,\hat{\mathbf{i}} + \frac{mg}{3}(\sin 30°)\,\hat{\mathbf{j}}$$
$$= \frac{mg\sqrt{3}}{6}\,\hat{\mathbf{i}} + \frac{mg}{6}\,\hat{\mathbf{j}}$$

$$|\vec{T}_2| = \frac{mg}{3}\,\hat{\mathbf{j}}$$

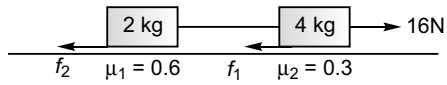
Force by clamp on pulley P



$$= \vec{T}_1 + \vec{T}_2$$
$$= \frac{mg\sqrt{3}}{6}\,\hat{\mathbf{i}} + \frac{mg}{6}\,\hat{\mathbf{j}} + \frac{mg}{3}\,\hat{\mathbf{j}}$$
$$= \frac{mg\sqrt{3}}{6}\,\hat{\mathbf{i}} + \frac{3mg}{6}\,\hat{\mathbf{j}} = \frac{mg}{6}(\sqrt{3}\,\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}})$$

Option (b) is correct.

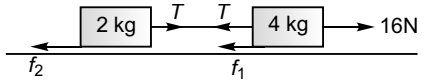
21. $f_1(\text{max}) = 0.3 \times 4 \times 10 = 12$ N



f_1 and f_2 are frictional forces.

$f_2(\text{max}) = 0.6 \times 2 \times 10 = 12$ N

As, $f_1(\text{max}) + f_2(\text{max}) < 16$ N (F_{ext})



The system will remain at rest.

For the equilibrium of 4 kg mass :

$\therefore \quad 16 = T + f_1 \quad \ldots(i)$

As f_1 will be at its maximum value

$$f_1 = 12\,\text{N}$$

$\therefore \quad T = 16 - 12$

$= 4\,\text{N}$ [from Eq. (i)]

Further, for the equilibrium of 2 kg mass.

$$T = f_1$$

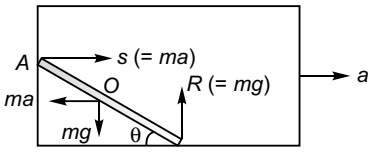
$\therefore \quad f_1 = 4\,\text{N}$

Option (c) is correct.

22. For the rotational equilibrium of rod

Taking moment about O.

$$R \times \frac{l}{2}\cos\theta = s\frac{l}{2}\sin\theta$$



or $\quad mg\cos\theta = ma\sin\theta$

$\Rightarrow \quad a = g\cot\theta$

Option (d) is correct.

23. $v = 2t^2$

$\therefore \quad a = \dfrac{dv}{dt} = \dfrac{d}{dt}(2t^2) = 4t$

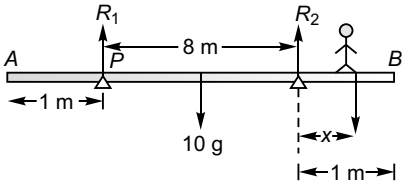
At $t = 1$ s, $a = 4\ \text{ms}^{-2}$

As $\quad a = \mu_s g$

$$\mu_s = \frac{a}{g} = \frac{4}{10} = 0.4$$

Option (c) is correct.

24. Just at the position of tipping off, R_1 will be zero.



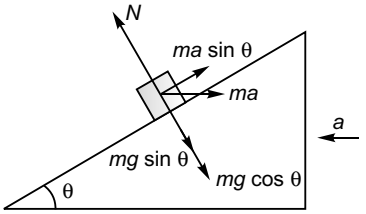
$\therefore$ Taking moment about point Q

$$(10g)(4) = (80g)(x)$$

$\therefore \quad x = \dfrac{1}{2}\,\text{m}$

Option (a) is correct.

25. $N = ma\sin\theta + mg\cos\theta \quad \ldots(i)$



Now, as the block does not slide

$$ma\cos\theta = mg\sin\theta$$

i.e., $\quad a = g\tan\theta$

Substituting the found value of a in Eq. (i)

$$N = m(g\tan\theta)\sin\theta + mg\cos\theta$$

$$= mg\left[\frac{\sin^2\theta}{\cos}\theta + \cos\theta\right] = mg\sec\theta$$

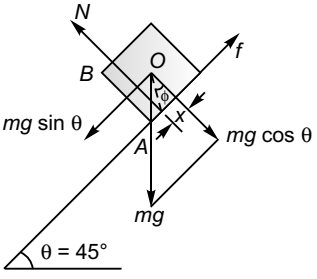
When, the block stops $a = 0$, the value of normal force will be

$$N' = mg\cos\theta$$

$\therefore \quad \dfrac{N'}{N} = \dfrac{mg\cos\theta}{mg\sec\theta}$

Option (c) is correct.

26. For the rotational equilibrium of the block



Taking moment about O.

$$Nx = f\frac{a}{2}$$

or $\quad (mg\cos\theta)\,x = (mg\sin\theta)\dfrac{a}{2}$

or $\quad x = \dfrac{a}{2}\tan\theta$

or $\quad \dfrac{x}{a/2} = \tan\theta$

or $\quad \tan\phi = \tan\theta$

or $\quad \phi = \theta$

Thus, the normal force (N) will pass through point A.

Option (a) is correct.

[Note : The cube will be just at the point of tilting (about point A). The cube will tilt if θ is made greater than 45°].

27. For the rotational equilibrium of the cube

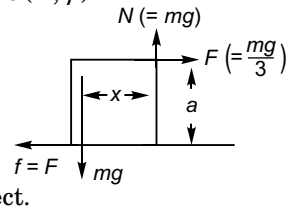
Moment of couple (N, mg)

= Moment of couple (F, f)

$\therefore \quad mgx = Fa$

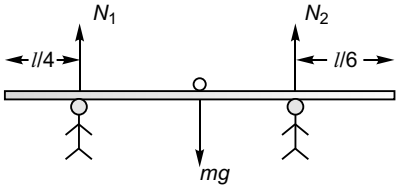
or $mgx = \frac{mg}{3} \cdot a$

i.e., $x = \frac{a}{3}$



Option (b) is correct.

28. Taking moment about point O.



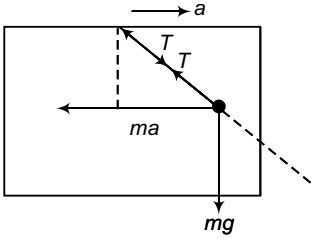
$$N_1\left(\frac{l}{2} - \frac{l}{4}\right) = N_2\left(\frac{l}{2} - \frac{l}{6}\right)$$

$$N_1 \frac{l}{4} = N_2 \frac{l}{3}$$

$\therefore \quad N_1 : N_2 = 4 : 3$

Option (c) is correct.

29.



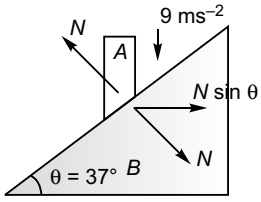
$(T / \sqrt{2}) = ma \rightarrow$ Box

$T + \frac{ma}{\sqrt{2}} = \frac{mg}{\sqrt{2}} \rightarrow$ Pendulum with respect to box

$\therefore \quad \sqrt{2}ma + \frac{ma}{\sqrt{2}} = \frac{mg}{\sqrt{2}}$

$\therefore \quad a = g / 3$

30. $N \sin\theta = m_B a_B$ ...(i)

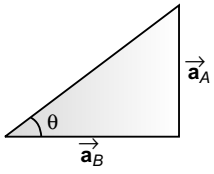


$a_A = a_B \tan\theta$ ...(ii)

$\Rightarrow \quad N \sin\theta = m_B \frac{a_A}{\tan\theta}$

$$N = \frac{m_B a_A}{\sin\theta \tan\theta}$$

$\therefore$ Force on rod by wedge



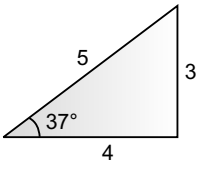
$a_A = |\vec{a}_A|$

$a_B = |\vec{a}_B|$

$$N \cos\theta = \frac{m_B a_A \cos\theta}{\sin\theta \tan\theta}$$

$$= \frac{m_B a_A}{\tan^2\theta}$$

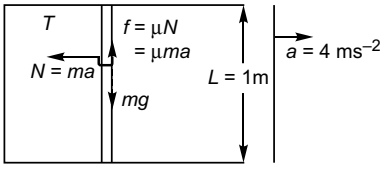
$$= \frac{10 \times 9}{\left(\frac{3}{4}\right)^2} = 160 \text{ N}$$



Option (c) is correct.

31. Net downward force on ring = $mg - \mu\, ma$

$= m(g - \mu a)$



$\therefore \quad g_{\text{eff}} = g - \mu\, a$

$$t = \sqrt{\frac{2L}{g_{\text{eff}}}}$$

$$= \sqrt{\frac{2L}{g - \mu\, a}}$$

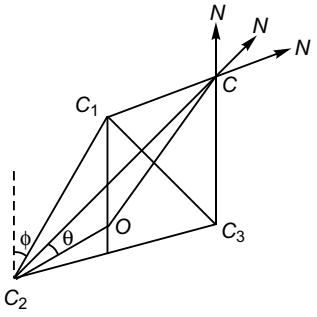
$$= \sqrt{\frac{2 \times 1}{10 - (0.5 \times 4)}}$$

$$= \frac{1}{2} = 0.5 \text{ s}$$

32. The direction of the normal reactions between any one hemisphere and the sphere will be along the centres of the two. The three centres of the hemisphere and that of sphere will form a tetrehadron of edge equal to $2R$.

In figure, C_1, C_2 and C_3 are the centres of the hemispheres and C is the centre of the sphere

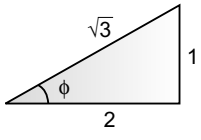
$$C_1C_2 = C_2C_3 = C_3C_1$$
$$= C_1C = C_2C = C_3C = 2R$$



$$\angle COC_2 = 90°$$
$$C_2O = \frac{2R}{\sqrt{3}}$$
$$\cos\theta = \frac{C_2O}{C_2C}$$
$$= \frac{2R/\sqrt{3}}{2R}$$
$$= \frac{1}{\sqrt{3}}$$

ϕ is the angle which any N makes with vertical

$$\phi = 90° - \theta$$
$$\sin\phi = \cos\theta = \frac{1}{\sqrt{3}}$$
$$\therefore \quad \cos\phi = \frac{2}{\sqrt{3}}$$



For the vertical equilibrium of the sphere.

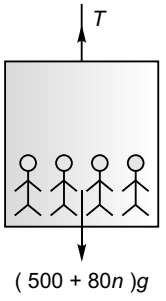
$$3N\cos\phi = mg$$

or $$3N \times \frac{2}{\sqrt{3}} = mg$$

or $$N = \frac{mg}{2\sqrt{3}}$$

Option (b) is correct.

33. $T - Mg = Ma$



i.e., $$T = M(g + a)$$

$\therefore$ $$2 \times 10^4 \geq M(g + a)$$

or $$2 \times 10^4 \geq (500 + 80n)(10 + 2)$$

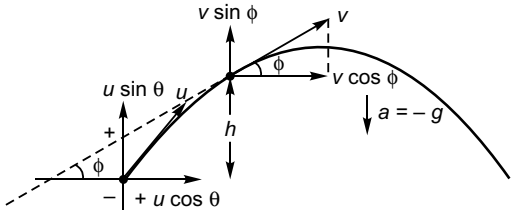
or $$14.58 \geq n$$

or $$n = 14$$

Option (b) is correct.

[Note : Tension in lift cable will increase when the lift is accelerated upwards].

34. Normal reaction between the surface and the particle will be zero throughout the motion if the path of the particle is that of a projectile motion (particle is free from surface).



$$v^2 = u^2 + 2as$$
$$(v\sin\phi)^2 = (u\sin\theta)^2 + 2(-g)h$$
$$v\sin\phi = \sqrt{u^2\sin^2\theta - 2gh}$$
$$= \sqrt{(20)^2(\sin^2 60°)^2 - 2\cdot 10\cdot 5}$$
$$= \sqrt{\left(400 \times \frac{3}{4}\right) - 100}$$
$$= 10\sqrt{2}$$
$$v\cos\phi = u\cos\theta = 20\cos 60° = 10$$
$$\frac{v\sin\phi}{v\cos\phi} = \frac{10\sqrt{2}}{10}$$

$$\tan\phi = \sqrt{2}$$
$$\phi = \tan^{-1}\sqrt{2}$$

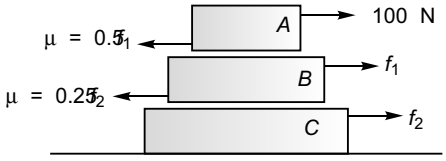
Option (c) is correct.

35. Acceleration of block B will be g throughout its motion while that of block A will increase from 0 to g and as such

$$t_A < t_B$$

Option (b) is correct.

36. $f_1(\text{max}) = 0.5 \times 10 \times 10 = 50\text{ N}$



Here, f_1 and f_2 are friction forces.

As, $f_1(\text{max.}) < F_{\text{ext.}}(100\text{ N})$, block A will move.

$f_2(\text{max.}) = 0.25(10+20)10 = 75\text{ N}$

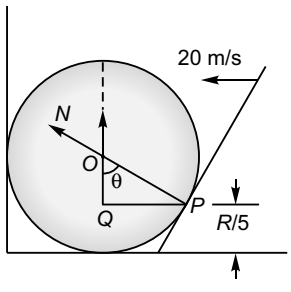
As, f_1 (max.), [driving force for block B] < f_2 (max.), the block will not slop over block C.

As, there is no friction between block C and surface below it, both the blocks B and block C will move together with acceleration

$$a = \frac{f_1(\text{max.})}{(\text{mass of } B + C)}$$
$$= \frac{50}{(20+30)}$$
$$= 1\text{ ms}^{-2}$$
$$\therefore \quad a_A = 1\text{ ms}^{-2}$$

Option (c) is correct.

37. $\cos\theta = \dfrac{R - \dfrac{R}{5}}{R} = \dfrac{4}{5}$



Velocity along $PQ = 20$ m/s.

$\therefore$ Velocity along $PO = 20\sin\theta$

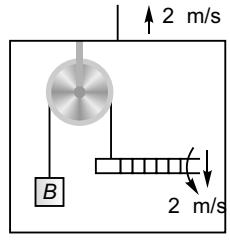
$$= 20 \times \frac{3}{5} = 12\text{ m/s}$$

Velocity of sphere (along vertical direction)

$$= \frac{12}{\cos\theta} = \frac{12}{4/5} = 15\text{ m/s}$$

Option (c) is correct.

38. String is winding on the motor shaft the block B will move up. Further, as shaft is also moving down, B will further.



Thus,

Velocity of block B

= Velocity of lift + Velocity of winding of string on shaft + Velocity of moving down of shaft

q= 2 m/s + 2 m/s + 2 m/s

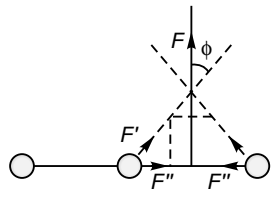
= 6 m/s.

Option (d) is correct.

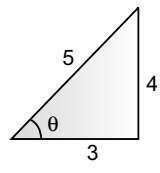
39. $F'' = F'\cos\theta$

F is resultant of two equal forces F' and F'

$$F = 2F'\cos\left(\frac{2\phi}{2}\right)$$



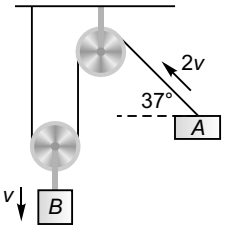
$$= 2F'\cos\phi$$
$$= 2F'\cos(90° - \theta)$$
$$= 2F'\sin\theta$$
$$= 2\left(\frac{F''}{\cos\theta}\right)\sin\theta$$



$$= 2F'' \tan\theta$$
$$= 2(ma'') \tan\theta$$
$$= 2 \times 03. \times \frac{5}{2} \times \frac{4}{3} = 2\,\text{N}$$

Option (b) is correct.

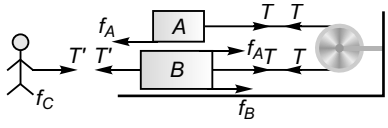
40. Velocity of block $A = 2v\cos 37°$



$$= 2 \times 10 \times \frac{4}{5}$$
$$= 16\,\text{m/s}$$

Option (d) is correct.

41. As the mass is applying maximum possible force without moving, the blocks would at the point of slipping,



$T' = f_C\,(\text{max}) = 0.5 \times 60 \times 10 = 300\,\text{N}$

$f_B\,(\text{max}) = 0.3 \times (60 + 60) \times 10$

$= 360\,\text{N}$

As, $T' < f_B\,(\text{max})$ the value of T will be zero.

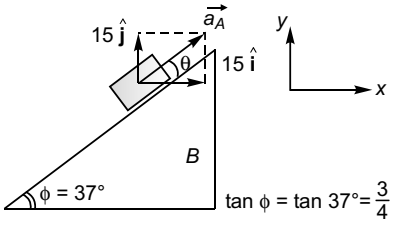
Option (d) is correct.

[Note : The value f_B will be 300 N and the values of T and f_A will be zero]

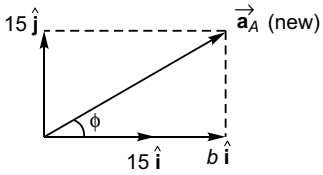
42. $\vec{\mathbf{a}}_A = |5\hat{\mathbf{i}} + 15\hat{\mathbf{j}}|$

$\therefore \quad \theta = 45°$

As, $\phi < \theta$ the block will leave contact with B.



For A to remain in contact with B, B must accelerate in the – ive x-direction.



Let acceleration of $B = -b\,\hat{\mathbf{i}}$

Due to this pseudo force with act on A in the + ive direction.

$$\tan\phi = \frac{15}{15 + b}$$

or $$\frac{3}{4} = \frac{15}{15 + b}$$

or $$45 + 3b = 60$$

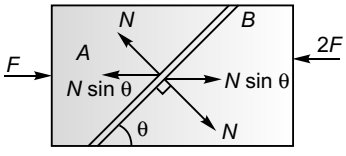
or $$3b = -15$$

or $$b = -5$$

$\therefore$ Acceleration of $B = -5\,\hat{\mathbf{i}}$

Option (d) is correct.

43. For block A not to slide on block B

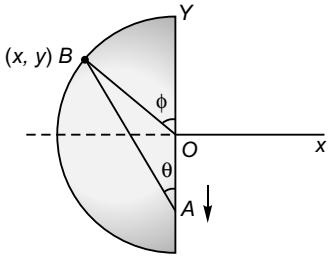


F_{net} on block $A = F_{\text{net}}$ on block B

$$F - N\sin\theta = N\sin\theta - 2F$$
$$2N\sin\theta = 3F$$
$$N = \frac{3F}{2\sin\theta}$$
$$= 3F \qquad (\text{as } \theta = 30°)$$

Option (d) is correct.

44. Equation to circle is



$$x^2 + y^2 = r^2 \text{ (where } r = OB) \quad \ldots\text{(i)}$$

$$\therefore \quad 2\frac{dx}{dt} + 2\frac{dy}{dt} = 0$$

$$\Rightarrow \quad \frac{dx}{dt} = -\frac{dy}{dt}$$

$$= -(-u) = +u$$

$$= u$$

Speed of bead B

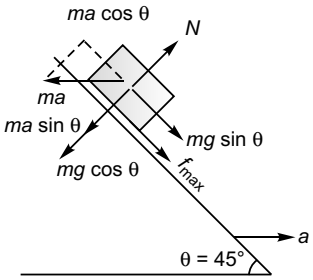
$$= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

$$= \sqrt{(u)^2 + (-u)^2}$$

$$= u\sqrt{2}$$

Option (a) is correct.

45. At maximum acceleration value of a, the block would be in a position to move upwards.



f = frictional force

$$N = ma\sin\theta + mg\cos\theta$$

$$ma\cos\theta = f_{\max} + mg\sin\theta$$

or $\quad ma\cos\theta = \mu N + mg\sin\theta$

or $\quad ma\cos\theta = \mu\,(ma\sin\theta + mg\cos\theta) + mg\sin\theta \quad (\because \theta = 45°)$

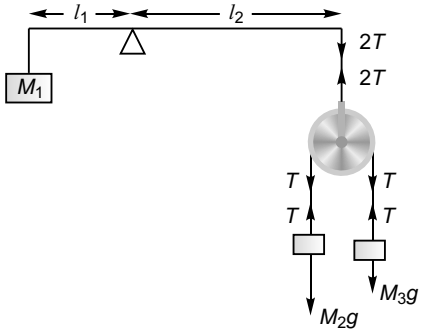
or $\quad a = \mu\,(a + g) + g$

or $\quad a\,(1 - \mu) = (1 + \mu)\,g$

i.e., $\quad a = \dfrac{1+\mu}{1-\mu}\,g$

Option (b) is correct.

46. For the beam to have no tendency to rotate



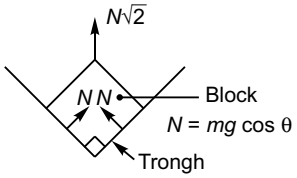
$$M_1 g l_1 = 2Tl_2$$

or $\quad M_1 g l_1 = 2\left(\dfrac{2M_1M_3}{M_2 + M_3}\,g\right) l_2$

or $\quad M_1 l_1 = 4\left(\dfrac{M_2 \cdot \dfrac{M_2}{3}}{M_2 + \dfrac{M_2}{3}}\right) 3\,l_1$

$\Rightarrow \quad \dfrac{M_1}{M_2} = 3$

47. $f_{\max} = \mu_k N\sqrt{2}$



$$= \mu_k \sqrt{2}\, mg\cos\theta$$

$\therefore \quad ma = mg\sin\theta - \sqrt{2}\,\mu_k\, mg\cos\theta$

i.e., $a = g\,(\sin\theta - \sqrt{2}\,\mu_k \cos\theta)$

Option (c) is correct.

48. $f_1\,(\max) = 0.5 \times 3 \times 10$

$= 15\text{ N}$

$f_2\,(\max) = 0.3 \times (3 + 2) \times 10$

$= 15\text{ N}$

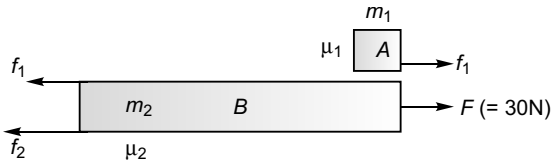
$f_3\,(\max) = 0.1 \times (3 + 2 + 1) \times 10$

$= 6\text{ N}$

Value of maximum frictional force is between block 1 kg and the ground. Increasing from zero when F attains 6 N, the block of mass 1 kg will be at the point of slipping over ground below it.

Option (c) is correct.

49. $f_2\,(\max) = \mu\,(m_1 + m_2)\,g$



$$= 0.5\,(1+2)\,10$$
$$= 15\text{ N}$$

a_S = Acceleration of both as one

a_A = Acceleration of A

$$f_1\,(\max) = \mu_1 m_1 g = 0.2 \times 1 \times 10$$
$$= 2\text{ N}$$

$$a_S = \frac{F - f_2\,(\max)}{m_1 + m_2} = \frac{30-15}{3} = 5\text{ m/s}^2$$

$$a_A = \frac{\mu_1 m_1 g}{m_1} = \mu_1 g = 0.2 \times 10 = 2\text{ m/s}^2$$

As, $\quad F > f_2\,(\max.)$

both will move.

Further, as $a_S > a_A$ both will not accelerate as one unit.

$$a_B = \frac{F - \mu_2\,(m_1+m_2)\,g - \mu_1 m_1 g}{m_2}$$

Acceleration of A w.r.t. B

$$a_{AB} = a_A - A_B$$
$$= \mu_1 g - \frac{f - \mu_2\,(m_1+m_2)\,g - \mu_1 m_1 g}{m_2}$$
$$= -\frac{\mu_1 m_2 g - F + \mu_2(m_1+m_2)\,g + \mu_1 m_1 g}{m_2}$$
$$= -\frac{F - (\mu_1+\mu_2)\,(m_1+m_2)\,g}{m_2}$$
$$= -\frac{30 - (0.2+0.5)\,(1+2)\,10}{2}$$
$$= -\frac{9}{2}\text{ m/s}^2$$

Negative sign indicates that the direction of a_{AB} will be opposite to that of a_A.

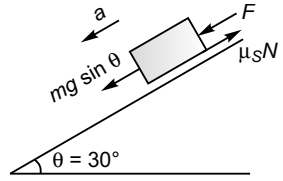
$$\therefore \text{Required time } t = \sqrt{\frac{2\text{s}}{a_{AB}}} = \sqrt{\frac{2.1}{9/2}} = \frac{2}{3}\text{ s}$$

Option (a) is correct.

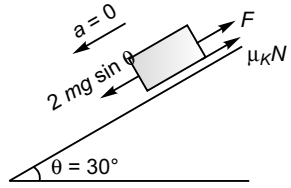
50. $$F = \mu_s mg\cos\theta - mg\sin\theta$$
$$= mg\left[\frac{4}{3\sqrt{3}}\,\frac{\sqrt{3}}{2} - \frac{1}{2}\right]$$
$$= \frac{mg}{6}$$

Option (b) is correct.

51. $\mu_s = 2\mu_k$



Force required to just slide block downwards



$$F = \mu_s N - mg\sin\theta$$
$$= \mu_s mg\cos\theta - mg\sin\theta$$
$$F = mg\sin\theta - \mu_k N$$
$$= mg\sin\theta - \mu_k mg\cos\theta$$

Thus, $\mu_s mg\cos\theta - mg\sin\theta$
$$= mg\sin\theta - \mu_k mg\cos\theta$$
$$(\mu_s + \mu_k)\,mg\cos\theta = 2mg\sin\theta$$
$$\left(\mu_s + \frac{\mu_s}{2}\right) = 2\tan\theta$$
$$\frac{3}{2}\mu_s = \frac{2}{\sqrt{3}} \Rightarrow \mu_s = \frac{4}{3\sqrt{3}}$$

Option (a) is correct.

52. $$a = \frac{F + mg\sin\theta - \mu_k\,mg\cos\theta}{m}$$
$$= \frac{\frac{mg}{6} + \frac{mg}{2} - \frac{2}{3\sqrt{3}}\cdot mg\,\frac{\sqrt{3}}{2}}{m}$$
$$= \frac{g}{3}$$

Option (d) is correct.

53. Minimum force required to start the motion upward

$$= mg\sin\theta + \mu_k\, mg\cos\theta$$
$$= mg\left[\frac{1}{2} + \frac{4}{3\sqrt{3}}\cdot\frac{\sqrt{3}}{2}\right]$$
$$= \frac{7}{6}mg$$

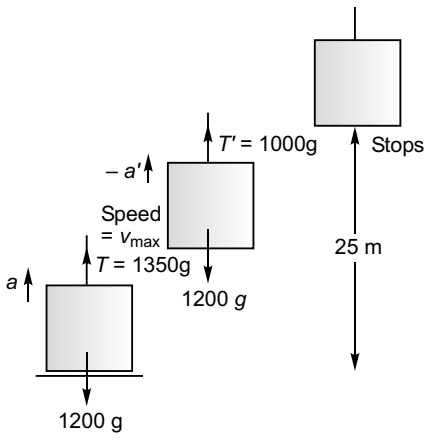
54. Minimum force required to move the block up the incline with constant speed

$$= mg\sin\theta + \mu_k\, mg\cos\theta$$
$$= mg\left[\frac{1}{2} + \frac{2}{3\sqrt{3}}\cdot\frac{\sqrt{3}}{2}\right]$$
$$= \frac{5}{6}mg$$

55. $S_1 = \dfrac{(5.22)^2}{2\cdot\left(\dfrac{9.8}{8}\right)} = 14.3$ m

Option (c) is correct.

56. $$a' = \frac{1200g - 1000g}{1200}$$
$$= \frac{g}{6}$$



$$a = \frac{1350g - 1200g}{1200}$$
$$= \frac{g}{8}$$

For accelerated motion

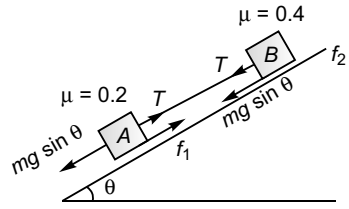
$$v_{max}^2 = 0^2 + 2as_1$$
$$\Rightarrow \quad s_1 = \frac{v_{max}^2}{2a}$$

For retarted motion

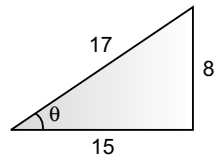
$$0^2 = v_{max}^2 - 2a's_2$$
$$\Rightarrow \quad s_2 = \frac{v_{max}^2}{2a'}$$
$$s_1 + s_2 = \frac{v_{max}^2}{2}\left[\frac{1}{a} + \frac{1}{a'}\right]$$
$$25 = \frac{v_{max}^2}{2}\left[\frac{8}{g} + \frac{6}{g}\right]$$
$$v_{max} = \sqrt{\frac{50\times 9.8}{14}} = 5.92\ \text{ms}^{-1}$$

Option (c) is correct.

57. $mg\sin\theta = 170\times 10\times\dfrac{8}{15} = 906.67$ N



$$f_1\,(\text{max}) = 0.2\times 170\times 10\times\frac{15}{17} = 300\ \text{N}$$
$$f_2\,(\text{max}) = 0.4\times 170\times 10\times\frac{15}{17}$$
$$= 600\ \text{N}$$



The whole system will accelerate as $mg\sin\theta$ is greater than both f_1 (max) and f_2 (max).

Total force of friction

$$= f_1\,(\text{max}) + f_2\,(\text{max})$$
$$= 900\ \text{N}$$

Option (a) is correct.

58. $mg\sin\theta - 300 - T = ma$ ...(i)

and $mg\sin\theta + T = ma$ ...(ii)

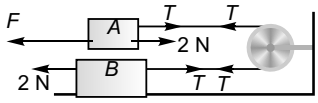
Substituting Eq. (i) by Eq. (ii),

$$2T + 300 = 0$$
$$T = -150\ \text{N}$$
$$= 150\ \text{N, compressive.}$$

Option (a) is correct.

More than One Correct Options

1. (a) Normal force between A and $B = m_2 g$



$$= 1 \times 10 = 10 \text{ N}$$

$\therefore$ Force of limiting friction by B on A (or by A on B)

$$= \mu \times 10 = 2 \text{ N}$$

Total force opposing applied external force

$$F = 2\text{N} + T$$
$$= 2\text{N} + 2\text{N}$$
$$= 4 \text{ N}$$

Thus, if $F \le 4$ N

The block A will remain stationary and so block B also. The system will be in equilibrium.

$\therefore$ Option (a) is correct.

(b) If $F > 4$ N

$F - T - 2 = 1a$

and $\quad T - 2 = 1a \quad$ ...(i)

Adding above equation

$$F - 4 = 2a \quad \text{...(ii)}$$

i.e., $\quad F = 4 + 2a$

For $F > 4$ N

$$2a + 4 > 4$$

or $\quad a > 0$

$\Rightarrow \quad T - 2 > 0$

i.e., $\quad T > 2\text{N}$

$\therefore$ Option (b) is incorrect.

(c) Block A will move over B only when $F > 4$ N and then the frictional force between the blocks will be 2 N if a is just 0 [as explained in (b)].

Option (c) is correct.

(d) If $F = 6$ N using Eq. (ii)

$$2a = 6 - 4$$

$\Rightarrow \quad a = 1 \text{ m/s}^2$

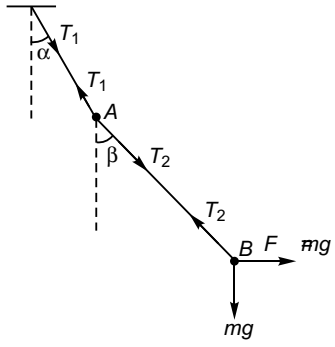
$\therefore$ Using Eq. (i),

$$T - 2 = 1$$

i.e., $\quad T = 3$ N

Option (d) is correct.

2. At point A



$T_1 \cos\alpha = T_2 \cos\beta + mg \quad$ ...(i)

and $T_1 \sin\alpha = T_2 \sin\beta \quad$ ...(ii)

At point B

$$T_2 \cos\beta = mg \quad \text{...(iii)}$$

$$T_2 \sin\beta = F - mg \quad \text{...(iv)}$$

Using Eq. (iii) in Eq. (i),

$$T_1 \cos\alpha = 2T_2 \cos\beta \quad \text{...(v)}$$

Dividing Eq. (ii) by Eq. (v),

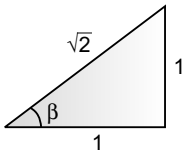
$$2\tan\alpha = \tan\beta \quad \text{...(vi)}$$

Option (a) is correct.

Squaring and adding Eqs. (iii) and (v),

$T_1^2 = 4T_2^2 \cos^2\beta + T_2^2 \sin^2\beta \quad$ ...(vii)

Dividing Eq. (iii) by Eq. (iv)



$$\tan\beta = 1$$

$\therefore \quad \cos\beta = \dfrac{1}{\sqrt{2}} = \sin\beta$

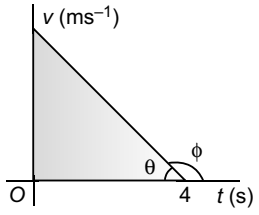
Substituting the values of $\sin\beta$ and $\cos\beta$ in Eq. (vii)

$$T_1^2 = 4T_2^2\left(\frac{1}{2}\right) + T_2^2\left(\frac{1}{2}\right)$$
$$= \frac{5}{2}T_2^2$$

$\Rightarrow \qquad \sqrt{2}T_1 = \sqrt{5}T_2$

Option (c) is correct.

3. Displacement of block in 4 s



S = Area under curve

$= 16$ m.

ΔK =Workdone by frictional force

$\frac{1}{2} \times 1 \times 4^2 = \mu \times 1 \times 10 \times 16$

$\Rightarrow \qquad \mu = 0.1$

Option (a) is correct.

Option (b) is incorrect.

Acceleration, $a = \tan \phi$

$= \tan(\pi - \theta)$

$= -\tan\theta$

$= -1 \text{ m/s}^2$

If half rough retardation $= 0.5 \text{ m/s}^2$

$\therefore \qquad 16 = 4t + \frac{1}{2}(-0.5)t^2$

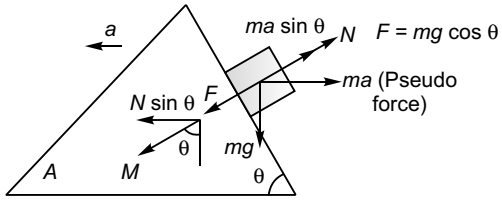
i.e., $\qquad t^2 - 16t + 64 = 0$

or $\qquad t = 8$ s

Option (d) is correct.

Option (c) is incorrect.

4. Let acceleration of wedge $(A) = a$



$N + ma\sin\theta = mg\cos\theta$

$N = mg\cos\theta - ma\sin\theta$

Acceleration of

$$a = \frac{N\sin\theta}{M}$$

or $Ma = (mg\cos\theta - ma\sin\theta)\sin\theta$

or $a(M + m\sin^2\theta) = mg\cos\theta\sin\theta$

i.e., $\quad a = \dfrac{mg\cos\theta\sin\theta}{M + m\sin^2\theta}$

$= \dfrac{0.6 \times g \times \cos 45° \sin 45°}{1.7 + (0.6 \times \sin^2 45°)}$

$= \dfrac{3g}{17+3}$

$= \dfrac{3g}{20}$

Let a_B = Acceleration of block B

Net force on B (along inclined plane)

$ma_B = ma\cos\theta + mg\sin\theta$

$\Rightarrow \qquad a_B = a\cos\theta + g\sin\theta$

Thus, $(a_B)_V = (a\cos\theta + g\sin\theta)\cos\theta$

$= a\cos^2\theta + g\sin\theta\cos\theta$

$= (a+g)\dfrac{1}{2}$

$= \left(\dfrac{3g}{20} + g\right)\dfrac{1}{2}$

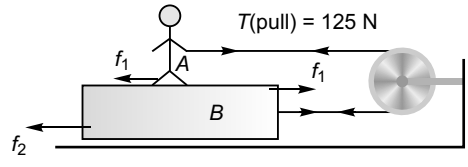
$= \dfrac{23g}{40}$

$(a_B)_H = (a\cos\theta + g\sin\theta)\sin\theta - a$

$= \dfrac{23g}{40} - \dfrac{3g}{20}$

$= \dfrac{17g}{40}$

5. f_1 (max.) $= \mu_1 m_A g$



$= 0.3 \times 60 \times 10$

$= 180$ N

F_{net} on $B = f_1$ (max.) $+ T$

$= 180 + 125$

$= 305$ N

A will remain stationary as

$T < f_1$ (max.)

$\therefore \qquad f_1 = 125$ N

Force of friction acting between A and $B = 125$ N

$\therefore$ Options (c) and (d) are incorrect.

$$f_2\,(\max) = \mu_2\,(m_A + m_B)\,g$$
$$= 0.2\,(60 + 40)\,10$$
$$= 200\text{ N}$$
$$f_1 + T = 125 + 125 = 250\text{ N}$$

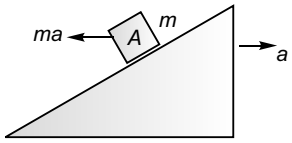
As, $f_1 + T > f_2$ (max. block B /along the A as A is stationary) will move towards right with acceleration.

Option (a) is correct.

$$a_B = \frac{(f_1 + T) - f_2\,(\text{max.})}{m_B + m_A}$$
$$= \frac{250 - 200}{40 + 60}$$
$$= 0.5\text{ m/s}^2$$

Option (b) is correct.

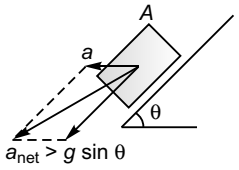
6. (See solution to Question no. 4).



$$N = mg\cos\theta - ma\sin\theta$$

Option (c) is correct and option (d) is incorrect.

As angle between the directions of a and $g\sin\theta$ will be less than 90°, acceleration of block A will be more than $g\sin\theta$.



Option (a) is correct and option (b) is incorrect.

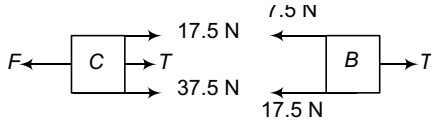
7. Maximum value of friction.

f_1 = between A and B
$= 0.25 \times 3 \times 10 = 7.5\text{ N}$

f_2 = between B and C
$= 0.25 \times 7 \times 10 = 17.5\text{ N}$

f_3 = between C and ground
$= 0.25 \times 15 \times 10 = 37.5\text{ N}$

(a)



$T = 17.5 + 7.5 = 25\text{ N}$

$F = T + 37.5 + 17.5 = 80\text{ N}$

(c) $T - 7.5 - 17.5 = 4a$ ...(i)

$F - T - 37.5 - 17.5 = 8a$ ...(ii)

$F = 200\text{ N}$ ...(iii)

Solving these equations we get,

$$a = 10\text{ m/s}^2$$

8. Maximum value of friction available to block is less than the maximum value of friction available to man.

9. $N\sin\theta = ma = 1 \times 5 = 5$ ...(i)

$N\cos\theta = mg = 1 \times 10 = 10$ ...(ii)

Solving these two equations we get, $\tan\theta = \frac{1}{2}$ and $N = 5\sqrt{5}$ N.

10. Let f_1 = friction between 2 kg and 4 kg

f_2 = friction between 4 kg and ground

$(f_{s1})_{\max} = 0.4 \times 2 \times 10 = 8\text{ N}$

$(f_{k1}) = 0.2 \times 2 \times 10 = 4\text{ N}$

$(f_{s2})_{\max} = 0.6 \times 6 \times 10 = 36\text{ N}$

$F_{k2} = 0.4 \times 6 \times 10 = 24\text{ N}$

(b) At $t = 1$ s, $F = 2\text{ N} < (f_{s2})_{\max}$

$\therefore$ Both the blocks are at rest.

$\therefore\ f_1 = 0$

(c) At $t = 4$ s, $F = 8\text{ N} < (f_{s2})_{\max}$

$\therefore$ Both the blocks are at rest.

$$f_2 = F = 8\text{ N},$$

11. $a = 0$, $T_1 = 10\text{ N}$,

$T_2 = 20 + T_1 = 30\text{ N}$,

$T_3 = 20\text{ N}$.

12. $f_{\max} = 0.3 \times 2 \times 10 = 6\text{ N}$

(a) At $t = 2$ s, $F = 2N$

$\therefore\ f = 2\text{ N}$

(b) At $t = 8$ s, $F = 8\text{ N} > 6\text{ N}$

$\therefore\ f = 6\text{ N}$

(c) At $t = 10$ s, $F = 10$ N and $f = 6$ N

$$\therefore \quad a = \frac{10-6}{2} = 2 \text{ m/s}^2$$

(d) Block will start at 6 s. After that, net impulse

$$= \frac{1}{2} \times 4 \times (6+10) + 2 \times 10 - 6 \times 6$$

$$= 16 \text{ N-s} = mv$$

$$\therefore \quad v = \frac{16}{2} = 8 \text{ m/s.}$$

13. $f_{max} = 0.4 \times 2 \times 10 = 8$ N

(b) At $t = 3$ s, $F = 6$ N

$\therefore$ Common acceleration

$$a = \frac{6}{6} = 1 \text{ m/s}^2$$

$\therefore$ Pseudo force on 2 kg

$$= 2 \times 1 = 2 \text{ N} \quad \text{(backward)}$$

14. $N = Mg - F \sin\theta$

$F \cos\theta = \mu N = \mu(Mg - F \sin\theta)$

$$\therefore \quad F = \frac{\mu Mg}{\cos\theta + \mu \sin\theta}$$

For F to be minimum,

$$\frac{dF}{d\theta} = 0$$

Match the Columns

1. Acceleration after $t = 4$ s

At $t = 4$ s, $F = 8$ N

$\therefore \quad F_{max} = 8$

i.e., $\mu_s mg = 8$

$$\Rightarrow \quad \mu_s = \frac{8}{mg} = \frac{8}{2 \times 10} = 0.4$$

$\therefore$ (a) $\rightarrow$ (r)

At $t = 4$ s, $a = 1 \text{ ms}^{-2}$

$t = 4$ s, $F = 8$ N

$F - \mu_k N = ma$

$$\text{i.e.,} \quad \mu_k = \frac{F - ma}{N} = \frac{F - ma}{mg}$$

$$= \frac{8 - (2 \times 1)}{2 \times 10} = 0.3$$

(b) = (q)

At $t = 0.1$ s, $F = 0.2$ N

$\therefore$ Force of friction (at $t = 0.1$ s) = 0.2 N

$\therefore$ (c) $\rightarrow$ (p)

At $t = 8$ s, $F = 16$ N

$$\therefore \quad a = \frac{F - \mu_k mg}{m}$$

$$= \frac{16 - (0.3 \times 2 \times 10)}{2} = 5$$

$$\text{i.e.,} \quad \frac{a}{10} = 0.5$$

$\therefore$ (d) $\rightarrow$ (s).

2. At $\theta = 0°$, dragging force = 0

$\therefore$ Force of force = 0

$\therefore$ (a) $\rightarrow$ (s)

At $\theta = 90°$

Normal force on block by plane will be zero.

$\therefore$ Force of friction = 0

$\therefore$ (b) $\rightarrow$ (s)

At $\theta = 30°$

Angle of repose $= \tan^{-1}\mu$

$= \tan^{-1}(1) = 45°$

As $\theta <$ angle of repose, the block will not slip and thus,

force of friction $= mg \sin\theta$

$= 2 \times 10 \times \sin 30° = 10$ N

$\therefore$ (c) $\rightarrow$ (p)

At $\theta = 60°$

As $\theta >$ angle of repose

Block will accelerate and thus force of friction $= \mu N$

$= 1 \times 2 \times 10 \times \cos 60°$

$= 10$ N

$\therefore$ (d) $\rightarrow$ (p).

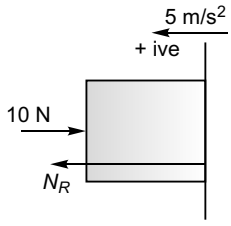
3. All contact forces (*e.g.,* force of friction and normal reaction) are electromagnetic in nature.

$\therefore$ (a) $\rightarrow$ (q), (r)

(b) $\rightarrow$ (q), (r).

Nuclear force is the force between nucleons (neutrons and protons). Between two protons field force also acts.

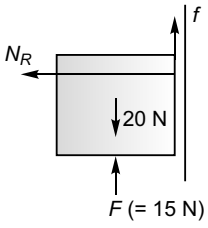
4. (a) $N_R - 10 = 2(+5)$



$\Rightarrow \qquad N_R = 20\,\text{N}$

$\therefore$ (a) $\rightarrow$ (f)

(b) $mg\ (= 20\text{N}) > F\ (= 15\text{N})$



Block would be slipping in the downward direction.

Force of friction will be in the upward direction.

Frictional force $= 20 - 15$

$= 5\,\text{N}$

$\therefore$ (b) $\rightarrow$ (p)

(c) If $F = 0$, the block will slip downwards due to $mg\ (= 20\text{N})$

Limiting friction $= \mu_s N_R$

$= 0.4 \times 20$

$= 8\,\text{N}$

Minimum value of F for stopping the block moving down $= 20 - 8$

$= 12\,\text{N}$

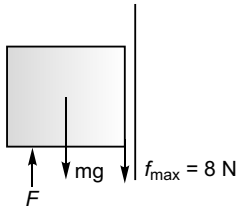
$\therefore$ (c) $\rightarrow$ (s).

(d) $F = mg + 8$

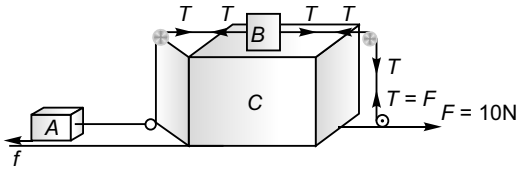
$= (2 \times 10) + 8$

$= 28\,\text{N}$

$\therefore$ (d) $\rightarrow$ (s)



5. (a) As block A is stationary, $f = 10\,\text{N}$



$\therefore$ (a) $\rightarrow$ (p)

(b) As block C is stationary force of friction between C and ground will be zero.

$\therefore$ (b) $\rightarrow$ (s)

(c) Normal force (N_C) on C from ground

$$N_C = N_B + m_C g$$
$$= m_B g + m_C g$$
$$= (m_B + m_c)\, g$$
$$= (1 + 1)\, 10 = 20\,\text{N}$$

$\therefore$ (c) $\rightarrow$ (q).

(d) As block A is stationary

$T = F$ (as shown in figure)

$= 10\,\text{N}$

$\therefore$ (a) $\rightarrow$ (p).

6. If friction force (f) is less than the applied force (F).

Net force on body $= F - f$

The body will be in motion and thus the friction will be kinetic.

$\therefore$ (a) $\rightarrow$ (q)

(b) If friction force (f) is equal to the force applied, the body will be at rest. If the body is at the point of slipping the force of force will be limiting too. Emphasis is being given to the word "may be" as when a body is moving and the external force is made just equal to the frictional force, the body would still be moving with friction force at its limiting (kinetic) value.

$\therefore$ (b) $\rightarrow$ (p), (r)

(c) If object is moving the friction would be kinetic as explained in (a).

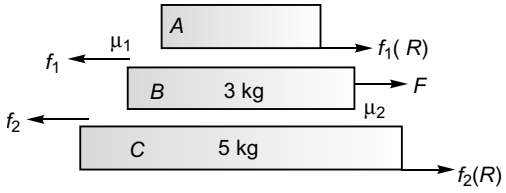
$\therefore$ (c) $\rightarrow$ (q).

(d) If the object is at rest, then friction may be static and limiting as explained in (b).

$\therefore$ (d) $\rightarrow$ (p), (r)

7. (a) Normal force between A and $B = m_A g$

f_1 (max.) $= \mu_1 m_A g$ (towards left)



$= 0.2 \times 2 \times 10$

$= 4$ N

Normal force between B and C

$= (m_A + m_B)\, g$

f_2 (max) $= \mu_2\, (m_A + m_B)\, g$

$= 0.1\,(2 + 3) \times 10$

$= 5$ N

Total friction force on 3 kg block

$= f_1$ (max) $+ f_2$ (max) $= 4 + 5$

$= 9$ N towards left

$\therefore$ (a) $\rightarrow$ (q), (s)

(b) Friction force on 5 kg block

$= f_2\,(R) = f_2$ (max)

$= 5$ N, towards right

$\therefore$ (b) $\rightarrow$ (p), (s)

(c) Friction force on 2 kg block due to 3 kg block $= f_1(R)$

$= f_1$ (max)

$= 4$ N, towards right

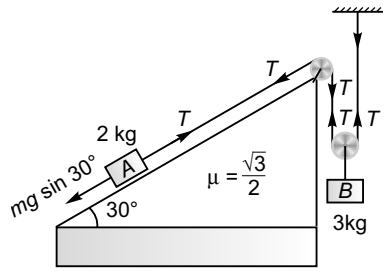
$\therefore$ (c) $\rightarrow$ (p), (s)

(d) Friction force on 3 kg block due to 5 kg block $= f_2$ (max)

$= 5$ N, towards left.

$\therefore$ (d) $\rightarrow$ (q), (s).

8. (a) and (b)



$\mu = \dfrac{\sqrt{3}}{2}$

$\therefore$ Angle of repose $= \tan^{-1} \dfrac{\sqrt{3}}{2} = 40.89°$

Now as, angle of incline (30°) < angle repose (= 40.89°)

The block A and so also B will remain stationary.

$\therefore$ (a) $\rightarrow$ (r),

(b) $\rightarrow$ (r)

(c) Tension (T) in the string connecting 2 kg mass $= mg \sin 30°$

$= 2 \times 10 \times \dfrac{1}{2}$

$= 10$ N

$\therefore$ (c) $\rightarrow$ (s)

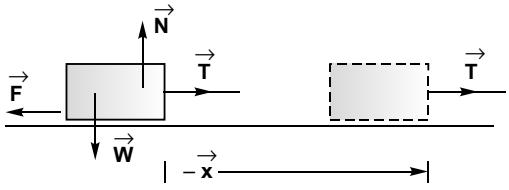
(d) Friction force on 2 kg mass = zero.

$\therefore$ (d) $\rightarrow$ (r).

6 Work, Energy and Power

Introductory Exercise 6.1

1. Work done by $\vec{\mathbf{F}} = \vec{\mathbf{F}} \cdot \vec{\mathbf{x}}$



$$|\vec{\mathbf{F}}| = F, |\vec{\mathbf{N}}| = N, |\vec{\mathbf{W}}| = W$$

$$|\vec{\mathbf{T}}| = T \text{ and } |\vec{\mathbf{x}}| = x$$

$$= |\vec{\mathbf{F}}||\vec{\mathbf{x}}| \cos \pi$$

$$= -F x$$

Work done by $|\vec{\mathbf{N}}| = \vec{\mathbf{N}} \cdot \vec{\mathbf{x}}$

$$= |\vec{\mathbf{N}}||\vec{\mathbf{x}}| \cos \frac{\pi}{2}$$

$$= 0$$

Work done by $\vec{\mathbf{W}} = \vec{\mathbf{W}} \cdot \vec{\mathbf{x}}$

$$= |\vec{\mathbf{W}}| \cdot |\vec{\mathbf{x}}| \cos \frac{\pi}{2}$$

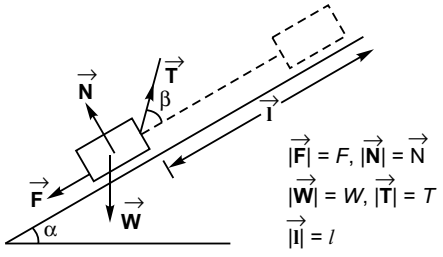
$$= 0$$

Work done by $\vec{\mathbf{T}} = \vec{\mathbf{T}} \cdot \vec{\mathbf{x}}$

$$= |\vec{\mathbf{T}}||\vec{\mathbf{x}}| \cos 0$$

$$= T x$$

2. Work done by $\vec{\mathbf{F}} = \vec{\mathbf{F}} \cdot \vec{\mathbf{l}}$



$$= |\vec{\mathbf{F}}| \cdot |\vec{\mathbf{l}}| \cos \pi$$

$$= -F l$$

Work done by $\vec{\mathbf{N}} = \vec{\mathbf{N}} \cdot \vec{\mathbf{l}}$

$$= |\vec{\mathbf{N}}| \cdot |\vec{\mathbf{l}}| \cos \frac{\pi}{2} = 0$$

Work done by $|\vec{\mathbf{W}}| = \vec{\mathbf{W}} \cdot \vec{\mathbf{l}}$

$$= |\vec{\mathbf{W}}| \, |\vec{\mathbf{l}}| \cos \left(\frac{\pi}{2} + \alpha\right)$$

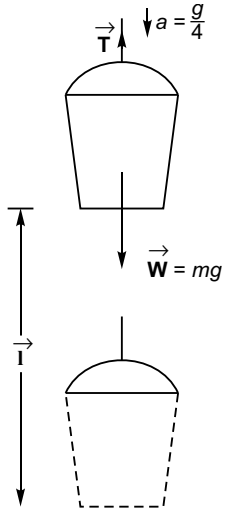
$$= -W l \sin \alpha$$

Work done by $|\vec{\mathbf{T}}| = \vec{\mathbf{T}} \cdot \vec{\mathbf{l}}$

$$= |\vec{\mathbf{T}}| \, |\vec{\mathbf{l}}| \cos \beta$$

$$= T l \cos \beta$$

3. $\vec{\mathbf{W}} - \vec{\mathbf{T}} = m\dfrac{\vec{\mathbf{g}}}{4}$



$|\vec{\mathbf{g}}| = g$ and $|\vec{\mathbf{l}}| = l$

$$\therefore \quad |\vec{\mathbf{T}}| = m\vec{\mathbf{g}} - \frac{m\vec{\mathbf{g}}}{4}$$

$$= \frac{3}{4} m\vec{\mathbf{g}}$$

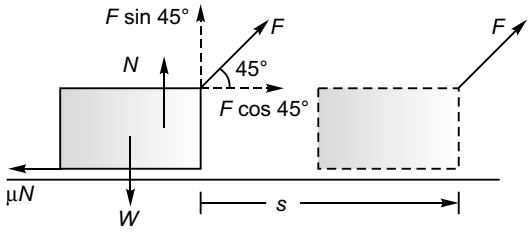
Work done by string $= \vec{\mathbf{T}} \cdot \vec{\mathbf{l}}$

$$= |\vec{\mathbf{T}}||\vec{\mathbf{l}}| \cos \pi$$

$$= \left|\frac{3}{4} m\vec{\mathbf{g}}\right| \vec{\mathbf{l}} \cos \pi$$

$$= -\frac{3}{4} mgl$$

4. $\mu N = F \cos 45°$ ...(i)



$N = |\vec{\mathbf{N}}|, W = |\vec{\mathbf{W}}|$

$s = |\vec{\mathbf{s}}|, F = |\vec{\mathbf{F}}|, g = |\vec{\mathbf{g}}|$

$N + F \sin 45° = W$ ...(ii)

Substituting value of N from Eq. (ii) in Eq. (i).

$\mu (W - F \sin 45°) = F \cos 45°$

or $\quad \dfrac{1}{4}\left(W - F\dfrac{1}{\sqrt{2}}\right) = F\dfrac{1}{\sqrt{2}}$

or $\quad W - \dfrac{F}{\sqrt{2}} = 4\dfrac{F}{\sqrt{2}}$

or $\quad \dfrac{5F}{\sqrt{2}} = W$

or $\quad F = \dfrac{\sqrt{2}}{5} W = \dfrac{\sqrt{2}}{5} mg$

Work done by force $F = \vec{\mathbf{F}} \cdot \vec{\mathbf{s}}$

$$= |\vec{\mathbf{F}}||\vec{\mathbf{s}}| \cos 45° = F s \frac{1}{\sqrt{2}}$$

$$= \left(\frac{\sqrt{2}}{5} mg\right) s \frac{1}{\sqrt{2}} = \frac{mgs}{5}$$

$$= \frac{1.8 \times 10 \times 2}{5}$$

$$= 7.2 \text{ J}$$

Work done by friction $= \mu \vec{\mathbf{N}} \cdot \vec{\mathbf{s}}$

$$= \mu |\vec{\mathbf{N}}||\vec{\mathbf{s}}| \cos \pi$$

$$= -\mu Ns$$

$$= -F \cos 45° s$$

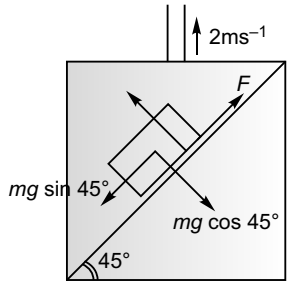
$$= -7.2 \text{ J}$$

Work done by gravity $= \vec{\mathbf{g}} \cdot \vec{\mathbf{s}}$

$$= |\vec{\mathbf{g}}| \cdot |\vec{\mathbf{s}}| \cos \frac{\pi}{2}$$

$$= 0$$

5. $F = mg \sin 45°$



$$= 1 \times 10 \times \frac{1}{\sqrt{2}}$$

$$= 5\sqrt{2} \text{ N}$$

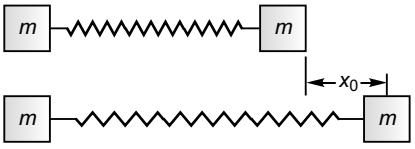
Displacement of lift in 1s $= 2$ m

Work done by force of friction $(F) = \vec{\mathbf{F}} \cdot \vec{\mathbf{s}}$

$= |\vec{\mathbf{F}}||\vec{\mathbf{s}}|\cos 45°$

$= F\,s\cos 45° = 5\sqrt{2} \cdot 2 \cdot \frac{1}{\sqrt{2}} = 10\,\text{Nm}$

6. Total work-done by spring on both masses



= PE of the spring when stretched by $2x_0$

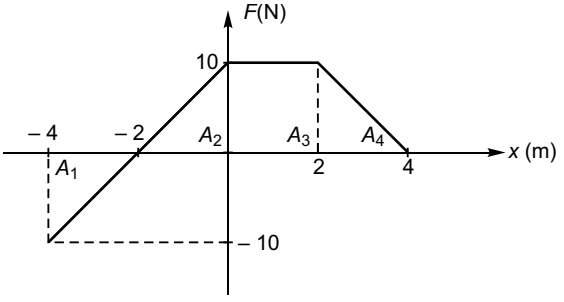
$= \frac{1}{2}k(2x_0)^2$

$= 2kx_0^2$

$\therefore$ Work done by spring on each mass

$= \frac{2kx_0^2}{2} = kx_0^2$

7. Work done = Area under the curve



$= A_1 + A_2 + A_3 + A_4$

$= \left[\frac{-2 \times -10}{2}\right] + \left[\frac{-2 \times 10}{2}\right] + [10 \times 2] + \left[\frac{2 \times 10}{2}\right]$

$= 30\,\text{Nm}$

Introductory Exercise 6.2

1. $a = \frac{-20\,\text{ms}^{-1}}{2s} = -10\,\text{ms}^{-2}$

$\therefore \quad F = ma = 2\,\text{kg} \times -10\,\text{ms}^{-2} = -20\,\text{N}$

s = Area under curve

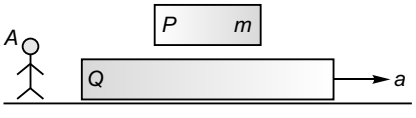
$= \frac{1}{2} \times 25 \times 20\,\text{ms}^{-1}$

$= 20\,\text{m}$

$\therefore$ Work done $= F\,s = (-20\,\text{N})(20\,\text{m})$

$= -400\,\text{Nm}$

2. According to A (inertial frame)



Acceleration of $P = a$

$\therefore$ Force on $P = ma$

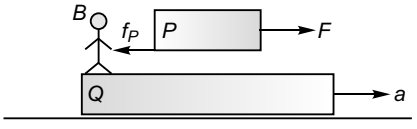
Work done over s displacement $= mas$

Now, $v^2 = u^2 + 2as$

$= 2as \qquad (\because u = 0)$

$\therefore$ Gain in KE $= \frac{1}{2}mv^2 = \frac{1}{2}m\,2as = mas$

$\therefore \quad \Delta K = W$ (Work -Energy theorem)



According to B (non-inertial frame) work done

= Work done by F + Work done by f_p (pseudo force)

$= (mas) + (-mas) = 0$

As P is at rest , $\Delta K = 0$

$\therefore \quad \Delta K = W$ (Work -Energy theorem)

Note *In inertial frames one has to also to consider work-done due to pseudo forces, while applying Work-energy theorem.*

3. $v = \alpha\sqrt{x}$

$\therefore \quad a = \frac{dv}{dt} = \alpha\frac{1}{2\sqrt{x}}\frac{dx}{dt}$

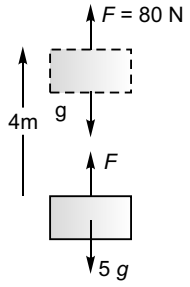
$= \alpha\frac{1}{2\sqrt{x}}\alpha\sqrt{x} = \frac{\alpha^2}{2}$

$F = ma = m\frac{\alpha^2}{2}$

$\therefore \quad W = \frac{m\alpha^2}{2}b$

4. ΔK = Work done by F

+ Work done by gravity



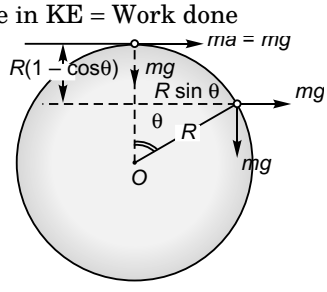
$$= 80 \cdot 4 \cdot \cos 0 + 5g \cdot 4 \cdot \cos \pi$$

$$= 320 + (-200)$$

or $\quad K_f - K_i = 120\,\text{J}$

or $\quad K_f = 120\,\text{J} \qquad (\text{as } K_i = 0)$

5. Change in KE = Work done



$$\frac{1}{2}mv^2 = mgR(1-\cos\theta) + mgR\sin\theta$$

$$\Rightarrow \quad v = \sqrt{2gR(1-\cos\theta+\sin\theta)}$$

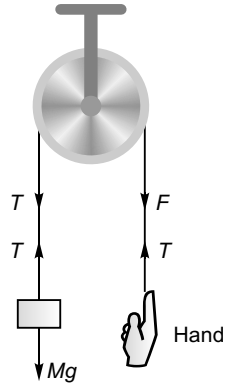
6. $\Delta K = W$

or $\quad 0 - \frac{1}{2}mv_0^2 = \int_0^x -Ax\,dx$

or $\quad -\frac{1}{2}mv_0^2 = -A\frac{x^2}{2}$

$$\Rightarrow \quad x = v_0\sqrt{\frac{m}{A}}$$

7. (a) If $T = mg$, the block will not get accelerated to gain KE. The value of T must be greater that Mg.



$\therefore$ **Ans.** False

(b) As some negative work will be done by Mg, the work done by T will be more that 40 J.

$\therefore$ **Ans.** False

(c) Pulling force F will always be equal to T, as T is there only because of pulling.

$\therefore$ **Ans.** True

(d) Work done by gravity will be negative **Ans.** False

Introductory Exercise 6.3

1. In Fig. 1

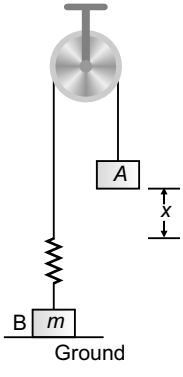


Fig. 1

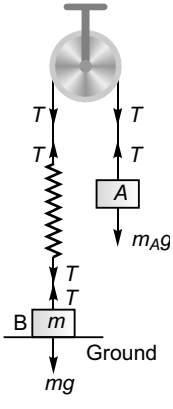


Fig. 2

Spring is having its natural length.

In Fig. 2 A is released. A goes down by x. Spring get extended by x. Decrease in PE of A is stored in spring as its PE.

$$\therefore \quad mAg\,x = \frac{1}{2}kx^2$$

Now, for the block B to just leave contact with ground

$$kx = mg$$

i.e., $\quad 2m_A g = mg$

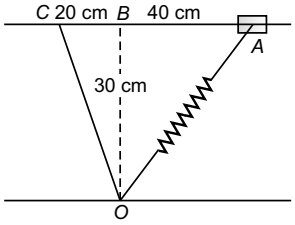
$$\Rightarrow \quad m_A = \frac{m}{2}$$

2. Decrease in PE $= mg\dfrac{l}{2}$

$\therefore \quad \dfrac{1}{2}mv^2 = mg\dfrac{l}{2}$

i.e., $\quad v = \sqrt{g\,l}$

3. $OA = 50$ cm



$\therefore$ Extension in spring (when collar is at A)

$= 50\text{ cm} - 10\text{ cm} = 0.4\text{ m}$

Extension in spring (collar is at B)

$= 30\text{ cm} - 10\text{ cm}$

$= 20\text{ cm} = 0.2\text{ m}$

KE of collar at B

$= \underset{\text{(collar at } A)}{\text{PE of spring}} - \underset{\text{(collar at } B)}{\text{PE of spring}}$

$= \dfrac{1}{2} \times K \times [(0.4)^2 - (0.2)^2]$

or $\dfrac{1}{m}mv_B^2 = \dfrac{1}{2} \times 500 \times 12$

or $\quad v_B = \sqrt{\dfrac{500 \times 0.12}{10}} = 2.45\text{ s}^{-1}$

Extension in spring (collar arrives at C)

$= [\sqrt{(30)^2 + (20)^2} - 10]\text{ cm} = 0.26\text{ m}$

KE of collar at C

$= \underset{\text{(Collar at } A)}{\text{PE of spring}} - \underset{\text{(Collar at } C)}{\text{PE of spring}}$

$= \dfrac{1}{2} \times 500 \times [(0.4)^2 - (0.26)^2]$

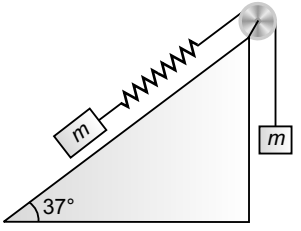
or $\dfrac{1}{2}mv_C^2 = \dfrac{1}{2} \times 500 \times 0.0924$

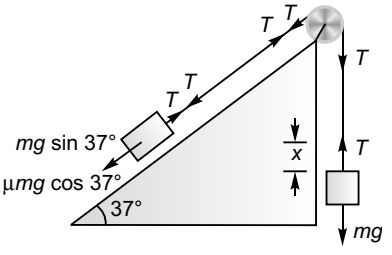
or $\quad v_c = \dfrac{500}{100} \times 0.0924 = 2.15\text{ m s}^{-1}$

4. Work done by man $= \dfrac{m}{2}gh + Mgh$

$= \left(\dfrac{m}{2} + M\right)gh$

5. When block of man M goes down by x, the spring gets extended by x. Decrease in PE of man M is stored in spring as its PE.





$\therefore \quad Mgx = \dfrac{1}{2}kx^2$

or $\quad kx = 2Mg$

For the block of man m to just slide

$kx = mg\sin 37° + \mu\, mg\cos 37°$

or $\quad 2Mg = mg\dfrac{3}{5} + \dfrac{3}{4}mg\dfrac{4}{5}$

or $\quad M = \dfrac{3}{5}m$

Introductory Exercise 6.4

1. Velocity at time $t = 2$ s

$v = g\,t = 10 \times 2 = 20\text{ ms}^{-1}$

Power = Force × velocity

$= mgv = 1 \times 10 \times 20 = 200\text{ W}$

2. Velocity at time $= a\,t = \dfrac{F}{m} \cdot t$

$\therefore v_{\text{av}} = \dfrac{Ft}{2m}$ (acceleration being constant)

$P_{\text{av}} = F \times v_{\text{av}} = \dfrac{F^2 t}{2m}$

Instantaneous power

$$= \text{Force} \times \text{instantaneous velocity}$$

$$= F \cdot \frac{Ft}{m} = \frac{F^2 t}{m}$$

3. Power $= \dfrac{\text{Energy}}{\text{Time}}$

$$P = \frac{\text{KE}}{t} \Rightarrow \text{KE} = P \cdot t$$

$$\therefore \quad \frac{1}{2}mv^2 = Pt$$

or $$v = \sqrt{\frac{2Pt}{m}}$$

or $$\frac{ds}{dt} = \sqrt{\frac{2P}{m}} \cdot t^{1/2}$$

or $$\int ds = \sqrt{\frac{2P}{m}} \int t^{1/2}\, dt$$

or $$s = \sqrt{\frac{2P}{m}} \frac{t^{3/2}}{3/2} + c$$

At $t = 0$, $s = 0$, $\therefore c = 0$

Thus, $$s = \sqrt{\frac{8P}{9m}}\, t^{3/2}$$

4. $P = 2t$

$$\text{KE} = \int P\, dt = \int 2t\, dt$$

$$= t^2 + c$$

As at $t = 0$, KE $= 0$, $c = 0$

$\therefore$ KE $= t^2$

i.e., $\dfrac{1}{2}mv^2 = t^2$ or $v = \sqrt{\dfrac{2}{m}}\, t$

$$P_{\text{av}} = \frac{2.0 + 2t}{2} = t$$

5. $U = -20 + (x-2)^2$

PE$(= U)$ is minimum at $x = 2$.

$\therefore$ Equilibrium position is at $x = 2$ m

$$\frac{dU}{dx} = 2(x-2)$$

$$\frac{d^2U}{dx^2} = 2$$

$\therefore$ Equilibrium is stable.

6. $F = x - 4$

For equilibrium, $F = 0$

i.e., $x - 4 = 0$

i.e., $x = 4$ m

As, $F = -\dfrac{dU}{dx}$

$$\frac{dU}{dx} = -(x-4)$$

$$\therefore \quad \frac{d^2U}{dx^2} = -1$$

Thus, equilibrium is unstable.

AIEEE Corner

Subjective Questions (Level I)

(a) **Work done by a constant force**

1. (a) Work done by a constant force

Work done by applied force $= F\, s \cos 0$

$$= 40 \times 2 = 80 \text{ Nm}$$

Work done by force of gravity $= mgs \cos \pi$

$$= 2 \times 10 \times 2 \times -1 = -40 \text{ Nm}$$

2. $\vec{\mathbf{r}}_{21} = \vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1$

$$= (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 4\hat{\mathbf{k}}) - (1\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 6\hat{\mathbf{k}})$$

$$= \hat{\mathbf{i}} - \hat{\mathbf{j}} - 10\hat{\mathbf{k}}$$

Work done $= \vec{\mathbf{F}} \cdot \vec{\mathbf{r}}_{21}$

$$= (6\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} - \hat{\mathbf{j}} - 10\hat{\mathbf{k}})$$

$$= 6 + 2 - 10 = -2 \text{ Nm}$$

3. $m_1 g - T = m_1 a$ ...(i)

$T - m_2 g = m_2 a$ ...(ii)

Solving Eq. (i) and Eq. (ii),

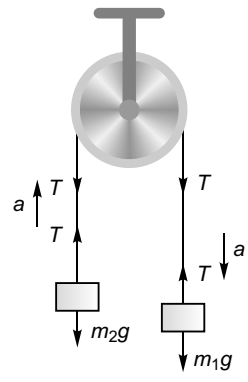
$$a = \frac{m_1 - m_2}{m_1 + m_2} g$$

$$= \frac{4-1}{4+1} g$$

$$\left(\begin{array}{l} \because m_1 = 4 \text{ kg} \\ \text{and } m_2 = 1 \text{ kg} \end{array}\right)$$

$$= \frac{3}{5} g$$

$$s = ut + \frac{1}{2}at^2$$



$$= \frac{1}{2}\left(\frac{3}{5}g\right)2^2$$

$$= 12 \text{ m}$$

Work done by gravity on 4 kg block

$$= 4g \times 12 \cos 0$$

$$= 480 \text{ Nm}$$

Solving Eq. (i) and Eq. (ii),

$$T = \frac{2m_1m_2}{m_1 + m_2}g$$

$$= 2 \times \frac{4}{5}g$$

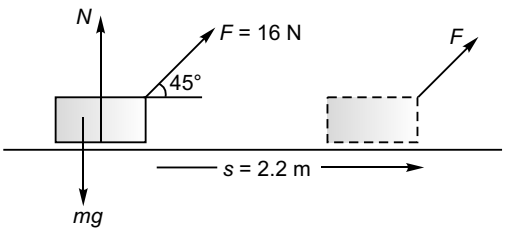
$$= 16 \text{ N}$$

Work done by string on 1 kg block

$$= 16 \times 12 \cos 0$$

$$= 192 \text{ Nm}$$

4. Work done by applied force $= F\,s \cos 45°$



$$= 16 \times 2 \cdot 2 \times \frac{1}{\sqrt{2}}$$

$$= 24.9 \text{ Nm}$$

Work done by normal force

$$= N\,s \cos 90° = 0$$

Work done by force of gravity

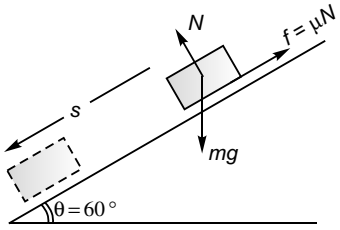
$$= mgs \cos 90° = 0$$

Total work done on the block

$$= 24.9 + 0 + 0$$

$$= 24.9 \text{ Nm}$$

5. Work done by gravity $= mgs \sin\theta$



$$= 2 \times 10 \times 2 \sin 60°$$

$$= 20\sqrt{3} \text{ Nm}$$

$$= 34.6 \text{ Nm}$$

Work done by force of friction

$$= f\,s \cos \pi$$

$$= \mu\, Ns \cos \pi$$

$$= -\mu\,(mg \cos\theta)\,s$$

$$= -\frac{1}{2}(2 \times 10 \times \cos 60°) \times 2$$

$$= -10 \text{ Nm}$$

(b) Work done by a variable force.

6. $$W = \int F\,dx$$

$$= \int_{x=2}^{x=-4} -2x\,dx$$

$$= -\left[2\frac{x^2}{2}\right]_{x=2}^{x=-4}$$

$$= -[(-4)^2 - (2)^2]$$

$$= -12 \text{ Nm}$$

7. $$W = \int F\,dx$$

$$= \int_{x=4}^{x=2} \frac{4}{x^2}\,dx$$

$$= \int_{x=4}^{x=2} 4x^{-2}\,dx$$

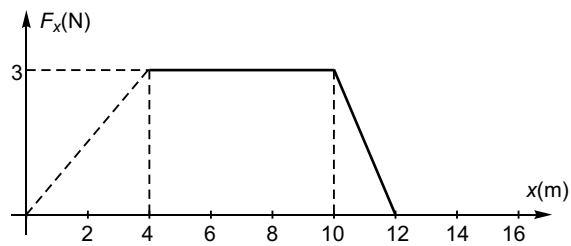
$$= \left[4\frac{x^{-2+1}}{-2+1}\right]_{x=4}^{x=2}$$

$$= -4\left[\frac{1}{x}\right]_{x=4}^{x=2}$$

$$= -4\left[\frac{1}{2} - \frac{1}{4}\right] = -1 \text{ Nm}$$

(c) Work done by area under *F-x* graph

8. (a) $W = 3 \times (5 - 10) = -15$ Nm



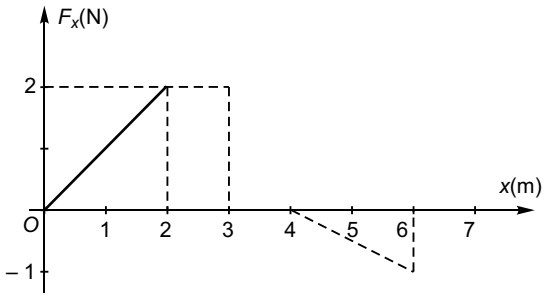
(b) $W = 3 \times (10 - 5) = +15$ Nm

(c) $W = \dfrac{3 \times (12 - 10)}{2} = 3$ Nm

(d) $W = \dfrac{(10 - 4) + (12 - 0)}{2} \times 3$

$= +27$ Nm

9. (a) $W = \dfrac{(3 - 2) + (3 - 0)}{2} \times 2$



$= 4$ Nm

(b) $W = 0$ Nm

(c) $W = \dfrac{1}{2}(6 - 4)(-1 - 0) = -1$ Nm

(d) $W = 4$ Nm $+ 0$ Nm $+ (-1)$ Nm

$= 3$ Nm.

Conservative force field and Potential Energy.

10. $$F = -\frac{d}{dr}U$$
$$= -\frac{d}{dr}Ar^{-1}$$
$$= (-)(-A)\,r^{-1-1} = \frac{A}{r^2}$$

11. $U = (x - 4)^2 - 16$

$\therefore$ PE (at $x = 6.0$ m)

$= (6 - 4)^2 - 16 = -12$ J

KE (at $x = 6.0$ m) $= 8$ J

(a) $\therefore$ Total mechanical energy

$= (-12) + (8) = -4$ J

(b) KE will be maximum where, PE, is minimum.

For U to be minimum,

$$\frac{dU}{dx} = 0$$

i.e., $\dfrac{d}{dx}[(x - 4)^2 - 16] = 0$

or $2(x - 4) = 0$

or $x = 4$ m

$\therefore$ minimum PE $= (4 - 4)^2 - 16$

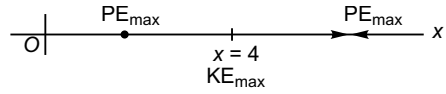
$= -16$J

Now, $(\text{KE})_{\max} + (\text{PE})_{\min} =$ Total mechanical energy

or $(\text{KE})_{\max} + (-16) = -4$

or $(\text{KE})_{\max} = 12$ J

(c) $\text{PE}_{\max} = \text{KE}_{\max} = 12$ J



$U = (x - 4)^2 - 16$

or $12 = (x - 4)^2 - 16$

or $x^2 - 8x - 12 = 0$

$\Rightarrow$ $x = 4 + 2\sqrt{3}$

and $= 4 - 2\sqrt{3}$

(d) $U = (x - 4)^2 - 16$

$$\frac{dU}{dx} = 2(x - 4)$$

$\therefore$ $F_x = -\dfrac{dU}{dx} = -2(x - 4)$

or $F_x = 8 - 2x$

(e) $F_x = 0$

i.e., $8 - 2x = 0$

or $x = 4$ m

Kinetic energy and Work-energy theorem

12. $K = \dfrac{p^2}{2m}$

$\therefore$ $$K' = \frac{\left(p + \frac{p}{2}\right)^2}{2m}$$

(K' is the KE when momentum p is increased by 50%)

or $K' = \dfrac{9}{4}\dfrac{p^2}{2m}$

or $K' = \dfrac{9}{4}K$

or $K' - K = \dfrac{9}{4}K - K = \dfrac{5}{4}K$

$\therefore$ $\dfrac{K' - K}{K} = \dfrac{5}{4}$

$= \dfrac{5}{4} \times 100\% = 125\%$

13. $p = (2mK)^{1/2}$

$\therefore \quad p' = [2m(K + 1\% \text{ of } K)]^{1/2}$

(p' is the momentum when KE *i.e.,* K is increased by 1%)

$$= [2mK(1 + 1\%)]^{\frac{1}{2}}$$

$$= (2mK)^{1/2}\left(1 + \frac{1}{100}\right)^{1/2}$$

i.e., $$p' = p\left(1 + \frac{1}{100}\right)^{1/2}$$

$$= p\left(1 + \frac{1}{2} \times \frac{1}{100}\right)$$

$$= p\left(1 + \frac{1}{2}\%\right)$$

$$p' = p + \frac{1}{2}\% \text{ of } p$$

$\therefore$ Increase in momentum = 0.5 %.

14. $s = (2t^2 - 2t + 10)$ m

$$\frac{ds}{dt} = 4t - 2$$

$$\frac{d^2s}{dt^2} = 4 \text{ ms}^{-2}$$

$$F = m\frac{d^2s}{dt^2}$$

$$= 2 \times 4 = 8 \text{ N}$$

s (at $t = 0$ s) = 10 m

s (at $t = 2$ s) = $2.2^2 - 2.2 + 10 = 14$ m

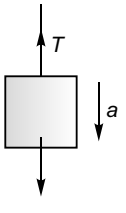
$\therefore \Delta s = 14 \text{ m} - 10 \text{ m} = 4 \text{ m}$

Work $= F\,\Delta s$

$= 8 \text{ N} \times 4 \text{ m}$

$= 32 \text{ Nm}$

15. $a = \dfrac{v^2}{2s} = \dfrac{(0.4)^2}{2 \times 2} = \dfrac{0.16}{4} = 0.04 \text{ m s}^{-2}$



$$\frac{mg - T}{m} = a$$

i.e., $T = m(g - a)$

$= 30(10 - 0.04)$

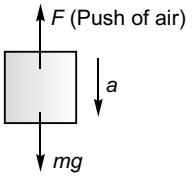
$= 298.8$ N

Work done by chain $= T\,s\cos\pi$

$= -(298.8 \times 2)$ Nm

$= -597.6$ Nm

16. $a = \dfrac{10^2}{2 \times 20} = 2.5 \text{ ms}^{-2}$



Now, $mg - F = ma$

$\therefore \quad F = m(g - a)$

$= 5(10 - 2.5)$

$= 37.5$ N

Work done by push of air

$= F\,s\cos\pi$

$= -(37.5 \times 20)$

$= -750$ Nm

17. (a) $W = \int F\,dx$

$$= \int_0^2 (2.5 - x^2)\,dx$$

$$= \left[2.5x - \frac{x^3}{2}\right]_0^2$$

$$= (2.5 \times 2) - \frac{2^3}{3}$$

$$= 2.33 \text{ Nm}$$

$\therefore \Delta K = 2.33$ J (1 Nm = 1 J)

i.e., KE (at $x = 2$) – KE (at $x = 0$) = 2.33 J

$\therefore$ KE (at $x = 2$) = 2.33 J

(b) Position of maximum KE

$$F = 2.5 - x^2$$

$\therefore F$ decreases as x increases and F is zero when $x = \sqrt{2.5}$ m

Thus, work will be +ive from $x = 0$ to $x = \sqrt{2.5}$ m an so KE will be maximum at $x = \sqrt{2.5}$ m.

KE (at $x = \sqrt{2.5}$ m)

$$= \int_0^{\sqrt{2.5}} (2.5 - x^2)\,dx$$

$$= \left[2.5x - \frac{x^3}{3}\right]_0^{\sqrt{2.5}}$$

$$= \left[2.5\sqrt{2.5} - \frac{2.5\sqrt{2.5}}{3}\right]$$

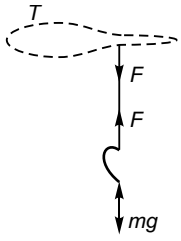
$$= \frac{2}{3} \times 2.5\sqrt{2.5}$$

$$= 2.635 \text{ J}$$

18. $a = \dfrac{F - mg}{m}$

or $\dfrac{g}{10} = \dfrac{F}{m} - g$

or $F = \dfrac{11}{10} mg$



(a) $\therefore$ Work done on astronaut by F

$= \dfrac{11mg}{10} \times 15$

$= \dfrac{11}{10} \times 72 \times 10 \times 15$

$= 11642.4$ Nm

(b) Work done on astronaut by gravitational force

$= mgh \cos \pi$

$= -72 \times 9.8 \times 15 = -10584$ Nm

(c) Net work done on astronaut

$= (11642.4) + (-10584) = 1058.4$ Nm

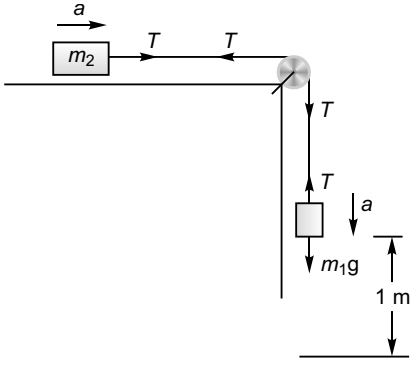
$\therefore$ KE $= 1058.4$ J

(d) $\dfrac{1}{2} mv^2 = 1058.4$

$\therefore \quad v = \sqrt{\dfrac{1058.4 \times 2}{72}} = 5.42 \text{ ms}^{-1}$

19. $T = m_2 a$ ...(i)

and $m_1 g - T = m_1 a$ ...(ii)



Adding Eq. (i) and Eq. (ii),

$m_1 g = (m_1 + m_2) a$

$\therefore \; a = \dfrac{m_1}{m_1 + m_2} g$

$= \dfrac{1}{1+4} g \quad (\because m_1 = 1 \text{ kg and } m_2 = 4 \text{ kg})$

$= \dfrac{g}{5} = 2 \text{ ms}^{-2}$

Now, $v^2 = u^2 + 2as$

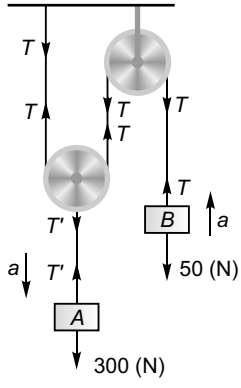
i.e., $v^2 = 2as \quad (\because u = 0 \text{ ms}^{-1})$

or $v = \sqrt{2as}$

$= \sqrt{2 \times 2 \times 1}$

$= 2 \text{ ms}^{-1}$

20. $T - 50 = \dfrac{50}{10} a$



or $T - 50 = 5a$ ...(i)

and $T' = 2T$

i.e., $T = \dfrac{T'}{2}$ ...(ii)

From Eq. (i) and Eq. (ii),

$\dfrac{T'}{2} - 50 = 5a$

or $T' - 100 = 10a$ ...(iii)

Also, $300 - T' = \dfrac{300}{10} a$

i.e., $300 - T' = 30a$ ...(iv)

Solving Eq. (iii) and Eq. (iv),

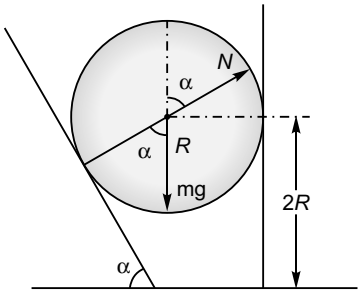
$a = 5 \text{ ms}^{-2}$

$T' = 100 + 10a = 150$ N

21. $NR\cos(180° - \alpha) + mgR = \frac{1}{2}mv_s^2$

$(mg\cos\alpha)R(-\cos\alpha) + mgR = \frac{1}{2}mv_s^2$

or $\frac{1}{2}mv_s^2 = mgR(1 - \cos^2\alpha)$



or $v_s^2 = 2gR\sin^2\alpha$

or $v_s = \sqrt{2gR}\sin\alpha$

(v_s is the speed with which sphere hits ground)

$$\frac{1}{2}mv_w^2 = mgR - \frac{1}{2}mv_s^2$$

$$= mgR - \frac{1}{2}m\,2gR\sin^2\alpha$$

$$= mgR(1 - \sin^2\alpha) = mgR\cos^2\alpha$$

$\therefore \quad v_w = \sqrt{2gR}\cos\alpha$

(v_w is the speed of wedge when the sphere hits ground)

22. For 45 kg mass to drop 12 mm, the increase in length of the spring will be 24 mm.

Now, decrease in PE of 45 kg mass

= Increase in KE of 45 kg mass + Increase in PE of spring

$45 \times 9.8 \times 12 \times 10^{-3} = \frac{1}{2} \times 45 \times v^2 + \frac{1}{2} \times 1050 \times [(75 + 24)^2 - 75^2] \times 10^{-6}$

i.e., $5.292 = 22.5v^2 + 2.192$

or $22.5v^2 = 3.0996$

or $v^2 = 0.13776$

or $v = 0.371\,\text{ms}^{-1}$

(b) With friction (when mechanical energy does not remain conserved)

23. Decrease in PE of 1 kg mass

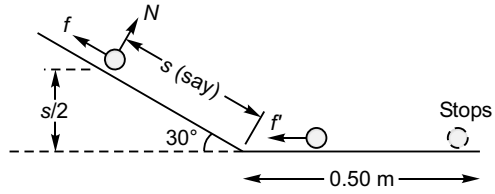
= Work done against friction due to 4 kg mass + Increase in KE of 1 kg mass

$1 \times 10 \times 1 = \mu_k \times 4 \times 10 \times 2 + \frac{1}{2} \times 1 \times (0.3)^2$

or $10 = 80\mu_k + 0.045$

or $\mu_k = \frac{10 - 0.045}{80} = 0.124$

24. f = force of friction while disc in slipping over inclined surface = $\mu\, mg\cos 30°$



f' = force of friction while disc is slipping over plane surface = $\mu\, mg$

Now, decreases in PE of disc = Work done against frictional force

$$mg\frac{s}{2} = fs + f'(0.5)$$

or $mg\frac{s}{2} = (\mu\, mg\cos 30°)s + \mu\, mg(0.5)$

or $mgs(0.5 - \mu\cos 30°) = \mu\, mg(0.5)$

$\Rightarrow \quad s = \frac{\mu \times 0.5}{0.5 - \mu\cos 30°}$

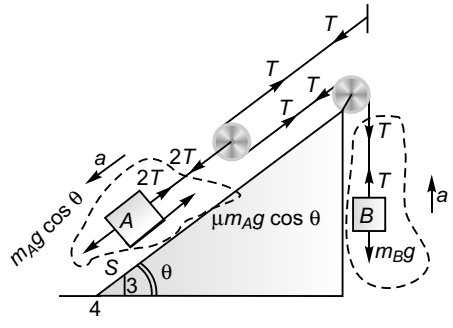
$$= \frac{0.15 \times 0.5}{0.5 - 0.15 \times \frac{\sqrt{3}}{2}} = 0.2027\text{ m}$$

Work performed by frictional forces over the whole distance

$$= -mg\frac{s}{2} = -\frac{50}{1000} \times 10 \times \frac{0.2027}{2}$$

$$= -0.051\text{ J}$$

25. $m_A g\sin\theta - \mu\, m_A g\cos\theta - 2T = m_A a$



or $300 \times \frac{3}{5} - 0.2 \times 300 \times \frac{4}{5} - 2T = 30a$

or $\quad 180 - 48 - 2T = 30a$

or $\quad 132 - 2T = 30a \quad \ldots(i)$

Also, $\quad T - m_B g = m_B a$

or $\quad T - 50 = 5a$

or $\quad 2T - 100 = 10a \quad \ldots(ii)$

Adding Eq. (i) and Eq. (ii),

$$32 = 40a$$

or $\quad a = 0.8 \text{ ms}^{-2}$

Speed (v) of block A after it moves 1 m down the plane

$$v^2 = 2as$$

or $\quad v^2 = 2 \times 0.8 \times 1$

or $\quad v = 1.12 \text{ ms}^{-1}$

26. Work done by frictional force acting on block

$$= -\mu mgs$$
$$= -0.25 \times 3.5 \times 9.8 \times 7.8$$
$$= -66.88 \text{ J}$$

$\therefore$ Increase in thermal energy of block-floor system

$$= 66.88 \text{ J}$$

As the block stopped after traversing 7.8 m on rough floor the maximum kinetic energy of the block would be 66.88 J (just before entering the rough surface).

Maximum PE of spring

$$= \text{Maximum KE of block}$$

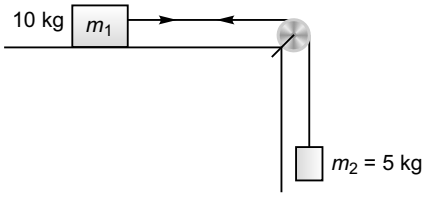
$$\frac{1}{2} k x_{\max}^2 = 66.88$$

$$\therefore \quad x_{\max} = \sqrt{\frac{2 \times 66.88}{640}} \text{ m}$$

Maximum compression in the spring

$$= 0.457 \text{ m}$$

27. Decrease in PE of mass m_2 = Work done against friction by mass m_1 + Increase KE of mass m_1 + Increase in KE of mass m_2



$$m_2 g \times 4 = \mu m_1 g \times 4 + \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v_2^2$$

$$5 \times 10 \times 4 = 0.2 \times 10 \times 10 \times 4 + \frac{1}{2}(10 + 5) v^2$$

Solving, $\quad v = 4 \text{ ms}^{-1}$

Three types of Equilibrium

28. (a) $F = -\dfrac{dU}{dr}$

Point	$\dfrac{dU}{dr}$	F
A	+ ive	– ive
B	+ ive	– ive
C	– ive	+ ive
D	– ive	+ ive
E	zero	zero

(b) $x = 2$ m point is of unstable equilibrium (U being + ive)

$x = 6$ m point is of stable equilibrium (U being lowest – ive)

29. $U = \dfrac{x^3}{3} - 4x + 6$

For U to be maximum (for unstable equilibrium) and minimum (for stable equilibrium)

$$\frac{dU}{dx} = 0$$

i.e., $\quad \dfrac{d}{dx}\left(\dfrac{x^3}{3} - 4x + 6\right) = 0$

or $\quad x^2 - 4 = 0$

or $\quad x = \pm 2$

$$\frac{d^2U}{dx^2} = \frac{d}{dx}(x^4 - 4) = 2x$$

At $x = +2$ m,

$$\frac{d^2U}{dx^2} = 2 \times (+2) = +4$$

$\therefore U$ is minium.

At $x = -2$ m,

$$\frac{d^2U}{dx^2} = 2 \times (-2) = -4$$

$\therefore U$ is maximum.

$\therefore x = +2$ m point of stable equilibrium.

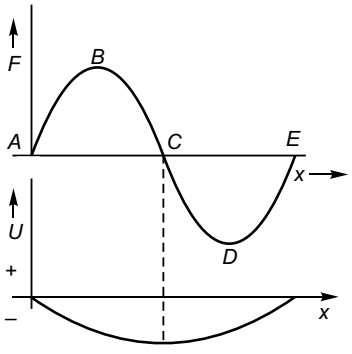
$x = -2$ m point of unstable equilibrium.

30. $F = -\dfrac{dU}{dx}$

i.e., $U = -\int F\,dx$

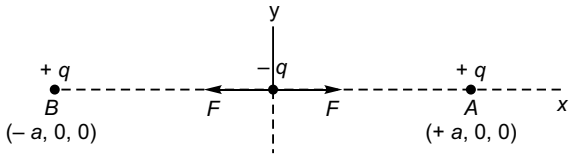
$= -$ (Area under F - x graph)

The corresponding U *Vs* x graph will be as shown in figure

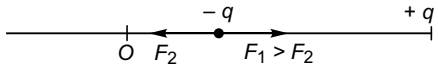


Thus, point C corresponds to stable equilibrium and points A and E correspond to unstable equilibrium.

31. $-q$ charge placed at origin is in equilibrium as is two equal and opposite forces act on it.



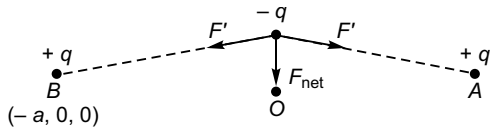
(a) But, if we displace it slightly say towards +ive x side, the force on it due to charge to B will decreases while that due to A will increase.



Due to net force on $-q$ towards right the change $-q$ will never come back to original O, its origin position.

Thus, the equilibrium of the charge $-q$ is **unstable** if it is slightly displaced along x-axis.

(b) If charge $-q$ is displaced slightly along Y axis, the net force on it will be along origin O and the particle will return to its original position. And as such the equilibrium of the $-q$ is stable.



32. (a) Velocity at $t = 0$ s is 0 ms^{-1}

Velocity at $t = 2$ s is 8 ms^{-1} (using $v = 0 + at$)

$\therefore v_{av} = 4$ m s^{-1} (as acceleration is constant)

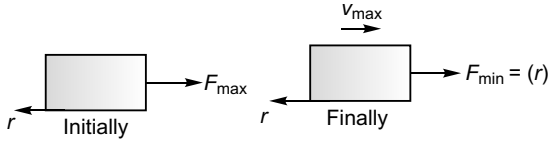
$$P_{av} = F \times v_{av}$$
$$= ma\,v_{av}$$
$$= 1 \times 4 \times 4 = 16\text{ W}$$

(b) Velocity at $t = 4$ s is 16 ms^{-1} (using $v = u + at$)

$\therefore$ Instantaneous power of the net force at $t = 4$ s will be

$$P = mav$$
$$= 1 \times 4 \times 16$$
$$= 64\text{ W}$$

33. Power $= F_{min}\,v_{max}$



$$P = r\,v_{max}$$

$\therefore$ $$v_{max} = \frac{P}{r}$$

Objective Questions (Level 1)

Single Correct Option

1. In KE $= \dfrac{1}{2}mv^2$

m is always +ive and v^2 is +ive.

(even if v is – ive).

$\therefore$ KE is always + ive.

Below reference level (PE = 0) PE is – ive.

$\therefore$ Mechanical energy which is sum of KE and PE may be – ive.

$\therefore$ Correct option is (a).

2. Yes, this is work-energy theorem.
$\therefore$ Correct option is (a).

3. On a body placed on a rough surface if an external force is applied the static body does not move then the work done by frictional force will be zero.

4. $W = \vec{\mathbf{F}} \cdot \vec{\mathbf{r}}_{21}$

$= \vec{\mathbf{F}} \cdot (\vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1)$

$= (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) \cdot [(\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) - (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})]$

$= (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) \cdot (-2\hat{\mathbf{j}} + \hat{\mathbf{k}})$

$= -4 + 3$

$= -1\,\text{J}$

$\therefore$ Correct option is (b).

5. $\therefore W = \int F\,dx$

$= \int_0^5 7 - 2x + 3x^2 dx$

$= [7x - x^2 + x^3]_0^5 = 135\,\text{J}$

$\therefore$ Correct option is (d).

6. $P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$

$= (10\hat{\mathbf{i}} + 10\hat{\mathbf{j}} + 20\hat{\mathbf{k}}) \cdot (5\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 6\hat{\mathbf{k}})$

$= 50 - 30 + 120$

$= 140\,\text{W}$

$\therefore$ Correct option is (c).

7. Work done in displacing the body

$=$ Area under the curve

$= (1 \times 10) + (1 \times 5) + (1 \times -5) + \left(\frac{1 \times 10}{2}\right)$

$= 15\,\text{J}$

$\therefore$ Correct option is (b).

8. $P = \frac{W}{t} = \frac{mgh + \frac{1}{2}mv^2}{t}$

$= \frac{800\,\text{kg} \times 10\,\text{ms}^{-2} \times 10\,\text{m} + \frac{1}{2} \times 800\,\text{kg} \times (20\,\text{ms}^{-1})^2}{1\,\text{min}}$

$= \frac{(800 \times 10 \times 10) + [400 \times (20)^2]}{60}$

$= \frac{80000 + 160000}{60}$

$= 4000\,\text{W}$

Option (c) is correct.

9. $x = \frac{t^3}{3}$

$\therefore \quad v = \frac{dx}{dt} = t^2$

and $\quad a = \frac{dv}{dt} = 2t$

$F = ma = 2 \times 2t = 4t$

$W = Fv = 4t \times t^2 = 4t^3$

$\therefore$ Work done by force in first two seconds

$= \int_{t=0}^{t=2} 4 \cdot t^3\,dt$

$= 4 \cdot \left[\frac{t^4}{4}\right]_0^2$

$= 16\,\text{J}$

$\therefore$ Correct option is (c).

10. Range $= 4 \times$ height

or $\quad \frac{u^2 \sin 2\theta}{g} = 4 \cdot \frac{u^2 \sin^2\theta}{2g}$

or $\quad \sin 2\theta = 2\sin^2\theta$

or $\quad 2\sin\theta\cos\theta - 2\sin^2\theta = 0$

or $\quad \sin\theta(\cos\theta - \sin\theta) = 0$

or $\quad \cos\theta - \sin\theta = 0 \quad (\text{as } \sin\theta \neq 0)$

or $\quad \tan\theta = 1$

i.e., $\quad \theta = 45°$

Now, K = KE at highest point

$= \frac{1}{2}m(u\cos\theta)^2$

$= \frac{1}{2}mu^2\cos^2\theta$

$= \frac{1}{2}mu^2\cos^2 45°$

$= \frac{1}{2}mu^2 \cdot \frac{1}{2}$

$\therefore \quad \frac{1}{2}mu^2 = 2K$

i.e., Initial KE $= 2K$

$\therefore$ Correct option is (b).

11. $P = 3t^2 - 2t + 1$

i.e., $\dfrac{dW}{dt} = 3t^2 - 2t + 1$

$$dW = (3t^2 - 2t + 1)\,dt$$

$$W = \int_2^4 (3t^2 - 2t + 1)\,dt$$

$$= \left[3 \cdot \frac{t^3}{3} - \frac{2t^2}{2} + t\right]_2^4$$

or $= [t^3 - t^2 + t]_2^4$

$= (4^3 - 4^2 + 4) - (2^3 - 2^2 + 2)$

$= 52 - 6$

$= 46$

$\Delta K = 46\,\text{J}$

Correct option is (b).

12. $K_i = \dfrac{1}{2} \times 10 \times 10^2 = 500\,\text{J}$

Work done by retarding force,

$$W = \int_{20}^{30} -0.1x\,dx$$

$$= -0.1\left[\frac{x^2}{2}\right]_{20}^{30}$$

$$= -0.05 \times [(30)^2 - (20)^2]$$

$$= -25\,\text{J}$$

Final kinetic energy $= K_i + W$

$= 500 + (-25)$

$= 475\,\text{J}$

$\therefore$ Correct option is (a).

13. KE of 12 kg mass : KE of 6 kg mass

$$= \frac{1}{2} m_{12} v^2 : \frac{1}{2} m_6 v^2$$

[Acceleration being same (equal to g) both will have same velocities]

$= m_{12} : m_6$

$= 12 : 6$

$= 2 : 1$

14. $W = \dfrac{1}{2} k [x_2^2 - x_1^2]$

$$= \frac{1}{2} \times 5 \times 10^3 \left[\left(\frac{10}{100}\right)^2 - \left(\frac{5}{100}\right)^2\right]$$

$$= \frac{5 \times 10^3}{2 \times 10^4}(100 - 25) = 18.75\,\text{Nm}$$

$\therefore$ Correct option is (c).

15. $x = 2.0$ m to 3.5 m

$$\frac{dU}{dx} = -\frac{6}{1.5} = -4$$

$\therefore$ $F = +4\,\text{N}$

$x = 3.5$ m to 4.5 m

$$\frac{dU}{dx} = \frac{2}{1} = 2$$

$\therefore$ $F = -2\,\text{N}$

$x = 4.5$ m to 5.0 m

$$\frac{dU}{dx} = 0$$

$\therefore$ $F = 0\,\text{N}$

Work done $= (4 \times 1.5) + (-2 \times 1) + 0$

$= 4\,\text{J}$

$\therefore$ $\dfrac{1}{2} mv^2 = 4$

or $\dfrac{1}{2} \times 1 \times v^2 = 4$

$\therefore$ $v = 2\sqrt{2}\,\text{ms}^{-1}$

16. KE at highest point

$$= \frac{1}{2} m (u \cos 45°)^2$$

$$= \frac{1}{2} mu^2 \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \frac{1}{2}\left(\frac{1}{2} mu^2\right)$$

$$= \frac{E}{2}$$

$\therefore$ Correct option is (a).

17. Work done by person

= – [Work done by gravitational pull on rope + gravitational pull on bucket]

$$= \left[\left(-mg\frac{h}{2}\right) + (-Mgh)\right]$$

$$= \left(M + \frac{m}{2}\right) gh$$

$\therefore$ Correct option is (a).

18. $\dfrac{1}{2} mv^2 = Fx$ ...(i)

$\dfrac{1}{2} mv'^2 = Fx'$ ...(ii)

$\therefore \dfrac{x'}{x} = \dfrac{v'^2}{v^2} = \dfrac{(2v)^2}{v^2} = 4 \Rightarrow x' = 4x$

$\therefore$ Correct option is (b).

19. Vertical velocity (initial) $= v_0 \sin\theta$

At an altitude h

(vertical velocity)$^2 = (v_0 \sin\theta)^2 + 2(-g)h$

(horizontal velocity)$^2 = (v_0 \cos\theta)^2$

(Net velocity)$^2 = v_0^2 - 2gh$

Net velocity $= \sqrt{v_0^2 - 2gh}$

20. Maximum power will be at 2 s

velocity (at $t = 2$ second) $= a \cdot 2$

$$= \frac{F}{m} \cdot 2$$

$\therefore$ Power (at $t = 2$ s) $= F \times \frac{2F}{m} = \frac{2F^2}{m}$

$\therefore$ Correct option is (d).

21.
$$v = 0 + at$$
$$= at = \frac{F}{m} \cdot t$$

Instantaneous power,

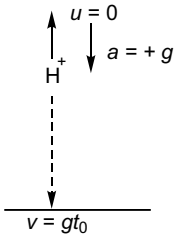
$$P = Fv$$
$$= F\frac{F}{m} \cdot t$$

or $$P = \frac{F^3}{m} \cdot t$$

or $P =$ constant t.

$\therefore$ Correct option is (b).

22. While ball comes down



at $t = t_0$ when ball strikes the surface

Using $$v = u + at$$
$$v = 0 + gt$$

i.e., $$v = gt.$$

$\therefore$ $$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}mg^2t^2$$
$$K = \phi t^2$$

where, $\phi = \frac{1}{2}mg^2 =$ constant.

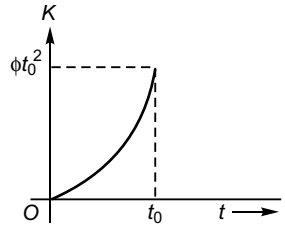
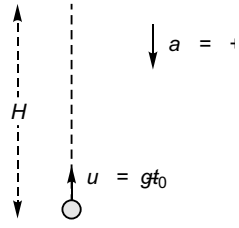


Fig. 1

While ball goes up after elastic collision with the surface it strikes



Using, $$v = u + at$$
$$v = (-gt_0) + (+g)(t - t_0)$$
$$v = -g(t - 2t_0)$$

$\therefore$ $$\text{KE} = \frac{1}{2}mg^2(t - 2t_0)^2$$

i.e., $$K = \phi(t - 2t_0)^2$$

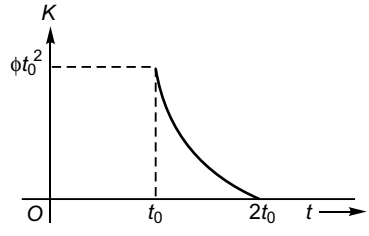


Fig. 2

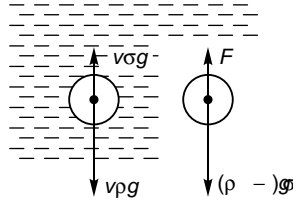
From the expression for KE

at $t = 2t_0$, $K = 0$

as shown in Fig. 2.

$\therefore$ Correct option is (b).

23. ρ (block) $= 3000$ kgm^{-3} and σ (water) $= 1000$ kgm^{-3}



i.e., $$\frac{\sigma}{\rho} = \frac{1}{3}$$

(External force applied to move the block upward with constant velocity).

Work done $= F\,s$

$$= v(\rho - \sigma)\,gs$$
$$= v(\rho - \sigma) \times 10 \times 3$$
$$= \frac{5}{\rho}(\rho - \sigma) \times 10 \times 3 \qquad (\because v\rho = 5)$$
$$= 5\left(1 - \frac{\sigma}{\rho}\right) \times 10 \times 3$$
$$= 5\left(1 - \frac{1}{3}\right) \times 10 \times 3 = 100\text{ J}$$

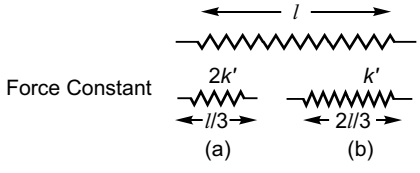
$\therefore$ Correct option is (a).

24. $$p_{av} = \frac{\text{Net work done}}{\text{Time of flight } (T)}$$
$$= \frac{mgH\cos\pi + mgH\cos 0}{T}$$

$H =$ Maximum height attained by the projectile

$$= \frac{(0)}{T} = \text{zero}$$

25. $$W(a) = \frac{1}{2}2k'(x)^2$$

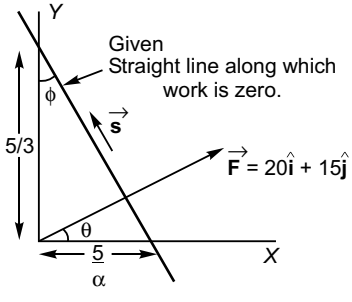


$$W(b) = \frac{1}{2}k'(x)^2$$

$\therefore \quad W(a) > W(b)$

$\therefore$ Correct option is (c).

26.



$$3y + \alpha x = 5$$
$$\frac{y}{5/3} + \frac{x}{5/\alpha} = 1$$

From above figure

$$\tan\theta = \frac{15}{20} = \frac{3}{4}$$

Work done will be zero, if

$$\phi = \theta$$

i.e., $\tan\phi = \tan\theta$

or $$\frac{5/\alpha}{5/3} = \frac{3}{4}$$

or $$\frac{3}{\alpha} = \frac{3}{4}$$

$\Rightarrow \quad \alpha = 4$

$\therefore$ Correct option is (d).

27. Let x be the elongation in the spring.

Increase in PE of spring = Decrease in PE of block

$\therefore \quad \frac{1}{2}kx^2 = mgx\sin\theta$

$\Rightarrow \quad x = \frac{2mg\sin\theta}{k}$

$\therefore$ Correct option is (a).

28. $$\vec{\mathbf{s}} = 2t^2\,\hat{\mathbf{i}} - 5\,\hat{\mathbf{j}}$$

$\therefore \quad \overrightarrow{\mathbf{ds}} = 4t\,\hat{\mathbf{i}}\,dt$

$$W = \int \vec{\mathbf{F}} \cdot \overrightarrow{\mathbf{ds}}$$
$$= \int_0^2 (3t\,\hat{\mathbf{i}} + 5\,\hat{\mathbf{j}}) \cdot 4t\,\hat{\mathbf{i}}\,dt$$
$$= \int_0^2 12t^2\,dt$$
$$= \left[12\frac{t^3}{3}\right]_0^2$$
$$= [4t^3]_0^2$$
$$= 32\text{ J}$$

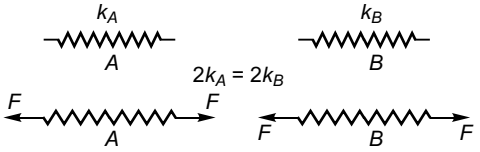
$\therefore$ Correct option is (b).

29. $$W = mgh + mgd$$
$$= mg(h + d)$$

or $F_{av}\,d = mg(h + d)$

$\therefore \quad F_{av} = mg\left(1 + \frac{h}{d}\right)$

30. Energy stored in $A = E = \frac{1}{2} k_A x_A^2$



$$= \frac{1}{2} k_A \left(\frac{F}{k_A}\right)^2$$

$$= \frac{F^2}{2k_A}$$

Energy stored in $B = \frac{F^2}{2k_B}$

$$= \frac{F^2}{4k_A} \qquad (\because k_B = 2k_A)$$

$$= \frac{1}{2}\frac{F^2}{2k_A} = \frac{E}{2}$$

$\therefore$ Correct option is (a).

31. $\frac{1}{2} kx^2 = \frac{1}{2} mv^2$

$$x = v\sqrt{\frac{m}{k}}$$

$$= 1.5\sqrt{\frac{0.5}{50}} = 0.15 \text{ m}$$

$\therefore$ Correct option is (a).

32. $\frac{1}{2} mv^2 = F(2x)$ ...(i)

$\frac{1}{2} m(2v)^2 = F(nx)$ ...(ii)

Dividing Eq. (i) by Eq. (ii),

$$\frac{(2v)^2}{v^2} = \frac{n}{2}$$

$\Rightarrow \qquad n = 8$

$\therefore$ Correct option is (d).

33. $mgh = \frac{1}{2} mv^2 + \frac{1}{2} kx^2$

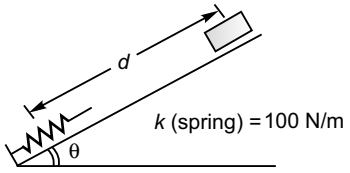
i.e., $\quad v^2 = \sqrt{2gh - \frac{k}{m} h^2}$

$$= \sqrt{2 \times 10 \times 0.15 - \frac{10}{0.1} \times (0.15)^2}$$

or $\quad v = 0.866 \text{ m s}^{-1}$

$\therefore$ Correct option is (b).

34. Decrease in PE of mass m



= Increase in PE of spring

i.e., $\quad mgd \sin\theta = \frac{1}{2} kx^2$

or $\quad d = \frac{kx^2}{2mg \sin\theta}$

$$= \frac{100 \times 2^2}{2 \times 10 \times 10 \times \sin 30°} = 4 \text{ m.}$$

$\therefore$ Correct option is (c).

35. PE of block will change into its KE and then the KE gained by the block will change into the PE of the spring.

Due to inertia the spring will not start compressing the moment the block just touches the spring and as such the block will still be in the process as increasing its KE. Thus v of the block will be maximum when it compresses the spring by some amount.

$\therefore$ Correct option is (b).

36. Work done by normal force is zero, being perpendicular to the displacement.

Work done by string is $+mgh$ while that due to gravity it is $-mgh$.

Net work done $= (+mgh) + (-mgh)$

$$= 0$$

$\therefore$ Correct option is (c).

37. Work done on floor = 0 (as displacement is zero).

38. Acceleration $= \frac{(0 - 20) \text{ ms}^{-1}}{(10 - 0) \text{ s}}$

$$= -2 \text{ ms}^{-2}$$

Therefore, net force on particle

$$= 2 \text{ kg} \times -2 \text{ m s}^{-2}$$

$$= -4 \text{ N}$$

i.e., the net force on the particle is opposite to the direction of motion.

$\therefore$ Correct option is (a).

Net work done $= F \times s$

$= F \times$ (Area under v-t graph)

$= -4 \times \left(\frac{1}{2} \times 20 \times 10\right) = -400\text{ J}$

Thus net work done may not be wholly due to frictional force only.

Further net force – 4 N may not be wholly due to friction only.

39. Height of bounce = (100 – 20)% of 10 m

= 8

∴ Correct option is (c).

KE just before bounce $= \frac{1}{2}mv^2 = mgh$

KE just after bounce = 80% of mgh

When the ball attains maxmum height after bounce

Gain in PE = mgh' = Loss of KE

or $mgh' = 80\%$ of mgh

or $h' = 80\%$ of $h = 80\%$ of 10 m

= 8 m

JEE Corner

Assertion and Reason

1. $P = Fv$

For power to be constant, the velocity must also be constant. Thus, assertion is false.

According to 2nd low of motion, net constant force will always produce a constant acceleration. Reason is true.

∴ Correct option is (d).

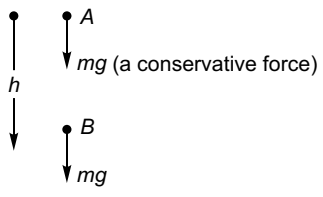
2. As displacement is opposite to force (reason) the work done by force will be negative.

Thus, Assertion is true. Further, as reason is the correct explanation of the Assertion.

∴ Correct option is (a).

3. Conservative force has nothing to do with kinetic energy. (If a non-conservative force acts on a particles, there would be loss of KE). Thus, assertion is false.

Work done by conservative force decreases PE (reason is true).



$W = mgh$

If PE at A is zero.

The PE at B would be $-mgh$.

∴ Correct option is (d).

4. In circular motion only the work done by The centripetal force is zero. Assertion is false, centripetal force acts towards centre while the velocity acts tangentially.

Reason is true.

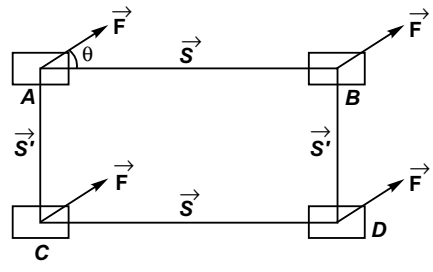
∴ Correct option is (d).

5. As the speed is increasing (slope of graph being increasing) there must be net force in the +ive direction of displacement.

Thus, work done to all forces will be positive, Assertion is true and also as explained above reason besides being true is the correct explanation for the Assertion.

∴ Correct option is (a).

6. Work done by constant force $\vec{\mathbf{F}}$ when the body shifts from A to B.



$$W_{AB} = \vec{\mathbf{F}} \cdot \vec{\mathbf{s}} = F\,s\cos\theta$$

Similarly, $W_{AC} = \vec{\mathbf{F}} \cdot \vec{\mathbf{s}}$

$= F\,s'\cos(90° + \theta)$

$= -F\,s'\sin\theta$

$$W_{CD} = \vec{\mathbf{F}} \cdot \vec{\mathbf{s}} = F\,s\cos\theta$$

$$W_{DB} = \vec{\mathbf{F}} \cdot \vec{\mathbf{s}} \cos(90° - \theta)$$
$$= F\, s' \sin\theta$$
$$W_{AC} + W_{CD} + W_{DB} = F\, s \cos\theta = W_{AB}$$

$\therefore$ Work done by a constant force is path independent. thus, Assertion is true.

The Reason is false. Kinetic frictional force remains constant but is a non-conservative force.

$\therefore$ Correct option is (c).

7. It is true that work-energy theorem can be applied to non-inertial frame also as explained in the answer to question no. 2 of introductory exercise 6.2.

Earth is non-inertial which is also true is a separate issue and has nothing to do with the assertion.

Thus, option (b) would be the answer.

8. When block is depressed the excess of upthrust force will act as restoring force and will bring the block up. The velocity gained by block will take the block above its equilibrium and the block will oscillate about its equilibrium position (as given in reason). Thus, the block will be in equilibrium in the vertical direction. Thus, assertion is also true and the reason being correct explanation of the assertion.

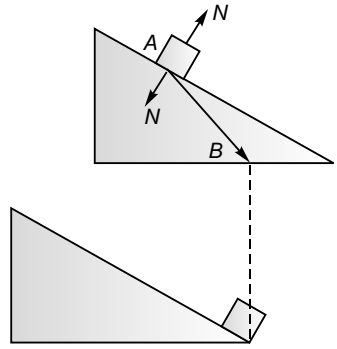
$\therefore$ Correct option is (a).

9. As displacement ($= s_2 - s_1$) is not equal to zero, the work done by all forces may not be zero. Therefore, assertion is false.

Work done by all the forces is equal to change in KE is as per work-energy theorem. Thus, reason is true.

$\therefore$ Correct option is (d).

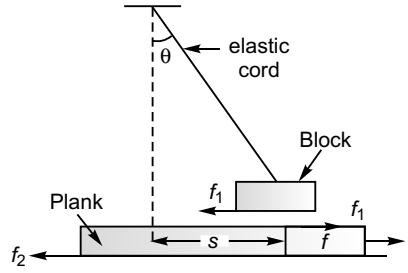
10. When the block comes down the wedge, the wedge will move towards left and the actually displacement of normal force will not be along the wedge (see chapter on CM). It will along AB as shown in figure. Thus, the angle between N (normal force on block) and its displacement AB will be greater than 90° and that between N (normal force on wedge) and its displacement will be less than 90° as given in reason.



For this the work done by N (or block) will be negative and that by N (on wedge) will be positive as given in assertion.

Reason also being the correct explanation of the assertion.

11. There will be increase in length of the elastic cord.



Work-done by static force

$$f_1 \text{ (on block)} = -f_1 s$$

Work done by static force

$$f_1 \text{ (on plank)} = +f_1 s$$

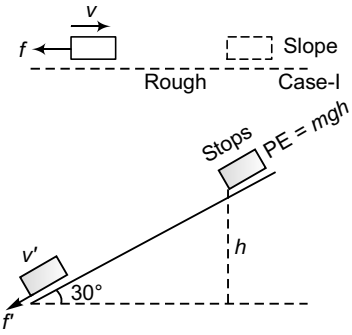
$\therefore$ Work done by static force f_1 on the system (block + plank + cord) = 0

Thus, reason is true.

The work done by the force F will be used up in doing work against friction f_2 and also increasing the elastic potential energy of the cord. Thus, assertion is true.

Further, as the reason is the correct explanation of the assertion.

12. Decrease in KE $= \frac{1}{2}mv^2 - 0$



$= \frac{1}{2}mv^2$

Change in mechanical energy

$$\text{PE} + \text{KE} = \frac{1}{2}mv^2$$

Decrease in ME is used up in doing work against friction.

In this case the mechanical energy is being used up in doing work against friction and in increasing the PE of the block.

$\therefore$ Change ME $= \frac{1}{2}mv^2 - mgh$

Thus, assertion is true.

As explained above the reason is false.

(μ does not change with the increase in angle of inclination)

$\therefore$ Correct option is (c).

Objective Questions (Level 2)

Single Correct Option

1. Increase in KE of bead = Work done by gravity + work done by force F

$\therefore \frac{1}{2}mv^2 = mgh + FR$

(Displacement of force F is R)

$$= \frac{1}{2} \times 10 \times 5 + 5 \times 5 = 50$$

$$\therefore \quad u = \sqrt{\frac{100}{m}}$$

$$= \sqrt{200} = 14.14 \text{ ms}^{-1}$$

$\therefore$ Correct option is (a).

2. $P = Fv$

$= mav$

$= m\left(v\frac{dv}{dx}\right)v \quad \left(a = \frac{dv}{dt} = \frac{dv}{ds} \cdot \frac{ds}{dt} = v\frac{dv}{ds}\right)$

$$\therefore \quad ds = \frac{m}{p}v^2 dv$$

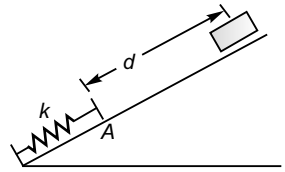
i.e., $\int ds = \frac{m}{p}\int_v^{2v} v^2\, dv$

or $\quad s = \frac{m}{p}\left[\frac{v^3}{3}\right]_v^{2v}$

$$= \frac{m}{3p}[(2v)^3 - (v)^3] = \frac{7mv^3}{3p}$$

$\therefore$ Correct option is (a).

3. Loss of PE of block = Gain in PE of spring

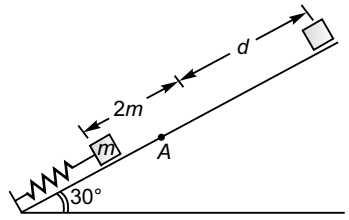


$$mg(d+2)\sin 30° = \frac{1}{2}k \times 2^2$$

or $\quad 10 \times 10 \times (d+2) \times \frac{1}{2} = \frac{1}{2} \times 100 \times 2^2$

$$d = 2 \text{ m}$$

Let, v_A = velocity of block when it just touches spring



$$\therefore \quad \frac{1}{2}mv_A^2 = mgd\sin 30°$$

$$v_A^2 = 10 \times 2 \times 2 \times \frac{1}{2}$$

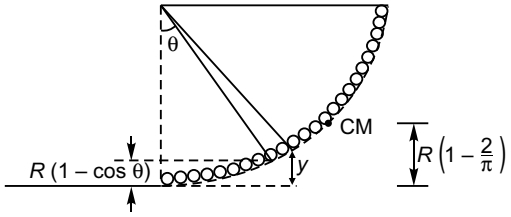
$$v_A = \sqrt{20} \text{ ms}^{-1}$$

Correct option is (a).

4. Mass per unit length of chain $= \dfrac{m}{\pi R/2}$

$$dm = R\,d\theta \frac{m}{\pi R/2}$$

$$y_{CM} = \frac{\int y\,dm}{\int dm}$$



$$= \frac{\int_0^{\pi/2} R(1-\cos\theta)\dfrac{m}{\pi R/2} R\,d\theta}{m}$$

$$= \frac{2R}{\pi}\int_0^{\pi/2}(1-\cos\theta)\,d\theta$$

$$= \frac{2R}{\pi}\left[\frac{\pi}{2}-1\right]$$

$$= R\left[1-\frac{2}{\pi}\right]$$

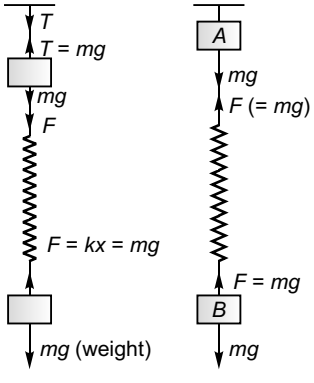
Now, $\frac{1}{2}mv^2 + mg\,y_{CM}$

or $\quad v^2 = 2g\,y_{COM}$

$$v = \sqrt{2gR\left(1-\frac{2}{\pi}\right)}$$

∴ Correct option is (c).

5. The moment string is cut



Net force on $A = 2mg$ (downward)

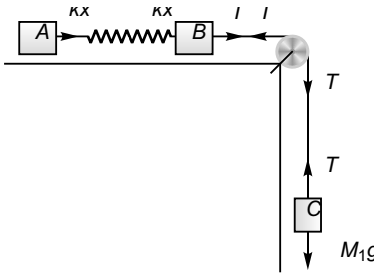
∴ $\quad a_1 = 2g$

Net force on $B = 0$

∴ $\quad a_2 = 0$

∴ Correct option is (b).

6. Let x be the expansion in the spring.



Increase in PE of spring = Decrease in PE of block C

$$\frac{1}{2}kx^2 = M_1 gx$$

i.e., $\quad kx = 2M_1 g$

For block A to remain at rest

$$kx = \mu_{min} Mg$$

or $\quad 2M_1 g = \mu_{min} Mg$

∴ $\quad \mu_{min} = \dfrac{2M_1}{M}$

∴ Correct option is (c).

7. $T_i = mg$

$$KX_i = \frac{2mg}{2} = mg$$

When one spring is cut. It means KX_i becomes zero. Downward acceleration,

$$a = \frac{KX_i}{2m} = \frac{mg}{2m} = \frac{g}{2}$$

Now drawing FBD of lower mass :

$$mg - T_f = m.a = \frac{mg}{2}$$

∴ $\quad T_f = \dfrac{mg}{2}$

or $\quad \Delta T = T_f - T_i = \dfrac{mg}{2}$

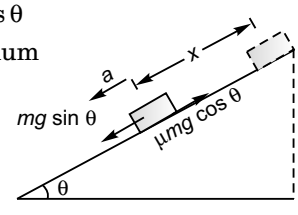
8. $a = \dfrac{mg\sin\theta - \mu\, mg\cos\theta}{m}$

$= g\sin\theta - \mu\, g\cos\theta$

For v to be maximum

$$\frac{dv}{dx} = 0$$

or $\quad v\dfrac{dv}{dx} = 0$



or $\frac{dx}{dt} \cdot \frac{dv}{dx} = 0$

or $\frac{dv}{dt} = 0$

or $a = 0$

i.e., $g \sin\theta = \mu\, g \cos\theta$

or $\sin\theta = \mu \cos\theta$

or $\frac{3}{5} = \frac{3}{10} x \cdot \frac{4}{5}$

$\Rightarrow$ $x = \frac{10}{4} = 2.5\,\text{m}$

$\therefore$ Correct option is (d).

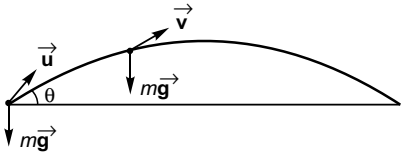
9. (a) Between points E and F, $\frac{dU}{dr}$ is – ive.

Now, $F = -\frac{dU}{dr}$, the force between E and F will be + ive *i.e.*, repulsive.

(b) At point C the potential energy is minimum. Thus, C is point of stable equilibrium.

$\therefore$ Correct option is (c).

10. Power $= \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$



$= m\,\vec{\mathbf{g}} \cdot (\vec{\mathbf{u}} + \vec{\mathbf{g}}\, t)$

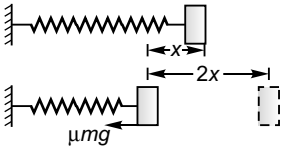
$= mg\, u \cos(90° + \theta) + m\,\vec{\mathbf{g}} \cdot \vec{\mathbf{g}}\, t$

i.e., $P = -\, mg\, u \sin\theta + mg^2 t$

$[|\vec{\mathbf{u}}| = u, |\vec{\mathbf{g}}| = g]$

Therefore, the graph between P and t will be as shown in option (c).

11.



PE of spring due to its compression by x

= | Work done by frictional force when displaced by $2x$ |

i.e., $\frac{1}{2} kx^2 = \mu\, mg\, 2x$

$\Rightarrow$ $x = \frac{4\mu\, mg}{x}$

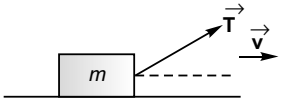
$\therefore$ Correct option is (c).

12. Power delivered by man $= \vec{\mathbf{T}} \cdot \vec{\mathbf{v}}$

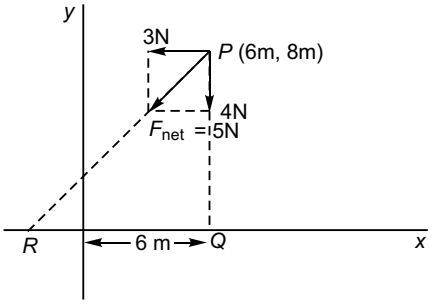
$= T v \cos\theta$

$|\vec{\mathbf{T}}| = T$

and $|\vec{\mathbf{v}}| = T$



13. $\phi = 3x + 4y$



$\therefore$ $F_x = -\frac{\partial\phi}{\partial x} = -3\,\text{N}$

and $F_y = -\frac{\partial\phi}{\partial y} = -4\,\text{N}$

$\frac{PR}{PQ} = \frac{5}{4}$

$\Rightarrow$ $PR = \frac{5}{4} \times PQ = 10\,\text{m}$

$\therefore$ Work done by the conservative force on the particle

$= F_{net} \times PR = 5\,\text{N} \times 10\,\text{m}$

$= 50\,\text{Nm} = 50\,\text{J}$

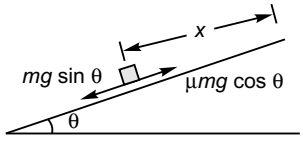
$\therefore$ Correct option is (c).

14. Both at $x = x_1$ and $x = x_2$ the force acting on the body is zero *i.e.*, it is in equilibrium.

Now, if the body (when at $x = x_1$) is moved towards right (*i.e.*, $x > x_1$) the force acting on it is + ive *i.e.*, the body will not come back and if the body (when at $x = x_2$) is moved toward rght (*i.e.*, $x > x_2$) the force acting on it is – ive *i.e.*, the body will return back. Then, $x = x_2$ is the position of stable equilibrium.

$\therefore$ Correct option is (b).

15. The man will stop when

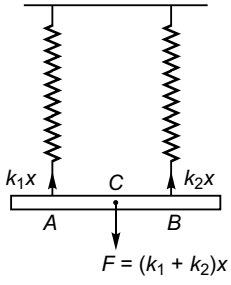


$$\mu\, mg \cos\theta = mg \sin\theta$$

or $\quad (\mu_0\, x)\, mg \cos\theta = mg \sin\theta$

or $\quad x = \dfrac{\tan\theta}{\mu_0}$

16. Taking moment about A



$$AC \cdot F = (k_2\, x)\, l$$

$\therefore \quad AC = \dfrac{(k_2\, x)\, l}{F}$

$$= \frac{(k_2\, x)\, l}{(k_1 + k_2)\, x}$$

$$= \frac{k_2}{k_1 + k_2}\, l$$

$\therefore$ Correct option is (d).

17. mgh + work done by the force of friction

$$= \frac{1}{2} mv^2$$

$\therefore$ Work done by the force of friction

$$= \frac{1}{2} mv^2 - mgh$$

$$= \left(\frac{1}{2} \times 1 \times 2^2\right) - (1 \times 10 \times 1)$$

$$= -8\text{J}$$

$\therefore$ Correct option is (c).

18. $U = \dfrac{a}{x^{12}} - \dfrac{b}{x^6}$

$U = ax^{-12} - bx^{-6}$

For stable equilibrium, $\dfrac{dU}{dx} = 0$

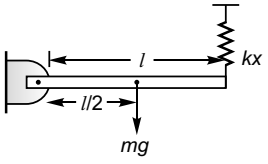
i.e., $\quad a(-12)\, x^{-13} - b(-6)\, x^{-7} = 0$

i.e., $\quad 6b\, x^{-7} = 12a\, x^{-12}$

i.e., $\quad x = \left(\dfrac{2a}{b}\right)^{\frac{1}{6}}$

$\therefore$ Correct option is (a).

19. For the rotational equilibrium of the rod



$$m\, g \cdot \frac{l}{2} = k\, x\, l$$

i.e., $\quad x = \dfrac{mg}{2k}$

$\therefore$ PE stored in the spring $= \dfrac{1}{2} kx^2$

$$= \frac{1}{2} k \left(\frac{mg}{2k}\right)^2 = \frac{(mg)^2}{8k}$$

$\therefore$ Correct option is (c).

20. $a = \dfrac{m_1 - m_2}{m_1 + m_2}\, g$

Speed (v) with which mass m_1 strikes the floor

$$= \sqrt{0^2 + 2gh} = \sqrt{2\left(\frac{m_1 - m_2}{m_1 + m_2}\right) gh}$$

$\therefore$ Correct option is (a).

21. $F = -ax + bx^2$

$\therefore \quad -\dfrac{dU}{dx} = -ax + bx^2$

or $\quad dU = (ax - bx^2)\, dx$

or $\quad U = \int (ax - bx^2)\, dx$

or $\quad U = \dfrac{ax^2}{2} - \dfrac{bx^3}{3} + c$

At $x = 0$, $F = 0$

$\therefore U = 0$

and so, $c = 0$

Thus, $\quad U = \dfrac{ax^2}{2} - \dfrac{bx^3}{3}$

$U = 0$, when $\dfrac{ax^2}{2} = \dfrac{bx^3}{3}$

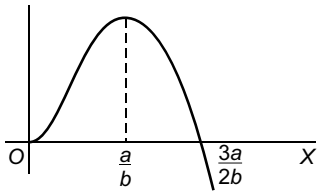
i.e., at $x = \dfrac{3a}{2b}$

$\frac{dU}{dx} = 0$, when $F = 0$

i.e., $\quad -ax + bx^2 = 0$

i.e., at $\quad x = \frac{a}{b}$

Graph between U and x will be



∴ Correct option is (c).

22. $\frac{W_A}{W_B} = \frac{Fs}{Fs} = \frac{1}{1}$

∴ Correct option is (c).

$$\frac{W_A}{W_B} = \frac{\frac{1}{2}mv_A^2}{\frac{1}{2}4mv_B^2} = 1$$

$$\Rightarrow \quad \frac{v_A^2}{v_B^2} = \frac{4}{1}$$

$$\Rightarrow \quad \frac{v_A}{v_B} = \frac{2}{1}$$

$$\frac{W_A}{W_B} = \frac{1}{1}$$

$$\therefore \quad \frac{K_A}{K_B} = \frac{1}{1}$$

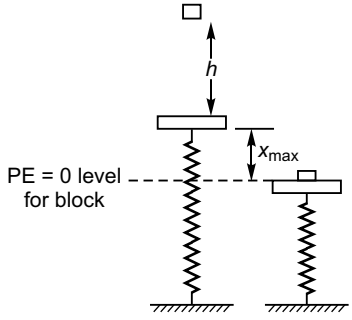
23. U_i at $(1, 1) = k(1 + 1) = 2k$

U_f at $(2, 3) = k(2 + 3) = 5k$

$$W = U_f - U_i$$
$$= 5k - 2k = 3k$$

∴ Correct option is (b).

24. Gain in PE of spring = Loss of PE of block



$$\therefore \quad \frac{1}{2}kx_{max}^2 = mg(h + x_{max}) \quad \ldots(i)$$

∴ From above Eq. (i),

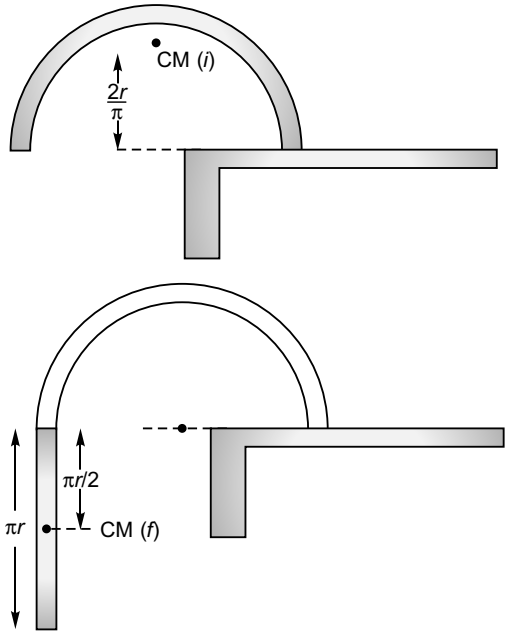
x_{max} depends upon h and also x_{max} depends upon k.

KE of the block will be maximum when it is just at the point of touching the plank and at this moment there would no compression in the spring.

Maximum KE of block = mgh

∴ Correct option is (c).

25. Gain in KE of chain = Decrease in PE of chain



When the whole chain has justcome out of the tube.

or $\quad \frac{1}{2}mv^2 = mg\left(\frac{2\pi}{\pi} + \frac{\pi r}{2}\right)$

$$\therefore \quad v + \sqrt{2gr\left(\frac{2}{\pi} + \frac{\pi}{2}\right)}$$

∴ Correct option is (b).

26. Acceleration of the block will decrease as the block moves to the right and spring expands the velocity (v) of block will be maximum, when

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

At this moment, $F = kx$

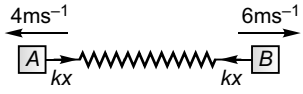
i.e., $$x = \frac{F}{k}$$

or $$v^2 = \frac{k}{m}\left(\frac{F}{k}\right)^2 = \frac{F^2}{km}$$

$\therefore$ $$v = \frac{F}{\sqrt{mk}}$$

$\therefore$ Correct option is (b).

27. Force on each block $= kx$



$$= 200 \times \frac{1}{10} = 20\,\text{N}$$

Power of $A = 20 \times 4 = 80$ W

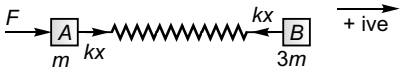
Power of $B = 20 \times 6 = 120$ W

$\therefore$ Total power $= 200$ W

i.e., rate of energy transfer $= 200\,\text{Js}^{-1}$

$\therefore$ Correct option is (c).

28. From O to x compression in the spring



Average acceleration of A

$$a_A = \frac{kx - F}{2m}$$

Average acceleration of B

$$a_B = \frac{kx}{2(3m)}$$

As at maximum compression of the spring both the blocks would be having same velocity.

$2a_A x = 2a_B x$ [using $v^2 = u^2 + 2as$]

i.e., $$a_A = a_B$$

$$\frac{kx - F}{2m} = \frac{kx}{6m}$$

or $$kx - F = \frac{kx}{3}$$

or $$F = \frac{2kx}{3}$$

i.e., $$x = \frac{3F}{2k}$$

$\therefore$ Correct option is (c).

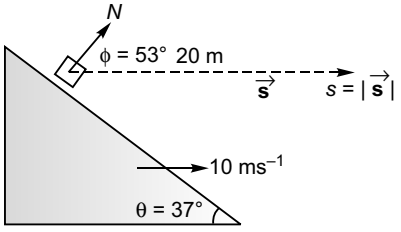
29. Work done on block A in ground frame

$$= 0.2 \times 45 \times 10 \times \frac{(50 - 30)}{100}$$

$$= 18\,\text{J}$$

$\therefore$ Correct option is (b).

30. $N = mg\cos\theta$



$$= 10 \times 10 \times \cos 37°$$

$$= 10 \times 10 \times \frac{4}{5}$$

$$= 80\ \text{N}$$

$\therefore$ Work done by N in 2 s

$$= N\cos\phi$$

$$= 80 \times 20 \times \cos 53°$$

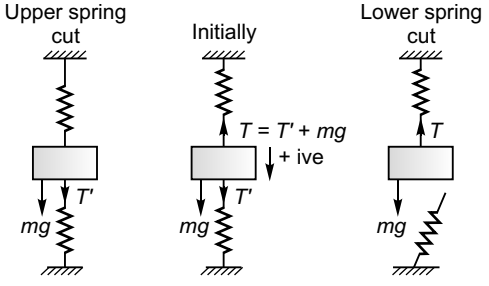
$$= 80 \times 20 \times \frac{3}{5}$$

$$= 960\,\text{Nm}$$

$$= 960\,\text{J}$$

$\therefore$ Correct option is (b).

31. $$\frac{T' + mg}{m} = 5 \qquad \text{(given)}$$



$T' + m \times 10 = 5m$

i.e., $T' = -5$ m

$$a = \frac{mg - T}{m}$$

$$= \frac{mg - (T' + mg)}{m}$$

$$= -\frac{T'}{m}$$

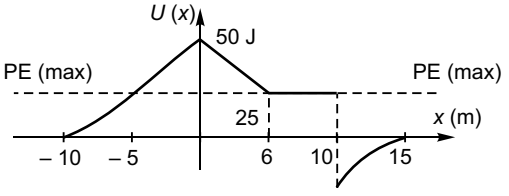
$$= -\frac{(-5m)}{m}$$

$$= 5\,\text{ms}^{-2}$$

$\therefore$ Correct option is (b).

32. Total ME = 25 J

$\therefore$ PE $(U)_{\max} = 25$ J [as KE can't be –ive.]



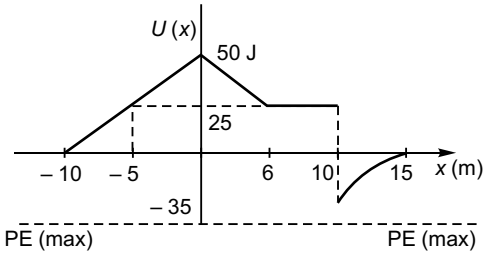
Particle can't be found in the region above PE (max) line.

$\therefore -10 < x < -5$ and $0 < x < 15$

$\therefore$ Correct option is (a).

33. Total ME = – 40 J

$\therefore$ PE (max) = – 40 J



Particle can't be found in the regions above PE (max) line.

$\therefore$ "It is not possible". Option (d).

More than One Correct Options

1. (i) Acceleration

$$U = 7x + 24y$$

$$\therefore \quad F_x = -\frac{\partial U}{\partial x} = -7$$

$$F_y = -\frac{\partial U}{\partial y} = -24$$

i.e., $$a_x = -\frac{7}{5}$$

i.e., $$a_y = -\frac{24}{5}$$

$$\therefore \quad \vec{\mathbf{a}} = -\frac{7}{5}\hat{\mathbf{i}} - \frac{24}{5}\hat{\mathbf{j}} \text{ m/s}^2$$

i.e., $$|\vec{\mathbf{a}}| = 5\,\text{ms}^{-2}$$

$\therefore$ Correct option is (b).

(ii) Velocity at $t = 4$ s

$$\vec{\mathbf{v}} = \vec{\mathbf{u}} + \vec{\mathbf{a}}\,t$$

$$= (8.6\hat{\mathbf{i}} + 23.2\hat{\mathbf{j}}) + \left[-\frac{7}{5}\hat{\mathbf{i}} - \frac{24}{5}\hat{\mathbf{j}}\right]4$$

$$= 8.6\hat{\mathbf{i}} + 23.2\hat{\mathbf{j}} - 5.6\hat{\mathbf{i}} - 10.2\hat{\mathbf{j}}$$

$$= 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$$

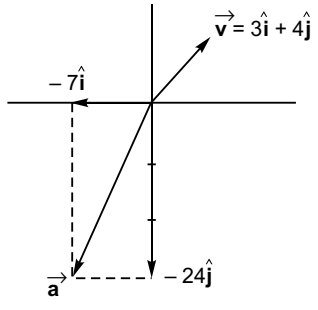
$$\therefore \quad |\vec{\mathbf{v}}| = 5\,\text{ms}^{-1}$$

$\therefore$ Correct option is (a).

(iii) As $\vec{\mathbf{a}} \cdot \vec{\mathbf{v}} \neq 0$, the path of the particle can't be a circle.

i.e., $\vec{\mathbf{a}}$ is not perpendicular to $\vec{\mathbf{v}}$.

Options (c) and (d) are incorrect.



2. $U = 100 - 5x + 100x^2$

$$\therefore \quad F_x = -\frac{\partial U}{\partial x} = -\frac{\partial}{\partial x}(100 - 5x + 100x^2)$$

$$= -[-5 + 200x]$$

$$= 5 - 200x$$

$$\therefore \quad a_x = \frac{F_x}{0.1} = \frac{5 - 200x}{0.1}$$

$$= 50 - 2000x$$

(i) At 0.05 m from origin

$x = +0.05$ m (first point)

$a_x = 50 - 2000(0.05)$

$= 50 - 100$

$= -50 \text{ m/s}^2$

$= 50 \text{ m/s}^2$ towards – ive x

∴ Correct option is (a).

(ii) At 0.05 m from origin

$x = -0.05$ m (second point)

$$a_x = 50 - 2000(-0.05)$$
$$= 50 + 100$$
$$= 150 \text{ m/s}^2$$

∴ Correct option is (c).

Mean Position

$a = 0$ at $x = \dfrac{50}{2000}$ m $= 0.025$ m

(iii) For point 0.05 m from mean position

$$x = (0.05 + 0.025) \text{ m}$$
$$= 0.075 \text{ m}$$

$$\therefore \quad a_x \text{ (at } x = 0.075 \text{ m)} = 50 - 2000(0.075)$$
$$= 50 - 150 = -100 \text{ m/s}^2$$
$$= 100 \text{ m/s}^2 \text{ towards – ve } x\text{-axis.}$$

∴ Correct option is (b).

(iv) For second point 0.05 m from mean position

$$x = 0.025 \text{ m} - 0.05 \text{ m}$$
$$= -0.025 \text{ m}$$

$$\therefore a_x = 50 - 2000(-0.025)$$
$$= 100 \text{ ms}^{-2}$$

∴ Option (d) is incorrect.

3. (i) If the spring is compressed by x, elastic potential energy equal to $\frac{1}{2}kx^2$ gets stored in the spring. Now, if the compressed spring is released the energy stored in the spring will be lost. When the spring attains to natural length.

Work done by spring = Energy stored in the spring

$$= \frac{1}{2}kx^2$$

∴ Correct option is (a).

(ii) If the spring is extended by x, energy stored in the spring would be $\frac{1}{2}kx^2$. If the extended spring is released the energy stored in the spring will be lost when the spring attains its natural length.

Work done by spring = Energy stored in the spring

$$= \frac{1}{2}kx^2$$

∴ Correct option is (b).

(iii) If spring is initially its natural length and finally compressed.

Work done on (not by) the spring $\left(= \frac{1}{2}kx^2\right)$ will be stored in the spring.

∴ Option (c) is incorrect.

(iv) If spring is initially at its natural length and finally extended.

Work done on (not by) the spring $\left(= \frac{1}{2}kx^2\right)$ will be stored in the spring.

∴ Option is (d) is incorrect.

4. (i) Work-Energy theorem states that W_{net} (Work done by all forces conservative or non-conservative, external or internal)

$$= \Delta \text{ (KE)}$$

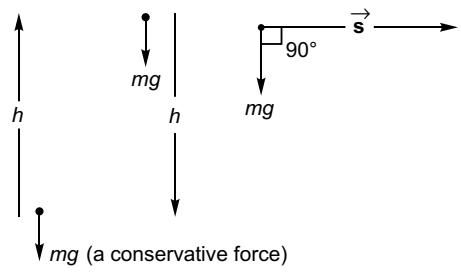
Correct option is (d) is incorrect.

∴ Correct option is (c).

(ii) Work done by non-conservative forces (*i.e.,* all forces except conservative forces) lead to decrease in KE and thus change in mechanical energy takes place.

∴ Correct option is (b).

(iii) Work done by a conservative force may be + ve, – ve or zero.



$$W = m\vec{\mathbf{g}} \cdot \vec{\mathbf{h}}$$
$$= mgh \cos \pi$$
$$= -mgh$$

(PE increases)

$$W = m\vec{\mathbf{g}}\cdot\vec{\mathbf{h}}$$
$$= mgh\cos 0$$
$$= +\,mgh$$

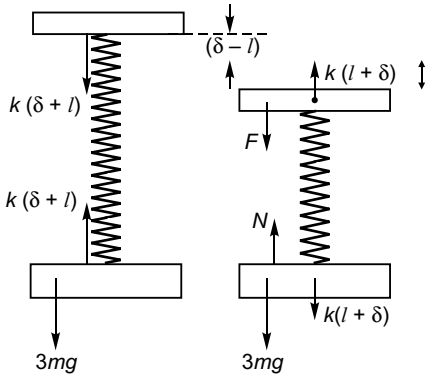
(PE increases)

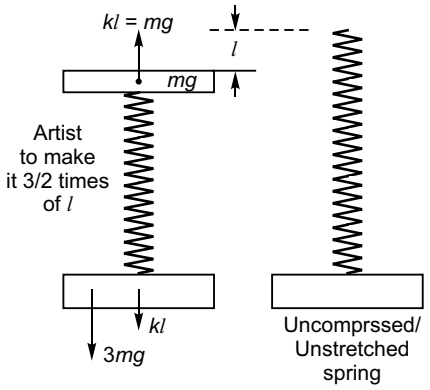
$$W = m\vec{\mathbf{g}}\cdot\vec{\mathbf{h}} = mgh\cos\frac{\pi}{2} = 0$$

(PE remains same)

Option (a) is incorrect.

5. F is the force by hand or upper disc





$$N = 3mg + k(l+\delta)$$
$$= 3mg + k\left(l + \frac{2mg}{k}\right)$$
$$= 3mg + kl + 2mg$$
$$= 3mg + mg + 2mg$$
$$= 6mg \quad \left(\text{for } \delta = \frac{2mg}{k}\right)$$

∴ Correct option is (b).

$$N = kl + 3mg$$
$$= mg + 3mg = 4mg$$

∴ Correct option is (c).

When F is removed, the upper disc accelerates upwards and when it attains the position as in figure 2, its acceleration reduces to zero and the velocity gained by it takes it further upwards. Restoring force on the upper plate now acts downwards and that on the lower plate acts in the upward direction and would lift it (lower plate) if

$$k(\delta - l) > 3mg$$

i.e., $k\delta > 3mg + kl$

or $k\delta > 3mg + mg$

or $\delta > \dfrac{4mg}{k}$

∴ Correct option is (d).

Correct option is (a) obviously being incorrect.

6. At maximum extension x :

decrease in potential energy of B = increase in spring energy

∴ $(2m)(g)(x) = \dfrac{1}{2}kx^2$

or $k = \dfrac{4mg}{k}$

7. Total work done by internal forces of a system, which constitute action and reaction pairs, is always zero and if it is not so the total work done will not zero.

∴ Correct options are (b) and (c).

8. (i) Work done by conservative forces may be + ive, – ive or zero as explained in the answer to question no. 4

∴ Option (a) is incorrect.

Correct option is (b).

Correct option is (c).

(ii) In pure rolling work done by frictional force (a non-conservative force) is always zero.

∴ Correct option is (d).

9. In moving from 1 to 2 work done by conservative force

$$= U_1 - U_2$$
$$= (-20) - (-10)$$
$$= -10\text{ J}$$

∴ Option (a) is incorrect.

Option (b) is correct.

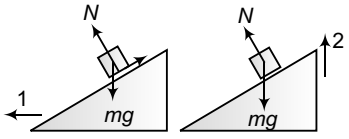
Work done by all forces

$$= (K_2 + U_2) - (K_1 + U_1)$$
$$= [20 + (-10)] - [10 + (-20)]$$
$$= 20 \text{ J}$$

∴ Correct option is (c).

Option (d) is incorrect.

10.



Work done by gravity

In motion 2 ⟶ – ive [as $\theta = \pi$]

In motion 1 ⟶ zero[as $\theta = \frac{\pi}{2}$]

∴ Correct option is (a).

Work done by Normal force (N) on block A

$$W = Ns\cos\theta$$

In motion 2 ⟶ + ve as $\theta < 90°$

In motion 1 ⟶ + ve as $\theta < 90°$

∴ Correct option is (b).

Work done by force of friction (f)

In motion 1 → may be – ive if f is directed upwards along the plane as shown in figure 1. (Motion 1 being retarded)

∴ Correct option is (c).

In motion 1 → may be + ive if f is directed downwards along the plane if motion 1 is accelerated.

∴ Correct option is (d).

Match the Columns

1. As body is displaced from $x = 4$ m to $x = 2$ m

$$\vec{\mathbf{s}} = -2\hat{\mathbf{i}} \text{ m}$$

(a) $\vec{\mathbf{F}} = 4\hat{\mathbf{i}}$

∴ $W = \vec{\mathbf{F}} \cdot \vec{\mathbf{s}} = 4\hat{\mathbf{i}} - 2\hat{\mathbf{i}}$

$= -8$ unit

(a) → (q)

and $|W| = 8$ unit

(a) → (s)

(b) $\vec{\mathbf{F}} = 4\hat{\mathbf{i}} - 4\hat{\mathbf{j}}$

∴ $W = \vec{\mathbf{F}} \cdot \vec{\mathbf{s}} = (4\hat{\mathbf{i}} - 4\hat{\mathbf{j}}) \cdot (-2\hat{\mathbf{i}}) = -8$ unit

(b) → (q, s)

(c) $\vec{\mathbf{F}} = -4\hat{\mathbf{i}}$

∴ $W = \vec{\mathbf{F}} \cdot \vec{\mathbf{s}} = 8\hat{\mathbf{i}}$ unit

and $|W| = 8$ unit

(c) → (p, s)

(d) $\vec{\mathbf{F}} = -4\hat{\mathbf{i}} - 4\hat{\mathbf{j}}$

∴ $W = \vec{\mathbf{F}} \cdot \vec{\mathbf{s}}$

$= (-4\hat{\mathbf{i}} - 4\hat{\mathbf{j}}) \cdot (-2\hat{\mathbf{i}}) = 8$ unit

and $|W| = 8$ unit

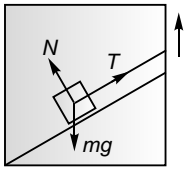
(d) → (p, s).

2. $W = Fs\cos\theta$

(a) Work done by N will be + ive as $\theta < 90°$

(a) → (p)

(b) Work done by mg will be – ive as $\theta = \pi$



(b) → (q)

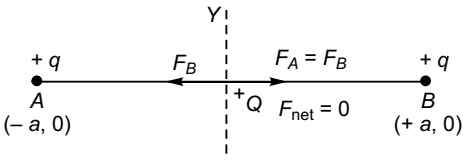
(c) Work done by force of friction (f) will be zero as f will be zero.

(c) → (r)

(d) Work done by tension (T) will be + ive as $\theta < 90°$

(d) → (p)

3.

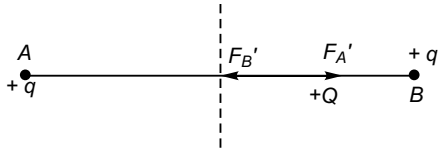


F_A = force on $+Q$ by $+q$ placed at A

F_B = force on $+Q$ by $+q$ placed at B

As $F_A = F_B$ charge $+Q$ will be in equilibrium.

(a)



Due to increase in distance between $+q$ (at A) and $+Q$

$$F_A' < F_A$$

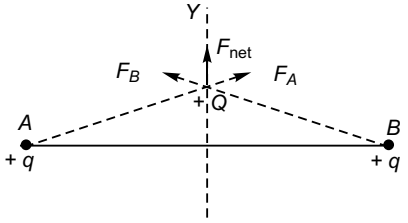
Due to decrease in distance between $+q$ (at B) and $+Q$

$$F_B' > F_B$$

Using $F_A = F_B$, we have $F_B' > F_A$. As there will net force on $+Q$ which will being $+Q$ to origin.

Thus, equilibrium will be a stable one.

(b)



As F_{net} will be along the increasing direction of Y, the charge $+Q$ will not return to origin.

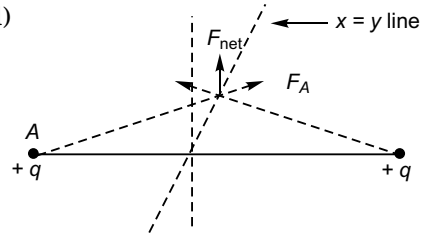
Thus, equilibrium will be an unstable one.

$\therefore$ (b) $\rightarrow$ (q)

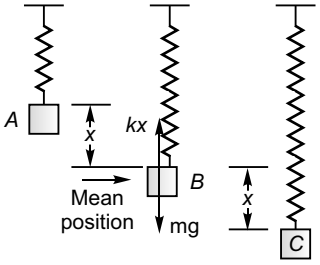
(c) As explained in (b) the equilibrium will be an unstable on.

$\therefore$ (c) $\rightarrow$ (q)

(d)



4. (a) From A to B :



Increase in spring PE $= \frac{1}{2}kx^2$

Decrease in gravitational. PE of block

$$= mgx$$
$$= (kx)\,x = kx^2$$

$\therefore$ (a) $\rightarrow$ (q)

(b) From A to B

Increase in KE of block

=Decrease in gravitational PE of block – Increase in spring PE

$$= kx^2 - \frac{1}{2}kx^2$$

$= \frac{1}{2}kx^2$ < Decrease in gravitational PE of block.

$\therefore$ (b) $\rightarrow$ (p)

(c) From B to C

Increase in spring PE $= \frac{1}{2}k(2x)^2 - \frac{1}{2}kx^2$

$$= \frac{3}{2}kx^2$$

Decrease in KE of block $= \frac{1}{2}kx^2$

(KE of block at C will be zero)

$\therefore$ (c) $\rightarrow$ (p).

(d) From B to C

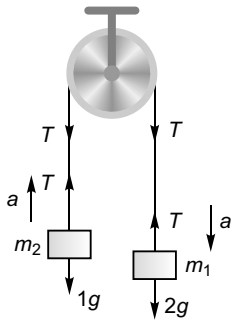
decrease in gravitational PE $= mgx$

$$= (kx)\,x$$
$$= kx^2$$

Increase in spring PE $= \frac{3}{2}kx^2$

$\therefore$ (d) $\rightarrow$ (p)

5. $a = \dfrac{m_1 - m_2}{m_1 + m_2} g = \dfrac{2-1}{2+1} g$



$$= \frac{1}{3} g$$

$$= \frac{10}{3} \text{ m s}^{-2}$$

$$T = \frac{2 m_1 m_2}{m_1 + m_2} g$$

$$= 2 \times \frac{1 \times 2}{1+2} \times 10$$

$$= \frac{40}{3} \text{ N}$$

Displacement of blocks,

$$s = ut + \frac{1}{2} at^2$$

$$= \frac{1}{2} at^2$$

$$= \frac{1}{2} \times \frac{10}{3} \times (0.3)^2$$

$$= 0.15 \text{ m}$$

(a) Work done by gravity on 2 kg block

$$= mgs \cos 0$$

$$= 2 \times 10 \times 0.15 \times (1)$$

$$= 3 \text{ J}$$

$\therefore$ (a) $\rightarrow$ (r)

(b) Work done by gravity on 1 kg block

$$= mgs \cos \pi$$

$$= 1 \times 10 \times 0.15 \times (-1)$$

$$= -1.5 \text{ J}$$

$\therefore$ (b) $\rightarrow$ (p).

(c) Work done by string on 2 kg block

$$= Ts \cos \pi$$

$$= \frac{40}{3} \times 0.15 \times (-1)$$

$$= -20 \text{ J}$$

$\therefore$ (c) $\rightarrow$ (s).

(d) Work done by string on 1 kg block

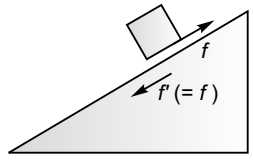
$$= Ts \cos 0$$

$$= \frac{40}{3} \times 0.15 \times 1$$

$$= 2 \text{ J}$$

$\therefore$ (d) $\rightarrow$ (q).

6. (a) Work done by friction force (f) (w.r.t. ground)



$$= f s \cos \pi$$

$$= -f s$$

$\therefore$ (a) $\rightarrow$ (q).

(b) Work done by friction force on incline (w.r.t. ground)

$$= f \times 0 \times \cos 0$$

$$= 0$$

[There being no displacement of incline w.r.t. ground]

$\therefore$ (b) $\rightarrow$ (r).

(c) Work done by a man in lifting a bucket

$= Ts \cos 0$ (T = Tension in rope)

$= a$ + ive quantity

$\theta = 0$ as T and s both would be in upward direction.

$\therefore$ (c) $\rightarrow$ (p).

(d) Total work done by friction force in (a) w.r.t. ground

$$= -f s + 0 = -f s$$

$\therefore$ (d) $\rightarrow$ (q).

7 Circular Motion

Introductory Exercise 7.1

1. In uniform circular motion the magnitude of acceleration $\left(=\frac{v^2}{r}\right)$ does not change while its direction (being always towards the centre of the circular path) changes.

2. If ω_0 and ω are in rad s^{-1} the value of α must be in rad s^{-2}. But, if ω_0 and ω are in degree s^{-1} the value of α must also be in degree s^{-2}. Thus, it is not necessary to express all angles in radian. One way change rad into degree using π rad = 180°.

3. During motion of an object along a curved path the speed and magnitude of its radial acceleration may remain constant. Due to change in direction of motion the velocity of the object will change even if its speed is constant. Further, the acceleration will also change even if the speed is constant.

4. (i) Radial acceleration (a_c)

$$= \frac{v^2}{r} = \frac{(2\,\text{cms}^{-1})^2}{1\,\text{cm}}$$

$$= 4\ \text{cms}^{-2}$$

(ii) Tangential acceleration (a_T)

$$= \frac{dv}{dt} = \frac{d}{dt}(2t)$$

$$= 2\,\text{cms}^{-2}$$

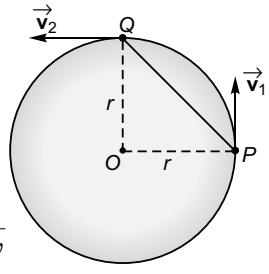
(iii) Magnitude of net acceleration

$$= \sqrt{(a_c)^2 + (a_T)^2}$$

$$= \sqrt{(4\ \text{cms}^{-2})^2 + (2\,\text{cms}^{-2})^2}$$

$$= 2\sqrt{5}\ \text{cms}^{-2}$$

5. $|\vec{\mathbf{v}}_1| = |\vec{\mathbf{v}}_2| = v$ (say)



$$T = \frac{2\pi r}{v}$$

$$|\vec{\mathbf{v}}_{\text{av}}| = \frac{|\overrightarrow{\mathbf{PQ}}|}{t_{\text{PQ}}}$$

$$= \frac{r\sqrt{2}}{T/4} = \frac{r\sqrt{2}}{\pi r/2v}$$

$$= \frac{2\sqrt{2}}{\pi}v$$

$$\therefore \frac{|\vec{\mathbf{v}}_{\text{av}}|}{v} = \frac{2\sqrt{2}}{\pi}$$

6. $\omega_t = 0 + 4t$

Centripetal acceleration

= tangential acceleration

$$r\omega_t^2 = r\alpha$$

$$\Rightarrow \quad \omega_t^2 = \alpha$$

$$(4t)^2 = 4$$

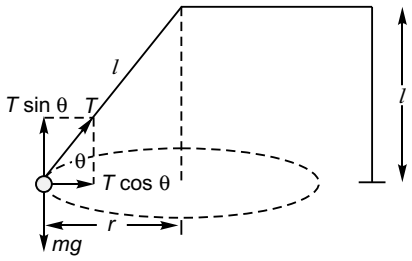
$$t = \frac{1}{2}\ \text{s}$$

Introductory Exercise 7.2

1. In uniform circular motion of a body the body is never in equilibrium as only one force (centripetal) acts on the body which forces the perform circular motion.

2. $v_{max} = \sqrt{\frac{gra}{h}} = \sqrt{\frac{9.8 \times 250 \times (1.5/2)}{1.5}}$

$= 35\,\text{ms}^{-1}$



3. (a) $T \sin\theta = mg$

$T \cos\theta = mr\omega^2$

$\therefore \quad \tan\theta = \frac{g}{r\omega^2}$

$\Rightarrow \quad \omega = \sqrt{\frac{g}{r\tan\theta}} = \sqrt{\frac{g}{(l\cos\theta)\tan\theta}}$

or $\quad \frac{2\pi}{T} = \sqrt{\frac{g}{l\sin\theta}} \Rightarrow T = 2\pi\sqrt{\frac{l\sin\theta}{g}}$

$= 2\pi\sqrt{\frac{\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)}{9.8}}$

$\therefore \quad f = \frac{1}{1} = \frac{\sqrt{9.8}}{2\pi}$ rve s^{-1}

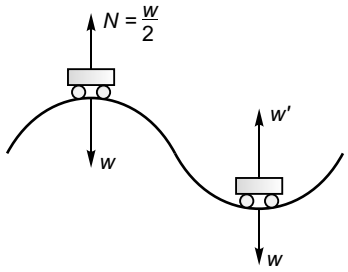
$= \frac{\sqrt{9.8}}{2\pi} \times 60$ rev min^{-1}

$= 29.9$ rev min^{-1}

(b) $T = \frac{mg}{\sin\theta} = \sqrt{2}\,mg$

$= \sqrt{2} \times 5 \times 9.8 = 69.3\text{N}$

4.



(a) At rest :

Required CPF $= w - N = \frac{w}{2}$

$\Rightarrow \quad \frac{mv^2}{r} = \frac{w}{2}$ ...(i)

At dip :

Required CPF $= N' - w$

$\frac{mv^2}{r} = N' - w$ ...(ii)

Comparing Eqs. (i) and (ii),

$N' - w = \frac{w}{2}$

$\therefore \quad N' = \frac{3w}{2} = \frac{3}{2} \times 16\,\text{kN}$

$= 24$ kN

(b) At crest on increasing the speed (v), the value of N will decrease and for maximum value of v the of N will be just zero.

Thus, $\quad \frac{mv_{max}^2}{r} = w - 0$

$\Rightarrow \quad v_{max} = \sqrt{\frac{wr}{m}}$

$= \sqrt{gr}$ (as $w = mg$)

$= \sqrt{10 \times 250}$

$= 50\,\text{ms}^{-1}$

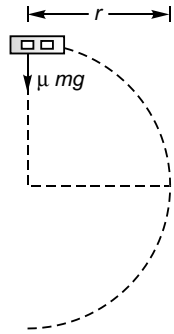
(c) At dip :

$N' = w + \frac{mv^2}{r}$

$= w + mg$

$= 2w = 32\,\text{kN}$

5. **Case I.** If the driver turns the vehicle



$$\frac{m v_1^2}{r} \le \mu\, mg$$

[where v_1 = maximum speed of vehicle]

$\Rightarrow \quad v_1 \le \sqrt{\mu\, gr}$

Case II. If the driver tries to stop the vehicle by applying breaks.

Maximum retardation $= \mu\, g$

$\therefore \quad v_2^2 = 0^2 + 2(\mu\, g)r$

$\Rightarrow \quad v_2 = \sqrt{2\mu\, gr}$

$= \sqrt{2}\, v_1$

As $v_2 > v_1$, driver should apply breaks to stop the vehicle rather than taking turn.

6. In the answer to question 3(a) if we replace θ by ϕ

$$\omega = \sqrt{\frac{g}{l \sin\phi}}$$

If θ is the angle made by the string with the vertical

$$\theta + \phi = 90°$$

i.e., $\quad \phi = 90° - \theta$

$\Rightarrow \quad \sin\phi = \cos\theta$

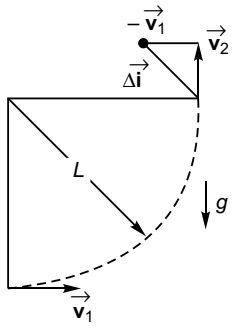
$\therefore \quad \omega = \sqrt{\frac{g}{l\cos\theta}}$

$\Rightarrow \quad \cos\theta = \frac{g}{l\omega^2}$

Introductory Exercise 7.3

1. $|\vec{\mathbf{v}}_1| = u$ and $|\vec{\mathbf{v}}_2| = v$ (say)

$\Delta\vec{\mathbf{v}} = \vec{\mathbf{v}}_2 + (-\vec{\mathbf{v}}_1)$



$= \vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1$

$\Delta\vec{\mathbf{v}} \cdot \Delta\vec{\mathbf{u}} = (\vec{\mathbf{u}}_2 - \vec{\mathbf{v}}_1)\cdot(\vec{\mathbf{v}}_2 - \vec{\mathbf{u}}_1)$

$= \vec{\mathbf{u}}_2 \cdot \vec{\mathbf{v}}_2 + \vec{\mathbf{u}}_1 \cdot \vec{\mathbf{v}}_1 - 2\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1$

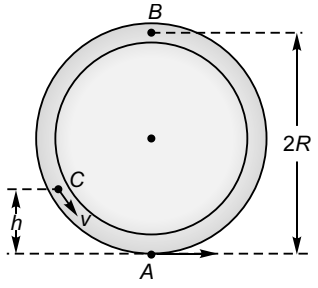
$|\vec{\mathbf{v}}_2|^2 + |\vec{\mathbf{v}}_1|^2 = v^2 + u^2$

$= u^2 - 2gL + u^2$

$|\Delta\vec{\mathbf{v}}|^2 = 2(u^2 - gL)$

$|\Delta\vec{\mathbf{v}}| = \sqrt{2(u^2 - gL)}$

2. Ball motion from A to B :



$0^2 = u_{\min}^2 + 2(-g)(2R)$

$\Rightarrow \quad u_{\min}^2 = 4gR$

Ball motion from C to A :

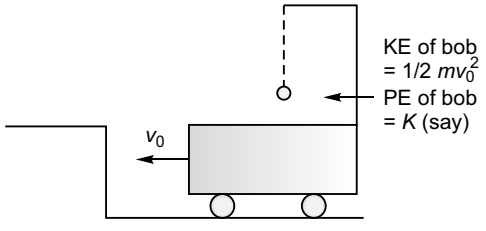
$v^2 = u_{\min}^2 + 2(-g)\,h$

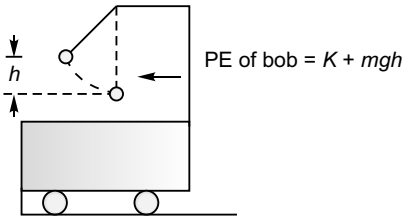
$= 4gR - 2gh$

$\Rightarrow \quad v = \sqrt{2g\,(2R - h)}$

3. Decrease in KE of bob

= Increase in PE of bob





$$\frac{1}{2}mv_0^2 = mgh$$

$$v_0^2 = 2gh$$

$$v_0 = \sqrt{2gl\,(1-\cos\theta)}$$

$$= \sqrt{2\times 9.8\times 5\times(1-\cos 60°)}$$

$$= 7\ \text{ms}^{-1}$$

AIEEE Corner

Subjective Questions (Level 1)

1. $v = 4t^2$

$\therefore \quad \dfrac{dv}{dt} = 8t$

i.e., $\quad a_T = 8\times 3 = 24\ \text{ms}^{-2}$

$v = 4\times 3^2 = 36\ \text{ms}^{-1}$ (at $t = 3$ s)

$\therefore \quad a_c = \dfrac{v^2}{4} = \dfrac{(36)^2}{54} = 24\ \text{ms}^{-2}$

Angle between $\vec{\mathbf{a}}_{\text{net}}$ and $\vec{\mathbf{a}}_t$

$$\theta = \tan^{-1}\frac{|\vec{\mathbf{a}}_c|}{|\vec{\mathbf{a}}_T|} = \tan^{-1}1 = 45°$$

2. $v = 16\ \text{ms}^{-1}$ and $r = 50$ m

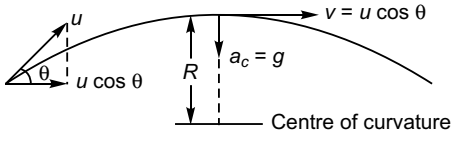
$\therefore \quad a_c = \dfrac{v^2}{r} = \dfrac{(16)^2}{50} = 5.12\,\text{ms}^{-2}$

$a_{\text{net}} = \sqrt{a_c^2 + a_T^2} = \sqrt{(5.12)^2 + 8^2}$

(given $a_T = 8\ \text{ms}^{-2}$)

$= 9.5\,\text{ms}^{-2}$

3. Speed (v) at the highest point (P)



$v = u\cos\theta$

Now, $\quad a_c = g$

$\therefore \quad \dfrac{v^2}{R} = g$

i.e., $\quad R = \dfrac{v^2}{g} = \dfrac{u^2\cos^2\theta}{g}$

4. (a) $a_c = a\cos 30° = 25\times\dfrac{\sqrt{3}}{2}$

$= 21.65\,\text{ms}^{-2}$

$\therefore \quad \dfrac{v^2}{R} = 21.65$

i.e., $\quad v = \sqrt{21.65\times 2.5}$ ($\because R = 2.5$ m)

$= 7.36\,\text{ms}^{-1}$

(b) $a_T = a\sin 30° = 25\times\dfrac{1}{2} = 12.5\,\text{ms}^{-2}$

5. (a) $v = 2.0t = 2.0\times 1$

$= 2.0\ \text{cms}^{-1}$

$\therefore \quad a_c = \dfrac{v^2}{R} = \dfrac{(2.0)^2}{1} = 4\ \text{cms}^{-2}$

(b) $v = 2.0t$

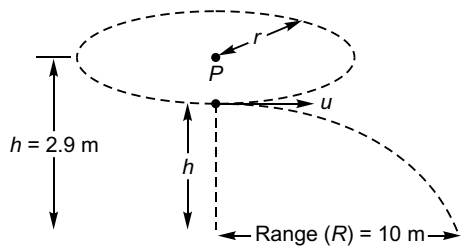
$\therefore \quad \dfrac{dv}{dt} = 2.0$

i.e., $\quad a_T = 2.0\,\text{cms}^{-2}$

(c) $a_{\text{net}} = \sqrt{a_C^2 + a_T^2}$

$= 4.47\ \text{cms}^{-2}$

6. $R = uT = u\sqrt{\dfrac{2h}{g}}$



$\therefore \quad u = R\sqrt{\dfrac{g}{2h}}$

i.e., $\quad u^2 = \dfrac{R^2 g}{2h}$

Thus, centripetal acceleration of the stone (at point P) while in circular motion

$$= \frac{u^2}{r} = \frac{R^2 g}{2h} \cdot \frac{1}{r}$$

$$= \frac{10^2 \times 9.8}{2 \times 2.9 \times 1.5}$$

$$= 112.6\,\text{ms}^{-2}$$

7. $v = 18\text{ km/h} = \frac{18.5}{18}\text{ms}^{-1} = 5\,\text{ms}^{-1}$

Angle of banking $(\theta) = \tan^{-1}\frac{v^2}{rg}$

$$= \tan^{-1}\frac{(5)^2}{10 \times 10}$$

$$= \tan^{-1}\frac{1}{40}$$

8. $\frac{mv^2}{r} = \mu\, mg$

i.e., $\mu = \frac{v^2}{gr} = \frac{(5)^2}{10 \times 10} = \frac{1}{4} = 0.25$

9. $v = \sqrt{r\,g\tan\theta}$

$$= \sqrt{50 \times 10 \times \tan 30^\circ}$$

$$= 17\text{ ms}^{-1}$$

10. $mr\,\omega_{\min}^2 = N$ ...(i)

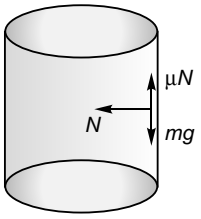
and $mg = \mu N$ ...(ii)

Solving Eqs. (i) and (ii),

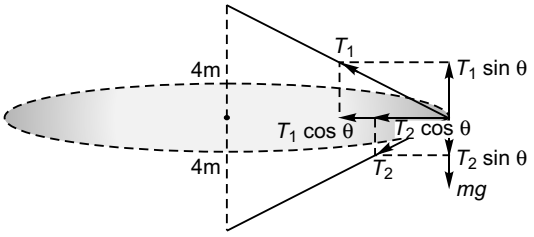
$$\omega_{\min} = \sqrt{\frac{g}{\mu r}}$$

$$= \sqrt{\frac{10}{0.15 \times 3}}$$

$$= 4.7\text{ rads}^{-1}$$



11. (a) $(T_1 + T_2)\cos\theta = mr\omega^2$ ...(i)



and $(T_1 - T_2)\sin\theta = mg$ ...(ii)

i.e., $T_2\cos\theta = mr\omega^2 - T_1\cos\theta$

and $T_2\sin\theta = T_1\sin\theta - mg$

$$\Rightarrow \quad \cos\theta = \frac{mr\omega^2 - T_1\cos\theta}{T_1\sin\theta - mg}$$

or $T_1\cos\theta - mg\cos\theta = mr\omega^2 - T_1\cos\theta$

or $\omega = \sqrt{\frac{2T_1\cos\theta - mg\cos\theta}{m\,r}}$

$$= \sqrt{\frac{2 \times 200 \times \left(\frac{3}{5}\right) - 4 \times 10 \times \left(\frac{3}{4}\right)}{4 \times 3}}$$

$$= 8.37\text{ rad s}^{-1}$$

$$= \frac{8.37 \times \frac{1}{2\pi}\text{ rev}}{\frac{1}{60}\text{ min}}$$

$$= 39.94\text{ rev min}^{-1}.$$

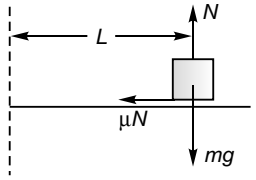
(b) From Eq. (ii),

$$T_2 = T_1 - \frac{mg}{\sin\theta}$$

$$= 200 - \frac{4 \times 100}{\left(\frac{4}{5}\right)}$$

$$= 150\text{ N}$$

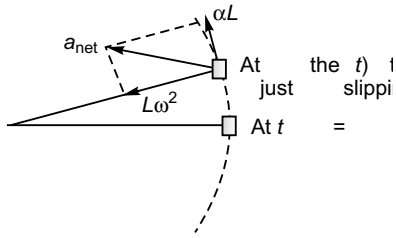
12. (a) For the block not to slip



$$mL\,\omega_{\max}^2 = \mu N = \mu\, mg$$

$$\therefore \quad \omega_{\max} = \sqrt{\frac{\mu g}{L}}$$

(b) If the angular speed is gradually increased, the block will also have translational acceleration $(= \alpha L)$ besides centripetal acceleration $(= L\omega^2)$.



The block will be at he point of slipping, when

$$ma_{\text{net}} = \mu\, mg$$

i.e., $\sqrt{(L\omega^2)^2 + (\alpha L)^2} = \mu\, g$

or $\quad L^2\omega^4 + \alpha^2 L^2 = \mu^2 g^2$

or $\quad \omega^4 + \alpha^2 = \dfrac{\mu^2 g^2}{L^2}$

or $\quad \omega = \left[\dfrac{\mu^2 g^2}{L^2} - \alpha^2\right]^{\frac{1}{4}}$

13. $F' = 2F\cos\dfrac{60°}{2} = F\sqrt{3}$

$PQ = QR = RP = a = 2PO\cos 30°$

$\therefore \quad a = r\sqrt{3}$

For the circular motion of P, Q or R

$$\frac{mv^2}{r} = F' = F\sqrt{3}$$

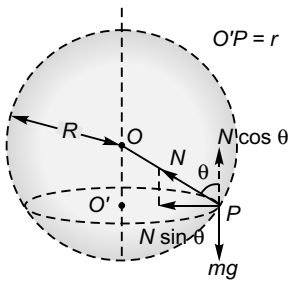
or $\quad \dfrac{mv^2}{r} = \dfrac{G\,mm}{(r\sqrt{3})^2} - \sqrt{3}$

or $\quad v = \left[\dfrac{Gm}{r\sqrt{3}}\right]^{\frac{1}{2}} = \left[\dfrac{GM}{a}\right]^{\frac{1}{2}}$

Now, $v = \dfrac{2\pi r}{T}$

$\therefore \quad T = \dfrac{2\pi r}{v} = 2\pi\dfrac{a/\sqrt{3}}{\left[\dfrac{GM}{a}\right]^{\frac{1}{2}}} = 2\pi\sqrt{\dfrac{a^3}{3\,Gm}}$

14. $N\sin\theta = mr\omega^2$



and $\quad N\cos\theta = mg$

$\therefore \quad \dfrac{\sin\theta}{\cos\theta} = \dfrac{r\omega^2}{g}$ or $\dfrac{\sin\theta}{\cos\theta} = \dfrac{(R\sin\theta)\,\omega^2}{g}$

i.e., $\quad \cos\theta = \dfrac{g}{R\omega^2}$

For $\quad \theta > 0° : \cos\theta < 1$

i.e., $\quad \dfrac{g}{R\omega^2} < 1$ or $\omega > \sqrt{\dfrac{g}{R}}$

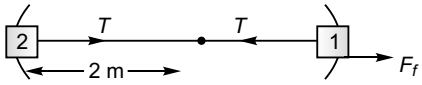
$\therefore$ if $\quad \omega \le \sqrt{\dfrac{g}{R}} : \theta = 0°$

i.e., bead will remain at the lowermost position.

For $\quad \omega = \sqrt{\dfrac{2g}{R}} : \cos\theta = \dfrac{g}{R\left(\dfrac{2g}{R}\right)} = \dfrac{1}{2}$

i.e., $\quad \theta = 60°$

15. (a) $T = 2\cdot 2\cdot\omega^2$



$= 4(4)^2$

$= 64$ N

$T - F_f = 1\cdot 1\cdot\omega^2$

$64 - F_f = (4)^2$

$\Rightarrow \quad F_f = 48$ N

(b) $\quad T = 2\cdot 2\cdot\omega_{\max}^2$ and

$T - F_{f_{\max}} = 1\cdot 1\cdot\omega_{\max}^2$

$\therefore \quad 4\omega_{\max}^2 - F_{f_{\max}} = 1\cdot\omega_{\max}^2$

i.e., $\quad 3\omega_{\max}^2 = \mu\, mg$

$\therefore \quad \omega_{\max} = \sqrt{\dfrac{\mu\, mg}{3}}$

$= \sqrt{\dfrac{0.8\times 1\times 10}{3}}$

$= 1.63$ rad s^{-1}

(c) $100 = 2\cdot 2\cdot\omega_{\max}^2$

$\therefore \quad \omega_{\max} = 5$ rad s^{-1}

Therefore, if the string can sustain a tension of 1100 N, the angular speed of the block system will be 5 rad s^{-1}.

In this case the frictional force (F_f) on the block of mass 1 kg will be given by the relation

$$100 - F_f = 1\cdot 1\cdot 5^2$$

$\Rightarrow \quad F_f = 75$ N

16. $\quad v_{\max} = \sqrt{\dfrac{grl}{h}}$

$= \sqrt{\dfrac{10\times 200\times 0.75}{1.5}} \qquad (2r = 1.5\text{ m})$

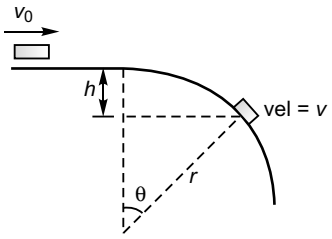
$= 31.6$ ms^{-1}

17. If the block just leaves the surface of sphere at point C

$$\frac{mv^2}{r} = mg\cos\theta \qquad \ldots(i)$$

[v = velocity of block at point C]

$$\Rightarrow \qquad v^2 = gr\cos\theta$$



Further, $v^2 = v_0^2 + 2gh$

$$= v_0^2 + 2gr(1-\cos\theta) \qquad \ldots(ii)$$

Comparing Eqs. (i) and (ii),

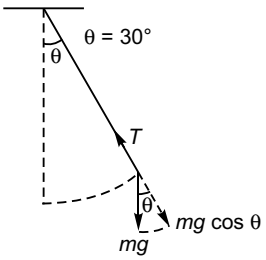
$$v_0^2 + 2gr(1-\cos\theta) = gr\cos\theta$$

or $(0.5\sqrt{gr})^2 + 2gr(1-\cos\theta) = gr\cos\theta$

$$\frac{1}{4} + 2(1-\cos\theta) = \cos\theta$$

i.e., $\theta = \cos^{-1}\left(\frac{3}{4}\right)$

18. $T - mg\cos\theta = \frac{mv^2}{r}$



$$\therefore \qquad 2.5\,mg - mg\cos\theta = \frac{mv^2}{r}$$

$$\therefore \qquad a_c = \frac{v^2}{r} = 25 - 5\sqrt{3}\ \text{ms}^{-2}$$

$$a_t = g\sin\theta = 5\,\text{ms}^{-2}$$

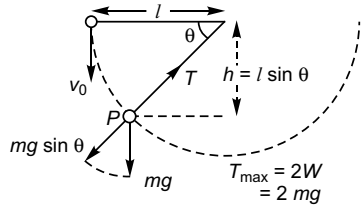
Thus, $a_{\text{net}} = \sqrt{(a_c)^2 + (a_t)^2}$

$$= \sqrt{(25-5\sqrt{3})^2 + (5)^2}$$

$$= \sqrt{625 + 75 + 25 - 10\sqrt{3}}$$

$$= 26.60\ \text{m/s}^2$$

19. At point P



$$T - mg\sin\theta = \frac{mv^2}{l}$$

$$= \frac{m}{l}[v_0^2 + 2gh]$$

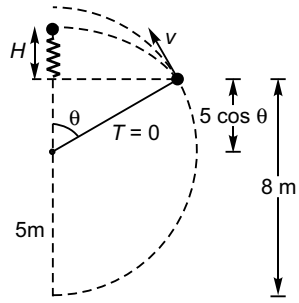
or $2mg - mg\sin\theta = \frac{m}{l}[v_0^2 + 2gl\sin\theta]$

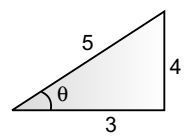
or $2g - g\sin\theta = \frac{5^2 + 2g \times 2\sin\theta}{2}$

Solving, $\sin\theta = \frac{1}{4}$

i.e,. $\theta = \sin^{-1}\left(\frac{1}{4}\right)$

20. $8 = 5 + 5\cos\theta$





$$\therefore \qquad \cos\theta = \frac{3}{5}$$

$$T + mg\cos\theta = \frac{mv^2}{r}$$

$$\Delta t\ \theta = \cos^{-1}\frac{3}{5}$$

T is just zero.

$$\therefore \qquad mg\cos\theta = \frac{mv^2}{r}$$

i.e,. $v^2 = 5g\cos\theta$

$$= 30$$

$$\therefore \qquad v = 5.42\,\text{ms}^{-1}$$

The height (H) to which the particle will rise further.

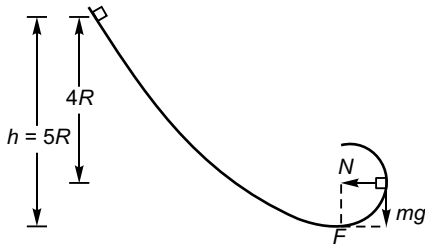
$$H = \frac{v^2 \sin^2 \theta}{2g} = \frac{30}{2 \times 10} \times \frac{16}{25}$$

$$= \frac{24}{25} = 0.96 \text{ m}$$

21. (a) $N = \frac{mv^2}{R} = m\frac{8gR}{R} = 8mg$

$$v^2 = 0^2 + 2.g.4R$$

$$= 8gR$$



$\therefore$ $F = \sqrt{N^2 + (mg)^2}$

$$= \sqrt{64\, m^2 g^2 + m^2 g^2}$$

$$= \sqrt{65}\, mg$$

(b) $N' + mg = \frac{mv'^2}{R}$

$$mg + mg = \frac{m}{R} 2g\,(h' - 2R)$$

or $2R = 2\,(h' - 2R)$

i.e., $h' = 3R$

Objective Questions (Level 1)

Single Correct Option

1. (a) As the speed (v) is increasing uniformly the magnitude of centripetal acceleration $\left(= \frac{v^2}{r}\right)$ will also keep on increasing besides its direction as usual.

(b) $v = kt$ (k = constant)

$\therefore$ $\frac{dv}{dt} = k$

i.e., $a_t = k$

Although the magnitude of the tangential acceleration will remain constant but its direction will keep on changing as the direction of velocity would be changing.

(c) $v = kt$

$$R\omega = kt$$

$$\omega = k' t$$

$$(k' = \frac{k}{R} = \text{constant})$$

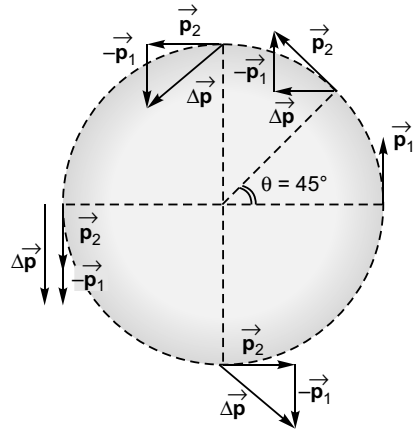
$$\frac{d\omega}{dt} = k'$$

$$\alpha = \text{constant.}$$

As the direction of α (angular acceleration) is perpendicular to the plane of rotation of the body, it will remain constant both in magnitude and direction.

Option (c) is correct.

2. $|\vec{\mathbf{p}}_1| = |\vec{\mathbf{p}}_2| = mv$



$\theta = 45°, |\Delta\vec{\mathbf{p}}| = 2\,mv \sin 22\frac{1}{2}^\circ$

$$= 0.765\, mv$$

$\theta = 90°, |\Delta\vec{\mathbf{p}}| = 2\,mv \sin 45°$

$$= 1.414\, mv$$

$\theta = 180°, |\Delta\vec{\mathbf{p}}| = 2\,mv \sin 90°$

$$= 2\,mv \qquad \text{(max)}$$

$\theta = 270°, |\Delta\vec{\mathbf{p}}| = 2\,mv \sin 135°$

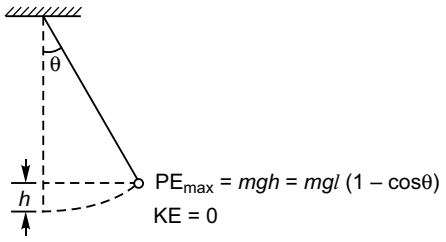
$$= 1.414\, mv$$

$\theta = 360°, |\Delta\vec{\mathbf{p}}| = 2mv \sin 180°$

$= 0$ (min)

Option (c) is correct.

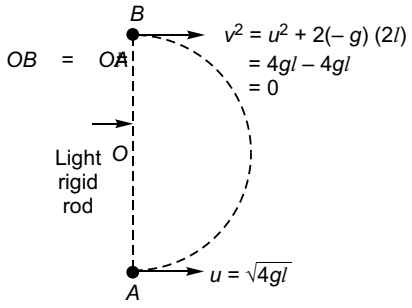
3.



$\therefore KE_{max} = mgl(1 - \cos\theta)$ [as here PE = 0]

Option (c) is correct.

4.



$= 4gl - 4gl$

$= 0$

If the mass m is given velocity $u\ (= \sqrt{4gl})$ at point A it will complete circle to reach point B (the highest point) with zero velocity.

[If in place of light rod these is light string the minimum value of u at A for the mass to reach point B will be $\sqrt{5gl}$ and the minimum velocity at B will be $\sqrt{g\,l}$.]

5. As explained in question 4 speed at lowest point

$$u = \sqrt{5gl}$$

At the lowest point

$$T - mg = \frac{mu^2}{l}$$

$$= \frac{m\,5gl}{l}$$

$$= 5mg$$

$$\Rightarrow \quad T = 6mg$$

Option (d) is correct.

6. Normal acceleration

= Tangential acceleration

$$\frac{v^2}{R} = 5\ (\text{cms}^{-2})$$

$\therefore \quad v = 10\,\text{cms}^{-1} \quad (\because R = 20\,\text{cm})$

Using, $\quad v = 0 + 5t$

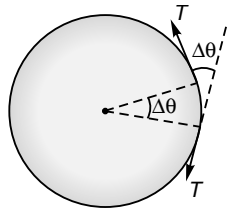
$10 = 5t$

$\Rightarrow \quad t = 2\,\text{s}$

Option (b) is correct.

7. Mass = 2π kg

$\therefore$ Mass per unit length $= \dfrac{2\pi\ \text{kg}}{2\pi\,(0.25)\ \text{m}}$



$$\Delta m = \frac{1}{0.25}\, r\,\Delta\theta = \Delta\theta \quad (\because r = 0.25)$$

$$= \frac{1}{0.25}\ \text{kgm}^{-1}$$

$$\omega = 300\,\frac{\text{rev}}{\text{min}}$$

$$= 300 \times \frac{2\pi\ \text{rad}}{60\,\text{s}}$$

$$= 10\pi\ \text{rad s}^{-1}$$

From figure

$$2T\cos\left(\frac{180° - \Delta\theta}{2}\right) = (\Delta m)\, r\omega^2$$

or $\quad 2T\sin\dfrac{\Delta\theta}{2} = (\Delta m)\, r\omega^2$

or $\quad 2\,T\,\dfrac{\Delta\theta}{2} = (\Delta m)\, r\omega^2$

or $\quad T\,\Delta\theta = (\Delta\theta)\, r\omega^2$

or $\quad T = r\omega^2$

$= 0.25 \times (10\pi)^2$

$\approx 250\,\text{N} \quad (\because \pi^2 \approx 10)$

Option (d) is correct.

8. Maximum speed of car $= \sqrt{\mu g r}$

$= \sqrt{0.3 \times 10 \times 300}$

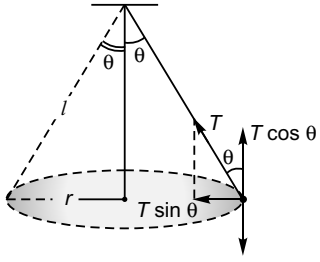
$= 30$ m/s

$$= 30 \times \frac{18}{5} \text{ km/h}$$

$$= 108 \text{ km/h}$$

Option (c) is correct.

9. $T \sin\theta = mr\omega^2 = m(l \sin\theta)\omega^2$



and $\quad T\cos\theta = mg$

$\therefore \quad \cos\theta = \dfrac{g}{l\omega^2}$

$$\omega l = \frac{2}{\pi} \text{ revs}^{-1}$$

$$= \frac{2}{\pi}(2\pi \text{ rad})\,\text{s}^{-1}$$

$$= 4 \text{ rads}^{-1}$$

$\therefore \quad \cos\theta = \dfrac{10}{1(4)^2} = \dfrac{5}{8}$

i.e., $\quad \theta = \cos^{-1}\left(\dfrac{5}{8}\right)$

Option (d) is correct.

10. From above question no. 9.

$$t = ml\omega^2$$

$$= \frac{100}{1000} \times 1 \times 4^2 = \frac{8}{5} \text{ N}$$

Option (b) is correct.

11. $\alpha = \dfrac{1}{3} \text{ rads}^{-1}$ and $R = 25$ cm

$\therefore \quad a_t = R\alpha = \dfrac{25}{3} \text{ cms}^{-2}$

Thus, $\quad \omega_t = 0 + \alpha t$

$$= \frac{1}{3} \times 2 \qquad [\because t = 2 \text{ s}]$$

$\therefore \quad a_N = R\omega_t^2 = 25 \times \left(\dfrac{2}{3}\right)^2 \text{ cms}^{-2}$

Thus, $a_{\text{net}} = \sqrt{a_N^2 + a_t^2} = \sqrt{\dfrac{725}{3}} \text{ cms}^{-2}$

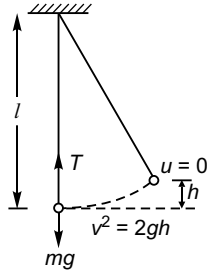
Frictional force $= ma_{\text{net}}$

$$= 0.36 \times 10^{-3} \text{ kg} \times \frac{\sqrt{725}}{3} \times 10^{-2} \text{ ms}^{-2}$$

$$= 32.4\ \mu\text{N}$$

Option (a) is correct.

12. At the lowermost point



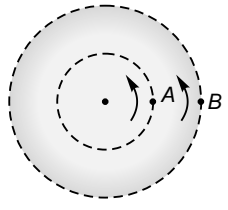
$$T - mg = \frac{mv^2}{l}$$

or $\quad T = \dfrac{m \cdot 2gh}{l} + mg$

$$= mg\left(1 + \frac{2h}{l}\right)$$

Option (d) is correct.

13. $\omega_A = \dfrac{2\pi}{3} \text{ rad min}^{-1}$



and $\quad \omega_B = 2\pi \text{ rad min}^{-1}$

$\therefore \quad \omega_{BA} = \omega_B - \omega_A$

$$= \left(2\pi - \frac{2\pi}{3}\right) \text{rad min}^{-1}$$

$$= \frac{4\pi}{3} \text{ rad min}^{-1}$$

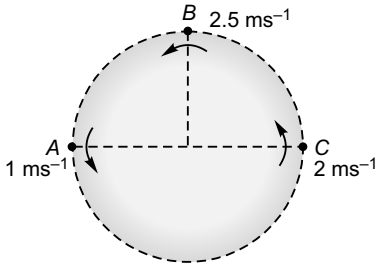
Time required for B to complete one revolution w.r.t. A

$$= \frac{2\pi}{4\pi/3} = \frac{3}{2} \text{ min}$$

$$= 1.5 \text{ min}$$

Option (c) is correct.

14. Let all the particles meet at time t (seconds)



$\therefore$ Distance travelled by B in t second :

Distance travelled by C in t second

$= 2.5t : 2t$

$= 5 : 4$

Option (c) is correct.

JEE Corner

Assertion and Reason

1. For stopping car :

Maximum retardation

$$= \frac{\text{Maximum frictional force}}{m}$$

$$= \frac{\mu N}{m} = \frac{\mu mg}{m} = \mu g$$

$$v^2 = u^2 + 2as$$

$$0^2 = v^2 + (2)(-\mu g)\, d \quad [\because \text{Initial velocity} = v]$$

$$\therefore \quad d = \frac{v^2}{2\mu g}$$

For circular turn of car :

Centripetal force

$$= \text{Maximum frictional force} = \mu mg$$

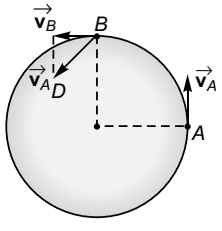
$$\therefore \quad \frac{mv^2}{d'} = \mu mg$$

i.e., safe radius $= d' = \dfrac{v^2}{\mu g} \Rightarrow d' = 2d$

Thus, Assertion and Reason are both correct and Reason is the correct explanation of the Assertion.

Option (a) is correct.

2. $\vec{\mathbf{a}}_{\text{av}} = \dfrac{\vec{\mathbf{v}}_B - \vec{\mathbf{v}}_A}{t_{AB}}$



i.e., $\quad |\vec{\mathbf{a}}_{\text{av}}| = \dfrac{|\overrightarrow{\mathbf{BD}}|}{t_{AB}} = \dfrac{v\sqrt{2}}{t_{AB}}$

Now, $\quad |\vec{\mathbf{v}}_{\text{av}}| = \dfrac{|\overrightarrow{\mathbf{AB}}|}{t_{AB}} = \dfrac{R\sqrt{2}}{t_{AB}}$

$$\therefore \quad \frac{|\vec{\mathbf{a}}_{\text{av}}|}{|\vec{\mathbf{v}}_{\text{av}}|} = \frac{v\sqrt{2}}{R\sqrt{2}} = \omega \text{ (angular velocity)}$$

Thus, assertion is correct.

In circular motion, when speed is constant, the angular velocity will obviously be constant; but this reason does not lead to the result as explained.

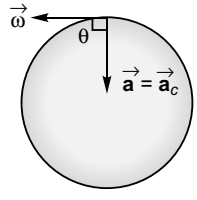
Option (b) is correct.

3. A frame moving in a circle with constant speed can never be an inertial frame as the frame is not moving with constant velocity (due to change in direction).

Reason that the frame is having constant acceleration is false.

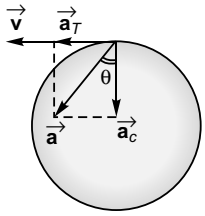
Option (c) is correct.

4. If speed is constant angle between $\vec{\mathbf{v}}$ and $\vec{\mathbf{a}}$ will always be 90°

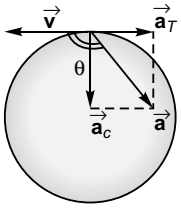


$$\therefore \quad \vec{\mathbf{v}} \cdot \vec{\mathbf{a}} = 0$$

If speed is increasing angle θ between $\vec{\mathbf{v}}$ and $\vec{\mathbf{a}}$ will be less than 90°.



∴ $\vec{\mathbf{v}} \cdot \vec{\mathbf{a}}$ will be positive.



If speed is decreasing angle between $\vec{\mathbf{v}}$ and $\vec{\mathbf{a}}$ will be greater than 90°.

∴ $\vec{\mathbf{v}} \cdot \vec{\mathbf{a}}$ will be negative.

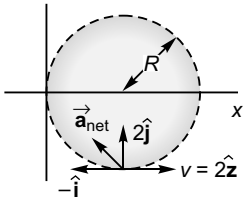
Assertion is correct.

Reason $\vec{\omega} \cdot \vec{\mathbf{v}} = 0$ as both are perpendicular to each other.

Reason is also true but not the correct explanation of the assertion.

Option (b) is correct.

5. $\vec{\mathbf{v}} = 2\,\hat{\mathbf{i}}\ \text{ms}^{-1}$



$$\vec{\mathbf{a}} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}}\ \text{ms}^{-2}$$

∴ $\vec{\mathbf{a}}_c = 2\hat{\mathbf{j}}\ \text{ms}^{-2}$

$$\vec{\mathbf{a}}_c = |\vec{\mathbf{a}}_c| = 2\ \text{ms}^{-2}$$

$$v = |\vec{\mathbf{v}}| = 2\ \text{ms}^{-1}$$

$$a_c = \frac{v^2}{R}$$

⇒ $$R = \frac{v^2}{a_c} = \frac{(2)^2}{2} = 2\ \text{m}$$

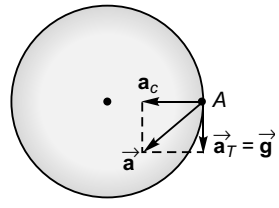
$$\vec{\mathbf{a}}_T = -\hat{\mathbf{i}}\ \text{ms}^{-2}$$

Speed is decreasing (as $\vec{\mathbf{a}}_T$ is –ive) at a rate of 1 ms^{-1} per second *i.e.,* 1 ms^{-2}.

Both Assertion and the Reason are correct but reason has nothing to do with the assertion.

Option (b) is correct.

6. $\vec{\mathbf{a}} = \vec{\mathbf{a}}_T + \vec{\mathbf{a}}_c$ (Reason)



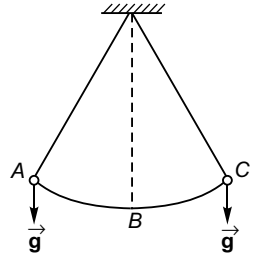
∴ $$|\vec{\mathbf{a}}| = \sqrt{\frac{v^2}{r} + g^2}$$

⇒ $|\vec{\mathbf{a}}| > g$ (Assertion)

We see that both Assertion and the Reason are correct and the reason is the correct explanation of assertion.

Option (a) is correct.

7. At points *A* and *C* : The bob is momentarily at rest.



i.e., $v = 0$ (Reason)

∴ $$|\vec{\mathbf{a}}_c| = \frac{v^2}{R} = 0$$

but net acceleration is not zero (see figure) *i.e.,* Assertion is false.

∴ Option (d) is correct.

8. v (speed) $= 4t - 12$

For $t < 3$ (time unit) speed is negative, which can't describe a motion. Thus, assertion is correct.

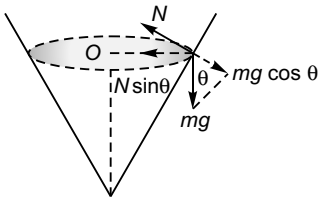
As speed can be changed linearly with time, the reason is false.

Option (c) is correct.

9. In circular motion the acceleration changes regularly where as in projectile motion it is constant. Thus, in circular motion we can't apply $\vec{\mathbf{v}} = \vec{\mathbf{u}} + \vec{\mathbf{a}}\, t$ directly, whereas in projectile motion we can say reason that in circular motion gravity has no role is wrong.

Option (c) is correct.

10. $N = mg\cos\theta$



Therefore, assertion is wrong.

Particle performs circular motion due to $N\sin\theta$

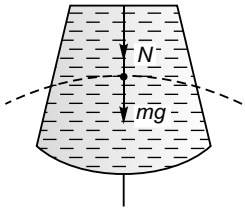
$$\therefore \quad \frac{mv^2}{r} = N\sin\theta$$

$N\cos\theta$ is balanced by mg (weight of particle).

Acceleration is not along the surface of the funnel. It is along the centre O of the circle. Thus, reason is true.

Option (d) is correct.

11. Centripetal force $\left(\dfrac{mv^2}{r}\right) = N + mg$



i.e., Centripetal force (reason) $\geq wt\,(mg)$ of water

for $\quad N = 0, v = \sqrt{gr}$

If at the top of the circular path $v \geq \sqrt{gr}$

i.e., if bucket moved fast, the water will not fall (Assertion).

As assertion and reason both are true and reason is the correct explanation of the assertion of the option would be (a).

Objective Questions (Level 2)
Single Correct Option

1. KE $\left(\dfrac{1}{2}mv^2\right) =$ Change in PE $\left[\dfrac{1}{2}k(\Delta x)^2\right]$

$\Delta x =$ Length of spring (Collar at B)
$\quad -$ Length of spring (Collar at A)

$$= \sqrt{(7+5)^2 + 5^2}\ \text{m} - 7\text{m} = 6\,\text{m}$$

Thus, $\quad \dfrac{1}{2} \times 2 \times v^2 = \dfrac{1}{2} \times 200 \times 6^2$

i.e., $\quad v^2 = 3600$

$\therefore$ Normal reaction $= \dfrac{mv^2}{r}$

$$= \frac{2 \times 3600}{5} = 1440\,\text{N}$$

Option (a) is correct.

2. The particle will remain in equilibrium till ω is constant. Any change in the value of ω will displace the particle up (if ω increase) and down (if ω decreases). Thus, the equilibrium is unstable.

Option (b) is correct.

3. Centripetal force $= \mu\, mg$

or $\quad mr\omega^2 = \mu\, mg$

or $\quad \dfrac{5a}{4}\omega^2 = \mu\, g$

or $\quad \omega^2 = \dfrac{4g}{15a} \qquad \left(\text{as } \mu = \dfrac{1}{3}\right)$

Option (d) is correct.

4. Acceleration at B = Acceleration at A

$$\therefore \quad \frac{v^2}{r} = g\sin\theta$$

or $$\frac{2gr(1-\cos\theta)}{r} = g\sin\theta$$

or $$2(1-\cos\theta) = \sin\theta$$

or $$[2(1-\cos\theta)]^2 = 1-\cos^2\theta$$

or $$(5\cos\theta - 3)(\cos\theta - 1) = 0$$

As $\cos\theta = 1$, *i.e.*, $\theta = 0°$ is not possible.

$$\cos\theta = \frac{3}{5}$$

i.e., $$\theta = \cos^{-1}\frac{3}{5}$$

Option (c) is correct.

5. At point P (for the circular motion)

$$mg\cos\theta - N = \frac{mv^2}{R}$$

If at point P skier leaves the hemisphere.

$$N = 0$$

$$\therefore \quad mg\cos\theta = \frac{mv^2}{R}$$

or $$mg\cos\theta = \frac{m}{R}2g\left(h + \frac{R}{4}\right)$$

or $$mg\cos\theta = \frac{m}{R}2g\left[R(1-\cos\theta) + \frac{R}{4}\right]$$

$$\Rightarrow \quad \cos\theta = 2\left[(1-\cos\theta) + \frac{1}{4}\right]$$

i.e., $$\theta = \cos^{-1}\frac{5}{6}$$

Option (c) is correct.

6. Velocity at B

$$v = \sqrt{2gh}$$
$$= \sqrt{2gR(\cos\theta_2 - \cos\theta_1)}$$
$$= \sqrt{2gR(\cos 37° - \cos 53°)}$$
$$= \sqrt{2gR\left(\frac{4}{3} - \frac{3}{5}\right)}$$
$$= \sqrt{\frac{2gR}{5}}$$

For the circular motion at B, when block just leaves the track

$$\frac{v^2}{R_1} = g\cos\theta_2$$

or $$R_1 = \frac{v^2}{g\cos\theta_2} = \frac{2R}{5\cos\theta_2}$$
$$= \frac{2R}{5\cdot\cos 37°}$$
$$= \frac{2R}{5\cdot\left(\frac{4}{5}\right)}$$
$$= \frac{R}{2}$$

Option (c) is correct.

7. $$mg\cos\theta = \frac{mv^2}{a}$$

or $$g\cos\theta = \frac{4^2 + 2\cdot g\cdot\frac{a}{4}}{a}$$

or $$g\frac{g - \frac{a}{4}}{a} = \frac{u^2 + 2\cdot g\cdot\frac{a}{4}}{a}$$

or $$g\cdot\frac{3}{4}a = u^2 + 2g\frac{a}{4}$$

$$\Rightarrow \quad u = \frac{\sqrt{ag}}{2}$$

Option (c) is correct.

8. $$\frac{v^2}{r} = \frac{4}{r^2}$$

$$\Rightarrow \quad v = \frac{2}{\sqrt{r}}$$

$$\therefore \quad \text{Momentum} = mv$$
$$= \frac{2m}{\sqrt{r}}$$

9. $$N\cos\theta = mr\omega^2$$

and $$N\sin\theta = mg$$

$$\therefore \quad N = m\sqrt{g^2 + r^2\omega^4}$$
$$= m\sqrt{g^2 + r^2\left(\frac{2\pi}{T}\right)^4}$$
$$= 10\sqrt{10^2 + (0.5)^2\left(\frac{2\pi}{1.5a}\right)^4}$$
$$= 128 \text{ N}$$

Option (b) is correct.

10. $$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$
$$= \left[\omega_0 + \frac{1}{2}\alpha t\right]t$$

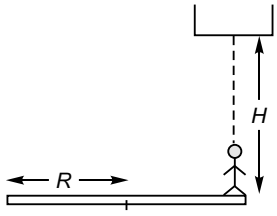
$$= \left[\frac{v}{R} + \frac{1}{2}\left(-\frac{1}{R}\cdot\frac{v^2}{4\pi R}\right)\left(\frac{4\pi R}{v}\right)\right]t$$

$$= \left[\frac{v}{R} - \frac{v}{2R}\right]\frac{4\pi R}{v}$$

$$= 4\pi$$

$$= 2 \text{ rev.}$$

11. $H = \frac{1}{2}gT^2$



$$\Rightarrow \quad T = \sqrt{\frac{2H}{g}}$$

$$\omega = \frac{2\pi}{T} = 2\pi\sqrt{\frac{g}{2H}}$$

$$= \pi\sqrt{\frac{2g}{H}}$$

12. ω (minute hand) $= \frac{2\pi}{3600}$ rad s^{-1}

ω (second hand) $= \frac{2\pi}{60}$ rad s^{-1}

For second hand to meet minute hand for the first time.

2π + Angle moved by minute hand in t second

=Angle moved by second hand in t second

or $\quad 2\pi + \frac{2\pi}{3600}t = \frac{2\pi}{60}t$

$$1 = \frac{t}{60} - \frac{t}{3600}$$

$$1 = \frac{t}{60} \times \frac{59}{60}$$

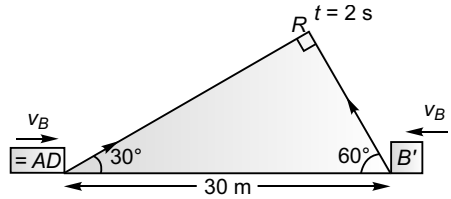
$$\Rightarrow \quad t = \frac{3600}{59} \text{ s}$$

Option (d) is correct.

13. $(PR)^2 + (QR)^2 = (PQ)^2$

$$(v_A \cdot 2)^2 + (v_B \cdot 2)^2 = 30^2$$

$$v_A^2 + v_B^2 = 225 \quad \ldots(i)$$



Further, $\quad \frac{PR}{PQ} = \cos 30°$

$$\frac{v_A \cdot 2}{30} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad v_A = 7.5\sqrt{3} \text{ ms}^{-1}$$

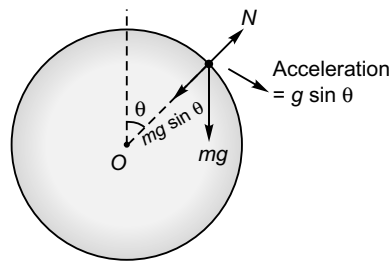
Substituting value of v_A in Eq. (i),

$$v_B = 7.5 \text{ ms}^{-1}$$

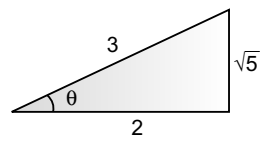
14. $mg\cos\theta - N = \frac{mv^2}{r}$

When breaks off $N = 0$

$$\therefore \quad mg\cos\theta = \frac{mv^2}{r}$$



or $\quad g\cos\theta = \frac{2gr(1-\cos\theta)}{r}$



or $\quad \cos\theta = 2(1-\cos\theta)$

or $\quad \cos\theta = \frac{2}{3}$

Acceleration of particle when it leaves sphere

$$= g\sin\theta$$

$$= \frac{g\sqrt{5}}{3}$$

Option (b) is correct.

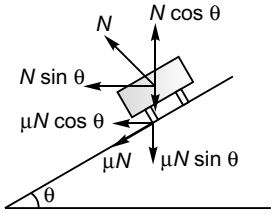
15. For minimum velocity (v) :

$$\tan\theta = \tan 45° = 1$$

$$\mu = 1 \qquad \text{(given)}$$

As $\qquad \mu = \tan\theta$

θ is the angle of repose.



Therefore, the automobile will be at the point of slipping when its velocity is zero

For maximum velocity (v')

$$N\sin\theta + \mu N\cos\theta = \frac{mv'^2}{r}$$

Also $\qquad N\cos\theta = mg + \mu N\sin\theta$

i.e., $\qquad N(\cos\theta - \mu\sin\theta) = mg \qquad \ldots\text{(ii)}$

Dividing Eq. (i) by Eq. (ii),

$$\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} = \frac{v'^2}{gr}$$

Now, as $\theta = 45°$ and $\mu = 1$

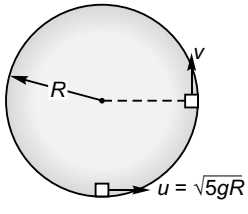
$$v' = \infty$$

Option (d) is correct.

16. For the particle to just complete the circle the value of

$$u = \sqrt{5gR}$$

Particle's velocity (v) when it is at B *i.e.,* when its velocity is vertical would be given by the relation



$$v^2 = u^2 + 2(-g)R$$
$$= 5gR - 2gR$$
$$= 3gR$$

At B :

$$a_c = \frac{v^2}{R} = 3g$$

$$a_T = g$$

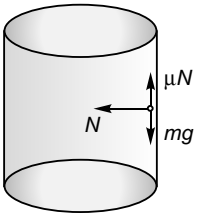
$$\therefore \quad a_{\text{net}} = \sqrt{a_c^2 + a_T^2}$$
$$= \sqrt{(3g)^2 + g^2} = g\sqrt{10}$$

Option (a) s correct.

17. $\qquad N = mr\omega^2$

and $\quad \mu N = mg$

$$\therefore \quad \mu = \frac{g}{r\omega^2}$$
$$= \frac{10}{0.2(5)^2} = 0.2$$



Option (c) is correct.

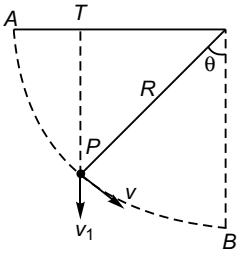
18. $v_{\max} = \sqrt{\dfrac{gra}{h}}$

$$= \sqrt{\frac{10 \times 200 \times 0.75}{1.5}}$$

$$= 31.62 \text{ m/s}$$

19. At point A : Velocity is zero and as such its vertical component will also be zero.

At point B : Velocity is completely horizontal and as such its vertical component will again be zero.



In figure, $TP = R\cos\theta$

At point P :

$$v^2 = 2g\,(R\cos\theta)$$
$$v = \sqrt{2gR}\cos^{1/2}\theta$$

$$\therefore \quad v_\perp = v\sin\theta = \sqrt{2gR}\cos^{1/2}\sin\theta$$

For $v_\perp$ to be maximum

$$\frac{d}{d\theta}v_\perp = 0$$

i.e., $\qquad \dfrac{d}{d\theta}\sqrt{2gR}\cos^{1/2}\theta\sin\theta = 0$

or $\cos^{1/2}\theta\cos\theta + \dfrac{1}{2}\cos^{-1/2}\theta(-\sin\theta)\sin\theta = 0$

or $\qquad \cos\theta\sqrt{\cos\theta} - \dfrac{\sin^2\theta}{2\sqrt{\cos\theta}} = 0$

or $$2\cos^2\theta - \sin^2\theta = 0$$

or $$3\cos^2\theta = 1$$

or $$\theta = \cos^{-1}\frac{1}{\sqrt{3}}$$

Option (b) is correct.

20. At any time

$$a_t = a_c$$

i.e., $$\frac{dv}{dt} = \frac{v^2}{R}$$

$\therefore$ $$\int v^{-2}dv = \int \frac{dt}{R} + k$$

i.e., $$\frac{v^{-2+1}}{-2+1} = \frac{t}{R} + k$$

or $$-\frac{1}{v} = \frac{1}{R} + k$$

At $t = 0, v = v_0$ (given)

$\therefore$ $$-\frac{1}{v_0} = \frac{0}{R} + k$$

i.e., $$k = -\frac{1}{v_0}$$

Thus, $$-\frac{1}{v} = \frac{t}{R} - \frac{1}{v_0}$$

or $$-\frac{1}{v} = \frac{tv_0 - R}{Rv_0}$$

or $$u = \frac{Rv_0}{R - tv_0}$$

or $$\omega = \frac{v_0}{R - tv_0}$$

i.e., $$\frac{d\theta}{dt} = \frac{v_0}{R - tv_0}$$

or $$d\theta = \frac{v_0}{R - tv_0}dt$$

$$\int_0^{2\pi} d\theta = \int_0^T \frac{v_0}{R - tv_0}dt$$

or $$2\pi = -[\log_e(R - tv_0)]_0^T$$
$$= \log_e R - \log_e(R - Tv_0)$$

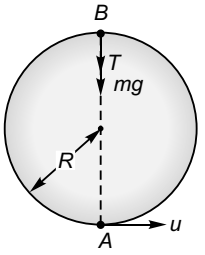
or $$\log_e = \frac{R - Tv_0}{R} = -2\pi$$

or $$1 - \frac{T}{r}v_0 = e^{-2\pi}$$

or $$T = \frac{R}{v_0}(1 - e^{-2\pi})$$

Option (c) is correct.

21. At highest point B



$$T + mg = \frac{mv^2}{l}$$

(where v = velocity at point B)

Thus, $$T = 0$$

if $$mg = \frac{mv^2}{l}$$

i.e., $$v^2 = gl$$

If u = velocity at the lowest point A

$$v^2 = u^2 + 2(-g)(2l)$$

or $$gl = u^2 - 4gh$$

$\Rightarrow$ $$u^2 = 5gl$$

i.e., $$u = \sqrt{5gl}$$

(If $T = 0, \vec{\mathbf{T}} \cdot \vec{\mathbf{a}} = 0$)

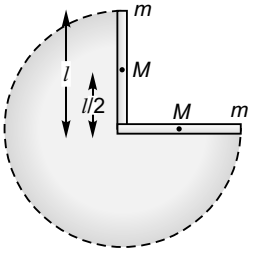
Option (b) is correct.

22. For any value of u at the lowest point both $\vec{\mathbf{T}}$ and $\vec{\mathbf{a}}$ will be towards the centre of the circle and thus

$\vec{\mathbf{T}} \cdot \vec{\mathbf{a}}$ will be positive.

Option (d) is correct.

23. Change in PE of the system



$$= Mg\frac{l}{2} + mgl$$

$$= \left(\frac{M}{2} + m\right)gl$$

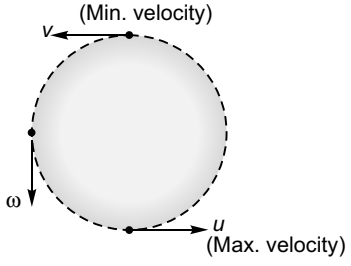
24. Decrease in KE = Increase in PE

i.e., $$\frac{1}{2}mu^2 = \left(\frac{M}{2} + m\right)gl$$

$\therefore$ Initial speed given to ball,

$$u = \sqrt{\left(\frac{M + 2m}{m}\right)gl}$$

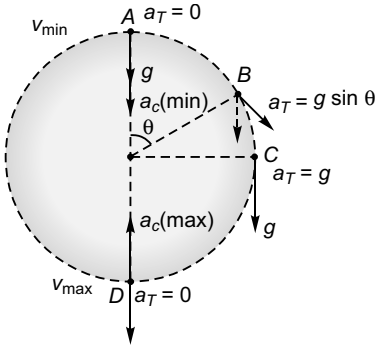
25. Maximum and minimum velocities will be respectively at the lowest and the highest points.



$$v^2 = u^2 + 2(-g)2L$$
$$= u^2 - 4gl$$

More than One Correct Options

1. At point C :



a_T is maximum and a_c is somewhere between maximum value (at D) and minimum value (at A).

$\therefore$ Option (b) is correct.

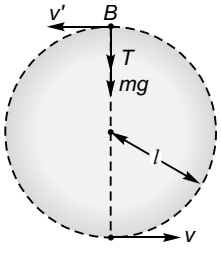
At point D :

a_c is maximum while be a_T is minimum.

$\therefore$ Option (d) is correct.

2. At point B :

$$T + mg = \frac{mv'^2}{l}$$



or $$2mg + mg = \frac{mv'^2}{l}$$

or $$v'^2 = 3gl$$

i.e., $$v' = \sqrt{3gl}$$

Option (d) is correct.

Now, $$v^2 = v'^2 + 2g(2l)$$

$\therefore$ $$v^2 = 3gl + 4gl$$
$$= 7gl$$

i.e., $$v = \sqrt{7gl}$$

Option (b) is correct.

$$= (2v)^2 - 4gL \qquad (\because u = 2v)$$

or $$3v^2 = 4gL$$

i.e., $$v = \frac{4}{3}gL = 2\sqrt{\frac{gL}{3}}$$

Option (b) is correct.

26. $u = 2v = 4\sqrt{\frac{gL}{3}}$

$\therefore$ KE at the lowermost position $= \frac{1}{2}mu^2$

$$= \frac{8mgL}{3}$$

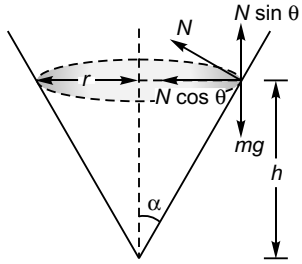
27. Let ω = velocity of the particle when moving downwards.

$\therefore$ $$\omega^2 = v^2 + 2gL$$
$$= \frac{4gl}{3} + 2gl$$

or $$\omega = \sqrt{\frac{10\,gl}{3}}$$

Option (a) is correct.

3. $N \sin \alpha = mg$



$$N \cos \alpha = mr\omega^2$$

$$\Rightarrow \quad \tan \alpha = \frac{g}{r\omega^2} = \sqrt{\frac{g}{r \tan \theta}}$$

i.e., $$\frac{2\pi}{T} = \sqrt{\frac{g}{r \tan \alpha}}$$

or $$T = 2\pi \sqrt{\frac{r \tan \alpha}{g}}$$

If α is increase r will also increase and as such T will increase.

Option (c) is correct.

$$T = 2\pi \sqrt{\frac{h \tan^2 \alpha}{g}}$$

Thus, if h is increase, T will increase.

Option (a) is correct.

4. Particle can't have uniform motion because of change in direction of motion *i.e.,* its velocity value. [Option (a)]

Particle can't have uniformly accelerated motion as acceleration changes direction even if speed is constant. [Option (b)]

Particle can't have not force equal to zero as centripetal force would be required for the circular motion. [Option (c)]

5. For this see figure in answer 3.

If ω is increased $N \cos \alpha$ will increase.

Thus, N will increase (as α is constant) [Option (b)]

And as such net force

$$= \sqrt{N^2 + m^2 g^2 + 2N \cos (90° + \alpha)}$$

on the block will increase.

$\therefore$ Option (a) is correct.

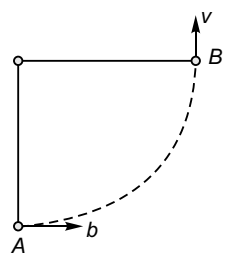
As N increases, the value of $N \sin \alpha$ (acting opposite to mg) will increase and the block will upwards *i.e.,* h will increase.

Match the Columns

1. At point B :

$$v^2 = u^2 - 2gl$$
$$= 12gl - 2gl$$

i.e., $$v = \sqrt{10gl}$$
$$= \sqrt{10 \times 10 \times 1}$$
$$= 10 \text{ ms}^{-1}$$



(a) → (p).

Acceleration of bob :

$$a_c = \frac{v^2}{l} = \frac{10g}{1} = 100 \text{ ms}^{-2}$$

$$a_T = g = 10 \text{ ms}^{-2}$$

$$a_{\text{net}} = \sqrt{a_c^2 + a_T^2}$$
$$= \sqrt{100^2 + 10^2} \text{ ms}^{-2}$$

(b) → (s).

Tension in string :

$$T = \frac{mv^2}{l} = \frac{1 \times 100}{1} = 100 \text{ N}$$

(c) → (r).

Tangential acceleration of bob :

$$a_T = g = 10 \text{ ms}^{-2}$$

(d) → (p).

2. $v = 2t$

$$\therefore \quad \frac{dv}{dt} = 2$$

i.e., $a = 2 \text{ ms}^{-2}$

At $t = 1$ s : $v = 2 \text{ ms}^{-1}$

$$\therefore \quad a_c = \frac{v^2}{r} = \frac{2^2}{2} = 2 \text{ ms}^{-2}$$

$$a_T = 2 \text{ ms}^{-2}$$

$$\therefore \quad a_{\text{net}} = \sqrt{2^2 + 2^2} = 2\sqrt{2} \text{ ms}^{-2}$$

Angle between $\vec{\mathbf{a}}_{\text{net}}$ and $\vec{\mathbf{v}} = 45°$

[As $a_c = a_T$]

(a) $\vec{\mathbf{a}} \cdot \vec{\mathbf{v}} = |\vec{\mathbf{a}}||\vec{\mathbf{v}}|\cos 45°$

$$= \vec{\mathbf{a}}_{\text{net}} \cdot v \frac{1}{\sqrt{2}}$$

$$= 2\sqrt{2} \cdot 2 \frac{1}{\sqrt{2}}$$

$$= 4 \text{ unit.}$$

Thus, (a) → (r).

(b) $|\vec{\mathbf{a}} \times \vec{\omega}| = |\vec{\mathbf{v}}||\vec{\omega}|\sin 90°$

[$\vec{\omega}$ will be perpendicular to the plane of circle]

$$= a_{\text{net}}\omega$$

$$= a_{\text{net}} \frac{v}{r}$$

$$= 2\sqrt{2} \cdot \frac{2}{\sqrt{r}}$$

$$= 2\sqrt{2} \text{ unit}$$

Thus, (b) → (p)

(c) $\vec{\mathbf{v}} \cdot \vec{\omega} = |\vec{\mathbf{v}}||\vec{\omega}|\sin 90°$

$$= v\omega = v \cdot \frac{v}{r}$$

$$= \frac{(2)^2}{2}$$

$$= 2 \text{ unit}$$

Thus, (c) → (q)

(d) $|\vec{\mathbf{v}} \times \vec{\mathbf{a}}| = |\vec{\mathbf{v}}||\vec{\mathbf{a}}|\sin 45°$

$$= v\, a_{\text{net}} \cdot \frac{1}{\sqrt{2}}$$

$$= 4 \text{ unit}$$

Thus, (d) → (r)

3. $N_1 = \frac{M}{2}\left[g - \frac{v^2 h}{ra}\right]$

On increase v, the value of N_1 will decrease.

Thus, (a) → (q).

$$N_2 = \frac{M}{2}\left[g + \frac{v^2 h}{ra}\right]$$

On increasing v, the value of N_2 will increase.

∴ (b) → (q).

As the centripetal force (F) would be provided by the frictional force (f)

$$F = f$$

i.e., $$\frac{F}{f} = 1$$

∴ (c) → (r).

If v is increased F will increase which will automatically increase the value of f.

∴ (d) → (p).

4. Speed of particle is constant

∴ $$|\vec{\mathbf{a}}| = \frac{|\vec{\mathbf{v}}|^2}{|\vec{\mathbf{r}}|}$$

$$\sqrt{(-6)^2 + (b)^2} = \frac{(4)^2 + (-a)^2}{\sqrt{(3)^2 + (-4)^2}}$$

$$\sqrt{36 + b^2} = \frac{16 + a^2}{5} \quad \text{...(i)}$$

(A) $\vec{\mathbf{r}} \cdot \vec{\mathbf{v}} = |\vec{\mathbf{r}}||\vec{\mathbf{v}}|\cos 90° = 0$

i.e., $(3\,\hat{\mathbf{i}} - 4\,\hat{\mathbf{j}}) \cdot (4\,\hat{\mathbf{i}} - a\,\hat{\mathbf{j}}) = 0$

$$12 + 4a = 0$$

$$a = -3$$

∴ (a) → (s).

(B) Substituting value of a (= –3) in Eq. (i)

$$\sqrt{36 + b^2} = 5$$

$$36 + b^2 = 25$$

$$b = \sqrt{25 - 36}$$

∴ (b) → (s).

(C) $\vec{\mathbf{r}} = 3\,\hat{\mathbf{i}} - 4\,\hat{\mathbf{j}}$

∴ $$r = |\vec{\mathbf{r}}| = 5$$

∴ (c) → (r).

(D) $(\vec{\mathbf{v}} \times \vec{\mathbf{a}}) = |\vec{\mathbf{v}}||\vec{\mathbf{a}}|\sin 90°\,\hat{\mathbf{k}}$

$$= |\vec{\mathbf{v}}||\vec{\mathbf{a}}|\,\hat{\mathbf{k}}$$

Thus, $\vec{\mathbf{r}} \cdot (\vec{\mathbf{v}} \times \vec{\mathbf{a}})$

$$= \vec{\mathbf{r}} \cdot |\vec{\mathbf{v}}||\vec{\mathbf{a}}|\,\hat{\mathbf{k}}$$

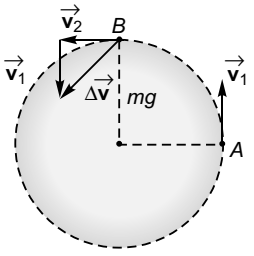
$$= (3\,\hat{\mathbf{i}} - 4\,\hat{\mathbf{j}}) \cdot |\vec{\mathbf{v}}|\,|\vec{\mathbf{a}}|\,\hat{\mathbf{k}} = 0$$

[as $\hat{\mathbf{i}} \cdot \hat{\mathbf{k}} = 0$, $\hat{\mathbf{j}} \cdot \hat{\mathbf{k}} = 0$]

∴ (d) → (s).

5. (A) As speed is constant

Average speed = 1 ms^{-1}



$\therefore$ (a) $\rightarrow$ (s)

(B) Modulus of average velocity $= \dfrac{AB}{t_{AB}}$

$$= \frac{R\sqrt{2}}{\dfrac{AB\text{ arc}}{\text{speed}}} = \frac{R\sqrt{2}}{\dfrac{\pi R}{2}} \cdot 1 = \frac{2\sqrt{2}}{\pi}$$

$\therefore$ (b) $\rightarrow$ (q)

(C) Modulus of average acceleration

$= |\vec{a}_c|$ [as speed is constant]

$$= \frac{|\vec{v}_2 - \vec{v}_1|}{t_{AB}} = \frac{|\Delta\vec{v}|}{t_{AB}} = \frac{v\sqrt{2}}{\pi\dfrac{R}{2}}$$

$= \sqrt{2}$ [$\because v = 1$ ms^{-1} and $R = \dfrac{2}{\pi}$]

$\therefore$ (c) $\rightarrow$ (r)

8 Centre of Mass, Conservation of Linear Momentum, Impulse and Collision

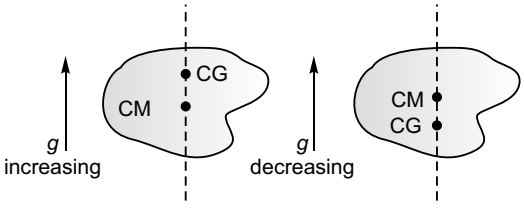
Introductory Exercise 8.1

1. If a body is placed in a uniform gravitational field, the CG of the body coincides with the CM of the body.

$$\vec{\mathbf{r}}_{\text{CM}} = \frac{\sum_{i=1}^{n} m_i \vec{\mathbf{r}}_i}{\sum_{i=1}^{n} m_i}$$

while $$\vec{\mathbf{r}}_{\text{CM}} = \frac{\sum_{i=1}^{n} m_i g_i \vec{\mathbf{r}}_i}{\sum_{i=1}^{n} m_i g_i}$$

If a body is placed in a uniformly increasing gravitational field ($\vec{\mathbf{g}}$) in the upward direction the CG of the body will be higher level than the CM. And, if the body is placed in a uniformly decreasing gravitational field in the upward direction the CG will be at a lower level the CM.



CG shifts from CM according to the magnitude and direction of the gravitational field (by some other agency *eg*, earth) in which the body is placed.

In zero gravitational field CG has no meaning while CM still exists, as usual.

2. The centre of mass of a rigid body may lie inside, on the surface and even outside the body. The CM of a solid uniform sphere is at its centre. The CM of a solid ring is at the centre of the ring which lies outside the mass of the body thus, the statement is false. (For further details see answer to 1 Assertion and Reason JEE corner).

3. Centre of mass always lies on the axis of symmetry of the body, if it exists. The statement is thus true.

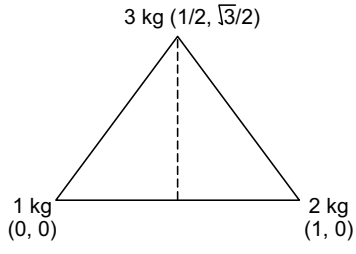
4. Statement is true.

5. As more mass is towards base.

$$\text{Distance} < \frac{r}{4}.$$

6. If two equal masses are kept at co-ordinates $(R, 0)$ and $(-R, 0)$, then their centre of mass will lie at origin.

7.



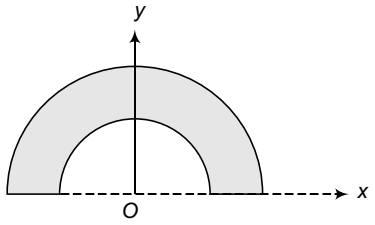
$$X_{\text{CM}} = \frac{m_1 X_1 + m_2 X_2}{m_1 + m_2}$$

$$Y_{\text{CM}} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$$

$$r = \sqrt{X_{\text{CM}}^2 + Y_{\text{CM}}^2}$$

= Distance of centre of mass from A

8.



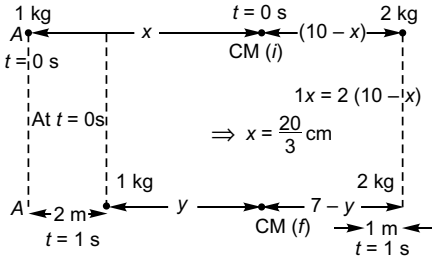
$$Y_{CM} = \frac{A_1y_1 - A_2y_2}{A_1 - A_2}$$

9. $A_1 = 4a^2, x_1 = a, y_1 = a$

$A_2 = a^2, x_2 = \frac{3a}{2}, y_2 = \frac{3a}{2}$

Introductory Exercise 8.2

1. Method 1



$$1y = 2(7 - y)$$

$$\Rightarrow \quad y = \frac{14}{3}\text{ cm}$$

Displacement of CM

= Position of CM (f) – Position of CM (i)

$$= (y + 2) - (x)$$

$$= \left(\frac{14}{3} + 2\right) - \left(\frac{20}{3}\right)$$

$$= 0$$

Method 2

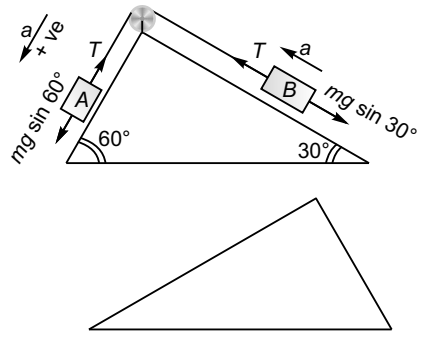
$$M\,v_{CM} = m_1v_1 + m_2v_2$$

$$= 1(2\text{ ms}^{-1}) + 2(-1\text{ ms}^{-1})$$

$$= 0$$

As velocity of CM is zero, there will not be any change in the position of the CM.

2.



$$mg\sin 60° - T = ma \quad \text{...(i)}$$

$$T - mg\sin 30° = ma \quad \text{...(ii)}$$

Adding above equations,

$$mg(\sin 60° - \sin 30°) = 2ma$$

$$\Rightarrow \quad a = g\left(\frac{\sqrt{3} - 1}{4}\right)$$

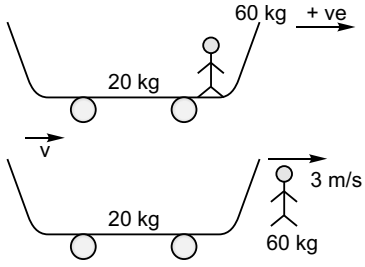
$\vec{a}_1$ and $\vec{a}_2$ are at right angle

$$\therefore\ \vec{a}_{CM} = \frac{m\vec{a}_1 + m\vec{a}_2}{2m} = \frac{1}{2}(a_1 + a_2)$$

or $$|\vec{a}_{CM}| = \frac{a}{\sqrt{2}} = \frac{g(\sqrt{3} - 1)}{4\sqrt{2}}$$

Introductory Exercise 8.3

1. $v_{CM} = \dfrac{20v + 60(+3)}{20 + 60} = 0$



[As there is no force along horizontal direction].

$\Rightarrow$ Velocity of trolley $(v) = -9\ \text{ms}^{-1}$

Total energy produced by man

= KE of man + KE of trolley

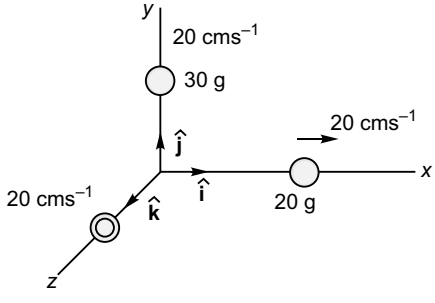
$= \dfrac{1}{2} \times 60 \times 3^2 + \dfrac{1}{2} \times 20 \times (-9)^2 = 1.08\ \text{kJ}$

2. On streching and then releasing the spring the restoring force on each block at instant will be same (according to Newton's 3rd law of motion). Now, as force is same momentum p of each block will also be same (Δt being same) [As according to Newton's second law of motion rate of change of momentum of a body is directly proportional to the net force applied on the body.]

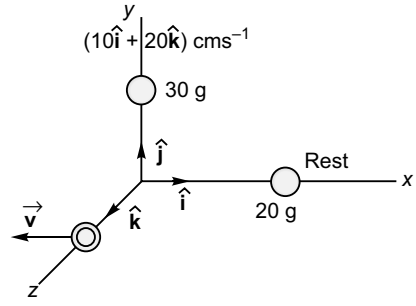
Now, as KE $= \dfrac{p^2}{2m}$,

KE of blocks at any instant will be inversely proportional to their respective masses.

3. As no external force acts on the system of particles, the velocity of CM shall not change. Thus,



$$v_{CM} = \frac{20 \times 20\,\hat{\mathbf{i}} + 30 \times 20\,\hat{\mathbf{i}} + 40 \times 20\,\hat{\mathbf{k}}}{20 + 30 + 40}$$



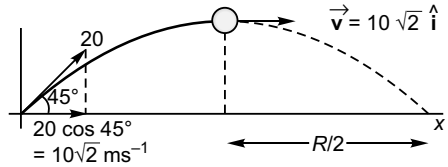
$$v_{CM} = \frac{20 \times 0 + 30\,(10\,\hat{\mathbf{i}} + 20\,\hat{\mathbf{k}}) + 40\,\vec{\mathbf{v}}}{20 + 30 + 40}$$

$$300\,\hat{\mathbf{i}} + 600\,\hat{\mathbf{j}} + 40\,\hat{\mathbf{k}} = 400\,\hat{\mathbf{i}} + 600\,\hat{\mathbf{j}} + 800\,\hat{\mathbf{k}}$$

$\Rightarrow \quad 40\,\vec{\mathbf{v}} = 100\,\hat{\mathbf{i}} + 600\,\hat{\mathbf{j}} + 200\,\hat{\mathbf{k}}$

i.e., $\quad \vec{\mathbf{v}} = (2.5\,\hat{\mathbf{i}} + 15\,\hat{\mathbf{j}} + 5\,\hat{\mathbf{k}})\ \text{cms}^{-1}$

4. Velocity of projectile at the highest point before explosion $= 10\sqrt{2}\,\hat{\mathbf{i}}\ \text{ms}^{-1}$



As no extra external force would be acting during explosion, the velocity of CM will not change

$$\frac{\frac{m}{2} \cdot \vec{\mathbf{0}} + \frac{m}{2}\vec{\mathbf{v}}}{m} = 10\sqrt{2}\,\hat{\mathbf{i}}$$

$\Rightarrow \quad \vec{\mathbf{v}} = 20\sqrt{2}\,\hat{\mathbf{i}}$

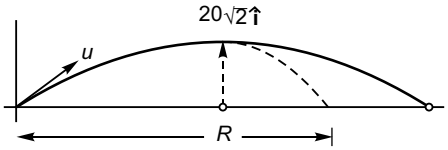
Range of rest half part $= \dfrac{R}{2} \times 2$

(as the velocity of the projectile has doubled at the highest point)

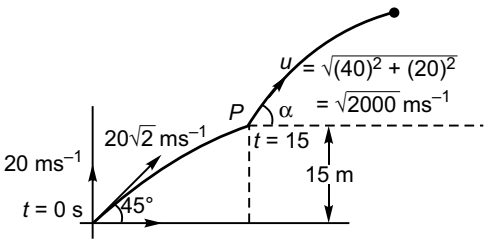
$$= R = \frac{u^2}{g} \qquad (\text{as } \alpha = 45°)$$

$$= \frac{(20)^2}{10} = 40\ \text{m}$$

Therefore, the rest half part will land at a distance of $\left(40+\frac{40}{2}\right)$ m *i.e.*, 60 m from the point of projection.



5. At point P



Horizontal velocity = Initial horizontal velocity

$$= 20\sqrt{2}\cos 45^\circ \text{ ms}^{-1}$$
$$= 20 \text{ ms}^{-1}$$

Vertical velocity

$$= 20\sqrt{2}\sin 45^\circ - g \cdot 1 \quad (\text{as } t = 1\text{s})$$
$$= 70 \text{ ms}^{-1}$$

$\therefore \vec{\mathbf{v}}$ (velocity of projectile at point P *i.e.*, at $t = 15$ just before explosion)

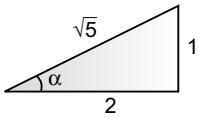
$$= 20\,\hat{\mathbf{i}} + 10\,\hat{\mathbf{j}}$$

Now, as the projectile breaks up into two equal parts and one part comes to rest, the velocity of other half part after explosion will be

$$\vec{\mathbf{u}} = 2\vec{\mathbf{v}} = 40\,\hat{\mathbf{i}} + 20\,\hat{\mathbf{j}} \text{ ms}^{-1}$$

Angle of projection (α) of 2nd half part after explosion at point P.

$$\alpha = \tan^{-1}\left[\frac{20}{40}\right]$$
$$= \tan^{-1}\left[\frac{1}{2}\right]$$



$$\sin\alpha = \frac{1}{\sqrt{5}}$$

Maximum height attained by second half part

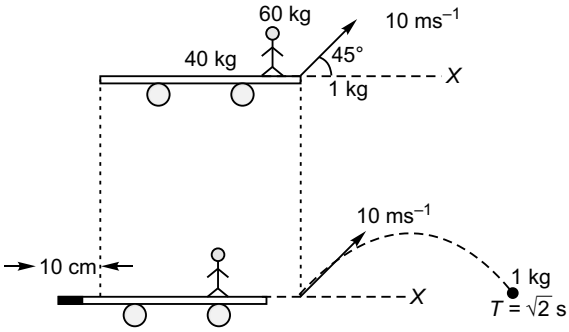
= Height of point P + Extra maximum height attained by second half part

$$= \left[20 \times 1 - \frac{1}{2} g \times 1^2\right] + \frac{(\sqrt{2000})^2 \sin^2\alpha}{2g}$$
$$= 15 + \frac{2000}{2 \times 10} \times \frac{1}{5}$$
$$= 15 + 20$$
$$= 35 \text{ m}$$

6. Momentum of platform + boy + stone along x-axis after throwing stone = before throwing stone

$$\therefore \quad (60 + 40)v + 1\left(10\frac{1}{\sqrt{2}}\right) = 0$$

$$\Rightarrow \quad v = -\frac{5\sqrt{2}}{100} \text{ ms}^{-1}$$

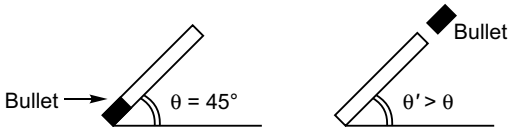


Time of flight of stone $\frac{2 \times 10 \sin 45^\circ}{g} = \sqrt{2}$ s

$\therefore$ Horizontal displacement of platform (+ boy)

$$= v\,T$$
$$= -\frac{5\sqrt{2}}{100} \times \sqrt{2} = -\frac{1}{10} \text{ m} = -10 \text{ cm}$$

7. Thrust due to the upward component of the velocity of the bullet will rotate the movable end of the barrel and thus the bullet leaving the barrel will travelling at an angle greater than 45° when it comes out of the barrel.



Introductory Exercise 8.4

1. To just lift rocket off the launching pad

$$\text{Thrust force} = \text{Weight}$$

or $$v\left(-\frac{dm}{dt}\right) = mg$$

or $$\left(-\frac{dm}{dt}\right) = \frac{mg}{v}$$

$$= \frac{(20+180)\times 9.8}{1.6\times 10^3}$$

$$= 1.225\ \text{kgs}^{-1}$$

(i) Rate of consumption of fuel $= 2\ \text{kgs}^{-1}$

$\therefore$ Time required for the consumption of fuel

$$t = \frac{180\ \text{kg}}{2\ \text{kgs}^{-1}} = 90\ \text{s}$$

Ultimate speed gained by rocket

$$v = u - gt + v\log_e\left(\frac{m_0}{m}\right) \quad \ldots\text{(i)}$$

Substituting $u = 0$, $v = 1.6\ \text{kms}^{-1}$, $m_0 = (20+180)$ kg and $m = 20$ kg in Eq. (i).

$$v = -9.8\times 90 + 1.6\times 10^3 \ln\left(\frac{200}{20}\right)$$

$$= 2.8\ \text{kms}^{-1}$$

(iii) Rate of consumption of fuel $= 20\ \text{kgs}^{-1}$

$$\therefore \quad t = \frac{180\ \text{kg}}{20\ \text{kgs}^{-1}} = 9\ \text{s}$$

i.e., $$v = -9.8\times 9 + 1.6\times 10^3 \ln\frac{200}{20}$$

$$= 3.6\ \text{kms}^{-1}$$

2. Mass at time t, $m = m_0 - \mu t$

$$\therefore \quad \frac{dm}{dt} = -\mu$$

ma = thrust force $- mg$

or $$ma = v\left(-\frac{dm}{dt}\right) - mg$$

or $$ma = \mu v - mg$$

or $$(m_0 - \mu t)\frac{d^2x}{dt^2} = \mu u - (m_0 - \mu t)g \quad (\because v = u)$$

3. $$v = u - gt + v\ln\left(\frac{m_0}{m}\right)$$

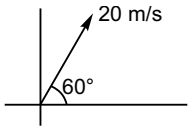
$$= 0 - gt + u\ln\frac{m_0}{m_0\left(1-\frac{t}{3}\right)}$$

$$= -g\,1 + u\ln\frac{3}{2} \quad (\text{at } t = 1\ \text{s})$$

$$= u\ln\frac{3}{2} - g$$

Introductory Exercise 8.5

1. $\vec{\mathbf{u}}$ (at $t = 0$ s) $= (10\sqrt{3}\,\hat{\mathbf{i}} + 10\,\hat{\mathbf{j}})\ \text{ms}^{-1}$



At $t = 1$ s

Horizontal velocity $= 10\sqrt{3}\,\hat{\mathbf{i}}\ \text{ms}^{-1}$

Vertical velocity $= (10 - g\cdot 1)\,\hat{\mathbf{j}}\ \text{ms}^{-1}$

$$= 0\,\hat{\mathbf{j}}\ \text{ms}^{-1}$$

$$\therefore \quad \vec{\mathbf{v}} = 10\sqrt{3}\,\hat{\mathbf{i}}\ \text{ms}^{-1}$$

Change in velocity, $\Delta\vec{\mathbf{v}} = \vec{\mathbf{v}} - \vec{\mathbf{u}}$

$$= (10\sqrt{3})\,\hat{\mathbf{i}} - (10\sqrt{3}\,\hat{\mathbf{i}} + 10\,\hat{\mathbf{j}})$$

$$= -10\,\hat{\mathbf{j}}\ \text{ms}^{-1}$$

i.e., $\Delta\vec{\mathbf{v}} = 10\ \text{ms}^{-1}$, downwards.

2. Impulse ($\vec{\mathbf{J}}$) imparted

= Change in momentum in the time interval

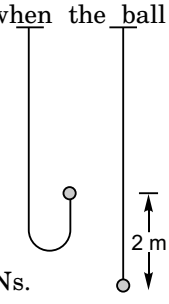
$t = 0$ s to $t = 2$ s

$= m$ [(Velocity at $t = 2$ s) − (Velocity at $t = 0$ s)]

$= 2$ kg $[(4\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}})\ \text{ms}^{-1} - (4\,\hat{\mathbf{j}})\ \text{ms}^{-1}]$

$= 8\,\hat{\mathbf{i}}$ Ns $\quad (\because 1\ \text{kg ms}^{-1} = 1\text{Ns})$

3. Spring will become taut when the ball would go down by 2 m.



$$v^2 = u^2 + 2\times g\times 2$$

or $$v = 2\sqrt{10}\ \text{ms}^{-1}$$

$$\Delta p = m\,\Delta v$$

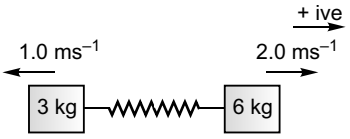
$$= 1(2\sqrt{10} - 0)\ \text{kg ms}^{-1}$$

$$= 2\sqrt{10}\ \text{kg ms}^{-1}$$

$\therefore$ Impulse imparted $= 2\sqrt{10}$ Ns.

Introductory Exercise 8.6

1. At maximum extension of the spring both blocks will move with same velocity v.



$\therefore \quad (-1.0)\times 3 + (+2.0)\times 6 = (3+6)v$

$\Rightarrow \quad v = +1\text{ m/s}$

Applying law of conservation of mechanical energy

$$\frac{1}{2}3(-1)^2 + \frac{1}{2}6(2)^2 = \frac{1}{2}9\times 1^2 + \frac{1}{k}x^2$$

Substituting $k = 200\text{ Nm}^{-1}$

$$x = 0.3\text{ m} = 30\text{ cm}$$

2. Kinetic energy of particle $= K$

$\Rightarrow \quad \frac{1}{2}mv^2 = K$

$\therefore \quad v = \sqrt{\frac{2K}{m}}$

During collision the EPE of the system would be at its maximum value when both the particles move with same velocity V given by the relation

$$mv + m\times 0 = (m+m)V$$

(Law of conservation of momentum)

$\Rightarrow \quad V = \frac{v}{2}$

$$= \frac{1}{2}\sqrt{\frac{2K}{m}}$$

Applying law of conservation of mechanical energy

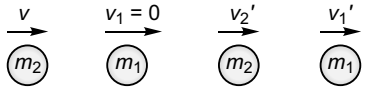
$$K + 0 = \frac{1}{2}(m+m)V^2 + \text{EPE}_{\max}$$

or $\quad K = mV^2 + \text{EPE}_{\max}$

or $\quad K = m\cdot\frac{1}{4}\left(\frac{2K}{m}\right) + \text{EPE}_{\max}$

or $\quad \text{EPE}_{\max} = \frac{K}{2}$

3. Let us consider the following case



$$v_2' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_2 + \left(\frac{2m_2}{m_1 + m_2}\right)v_1$$

$$= \frac{m_1 - m_2}{m_1 + m_2}v_2 \qquad (\text{as } v_1 = 0)$$

ΔK (Transfer of KE of particle of mass m_2)

$$= \frac{1}{2}m_2 v_2^2 - \frac{1}{2}m_2 v_2'^2$$

$$= \frac{1}{2}m_2 v_2^2 - \frac{1}{2}m_2\left[\frac{m_1 - m_2}{m_1 + m_2}v_2\right]^2$$

$$= \frac{1}{2}m_2 v_2^2\left[1 - \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2\right]$$

From above we conclude that ΔK will be maximum when

$$m_1 - m_2 = 0$$

i.e., $\quad \frac{m_2}{m_1} = 1$

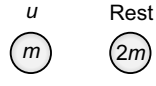
4. Continuing from the previous answer

$$\frac{\Delta K}{K} = 1 - \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$$

$$= \frac{(m_1 + m_2)^2 - (m_1 - m_2)^2}{(m_1 + m_2)^2}$$

$$= \frac{4m_1 m_2}{(m_1 + m_2)^2}$$

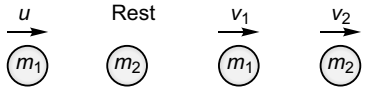
5. Substituting $m_2 = m$ and $m_1 = 2m$ in the result of the previous question no. 4.



$$\frac{\Delta K}{K} = \frac{4(2m)(m)}{(2m + m)^2}$$

$$= \frac{8}{9}$$

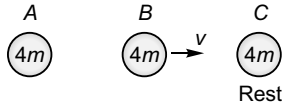
6. As the collision is elastic



$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} u$$

and
$$v_2 = v_1 + u$$
$$= \frac{m_1 - m_2}{m_1 + m_2} u + u$$
$$= \frac{2m_1}{(m_1 + m_2)} u$$

Before first collision of B with C

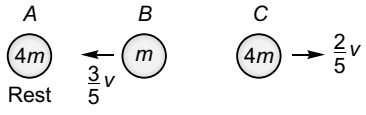


After collision of B with C

$$v_{B_1} = \frac{m - 4m}{m + 4m} v$$
$$= -\frac{3}{5} v$$

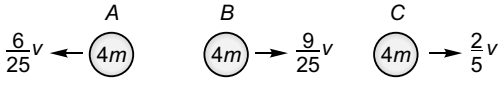
and
$$v_{C_1} = -\frac{3}{5} v + v = \frac{2}{5} v$$

Before second collision of B with A



$$v_{B_2} = \frac{m - 4m}{m + 4m}\left(-\frac{3}{5} v\right)$$
$$= \frac{9}{25} v$$
$$v_{A_2} = \frac{9}{25} v + \left(-\frac{3}{5} v\right) = -\frac{6}{25} v$$

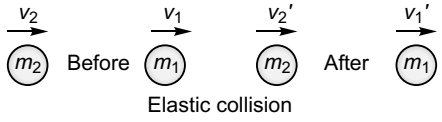
After second collision : of B with A



As velocity of C is greater than that of B, no further collision will take place.

$\therefore$ Total number collisions between the balls = 2.

7. $v'_2 = \dfrac{m_2 - m_1}{m_1 + m_2} v_2 + \dfrac{2m_1}{m_1 + m_2} v_1$



$$v'_1 = \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2$$

We see that $v'_2 \neq v_1$ and $v'_1 \neq v_2$.

However, if $m_1 = m_2$

$$v'_2 = v_1 \text{ and } v'_1 = v_2$$

Ans. No.

8. In elastic collision of two bodies of equal masses, the velocities get interchanged.

Before collision of A with B

velocity of $A = +v$

velocity of B = zero

Therefore, after collision of A with B

velocity of A = zero

velocity of $B = +v$

After collision of B with wall

velocity of $B = -v$

Before collision of B with A

velocity of A = zero

velocity of $B = -v$

Therefore, after collision of B with A

velocity of $A = -v$

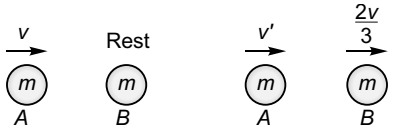
velocity of B = zero

Now, no more collision will take place as A will move towards left with speed v leaving B at rest to its right side.

Thus, we see that speeds of the balls remains unchanged after all the possible collisions have taken place.

Introductory Exercise 8.7

1. Applying law of conservation of momentum



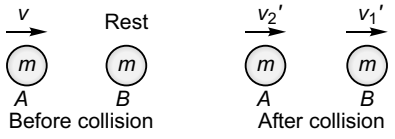
$$mv = mv' + m \cdot \frac{2v}{3}$$

i.e., $$v' = \frac{v}{3}$$

$$e = \frac{\text{velocity of separation}}{\text{velocity of approach}}$$

$$= \frac{\frac{2v}{3} - v'}{v - 0} = \frac{\frac{2v}{3} - \frac{v}{3}}{v} = \frac{1}{3}$$

2. $v'_1 = \left(\frac{1+e}{2}\right)v$ and $v'_2 = \left(\frac{1-e}{2}\right)v$



KE after collision = 0.2 J (given)

$\therefore$ $$\frac{1}{2}mv_2'^2 + \frac{1}{2}mv_1'^2 = 0.2$$

or $$v_2'^2 + v_1'^2 = 4 \quad (\because m = 0.1 \text{ kg})$$

or $$\left(\frac{1-e}{2}\right)^2 v^2 + \left(\frac{1+e}{2}\right)^2 v^2 = 4$$

or $$\frac{v^2}{4}(2 + 2e^2) = 4$$

Extreme cases

If collision is perfectly in elastic *i.e.*, $e = 0$

$$\frac{v^2}{4}(2) = 4$$

i.e., $$v = 2\sqrt{2} \text{ m/s}$$

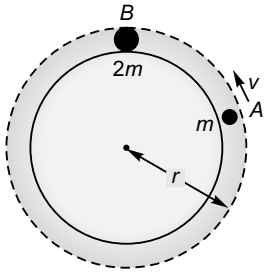
If collision is perfectly elastic *i.e.*, $e = 1$

$$\frac{v^2}{4}(2 + 2) = 4$$

i.e., $$v = 2 \text{ m/s}$$

$$\left.\begin{aligned} \therefore\ v_{\min} &= 2 \text{ m/s} \\ v_{\max} &= 2\sqrt{2} \text{ m/s} \end{aligned}\right]$$

3. $A(m)$ hitting $B(2m)$



A collision is elastic

$$v_B = \frac{2(m)}{2m + m}v = \frac{2}{3}v$$

and $$v_A = \frac{m - 2m}{2m + m}v = -\frac{v}{3}$$

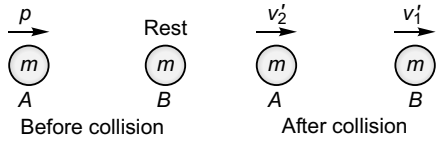
$\therefore$ Velocity of B w.r.t. $A = v_B - v_A$

$$= \frac{2}{3}v - \left(-\frac{v}{3}\right) = v$$

$\therefore$ Next collision between the balls will take place after time

$$t = \frac{2\pi r}{v}$$

4. As collision is elastic



$$v'_2 = \left(\frac{1-e}{2}\right)\frac{p}{m}$$

Now, Momentum of A before impact

– Impulse given by B on A

= Momentum of A after impact.

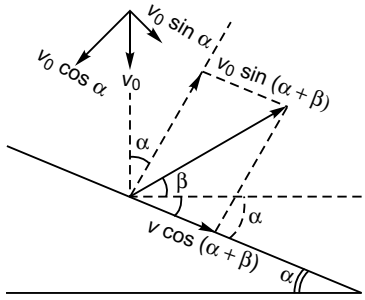
$\therefore$ $p - J = m v'_2$

or $$p - J = m\left(\frac{1-e}{2}\right)\frac{p}{m}$$

or $$e = \frac{2J}{p} - 1$$

Introductory Exercise 8.8

1. $v_0 \sin\alpha = v\cos(\alpha+\beta)$



$v_0 \cos\alpha = \sin(\alpha+\beta)$ (Impact being elastic)

$\therefore \quad \tan\alpha = \cot(\alpha+\beta)$

or $\quad \cot\dfrac{\pi}{2} - \alpha = \cot(\alpha+\beta)$

or $\quad \dfrac{\pi}{2} - \alpha = \alpha + \beta$

or $\quad \beta = \dfrac{\pi}{2} - 2\alpha$

2. Speed after n impacts

Speed before first impact $= u\,(= \sqrt{2gh})$

Speed before one impact $= e\,u$

Speed after 2 impacts $= e\,(eu) = e^2u$

....

Speed after n impacts $= e^n u$

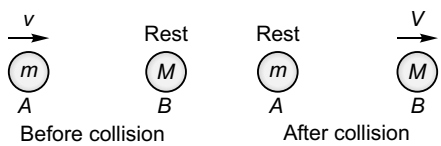
Height (H) upto which the ball rebounds after nth rebound

$$H = \frac{(e^n u)^2}{2g}$$

$$= \frac{e^{2n}u^2}{2g}$$

$$= e^{2n}\cdot h$$

3. $mv + 0 = 0 + MV$



$\Rightarrow \quad V = \dfrac{m}{M}v$

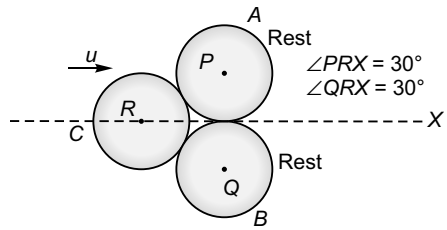
$$e = \frac{\text{Velocity of separation}}{\text{Velocity of approach}}$$

$$= \frac{V-0}{v-0}$$

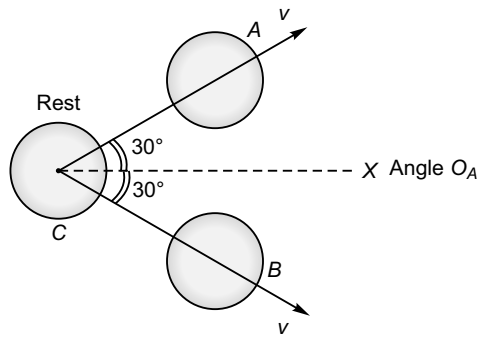
$$= \frac{V}{v}$$

$$= \frac{m}{M}$$

4.



As the balls are of same size, the centres of the balls P, Q and R will be at vertices of an equilateral triangle when ball C just strikes balls A and B symmetrically and as such the balls A and B will follow the path as shown below



Applying law of conservation of momentum

$mu =$ Resultant momentum of A and B balls along the axis of X.

or $\quad mu = 2mv\cos 30°$

or $\quad u = 2v\dfrac{\sqrt{3}}{2}$

or $\quad u = v\sqrt{3}$

As the ball C will strike ball A (and as well as ball B) with velocity $u\cos 30°$

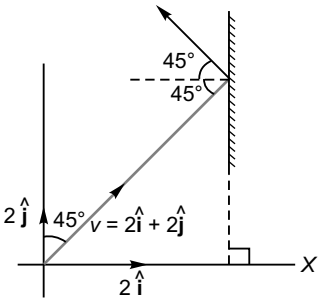
Velocity of approach of ball C towards ball A

$$= u\cos 30° - 0$$
$$= u\cos 30°$$
$$= u\sqrt{3}$$

Velocity of separation of ball A away from ball $C = v$

$$\therefore \quad e = \frac{v}{u\dfrac{\sqrt{3}}{2}} = \frac{v}{(v\sqrt{3})\dfrac{\sqrt{3}}{2}} = \frac{2}{3}$$

5. x-component of velocity



before impact $= 2\hat{\mathbf{i}}$

$\therefore$ after impact $= -e\,2\hat{\mathbf{i}}$

$$= -\frac{1}{2}\times 2\hat{\mathbf{i}}$$
$$= -\hat{\mathbf{i}}$$

y-component of velocity

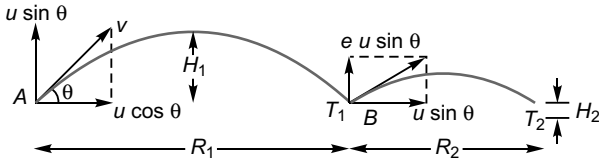
before impact $= 2\hat{\mathbf{j}}$

$\therefore$ after impact $= 2\hat{\mathbf{j}}$

$\therefore$ Velocity after impact $= -\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$

6. At A, $u_{\parallel} = u\cos\theta$

$$u_{\perp} = u\sin\theta$$



$\therefore$ At B

$$u_{\parallel} = u\cos\theta$$
$$u_{\perp} = e\,u\sin\theta$$

$$\therefore \quad \frac{T_1}{T_2} = \frac{\dfrac{2u\sin\theta}{g}}{\dfrac{2e\,u\sin\theta}{g}} = \frac{1}{e} \qquad \left[\text{Using } T = \frac{2u_{\perp}}{g}\right]$$

$$\frac{R_1}{R_2} = \frac{\dfrac{2(u\sin\theta)(u\cos\theta)}{2}}{\dfrac{2(eu\sin\theta)u\cos\theta}{g}} \qquad \left[\text{Using } R = \frac{2u_{\perp}u_{\parallel}}{g}\right]$$

$$= \frac{1}{e}$$

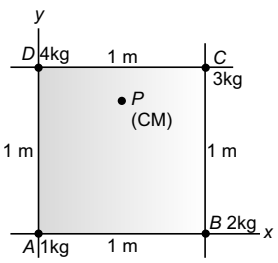
$$\frac{H_1}{H_2} = \frac{\dfrac{(u\sin\theta)^2}{2g}}{\dfrac{(e\,u\sin\theta)^2}{2g}} \qquad \left[\text{Using } H = \frac{u_{\perp}^2}{2g}\right]$$

$$= \frac{1}{e^2}$$

AIEEE Corner

Subjective Questions (Level 1)

1.



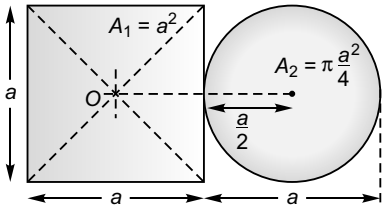
$$x_{CM} = \frac{1\cdot 0 + 2\cdot 1 + 3\cdot 1 + 4\cdot 0}{1+2+3+4} = \frac{5}{10}\text{ m}$$

$$y_{CM} = \frac{1\cdot 0 + 2\cdot 0 + 3\cdot 1 + 4\cdot 1}{1+2+3+4} = \frac{7}{10}\text{ m}$$

$$AP^2 = x_{CM}^2 + y_{CM}^2 = \left(\frac{5}{10}\right)^2 + \left(\frac{7}{10}\right)^2$$

$$= 0.74 \text{ m}^2$$

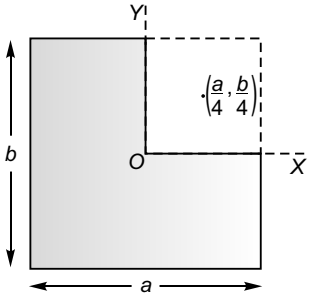
2. $x_{CM} = \dfrac{A_1x_1 + A_2x_2}{A_1 + A_2}$



$$= \frac{a^2 \cdot 0 + \frac{\pi a^2}{4} \cdot (a)}{a^2 + \frac{\pi a^2}{4}}$$

$$= \frac{\pi}{4 + \pi} a$$

3. Let A = area of rectangle



$$x_{CM} = \frac{A \cdot 0 + \left(-\frac{A}{4}\right)\frac{a}{4}}{A + \left(-\frac{A}{4}\right)}$$

$$= -\frac{A}{4} \cdot \frac{4}{3A} \cdot \frac{a}{4}$$

$$= -\frac{a}{12}$$

$$y_{CM} = \frac{A \cdot 0 + \left(-\frac{A}{4}\right)\frac{b}{4}}{A + \left(-\frac{A}{4}\right)}$$

$$= -\frac{b}{12}$$

Centre of mass $\left(-\frac{a}{12}, -\frac{b}{12}\right)$

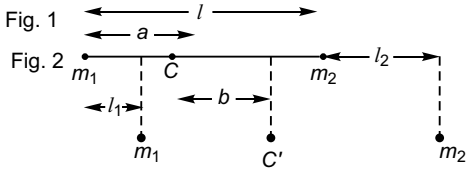
4. $$x_{CM} = \frac{V \cdot 0 + \left(-\frac{4}{3}\pi a^3\right)b}{V - \frac{4}{3}\pi a^3}$$

$$= \frac{-\frac{4}{3}\pi a^3 b}{\frac{4}{3}\pi R^3 - \frac{4}{3}\pi a^3}$$

$$= -\left(\frac{a^3}{R^3 - a^3}\right)b$$

By symmetry $y_{CM} = 0$

5. In Fig. 1, C is CM.



$\therefore$ $$m_1a = m_2(l - a) \quad \ldots\text{(i)}$$

In Fig. 2, C' is CM.

$$m_1(a + b - l_1) = m_2(l - a + l_2 - b) \ldots\text{(ii)}$$

Substituting Eq. (i) in Eq. (ii),

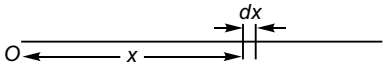
$$m_1(b - l_1) = m_2(l_2 - b)$$

or $$m_1b - m_1l_1 = m_2l_2 - m_2b$$

or $$(m_1 + m_2)\,b = m_1l_1 + m_2l_2$$

or $$b = \frac{m_1l_1 + m_2l_2}{m_1 + m_2}$$

6. $x_{CM} = \dfrac{\int x\,dm}{\int dm}$



where, dm = mass of element of length dx.

$$= \frac{\int x\rho\,dx\,A}{\int \rho\,dx\,A}$$

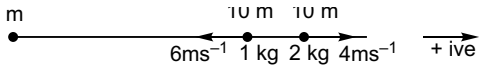
$$= \frac{\int_0^l x\left(\rho_0 \frac{x^2}{l^2}\right)dx}{\int_0^l \left(\rho_0 \frac{x^2}{l^2}\right)a_x} \quad \left(\because \rho = \rho_0 \frac{x^2}{l^2}\right)$$

$$= \frac{\int_0^l x^3 dx}{\int_0^l x^2 dx}$$

$$= \left[\frac{3}{x^3} \cdot \frac{x^4}{4}\right]_0^l$$

$$= \frac{3}{4} l$$

7. $x_{CM} = \dfrac{1 \times 10 + 2 \times 12}{1 + 2}$



$$= \frac{34}{3} \text{ m}$$

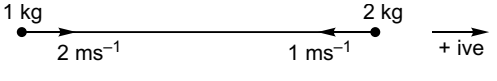
$$v_{CM} = \frac{1 \times (-6) + 2 \times (+4)}{1 + 2} = +\frac{2}{3} \text{ ms}^{-1}$$

x'_{CM} (new position of CM)

$$= \frac{34}{3} \text{m} + 2\text{s}\left(\frac{2}{3} \text{ms}^{-1}\right)$$

$$= \frac{38}{3} \text{ m} = 12.67 \text{ m}$$

8. $v_{CM} = \dfrac{1 \times 2(+2) + 2 \times (-1)}{1 + 2}$



$$= 0 \text{ ms}^{-2}$$

$\therefore$ Displacement of CM in 1 s = 0 m.

9. Acceleration $(\vec{\mathbf{a}}) = -10\,\hat{\mathbf{j}}\,\text{ms}^{-2}$

$$\vec{\mathbf{v}} = \vec{\mathbf{u}} + \vec{\mathbf{a}}\,t$$

$$= \frac{(1)(0) + (2)(10\hat{\mathbf{i}} + 10\,\hat{\mathbf{j}})}{3} - 10\,\hat{\mathbf{j}}$$

$$= \left(\frac{20}{3}\hat{\mathbf{i}} - \frac{10}{3}\hat{\mathbf{j}}\right) \text{m/s}$$

New position vector $(\vec{\mathbf{r}}_1)$ of particle A

$$(\vec{\mathbf{r}}_1) = \vec{\mathbf{s}}_0 + \vec{\mathbf{u}}\,t + \frac{1}{2}\vec{\mathbf{a}}\,t^2$$

$$= (10\,\hat{\mathbf{i}} + 20\,\hat{\mathbf{j}}) + \vec{0} + \frac{1}{2}(-10\,\hat{\mathbf{j}}) \cdot 1^2$$

$$= 10\,\hat{\mathbf{i}} + 15\,\hat{\mathbf{j}}\,\text{m}$$

New position vector $(\vec{\mathbf{r}}_2)$ of particle B

$$\vec{\mathbf{r}}_2 = (20\,\hat{\mathbf{i}} + 40\,\hat{\mathbf{j}}) + (10\,\hat{\mathbf{i}} + 10\,\hat{\mathbf{j}}) \cdot 1 + \frac{1}{2}(-10\,\hat{\mathbf{j}}) \cdot 1^2$$

$$= 30\,\hat{\mathbf{i}} + 45\,\hat{\mathbf{j}}$$

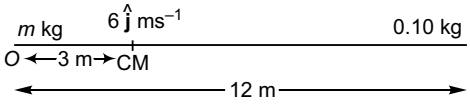
New position of CM

$$\vec{\mathbf{R}} = \frac{\vec{\mathbf{r}}_1(1) + \vec{\mathbf{r}}_2(2)}{1 + 2}$$

$$= \frac{10\,\hat{\mathbf{i}} + 15\,\hat{\mathbf{j}} + 60\,\hat{\mathbf{i}} + 90\,\hat{\mathbf{j}}}{3}$$

$$= \frac{70\,\hat{\mathbf{i}} + 105\,\hat{\mathbf{j}}}{3} \text{ m}$$

10.



(a) $3 = \dfrac{m \cdot 0 + 0 \cdot 1 \times 12}{m + 0.1}$

$\Rightarrow \qquad m = 0.3$ kg

(b) Momentum of system = Momentum of CM

$$= (m + 0.1) \text{ kg} \times 6\,\hat{\mathbf{j}}\,\text{ms}^{-1}$$

$$= 2.4\,\hat{\mathbf{j}} \text{ kg ms}^{-1}$$

(c) $\vec{\mathbf{v}}_{CM} = \dfrac{\vec{\mathbf{v}}_m(0.3) + \vec{0}(0.10)}{0.3 + 0.1}$

$$\vec{\mathbf{v}}_m = \frac{4}{4}\vec{\mathbf{v}}_{CM}$$

$$= \frac{4}{3} \times 6\,\hat{\mathbf{j}}\,\text{ms}^{-1}$$

$$= 8\,\hat{\mathbf{j}}\,\text{m s}^{-1}$$

11. $A \overset{m}{\bullet}\ t = 0$ s

$B \overset{2m}{\bullet}\ (t = 100 \text{ ms})$

Position of 1st particle (A) at $t = 300$ ms

$$s_1 = \frac{1}{2} \times 10 \times (300 \times 10^{-3})^2$$

$$= 0.45 \text{ m}$$

Position of 2nd particle (B) at $t = 300$ ms

(B is at the position of A at $t = 100$ ms)

$$s_2 = \frac{1}{2} \times 100 \times (200 \times 10^{-3})^2$$

$$= 0.20 \text{ m}$$

$\therefore$ Position of CM $= \dfrac{2m \times 0.2 + m \times 0.45}{2m + m}$

$$= 28.3 \text{ cm}$$

Velocity of 1st particle (A) at $t = 300$ ms

$$v_1 = 10 \times 300 \times 10^{-3}$$
$$= 3 \text{ ms}^{-1}$$

Velocity of 2nd particle at $t = 300$ ms

$$v_2 = 10 \times 200 \times 10^{-3} \text{ ms}^{-1}$$
$$= 2 \text{ ms}^{-1}$$

$$\therefore \quad v_{CM} = \frac{2m \times 2 + m \times 3}{2m + m}$$
$$= \frac{7}{3} = 2.33 \text{ ms}^{-1}$$

12. $24 = \dfrac{m_A \cdot 0 + m_B \cdot 80}{m_A + m_B}$

or $\quad 24(m_A + 0.6) = 80 \times 0.6$

or $\quad m_A = 1.4$ kg

$\therefore$ Total mass of system $= 1.4$ kg $+ 0.6$ kg

$= 2.0$ kg

$$v_{CM} = 6.0\,t^2\,\hat{\mathbf{j}}$$

$$\therefore \quad a_{CM} = 12\,t\,\hat{\mathbf{j}} \text{ ms}^{-2}$$

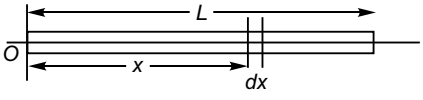
Net force acting on system (at $t = 3$ s)

$=$ Total mass of system $\times$ (a_{COM} at $t = 3$ s)

$= 2.0 \text{ kg} \times 36\,\hat{\mathbf{j}} \text{ ms}^{-2}$

$= 72 \text{ N}\,\hat{\mathbf{j}}$

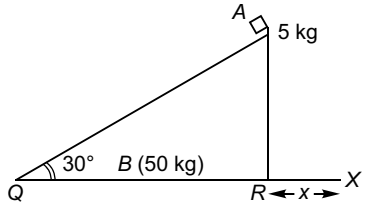
13.



$$x_{CM} = \frac{\int x\,dm}{\int dm}$$
$$= \frac{\int_0^L x\,(Ax\,dx)}{\int_o^L Ax\,dx}$$
$$= \frac{\int_0^L x^2\,dx}{\int_0^L x\,dx}$$
$$= \frac{L^3/3}{L^2/2}$$
$$= \frac{2}{3}L$$

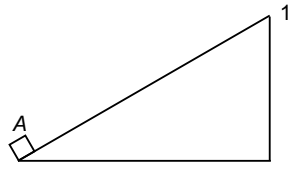
14. Let $x =$ displacement of wedge (30 kg) towards right.

$\therefore$ Displacement of block A towards right (along x-axis) when it arrives at the bottom of the wedge



$$= QR - x$$
$$= 0.5 - x$$

Now, as net force on the system (wedge + block) along x-axis is zero, the position of CM of the system, along x-axis, will not change

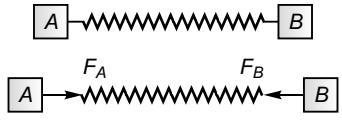


$$\therefore \quad 5(0.5 - x) = 30x$$

i.e., $\quad x = \dfrac{0.5}{7}$ m

$= 71.4$ mm

15. As no external force acts on the system, the velocity of CM will be zero.



i.e., $\quad \dfrac{m_A v_A + m_B v_B}{m_A + m_B} = 0$

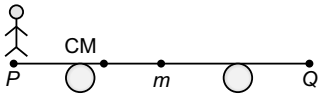
i.e., $\quad \dfrac{v_A}{v_B} = -\dfrac{m_B}{m_A} = -\dfrac{2}{1}$

(a) $\therefore$ Ratio of speeds $= 2$

(b) $\dfrac{p_A}{p_B} = \dfrac{m_A v_A}{m_B v_B} = \dfrac{m_A}{m_B}\left(-\dfrac{m_B}{m_A}\right) = -1$

(c) $\dfrac{K_A}{K_B} = \dfrac{p_A^2/2m_A}{p_B^2/2m_B} = \dfrac{p_A^2}{p_B^2} \times \dfrac{m_B}{m_A} = \dfrac{2}{1}$

16. While man travels from P to Q



$$v_{CM} = \frac{m\left(\frac{3v}{2} - v\right) + mv}{m + m} = \frac{3v}{4}$$

Displacement of CM (along horizontal) $= \frac{L}{2}$.

$\therefore$ Time taken by man to reach point Q starting from point P

$$t_{PQ} = \frac{L/2}{3v/4} = \frac{2L}{3v}$$

While man travels from Q to P

$$v_{CM} = \frac{m\left(-\frac{3v}{2} - v\right) + mv}{m + m}$$

$$= -\frac{3v}{4}$$

Displacement of CM $= -\frac{1}{2}$

$\therefore$ Time taken by man to reach point P starting from point Q

$$t_{QP} = \frac{-L/2}{-3v/4} = \frac{2L}{3v}$$

$\therefore$ Total time $= t_{PQ} + t_{QP}$

$$= \frac{2L}{3v} + \frac{2L}{3v}$$

$$= \frac{4L}{3v}$$

$\therefore$ Net displacement of trolley $= \frac{4L}{3v} \times v$

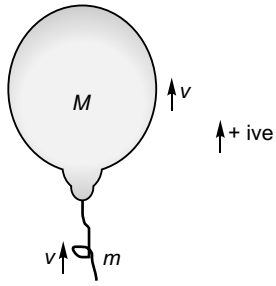
$$= \frac{4L}{3}$$

17. (a) While the man climbs in the rope, no extra external net force acts on the system (balloon along with rope + man). Force applied by man to gain velocity to climb up is an internal force and as such the velocity of the CM of the system will remain stationary.

$$mv + (M + m)V = 0$$

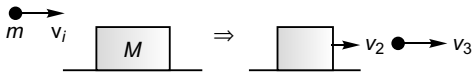
or $$V = -\frac{m}{(M + m)}v$$

Negative sign shows that the balloon will move downwards.



(b) If the man slops climbing *i.e.*, $v = 0$, then according to the above relation the value of v will also be zero. Thus, the balloon will also slop.

18.



$$mv_1 = Mv_1 + mv_3 \quad \ldots(i)$$

$$m = 4 \times 10^{-3}\ \text{kg}$$

$$M = 1\,\text{kg}$$

$$v_3 = 100\ \text{m/s}$$

$$v_1 = 500\ \text{m/s}$$

Find v_2

(a) $0 = v_2^2 - 2as = v_2^2 - 2(\mu g)s$

$\therefore$ $$\mu = \frac{v_2^2}{2gs}$$

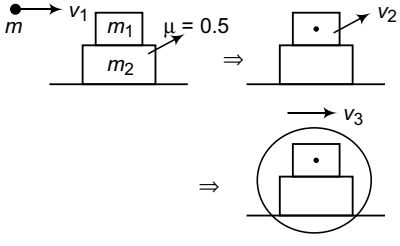
Here $s = 0.30\ \text{m}$

(b) Decrease in kinetic energy of bullet

$$= \frac{1}{2}m(v_1^2 - v_3^2)$$

(c) KE of block $= \frac{1}{2}mv_2^2$

19.



$$(m + m_1)v_2 = mv_1$$

$\therefore \qquad v_2 = \dfrac{mv_1}{m + m_1} \qquad \ldots(i)$

Common velocity $v_3 = \dfrac{mv_1}{m + m_1 + m_2} \qquad \ldots(ii)$

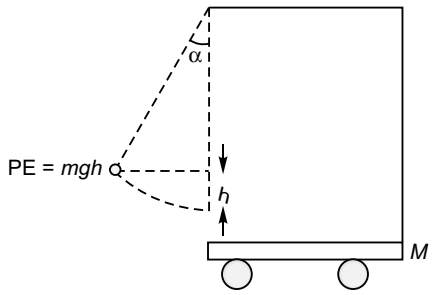
For m_1 : $v_3^2 = v_2^2 - 2a_1s_1$ and Find s_1

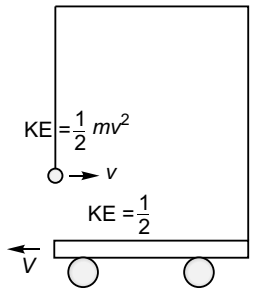
For m_2 : $v_3^2 = 2a_2s_2$ and Find s_2

Here : $a_1 = \dfrac{\mu m_1 g}{m_1} = \mu g$

and $\qquad a_2 = \dfrac{\mu m_1 g}{m_2}$

20. Let v = Velocity of pendulum bob when the pendulum is vertical





V = Velocity of wagon when pendulum is vertical

$\therefore \qquad mv = MV$

or $\qquad v = \dfrac{M}{m} V$

KE of wagon + KE of bob = PE of bob

$$\frac{1}{2}MV^2 + \frac{1}{2}mv^2 = mgh$$

or $\quad \dfrac{1}{2}MV^2 + \dfrac{1}{2}m\left(\dfrac{M}{m}V\right)^2 = mgh$

or $\quad \dfrac{1}{2}MV^2\left[1 + \dfrac{M}{m}\right] = mgh$

or $\quad \dfrac{1}{2}MV^2(M + m) = m^2gh$

or $\quad V^2 = \dfrac{2m^2gh}{M(M + m)}$

or $\quad V^2 = \dfrac{2m^2g\,l\,(1 - \cos\alpha)}{M\,(M + m)}$

or $\quad V^2 = \dfrac{2m^2g\,l\,2\sin^2\dfrac{\alpha}{2}}{M\,(M + m)}$

or $\quad V = 2m\sin\dfrac{\alpha}{2}\sqrt{\dfrac{g\,l}{M\,(M + m)}}$

21. Let the track shift by x (to the right) when the cylinder reaches the bottom of the track.

$\therefore \qquad x' + x = R - r$

or $\qquad x' = (R - r) - x$

(x' = horizontal displacement of cylinder w.r.t. ground)

As no force would be acting along horizontal direction, for no shift in CM along horizontal. We would have

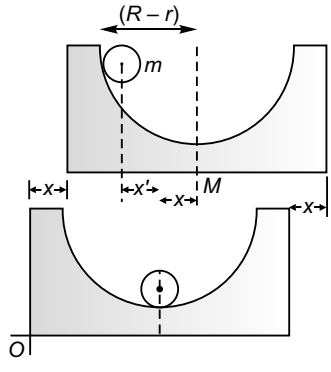
$$mx' - Mx = 0$$

or $\quad m\,[(R - r) - x] - Mx = 0$

or $\quad (M + m)\,x = m\,(R - r)$

$\Rightarrow \qquad x = \dfrac{m\,(R - r)}{M + m}$

Now, as the PE of the cylinder would change into the kinetic energies of the



cylinder and the track we have,

$$mg\,(R - r) = \frac{1}{2}mv^2 + \frac{1}{2}MV^2 \qquad \ldots(i)$$

where, v = velocity of cylinder

and $\quad V$ = velocity of track.

(when cylinder just reaches bottom of the track)

Applying law of conservation of momentum

$$mv + MV = 0$$

or $$v = -\frac{M}{m}V$$

Thus, Eq. (i) becomes

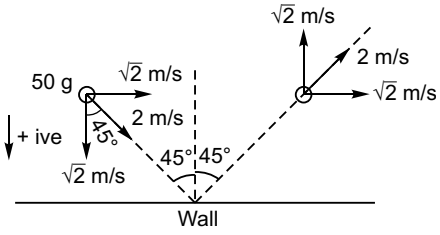
$$mg(R-r) = \frac{1}{2}m\left(-\frac{M}{m}V\right)^2 + \frac{1}{2}MV^2$$

or $$mg(R-r) = \frac{1}{2}MV^2\left[\frac{M}{m}+1\right]$$

or $$V^2 = \frac{2m^2g(R-r)}{M(M+m)}$$

or $$V = m\sqrt{\frac{2g(R-r)}{M(M+m)}}$$

22. No change in momentum of ball and also that of wall along horizontal direction.



Along perpendicular direction :

Momentum of ball after reflection

$$(\vec{\mathbf{p}}_f) = -50\times10^{-3}\sqrt{2}\ \text{kg ms}^{-1}$$

Momentum of ball before reflection

$$(\vec{\mathbf{p}}_i) = +50\times10^{-3}\sqrt{2}\ \text{kg ms}^{-1}$$

$\therefore$ $$\Delta\vec{\mathbf{p}} = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i$$

$$= -100\times10^{-3}\sqrt{2}\ \text{kg ms}^{-1}$$

$$= -0.14\ \text{kg ms}^{-1}$$

i.e., $$|\Delta\vec{\mathbf{p}}| = 0.14\ \text{kg ms}^{-1}$$

Momentum of wall (when ball strikes wall)

$$\vec{\mathbf{p}}_i = 50\times10^{-3}\times\sqrt{2}\ \text{kg ms}^{-1}$$

$\therefore$ $$|\vec{\mathbf{p}}_i| = 50\times10^{-3}\times\sqrt{2}\ \text{kg ms}^{-1}$$

Momentum of wall (when ball rebounds from the wall)

$$\vec{\mathbf{p}}_f = -50\times10^{-3}\times\sqrt{2}\ \text{kg ms}^{-1}$$

$\therefore$ $$|\vec{\mathbf{p}}_f| = 50\times10^{-3}\times\sqrt{2}\ \text{kg ms}^{-1}$$

$\therefore$ Change in the magnitude of the momentum of the wall $= |\vec{\mathbf{p}}_f| - |\vec{\mathbf{p}}_i|$

$$= 0$$

23. $m_0 = 40$ kg

$$m = (40+160)\ \text{kg}$$

$$= 200\ \text{kg}$$

$v_i = 2$ km/s $= 2\times10^3$ kg^{-1}

Rate of consumption of fuel = 4 kgs^{-1}

$\therefore$ Time (t) required to completely brunt out of the fuel $= \dfrac{160\ \text{kg}}{4\ \text{kgs}^{-1}} = 40$ s.

Ultimate vertical speed gained by the rocket

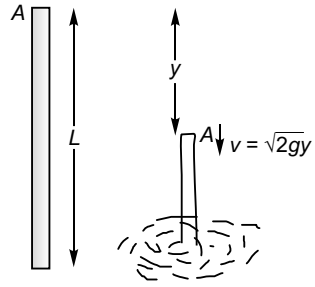
$$= -gt + v_i \ln\frac{m}{m_0}$$

$$= -10\times40 + 2\times10^3\times\ln\frac{200}{40}$$

$$= -400 + 3218 = 2818\ \text{m/s}$$

$$= 2.82\ \text{kms}^{-1}$$

24. When y length of rope has fallen on table top



$$F_{\text{th}} = v_{\text{rel}}\frac{dm}{dt}$$

$$= v\frac{dm}{dy}\cdot\frac{dy}{dt}$$

$$= v^2\frac{dm}{dy} = v^2\frac{M}{L}$$

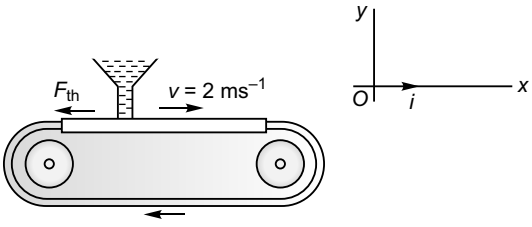
$$W = \frac{M}{L}g$$

$$F_{\text{net}} = W + F_{\text{th}}$$

$$\therefore \quad F_{net} = \frac{M}{L} yg + v^2 \frac{M}{L}$$
$$= \frac{M}{L} yg + 2gy \frac{M}{L}$$
$$= 3 \frac{M}{L} yg$$

= Weight due to $3y$ length of the rope.

25. Horizontal velocity of incoming sand



$$\vec{\mathbf{v}} = \vec{0}$$

Horizontal velocity of the conveyer belt

$$= (\vec{\mathbf{v}})\ (\text{say}) = v\ \hat{\mathbf{i}}$$

$$\therefore \quad \vec{\mathbf{u}}_{rel} = \vec{\mathbf{v}}_1 - \vec{\mathbf{v}}_2$$
$$= \vec{0} - (v\ \hat{\mathbf{i}})$$
$$= -v\ \hat{\mathbf{i}}$$

$$\therefore \quad F_{th} = \vec{\mathbf{v}}_{rel} \frac{dm}{dt}$$
$$= -\left(v \cdot \frac{dm}{dt}\right) \hat{\mathbf{i}}$$

As $\frac{dm}{dt} > 0$, the falling sand particles exert thrust force which decelerates the conveyer belt.

Force required to keep the belt moving

$$\vec{\mathbf{F}} = -\vec{\mathbf{F}}_{th}$$
$$= v \frac{dm}{dt} \hat{\mathbf{i}}$$
$$= (2\,ms^{-1} \times 5\,kgs^{-1})\ \hat{\mathbf{i}}$$
$$= 10\,N\ \hat{\mathbf{i}}$$

Power delivered by motor to drive belt at 2 m/s

$$= |\vec{\mathbf{F}}|\ |\vec{\mathbf{v}}| = 10\,N \times 2\,ms^{-1}$$
$$= 20\ W$$

26. $$ma = F_{th} - mg$$

or $$ma = F_{th}$$

(neglecting mg as compare to F_{th})

or $$ma = u\left(-\frac{dm}{dt}\right)$$

or $$\frac{dm}{m} = -\frac{a}{u} dt$$

or $$\int \frac{dm}{m} = \int -\frac{a}{u} dt$$

or $$\log_e m = -\frac{a}{u} t + K$$

Now, as $t = 0$, $m = m_0$ (given)

$$\log_e m_0 = K$$

Thus, $$\log_e m = -\frac{a}{u} t + \log_e m_0$$

or $$\log_e \frac{m}{m_0} = -\frac{a}{u} t$$

or $$m = m_0 e^{-\frac{a}{u} t}$$

27. $u = 100\,ms^{-1}$, $v = 0\,ms^{-1}$, $s = 6$ cm

$$v^2 = u^2 + 2as$$

$$\Rightarrow \quad a = -\frac{u^2}{2s}$$

(a) Now, $v = u + at$

$$\therefore \quad t = -\frac{u}{a} = -\frac{u}{\frac{u^2}{2s}} = \frac{2s}{u}$$
$$= \frac{2 \times 6 \times 10^{-2}}{100} = 1.2\,ms$$

(b) Impulse on log = – Change in momentum of bullet

$$= -m(0 - u)$$
$$= mu$$
$$= 5 \times 10^{-3} \times 100$$
$$= 0.5\,Ns$$

(c) Average force experienced by the log

$$= \frac{\text{Impulse}}{\text{time}} = \frac{0.5}{1.2 \times 10^{-3}}$$
$$= 416.67\ N$$

28. Let us consider right direction as positive.

Impulse of the spring on the block

= Change in momentum of block

$$= m(v - u)$$

$= 3[(+40) - (-50)]$

$= +270\,\text{Ns}$

$= 270\,\text{Ns}$ (to the right)

Average force on block $= \dfrac{\text{Impulse}}{\Delta t}$

$= \dfrac{270}{0.02}$

$= 13500\,\text{N}$

$= 13.5\,\text{kN}$ (to the right)

29. $\vec{\mathbf{p}} = m\vec{\mathbf{v}} = 2(2t\,\hat{\mathbf{i}} - 4\,\hat{\mathbf{j}})$

$\therefore \quad \dfrac{d\vec{\mathbf{p}}}{dt} = 4\,\hat{\mathbf{i}}$

or $\quad d\vec{\mathbf{I}} = 4\,\hat{\mathbf{i}}\,dt$ (where, $\vec{\mathbf{I}}$ = Impulse)

or $\quad \int d\vec{\mathbf{I}} = \int_0^2 4\,\hat{\mathbf{i}}\,dt$

or $\quad \vec{\mathbf{I}} = 4\,\hat{\mathbf{i}}(2-0)$

$= 8\,\hat{\mathbf{i}}\ \text{kg-ms}^{-1}$

30. $m\,\Delta v = F\,\Delta t$

or $\quad m\,\Delta v =$ Area under F-t graph

or $\quad m\,\Delta v = \dfrac{16+8}{2} \times 20000$

or $\quad m\,\Delta v = 240000$

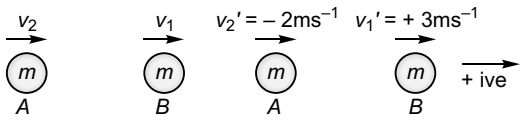
$\therefore \quad \Delta v = \dfrac{240000}{1200} = 200\,\text{m/s}$

$v - u = 200$

$\therefore \quad v = 200\,\text{ms}^{-1}$ as $u = 0$

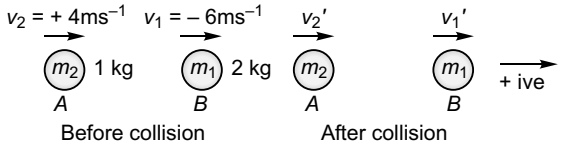
31. $v_1 = v_2' = -2\,\text{ms}^{-1}$

$v_2 = v_1' = +3\,\text{ms}^{-1}$



(When two bodies of equal masses collide elastically they interchange their velocities)

32. $m_1 = 1\,\text{kg}$ and $m_2 = 1\,\text{kg}$ (given)



$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_1 + \left(\frac{2m_2}{m_1 + m_2}\right)v_2$$

$$= \left(\frac{2-1}{2+1}\right)(-6) + \left(\frac{2\times 1}{2+1}\right)(4)$$

$$= -2 + \frac{8}{3}$$

$$= +\frac{2}{3}\,\text{ms}^{-1}$$

$$= \frac{2}{3}\,\text{ms}^{-1} \text{ (in + ive } x \text{ direction)}$$

$$v_2' = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_2 + \left(\frac{2m_1}{m_1 + m_2}\right)v_1$$

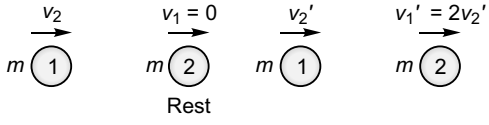
$$= \left(\frac{1-2}{2+1}\right)(+4) + \left(\frac{2\times 2}{2+1}\right)(-6)$$

$$= -\frac{4}{3} - 8$$

$$= -\frac{28}{3}\,\text{ms}^{-1} = \frac{28}{3}\,\text{ms}^{-1}$$

(in – ive x direction).

33. $v_1' = \dfrac{1+e}{2}v_2$ and $v_2' = \dfrac{1-e}{2}v_2$



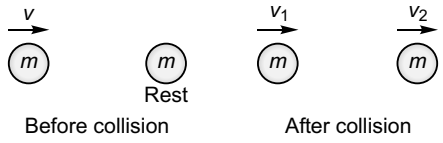
$\Rightarrow \quad \dfrac{v_1'}{v_2'} = \dfrac{1+e}{1-e}$

or $\quad 2 = \dfrac{1+e}{1-e} \quad (\because v_1' = 2v_2')$

or $\quad 2 - 2e = 1 + e$

or $\quad e = \dfrac{1}{3}$

34. $mv = mv_1 + mv_2$



$\Rightarrow \quad v = v_1 + v_2$

Also, $$\frac{\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2}{\frac{1}{2}mv^2} = \frac{3}{4}$$

or $$(v_1^2 + v_2^2) = \frac{3}{4}v^2 \quad \ldots(i)$$

or $$(v_1 + v_2)^2 - 2v_1v_2 = \frac{3}{4}v^2$$

or $$v^2 - 2v_1v_2 = \frac{3}{4}v^2$$

or $$2v_1v_2 = \frac{1}{4}v^2$$

$$e = \frac{\text{Velocity of separation}}{\text{Velocity of approach}}$$

or $$e = \frac{v_2 - v_1}{v - 0}$$

$$e = \frac{v_2 - v_1}{v}$$

$$e^2 = \left(\frac{v_2 - v_1}{v}\right)^2$$

or $$e^2 = \frac{v_2^2 + v_1^2 - 2v_1v_2}{v^2}$$

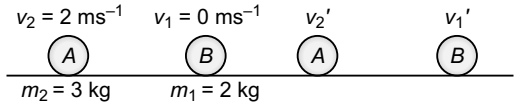
or $$e^2 = \frac{(v_1 + v_2)^2 - 4v_1v_2}{v^2}$$

or $$e^2 = \frac{v^2 - \frac{v^2}{2}}{v^2}$$

or $$e^2 = \frac{1}{2}$$

or $$e = \frac{1}{\sqrt{2}}$$

35. $v_1' = \left(\frac{2m_2}{m_1 + m_2}\right)v_2$



$$= \frac{2 \times 3}{2 + 3} \times 2$$

$$= \frac{12}{5} = 2.4 \text{ ms}^{-1}$$

and $$v_2' = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_2$$

$$= \frac{3 - 2}{2 + 3} \times 2$$

$$= \frac{2}{5} = 0.4 \text{ ms}^{-1}$$

Distance between blocks when they stop sliding

$$= \frac{v_1'^2}{2\mu_k g} - \frac{v_2'^2}{2\mu_k g}$$

$$= \frac{1}{2\mu_k g}(v_1'^2 - v_2'^2)$$

$$= \frac{1}{2 \times 0.3 \times 10}[(2.4)^2 - (0.4)^2]$$

$$= \frac{2.8}{3} = 0.933 \text{ m}$$

Objective Questions (Level 1)

Single Correct Option

1. Momentum remains conserved. Decrease in momentum of the ball is transferred to sand while KE does not remain conserved as it gets used up in doing work against friction.

2. $F_{\text{ext}}^{\text{net}} = M \times a_{\text{CM}}$

   $\therefore$ If $F_{\text{ext}}^{\text{net}} = 0$, $\quad a_{\text{CM}} = 0$

   i.e., $\quad \dfrac{d}{dt} v_{\text{CM}} = 0$

   or $\quad v_{\text{CM}} = \text{constant}$

   Option (a) is correct.

3. The forces acting on the blocks would be equal and opposite as per Newton's 3rd law of motion. Acceleration of the blocks will depend upon their masses as per Newton's 2nd law of motion. Accelerations being different velocities will be unequal.

   Option (c) is correct.

4. While colliding the balls will apply equal and opposite impulsive force on each other. Impulsive forces will change the momentum of the balls but the total momentum of the system of 2 balls will remain conserved impulsive forces being internal ones. Change in momentum of the system will definitely be due to external gravitational forces on the balls but as the time of impact shall be very less the impulsive force will over shadow the weak gravitational force.

5. External force acting on the cannon shell before explosion is the gravitational force. Now, as no extra net external force would be act on the shell during collision the momentum of the system shall remain conserved and the CM of the system (now broken in pieces) will also keep on following the path which the shell would have followed had the explosion not taken place. Further, as the explosion would never be super-elastic, the KE of the system can't increase after explosion.

   Option (d) is correct.

6. $\dfrac{\text{Velocity of separation}}{\text{Velocity of approach}} = e$

   As in an elastic collision $e < 1$

   Velocity of separation < velocity of approach

   (when $e = 0$, the velocity of separation in zero and the colliding bodies do not separate from each other.)

   Further, whether the collision in elastic or inelastic the law of conservation of momentum always hold gord.

   $\therefore$ Option (d) is correct.

7. $\vec{\mathbf{p}} = M\,\vec{\mathbf{v}}_{\text{CM}}$, Option (a) is correct.

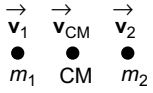
   $\vec{\mathbf{p}} = \vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2 + \vec{\mathbf{p}}_3 + \ldots,$

   Option (b) is correct.

   Further, we define momentum for every type of motion.

   $\therefore$ Option (d) is correct.

8. Let us consider a system of two masses as shown in figure.

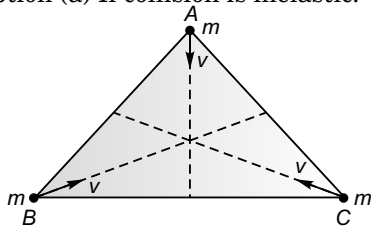


   Momentum of system about CM

$$= m_1(\vec{\mathbf{v}}_1 - \vec{\mathbf{v}}_{\text{CM}}) + m_2(\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_{\text{CM}})$$
$$= m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2 - (m_1 + m_2)\vec{\mathbf{v}}_{\text{CM}}$$
$$= (m_1 + m_2)\vec{\mathbf{v}}_{\text{CM}} - (m_1 + m_2)\vec{\mathbf{v}}_{\text{CM}}$$
$$= \vec{0}$$

   Option (c) is correct.

9. Option (a) If collision is inelastic.



   Option (b) If collision is perfectly inelastic

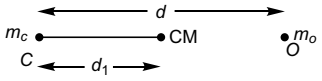
   Option (c) If the dimensions of the particles $\to 0$

   $\therefore$ Option (d) would be the answer.

10. $F\,\Delta t = m\,\Delta v$

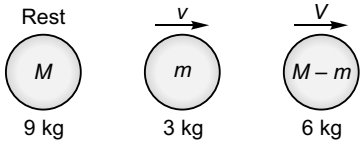
$$\Rightarrow \quad F = \frac{m\,\Delta v}{\Delta t} = \frac{5 \times (65 - 15)}{2} = 125\,\text{N}$$

11. $m_c \times d_1 = m_o \times (d - d_1)$



$$\Rightarrow \quad d_1 = \frac{m_o}{m_o + m_c} d = \frac{8}{8+6} d = \frac{4}{7} d = \frac{4}{7} \times (1.2 \times 10^{-10}\ \text{m}) = 0.64 \times 10^{-10}\ \text{m}$$

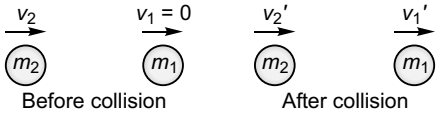
12. $M \times 0 = mv + (M - m)\,V$



$$\Rightarrow \quad V = -\frac{m}{(m - m)} v = -\frac{3}{9-3} \times 16 = -8\ \text{ms}^{-1}$$

$\therefore$ KE of 6 kg mass $= \frac{1}{2} \times 6 \times (-8)^2 = 192\ \text{J}$

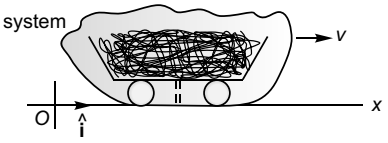
13. $v_1' = \dfrac{2m_2}{m_1 + m_2} v_2$



$$= 2v_2 \qquad (\text{as } m_1 << m_2)$$

Option (b) is correct.

14. Horizontal velocity of the leaving coal :



$$\vec{\mathbf{v}}_1 = +v\,\hat{\mathbf{i}}$$

Horizontal velocity of the system

$$\vec{\mathbf{v}}_2 = +\,v\,\hat{\mathbf{i}}$$

$$\therefore \quad \vec{\mathbf{U}}_{\text{rel}} = \vec{\mathbf{v}}_1 - \vec{\mathbf{v}}_2 = \vec{0}_1$$

$$\therefore \quad \vec{\mathbf{F}}_{\text{th}} = \vec{\mathbf{U}}_{\text{rel}} \frac{dm}{dt} = \vec{0}$$

As, the leaving coal does not exert any thrust force on the wagon, the speed of the wagon won't change.

Option (a) is correct.

15. If n be the number of bullet shots per second

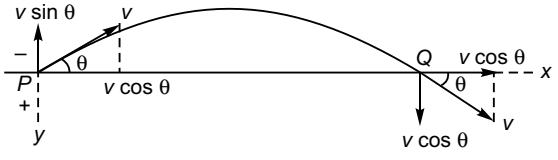
$n \times$[change in momentum per second] $\le F$

i.e., $\quad n\left[\dfrac{40}{100} \times (1200 - 0)\right] \le 144$

or $\quad n \le \dfrac{144}{48}$ or $n \le 3$

$\therefore$ Option (a) is correct.

16. Change in momentum along x-axis



$$= m\,(v\cos\theta - v\cos\theta) = 0$$

$\therefore$ Net change in momentum

= Change in momentum along y-axis

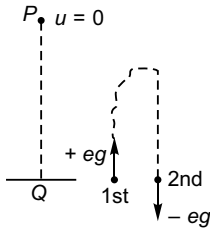
$= m[(+v\sin\theta) - (-v\sin\theta)]$

$= 2mv\sin\theta$

$= mv\sqrt{2} \qquad (\text{as } \theta = 45°)$

Option (a) is correct.

17. Velocity of ball before first impact *i.e.*, when it reaches point Q of the horizontal plane



$$v = 0 + (g) \cdot 1 = g$$

$\therefore$ Velocity of ball after 1st impact

$$= ev = eg$$

time elapsed between 1st and 2nd impact with the horizontal plane

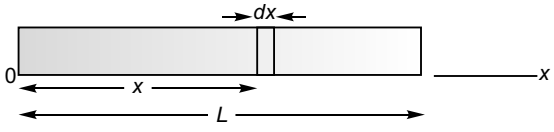
$$(+ e\,g) = (+ e\,g) + (- g)\,t$$

$$\Rightarrow \qquad t = 2e$$

$$= \frac{4}{3}\,s \qquad \left(\text{as, } e = \frac{2}{3}L\right)$$

Option (c) is correct.

18. $x_{CM} = \dfrac{\int x\,dm}{\int dm} = \dfrac{\int x\,m\,dx}{\int m\,dx}$



(where m = mass per unit length)

$$= \frac{\int x \dfrac{Ax^2}{L}\,dx}{\int \dfrac{Ax^2}{L}dx} \qquad \left(\because m = \frac{Ax^2}{L}\right)$$

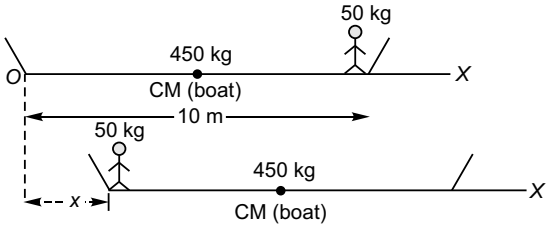
$$= \frac{\int_0^L x^3 dx}{\int_0^L x^2 dx}$$

$$= \frac{L^4}{4} \times \frac{3}{L^3}$$

$$= \frac{3}{4}L$$

Option (a) is correct.

19. As there is no net external force acting on the system in the horizontal direction, the CM of the system shall not shift along x-axis.



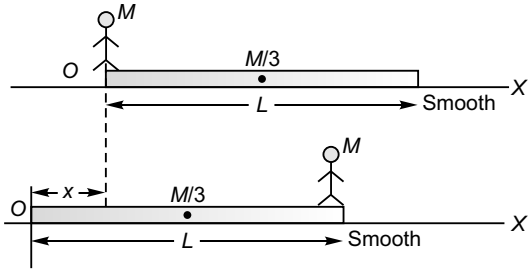
$$\therefore \frac{50 \times 10 + 450 \times 5}{50 + 450} = \frac{50 \times (x) + 450 \times (x + 5)}{50 + 450}$$

(Initially) (Finally)

$$\Rightarrow \qquad x = 1\text{ m}$$

Option (b) is correct.

20. As discussed in the answer to previous question no. 19.



$$\frac{Mx + \dfrac{M}{3}\left(x + \dfrac{L}{2}\right)}{M + \dfrac{M}{3}} = \frac{ML + \dfrac{M}{3}\left(\dfrac{L}{2}\right)}{M + \dfrac{M}{3}}$$

(Initially) (Finally)

$$\Rightarrow \qquad x\left[M + \frac{M}{3}\right] = ML$$

$$x = \frac{3}{4}L$$

i.e., the distance that plank moves $= \dfrac{3}{4}L$

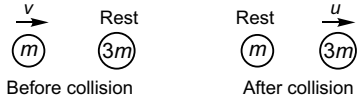
$\therefore$ The distance that the man moves

$$= L - \frac{3}{4}L$$

$$= \frac{L}{4}$$

Option (b) is correct.

21.



$$mv + 3m \cdot 0 = m \cdot 0 + 3\,mu$$

$\therefore \quad u = \frac{v}{3}$

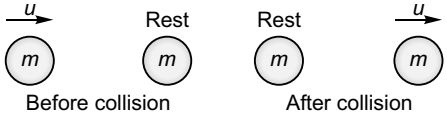
$$e = \frac{\text{Velocity of separation}}{\text{Velocity of approach}}$$

$$= \frac{u-0}{v-0} = \frac{u}{v}$$

$$= \frac{1}{3}$$

Option (d) is correct.

22. Change in momentum of A or $B = mu$



(As collision is elastic)

Impulse = Change in momentum

$$\frac{Ft_0}{2} = mu$$

$$\Rightarrow \quad F = \frac{2mu}{t_0}$$

Option (b) is correct.

23. Acceleration of block A

$$a_0 = \frac{F'}{m}$$

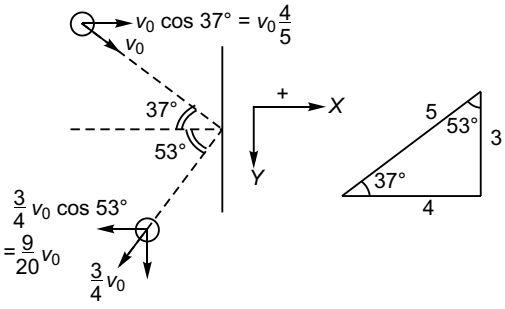
$$\Rightarrow \quad F' = ma_0$$

Acceleration of block B :

$$a_B = \frac{F - F'}{m}$$

$$= \frac{F - ma_0}{m} = \frac{F}{m} - a_0$$

24.



Option (c) is correct.

Impulse on ball

= Change in momentum of ball

$$= \left(-\frac{9}{20}mv_0\right) - \left(+\frac{4}{5}mv_0\right)$$

$$= -\frac{5}{4}mv_0$$

25. If a ball dropped from height h rebounds to a height h', then speed of ball just before 1st impact, $u = \sqrt{2gh}$

Just after 1st impact $u' = \sqrt{2gh'}$

$$\therefore \quad e = \frac{u'}{u} = \sqrt{\frac{h'}{h}} = \sqrt{\frac{64}{100}}$$

$$= 0.8$$

i.e., $\quad h' = e^2 h$

Height attained after nth impact

$$= e^{2n}h$$

$$= (0.8)^{2n} \cdot 1 \qquad (\text{as } h = 1\,\text{m})$$

$$= (0.8)^{2n}$$

Option (d) is correct.

26. Momentum of car (+ block) before throwing block

= Momentum of car after throwing block + Momentum of block

$$500 \times 1\,\hat{\mathbf{i}} = (500 - 25)\vec{\mathbf{v}} + 25(20\,\hat{\mathbf{k}})$$

or $\quad 475\vec{\mathbf{v}} = 500(\hat{\mathbf{i}} - \hat{\mathbf{k}})$

or $\quad \vec{\mathbf{v}} = \frac{20}{19}(\hat{\mathbf{i}} - \hat{\mathbf{k}})$

Option (c) is correct.

27. While force is increasing with time

$F = kt$ (where k is + ive constant)

or $\quad ma = kt$

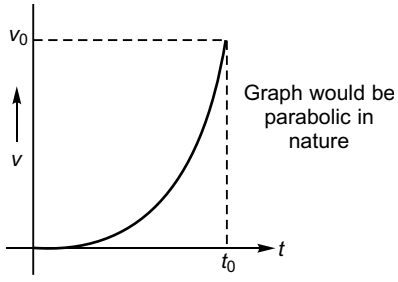
or $\quad \frac{dv}{dt} = \frac{k}{m}t$

$\therefore \quad v = \frac{k}{m}\frac{t^2}{2} + C$

or $\quad v = \frac{k}{m} \cdot \frac{t^2}{2}$

(If at $t = 0, v = 0$)

Thus, graph between v and t would be



While force is decreasing with time.

$F = -kt$ (where k is + ive constant)

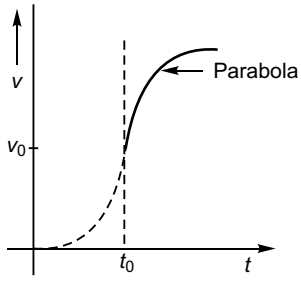
$$\Rightarrow \qquad v = -\frac{k}{m}\frac{t^2}{2} + C'$$

At $t = 0, v = v_0$

$$\therefore \qquad C' = v_0 + \frac{k}{m}\frac{t_0^2}{2}$$

Thus, $$v = v_0 + \frac{k}{2m}(t_0^2 - t^2)$$

Thus, graph between v and t would be



$\therefore$ Option (a) is correct.

28. $$x_{CM} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2}$$

$$0 = \frac{[\pi(3R)^2 - \pi R^2]x_1 + [-\pi R^2]2R}{\pi(3R)^2 - \pi R^2}$$

or $8\pi R^2 x_1 = 2\pi R^2 R$ or $8x_1 = 2R$

or $$x_1 = \frac{R}{4}$$

Option (c) is correct.

29. $$x_{CM} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1 + A_2 + A_3}$$

$$\therefore \quad 0 = [\pi(4R)^2 - \pi R^2 - \pi R^2]x_1 + [-\pi R^2]3R + [-\pi R^2]\cdot 0$$

or $$14x_1 = 3R$$

or $$x_1 = \frac{3}{14}R$$

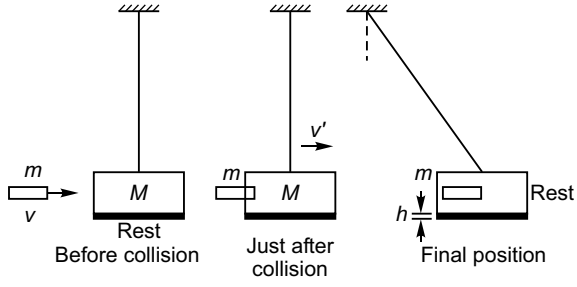
Option (d) is correct.

30. As no external force is acting along the horizontal direction on the system (wedge + block). The CM of the system shall not change along horizontal when the block moves over the wedge but would change along vertical as net force (= gravitational force) is acting on the block.

Further, as no non-conservative force is acting on the system, its total energy will not change.

Option (d) is correct.

31.



$$mv = (M + m)v' \qquad \ldots(i)$$

From final position, $v' = \sqrt{2gh}$

From Eq. (i),

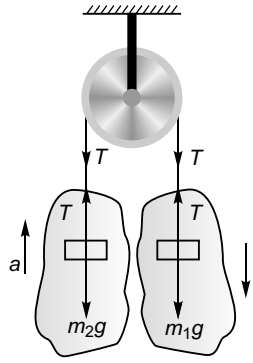
or $$\frac{m}{M + m}v = \sqrt{2gh}$$

or $$v = \left(1 + \frac{M}{m}\right)\sqrt{2gh}$$

Option (c) is correct.

32. As no net extra external force is acting on the system the CM of the gun and the bullet system remains at rest. The force exerted by the trigger of the gun on the bullet is an interval one.

33. $m_1g - T = m_1a$



$T - m_2 g = m_2 a$

$\therefore \quad (m_1 - m_2)\, g = (m_1 + m_2)\, a$

or $$a = \frac{m_1 - m_2}{m_1 + m_2} g$$

$$a_{CM} = \frac{m_1 a + m_2 (-a)}{m_1 + m_2}$$

$$= \frac{m_1 - m_2}{m_1 + m_2} a$$

$$= \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 g$$

Option (b) is correct.

34. $v = u - gt + v_i \ln \frac{m_0}{m}$

or $$v = u + v_2 \ln \frac{m_0}{m}$$

(neglecting gravity as given)

or $$v = v_2 \ln \frac{m_0}{m} \quad \text{(Taking } u = 0\text{)}$$

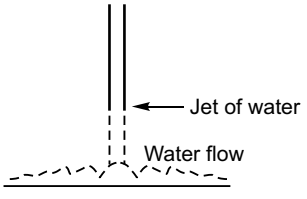
or $$v = v \ln \frac{m_0}{m} \quad (\text{as } v_i = v)$$

$\therefore \quad \log_e \frac{m_0}{m} = 1$

$\Rightarrow \quad \frac{m_0}{m} = e^1 = 2.718$

Option (a) is correct.

35. Velocity of water after striking the plate would be almost zero as then it flows parallel to the plate.



Force exerted on plate

= – [Rate of change of momentum of water]

$$= -\left[\frac{m \cdot 0 - mu}{t}\right]$$

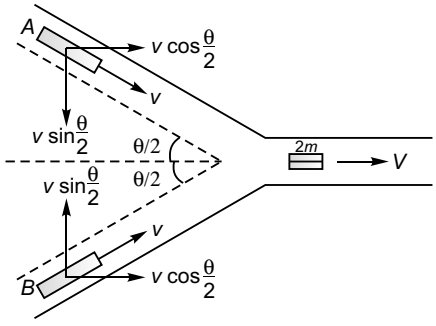
$$= \frac{mu}{t} = \frac{m}{t} \cdot u$$

$$= 0.5 \frac{\text{kg}}{\text{s}} \times 1 \text{ms}^{-1}$$

$$= 0.5 \text{ N}$$

Option (c) is correct.

36. Velocity of separation would be zero as the collision is completely inelastic.



Velocity of approach = Velocity of A w.r.t. B or velocity of B w.r.t. A

$$= \left(+ v \sin \frac{\theta}{2}\right) - \left(- v \sin \frac{\theta}{2}\right)$$

$$= 2v \sin \frac{\theta}{2}$$

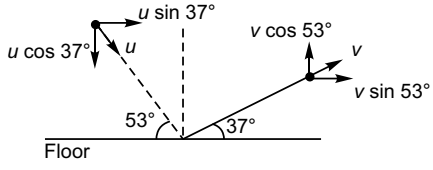
Common velocity (V) after collision

$$mv \cos \frac{\theta}{2} + mv \cos \frac{\theta}{2} = 2mV$$

$\therefore \quad V = v \cos \frac{\theta}{2}$

Option (d) is correct.

37. $v \sin 53° = u \sin 37°$



$$v \times \frac{4}{5} = u \times \frac{3}{5}$$

i.e., $$v = \frac{3}{4} u$$

Impulse exerted on floor

= – [Change in momentum of ball]

$= -[(-mv \cos 53°) - (+mu \cos 37°)]$

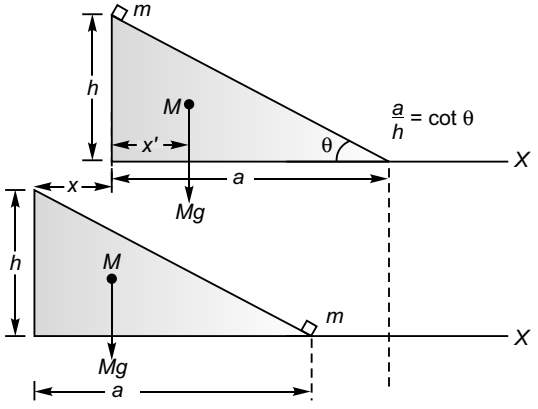
$= m[v \cos 53° + u \cos 37°]$

$$= m\left[\frac{3}{4} u \times \frac{3}{5} + u \times \frac{4}{5}\right]$$

$$= \frac{5}{4} mu$$

$$= \frac{5}{4} \times 1 \times 5 = 6.25 \text{ Ns}$$

38. As the CM will not change along x-axis, for no net force acting on system (wedge + block) along x-axis.



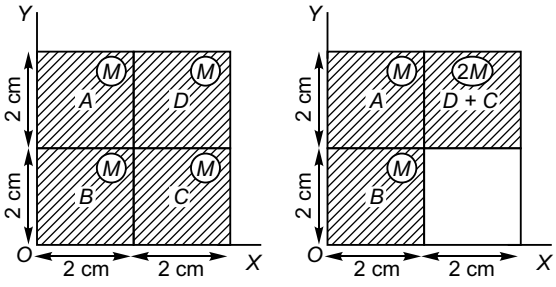
$$mx + M(x + x') = Ma + Mx'$$

$$\therefore \quad mx + Mx = ma \text{ or } x = \frac{m}{M+m}a$$

$$= \frac{m\,h\cot\theta}{M+m}$$

Option (b) is correct.

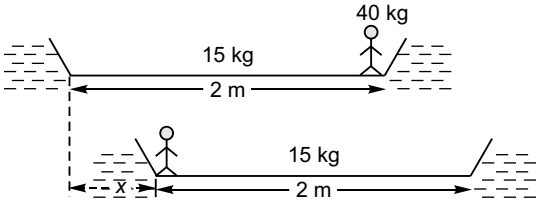
39. $x_{CM} = \dfrac{2(4M) + 3(-M) + 3(+M)}{4M + (-M) + (+M)} = 2\text{ cm}$



$$y_{CM} = \frac{2(4M) + 1(-M) + 3(+M)}{4M + (-M) + (+M)} = 2.5\text{ cms}$$

Option (c) is correct.

40. As explained in the answer to question no. 20 and 19.



x = Displacement of boat

$$x \cdot 40 + (1 + x)15 = 2 \times 40 + 1 \times 15$$

i.e., $\quad x = 1.46\text{ m}$

(The frictional forces on the boat by the boy and that by the boy on the boat are internal forces).

41. $$\vec{\mathbf{v}} = \frac{m\vec{\mathbf{v}}_1 + m\vec{\mathbf{v}}_2 + m\vec{\mathbf{v}}_3}{m + m + m}$$

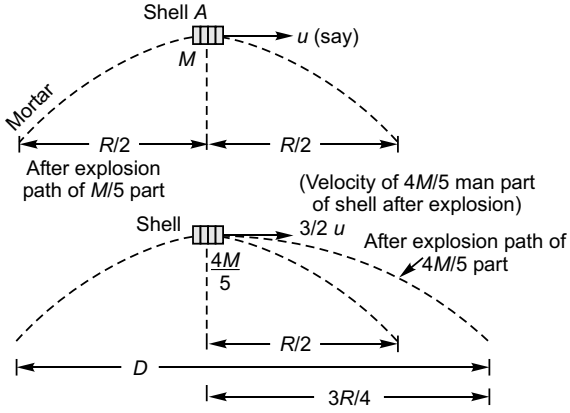
$$= \frac{1}{3}[\vec{\mathbf{v}}_1 + \vec{\mathbf{v}}_2 + \vec{\mathbf{v}}_3]$$

$$= \frac{1}{3}[v_0\,\hat{\mathbf{i}} + (-3v_0\,\hat{\mathbf{j}}) + (5v_0\,\hat{\mathbf{k}})]$$

$$= \frac{v_0}{3}[\hat{\mathbf{i}} - 3\,\hat{\mathbf{j}} + 5\,\hat{\mathbf{k}}]$$

Option (d) is correct.

42. The shell explodes at A, the highest point. As piece of mass $\dfrac{M}{5}$ falls very close to mortar, its velocity after explosion must be $-u$.



Thus, $Mu = \dfrac{M}{5}(-u) + \dfrac{4M}{5}(v)$

i.e., $\quad v = \dfrac{3}{2}u$

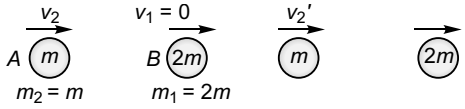
As the velocity has increased $\dfrac{3}{2}$ times the range of this part will be

$$\frac{3}{2} \text{ of } \frac{R}{2} \text{ i.e., } \frac{3R}{4}$$

$$\therefore \quad D = \frac{R}{2} + \frac{3R}{4} = \frac{5R}{4}$$

Option (c) is correct.

43. $v_2' = \dfrac{m_2 - m_1}{m_2 + m_1} v_2$



$$= \frac{m - 2m}{m + 2m} v_2$$

$$= -\frac{1}{3} v_2$$

Fraction of KE lost by colliding particle A

$$= \frac{\text{KE lost}}{\text{Initial KE}}$$

$$= \frac{\frac{1}{2} m v_2^2 - \left[\frac{1}{2} m \left(-\frac{1}{3} v_2\right)^2\right]}{\frac{1}{2} m v_2^2}$$

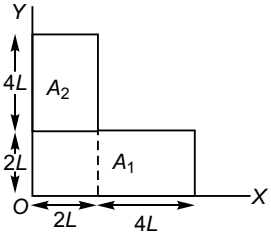
$$= \frac{\frac{1}{2} - \frac{1}{18}}{\frac{1}{2}}$$

$$= \frac{8}{9}$$

JEE Corner

Assertion and Reason

1. To answer this question, let us find the centre of mass of an L-shaped rigid body (as shown in figure) of uniform thickness.



$$x_{CM} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2}$$

$$= \frac{(6L \times 2L)\, 3L + (4L \times 2L)\, L}{(6L \times 2L) + (4L \times 2L)}$$

$$= \frac{36L^3 + 8L^3}{12L^2 + 8L^2}$$

$$= \frac{44L^3}{20L^2}$$

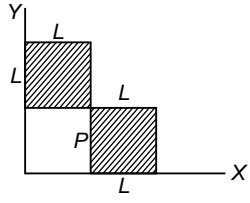
$$= 2.2\,L$$

Similarly $y_{CM} = 2.2\,L$

Thus, we see that CM is lying outside.

Centre of mass of the rigid body of uniform thickness as shown in figure would be at point P which is neither outside nor inside.

The centre of mass of a uniform plate lies at its centre. The CM of a uniform sphere is at its centre.

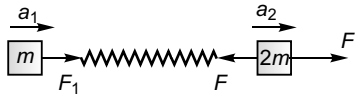


These examples show that assertion is false.

It is correct that centre of mass and centre of gravity of a body coincide if the body is placed in a uniform gravitational field.

Option (d) is correct.

2. $a_1 = \dfrac{F_1}{m}$, $a_2 = \dfrac{F - F_1}{2m}$



$$\therefore \quad a_{CM} = \frac{m a_1 + 2m a_2}{m + 2m}$$

$$= \frac{F_1 + (F - F_1)}{3m}$$

$$= \frac{F}{3m}$$

As F is constant, the value of a_{CM} will remain constant (Reason).

As CM is accelerated the velocity of CM will obviously increase (Assertion).

Option (a) is correct.

3. As per assertion if force is applied on a system the linear momentum of the system must not remain conserved. But, it will not be true if we apply two equal and opposite external forces on the system as then net external force on the system will be zero and the linear momentum will remain conserved as given under Reason which is correct.
Option (d) is correct.

4. A rocket moves forward due to the thrust force produced on it as per Newton's 3rd law of motion (as given under Reason) when gas inside (not the surrounding air as given under assertion) it is pushed backwards.

Option (d) is correct.

5. Linear momentum of a system remains conserved when no net external force acts on the system *i.e.*, only a net external force on a system can change its linear momentum. Inside a system internal forces are always in pairs and as such can't change linear momentum.

If two blocks connected by a spring placed on a smooth surface are stretched apart the internal restoring forces acting on blocks will definitely increase the KE of the system but this argument under Reason has nothing to do with the nothing under Assertion that internal forces can't change linear momentum.

Option (b) is correct.

6. $\text{KE} = \dfrac{p^2}{2m}$

i.e., $\text{KE} \propto \dfrac{1}{m}$ (if momentum p is constant).

$\therefore$ Nothing under reason is correct.

When bullet is fired and comes out of the gun.

Linear momentum of gun

$$= \text{Linear momentum of bullet} = p$$

(In magnitude)

$$\therefore \quad \frac{(\text{KE}) \text{ gun}}{(\text{KE}) \text{ bullet}} = \frac{\text{Mass of bullet}}{\text{Mass of gun}}$$

(as given under Reason)

This is what is given in Assertion.

Thus, both **Assertion and Reason** are true and also reason is the correct explanation of the assertion.

Option (a) is correct.

7. As no net external force is acting there on the system along horizontal direction the momentum of the system remains conserved along horizontal direction but as gravitational force (a net external force) acts on block in the vertical direction (downwards) the momentum of the system does not remain constant along vertical direction. As overall momentum of the system does not remain constant, **Assertion is true**.

As wedge is at rest, **Reason is false**.

Option (c) is correct.

8. In any collision, there is not change in the momentum of the system as given in **reason**, which is **true**.

$$\therefore \quad \Delta\vec{\mathbf{p}}_1 + \Delta\vec{\mathbf{p}}_2 = \vec{0}$$

i.e., $$\Delta\vec{\mathbf{p}}_1 = -\Delta\vec{\mathbf{p}}_2 \quad \ldots(\text{i})$$

$\therefore$ **Assertion is false**.

Option (d) is correct.

9. **Reason is true** as explained in the answer to question no. 6 and also in Eq. (i) in the answer to question no. 8.

As KE is inversely proportional to mass, the KE of the block of man $2m$ will be $\dfrac{K}{2}$ when the KE of the block of mass m is K.

$\therefore$ **Assertion is true.**

Further, as Reason is the correct explanation of the Assertion. Option would be (a).

10. **Assertion is false** as for example heat energy can be given to a system without any increase in momentum of the system while KE given to a system increases its momentum.

$\left(KE = \frac{p^2}{2m}\right)$ as given is Reason which is true.

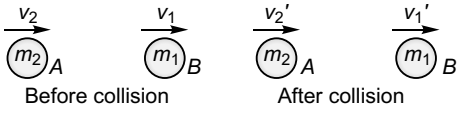
Option (d) is correct.

11. As no external force would be acting on the system of electron and proton along the line joining electron and proton the CM of electron and proton will remain at rest. Therefore, **Assertion is false**.

Further, as proton is heavier than electron **the reason is true**.

Option (d) is correct.

12. $v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2$



and $v_2' = \frac{m_2 - m_1}{m_1 + m_2} v_2 + \frac{2m_1}{m_1 + m_2} v_1$

$$v_2' - v_1' = \frac{m_2 - m_1 - 2m_2}{m_1 + m_2} v_2 + \frac{2m_1 - m_1 + m_2}{m_1 + m_2} v_1$$

$$= -v_2 + v_1$$

$$= -(v_2 - v_1)$$

i.e., $v'_{21} = -v_{21}$

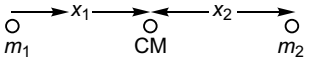
i.e., relative velocity of A w.r.t. B after collision

= – (relative velocity of A w.r.t. B before collision)

∴ Reason is true and Assertion is false.

Option (d) is correct.

13. As explained in the answer to question no. 11, the CM of the objects will remain at rest. Therefore, assertion is false.



$$m_1 x_1 = m_2 x_2$$

$$\therefore \quad \frac{x_2}{x_1} = \frac{m_1}{m_2}$$

$$\therefore \quad x_2 > x_1 \text{ as } m_2 > m_1$$

Reason is true.

Option (d) is correct.

14. $\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt}$ (Newton's second law of motion)

$\vec{\mathbf{a}} = \frac{\vec{\mathbf{F}}}{m}$ (outcome of the above)

∴ **Reason is true**.

First equation tells that if same force $\vec{\mathbf{F}}$ is applied on different masses the rate of change of momentum *i.e.*, $\frac{d\vec{\mathbf{p}}}{dt}$ of each mass will be same. Second equation tells that if same force $\vec{\mathbf{F}}$ is applied on different masses the $\vec{\mathbf{a}}$ produced in each will be different.

∴ **Assertion is true.**

Further, as **Reason is the correct explanation of the Assertion.**

Option (a) is correct.

15. **Assertion is false** as explained in the answer to question no. 12.

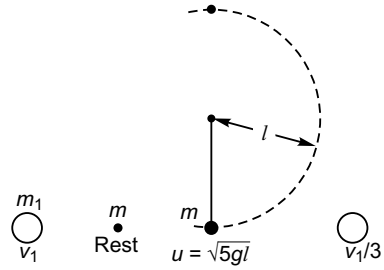
In every type of collision the linear momentum of the system remains conserved. Therefore, **Reason is true.**

Option (d) is correct.

Objective Questions (Level 2)

Single Correct Option

1. $m_1v_1 + m \times 0 = m\sqrt{5g\,l} + m_1\dfrac{v_1}{3}$

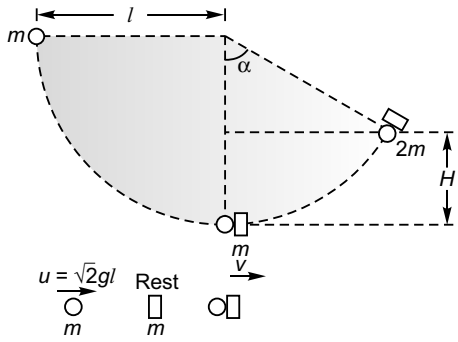


$$m_1 \cdot \frac{2v_1}{3} = m\sqrt{5g\,l}$$

$$\therefore \qquad v_1 = \frac{3}{2}\frac{m}{m_1}\sqrt{5g\,l}$$

Option (b) is correct.

2. $mu = 2mv$



$$\Rightarrow \qquad v = \frac{u}{2}$$

$$0^2 = v^2 + 2(-g)H$$

$$\Rightarrow \qquad H = \frac{v^2}{2g}$$

$$= \frac{(u/2)^2}{2g}$$

$$= \frac{u^2}{8g}$$

or $$H = \frac{2g\,l}{8g}$$

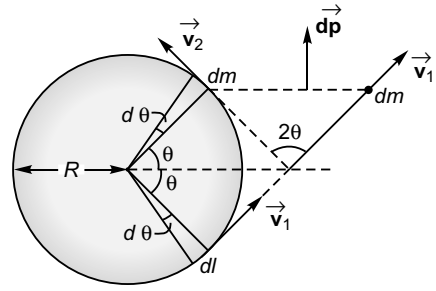
or $$l(1 - \cos\alpha) = \frac{l}{4}$$

or $$1 - \cos\alpha = \frac{1}{4}$$

or $$\alpha = \cos^{-1}\frac{3}{4}$$

Option (c) is correct.

3. Here $|\vec{\mathbf{v}}_2| = |\vec{\mathbf{v}}_1| = v$ say



Net momentum of the two elements as shown in figure $= dp = |\vec{\mathbf{dp}}|$

$$= 2v\,dm\sin\theta$$

$$= 2v\left(\frac{M}{\pi R}\right)(R\,d\theta)\sin\theta$$

$$= \frac{2M\,v}{\pi}\sin\theta\,d\theta$$

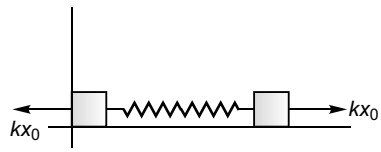
$$p = \int_0^{\pi/2}\frac{2Mv}{\pi}\sin\theta\,d\theta$$

$$= -\frac{2M\,v}{\pi}[\cos\theta]_0^{\pi/2}$$

$$= -\frac{2M\,v}{\pi}[0 - 1] = \frac{2Mv}{\pi}$$

Option (b) is correct.

4. $\dfrac{1}{2}(2m)v^2 = \dfrac{1}{2}kx_0^2$



or $$v = \sqrt{\frac{k}{2m}}x_0$$

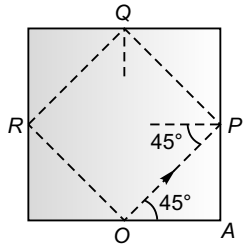
or $$2mv = \sqrt{2mk}\,x_0$$

or $$F_{av} \times \Delta t = \sqrt{2mk}\,x_0$$

or $$F_{av} = \frac{\sqrt{2mk}}{\Delta t}x_0$$

Option (b) is correct.

5. As the collisions of the striker and the walls of the carrom are perfectly elastic the striker will follow the path *OPQROP* ...



Change in KE = work done against friction

$$\frac{1}{2}mv^2 - 0 = \mu\, mgs$$

(m = mass of striker, s = displacement of the striker)

$$\Rightarrow \qquad s = \frac{v^2}{2\mu\, g}$$

$$= \frac{(2)^2}{2 \times 0.2 \times 10} = 1\text{ m}$$

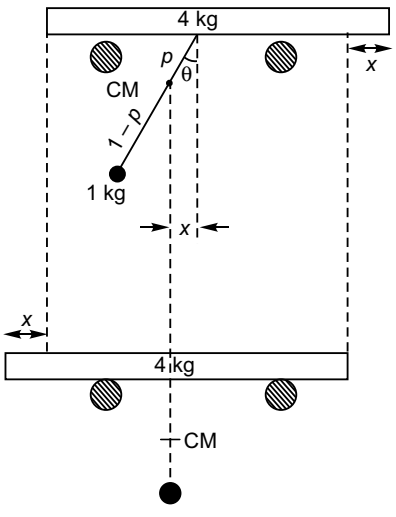
$$PQ = OP = OA\sqrt{2} = \left(\frac{1}{2\sqrt{2}}\right)\sqrt{2} = \frac{1}{2}\text{ m}$$

$$OP + PQ = \frac{1}{2} + \frac{1}{2} = 1\text{ m}$$

$\therefore$ Striker will stop at point Q where co-ordinates are $\left(\frac{1}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$.

Option (a) is correct.

6. As no force is acting on the system along horizontal, the CM of the system will not shift horizontally.



$$4p = 1(1 - p)$$

$$\Rightarrow \qquad p = \frac{1}{5}\text{ m}$$

Displacement (x) of bar when pendulum becomes vertical

$$\frac{x}{p} = \sin\theta$$

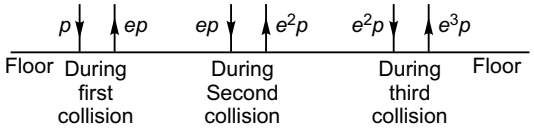
$$x = p\sin\theta$$

$$= \frac{1}{5}\sin 30° = \frac{1}{10}\text{ m} = 0.1\text{ m}$$

When the ball reaches the other extreme end the bar will further shift to the left by distance x and as such the net displacement of the bar will be $2x$ *i.e.*, 0.2 m.

Option (b) is correct.

7.



Momentum imparted to the floor in

1st collision $= p - (-ep) = p(1 + e)$

2nd collision $= ep - (-e^2 p) = ep(1 + e)$

3rd collision $= e^2 p - (-e^3 p) = e^2 p(1 + e)$

As theoretically there will be infinite collision, total momentum imparted to floor

$$= p(1 + e) + ep(1 + e) + e^2 p(1 + e) + \ldots\infty$$

$$= p(1 + e)[1 + e + e^2 + \ldots\infty]$$

$$= p(1 + e)\frac{1}{1 - e} = p\left(\frac{1 + e}{1 - e}\right)$$

Option (d) is correct.

8. Let F be the frictional force applied by plate when bullet enters into it

$$\therefore \qquad \frac{1}{2}mu^2 = Fh \qquad \ldots\text{(i)}$$

If plate was free to move

$$mu + 0 = (M + m)u'$$

$$\therefore \qquad u' = \frac{m}{M + m}u$$

New KE of bullet $= \frac{1}{2}mu^2 - \frac{1}{2}(M + m)u'^2$

(entering plate)

$$= \frac{1}{2}mu^2 - \frac{1}{2}(M+m)\left(\frac{m}{M+m}u\right)^2$$

$$= \frac{1}{2}mu^2\left[1 - \frac{m}{M+m}\right] = \frac{1}{2}mu^2\left(\frac{M}{M+m}\right)$$

$$= Fh' \qquad \ldots(ii)$$

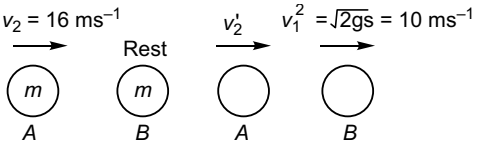
Dividing Eq. (ii) by Eq. (i),

$$\frac{h'}{h} = \frac{M}{M+m}$$

i.e., $$h' = \left(\frac{M}{M+m}\right)h$$

Option (a) is correct.

9.



Now, $v_1' = \frac{1+e}{2}v_2$ (entering plase)

$\therefore$ $$10 = \frac{1+e}{2}\cdot 16$$

i.e., $$e = \frac{1}{4}$$

Option (b) is correct.

10. $$y_{CM} = \frac{(l\cdot b)\frac{l}{2} + \left(\frac{1}{2}L\cdot b\right)\left(l + \frac{L}{3}\right)}{(l\cdot b) + \left(\frac{1}{2}L\cdot b\right)}$$

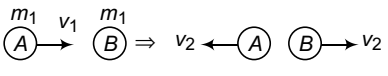
$$l = \frac{\frac{bl^2}{2} + \frac{bLl}{2} + \frac{L^2 b}{6}}{bl + \frac{bL}{2}} \qquad \left[\begin{array}{l} y_{CM} = l \\ \text{according} \\ \text{to question} \end{array}\right]$$

or $$l\left[l + \frac{L}{2}\right] = \frac{l^2}{2} + \frac{Ll}{2} + \frac{L^2}{6}$$

or $$\frac{l^2}{2} = \frac{L^2}{6}$$

or $$l = \frac{L}{\sqrt{3}}$$

11.



$$m_1v_1 = m_2v_2 - m_1v_2 \qquad \ldots(i)$$

$$e = 1$$

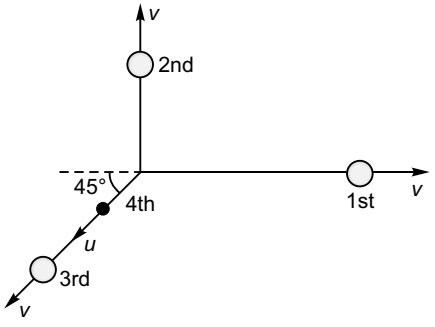
$\therefore$ $$2v_2 = v_1$$

$\therefore$ $$m_1(2v_2) = m_2v_2 - m_1v_2$$

$\therefore$ $$3m_1 = m_2$$

or $$\frac{m_1}{m_2} = \frac{1}{3}$$

12. As the resultant of the velocities of 1st and 2nd are just opposite to that of 3rd, the 4th particle will travel in the line in which 3rd is travelling.



Let the velocity of 4th particle is u as shown in figure.

$\therefore$ $$u\cos 45° + v\cos 45° = v$$

i.e., $$u = v(\sqrt{2} - 1)$$

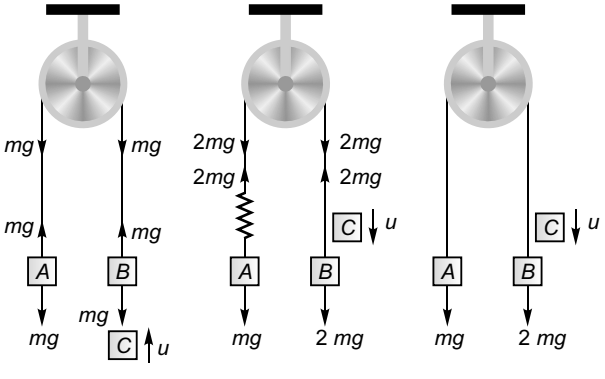
Total energy released

$$= \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}m[v(\sqrt{2}-1)]^2$$

$$= \frac{1}{2}mv^2[3 + (\sqrt{2}-1)^2] = mv^2[3 - \sqrt{2}]$$

Option (a) is correct.

13.



$mu = 2mv_1$

i.e., $v_1 = \frac{u}{2}$

C will increase the tension in the string.

$mu = 2mv_2$

i.e., $v_2 = \frac{u}{2}$

C will also increase the weight of B when collision takes place. Thus,

$mu = (2m + m)v_3$

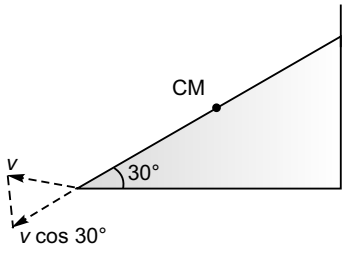
i.e., $v_3 = \frac{u}{3}$

$\therefore\ v_1 : v_2 : v_3$

$$= \frac{u}{2} : \frac{u}{2} : \frac{u}{3} = 3 : 3 : 2$$

Option (b) is correct.

14.



$$v_{CM} = v \cos 30°$$
$$= v \frac{\sqrt{3}}{2}$$

Option (a) is correct.

15. $\vec{\mathbf{r}}_{CM} = \hat{\mathbf{i}}$

$\vec{\mathbf{r}}_1$ (position vector of lighter piece)

$$= 3\,\hat{\mathbf{i}} + 2\,\hat{\mathbf{j}} - 4\,\hat{\mathbf{k}}$$

$$\vec{\mathbf{r}}_{CM} = \frac{m_1 \vec{\mathbf{r}}_1 + m_2 \vec{\mathbf{r}}_2}{m_1 + m_2}$$

$$\vec{\mathbf{r}}_2 = \frac{(m_1 + m_2)\vec{\mathbf{r}}_{CM} - m_1 \vec{\mathbf{r}}_1}{m_2}$$

$$= \frac{2\,\hat{\mathbf{i}} - \frac{2}{3}(3\,\hat{\mathbf{i}} + 2\,\hat{\mathbf{j}} - 4\,\hat{\mathbf{k}})}{\frac{4}{3}}$$

$$= \frac{1}{4}[6\,\hat{\mathbf{i}} - 2(3\,\hat{\mathbf{i}} + 2\,\hat{\mathbf{j}} - 4\,\hat{\mathbf{k}})]$$

$$= \frac{1}{4}[-4\,\hat{\mathbf{j}} + 8\,\hat{\mathbf{k}}]$$

$$= -\,\hat{\mathbf{j}} + 2\,\hat{\mathbf{k}}$$

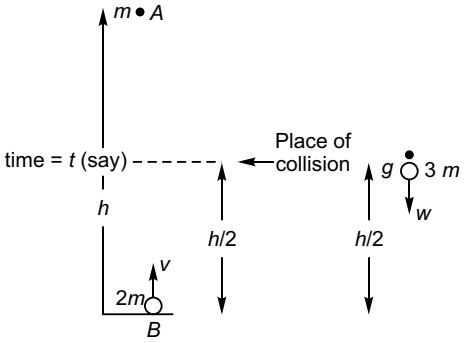
$\therefore$ The heavier part will be at $(0, -1, 2)$.

Option (d) is correct.

16. Motion of A :

$\frac{h}{2} = \frac{1}{2} g t^2$, v_A (at time t) $= g\,t = \sqrt{gh}$

i.e., $$t = \sqrt{\frac{h}{g}}$$



Motion of B :

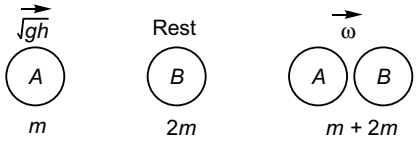
$$\frac{h}{2} = v\frac{\sqrt{h}}{g} - \frac{1}{2} g \frac{h}{g}$$

i.e., $$h = v\sqrt{\frac{h}{g}}$$

$\therefore$ $$v = \sqrt{gh}$$

$$v_B = \sqrt{gh} - g \cdot \sqrt{\frac{h}{g}} = 0$$

Collision of A and B at time t :



$$m\sqrt{gh} = 3m\,\omega$$

$\therefore$ $$\omega = \frac{\sqrt{gh}}{3}$$

Velocity of the combined mass when it reach ground

$$v'^2 = \omega^2 + 2gh$$
$$= \frac{gh}{g} + 2gh$$

i.e., $$v' = \frac{\sqrt{19\,gh}}{3}$$

Option (d) is correct.

17. u = velocity of man w.r.t. cart

Let v = velocity of cart w.r.t. ground

$\therefore$ Velocity of man w.r.t. ground $= u + v$

$$m(u + v) + 2mv = 3m \cdot 0$$

$\therefore$ $$v = -\frac{u}{3}$$

Work done = KE gained by man and cart

$$= \frac{1}{2} m (u + v)^2 + \frac{1}{2} 2mv^2$$

$$= \frac{1}{2} m \left(u - \frac{u}{3}\right)^2 + \frac{1}{2} 2m \left(-\frac{u}{3}\right)^2$$

$$= \frac{1}{2} m \cdot \frac{4u^2}{3} + \frac{1}{2} 2m \frac{u^2}{9}$$

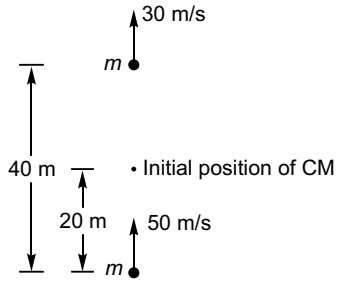
$$= \frac{2mu^2}{3} + \frac{mu^2}{9}$$

$$= \frac{7}{9} mu^2$$

Option (d) is correct.

18. $v_{CM} = \dfrac{30\,m + 50\,m}{m + m}$

$= 40$ m/s upwards.



If the velocity of CM becomes zero at displacements

$$0^2 = 40^2 + 2(-10)\,s$$

$\Rightarrow \quad s = 80$ m

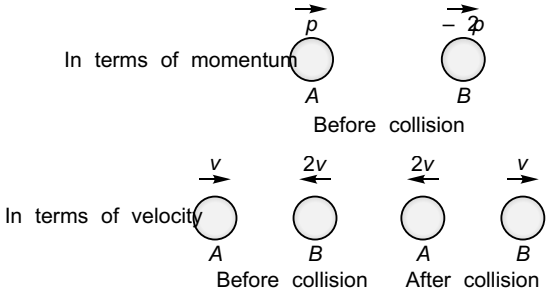
$\therefore$ Maximum height attained by CM

$= 20$ m $+ 80$ m

$= 100$ m

Option (c) is correct.

19. As the masses are equal and the collision is elastic, the particles will exchange their velocities as shown in figure.



Gain in KE of 1st particle

$$= \frac{1}{2} m(-2v)^2 - \frac{1}{2} mu^2$$

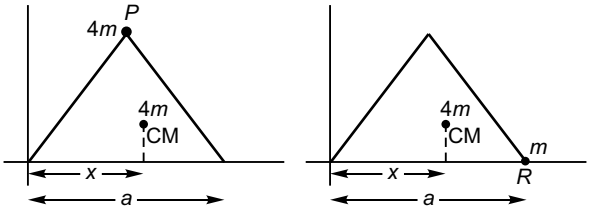
$$= 2mv^2 - \frac{1}{2} mv^2$$

$$= \frac{3}{2} mv^2$$

$$= \frac{3p^2}{2m}$$

Option (c) is correct.

20. According to question



$$\frac{(4m)\,x + (4m)\,\frac{a}{2}}{4m + 4m} = \frac{(4m)\,x + (m)\,a}{4m + m}$$

or $\quad \dfrac{x}{2} + \dfrac{a}{4} = \dfrac{4x + a}{5}$

i.e., $\quad x = \dfrac{a}{6}$

Option (b) is correct.

21. x_{CM} of Fig. 1 will as that of Fig. 2.

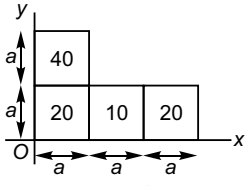


Fig. 1

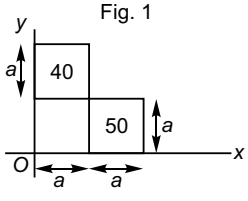


Fig. 2

$$x_{CM} = \frac{40 \cdot \frac{a}{2} + 50 \cdot \frac{3a}{2}}{40 + 50}$$

$$= \frac{20a + 75a}{90}$$

$$= \frac{95a}{90} = \frac{19a}{18}$$

$$y_{CM} = \frac{40 \cdot \frac{3a}{2} + 50 \cdot \frac{a}{2}}{40 + 50} \quad \left[\begin{array}{l}\text{There is no}\\ \text{need to find}\\ \text{the value of } y_{CM}\end{array}\right]$$

$$= \frac{17a}{18}$$

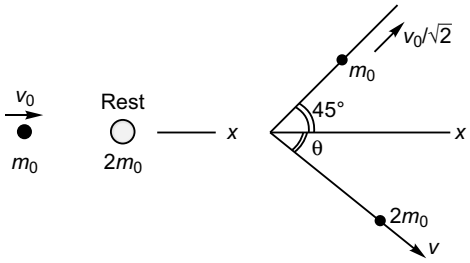
Option (a) is correct.

22. Conservation of momentum along y-axis

$$m_0 \frac{v_0}{\sqrt{2}} \sin 45° = 2 m_0 v \sin\theta$$

i.e., $$2v \sin\theta = \frac{v_0}{2} \quad \ldots(i)$$

Conservation of momentum along x-axis.



$$m_0 \frac{v_0}{\sqrt{2}} \cos 45° + 2 m_0 v \cos\theta = m_0 v_0$$

i.e., $$2v \cos\theta = \frac{v_0}{2} \quad \ldots(ii)$$

Squaring and adding Eqs. (i) and (ii),

$$2v = \frac{v_0}{2}\sqrt{2}$$

$$\therefore \quad v = \frac{v_0}{2\sqrt{2}}$$

Option (b) is correct.

23. $$\frac{1}{2} k x_0^2 = \frac{1}{2} m_2 v_2^2$$

i.e., $$v_2 = x_0 \sqrt{\frac{k}{m_2}}$$

$$v_{CM} = \frac{m_1 \times 0 + m_2 v_2}{m_1 + m_2}$$

(When wall just breaks off the velocity of mass m_1 would be zero)

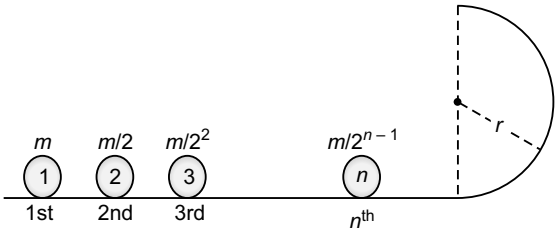
$$= \frac{m_2}{m_1 + m_2} v_2$$

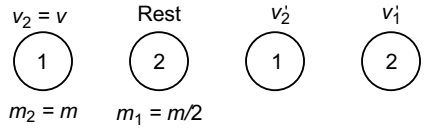
$$= \frac{m_2}{m_1 + m_2} x_0 \sqrt{\frac{k}{m_2}}$$

$$= \frac{x_0}{m_1 + m_2} \sqrt{k m_2}$$

Option (b) is correct.

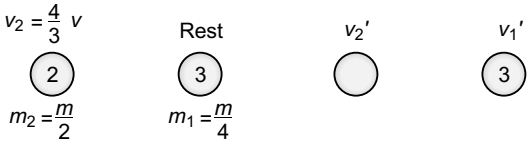
24. Velocity of 2nd ball when 1st with velocity v strikes 2nd at rest





$$v_1' = \frac{2m_2}{m_1 + m_2} v_2 = \frac{2m}{\frac{m}{2} + m} v = \frac{4}{3} v$$

Velocity of 3rd ball when 2nd with velocity $\frac{4}{3} v$ strikes 3rd at rest



$$v_1' = \frac{2m_2}{m_1 + m_2} v_2 = \frac{2 \cdot \frac{m}{2}}{\frac{m}{4} + \frac{m}{2}} \frac{4}{3} v$$

$$= \frac{4}{3} \cdot \frac{4}{3} v$$

$$= \left(\frac{4}{3}\right)^2 v$$

Velocity of 3rd ball $= \left(\frac{4}{3}\right)^{3-1} v$

As in every collision $\frac{m_2}{m_1 + m_2} = \frac{4}{3}$

The velocity of nth ball $= \left(\frac{4}{3}\right)^{n-1} v$

Now, this must be equal to $\sqrt{5gr}$ for it to complete the circle

$$\therefore \quad \left(\frac{4}{3}\right)^{n-1} v = \sqrt{5gr}$$

i.e., $$v = \left(\frac{3}{4}\right)^{n-1} \sqrt{5gr}$$

Option (a) is correct.

25. Impulse given to the block will also release it from abstraction besides giving and then imparting the restoring force on it due to velocity to it 5cm of the spring expansion will accelerate it.

Impulse = 4 kg ms^{-1}

Initial velocity $(u) = \dfrac{4 \text{ kg ms}^{-1}}{2 \text{ kg}} = 2 \text{ ms}^{-1}$

Average Acceleration (a)

$$= \frac{kx}{2m} = \frac{4000 \times \left(\frac{5}{100}\right)}{2 \times 2}$$

$$= 50 \text{ m/s}^2$$

Displacement $(s) = x = \dfrac{5}{100}$ m

$$\therefore \quad v^2 = u^2 + 2as$$

$$= 2^2 + 2 \times 50 \times \frac{5}{100}$$

$$= 4 + 5$$

$$= 9$$

$$\therefore \quad v = 3 \text{ ms}^{-1}$$

Option (b) is correct.

26. **Compression in spring** Velocity gained by block when the spring is at its natural length will compress the spring.

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$$

$$x = v\sqrt{\frac{m}{k}}$$

$$= 3\sqrt{\frac{2}{4000}}$$

$$= \sqrt{\frac{2 \times 9 \times 5}{4000 \times 5}}$$

$$= \sqrt{\frac{45}{10000}}$$

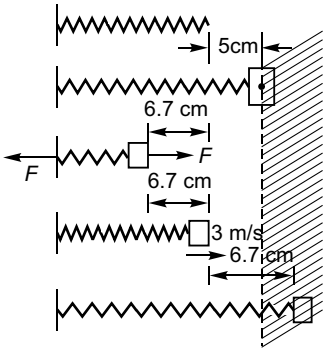
$$= \frac{6.7}{100} \text{ m}$$

$$= 6.7 \text{ cm}$$

Spring at its natural length.

Block at rest first time due to some reason.

Restoring force brings the spring to its natural length and block attains a velocity of 3 ms^{-1}.



KE of the block moves blocks ahead and the spring stretches by 6.7 cm but the block does not return due to same region. Block is now at rest for the second time.

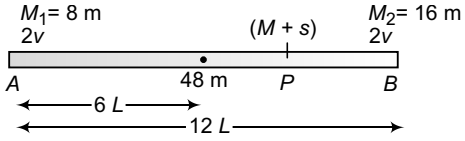
$\therefore$ Distance travelled by block when it comes to rest for the second time

$$= (5 + 6.7 + 6.7 + 6.7) \text{ cm}$$

$$= 25 \text{ cm approx.}$$

Option (b) is correct.

27.



$$8m \times AP = 16m \times (12L - AP)$$

$$AP = 24L - 2AP$$

$$3AP = 24\,L$$

$$AP = 8L$$

As the CM of M and S does not change, the CM of the bar shall also not change *i.e.*, the displacement of bar will be zero.

Let x be the displacement of rod.

$$\therefore \; x \times 8\,m + (x + 6L)\,48\,m + (x + 12L)\,16m$$

$$= (6L \times 48\,m) + (8L \times 24\,m)$$

i.e., $\quad x = 0$ m

i.e., no displacement of bar.

Option (d) is correct.

28. v_{CM} of M and $S = \dfrac{8\,m\,(2v) + 16\,m\,(-v)}{8\,m + 16\,m} = 0$

There CM of M and S will not change while they move *i.e.*,the point P (where they meet) is at the edge of the table supporting the end B.

Option (b) is correct.

29. When the spider eats up the moth and travels towards A with velocity $\frac{v}{2}$ relative to rod.

$$24\,m\left(\frac{v}{2}+v_R\right)+48\,m\,v_R=0$$

[v_R = velocity (absolute) of rod]

$$\Rightarrow \qquad v_R=-\frac{v}{6}$$

Option (c) is correct.

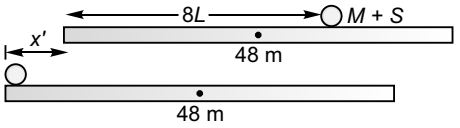
30. Time taken by spider to reach point A starting from point B

$$=\frac{4L}{v}+\frac{8L}{v/2}$$

$$=\frac{20L}{v}=\frac{20L}{L/T}=20T$$

$$=20\times 4=80\text{ s}$$

31. Form CM not to shift



$$\Rightarrow \qquad (x'+8L)\,24\,m+(x'+6L)\,48\,m$$
$$=64\times 48\,m$$

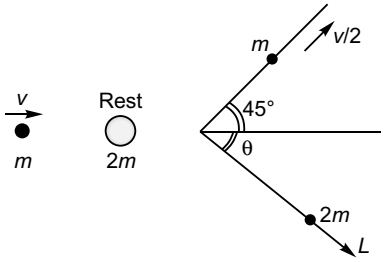
i.e., $x'=-\frac{8L}{3}$

Option (a) is correct.

More than One Correct Options

1. Along vertical : $2\,mV\sin\theta=m\cdot\frac{v}{2}\sin 45°$

i.e., $$2\,V\sin\theta=\frac{v}{2\sqrt{2}} \qquad \text{...(i)}$$



Along horizontal :

$$mv=m\frac{v}{2}\cos 45°+2mV\cos\theta$$

i.e., $$2\,V\cos\theta=v\left(1-\frac{1}{2\sqrt{2}}\right) \qquad \text{...(ii)}$$

Squaring and adding Eq. (i)and (ii),

$$8V^2=\left(\frac{v}{2\sqrt{2}}\right)^2+\left(v-\frac{v}{2\sqrt{2}}\right)^2$$
$$=\frac{v^2}{8}+v^2+\frac{v^2}{7}-2\cdot v\cdot\frac{v}{2\sqrt{2}}$$
$$=\frac{5v^2}{4}-\frac{v^2}{\sqrt{2}}$$

Dividing Eq. (ii) by Eq. (i)

$$\tan\theta=\frac{\frac{1}{2\sqrt{2}}}{\frac{2\sqrt{2}-1}{2\sqrt{2}}}$$

$$=\frac{1}{2\sqrt{2}-1}<1$$

$\therefore \qquad \theta<45°$

Thus, the divergence angle between the particles will be less than $\frac{\pi}{2}$.

Option (b) is correct.

Initial KE $=\frac{1}{2}mv^2$

Final KE $=\frac{1}{2}m\left(\frac{v}{2}\right)^2+\frac{1}{2}2\,mV^2$

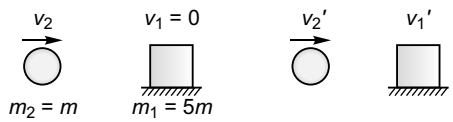
$$=\frac{1}{2}mv^2\left[\frac{1}{4}+2\cdot\frac{5}{32}-2\cdot\frac{1}{8\sqrt{2}}\right]$$

As Final KE < Initial KE

Collision is inelastic.

Option (d) is correct.

2. $v_2'=\frac{m_2-m_1}{m_1+m_2}v_2$



$$=\frac{m-5m}{5m+m}v_2$$
$$=-\frac{2}{3}v_2$$
$$=-\frac{2}{3}\sqrt{2g\,l}$$

$$T - \frac{mv_2'^2}{l} = mg$$

or $$T = \frac{mv_2'}{l} + mg$$

$$= \frac{m\,8g}{9} + mg$$

$$= \frac{17\,mg}{9}$$

Option (a) is correct.

Velocity of block

$$v_1' = \frac{2m_2}{m_1 + m_2} v_2$$

$$= \frac{2m}{5m + m} \cdot \sqrt{2g\,l}$$

$$= \frac{1}{3}\sqrt{2g\,l}$$

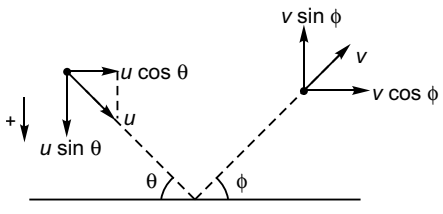
Option (c) is correct.

Maximum height attained by pendulum bob

$$= \frac{v_2'^2}{2g} = \frac{8g\,l/9}{2g} = \frac{4\,l}{g}$$

Option (d) is correct.

3. $v\cos\phi = u\cos\theta \Rightarrow \frac{v}{u} = \frac{\cos\theta}{\cos\phi}$



and $$e = \frac{v\sin\phi}{u\sin\theta}$$

$$= \frac{\cos\theta}{\cos\phi} \times \frac{\sin\phi}{\sin\theta}$$

$$= \frac{\tan\phi}{\tan\theta}$$

Option (b) is correct.

Change in momentum of particle

$$= (-\,mv\sin\phi) - (+\,mu\sin\theta)$$

$\therefore$ Impulse delivered by floor to the particle

$$= mv\sin\phi + mu\sin\theta$$

$$= mv\sin\theta\left[\frac{\sin\phi}{\sin\theta} + \frac{u}{v}\right]$$

$$= mv\sin\theta\left[\frac{u}{v}e + \frac{u}{v}\right]$$

$$= mu\sin\theta(1 + e)$$

Option (d) is correct.

$$u\sqrt{1 - (1 - e^2)\,\sin^2\theta}$$

$$= u\sqrt{1 - \sin^2\theta + e^2\sin^2\theta}$$

$$= u\sqrt{\cos^2\theta + e^2\sin^2\theta}$$

$$= u\sqrt{\cos^2\theta + \frac{v^2}{u^2}\sin^2\phi}$$

$$= \sqrt{u^2\cos^2\theta + v^2\sin^2\phi}$$

$$= \sqrt{v^2\cos^2\phi + v^2\sin^2\phi}$$

$$= v$$

Option (c) is correct.

$$\cos^2\theta + e^2\sin^2\theta$$

$$= \cos^2\theta + \frac{\tan^2\phi}{\tan^2\theta}\sin^2\theta$$

$$= \cos^2\theta(1 + \tan^2\phi)$$

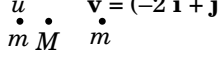
$$= \cos^2\theta\,\sec^2\phi$$

$$= \frac{\cos^2\theta}{\cos^2\phi}$$

$$= \frac{v^2}{u^2} = \frac{\text{Final KE}}{\text{Initial KE}}$$

Option (d) is correct.

4. $\vec{\mathbf{u}} = (3\,\hat{\mathbf{i}} + 2\,\hat{\mathbf{j}})\ \text{ms}^{-1}$



Impulse received by particle of mass m

$$= -\,m\,\vec{\mathbf{u}} + m\,\vec{\mathbf{v}}$$

$$= -\,m\,(3\,\hat{\mathbf{i}} + 2\,\hat{\mathbf{j}}) + m(-\,2\,\hat{\mathbf{i}} + \hat{\mathbf{j}})$$

$$= -\,m\,(5\,\hat{\mathbf{i}} + \hat{\mathbf{j}})\ \text{unit}$$

Option (b) is correct.

Impulse received by particle of mass M

= –(impulse received by particle of mass m)

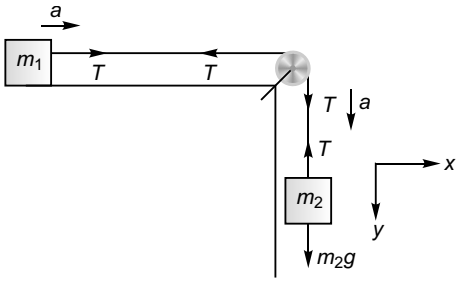
$$= m\,(5\,\hat{\mathbf{i}} + \hat{\mathbf{j}})$$

Option (d) is correct.

5. $T = m_1 a$

and $\quad m_2 g - T = m_2 a$

Solving, $\quad a = \dfrac{m_2}{m_1 + m_2} g$



$$(a_{CM})_x = \frac{m_1 a + m_2 0}{m_1 + m_2}$$
$$= \frac{m_1}{m_1 + m_2} a$$
$$= \frac{m_1 m_2}{(m_1 + m_2)^2} g$$

Option (b) is correct.

$$(a_{CM})_y = \frac{m_1 0 + m_2 a}{(m_1 + m_2)}$$
$$= \frac{m_2}{m_1 + m_2} a$$
$$= \left(\frac{m}{m_1 + m_2}\right)^2 g$$

Option (c) is correct.

6. As the block comes down, the CM of the system will also come down *i.e.*, it does not remain stationary.

$$a_{CM} = \frac{mg}{m + M} \neq g$$

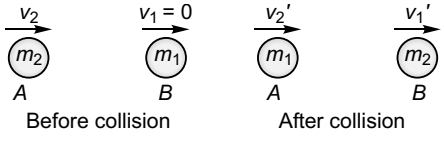
a_{CM} is downwards and also $a_{CM} < g$.

Option (d) is correct.

As no force acts along horizontal direction, the momentum of the system will remain conserved along horizontal direction.

Option (c) is correct.

7. Velocity of B after collision :



$$v_1' = \left(\frac{1+e}{2}\right) v_2$$
$$= \frac{3}{4} v \text{ [as } e = \frac{1}{2} \text{ and } v_2 = v \text{ (given)]}$$
$$\neq \frac{v}{2}$$

Impulse given by A to B

$$= \text{ change in momentum of } B$$
$$= m\left(\frac{3}{4} v\right) - m \cdot 0$$
$$= \frac{3}{4} mv$$

Option (b) is correct.

Velocity of A after collision

$$v_2' = \left(\frac{1-e}{2}\right) v_2$$
$$= \frac{v}{4}$$

Loss of KE during collision

$$= \frac{1}{2} m v_2^2 - \frac{1}{2} m (v_1'^2 + v_2'^2)$$
$$= \frac{1}{2} m \left[v^2 - \left(\frac{v}{4}\right)^2 - \left(\frac{3v}{4}\right)^2\right]$$
$$= \frac{3}{16} mv^2$$

Option (c) is correct.

8. As the mass of the system keeps on decreasing momentum of the system does not remain constant.

Thrust force is developed on the rocket due to Newton's 3rd law of motion.

Option (b) is correct.

As, $\quad a = \dfrac{dv}{dt} = \dfrac{v_i}{m}\left(-\dfrac{dm}{dt}\right) - g$

The value of a will remain constant if v_i and $-\dfrac{dm}{dt}$ are constant.

Option (c) is correct.

$F_{net} = F_t$ (Thrust force due to gas ejection) $- W$ (weight of rocket)

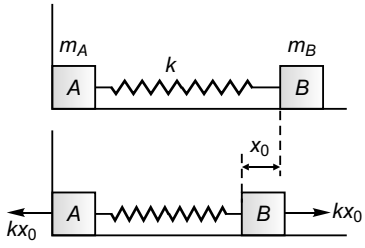
$$a = \frac{F_{net}}{m}$$

Thus, Newton's 2nd law is applied.

Option (d) is correct.

Match the Columns

1. If x_0 is the compression made in the spring, the restoring force on B will decrease from kx_0 to zero as the spring regains its original length. Thus, the acceleration of B will also decrease from $\frac{kx_0}{m_B}$ to zero.



So, the a_{CM} will also decrease from $\frac{kx_0}{m_A + m_B}$ to zero.

$\therefore$ (a) $\rightarrow$ (r)

When spring is released after compressing it, the restoring on B will accelerate it towards right while the reaction force on A will apply a force on the wall which in turn will apply equal and opposite force on A and consequently A will travel towards right. As both travel towards right the velocity of CM will be maximum in the beginning.

After this A will start compressing the spring and at a certain instant when the spring is compressed to maximum value both the blocks will travel towards right with a constant velocity and then the velocity of CM will become constant.

$\therefore$ (b) $\rightarrow$ (q)

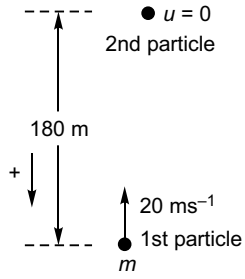
As the blocks will never move along y-axis, the y-component of the CM of the two blocks will not change.

$\therefore$ (d) $\rightarrow$ (p)

As the two blocks will keep on moving towards right (surface below being smooth) the x-coordinate of the CM of the blocks will keep on increasing.

$\therefore$ (c) $\rightarrow$ (s)

2. Initial $a_{CM} = \frac{m(+g) + m(+g)}{m + m}$



$$= +g$$
$$= +10 \text{ SI unit}$$

$\therefore$ (a) $\rightarrow$ (q)

Initial $v_{CM} = \frac{m(-20) + m \times 0}{m + m}$

$$= -10$$

$\therefore \quad |v_{CM}| = 10$ SI unit

$\therefore$ (b) $\rightarrow$ (q)

For the time taken by the first particle to return to ground

$$s = ut + \frac{1}{2}at^2$$
$$0 = (-20)t + 5t^2$$
$$\Rightarrow \quad t = 4 \text{ s}$$

Now, as the collision of the first particle with the ground is perfectly inelastic, the first particle will remain on ground at rest.

Now, let us find the position of 2nd particle at $t = 5$ s

$$s = (0)5 + \frac{1}{2}(10)5^2$$
$$= 125 \text{ m}$$

The particle (2nd) will still be in space moving downwards.

$$a_{CM} = \frac{m \cdot 0 + m \cdot g}{m + m}$$
$$= \frac{g}{2} = 5 \qquad \text{(SI unit)}$$

$\therefore$ (c) $\rightarrow$ (p)

Velocity of 2nd particle at $t = 5$ s

$$v = 0 + 10 \times 5$$
$$= 50 \text{ ms}^{-1}$$

$\therefore$ At $t = 5$ s

$$v_{CM} = \frac{m \cdot 0 + m \cdot 50}{m + m}$$

$$= 25 \qquad \text{(SI unit)}$$

$\therefore$ (d) $\rightarrow$ (s)

3. Initial KE of block $B = 4$ J

$$\therefore \qquad \frac{1}{2} \times 0.5 \times u^2 = 4$$

$$\Rightarrow \qquad u = 4 \text{ ms}^{-1}$$

$\therefore$ Initial momentum of $B = 0.5 \times 4$

$$= 2 \text{ kg ms}^{-1}$$

$\therefore$ (a) $\rightarrow$ (r)

Initial momentum

$$p_{CM} = p_A + p_B$$
$$= 0 + 2$$
$$= 2 \text{ kg ms}^{-1}$$

$\therefore$ (b) $\rightarrow$ (r)

Velocity given to block B will compress the spring and this will gradually increase the velocity of A. When the spring gets compressed to its maximum both the blocks will have the same velocities *i.e.*, same momentum as both have same mass.

$$p_A = p_B$$

(at maximum compression of the spring)

But, $p_A + p_B$ = initial momentum of B.

$$\therefore \qquad p_A + p_A = 2$$

i.e., $\qquad p_A = 1 \text{ kgms}^{-1}$

$\therefore$ (c) $\rightarrow$ (q)

After the maximum compression in the spring, the spring will gradually expand but now the velocity of block A will increase and that of B will decrease and when the spring attains maximum expansion the velocity of B will be zero and so will be its momentum.

$\therefore$ (d) $\rightarrow$ (p)

4. If collision is elastic, the two blocks will interchange there velocities (mass of both balls being equal).

Thus, velocity of A after collision $= v$

$\therefore$ (a) $\rightarrow$ (r)

If collision is perfectly inelastic, the two balls will move together (with velocities V).

$$\therefore \qquad mv = (m + m)V$$

$$\Rightarrow \qquad V = \frac{v}{2}$$

$\therefore$ (b) $\rightarrow$ (s)

If collision is inelastic with $e = \frac{1}{2}$,

$$v_1' = \frac{1+e}{2} \cdot v_2$$

$$= \frac{1 + \frac{1}{2}}{2} \cdot v \quad [\because v_2 = v \text{ (given)}]$$

$$= \frac{3}{4} v$$

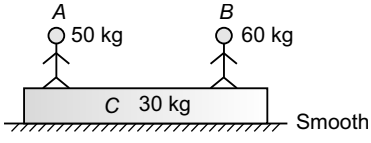
$\therefore$ (c) $\rightarrow$ (p)

If collision is inelastic with $e = \frac{1}{4}$,

$$v_1' = \frac{\left(1 + \frac{1}{4}\right)}{2} v = \frac{5}{8} v$$

$\therefore$ (d) $\rightarrow$ (q).

5. If A moves x towards right



Let plank (along with B) move by x' to the right.

$$\therefore \qquad \frac{x \times 30 + x'(60 + 30)}{30 + (60 + 30)} = 0$$

i.e., $\qquad x' = -\frac{x}{3}$

$$= \frac{x}{3}, \text{ towards left.}$$

$\therefore$ (a) $\rightarrow$ (r)

If B moves x towards left

Let plank (along with A) move x' to the left

$$\therefore \qquad \frac{x \cdot 60 + x'(30 + 30)}{60 + (30 + 30)} = 0$$

i.e., $\qquad x' = -x$

$$= x, \text{ towards right}$$

$\therefore$ (b) $\rightarrow$ (p)

If A moves x towards right and B moves x towards left.

Let plank moves x' towards right

$$\therefore \quad \frac{30 \times x + 60(-x) + 30(x')}{30 + (60) + (30)} = 0$$

i.e., $x' = x$

$= x$, towards right

$\therefore$ (c) $\rightarrow$ (p)

If A and B both move x towards right.

Let plank moves x' towards right

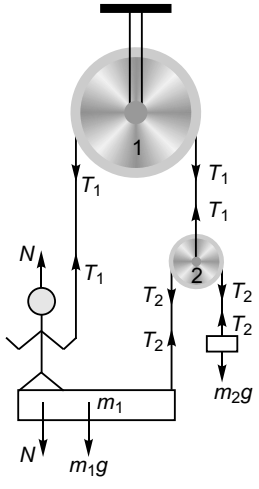
$$\therefore \quad \frac{(30+60)x + 30x'}{(30+60)+30} = 0$$

i.e., $x' = -3x$

$= 3x$, towards right

$\therefore$ (d) $\rightarrow$ (s)

6. For man to be in equilibrium



$$N + T_1 = W \quad \ldots(i)$$

For the block of mass m_1 to be in quilibrium

$$T_2 = N + m_1 g \quad \ldots(ii)$$

For the block of mass m_2 to be in quilibrium

$$T_2 = m_2 g \quad \ldots(iii)$$

For the equilibrium of pulley 2

$$T_1 = 2T_2 \quad \ldots(iv)$$

Solving Eqs. (i), (ii), (iii) and (iv)

$$W = 3m_2 g - m_1 g$$
$$= (3m_2 - m_1)\, g$$
$$= (60 - 10)\, 10$$
$$= 500 \text{ N}$$

$\therefore$ (a) $\rightarrow$ (r)

For the equilibrium of man

$$N = W - T_1$$
$$= W - 2T_2$$
$$= W - 2(N + m_1 g)$$

i.e., $3N = W - 2m_1 g$

$$N = 100 \text{ N}$$

$\therefore$ (d) $\rightarrow$ (s)

Force exerted by man on string to accelerate the centre of mass of the system upwards

Centre of mass of the system will move upwards if man move upward.

i.e., when $T_1 > W - N$

$> 500 - 10$

$> 400 \text{ N}$

Options are 500 N and 600 N.

$\therefore$ (b) $\rightarrow$ (r) and (s).

Force (T_1) exerted by man on string to accelerate the centre of mass of the system.

Centre of mass of the system will move downward if man moves downward.

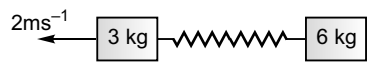
i.e., when $T_1 < W - N$

$< 400 \text{ N}$

Options are 100 N and 150 N.

$\therefore$ (c) $\rightarrow$ (q).

7. $v_{CM} = \dfrac{2 \times 3 + 0 \times 6}{3 + 6}$



$$= \frac{2}{3} \text{ ms}^{-1}$$

When both the blocks move with same velocity (say v) deformation in the spring will be maximum

$$2 \times 3 = 3v + 6v$$

i.e., $v = \dfrac{2}{3} \text{ ms}^{-1}$

$=$ velocity of A

$=$ velocity of B

$$v_{CM} = \frac{3 \times \frac{2}{3} + 6 \times \frac{2}{3}}{3+6}$$

$$= \frac{2}{3}\,\text{ms}^{-1}$$

When both the blocks move with same velocities, each will be at rest w.r.t. the other

$\therefore$ (a) $\rightarrow$ (p), (r), (s)

(b) $\rightarrow$ (p), (r), (s)

Minimum speed of 3 kg block will be $\frac{2}{3}\,\text{ms}^{-1}$ and at that moment velocity of CM will be $\frac{2}{3}\,\text{ms}^{-1}$.

$\therefore$ (c) $\rightarrow$ (p)

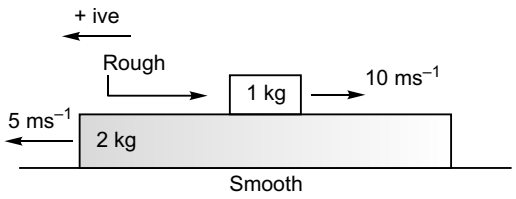
Initial velocity of 6 kg block is zero.

When the spring is compressed to maximum value the velocity of 6 kg will be maximum and the velocity of CM will be $\frac{2}{3}\,\text{ms}^{-1}$ (as explained above).

$\therefore$ (c) $\rightarrow$ (p).

8. $v_{CM} = \frac{2 \times 5 + 1(-10)}{2+1}$

$= 0\,\text{ms}^{-1}$



$\therefore$ (a) $\rightarrow$ (r)

Momentum of CM $= 2(+5) + 1(-10)$

$= 0\,\text{kg ms}^{-1}$

$\therefore$ (b) $\rightarrow$ (r)

Velocity and so the momentum of 1 kg block will decrease to zero as the surface below is rough.

$\therefore$ (c) $\rightarrow$ (q)

Velocity and so the KE of 2 kg block will decrease to zero when the velocity of 1 kg block becomes zero (according to law of conservation of momentum).

$\therefore$ (d) $\rightarrow$ (q).

Solutions
of
Mechanics Part-II

Lesson 9th to 13th

By DC Pandey

9 Mechanics of Rotational Motion

Introductory Exercise 9.1

1. The mass distribution is at minimum separation from the diagonal passing through centre to opposite corners of the cube, so, moment of inertia is minimum about that axis.

2. $I_1 = \dfrac{ml^2}{12}$

while $I_2 = mr^2 = m\left(\dfrac{l}{2\pi}\right)^2$

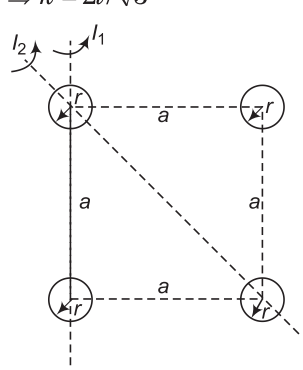
$$= \frac{ml^2}{4\pi^2}$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{4\pi^2}{12} = \frac{\pi^2}{3}$$

3. $I = \dfrac{m\,(2l)^2}{3} = \dfrac{4ml^2}{3} = mk^2$

$\Rightarrow k = 2l/\sqrt{3}$

4.



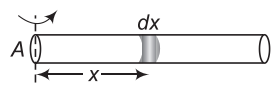
(i) $I_1 = 2 \times \dfrac{2}{5}mr^2 + 2\left(\dfrac{2}{5}mr^2 + ma^2\right)$

$$= \frac{8}{5}mr^2 + 2ma^2$$

(ii) $I_2 = 2 \times \dfrac{2}{5}mr^2 + 2\left(\dfrac{2}{5}mr^2 + m\left(\dfrac{a}{\sqrt{2}}\right)^2\right)$

$$= \frac{8}{5}mr^2 + ma^2$$

5.



(a) $I = \int dI = \int_0^{2l} dmx^2 = m\int_0^{2l} x^3 dx$

$$= \frac{1}{4}\,m(2l)^4 = 4\,ml^4$$

$$M = \int_0^{2l} mx\,dx = \frac{1}{2}\,m\,(2l)^2 = 2ml^2$$

$$\Rightarrow m = \frac{M}{2l^2}$$

$$\therefore \quad I = 4 \cdot \frac{M}{2l^2} \cdot l^4 = 2Ml^2$$

(b) $X_{\text{CM}} = \dfrac{\int xdm}{\int dm} = \dfrac{\int_0^{2l} x\,mx\,dx}{\int_0^{2l} mx\,dx} = \left.\dfrac{\frac{1}{3}x^3}{\frac{1}{2}x^2}\right|_0^{2l}$

$$= \frac{2}{3}x\Big|_0^{2l} = \frac{4}{3}l$$

$$\therefore I_{\text{CM}} = I_A - M\left(\frac{4}{3}l\right)^2$$

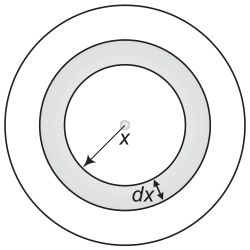
$$= 2Ml^2 - \frac{16}{9}Ml^2 = \frac{2}{9}Ml^2$$

$$\therefore \quad I_B = I_{\text{CM}} + M\left(\frac{1}{3}l\right)^2$$

$$= \frac{2}{9}Ml^2 + \frac{1}{9}Ml^2$$

$$= \frac{3}{9}Ml^2 = \frac{1}{3}Ml^2$$

6.



$$I = \int dI = \int dmx^2 = \int_0^a kx^2 \cdot 2\pi x\, dx \cdot x^2$$

$$= 2\pi k \int_0^a x^5\, dx = \frac{\pi k}{3} a^6$$

$$M = \int dm = \int kx^2 \cdot 2\pi x\, dx = 2\pi k \int_0^a x^3 dx$$

$$= \frac{\pi k}{2} a^4 \Rightarrow k = \frac{2M}{\pi a^4}$$

$$\therefore I = \frac{\pi}{3} \cdot \frac{2M}{\pi a^4} a^6 = \frac{2}{3} Ma^2$$

7. $I' = \frac{1}{2} Mr^2 - \frac{1}{2} \cdot \frac{M}{6} r^2 = \frac{5}{6} \cdot \frac{1}{2} Mr^2$

$$= \frac{5}{6} \times 0.6 \text{ kg-m}^2 = 0.5 \text{ kg-m}^2$$

8. $I = (1 \times 49^2 + 2 \times 48^2 + 3 \times 47^2 + \ldots + 49 \times 1^2)$

$$+ 50 \times 0^2 + (100 \times 50^2 + 99 \times 49^2 + 98 \times 48^2 + \ldots + 51 \times 1^2)$$

$$= 100 \times 50^2 + 100 \times 49^2 + \ldots + 100 \times 1^2$$

$$= 100(1^2 + 2^2 + 3^2 + \ldots + 50^2)$$

$$= 100 \times \frac{50\,(50+1)\,(100+1)}{6}$$

$$= \frac{50 \times 51 \times 100 \times 101}{6} \text{ g-cm}^2$$

$$= \frac{0.5 \times 5.1 \times 1.01}{6} \text{ kg-m}^2$$

$$= 0.43 \text{ kg-m}^2$$

9. $I = \frac{1}{2} MR^2 + Mx^2 = MR^2$

$$\Rightarrow \quad x^2 = \frac{R^2}{2}$$

$$\Rightarrow \quad x = \frac{R}{\sqrt{2}}$$

10. The disk with smaller density will have larger radius and as $I = \frac{1}{2} Mr^2$, so the disk with larger radius will have higher moment of inertia.

Introductory Exercise 9.2

1. $\omega_2^2 = \omega_1^2 + 2\,\alpha\theta$

$$\Rightarrow \theta = \frac{\omega_2^2 - \omega_1^2}{2\alpha}$$

$$= \frac{15^2 - 5^2}{2 \times 1} = \frac{5^2 \times 8}{2} = 100 \text{ rad}$$

2. $\theta = \theta_1 + \theta_2 + \theta_3 = \left(\frac{1}{2}\alpha t^2\right) + (\alpha t \cdot t)$

$$+ \left(\alpha t \cdot t - \frac{1}{2}\alpha t^2\right) = 2\alpha t^2$$

$$= 2 \times 4 \times 10^2 = 800 \text{ rad}$$

3. $0 = \omega - \alpha t \Rightarrow \alpha = \frac{\omega}{t} = \frac{10}{10} = 1 \text{ rad/s}^2$

$$\Rightarrow \tau = I\,\alpha = 5 \text{ kg-m}^2 \times 1 \text{ rad/s}^2$$

$$= 5 \text{ N-m}$$

4. $0^2 = \omega^2 - 2\alpha\theta \Rightarrow \alpha = \frac{\omega^2}{2\theta}$

$$= \frac{\left(100 \times \frac{2\pi}{60}\right)^2}{2 \times 10 \times 2\pi}$$

$$= \frac{100\pi^2}{9 \times 40\pi}$$

$$= \frac{5\pi}{18} \text{ rad/s}^2$$

$$F = \frac{\tau}{r} = \frac{I\alpha}{r} = \frac{\frac{1}{2} mr^2 \alpha}{r}$$

$$= \frac{1}{2} mr\alpha = \frac{1}{2} \times 10 \times 0.2 \times \frac{5\pi}{18}$$

$$= 0.87 \text{ N}$$

5. $\theta = 6t - 2t^3$, $\omega = 6 - 6t^2$

$$\Rightarrow \omega = 0 \text{ at } t = 1 \text{ s}$$

(a) $\omega_{av} = \frac{\int_0^1 \omega dt}{\int_0^1 dt} = \int_0^1 (6 - 6t^2)\, dt$

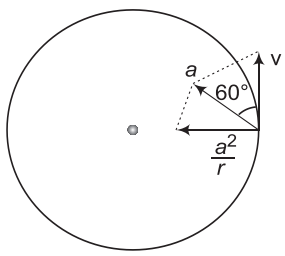
$$= 6t - 2t^2\Big|_0^1 = 4 \text{ rad/s}$$

$$\alpha_{av} = \frac{\int_0^1 \alpha dt}{\int_0^1 dt} = \int_0^1 -12\, t\, dt$$

$$= -12 \times \frac{1}{2} t^2\Big|_0^1 = -6 \text{ rad/s}^2$$

(b) $\alpha\,(1s) = -12t = -12 \text{ rad/s}^2$

6.



$$\tan 60° = \frac{v^2/r}{r\alpha} = \frac{v^2}{r^2\alpha} = \frac{\omega^2}{\alpha}$$

$$d\omega = \alpha dt$$

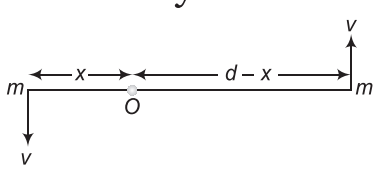
$$\Rightarrow \quad \omega = \int 2 \times 10^{-2}\, t\, dt = 10^{-2} t^2$$

$$\therefore \tan 60° = \frac{10^{-4}\ t^4}{2 \times 10^{-2}\, t} = \sqrt{3}$$

$$\Rightarrow t^3 = 200\sqrt{3} \Rightarrow t = 7 \text{ s}$$

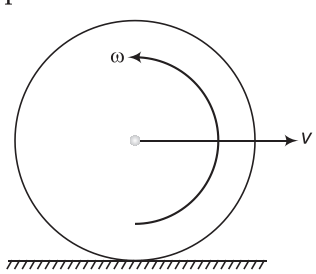
Introductory Exercise 9.3

1.



$L_0 = mvx + mv\,(d - x) = mvd$ is a constant and is independent of x, *i.e.*, position of O.

2.



$$\vec{\mathbf{L}} = \vec{\mathbf{L}}_{CM} + m\,(\vec{\mathbf{r}}_0 \times \vec{\mathbf{v}}_0)$$

$$= I\omega + (-\,mvR)$$

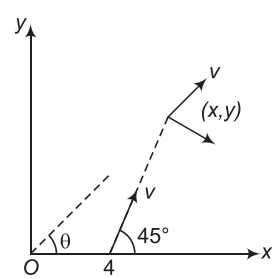
$$= \frac{1}{2} mR^2 \frac{v}{R} - mvR$$

$$= \frac{1}{2} mvR - mvR = -\frac{1}{2} mvR$$

$$\Rightarrow L = \frac{1}{2}\, mvR$$

3. $\vec{\mathbf{L}} = m\,(\vec{\mathbf{r}} \times \vec{\mathbf{v}}) = m\,(x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}})$

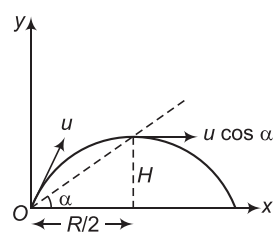
$$\times\,(v \cos 45°\,\hat{\mathbf{i}} + v \sin 45°\,\hat{\mathbf{j}})$$



$$= m\left(\frac{vx}{\sqrt{2}}\,\hat{\mathbf{k}} - \frac{vy}{\sqrt{2}}\,\hat{\mathbf{k}}\right) = \frac{mv}{\sqrt{2}}\,(x - y)\,\hat{\mathbf{k}}$$

$$= \frac{mv}{\sqrt{2}}\,(x - x + 4)\,\hat{\mathbf{k}} = 2\sqrt{2}\, mv\,\hat{\mathbf{k}}$$

4. $\vec{\mathbf{L}} = m\left(\frac{R}{2}\,\hat{\mathbf{i}} + H\,\hat{\mathbf{j}}\right) \times (u \cos\alpha\,\hat{\mathbf{i}})$



$$= -\,mHu \cos\alpha\,\hat{\mathbf{k}}$$

$$= -\,m \cdot \frac{u^2 \sin^2\alpha}{2g} \times u \cos\alpha\,\hat{\mathbf{k}}$$

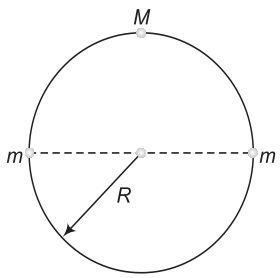
$$= -\,\frac{mu^3 \cos\alpha \sin^2\alpha}{2g}\,\hat{\mathbf{k}}$$

$$\therefore \qquad L = \frac{mu^3 \cos\alpha \sin^2\alpha}{2g}$$

5. It depends upon the distance, velocity and angle from the axis, which may not given zero from a different point.

Introductory Exercise 9.4

1.



$$I_1\omega_1 = I_2\omega_2 \quad \Rightarrow \quad \omega_2 = \frac{I_1\omega_1}{I_2}$$

$$= \frac{MR^2\omega_0}{MR^2 + 2mR^2} = \frac{M\omega_0}{M + 2m}$$

2. As ice melts at pole and flows to equator, it gets distributed away from centre (as equitorial radius is greater than polar radius) such that moment of inertia about axis increases which results in decrease in angular speed (as $I\omega =$ constant), this leads in increase in time period $\left(\text{as } T = \frac{2\pi}{\omega}\right)$ or duration of day and night.

3. True : (Reference previous problem).

Introductory Exercise 9.5

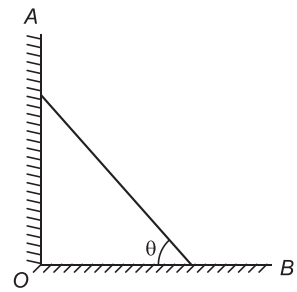
1. $x = \frac{1}{2}a_0t^2 \Rightarrow t = \sqrt{\frac{2x}{a_0}}$

$$\therefore\ y = \frac{v}{\omega} = \frac{a_0t}{\omega} = \frac{a_0}{\omega}\sqrt{\frac{2x}{a_0}} = \frac{\sqrt{2xa_0}}{\omega}$$

$$\Rightarrow y^2 = \frac{2a_0}{\omega^2}x$$

2. $mgh = \frac{1}{2}I\,\omega^2$

$$mg\,\frac{l}{2}(1 - \sin\theta) = \frac{1}{2}\cdot\frac{ml^2}{3}\cdot\omega^2$$



$$\omega = \sqrt{\frac{3g}{l}(1 - \sin\theta)}$$

Introductory Exercise 9.6

1. $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

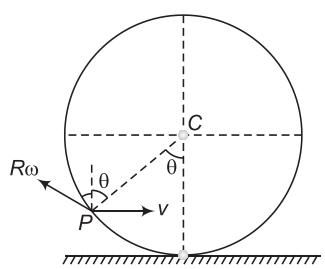
$$= \frac{1}{2}mv^2 + \frac{1}{2}\times\frac{2}{5}mv^2$$

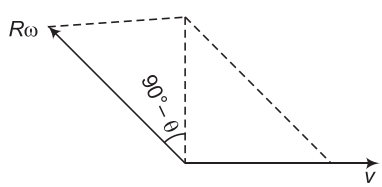
$$= \frac{7}{10}mv^2 = \frac{7}{5}\cdot\frac{1}{2}mv^2$$

$\therefore$ Rotational KE $= mgh - \frac{1}{2}mv^2$

$$= mgh - \frac{5}{7}mgh = \frac{2}{7}mgh$$

2.





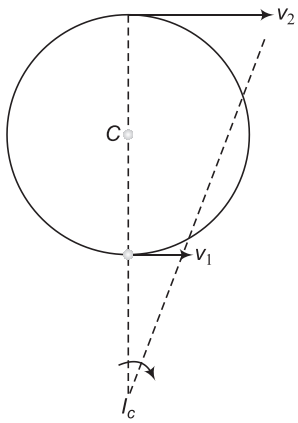
$$\tan(90° - \theta) = \frac{v}{R\omega\cos(90° - \theta)}$$

$$\sin(90° - \theta) = \frac{v}{R\omega}$$

$$\cos\theta = \frac{v}{R\omega}$$

$$\theta = \pm\cos^{-1}\left(\frac{v}{R\omega}\right)$$

3.



$\omega x = v_1$ and $\omega(x + 2R) = v_2$

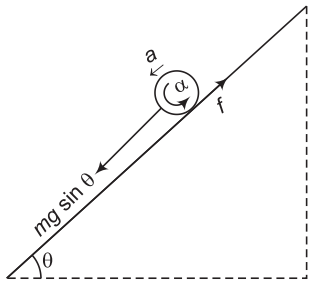
$$\therefore \quad \omega\left[\frac{v_1}{\omega} + 2R\right] = v_2$$

$$2R\omega = v_2 - v_1$$

$$\omega = \frac{v_2 - v_1}{2R}$$

Introductory Exercise 9.7

1.



$f = f_2 = \mu N = \mu mg \cos\theta$

(a) $ma = mg \sin\theta - \mu\, mg \cos\theta$

$a = g(\sin\theta - \mu\cos\theta)$

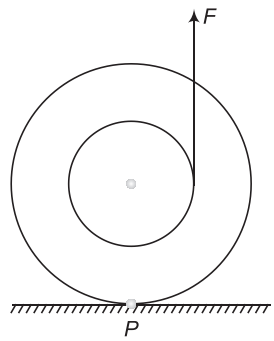
(b) $Rf = I\alpha$

$$\mu Rmg\cos\theta = \frac{2}{5}mR^2\alpha$$

$$\therefore \quad \alpha = \frac{5\mu g\cos\theta}{2R}$$

2. Work done by friction is zero only in uniform pure rolling but not in accelerated pure rolling. So, the statement is false.

3.



As the torque due to applied force is anti-clockwise, so the point of contact tries to slip rightwards and friction tries to prevent it and so acts leftwards.

4. As during rolling down, friction acts upward which exerts a torque, thus the angular momentum of the system is not conserved. Even if we take axis of rotation at point of contact, then component of weight exerts torque, so, statement is false.

5. Force of friction will be zero, only when there is uniform pure rolling, *i.e.*, there is no external unbalanced torque.

Thus, $a = R\alpha$

$$a = \frac{F_1 + F_2}{M} \quad \text{and} \quad \alpha = \frac{2rF_1 - rF_2}{I}$$

as, $a = 2r\alpha$

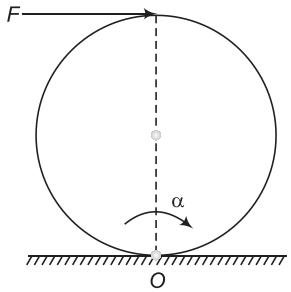
$$\therefore \quad \frac{F_1 + F_2}{M} = \frac{2r^2}{I}(2F_1 - F_2)$$

$$\Rightarrow \quad F_1\left(1 - \frac{4Mr^2}{I}\right) = -F_2\left(1 + \frac{2Mr^2}{I}\right)$$

$$\therefore \quad \frac{F_1}{F_2} = \frac{1 + \dfrac{2Mr^2}{I}}{\dfrac{4Mr}{I} - 1}$$

$$= \frac{2Mr^2 + I}{4Mr^2 - I}$$

6.



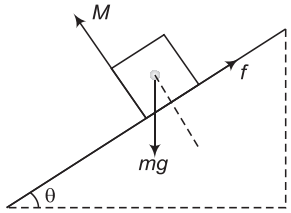
$\tau = 2RF = I\alpha \Rightarrow \alpha = \frac{2R}{I}F$; $\alpha \geq 0$ for $F \geq 0$

i.e., even a slightest amount of force can initiate motion of the disk.

7. Acceleration of point of contact is zero only when there is uniform pure rolling and no slipping.

Introductory Exercise 9.8

1.



For sliding,

$mg \sin\theta > \mu\, mg \cos\theta$

$\therefore \tan\theta > \mu$

And for toppling,

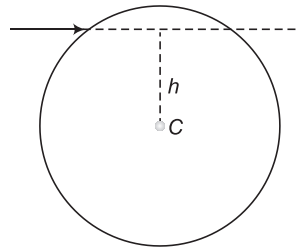
$mg \sin\theta \frac{a}{2} > mg \cos\theta \frac{a}{2}$

$\therefore \tan\theta > 1$

(a) Cube will slide before toppling if $\tan\theta < 1$, *i.e.*, $\mu < 1$ and

(b) Cube will topple before sliding if $\tan\theta > 1$, *i.e.*, $\mu > 1$

2.



For rolling without slipping on horizontal plane, $f = 0$

Impulse, $I_m = \Delta p = mv$

Angular Impulse, $I_m h = \Delta L$

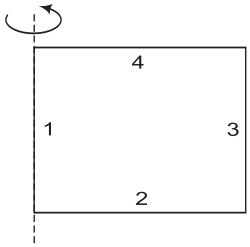
$$= I\omega = \frac{2}{5}mR^2\omega = \frac{2}{5}mRv$$

$$\therefore \quad mvh = \frac{2}{5}mRv \quad \Rightarrow \quad h = \frac{2R}{5}$$

AIEEE Corner

Subjective Questions (Level 1)

1.

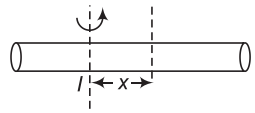


$$I = I_1 + I_2 + I_3 + I_4$$
$$= 0 + \frac{ml^2}{3} + ml^2 + \frac{ml^2}{3} = \frac{5ml^2}{3}$$

2. $I = I_1 + I_2$

$$= 1 \times (1^2 + 2^2) + 2(3^2 + 4^2) = 55 \text{ kg-m}^2$$

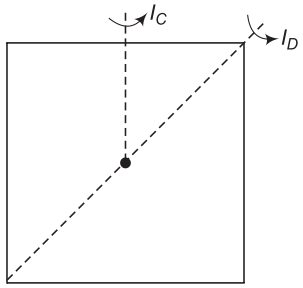
3.



$$I = \frac{ml^2}{12} + mx^2 = \frac{7}{12}ml^2$$

$$\therefore \quad x^2 = \left(\frac{7}{12} - \frac{1}{12}\right)l^2 = \frac{l^2}{2} \Rightarrow x = \frac{l}{\sqrt{2}}$$

4.



$$I_C = \frac{Ma^2}{6} \quad \text{and} \quad I_D = \frac{1}{2} I_C = \frac{Ma^2}{12}$$

5.
$$k = \sqrt{\frac{I}{M}} = \sqrt{\frac{I_{CM} + Mx^2}{M}}$$

$$\Rightarrow \quad k^2 = \frac{I_{CM}}{M} + x^2$$

$$\therefore \quad \frac{I_{CM}}{M} = k^2 - x^2$$

$$\Rightarrow \quad k_{CM} = \sqrt{\frac{I_{CM}}{M}} = \sqrt{k^2 - x^2}$$

$$= \sqrt{10^2 - 6^2} = 8 \text{ cm}$$

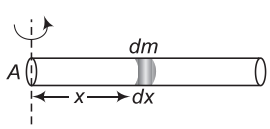
6. $r_1 = \left(\frac{m_2}{m_1 + m_2}\right) r$ and $r_2 = \left(\frac{m_1}{m_1 + m_2}\right) r$

$$I = m_1 r_1^2 + m_1 r_2^2$$

Substituting the values we get,

$$I = \left(\frac{m_1 m_2}{m_1 + m_2}\right) r^2$$

7.



$$dI = dmx^2 = x^2 \lambda \, dx = x^2 \, (\alpha x + \beta) \, dx$$

$$= (\alpha x^3 + \beta x^2) \, dx$$

$$\therefore \; I = \int_0^l (\alpha x^3 + \beta x^2) \, dx$$

$$= \frac{1}{4} \alpha l^4 + \frac{1}{3} \beta l^3$$

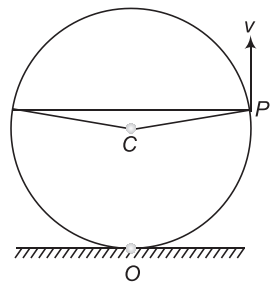
8. In second's clock, it takes 60 s by the seconds hand to rotate by 2π.

So, $\omega = \frac{2\pi}{60}$ rad/s $= \frac{\pi}{30}$ rad/s.

9.
$$\omega = \frac{1}{r^2} (\vec{\mathbf{r}} \times \vec{\mathbf{v}}) = \frac{1}{25} \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 3 & 4 & 0 \\ 4 & -3 & 0 \end{vmatrix}$$

$$= \frac{1}{25} (-9 - 16) \, \hat{\mathbf{k}} = -\hat{\mathbf{k}} \text{ rad/s}$$

10.



$$\omega_c = 2\omega_o$$

$$\Rightarrow \; \omega_o = \frac{1}{2} \omega_c$$

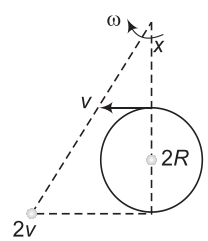
$$= \frac{1}{2} \cdot \frac{v}{R} = \frac{1}{2} \times \frac{2}{0.1} = 10 \text{ rad/s}$$

11.
$$\omega = \frac{v}{x} = \frac{2v}{x + 2R}$$

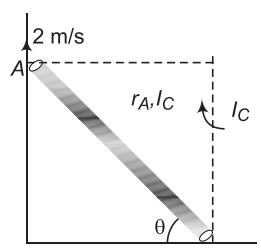
$$\Rightarrow \; x + 2R = 2x$$

$$\Rightarrow \; x = 2R$$

$$\therefore \; \omega = \frac{v}{2R}$$



12.



$$\omega = \frac{v_A}{r_A, I_C} = \frac{2}{2 \cos 60°} = 2 \text{ rad/s}$$

13.
$$\tau = \vec{\mathbf{r}} \times \vec{\mathbf{F}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 4 & -2 \\ 2 & 3 & -2 \end{vmatrix}$$

$$= \hat{\mathbf{i}} (-8 + 6) - \hat{\mathbf{j}} (-4 + 4) + \hat{\mathbf{k}} (6 - 8)$$

$$= -2 \, \hat{\mathbf{i}} - 2 \, \hat{\mathbf{k}} = -2 (\hat{\mathbf{i}} + \hat{\mathbf{k}}) \text{ Nm}$$

14. $\tau = \vec{\mathbf{r}} \times \vec{\mathbf{F}} = \frac{R}{2} \cdot mg = \frac{u^2 \sin 2\theta}{2g} \cdot mg$

$= mu^2 \sin\theta \cos\theta = \frac{1}{2} mu^2$

$= \frac{1}{2} \times 1 \times (20\sqrt{2})^2 = 400\,\text{Nm}$

15. $\tau = \tau_{10} + \tau_{20} + \tau_{30}$

$= 0 + 20 \sin 45° \times 0.1 + 30 \sin 60° \times 0.05$

$= \sqrt{2} + 1.5 \times \frac{\sqrt{3}}{2} = \sqrt{2} + \frac{3\sqrt{3}}{4} = 2.71\,\text{Nm}$

16. $\tau = 12 \cos 60° \times 0.1 - 10 \times 0.25 - 9 \times 0.25$

$= 0.6 - \frac{19}{4} = \frac{2.4 - 19}{4} = \frac{16.6}{4} = \frac{83}{20}\,\text{Nm}$

17. $\theta = \frac{1}{2}\alpha t^2 \Rightarrow \alpha = \frac{2\theta}{t^2} = \frac{2 \times 50 \times 2\pi}{5^2}$

$= 8\pi\,\text{rad/s}^2$

$\therefore \omega = \alpha t = 8\pi\,\text{rad/s}^2 \times 5s = 40\,\text{rad/s}$

18. $\theta = \theta_1 + \theta_2 + \theta_3$

$= \frac{1}{2}\alpha t_1^2 + \omega t_2 + \omega t_1 - \frac{1}{2}\alpha t_1^2$

$= \alpha t_1 (t_1 + t_2) = 2 \times 5 (5 + 2) = 70\,\text{rad}$

19. $\omega = \alpha_1 t_1$

$\Rightarrow \alpha_1 = \frac{\omega}{t_1} = \frac{20}{5} = 4\,\text{rad/s}^2$

$\alpha_2 = \frac{\omega}{t_2} = \frac{20}{60} = \frac{1}{3}\,\text{rad/s}^2,$

$\tau_2 = I\alpha_2 = 0.03 \times \frac{1}{3} = 0.01\,\text{Nm}$

is the torque due to friction. Again,

$\tau_1 - \tau_2 = I\alpha_1$

$\Rightarrow \quad \tau_1 = \tau_2 + I\alpha_1$

or $\quad \tau_1 = 0.01 + 0.03 \times 4 = 0.13\,\text{Nm}$

20. $\omega = \omega_0 + \alpha t$

$\Rightarrow \quad -20\,\text{rad/s}$

$= 20\,\text{rad/s} - 2\,\text{rad/s}^2 \times t$

$\Rightarrow \quad t = \frac{40}{2}\,\text{s} = 20\,\text{s}$

(As KE $= \frac{1}{2} I\omega^2$ is same for $\pm\,\omega$)

21. $\tau = I\alpha = \frac{1}{2} MR^2 \cdot \frac{\omega}{t}$

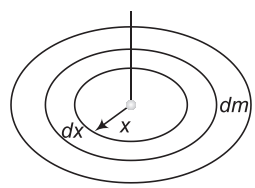
$= \frac{1}{2} \times 20 \times \left(\frac{1}{2}\right)^2 \times \frac{\frac{240}{\pi} \times \frac{2\pi}{60}}{3}$

$= \frac{5}{2} \times \frac{8}{3} = \frac{20}{3}\,\text{Nm}$

$\tau' = I\alpha' = \frac{1}{2} MR^2 \frac{\omega}{t'} = RF$

$\Rightarrow t' = \frac{MR\omega}{2F} = \frac{20 \times \frac{1}{2} \times 8}{2 \times 10} = 4\,\text{s}$

22.



(a) $d\tau = \mu \cdot dm \cdot g \cdot x = \mu g x \cdot \frac{M}{\pi R^2} \cdot 2\pi x\, dx$

$= \frac{2\mu Mg}{R^2} x^2 dx$

$\tau = \frac{2\mu Mg}{R^2} \int_0^R x^2 dx = \frac{2}{3} \mu MgR$

$= I\,\alpha = \frac{1}{2} MR^2 \alpha$

$\Rightarrow \alpha = \frac{4}{3} \frac{\mu g}{R}, 0 = \omega_0 - \alpha t$

$\Rightarrow t = \frac{\omega_0}{\alpha} = \frac{3R\omega_0}{4\mu g}$

(b) $\theta = \omega_0 t - \frac{1}{2}\alpha t^2$

$= \frac{3R\omega_0^2}{4\mu g} - \frac{1}{2} \cdot \frac{4\mu g}{3R} \cdot \frac{9R^2\omega_0^2}{16\mu^2 g^2}$

$= \frac{3R\omega_0^2}{4\mu g} - \frac{3R\omega_0^2}{8\mu g} = \frac{3R\omega_0^2}{8\mu g}$

23. (a) $\tau = I\,\alpha$ and $0 = \omega_0 - \alpha t$

$\Rightarrow t = \frac{\omega_0}{\alpha} = \frac{I\omega_0}{\tau} = \frac{16 \times 9}{4} = 36\,\text{s}$

(b) $\tau = kt = I \frac{d\omega}{dt} \Rightarrow d\omega = \frac{k}{I} t\, dt$

$\Rightarrow \quad \int_{\omega_0}^{0} d\omega = \int_0^t \frac{k}{I} t\, dt$

$$\therefore\ \omega_0 = \frac{k}{2I}t^2 \Rightarrow t = \sqrt{\frac{2\omega_0 I}{k}}$$

$$= \sqrt{\frac{2\times 9\times 16}{k}} = 12\sqrt{\frac{2}{k}}$$

24. $\frac{d\omega}{dt} = \alpha = -10 - 5t$

$$\Rightarrow \omega = \int(-10-5t)\,dt = -10\,t - \frac{5}{2}t^2 + c$$

at $t = 0, \omega = c = 65$ rad/s

$$\therefore \quad \omega = 65 - 10\,t - \frac{5}{2}\,t^2$$

(a) $\omega(3\text{ s}) = 65 - 10\times 3 - \frac{5}{2}\times 3^2$

$$= 35 - \frac{45}{2} = \frac{25}{2}\text{ rad/s}$$

$$= 12.5\text{ rad/s}$$

(b) $\frac{d\theta}{dt} = \omega \Rightarrow d\theta = \omega dt$

$$\Rightarrow \quad \theta = 65t - 5t^2 - \frac{5}{6}t^3$$

$$\therefore \quad \theta = 65\times 3 - 5\times 3^2 - \frac{5}{6}\times 3^3$$

$$= 195 - 45 - \frac{45}{2} = 127.5\text{ rad.}$$

25. $\Delta\theta = \int_0^3 \omega dt = \int_0^3 (12 - 3t^2)\,dt$

$$= 12t - t^3\Big|_0^3 = 36 - 9 = 27\text{ rad}$$

$$N = \frac{\Delta\theta}{2\pi} = \frac{27\text{ rad}}{2\pi\text{ rad}} = 4.3;$$

$$\Delta\theta\,(2\text{ s}) = 12\times 2 - 2^3 = 16\text{ rad}$$

$$\therefore\ \theta_3 - \theta_2 = 27 - 16 = 9\text{ rad}$$

So, in third second, $N' = \frac{9\text{ rad}}{2\pi\text{ rad}} = 1.43$

26. $\theta = 6t - 2t^3 \Rightarrow \omega = \frac{d\theta}{dt} = 6 - 6t^2$

$$\Rightarrow \quad \omega = 0 \text{ at } t = 1\text{ s}$$

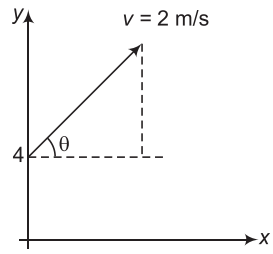
$$\therefore\ \theta(0) = 0 \quad \text{and} \quad \theta(1) = 6 - 2 = 4\text{ rad}$$

$$\therefore\ \omega_{av} = \frac{\theta(1) - \theta(0)}{t} = \frac{4-0}{1} = 4\text{ rad/s}$$

$$\alpha_{av} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{0 - (6 - 6\times 0)}{1 - 0}$$

$$= -6\text{ rad/s}^2$$

27.



$$L = m\cdot v\cos\theta\cdot r,\ \tan\theta = \frac{dy}{dx} = 1$$

$$= 1\times 2\times\frac{1}{\sqrt{2}}\times 4 \Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$$

$$= 4\sqrt{2}\text{ kg-m}^2/\text{s}$$

28. $I = I_1 + I_2 = \frac{\frac{m}{3}\times l^2}{3} + \frac{\frac{2m}{3}\times 4l^2}{3}$

$$= \frac{ml^2}{9} + \frac{8ml^2}{9} = ml^2$$

$$L = I\omega = ml^2\omega$$

29. $L = mvr + I\omega$

$$= -\left(mvR + \frac{2}{5}mR^2\omega\right)\hat{\mathbf{k}}$$

$$= -\frac{7}{5}mvR\,\hat{\mathbf{k}}$$

30. $L = mvr + I\omega$

$$= -\left[2mvR + \left(mR^2 + \frac{1}{3}mR^2\right)\omega\right]\hat{\mathbf{k}}$$

$$= -\left[2mvR + \frac{4}{3}mvR\right]\hat{\mathbf{k}}$$

$$= -\frac{10}{3}mvR\,\hat{\mathbf{k}}$$

31. As $I\omega = \text{constant} \Rightarrow T \propto I$.

With increase in R, I increases and so is T. So, length of day will increase.

32. $I_1\omega_1 = I_2\omega_2 \Rightarrow \omega_2 = \frac{I_1}{I_2}\omega_1$

$$= \frac{\frac{1}{2}MR^2}{\frac{1}{2}MR^2 + mR^2\sin^2\theta}\cdot\omega_1$$

$$= \frac{M}{M + 2m\sin^2\theta}\omega_1$$

$$\therefore\ \omega_2 = \frac{1}{1 + 2 \times 0.2 \times \frac{1}{2}} \times 5 \text{ rad/s}$$

$$= \frac{5}{1.2} \text{ rad/s} = \frac{25}{6} \text{ rad/s}$$

33. $I_1\omega_1 = (I_1 + mr^2)\ \omega_2$

$$\Rightarrow \quad I_1 = \frac{mr^2\omega_2}{\omega_1 - \omega_2}$$

$$= \frac{10 \times 10^{-3} \times 81 \times 10^{-4} \times 90}{100 - 90}$$

$$= 7.29 \times 10^{-4} \text{ kg-m}^2$$

34. (a) $\omega_2 = \frac{I_1}{I_2}\ \omega_1$

$$= \frac{I_0 + 2mr_1^2}{I_0 + 2mr_2^2} \cdot \omega_1$$

$$= \frac{1.6 + 8 \times 0.81}{1.6 + 8 \times 0.0225} \times 0.5 \text{ rev/s}$$

$$= \frac{8.08}{1.78} \times 0.5 \text{ rev/s}$$

$$= 2.27 \text{ rev/s}$$

(b) $E_i = \frac{1}{2} I_1 \omega_1^2$

$$= \frac{1}{2}(1.6 + 6.48)(\pi)^2 = 39.9 \text{ J}$$

$$E_f = \frac{1}{2} I_2 \omega_2^2$$

$$= \frac{1}{2}(1.6 + 0.18)(2.27 \times 2\pi)^2 = 181 \text{ J}$$

(c) $\Delta W = E_f - E_i = 181 \text{ J} - 39.9 \text{ J}$

$$= 141.1 \text{ J}$$

35. (a) $\left(\frac{1}{2}MR^2 + mR^2\right)\omega_0 = \frac{1}{2}MR^2\omega$

$$\Rightarrow \quad \omega = \frac{\frac{1}{2}M + m}{\frac{1}{2}M}\omega_0$$

$$= \frac{M + 2m}{M}\ \omega_0$$

(b) $E_i = \frac{1}{2}\left(\frac{1}{2}MR^2 + mR^2\right)\omega_0^2$

and $E_f = \frac{1}{2} \cdot \frac{1}{2} MR^2 \left(1 + \frac{2m}{M}\right)^2 \omega_0^2$

$$\therefore \quad W = E_f - E_i$$

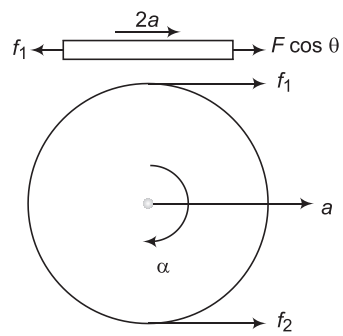
$$= \frac{1}{2}\ \omega_0^2$$

$$\left[\frac{1}{2}MR^2\left(1 + \frac{4m}{M} + \frac{4m^2}{M^2}\right) - \frac{1}{2}MR^2 - mR^2\right]$$

$$= \frac{1}{2}\omega_0^2\left[mR^2 + \frac{2m^2R^2}{M}\right]$$

$$= \frac{1}{2}m\omega_0^2R^2\left(1 + \frac{2m}{M}\right)$$

36.



$F\cos\theta - f_1 = m \cdot 2a$ ...(i)

$(f_1 - f_2)\,R = I\alpha = \frac{1}{2}MR^2 \cdot \frac{a}{R} = \frac{1}{2}MRa$

or $f_1 - f_2 = \frac{1}{2}Ma$ ...(ii)

While, $f_1 + f_2 = Ma$ ...(iii)

From Eqs. (ii) and (iii),

$$f_1 = \frac{3}{4}Ma \quad \text{and} \quad f_2 = \frac{1}{4}Ma$$

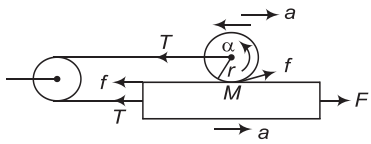
$$\therefore\ F\cos\theta - \frac{3}{4}Ma = 2ma$$

$$\Rightarrow a = \frac{4F\cos\theta}{3M + 8m}$$

$$f_1 = \frac{3M}{4} \cdot \frac{4F\cos\theta}{3M + 8m} = \frac{3MF\cos\theta}{3M + 8m}$$

and $f_2 = \frac{M}{4} \cdot \frac{4F\cos\theta}{3M + 8m} = \frac{MF\cos\theta}{3M + 8m}$

37.



$$F - T - f = Ma$$
$$T - f = ma \quad \Rightarrow \quad F - 2f = (M + m)\,a$$

$$fR = I\alpha = \frac{1}{2}mR^2\alpha \;\Rightarrow\; f = \frac{1}{2}mR\alpha$$

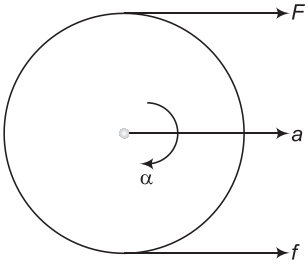
$$R\alpha - a = a \;\Rightarrow\; R\alpha = 2a \;\Rightarrow\; f = ma$$

$$\therefore\; F - 2ma = (M + m)\,a$$

$$\Rightarrow\; F = (M + 3m)\,a$$

$$\therefore\; a = \frac{F}{M + 3m}$$

38.



$$F + f = ma \qquad \ldots(i)$$

$$(F - f)\,R = I\alpha = \frac{1}{2}mR^2\alpha = \frac{1}{2}mRa$$

$$\therefore \qquad F - f = \frac{1}{2}ma \qquad \ldots(ii)$$

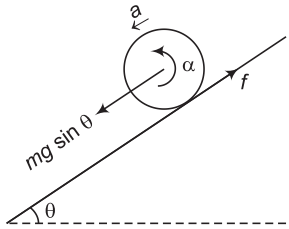
$$\therefore\; F = \frac{3}{4}ma \text{ and } f = \frac{1}{4}ma$$

$$f_{\max} = \mu mg = \frac{1}{4}ma \;\Rightarrow\; a = 4\,\mu g$$

$$\Rightarrow F_{\max} = \frac{3}{4}ma = \frac{3}{4}m4\mu g$$

$$= 3\mu mg = 3 \times 0.6 \times 4 \times 10 = 72\,\text{N}$$

39.



$$fR = I\alpha = \frac{2}{5}mR^2\alpha = \frac{2}{5}mRa$$

$$\Rightarrow f = \frac{2}{5}ma$$

$$mg\sin\theta - f = ma \;\Rightarrow\; mg\sin\theta = \frac{7}{5}ma$$

$$\therefore\; a = \frac{5}{7}g\sin\theta \text{ and } f = \frac{2}{7}mg\sin\theta$$

(a) For minimum value of μ,

$$f = \mu mg\cos\theta = \frac{2}{7}mg\sin\theta$$

$$\text{or } \mu = \frac{2}{7}\tan\theta = \frac{2}{7}\tan 30^\circ = \frac{2}{7\sqrt{3}} = \mu_{\min}$$

(b) $a = \frac{5}{7} \times 10 \times \sin 30^\circ = \frac{25}{7}\,\text{m/s}^2$

(c) For, $\mu = \frac{\mu_{\min}}{2} = \frac{1}{7\sqrt{3}}$

$$\Rightarrow \qquad f = \mu mg\cos\theta$$

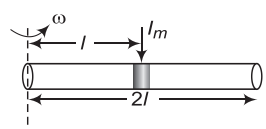
$$\therefore \qquad a = g\sin\theta - \mu g\cos\theta$$

$$= g\,(\sin\theta - \mu\cos\theta)$$

$$= g\left(\frac{1}{2} - \frac{1}{7\sqrt{3}}\,\frac{\sqrt{3}}{2}\right)$$

$$= 5\left(1 - \frac{1}{7}\right) = \frac{30}{7}\,\text{m/s}^2$$

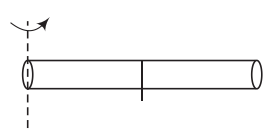
40.



$$I_m l = I\omega = \frac{m(2l)^2}{3}\omega = \frac{4ml^2}{3}\omega$$

$$\therefore\; I_m = \frac{4}{3}ml\omega$$

41.



(a) $mv\frac{L}{2} = \left(\frac{1}{3}ML^2 + m\frac{L^2}{4}\right)\omega$

$$\therefore \qquad \omega = \frac{mv\frac{L}{2}}{\frac{1}{3}ML^2 + \frac{1}{4}mL^2}$$

$$= \frac{\frac{M}{6} v \times \frac{L}{2}}{\frac{1}{3}ML^2 + \frac{1}{24}ML^2}$$

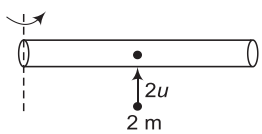
$$= \frac{\frac{v}{12}}{\frac{9L}{24}} = \frac{2v}{9L}$$

(b) $$\frac{K_f}{K_i} = \frac{\frac{1}{2}\left(\frac{1}{3}ML^2 + m\frac{L^2}{4}\right)\omega^2}{\frac{1}{2}mv^2}$$

$$= \frac{\frac{1}{2}\left(\frac{ML^2}{3} + \frac{ML^2}{24}\right)\cdot\frac{4v^2}{81L^2}}{\frac{1}{12}Mv^2}$$

$$= \frac{9 \times 12 \times 2}{24 \times 81} = \frac{1}{9}$$

42.



3 m, 2 l

(a) $\frac{3m \cdot 4l^2}{3}\ \omega = 2m\ (2u + v)\ l$

where, $2u = l\omega + v \Rightarrow v = -l\omega + 2u$

$\therefore \quad 4ml^2\omega = 2m\ (4u - l\omega)\ l$

$6ml^2\omega = 8mul \Rightarrow \omega = \frac{8u}{6l} = \frac{4u}{3l}$

$\therefore \quad v = -\frac{4u}{3} + 2u = \frac{2u}{3}$

(b) $\frac{3m \cdot 4l^2}{3}\ \omega = 2m\ (\sqrt{3}u + v)\ l$

where, $\sqrt{3}u = l\omega + v$

$\therefore \quad 4ml^2\omega = 2m\ (2\sqrt{3}u - l\omega)\ l$

$\Rightarrow \quad 6ml^2\omega = 4\sqrt{3}mul$

$\Rightarrow \quad \omega = \frac{\sqrt{3} \cdot 4u}{6l}$

$= \frac{\sqrt{3} \cdot 2u}{3l} = \frac{2u}{\sqrt{3}l}$

$\therefore \quad v = \sqrt{3}u - l\omega = \sqrt{3}u - \frac{2u}{\sqrt{3}} = \frac{u}{\sqrt{3}}$

Objective Questions (Level 1)

1. $I = Mk^2$, which depends upon, mass and its distribution about an axis.

2. $k = \sqrt{\frac{I}{M}} = \sqrt{\frac{\frac{1}{2}MR^2}{M}} = \frac{R}{\sqrt{2}} = \frac{25}{\sqrt{2}} = 18$ cm

3. $N = \frac{\theta}{2\pi} = \frac{\omega t - \frac{1}{2}\alpha t^2}{2\pi}$

$= \frac{\omega t - \frac{1}{2}\omega t}{2\pi} = \frac{\omega t}{4\pi}$

$= \frac{1725 \times \frac{2\pi}{60} \times 20}{4\pi} = \frac{1725}{6} = 287$

4. In pulling hands closes, external torque is zero, so angular momentum remains constant. Due to decrease in moment of inertia, angular velocity increases and KE increases.

5. $I = \frac{1}{2}Mr^2$

6. Masses cannot be compared as distribution is not given.

7. As the displacement of point of contact is zero, so work done by friction is zero.

8. Due to melting of ice, water spreads on outer side of the pan, increasing moment of inertia, which leads to a decrease in angular speed due to conservation of angular momentum.

9. $\frac{\text{Rotational KE}}{\text{Translational KE}} = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}Mv^2}$

$= \frac{\frac{1}{2}\cdot MR^2 \cdot \frac{v^2}{r^2}}{\frac{1}{2}Mv^2} = \frac{R^2}{r^2}$

10. As, $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}\left(m + \frac{I}{r^2}\right)v^2$

$\Rightarrow v =$ constant, irrespective of the inclination, but time is different because

of different acceleration due to different inclination.

11. Moment of inertia is maximum, when masses are at maximum distance from axis.

12. $mg\sin\theta - f = ma$ and $fR = I\alpha = \frac{I}{R}a$

$$\Rightarrow f = \frac{I}{R^2}a$$

$$\therefore\ mg\sin\theta = f + \frac{fmR^2}{I} = \left(1 + \frac{mR^2}{I}\right)f$$

$$\Rightarrow f = \frac{mg\sin\theta}{1 + \frac{mR^2}{I}}$$

$$\frac{f_D}{f_S} = \frac{1 + \frac{mR^2}{I_S}}{1 + \frac{mR^2}{I_D}}$$

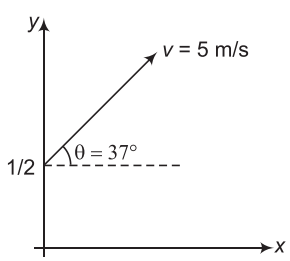
$$= \frac{1 + \frac{5}{2}}{1 + 2} = \frac{7}{6}$$

13. As, there is no external torque, so angular momentum is conserved.

14. As torque due to friction about any point on horizontal surface is zero, so angular moment about that point is conserved.

15. $y = \frac{3}{4}x + \frac{1}{2}$, $\tan\theta = \frac{3}{4}$

$$\Rightarrow \theta = 37^\circ \Rightarrow \cos\theta = \frac{4}{5}$$



$$L = mvr = 3 \times \left(5 \times \frac{4}{5}\right) \times \frac{1}{2} = 6\ \text{kg-m}^2/\text{s}$$

16. Due to elastic collision, velocity will reverse and not angular velocity, so it will take some time to again settle to uniform rolling. During this time there will be sliping and its rotation will stop for a moment.

17. Moment of inertia about an axis perpendicular to the plane and passing through centre of a square plate is $\frac{Ma^2}{6}$ and about an axis passing through its plane is $\frac{1}{2}\cdot\frac{Ma^2}{6} = \frac{Ma^2}{12} = I_0$ is same for AA' and BB'.

18. Due to clockwise torque, the spool will move rightward and so, will the centre of mass.

19. $I\omega_0 = 2I\omega \Rightarrow \omega = \frac{\omega_0}{2}$

$$K_i = \frac{1}{2}I\omega_0^2 = K_0 \text{ and}$$

$$K_f = \frac{1}{2} \times 2I \times \omega^2$$

$$= \frac{1}{4}I\omega_0^2$$

$$\therefore\ \Delta K = K_f - K_i = \frac{1}{4}I\omega_0^2 - \frac{1}{2}I\omega_0^2$$

$$= \frac{1}{4}I\ \omega_0^2$$

$$= -\frac{1}{2}\cdot\frac{1}{2}I\ \omega_0^2 = -\frac{K_0}{2}$$

20. $I = \int x^2 dm = \int_R^{2R} x^2 \cdot 4\pi x^2 dx\ \rho$

$$= 4\pi\rho \int_R^{2R} x^4 dx = \frac{4\pi}{5}\rho x^4\Big|_R^{2R}$$

$$= \frac{4}{5}\pi\rho\,(32R^5 - R^5) = \frac{124}{5}\pi\rho R^5$$

$$M = \int_R^{2R} \rho \cdot 4\pi x^2 dx = 4\pi\rho\frac{1}{3}x^3\Big|_R^{2R}$$

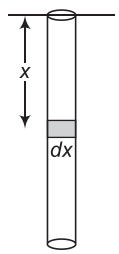
$$= \frac{4}{3}\pi\rho\,(8R^3 - R^3) = \frac{28}{3}\pi\rho R^3$$

$$\therefore\ \rho = \frac{3M}{28\pi R^3} \Rightarrow I = \frac{124}{5}\pi\cdot\frac{3M}{28\pi R^3}\cdot R^5$$

$$= \frac{91}{35}MR^2$$

21. $\lambda(x) = \lambda + \frac{\lambda}{l}x = \lambda\left(1 + \frac{x}{l}\right)$

$$I = \int x^2 dm = \int_0^l x^2 \lambda\left(1 + \frac{x}{l}\right) dx$$



$$= \lambda \int_0^l \left(x^2 + \frac{x^3}{l}\right) dx$$

$$= \lambda\left(\frac{1}{3}x^3 + \frac{x^4}{4l}\right)\Bigg|_0^l$$

$$= \lambda\left(\frac{1}{3}l^3 + \frac{1}{4}l^3\right) = \frac{7}{12}\lambda l^3$$

$$M = \int_0^l \lambda\left(1 + \frac{x}{l}\right) dx = \lambda\left(x + \frac{x^2}{2l}\right)\Bigg|_0^L$$

$$= \frac{3}{2}\lambda L \Rightarrow \lambda = \frac{2M}{3L}$$

$$\therefore\ I = \frac{7}{12}\cdot\frac{2M}{3L}\cdot L^3 = \frac{7}{18}ML^2$$

22. $I_1 = \frac{1}{2}\cdot\frac{Ma^2}{6} = \frac{Ma^2}{12}$

and $I_2 = I_1 + M\left(\frac{a}{\sqrt{2}}\right)^2 = I_1 + \frac{Ma^2}{2}$

$$= I_1 + 6\frac{Ma^2}{12} = 7I_1$$

$$\therefore\ I_1 : I_2 = I_1 : 7I_1 = 1 : 7$$

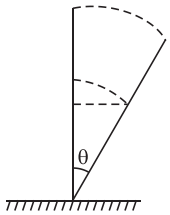
23. $I = \frac{ML^2}{12} + M\left(\frac{L}{2} - \frac{L}{4}\right)^2$

$$= \frac{ML^2}{12} + \frac{ML^2}{16}$$

$$= \frac{4+3}{48}ML^2$$

$$= \frac{7}{48}ML^2$$

24.



$$mg\frac{L}{2}(1 - \cos\theta) = \frac{1}{2}\cdot\frac{mL^2}{3}\cdot\omega^2$$

$$g\cdot 2\sin^2\frac{\theta}{2} = \frac{1}{3}L\omega^2$$

$$\Rightarrow \quad \omega = \sqrt{\frac{6g}{L}}\cdot\sin\frac{\theta}{2}$$

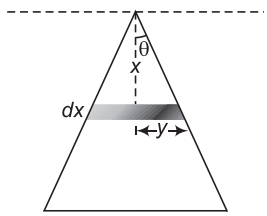
25. $r_1 = \left(\frac{m_1}{m_1 + m_2}\right)r_2$ and $r_2 = \left(\frac{m_1}{m_1 + m_2}\right)r_1$

$$I = m_1 r_1^2 + m_2 r_2^2$$

Substituting the values we get,

$$I = \left(\frac{m_1 m_2}{m_1 + m_2}\right)r^2$$

26.



$$I = \int dI = \int x^2 dm = \int x^2\cdot\sigma\cdot 2y\ dx$$

$$= 2\sigma\int x^2\cdot x\tan\theta\cdot dx$$

$$= 2\sigma\tan\theta\cdot\int x^3 dx = 2\sigma\tan\theta\cdot\frac{1}{4}x^4\Bigg|_0^{L\cos\theta}$$

$$= \frac{1}{2}\sigma L^4\tan\theta\cos^4\theta$$

$$= \frac{1}{2}\sigma L^4\cdot\frac{1}{\sqrt{3}}\cdot\frac{9}{16} = \frac{3\sqrt{3}}{32}\sigma l^4$$

$$M = \int dm = \int \sigma\cdot 2y\ dx$$

$$= 2\sigma\tan\theta\int x dx = 2\sigma\tan\theta\cdot\frac{1}{2}x^2\Bigg|_0^{L\cos\theta}$$

$$= \sigma\tan\theta\cdot L^2\cos^2\theta$$

$$= \sigma L^2\cdot\frac{1}{\sqrt{3}}\cdot\frac{3}{4} = \sigma L^2\cdot\frac{\sqrt{3}}{4}$$

$$\therefore\ \sigma = \frac{4M}{\sqrt{3}L^2}$$

$$\Rightarrow \quad I = \frac{3\sqrt{3}}{32}\times\frac{4M}{\sqrt{3}L^2}\cdot L^4 = \frac{3}{8}ML^2$$

27. $I = 2\cdot\frac{M(L\sin 45°)^2}{3}$

$$+ 2\left[\frac{M(L\sin 45°)^2}{12} + M\left(\frac{3L}{2}\sin 45°\right)^2\right]$$

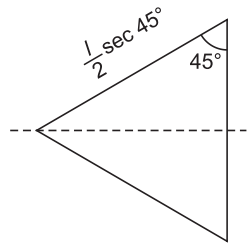
$$= \frac{ML^2}{3} + 2\left[\frac{ML^2}{24} + \frac{9}{8}ML^2\right]$$
$$= ML^2\left(\frac{1}{3} + \frac{1}{12} + \frac{9}{4}\right)$$
$$= \frac{4+1+27}{12}ML^2$$
$$= \frac{32}{12}ML^2$$
$$= \frac{8}{3}ML^2$$

28. $I = 2\cdot\frac{Ml^2}{3} + 2\left[\frac{Ml^2}{12} + M\left(\sqrt{l^2 + \frac{l^2}{4}}\right)^2\right]$

$$= \frac{2}{3}Ml^2 + \frac{1}{6}Ml^2 + \frac{5}{2}Ml^2$$
$$= \frac{4+1+15}{6}Ml^2 = \frac{10}{3}Ml^2$$

29. Is both have same mass and same radius, *i.e.*, same distribution of mass, that is why both of them have same moment of inertia. $\left(\text{As } I = \frac{3}{10}MR^2\right)$

30.



$$I = 2\times\frac{1}{3}M\left(\frac{l}{2}\sec 45°\right)^2\cdot\sin^2 45°$$
$$= \frac{2}{3}Ml^2\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{Ml^2}{6}$$
$$M = x\cdot\frac{l}{2}\sec 45° = \frac{xl}{\sqrt{2}}$$
$$\therefore\ I = \frac{xl}{\sqrt{2}}\cdot\frac{l^2}{6} = \frac{xl^3}{6\sqrt{2}}$$

JEE Corner

Assertion and Reason

1. It depends upon the axis, whether it is parallel or perpendicular to the plane of rotation. So, assertion is not always true.

2. Due to anti-clockwise rotation, friction will start acting leftward, for which translational speed will increase and angular speed will decrease till the moment, when pure rolling starts. So, assertion and reason are both true but not complete explanation.

3. Hollow sphere has larger moment of inertia, such that it has lesser rotational kinetic energy, so assertion is false.

$$R_S + K_S = R_H + K_H$$
$$\frac{1}{2}\times\frac{2}{5}MR^2\omega^2 + \frac{1}{2}Mv^2$$
$$= \frac{1}{2}\times\frac{2}{3}mr^2\omega_1^2 + \frac{1}{2}mv_1^2$$
$$\frac{7}{5}Mv^2 = \frac{5}{3}mv_1^2$$
$$Mv^2 = \frac{25}{21}mv_1^2$$
$$\frac{1}{2}Mv^2 = \frac{25}{42}\cdot mv_1^2$$
$$K = \frac{25}{21}\cdot\frac{1}{2}mv_1^2$$
$$\therefore\ \frac{1}{2}mv_1^2 = \frac{21}{25}K$$
$$\Rightarrow\quad \frac{1}{2}mv_1^2 < K$$

4. Reason is true explanation of assertion.

5. Reason is true explanation of assertion.

6. Reason is true explanation of assertion.

7. As one goes from A to D, distance of CM first decrease and then increase, so moment of inertia also first decrease and then increase by applying parallel axes theorem. So, assertion is true but reason

is false, as perpendicular axis theorem is also applicable.

8. If linear momentum is constant, then angular momentum from a particular axis remains constant as the projection of distance remains constant. So, assertion and reason are both true but not correct explanation.

9. $v_C = v + R\omega, v_B = \sqrt{v^2 + R^2\omega^2}$

and $\quad v_A = v - R\omega$

$\Rightarrow \quad v_C > v_B > v_A$

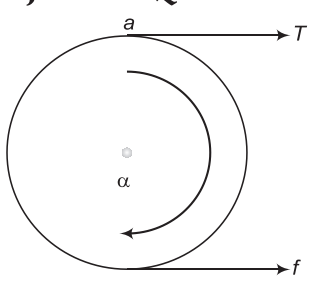
So, assertion and reason are both true but not correct explanation.

10. Reason is correct explanation of assertion.

11. Here, reason is false as when plank was at rest then work done by friction was not zero.

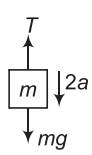
Objective Questions (Level 2)

1.



$T + f = ma \Rightarrow T = ma, f = 0,$

$T - f = \frac{I}{r}\alpha = ma$



$mg - T = m \cdot 2a$

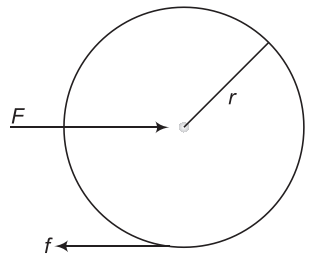
$\Rightarrow \quad mg = 3ma$

$\Rightarrow \quad a = \frac{g}{3}$

i.e., acceleration of the ring is $\frac{g}{3}$ while that of block is $\frac{2g}{3}$.

Here, $f = 0$

2.



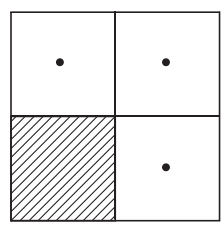
$F - f = ma$

$f = \frac{2}{5}ma \Rightarrow F = \frac{7}{5}ma$

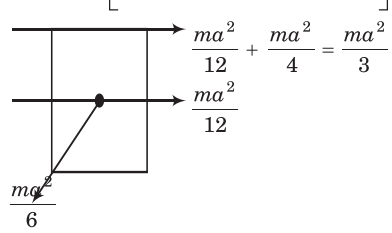
$\Rightarrow ma = \frac{5}{7}F = 5\text{ N}$

$\therefore \; f = \frac{2}{5}ma = \frac{2}{5} \times 5\text{ N} = 2\text{ N}$

3.



$$I = 2 \times \frac{1}{3} \cdot \frac{m}{4} \cdot \left(\frac{a}{2}\right)^2 + \left[\frac{\frac{m}{4} \cdot \left(\frac{a}{2}\right)^2}{12} + \frac{m}{4} \cdot \left(\frac{3a}{4}\right)^2\right]$$



$$= ma^2\left(\frac{2}{3} \times \frac{1}{16} + \frac{1}{12} \times \frac{1}{16} + \frac{9}{4} \times \frac{1}{16}\right)$$

$$= \frac{ma^2}{16}\left[\frac{2}{3} + \frac{1}{12} + \frac{9}{4}\right]$$

$$= \frac{ma^2}{16} \times \frac{8 + 1 + 27}{12}$$

$$= \frac{3ma^2}{16}$$

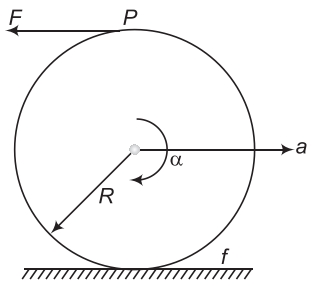
4. $mgR = \frac{1}{2}mv^2 + \frac{1}{2}\cdot\frac{2}{5}mv^2 = \frac{7}{10}mv^2$

$\Rightarrow \frac{mv^2}{R} = \frac{10}{7}mg$

$N = mg + \frac{mv^2}{R} = mg + \frac{10}{7}mg = \frac{17}{7}mg$

5. $F + f = Ma$

$F - f = \frac{I}{R}\alpha = \frac{1}{2}Ma$

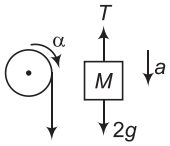


$\Rightarrow 2F = \frac{3}{2}Ma \Rightarrow a = \frac{4F}{3M}$

6. $f = F - \frac{1}{2}Ma = F - \frac{2}{3}F = \frac{1}{3}F$, towards right.

7. $2g - T = 2a$ and $TR = I\alpha = I\frac{a}{R}$

$\Rightarrow T = \frac{Ia}{R^2}$



$2g = \left(2 + \frac{I}{R^2}\right)a$

$\Rightarrow a = \frac{2g}{2 + \frac{I}{R^2}}$

$= \frac{2\times 10}{2 + \frac{0.32}{0.04}} = 2\,\text{m/s}^2$

8. $p(h+R) = \left(\frac{3}{2}mR^2\right)\cdot\omega$

$\therefore \quad \omega = \frac{2p(h+R)}{3mR^2}$

Further, $p = mv = m(R\omega) = \frac{2p(h+R)}{3R}$

Solving we get, $h = \frac{R}{2}$.

9. $T_i = \frac{mg}{2}$ and $T_f = \frac{mg}{4}$

$\therefore\ T_f < T_i$

10. $Mg - T = Ma_1$

While, $MgR = \frac{1}{2}MR^2\alpha_2$

$\Rightarrow \quad \alpha_2 = \frac{2g}{R}$

$TR = \frac{1}{2}MR^2\alpha_1$

$\therefore\ Mg = \frac{3}{2}Ma_1$

$a_1 = \frac{2}{3}g$

$\alpha_1 = \frac{2g}{3R}$

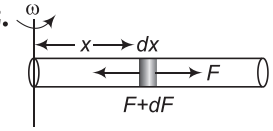
$\therefore\ \alpha_1 < \alpha_2$

or $\alpha_1 \neq \alpha_2$

11. $\frac{1}{2}kx^2 = Mgx\sin\theta$

$\Rightarrow x = \frac{2Mg\sin\theta}{k}$

12.



$\therefore\ F + dF - F = dm\omega^2 x = \frac{m}{l}dx\cdot\omega^2 x$

$\therefore\ dF = \frac{m\omega^2}{l}x\,dx$

$F = \frac{m\omega^2}{l}\int_x^l x\,dx = \frac{1}{2}\frac{m\omega^2}{l}(l^2 - x^2)$

$= \frac{1}{2}m\omega^2 l\left(1 - \frac{x^2}{l^2}\right)$

13. $v = r\omega = l\,(a + bt) = la + lbt$

$v\,(t = 0) = la = 1\,\text{m} \times 10\,\text{rad/s} = 10\,\text{m/s}$,

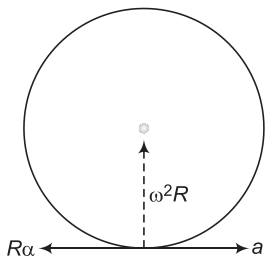
$\vec{\mathbf{v}} = 10\,\hat{\mathbf{j}}\,\text{m/s}$

$\vec{\mathbf{a}} = a_n \hat{\mathbf{j}} + a_n\ (-\hat{\mathbf{i}}) = l\ \alpha\ \hat{\mathbf{j}} - l\omega^2\ \hat{\mathbf{i}}$

$= lb\ \hat{\mathbf{j}} - l\ (a + bt)^2\ \hat{\mathbf{i}}$

$= 5\ \text{m/s}^2\ \hat{\mathbf{j}} - 100\ \text{m/s}^2\ \hat{\mathbf{i}}$

14.



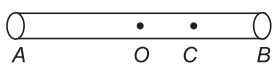
For pure rolling, $\alpha = \dfrac{a}{R}$

Here, $\vec{\mathbf{a}}_{\text{net}} = a\ \hat{\mathbf{i}} - R\ \alpha\ \hat{\mathbf{i}} + \omega^2 R\ \hat{\mathbf{j}}$

$= \omega^2 R\ \hat{\mathbf{j}}$

$\therefore\ a_{\text{net}} = \omega^2 R$

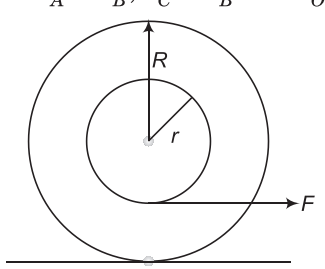
15.



Here, $I_C < I_O < I_B < I_A$

$\Rightarrow I_A > I_B, I_C < I_B$ and $I_O > I_C$

16.



Due to anti-clockwise torque by F and rightward F, the point of contact has the tendency to move rightward, so, frictional force acts leftward.

For, no rotation of the spool, $fR = Fr$

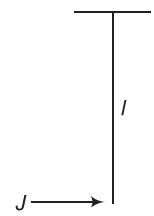
$\Rightarrow f = \dfrac{r}{R} F \Rightarrow f < F$

17. From conservation of angular momentum,

$mvR = 3mR^2\omega \Rightarrow\ \omega = \dfrac{v}{3R}$

18. As N provides clockwise torque about instantaneous centre of rotation and rod also rotates in clockwise sense with increasing angular speed, so, normal reaction does positive rotational work.

19. $L = Jl = I\omega = \dfrac{1}{3} ml^2 \omega$



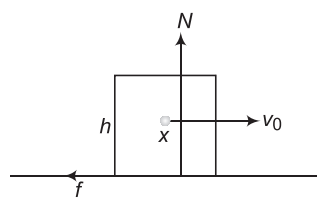
$\Rightarrow \omega = 3\ \text{J}$

$K = \dfrac{1}{2} I\omega^2$

$= \dfrac{1}{2} \times \dfrac{1}{3}\ ml^2 \times \dfrac{9J^2}{m^2 l^2}$

$= \dfrac{3J^2}{2m}$

20.

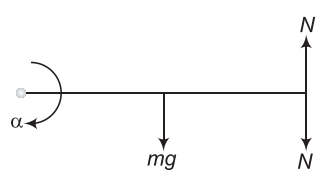


$f_L = \mu mg$

$\therefore\ \tau = Nx - f\dfrac{h}{2} = 0$, as the block is not toppling till, $x > b$ *i.e.*, $\dfrac{\mu h}{2} > b$

or $\mu > \dfrac{2b}{h}$

21.



Immediately after release, the free end is in free fall and so, is the coin, thus reaction on the coin is zero.

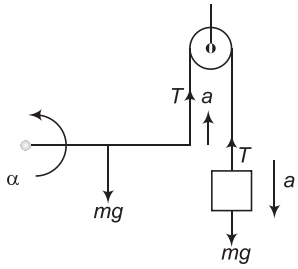
22. $Fr = fR$ and $F\cos\theta = f$

$F\cos\theta = F\dfrac{r}{R}$

$\Rightarrow\ \cos\theta = \dfrac{r}{R}\ \Rightarrow\ \sin\theta = \sqrt{1 - \dfrac{r^2}{R^2}}$

$\Rightarrow\ \theta = \sin^{-1}\sqrt{1 - \dfrac{r^2}{R^2}}$

23.



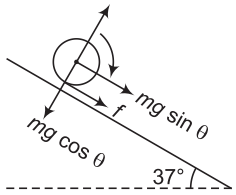
$$mg - T = ma \quad \text{...(i)}$$

$$Tl - mg\frac{l}{2} = \frac{1}{3}ml^2\alpha$$

$$T - \frac{1}{2}mg = \frac{1}{3}ma \quad \text{...(ii)}$$

$$\frac{1}{2}mg = \frac{4}{3}ma \Rightarrow a = \frac{3g}{8}$$

24.



$$\alpha = \frac{fr}{I} = \frac{fr}{\frac{1}{2}mr^2} = \frac{2f}{mr}$$

and $\quad a = g\sin\theta + \mu g\cos\theta$

$$v = at = r\omega = r(\omega_0 - \alpha t)$$

$$g(\sin\theta + \mu\cos\theta)\,t + r\,\alpha\,t = r\omega_0$$

$$\therefore\ t = \frac{r\omega_0}{r\alpha + g(\sin\theta + \mu\cos\theta)}$$

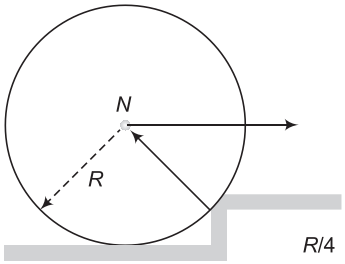
$$= \frac{0.4 \times 54}{0.4 \times \dfrac{2 \times \frac{1}{2} \times g \times \cos 37°}{0.4} + g\left(\sin 37° + \frac{1}{2}\cos 37°\right)}$$

$$= \frac{0.4 \times 54}{10\left(\frac{4}{5} + \frac{3}{5} + \frac{2}{5}\right)}$$

$$= \frac{0.4 \times 54}{2 \times 9}$$

$$= 1.2\text{ s}$$

25.



26. $a = \dfrac{f}{m} = \dfrac{\mu mg}{m} = \mu g$ and

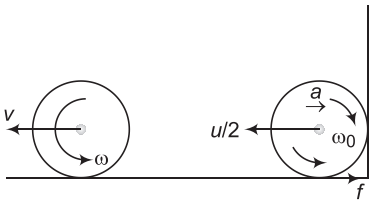
$$v = \frac{u}{2} - at = \frac{u}{2} - \mu gt \quad \text{...(i)}$$

$$\alpha = \frac{\tau}{I} = \frac{\mu mgR}{\frac{2}{5}mR^2} = \frac{5\mu g}{2R}$$

$$\omega = -\omega_0 + \alpha t$$

$$v = -\omega_0 R + R\alpha t$$

$$v = -u + \frac{5}{2}\mu gt \quad \text{...(ii)}$$



$$\therefore\ \frac{u}{2} - \mu gt = -u + \frac{5}{2}\mu gt$$

$$\frac{3u}{2} = \frac{7}{2}\mu gt$$

$$\Rightarrow \quad t = \frac{3u}{7\mu g}$$

27. $\text{KE} = \dfrac{1}{2}\cdot 2mv^2 + \dfrac{1}{2}\cdot\dfrac{2}{3}mr^2\omega^2$

$$= mv^2 + \frac{1}{3}mv^2$$

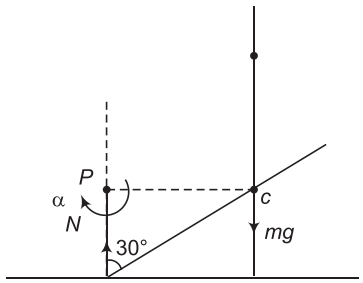
$$= \frac{4}{3}mv^2$$

28. $\tau = I\,\alpha$

$$\Rightarrow f \cdot R = \frac{1}{2}mR^2 \cdot \alpha$$

$$\Rightarrow \quad f = \frac{1}{2}mR\,\alpha = \frac{1}{2}ma$$

29.



$$PC = \frac{l}{2}\sin 30° = \frac{l}{4}$$

Instantaneous axis of rotation is at points P.

$$\alpha = \frac{\tau}{I} = \frac{(mg)(PC)}{I}$$

Further,

$$mg - N = ma = m(PC)\alpha$$

$$\therefore \quad N = mg - m(PC)\alpha$$

$$= mg\left[1 - \frac{m(PC)^2}{I}\right]$$

$$= mg\left[1 - \frac{m(l/4)^2}{\left(\frac{ml^2}{12}\right) + m\left(\frac{l}{4}\right)^2}\right]$$

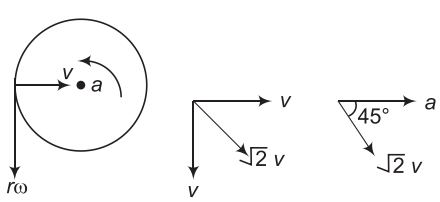
$$= \frac{4}{7}mg$$

30. $a = r\alpha = \left(\frac{l}{2}\cos 30°\right)\left[\frac{mg(PC)}{I}\right]$

$$= \left(\frac{l}{2}\right)\left(\frac{\sqrt{3}}{2}\right)\left[\frac{mg\left(\frac{l}{4}\right)}{\frac{ml^2}{12} + m\left(\frac{l}{4}\right)^2}\right]$$

$$= \frac{3\sqrt{3g}}{7}$$

31.



So, acceleration is towards centre and resultant velocity is at an angle of 45° with it.

32. $F = ma$ and $F\frac{L}{2} = \frac{mL^2}{12}\alpha$

or $ma\frac{L}{2} = \frac{mL^2}{12}\alpha$

or $L\alpha = 6a$

$$\therefore \; a_B = \frac{L\alpha}{2} - a = 3a - a = 2a = 2\frac{F}{m}$$

33. Instantaneous centre of rotation forms an equilateral triangle, so, $IA = L$

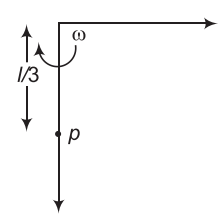
$$\therefore \; L\omega = v \quad \text{or} \quad \omega = \frac{v}{L}$$

34. $L = mvL\cos\theta = \text{constant} = I\omega$

$$\Rightarrow \omega = \frac{mvL\cos\theta}{I}$$

or $\omega \propto \frac{1}{I}$, with increase of distance of particle from origin, moment of inertia increases, so, angular velocity decreases.

35. $Mg \cdot \frac{L}{2} = \frac{1}{2} \cdot \frac{ML^2}{3} \cdot \omega^2$



$$\Rightarrow \omega = \sqrt{\frac{3g}{L}}$$

$$dT = gdm + \omega^2 xdm$$

$$= \left(g + \frac{3g}{L}x\right)\frac{M}{L}dx$$

$$= \frac{Mg}{L}\left(1 + \frac{3x}{L}\right)dx$$

$$T = \int_{L/3}^{L} \frac{Mg}{L}\left(1 + \frac{3x}{L}\right)dx$$

$$= \frac{Mg}{L}\left(x + \frac{3x^2}{2L}\right)\Bigg|_{L/3}^{L}$$

$$= \frac{Mg}{L}\left[\left(L - \frac{L}{3}\right) + \frac{3}{2L}\left(L^2 - \frac{L^2}{9}\right)\right]$$

$$= \frac{Mg}{L}\left[\frac{2L}{3} + \frac{3}{2L} \cdot \frac{8L^2}{9}\right]$$

$$= 2\,Mg$$

36. $mv\dfrac{L}{2} = \dfrac{mL^2}{3}\omega$

$\therefore \quad \omega = \dfrac{3}{2}\cdot\dfrac{v}{L}$

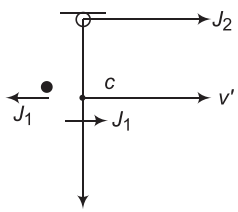
$J_1 = mv$

$v' = \dfrac{L}{2}\cdot\omega = \dfrac{3}{4}v$

$J_1 + J_2 = mv'$

$\therefore\ J_2 + mv = \dfrac{3}{4}mv$

or $\quad J_2 = -\dfrac{1}{4}mv$



37. $L = I\omega + m\cdot R\omega\cdot R = I\omega' + mvR$

$\therefore\ \omega' = \dfrac{I\omega + mR^2\omega - mvR}{I}$

$= \dfrac{(I + mR^2)\,\omega - mvR}{I}$

38. As the point of contact has a net forward velocity, friction acts in backward direction, which tries to bring back the disk at its initial position.

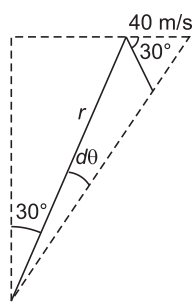
39. $r\dfrac{d\theta}{dt} = v\cos 30°$

$d\sec 30°\,\omega = v\cos 30°$

$\Rightarrow \omega = \dfrac{v}{d}\cos^2 30°$

$= \dfrac{40\text{ m/s}}{30\text{ m}}\cdot\dfrac{3}{4}$

$= 1\text{ rad/s}$



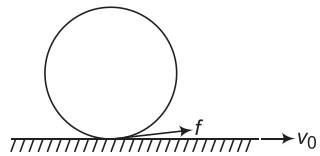
40. Locus of all other points is an ellipse. But locus of a point P such that, $OP = \dfrac{l}{4}$ is a circle.

41. When A hits ground, velocity of all points except O is vertically downward with no horizontal component. And O_4 at rest.

42. $mg\dfrac{l}{2} = \dfrac{1}{2}\cdot\dfrac{ml^2}{12}\cdot\omega^2 \Rightarrow \quad \omega = 2\sqrt{\dfrac{3g}{l}}$,

$v_A = \omega\dfrac{l}{2} = \dfrac{l}{2}\cdot 2\sqrt{\dfrac{3g}{l}} = \sqrt{3gl}$

43.



$ft = mv$ ...(i)

and $\quad fRt = I\omega = \dfrac{2}{5}mR^2\omega$ ...(ii)

Friction will stop acting when,

$v + R\omega = v_0$ ...(iii)

From Eqs. (i) and (ii),

$mvR = \dfrac{2}{5}mR^2\omega \Rightarrow R\omega = \dfrac{5}{2}v$

From Eq. (iii),

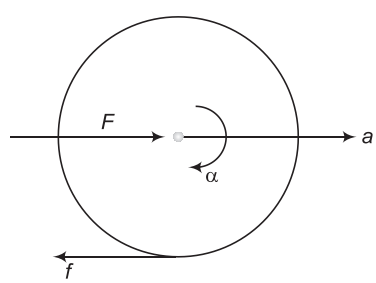
$v + \dfrac{5}{2}v = v_0 \Rightarrow v = \dfrac{2}{7}v_0$

$a = \dfrac{f}{m} = \dfrac{\mu mg}{m} = \mu g = \dfrac{2}{7}g$

As, $v^2 = 2as \Rightarrow s = \dfrac{v^2}{2a} = \dfrac{\left(\dfrac{2}{7}v_0\right)^2}{2\left(\dfrac{2}{7}g\right)} = \dfrac{v_0^2}{7g}$

48. In case of pure rolling, displacement of point contact is zero. So, work done by friction is also zero.

49.



$F - f = ma$

$fR = I\,\alpha = \dfrac{1}{2}mR^2\alpha$

$= \dfrac{1}{2}mRa \Rightarrow f = \dfrac{1}{2}ma$

50. $f = \dfrac{1}{2}ma$ and $F = ma + f = \dfrac{3}{2}ma.$

Angular momentum is conserved about a point which net torque is zero. Let this

point is at a distance x above point of contact, so

$$f(x) = F(x - R)$$

or $$\frac{1}{2}ma \cdot x = \frac{3}{2}ma(x - R)$$

or $$x = 3(x - R)$$

or $$3R = 2x$$

or $$x = \frac{3R}{2}$$

51. $$mgh = \frac{1}{2}\left(\frac{2}{3}mr^2\right)\omega^2 + \frac{1}{2}mv^2$$

$$= \frac{1}{3}mv^2 + \frac{1}{2}mv^2 = \frac{5}{6}mv^2$$

$$\therefore\ v = \sqrt{\frac{6gh}{5}}$$

52. $$H = \frac{v^2 \sin^2\theta}{2g}$$

$$= \frac{\frac{6gh}{5} \times \frac{9}{25}}{2g} = \frac{27h}{125}$$

53. $$x = \frac{v^2 \sin^2\theta}{g}$$

$$= \frac{\frac{6gh}{5} \times 2 \times \frac{3}{5} \times \frac{4}{5}}{g} = \frac{144h}{125}$$

More than One Correct Options

1. $$\frac{1}{2}mv^2 + \frac{1}{2} \cdot \frac{I}{R^2}v^2$$

$$= mg \cdot \frac{3}{4} \cdot \frac{v^2}{g}$$

$$\Rightarrow \frac{I}{2R^2} = \left(\frac{3}{4} - \frac{1}{2}\right)m$$

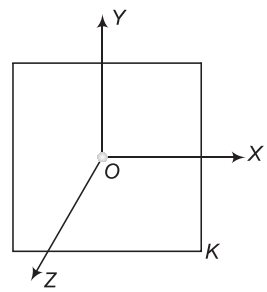
$$= \frac{1}{4}m$$

$$\Rightarrow\ I_{CM} = \frac{1}{2}mR^2 \Rightarrow \text{Disc or cylinder}$$

$$I_0 = I_{CM} + mR^2$$

$$= \frac{3}{2}mR^2$$

2.



$$I_{XX'} = 2 \cdot \frac{ml^2}{12} + 2 \cdot m\left(\frac{l}{2}\right)^2$$

$$= ml^2\left(\frac{1}{6} + \frac{1}{2}\right)$$

$$= \frac{4}{6}ml^2 = \frac{2}{3}ml^2$$

$$I_{ZZ'} = 4 \cdot \left[\frac{ml^2}{12} + \frac{ml^2}{4}\right]$$

$$= ml^2\left(1 + \frac{1}{3}\right) = \frac{4}{3}ml^2$$

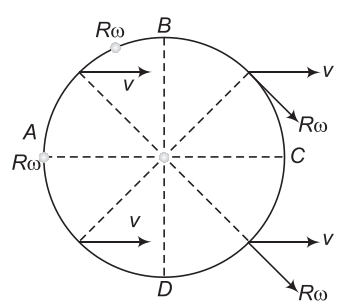
$$I_{KZZ'} = I_{ZZ'} + 4m\left(\frac{l}{\sqrt{2}}\right)^2$$

$$= \frac{4}{3}ml^2 + 2ml^2 = \frac{10}{3}ml^2$$

$$I_{KXX'} = I_{XX'} + 4m\left(\frac{l}{2}\right)^2$$

$$= \frac{2}{3}ml^2 + ml^2 = \frac{5}{3}ml^2$$

3.



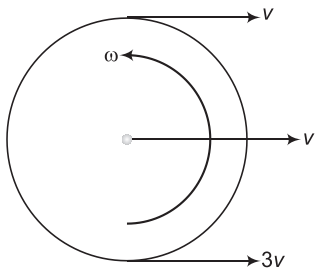
Resultant velocity of particles in region ABC is greater than that in CDA.

So, $(KE)_{ABC} > (KE)_{CDA}$

Similarly, $v_{BC} > v_{CD} \Rightarrow K_{BC} > K_{CD}$

and $v_{CD} = v_{DA} \Rightarrow K_{CD} = K_{DA}$

4.

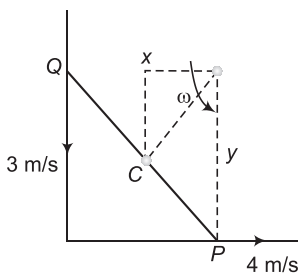


$u - R\omega = v$

$u + R\omega = 3v$

$\Rightarrow \quad u = 2v$ and $R\omega = v$

5.



$\frac{3}{x} = \frac{4}{y}$ and $l = \sqrt{x^2 + y^2} = x\sqrt{1 + \frac{16}{9}}$

$= \frac{5}{3}x \Rightarrow x = \frac{3}{5}l = 0.3$ m

$\omega = \frac{v}{r} = \frac{3 \text{ m/s}}{0.3 \text{ m}} = 10$ rad/s

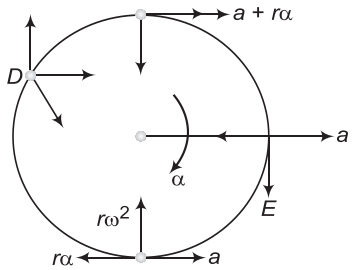
$v_c = \frac{l}{2}\omega = \frac{1}{4} \times 10 \text{ m/s} = 2.5 \text{ m/s}$

$\text{KE} = \frac{1}{2}\left[\frac{ml^2}{12} + \frac{ml^2}{4}\right]\omega^2$

$= \frac{4}{24}ml^2\omega^2 = \frac{4}{24} \times 2 \times \frac{1}{4} \times 100$

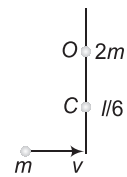
$= \frac{200}{24} = \frac{25}{3}$ J

6.



Net acceleration at A is vertical, at B it may be vertical, at C it cannot be horizontal and at E it may be horizontal leftward.

7. $mv\left(\frac{l}{2} - \frac{l}{6}\right) = \left[\left(\frac{2ml^2}{12} + \frac{2ml^2}{36}\right) + m\left(\frac{l}{2} - \frac{l}{6}\right)^2\right]\omega$



$mv\frac{l}{3} = ml^2\left(\frac{1}{6} + \frac{2}{36} + \frac{4}{36}\right)\omega$

$= ml^2\omega\frac{6 + 2 + 4}{36} = \frac{12}{36}ml^2\omega$

$= \frac{ml^2\omega}{3}$

$\Rightarrow \quad l\omega = v$

$\Rightarrow \quad \omega = \frac{v}{l}$

$\Delta K = \frac{1}{2}mv^2 - \frac{1}{2}I\omega^2 - \frac{1}{2}\cdot 3mu^2;$

$mv = 3mu$

$\Rightarrow u = \frac{v}{3}$

$= \frac{1}{2}mv^2 - \frac{1}{2}\frac{v^2}{l^2}\left[\frac{1}{3}ml^2\right] - \frac{1}{6}mv^2$

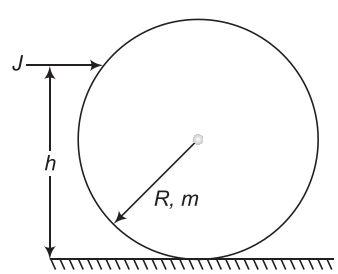
$= \left(\frac{1}{2} - \frac{1}{3}\right)mv^2$

$= \frac{1}{6}mv^2$

8. $I = mk^2 = \frac{1}{4}mR^2$

$J = mv$ and $J(h - R) = I\omega$

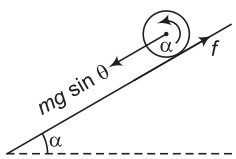
$\therefore \; mv(h - R) = \frac{1}{4}mR^2\omega$



For, $v = R\omega$, $h - R = \frac{1}{4}R \Rightarrow h = \frac{5}{4}R$ for pure rolling.

If impulse is through centre of mass, $h = R$, then there is no angular impulse and no rotation.

9.



$$fR = I\alpha = \frac{2}{3}mR^2\alpha$$

$$= \frac{2}{3}R\,ma \Rightarrow f = \frac{2}{3}ma \quad \ldots(i)$$

$$mg\sin\alpha - f = ma \Rightarrow mg\sin\alpha$$

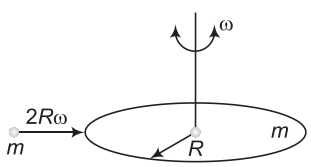
$$= \frac{5}{3}ma \Rightarrow a = \frac{3}{5}g\sin\alpha \quad \ldots(ii)$$

$$f \le \mu mg\cos\alpha;\ f = \frac{2}{3}ma = \frac{2}{3}m\cdot\frac{3}{5}g\sin\alpha$$

$$\mu \ge \frac{2mg\sin\alpha}{5mg\cos\alpha}$$

$$\Rightarrow \mu \ge \frac{2}{5}\tan\alpha$$

10.



$$\frac{1}{2}mR^2\omega = \left[\frac{1}{2}mR^2 + mR^2\right]\omega'$$

$$= \frac{3}{2}mR^2\omega'$$

$$\Rightarrow \omega' = \frac{\omega}{3}$$

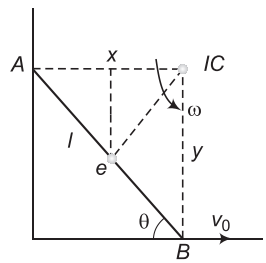
$$\vec{\mathbf{v}}_f = R\omega'\,\hat{\mathbf{j}},\ \vec{\mathbf{v}}_i = 2R\omega'\,\hat{\mathbf{i}} = \frac{1}{3}R\omega\,\hat{\mathbf{j}}$$

$$\therefore\ \Delta v = R\omega\sqrt{\frac{1}{9} + 4} = \frac{\sqrt{37}}{3}R\omega$$

$$\Rightarrow \Delta p = m\Delta v = \frac{\sqrt{37}}{3}mR\omega$$

$$= \text{Impulse}$$

11.



$$\frac{v_A}{x} = \frac{v_0}{y}$$

$$\frac{v_A}{l\cos\theta} = \frac{v_0}{l\sin\theta} \Rightarrow v_A = v_0\cot\theta$$

$$= v_0\cot 37^\circ = \frac{4}{3}v_0$$

$$\omega = \frac{v_0}{y} = \frac{v_0}{l\sin 37^\circ} = \frac{5v}{3l}$$

Match the Columns

1. $I_S = \frac{2}{5}mr^2, I_H = \frac{2}{3}mr^2, I_D = \frac{1}{2}mr^2$

(a) As I_H is maximum for hollow sphere, it takes maximum time to reach at the bottom. $\rightarrow q$

(b) As fall in height is same for all, so is their KE $\rightarrow s$

(c) Rotational KE is also maximum for I_H. $\rightarrow q$

(d) As rotational KE is minimum for I_S, so, its translational KE is maximum. $\rightarrow p$

2. (a) As D is at maximum distance from point of contact, so, angular impulse is maximum for $A \rightarrow p$

(b) Linear speed acquired will be same for any point $\rightarrow s$

(c) There can be pure rolling, if angular impulse is such that velocity of point of contact is zero. It is possible when impulse is applied at $A \rightarrow p$

(d) Forward slipping will be there for sure, if impulse is provided below E *i.e.*, at $C \rightarrow r$

3. (a) As the point of contact tries to move downward, so, friction acts upward. And the value of friction will be self adjusting $\rightarrow p, s$

(b) As the point of contact is slipping downward, so, limiting friction is acting upward $\rightarrow p, r$

(c) As the sphere is slipping upward, so, limiting friction is acting downward $\rightarrow q, r$

(d) As the sphere is slipping downward, so limiting friction is acting upward $\rightarrow p, r$

4. Radius of gyration $K = \sqrt{\dfrac{I}{m}}$

$$K_1 = \sqrt{\frac{m(2a)^2/3}{m}} = \frac{2a}{\sqrt{3}}$$

$$K_2 = \sqrt{\frac{m(a)^2/12}{m}} = \frac{a}{\sqrt{3}}$$

$$K_3 = \sqrt{\frac{ma^2/3}{m}} = \frac{a}{\sqrt{3}}$$

$$K_4 = \sqrt{\frac{ma^2/12}{m}} = \frac{a}{2\sqrt{3}}$$

5. $mgh = \dfrac{1}{2}\cdot\dfrac{2}{5}mr^2\omega^2 + \dfrac{1}{2}mv^2$

$= \dfrac{7}{10}mv^2 = \dfrac{7}{5}\cdot\dfrac{1}{2}mv^2$

$\therefore \dfrac{1}{2}mv^2 = \dfrac{5}{7}mgh = (\text{Translational KE})_2$

$\dfrac{1}{5}mv^2 = \dfrac{2}{5}\cdot\dfrac{1}{2}mv^2 = \dfrac{2}{5}\cdot\dfrac{5}{7}mgh$

$= \dfrac{2}{7}mgh = (\text{Rotational KE})_2 \Rightarrow a \rightarrow q$

$mg\dfrac{h}{2} = \dfrac{1}{2}\cdot\dfrac{2}{5}mr^2\omega^2 + \dfrac{1}{2}mu^2$

$= \dfrac{7}{10}mu^2 = \dfrac{7}{5}\cdot\dfrac{1}{2}mu^2$

$\dfrac{1}{2}mu^2 = \dfrac{5}{7}\cdot mg\dfrac{h}{2}$

$= \dfrac{5}{14}mgh = (\text{Translational KE})_3$

$\Rightarrow b \rightarrow s$

$(\text{Rotational KE})_4 = (\text{Rotational KE})_3$

$= \dfrac{1}{5}mr^2\omega'^2$

$= \dfrac{1}{5}mu^2 = \dfrac{2}{5}\cdot\dfrac{1}{2}mu^2$

$= \dfrac{2}{5}\cdot\dfrac{5}{14}mgh = \dfrac{1}{7}mgh \Rightarrow c \rightarrow p$

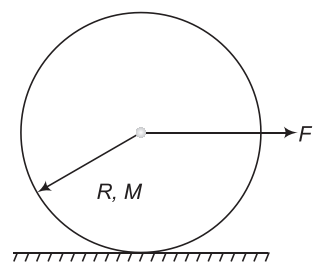
$(\text{Translational KE})_4$

$= mgh - \dfrac{1}{7}mgh = \dfrac{6}{7}mgh$

$\Rightarrow d \rightarrow s$

6. $F = \dfrac{80}{8}t = 10t$

$f_L = \mu Mg = 0.2 \times 10 \times 10 = 20\text{ N}$



$F - f = ma,\ fR = \dfrac{1}{2}mR^2\alpha$

$f = \dfrac{1}{2}mR\alpha$

For, $R\alpha = a$, $F = \dfrac{3}{2}ma$ and $f = \dfrac{1}{2}ma$

$f_{\max} = \dfrac{1}{2}ma_{\max} = 20\text{ N}$

$\Rightarrow F = 60\text{ N} = 10\dfrac{N}{5}\cdot t$

$\Rightarrow t = 6\text{ s}$, *i.e.*, there will be pure rolling till

$t = 6\text{ s} \Rightarrow a \rightarrow q, r$

$b \rightarrow p$

$c \rightarrow s$

$f = \dfrac{1}{2}ma = \dfrac{F}{3} = \dfrac{10t}{3} = 10\text{ N} \Rightarrow t = 3\text{ s}$

$d \rightarrow q$

7. (a) $I = \dfrac{1}{2}\cdot\dfrac{1}{2}mr^2 + mr^2 = \dfrac{5}{4}mr^2 \rightarrow r$

(b) $I = \dfrac{2}{5}mr^2 + mr^2 = \dfrac{7}{5}mr^2 \rightarrow q$

(c) $I = \dfrac{1}{2}mr^2 + mr^2 = \dfrac{3}{2}mr^2 \rightarrow s$

(d) $I = \dfrac{1}{2}mr^2 \rightarrow p$

10 Gravitation

Introductory Exercise 10.1

1. $g' = g - R\omega^2 \cos^2 \phi$

$$= g - 6.37 \times 10^6 \times \frac{4\pi^2}{(8.64 \times 10^4)^2} \times \cos^2 45°$$

$$= g - \frac{2\pi^2 \times 6.37 \times 10^{-2}}{(8.64)^2}$$

$$= g - 0.0168 \text{ m/s}^2$$

$$\therefore \Delta g = g - g' = 0.0168 \text{ m/s}^2$$

2. $g' = g - R\omega^2 = \frac{3}{5} g$

$$\Rightarrow \quad \frac{2}{5} g = R\omega^2$$

$$\text{or } \omega = \sqrt{\frac{2g}{5R}} = \sqrt{\frac{2 \times 10}{5 \times 6400 \times 10^3}}$$

$$= \sqrt{\frac{4}{6400 \times 10^3}} = \frac{1}{\sqrt{1600 \times 10^3}}$$

$$= \frac{1}{40 \times 10\sqrt{10}} = 7.9 \times 10^{-4} \text{ rad/s}$$

3. $g' = \dfrac{g}{\left(1 + \dfrac{h}{R}\right)^2} = 0.64g$

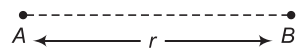
$$\Rightarrow 1 = 0.64\left(1 + \frac{h}{R}\right)^2$$

$$\text{or} \quad 0.8\left(1 + \frac{h}{R}\right) = 1$$

$$\Rightarrow \quad 1 + \frac{h}{R} = \frac{5}{4}$$

$$\Rightarrow \quad h = \frac{R}{4} = 1600 \text{ km}$$

4.



$$F = \frac{GM_A M_B}{r^2}$$

$$\Rightarrow \quad a = \frac{F}{M_A} = \frac{GM_B}{r^2}$$

$$\text{For } F' = \frac{GM_A M_B}{r^4}$$

$$\Rightarrow \quad a_{cc} = \frac{F'}{m_A} = \frac{GM_B}{r^4} = \frac{a}{r^2}$$

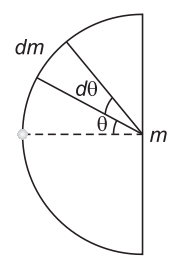
5. $F = \int dF \cos\theta = \int \dfrac{GMdm}{r^2} \cos\theta$

$$= \frac{GM}{r^2} \int_{-\pi/2}^{\pi/2} \frac{m}{\pi r} . rd\theta . \cos\theta$$

$$= \frac{GMm}{\pi r^2} \int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta$$

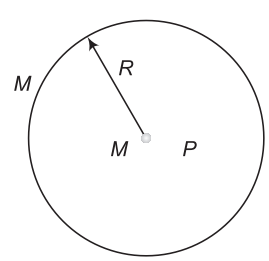
$$= \frac{2GMm}{\pi r^2}$$

$$= \frac{2GMm}{\pi\left(\dfrac{L}{\pi}\right)^2} = \frac{2\pi GMm}{L^2}$$



Introductory Exercise 10.2

1.



$$V = V_e + V_s = -\frac{GM}{R/2} - \frac{GM}{R} = -\frac{3GM}{R}$$

2. $\vec{\mathbf{g}} = \frac{\vec{\mathbf{F}}}{m} = \frac{4\hat{\mathbf{i}}}{20 \times 10^{-3}} = 200 \text{ m/s}^2$

$= 200 \text{ N/kg } \hat{\mathbf{i}}$

3. $\vec{\mathbf{E}} = -\left(\frac{\partial V}{\partial x}\hat{\mathbf{i}} + \frac{\partial V}{\partial y}\hat{\mathbf{j}} + \frac{\partial V}{\partial z}\hat{\mathbf{k}}\right)$

$= -(6\,xy\hat{\mathbf{i}} + 3x^2\hat{\mathbf{j}} + 3y^2 z\hat{\mathbf{j}} + y^3\hat{\mathbf{k}})$

4. Only the variation is given along x-axis, nothing is about y and z axis, so, the statement is false.

5. $\vec{\mathbf{E}} = -\vec{\nabla} V = -\left(\frac{\partial V}{\partial x}\hat{\mathbf{i}} + \frac{\partial V}{\partial y}\hat{\mathbf{j}} + \frac{\partial V}{\partial z}\hat{\mathbf{k}}\right)$

$20\,(x + y) = -20\,[\hat{\mathbf{i}} + \hat{\mathbf{j}}]$

$\vec{\mathbf{F}} = M\,\vec{\mathbf{E}} = \frac{1}{2}[-20\,(\hat{\mathbf{i}} + \hat{\mathbf{j}})]$

$= -10\,(\hat{\mathbf{i}} + \hat{\mathbf{j}})$

$\Rightarrow \quad F = 10\sqrt{2}$ N

6. $W = \int dW = \int \vec{\mathbf{F}}.\vec{\mathbf{ds}} = \int m\,\vec{\mathbf{E}}.(dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}})$

$= m\int E_x dx + m\int E_y dy$

$= 1\int_1^{-2} 2dx + 1\int_1^{3} 3dy$

$= 0$

Introductory Exercise 10.3

1. Escape velocity is given to overcome the potential barrier and just free the particle from a system, such that its total mechanical energy is just zero at infinity. So, the statement is true.

2. $K = \frac{GMm}{R} = \frac{GM}{R^2}.Rm = gmR.$

3. $Gm_1m_2\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2.$

$= \frac{1}{2}m_1v_1^2 + \frac{1}{2}\frac{m_1^2}{m_2}v_1^2$

$= \frac{1}{2}m_1v_1^2\left(1 + \frac{m_1}{m_2}\right)$

$\therefore\ Gm_2\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = \frac{1}{2}v_1^2\left(1 + \frac{m_1}{m_2}\right)$

or $\quad v_1 = \sqrt{\frac{2Gm_2\left(\frac{1}{r_2} - \frac{1}{r_1}\right)}{1 + \frac{m_1}{m_2}}}$

$= m_2\sqrt{\frac{2G\left(\frac{1}{r_2} - \frac{1}{r_1}\right)}{m_1 + m_2}}$

$= 10\sqrt{\frac{2 \times 6.67 \times 10^{-11}\,(2-1)}{20 + 30}}$

$= 10 \times 10^{-6}\sqrt{\frac{2 \times 6.67}{5}}$

$= \sqrt{\frac{8}{3}} \times 10^{-5}$ m/s

$= 1.63 \times 10^{-5}$ m/s

$v_2 = \frac{m_1}{m_2}v_1 = 2\,v_1$

$= 3.3 \times 10^{-5}$ m/s

4. $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - \frac{GMm}{R}$

$\Rightarrow \quad v = \sqrt{u^2 - \frac{2GM}{R}}$

$\therefore \quad v = \sqrt{u^2 - v_e^2}$

$= \sqrt{15^2 - (11.2)^2}$ km/s

$= 10$ km/s

5. (i) $\Delta U = \frac{GMm}{R} - \frac{GMm}{(1+n)R}$

$= \frac{GMm}{R}\left(1 - \frac{1}{1+n}\right)$

$= \frac{n}{n+1}.\frac{GM}{R^2}.mR = \left(\frac{n}{n+1}\right)mgR$

(ii) $K = \Delta U = \frac{1}{2}mv^2$

$\Rightarrow \quad v = \sqrt{\frac{2\Delta U}{m}} = \sqrt{\frac{2ngR}{n+1}}$

Introductory Exercise 10.4

1. $\Delta K = \Delta U$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{GMm}{R} - 0 = \frac{GM}{R^2}Rm = mgR$$

$$\Rightarrow v = \sqrt{2Rg}$$

$$\therefore\ v = \sqrt{2 \times 6400 \times 10^3 \times 10}$$

$$= 8\sqrt{2} \times 10^3 \text{ m/s} = 8\sqrt{2} \text{ km/s} = 11.2 \text{ km/s}$$

2. In planetary motion areal velocity, *i.e.*, angular momentum and total mechanical energy is conserved.

3. $\dfrac{mv_1^2}{2R} = \dfrac{GMm}{(2R)^2}$

$$\Rightarrow \quad \frac{1}{2}mv_1^2 = \frac{GMm}{4R}$$

and $\quad \dfrac{mv_2^2}{4R} = \dfrac{GMm}{(4R)^2}$

$$\Rightarrow \quad \frac{1}{2}mv_2^2 = \frac{GMm}{8R}$$

$$\Rightarrow \quad \frac{K_1}{K_2} = \frac{1/4}{1/8} = \frac{2}{1} \Rightarrow 2:1$$

$$\frac{U_1}{U_2} = \frac{\dfrac{-GMm}{2R}}{\dfrac{-GMm}{4R}} = \frac{2}{1} \Rightarrow 2:1$$

4. $T = \dfrac{2\pi r}{v}; \dfrac{mv^2}{r} = \dfrac{GMm}{r^2}$

$$\Rightarrow v = \sqrt{\frac{GM}{r}}$$

$$\therefore\ T = \frac{2\pi r}{\sqrt{\dfrac{GM}{r}}} = \frac{2\pi r\sqrt{r}}{\sqrt{G\rho\dfrac{4}{3}\pi R^3}}$$

$$\Rightarrow \quad T^2 = \frac{4\pi^2 r^3}{G\rho\dfrac{4}{3}\pi R^3}$$

$$\Rightarrow \quad \rho T^2 = \frac{3\pi r^3}{GR^3}$$

For $r \simeq R, \rho T^2 = \dfrac{3\pi}{G} = \text{constant}$

5. $\dfrac{m^2 v_o^2}{r} = \dfrac{GMm}{r^2}$

$$\Rightarrow \quad v_o = \sqrt{\frac{GM}{r}}$$

(a) $v = \sqrt{1.5}\,v_o = 1.22\,v_o$

While, $v_e = \sqrt{2}\,v_o = 1.41 v_o$, so, the satellite will not escape from the planet, rather it will revolve in elliptical orbit.

(b) As, $v_e = \sqrt{2}\,v_o$, while, $v = 2\,v_o$, *i.e.*, the satellite will escape.

AIEEE Corner

Subjective Questions (Level 1)

1. $a_1 = \dfrac{F}{m_1} = \dfrac{GM_2}{r^2}$

$$= \frac{6.67 \times 10^{-11} \times 2}{(1/2)^2}$$

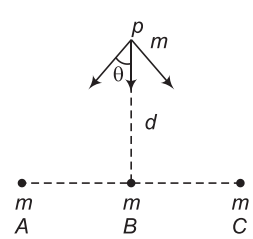
$$= 8 \times \frac{20}{3} \times 10^{-11} \text{ m/s}^2$$

$$= 5.3 \times 10^{-10} \text{ m/s}^2$$

$$a_2 = \frac{F}{m_2} = \frac{GM_1}{r^2} = \frac{1}{2}.a_1$$

$$= 2.65 \times 10^{-10} \text{ m/s}^2$$

2.

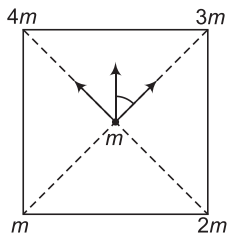


$$F = \frac{Gm^2}{d^2} + 2.\frac{Gm^2}{(\sqrt{2}d)^2}.\cos 45°$$

$$= \frac{Gm^2}{d^2}\left(1 + \frac{1}{\sqrt{2}}\right)$$

And the net force is directed along *PB*.

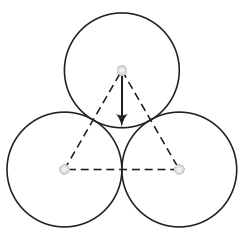
3.



$$F = 2 \cdot \frac{G.m.2m}{(a/\sqrt{2})^2} \cos 45°$$

$$= \frac{4\sqrt{2}\, Gm^2}{a^2}$$

4.



$$F = 2\frac{G.MM}{(2a)^2} \cos 30° = \frac{\sqrt{3}GM^2}{4a^2}$$

5. $F = \int_d^{d+l} \frac{G.M.\lambda dx}{x^2} = GM\lambda\left(-\frac{1}{x}\right)_d^{d+l}$

$$= GM\lambda\left(\frac{1}{d} - \frac{1}{d+l}\right) = \frac{GM\lambda l}{d(d+l)}$$

6. $g = \frac{GM}{R^2}$,

$$g' = \frac{G2M}{4R^2} = \frac{1}{2}\frac{GM}{R^2} = \frac{g}{2} = 4.9 \text{ m/s}^2$$

7. (a) $g = \frac{GM}{R^2}$, $g' = \frac{GM}{(R+R)^2} = \frac{1}{4}\frac{GM}{R^2}$

$$= \frac{1}{4} g = 2.45 \text{ m/s}^2$$

(b) $g = \frac{GM}{R^2}$ and

$$g' = \frac{GM}{R^3} \cdot r = \frac{GM}{R^3}(R - R/2)$$

$$= \frac{1}{2}\frac{GM}{R^2} = \frac{g}{2} = 4.9 \text{ m/s}^2$$

8. $\frac{GM}{R^3}(R - x) = \frac{GM}{(R+x)^2}$

$$\Rightarrow (R - x)(R + x)^2 = R^3$$

$$\Rightarrow (R^2 - x^2)(R + x) = R^3$$

or $R^3 + R^2x - x^2R - x^3 = R^3$

or $x^3 + x^2 R - xR^2 = 0$

or $x^2 + Rx - R^2 = 0$

$$\therefore\ x = \frac{1}{2}[-R \pm \sqrt{R^2 + 5R^2}]$$

$$= \frac{1}{2}[-R + \sqrt{5}R]$$

$$\therefore\ x = \frac{(\sqrt{5} - 1)}{2} R$$

9. $w_p = mg$ and $w_e = mg - m\omega^2 R$

$$\therefore\ w_e = mg\left[1 - \frac{\omega^2 R}{g}\right] = mg\left[1 - \frac{4\pi^2 R}{T^2 g}\right]$$

$$= 1000\left[1 - \frac{4 \times 6400 \times 10^3}{(86400)^2}\right]$$

$$= 1000\left[1 - \frac{4 \times 6.4 \times 10^{-2}}{(8.64)^2}\right]$$

$$= 1000 \times 0.997 = 997 \text{ N}$$

10. $g_{\text{apparent}} = g - \omega^2 R = 0$

$$\Rightarrow \omega = \sqrt{g/R} = \frac{2\pi}{T}$$

$$\therefore\ T = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{6400 \times 10^3}{10}}$$

$$= 16\pi \times 10^2$$

$$= 5026.5 \text{ s} = 1.4 \text{ h}$$

and $\omega = \sqrt{g/R} = \sqrt{\frac{10}{6400 \times 10^3}}$

$$= \frac{1}{8 \times 10^2} = 1.25 \times 10^{-3} \text{ rad/s}$$

11. $g(h) = \frac{g}{\left(1 + \frac{h}{R}\right)^2} = g - \omega^2 R$

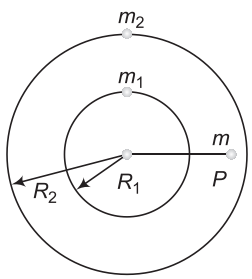
$$\therefore\ \omega^2 R = g\left[1 - \left(1 + \frac{h}{R}\right)^{-2}\right] = g\left[\frac{2h}{R}\right]$$

or $h = \frac{\omega^2 R^2}{2g} = \frac{4\pi^2 R^2}{2T^2 g} = \frac{2R^2}{T^2}$

$$= 2\left(\frac{6400 \times 10^3}{86400}\right)^2$$

$$= 2\left(\frac{640}{8.64}\right)^2 = 10.97 \times 10^3 \text{m} \simeq 11 \text{ km}$$

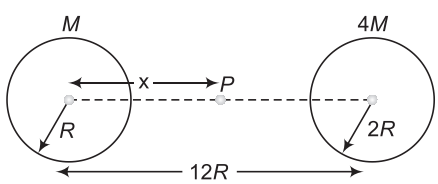
12.



There is no gravitational field at P due to outer shell, so it has no effect. Only m_1 has an effect at P. Thus,

$$F = \frac{Gm_1 m}{\left(\frac{R_1 + R_2}{2}\right)^2} = \frac{4Gm_1 m}{(R_1 + R_2)^2}$$

13.



(a) $\frac{GM}{x^2} = \frac{G.4M}{(12R - x)^2}$ gives field to be zero at P.

$$\therefore \quad \frac{12R - x}{x} = 2$$

$$\Rightarrow 12R = 3x \Rightarrow x = 4R$$

(b) At the surface of second sphere,

$$V = -\frac{G.4M}{2R} - \frac{GM}{10R}$$

$$= -\frac{GM}{R}\left(2 + \frac{1}{10}\right)$$

$$= -\frac{2.1\,GM}{R}$$

At a distance y from the smaller sphere,

$$V' = -\frac{GM}{y} - \frac{G4M}{(12R - y)}$$

$$= -\frac{GM}{y}\left(1 + \frac{4y}{12R - y}\right)$$

as, $V' = \frac{V}{2}$

$$\Rightarrow -\frac{GM}{y}\left[1 + \frac{4y}{12R - y}\right]$$

$$= \frac{1}{2}\left(-\frac{2.1\,GM}{R}\right)$$

$$\frac{12R + 3y}{y(12R - y)} = \frac{1.05}{R}$$

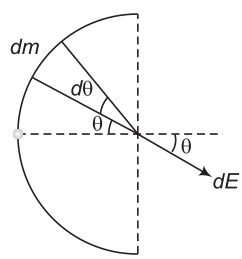
$$\therefore \quad 12R^2 + 3yR = 12.6yR - 1.05y^2$$

or $\quad 1.05y^2 - 9.6Ry + 12R^2 = 0$

$$\therefore \quad y = \frac{R}{2.1}[9.6 \pm \sqrt{(9.6)^2 - 4 \times 1.05 \times 12)}]$$

$$= \frac{R}{2.1}[9.6 \pm 6.5] = 7.7R,\ 1.5\,R$$

14.



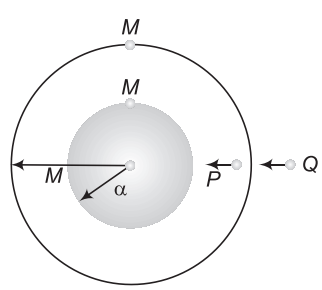
$$E = \int dE \cos\theta$$

$$= \int \frac{Gdm}{r^2}\cos\theta = \int_{-\pi/2}^{\pi/2} \frac{G}{r^2}.\ \lambda r d\theta.\cos\theta$$

$$= \frac{2G\lambda}{r} = \frac{2G}{r}.\frac{M}{\pi r} = \frac{2GM}{\pi r^2} = \frac{2GM}{\pi(L/\pi)^2}$$

$$= \frac{2\pi GM}{L^2}$$

15.



(a) $E_P = \dfrac{GM}{\left(\frac{3}{2}a\right)^2} = \dfrac{4GM}{9a^2}$ and

(b) $E_Q = \dfrac{G2M}{\left(\frac{5}{2}a\right)^2} = \dfrac{8GM}{25a^2}$

16. $M = \int dm = \int_0^a \rho.4\pi r^2 dr = \int_0^a \dfrac{\rho_0 a}{r}.4\pi r^2 dr$

$= 4\pi\rho_0 a\int_0^a rdr$

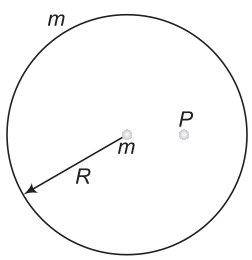
$= 4\pi\rho_0 \; a \times \dfrac{1}{2}a^2$

$= 2\pi\rho_0 a^3$

$E = \dfrac{GM}{(2a)^2} = \dfrac{G2\pi\rho_0 a^3}{4a^2}$

$= \dfrac{\pi G\rho_0 a}{2}$

17.



$V_P = V_{\text{point}} + V_{\text{shell}}$

$= -\dfrac{Gm}{R/2} - \dfrac{Gm}{R}$

$= -\dfrac{3Gm}{R}$

18. $\dfrac{1}{2}mv^2 = \dfrac{1}{2}m\,(2\sqrt{gR})^2 - \dfrac{GMm}{R}$

$= \dfrac{1}{2}m\,4gR - gRm$

$= mgR$

$v = \sqrt{2Rg}$

19. (a) $2 \times \dfrac{1}{2}mu^2 = GM^2\left(\dfrac{2}{r} - \dfrac{1}{r}\right) = \dfrac{GM^2}{r}$

$u = \sqrt{\dfrac{GM}{r}} = \sqrt{\dfrac{6.67 \times 10^{-11} \times 10^{30}}{10^{10}}}$

$= 10^4\sqrt{66.7}$ m/s $= 8.17 \times 10^4$ m/s

$= 82$ km/s

(b) $2 \times \dfrac{1}{2}Mv^2 = GM^2\left(\dfrac{1}{2 \times 10^5} - \dfrac{1}{10^{10}}\right)$

$v = \sqrt{GM\dfrac{10^{10} - 2 \times 10^5}{2 \times 10^{15}}}$

$\simeq \sqrt{\dfrac{6.67 \times 10^{-11} \times 10^{30}}{2 \times 10^5}}$

$= 10^7\sqrt{\dfrac{6.6.7}{2}}$

$= 1.83 \times 10^7$ m/s $= 1.8 \times 10^4$ km/s

20. $h = \dfrac{v^2}{2g - v^2/R}$

$= \dfrac{(10 \times 10^3)^2}{2 \times 9.81 - \dfrac{(10 \times 10^3)^2}{(6400 \times 10^3)}} = 2.5 \times 10^7$ m

$= 2.5 \times 10^4$ km

21. $\dfrac{1}{2}mv^2 = \dfrac{GMm}{R + R}$

$\Rightarrow v = \sqrt{\dfrac{GM}{R}} = \sqrt{Rg} = \sqrt{64 \times 10^5 \times 10}$

$= 8 \times 10^3$ m/s = 8 km/s

22. (a)

$U_A - U_B = \left(\dfrac{G \times 400 \times 1}{2} + \dfrac{G \times 100 \times 1}{8}\right)$

$-\left(\dfrac{G \times 400 \times 1}{8} + \dfrac{G \times 100 \times 1}{2}\right)$

$= 100G\left(2 + \dfrac{1}{8}\right) - 100G\left(\dfrac{1}{2} + \dfrac{1}{2}\right)$

$= 100G \times \dfrac{9}{8} = \dfrac{900}{8} \times 6.67 \times 10^{-11}$ J

$= 7.5 \times 10^{-9}$ J

(b) For $E = 0$,

$\dfrac{G \times 400}{x^2} = \dfrac{G \times 100}{(10 - x)^2}$

$\Rightarrow \dfrac{x}{10 - x} = 2 \Rightarrow x = \dfrac{20}{3}$ m, *i.e.*, the particle has to be given such a velocity, that it

can travel from $2m(A)$ to $\frac{20}{3}$ m, then after it will reach at B

$$\therefore K = \Delta U = \left(\frac{G \times 400 \times 1}{2} + \frac{G \times 100 \times 1}{8}\right) - \left(\frac{G \times 400 \times 1}{20/3} + \frac{G \times 100 \times 1}{10/3}\right)$$

$$= 100\,G\left(2 + \frac{1}{8}\right) - 100\,G\left(\frac{6}{10} + \frac{3}{10}\right)$$

$$= 122.5\,G = 8.17 \times 10^{-9}\ \text{J}$$

23. (a)

$$\text{TE} = \Delta U + K = \frac{GMm}{R} - \frac{GMm}{2R} + \frac{1}{2}mv^2$$

$$= \frac{GMm}{2R} + \frac{1}{2}\cdot\frac{GMm}{2R} = \frac{3}{4}\cdot\frac{GMm}{R} = \frac{3}{4}mgR$$

$$= \frac{3}{4} \times 2 \times 10^3 \times 10 \times 6400 \times 10^3$$

$$= \frac{3}{2} \times 6.4 \times 10^{10}\ \text{J} = 9.6 \times 10^{10}\ \text{J}$$

(b) $\text{TE}' = \Delta U' + K'$

$$= \frac{GMm}{R} - \frac{GMm}{3R} + \frac{1}{2}\frac{GMm}{3R}$$

$$= \frac{GMm}{R}\left(1 - \frac{1}{3} + \frac{1}{6}\right)$$

$$= \frac{5}{6}mgR = \frac{5}{6} \times 2 \times 10^3 \times 10 \times 6400 \times 10^3$$

$$= \frac{5}{3} \times 6.4 \times 10^{10}\ \text{J} = 10.7 \times 10^{10}\ \text{J}$$

$\therefore$ Energy needed

$= (10.7 - 9.6) \times 10^{10}\ \text{J} = 1.1 \times 10^{10}\ \text{J}$

24. (a) $F = \frac{GM^2}{4R^2} = \frac{mv^2}{R}$

(b) $v = \sqrt{\frac{GM}{4R}}$,

$$T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{GM/4R}} = \frac{4\pi R\sqrt{R}}{\sqrt{GM}}$$

(c) $E = \frac{GMM}{2R} = \frac{GM^2}{2R}$

25. (a) $\text{KE} = \frac{1}{2}mv^2 = \frac{r}{2}\frac{mv^2}{r}$

$$= \frac{r}{2}\cdot\frac{GMm}{r^2} = \frac{GMm}{2r}$$

$$\text{PE} = -\frac{GMm}{r}$$

$$\text{TE} = 2\,(\text{KE} + \text{PE}) = 2\left(\frac{GMm}{2r} - \frac{GMm}{r}\right)$$

$$= -\frac{GMm}{r}$$

(b) $\text{PE} = -\frac{GM\,2m}{r}$

$\therefore$ $\text{TE} = -\frac{2GMm}{r}$ and the satellites will start falling towards the centre of earth.

26. $T = \frac{2\pi R}{v}$

where, $\frac{mv^2}{R} = \frac{GMm}{R^2} \Rightarrow v = \sqrt{\frac{GM}{R}}$

and $T = \frac{2\pi R^{3/2}}{\sqrt{GM}}$ as, $T \propto R^{3/2}$

$$\therefore \quad \frac{T_B}{T_A} = \left(\frac{R_B}{R_A}\right)^{3/2} = 2^{3/2} = 2\sqrt{2}$$

$$\Rightarrow \quad T_B = 56\sqrt{2}\ \text{h}$$

$$\frac{v_B}{v_A} = \sqrt{\frac{R_A}{R_B}} = \frac{1}{\sqrt{2}}$$

$$T = \frac{2\pi R\sqrt{R}}{\sqrt{GM}}$$

$$\Rightarrow \quad \sqrt{\frac{GM}{R}} = \frac{2\pi R}{T} = v$$

$$\therefore v_A = \frac{2\pi \times 10^4}{28}\ \text{km/h} = 2244\ \text{km/h}$$

$$v_B = \frac{v_A}{\sqrt{2}} = 1587\ \text{km/h}$$

$$v_{\text{rel}} = v_A + v_B = 3831\ \text{km/h}$$

27. (a) $\frac{mv^2}{r} = \frac{GMm}{r^2}$, $v = \sqrt{\frac{GM}{r}}$

$$= \sqrt{\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{8400 \times 10^3}}$$

$$= 6.9 \times 10^3\ \text{m/s} = 6.9\ \text{km/s}$$

(b) $K = \frac{1}{2}mv^2 = \frac{1}{2} \times 10^3 \times (6.9)^2 \times 10^6\ \text{J}$

$$= 2.38 \times 10^{10}\ \text{J}$$

(c) $U = -\dfrac{GMm}{r}$

$$= -\frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 10^3}{8400 \times 10^3}$$

$$= -4.76 \times 10^{10}\ \text{J}$$

(d) $T = \dfrac{2\pi r}{v} = \dfrac{2\pi \times 8400}{6.9}\ \text{s} = 2.12\ \text{h}$

28. $\dfrac{Mv^2}{R/2} = \dfrac{GMm}{R^2}$

$$\Rightarrow v = \sqrt{\frac{GM}{2R}},\ T = \frac{2\pi R/2}{v} = \frac{\sqrt{2}\pi R\sqrt{R}}{\sqrt{GM}} \quad \text{...(i)}$$

For earth-sun system, $\dfrac{mv_0^2}{R} = \dfrac{GMm}{R^2}$

$$\Rightarrow \quad v_0 = \sqrt{\frac{GM}{R}} \text{ and}$$

$$T_0 = \frac{2\pi R}{v_0} = \frac{2\pi R\sqrt{R}}{\sqrt{GM}} \quad \text{...(ii)}$$

From Eqs. (i) and (ii)

$$T = \frac{1}{\sqrt{2}}\frac{2\pi R\sqrt{R}}{\sqrt{GM}} = \frac{T_0}{\sqrt{2}} = \frac{1}{\sqrt{2}}\ \text{yr} = 0.71\ \text{yr}$$

29. (a)

$$E_1 = \frac{GMm}{R} - \frac{GMm}{R+h} = \frac{GMm}{R}\left(1 - \frac{R}{R+h}\right)$$

$$= \frac{GMm}{R}\left(1 - \frac{6400}{7900}\right) = \frac{15}{79}\cdot\frac{GMm}{R}$$

$$E_2 = \frac{1}{2}mv^2 = \frac{r}{2}\cdot\frac{mv^2}{r} = \frac{r}{2}\cdot\frac{GMm}{r^2}$$

$$= \frac{GMm}{2(R+h)}$$

$$= \frac{GMm}{R}\left(\frac{R}{2(R+h)}\right) = \frac{GMm}{R}\left(\frac{6400}{2 \times 7900}\right)$$

$$= \frac{32}{79}\cdot\frac{GMm}{R}$$

$\Rightarrow E_2 > E_1$

So, it requires less energy to take it to 1500 km above, than to put it in circular orbit.

(b) For, $E_1 = E_2 \cdot \dfrac{GMm}{R}$

$$\frac{h}{R+h} = \frac{GMm}{2(R+h)}$$

$\Rightarrow h = R/2 = 6370/2 = 3185$ km. Thus for 3185 km height, energy required to lift and that required to put in orbit are equal.

(c) Similarly for 4500 km, energy required to lift will be more than that to put it in orbit.

Objective Questions (Level 1)

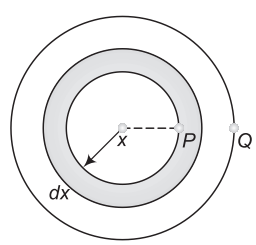
1. Centripetal force is required for circular motion, which is provided gravitational pull by earth on satellite.

2. In planetary motion total mechanical energy and angular momentum (areal velocity) are conserved.

3. Due to rotation and earth, $g_{\text{eff}} = g - R\omega^2\cos^2\phi$, where $\phi = 90°$ at poles, such that at poles $g_{\text{eff}} - g$, which do not depend on rotation of earth.

4. The particle has to be fired with escape velocity, which is independent of angle of projection.

5.



$$V_Q = -\frac{GM}{R},\ dV = -\frac{G.\dfrac{M}{\frac{4}{3}\pi R^3}\cdot\frac{4}{3}\pi x^3}{x^2}dx$$

$$= -\frac{GM}{R^3}\,.\,x\,dx$$

$$\Delta V = +\frac{GM}{R^3}\int_R^r x dx = -\frac{MG}{R^3}\frac{1}{2}(R^2 - r^2)$$

$$\therefore\ V_P = V_Q + \Delta V = -\frac{GM}{R} - \frac{GM}{2R} + \frac{GM}{2R^3}.r^2$$

$$= -\frac{3GM}{2R} + \frac{GM}{2R^3}\,.\,r^2,$$

is the equation of a parabola

(assuming earth to be of uniform density)

6. In electrostatics, $\oint \vec{\mathbf{E}} \cdot \vec{\mathbf{ds}} = \frac{q}{\varepsilon_0}$ and in gravitation, $-\oint \vec{\mathbf{g}}.\vec{\mathbf{ds}} = 4\pi Gm$

as $q = m$ and $\frac{1}{4\pi\varepsilon_0} = G$.

7. In centre of mass system, $\omega_1 = \omega_2$

$\Rightarrow \quad T_1 = T_2$

8. Field inside a shell is zero, so, a mass placed inside it do not experience any force.

9. $T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{\frac{GM}{R}}} = \frac{2\pi R^{3/2}}{\sqrt{GM}}$

$\Rightarrow \quad \frac{T_2}{T_1} = \left(\frac{R_2}{R_1}\right)^{3/2} = \left(\frac{1}{2}\right)^{3/2} = \frac{1}{2\sqrt{2}}$

$\therefore \quad T_2 = \frac{365}{2\sqrt{2}} = 129$ day

10. In planetary motion, $\frac{dA}{dt} = \text{constant}$

$\Rightarrow \quad \Delta A \propto \Delta T$

$\frac{\Delta A_{AB}}{\Delta A_{CD}} = \frac{T_{AB}}{T_{CD}}$

$\Rightarrow \quad \frac{A}{2A} = \frac{T_1}{T_2}$

$\Rightarrow \quad T_1 = 0.5\, T_2$

11. $T \propto \frac{1}{\sqrt{g}}$, $g_{\text{eff}} = g\left(1 - \frac{d}{R}\right)$

$\frac{T_2}{T_1} = \sqrt{\frac{g_1}{g_2}} = \sqrt{\frac{g}{g\left(1 - \frac{d}{R}\right)}}$

$= \left(1 - \frac{d}{R}\right)^{-1/2} \simeq 1 + \frac{d}{2R}$

$1 + \frac{0.5}{100} = 1 + \frac{d}{2R}$

$\Rightarrow \quad d = \frac{R}{100} = 64$ km.

12. $g = \frac{GM}{R^2}$

$\Rightarrow \quad \frac{g}{G} = \frac{M}{R^2}$

13. $g_{\text{eff}} = g\left(1 + \frac{h}{R}\right)^{-2} = \frac{g}{100}$

$\Rightarrow \quad \left(1 + \frac{h}{R}\right)^2 = 100$

$\Rightarrow \quad 1 + \frac{h}{R} = 10$

or $\quad h = 9R$

14. $T = \frac{2\pi R^{3/2}}{\sqrt{GM}}$, $\frac{T_2}{T_1} = \left(\frac{R_2}{R_1}\right)^{3/2} = \left(\frac{4R}{R}\right)^{3/2}$

$= 2^3 = 8$

$\Rightarrow \quad T_2 = 8T_1$

$\therefore \quad T_2 = (84) \times 8$ min

15. $g_{\text{eff}} = g - \omega^2 R = g/2$

$\Rightarrow \quad \omega^2 R = g/2$

$\Rightarrow \quad \omega = \sqrt{\frac{g}{2R}}$

$\therefore \quad \omega = \sqrt{\frac{10}{2 \times 6400 \times 10^3}}$

$= \frac{1}{8\sqrt{2} \times 10^{+2}}$ rad/s

$= 8.8 \times 10^{-4}$ rad/s

16. At centre, $V_c = -\frac{3GM}{2R}$ and $V_h = -\frac{GM}{R+h}$

$V_h = \frac{1}{2} V_c$

$\frac{GM}{R+h} = \frac{3}{4} \cdot \frac{GM}{R}$

$\Rightarrow \quad 4R = 3\,(R + h)$

$\Rightarrow \quad h = R/3$

17. $U_c = -\frac{GMm}{R}$, $U_h = -\frac{GMm}{4R}$

$\Rightarrow \Delta U = \frac{GMm}{R} - \frac{GMm}{4R} = \frac{3}{4} \cdot \frac{GMm}{R}$

$= \frac{3}{4} mgR$

18. $T = \frac{2\pi R^{3/2}}{\sqrt{GM}}$ is independent of the radius of planet.

19. $v_e = \sqrt{\frac{2GM}{R}}$

$= \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 7.34 \times 10^{22}}{1.7 \times 10^6}}$ m/s

$= 10^3 \sqrt{\frac{2 \times 6.67 \times 7.34}{17}}$ m/s

$= 2.4$ km/s $= 2.4 \times 10^3$ m/s

20. $g = \frac{GM}{R^2} = \frac{G\frac{4}{3}\pi R^3 \rho}{R^2}$

$= \frac{4}{3}\pi G\rho R \propto \rho R$

$\therefore \rho_1 R_1 = \rho_2 R_2 = 2\rho_1 R_2$

$\Rightarrow \quad R_2 = \frac{R_1}{2} = R/2$

21. $g_{\text{eff}} = g - \omega^2 R$ for $g_{\text{eff}} = 0$,

$\omega = \sqrt{g/R} = \sqrt{\frac{10}{6400 \times 10^3}}$ rad/s

$\omega = \frac{1}{800}$ rad/s, presently,

$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{86400}$ rad/s

$\frac{\omega}{\omega_0} = \frac{86400}{1600\pi} = 17.2$

$\Rightarrow \quad \omega = 17.2\,\omega_0 = x\omega_0$

$\Rightarrow \quad x = 17.2$

22. $\omega_{\text{rel}} = \omega_s + \omega_e$

$\frac{1}{T_{\text{rel}}} = \frac{1}{T_s} + \frac{1}{T_e}$

$\Rightarrow \quad \frac{1}{T_s} = \frac{1}{T_{\text{rel}}} - \frac{1}{T_e} = \frac{1}{6} - \frac{1}{24}$

$= \frac{3}{24} = \frac{1}{8} \Rightarrow T_s = 8$ h

$\therefore \quad \omega_s = \frac{2\pi}{T_s} = \frac{2\pi}{8} = \frac{\pi}{4}$

23. According to Keplar's law, $T^2 \propto a^3$

24. External shell do not produce any field inside, so, $E = \frac{GM_1}{a^2}$

25. $t = \frac{T}{2} = \frac{84 \text{ min}}{2} = 42$ min.

26. $U = U_1 + U_2 + U_3$

$= 0 - \frac{Gm^2}{a} - \frac{2Gm^2}{a} = -\frac{3Gm^2}{a}$

$\Delta U = \frac{3Gm^2}{l} - \frac{3Gm^2}{2l} = \frac{3Gm^2}{2l}$

27. $v_e = \sqrt{\frac{2GM}{R}}, \frac{1}{2}mv^2$

$= \frac{GMm}{R} - \frac{GMm}{2R}$

$= \frac{GMm}{2R}$

$\Rightarrow \quad v = \sqrt{\frac{GM}{R}}$

$\therefore \quad \frac{v}{v_e} = \frac{1}{\sqrt{2}}$

28. $U = -\frac{GMm}{r}, g_{2r} = -\frac{GM}{4r^2}$

$= -\frac{1}{4mr} \cdot \frac{GMm}{r} = \frac{U}{4mr}$

$\therefore \quad w = mg_{2r} = \frac{U}{4r}$

JEE Corner

Assertion and Reason

1. As PE is proportional to negative of inverse of distance that is why PE decreases with decrease in separation. Thus assertion and reason are true but not the correct explanation.
2. Same explanation as Answer 1
3. While going from A to B, PE first increases, becomes maximum at middle and again decreases. Assertion and reason are true but not correct explanation.
4. $E = -\frac{dV}{dx}$, at zero, slope of V - x graph E is zero if E is a function of x only, otherwise $\vec{\mathbf{E}} = -\vec{\nabla} V$. So, assertion is not always true.
5. $\frac{1}{2}mv^2 = \frac{GMm}{R}\left(1 - \frac{R}{R+h'}\right)$ and

$$\frac{1}{2}m4v^2 = \frac{GMm}{R}\left(1 - \frac{R}{R+h'}\right)$$

$$\therefore \quad 4 = \frac{1 - \frac{R}{R+h'}}{1 - \frac{R}{R+h}} = \frac{h'(R+h)}{h(R+h')}$$

$$\Rightarrow \quad 4hR + 4hh' = Rh' + hh'$$

$$\Rightarrow 4\,hR = Rh' - 3hh' = (R - 3h)h'$$

$$\Rightarrow h' = 4h.\frac{R}{R-3h} \neq 4h$$

So, assertion is false.

6. In planetary motion, angular momentum of the planet is constant but it is not true for the system. So, assertion is false.
7. Only geo-stationary satellites are on equatorial plane, not all. So, assertion is false.
8. Reason is true for explanation of assertion.
9. Assertion is true but reason is false (and even Moscow is not at equator).
10. Both assertion and reason are true but not correct explanation.

As, $U = -\frac{GMm}{r}$, $K = \frac{1}{2}mv^2$ and

$$E = U + K = -\frac{GMm}{2r}$$

11. $g = \frac{GM}{R^2}$, so as R is decreased, g increase, thus assertion is false.

Objective Questions (Level 2)

1. $E_1 = -\frac{GMm}{2R}, \frac{mv^2}{2R} = \frac{GMm}{4R^2}$

$$\Rightarrow \quad v = \sqrt{\frac{GM}{2R}}$$

Finally, $E_2 = -\frac{GMm}{2R} + \frac{1}{2}.\frac{m}{2}.4v^2$

$$= -\frac{GMm}{2R} + mv^2$$

$$\therefore \quad \Delta E = E_2 - E_1 = -\frac{GMm}{2R} + mv^2 + \frac{GMm}{2R} = mv^2$$

$$= m.\frac{GM}{2R} = \frac{1}{2}.m\frac{GM}{R^2}R = \frac{1}{2}mgR$$

2. **See : Q. 5, P-92**
3. **See : Q. 5, P-123**
4. $F = \int dF \cos\theta = \int \frac{G.Mdm}{(\sqrt{3a^2 + a^2})^2}.\frac{\sqrt{3}a}{2a}$

$$= \frac{\sqrt{3}}{8}.\frac{GMm}{a^2}$$

5. $\frac{Mv^2}{R} = 2.\frac{GM^2}{(\sqrt{2}R)^2}\cos 45° + \frac{GM^2}{4R^2}$

$$= \frac{GM^2}{R^2}\left(\frac{1}{\sqrt{2}} + \frac{1}{4}\right)$$

$$v = \sqrt{\frac{GM}{R}\frac{4+\sqrt{2}}{4\sqrt{2}}} = \sqrt{\frac{GM}{R}\frac{2\sqrt{2}+1}{4}}$$

6. $\frac{1}{2}mv^2 = \frac{GMm}{R}\left(1-\frac{R}{r}\right)$

or $\frac{1}{2}mh^2\frac{2GM}{R} = \frac{GMm}{R}\left(1-\frac{R}{r}\right)$

$\frac{R}{r} = 1-h^2$

or $r = \frac{R}{1-h^2}$

7. $g = \frac{GM'}{y^2}$

$= \frac{G}{y^2}\cdot\frac{M}{\frac{4}{3}\pi R^3}\cdot\frac{4}{3}\pi y^3 = \frac{4}{3}\pi\rho G y$

8. $N = mg_{\text{eff}} - \frac{mv^2}{R}$

$= m(g-\omega'^2 R) - \frac{mv^2}{R}$

$= mg - mg\frac{(\omega')^2 R}{g} - mg\frac{v^2}{Rg}$

$= mg\left[1-\frac{\omega'^2 R}{g}-\frac{v^2}{Rg}\right];$

$(\omega')^2 R = \omega^2 R - 2\frac{v}{R}.\omega R$

$= \omega^2 R - 2v\omega$

$= mg\left[1-\frac{(\omega R-2v)\omega}{g}-\frac{v^2}{Rg}\right]$

9. According to Gauss' law field inside the cavity is uniform and depends upon the mass of sphere of radius a.

10. $U_c = -\frac{3GMm}{2R}$

$\therefore \quad \frac{1}{2}mv_e'^2 = \frac{3GMm}{2R}$

or $v_e' = \sqrt{\frac{3GM}{R}} = \sqrt{3\left(\frac{v_e^2}{2}\right)} = \sqrt{\frac{3}{2}}v_e$

11. $g = -\frac{GM}{r^{2.5}} = \frac{-dV}{dr}$

$\Rightarrow \quad v = GM\int_{\infty}^{r}\frac{dr}{r^{2.5}}$

$= +GM\left.\frac{r^{-2.5+1}}{-2.5+1}\right|_{\infty}^{r}$

$= \frac{-GM}{1.5r^{1.5}} = \frac{-2GM}{3r^{1.5}}$

12. $\frac{Mv^2}{R} = 2\cos 30^\circ\frac{GM^2}{(\sqrt{3}R)^2}$

$= \frac{GM^2}{\sqrt{3}R^2}$

$\Rightarrow \quad v = \sqrt{\frac{GM}{\sqrt{3}R}}$

13. $T^2 \propto R^3$ or $T \propto R^{3/2}$ or $T^{2/3} \propto R$ or $T^2 \propto \frac{1}{1/R^3}$

So, graph in (a), (b) and (c) are correct.

14. $\Delta W = \Delta U + \Delta K$

$\Rightarrow \Delta U = \Delta W - \Delta K$

$= -5.5 - \frac{1}{2}\times 1\times 3^2$

$= -10$ J

$\Delta V = \frac{\Delta U}{m} = -10$ J/kg

15. $\frac{1}{2}mv^2 = 2.\frac{GMm}{\sqrt{2}R}$

$\Rightarrow v = \sqrt{\frac{2\sqrt{2}GM}{R}}$, it is independent of angle of projection.

16. $U_1 = 0 - \frac{Gm2m}{a} - \frac{Gm3m}{a} - \frac{G3m.2m}{a}$

$= -\frac{5Gm^2}{a} - \frac{3\sqrt{2}m^2}{a}$

$= -\frac{Gm^2}{a}(5+3\sqrt{2})$

$U_2 = 0 - \frac{Gm3m}{a} - \frac{Gm2m}{a} - \frac{G3m2m}{a}$

$= -\frac{11Gm^2}{a}$

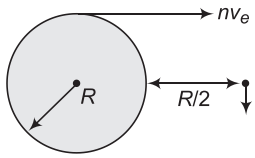
$\Delta W = \Delta U = \frac{Gm^2}{a}(11-5-3\sqrt{2})$

$= \frac{6Gm^2}{a}\left(1-\frac{1}{\sqrt{2}}\right)$

17. $\frac{1}{2}mv^2 = -\frac{GMm}{R} + \frac{3GMm}{2R} = \frac{GMm}{2R}$

$\Rightarrow \quad v = \sqrt{\frac{GM}{R}}$

18.



$Mnv_eR = mv\left(R + \frac{R}{2}\right)$

$\Rightarrow \quad v = \frac{2}{3}nv_e; v_e = \sqrt{\frac{2GM}{R}}$

$\frac{GMm}{R} - \frac{GMm}{\frac{3}{2}R} = \frac{1}{2}m(nv_e)^2 - \frac{1}{2}mv^2$

or $\quad \frac{1}{3}.\frac{GMm}{R} = \frac{1}{2}mn^2v_e^2\left(1 - \frac{4}{9}\right)$

$\Rightarrow \quad \frac{1}{3}\frac{GMm}{R} = \frac{1}{2}mn^2\frac{2GM}{R}.\frac{5}{9}$

or $\quad n = \sqrt{\frac{3}{5}} = \sqrt{0.6}$

19. $2mu - mu = 3mv$

$\Rightarrow v = u/3\ ;\ \frac{1}{2}mu^2 = \frac{3GMm}{2R} - \frac{GMm}{R}$

$u = \sqrt{\frac{Gm}{R}}$

$\Delta U(r) = +\frac{GM \times 3m}{2R^3}A^2 = \frac{1}{2} \times 3\,m \cdot v^2$

or $\frac{GM}{2R^3}A^2 = \frac{1}{2}.\frac{1}{9}.\frac{GM}{R}$

$\therefore \quad \frac{A^2}{R^2} = \frac{1}{9} \Rightarrow A = \frac{R}{3}$

20. $v_e = \sqrt{2gR}$ and

$\frac{v_{e_1}}{v_{e_2}} = \sqrt{\frac{R_1\ g_1}{R_2 g_2}} = \sqrt{kg} = (kg)^{1/2}$

21. $F = -\frac{dU}{dr} = -6r^2 = -\frac{mv^2}{r}$

$mv^2 = 6r^3$ and $\frac{1}{2}mv^2 = 3r^3$

$\therefore \quad E = U + K = 2r^3 + 3r^3 = 5r^3$

$= 5 \times 5^3 = 625\text{J}$

22. $g_{\text{eff}} = g\left(1 + \frac{h}{R}\right)^{-2} = g\left(1 + \frac{1}{2}\right)^{-2} = \frac{4g}{9};$

$F = mg_{\text{eff}} = 200 \times \frac{4}{9} \times 10 = 889\text{ N}$

23. $mg\left(1 + \frac{x}{R}\right)^{-2} = \frac{mv^2}{R + x}$

$\Rightarrow \quad v = \sqrt{g(R + x)\frac{R^2}{(R + x)^2}}$

$= \sqrt{\frac{gR^2}{R + x}}$

24. $F_1 = \frac{G.\frac{4}{3}\pi R^3\rho m}{4R^2} = \frac{1}{3}\pi\rho GmR$

$F_2 = \frac{G.\frac{4}{3}\pi R^3\rho m}{4R^2} - \frac{G\frac{4}{3}\pi\frac{R^3}{8}\rho m}{\frac{9}{4}R^2}$

$= \frac{1}{3}\pi\rho GmR - \frac{2}{27}\pi\rho GmR$

$\therefore \frac{F_1}{F_2} = \frac{1/3}{\frac{1}{3} - \frac{2}{27}} = \frac{1}{1 - \frac{2}{9}} = \frac{9}{7}$

$\Rightarrow \quad \frac{F_2}{F_1} = \frac{7}{9}$

More than One Correct Options

1. $g = \frac{GM}{R^2} = \frac{G\frac{4}{3}\pi R^3\rho}{R^2} = \frac{4}{3}\pi\rho GR$

$\Rightarrow g \propto R$

$\therefore g_1 : g_2 : g_3 = R_1 : R_2 : R_3 = 1 : \frac{1}{2} : \frac{1}{3}$

$\Rightarrow g_1/g_2 = \frac{1}{1/2} = 2$

$g_1/g_3 = \frac{1}{1/3} = 3$

$v_e = \sqrt{2gR}$

$\Rightarrow v_1 : v_2 : v_3 = \sqrt{g_1R_1} : \sqrt{g_2R_2} : \sqrt{g_3R_3}$

$$= 1 : \frac{1}{2} : \frac{1}{3}$$

$$\therefore \frac{v_1}{v_2} = \frac{1}{1/2} = 2 \text{ and } \frac{v_1}{v_3} = \frac{1}{1/3} = 3$$

2. A geo-stationary satellite revolves at a height 36,000 km on equatorial plane with a time period of 24 h.

3. Particle performs SHM inside tunnel but not outside surface, so, if just performs periodic motion with amplitude, $(R + h)$

then

$$\frac{1}{2}mv^2 = \frac{3GMm}{2R} - \frac{GMm}{2R} = \frac{GMm}{R}$$

$$\Rightarrow \quad v = \sqrt{\frac{2GM}{R}}$$

4. $E(x) = \dfrac{G2m}{(r-x)^2} - \dfrac{Gm}{x^2}$

$$= Gm\left[\frac{2}{(r-x)^2} - \frac{1}{x^2}\right] \text{ at } x = 0 \text{ and } r, E = \infty$$

So, field first decreases and then increases.

$$V(x) = -\left(\frac{G2m}{r-x} + \frac{Gm}{x}\right)$$

$$= -Gm\left[\frac{2}{r-x} + \frac{1}{x}\right]$$

So, V first increases and then decreases.

5. As there is no mass inside B, So. field between A to B is zero, *i.e.*, potential between A to B is constant.

$$\frac{Gm}{x^2} = \frac{G3m}{y^2}$$

Potential between B and C is not zero at any point.

6. On exchanging masses, potential at centre remains unchanged, but field strength increases and its direction also changes.

7. 1 will stop at minimum height while 2 will have a tangential velocity. (Reference : Q. 18; P 132); and particle 2 will rise lesser height than 1, as 2 do not looses its entire KE. And particle 1 returns earlier.

8. While going from A to B, KE increases and value of PE decreases, where speed is minimum at A. Angular momentum remains unchanged.

9. $E_1 = -\dfrac{GMm}{R} = U_1$,

$$U_2 = -\frac{GMm}{2R}$$

$$= \frac{E_1}{2}; E_2 = U_2 + K_2 = \frac{1}{2}U_2$$

$$= -\frac{GMm}{4R} = \frac{E_1}{4}$$

$$K_2 = -\frac{1}{2}U_2 = -E_2$$

10. $v \propto \sqrt{m}$, so, $v_2 = 2v_1$, $T \propto \dfrac{1}{v}$

$\Rightarrow T_2 = \dfrac{1}{2}T_1$ as $v \propto \sqrt{G}$ so with increase in G, v increases and T decreases.

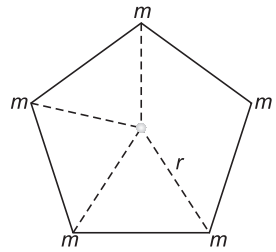
Match the Columns

1. $U(r) = -\dfrac{GMm}{R}$ for $r \geq R$, *i.e.*, PE decreases continuously from A to B. $a \to q$

$U(r) =$ *i.e.*, From B to C, PE remains constant $= -\dfrac{GMm}{R}$ for $r < R \Rightarrow b \to s$

Speed from B to C remains constant. $c \to s$ As between B to C there is no field, So acceleration becomes zero and it increases from A to B. $d \to r$

2.



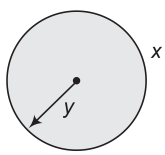
Gravitational field at centre is zero, $a \to r$. Gravitational potential at centre $= -\dfrac{5\,Gm}{r} \Rightarrow b \to s$

With four masses, $E = \dfrac{Gm}{r^2}$

$\Rightarrow c \to p$ and $V = -\dfrac{4Gm}{r}$

$\Rightarrow d \to s$

3.



$x = -\dfrac{GM}{y}$

(a) $g = -\dfrac{GM}{4y^2} = -\dfrac{-xy}{4y^2} = \dfrac{x}{4y} \to r$

(b) $V(y/2) = -\dfrac{3GM}{2y} + \dfrac{GM}{2y^3}\cdot\dfrac{y^2}{4}$

$= -\dfrac{GM}{y}\left(\dfrac{3}{2} - \dfrac{1}{8}\right) = -\dfrac{GM}{y}\cdot\dfrac{11}{8}$

$= \dfrac{11}{8}x \to s$

(c) $g(y/2) = g\left(1 - \dfrac{d}{R}\right) = g\,\dfrac{1}{2} = \dfrac{x}{8y} \to s$

(d) $V(2y) = -\dfrac{GM}{2y} = \dfrac{xy}{2y} = \dfrac{1}{2}x \to q$

4. $\Delta W = \Delta U = \dfrac{GMm}{R} - \dfrac{GMm}{2R}$

$= \dfrac{1}{2}\cdot\dfrac{GM}{R^2}.mR$

$= \dfrac{1}{2}mgR \Rightarrow a \to r$

$\dfrac{mv^2}{2R} = \dfrac{GMm}{4R^2} \Rightarrow \dfrac{1}{2}mv^2 = \dfrac{1}{4}\dfrac{GMm}{R}$

$= \dfrac{1}{4}mgR \Rightarrow b \to p$

$\Delta E = \dfrac{1}{2}\Delta U = \dfrac{1}{2}\left(\dfrac{GMm}{2R} - \dfrac{GMm}{3R}\right)$

$= \dfrac{1}{12}\dfrac{GMm}{R} = \dfrac{1}{12}mgR \Rightarrow c \to s$

$\Delta K = \Delta U = \dfrac{GMm}{R} - \dfrac{GMm}{2R}$

$= \dfrac{1}{12}\dfrac{GM}{R^2}.\,mR = \dfrac{1}{2}mgR \Rightarrow d \to r$

5. g = maximum at $r = R \Rightarrow a \to q$

$g = 0$ at $r = 0$ and $r = \infty \Rightarrow b \to p$

V = minimum at $r = 0 \Rightarrow c \to p$

$= -\dfrac{3GM}{2R}$

$V = 0$ x $r = \infty \Rightarrow d \to s$

11 Simple Harmonic Motion

Introductory Exercise 11.1

1. $F(x) = x^2 - 6x$

$$F_r = F(x + \Delta x) - F(x)$$
$$= (x + \Delta x)^2 - 6(x + \Delta x) - x^2 + 6x$$
$$= 2x\,\Delta x - 6\Delta x$$
$$= -(6 - 2x)\,\Delta x$$
$$= -6\Delta x \quad \text{for} \quad x = 0$$
$$= -k\Delta x \quad \Rightarrow \quad k = 6\ \text{N/m}$$

2. $v = \omega\sqrt{A^2 - x^2}$

$$= \omega\sqrt{A^2 - \frac{A^2}{4}} = \frac{\sqrt{3}}{2}A\omega$$

$$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}\cdot m\cdot\frac{3}{4}A^2\omega^2$$
$$= \frac{3}{4}\cdot\frac{1}{2}mA^2\omega^2$$
$$= \frac{3}{4}\ \text{(Translational energy)}$$

$$\text{PE} = \text{Translational energy} - \text{KE}$$
$$= \text{Translational energy} - \frac{3}{4}\text{(TE)}$$
$$= \frac{1}{4}\text{(Translational energy)}$$

3. $v = \omega\sqrt{A^2 - x^2}$

$$\Rightarrow \quad A = \sqrt{\frac{v^2}{\omega^2} + x^2}$$
$$= \sqrt{\left(\frac{2.18 \times 0.25}{2\pi}\right)^2 + (0.05)^2}$$
$$= \sqrt{0.0075 + 0.0025}$$
$$= \sqrt{0.01} = 0.1\ \text{m}$$
$$= 10\ \text{cm}$$

$$x = A\sin(\omega t + \theta)$$
$$\Rightarrow \quad \frac{x}{A} = \frac{5}{10} = \frac{1}{2}$$
$$= \sin(\omega t + \theta) = \sin\frac{\pi}{6}$$
$$\therefore \quad \phi = \omega t + \theta = \frac{\pi}{6}$$

4. (a) Maximum displacement

$= 15\ \text{cm} = \text{Amplitude.}$

(b) $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{m}{k}}$

$$= 2\pi\sqrt{\frac{2}{150}} = \frac{2\pi}{5\sqrt{3}} = 0.73\ \text{s}$$

(c) $\nu = \dfrac{1}{T} = 1.38\ \text{Hz}$

(d) $\text{TE} = \dfrac{1}{2}m\,\omega^2 A^2$

$$= \frac{1}{2}\times 2\times\frac{150}{2}\times(0.15)^2 = 1.69\ \text{J}$$

(e) $v_{\max} = A\omega = 0.15\times\sqrt{\dfrac{150}{2}}$

$$= 0.15\times 5\sqrt{3} = 0.75\sqrt{3} = 1.3\ \text{m/s}$$

5. $A = 10\ \text{cm}$

$8\ \text{cm} = 10\sin\omega t_1$ and $6\ \text{cm} = 10\sin\omega t_2$

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.1}{4\times 10^3}} = \frac{\pi}{100}\ \text{s}$$

$$\therefore\ \omega t_1 = \sin^{-1}\left(\frac{4}{5}\right) = 53^\circ = \frac{53\pi}{180}$$

and $\omega t_2 = \sin^{-1}\left(\dfrac{3}{5}\right) = 37^\circ = \dfrac{37\pi}{180}$

$$\therefore\ t_1 - t_2 = \frac{1}{\omega}\left(\frac{53\pi}{180} - \frac{37\pi}{180}\right)$$

$$= \frac{T}{2\pi} \times \frac{16\pi}{180} = \frac{8T}{180} = \frac{2T}{45} = \frac{2\pi}{4500} \text{ s}$$

$$= 1.4 \times 10^{-3} \text{ s}$$

6. (a) $\text{TE} = \frac{1}{2} m\,\omega^2 A^2$

$$= \frac{1}{2} \times 2 \times \left(\frac{\pi}{4}\right)^2 \cdot (1.5)^2$$

$$= \frac{\pi^2}{16} \times \frac{9}{4} = \frac{9\pi^2}{64} = 1.4 \text{ J}$$

(b) $0.5 = 1.5 \sin\left(\frac{\pi t_1}{4} + \frac{\pi}{6}\right)$

$$\Rightarrow \quad \frac{\pi t_1}{4} + \frac{\pi}{6} = \sin^{-1}\left(\frac{1}{3}\right)$$

$$0.75 = 1.5 \sin\left(\frac{\pi t_2}{4} + \frac{\pi}{6}\right)$$

$$\Rightarrow \quad \frac{\pi t_2}{4} + \frac{\pi}{6} = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

$$\Rightarrow \quad t_2 = 0$$

7. (a) $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.2}{100}}$

$$= \frac{2\pi}{10}\sqrt{0.2} = 0.28 \text{ s}$$

(b) $(F_{\text{actual}})_{\text{max.}} = -100 \times \frac{4}{100} + 10 \times \left(\frac{4}{100}\right)^2 = -4 + \frac{16}{1000}$

$$(F_{\text{approx.}})_{\text{max.}} = -4$$

$$\% \text{ Error} = \frac{-4 + \frac{16}{1000} + 4}{4} \times 100\% = \frac{4}{10}\%$$

$$= 0.4\%$$

Introductory Exercise 11.2

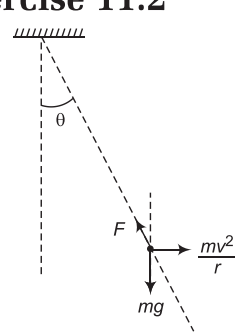
1. At equilibrium,

$$F \sin\theta = \frac{mv^2}{r}$$

$$F \cos\theta = mg$$

$$\Rightarrow \tan\theta = \frac{v^2}{rg}$$

$$\text{or } \theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$



$$g_{\text{eff}} = \sqrt{g^2 + \left(\frac{v^2}{r}\right)^2}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{l}{g_{\text{eff}}}} = 2\pi\sqrt{\frac{l}{\left(g^2 + \frac{v^4}{r^2}\right)^{1/2}}}$$

2. (a) $g_{\text{eff}} = g + a$

$$\Rightarrow T = 2\pi\sqrt{\frac{l}{g+a}}$$

(b) $g_{\text{eff}} = g - a$

$$\Rightarrow T = 2\pi\sqrt{\frac{l}{g-a}}$$

(c) $g_{\text{eff}} = 0 \Rightarrow T \to \infty$

(d) $g_{\text{eff}} = \sqrt{g^2 + a^2}$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{l}{(g^2 + a^2)^{1/2}}}$$

3. $T \propto l^{1/2} \Rightarrow \frac{\Delta T}{T} = \frac{1}{2}\alpha\Delta\theta$

$$\Rightarrow \quad \Delta t = \frac{1}{2}\alpha t \Delta\theta$$

$\therefore$ With increase in temperature, pendulum clock becomes slow.

$$\Delta t = \frac{1}{2} \times 12 \times 10^{-6} \times 86400 \times 20$$

$$= 1.2 \times 8.64 \text{ s}$$

$$= 10.37 \text{ s}$$

4. $g_{\text{eff}} = \frac{W - B}{m} = \frac{\rho - \sigma}{\rho} g$

$$= \left(1 - \frac{\sigma}{\rho}\right) g = \left(1 - \frac{1}{10}\right) g = \frac{9}{10} g$$

$$T \propto \frac{1}{\sqrt{g}}$$

$$\Rightarrow T_2 = T_1\sqrt{\frac{g_1}{g_2}}$$

$$= T\sqrt{\frac{g}{9/10\, g}}$$

$$= T\frac{\sqrt{10}}{3} = \sqrt{\frac{10}{9}}\, T$$

Introductory Exercise 11.3

1. $\frac{1}{k_{\text{eff}}} = \frac{1}{2k} + \frac{1}{k+k} = \frac{2}{2k} = \frac{1}{k}$

$\Rightarrow \quad T = 2\pi\sqrt{\frac{m}{k}}$

2. $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.2}{80}}$

$= \frac{2\pi}{\sqrt{400}} = \frac{2\pi}{20} = \frac{\pi}{10} = 0.314 \text{ s}$

3. $v = \frac{mv_o}{M+m} \Rightarrow \frac{1}{2}(M+m)v^2 = \frac{1}{2}kA^2$

$\Rightarrow \quad A = v\sqrt{\frac{M+m}{k}}$

$\therefore \quad A = \frac{mv_o}{M+m}\sqrt{\frac{M+m}{k}}$

$= \frac{mv_o}{\sqrt{k(M+m)}}$

4. $T_1 = 2\pi\sqrt{\frac{m}{k}}; \frac{1}{k_{\text{eff}}} = \frac{1}{3k} + \frac{1}{3k+3k}$

$= \frac{1}{3k} + \frac{1}{6k}$

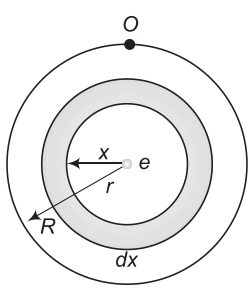
$= \frac{2+1}{6k} = \frac{1}{2k}$

$T_2 = 2\pi\sqrt{\frac{m}{2k}} = \frac{1}{\sqrt{2}}T_1,$

i.e., time period becomes $\frac{1}{\sqrt{2}}$ times.

Introductory Exercise 11.4

1.



$I_e = \int dI_c = \int x^2 dm = \int x^2 \cdot \sigma\, 2\pi x dx$

$= 2\pi\sigma\int_r^R x^3 dx = \frac{\pi\sigma}{2}(R^4 - r^4)$

$m = \int dm = \int \sigma \cdot 2\pi x dx = 2\pi\sigma\int_r^R x dx$

$= \pi\sigma(R^2 - r^2)$

$\therefore \; I_0 = I_c + mR^2 = \frac{\pi\sigma}{2}(R^4 - r^4)$

$+ \pi\sigma R^2(R^2 - r^2)$

$\therefore \; T = 2\pi\sqrt{\frac{I}{mgl}}$

$= 2\pi\sqrt{\frac{\frac{\pi\sigma}{2}(R^4 - r^4) + \pi\sigma R^2(R^2 - r^2)}{\pi\sigma(R^2 - r^2)g \cdot R}}$

$= 2\pi\sqrt{\frac{\frac{1}{2}(R^2 + r^2) + R^2}{Rg}}$

$= 2\pi\sqrt{\frac{\frac{3R}{2} + \frac{r^2}{2R}}{g}}$

$\therefore \; l_{\text{eff}} = \frac{3R}{2} + \frac{r^2}{2R},$

For $r \to 0 \Rightarrow l_{\text{eff}} = \frac{3R}{2}$

and for $r \to R \Rightarrow l_{\text{eff}} = 2R$

2. Here, $l_{\text{eff}} = 35$ cm

and $l = 20$ cm

As, $T = 2\pi\sqrt{\frac{I}{mgl}} = 2\pi\sqrt{\frac{l_{\text{eff}}}{g}}$

$\Rightarrow \frac{I}{mgl} = \frac{l_{\text{eff}}}{g} \Rightarrow I = ml\, l_{\text{eff}}$

$\therefore$ 200 g × 20 cm × 35 cm

$= 1.4 \times 10^5$ g-cm^2

Introductory Exercise 11.5

1. Here, $a_1 = 4.0$ cm and $a_2 = 3.0$ cm

(a) For, $\Delta\phi = 0$, $a = a_{\max} = a_1 + a_2$

$= 4.0 + 3.0 = 7.0$ cm

(b) For, $\Delta\phi = 60°$,

$a = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\Delta\phi}$

$= \sqrt{4^2 + 3^2 + 12} = \sqrt{37} = 6.1$ cm

(c) For, $\Delta\phi = 90°$, $a = \sqrt{a_1^2 + a_2^2}$
$= \sqrt{4^2 + 3^2} = 5.0$ cm

(d) For, $\Delta\phi = 180°$, $a = a_{min} = a_1 - a_2$
$= 4.0 - 3.0 = 1.0$ cm

2. $x = x_1 + x_2 = 4 \sin (100\pi t) + 3 \sin \left(100\pi t + \frac{\pi}{3}\right)$

$$= \sqrt{4^2 + 3^2 + 2 \times 4 \times 3 \cos \frac{\pi}{3}}$$

$$\sin \left[100\pi t + \frac{3 \sin \frac{\pi}{3}}{4 + 3 \cos \frac{\pi}{3}}\right]$$

$$= \sqrt{37} \sin \left(100\pi t + \frac{3\sqrt{3}}{11}\right)$$

(a) At $t = 0$, $x = 4 \sin 0 + 3 \sin \frac{\pi}{3}$
$= \frac{3\sqrt{3}}{2}$ units

(b) $v_{max} = a\omega = 100\pi\sqrt{37}$ unit

(c) $a_{max} = \omega^2 a = (100\pi)^2 \sqrt{37}$ unit

AIEEE Corner

Subjective Questions (Level 1)

1. (a) $20 \times 10^{-3} g = k \times 7 \times 10^{-2}$

$\Rightarrow k = \frac{2g}{7} = \frac{20}{7} = 2.8$ N/m as $\Delta mg = k\Delta x$

(b) $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{50 \times 10^{-3}}{20/7}}$
$= 2\pi\sqrt{17.5 \times 10^{-3}} = 0.84$ s

2. $k = \frac{\Delta m \cdot g}{\Delta x} = \frac{\Delta F}{\Delta x}$
$= \frac{9}{0.05} = 180$ N/m

$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{F}{k \cdot g}}$
$= 2\pi\sqrt{\frac{27}{1800}} = 0.78$ s

3. $v_{max} = A\omega = 2\pi A\nu$
$= 2\pi \times 8 \times 10^{-3} \times 2$ m/s
$= 3.2\,\pi$ cm/s $= 0.101$ m/s

$a_{max} = A\omega^2 = 4\pi^2\nu^2 A$
$= 4 \times \pi^2 \times 4 \times 8 \times 10^{-3}$
$= 1.26$ m/s^2

$F_{max} = kA = ma_{max} = 0.5 \times 1.264$
$= 0.632$ N

4. $\omega = \frac{A\omega^2}{A\omega} = \frac{a_{max}}{v_{max}} = \frac{8\pi}{1.6} = 5\pi$

$T = \frac{2\pi}{\omega} = \frac{2\pi \times 1.6}{8\pi} = 0.4$ s

$A = \frac{v_{max}}{\omega} = \frac{1.6}{5\pi}\pi = 0.102$ m

5. $v = \omega\sqrt{a^2 - x^2} = 2\sqrt{(0.2)^2 - (0.05)^2}$
$= 2\sqrt{0.04 - 0.0025}$
$= 0.39$ m/s

$a = -\omega^2 x = -2^2 \times 0.05 = -0.2$ m/s^2

For, $x = 0$, $v = v_{max} = A\omega = 0.2 \times 2$
$= 0.4$ m/s

and $a = a_{min} = 0$

6. $x = A \sin (\omega t + \delta)$

$\Rightarrow \frac{A}{2} = A \sin (0 + \delta)$

$\Rightarrow \sin \delta = \frac{1}{2}$

or $\delta = \frac{\pi}{6}$

7. $\theta = \frac{\pi}{10} \sin \left(\frac{2\pi}{0.05} t + \delta\right)$

$\Rightarrow \frac{\pi}{10} = \frac{\pi}{10} \sin (0 + \delta)$

$\Rightarrow \sin \delta = 1$

or $\delta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{10} \sin\left(\frac{2\pi}{0.05} t + \frac{\pi}{2}\right)$

$= \frac{\pi}{10} \cos(40\pi t)$

8. $x = 5 \sin\left(20t + \frac{\pi}{3}\right)$

(a) $v = 0 \Rightarrow \cos\left(20t + \frac{\pi}{3}\right) = 0$

$\Rightarrow \quad 20t = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6} \Rightarrow t = \frac{\pi}{120}$ s

(b) $a = 0 = \sin\left(20t + \frac{\pi}{3}\right) = \sin \pi$

$\Rightarrow \quad 20t = \pi - \frac{\pi}{3} = \frac{2\pi}{3}, t = \frac{\pi}{30}$ s

(c) $v = v_{max}$ for, $a = 0$ *i.e.*, at $t = \frac{\pi}{30}$

9. $x = A \sin\left(\frac{2\pi}{T} t\right)$

and $\quad v = A \cdot \frac{2\pi}{T} \cos\left(\frac{2\pi}{T} t\right)$

$2 = A \cdot \frac{2\pi}{16} \cos\left(\frac{2\pi}{16} \cdot 2\right)$

or $\frac{16}{\pi} = A \cos\frac{\pi}{4} \Rightarrow A = \frac{16\sqrt{2}}{\pi} = 7.2$ m

10. $x = A \sin \omega t$

(a) $x(4) = A \sin \frac{2\pi}{8} \times 4 = 0,$

So, distance travelled is $2A$. In the next 4s it again travels $2A$ but from other side.

(b) $x(2) = A \sin \frac{2\pi}{8} \times 2 = A,$

i.e., distance is travelled A in first 2s and again A in next 2s but from other side.

11. (a) $A = 0.08$ m

(b) $T = 4$s, $\omega = \frac{2\pi}{T} = \frac{\pi}{2}$ s^{-1} $= 1.57$ rad/s

(c) $T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow k = \frac{4\pi^2 m}{T^2}$

$= \frac{4 \times \pi^2 \times 0.8}{4^2} = 1.97$ N/m

(d) $v = A\omega \cos \omega t = 0.08 \times 1.57 \cos\left(\frac{\pi}{2} \times 1\right)$

$= 0$

(e) $a = \omega^2 x = \omega^2 A = (1.57)^2 \times 0.08$ m/s^2

$= 0.197$ m/s^2

12. (a) $u = \omega\sqrt{A^2 - x_1^2}$ and $v = \omega\sqrt{A^2 - x_2^2}$

$\Rightarrow \frac{u}{v} = \sqrt{\frac{A^2 - x_1^2}{A^2 - x_2^2}}$

$\frac{u^2}{v^2} = \frac{A^2 - x_1^2}{A^2 - x_2^2}$

$\Rightarrow \quad u^2(A^2 - x_2^2) = v^2(A^2 - x_1^2)$

$\Rightarrow \quad (u^2 - v^2) A^2 = (u^2 x_2^2 - v^2 x_1^2)$

$\Rightarrow \quad A = \left[\frac{v^2 x_1^2 - u^2 x_2^2}{v^2 - u^2}\right]^{\frac{1}{2}}$

(b) $u_1 = \omega\sqrt{A^2 - x_1^2}$; $u_2 = \omega\sqrt{A^2 - x_2^2}$

$\Rightarrow \quad A = \left[\frac{u_2^2 x_1^2 - u_1^2 x_2^2}{u_2^2 - u_1^2}\right]^{\frac{1}{2}}$

$\Rightarrow \quad \omega = \frac{u_1}{\sqrt{A^2 - x_1^2}}$

$= \frac{u_1}{\left[\frac{u_2^2 x_1^2 - u_1^2 x_2^2}{u_2^2 - u_1^2} - x_1^2\right]^{1/2}}$

$= \frac{u_1\sqrt{u_2^2 - u_1^2}}{[u_1^2 x_1^2 - u_1^2 x_2^2]^{\frac{1}{2}}}$

$= \sqrt{\frac{u_2^2 - u_1^2}{x_1^2 - x_2^2}}$

$\Rightarrow \quad T = \frac{2\pi}{\omega}$

$= 2\pi\sqrt{\frac{x_2^2 - x_1^2}{u_1^2 - u_2^2}}$

13. $E_0 = \frac{1}{2} k x_0^2$

(a) $U = \frac{1}{2} k x^2 = \frac{1}{2} k \left(\frac{x_0}{2}\right)^2$

$$= \frac{1}{4} \cdot \frac{1}{2} kx_0^2 = \frac{1}{4} E_0$$

$$k = E_0 - U = E_0 - \frac{1}{4} E_0 = \frac{3}{4} E_0$$

(b) $k = U = \frac{1}{2} E_0 = \frac{1}{2} \cdot \frac{1}{2} kx_0^2$

$$= \frac{1}{2} k \left(\frac{x_0}{\sqrt{2}} \right)^2 = \frac{1}{2} kx^2$$

$$\Rightarrow \quad x = \frac{x_0}{\sqrt{2}}$$

14. At equilibrium, $mg = kx_0$

$$\Rightarrow \quad x_0 = \frac{mg}{k}$$

At a distance y below equilibrium position, spring energy and gravitational energy, is given by

$$U = U_s + U_g = \frac{1}{2} k (y + x_0)^2 - mgy$$

$$= \frac{1}{2} ky^2 + kyx_0 + \frac{1}{2} kx_0^2 - mgy$$

$$= \frac{1}{2} ky^2 + k \cdot y \cdot \frac{mg}{k} + \frac{1}{2} kx_0^2 - mgy$$

$$= \frac{1}{2} ky^2 + \frac{1}{2} kx_0^2$$

If equilibrium position is taken as reference frame, then $\frac{1}{2} kx_0^2$ can be considered to be zero, such that

$$U = \frac{1}{2} ky^2$$

15. At mean position,

$$\frac{1}{2} kd^2 = \frac{1}{2} (m_1 + m_2) v^2$$

$$\Rightarrow \quad v = \sqrt{\frac{kd^2}{m_1 + m_2}} = v_{\max}$$

$$\therefore \quad v_{\max} = A\omega = A\sqrt{\frac{k}{m_1}}$$

$$\Rightarrow \quad A = \sqrt{\frac{m_1}{k}} \cdot v_{\max}$$

$$= \sqrt{\frac{m_1}{k}} \sqrt{\frac{kd^2}{m_1 + m_2}}$$

or $\quad A = d\sqrt{\frac{m_1}{m_1 + m_2}}$

16. (a) At equilibrium, $F = kx_0$

$$\Rightarrow \quad x_0 = \frac{F}{k} = A \text{ and } T = 2\pi\sqrt{\frac{M}{k}}$$

(b) $W = \int dW = \int_0^{x_0} (F - kx)\, dx$

$$= Fx_0 - \frac{1}{2} kx_0^2 = \frac{F^2}{k} - \frac{F^2}{2k} = \frac{F^2}{2k} = U$$

(c) $k = Fx_0 - W = \frac{F^2}{k} - \frac{F^2}{2k} = \frac{F^2}{2k}$

17. (a) $x_0 = \frac{F}{k} = \frac{10}{100} = 0.1 \text{ m} = 10 \text{ cm}$

(b) $E = U + K = \frac{1}{2} kx_0^2 + \frac{1}{2mv^2}$

$$= \frac{1}{2} \times 100 \times (0.1)^2 + \frac{1}{2} \times 1 \times 2^2 = 2.5 \text{ J}$$

(c) $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1}{100}} = \frac{\pi}{5} \text{ s}$

(d) $\frac{1}{2} k (A)^2 = \frac{1}{2} mv^2$

$$\Rightarrow \quad A = v\sqrt{\frac{k}{m}}$$

$$= 2\sqrt{\frac{100}{1}} = 20 \text{ cm}$$

(e) $U = \frac{1}{2} k (x_0 + A)^2$

$$= \frac{1}{2} \times 100 \times (0.3)^2 = 4.5 \text{ J}$$

(f) $U = \frac{1}{2} k (A - x_0)^2$

$$= \frac{1}{2} \times 100 \times (0.1)^2 = 0.5 \text{ J}$$

18. $K_{\max} = \frac{1}{2} mv_{\max}^2 = \frac{1}{2m} \omega^2 A^2$

$$\Rightarrow \omega = \sqrt{\frac{2k}{mA^2}} = \sqrt{\frac{2 \times 8 \times 10^{-3}}{0.1 \times (0.1)^2}}$$

$$= 4 \text{ rad/s}$$

$$x = A \sin\left(\omega t + \frac{\pi}{4}\right)$$

$$= 0.1 \sin\left(4t + \frac{\pi}{4}\right)$$

19. $U = 10 + (x-2)^2$

$\Rightarrow \quad F = -\dfrac{dU}{dx} = -2(x-2) = -2x + 4$

$F_r = -2\Delta x$

$\Rightarrow \quad k = 2$

(a) $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{2}{2}} = 1$ rad/s

(b) At mean position

$U = 10$ J and $K = 26 - 10 = 16$ J

At extreme position

$U = 26$ J and $K = 0$

(c) $v_{\max} = \sqrt{\dfrac{2K}{m}}$

$= \sqrt{\dfrac{2 \times 16}{2}} = 4$ m/s

$= A\omega$

$\Rightarrow \quad A = 4$ m

(d) $U_{\max} = 26 = 10 + (x_1 - 2)^2$

$\Rightarrow \quad x_1 = 4 + 2 = 6$ m

$U_{\max}, \dfrac{dU}{dx} = 0 = -2(x_2 - 2)$

$\Rightarrow \quad x_2 = 2$ m

$\therefore$ particle oscillates between

$x = 2$ m to 6m

20. $T' = \dfrac{T_1}{2} + \dfrac{T_2}{2}$

$= \dfrac{1}{2} \times 2\pi\sqrt{\dfrac{l/4}{g}} + \dfrac{1}{2} \times 2\pi\sqrt{\dfrac{l}{g}}$

$= \left(\dfrac{\pi}{2} + \pi\right)\sqrt{\dfrac{l}{g}}$

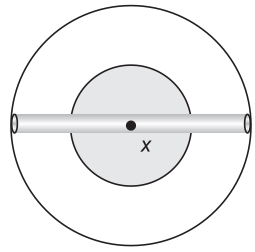
$= \dfrac{3\pi}{2}\sqrt{\dfrac{l}{g}} = \dfrac{3}{4} \cdot 2\pi\sqrt{\dfrac{l}{g}} = \dfrac{3}{4}T$

21. $K_{eq} = 2K + 2K = 4K$ and $T \propto \dfrac{1}{\sqrt{K}}$

$\therefore \quad \dfrac{T'}{T} = \sqrt{\dfrac{K}{4K}} = \dfrac{1}{2}$

$\Rightarrow \quad T' = \dfrac{T}{2}$

22.



$g(x) = \dfrac{G \cdot \dfrac{4}{3}\pi x^3 \rho m}{mx^2} = \dfrac{4}{3}\pi\rho G x$

$F(x) = -mg(x) = -\dfrac{4}{3}\pi\rho G m x$

$= -kx \Rightarrow k = \dfrac{4}{3}\pi\rho G m$

$\Rightarrow F \propto -x$

i.e., the particle will execute SHM

$T = 2\pi\sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{3}{4\pi\rho G}}$

$= 2\pi\sqrt{\dfrac{1}{\dfrac{4}{3}\pi \cdot \dfrac{M}{\dfrac{4}{3}\pi R^3}G}} = 2\pi\sqrt{\dfrac{R^3}{MG}}$

23. $g'(2R) = \dfrac{GM}{4R^2} = \dfrac{g}{4}$;

$T = 2\pi\sqrt{\dfrac{l}{g}}$

$= 2\pi\sqrt{\dfrac{4l}{g}} = 2\pi\sqrt{\dfrac{4 \times 1}{\pi^2}} = 2\pi \times \dfrac{2}{\pi} = 4$ s

24. $mg = kx_0$; $F_r = F(x + \Delta x) - F(x)$

$= k(x + \Delta x) - kx = k\Delta x$

$\Rightarrow T = 2\pi\sqrt{\dfrac{m}{k}}$

25. $F_r = -(k\Delta x + A\Delta x\sigma g)$

$= -(k + A\sigma g)\Delta x$

$T = 2\pi\sqrt{\dfrac{m}{k + A\sigma g}}$

$= 2\pi\sqrt{\dfrac{10}{100 + 20 \times 10^{-4} \times 10^3 \times 10}}$

$= \dfrac{2\pi}{\sqrt{12}} = \dfrac{\pi}{\sqrt{3}} = 1.8$s

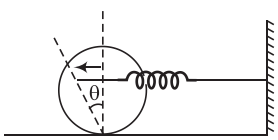
26. $T_A = 2\pi\sqrt{\dfrac{I}{mgl}} = 2\pi\sqrt{\dfrac{\frac{1}{3}md^2}{mg\,\frac{d}{2}}}$

$= 2\pi\sqrt{\dfrac{2d}{3g}}$

and $T_B = 2\pi\sqrt{\dfrac{d}{g}}$

$\Rightarrow \quad \dfrac{T_A}{T_B} = \sqrt{\dfrac{2}{3}} = 0.816$

27.



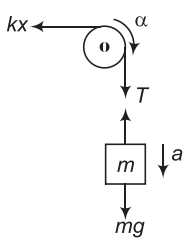
$\tau = I\alpha$

$k \cdot r\theta \cdot r = -\left(\dfrac{1}{2}mr^2 + mr^2\right)\alpha$

$\alpha = -\dfrac{2k}{3m}\theta = -\omega^2\theta$

$\Rightarrow \quad T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{3m}{2k}}$

28.



$(kx - T)\,r = I\alpha \Rightarrow kx - T = \dfrac{I}{r}\cdot\dfrac{a}{r}$

and $\quad T - mg = ma$

$kx - mg = \left(\dfrac{I}{r^2} + m\right)a$

$\therefore\ -k\Delta x = \left(\dfrac{I}{r^2} + m\right)a$

$\nu = \dfrac{\omega}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m + \frac{I}{r^2}}}$

$= \dfrac{1}{2\pi}\sqrt{\dfrac{20}{0.5 + 0.6 \times 5}}$

$= \dfrac{1}{2\pi}\sqrt{\dfrac{20}{3.5}} = \dfrac{1}{\pi\sqrt{0.7}} = 0.38$ Hz

29. $x = A\sin\omega t$ and $y = A\sin\left(2\,\omega t + \dfrac{\pi}{2}\right)$

$= A\cos 2\,\omega t$

$= A\,(1 - 2\sin^2\omega t) = A\left(1 - \dfrac{2x^2}{A^2}\right)$

$\therefore\ y = A - \dfrac{2}{A}x^2 \rightarrow$ equation of parabola.

30. $x = A\sin\left(\omega t - \dfrac{\pi}{3}\right) + A\sin\omega t$

$+ A\sin\left(\omega t + \dfrac{\pi}{3}\right)$

$= A\sin\omega t + 2A\sin\omega t \cdot \cos\dfrac{\pi}{3}$

$= 2A\sin\omega t$

So, resultant amplitude is 2 A.

31. $A = \sqrt{A^2 + A^2 + 2AA\cos\theta}$

$= 2A\cos\dfrac{\theta}{2} \Rightarrow \cos\dfrac{\theta}{2} = \dfrac{1}{2} = \cos\dfrac{\pi}{3}$

$\therefore\ \Delta\phi = \theta = \dfrac{2\pi}{3}$

32. $x = x_1 + x_2 = 2\sin 100\pi t$

$+ 2\sin\left(120\pi t + \dfrac{\pi}{3}\right)$

(a) $x = 2\sin 1.25\,\pi + 2\sin\left(1.5\pi + \dfrac{\pi}{3}\right)$

$= 2\sin\left(\pi + \dfrac{\pi}{4}\right) + 2\sin\left(2\pi - \dfrac{\pi}{6}\right)$

$= -2\sin\dfrac{\pi}{4} - 2\sin\dfrac{\pi}{6}$

$= -\sqrt{2} - 1 = -2.414$ cm

(b) $x = 2\sin 2.5\,\pi + 2\sin\left(3\pi + \dfrac{\pi}{3}\right)$

$= 2\sin\left(2\pi + \dfrac{\pi}{2}\right) + 2\sin\left(3\pi + \dfrac{\pi}{3}\right)$

$= 2\sin\dfrac{\pi}{2} - 2\sin\dfrac{\pi}{3}$

$= 2 - \sqrt{3}$ cm

$= 0.27$ cm

Objective Questions (Level 1)

1. $x = A\sin\omega t \Rightarrow \frac{A}{2} = A\sin\omega t$

$\Rightarrow \sin\frac{2\pi}{T}\cdot t = \frac{1}{2} = \sin\frac{\pi}{6}$

$\therefore \frac{2\pi}{T}\cdot t = \frac{\pi}{6} \Rightarrow t = \frac{T}{12}$

$\therefore \Delta t = \frac{T}{4} - \frac{T}{12} = \frac{3T - T}{12} = \frac{T}{6}$

2. Potential energy becomes maximum to maximum in time $\frac{T}{2}$, whereas oscillation takes a time T.

So, frequency of oscillation is half of frequency of potential energy.

i.e., $\frac{f}{2}$ as $f \propto \frac{1}{T} \Rightarrow \frac{f_P}{f_U} = \frac{T_U}{T_p} = \frac{T/2}{T} = \frac{1}{2}$

3. $v = \omega\sqrt{A^2 - x^2} \Rightarrow v^2 = \omega^2(A^2 - x^2)$

$\Rightarrow v^2 + \omega^2 x^2 = \omega^2 A^2$

or $\frac{v^2}{\omega^2 A^2} + \frac{x^2}{A^2} = 1$, is an equation of ellipse.

4. As initial and final distance of centre of mass from the point of suspension is same that is why the time period remains same.

5. $\Delta\phi = \frac{2\pi}{3} + \phi - \frac{\pi}{2} - \phi$

$= \frac{4\pi - 3\pi}{6} = \frac{\pi}{6}$

6. $v = \omega\sqrt{A^2 - x^2} = \frac{1}{2}\omega A$

$\Rightarrow \quad A^2 - x^2 = \frac{A^2}{4}$

$\Rightarrow \quad x^2 = \frac{3}{4}A^2$

or $x = \frac{\sqrt{3}}{2}A$

7. $T \propto \sqrt{l} \Rightarrow \frac{T_2}{T_1} = \sqrt{\frac{l_2}{l_1}}$

$= \sqrt{\frac{0.79\, l_1}{l_1}} = 0.89$

$\Rightarrow \quad T_2 = 0.89\, T_1$

i.e., T_2 is 11% lesser than T_1

8. As in (b) two parts of the function has different frequencies their addition will lead to periodic motion Lissajous figures but not simple harmonic motion.

9. $T = 2\pi\sqrt{\frac{M}{k}} = 2$ and $2 + 1 = 2\pi\sqrt{\frac{M+4}{k}}$

$\Rightarrow \frac{3}{2} = \sqrt{\frac{M+4}{M}} \Rightarrow \frac{M+4}{M} = \frac{9}{4}$

$\Rightarrow 16 = 5M$

$\therefore M = \frac{16}{5} = 3.2\,\text{kg}$

10. $T_1 = 2\pi\sqrt{\frac{400}{k}}$ and $T_2 = 2\pi\sqrt{\frac{100}{k}}$

$\Rightarrow \frac{T_2}{T_1} = \sqrt{\frac{100}{400}} = \frac{1}{2}$

$T_2 = \frac{T_1}{2} = 1\,\text{s}$

11. $y = 4\cos\pi t + 4\sin\pi t$

$= \sqrt{4^2 + 4^2 + 2\times 4\times 4\cos\frac{\pi}{2}}\sin(\pi t + \theta)$

$= 4\sqrt{2}\sin(\pi t + \theta)$

12. $T_1 = 2\pi\sqrt{\frac{m}{k_1}}$, $T_2 = 2\pi\sqrt{\frac{m}{k_2}}$

and $\quad T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$

$\frac{1}{T_1^2} = \frac{k_1}{4\pi^2 m}$; $\frac{1}{T_2^2} = \frac{k_2}{4\pi^2 m}$ and

$\frac{1}{T^2} = \frac{k_1 + k_2}{4\pi^2 m}$

$\therefore \frac{1}{T^2} = \frac{1}{T_1^2} + \frac{1}{T_2^2} \Rightarrow T^{-2} = T_1^{-2} + T_2^{-2}$

13. $a = -px = -\omega^2 x$

$\Rightarrow \omega = \sqrt{p}$

14. Let the time in which X completes n oscillations is t, then during that time Y completes $(n - 1)$ oscillations to come in same phase.

Therefore, $t = 4n = 4.2\,(n-1)$

$\Rightarrow 4.2 = 0.2\,n \Rightarrow n = 21$

15. $kx = mg \Rightarrow k = \dfrac{mg}{x}$

$$T = 2\pi\sqrt{\frac{M+m}{k}} = 2\pi\sqrt{\frac{M+m}{\frac{mg}{x}}}$$

$$= 2\pi\sqrt{\frac{(M+m)\,x}{mg}}$$

16. $v = \omega_1 A_1 = \omega_2 A_2$

$$\Rightarrow \frac{A_1}{A_2} = \frac{\omega_2}{\omega_1} = \frac{\sqrt{\frac{k_2}{m}}}{\sqrt{\frac{k_1}{m}}} = \sqrt{\frac{k_2}{k_1}}$$

17. $|a| = \omega^2 x \Rightarrow \omega = \sqrt{\dfrac{|a|}{x}} = \sqrt{\dfrac{1.25}{5\times 10^{-2}}}$

$= \sqrt{25} = 5$ rad/s

$T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{5}$ s

18. $T = 2\pi\sqrt{\dfrac{I}{mgl}}$

$$= 2\pi\sqrt{\frac{\frac{3}{2}mr^2}{mgr}} = 2\pi\sqrt{\frac{3r}{2g}}$$

19. $U = \dfrac{1}{2}kx^2 \Rightarrow y = mx$, is a straight line passing through origin. The answer will be (d) as x^2 cannot have negative value (for real x).

20. $T = 2\pi\sqrt{\dfrac{1}{g\left(\dfrac{1}{l}+\dfrac{1}{R}\right)}}$

$$= 2\pi\sqrt{\frac{1}{g\left(\frac{1}{R}+\frac{1}{R}\right)}}$$

$$= 2\pi\sqrt{\frac{R}{2g}}$$

21. At positive maximum displacement force is also maximum but negative, as $F = -m\omega^2 x$, so, the $F - t$ graph will be as in (b).

22. $T = 2\pi\sqrt{\dfrac{M}{k}}$ and $\dfrac{5T}{3} = 2\pi\sqrt{\dfrac{M+m}{k}}$

$\Rightarrow \dfrac{5}{3} = \sqrt{\dfrac{M+m}{M}}$

$\Rightarrow = \dfrac{M+m}{M} \Rightarrow \dfrac{25}{9}$

$\Rightarrow 9\,m = 16\,M \Rightarrow \dfrac{m}{M} = \dfrac{16}{9}$

23. $T = 2\pi\sqrt{\dfrac{m}{k}}$; $mg\sin\theta = kx_0$

$\Rightarrow x_0 = \dfrac{mg\sin\theta}{k}$

24. $\dfrac{d^2x}{dt^2} = -\pi^2 x = -\omega^2 x$

$\Rightarrow \omega = \pi$;

$\nu = \dfrac{\omega}{2\pi} = \dfrac{1}{2}$ Hz

25. $\dfrac{1}{m_{eq}} = \dfrac{1}{M} + \dfrac{1}{m}$

$\Rightarrow m_{eq} = \dfrac{mM}{M+m}$,

$$\omega = \sqrt{\frac{k}{m_{eq}}} = \sqrt{\frac{k\,(M+m)}{mM}}$$

$$\Rightarrow \nu = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k\,(M+m)}{mM}}$$

26. $v = \omega\sqrt{2^2 - 1^2} = \omega\sqrt{3}$;

$|a| = \omega^2 x = \omega^2$

$\Rightarrow \omega\sqrt{3} = \omega^2 \Rightarrow \omega = \sqrt{3}$, $\nu = \dfrac{\omega}{2\pi} = \dfrac{\sqrt{3}}{2\pi}$

27. $g = \dfrac{GM}{R^2}$ and $g' = \dfrac{G\cdot 2M}{4R^2}$

$= \dfrac{1}{2}g$

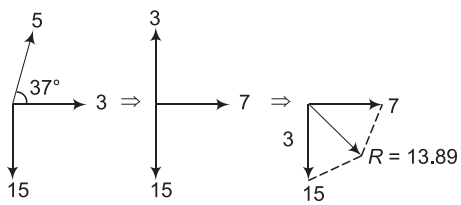
$T \propto \dfrac{1}{\sqrt{g}} \Rightarrow \dfrac{T_2}{T_1} = \sqrt{\dfrac{g_1}{g_2}}$

$T = \sqrt{\dfrac{g}{g/2}} = \sqrt{2}$

$\therefore\ T_2 = \sqrt{2}T_1 = 2\sqrt{2}$ s

28. $x = 3\sin\omega t + 5\sin(\omega t + 37°)$

$$= 3\sin\omega t + 15\sin\left(\omega t - \frac{\pi}{2}\right) + 5\sin(\omega t + 37°) - 15\cos\omega t$$



29. When the two SHM are in same direction, the resultant will also be SHM and when they are in perpendicular directions, they will form Lissajous figures performing periodic motion but not SHM. But at 37°, particle will not perform SHM as well as not travel in straight line.

30. $K_{max} = U_{max} = \frac{1}{2}k \times 10^2 = 50\,k$

$$K = K_{max} - U = 50\,k - \frac{1}{2} \times k \times 6^2$$

$$= 32\,k$$

$$= \frac{32}{50}K_{max} = 0.64\,K_{max}$$

$$v = \omega\sqrt{A^2 - x^2} = \omega\sqrt{A^2 - \frac{A^2}{4}}$$

$$= \frac{\sqrt{3}}{2}\omega A = \frac{\sqrt{3}}{2}v_{max}$$

(a) is correct while (b) is wrong.

31. $k \times 4 \times 10^{-2} = m \times 0.5$

$$\Rightarrow \frac{k}{m} = \frac{0.5}{4 \times 10^{-2}}$$

$$= \frac{50}{4} = \frac{25}{2}$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{25}{2}} = \frac{5}{\sqrt{2}}$$

$$= 3.53 \text{ rad/s}$$

32. $Ma_0 = -kx$

$$x = \frac{Ma_0}{k} = \frac{1 \times 2}{100} = 0.02 \text{ m}$$

JEE Corner

Assertion and Reason

1. For, $x = A\cos\omega t$, particle is at $x = A$ at $t = 0$, but the displacement is measured from mean position. So, assertion is false.

2. Assertion and reason are both true but reason is not correct explanation.

3. Assertion is true while time period of oscillation is $2\pi\sqrt{\frac{m}{k}}$, so, the reason is false.

4. $\frac{\sqrt{3}}{2}A = A\sin\omega t$

$$\Rightarrow \quad \sin\omega t = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \frac{2\pi}{T} \cdot t = \frac{\pi}{3}$$

$$\Rightarrow \quad t = \frac{T}{6}$$

So, for A to $\frac{\sqrt{3}}{2}A$, $\Delta t_1 = \frac{T}{4} - \frac{T}{6} = \frac{T}{12}$

For, $\frac{A}{2} = A\sin\omega t'$

$$\Rightarrow \quad \frac{2\pi}{T}t' = \frac{\pi}{6}$$

$$\Rightarrow \quad t' = \frac{T}{12}$$

So, for $\frac{\sqrt{3}}{2}A$ to $\frac{1}{2}A$

$$\Rightarrow \quad \Delta t_2 = \frac{T}{6} - \frac{T}{12} = \frac{T}{12}$$

Reason correctly explains assertion.

5. In uniform circular motion also particle performs SHM, so assertion is false.

6. If spring is halved, then spring constant becomes twice and as mass is halved, then $\omega' = \sqrt{\frac{2k}{\frac{m}{2}}} = 2\sqrt{\frac{k}{m}} = 2\omega$, *i.e.*, angular frequency gets double. Thus, assertion is false.

7. Assertion is false as if $F \not\propto -x$, then motion will not be SHM.

8. For $x = A\cos\omega t$ between $t = 0$ to $\frac{\pi}{2\omega}$, particle moves from one extreme to mean position such that velocity and acceleration are in same direction, *i.e.*, angle between them is zero. Such that $\vec{\mathbf{a}}\cdot\vec{\mathbf{v}} = -\omega^2 A\cos\omega t \times (-\omega A\sin\omega t)\cos\theta$ $\cdot\cos\theta = \omega^3 A^2 \sin\omega t \quad \cos\omega t\cos\theta$ as $\cos\theta = \cos 0° = +1$,

then $\vec{\mathbf{a}}\cdot\vec{\mathbf{v}}$ = positive. Thus, reason is true explanation of assertion.

9. In SHM, $a = -\omega^2 x$,

i.e., $\omega = \sqrt{\left|\frac{\text{acceleration}}{\text{displacement}}\right|}$ and $T = \frac{2\pi}{\omega}$

So, reason is correct explanation of assertion.

10. Only uniform circular motion can be called SHM, so, assertion is false.

Objective Questions (Level 2)

1. $U = \frac{1}{2}kx^2$

$$\Rightarrow \quad k = \frac{2U}{x^2} = \frac{2\times 1}{(0.4)^2} = \frac{200}{16} = 12.5$$

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{2}{12.5}}$$

$$= 2\pi\sqrt{\frac{4}{25}} = 2\pi\frac{2}{5} = \frac{4\pi}{5}\text{ s}$$

2. $T = \frac{1}{2}\cdot 2\pi\sqrt{\frac{m}{k}} = \pi\sqrt{\frac{m}{k}}$, and $\frac{1}{2}ka^2 = \frac{1}{2}mv^2$

$$\Rightarrow \quad v = a\sqrt{\frac{k}{m}}$$

$$F = \frac{dP}{dt} = \frac{2m\cdot v}{T} = \frac{2ma\sqrt{\frac{k}{m}}}{\pi\sqrt{\frac{m}{k}}}$$

$$= \frac{2ma}{\pi}\cdot\frac{k}{m} = \frac{2ak}{\pi}$$

3. $y_2 = 10\sin\omega t + 10C\cos\omega t$

$$= \sqrt{10^2 + 10^2C^2}\sin(\omega t + \theta)$$

$$= 10\sqrt{1 + C^2}\sin(\omega t + \theta)$$

So, $10\sqrt{1 + C^2} = 40$

$\Rightarrow 1 + C^2 = 16$

$\Rightarrow C = \sqrt{15}$

4. $x = 2\sin(5\pi t)$

$\Rightarrow x(0.3) = 2\sin(1.5\pi)$

$$= 2\sin\left(\pi + \frac{\pi}{2}\right) = -2$$

$$v = \omega\sqrt{A^2 - x^2} = 5\pi\sqrt{2^2 - 2^2} = 0$$

5. $x = A\sin\left(\frac{2\pi}{16}t\right) = A\sin\left(\frac{\pi t}{8}\right)$;

$$x(2) = A\sin\left(\frac{\pi\times 2}{8}\right) = \frac{A}{\sqrt{2}}$$

$$v = \omega\sqrt{A^2 - x^2} = \frac{2\pi}{16}\sqrt{A^2 - \frac{A^2}{2}}$$

$$= \frac{\pi A}{8\sqrt{2}} = 1$$

$$\Rightarrow A = \frac{8\sqrt{2}}{\pi}\text{ m}$$

6. $T' = 2\pi\sqrt{\frac{l}{g_{\text{eff}}}} = 2\pi\sqrt{\frac{l}{g}}\cdot\sqrt{\frac{g}{g_{\text{eff}}}}$

$$T = \sqrt{\frac{10}{\sqrt{10^2 + 4^2}}} = 2\sqrt{\frac{10}{\sqrt{116}}}\text{ s}$$

$$= 1.93\text{ s}$$

7. $x = A\sin\omega t \Rightarrow v = A\omega\cos\omega t$ and $a = -\omega^2 A\sin\omega t$

$$\frac{v^2}{a^2} = \frac{A^2\omega^2\cos^2\omega t}{A^2\omega^4\sin^2\omega t}$$

or $\dfrac{v^2}{a^2} = \dfrac{1}{\omega^2} \cdot \dfrac{\dfrac{v^2}{A^2\omega^2}}{1 - \dfrac{v^2}{A^2\omega^2}}$

$\dfrac{v^2}{a^2} = \dfrac{1}{\omega^2} \cdot \dfrac{v^2}{A^2\omega^2 - v^2}$

or $v^2\omega^2 (A^2\omega^2 - v^2) = a^2\omega^2 v^2$

or $A^2\omega^2 - v^2 = a^2$

or $v^2 = -a^2 + A^2\omega^2$

or $y = -mx + C$

i.e., a straight line with negative slope and positive intercept.

8. $l = \dfrac{1}{2} \times \dfrac{1000}{20}$ cm $= 25$ cm

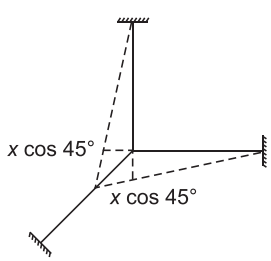
$T = 2\pi\sqrt{\dfrac{l}{g}} = 2\pi\sqrt{\dfrac{0.25}{10}} = \dfrac{\pi}{\sqrt{10}} \simeq 1$ s

9. $F_r = -A\rho g x$

$\Rightarrow a_{cc} = -\dfrac{A\rho g}{\sqrt{\rho_0}} x = -\omega^2 x$

$T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{V\rho_0}{A\rho g}} = 2\pi\sqrt{\dfrac{a\rho_0}{\rho g}}$

10. $\therefore F_r = -[kx + (2kx \cos 45°) \cos 45°]$



$= -2kx$

$a = -\dfrac{2k}{m} x = -\omega^2 x$

$T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{m}{2k}}$

11. $I = \dfrac{ml^2}{12} + md^2$,

$T = 2\pi\sqrt{\dfrac{I}{mgd}} = 2\pi\sqrt{\dfrac{\dfrac{l^2}{12} + d^2}{gd}}$

For $T =$ minimum, $\dfrac{\partial T}{\partial d} = 0 = \dfrac{\partial}{\partial d}\left[\dfrac{\dfrac{l^2}{12} + d^2}{d}\right]^{\frac{1}{2}}$

or $= \dfrac{\partial}{\partial d}\left[\dfrac{l^2}{12d} + d\right]^{\frac{1}{2}} = \dfrac{l^2}{12} \cdot \left(-\dfrac{1}{d^2}\right) + 1$

or $\dfrac{l^2}{d^2} = 12 \Rightarrow \dfrac{d}{l} = \dfrac{1}{\sqrt{12}}$

12. $T_A = 2\pi\sqrt{\dfrac{m}{k}}$; $T_B = 2\pi\sqrt{\dfrac{m}{\frac{k}{2}}}$; $T_C = 2\pi\sqrt{\dfrac{m}{2k}}$

$\therefore T_A : T_B : T_C = 1 : \sqrt{2} : \dfrac{1}{\sqrt{2}}$

$\Rightarrow T_B > T_A > T_C \Rightarrow T_2 > T_1 > T_3$

13. $T_B = 2\pi\sqrt{\dfrac{m}{k}}$,

$T_A = 2\pi\sqrt{\dfrac{\frac{m}{3}}{k}} = 2\pi\sqrt{\dfrac{m}{3k}}$;

$T_C = 2\pi\sqrt{\dfrac{\dfrac{m}{2} + \dfrac{\frac{m}{2}}{3}}{k}} = 2\pi\sqrt{\dfrac{4m}{6k}}$

$\therefore T_A : T_B : T_C = \dfrac{1}{\sqrt{3}} : 1 : \sqrt{\dfrac{2}{3}}$

$\Rightarrow T_B > T_C > T_A \Rightarrow T_2 > T_3 > T_1$

or $T_1 < T_3 < T_2$

14. $T = \dfrac{1}{2}(T_1 + T_2) = \dfrac{1}{2}\left(2\pi\sqrt{\dfrac{M}{k}} + 2\pi\sqrt{\dfrac{M}{4k}}\right)$

$= \dfrac{1}{2} \cdot 2\pi\sqrt{\dfrac{M}{k}}\left(1 + \dfrac{1}{2}\right) = \dfrac{3\pi}{2}\sqrt{\dfrac{M}{k}}$

15. $U = 2 - 20x + 5x^2$; $F = -\dfrac{\partial U}{\partial x} = 20 - 10x$

$F = 0$ at $x = 2$, *i.e.*, $x = 2$ is the equilibrium position.

As particle is released from $x = -3$, so amplitude is 5 m.

Thus, maximum x coordinate is

2 m + 5 m = 7 m.

16. $T = 2\pi\sqrt{\frac{m}{k}} = \frac{1}{f}$;

$T_1 = 2\pi\sqrt{\frac{m(1-n)}{\frac{k}{n}}}$ and $T_2 = 2\pi\sqrt{\frac{nm}{\frac{k}{(1-n)}}}$

$\frac{f_1}{f_2} = \frac{T_2}{T_1} = \sqrt{\frac{nm(1-n)}{k}} \cdot \sqrt{\frac{k}{nm(1-n)}} = 1$

17. $x = R \sin \omega t$

$U_B = U_D = \frac{3}{4} \times \frac{1}{2} kR^2 = \frac{1}{2} kx^2$

$\Rightarrow x = \frac{\sqrt{3}}{2} R$

$\therefore BD = 2x = \sqrt{3} R$

18. $F_{\text{elastic}} = \frac{Y\Delta LA}{l}$

$\Rightarrow F_Q = \frac{-Y \cdot \Delta LA}{L}$

$\Rightarrow a_Q = \frac{-YA}{Lm} \cdot \Delta L$

$\frac{1}{2} T_Q = \frac{1}{2} \cdot 2\pi\sqrt{\frac{Lm}{YA}}$

Similarly, $F_P = -\frac{2Y \cdot \frac{A}{2}}{L} \Delta L = -\frac{YA}{L} \Delta L$

$a_P = -\frac{YA}{Lm} \Delta L$

$\therefore \frac{1}{2} T_P = \frac{1}{2} 2\pi\sqrt{\frac{Lm}{YA}}$

$\therefore T = t + \frac{1}{2} T_P + \frac{1}{2} T_Q$

$= \frac{2L}{v} + 2\pi\sqrt{\frac{Lm}{YA}}$

19. $5 = 10 \sin \omega t$

$\Rightarrow \sin \omega t = \frac{1}{2} = \sin \frac{\pi}{6}$

$\Rightarrow \frac{2\pi}{T} \cdot t = \frac{\pi}{6}$

$\Rightarrow t = \frac{T}{12}$

$\therefore T' = \frac{T}{2} + 2 \times \frac{T}{12} = \left(\frac{1}{2} + \frac{1}{6}\right) T$

$= \frac{4}{6} T = \frac{2}{3} T$

$= \frac{2}{3} \times 2\pi\sqrt{\frac{0.1}{100}}$

$= \frac{4\pi}{30\sqrt{10}} = 0.133$ s

20. $x = A \cos \omega t$

$\Rightarrow 5 \text{ cm} = 8 \cos \omega t$

$\Rightarrow \omega t = 0.9$

$\frac{2\pi}{1.2} t = 0.9$

$\Rightarrow t = \frac{1.08}{2\pi}$ s $= 0.17$ s

21. $I = \frac{m}{3} \cdot \frac{a^2}{3} + \frac{m}{3} \frac{a^2}{3}$

$+ \left(\frac{ma^2}{3 \times 12} + \frac{m}{3}\left(\frac{\sqrt{3}}{2} a\right)^2\right)$

$= \frac{2ma^2}{9} + \frac{ma^2}{36} + \frac{ma^2}{4}$

$\therefore I = \frac{8+1+9}{36} ma^2 = \frac{1}{2} ma^2$

$T = 2\pi\sqrt{\frac{I}{mgl}} = 2\pi\sqrt{\frac{\frac{1}{2} ma^2}{mg \frac{a}{\sqrt{3}}}}$

$= 2\pi\sqrt{\frac{\sqrt{3}\, a}{2g}} = \frac{2\pi}{\sqrt{20}} = \frac{\pi}{\sqrt{5}}$ s

22. $t = \frac{2.5\,\pi}{\omega} = 1.25\left(\frac{2\pi}{\omega}\right) = 1.25T = T + \frac{T}{4}$

In one time period distance travelled $d_1 = 4A$. In one time period distance travelled $d_2 = 4A$.

In the remaining time $\frac{T}{4}$, distance travelled $d_2 = A$.

$\therefore$Total distance travelled $= 5$ A

23. $T = 2\pi\sqrt{\frac{l}{g}}$,

$T' = \frac{T}{2} + \frac{T_1}{2}$

$$= \frac{1}{2} \times 2\pi\sqrt{\frac{l}{g}} + \frac{1}{2} \times 2\pi\sqrt{\frac{l/4}{g}}$$

$$= \left(\frac{1}{2} + \frac{1}{4}\right) 2\pi\sqrt{\frac{l}{g}} = \frac{3}{4}T$$

24. $A = \frac{d}{2} = 0.4$ m, $\omega = 30$ rev./min

$$= \frac{30 \times 2\pi}{60} = \pi \text{ rad/s}$$

$$\therefore \quad T = \frac{2\pi}{\omega} = 2 \text{ s}$$

25. $y = -2\cos\omega t = 2\sin\left(\omega t - \frac{\pi}{2}\right)$

$$= 2\sin\left(\frac{2\pi}{0.6\pi}t - \frac{\pi}{2}\right) = 2\sin\left(\frac{10}{3}t - \frac{\pi}{2}\right)$$

26. $x = A\cos\omega t \Rightarrow \frac{A}{2} = A\cos\omega t$

$$\Rightarrow \quad \cos\omega t = \frac{1}{2} = \cos\frac{\pi}{3}$$

$$\frac{2\pi}{T}t = \frac{\pi}{3}$$

$$\Rightarrow \quad t = \frac{T}{6}$$

$$v_{av} = \frac{a/2}{t} = \frac{a/2}{T/6} = \frac{3a}{T}$$

27. $|a| = \omega^2 A = g$ for weightlessness at highest point

$$\omega = \sqrt{\frac{g}{A}} = 2\pi f$$

$$\Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{g}{A}} = \frac{1}{2\pi}\sqrt{\frac{g}{0.5}} = \frac{\sqrt{2g}}{2\pi}$$

28. $U = K = \frac{1}{2}U_{max}$

$$= \frac{1}{2} \cdot \frac{1}{2}KA^2 = \frac{1}{2}kx^2$$

$$\Rightarrow \quad x = \frac{A}{\sqrt{2}}$$

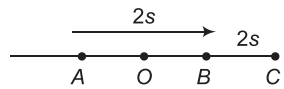
$$\frac{A}{\sqrt{2}} = A\sin\frac{2\pi}{T} \cdot t$$

$$\Rightarrow \quad \frac{2\pi}{T} \cdot t = \frac{\pi}{4}$$

$$\Rightarrow \quad t = \frac{T}{8}$$

$$\therefore \quad v_{av} = \frac{2x}{2t} = \frac{x}{t} = \frac{A\sqrt{2}}{T/8} = \frac{4\sqrt{2}A}{T}$$

29.



$$\Rightarrow T = 8 \text{ s}$$

$$OB = x_B = A\sin\frac{2\pi}{T} \cdot 1 \text{ s}$$

$$= A\sin\frac{\pi}{4} = \frac{A}{\sqrt{2}}$$

$$\therefore OB : A = \frac{A}{\sqrt{2}} : A = 1 : \sqrt{2}$$

30. $x = A\cos\omega t$

At $t = 0$, $x = A$ and $t = \frac{\pi}{6\omega}$,

$$x = A\cos\omega \cdot \frac{\pi}{6\omega} = \frac{\sqrt{3}A}{2}$$

$$\therefore \quad v_{av} = \frac{A - \frac{\sqrt{3}}{2}A}{\frac{\pi}{6}\omega}$$

$$= \frac{6\omega A}{\pi}\left(1 - \frac{\sqrt{3}}{2}\right)$$

$$= \frac{3\omega A}{\pi}(2 - \sqrt{3})$$

31. $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{400}{2}} = \sqrt{200}$

$$= 10\sqrt{2} \text{ rad/s}$$

32. $ma = kx_0$

$$\Rightarrow x_0 = \frac{ma}{k} = \frac{2 \times 5}{400} \text{ m}$$

$$= \frac{10}{4} \text{ cm} = 2.5 \text{ cm}$$

More than One Correct Options

1. At equilibrium, $T\sin\theta = ma$ and $T\cos\theta = mg$

$\Rightarrow \quad \tan\theta = \frac{a}{g}$

$T = \sqrt{T^2 \sin^2\theta + T^2 \cos^2\theta}$

$= m\sqrt{g^2 + a^2}$

2. $x = A\cos(\omega t + \theta)$

and $v = -\omega A \sin(\omega t + \theta)$

at $t = 0, x = A\cos\theta$ and $v = -\omega A \sin\theta$

$\therefore \quad \frac{A}{2} = A\cos\theta$

$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

$\therefore \quad x = A\cos\left(\omega t + \frac{\pi}{3}\right)$

$= A\cos\left(\frac{2\pi}{T}\cdot t + \frac{\pi}{3}\right)$

$= A\sin\left(\frac{2\pi}{T}t + \frac{\pi}{2} + \frac{\pi}{3}\right)$

$= A\sin\left(\frac{2\pi}{T}t + \frac{5\pi}{6}\right)$

3. At equilibrium, $Mg = kx_0$

$\Rightarrow x_0 = \frac{Mg}{k} = \frac{1\times 10}{500}\text{ m} = 2\text{ cm}$

$\Rightarrow A = x - x_0 = 45 - 42 = 3\text{ cm}$

$\frac{1}{2}\times 1\times v^2 = \frac{1}{2}\times 500\,(0.05^2 - 0.02^2)$

$- 1\times 10\times 0.03$

$v = \sqrt{500\,(0.0025 - 0.0004 - 0.6)}$

$= \sqrt{1.05 - 0.6}$ m/s

$= \sqrt{0.45}$ m/s

$= 10\sqrt{45}$ cm/s

$= 30\sqrt{5}$ cm/s

$a_{\max} = \omega^2 A = \frac{K}{m}\cdot A = \frac{500}{1}\times 0.03\text{ m/s}^2$

$= 15\text{ m/s}^2$

At natural length, $U = 0$

4. $T = 2\pi\sqrt{\frac{m_{\text{eff}}}{k}} = 2\pi\sqrt{\frac{2}{800}}$

$= \frac{2\pi}{20} = \frac{\pi}{10}$ s; $\frac{1}{m_{\text{eff}}} = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$

$3x_1 = 6x_2 \Rightarrow x_1 = 2x_2$ and $x_1 + x_2 = 6$ cm

$\Rightarrow \quad 3x_2 = 6$ cm

$\Rightarrow \quad x_2 = 2$ cm

and $\quad x_1 = 4$ cm

$\frac{1}{2}3v_1^2 + \frac{1}{2}6\,v_2^2 = \frac{1}{2}k\times(0.06)^2$

$3v_1^2 + 6v_2^2 = 1.8$

$\Rightarrow \quad v_1^2 + 2v_2^2 = 0.6$

Again, $\quad 3v_1 = 6v_2$

$\Rightarrow \quad v_1 = 2v_2;$

$\therefore \quad 6v_2^2 = 0.6, v_2 = \sqrt{0.1}$

$\therefore \quad P_2 = m_2 v_2 = 6\sqrt{0.1}$

$= \frac{6}{\sqrt{10}} = 1.9$ kg-m/s

$T_1 : T_2 = 1 : 1$

5. At, $t = T$, $y =$ maximum $\Rightarrow a =$ maximum

At, $t = \frac{3T}{4}$, $y = 0 \Rightarrow a = 0 \Rightarrow F = 0$

6. $v = \omega\sqrt{A^2 - x^2}$

$\Rightarrow v^2 = \omega^2 A^2 - \omega^2 x^2$

$\Rightarrow v^2 + \omega^2 x^2 = \omega^2 A^2$

i.e., v-x graph is an ellipse.

$a = -\omega^2 x$

$\Rightarrow y = -mx \rightarrow$ straight line

7. $a = 0$ at $x = \frac{1}{2}$, so, $A = 2 - \frac{1}{2} = 1.5$ m

$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{100}} = \frac{2\pi}{10} = \frac{\pi}{5}$ s $= 0.63$ s

$v_{\max} = A\omega = 1.5\sqrt{100} = 15$ m/s

8. Same phase means a phase difference of $2n\pi$, *i.e.*, their displacement, velocity, acceleration are equal.

9. $y = 3\sin 100\pi t + 4(1 - \cos 100\pi t) - 6$

$= 3\sin 100\pi t - 4\cos 100\pi t - 2$

$= 5\sin(100\pi t + \theta) - 2$

Match the Columns

1. $x = 2\cos\omega t + 2$

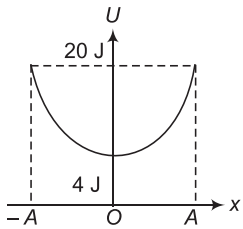
$\therefore$ Mean position is $x = 2$,

Amplitude is $x = 2$

Extreme positions are, $x = 4$ and $x = 0$ where, PE is also maximum.

While, PE is not zero between $x = 0$ to $x = 4$.

2.



$U = 4 + ax^2$, where, $20 = 4 + aA^2$

$\Rightarrow \quad a = \dfrac{16}{A^2}$

$\therefore \quad U = 4 + \dfrac{16}{A^2} x^2$

$$U\left(\frac{A}{2}\right) = 4 + \frac{16}{A^2} \cdot \frac{A^2}{4} = 4 + 4 = 8 \text{ J}$$

$$U\left(\frac{A}{4}\right) = 4 + \frac{16}{A^2} \cdot \frac{A^2}{16} = 5 \text{ J}$$

$$K\left(\frac{A}{4}\right) = 20 - U\left(\frac{A}{4}\right) = 15 \text{ J}$$

$U(0) = 4$ J $\Rightarrow$ $K(0) = 20 - 4 = 16$ J

$$K\left(\frac{A}{2}\right) = 20 - U\left(\frac{A}{2}\right) = 20 - 8 = 12 \text{ J}$$

3. At t_1, $a \neq 0$, $v \neq 0$ and $x \neq 0$ but a = positive

$\Rightarrow x = -$ve and $v = -$ve

At t_2, $a = 0, v \neq 0$ and $x = 0$ at t_2, area is –ve, so, $v =$ –ve

and at mean position velocity is maximum.

4. $x = 2 \sin 4\pi t$, $\quad v = 8\pi \cos 4\pi t$ $\quad$ and $a = -32\pi^2 \sin 4\pi t$

$v_{\max} = A\omega = 8\pi$ m/s $= 25.1$ m/s.

So, velocity never becomes 30 m/s

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{4\pi} = \frac{1}{2} \text{ s},$$

So, in 1 s particle completes 2 oscillations.

So, speed becomes 10 m/s, 8 times in 2 oscillations and velocity 4 times.

$$K_{\max} = \frac{1}{2} m v_{\max}^2 = \frac{1}{2} \times 2 \times 64\pi^2 = 640 \text{ J}$$

So, it becomes 400 J, 8 times in two oscillations.

$a_{\max} = 32\pi^2 = 320$ m/s, in two oscillations it becomes – 100, 4 times.

5. $x = 4 + 6 \sin \pi t$.

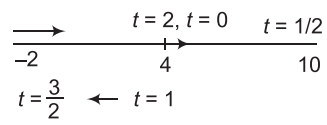
$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2 \text{ s}.$$

$A = 6$ m and $-2 \le x \le 10$

at $t = 0, x = 4$ m,

at $t = \dfrac{1}{2}$ s, $x = 10$ m.

at $t = 1$ s, $x = 4$ m.



at $t = \dfrac{3}{2}$ s, $x = -2$ m.

and $t = 2$ s, $x = 4$ m.

So,

(a) 10 m $\to$ 4 m $\quad \dfrac{1}{2} \le t \le 1 \to$ q

(b) 10 m $\to$ 7 m $\quad \dfrac{1}{2} \le t < 1 \to$ q

(c) 7 m $\to$ 1 m $\quad \dfrac{1}{2} < t < \dfrac{3}{2} \to$ r

(d) 10 m $\to -2$m $\quad \dfrac{1}{2} \le t \le \dfrac{3}{2} \to$ r

12 Elasticity

Introductory Exercise 12.1

1. $Y = \frac{F/A}{\Delta l / l} = \frac{Fl}{A\,\Delta l}$

$\Rightarrow \Delta l = \frac{Fl}{YA}$

$\Rightarrow \Delta l \propto \frac{1}{Y}$ as Young's modulus of A is twice of B, so elongation of A will be half of B.

2. $\Delta l = \frac{Fl}{YA} = \frac{10^4 \times 1}{2 \times 10^{10} \times 4 \times 10^{-4}} = \frac{1}{800}$ m

$= \frac{1}{8}$ cm $= 0.125$ cm

3. $\frac{F}{A} = Y\,\frac{\Delta l}{l} = 90 \times 10^6$ N/m^2

$\Rightarrow A = \frac{1.6 \times 10^6}{90 \times 10^6} = \frac{1.6}{90} = \frac{\pi}{4}(d_2^2 - d_1^2)$

$d_2 = \sqrt{\frac{4 \times 1.6}{90\,\pi} + (0.2)^2} = \sqrt{0.063}$

$= 0.250 = 250$ mm

AIEEE Corner

Subjective Question (Level 1)

1. $Y = \frac{Fl}{A\Delta l} \Rightarrow A = \frac{Fl}{Y\,\Delta l} \Rightarrow \pi\frac{d^2}{4} = \frac{Fl}{Y\,\Delta l}$

$\Rightarrow d = \sqrt{\frac{4Fl}{\pi\,Y\Delta l}}$

$\therefore d = \sqrt{\frac{4 \times 400 \times 3}{2.1 \times 10^{11} \times 3.14 \times 2 \times 10^{-3}}}$ m

$= 10^{-3}\sqrt{\frac{4.8}{4.2 \times 3.14}}$ m

$= 1.9 \times 10^{-3}$ m $= 1.9$ mm

2. $\left(\frac{F}{A}\right)_{max} = \frac{1}{3} \times 3 \times 10^8$ N/m^2

$= \frac{900\,(g + a)}{4 \times 10^{-4}}$

$\Rightarrow g + a = \frac{4 \times 10^4}{900} = \frac{400}{9}$

$\Rightarrow a_{max} = \frac{400}{9} - g = 34.64$ m/s^2

3. $\left(\frac{F}{A}\right)_{max} = 1.5 \times 10^8$ N/m^2

$\Rightarrow \pi\frac{d^2}{4} = \frac{F}{1.5 \times 10^8}$

$\Rightarrow d_{min} = \sqrt{\frac{4F}{\pi \times 1.5 \times 10^8}}$

$$= \sqrt{\frac{4 \times 100}{3.14 \times 15 \times 10^8}}$$

$$= 10^{-3} \sqrt{\frac{4}{3.14 \times 1.5}} = 0.92 \text{ mm}$$

4. $dl = \frac{\mu\ x\ g + dx}{YA} = \frac{\rho g}{Y}\ x\ dx$

$$\Rightarrow \Delta l = \frac{\rho g}{Y} \int_0^l x\ dx = \frac{\rho g l^2}{2Y}$$

$$= \frac{8 \times 10^3 \times 9.8 \times 25}{2 \times 2 \times 10^{11}} = 5 \times 9.8 \times 10^{-7} \text{ m}$$

$$= 4.9 \times 10^{-6} \text{ m}$$

5. $\left(\frac{F}{A}\right)_0 = \frac{(m + m_1 + m_2)\ g}{A_0} = 8 \times 10^8$

$$\Rightarrow m = \frac{8 \times 10^8 \times 6 \times 10^{-7}}{10} - 30$$

$$= 48 - 30 = 18 \text{ kg}$$

$$\left(\frac{F}{A}\right)_L = \frac{(m + m_1)\ g}{A_L} = 8 \times 10^8$$

$$\Rightarrow m = \frac{8 \times 10^8 \times 3 \times 10^{-7}}{10} - 10 = 14 \text{ kg}$$

So, lower wire will break if $m > 14$ kg. *i.e.*, maximum load which can be put on hanger without breaking wire is 14 kg

6. $\frac{F}{A}$ is same as both have same cross-section area and subjected under same tension.

$$\frac{\Delta l}{l} = \frac{F}{YA} \Rightarrow \frac{\Delta l}{l} \propto \frac{1}{Y}$$

$$\therefore \left(\frac{\Delta l}{l}\right)_{St} \Big/ \left(\frac{\Delta l}{l}\right)_{Cu} = \frac{Y_{Cu}}{Y_{St}} = \frac{1.3}{2} = \frac{13}{20}$$

7. $B = \frac{\Delta p}{\frac{\Delta l}{V}} = \frac{-\Delta p}{\frac{\Delta p}{\rho}}$

$$\Rightarrow \Delta\rho = -\frac{\rho \Delta p}{B} = \frac{\rho\ h\ \rho\ g}{B}$$

$$\Delta\rho = \frac{400 \times (1030)^2 \times 10}{2 \times 10^9}$$

$$= \frac{4 \times (1.03)^2}{2} = 2.12 \text{ kg/m}^3$$

8. $\Delta l_1 = \frac{Mgl}{YA} = \frac{V\rho gl}{YA} \therefore \Delta l_2 = \frac{V\ (\rho - \sigma)\ gl}{YA}$

$$\Rightarrow \Delta l = \Delta l_2 - \Delta l_1 = -\frac{V\sigma gl}{YA}$$

$$= -\frac{10^{-3} \times 800 \times 10 \times 3}{\pi \times (2 \times 10^{-4})^2 \times 8 \times 10^{10}}$$

$$= -\frac{30}{4\pi} \times 10^{-3} \text{ m}$$

$$= -2.39 \times 10^{-3} \text{ m}$$

9. $B = -\frac{\Delta p}{\Delta V / V} = \frac{-h\rho g}{\Delta V / V} = \frac{180 \times 10^3 \times 9.8}{10^{-3}}$

$$= 1.76 \times 10^9 \text{ N/m}^2$$

10. $Y = \frac{F / A}{\Delta l / l}$

$$\Rightarrow F = \frac{YA\ \Delta l}{l} = mg + \frac{mv^2}{l}$$

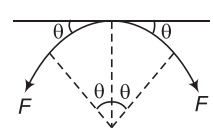
$$\Rightarrow \quad v = \sqrt{\frac{l}{m}\left(\frac{YA\Delta l}{l} - mg\right)}$$

$$v = \sqrt{\frac{5}{25}\left(\frac{2 \times 10^{11} \times \pi \times 4 \times 10^{-6} \times 10^{-2}}{5} - 250\right)}$$

$$= \sqrt{\frac{1}{5}\left(\frac{8\pi}{5} \times 10^3 - 250\right)} = 30.9 \text{ m/s}$$

11. $2F \sin\theta = \frac{mv^2}{R} = \frac{\mu R \cdot 2\theta \cdot v^2}{R}$

$$\sin\theta \approx \theta$$



$$\Rightarrow F = \mu v^2$$

$$F = \frac{m}{2\pi R} R^2\ \omega^2$$

$$= \frac{4\pi^2 \nu^2\ mR^2}{2\ \pi\ R}$$

$$= 2\pi\ \nu^2\ mR$$

$$Y = \frac{Fl}{A\ \Delta l} = \frac{2\pi\ \nu^2\ mR \times 2\ \pi R}{\pi r^2 \Delta l}$$

$$\Rightarrow \Delta l = \frac{4\pi^2\ \nu^2\ mR^2}{Y\ \pi r^2}$$

$$l = 2\pi R$$

$$\therefore \Delta R = \frac{\Delta l}{2\pi} = \frac{2\pi\nu^2 mR^2}{\pi Y r^2}$$

But $m = (2\pi R)(\pi r^2)(\rho)$

Substituting the values we get,

$$\Delta R = \frac{4\pi^2 \nu^2 \rho R^3}{Y}$$

12. $\Delta l = \dfrac{Fl}{YA} = \dfrac{(mg + m\omega^2\ l)\ l}{YA}$

$$= \frac{(60 + 6 \times 4 \times 4\pi^2 \times 0.6) \times 0.6}{2 \times 10^{11} \times 5 \times 10^{-6}}$$

$$= 3.8 \times 10^{-4}\ \text{m}$$

13. $2T_c + T_s = mg$ ...(i)

$\Delta l_s = \Delta l_c$

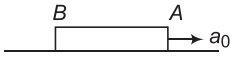
$$\frac{T_s l}{AY_s} = \frac{T_c l}{AY_c}$$

$$T_c = \frac{T_s}{2} \quad \text{...(ii)}$$

Solving these two equations we get,

$T_s = \dfrac{mg}{2}$ and $T_c = \dfrac{mg}{4}$

14.



(a) **At centre :**

$$T = \frac{m}{2} \cdot a_0 = \left(\frac{L}{2} s\rho\right) a_0$$

$$\therefore \text{Stress} = \frac{T}{S} = \frac{L}{2}\rho a_0$$

(b) **At distance *x* from end *B* :**

$$T_x = m_x a_0 = (xs\rho) a_0$$

$$\Delta l = \int_0^L \frac{T_x dx}{SY} = \frac{\rho_0 a_0 L^2}{2Y}$$

15. Energy stored in stretched wire = $ms\Delta Q$

Objective Questions (Level-1)

1. $\theta = -\dfrac{\Delta p}{\dfrac{\Delta V}{V}}$ as $\Delta V = \Delta \Rightarrow B = \infty$

2. Young's modulus do not depend upon length of radius of wire. It is constant for a material.

3. $F = \dfrac{YA\ \Delta l}{l} \Rightarrow F \propto \dfrac{1}{l}$

$$\Rightarrow F_2 = F_1 \frac{l_1}{l_2} = 2F_1 = 2W$$

4. During free fall, stress developed in the rod is zero, so there is no elongation.

5. $B = -\dfrac{\Delta p}{\Delta V / V} = \dfrac{1.2 \times 10^5 \times 10^{-3}}{0.3 \times 10^{-3}}$

$$= 4 \times 10^5\ \text{N/m}^2$$

6. $\dfrac{F}{A} = Y\dfrac{\Delta l}{l} = \dfrac{mv^2}{lA}$

$$\Rightarrow v = \sqrt{\frac{l}{m}\frac{F}{A}A} = \sqrt{\frac{0.3}{10} \times 4.8 \times 10^7 \times 10^{-6}}$$

$$= \sqrt{3 \times 48 \times 10^4 \times 10^{-6}} = 1.2\ \text{m/s}$$

$$\omega = \frac{v}{l} = \frac{1.2}{0.3} = 4\ \text{rad/s}$$

7. $dl = \dfrac{\mu gx\ dx}{YA} \Rightarrow \dfrac{F}{A} = \dfrac{\mu gx}{A} = \rho gx$ for $x = \dfrac{1}{2}$

$$\Rightarrow \frac{F}{A} = \frac{1}{2}\ pgL$$

8. $\dfrac{\Delta V}{V} = -\dfrac{\Delta p}{p} = -10^{-3}$

$$\Rightarrow \Delta p = -B\frac{\Delta V}{V} = 2 \times 10^9 \times 10^{-3}$$

$$= 2 \times 10^6\ \text{N/m}^2$$

9. System is bound or force is attractive when potential energy is negative.

10. $Y = \dfrac{Fl}{A\ \Delta l} = \dfrac{T_1 l}{A\ (l_1 - l)} = \dfrac{T_2 l}{A\ (l_2 - l)}$

$$\Rightarrow \quad T_1\ (l_2 - l) = T_2\ (l_1 - l)$$

$$\Rightarrow \quad T_1 l_2 - T_2 l_1 = (T_1 - T_2)\ l$$

$$\Rightarrow \quad l = \frac{T_1 l_2 - T_2 l_1}{T_1 - T_2}$$

11. $\Delta l = \dfrac{Fl}{YA} = \dfrac{mgl}{YA} = \dfrac{mgl\ .\ l}{YAl} = \dfrac{\rho g l^2}{Y}$

$$\frac{\Delta l_2}{\Delta l_1} = \left(\frac{l_2}{l_1}\right)^2 = 4 \Rightarrow \Delta l_2 - 4\Delta l_1 = 4\Delta l$$

12. $\Delta l = l\alpha\ \Delta Q, U = \frac{1}{2} \times F \times \Delta l = \frac{1}{2} YA\ \frac{\Delta l}{l} \cdot \Delta l$

$= \frac{YA\ (\Delta l)^2}{2l} = \frac{YAl^2\ d^2\ (\Delta l)^2}{2l}$

$\therefore\ u = \frac{U}{V} = \frac{\frac{1}{2}\ YAl\ \alpha^2\ (\Delta\theta)^2}{Al}$

$= \frac{1}{2}\ Y\ \alpha^2\ (\Delta\theta)^2$

$= \frac{1}{2} \times 10^{11} \times 144 \times 10^{-12} \times 400$

$= 72 \times 40\ \text{J} = 2880\ \text{J}$

13. $\Delta l_t = l\ \alpha\ \Delta\theta = 1 \times 10^{-4} \times 20\ \text{m} = 2\ \text{m}$

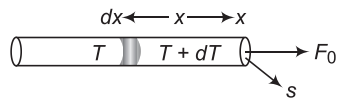
$\Rightarrow \quad \Delta l_e = 1\ \text{mm}$

$\frac{F}{A} = Y\ \frac{\Delta l}{l}$

$= 10^{11} \times \frac{10^{-3}}{1} = 10^8\ \text{N/m}^2$

14. $dT = \mu\ dx \cdot a = \rho S\ dx \cdot \frac{F_0}{m} = \frac{\rho S F_0}{PSL} dx$

$= \frac{F_0}{L} dx$



$dl = \frac{T \cdot dx}{YS} = \frac{F_0\ x\ dx}{YSL}$

$\Rightarrow\ \Delta l = \frac{F_0 L^2}{2YSL}$

$= \frac{1}{2} \cdot \frac{F_0}{YS} \cdot L$

$\therefore\ \frac{\Delta l}{L} = \frac{F_0}{2YS}$

JEE Corner

Assertion and Reason

1. Elasticity depends upon Young's modulus, which is more for steel as more stress is required to produce same strain, *i.e.*, reason is true explanation of assertion.
2. Assertion and reason are both false as Bulk modulus of gas depends upon thermodynamic process, while the relation between pressure and temperature also depends upon thermodynamic process.
3. Assertion is false as Young's modulus do not depend upon length while reason is true.
4. Assertion and reason are both true but not correct explanation.
5. Reason truly explains assertion.
6. Assertion is not true as it is true only with in elastic limit.
7. $\frac{B_{\text{iso}}}{B_{\text{adio}}} = \frac{p}{rp} = \frac{1}{r} = \frac{1}{5/9} = \frac{3}{5}$ is true and reason is also true but not correct explanation.

More than one Correct Options

1. $dW = F \cdot dx = kx\ dx = \frac{YA}{L} \cdot x\ dx$

$W = \frac{YA}{l} \int_0^l x\ dx = \frac{YA}{L} \cdot \frac{1}{2} l^2 = \frac{YAl^2}{2L} = U$

2. $F = k\ \Delta x = \frac{YA}{L}\ \Delta x = m\Delta x$

From graph, $m_A < m_B$

$\Rightarrow \quad \left(\frac{YA}{L}\right)_A < \left(\frac{YA}{L}\right)_B$

it is possible when, $Y_A < Y_B$ or $A_A < A_B$ or $L_A > L_B$ but as $Y_A = Y_B$ and $L_A = L_B \Rightarrow A_A < A_B$

3. $W_g = Mg \cdot l, \qquad U = \frac{1}{2} kB^2 = \frac{1}{2} mgl,$

$Q = W_g - U = \frac{1}{2} Mgl$

Match the Columns

1. (a) $[\text{stress}] = [F]/[A] = [\text{MLT}^{-2}\text{L}^{-2}]$

$= [\text{ML}^{-1}\text{T}^{-2}]$

$u = \frac{U}{V} = \frac{[\text{MLT}^{-2}\text{L}^{-1}]}{[\text{L}]^3}$

$= [\text{ML}^{-1}\text{T}^{-2}] \longrightarrow r^2$

(b) $[\text{strain}] = \Delta l / l = [\text{M}^0\text{L}^0\text{T}^0] \rightarrow p, q$

(c) $[Y] = \frac{[F/A]}{[\Delta l / l]} = [\text{stress}]$

$= [\text{ML}^{-1}\text{T}^{-2}] \rightarrow r$

(d) $F = kx$

$\Rightarrow [h] = \frac{[F]}{[x]}$

$= [\text{MLT}^{-2}\text{L}^{-1}]$

$= [\text{MT}^{-2}] \rightarrow s$

2. $Y = \frac{Fl}{A\Delta l}$ or $Fl = YA\Delta l$

(a) $F\uparrow \Rightarrow \frac{F}{A}\uparrow \Rightarrow \Delta l\uparrow \rightarrow p, q$

(b) $l\uparrow \Rightarrow \Delta l\uparrow \rightarrow p$

(c) $A\uparrow \Rightarrow \frac{F}{A}\downarrow \Rightarrow \Delta l\downarrow \rightarrow r, s$

(d) $Y\uparrow \Rightarrow \Delta l\downarrow \rightarrow q, r$

13 Fluid Mechanics

Introductory Exercise 13.1

1. $F_A = p_A\ A = h_A\ \rho g A$ and $F_B = h_B\ \rho g A$ as $h_A < h_B \Rightarrow F_A < F_B$

2. $p = \dfrac{F}{A} = \dfrac{3 \times 10^4 \times 9.8}{425 \times 10^{-4}}$ N/m^2

$= 6.92 \times 10^5$ N/m^2

3. $h_1\ \rho_1\ g = h_2\ \rho_2\ g$

$\Rightarrow \dfrac{\rho_2}{\rho_1} = \dfrac{h_1}{h_2} = \dfrac{100}{12.5} = \dfrac{4}{5} = 0.8$

4. $(10+15)\ \rho_1 g + l_1 \rho g = (12.5 + 15)\rho_2 g + l_2 \rho g$

$\Rightarrow (l_2 - l_1)\ \rho = 25\rho_1 - 27.5\rho_2$

$\therefore \quad l_2 - l_1 = \dfrac{25 \times 1}{13.6} - \dfrac{27.5 \times 0.8}{13.6}$

$= \dfrac{25 - 22}{13.6}$ cm $= 0.22$ cm

5. (i) $p_a = pxr_0 + h_1 \rho g = (76 + 20)\ \rho g$

$= 96$ cm of Hg and $p_a - p_0 = 20$ cm of Hg

$p_b = p_0 - h_2 \rho g = (76 - 18)\ \rho g = 58$ cm of Hg and $p_0 - p_b = 18$ cm of Hg

(ii) 13.6 cm of water ≡ 1 cm of Hg, *i.e.*, Hg in left limb will rise by 0.5 cm and on right limb, it will fall by 0.5 cm. Thus difference in Hg level will become 19 cm.

Introductory Exercise 13.2

1. $V\rho_s g = \dfrac{4}{5}\ V\ \rho_w\ g + \dfrac{1}{5}\ V\rho_l g$

$\Rightarrow \rho_s = \dfrac{4}{5}\ \rho_w + \dfrac{1}{5}\ \rho_l$

$= \dfrac{4}{5} \times 10^3 + \dfrac{1}{5} \times 13.5 \times 10^3$

$\rho_s = \dfrac{17.5}{5} \times 10^3$ kg/m^3 $= 3.5 \times 10^3$ kg/m^3

2. $V\rho_s = 210$ g, $V(\rho_s - \rho_w) = 180$ g and $V(\rho_s - \rho_l) = 120$ g

$\dfrac{\rho_s - \rho_l}{\rho_s} = \dfrac{180}{210} = \dfrac{6}{7}$

$\Rightarrow 1 - \dfrac{6}{7} = \dfrac{\rho_w}{\rho_s}$

$\Rightarrow \rho_s = 7\rho_w = 7 \times 10^3$ kg/m^3

$\dfrac{\rho_s - \rho_l}{\rho_s} = \dfrac{120}{210} = \dfrac{4}{7}$

$\Rightarrow \quad 1 - \dfrac{4}{7} = \dfrac{\rho_l}{\rho_s}$

$\Rightarrow \quad \rho_l = \dfrac{3}{7}\rho_s = 3 \times 10^3$ kg/m^3

3. As in equilibrium buoyant force and weight balance, so in accelerating lift both buoyant force and weight increase such that there is no change in volume of submerged wood.

4. As $w_{\text{ap}} = w - B$ as water is placed in water, so, apparent weight is zero.

5. If a body is placed on water, then it cannot displace water of more weight than its own weight as in such case, buoyant force will be more than weight and the case buoyant jump up the liquid, which is not possible.

6. $w + V\rho g = V\sigma g$

$$m = V(\sigma - \rho)$$

$$\Rightarrow \frac{m'}{\rho}(\sigma - \rho) = 120\left(\frac{\sigma}{\rho} - 1\right)$$

$$= 120 \text{ kg}\left(\frac{1}{0.6} - 1\right) = 120 \times \frac{0.4}{0.6} \text{ kg} = 80 \text{ kg}$$

7. The amount of water drink and the volume of water displaced due to weight of drank water remains same such that water level do not change.

$$V\rho g + mg = V\sigma g$$

$$\Rightarrow V(\sigma - \rho) = m$$

$$\Rightarrow a^3(\sigma - \rho) = m$$

$$\therefore \quad a = \left[\frac{m}{\sigma - \rho}\right]^{1/3}$$

$$= \left[\frac{0.5}{0.1 \times 10^3}\right]^{1/3}$$

$$= 10^{-1} \times 5^{1/3}$$

$$= 10 \times 5^{1/3} \text{ cm} = 17 \text{ cm}$$

9. $V\rho g = 0.6\, V\sigma_1 g$

$$\Rightarrow \rho = 0.6\,\sigma_1 = 600 \text{ kg/m}^3$$

$$V\rho g = 0.85\, V\sigma_2 g$$

$$\Rightarrow \sigma_2 = \frac{\rho}{0.85} = \frac{600}{0.85} \text{ kg/m}^3$$

$$= 705.88 \text{ kg/m}^3$$

10. $V\sigma g = mg$

$$\Rightarrow m = V\sigma = A h \sigma = \pi r^2 h \sigma$$

$$= \pi \times (0.8 \times 10^{-2})^2 \times 3 \times 10^{-2} \times 10^3 \text{ kg}$$

$$= \pi \times 0.64 \times 3 \text{ g}$$

$$= 6.03 \text{ g}$$

Introductory Exercise 13.3

1. $\Delta W = F \times s = 2\,Tl \times 0.5$

$$= 2 \times 7.2 \times 10^{-2} \times 10^{-1} \times 10^{-3}$$

$$= 1.44 \times 10^{-5} \text{ J}$$

2. $p = \dfrac{4T}{r}$, $dW = p\,dV = \dfrac{4T}{r} \cdot 4\pi r^2 dr$

$$= 16\pi\, Tr\, dr$$

$$W = \int_0^R 16\pi\, T\, r\, dr = 8\pi\, TR^2$$

3. $\dfrac{4}{3}\pi r^3 = 27 \times \dfrac{4}{3}\pi R^3$

$$\Rightarrow r = 3R$$

$$\Rightarrow R = r/3$$

$$\therefore \Delta S = 27 \times 4\pi R^2 - 4\pi r^2$$

$$= 4\pi\left(27 \times \frac{r^2}{9} - r^2\right) = 8\pi r^2$$

$$\Delta W = T\,\Delta S = 8\,\pi r^2 T$$

4. As wax seals thread capillaries which sucks water, that's why water cannot spread over cloth and cloth becomes waterproof.

5. (a) $v = \dfrac{p_1 - p_2}{4\,\pi L}(R^2 - r^2)$

$$v_0 = \frac{(p_1 - p_2)R^2}{4\,\eta\, l} = 3 \text{ m/s}$$

$$= v_0\left(1 - \frac{r^2}{R^2}\right)$$

$$= 3\left(1 - \frac{100}{400}\right) = \frac{9}{4} = 2.25 \text{ m/s}$$

(b) At $r = R \Rightarrow v = 0$

6. (a) $\dfrac{dV}{dt} = \dfrac{\pi}{8}\left(\dfrac{R^4}{\eta}\right)\left(\dfrac{p_1 - p_2}{L}\right)$

$$= \frac{\pi}{8} \times \frac{4^4 \times 10^{-8}}{1.005 \times 10^{-1}} \times \frac{1400}{20}$$

$$= \frac{32\pi \times 70 \times 10^{-7}}{1.005}$$

$$= 7.005 \times 10^{-4} \text{ m}^3/\text{s}$$

(b) $\Delta p \propto \dfrac{1}{R^4}$

$$\Rightarrow \frac{\Delta p_2}{\Delta p_1} = \frac{R_1^4}{R_2^4} = 2^4$$

$$= 16$$

$$\Rightarrow \Delta p_2 = 16\Delta p_1 = 2.24 \times 10^4 \text{ Pa}$$

(c) $\frac{dV}{dt} \propto \frac{1}{\eta}$

$\therefore$ For new value of η.

$$\frac{dV}{dt} = \left(\frac{1.005}{0.469}\right)(7.0 \times 10^{-4})\ \text{m}^3/\text{s}$$

$$= 1.5 \times 10^{-3}\ \text{m}^3/\text{s}$$

AIEEE Corner

Subjective Questions (Level 1)

1. $mg = V\sigma g$

$\Rightarrow V\rho_1 g = \frac{3}{4} V\rho_2 g$

$\Rightarrow \rho_2 = \frac{4}{3}\rho_1$

2. $V\rho g = 0.095$ N and $V(\rho - \sigma) g = 0.071$

$\Rightarrow \frac{\rho - \sigma}{\rho} = \frac{71}{96}$

$\frac{\sigma}{\rho} = 1 - \frac{71}{96} = \frac{25}{96}$

$\Rightarrow \rho = \frac{96}{25}\sigma = 3840\ \text{kg/m}^3$

3. $(V + 2)\sigma g = m_1 g + m_2 g$

$\Rightarrow V = \frac{m_1 + m_2}{\sigma} - 2 = \frac{30\ \text{g}}{10/\text{cm}^3} - 2\ \text{cm}^3$

$= 28\ \text{cm}^3$

4. $T = B - W = V(\sigma - \rho) g = mg\left(\frac{\sigma}{\rho} - 1\right)$

$= 71.2\left(\frac{1}{0.75} - 1\right)\text{N}$

$= \frac{71.2}{3}\ \text{N} = 23.73\ \text{N}$

5. $V\sigma g = V\rho g + mg$

$\Rightarrow V = \frac{m}{\sigma - \rho} = \frac{50}{1000 - 850}\ \text{m}^3$

$= \frac{1}{3}\ \text{m}^3 = 0.3\ \text{m}^3$

6. (a) $\rho V = 10$ g, $(\rho - \sigma) V = 8$ g

$\Rightarrow 1 - \frac{\sigma}{\rho} = 0.8 \Rightarrow \frac{\sigma}{\rho} = 0.2$

or $\rho = \frac{\sigma}{0.2} = 5\sigma$

$= 5000\ \text{kg/m}^3$

$= 6\ \text{g/cm}^3$

$\therefore V = \frac{10\ \text{g}}{p} = 2\ \text{cm}^3 = 2 \times 10^{-6}\ \text{m}^3$

(b) $(\rho - \sigma') V = 8.5\ \text{g} \Rightarrow 1 - \frac{\sigma'}{\rho} = 0.85$

$\Rightarrow \frac{\sigma'}{\rho} = 0.15$

$\Rightarrow \sigma' = 0.15\rho = 750\ \text{kg/m}^3$

7. $B = V\sigma g = 1\ \text{cm}^3 \times 1\ \text{g/cm}^3$

$\times 980\ \text{cm/s}^2 = 980$ dyne

$W_{ap} = 20 \times 980$ dyne $+ 980$ dyne

$= 20580$ dyne $= 0.206$ N

8. $a^3 \rho_i g = a^2 x \sigma_m g + a^2 (a - x)\sigma_w g$

$a\rho_i = x\sigma_m + (a - x)\sigma_w$

$\Rightarrow a(\rho_i - \sigma_w) = x(\sigma_m - \sigma_w)$

$\Rightarrow \frac{x}{a} = \frac{\rho_i - \sigma_w}{\sigma_m - \sigma_w} = \frac{7.7 - 1}{13.6 - 1} = \frac{6.7}{12.6} = 0.532$

$\Rightarrow x = 31.9$ mm is in mercury and 28.1 mm in water.

9. $a = \frac{F}{m} = \frac{B - W}{m}$

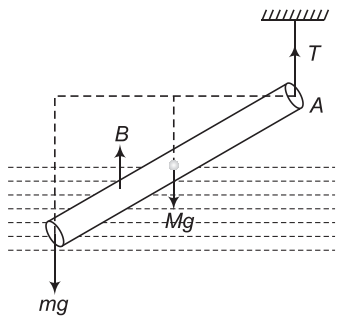
$= \frac{V(\sigma - \rho) g}{m}$

$= \left(\frac{\sigma}{\rho} - 1\right) g = \left(\frac{1}{0.4} - 1\right) g$

$= \frac{3}{2} g = 14.7\ \text{m/s}^2$

$t = \sqrt{\frac{2S}{a}} = \sqrt{\frac{2 \times 2.9}{14.7}} = 0.63$ s

10. $B + T = (M + m)\, g \Rightarrow T = 18g - B$ ...(i)



Equating the torques above point A, we have

$$(12\,g)\left(\frac{l}{2}\right) + (6g)(l) = B\left(\frac{3l}{4}\right)$$

or $B = 16g$ ...(ii)

Substituting in Eq. (i) we have

$$T = 2g = 19.6\,\text{N}$$

$$B = \frac{V}{2} \times (1000)(9.8) = 16 \times 9.8$$

$$\therefore\ V = 32 \times 10^{-3}\ \text{m}^3$$

11. (a) $T = B - W = V\,(\sigma - \rho)\,(g + a)$

$$= \frac{m}{\rho}\,(\sigma - \rho)\,(g + a)$$

$$= M\left(\frac{\sigma}{\rho} - 1\right)(g + a)$$

$$= 2\,(2 - 1)\,(10 + 2) = 24\ \text{N}$$

(b) $a = \dfrac{F}{m} = \dfrac{B - W}{m} = \dfrac{24\,\text{N}}{2\ \text{kg}} = 12\ \text{m/s}^2$

12. $F = kx \Rightarrow p = \dfrac{kx}{A}$ or $x = \dfrac{pA}{k}$

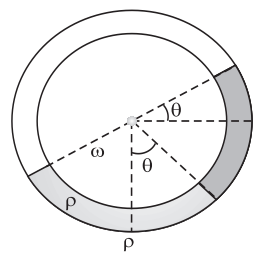
(a) $x_1 = \dfrac{131 \times 10^3 \times 0.5 \times 10^{-4}}{60}\ \text{m}$

$$= \frac{131 \times 0.5}{6}\ \text{cm} = 10.9\ \text{cm}$$

(b) $x_2 = \dfrac{30 \times 10^3 \times 0.5 \times 10^{-4}}{60}\ \text{m} = 2.5\ \text{cm}$

13. $(r - r\sin\theta)\,\rho g = (r - r\cos\theta)\,\rho g$

$$+ (r\sin\theta + r\cos\theta)\,\sigma\, g$$



$$(\cos\theta - \sin\theta)\,\rho = (\cos\theta + \sin\theta)\,\sigma$$

$$\cos\theta\,(\rho - \sigma) = \sin\theta\,(\rho + \sigma)$$

$$\therefore \quad \tan\theta = \frac{\rho - \sigma}{\rho + \sigma}$$

14. $p_L = p_R \Rightarrow \dfrac{600\,g}{800 \times 10^{-4}}$

$$= 8\sigma\, g + \frac{F}{25 \times 10^{-4}}$$

$$\therefore F = 25 \times 10^{-4}\left[\frac{600}{800 \times 10^{-4}} - 8\,\sigma\right] g$$

$$= \left(\frac{600}{32} - 2 \times 10^{-2} \times 780\right) g$$

$$= (18.75 - 15.6)\ g = 31\ \text{N}$$

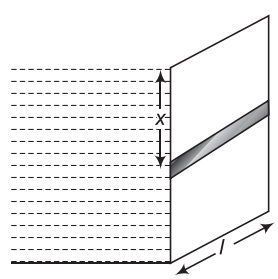
15. $(h_1 + l + h_2)\rho_w g = l\,\rho_w\, g + h_2\,\rho_m\, g$

$$h_1\rho_w = h_2\,(\rho_m - \rho_w)$$

$$\Rightarrow \quad h_1 = \frac{\rho_m - \rho_w}{\rho_w}\, h_2$$

$$= \frac{13.6 - 1}{1} \times 1\ \text{cm} = 12.6\ \text{cm}$$

16.



$$p(x) = x\,\rho g$$

(a) $dF\,(x) = \rho g\ x\ .\ l dx = \rho g l x\ dx$

$$F = \rho g l \int_0^h x\ dx = \frac{1}{2}\,\rho g\ l h^2$$

(b) $d\tau(x) = dF(x) \,.\, (h - x)$

$$= \rho g l\,(h - x)\,x\,dx$$

$$= \rho g l\,(hx - x^2)\,dx$$

$$\tau = \rho g l \int_0^h (hx - x^2)\,dx$$

$$= \rho g l \left(\frac{1}{2}h^3 - \frac{1}{3}h^3\right)$$

$$= \frac{1}{6}\rho g l h^3$$

(c) $\frac{1}{2}\rho g l h^2 \times y = \frac{1}{6}\rho g l h^3$

$$\Rightarrow \quad y = \frac{2}{6}h = \frac{1}{3}h$$

17. $15\rho_w g = x\rho_m g$

$$\Rightarrow \quad x = \frac{15\rho_w}{\rho_m} = \frac{15}{13.6}\text{ cm} = 1.10\text{ cm}$$

$$h = (15 - 1.10)\text{ cm} = 13.90\text{ cm}$$

(a) $\Delta p = 15\rho_w g$

$= 15 \times 10^{-2}\text{ m} \times 10^3\text{ kg/m}^3 \times 9.8\text{ m/s}^2$

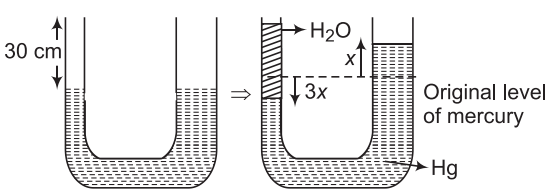
$= 1470\text{ N/m}^2 = 1470\text{ Pa}$

18. $h_1\rho_w g = h_2\rho_0 g$

$$\Rightarrow \quad h_1 = h_2\,\frac{\rho_0}{\rho_w}$$

$$= 20 \times \frac{0.7}{1} = 18\text{ cm}$$

19.



$h_1\rho_1 = h_2\rho_2$

$\therefore \quad (30 + 3x)(1) = (4x)(13.6)$

Solving we get, $x = 0.584$ cm.

20. $p + \rho g h + \frac{1}{2}\rho v^2 = \text{constant}$

$$20\text{kPa} + 0 + \frac{1}{2}\rho \times 3^2 = p + \rho g \times 1 + \frac{1}{2}\rho \times 4^2$$

$$p = 20\text{kPa} - \rho g + \frac{1}{2}\rho\,(3^2 - 4^2)$$

$$= 20 \times 10^3 - 9.8 \times 10^3 + \frac{1}{2} \times 10^3\,(-7)$$

$$= (20 - 13.3) \times 10^3\text{ Pa}$$

$$= 6.7\text{ kPa}$$

$$18\text{kPa} + 0 + \frac{1}{2}\rho \times 0^2$$

$$= p' + pg \times 1 + \frac{1}{2}\rho \times 0^2$$

$$p' = 18\text{ kPa} - 9.8\text{ kPa} = 8.2\text{ kPa}$$

21. $v = \sqrt{2gd}$, $R = vt = t\sqrt{2gd}$ and $h = \frac{1}{2}gt^2$

$$\therefore \; h = \frac{1}{2}g\,\frac{R^2}{2gd} = \frac{R^2}{4d}$$

$$\Rightarrow d = \frac{R^2}{4h}$$

22. $Au = av$ and $Fu\,dt = \frac{1}{2}mv^2$

$$= \frac{1}{2}\rho\,.\,a\,v\,dt\,.\,v^2$$

$$Fu = \frac{1}{2}\rho a\,v^3$$

$$F\,\frac{av}{A} = \frac{1}{2}\rho\,a\,v^3 \Rightarrow v^2 = \frac{2F}{\rho A}$$

$$\Rightarrow \; v = \sqrt{\frac{2F}{\rho A}}$$

23. $p_0 = p_0 + \rho_w g h - \frac{1}{2}\rho_a\,v^2$

$$\Rightarrow v = \sqrt{\frac{2\rho_N g h}{\rho_a}}$$

$$\therefore \; v = \sqrt{\frac{2 \times 10^3 \times 10^{-2} \times 9.8}{1.3}} = 12.28\text{ m/s}$$

24. $\frac{1}{2}\rho v^2 = \frac{F}{A} + \rho g h + \frac{1}{2}\rho u^2 = \frac{F}{A} + \rho g h + \frac{1}{2}\rho\,\frac{a^2}{A^2}\,v^2$

$$\frac{1}{2}\rho\left(1 - \frac{a^2}{A^2}\right)v^2 = \frac{F}{A} + \rho g h$$

$$v = \sqrt{\frac{2}{\rho\left(1 - \frac{a^2}{A^2}\right)} - \left(\frac{F}{A} + \rho g h\right)}$$

$$= \sqrt{\frac{2}{10^3\left(1-\frac{10^{-4}}{0.5}\right)}\left(\frac{20}{0.5}+10^3\times 10\times 0.5\right)}$$

$$= \sqrt{\frac{2\times 5.04}{1-0.002}} = \sqrt{\frac{2\times 5.04}{0.998}} = 3.18 \text{ m/s}$$

25. $\frac{1}{2}\rho v_A^2 + \rho g h_A = \frac{1}{2}\rho v_B^2 + h_B \rho g$

$$v_A \cdot A_A = v_B \cdot A_B \quad \text{...(i)}$$

$$v_A^2 - v_B^2 = 2(h_B - h_A)g \quad \text{...(ii)}$$

$$v_A^2\left(1-\frac{A_B^2}{A_A^2}\right) = 2(h_D - h_A)\,g$$

$$v_A = \sqrt{\frac{2\Delta hg}{1-\frac{A_B^2}{A_B^2}}} = \sqrt{\frac{2\times 2\times 10^{-2}\times 10}{1-\frac{2^2}{4^2}}}$$

$$= \sqrt{\frac{0.4}{3/4}} = \sqrt{\frac{1.6}{3}}$$

$$= 0.73 \text{ m/s}$$

$$v_B = v_A \cdot \frac{A_A}{A_B} = 0.73 \text{ m/s} \times \frac{4}{2} = 1.46 \text{ m/s}$$

26. $\frac{dV}{dt} = Av = 500 \text{ cm}^3/\text{s}$

$$\therefore v_1 = 100 \text{ cm/s} = 1 \text{ m/s}$$

$$v_2 = 250 \text{ cm/s}$$

$$= 2.5 \text{ m/s}$$

$$\rho_m \Delta h = \frac{1}{2}\rho_w (v_2^2 - v_1^2)$$

$$\Delta h = \frac{v_2^2 - v_1^2}{2g\rho_m\rho_w}$$

$$= \frac{6.25-1}{2\times 9.8\times 13.6} = 1.97 \text{ cm}$$

27. $v_t = \frac{2}{9}\cdot\frac{r^2(\rho-\sigma)g}{\eta}$

$$= \frac{2}{9}\cdot\frac{4\times 10^{-10}(2-1)\times 10^3\times 10}{1\times 10^{-3}}$$

$$= \frac{8}{9}\times 10^{-3} \text{ m/s} = 0.89 \text{ mm/s}$$

28. $Q = \frac{\pi}{8}\cdot\frac{R^4}{\eta}\frac{\Delta p}{L} \Rightarrow \Delta p = \frac{8\eta LQ}{\pi R^4}$

$$= \frac{8\times 4\times 10^{-3}\times 10^{-3}\times 0.66\times 10^{-3}}{\pi\times 16\times 10^{-24}}$$

$$h\rho g = \frac{2\times 0.66}{\pi}\times 10^{+15}$$

$$\Rightarrow h = 3.15\times 10^9 \text{ m} = 3.15\times 10^{12} \text{ cm}$$

29. $v_t \propto r^2$

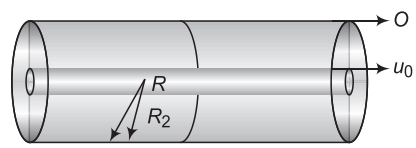
$$\frac{4}{3}\pi R^3 = 2\times\frac{4}{3}\pi r^3 \Rightarrow R = 2^{1/3}\,r$$

$$\therefore \quad \frac{v_2}{v_1} = \left(\frac{r_2}{r_1}\right)^2$$

$$= \left(\frac{R}{r}\right)^2 = 2^{2/3}$$

$$\Rightarrow \quad v_2 = 2^{2/3}\,v$$

30. $v = \frac{\Delta p}{4\eta L}(R_2^2 - r^2)$



$$u_0 = \frac{\Delta p}{4\eta L}(R_2^2 - R^2)$$

$$\therefore v = \frac{u_0}{R_2^2 - R^2}\cdot(R_2^2 - r^2)$$

31. $F = -\eta A \cdot \frac{dv}{dx} = +10^{-2}\times 0.1\times 10\times\frac{2}{1}$

$$= 0.02 \text{ N}$$

32. Stress $= \frac{F}{A} = \eta\frac{dv}{dx} = 10^{-2}\times 0.1\times\frac{5}{5}$

$$= 10^{-3} \text{ N/m}^2$$

33. $h = \frac{2T\cos\theta}{r\rho g}$

$$\Rightarrow \frac{T_2}{T_1} = \frac{h_2\rho_2/\cos\theta_2}{h_1\rho_1/\cos\theta_1}$$

$$= \frac{3.4\times 13.6\times 6.71}{9\times 1/1}$$

$$= \frac{3.4\times 13.6}{9\times 0.71} = 7.24$$

34. $p_a = p_0 + \frac{4T}{a}$, $p_b = p_0 + \frac{4T}{b}$

$p_a V_a = nRT$ and $p_b V_b = 2nRT$

$$\left(p + \frac{4T}{a}\right) \times \frac{4}{3}\pi a^3 \times 2 = \left(p + \frac{4T}{b}\right) \times \frac{4}{3}\pi b^3$$

$$2a^3\left(p + \frac{4T}{a}\right) = b^3\left(p + \frac{4T}{b}\right)$$

$$p(2a^3 - b^3) = 4T(b^2 - 2a^2)$$

$$\therefore \quad T = \frac{p(2a^3 - b^3)}{4(b^2 - 2a^2)}$$

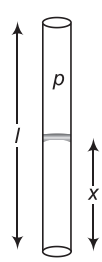
35. $p_0 = p + \frac{2T}{R} = p + h\rho g$

$$\Rightarrow R = \frac{2T}{h\rho g} = \frac{2 \times 0.07}{10^{-2} \times 10^3 \times 10}\text{ m}$$

$$= 1.4 \times 10^{-3}\text{ m} = 1.4\text{ mm}$$

36. $p = p_0 + \frac{2T}{R}$ and $p_0\, l\, A = p(l - x)\, A$

$$\therefore \quad p = p_0 \frac{l}{l - x}$$



$$\therefore \quad p_0 \frac{l}{l - x} - p_0 = \frac{2T}{R}$$

$$p_0\left[\frac{l - l + x}{l - x}\right] = \frac{2T}{R}$$

$$\frac{p_0 R}{2T} = \frac{l - x}{x} = \frac{l}{x} - 1$$

or $$x = \frac{l}{\frac{p_0 R}{2T} + 1} = \frac{0.11\text{ m}}{\frac{10^5 \times 1 \times 10^{-5}}{2 \times 5.06 \times 10^{-2}} + 1}$$

$$= \frac{0.11}{\frac{100}{10.12} + 1} = 1.01\text{ cm}$$

If seal is broken then water will start rising in the capillary and will go upto a height, $h = \frac{2T\cos\theta}{r\rho g}$

$$h = \frac{2 \times 5.06 \times 10^{-2}\cos\theta}{10^{-5} \times 10^3 \times 10} = 1.01\cos\theta,$$

above *i.e.*, it will make convex meniscus as the tube is of insufficient length.

Objective Questions (level 1)

1. Net force is zero, when a body moves with constant velocity.

2. $[x] = \frac{[\eta]}{[P]} = \frac{[\text{MLT}^{-2}][\text{L}^{-2}][\text{T}]}{[\text{ML}^{-3}]}$

$$= [\text{L}^2\text{T}^{-1}] = [\text{M}^0\text{L}^2\text{T}^{-1}]$$

3. $\eta \propto \frac{1}{T}$ as with increase in temperature fluidity increases.

4. $$\left(p_0 + \frac{4T}{a}\right) \times \frac{4}{3}\pi a^3 + \left(p_0 + \frac{4T}{b}\right) \times \frac{4}{3}\pi b^3 = \left(p_0 + \frac{4T}{c}\right) \times \frac{4}{3}\pi c^3$$

$$p_0(a^3 + b^3) + 4T(a^2 + b^2) = p_0 c^3 + 4Tc^2$$

(a) In vacuum, $p_0 = 0$

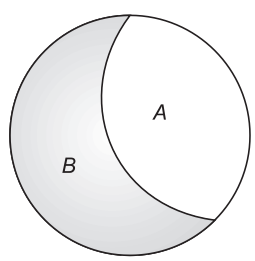
$$\Rightarrow c = \sqrt{a^2 + b^2} = \sqrt{3^2 + 4^2} = 5\text{ cm}$$

5. $p_B < p_0$ while, $p_A = p_C = p_0$

6. $ku + V\sigma g = V\rho g \Rightarrow ku = V(\rho - \sigma)g$

$$\therefore ku = m\left(1 - \frac{\sigma}{\rho}\right)g \Rightarrow u = \frac{mg}{k}\left(1 - \frac{\sigma}{\rho}\right)$$

7. Because of the pull due to surface lenses of liquid at side *B*, the film will take a concave shape in side *A*.



8. $3p_0 = p_a + \frac{1}{2}\rho v^2$

$$\Rightarrow v = \sqrt{\frac{4p_0}{\rho}} = \sqrt{\frac{4 \times 10^3}{10^3}} = \sqrt{400}\text{ m/s}$$

9. $v \propto r^2$ and $m \propto r^3 \Rightarrow v \propto m^{2/3}$

$$\therefore \frac{v_2}{v_1} = \left(\frac{m_2}{m_1}\right)^{2/3} = \left(\frac{8m}{m}\right)^{2/3} = 4 \Rightarrow v_2 = 4v$$

10. $F = -\eta A \dfrac{dv}{dy}$

11. $W = T\,\Delta A = \sigma(n\,4\,\pi r^2 - 4\pi R^2)$

$= 4\pi\sigma(nr^2 - R^2)$

Again $R^3 = nr^3$,

$W = 4\pi\,\sigma(n\,.\,n^{-2/3}\,R^2 - R^2)$

$\therefore \quad W = 4\pi R^2\,(N^{1/3} - 1)\,\sigma$

12. $p_1 = p_0 + \dfrac{4T}{R_1}$ and $p_2 = p_0 + \dfrac{4T}{R_2}$

$\Rightarrow p_2 - p_1 = 4T\left(\dfrac{1}{R_2} - \dfrac{1}{R_1}\right) = \dfrac{4T}{R}$

$\therefore \quad \dfrac{1}{R} = \dfrac{1}{R_2} - \dfrac{1}{R_1}$

13. Pressure decreases along the direction of flow for viscous fluid even in case of uniform cross-section and here as velocity increases with decreasing cross-section presses further decreases. So, correct graph is (c).

14. $mg = 6\,\pi\,\eta\,r\,v_t$

$\dfrac{4}{3}\,\pi\,r^3\,\rho g = 6\pi\,\eta\,r\,v_t$

$r = \sqrt{\dfrac{9\eta\,v_t}{2\,\rho\,g}} = \sqrt{\dfrac{9 \times 1.8 \times 10^{-5} \times 30}{2 \times 10^3 \times 10}}$

$= \sqrt{27 \times 0.9 \times 10^{-8}} = 4.93 \times 10^{-4}$ m

$\simeq 0.5$ mm

15. $h = \dfrac{2T\cos\theta}{r\,\rho\,g} = 0 \Rightarrow \cos\theta = 0 \Rightarrow \theta = 90°$

16. $v_t \propto r^2 \Rightarrow \dfrac{v_2}{v_1} = \left(\dfrac{r_2}{r_1}\right)^2 = \left(\dfrac{1}{2}\right)^2 = \dfrac{1}{4}$

$\Rightarrow v_2 = \dfrac{v_1}{4} = 5$ cm/s

17. $p_0 = h\,\rho\,g$

$\Rightarrow p_x + \dfrac{h}{5}\rho g = p_0 = h\rho g$

$\Rightarrow p_x = \dfrac{4}{5}\,h\,\rho g = 0.8\,p_0 = 0.8 \times 10^5$ Pa

18. $V\rho G = \dfrac{2}{3}\,V\,\sigma_w\,g \quad \Rightarrow \quad \rho = \dfrac{2}{3}\,\sigma_w$ and

$V\rho g = \dfrac{1}{3}\,V\,\sigma_l g \quad \Rightarrow \quad \sigma_l = 3\rho$ or

$\sigma_l = 3 \times \dfrac{2}{3}\,\sigma_w = 2\sigma_w$

19. In sugar solution ice was less immersed due to high density of liquid. But with melting ice, density of sugar solution gets decreased and ice gets more immersed, displacing more liquid. So, liquid level will increase.

20. As apparent weight of the downward accelerating body decreases. So, that of water also decreases and this leads to decrease in magnitude of buoyant force.

21. Two limbs will have liquids at same level only when their densities are equal *i.e.*, $\rho = \rho_w$

22. As $F_1 > F_2$ and $F_1 = F_2 + C$, again, $f_2 = F_2 + B$ and $f_1 = F_1$ thus $f_1 > f_2$ only when, $C > B$ but not in all other cases. Thus, $f_1 > f_2$ is not always correct.

23. $p_1 = p_2 + \dfrac{1}{2}\rho v^2 \Rightarrow v = \sqrt{\dfrac{2(p_1 - p_2)}{\rho}}$

$v = \sqrt{\dfrac{2 \times \frac{1}{2} \times 10^5}{10^3}} = 10$ m/s

24. $Mg - V\,\sigma\,g = Ma,$

$V\,\sigma\,g - (M - m)\,g = (M - m)\,a_0$

$M\,(g - a_0) = V\,\sigma\,g = (M - m)\,(g + a_0)$

$M\,(g - a_0) - M(g + a_0) = -\,m\,(g + a_0)$

$-\,2Ma_0 = -\,m\,(g + a_0)$

$\Rightarrow \quad m = \dfrac{2Ma_0}{g + a_0}$

25. $W = Mgh$

$\Rightarrow \quad M = \dfrac{W}{gh} = \dfrac{500}{10 \times 0.5 \times 10^{-2}} = 10^4$ kg

26. $V\sigma_1 + V\sigma_2 = 2V \times 4 \times \sigma_w$

$\Rightarrow \quad \sigma_1 + \sigma_2 = 8\sigma_N$ ...(i)

$V_1\sigma_1 = V_2\sigma_2;\ V_1\sigma_1 + V_2\sigma_2$

$= (V_1 + V_2)\,.\,3\sigma_w$

$\Rightarrow \quad 2V_1\sigma_1 = \left(V_1 + \dfrac{V_1\sigma_1}{\sigma_2}\right)3\sigma_w$

$$2\sigma_1 = 3\sigma_w\left(1+\frac{\sigma_1}{\sigma_2}\right) \quad \text{...(ii)}$$

$$2(8\sigma_w - \sigma_2) = 3\sigma_w\left(1+\frac{(8\sigma_w - \sigma_2)}{\sigma_2}\right)$$

$$16\sigma_w - 2\sigma_2 = 3\sigma_w\left(8\frac{\sigma_w}{\sigma_2}\right)$$

$$\Rightarrow 8\sigma_w\ \sigma_2 - \sigma_2^2 = 12\sigma_w^2$$

$$\therefore\ \sigma_2^2 - 8\sigma_w\sigma_2 + 12\sigma_w^2 = 0$$

$$\sigma_2 = \frac{1}{2}[8\sigma_w \pm \sqrt{64\sigma_w^2 - 48\sigma_w^2}\,]$$

$$= \frac{1}{2}[8\sigma_w \pm 4\sigma_w]$$

$= 6\sigma_w$ or $2\sigma_w$ Similarly for, $\sigma_1 = 2\sigma_w$ or $6\sigma_w$

$\therefore$ Specific gravity of them could be 6 and 2.

27. $A \times 0.5 \times 0.9 \times 10^3 \times g + 100g$

$$= A \times 0.5 \times 10^3 \times g$$

$$\Rightarrow 100 = A \times 0.5 \times 0.1 \times 10^3$$

$$\Rightarrow A = \frac{100}{50} = 2\ \text{m}^2$$

28. $(V - v)\rho_g = 38.2$

while $V\sigma = 38.2 - 36.2 = 2$ g

$$\frac{2}{\sigma}\rho - V\rho = 38.2$$

or $\dfrac{2 \times 19.3}{1} - 38.2 = V \times 19.3$

or $V = \dfrac{0.4}{19.3}\ \text{cm}^3 = 0.02\ \text{cm}^3$

29. $F = \dfrac{1}{2}\rho\, gbh^2 \Rightarrow \dfrac{F}{b} = \dfrac{1}{2}\rho gh^2 = \dfrac{1}{2}\rho gD^2$

30. $av =$ constant and $v = \sqrt{\dfrac{2gh}{\dfrac{p^2}{a^2} - 1}}$. So v do not depend upon p.

31. $W = F\,.\,v = p\,.\,A\,.\,\dfrac{s}{t} = \dfrac{pV}{t}$

$$= \frac{100 \times 10^{-3} \times 13.6 \times 10^3 \times 10 \times 60 \times 10^{-6}}{60/72}$$

$$= 1.36 \times 0.72\ \text{W} = 0.98\ \text{W}$$

32. Accordings to continuity, $Q_{\text{in}} = Q_{\text{out}}$

$$4 \times 10^{-6} = 6 \times 10^{-6} + 5 \times 10^{-6} + 2 \times 10^{-6} + 4 \times 10^{-6} + 8 \times 10^{-6} + Q$$

$$Q = -21 \times 10^{-6}\ \text{m}^3/\text{s}$$

33. $Mg = a^2 \times \dfrac{3}{4}a \times p \times g$

$$\Rightarrow\ a^3 = 4M/3\rho$$

$$\Rightarrow\ a = (4M/3\rho)^{1/3}$$

34. $h = \dfrac{2T\cos\theta}{r\rho g}$

$$\Rightarrow\ \frac{T_w}{T_m} = \frac{rh\rho g/2\cos\theta}{rh'\rho' g/2\cos\theta'} = \frac{h\rho\cos\theta'}{h'\rho'\cos\theta}$$

$$= \frac{10 \times 1 \times \cos 135^\circ}{-3.42 \times 13.6 \times \cos 0^\circ}$$

$$= \frac{5r_2}{3.42 \times 13.6} = \frac{1}{6.5}$$

35. $h = \dfrac{2T\cos\theta}{r\rho g} = \dfrac{2T\cos\theta}{\sqrt{A/\pi}\,\rho g}$

$$\Rightarrow\ h \propto \frac{1}{\sqrt{A}}$$

$$\Rightarrow \frac{h_2}{h_1} = \sqrt{\frac{A_1}{A_2}} = \sqrt{\frac{A_1}{A/2}} = \sqrt{2}$$

$$\Rightarrow h_2 = \sqrt{2}h_1 = 20\sqrt{2}\ \text{cm}$$

36. Volume flow rate $= av = a\sqrt{2gh}$

$\therefore$In steady state,

$$10^4 = (1)\sqrt{2 \times 1000 \times h}$$

$$\therefore\quad h = 5\ \text{cm}$$

37. $v_2 = \dfrac{A_1v_1}{A_2} = \dfrac{10^{-2} \times 2}{0.5 \times 10^{-2}} = 4$ m/s

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

$$\Rightarrow p_2 = 8000\text{Pa} + \frac{1}{2} \times 10^3 \times 4 - \frac{1}{2} \times 10^3 \times 16$$

$$p_2 = 8000\ \text{Pa} + 2000\ \text{Pa} - 8000\ \text{Pa}$$

$$= 2000\ \text{Pa}$$

38. $8 \times 4/3\ \pi r^3 = 4/3\ \pi R^3$

$$\Rightarrow R = 2r, v_r \propto r^2$$

$$\Rightarrow\ v_2 = 4V_1 = 24\ \text{cm/s}$$

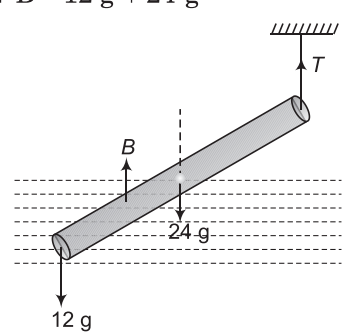
JEE Corner

Assertion and Reason

1. The magnitude of normal force per unit area is defined as pressure and so it is scalar quantity, not a vector, so assertion is false.
2. Assertion and reason are true with correct explanation.
3. Assertion is false as $p(h) = p_0 + h\rho g$ while $p(2h) = p_0 + 2h\rho y$
   i.e., $p(2h) \neq 2p(h)$
4. $V\rho g = W$ while, $V(\rho - \sigma) g = \frac{2W}{3}$
   $\Rightarrow \frac{\rho - \sigma}{\rho} = \frac{2}{3}$
   or $1 - \frac{2}{3} = \frac{\sigma}{\rho}$
   $\Rightarrow \rho = 3\sigma$. Assertion is true and reason is correct explanation as
   $$\frac{\rho}{\sigma} = \frac{W}{W - \frac{2}{3}W} = 3$$
5. Assertion is false as due to different cross-sectional area and height of oil columns will be different producing different pressures which lead to different water levels in the limbs of U-tube.
6. Assertion is not always true as it also depends upon the height from ground.
7. Assertion and reason are both true but not correct explanation as v depends upon P while R depends upon v.
8. Assertion is true and reason is correct explanation as acceleration or retardation depends upon force of buoyancy which in turn depends upon densities of ball and liquid.
9. Assertion is false as barometer measures air pressure but on moon, there is no atmosphere to exert pressure, the barometer reading will be zero.
10. At P, liquid is in motion such that at that point pressure is less than atmospheric pressure. So, assertion is false.
11. Assertion and reason are both false as force of buoyancy can be negligible only when relative density of the body in very high in comparison to that of air, it is not true.

Objective Questions (Level 2)

1. $\tan\theta = \frac{a}{g} = \frac{h}{h/2} = 2$
   $\Rightarrow a = 2g$
2. As ice melts, its volume decreases, so oil level will decrease, and as volume of water increases, so level of interface will rise.
3. As water is under free fall such that it do not exert pressure on side wall, so water do not come out of the hole.
4. For translational equilibrium $T + B = 12\text{ g} + 24\text{ g}$



$\therefore T = 36g - B \Rightarrow T < 36g$

$24g \times 6 + 12g \times 12 - B \times 9 = 0$

$\Rightarrow \quad B = \frac{2 \times 24 \times 6g}{9} = 32\,g \Rightarrow T = 4\text{ g}$

$B = \frac{V}{2}\sigma g = 32g$

$\Rightarrow \quad V = \frac{64}{\sigma} = 6.4 \times 10^{-2}\text{ m}^3$

5. $Q = Av \Rightarrow v = \dfrac{Q}{A} = \dfrac{120 \times 10^3 \text{ cm}^3 / 120 \text{ s}}{5 \text{ m}^2}$

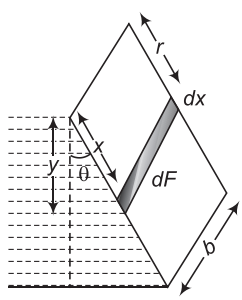
$= 200 \text{ cm/s} = 2 \text{ m/s}$

$v_2 = \dfrac{A_1 v_1}{A_2} = \dfrac{5 \times 2}{1} = 10 \text{ m/s}$

$t = \sqrt{\dfrac{2h}{g}}$ and $R = vt = 10 \text{ m/s}$

$\sqrt{\dfrac{2 \times 1}{10}} \text{ s} = 2\sqrt{5} \text{ m} = 4.47 \text{ m}$

6. $dF = \rho g y \, b \, dx$



$F = \int_0^{h \sec \theta} \rho g \, x \cos \theta \, b \, dx$

$= \rho \, g \, b \cos \theta \, \dfrac{1}{2} x^2 \Big|_0^{h \sec \theta}$

$= \dfrac{1}{2} \rho \, g \, b \, h^2 \sec \theta$

7. $v^2 = 2 \dfrac{(\sigma - \rho) \, g \, d}{\rho} = 2gh$

$\Rightarrow \quad d = \dfrac{\rho h}{\sigma - \rho}$

8. $\tan \theta = \dfrac{a}{g} = \dfrac{\Delta h}{L}$

$\Delta h = \dfrac{aL}{g}$

9. $p_A = p_{\text{atm}} - \dfrac{2\sigma}{r} = p_{\text{atm}} - \dfrac{2\sigma \cos \theta}{R}$ and

$p_B = p_A + h \rho g = p_{\text{atm}}$

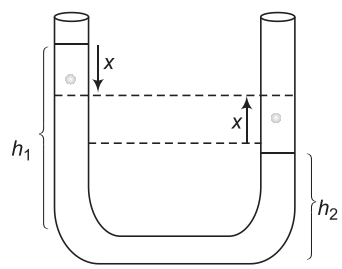
10. $Q = A_1 v_1 = A_2 v_2$

$\Rightarrow L^2 \sqrt{2gh} = \pi R^2 \sqrt{2gh4h}$

$\Rightarrow \quad L^2 = 2\pi R^2$

$\therefore \quad R = L / \sqrt{2\pi}$

11. $x = \dfrac{h_1 - h_2}{2}$



$W_g = Ax\rho g \, . \, x = A\rho g x^2$

$= \dfrac{1}{4} A\rho g \, (h_1 - h_2)^2$

12. $\Delta p = h_1 \rho_1 g + h_2 \rho_2 g$

$= 0.1 \times 0.6 \times 10^3 \times 10 + 0.02 \times 10^3 \times 10$

$= 600 + 200 = 800 \text{ Pa}$

13. $\Delta p = \dfrac{1}{2} \rho v^2$

$\Rightarrow v = \sqrt{\dfrac{2\Delta p}{p}} = \sqrt{\dfrac{2 \times 5 \times 10^5}{10^3}}$

$= 10\sqrt{10} = 32 \text{ m/s}$

14. $v_A = v_B = v_0$

$\Rightarrow p_A = p_B$

$\Rightarrow h = 0$

15. $F = \dfrac{dp}{dt} = 2v \dfrac{dm}{dt} = 2v \dfrac{d}{dt} (\rho V)$

$= 2v \, \rho v \, A = 2\rho A \, v^2$

16. $p_A = p_0 + \rho g h$

$\Rightarrow \Delta p = \rho g h$

$p_A - p_0 = \rho a l$

$\Rightarrow \Delta p = \rho a l$

17. $\pi \dfrac{d^2}{4} (2L - 2 \text{ cm/h } t) \rho g = B = \pi \dfrac{d^2}{4} \, . \, y \, \sigma \, g$

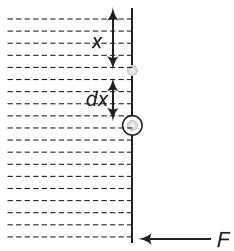
$\Rightarrow y = \dfrac{\rho}{\sigma} (2L - 2 \text{ cm/h } t)$

$= \dfrac{1}{2} (2L - 2 \text{ cm/h } t)$

$= L - 1 \text{ cm/h } t$

$\Rightarrow \dfrac{dy}{dt} = -1 \text{ cm/h}$

18. $dF = \rho g y \,.\, b dy = \rho g b\, y dy$



$$\Rightarrow F(0 \to h/2) = \rho g b \int_0^{h/2} y dy = \frac{1}{8} \rho g\, b h^2$$

$$\tau\,(0 \to h/2) = \int dF > (h/2 - y)$$

$$= \int_0^{h/2} \left(\frac{h}{2} - y\right) \cdot \rho g b y\, dy$$

$$= \rho g b \int_a^{h/2} \left(\frac{h}{2}\, y - y^2\right) dy$$

$$= \rho g b \left[\frac{h}{2}\,\frac{y^2}{2} - \frac{y^3}{3}\right]_0^{h/2}$$

$$= \rho g b h^3 \left(\frac{1}{16} - \frac{1}{24}\right) = \frac{1}{48} \rho g b h^3$$

$$\tau\,(h/2 - h) = \int_{h/2}^{h} (y - h/2) \rho g b y\, dy$$

$$= \rho g b \left[\frac{y^3}{3} - \frac{h}{2} \cdot \frac{y^2}{2}\right]_{h/2}^{h}$$

$$= \rho g b \left[\frac{1}{3}\left(h^3 - \frac{h^3}{8}\right) - \frac{h}{4}\left(h^2 - \frac{h^2}{4}\right)\right]$$

$$= \rho g b h^3 \left(\frac{1}{3} \cdot \frac{7}{8} - \frac{3}{16}\right)$$

$$= \rho g b h^3\, \frac{14 - 9}{48} = \frac{5 \rho g b h^3}{48}$$

$$\therefore \left(\frac{5}{48} - 1\right) \rho g b h^3 = F \times \frac{h}{2}$$

$$\frac{1}{12} \rho g b h^3 = \frac{1}{2} F h \Rightarrow F = \frac{1}{6} \rho g b h^2 = \frac{1}{6} \rho g$$

19. $(r - r\cos 60°)\, \rho_1 g = (r - r \cos 30°)\, 0$

$$(\cos 30° - \cos 60°) \rho_1 = (\cos 30° + \cos 60°) \rho_2$$

$$\frac{\rho_1}{\rho_2} = \frac{\cos 30° + \cos 60°}{\cos 30° - \cos 60°} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

20. $F = \dfrac{dp}{dt} = \dfrac{d}{dt}(mv) = (v_1 + v_2)\, \dfrac{dm}{dt}$

$$= (v_1 + v_2) \rho V$$

21. $v_t = \dfrac{2r^2\, (\rho - \sigma)\, g}{9\eta}$

$$= \frac{2 \times 9 \times 10^{-6}\, (2 \times 1260 - 1260) \times 10}{9 \times 1.26}$$

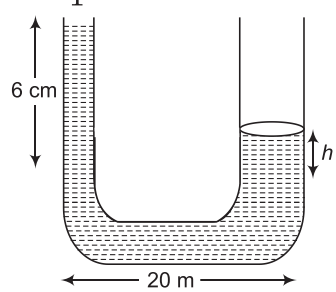
$$= 2 \times 10^{-2} \text{ m/s} = 2 \text{ cm/s}$$

$$t = \frac{d}{v_1} = \frac{(20 - 10) \text{ cm}}{2 \text{ cm/s}} = 5 \text{ s}$$

22. $(2\rho)(g)\left(\dfrac{h^2}{2}\right) = (3\rho)(g)\left(\dfrac{R^2}{2}\right)$

$$\therefore \quad h = \sqrt{\frac{3}{2}} R$$

23. $l_w = \dfrac{60}{1^2} = 60 \text{ cm}$



$$60 \rho_w g = h \rho_l g \;\Rightarrow h = \frac{60 \times 1}{4} = 15 \text{ cm}$$

$$\therefore \quad l_1 = 15 \text{ cm} + 20 \text{ cm} = 35 \text{ cm}$$

$$V_L = liA = 35 \text{ cc}$$

24. $mg \sin 37° = \eta\, A \,.\, \dfrac{v}{t}$

$$a^3 \rho g\, \frac{3}{5} = \eta\, a^2\, \frac{v}{t} \Rightarrow \eta = \frac{3 a \rho g t}{5 v}$$

25. $\sqrt{2gH}\; t = R$ and $\dfrac{1}{2} g t^2 = h$

$$\sqrt{2gH} \,.\, \sqrt{\frac{2h}{g}} = R \text{ or } 4hH = R^2 \Rightarrow H = \frac{R^2}{4h}$$

26. $w' + w + B$

$$\Rightarrow m' = m + \frac{B}{g} = m + \frac{V}{2}\, \sigma = m + \frac{m_0}{2\rho}\, \sigma$$

$$\therefore \quad m' = 10 \text{ kg} + \frac{7.2}{2 \times 7.2} = 10.5 \text{ kg}$$

27.

	P	v	H
A	2	3	1
B	1	2	2
C	4	1	2

For A, $2 + \frac{1}{2} \times 1 \times 3^2 + 1 \times 1 \times 10 = 16.5$

For B, $1 + \frac{1}{2} \times 1 \times 2^2 + 2 \times 1 \times 10 = 23$

For C, $4 + \frac{1}{2} \times 1 \times 1^2 + 2 \times 1 \times 10 = 24.5$

So, Bernoulli's equation gives different values for A, B and C. Thus, they do not lie on same stream line.

28. $a = \frac{F}{m} = \frac{W - B}{m} = \frac{V\rho g - V\sigma g}{VP}$

$= \left(1 - \frac{\sigma}{\rho}\right) g$, downward

$= -\left(\frac{\sigma}{\rho} - 1\right) g$, upward

29. $v = \sqrt{2g\,(3H - x)}$, $R = v \cdot t$, and $x = \frac{1}{2} gt^2$

$\therefore\ R = \sqrt{2g(3H - x)}\ \sqrt{\frac{2x}{g}} = 2\sqrt{(3Hx - x^2)}$

For $R = \max$, $\frac{dR}{dx} = 0$

$= \frac{d}{dx}\sqrt{3Hx - x^2}$

$= \frac{1}{2\sqrt{3Hx - x^2}}\,(3H - 2x)$

$\Rightarrow x = \frac{3}{2} H = 1.5\,H$

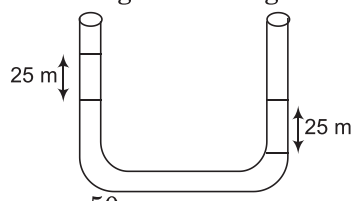
30. $T = \eta A \cdot \frac{v}{H}$ and $F_0 - T = \eta A_2 \frac{v}{H}$

$\Rightarrow F_0 = \eta\,(A_1 + A_2)\,\frac{v}{H}$

$\Rightarrow \quad \eta \frac{v}{H} = \frac{F_0}{A_2 + A_2}$

$\therefore\ T = \frac{A_1 F_0}{A_1 + A_2}$

31. $h \times 0.8 \times g = 50 \times 1 \times g$



$\Rightarrow h = \frac{50}{0.8} = 62.5$ cm

$\Delta h = (62.5 - 50)$ cm $= 12.5$ cm

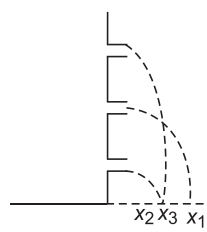
32. $v = \sqrt{2gh}$

33. $R = 2\sqrt{h(H - h)}$

$\Rightarrow x_3 = 2\sqrt{a\,(4a - a)} = 2\sqrt{3}a$

34. $x_1 = 2\sqrt{3a\,(4a - 3a)} = 2\sqrt{3}a = x_3$ and

$x_2 = 2\sqrt{2a\,(4a - 2a)} = 4a$



$x_1 = x_3 < x_2$

35. $L \cdot \frac{A}{5}\rho g = \frac{L}{4} \cdot \frac{A}{5} \cdot 2d \cdot g + \frac{3L}{4} \cdot \frac{A}{5} \cdot d \cdot g$

$\rho = \frac{2d}{4} + \frac{3d}{4} = \frac{5d}{4}$

36. $p = p_0 + \left(\frac{H}{2} + \frac{\frac{A}{5} \times \frac{L}{4}}{A}\right) \cdot 2d \cdot g$

$+ \left(\frac{H}{2} + \frac{\frac{A}{5} \times \frac{3L}{4}}{A}\right) \cdot dg$

$= p_0 + \left(\frac{H}{2} \times 2 + \frac{1}{20} \times 2 + \frac{H}{2} + \frac{3L}{20}\right) dg$

$= p_0 + \left(\frac{3H}{2} + \frac{L}{4}\right) dg$

$= p_0 + \frac{(6H + L)}{4}\, dg$

37. $p_0 + \frac{H}{2} dg + \left(\frac{H}{2} - h\right) 2dg = p_0 + \frac{1}{2} \times 2dv^2$

$\left(\frac{H}{2} + H - 2h\right) g = v^2$

or $v = \sqrt{\frac{g}{2}(3H - 4h)}$

38. $R = vt = \sqrt{\frac{g}{2}(3H - 4h)} \cdot \sqrt{\frac{2h}{g}} = \sqrt{h(3H - 4h)}$

More than One Correct Options

1. $\frac{dv}{dx} = \frac{2}{1} = 2$ m/s

$F = \eta A \frac{dv}{dx} = 10^{-3} \times 10 \times 2 = 0.02$ N

2. For liquids viscosity decreaese with increase in temperature as adhesive and coheive forces decrease while for gases due to in random motion with temperature viscosity increases. Surface tension decreases due to decrease in adhesive force with increase in temperature.

3. In opposite direction to the displacement and it is possible only vertical direction.

As, $F_r = -\Delta B = -\sigma A x\, g$

$\Rightarrow F_r \propto -x$ irrespective of the value of x.

4. $A_1 v_1 = A_2 v_2$, as $A_2 < A_1 \Rightarrow v_2 > v_1$

$Q = \frac{dv}{dt} = -\frac{dv}{dt}$

So, $p + \rho g h + \frac{1}{2}\rho p v^2 = \text{constant}$

$\Rightarrow \quad p_2 < p_1$

5. $a^3 \rho g = a^2 l g \sigma \Rightarrow \frac{l}{a} = \frac{\rho}{\sigma} = f$

f is independent of atmospheric pressure, f increases with decrease in σ and f decreases with decreases in ρ

6. $\frac{v_1}{v_2} = \frac{\sqrt{2gh}}{\sqrt{2g4h}} = \frac{1}{2}, \frac{t_1}{t_2} = \frac{\sqrt{\frac{2 \times 4h}{g}}}{\sqrt{\frac{2 \times h}{g}}} = \frac{2}{1}$

$\frac{R_1}{R_2} = \frac{2\sqrt{h(5h - h)}}{2\sqrt{4h(5h - h)}} = \frac{\sqrt{h \cdot 4h}}{\sqrt{4h \cdot h}} = 1$

7. $P_A > P_B$ and $P_D > P_C$

8. $W = mg = V\rho g$, $B = V \,.\, 2\rho g = 2W$

$a_{\text{air}} = g \downarrow$,

$a_{\text{liquid}} = \frac{B - W}{m} = \frac{2W - W}{m} = g \uparrow$

a, c and d are correct.

9. $A_1 v_1 = \pi 4r^2 \,.\, \sqrt{2gh}$ and

$A_2 v_2 = \pi r^2 \sqrt{2g16h} = 4\pi r^2 \sqrt{2gh} = A_1 v_1$

$A_1 v_1 = A_2 v_2 = Q$

After some time $Q_1 = 0$ and $Q_2 \neq 0$

10. $V\rho g = V \sigma g \Rightarrow \frac{V'}{V} = \frac{\rho}{\sigma} = f =$ constant

Respective of shape.

$\therefore \; f_1 = f_2 = f_3$

as for 3rd area is uniform, h_3 is minimum.

for 2nd area is decreasing faster, h_2 is Moderate.

And for 1st area is slowly, h_1 is maximum

$\therefore \quad h_3 < h_1$ and $h_3 < h_2$

Match the Columns

1. $v = \sqrt{2gh_{\text{Top}}} \therefore v \propto \sqrt{h_{\text{Top}}}$.

R is maximum when $h = \frac{H}{2}$

2. With increase in temperature, density of liquid decreases. Furhter,

Upthrust $F = V\rho_e g$

or $\quad F \propto \rho_e$

3. Upthrust $F = V\rho_e g = V(\propto h)g$

$\therefore \quad F \propto h$

4. No solution is required.

5. $F_2 - F_1 =$ Net upthrust

$= V_1\rho_1 g + V_2\rho_2 g$

= Weight of cylinder in equilibrium.

Solutions
of
Waves & Thermodynamics

Lesson 14th to 19th

By DC Pandey

14. Wave Motion

Introductory Exercise 14.1

1. A function, f can represent wave equation, if it satisfy

$$\frac{\partial^2 f}{\partial t^2} = v^2 \frac{\partial^2 f}{\partial x^2}$$

For, $y = a \sin \omega t$,

$$\frac{\partial^2 y}{\partial t^2} = -\omega^2 a \sin \omega t = -\omega^2 y$$

but, $\dfrac{\partial^2 y}{\partial x^2} = 0$

So, y do not represent wave equation.

2. $y(x, t) = ae^{-(bx - et)^2} = ae^{-(kx - \omega t)^2}$

$\Rightarrow k = b$ and $\omega = e \Rightarrow v = \dfrac{\omega}{k} = \dfrac{c}{b}$

3. $y(x, t) = \dfrac{1}{1 + (4x + \omega t)^2}$ represent the given pulse, where,

$$y(x, 0) = \frac{1}{1 + k^2 x^2} = \frac{1}{1 + x^2}$$

$\Rightarrow \quad k = 1$

and $y(x, z) = \dfrac{1}{1 + (x - 2\omega)^2} = \dfrac{1}{1 + (x - 1)^2}$

$\Rightarrow \quad \omega = \dfrac{1}{2}$

$\therefore \quad v = \dfrac{\omega}{k} = \dfrac{1/2}{1} = 0.5 \text{ m/s}$

4. $y = \dfrac{10}{5 + (x + 2t)^2} = \dfrac{a}{b + (kx + \omega t)^2}$

Amplitude, $y_{\max} = \dfrac{a}{b} = \dfrac{10}{5} = 2 \text{ m}$

and $k = 1; \omega = 2$

$v = \dfrac{\omega}{k} = 2 \text{ m/s}$ and is travelling in $(-)\, x$ direction.

5. $y = \dfrac{10}{(kx - \omega t)^2 + 2}$

$$y(x, 0) = \frac{10}{k^2 x^2 + 2} = \frac{10}{x^2 + 2} \Rightarrow k = 1$$

$$\omega = vk = 2 \text{ m/s} \times 1 \text{ m}^{-1} = 2 \text{ rad/s}$$

$$\Rightarrow \quad y = \frac{10}{(x - 2t)^2 + 2}$$

Introductory Exercise 14.2

1. $y(x, t) = 0.02 \sin\left(\dfrac{x}{0.05} + \dfrac{t}{0.01}\right) \text{m}$

$= A \sin(kx + \omega t) \text{ m}$

$\Rightarrow \quad A = 0.02 \text{ m}, k = \dfrac{1}{0.05} \text{ m}^{-1}, \omega = \dfrac{1}{0.01} \text{ s}^{-1}$

(a) $v = \dfrac{\omega}{k} = \dfrac{0.05}{0.01} \text{ m/s} = 5 \text{ m/s}$

(b) $v_p = \dfrac{\partial y}{\partial t} = A\omega \cos(kx + \omega t)$

$v_p(0.2, 0.3) = 0.02 \times \dfrac{1}{0.01} \cos\left(\dfrac{0.2}{0.5} + \dfrac{0.3}{0.01}\right)$

$= 2 \cos(4 + 30)$

$= 2 \cos 34$

$= 2(-0.85)$

$= -1.7 \text{ m/s}$

2. Yes, $(v_p)_{\max} = A\omega = Ak \cdot \dfrac{\omega}{k} = (Ak)v$

3. $\lambda = 4$ cm, $v = 40$ cm/s (given)

(a) $\nu = \dfrac{v}{\lambda} = \dfrac{40 \text{ cm/s}}{4 \text{ cm}} = 10 \text{ Hz}$

(b) $\Delta\phi = \frac{2\pi}{\lambda}\Delta x$

$= \frac{2\pi}{4\text{ cm}} \times 2.5\text{ cm} = \frac{5\pi}{4}\text{ rad}$

(c) $\Delta t = \frac{T}{2\pi}\Delta\phi = \frac{1}{2\pi\nu}\Delta\phi$

$= \frac{1}{2\pi \times 10} \times \frac{\pi}{3}$

$= \frac{1}{60}\text{ s}$

(d) $v_p = (v_p)_{\max}$

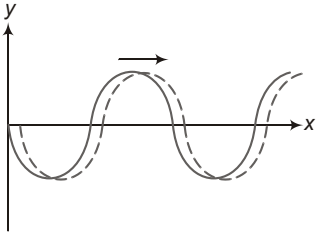
$= -A\omega = -2\pi A\nu$

$= -2\pi \times 2\text{ cm} \times 10\text{ s}^{-1}$

$= -40\pi\text{ cm/s}$

$= -1.26\text{ cm/s}$

4. (a)



$y = A\sin(\omega t - kx)$

$= A\sin\left(v \cdot \frac{2\pi}{\lambda}t - \frac{2\pi}{\lambda}x\right)$

$= 0.05\sin\left(12 \times \frac{2\pi}{0.4}t - \frac{2\pi}{0.4}x\right)$

$= 0.05\sin(60\pi t - 5\pi x)$

(b) $y(0.25, 0.15)$

$= 0.05\sin(60\pi \times 0.15 - 5\pi \times 0.25)$

$= 0.05\sin(9\pi - 1.25\pi)$

$= 0.05\sin(7.75\pi) = 0.05\sin(1.75\pi)$

$= -0.0354\text{ m} = -3.54\text{ cm}$

(c) $\Delta t = \frac{T}{2\pi}\Delta\phi = \frac{\Delta\phi}{\omega} = \frac{0.25\pi}{60\pi}$

$= \frac{1}{240}\text{ s} = 4.2\text{ ms}$

Introductory Exercise 14.3

1. $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{m/l}} = \sqrt{\frac{Tl}{m}}$

$= \sqrt{\frac{500 \times 2}{0.06}} = \frac{100\sqrt{5}}{\sqrt{3}} = 129.1\text{ m/s}$

2. $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\rho \cdot A}}$

$= \sqrt{\frac{0.98}{9.8 \times 10^3 \times 10^{-6}}} = 10\text{ m/s}$

Introductory Exercise 14.4

1. $I = \frac{P}{4\pi r^2} = \frac{1\text{ W}}{4\pi \times (1\text{ m})^2} = \frac{1}{4\pi}\text{ W/m}^2$

2. For line source, $I = \frac{\rho}{2\pi r l}$

$\Rightarrow I \propto \frac{1}{r}$ and as $I \propto A^2$

$\Rightarrow A \propto \frac{1}{\sqrt{r}}$

AIEEE Corner

Subjective Questions (Level 1)

1. $y(x, t) = 6.50 \text{ mm} \cos 2\pi \left(\frac{\pi}{28.0 \text{ cm}} - \frac{t}{0.0360 \text{s}}\right)$

$= A \cos 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$

$\Rightarrow \quad A = 6.50 \text{ mm}, \ \lambda = 28.0 \text{ cm},$

$\nu = \frac{1}{T} = \frac{1}{0.036} \text{s}^{-1} = 27.78 \text{ Hz}$

$v = \nu\lambda = 28.0 \text{ cm} \times 27.78 \text{ s}^{-1} = 778 \text{ cm/s}$

$= 7.78 \text{ m/s}$

The wave is travelling along (+)ve x-axis.

2. $y = 5 \sin 30\pi\left(t - \frac{x}{240}\right)$

$= 5 \sin\left(30\pi t - \frac{\pi}{8}x\right) = A \sin(\omega t - kx)$

(a) $y(2, 0) = 5 \sin\left(3\pi \times 0 - \frac{\pi}{8} \times 2\right)$

$= -5 \sin \frac{\pi}{4} = -\frac{5}{\sqrt{2}} = -3.535 \text{ cm}$

(b) $\lambda = \frac{2\pi}{k} = \frac{2\pi}{\pi/8} = 16 \text{ cm}$

(c) $v = \frac{\omega}{k} = \frac{30\pi}{\pi/8} = 240 \text{ cm/s}$

(d) $\nu = \frac{\omega}{2\pi} = \frac{30\pi}{2\pi} = 15 \text{ Hz}$

3. $y = 3 \text{ cm} \sin(3.14 \text{ cm}^{-1} x - 314 \text{ s}^{-1} t)$

$= 3 \text{ cm} \sin(\pi \text{ cm}^{-1} x = 100\pi \text{s}^{-1} \text{t})$

$= A \sin(kx - \omega t)$

(a) $(v_p)_{\max} = A\omega = 3 \text{ cm} \times 100\pi \text{ s}^{-1}$

$= 300\pi \text{ cm/s} = 3\pi \text{ m/s} = 9.4 \text{ m/s}$

(b) $a = -\omega^2 y = -(100\pi \text{s}^{-1})^2 \times 3 \text{ cm} \sin(6\pi - 111\pi)$

$= -300\pi \sin(-105\pi) = 0$

4. (a) $\Delta x = \frac{\lambda}{2\pi}\Delta\phi = \frac{v/\nu}{2\pi}\Delta\phi = \frac{350}{500 \times 2\pi} \times \frac{\pi}{3}$

$= \frac{7\pi}{60\pi} = \frac{7}{50} \text{m} = 0.166 \text{ m}$

(b) $\Delta\phi = \frac{2\pi}{T}\Delta t = 2\pi\nu\,\Delta t = 2\pi \times 500 \times 10^{-3}$

$= \pi = 180°$

5. $y(x, t) = \frac{6}{(kx + \omega t)^2 + 3}$

$y(x, 0) = \frac{6}{k^2x^2 + 3} = \frac{6}{x^2 + 3}$

$\Rightarrow \quad k = 1 \text{ m}^{-1}$

$\Rightarrow \omega = vk = 4.5 \text{ m/s} \times 1 \text{ m}^{-1} = 4.5 \text{ rad/s}$

$\Rightarrow y(x, t) = \frac{6}{(x - 4.5t)^2 + 3}$

6. $y = 1.0 \sin \pi\left(\frac{x}{2.0} - \frac{t}{0.01}\right)$

$= 1.0 \sin 2\pi\left(\frac{x}{4.0} - \frac{t}{0.02}\right)$

$= A \sin 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$

(a) $A = 1.0 \text{ mm}, \lambda = 4.0 \text{ cm}, T = 0.02\text{s}$

(b) $v_p = \frac{\partial y}{\partial t} = -\omega A \cos 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$

$= -\frac{2\pi A}{T} \cos 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$

$= -\frac{2\pi \times 1.0 \text{ mm}}{0.02\text{s}} \cos 2\pi\left(\frac{x}{4.0} - \frac{t}{0.02\text{s}}\right)$

$= -\frac{\pi}{10} \text{m/s} \cos \pi\left(\frac{x}{2.0 \text{ cm}} - \frac{t}{0.01\text{s}}\right)$

$v_p(1.0 \text{ cm}, 0.01\text{s}) = -\frac{\pi}{10} \text{m/s} \cos \pi\left(\frac{1}{2} - \frac{0.01}{0.01}\right)$

$= -\frac{\pi}{10} \text{m/s} \cos \frac{\pi}{2} = 0 \text{ m/s}$

(c) $v_p(3.0, 0.01)$

$= -\frac{\pi}{10} \cos \pi\left(\frac{3}{2} - 1\right) = 0 \text{ m/s}$

$$v_p\,(5.0\text{ cm}, 0.01\text{s}) = -\frac{\pi}{10}\text{ms}\cos\pi\left(\frac{5}{2}-1\right)$$
$$= 0\text{ m/s}$$
$$v_p(7.0\text{ cm}, 0.01\text{s}) = -\frac{\pi}{10}\text{m/s}\cos\pi\left(\frac{7}{2}-1\right)$$
$$= 0\text{ m/s}$$

(d) $v_p(1.0\text{ cm}, 0.011\text{s})$
$$= -\frac{\pi}{10}\text{m/s}$$
$$\cos\pi\left(\frac{1}{2}-\frac{0.011}{0.01}\right)$$
$$= -\frac{\pi}{10}\cos\pi\left(\frac{1}{12}-1.1\right)$$
$$= -\frac{\pi}{10}\cos 0.6\pi = -\frac{\pi}{10}\cos\frac{3\pi}{5} = 9.7\text{ cm/s}$$

$v_p(1.0\text{ cm}, 0.012\text{s})$
$$= -\frac{\pi}{10}\text{m/s}\cos\left(\frac{1}{2}-\frac{0.012}{0.01}\right)$$
$$= -\frac{\pi}{10}\cos\pi(0.5-1.2)$$
$$= -\frac{\pi}{10}\cos 0.7\pi = 18.5\text{ cm/s}$$

$$v_p(1.0\text{ cm}, 0.013\text{ s}) = -\frac{\pi}{10}\text{m/s}$$
$$\cos\pi\left(\frac{1}{2}-\frac{0.013}{0.01}\right) = -\frac{\pi}{10}\cos 0.8\pi$$
$$= 25.4\text{ cm/s}$$

7. (a) $k = \frac{2\pi}{\lambda} = \frac{2\pi}{40\text{ cm}} = \frac{\pi}{20}\text{cm}^{-1}$
$$= 0.157\text{ rad/cm}$$
$$T = \frac{1}{\nu} = \frac{1}{8}\text{s} = 0.125\text{s}$$
$$\omega = 2\pi\nu = 16\pi\text{ rad/s} = 50.26\text{ rad/s}$$
$$v = \nu\lambda = 8\,\text{s}^{-1}\times 40\text{ cm} = 320\text{ cm/s}$$

(b) $y(x, t) = A\cos(kx - \omega t)$
$$= 15.0\text{ cm}\cos(0.157x - 50.3\,t)$$

8. $A = 0.06\text{m}$ and $2.5\lambda = 20\text{ cm}$

$$\Rightarrow \quad \lambda = \frac{20}{2.5}\text{cm} = 8\text{ cm}$$
$$\nu = \frac{v}{\lambda} = \frac{300\text{ m/s}}{8\text{ cm}} = 3750\text{ Hz}$$
$$y = A\sin(kx - \omega t) = 0.06\text{m}$$
$$\sin\left(\frac{2\pi}{0.08}x - 2\pi\times 3750\,t\right)$$
$$= 0.06\text{m}\sin(78.5\text{ m}^{-1}x - 23561.9\text{ s}^{-1}t)$$

9. (a) $\nu = \frac{v}{\lambda} = \frac{8.00\text{ m/s}}{0.32\text{ m}} = 25\text{ Hz}$
$$T = \frac{1}{\nu} = \frac{1}{15}\text{s} = 0.043\text{ Hz}$$
$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.32\text{m}} = 19.63\text{ rad/m}$$

(b) $y = A\cos(kx + \omega t) = A\cos 2\pi\left(\frac{x}{\lambda}+\frac{t}{T}\right)$
$$= 0.07\text{ m}\cos 2\pi\left(\frac{x}{0.32\text{m}}+\frac{t}{0.04\text{ s}}\right)$$

(c) $y = 0.07\text{ m}\cos 2\pi\left(\frac{0.36}{0.32}+\frac{0.15}{0.04}\right)$
$$= 0.07\text{ m}\cos 2\pi\left(\frac{9}{8}+\frac{30}{8}\right)$$
$$= 0.07\text{ m}\cos\frac{39}{4}\pi$$
$$= 0.07\text{ m}\cos\left(10\pi - \frac{\pi}{4}\right)$$
$$= 0.07\text{ m}\cos\frac{\pi}{4} = 0.0495\text{ m}$$

(d) $\Delta t = \frac{T}{2\pi}\Delta\phi = \frac{\Delta\phi}{2\pi\nu} = \frac{\pi+\pi/4}{2\pi\times 25}$
$$= \frac{3}{200}\text{s} = 0.015\text{s}$$

10. $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\rho A}} = \sqrt{\frac{Mg}{\rho A}}$
$$= \sqrt{\frac{2\times 9.8}{8920\times 3.14\times(1.2\times 10^{-3})^2}}$$
$$= \sqrt{\frac{2\times 9.8\times 10^4}{89.2\times 3.14\times 1.44}} = 22\text{ m/s}$$

11. $\lambda \propto \nu \propto \sqrt{T} \propto \sqrt{M}$

$$\Rightarrow \quad \frac{\lambda_2}{\lambda_1} = \sqrt{\frac{M_2}{M_1}}$$
$$= \sqrt{\frac{8}{2}} = \sqrt{4} = 2.$$
$$\Rightarrow \quad \lambda_2 = 2\lambda_1$$
$$= 0.12\text{ m.}$$

12. $T(x) = \mu(L-x)g,\ v(x) = \sqrt{\dfrac{T(x)}{\mu}}$

$$= \sqrt{g(L-x)}$$

$$\frac{dx}{\sqrt{g(L-x)}} = dt\,;$$

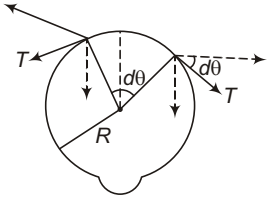
Let, $L - x = y$

$$dx = -dy$$

$$\int_L^0 \frac{-dy}{\sqrt{g}\sqrt{y}} = t$$

$$\therefore \qquad t = \frac{1}{\sqrt{g}} \left.\frac{-\sqrt{y}}{1/2}\right|_1^0 = 2\sqrt{\frac{L}{g}}$$

13. (a) $dm\ \omega^2 R = 2T \sin d\theta$



$$\mu R\, 2d\theta\ \omega^2 R = 2T\, d\theta$$

$$\Rightarrow \qquad \omega^2 R^2 = \frac{T}{\mu}$$

$\therefore$ Wave speed, $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\omega^2 R^2} = R\omega$

(b) Kink remains stationary when rope and kink moves in opposite sence *i.e.*, if rope is rotating anticlockwise then kink has to move clockwise.

14. x is being measured from lover end of the string

$$\therefore \quad m(x) = \int dm = \int_0^x \mu_0\, x\, dx = \frac{1}{2}\mu_0 x^2$$

$$\therefore \quad v(x) = \sqrt{\frac{T(x)}{\mu}} = \sqrt{\frac{m(x)g}{\mu}}$$

$$= \sqrt{\frac{\frac{1}{2}\mu_0 x^2 g}{\mu_0 x}} = \sqrt{\frac{1}{2}gx}$$

$$\Rightarrow \qquad \int_0^l \frac{dx}{\sqrt{\frac{1}{2}gx}} = \int_0^t dt$$

$$\Rightarrow \qquad t = \sqrt{\frac{2}{g}} \times 2\sqrt{l_0}$$

$$\therefore \qquad t = \sqrt{\frac{8l_0}{g}}$$

15. $\mu = \dfrac{dm}{dx} = kx$

$$\Rightarrow M = \int dm = \int_0^2 kx\, dx = \frac{1}{2}kL^2$$

$$\Rightarrow \quad k = \frac{2M}{L^2}$$

$$v(x) = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{kx}} = \sqrt{\frac{TL^2}{2Mx}} = \frac{dx}{dt}$$

$$\therefore \quad t = \int dt = \int_0^L \sqrt{\frac{2Mx}{TL^2}}\, dx = \sqrt{\frac{2M}{TL^2}}\ \frac{L^{\frac{1}{2}+1}}{\frac{1}{2}+1}$$

$$= \frac{2}{3}\sqrt{\frac{2ML^3}{TL^2}} = \frac{2}{3}\sqrt{\frac{2ML}{T}}$$

16. (a) $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{Mg}{\mu}}$

$$= \sqrt{\frac{1.5 \times 9.8}{0.055}} = 16.3 \text{ m/s}$$

(b) $\lambda = \dfrac{v}{\nu} = \dfrac{16.3 \text{ m/s}}{120/\text{s}} = 0.136\,\text{m}$

(c) $\lambda \propto v \propto \sqrt{T} \propto \sqrt{M}$ *i.e.*, if M is doubled both speed and wavelength increases by a factor of $\sqrt{2}$.

17. $E = I\,At = 2\pi^2\nu^2 a^2 \rho v A t$

$$= 2\pi^2\nu^2 a^2 (\rho A)(v.t)$$

$$= 2\pi^2\nu^2 a^2 \mu.l$$

$$= 2\pi^2\nu^2 a^2 m$$

$$= 2 \times (3.14)^2 \times (120)^2 \times (0.16 \times 10^{-3})^2 \times 80 \times 10^{-3}$$

$$= 582 \times 10^{-6}\ \text{J} = 582\ \mu\text{J} = 0.58\,\text{mJ}$$

18. $P = \dfrac{E}{t} = IA = 2\pi^2\nu^2 a^2 \rho\nu A = 2\pi^2\nu^2 a^2 \mu v$

$$= 2\pi^2\nu^2 a^2 \sqrt{T\mu}$$

$$= 2 \times (3.14)^2 \times (60)^2 \times (6 \times 10^{-2})^2 \sqrt{80 \times 5 \times 10^{-2}}$$

$$= 4(3.14 \times 60 \times 0.06)^2 = 511.6\ \text{W}$$

19. $P = IA = 2\pi^2 \nu^2 a^2 \sqrt{T\mu}$

$= 2 \times (3.14)^2 \times (200)^2 \times 10^{-6} \sqrt{60 \times 6 \times 10^{-3}}$

$= 8 \times (3.14)^2 \times 10^{-2} \times 6 \times 10^{-1}$ W

$= 0.474$ W

$E = Pt = P \cdot \dfrac{l}{v}$

$= \dfrac{0.474 \times 2}{\sqrt{\dfrac{60}{6 \times 10^{-3}}}} = \dfrac{0.474 \times 2}{100}$ J $= 9.48$ mJ

20. $P = 2\pi^2 v^2 a^2 \rho v A = 2\pi^2 v^2 a^2 \mu v ; v = \sqrt{\dfrac{T}{\mu}}$

$\mu = \dfrac{T}{v^2}$

$= 2\pi^2 \nu^2 a^2 \dfrac{T}{v^2} \cdot v = 2\pi^2 \nu^2 a^2 \dfrac{T}{v}$

$= \dfrac{2 \times (3.14)^2 \times (100)^2 (0.5 \times 10^{-3})^2 \times 100}{100}$

$= 2 \times (3.14)^2 \times 10^4 \times 0.25 \times 10^{-6}$

$= 4.93 \times 10^{-2}$ W $= 49$ mW

■ Objective Questions (Level 1)

1. $\omega = \dfrac{150 \times 2\pi}{60} = 5\pi$ rad/s, $A = 0.04$ m and $\theta = \dfrac{\pi}{4}$

$\therefore y = A \sin(\omega t + \theta) = 0.04 \sin\left(5\pi t + \dfrac{\pi}{4}\right)$

2. $\omega = 600\pi, v = 300 \Rightarrow k = \dfrac{\omega}{v} = 2\pi$

$\Rightarrow y = A \sin(\omega t - kx)$

$= 0.04 \sin(600\pi t - 2\pi x)$

$y(0.75, 0.01) = 0.04 \sin\left(600\pi \times 0.01 - 2\pi \times \dfrac{3}{4}\right)$

$= 0.04 \sin\left(6\pi - \dfrac{3\pi}{2}\right)$

$= 0.04 \sin\left(4\pi + \dfrac{\pi}{2}\right) = 0.04$ m

3. $y(x, t) = \dfrac{1}{2 + 3(kx - \omega t)^2}$

$y(x, 0) = \dfrac{1}{2 + 3k^2 x^2} = \dfrac{1}{2 + 3x^2}$

$\Rightarrow \quad k = 1$

$y(x, 2) = \dfrac{1}{2 + 3(x - 2\omega)^2} = \dfrac{1}{2 + 3(x - 2)^2}$

$\Rightarrow \quad \omega = 1 \quad \therefore v = \dfrac{\omega}{k} = 1$ m/s

4. $y = A \sin(\omega t - kx) = \dfrac{A}{2}$

$\Rightarrow \quad \omega t - kx = \dfrac{\pi}{6}$

$\therefore \quad \dfrac{2\pi}{T} \cdot \dfrac{T}{6} - \dfrac{\pi}{6} = kx$

$\Rightarrow \quad \dfrac{\pi}{6} = \dfrac{2\pi}{\lambda} \times 0.04$

$\therefore \quad \lambda = 12 \times 0.04 = 0.48$ m.

5. $\lambda = \dfrac{v}{\nu} = \dfrac{300 \text{ m/s}}{25 \text{ Hz}} = 12$m

$\Delta\phi = \dfrac{2\pi}{\lambda} \Delta x = \dfrac{2\pi}{12\text{m}} (16 - 10)\text{m} = \pi$

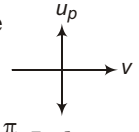
6. $y = 0.02 \sin(x + 30t) = A \sin(kx + \omega t)$

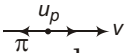
$\Rightarrow \quad k = 1, \omega = 30$

$v = \dfrac{\omega}{k} = 30$ m/s $= \sqrt{\dfrac{T}{\mu}}$

$\Rightarrow T = \mu v^2 = 1.3 \times 10^{-4} \times 900 = 0.117$ N

7. $v_p = \dfrac{\partial y}{\partial t} = \dfrac{\partial y}{\partial x} \cdot \dfrac{\partial x}{\partial t} = v \dfrac{\partial y}{\partial x} = \text{slope} \times v$

In transverse wave 

they are perpendicular *i.e.*, $\dfrac{\pi}{2}$. In longitudinal wave 

, they are either at 0 or π so, 0, $\dfrac{\pi}{2}$ and π are the possible angles between v_p and v.

8. $\omega = 2\pi\nu = 200\pi$ rad/s,

$k = \dfrac{\omega}{v} = \omega\sqrt{\dfrac{\mu}{T}} = 200\pi\sqrt{\dfrac{3.5 \times 10^{-3}}{35}}$

$= 2\pi$ rad/m

$y = A\cos(\omega t - kx) = A\cos(200\pi t - 2\pi x)$

$\frac{\partial y}{\partial x} = 2\pi A \sin(200\pi t - 2\pi x)$

When, $y = 0$

$\Rightarrow \quad \sin(200\pi t - 2\pi x) = 0$

$\Rightarrow \quad \sin(200\pi t - 2\pi x) = 1$

$\therefore \quad 2\pi A = \frac{\pi}{20} \Rightarrow A = \frac{1}{40} = 0.025\,\text{m}$

$\therefore \quad y = 0.025\cos(200\pi t - 2\pi x)$

9. $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.25} = 8\pi$ rad/s;

$k = \frac{\omega}{v} = \frac{8\pi}{48} = \frac{\pi}{6}$ rad/cm

$y = A\sin(\omega t - kx)$

$= A\sin\left(8\pi \times 1 - \frac{\pi}{6} \times 67\right)$

$= A\sin\frac{\pi}{6} = A\sin 30° = \frac{A}{2} = 3$ cm

$\Rightarrow A = 6$ cm

10. $\frac{v_A}{v_B} = \sqrt{\frac{T_A}{\rho A_A} \cdot \frac{\rho A_B}{T_B}} = \sqrt{\frac{T_A \pi d_B^2}{T_B \pi d_A^2}}$

$= \frac{d_B}{d_A}\sqrt{\frac{T_A}{T_B}} = \frac{d_B}{d_B/2}\sqrt{\frac{T_B/2}{T_B}}$

$= 2 \times \frac{1}{\sqrt{2}} = \sqrt{2}$

11. $E \propto A^2\nu^2$ for E to constant, $A\nu =$ constant

$A_A\nu_A = A_B\nu_B \Rightarrow A_A 4\nu_B = A_B\nu_B$

$\Rightarrow A_B = 4A_A$

12. $k = 1$ rad/m; $v = 4$ m/s

$\Rightarrow \quad \omega = vk = 4$ rad/s

$\therefore \quad y = \frac{6}{(kx - \omega t)^2 + 3} = \frac{6}{(x - 4t)^2 + 3}$

13. $v_l = \sqrt{\frac{Y}{\rho}}$ and $v_t = \sqrt{\frac{Y\frac{\Delta l}{l}}{\rho}} = v_l\sqrt{\frac{\Delta l}{l}}$

$\Rightarrow \quad \sqrt{\frac{l}{\Delta l}} = \frac{v_l}{v_t} = 10 \quad \therefore \quad \frac{\Delta l}{l} = \frac{1}{100}$

Stress $= Y\frac{\Delta l}{l} = E\frac{\Delta l}{l} = \frac{E}{100}$

14. $A = 4$ m, $\omega = \frac{\pi}{5}$, $k = \frac{\pi}{9}$, $\theta = \frac{\pi}{6}$

$\therefore \quad v = \frac{\omega}{k} = \frac{\pi/5}{\pi/9} = \frac{9}{5}$ m/s

$\lambda = \frac{2\pi}{k} = \frac{2\pi}{\pi/9} = 18$ m

$\nu = \frac{\omega}{2\pi} = \frac{\pi/5}{2\pi} = \frac{1}{10}$ Hz

15. $\omega = 10\pi$ and $k = 0.1\pi$

$\Rightarrow \quad \lambda = \frac{2\pi}{k} = \frac{2\pi}{0.1\pi} = 20$ m

$\therefore \quad \Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{20} \times 10 = \pi$

16. $y = \frac{2}{(2x - 6.2t)^2 + 20}$

$\Rightarrow \quad A = \frac{2}{20} = 0.1$ m, $k = 2$ rad/m

and $\quad \omega = 6.2$ rad/s

$\therefore \quad v = \frac{\omega}{k} = \frac{6.2}{2} = 3.1$ m/s

$\nu = \frac{\omega}{2\pi} = \frac{6.2}{2 \times 3.1} = 1$ Hz

$\lambda = \frac{2\pi}{k} = \frac{2\pi}{2} = \pi$ m

17. $I = 2\pi^2\nu^2 A^2\rho v = \frac{1}{2}\omega^2 A^2\rho v$

$u = \frac{E}{V} = \frac{IST}{V} = \frac{2\pi^2\nu^2 A^2\rho v\, St}{V}$

$= 2\pi^2\nu^2 A^2\rho = \frac{1}{2}\rho\omega^2 A^2$

$P = \frac{E}{t} = I.S = 2\pi^2\nu^2 A^2\rho v \cdot S$

$= \frac{1}{2}\rho\omega^2 A^2 v \cdot S$

$E = Pt \Rightarrow P = \frac{E}{t} = IS \Rightarrow I = \frac{P}{S}$

18. $y = A\sin(\pi x + \pi t)$

$y(x, 0) = A\sin(\pi x) \Rightarrow y = 0$ for $x = 0$ and 1

$a = -\omega^2 y = -\omega^2 A\sin(\pi x)$

$\Rightarrow a = \pm\omega^2 A$ at $x = \frac{1}{2}$ and $\frac{3}{2}$

$v_P = \pi A \cos(\pi x) \Rightarrow v_p = 0$ for $x = \frac{1}{2}$ and $\frac{3}{2}$

So all the above options are correct.

19. $y = A \sin \frac{2\pi}{a}(x - bt) = A \sin(kx - \omega t)$

$k = \frac{2\pi}{a}, \omega = \frac{2\pi b}{a}$

$\Rightarrow v = \frac{\omega}{k} = \frac{2\pi b/a}{2\pi/a} = b$

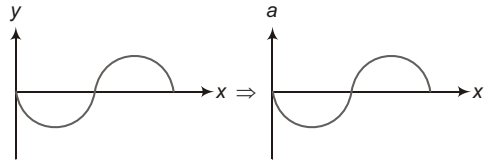
$\lambda = \frac{2\pi}{k} = \frac{2\pi}{2\pi/a} = a$

20. $y = A \sin 2\pi\left(\frac{x}{a} - \frac{t}{b}\right) = A \sin 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$

$\Rightarrow \quad \lambda = a, T = b$

$\Rightarrow \quad v = \nu\lambda = \frac{\lambda}{T} = \frac{a}{b}$

21.



as $\quad a = -\omega^2 y$

JEE Corner

■ Assertion and Reason

1. For propagation of transverse waves medium require tension which is possible due to modulus of rigidity. And in gases there is no such Young's modulus or surface tension. So the reason given is correct explanation.

2. Surface tension of water plays the role of modulus of rigidity and that is why transverse waves can travel on liquid surface.

3. Both the waves are travelling in same direction with a phase difference of π. So reason is false.

4. $v = f\lambda$ is constant for a particular medium so if frequency is doubled wavelength becomes half, and speed remains constant. Thus assertion is false.

5. Sound is mechanical wave which requires material medium for propagation and as on moon there is no atmosphere, sound cannot travel.

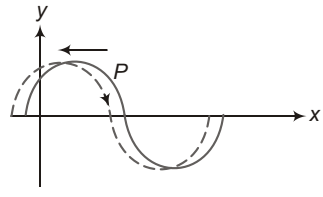
6. Angular wave number, $\bar{\nu} = \frac{2\pi}{\lambda}$ while wave number, $k = \frac{1}{\lambda}$ which is defined as the number of waves per unit length.

7. Electromagnetic wave are non-mechanical, they travel depending upon electric and magnetic properties of medium. They can travel in medium as well as an vacuum. So reason is false.

8. As speed, $v = \sqrt{\frac{T}{\mu}} \Rightarrow v \propto \frac{1}{\sqrt{\mu}}$ in second string μ is more (by looking) so v will be less. Thus reason is true explanation of assertion.

9. At point A both v_p and Δl is zero *ie*, K.E. and P.E. are minimum while at B both v_p and Δl are maximum *i.e.*, both K.E. and P.E. are maximum. Thus both assertion and reason are true but not correct explanation.

12.



If P is moving downword then it shows that the wave is travelling in (–) ve x direction. So assertion is false.

11. $A = 2a \cos \frac{\Delta\phi}{2}$, for $A = a$

$\Rightarrow \quad \cos \frac{\Delta\phi}{2} = \frac{1}{2} = \cos \frac{\pi}{3}$

$\therefore \quad \Delta\phi = \frac{2\pi}{3} = \frac{360°}{3} = 120°$

Assertion is true but the reason is false.

Match the Columns

1. $y = a \sin (bt - cx) = A \sin (\omega t - kx)$

(a)	$v = \frac{\omega}{k} = \frac{b}{c}$	r
(b)	$(v_p)_{max} = A\omega = ab$	s
(c)	$\nu = \frac{\omega}{2\pi} = \frac{b}{2\pi}$	p
(d)	$\lambda = \frac{2\pi}{k} = \frac{2\pi}{c}$	s

2. $y = 4 \text{ cm} \sin (\pi t + 2\pi x)$

$v_p = 4\pi \text{ cm/s} \cos (\pi t + 2\pi x)$

$a = -4\pi^2 \text{ cm/s}^2 \sin (\pi t + 2\pi x)$

(a)	$v_p (0, t) = 4\pi \text{ cm s} \cos \pi t$ $= \pm 4\pi$ for $\cos \pi t = \pm 1$ or $\pi t = n\pi \Rightarrow t = n = 0, 1, 2, 3, \rightarrow$	q, r
(b)	$a(0, t) = -4\pi^2 \text{ cm/s}^2 \sin \pi t$ $= \pm 4\pi^2$ for $\sin \pi t = \pm 1$ or $\pi t = (2n + 1) \frac{\pi}{2}$ $\Rightarrow t = n + \frac{1}{2} = 0.5, 2.5 \rightarrow$	p, s
(c)	$v_p (0.5, t) = 4\pi \text{ cm/s} \cos (\pi t + \pi)$ $= \pm 4\pi$ for $\pi t + \pi = n\pi$ or $t = n - 1 = 0, 1, 2, 3, \rightarrow$	q, r
(d)	$a (0.5, t) = -4\pi^2 \text{ cm/s}^2$ $= \pm 4\pi^2 \Rightarrow \pi t + \pi = (2n + 1) \frac{\pi}{2}$ or $t = n - \frac{1}{2} = 0.5, 1.5, 2.5 \rightarrow$	p

3. $y = A \sin (\omega t \pm kx)$ at $t = 0$

$\Rightarrow y = \pm A \sin kx$

$v_p = \pm \omega A \cos kx$ and $a = -\omega^2 y$

(a)	$v_p = \pm \omega A \cos kx \rightarrow$	s
(b)	$a_A = (+) v_e$ as y_A is negative $\rightarrow$	p
(c)	$v_B = \pm A\omega \rightarrow$	s
(d)	$a_B = 0$ ve $y_0 = 0 \rightarrow$	r

4. (a) $u = \frac{E}{V} = \frac{IST}{V} = \frac{2\pi^2 \nu^2 A^2 \rho v s t}{V}$

$= 2\pi^2 \nu^2 A^2 \rho = \frac{1}{2} \rho \omega^2 A^2$

$[u] = \frac{[ML^2T^{-2}]}{[L^3]} = [ML^{-1}T^{-2}] \rightarrow s$

(b) $P = \frac{E}{t} = \frac{IST}{t} = IS = 2\pi^2 \nu^2 A^2 \rho v s$

$= \frac{1}{2} \omega^2 A^2 \rho v s = \frac{1}{2} \rho \omega^2 A^2 s v \rightarrow q$

$[P] = \frac{E}{t} = \frac{[ML^2T^{-2}]}{[T]} = [ML^2T^{-3}] \rightarrow p$

(c) $I = \frac{E}{St} = \frac{[ML^2T^{-2}]}{[L^2T]}$

$= [MT^{-3}] = [ML^0T^{-3}] \rightarrow s$

(d) $\frac{1}{\lambda} = [L^{-1}] = [M^0L^{-1}T^0] \rightarrow s$

5.	(a)	$y = A \sin (\omega t - kx) \rightarrow$	p
		$v_p = \omega A \cos (\omega t - kx) \rightarrow$	r
		$a = -\omega^2 A \sin (\omega t - kx)$	
	(b)	$y = A \sin (kx - \omega t) \rightarrow$	p
		$v_p = -\omega A \cos (kx - \omega t)$	
		$a = -\omega^2 A \sin (kx - \omega t)$	
	(c)	$y = -A \cos (\omega t + kx) \rightarrow$	q
		$v_p = \omega A \sin (\omega t + kx)$	
		$a = \omega^2 A \cos (\omega t + kx) \rightarrow$	s
	(d)	$y = -A \cos (kx - \omega t) \rightarrow$	p
		$v_p = -A\, \omega \sin (kx - \omega t)$	
		$a = \omega^2 A \cos (kx - \omega t) \rightarrow$	d s

15. Superposition of Waves

Introductory Exercise 15.1

1.

When displacement of all the particles is momentarily zero, then there is no elastic potential energy stored in the string and as the speed is maximum at mean position, so entire energy is purely kinetic.

2. (a) $v = \sqrt{\dfrac{T}{\mu}}$

$$\Rightarrow \quad \frac{v_2}{v_1} = \sqrt{\frac{\mu_1}{\mu_2}}$$

$$= \sqrt{\frac{\mu_1}{0.25\,\mu_1}} = \frac{1}{\sqrt{0.25}}$$

$$= \frac{1}{0.5} = 2$$

$$\Rightarrow \quad v_2 = 2v_1 = 20\text{ cm/s}$$

(b) $a_t = \dfrac{2v_2}{v_1 + v_2}\, a_i$

$$= \frac{2 \times 20}{10 + 20}\, a_i = \frac{4}{3}\, a_i$$

and $a_r = \dfrac{v_2 - v_1}{v_2 + v_1}\, a_i$

$$= \frac{20 - 10}{20 + 10}\, a_i = \frac{1}{3}\, a_i$$

3. (a) For fixed end, a phase change of π takes place in reflected wave and direction becomes opposite.

as $\quad Y_i = 0.3 \cos(2x - 40t)$

$\Rightarrow \quad Y_r = 0.3 \cos(2x + 40t + \pi)$

(b) For free end, there is no change in phase for reflected wave and direction becomes opposite.

as $\quad Y_i = 0.3 \cos(2x - 40t)$

$\Rightarrow \quad Y_r = 0.3 \cos(2x + 40t)$

4. $v_1 = \dfrac{\omega}{k_1} = \dfrac{50}{2} = 25\text{ m/s}$ and $v_2 = 50\text{ m/s}$

$$\Rightarrow \quad a_t = \frac{2v_2}{v_1 + v_2}\, a_i = \frac{2 \times 50}{25 + 50}\, a_i$$

$$= \frac{4}{3} \times 2 \times 10^{-3}\text{ m} = \frac{8}{3}\text{ mm.}$$

$$a_r = \frac{v_2 - v_1}{v_2 + v_1}\, a_i = \frac{50 - 25}{50 + 25}\, a_i$$

$$= \frac{1}{3} \times 2 \times 10^{-3}\text{ m} = \frac{2}{3}\text{ mm.}$$

as $v_2 > v_1 \Rightarrow$ the boundary is rearer and there is no phase change.

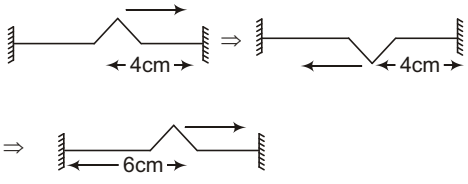
$$k_2 = \frac{\omega}{v_2}$$

$$= \frac{50\,\pi}{50} = \pi$$

$$\therefore \quad y_r = \frac{2}{3} \times 10^{-3} \cos \pi(0.2\,x + 50t)$$

and $y_t = \dfrac{8}{3} \times 10^{-3} \cos \pi(1.0\,x - 50t)$

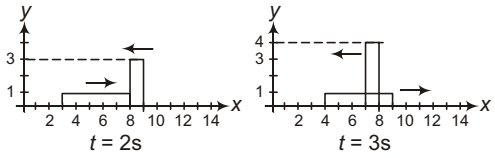
5. $t_1 = \dfrac{2 \times 40\text{ cm}}{1\text{ cm/s}} = 8\text{ s}$, inverted

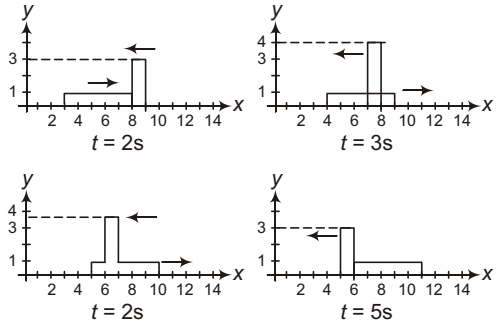


$$t_2 = \frac{4+10+6}{1}$$

$= 20$ s upright

6.





Introductory Exercise 15.2

1. $y = 5 \sin \frac{\pi x}{3} \cos 40\pi t = 2a \sin kx \cos \omega t$

$a = \frac{5}{2} = 2.5$ cm, $k = \frac{\pi}{3}$ cm^{-1}, $\omega = 40\ \pi$s^{-1}

$$v = \frac{\omega}{k} = \frac{40\pi}{\pi/3} = 120 \text{ cm/s}$$

$$\Delta x = \frac{\lambda}{2} = \frac{1}{2} \cdot \frac{2\pi}{k} = \frac{\pi}{k} = \frac{\pi}{\pi/9} \text{ cm} = 3 \text{ cm}$$

$$v_P = \frac{dy}{dt} = -200\ \pi \sin \frac{\pi x}{3} \sin 40\pi t$$

$$v_P\left(1.5, \frac{9}{8}\right) = -200\ \pi \sin\left(\frac{\pi}{3} \cdot \frac{3}{2}\right) \sin\left(40\ \pi \times \frac{9}{8}\right)$$

$$= -200\ \pi \sin\left(\frac{\pi}{2}\right) \sin(45\ \pi)$$

$$= -200\ \pi \times 1 \times 0$$

$$= 0 \text{ cm/s}$$

2. Two waves with different amplitudes can produce partial stationary waves with amplitude of antinodes being $a_1 + a_2$ and amplitude of nodes being $a_1 \sim a_2$. As here node is not stationary that is why energy is also transported through nodes.

3. (a) $\frac{\lambda}{2} = 2 \text{ m} \Rightarrow \lambda = 4$ m,

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{100}{4 \times 10^{-2}}} = \frac{10^2}{2} = 50 \text{ m/s}$$

$$\nu = \frac{v}{\lambda} = \frac{50}{4} = 12.5 \text{ Hz}$$

and is fundamental tone or first harmonic.

$$y = 0.1 \sin \frac{2\pi}{\lambda} x \sin 2\pi\nu t$$

$$= 0.1 \sin \frac{2\pi}{4} x \sin 2\pi \times 12.5\ t$$

$$= 0.1 \sin \frac{\pi}{2} x \sin 25\ \pi t$$

(b) $3\ \frac{\lambda}{2} = 2 \text{ m} \Rightarrow \lambda = \frac{4}{3}$ m and $v = 50$ m/s

$\nu = \frac{v}{\lambda} = \frac{50}{4/3}$ Hz $= 37.5$ Hz and is 2nd overtone or 3rd harmonic.

$$y = 0.04 \sin \frac{2\pi}{4/3} x \sin 2\pi \times 37.5\ t$$

$$= 0.04 \sin \frac{3\pi}{2} x` \sin 75\ \pi t$$

4. $v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{Fl}{m}} = \sqrt{\frac{400 \times 4}{160 \times 10^{-3}}}$

$$= \sqrt{\frac{1600}{16 \times 10^{-2}}} = 10^2 = 100 \text{ m/s}$$

(a) $\frac{\lambda_0}{4} = l \Rightarrow \lambda_0 = 4l = 16$ m

$$\frac{3\lambda_1}{4} = l \Rightarrow \lambda_1 = \frac{4l}{3} = \frac{16}{3}\text{ m} = 5.33\text{ m}$$

and $\frac{5\lambda_2}{4} = l \Rightarrow \lambda_2 = \frac{4l}{5} = \frac{16}{5}\text{ m} = 3.2\text{ m}$

(b) $\nu_0 = \frac{v}{\lambda_0} = \frac{100}{16} = 6.25\text{ Hz}$

$\nu_1 = \frac{v}{\lambda_1} = \frac{100}{16/3} = 18.75\text{ Hz}$

$\nu_3 = \frac{v}{\lambda_3} = \frac{100}{16/5} = 31.25\text{Hz}$

5. $l = n\frac{\lambda}{2} = \frac{0.54}{2}n = 0.27\,n$

and $l = (n+1)\frac{\lambda'}{2} = \frac{0.48}{2}(n+1)$

$= 0.24\,(n+1)$

$\Rightarrow 0.27\,n = 0.24\,n + 0.24$

$\Rightarrow 0.03\,n = 0.24 \Rightarrow n = 8$

(a) These are 8th and 9th harmonic

(b) $l = 0.27\,n = 0.27 \times 8 = 2.16\text{ m}$

(c) $\frac{\lambda_0}{2} = l \Rightarrow \lambda_0 = 2l = 4.32\text{ m}$

6. $5\nu_0 - 2\nu_0 = 54\text{ Hz}$

$\Rightarrow 3\nu_0 = 54\text{ Hz} \Rightarrow \nu_0 = 18\text{ Hz}$

7. $\nu_0 = \frac{1}{2l}\sqrt{\frac{F}{\mu}}$

$\Rightarrow \frac{\nu_2}{\nu_1} = \sqrt{\frac{F_2}{F_1}}$

$= \sqrt{\frac{M+2.2}{2.2}} = \frac{260}{220} = \frac{13}{11}$

$\Rightarrow \frac{M+2.2}{2.2} = \frac{169}{121} = 1 + \frac{48}{121}$

$= 1 + \frac{M}{2.2}$

$\Rightarrow M = \frac{48 \times 2.2}{121} = \frac{9.6}{11} = 0.873\text{ kg}$

8. $n\nu_0 = 250\text{ Hz}$ and $(n+1)\nu_0 = 300\text{ Hz}$

$\Rightarrow \nu_0 = 50\text{ Hz}$

and $n = 5 \Rightarrow$ So these are 5th and 6th harmonics.

$\nu_0 = \frac{1}{2l}\sqrt{\frac{F}{\mu}}$

$\Rightarrow F = 4l^2 v_0^2 \mu = 4 \times 50^2 \times \frac{36 \times 10^{-3}}{1} = 360\text{ N}$

AIEEE Corner

Subjective Question (Level 1)

1. $A = \sqrt{A_1^2 + A_1^2 + 2A_1 A_1 \cos 90°}$

$= A_1\sqrt{2} = 4\sqrt{2}\text{ cm} = 5.66\text{ cm}$

2. $v_2 = 2v_1$

$A_r = \frac{v_2 - v_1}{v_2 + v_1}A = \frac{v_1}{3v_1}A = \frac{1}{3}A$

$A_t = \frac{2v_2}{v_2 + v_1}A = \frac{4v_1}{3v_1}A = \frac{4}{3}A$

$\frac{I_r}{I_i} = \left(\frac{A_r}{A}\right)^2 = \frac{1}{9}$

and $\frac{I_t}{I_i} = 1 - \frac{1}{9} = \frac{8}{9}$

$A = \sqrt{10^2 + 20^2 + 2 \times 10 \times 20 \cos\frac{\pi}{3}}$

$= \sqrt{100 + 400 + 200} = \sqrt{700} = 10\sqrt{7}$

$= 26.46\text{ units}$

$\tan\theta = \frac{20\sin\frac{\pi}{3}}{10 + 20\cos\frac{\pi}{3}}$

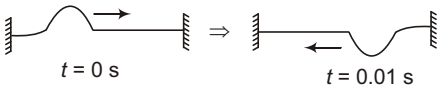
$= \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right) = 0.714\text{ rad}$

$\therefore$ Phase $= 5x + 25t + 0.714$ rad.

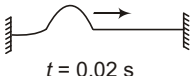
4. $y_1 = 1\text{ cm}\sin(\pi\text{ cm}^{-1}x - 50\,\pi\text{s}^{-1}t)$

$$y_2 = 1.5\text{ cm}\sin\left(\frac{\pi}{2}\text{ cm}^{-1}x - 100\,\pi\text{s}^{-1}t\right)$$



$\Rightarrow$

$$y_1\,(4.5, 5\times10^{-3}) = 1\text{ cm}\sin\left(4.5\,\pi - \frac{250\,\pi}{1000}\right)\Rightarrow$$



$$= 1\sin\left(\frac{9}{2}\pi - \frac{\pi}{4}\right)$$

$$= 1\sin\left(\frac{17\pi}{4}\right)$$

$$= 1\text{ cm}\sin\left(4\pi + \frac{\pi}{4}\right)$$

$$= 1\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}\text{ cm and}$$

$$y_2\,(4.5, 5\times10^{-3}) = 1.5\text{ cm}\sin\left(\frac{9\pi}{4} - \frac{500\,\pi}{1000}\right)$$

$$= 1.5\text{ cm}\sin\left(\frac{9\pi}{4} - \frac{\pi}{2}\right)$$

$$= 1.5\sin\left(\frac{5\pi}{4}\right)$$

$$= 1.5\sin\left(\pi + \frac{\pi}{4}\right)$$

$$= -1.5\sin\left(\frac{\pi}{4}\right)$$

$$= -\frac{1.5}{\sqrt{2}}\text{ cm}$$

$$\therefore\quad y = y_1 + y_2 = \frac{1}{\sqrt{2}} - \frac{1.5}{\sqrt{2}}$$

$$= -\frac{0.5}{\sqrt{2}} = -\frac{1}{2\sqrt{2}}\text{ cm}$$

5. $$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{16\text{N}}{0.4\times10^{-3}\times10^{2}\ \text{kg/N}}}$$

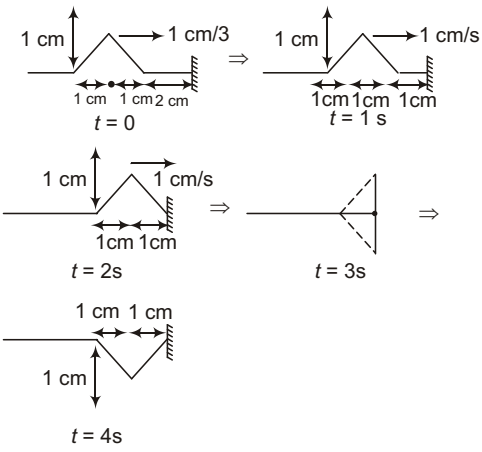
$$= \sqrt{\frac{16\times10^{2}}{4}} = 20\text{ m/s}$$

(a) For same shape, time,

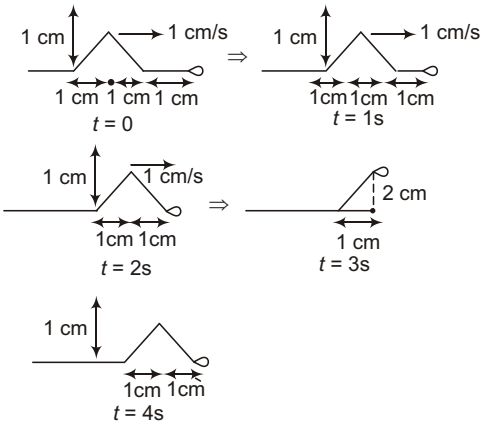
$$t = \frac{2l}{v} = \frac{2\times0.2}{20}\text{ s} = 0.02\text{ s}$$

(b)

6. (a)



(b)



7. $y = 1.5\sin(0.4\,x)\cos(200t)$

$$= 2A\sin kx\cos\omega t$$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.4} = 5\pi\text{ m} = 15.7\text{ m}$$

$$\nu = \frac{\omega}{2\pi} = \frac{200}{2\pi} = \frac{100}{\pi}\text{ Hz} = 31.8\text{ Hz}$$

$$v = \frac{\omega}{k} = \frac{200}{0.4} = 500\text{ m/s}$$

8. $y = y_1 + y_2 = 3 \text{ cm} \sin(\pi x + 0.6\,\pi t)$
$+ 3 \text{ cm} \sin(\pi x - 0.6\,\pi t)$
$= 6 \text{ cm} \sin \pi x \cos 0.6\,\pi t = R \cos 0.6\,\pi t$
where, $R = 6 \text{ cm} \sin \pi x$.

(a) $R(0.25) = 6 \text{ cm} \sin \pi \times \frac{1}{4}$
$= \frac{6}{\sqrt{2}} = 3\sqrt{2} \text{ cm} = 4.24 \text{ cm}$

(b) $R(0.50) = 6 \text{ cm} \sin \pi \times \frac{1}{2} = 6 \text{ cm}$

(c) $R(1.50) = 6 \text{ cm} \sin \frac{3\pi}{2} = -6 \text{ cm}$
$\Rightarrow \quad |R| = 6 \text{ cm}$

(d) For antinodes, $R = \pm 6$ cm
$\Rightarrow \quad \sin \pi x = \pm 1 \Rightarrow \pi x = (2n + 1)\frac{\pi}{2}$
or $x = n + \frac{1}{2} = 0.5 \text{ cm}, 1.5 \text{ cm}, 2.5 \text{ cm}$

9. $\lambda = \frac{2\pi}{4} = \frac{2\pi}{\pi/2} = 4 \text{ cm}$

(a) Distance between successive antinodes $= \frac{\lambda}{2} = 2 \text{ cm}$

(b) $R(x) = 2A \sin kx$
$= 2 \times \pi \text{ cm} \sin \frac{\pi}{2} \times 0.5$
$= 2\pi \sin \frac{\pi}{4}$
$= \frac{2\pi}{\sqrt{2}} = \sqrt{2}\,\pi \text{ cm}$

10. $\nu_n = \frac{n+1}{2l}\sqrt{\frac{T}{\mu}} = \frac{n+1}{2 \times 20}\sqrt{\frac{20}{9 \times 10^{-3}}}$
$= \frac{n+1}{60} \times \frac{100\sqrt{2}}{3} = \frac{5\sqrt{2}}{9} = \frac{5\sqrt{2}}{9}(n+1)$
$= 0.786\,(n+1)$
$= 0.786 \text{ Hz}$
1.57 Hz, 2.36 Hz, 3.14 Hz

11. (a) $T = \mu v^2 = \mu \nu^2 \lambda^2$
$= \frac{1.2 \times 10^{-3}}{0.7} \times (220)^2 \times (1.4)^2$
$= 162.6 \text{ N}$

(b) $\nu_2 = 3\,\nu_0 = 3 \times 220 \text{ Hz} = 660 \text{ Hz}$

12. $\nu_n = \frac{n+1}{2l}\sqrt{\frac{T}{\mu}} = \frac{n+1}{2 \times 0.6}\sqrt{\frac{50}{0.01}}$
$= \frac{50\sqrt{2}}{1.2}(n+1) = 58.93\,(n+1) \text{ Hz}$
$\nu_n \le 20{,}000 \text{ Hz} \Rightarrow n = 338$
$\therefore \nu_{338} = 339 \times 58.93 = 19975.8 \text{ Hz}$
$= 19.976 \text{ kHz}$

13. $n\nu_0 = 420 \text{ Hz}$ and $(n+1)\,\nu_0 = 490 \text{ Hz}$
$\Rightarrow \nu_0 = 70 \text{ Hz}$ and $n = 6$
$\therefore \nu_0 = \frac{1}{2l}\sqrt{\frac{T}{\mu}} \Rightarrow l = \frac{1}{2\nu_0}\sqrt{\frac{T}{\mu}} = \sqrt{\frac{\frac{450}{0.005}}{2 \times 70}}$
$= \frac{300}{140} = 2.143 \text{ m}$

14. $\lambda = \frac{v}{\nu} = \frac{400 \text{ m/s}}{800 \text{ Hz}} = \frac{1}{2} \text{ m},$
$l = 4\frac{\lambda}{2} = 2\lambda = 1 \text{ m}$

(a) $4\nu_0 = 400 \text{ Hz} \Rightarrow \nu_0 = 100 \text{ Hz}$

(b) $7\nu_0 = 700 \text{ Hz}$

16. $\nu_0 = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$
$\Rightarrow \quad \nu_0 \propto \frac{1}{l}$
$\nu_1 : \nu_2 : \nu_3 = 1 : 2 : 3$
$= \frac{1}{l_1} : \frac{1}{l_2} : \frac{1}{l_3}$
$\Rightarrow \quad l_1 : l_2 : l_3 = \frac{1}{1} : \frac{1}{2} : \frac{1}{3}$
$= 6 : 3 : 2 = 6x : 3x : 2x$
$6x + 3x + 2x = 1 \text{ m}$
$\Rightarrow \quad x = \frac{1}{11} \text{ m}$
$\therefore$ position of first bridge $= 6x = \frac{6}{11} \text{ m}$
and position of second bridge
$= 6x + 3x = 9x = \frac{9}{11} \text{ m}$
From the same end or $1 - \frac{9}{11} = \frac{2}{11} \text{ m}$ from other end .

17. $\nu_0 = \dfrac{v}{2l}$

$\Rightarrow \quad \nu_0' = \dfrac{v}{2l'}$

$\Rightarrow \quad l' = \dfrac{\nu_0}{\nu_0'} l = \dfrac{124}{186} \times 90 \text{ cm} = 60 \text{ cm}$

Thus length of the vibrating string has to be 60 cm.

18. $\dfrac{\lambda}{2} = 15 \text{ cm} \Rightarrow \lambda = 30 \text{ cm},$

$R_{max} = 2A = 0.85 \text{ cm},$

$T = 0.075 \text{ s}$

(a) $y = 2A \sin kx \sin \omega t$

$= 0.85 \text{ cm} \sin\left(\dfrac{2\pi}{0.3 \text{ m}} x\right) \sin\left(\dfrac{2\pi}{0.075 \text{ s}} t\right)$

(b) $v = \dfrac{\omega}{k} = \dfrac{2\pi/0.075}{2\pi/0.3} = \dfrac{0.3}{0.075} = 4 \text{ m/s}$

(c) $\dfrac{\lambda}{4} = \dfrac{30}{4} = 7.5 \text{ cm}$

$\therefore \quad R(7.5 + 3) = 2A \sin kx$

$= 0.85 \sin \dfrac{2\pi}{30} \times 10.5$

$\therefore \quad R(11.5 \text{ cm}) = 0.85 \sin\left(\dfrac{21\pi}{30}\right)$

$= 0.85 \sin (0.7\ \pi)$

$= 0.85\,(126°) = 0.688 \text{ cm}$

19. $\nu_0 = \dfrac{v}{2l} = \dfrac{48}{2 \times 1.5} = 16 \text{Hz}$

and $\quad \lambda_0 = 2l = 3 \text{ m}$

$\nu_2 = 3\nu_0 = 48 \text{ Hz and } \lambda_2 = \dfrac{v}{\nu_2} = \dfrac{48}{48} = 1 \text{ m}$

$\nu_3 = 4\ \nu_0 = 64 \text{Hz}$

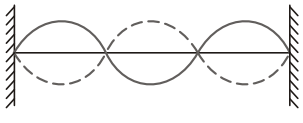
and $\lambda_3 = \dfrac{v}{\nu_3} = \dfrac{48}{64} = \dfrac{3}{4} = 0.75 \text{ m}$

20. $y = 5.60 \text{ cm} \sin (0.340 \text{ rad/cm } x)$

$\sin (50.0 \text{ rad/s } t)$

$= 2A \sin (kx) \sin (\omega t)$

(a)



(b) $2A = 5.60 \text{ cm} \Rightarrow A = 2.80 \text{ cm}$

(c) $l = 3\dfrac{\lambda}{2} = \dfrac{3}{2} \cdot \dfrac{2\pi}{k} = \dfrac{3\pi}{k}$

$= \dfrac{3\pi}{0.0340} \text{ cm} = 277.2 \text{ cm}$

(d) $\lambda = \dfrac{2\pi}{k} = \dfrac{2\pi}{0.0340} \text{ cm} = 184.8 \text{ cm}$

$\nu = \dfrac{\omega}{2\pi} = \dfrac{50}{2\pi} = 7.96 \text{Hz}$

$T = \dfrac{1}{\nu} = \dfrac{1}{7.96} \text{ s} = 0.216 \text{ s}$

$v = \nu\lambda = 7.96 \text{ Hz} \times 184.8 \text{ cm}$

$= 1470 \text{ cm/s}$

(e) $(v_p)_{max} = R_{max}\ \omega = 2A\omega$

$= 5.60 \text{ cm} \times 50 \text{ rad/s}$

$= 280 \text{ cm/s}$

(f) for eight harmonic,

$8\dfrac{\lambda}{2} = l \Rightarrow \lambda' = \dfrac{l}{4} = \dfrac{277.2}{4} = 69.3 \text{ cm}$

$k = \dfrac{2\pi}{\lambda'} = \dfrac{2 \times 3.14}{69.3} = 0.0907 \text{ rad/cm}$

$v' = 8\ v_0 = \dfrac{8}{3} \cdot v = \dfrac{8}{3} \times 7.96 \text{Hz}$

$= 21.22 \text{ Hz}$

$\omega' = 2\pi\nu = 133.4 \text{ rad/s}$

$\Rightarrow y = 5.60 \text{ cm} \sin (0.0907 \text{ rad/s} \cdot x)$

$\sin \cdot (133 \text{ rad/s} \cdot t)$

21. (a) $v = \nu\lambda = \nu_0 \cdot 2l = 60 \times 2 \times 0.8$

$= 96 \text{ m/s}$

(b) $T = \mu v^2 = \dfrac{40 \times 10^{-3}}{80 \times 10^{-2}} \times (96)^2$

$= \dfrac{96^2}{20} = 460.8 \text{ N}$

(c) $(v_p)_{max} = R_{max}\ \omega$

$= 0.3 \text{ cm} \times 2\pi$

$\times 60 \text{ rad/s} = 113 \text{ cm/s} = 1.13 \text{ m/s}$

$a_{max} = \omega^2 R_{max} = (120\ \pi)^2 \times 0.3 \text{ cm/s}^2$

$= 426.4 \text{ m/s}^2$

■ Objective Questions (Level 1)

1. $\frac{\nu_2}{\nu_1} = \sqrt{\frac{T_2}{T_1}} \Rightarrow \frac{3}{2} = \sqrt{\frac{T + 2.5}{T}}$

$\Rightarrow \quad 9T = 4\,(T + 2.5)$

$\Rightarrow \quad 5T = 10 \Rightarrow T = 2\text{N}$

2. $\nu = \frac{n+1}{2l}\sqrt{\frac{T}{\mu}} = \frac{n+1}{2l}\sqrt{\frac{T}{\pi r^2 \rho}}$

$= \frac{n+1}{2lr}\sqrt{\frac{T}{\pi\rho}} = \frac{n+1}{ld}\sqrt{\frac{T}{\pi\rho}} = \text{constant.}$

$n + 1 \propto \frac{ld}{\sqrt{T}}$

$\frac{n_1 + 1}{n_2 + 1} = \frac{l_1}{l_2} \cdot \frac{d_1}{d_2}\sqrt{\frac{T_2}{T_1}}$

$= \frac{1}{2} \times \frac{1}{3} \times \sqrt{2}$

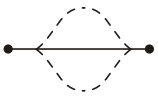
$= \frac{1}{3\sqrt{2}} = 1 : 3\sqrt{2}$

or $\frac{n_2 + 1}{n_1 + 1} = 3\sqrt{2}$

3. $f \propto \frac{1}{l}; l = l_1 + l_2 + l_3$

$\Rightarrow \frac{1}{f_0} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$

4. During overlapping the displacement of particles is zero while velocity is maximum. So the entire energy is purely kinetic.



5. $y\,(x, y) = y_1 + y_2 = a\cos(kx + \omega t) + y_2$

$= -2a \sin kx \sin \omega t$ is necessary for a node at $x = 0$. Thus,

$y_2 = 2a \sin kx \sin \omega t - a \cos(kx + \omega t)$

$= -2a \sin kx \sin \omega t - a \cos kx \cos \omega t + a \sin kx \sin \omega t$

$= -a[\cos kx \cos \omega t + \sin kx \sin \omega t]$

$= -a \cos(kx - \omega t)$

6. In transverse stationary wave, longitudinal strain is maximum at node. While in longitudinal stationary wave at displacement node pressure and density are maximum. So all are correct.

7. In stationary wave all particles errors the mean position simultaneously and are at their maximum displacement simultaneously at different instant at this time all of them are at rest. So all are correct.

8. Maximum displacement

$y_{\max} = 3A - A + 2A = 4A$

$\frac{v_t}{v_l} = \frac{v_t}{v_l} = \frac{\sqrt{\frac{\frac{Y\,\Delta l}{l}}{\rho}}}{\sqrt{\frac{Y}{\rho}}} = \sqrt{\frac{\Delta l}{l}}$

$= \sqrt{\frac{\frac{1}{\eta}\,l}{l}} = \frac{1}{\sqrt{\eta}}$

10. $f_n = \frac{n+1}{2 \times 1}\sqrt{\frac{100}{0.01}} = 50\,(n + 1)$

$= 50\text{ Hz}, 100\text{ Hz}, 150\text{ Hz}$

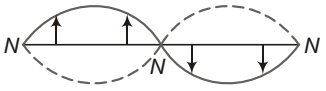
$n_n = \frac{2n+1}{4 \times 1}\sqrt{\frac{100}{0.01}} = 25\,(2m + 1)$

$= 25\text{ Hz}, 75\text{Hz}, 125\text{ Hz}$

$\therefore \quad n_2 = 75\text{ Hz} = \frac{f_1 + f_2}{2}$

$= \frac{50\text{ Hz} + 100\text{Hz}}{2} = 75\text{ Hz}$

11. In stationary waves all particles perform SHM such that they are at their positive and negative extremes one time each in a time period, where they come to rest. Particles between two successive nodes are in phase while beside node are in opposite phase. So all the particles cannot be at positive extreme simultaneously.



12. The question is wrong, string has to be fix at one end and free at other. Then

$(2n + 1)\,\nu_0 = 90\text{ Hz}, (2n + 3)\,\nu_0 = 50\text{ Hz}$

and $(2n + 5)\,\nu_0 = 210\text{ Hz}$

$\Rightarrow 2\nu_0 = 60$ Hz or $\nu_0 = 30$ Hz and $n = 1$

i.e., vibrations are 3rd, 5th and 7th harmonic.

$$\lambda_0 = 2l = 1.6\text{ m}$$

$$\therefore \quad v = \nu_0\lambda_0 = 30\text{ Hz} \times 1.6\text{ m} = 48\text{ m/s}$$

13. $y = y_1 + y_2 + y_3 = 12\sin\left(\theta - \frac{\pi}{2}\right) + 6\sin(\theta + 0) + 4\sin\left(\theta + \frac{\pi}{2}\right)$

$$= 6\sin\theta - 12\cos\theta + 4\cos\theta$$

$$= 6\sin\theta - 8\cos\theta$$

$$\Rightarrow R = \sqrt{6^2 + 8^2 + 2 \times 6 \times 8\cos\frac{\pi}{2}}$$

$$= \sqrt{100} = 10\text{ mm}$$

14. $(2n + 1)\nu_0 = 105$ Hz

and $\quad (2n + 3)\nu_0 = 175$ Hz

$\Rightarrow \quad 2\nu_0 = 70$ Hz

$\Rightarrow \quad \nu_0 = 35$ Hz

15. $l_1 : l_2 : l_3 = \frac{1}{\nu_1} : \frac{1}{\nu_2} : \frac{1}{\nu_3} = \frac{1}{1} : \frac{1}{3} : \frac{1}{4}$

$$= 12 : 4 : 3$$

$\therefore \quad 12x + 4x + 3x = 114$ cm

$\Rightarrow \quad x = \frac{114}{19}$ cm $= 6$ cm

$\therefore \quad l_1 = 12x = 72$ cm, $l_2 = 4x = 24$ cm, $l_3 = 3x = 18$ cm

16. $f \propto \sqrt{T}$

$$\frac{f/2}{f} = \sqrt{\frac{V\rho g - V\sigma_1 g}{V\rho g}} = \sqrt{1 - \frac{\sigma_2}{\rho}} = \frac{1}{2}$$

$$\Rightarrow 1 - \frac{\sigma_1}{\rho} = \frac{1}{4} \Rightarrow \frac{\sigma_1}{\rho} = \frac{3}{4}$$

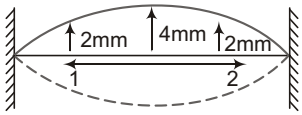
$$\frac{f/3}{f} = \sqrt{\frac{V\rho g - V\sigma_2 g}{V\rho g}} = \sqrt{1 - \frac{\sigma_2}{\rho}} = \frac{1}{3}$$

$$\Rightarrow \quad 1 - \frac{\sigma_2}{\rho} = \frac{1}{q} \Rightarrow \frac{\sigma_2}{\rho} = \frac{8}{9}$$

$$\therefore \quad \frac{\sigma_1}{\sigma_2} = \frac{3/4}{8/9} = \frac{27}{32} \Rightarrow \frac{\sigma_2}{\sigma_1} = \frac{32}{27} = 1.18$$

where σ_1 is density of water and σ_2 is density of the other liquid.

17. $R = 2A\sin Kx = 4\text{ mm}\sin\frac{2\pi}{\lambda}x$



$$= 4\text{ mm}\sin\frac{2\pi x}{3\text{ m}}$$

$$2\text{ mm} = 4\text{ mm}\sin\frac{2\pi x}{3}$$

$$\Rightarrow \quad \frac{2\pi x}{3} = \frac{\pi}{3}$$

$$\Rightarrow \quad x = 0.5\text{ m}$$

Thus points 1 and 2 are at 0.5 m from their nearest boundary. So separation between them is

$1.5\text{ m} - 2 \times 0.5\text{ m} = 0.5\text{ m} = 50\text{ m}$

18. $y = -A\sin(\omega t - kx)$

$$= -A\sin\left(2\pi\nu t - \frac{2\pi\nu}{v}x\right)$$

$$= -A\sin(6\pi t - 2\pi x)$$

$$y(3, t) = +A = -A\sin(6\pi t - 6\pi)$$

$$= A\sin(6\pi - 6\pi t)$$

$$\therefore \quad 6\pi - 6\pi t = \frac{\pi}{2} \Rightarrow \frac{11}{2} = 6t$$

$$\Rightarrow \quad t = \frac{11}{12}\text{ s}$$

19. $\Delta\phi_1 = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda} \cdot \frac{3}{2}\lambda = 3\pi$

and $\Delta\phi_2 = -\frac{\pi}{2} \Rightarrow \Delta\phi = 3\pi - \frac{\pi}{2} = \frac{5\pi}{2}$

20. $n\nu_0 = 400$ Hz, $(n + 1)\nu = 450$ Hz

$\Rightarrow \nu_0 = 50$ Hz and $n = 8$

$$\nu_0 = \frac{1}{2l}\sqrt{\frac{T}{\mu}} \Rightarrow l = \frac{1}{2\nu_0}\sqrt{\frac{T}{\mu}}$$

$$= \frac{1}{2 \times 50}\sqrt{\frac{490}{0.1}} = \frac{70}{100} = 0.7\text{ m}$$

21. $3 \cdot \frac{\lambda}{2} = 1\text{ m}, \lambda = \frac{2}{3}\text{ m}$

$$v = \nu\lambda = 300\text{ Hz} \times \frac{2}{3}\text{ m} = 200\text{ m/s}$$

22. $l = \frac{\lambda_1}{2}, \frac{2\lambda_2}{2}, \frac{3\lambda_3}{2}$

$$\Rightarrow \quad \lambda_1 = 2l,\ \lambda_2 = \frac{2l}{2},\ \lambda_3 = \frac{2l}{3}$$

$$\therefore \quad \lambda_1 : \lambda_2 : \lambda_3 = 2l : \frac{2l}{2} : \frac{2l}{3}$$

$$= 1 : \frac{1}{2} : \frac{1}{3}$$

23. $\Delta\phi = \frac{2\pi}{\lambda}\Delta x$

$$= \frac{2\pi}{vT}\Delta x = \frac{2\pi}{300 \times 0.04} \times (16 - 10)$$

$$= \frac{2\pi}{12} \times 6 = \pi$$

24. $v = \frac{\omega}{k} = \frac{30}{1} = 30\ \text{m/s}$

$$= \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\rho A}}$$

$$\therefore T = \rho A v^2 = 8000 \times 10^{-6} \times 900 = 7.2\,\text{N}$$

25. $5\nu_0 = 480\ \text{Hz},\ 2\nu_0 = \frac{2}{5} \times 480\ \text{Hz}$

$$= 192\ \text{Hz}$$

26. $\frac{I_r}{I_i} = 0.64 = \left(\frac{A_r}{A_i}\right)^2$

$$\Rightarrow \quad \frac{A_r}{A_i} = 0.8 \Rightarrow A_r = 0.8\ A_i$$

$$A_r = \frac{v_2 - v_1}{v_2 + v_1} A_i = 0.8\ A_i = \frac{4}{5}\text{A}_i$$

$$\Rightarrow \quad 5v_2 - 5v_1 = \pm(4v_2 + 4v_1)$$

$$\Rightarrow \quad v_2 = 9v_1,\ \frac{1}{9}v_1$$

For, $v_2 > v_1$ the boundary is rarer and there will not be any change in phase of reflected wave and for $v_2 < v_1$ a phase change of 180° takes place.

$$\therefore Y_r = 0.8\ A \sin(kx + \omega t + 30° + 180°)$$

27. $\nu_A = \frac{1}{2l}\sqrt{\frac{T}{\frac{\rho\pi d^2}{4}}} = \frac{1}{ld}\sqrt{\frac{T}{\pi\rho}}$

$$\nu_B = \frac{1}{2 \times 2l}\sqrt{\frac{2T}{2\rho\ \pi\frac{4d^2}{4}}}$$

$$= \frac{1}{4ld}\sqrt{\frac{T}{\pi\rho}} = \frac{1}{4}\nu_A$$

$\therefore$ Third overtone of $\nu_B = 4\nu_B = \nu_A$

■ Passage (Q 28 to 30)

$$I_r = (100\% - 36\%)\ I_i = 64\%\ I_i = 0.64\ I_i$$

$$\therefore \quad \frac{A_r}{A_i} = \sqrt{\frac{I_r}{I_i}} = \sqrt{0.64} = 0.8 = \pm\frac{v_2 - v_1}{v_2 + v_1}$$

$$\Rightarrow \quad 0.8\ v_2 + 0.8\ v_1 = \pm(v_2 - v_1)$$

$$\Rightarrow \quad -0.2\ v_2 = 1.8\ v_1$$

$$\Rightarrow \quad v_2 = 9v_1$$

for rarer boundary

or $1.8\ v_2 = 0.8\ v_1 \Rightarrow v_2 = \frac{1}{9}v_1$

for danser boundary

28. $A_r = 0.8\ A$

29. $Y = A\sin\left(ax + bt + \frac{\pi}{2}\right) + 0.8\ A\sin\left(ax - bt + \frac{\pi}{2} + \pi\right)$

$$= A\sin\left(ax + bt + \frac{\pi}{2}\right) - 0.8\ A\sin\left(ax - bt + \frac{\pi}{2}\right)$$

$$= A\cos(ax + bt) - 0.8\ A\cos(ax - bt)$$

$$= A\cos ax\cos bt - A\sin ax\sin bt - 0.8\ A\cos ax\cos bt - 0.8\ A\sin ax\sin bt$$

$$= 0.2\ A\cos ax\cos bt - 1.8\ A\sin ax\sin bt$$

$$= 0.2\ A\cos ax\cos bt - 0.2\ A\sin ax\sin bt - 1.6\ A\sin ax\sin bt$$

$$= 0.2\ A\cos(ax + bt) - 1.6\ A\sin(ax)\sin(bt)$$

$$= cA\cos(ax + bt) - 1.6\ A\sin ax\sin bt$$

$$\Rightarrow e = 0.2$$

30. For antinodes, $\sin ax = \pm 1$

$$\Rightarrow \quad ax = (2n + 1)\frac{\pi}{2}$$

$$x = (2n + 1)\frac{\pi}{2a} = \frac{\pi}{2a}, \frac{3\pi}{2a}, \frac{5\pi}{2a}$$

So for second antinode, $x = \frac{3\pi}{2a}$

31. $\frac{\nu_0 + 15}{\nu_0} = \sqrt{\frac{1+0.21}{1}} = 1 \cdot 1$

$\Rightarrow \quad 15 = 0.1\,\nu_0 \Rightarrow \nu_0 = 150$ Hz

$\frac{\nu_2}{\nu_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{1.21}{1}} = 1.1$

$\Rightarrow \quad \nu_2 = 110\%$ of v_1

$\lambda_0 = 2l$ which do not change

So, (a), (c) and (d) are correct.

32. For interference, sources must be coherent there frequency has to be equal and phase different has to be constant. So, (a) and (d) are correct.

33. Stationary waves are formed due to superposition **(here use of the term 'interference' is literary and not scientific because interference is a different phenomenon than stationary waves)** of waves having some amplitude, same frequency and travelling opposite direction. Here nodes are the points who always remain at rest. Total energy is always conserved.

34. A medium is said to be rarer if speed of wave in it is higher. And as frequency is constant, wavelength increases while frequency is constant, wavelength increases while phase do not change during change in medium.

35. $Y = A \sin kx \cos \omega t = 2a \sin kx \cos \omega t$

$a = \frac{A}{2}$, third overtone means fourth harmonic and wire oscillate with four loops.

$l = 4\frac{\lambda}{2} = 2\lambda = 2 \cdot \frac{2\pi}{k} = \frac{4\pi}{k}$

and stationary wave do not propagate.

36. For stationary waves, frequency and amplitude has to be same and direction has to be opposite with constant phase difference.

It is satisfied in (b) and (d) only.

37. $y = y_1 + y_2 = 2A \cos kx \sin \omega t$

$= R \sin \omega t$

$R = 2A \cos kx$ so at $x = 0$ there is antinode.

$\therefore \quad \cos kx = \pm 1$

$\Rightarrow \quad kx = n\pi, x = \frac{n\pi}{k} = 0, \frac{\pi}{4}, \frac{2\pi}{x},$

are antinodes.

JEE Corner

Assertion and Reason

1. $y_1 + y_2 = A \sin(\omega t + kx) + A \cos(\omega t - kx)$

$= A \sin(\omega t + kx) + A \sin\left(\frac{\pi}{2} - \omega t + kx\right)$

$= 2A \sin \frac{\omega t + kx + \frac{\pi}{2} - \omega t + kx}{2} \cos \frac{\omega t + kx - \frac{\pi}{2} + \cot - kx}{2}$

$= 2A \sin\left(kx + \frac{\pi}{4}\right) \cos\left(\omega t - \frac{\pi}{4}\right)$

$= R \cos\left(\omega t - \frac{\pi}{4}\right)$

where, $R = 2A \sin\left(kx + \frac{\pi}{4}\right)$;

$R(0) = 2A \sin \frac{\pi}{4} = A\sqrt{2}$

So, at $x = 0$, node is not present, *i.e.*, Assertion is false.

2. In stationary waves only nodes are at rest and not other particles. It is so

called as energy is not transmitted, thus assertion is false.

3. In rarer medium speed of wave is higher and as

$$A_t = \frac{2v_2}{v_1 + v_2} A_i$$

$$\Rightarrow \quad A_t > A_i$$

so reason is correct explanation to assertion.

4. In second overtone or third harmonic there are three loops or three antinodes or four nodes. And length of the string, $l = 3\frac{\lambda}{2}$ so, assertion and reason are both true.

5. As speed of wave is constant in stretched wire, and $v = f\lambda$, so with increase in frequency, wavelength decreases. So reason is correct explanation of assertion.

6. In stationary waves, amplitude of nodes is zero and it is possible only when superposing waves has same amplitude. But it is not the only condition, there has to be same frequency, opposite direction of propagation and constant phase difference. So assertion is not completely true.

7. Energy lying between conservative node and antinode is constant where it moves to and fro between node and antinode.

8. $\frac{I_{max}}{I_{min}} = \frac{25}{1} = \left(\frac{5}{1}\right)^2 = \left(\frac{A_1 + A_2}{A_1 - A_2}\right)^2$

$$\Rightarrow \quad 5(A_1 - A_2) = A_1 + A_2$$

$$\Rightarrow \quad 4A_1 = 6A_2 \Rightarrow A_1 : A_2 = 3 : 2.$$

Thus reason is the correct explanation of assertion.

9. $y = A \sin\left(\theta - \frac{\pi}{2}\right) + A \sin\theta$

$$+ A \sin\left(\theta + \frac{\pi}{2}\right)$$

$$= -A\cos\theta + A\sin\theta + A\cos\theta$$

$$= A\sin\theta$$

$$\therefore \quad R = A \Rightarrow I_f = I_i$$

Assertion and reason are both true but reason do no explain assertion.

10. For two coherent sources phase difference has to be constant and that constant be same at all points as $\Delta\phi \neq \Delta\phi(t)$. Different light sources can never be coherent. So phase difference must be same, thus assertion is false.

Match the Columns

1. $v_1 = \sqrt{\frac{T}{\mu}}$ and $v_2 = \sqrt{\frac{T}{9\mu}} = \frac{v_1}{3}$

$$\Rightarrow \quad \frac{v_1}{v_2} = 3$$

$$A_r = \frac{v_1 - v_1/3}{v_1 + v_1/3} A_i = \frac{2/3}{4/3} A_i = \frac{1}{2} A_i$$

and $A_t = \frac{2v_2}{v_1 + v_2} A_i$

$$= \frac{2v_1/3}{v_1 + v_1/3} A_i = \frac{1}{2} A_i$$

(a) $\frac{A_1}{A_2} = \frac{A_r}{A_t} = \frac{1/2\, A_i}{1/2\, A_i} = 1 \rightarrow$ q

(b) $\frac{v_1}{v_2} = 3 \rightarrow$ r

(c) $\frac{I_r}{I_i} = \left(\frac{A_r}{A_i}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$ and

$$\frac{I_t}{I_i} = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{I_r}{I_t} = \frac{I_r/I_i}{I_t/I_i} = \frac{1/4}{3/4} = \frac{1}{3} \rightarrow \text{s}$$

(d) $P = IS = 2\pi^2 \nu^2 A^2 \rho v \rho$

$$= \frac{1}{2}\omega^2 A^2 \mu \sqrt{\frac{T}{\mu}}$$

$$= \frac{1}{2}\omega^2 A^2 \sqrt{T\mu}$$

$$\frac{P_1}{P_2} = \frac{\frac{1}{2}\omega^2 A_1^2 \sqrt{T\mu_1}}{\frac{1}{2}\omega^2 A_2^2 \sqrt{T\mu_2}}$$

$$= \frac{A_1^2}{A_2^2}\sqrt{\frac{\mu_1}{\mu_2}} = \sqrt{\frac{\mu_1}{9\mu_1}}$$

$$= \frac{1}{3} \rightarrow s$$

2. (a) $\frac{\nu_2}{\nu_4} = \frac{\frac{3}{2l}v}{\frac{5}{2l}v} = \frac{3}{5} \rightarrow r$

(b) Number of nodes in 3rd harmonic is 4 and in Fifth harmonic 6,

so, $\frac{4}{6} = \frac{2}{3} \rightarrow p$

(c) Number of antnodes in 3rd harmonic is 3 and in fifth harmonic 5, 50, $\frac{3}{5} \rightarrow r$

(d) $\frac{\lambda_2}{\lambda_4} = \frac{\nu_4}{\nu_2} = \frac{5}{3} \rightarrow s$

3. In danser medium speed of wave is lesser and in rarer medium it is greater.

(a) When wave goes from denser to rarer medium its speed increases $\rightarrow p$

(b) As frequency do not change with change in medium then with increase in speed wavelength increases $\rightarrow p$

(c) As $v_t > v_i$ then $A_t > A_i \rightarrow p$

(d) Frequency remains unchanged $\rightarrow r$

4. $R = \sqrt{A^2 + A^2 + 2 \cdot A \cdot A \cdot \cos\theta}$

$= 2A \cos\frac{\theta}{2}$

(a) $R(60°) = 2A \cos\frac{60°}{2}$

$= 2A \cos 30° = 2A \cdot \frac{\sqrt{3}}{2} = A\sqrt{3} \rightarrow s$

(b) $R(120°) = 2A \cos 120°/2 = 2A \cos 60°$

$= 2A \cdot \frac{1}{2} = A \rightarrow s$

(c) $R(90°) = 2A \cos 90°/2$

$= 2A \cos 45° = A\sqrt{2}$

$\Rightarrow I_R = 2A^2 = 2I_i \rightarrow p$

(d) $R(0°) = 2A \cos(0°/2) = 2A$

$\Rightarrow I_R = 4A^2 = 4I_i \rightarrow r$

5. $\nu_2 = 3\nu_0 = 210\text{ Hz} \Rightarrow \nu_0 = 70\text{ Hz}$

(a) $\nu_0 = 70\text{ Hz} \rightarrow s$

(b) $\nu_2 = 3\nu_0 = 210\text{ Hz} \rightarrow p$

(c) $\nu_3 = 4\nu_0 = 4 \times 70\text{ Hz} = 280\text{ Hz} \rightarrow r$

(d) $\nu_1 = 2\nu_0 = 140\text{ Hz} \rightarrow s$

16. Sound Waves

Introductory Exercise 16.1

1. $P_0 = S_0 kB$

$$\Rightarrow \quad B = \frac{P_0}{S_0 k} = \frac{P_0 \lambda}{2\pi S_0}$$

$$= \frac{14 \times 0.35}{2 \times 3.14 \times 5.5 \times 10^{-6}}$$

$$= 1.4 \times 10^5 \text{ N/m}^2$$

2. $\lambda = \frac{v}{\nu} \Rightarrow \lambda_{max} = \frac{1450 \text{ m/s}}{20 \text{ Hz}} = 72.5 \text{ m},$

$$\lambda_{min} = \frac{1450 \text{ m/s}}{20000 \text{ Hz}} = 7.25 \text{ cm}$$

3. Pressure wave and displacement wave has a phase difference of $\frac{\pi}{2}$, so,

(a) When pressure is maximum, displacement is minimum *i.e.*, zero.

(b) $S_0 = \frac{p_0}{kB} = \frac{p_0}{\frac{2\pi}{\lambda} \cdot \rho v^2}$

$$= \frac{p_0 \lambda}{2\pi \rho v^2} = \frac{p_0 \, v\lambda}{2\pi \nu \rho v^2} = \frac{p_0}{2\pi \nu \rho v}$$

$$= \frac{10}{2 \times 3.14 \times 10^3 \times 1.29 \times 340} \text{ m}$$

$$= 3.63 \times 10^{-6} \text{ m}$$

4. $S_0 = \frac{P_0}{kB} = \frac{P_0}{2\pi \nu \rho v} = \frac{P_0}{\omega \rho v}$

$$= \frac{P_0 k}{\rho \omega^2}$$

$$= \frac{12 \times 8.18}{1.29 \times (2700)^2}$$

$$= 1.04 \times 10^{-5} \text{ m}$$

Introductory Exercise 16.2

1. $\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} = 2 \Rightarrow T_2 = 4T_1 = 4 \times 273 \text{ K}$

$$= 3 \times 273°\text{C} = 819°\text{C}$$

2. $v_t = v_0 \left(1 + \frac{t}{273}\right)^{1/2} = v_0 \left(1 + \frac{t}{546}\right)$

$$v_{30} - v_3 = v_0 \left[1 + \frac{30}{546} - 1 + \frac{3}{546}\right]$$

$$= v_0 \left(\frac{33}{546}\right)$$

$$= 332 \times \frac{33}{546} = 20.06 \text{ m/s}$$

3. $v = \nu\lambda = 250 \times 8 = 2000 \text{ m/s}$

$$B = \rho v^2 = 900 \times (2000)^2$$

$$= 36 \times 10^8 \text{ N/m}$$

$$= 3.6 \times 10^9 \text{ Pa}$$

4. $v = \sqrt{\frac{\gamma R t}{M}} = \sqrt{\frac{\frac{7}{5} \times 8.314 \times 273}{32 \times 10^{-3}}}$

$$= 315 \text{ m/s}$$

Introductory Exercise 16.3

1. $P_0 = S_0 kB = 2\pi\,\nu\rho v\, S_0$

$= 2 \times 3.14 \times 300 \times 1.2 \times 344 \times 6 \times 10^{-6}$

$= 4.67$ Pa

$I = \dfrac{P_0^2}{2\rho v} = \dfrac{(4.67)^2}{2 \times 1.2 \times 344}$

$= 2.64 \times 10^{-2}$ W/m^2

$L = 10 \log \dfrac{I}{I_0} = 10 \log \dfrac{2.64 \times 10^{-2}}{10^{-12}}$

$= 104$ dB

2. $2L - L = 10 \log \dfrac{\eta I}{I_0} - 10 \log \dfrac{I}{I_0}$

$= 10 \log (\eta) = 9$ dB

$\Rightarrow \log \eta = 0.9, \quad \eta = 10^{0.9} = 7.9$

3. $I \propto \dfrac{1}{r^2} \Rightarrow I = \dfrac{k}{r^2}$

$L_F - L_M = 10 \log \dfrac{I_F}{I_M}$

$= 10 \log \left(\dfrac{r_M}{r_F}\right)^2$

$= 20 \log \left(\dfrac{3}{0.3}\right) = 20$ dB

4. (a) $I = \dfrac{P_0^2}{2\rho v}; I_{\max} = \dfrac{(28)^2}{2 \times 1.29 \times 345}$

$= 0.881$ W/m^2

$L_{\max} = 10 \log \dfrac{0.881}{10^{-12}} = 119.45$ dB

$I_{\min} = \dfrac{(2 \times 10^{-5})^2}{2 \times 1.29 \times 345}$

$= 4.49 \times 10^{-13}$ W/m^2

$L_{\min} = 10 \log \dfrac{4.49 \times 10^{-13}}{10^{-12}}$ dB

$= -3.48$ dB

(b) $S_0 = \dfrac{\text{Po}}{kB} = \dfrac{\text{Po}}{2\pi\nu\rho v}$

$(S_0)_{\max} = \dfrac{28}{2 \times 3.14 \times 500 \times 1.29 \times 345}$

$= 2 \times 10^{-5}$ m

$(S_0)_{\min} = \dfrac{2 \times 10^{-5}}{2 \times 3.14 \times 500 \times 1.29 \times 345}$

$= 1.43 \times 10^{11}$ m

Introductory Exercise 16.4

1. $(2n - 1)\dfrac{\lambda}{2} = 12$ cm

and $(2n + 1)\dfrac{\lambda}{2} = 36$ cm

$\Rightarrow \lambda = 36 - 12 = 24$ cm

$\nu = \dfrac{v}{\lambda} = \dfrac{330 \text{ m/s}}{0.24 \text{ m}} = 1375$ Hz

2. $\Delta x = \dfrac{\lambda}{2\pi}\,\Delta\phi = \dfrac{\lambda}{2\pi} \cdot \dfrac{\pi}{3} = \dfrac{\lambda}{6}$

$= \dfrac{v}{6\nu} = \dfrac{350}{6 \times 500} = 0.117$ m $= 11.7$ cm

$\Delta\phi = \dfrac{2\pi}{T}\,\Delta t = 2\pi\,\nu\Delta t = 2\pi \times 500 \times 10^{-3}$

$= \pi$ rad $= 180°$

3. $\Delta x_1 = 2\sqrt{H^2 + \dfrac{d^2}{4}} - d = n\lambda$

and

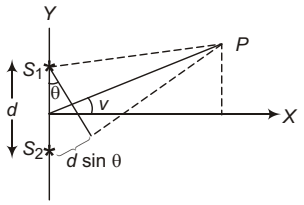
$\Delta x_2 = 2\sqrt{(H + h)^2 + \dfrac{d^2}{4}} - d = \left(n + \dfrac{1}{2}\right)\lambda$

$\Rightarrow \dfrac{\lambda}{2} = 2\sqrt{(H + h)^2 + \dfrac{d^2}{4}} - 2\sqrt{H^2 + \dfrac{d^2}{4}}$

or $\lambda = 4\sqrt{(H + h)^2 + \dfrac{d^2}{4}} - 4\sqrt{H^2 + \dfrac{d^2}{4}}$

$\lambda = 2\sqrt{4(H + h)^2 + d^2} - 2\sqrt{4H^2 + d^2}$

4. $\Delta x_p = d \sin\theta = \left(n + \dfrac{1}{2}\right)\lambda$ for minima



(a) $\therefore\ d \sin\theta = \dfrac{\lambda}{2}$ for first minima

$$\theta = \sin^{-1}\left(\frac{\lambda}{2d}\right) = \sin^{-1}\left(\frac{v}{2\nu d}\right)$$

$$= \sin^{-1}\left(\frac{340}{2 \times 600 \times 2}\right)$$

$$= \sin^{-1}(0.142) = 0.142\,\text{rad}$$

$$= 8.14°$$

(b) For, first maxima $d \sin\theta = \lambda$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{\lambda}{d}\right) = \sin^{-1}\left(\frac{340}{1200}\right)$$

$$= 16.46°$$

(c) $\Delta x_{\max} \le d \Rightarrow n\lambda \le d,\ n \le \dfrac{d}{\lambda}$

$$= \frac{2 \times 600}{340} = 3.53$$

$$\Rightarrow \quad n = 3 \text{ maxima.}$$

5. (a) For coherent speakers in phase,

$$I_R = 4I_0 \cos^2\frac{\theta}{2}$$

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}\cdot\frac{\lambda}{2} = \pi = \theta$$

$$\Rightarrow \quad I_R = 4I_0 \cos\frac{\pi}{2} = 0$$

(b) For incoherent sources,

$$I_R = I_1 + I_2 = I_0 + I_0 = 2I_0$$

(c) For coherent speakers with a phase difference 180°.

$$\Delta\phi' = 180° + \Delta\phi = \pi + \pi = 2\pi$$

$$\Rightarrow \quad I'_R = 4I_0 \cos^2\frac{2\pi}{2} = 4I_0$$

6. $60\,\text{dB} = 10 \log \dfrac{I_0}{10^{-12}}$

$$\Rightarrow \quad 10^6 \times 10^{-12} = I_0$$

$$\Rightarrow \quad I_0 = 10^{-6}\ \text{W/m}^2$$

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi\nu}{v}\Delta x$$

$$= \frac{2\pi \times 170}{340} \times (11 - 8) = 3\pi = \theta$$

(a) $\therefore I_R = 4I_0 \cos^2\dfrac{\theta}{2} = 4I_0 \cos^2\dfrac{3\pi}{2} = 0$

(b) $\Delta\phi' = 3\pi + \pi = 4\pi$

$$\Rightarrow \quad I'_R = 4I_0 \cos\frac{4\pi}{2} = 4I_0$$

$$= 4 \times 10^{-6}\ \text{W/m}^2$$

$$L'_R = 10 \log \frac{4 \times 10^{-6}}{10^{-12}}$$

$$= 10 \log 10^6\ \text{dB} + 10 \log 4$$

$$= 60\,\text{dB} + 2 \log 2\,\text{dB}$$

$$= 60\,\text{dB} + 6\,\text{dB} = 66\,\text{dB}$$

(e) $\Delta\phi'' = \dfrac{2\pi\nu}{v}\cdot\Delta x = \dfrac{2\pi \times 85}{340} \times (11 - 8)$

$$= \frac{3\pi}{2} = \theta$$

$$I_R'' = 4I_0 \cos^2\frac{3\pi}{4}$$

$$= 4I_0 \cos^2\left(\pi - \frac{\pi}{4}\right) = 2I_0$$

$$\Rightarrow L_R'' = 10 \log \frac{2 \times 10^{-6}}{10^{-12}} = 63\ \text{dB}$$

7. (a) $I_1 = \dfrac{10^{-3}}{4\pi \times 2^2} = \dfrac{10^{-3}}{16\pi}$

$$= 19.9 \times 10^{-6}\ \text{W/m}^2$$

$$= 19.9\ \mu\text{W/m}^2$$

$$I_2 = \frac{10^{-3}}{4\pi \times 3^2} = \frac{10^{-3}}{36\pi}$$

$$= 8.84 \times 10^{-6}\ \text{W/m}^2$$

$$= 8.84\ \mu\text{W/m}^2$$

(b) $(I_P)_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2$

$$= (4.46 + 2.97)^2$$

$$= 55.27\ \mu\text{W/m}^2$$

(c) $(I_P)_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2$

$$= (4.46 - 2.97)^2$$

$$= 2.22\ \mu\text{W/m}^2$$

(d) $I_P = I_1 + I_2 = 28.7\ \mu\text{W/m}^2$

Introductory Exercise 16.5

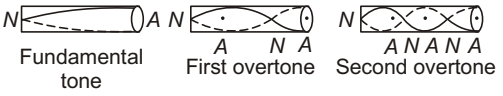
1. (a) $\nu_0 = \dfrac{v}{4l_c} \Rightarrow l_c = \dfrac{v}{4\nu_0} = \dfrac{345\text{ m/s}}{4 \times 220\text{ Hz}}$

$= 0.392\text{ m}$

(b) $\dfrac{3v}{2l_0} = 5\nu_0$

$\Rightarrow \quad l_0 = \dfrac{3v}{10\nu_0} = \dfrac{3 \times 345}{10 \times 220} = 0.470\text{ m}$

2. (a)



$d_A = l = 0.8$ m $\quad d_A = \dfrac{l}{3}, l = \dfrac{0.8}{3}$ m, $\quad d_A = \dfrac{l}{5}, \dfrac{3l}{5}, l$

$= \dfrac{0.8}{5}$ m, $\dfrac{2.4}{4}$ m, 0.8 m

(b)

$d_A = 0_n \quad d_A = 0, \dfrac{2l}{3} \quad d_A = 0, \dfrac{2l}{5}, \dfrac{4l}{5}$

$= 0, 0.533$ m $\quad = 0$ m, 0.32 m, 0.64 m

3.

2	400, 560
0	
4	20, 28

$\Rightarrow$ HCF of the two shows, 80 and the values, 400 Hz and 560 Hz are odd multiples of 80. These conservative harmonics are odd, which can be seen in closed organ pipe only.

(b) These are 5th and 7th harmonic.

(c) $\nu_0 = \dfrac{v}{4l_c}$

$\Rightarrow \quad l_c = \dfrac{v}{4\nu_0} = \dfrac{344}{4 \times 80} = 1.075\text{ m}$

4. $v = \nu\lambda = 1000 \times 2 \times 6.77 \times 10^{-2}$ m/s

$= 135.4$ m/s

$v = \sqrt{\dfrac{\gamma RT}{M}} \Rightarrow r = \dfrac{Mv^2}{RT}$

$= \dfrac{n \times 127 \times 10^{-3} \times (135.4)^2}{8.314 \times 400} = 0.7\ n$

As $1 < r < 2$

$\Rightarrow n = 2 \Rightarrow r = 0.7 \times 2 = 1.4 = \dfrac{7}{5}$ diatonic

5. $\nu = \dfrac{(2n+1)v}{4l_1} = \dfrac{(2n+3)v}{4l_2}$

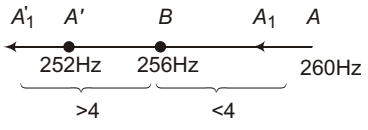
$\dfrac{2n+3}{2n+1} = \dfrac{l_2}{l_1} = \dfrac{100}{60} = \dfrac{5}{3}$

$\Rightarrow \quad n = 1$

$\therefore v = \dfrac{4l_1\ \nu}{2n+1} = \dfrac{4 \times 0.6 \times 440}{3} = 352\text{ m/s}$

Introductory Exercise 16.6

1.



$\Rightarrow \quad \nu_A = 252$ Hz

$\nu_A = (256 \pm 4)$ Hz

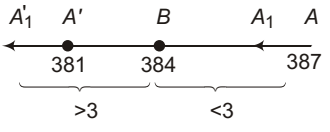
and $\quad \nu_A - n = (256 \pm 6)$ Hz

$\therefore \quad 256 \pm 4 - n = 256 \pm 6$

$\pm 4 \mp 6 = n \Rightarrow n = -4 + 6 = 2$

$\therefore \quad \nu_A = 256 - 4 = 252$ Hz

2.



$\Rightarrow \quad \nu_A = 387$ Hz

$\nu_A = (384 \pm 3)$ Hz

and $\quad \nu_A - n = 384 \pm$ m, m < 3

$\therefore \quad 384 \pm 3 - n = 385 \pm$ m

$\Rightarrow \quad \pm 3 - n = \pm m$

$\Rightarrow \quad \pm 3 \mp m = n = (+)$ ve

$\Rightarrow \quad n = +3 - m$

$\Rightarrow \quad \nu_A = 384 + 3 = 387$ Hz

$$6 \text{ Hz} = 600 \text{ Hz} = \frac{1}{2l}\sqrt{\frac{T_A}{\mu}}$$

and $$600 \text{ Hz} = \frac{1}{2l}\sqrt{\frac{T_B}{\mu}}$$

$$\Rightarrow \quad \frac{606}{600} = \sqrt{\frac{T_A}{T_B}} = 1.01$$

$$\Rightarrow \quad \frac{T_A}{T_B} = 1.02$$

4. $256 \pm 4 = \dfrac{v}{2 \times 0.25}$

and $$256 = \frac{v}{2 \times (0.25 - x)}$$

$$\frac{256}{252} = \frac{2 \times 0.25}{2(0.25 - x)} = \frac{1}{1 - 4x}$$

$$256 - 4 \times 256\, x = 252$$

$$\Rightarrow \quad 4 = 4 \times 256\, x$$

$$x = \frac{1}{256} \text{ m} = \frac{100}{256} \text{ cm} = 0.4 \text{ cm}$$

Introductory Exercise 16.7

1. When source is moving,

$$\nu_s' = \frac{v}{v + v_s}\nu = \frac{1}{1 \mp \frac{v_s}{v}}\nu$$

$$= \left(1 \mp \frac{v_s}{v}\right)^{-1}\nu$$

$$= \left(1 \pm \frac{v_s}{v}\right)\nu = \left(1 \pm \frac{u}{v}\right)\nu$$

When observer is moving, $\nu_0 = \dfrac{v \pm v_0}{v}\nu$

$$= \left(1 \pm \frac{v_0}{v}\right)\nu = \left(1 \pm \frac{v}{v}\right)\nu$$

So, it can be seen that, ν_0 and ν_s are equal if $u << v$.

2. $\lambda = \dfrac{340}{200} = 1.7$ m

(a) $\lambda' = \lambda - uT = 1.7 \text{ m} - \dfrac{80}{200} = 1.7 \text{ m}$

$- 0.4 \text{ m} = 1.3 \text{ m}$

(b) $\nu' = \dfrac{v}{\lambda'} = \dfrac{340 \text{ m/s}}{1.3 \text{ m}}$

$= 262$ Hz

3. For doppler effect there has to be relative motion between source and receiver, but as they are at rest relative to each other that's why there is no shift in wavelength and frequency.

4. $\lambda = \dfrac{v}{\nu} = \dfrac{344}{500} = 0.688$ m

(a) $\lambda_{\text{front}} = \lambda - uT = 0.688 - \dfrac{30}{500}$

$= 0.688 - 0.060 = 0.628$ m

(b) $\lambda_{\text{behind}} = \lambda + uT = 0.688 + 0.060$

$= 0.748$ m

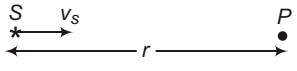
(c) $\nu_{\text{front}} = \dfrac{344}{0.628} = 547.8$ Hz

(d) $\nu_{\text{behind}} = \dfrac{344}{0.748} = 459.9$ Hz

5. $\nu' = \dfrac{v - w - v_0}{v - w + v_s}\nu = \dfrac{340 - 5 - 20}{340 - 5 + 10} \times 300$ Hz

$= \dfrac{315}{345} \times 300 \text{ Hz} = 273.9$ Hz

6.



AIEEE Corner

Subjective Questions (Level 1)

1. $d = d_1 + d_2 = v\,\frac{t_1}{2} + v\,\frac{t_2}{2}$

$= \frac{v}{2}(t_1 + t_2) = \frac{332}{2}\left(\frac{3}{2} + \frac{5}{2}\right)$

$= 332 \times 2 = 664$ m

The time for third eco is,

$t = t_1 + t_2 = \frac{3}{2} + \frac{5}{2} = 4$ s

2. $v = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{\frac{7}{5} \times 8.314 \times 300}{2 \times 10^{-3}}}$

$= \sqrt{21 \times 8.314 \times 10^4} = 1321$ m/s

3. $v = \sqrt{\frac{\gamma p}{\rho}}$

$= \sqrt{\frac{\frac{5}{3} \times 76 \times 10^{-2} \times 13.6 \times 10^3 \times 9.8}{0.179}}$

$= \sqrt{\frac{5 \times 76 \times 136 \times 9.8}{3 \times 0.179}} = 971$ m/s

4. (a) $B = \rho v^2 = \rho \nu^2 \lambda^2$

$= 1300 \times 16 \times 10^4 \times 64$

$= 1.33 \times 10^{10}$ N/m^2

(b) $Y = \rho v^2 = \frac{\rho l^2}{t^2} = \frac{6400 \times (1.5)^2}{(3.9 \times 10^{-4})^2}$

$= 9.47 \times 10^{10}$ Pa

5. $v_t = \sqrt{\frac{\Delta l}{l}} v_l \Rightarrow \frac{\Delta l}{l} = \left(\frac{v_t}{v_l}\right)^2 ; \frac{F}{A} = Y\,\frac{\Delta l}{l}$

$= Y\left(\frac{v_t}{t_l}\right)^2 = Y\left(\frac{1}{30}\right)^2 = \frac{Y}{900}$

6. $M_{\text{mix}} = \frac{2 \times 2 + 1 \times 14}{2 + 1} = 6$ m/mole

$\frac{v_{\text{mix}}}{v_{H_2}} = \sqrt{\frac{M_{H_2}}{M_{\text{mix}}}} = \sqrt{\frac{2}{6}} = \frac{1}{\sqrt{3}}$

$v_{\text{mix}} = \frac{1}{\sqrt{3}} v_{H_2} = \frac{1}{\sqrt{3}} v_0 \sqrt{\frac{T_2}{T_1}} = \frac{v_0}{\sqrt{3}} \sqrt{\frac{300}{273}}$

$= \frac{v_0}{\sqrt{2.73}} = \frac{1300}{\sqrt{2.73}} = 787$ m/s

7. $L_1 = 10 \log \frac{10^{-6}}{10^{-12}} = 60 \log 10 = 60$ dB

$L_2 = 10 \log \frac{10^{-9}}{10^{-12}} = 30 \log 10 = 30$ dB

$\Rightarrow L_1 = 2L_2$

8. $100 \text{ dB} = 10 \log \frac{I}{I_0}$ dB

$\Rightarrow \quad I = 10^{10} I_0 = 10^{-2}$ W/m^2

$P = 4\pi r^2 I = 4\pi \times (40)^2 \times 10^{-2}$

$= 64\,\pi$ W $= 201$ W

9. (a) $60 \text{ dB} = 10 \log \frac{I}{I_0}$ dB

$\Rightarrow \quad I = 10^6 I_0 = 10^{-6}$ W/m^2

(b) $P = AI = 120 \times 10^{-4} \times 10^{-6}$ W

$= 1.2 \times 10^{-8}$ watt

10. (a) $\Delta L = 13 \text{ dB} = 10 \log \frac{I_2}{I_1}$ dB

$\Rightarrow \quad I_2 = 10^{1.3} I_1 = 20 I_1$

(b) As with doubling the intensity, loudness increases by 3 dB irrespective of the initial intensity.

11. $I = \frac{P}{4\pi r^2} = \frac{5}{4\pi(20)^2} = \frac{5}{4\pi \times 400}$

$= \frac{1}{320\pi}$ W/m^2 $= 9.95 \times 10^{-4}$ W/m^2

(b) $I = 2\pi^2 \nu^2 a^2 \rho v \Rightarrow a = \frac{1}{\pi \nu}\sqrt{\frac{I}{2\rho v}}$

$= \frac{1}{300\,\pi}\sqrt{\frac{1}{320\,\pi \times 2 \times 1.29 \times 330}}$

$= \frac{1}{300\,\pi \times 10^{12}}\sqrt{\frac{1}{85.5}}$

$= 1.15 \times 10^{-6}$ m

12. $60\text{ dB} = 10 \log \dfrac{I}{10^{-12}}\text{ dB}$

$\Rightarrow I = 10^{-6}\ \text{W/m}^2$ and $a = \dfrac{1}{\pi\nu}\sqrt{\dfrac{T}{2\rho v}}$

$= \dfrac{1}{800\,\pi}\sqrt{\dfrac{10^{-6}}{2 \times 1.29 \times 330}} = 13.6 \times 10^{-9}\ \text{m}$

13. $102\text{ dB} = 10 \log \dfrac{I}{I_0}\text{ dB}$

$\Rightarrow \quad I = 10^{10.2}\ I_0 = 10^{10.2-12}$

$= 10^{-1.8}\ \text{W/m}^2$

$P = 4\pi r^2\ I$

$= 4 \times 3.14 \times (20)^2 \times 10^{-1.8}$

$= 80\text{ W}$

14. $I = 2\pi^2 v^2 a^2\ \rho v$

$= 2 \times (3.14)^2 \times (300)^2$

$\times (0.2 \times 10^{-3})^2 \times 1.29 \times 330\ \text{W/m}^2$

$= 30.25\ \text{W/m}^2$

$L = 10 \log \dfrac{I}{I_0}\text{ dB} = 10 \log \dfrac{30.25}{10^{-12}}\text{ dB}$

$= 134.8\text{ dB}$

15. (a) $v_w = \sqrt{\dfrac{B}{\rho}} = \sqrt{\dfrac{2.18 \times 10^9}{10^3}}$

$= 1.48 \times 10^3\ \text{m/s}$

$A_w = \sqrt{\dfrac{I}{2\pi^2\ \nu^2\ \rho v}} = \dfrac{1}{\pi\nu}\sqrt{\dfrac{I}{2\rho v}}$

$= \dfrac{1}{3400\,\pi}\sqrt{\dfrac{3 \times 10^{-6}}{2 \times 10^3 \times 1.48 \times 10^3}}$

$= 9.44 \times 10^{-11}\ \text{m}$

$\lambda_w = \dfrac{v_N}{\nu} = \dfrac{1.48 \times 10^3}{3400} = 0.43\text{ m}$

(b) $v_a = \sqrt{\dfrac{\gamma p}{\rho}} = \sqrt{\dfrac{1.4 \times 10^5}{1.2}} = 341.6\text{ m/s}$

$A_a = \dfrac{1}{3400\,\pi}\sqrt{\dfrac{3 \times 10^{-6}}{2 \times 1.2 \times 341.6}}$

$= 5.66 \times 10^{-9}\ \text{m}$

$\lambda_a = \dfrac{341.6}{3400} = 0.1\text{ m}$

(c) $A_a > A_w;\ \dfrac{A_a}{A_w} = \dfrac{5.66 \times 10^{-9}}{9.44 \times 10^{-11}} = 60$

As bulk modulus of water is much larger than air, such that displacement of particles of medium becomes less.

16. $I = \dfrac{p_0^2}{2\rho v} = \dfrac{(6 \times 10^{-5})^2}{2 \times 1.29 \times 343}\ \text{W/m}^2$

$= 4 \times 10^{-12}\ \text{W}$

$\therefore\ L = 10 \log \dfrac{I}{I_0} = 10 \log \dfrac{4 \times 10^{-12}}{10^{-12}}$

$= 20 \log 2 = 6\text{ dB}$

17. $\nu_o = \dfrac{v}{2l} = 594\text{ Hz};$

$\nu_c = \dfrac{v}{4l} = \dfrac{\nu_0}{2} = \dfrac{594}{2}\text{ Hz} = 297\text{ Hz}$

18. $\nu_0 = \dfrac{(n+1)v}{2l} = (n+1)\dfrac{344}{2 \times 0.45}$

$= (n+1) \times 382.2\text{ Hz}$

$= 382.2\text{ Hz}, 764.4\text{ Hz}, 1146.7\text{ Hz},$

$\nu_t = \dfrac{(2n+1)v}{2l}$

$= (2n+1)\dfrac{344}{2 \times 0.45}$

$(2n+1) \times 191.1\text{ Hz}$

$= 191.1\text{ Hz}, 573.3\text{ Hz}, 955.6\text{ Hz}$

19. $\nu_c = \dfrac{v}{4l}$

$\Rightarrow\ v = 4l\ \nu_c = 4 \times 0.15 \times 500 = 300\text{ m/s}$

$\nu_o = \dfrac{v}{2l_o} = \dfrac{300}{2 \times 0.6} = 250\text{ Hz}$

20. $y = A \cos kx \cos \omega t$

$= A \cos \dfrac{2\pi}{1.6}x \cos 2\pi \dfrac{330}{1.6}t$

$= A \cos 3.93x \cos 1296\,t$

21. $\nu = \dfrac{2n+1}{4 \times 0.5}v = \dfrac{2n+3}{4 \times 0.84}v$

$\Rightarrow \quad \dfrac{2n+3}{2n+1} = \dfrac{84}{50} = 1.68$

$\Rightarrow \quad 3 - 1.68 = 2n \times 0.68$

$\therefore \quad n = 0.97 = 1$ as n is an integer

$$v = \frac{4lv}{2n+1} = \frac{4 \times 0.5 \times 512}{3} \text{ m/s}$$

$$= 341.3 \text{ m/s}$$

$$\nu = \frac{2n+5}{4l}\nu$$

$$\Rightarrow \quad l = \frac{2n+5}{4\nu}v = \frac{7}{4 \times 512} \times 341.3$$

$$= 1.167 \text{ m} = 116.7 \text{ cm}$$

22. $\nu_c = \dfrac{v}{4l} = \dfrac{340}{4 \times 1} = 85 \text{ Hz}$

$$\nu_s = \frac{v}{\lambda} = \frac{1}{0.4}\sqrt{\frac{F}{\mu}} = 85$$

$$\Rightarrow F' = (85 \times 0.4)^2 \ \mu = (34)^2 \times \frac{4 \times 10^{-3}}{0.4}$$

$$= 11.65 \text{ N}$$

23. $\nu_c = \dfrac{v}{4(l+e)} \Rightarrow v = 4\nu\,(l + l)$

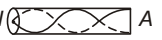
$$= 4\nu(l + 0.3d)$$

$$= 4 \times 480\,(0.16 + 0.3 \times 0.05) = 336 \text{ m/s}$$

24. (a) $\nu_e = \dfrac{(2n+1)\,\nu}{4l}$

$$\Rightarrow \quad 440 = \frac{5 \times 330}{4l}$$

$$\Rightarrow \quad l = \frac{5 \times 330}{4 \times 440} = \frac{15}{16} \text{ m}$$

(b) 

$$\frac{5\lambda}{4} = l = \frac{15}{16}$$

$$\Rightarrow \quad \lambda = \frac{15}{16} \times \frac{4}{5} = \frac{3}{4} \text{ m}$$

$$\Delta p = \Delta p_0 \cos kx = \Delta p_0 \cos \frac{2\pi}{3/4} \times \frac{15}{32}$$

$$= \Delta p_0 \cos \frac{15\pi}{12} = \Delta p_0 \cos \frac{5\pi}{4} = \frac{\Delta p_0}{\sqrt{2}}$$

(c) At open end there is pressure node, so, $p_{\max} = \Delta p_{\min} = \Delta p_0$

(d) At closed end there is pressure antinode, such that, $p_{\max} = p_0 + \Delta p_0$ and $p_{\min} = p_0 - \Delta p_0$

25. (a) $\nu_c = \dfrac{v}{4l_c}$

$$\Rightarrow l_c = \frac{v}{4\nu_c} = \frac{345}{4 \times 220} = 0.392 \text{ m}$$

(b) $l_0 = \dfrac{5\lambda}{4}$, $l_0 = \dfrac{3\lambda}{2} = \dfrac{3}{2} \cdot \dfrac{4}{5}\, l_c$

$$= \frac{6}{5}\, l_c = \frac{6}{5} \times 0.392 \text{ m} = 0.47 \text{ m}$$

26. $\nu_s = \nu_c \Rightarrow \dfrac{v_s}{2 \times 0.8\, l_c} = \dfrac{v_s}{4\, l_c}$

$$\Rightarrow \quad \frac{v_s}{v_a} = \frac{1.6}{4} = 0.4$$

27. (a) $\lambda_s = \dfrac{v}{\nu} = \dfrac{340}{300} = \dfrac{17}{15} \text{ m} = 1.13 \text{ m}$

(b) $\lambda_a = \lambda - v_s T \Rightarrow \dfrac{v - v_s}{\nu}$

$$= \frac{340 - 30}{300} = \frac{31}{30} = 1.03 \text{ m}$$

$$\lambda_b = \lambda + v_s T = \frac{v + v_s}{\nu}$$

$$= \frac{340 + 30}{300} = \frac{37}{30} = \frac{37}{30} = 1.23 \text{ m}$$

28. $\nu = \dfrac{1}{2l}\sqrt{\dfrac{F}{\mu}} \Rightarrow \dfrac{\Delta\nu}{\nu} = \dfrac{1}{2}\,\dfrac{\Delta F}{F}$

$$\Rightarrow \frac{\Delta F}{F} = 2\,\frac{\Delta\nu}{\nu} = 2 \times \frac{1.5}{440} = \frac{3}{440} = \pm\, 0.68\%$$

$$\nu' = \nu + \Delta\nu = 440 \pm 1.5$$

$$= 438.5 \text{ Hz or } 441.5 \text{ Hz}$$

29. $v = 0.32$ m/s;

$$\lambda = vT = 0.32 \times 1.6 \text{ m} = 0.512 \text{ m.}$$

$$\lambda'_a = \lambda - v_s T \Rightarrow v_s = \frac{\lambda - \lambda'}{T} = v - \frac{\lambda'}{T}$$

$$= 0.32 - \frac{0.12}{1.6} = 0.245 \text{ m/s}$$

$$\lambda_b'' = \lambda + v_s T = 0.512 \text{ m} + 0.245 \times 1.6$$

$$= 0.512 + 0.392 = 0.904 \text{ m.}$$

30. (a) $\nu_a = \dfrac{v - v_0}{v - v_s}\nu = \dfrac{340 + 18}{240 - 30} \times 262 \text{ Hz}$

$$= \frac{358}{310} \times 262 \text{ Hz} = 302.5 \text{ Hz}$$

(b) $\nu_r = \dfrac{v - v_0}{v + v_s}\nu = \dfrac{340 - 18}{340 + 30} \times 262$

$$= \frac{322}{370} \times 262 \text{ Hz} = 228 \text{ Hz}$$

31. $\Delta\nu = \frac{v}{v - v_s}\nu - \frac{v}{v + v_s}\nu$

$$= \frac{2vv_s\nu}{v^2 - v_s^2} \simeq \frac{2v_s\nu}{v} \Rightarrow v = \frac{v\Delta\nu}{2v_s}$$

$$\therefore \quad \nu = \frac{340 \times 4}{2 \times 1} = 680 \text{ Hz}$$

32. $\nu_0 = \nu_c \pm \Delta\nu = 110 \pm 2.2; \nu_c = \frac{v}{4l_c}$

$$\Rightarrow \quad l_c = \frac{v}{4\nu_c} = \frac{330}{4 \times 110}$$

$$\therefore \quad l_c = \frac{3}{4}\text{m}; \nu_0 = \frac{2v}{2l_0}$$

$$\Rightarrow \quad l_0 = \frac{2v}{2\nu_0} = \frac{2 \times 330}{2(330 \pm 2.2)}$$

$$= 0.993 \text{ m or } 1.007 \text{ m}$$

33. $\nu_p = \nu_Q \pm \frac{7}{2}$ and $\nu_P < \nu_Q$ as beat frequency increases waxing of P.

$$\nu_Q + 5 = \frac{v}{v - v_s}\nu_Q = \frac{332}{332 - 5}\nu_Q$$

$$= \frac{332}{327}\nu_Q \Rightarrow 5 = \frac{5}{327}\nu_Q$$

$$\Rightarrow \quad \nu_Q = 327 \text{ Hz and}$$

$$\nu_P = 327 - \frac{7}{2} = 323.5 \text{ Hz}$$

When Q gives 5 beats with its own echo.

OR

$$\nu_P = \nu_Q - \frac{7}{2} = \nu'_q - 5 = \frac{332}{327}\nu_Q - 5$$

$$\Rightarrow \quad 5 - \frac{7}{2} = \frac{5}{327}\nu_Q$$

$$\Rightarrow \quad \nu_Q = \frac{327 \times 1.5}{5} = 98.1 \text{ Hz}$$

$$\Rightarrow \quad \nu_P = 98.1 - 2.5 = 94.6 \text{ Hz}$$

When P gives 5 beats with the echo of Q.

34. $\Delta\nu = \frac{v}{v - v_s}\nu - \frac{v}{v + v_s}\nu \Rightarrow \frac{2vv_s\nu}{v^2 - v_s^2} \simeq \frac{2v_s\nu}{v}$

$$\Rightarrow v_s = \frac{v\nu}{2\nu} = \frac{340 \times 2}{2 \times 680}$$

$$\therefore \quad v_s = \frac{1}{2}\text{m/s}$$

35. $(2n + 1)\frac{\lambda}{2} = 11.5 \text{ cm}$

$$(2n + 3)\frac{\lambda}{2} = 34.5 \text{ cm}$$

$$\Rightarrow \frac{2n + 3}{2n + 1} = \frac{34.5}{11.5} = 3 \Rightarrow 4n = 0 \Rightarrow n = 0$$

$$\therefore \quad \frac{\lambda}{2} = 11.5 \text{ cm} \Rightarrow \lambda = 23 \text{ cm}$$

$$\nu = \frac{v}{\lambda} = \frac{331.2 \text{ m/s}}{0.23 \text{ m}} = 1440 \text{ Hz}$$

36. $\lambda = \frac{v}{\nu} = \frac{330}{220} = 1.5 \text{ m}$

$$\Delta x = S_2P - S_1P = 3 - \frac{3}{4} = \frac{9}{4}\text{m}$$

$$= \frac{3}{2} \cdot \frac{3}{2} = \frac{3}{2}\lambda = (2n + 1)\frac{\lambda}{2}$$

Here, $S_1P = \frac{3}{4} = \frac{1}{2}\lambda$

$$\Rightarrow \quad \phi_1 = \frac{2\pi}{\lambda}S_1P = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} = \pi$$

and $\quad S_2P = 3 = 2 \cdot \frac{3}{2} = 2\lambda$

$$\Rightarrow \quad \phi_2 = \frac{2\pi}{\lambda} \cdot 2\lambda = 4\pi$$

Destructive interference will take place at P.

$$\therefore \quad P_P = P_{\min} = (\sqrt{P_1} - \sqrt{P_2})^2$$

$$= (\sqrt{1.8 \times 10^{-3}} - \sqrt{1.2 \times 10^{-3}})^2$$

$$= 0.6 \times 10^{-3}(\sqrt{3} - \sqrt{2})^2$$

$$= 0.6 \times 10^{-3} \times 0.1 = 6 \times 10^{-5} \text{ W}$$

37. $\Delta x = 2\sqrt{2^2 + \left(\frac{x}{2}\right)^2} - x = n\lambda = 1 \times \lambda$

$$= \frac{360 \text{ m/s}}{360 \text{ Hz}} = 1 \text{ m}$$

$$2\sqrt{4 + \frac{x^2}{4}} = 1 + x$$

or $\quad 4\left(4 + \frac{x^2}{4}\right) = 1 + 2x + x^2$

$$16 - 1 = 2x$$

$$x = 7.5 \text{ m}$$

Objective Questions (Level 1)

1. Sound cannot travel in vacuum, as it is mechanical wave.

2. Longitudinal waves can travel through all mechanical mediums.

3. $\sqrt{\dfrac{\gamma RT}{32}} = \sqrt{\dfrac{\gamma R \times 288}{28}}$

$T = \dfrac{32}{28} \times 288\text{ K} = \dfrac{8}{7} \times 288\text{ K} = 56°\text{C}$

4. Third overtone is 7th harmonic *ie,* there 4 nodes and 4 antinodes.

5. $\nu \propto \dfrac{v}{l} \Rightarrow \nu \propto \sqrt{T}$ so with increase in temperature, frequency increases.

6. For sound water is rarer medium and air is densor medium so, it bends towards normal while going from water to air.

7. $\nu_c = \dfrac{v}{4l_c} = \nu_o = \dfrac{v}{2l_o} \Rightarrow \dfrac{l_c}{l_o} = \dfrac{2}{4} = 1:2$

8. $\dfrac{\nu_2}{\nu_1} = \sqrt{\dfrac{F_2}{F_1}}$

$\Rightarrow F_2 = \left(\dfrac{\nu_2}{\nu_1}\right)^2 F_1$

$\Rightarrow M_2 = \left(\dfrac{\nu_2}{\nu_1}\right)^2 M_1 = \left(\dfrac{256}{320}\right)^2 \times 10\text{ kg}$

$= 6.4\text{ kg}$

$\therefore\ OM = M_2 - M_1 = 6.4 - 10 = -3.6\text{ kg}$

i.e., Mass has to be decreased by 3.6 kg

9. $\nu_{\text{direct}} = \dfrac{v}{v - v_s}\nu$ and $\nu_{\text{reflected}} = \dfrac{v}{u - v_s}\nu$

as $\nu_D = \nu_R$ so there will be no beats *i.e.,* beat frequency will be zero.

10. $\Delta\nu = \nu_2 - \nu_1 = \dfrac{v}{\lambda_2} - \dfrac{v}{\lambda_1} = v\dfrac{(\lambda_1 - \lambda_2)}{\lambda_1\lambda_2}$

$\Rightarrow \quad v = \dfrac{\lambda_1\lambda_2\Delta\nu}{\Delta\lambda}$

$\therefore \quad v = \dfrac{1 \times 1.01 \times \dfrac{10}{3}}{0.01} = 337\text{ m/s}$

11. $\nu' = \dfrac{v}{v + v}\nu = \dfrac{1}{2}\nu = n\nu \Rightarrow n = \dfrac{1}{2} = 0.5$

12. $I_{\max} = (\sqrt{I} + \sqrt{I})^2 = 4I = NI \Rightarrow N = 4$

13. $\nu = \dfrac{v}{4(l_1 + e)} = \dfrac{3v}{4(l_2 + e)}$

$\Rightarrow \quad l_2 + e = 3l_1 + 3e$

$\Rightarrow \quad e = \dfrac{l_2 - 3l_1}{2}$

$\therefore \quad e = \dfrac{42 - 3 \times 17}{2}\text{ cm} = 0.5\text{ cm}$

$v = 4\nu(l_1 + e) = 4 \times 500(17 + 8.5) \times 10^{-2}$

$= 20 \times 17.5 = 350\text{ m/s}$

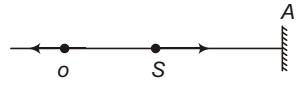
14. At the moment when velocity of source is perpendicular to the line joining source and observer then there is no Doppler effect *i.e.,* $n + n_1 = n \Rightarrow n_1 = 0$

15. $\nu = \dfrac{(n+1)v}{4l} = (2n+1)\dfrac{340}{4 \times 1} = 85(2n+1)$

$= 85, 255, 425, 595, 765, 935$

$\therefore$ 6 frequencies below 1 kHz.

16. $\Delta\nu = \dfrac{v - v_0}{v - v_s}\nu - \dfrac{v - v_0}{v + v_s}\nu = \nu\left(1 - \dfrac{v + v_0}{v + v_s}\right)$

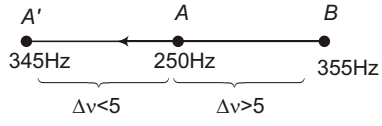


$= \nu \cdot \dfrac{v_s + v_0}{v + v_s} = \dfrac{10}{360} \times 180 = 5\text{ Hz}$

17. $\Delta\nu = n = \nu_1 - \nu_2 = \dfrac{v}{\lambda_1} - \dfrac{v}{\lambda_2}$

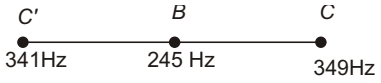
$= \dfrac{v(\lambda_2 - \lambda_1)}{\lambda_1\lambda_2} \Rightarrow v = \dfrac{n\lambda_1\lambda_2}{\lambda_1 - \lambda_2}$

18.



As beat frequency between A and B decreases on loading A.

i.e., $\nu_B < \nu_A \Rightarrow \nu_B = 345$ Hz



After loading A, $\nu_A' = 345 + 2 = 247$ Hz

and $\nu_A' - \nu_c = \pm 6 \Rightarrow \nu_c = \lambda_A' \mp 6$

$= 347 \mp 6$

$= 341$ or 353 Hz.

As possible frequency of C are 341 Hz and 249 Hz then only 341 Hz is justified.

19. $e = \dfrac{l_2 - 3l_1}{2} = \dfrac{122 - 3 \times 40}{2}$ cm $= 1$ cm

So, $\dfrac{v}{l(l_1 + e)} = \dfrac{5v}{4(l_1 + e)}$

$\Rightarrow \quad l_3 = 5l_1 + 4l$

$= 5 \times 40 + 4 \times 1 = 204$ cm

20. $\dfrac{\Delta\nu}{\nu} = \dfrac{1}{2}\dfrac{\Delta F}{F} \Rightarrow \Delta\nu = \dfrac{1}{2}\nu\dfrac{\Delta F}{F}$

$= \dfrac{1}{2} \times 200 \times \dfrac{1}{100} = 1$ Hz

21. $\nu = \dfrac{2n+1}{4l}v \quad \Rightarrow \quad l = \dfrac{2n+1}{4\nu}v$

$= (2n+1)\dfrac{340}{4 \times 340} = \dfrac{2n+1}{4}$ m

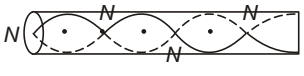
$= \dfrac{1}{4}$ m, $\dfrac{3}{4}$ m, $\dfrac{5}{4}$ m.

As, $l_{max} = 120$ cm $\Rightarrow l = 25$ cm 75 cm.

$\therefore$ Height of water column

$= 120$ cm $- 75$ cm $= 45$ cm

22. $7\dfrac{\lambda}{4} = 105$ cm $\Rightarrow \lambda = \dfrac{105 \times 4}{7} = 60$ cm



$\Rightarrow \quad \dfrac{\lambda}{4} = \dfrac{60}{4} = 15$ cm

So, nodes are at, $\dfrac{\lambda}{4}, 3\dfrac{\lambda}{4}, 5\dfrac{\lambda}{4}$ and $7\dfrac{\lambda}{4}$ from closed end *i.e.*, they are at, 15 cm, 45 cm, 75 cm and 105 cm.

23. $\nu_c = \dfrac{v}{4l} = 512$ Hz, $\nu_o = \dfrac{v}{2l} = 2\dfrac{v}{4l}$

$= 2\nu_c = 2 \times 512$ Hz $= 1024$ Hz

24. $M_{min} = \dfrac{1 \times 32 + 1 \times 2}{1 + 1} = 17$

$\dfrac{v_{min}}{v_{H_2}} = \sqrt{\dfrac{M_{H_2}}{M_{min}}} = \sqrt{\dfrac{2}{17}}$

25. $\nu_a > f$ and $\nu_r < f$ but $\nu_a =$ constant and $\nu_r =$ cosntant.

So, curve in (b) represents correctly.

26. $u = \dfrac{(2n+1)v}{4l} - \dfrac{(m+1)v}{2l}$

How, $\dfrac{(2n+1)v}{4 \times 2l} - \dfrac{(m+1)v}{4 \times 2l}$

$= \dfrac{4}{2} = 2$ beat/s

27. $\Delta\nu = \nu_a - \nu_r = \dfrac{v}{v - v_1}\nu - \dfrac{v}{v + v_1}\nu$

$= \dfrac{2vv_1\nu}{(v - v_1)(v + v_1)}$

$= \dfrac{2 \times 320 \times 4 \times 243}{316 \times 324} = 6$ Hz

28. $\nu_c = \dfrac{(2n+1)v}{4l_c} = \dfrac{320}{4 \times 1}(2n+1)$

$= (2n+1) \times 80$ Hz $= 80$ Hz, 240 Hz, 400 Hz,...

$\nu_0 = \dfrac{(n+1)v}{2l_0} = \dfrac{320}{2 \times 1.6}(n+1)$

$= (n+1) \times 100$ Hz $= 100$ Hz, 200 Hz, 300 Hz, 400 Hz,...

$\therefore \quad \nu_c = \nu_o = 400$ Hz

29. $I_{max} = 4I_0$

and $\quad I'_{max} = 4I_{max} = 16I_0$

$L' = 10$dB $+ 10\log 16$

$= 10$ dB $+ 40\log 2$ dB $= 22$ dB

30. $\lambda = \dfrac{2\pi}{k} = \dfrac{2\pi}{\pi/2} = 4$ m

$l = 5\dfrac{\lambda}{4} = 5 \times \dfrac{4}{4}$ m $= 5$ m

31. $d = (2n+1)\frac{\lambda}{4} = \frac{(2n+1)}{4}\cdot\frac{330}{660}\text{m}$

$= \frac{330}{24}(2n+1)\text{ cm} = (2n+1)\times 13.75\text{ cm}.$

$= 13.75\text{ cm}, 41.25\text{ cm}, 68.75\text{ cm}, 96.25\text{ cm}$ etc.

32. $\Delta\nu = \frac{v}{\lambda_1} - \frac{v}{\lambda_2} = \frac{v(\lambda_2 - \lambda_1)}{\lambda_1\lambda_2}$

$= \frac{332\times 1\times 10^{-2}}{0.49\times 0.5} = 13.15\text{ Hz}$

33. $\Delta\nu_B = \nu_A - \nu_B = \frac{300}{300-30}\times 300 - 300$

$= 33.33\text{ Hz and } \nu'_A \neq \nu'_B$

So both (a) and (b) options are wrong.

34. $f_a = \frac{v+v_0}{v}f = \left(1+\frac{v_0}{v}\right)f$ and

$f_r = \left(1-\frac{v_0}{v}\right)f$

$$\frac{f_a}{f_r} = \frac{v+v_0}{v-v_0}$$

$$\Rightarrow \quad (f_a - f_r)v = (f_a + f_r)v_0$$

$$\Rightarrow \quad \frac{v}{v_0} = \frac{f_a + f_r}{f_a - f_r}.$$

and

$$f_a - f_r = \frac{2v_0}{v}f = 2\left(\frac{f_a - f_r}{f_a + f_r}\right)f$$

$$\Rightarrow \quad f = \frac{f_a + f_r}{2}.$$

JEE Corner

Assertion and Reason

1. $\nu_c = (2n+1)\frac{v}{4l} = \frac{v}{4l}, \frac{3v}{4l}, \frac{5v}{4l}, \ldots$

while, $\nu_0 = \frac{(n+1)v}{2l} = \frac{v}{2l}, \frac{2v}{2l}, \frac{3v}{2l}, \frac{4v}{2l}, \ldots$

it can be seen that $\nu_c \neq \nu_o$ at all situation and $\nu_c = \frac{1}{2}\nu_o$ so assertion is true but reason is false.

2. Apparent frequency is constant for constant relative velocity so assertion is false.

3. At a point of minimum displacement pressure amplitude is maximum *i.e.*, pressure difference is maximum not pressure. So assertion is false.

4. The deriver receiver two sounds one direct, $\nu_0 = \nu$ and other $\nu_R = \frac{v+u}{v-u}\nu$ such that be detects beats. So reason is true explanation of assertion.

5. With increase in intensity sound level increases in lograthmic order so assertion is false.

6. Speed of sound $v = \sqrt{\frac{\gamma p}{\rho}}$, with increase in only pressure density increases such that p/ρ remains constant. Again $v = \sqrt{\frac{\gamma RT}{M}}$ so both assertion and reason are true but reason is not correct explanation of assertion.

7. $\nu_A = \nu_B + 4$ when A is loaded with little wax then ν_A sightly decreases and then beat frequency decreases, but if it is heavily loaded with wax then its frequency goes much below ν_B such that beat frequency increases. So, assertion and reason are both true but reason is not correct explanation of assertion.

8. $150 \,|\, 450, 750,$
$\quad\;\; 3, \quad 5$

The frequencies are odd harmonics then the pipe is closed and fundamental frequency is also 150 Hz. So assertion and reason are both true but reason is not correct explanation of assertion.

9. $\nu \propto \dfrac{1}{l+e}$ with increase in diameter end correction, e, increases and ν decreases. So reason is correct explanation of assertion.

10. With increasing length of air column, number of overtone increases and not the wavelength so assertion is false.

■ Objective Questions (Level 2)

1. At the boundary between two mediums, one part of incident wave gets reflected and other part gets transmitted or refracted.

2. $\dfrac{3\lambda}{2} = 3.9\,\pi \Rightarrow \lambda = \dfrac{3.9\,\pi}{1.5}$

$$\Rightarrow k = \frac{2\pi}{\lambda} = \frac{2\pi}{\frac{3.9\,\pi}{1.5}} = \frac{3}{3.9}\rho_0 = S_0\, kB$$

$$\Rightarrow \quad S_0 = \frac{\rho_0}{kB} = \frac{10^{-2} \times 10^5}{\frac{3}{3.9} \times 1.3 \times (200)^2}$$

$$= \frac{3.9 \times 10^{-1}}{12 \times 1.3} = 0.025 \text{ m} = 2.5 \text{ cm}$$

3. $\dfrac{A_t}{A_i} = \dfrac{2v_2}{v_1 + v_2} = \dfrac{2 \times 100}{200 + 100} = \dfrac{2}{3}$

4. $\Delta\nu = \dfrac{v}{v + v_2}\nu - \dfrac{v}{v + v_1}\nu$

$$= \frac{v\nu(v_1 - v_2)}{(v + v_2)(v + v_1)} \simeq \frac{\nu(v_1 - v_2)}{v}$$

$$\therefore\; v_1 - v_2 = \frac{v\Delta\nu}{\nu} = \frac{340 \times 10}{1700} = 2 \text{ m/s}$$

5. $v_s = gt = 10$ m/s

$$\Delta\nu = \frac{v + v_0}{v - v_s}\nu - \frac{v - v_0}{v + v_s}\nu$$

$$= \left(\frac{300 + 2}{300 - 10} - \frac{300 - 2}{300 + 10}\right) \times 150 \text{ Hz}$$

$$= \left(\frac{302}{290} - \frac{298}{310}\right) \times 150 = 12 \text{ Hz}$$

6. $7\dfrac{\lambda}{2} = L \Rightarrow \lambda = \dfrac{2L}{7}$

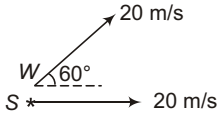
$$A = a\cos kx = a\cos\frac{2\pi}{2\frac{L}{7}} \cdot \frac{L}{7} = a\cos\pi = -a$$

7. For maxima, $n\lambda = 3$

$$\Rightarrow \lambda = \frac{3}{n};\; \nu = \frac{v}{\lambda} = \frac{nv}{3} = 110\, n.$$

$\therefore \nu = 110, 220, 330$ Hz, ..etc. maxima will be formed so maximum will not be formed at 120 Hz and 100 Hz.

8.



$$\nu' = \frac{v + \omega\cos 60°}{v + w\cos 60° - v_s}$$

$$= \frac{300 + 10}{300 + 10 - 20} \times 500 \text{ Hz}$$

$$= \frac{310}{290} \times 500 = 534 \text{ Hz}$$

9. $\Delta\nu = \nu_R - \nu_0 = \left[\dfrac{v + 20}{v - 10} - \dfrac{v + 20}{v + 10}\right] \times 500$

$$= \left(\frac{360}{300} - \frac{360}{350}\right) \times 500 \text{ Hz} = 31 \text{ Hz}$$

10. $\Delta\nu = \dfrac{404\pi}{2\pi} - \dfrac{400\pi}{2\pi} = 202 - 200 = 2$ Hz

$$\frac{I_{max}}{I_{min}} = \left(\frac{2 + 1}{2 - 1}\right)^2 = 9 : 1$$

11. $3\frac{\lambda}{4} = 34 \text{ cm} \Rightarrow \lambda = \frac{4}{3} \times 34 \text{ cm}$

$\Rightarrow \nu = \frac{v}{\lambda} \Rightarrow v_{51} = \nu\lambda = \frac{136}{3}\nu$

$\frac{v_{16}}{v_{51}} = \sqrt{\frac{273+16}{273+51}} = \sqrt{\frac{289}{324}} = \frac{1}{\sqrt{1.121}} = \frac{1}{1.1}$

$\Rightarrow \nu\lambda_{16} = \frac{\nu\lambda_{51}}{1.1}$

$\therefore \quad \lambda_{16} = \frac{136}{3 \times 1.1} = 41.21 \text{ cm}$

12. $176 \times \frac{v - v}{v - 22} = 165 \times \frac{v + v}{v}$

$\Rightarrow \quad 176\, v(v - v) = 165(v + v)(v - 22)$

$\therefore$

$176 \times 330(330 - v) = 165(330 + v)(330 - 22)$

or $\quad 1.143\,(330 - v) = 330 + v$

or $\quad 0.143 \times 330 = 2.143\, v \Rightarrow v = 22 \text{ m/s}$

13. $M_{\min} = \frac{2 \times 32 + 3 \times 48}{2 + 3} = 41.6$

$\frac{\nu_2}{\nu_1} = \frac{v_2}{v_1} = \sqrt{\frac{\mu_1}{\mu_2}} = \sqrt{\frac{32}{41.6}} = \sqrt{0.77}$

$= 0.875 = 175 \text{ Hz}$

14. $v_0 = gt = 30 \text{ m/s}$

$1100 = \frac{v + 30}{v} \times 1000, 1.1v = v + 30$

$0.1\, v = 30 \Rightarrow v = 300 \text{ m/s}$

Passage (Q 5 to 17)

$v_m + v_p = 8 \text{ m/s}, \; 50v_m = 150v_p$

$\Rightarrow \quad v_m = 3v_p, 4v_p = 8 \text{ m/s}$

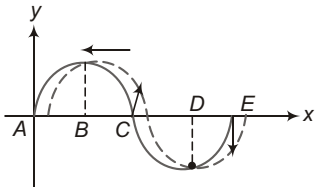
$v_p = 2 \text{ m/s and } v_m = 6 \text{ m/s}$

15. $\nu' = \frac{v + 2}{v - 6} f_0 = \frac{332}{324} f_0 = \text{constant}$

16. $\nu'' = \frac{v - 2}{v + 6} f_0 = \frac{328}{336} f_0 = \text{constant}$

17. $\nu'' < f_0 < v'$ and graph is (a)

18.



Both (a) and b are correct.

More Than One

19. $\nu = \frac{(2n+1)v}{4l}$

$\Rightarrow l = \frac{v}{4\nu}(2n + 1) = \frac{330}{4 \times 264}(2n + 1) \text{ m}$

$= (2n + 1) \times 31.25 \text{ cm}$

$= 31.25 \text{ cm}, 93.75 \text{ cm}, 156.25 \text{ cm}$

20. (a) $v \propto p^0$, (b) $v \propto \sqrt{T} \Rightarrow v^2 \propto T$,

where T is absolute temperature.

(c) $v \propto \sqrt{F}$ (d) $\nu \propto \frac{1}{l}$

$\therefore$(c) and (d) are correct.

21. $P_0 = BA\,k; \; B = -\frac{\Delta p}{\frac{\Delta V}{V}} - \frac{\Delta p}{\frac{\Delta p}{p}}$

$\therefore \quad \Delta\rho = \frac{\Delta p \rho}{B} = BA\,k\frac{\rho}{B} = \rho A\,k$

$\rho \propto p;$

Pressure and density equations are in opposite phase *i.e.*, $\Delta\phi = \frac{\pi}{2}$ and not π.

So, (a), (b) and (c) are correct.

22. $\frac{5v}{4l_c} = \frac{3v}{2l_o} \Rightarrow \frac{1.25}{l_c} = \frac{2}{l_o} \Rightarrow \frac{l_o}{l_c} = \frac{2}{1.25} = \frac{8}{5}.$

(a) $\nu_c = \frac{v}{4l_c} = \frac{v}{4 \times \frac{5}{8} l_o} = \frac{2v}{5l_o}$

$= \frac{4}{5} \cdot \frac{v}{2l_o} = \frac{4}{5}\nu_o \Rightarrow \nu_c < \nu_o$

(b) $\nu_c = \frac{3v}{4l_c} = \frac{3v}{4 \times \frac{5}{8} l_0} = \frac{12}{5} \cdot \frac{v}{2l_0}$

$= \frac{6}{5} \cdot \frac{2v}{2l_o} = \frac{6}{5} \cdot \nu_o \Rightarrow \nu_c > \nu_o.$

(c) $\nu_c = \frac{15v}{4l_c} = \frac{15v}{4 \times \frac{5}{8} l_o} = \frac{6v}{l_o} = 12\frac{v}{2l_o}$

$= 12v_0$ twelbth harmonic.

(d) Closed organ pipe cannot have tenth harmonic it only has odd harmonics.

23. $f = \frac{v}{4(l+e)} = \frac{1}{4(l+e)}\sqrt{\frac{\gamma RT}{M}}$

(a) increase in $r \Rightarrow$ increase in $e \Rightarrow$ decrease in f

(b) increase in $T \Rightarrow$ increase in $v \Rightarrow$ increase in f

(c) increase in $M \Rightarrow$ decrease in $v \Rightarrow$ decrease in f

(d) increase in $P \Rightarrow$ increase in $\rho \Rightarrow$ no change in $v \Rightarrow$ no change in f

24. $f_a = \frac{v}{v - v_s} f$ and $f_r = \frac{v}{v + v_s} f$ are constants during approach and received.

■ Match the Columns

1. $\nu_o = \frac{v}{2l} = f$

(a) $\nu_c = \frac{v}{4 \times 2l} = \frac{f}{4} = 0.25f \rightarrow s$

(b) $\nu_{c2} = \frac{5v}{4 \times 2l} = \frac{5}{4} \cdot f = 1.25f \rightarrow p$

(c) $\nu_{c1} = \frac{3v}{4 \times 2l} = \frac{3}{4} f = 0.75f \rightarrow r$

(d) $\nu_{c1} = \frac{3v}{4 \times 2l} = 0.75f \rightarrow r$

2. $\Delta\nu_1 = \left(\frac{v}{v - v_s} - \frac{v}{v + v_s}\right) f = \frac{2vv_f}{v^2 - v_s^2} f$

$= \frac{2v\frac{v}{4}}{v^2 - \frac{v^2}{16}} f = \frac{16}{15} \times \frac{1}{2} f = \frac{8}{15} f$

$\Delta\nu_2 = \left(\frac{v + v_s}{v - v_s} - 1\right) f = \frac{2v_s}{v - v_s} f$

$= \frac{2v/4}{v - v/4} f = \frac{2}{3} f$

$\Delta\nu_3 = \left(\frac{v}{v - v_5} - \frac{v}{v - v_5}\right) f = 0$

(a) $\rightarrow \Delta\nu_1 = \frac{8}{15} f \rightarrow q$

(b) $\rightarrow \Delta\nu_2 = \frac{2}{3} f \rightarrow p$

(c) $\Delta\nu_3 = 0 \rightarrow s$

(d) $\Delta\nu_3' = 0 \rightarrow s$

3. $f = f_T - f_S$

(a) If tuning fork is loaded f_T decreases such that beat frequency may increase or decrease depending upon amount of wax $\rightarrow r, s$

(b) If prongs are filed, beat frequency must increase $\rightarrow p$

(c) If tension is increased beat frequency may increase or decrease depending upon the amount of change in tension. $\rightarrow r, s$

(d) If tension is decreased, beat frequency must increase $\rightarrow p$

4. (a) For point source, $I \propto \frac{1}{r}$, and $A \propto \frac{1}{r} \rightarrow r$

(b) $\rightarrow q$

(c) For line source, $I \propto \frac{1}{r}$ and

$A \propto \frac{1}{\sqrt{r}} \rightarrow q$

(d) $\rightarrow p$

5. $\lambda = \frac{2\pi}{k} = \frac{2\pi}{\pi} = 2$ m

$\lambda = 5\frac{\lambda}{4} = \frac{5}{2}$m $= 2.5$ m

(a) $l = 2.5$ m $\rightarrow s$

(b) $\lambda = 2$ m $\rightarrow r$

(c) $\frac{\lambda}{2}, \frac{2\lambda}{2} = 1$m, 2m $\rightarrow p, r$

(d) $\frac{\lambda}{4}, 3\frac{\lambda}{4} = 0.5$ m, 1.5 m $\rightarrow q$

17. Thermometry, Thermal Expansion & Kinetic Theory of Gases

Introductory Exercise 17.1

1. (a) $\frac{C}{5} = \frac{F-32}{9}$ for $F = 0, C = -\frac{5}{9} \times 32$

$= -17.8°C$

(b) $\frac{K - 273.15}{5} = \frac{F-32}{9}$ for $K = 0$,

$F = -\frac{9}{5} \times 273.15 + 32 = -459.67°F$

2. (a) $\frac{x}{5} = \frac{2x-32}{9} \Rightarrow x = \frac{10x}{9} - 17.8$

$\Rightarrow \quad 17.8 = \left(\frac{10}{9} - 1\right)x$

$\Rightarrow \quad x = 17.8 \times 9 = 160.2\ °C$

(b) $\frac{x}{5} = \frac{x/2 - 32}{9} \Rightarrow x = \frac{5}{18}x - 17.8$

$\Rightarrow 17.8 = -\frac{13}{18}x \Rightarrow x = -24.65\ °C$

3. $\frac{C-5}{99-5} = \frac{F-32}{212-32}$

$\Rightarrow \quad \frac{C-5}{94} = \frac{F-32}{180}$

$\Rightarrow \quad \frac{52-5}{94} = \frac{F-32}{180}$

$\Rightarrow \quad F = 32 + \frac{180}{94} \times 47 = 122\ °F$

4. $\frac{K - 273.15}{5} = \frac{F-32}{9}$

$\Rightarrow \quad x - 273.15 = \frac{5}{9}x - 17.8$

$\Rightarrow \quad \frac{4}{9}x = 255.35 \Rightarrow x = 574.54$

5. $\frac{C}{5} = \frac{F-32}{9} \quad \Rightarrow \frac{9}{5}x = x - 32$

$\Rightarrow \quad \frac{4}{5}x = -32$

$\Rightarrow \quad x = -\frac{5}{4} \times 32 = -40\ °C$

6. $\Delta t = \frac{1}{2}\alpha t\ \Delta\theta$

$= \frac{1}{2} \times 1.2 \times 10^{-5} \times 86400 \times 30$

$= 1.5 \times 1.2 \times 8.64\ s = 15.55\ s$ given.

7. As from 0°C to 4°C, density of water increases so the volume of wooden block above water level increases and as from 4°C to 10°C density of water decreases so the volume of block above water decreases.

8. $V_1\rho_1 g = V_2'\sigma_1 g$ and $V_2\rho_2 g = V_2'\sigma_2 g$

$\Rightarrow \quad \frac{\Delta V_1}{V_1} = 1 - \frac{V_1'}{V_1} = -1 - \frac{\sigma_1}{\rho_1}$

and $\frac{\Delta V_2}{V_2} = 1 - \frac{\sigma_2}{\rho_2}$

$\therefore \quad \frac{\Delta V_2}{V_2} - \frac{\Delta V_1}{V_1}$

$= \left(1 - \frac{\sigma_2}{\rho_2}\right) - \left(1 - \frac{\sigma_1}{\rho_1}\right) = \frac{\sigma_1}{\rho_1} - \frac{\sigma_2}{\rho_2}$

$= \frac{\sigma_1}{\rho_1} - \frac{\sigma_1\ (1 - \gamma_2 \Delta T)}{\rho_1\ (1 - \gamma_1 T)}$

$$= \frac{\sigma_1}{\rho_1}\left(\frac{\gamma_2 - \gamma_1}{1-\gamma_1}\right)_T^{\Delta T}$$

$$= \frac{\sigma_1}{\rho_1} - \frac{\sigma_1(1-\gamma_2\Delta T)}{\rho_1(1-\gamma_1 T)}$$

$$= \frac{\sigma_1}{\rho_1}\left(\frac{\gamma_2 - \gamma_1}{1-\gamma_1}\right)_T^{\Delta T}$$

9. On cooling brass contracts more than iron ($\alpha_B > \alpha_{Fe}$) such that brass disk gets loosen from hole of iron.

10. $V \propto T \Rightarrow V = kT \Rightarrow \ln V = \ln k + \ln T$

$$\Rightarrow \frac{\Delta V}{V} = \frac{\Delta T}{T} \Rightarrow \frac{\Delta V}{V\Delta T} = \frac{1}{T} = \gamma$$

Introductory Exercise 17.2

1. For ideal gases, $pV = nRT$

$$\Rightarrow \quad T = \frac{V}{nR}p = \frac{VM}{mR}p$$

$$\text{Slope} = \frac{VM}{mR}$$

As slope $\propto \frac{1}{m} \Rightarrow m_2 < m_1$

2. $pV = nRT \Rightarrow \frac{p_2}{p_1} = \frac{T_2}{T_1}$

$$= \frac{360}{300} = \frac{6}{5}$$

$$\Rightarrow p_2 = \frac{6}{5}p_1 = \frac{6}{5} \times 10 \text{ atm} = 12 \text{ atm}$$

3. $$M_{mix} = \frac{\frac{1}{4}\times 28 + \frac{1}{4}\times 44}{\frac{1}{4}+\frac{1}{4}} = \frac{7+11}{\frac{1}{2}} = 36$$

$$pV = nRT = \frac{m}{M}RT$$

$$\Rightarrow pM = \frac{m}{V}RT = \rho RT \Rightarrow \rho = \frac{pM}{RT}$$

$$\therefore \quad \rho = \frac{1.01\times 10^5 \times 36 \times 10^{-3}}{8.31 \times 290}$$

$$= \frac{1.01 \times 36}{8.31 \times 2.9} = 1.5 \text{ kg/m}^3$$

4. $pV = nRT = \frac{N}{N_A}RT$

$$\Rightarrow N = \frac{pVN_A}{RT}$$

$$= \frac{10^{-6} \times 13.6 \times 10^3 \times 10 \times 250 \times 10^{-6} \times 6.02 \times 10^{23}}{8.31 \times 300}$$

$$= \frac{13.6 \times 5 \times 6.02}{8.31 \times 6} \times 10^{15} = 8.21 \times 10^{15}$$

5. $pV = nRT$

$$\Rightarrow \quad V = \frac{nR}{p}\cdot T$$

$$\text{Slope} \propto \frac{1}{\rho}$$

$$\Rightarrow \quad p_1 > p_2$$

6. $pV = nRT \Rightarrow p = (nRT)\frac{1}{V}$

$\Rightarrow y = mx$ is a straight line passing through origin.

Introductory Exercise 17.3

1. Average velocity depends on the direction of motion of gas molecules and as container do not move such that their net effect becomes zero, due to the reason that some molecules are moving in one direction while other are moving in opposite direction. But in case of average speed only magnitudes are in use which do not cancel each other.

2. $\text{KE} = \frac{3}{2}kT = \frac{3}{2} \times \frac{8.31}{6 \times 10^{23}} \times 300 \text{ J}$

$= \frac{3}{4} \times 8.31 \times 10^{-21} \text{ J}$

$= 6.21 \times 10^{-21} \text{ J}$

3. $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$,

$v_{\text{He}} = \sqrt{\frac{3 \times 8.31 \times 300}{4 \times 10^{-3}}}$

$= 1.37 \times 10^3 \text{ m/s}$

$v_{\text{Ne}} = \sqrt{\frac{3 \times 8.31 \times 300}{20.2 \times 10^{-3}}} = 608.5 \text{ m/s}$

$\text{KE} = \frac{3}{2}kT = 6.21 \times 10^{-21} \text{ J}$

4. $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

$\Rightarrow T = \frac{Mv_{\text{rms}}^2}{3R} = \frac{4 \times 10^{-3} \times 10^6}{3 \times 8.31}$

$= 160.45 \text{ K}$

5. $\rho = \frac{n_1\rho_1 + n_2\rho_2}{n_1 + n_2} = \frac{(1 - n_2)\rho_1 + n_2\rho_2}{1 - n_2 + n_2}$

$= \rho_1 + n_2(\rho_2 - \rho_1)$

$\Rightarrow n_2 = \frac{\rho - \rho_1}{\rho_2 - \rho_1} = \frac{1.293 - 1.429}{1.251 - 1.429}$

$= \frac{136}{178} = 0.764 = 76.4\%$ by mass

6. $\frac{V_2}{V_1} = \frac{p_1T_2}{p_2T_1} = \frac{(p_0 + h\rho g)}{p_0 \times 277}$

$= \frac{(1.01 \times 10^5 + 40 \times 10^3 \times 10) \times 293}{1.01 \times 10^5 \times 277}$

$= \frac{5.01 \times 293}{1.01 \times 277} = 5.25$

$\Rightarrow V_2 = 5.25\, V_1 = 105 \text{ cm}^3$

7. $N = nN_A = \frac{1}{18} \times 6 \times 10^{23}$

$= \frac{1}{3} \times 10^{23}$;

$S = 4\pi R^2 = 4 \times 3.14 \times (6400 \times 10^3 \times 10^2)^2$

$= 5.14 \times 10^{18} \text{ cm}^2$

$\therefore \quad \frac{N}{S} = \frac{10^{23}}{3 \times 5.14 \times 10^{18}}$

$= 6.5 \times 10^3 \text{ molecules/cm}^2$

(a) $nC_V = \frac{3}{2}nR = 35 \text{ J/K}$

$\Rightarrow \quad n = \frac{70}{3R} = 2.8 \text{ mole}$

(b) $U = \frac{3}{2}nRT = 35 \text{ J/K} \times 273 \text{ K} = 9555 \text{ J}$

(c) $C_p = C_V + R = \frac{5}{2}R = 20.8 \text{ J/K mole}$

8. (a) $n(C_p - C_V) = nR = 29.1 \text{ J/K}$

$\Rightarrow n = \frac{29.1}{8.314} \text{ mole} = 3.5 \text{ mole}$

(b) $C_V = nc_V = n\frac{3}{2}R = 3.5 \times 1.5 \times 8.314$

$= 43.65 \text{ J/K}$

$C_p = nc_p = n\frac{5}{2}R = C_V + nR$

$= 43.65 + 3.5 \times 8.314$

$= 72.75 \text{ J/K}$

(c) $C_V' = nc_V = n \times \frac{5}{3}R = 72.75 \text{ J/K}$

$C_p' = nc_p = n \times \frac{7}{2}R$

$= 72.75 + 3.5 \times 8.314$

$= 101.85 \text{ J/K}$

10. $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$ and $v_{\text{av}} = \sqrt{\frac{8RT}{\pi M}}$

Here $3 > \frac{8}{\pi} \Rightarrow v_{\text{rms}} > v_{\text{av}}$,

i.e., the statement is true.

AIEEE Corner

Subjective Questions (Level 1)

1. $\frac{C'}{5} = \frac{68-32}{9} = \frac{36}{9} = 4$

$\Rightarrow C' = 20°\text{C}$; $\frac{K'-273}{5} = \frac{68-32}{9} = 4$

$\Rightarrow K' = 293\text{ K}$

$\frac{C'}{5} = \frac{5-32}{9} = -\frac{27}{9} = -3$

$\Rightarrow C' = -15°\text{C}$; $\frac{K'-273}{5} = \frac{5-32}{9} = -3$

$\Rightarrow K' = 258\text{ K}$

$\frac{C'}{5} = \frac{176-32}{9} = \frac{144}{9} = 16$

$\Rightarrow C' = 80°\text{C}$; $\frac{K'-273}{5} = 16$

$\Rightarrow K' = 353\text{ K}$

2. $\frac{30}{5} = \frac{F'-32}{9} \Rightarrow F' = 54 + 32 = 86°\text{F}$

$= 546°\text{R}$

$\frac{5}{5} = \frac{F'-32}{9} \Rightarrow F' = 9 + 32 = 41°\text{F} = 501°\text{R}$

$-\frac{20}{5} = \frac{F'-32}{9}$

$\Rightarrow \quad F' = -36 + 32 = -41°\text{F}$

$= 456°\text{R}$

3. $\frac{x}{5} = \frac{x-32}{9} \Rightarrow 32 = x - \frac{9}{5}x = -\frac{4}{5}x$

$\Rightarrow \quad x = -\frac{5}{4} \times 32 = -40°$

$\Rightarrow \quad -40°\text{C} = -40°\text{F}$

4. $\frac{\Delta C}{5} = \frac{\Delta F}{9} \Rightarrow \Delta F = \frac{9}{5}\Delta C = \frac{9}{5} \times 40 = 72°$

$\therefore \; F_2 = F_1 + 72° = 140.2°\text{F}$

5. $\frac{32-20}{80-20} = \frac{C'-0}{100-0}$

$\Rightarrow \frac{12}{60} = \frac{C'}{100} \Rightarrow C' = \frac{12 \times 100}{60} = 20°\text{C}$

6. $\frac{T_2}{T_1} = \frac{p_2}{p_1} = \frac{160}{80} = 2 \Rightarrow T_2 = 2T_1$

$\therefore \; T_2 = 2 \times 273.15\text{ K} = 546.30\text{ K}$

7. $R_t = R_0(1 + \alpha\Delta\theta)$

$\Rightarrow 3.50 = 2.50\,(1 + 100\alpha) \Rightarrow 1 = 250\text{ K}$

or $\alpha = \frac{1.0}{250} = 4 \times 10^{-3}/°\text{C}$

$\therefore \; 6.50 = 2.50\,(1 + 4 \times 10^{-3}\Delta\theta)$

$\Rightarrow \quad 4 = 10^{-2} \times \Delta\theta$

$\Rightarrow \quad \Delta\theta = 400 \Rightarrow \theta_2 = 400°\text{C}$

$\Rightarrow \quad \Delta\theta = 400 \Rightarrow \theta_2 = 400°\text{C}$

i.e., boiling point of sulphur is 400°C.

8. $\frac{T_2}{T_1} = \frac{p_2}{p_1} = \frac{75+45}{75+5} = \frac{120}{80} = \frac{3}{2}$

$T_2 = \frac{3}{2}T_1 = \frac{3}{2} \times 300.15\text{ K}$

$= 450.225\text{ K} = 177.08°\text{C}$

9. $\Delta\gamma = \gamma(\alpha_{Br} - \alpha_{Fe})\,\Delta\theta$

$\Rightarrow \quad \Delta\theta = \frac{\Delta\gamma}{\gamma} \cdot \frac{1}{\alpha_{Br} - \alpha_{Fe}}$

$= \frac{0.01 \times 10^{-3}}{6 \times 10^{-2}} \cdot \frac{1}{\alpha_{Br} - \alpha_{Fe}}$

$= \frac{10^{-3}}{6(\alpha_{Br} - \alpha_{Fe})}$

$\therefore \quad \theta_2 = \theta_1 + \frac{10^{-3}}{6(\alpha_{Br} - \alpha_{Fe})}$

$= 30°\text{C} + \frac{10^{-3}}{6(\alpha_{Br} - \alpha_{Fe})} = 30°\text{C} + \frac{100}{6 \times 0.63}$

$= 57.78°\text{C}.$

10. (a) $\Delta l = l\,\alpha\Delta\theta \simeq 88.42 \times 2.4 \times 10^{-5} \times 30$

$= 0.064\text{ cm}$

(b) $\Delta l = l\,(\alpha_{Al} - \alpha_{St})\,\Delta\theta$

$= 88.42\,(2.4 - 1.2) \times 10^{-5} \times 30$

$= 0.032\text{ cm}$

$l_S = l + \Delta l = 88.42 + 0.032\text{ cm}$

$= 88.45\text{ cm}$

11. $\frac{\Delta l}{l} \times 100\% = \alpha \Delta\theta \times 100\%$

$= -1.2 \times 10^{-5} \times 35 \times 100\%$

$= -0.042\%$

12. $F = YA \frac{\Delta l}{l} = YA\ \alpha\ \Delta\theta$

$= 2 \times 10^{11} \times 2 \times 10^{-6} \times 1.2 \times 10^{-5} \times 40$

$= 4 \times 1.2 \times 40\ \text{N} = 160 \times 1.2\ \text{N} = 192\ \text{N}$

13. $V\sigma g = (50 - 45) \times 10^{-3}$ kg

$= 5 \times 10^{-3}$ kg

$V'\sigma' g = (50 - 45.1) \times 10^{-3}$ kg

$= 4.9 \times 10^{-3}$ kg

$V(1 + \gamma_s \Delta\theta) \frac{\sigma}{1 + \gamma_l\ \Delta\theta} g = 4.9 \times 10^{-3}$

$\frac{1 + \gamma_s\ \Delta\theta}{1 + \gamma_l\ \Delta\theta} = \frac{4.9}{5}$

$\Rightarrow 5 + 5\gamma_s \Delta\theta = 4.9 + 4.9\ \gamma_e\ \Delta\theta$

$\gamma_l = \frac{0.1 + 5\gamma_s\ \Delta\theta}{4.9\ \Delta\theta} = \frac{1}{49\Delta\theta} + \frac{5}{4.9}\gamma_s$

$\gamma_s = \frac{1}{49 \times 75} + \frac{5}{4.9} \times 12 \times 10^{-6}$

$= 272.1 \times 10^{-6} + 12.2 \times 10^{-6}$

$= 2.84 \times 10^{-4}\ ^\circ\text{C}$

14. $M = 14 + 3 = 17$ g/mole

$= 17 \times 10^{-3}$ kg/mole

$\Rightarrow M = \frac{17 \times 10^{-3}}{6.033 \times 10^{-23}}$ kg/molecule

$= 2.82 \times 10^{-26}$ kg/molecule

15. $n = \frac{pV}{RT} = \frac{1.52 \times 10^6 \times 10^{-2}}{8.314 \times 298.15} = 6.13$

$\rho = \frac{m}{V} = \frac{nM}{V} = \frac{6.13 \times 2 \times 10^{-3}}{10^{-2}}$

$= 1.23\ \text{kg/m}^3$

$\rho' = \frac{m'}{V} = \frac{nM'}{V} = \frac{16\ nM}{V} = 16\rho$

$= 19.62\ \text{kg/m}^3$

16. $p_2 = p_1 \frac{V_1}{V_2} = 1\ \text{atm} \times \frac{76}{6}$

$= 12.7$ atm

17. $V_2 = \frac{p_1 V_1}{T_1} \cdot \frac{T_2}{p_2} = \frac{p_1}{p_2} \cdot \frac{T_2}{T_1} \cdot V_1$

$= \frac{1}{0.5} \times \frac{270}{300} \times 500\ \text{m}^3 = 900\ \text{m}^3$

18. $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$

$\Rightarrow \frac{\left(\frac{mg}{A} + p_0\right) A \cdot h_i}{293} = \frac{\left(\frac{mg}{A} + p_0\right) A h_f}{273}$

$\Rightarrow h_f = \frac{373}{293} h_i = \frac{373}{293} \times 4\ \text{cm} = 50.9\ \text{cm}$

19. $p_1 = p_2 \Rightarrow \frac{n_1}{V_1} = \frac{n_2}{V_2} = \frac{25/28}{L_1 A} = \frac{40/4}{L_2 A}$

$\Rightarrow \frac{L_1}{L_2} = \frac{25}{28} \times \frac{1}{10} = \frac{5}{56} = 0.089$

$\frac{n_1}{n_2} = \frac{25/28}{40/4} = \frac{25}{280} = \frac{5}{56} = 0.089$

20. $n = n_1 + n_2$

$\Rightarrow p(V_1 + V_2) = p_1 V_1 + p_2 V_2$

$\Rightarrow p = \frac{p_1 V_1 + p_2 V_2}{V_1 + V_2}$

$\therefore p = \frac{1.38 \times 0.11 + 0.69 \times 0.16}{0.11 + 0.16}\ \text{MP}_\text{a}$

$= \frac{0.1518 + 0.1104}{0.27} = \frac{0.2622}{0.27} = 0.97\ \text{MP}_\text{a}$

21. $\frac{pV_1}{T} + \frac{pV_2}{T} = \frac{p_1 V_1}{T_1} + \frac{p_1 V_2}{T_2}$

$\frac{1\ \text{atm}}{293\ \text{K}} \times 600\ \text{cm}^3$

$= p_1 \left(\frac{400\ \text{cm}^3}{373\ \text{K}} + \frac{200\ \text{cm}^3}{273\ \text{K}}\right)$

$\Rightarrow p_1 = \frac{600/293}{\frac{400}{373} + \frac{200}{273}}$ atm

$\therefore p_1 = \frac{3}{293\left(\frac{2}{373} + \frac{1}{273}\right)}$ atm

$= \frac{3}{1.57 + 1.07} = \frac{3}{2.64}$ atm

$= 1.136$ atm

22. $V = \frac{nRT}{p} = \frac{1 \times 8.314 \times 273.15}{1.013 \times \times 10^5}\ \text{m}^3$

$= 0.02242\ \text{m}^3 = 22.42$ litre

23. $p_2 = \frac{p_1 V_1}{T_1} \cdot \frac{T_2}{V_2} = p_1 \frac{V_1}{V_2} \cdot \frac{T_2}{T_1}$

$= 1.5 \times 10^5 \times \frac{0.75}{0.48} \times \frac{430}{300}$

$= 3.36 \times 10^5$ Pa

24. $\rho = p_1 + p_1' + p_2$

$= \frac{n_1 RT}{V} + \frac{n_1' RT}{V} + \frac{n_2 RT}{V}$

$= \left[0.7 \times \frac{1.4}{28} + 0.3 \times \frac{1.4}{14} + \frac{0.4}{4}\right] \frac{RT}{V}$

$= \left[\frac{0.7}{20} + \frac{0.3}{10} + \frac{1}{10}\right] \times \frac{8.314 \times 1500}{5 \times 10^{-3}}$

$= \frac{3.3}{20} \times 8.314 \times 3 \times 10^5$ Pa

$= 4.11 \times 10^5$ Pa

25. $\text{RKE} = 2 \times \frac{1}{2} kT = \frac{1}{2} I\omega^2$

$\Rightarrow \omega = \sqrt{\frac{2kT}{I}} = \sqrt{\frac{2 \times 1.38 \times 10^{-23} \times 300}{8.28 \times 10^{-38} \times 10^{-7}}}$

$\therefore \omega = 10^{12} \sqrt{\frac{6 \times 1.38}{8.28}} = 10^{12}$ rad/s

26. $v = \sqrt{\frac{\gamma p}{\rho}}$

$\Rightarrow \gamma = \frac{\rho v^2}{p} = \frac{1.3 \times (330)^2}{1.013 \times 10^5} = 1.398$

$= \frac{f+2}{f} \Rightarrow f = \frac{2}{0.398} \simeq 5$

27. $\gamma = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 C_{V_1} + n_2 C_{V_2}} = \frac{3 \times \frac{5}{2} + 2 \times \frac{7}{2}}{3 \times \frac{3}{2} + 2 \times \frac{5}{2}}$

$= \frac{15+14}{9+10} = \frac{29}{19} = 1.53$

28. $K = \frac{3}{2} pV$

$\Rightarrow \frac{K_2}{K_1} = \frac{\frac{3}{2} p_2 V_2}{\frac{3}{2} p_1 V_1} = \frac{3}{2} \cdot \frac{15}{5} = 4.5$

$\Rightarrow K_2 = 4.5$ K

29. $C_p = \frac{f+2}{2} R = 29 \Rightarrow f = \frac{58}{R} - 2 = 5$

$pT = p \cdot \frac{pV}{nR}$

$\Rightarrow p^2 V = \text{constant}$

$\Rightarrow pV^{1/2} = \text{constant} \Rightarrow a = \frac{1}{2},$

$c = \frac{f}{2} R + \frac{R}{1 - \frac{1}{2}} = \frac{f+4}{2} R = 29$

$f = \frac{58}{R} - 4 = 3$

30. TKE $= \frac{3}{5}$ of total energy and RKE $= \frac{2}{5}$ of total energy, so the gas is diatomic.

TKE $= \frac{3}{2} kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 300$ J

$= 6.21 \times 10^{-21}$ J/molecule

$\Delta Q = nC_V\ \Delta T = 1 \cdot \frac{5}{2} \times 8.314 \times 1 = 20.8$ J

31. $C_p = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 + n_2}$

$= \frac{2.5R + 3.5R}{1+1} = 3R$

$C_V = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2}$

$= \frac{1.5R + 2.5R}{1+1} = 2R$

$\gamma = \frac{C_P}{C_V} = \frac{3R}{2R} = 1.5$

32. $\gamma = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 C_{V_1} + n_2 C_{V_2}} = \frac{(n_1 + n_2) C_{p_1}}{(n_1 + n_2) C_{V_1}}$

$= \frac{C_{p_1}}{C_{V_1}} = \gamma$

33. $p = aV^b \Rightarrow pV^{-b} = \text{constant}$

$C = \frac{\Delta Q}{n \Delta T} = 0$ for adiabatic process for which pV^γ = constant comparing, we get, $b = -\gamma$

34. $p = kV \Rightarrow pV^{-1} = \text{constant}$

$\Rightarrow pV = a \text{ constant} \Rightarrow a = -1$

$C = C_V + \frac{R}{1-a} = C_V + \frac{R}{1+1} = C_V + \frac{R}{2}$

35. $v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.314 \times 373.15}{2 \times 10^{-3}}}$

$= 2.16 \times 10^3 \text{ m/s}$

$= 2.16 \text{ km/s}$

36. $v_{\text{rms}} = \sqrt{\frac{(500)^2 + (600)^2 + (700)^2 + (800)^2 + (900)^2}{5}}$

$= \frac{100}{\sqrt{5}} \sqrt{25 + 36 + 49 + 64 + 81}$

$= 714 \text{ m/s}$

$v_{\text{av}} = \frac{500 + 600 + 700 + 800 + 900}{5}$

$= 20\,(5 + 6 + 7 + 8 + 9) = 700 \text{ m/s}$

$v_{\text{rms}} \neq v_{\text{av}}$

37. $\text{KE} = \frac{3}{2} pV$

$\Rightarrow N \times 6 \times 10^{-26} = 1.5 \times 2 \times 10^5 \times 100 \times 10^{-3} \times 10^{-3}$

$\therefore \quad N = \frac{3 \times 10}{6 \times 10^{-26}} = 5 \times 10^{26}$

$= \frac{5000}{6.023} \times 6.023 \times 10^{23} = 830.15 \text{ Na}$

$\therefore \quad n = 8300.15 \text{ moles}$

38. Frequency of collision, $\nu = \frac{v}{2\sqrt{3}\,l} = \frac{v}{2\sqrt{3} \cdot V}$

$= \frac{1}{2\sqrt{3}V} \sqrt{\frac{3RT}{M}}$

$\therefore \nu = \sqrt{\frac{RT}{4VM}} = \sqrt{\frac{RT}{4 \cdot \frac{nRT}{p} \cdot M}} = \sqrt{\frac{p}{4nM}}$

$= \sqrt{\frac{2 \times 10^{+5}}{4 \times 1 \times 46 \times 10^{-3}}}$

$= 41.04 \times 10^3 /\text{s}$

39. $\text{KE} = \frac{3}{2} pV = \frac{3}{2} \times 10^5 \times 2 \times 10^{-6} = 0.3 \text{ J}$

$N = \frac{m}{m_1} = \frac{50 \times 10^{-6}}{8 \times 10^{-26}} = 6.25 \times 10^{20}$

$\therefore K_1 = \frac{K}{N} = \frac{0.3}{6.25 \times 10^{20}} = \frac{30}{6.25} \times 10^{-22} \text{ J}$

$= 4.8 \times 10^{-22} \text{ J}$

40. $v_0 = \sqrt{\frac{3RT_0}{M_0}}$

(a) $\frac{v}{v_0} = \sqrt{\frac{T}{T_0}} = \sqrt{\frac{573}{293}} = 1.4 \Rightarrow v = 1.4\, v_0$

(b) $v = v_0$ as RMS speed changes with temperature and not with pressure.

(c) $\frac{v}{v_0} = \sqrt{\frac{M_0}{M}} = \sqrt{\frac{M_0}{3M_0}} = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad v = \frac{v_0}{\sqrt{3}} = 0.58\, v_0$

41. $\sqrt{\frac{\gamma RT}{M_{H_2}}} = \sqrt{\frac{\gamma RT'}{M_{O_2}}} \Rightarrow T = \frac{M_{H_2}}{M_{O_2}} \cdot T'$

$= \frac{2}{32} \times 320 = 20 \text{ K} = -253°\text{C}$

42. $\frac{1}{2} mv_e^2 = \frac{GMm}{R_e} = g_e R_e m$

$\Rightarrow v_e = \sqrt{2g_e R_e} = v_{H_2} = \sqrt{\frac{3RT}{M}}$

$\Rightarrow T_e = \frac{2g_e R_e M}{3R}$

$= \frac{2 \times 9.8 \times 6.367 \times 10^6 \times 2 \times 10^{-3}}{3 \times 8.314}$

$= 10007 \text{ K}$

and $T_m = \frac{2g_m R_m M}{3R}$

$= \frac{2 \times 1.6 \times 1.75 \times 10^6 \times 2 \times 10^{-3}}{3 \times 8.314}$

$= 449 \text{ K}$

43. (a) $\text{KE} = \frac{3}{2} kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 300 \text{ J}$

$= 6.21 \times 10^{-21} \text{ J}$

(b) $\text{KE}' = \frac{3}{2} kT \cdot N_a = 6.023 \times 10^{23} \times 6.21 \times 10^{-21} \text{ J}$

$= 3740 \text{ J}$

(c) $v_{\text{rms}} = \sqrt{\dfrac{3RT}{M}} = \sqrt{\dfrac{3 \times 8.314 \times 300}{32 \times 10^{-3}}}$

$= 483.6 \text{ m/s}$

Objective Questions (Level 1)

1. $v_{\text{av}} = \sqrt{\dfrac{8RT}{\pi M}}$

2. $v_{\text{rms}} = \sqrt{\dfrac{1^2 + 0^2 + 2^2 + 3^2}{4}}$

$= \sqrt{\dfrac{14}{4}} = \sqrt{3.5} \text{ m/s}$

3. $\dfrac{\Delta l}{l} = -\alpha\Delta\theta = -12 \times 10^{-6} \times 50$

$= -600 \times 10^{-6} = -6 \times 10^{-4}$

4. $V \propto T \Rightarrow \dfrac{V_2}{V_1} = \dfrac{T_2}{T_1} = 2;$

$\dfrac{\Delta V}{V} = \dfrac{V_2 - V_1}{V_1} = 2 - 1 = 1 = 100\%$

5. $\text{KE} \propto T \Rightarrow \dfrac{K_2}{K_1} = \dfrac{T_2}{T_1} = \dfrac{2E}{E} = 2$

$\Rightarrow T_2 = 2T_1 = 2 \times 283 \text{ K}$

$\therefore T_2 = 566 \text{ K} = 293°\text{C}$

6. $\text{TE} = \dfrac{f}{2} kT = \dfrac{n}{2} kT.$

7. $\dfrac{v_2}{v_1} = \sqrt{\dfrac{T_2}{T_1}} = \sqrt{\dfrac{1200}{300}} = 2 \Rightarrow v_2 = 2v_1$

8. (a) $p_{\text{av}} = m_1 v$ is different for different m_1

(b) $(\text{KE})_{\text{molecule}} = \dfrac{3}{2} kT$ is same for any gas.

(c) $(\text{KE})/V = \dfrac{3}{2}\dfrac{pV}{V} = \dfrac{3}{2} p$ is different as p is different for different.

(d) $(\text{KE})_m = \dfrac{3}{2}\dfrac{pV}{m} = \dfrac{3}{2}\dfrac{p}{\rho}$ is different as $\dfrac{p}{\rho}$ is different.

9. $\dfrac{p_1V_1}{RT_1} + \dfrac{p_2V_2}{RT_2} = \dfrac{pV_1}{RT} + \dfrac{pV_2}{RT} = \dfrac{p(V_1 + V_2)}{RT}$

$$T = \frac{p(V_1 + V_2)}{\dfrac{p_1V_1}{T_1} + \dfrac{p_2V_2}{T_2}} = \frac{T_1T_2\ p(V_1 + V_2)}{p_1V_1T_2 + p_2V_2T_1}$$

10. $\alpha = \dfrac{\Delta l}{l\,\Delta\theta} = \dfrac{0.08 \times 10^{-3}}{10 \times 10^{-2} \times 100}$

$= 8 \times 10^{-6}/°\text{C}$

$\Delta V = V\gamma\Delta\theta = 3V\alpha\Delta\theta = 3 \times 100 \text{ cc} \times 8 \times 10^{-6} \times 100$

$= 0.24 \text{ cc}$

$\Rightarrow V' = 100 \text{ cc} + 0.24 \text{ cc} = 100.24 \text{ cc}$

11. $T = T_0 + \tan 45° V = T_0 + V$

$pV = nRT = nR(T_0 + V) = nRT_0 + nRV$

or $p = nR + \dfrac{nRT_0}{V} \Rightarrow p = a + \dfrac{b}{V}$

ie, p *versus* V graph will be hyperbola.

12. $p^2V =$ constant

$\Rightarrow \left(\dfrac{nRT}{V}\right)^2 V = \text{constant.}$

$\Rightarrow T^2 \propto V$

$\Rightarrow \dfrac{T_2}{T_1} = \sqrt{\dfrac{V_2}{V_1}} = \sqrt{\dfrac{3V_0}{V_0}} = \sqrt{3}$

$\Rightarrow T_2 = \sqrt{3}\ T_1 = \sqrt{3}\ T_0$

13. $p = \dfrac{1}{3}\rho v_{\text{rms}}^2 = \dfrac{1}{3}\dfrac{m}{V} v_{\text{rms}}^2$

$\Rightarrow mT = \text{constant}$

$\dfrac{m_1}{m_2} = \dfrac{T_2}{T_1} = \dfrac{310}{280} = 1.1$

14. As temperature of vessels A and B are some so is average velocity of O_2, *i.e.*, u.

15. $N = nN_a = \dfrac{pV}{RT} N_a$

$= \dfrac{10^{-13} \times 10^{-6}}{8.314 \times 300} \times 6.023 \times 10^{+23}$

$$= \frac{602.3}{8.314 \times 3} = 24$$

16. $\frac{1}{2} mv^2 = \frac{GMm}{R} - \frac{GMm}{R+h} = mgh$

$\Rightarrow \quad h \simeq \frac{v^2}{2g} = \frac{3RT}{2gM}$

$\therefore \quad h = \frac{3 \times 8.314 \times 273}{2 \times 10 \times 28 \times 10^{-3}}$ m

$= 12.16 \times 10^3$ m ≈ 12 km

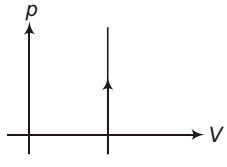
17. $\frac{l_1}{l_2} = \frac{\alpha_2}{\alpha_1} = \frac{19}{11}$ and $l_1 - l_2 = 30$ cm

$\Rightarrow \quad l_1 = \frac{11}{19} l_1 = 30$ cm

$\Rightarrow \quad l_1 = \frac{19}{8} \times 30$ cm $= 71.25$ cm

and $l_2 = \frac{11}{19} l_1 = 41.25$ cm

18. $p \propto V \propto T \Rightarrow V =$ constant



19. $V = V_0 + \tan\theta \cdot T$, $pV = nRT = \frac{m}{M} RT$

$\therefore \quad p(V_0 + \tan\theta \times T) = \frac{m}{M} RT$

$\Rightarrow \quad \tan\theta = \frac{1}{pT}\left[\frac{m}{M} RT - pV_0\right]$

or $\quad \tan\theta = \frac{mR}{p_M} - \frac{V_0}{T}$

$\therefore \tan\theta$ remains same when $m \to 2m$ and $p \to 2p$

20. $n_1 = \frac{p_1 V}{RT_1}$ and $n_2 = \frac{p_2 V}{RT_2}$

$\frac{n_1}{n_2} = \frac{p_1}{T_1} \cdot \frac{T_2}{p_2} = \frac{10 \times 300}{5 \times 330} = \frac{600}{330} = \frac{20}{11}$

$\therefore \; n_2 = \frac{11}{20} n_1$

$\therefore \Delta m = m_1 - m_2 = m_1 = \frac{11}{20} m_1$

$= \frac{9}{20} \times 28$ g $= \frac{63}{5}$ g

21. $\rho = \frac{m}{V} = \frac{m}{(n_1 + n_2)\frac{RT}{p}} = \frac{mp}{(n_1 + n_2)RT}$

$= \frac{12 \times 1.01 \times 10^5 \times 10^{-3}}{(2+2) \times 8.314 \times 300} = 0.12$ kg/m^3

22. $p = k\rho = \frac{kM}{V}$

$\Rightarrow pV =$ constant is for isothermal process, *i.e.*, $T =$ constant

23. $\frac{p^2}{\rho} =$ constant

$\Rightarrow \; p^2 V$constant $\Rightarrow \; pT =$ constant

$\frac{p_2}{p_1} = \sqrt{\frac{\rho_2}{\rho_1}} = \sqrt{\frac{\rho/2}{\rho}} = \frac{1}{\sqrt{2}}$

$\Rightarrow \quad p_2 = \frac{p_1}{\sqrt{2}}$

$\frac{T_2}{T_1} = \frac{p_1}{p_2} = \sqrt{2}$

$\Rightarrow \quad T_2 = \sqrt{2}T_1 = \sqrt{2}T$

as $pT =$ constant $\Rightarrow p \propto \frac{1}{T}$

i.e., p - T graph is hyperbola.

24. $p^2 V =$ constant

$\Rightarrow PT =$ constant and $T^2 V^{-1} =$ constant.

$\frac{p_2}{p_1} = \sqrt{\frac{V_1}{V_2}} = \sqrt{\frac{V}{4V}} = \frac{1}{2}$

$\Rightarrow \quad p_2 = \frac{p_1}{2} = \frac{p}{2}$

$\frac{T_2}{T_1} = \sqrt{\frac{V_2}{V_1}} = \sqrt{\frac{4V}{V}} = 2$

$\Rightarrow \quad T_2 = 2T_1 = 2T$

as $p \propto \frac{1}{T} \Rightarrow$ p-T graph is hyperbola.

25. $\frac{C - 0}{100 - 0} = \frac{F - 32}{212 - 32} = \frac{F - \text{MP}}{\text{BP} - \text{MP}}$

$\Rightarrow$ ice point $= 32°$F and steam point $= 212°$F

26. $p \propto V$ at 'a' $p = p_0$ and $V = V_0$ and at 'b',
$p = 2p_0$ and $V = 2V_0$, $\rho = \frac{m}{V}$

$$\Rightarrow \quad \frac{\rho_b}{\rho_a} = \frac{V_a}{V_b} = \frac{V_0}{2V_0} = \frac{1}{2}$$

$$\Rightarrow \quad \rho_b = \frac{1}{2}\text{ Pa}$$

$$\frac{T_b}{T_a} = \frac{P_b V_b}{P_a V_a} = \frac{2p_0 \cdot 2V_0}{P_0 V_0} = 4$$

$$\Rightarrow \quad T_b = 4T_a$$

$$U \propto T \Rightarrow U_b = 4U_a$$

as $\quad P \propto V \Rightarrow T \propto V^2$

$\Rightarrow$ Parabola passing through origin

27. (a) TKE $= \frac{3}{2} nRT$ is independent of type of gas $\rightarrow$ true.

(b) In one degree of freedom for one mole of gas, $V = \frac{1}{2} RT$

(c) false

(d) false

28. $V \propto T \Rightarrow V = \tan\theta \cdot T$

$$pV = nRT = \frac{m}{M} RT$$

$$\Rightarrow p \cdot T \tan\theta = \frac{mRT}{M} \Rightarrow \tan\theta + \frac{mR}{MP}$$

$$\tan\theta_1 > \tan\theta_2 \Rightarrow \frac{m_1}{p_1} > \frac{m_2}{p_2}$$

$\Rightarrow$ all a, b, c and d are possible.

29. $pV = nRT$

$$\Rightarrow p = \frac{n}{V} RT = \frac{N_a m/V}{M} RT = \frac{m}{M} RT$$

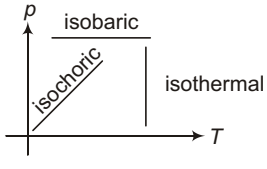
$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3kN_a T}{M}} = \sqrt{\frac{3kT}{M}}$$

$\Rightarrow$ (a) and (d) are correct.

JEE Corner

■ Assertion & Reason

1. Assertion is false.



2. Assertion and reason are both true but reason is not correct explanation of assertion. As at low temperature atoms in molecules are tightly bound such that they cannot oscillate.

3. $pV = nRT = \frac{2}{3} \cdot \text{KE}$

$$\Rightarrow p = \frac{2}{3}\frac{\text{KE}}{V} \Rightarrow p = \frac{2}{3}E.$$

Assertion and reason are both true but reason cannot explain assertion.

4. Internal energy remains same in train frame of reference, so temperature do not change, but KE of gas molecules in ground frame increases.

5. According to equipartition theory, energy is equally distributed for each degree of freedom, so assertion is false.

6. At high temperature and low pressure intermolecular distance is much larger than size of the molecules and intermolecular forces can be neglected. So, assertion and reason are both true but not correct explanation.

7. At 4°C, volume is minimum or density is maximum *i.e.*, liquid will overflow on increasing or decreasing temperature This reason is false.

8. Temperature remains constant as pressure is double and volume is halfed, so internal energy remains constant. So reason partially explains assertion.

9. Assertion and reason are both true but not correct explanation.

10. $V = \frac{nR}{\rho} T = \left(\frac{MR}{Mp}\right) T \Rightarrow$ slope $\propto m$; reason is correct explanation.

Match The Columns

1.

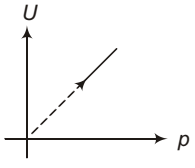
(a) TKE $= \frac{3}{2} nRT = \frac{3}{2} \cdot 2RT = 3RT \rightarrow$ r

(b) RKE $= \frac{2}{2} nRT = \frac{2}{2} \cdot 2RT = 2RT \rightarrow$ p

(c) PE $= \rightarrow$ s

(d) TKE $= \frac{5}{2} nRT = 5RT \rightarrow$ s

2.



$U = p \Rightarrow T \propto \frac{1}{V} \Rightarrow VT =$ constant

$\Rightarrow pT^2 =$ constant and $pV^2 =$ constant

(a) U increases $\Rightarrow T$ increases $\Rightarrow P$ decreases $\rightarrow r$

(b) p increase $\Rightarrow V$ decreases $\rightarrow r$

(c) U increases $\Rightarrow T$ increases $\rightarrow q$

(d) $\frac{T}{V} = \frac{TV}{V^2} = \frac{\text{constant}}{V^2} =$ increase as V decreases $\rightarrow q$

3. $x_1 = 3$, $x_2 = \frac{8}{\pi}$, $x_3 = 2$ and $x_4 = r$

(a) $\rightarrow r$, (b) $\rightarrow s$, (c) $\rightarrow q$, (d) $\rightarrow s$

4.

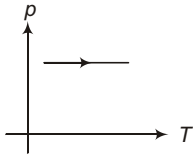
(a) density of water is maximum of 4°C $\rightarrow$ s

(b) depends of change in density of solid and liquid $\rightarrow$ s

(c) depends of change in density of solid and liquid $\rightarrow$ s

(d) $\Delta T = \frac{1}{2} \alpha T \Delta\theta$ increases with $\rightarrow$ increasing temperature p

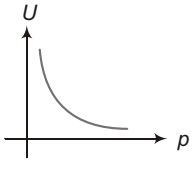
5. (a) $p =$ constant $\rightarrow q$



$\longrightarrow q$

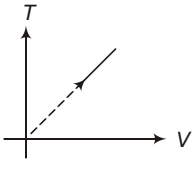
(b) $V \propto T$

$\Rightarrow \frac{1}{\rho} \propto U$



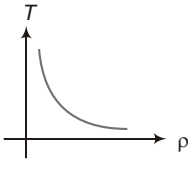
$\longrightarrow R$

(c)



$\longrightarrow p$

(d) $V \propto T \Rightarrow \frac{1}{\rho} \propto T$



$\longrightarrow r$

18. First Law of Thermodynamics

Introductory Exercise 18.1

1. (a) $\Delta W = p\Delta V = -1.7 \times 10^5 (1.2 - 0.8)$ J

$= -6.8 \times 10^4$ J

(b) $\Delta V = 1.1 \times 10^5$ J

$\Rightarrow \Delta Q = \Delta U + \Delta W = -17.8 \times 10^4$ J

i.e., 1.78×10^5 J of heat has flown out of the gas.

(c) No, it is independent of the type of the gas.

2. (a) In p - V graph of cyclic process, clockwise rotation gives positive work and anticlockwise gives negative work. And as loop 1 has greater area than loop 2, that is why total work done by the system is positive.

(b) As in cyclic process change in internal energy is zero, that's why for positive work done by the system, heat flows into the system.

(c) In loop '1' work done is positive so, heat flows into the system and in loop '2' work done is negative so heat flows out of the system.

3. As the box is insulated *i.e.*, no heat exchange takes place with surrounding and as the gas expands against vacuum *i.e.*, zero pressure that's why no work has been done and there is no change in internal energy. Thus, temperature do not change, internal energy and gas does not do any work.

4. $U = \frac{f}{2} nRT = \frac{3}{2} nRT$

$$\Rightarrow \quad n = \frac{2U}{3RT} = \frac{2 \times 100}{3 \times 8.314 \times 300}$$

$= 0.0267$ mole.

5. $\Delta Q = ms\,\Delta\theta = 1 \times 387 \times 30\,\text{J} = 11610\,\text{J}$

$$\Delta V = V\gamma\,\Delta\theta = \frac{m}{\rho} \times 3\,\alpha\,\Delta\theta$$

$$= \frac{1}{8.92 \times 10^3} \times 3 \times 7 \times 10^{-6} \times 30$$

$= 7.1 \times 10^{-8}\ \text{m}^{-3}$

$\Delta W = p\Delta V = 1.01 \times 10^5 \times 7.06 \times 10^{-8}$

$= 7.13 \times 10^{-3}$ J

$\Delta U = \Delta Q - \Delta W = 11609.99$ J

Introductory Exercise 18.2

1. (a) At constant volume,

$\Delta U = 0 \Rightarrow \Delta W = 0$

$\Delta Q = nC_V\,\Delta T$

$$\Rightarrow \quad \Delta = \frac{\Delta Q}{nC_V} = \frac{200}{1 \times \frac{3}{2} \times 8.314} = 16.04\,\text{K}$$

$\therefore T_f = T_i + \Delta T = 300 + 16.04 = 316.04$ K

(b) At constant pressure,

$$\Delta T = \frac{\Delta Q}{nC_p} = \frac{200}{1 \times \frac{5}{2} \times 8.314} = 9.62\ \text{K}$$

$\Rightarrow T_f = 300 + 9.62\,\text{K} = 309.62\,\text{K}$

2. For adiabatic process,

$p V\gamma = \text{constant} = c$ (say)

$$\therefore \int_{V_i}^{V_f} p\,dV = \int_{V_i}^{V_f} \frac{c}{V^\gamma}\,dV = c\int_{V_i}^{V_f} V^{-\gamma}\,dV$$

$$= c\left.\frac{V^{-\gamma+1}}{-\gamma+1}\right|_{V_i}^{V_f} = c\,\frac{V_f^{-\gamma+1} - V_i^{-\gamma+1}}{1-\gamma}$$

$$= \frac{p_f V_f \cdot V_f^{-\gamma+1} - p_i V_i^{\gamma} \cdot V_f^{-\gamma+1}}{1-\gamma}$$

$$= \frac{p_f V_f - p_i V_i}{1-\gamma} = \frac{p_i V_i - p_f V_f}{\gamma - 1}$$ **(Proved)**

3. $\Delta W_{AB} = +500\,\text{J}, \Delta Q_{AB} = +250\,\text{J}$

$\Rightarrow \Delta U_{AB} = -250\,\text{J}$

$\Delta W_{AC} = +700\,\text{J}, \Delta Q_{AC} = +300\,\text{J}$

$\Rightarrow \Delta U_{AC} = -400\,\text{J}$

(a) Path BC is isochoric process, *i.e.*,

$\Delta W_{BC} = 0$

$\therefore \quad \Delta Q_{BC} = \Delta U_{BC} = \Delta U_{AC} - \Delta U_{AB}$

$= -400\,\text{J} - (-250\,\text{J}) = -150\,\text{J}$

(b) $\Delta W_{CDA} = \Delta W_{CD} + \Delta W_{DA}$

$= -800\text{J} + 0 = -800\,\text{J}$

(Work is negative as volume is decreasing)

$\Delta U_{CDA} = \Delta U_{AC} = -\Delta U_{AC} = 400\,\text{J}$

$\Rightarrow \Delta Q_{CDA} = \Delta W_{CDA} + \Delta U_{CDA}$

$= -800\,\text{J} + 400\,\text{J} = -400\,\text{J}$

4. (a) $T = \dfrac{pV}{nR} = \dfrac{1\times10^{-2}\times2\times10^{5}}{1\times8.314}$

$= 240.6\,\text{K}$

(b) $\Delta W = \dfrac{p\Delta V}{\gamma-1} = \dfrac{2\times10^{5}\times5\times10^{-3}}{\frac{5}{3}-1}$

$= \dfrac{10^3}{2/3}\,\text{J}$

5. (a)

$$\Delta K = \frac{p^2}{2m_i} - \frac{p^2}{2m_f} = \frac{p^2}{2}\left(\frac{1}{m_i} - \frac{1}{m_f}\right)$$

$$= \frac{(10\times10^{-3}\times200)^2}{2}\left[\frac{1}{10\times10^{-3}} - \frac{1}{2.01}\right]$$

$$= 2\left[100 - \frac{1}{2}\right] = 199\,\text{J}$$

(b) $\Delta Q = nC_V\,\Delta T \Rightarrow \Delta T = \dfrac{\Delta Q}{nC_V} = \dfrac{\Delta Q}{\frac{m}{M}C_V}$

$$= \frac{M\,\Delta Q}{m\times3R} = \frac{200\times199}{2010\times3\times8.314}$$

$= 0.8\ ^\circ\text{C}$

6. $\Delta W = \Delta W_{AB} + \Delta W_{BC} + \Delta W_{CD} + \Delta W_{DA}$

$$= nRT_1 m\left(\frac{p_1}{p_2}\right) + p_2(V_C - V_{BC})$$

$$+ nRTm\left(\frac{p_2}{p_1}\right) + p_1(V_1 - V_2)$$

$$= nR(T_2 - T_1)\ln\left(\frac{p_2}{p_1}\right) + p_1V_2 - p_1V_1$$

$$+ p_1V_1 - p_1V_2$$

$$= (p_2V_2 - p_1V_1)\ln\left(\frac{p_2}{p_1}\right)$$

7. $\Delta W_{ABCA} = (+)\text{ve} \Rightarrow \Delta W_{AB} = (+)\text{ve},$

$\Delta W_{BC} = 0, \Delta W_{CA} = (-)\text{ve}$

For BC, $\Delta Q = (-)\text{ve} \Rightarrow \Delta U_{BC} = (-)\text{ve}$ and

$\Delta W_{BC} = 0$

For CA, $\Delta U = (-)\text{ve} \Rightarrow \Delta Q_{CA} = (-)\text{ve}$ as $\Delta W_{CA} = (-)\text{ve}$.

	ΔU	ΔW	ΔQ
AB	+	+	+
BC	–	0	–
CA	–	–	–
Total	0	+	+

For AB, as $\Delta U_{ABCA} = 0$ and $\Delta U_{BC} = (-)\text{ve}$,

$\Delta U_{CA} = (-)\text{ve}$

$\Rightarrow \quad \Delta U_{AB} = (-)\text{ve}$

As $\quad \Delta Q_{ABCA} = \Delta W_{ABCA} = (+)\text{ve}$ and

$\Delta Q_{BC} = (-)\text{ve}$

$\Delta Q_{CA} = (-)\text{ve} \Rightarrow \Delta Q_{AB} = (+)\text{ve}$

In isobaric process, $\Delta W = p\Delta V = nR\,\Delta T$

$= 0.2\times8.314\times(300-200) = 166.3\,\text{J}$

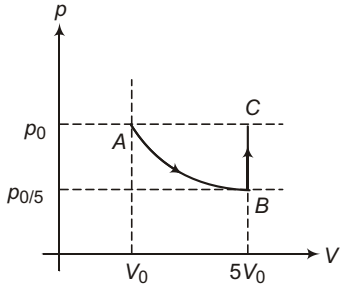
9. $\Delta W = \int p\,dV = \int \alpha V^2 dV = \dfrac{1}{3}\alpha V^3$

$= \dfrac{1}{3}\times5\times1.01\times10^5\times(2^3 - 1^3)$

$= 1.18\times10^6\,\text{J}$

Introductory Exercise 18.3

1.



$$\Delta W_{BB} = nRT \ln\left(\frac{V_B}{V_A}\right) = 3R \times 273 \ln 5$$

$$= 10959 \text{ J}$$

$$\Delta W_{BC} = 0$$

$$\Delta Q = \Delta U + \Delta W$$

$$\Delta U = \Delta Q - \Delta W$$

$$= 80000 - 10959$$

$$= 69041$$

$$T_f = 5T_i = 5 \times 273 \text{ K} = 1365 \text{ K}$$

$$\Delta Q_{ABC} = \Delta Q_{AB} + \Delta Q_{BC}$$

$$= \Delta W_{BC} + 0 + 0 + \Delta U_{BC}$$

$$\Delta Q_{BC} = nC_V \Delta T \Rightarrow C_V = \frac{\Delta Q_{BC}}{n\Delta T}$$

$$= \frac{69041}{3 \times 4 \times 273} = 21.07$$

$$C_p = C_V + R = 29.39$$

$$\Rightarrow \quad \gamma = \frac{C_p}{C_V} = \frac{29.39}{21.07} = 1.4$$

2. $\Delta Q = \Delta U + \Delta W; \Delta Q = nC_p \, \Delta T$

$$\Rightarrow \quad 1600 = 1 \cdot C_p \cdot 72$$

$$\Rightarrow \quad C_p = 22.22$$

$$C_V = C_p - R = 13.9 \Rightarrow \gamma = \frac{C_p}{C_V} = 1.6$$

$$\Delta W = \Delta Q - \Delta U = 1600 - nC_V \, \Delta T$$

$$= 1600 - 1 \times 13.9 \times 72$$

$$= 1600 - 1000.8 \text{ J}$$

$$= 599.2 \text{ J}$$

and $\Delta U = nC_V \Delta T = 1 \times 13.9 \times 72$

$$= 1001$$

$$= 1 \text{ kJ}$$

3. $\Delta W = \frac{1}{2} \Delta p \Delta V$

$$= \frac{1}{2} \times 20 \times 1.01 \times 10^5 \times 1 \times 10^{-3}$$

$$= 10 \times 101 = 1010 \text{ J}$$

$$\therefore \; p = \frac{n\Delta W}{\Delta t} = \frac{100 \times 1010 \text{ J}}{60 \text{ s}}$$

$$= 1.68 \text{ kW}$$

AIEEE Corner

■ Subjectve Questions (Level 1)

1. $\Delta U = \Delta Q - \Delta W = 254 \text{ J} + 73 \text{ J}$

$$= 327 \text{ J}$$

2. (a) $\Delta T = \frac{\Delta Q}{nC_V} = \frac{2\Delta Q}{3nR} = \frac{2 \times 200}{2 \times 1 \times 8.314}$

$$= 16 \text{ K}$$

$$\Rightarrow \quad T_f = T_i + \Delta T = 316 \text{ K}$$

(b) $\Delta T' = \frac{\Delta Q}{nC_p} = \frac{2\Delta Q}{5nR} = \frac{2 \times 200}{5 \times 1 \times 8.314}$

$$= 9.6 \text{ K}$$

$$\Rightarrow T'_f = T_i + \Delta T' = 309.6 \text{ K}$$

3. $\Delta U = nC_V \Delta T$, in adiabatic process,

$\Delta Q = 0$ and $\Delta U = -\Delta W$

where, $\Delta W = \dfrac{nR\,\Delta T}{1-\gamma}$

$\Rightarrow \qquad \Delta U = \dfrac{nR\Delta T}{\gamma - 1}$ for all process.

4. $\Delta V = 0 \Rightarrow \Delta W = 0$

$\therefore \Delta Q = \Delta U = nC_V\,\Delta T = n \cdot \dfrac{5}{2} R\,\Delta T$

$= \dfrac{5}{2}(p_f V_f - p_i V_i) = \dfrac{5}{2}(p_f - V_i)V$

$= \dfrac{5}{2}(5 \times 10^5 - 10^5) \times 10 \times 10^{-3}$

$= \dfrac{5}{2} \times 4 \times 10^5 \times 10^{-2} = 10^4$ J

5. $\Delta Q_1 = \Delta U_1 = nC_V \Delta T = n \cdot \dfrac{5}{2} R\,(3T_i - T_i)$

$= 5\,nRT_i$

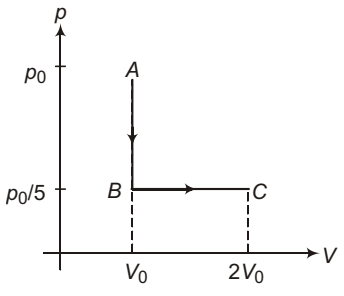
$\Delta Q_2 = nC_p \Delta T = n \cdot \dfrac{5}{2} R\,(6T_i - 3T_i)$

$= 7.5\,nRT_i$

$\therefore c = \dfrac{\Delta Q}{n\,\Delta T} = \dfrac{12.5\,nRT_i}{n\,(6T_i - T_i)} = \dfrac{12.5R}{5} = 2.5R$

6. $\Delta W_{AB} = 0, \Delta W_{BC} = \dfrac{p_0}{2} \times V_0 = \dfrac{1}{2} p_0 V_0$

$= \dfrac{1}{2} \cdot nRT_0 = 300\,R$



$\Delta Q = (\Delta U + \Delta W)_{AB} + (\Delta U + \Delta W)_{BC}$

$= \Delta U_{AB} + \Delta U_{BC} + \Delta W_{BC}$

$= 0 + 300\,R$

(As $T_A = T_C$)

$= 2.49 \times 10^3$ J $= 2.49$ kJ

7. $\Delta U = \Delta Q - \Delta W = 1200$ J $- 2100$ J

$= 900$ J

$\Delta T = \dfrac{\Delta U}{nC_V} = \dfrac{900}{5 \times \dfrac{3}{2} \times 8.314} = 14.43$

$\Rightarrow T_f = T_i - \Delta T = 127°\text{C} - 14.43°\text{C}$

$= 112.6°\text{C}$

8. When gas expands it does positive work on the surrounding and for this purpose heat has to be supplied into the system.

9. $\Delta W = \rho\Delta V = \rho(V_f - V_i)$

$= \rho m\left(\dfrac{1}{\rho_f} - \dfrac{1}{\rho_i}\right) = \rho m\left(\dfrac{1}{1000} - \dfrac{1}{999.9}\right)$

$= -\dfrac{10^5 \times 2 \times 0.1}{1000 \times 999.9} = -0.02$ J

(work done is negative as volume decreases)

$\Delta Q = ms\,\Delta\theta = 2 \times 4200 \times 4$

$= 33600$ J

$\Delta U = \Delta Q - \Delta W = 33600.02$ J

10. $\Delta W = p\Delta V \approx pV_f = p\dfrac{m}{\rho}$

$= \dfrac{10^5 \times 10 \times 10^{-3}}{0.6} = 1666.67$ J

$\Delta Q = ms\Delta\theta + mL$

$= 10^{-2} \times 4200 \times 100 + 10^{-2} \times 2.5 \times 10^6$

$= 4200$ J $+ 25000$ J $= 29200$ J

$\Delta U = \Delta Q - \Delta W = 29200$ J $- 1666.67$ J

$= 27533.33$ J $= 2.75 \times 10^4$ J

11. $\Delta W = p\,\Delta V$

$= 1.013 \times 10^5 \times 1670 \times 10^{-6}$

$= 1.013 \times 167$ J $= 169.2$ J

$\Delta Q = mL = 10^{-3} \times 2.256 \times 10^6$ J

$= 2256$ J

$\therefore \Delta U = \Delta Q - \Delta W = (2256 - 169.2)$ J

$= 2086.8$ J ≈ 2087 J

12. $\Delta W = p\Delta V = -2.3 \times 10^5 \times 0.5$

$= -1.15 \times 10^5$ J

$\Delta U = -1.4 \times 10^5$ J

$\Rightarrow \Delta Q = \Delta U + \Delta W$

$$= -(1.4 + 1.15) \times 10^5 \text{ J}$$
$$= -2.55 \times 10^5 \text{ J}$$

Thus, 2.55×10^5 J of heat flows out of the system and it is independent of the type of the gas.

13. In a cyclic process, $U = 0 \Rightarrow \Delta Q = \Delta W$

(a) $\therefore W_\eta = (Q_1 + Q_2 + Q_3 + Q_4) - (W_1 + W_2 + W_3)$

$= (5960 - 5585 - 2980 + 3645) - (2200 - 825 - 1100)$

$= 1040 - 275 = 765 \text{ J}$

(b) $\eta = \dfrac{\text{work done}}{\text{heat supplied}} = \dfrac{1040}{9605} = 10.83\%$

14. (a) $\Delta W = \dfrac{1}{2}\overrightarrow{\mathbf{AB}} \times \overrightarrow{\mathbf{AC}} = \dfrac{1}{2} \times 2p_0 \times V_0$

$= p_0 V_0$

(b) $T_C = \dfrac{2\, p_0 V_0}{nR}$ and $T_A = \dfrac{p_0 V_0}{nR}$

$\Rightarrow \Delta Q_{CA} = nC_p \Delta T = -nC_p \cdot \dfrac{p_0 V_0}{nR}$

$= -\dfrac{5}{2} R \cdot \dfrac{p_0 V_0}{R} = -\dfrac{5}{2} p_0 V_0$

$T_B = \dfrac{3 p_0 V_0}{nR}$, $\Delta Q_{AB} = nC_V \Delta T$

$= n \times \dfrac{3}{2} R \left(\dfrac{3 p_0 V_0}{nR} - \dfrac{p_0 V_0}{nR} \right)$

$= \dfrac{3}{2} \times 2\, p_0 V_0 = 3 p_0 V_0$

(c) $\Delta Q_{AB} + \Delta Q_{BC} + \Delta Q_{CA} = \Delta W$

$= 3 p_0 V_0 + \Delta Q_{BC} - \dfrac{5}{2} p_0 V_0 = p_0 V_0$

$\Rightarrow \quad \Delta Q_{BC} = \dfrac{p_0 V_0}{2}$

(d) Temperature is maximum at a point D lying somewhere between B and C where the product pV is maximum.

$$p = -\frac{2p_0}{V_0} + 5p_0$$

$$\Rightarrow \quad pV = \left(\frac{-2p_0}{V_0} V + 5p_0 \right) V$$

$$= -\frac{2p_0}{V_0} V^2 + 5p_0 V$$

For $pV = \text{maximum}$ $\dfrac{d}{dV}(pV) = 0$

$$\Rightarrow \quad -2V \cdot \frac{2p_0}{V_0} + 5p_0 = 0$$

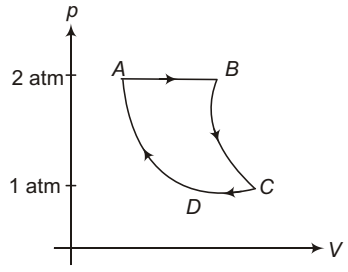
$$\Rightarrow \quad V = \frac{5V_0}{4}$$

$$p = -\frac{2p_0}{V_0} \cdot \frac{5V_0}{4} + 5p_0 = \frac{5}{2} p_0$$

$$\therefore \quad T_{\max} = \frac{(pV)_{\max}}{nR}$$

$$\therefore \quad p = \frac{\frac{5}{2} p_0 \cdot \frac{5}{4} V_0}{1 \cdot R} = \frac{25 p_0 V_0}{8R}$$

15. $V_A = \dfrac{nRT_A}{p_A} = \dfrac{2R \times 300}{2 \times 10^5} = 3 \times 10^{-3} R$



$V_B = \dfrac{2R \times 400}{2 \times 10^5} = 4 \times 10^{-3} R$,

$V_C = \dfrac{2R \times 400}{10^5} = 8 \times 10^{-3} R$

$V_0 = \dfrac{2R \times 300}{10} = 6 \times 10^{-3} R$

$\Delta W = 2 \times 10^5 (4 - 3) \times 10^{-3} R + 2R \times 400 \ln\left(\dfrac{8}{4}\right) + 1 \times 10^5 (6 - 8) \times 10^{-3}$ K $+ 2R \times 300 \ln\left(\dfrac{3}{6}\right)$

$\therefore \ \Delta W = 200R + 800R \ln 2 - 200R - 600R \ln 2$

$= 2000R \ln 2 = 1153 \text{ J}$

As $\Delta Q = \Delta W = 1153$ J and $\Delta U = 0$ cyclic process.

16. $\Delta W = \frac{1}{2}\left(\frac{3V_0}{2} - \frac{V_0}{2}\right)(p_B - p_0)$

$$+ \frac{1}{2}\left(\frac{3V_0}{2} - \frac{V_0}{2}\right)(p_0 - p_0)$$

$$= \frac{1}{2}V_0(p_B - p_0) + \frac{1}{2}V_0(p_0 - p_D)$$

$$= \frac{1}{2}V_0(p_B - p_D)$$

where, $p_B = p_0 + \frac{p_0}{V_0} \cdot \frac{V_0}{2} = \frac{3p_0}{2}$

and $p_D = p_0 - \frac{p_0}{V_0} \cdot \frac{V_0}{2} = \frac{p_0}{2}$

$$\therefore \Delta W = \frac{1}{2}V_0\left(\frac{3}{2}p_0 - \frac{1}{2}p_0\right) = \frac{1}{2}p_0V_0$$

$$\Delta W_{ABC} = p_0\left(\frac{3V_0}{2} - \frac{V_0}{2}\right) + \frac{1}{2}\left(\frac{3V_0}{2} - \frac{V_0}{2}\right)$$

$$(p_B - p_0) = p_0V_0 + \frac{1}{2}V_0\left(\frac{3}{2}p_0 - p_0\right)$$

$$= \frac{5}{4}p_0V_0$$

$$\Delta U_{ABC} = nC_V(T_C - T_A)$$

$$= n \cdot \frac{3}{2}R\left[\frac{p_0\frac{3V_0}{2}}{nR} - \frac{p_0\frac{V_0}{2}}{nR}\right] = \frac{3}{2}p_0V_0$$

$$\therefore \Delta Q_{\text{supplied}} = \frac{5}{4}p_0V_0 = \frac{11}{4}p_0V_0$$

$$\eta = \frac{\frac{1}{2}p_0V_0}{\frac{11}{4}p_0V_0} = \frac{2}{11} = 0.1818 = 18.18\%$$

17. (a) As the cyclic process is clockwise *i.e.,* work done is positive, so heat is absorbed by the system.

(b) In cyclic process work done is equal to the net heat absorbed (as change in internal energy is zero) so, work done in one cycle is 7200 J.

(c) In anticlockwise rotation, work done is negative and heat is liberated by the system, and its magnitude is 7200 J.

18. (a) As area under clockwise loop is more than that at anticlockwise loop, so network done is positive.

(b) In loop I work done is positive and in loop II work done is negative.

(c) As network done in one cycle is positive so heat flows into the system.

(d) In loop I heat flows into the system and in loop II heat flows out of the system.

19. $T_A = \frac{p_AV_A}{nR} = \frac{1.01 \times 10^5 \times 22.4}{10^3 \times 8.314}$

$$= 273\text{ K}$$

$$T_B = \frac{p_BV_A}{nR} = \frac{2p_AV_A}{nR} = 2T_A$$

$$= 546\text{ K} = T_c$$

$$V_c = \frac{nRT_c}{p_c} = \frac{nRT_c}{p_A} = \frac{nRT_B}{p_A}$$

$$= \frac{2nRT_A}{p_A} = 2V_A = 44.8\text{ m}^3$$

20. (a) $\Delta W = AB \times BC$

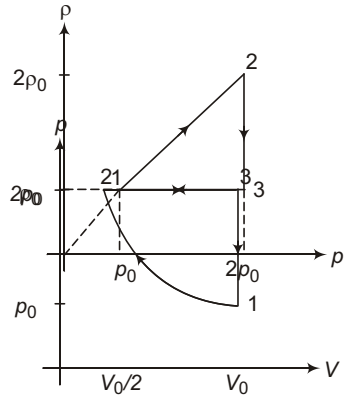
$$= (4 - 1.5) \times 10^{-6} \times (4 - 2) \times 10^5$$

$$= 2.5 \times 0.2 = 0.5\text{ J}$$

(b) $\Delta Q = \Delta W$ as $\Delta U = 0$ in a cycle

$\Rightarrow \Delta Q = 0.5\text{ J}$

21. As $\rho \propto \frac{1}{V}$



(a) $\Delta W_{12} = nRT_0 \ln\left(\frac{V_0/2}{V_0}\right)$

$= -p_0V_0 \ln 2 = -p_0 \frac{M}{\rho_0} \ln 2$

$\Delta W_{23} = 2p_0\left(V_0 - \frac{V_0}{2}\right) = p_0V_0$

$= p_0 \frac{M}{\rho_0}; \Delta W_{31} = 0$

(b) $\Delta Q_{231} = \Delta Q_{23} + \Delta Q_{31}$

$= nC_V\Delta T_{23} + \Delta W_{23} + nC_V\Delta T_{31}$

$= n \times \frac{3}{2}R\left(\frac{2p_0V_0}{nR} - \frac{2p_0\frac{V_0}{2}}{nR}\right) + p_0V_0$

$+ n \times \frac{3}{2}R\left(\frac{p_0V_0}{nR} - \frac{2p_0 \times \frac{V_0}{2}}{nR}\right) +$

$p_0V = \frac{3}{2}p_0V_0 + p_0V_0 = \frac{5}{2}p_0V_0$

$\therefore$ Heat rejected $= \Delta Q_{231} - \Delta W$

$= \frac{5}{2}p_0V_0 - p_0V_0 + p_{V_0} \ln 2$

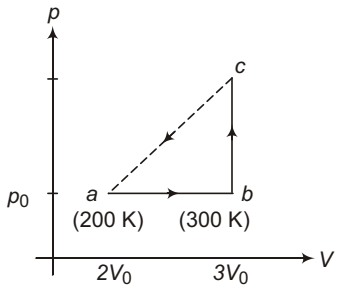
$= \frac{3}{2}p_0V_0 + p_0V_0 \ln 2$

$= p_0V_0\left(\frac{5}{2} + \ln 2\right) - \frac{p_0M}{p_0}\left(\frac{3}{2} - \ln 2\right)$

(c) $\eta = \frac{\text{work done}}{\text{heat supplied}} = \frac{\Delta W}{\Delta Q_{231}}$

$= \frac{p_0V_0 - p_0V_0 \ln 2}{\frac{5}{2}p_0V_0} = \frac{2}{3}(1 - \ln 2)$

22. $\Delta W_{AB} = p_0(3V_0 - 2V_0) = p_0V_0;$



$\Delta W_{BC} = 0, \Delta W_{CA} = ?$

$\Delta Q = \Delta W_{AB} + \Delta W_{BC} + \Delta W_{CA}$

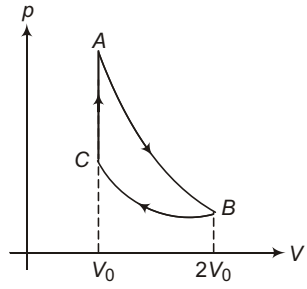
$-800 \text{ J} = P_0V_0 + 0 + \Delta W_{CA}$

$\Rightarrow \Delta W_{CA} = -800 \text{ J} - p_0V_0$

$= -800 \text{ J} - \frac{1}{2}nRT_A$

$\therefore \Delta W_{CA} = -800 \text{ J} - 200R = -2463 \text{ J}$

23. $\Delta W_{AB} = \frac{p_BV_B - p_AV_A}{1-\gamma}$



$= \frac{3}{2}(p_AV_A - p_BV_B)$

$= \frac{3}{2}nR(T_A - T_B) = \frac{3}{2}nRT_B\left(\frac{T_A}{T_B} - 1\right); TV^{\gamma-1}$

$= \frac{3}{2}nRT_B\left(\left(\frac{2}{T}\right)^{\frac{5}{3}-1} - 1\right) = \frac{3}{2}nRT_B(2^{2/3} - 1)$

$\Delta W_{BC} = nRT_B \ln\left(\frac{V_0}{2V_0}\right)$

$= -nRT_B \ln 2$ and $\Delta W_{CA} = 0$

Heat Supplied

$\Delta Q_{CA} = \Delta U_{CA} = \frac{3}{2}nR(T_A - T_C)$

$= \frac{3}{2}nR(T_A - T_B)$

$= \frac{3}{2}nR\,T_B\left(\frac{T_A}{T_D} - 1\right) = \frac{3}{2}nRT_B(2^{2/3} - 1)$

$\therefore \quad \eta = \frac{\Delta W}{\Delta Q_{CA}}$

$= \frac{\frac{3}{2}nRT_B(2^{2/3} - 1) - nRT_B \ln 2 + 0}{\frac{3}{2}nRT_B(2^{2/3} - 1)}$

$$\eta = 1 - \frac{2}{3} \cdot \frac{\ln 2}{2^{2/3} - 1} = 1 - 0.7867 = 0.213$$

$$= 21.3\%$$

■ Objective Questions (Level 1)

1. $U = nC_V T = \frac{3}{2} \times 1 \times RT = \frac{3}{2} RT$

$\Rightarrow \quad T = \frac{2U}{3R}$

$T_D = \frac{2V_0}{3R} = 300\,\text{K} \Rightarrow U_0 = 450R,$

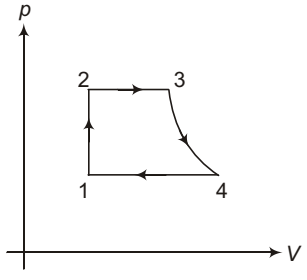
$T_A = \frac{4V_0}{3R} = 600\,\text{K}$

$\Delta W = \Delta W_{AB} + W_{CD} = nRT_A \ln\left(\frac{2V_0}{V_0}\right)$

$+ nRT_D \ln\left(\frac{V_0}{2V_0}\right) = nR(T_A - T_D)\ln 2$

$= 1 \times R \times (600 - 300)\ln 2$

$= 300R \ln 2 = \Delta Q$



2. $\Delta W_{12} = p\Delta V = nR\Delta T$

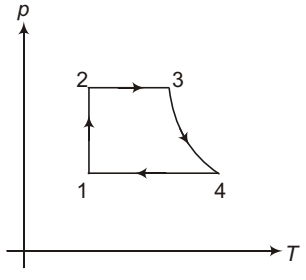
$= 2R \times 300 = 600R$

$\Delta W_{23} = ?; \Delta W_{31} = 0.$

As, $\Delta Q = \Delta W_{12} + \Delta W_{23} + \Delta W_{31}$

$= -300\text{ J} = 600R + \Delta W_{23} + 0$

$\Rightarrow \Delta W_{23} = -300\text{ J} - 600R = -5288\text{ J}$



3. $nC_p \Delta T_1 = nC_V \Delta T_2$

$\Rightarrow \quad \frac{7}{2} \times 30 = \frac{5}{2} \Delta T_2$

$\Rightarrow \quad \Delta T_2 = 42\,\text{K}$

4. $TV^{n-1} = \text{constant}$

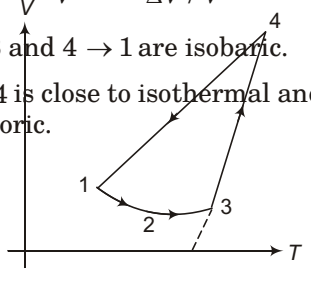
$\Rightarrow pV \cdot V^{n-1} = pV^n = \text{constant}$

$\ln p + n \ln V = \ln c$

$\frac{\Delta p}{p} = -n\frac{\Delta V}{V} \Rightarrow -\frac{\Delta p}{\Delta V / V} = np = B$

5. $2 \to 3$ and $4 \to 1$ are isobaric.

$3 \to 4$ is close to isothermal and $1 \to 2$ is isochoric.



6. $W = \int p\,dV = \int kV\,dV = \frac{k}{2}V^2$

$= \frac{1}{2}pV = \frac{1}{2}nR(T_2 - T_1) = \frac{R}{2}(T_2 - T_1)$

7. $p \propto V^2, W = \int p\,dV = \int kV^2\,dV$

$= \frac{1}{3}kV^3 = \frac{1}{3}pV$

$= \frac{1}{3}nR(T_f - T_i) = (+)\text{ve}$

8. $\Delta W = -nRT \ln\left(\frac{V_f}{V_i}\right)$

$= -nRT \ln\left(\frac{1}{2}\right) = nRT \ln 2$

9. $\Delta U = 600\text{ J} - 150\text{ J} = 450\text{ J}$

$= nC_V \Delta T = \frac{3}{2}R \cdot n\Delta T$

$$C = \frac{\Delta Q}{n\Delta T} = \frac{600 \text{ J}}{\frac{450 \text{ J}}{\frac{3}{2}R}} = \frac{3}{2}R \times \frac{600}{450}$$

$$= \frac{3}{2}R \times \frac{4}{3} = 2R$$

10. $\Delta W_1 = (+)$ ve, $\Delta W_2 = 0$, $\Delta W_3 = (-)$ ve

and $\Delta U_1 = \Delta U_2 = \Delta U_3$

as $\Delta Q = \Delta U + \Delta W \Rightarrow Q_1 > Q_2 > Q_3$

11. $U = 2p_0 2V_0 - 2p_0 V_0 = 2p_0 V_0$

and $\Delta W = p_0(2V_0 - V_0) = p_0 V_0$

$\therefore \quad \Delta Q = \Delta U + \Delta W = 3p_0 V_0$

12. In adiabatic compression, temperature of the gas increases and as $pV \propto T$ so, pV increases.

13. As $\Delta W_1 < \Delta W_2$ while $\Delta U_1 = \Delta U_2$

$\Rightarrow \quad \Delta Q_1 < \Delta Q_2$

$\Rightarrow \quad C_1 < C_2$

$\Rightarrow \quad \frac{C_1}{C_2} < 1$

14. $\Delta W = nR(4T - T) + \frac{nR(5T - 4T)}{1-\gamma}$

$+ nR(3T - 5T) + \frac{nR(T - 3T)}{1-\gamma}$

$= 3nRT - 2nRT + \frac{nRT}{1-\gamma} - \frac{2nRT}{1-\gamma}$

$= nRT + \frac{nRT}{\gamma - 1}$

$= \frac{nRT}{\gamma - 1}(\gamma - 1 + 1) = \frac{\gamma}{\gamma - 1}nRT$

$= \frac{5/3}{5/3 - 1} \cdot 1RT = 2.5RT$

15. $Up = \text{constant}$

$= \frac{3}{2}nRT \cdot \frac{nM}{V} = \frac{3}{2}n^2 MR\frac{T}{V}$

$\Rightarrow T \propto V$ *i.e.,* isobaric process.

$\frac{\Delta U}{\Delta W} = \frac{\Delta U}{\Delta Q - \Delta U} = \frac{3/2}{\frac{5}{2} - \frac{3}{2}} = \frac{3}{2}$

$= \frac{C_V}{C_p - C_V} = \frac{C_V}{R}$

16. $\Delta W = 50 \times (0.4 - 0.1) + \frac{1}{2} \times 50 \times (0.2 - 0.1)$

$= 15 + 2.5 = 27.5$ J

$\Delta U = 2.5$ J

$\Rightarrow \quad \Delta Q = \Delta U + \Delta W = 20$ J

17. $W_1 = \int_{V_0}^{2V_0} pdV = p(2V_0 - V_0) = pV_0$

$W_2 = \int_{V_0}^{2V_0} kVdV = \frac{1}{2}kV^2$

$= \frac{1}{2}k(4V_0^2 - V_0^2) = \frac{3}{2}kV_0^2 = \frac{3}{2}PV_0$

$\Rightarrow \quad W_1 < W_2$

18. $\Delta W = \pi r_1 r_2 = \pi ab$

$= \pi\frac{r_2 - r_1}{2} \cdot \frac{(p_2 - p_1)}{2}$

$= \frac{\pi}{4}(p_2 - p_1)(V_2 - V_1)$

19. $W = \int PdV = \int \frac{nRT}{V - b}dV = nRT\int \frac{dx}{x}$

$x = V - b$

$dx = dV$

$= nRT \ln x = nRT \ln (V - b)\Big|_V^{2V}$

$= nRT [\ln (2V - b) - \ln (V - b)]$

$= nRT \ln \left|\frac{2V - b}{V - b}\right| = RT \ln \left|\frac{2V - b}{V - b}\right|$

as $n = 1$ mole

20. AB is isochoric process, so, $\Delta W_{AB} = 0$

BC is isothermal process, so,

$\Delta W_{BC} = nRT_2 \ln\left(\frac{V_2}{V_1}\right) = RT_2 \ln\left(\frac{V_2}{V_1}\right)$

CA is close to isobaric process, so,

$\Delta W_{CA} = nRT = nR(T_1 - T_2)$

$= R(T_1 - T_2)$

21. $\Delta Q = \Delta U + \Delta W = -\Delta Q + \Delta W$

$\Rightarrow \Delta W = 2\Delta Q$

$\Delta U = nC_V \Delta T = n\frac{f}{2}R\Delta T = \frac{n}{\gamma - 1}R\Delta T;$

$$\gamma = \frac{f+2}{f} = 1 + \frac{2}{f}$$

$$f = \frac{2}{\gamma - 1}$$

$$\Delta W = \int p\,dV = 2\,\Delta Q - \frac{2n}{\gamma - 1} R\Delta T$$

$$= \frac{2\,nR\Delta T}{1-\gamma} = \frac{nR\Delta T}{1-a} \text{ for polytropic}$$

process with $pV^a = \text{constant}$

$$\therefore \quad \frac{2}{1-\gamma} = \frac{1}{1-a} \Rightarrow 1 - a = \frac{1-\gamma}{2}$$

$$\Rightarrow \quad 1 - \frac{1}{2} + \frac{\gamma}{2} = a$$

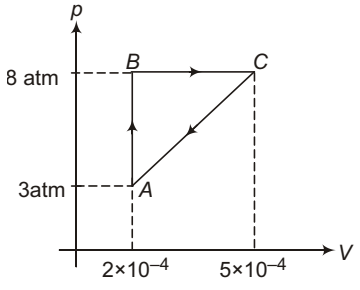
$$\text{or} \quad a = \frac{1}{2} + \gamma = \frac{1+\gamma}{2}$$

$$\therefore \quad pV^a = \text{constant}$$

$$= TV^{a-1} = TV^{\frac{1+\gamma}{2} - 1}$$

$$= TV^{\frac{\gamma - 1}{2}} = \text{constant}$$

22. $\Delta W_{AB} = 0, \Delta U_{AB} = 600$ J



$$\Delta W_{BC} = 8 \times 10^5 (5-2) \times 10^{-4} = 240 \text{ J}$$

$$\Delta U_{BC} = Q_{BC} - W_{BC} = 200 - 240$$

$$= -40 \text{ J}$$

$\Delta U = \Delta U_{AB} + \Delta U_{BC} + \Delta U_{CA} = 0$ in cyclic process.

$$\therefore \Delta U_{CA} = -\Delta U_{AB} - \Delta U_{BC}$$

$$= -600 \text{ J} + 40 \text{ J} = -560 \text{ J}$$

23. Starting and ending points along x-axis in graph are not clear, so nothing can be said about the magnitude of work.

It can only be said that work done in ABC is negative and that in DEF is positive. Looking at the graph, area can be assumed to be equal so,

$$W_{DEF} = -W_{ABC}.$$

24. $\Delta W_{\text{isobaric}} = p\Delta V = p(2V - V) = pV$

$$\Delta W_{\text{isothermal}} = nRT \ln\left(\frac{2V}{V}\right) = pV \ln 2$$

$$= 0.693\, pV$$

$$\Delta W_{\text{adiabatic}} = \frac{p_f \cdot 2V - p_i V}{1-\gamma}$$

$$= \frac{p_i\left(\frac{V}{2V}\right)^r \cdot 2V - p_i V}{1-\gamma} = \frac{pV(2^{1-r} - 1)}{1-r}$$

$$= pV\left(\frac{1 - 2^{1-r}}{r-1}\right) = pV\left(\frac{1 - 4^{-1/3}}{2/3}\right)$$

$$= 0.55\, pV$$

So, work done is minimum in adiabatic process.

25. $\Delta Q = \Delta U + \Delta W$

$$\frac{7}{2} RT_0 = 10 \times \frac{5}{2} R\Delta T + 10R\Delta T = 35R\Delta T$$

$$T_0 = 100T = 10(T - T_0)$$

$$\Rightarrow \quad 11T_0 = 10T$$

$$\Rightarrow \quad T = 1.1T_0$$

$$\frac{pV_0}{RT_0} = \frac{pV}{R \times 1.1T_0}$$

$$\Rightarrow \quad V = \frac{11}{10} V_0 = 1.1V_0$$

26. $\Delta W = (3p_0 - p_0)(2V_0 - V_0) = 2p_0V_0$

$$\Delta Q_{\text{supplied}} = n\frac{3}{2}R\left(\frac{3p_0V_0}{nR} - \frac{p_0V_0}{nR}\right)$$

$$+ n \cdot \frac{5}{2}R\left(\frac{3p_0 \cdot 2V_0}{nR} - \frac{3p_0V_0}{nR}\right)$$

$$= \frac{3}{2}nR \cdot \frac{2p_0V_0}{nR} + \frac{5}{2}nR\frac{3p_0V_0}{nR}$$

$$= 3p_0V_0 + \frac{15}{2}p_0V_0 = \frac{21}{2}p_0V_0$$

$$\eta = \frac{\Delta W}{\Delta r} = \frac{2p_0V_0}{\frac{21}{2}p_0V_0} = \frac{4}{21}$$

27. $\Delta W_{12} < \Delta W_{13}$ can be seen from area under the curve, while $\Delta V_1 = \Delta V_2$

$\Rightarrow \quad \Delta Q_{12} < \Delta Q_{13}$

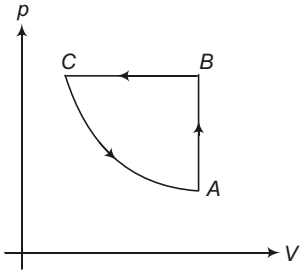
$\Rightarrow \quad Q_2 < Q_1$ or $Q_1 > Q_2$

28. $\Delta W_{CA} = p_0(V_0 - 2V_0) = -p_0 V_0$

and $\quad \Delta U_{CA} = -\frac{3}{2} p_0 V_0$

$\Rightarrow \quad \Delta Q_{CA} = -\frac{5}{2} p_0 V_0$

29. $\Delta Q_{AB} = 200\,\text{kJ} = nC_V \Delta T$;



$\Delta U_{BC} = -100\,\text{kJ}$ and $\Delta W_{BC} = -50\,\text{kJ}$

$\Delta W_{AB} = 0 \Rightarrow \Delta U_{AB} = 200\,\text{kJ}, \Delta Q_{CA} = 0$

$\Delta U_{ABC} = \Delta U_{AB} + \Delta U_{BC} + \Delta U_{CA} = 0$

or $200\text{ kJ} - 100\text{ kJ} + \Delta U_{CA} = 0$

$\Delta U_{CA} = -100\text{ kJ}$

$\Delta Q_{AB} + \Delta Q_{BC} + \Delta Q_{CA}$

$= 200\text{ kJ} + (-100\text{ kJ} - 50\text{ kJ}) + 0$

$= 50\,\text{kJ}$

$\Delta W_{AB} + \Delta W_{BC} + \Delta W_{CA}$

$= 0 + 200\text{ kJ} + \Delta W_{CA}$

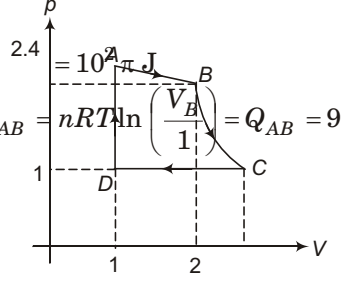
$= \Delta Q_{ABC} = 50\text{ kJ}$

$\therefore \Delta W_{CA} = -150\text{ kJ}$

30. $\Delta Q = \Delta W = \pi ab = \pi \times \frac{20 \times 10^{-3}}{2} \times \frac{20 \times 10^3}{2}$

$= 10^2 \pi$ J

31. $\Delta W_{AB} = nRT \ln\left(\frac{V_B}{1}\right) = Q_{AB} = 9 \times 10^4$ J



$\Rightarrow \quad 800\,T \ln V_B = 9 \times 10^4$ J

$\Rightarrow \quad T \ln V_D = \frac{225}{2}$

$\Delta W_{ABCD} = \Delta W_{AB} + \Delta W_{BC} + \Delta W_{CD} + \Delta W_{DA}$

$= nRT \ln\left(\frac{V_B}{V_A}\right) + \frac{p_C p_C - p_B p_B}{1-\gamma} + p_C(V_D - V_C) + 0$

$= 9 \times 10^4 + \frac{10^5 - nRT_B}{1 - \frac{5}{3}} + 10^5(2-1)$

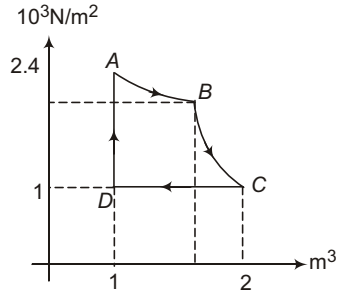
$= 19 \times 10^4 - \frac{3}{2}(10^5 - 800\,T_B)$

$= 4 \times 10^4 + 1200\,T_B$

$= 4 \times 10^4 + 1200 \times \frac{2.4 \times 10^5 \times 1}{100 \times 8}$

$= 4 \times 10^5$ J

31. $\Delta W = \Delta W_{AB} + \frac{p_C V_C - p_B V_B}{1-\gamma} + \Delta W_{CD}$



$= 9 \times 10^4 + \frac{2 \times 10^5 - 9 \times 10^4}{1 - 5/3} - 1 \times 10^5$

$= 9 \times 10^4 + \frac{3}{2} \times 11 \times 10^4 - 10 \times 10^4$

$= \left(\frac{33}{2} - 1\right) \times 10^4 = 15.5 \times 10^4$

32. $\Delta W = \int p\,dV = \int kV\,dV = \frac{1}{2} kV^2$

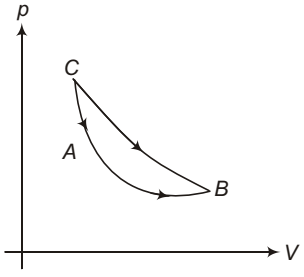
$$= \frac{1}{2} pV = \frac{1}{2} nRT_0 = \frac{1}{2} RT_0$$

$$\Delta U = nC_V \Delta T = 1 \cdot \frac{3}{2} RT_0$$

$$\Rightarrow \Delta Q = \left(\frac{3}{2} + \frac{1}{2}\right) RT_0 = 2RT_0$$

33. $pT = \text{constant} = p\dfrac{pV}{nR} = \dfrac{p^2 V}{nR}$

$$\Rightarrow \quad p^2 V = \text{constant}$$



$$\therefore \quad p_0^2 V_0 = \left(\frac{p_0}{2}\right)^2 V \Rightarrow V = 4V_0$$

$$\Rightarrow \quad T = \frac{\frac{p_0}{2} \cdot 4V_0}{nR} = 2\frac{p_0 V_0}{nR} = 2T_0$$

$$\therefore \quad \Delta U = nC_V \Delta T = 2 \times \frac{3}{2} R\,(2T_0 - T_0)$$

$$= 3R \cdot \frac{p_0 V_0}{2R} = \frac{3}{2} p_0 V_0$$

35. $\Delta W_{BC} = nRT_0 \ln\left(\dfrac{V_C}{V_B}\right)$

$$= nRT_0 \ln\left(\frac{p_B}{p_C}\right) = 2 \cdot nRT_0 \ln\left(\frac{V_B}{V_A}\right)$$

$$= 2nRT_0 \ln\left(\frac{p_A}{p_B}\right)$$

$$\therefore \quad \ln\left(\frac{p_B}{p_C}\right) = \ln\left(\frac{p_0}{p_0/2}\right)^2 = \ln 4$$

$$\Rightarrow \quad p_B = 4p_C$$

$$\Rightarrow \quad p_C = \frac{p_B}{4} = \frac{p_0}{8}$$

36. As, $\Delta W_a > \Delta W_b \Rightarrow \Delta W_1 > \Delta W_2$

while, $\Delta U_1 = \Delta U_2 \Rightarrow \Delta Q_1 > \Delta Q_2$

37. $\eta = 1 - \dfrac{T_{\text{sink}}}{T_{\text{source}}} = 1 - \dfrac{300}{600}$

$$= 1 - \frac{1}{2} = 0.5 = 50\%$$

38. As the volume is adiabatically decreased, temperature of the gas increases and as the time elapsed, temperature normalizes *i.e.*, decreases and so pressure also decreases.

39. As the compression is quick , the process is adiabatic while leads to heating of the gas.

40. $pV^\gamma = \text{constant}$

$$= \frac{nRT}{V} V^\gamma = nRTV^{\gamma - 1}$$

$$\Rightarrow TV^{\gamma - 1} = \text{constant}$$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma - 1} = \left(\frac{L_2}{L_1}\right)^{\frac{5}{3} - 1} = \left(\frac{L_2}{L_1}\right)^{\frac{2}{3}}$$

41. $pV^\gamma = \text{constant} = p\left(\dfrac{n\gamma T}{p}\right)^\gamma$

$$\Rightarrow \quad p^{1-\gamma} T^\gamma = \text{constant}$$

$$\Rightarrow \quad p^{\gamma - 1} \propto T^\gamma$$

$$\Rightarrow \quad p \propto T^{\frac{\gamma}{\gamma - 1}}$$

As $\dfrac{\gamma}{\gamma - 1} = \dfrac{7/5}{\frac{7}{5} - 1} = \dfrac{7}{2}$ for diatom gases.

$$\therefore \quad p \propto T^{3.5} \Rightarrow \alpha = 3.5$$

42. $pV^x = \text{constant}$, $\Delta W = \dfrac{nR\Delta T}{1 - x}$,

$$\Delta U = n \cdot \frac{5}{2} R\Delta T$$

$$C = \frac{\Delta Q}{n\Delta T} = \frac{\frac{nR\Delta T}{1 - x} + \frac{5}{2} nR\Delta T}{n\Delta T}$$

$$= \frac{5}{2} R + \frac{R}{1 - x} < 0$$

$$\frac{5}{2} R < \frac{R}{x - 1} \Rightarrow x - 1 < \frac{2}{5}$$

$x < \frac{7}{5} \Rightarrow x < 1.4$ but $x > 1$ as for $x < 1$,

C will become positive.

$\therefore \quad 1 < x < 1.4$

43. $C_V = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2} = \frac{13}{6}R$

(a) $\frac{2 \times \frac{5}{2}R + 4 \times \frac{5}{2}R}{2+4} = \frac{15}{6}R$

(b) $\frac{2 \times \frac{5}{2}R + 4 \times \frac{3}{2}R}{2+4} = \frac{11}{6}R$

(c) $\frac{2 \times \frac{3}{2}R + 4 \times \frac{5}{2}R}{2+4} = \frac{13}{6}R$ and

(d) $\frac{2 \times \frac{6}{2}R + 4 \times \frac{3}{2}R}{2+4} = \frac{12}{6}R$

Passage 44 & 45

44. $\Delta W_{ABCA} = \frac{1}{2} \times p \times V = \frac{pV}{2} = \Delta Q_{\text{net}}$

45. $CA \rightarrow$ isobaric and $BC \rightarrow$ isochoric,

$\therefore \quad \frac{C_p}{C_v} = \gamma = \frac{5}{3}$

46. $pV^{\gamma} = \text{constant} = p\left(\frac{nRT}{p}\right)^{\gamma}$

$\Rightarrow \quad p^{1-\gamma}T^{\gamma} = \text{constant}$

$\Rightarrow \quad T \propto p^{\frac{\gamma-1}{\gamma}}$

$\therefore \quad T \propto p^{\frac{5/3-1}{5/3}} \Rightarrow T \propto p^{2/5}$

$\therefore \quad \frac{T_B}{T_A} = \left(\frac{p_B}{p_A}\right)^{2/5} = \left(\frac{2p_c}{3p_c}\right)^{2/5} = 0.85$

$\therefore \quad T_B = 0.85 T_A = 850 \text{ K}$

47. $\Delta W_{AB} = \frac{nRT}{1-\gamma} = \frac{1 \times \frac{25}{3} \times 150}{\frac{5}{3} - 1}$

$= 75 \times 25 \text{ J} = 1875 \text{ J}$

48. $\Delta W_{BC} = 0, \Delta Q_{BC} = \Delta U_{BC}$

$= n \times \frac{3}{2}R(T_C - T_B)$

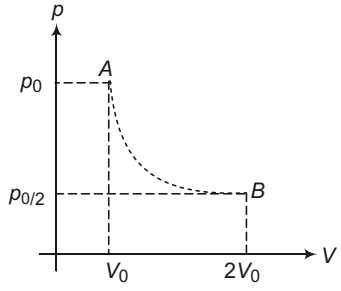
$= n \times \frac{3}{2}R\left(\frac{p_C V}{nR} - \frac{p_B V}{nR}\right)$

$= \frac{3}{2}\left(\frac{1}{3}p_A - \frac{2}{3}p_A\right)V$

$= -\frac{1}{2}p_A V = -\frac{1}{2} \times \frac{3}{2}p_B V = -\frac{3}{4} \cdot nRT_B$

$= -\frac{3}{4} \times 1 \times \frac{25}{3} \times 850 = -5312.5 \text{ J}$

49. $\Delta W_{AB} = (+)\text{ve}, T_A = T_B$



$p = -\frac{p_0}{2V_0}V + \frac{3}{2}p_0$

$\Rightarrow \quad \frac{nRT}{V} = -\frac{p_0}{2V_0}V + \frac{3}{2}p_0$

or $\quad T = -\frac{p_0}{2nRV_0}V^2 + \frac{3p_0}{2nR}V_0$

$\Rightarrow \quad y = ax^2 + bx$ is parabola .

Again, $p = -\frac{p}{2V_0} \cdot \frac{nRT}{p} + \frac{3}{2}p_0$

$\Rightarrow$ is also equation of parabola.

While going from A to B temperature first increases ad than decreases.

50. $pV^2 = \text{constant}$

$\Delta W = \int p\,dV = \int \frac{k}{V^2}dV = k\left(-\frac{1}{V}\right)$

$= -pV\Big|_i^f = p_i V_i - p_f V_f$

$= nR(T_i - T_f) = -nR(T_f - T_i) = (-)\text{ve}$

as $T_f > T_i$

as $T_i < T_f \Rightarrow U_i < U_f$

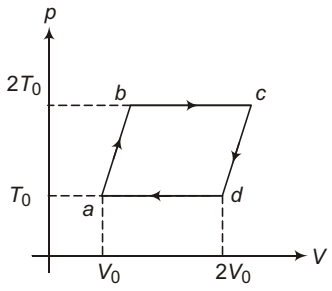
$\Rightarrow \Delta U = (+)\text{ve}$

$\Delta Q = nC_V \Delta T - nR\Delta T = n(C_V - R)\ \Delta T$

$= (+)$ ve as $C_V > R$

i.e., heat is given to the system.

51. In cyclic process, $\Delta U = 0$



$$\Delta W = 0 + nR2T_0 \ln\left(\frac{2V_0}{V_0}\right) + 0 + nRT_0 \ln\left(\frac{V_0}{2V_0}\right)$$

$= 2nRT_0 \ln 2 - nRT_0 \ln 2$

$= nRT_0 \ln 2 = (+)$ ve

i.e., $\Delta W > 0$

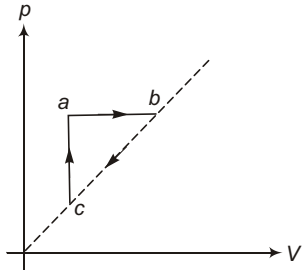
$\Delta Q_{\text{supplied}} = \Delta U_{ab} + \Delta W_{bc}$

$= nC_V(2T_0 - T_0) + nR2T_0 \ln\left(\frac{2V_0}{V_0}\right)$

$= 2 \times \frac{3}{2} RT_0 + 4RT_0 \ln 2$

$= 3RT_0 + 4RT_0 \ln 2$

52. $ab \rightarrow$ isochoric, $bc \rightarrow$ isobaric and $ca \rightarrow$ isothermal.



$\Delta W_{ab} = 0, \Delta U_{ca} = 0$

as in ca density is increasing, so volume is decreasing *i.e.,*

$\Delta W_{ca} = (-)$ ve, *i.e.*, $\Delta W_{ca} < 0$

in isochoric process ΔQ_{ab} is positive for increase in temperature.

53. In isochoric process $\Delta W = 0$.

and in adiabatic process

$\Delta Q = 0 \Rightarrow Q_3$ to be minimum

$\Rightarrow \quad Q_2 > Q_1 > Q_3$

JEE Corner

■ Assertion & Reasons

1. In adiabatic expression, $\Delta W = (+)$ ve while $\Delta Q = 0$ and as according to first law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W \Rightarrow \Delta U = -\Delta W$$

i.e., $\Delta U = (-)$ ve this implies decrease in temperature. So, Assertion and reason are both true but not correct explanation.

2. Assertion is false, as work done is a path function and not a state function i.e., it depends on the path through which the gas was taken from initial to find state.

3. Assertion is false, as first law can be applied for both real and ideal gases.

4. During melting of ice its volume decreases, so work done by it is negative and that by atmosphere is positive. So, reason is true explanation of assertion.

5. As $\Delta Q = \Delta U + \Delta W \Rightarrow \Delta U = \Delta Q - \Delta W$, where ΔU is state function while ΔQ and ΔW are path function as for definite

initial and final state ΔU is constant and so is $Q - W$. Thus assertion and reason are both true but not correct explanation.

6. Carnot's engine is ideal heat engine with maximum efficiency but it is not also 100%. So assertion and reason are both true but not correct explanation.

7. $pT = \text{constant} = p \cdot \frac{pV}{R} = \frac{p^2 V}{nR}$

$\Rightarrow p^2 V = \text{constant}$

$\therefore \Delta W = \int p\,dV = \int \sqrt{k}\frac{dV}{\sqrt{V}} = \sqrt{k} \cdot \frac{V^{1/2}}{1/2}$

$= 2\sqrt{k}\sqrt{V} = 2\sqrt{kV} = 2\sqrt{p^2/V}$

$= 2pV = 2nR(T_f - T_i) = 2nRT\,\Delta T$

$\therefore \Delta W = (+)$ ve for $\Delta T = (+)$ ve

and $\frac{nRT}{V} T = \text{constant}$.

$\Rightarrow \quad T^2 \propto V$

or, $\quad V \propto T^2$

Thus assertion is true but reason is false.

8. In adiabatic changes for free expansion, $Q = 0, W = 0$ and $\Delta U = 0$

as in free expansion no work is done against any force.

For ideal gases $pV = \text{constant}$ as $\Delta U = 0$ $\Rightarrow T = \text{constant}$ So, assertion and reason are both true but not correct explanation.

9. Assertion and reason are both true and correct explanation.

10. Assertion and reason are both true and correct explanation.

Match the Columns

1. (a) $\Delta W = p\int dV = pV = nR(T_f - T_i)$

$= nRT = 2RT \longrightarrow r$

(b) $\Delta U = nC_V T = 2 \times \frac{3}{2} R(2T - t)$

$= 3RT \longrightarrow p$

(c) $\Delta W = \frac{nR(2T - T)}{1 - 5/3} = -\frac{3}{2} \times 2RT$

$= -3RT \longrightarrow s$

(d) $\Delta U = nC_V \Delta T = 3RT \longrightarrow p$

2. (a) In ab slope is more so, pressure is less as $V = \frac{nR}{p} \cdot T$, but is constant and in isobaric process. $\Delta W = p\Delta V = nR\Delta T$ and as ΔT is same in both process so, ΔW is same for both $\longrightarrow r$

(b) As $\Delta U = -nC_V \Delta T$ is same for both process $\longrightarrow r$

(c) As $\Delta Q = \Delta U + \Delta W$, it is also same for both process $\longrightarrow s$

(d) Nothing can be said about molar heat capacity $\rightarrow s$

3. (a) $\Delta W = \int pdV$

$= \int \sqrt{\frac{k}{V}} dV = \sqrt{k}\int \frac{dV}{\sqrt{V}}$

$= 2\sqrt{kV} = 2pV = 2nR\Delta T \longrightarrow p$

(b) $\Delta U = nC_V \Delta T = \frac{3}{2} nR\Delta T \longrightarrow s$

(c) $\Delta Q = 2nR\Delta T + \frac{3}{2} nR\Delta T$

$= \frac{7}{2} nR\Delta T \longrightarrow s$

(d) $\longrightarrow s$

4. (a) $\Delta W = p\Delta V = nR\Delta T$ and $\Delta U = nC_V \Delta T$

$\Rightarrow \Delta W < \Delta U \longrightarrow q$

(b) $\Delta W = 0 \Rightarrow \Delta Q = \Delta U, \Delta U = (-)$ ve $\rightarrow p, r$

(c) $\Delta W = (+)$ ve, $\Delta U = (-)$ ve, $\Delta Q = 0 \rightarrow p$

(d) $\Delta W = (+)$ ve, $\Delta U = 0$, $\Delta Q = (+)$ ve $\rightarrow p$

5. (a) $\Delta W_{AB} = p_0 V_0 + \frac{1}{2} p_0 V_0$

$$= \frac{3}{2} p_0 V_0 \rightarrow s$$

(b) $\Delta U_{AB} = \Delta Q - \Delta W$

$$= +6p_0V_0 - \frac{3}{2} p_0 V_0 = + \frac{9}{2} p_0 V_0 \rightarrow s$$

(c) $\Delta Q = +6p_0V_0$

$$= nC\left(\frac{4p_0V_0}{nR} - \frac{p_0V_0}{nR}\right)$$

$$= \frac{3p_0V_0}{R} C$$

$$\Rightarrow \quad C = 2R \rightarrow p$$

(d) ΔU

$$= nC_V \left(\frac{4p_0V_0}{nR} - \frac{p_0V_0}{nR}\right)$$

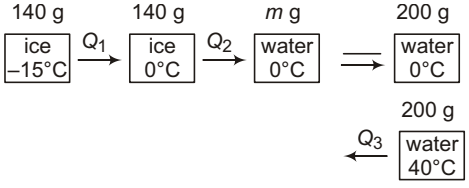
$$= 3C_V \frac{p_0V_0}{R} = \frac{9}{2} p_0 V_0$$

$$\Rightarrow C_V = \frac{3}{2} R \rightarrow s$$

19. Calorimetry and Heat Tansfer

Introductory Exercise 19.1.

1.



As Heat gain = Heat loss

$$Q_1 + Q_2 = Q_3$$

$\Rightarrow 140 \times 0.53 \times 15 + m \times 80$

$= 200 \times 1 \times 40$

$\Rightarrow m = \dfrac{8000 - 1113}{80} = 86$ g is the mass of ice melt

$\therefore$ Mass of water

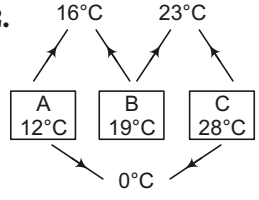
$= 200 \text{ g} + 86 \text{ g} = 286 \text{ g}$

and mass of ice

$= 140 \text{ g} - 86 \text{ g} = 54 \text{ g}$

while final temperature of mixture is 0°C.

2.



$ms_A\,(16 - 12) = ms_B\,(19 - 16)$

$\Rightarrow \quad 4s_A = 3s_B$

$ms_B\,(23 - 19) = ms_C\,(28 - 23)$

$\Rightarrow \quad 4s_B = 5\,s_C$

$\therefore \quad ms_A\,(\theta - 12) = \dfrac{4}{5}\,ms\,(28 - \theta)$

or $\quad \dfrac{3}{4}\,s_B\,(\theta - 12) = \dfrac{4}{5}\,s_B\,(28 - \theta)$

or $\quad 15\,(\theta - 12) = 16\,(28 - \theta)$

or $\quad 31\theta = 448 + 180$

$\Rightarrow \quad \theta = 20.26°\text{C}$

3. $mL = ms\Delta\theta$

$\Rightarrow 80 \text{ cal} = 1 \text{ cal/°C}\,(\theta - 0°\text{C})$

$\Rightarrow \theta = 80°\text{C}$

4. As Heat gain = Heat loss

$\Rightarrow (100 - m) \times 529 = m \times 80$

$\therefore \quad 100 \times 529 = 609\text{ m}$

$\Rightarrow m = \dfrac{100 \times 529}{609}$ g = 86.86 g of ice will be formed.

5. $P = \dfrac{d\theta}{dt} = \dfrac{d}{dt}(ms\Delta\theta) = \dfrac{dm}{dt}\,s\Delta\theta$

$\Rightarrow \quad \dfrac{dm}{dt} = \dfrac{P}{s\Delta\theta}$

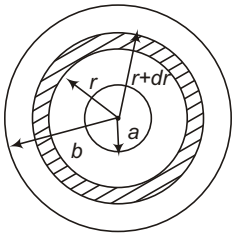
$\therefore \quad \dfrac{dm}{dt} = \dfrac{500 \times 10^6 \text{ J/s}}{4200 \text{ J/kg °C} \times 10°\text{C}}$

$= \dfrac{5}{4.2} \times 10^4 \text{ kg/s} = 1.2 \times 10^4 \text{ kg/s}$

Introductory Excersise 19.2

1. Rest of the liquid will be heated due to conduction and not convection.

2. $\dfrac{dQ}{dt} = \dfrac{k \cdot 4\pi r^2 \ (-d\theta)}{dr}$



$\therefore \ \dfrac{dQ}{dt} \cdot \dfrac{dr}{r^2} = -4\pi k \, d\theta$

or $\dfrac{dQ}{dt} \displaystyle\int_a^b \frac{dr}{r^2} = -4\pi k \int_{T_1}^{T_2} d\theta$

or $\dfrac{dQ}{dt}\left(\dfrac{1}{a} - \dfrac{1}{b}\right) = -4\pi k\,(T_2 - T_1)$

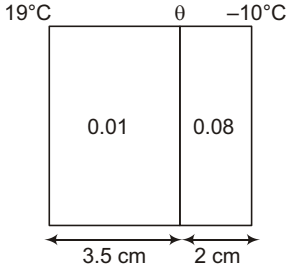
$\Rightarrow \dfrac{dQ}{dt} = \dfrac{4\pi k\,(T_1 - T_2)}{\dfrac{1}{a} - \dfrac{1}{b}} = 4\pi kab\,\dfrac{T_1 - T_2}{b - a}$

3. $\dfrac{dQ}{dt} = \dfrac{kA\Delta\theta}{t}$

$\Rightarrow \qquad k = \dfrac{dQ}{dt} \cdot \dfrac{t}{A\Delta\theta}$

$\therefore$ Unit of k = watt $\dfrac{\text{m}}{\text{m}^2\text{-}k}$ = W/m-K

4. $\dfrac{K_1 A \ \Delta\theta_1}{l_1} = \dfrac{K_2 \ A\Delta\theta_2}{l_2}$



$\dfrac{0.01(19 - \theta)}{3.5} = \dfrac{0.08\,(\theta + 10)}{2}$

or $2(19 - \theta) = 28\,(\theta + 10)$

or $38 - 280 = 30\theta$

or $\theta = -\dfrac{242}{30} = -8.07°\text{C}$

$\dfrac{dQ}{dt} = \dfrac{0.01 \times 1 \times (19 + 8.1)}{3.5 \times 10^{-2}}$

$= 7.74 \ \text{W/m}^2$

5. $\dfrac{dQ}{dt} = \dfrac{dm}{dt} \cdot L = \dfrac{0.44 \ \text{kg}}{300 \ \text{s}} \times 2.256 \times 10^6 \text{J/kg}$

$= 3308.8$ J/s

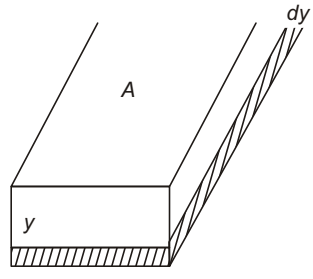
$= \dfrac{kA \ \theta}{t} = \dfrac{50.2 \times 0.15 \times (\theta - 100)}{1.2 \times 10^{-2}}$

$= 627.5\,(\theta - 100)$

$\Rightarrow \theta - 100 = \dfrac{3308.8}{627.5} = 5.27$

$\Rightarrow \qquad \theta = 105.27°\text{C}$

6. $\dfrac{dQ}{dt} = \dfrac{kA\,[0 - (-\theta)]}{y} = \dfrac{dm}{dt} \cdot L$



$= \rho \dfrac{dV}{dt} \cdot L = \rho A \dfrac{dy}{dt} \cdot L$

$\Rightarrow \dfrac{kA\theta}{y} = \rho AL \dfrac{dy}{dt}$

$\Rightarrow \dfrac{dy}{dt} = \dfrac{k\theta}{L\rho y}$ (Proved)

7. $\dfrac{dQ}{dt} = e\sigma AT^4$

$= 4 \times 5.67 \times 10^{-8} \times 4\pi\,(4 \times 10^{-2})^2 \times (3000)^4$

$= 0.4 \times 4\,\pi \times 5.67 \times 4^2 \times 3^4$ J/s

$= 3.7 \times 10^4$ watt

8. $\dfrac{dQ}{dt} = \dfrac{\Delta\theta}{R_{\text{th}}} \Rightarrow R_{\text{th}} = \dfrac{\Delta\theta}{\dfrac{d\theta}{dt}} = \dfrac{K}{W} = \text{KW}^{-1}$

AIEEE Corner

Subjective Questions (Level-1)

1. $\underset{0°C}{\text{ice}} \xrightarrow{Q_1} \underset{0°C}{\text{Water}} \xrightarrow{Q_2} \underset{100°C}{\text{Water}} \xrightarrow{Q_3} \underset{100°C}{\text{steam}}$

$Q = Q_1 + Q_2 + Q_3 = mL_f + ms\Delta\theta + mL_v$

$= 10\,[80 + 1 \times 100 + 540]$

$= 10 \times 720 \text{ cal} = 7200 \text{ cal}$

2. 10 g of water at 40°C do not have sufficient heat energy to melt 15 g of ice at 0°C , so there will be a mixture of ice-water at 0°C. Let the mass of ice left is *mg*.

$\therefore \quad (15 - m) \times 80 = 10 \times 1 \times 40$

$15 - m = 5 \Rightarrow m = 10 \text{ g}$

$\therefore$Mass of ice = 10 g

and mass of water = (10 + 5) g = 15 g

3. $4 \times s_P\ (60 - 55) = 1 \times s_R \times (55 - 50)$

$\Rightarrow \quad 4s_P = s_R$

$1 \times s_P\ (60 - 55) = 1 \times s_Q\ (55 - 50)$

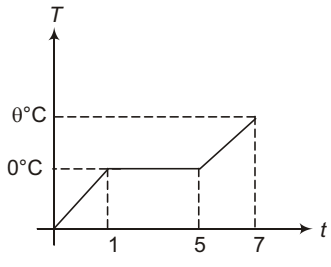
$\Rightarrow \quad s_P = s_Q$

$1 \times s_Q\ (60 - \theta) = 1 \times s_R\ (\theta - 50)$

or $s_P\ (60 - \theta) = 4s_P\ (\theta - 50)$

$260 = 5\theta \Rightarrow \theta = \dfrac{260}{5} = 52°C$

4. $\dfrac{dQ}{dt} = \dfrac{m \times 336 \times 10^3 \text{ J/kg}}{4 \times 60 \text{ s}}$



$= 1400 \text{ J/kg}$

$= 1400\ m\text{W/kg}$

$= \dfrac{m \cdot s\Delta\theta}{t} = \dfrac{m \times 4200\,(\theta - 0)\text{ c}}{2 \times 60 \text{ s}}$

$\therefore \quad \dfrac{1400 \times 2 \times 60}{4200} = \theta$

$\Rightarrow \quad \theta = 40°C$

5. $Q = \dfrac{1}{2} \times \dfrac{1}{2}\, mv^2 = ms\ \Delta\theta + mL$

$\Rightarrow v = \sqrt{4\,(s\Delta\theta + L)}$

$\therefore \quad v = \sqrt{4\,(125 \times 300 + 2.5 \times 10^4)}$

$= \sqrt{4 \times (3.75 + 2.5) \times 10^4}$

$= \sqrt{4 \times 6.25 \times 10^4} = 500 \text{ m/s}$

6. $\eta\ mg\ \Delta h = ms\Delta\theta$

$\therefore \quad \Delta\theta = \dfrac{\eta g \Delta h}{s} = \dfrac{0.4 \times 10 \times 0.5}{800} = \dfrac{1}{400}\ °C$

$= 2.5 \times 10^{-3}\ °C$

7. $\dfrac{K_1 A\ (\theta - 0)}{l} = \dfrac{K_2 A(100 - \theta)}{l}$

$\Rightarrow (K_1 + K_2)\ \theta = 100\ K_2$

$\therefore \quad \theta = \dfrac{100\ K_2}{K_1 + K_2} = \dfrac{100 \times 46}{390 \times 46} = 10.55°C$

8. $i_{CD} = i_{AC} - i_{CB}$

$\dfrac{KA(\theta - 25)}{l} = \dfrac{KA(100 - \theta)}{l/2} - \dfrac{KA(\theta - 0)}{l/2}$

or $\quad \theta - 25 = 2(100 - \theta) - 2\theta$

or $\quad 5\theta = 225 \Rightarrow \theta = 45°C$

$\therefore \quad i_{CD} = \dfrac{\Delta\theta}{R_{\text{th}}} = \dfrac{45 - 25}{5} = 4 \text{ W}$

9. $i_A = i_C + i_D$

$\dfrac{KA\ (T_1 - \theta)}{l} = \dfrac{KA(\theta - T_3)}{3l/2} + \dfrac{KA(\theta - T_2)}{3l/2}$

$\Rightarrow T_1 - \theta = \dfrac{2}{3}\,(\theta - T_3) + \dfrac{2}{3}\,(\theta - T_2)$

or $T_1 + \dfrac{2}{3}\,(T_2 + T_3) = \theta\left(1 + \dfrac{4}{3}\right)$

$\Rightarrow \quad \theta = \dfrac{T_1 + \dfrac{2}{3}\,(T_2 + T_3)}{7/3}$

$= \dfrac{3T_1 + 2(T_2 + T_3)}{7}$

10. $\dfrac{KA(200 - \theta_1)}{l} = \dfrac{2KA(\theta_1 - \theta_2)}{l}$

$$= \frac{3KA(\theta_2 - 100)}{l}$$

$\therefore\ 200 - \theta_1 = 2(\theta_1 - \theta_2) = 3(\theta_2 - 100)$

$\Rightarrow\ 3\theta_1 - 2\theta_2 = 200$

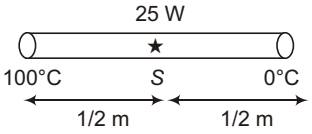
$\theta_1 + 3\theta_2 = 500$

$-11\theta_2 = -1300$

$\Rightarrow\ \theta_2 = \frac{1300}{11} = 118.2°C$

$\theta_1 = \frac{1}{3}[200 + 2\theta_2] = 145.45°C$

11. $25 = \frac{400 \times 10^{-4}\ (\theta - 100)}{1/2} + \frac{400 \times 10^{-4}\ (\theta - 0)}{1/2}$



$25 = 8 \times 10^{-2}\ [\theta - 100 + \theta]$

or $\quad 312.5 = 2\theta - 100$

$\Rightarrow\ \theta = \frac{412.5}{2} = 206.25$

$\therefore\ \Delta\theta_1 = 106.25$ and $\Delta\theta_2 = 206.25$

$\therefore\ \frac{\Delta\theta_1}{\Delta l} = \frac{106.25°C}{1/2\ m} = 212.5°C/m$

and $\quad \frac{\Delta\theta_2}{\Delta l} = \frac{206.25\ °C}{1/2\ m} = 412.5\ °C/m$

12. $\frac{dQ}{dt} = e\sigma AT^4 = 0.6 \times 5.67 \times 10^{-8} \times 2 \times (0.1)^2 \times (1073)^4$

$= 0.6 \times 5.67 \times (10.73)^4 \times 10^{-2} \times 2$

$= 902\ W$

13. $\left(\frac{dQ}{dt}\right)_1 = e\sigma AT^4$ and $\left(\frac{dQ}{dt}\right)_2 = \sigma AT^4$

$\Rightarrow e = \frac{(dQ/dt)_1}{(dQ/dt)_2} = \frac{210}{700} = 0.3$

14. $\frac{(80-50)_c}{5} = \left(\frac{80+50}{2} - 20\right)c$

$\Rightarrow K = \frac{6}{45};$

$\frac{(60-30)}{t} = \frac{6}{45}$

$\left(\frac{60+30}{2} - 20\right) \Rightarrow t = 9\ min$

■ Objective Questions (Level-1)

1. $\frac{3KA(35-\theta)}{10} = \frac{KA(\theta - 0)}{20}$

$\Rightarrow\ 6(35 - \theta) = \theta$

$\Rightarrow\ \theta = \frac{6 \times 35}{7} = 30°C$

$\therefore\ \Delta\theta_A = 35 - 30 = 5°C$

2. $\frac{T_S}{T_N} = \frac{\lambda_N}{\lambda_S} = \frac{350}{510} = 0.69$

According to Wien's law

3. $\left(\frac{dQ}{dt}\right)_2 \Big/ \left(\frac{dQ}{dt}\right)_1 = \frac{\frac{1}{4}\frac{K \cdot 4A\ \Delta\theta}{l/2}}{\frac{K\Delta\theta}{l}} = 2$

$\Rightarrow \left(\frac{dm}{dt}\right)_2 = 2\left(\frac{dm}{dt}\right)_1 = 0.2\ g/s$

4. $\frac{dQ}{dt} = \frac{4\pi K(\theta - 0)}{\frac{2a - a}{a \cdot 2a}} = \frac{4\pi\ K(100 - \theta)}{\frac{3a - 2a}{3a \cdot 2a}}$

$\Rightarrow\ 2\theta = 6(100 - \theta)$

$\Rightarrow\ \theta = \frac{6}{8} \times 100 = 75°C$

5. $\frac{K_1 A(T_2 - T_1)}{d} = \frac{K_2 A(T_3 - T_2)}{3d}$

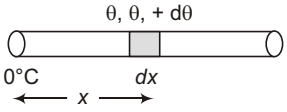
$\Rightarrow\ K_1(T_2 - T_1) = \frac{1}{3}K_2(T_3 - T_2)$

$\Rightarrow\ K_1 = \frac{1}{3}K_2 \Rightarrow K_1 : K_2 = 1:3$

6. $\left(\frac{dQ}{dt}\right)_2 \Big/ \left(\frac{dQ}{dt}\right)_1 = \left[\frac{2K \cdot 2A \cdot \Delta\theta}{2l}\right] \Big/ \left[\frac{KA\ \Delta\theta}{l}\right] = 2$

$\Rightarrow \left(\frac{dQ}{dt}\right)_2 = 2\left(\frac{dQ}{dt}\right)_1 = 8\ cal/s$

7.



$$P = \frac{dQ}{dt} = \frac{K \cdot A d\theta}{dx} = \frac{K_0(1 + ax)A\, d\theta}{dx}$$

$$\therefore \int_0^l \frac{dx}{1 + ax} = \frac{K_0 A}{P} \int_0^{100} d\theta$$

$$\frac{1}{a} \ln (1 + ax) |_0^l = \frac{10^2 \times 10^{-4}}{1} \cdot \theta |_0^{100} = 1$$

$$\Rightarrow \quad \ln (1 + al) - \ln 1 = 1$$

$$\Rightarrow \ln (1 + al) - \ln 1 = 1$$

$$\Rightarrow \ln (1 + al) = 1$$

or $1 + al = e^1$

or $l = \frac{1}{a}(e - 1) = e - 1 = 1.7$ m

8. $\frac{\lambda_2}{\lambda_1} = \frac{T_1}{T_2} = \frac{2}{3} \Rightarrow \lambda_2 = \frac{2}{3}\lambda_m$

9. Heat required to boil 1 g of ice is 180 cal while 1 g of steam can release 540 cal during condenstion. So, temperture of the mixture will be 100°C with 2/3 g steam and 4.3 g water.

10. $T_1 < T_2 < T_3$ as temperature of a body decreases in rate of cooling also decreases such that time increases for equal temperature difference.

11. Conduction is maximum for which thermal resistance is minimum, as $R_{th} \propto \frac{l}{r^2}$ then for

(a) 50 (b) 25 (c) 100 (d) 33.33,

So option 'b' has minimum resistance.

12. Slope of temperature *versus* heat graph gives increase of specific heat or heat capacity and the portion *DE* is the gaseous state.

13. $dQ = m\, sdt = maT^3 dT$

$$\Rightarrow \frac{Q}{m} = \frac{a}{4} T^4 |_1^2 = \frac{a}{4}(16 - 1) = \frac{15a}{4}$$

14. Resistance becomes 1/4th in parallel of that in series, so times taken will also become 1/4th *ie*, 12/4 = 3 min.

15. $ms_1 \times 12 = ms_2 \times 8 \Rightarrow s_1 : s_2 = 2 : 3$

16. $\frac{KA(T - T_c)}{\sqrt{2}\, l} = \frac{KA(T_c - \sqrt{2}\, T)}{l}$

$$\Rightarrow \quad \frac{T}{\sqrt{2}} + \sqrt{2}T = T_c + \frac{T_c}{\sqrt{2}}$$

$$\Rightarrow \quad \frac{3}{\sqrt{2}} T = \frac{1 + \sqrt{2}}{\sqrt{2}} T_c$$

$$\Rightarrow \quad T_c = \frac{3}{1 + \sqrt{2}} T$$

17. $P = (1000 - 160)\text{ W} = 840\text{ W}$

$$= \frac{2 \times 4200 \times 50}{t}$$

$$\therefore \quad t = \frac{42 \times 10^4}{840} = 500\text{ s} = 8\text{ min } 20\text{ s}$$

18. $\frac{dQ}{dt} = \frac{KA(T_2 - T)}{x} = \frac{2KA(T - T_1)}{4x}$

$$\Rightarrow \quad T_2 - T = \frac{1}{2}T - \frac{1}{2}T_1$$

$$\Rightarrow \quad T_2 + \frac{1}{2}T_1 = \frac{3}{2}T$$

$$\Rightarrow \quad T = \frac{2}{3}\left(T_2 + \frac{1}{2}T_1\right) = \frac{1}{3}(2T_2 + T_1)$$

$$\therefore \quad \frac{dQ}{dt} = \frac{KA}{x}\left[T_2 - \frac{1}{3}(2T_2 + T_1)\right]$$

$$= \frac{KA}{x}[3T_2 - 2T_2 - T_1] \times \frac{1}{3}$$

$$= \frac{KA}{x}(T_2 - T_1) \times \frac{1}{3}$$

$$\Rightarrow \quad f = \frac{1}{3}$$

19. $\Delta\theta \propto \frac{1}{K}$

$$\Rightarrow \quad \frac{\Delta\theta_A}{\Delta\theta_B} = \frac{K_B}{K_A} = \frac{1}{2}$$

$$\Rightarrow \quad \Delta\theta_A = \frac{1}{2}\Delta\theta_B = 18°\text{C}$$

■ More than One Correct Options

20. Amount of heat radiated or absorbed depends upon. Surface type, surface area, surface temperature and temperature of surrounding, so (a) and (b) are correct.

21. $\frac{KA(40-\theta)}{l} = \frac{KA(\theta-30)}{l} + \frac{KA(\theta-20)}{l}$

$\Rightarrow \quad 40-\theta = 2\theta - 50$

$\Rightarrow \quad 3\theta = 90°$

or $\quad \theta = 30°C$

So, (b) and (d) are correct.

22. $m \times s \times (2\theta - \theta_0) = m \times 2s \times (\theta_0 - \theta)$

$\Rightarrow \quad 4\theta = 3\theta_0 \Rightarrow \theta_0 = \frac{4}{3}\theta$

$c_1 : c_2 = m_1 : s_2 = s_1 : s_2 = 1:2$

So, (b) and (c) are correct.

23. In series rate of $R = R_1 + R_2$

$\Rightarrow \frac{1}{q} = \frac{1}{q_1} + \frac{1}{q_2} \Rightarrow q = \frac{q_1 q_2}{q_1 + q_2}$

In parallel $\quad \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

$\Rightarrow q = q_1 + q_2$ as $q \propto \frac{1}{R}$

So, (b) and (c) are correct.

24. (a), (c) and (d) are correct.

JEE Corner

Assertion and Reason

1. Assertion is false.
2. According to Wien's law assertion and reason are correct.
3. Assertion and reason are true but not correct explanation.
4. Assertion is true but reason is false as resistance becomes 1/4th.
5. Assertion and reason are both false.
6. Assertion is false as this statement was not given by Newton.
7. Assertion and reason are both true with correct explanation.
8. Both are true but not correct explanation.
9. Assertion is false as temperature at different points become different.
10. As mass of follow sphere is less so cooling will be faster. So, both are true with correct explanation.

Match the Columns

1.

(a)	$\sigma = \frac{(dQ/dt)}{AT^4} = \frac{ML^2T^{-2}T^{-1}}{L^2\theta^4} = [MT^{-3}\theta^{-4}]$	s
(b)	$b = \lambda T = L\theta$	p
(c)	$\varepsilon = \frac{E}{At} = \frac{[ML^2T^{-2}]}{[L^2T]} = [MT^{-3}]$	r
(d)	$R\text{th} = \frac{d\theta}{dQ/dt} = \frac{\theta}{[ML^2T^{-2}T^{-1}]} = [M^{-1}L^{-2}T^3\theta]$	s

2.

(a)	Slope of line ab	s
(b)	Length of line $bc \propto m$	r
(c)	Solid + liquid $\rightarrow bc$	s
(d)	Only liquid $\rightarrow cd$	q

3. $\frac{KA(100-\theta_b)}{l} = \frac{KA(\theta_b - \theta_d)}{l} = \frac{KA(\theta_d + 80)}{l}$

$\therefore \quad 100 - \theta_b = \theta_b - \theta_d$ and

$100 - \theta_b = \theta_d + 80$

$\therefore \quad \left.\begin{array}{l}\theta_d - 2\theta_b = -100 \\ \theta_d + \theta_b = 20\end{array}\right\} \Rightarrow -3\theta_b = -120$

$\Rightarrow \quad \theta_b = 40°C \Rightarrow \theta_d = -20°C$

$\theta_c = \theta_f = \frac{40-20}{2} = 10°C$

$\therefore$ (a) $\rightarrow q$, (b) $\rightarrow p$, (c) $\rightarrow p$, (d) $\rightarrow r$

4. (a) $ms(\theta_1 - \theta) = 2ms(2\theta - \theta_1)$

$\Rightarrow 3\theta_1 = 5\theta \Rightarrow \theta_1 = \frac{5}{3}\theta \rightarrow q$

(b) $ms(\theta_2 - \theta) = 3ms(3\theta - \theta_2)$

$\Rightarrow 4\theta_2 = 10\theta \Rightarrow \theta_2 = \frac{5}{2}\theta \rightarrow p$

(c) $2ms(\theta_3 - 2\theta) = 3ms(3\theta - \theta_3)$

$\Rightarrow 5\theta_3 = 13\theta \Rightarrow \theta_3 = \frac{13}{2}\theta \rightarrow s$

(d) $ms(\theta_4 - \theta) + 2ms(\theta_4 - 2\theta)$

$= 3ms(3\theta - \theta_4)$

$\Rightarrow 6\theta_4 = 14\theta \Rightarrow \theta_4 = \frac{7}{3}\theta \rightarrow r$

5.

(a)	$s = \frac{1}{m}\frac{dQ}{d\theta} = \frac{\text{J}}{\text{kg}^\circ\text{C}} \longrightarrow$	q
(b)	$c = ms = m\frac{dQ}{md\theta} = \text{J}^\circ/\text{C} \longrightarrow$	s
(c)	$i = \frac{dQ}{dt} = \text{J/s} \longrightarrow$	r
(d)	$L = \frac{E}{m} = \text{J/kg} \longrightarrow$	s

Solutions
of
Electricity & Magnetism

Lesson 20th to 25th

By DC Pandey

20 Current Electricity

Introductory Exercise 20.1

1. $i = \frac{q}{t}$, here $q = e$, $t = \frac{2\pi r}{v}$

$\therefore \quad i = \frac{ev}{2\pi r}$

$= \frac{1.6 \times 10^{-19} \times 2.2 \times 10^{6}}{2 \times 3.14 \times 5 \times 10^{-11}}$

$= 1.12 \times 10^{-3}$ A

$= 1.12$ mA

2. No. of atoms in 63.45 g of Cu $= 6.023 \times 10^{23}$

$\therefore$ No. of atoms in 1 cm^3 (8.89 g) of Cu

$= \frac{6.023 \times 10^{23}}{63.54} \times 8.89$

$= 8.43 \times 10^{22}$

As one conduction electron is present per atoms,

$n = 8.43 \times 10^{22}\ cm^{-3}$ or $8.43 \times 10^{28}\ m^{-3}$

As $i = neAv_d$

$\Rightarrow \quad v_d = \frac{i}{neA}$

$= \frac{2.0}{8.43 \times 10^{28} \times 1.6 \times 10^{-19} \times 3.14 \times (0.5 \times 10^{-3})^2}$

$= 1.88 \times 10^{-6}\ ms^{-1}$

3. Yes.

As current always flows in the direction of electric field.

4. False.

In the absence of potential difference, electrons passes random motion.

5. Current due to both positive and negative ions is from left to right, hence, there is a net current from left to right.

6. $i = 10 + 4t \Rightarrow \frac{dq}{dt} = 10 + 4t$

$\Rightarrow \quad \int_0^q dq = \int_0^{10} (10 + 4t)\, dt$

$\Rightarrow \quad q = [10t + 2t^2]_0^{10} = 300$ C

Introductory Exercise 20.2

1. $R = \frac{\rho L}{A}$

$= 1.72 \times 10^{-8} \times \frac{35}{3.14 \times \left(\frac{2.05}{2} \times 10^{-3}\right)^2}$

$= 0.57\ \Omega$

2. (a) $J = \frac{E}{\rho}$

$\Rightarrow \quad i = JA = \frac{EA}{\rho}$

$= \frac{0.49 \times 3.14 \times (0.42 \times 10^{-3})^2}{2.75 \times 10^{-8}}$

$= 9.87$ A

(b) $V = EL = 0.49 \times 12 = 5.88$ V

(c) $R = \frac{V}{i} = \frac{5.88}{9.87} = 0.6\ \Omega$

3. Let us consider the conductor to be made up of a number of elementary discs. The conductor is supposed to be extended to form a complete cone and the vertex O of the cone

is taken as origin with the conductor placed along x-axis with its two ends at $x = r$ and $x = l + r$. Let θ be the semi-vertical angle of the cone.

Consider an elementary disc of thickness dx at a distance x from origin.

Resistance of this disc,

$$dR = \rho \frac{dx}{A}$$

If y be the radius of this disc, then

$$A = \pi y^2$$

But $y = x \tan\theta$

$$dR = \rho \frac{dx}{\pi x^2 \tan^2\theta}$$

$\therefore$ Resistance of conductor

$$R = \int dR = \int_r^{l+r} \frac{\rho dx}{\pi x^2 \tan^2\theta}$$

$$R = \frac{\rho}{\pi \tan^2\theta}\left[-\frac{1}{x}\right]_r^{l+r}$$

$$= \frac{\rho}{\pi \tan^2\theta}\left[\frac{1}{r} - \frac{1}{l+r}\right]$$

$$R = \frac{\rho l}{\pi r(l+r)\tan^2\theta}$$

But, $r \tan\theta = a$

$(r + l)\tan\theta = b$

$$R = \frac{\rho l}{\pi ab}$$

4. True. $\rho = \frac{1}{\sigma}$

$\therefore$ $\rho \times \sigma = \frac{1}{\sigma} \times \sigma = 1$

5. $R_{Cu} = R_{Fe}$

$$4.1(1 + \alpha_{Cu}\Delta T) = 3.9(1 + \alpha_{Fe}\Delta T)$$

$$4.1[1 + 4.0 \times 10^{-3}(T - 20)] = 3.9[1 + 5.0 \times 10^{-3}(T - 20)]$$

$$4.1 + 16.4 \times 10^{-3}(T - 20) = 3.9 + 19.5 \times 10^{-3}(T - 20)$$

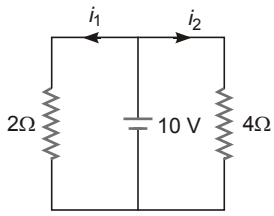
$$3.1 \times 10^{-3}(T - 20) = 0.2$$

$$T - 20 = \frac{0.2}{3.1 \times 10^{-3}} = 64.5°\text{C}$$

$$T = 84.5°\text{C}$$

Introductory Exercise 20.3

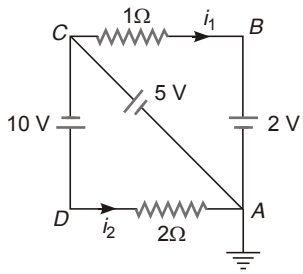
1. Potential difference across both the resistors is 10 V.



Hence, $i_1 = \frac{10}{2} = 5$ A

and $i_2 = \frac{10}{4} = 2.5$ A

2. As A is grounded, $V_A = 0$



$$V_B = V_A + 2 = 2 \text{ V}$$

$$V_C = V_A + 5 = 5 \text{ V}$$

$$V_D = V_C + 10 = 15 \text{ V}$$

$\therefore$ $i_1 = \frac{V_C - V_B}{1} = 3$ A

and $i_2 = \frac{V_D - V_A}{2} = \frac{15}{2} = 7.5$ A

3. Current in the given loop is

$$i = \frac{E + 15}{8}$$

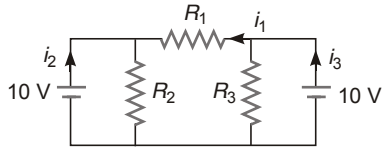
$$V_{AB} = E - 2i = E - 2 \cdot \left(\frac{E + 15}{8}\right) = 0$$

$\Rightarrow$ $E = 5$ V

4. Effective emf,

$$E = 8 \times 1 - 2 \times 1 = 6 \text{ V}$$

Effective resistance of circuit



$$R = R_{\text{external}} + 10r = 2 + 10 \times 1 = 12\,\Omega$$

$$\therefore \quad i = \frac{E}{R} = \frac{6}{12} = 0.5 \text{ A}$$

5. As $R_2 = R_3$ and $V_1 = V_2$

Potential difference across R_1 is zero.

Hence, current through $R_1 \Rightarrow i_1 = 0$

and current through R_2

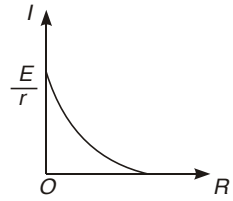
$$\Rightarrow \quad i_2 = \frac{V_1}{R_2} = \frac{10}{10} = 1 \text{ A}$$

6. $i = \dfrac{E}{R + r}$

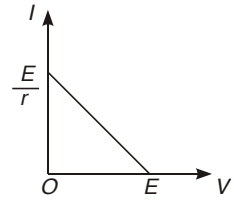
Also, $V = E - ir$

$$i = \frac{E - V}{r}$$

(a)

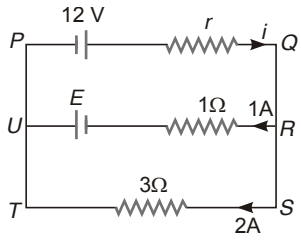


(b)



Introductory Exercise 20.4

1.



Applying KCL at junction R

$$i = 1 + 2 = 3 \text{ A}$$

$$V_{ST} = V_{RU} = V_{QP}$$

Taking $V_{ST} = V_{RU}$

$$6 = 1 - E$$

$$E = -5 \text{ V}$$

And from

$$V_{ST} = V_{QP}$$

$$6 = -ir + 12$$

$$r = \frac{12 - 6}{i} = \frac{6}{3} = 2\,\Omega$$

2. Power delivered by the 12 V power supply,

$$P_1 = Vi = 12 \times 3 = 36 \text{ W}$$

and power dissipated in 3 Ω resister,

$$P_3 = i_3^2 R_3 = 2^2 \times 3 = 12 \text{ W}$$

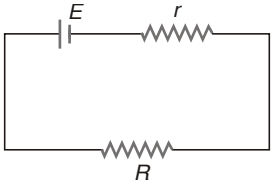
Introductory Exercise 20.5

1. $$E = \frac{\dfrac{E_1}{r_1} + \dfrac{E_2}{r_2} + \dfrac{E_3}{r_3}}{\dfrac{1}{r_1} + \dfrac{1}{r_2} + \dfrac{1}{r_3}} = \frac{\dfrac{10}{1} + \dfrac{4}{2} + \dfrac{6}{2}}{\dfrac{1}{1} + \dfrac{1}{2} + \dfrac{1}{2}} = \frac{10 + 2 + 3}{2} = 7.5 \text{ V}$$

and $$\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{1} + \frac{1}{2} + \frac{1}{2} = 2$$

$$\Rightarrow \quad r = \frac{1}{2} = 0.5\,\Omega$$

2. $i = \dfrac{E}{R + r}$



Rate of dissipation of energy

$$P = i^2 R = \frac{E^2 R}{(R + r)^2}$$

For maximum or minimum power

$$\frac{dP}{dR} = 0$$

$$\Rightarrow \quad E^2 \left[\frac{(R + r)^2 - 2R(R + r)}{(R + r)^4} \right] = 0$$

$$\Rightarrow \quad E^2 \frac{(R+r)(r-R)}{(R+r)^4} = 0$$

$$\Rightarrow \quad \frac{E^2(r-R)}{(R+r)^3} = 0$$

$$\Rightarrow \quad R = r$$

$$\frac{d^2P}{dR^2} = E^2\left[\frac{(R+r)^3(-1) - 3(r-R)(R+r)^2}{(R+r)^6}\right]$$

$$= \frac{-E^2(4r-2R)}{(R+r)^4}$$

Clearly $\frac{d^2P}{dR^2}$ is negative at $R = r$.

Hence, P is maximum at $R = r$

and $P_{\max} = \frac{E^2 r}{(r+r)^2} = \frac{E^2}{4r}$

3. When the batteries are connected in series

$$E_{\text{eff}} = 2E = 4\text{V}, \; r_{\text{eff}} = 2r = 2\,\Omega$$

For maximum power

$$R = r_{\text{eff}} = 2\Omega$$

and $P_{\max} = \frac{E_{\text{eff}}^2}{4r_{\text{eff}}} = \frac{(4)^2}{4 \times 2} = 2$ W

4. $I_g = 5$ mA, $G = 1\,\Omega$, $V = 5$ V

$$R = \frac{V}{I_g} - G = \frac{5}{5 \times 10^{-3}} - 1$$

$$= 999\,\Omega$$

A 999 Ω resistance must be connected in series with the galvanometer.

5. $G = 100\,\Omega$, $i_g = 50\,\mu\text{A}$, $i = 5$ mA

$$\therefore \quad S = \frac{i_g G}{i - i_g} = \frac{50 \times 10^{-6} \times 100}{5 \times 10^{-3} - 50 \times 10^{-6}}$$

$$= \frac{1}{1 - 0.01} = \frac{1}{0.99}$$

$$= \frac{100}{99}\,\Omega$$

By connecting a shunt resistance of $\frac{100}{99}\,\Omega$.

6. $i_g = \frac{V}{G}$

and $R = \frac{nV}{i_g} - G = (n-1)G$

7. $V_{AB} = \frac{15}{16}E$

Potential gradient

$$k = \frac{V_{AB}}{L} = \frac{15E}{16 \times 600}$$

$$= \frac{E}{640}\text{ V/cm}$$

(a) $\frac{E}{2} = kL \Rightarrow L = \frac{E}{2k} = 320$ cm

(b) $V = kl = \frac{E}{640} \times 560 = \frac{7E}{8}$

Also, $V = E - ir$

$$\therefore \quad E - ir = \frac{7E}{8}$$

$$i = \frac{E}{8r}$$

AIEEE Corner

Subjective Questions (Level 1)

1. $i = \frac{q}{t} = \frac{ne}{t}$

Given,

$$i = 0.7\,,\; t = 1\text{ s},\; e = 1.6 \times 10^{-19}\text{ C}$$

$$\therefore \quad n = \frac{it}{e} = \frac{0.7 \times 1}{1.6 \times 10^{-19}}$$

$$= 4.375 \times 10^8$$

2. $q = it = 3.6 \times 3 \times 3600$

$$= 38880\text{ C}$$

3. (a) $q = it = 7.5 \times 45 = 337.5$ C

(b) $q = ne \Rightarrow n = \frac{q}{e}$

$$= \frac{337.5}{1.6 \times 10^{-19}} = 2.11 \times 10^{21}$$

4. $T = \frac{2\pi r}{v} \Rightarrow f = \frac{1}{T} = \frac{v}{2\pi r}$

$$= \frac{2.2 \times 10^6}{2 \times 3.14 \times 5.3 \times 10^{-11}}$$

$$= 6.6 \times 10^{19}\text{ s}^{-1}$$

$$I = \frac{q}{T} = ef$$

$$= 1.6 \times 10^{-19} \times 6.6 \times 10^{19}$$

$$= 10.56\text{ A}$$

5. (a) $I = 55 - 0.65\,t^2$

$$I = \frac{dq}{dt}$$

$$\Rightarrow \quad dq = Idt$$

$$\Rightarrow \quad q = \int I\,dt$$

$$\therefore \quad q = \int_0^8 Idt = \int_0^8 (55 - 0.65\,t^2)\,dt$$

$$= 55[t]_0^8 - 0.65\left[\frac{t^2}{2}\right]_0^8$$

$$= 440 - 20.8 = 419.2\text{ C}$$

(b) If current is constant

$$I = \frac{q}{t} = \frac{419.2}{8} = 52.4\text{ A}$$

6. $i \propto v_d$

$$\therefore \quad \frac{v_{d_2}}{v_{d_1}} = \frac{i_2}{i_1}$$

$$\Rightarrow \quad v_{d_2} = \frac{i_2}{i_1} v_{d_1} = \frac{6.00}{1.20} \times 1.20 \times 10^{-4}$$

$$= 6.00 \times 10^{-4}\text{ ms}^{-1}$$

7. $$v_d = \frac{i}{neA}$$

$$= \frac{1}{8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times 1 \times 10^{-4}}$$

$$= 0.735 \times 10^{-6}\text{ ms}^{-1}$$

$$= 0.735\,\mu\text{m/s}$$

$$t = \frac{l}{v_d} = \frac{10^3}{0.735 \times 10^{-6}}$$

$$= 1.36 \times 10^9\text{ s} = 43\text{ yr}$$

8. Distance covered by one electron in 1 s

$$= 1 \times 0.05 = 0.05\text{ cm}$$

Number of electrons in 1 cm of wire

$$= 2 \times 10^{21}$$

$\therefore$ Number of electrons crossing a given area per second

= Number of electrons in 0.05 cm of wire

$$= 0.05 \times 2 \times 10^{21} = 10^{20}$$

$$i = \frac{q}{t} = \frac{ne}{t}$$

$$= \frac{10^{20} \times 1.6 \times 10^{-19}}{1} = 1.6 \times 10 = 16\text{ A}$$

9. $R = \rho\dfrac{L}{A}$

Given,

$$\rho = 0.017\,\mu\Omega\text{-m}$$

$$= 1.7 \times 10^{-8}\ \Omega\text{-m}$$

$$l = 24.0\text{ m}$$

$$A = \pi\left(\frac{d}{2}\right)^2 = 3.14 \times \left(\frac{2.05}{2} \times 10^{-3}\right)^2$$

$$= 3.29 \times 10^{-6}\text{ m}^2$$

$$R = 1.7 \times 10^{-8} \times \frac{24.0}{3.29 \times 10^{-6}}$$

$$= 0.12\,\Omega$$

10. $$R = \rho\frac{L}{A}$$

$$A = \frac{\rho L}{R}$$

If D is density, then

$$m = DV = DA\,L = \frac{D\rho L^2}{R}$$

$$= \frac{8.9 \times 10^3 \times 1.72 \times 10^{-8} \times (3.5)^2}{0.125}$$

$$= 1.5 \times 10^{-2}\text{ kg} = 15\text{ g}$$

11. At 20° C,

$R_1 = 600\,\Omega$, $R_2 = 300\,\Omega$

At 50° C,

$$R_1' = R_1(1 + \alpha_1\Delta t)$$

$$= 600(1 + 0.001 \times 30) = 600 \times 1.03$$

$$= 618\,\Omega$$

$$R_2' = R_2(1 + \alpha_2\Delta t)$$

$$= 300(1 + 0.004 \times 30) = 336$$

$$\therefore \quad R' = R_1' + R_2' = 618 + 336$$

$$= 954\,\Omega$$

$$\alpha = \frac{R' - R}{R \times \Delta t} = \frac{954 - 900}{900 \times 30}$$

$$R = 600 + 300 = 900\,\Omega$$

$$= 0.002\ ^\circ\text{C}^{-1}$$

12. As both the wires are connected in parallel,

$$V_{\text{Al}} = V_{\text{Cu}}$$

$$i_{\text{Al}}R_{\text{Al}} = i_{\text{Cu}}R_{\text{Cu}}$$

$$i_{\text{Al}}\rho_{\text{Al}}\frac{L_{\text{Al}}}{\pi d_{\text{Al}}^2} = i_{\text{Cu}}\rho_{\text{Cu}}\frac{L_{\text{Cu}}}{\pi d_{\text{Cu}}^2}$$

$$\Rightarrow \quad d_{\text{Cu}} = d_{\text{Al}}\sqrt{\frac{i_{\text{Cu}}\,\rho_{\text{Cu}}\,L_{\text{Cu}}}{i_{\text{Al}}\rho_{\text{Al}}L_{\text{Al}}}}$$

$$= 1 \times 10^{-3}\sqrt{\frac{2 \times 0.017 \times 6}{3 \times 0.028 \times 7.5}}$$

$$= 0.569 \times 10^{-3}\text{ m}$$

$$= 0.569\text{ mm.}$$

13. (a) $E = \frac{V}{L} = \frac{0.938}{75 \times 10^{-2}} = 1.25$ V/m

(b) $J = \frac{E}{\rho} \Rightarrow \rho = \frac{1.25}{4.4 \times 10^{7}}$

$$\rho = 2.84 \times 10^{-8}\ \Omega\text{-m}$$

14. (a) $J = \frac{E}{\rho} = \frac{V}{\rho L}$

Current density is maximum when L is minimum, *ie*, $L = d$, potential difference should be applied to faces with dimensions $2d \times 3d$.

$$J_{\text{min.}} = \frac{V}{\rho d}.$$

(b) $i = \frac{V}{R} = \frac{VA}{\rho L}$

Current is maximum when L is minimum and A is maximum.

Hence, in this case also, V should be applied to faces with dimensions $2d \times 3d$

and $i_{\max} = \frac{V(2d \times 3d)}{\rho(d)} = \frac{6Vd}{\rho}$.

15. (a) $R = \rho \frac{L}{A}$

$$\rho = \frac{RA}{L}$$

$[r = \frac{d}{2} = 1.25 \text{ mm} = 1.25 \times 10^{-3} \text{ m}]$

$$= \frac{0.104 \times 3.14 \times (1.25 \times 10^{-3})^2}{14.0}$$

$$= 3.64 \times 10^{-7}\ \Omega\text{-m}$$

(b) $i = \frac{V}{R} = \frac{EL}{R} = \frac{1.28 \times 14}{0.104} = 172.3$ A

(c) $i = neAv_d$

$$v_d = \frac{i}{neA}$$

$$= \frac{172.3}{8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times 3.14 \times (1.25 \times 10^{-3})^2}$$

$$= 2.58 \times 10^{-3} \text{ ms}^{-1}$$

16. For zero thermal coefficient of resistance,

$$\Delta R = 0$$

$$R_C \alpha_C \Delta T + R_{Fe} \alpha_{Fe} \Delta T = 0$$

$$\frac{R_1}{R_2} = \frac{-\alpha_{Fe}}{\alpha_C} = \frac{-5.0 \times 10^{-3}}{-0.5 \times 10^{-3}} = 10$$

$$R_1 = 10R_2$$

Also, $R_1 + R_2 = 20$

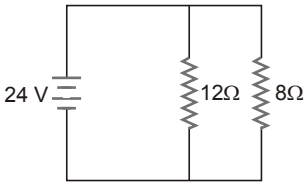
$\Rightarrow$ $10R_2 + R_2 = 20$

$\Rightarrow$ $R_2 = \frac{20}{11}\Omega = 1.82\ \Omega$

and $R_1 = 20 - R_2 = 20 - 1.82\ \Omega$

$$= 18.2\ \Omega$$

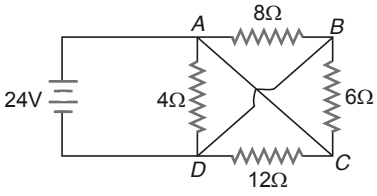
17. The circuit can be redrawn as



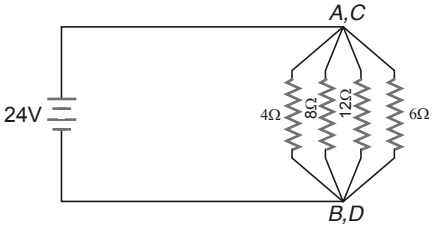
$$R_{\text{eff}} = \frac{8 \times 12}{12 + 8} = 4.8\ \Omega$$

$$I = \frac{V}{R_{\text{eff}}} = \frac{24}{4.8} = 5 \text{ A}$$

18. Here, A and C are at same potential and B and D are at same potential,



Hence, the circuit can be redrawn as



$$\therefore \quad \frac{1}{R} = \frac{1}{4} + \frac{1}{8} + \frac{1}{12} + \frac{1}{6}$$

$$= \frac{6 + 3 + 2 + 4}{24}$$

$$= \frac{15}{24} = \frac{5}{8}$$

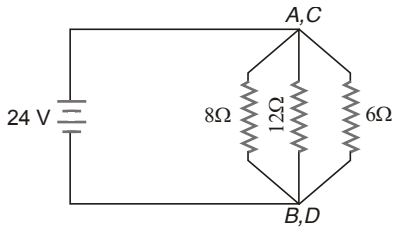
$$R = \frac{8}{5}\Omega$$

$$= 1.6\ \Omega$$

$$i = \frac{V}{R} = \frac{24}{1.6}$$

$$= 15 \text{ A}$$

19. Given circuit is similar to that in previous question but $4\ \Omega$ resistor is removed. So the effective circuit is given by



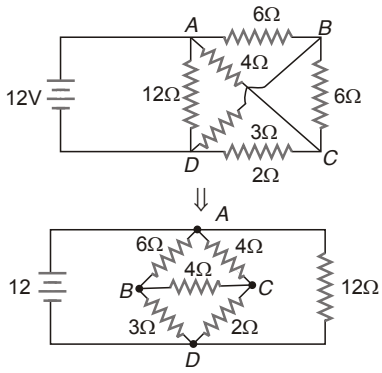
$$\frac{1}{R} = \frac{1}{8} + \frac{1}{12} + \frac{1}{6}$$

$$\frac{1}{R} = \frac{3+2+4}{24} = \frac{9}{24} = \frac{3}{8}$$

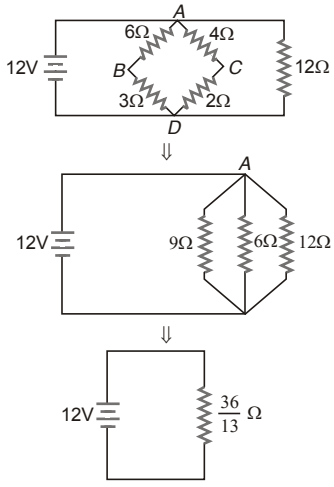
$$R = \frac{8}{3}\,\Omega = 2.67\,\Omega$$

$$i = \frac{V}{R} = \frac{24}{2.67} = 9\text{ A}$$

20.



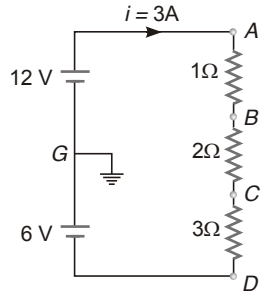
Wheatstone bridge is balanced, hence $4\,\Omega$ resistance connected between B and C be removed and the effective circuit becomes



$$i = \frac{V}{R} = \frac{12}{36/13}$$

$$= \frac{13}{3}\text{ A}$$

21. (a) $i = \dfrac{12+6}{1+2+3} = 3\text{ A}$



$$V_G = 0$$

$$V_A = V_G + 12 = 12\text{ V}$$

$$V_A - V_B = 3\text{ V}$$

$$V_B = 12 - 3 = 9\text{ V}$$

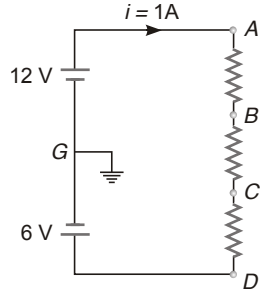
$$V_B - V_C = 6\text{ V}$$

$$V_C = 9 - 6 = 3\text{ V}$$

$$V_G - V_D = 6\text{ V},\ V_D = -6\text{ V}$$

(b) If 6 V battery is reversed

$$i = \frac{12-6}{1+2+3} = 1\text{ A}$$



$$V_G = 0,$$

$$V_A - v_G = 12\text{ V},\ \ V_A = 12\text{ V}$$

$$V_A - V_B = 1\text{ V}$$

$$\Rightarrow \quad V_B = 11\text{ V}$$

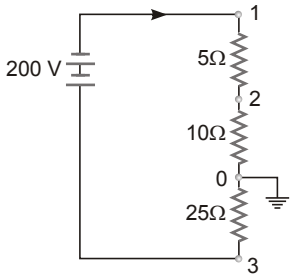
$$V_B - V_C = 2\text{ V}$$

$$\Rightarrow \quad V_C = 9\text{ V}$$

$$V_D - V_G = 6\text{ V}$$

$$\Rightarrow \quad V_D = 6\text{ V}$$

22. $i = \dfrac{200}{5 + 10 + 25} = 5$ A



(i) $V_3 - V_0 = 5 \times 25$

$\Rightarrow \quad V_3 = 125$ V

(ii) $V_0 - V_2 = 5 \times 10$

$V_2 = -50$ V

(iii) $V_2 - V_1 = 5 \times 5$

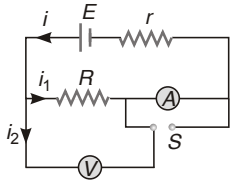
$V_1 = -75$ V

(iv) $V_{3-2} = 5 \times 35 = 175$ V

(v) $V_{1-2} = -5 \times 5 = -25$ V

(vi) $V_{1-3} = -200$ V

23. (a)



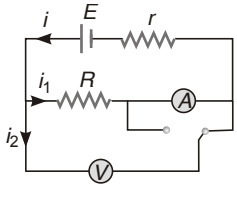
$$R_{\text{eff}} = R \,||\, R_v + R_a + r$$
$$= \frac{50 \times 200}{50 + 200} + 2 + 1$$
$$= 43\ \Omega$$
$$i = \frac{E}{R_{\text{eff}}} = \frac{4.3}{43} = 0.1 \text{ A}$$

$\therefore$ Reading of ammeter, $i = 0.1$ A

and reading of voltmeter $= i(R \,||\, R_v)$

$= 0.1 \times 40 = 4$ V

(b)



$$R_{\text{eff}} = (R_a + R) \,||\, R_v + r$$
$$= \frac{52 \times 200}{52 + 200} + 1$$
$$= 42.26\ \Omega$$
$$i = \frac{E}{R_{\text{eff}}} = 0.102\ \Omega$$

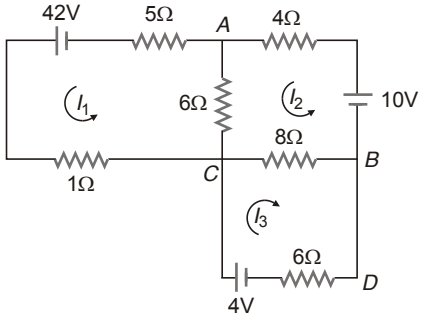
Reading of voltmeter

$$V = E - ir$$
$$= 4.3 - 0.102 \times 1$$
$$\approx 4.2\ \Omega$$

Reading of Ammeter,

$$i_1 = \frac{V}{R + R_a} = \frac{4.2}{42} = 0.08 \text{ A}$$

24. Consider the directions of current as shown in figure.



Applying KVL in loop 1, 2 and 3, we respectively get,

$I_1 + 6(I_1 - I_2) + 5I_1 = 42$

$\Rightarrow \quad 12I_1 - 6I_2 = 42$

$\Rightarrow \quad 2I_1 - I_2 = 7 \quad$ ...(i)

$4I_2 + 6(I_2 - I_1) + 8(I_2 + I_3) = 10$

$\Rightarrow \quad 9I_2 - 3I_1 + 4I_3 = 5 \quad$ ...(ii)

$8(I_2 + I_3) + 16I_3 = 4$

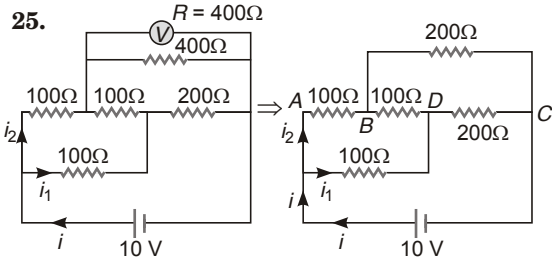
$2I_2 + 6I_3 = 1 \quad$ ...(iii)

On solving, we get,

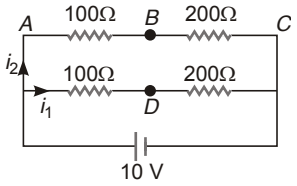
$I_1 = 4.7$ A, $I_2 = 2.4$ A, $I_3 = 0.5$ A

Resistor	5Ω	1Ω	4Ω	6Ω	8Ω	16Ω
Current	4.7 A	4.7 A	2.4 A	2.3 A	2.9 A	0.5 A

25.



As Wheatstone bridge is balanced, $100\,\Omega$ resistance between B and D can be removed, *ie*,



$$\therefore \qquad i_1 = i_2 = \frac{10}{300} = \frac{1}{30}\text{ A}$$

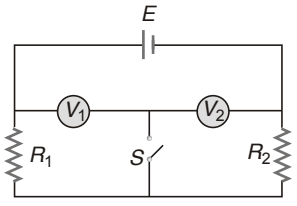
Hence, reading of voltmeter

$= $ Potential difference between B and C

$$= 200 \times i_2 = \frac{20}{3}\text{ V}$$

$$= 6.67\text{ V}$$

26. (a) (i) When S is open.



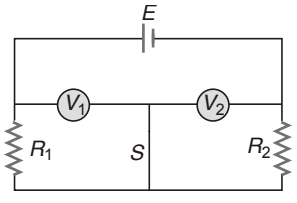
$$V_1 = \frac{R_1}{R_{V_1} + R_{V_2}} E = \frac{3000}{5000} \times 200$$

$$= 120\text{ V}$$

$$V_2 = \frac{R_{V_2}}{R_{V_1} + R_{V_2}} E = \frac{2000}{5000} \times 200$$

$$= 80\text{ V}$$

(ii) When S is closed,

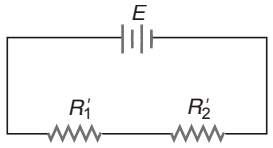


Now, R_1 and V_1 are in parallel and their effective resistance

$$R_1' = \frac{R_1 R_{V_1}}{R_1 + R_{V_1}} = \frac{6000}{5} = 1200\,\Omega$$

Similarly,

R_2 and V_2 are in parallel with their effective resistance,



$$R_2' = \frac{R_2 R_{V_2}}{R_2 + R_{V_2}} = \frac{6000}{5} = 1200\,\Omega$$

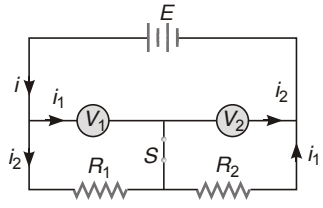
As $\qquad R_1' = R_2'$

Hence,

reading of V_1 = reading of V_2

$$= \frac{1200}{1200 + 1200} \times 200 = 100\text{ V}$$

(b) Current distribution is shown in figure



$$i = \frac{E}{R_1' + R_2'}$$

$$i = \frac{200}{2400} = \frac{1}{12}\text{ A}$$

$$i_1 = \frac{R_{V_1}}{R_1 + R_{V_1}} i = \frac{3000}{5000} \times \frac{1}{12}$$

$$= \frac{1}{20}\text{ A}$$

$$i_2 = \frac{R_1}{R_1 + R_{V_1}} i = \frac{2000}{5000} \times \frac{1}{12}$$

$$= \frac{1}{30}\text{ A}$$

$\therefore$ Current flowing through

$$S = i_1 - i_2 = \frac{1}{20} - \frac{1}{30}$$

$$= \frac{1}{60}\text{ A}$$

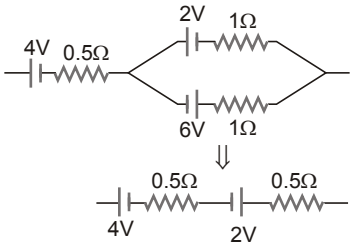
27. Effective emf of 2 V and 6 V batteries connected in parallel

$$E' = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2} = \frac{2 \times 1 - 6 \times 1}{1 + 1}$$

$$= -2\text{ V}$$

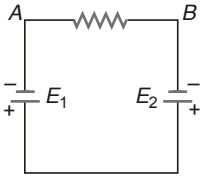
and $\qquad r' = \dfrac{r_1 r_2}{r_1 + r_2} = \dfrac{1}{2}\,\Omega$

$$= 0.5\,\Omega$$



Net emf, $E = 4 - 2 = 2$ V

28. (a)



As $E_1 > E_2$

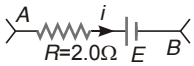
Current will flow from B to A.

(b) E_1 is doing positive work

(c) As current flows from B to A through resistor, B is at higher potential.

29. $i^2R = 2\text{ W} < 5\text{ W}$

Clearly X is doing negative work.

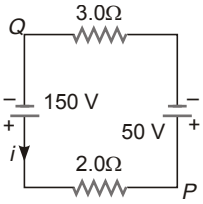


(a) $P = Vi \Rightarrow V = \dfrac{P}{i} = \dfrac{0.5}{1.0} = 5.0$ V

(b) $E = V - iR = 5 - 2 = 3$ V

(c) It is clear from figure that positive terminal of X is towards left.

30. $i = \dfrac{150 - 50}{3 + 2} = 20$ A



$$V_P - V_Q = 50 + 3.0\,i$$
$$V_Q = 100 - (50 + 60)$$
$$= -10\text{ V}$$

31. (a) As voltmeter is ideal, it has infinite resistance, therefore current is zero.

(b) $V = E - ir \Rightarrow E = 5.0$ V

(c) Reading of voltmeter $\Rightarrow V = 5.0$ V

32. $V_1 = E - i_1 r \Rightarrow E - 1.5\,r = 8.4$ ...(i)

$V_2 = E + i_2 r \Rightarrow E + 3.5\,r = 9.4$ ...(ii)

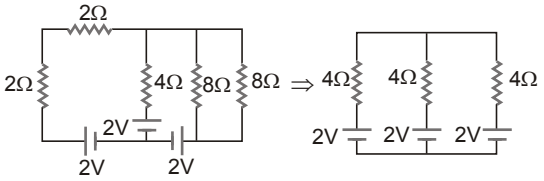
On solving, we get

$$r = 0.2\,\Omega$$
$$E = 8.7\text{ V}$$

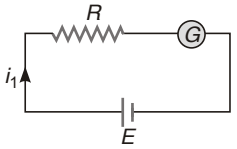
33. In case of charging

$V = E + i\,r = 2 + 5 \times 0.1 = 2.5$ V

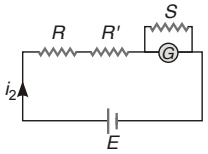
34. Clearly current through each branch is zero.



35. $i_1 = \dfrac{E}{R + G}$



On shunting the galvanometer with resistance S,



$$i_2 = \frac{E}{R + R' + \dfrac{GS}{S + G}}$$

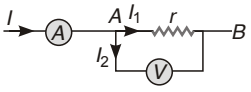
As $i_1 = i_2$

$$\frac{E}{R + G} = \frac{E}{R + R' + \dfrac{GS}{G + S}}$$

$$\Rightarrow \quad R + R' + \frac{GS}{G + S} = R + G$$

$$R' = \frac{G^2}{G + S}$$

36.



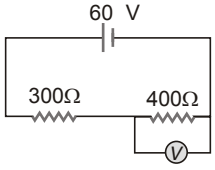
$$I_2 = \frac{r}{R + r} I = \frac{V}{R}$$

$$\Rightarrow \quad \frac{r}{R+r} = \frac{V}{IR}$$

$$\Rightarrow \quad \frac{R}{r} = \frac{IR - V}{V} = \frac{5 \times 2500 - 100}{100}$$

$$r = \frac{100}{12400} \times 2500 = 20.16\,\Omega$$

37.

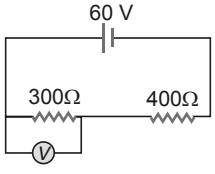


Let R be the resistance of voltmeter

As reading of voltmeter is 30 V,

$$\frac{1}{R} + \frac{1}{400} = \frac{1}{300} \Rightarrow R = 1200\,\Omega$$

If voltmeter is connected across $300\,\Omega$ resistor,



Effective resistance of $300\,\Omega$ resistor and voltmeter

$$R' = \frac{300 \times 1200}{300 + 1200} = 240\,\Omega$$

$$i = \frac{60}{400 + 240}$$

$$= \frac{60}{640}\text{ A}$$

$$= \frac{3}{32}\text{ A}$$

$\therefore$ Reading of voltmeter,

$$V = iR' = \frac{3}{32} \times 240$$

$$= 22.5\text{ V}$$

38. $V_2 = \dfrac{R'}{R_1 + R_2'} V,$

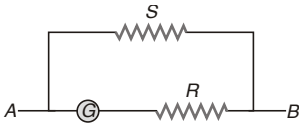
$$R_2' = \frac{rR_2}{r + R_2} = \frac{120}{3}$$

$$= 40\,\Omega$$

$$V_2 = \frac{40}{60 + 40} 120$$

$$= 48\text{ V}$$

39. $S = \dfrac{i_g}{i - i_g}(G + R)$



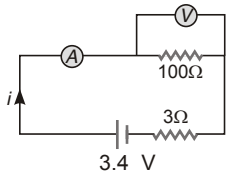
$$R = \frac{i - i_g}{i_g} S - G$$

$$= \frac{20 - 10^{-3}}{10^{-3}} \times 0.005 - 20$$

$$= 79.995\,\Omega$$

40. $r = \dfrac{L_1 - L_2}{L_2} R = \dfrac{0.52 - 0.4}{0.4} \times 5 = 1.5\,\Omega$

41. Let R be the resistance of voltmeter



$$R_e = 3 + 2 + \frac{100R}{100 + R}$$

$$= 5 + \frac{100R}{100 + R}$$

$$i = \frac{3.4}{5 + \dfrac{100R}{100 + R}} = 0.04$$

$$\Rightarrow \quad 0.2 + \frac{4R}{100 + R} = 3.4$$

$$\Rightarrow \quad R = 400\,\Omega$$

Reading of voltmeter,

$$V = i \times \frac{100R}{100 + R} = 0.04 \times \frac{100 \times 400}{100 + 400}$$

$$= 3.2\text{ V}$$

If the voltmeter had been ideal,

Reading of voltmeter

$$= \frac{100}{105} \times 3.4 = 3.24\text{ V}$$

42. $\dfrac{L_1}{L_2} = \dfrac{R_1}{R_2}$

$$\Rightarrow \quad \frac{L_1}{40 - L_1} = \frac{8}{12} \qquad (L_1 + L_2 = 40\text{ cm})$$

$$\Rightarrow \quad L_1 = 16\text{ cm} \quad \text{from } A.$$

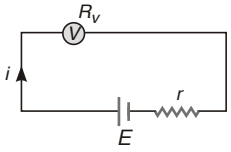
43. $S = \frac{i_g}{i - i_g}(G + R)$

$$\Rightarrow \quad R = \frac{i - i_g}{i_g} S - G$$

$$= \frac{20 - 0.0224}{0.0244} \times 0.0250 - 9.36$$

$$= 12.94\,\Omega$$

44. (a) $i = \frac{E}{R_V + r}$



$$V = iR_V = \frac{E}{R_V + r} R_V$$

(b) $\frac{r}{R_V + r} = 1\% = \frac{1}{100}$

$$R_V = 99r = 99 \times 0.45$$

$$= 44.55\,\Omega$$

(c) $$\frac{V}{E} = \frac{R_V}{R_V + r}$$

As R_V decreases, V decreases, decreasing accuracy of voltmeter.

45. (a) When ammeter is connected

$$I_A = \frac{E}{R_A + R + r}$$

When ammeter is removed

$$I = \frac{E}{R + r} = \frac{R_A + R + r}{R + r} I_A$$

(b) $\frac{I_A}{I} = 99\%$

$$\frac{R + r}{R_A + R + r} = \frac{99}{100}$$

$$\Rightarrow \quad R_A = \frac{1}{99}(R + r) = \frac{1}{99}(3.8 + 0.45)$$

$$R_A = 0.043\,\Omega$$

(c) As $\frac{I_A}{I} = \frac{R + r}{R_A + R + r}$, as R_A increases, I_A decreases, decreasing the accuracy of ammeter.

46. $I_{max} = \sqrt{\frac{\rho_{max}}{R}} = \sqrt{\frac{36}{2.4}} = \sqrt{15}$ A

For the given circuit

$$R_e = \frac{1}{2} R + R = \frac{3}{2} R$$

Maximum power dissipated by the circuit

$$P'_{max} = I^2_{max} R_e$$

$$= 15 \times \frac{3}{2} \times 2.4 = 54\text{ W}$$

47. Total power of the circuit, $P = P_1 + P_2 + P_3$

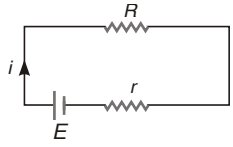
$$= 40 + 60 + 75$$

$$= 175\text{ W}$$

As $P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P}$

$$= \frac{(120)^2}{175} = 82.3\,\Omega$$

48. Thermal power generated in the battery



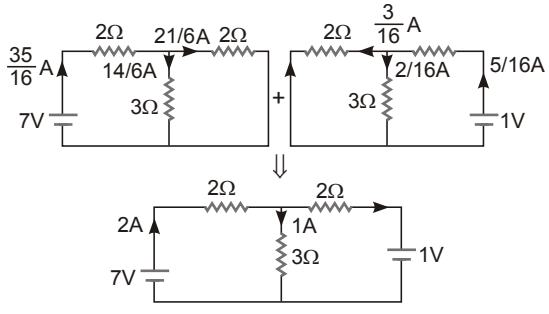
$$P_1 = i^2 r = i(E - V)$$

$$= 0.6\text{ W}$$

Power development in the battery by electric forces

$$P_2 = IE = 2.6\text{ W}$$

49. The given circuit can be considered as the sum of the circuit as shown.



$\therefore$ $$P_1 = 7 \times 2 = 14\text{ W},$$

$$P_2 = -1 \times 1 = 1\text{ W}$$

50. (a) $i = \frac{E_1 - E_2}{R_1 + R_2} = \frac{12 - 6}{4 + 8} = 0.5$ A

(b) Power dissipated in $R_1 = I^2 R_1 = 1$ W

and power dissipated in $R = I^2 R_2 = 2$ W

(c) Power of battery $E_1 = E_1 I$

$$= 12 \times 0.5 = 6\text{ W (supplied)}$$

Power of battery $E_2 = E_2 I$

$$= -6 \times 0.6 = -3\text{ W (absorbed)}$$

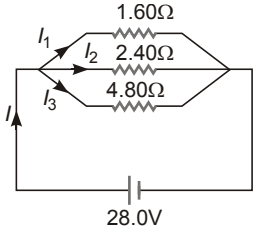
51. $I = \dfrac{E}{R+r} = \dfrac{12}{5+1} = 2\text{ A}$

(a) $P = EI = 12 \times 2 = 24\text{ W}$

(b) $P_1 = I^2R = 2^2 \times 5 = 20\text{ W}$

(c) $P_2 = I^2r = 2^2 \times 1 = 4\text{ W}$

52. (a)



$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R} = \frac{1}{1.60} + \frac{1}{2.40} + \frac{1}{4.80}$$

$$R = 0.80\,\Omega$$

(b) $I_1 = \dfrac{V}{R_1} = \dfrac{28.0}{1.60} = 17.5\text{ A}$

$$I_2 = \frac{V}{R_2} = \frac{28.0}{2.40} = 11.67\text{ A}$$

$$I_3 = \frac{V}{R_3} = \frac{28.0}{4.80} = 5.83\text{ A}$$

(c) $I = I_1 + I_2 + I_3 = 35.0\text{ A}$

(d) As all the resistance connected in parallel, voltage across each resistor is 28.0 V.

(e) $P_1 = \dfrac{V^2}{R_1} = \dfrac{(28)^2}{1.6} = 490\text{ W}$

$$P_2 = \frac{V^2}{R_2} = \frac{(28)^2}{2.4} = 326.7\text{ W}$$

$$P_3 = \frac{V^2}{R_3} = \frac{(28)^2}{4.8} = 163.3\text{ W}$$

(f) As, $P = \dfrac{V^2}{R}$

Resistor with least resistance will dissipate maximum power.

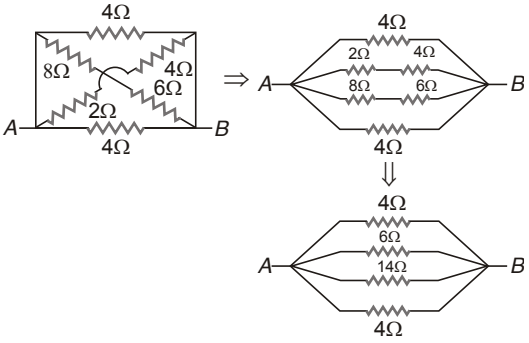
53. (a) $P = \dfrac{V^2}{R} \Rightarrow V = \sqrt{PR}$

$$= \sqrt{5 \times 15 \times 10^3} = 2.74 \times 10^2$$

$$= 274\text{ V}$$

(b) $P = \dfrac{V^2}{R} = \dfrac{(120)^2}{9 \times 10^3} = 1.6\text{ W}$

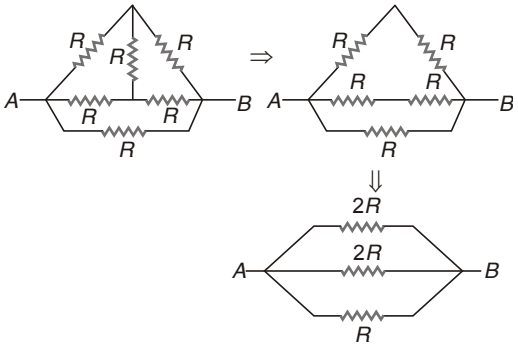
54. (a) $\dfrac{1}{R} = \dfrac{1}{4} + \dfrac{1}{6} + \dfrac{1}{14} + \dfrac{1}{4}$



$$\frac{1}{R} = \frac{31}{42}$$

$$R = \frac{42}{31}$$

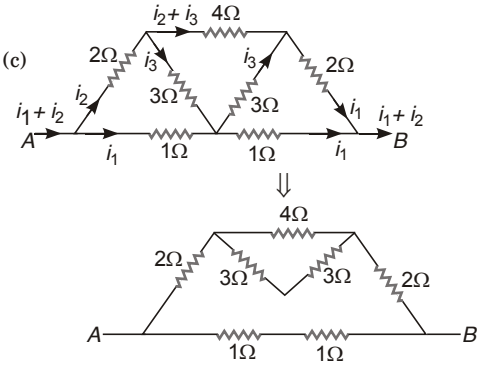
(b) $\dfrac{1}{R_e} = \dfrac{1}{2R} + \dfrac{1}{2R} + \dfrac{1}{R}$

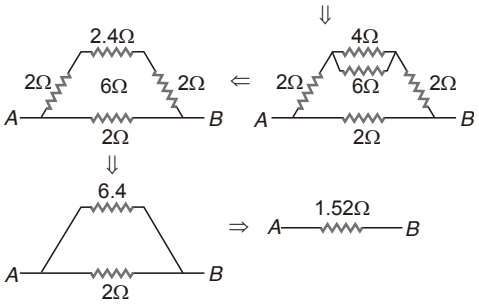


Wheatstone bridge is balanced

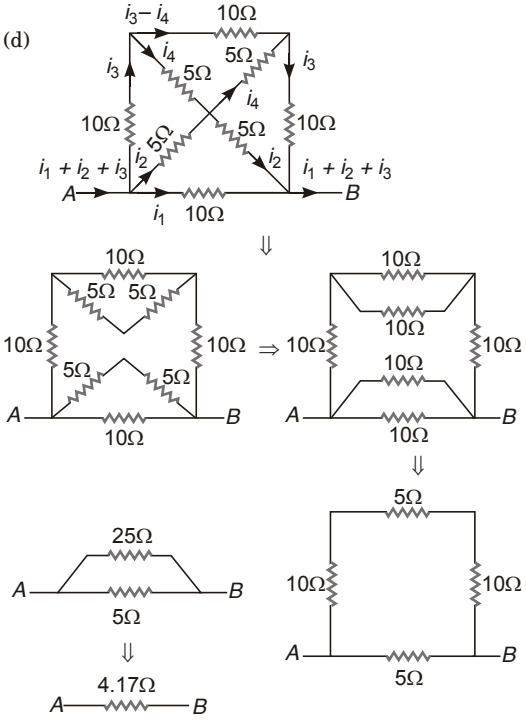
$\Rightarrow \qquad R_e = \dfrac{R}{2}$

(c)

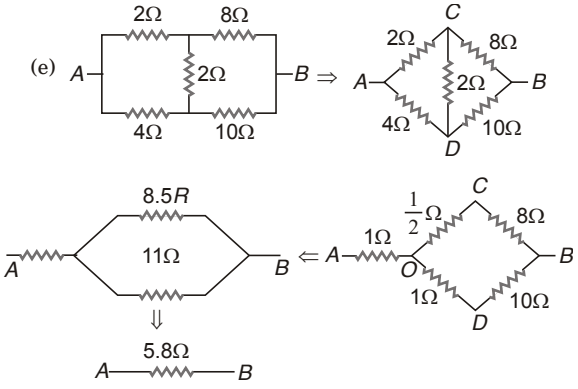




(d)



(e)



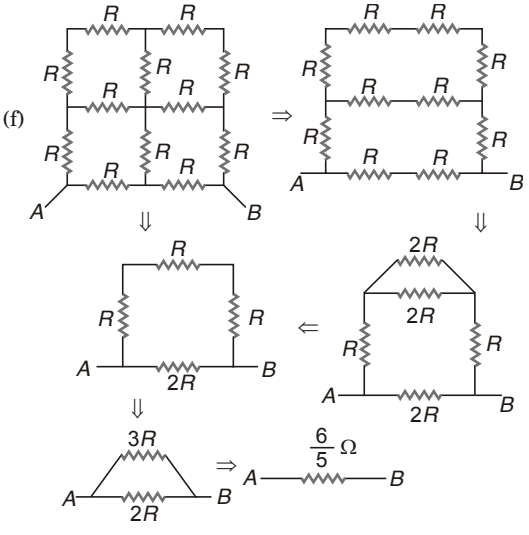
By Star-Delta Method

$$R_A = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

$$R_B = \frac{R_1 R_3}{R_1 + R_2 + R_3}$$

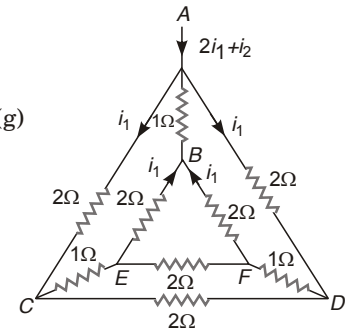
$$R_C = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

(f)



As circuit is symmetrical about perpendicular bisector of AB, lying on it are at same potential.

(g)



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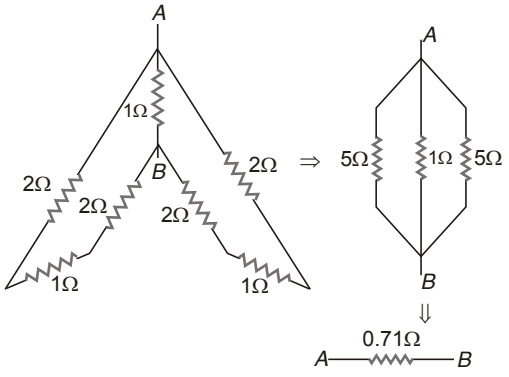
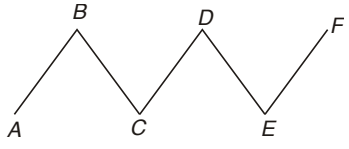


Clearly C and D, E and F are at same potential.

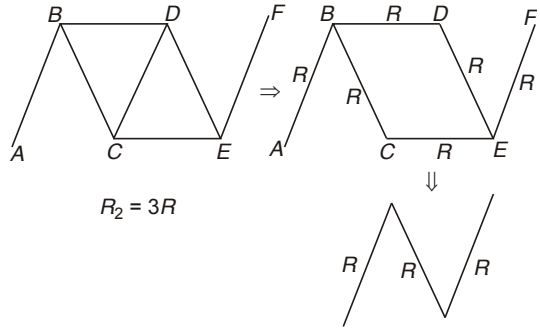
55.



Let R be the resistance of each conductor, and R_1 be the effective resistance between A and F in first case then,

$\therefore \qquad R_1 = 5R$

If R_2 be effective resistance between A and F in second case then,

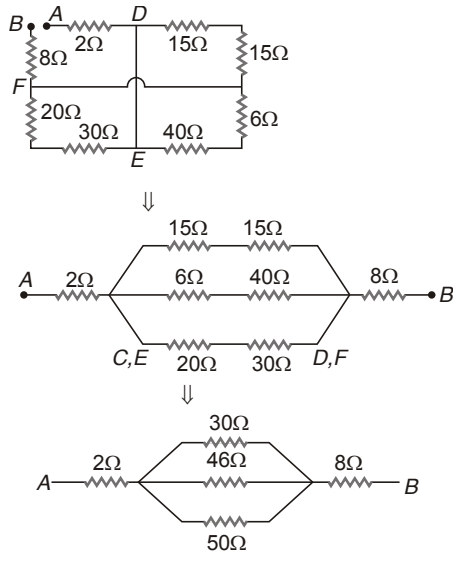


$R_2 = 3R$

$$\therefore \qquad \frac{R_2}{R_1} = \frac{3R}{5R} = 0.6$$

$$\Rightarrow \qquad R_2 = 0.6R_1$$

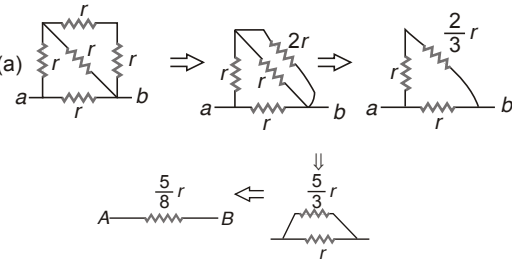
56.



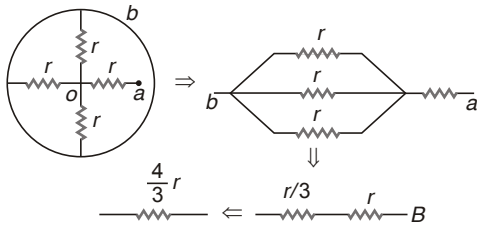
Here, C and E , D and F are at same potential.

$$R_e = 23.3\ \Omega$$

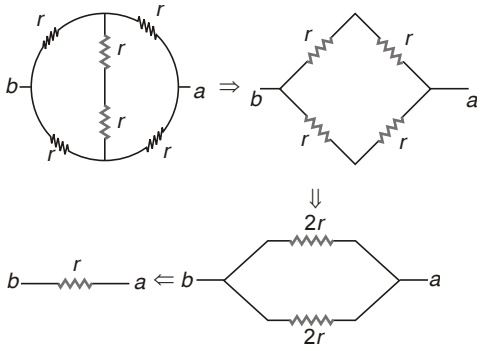
57. (a)



(b)

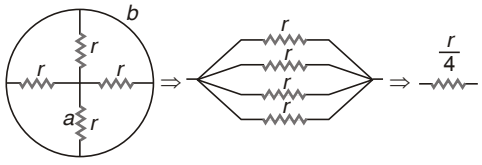


(c)

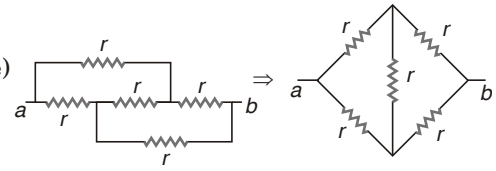


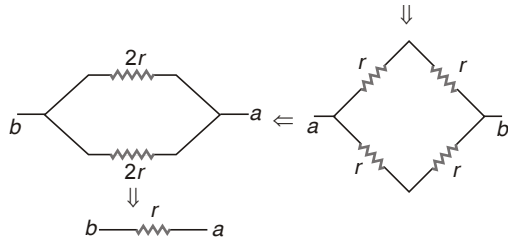
As Wheatstone bridge is balanced

(d)



(e)

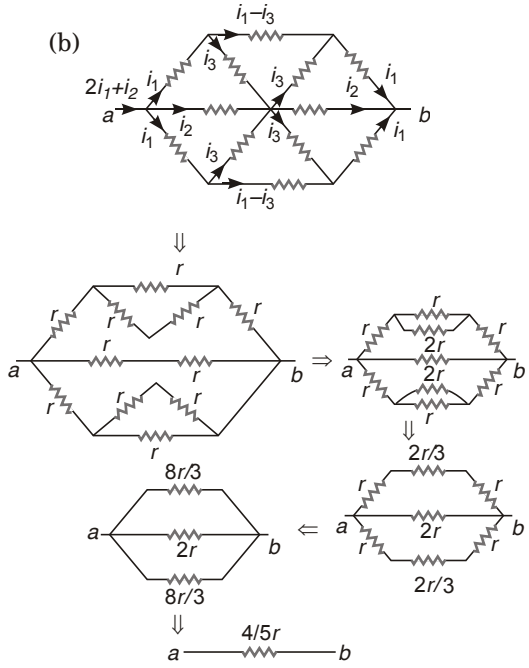




As Wheatstone bridge is balanced.

58. $R_e = \frac{r}{2}$

(b)



Objective Questions (Level-1)

1. When ammeter is connected in series

$$R_e = R + R_A$$

Hence, net current decreases. So R_A should be very low.

2. Amount of charge entering per second from one face is equal to the amount of charge leaving per second at the other, hence I is constant.

Again,

$$v_d = \frac{I}{neA} = \text{not constant.}$$

As $$v_d = \frac{eF}{m}\tau$$

$$\Rightarrow \quad E = \frac{mv_d}{e\tau} = \text{not constant}$$

3. $R = \frac{V}{I}$

$$\Rightarrow \quad [R] = \frac{[V]}{[I]} = \frac{[ML^2T^{-3}I^{-1}]}{[I]}$$

$$= [ML^2T^{-3}I^{-2}]$$

4. $\sigma = \frac{1}{\rho}$

As unit of resistivity is ohm-m and unit of σ is ohm^{-1}- m^{-1}.

5. Fact.

6. $E = I(R + r)$

Case I

$$E = 0.5(3.75 + r)$$

Case II

$$E = 0.4(4.75 + r)$$

On solving

$$r = 0.25\,\Omega,\ E = 2\text{ V}$$

7. $\dfrac{I}{I_g} = \dfrac{50}{20} \Rightarrow I = \dfrac{5}{2} I_g$

$$S = \frac{I_g}{I - I_g} G \Rightarrow G = \frac{I - I_g}{I_g} S$$

$$= \frac{3}{2} \times 12$$

$$= 18\,\Omega$$

8. $I_g = 2\% I = \dfrac{1}{50} I$

$$S = \frac{I_g}{I - I_g} G = \frac{G}{49}$$

9. $P = \dfrac{V^2}{R}$

$$P + \Delta P = \frac{V^2}{R + \Delta R}$$

As $R \propto l$

$$\Delta R = -10\% R$$

$$\Delta P = \frac{V^2}{0.9R} - \frac{V^2}{R} = \left(\frac{1}{0.9} - 1\right) P$$

$$= \frac{10}{9} P \approx 11\% P$$

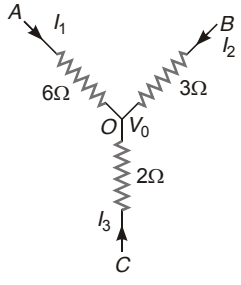
10. Potential difference between any two points is zero.

11. $$r = \frac{l_1 - l_2}{l_2} R$$

$$= \frac{75 - 60}{60} \times 10$$

$$= 2.5\,\Omega$$

12. (b) By applying KCL at O



$$I_1 + I_2 + I_3 = 0$$

$$\frac{6 - V_0}{6} + \frac{3 - V_0}{3} + \frac{2 - V_0}{2} = 0$$

$$\Rightarrow 6 - V_0 + 2(3 - V_0) + 3(2 - V_0) = 0$$

$$V_0 = 3\text{ V}$$

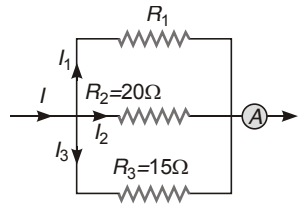
13. $v_d = \dfrac{I}{neA} = \dfrac{I}{ne\,\pi r^2}$

$$v_d' = \frac{2I}{ne\pi(2r)^2} = \frac{v_d}{2} = \frac{v}{2}$$

14. Voltmeter has higher resistance than ammeter.

Again higher the range of voltmeter, higher will be its resistance.

15. $I_2 = \dfrac{\dfrac{1}{R_2}}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}} I$



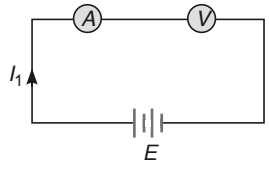
$$\Rightarrow \quad \frac{1}{R_1} = \frac{I}{I_2 R_2} - \left(\frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$= \frac{0.8}{0.3 \times 20} - \left(\frac{1}{20} + \frac{1}{15}\right)$$

$$= \frac{1}{60}$$

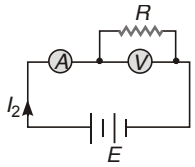
$$R_1 = 60\,\Omega$$

16. (d) $I_1 = \dfrac{E}{R_A + R_V}$, $V_1 = I_1 R_V$



$$= E - I_1 R_A$$

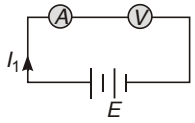
If resistance is connected in parallel with voltmeter,



$$I_2 = \frac{E}{R_A + \dfrac{RR_V}{R + R_V}} > I_1$$

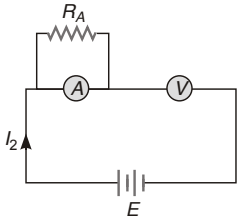
and $\quad V_2 = E - I_2 R_A < V_1$

17. Before connectivity resistance is parallel with ammeter



$$I_1 = \frac{E}{R_A + R_V},\ V_1 = I_1 R_V$$
$$= E - I_1 R_A$$

After connecting resistance in parallel to the ammeter.



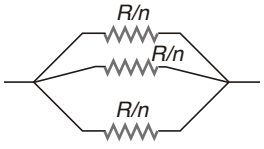
$$I_2 = \frac{E}{\dfrac{R_A}{2} + R_V},$$

Reading of ammeter $= \dfrac{1}{2} I_2$

$$= \frac{E}{R_A + 2R_V} > \frac{1}{2} I_1$$

$$V = I_2 R_V = \frac{2E}{R_A + 2R_V} < 2V_1$$

18. $R_e = \dfrac{R}{n^2}$



$$P = \frac{V^2}{R}$$

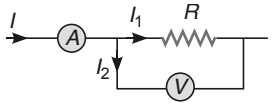
$$P_e = \frac{V^2}{R_e} = n^2 P$$

19. As bulb A is in series with entire circuit.

20. $I = \dfrac{E_1 + E_2}{R + r_1 + r_2} = \dfrac{18}{R + 3}$

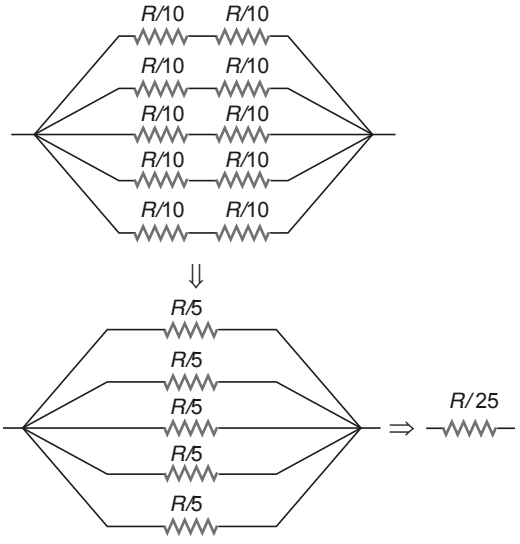
$$V_{ab} = E_2 - Ir_2 = 0$$
$$3 - \frac{18}{R+3} \times 1 = 0$$
$$\Rightarrow \qquad R = 3\ \Omega$$

21. $I_2 = \dfrac{R}{R + R_V} I$



$$\frac{V}{R_V} = \frac{R}{R + R_V} I$$
$$\frac{100}{2500} = \frac{R}{R + 2500} 5$$
$$R + 2500 = 125\,R$$
$$R = \frac{2500}{24}\,\Omega \approx 100\,\Omega$$

22.



23. $\dfrac{R_1}{R_2} = \dfrac{20}{80} = \dfrac{1}{4}$ ...(i)

$$\frac{R_1 + 15}{R_2} = \frac{40}{60} = \frac{2}{3}$$

$$\frac{R_1}{R_2} + \frac{15}{R_2} = \frac{2}{3}$$

$$\frac{15}{R_2} = \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$$

$$\Rightarrow \quad R_2 = 36\,\Omega,$$

$$R_1 = \frac{R_2}{4} = 9\Omega$$

24. (b) As $V_1 = \frac{V}{2}$, $R_1 = R_2$

$$\frac{R_V \times 100}{100 + R_V} = 50$$

$$R_V = 100\,\Omega$$

25. (d) $\frac{E}{R_{PB} + r} = \frac{2}{4+1} = 0.4\,\Omega$

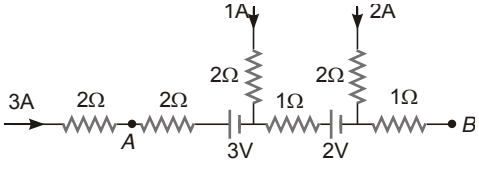
$$V_{AB} = IR_{AB} = 1.6\,\Omega$$

$$K = \frac{V_{PB}}{L} = \frac{1.6}{100} = 0.016\text{ V/cm}$$

$$L = \frac{E_1}{K} = \frac{1.2}{0.016} = 75\text{ cm}$$

26. (d) $V_{AB} = 3 \times 2 + 3 + 1 \times 4 - 2 + 6 \times 1$

$= 17$ V



27. (c) $E_e = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2} = 2$ V

$$r_e = \frac{r_1 r_2}{r_1 + r_2} = 0.5\,\Omega$$

For maximum power $R = r_e$

and $\quad P_{\max} = \frac{E_e^2}{4r_e} = \frac{(2)^2}{4 \times 0.5} = 2$ W

28. (a) $V = \frac{R}{R+r}E$

$$r = \left(\frac{E}{V} - 1\right)R = \left(\frac{2.2}{1.8} - 1\right)5$$

$$= \frac{10}{9}\,\Omega$$

29. (d) $I = \frac{E_1 - E_2}{R_1 + R_2 + r_1 + r_2}$

$$= \frac{10 - 5}{25 + 15 + 2.5 + 2.5} = \frac{1}{9}\text{ A}$$

$$V_{AB} = -I(25 + 15)$$

$$= -\frac{1}{9} \times 40 \approx -4\text{ V}$$

30. (a) $V_{AB} = kL = 0.2 \times 100 = 20$ mV

$$V_{AB} = \frac{R_{AB}}{R_{AB} + R}E$$

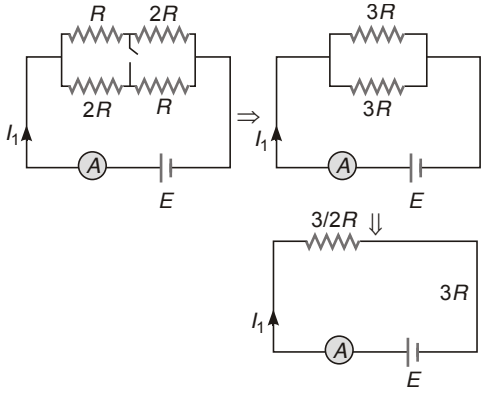
$$\Rightarrow \quad 0.02 = \frac{R_{AB}}{R_{AB} + 490} \times 2$$

$$\Rightarrow \quad R_{AB} + 490 = 100R_{AB}$$

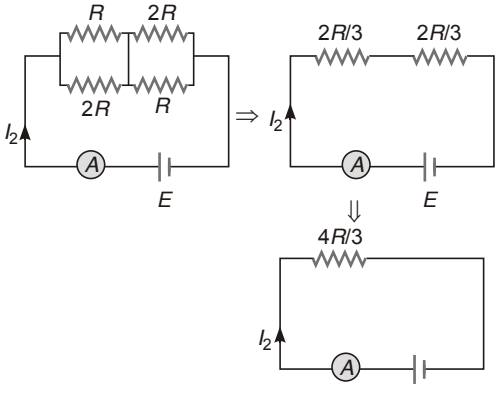
$$R_{AB} = \frac{490}{99} \approx 4.9\,\Omega$$

31. (c) When key is open,

$$I_1 = \frac{2E}{3R}$$



When key is closed



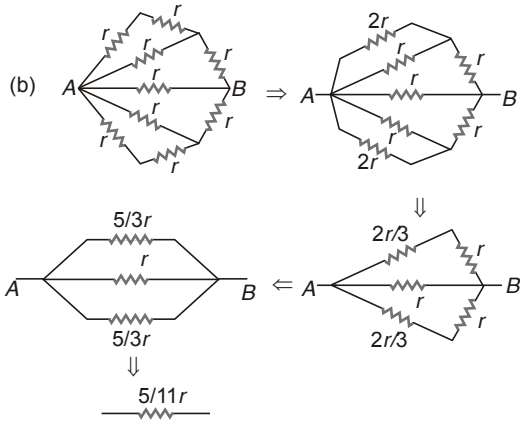
$$I_2 = \frac{3E}{4R}$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{8}{9}$$

32. (b) $S = \dfrac{I_g}{I - I_g} G = \dfrac{\frac{1}{34} I}{\frac{33}{34} I} \times 3663$

$= 111\,\Omega$

33. (b)

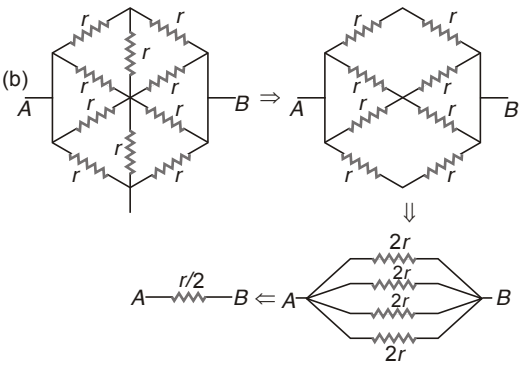


$R_e = \dfrac{5}{11} r$

$r = \dfrac{11}{5} \times 1.5$

$= 3.3\ \Omega$

34. (b)

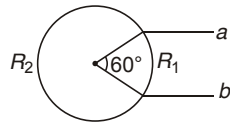


As the circuit is symmetrical about perpendicular bisector of AB, all points lying on it are at same potential.

35. (c) $R_1 = \dfrac{L_1}{L_1 + L_2} R$

$\Rightarrow \quad R_1 = \dfrac{R}{6} = 3\ \Omega$

$R_2 = \dfrac{l_2}{l_1 + l_2} \Rightarrow R_2 = 15\ \Omega$

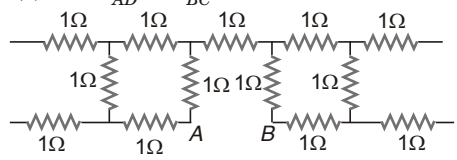


Hence R_1 and R_2 are in parallel

$R_e = \dfrac{R_1 R_2}{R_1 + R_2}$

$= 2.5\Omega$

36. (c) Let $R_{AD} = R_{BC} = x$



Clearly $x < 1$ as $1\,\Omega$ resistor is in parallel with some combination.

Now $\quad R_{AB} = x + 1 + x$

$= 2x + 1$

As $x < 1$

$1 < R_{AB} < 3$

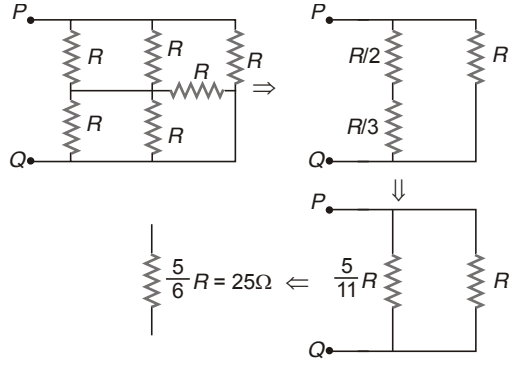
37. (d) $R_{AB} = R + \dfrac{R(R + R_0)}{2R + R_0} = R_0$

$\Rightarrow \quad 2R^2 + RR_0 + R^2 + RR_0 = 2RR_0 + R_0^2$

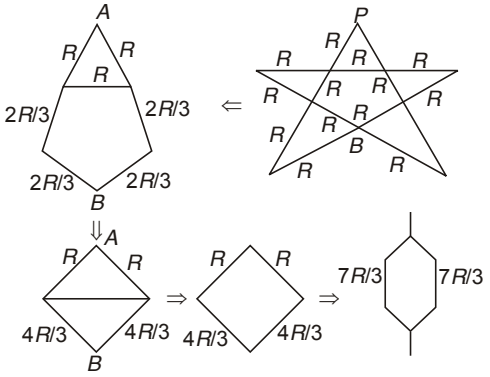
$\Rightarrow \quad 3R^2 = R_0^2$

$\Rightarrow \quad R = \dfrac{R_0}{\sqrt{3}}$

38.



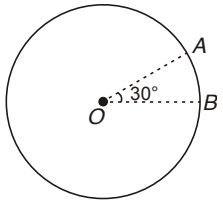
39. Wheatstone bridge is balanced.



$$\Rightarrow \quad R_e = \frac{7}{6} R$$

40. (d) $R_1 = \dfrac{L_1}{L_1 + L_2} R = \dfrac{1}{12} R$

$$= 3\ \Omega$$



$$R_2 = \frac{L_2}{L_1 + L_2} = \frac{11}{12} R = 33\ \Omega$$

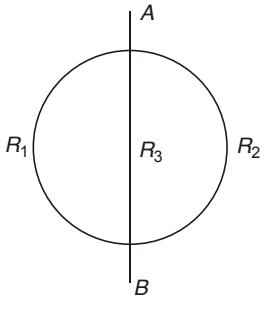
R_1 and R_2 are in parallel,

$$R_e = \frac{R_1 R_2}{R_1 + R_2} = \frac{3 \times 33}{3 + 33}$$

$$= 2.75\ \Omega$$

41. (a) Resistance per unit length of wire

$$= \frac{4}{2\pi r}$$



$$R_1 = \frac{4}{2\pi r} \times \pi r = 2 = R_2$$

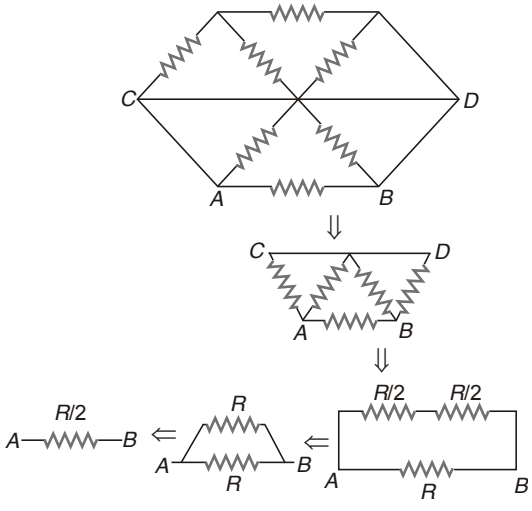
$$R_3 = \frac{4}{2\pi r} \times 2r = \frac{4}{\pi}$$

$$\therefore \quad \frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

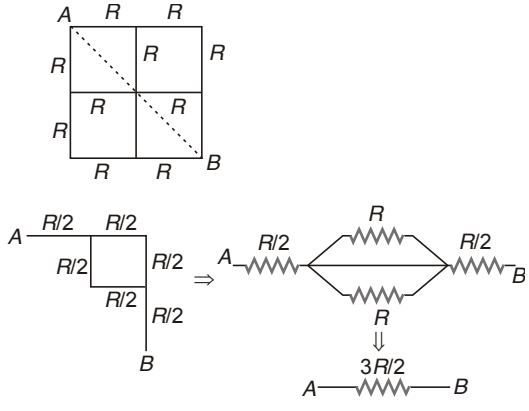
$$= \frac{1}{2} + \frac{1}{2} + \frac{\pi}{4} = \frac{4 + \pi}{4}$$

$$R_e = \frac{4}{4 + \pi}\ \Omega$$

42. (d) Points C and D are shorted hence the portion above line CD can be removed.



43. (b) As AB is line of symmetry, we can fold the network about AB.



JEE Corner

Assertion and Reason

1. (d) $V = IR$, If $V = 0$ either $I = 0$ or $R = 0$
2. (b) As all the resistors are in parallel potential difference is same, hence $P = \frac{V^2}{R}$ is maximum if R is minimum.
3. (b) $dH = I^2 dRt = \frac{I^2 t \rho}{A} dH$

   I is same everywhere, hence portion having less area is more heated.

   Again $J = \frac{I}{A}$

   $\therefore \quad J_A > J_B$.

   Reason is also correct but does not explain assertion.
4. (b) Both assertion and reason are correct but reason does not explain the cause of decrease in voltmeter reading.
5. (b) As $R_A < R_V$, more current passes through ammeter when positions of ammeter and voltmeter are interchanged and potential difference across voltmeter becomes less that emf of cell.
6. (c) During charging current inside the battery flows from positive terminal to negative terminal. Reason is false while assertion is true.
7. (d) $I = \frac{E}{R + r}$ is maximum when R is zero hence reason is false.

   $P = \frac{E^2 R}{(R + r)^2}$ is maximum at $R = r$.
8. (c) $I = \frac{V}{R}$, $P = \frac{V^2}{R}$ both I and P are inversly proportional to R hence both decrease with increase in R which increases with temperature.

   According to Ohm's law $V \propto I$ not $V = IR$.

   As R can be variable also.
9. (d) Drift velocity is average velocity of all the electrons but velocity of all electrons is not constant.
10. (a) $R = \rho \frac{L}{A}$

    $$\rho = \frac{m}{ne^2 \tau}$$

    with increase in temperature, electron collide more frequently, *i.e.*, τ decreases, increasing ρ and hence R.
11. (d) $E = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2} \quad \therefore \; E_1 < E < E_2$

    If $E_1 < E_2$

    $r = \frac{r_1 r_2}{r_1 + r_2}$, $r < r_1$, $r < r_2$
12. (d) $\frac{R_1}{R_2} = \frac{L_1}{L_2}$

    Hence there is no effect of one while measuring using meter bridge.

Objective Questions (Level-2)

1. (b) $I = \frac{E_2 - E_1}{r_1 + r_2} = \frac{1.5 - 1.3}{r_1 + r_2}$

   $$= \frac{0.2}{r_1 + r_2} \quad \ldots(i)$$

   $V = E_1 + Ir_1$

   $\Rightarrow \quad 1.45 = 1.3 + \frac{0.2}{r_1 + r_2} r_1$

   $\frac{r_1}{r_1 + r_2} = \frac{0.15}{0.2} \Rightarrow 0.2 r_1 = 0.15 r_1 + 0.15 r_2$

   $0.05 r_1 = 0.15 r_2 \;\; r_1 = 3 r_2$
2. (c) Let R = Resistance of voltmeter,

   $$V_1 = \frac{ER}{R_1 + R} = 198 \text{ V} \quad \ldots(i)$$

   $$V_2 = \frac{ER}{R_2 + R} = \frac{ER}{2R_1 + R} = 180 \text{ V} \quad \ldots(ii)$$

   $$\frac{2R_1 + R}{R_1 + R} = \frac{198}{180} = \frac{11}{10}$$

   $$20R_1 + 10R = 11R_1 + 11R$$

   $$9R_1 = R$$

From Eq. (i),

$$\frac{ER}{R_1 + R} = 198$$

$$E = 198 \times \frac{10R_1}{9R_1} = 220\text{ V}$$

3. (b) $P = I^2R$

As R is same for all bulbs and maximum current passes through bulb A, it will glow most brightly.

4. (c) $R + R_A = \frac{V}{I} = 5\,\Omega$

$$R = 5 - R_A < 5\,\Omega$$

5. (a) $r = \frac{L_1 - L_2}{L_2} R = \frac{10}{60} \times 132.40$

$$\approx 22.1\,\Omega$$

6. (b) Current through R when S is open.

$$I_1 = \frac{E_1 + E_2}{R + r_1 + r_2}$$

Current through R when S is closed

$$I_2 = \frac{E_1}{R + r_1}$$

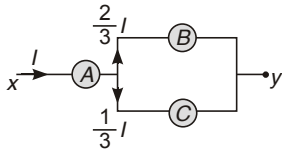
$$\Delta I = I_2 - I_1$$

$$= \frac{E_1}{R + r_1} - \frac{E_1 + E_2}{R + r_1 + r_2}$$

$$= \frac{E_1 r_2 - E_2(R + r_1)}{(R + r_1)(R + r_1 + r_2)}$$

$\Delta I = +$ ve if $E_1 r_2 > E_2(R + r_1)$

7. (a) $V_A = IR$



$$V_B = \frac{2}{3} I \times 1.5R = IR$$

$$V_C = \frac{1}{3} I \times 3R = IR$$

$\therefore$ $V_A = V_B = V_C$

8. (d) Current through $15\,\Omega$ resistor

$$= \frac{30}{15} = 2\text{ A}$$

$$V_{BC} = (2 + 5) \times 5 = 35\text{ V}$$

Voltage drop across $R = 100 - (30 + 55)$

$$= 35\text{ V}$$

$\therefore$ Required ratio $= \frac{35}{35} = 1$

9. (a) $r = \frac{L_1 - L_2}{L_2} R = \frac{x - y}{y} R$

10. (d) $\frac{R - 20}{40 - 20} = \frac{t - 10}{30 - 20}$

$\Rightarrow$ $R = t + 10$

$$I = \frac{E}{R} = \frac{10}{t + 10}$$

$$\frac{dq}{dt} = \frac{10}{t + 10}$$

$$q = \int_{10}^{30} \frac{10}{t + 10} dt = 10\left[\log_e(t + 10)\right]_{10}^{30}$$

$$= 10 \log_e 2$$

11. (b) Let l_1 length is kept fixed and l_2 is stretched,

$$R_1 = \rho \frac{l_1}{A},\ R_2 = \rho \frac{l_2}{A}$$

Initial resistance,

$$R = R_1 + R_2 \qquad \text{...(i)}$$

Now full is stretched $\frac{3}{2}$ times, *ie*,

$$l_2' = \frac{3}{2}(l_1 + l_2) - l_1$$

$$= \frac{1}{2}(l_1 + 3l_2)$$

$$A_2' = \frac{A_2' l_2'}{l_2} = \frac{2Al_2}{l_1 + 3l_2}$$

$$R_2' = \rho \frac{\frac{1}{2}(l_1 + 3l_2)^2}{2Al_2}$$

$$R_2' = \rho \frac{(l_1 + 3l_2)^2}{4Al_2}$$

Now, $R' = 4R$

$$R_1 + R_2' = 4(R_1 + R)$$

$$l_1 + \frac{(l_1 + 3l_2)^2}{4l_2} = 4(l_1 + l_2)$$

$\Rightarrow$ $\frac{l_2}{l_1} = \frac{1}{7}$

$\Rightarrow$ $\frac{l_2}{l_1 + L_2} = \frac{1}{8}$

12. (b) $\frac{X}{R} = \frac{l_1}{100 - l_1} \Rightarrow l_1 = 40$ cm

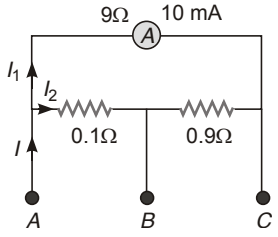
If $R' = 8\,\Omega$

$$\frac{X}{R'} = \frac{l_1'}{100 - l_1'}$$

$$\Rightarrow \quad l_1' = 60 \text{ cm}$$

$$l_1' - l_1 = 20 \text{ cm}$$

13. (c) $I_1 = \frac{0.1}{0.1 + 9.9} I$



But $I_1 = 10$ mA

$$I = \frac{10}{0.1} \times 10 \text{ mA} = 1000 \text{ mA}$$

$$= 1 \text{ kA}$$

14. (d) Effective emf of two cells

$$E = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2} = \frac{2 \times 6 + 4 \times 2}{2 + 6}$$

$$= \frac{20}{8} = 2.5 \text{ V}$$

$$R_{AB} = 4 \times 4 = 16\,\Omega$$

$$V_{AB} = \frac{R_{AB}}{R + R_{AB}} E_0 = \frac{16}{4 + 16} \times 12$$

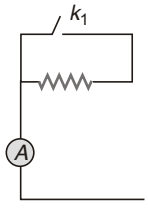
$$= 9.6 \text{ V}$$

$$k = \frac{V_{AB}}{L} = 2.4 \text{ V/m}$$

Now, $E = kl$

$$\Rightarrow \quad L = \frac{E}{k} = \frac{2.5}{2.4} = \frac{25}{24}$$

15. When k_1 and k_2 both are closed, the resistance R_1 is short circuited. Therefore net resistance is



$$R_{\text{net}} = r + \frac{100 \times 100}{100 + 100} = r + 50$$

$$\therefore \quad I_0 = \frac{E}{r + 50} \quad \text{...(i)}$$

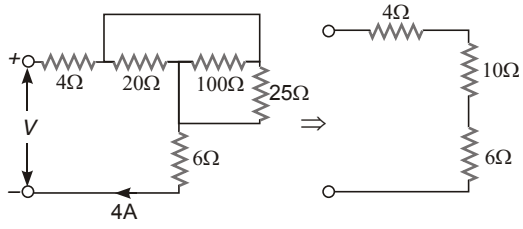
when, k_1 is open and k_2 is closed, net resistance is

$$R_{\text{net}} = r + R_1 + \frac{100 \times 100}{100 + 100} = (r + R_1 + 50)$$

$$\therefore \quad \frac{I_0}{2} = \frac{E}{r + R_1 + 50} \quad \text{...(ii)}$$

The above two equations are satisfied if $r = 0$ and $R_1 = 50\,\Omega$.

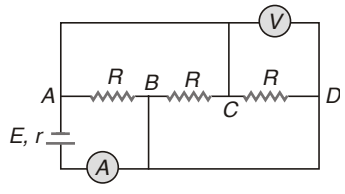
16. (b) $20\,\Omega, 100\,\Omega$ and $25\,\Omega$ resistors are in parallel.



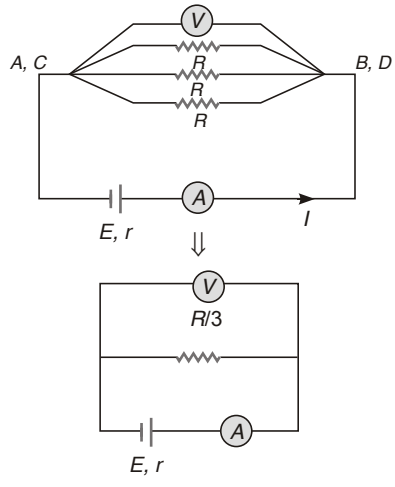
$$R = 20\,\Omega$$

$$V = IR = 80 \text{ V}$$

17. (a) Hence, points A and C, B and D are at same potential.



The equivalent circuit is given by



$$I = \frac{E}{\frac{R}{3} + r} = 1\text{ A}$$

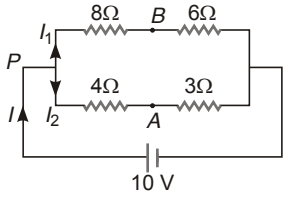
$$V = I \times \frac{R}{3} = 3\text{ V}$$

18. (c) $S = \frac{I_g}{I - I_g} G$, $G = r$, $S = \frac{r}{4}$

$$I_g = \frac{1}{4}(I - I_g) \Rightarrow I_g = \frac{1}{5} I$$

$$= 0.006\text{ A}$$

19. (d) $I_1 = \frac{10}{14} = \frac{5}{7}$ A



$$I_2 = \frac{10}{7}\text{ A}$$

$$V_P - V_A = I_2 \times 4$$

$$= \frac{40}{7}\text{ A}$$

$$V_P - V_B = I_1 \times 8 = \frac{40}{7}\text{ A}$$

$$V_A - A_B = 0$$

Another method

As, $\frac{R_1}{R_2} = \frac{R_3}{R_4}$, $V_B = V_A$

20. (b) For series connection

$$\frac{V_1}{V_2} = \frac{R_1}{R_2}$$

$$\therefore \quad \frac{R_1}{R_2} = \frac{3}{2}$$

Now, $R_1 = \rho \frac{L_1}{A_1} = \frac{\rho L_1}{\pi r_1^2}$,

$$R_2 = \delta \frac{L_2}{A_2} = \frac{\rho L_2}{\pi r_2^2}$$

$$\therefore \quad \frac{R_1}{R_2} = \frac{L_1}{L_2} \times \left(\frac{r_2}{r_1}\right)^2$$

$$\frac{r_1}{r_2} = \sqrt{\frac{R_2 L_1}{R_1 L_2}} = \sqrt{\frac{2 \times 6}{3 \times 1}} = 2$$

$$\frac{r_2}{r_1} = \frac{1}{2}$$

21. (a) Voltage sensitivity of voltmeter

$$\propto \frac{1}{\text{Resistance of voltmeter}}$$

$$\therefore \quad \frac{V_{s_1}}{V_{s_2}} = \frac{R_2 + G}{R_1 + G}$$

$$\frac{30}{20} = \frac{R_2 + 50}{2950 + 50}$$

$$R_2 + G = \frac{30 \times 3000}{20} = 4500$$

$$\Rightarrow \quad R_2 = 4450\ \Omega$$

22. (b) For $x = 0$

$$V_{AB} = E$$

$$k_1 = \frac{E}{L}$$

$$E_0 = k_1 L_1 = \frac{E L_1}{L} \quad \text{...(i)}$$

For $x = x$ (say)

$$V_{AB} = \frac{R_{AB}}{R_{AB} + x} E$$

$$k_2 = \frac{R_{AB} E}{(R_{AB} + x) L}$$

$$E_0 = k_2 L_2 = \frac{R_{AB} E L_2}{(R_{AB} + x) L} \quad \text{...(ii)}$$

From Eqs. (i) and (ii),

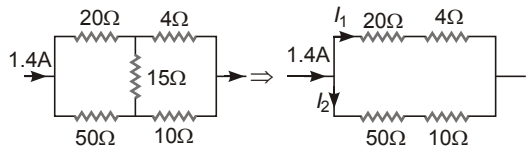
$$L_1 = \frac{R_{AB} + L_2}{(R_{AB} + x)}$$

$$\Rightarrow \quad 20 = \frac{10 \times 30}{10 + x}$$

$$\Rightarrow \quad x = 5\ \Omega$$

23. (d) To obtain null point similar terminal of both the batteries should be connected.

24. (c) Wheatstone bridge is balanced.

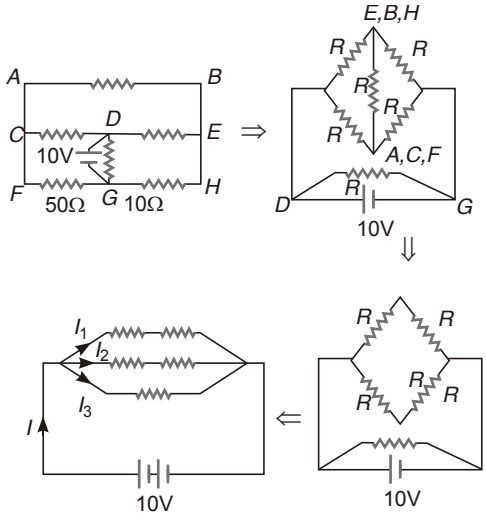


$$\therefore \quad I_1 = \frac{R_2}{R_1 + R_2} \times I$$

$$= \frac{60}{84} \times 1.4$$

$$= 1\text{ A}$$

25. (b)



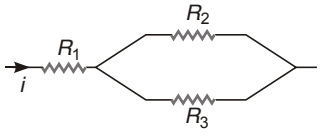
$$I_1 = \frac{10}{2 \times 2} = 2.5 \text{ A}$$

26. (b) Effective resistance of voltmeter and 3 kΩ resistor,

$$R_1 = \frac{3 \times 6}{2 + 6} = 2 \text{ k}\Omega$$

$$V_1 = \frac{R_1}{R_1 + R_2} E = \frac{2}{4} \times 10 = 5 \text{ V}$$

27. (d) $P_1 = P_2 = P_3$, Clearly $R_2 = R_3$



$$\therefore \quad i_2 = i_3 = \frac{i}{2}$$

$$P_1 = i^2 R_1, P_2 = \left(\frac{i}{2}\right)^2 \times R_2 = \frac{1}{4} i^2 R_2$$

$$P_3 = \frac{1}{4} i^2 R_3$$

$$R_2 = 4R_1, R_3 = 4R_1$$

$$\therefore \quad R_1 : R_2 : R_3 = 1 : 4 : 4$$

28. As $E = kL_1 \Rightarrow k = \frac{E}{L_1} = \frac{2}{500} = 250$

$$= \frac{1}{250} \text{ V/cm}$$

$$V = kL_2 = \frac{1}{250} \times 490 \text{ cm}$$

$$= 1.96 \text{ V}$$

29. (c) $r = \frac{L_1 - L_2}{L_2} R$

$$\Rightarrow \quad R = \frac{L_2 r}{L_1 - L_2}$$

$$= \frac{490 \times 10}{10} = 490 \, \Omega$$

More than One Correct Options

30. $H = P_1 t_1 = P_2 t_2$

$$\Rightarrow \quad t_1 = \frac{H}{P_1}, t_2 = \frac{H}{P_2}$$

If connected in series

$$\frac{1}{P} = \frac{1}{P_1} + \frac{1}{P_2}$$

$$\Rightarrow \quad t = t_1 + t_2$$

If connected in parallel

$$P = P_1 + P_2$$

$$\Rightarrow \quad t = \frac{t_1 t_2}{t_1 + t_2}$$

31. $E = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2} = \frac{6 \times 3 + 5 \times 2}{2 + 3}$

$$= 5.6 \text{ V}$$

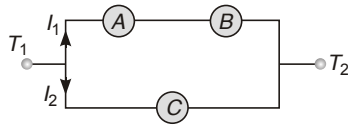
As there is no load.

$$V = E = 5.6 \text{ V}$$

If $E_1 = E_2, \; I = 0$

$$I = \frac{E_1 - E_2}{r_1 + r_2} = \frac{6 - 5}{2 + 3} = 0.2 \text{ A}$$

32. Let V = Potential difference between T_1 and T_2.



$$I_1 = \frac{V}{R_A + R_B}$$

$$I_2 = \frac{V}{R_C}$$

Now,. $I_A = I_B = I_1$

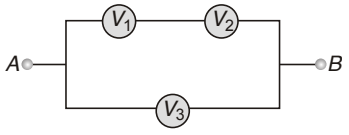
$$I_C = I_2$$

Also, $V = I_C R_C = I_1 (R_A + R_B)$

$$= I_A R_A + I_D R_B$$

$$\frac{I_B}{I_C} = \frac{I_1}{I_2} = \frac{R_C}{R_A + R_B}$$

33. As $R_1 \neq R_2$



$$V_1 \neq V_2$$

Also, $$V_3 = V_1 + V_2$$

34. As $R_1 = R_2$

$$V_1 = V_2$$

$$R = \rho \frac{L}{A}$$

But

$$L_2 = 2L_1$$

and $$R_1 = R_2$$

$\therefore$ $$A_2 = 2\,A_1$$

Also, $v_d \propto \dfrac{1}{A}$ (For constant current)

$$v_{d_2} = \frac{1}{2} v_{d1} \Rightarrow v_{d_1} = 2\,v_{d_2}$$

Again, $v_d \propto E$

$\therefore$ $$E_1 = 2\,E_2$$

35. If $E > 18$ V current will flow from B to A and *vice-versa*.

36. $V = kl$

If Jockey is shifted towards right, I and hence k will decreases as $k \propto I$.

Hence L will increase.

If E_1 is increased, k will increase, hence L will decrease.

If E_2 is increased L will increase as V will increase.

If ρ is closed V will decrease hence L will decrease.

37. $I_e = \dfrac{E}{R_e + r_e}$, Initially, $I = \dfrac{E}{R + r}$

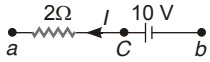
If S_1 is closed

$$I_e = \frac{E}{\frac{R}{2} + r} > I$$

If S_2 is closed

$$I_e = \frac{E}{R + \frac{r}{2}} > I$$

38. $V_b - V_a = -10 + 2I = 2$ V

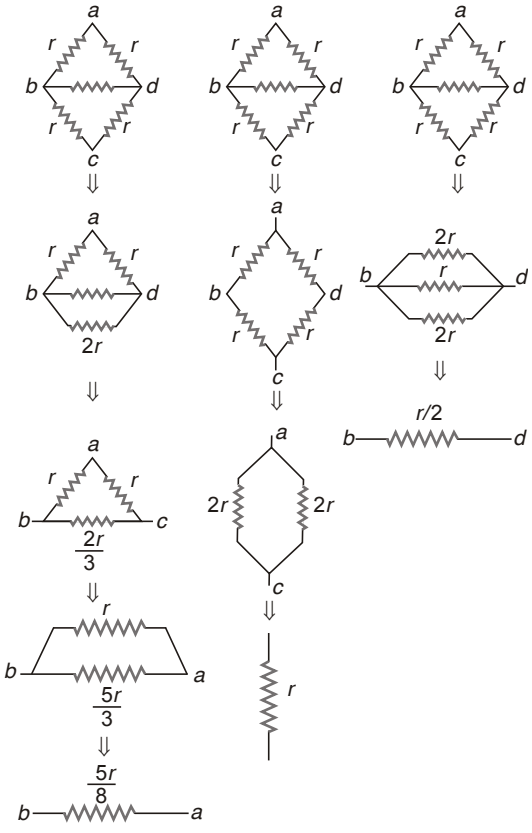


$$I = 6\text{ A}$$

From b to a.

$$V_c - V_a = 2 \times 6 = 12\text{ V}$$

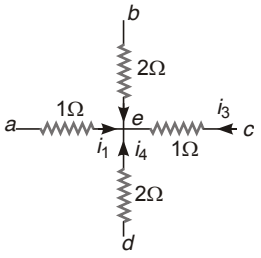
39.



Match the Columns

1. By applying KCL at e

$$i_1 + i_2 + i_3 + i_4 = 0$$



$$\frac{2 - V_e}{1} + \frac{4 - V_e}{2} + \frac{6 - V_e}{1} + \frac{4 - V_e}{2} = 0$$

$$V_e = 4\text{ V},\ I_1 = -2\text{ A},\ i_2 = 0,\ i_3 = 2\text{ A},\ i_4 = 0$$

2. Current is same at every point and $A_1 < A_2$

$$J = \frac{i}{A} \Rightarrow J_1 > J_2$$

$$v_d = \frac{i}{neA} \Rightarrow v_{d_1} > v_{d_2}$$

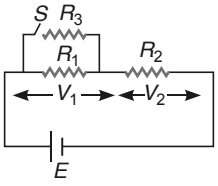
$$r = \frac{R}{L} = \frac{\rho}{A} \Rightarrow r_1 > r_2$$

$$k = \frac{V}{L} \Rightarrow k_1 > k_2$$

3. When switch S is closed

V_1 decreases, V_2 increases,

$\therefore$ Current through R_1 decreases and through R_2 increases.



4. $[\text{R}] = \dfrac{[\text{V}]}{[\text{I}]} = \dfrac{[\text{ML}^2\text{T}^{-3}\text{A}^{-1}]}{[\text{A}]}$

$$= [\text{ML}^2\text{T}^{-3}\text{A}^{-2}]$$

$$[\text{V}] = \frac{[W]}{[q]} = \frac{[\text{ML}^2\text{T}^{-2}]}{[\text{AT}]}$$

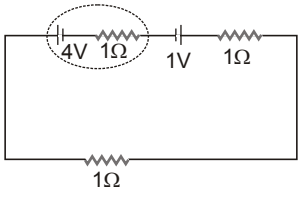
$$= [\text{ML}^2\text{T}^{-3}\text{A}^{-1}]$$

$$[\sigma] = \frac{[R][A]}{[L]} = \frac{[\text{ML}^2\text{T}^{-3}\text{A}^{-2}][\text{L}^2]}{[\text{L}]}$$

$$= [\text{ML}^3\text{T}^{-3}\text{A}^{-2}]$$

$$[\sigma] = \frac{1}{[\rho]} = [\text{M}^{-1}\text{L}^{-3}\text{T}^3\text{A}^2]$$

5. $I = \dfrac{E_A - E_B}{R + r_A + r_A} = 1\text{ A}$



$$V_A = E_A - Ir_A = 3\text{ V}$$

$$V_B = E_B + Ir_B = 2\text{ V}$$

$$P_A = IV_A = 3\text{ W}$$

$$P_B = IV_B = 2\text{ W}$$

21 Electrostatics

Introductory Exercise 21.1

1. No, because charged body can attract an uncharged by inducing charge on it.
2. Yes.
3. On clearing, a phonograph record becomes charged by friction.
4. No. of electrons in 3 g mole of hydrogen atom

$$= 3 \times 6.022 \times 10^{23}$$

$$\therefore \quad q = ne = 3 \times 6.022 \times 10^{23} \times 1.6 \times 10^{-19}$$

$$= 2.9 \times 10^{5}\ \text{C}$$

Introductory Exercise 21.2

1. $$F_e = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r^2}$$

$$F_g = \frac{G m_1 m_2}{r^2}$$

$$\frac{F_e}{F_g} = \frac{e^2}{4\pi\varepsilon_0 \cdot G m_1 m_2}$$

$$= \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{6.67 \times 10^{-11} \times 9.11 \times 10^{-31} \times 1.67 \times 10^{-27}}$$

$$= 2.27 \times 10^{39}$$

2. $$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

$$\varepsilon_0 = \frac{q_1 q_2}{4\pi F r^2}$$

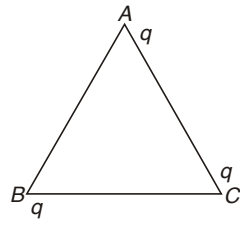
$$[\varepsilon_0] = \frac{[q_e]^2}{[F][r]^2}$$

$$= \frac{[\text{IT}]^2}{[\text{MLT}^{-2}][\text{L}]^2}$$

$$= [\text{M}^{-1}\text{L}^{-3}\text{T}^{4}\text{I}^{2}]$$

SI units of $\varepsilon_0 = \text{C}^2\,\text{N}^{-1}\text{m}^{-2}$.

3. Let us find net force on charge at A.

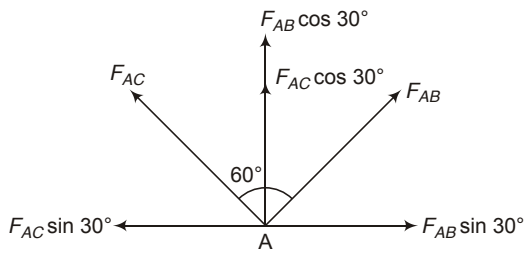


$$F_{AB} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{a^2} \quad F_{AC} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{a^2}$$

Net force on charge at A

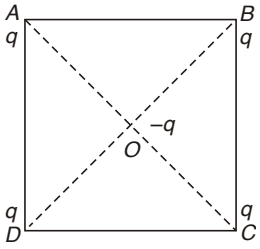
$$F_A = F_{AB}\cos 30° + F_{AC}\cos 30°$$

$$= \frac{\sqrt{3}q^2}{4\pi\varepsilon_0 \cdot a^2}$$



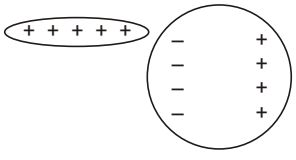
4. $\vec{\mathbf{F}}_{OA} = \vec{\mathbf{F}}_{OC}$

and $\vec{\mathbf{F}}_{OB} = -\vec{\mathbf{F}}_{OD}$



Hence, net force on charge at centre is zero.

5. No. In case of induction while charge comes closer and like charge moves further from the source.



The cause of attraction is more attractive force due to small distance. But if electrostatic force becomes independent of distance, attractive force will become equal to repulsive force, hence net force becomes zero.

6. When the charged glass rod is brought near the metal sphere, negative induces on the portion of sphere near the charge, hence it get attracted. But when the sphere touches the rod it becomes positively charged due to conduction and gets repelled by the rod.

7. Yes as $q_{\min} = e$

$$F_{\min} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r^2}$$

8. No. Electrostatic force is independent of presence or absence of other charges.

9. $\vec{F}_{21} = -\vec{F}_{12} = (-4\,\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}})$ N.

Introductory Exercise 21.3

1. False. $E = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q}{r^2}$

2. $V_A > V_B$ as electric lines of force move from higher potential to lower potential.

3. False. Positively charged particle moves in the direction of electric field while negatively charged particle moves opposite to the direction of electric field.

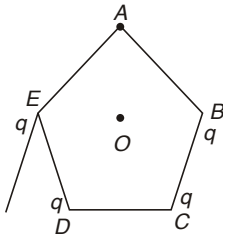
4. False. Direction of motion can be different from direction of force.

5. $E = \dfrac{\sigma}{\varepsilon_0} \Rightarrow \sigma = E\varepsilon_0 = 3.0 \times 8.85 \times 10^{-12}$

$$= 2.655 \times 10^{-11}\ \text{C/m}^2$$

6. q_1 and q_3 are positively charged as lines of force are directed away from q_1 and q_3. q_2 is negatively charged because electric field lines are towards q_2.

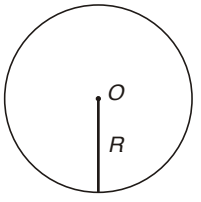
7. If a charge q is placed at A also net field at centre will be zero.



Hence net field at O is same as produced by A done but in opposite direction,02 *i.e.*,

$$E = \frac{1}{4\pi\varepsilon} \cdot \frac{q}{a^2}$$

8. Net field at the centre (O) of wire is zero. If a small length of the wire is cut-off, net field will be equal to the field due to cut-off portion, *i.e.*,



$$dE = \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq}{R^2}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{\dfrac{q}{2\pi R}\,dl}{R^2}$$

$$= \frac{q\,dl}{8\pi^2\varepsilon_0 R^3}$$

9. $$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^3}\vec{\mathbf{r}}$$
$$= -\frac{9\times10^9\times2\times10^{-6}}{(3^2+4^2)^{3/2}}(3\,\hat{\mathbf{i}}+4\,\hat{\mathbf{j}}) = -144\,(3\,\hat{\mathbf{i}}+4\,\hat{\mathbf{j}})\ \text{N/C}$$

Introductory Exercise 21.4

1. Gain in KE = loss of PE
$$\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\cdot q_1q_2\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$
$$\frac{1}{2}\times10^{-4}v^2$$
$$= -1\times10^{-6}\times2\times10^{-6}\times9\times10^9\left(\frac{1}{1}-\frac{1}{0.5}\right)$$
$$v^2 = 360$$
$$v = 6\sqrt{10}\ \text{ms}^{-1}$$

2. $W = q(V_A - V_B)$
$$= 2\times10^{-6}\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{-1\times10^{-6}}{1}-\frac{1}{4\pi\varepsilon_0}\cdot\frac{-1\times10^{-6}}{2}\right)$$
$$= -9\times10^{-3}\ \text{J}$$
$$= -9\ \text{m J}$$

3. Whenever work is done by electric force, potential energy is decreased.
$$W = -\Delta U$$
$$U_2 = U_1 - W = -8.6\times10^{-8}\ \text{J}$$

4. No. As $U = \dfrac{q_1q_2}{4\pi\varepsilon_0 r}$

If there are three particles
$$U = \frac{1}{4\pi\varepsilon_0}\cdot\left[\frac{q_1q_2}{r_{12}}+\frac{q_2q_3}{r_{23}}+\frac{q_3q_1}{r_{31}}\right]$$
Here U may be zero.

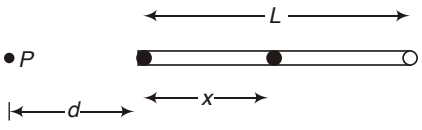
In case of more than two particles PE of systems may same as if they were separated by infinite distance but not in case of two particles.

Introductory Exercise 21.5

1. $V_{ba} = \dfrac{W_{a\to b}}{q} = 12\times10^2 = 1200\ \text{V}$

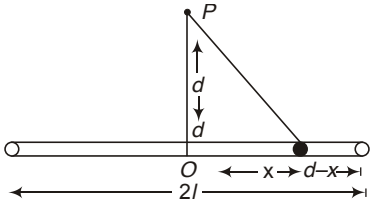
2. $\lambda = \alpha x$

(a) SI Units of λ = C/m
$$\alpha = \frac{\lambda}{x}$$
Hence SI unit of $\alpha = \dfrac{\text{C/m}}{\text{m}} = \text{C/m}^2$.



(b) Consider an elementary portion of rod at a distance x from origin having length dx. Electric potential at P due to this element.
$$dV = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\lambda\,dx}{x+d}$$
Net electric potential at P
$$V = \int_0^L\frac{1}{4\pi\varepsilon_0}\cdot\frac{\lambda\,dx}{x+d}$$
$$\Rightarrow\quad \frac{\alpha}{4\pi\varepsilon_0}\cdot\int_0^L\frac{x\,dx}{x+d}$$
$$= \frac{\alpha}{4\pi\varepsilon_0}\cdot\left[\int_0^L dx - d\int_0^L\frac{dx}{x+d}\right]$$
$$= \frac{\alpha}{4\pi\varepsilon_0}\cdot[[x]_0^L - d\,[\ln(x+d)]_0^L]$$
$$= \frac{\alpha}{4\pi\varepsilon_0}\cdot\left[L - d\ln\frac{L+d}{d}\right]$$

3. Consider an elementary portion of length dx at a distance x fro my centre O of the rod.



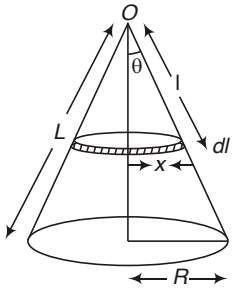
Electric potential at P due to this element,
$$dV = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\lambda\,dx}{\sqrt{d^2+x^2}}$$
$$V = \frac{\lambda}{4\pi\varepsilon_0}\cdot\int_{-l}^{l}\frac{dx}{\sqrt{d^2+x^2}}$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \cdot \left[\sin^{-1}\frac{x}{d}\right]_{-l}^{l}$$

$$= \frac{q}{4\pi\varepsilon_0 \cdot 2l} \times 2\sin^{-1}\frac{x}{d}$$

$$V = \frac{q}{4\pi\varepsilon_0\, l}\sin^{-1}\frac{x}{d}$$

4. Consider the cone to be made up of large number of elementary rings.



Consider one such ring of radius x and thickness dl. Let θ be the semi-vertical angle of cone and R be the radius of cone.

Charge on the elementary ring;

$$dQ = \sigma\, dA = \frac{Q}{\pi RL} \cdot 2\pi x\, dl$$

or $$dQ = \frac{2Ql\sin\theta}{RL}\, dl$$

Potential at O due to this ring

$$dV = \frac{1}{4\pi\varepsilon_0} \cdot \frac{dQ}{l}$$

$$= \frac{Q\sin\theta}{2\pi\,\varepsilon_0 RL}\, dl$$

Total potential at O

$$V = \frac{Q\sin\theta}{2\pi\varepsilon_0\, RL}\int_0^L dl = \frac{QL\sin\theta}{2\pi\varepsilon_0\, RL}$$

$$= \frac{Q}{2\pi\varepsilon_0\, L}\ [L\sin\theta = R]$$

$$U = qV$$

$$= \frac{Qq}{2\pi\varepsilon_0\, L}$$

Introductory Exercise 21.6

1. (a) $V = a\,(x^2 - y^2)$

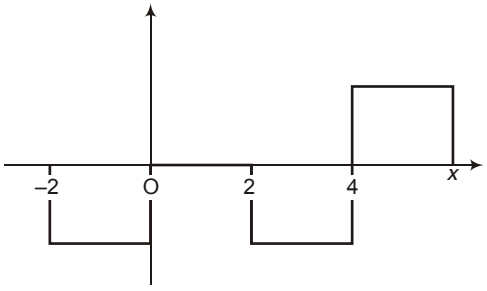
$$E = -\left(\frac{\partial v}{\partial x}\hat{\mathbf{i}} = \frac{\partial v}{\partial y}\hat{\mathbf{j}}\right) = -2ax\,\hat{\mathbf{i}} + 2y\,\hat{\mathbf{j}}$$

(b) $V = axy$

$$E = -\left(\frac{\partial v}{\partial x}\hat{\mathbf{i}} = \frac{\partial v}{\partial y}\hat{\mathbf{j}}\right) = -a\,(y\,\hat{\mathbf{i}} + x\,\hat{\mathbf{j}})$$

2. From $x = -2$ to $x = 0$ & $x = 2$ to $x = 4$

V is increasing uniformly.



Hence, E is uniform and negative

From $x = 0$ to $x = 2$

V is constant hence E is zero.

For $x > 4$

V is decreasing at constant rate, hence E is positive.

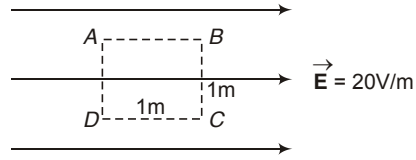
3. $E = -\dfrac{dV}{dr} = -\dfrac{(50-100)}{5-0} = 10$ V/m

True.

4. (a) $V_P - V_D = \vec{\mathbf{E}} \cdot \vec{\mathbf{l}} = 0$

(b) $V_P - V_C = \vec{\mathbf{E}} \cdot \vec{\mathbf{l}} = 20 \times 1 \times \cos 0°$

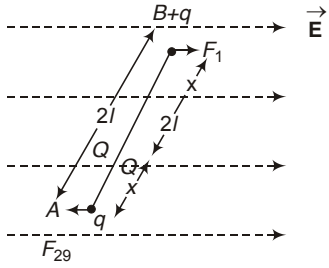
$= 20$ V



(c) $V_B - V_D = -20 \times 1 = -20$ V

(d) $V_C - V_D = -20 \times 1 = -20$ V

Introductory Exercise 21.7



1. $F_1 = qE$ towards right

$F_2 = qE$ towards left

Net torque about θ,

$$\tau = qE(2l - x)\sin\theta + qEx\sin\theta$$
$$= q(2l)E\sin\theta = pE\sin\theta$$
$$\vec{\tau} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}$$

2.
$$E_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{(\sqrt{y^2+a^2})^2}$$
$$E_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{(\sqrt{y^2+a^2})^2}$$
$$E_3 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2q}{y^2}$$

Net field at P

$$\vec{\mathbf{E}} = -(E_3 - E_1\cos\theta - E_2\cos\theta)\hat{\mathbf{j}}$$
$$= \frac{-q}{4\pi\varepsilon_0}\left[\frac{2}{y^2} - \frac{\cos\theta}{y^2+a^2} - \frac{\cos\theta}{y^2+a^2}\right]\hat{\mathbf{j}}$$
$$= -\frac{2q}{4\pi\varepsilon_0}\left[\frac{1}{y^2} - \frac{y}{(y^2+a^2)^{3/2}}\right]\hat{\mathbf{j}}$$
$$= -\frac{2q}{4\pi\varepsilon_0}\left[\frac{(y^2+a^2)^{3/2} - y^3}{y^2(y^2+a^2)^{3/2}}\right]\hat{\mathbf{j}}$$
$$= -\frac{2q}{4\pi\varepsilon_0}\left[\frac{y^3\left(1+\frac{q^2}{y^2}\right)^{3/2} - y^3}{y^2(y^2+a^2)^{3/2}}\right]\hat{\mathbf{j}}$$

As $y >> a$

$$E = -\frac{2q}{4\pi\varepsilon_0}\cdot\left[\frac{y^3\left(1+\frac{3q^2}{2y^2}\right) - y^3}{y^5}\right]\hat{\mathbf{j}}$$
$$E = -\frac{3qa^2}{4\pi\varepsilon_0 y^4}\hat{\mathbf{j}}$$

Introductory Exercise 21.8

1. (a) Charge q is completely the hemisphere hence flux through hemisphere is zero.

(b) Charge inside the sphere is q hence flux through hemisphere
$$\phi = \frac{q}{\varepsilon_0}$$

(c) As charge q is at the surface, net flux through hemisphere
$$\phi = \frac{q}{2\varepsilon_0}$$

2. When charge is at any of the vertex, net flux through the cube,
$$\phi = \frac{q}{8\varepsilon_0}$$

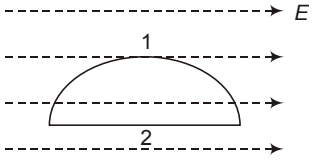
If charge q is at D,

flux through three faces containing D is zero and the flux ϕ is divided equal among other three faces, hence
$$\phi_{EFGH} = \frac{1}{\phi} = \frac{q}{2\pi\varepsilon_0}$$
and $\phi_{AEHD} = 0$

3. True. As electric field is uniform, flux entering the cube will be equal to flux leaving it.

$$\therefore \quad \phi_{net} = 0 \Rightarrow \phi_{net} = \frac{q}{\varepsilon_0}$$
$$\Rightarrow q = 0$$

4. (a) As net charge inside hemisphere is zero,



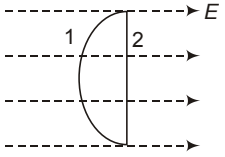
$$\phi_1 + \phi_2 = 0$$

But E is parallel to surface 2.

$\therefore \qquad \phi_2 = 0$

Hence, $\phi_1 = 0$

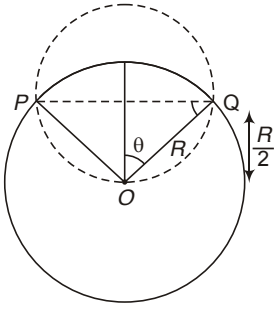
(b) Again, $\phi_1 + \phi_2 = 0$



$$\phi_2 = E \times \pi R^2 = \pi R^2 E$$

$$\therefore \qquad \phi_1 = -\phi_2 = -\pi R^2 E$$

5. $\cos\theta = \dfrac{R/2}{R} = \dfrac{1}{2}$, $\theta = 60°$



$\angle POQ = 2\theta = 120°$

$\therefore$ Length of arc $PQ = \dfrac{2\pi}{3} R$

Charge inside sphere,

$$q = \frac{q_0}{2\pi R} \times \frac{2\pi}{3} R \Rightarrow \frac{q_0}{3}$$

$\therefore$ Flux through the sphere

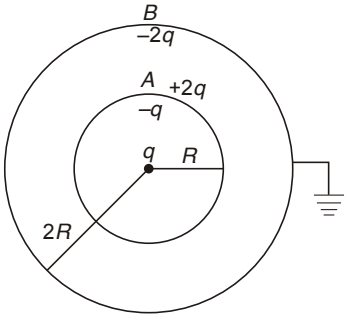
$$\phi = \frac{q}{\varepsilon_0} = \frac{q_0}{3\,\varepsilon_0}$$

6. Net charge inside the cube = 0.

$\therefore$ Net flux through the cube = 0.

Introductory Exercise 21.9

1. $V_B = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q + q + q_B}{2R} = 0$



$$q_B = -2q$$

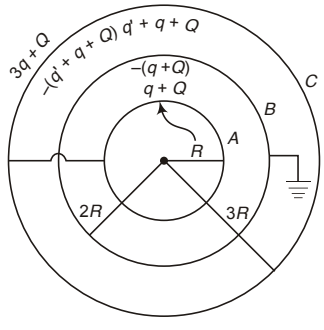
Total charge inside a conducting sphere appears on its outer surface,

$\therefore$ Charge on outer surface of $A = 2q$

and charge on outer surface of B

$$= 2q - 2q = 0$$

2. Let q' = charge on sphere B and charge ϕ flows from sphere C to A.



$$V_B = \frac{1}{4\pi\varepsilon_0} \cdot \left(\frac{q' + q + Q}{2R} + \frac{2q - Q}{3R} \right) = 0$$

$$\Rightarrow \qquad 3q' + q + \phi = 0 \qquad \ldots(i)$$

Again, $V_P = V_C$

$$\frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{q + Q}{R} + \frac{q}{2R} + \frac{2q - Q}{3R} \right] = \frac{1}{4\pi\varepsilon_0} \cdot \frac{3q + q'}{3R}$$

$$6(q + Q) + 3q' + 2(2q - Q) = 2(3q + q')$$

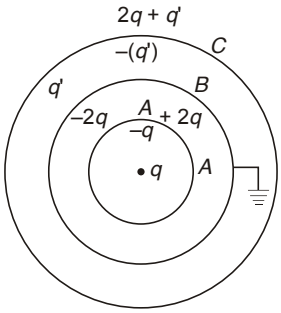
$$4q + 4q + q' = 0$$

On solving

$$Q = -\frac{5}{11}q,\ q' = -\frac{24}{11}q$$

	A	B	C
Charge on inner surface	0	$-(q+Q) = -\frac{6}{11}q$	$-(q'+q+Q) = \frac{18}{11}q$
Charge on outer surface	$q+Q = \frac{6}{11}$	$q'+q+Q = -\frac{18}{11}q$	$3q+q = \frac{9}{11}q$

3.



	A	B	C
Charge on inner surface	$-q$	$-2q$	$+\frac{4}{3}q$
Charge on outer surface	$+2q$	$-\frac{4q}{3}$	$+\frac{2}{3}q$

AIEEE Corner

Subjective Questions (Level-1)

1. $F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q(Q-q)}{r^2}$

For maximum force

$$\frac{dF}{dq} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q-2q}{r^2} = 0$$

$$q = \frac{Q}{2}$$

$$\frac{d^2F}{dq^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{-2}{r^2} < 0$$

Hence F is maximum at $q = \frac{Q}{2}$.

2. Minimum possible charge on a particle $= e$.

$$\therefore F_{\min} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{r^2} = \frac{9\times10^9\times(1.6\times10^{-19})^2}{(1\times10^{-2})^2}$$

$$= 2.3\times10^{-24}\ \text{N}$$

3. $F_e = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r^2}$ ...(i)

$F_g = \frac{Gm_1m_2}{r^2}$ ...(ii)

$$\frac{F_e}{F_g} = \frac{q_1q_2}{4\pi\varepsilon_0 Gm_1m_2}$$

$$= \frac{(3.2\times10^{-19})^2\times9\times10^9}{6.67\times10^{-11}\times(6.64\times10^{-27})^2}$$

$$= 3.1\times10^{35}$$

4. $F_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r^2}$ ...(i)

$F_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{r^2}$ ...(ii)

[As both the spheres are identical, find charge on both the spheres will be equal]

$$q = \frac{q_1-q_2}{2}$$

$\Rightarrow$ $q_1 - q_2 = 2q$

From Eq. (ii),

$$q^2 = 4\pi\varepsilon_0 r^2F_2$$

$$= \frac{(50\times10^{-2})^2\times0.036}{9\times10^9} = 10^{-12}$$

$$q = 10^{-6}\ \text{C} = 1\ \mu\text{C}$$

From Eq. (i),

$$q_1q_2 = 4\pi\varepsilon_0 r^2F_1 = \frac{(50\times10^{-2})^2\times0.108}{9\times10^9}$$

$$= 3\times10^{-12}$$

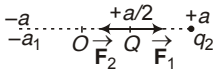
Also, $q_1 + q_2 = 2q = 2\times10^{-6}$

On solving

$$q_1 = \pm 3\ \mu\text{C}$$

and $q_2 = \mp 1\ \mu\text{C}$

5. (a) $F_1 = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q_1 Q}{(3a/2)^2}$



$$F_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{4q_1 Q}{9a^2} \quad \text{...(i)}$$

$$F_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2 Q}{(a/2)^2}$$

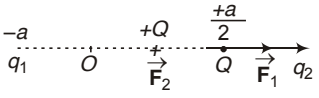
$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{4q_2 Q}{a^2} \quad \text{...(ii)}$$

For net force on Q to be zero

$$F_1 = F_2$$

or $$q_1 = 9\,q_2$$

(b) $F_1 = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{4q_1 Q}{25a^2}$



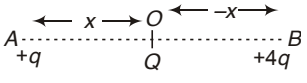
$$F_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{4q_2 Q}{9a^2}$$

For net force on Q to be zero.

$$F_1 + F_2 = 0$$

$$\Rightarrow \quad \frac{q_1}{q_2} = \frac{25}{9}$$

6. (a) In order to make net force on charge at A and B zero, Q must have negative sign.



Let the charge Q is planed at a distance x from A (+ Q charge)

$$F_{OA} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qQ}{x^2}$$

$$F_{OB} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{4qQ}{(x-x)^2}$$

For net force on Q to be zero.

$$F_{OA} = F_{OB}$$

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{qQ}{x^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{4qQ}{(L-x)^2}$$

$$(L-x)^2 = (2x)^2$$

$$x = \frac{L}{3}$$

Force on A,

$$F_{AB} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{4q^2}{L^2}$$

$$F_{AO} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qQ}{x^2}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{qqQ}{L^2}$$

For net force on Q to be zero.

$$F_{AB} = F_{AO}$$

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{4q^2}{L^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qqQ}{L^2}$$

$$\Rightarrow \quad q = \frac{9}{4}Q$$

$$\Rightarrow \quad Q = \frac{4}{9}q$$

As Q is negative $\Rightarrow q = \dfrac{-4}{9}q$

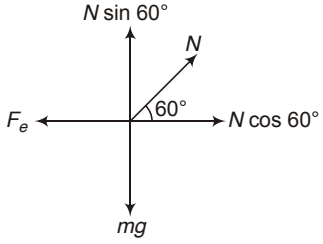
(b) PE of the system

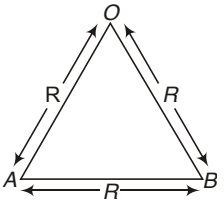
$$U = \frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{4q^2}{L} + \frac{qQ}{x} + \frac{4qQ}{L-x}\right]$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{4q^2}{L} - \frac{4qQ}{3L} - \frac{8qQ}{3L}\right] = 0$$

Hence, equilibrium is unstable.

7. FBD of af placed at left can be given by





ΔABD is equilateral

As beads are in equilibrium

$$mg = N\sin 60°$$

$$F_e = N\cos 60°$$

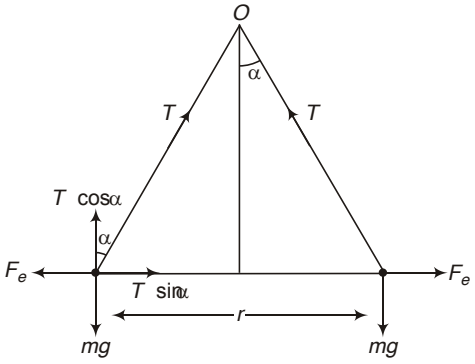
$$\frac{F_e}{mg} = \cot 60°$$

$$q^2 = 4\pi\varepsilon_0 R^2\, mg \cot 60°$$

$$q = \sqrt{\frac{4\pi\varepsilon_0 R^2\, mg}{3}}$$

$$= 2R\sqrt{\frac{6\pi\varepsilon_0 mg}{\sqrt{3}}}$$

8. As ball are in equilibrium



$$F_e = T \sin \alpha$$
$$mg = T \cos \alpha$$
$$F_e = mg \tan \alpha$$
$$q^2 = 4\pi\varepsilon_0 r^2 \tan \alpha$$

Here, $r = 2\,l \sin \alpha$

$$q^2 = 16\pi\varepsilon_0\, l^2 \sin^2 \alpha \tan \alpha$$
$$q = 3.3 \times 10^{-8} \text{ C.}$$

9. Same as Q.7. Introductory Exercise 21.3.

10. See Q.7. Introductory Exercise 21.3.

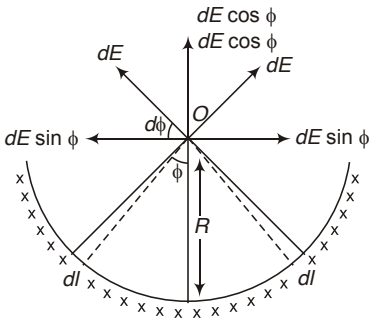
11. $$E = \frac{1}{4\pi\varepsilon_0} \cdot 1 \frac{q}{r^3} \vec{\mathbf{r}}$$

$$= \frac{9 \times 10^9 \times (-8.0 \times 10^{-9})}{((1.2)^2 + (1.6)^2)^{3/2}} (1.2\,\hat{\mathbf{i}} - 1.6\,\hat{\mathbf{j}})$$

$$= -18\sqrt{2}\,(1.2\,\hat{\mathbf{i}} - 1.6\,\hat{\mathbf{j}}) \text{ N/C.}$$

12. Consider an elementary portion on the ring of length dl subtending angle $d\phi$ at centre 'O' of the ring.

Charge on this portion,



$$dq = \lambda\, dl = \lambda\, R d\phi$$

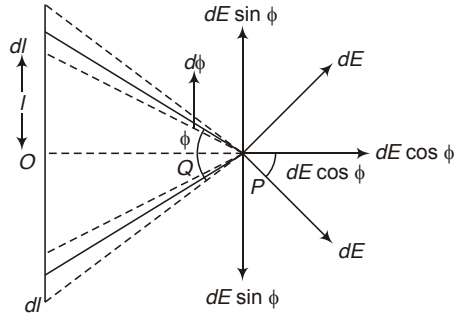
$$\therefore \quad dE = \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq}{R^2} = \frac{1}{4\pi\varepsilon_0} \frac{\lambda\, d\phi}{R}$$

Here, $dE \sin \phi$ components of field will cancel each other.

Hence, Net field at O

$$E = \int dE \cos \phi = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\lambda}{R} \int_{-\pi/2}^{\pi/2} \cos \phi\, d\phi$$
$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{2\lambda}{R}$$

13. Consider elementary portion of the rod of length dl at a distance l from the centre O of the rod.



Charge on this portion

$$dq = \lambda\, dl = \frac{Q}{L} dl$$

$$\therefore \quad dE = \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq}{(a \sec \phi)^2}$$
$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q\, dl}{La^2 \sec^2 \phi}$$

Now,

$$l = a \tan \phi$$
$$\Rightarrow \quad dl = a \sec^2 \phi\, d\phi$$
$$\therefore \quad dE = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q\, d\phi}{La}$$

Net Electric field at P.

$$E = \int dE \cos \phi$$

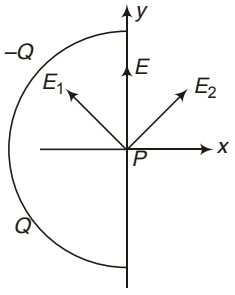
[$dE \sin \phi$ components will cancel each other as rod in symmetrical about P.]

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{La} \int_{-\theta}^{\theta} \cos \phi\, d\phi$$
$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Q \sin \theta}{La}$$

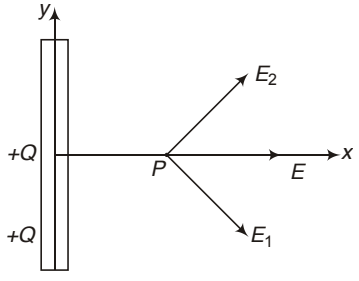
But $\sin \theta = \dfrac{L}{2\sqrt{a^2 + \left(\dfrac{L}{2}\right)^2}} = \dfrac{L}{\sqrt{4a^2 + L^2}}$

$$\therefore \quad E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Q}{a\sqrt{4a^2 + L^2}}$$

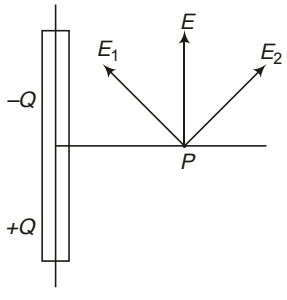
14. (a) As shown in figure, direction of electric field at P will be along + ve y-axis.



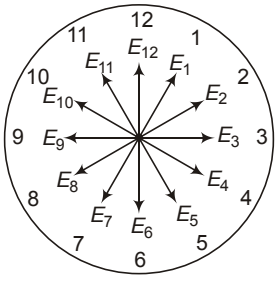
(b) Positive x-axis.



(c) Positive y-axis.

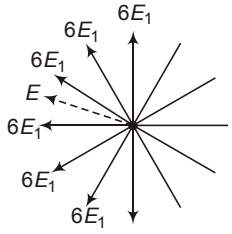


15. Let $E_1 = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q}{R^2}$



Resultant fields of two opposite charges can be shown as given in figure.

Clearly resultant field is along angle bisector of field towards 9 and 10.



Hence time shown by clock in the direction of electric field is 9 : 30.

16. (a) $a = \dfrac{F}{m} = \dfrac{-eE}{m}$

$$= \frac{-1.6 \times 10^{-19} \times 1 \times 10^3}{9.1 \times 10^{-31}}$$

$$= -1.76 \times 10^{14} \text{ ms}^{-2}$$

$u = 5.00 \times 10^8$ cm/s $= 5 \times 10^6$ ms^{-1}

$v = 0$

$v^2 - u^2 = 2as$

$$s = \frac{(5 \times 10^6)^2}{2 \times 1.7 \times 10^{14}} = 1.4 \times 10^{-2} = 1.4 \text{ cm}$$

(b) $v = u + at$

$$t = \frac{5 \times 10^6}{1.76 \times 10^{14}} = 2.8 \times 10^{-8} = 28 \text{ ns.}$$

(c) Δk = work done by electric field.

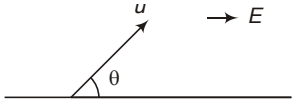
$= F \cdot x = -eEx$

$= -1.6 \times 10^{-19} \times 1 \times 10^3 \times 8 \times 10^{-3}$

$= -1.28 \times 10^{-18}$ J

Loss of KE $= 1.28 \times 10^{-18}$ J

17. Here, $u_x = u \cos 45° = \dfrac{25}{\sqrt{2}}$ ms^{-1}



$$u_y = u \sin 45° = \frac{25}{\sqrt{2}} \text{ ms}^{-1}$$

$$a_x = qE = 2 \times 10^{-6} \times 2 \times 10^7$$

$$= 40 \text{ ms}^{-1}$$

$$a_y = -10 \text{ ms}^{-1}$$

$$y = u_{yt} + \frac{1}{a_y} t^2$$

$$y = \frac{25}{\sqrt{2}} t - 5t^2$$

at the end of motion,

$$t = T \text{ and } y = 0$$

$$\therefore \quad T = \frac{5}{\sqrt{2}} \text{ s}$$

Also at the end of motion,

$$x = R$$

$$\therefore \quad x = u_x t + \frac{1}{2} a_x t^2$$

$$R = \frac{25}{\sqrt{2}} \times \frac{5}{\sqrt{2}} + 20 \times \left(\frac{5}{\sqrt{2}}\right)^2$$

$$= 312.5 \text{ m}$$

18. (a) $$R = \frac{\mu^2 \sin 2\theta}{qE}$$

$$\sin 2\theta = \frac{qER}{mu^2}$$

$$= \frac{1.6 \times 10^{-19} \times 720 \times 1.27 \times 10^{-3}}{1.67 \times 10^{-27} \times (9.55 \times 10^3)^2}$$

$$= 0.96$$

$$2\theta = 88° \text{ or } 92°$$

$$\theta = 44° \text{ or } 46°$$

$$T = \frac{2mh \sin \theta}{2E}$$

$$= \frac{2 \times 9.55 \times 10^3 \times \frac{1}{\sqrt{2}} \times 1.67 \times 10^{-31}}{1.6 \times 10^{-19} \times 720}$$

$$= 1.95 \times 10^{-11} \text{ s}$$

19. (a) $$\vec{\mathbf{a}} = -\frac{e\vec{\mathbf{E}}}{m} = -\frac{1.6 \times 10^{-19} \times 120}{9.1 \times 10^{-31}} \hat{\mathbf{j}}$$

$$= -2.1 \times 10^{13} \hat{\mathbf{i}} \text{ m/s}$$

(b) $$t = \frac{\Delta x}{u_x} = \frac{2 \times 10^{-2}}{1.5 \times 10^5} = \frac{4}{3} \times 10^{-7} \text{ s}$$

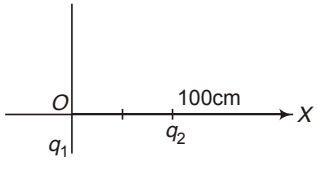
$$v_y = u_y + a_y t$$

$$= 3.0 \times 10^6 \times 2.1 \times 10^{13} \times \frac{4}{3} \times 10^{-7}$$

$$= 0.2 \times 10^6 \text{ m/s}$$

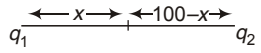
$$\vec{\mathbf{v}} = (1.5 \times 10^5)\hat{\mathbf{i}} + (0.2 \times 10^6)\hat{\mathbf{j}}$$

20. Absolute potential can be zero at two points on the x-axis. One in between the charges and other on the left of charge a_1 (smaller in magnitude).



Case I.

In between two charges : let potential is zero at a distance x from q_1 towards q_2.



$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{x} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2}{100 - x} = 0$$

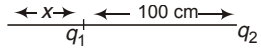
$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{2 \times 10^{-6}}{x} - \frac{1}{4\pi\varepsilon_0} \cdot \frac{3 \times 10^{-6}}{100 - x} = 0$$

$$\Rightarrow \quad 200 - 2x = 3x$$

$$x = 20 \text{ cm}$$

Case II.

Consider the potential is zero at a distance x from charge q, on its left.



$$\therefore \quad V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{x} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2}{100 + x} = 0$$

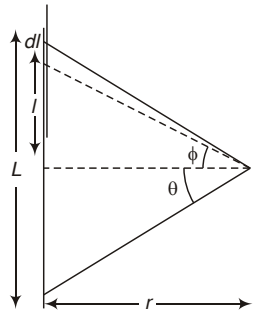
$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{2 \times 10^{-6}}{x} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{-3 \times 10^{-6}}{100 + x} = 0$$

$$200 + 2x = 3x$$

$$x = 200 \text{ cm}$$

21. Let us first find the potential at a point on the perpendicular bisector of a line charge.

Consider a line of carrying a line charge density λ having length L.



Consider an elementary portion of length dl on the rod.

Charge on this portion

$$dq = \lambda \, dl$$

$$\therefore \quad dV = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\lambda \, dl}{r \sec \phi}$$

Now, $$l = r \tan \phi$$

$$dl = r \sec^2 \phi \, d\phi$$

$$\therefore \quad dV = \frac{\lambda \sec\phi \, d\phi}{4\pi\varepsilon_0}$$

$$\therefore \quad V = \int dV = \frac{\lambda}{4\pi\varepsilon_0} \cdot \int_{-\theta}^{\theta} \sec^2\phi \, d\phi$$

$$= \frac{\lambda}{4\pi\varepsilon_0} [\ln|\sec\theta + \tan\theta|]_{-\theta}^{\theta}$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \left[\ln \left| \frac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta} \right| \right]$$

$$= \frac{2\lambda}{4\pi\varepsilon_0} \ln|\sec\theta + \tan\theta|$$

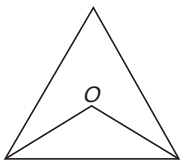
In the given condition

$$\theta = 60°$$

Potential due to one side

$$V_1 = V_2 = V_3 = \frac{2\lambda}{4\pi\varepsilon_0} \cdot \ln|\sec 60° + \tan 60°|$$

$$= \frac{2\lambda}{4\pi\varepsilon_0} \cdot \ln|2 + \sqrt{3}|$$



Total potential at O

$$V = 3V_1 = \frac{6\lambda}{4\pi\varepsilon_0} \cdot \ln|2 + \sqrt{3}|$$

$$= \frac{Q}{2\pi\varepsilon_0 a} \cdot \ln|2 + \sqrt{3}|$$

22. (a) $V_2 - V_1 = -\vec{\mathbf{E}} \cdot \vec{\mathbf{d}} = -250 \times 20 \times 10^{-2}$

$$= -50 \text{ V}$$

$$W = \Delta V = q(V_2 - V_1)$$

$$= 12 \times 10^{-6} \times -50 = -0.6 \text{ mJ}$$

(b) $V_2 - V_1 = -50$ V

23. By work energy theorem

$$W = \Delta K$$

$$q(V_1 - V_2) = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

$$-5 \times 10^{-6} (20 - 800)$$

$$= \frac{1}{2} \times 2 \times 10^{-4} (V_2^2 - (5)^2)$$

$$v_2^2 = 55$$

$$v_2 = \sqrt{55} = 7.42 \text{ ms}^{-1}$$

When a particle is released in electric field it moves in such a way that, it decreases its PE and increases KE

Hence, particle at B is faster than that at A.

24. Centre of circle is equidistant from every point on its periphery,

Hence, $$V_0 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R},$$

where $q = Q_1 + Q_2 = -SQ$

$$\therefore \quad V_0 = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{5Q}{R}$$

Similarly, $$V_p = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{\sqrt{R^2 + Z^2}}$$

$$= -\frac{1}{4\pi\varepsilon_0} \cdot \frac{SQ}{\sqrt{R^2 + Z^2}}$$

25. Initial PE

$$U_i = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_1}$$

$$U_f = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_2}$$

Work done by electric force

$$W = -\Delta U = -(U_f - U_i)$$

$$= -\frac{1}{4\pi\varepsilon_0} \cdot q_1 q_2 \left(\frac{1}{r_2} - \frac{1}{r_1} \right)$$

$$\Rightarrow W = -9 \times 10^9 \times 2.4 \times 10^{-6} \times (-4.3 \times 10^{-6}) \left(\frac{1}{0.25\sqrt{2}} - \frac{1}{0.15} \right)$$

$$W = -0.356 \text{ mJ}$$

26. (a) $$U = \frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_3 q_1}{r_{31}} \right]$$

$$= 9 \times 10^9 \left[\frac{4 \times 10^{-9} \times (-3 \times 10^{-9})}{0.2} + \frac{(-3 \times 10^{-9}) \times (2 \times 10^{-9})}{0.1} + \frac{4 \times 10^{-9} \times 2 \times 10^{-9}}{0.1} \right]$$

$$U = 9 \times 10^{-8} [-6 - 6 + 8] = -360 \text{ nJ}$$

(b) Let the distance of q_3 from q_1 is x cm. Then

$$U = \frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{q_1 q_2}{0.2} + \frac{q_2 q_3}{0.2 - x} + \frac{q_3 q_1}{x} \right] = 0$$

$$\Rightarrow \quad 9 \times 10^9 \left[\frac{4 \times 10^{-9} \times (-3 \times 10^{-9})}{20 \times 10^{0-2}} + \frac{(-3 \times 10^{-9}) \times 2 \times 10^{-9}}{(20 - x) \times 10^{-2}} \right.$$

$$+\frac{2\times10^{-9}\times4\times10^{-9}}{x\times10^{-2}}\Bigg]=0$$

$$\Rightarrow \quad -\frac{6}{10}-\frac{6}{20-x}+\frac{8}{x}=0$$

$$\Rightarrow \quad x=6.43 \text{ cm}$$

27. Let Q be the third charge

$$U=\frac{1}{4\pi\varepsilon_0}\cdot\left[\frac{q^2}{d}+\frac{qQ}{d}+\frac{qQ}{d}\right]=0$$

$$Q=-\frac{q}{2}$$

28. $V=-\vec{\mathbf{E}}\cdot\vec{\mathbf{r}}$

(a) $\vec{\mathbf{r}}=5\hat{\mathbf{k}}$

$$V=-(5\hat{\mathbf{i}}-3\hat{\mathbf{j}})-(5\hat{\mathbf{k}})=0$$

(b) $\vec{\mathbf{r}}=4\hat{\mathbf{i}}+3\hat{\mathbf{k}}$

$$V=-(5\hat{\mathbf{i}}-3\hat{\mathbf{j}})-(4\hat{\mathbf{i}}+3\hat{\mathbf{j}})$$

$$=-20 \text{ kV}$$

29. $\vec{\mathbf{E}}=400\hat{\mathbf{j}}$ V/m

(a) $\vec{\mathbf{r}}=20\hat{\mathbf{j}}$ cm $=(0.2\hat{\mathbf{j}})$ m

$$V=-\vec{\mathbf{E}}\cdot\vec{\mathbf{r}}=-80 \text{ V}$$

(b) $\vec{\mathbf{r}}=(-0.3\hat{\mathbf{j}})$ m

$$V=-\vec{\mathbf{E}}\cdot\vec{\mathbf{r}}=120 \text{ V}$$

(c) $\vec{\mathbf{r}}=(0.15\hat{\mathbf{k}})$

$$V=0$$

30. $\vec{\mathbf{E}}=20\hat{\mathbf{i}}$ N/C

(a) $\vec{\mathbf{r}}=(4\hat{\mathbf{i}}+2\hat{\mathbf{j}})$ m

$$V=-\vec{\mathbf{E}}\cdot\vec{\mathbf{r}}=-80 \text{ V}$$

(b) $\vec{\mathbf{r}}=(2\hat{\mathbf{i}}+3\hat{\mathbf{j}})$ m

$$V=-\vec{\mathbf{E}}\cdot\vec{\mathbf{r}}=-40 \text{ V}$$

31. (a) $[A]=\dfrac{[V]}{[xy+yz+zx]}=\dfrac{[\mathrm{ML^2T^{-3}I^{-1}}]}{[\mathrm{L^2}]}$

$$=[\mathrm{ML^0T^{-3}I^{-1}}]$$

(b) $E=-\vec{\nabla}V=-\left(\dfrac{\partial v}{\partial x}\hat{\mathbf{i}}+\dfrac{\partial v}{\partial y}\hat{\mathbf{j}}+\dfrac{\partial v}{\partial z}\hat{\mathbf{k}}\right)$

$$=-A[(y+z)\hat{\mathbf{i}}+(z+x)\hat{\mathbf{j}}+(x+y)\hat{\mathbf{k}}]$$

(c) at (1m, 1m, 1m)

$$E=-10(2\hat{\mathbf{i}}+2\hat{\mathbf{j}}+2\hat{\mathbf{k}})$$

$$=-20(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})$$

32. $V_B-V_0=-\vec{\mathbf{E}}\cdot\vec{\mathbf{r}}$

$$\Rightarrow \quad V-0=-(40+60)$$

$$\Rightarrow \quad V=-100$$

33. (a) $E_x=-\dfrac{\partial v}{\partial x}=-(Ay-2Bx)$

$$E_y-\frac{\partial V}{\partial y}=-(Ax+C)$$

$$E_z=-\frac{\partial V}{\partial Z}=0$$

(b) For $E=0$

$$E_x=0 \text{ and } E_y=0$$

Hence, $E_y=0$

$$Ax+C=0$$

$$x=-\frac{C}{A}$$

$E_x=0$

$$Ay-2B\left(-\frac{C}{A}\right)=0$$

$$y=-\frac{2BC}{A^2}$$

Hence, E is zero at $\left(-\dfrac{C}{A},-\dfrac{2BC}{A^2}\right)$.

34. $\phi=\dfrac{q}{\varepsilon_0}$

$$q=\varepsilon_0\phi=8.8\times10^{-12}\times360$$

$$=3.18\times10^{-9} \text{ C}$$

$$=3.186 \text{ nC}$$

36. (a) $\phi=\dfrac{q}{\varepsilon_0}=-\dfrac{3.60\times10^{-6}}{8.85\times10^{-12}}$

$$=4.07\times10^5 \text{ V-m.}$$

(b) $\phi=\dfrac{q}{\varepsilon_0}\Rightarrow q=\varepsilon_0\phi$

$$=8.85\times10^{-12}\times780=6.903\times10^{-9}$$

$$q=6.903 \text{ nC}$$

(c) No.
Net flux through a closed surface does not depend on position of charge.

36. $\vec{\mathbf{E}}=\left(\dfrac{3}{5}E_0\hat{\mathbf{i}}+\dfrac{4}{5}E_0\hat{\mathbf{j}}\right)$

$$\vec{\mathbf{S}}=0.2\hat{\mathbf{j}}\,\mathrm{m}^2=\frac{1}{5}\hat{\mathbf{j}}\,\mathrm{m}^2$$

$$\therefore \quad \phi=\vec{\mathbf{E}}\cdot\vec{\mathbf{S}}=\frac{4}{25}\,\mathrm{Nm^2/C}$$

$$= \frac{4}{25} \times 2.0 \times 10^3 \text{ N-m}^2/\text{C}$$
$$= 320 \text{ N-m}^2/\text{C}$$

37. $\vec{\mathbf{E}} = \frac{E_0 x}{l} \hat{\mathbf{i}}$ $x_1 = 0$

$$\vec{\mathbf{E}}_1 = 0$$
$$x_2 = a$$
$$\vec{\mathbf{E}}_2 = \frac{E_0 a}{l} \hat{\mathbf{i}}$$

Flux entering the surface

$$\phi_1 = 0$$

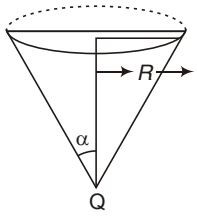
Flux leaving the surface

$$\phi_2 = E_2 a^2 = \frac{E_0 a^3}{l}$$
$$= \frac{5 \times 10^3 \times (1 \times 10^{-2})^3}{2 \times 10^{-2}}$$
$$= 0.25 \text{ N-m}^2/\text{C}$$

Net flux, $\phi = \frac{q}{\varepsilon_0}$

$$\phi_2 - \phi_1 = \frac{q}{\varepsilon_0}$$
$$q = \varepsilon_0 (\phi_2 - \phi_1)$$
$$= 8.85 \times 10^{-12} \times 0.25$$
$$= 2.21 \times 10^{-12} \text{ C} = 2.21 \text{ pC}$$

38. Consider the charge is placed at vertex of the cone of height b and radius R.



Let α be the semi-vertical angle of the cone, then solid angle subtended by the cone.

$$\Omega = 2\pi (1 - \cos \alpha)$$

Flux passing through cone

$$\phi = \frac{\Omega}{4\pi} \cdot \phi_{\text{total}}$$

But $\phi = \frac{1}{4} \phi_{\text{total}}$ (Given)

$$\Omega = \pi$$
$$2\pi (1 - \cos \alpha) = \pi$$
$$1 - \cos \alpha = \frac{1}{2}$$
$$\cos \alpha = \frac{1}{2} \Rightarrow \alpha = \frac{\pi}{3}$$
$$R = b \tan \alpha = \sqrt{3} b$$

Hence proved

39. $\vec{\mathbf{E}} = -B\hat{\mathbf{i}} + C\hat{\mathbf{j}} - D\hat{\mathbf{k}}$, $\vec{\mathbf{S}}_1 = -L^2 \hat{\mathbf{i}}$, $\vec{\mathbf{S}}_2 = -L^2 \hat{\mathbf{j}}$,

$\vec{\mathbf{S}}_3 = -L^2 \hat{\mathbf{i}}$, $\vec{\mathbf{S}}_4 = -L^2 \hat{\mathbf{j}}$, $\vec{\mathbf{S}}_4 = -L^2 \hat{\mathbf{k}}$,

$\vec{\mathbf{S}}_6 = -L^2 \hat{\mathbf{k}}$

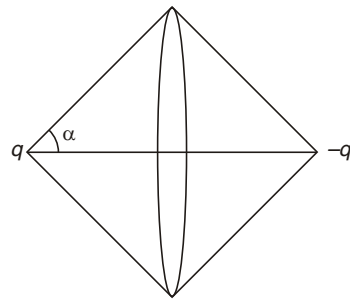
$\therefore$ $\phi_1 = \vec{\mathbf{E}} \cdot \vec{\mathbf{S}}_1 = BL^2$, $\phi_2 = \vec{\mathbf{E}} \cdot \vec{\mathbf{S}}_2 = CL^2$,

$\phi_3 = \vec{\mathbf{E}} \cdot \vec{\mathbf{S}}_3 = -BL^2$, $\phi_4 = \vec{\mathbf{E}} \cdot \vec{\mathbf{S}}_4 = -CL^2$,

$\phi_5 = \vec{\mathbf{E}} \cdot \vec{\mathbf{S}}_5 = -DL^2$, $\phi_6 = \vec{\mathbf{E}} \cdot \vec{\mathbf{S}}_6 = DL^2$

(b) $\phi = \phi_1 + \phi_2 + \phi_3 + \phi_4 + \phi_5 + \phi_6 = 0$

40. $\Omega = 2\pi (1 - \cos \phi)$

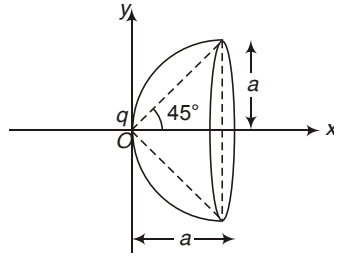


$$\phi_1 = \phi_2 = \frac{\Omega}{4\pi} \cot \phi_{\text{total}} = \frac{2\pi (1 - \cos \alpha)}{4\pi} \cdot \frac{q}{\varepsilon_0}$$
$$= \frac{1}{2\varepsilon_0} (1 - \cos \alpha)$$

Total flux through the ring

$$\phi = \phi_1 + \phi_2$$
$$= \frac{q}{\varepsilon_0} (1 - \cos \alpha) = \frac{q}{\varepsilon_0} \left(1 - \frac{l}{\sqrt{R^2 + l^2}} \right)$$

41. From the given equation,



radius of hemisphere $= a$

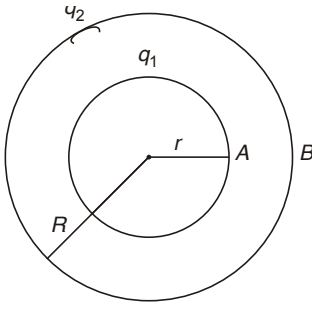
and its centre is at $(a, 0, 0)$

$$\Omega = 2\pi(1 - \cos\alpha) = 2\pi\left(1 - \frac{1}{\sqrt{2}}\right)$$

$$\phi = \frac{\Omega}{4\pi}\phi_{\text{total}} = \frac{2\pi\left(1 - \frac{1}{\sqrt{2}}\right)}{4\pi} \cdot \frac{q}{\varepsilon_0}$$

$$\phi = \frac{q}{2\varepsilon_0}\left(1 - \frac{1}{\sqrt{2}}\right)$$

42. $q_1 = \sigma(4\pi r^2)$, $q_2 = \sigma(4\pi R^2)$



But, $q_1 + q_2 = Q \Rightarrow \sigma = \dfrac{Q}{4\pi(r^2 + R^2)}$

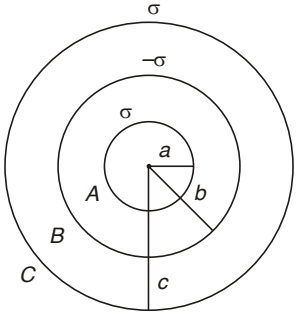
$$V_A = \frac{1}{4\pi\varepsilon_0} \cdot \left(\frac{q_1}{r} + \frac{q_2}{R}\right)$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \left(\frac{\sigma(4\pi r^2)}{r} + \frac{\sigma(4\pi R^2)}{R}\right)$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q(r + R)}{r^2 + R^2}$$

$$V_B = \frac{1}{4\pi\varepsilon_0} \cdot \left(\frac{q_1 + q_2}{R}\right) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R}$$

43. $q_A = \sigma(4\pi a^2)$, $q_B = -\sigma(4\pi b^2)$



$$q_C = \sigma(4\pi c^2)$$

$$V_A = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_A}{a} + \frac{q_B}{b} + \frac{q_C}{c}\right)$$

$$= \frac{\sigma}{\varepsilon_0}(a + b + c)$$

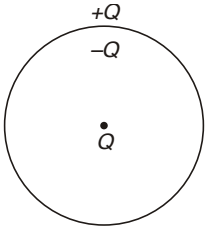
$$V_B = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_A + q_B}{b} + \frac{q_C}{c}\right)$$

$$= \frac{\sigma}{\varepsilon_0}\left(\frac{a^2}{b} - b + c\right)$$

$$V_C = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_A + q_B + q_C}{c}\right)$$

$$= \frac{\sigma}{\varepsilon_0}\left(\frac{a^2 - b^2 + c^2}{c}\right)$$

44. (a) As charge Q is placed at the centre of the sphere, charge $-Q$ will appear on the inner surface and Q on its outer surface.



Hence, $\sigma_{\text{in}} = \dfrac{-Q}{4\pi a^2}$

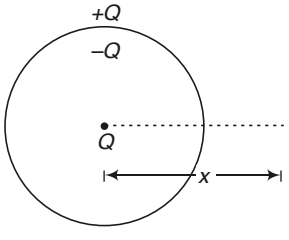
and $\sigma_{\text{out}} = \dfrac{Q}{4\pi a^2}$

(b) Entire charge inside the sphere appears on its outer surface, hence

$$\sigma_{\text{in}} = -\frac{Q}{4\pi a^2} \text{ and } \sigma_{\text{out}} = \frac{Q + q}{4\pi a^2}$$

(c) In case (a)

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{x^2}$$



In case (b)

$E = E_1 + E_2$

E_1 = Field due to charge Q.

E_2 = Field due to charge on shell.

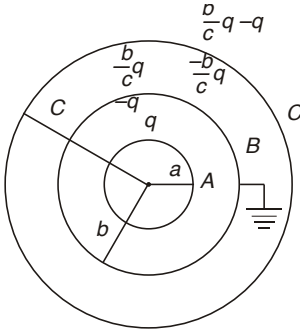
$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{x^2}$$

for $x < a$

As field due to shell is zero for $x < a$.

and $\quad E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q+q}{x^2}$, for $x > a$

45. Let Q be the charge on the shell B,

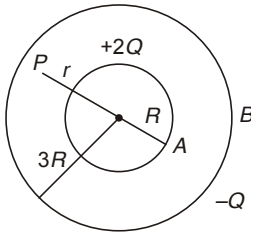


$$V_B = \frac{1}{4\pi\varepsilon_0}\left[\frac{q+Q}{b} + \frac{-q}{c}\right] = 0$$

$$Q = q\left(\frac{b-c}{c}\right)$$

Charge distribution on different surfaces is shown in figure.

46. (a) Let E_1 and E_2 be the electric field at P due to inner shell and outer shell respectively.



Now, $E_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Q}{r}$ and $E_2 = 0$

$\therefore \quad E = E_1 + E_2 = E_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Q}{r}$

(b) $V_A = \frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{2Q}{R} - \frac{Q}{3R}\right]$

$$V_B = \frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{2Q-Q}{3R}\right]$$

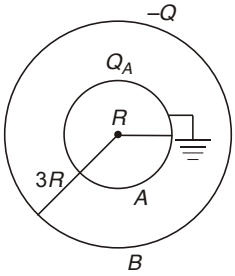
$$V_A - V_B = \frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{2Q}{R} - \frac{2Q}{3R}\right]$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{4Q}{3R}$$

(c) Whenever two concentric conducting spheres are joined by a conducting wire entire charge flows to the outer sphere.

$\therefore \quad Q_A = 0, Q_B = 0$

(d) Let Q_A be the charge on inner sphere.



$$V_A = \frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{Q_A}{R} - \frac{Q}{3R}\right] = 0$$

$$Q_A = \frac{Q}{3}$$

47. (a) At $r = R$

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{Q}{R} + \frac{-2Q}{2R} + \frac{3Q}{3R}\right]$$

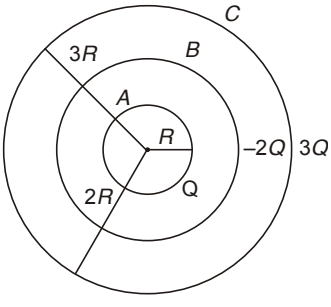
$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R}$$

At $r = 3R$

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{Q - 2Q + 3Q}{3R}\right]$$

$$= \frac{1}{4\pi\varepsilon_9} \cdot \frac{2Q}{3R}$$

(b) Let E_1, E_2 and E_3 be the electric fields at $r = \frac{5}{2}R$ due to shells A, B and C respectively.



$\therefore \quad E_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{\left(\frac{5}{2}R\right)^2}$

$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{4Q}{25R}$ (outwards)

$$E_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Q}{\left(\frac{5}{2}R\right)^2}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{8Q}{25R} \quad \text{(inward)}$$

$$E_3 = 0$$

Net field at $r = \frac{5}{2}R$

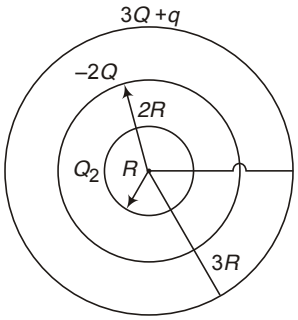
$$E = E_2 + E_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{4Q}{25R} \quad \text{(inward)}$$

(c) Total electrostatic energy of system is the sum of self-energy of three shell and the energy of all possible pairs *i.e.*,

$$U = \frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{Q^2}{2R} + \frac{(-2Q)^2}{2 \times 2R} + \frac{(3Q)^2}{2 \times 3R} + \frac{Q(-2Q)}{2R} + \frac{(-2Q) \times 3Q}{3R} + \frac{Q \times 3Q}{3R} \right]$$

$$U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R}$$

(d) Let q charge flows from innermost shell to outermost shell on connecting them with a conducting wire.



$$V_A = \frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{Q-q}{R} + \frac{-2Q}{2R} + \frac{3Q+q}{3R} \right]$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{3Q-2q}{3R}$$

$$V_B = \frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{Q-q-2Q+3Q+q}{3R} \right]$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Q}{3R}$$

But $V_A = V_B$

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{3Q-2q}{3R} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Q}{3R}$$

$$\Rightarrow \quad q = \frac{Q}{2}$$

$\therefore$ Charge on innermost shell $= Q - q = \frac{Q}{2}$

and charge on outermost shell $= 3Q + q = \frac{7Q}{2}$

and $$V_A = \frac{1}{4\pi\varepsilon_0} \cdot \frac{3Q-2q}{3R}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Q}{3R}$$

(c) In this case

$$E_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{2R\left(\frac{5}{2}R\right)^2}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Q}{25R} \quad \text{(outward)}$$

$$E_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Q}{\left(\frac{5}{2}R\right)^2}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{8Q}{25R} \quad \text{(inward)}$$

Net electric field at $r = \frac{5}{2}R$

$$E = E_2 - E_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{6Q}{25R} \quad \text{(inward)}$$

Objective Questions (Level-1)

1. $\phi = E \cdot A$

Units of $\phi = \text{N/C} \times \text{m}^2 = \text{N-m}^2/\text{C}$

or $\text{V/m} \times \text{m}^2 = \text{V-m}$

2. Net force

$$F' = mg - qE$$

$$g' = g - \frac{qE}{m}$$

$$T' = 2\pi\sqrt{\frac{l}{g_1}} > T$$

3. Electric lines of force terminate at negative charge.

4. $F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{l^2}$

Initial PE

$$U_i = \frac{1}{4\pi\varepsilon_0} \cdot \left(\frac{q^2}{l} + \frac{q^2}{l} + \frac{q^2}{l} \right) = 3Fl$$

Find PE

$$U_f = \frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{q^2}{2l}+\frac{q^2}{2l}+\frac{q^2}{2l}\right)=\frac{3}{2}Fl$$

$$W = U_f - U_i = -\frac{3}{2}Fl$$

5. $\text{KE} = qV$

$$\frac{1}{2}mv^2 = qV$$

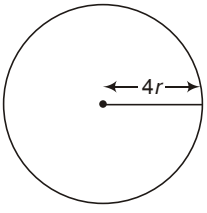
$$v = \sqrt{\frac{2qV}{m}}$$

$$\Rightarrow \quad V_1 : V_2 : V_3 = \sqrt{\frac{q_1V_1}{m_1}} : \sqrt{\frac{q_2V_2}{m_2}} : \sqrt{\frac{q_3V_3}{m_3}}$$

$$\Rightarrow \quad V_1 : V_2 : V_3 = \sqrt{\frac{e\times 1}{m}} : \sqrt{\frac{e\times 2}{2m}} : \sqrt{\frac{2e\times 4}{4m}}$$

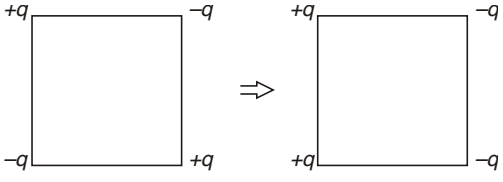
$$V_1 : V : V_3 = 1 : 1 : \sqrt{2}$$

6. $V = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q}{r}$



$$V' = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2q}{4r} = \frac{V}{2}$$

7. $U_i = \dfrac{1}{4\pi\varepsilon_0}\left[-\dfrac{q^2}{a}\times 4 + \dfrac{q^2}{\sqrt{2}\,a}\times 2\right]$



$$= \frac{-q^2}{4\pi\varepsilon_0\, a}[4-\sqrt{2}]$$

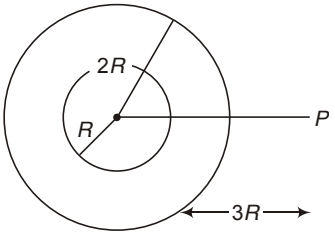
$$U_f = \frac{1}{4\pi\varepsilon_0}\cdot\left[-\frac{q^2}{a}\times 2 + \frac{q^2}{a}\times 2 - \frac{q^2}{\sqrt{2}}\times 2\right]$$

$$= -\frac{\sqrt{2}\,q^2}{4\pi\varepsilon_0\, a}$$

$$W = U_f - U_i = -\frac{\sqrt{2}\,q^2}{4\pi\varepsilon_0\, a} + \frac{q^2}{4\pi\varepsilon_0\, a}[4-\sqrt{2}]$$

$$= \frac{q^2}{4\pi\varepsilon_0 a}[4-2\sqrt{2}]\ \text{J}$$

8. Potential at point P



$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1+q_2}{3R}$$

$$\Rightarrow \quad \frac{9\times 10^9\times 3\times 10^{-6}}{3R} = 9000$$

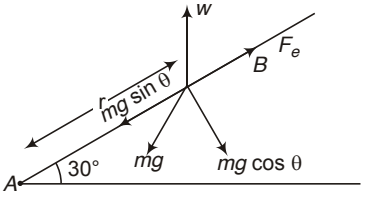
$$R = 1\ \text{m}$$

9. As distance of every point of ring from axis is same.

$$V = \frac{kq}{\sqrt{R^2+x^2}}, \text{ But } x = 2\sqrt{R}$$

$$= \frac{kq}{3R}$$

10. For equilibrium,



$$mg\sin 30° = F_e$$

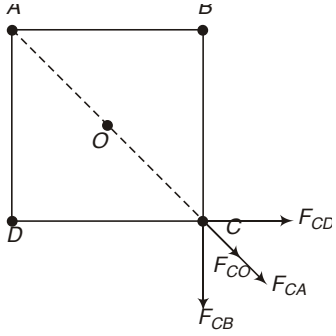
$$mg\sin 30° = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{r^2}$$

$$r = q\sqrt{\frac{1}{4\pi\varepsilon_0\, mg\sin 30°}}$$

$$= 2.0\times 10^{-6}\sqrt{\frac{91\times 10^9}{0.1\times 10\times\frac{1}{2}}}$$

$$\approx 20\ \text{cm}$$

11. Net force on $C = 0$



$$F_{CB} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(2\sqrt{2}-1)^2 Q^2}{a^2}$$

$$F_{CD} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(2\sqrt{2}-1)^2 Q^2}{a^2}$$

$$F_{CA} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(2\sqrt{2}-1)^2 Q^2}{2a^2}$$

$$F_{CO} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2(2\sqrt{2}-1)Q^2}{a^2}$$

Net force on C

$$F = F_{CA} + F_{CO} + F_{CB}\cos 45° + F_{CO}\cos 45°$$

$$= \frac{(2\sqrt{2}-1)Q}{a^2}\left[\frac{(2\sqrt{2}-1)Q}{2} + \frac{(2\sqrt{2}-1)Q}{\sqrt{2}} + \frac{(2\sqrt{2}-1)Q}{\sqrt{2}} + \frac{2q}{1}\right] = 0$$

$$q = -\frac{7Q}{4}$$

12. $E = \dfrac{\sigma}{\varepsilon_0}$

$$F = eF = \frac{e\sigma}{\varepsilon_0}$$

Acceleration of proton

$$a = \frac{F}{m} = \frac{\sigma e}{m\varepsilon_0}$$

$$s = ut + \frac{1}{a}t^2$$

$$u = 0$$

$$t = \sqrt{\frac{25}{a}} = \sqrt{\frac{25\, m\varepsilon_0}{\sigma e}}$$

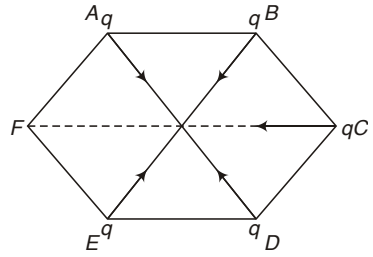
$$= \sqrt{\frac{2 \times 0.1 \times 1.67 \times 10^{-27} \times 8.8 \times 10^{-12}}{2.21 \times 10^{-9} \times 1.6 \times 10^{-19}}}$$

$$= 2\sqrt{2}\ \mu\text{s}$$

13. Data is not sufficient.

14. If the charges have opposite sign, electric field is zero on the left of smaller charge.

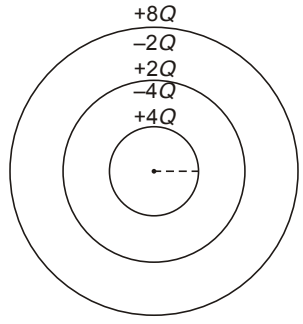
15. Net field is only due to charge on C.



$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(2a)^2} = \frac{q}{16\pi\varepsilon_0 a^2}$$

16. On touching two spheres, equal charge will appear on both the spheres and for a given total charge, force between two spheres is maximum if charges on them are equal.

17. Charge distribution is shown in figure.



18. $V = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q}{r}$

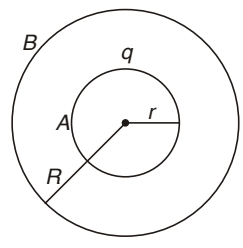
If drops coalesce, total volume remains conserved,

$$\frac{4}{3}\pi R^3 = 1000 \times \frac{4}{3}\pi r^3$$

$R = 10r$

$$V' = \frac{1}{4\pi\varepsilon_0} \cdot \frac{1000q}{10q} = 10\text{V}$$

19. $V_A = \dfrac{1}{4\pi\varepsilon_0} \cdot \left[\dfrac{q}{r} + \dfrac{Q}{R}\right]$



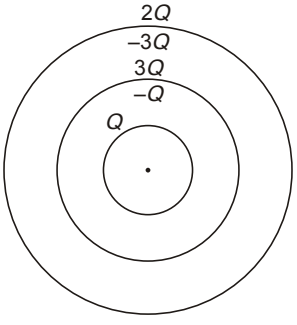
$$V_B = \frac{1}{4\pi\varepsilon_0}\cdot\left[\frac{q+Q}{R}\right]$$

$$V_A - V_B = \frac{q}{4\pi\varepsilon_0}\cdot\left[\frac{1}{r}-\frac{1}{R}\right]$$

$\therefore \qquad V_A - V_B \propto q$

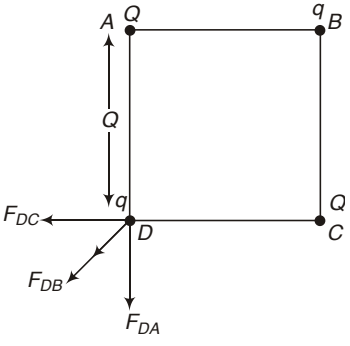
If q is doubled, $V_A - V_B$ will become double.

20. Charge distribution is shown in figure.



21. $\phi = \vec{\mathbf{E}}\cdot\vec{\mathbf{S}} = (5\,\hat{\mathbf{i}} + 2\,\hat{\mathbf{j}})\cdot(\hat{\mathbf{i}}) = 5$ V-m.

22. $F_{DA} = F_{DC} = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{Qq}{a^2}$



$$F_{DB} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{2a^2}$$

Net force on charge at D

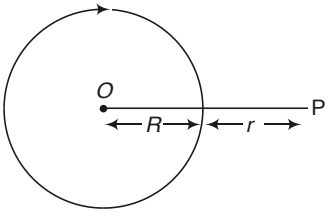
$$\vec{\mathbf{F}}_0 = F_{DB} + F_{DA}\cos 45° + F_{DB}\cos 45° = 0$$

$$\Rightarrow \qquad \frac{1}{4}4\pi\varepsilon_0\cdot\frac{q}{a^2}\left[\frac{q}{2}+\frac{Q}{\sqrt{2}}+\frac{Q}{\sqrt{2}}\right] = 0$$

$$q = -2\sqrt{2}\,Q$$

23. As $V_B = 0$, Total charge inside B must be zero and hence charge on its outer surface is zero and on its inner surface is $-q$.

24. $V_p = \dfrac{1}{V_0}$



$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R+r} = \frac{1}{2}\cdot\frac{1}{4\pi\varepsilon_0}\cdot\frac{3q}{2R}$$

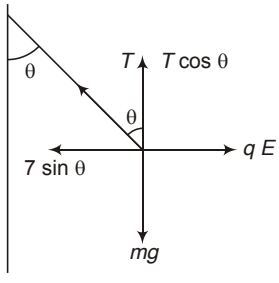
$$\frac{1}{R+r} = \frac{3°}{4R}$$

$$4R = 3R + 3r$$

$$r = \frac{R}{3}$$

25. Net charge on any dipole is zero.

26. For net force to be zero.



$T\cos\theta = mg \Rightarrow T = \dfrac{mg}{\cos\theta}$

or $T\sin\theta = qE \Rightarrow T = \dfrac{qE}{\sin\theta}$

27. $$E_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{a} = \frac{V_1}{a}$$

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{b^2} = \frac{V_2}{b}$$

But $\qquad E_1 = E_2$

$$\frac{V_1}{a} = \frac{V_2}{b}$$

$$\Rightarrow \qquad \frac{V_1}{V_2} = \frac{a}{b}$$

28. Electric field on equatorial lines of dipole is opposite to dipole moment.

29. Potential difference between two concentric spheres is independent of charge on outer sphere.

30. $E = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q}{r^2}$

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r} = Er$$

$$r = \left|\frac{V}{E}\right| = \frac{3000}{500} = 6 \text{ m}$$

$$q = 4\pi\varepsilon_0 rV = \frac{6 \times (-3000)}{9 \times 10^9} = -2\ \mu\text{C}$$

31. $F_1 = F_2$

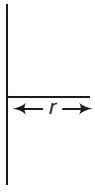
$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_1^2} = \frac{1}{4\pi K\varepsilon_0} \cdot \frac{q_1 q_2}{r_2^2}$$

$$r_2 = \frac{r_1}{\sqrt{K}} = \frac{50}{\sqrt{5}} = 10\sqrt{5} \text{ m}$$

$$\approx 22.3 \text{ m}$$

32. Electric field at a distance r from infinite line charge

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$



$$dV = -E\,dr$$

$$\int_{V_1}^{V_2} dV = -\int_a^b E\,dr$$

$$\Rightarrow \quad V_2 - V_1 = \frac{\lambda}{2\pi\varepsilon_0} \cdot \ln\frac{1}{2}$$

$$W = q(V_2 - V_1) = \frac{q\lambda}{2\pi\varepsilon_0} \ln\frac{1}{2}$$

33. As negative charge is at less distance from the line charge, it is attracted towards the line charge.

34. $r = \sqrt{(4-1)^2 + (2-2)^2 + (0-4)^2} = 5 \text{ m}$

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r} = \frac{9 \times 10^9 \times 2 \times 10^{-8}}{5} = 36 \text{ V}$$

(b) and (c) are wrong.

35. $V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R}$

At a distance r from the centre,

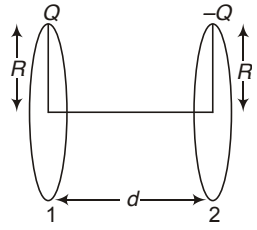
$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} = \frac{VR}{r^2}$$

36. When outer sphere is earthed field between the region of two spheres in non-zero and is zero in all other regions.

37. $W = \vec{\mathbf{F}} \cdot \vec{\mathbf{s}} = qEs\cos\theta$

$$E = \frac{W}{qs\cos\theta} = \frac{4}{0.2 \times 2 \times \cos 60^\circ} = 20 \text{ N/C}$$

38. $V_1 = \frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{Q}{R} - \frac{Q}{\sqrt{d^2 + R^2}}\right]$



$$V_2 = \frac{1}{4\pi\varepsilon_0}\left[-\frac{Q}{R} + \frac{Q}{\sqrt{d^2 + R^2}}\right]$$

$$V_1 - V_2 = \frac{1}{4\pi\varepsilon_0}\left[-\frac{2Q}{R} - \frac{2Q}{\sqrt{d^2 + R^2}}\right]$$

$$V_1 - V_2 = \frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{R} + \frac{1}{\sqrt{d^2 + R^2}}\right]$$

39. Electric field inside a hollow sphere is always zero.

40. $W = \vec{\mathbf{F}} \cdot \vec{\mathbf{r}} = q\,\vec{\mathbf{E}} \cdot \vec{\mathbf{r}}$

$$= q(E_1\,\hat{\mathbf{i}} + E_2\,\hat{\mathbf{j}}) - (a\,\hat{\mathbf{i}} + b\,\hat{\mathbf{j}})$$

$$= q(aE_1 + bE_2)$$

JEE Corner

Assertion and Reason

1. Negative charge always moved towards increasing potential.

On moving from A to B potential energy of negative charge decreases hence its KE increases.

2. $U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r}$

If q_1 and q_2 have opposite sign, U decreases with decrease in r.

$F = -\frac{dU}{dr} \Rightarrow$ work done by conservative force always decreases PE.

3. $E = -\frac{dV}{dr} = -(10) = 10$ V/m along x-axis.

4. $V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R}$

Inside the solid sphere.

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qr}{R^3}$$

at $r = \frac{R}{2}$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{2R^2} = \frac{V}{2R}$$

Assertion is correct.

Reason is false as electric field inside the sphere is directly proportional to distance from centre but not outside it.

5. Gauss theorem is valid only for closed surface but electric flux can be obtained for any surface.

6. Let V_0 = Potential at origin,

$$V_A = -(4\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}}) \cdot (4\,\hat{\mathbf{i}}) = -16\text{ V}$$

$$V_B = -(4\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}}) \cdot (4\,\hat{\mathbf{i}}) = -16\text{ V}$$

$$V_A = V_B$$

Hence, Assertion is false.

7. In the line going A and B, the energy of third charge is minimum at centre.

8. Dipole has both negative and positive charges hence work done is not positive.

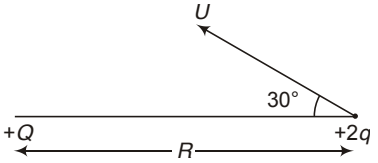
9. Charge outside a closed surface can produce electric field but cannot produce flux.

10. $E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qx}{(x^2 + a^2)^{3/2}}$ is maximum at $x = \frac{a}{\sqrt{2}}$

But $V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{\sqrt{a^2 + x^2}}$ is maximum at $x = 0$.

Objective Questions (Level-2)

1. Electrostatic force always acts along the line joining the two charges, hence net torque on charge + 2q is always zero.

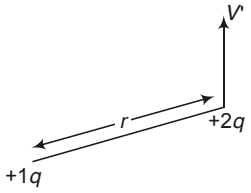


As net torque is zero angular momentum of charge remains conserved.

Initial angular momentum

$$L_i = m(V \sin 30°)R$$

When the separation between the charges become minimum, direction of motion of charge + 2q become perpendicular to the line joining the charges.



$\therefore$ find angular momentum

$$L_f = mv'r = \frac{mvr}{\sqrt{2}}$$

By conservation of angular momentum

$$L_i = L_f \Rightarrow r = \frac{\sqrt{3}}{2} R$$

2. $\vec{\mathbf{v}}_1 = v\,\hat{\mathbf{j}}$, $\vec{\mathbf{v}}_2 = 2v\cos 30°\,\hat{\mathbf{i}} + 2v\sin 30°\,\hat{\mathbf{j}}$

$$= \sqrt{3}\,\hat{\mathbf{i}} + v\,\hat{\mathbf{j}}$$

As velocity along y-axis is unchanged, electric field along x-axis is zero.

For motion along x-axis,

$$v_x^2 - u_x^2 = 2a_x(x - x_0)$$

$$a_x = \frac{(\sqrt{3}v)^2 - 0}{2a} = \frac{3v^2}{2a}$$

$$F_x = ma_x = \frac{3mv^2}{2a}$$

$$\Rightarrow \quad \vec{\mathbf{F}} = \frac{3mv^2}{2a}\,\hat{\mathbf{i}}$$

Also, $\vec{\mathbf{F}} = -e\,\vec{\mathbf{E}}$

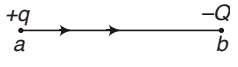
$$\vec{\mathbf{E}} = -\frac{3mv^2}{2ea}\,\hat{\mathbf{i}}$$

Rate of work done by electric field at B

$$P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}} = \left(\frac{3mv}{2a}\,\hat{\mathbf{i}}\right) \cdot (\sqrt{3}v\,\hat{\mathbf{i}} + v\,\hat{\mathbf{j}})$$

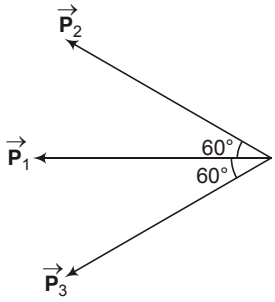
$$= \frac{3\sqrt{3}\,mv^3}{2a}$$

3. Electric field is always possible, hence a must be positive and b must be negative.



4. The system can be assumed as a combination of three identical dipoles as shown in figure.

Here, $P_1 = P_2 = P_3 = Q(2a)$



Net dipole moment of the system

$$P = P_1 + P_2 \cos 60° + P_3 \cos 60°$$
$$= 2p = 4Qa$$

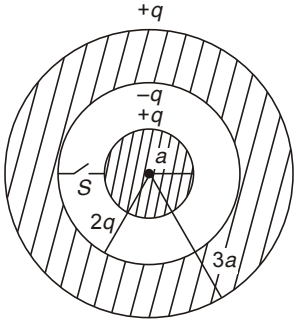
Electric field on equatorial lines of short dipole is given by

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{P}{x^3}$$
$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{4Qa}{x^3} = \frac{Qa}{\pi\varepsilon_0 x^3}$$

5. Potential at centre will be same as potential at the surface of inner shell *i.e.*, 10 V.

6. Initial charge distribution is shown in figure,

Initial energy of system



$$U_i = \frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{q^2}{2a} + \frac{(-q)^2}{2 \times 2a} + \frac{q^2}{2 \times 3a} + \frac{q(-q)}{2a} + \frac{q(q)}{3a} + \frac{q(-q)}{3a} \right]$$
$$= \frac{5a^2}{48\pi\varepsilon_0 a}$$

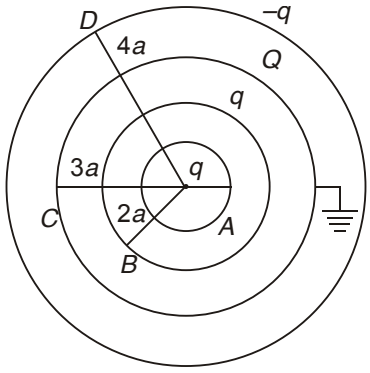
When switch S is closed, entire charge flows to the outer surface of outer shell,

$$U_f = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{2 \times 3a} = \frac{q^2}{24\pi\varepsilon_0 a}$$

Heat produced $= U_i = U_f = \dfrac{q^2}{8\pi\varepsilon_0 a}$

$$= \frac{kq^2}{2a}$$

7. Let Q charge flows to C



$$V_C = \frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{q+Q}{3a} + \frac{(-q)}{4a} \right] = 0$$

$$Q = -\frac{q}{4}$$

$$\therefore \quad V_A = \frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{q}{2a} + \frac{Q}{3a} + \frac{-q}{4a} \right]$$
$$= \frac{1}{4\pi\varepsilon_0} \left[\frac{q}{\sigma a} \right]$$

$$V_A - V_C = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{\sigma a}$$
$$= \frac{kq}{\sigma a}$$

8. $V_S = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q}{R}$

and $\quad V_C = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{3q}{2R}$

$$\therefore \quad V_C - V_S = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{2R}$$
$$= \frac{1}{4p\varepsilon_0} \cdot \frac{\frac{4}{3}\pi\,\delta R^3}{2R} = \frac{\delta R^2}{\sigma E_0}$$

9. As particle comes to rest, force must be repulsive, hence it is positively charged.

Again on moving down its KE first increases than decreases, PE will first decrease than increase.

10. (1) is correct as the points having zero potential are close to Q_2, $|Q_2| < |Q_1|$.

Again as potential near Q_1 is positive, Q_1 is positive, hence (2) is correct.

At point A and B potential is zero not field, hence they may or may not be equilibrium point.

Hence (3) is wrong.

At point C potential is minimum, Q positive charge placed at this point will have unstable equilibrium but a negative charge will be in stable equilibrium at this position.

Hence, (4) is wrong.

11. V_1 is always negative and V_2 is always positive.

12. Electric field between the two points is positive near q_1 and negative near q_2, hence q_1 is positive and q_2 is negative.

Again neutral point is closer to q_1, hence $q_1 < q_2$.

13. Electric field due to a conductor does not depend on position of charge inside it.

14. $\vec{\mathbf{E}} = 400\cos 45^\circ\,\hat{\mathbf{i}} + 4000\sin 45^\circ\,\hat{\mathbf{j}}$

$$= 200\sqrt{2}\,(\hat{\mathbf{i}} + \hat{\mathbf{j}})$$

$$\vec{\mathbf{V}}_A - \vec{\mathbf{V}}_B = -\vec{\mathbf{E}}\cdot\vec{\mathbf{r}}_{AB}$$

$$= -200\sqrt{2}\,(\hat{\mathbf{i}} + \hat{\mathbf{j}})\cdot(2\,\hat{\mathbf{j}} - 3\,\hat{\mathbf{i}})\times 10^{-2}$$

$$= 2\sqrt{2}\ \text{V} \approx 2.8\ \text{V}$$

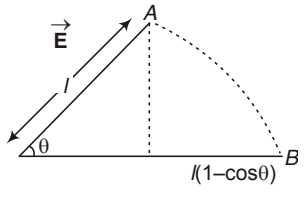
15. Potential difference between two concentric spherical shells does not depend on charge of outer sphere. Hence,

$$V_A' - V_B' = V_A - V_B$$

But $V_B' = 0$

$\therefore \qquad V_A' = V_A - V_B.$

16. By work energy theorem,



Work done by electric field

= charge is KE

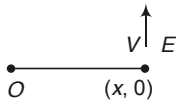
$$qE\,l(1-\cos\theta) = \frac{1}{2}mv^2 - 0$$

$$v = \sqrt{\frac{qE\,l}{m}}$$

At point B

$$T = qE + \frac{mv^2}{r} = 2\,qE$$

17. Velocity of particle at any instant



$$V = at = \frac{qE}{m}t$$

$\therefore \qquad L = mvr = qE\,x_0 t$

Hence, angular momentum of the particle increases with time.

18. By work energy theorem

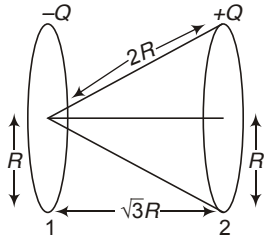
$$W = \Delta K$$

$\Rightarrow \qquad q(V_S - V_C) = 0 - \frac{1}{2}mv^2$

$\Rightarrow \qquad q\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R} - \frac{1}{4\pi\varepsilon_0}\cdot\frac{3Q}{2R}\right) = -\frac{1}{2}mv^2$

$\Rightarrow \qquad u = \sqrt{\frac{Q\,q}{4\pi\varepsilon_0 mR}}$

19. Potential at the centre of negatively charged ring



$$V_1 = \frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{-Q}{R} + \frac{Q}{2R}\right)$$

$$= -\frac{Q}{8\pi\varepsilon_0 R}$$

Potential at the centre of positively charged ring

$$V_2 = \frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{Q}{R} - \frac{Q}{2R}\right)$$

$$= \frac{Q}{8\pi\varepsilon_0 R}$$

Kinetic energy required = Work done required

$$= q(V_2 - V_1) = \frac{Q}{4\pi\varepsilon_0 R}$$

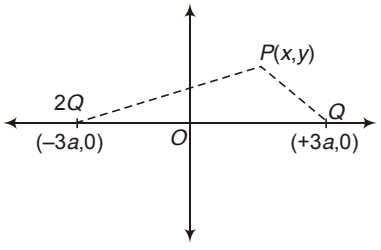
20. $E_x = -\dfrac{V_{x_2} - V_{x_1}}{x_2 - x_1} = -\dfrac{16-4}{-2-2} = 3$ V/m

$E_y = -\dfrac{V_{y_3} - V_{y_1}}{y_3 - y_1} = -\dfrac{12-4}{4-2} = -4$ V/m

$\therefore E = E_x\,\hat{\mathbf{i}} + E_y\,\hat{\mathbf{j}} = (3\,\hat{\mathbf{i}} - 4\,\hat{\mathbf{j}})$ V/m.

21. Consider a point $P(x, y)$ where potential is zero.

Now, V_P



$$V_P = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{\sqrt{(3a-x)^2+y^2}} + \frac{-2Q}{\sqrt{(x+3a)^2+y^2}}\right)$$

$= 0$

$\Rightarrow \quad (x+3a)^2 + y^2 = 4[(3a-x)^2 + y^2]$

$\Rightarrow \quad 3x^2 + 3y^2 - 30ax + 27a^2 = 0$

$\Rightarrow \quad x^2 + y^2 - 10ax + 9a^2 = 0$

The equation represents a circle with radius

$$= \sqrt{\left(\frac{10a}{2}\right)^2 - 9a^2} = 4a$$

and centre at $\left(\dfrac{10}{2}a, 0\right) = (5a, 0)$

Clearly points $x = a$ and $x = 9a$ lie on this circle.

22. Work done $= qEy =$ Charge in KE

Hence, $\quad K_f = \dfrac{1}{2}mv^2 + qEy$

All other statements are correct.

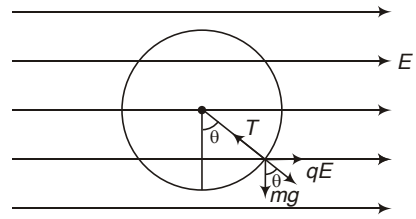
23. Electrostatic force of attraction provides necessary centripetal force.

ie, $\quad \dfrac{mv^2}{r} = \dfrac{\lambda q}{2\pi\varepsilon_0 r}$

$\Rightarrow \quad V = \sqrt{\dfrac{\lambda q}{2\pi\varepsilon_0 m}}$

$$T = \frac{2\pi r}{V} = 2\pi r\sqrt{\frac{2\pi\varepsilon_0 m}{\lambda q}} = 2\pi r\sqrt{\frac{m}{2K\lambda q}}$$

24. $T - mg\cos\theta - qE\sin\theta = \dfrac{mv^2}{l}$



Tension will be minimum when velocity is minimum.

Minimum possible in the string is zero.

ie, $\quad \dfrac{mv^2}{l} = -(mg\cos\theta + qE\sin\theta)$

Diff. both sides w.r.t. θ

$$\frac{2mv}{l}\frac{dv}{d\theta} = mg\sin\theta - qE\sin\theta \qquad \ldots(i)$$

For minima or maxima

$$\frac{dv}{d\theta} = 0 \Rightarrow \theta = \tan^{-1}\frac{qE}{mg}$$

or $\quad \pi + \tan^{-1}\dfrac{qE}{mg}$

Differentiating Eq. (i) again,

$$\frac{2mv}{l}\frac{d^2v}{d\theta^2} + \frac{2m}{l}\cdot\left(\frac{dv}{d\theta}\right)^2 = mg\sin\theta + qE\sin\theta$$

$\therefore \dfrac{d^2v}{d\theta^2} = +$ ve for $\theta = \tan^{-1}\dfrac{qE}{mg}$

and –ve for $\theta = \pi + \tan^{-1}\dfrac{qE}{mg}$

25. $q_A = \sigma(4\pi a^2)$, $q_B = -\sigma(4\pi b^2)$ and $q_C = -\sigma(4\pi c^2)$

$$V_B = \frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{q_A + q_B}{b} + \frac{q_C}{c}\right)$$

$$= \frac{\sigma}{\varepsilon_0}\left[\frac{a^2}{b^2} - b + c\right]$$

26. $U_i = \dfrac{1}{4\pi\varepsilon_0}\cdot\left[\dfrac{q^2}{a} - \dfrac{-q^2}{a} + \dfrac{q^2}{a} - \dfrac{q^2}{a} - \dfrac{q^2}{\sqrt{2}a}\times 2\right]$

$= \dfrac{-\sqrt{2}\,q^2}{4\pi\varepsilon_0 a}$

$U_f = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q^2}{a}$

$W = U_f - U_i = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q^2}{a}(\sqrt{2}+1)$

27. $q_A = \sigma(4\pi a^2)$, $q_B = -\sigma(4\pi b^2)$, $q_C = \sigma(4\pi c^2)$

Given, $V_A = V_C$

$$\frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{q_A}{a}+\frac{q_B}{b}+\frac{q_C}{c}\right)=\frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{q_A+q_B+q_C}{c}\right)$$

$$\Rightarrow \quad a-b+c=\frac{a^2-b^2}{c}+c$$

$$\Rightarrow \quad a+b=c$$

28. Potential at minimum at mid-point in the region between two charges, and is always positive.

29. $U_i = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{r} = U$

$U_f = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{r}\times 3 = 3U$

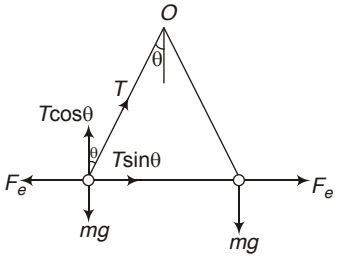
$\therefore W = U_f - U_i = 2U$

30. Loss of KE = Gain in PE

$$\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qQ}{r}$$

$$r \propto \frac{1}{v^2}$$

31. When the spheres are in air

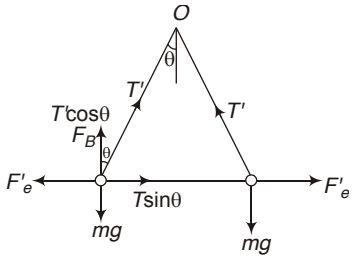


$$T\cos\theta = mg$$

$$T\sin\theta = F_e$$

$\therefore \quad F_e = mg\tan\theta \quad$ ...(i)

When the spheres are immersed in liquid



$$F_1' = T'\sin\theta$$

$$g - F_B = T'\cos\theta$$

$\therefore \quad F_e' = (mg - F_B)\tan\theta \quad$ ...(ii)

On dividing Eq. (ii) by Eq. (i),

$$\frac{F_e'}{F_e} = \frac{mg - F_B}{mg}$$

$$\frac{1}{K} = 1 - \frac{F_B}{mg} = 1 - \frac{0.8}{1.6} = \frac{1}{2}$$

$$K = 2$$

32. $V_P = \frac{1}{4\pi\varepsilon_0}\cdot\left[\frac{q}{a}\times 2 - \frac{q}{\sqrt{a^2+b^2}}\times 2\right]$

$$= \frac{2q}{4\pi\varepsilon_0}\cdot\left[\frac{\sqrt{a^2+b^2}-a}{a\sqrt{a^2+b^2}}\right]$$

$$= \frac{2q}{4\pi\varepsilon_0}\left[\frac{a\left(1+\frac{b^2}{a^2}\right)^{1/2}-a}{a\sqrt{a^2+b^2}}\right] = \frac{2q}{4\pi\varepsilon_0}\cdot\frac{b^2}{a^3}$$

[As $b << a$]

33. In any case electric field at origin is $\frac{1}{4\pi\varepsilon_0}\cdot\frac{5q}{r^2}$ along x-axis and $\frac{1}{4\pi\varepsilon_0}\cdot\frac{5q}{r^2}$ along y-axis.

34. $< u > = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^2}\right)^2$

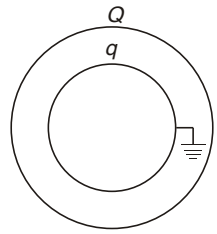
$$= \frac{1}{2}\varepsilon_0\left[\frac{9\times 10^9\times\frac{1}{9}\times 10^{-9}}{12}\right]$$

$$= \frac{\varepsilon_0}{2}\ \text{J/m}^3$$

35. If Q is initial charge on B

then, $V_A - V_B = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{b} = V$

Now, if A is earthed, let charge q moves on A from ground, then



$$V_A = \frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{q}{a}+\frac{Q}{b}\right) = 0$$

$$q = -\frac{a}{b}Q$$

$$V_B = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q+Q}{b}$$
$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{b}\left(1-\frac{a}{b}\right) = V\left(1-\frac{a}{b}\right)$$

36. $\vec{\mathbf{E}} = -\left(\frac{\partial v}{\partial x}\hat{\mathbf{i}} + \frac{\partial v}{\partial y}\hat{\mathbf{j}} + \frac{\partial v}{\partial z}\hat{\mathbf{k}}\right)$

$$= -\left(\frac{-2}{1}\hat{\mathbf{i}} + \frac{-2}{1}\hat{\mathbf{j}} + \frac{-2}{1}\hat{\mathbf{k}}\right)$$
$$= 2(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})\ \text{N/C}$$

If V_P is potential at P, then

$$V_P - V_0 = -\vec{\mathbf{E}}\cdot\vec{\mathbf{r}}$$
$$V_P - 10 = -2(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})\cdot(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) = -6$$
$$V_P = 4\ \text{V}$$

37. On touching two spheres, charge is equally divided among them, then due to induction a charge $\left(-\frac{q}{2}\right)$ appears on the earthed sphere.

38. Negative charge will induce on the conductor near P.

39. $\begin{cases} 0 & \text{for} \quad r < r_A \\ \frac{kQ_P}{r} & \text{for} \quad r_P < r < r_B \\ \frac{k(Q_A - Q_B)}{r} & \text{for} \quad r > r_B \end{cases}$

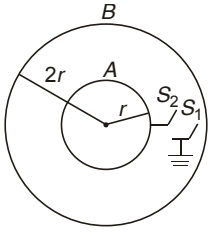
As $|Q_B| > |Q_A|$

E is –ve for $r > r_B$.

40. $\vec{\mathbf{E}} = -\left(\frac{\partial v}{\partial x}\hat{\mathbf{i}} + \frac{\partial v}{\partial y}\hat{\mathbf{j}}\right)$

$$= k(y\,\hat{\mathbf{i}} + x\,\hat{\mathbf{j}})$$
$$|\vec{\mathbf{E}}| = k\sqrt{y^2 + x^2} = kr$$

41. Let charge on outer shell becomes q.



$$V_B = \frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{Q+q}{2r}\right) = 0$$
$$q = -Q$$

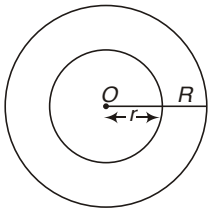
42. Let charge q' flows through the switch to the ground, then

$$\frac{1}{4\pi\varepsilon_0}\cdot\left[\frac{Q-q'}{r} - \frac{Q}{2r}\right] = 0$$
$$q' = \frac{1}{2}Q$$

43. After n steps

$$q' = \frac{1}{2^n}Q \text{ and } q = \frac{-1}{2^{n-1}}Q$$
$$\therefore \quad V_A = \frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{q'}{r} + \frac{q}{2r}\right) = 0$$
$$V_B = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q'+q}{2r}$$
$$= \frac{1}{2^{n+1}}\left[\frac{Q}{4\pi\varepsilon_0 r}\right]$$

44. Consider a spherical Gaussian surface of radius $r\,(< R)$ and concentric with the sphere,



Charge on a small sphere of radius r

$$dq = \delta\, dV = 4\pi r^2 \delta\, dr$$
$$= r\,\pi\delta_0\left(r^2 - \frac{r^3}{R}\right)dr$$

Total charge inside the Gaussian surface,

$$q = 4\pi\,\delta_0\int_0^r\left(r^2 - \frac{r^3}{R}\right)dr$$
$$= 4\pi\delta_0\left[\frac{r^3}{3} - \frac{r^4}{4R}\right]$$
$$\therefore E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2} = \frac{\delta_0}{\varepsilon_0}\left[\frac{r}{3} - \frac{r^2}{4R}\right]$$

45. Total charge inside the surface.

$$Q = 4\pi\delta_0\left[\frac{R^3}{3} - \frac{R^3}{r}\right] = \frac{1}{3}\pi\delta_0 R^3$$
$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2} = \frac{\delta_0 R^3}{12\varepsilon_0 r^2}$$

46. $E = \frac{\delta_0}{\varepsilon_0}\left[\frac{r}{3} - \frac{r^2}{4R}\right]$

For maximum intensity of electric field

$$\frac{dE}{dr} = \frac{\delta_0}{\varepsilon_0}\left[\frac{1}{3} - \frac{r}{2R}\right] = 0$$

$$\Rightarrow \qquad r = \frac{2}{3}R$$

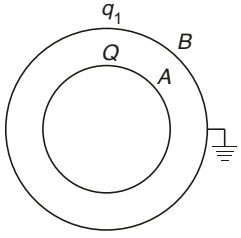
$$\frac{d^2E}{dr^2} = -\frac{\delta_0}{2R\varepsilon_0} = -\text{ve},$$

hence E is maximum at $r = \frac{2}{3}R$.

47. $$E_{max} = \frac{\rho_0}{\varepsilon_0}\left[\frac{\frac{2\lambda}{3}}{3} - \frac{\left(\frac{2R}{3}\right)^2}{4R}\right] = \frac{\rho_0 R}{q\varepsilon_0}$$

48. Potential difference between two concentric spheres do not depend on the charge on outer sphere.

49. When outer sphere B is earthed



$$V_B = \frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{Q + q_1}{b}\right) = 0$$

$$q_1 = -Q$$

Now, if A is earthed

$$q_A = \frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{q_2}{a} + \frac{q_1}{b}\right) = 0$$

$$q_2 = -\frac{a}{b}q_1 = \frac{a}{b}Q$$

50. When connected by conducting wires, entire charge from inner sphere flows to the outer sphere, *ie*,

$$q_3 = q_1 + q_2\left(\frac{a}{b} - 1\right)Q$$

$$= \frac{a-b}{b}Q$$

More than One Correct Options

1. Before earthing the surface B,

$$V_A = \frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{q_A}{R} + \frac{q_B}{2R}\right) = 2\text{ V}$$

$$V_B = \frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{q_A + q_B}{2R}\right) = \frac{3}{2}\text{ V}$$

$$\Rightarrow \qquad \frac{q_A}{q_B} = \frac{1}{2}$$

On earthing the sphere B,

$$q_A' = q_A$$

$$V_B = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_A' + q_B'}{2R} = 0$$

$$q_B' = -q_A'$$

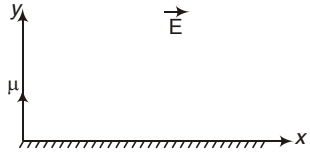
$$\Rightarrow \qquad \frac{q_A'}{q_B'} = -1$$

As potential difference does not depend on charge on outer sphere,

$$V_A' - V_B' = V_A - V_B = \frac{V}{2}$$

$$V_A' = \frac{1}{2}V$$

2. For the motion of particle



$u_x = 0, v_x = v, a_x = \frac{qE}{m}, a_y = -g,$

$x_0 = 0, y_0 = 0$

$$x = x_0 + u_x t + \frac{1}{2}a_x t^2$$

$$x = \frac{qE}{2m}t^2 \qquad \text{...(i)}$$

$$y = y_0 + u_y t + \frac{1}{2}a_y t^2$$

$$= ut - \frac{1}{2}gt^2 \qquad \text{...(ii)}$$

At the end of motion

$$t = T,\ y = 0,\ x = R$$

From Eq. (ii),

$$0 = \left(u - \frac{1}{2}gT\right)T$$

$$T = \frac{2u}{g} = \frac{2 \times 10}{10} = 2 \text{ s}$$

From Eq. (i),

$$R = \frac{qE}{2m}T^2$$

$$= \frac{1 \times 10^{-3} \times 10^4}{2 \times g} \times 4 = 10 \text{ m}$$

Now, $v_y^2 - u_y^2 = 2a_y(y - y_0)$

At highest point (*i.e.*, $y = H$), $v_y = 0$

$$0 - (10)^2 = -2 \times 10(H - 0)$$

$$H = 5 \text{ m}$$

3. Let R be the radius of the sphere

$$V_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R + r_1}$$

$$\Rightarrow \quad \frac{q \times 10^9 \times q}{(R + S) \times 10^{-2}} = 100 \quad \text{...(i)}$$

$$V_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R + r_2}$$

$$\Rightarrow \quad \frac{9 \times 10^9 \times q}{(R + 10) \times 10^{-2}} = 75 \quad \text{...(ii)}$$

On solving,

$R = 10$ cm,

$$q = \frac{5}{3} \times 10^{-9} \text{ C} = \frac{50}{3} \times 10^{-10} \text{ C}$$

Electric field on surface,

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^2} = \frac{9 \times 10^9 \times \frac{5}{3} \times 10^{-9}}{(10 \times 10^{-2})}$$

$$= 1500 \text{ V/m}$$

Potential at surface,

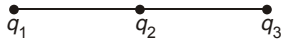
$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R} \quad \frac{9 \times 10^9 \times \frac{5}{3} \times 10^{-9}}{10 \times 10^{-2}}$$

$$= 150 \text{ V}$$

Potential at Centre

$$V_C = \frac{3}{2}V_S = 225 \text{ V}$$

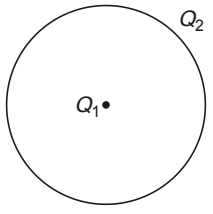
4. For all charges to be in equilibrium, force experienced by either charge must be zero *ie*., force due to other two charges must be equal and opposite.



Hence all the charges must be collinear, charges q_1, and q_3 must have same sign and q_2 must have opposite sign, q_2 must have maximum magnitude.

Such on equilibrium is always unstable.

5. Flux through any closed surface depends only on charge inside the surface but electric field at any point on the surface depends on charges inside as well as outside the surface.



6. As net charge on an electric dipole is zero, net flux through the sphere is zero.

But electric field at any point due to a dipole cannot be zero.

7. Gauss's law gives total electric field and flux due to all charges.

8. If two concentric spheres carry equal and opposite charges, Electric field is **non-zero** only in the region between two sphere and potential is is *zero* only outside both the spheres.

9. As force on the rod due to electric field is towards right, force on the rod due to hinge must be left.

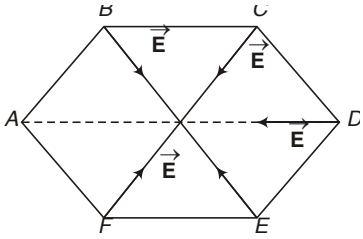
The equilibrium is clearly neutral.

10. If moved along perpendicular bisector, for all identical charges, electrostatic potential energy is maximum at mid point and if moved along the line joining the particles, electrostatic potential energy is minimum at the mid-point.

Match the Columns

1. (a → s), (b → q), (c → r), (d → p).

If charge at B is removed



$$E_{\text{net}} = E_D \cos 30° + E_E \cos 30°$$
$$= \sqrt{3}E$$

If charge at C is removed

$$E_{\text{net}} = E_D \cos 60° + E_f \cos 60°$$
$$= E$$

If charge at D is removed

$$\vec{\mathbf{E}}_{\text{net}} = 0 \text{ and } \vec{\mathbf{E}}_B = -\vec{\mathbf{E}}_E$$

and $$\vec{\mathbf{E}}_E = -\vec{\mathbf{E}}_E$$

If charge at B and C both are removed,

$$E_{\text{net}} = E_E + E_D \cos 60° + E_F \cos 60°$$
$$= 2E$$

2. (a → q), (b → p), (c → s), (d → r).

$$V = -\vec{\mathbf{E}} \cdot \vec{\mathbf{r}}$$

If $\vec{\mathbf{r}} = 4\hat{\mathbf{i}}$, $V = -8\,\text{V}$,

If $\vec{\mathbf{r}} = -4\hat{\mathbf{i}}$, $V = 8\,\text{V}$

If $\vec{\mathbf{r}} = 4\hat{\mathbf{j}}$, $V = -16\,\text{V}$,

If $\vec{\mathbf{r}} = -4\hat{\mathbf{j}}$, $V = 16\,\text{V}$

3. For a solid sphere

$$V_{\text{in}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{2R^3}(3R^2 - r^2)$$

at $$r = \frac{R}{2}$$

$$V_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{2R^3}\left(3R^2 - \frac{R^2}{4}\right)$$
$$= \frac{11}{8}V$$

$$V_{\text{out}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$

at $r = 2R$

$$V_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{2R} = \frac{V}{2}$$

$$\text{E}_{\text{in}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qr}{R^3}$$

at $r = \frac{R}{2}$

$$E_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{2R^2} = \frac{V}{2R} = \frac{V}{2} \qquad (\because R = 1\,\text{m})$$

$$\text{E}_{\text{out}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

at $r = 2R$

$$E_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(2R)^2} = \frac{V}{4R} = \frac{V}{4}$$

$\therefore (a \to s), (b \to q), (c \to q), (d \to p)$.

4. $(a \to r), (b \to q), (c \to q), (d \to s)$

5. $(a \to p), (b \to q), (c \to r), (d \to s)$

For a spherical shell,

$$E = \begin{cases} 0 & \text{for } r < R \\ \dfrac{Kq}{r^2} & \text{for } r \geq R \end{cases}$$

$$V = \begin{cases} \dfrac{Kq}{R} & \text{for } r \leq R \\ \dfrac{Kq}{r} & \text{for } r \geq R \end{cases}$$

For a solid sphere,

$$E = \begin{cases} \dfrac{Kqr}{R^3} & \text{for } r \leq R \\ \dfrac{Kq}{r^2} & \text{for } r \geq R \end{cases}$$

$$V = \begin{cases} \dfrac{Kq}{2R^2}(2R^2 - r^2) & \text{for } r \leq R \\ \dfrac{Kq}{r^2} & \text{for } r \geq R \end{cases}$$

22 Capacitors

Introductory Exercise 22.1

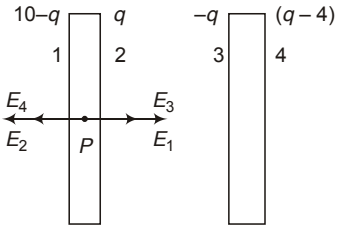
1. $C = \dfrac{q}{V} = [C] = \dfrac{[\mathrm{AT}]}{[\mathrm{ML^2T^{-3}A^{-1}}]}$

$= [\mathrm{M^{-1}L^{-2}T^4A^2}]$

2. False.

Charge will flow if there is potential difference between the conductors. It does not depend on amount of charge present.

3. Consider the charge distribution shown in figure.



Electric field at point P

$$E_P = E_1 + E_3 - E_2 - E_4$$

$$= \frac{10-q}{2\varepsilon_0 A} - \frac{q}{2\varepsilon_0 A} + \frac{q}{2\varepsilon_0 A} - \frac{q-4}{2\varepsilon_0 A}$$

But P lies inside conductor

$\therefore E_P = 0$

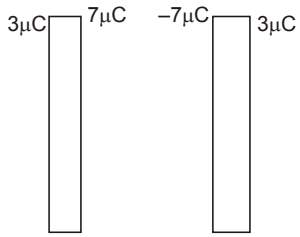
$\Rightarrow \quad 10 - q - q + q - q + 4 = 0$

$\Rightarrow \quad q = 7\ \mu\mathrm{C}$

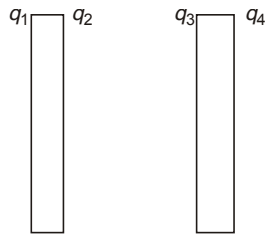
Hence, the charge distribution is shown in figure.

Sort-cut Method

Entire charge resides on outer surface of conductor and will be divided equally on two outer surfaces.



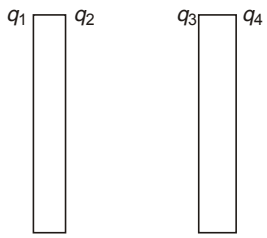
Hence, if q_1 and q_2 be charge on two plates then.



$$q_1 = q_4 = \frac{q_1 + q_2}{2} = 3\ \mu\mathrm{C}$$

$$q_2 = \frac{q_1 - q_2}{2} = 7\ \mu\mathrm{C}$$

$$q_3 = \frac{q_2 - q_1}{2} = -7\ \mu\mathrm{C}$$



4. Charge distribution is shown in figure.

$$q_1 = q_4 = \frac{q_1 + q_2}{2} = -\frac{q}{2}$$

$$q_2 = \frac{q_1 - q_2}{2} = \frac{5q}{2}$$

$$q_3 = \frac{q_2 - q_1}{2} = -\frac{5q}{2}$$

$\therefore$ Charge on capacitor = Charge on inner side of positive plate.

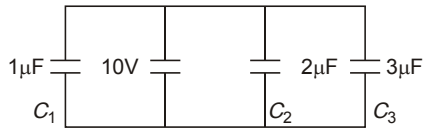
$$q = \frac{5q}{2}$$

and $$C = \frac{\varepsilon_0 A}{d}$$

$\therefore$ $$V = \frac{q}{C} = \frac{5q\,d}{2\varepsilon_0 A}$$

Introductory Exercise 22.2

1. All the capacitors are in parallel

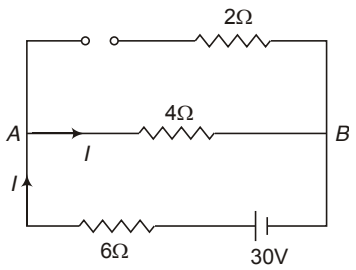


$$q_1 = C_1V = 1 \times 10 = 10\ \mu\text{C}$$
$$q_2 = C_2V = 2 \times 10 = 20\ \mu\text{C}$$
$$q_3 = C_3V = 3 \times 10 = 30\ \mu\text{C}$$

2. Potential difference across the plates of capacitor

$$V = 10\ \text{V}$$
$$q = CV = 4 \times 10 = 40\ \mu\text{C}$$

3. In the steady state capacitor behaves as open circuit.



$$I = \frac{30}{6+4} = 3\ \text{A}$$

Potential difference across the capacitor,

$$V_{AB} = 4 \times I = 4 \times 3 = 12\ \text{V}$$

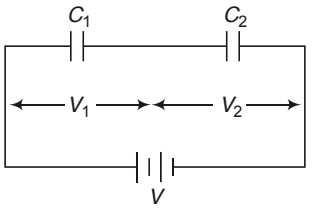
$\therefore$ Charge on capacitor

$$q = CV_{AB} = 2 \times 12 = 24\ \mu\text{C}$$

4. (a) $\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{1} + \frac{1}{2}$

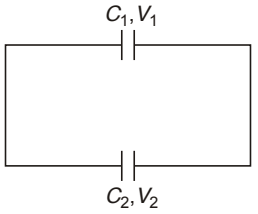
$$C_e = \frac{2}{3}\ \mu\text{C}$$

$$q = C_eV = \frac{2}{3} \times 1200 = 800\ \mu\text{C}$$



(b) $V_1 = \frac{q}{C_1} = \frac{800}{1} = 800\ \text{V}$

$$V_2 = \frac{q}{C_2} = \frac{800}{2} = 400\ \text{V}$$



Now, if they are connected in parallel,

Common potential, $\left(V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right)$

$$= \frac{1 \times 800 \times 2 \times 400}{1+2} = \frac{1600}{3}\ \text{V}$$

$$q_1 = C_1V = \frac{1600}{3}\ \mu\text{C},\ q_2 = C_2V = \frac{3200}{3}\ \mu\text{C}$$

5. Common potential

$$V = \frac{C_1V_2 + C_2V_2}{C_1 + C_2}$$

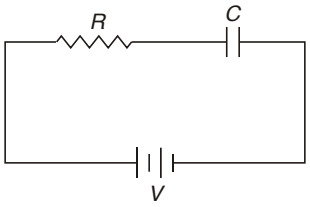
But $V = 20,\ V_2 = 0,\ V_1 = 100\ \text{V},\ C_1 = 100\ \mu\text{C}$

$\therefore$ $$\frac{100 \times 100 + C_2 \times 0}{400 + C_2} = 20$$

$\Rightarrow$ $$C_2 = 400\ \mu\text{C}$$

Introductory Exercise 22.3

1. Let q be the final charge on the capacitor, work done by battery



$$W = qV$$

Energy stored in the capacitor

$$U = \frac{1}{2}qV$$

$\therefore$ Energy dissipated as heat

$$H = U - W = \frac{1}{2}qV = U$$

2. We have

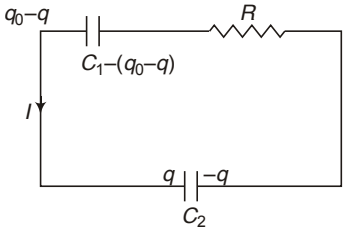
$$I = I_0 e^{-t/\tau}$$

$$\frac{I_0}{2} = I_0 e^{-t/\tau} \Rightarrow e^{-t/\tau} = \frac{1}{2}$$

$$t = \tau \ln 2 = 0.693\ \tau$$

$t = 0.693$ time constant.

3. Let capacitor C_1 is initially charged and C_2 is uncharged.



At any instant, let charge on C_2 be q, charge on C_1 at that instant $= q_0 - q$

By Kirchhoff's voltage law,

$$\frac{(q_0 - q)}{C} - IR - \frac{q}{C} = 0$$

$$\Rightarrow \quad \frac{dq}{dt} = \frac{q_0 - 2q}{RC}$$

$$\Rightarrow \quad \int_0^q \frac{dq}{q_0 - 2q} = \int_0^t \frac{dt}{RC}$$

$$\Rightarrow \quad \frac{[\ln(q_0 - 2q)]_0^q}{-2} = \frac{1}{RC}[t]_0^t$$

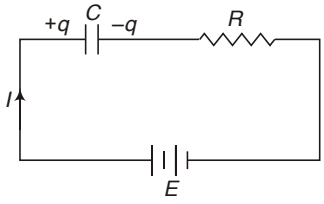
$$\Rightarrow \quad q = \frac{q_0}{2}(1 - e^{-t/\tau})$$

$\therefore$ At time t,

Charge on $C_1 = q = \frac{q_0}{2}(1 - e^{-t/\tau})$

Charge on $C_2 = q_0 - q = \frac{q_0}{2}(1 + e^{-t/\tau})$

4. Let q be the charge on capacitor at any instant t



By Kirchhoff's voltage law

$$\frac{q}{C} + IR = E$$

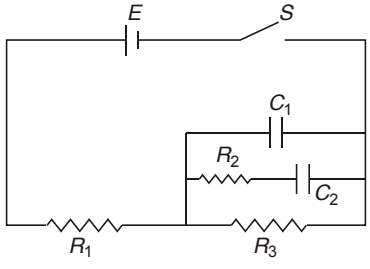
$$\frac{dq}{dt} = \frac{CE - q}{RC}$$

$$\int_{q_0}^{q} \frac{dq}{CE - q} = \int_0^t \frac{dt}{RC}$$

$$\Rightarrow \quad q_0 = CE(1 - e^{-t/\tau}) + q_0 e^{-t/\tau}$$

where, $\tau = RC$

5. (a) When the switch is just closed, Capacitors behave like short circuit.



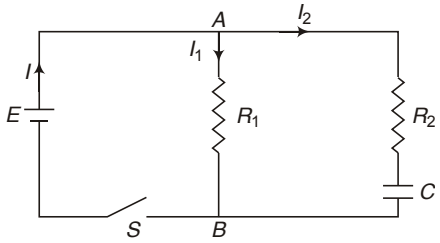
$\therefore$ Initial current

$$I_i = \frac{E}{R_1}$$

(b) After a long time, *i.e.*, in steady state, both the capacitors behaves open circuit,

$$I_f = \frac{E}{R_1 + R_3}$$

6. (a) Immediately after closing the switch, capacitor behaves as short circuit,



$\therefore \quad I_1 = \dfrac{E}{R_1}$ and $I_2 = \dfrac{E}{R_2}$

(b) In the steady state, capacitor behaves as open circuit,

$\therefore \quad I_1 = \dfrac{E}{R_1}, I_2 = 0$

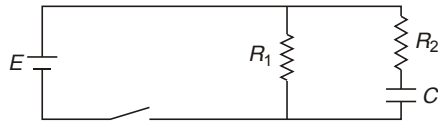
(c) Potential difference across the capacitors in the steady state,

$$V = E$$

$\therefore$ Energy stored in the capacitor

$$U = \frac{1}{2}CE^2$$

(d) After the switch is open



$$R_e = R_1 + R_2$$
$$\tau = R_3C = (R_1 + R_2)C$$

AIEEE Corner

Subjective Questions (Level-1)

1. $C = \dfrac{\varepsilon_0 A}{d} \Rightarrow A = \dfrac{Cd}{\varepsilon_0} = \dfrac{1 \times 1 \times 10^{-3}}{8.85 \times 10^{-12}}$

$= 1.13 \times 10^8 \text{ m}^2$

2. $C_1 = \dfrac{\varepsilon_0 A_1}{d}$ and $C_2 = \dfrac{\varepsilon_0 A_2}{d}$

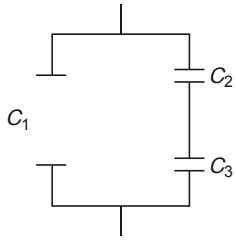
If connected in parallel

$$C = C_1 + C_2 = \frac{\varepsilon_0 A_1}{d} + \frac{\varepsilon_2 A_2}{d}$$
$$= \frac{\varepsilon_0 (A_1 + A_2)}{d} = \frac{\varepsilon_0 A}{d}$$

where, $A = A_1 + A_2 =$ effective area.

Hence proved.

3. The arrangement can be considered as the combination of three different capacitors as shown in figure, where



$$C_1 = \frac{k_1\varepsilon_0 \dfrac{A}{2}}{2d} = \frac{k_1\varepsilon_0 A}{4d}$$

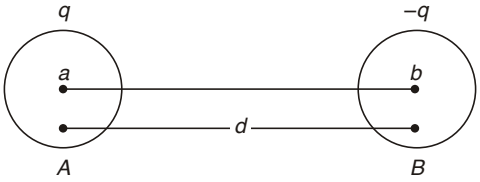
$$C_2 = \frac{k_2\varepsilon_0 \dfrac{A}{2}}{2d/2} = \frac{k_2\varepsilon_0 A}{2d}$$

$$C_3 = \frac{k_3\varepsilon_0 \dfrac{A}{2}}{2d/2} = \frac{k_3\varepsilon_0 A}{2d}$$

Therefore, the effective capacitance,

$$C = C_1 + \frac{C_2C_3}{C_2 + C_3}$$
$$= \frac{\varepsilon_0 A}{2d}\left(\frac{k_1}{2} + \frac{k_1k_3}{k_2 + k_3}\right)$$

4. (a) Let the spheres A and B carry charges q and $-q$ respectively,



$$\therefore \quad V_A = \frac{1}{4\pi\varepsilon_0}\cdot\left[\frac{q}{a} - \frac{q}{d}\right]$$
$$V_B = \frac{1}{4\pi\varepsilon_0}\cdot\left[-\frac{q}{b} + \frac{q}{d}\right]$$

Potential difference between the spheres,

$$V = V_A - V_B = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{a} + \frac{1}{b} - \frac{2}{d}\right]$$
$$C = \frac{q}{V} = \frac{4\pi\varepsilon_0}{\dfrac{1}{a} + \dfrac{1}{b} - \dfrac{2}{d}}$$

Hence proved.

(b) If $d \to \infty$

$$C = \frac{4\pi\varepsilon_0}{\frac{1}{a} + \frac{1}{b}} = \frac{4\pi\varepsilon_0 \cdot ab}{a + b}$$

If two isolated spheres of radii a and b are connected in series,

then,

$$C' = \frac{C_1 C_2}{C_1 + C_2}$$

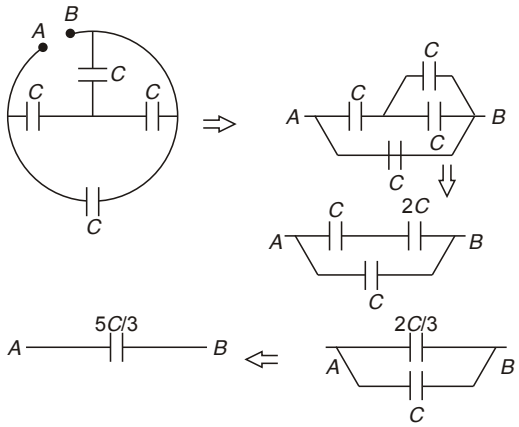
where, $C_1 = 4\pi\varepsilon_0 a$, $C_2 = 4\pi\varepsilon_0 b$

$$\therefore \quad C' = \frac{4\pi\varepsilon_0 \cdot ab}{a + b}$$

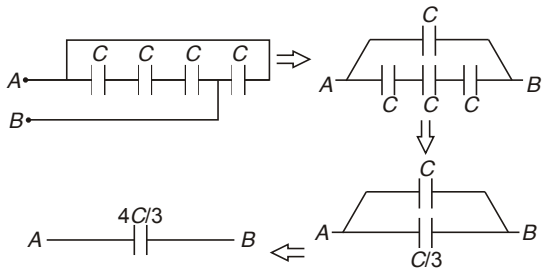
$$\therefore \quad C' = C$$

Hence proved.

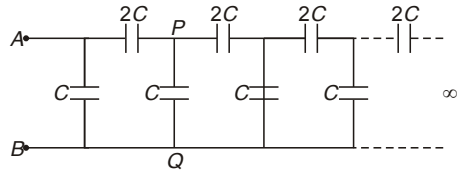
5. (a)



(b)



(c)



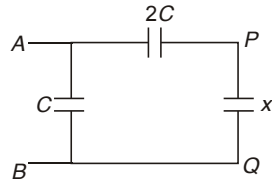
Let effective capacitance between A and B

$$C_{AB} = x$$

As the network is infinite,

$$C_{PQ} = C_{AB} = x$$

Equivalent circuit is shown in figure,



$$R_{AB} = C + \frac{2Cx}{2C + x} = x$$

$$\Rightarrow \quad 2C^2 + Cx + 2Cx = 2Cx + x^2$$

$$\Rightarrow \quad x^2 - Cx - 2C^2 = 0$$

On solving, $x = 2C$ or $-C$

But x cannot be negative,

Hence, $x = 2C$

6. $q = CV = 7.28 \times 25 = 182\ \mu\text{C}$

7. (a) $V = \frac{q}{C} = \frac{0.148 \times 10^{-6}}{245 \times 10^{-12}} = 604\ \text{V}$

(b) $C = \frac{\varepsilon_0 A}{d} \Rightarrow A = \frac{Cd}{\varepsilon_0}$

$$= \frac{245 \times 10^{-12} \times 0.328 \times 10^{-3}}{8.85 \times 10^{-12}}$$

$$= 9.08 \times 10^{-3}\ \text{m}^2$$

$$= 90.8\ \text{cm}^2$$

(c) $\sigma = \frac{q}{A} = \frac{0.148 \times 10^{-6}}{9.08 \times 10^{-3}} = 16.3\ \mu\text{C/m}^2$

8. (a) $E_0 = 3.20 \times 10^5$ V/m

$E = 2.50 \times 10^5$ V/m

$$k = \frac{E_0}{E} = \frac{3.20 \times 10^5}{2.50 \times 10^5} = 1.28$$

(b) Electric field between the plates of capacitor is given by

$$E = \frac{\sigma}{\varepsilon_0}$$

$$\Rightarrow \quad \sigma = \varepsilon_0 E = 8.8 \times 10^{-12} \times 3.20 \times 10^5$$

$$= 2.832 \times 10^{-6}\ \text{C/m}^2$$

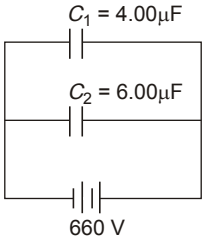
$$= 2.832\ \mu\text{C/m}^2$$

9. (a) $q_1 = C_1 V = 4 \times 660 = 2640\ \mu\text{C}$

$q_2 = C_2 V = 6 \times 660 = 3960\ \mu\text{C}$

As C_1 and C_2 are connected in parallel,

$$V_1 = V_2 = V = 660 \text{ V}$$



(b) When unlike plates of capacitors are connected to each other,

Common potential

$$V = \frac{C_2V_2 - C_1V_1}{C_1 + C_2} = \frac{6 \times 660 - 4 \times 660}{6}$$

$$= 220 \text{ V}$$

$$q_1 = C_1V = 4 \times 220 = 880 \ \mu\text{C}$$

$$q_2 = C_2V = 6 \times 220 = 1320 \ \mu\text{C}$$

10. $E = \frac{V}{d} = \frac{400}{5 \times 10^{-3}} = 8 \times 10^4 \text{ V/m}$

Energy density,

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2} \times 8.85 \times 10^{-12} \times (8 \times 10^{-4})^2$$

$$= 2.03 \times 10^{-2} \text{ J/m}^3$$

$$= 20.3 \text{ mJ/m}^3$$

11. Dielectric strength = maximum possible electric field

$$E = \frac{V}{d} \Rightarrow d = \frac{V}{E}$$

$$= \frac{5500}{1.6 \times 10^7} = 3.4 \times 10^{-4} \text{ m}$$

$$C = \frac{k\varepsilon_0 A}{d} \Rightarrow A = \frac{Cd}{k\varepsilon_0}$$

$$= \frac{1.25 \times 10^{-9} \times 3.4 \times 10^{-4}}{3.6 \times 8.85 \times 10^{-12}}$$

$$= 1.3 \times 10^{-2} \text{ m}^2$$

$$= 0.013 \text{ m}^2$$

12. Let C_P and C_S be the effective capacitance of parallel and series combination respectively.

For parallel combination,

$$U_P = 0.19 \text{ J}$$

$$U_P = \frac{1}{2}C_PV^2$$

$$\Rightarrow \quad C_P = \frac{2U_P}{V^2} = \frac{2 \times 0.1}{(2)^2} = 0.05 \text{ F}$$

$$= 50 \text{ mF}$$

For series combination,

$$U_S = 1.6 \times 10^{-2} \text{ J} = 0.016 \text{ J}$$

$$U_S = \frac{1}{2}C_SV^2$$

$$\Rightarrow \quad C_S = \frac{2U_S}{V^2} = \frac{2 \times 0.016}{(2)^2} = 0.008 \text{ F}$$

$$= 8 \text{ mF}$$

Now, $C_P = C_1 + C_2 = 5$ mF

or $\quad C_2 = (5 - C_1)$ mF

and $\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{8}$

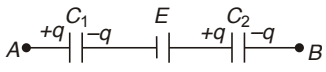
$$\Rightarrow \quad \frac{1}{C_1} + \frac{1}{S - C_1} = \frac{1}{8}$$

On solving,

$C_1 = 40$ mF, $C_2 = 10$ mF or *vice-versa*.

13. In the given circuit,

$$V_A - V_B = \frac{q}{C_1} - E + \frac{q}{C_2} = 5$$



$$\Rightarrow \quad \frac{q}{10^{-6}} - 10 + \frac{q}{2 \times 10^{-6}} = 5$$

$$\Rightarrow \quad q = 10 \times 10^{-6} \text{ C} = 10 \ \mu\text{C}$$

$$\therefore \quad V_1 = \frac{q}{C_1} = 10 \text{ V}, \ V_2 = \frac{q}{C_2} = 5 \text{ V}$$

14. (a) In order to increase voltage range n times, n-capacitors must be connected in series.

Hence, to increase voltage range to 500V, 5 capacitors must be connected in series.

Now, effective capacitance of series combination,

$$C_S = C_n = \frac{10}{5} = 2 \text{ pF}$$

Hence, no parallel grouping of such units is required.

Hence, a series grouping of 5 such capacitors will have effective capacitance 2 pF and can withstand 500 V.

(b) If n capacitors are connected in series and m such units are connected in parallel,

$$V_e = nV$$

$$C_e = \frac{mC}{n}$$

Here, $V = 100$ V

$$V_e = 300 \text{ V}$$

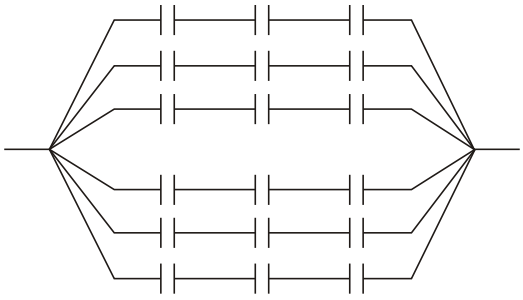
$$\therefore \quad n = \frac{V_e}{V} = 3$$

$C = 10$ pF

$C_e = 20$ pF

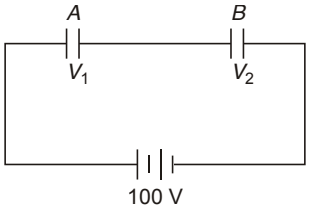
$$m = \frac{nC_e}{C} = \frac{3 \times 20}{10} = 6$$

Hence, the required arrangement is shown in figure.



15. Case I.

$$V_1 = \frac{C_2}{C_1 + C_2} V = 60 \text{ V}$$



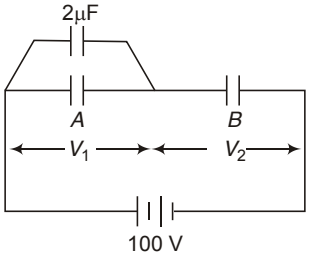
$$V_2 = \frac{C_1}{C_1 + C_2} V = 40 \text{ V}$$

$$\Rightarrow \quad \frac{C_1}{C_2} = \frac{2}{3}$$

$$\Rightarrow \quad C_2 = \frac{3}{2} C_1$$

Case II.

$$V_1 = \frac{C_2}{C_1 + C_2 + 2} = 10 \text{ V}$$



$$V_2 = \frac{C_1 + 2}{C_1 + C_2 + C} = 90 \text{ V}$$

$$\Rightarrow \quad \frac{C_1 + 2}{C_2} = \frac{90}{10} = 9$$

$$\Rightarrow \quad C_1 + 2 = 9C_2$$

$$\Rightarrow \quad C_1 + 2 = 9 \times \frac{3}{2} C_1$$

$$\Rightarrow \quad \frac{25}{2} C_1 = 2 \Rightarrow C_1 = \frac{4}{25} \mu\text{F}$$

$$= 0.16 \mu\text{F}$$

$$C_2 = \frac{3}{2} C_1 = 0.24 \mu\text{F}$$

16. (a) $q = CV = 10 \times 12 = 120 \mu\text{C}$

(b) $C = \dfrac{\varepsilon_0 A}{d}$

If separation is doubled, capacitance will become half. *i.e.*,

$$C' = \frac{C}{2}$$

$$q' = E'V = \frac{C}{2} V = 60 \mu\text{C}$$

(c) $C = \dfrac{\varepsilon_0 A}{d} = \dfrac{\pi \varepsilon_0 r^2}{d}$

If r is doubled, C will become four times, *i.e.*,

$$C' = 4C$$

$$q' = C'V = 480 \mu\text{C}$$

17. Heat produced = Energy stored in the capacitor

$$H = \frac{1}{2} CV^2 = \frac{1}{2} \times 450 \times 10^{-6} \times (295)^2$$

$$= 19.58 \text{ J}$$

18. (a) $C = \dfrac{\varepsilon_0 A}{d} = \dfrac{8.85 \times 10^{-12} \times 2}{5 \times 10^{-3}}$

$$= 3.54 \times 10^{-6} \text{ F}$$

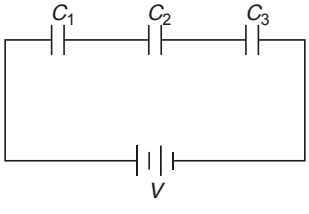
$$= 3.54 \mu\text{F}$$

(b) $q = CV = 3.54 \times 10^{-9} \times 10000$

$$= 35.4 \times 10^{-6} = 35.4 \mu\text{C}$$

(c) $E = \dfrac{V}{d} = \dfrac{10000}{5 \times 10^{-3}} = 2 \times 10^6$ V/m

19. Given,



$C_1 = 8.4 \mu\text{F}$, $C_2 = 8.2 \mu\text{F}$

$C_3 = 4.2 \mu\text{F}$, $V = 36$ V

(a) Effective capacitance,

$$\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$= \frac{1}{8.4} + \frac{1}{8.2} + \frac{1}{4.2} \Rightarrow C_e = 2.09\ \mu\text{F}$$

$$q = C_e V = 2.09 \times 36 = 75.2\ \mu\text{C}$$

As combination is series, charge on each capacitor is same, *i.e.*, 75.2 μC.

(b) $U = \frac{1}{2} qV = \frac{1}{2} \times 75.2 \times 36 \times 10^{-6}$

$$= 1.35 \times 10^{-3}\ \text{J} = 1.35\ \text{mJ}$$

(c) Common potential,

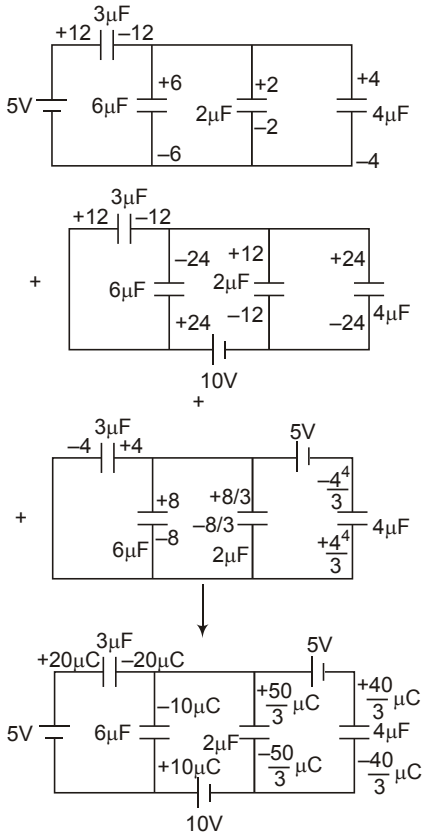
$$V = \frac{C_1V_1 + C_2V_2 + C_3V_3}{C_1 + C_2 + C_3} = 10.85\ \text{V}$$

(d) $U' = \frac{1}{2}(C_1 + C_2 + C_3)V^2$

$$= \frac{1}{2} \times (8.4 + 8.2 + 4.2) \times (10.85)^2 \times 10^{-6}$$

$$= 1.22 \times 10^{-3}\ \text{J} = 1.22\ \text{mJ}$$

20. The Given circuit can be considered as the sum of three circuits as shown

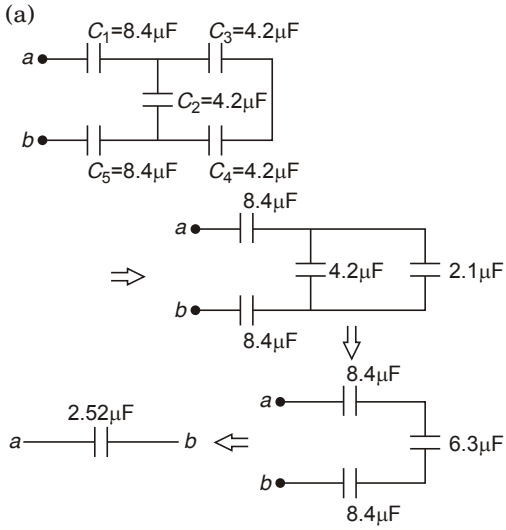


(Charge is shown in μC).

Hence, charge on 6 μF capacitor = 10 μC

and Charge on 4 μF capacitor = $\frac{40}{3}$ μC

21. (a)



(b) Charge supplied by the source of emf

$$q = CV = 2.52 \times 10^{-6} \times 220$$

$$= 554.4\ \mu\text{C}$$

$$q_1 = q_5 = q = 554.4\ \mu\text{C}$$

$$q_2 = \frac{4.2}{4.2 + 2.1} q$$

$$= \frac{4.2}{6.3} \times 554.4\ \mu\text{C} = 369.6\ \mu\text{C}$$

and $q_3 = q_4 = \frac{2.1}{4.2 + 2.1} q = \frac{2.1}{6.3} \times 554.4\ \mu\text{C}$

$$= 184.8\ \mu\text{C}$$

$$V_1 = \frac{q_1}{C_1} = \frac{554.4}{8.4} = 66\ \text{V} = V_5$$

$$V_2 = \frac{q_2}{C_2} = \frac{369.6}{4.2} = 88\ \text{V}$$

$$V_3 = V_4 = \frac{q_3}{C_3} = \frac{184.8}{4.2} = 44\ \text{V}$$

22. Let C_1 and C_2 be the capacitances of A and B respectively.

$$\therefore \quad C_1 = \frac{k_1 \varepsilon_0 A_1}{d_1}, C_2 = \frac{k_1 \varepsilon_0 A_2}{d_2}$$

Now, $\quad V_1 = \frac{C_2}{C_1 + C_2} V$

$$\Rightarrow \quad \frac{C_2}{C_1 + C_2} = \frac{130}{230} = \frac{13}{23} \qquad \text{...(i)}$$

$$V_2 = \frac{C_1}{C_1 + C_2} V$$

$$\Rightarrow \quad \frac{C_1}{C_1 + C_2} = \frac{10}{23} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii),

$$\frac{C_1}{C_2} = \frac{10}{13}$$

If dielectric slab of C_1 is replaced by one for which $k = 5$ then,

$$C_1' = \frac{5\varepsilon_0 A_1}{d_1} = \frac{5}{2} C_1$$

$$\therefore \quad \frac{V_2'}{V_1'} = \frac{C_1'}{C_2} = \frac{5C_1}{2C_2} = \frac{50}{26}$$

$$V_2' + \frac{50}{26} V_1' = 230$$

Also,

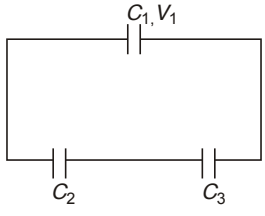
$$V_1' + V_2' = 230$$

$$V_1' = \frac{50}{26} V_1'$$

$$V_1' = 78.68 \text{ V}$$

and $\quad V_2 = 151.32 \text{ V}$

23. In this case



Common potential,

$$V = \frac{C_1 V_1}{C_1 + \dfrac{C_2 C_3}{C_2 + C_3}}$$

$$V = \frac{1 \times 110}{1 + 1.2}$$

$$= \frac{110}{2.2}$$

$$= 50 \text{ V}$$

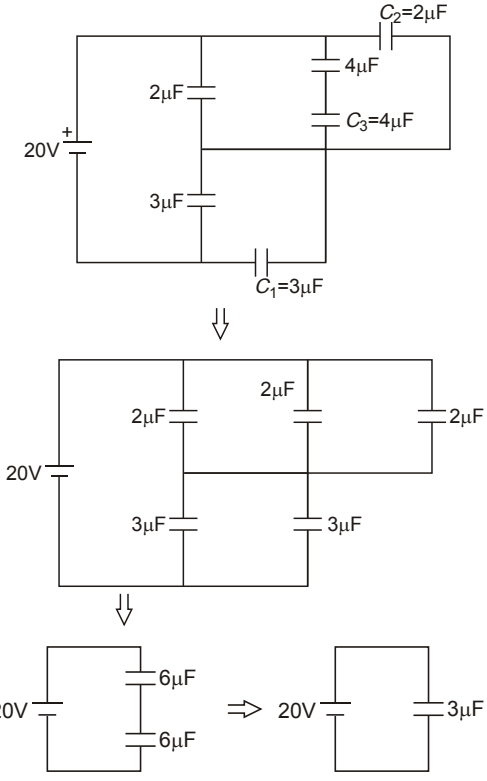
Charge flown through connecting wires,

$$= \frac{C_2 C_3}{C_2 + C_3} V$$

$$= 1.2 \times 50$$

$$= 60 \ \mu\text{C}$$

24. (a) Hence, effective capacitance across the battery is 3 μF.



(b) $q = CV = 3 \times 20 = 60 \ \mu\text{C}$

(c) Potential difference across C_1

$$V_1 = \frac{6}{6 + 6} \times 20 = 10 \text{ V}$$

$$q_1 = C_1 V_1 = 3 \times 10 = 30 \ \mu\text{C}$$

(d) Potential difference across C_2

$$V_2 = \frac{6}{6 + 6} \times 20 = 10 \text{ V}$$

$$q_2 = C_2 V_2 = 2 \times 10 = 20 \ \mu\text{C}$$

(e) Potential difference across C_3

$$V_3 = \frac{4}{4 + 4} \times V_2 = 5 \text{ V}$$

$$q_3 = C_3 V_3 = 4 \times 2 = 20 \ \mu\text{C}$$

25. (a) When switch S_2 is open, C_1 and C_3 are in series, C_2 and C_4 are in series their effective capacitances are in parallel with each other.

Hence,

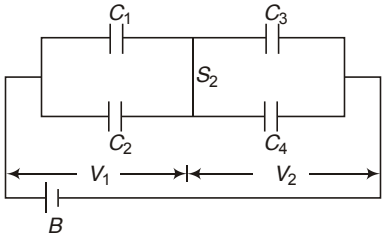
$$q_1 = q_3 = \frac{C_1 C_3}{C_1 + C_3} V$$

$$= \frac{1 \times 3}{1+3} \times 12 = 9\ \mu\text{C}$$

$$q_2 = q_4 = \frac{C_2 C_4}{C_2 + C_4}$$

$$= \frac{2 \times 4}{2+4} \times 12 = 16\ \mu\text{C}$$

(b) When S_2 is closed, C_1 is in parallel with C_2 and C_3 is in parallel with C_4.



Therefore,

$$V_1 = V_2 = \frac{C_3 + C_4}{C_1 + C_2 + C_3 + C_4}\ \text{V}$$

$$= \frac{7}{10} \times 12 = 8.4\ \text{V}$$

$$V_3 = V_4 = \frac{C_1 + C_2}{C_1 + C_2 + C_3 + C_4}\ \text{V}$$

$$= \frac{3}{10} \times 12 = 3.6\ \text{V}$$

$q_1 = C_1 V_1 = 1 \times 8.4 = 8.4\ \mu\text{C}$

$q_2 = C_2 q_2 = 2 \times 8.4 = 16.8\ \mu\text{C}$

$q_3 = C_3 V_3 = 3 \times 3.6 = 10.8\ \mu\text{C}$

$q_4 = C_4 V_4 = 4 \times 3.6 = 14.4\ \mu\text{C}$

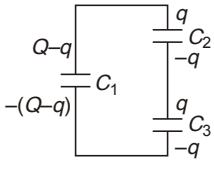
26. Initial charge on C_1

$$Q = C_1 V_0$$

Now, if switch S is thrown to right.

Let charge q flows from C_1 to C_2 and C_3.

By Kirchhoff's voltage law,



$$\frac{q}{C_2} + \frac{q}{C_3} - \frac{Q-q}{C_1} = 0$$

$$q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) = \frac{Q}{C_1}$$

$$q = \frac{C_2 C_3 Q}{C_1 C_2 + C_2 C_3 + C_3 C_1}$$

$$= \frac{C_1 C_2 C_3 V}{C_1 C_2 + C_2 C_3 + C_3 C_1}$$

$$\therefore \quad q_1 = Q - q = \frac{C_1^2 (C_2 + C_3) V}{C_1 C_2 + C_2 C_3 + C_3 C_1}$$

$$q_2 = q_3 = q = \frac{C_1 C_2 C_3 V}{C_1 C_2 + C_2 C_3 + C_3 C_1}$$

27. $C = \frac{\varepsilon_0 A}{d}$, $q = CV = \frac{\varepsilon_0 A V}{d}$

(a) $C' = \frac{\varepsilon_0 A}{2d}$, $q' = q$

(As battery is disconnected)

$$V' = \frac{q'}{C'} = 2\ \text{V}$$

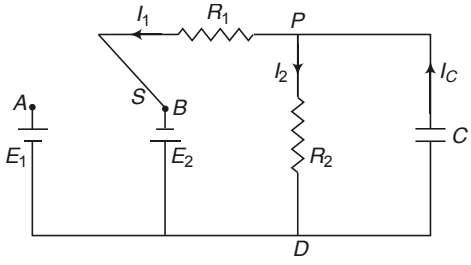
(b) $V_i = \frac{1}{2C} V^2 = \frac{\varepsilon_0 A V^2}{2d}$

$$U_f = \frac{1}{2} C' V'^2 = \frac{1}{2} \cdot \frac{\varepsilon_0 A}{2d} (2V)^2$$

$$= \frac{\varepsilon_0 A V^2}{d}$$

(c) $W = U_f - U_i = \frac{\varepsilon_0 A V^2}{2d}$

28. In the steady state, capacitor behaves as open circuit,



$$I_1 = I_2 = \frac{E_1}{R_1 + R_2} = 1\ \text{mA and } I_C = 0$$

$$V_{PD} = I_2 R_2 = \frac{E_1 R_2}{R_1 + R_2}$$

When switch is shifted to B,

At this instant,

$$V_{PD} = \frac{E_1 R_2}{R_1 + R_2}$$

$$I_2 = \frac{V_{PD}}{R_2} = \frac{E_1}{R_1 + R_2} = 1\ \text{mA}$$

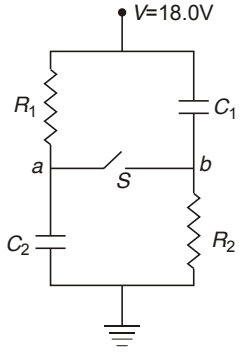
$$I_1 = \frac{E_2 + V_{PD}}{R_1} = \frac{E_2 + \dfrac{E_1 R_2}{R_1 + R_2}}{R_1}$$

$$= \frac{(R_1 + R_2)E_2 + E_1 R_2}{R_1}$$

$$= 2 \text{ mA}$$

$$I_C = I_1 + I_2 = 1 + 2 = 3 \text{ mA}$$

29. (a) When switch S is open, no current pass through the circuit,



Hence,

$$V_b - 0 = 0$$

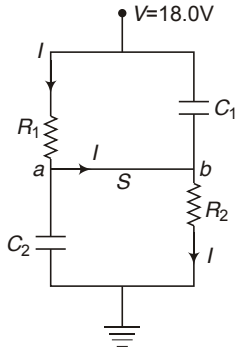
$$V_b = 0$$

$$18 - V_a = 0 \Rightarrow V_a = 18 \text{ V}$$

$$\Rightarrow \quad V_a - V_b = 18 \text{ V}$$

(b) a is at higher potential.

(c) When switch S is closed,



$$I = \frac{V}{R_1 + R_2} = 2 \text{ A}$$

$$V_b - 0 = IR_2 = 2 \times 3$$

$$V_b = 6 \text{ V}$$

(d) $q_1 = C_1 V = 6 \times 18 = 108 \ \mu\text{C}$

$$q_2 = C_2 V = 3 \times 18 = 54 \ \mu\text{C}$$

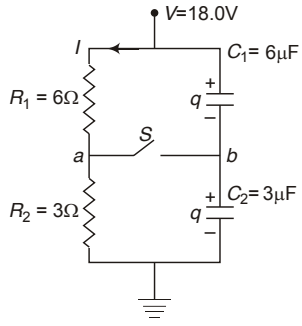
After closing the switch,

$$q_1' = C_1 V_1 = 6 \times 12 = 72 \ \mu\text{C}$$

$$q_2' = C_2 V_2 = 3 \times 6 = 18 \ \mu\text{C}$$

$\Delta q_1 = 18 \ \mu\text{C}$, $\Delta q_2 = -36 \ \mu\text{C}$

30. (a) $I = \dfrac{V}{R_1 + R_2} = \dfrac{18}{9} = 2 \text{ A}$



$$q = \frac{C_1 C_2}{C_1 + C_2} V = 2 \times 18 = 36 \ \mu\text{C}$$

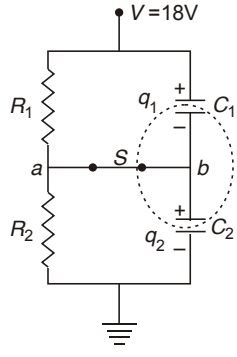
Now, $\quad V_a - 0 = IR_2 \Rightarrow V_a = 6 \text{ V}$

and $\quad V_b - 0 = \dfrac{q}{C_2} = \dfrac{36}{3} = 12 \text{ V}$

$$V_a - V_b = -6 \text{ V}$$

(b) b is at higher potential.

(c) When switch S is closed, in steady state,



$$V_a - v_b = 6 \text{ V}$$

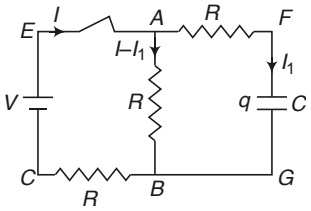
$$q_1 = C_1 V_1 = 6 \times 12 = 72 \ \mu\text{C}$$

$$q_2 = C_2 V_2 = 3 \times 6 = 18 \ \mu\text{C}$$

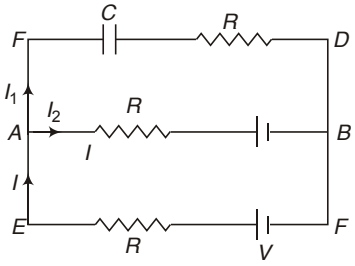
Charge flown through S

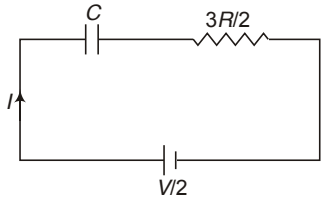
$$= q_1 - q_2 = 72 - 18 = 54 \ \mu\text{C}$$

31.



(a) Consider the circuit as combination of two cells of emf E and OV.





$$E_e = \frac{E_1R_2 + E_2R_1}{R_1 + R_2} = \frac{V}{2}$$

$$R_e = R + \frac{R}{2} = \frac{3R}{2}$$

$$q = q_0(1 - e^{-t/\tau})$$

$$q_0 = \frac{CE}{2}$$

$$\tau = \frac{3RC}{2}$$

$$\therefore \quad q = \frac{CE}{2}(1 - e^{-2t/3RC})$$

(b) $I_1 = \dfrac{dq}{dt} = \dfrac{E}{3R}e^{-2t/3RC}$

In loop $EDBA$ $\dfrac{q}{C} + I_1R - I_2R = 0$

$$I_2 = \frac{q}{RC} + I_1$$

$$= \frac{E}{2R}(1 - e^{-2t/3RC}) + \frac{E}{3R}e^{-2t/3RC}$$

$$= \frac{E}{6R}(3 - e^{-2t/3RC})$$

Objective Questions (Level 1)

1. $F = \dfrac{Q^2}{2\varepsilon_0 A}$ is independent of d.

2. $C = \dfrac{q}{V}$

On connecting the plates V becomes zero.

3. The system can assumed to a parallel combination of two spherical conductors.

$$C = C_1 + C_2 = 4\pi\varepsilon_0 a + 4\pi\varepsilon_0 b$$
$$= 4\pi\varepsilon_0(a + b)$$

4. $V = \dfrac{q}{C}$

On connecting in series

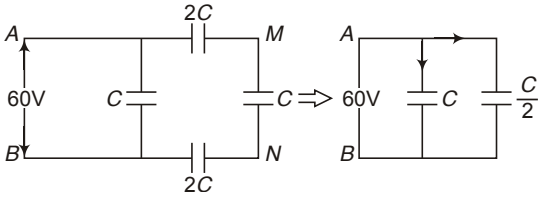
$q' = q =$ Charge on any capacitor

$$C' = \frac{C}{n}$$

$$\therefore \quad V' = \frac{nq}{C} = nV$$

5. Incorrect diagram.

6. Charge on capacitor of capacitance



$$\frac{C}{2} = \frac{C}{2}V = 30\text{ C}$$

$$V_{MN} = \frac{q}{C} = 30\text{ V}$$

7. For equilibrium,

$$qE = mg$$

$$q\frac{V}{d} = \frac{4}{3}\pi r^3\rho g$$

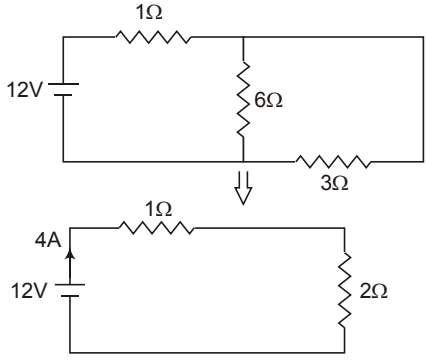
$$V \propto \frac{r^3}{q}$$

$$\frac{V_2}{V_1} = \left(\frac{r_2}{r_1}\right)^3 \times \frac{q_1}{q_2}$$

$$\Rightarrow \qquad V_2 = 4\text{ V}$$

8. Electric field between the plates is uniform but in all other regions it is zero.

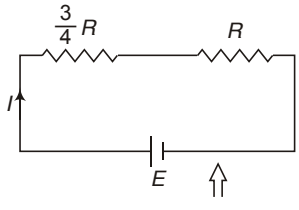
9. Initially the capacitor offers zero resistance.

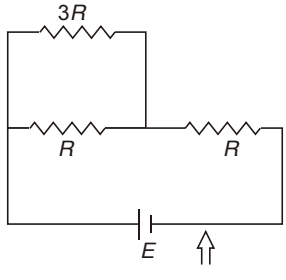


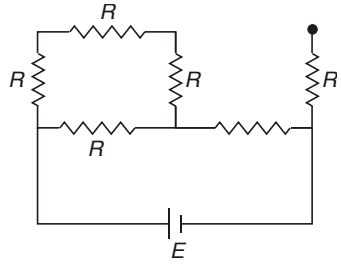
$$i = \frac{12}{1+2} = 4\text{ A}$$

10. $q = CV = CE$

11. In the steady state, capacitor behaves as open circuit. the equivalent diagram is given by







$$I = \frac{E}{R + \frac{3}{4}R} = \frac{4E}{7R}$$

But potential difference across capacitor,

$$V = IR$$

$$\Rightarrow \qquad 10 = \frac{4E}{7R}R$$

$$\Rightarrow \qquad E = 17.5\text{ V}$$

12. As all the capacitors are connected in series potential difference across each capacitor is

$$V = \frac{E}{4} = \frac{10}{4} = 2.5\text{ V}$$

$$V_A - V_N = 3\text{V} = 7.5\text{ V}$$

$$V_A = 7.5\text{ V}$$

$$V_N - V_B = 2.5\text{ V}$$

$$V_B = -2.5\text{ V}$$

13. Heat produced = Loss of energy

$$= \frac{C_1C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$$

$$= \frac{2 \times 10^{-6} \times 2 \times 10^{-6}}{2(2+2) \times 10^{-6}}(100 - 0)$$

$$= 5 \times 10^{-3}\text{ J} = 5\text{ mJ}$$

14.

$$q = q_0 e^{-t/\eta}$$

$$I = I_0 e^{-t/\eta}$$

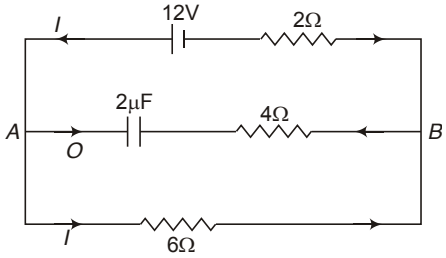
$$P = I^2R = I_0^2 e^{-2t/\eta}R = P_0 e^{-2t/\eta}$$

$$\Rightarrow \qquad \eta' = \frac{\eta}{2}$$

15. Common potential $= \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2} = \dfrac{E}{2}$

16. $V_A - V_B = 6 + 3 \times 2 - \dfrac{9}{1} + 3 \times 3 = 12\text{ V}$

17. In the steady state, current through battery



$$I = \frac{12}{6+2} = \frac{3}{2}\text{ A}$$

Potential difference across the capacitor,

$$V_{AB} = 6 \times \frac{3}{2} = 9\text{ N}$$

$\therefore \qquad q = CV_{AB} = 2 \times 9 = 18\ \mu\text{C}$

18. C_2 and C_3 are in parallel

Hence, $V_2 = V_3$

Again Kirchhoff's junction rule

$$-q_1 + q_2 + q_3 = 0$$

$\Rightarrow \qquad q_1 = q_2 + q_3$

19. For the motion of electron

$$R = \frac{mu^2 \sin 2\theta}{eE} = l \qquad \text{...(i)}$$

and $$H = \frac{mu^2 \sin^2\theta}{2eE} = d \qquad \text{...(ii)}$$

Dividing Eq. (ii) by Eq. (i),

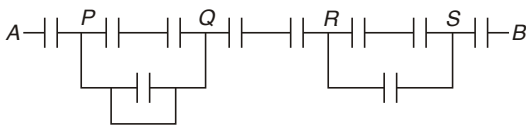
$$\tan\theta = \frac{4d}{l}$$

20. $V = Ed \Rightarrow d = \frac{V}{E} = \frac{2V\varepsilon_0}{6}$

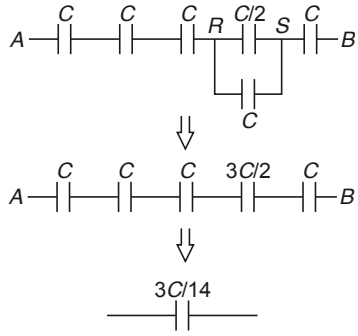
$$= \frac{2 \times 5 \times 8.85 \times 10^{-12}}{10^{-7}}$$

$$= 8.85 \times 10^{-4} = 0.88\text{ mm}$$

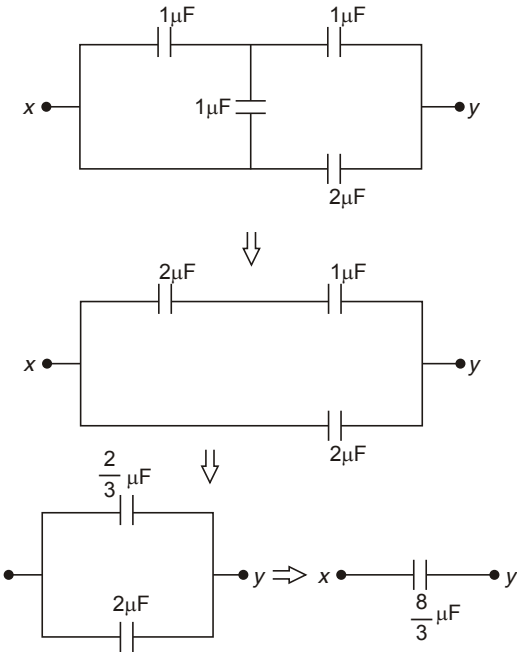
21.



P and Q are at same potential, hence capacitor connected between them have no effect on equivalent capacitance.



22.



23. $C_1 = \frac{k_1\varepsilon_0 A}{2d} + \frac{k_2\varepsilon_0 A}{2d}$

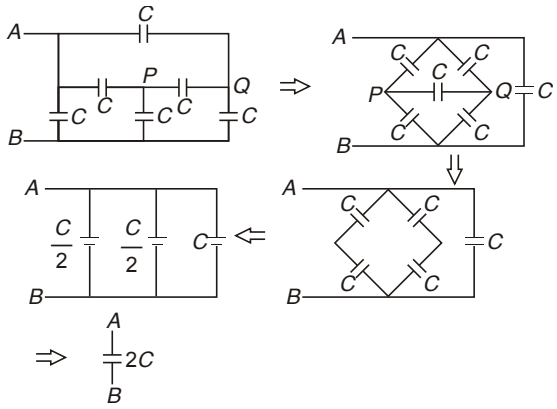
$$= \frac{(k_1 + k_2)\varepsilon_0 A}{2d} \qquad \text{(Parallel grouping)}$$

$$\frac{1}{C_2} = \frac{d}{2k_1\varepsilon_0 A} + \frac{d}{2k_2\varepsilon_0 A} \qquad \text{(Series grouping)}$$

$$C_2 = \frac{2k_1k_2}{k_1 + k_2}\frac{\varepsilon_0 A}{d}$$

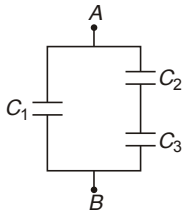
$$\frac{C_1}{C_2} = \frac{(k_1 + k_2)^2}{4k_1k_2} = \frac{(2+3)^2}{4 \times 2 \times 3} = \frac{25}{24}$$

24.



25. Cases (a), (b) and (c) are balanced Wheatstone bridge.

26. The given arrangement can be considered as the combination of three capacitors as shown in figure.



Hence, $$C_1 = \frac{k_1 \varepsilon_0 A}{2d}$$

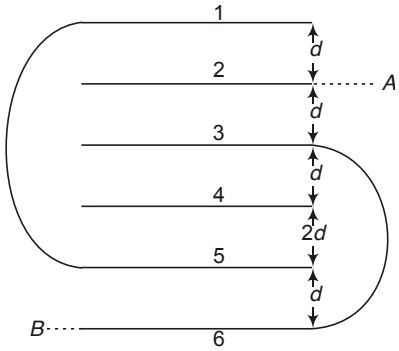
$$C_2 = \frac{k_2 \varepsilon_0 \frac{A}{2}}{d/2} = \frac{k_2 \varepsilon_0 A}{d}$$

$$C_3 = \frac{k_3 \varepsilon_0 \frac{A}{2}}{d/2} = \frac{k_3 \varepsilon_0 A}{d}$$

Effective capacitance,

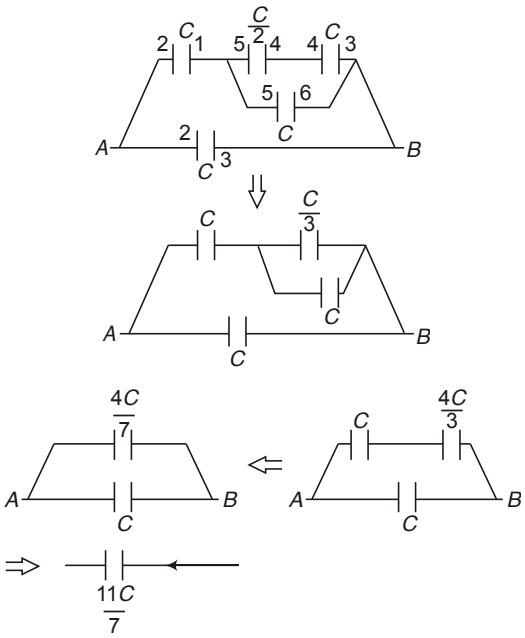
$$C = C_1 + \frac{C_2 C_3}{C_1 + C_2} = \frac{\varepsilon_0 A}{d}\left[\frac{k_1}{2} + \frac{k_2 k_3}{k_2 + k_3}\right]$$

27. Here, plate 1 is connected to plate 5 and plate 3 is connected to plate 6.



Capacitance of all other capacitance is same, i.e., $C = \frac{\varepsilon_0 A}{d}$ but that of formed by plates 4 and 5 is $\frac{C}{2}$ as distance between these two plates is $2d$.

The equivalent circuit is shown in figure.



$$\therefore C_{eq} = \frac{11}{7}C = \frac{11\varepsilon_0 A}{7d} = \frac{11}{7} \times 7\ \mu\text{F} = 11\ \mu\text{F}$$

JEE Corner

Assertion and Reason

1. Capacitance $= \frac{q}{V}$ is constant for a given capacitor.
2. Reason correctly explains the assertion.
3. $U = \frac{1}{2} qV, W = qV$
4. For discharging of capacitor
$$q = q_0 e^{-t/\tau}$$
$$\frac{dq}{dt} = -\frac{q_0}{\tau} e^{-t/\tau}$$
$$= -\frac{q_0}{RC} e^{-t/\tau}$$
Hence, more is the resistance, less will be the slope.
5. Charge on two capacitors will be same only if both the capacitors are initially uncharged.
6. As potential difference across both the capacitors is same, charge will not flow through the switch.
7. C and R_2 are shorted.
8. Time constant for the circuit,
$$\tau = RC$$
9. In series, charge remains same
and $$U = \frac{q^2}{2C} \Rightarrow U \propto \frac{1}{C}$$
10. In series charge remains same
$$\therefore \quad V_1 = \frac{q}{C_1}, V_2 = \frac{q}{C_2}$$
On inserting dielectric slab between the plates of the capacitor, C_2 increases and hence, V_2 decreases. So more charge flows to C_2.

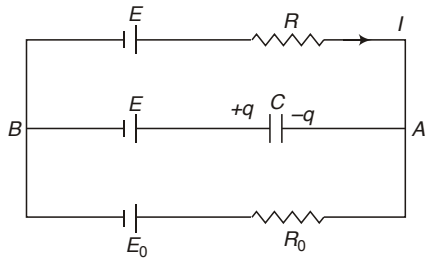
Objective Questions (Level 2)

1. $$E = \begin{cases} -\frac{4Q}{\varepsilon_0 A}\hat{\mathbf{i}} & \text{for } x < d \\ -\frac{2Q}{\varepsilon_0 A}\hat{\mathbf{i}} & \text{for } d < x < 2d \\ \frac{4Q}{\varepsilon_0 A}\hat{\mathbf{i}} & \text{for } 2d < x < 3d \end{cases}$$
2. Let E_0 = external electric field
and E = electric field due to sheet
$$\therefore \quad E_1 = E_0 - E = 8$$
$$E_2 = E_0 + E = 12$$
$$\Rightarrow \quad E = 2\text{ V/m} \Rightarrow \frac{\sigma}{2\varepsilon_0} = 2$$
$$\sigma = 4\varepsilon_0$$
3. When the switch is just closed, capacitors behave like short circuit, no current pass through either 6 Ω or 5 Ω resistor.
4. For charging of capacitor
$$I = I_0 e^{-t/\tau}$$
$$\ln I = \log I_0 - \frac{t}{\tau}$$
$$\ln I = \ln \frac{V}{R} - \frac{t}{RC}$$
But, $I_{01} = I_{02}$
$$\Rightarrow \quad \frac{V_1}{R_1} = \frac{V_2}{R_2}$$
Also, $\frac{1}{R_1C_1} > \frac{1}{R_2C_2}$
$$\Rightarrow \quad R_2C_2 > R_1C_1$$
As only two parameters can be different,
$$C_1 = C_2$$
$$R_2 > R_1$$
and $$V_2 > V_1$$
5. Charge on capacitor at the given instant.
$$q = \frac{q_0}{2} = \frac{CE}{2}$$
Heat produced = Energy stored in capacitor
$$= \frac{q^2}{2C} = \frac{CE^2}{8}$$
Heat liberated inside the battery,
$$= \frac{r}{r + 2r} \times \text{Total heat produced}$$
$$= \frac{CE^2}{24}$$

6. Capacitor is not inside any loop.

7. $I = \dfrac{E - E_0}{R + R_0}$

$$V_{BA} = \frac{q}{C} - E$$



$$-E + IR = \frac{q}{C} - E$$

$$q = IRC + \frac{(E - E_0)RC}{R + R_0}$$

8. $C = \dfrac{C_1C_2}{C_1 + C_2}$

$$C_1 = C_2 = \frac{\varepsilon_0 A}{d}$$

$$C = \frac{\varepsilon_0 A}{2d}$$

$$C_1' = \frac{2\varepsilon_0 A}{d}, C_2' = \frac{\varepsilon_0 A}{2d}$$

$$C' = \frac{C_1'C_2'}{C_1' + C_2'} = \frac{2\varepsilon_0 A}{5d} < C$$

9. $R_e = \dfrac{R}{3}$

$$\tau = R_e C = \frac{RC}{3}$$

$$q = q_0(1 - e^{-t/\tau})$$

$$= CV(1 - e^{-3t/RC})$$

10. Energy loss $= \dfrac{C_1C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$

$$= \frac{2 \times 4}{2 \times (2 - 14)}(100 - 50)^2 \times 10^{-6}$$

$$= 1.7 \times 10^{-3}\ \text{J}$$

11. $q = q_0 e^{-t/RC}$

$$I = -\frac{dq}{dt} = \frac{q_0}{RC} e^{-t/RC}$$

at $t = 0$

$$I = \frac{q_0}{RC} = 10$$

$$V_0 = \frac{q_0}{C} = 10 \times R = 10 \times 10 = 100\ \text{V}$$

12. $V_A = -V_B$

ie, $V_A - V_P = V_P - V_B \qquad [V_P = 0]$

$$\Rightarrow \quad \frac{q}{C_{123}} = \frac{q}{C_n}$$

$$\Rightarrow \quad C_n = C_{123}$$

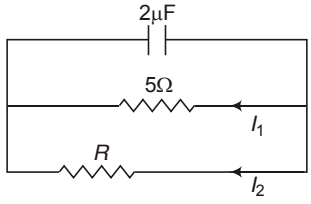
$$\Rightarrow \quad \frac{1}{C_n} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

13. When connected with reverse polarity

$$H = \frac{C_1C_2}{2(C_1 + C_2)}(V_1 + V_2)^2$$

$$= \frac{C \times 2C}{2(C + 2C)} \times (V + 4V)^2 = \frac{25}{3}CV^2$$

14. $\dfrac{H_1}{H_2} = \dfrac{R_2}{R_1} = \dfrac{R}{S}$



Also, $H_1 + H_2 = \dfrac{1}{2}CV^2 = \dfrac{1}{2} \times 2 \times 10^{-6} \times (5)^2$

$$H_1 + H_2 = 25\ \mu\text{J}$$

$$H_2 = 25 - 10 = 15\ \mu\text{J}$$

$$\frac{10}{15} = \frac{R}{5} \Rightarrow R = \frac{10}{3}\ \Omega$$

15. When current in the resistor is 1 A.

$$IR + \frac{q}{C} = E$$

$$1 \times 5 + \frac{q}{2} = 10$$

$$\Rightarrow \quad q = 10\ \mu\text{C}$$

When the switch is shifted to position 2. In steady state, charge on capacitor

$$q' = 5 \times 2 = 10\ \mu\text{C}$$

but with opposite polarity.

$\therefore$ Total charge flown through 5 V battery,

$$= q + q' = 20\ \mu\text{C}$$

Work done by the battery $= 20 \times 5$

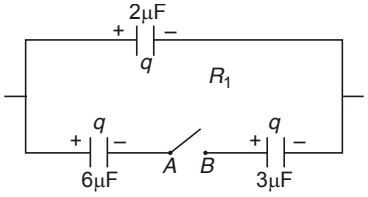
$$= 100\ \mu\text{J}$$

Heat produced $= W - \Delta U$

But, $\quad \Delta U = 0$

$\therefore \quad H = W = 100\ \mu\text{J}$

16. $V_A - V_B = -\frac{q}{6} + \frac{q}{2} - \frac{q}{3} = 0$



Hence, no charge will flow from A to B.

17. As potential difference across both the capacitors is same, they are in parallel.

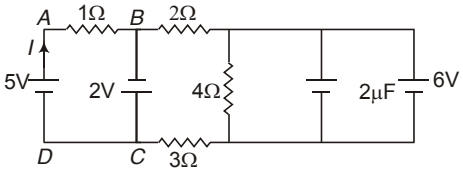
Hence, effective capacitance,

$$C = \frac{2\varepsilon_0 A}{d}$$

$$U = \frac{1}{2}CV^2 = \frac{\varepsilon_0 A}{d}V^2$$

18. Rate of charging decreases as it just charged.

19. Potential difference across capacitor = 6 V



$$q = CV = 2 \times 6 = 12\ \mu\text{C}$$

In loop $ABCD$,

$$I \times 1 - 2 - 5 = 0 \Rightarrow I = 7\ \text{A}$$

20. While charging

$$R_e = R \Rightarrow \tau = RC$$

While discharging

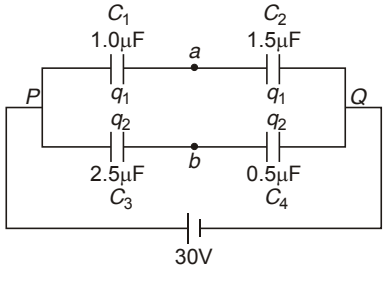
$$R_e = 2R \Rightarrow \tau = 2RC$$

21. Common potential,

$$V = \frac{C_2V_2 - C_1V_1}{C_1 + C_2} = \frac{3 \times 100 - 1 \times 100}{1 + 3}$$

$$= 25\ \text{V}$$

22. $q_1 = \frac{1 \times 1.5}{1 + 1.5} \times 30 = 18\ \mu\text{C}$



$$q_2 = \frac{2.5 \times 0.5}{2.5 + 0.5} \times 30$$

$$= 12.5\ \mu\text{C}$$

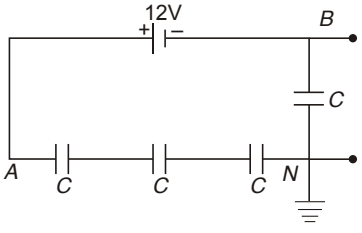
$$\therefore\ V_p - V_a = \frac{q_1}{C_1} = 18\ \text{V}$$

$$V_p - V_b = \frac{q_2}{C_3}$$

$$= \frac{12.5}{2.5} = 5\ \text{V}$$

$$V_b - V_a = 13\ \text{V}$$

23. As all the capacitors are identical, potential difference across each capacitor,



$$V = \frac{E}{4} = 3\ \text{V}$$

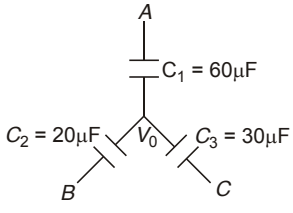
$$V_N - V_B = 3\ \text{V}$$

$$V_B = -3\ \text{V}$$

$$V_A - V_B = 12\ \text{V}$$

$$\Rightarrow \quad V_A = 9\ \text{V}$$

24. By Kirchhoff's voltage law,



$$q_1 + q_2 + q_3 = 0$$

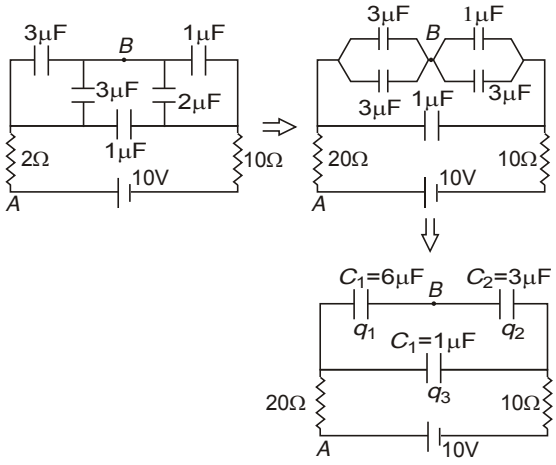
$$C_1(C_A - V_0) + C_2(V_B - V_0) + C_3(V_C - V_0) = 0$$

$$V_0 = \frac{C_1V_A + C_2V_B + C_3V_C}{C_1 + C_2 + C_3}$$

$$V_0 = \frac{60 \times 6 + 2 \times 20 + 3 \times 30}{60 + 20 + 30}$$

$$= \frac{49}{11}\ \text{V}$$

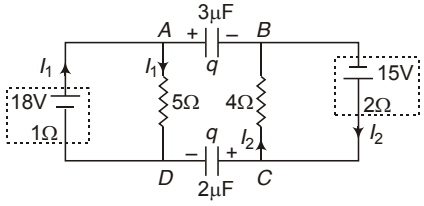
25. In the steady state, there will be no current in the circuit.



$$\therefore \quad V_1 = \frac{C_2}{C_1 + C_2} V = \frac{3}{3+6} 10 = \frac{10}{3} \text{ V}$$

26. $I_1 = \dfrac{E_1}{R_1 + r_1} = \dfrac{18}{5+1} = 3 \text{ A}$

$$I_2 = \frac{E_2}{R_2 + r_2} = \frac{15}{4+2} = 2.5 \text{ A}$$



In loop $ABCD$,

$$\frac{q}{3} - I_2 R_2 + \frac{q}{2} - I_1 R_1 = 0$$

$$\Rightarrow \quad \frac{5q}{6} = 3 \times 5 + 2.5 \times 4 \Rightarrow q = 30 \ \mu\text{C}$$

27. During discharging

$$q = q_0 \, e^{-t/\tau}$$

$$q_0 = CE = 10 \ \mu\text{C}$$

at $t = 12$ s,

$$q = 10 e^{-12/6} = 10 e^{-2}$$

$$= (0.37)^2 10 \ \mu\text{C}$$

28. $q = \dfrac{C_1 C_2}{C_1 + C_2}(E_1 - E_2)$

$$V_{ap} = \frac{q}{C_2} = \frac{C_1}{C_1 + C_2}(E_1 - E_2)$$

$$= \left(\frac{E_1 - E_2}{C_1 + C_2}\right) C_1$$

29. $H = \dfrac{C_1 C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$ is independent of resistance.

30. Immediately after switch is closed, capacitor behaves like short circuit.

31. $i_1 = \dfrac{V}{2R} e^{-t/RC}, \ i_2 = \dfrac{V}{R} e^{-t/RC}$

$$\frac{i_1}{i_2} = \frac{1}{2} e^{\frac{5t}{6RC}}$$

Increases with time.

32. $R = \dfrac{\rho d}{A} = \dfrac{d}{\sigma A}$

$$C = \frac{k \varepsilon_0 A}{d}$$

$$\tau = RC = \frac{d}{\sigma A} \times \frac{k \varepsilon_0 A}{d}$$

$$= \frac{\varepsilon_0}{6} = \frac{8.85 \times 10^{-12}}{7.4 \times 10^{-12}} = 6 \text{ s}$$

33. $i = i_0 e^{-t/\tau}$

$$\Rightarrow \quad \frac{i_0}{2} = i_0 e^{\frac{-\ln 4}{RC}}$$

$$\Rightarrow \quad \frac{\ln 4}{RC} = \ln 2$$

$$\Rightarrow \quad \ln 4 = \ln 2^{RC}$$

$$\Rightarrow \quad RC = 2$$

$$\Rightarrow \quad R = \frac{2}{C} = \frac{2}{0.5} = 4 \ \Omega$$

34. Potential difference across each capacitor is equal, hence they are in parallel,

charge on each capacitor

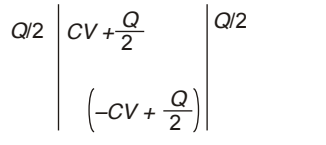
$$q = C_e V = 2 \times 10 = 20 \ \mu\text{C}$$

As plate C contributed to two capacitors,

charge on plate,

$$C = 2q = +40 \ \mu\text{C}$$

35. Charge distribution on the plates of the capacitor is shown in figure

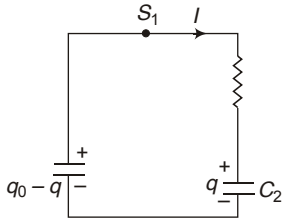


$$\therefore \quad V' = \frac{Q'}{C} = \frac{CV + \frac{Q}{2}}{C}$$

$$= V + \frac{Q}{2C}$$

36. Let q be the charge on C_2 (or charge flown through the switches at any instant of time)

By Kirchhoff's law



$$IR + \frac{q}{C_2} - \frac{q_0 - q}{C_1} = 0$$

$$\frac{dq}{dt} = \frac{C_2 q_0 - (C_1 + C_2)q}{C_1 C_2 R}$$

$$\int_0^q \frac{dq}{C_2 q_0 - (C_1 + C_2)q} = \int_0^t \frac{dt}{C_1 C_2 R}$$

$$= \frac{1}{C_1 + C_2}\left[\ln |C_2 q_0 - (C_1 + C_2)q\right]_0^q$$

$$= \frac{1}{C_1 C_2 R} t$$

$$q = \frac{C_2 q_0}{C_1 + C_2}\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$\tau = \frac{C_1 + C_2}{C_1 C_2 R}$$

or $$q = \frac{C}{C_1} q_0 \left(1 - e^{-\frac{t}{RC}}\right)$$

where, $$C = \frac{C_1 C_2}{C_1 + C_2}$$

37. $$H = \frac{C_1 C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$$

$$= \frac{C_1 C_2}{2(C_1 + C_2)}\left(\frac{q_0}{C_1}\right)^2$$

$$= \frac{C_2 q_0^2}{2C_1(C_1 + C_2)} = \frac{C q_0^2}{2C_1^2}$$

38. Electric field in the gap will remain same.

39. Electric field inside the dielectric slab

$$E' = \frac{E}{k} = \frac{V}{kd}.$$

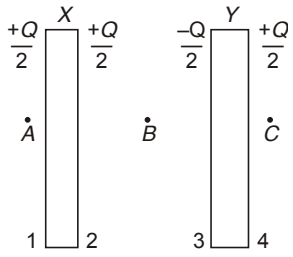
More Than One Correct Options

1. Charge distribution is shown in figure

$$E_B = E_1 + E_2 + E_3 + E_4$$

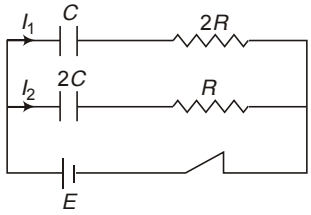
$$= \frac{Q}{4\varepsilon_0 A} + \frac{Q}{4\varepsilon_0 A} + \frac{Q}{4\varepsilon_0 A} - \frac{Q}{4\varepsilon_0 A}$$

$$= \frac{Q}{2\varepsilon_0 A}$$



$|E_A| = |E_C| = |E_1 + E_2 - E_3 + E_4|$ but E_A and E_C have opposite direction.

2. $$q_1 = CE\left(1 - e^{-\frac{t}{RC}}\right)$$



$$q_2 = 2CE\left(1 - e^{-\frac{t}{RC}}\right)$$

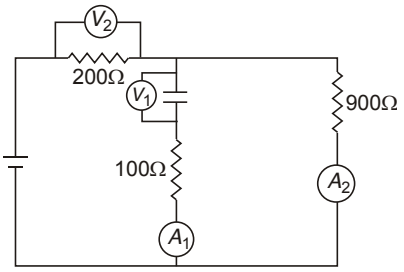
$$\frac{dq_1}{dt} = \frac{E}{R} e^{-\frac{t}{RC}}$$

$$\frac{dq_2}{dt} = \frac{2E}{R} e^{-\frac{t}{2RC}}$$

$$\frac{q_1}{q_2} = \frac{1}{2} \Rightarrow \frac{q_{0_1}}{q_{0_2}} = \frac{1}{2}$$

$$\tau_1 = \tau_2 = RC$$

3. $$V_1 = \frac{q}{C} = \frac{4 \times 10^{-3}}{100 \times 10^{-6}} = 40\text{V}$$



$$I_1 = 0$$

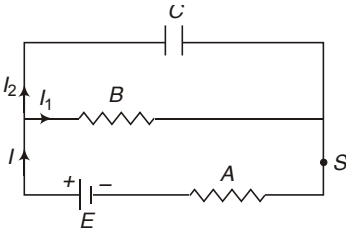
$$I_2 = \frac{V_1}{900} = \frac{40}{900} = \frac{2}{45}\text{ A}$$

$$V_2 = I_2 \times 200 = \frac{2}{45} \times 200 = \frac{80}{9}\text{ V}$$

$$E = V_1 + V_2 = 40 + \frac{80}{9}$$

$$= \frac{440}{9}\text{ V}$$

4. Initially $I_1 = 0, I_2 = I = \frac{E}{R}$



As the capacitor starts charging,

I_2 decreases and I_1 increases,

In the steady state

$$I_1 = I = \frac{E}{R}, I_2 = 0$$

At any instant

$$P_1 = I_1^2 R, P_2 = I_2^2 R$$

Steady state potential difference across the capacitor,

$$V = \frac{E}{2}$$

$$U = \frac{1}{2}CV^2 = \frac{CE^2}{8}$$

5. $F = \frac{Q^2}{2\varepsilon_0 A}$ independent of d.

$E = \frac{Q}{\varepsilon_0 A}$ independent of d.

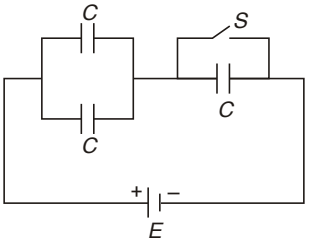
$$U = \frac{Q^2}{2C} = \frac{Q^2 d}{2\varepsilon_0 A}$$

$$\Rightarrow \quad U \propto d$$

$$V = \frac{Q}{C} = \frac{Qd}{\varepsilon_0 A}$$

$$\Rightarrow \quad V \propto d$$

6. When switch S is open



$$C_e = \frac{C \times 2C}{C + 2C} = \frac{2}{3}C$$

$$q_1 = \frac{2}{3}CE$$

When switch S is closed

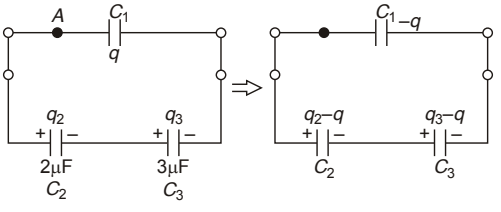
$$C_e' = 2C$$

$$q_2 = 2CE$$

Charge flown through the battery

$$\Delta q = q_2 - q_1 = \frac{4}{3}CE = \text{ positive}$$

7. Let charge q flows to C_1 at it falls to the free end of the wire.



By Kirchhoff's voltage law,

$$\frac{q_2 - q}{C_2} + \frac{q_3 - q}{C_3} - \frac{q}{C_1} = 0$$

$$q = \frac{\frac{q_2}{C_2} + \frac{q_3}{C_3}}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{q}{C_3}}$$

$$= \frac{V_2 + V_3}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

$$q = \frac{150 + 120}{\frac{1}{2} + \frac{1}{3} + \frac{1}{1.5}} = 180\ \mu\text{C}$$

$$q_2' = q_2 - q = 150 \times 2 - 180$$

$$= 120\ \mu\text{C}$$

8. $C = \dfrac{\varepsilon_0 A}{d}$

$$U = \frac{Q^2}{2C} = \frac{Q^2 d}{2\varepsilon_0 A} \Rightarrow U \propto d$$

$$V = \frac{Q}{C} = \frac{Qd}{\varepsilon_0 A} \Rightarrow V \propto d$$

$$C = \frac{\varepsilon_0 A}{d} \Rightarrow C \propto \frac{1}{d}$$

$E = \dfrac{Q}{\varepsilon_0 A} \Rightarrow E$ is independent of d.

9. $R = 1 + 2 = 3\ \Omega, C = 2\ \text{F}$

$$q_0 = CV_0 = 2 \times 6 = 12\text{C}$$

At any instant

$$q = q_0 \left(e^{\frac{-t}{RC}} \right)$$

$$I = \frac{dq}{dt} = \frac{q_0}{RC} e^{\frac{-t}{RC}}$$

at $t = 0$

$$I = \frac{q_0}{RC} = \frac{12}{3 \times 2} = 2\ \text{A}$$

at $t = 6 \ln 2$

$$I = \frac{q_0}{RC} e^{\frac{-6 \ln 2}{6}} = \frac{12}{6} \times \frac{-1}{2}$$

$$= 2 \times \frac{1}{2} = 1\ \text{A}$$

Potential difference across $1\ \Omega$ resistor

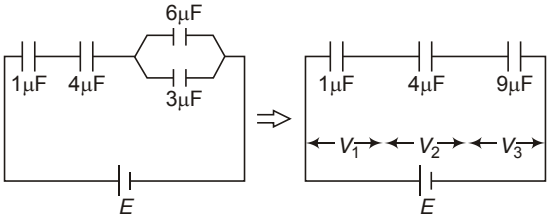
$\Rightarrow \quad 1 \times 1 = 1\ \text{V}$

Potential difference across $2\ \Omega$ resistor

$\Rightarrow \quad 1 \times 2 = 2\ \text{V}$

$\therefore$ By Kirchhoff's voltage law, potential difference across capacitors $= 1 + 2 = 3$ V.

10. $q = C_1 V_1 = 1 \times 10 = 10\ \mu\text{F}$



$$V_2 = \frac{q}{C_2} = \frac{10}{4} = 2.5\ \text{V}$$

$$V_3 = \frac{q}{C_3} = \frac{10}{9}\ \text{V}$$

Match the Columns

1. $C_1' = kC_1 = 8\ \mu\text{F},$

$$C_2' = \frac{C_2}{k} = 2\ \mu\text{F}$$

$$q' = \frac{C_1' C_2'}{C_1' + C_2'} V = (1.6V)\ \mu\text{C}$$

$$q = (2V)\ \mu\text{C}$$

$\therefore q' < q$

$$U_2' = \frac{q'^2}{C_2'} = \frac{(1.6\ V)^2}{2 \times 2} = 0.64V^2,$$

$$U_2 = \frac{q^2}{2C_2} = \frac{(2)^2}{2 \times 2} = (1\ V)\ \mu\text{C}$$

$U_2' < U_2$

$$V_2' = \frac{q'}{C_2'} = \frac{1.6V}{8} = 0.2V,$$

$$V_2 = \frac{q}{C_2} = \frac{(2V)}{4} = 0.5V$$

$V_2' < V$

$$E_2' = \frac{q'}{\varepsilon_0 A} = \frac{1.6V}{\varepsilon_0 A},$$

$$E_2 = \frac{q}{k\varepsilon_0 A} = \frac{2V}{2 \times \varepsilon_0 A} = \frac{V}{\varepsilon_0 A}$$

$E_2' > E_2$

(a $\rightarrow$ q), (b $\rightarrow$ q), (c $\rightarrow$ q), (s $\rightarrow$ p).

2. Before switch S is closed, charge distribution is shown in figure (1).

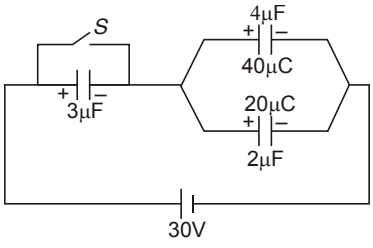


Fig. 1

After switch S is closed, charge distribution is shown in figure (2).

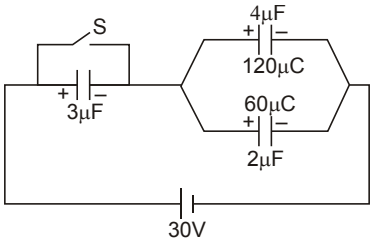
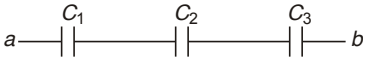


Fig. 2

(a → s), (b → p), (c → q), (d → s).

3. (a → q), (b → p, r), (c → q), (d → p, π)

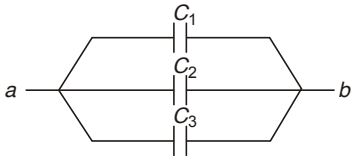
(p)



$$V_1 = V_2 = V_3 = \frac{V}{3}$$

$$q_1 = q_2 = q_3 = \frac{CV}{3}$$

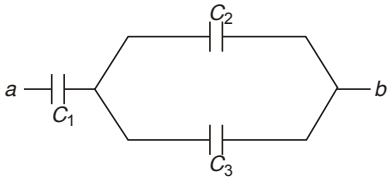
(q)



$$V_1 = V_2 = V_3 = V$$

$$q_1 = q_2 = q_3 = CV$$

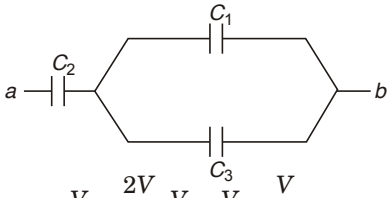
(r)



$$V_1 = \frac{2V}{3},\ V_2 = V_3 = \frac{V}{3}$$

$$q_1 = \frac{2CV}{3},\ q_2 = q_3 + \frac{CV}{3}$$

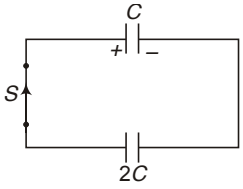
(s)



$$V_2 = \frac{2V}{3},\ V_1 = V_2 = \frac{V}{3}$$

$$q_2 = \frac{2CV}{3},\ q_1 = q_3 = \frac{CV}{3}$$

4. Common potential



$$V' = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{V}{3}$$

$$U_1 = \frac{1}{2}C_1V'^2 = \frac{1}{18}CV^2$$

$$U_2 = \frac{1}{2}C_2V'^2 = \frac{1}{9}CV^2$$

$$\Delta U = \frac{C_1C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$$

$$= \frac{C \times C}{2(C_1 + C_2)}(V - 0)^2$$

$$= \frac{1}{6}CV^2$$

(a → r), (b → p), (c → q).

5.

$$C_1 = \frac{k\varepsilon_0 A}{2d} + \frac{\varepsilon_0 A}{2d}$$

$$= (k + 1)\frac{\varepsilon_0 A}{2d} = \frac{3\varepsilon_0 A}{2d}$$

$$= \frac{1}{C_2} = \frac{d}{2k\varepsilon_0 A} + \frac{d}{2\varepsilon_0 A}$$

$$= \frac{d}{2\varepsilon_0 A}\left(\frac{1 + k}{k}\right)$$

$$\Rightarrow \quad C_2 = \frac{2k\varepsilon_0 A}{d\,(1 + k)} = \frac{4\varepsilon_0 A}{3d}$$

$$\frac{C_1}{C_2} = \frac{9}{8}$$

As combination is series, $q_1 = q_2$

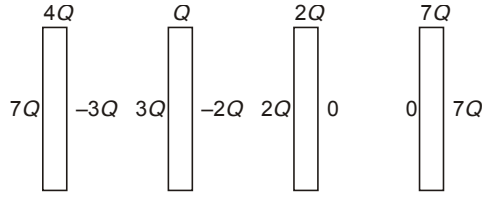
$$\Rightarrow \quad \frac{q_1}{q_2} = 1$$

$$\frac{U_1}{U_2} = \frac{C_2}{C_1} = \frac{8}{9}$$

(a → s), (b → s), (c → s).

6. Charge distribution is shown in figure.

(a → p), (b → p, q), (c → s), (d → p, q, r).



23 Magnetics

Introductory Exercise 23.1

1. $[F_e] = [F_m]$

$$[qE] = [qvB]$$

$$\Rightarrow \quad \left[\frac{E}{B}\right] = [v] = [\mathrm{L\,T^{-1}}]$$

2. $\vec{\mathbf{F}} = q(\vec{\mathbf{v}} \times \vec{\mathbf{B}})$

$\therefore \quad \vec{\mathbf{F}} \perp \vec{\mathbf{v}}$ and $\vec{\mathbf{F}} \perp \vec{\mathbf{B}}$

Because cross product of any two vectors is always perpendicular to both the vectors.

3. No. As $\vec{F}_m = q(\vec{\mathbf{v}} \times \vec{\mathbf{B}})$

$$\Rightarrow \quad |\vec{\mathbf{F}}_m| = qvB\sin\theta$$

If $F_m = 0$, either $B = 0$ or $\sin\theta = 0$,

i.e., $\theta = 0$

4. $\vec{\mathbf{F}} = q(\vec{\mathbf{v}} \times \vec{\mathbf{B}})$

$$= -4 \times 10^{-6} \times 10^{-6} \times 10^{-2}[(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}) \times (2\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 3\hat{\mathbf{k}})]$$

$$= -4 \times 10^{-2}(4\hat{\mathbf{i}} + 8\hat{\mathbf{j}} + 16\hat{\mathbf{k}})$$

$$= -16(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) \times 10^{-2}\ \mathrm{N}$$

Introductory Exercise 23.2

1. As magnetic field can exert force on charged particle, it can be accelerated in magnetic field but its speed cannot increases as magnetic force is always perpendicular to the direction of motion of charged particle.

2. $\vec{\mathbf{F}}_m = -e(\vec{\mathbf{v}} \times \vec{\mathbf{B}})$

By Fleming's left hand rule, $\vec{\mathbf{B}}$ must be along positive z-axis.

3. As magnetic force provides necessary centripetal force to the particle to describe a circle.

$$qvB = \frac{mv^2}{r}$$

$$\Rightarrow \quad r = \frac{mv}{qB}$$

(a) $\quad r = \dfrac{mv}{qB}$

$$\Rightarrow \quad r \propto m$$

Hence, electron will describe smaller circle.

(b) $\quad T = \dfrac{2\pi r}{v} = \dfrac{2\pi m}{qB}$

$$f = \frac{1}{T} = \frac{qB}{2\pi m}$$

$$\Rightarrow \quad f \propto \frac{1}{m}$$

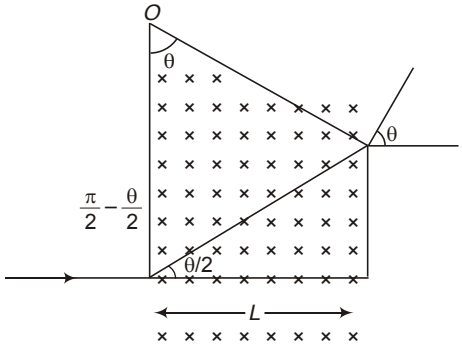
$\therefore$ electron have greater frequency.

4. Electrons are refocused on x-axis at a distance equal to pitch, *i.e.*,

$$n = p = v_{\parallel}T$$

$$= \frac{2\pi mv\cos\theta}{eB}$$

5. (a) If $L \geq r = \dfrac{mv}{qB}$,



(b) The particle will describe a semi-circle.

Hence, $\theta = \pi$

(c) $$\frac{L}{l} = \cos\frac{\theta}{2}$$

$$\Rightarrow \quad \frac{L}{2R\sin\frac{\theta}{2}} = \cos\frac{\theta}{2}$$

$$\frac{L}{R} = \sin\theta \Rightarrow \sin\theta = \frac{1}{2}$$

$$\Rightarrow \quad \theta = \frac{\pi}{6}$$

6. $$r = \frac{mv}{eB} = \sqrt{\frac{2mk}{eB}}$$

$$= \frac{\sqrt{2m\,eV}}{eB} = \frac{1}{B}\sqrt{\frac{2mV}{e}}$$

For electron,

$$r = \frac{1}{0.2}\sqrt{\frac{2 \times 9.1 \times 10^{-31} \times 100}{1.6 \times 10^{-19}}}$$

$$= 1.67 \times 10^{-4}\ \text{m} = 0.0167\ \text{cm}$$

For proton

$$r = \frac{1}{0.2}\sqrt{\frac{2 \times 1.67 \times 10^{-27} \times 100}{1.6 \times 10^{-19}}}$$

$$= 7 \times 10^{-3}\ \text{m} = 0.7\ \text{cm}$$

7. $$r = \frac{mv}{qB} = \frac{\sqrt{2m\,k}}{qB}$$

$$\therefore \quad r \propto \frac{\sqrt{m}}{q}$$

$$\therefore \quad r_p : r_d : r_\alpha = \frac{\sqrt{1}}{1} : \frac{\sqrt{2}}{1} : \frac{\sqrt{4}}{2}$$

$$= 1 : \sqrt{2} : 1$$

Introductory Exercise 23.3

1. Let at any instant

$$\vec{\mathbf{V}} = V_x\,\hat{\mathbf{i}} + V_y\,\hat{\mathbf{j}} + V_z\,\hat{\mathbf{k}}$$

Now, $V_x^2 + V_y^2 = V_0^2 = \text{constant}$

and $$V_2 = V_0 - \frac{qE}{m}f$$

$\vec{\mathbf{V}}$ is minimum when $V_2 = 0$

at $$f = \frac{mv_0}{qE}$$

and $$V_{\min} = V_0$$

2. After one revolution, $y = 0$,

$x = p =$ pitch of heating

$$= \frac{2\pi\,mv\sin\theta}{qB}$$

Hence, coordination of the particle,

$$= (x, b) = \left(0, \frac{2\pi\,mv\sin\theta}{qB}\right)$$

3. $$\vec{\mathbf{F}} = i(\vec{\mathbf{l}} \times \vec{\mathbf{B}}) = ilB\,[\hat{\mathbf{i}} \times (\hat{\mathbf{j}} + \hat{\mathbf{k}})]$$

$$(\vec{\mathbf{F}}) = \sqrt{2}\,ilB$$

4. No. as $\hat{\mathbf{i}} \times (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) = \hat{\mathbf{i}} \times \hat{\mathbf{j}} + \hat{\mathbf{i}} \times (\hat{\mathbf{j}} \times \hat{\mathbf{k}})$

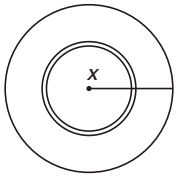
But $\hat{\mathbf{i}} \times \hat{\mathbf{j}} = 0$

$\therefore\ \hat{\mathbf{i}} \times (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) = \hat{\mathbf{i}} \times (\hat{\mathbf{j}} \times \hat{\mathbf{k}})$

Introductory Exercise 23.4

1. Consider the disc to be made up of large number of elementary concentric rings. Consider one such ring of radius x and thickness dx.

Charge on this ring



$$dq = \frac{q}{\pi R^2} \cdot dA + \frac{q}{\pi R^2} \times 2\pi x\, dx$$

$$dq = \frac{2qx\, dx}{R^2}$$

Current in this ring,

$$di = \frac{dq}{T} + \frac{\omega\, dq}{2\pi} = \frac{\omega qx\, dx}{\pi R^2}$$

$\therefore$ Magnetic moment of this ring,

$$dM = di \times A = \frac{\omega qx\, dx}{\pi R^2} \times \pi x^2$$

$$= \frac{\omega q}{R^2} x^3 dx$$

$\therefore$ Magnetic moment of entire disc,

$$M = \int dM = \frac{\omega q}{R^2} \int_0^R x^3 dx$$

$$= \frac{\omega q}{R^2}\left[\frac{R^4}{4}\right] = \frac{1}{4}\omega q R^2$$

2. $\vec{\mathbf{M}} = i \times [(\vec{\mathbf{OA}} \times \vec{\mathbf{AB}})]$

$\vec{\mathbf{OA}} = OA \cos\theta\, \hat{\mathbf{j}} + OA \sin\theta\, \hat{\mathbf{k}}$

$\vec{\mathbf{AB}} = AB\, \hat{\mathbf{i}}$

$\therefore \quad \vec{\mathbf{M}} = i \times OA \cdot AB\, [(\cos\theta\, \hat{\mathbf{j}} + \sin\theta\, \hat{\mathbf{k}}) \times \hat{\mathbf{i}}]$

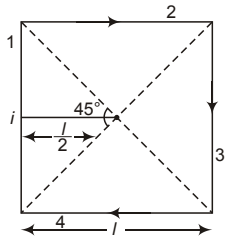
$$= 4 \times 0.2 \times 0.1 \left[\left(\frac{\sqrt{3}}{2}\hat{\mathbf{j}} + \frac{1}{2}\hat{\mathbf{k}}\right) \times \hat{\mathbf{i}}\right]$$

$$= (0.04\, \hat{\mathbf{j}} - 0.07\, \hat{\mathbf{k}})\ \text{A-m}^2$$

Introductory Exercise 23.5

1. (a) $B_1 = B_2 = B_3 = B_4$

$$= \frac{\mu_0}{4\pi} \cdot \frac{i}{l/2}[\sin 45° + \sin 45°]$$



$$= \frac{\mu_0}{4\pi} \cdot \frac{2\sqrt{2}i}{l}$$

Net magnetic field at the centre of the square,

$$B = B_1 + B_2 + B_3 + B_4 = \frac{\mu_0}{4\pi} \cdot \frac{8\sqrt{2}i}{l}$$

$$= \frac{2\sqrt{2}\,\mu_0 i}{\pi l} = 28.3\ \mu\text{T (inward)}$$

(b) If the conductor is converted into a circular loop, then

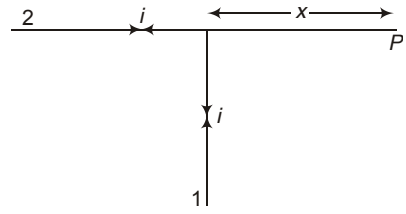
$$2\pi r = 4l \Rightarrow r = \frac{2l}{\pi}$$

$$B = \frac{\mu_0 i}{2} r = \frac{\pi \mu_0 i}{4l} = 24.7\ \mu\text{T (inward)}$$

2. $B = \frac{\mu_0}{4\pi} \cdot \frac{i}{x}$

(As P is lying near one end of conductor 1)

$B_2 = 0$ (Magnetic field on the axis of a current carrying conductor is zero)

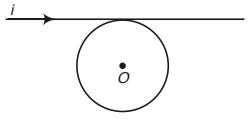


$$B = B_1$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{i}{x}$$

By right hand thumb rule, direction of magnetic field at P is inward.

3. Magnetic field due to straight conductor at O



$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2i}{R}$$

Magnetic field at O due to circular loop

$$B_2 = \frac{\mu_0 i}{2R}$$

By right hand thumb rule, both the filds are acting inward.

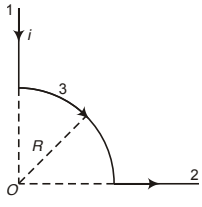
Hence,

$$B = B_1 + B_2 = \frac{\mu_0 i}{2\pi R}(1 + \pi)$$

$$= \frac{4\pi \times 10^{-7} \times 7}{2\pi \times 10 \times 10^{-2}}\left(1 + \frac{22}{7}\right)$$

$$= 58 \times 10^{-6}\ \text{T} = 58\ \mu\text{T (inward)}.$$

4. $B_1 = B_2 = 0$ (Magnetic field on the axis of current carrying conductor is zero)



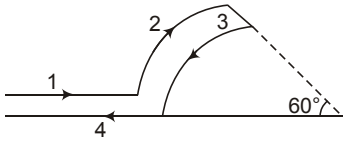
$$B_3 = \frac{1}{4} \cdot \frac{\mu_0 i}{2R} = \frac{\mu_0 i}{8R}$$

$$= \frac{4\pi \times 10^{-7} \times 5}{8 \times 3 \times 10^{-2}}$$

$$= 2.62 \times 10^{-5}\ \text{T}$$

$$= 26.2\ \mu\text{T (inward)}.$$

5. $B_1 = B_2 = 0$ (Magnetic field on the axis of straight conductor is zero)



$$B_2 = \frac{60°}{360°} \cdot \frac{\mu_0 i}{2b} = \frac{\mu_0 i}{12b}\ \text{(inward)}$$

$$B_3 = \frac{60°}{360°} = \frac{\mu_0 i}{2a} = \frac{\mu_0}{12a}\ \text{(outward)}$$

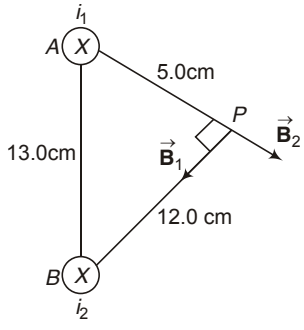
As $B_3 > B_2$,

Net magnetic field at P,

$$B = B_3 - B_2$$

$$= \frac{\mu_0 i}{12}\left[\frac{1}{a} - \frac{1}{b}\right]$$

6. AB, AP and BP from Pythagorus triplet, hence $\angle APB = 90°$



$$\therefore \quad \vec{\mathbf{B}}_1 = \frac{\mu_0}{4\pi} \cdot \frac{2i_1}{r_1}\hat{\mathbf{PB}}$$

$$\vec{\mathbf{B}}_2 = \frac{\mu_0}{4\pi} \cdot \frac{2i_2}{r_2}\hat{\mathbf{AP}}$$

$$B = \sqrt{B_1^2 + B_2^2}$$

$$= \frac{\mu_0}{2\pi}\sqrt{\left(\frac{i_1}{r_1}\right)^2 + \left(\frac{i_2}{r_2}\right)^2}$$

$$= \frac{4\pi \times 10^{-7}}{2\pi} \times \sqrt{\left(\frac{3}{0.05}\right)^2 + \left(\frac{3}{0.12}\right)^2}$$

$$= 1.3 \times 10^{-5}\ \text{T}$$

$$= 13\ \mu\text{T}$$

7. $\tau = NIAB\cos\theta$

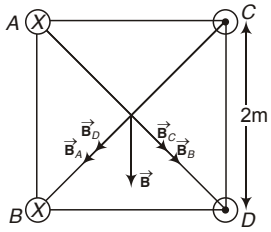
$$= 100 \times 1.2 \times 0.4 \times 0.3 \times 0.8 \times \cos 30°$$

$$= 9.98\ \text{N-m}$$

Rotation will be clockwise as seen from above.

Introductory Exercise 23.6

1. By right hand thumb rule, direction of magnetic field due to conductor A, B, C and D are as shown in figure.



$$B_A = B_B = B_C = B_D = \frac{\mu_0}{4\pi} \cdot \frac{2I}{r}$$

Here, $I = 5$ A

$$r = \frac{a}{\sqrt{2}} = \frac{0.2}{\sqrt{2}} = 0.14$$

$\therefore$ Net magnetic field at P

$$B = \sqrt{(B_A + B_D)^2 + (B_B + B_C)^2}$$

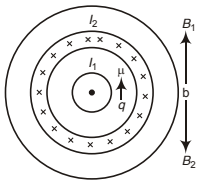
$$= \frac{\mu_0}{4\pi} \cdot \frac{4\sqrt{2}\, I}{r}$$

$$= \frac{10^{-7} \times 4\sqrt{2} \times 5}{0.2/\sqrt{2}} = 20 \times 10^{-6} \text{ T}$$

$$= 20\ \mu\text{T}$$

Clearly resultant magnetic field is downward.

2. At point A



$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{I_1}{r_1}$$

$B_2 = 0$ (Magnetic field inside a current carrying hollow cylinder is zero)

$$\therefore B_a = B_1 + B_2 = \frac{\mu_0}{4\pi} \cdot \frac{I_1}{r_1}$$

$$= \frac{10^{-7} \times 1}{1 \times 10^{-3}} - 10^{-4} \text{ T}$$

$$= 100\ \mu\text{T (upward)}$$

At point B

$$B_1 = \frac{\mu_0}{4\pi\varepsilon_0} \cdot \frac{I_1}{r_2} \uparrow,\ B_2 = \frac{\mu_0}{4\pi} \cdot \frac{I_2}{r_2} \downarrow$$

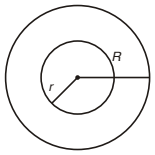
Net field at B

$$B = B_2 - B_1 = \frac{\mu_0}{4\pi} \cdot \frac{1}{r_2}(I_2 - I_1)$$

$$= \frac{10^{-7}}{3 \times 10^{-3}} \times (3 - 2) = 0.67 \times 10^{-4} \text{ T}$$

$$= 67\ \mu\text{T}$$

3. Consider the cylinder to be made up of large number of elementary hollow cylinders.



Consider one such cylinder of radius r and thickness dr.

Current passing through this hollow cylinder,

$$di = jdA = j(2\pi r\, dr)2\pi\, br^2 dr$$

(a) Total current inside the portion of radius r_1,

$$I_1 = \int di = 2\pi b \int_0^{r_1} r^2 dr$$

$$= 2\pi b \left[\frac{r_1^3}{3}\right]_0^{r_1}$$

$$= \frac{2}{3}\pi\, br_1^3$$

By ampere's circuital law,

$$\oint_{2\pi r_1} B \cdot dl = \mu_0 i_1$$

$$B_1 \times 2\pi r_1 = \mu_0 \left(\frac{2}{3}\pi\, br_1^3\right)$$

$$\Rightarrow \qquad B_1 = \frac{\mu_0\, br_1^2}{3}$$

(b) Total current inside the cylinder

$$i = 2\pi b \int_0^R r^2 dr$$

$$= \frac{2}{3}\pi\, bR^3$$

$$B_2 = \frac{\mu_0}{4\pi} \frac{2i}{r_2} = \frac{\mu_0\, bR^3}{3r_2}$$

AIEEE Corner

Subjective Questions (Level-1)

1. Positive. By Flemings left hand rule.

2. $F_m = evB\sin\theta$

$$\Rightarrow \quad v = \frac{F_e}{eB\sin\theta}$$

$$= \frac{4.6\times10^{-15}}{1.6\times10^{-19}\times3.5\times10^{-3}\times\sin 60^\circ}$$

$$= 9.46\times10^{6}\ \text{m/s}$$

3. $F_m = qvB\sin\theta$

$$= (2\times1.6\times10^{-19})\times10^{5}\times0.8\times1$$

$$= 2.56\times10^{-14}\ \text{N}$$

4. (a) $\vec{\mathbf{F}}_m = e(\vec{\mathbf{v}}\times\vec{\mathbf{B}})$

$$= -1.6\times10^{-19}\,[(2.0\times10^{6})\hat{\mathbf{i}} + (3.0\times10^{6})\hat{\mathbf{j}}]\times(0.03\,\hat{\mathbf{i}} + 0.15\,\hat{\mathbf{j}})$$

$$= -(6.24\times10^{-4}\ \text{N})\hat{\mathbf{k}}$$

(b) $= \vec{\mathbf{F}}_m\, e(\vec{\mathbf{v}}\times\vec{\mathbf{B}}) = -(6.24\times10^{-4}\text{N})\hat{\mathbf{k}}$

5. $\vec{\mathbf{F}}_m = e(\vec{\mathbf{v}}\times\vec{\mathbf{B}})$

$$(6.4\times10^{-19})\hat{\mathbf{k}} = -1.6\times10^{-19}[(2\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}})\times(B_x\,\hat{\mathbf{i}} + 3B_x\,\hat{\mathbf{j}})]$$

$$6.4\times10^{-19}\,\hat{\mathbf{k}} = -1.6\times10^{-19}[2B_x\,\hat{\mathbf{k}}]$$

$$B_x = \frac{6.4\times10^{-19}}{-3.2\times10^{-19}} = -2.0\ \text{T}$$

6. (a) As magnetic force always acts perpendicular to magnetic field, magnetic field must be along x-axis.

$$F_1 = qv_1B\sin\theta_1$$

$$\Rightarrow \quad B = \frac{F_1}{qv_1B\sin\theta_1} = \frac{5\sqrt{2}\times10^{-3}}{1\times10^{-6}\times10^{6}\times\frac{1}{\sqrt{2}}}$$

$$\Rightarrow \quad B = 10^{-3}\ \text{T}$$

or $\quad \vec{\mathbf{B}} = (10^{-3}\ \text{T})\hat{\mathbf{i}}$

(b) $F_2 = qv_2B\sin\theta_2$

$$= 1\times10^{-6}\times10^{6}\times10^{-3}\times\sin 90^\circ$$

$$= 10^{-3}\ \text{N}$$

$$F_2 = 1\ \text{mN}$$

7. Let $\vec{\mathbf{B}} = B_x\,\hat{\mathbf{i}} + B_y\,\hat{\mathbf{j}} + B_z\,\hat{\mathbf{k}}$

(a) $\vec{\mathbf{F}} = q(\vec{\mathbf{v}}\times\vec{\mathbf{B}})$

$$7.6\times10^{-3}\,\hat{\mathbf{i}} - 5.2\times10^{-3}\,\hat{\mathbf{k}} = -7.8\times10^{-6}\times3.8\times10^{3}(B_z\,\hat{\mathbf{i}} - B_x\,\hat{\mathbf{k}})$$

$$\Rightarrow \quad B_x = -0.175\ \text{T},\ B_z = -0.256\ \text{T}$$

(b) Cannot be determined by this information.

(c) As $\vec{\mathbf{F}} = q(\vec{\mathbf{v}}\times\vec{\mathbf{B}})$

$$\vec{\mathbf{F}} \perp \vec{\mathbf{B}}$$

Hence, $\vec{\mathbf{B}}\cdot\vec{\mathbf{F}} = 0$

8. $\vec{\mathbf{B}} = B\,\hat{\mathbf{i}}$

(a) $\vec{\mathbf{v}} = v\,\hat{\mathbf{j}}$

$$\vec{\mathbf{F}} = q(\vec{\mathbf{v}}\times\vec{\mathbf{B}}) = -qvB\,\hat{\mathbf{k}}$$

(b) $\vec{\mathbf{v}} = v\,\hat{\mathbf{j}}$

$$\vec{\mathbf{F}} = q(\vec{\mathbf{v}}\times\vec{\mathbf{B}}) = qvB\,\hat{\mathbf{j}}$$

(c) $\vec{\mathbf{v}} = -v\,\hat{\mathbf{i}}$

$$\vec{\mathbf{F}} = q(\vec{\mathbf{v}}\times\vec{\mathbf{B}}) = 0$$

(d) $\vec{\mathbf{v}} = v\cos45^\circ\,\hat{\mathbf{i}} - v\cos45^\circ\,\hat{\mathbf{k}}$

$$\vec{\mathbf{F}} = q(\vec{\mathbf{v}}\times\vec{\mathbf{B}}) = -\frac{qvB}{\sqrt{2}}\,\hat{\mathbf{j}}$$

(e) $\vec{\mathbf{v}} = v\cos45^\circ\,\hat{\mathbf{j}} - v\cos45^\circ\,\hat{\mathbf{k}}$

$$\vec{\mathbf{F}} = q(\vec{\mathbf{v}}\times\vec{\mathbf{B}}) = \frac{qvB}{\sqrt{2}}(-\hat{\mathbf{j}} - \hat{\mathbf{k}})$$

$$= -\frac{qvB}{\sqrt{2}}(\hat{\mathbf{j}} + \hat{\mathbf{k}})$$

9. $r = \dfrac{mv}{qB} = \dfrac{\sqrt{2mk}}{eB} = \dfrac{\sqrt{2meV}}{eB}$

$$\Rightarrow \quad B = \sqrt{\frac{2mV}{e}}\,r$$

$$= \sqrt{\frac{2\times9.1\times10^{-31}\times2\times10^{3}}{1.6\times10^{-19}}}\times0.180$$

$$= 0.36 \times 10^{-4} \text{ T}$$

$$B = 3.6 \times 10^{-4} \text{ T}$$

10. (a) $r = \dfrac{mv}{qB} \Rightarrow v = \dfrac{qBr}{m}$

$$= \frac{1.6 \times 10^{-19} \times 2.5 \times 6.96 \times 10^{-3}}{3.34 \times 10^{-27}}$$

$$= 8.33 \times 10^{5} \text{ ms}^{-1}$$

(b) $t = \dfrac{T}{2} = \dfrac{\pi m}{qB}$

$$= \frac{3.14 \times 3.34 \times 10^{-27}}{1.6 \times 10^{-19} \times 2.5}$$

$$= 2.62 \times 10^{-8} \text{ s}$$

(c) $k = eV = \dfrac{1}{2} mv^2$

$$\Rightarrow V = \frac{mv^2}{2e}$$

$$= \frac{3.34 \times 10^{-27} \times (8.33 \times 10^{5})^2}{2 \times 1.6 \times 10^{-19}}$$

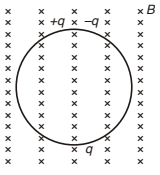
$$= 7.26 \times 10^{3} \text{ V}$$

$$= 7.26 \text{ kV}$$

11. (a) $-q$. As initially particle is neutral, charge on two particles must be equal and opposite.

(b) The will collide after completing half rotation, *i.e.*,

$$t = \frac{T}{2} = \frac{\pi m}{qB}$$



12. Here, $r = \dfrac{10.0}{2} = 5.0$ cm,

(a) $r = \dfrac{mv}{qB} \Rightarrow B = \dfrac{mv}{qr}$

$$= \frac{9.1 \times 10^{-31} \times 1.41 \times 10^{6}}{1.6 \times 10^{-19} \times 5 \times 10^{-2}}$$

$$= 1.6 \times 10^{-4} \text{ T}$$

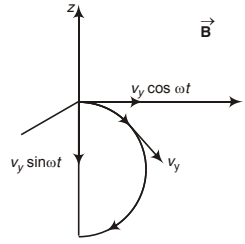
By Fleming's left hand rule, direction of magnetic field must be inward.

(b) $t = \dfrac{T}{2} = \dfrac{\pi m}{qB}$

$$= \frac{3.14 \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19} \times 1.6 \times 10^{-4}}$$

$$= 1.1 \times 10^{-7} \text{ s}$$

13. The component of velocity along the magnetic field (*i.e.*, v_x) will remain unchanged and the proton will move in a helical path.



At any instant,

Components of velocity of particle along Y-axis and Z-axis

$$v'_y = v_y \cos\theta = v_y \cos \omega t$$

and $$v'_z = -v_z \sin\theta = v_z \sin \omega t$$

where, $$\omega = \frac{qB}{m}$$

$$\therefore \vec{\mathbf{v}} = v_x \hat{\mathbf{i}} + v_y \cos \omega t \, \hat{\mathbf{j}} - v_z \sin \omega t \, \hat{\mathbf{k}}$$

14. For the electron to hit the target, distance GS must be multiple of pitch, *i.e.*,

$$GS = np$$

For minimum distance, $n = 1$

$$\Rightarrow \quad GS = p = \frac{2\pi mv \cos\theta}{qB}$$

$$\Rightarrow \quad p = \frac{2\pi\sqrt{2mk}\cos 60^\circ}{qB} \qquad (mv = \sqrt{2mk})$$

$$\Rightarrow \quad B = \frac{2\pi\sqrt{2mk}\cos 60^\circ}{qp}$$

$$= \frac{2 \times 3.14 \times \sqrt{2 \times 9.1 \times 10^{-31} \times 2 \times 1.6 \times 10^{-16} \times \frac{1}{2}}}{1.6 \times 10^{-19} \times 0.1}$$

$$\Rightarrow \quad B = 4.73 \times 10^{-4} \text{ T}$$

15. (a) From Question 5 (c)

Introductory Exercise 23.2

$$\frac{L}{R} = \sin\theta \Rightarrow L = R\sin\theta$$

$$R\sin 60^\circ = \frac{R}{2}$$

$$\Rightarrow \quad L = \frac{mv}{2qB} = \frac{mv_0}{2qB_0}$$

(b) Now, $L' = 2.1L = 1.05R$

As $L' > R$,

Particle will describe a semicircle and move out of the magnetic field moving in opposite direction, *i.e.,*

$$v' = -v = -v_0\,\hat{\mathbf{i}}$$

and $$t = \frac{T}{2} = \frac{\pi m}{qB_0}$$

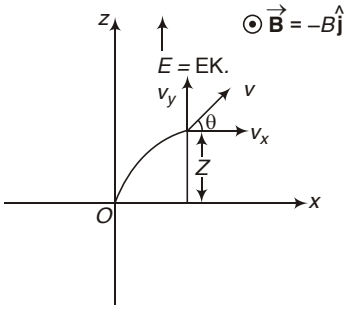
16. $\vec{\mathbf{v}} = (50\ \text{ms}^{-1})\,\hat{\mathbf{i}},\ \vec{\mathbf{B}} = (2.0\ \text{mT})\,\hat{\mathbf{j}}$

As particle move with uniform velocity,

$$\vec{\mathbf{F}} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}) = 0$$

$$\Rightarrow \quad \vec{\mathbf{E}} = \vec{\mathbf{B}} \times \vec{\mathbf{v}} = -(0.1\ \text{N/C})\hat{\mathbf{k}}$$

17. If v be the speed of particle at point $(0, y, z)$ then by work-energy theorem,



$$W = \Delta K = \frac{1}{2}mv^2$$

But work done by magnetic force is zero,

hence, network done = work done by electric force

$$= qEZ$$

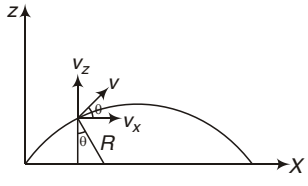
$$\therefore \quad qE_0Z = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{2qE_0Z}{m}}$$

As the magnetic field is along Y-axis, particle will move in XZ-plane.

The path of particle will be a cycloid. In this case, instantaneous centre of curvature of the particle will move along X-axis.

As magnetic force provides centripetal force to the particle,



$$qvB_0 = \frac{mv^2}{R}$$

$$v = \frac{qB_0R}{m}$$

$$v_x = v\cos\theta = \frac{qB_0R\cos\theta}{m}$$

$$= \frac{qB_0Z}{m} \quad (\because R\cos\theta = Z)$$

Now, $v_z = \sqrt{v^2 - v_x^2} = \sqrt{\dfrac{2qE_0Z}{m} - \dfrac{q^2B_0^2Z^2}{m^2}}$

18. Given, $\vec{\mathbf{E}} = E\,\hat{\mathbf{j}},\ \vec{\mathbf{B}} = B\,\hat{\mathbf{k}}$,

$$\vec{\mathbf{v}} = v\cos\theta\,\hat{\mathbf{j}} + v\sin\theta\,\hat{\mathbf{k}}$$

As protons are moving undeflected,

$$\vec{\mathbf{F}} = 0 \Rightarrow e(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}) = 0$$

$$\Rightarrow \quad e(E\,\hat{\mathbf{j}} - vB\cos\theta\,\hat{\mathbf{j}}) = 0$$

or $$v = \frac{E}{B\cos\theta}$$

Now, if electric field is switched off

$$p = \frac{2\pi mv\sin\theta}{qB} = \frac{2\pi mE\tan\theta}{qB^2}$$

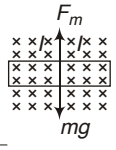
(Component of velocity along magnetic field $= v_z = v\sin\theta$)

19. $F = I\,l\,B\sin\theta$

$$I = \frac{F}{lB\sin\theta} = \frac{0.13}{0.2 \times 0.067 \times \sin 90°}$$

$$= 9.7\ \text{A}$$

20. For no tension in springs



$$F_m = mg$$

$$\Rightarrow \quad I\,lB = mg$$

$$I = \frac{mg}{lB} = \frac{13.0 \times 10^{-3} \times 10}{62.0 \times 10^{-2} \times 0.440}$$

$$= 0.48\ \text{A}$$

By Fleming left hand rule, for magnetic force to act in upward direction, current in the wire must be towards right.

21. (a) FBD of metal bar is shown in figure, for metal to be in equilibrium,

$$F_m + N = mg$$

$$\Rightarrow \quad F_m = mg - N$$

$$\Rightarrow \quad I\,lB = m - N$$

$$\Rightarrow \quad \frac{V}{R}lB = mg - N$$

$$\Rightarrow \quad V = \frac{R}{lB}(mg - N)$$

For largest voltage,

$$N = 0$$

$$V = \frac{R\,mg}{l\,B} = \frac{25 \times 750 \times 10^{-3} \times 9.8}{50.0 \times 10^{-2} \times 0.450}$$

$$= 817.5 \text{ V}$$

(b) If $I\,lB > mg$

$$I\,lB - mg = ma$$

$$a = \frac{I\,lB - mg}{m} = \frac{V\,lB}{Rm} - g$$

$$= \frac{817.5 \times 50 \times 10^{-2} \times 0.45}{2 \times 750 \times 10^{-3}} - 9.8$$

$$= 112.8 \text{ m/s}^2$$

22. $I = 3.50$ A, $l = -(1.00 \text{ cm})\hat{\mathbf{i}}$

$$\Rightarrow \qquad l = -(1.00 \times 10^{-2} \text{ m})\hat{\mathbf{i}}$$

(a) $\vec{\mathbf{B}} = -(0.65 \text{ T})\hat{\mathbf{j}}$

$$\vec{\mathbf{F}}_m = I(\vec{\mathbf{l}} \times \vec{\mathbf{B}}) = -(0.023 \text{ N})\hat{\mathbf{k}}$$

(b) $\vec{\mathbf{B}} = +(0.56 \text{ T})\hat{\mathbf{k}}$

$$\vec{\mathbf{F}}_m = I(\vec{\mathbf{l}} \times \vec{\mathbf{B}}) = (0.0196 \text{ N})\hat{\mathbf{j}}$$

(c) $\vec{\mathbf{B}} = -(0.33 \text{ T})\hat{\mathbf{i}}$

$$\vec{\mathbf{F}}_m = I(\vec{\mathbf{l}} \times \vec{\mathbf{B}}) = 0$$

(d) $\vec{\mathbf{B}} = (0.33 \text{ T})\hat{\mathbf{i}} - (0.28 \text{ T})\hat{\mathbf{k}}$

$$\vec{\mathbf{F}}_m = I(\vec{\mathbf{l}} \times \vec{\mathbf{B}}) = -(0.0098 \text{ N})\hat{\mathbf{j}}$$

(e) $\vec{\mathbf{B}} = +(0.74 \text{ T})\hat{\mathbf{j}} - (0.36 \text{ T})\hat{\mathbf{k}}$

$$\vec{\mathbf{F}}_m = I(\vec{\mathbf{l}} \times \vec{\mathbf{B}}) = -(0.0259 \text{ N})\hat{\mathbf{k}} + (0.0126 \text{ N})\hat{\mathbf{j}}$$

$$= (0.0126 \text{ N})\hat{\mathbf{j}} - (0.0259 \text{ N})\hat{\mathbf{K}}$$

23. $\vec{\mathbf{B}} = (0.020 \text{ T})\hat{\mathbf{j}}$

$$\vec{\mathbf{l}}_1 = \vec{\mathbf{ab}} = -(40.0 \text{ cm})\hat{\mathbf{j}}$$

$$= -(40.0 \times 10^{-2} \text{ m})\hat{\mathbf{j}}$$

$$\vec{\mathbf{F}}_1 = I(\vec{\mathbf{l}}_1 \times \vec{\mathbf{B}}) = 0$$

$$\vec{\mathbf{l}}_2 = \vec{\mathbf{bc}} = (40.0 \text{ cm})\hat{\mathbf{k}}$$

$$= -(400 \times 10^{-2} \text{ m})\hat{\mathbf{k}}$$

$$\vec{\mathbf{F}}_2 = I(\vec{\mathbf{l}}_2 \times \vec{\mathbf{B}}) = (0.04 \text{ N})\hat{\mathbf{i}}$$

$$\vec{\mathbf{l}}_3 = \vec{\mathbf{cd}} = -(40 \times 10^{-2})\hat{\mathbf{i}} + (40 \times 10^{-2} \text{ m})\hat{\mathbf{j}}$$

$$\vec{\mathbf{F}}_3 = I(\vec{\mathbf{l}}_3 \times \vec{\mathbf{B}}) = -(0.04 \text{ N})\hat{\mathbf{k}}$$

$$\vec{\mathbf{l}}_4 = \vec{\mathbf{da}} = (40 \times 10^{-2} \text{ m})\hat{\mathbf{i}} - (40 \times 10^{-2} \text{ m})\hat{\mathbf{k}}$$

$$\vec{\mathbf{F}}_4 = I(\vec{\mathbf{l}}_4 \times \vec{\mathbf{B}}) = (0.04 \text{ N})\hat{\mathbf{i}} + (0.04 \text{ N})\hat{\mathbf{k}}$$

24. $\vec{\mathbf{M}} = IA\,\hat{\mathbf{M}}$

$$= 0.20 \times \pi(8.0 \times 10^{-2})^2(0.60\,\hat{\mathbf{i}} - 0.80\,\hat{\mathbf{j}})$$

$$= (40.2 \times 10^{-4})(0.60\,\hat{\mathbf{i}} - 0.80\,\hat{\mathbf{j}})\text{A-m}^2$$

$$\vec{\mathbf{B}} = (0.25 \text{ T})\hat{\mathbf{i}} + (0.30 \text{ T})\hat{\mathbf{k}}$$

(a) $\vec{\tau} = \vec{\mathbf{M}} \times \vec{\mathbf{B}}$

$$= (40.2 \times 10^{-4})(-0.24\,\hat{\mathbf{i}} - 0.18\,\hat{\mathbf{j}} + 0.2\,\hat{\mathbf{k}})$$

$$= (-9.6\,\hat{\mathbf{i}} - 7.2\,\hat{\mathbf{j}} + 8.0\,\hat{\mathbf{k}}) \times 10^{-4} \text{ N-m.}$$

(b) $U = -\vec{\mathbf{M}} \cdot \vec{\mathbf{B}} = -(40.2 \times 10^{-4})(0.15)$ J

$$\approx -6.0 \times 10^{-4} \text{ J}$$

25. Consider the wire is bent in the form of a loop of N turns,

Radius of loop, $\quad r = \dfrac{L}{2\pi N}$

Magnetic dipole moment associated with the loop

$$M = NiA = Ni \times \pi r^2 = \frac{i\,L^2}{4\pi N^2}$$

$$\tau = MB \sin 90° = \frac{i\,L^2 B}{4\pi N}$$

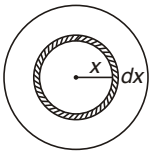
Clearly τ is maximum, when $N = 1$

and the maximum torque is given by

$$\tau_m = \frac{i\,L^2 B}{4\pi}$$

26. Consider the disc to be made up of large number of elementary rings. Consider on such ring of radius x and thickness dx.

Charge on this ring,



$$dq = \frac{q}{\pi R^2} \times 2\pi x\,dx = \frac{2q}{R^2}\,x\,dx$$

Current associated with this ring,

$$di = \frac{dq}{T} = \frac{\omega\, dq}{2\pi} = \frac{\omega q}{\pi R^2} x\, dx$$

Magnetic moment of this ring

$$dM = \pi x^2 di = \frac{\omega q}{R^2} x^3 dx$$

Magnetic moment of entire disc,

$$M = \int dM = \frac{\omega q}{R^2}\int_0^R x^3 dx = \frac{1}{4}\omega q R^2 \quad \ldots(i)$$

Magnetic field at the centre of disc due to the elementary ring under consideration

$$dB = \frac{\mu_0\, di}{2x} = \frac{\mu_0 \omega q^2}{2\pi R^2} dx$$

Net magnetic field at the centre of the disc,

$$B = \int dB = \frac{\mu_0\, \omega q}{2\pi R^2}\int_0^R dx = \frac{\mu_0 \omega q}{2\pi R}$$

$$\therefore \quad \frac{M}{B} = \frac{\pi R^3}{2\mu_0}$$

27. (a) By principle of conservation of energy,

Gain in KE = Loss in PE

$$\text{KE} = -\text{PE}\cos\theta + \text{ME}$$

$$f\cos\theta = 1 - \frac{K}{\text{ME}} = 1 - \frac{0.80\times10^{-3}}{0.02\times52\times10^{-3}}$$

$$= \frac{10}{13}$$

$$\theta = \cos^{-1}\frac{10}{13} = 76.7°$$

(b) $\theta = \cos^{-1}\frac{10}{13} = 76.7°$

Entire KE will again get converted into PE

28. $\Delta U = U_2 - U_1 = -MB - (+MB)$

$$= -2MB$$

$$= -2\times1.45\times0.835 = -2.42\text{ J}$$

29. (a) $T = \frac{2\pi r}{v} = \frac{2\times3.14\times5.3\times10^{-11}}{2.2\times10^6}$

$$= 1.5\times10^{-16}\text{ s}$$

(b) $i = \frac{e}{T} = \frac{1.6\times10^{-19}}{1.5\times10^{-16}} = 1.1\times10^{-3}\text{A}$

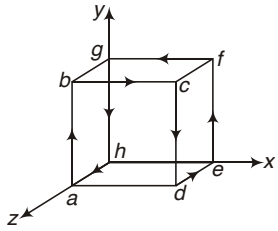
$$= 1.1\text{ mA}$$

(c) $M = \pi r^2 i$

$$= 3.14\times(5.3\times10^{-11})^2\times1.1\times10^{-3}$$

$$= 9.3\times10^{-24}\text{ A-m}^2$$

30. Suppose equal and opposite currents are flowing in sides $a\,d$ and $e\,h$, so that three complete current carrying loops are formed,



$$\vec{\mathbf{M}}_{abcd} = -i\,l^2\,\hat{\mathbf{k}}$$

$$\vec{\mathbf{M}}_{efgh} = i\,l^2\,\hat{\mathbf{k}}$$

$$\vec{\mathbf{M}}_{adeh} = i\,l^2\,\hat{\mathbf{j}}$$

$\therefore$ Total magnetic moment of the closed path,

$$\vec{\mathbf{M}} = \vec{\mathbf{M}}_{abcd} + \vec{\mathbf{M}}_{efgh} + \vec{\mathbf{M}}_{adeh} = i\,l^2\,\hat{\mathbf{j}}$$

31. Circuit is same as in Q.30

$$\vec{\mathbf{M}} = i\,l^2\,\hat{\mathbf{j}} = \hat{\mathbf{j}}$$

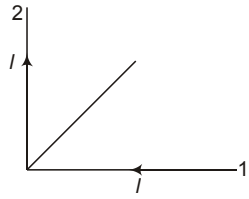
$$\vec{\mathbf{B}} = 2\,\hat{\mathbf{j}}$$

$$\vec{\tau} = \vec{\mathbf{M}}\times\vec{\mathbf{B}} = 0$$

32. $B_1 = \frac{\mu_0}{4\pi}\cdot\frac{I}{r}$

$$B_2 = \frac{\mu_0}{4\pi}\cdot\frac{I}{r}$$

Here, B_1 and B_2 are perpendicular to each other, hence,



$$B = \sqrt{B_1^2 + B_2^2}$$

$$= \frac{\mu_0}{4\pi}\cdot\frac{\sqrt{2}I}{r} = \frac{10^{-7}\times\sqrt{2}\times5}{35\times10^{-2}}$$

$$= 2.0\times10^{-6}\text{ T}$$

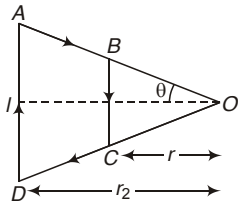
$$= 2.0\,\mu\text{T}$$

33. Clearly $\Delta BOC \sim \Delta AOB$

$$\therefore \quad \frac{r_2}{r_6} = \frac{AD}{BC}$$

$$\Rightarrow \quad r_2 = 2r$$

$$= 100\text{ mm}$$



and $AD = 2BC = 200$ mm

$$\theta = \cos^{-1}\frac{r}{\frac{BC}{2}} = 45°$$

$$B_{BC} = \frac{\mu_0}{4\pi}\frac{I}{r}[\sin 45° + \sin 45°]\frac{\sqrt{2}\,I}{r}$$

$$= \frac{\mu_0}{4\pi} \qquad \text{(outwards)}$$

$$B_{AD} = \frac{\mu_0}{4\pi}\cdot\frac{I}{r_2}(\sin 45° + \sin 45°)$$

$$= \frac{\mu_0}{4\pi}\cdot\frac{\sqrt{2}\,I}{r_2} \qquad \text{(inwards)}$$

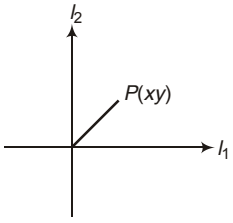
Net magnetic field at O.

$$B = B_{BC} - B_{AD} = \frac{\sqrt{2}\,\mu_0 I}{4\pi}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$= \sqrt{2}\times 10^{-7}\times\sqrt{2}\left[\frac{1}{50\times 10^{-3}} - \frac{1}{100\times 10^{-3}}\right]$$

$$= 2\times 10^{-6}\ \text{T} = 2\ \mu\text{T} \qquad \text{(outwards)}$$

34. Let us consider a point $P(x, y)$ where magnetic field is zero. Clearly the point must lie either in 1st quadrant or in 3rd quadrant.



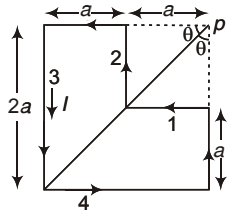
$$B = \frac{\mu_0}{4\pi}\cdot\frac{2I_1}{y} - \frac{\mu_0}{4\pi}\cdot\frac{2I_2}{x} = 0$$

$$\Rightarrow \qquad I_1 x = I_2 y$$

$$\Rightarrow \qquad y = \left(\frac{I_1}{I_2}\right)x$$

35. $\theta = 45°$

$$B_1 = B_2 = \frac{\mu_0}{4\pi}\cdot\frac{I}{a}(\sin\theta + \sin\theta)$$



$$= \frac{\mu_0}{4\pi}\cdot\frac{I}{a\sqrt{2}} \qquad \text{(inwards)}$$

$$B_3 = B_4 = \frac{\mu_0}{4\pi}\frac{I}{\sqrt{2}a}(\sin 0 + \sin 0)$$

$$= \frac{\mu_0}{4\pi}\cdot\frac{I}{2a\sqrt{2}} \qquad \text{(outwards)}$$

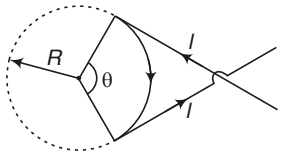
Net magnetic field at P

$$B = B_1 + B_2 - (B_3 + B_4)$$

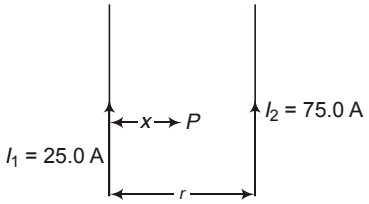
$$= \frac{\mu_0}{4\pi}\cdot\frac{I}{\sqrt{2}a} \qquad \text{(inwards)}$$

36. $B = 2\times\frac{\mu_0}{4\pi}\cdot\frac{I}{R} - \frac{\mu_0 I}{2R}\times\frac{\theta}{2\pi} = 0$

$$\Rightarrow \qquad \theta = 2\ \text{rad.}$$



37. (a) Consider a point P in between the two conductors at a distance x from conductor carrying current I_1 (= 25.0 A),



Magnetic field at P

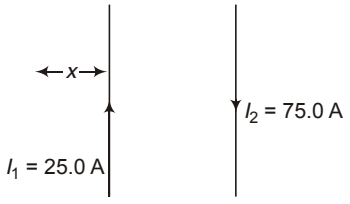
$$B = \frac{\mu_0}{4\pi}\cdot\frac{I_1}{x} - \frac{\mu_0}{4\pi}\cdot\frac{I_2}{r-x} = 0$$

$$\Rightarrow \qquad \frac{I_1}{x} = \frac{I_2}{r-x}$$

$$\Rightarrow \qquad \frac{r-x}{x} = \frac{I_2}{I_1}$$

$$\Rightarrow \qquad x = \frac{I_1}{I_1 + I_2}r = \frac{25.0}{100.0}\times 40 = 10\ \text{cm}$$

(b) Consider a point Q lying on the left of the conductor carrying current I_1 at a distance x from it.



$$B = \frac{\mu_0}{4\pi}\cdot\frac{I_1}{x} - \frac{\mu_0}{4\pi}\cdot\frac{I_2}{r+x} = 0$$

$$\Rightarrow \quad \frac{I_1}{x} = \frac{I_2}{r+x}$$

$$\Rightarrow \quad x = \frac{I_1}{I_2 - I_1} r = \frac{25.0}{50.0}\times 40$$

$$= 20 \text{ cm}$$

38. $B = \frac{\mu_0}{4\pi}\cdot\frac{2N\pi r^2 I}{(r^2+x^2)^{3/2}}$

But, $x = R$

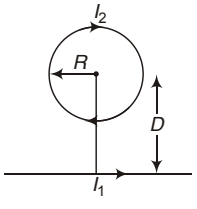
$$B = \frac{\mu_0 NI}{4\sqrt{2}r} \Rightarrow N = \frac{4\sqrt{2}\,Br}{\mu_0 I}$$

$$= \frac{4\sqrt{2}\times 6.39\times 10^{-4}\times 6\times 10^{-2}}{4\pi\times 10^{-7}\times 2.5}$$

$$\Rightarrow \quad N = 69$$

39. For magnetic field at the centre of loop to be zero, magnetic field due to straight conductor at centre of loop must be outward, hence I_1 must be rightwards.

At the centre of the loop



$$B = B_1 - B_2$$

$$= \frac{\mu_0}{4\pi}\cdot\frac{2I_1}{D} - \frac{\mu_0 I_2}{2R} = 0$$

$$I_1 = \frac{\pi D}{R} I_2$$

40. (a) $B = \frac{\mu_0 NI}{2R} \Rightarrow I = \frac{2BR}{\mu_0 N}$

$$= \frac{2\times 0.0580\times 2.40\times 10^{-2}}{4\pi\times 10^{-7}\times 800}$$

$$\Rightarrow \quad I = 2.77 \text{ A}$$

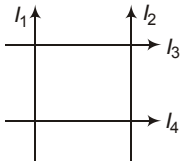
(b) On the axis of coil,

$$B = \frac{\mu_0}{4\pi}\cdot\frac{2NIA}{(r^2+x^2)^{3/2}}$$

$$\frac{B_C}{B} = \frac{(r^2+x^2)^{3/2}}{r^3} \Rightarrow \left(\frac{r^2+x^2}{r^2}\right)^{3/2} = 2$$

$$\Rightarrow \quad x = 0.0184 \text{ m}$$

41. Let the current $I_2 (= I)$ upwards



$$B = -B_1 + B_2 - B_3 + B_4$$

$$= \frac{\mu_0}{4\pi}\cdot\frac{2}{r}[-I_1 + I_2 - I_3 + I_4] = 0$$

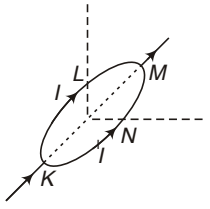
$$I_2 = I_1 + I_3 - I_4$$

$$= 10 + 8 - 20$$

$$= -2 \text{ A}$$

Negative sign indicates that current I is directed downwards.

42. $\vec{\mathbf{B}}_{KLM} = -\frac{\mu_0 I}{4R}\hat{\mathbf{i}}$



$$\vec{\mathbf{B}}_{KNM} = \frac{\mu_0 I}{4R}\hat{\mathbf{j}}$$

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_{KLM} + \vec{\mathbf{B}}_{KNM} = \frac{\mu_0 I}{4R}(-\hat{\mathbf{i}} + \hat{\mathbf{j}})$$

(a) $\vec{\mathbf{F}} = q(\vec{\mathbf{v}}\times\vec{\mathbf{B}}) = -\frac{\mu_0 Iqv}{4R}\hat{\mathbf{k}}$

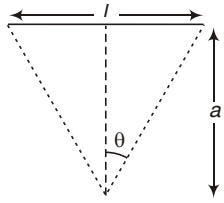
(b) $\vec{\mathbf{l}}_1 = \vec{\mathbf{l}}_2 = -2R\hat{\mathbf{k}}$

$$\vec{\mathbf{F}}_1 = I(\vec{\mathbf{l}}_1\times\vec{\mathbf{B}}) = 2IRB\hat{\mathbf{i}}$$

$$\vec{\mathbf{F}}_2 = I(\vec{\mathbf{l}}_2\times\vec{\mathbf{B}}) = 2IRB\hat{\mathbf{i}}$$

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = 4IRB\hat{\mathbf{i}}$$

43. (a) Length of each side



$$l = \frac{2\pi r}{n}$$

$$\theta = \frac{\pi}{n}$$

$$a = \frac{l}{2}\cot\theta = \frac{\pi r}{n}\cot\theta$$

$$B = n \times \frac{\mu_0}{4\pi}\cdot\frac{i}{a}(2\sin\theta)$$

$$= \frac{\mu_0}{4\pi}\cdot\frac{2n^2\sin\theta}{\pi r\cot\theta}$$

$$= \frac{\mu_0 i n^2 \sin^2\frac{\pi}{n}}{2\pi^2 r\cos\frac{\pi}{n}}$$

(b) $\lim_{n\to\infty} B = \lim_{n\to\infty}\frac{\mu_0 i n^2\sin^2\frac{\pi}{n}}{2\pi^2 r\cos\frac{\pi}{n}}$

$\Rightarrow \quad \lim_{n\to 0} = \frac{\mu_0 i}{2r}$

44. $\oint \vec{\mathbf{B}}\cdot\vec{\mathbf{dl}} = 3.83\times10^{-7}$ T-m

(a) By Ampere's circuital law

$\oint \vec{\mathbf{B}}\cdot\vec{\mathbf{dl}} = \mu_0 I$

$\Rightarrow I = \frac{1}{\mu_0}\oint\vec{\mathbf{B}}\cdot\vec{\mathbf{dl}} = \frac{1}{4\pi\times10^{-7}}\times3.83\times10^{-7}$

$= 0.3\text{A}$

(b) If we integrate around the curve in the opposite direction, the value of line integral will become negative, *i.e.*,

-3.83×10^{-7} T-m.

45. $\oint\vec{\mathbf{B}}\cdot\vec{\mathbf{dl}} = \mu_0 I$

As the path is taken counter-clockwise direction, $\oint\vec{\mathbf{B}}\cdot\vec{\mathbf{dl}}$ will be positive if current is outwards and will be negative if current is inwards.

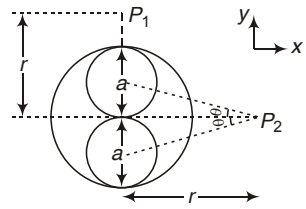
$\oint_a \vec{\mathbf{B}}\cdot\vec{\mathbf{dl}} = 0$

$\oint_b \vec{\mathbf{B}}\cdot\vec{\mathbf{dl}} = -\mu_0 I_1 = -5.0\times10^{-6}$ T-m

$\oint_c \vec{\mathbf{B}}\cdot\vec{\mathbf{dl}} = \mu_0(I_2 - I_1) = 2.5\times10^{-6}$ T-m

$\oint_d \vec{\mathbf{B}}\cdot\vec{\mathbf{dl}} = \mu_0(I_2 + I_3 - I_1) = 5.0\times10^{-6}$ T-m

46.



Current density

$$J = \frac{I}{\pi a^2 - 2\pi\left(\frac{a}{2}\right)^2} = \frac{2I}{\pi a^2}$$

Let us consider both the cavities are carrying equal and opposite currents with current density J.

Let B_1, B_2 and B_3 be magnetic fields due to complete cylinder, upper and lower cavity respectively.

(a) At point P_1

$$\vec{\mathbf{B}}_1 = -\frac{\mu_0}{4\pi}\cdot\frac{2I_1}{r}\hat{\mathbf{i}} = -\frac{\mu_0}{4\pi}\cdot\frac{2J\times\pi a^2}{r}\hat{\mathbf{i}}$$

$$= -\frac{\mu_0 I}{\pi r}\hat{\mathbf{i}}$$

$$\vec{\mathbf{B}}_2 = \frac{\mu_0}{4\pi}\cdot\frac{2I_2}{r-\frac{a}{2}}\hat{\mathbf{i}} = \frac{\mu_0}{4\pi}\cdot\frac{2J\times\pi\left(\frac{a}{2}\right)^2}{r-\frac{a}{2}}\hat{\mathbf{i}}$$

$$= -\frac{\mu_0 I}{4\pi\left(r-\frac{a}{2}\right)}\hat{\mathbf{i}}$$

$$\vec{\mathbf{B}}_3 = \frac{\mu_0}{4\pi}\cdot\frac{2I_3}{r+\frac{a}{2}}\hat{\mathbf{i}} = \frac{\mu_0}{4\pi\left(r+\frac{a}{2}\right)}\hat{\mathbf{i}}$$

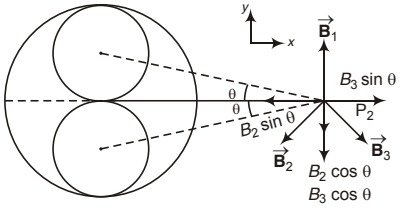
$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 + \vec{\mathbf{B}}_3$$

$$= \frac{\mu_0 I}{4\pi}\left[-\frac{4}{r} + \frac{1}{r-\frac{a}{2}} + \frac{1}{r+\frac{a}{2}}\right]\hat{\mathbf{i}}$$

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi r}\left[\frac{2r^2 - a^2}{4r^2 - a^2}\right]\hat{\mathbf{i}}$$

$$\therefore (\vec{\mathbf{B}}) = \frac{\mu_0 I}{4\pi r}\left[\frac{2r^2 - a^2}{4r^2 - a^2}\right], \text{ towards left.}$$

(b) At point P_2



$$\vec{\mathbf{B}}_1 = \frac{\mu_0}{4\pi}\cdot\frac{2I_1}{r}\hat{\mathbf{j}} = \frac{\mu_0 I}{\pi r}\hat{\mathbf{j}}$$

$$\vec{\mathbf{B}}_2 = \frac{\mu_0}{4\pi}\cdot\frac{2I_2}{\sqrt{r^2 + \frac{a^2}{4}}}[-\sin\theta\,\hat{\mathbf{i}} - \cos\theta\,\hat{\mathbf{j}}]$$

$$= \frac{-\mu_0 I}{2\pi\sqrt{4r^2 + a^2}}[\sin\theta\,\hat{\mathbf{i}} + \cos\theta\,\hat{\mathbf{j}}]$$

$$\vec{\mathbf{B}}_3 = \frac{\mu_0}{4\pi}\cdot\frac{2I_3}{\sqrt{r^2 + \frac{a^2}{4}}}[\sin\theta\,\hat{\mathbf{i}} - \cos\theta\,\hat{\mathbf{j}}]$$

$$= \frac{\mu_0 I}{2\pi\sqrt{4r^2 + a^2}}[\sin\theta\,\hat{\mathbf{i}} - \cos\theta\,\hat{\mathbf{j}}]$$

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 + \vec{\mathbf{B}}_3$$

$$= \frac{\mu_0 I}{2\pi}\left[\frac{2}{r} - \frac{2\cos\theta}{\sqrt{4r^2 + a^2}}\right]\hat{\mathbf{j}}$$

but, $\cos\theta = \dfrac{r}{\sqrt{r^2 + \frac{a^2}{4}}} = \dfrac{2r}{\sqrt{4r^2 + a^2}}$

$$\therefore \quad \vec{\mathbf{B}} = \frac{\mu_0 I}{2\pi}\left[\frac{2}{r} - \frac{4r}{4r^2 + a^2}\right]\hat{\mathbf{j}}$$

$$= \frac{\mu_0 I}{4\pi r}\left[\frac{2r^2 + a^2}{4r^2 + a^2}\right]\hat{\mathbf{j}}$$

$$(\vec{\mathbf{B}}) = \frac{\mu_0 I}{4\pi r}\left[\frac{2r^2 + a^2}{4r^2 + a^2}\right], \text{ upwards.}$$

47. Let us first find magnetic field due a current carrying infinite plate.

Consider a rectangular amperian loop (*WXYZ*) as shown in Fig. 1.

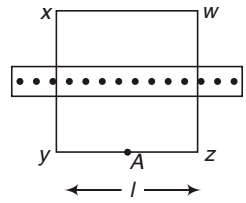


Fig.1

$$\oint_{WXYZ} \vec{\mathbf{B}}\cdot\vec{\mathbf{dl}} = \mu_0 \lambda l$$

$$\Rightarrow \int_{W\to X}\vec{\mathbf{B}}\cdot\vec{\mathbf{dl}} + \int_{X\to Y}\vec{\mathbf{B}}\cdot\vec{\mathbf{dl}} + \int_{Y\to Z}\vec{\mathbf{B}}\cdot\vec{\mathbf{dl}} + \int_{Z\to W}\vec{\mathbf{B}}\cdot\vec{\mathbf{dl}} = \mu_0\lambda l$$

$$B\,l + 0 + B\,l + 0 = \mu_0\lambda l$$

$$B = \frac{1}{2}\mu_0\lambda$$

In Fig. 2.

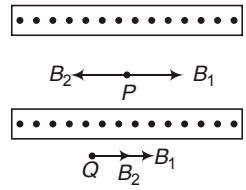


Fig.2

At point P,

$$B_1 = B_2 = \frac{1}{2}\mu_0\lambda$$

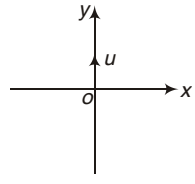
$$B = B_1 - B_2 = 0,$$

At point Q,

$$B_1 = B_2 = \frac{1}{2}\mu_0\lambda$$

$$B = B_1 + B_2 = \mu_0\lambda$$

48. $\vec{\mathbf{B}} = \dfrac{\mu_0}{4\pi}\cdot\dfrac{I\,\vec{\mathbf{dl}}\times\vec{\mathbf{r}}}{r^3} = \dfrac{\mu_0}{4\pi}\cdot\dfrac{q(\vec{\mathbf{v}}\times\vec{\mathbf{r}})}{r^3}$



$$\vec{\mathbf{v}} = (8.00\times10^6\ \text{ms}^{-1})\hat{\mathbf{j}}$$

(a) $\vec{\mathbf{r}} = (0.500\ \text{m})\hat{\mathbf{i}}$

$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\cdot\frac{q(\vec{\mathbf{v}}\times\vec{\mathbf{r}})}{r^3}$$

$$= \frac{10^{-7} \times 6.00 \times 10^{-6}[(8.00 \times 10^{6}\,\hat{\mathbf{j}}) \times (0.500)\,\hat{\mathbf{i}}\,]}{(0.500)^3}$$

$$\Rightarrow \vec{\mathbf{B}} = -(1.92 \times 10^{-5}\ \text{T})\,\hat{\mathbf{k}}$$

(b) $\vec{\mathbf{r}} = -(0.500\ \text{m})\,\hat{\mathbf{j}}$

$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \cdot \frac{q(\vec{\mathbf{v}} \times \vec{\mathbf{r}})}{r^3} = 0$$

(c) $\vec{\mathbf{r}} = +(0.500\ \text{m})\,\hat{\mathbf{k}}$

$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \cdot \frac{q(\vec{\mathbf{v}} \times \vec{\mathbf{r}})}{r^3} = (1.92 \times 10^{-5}\ \text{T})\,\hat{\mathbf{i}}$$

(d) $\vec{\mathbf{r}} = -(0.50\ \text{m})\,\hat{\mathbf{j}} + 0.500\ \text{m}\,\hat{\mathbf{k}}$

$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \cdot \frac{q(\vec{\mathbf{v}} \times \vec{\mathbf{r}})}{r^3} = (1.92 \times 10^{-5}\ \text{T})\,\hat{\mathbf{i}}$$

49. $q = -4.80\ \mu\text{C} = -4.80 \times 10^{-6}\ \text{C}$

$\vec{\mathbf{v}} = (6.80 \times 10^{5}\ \text{m/s})\,\hat{\mathbf{i}}$

(a) $\vec{\mathbf{r}} = (0.500\ \text{m})\,\hat{\mathbf{i}}$

$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \cdot \frac{q(\vec{\mathbf{v}} \times \vec{\mathbf{r}})}{r^3} = 0$$

(b) $\vec{\mathbf{r}} = (0.500\ \text{m})\,\hat{\mathbf{j}}$

$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \cdot \frac{q(\vec{\mathbf{v}} \times \vec{\mathbf{r}})}{r^3} = -(1.3 \times 10^{-6}\ \text{T})\,\hat{\mathbf{k}}$$

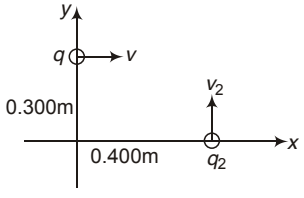
(c) $\vec{\mathbf{r}} = (0.500\ \text{m})\,\hat{\mathbf{i}} + (0.500\ \text{m})\,\hat{\mathbf{j}}$

$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \cdot \frac{q(\vec{\mathbf{v}} \times \vec{\mathbf{r}})}{r^3} = -(1.31 \times 10^{-6}\ \text{T})\,\hat{\mathbf{k}}$$

(d) $\vec{\mathbf{r}} = (0.500\ \text{m})\,\hat{\mathbf{k}}$

$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \cdot \frac{q(\vec{\mathbf{v}} \times \vec{\mathbf{r}})}{r^3} = (1.31 \times 10^{-6}\ \text{T})\,\hat{\mathbf{j}}$$

50. $\vec{\mathbf{B}}_1 = \dfrac{\mu_0}{4\pi} \cdot \dfrac{q_1(\vec{\mathbf{v}}_1 \times \vec{\mathbf{r}}_2)}{r_1^3}$



$$\Rightarrow \vec{\mathbf{B}}_1 = \frac{10^{-7} \times 4.00 \times 10^{-6}\,[(2.00 \times 10^{5}\,\hat{\mathbf{i}}) \times (-0.300\,\hat{\mathbf{j}})]}{(0.300)^3}$$

$$= -(8.89 \times 10^{-7}\ \text{T})\,\hat{\mathbf{k}}$$

$$\vec{\mathbf{B}}_2 = \frac{\mu_0}{4\pi} \cdot \frac{q_2(\vec{\mathbf{v}}_2 \times \vec{\mathbf{r}}_2)}{r_2^3}$$

$$\Rightarrow \vec{\mathbf{B}}_2 = \frac{10^{-7} \times (-1.5 \times 10^{-6})[(8.00 \times 10^{5}\,\hat{\mathbf{i}}) \times (-0.400\,\hat{\mathbf{j}})]}{(0.400)^2}$$

$$= -(7.5 \times 10^{-7}\ \text{T})\,\hat{\mathbf{k}}$$

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 = -(16.4 \times 10^{-6}\ \text{T})\,\hat{\mathbf{k}}$$

$$= -(1.64 \times 10^{-6}\ \text{T})\,\hat{\mathbf{k}}$$

or $\quad B = 1.64 \times 10^{-6}\ \text{T}$ (inwards)

51. Magnetic force per unit length on the conductor AB,

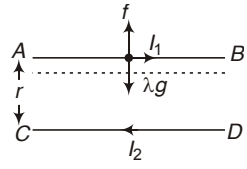
$$f = \frac{\mu_0}{4\pi} \cdot \frac{2I_1I_2}{r}$$

For equilibrium

$$f = \frac{m}{l}\,g = \lambda\,g$$

$$\Rightarrow \quad \lambda g = \frac{\mu_0}{4\pi} \cdot \frac{2I_1I_2}{r} \qquad \ldots\text{(i)}$$

Suppose wire AB is depressed by x,



Net force on unit length of wire AB

$$\lambda a = \lambda g - f'$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{2I_1I_2}{r} - \frac{\mu_0}{4\pi} \cdot \frac{2I_1I_2}{r - x}$$

$$= -\frac{\mu_0}{4\pi} \cdot \frac{2\,I_1I_2 \cdot x}{r(r - x)}$$

If $x << r$

$$\lambda a = -\frac{\mu_0 I_1 I_2}{2r^2}\,x$$

$$a = -\frac{\mu_0 I_1 I_2}{2\lambda\,r^2}\,x \qquad \ldots\text{(ii)}$$

General equation of SHM

$$a = -\omega^2 x \quad \text{...(ii)}$$

Hence, motion of wire AB will be simple harmonic.

From Eqs. (i) and (ii),

$$\omega = \sqrt{\frac{\mu_0 I_1 I_2}{2\lambda r^2}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2\lambda r^2}{\mu_0 I_1 I_2}} = 2\pi\sqrt{\frac{r}{g}}$$

$$= 2 \times 3.14\sqrt{\frac{0.01}{9.8}}$$

$$= 0.2 \text{ s}$$

52. (a) $f = \frac{\mu_0}{4\pi} \cdot \frac{2I_1 I_2}{r}$

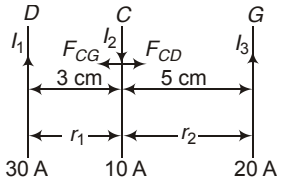
$$I_2 = \frac{f\,r}{\frac{\mu_0}{4\pi} \cdot 2I_1}$$

$$= \frac{4.00 \times 10^{-5} \times 2.50 \times 10^{-2}}{10^{-7} \times 2 \times 0.600}$$

$$= 8.33 \text{ A}$$

(b) As the wires repel each other, current must be in opposite directions.

53. $f_{CD} = \frac{\mu_0}{4\pi} \cdot \frac{2I_1 I_2}{r_1}$



$$f_{CG} = \frac{\mu_0}{4\pi} \cdot \frac{2I_2 I_3}{r_2}$$

$$f = f_{CD} - f_{CG}$$

$$f = f_{CD} - f_{CG} = \frac{\mu_o}{4\pi} \cdot 2I_2 \left[\frac{I_1}{r_1} - \frac{I_3}{r_2}\right]$$

$$= 10^{-7} \times 2 \times 10 \left[\frac{30}{3 \times 10^{-2}} - \frac{20}{5 \times 10^{-2}}\right]$$

$$f = 12 \times 10^{-4} \text{ N} = 1.2 \times 10^{-3} \text{ N/m}$$

$$F = f\,l = 1.2 \times 10^{-3} \times 25 \times 10^{-2}$$

$$= 3 \times 10^{-4} \text{ N}$$

54. Force per unit length on wire MN

$$f_{MN} = \frac{\mu_0}{4\pi} \cdot \frac{2I_1 I_2}{a}$$

$$F = f_{MN} \times L = \frac{\mu_0 I_1 I_2 L}{2\pi a}$$

Torque acting on the loop is zero because magnetic field is parallel to the area vector.

Objective Questions (Level 1)

1. Fact

2. $T = \frac{2\pi m}{qB}$ is independent of speed.

3. Outside the wire

$B = \frac{\mu_0}{4\pi} \cdot \frac{2I}{r}$ where, r is distance from the centre.

4. The path will be parabola if force acting on the particle is constant in magnitude as well as in direction.

5. $B = \frac{\mu_0}{4\pi} \cdot \frac{2I}{r}$

$$\mu_0 = \frac{4\pi r B}{2I}$$

$$\text{Units of } \mu_0 = \frac{\text{m} \times \text{Wb/m}^2}{\text{A}}$$

$$= \text{Wbm}^{-1}\text{A}^{-1}$$

6. Fact

7. $\vec{\mathbf{M}} = i\,\vec{\mathbf{A}}$, where $\vec{\mathbf{A}}$ = Area vector.

8. Force acting on a closed current carrying loop is always zero.

9. $M = NIA$

10. $\vec{\mathbf{a}} \perp \vec{\mathbf{B}} \Rightarrow \vec{\mathbf{a}} \cdot \vec{\mathbf{B}} = 0$

$$(x\,\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) \cdot (2\,\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}} + 4\,\hat{\mathbf{k}}) = 2x + 3 - 4 = 0$$

$$\Rightarrow x = 0.5$$

12. A current carrying closed loop never experiences a force magnetic field.

13. $r = \frac{mv}{qB} = \frac{P}{qB}$,

$$P = mv = \text{momentum.}$$

$$\therefore \quad r \propto \frac{1}{q}$$

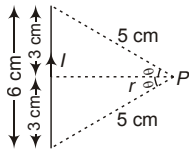
$$\therefore \quad \frac{r_p}{r_\alpha} = \frac{q_\alpha}{q_p} \Rightarrow r_P : r_\alpha = 2:1$$

14. $W = MB(\cos\theta_1 - \cos\theta_2)$

Here, $\theta_1 = \pi$, $\theta_2 = \pi - \theta$

$$W = MB(\cos\pi - \cos(\pi - \theta))$$
$$= -MB(1 - \cos\theta)$$

15. $B_P = \frac{\mu_0}{4\pi} \cdot \frac{I}{r}(2\sin\theta)$



$$r = \sqrt{5^2 - 3^2} = 4 \text{ cm}$$
$$= 4 \times 10^{-2} \text{ m}$$

$\sin\theta = \frac{3}{5}$

$$\therefore \quad B_P = \frac{10^{-7} \times 50 \times 2 \times \frac{3}{5}}{4 \times 10^{-2}}$$
$$= 1.5 \times 10^{-4} \text{ T}$$
$$= 1.5 \text{ gauss.}$$

16. Magnetic field on the axis of current carrying circular loop,

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2M}{(r^2 + x^2)^{3/2}} \quad \ldots(i)$$

Magnetic field at the centre of current carrying circular loop,

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3} \quad \ldots(ii)$$

From Eqs. (i) and (ii),

$$\frac{B_2}{B_1} = \frac{(r^2 + x^2)^{3/2}}{r^3}$$
$$= \frac{(3^2 + 4^2)^{3/2}}{3^3}$$
$$= \frac{125}{27}$$
$$\Rightarrow \quad B_2 = \frac{125}{27} \times 54 = 150\ \mu\text{T}$$

17. $\vec{\mathbf{F}} = I(\vec{\mathbf{l}} \times \vec{\mathbf{B}}) = I(\vec{\mathbf{ba}} \times \vec{\mathbf{B}})$

$$= -I(\vec{\mathbf{ab}} \times \vec{\mathbf{B}}) = I(\vec{\mathbf{B}} \times \vec{\mathbf{ab}})$$

18. Kinetic energy of electron,

$$K = \frac{1}{2}mv^2 = eV$$

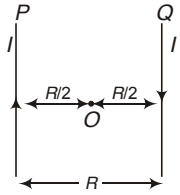
$$\Rightarrow \quad v = \sqrt{\frac{2eV}{m}}$$

Magnetic force,

$$F_m = evB\sin\theta$$
$$F_m \propto v \Rightarrow F_m \propto \sqrt{v}$$

Hence, if potential difference is doubled, force will become $\sqrt{2}$ times.

19. Magnetic field at O due to P,



$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2I}{R/2} = \frac{\mu_0 I}{\pi R} \quad \text{(inwards)}$$

Magnetic field at O due to Q,

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2I}{R/2} = \frac{\mu_0 I}{\pi R} \quad \text{(inwards)}$$

Net magnetic field at O,

$$B = B_1 + B_2 = \frac{2\mu_0 I}{\pi R}$$

20. As solved in Question 16,

$$\frac{B_2}{B_1} = \left(\frac{x^2 + R^2}{R^2}\right)^{3/2}$$
$$\Rightarrow \quad \left(\frac{x^2 + R^2}{R^2}\right)^{3/2} = 8$$
$$\Rightarrow \quad \frac{x^2 + R^2}{R^2} = 4$$
$$\Rightarrow \quad x = \sqrt{3}\,R$$

21. Component of velocity of particle along magnetic field, *i.e.*,

$$v_y = \frac{qE}{m}t = \alpha E t$$

is not constant, hence pitch is variable.

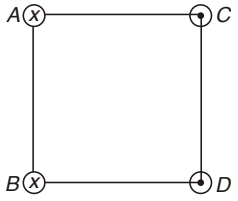
22. $r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$

Now, $R = \frac{\sqrt{2mK}}{eB}$

$$R' = \frac{\sqrt{2m(2K)}}{e(3R)} = \frac{\sqrt{2}}{3}R$$

23. Same as question 1. Introductory exercise 23.6.

Note. Her diagram is wrong correct diagram should be



24. $r = \dfrac{mv}{qB} = \dfrac{\sqrt{2mK}}{qB} = \dfrac{\sqrt{2mqV}}{qB}$ $[K = qV]$

$\Rightarrow \qquad r = \sqrt{\dfrac{2mV}{q}}\left(\dfrac{1}{B}\right)$

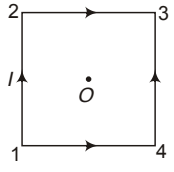
25. Magnetic field due to a conductor of finite length.

$$B = \frac{\mu_0}{4\pi}\cdot\frac{I}{r}(\sin\alpha + \sin\beta)$$

Here, $\alpha = -\theta_2, \beta = \theta_1$ and $r = a$

$\therefore \qquad B = \dfrac{\mu_0 I}{2a}(\sin\theta_1 - \sin\theta_2)$

26. In case C, magnetic field of conductor 1-2 and 2-3 at O is inward while those of 3-4 and 4-1 at O is outward, hence net magnetic field at O in this case is zero.

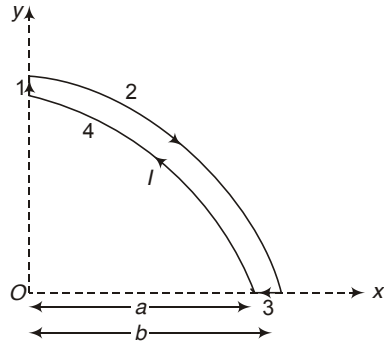


27. $\vec{\mathbf{dF}} = I(\vec{\mathbf{dl}} \times \vec{\mathbf{B}})$

But $\vec{\mathbf{B}} \,||\, \vec{\mathbf{dl}}$ at every point,

hence, $\vec{\mathbf{dF}} = 0$.

28. $B_1 = B_3 = 0$ (Magnetic field on the axis of current carrying straight conductor is zero)



$$\vec{\mathbf{B}}_2 = \frac{-1}{4}\left(\frac{\mu_0 I}{2b}\right)\hat{\mathbf{k}} = -\frac{\mu_0 I}{8b}\hat{\mathbf{k}},$$

$$\vec{\mathbf{B}}_3 = \frac{1}{4}\left(\frac{\mu_0 I}{2a}\right)\hat{\mathbf{k}} = \frac{\mu_0 I}{8a}\hat{\mathbf{k}}$$

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 + \vec{\mathbf{B}}_3 + \vec{\mathbf{B}}_4$$

$$= \frac{\mu_0 I}{8}\left[\frac{1}{a} - \frac{1}{b}\right]\hat{\mathbf{k}}$$

29. Current associated with electron,

$$I = \frac{q}{T} = ef$$

$$B = \frac{\mu_0 I}{2R} = \frac{\mu_0 ef}{2R}$$

30. Same as question 1(a). Introductory Exercise 23.5.

31. At point 1,

Magnetic field due to inner conductor is non-zero, but due to outer conductor is zero.

Hence, $B_1 \neq 0$

At point 2,

Magnetic field due to both the conductors is equal and opposite.

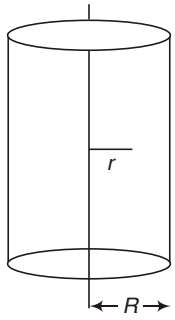
Hence, $B_2 = 0$

32. Apply Fleming's left hand rule or right hand thumb rule.

33. Magnetic field due to straight conductors at O is zero because O lies on axis of both the conductors.

Hence, $B = \dfrac{\phi}{2\pi}\cdot\dfrac{\mu_0 I}{2x} = \dfrac{\mu_0 I\phi}{4\pi x}$

34. Inside a solid cylinder having uniform current density,



$$B = \frac{\mu_0 I r}{2\pi R^2}$$

Here, $r = R - x$

$$\therefore \quad B = \frac{\mu_0 I (R - x)}{2\pi R^2}$$

35. Magnetic force is acting radially outward on the loop.

JEE Corner

Assertion and Reason

1. For parabolic path, acceleration must be constant and should not be parallel or antiparallel to velocity.

2. By Fleming's left hand rule.

3. Magnetic force on upper wire must be in upward direction, hence current should be in a direction opposite to that of wire 1.

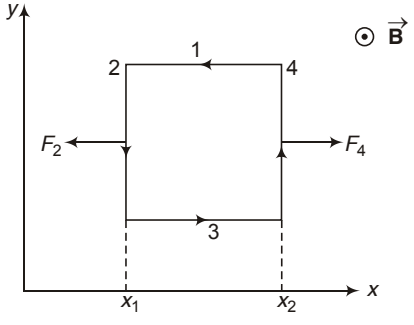
Reason is also correct but does not explain Assertion.

4. $\tau = MB \sin \alpha$

$\alpha = 90°$

$\therefore \tau = MB \neq 0$

5. $F_2 = I\, l B_O x_1$



$$F_4 = I\, l B_0 x_2$$

$$\therefore \quad F_4 > F_2$$

Hence, net force is along X-axis.

6. Radii of both is different because mass of both is different

$$r = \frac{mv}{qB} = \frac{\sqrt{2meV}}{eB}$$

7. For equilibrium

$\vec{\mathbf{F}}_e + \vec{\mathbf{F}}_m = 0$

$$\Rightarrow \quad q\vec{\mathbf{E}} = -q(\vec{\mathbf{v}} \times \vec{\mathbf{B}})$$

$$\Rightarrow \quad \vec{\mathbf{E}} = -\vec{\mathbf{v}} \times \vec{\mathbf{B}} = \vec{\mathbf{B}} \times \vec{\mathbf{v}}$$

8. $P_m = \vec{\mathbf{F}}_m \cdot \vec{\mathbf{v}}$

As $\vec{\mathbf{F}}_m$ is always perpendicular to $\vec{\mathbf{v}}$,

$$\vec{\mathbf{P}}_m = 0$$

Again, $P_e = \vec{\mathbf{F}}_e \cdot \vec{\mathbf{v}}$, may or may not be zero.

9. Reason correctly explains Assertion.

10. Magnetic force cannot change speed of particle as it is always perpendicular to the speed of the particle.

11. $a = \dfrac{v^2}{R}$

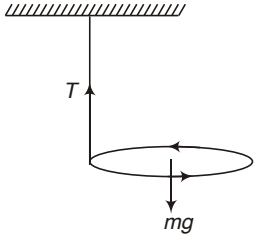
but R also depends on v.

$$\therefore \quad a = \frac{F_m}{m} = \frac{qvB}{m}$$

$$\Rightarrow \quad a \propto v$$

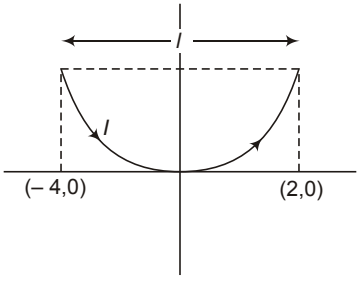
Objective Questions (Level 2)

1. For net torque to be zero.



$$IAB_0 = mgR$$
$$I = \frac{mgR}{AB_0} = \frac{mgR}{\pi R^2 B_0}$$
$$= \frac{mg}{\pi RB_0}$$

2. As it is clear from diagram,



Effective length of wire,

$$\vec{\mathbf{l}} = (4\text{ m})\hat{\mathbf{i}}$$
$$\vec{\mathbf{F}} = I(\vec{\mathbf{l}} \times \vec{\mathbf{B}})$$
$$\vec{\mathbf{a}} = \frac{\vec{\mathbf{F}}}{m} = \frac{I}{m}(\vec{\mathbf{l}} \times \vec{\mathbf{B}})$$
$$= \frac{2}{0.1}(4\hat{\mathbf{i}} \times (-0.02\hat{\mathbf{k}})) = 1.6\hat{\mathbf{j}}\text{ m/s}^2$$

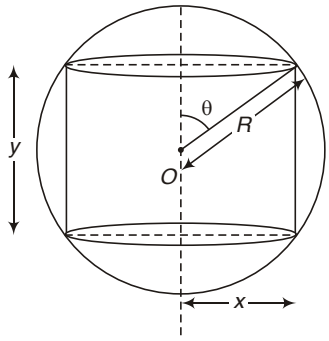
3. Impulse = Change in momentum

$$\int I\,lB\,dt = mv - 0$$
$$lB\int dq = mv$$
$$dq = \frac{mv}{lB} = \frac{m\sqrt{2gh}}{lB}$$

4. Consider the sphere to be made up of large number of hollow, coaxial cylinder of different height and radius. Consider one such cylinder of radius x, height y and thickness.

Now, $y = 2R\cos\theta$, $x = R\sin\theta$, $dx = R\cos\theta\, d\theta$

Charge on this cylinder,



$$dq = \frac{q}{\frac{4}{3}\pi R^3}\cdot(2\pi\, yx\, dx)$$
$$= 3q\cos^2\theta\sin\theta\, d\theta$$

Current associated with this cylinder,

$$di = \frac{dq}{T} = \frac{\omega\, dq}{2\pi} = \frac{3\omega q}{2\pi}\cos^2\theta\sin\theta\, d\theta$$

Magnetic moment associated with this cylinder,

$$dM = di\,A = \frac{3q\omega}{2\pi}\cos^2\theta\sin\theta\, d\theta \times \pi x^2$$
$$dM = \frac{3}{2}R^2\omega qA\cos^2\theta\sin^3\theta\, d\theta$$
$$M = \int dM = \frac{3}{2}R^2 q\int_{\pi 2}^{0}\cos^2\theta\sin^3\theta\, d\theta$$
$$= \frac{3}{2}R^2\omega q\int_{\pi/2}^{0}\cos^2\theta(1-\cos^2\theta)\sin\theta\, d\theta$$
$$= \frac{3}{2}R^2\omega q\left[\frac{\cos^3\theta}{3} - \frac{\cos^5\theta}{5}\right]_{\pi/2}^{0}$$
$$= \frac{1}{5}R^2\omega q$$

5. As solved in question 5(c). Introductory Exercise 23.2.

$$\frac{L}{R} = \sin\theta$$

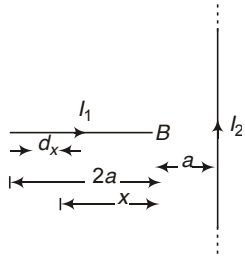
Here, $L = d$, $R = \frac{mV}{qB}$

$\therefore$ $$\frac{qB\,d}{mV} = \sin\theta$$

or $$\frac{q}{m} = \frac{V\sin\theta}{Bd}$$

6. Force on portion AC will more compared to that on portion CB.

7. Consider an elementary portion of the wire carrying current I_1 of length dx at a distance x from end B.



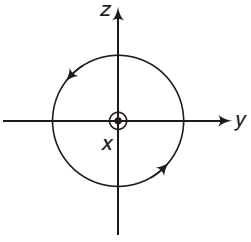
Force on this portion

$$dF = I_1 dx\, B$$
$$= \frac{\mu_0}{4\pi} \cdot \frac{2 I_1 I_2}{a + x} dx$$

Total force on wire AB

$$F = \int dF = \frac{\mu_0}{4\pi} \cdot 2 I_1 I_2 \int_a^{2a} \frac{dx}{a + x}$$
$$= \frac{\mu_0 I_1 I_2}{2\pi} \ln 3$$

8. Magnetic field line due to current carrying conductor is shown in figure.



9.
$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2 I A_1}{(x_1^2 + r_1^2)^{3/2}}$$
$$= \frac{\mu_0}{4\pi} \cdot \frac{2I \cdot \pi r_1^2}{(x_1^2 + r_1^2)^{3/2}}$$
$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2I \cdot \pi r_2^2}{(x_2^2 + r_2^2)^{3/2}}$$
$$\frac{B_1}{B_2} = \frac{r_1^2 (x_2^2 + r_2^2)^{3/2}}{r_2^2 (x_1^2 + r_1^2)^{3/2}}$$

But, $r_1 = x_1 \tan\theta$

and $r_2 = x_2 \tan\theta$

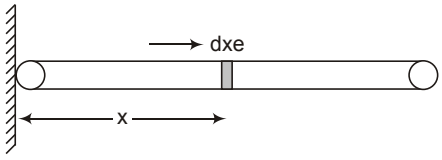
$\therefore \quad \frac{B_1}{B_2} = 2$

10. $b - a$ must be less than or equal to radius of circular path,

i.e., $$b - a \le \frac{mv}{qB}$$

or $$v \ge \frac{qB(b - a)}{m}$$

11. Consider an elementary portion of length dx at a distance x from the pivoted end.



Charge on this portion

$$dq = \frac{q}{l} dx$$

Current associated with this portion

$$di = \frac{dq}{T} = \frac{qf}{l} dx$$

Magnetic moment of this portion

$$dM = \pi x^2 di = \frac{\pi q f}{l} x^2 dx$$
$$M = \frac{\pi q f}{l} \int_0^l x^2 dx = \frac{1}{3} \pi q f l^2$$

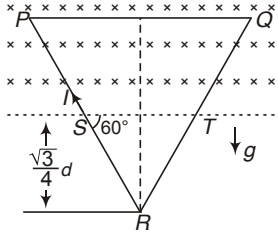
12. At $x = 0$, $y = \pm 2$ m

Effective length of wire

$$l = (4 \text{ m})\hat{\mathbf{j}}$$

$$\therefore \quad \vec{\mathbf{F}}_m = I(\vec{\mathbf{l}} \times \vec{\mathbf{B}}) = 3(4\hat{\mathbf{j}} \times 5\hat{\mathbf{k}})$$
$$= 60\hat{\mathbf{i}} \text{ N}$$

13. Effective length of wire,



$$l = ST = 2 \times \frac{\sqrt{3}}{4} a \times \cot 60°$$
$$= \frac{a}{2}$$

For equilibrium, $I\,lB = Mg$

$\Rightarrow \quad I = \frac{2Mg}{lB}$

14. For particle not collide with the solenoid, radius of path of particle ≤ half or radius of solenoid.

$$\frac{mv}{qB} \geq \frac{r}{2}$$

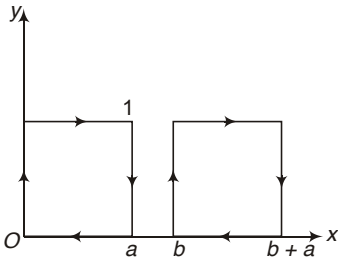
But $B = \mu_0 n i$

$$\Rightarrow \qquad v > \frac{rqB}{2m} = \frac{\mu_0 qr\, n\, i}{2m}$$

16. Magnetic force cannot do work on charged particle, hence its energy will remain same, so that θ remains same.

Again, magnetic force is always along the string, it will never produce a torque hence, T will also remain same.

17. Let the x-coordinates of loops be as shown in figure,

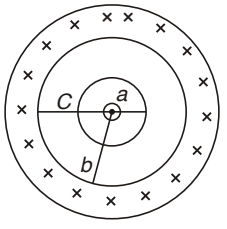


then, $\quad F_1 = Ia(B_0 a) - 0 = I a^2 B_0$

$$F_2 = Ia(B_0(b+a)) - Ia(B_0 b)$$
$$= I a^2 B_0$$

$$\therefore \qquad F_1 = F_2 \neq 0$$

18. Consider an amperian loop of radius $x\,(b < x < c)$, threaded by current the amperian loop,



$$I' = I - \frac{x^2 - b^2}{c^2 - b^2} I$$

$$= \frac{c^2 - x^2}{c^2 - b^2} I$$

$$I = \frac{\mu_0 I'}{2\pi x} = \frac{\mu_0 I (c^2 - x^2)}{2\pi x (c^2 - b^2)}$$

19. As $\vec{\mathbf{E}} = -\vec{\mathbf{v}} \times \vec{\mathbf{B}}$

Net force on the particle must be zero.

20. Consider an elementary portion of length dy at $y - y$ on the wire.

Force on this portion,

$$dF = I(\vec{\mathbf{dy}} \times \vec{\mathbf{B}})$$

Here, $\vec{\mathbf{dy}} = -dy\,\hat{\mathbf{j}}$ (Current is directed along negative y-axis).

$$dF = -I\{dy\,\hat{\mathbf{j}}(0.3y\,\hat{\mathbf{i}} + 0.4y\,\hat{\mathbf{j}})\}$$

$$= -2 \times 10^{-3}(-0.3y\,dy\,\hat{\mathbf{k}})$$

Total force on the wire,

$$F = \int dF = -2 \times 10^{-3} \int_0^1 (-0.3y\,dy\,\hat{\mathbf{k}})$$

$$F = (3 \times 10^{-4}\,\hat{\mathbf{k}})\ \text{N}$$

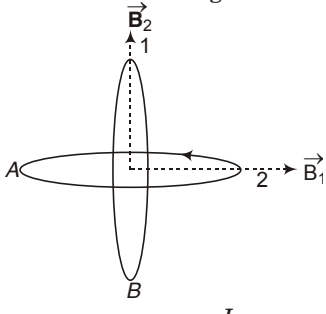
21. $\vec{\mathbf{E}} = -\vec{\mathbf{v}} \times \vec{\mathbf{B}}$

$$|\vec{\mathbf{E}}| = vB = \frac{rqB}{m} B$$

$$= \frac{(5 \times 10^{-2})(20 \times 10^{-6})(0.1)^2}{(20 \times 10^{-9})}$$

$$\therefore \vec{\mathbf{E}} = 0.5\ \text{V/m} \qquad (1\ \mu\text{g} = 10^{-9}\text{kg})$$

22. Condition is shown in figure.



$$B_1 = \frac{\mu_0 I_1}{2R_1}$$

$$B_2 = \frac{\mu_0 I_2}{2R_2}$$

$$\therefore \qquad B = \sqrt{B_1^2 + B_2^2}$$

$$= \frac{\mu_0}{2}\sqrt{\left(\frac{I_1}{R_1}\right)^2 + \left(\frac{I_2}{R_2}\right)^2}$$

$$= \frac{4\pi \times 10^{-7}}{2} \times \sqrt{\left(\frac{5}{\frac{5}{\sqrt{2}} \times 10^{-2}}\right)^2 + \left(\frac{5\sqrt{2}}{5 \times 10^{-2}}\right)^2}$$

$$= \frac{4\pi \times 10^{-7} \times 2}{2 \times 10^{-2}} = 4\pi \times 10^{-5}\ \text{T}$$

23. Initially, net force on the particle is zero. Hence,

$$V = \frac{E}{B}$$

Now, if electric field is switched off.

$$r = \frac{mv}{qB} = \frac{E}{SB^2} \qquad \left(\frac{q}{m} = S\right)$$

24. For equilibrium,

$f = \frac{mg}{l}$ [f = magnetic force per unit length on the conductors]

$$\Rightarrow \qquad \frac{\mu_0}{4\pi} \cdot \frac{2I_1I_2}{r} = \lambda g$$

$$\Rightarrow \qquad r = \frac{\mu_0}{4\pi} \cdot \frac{2I_1I_2}{\lambda g}$$

$$= \frac{10^{-7} \times 2 \times 100 \times 50}{0.01 \times 10}$$

$$= 0.01\ \text{m}$$

Clearly, equilibrium of conductor B is unstable.

25. If $\vec{\mathbf{B}}_1, \vec{\mathbf{B}}_2$ and $\vec{\mathbf{B}}_3$ be magnetic fields at the given point due to the wires along x, y and z axis respectively, then

$$\vec{\mathbf{B}}_1 = \frac{\mu_0}{4\pi} \cdot \frac{2I}{a} \hat{\mathbf{j}}$$

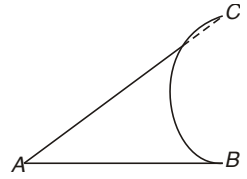
$$\vec{\mathbf{B}}_2 = -\frac{\mu_0}{4\pi} \cdot \frac{2I}{a} \hat{\mathbf{i}}$$

$$\vec{\mathbf{B}}_3 = 0$$

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 + \vec{\mathbf{B}}_3 = \frac{\mu_0 i}{2\pi a}(\hat{\mathbf{j}} - \hat{\mathbf{i}})$$

26. Effective length, $l = AC = \sqrt{4^2 + 3^2}$

$= 5$ m



$$F = I\,lB = 2 \times 5 \times 2 = 20\ \text{N}$$

27. At point P,

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qx}{(R^2 + x^2)^{3/2}}$$

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2iA}{(R^2 + x^2)^{3/2}}$$

Hence, $i = \frac{q}{T} = \frac{qv}{2\pi R}$

and $A = \pi R^2$

$$\therefore \qquad \frac{E}{B} = \frac{1}{\mu_0\varepsilon_0} \cdot \frac{1}{v} = \frac{c^2}{v} \qquad \left[c = \frac{1}{\mu_0\varepsilon_0}\right]$$

More than One Correct Options

1. $B_1 = \frac{\mu_0 N_1 I_1}{2R_1} = \frac{4\pi \times 10^{-7} \times 50 \times 2}{2 \times 5 \times 10^{-2}}$

$$= 4\pi \times 10^{-4}\ \text{T}$$

$$B_2 = \frac{\mu_0 N_2 I_2}{2R_2} = \frac{4\pi \times 10^{-7} \times 100 \times 2}{2 \times 10 \times 10^{-2}}$$

$$= 4\pi \times 10^{-4}\ \text{T}$$

If current is in same sense,

$$B = B_1 + B_2 = 8\pi \times 10^{-4}\ \text{T}$$

And if current is in opposite sense,

$$B = B_1 - B_2 = 0$$

2. $\vec{\mathbf{F}} = \vec{\mathbf{F}}_e + \vec{\mathbf{F}}_m = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}})$

If $\vec{\mathbf{F}} = 0$

Either, $\vec{\mathbf{E}} = -\vec{\mathbf{v}} \times \vec{\mathbf{B}}$,

$$\vec{\mathbf{E}} \neq 0, \vec{\mathbf{B}} = 0$$

or $\qquad \vec{\mathbf{E}} = 0$, or $\vec{\mathbf{v}} \times \vec{\mathbf{B}} = 0$

Again, If $\vec{\mathbf{v}} \times \vec{\mathbf{B}} = 0$

Either $\vec{\mathbf{B}} = 0$

or $\theta = 0°$, *i.e.*, $\vec{\mathbf{v}} \parallel \vec{\mathbf{B}}$.

3. The particle will describe a circle in x-y plane with radius,

$$r = \frac{mv}{qB} = \frac{1 \times \sqrt{8^2 + 6^2}}{1 \times 2} = 5 \text{ m}$$

and $T = \frac{2\pi m}{qB} = \pi \text{ s} = 3.14 \text{ s}$

4. $\vec{\tau} = MB \sin\theta$

$U = -pE\cos\theta$

$\theta = 80°$

Hence, $\tau = 0, U = pE =$ maximum.

As PE (U) is maximum, equilibrium is unstable.

5. Fact.

6. Upward and downward components of force will cancel each other while leftward force is more than rightward force, hence net force is leftwards.

7. $\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q(\vec{\mathbf{v}} \times \vec{\mathbf{B}})$

$$= q\{E_0 \hat{\mathbf{k}} + (v\hat{\mathbf{j}}) \times (B_0 \hat{\mathbf{i}})\}$$

$$= q(E_0 - vB_0)\hat{\mathbf{k}}$$

If $v < \frac{E_0}{B_0}$, particle will deflect towards positive z-axis.

If $v > \frac{E_0}{B_0}$, particle will deflect towards negative z-axis.

If $v = \frac{E_0}{B_0}$, particle will move undeflected and its KE will remain constant.

8. $K = eV \Rightarrow K \propto V$ will become double

$R = \frac{\sqrt{2mK}}{qB} \Rightarrow R \propto \sqrt{K}$ will become $\sqrt{2}$ times.

$\omega = \frac{qB}{2\pi m}$ is independent of kinetic energy.

9. Use right hand thumb rule.

10. For cd to be in equilibrium, force on it must be repulsive while for ab to be in equilibrium, force on it must be attractive.

Equilibrium of cd will be stable while that of ab will be unstable.

Match the Columns

1. (a $\to$ r), (b $\to$ q), (c $\to$ p), (d $\to$ r)

$\vec{\mathbf{F}}_m = q(\vec{\mathbf{v}} \times \vec{\mathbf{B}}) = -e(\vec{\mathbf{v}} \times \vec{\mathbf{B}})$

and $\vec{\mathbf{F}}_m = q\vec{\mathbf{E}} = -e\vec{\mathbf{E}}$

2. (a $\to$ r), (b $\to$ s), (c $\to$ q), (d $\to$ p)

As $\vec{\mathbf{F}}_m = q(\vec{\mathbf{v}} \times \vec{\mathbf{B}})$

By Fleming's left hand rule, positively charged particles deflects towards left and negatively charged particles deflects towards right.

Again, $r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$

$\Rightarrow \quad r \propto \frac{\sqrt{m}}{q}$

3. (a $\to$ p, s), (b $\to$ p, q), (c $\to$ p, r), (d $\to$ p, s)

Whenever a closed current carrying loop is placed in uniform magnetic field, net force experienced by it is zero.

Also $\tau = PE\sin\alpha$

is maximum if $\alpha = 90°$, *i.e.*, in case (b) only.

And $U = -PE\cos\alpha$

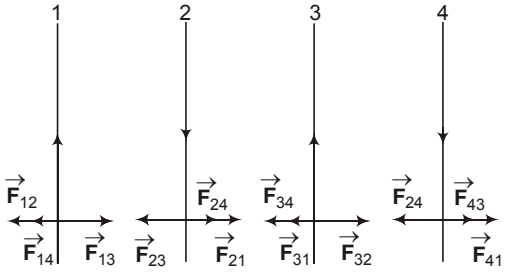
U is positive if α is obtuse, *i.e.*, in cases (a) and (d).

and U is minimum if $\alpha = 0$, *i.e.*, in case (c).

4. (a $\to$ q), (b $\to$ r), (c $\to$ s), (d $\to$ s)

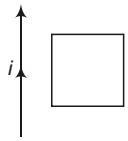
Use right hand thumb rule.

5. (a $\to$ q), (b $\to$ r), (c $\to$ q), (d $\to$ r)



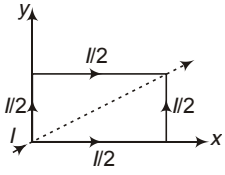
Direction of different forces on different wires is shown in figure.

6. (a $\rightarrow$ q, s), (b $\rightarrow$ p, r), (c $\rightarrow$ p, r), (d $\rightarrow$ q, s)



When the current is increased or the loop is moved towards the wire, magnetic flux linked with the loop increases. As a result of this, induced current will produce in the loop to decrease the magnetic field. Because initial magnetic flux linked with the loop is inward, induced magnetic flux will be outward and induced current will be anti-clockwise and *vice-versa*.

7. (a $\rightarrow$ r, s), (b $\rightarrow$ r, s), (c $\rightarrow$ q, r), (d $\rightarrow$ p, r)



Effective lengths of two conductors,

$$l_1 = l_2 = l\,\hat{\mathbf{i}} + l\,\hat{\mathbf{j}}$$

If $\quad \vec{\mathbf{B}} = B_0\,\hat{\mathbf{i}}$

$$\vec{\mathbf{F}} = \frac{I}{2}(\vec{\mathbf{l}}_1 \times \vec{\mathbf{B}}) + \frac{I}{2}(\vec{\mathbf{l}}_2 \times \vec{\mathbf{B}}) = -B_0 I\,l\,\hat{\mathbf{k}}$$

$\vec{\tau} = 0$, because lines of action of force on the two wires are equal and opposite.

If $\quad \vec{\mathbf{B}} = B_0\,\hat{\mathbf{j}}$

$$\vec{\mathbf{F}} = B_0 I\,l\,\hat{\mathbf{k}}$$

Again, lines of action of force on the two wires are equal and opposite.

$$\tau = 0$$

If $\quad \vec{\mathbf{B}} = B_0(\hat{\mathbf{i}} + \hat{\mathbf{j}})$

$$\vec{\mathbf{F}} = 0$$

$$\vec{\tau} = 0$$

If $\quad \vec{\mathbf{B}} = B_0\,\hat{\mathbf{k}}$

$$\vec{\mathbf{F}} = B_0 I\,l(\hat{\mathbf{i}} - \hat{\mathbf{j}})$$

$$\Rightarrow |\vec{\mathbf{F}}| = \sqrt{2}B_0 I\,l$$

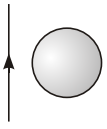
$$\tau = 0$$

24 Electromagnetic Induction

Introductory Exercise 24.1

1. Magnetic field inside the loop due to current carrying conductor is inwards.

   As the current in the conductor increases, magnetic flux linked with the loop increases as a result of which, induced current will produce in the loop to produce an outward magnetic field, *i.e.*, induced current will be anti-clockwise.



2. No.

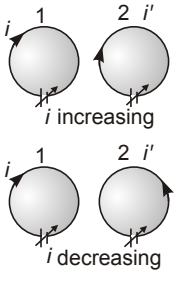
   Emf is induced if the field is time varying.

3. $\frac{d\phi_B}{dt}$ = induced emf

   $\therefore \quad \left[\frac{d\phi_B}{dt}\right] = [V] = [\mathrm{ML^2T^{-3}I^{-1}}]$

Introductory Exercise 24.2

1. If the outward magnetic flux increases, induced current will be in such a way that it produces inwards magnetic flux, *i.e.,* it will be clockwise.
2. Magnetic flux linked with the coil will not change, hence induced current will be zero.
3. If the current in coil 1 (clockwise) increases, outward magnetic flux linked with the coil 2 increases and the coil 2 will produce induced current in clockwise direction to oppose the change in magnetic flux linked with it.



Hence, if the current in coil 1 increases, induced current will be in same sense and *vice-versa*.

Introductory Exercise 24.3

1. $\phi_B = BS = B_0 S\, e^{-at}$

   $$e = -\frac{d\phi_B}{dt} = a\, B_0 S\, e^{-at}$$

2. No.

   As, $\quad F_m = i\,lB = 0$

   Because, $i = 0$ as the circuit is not closed. As net force acting on the bar is zero, no external force is required to move the bar with constant velocity.

3. $|e| = \dfrac{\phi_2 - \phi_1}{t}$

But, $\phi_1 = NB_1A\cos\theta$, $\phi_2 = NB_2A\cos\theta$

$\therefore \quad |e| = \dfrac{NA\cos\theta(B_2 - B_1)}{t}$

$\Rightarrow A = \dfrac{|e|t}{N(B_2 - B_1)\cos\theta}$

$= \dfrac{80.0 \times 10^{-3} \times 0.4}{50 \times (600 \times 10^{-6} - 200 \times 10^{-6}) \times \dfrac{\sqrt{3}}{2}}$

$= 1.85 \text{ m}^2$

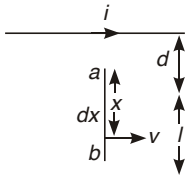
Side of square, $a = \sqrt{A} = 1.36$ m

Total length of wire $= 50 \times 4a$

$= 50 \times 4 \times 1.36 = 272$ m

4. (a) Consider an elementary portion of length dx of the bar at a distance x from end a.

Magnetic field at this point,



$$B = \frac{\mu_0}{4\pi} \cdot \frac{2i}{a + x}$$

Induced emf in this portion,

$$de = B\,dxv = \frac{\mu_0}{4\pi} \cdot \frac{2vi}{d + x}\,dx$$

5. (a) EMF induced in the bar ab,

$$e = \int de = \frac{\mu_0}{4\pi} \cdot 2vi \int_0^l \frac{dx}{d + x}$$

$$= \frac{\mu_0}{4\pi} 2vi[\ln(d + x)]_0^l$$

$$= \frac{\mu_0 vi}{2\pi} \ln \frac{d + l}{d}$$

$$= \frac{\mu_0 vi}{2\pi} \ln\left(1 + \frac{l}{d}\right)$$

(b) Magnetic field in the region ab is inwards, hence by Fleming's left hand rule, positive charge will move up and a will be at higher potential.

Or

Use Fleming's right hand rule.

(c) No.

As flux linked with the square loop will remain same.

Introductory Exercise 24.4

1. Potential difference across an inductor,

$$V = L\frac{di}{dt} = L\frac{d}{dt}(3t\sin t)$$

$$= 3L[\sin t + t\cos t]$$

Introductory Exercise 24.5

1. (a) Total number of turns on the solenoid,

$$N = \frac{l}{d} = \frac{40 \times 10^{-2}}{0.10 \times 10^{-2}}$$

$$= 400$$

$$L = \frac{\mu_0 N^2 A}{l}$$

$$= \frac{4\pi \times 10^{-7} \times (400)^2 \times 0.90 \times 10^{-4}}{40 \times 10^{-2}}$$

$$= 4.5 \times 10^{-5} \text{ H}$$

(b) $e = -L\dfrac{di}{dt}$

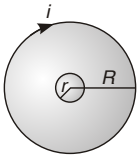
$$= -4.5 \times 10^{-5} \times \frac{0 - 10}{0.10}$$

$$= 4.5 \times 10^{-3} \text{ V}$$

$$= 4.5 \text{ mV}$$

Introductory Exercise 24.6

1. Consider a current i is flowing in the outer loop.



Magnetic field at the centre of the loop.

$$B = \frac{\mu_0 i}{2R}$$

As $R >> r$, magnetic field inside smaller loop may assumed to be constant.

Hence, magnetic flux linked with the smaller loop,

$$\phi_m = B \times \pi r^2 = \frac{\mu_0 \pi r^2 i}{2R}$$

$$M = \frac{\phi_m}{i} = \frac{\pi \mu_0 r^2}{2R}$$

Introductory Exercise 24.7

1. (a) $V_0 = i_0 R = 36 \times 10^{-3} \times 175 = 6.3$ V

(b) $i = i_0 (1 - e^{-t/\tau})$

where, $\tau = \dfrac{L}{R}$

Now, at $t = 58\ \mu s$

$$i = 4.9 \text{ mA}$$

$\therefore\ 4.9 = 36(1 - e^{-58/\tau})$

$\Rightarrow \quad e^{-58/\tau} = \dfrac{31.1}{36}$

$\Rightarrow \quad \tau = 397\ \mu s$

$$\frac{L}{R} = 397\ \mu s$$

$\Rightarrow \quad L = 175 \times 397 \times 10^{-6}$

$$= 69 \text{ mH}$$

(c) $\tau = 397\ \mu s$

2. $$[L] = \frac{[e]}{\left[\dfrac{di}{dt}\right]} = \frac{[V][t]}{[i]}$$

and $$[R] = \frac{[V]}{[i]}$$

$\therefore \quad \left[\dfrac{L}{R}\right] = \dfrac{[L]}{[R]} = [\text{T}]$

3. (a) Initially

$$E = L\frac{di}{dt}$$

$\Rightarrow \quad \dfrac{di}{dt} = \dfrac{E}{L}$

$$= \frac{12.0}{3.00} = 4 \text{ A/s}$$

(b) $E = V_L + V_R$

$\Rightarrow \quad E = L\dfrac{di}{dt} + iR$

$\Rightarrow \quad \dfrac{di}{dt} = \dfrac{1}{L}[E - iR]$

$$= \frac{1}{3.00} \times [12 - 1 \times 7]$$

$\Rightarrow \quad \dfrac{di}{dt} = \dfrac{5}{3} = 1.67$ A/s

(c) $\tau = \dfrac{L}{R} = \dfrac{3}{7}$

$$i = i_0 (1 - e^{-t/\tau})$$

$$= \frac{E}{R}(1 - e^{-t/\tau}) = \frac{12}{7}(1 - e^{-1.4/3})$$

$\Rightarrow \quad i = 0.639$ A

(d) $i_0 = \dfrac{E}{R} = \dfrac{12}{7} = 1.71$ A

4. (a) $P = Ei = \dfrac{E^2}{R}(1 - e^{-t/\tau})$

$$= \frac{(12)^2}{7}(1 - e^{-7t/3}) = 20.6(1 - e^{-2.33t})\text{ W}$$

(b) Rate of dissipation of energy,

$$P_R = i^2 R = i_0^2 R(1 - e^{-7t/R})^2$$

$$= 20.6(1 - e^{-2.33t})^2 \text{ W}$$

(c) Rate of increase of magnetic energy

$$P_L = ei = L\frac{di}{dt}i$$

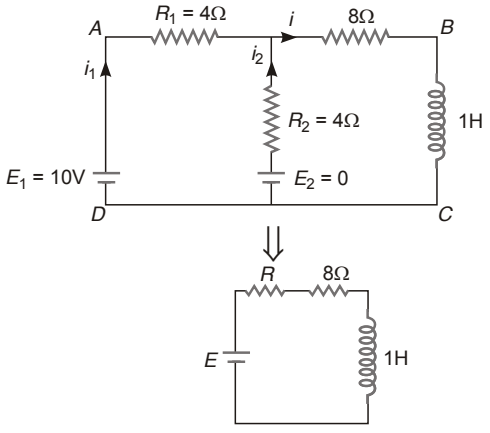
$$= 20.6(e^{-2.33t} - e^{-4.67t})\text{ W}$$

(d) Clearly, $P = P_R + P_L$

5. No.

$E = V_L + V_R$ and V_R cannot be negative in RL circuit.

6. Consider the system as a combination of two batteries ($E_1 = 10$ V and $E_2 = 0$) as shown



$$E = \frac{E_1R_2 + E_2R_1}{R_1 + R_2} = 5 \text{ V}$$

$$R = \frac{R_1R_2}{R_1 + R_2} = 2\Omega$$

$$i_0 = \frac{E}{R+8} = \frac{5}{10} = 0.5 \text{ A}$$

$$\tau = \frac{L}{R+8} = \frac{1}{10}$$

$$i = i_0(1 - e^{-t/\tau})$$

$$i = 0.5(1 - e^{-10t}) \text{ A}$$

$\therefore$ Current through inductor

$$i = 2.5(1 - e^{-10t}) \text{ A}$$

In loop *ABCDA*

$$i_1R_1 + 8i + L\frac{di}{dt} - E_1 = 0$$

$$i_1 \times 4 + 8 \times 0.5(1 - e^{-10t}) + 1(5e^{-10t}) - 10 = 0$$

$$i_1 = (1.5 - 0.25\,e^{-10t}) \text{ A}$$

Introductory Exercise 24.8

1. $$[C] = \frac{[q]}{[V]} = \frac{[i][T]}{[V]}]$$

$$[L] = \frac{[e]}{\left[\frac{di}{dt}\right]} = \frac{[V][T]}{[i]}$$

$$\Rightarrow [\sqrt{LC}] = [\sqrt{L}\sqrt{C}] = [\text{T}]$$

2. In LC oscillations, magnetic energy is equivalent to kinetic energy in spring block system.

$$i = \frac{dq}{dt} \Rightarrow v = \frac{dx}{dt}$$

Also L is equivalent to inertia (m) in electricity, hence

Magnetic energy $= \frac{1}{2}Li^2$ is equivalent to kinetic energy $= \frac{1}{2}mv^2$.

3. In LC oscillations,

(a) $\frac{di}{dt} = -\frac{1}{LC}q \Rightarrow q = -LC\frac{di}{dt}$

$$\Rightarrow \quad |q| = 18 \times 10^{-6} \times 0.75 \times 3.40$$

$$= 46.5 \times 10^{-6} \text{ C}$$

$$= 46.5\ \mu\text{C}$$

(b) $e = -L\frac{di}{dt} = -L\left(-\frac{1}{LC}q\right)$

$$= \frac{q}{C} = \frac{4.8 \times 10^{-4}}{18 \times 10^{-6}} = 23.3 \text{ V}$$

4. $i_0 = \omega q_0$

where, $\omega = \frac{1}{\sqrt{LC}}$

$$\Rightarrow \quad V_0 = \frac{q_0}{C} = \frac{i_0}{\omega C}$$

$$V_0 = i_0\sqrt{\frac{L}{C}} = 0.1 \times \sqrt{\frac{20 \times 10^{-3}}{0.5 \times 10^{-6}}}$$

$$= 20 \text{ V}$$

Introductory Exercise 24.9

1. (a) $B = \mu_0 ni$

$$\phi_m = NBA = \mu_0 n\,NAi$$

$$e = -\frac{d\phi_m}{dt} = -\mu_0 nNA\frac{di}{dt}$$

$$= -4\pi \times 10^{-7} \times \frac{25}{0.01} \times 10 \times 5.0 \times 10^{-4} \times (-0.2)$$

$= 3.14 \times 10^{-6}$ V

$= 3.14\ \mu$V

(b) $E = \dfrac{e}{2\pi R} = \dfrac{3.14 \times 10^{-6}}{2 \times 3.14 \times 25 \times 10^{-2} \times 10}$

$= 2 \times 10^{-7}$ V/m

2. $B = (2.00t^3 - 4.00t^2 + 0.8)$ T

$\dfrac{dB}{dt} = (6.00t^2 - 8.00t)$ T/s

From, $t = 0$ to $t = 1.33$ s, $\dfrac{dB}{dt}$ is negative, hence B is decreasing in that interval.

For $t > 1.33$ s, $\dfrac{dB}{dt}$ is positive, hence B is increasing for $t > 1.33$ s.

(a) For point P_2,

induced emf, $V_2 = -\dfrac{d\phi_{m_2}}{dt} = -\pi R^2 \dfrac{dB}{dt}$

Induced electric field at P_2,

$E = \dfrac{V_2}{2\pi r_2} = -\dfrac{R^2}{2r_2} \cdot \dfrac{dB}{dt}$

$= -\dfrac{R^2}{2r_2}(6.00t^2 - 8.00t)$

$F = -eE = \dfrac{R^2}{2r_2}(6.00t^2 - 8.00t)$

$= 8.0 \times 10^{-21}$ N

As magnetic field is increasing in this region, induced electric field will be anti-clockwise and hence, electron will experience force in clockwise sense, *i.e.*, downward at P_2.

(b) For point P_1,

Induced emf, $V_1 = -\dfrac{d\phi_{m_1}}{dt} = -\pi r_1^2 \dfrac{dB}{dt}$

Induced electric field at P_1,

$E = -\dfrac{V_1}{2\pi r_1} = -\dfrac{1}{2} r_1 \dfrac{dB}{dt}$

$= -\dfrac{1}{2} r_1 (6.00t^2 - 8.00t) = 0.36$ V/m

At, $t = 2.00$ s

magnetic field is increasing, hence, induced electric field will be anti-clockwise, *i.e.*, upward at P_1 and perpendicular to r_1.

AIEEE Corner

Subjective Questions (Level 1)

1. $< e > = -\dfrac{\phi_2 - \phi_1}{t} = -\dfrac{B(A_2 - A_1)}{t}$

$A_1 = \pi r^2 = 3.14 \times (0.1)^2$

$= 3.14 \times 10^{-2} = 0.0314$

$A_2 = a^2 = \left(\dfrac{2\pi r}{4}\right)^2$

$= \left(\dfrac{2 \times 3.14 \times 0.1}{4}\right)^2 = 0.025$

$\therefore \quad < e > = -\dfrac{100(0.025 - 0.0314)}{0.1}$

$= 6.4$ V

2. $\phi_1 = NBA = 500 \times 0.2 \times 4 \times 10^{-4}$

$= 0.04$ Wb

$\phi_2 = -NBA = -0.04$ Wb

Average induced emf,

$< e > = -\dfrac{(\phi_2 - \phi_1)}{t}$

Average induced current,

$< i > = \dfrac{< e >}{R} = -\dfrac{(\phi_2 - \phi_1)}{Rt}$

Charge flowing through the coil

$q = < i > t$

$\Rightarrow q = -\dfrac{(\phi_2 - \phi_1)}{R} = -\dfrac{(-0.04 - 0.04)}{50}$

$= \dfrac{0.08}{50} = 1.6 \times 10^{-3}$ C

$= 1.6$ mC $= 1600\ \mu$C

3. $\phi_1 = NBS$, $\phi_2 = -NBS$

Induced emf,

$< e > = -\dfrac{(\phi_2 - \phi_1)}{t} = \dfrac{2NBS}{t}$

Induced current

$< i > = \dfrac{< e >}{R} = \dfrac{2NBS}{Rt}$

Charge flowing through the coil,

$q = < i > t = \dfrac{2NBS}{R}$

$\Rightarrow \quad B = \dfrac{qR}{2NS} = \dfrac{4.5 \times 10^{-6} \times 40}{2 \times 60 \times 3 \times 10^{-6}}$

$$= 0.5 \text{ T}$$

4. $\vec{\mathbf{B}} = (4.0\,\hat{\mathbf{i}} - 1.8\,\hat{\mathbf{k}}) \times 10^{-3}$ T,

$\vec{\mathbf{S}} = (5.0 \times 10^{-4}\,\hat{\mathbf{k}})\ \text{m}^2$

$\phi = \vec{\mathbf{B}} \cdot \vec{\mathbf{S}} = -9.0 \times 10^{-7}$ Wb

5. $e = Blv = 1.1 \times 0.8 \times 5 = 4.4$ V

By Fleming's right hand rule, north end of the wire will be positive.

6. $A = \pi r^2 = 3.14 \times (12 \times 10^{-2})^2 = 0.045\ \text{m}^2$

(a) For $t = 0$ to $t = 2.0$ s

$$\frac{dB}{dt} = \text{slope} = \frac{0.5 - 0}{2.0 - 1} = 0.25 \text{ T/s}$$

$$e = -\frac{d\phi_m}{dt} = -A\frac{dB}{dt}$$

$$= -0.045 \times 0.25 = -0.011 \text{ V}$$

$$|e| = 0.011 \text{ V}$$

(b) For, $t = 2.0$ s to $t = 4.0$ s

$$\frac{dB}{dt} = \text{slope} = 0 \quad e = 0$$

(c) For, $t = 4.0$ s to $t = 6.0$ s

$$\frac{dB}{dt} = \text{slope} = \frac{0 - 0.5}{6.0 - 4.0} = -0.25$$

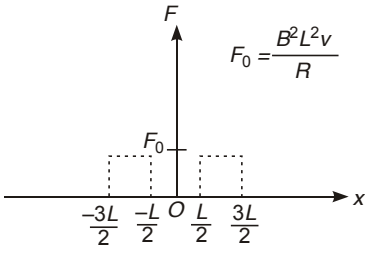
$$e = -\frac{d\phi_m}{dt} = -A\frac{dB}{dt} = 0.11 \text{ V}$$

7. (a) When magnetic flux linked with the coil changes, induced current is produced in it, in such a way that, it opposes the change.

Magnetic flux linked with the coil will change only when coil is entering in (from $x = -\frac{3L}{2}$ to $x = -\frac{L}{2}$) or moving (from $x = \frac{L}{2}$ to $x = \frac{3L}{2}$) of the magnetic field.

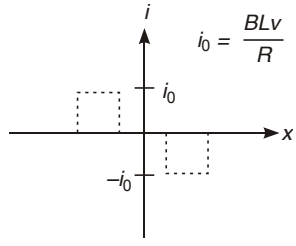
Because, of induced current, an opposing force act on the coil, which is given by

$$F = ilB = \frac{BLv}{R}BL = \frac{B^2L^2v}{R}$$

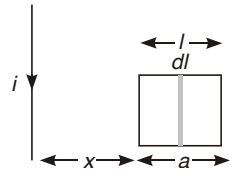


Hence, equal force in direction of motion of coil is required to move the block with uniform speed.

(b) When the coil is entering into the magnetic field, magnetic flux linked with the coil increases and the induced current will produce magnetic flux in opposite direction and will be counter-clockwise and *vice-versa*.



8. Consider an elementary section of length dl of the frame as shown in figure. Magnetic flux linked with this section,



$$d\phi_m = BdB = \frac{\mu_0}{4\pi} \cdot \frac{2i}{x + l} adl$$

Total magnetic flux linked with the frame,

$$\phi_m = \int d\phi_m = \frac{\mu_0 ai}{2\pi}\int_0^a \frac{dl}{x + l}$$

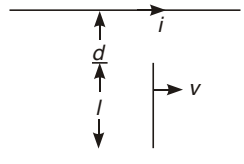
$$= \frac{\mu_0 ai}{2\pi}[\ln(x + a) - \ln x]$$

Induced emf

$$e = \frac{-d\phi_m}{dt} = -\frac{\mu_0 ai}{2\pi} \cdot \left[\frac{1}{x + a} - \frac{1}{x}\right]\frac{dx}{dt}$$

$$= \frac{\mu_0 a^2 i}{2\pi x(x + a)}v = \frac{\mu_0 a^2 iv}{2\pi x(x + a)}$$

9. As solved in Qusetion 4. Introductory Exercise 24.3.



$$e = \frac{\mu_0 iv}{2\pi} \ln\left(1 + \frac{l}{d}\right)$$

Here, $i = 10$ A

$v = 10 \text{ ms}^{-1}$

$l = 10.0 \text{ cm} - 1.0 \text{ cm} = 9.0 \text{ cm}$

$d = 1.0$ cm

$$e = \frac{4\pi \times 10^{-7} \times 10 \times 10}{2\pi} \ln\left(1 + \frac{9.0}{1.0}\right)$$

$e = (2 \times 10V) \ln(10)$ V

10. Induced current

$$i = \frac{e}{R} = \frac{Blv}{R}$$

Force needed to move the rod with constant speed = Magnetic force acting on the rod

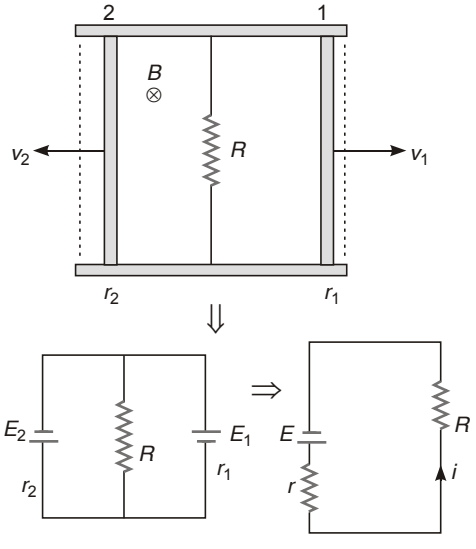
ie., $$F = i\,lB = \frac{Blv}{R} lB$$

$$= \frac{B^2 l^2 v}{R} = \frac{(0.15)^2 \times (50 \times 10^{-2})^2 \times 2}{3}$$

$\Rightarrow$ $F = 0.00375$

11. Suppose the magnetic field is acting into the plane of paper.

Rods 1 and 2 can be treated as cells of emf $E_1 (= Blv_1)$ and $E_2 (= Blv_2)$ respectively.



Now, $E_1 = Blv_1 = 0.010 \times 10.0 \times 10^{-2} \times 4.00$

$= 0.004$ V

$E_2 = Blv_2 = 0.010 \times 10 \times 0 \times 10^{-2} \times 8.00$

$= 0.008$ V

Effective emf

$$E = \frac{E_2 r_1 - E_1 r_2}{r_1 + r_2}$$

$$= \frac{0.008 \times 15.0 - 0.004 \times 10.0}{15.0 + 10.0}$$

$= 0.0032$ V

$$r = \frac{r_1 r_2}{r_1 + r_2} = \frac{15 \times 10}{25} = 6\,\Omega$$

$$i = \frac{E}{R + r} = \frac{0.0032}{5 + 6} = 0.003 \text{ A} = 0.3 \text{ mA}$$

12. (a) $e = -L\dfrac{di}{dt} = -0.54 \times (-0.030)$

$= 1.62 \times 10^{-2}$ V

(b) Current flowing from b to a is decreasing, hence, a must be at higher potential.

13. (a) $i = 5 + 16t$, $|e| = 10\text{mV} = 10 \times 10^{-3}$ V

$$|e| = L\frac{di}{dt} \Rightarrow 10 \times 10^{-3} = L\frac{d}{dt}(5 + 16t)$$

$$L = \frac{10 \times 10^{-3}}{16} = 0.625 \text{ mH}$$

(b) at $t = 1$ s

$i = 5 + 16(1) = 21$ A

Energy stored in the inductor,

$$U = \frac{1}{2}Li^2 = \frac{1}{2} \times 0.625 \times 10^{-3} \times (21)^2$$

$= 0.138$ J

$$P = \frac{dU}{dt} = Li\frac{di}{dt} = 0.625 \times 10^{-3} \times 21 \times 16$$

$= 0.21$ W

14. From $t = 0$ to $t = 2.0$ ms

$$\frac{V - 0}{t - 0} = \frac{5.0 - 0}{2.0 \times 10^{-3}} = 0$$

$\Rightarrow$ $V = 2500\,t$

$$L\frac{di}{dt} = 2500\,t$$

$$\int di = \frac{2500}{L}\int t\,dt$$

$$\Rightarrow \int_0^i di = \frac{2500}{L}\int_0^t t\,dt$$

$$i = \frac{1250}{L}t^2$$

at $t = 2.0$ ms

$$i = \frac{1250}{150 \times 10^{-3}} \times (2.0 \times 10^{-3})^2$$

$= 3.33 \times 10^{-2}$ A

From $t = 2.0$ ms to $t = 4.0$ ms

$$\frac{V - 5.0}{t - 2.0 \times 10^{-3}} = \frac{0 - 0.50}{(4.0 - 2.0) \times 10^{-3}}$$

$$V = -2500(t - 2.0 \times 10^{-3}) + 5.0$$
$$= -2500\,t + 10.0$$
$$L\frac{di}{dt} = -2500\,t + 10.0$$
$$di = \frac{1}{L}(-2500\,t + 10.0)\,dt$$
$$i = \frac{1}{L}[-1250\,t^2 + 10.0\,t]$$

at $t = 4$ s

$$i = \frac{1}{150 \times 10^{-3}}[-1250 \times (4.0 \times 10^{-3})^2 + 10.0(4.0 \times 10^{-3})]$$
$$= 3.33 \times 10^{-2} \text{ A}$$

15. (a) $|e| = L\dfrac{di}{dt} \Rightarrow L = \dfrac{|e|}{di/dt} = \dfrac{0.0160}{0.0640}$

$$= 0.250 \text{ H}$$

(b) Flux per turn

$$\phi = \frac{Li}{N} = \frac{0.250 \times 0.720}{400}$$
$$= 4.5 \times 10^{-4} \text{ Wb}$$

16. $|e| = M\dfrac{di}{dt} = M\dfrac{i_2 - i_1}{t}$

$$\Rightarrow \quad 50 \times 10^{-3} = M \cdot \frac{12 - 4}{0.5}$$
$$M = \frac{50 \times 10^{-3} \times 0.5}{8} = 3.125 \times 10^{-3} \text{ H}$$
$$= 3.125 \text{ mH}$$

If current changes from 3 A to 9 A in 0.02 s.

$$|e| = M\frac{di}{dt} = M\frac{i_2 - i_1}{t}$$
$$= 3.125 \times 10^{-3} \times \frac{9 - 3}{0.02}$$
$$= 0.9375 \text{ V}$$

17. (a) Magnetic flux linked with secondary coil,

$$\phi_{m_2} = M\,i_1$$
$$M = \frac{\phi_2}{i_1} = \frac{6.0 \times 10^{-3} \times 1000}{3} = 2 \text{ H}$$

(b) $$e = -\frac{d\phi_{m_2}}{dt} = -M\frac{d\,i_1}{dt}$$
$$= -2 \times \frac{0 - 3}{0.2} = 30 \text{ V}$$

(c) $L = \dfrac{\phi_{m_1}}{i_1} = \dfrac{600 \times 5 \times 10^{-3}}{3} = 1 \text{ H}$

18. (a) $|e| = M\dfrac{di}{dt} = 3.25 \times 10^{-4} \times 830$

$$= 0.27 \text{ V}$$

As, $\dfrac{di}{dt}$ is constant, induced emf is constant.

(b) Coefficient of mutual induction remains same whether current flows in first coil or second.

Hence, $|e| = M_1\dfrac{di}{dt} = 0.27$ V

19. (a) Magnetic flux linked with the secondary coil,

$$\phi_2 = Mi_1$$
$$M = \frac{\phi_2}{i_1} = \frac{0.0320 \times 400}{6.52}$$
$$= 1.96 \text{ H}$$

(b) $\phi_1 = Mi_2 = 1.96 \times 2.54 = 4.9784$ Wb

Flux per turn through primary coil

$$= \frac{\phi_1}{N_1} = \frac{4.9784}{700}$$
$$= 7.112 \times 10^{-3} \text{ Wb/turn.}$$

20. Same as Question 2. Introductory Exercise 24.4

21.

$$i = i_0(1 - e^{-t/\tau})$$
$$= \frac{E}{R}(1 - e^{-\frac{Rt}{L}})$$
$$\frac{di}{dt} = \frac{E}{L}e^{-\frac{Rt}{L}}$$

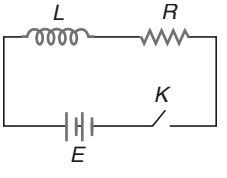
Power supplied by battery,

$$P = Ei = \frac{E^2}{R}(1 - e^{-\frac{Rt}{L}})$$

Rate of storage of magnetic energy

$$P_1 = Li\frac{di}{dt} = \frac{E^2}{R}(1 - e^{-\frac{Rt}{L}})e^{-\frac{Rt}{L}}$$
$$\frac{P_1}{P} = e^{-\frac{Rt}{L}} = e^{-\frac{10 \times 0.1}{1}} = e^{-1} = 0.37$$

22. (a) $\tau = \dfrac{L}{R} = \dfrac{2}{10} = 0.2$ s



(b) $i_0 = \frac{E}{R} = \frac{100}{10} = 10$ A

(c) $i = i_0(1 - e^{-\frac{t}{\tau}})$

$$i = 10(1 - e^{-\frac{1}{0.2}})$$

$$= 10(1 - e^{-5}) = 9.93 \text{ A}$$

23. (a) Power delivered by the battery,

$$P = Ei = \frac{E^2}{R}(1 - e^{-\frac{Rt}{L}})$$

$$= \frac{(3.24)^2}{12.8}(1 - e^{\frac{12.8 \times 0.278}{3.56}})$$

$$= 0.82(1 - e^{-1}) = 0.518 \text{ W}$$

$$= 518 \text{ mW}$$

(b) Rate of dissipation of energy as heat

$$P_2 = i^2R = \frac{E^2}{R}(1 - e^{-\frac{Rt}{L}})^2$$

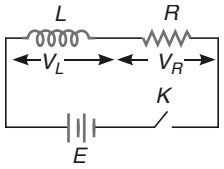
$$= 0.82(1 - e^{-1})^2 = 0.328 \text{ W}$$

$$= 328 \text{ mW}$$

(c) Rate of storage of magnetic energy

$$P_1 = P - P_2 = 190 \text{ mW}$$

24. $E = V_L + V_R = L\frac{di}{dt} + iR$



(a) Initially, $i = 0$

$$\therefore \quad \frac{di}{dt} = \frac{E}{L} = \frac{6.00}{2.50} = 2.40 \text{ A/s}$$

(b) When, $i = 0.500$ A

$$\frac{di}{dt} = \frac{E - iR}{L} = \frac{6.00 - 0.500 \times 8.00}{2.50}$$

$$= 0.80 \text{ A/s}$$

(c) $i = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$

$$= \frac{6.00}{8.00}\left(1 - e^{-\frac{8.00 \times 0.250}{2.5}}\right)$$

$$= 0.750(1 - e^{-0.8}) = 0.413 \text{ A}$$

(d) $i_0 = \frac{E}{R} = \frac{6.00}{8.00} = 0.750$ A

25. (a) $i = i_0\left(1 - e^{-\frac{Rt}{L}}\right)$

But $i = \frac{i_0}{2}$

$$\frac{i_0}{2} = i_0\left(1 - e^{-\frac{Rt}{L}}\right)$$

$$e^{-\frac{Rt}{L}} = \frac{1}{2}$$

$$t = \frac{L}{R}\ln 2 = \frac{1.25 \times 10^{-3}}{50.0} \times 0.693$$

$$= 17.3 \times 10^{-6} = 17.3 \ \mu\text{s}$$

(b) $U = \frac{1}{2}Li^2 = \frac{1}{2}\left(\frac{1}{2}L\, i_0^2\right)$

$$i = \frac{1}{\sqrt{2}}i_0$$

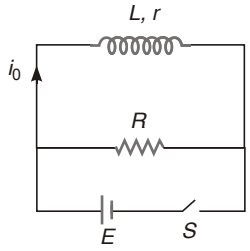
$$i_0\left(1 - e^{-\frac{Rt}{L}}\right) = \frac{i_0}{\sqrt{2}}$$

$$\Rightarrow \quad e^{-\frac{Rt}{L}} = \frac{\sqrt{2} - 1}{\sqrt{2}}$$

$$\Rightarrow \quad t = \frac{L}{R}\ln\frac{\sqrt{2}}{\sqrt{2} - 1}$$

$$= 30.7 \ \mu\text{s}.$$

26. Steady state current through the inductor



$$i_0 = \frac{E}{r}$$

When the switch S is open

$$\tau = \frac{L}{R + r}$$

(a) $i = i_0 e^{-t/\tau}$

$$\Rightarrow \quad i = \frac{E}{r}e^{-\left(\frac{(R+r)}{L}\right)}$$

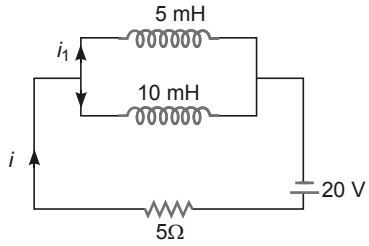
(b) Amount of heat generated in the solenoid

$$H = \int_0^\infty i^2r\,dt = i_0^2r\int_0^\infty e^{-2t/\tau}\,dt$$

$$= \frac{E^2}{r}\left\{-\frac{\tau}{2}[e^{-2t/\tau}]_0^\infty\right\}$$

$$= \frac{(R + r)E^2}{2rL}$$

27. At any instant of time,



$$L_1 \frac{di_1}{dt} = L_2 \frac{di_2}{dt}$$

$$\Rightarrow \quad L_1 i_1 = L_2 i_2$$

$$i_1 = 2i_2 \qquad \ldots(i)$$

In steady state,

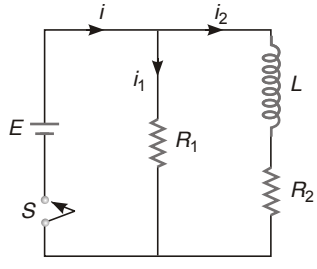
inductors offer zero resistance, hence

$$i = \frac{20}{5} = 4 \text{ A}$$

But $\quad i_1 + i_2 = i$

$$i_2 = \frac{4}{3} \text{A}, \; i_1 = \frac{8}{3} \text{A}$$

28. When the switch is closed,

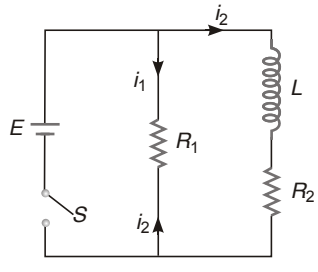


$$i_2 = \frac{E}{R_2}(1 - e^{-R_2 t/L})$$

$$\frac{di_2}{dt} = \frac{E}{L} e^{-R_2 t/L}$$

Potential difference across L

$$V + L\frac{di_2}{dt} = E\, e^{-R_2 t/L} = (12e^{-5t})\,\text{V}$$



When the switch S is open, current i_2 flows in the circuit in clockwise direction and is given by

$$i_2 = i_0 e^{-t/\tau}$$

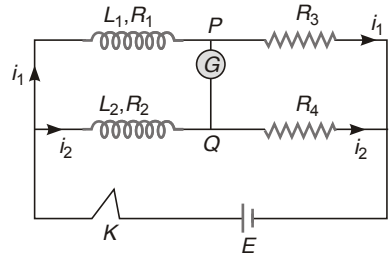
Here, $\quad i_2 = \dfrac{E}{R_2}$

$$\tau = \frac{L}{R_1 + R_2}$$

$$i_2 = \frac{E}{R_2} e^{-\left(\frac{R_1 + R_2}{L}\right)t}$$

$$= \frac{12}{2} e^{-10t} = (6\, e^{-10t})\,\text{A}$$

29. For current through galvanometer to be zero,



$$V_P = V_Q$$

$$L_1 \frac{di_1}{dt} + i_1 R_1 = L_2 \frac{di_2}{dt} + i_2 R_2 \qquad \ldots(i)$$

Also, $\quad i_1 R_3 = i_2 R_4 \qquad \ldots(ii)$

From Eqs.(i) and (ii),

$$\frac{L_1 \frac{di_1}{dt} + i_1 R_1}{i_1 R_3} = \frac{L_2 \frac{di_2}{dt} + i_2 R_2}{i_2 R_4} \qquad \ldots(iii)$$

In the steady state,

$$\frac{di_1}{dt} = \frac{di_2}{dt} = 0$$

$$\therefore \quad \frac{R_1}{R_3} = \frac{R_2}{R_4} \Rightarrow \frac{R_1}{R_2} = \frac{R_3}{R_4}$$

Again as current through galvanometer is always zero.

$$\frac{i_1}{i_2} = \text{constant}$$

or $\quad \dfrac{di_1 / dt}{di_2 / dt} = \text{constant}$

or $\quad \dfrac{\frac{di_1}{dt}}{\frac{di_2}{dt}} = \dfrac{i_1}{i_2} \qquad \ldots(iv)$

From Eqs. (iii) and (iv),

$$\frac{L_1}{L_2} = \frac{R_3}{R_4} = \frac{R_1}{R_2}$$

30. (a) In LC circuit

Maximum electrical energy = Maximum magnetic energy

$$\Rightarrow \quad \frac{1}{2}CV_0^2 = \frac{1}{2}Li_0^2$$

$$L = C\left(\frac{V_0}{i_0}\right)^2 = 4 \times 10^{-6}\left(\frac{1.50}{50 \times 10^{-3}}\right)^2$$

$$= 3.6 \times 10^{-3} \text{ H}$$

$$\Rightarrow \quad L = 3.6 \text{ mH}$$

(b) $f = \dfrac{1}{2\pi\sqrt{LC}}$

$$= \frac{1}{2 \times 3.14\sqrt{3.6 \times 10^{-3} \times 4 \times 10^{-6}}}$$

$$= 1.33 \times 10^3 \text{ Hz}$$

$$= 1.33 \text{ kHz}$$

(c) Time taken to rise from zero to maximum value,

$$t = \frac{T}{4} = \frac{1}{4f} = \frac{1}{4 \times 1.33 \times 10^3}$$

$$= 3 \times 10^{-3} \text{ s} = 3 \text{ ms.}$$

31. (a) $\omega = 2\pi f = 2 \times 3.14 \times 10^3$

$$= 6.28 \text{ rad/s}$$

$$T = \frac{1}{f} = \frac{1}{10^3} = 10^{-3} \text{ s} = 1 \text{ ms}$$

(b) As initially charge is maximum, (*i.e.., it is extreme position for charge).*

$$q = q_0 \cos \omega t$$

$$q_0 = CV_0 = 1 \times 10^{-6} \times 100$$

$$= 10^{-4}$$

$$\therefore \quad q = [10^{-4} \cos(6.28 \times 10^3)t] \text{ C.}$$

(c) $\omega = \dfrac{1}{\sqrt{LC}}$

$$\Rightarrow L = \frac{1}{\omega^2 C} = \frac{1}{(6.28 \times 10^3)^2 \times 10^{-6}}$$

$$= 2.53 \times 10^{-3}$$

$$\Rightarrow \quad L = 2.53 \text{ mH}$$

(d) In one quarter cycle, entire charge of the capacitor flows out.

$$< i > = \frac{q}{t} = \frac{4CV}{T}$$

$$= \frac{4 \times 10^{-6} \times 100}{10^{-3}} = 0.4 \text{ A}$$

32. (a) $V_0 = \dfrac{q_0}{C} = \dfrac{5.00 \times 10^{-6}}{4 \times 10^{-4}}$

$$= 1.25 \times 10^{-2} \text{ V} = 12.5 \text{ mV}$$

(b) Maximum magnetic energy = Maximum electric energy

$$\frac{1}{2}Li_0^2 = \frac{q_0^2}{2C}$$

$$\Rightarrow \quad i_0 = \frac{q_0}{\sqrt{LC}}$$

$$\Rightarrow \quad i_0 = \frac{5.00 \times 10^{-6}}{\sqrt{0.090 \times 4 \times 10^{-4}}} = 8.33 \times 10^{-4} \text{ A}$$

(c) Maximum energy stored in inductor,

$$= \frac{1}{2}L\, i_0^2$$

$$= \frac{1}{2} \times 0.0900 \times (8.33 \times 10^{-4})^2$$

$$= 3.125 \times 10^{-8} \text{ J}$$

(d) By conservation of energy,

$$\frac{q^2}{2C} + \frac{1}{2}Li^2 = \frac{1}{2}Li_0^2$$

But $i = \dfrac{i_0}{2}$

$$\frac{q^2}{2C} = \frac{3}{8}Li_0^2$$

$$q = \frac{i_0}{2}\sqrt{3LC} = \frac{\sqrt{3}}{2}q_0$$

$$= \frac{1.732}{2} \times 5.00 \times 10^{-6}$$

$$= 4.33 \times 10^{-6} \text{ C}$$

$$U_m = \frac{1}{2}Li^2 = \frac{1}{4}\left(\frac{1}{2}Li_0^2\right)$$

$$= 7.8 \times 10^{-9} \text{ J}$$

33. (a) $\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{2.0 \times 10^{-3} \times 5.0 \times 10^{-6}}}$

$$= 10^4 \text{ rad/s}$$

$$\left|\frac{di}{dt}\right| = \omega^2 Q$$

$$= (10^4)^2 \times 100 \times 10^{-6} = 10^4 \text{ A/s}$$

(b) $i = \omega\sqrt{Q_0^2 - Q^2}$

$$= 10^4\sqrt{(200 \times 10^{-6})^2 - (200 \times 10^{-6})^2} = 0$$

(c) $i_0 = \omega Q_0 = 10^4 \times 200 \times 10^{-6} = 2 \text{ A}$

(d) $i = \omega\sqrt{Q_0^2 - Q^2}$

$$\Rightarrow \quad \frac{i_0}{2} = \omega\sqrt{Q_0^2 - Q^2}$$

$$\Rightarrow \quad \frac{\omega Q_0}{2} = \omega\sqrt{Q_0^2 - Q^2}$$

$$Q = \frac{\sqrt{3}}{2} Q_0 = \frac{1.73 \times 200 \times 10^{-6}}{2}$$

$$= 173\ \mu\text{C}$$

34. As initially charge is maximum

$$q = q_0 \cos \omega t$$

and $$|i| = i_0 \sin \omega t$$

where, $$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{3.3 \times 840 \times 10^{-6}}}$$

$$\approx 19 \text{ rad./s}$$

$$i_0 = \omega q_0 = 19 \times 105 \times 10^{-6}$$

$$\approx 2.0 \times 10^{-3} \text{ A} = 2.0 \text{ mA}$$

At $t = 2.00$ ms

(a) $$U_e = \frac{q^2}{2C} = \frac{q_0^2}{2C}(\cos^2 \omega t)$$

$$= \frac{(105 \times 10^{-6})^2}{2 \times 840 \times 10^{-6}}[\cos^2 (38 \text{ rad})]$$

$$U_e = 6.55 \times 10^{-6} \text{ J} = 6.55\ \mu\text{J}$$

(b) $$U_m = \frac{1}{2} Li^2 = \frac{1}{2} Li_0^2 (\sin \omega t)$$

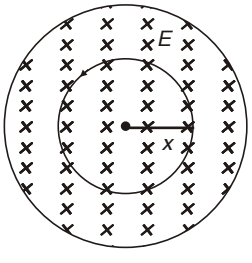
$$= \frac{1}{2} \times 3.3 \times (2 \times 10^{-3})^2 \sin^2(38 \text{ rad})$$

$$= 0.009 \times 10^{-6} \text{ J} = 0.009\ \mu\text{j}$$

(c) $$U = \frac{q_0^2}{2C} = \frac{1}{2} Li_0^2$$

$$= 6.56 \times 10^{-6} \text{ J} = 6.56\ \mu\text{J}$$

35. As the inward magnetic field is increasing, induced electric field will be anticlockwise.



At a distance x from centre of the region,

Magnetic flux linked with the imaginary loop of radius x

$$\phi_m = \pi x^2 B$$

$$e = \frac{-d\phi_m}{dt} = -\pi x^2 \frac{dB}{dt}$$

Induced electric field,

$$E = \frac{e}{2\pi x} = \frac{1}{2} x \frac{dB}{dt}$$

At a,

$$E = \frac{1}{4} r \frac{dB}{dt}, \text{ towards left.}$$

At b ,

$$E = \frac{1}{2} r \frac{dB}{dt}, \text{ upwards.}$$

At c,

$$E = 0$$

36. Inside the solenoid,

$$B = \mu_0 n i$$

$$\frac{dB}{dt} = \mu_0 n \frac{di}{dt}$$

Inside the region of varying magnetic field

$$E = \frac{1}{2} r \frac{dB}{dt} = \frac{1}{2} \mu_0 n r \frac{di}{dt}$$

(a) $r = 0.5$ cm $= 5.0 \times 10^{-3}$ m

$$E = \frac{1}{2} \mu_0 r n \frac{di}{dt}$$

$$= \frac{1}{2} \times 4\pi \times 10^{-7} \times 5.0 \times 10^{-3} \times 900 \times 60$$

$$= 1.7 \times 10^{-4} \text{ V/m}$$

(b) $r = 1.0$ cm $= 1.0 \times 10^{-2}$ m

$$E = \frac{1}{\mu_0} r n \frac{di}{dt}$$

$$= \frac{1}{2} \times 4\pi \times 10^{-3} \times 5.0 \times 10^{-3} \times 900 \times 60$$

$$= 3.4 \times 10^{-4} \text{ V/m}$$

AIEEE Corner

Objective Questions (Level 1)

1. $V = L\dfrac{di}{dt}$

$$[L] = \frac{[V][T]}{[i]} = \frac{[\mathrm{ML^2T^{-3}A^{-1}}][\mathrm{T}]}{[\mathrm{A}]}$$
$$= [\mathrm{ML^2T^{-2}A^{-2}}]$$

2. $M \propto n_1 n_2$

3. Both will tend to oppose the magnetic flux changing with them by increasing current in opposite direction.

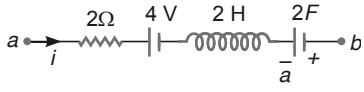
4. Moving charged particle will produced magnetic field parallel to ring, Hence

$$\phi_m = 0$$

Velocity of particle increases continuously due to gravity.

5. Induced electric field can exist at a point where magnetic field is not present, i.e., outside the region occupying the magnetic field.

6. At, $t = 1$ s



$$q = 4t^2 = 4\,\mathrm{C}$$
$$i = \frac{dq}{dt} = 8t = 8\,\mathrm{A}$$
$$\frac{di}{dt} = 8\,\mathrm{A/s}$$

As, $$\frac{di}{dt} = \frac{d^2q}{dt} = \text{Positive}$$

Charge in capacitor is increasing, current i must be towards left.

$$V_{ab} = -2I + 4 - L\frac{di}{dt} - \frac{q}{C}$$
$$= -2\times 8 + 4 - 2\times 8 - \frac{4}{2} = -30\,\mathrm{V}$$

7. $|e| = M\dfrac{di}{dt} = M\dfrac{d}{dt}(i_0 \sin\omega t)$

$$= \omega M i_0 \cos\omega t$$

Maximum induced emf $= \omega M i_0$

$$= 100\pi \times 0.005 \times 10$$
$$= 5\pi$$

8. $\dfrac{1}{2}Li_0^2 = \dfrac{1}{2}CV_0^2 \Rightarrow i_0\sqrt{\dfrac{L}{C}} = 2\times\sqrt{\dfrac{2}{4\times 10^{-6}}}$

$$= \sqrt{2}\times 10^3\,\mathrm{V}$$

9. $e = \dfrac{1}{2}Bl^2\omega$, is independent of t.

10. $|e| = \dfrac{d\phi}{dt} = \dfrac{\Delta\phi}{t}$

$$\Delta\phi = |e|t = iRt$$
$$= 10\times 10^{-3}\times 0.5\times 5$$
$$= 25\times 10^{-3}\,\mathrm{Wb}$$
$$= 25\,\mathrm{mWb}.$$

11. As inward magnetic field is increasing, induced electric field must be anti-clockwise. Hence, direction of induced electric field at P will be towards and electron will experience force towards right (opposite to electric field).

12. $\phi = at(\tau - t) = a\tau t - at^2$

$$|e| = \frac{d\phi}{dt} = a\tau - 2at$$
$$i = \frac{|e|}{R} = \frac{a\tau - 2at}{R}$$
$$H = \int_0^\tau i^2 R\,dt = \int_0^\tau \frac{(a\tau - 2at)^2}{R}\,dt$$
$$= \frac{1}{R}\left[\frac{(a\tau - 2at)^3}{3\times(-2a)}\right]_0^\tau$$
$$= \frac{1}{-6Ra}[-a^3\tau^3 - a^3\tau^3]$$
$$= \frac{a^2\tau^3}{3R}$$

13. $E = -L\dfrac{di}{dt}$

14. $V_{BA} = -L\dfrac{di}{dt} + 15 - iR$

$$= -5\times 10^{-3}(-10^3) + 15 - 5\times 1$$
$$= 15\,\mathrm{V}$$

15. $\dfrac{di}{dt} = 10$ A/s, at $t = 0$, $i = 5$A

$$\frac{di}{dt} = 10\,\mathrm{A/s}$$

$$V_A - V_B = iR + L\frac{di}{dt} - E = 0$$

$$= 5 \times 3 + 1 \times 10 - 10 = 15 \text{ V}$$

16. $\left(\frac{di}{dt}\right)_{max} = \left(\frac{d^2q}{dt}\right)_{max} = \omega^2 q_0 = \frac{q_0}{LC}$

17. $V = L\frac{di}{dt}$

18. $\phi_m = BA\cos\theta$

$$\Rightarrow \quad e = -\frac{d\phi_m}{dt} = BA\sin\theta\frac{d\theta}{dt}$$

$$\Rightarrow \quad iR = BA\sin\theta\frac{d\theta}{dt}$$

$$\Rightarrow \quad \frac{dq}{dt}R = BA\sin\theta\frac{d\theta}{dt}$$

$$\Rightarrow \quad dq = \frac{BA}{R}\sin\theta\, d\theta$$

$$\Rightarrow \quad q = \frac{BA}{R}\int_{\pi/2}^{3\pi/2}\sin\theta\, d\theta = 0$$

19. $\vec{\mathbf{A}} = ab\,\hat{\mathbf{k}}, \vec{\mathbf{B}} = 20t\,\hat{\mathbf{i}} + 10t^2\,\hat{\mathbf{j}} + 50\,\hat{\mathbf{k}}$

$$\phi_m = \vec{\mathbf{B}}\cdot\vec{\mathbf{A}} = 50\,ab$$

$$e = -\frac{d\phi_m}{dt} = 0$$

20. $E = V_b + iR$

$V_b = E - iR = 200 - 20 \times 1.5 = 170$ V

21. $\frac{V_s}{V_p} = \frac{N_s}{N_p} \Rightarrow V_s = \frac{1}{2} \times 290 = 10$ V

$$\frac{i_p}{i_s} = \frac{N_s}{N_p}$$

$$\Rightarrow \quad i_s = \frac{N_p}{N_s}i_p = 2 \times 4 = 8 \text{ A}$$

22. $V_r = 0$, hence magnetic flux linked with the coil remain same.

$\therefore e = \frac{-d\phi}{dt} = 0$

23. $s = \frac{1}{2}at^2$

Due to change in magnetic flux linked with the ring, magnet experiences an upward force, hence,

$$a < g$$

$$s < \frac{1}{2}gt^2 \Rightarrow s < 5 \text{ m}$$

24. $V_A - V_B = L\frac{di}{dt} = -\alpha t$

25. $i_0 = \frac{E}{R} = \frac{12}{0.3} = 40$ A

$$U_0 = \frac{1}{2}Li_0^2 = \frac{1}{2} \times 50 \times 10^{-3} \times (40)^2$$

$$= 40 \text{ J}$$

26. $i = i_0\left(1 - e^{-\frac{t}{\tau}}\right) = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$

$$\frac{di}{dt} = \frac{E}{L}e^{-\frac{Rt}{L}}$$

$$V_L = L\frac{di}{dt} = Ee^{-\frac{Rt}{L}}$$

at $t = 0$

$$V_L = E = 20 \text{ V}$$

at $t = 20$ ms

$$V_L = Ee^{-\frac{R \times 20 \times 10^{-3}}{L}} \Rightarrow 5 = 20e^{-\frac{R}{50L}}$$

$$\frac{R}{50L} = \ln 4 \Rightarrow R = (100\ln 4)\,\Omega$$

27. $|i| = \frac{|e|}{R} = \frac{1}{R}\cdot\frac{d\phi}{dt} = \frac{1}{R}NA\frac{dB}{dt}$

$$= \frac{10 \times 10 \times 10^{-4}}{20} \times 10^8 \times 10^{-4}$$

$$= 5 \text{ A}$$

28. In the steady state, inductor behaves as short circuit, hence entire current flows through it.

29. $\phi_m = AB\cos\theta$

But, $\theta = 90°$

$$\therefore \quad \phi_m = 0$$

30. $i = \frac{|e|}{R} = -\frac{1}{R}\frac{d\phi_m}{dt}$

$$\Rightarrow \quad \frac{dq}{dt} = \frac{-nBA}{R}\frac{d}{dt}(\cos\theta)$$

$$= \frac{nBA}{R}\sin\theta\frac{d\theta}{dt}$$

$$\Rightarrow \quad dq = \frac{nBA}{R}\sin\theta\, d\theta$$

$$Q_1 = \frac{nBA}{R}\int_0^{\pi}\sin\theta\, d\theta = \frac{2nBA}{R}$$

$$Q_2 = \frac{nBA}{R}\int_0^{2\pi}\sin\theta\, d\theta = 0$$

$$\therefore \quad \frac{Q_2}{Q_1} = 0$$

31. According to Lenz's law, induced current always opposes the cause producing it.

32. $i = i_0\left(1 - e^{-\frac{t}{\tau}}\right) = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$

$$= \frac{15}{5}\left(1 - e^{-\frac{5\times 2}{10}}\right) = 3\,(1 - e^{-1})$$

$$= 3\left(1 - \frac{1}{e}\right)\text{A}$$

33. Velocity of AB is parallel to its length.

34. Velocity of rod is parallel to its length.

35. $V_c - V_a = V_c - V_b = BRV$

and $V_a - V_b = 0$

36. Induced current always opposes the cause producing it.

37. $E = -\frac{d\phi}{dt}$

38. Magnetic flux linked with the coil does not change, hence

$$i = \frac{e}{R} = \frac{-1}{R}\cdot\frac{d\phi}{dt} = 0$$

39. $e = Blv\cos\theta = \frac{1}{2}Bl^2\omega\cos\theta \qquad \left(\because v = \frac{l}{2}\omega\right)$

As $|\cos\theta|$ varies from 0 to 1

e varies from 0 to $\frac{1}{2}B\,l^2\omega$.

JEE Corner

Assertion and Reason

1. Magnetic flux linked with the coil is not changing with time, hence induced current is zero.

2. Both Assertion and Reason are correct but Reason does not explain Assertion.

3. Induced electric field is non-conservative but can exert force on charged particles.

4. $i = 2t - 8$

$\frac{di}{dt} = 2$

$V_a - V_b = L\frac{di}{dt} = 2\times 2 = 4\text{ V}$

5. $\left(\frac{di}{dt}\right)_{\max} = (i_{\max})\,\omega = 1\times 2 = 2\text{ A/s}$

6. $V_a - V_b = V_c - V_a$

$V_c > V_a > V_b$

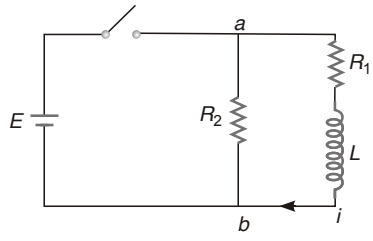
7. Fact.

8. $L = \mu_r\mu_0 n^2 lA$, for ferromagnetic substance,

$$\mu_r > g$$

and L does not depends on i.

9. As soon as key is opened



$$i = i_0 = \frac{E}{R_1}$$

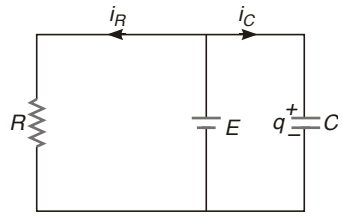
10. Inductors oppose change in current while resistor does not.

Objective Questions (Level 2)

1. By conservation of energy

$$\frac{1}{2}L\,i_0^2 = \frac{1}{2}mv_0^2$$

$$i_0 = \sqrt{\frac{m}{k}}v_0$$



2. Wire AB behaves as a cell of emf, $E = Blv$

$$i_R = \frac{E}{R} = \frac{Blv}{R}$$
$$i_c = 0$$
$$U_c = \frac{1}{2}CE^2 = \frac{1}{2}CB^2l^2v^2$$

3. Apply Fleming's left hand rule.

4. For SHM,

$$v = \omega A \cos \omega t$$
$$e = Blv = Bl\omega A \cos \omega t$$
$$e = \begin{cases} e_0 \cos \omega t & \text{for } nT < t < (2n-1)\dfrac{T}{2} \\ -e_0 \cos \omega t & \text{for } \dfrac{(2n-1)T}{2} > t > nT \end{cases}$$

5. $\phi_m = BA$

At any instant when wires have moved through a distance x,

$$A = (a + 2x)^2$$
$$\phi_m = B(a + 2x)^2$$
$$|e| = \frac{d\phi_m}{dt} = 4B(a + 2x)\frac{dx}{dt}$$
$$= 4B(a + 2x)v_0$$
$$|i| = \frac{|e|}{R} = \frac{4B(a + 2x)v_0}{\lambda \times 4(a + 2x)} = \frac{Bv_0}{\lambda}$$

6. $A = l^2$

$$\frac{dA}{dt} = 2l\frac{dl}{dt} = -2l\alpha \qquad \left(\alpha = -\frac{dl}{dt}\right)$$
$$\phi_m = BA$$
$$e = -\frac{d\phi_m}{dt} = -B\frac{dA}{dt} = 2Bl\alpha$$

at $\quad l = a$

$$e = 2a\,\alpha B$$

7. At this instant, direction of motion of wire PQ is perpendicular to its length.

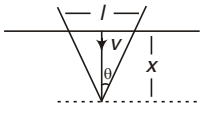
$\therefore \qquad e = Blv$

8. $q = CV = CBlv$

$$= 20 \times 10^{-6} \times 0.5 \times 0.1 \times 0.2$$
$$= 0.2\ \mu\text{C}$$

Plate A is positive while plate B is negative.

9. $\phi_m = BA = B\left(\frac{1}{2}lx\right)$



But $l = 2x \tan\theta$

$$\therefore \quad \phi_m = B \tan\theta\, x^2$$
$$e = -\frac{d\phi_m}{dt} = -2B \tan\theta\, x\frac{dx}{dt}$$
$$= 2B \tan\theta\, vx$$
$$R = r\,l = r(2x \tan\theta)$$

where, r = resistance per unit length of the conductor.

$$\therefore \qquad i = \frac{e}{R} = \frac{Bv}{r} = \text{constant.}$$

10. $\phi_m = BA\cos\theta = BA\cos\omega t$

$$e = -\frac{d\phi_m}{dt} = \omega BA \sin\omega t$$

But $\qquad A = b^2$

$\therefore \qquad e = b^2B\omega \sin\omega t$

11. Induced emf

$$e = a^2\frac{dB}{dt} = (1)^2 \times 2 \times 10^{-3}$$
$$= 2 \times 10^{-3}\ \text{V}$$
$$W = qe = 1 \times 10^{-6} \times 2 \times 10^{-3}$$
$$= 2 \times 10^{-9}\ \text{J}$$

12. In the steady state, current through capacitor = 0.

$$i_L = \frac{20}{5} = 4\ \text{A}$$
$$\phi_1 = 0,\ \phi_2 = i_L L = 4 \times 500 \times 10^{-2}$$
$$= 2\ \text{Wb}$$
$$\Delta\phi = \phi_2 - \phi_1 = 2\ \text{Wb.}$$

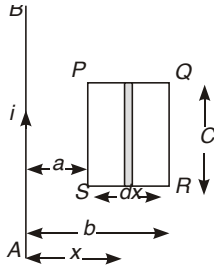
13. $\frac{1}{2}Li^2 = \frac{1}{2}\left(\frac{1}{2}Li_0^2\right)$

$$i = \frac{i_0}{\sqrt{2}}$$
$$i_0\left(1 - e^{-\frac{t}{\tau}}\right) = \frac{i_0}{\sqrt{2}}$$
$$e^{-t/\tau} = \frac{\sqrt{2} - 1}{\sqrt{2}}$$
$$t = \tau \ln\left(\frac{\sqrt{2}}{\sqrt{2} - 1}\right)$$
$$= \frac{L}{R}\ln\left(\frac{\sqrt{2}}{\sqrt{2} - 1}\right)$$

14. $B = \frac{\mu_0 i}{2\pi a}$

$$F = qvB = \frac{\mu_0 iqv}{2\pi a}$$

15. Consider an elementary section of loop of width dx at a distance x from wire AB

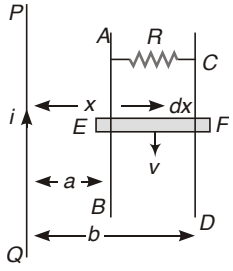


$$d\phi_m = BdA = \frac{\mu_0 i}{2\pi x} C\, dx$$

$$\phi_m = \frac{\mu_0 iC}{2\pi}\int_a^b \frac{dx}{x} = \frac{\mu_0 iC}{2\pi}\ln\frac{b}{a}$$

$$M = \frac{\phi_m}{i} = \frac{\mu_0 C}{2\pi}\ln\frac{b}{a}$$

16. From previous question



$$\phi_m = \frac{\mu_0 iy}{2\pi}\ln\frac{b}{a}$$

$$e = \frac{d\phi_m}{dt} = \frac{\mu_0 i \ln\frac{b}{a}}{2\pi}\frac{dy}{dt}$$

$$e = \frac{\mu_0 iv}{2\pi}\ln\frac{b}{a}$$

$$i = \frac{e}{R} = \frac{\mu_0 iv}{2\pi R}\ln\frac{b}{a}$$

Consider an elementary portion of length dx of the rod at a distance, x from the wire PQ.

Force on this portion,

$$dF = i\,dxB$$

$$= i\frac{\mu_0}{4\pi}\cdot\frac{2i}{x}dx$$

$$F = i\frac{\mu_0}{4\pi}\cdot 2i\int_a^b \frac{dx}{x}$$

$$= \left[\frac{\mu_0 iv}{2\pi R}\ln\frac{b}{a}\right]\frac{\mu_0 i}{2\pi}\ln\frac{b}{a}$$

$$= \frac{1}{vR}\left[\frac{\mu_0 iv}{2\pi}\ln\frac{b}{a}\right]^2$$

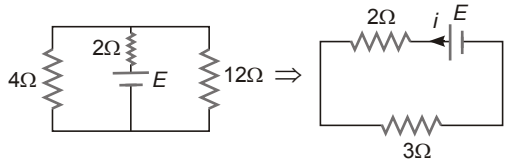
17. $E = \frac{1}{2}r\frac{dB}{dt} \Rightarrow E \propto r$

18. Induced current opposes change in magnetic flux.

19. $V_L = E - iR$

20. The rod can be assumed as a cell of emf $E = Blv$

The equivalent circuit is shown in figure,



$$i = \frac{E}{2+3} = \frac{Blv}{5} = \frac{0.50\times 0.25\times 4}{5}$$

$$= 0.1\text{ A}$$

21. Outside the region of magnetic field, induced electric field,

$$E = \frac{r^2}{2R}\frac{dB}{dt} = \frac{Br^2}{2R}$$

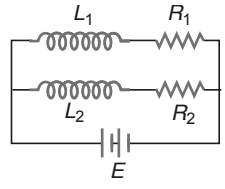
$$F = qE$$

$$\tau = qER = \frac{1}{2}qBr^2$$

22. $V_A - V_0 = B(2R)V$

$\Rightarrow \quad V_A - V_0 = 2BRV$

23. $L_1 = \frac{\eta L}{\eta+1}, L_2 = \frac{L}{\eta+1}$



$$R_1 = \frac{\eta R}{\eta+1}, R_2 = \frac{\eta R}{\eta+1}$$

$$\frac{1}{L_e} = \frac{1}{L_1} + \frac{1}{L_2}$$

$$= \frac{\eta+1}{\eta L} + \frac{\eta+1}{L}$$

$$= \frac{(\eta+1)}{L}\left[\frac{1}{\eta} + \frac{1}{1}\right]$$

$$L_e = \frac{\eta L}{(\eta+1)^2}$$

Similarly, $R_e = \frac{\eta R}{(\eta+1)^2} \quad \therefore \quad \tau = \frac{L_e}{R_e} = \frac{L}{R}$

24. $i = i_0 e^{-t/\tau}$

$$Bi_0 = i_0 e^{-T/\tau}$$

$$\tau = \frac{T}{\ln\left(\frac{1}{B}\right)}$$

25. Given, $i_0^2 R = P, \frac{L}{R} = \tau$

when, choke coil is short circuited,

Total heat produced = Magnetic energy stored in the choke coil

$$= \frac{1}{2} L i_0^2 = \frac{1}{2}(R\tau)\left(\frac{P}{R}\right) = \frac{1}{2} P\tau$$

26. $i = i_0 e^{-\frac{Rt}{L}}$

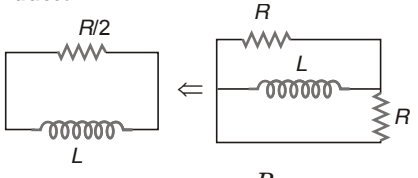
For current to be constant

$$i = i_0$$

$$e^{-\frac{Rt}{L}} = 1$$

$\frac{Rt}{L} = 0 =$ not possible.

27. To final time constant, short the battery and find effective resistance in series with inductor

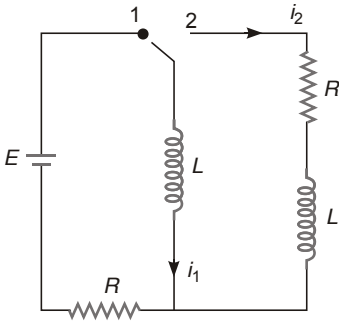


$$R_e = \frac{R}{2}$$

$$\tau = \frac{L}{R_e} = \frac{2L}{R}$$

28. When switch is at position 1.

In steady state,



$$i_1 = \frac{E}{R}$$

$$i_2 = 0$$

When switch is thrown to position 2.

$$i_1 = \frac{E}{R}, i_2 = -\frac{E}{R}$$

29. $\frac{1}{2} L i^2 = \frac{1}{4}\left(\frac{1}{2} L i_0^2\right)$

$$i = \frac{i_0}{2}$$

$$i_0\left(1 - e^{-\frac{t}{\tau}}\right) = \frac{i_0}{2}$$

$$t = \tau \ln 2$$

$$\Rightarrow \quad t = \frac{L}{2} \ln 2$$

30. At the moment when switch is thrown to position 2,

current in capacitor = current in inductor just before throwing the switch to position 2,

$$\Rightarrow \quad i_c = \frac{E}{R}$$

31. Initially, inductor offers infinite resistance, hence,

$$i = 0 \text{ and } \frac{di}{dt} = \text{maximum}$$

$\therefore \quad E = V_L + V_C + V_R$

But $\quad V_C = V_R = 0$

$\Rightarrow \quad V_L = E$

32. Same as Q.12 objective Questions (Level 2).

33. Let V_0 = Potential of metallic rod,

$$V_B - V_0 = B(2R)V = 2BR^2\omega \quad \text{...(i)}$$

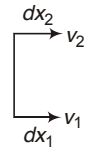
$$V_0 - V_C = B(2R)V = 2BR^2\omega \quad \text{...(ii)}$$

Adding Eqs. (i) and (ii), we get

$$V_B - V_C = 4B\omega R^2$$

34. $e = Blv_c$

$$v_c = \frac{v_1 + v_2}{2}$$



$$\therefore \quad e = \frac{1}{2} Bl(v_1 + v_2)$$

or

$$e = B\frac{dA}{dt}$$

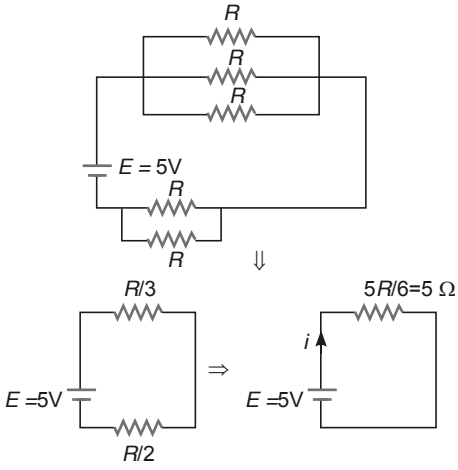
$$dA = \frac{1}{2} l(dx_1 + dx_2)$$

$$e = \frac{1}{2} Bl\left(\frac{dx_1}{dt} + \frac{dx_2}{dt}\right)$$

$$= \frac{1}{2} Bl(v_1 + v_2)$$

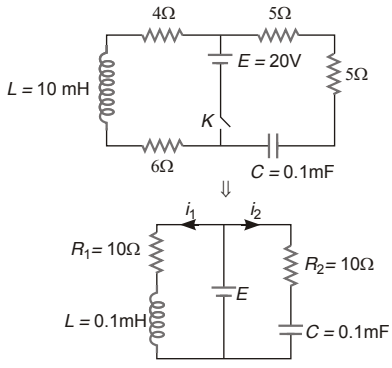
35. Initially, capacitor offer zero resistance and inductor offers infinite resistance.

Effective circuit is given by



$$i = \frac{E}{R} = 1\text{ A}$$

36 . $i_1 = \frac{E}{R_1}\left(1 - e^{-\frac{R_1 t}{L}}\right),\ i_2 = \frac{E}{R_2}\left(e^{-\frac{t}{R_2 C}}\right)$



$$i = i_1 + i_2$$

$$= \frac{E}{R_1}\left(1 - e^{-\frac{R_1 t}{L}}\right) + \frac{E}{R_2} e^{-\frac{t}{R_2 C}}$$

at $t = 10^{-3}\ \ln 2$

$$i = \frac{20}{10}\left(1 - e^{-\frac{10\times10^{-3}\ \ln 2}{10\times10^{-3}}}\right) + \frac{20}{10} e^{-\frac{10^3\ \ln 2}{10\times0.1\times10^{-3}}}$$

$$= 2\left(1 - \frac{1}{2}\right) + 2\left(\frac{1}{2}\right) = 2\text{ A}$$

37. $|i| = \frac{|e|}{R} = \frac{A}{R}\frac{dB}{dt}$

$$= \frac{B_0 A}{R} = \frac{B_0[(2b)^2 - \pi a^2]}{R}$$

$$= \frac{B_0(4b^2 - \pi a^2)}{R}$$

As inward magnetic field is increasing, net current must be anticlockwise. Hence current in inner circle will be clockwise.

38. From Q. 48 Subjective Questions (Level 1).

$$\phi_m = \frac{\mu_0 ai}{2\pi}\ln\left(1 + \frac{a}{x}\right)$$

Case 1

$$x = b,\ a = a$$

$$\phi_{m_1} = \frac{\mu_0 ai}{2\pi}\ln\left(1 + \frac{a}{b}\right)$$

$$= \frac{\mu_0 ai}{2\pi}\ln\left(\frac{b+a}{b}\right)$$

Case 2

$$x = b - a$$

$$a = a$$

$$\phi_{m_2} = \frac{\mu_0 ai}{2\pi}\ln\left(1 + \frac{a}{b-a}\right)$$

$$= \frac{\mu_0 ai}{2\pi}\ln\left(\frac{b}{b-a}\right)$$

$$< e > = -\frac{\phi_{m_2} - \phi_{m_1}}{t}$$

$$< e > = \frac{< e >}{R} = -\frac{\phi_{m_2} - \phi_{m_1}}{R\,t}$$

$$q = < i > t = -\frac{\phi_{m_2} - \phi m_1}{R}$$

$$= -\frac{\mu_0 ai}{2\pi R}\left[\ln\left(\frac{b+a}{b}\right) - \left(\ln\frac{b}{b-a}\right)\right]$$

$$= \frac{-\mu_0 ai}{2\pi R}\ln\left(\frac{b}{b^2 - a^2}\right)$$

$$|q| = \frac{\mu_0 ai}{2\pi R}\ln\left(\frac{b}{b^2 - a^2}\right)$$

39. Magnetic flux linked with the coil.

$$\phi_m = nBA = \frac{\mu_0 n\, iA}{2r}$$

$$|e| = \frac{d\phi_m}{dt}$$

$$\Rightarrow \quad iR = \frac{d\phi_m}{dt}$$

$$\frac{dq}{dt}R = \frac{d\phi_m}{dt} \Rightarrow dq = \frac{1}{R}d\phi_m$$

$$q = \frac{\mu_0 nA}{2rR}\int_0^i di = \frac{\mu_0 n\, iA}{2rR}$$

40. Induced electric field inside the region of varying magnetic fields,

$$E = \frac{1}{2}r\frac{dB}{dt} = \frac{1}{2}r(6t^2 + 2x) = 3r(t^2 + x)\text{ V/m}$$

At, $t = 2.0$ s and $r = \frac{R}{2} = 1.25$ cm

$$= 1.25 \times 10^{-2}\text{ m}$$

$$E = 3 \times 1.25 \times 10^{-2} \times (4 + x)$$

$$= 0.3\text{ V/m}$$

$$F = eE = 1.6 \times 10^{-19} \times 0.3$$

$$= 48 \times 10^{-21}\text{ N}$$

41. $E = \frac{1}{2}r\frac{dB}{dt} \Rightarrow E \propto r$

42. As inward magnetic field is increasing, induced electric field must be anticlockwise.

43. $e = \frac{d\phi_m}{dt} = \pi a^2\frac{dB}{dt} = \pi a^2 B_0$

44. $E = \frac{e}{2\pi a} = \frac{1}{2}aB_0$

45. $\tau = qEa = i\alpha$

$$\alpha = \frac{qEa}{ma^2} = \frac{q \times \frac{1}{2}aB_0 a}{ma^2}$$

$$= \frac{qB_0}{2m}$$

46. $P = \tau\omega = \tau(\alpha t) = i\alpha^2 \cdot t$

$$= ma^2 \times \frac{q^2B_0^2}{\mu\, m^2}t$$

At $t = 1$ s

$$P = \frac{q^2B_0^2a^2}{4m}$$

47. $i = \frac{e}{R} = \frac{A}{R}\cdot\frac{dB}{dt}$

$$\frac{dB}{dt} = 2\text{ T/s},\ A = 0.2 \times 0.4 = 0.08\text{ m}^2$$

$$\therefore\quad i = \frac{0.08}{1 \times 1.0} \times 2 = 16\text{ A} \qquad [\because R = r \times (b + 2l)]$$

As outward magnetic field is increasing, induced current must be clockwise.

48. $e = B\frac{dA}{dt} + A\frac{dB}{dt} = Blv + A\frac{dB}{dt}$

At $t = 2$ s,

$$B = 4\text{ T},\ A = 0.2 \times (0.4 - vt) = 0.06\text{ m}^2$$

$$v = 5\text{ cm/s} = 0.05\text{ m/s}$$

$$\therefore\quad e = -4 \times 0.2 \times 0.05 + 0.06 \times 2$$

$$= -0.04 + 0.12 = 0.08\text{ V}$$

49. $F = ilB = \frac{e}{R}lB$

$$= \frac{0.08}{1 \times 0.8} \times 0.2 \times 4$$

$$= 0.008\text{ N}$$

50. When terminal velocity is attained,

power delivered by gravity = power dissipated in two resistors

$$mgv = 0.76 + 1.2$$

$$v = \frac{1.96}{0.2 \times 9.8} = 1\text{ m/s}$$

51. $e = Blv = 0.6 \times 1 \times 1 = 0.6$ V

$$P_1 = \frac{e^2}{R_1}$$

$$\Rightarrow \qquad R_1 = \frac{e^2}{P_1} = \frac{(0.6)^2}{0.76} = 0.47\ \Omega$$

52. $P_2 = \frac{e^2}{R_2}$

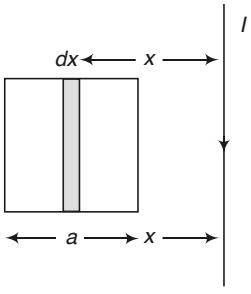
$$\Rightarrow \qquad R_2 = \frac{e^2}{P_2} = \frac{(0.6)^2}{1.2} = 0.3\ \Omega$$

More than One Correct Options

1. $e = B\left(\frac{1}{2}\right)v = \frac{1}{2}BLv$

By Fleming's left hand rule, P must be positive w.r.t. Q.

2. $d\phi_m = BdA = Ba\,dx$



$$= \frac{\mu_0 a\, i}{2\pi x}dx$$

$$\phi_m = \frac{\mu_0 a\, i}{2\pi i}\ln 2$$

$$M = \frac{\phi_m}{i} = \frac{\mu_0 a}{2\pi}\ln 2$$

If the loop is brought close to the wire, upward magnetic flux linked with the loop increases, hence induced current will be clockwise.

3. $\phi = Li =$ Henry-Ampere.

$$L = \frac{V}{di/dt} = \frac{V\,dt}{di} = \frac{\text{Volt-second}}{\text{Ampere}}$$

4. $\tau = \frac{L}{R} = 1\text{ s}$

$$i = i_0(1 - e^{-t/\tau}) = \frac{E}{R}(1 - e^{-t/\tau})$$

$$= 4(1 - e^{-t})$$

At $t = \ln 2$,

$$i = 2A$$

Power supplied by battery, $P = EI = 16$ J/s.

Rate of dissipation of heat in across resistor

$$= i^2R = 8\text{ J/s}$$

$$V_R = iR = 4\text{ V}$$

$$V_a - V_b = E - V_R = 4\text{ V}$$

5. In both the cases, magnetic flux linked with increases, so current i_2 decreases in order to oppose the change.

6. $\phi_1 = BA = 4 \times 2 = 8$ Wb, $\phi_2 = 0$

$$e = \frac{\phi_2 - \phi_1}{t} = \frac{8}{0.1} = 80\text{ V}$$

$$i = \frac{e}{R} = \frac{80}{4} = 20\text{ A}$$

$$q = it = 20 \times 0.1 = 2\text{ C}$$

Current is not given as a function of time, hence heat produced in the coil cannot be determined.

7. In LC oscillations,

$$\omega = \frac{1}{\sqrt{LC}},\ f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

$$T = \frac{1}{f} = 2\pi\sqrt{LC}$$

$$i_0 = \omega q_0 = \frac{q_0}{\sqrt{LC}}$$

$$\left(\frac{di}{dt}\right)_{\max} = \omega q_0 = \frac{q_0}{LC}$$

$$(V_L)_{\max} = L\left(\frac{di}{dt}\right)_{\max} = \frac{q_0}{C}$$

8. If magnetic field increases, induced electric field will be anticlockwise and *vice-versa*.

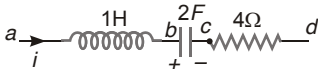
9. $q = 2t^2$

$$i = \frac{dq}{dt} = 4t$$

$$\frac{di}{dt} = 4\text{ A/s}$$

As $\frac{dq}{dt} =$ Positive

Charge on the capacitor is increasing, hence current flows from a to b.



$$t = 1\text{ s},\ q = 2\text{ C},\ i = 4\text{ A}$$

$$\frac{di}{dt} = 4\text{ A/s}$$

$$V_a - V_b = L\frac{di}{dt} = 1 \times 4 = 4\text{ V}$$

$$V_b - V_c = \frac{q}{c} = \frac{2}{2} = 1\text{ V}$$

$$V_c - V_d = iR = 4 \times 4 = 16\text{ V}$$

$$V_a - V_d = 4 + 1 + 16 = 21\text{ V}$$

10. $V_a - V_b = \frac{1}{2}Bl^2\omega$

$$V_c - V_b = \frac{1}{2}Bl^2\omega$$

$$V_a - V_c = 0$$

[Direction of velocity of rod a-c is parallel to length a-c]

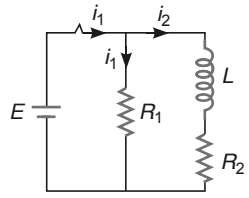
Match the Columns

1. $$[B] = \frac{[F]}{[i][l]} = \frac{[\mathrm{ML\,T^{-2}}]}{[\mathrm{A}][\mathrm{L}]}$$
$$= [\mathrm{ML^0\,T^{-2}\,A^{-1}}]$$
$$[L] = \frac{[V][dt]}{[di]} = \frac{[\mathrm{ML^2\,T^{-3}}][\mathrm{T}]}{[\mathrm{A}]}$$
$$= [\mathrm{ML^2\,T^{-2}\,A^{-2}}]$$
$$[LC] = [\mathrm{T}^2]$$
$$[\phi_m] = [B][S]$$
$$= [\mathrm{ML^0\,T^{-2}\,A^{-1}}][\mathrm{L}^2] = [\mathrm{ML^2\,T^{-2}\,A^{-1}}]$$

2. $i = i_0\,(1 - e^{-t/\tau})$
$$\tau = \frac{L}{R} = 1\ \mathrm{s}$$
$$i_0 = \frac{E}{R} = 5\ \mathrm{A}$$
$$V_R = iR = E(1 - e^{-t})$$
$$V_L = E - V_R = Et^{-t}$$
At $t = 0$,
$$V_L = E = 10\ \mathrm{V},\ V_R = 0$$
at $t = 1$ s
$$V_L = E(1 - e^{-1}) = \left(1 - \frac{1}{e}\right)10\ \mathrm{V}$$
$$V_R = \frac{10}{e}\ \mathrm{V}$$

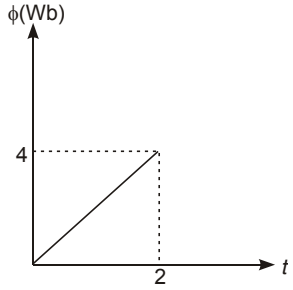
3. In LC oscillations,
$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \times \frac{1}{4}}} = 2\ \mathrm{rad/s}$$
$$q_0 = 4\ \mathrm{C}$$
$$i_0 = \omega q_0 = 8\ \mathrm{A}$$
$$\left(\frac{di}{dt}\right)_{\max} = \omega^2 q_0 = 16\ \mathrm{A/s}.$$
When, $q = 2$ C
$$V_L = V_C = \frac{q}{C} = 8\ \mathrm{V}$$
When, $$\left(\frac{di}{dt}\right) = \frac{1}{2}\left(\frac{di}{dt}\right)_{\max} = 8\ \mathrm{A/s}.$$
$$V_C - V_L = L\frac{di}{dt} = 1 \times 8 = 8\ \mathrm{V}$$

4. $i_1 = \dfrac{E}{R_1} = \dfrac{9}{6} = 1.6$ A



$$i_2 = \frac{E}{R_2}(1 - e^{-\frac{R_2 t}{L}}) = 3\,(1 - e^{-t/3})$$
At $t = (\ln 2)$ s
$$V_L = E - i_2 R_2 = qe^{-t/3} = \frac{q}{2^{1/3}}$$
$$V_{R_2} = i_2 R_2 = q\,(1 - e^{-t/3})$$
$$= q\left(1 - \frac{1}{2^{1/3}}\right)$$
$$V_{R_1} = i_1 R_1 = 9\ \mathrm{V}$$
$$V_{bc} = V_L + V_{R_2} = 9\ \mathrm{V}$$
(a $\rightarrow$ s), (b $\rightarrow$ s), (c $\rightarrow$ p), (d $\rightarrow$ p).

5. Induced emf



$|e|$ = slope of ϕ - t graph
$$= \frac{4 - 0}{2 - 0} = 2\ \mathrm{V}$$
$$|i| = \frac{|e|}{R} = \frac{2}{2} = 1\ \mathrm{A}$$
$$|q| = |i|t = 1 \times 2 = 2\ \mathrm{C}$$
As current i is constant
$$H = i^2 Rt = (1)^2 \times 2 \times 2 = 4\mathrm{J}$$

25 Alternating Current

Introductory Exercise 25.1

1. $$R = \frac{V_{DC}}{I} = \frac{100}{10} = 10\,\Omega$$

$$Z = \frac{V_{AC}}{I} = \frac{150}{10} = 15\,\Omega$$

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{(15)^2 - (10)^2}$$

$$= 5\sqrt{5}\,\Omega$$

$$L = \frac{X_L}{\omega} = \frac{X_L}{2\pi f} = \frac{5\sqrt{5}}{2 \times 3.14 \times 50}$$

$$\approx 0.036\,\text{H}$$

$$\therefore \quad V_L = IX_L = 50\sqrt{5}\,\text{V}$$

$$= 111.8\,\text{V}$$

2. For phase angle to be zero,

$$X_L = X_C$$

$$\Rightarrow \quad \omega L = \frac{1}{\omega C}$$

$$\Rightarrow \quad L = \frac{1}{\omega^2 C} = \frac{1}{(2\pi f)^2 C}$$

$$= \frac{1}{(360)^2 \times 10^{-6}}$$

$$\approx 7.7\,\text{H}$$

As $X_L = X_C$

$\therefore$ $Z = R$

$$I = \frac{V}{Z} = \frac{120}{20} = 6\,\text{A}$$

Introductory Exercise 25.2

1. Resonating frequency,

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.03 \times 2 \times 10^{-6}}}$$

$$= \frac{10^4}{\sqrt{6}}$$

$$f_r = \frac{\omega_r}{2\pi} = \frac{10^4}{2 \times 3.14 \times \sqrt{6}}$$

$$\approx 1105\,\text{Hz}$$

Phase angle at resonance is always 0°.

2. Resistance of arc lamp,

$$R = \frac{V_{DC}}{I}$$

$$= \frac{40}{10} = 4\,\Omega$$

Impedance of series combination,

$$Z = \frac{V_{AC}}{I} = \frac{200}{10} = 20\,\Omega$$

Power factor $= \cos\phi = \frac{R}{Z} = \frac{4}{20} = \frac{1}{5}$

AIEEE Corner

Subjective Questions (Level-1)

1. (a) $X_L = \omega L = 2\pi f L$

$= 2 \times 3.14 \times 50 \times 2$

$= 628\ \Omega$

(b) $X_L = \omega L \Rightarrow L = \dfrac{X_L}{\omega}$

$= \dfrac{X_L}{2\pi f} = \dfrac{2}{2 \times 3.14 \times 50}$

$= 6.37$ mH

(c) $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C}$

$= \dfrac{1}{2 \times 3.14 \times 50 \times 2 \times 10^{-6}}$

$= 1592\ \Omega = 1.59\ \text{k}\Omega$

(d) $X_C = \dfrac{1}{\omega C} \Rightarrow C = \dfrac{1}{\omega X_C}$

$= \dfrac{1}{2 \times 3.14 \times 50 \times 2} = 1.59$ mF

2. (a) $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$

$= \sqrt{(300)^2 + \left(400 \times 0.25 - \dfrac{1}{400 \times 8 \times 10^{-6}}\right)^2}$

$= 367.6\ \Omega$

$I_0 = \dfrac{V_0}{Z} = \dfrac{120}{367.6} = 0.326$ A

(b) $\phi = \tan^{-1} \dfrac{X_L - X_C}{R}$

$= \tan^{-1} \dfrac{212.5}{300} \approx -35.3°$

As $X_C > X_L$ voltage will lag behind current by 35.3°.

(c) $V_R = I_0 R = 0.326 \times 300 = 97.8$ V,

$V_L = I_0 X_L = 32.6$ V

$V_C = I_0 X_C = 0.326 \times 312.5$

$= 101.875\ \text{V} \approx 120\ \text{V}$

3. (a) Power factor at resonance is always 1, as $Z = R$, Power factor $= \cos\phi = \dfrac{R}{Z} = 1$.

(b) $P = \dfrac{I_0 E_0 \cos\phi}{2} = \dfrac{E_0^2}{2R}$

$= \dfrac{(150)^2}{3 \times 150} = 75$ W

(c) Because resonance is still maintained, average power consumed will remain same, *i.e.*, 75 W.

4. (a) As voltage is lag behind current, inductor should be added to the circuit to raise the power factor.

(b) Power factor $= \cos\phi = \dfrac{R}{Z}$

$\Rightarrow \quad Z = \dfrac{R}{\cos\phi} = \dfrac{60}{0.720} = \dfrac{250}{3}\ \Omega$

$X_C = \sqrt{Z^2 - R^2}$

$= \sqrt{\left(\dfrac{250}{3}\right)^2 - (60)^2}$

$= 58\ \Omega$

$C = \dfrac{1}{\omega X_C}$

$= \dfrac{1}{2\pi f\, X_C}$

$= \dfrac{1}{2 \times 3.14 \times 50 \times 58}$

$= 54\ \mu\text{F}$

For resonance,

$\omega_r = \dfrac{1}{\sqrt{LC}}$

$\Rightarrow \quad L = \dfrac{1}{\omega_r^2 C}$

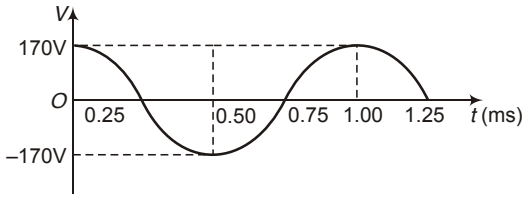
$= \dfrac{1}{(2\pi f)^2 C}$

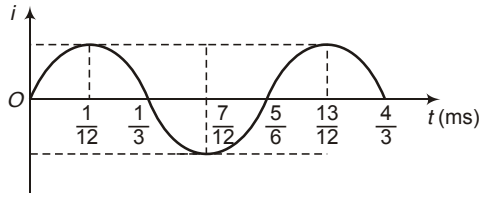
$\Rightarrow \quad L = \dfrac{1}{(2 \times 3.14 \times 50)^2 \times 54 \times 10^{-6}}$

$= 0.185$ H

5. $V(t) = 170 \sin(6280t + \pi/3)$ volt

$i(t) = 8.5 \sin(6280t + \pi/2)$ amp.





(b) $f = \dfrac{\omega}{2\pi} = \dfrac{6280}{2 \times 3.14} = 1000$ Hz

$= 1$ kHz

(c) $\phi = \dfrac{\pi}{2} - \dfrac{\pi}{3} = \dfrac{\pi}{6}$

$\Rightarrow \quad \cos\phi = \cos\dfrac{\pi}{6} = \dfrac{\sqrt{3}}{2}$

As phase of i is greater than V, current is leading voltage.

(d) Clearly the circuit is capacitive in nature, we have

$$\cos\phi = \frac{R}{Z}$$

$\Rightarrow \quad \dfrac{\sqrt{3}}{2} = \dfrac{R}{Z} \Rightarrow Z = \dfrac{2}{\sqrt{3}} R$

Also, $\quad Z = \dfrac{V_0}{i_0} = \dfrac{170}{8.5} = 20\,\Omega$

$$R = \frac{\sqrt{3}}{2} Z = 10\sqrt{3}\,\Omega$$

Again, $\quad Z = \sqrt{R^2 + X_C^2} \Rightarrow X_C = \sqrt{Z^2 - R^2}$

$= \sqrt{400 - 300} = 10\,\Omega$

$X_C = \dfrac{1}{\omega C} \Rightarrow \dfrac{1}{\omega X_C} = \dfrac{1}{6280 \times 10}$

$= 15.92\,\mu$F

6. $I = \dfrac{V}{X_L} = \dfrac{V}{\omega L}$

(a) $\omega = 100$ rad/s

$\therefore \quad I = \dfrac{60}{100 \times 5} = 0.12$ A

(b) $\omega = 1000$ rad/s

$\therefore \quad I = \dfrac{60}{1000 \times 5} = 1.2 \times 10^{-2}$ A

(c) $\omega = 10000$ rad/s

$\therefore \quad I = \dfrac{60}{10000 \times 5} = 1.2 \times 10^{-3}$ A

7. $V_R = (2.5\text{ V}) \cos[(950 \text{ rad/s})\, t]$

(a) $I = \dfrac{V_R}{R}$

$= \dfrac{(2.5\text{ V}) \cos[(950 \text{ rad/s})\, t]}{300}$

$= (8.33 \text{ mA}) \cos[(950 \text{rad/s})\, t]$

(b) $X_L = \omega L = 950 \times 0.800$

$= 760\,\Omega$

(c) $V_L = I_0 X_L \cos(\omega t + \pi/2)$

$\Rightarrow \quad V_L = -I_0 X_L \sin \omega t$

$= -6.33 \sin[(950 \text{ rad/s})\, t]$ V

8. Given, $L = 0.120$ H, $R = 240\,\Omega$, $C = 7.30\,\mu$F, $I_{\text{rms}} = 0.450$ A, $f = 400$ Hz

$X_L = \omega L = 2\pi f L$

$= 2 \times 3.14 \times 400 \times 0.120$

$= 301.44\,\Omega$

$X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C}$

$= \dfrac{1}{2 \times 3.14 \times 400 \times 7.3 \times 10^{-6}}$

$= 54.43\,\Omega$

(a) $\cos\phi = \dfrac{R}{Z} = \dfrac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$

$= \dfrac{240}{\sqrt{(240)^2 + (301.44 - 54.43)^2}}$

$= 0.697$

$\phi = \cos^{-1}(0.697) \approx 45.8°$

(b) $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{(240)^2 + (301.44 - 54.43)^2} = 344\,\Omega$

(c) $V_{\text{rms}} = I_{\text{rms}} Z = 0.450 \times 344$

$= 154.8 \text{ V} \approx 155 \text{ V}$

(d) $P_{av} = V_{\text{rms}} I_{\text{rms}} \cos\phi$

$= 155 \times 0.450 \times 0.697 = 48.6$ W

(e) $P_R = I_{\text{rms}}^2 R = (0.450)^2 \times 240 = 48.6$ W

(f) and (g) Average power associated with inductor and capacitor is always zero.

Objective Questions (Level-1)

1. In an AC circuit, $\cos\phi$ is called power factor.
2. DC ammeter measures charge flowing in the circuit per unit time, hence it measures average value of current, but average value of AC over a long time is zero.
3. $$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$
   Hence, for $X_L < X_C$, Z decreases with increase in frequency and for $X_L > X_C$, Z increases with increase in frequency.
4. As voltage leads current and $\phi < \frac{\pi}{2}$, hence either circuit contains inductance and resistance or contains inductance, capacitance and resistance with $X_L > X_C$.
5. RMS value of sine wave AC is $0.707\, I_0$, but can be different for different types of AC's.
6. $P = I_v E_v \cos\phi = 0$
7. $Z = \sqrt{R^2 + (X_L - X_C)^2}$
8. $P = \frac{V_0 I_0}{2}$ [V_0 and I_0 are peak voltage and current through resistor only]
9. $V_{rms} = \frac{V_0}{\sqrt{2}} = 170\text{ V}$
   $$f = \frac{\omega}{2\pi} = \frac{120}{2 \times 3.14} \approx 19\text{ Hz}$$
10. Current is maximum at
    $$\omega = \omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.5 \times 8 \times 10^{-6}}} = 500\text{ rad/s.}$$
11. $P = \frac{I_0 E_0 \cos\phi}{2}$
    $$= \frac{100 \times 100}{2} \cos\frac{\pi}{3} \times 10^{-3} = 2.5\text{ W}$$
12. $X_C = \frac{1}{\omega C} = \infty$ if $\omega = 0$, *i.e.,* for DC
13. $V = 10\cos 100\pi t$
    at $t = \frac{1}{600}$ s,
    $$V = 10\cos 100\pi \frac{1}{600} = 10\cos\frac{\pi}{6} = 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3}\text{ V}$$
14. For purely resistive circuit $\phi = 0$.
15. $X_C = \frac{1}{\omega C} \Rightarrow X_C \propto \frac{1}{\omega}$ or $X_C \propto \frac{1}{f}$
16. $$\sin\phi = \frac{X}{Z} = \frac{1}{\sqrt{3}}$$
    $$\Rightarrow \quad \phi = \sin^{-1}\left[\frac{1}{\sqrt{3}}\right]$$
17. $\phi = \frac{3\pi}{2}$, $P = \frac{I_0 E_0}{2}\cos\phi = 0$
18. $R = \frac{V_{DC}}{I_{DC}} = 100\,\Omega$
    $$Z = \frac{V_{AC}}{I_{AC}} = \frac{100}{0.5} = 200\,\Omega$$
    $$X_L = \sqrt{Z^2 - R^2} = 100\sqrt{3}\,\Omega$$
    $$L = \frac{X_L}{\omega} = \frac{X_L}{2\pi f} = \frac{100\sqrt{3}}{2\pi \times 50} = \left(\frac{\sqrt{3}}{\pi}\right)\text{H}$$
19. $I_{rms} = \frac{V_{rms}}{X_C} = \omega C V_{rms}$
    $$= 100 \times 1 \times 10^{-6} \times \frac{200\sqrt{2}}{\sqrt{2}}$$
    $$I_{rms} = 20\text{ mA}$$
20. $V = \sqrt{V_R^2 + V_L^2} = \sqrt{(20)^2 + (15)^2}$
    $= 25\text{ V}$, $V_0 = 25\sqrt{2}\text{ V}$
21. $P = \frac{I_0 V_0 \cos\phi}{2} = 0$
    $$\Rightarrow \quad \cos\phi = 0$$
    $$\Rightarrow \quad \phi = 90^\circ$$
22. R is independent of frequency.
23. L is very high so that circuit consumes less power.

24. $\tan\phi = \frac{X_L}{R} \Rightarrow \tan 45° = \frac{X_L}{100}$

$\Rightarrow \quad X_L = 100\,\Omega$

$\omega L = 100\,\Omega$

$L = \frac{100}{2 \times 3.14 \times 10^3} \approx 16\,\text{mH}$

25. The minimum time taken by it in reaching from zero to peak value $= \frac{T}{4}$

$= \frac{1}{4f} = \frac{1}{4 \times 50} = \frac{1}{200} = 5\,\text{ms}$

26. $\phi = 60°$

$P = \frac{I_0 V_0 \cos\phi}{2} = \frac{4 \times 220 \times \frac{1}{2}}{2}$

$= 220\,\text{W}$

JEE Corner

Assertion and Reasons

1. X_C and X_L can be greater than Z because

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Hence, $V_C = IX_C$ and $V_L = IX_L$ can be greater than $V = IZ$.

2. At resonance $X_L = X_C$, with further increase in frequency, X_L increases but X_C decreases hence voltage will lead current.

3. $f_r = \frac{1}{2\pi\sqrt{LC}}$, if dielectric slab is inserted between the plates of the capacitor, its capacitance will increase, hence, f_r will decrease.

4. $q =$ Area under graph

$= \frac{1}{2} \times 4 \times (2+3) + \frac{1}{2} \times 4 \times (2+4)$

$= 22\,\text{C}$

Average current $= \frac{q}{t} = \frac{22}{6} = 3.6\,\text{A}$

5. On inserting ferromagnetic rod inside the inductor, X_L and hence V_L increase. Due to this current will increase if it is lagging and *vice-versa*.

6. $V_R = V_L = V_C \Rightarrow R = X_L = X_C$

Hence, $\phi = 0$ and I is maximum.

as $Z = \sqrt{R^2 + (X_L - X_C)^2}$ is minimum.

7. $I = I_L - I_C = 0$

8. $P = I_{\text{rms}}^2 R = (\sqrt{2})^2 \times 10 = 20\,\text{W}$

9. Inductor coil resists varying current.

10. $I_0 = \frac{E_0}{\sqrt{R^2 + \omega^2 L^2}}$, $\phi = \tan^{-1}\frac{\omega L}{R}$

11. At resonance, current and voltage are in same phase and $I_0 = \frac{V_0}{R}$. Hence, I_0 depends on R.

Objective Questions (Level-2)

Single Correct Options

1. For parallel circuit

$$\phi = \tan^{-1}\left[\frac{1/X_L}{1/R}\right] = \tan^{-1}\frac{4}{3}$$

$$= 53°$$

2. Current will remain same in series circuit given by

$$I = I_0 \sin(\omega t - \phi)$$

$$= I_0 \sin\left(\omega t - \tan^{-1}\frac{X_L}{R}\right)$$

3. $R = R_1 + R_L = 10\,\Omega$

$X_L = \omega L = 10\,\Omega$,

$X_C = \frac{1}{\omega C} = 10\,\Omega$

Reading of ammeter

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{10\sqrt{2}}{10}$$
$$= \sqrt{2}\text{ A} = 1.4\text{ A}$$

Reading of voltmeter,

$$V = I_{\text{rms}} R_L = 5.6\text{ V}$$

4. $X_C = \frac{1}{\omega C} = \frac{1}{2\pi \times 5 \times 10^3 \times \frac{1}{\pi} \times 10^{-6}}$

$$= 100\,\Omega$$
$$I_R = \frac{V}{R} = \frac{200}{100} = 2\text{ A}$$
$$I_C = \frac{V}{X_C} = \frac{200}{100} = 2\text{ A}$$

[Question is wrong. It should be choose the correct statement].

5. Let $i = i_1 + i_2$

where, $i_1 = 5$ A, $i_2 = 5 \sin 100\,\omega t$ A

Average value of $i_1 = 5$ A

Average value of $i_2 = 0$

$\therefore$ Average value of $i = 5$ A

Another method

$$i = 5\left[1 + \cos\left(\frac{\pi}{2} + 100\,\omega t\right)\right]$$
$$= 5\left[2\cos^2\left(\frac{\pi}{4} + 50\,\omega t\right)\right]$$
$$= 10\cos^2\left[\frac{\pi}{4} + 50\,\omega t\right]$$

Average value of $\cos^2\left(\frac{\pi}{4} + 50\,\omega t\right) = \frac{1}{2}$

$\therefore$ average value of $i = \frac{10}{2} = 5$ A.

6. As voltage is leading with current, circuit is inductive, and as $\phi = \frac{\pi}{4}, X_L = R$

or $$L = \frac{R}{\omega} = \frac{R}{100}$$

7. As $X_C > X_L$ voltage will lag with current.

Again $V = \sqrt{V_R^2 + (V_L - V_C)^2} = 10$ V

$\therefore$ $$V < V_C$$

and $$\cos\phi = \frac{R}{Z} = \frac{V_R}{V} = \frac{4}{5}$$

Hence, a, b and c are wrong.

8. For parallel RLC circuit,

$$I = \sqrt{I_R^2 + (I_C - I_L)^2}$$

$$\Rightarrow \quad I = \sqrt{\left(\frac{V_0}{R}\right)^2 + \left(\frac{V_0}{X_C} - \frac{V_0}{X_L}\right)^2}$$
$$= V_0\sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

9. $$V = \sqrt{V_L^2 + V_R^2} = 72.8\text{ V}$$
$$\phi = \tan^{-1}\frac{V_L}{V_R} = \tan^{-1}\left(\frac{2}{7}\right)$$

10. Clearly P is capacitor and Q is resistor, as, $V_P = V_Q$, $X_C = R$.

$\therefore$ When connected in series,

$$Z = \sqrt{X_C^2 + R^2} = \sqrt{2}\,R$$

and $\phi = \frac{\pi}{4}$, leading.

$\therefore I = \frac{1}{4\sqrt{2}}$ A, leading in phase by $\frac{\pi}{4}$.

11. $I = \sqrt{I_R^2 + (I_C - I_L)^2}$

Here, $I_C < I$ or $I_L > I$

12. $I = I_L - I_C = 0.2$ A

13. For a pure inductor voltage leads with current by $\frac{\pi}{2}$.

14. $V_R = IR = 220$V

Hence it is condition of resonance, i.e.,

$$V_L = V_C = 200\text{ V}$$

15. $\frac{H_1}{H_2} = \frac{I_{\text{DC}}^2 R}{I_{\text{rms}}^2 R} = \frac{I^2}{(I/\sqrt{2})^2} = 2$

16. $H = I_{\text{rms}}^2 R = \frac{I_0^2 R}{2} = \frac{V_0^2 R}{2(R^2 + \omega^2 L^2)}$

17. $$V_L = IX_L = I\omega L$$
$$V_C = IX_C = \frac{I}{\omega C}$$

If ω is very small,

$$V_L \approx 0, V_C \approx V_0.$$

18. Resistance of coil, $R = \frac{V}{I} = 4\,\Omega$

When connected to battery

$$I = \frac{V}{R + r} = \frac{12}{4 + 4} = 1.5\text{ A}$$

19. $V_R = \sqrt{V^2 - V_C^2} = 6\text{ V}$

$$\phi = \tan^{-1}\frac{V_C}{V_R} = \tan^{-1}\frac{4}{3}$$

20. $V_C = \sqrt{V^2 - V_R^2} = 16\text{ V}$

21. $$I = I_0 \sin\left(\frac{\pi}{2}t + \pi\right)$$

$$I = I_0 \text{ at } \frac{\pi}{2} + \pi = \frac{3\pi}{2}$$

$$t = 1\text{ s}$$

22. $$I_0 = \frac{V_0}{\sqrt{2}R}$$

$$X_C' = \frac{\sqrt{3}}{\omega C} = \sqrt{3}R$$

$$\Rightarrow \quad I_0 = \frac{V_0}{2R} = \frac{I_0}{\sqrt{2}}$$

23. $R = \dfrac{V_{DC}}{I_{DC}} = \dfrac{12}{4} = 3\,\Omega$

24. $X_L = \sqrt{Z_1^2 - R^2} = \sqrt{(5)^2 - (3)^2}$

$$= 4\,\Omega$$

$$X_C = \frac{1}{\omega C}$$

$$= \frac{1}{50 \times 2500 \times 10^{-6}} = 8\,\Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = 5\,\Omega$$

Average power $= I_{rms}^2 R = \dfrac{V_{rms}^2 R}{Z^2}$

$$= \frac{(12)^2 \times 3}{(5)^2} = 17.28\text{ W}$$

25. Already $X_C > X_L$, with increase in ω, X_C further decrease in ω, X_C increases and X_L decreases, hence, I will decrease.

26. For maximum current

$$\omega = \omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \times 10^{-6} \times 4.9 \times 10^{-3}}}$$

$$= \frac{10^5}{7}\text{ rad/s.}$$

27. In resonance,

$$Z = \sqrt{R_P^2 + X_C^2} \approx 77\,\Omega$$

28. In resonance, $\cos\phi = 1$.

More than One Correct Answers

1. $V_R^2 + V_L^2 = 10000$ ...(i)

$V_L - V_C = 120$ ...(ii)

$V_R^2 + (V_L - V_C)^2 = (130)^2 = 16900$ ...(iii)

On solving

$$V_r = 50\text{ V},\ V_L = 86.6\text{ V},\ V_C = 206.6\text{ V}$$

and $\cos\phi = \dfrac{V_R}{V} = \dfrac{50}{130} = \dfrac{5}{15}$

As $V_C > V_L$, circuit is capacitive in nature.

2. $i = 3\sin\omega t + 4\cos\omega t$

$= R\sin(\omega t + \phi)$

$R = 5$ and $\phi = \tan^{-1}\dfrac{4}{3}$

$$i_m = \frac{2i_0}{\pi} = \frac{2R}{\pi} = \frac{10}{\pi}$$

If $V = V_m \sin\omega t$

current will lead with the voltage.

If $V = V_m \cos\omega t$

current will lag with voltage.

3. $I = \dfrac{P}{V} = 1\text{ A},\ R = \dfrac{V}{I} = 60\,\Omega$

For AC,

$$Z = \frac{100}{1} = 100\,\Omega$$

X_C or $X_L = \sqrt{Z^2 - R^2} = 80\,\Omega$

$$L = \frac{X_L}{\omega} = \frac{80}{2\pi \times 50} = \frac{4}{5\pi}\text{ H}$$

or $C = \dfrac{1}{\omega X_C} = \dfrac{1}{2\pi \times 50 \times 80} = \dfrac{125}{\pi}\,\mu\text{F}$

or $R' + R = \dfrac{V}{I}$

$\Rightarrow \quad R' = 100 - 60 = 40\,\Omega$

4. $\cos\phi = \dfrac{R}{Z} = \begin{cases} 1 & \text{if } R = Z \\ 0 & \text{if } R = 0 \end{cases}$

5. As $X_L > X_C$, voltage will lead with the current.

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = 10\sqrt{2}\,\Omega$$

$$\phi = \tan^{-1}\frac{X_L - X_C}{R} = \frac{\pi}{4} = 45°$$

$$\cos\phi = \frac{R}{Z} = \frac{1}{\sqrt{2}}$$

6. As $X_L > X_C$, $\omega > \omega_r$

with increase in ω, X_L and hence, Z will increase while with decrease in ω, Z will first decrease and then increase.

7. $X_c = \frac{V_C}{I} = 50\,\Omega$

$$V_R = IR = 80\text{ V}$$

$$V_L = IX_L = 40\text{ V}$$

$$V_{\text{rms}} = \sqrt{V_R^2 + (V_L - V_C)^2} = 100\text{ V},$$

$$V_0 = 100\sqrt{2}\text{ V}$$

8. $I = \dfrac{V}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$

with change in L or C I may decrease or increase depending on effect on $\omega L - \frac{1}{\omega C}$.

Match the Columns

1. (a) $\rightarrow$ (p, r), (b) $\rightarrow$ (q, r), (c $\rightarrow$ s), (d) $\rightarrow$ (p)

Concept based insertion.

2. (a) $\rightarrow$ (p, s) current and voltage are in same phase so either $X_C = 0$, $X_L = 0$

or $\quad X_C = X_L \neq 0.$

(b) $\rightarrow$ (q)

$$I = -I_0 \cos\omega t$$

$$= I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$\phi = 90° \Rightarrow R = 0$$

(c) $\rightarrow$ (r, s) current is leading with voltage by $\frac{\pi}{6}$, either $X_L = 0$ or $X_C > X_L$

but X_C and R are non-zero.

(d) $\rightarrow$ (s) current lags with voltage by $\frac{\pi}{6}$, R and X_L are both non-zero.

3. (a) $\rightarrow$ (q, s), (b) $\rightarrow$ (r, s), (c) $\rightarrow$ (r, s), (d) $\rightarrow$ (r, s).

$$I = \frac{V}{Z} \text{ and } P = \frac{V^2 r}{Z^2}$$

with increase in L, C or f, Z may increase or decrease, hence power and current.

4. (a $\rightarrow$ q),

$$R = \frac{V_R}{I} = \frac{40}{2} = 20\,\Omega$$

(b $\rightarrow$ p)

$$V_C = IX_C = 2 \times 30 = 60\text{ V}$$

(c $\rightarrow$ r)

$$V_L = IX_C = 2 \times 15 = 30\text{ V}$$

(d $\rightarrow$ s)

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$= 50\text{ V}$$

5. (a $\rightarrow$ s) R is independent of f.

(b $\rightarrow$ p) $X_C \propto \frac{1}{f}$

(c $\rightarrow$ r) $X_L \propto \frac{1}{f}$

(d $\rightarrow$ q)

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

i.e., first decreases then increases.

Solutions
of
Optics &
Modern Physics

Lesson 26th to 30th

By DC Pandey

26. Reflection of Light

Introductory Exercise 26.1

1. Since $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ where c is the speed of light in vacuum hence unit of $\frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ is m/s.

2. Hence

$B_y = 2 \times 10^{-7}$ T $\sin [500x + 1.5 \times 10^{11} t]$

Comparing this equation with the standard wave eqution $B_y = B_0 \sin [kx + \omega t]$

$$k = 500 \text{ m}^{-1} \Rightarrow k = \frac{2\pi}{\lambda}$$

$$\Rightarrow \quad \lambda = \frac{2\pi}{k} \text{ m}$$

$$\Rightarrow \quad \lambda = \frac{2\pi}{500} \text{ m} = \frac{\pi}{250} \text{ metre}$$

$$\omega = 1.5 \times 10^{11} \text{ rad/s}$$

$$\Rightarrow \quad 2\pi n = 1.5 \times 10^{11}$$

$$\Rightarrow \quad n = \frac{1.5}{2\pi} \times 10^{11} \text{ Hz}$$

Speed of the wave $v = \frac{\omega}{k} = \frac{1.5 \times 10^{11}}{500}$

$$= 3 \times 10^8 \text{ m/s}$$

Let E_0 be the amplitude of electric field.

Then $E_0 = cB_0 = 3 \times 10^8 \times 2 \times 10^{-7}$

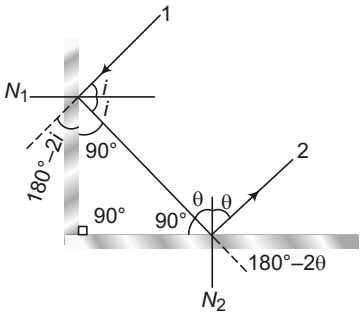
$$= 60 \text{ V/m}$$

Since wave is propagating along x-axis and B along y-axis, hence E must be along z-axis

$\Rightarrow E = 60 \text{ V/m } \sin [500x + 1.5 \times 10^{11} t]$

Introductory Exercise 26.2

1. Total deviation produced



$$\delta = 180° - 2i + 180° - 2\theta$$

$$\delta = 360° - 2(i + \theta)$$

From figure

$$\theta = 90° - i$$

$$\Rightarrow \quad \delta = 360° - 2[i + 90 - i]$$

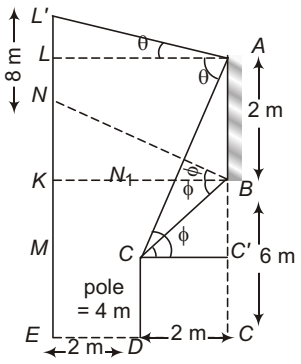
$$= 180°$$

Hence rays 1 and 2 are parallel (anti-parallel).

2. $v_0 = 2$ m/s for plane mirror $v_i = 2$ m/s.

Velocity of approach $= v_0 + v_i = 4$ m/s.

3. In figure, AB is mirror, G is ground, CD is pole and M is the man. The minimum height to see the image of top of pole is $= EN$



$= EK + KN = 6 + KN$

Now in ΔNKB,

$$\frac{NK}{KB} = \tan\phi \Rightarrow NK = KB\tan\phi$$

$$= 4\tan\phi$$

In $\Delta BC'C$ we get,

$$\tan\phi = \frac{BC'}{CC'} = \frac{2}{2} = 1$$

$$\phi = 45°$$

So, $NK = 4 \times \tan 45° = 4$ m

Hence in minimum height
$= 6\text{ m} + 4\text{ m} = 10\text{ m}$

In $\Delta AC'C$

$$\tan\theta = \frac{4}{2} = 2$$

In $\Delta L'LA$ we get,

$$\frac{LL'}{LA} = \tan\theta$$

$$\Rightarrow \quad \frac{LL'}{4} = 2$$

$$\Rightarrow \quad LL' = 8\text{ m}$$

Maximum height $= CA + LL' = 8 + 8 = 16$ m

Introductory Exercise 26.3

1. Here $f = -10$ cm (concave mirror)

(a) $u = -25$ cm

Using mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{10} + \frac{1}{25}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{-5+2}{50}$$

$$\Rightarrow \quad v = -\frac{50}{3} = -16.7\text{ cm}$$

Hence image is real, inverted and less height of the object.

(b) Since $u = -10$ cm,

Hence object is situated on focus of the image formed at ∞.

(c) $u - 5, f = -10$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{10} + \frac{1}{5}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{-1+2}{10}$$

$$\Rightarrow \quad v = 10\text{ cm}$$

Hence, image is virtual, erect and two time of the object.

2. Here $u = -3$ m, $f = -\frac{1}{2}$ m,

we have,

(a) $\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

$$\Rightarrow \quad \frac{1}{v} = -2 + \frac{1}{3}$$

$$\Rightarrow \quad v = -0.6\text{ m}$$

As ball moves towards focus the image moves towards $-\infty$ and image is real as the distance decreases by focal length image become virtual which moves from $+\infty$ to zero.

(b) The image of the ball coincide with ball, when $u = -R = -1$ m

Using $h = ut + \frac{1}{2}gt^2$

$$\Rightarrow \quad t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 2}{9.8}}$$

$$= 0.639 \text{ s}$$

Similarly again images match at $t = 0.78$ s.

3. Since image is magnified, hence the mirror is concave.

Here, $$m = \frac{-v}{u} \Rightarrow \frac{-v}{u} = 5$$

$$\Rightarrow \quad v = -5u \qquad \ldots(i)$$

Let distance between mirror and object is x.

Since image is formed at a distance 5 m from mirror

$$v = -(5 + x) \qquad \ldots(ii)$$

From Eqs.(i) and (ii), we get

$$-(5 + x) = -5x$$

$$\Rightarrow \quad 4x = 5$$

$$\Rightarrow \quad x = 1.25$$

Hence mirror is placed at 1.25 m on right side of the object by mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f},$$

we have

$$\frac{1}{f} = -\frac{1}{6.25} - \frac{1}{1.25}$$

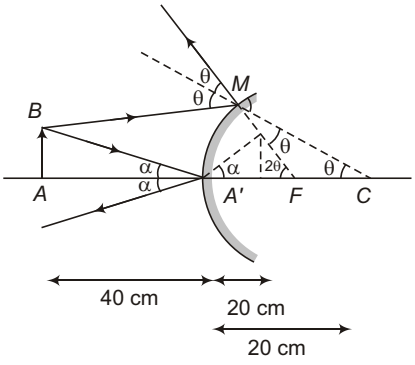
$$\Rightarrow \quad f = \frac{-6.25}{6},$$

Hence $R = 2f \Rightarrow R = -\frac{6.25}{3} = -2.08$ m

Thus mirror is concave mirror of radius of curvature 2.08 m.

4. Since the incident rays and reflected rays are parallel to each other therefore mirror is plane mirror.

5. Let us solve the first case :



By applying the geometry we can prove that,

$$PA' = v = \frac{40}{3} \text{ cm}$$

Further, in triangles ABP and $PA'B'$ we have,

$$\frac{AB}{40} = \frac{A'B'}{(40/3)}$$

$$\therefore \quad A'B' = \frac{AB}{3} = \frac{2}{3} \text{ cm}$$

Similary, we can solve other parts also.

6. Simply apply :

$$\frac{1}{v} = \frac{1}{u} = \frac{1}{f}$$

and $m = \frac{I}{o} = \frac{-v}{u}$ for lateral magnification. If magnitfication is positive, image will be virtual. If magnification is negative, image will be real.

AIEEE Corner

■ Subjective Questions (Level 1)

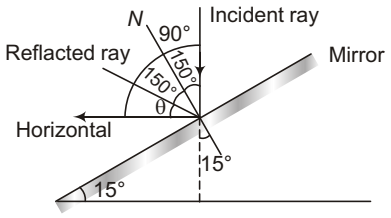
1. Here $v = 39.2$ cm, hence $v = -39.2$ cm

and magnification $m = 1$

$\Rightarrow \qquad h_i = h_o = 4.85$

Hence image is formed at 39.2 cm behind the mirror and height of image is = 4.85 cm.

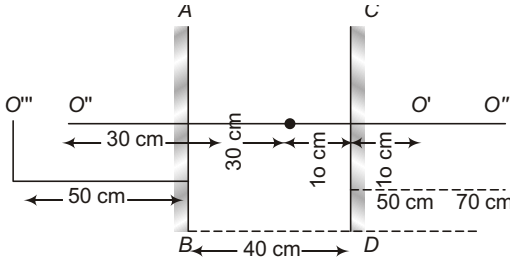
2. From figure, angle of incident = 15°



Let reflected ray makes an angle θ with the horizontal, then

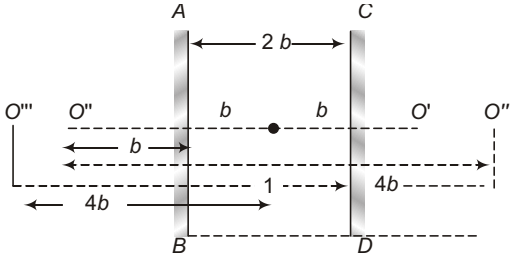
$$\theta + 15° + 15° = 90° \Rightarrow \theta = 60°$$

3.



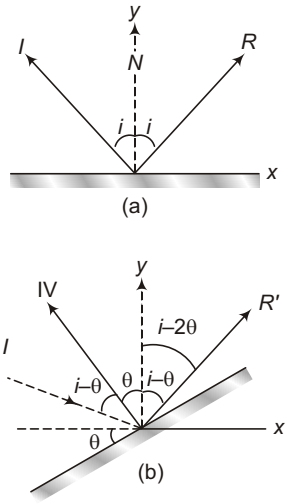
Since mirror are parallel to each other ∞ image are formed the distance of five closet to object are 20 cm, 60 cm, 80 cm, 100 cm and 140 cm.

4. The distance of the object from images are $2b$, $4b$, $6b$..... etc.



Hence the images distance are $2nb$, where $n = 1, 2, \ldots$. Ans.

5. Suppose mirror is rotated at angle θ about its axis perpendicular to both the incident ray and normal as shown in figure



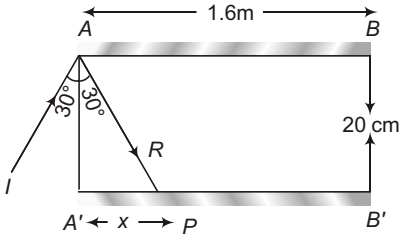
In figure (b) I remain unchanged N and R shift to N' and R'.

From figure (a) angle of rotation = i,

From figure (b) it is $i - 2\theta$

Thus, reflected ray has been rotated by angle 2θ.

6. I is incident ray $\angle i = 30° = \angle r$



From $\Delta\, PA'A$, we get

$$\frac{x}{20} = \tan 30° \Rightarrow x = 20 \tan 30°$$

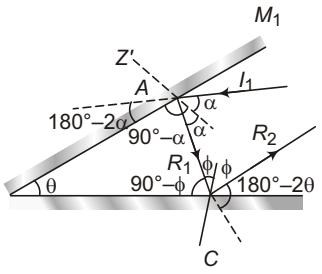
No. of reflection $= \frac{AB}{x} = \frac{160 \text{ cm}}{20 \text{ cm} \times \tan 30°}$

$$= 8\sqrt{3} \approx 14$$

Hence the reflected ray reach other end after 14 reflections.

7. The deviation produced by mirror M_1 is

$$= 180° - 2\alpha$$



and the deviation produced by mirror M_2 is

$$= 180 - 2$$

Hence total deviation

$$= 180 - 2\alpha + 180 - 2\phi$$

$$= 360 - 2(\alpha + \phi)$$

In $\Delta\, ABC$ we get,

$$90 - \alpha + \theta + 90 - \phi = 180$$

$$\Rightarrow \quad \alpha + \phi = \theta$$

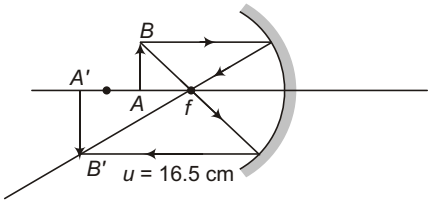
Hence deviation produces $= 180 - 2\theta$.

8. Here $f = -\frac{R}{2} = -\frac{22}{2} = -11$ cm

Object height $h_0 = 6$ mm

$$u = -16.5 \text{ cm}$$

(a) The ray diagram is shown in figure



Using mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$\Rightarrow \quad \frac{1}{v} = -\frac{1}{11} + \frac{1}{16.5} = \frac{-165 + 11}{16.5 \times 11}$$

$$\Rightarrow \quad v = -\frac{16.5 \times 11}{5.5} = -33 \text{ cm}$$

Hence the image is formed at 33 cm from the pole (vertex) of mirror on the object side the image is real, inverted and magnified. The absolute magnification

$$|m| = \left|\frac{v}{u}\right| = \frac{33}{16.5} = 2$$

Hence size of image is $h_i = 2 \times h_0$

$$= 2 \times 6 = 12 \text{ mm.}$$

9. Here $u = -12$ cm, $f = +\frac{R}{2} = +10$ cm

Using mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

we get

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{10} + \frac{1}{12}$$

$$= \frac{6+5}{60}$$

$$\Rightarrow \quad v = \frac{60}{11} \text{ cm} = 5.46 \text{ cm}$$

The image is formed on right side of the vertex at a distance $\frac{60}{11}$ cm. the image is virtual and erect the absolute magnification is given by $|m| = \left|\frac{v}{u}\right|$

$$\Rightarrow \quad |m| = \left|\frac{60}{11 \times (-12)}\right| = \frac{5}{11}$$

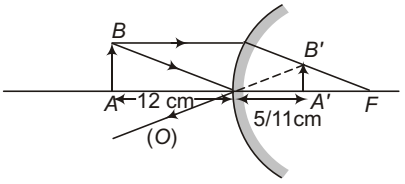
$$\because \quad m < 1$$

Hence image is de-magnified.

Height of image $h_i = |m| \times h_0$

$\Rightarrow h_i = \frac{5}{11} \times 9 = \frac{45}{11} = 4.09$ mm

The ray diagram is shown in figure



10. Here $f = -18$ cm

Let distance of object from vertex of concave mirror is u. Since image is real hence image and object lie left side of the vertex.

Magnification $m = -\frac{v}{u} = \frac{1}{9}$

$\Rightarrow \quad v = -\frac{u}{9}$

By mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we have

$$-\frac{1}{u/9} - \frac{1}{u} = -\frac{1}{18} \Rightarrow -\frac{10}{u} = -\frac{1}{18}$$

$\Rightarrow u = 180$ cm (left side of the vertex).

11. Here $u = -30$ cm, since image is inverted.

Hence the mirror is concave.

$$m = \frac{1}{2} = \frac{-v}{u} \Rightarrow v = -\frac{u}{2}$$

Using mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we get

$$-\frac{2}{u} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{-3}{u} = \frac{1}{f}$$

$\Rightarrow \quad f = -\frac{u}{3} = -\frac{30}{3} = -10$ cm

Hence mirror is concave of focal length 10 cm.

12. Here $f = -\frac{24}{2}$ cm $= -12$ cm

(a) Since image is virtual

$\Rightarrow \quad m = \frac{v}{u} \Rightarrow v = mu$

$\Rightarrow \quad v = 3 \times u$

$\Rightarrow \quad v = 3u$ and v is +ve

By mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{3u} - \frac{1}{u} - \frac{1}{12}$$

$\Rightarrow \quad \frac{1-3}{3u} = -\frac{1}{12} \Rightarrow u = 8$ cm

(b) Since image is real

$\Rightarrow \quad m = -\frac{v}{u} = 3 \Rightarrow v = -3u$

By using $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we get

$$-\frac{1}{e} - \frac{1}{u} = -\frac{1}{12} \Rightarrow \frac{-4}{3u} = -\frac{1}{12}$$

$\Rightarrow \quad u = 16$ cm

(c) Here $m = \frac{-v}{m} = \frac{1}{3} \Rightarrow v = -\frac{u}{3}$

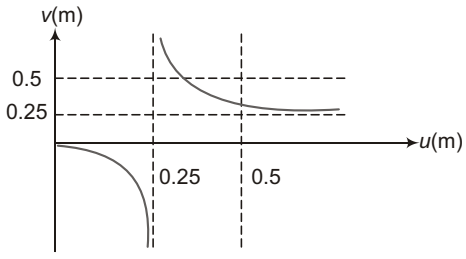
$\Rightarrow \quad -\frac{1}{u/3} - \frac{1}{u} = -\frac{1}{12}$

$\Rightarrow \quad -\frac{4}{u} = -\frac{1}{12} \Rightarrow u = 48$ cm

13. We have $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$\Rightarrow v = \frac{uf}{u-f}$ at $u = f, v = \infty$

The variation is shown in figure



Hence focal length if assymtote of the curve.

When $u < f$, Image is virtual. It means v is negative.

When $\quad u = 2f$

$v = 2f$

$u \to 0, v \to 0$

14. Here $f = 21$ cm $\Rightarrow R = 2f = 42$ cm

Since the object is placed on C. Hence its image by concave mirror is formed on C. This image acts as a virtual objet for plane mirror the distance between plane mirror and virtual object = 21 cm.

Hence plane mirror forms its real image in front of plane mirror at 12 cm.

15. Let u is the object distance from vertex, v is the image distance for vertex and f is the focal length then distance between object and focus is $u - f$ and distance between image and focus is $v - f$ *ie*,

$$(u - f)(v - f) = uv - (u + v)f + f^2 \quad \ldots\text{(i)}$$

Using $\frac{1}{v} + \frac{1}{u} + \frac{1}{f}$, we get

$$uv = (u + v)f \quad \ldots\text{(ii)}$$

Putting the value of uv in RHS of Eq. (i), we get

$$(u - f)(u - f) = (v + u)f - (v + u)f + f^2$$

$$(u - f)(v - f) = f^2$$

Hence proved.

16. Let object is placed at a distance x from the convex mirror then for convex mirror

$$u - x \text{ and } f = +\frac{R}{2}$$

Let v be the distance of the image from pole (vertex) of convex mirror.

Using $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{v} - \frac{1}{x} = \frac{2}{12} \Rightarrow v = \frac{xR}{2x + R}$$

For concave mirror

$$u' = -\left[2R + \frac{xR}{2x + R}\right] = -\left[\frac{2R^2 + 5xR}{2x + R}\right]$$

$$v' = -(2R - x) \text{ and } f' = -\frac{R}{2}$$

Using $\frac{1}{v'} + \frac{1}{u'} = \frac{1}{f'}$, we get

$$-\frac{1}{(2R - x)} - \frac{(2x + R)}{(2R^2 + 5xR)} - \frac{2}{R}$$

$$\Rightarrow \quad 4R^3 - 2x^2R + 8xR^2$$

$$= 8R^3 + 16xR^2 - 10x^2R$$

$$\Rightarrow \quad 4R^3 + 8xR^2 - 8x^2R = 0$$

$$\Rightarrow \quad 4R[R^2 + 2xR - 2x^2] = 0$$

$$\Rightarrow \quad 2x^2 - 2xR - R^2 = 0$$

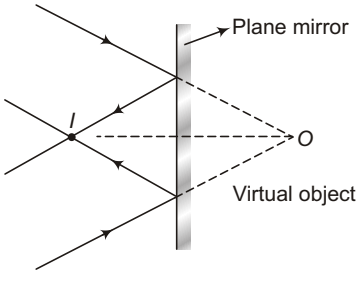
$$\because \quad R \neq 0$$

$$\Rightarrow \quad x = \frac{2R \pm 2\sqrt{3}R}{4} = \frac{[1 \pm \sqrt{3}]}{2}R$$

$$\Rightarrow \quad x = \left(\frac{1 + \sqrt{3}}{2}\right)R$$

■ Objective Questions (Level 1)

1. When convergent beam incident on a plane mirror, then mirror forms real image



2. When an object lies at the focus of a concave mirror $u = -f$ focal length of a concave mirror is negative.

Using mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

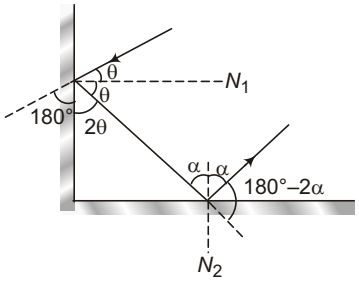
we get,

$$\frac{1}{v} - \frac{1}{f} = -\frac{1}{f} \Rightarrow v = \infty$$

also magnification $m = -\frac{v}{u} = \infty$.

Hence, correct option is (c) ∞, ∞.

3. Total deviation, $\delta = \delta_1 + \delta_2$



$$= 180 - 2\theta + 180 - 2\alpha$$

but $\alpha = 90 - \theta$

$\Rightarrow$ $\delta = 180 - 2\theta + 180 - 2(90 - \theta)$

$\Rightarrow$ $\delta = 180°$

Hence, option (a) is correct.

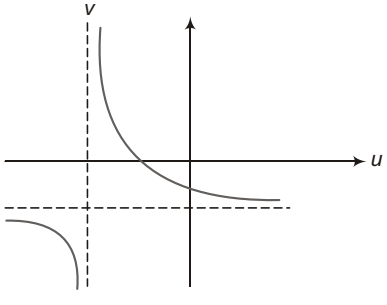
4. A concave mirror cannot from a virtual image of a virtual object.

Hence option (a) is correct.

5. For *a* concave mirror for normal sign convention if $u = -f \Rightarrow v = \infty$

and at $u = -\infty, v = -f$

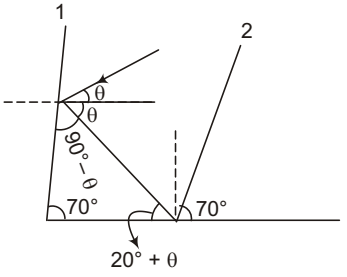
graph between u and v is



The dotted lines are the asymptotes (tangent at ∞) of the curve.

Hence correct option is (b).

6. From figure



$$20 + \theta = 70°$$

$$\theta = 70° - 20°$$

$$\theta = 50°$$

Here (1) and (2) are paralledl 11 to each other.

Hence the correct option is (a) = 50°.

7. The radius of curvature of convex mirror

$$R = +60 \text{ cm}.$$

Its focal length $f = \frac{R}{2} = +30$ cm

Magnification $m = \frac{v}{u} = \frac{1}{2}$

$\Rightarrow$ $v = \frac{u}{2}$

Using mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$,

we get, $-\frac{1}{u/2} - \frac{1}{u} = \frac{1}{30}$

$\Rightarrow$ $\frac{-3}{u} = \frac{1}{30}$

$\Rightarrow$ $u = -90$ cm

$v = \frac{u}{2} = -45$ cm

Hence distance between A and B is

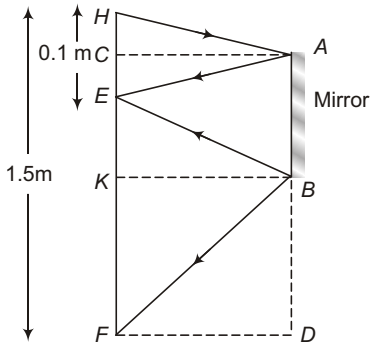
$= 90 - 45$

$= 45$ cm

Hence the correct option is (c).

8. Here it is given that height of the boy

$$HF = 1.5 \text{ m}$$



Length of mirror $= AB = 0.75$ m

The ray diagram is shown in above figure.

H is the Head of the boy and F is the feet. It also shows the paths of the rays that leaves the head of the man enter his eyes (E). After reflection from the mirror at point A, and the rays that leave his feet and enter his eyes after reflected at point B.

From figure $CE = \frac{1}{2} HE = 0.05$ m

$$CF = HF - HC = HF = CE$$
$$= 1.50 - 0.05 = 1.45 \text{ m}$$

The distance of the bottom edge of mirror above the floor is

$$BP = KF = CF - KC = CD - AB$$
$$= 1.45 - 0.75 = 0.7 \text{ m}$$

But according to question $BD = 0.8$ m (given) which is greater than 0.7 m, the height required to see full image. Hence the boy cannot see his feet.

Option (c) is correct

9. Since the image is magnified hence mirror is concave mirror.

Here $\quad m = -\frac{v}{u} = 3 \Rightarrow v = -3u$

$\Rightarrow \quad |v| = |-3u| = 3u$

but $\quad |v - u| = 80$

$\Rightarrow \quad |3u - u| = 80 \Rightarrow \quad u = 40$ cm'

Using mirror formula, we get

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$-\frac{1}{3u} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad f = \frac{-3u}{4}$$

$$\Rightarrow \quad f = \frac{-3 \times 40}{2} = -30 \text{ cm}$$

Mirror is concave and focal length is 30 cm.

Correct option is (a).

10. Here $m = +\frac{1}{n} = -\frac{v}{u}$

$$\Rightarrow \quad v = -\frac{u}{n}$$

From mirror formula $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$,

we get,

$$\frac{1}{f} = \frac{1}{(-u/n)} + \frac{1}{u}$$

$$\Rightarrow \quad u = -(n-1)f$$

Hence the correct option is (d).

11. Differentiating mirror formula, we get

$$\frac{dv}{dt} = \frac{v^2}{u^2}\frac{du}{dt} \quad [\because \text{ here } \frac{du}{dt} \text{ is } -\text{ve}]$$

Using mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f},$$

we get $\quad \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

Here $\quad u = -60$ cm, $f = -24$ cm

Putting these we get, $\quad v = 40$ cm

Hence, $\quad \frac{dv}{dt} = \frac{40^2}{60^2} \times 9 = 4$ cm/s

Hence the speed of the image is 4 cm/s away from the mirror.

Hence correct option is (c).

12. The wrong statement is (d)

13. Let v_m is the speed of mirror, v_p is the speed of particle and v_p is the speed of the observer, then speed of the image measured by observer is given by

$$v_{op} = 2[v_m + v_p] - v_o$$

$$\Rightarrow \quad v_{op} = 2[10 + 4] - 2$$

$$= 28 - 2 = 26 \text{ cm/s}$$

Hence correct option is (d).

■ Assertion and Reason

1. Assertion is wrong since when a virtual object is placed at a distance less than the focal length its real image is formed.

Hence answer is (d).

2. Using mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ we get

$\frac{1}{v} - \frac{1}{20} = \frac{1}{20} \Rightarrow v = 10$ cm

ie image is virtual exect and since $m = \frac{v}{u} = \frac{1}{2}$.

Hence image is diminished, thus assertion is true.

If $u = +20$ cm for virtual object $v = \infty$ hence reason is true but reason is not correct explanation of assertion. Hence answer is (b).

3. Using mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ we get

$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

If u is front of mirror u is negative and f is negative for concave mirror.

$$\Rightarrow \quad \frac{1}{v} = -\frac{1}{f} + \frac{1}{u} \Rightarrow \quad v = \frac{uf}{f-u}$$

$$\Rightarrow \quad u \to f \Rightarrow v \to \infty$$

Hence assertion is true also in refractive image and object moves in opposite direction. Hence both assertion and reason are true and reason correctly explain the assertion Correct answer is (a).

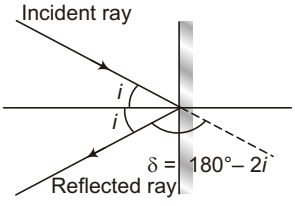
4. Real view mirror of vehicles is convex mirror, hence assertion is true.

It never makes real image of real object reason is also true but convex mirror is used because since its field of view is greatest. Hence both assertion and reason are true but reason is not correct explanation of assertion. Correct answer is (b).

5. Since $m = -2$ hence it is definitely a concave mirror since only concave mirror form magnified image. Since concave mirror form only real image of real object hence reason is also true. Hence it may true but when object is placed between C and F, $m < 1$.

Hence correct answer be (a) or (b).

6.

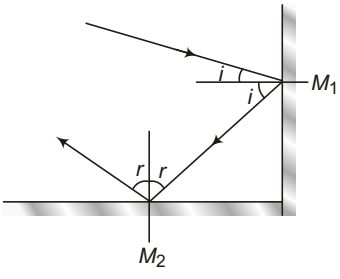


Hence assertion is true.

For normal incidence $i = 0$ hence $\delta = 180°$.

hence assertion is true but reason is false. hence correct option is (c).

7.



Deviation produced by $M_1 = 180° - 2i$
Deviation produced by $M_2 = 180° - 2r$

Total deviation produced $= 360° - 2(i + r)$

But from figure $i + r = 90°$, hence deviation $= 180°$ for any value of i.

Hence assertion is true but reason is false. Correct option is (c).

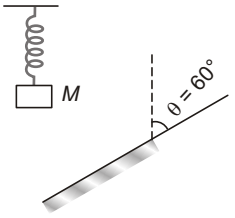
8. The correct option is (b).

9. The correct option is (a, b).

10. The correct option is (b).

■ Objective Questions (Level 2)

1. $v_{max} = \omega A$



$$\Rightarrow \quad v_{max} = \sqrt{\frac{k}{m}}\, A$$

$$\because \quad \omega^2 = \frac{k}{m} \text{ for SHM}$$

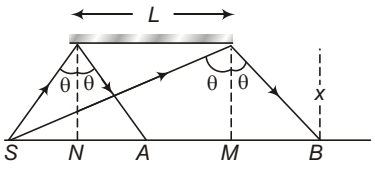
Maximum speed of insect relative to its image

$$= 2v_{max}\perp = 2v_{max} \sin 60°$$
$$= A\sqrt{3}\sqrt{\frac{k}{m}}$$

Hence correct option is (c).

2. $au^n = g \downarrow$

Height $= x$



Let after time t paperndicular distance between mirror and source is x' we have from figure

$$AB = AM + MB = SM - SA + MB$$

but $SM = MB$

$$AB = 2MB - SA = 2x' \tan\phi - SA$$
$$= 2x' \tan\phi - 2x' \tan\theta$$
$$\Rightarrow \quad AB = 2x'[\tan\phi - \tan\theta]$$
$$= 2x'\left[\frac{SM}{x'} - \frac{SN}{x'}\right] = 2[SM - SN]$$
$$\Rightarrow \quad AB = 2 \times L,$$

where $SM - SN = L =$ Length of mirror

$$\Rightarrow \quad \frac{d}{dt}[AB] = \frac{d}{dt}(2L) = 0$$

$\because$ Length of mirror is constant.

Hence the correct option is (d).

3. Here $u = -10$ cm and $v = -20$ cm

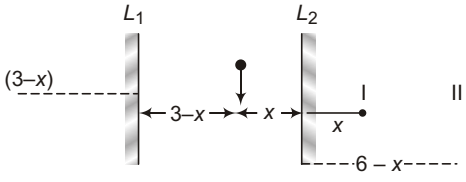
Using mirror formula

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \text{ we get} = \frac{dv}{v^2} - \frac{du}{u^2} = 0$$
$$\Rightarrow \quad \frac{dv}{du} = -\frac{v^2}{u^2} = -\frac{20^2}{10^2} = -4$$
$$\Rightarrow \quad dv = -4du$$
$$\Rightarrow \quad dv = -4 - (-0.1), \text{ here } du = -0.1$$
$$\Rightarrow \quad dv = 0.4 \text{ cm},$$

ie, 0.4 cm away from the mirror.

Hence the correct option is (a).

4. The first and second images are shown in figure but according to question

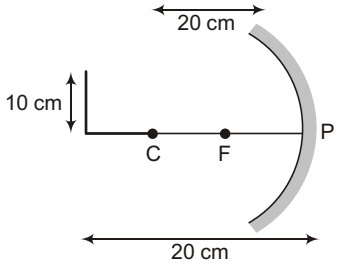


$$(6-x)-x=4$$

$\Rightarrow$ $$2=2x \Rightarrow x=1\text{ m}$$

Hence the correct option is (c).

5. For vertical part $-\frac{1}{20}+\frac{1}{v}=-\frac{1}{5}$



$\Rightarrow$ $$v=-\frac{20}{3}$$

$$|m_v|=\left|\frac{v}{u}\right|=\left|\frac{20/3}{20}\right|=\frac{1}{3}$$

$\Rightarrow$ $$L_v=\frac{10}{3}\text{ cm}$$

For horizontal part first end is at C hence its image is also at C *ie* at $v=-10$ cm, for other end

$$-\frac{1}{-20}+\frac{1}{v}=-\frac{1}{5}\Rightarrow v=-\frac{20}{3}$$

$\Rightarrow$ $$|v|=\frac{20}{3}$$

$$L_H=|v-u|=\left|\frac{20}{3}-10\right|=\frac{10}{3}$$

$\Rightarrow$ $$L_H=\frac{10}{3}$$

The ratio $L_V : L_H = 1:1$.

Hence correct option is (c) 1 : 1.

6. Here $u=-15$ cm, $f=-10$ cm

Using $$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

We get, $v=-30$ cm

We have $$m=\frac{v}{m}=\frac{(v_2-v_1)}{u_2-u_1}$$

$$=\frac{dv}{du}=2$$

$\Rightarrow$ $$dv=2\,du,$$

$$AB=du=4\text{ mm}$$

$\Rightarrow$ $$dv=2\times 4\text{ mm} \Rightarrow dv=8\text{ mm}$$

Hence the correct option is (c).

7. If the mirror is rotated by an angle θ in anticlock, wise direction about an axis $\infty\perp$ to the plane mirror, the new angle of incidence becomes $i-\theta$ and angle of reflection also $i-2\theta$.

According to problem

$$i+i-2\theta=45°$$

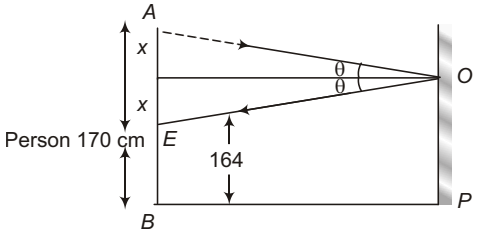
$$2i=45°+2\theta=45°+2\times 20°=85°$$

But angle of incidence = angle of reflection.

Hence the angle between origial incident and reflected ray was 85°. Similarly is the mirror is rotated clockwise the angle became 5°.

Hence correct option is (c) 85° or 5°.

8. The person see his hair if the incident ray statics from point A after reflected by mirror reach his eyes. Let O is point at minimum at a distance x below the point A.



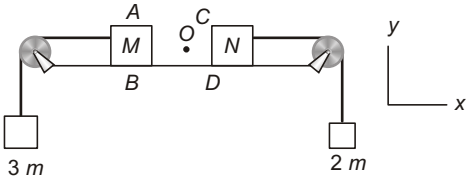
We have $2x=60\text{ cm}\Rightarrow x=3$ cm

The distance of O from P is

$$=170-3=167\text{ cm}$$

Hence correct option is (a).

9. Acceleration of block



$$a_{AB} = \frac{3mg}{3m + m} = \frac{3g}{4}$$

Acceleration of block CD :

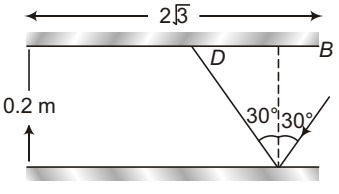
$$a_{CD} = \frac{2mg}{2m + m} = \frac{2g}{3}$$

Since the accelerations are in opposite directions relative acceleration of one image with respect to other is given by

$$a_{AB} + a_{CD} = \frac{3g}{4} + \frac{2g}{13} = \frac{17g}{12}$$

Hence the correct option is (c).

10. Here $\frac{BD}{0.2} = \tan 30°$

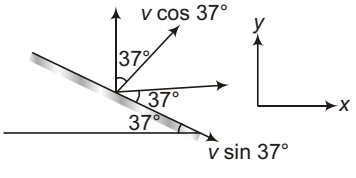


$$\Rightarrow \qquad BD = 0.2 \times \frac{1}{\sqrt{3}}$$

No. of reflections $= \frac{2\sqrt{3}}{0.2/\sqrt{3}} = 30$

Hence, the correct option is (b).

11. Resolving velocity along parallel to mirror and perpendicular to mirror, we get



$v_{\parallel} = v \sin 37°$ and $v_{\perp} = v \cos 37°$

From figure, we get

$$v_x = v \cos 37° \sin 37° + v \sin 37° \cos 37°$$

$$= 2v \cos 37° \sin 37°$$

$$v_x = 2 \times 5 \times \frac{4}{5} \times \frac{3}{5} = \frac{24}{5} = 4.8$$

$$v_y = v \cos 37° \times \cos 37° - v \sin 37° \times \sin 37°$$

$$v_y = v\,[\cos^2 37° - \sin^2 37°]$$

$$= 5 \times \left(\frac{4}{5} + \frac{3}{5}\right)\left(\frac{4}{5} - \frac{3}{5}\right)$$

$$= 5 \times \frac{7}{5} \times \frac{1}{5} = \frac{7}{5} = 1.4$$

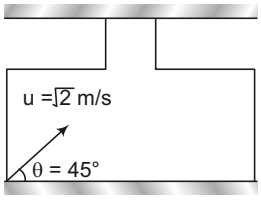
Hence velocity of image is given by

$$\vec{\mathbf{v}} = v_x\,\hat{\mathbf{i}} + v_y\,\hat{\mathbf{j}}$$

$$\Rightarrow \qquad \vec{\mathbf{v}} = 4.8\,\hat{\mathbf{i}} + 1.4\,\hat{\mathbf{j}}$$

Hence the correct option is (c).

12. Since elevator start falling freely, the relative acceleration of the particle in elevator frame $= g - g = 0$



Hence, in elevator frame path of the particle is a straight line.

The vertical component of velocity is

$$u \sin 45° = \sqrt{2} \times \frac{1}{\sqrt{2}} = 1 \text{ m/s}$$

The separation between mirror and particle in 0.5 s is

$$y = v_y t = 1 \times 0.5 = 0.5 \text{ m}$$

The separation between image of particle and particle at this moment

$$= 2y = 2 \times 0.5 \text{ m} = 1 \text{ m}$$

Hence the correct option is (b).

13. Here velocity of mirror

$$\vec{\mathbf{v}}_m = 4\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}} + 8\,\hat{\mathbf{k}}$$

and velocity of object

$$\vec{\mathbf{v}}_o = 3\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}} + 5\,\hat{\mathbf{k}}$$

Since $\hat{\mathbf{k}}$ is normal to the mirror hence $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ components of image velocity remain unchanged *ie*, velocity of image can be written as

$$\vec{\mathbf{v}}_i = 3\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}} + v_{iz}\,\hat{\mathbf{k}}$$

but $\quad v_{iz} = 2u_{mz} - v_{oz} = 2 \times 8 - 5 = 11$

Hence, we get

$$\vec{\mathbf{v}}_i = 3\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}} + 11\,\hat{\mathbf{k}} \ (\textit{wrt} \text{ ground})$$

Hence, the correct option is (b).

$$3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 11\hat{\mathbf{k}}$$

14. Only option (b) satisfy the given condition.

Here $X_0 = -2, X_i = +10$

Using $\dfrac{1}{X_0} + \dfrac{1}{X_i} = \dfrac{1}{f}$

we get

$$\frac{1}{10} - \frac{1}{2} = \frac{1}{f}$$

$$\Rightarrow \qquad f = -2.5 \text{ cm}$$

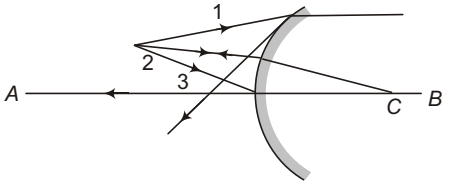
Hence, the mirror is concave.

We know that $y_i = \dfrac{fy_0}{f - x_0}$

$$= \frac{-2.5 \times 1}{-2.5 + 2} = 5 \text{ cm}$$

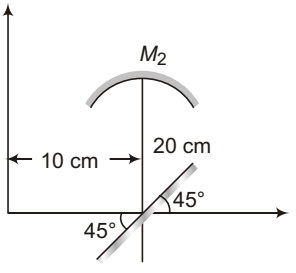
Hence, the correct option is (b).

16. There are two mistakes one in ray (1) and other in ray (3).



Hence correct option is (b).

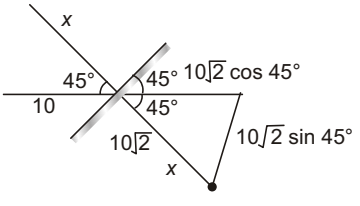
17. The image formation by plane mirror is shown as



$$\frac{10}{x} = \sin 45° \quad \Rightarrow \ x = 10\sqrt{2}$$

The x-coordinate is $10\sqrt{2}\cos 45° = 10$

and y-coordinate is $-10\sqrt{2}\sin 45° = -10$



Hence, the convert option if (c), (10, – 10).

18. $x_i = \dfrac{fx_0}{x_0 - f} = \dfrac{-10 \times 10}{-10 - 10} = +5 \text{ cm}$

For concave mirror $f = -10$ cm.

$$y_i = \frac{fy_0}{f - x_0} = \frac{-10 \times -20}{-10 - 10} \text{ cm}$$

$$= +10 \text{ cm}$$

Hence the coordinates of image are (5, 10).

Therefore, the correct option is (d).

19. For convex mirror $f = +10$ cm

$$x_i = \frac{fx_0}{x_0 - f} = \frac{10 \times 10}{10 - 10} = \infty$$

$$y_i = \frac{fy_0}{f - y_0} = \frac{10 \times -20}{10 - 10} = -\infty$$

Hence the correct option is (d).

20. It concave mirror is replaced by plane mirror the coordinates are (0, + 40).

Hence the correct option is (d).

■ More than one options are correct

1. Here $f = -20$ cm

Case 1. (if image is real) u, v and f all are –ve.

Here $m = 2 \Rightarrow v = -2u$

using mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

we get, $$-\frac{1}{2u} - \frac{1}{u} = -\frac{1}{20}$$

$$\Rightarrow \quad \frac{3}{2u} = \frac{1}{20} \Rightarrow u = +30 \text{ cm}$$

Case 2. (if image is virtual)

u and f are –ve, while v is +ve

$$\Rightarrow \quad \frac{1}{2u} - \frac{1}{u} = -\frac{1}{20}$$

$$\Rightarrow \quad u = +10 \text{ cm}$$

Hence possible values of u are 10 cm, 30 cm.

The correct options are (a) and (b).

2. Magnitude of focal length spherical mirror is f and linear magnification is $\frac{1}{2}$

Since concave mirror fro inverted real image and magnification is less than unity, therefore $u > 2f$.

Hence option (a) is correct.'

If image is erect than it is a convex mirror.

Let mirror is concave hence focal length $= -f$.

Here $m = \frac{1}{2} = -\frac{v}{u}$

$$\Rightarrow \quad v = -\frac{u}{2}$$

Using mirror formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$, we get

$$-\frac{1}{u/2} - \frac{1}{u} = -\frac{1}{f}$$

$$\Rightarrow \quad \frac{-3}{u} = -\frac{1}{f}$$

$$\Rightarrow \quad u = -3f$$

Hence, if the mirror is concave the object distance will be $3f$.

Let mirror is convex, then

$$m = \frac{v}{u} = \frac{1}{2} \Rightarrow u = \frac{v}{2}$$

Using mirror formula, we get

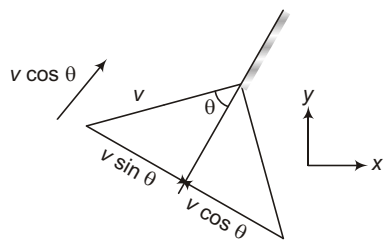
$$\frac{1}{u/2} - \frac{1}{u} = \frac{1}{f} \Rightarrow u = f$$

Hence, if mirror is convex the object distance will be f.

Hence correct options are (a), (b), (c) and (d).

3. Since by a plane mirror

speed of image = speed of object



Hence speed of image also v.

Horizontal component (along mirror)

$$= v \cos \theta$$

Vertical component ($\perp$ to mirror)

$$= v \sin \theta$$

Hence image velocity also make an angle θ with the mirror.

Resolving velocity along (y-axis *ie*, parallel to mirror) and (x-axis *ie* perpendicular to mirror).

$$\vec{\mathbf{v}}_0 = v \sin \hat{\mathbf{i}} + v \cos \hat{\mathbf{j}}$$

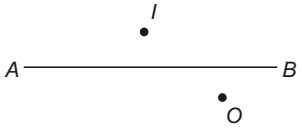
$$\vec{\mathbf{v}}_i = -v \sin \hat{\mathbf{i}} + v \cos \hat{\mathbf{j}}$$

Relative velocity of object w.r.t. image is

$$\vec{\mathbf{v}}_{0i} = \vec{\mathbf{v}}_0 - \vec{\mathbf{v}}_i = 2v \sin \theta \, \hat{\mathbf{i}}$$

Hence, correct options are (a), (b) and (d).

4.



As image is on opposite side of the principle axis (inverter image) hence the mirror is concave because convex mirror always form erect image.

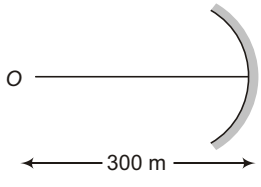
The mirror is lying to the right of O and the O lies between C and F.

If centre of curvature lies to the right hand side of O then $v < u$.

Hence, this option is incorrect.

Hence, the correct options are (a), (b) and (d).

5. Here $f = -20$ cm, $u = -30$ cm



Using mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

we get,

$$v = -60$$

Different this w.r.t. time, we get

$$-\frac{1}{v^2}\frac{dv}{dt} - \frac{1}{u^2}\frac{du}{dt} = 0$$

$$\Rightarrow \quad \frac{dv}{dt} = -\frac{v^2}{u^2}\left(\frac{du}{dt}\right)$$

Hence in event (1),

$$\frac{du}{dt} = -v$$

$$\Rightarrow \quad \frac{dv}{dt} = -\frac{60^2}{30^2} \times -v = 4v$$

Hence, speed of image in event (1) is $4v$. after time y coordinate of object $y_0 = v\,t$

but $x_0 = -30$

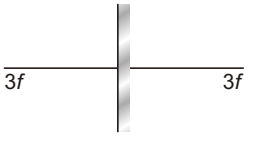
then $$y_i = \left|\frac{fy_0}{f - x_0}\right| = \left|\frac{-20 \times v\,t}{-20 + 30}\right|$$

$$y_i = |-2vt| = 2vt$$

$$\Rightarrow \quad \frac{dy_i}{dt} = +2v$$

Hence, option (b) and (c) are correct.

6. For plane mirror



$$u = 3f \Rightarrow v = 3f$$

For concave mirror

$$u = -3f$$

Using mirror formula

$$\frac{1}{v} = \frac{1}{3f} - \frac{1}{f} = -\frac{2}{3f}$$

$$\Rightarrow \quad v = -1.5f$$

$$\Rightarrow \quad |v| = 1.5f$$

For convex mirror,

$$\frac{1}{v}=\frac{1}{3f}+\frac{1}{f}=\frac{4}{3f}$$

$\Rightarrow \quad v=0.75f$

Hence maximum distance in event (1) if image is from plane mirror and minimum distance from convex mirror

When $\quad v=1.5f$, then $v=1.5f$

by plane mirror

For concave mirror

$$\frac{1}{u}=\frac{1}{1.5f}-\frac{1}{f}=\frac{2}{3f}-\frac{1}{f}$$

$$=\frac{2-3}{3f}=-\frac{1}{3f}$$

$\Rightarrow \quad v=-3f$

$\Rightarrow \quad |v|=3f$

For convex mirror

$$\frac{1}{v}=\frac{2}{3f}+\frac{1}{f}=\frac{5}{3f}$$

$\Rightarrow \quad v=0.6f$

Hence, in event (2) maximum distance of image from the concave mirror.

Hence, correct options are (a), (b) and (c).

■ Match the Columns

1. (a) $m=-2$, since $|m|=2>1$.

Therefore mirror is concave and $\because m$ is –ve.

Hence image is real [for concave mirror m is $=-$ve]

Therefore,

(a) $\rightarrow$ q, r

(b) Since $m=-\frac{1}{2}, \because m$ is – ve

Hence mirror is concave and image is real.

(b) $\rightarrow$ q, r

(c) $m=+2, \because m>1$

Hence mirror is concave and $\because m$ is + ve

Hence image is virtual.

(c) $\rightarrow$ q, s

(d) $\because 1\,\text{m}=+\frac{1}{2}<1$ and + ve

Hence the mirror is convex and image is virtual.

(d) $\rightarrow$ p, s

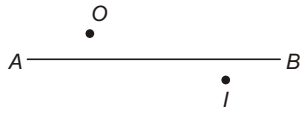
2. Plane mirror (for virtual object) $\rightarrow$ only real image

$\Rightarrow$ (a) $\rightarrow$ p

(b) $\rightarrow$ r

(c) $\rightarrow$ p

3. (a) Since object and its image are on opposite side of principle axis.



Hence mirror is concave

$\Rightarrow$ (a) $\rightarrow$ r.

(b) Similarly as for option (a).

(b) $\rightarrow$ r

(c) Since image and object are of same height from AB.

Hence mirror is plane mirror.

(c) $\rightarrow$ p

(d) Since image is magnified.

Hence mirror is concave [D is. distance between O and mirror is less than the focal length].

Hence

(d) $\rightarrow$ r.

4. (a) For concave mirror M_1 focal length

$$= -20 \text{ cm}$$

When $x = 20$ cm, Mirror is M_1

$$v = \infty \text{ and magnified}$$

(a) $\rightarrow$ p, s

(b) For convex mirror M_2 of focal length $+20$ cm if X (distance of object from pole) $= 20$

Using mirror formula $\frac{1}{v} + \frac{1}{v} = \frac{1}{f}$

we get

$$\frac{1}{v} = \frac{1}{20} + \frac{1}{20} = \frac{1}{10}$$

$\Rightarrow$ $v = 10$ cm

Hence image is virtual.

(b) $\rightarrow$ r

(c) $u = -30$ cm, $f = -20$ cm

$$\frac{1}{v} = \frac{1}{30} - \frac{1}{20} = \frac{2-3}{60} = -\frac{1}{60}$$

$v = -60$ cm

Hence image is real.

$$m = -\frac{60}{30} = -2$$

Hence image is magnified (2 times).

$\Rightarrow$ (c) $\rightarrow$ q, s

(d) for mirror M_2 (convex) at $X = +30$ cm

image again virtual.

(d) $\rightarrow$ r

5. (a) For concave mirror $f = -20$ cm

Case I. Image is real.

$$m = 2 = -\frac{v}{u} \Rightarrow v = -2u$$

Using $\frac{1}{u} - \frac{1}{u} = \frac{1}{f}$

we get, $-\frac{1}{2u} - \frac{1}{u} = -\frac{1}{20}$

$\Rightarrow$ $\frac{3}{2u} = \frac{1}{20}$

$\Rightarrow$ $u = 30$ cm

If image is virtual $v = 2v$

$$\frac{1}{2u} - \frac{1}{u} = -\frac{1}{20}$$

$\Rightarrow$ $u = 10$ cm

Hence correct option are as

(a) $\rightarrow$ p, q

(b) Here $m = \frac{1}{2} < 1$

Hence image is real.

$\Rightarrow$ $\frac{1}{2} = -\frac{v}{u} \rightarrow v = -\frac{u}{2}$

Using $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we get

$$-\frac{1}{u/2} - \frac{1}{u} = -\frac{1}{20}$$

$\Rightarrow$ $\frac{3}{4} = \frac{1}{20} \Rightarrow u = 60$ cm

Hence correct option is none of these.

$\Rightarrow$ (b) $\rightarrow$ s

(c) if $m = 1$, than $u = 2f$

$$u = -40 \text{ cm}$$

Hence correct option is none of these.

$\Rightarrow$ (c) $\rightarrow$ (s)

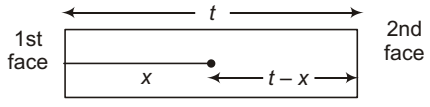
(d) Similarly as in part (b) we see that answer is none of these.

$\Rightarrow$ (d) $\rightarrow$ (s)

27 Refraction of Light

Introductory Exercise 27.1

1. Let real depth of dust particle is x and thickness of slab is t



From Ist surface

$$\mu = \frac{\text{Real depth}}{\text{App. depth}}$$

$$1.5 = \frac{x}{6\text{ cm}} \Rightarrow x = 9\text{ cm} \qquad \ldots\text{(i)}$$

From other face

$$\mu = \frac{t-x}{4} \Rightarrow t - x = 4 \times 1.5$$

$$\Rightarrow \qquad t = x + 6 = 9 + 6 = 16\text{ cm}$$

2. $${}_1\mu_2 = \frac{4}{3} \Rightarrow \frac{\mu_2}{\mu_1} = \frac{4}{3} \qquad \ldots\text{(i)}$$

$${}_2\mu_3 = \frac{3}{2} \Rightarrow \frac{\mu_3}{\mu_2} = \frac{3}{2} \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{\mu_3}{\mu_1} = \frac{4}{3} \times \frac{3}{2} = 2$$

3. Frequency remain same.

Let v_1 is velocity in medium (1) and v_2 in Medium (2)

We have

$$\mu_1 = \frac{c}{v_1} \text{ and } \mu_2 = \frac{c}{v_2}$$

$$\Rightarrow \qquad \frac{\mu_1}{\mu_2} = \frac{v_2}{v_1} \Rightarrow v_2 = \frac{\mu_1}{\mu_2} v_1$$

Similarly, wavelength $\lambda_2 = \frac{\mu_1}{\mu_2}\lambda_1$

4. From $v_a = n\lambda_a$

$$\Rightarrow \qquad \lambda_a = \frac{v_a}{n_a} = \frac{3 \times 10^8}{6 \times 10^{+14}} = 5 \times 10^{-7}\text{ m}$$

$$= 50\text{ nm}$$

$$\mu = \frac{\lambda_a}{\lambda_m} = \frac{500}{300} = \frac{5}{3} = 1.67$$

Introductory Exercise 27.2

1. Since light rays are coming from glass to air applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$$\Rightarrow \qquad \frac{1}{v} - \frac{1.5}{10} = \frac{1 - 1.5}{-1.5}$$

$$\Rightarrow \qquad \frac{1}{v} = -\frac{1}{30} + \frac{1.5}{10} = \frac{-1 + 4.5}{30}$$

$$\Rightarrow \qquad v = \frac{30}{3.5} = 8.57\text{ cm}$$

2. $$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

(a) $$\frac{1.5}{\mu} - \frac{1}{(-20)} = \frac{0.5}{6}$$

On solving $v = 45\text{ cm}$

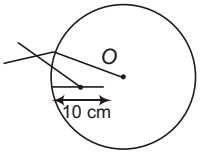
(b) $$\frac{1.5}{v} - \frac{1}{(-10)} = \frac{0.5}{6}$$

On solving we get $v = -90\text{ cm}$

(c) $\frac{1.5}{v} - \frac{1}{(-3)} = \frac{0.5}{6}$

On solving $v = -6.0$ cm

3. Light rays are coming from glass to air



$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1}{v} - \frac{4}{3(-10)} = \frac{1 - 4/3}{(-15)}$$

$\frac{1}{v} + \frac{4}{30} = \frac{1}{45}$ on solving $v = -9$ cm

4. Applying $\frac{u_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$$\Rightarrow \quad \frac{1.44}{v} - \frac{1}{\infty} = \frac{0.44}{1.25}$$

On solving $v = 0.795$ cm

5. $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$$\frac{1.635}{v} - \frac{1}{(-9)} = \frac{0.635}{(-2.50)}$$

on solving $v = 6.993$ cm

Lateral magnification $m = -\frac{v}{u}$

$$\Rightarrow \quad = -\frac{6.993}{9} = -0.777$$

Introductory Exercise 27.3

1. We have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\Rightarrow \quad -\frac{1}{20} + \frac{1}{60} = (1.65 - 1)\left[-\frac{1}{R} - \frac{1}{R}\right]$$

$$\Rightarrow \quad \frac{-3+1}{60} = 0.65 \times \frac{-2}{R}$$

$$\Rightarrow \quad R = 60 \times 0.65 = 39 \text{ cm}$$

2. Using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{-50} - \frac{1}{x} = \frac{1}{30} \Rightarrow -\frac{1}{x} = \frac{1}{30} + \frac{1}{50}$$

On solving $x = -18.75$ cm

$$m = \frac{-v}{u} = \frac{50}{18.75}$$

Height of filament image $= 2 \times \frac{50}{18.75}$

$= 5.3$ cm

3. $\frac{1}{f} = (\mu - 1)\left[\frac{1}{R} + \frac{1}{R}\right]$

If lens faces becomes opposite three is no change in radius of curvature hence focal length does not change.

4. Using formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ when $u \to 0_1\ v \to 0$

when $u \to f, v \to \infty$ hence image moves from surface to ∞.

5. $\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

$$\Rightarrow \quad \frac{1}{f} = (1.3 - 1)\left[\frac{-1}{R} - \frac{1}{R}\right] = 0.3 \times \frac{-2}{20}$$

$$\Rightarrow \quad f = -\frac{100}{3} \text{ cm}$$

(a) When immersed in a liquid of 1.8 refractive index

$$\frac{1}{f_1} = \left(\frac{1.3}{1.8} - 1\right)\left[\frac{-2}{R}\right] = \frac{-0.5}{1.8} \times \frac{-2}{20}$$

$f' = 36$ cm

(b) The minimum distance is equal to the focal length = 36 cm

6. Using $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$$\frac{1}{v}-\frac{1}{(-20)}=\frac{12}{10}$$

On solving $v=20$ cm

Magnification $=-\frac{v}{u}=-1$

Hence the image of same size and inverted. Let the distance between second lens is x Since magnification is unity image distance also x using again

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

we get

$$\frac{1}{x}-\frac{1}{(-x)}=\frac{1}{f}=\frac{1}{10}\Rightarrow x=20\text{ cm}$$

Hence the distance between two lenses

$$=20\text{ cm}+20\text{ cm}=40\text{ cm}$$

7. $\frac{1}{v}=\frac{1}{f}+\frac{1}{u}$ ...(i)

$\frac{1}{v_1}=\frac{1}{f}+\frac{1}{u+du}$ ...(ii)

$\frac{1}{v'}-\frac{1}{v}=\frac{(u+du-u)}{(u+du)u}$ on solving, we get

$\frac{v-v'}{v'}=\frac{du}{u(u+du)}$ thickness $dv=\frac{-v^2}{u^2}du$

10. Size of image $=\sqrt{6\times\frac{2}{3}}=2$ cm.

11. Let image distance is u

$$|m|=3\Rightarrow v=3u$$

$$\frac{1}{3u}+\frac{1}{u}=\frac{1}{12}\Rightarrow v=16\text{ cm}$$

12. Since image is upright and diminished hence lens is concave. Now

$u-v=20$ ...(i)

$m=\frac{v}{u}=\frac{1}{2}$

$\Rightarrow$ $\frac{1}{2}=\frac{u-20}{u}$

$\Rightarrow$ $u=40$ cm and $v=20$ cm

$\Rightarrow$ Using $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$$-\frac{1}{20}+\frac{1}{40}=\frac{1}{f}\Rightarrow f=-40\text{ cm}$$

13. The image coincide itself if light falls normally on plane mirror hence object must be on focus *i.e.* + 10 cm.

8. $\frac{1}{v}+\frac{1}{u}=\frac{2(\mu_2/\mu_1)}{R_2}-\frac{2(\mu_2/\mu_1-1)}{R_1}$

$$\frac{1}{v}+\frac{1}{0.2}=\frac{2(4/3)}{0.4}-\frac{2(4/3-1)}{0.4}$$

On solving $v=12$ cm

9. Since shift in position $\Delta t=0.1$ m

Hence real depth $=(0.1+0.2)$ m

$=0.3$ m

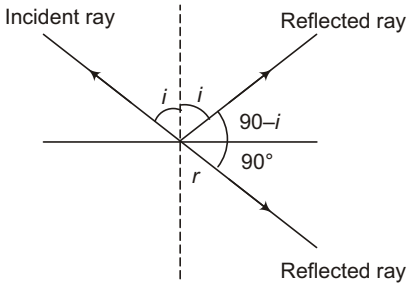
and apparent depth$=0.2$ m

$$\mu=\frac{\text{real depth}}{\text{apparent depth}}$$

$$=\frac{0.3}{0.2}=1.5$$

AIEEE Corner

1.



We have $r + 90° + i = 180° \Rightarrow r = 90 - i$

From Snell's law $1.5 = \dfrac{\sin i}{\sin r} = \dfrac{\sin i}{\sin(90 - i)}$

$\Rightarrow \quad \tan i = 1.5$

$\Rightarrow \quad i = \tan^{-1}(1.5)$

2. $n_w = \dfrac{v_{\text{air}}}{u_w} = \dfrac{343}{1498} = 0.229$

Critical angle $\theta = \sin^{-1}(0.229) = 13.2°$

3. Sped in glycrine $v_g = \dfrac{c}{n_g} = \dfrac{3 \times 10^8}{1.47}$

$$t_1 = \frac{20}{v_g} = \frac{20 \times 1.47}{3 \times 10^8} = 9.8 \times 10^{-8}\,\text{s}$$

Speed in glycrine $v_g = \dfrac{c}{n_g} = \dfrac{3 \times 10^8}{1.63}$

$$t_2 = \frac{20}{v_c} = \frac{20 \times 1.63}{3 \times 10^8} \simeq 10.8 \times 10^{-8}$$

$$t_2 - t_1 = (10.86 - 9.8) \times 10^{-8}$$

$$= 1.67 \times 10^{-8}\,\text{s}$$

4. (a) $t_1 = \dfrac{1 \times 10^{-6}\ \text{m}}{v_1} = \dfrac{1 \times 10^{-6}\ \text{m}}{3 \times 10^8/1.2}$

$$= \frac{1.2 \times 10^{-6}}{3 \times 10^8}$$

$\Rightarrow \quad t_1 = 0.4 \times 10^{-14}\,\text{s}$

$$t_2 = \frac{1.5 \times 10^{-6}}{3 \times 10^8} = 0.5 \times 10^{-14}\ \text{s}$$

$$t_3 = \frac{1 \times 10^{-6}}{3 \times 10^8/1.8} = \frac{1.8 \times 10^{-6}}{3 \times 10^8} = 0.6 \times 10^{-14}$$

Hence t_1 is least and $t_1 = 0.4 \times 10^{-14}$ s

(b) Total number of wavelengths

$$= \frac{1\,\mu\text{m}}{\lambda/n_1} + \frac{1.5\,\mu\text{m}}{\lambda/n_2} + \frac{1\,\mu\text{m}}{\lambda/n_3}$$

$$= \frac{1000 \times 1.2\ \text{nm}}{600\ \text{nm}} + \frac{1.5 \times 100\ \text{nm}}{600\ \text{nm}} + \frac{1 \times 1.8 \times 1000\ \text{nm}}{600\ \text{nm}}$$

$$= \frac{4500}{600} = 7.5$$

5. The given wave equation is

$$E_x(y, t) = E_{ax} \sin\left[\frac{2\pi y}{5 \times 10^{-7}} - 3 \times 10^{14} \times 2\lambda t\right]$$

Comparing with standard equation

$$E_x(y, t) = E_0 \sin[ky - \omega t]$$

$$k = \frac{2\pi}{5 \times 10^{-7}},\ \omega = 2\pi \times 3 \times 10^{14}$$

$$v = \frac{\omega}{k} = \frac{2\pi \times 3 \times 10^{14}}{2\pi/5 \times 10^{-7}} = 1.5 \times 10^8\ \text{m/s}$$

Refractive index $n = \dfrac{c}{v} = \dfrac{3 \times 10^8}{1.5 \times 10^8} = 2$

Wavelength in this way $\lambda_n = \dfrac{2\pi}{k}$

$\Rightarrow \quad \lambda_n = \dfrac{2\pi}{25/5 \times 10^7} = 5 \times 10^{-7}\,\text{m}$

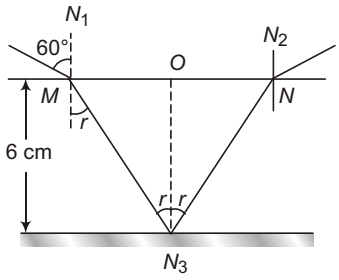
$\Rightarrow \quad \lambda_n = 500\,\text{nm}$

If vacuum, wavelength is λ then

$$\lambda_n = \frac{\lambda}{n}$$

$\Rightarrow \quad \lambda = n\lambda_n = 2 \times 500 = 1000\,\text{nm}$

6. Refraction from plane and spherical surfaces



We have $\dfrac{\sin 60°}{\sin r} = 1.8$

$\Rightarrow \quad \sin r = \dfrac{\sin 60°}{1.8}$

$\Rightarrow \quad \sin r = \dfrac{\sqrt{3}}{2 \times 1.8} = 0.48$

$\Rightarrow \quad r = \sin^{-1}(0.48)$

$\Rightarrow \quad r \simeq 28.7°$

Now $\quad \dfrac{MO}{6} = \tan r$

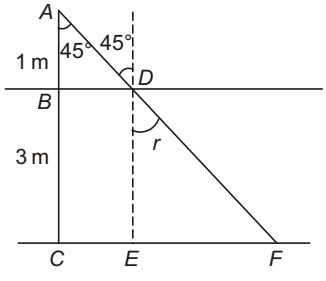
$\Rightarrow \quad MO = 6 \tan r$

Similarly $ON = 6 \tan r$

$\Rightarrow MN = MO + ON = 12 \tan r = 12 \tan(28.7°)$

$\Rightarrow MN = 6.6 \text{ cm}$

7. From Snell's law $\dfrac{4}{3} = \dfrac{\sin 45°}{\sin r}$



Solving we get $r = 32°$

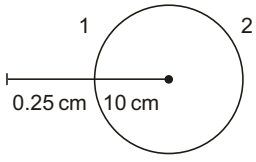
$EF = DE \tan r = 3 \tan 32°$

$= 1.88 \text{ m}$

Total length of shadow $= 1 + 1.88$

$= 2.88 \text{ m}$

8. The situation is shown in figure



For first surface $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$

$\Rightarrow \quad \dfrac{1.5}{v} - \dfrac{1}{(-2.5)} = \dfrac{0.5}{10}$

$\Rightarrow \quad \dfrac{1.5}{v} = \dfrac{1}{20} - \dfrac{1}{2.5} = -\dfrac{7}{20}$

$\Rightarrow \quad v = -\dfrac{30}{7} \text{ cm}$

This image acts as a virtual object for 2nd surface

$$u_2 = -\left(20 + \frac{30}{7}\right) = -\frac{170}{7} \text{ cm}$$

and $\quad R = -10 \text{ cm}$

$\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{r}$

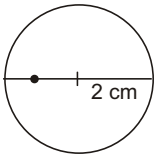
$\Rightarrow \quad \dfrac{1}{v} + \dfrac{1.5}{170/7} = \dfrac{-0.5}{-10}$

$\Rightarrow \quad \dfrac{1}{v} = \dfrac{1}{20} - \dfrac{10.5}{170}$

$\Rightarrow \quad v = -85 \text{ cm}$

Hence final image will produced at -65 cm from Ist surface.

9. Here $v = -1$ cm



$R = -2$ cm

Applying $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$

$\Rightarrow \dfrac{1}{-1} - \dfrac{1.5}{x} = \dfrac{1 - 1.5}{-2} = \dfrac{-0.5}{-2} = \dfrac{1}{4}$

$\Rightarrow \quad \dfrac{-1.5}{x} = \dfrac{5}{4}$

$\Rightarrow \quad x = \dfrac{-6}{5} = -1.2 \text{ cm}$

10.

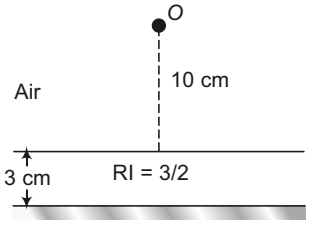


Image formed by refection acts the virtual object for the mirror.

Here shift $= t\left(1 - \frac{1}{\mu}\right)$

$$= 3\left(1 - \frac{1}{3/2}\right) = 1 \text{ cm}$$

Hence object appear to the mirror

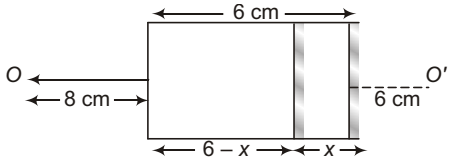
$$= (10 + 1) \text{ cm}$$

$$= 11 \text{ cm}$$

The image formed by mirror $= -11$ cm

Hence image formed by the mirror at 11 cm behind the mirror.

11.



Step. Let shift in mirror is x then the distance of object.

From the mirror is $= 8 + (6 - x)$.

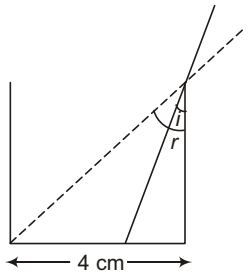
Step II. Plane mirror form image behind the mirror at same distance as the distance object from mirror hence

$$8 + (6 - x) = x + 6 \Rightarrow x = 4 \text{ cm}$$

Step III. $\mu = \frac{\text{real depth}}{\text{app. depth}} = \frac{6}{4} = 1.5$

hence real position of the bubble inside sphere is 1.2 cm from the surface.

12.



Here $\sin r = \frac{4}{\sqrt{4^2 + n^2}}$

$$\sin i = \frac{2}{\sqrt{2^2 + n^2}}$$

$$\Rightarrow \quad u = \frac{\sin i}{\sin r} \Rightarrow \frac{4}{3} = \frac{4\sqrt{2^2 + n^2}}{2\sqrt{4^2 + n^2}}$$

$$\Rightarrow \quad \frac{4}{9} = \frac{4 + n^2}{16 + n^2} \Rightarrow 5n^2 = 28$$

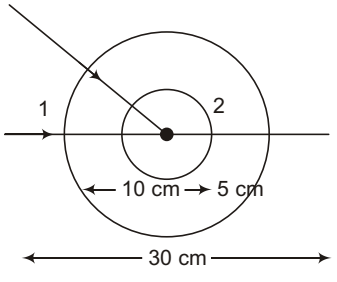
$$\Rightarrow \quad n^2 = \sqrt{\frac{28}{5}} \Rightarrow n = 2.4 \text{ cm}$$

13. Using $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$$\frac{1}{v} - \frac{1.5}{\infty} = \frac{1 - 1.5}{-5}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{10} \Rightarrow v = 10 \text{ cm}$$

14. For first surface $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$



$$\Rightarrow \quad \frac{1.5}{v} - \frac{1}{\infty} = \frac{0.5}{10} \Rightarrow v = 30 \text{ cm}$$

For 2nd surface

$$\frac{1}{v}-\frac{1.5}{(-15)}=\frac{-0.5}{5}$$

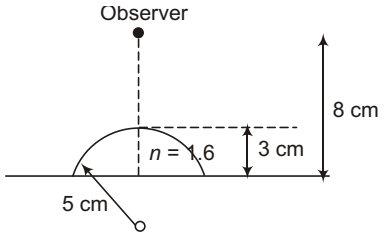
$$\Rightarrow \quad \frac{1}{v}=-\frac{1}{10}-\frac{1}{10}\Rightarrow v=-5\text{ cm}$$

Hence the distance from first face is

$$=(10+5)\text{ cm}$$

$$=15\text{ cm}$$

15. Since rays goes from paper weight ($n=1.6$) to air hence



$$\frac{1}{v}-\frac{1.6}{(-3)}=\frac{1-1.6}{5}$$

$$\Rightarrow \quad \frac{1}{v}=-\frac{0.6}{5}-\frac{1.6}{3}$$

On solving we get $v=-0.58\Rightarrow |v|=0.58$ cm

Hence the distance between observer and table top is $=(8-0.58)$ cm $=7.42$ cm.

16. Let real velocity of bird $=v_B$ cm/s

Velocity of bird w.r.t. fish $=16$ cm/s

Velocity of bird w.r.t. water $=\mu v_B$

But $\mu v_B+v_f=16$ cm/s

Here $v_f=4$ cm/s

$$\Rightarrow \quad \left(\frac{4}{3}v_B+4\right)=16$$

$$\Rightarrow \quad \frac{4}{3}v_B=12\text{ cm/s}$$

$$\Rightarrow \quad v_B=9\text{ cm/s}$$

17. Let the distance between the object and screen is d and let distance between object and lens is x

Using lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ we get

$$\frac{1}{d-x}-\frac{1}{(-x)}=\frac{1}{16}$$

$$\Rightarrow \quad x^2-xd+16d=0$$

$$\Rightarrow \quad x=\frac{d\pm\sqrt{d^2-64d}}{2}$$

Let $\quad x_1=\frac{d+\sqrt{d^2-64d}}{2}$

and $\quad x_2=\frac{d-\sqrt{d^2-64d}}{2}$

$$\Rightarrow \quad x_1-x_2=\sqrt{d^2-64d}$$

But $x_1-x_2=60\Rightarrow d^2-64d-3600=0$

$$\Rightarrow \quad (d-100)(d+36)=0$$

$$\Rightarrow \quad d=100 \because d\neq -36$$

Hence the distance between object and screen is 100 cm.

18. $\because \frac{1}{f}=(n-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$

For identical double convex lens

$$|R_1|=|R_2|=R$$

but $\quad R_1=R$ and $R_2=-R$

$$\Rightarrow \frac{1}{f}=(n-1)\left[\frac{1}{R}+\frac{1}{R}\right]=(n-1)\frac{2}{R}$$

$$\Rightarrow f=\frac{R}{2(n-1)} \text{ hence } f_1=\frac{R}{2(1.5-1)}=R$$

and $f_2=\frac{R}{2(1.7-1)}=\frac{R}{1.4}$

(a) $\Rightarrow \frac{f_1}{f_2}=1.4\Rightarrow f_1:f_2=1.4:1$

(b) For first lens

$$f_i=\frac{R}{2\left[\frac{1.5}{1.6}-1\right]}=-\frac{1.6R}{2\times 1}=-8R$$

hence first lens become concave (diverging)

For 2nd lens

$$f_2 = \frac{R}{2\left[\frac{1.7}{1.6} - 1\right]} = \frac{1.6R}{2 \times 0.1} = 8R$$

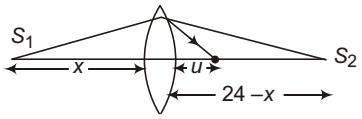
Hence 2nd lens remain **convex.**

19. Here $u = -10$ cm acts the virtual object for the lens $v = -15$ cm using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \quad \frac{1}{-15} + \frac{1}{10} = \frac{1}{f}$$

On solving we get $f = -30$ cm.

20. Situation is shown in figure.



For Ist source direction left to right is + ve
Using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{v} - \frac{1}{(-x)} = \frac{1}{9} \Rightarrow \frac{1}{v} + \frac{1}{x} = \frac{1}{9} \quad \text{...(i)}$$

For 2nd source direction right to left is + ve

hence $\frac{-1}{v} + \frac{1}{24 - x} = \frac{1}{9}$ ...(ii)

On adding Eqs. (i) and (ii) we set

$$\frac{1}{x} + \frac{1}{24 - x} = \frac{2}{9} \Rightarrow x^2 - 24x + 108 = 0$$

$$\Rightarrow \quad x = 6, 18$$

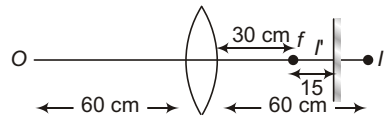
hence lens can be placed at a distance of 6 cm from any source.

21. Since the object is placed at c of first lens hence image also form at c and of same magnification *i.e.* $v = 2f$. Since two lens are separated by distance f hence the distance between 2nd lens and image is f. This image acts a virtual object for this lens using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get $\frac{1}{v} - \frac{1}{f} \Rightarrow v = \frac{f}{2}$

and magnification $m_2 = -\frac{v}{u} = \frac{-f/2}{f} = \frac{-1}{2}$

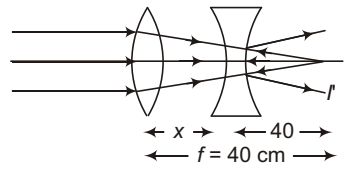
Hence image is formed at $\frac{f}{2}$ right of 2nd lens.

22. Since the object is placed at $2f$ hence image also form $2f$ by lens *i.e.*, at 60 cm. The mirror must be placed at that place that it made the final image at focus of lens. The difference is shown below.



Hence the distance between lens and mirror = 40 cm + 15 cm = 45 cm

23. The diagram is shown in figure.

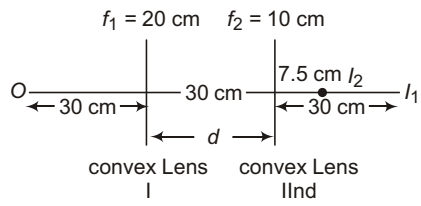


The parallel ray after refraction on convergent lens meet at focus = 40 cm. Let distance between two lenses is x then using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ for diverging lens

$$\frac{1}{\infty} - \frac{1}{(40 - x)} = \frac{-1}{15}$$

$$\Rightarrow \quad x = 25 \text{ cm}$$

24. Here $f_1 = 20$ cm $f_2 = 10$ cm and $d = 30$ cm



For first lens using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ we get

$$\frac{1}{v} + \frac{1}{30} = \frac{1}{20} \Rightarrow \frac{1}{v} = \frac{1}{20} - \frac{1}{30} \Rightarrow v = 60 \text{ cm}$$

For 2nd lens

$$\frac{1}{v}-\frac{1}{30}=\frac{1}{10}$$

$$\Rightarrow \quad \frac{1}{v}=\frac{1}{30}+\frac{1}{10}$$

$$\Rightarrow \quad \frac{1}{v}=\frac{4}{30}$$

$$\Rightarrow \quad v=7.5\text{ cm}$$

Hence image formed at 7.5 cm from 2nd lens.

25. For lens $u=-40$ cm $f=+20$ cm using $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$$\frac{1}{v}=\frac{1}{f}+\frac{1}{u}$$

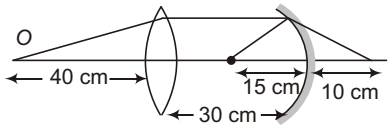
$$\Rightarrow \quad \frac{1}{v}=\frac{1}{20}-\frac{1}{40}=\frac{2-1}{40}=\frac{1}{40}$$

$$\Rightarrow \quad v=40\text{ cm}$$

Hence the image is at 40 cm right from the lens. Since distance between mirror and lens is 30 cm. Hence for mirror $v=+10$ cm, $f=-10$ cm.

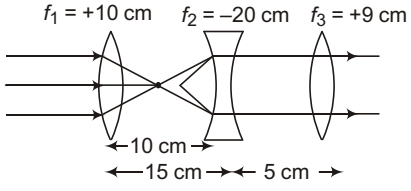
Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{v}=\frac{1}{f}-\frac{1}{u}$

$$\Rightarrow \quad \frac{1}{v}=-\frac{1}{10}-\frac{1}{10}\Rightarrow v=-5\text{ cm}$$



Hence image is formed at 5 cm from the mirror toward lens.

26.



For first lens $v=+10$ cm

For 2nd lens $u=-5$ cm, $f=-20$ cm

Using $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$$\Rightarrow \quad \frac{1}{v}=-\frac{1}{20}-\frac{1}{5}$$

$$\Rightarrow \quad v=-4\text{ cm}$$

For third lens $v=-9$ cm and $f=9$ cm

$$\Rightarrow \quad \frac{1}{v}-\frac{1}{(-9)}=\frac{1}{9}$$

$$\Rightarrow \quad v=\infty$$

Hence the image formed at ∞ or rays become parallel.

27. Situation is shown in figure when space between two convex lenses is filled with refractive index 1.3 it become a concave lens of radii $R_1=-30$ cm and $R_2=+70$ cm hence it focal length is

$$\frac{1}{f}=(n-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

$$\Rightarrow \quad \frac{1}{f}=(1.3-1)\left[\frac{-1}{30}-\frac{1}{70}\right]$$

$$\Rightarrow \quad f=-70\text{ cm}$$

Hence the equivalent focal length of combination $\frac{1}{F}=\frac{1}{30}-\frac{1}{70}+\frac{1}{70}$

$\Rightarrow$ $F=30$ cm if

$u=-90$ cm then using

$$\frac{1}{F}=\frac{1}{v}-\frac{1}{u}$$

we get $\frac{1}{30}=\frac{1}{v}+\frac{1}{90}$

$$\Rightarrow \quad \frac{1}{v}=\frac{1}{30}-\frac{1}{90}=\frac{2}{90}=\frac{1}{45}$$

$$\Rightarrow \quad v=45\text{ cm}$$

28. $n_{\text{ice}} = \frac{c}{v} = \frac{3 \times 10^8}{2.3 \times 10^8} = 1.30$

$$\theta_c = \sin^{-1}\left(\frac{1}{n_{\text{ice}}}\right) = \sin^{-1}\left(\frac{1}{1.30}\right)$$

$$= \sin^{-1}(0.77)$$

29. (a) Let angle of refraction in material 2 is r then $\frac{\sin\theta}{\sin r} = \frac{1.8}{1.6} = \frac{18}{16}$ ...(i)

For (2) to (3) interface

$$\frac{\sin r}{\sin 90°} = \frac{1.3}{1.8} = \frac{13}{18}$$

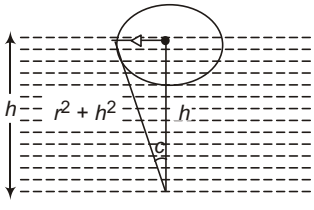
$\Rightarrow$ $\sin r = \frac{13}{18}$ ...(ii)

From (i) and (ii) $\sin\theta = \frac{18}{16} \times \frac{13}{18}$

$\Rightarrow$ $\theta = \sin^{-1}\left(\frac{13}{16}\right)$

(b) Yes, if θ decreases r also decreases and become less than the critical angle and hence light goes into material 3.

30. Let maximum height of liquid is h. From figure for critical angle C



$$\mu = \frac{1}{\sin C} = \frac{\sqrt{r^2 + h^2}}{r} \quad \text{...(i)}$$

Here $r = 1$ cm and $\mu = \frac{4}{3}$ putting these values in Eq. (i). Solving we get $h = \frac{4}{3}$ cm

Here $\sin\theta_c = \frac{1}{\mu_g} = \frac{2}{3}$

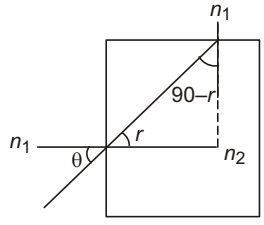
Now if water film is poured on the glass air surface. Let emergent angle at glass water surface is r, then

$$\frac{\sin\theta_c}{\sin r} = \frac{\mu_w}{\mu_g} = \frac{4 \times 2}{3 \times 3}$$

$\Rightarrow$ $\sin r = \frac{9}{8}\sin\theta_c = \frac{9}{8} \times \frac{2}{3} = \frac{3}{4}$

$\Rightarrow$ $r = \sin^{-1}\left(\frac{3}{4}\right)$

32. For total internal reflection at top surface



$$\frac{\sin(90° - r)}{\sin 90°} = \frac{n_1}{n_2}$$

$\Rightarrow$ $\cos r = \frac{n_1}{n_2}$

and $\frac{\sin\theta}{\sin r} = \frac{n_2}{n_1}$

$\Rightarrow$ $\sin\theta = \frac{n_2}{n_1}\sin r$

$\Rightarrow$ $\sin\theta = \frac{n_2}{n_1}\sqrt{1 - \cos^2 r}$

$$= \frac{n_2}{n_1}\sqrt{1 - \left(\frac{n_1}{n_2}\right)^2}$$

$\Rightarrow$ $\sin\theta = \sqrt{\frac{n_2^2 - n_1^2}{n_1^2}}$

$\Rightarrow$ $\theta = \sin^{-1}\sqrt{\left(\frac{n_2}{n_1}\right)^2 - 1}$

33. The deviation angle vary from 0° to $\theta°$ where $\theta = 90° - c$ ...(i)

where C is the critical angle

Now, $\sin c = \frac{\mu_w}{\mu_g} = \frac{4/3}{3/2} = \frac{8}{9}$

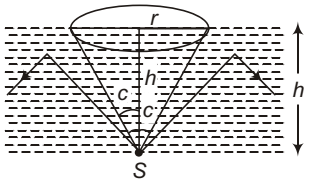
From Eq. (i) $\cos\theta = \sin C$

$\Rightarrow$ $\cos\theta = \frac{8}{9}$

$\Rightarrow \qquad \theta = \cos^{-1}\left(\frac{8}{9}\right)$

Hence deviation angle vary from 0° to $\cos^{-1}\left(\frac{8}{9}\right)$.

34.



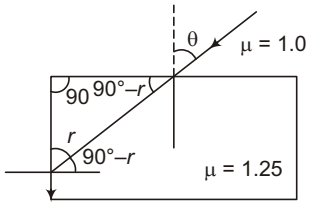
(a) Only circular patch light escapes because only those rays which are incident within a cone of semivertex angle C [Critical angle] are refracted out of the water surface. All other rays are totally internally reflected as shown in figures

(b) Now $\mu = \frac{1}{\sin C} = \frac{\sqrt{r^2 + h^2}}{r}$

or $\quad C = \sin^{-1}\left(\frac{1}{\mu}\right)$

$$= \sin^{-1}\left(\frac{r}{\sqrt{r^2 + h^2}}\right)$$

35.



For maximum angle θ the angle $90 - r$ at left surface must be equal to critical angle

$\Rightarrow \sin(90° - r) = \frac{1}{1.25} = \frac{100}{125} = \frac{4}{5}$

$\Rightarrow \qquad \cos r = \frac{4}{5}$

$\Rightarrow \qquad \sin r = \frac{3}{5}$

Now, $\qquad \frac{\sin\theta}{\sin r} = 1.25 = \frac{5}{4}$

$\Rightarrow \qquad \sin\theta = \frac{5}{4}\sin r = \frac{5}{4} \times \frac{3}{5}$

$\Rightarrow \qquad \theta = \sin^{-1}\left(\frac{3}{4}\right)$

36. $\mu = \dfrac{\sin\left(\dfrac{A + \delta m}{2}\right)}{\sin\dfrac{A}{2}}$

$$\sqrt{3} = \frac{\sin\left(\dfrac{A + \sin}{2}\right)}{\sin\dfrac{A}{2}} = \frac{2\sin\dfrac{A}{2}\cos\dfrac{A}{2}}{\sin\dfrac{A}{2}}$$

$\Rightarrow \cos\frac{A}{2} = \frac{\sqrt{3}}{2} \Rightarrow \frac{A}{2} = 30°$

$\Rightarrow \qquad A = 60°$

37. Here $i_1 = r_1 = 0°$.

Now, let other face angle of incidence is r_2

$\because \qquad r_1 + r_2 = A \Rightarrow 0 + r_2 = A$

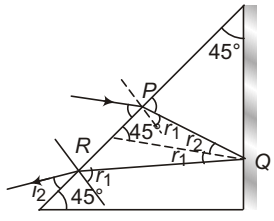
$\Rightarrow \qquad r_2 = A = 30°$

$\frac{\sin r_2}{\sin i_2} = \frac{1}{1.5} \Rightarrow \sin i_2 = 1.4 \sin r_2$

$\Rightarrow \qquad \sin i_2 = 1.5 \times \sin 30°$

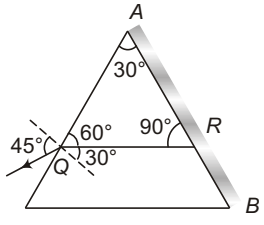
$\Rightarrow \qquad i_2 = \sin^{-1}(0.75) = 19°$

38. From figure $\sin \angle OQP = \angle OQR$



Hence the ray retrace its path.

39.



The ray retrace its path from ref. by surface AB hence $\angle AR\theta = 90°$ from geometry it is clear that $r = 30°$

$$\mu = \frac{\sin i}{\sin r}$$

$$\Rightarrow \mu = \frac{\sin 45°}{\sin 30°} \Rightarrow \mu = \frac{1/\sqrt{2}}{1/2} \Rightarrow \mu = \sqrt{2}$$

40. Depends on formula.

41. The maximum angle will be $A = 2C$ where C is the critical angle

Now, $C = \sin^{-1}\left(\frac{1}{1.5}\right) = 41.81$

Hence $A = 2C = 2 \times 41.81 = 83.62$

42. $\mu = \dfrac{\sin\left(\dfrac{A + \delta_m}{2}\right)}{\sin \dfrac{A}{2}}$ here $A = 60°$

$$1.5 = \frac{\sin\left(\dfrac{60° + \delta_m}{2}\right)}{\sin 30°}$$

$$0.75 = \sin\left(\frac{60° + \delta_m}{2}\right)$$

$$\Rightarrow 60° + \delta_m = 2\sin^{-1}(0.75)$$

$$\Rightarrow \delta_m = 22.8$$

and not deviation $= 180° - 22.8 = 157.2°$

(b) If the system is placed in water

$$\mu = \frac{1.5}{4/3} = \frac{4.5}{4}$$

$$\Rightarrow 60° + \delta'_m = 2\sin^{-1}(1.125 \times \sin 30°)$$

$$\Rightarrow \delta'_m = 2\sin^{-1}\left[\frac{1.125}{2}\right] - 60°$$

Net deviation $= 180 - \delta_m = 128.4°$

43. $\omega = \dfrac{\mu_V - \mu_R}{\mu_Y - 1}$

$$0.0305 = \frac{1.665 - 1.645}{\mu_y - 1}$$

On solving we get $\mu y = 1.656$

44. $\dfrac{\omega_1}{f_1} + \dfrac{\omega_2}{f_2} = 0$

$$\Rightarrow \frac{0.18}{20} + \frac{\omega_2}{-30} \Rightarrow \omega_2 = \frac{0.18 \times 30}{20}$$

$$\Rightarrow \omega_2 = 0.27$$

Now, $\dfrac{1}{F} = \dfrac{1}{f_1} + \dfrac{1}{f_2} \Rightarrow \dfrac{1}{F} = \dfrac{1}{20} - \dfrac{1}{30}$

$$\Rightarrow F = 60 \text{ cm}$$

45. $\dfrac{\omega_1}{f_1} + \dfrac{\omega_2}{f_2} = 0$

$$\Rightarrow \frac{\omega_1}{\omega_2} = -\frac{f_1}{f_2}$$

$$\frac{3}{2} = -\frac{f_1}{f_2}$$

$$\Rightarrow f_1 = -\frac{3}{2} f_2$$

Now, $\dfrac{1}{F} = \dfrac{1}{f_1} + \dfrac{1}{f_2}$

$$\Rightarrow \frac{1}{150} = \frac{1}{f_2} - \frac{2}{3f_2}$$

$f_2 = 50$ cm and $f_1 = -75$ cm

46. Applying $\mu = \dfrac{\sin i_1}{\sin r_1}$

Find angle r_1 for two different refraction indices. Because $i_1 = 65°$ from both the cases.

Then again apply

$\mu = \dfrac{\sin i_2}{\sin r_2}$ and find i_2. Because $r_2 = A - r_1$.

Then apply :

$\delta = (i_1 + i_2) - A$

for two refraction indices. Then difference in deviations is :

$\Delta\delta = \delta_1 - \delta_2$

Objective Questions (Level-1)

1. Endoscope is bases on total internal refraction Hence, correct option is (c)

2. Here $\mu = A + \frac{B}{\lambda^2}$

$\because$ μ is dimensionless.

$\Rightarrow$ $\frac{B}{\lambda_2} =$ dimension of μ

$\Rightarrow B = \lambda^2 \Rightarrow B$ has dimension of Area

Hence, correct option is (d).

3. Shift $= \left(1 - \frac{1}{\mu}\right)$

$\because \mu_R$ is minimum. than other visible colour. Red colour least raised.

correct option is (c)

4. Critical angle $\theta_C = \sin^{-1}\left(\frac{1}{\mu}\right)$

$\because \mu_0$ is maximum for violet colour hence θ_c for violet colour is least.

Hence correct option is (d)

5. We have $P = \frac{1}{f(\text{metre})}$

$= \frac{100}{f(\text{cm})} = 100 \times (n-1)\left[\frac{1}{R} + \frac{1}{r}\right]$

$\Rightarrow P = \frac{100 \times 0.6 \times 2}{10} = +12$

Hence, correct option is (a).

6. Speed of light in water $= \frac{c}{\mu_w}$

$\Rightarrow v_w = \frac{3 \times 10^8}{4/3} = 2.25 \times 10^8$ m/s

Hence correct option os (c).

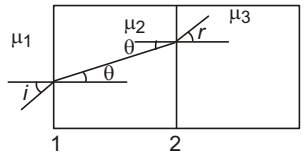
7. Due to TIR emergent beam will turn into black.

Hence correct option is (c).

8. $\because v = n\lambda$ but frequency n remain constant and v decreases hence λ decreases.

Hence correct option is (b).

9. Using Snell's law



On first and 2nd interface

$$\frac{\sin i}{\sin \theta} = \frac{\mu_2}{\mu_1} \quad \ldots(i)$$

and

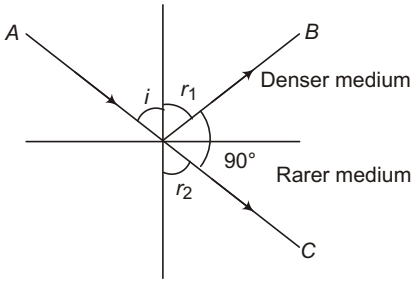
$$\frac{\sin \theta}{\sin r} = \frac{\mu_3}{\mu_2} \quad \ldots(ii)$$

Multiplying (i) and (ii), we get

$$\frac{\sin i}{\sin r} = \frac{\mu_3}{\mu_1}$$

Hence correct option is (b).

10. We have $i = r_1$ and $r_2 = 90 - i$



Now $\frac{\sin i}{\sin r_2} = \frac{1}{\mu}$

$\Rightarrow$ $\frac{\sin i}{\sin(90° - i)} = \frac{1}{\mu}$

$\Rightarrow$ $\tan i = \frac{1}{\mu}$ ...(i)

If C is the critical angle then $C = \sin^{-1}\left(\frac{1}{\mu}\right)$

$\Rightarrow$ $C = \sin^{-1}(\tan i)$

Hence correct option is (a).

11. Let angle of minimum deviation is δ_m we know that

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\Rightarrow \quad \sqrt{2} = \frac{\sin\frac{(60^\circ+\delta_m)}{2}}{\sin 30^\circ}$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \sin\left(\frac{60^\circ+\delta_m}{2}\right)$$

$$\Rightarrow \quad \frac{60+\delta_m}{2} = 45^\circ \Rightarrow \delta_m = 30^\circ$$

Hence correct option is (a).

12. We know that

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{a^2} = (1.5-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \frac{1}{0.2} = (0.5-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{...(i)}$$

Let refractive index of the liquid s n_l

$$\Rightarrow \frac{1}{f_{cm}} = \left(\frac{n}{n_l} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

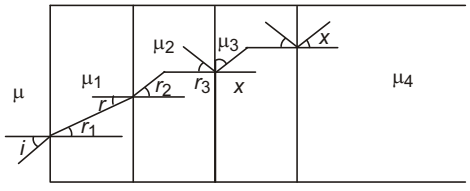
$$\Rightarrow -\frac{1}{0.5} = \left(\frac{1.5}{n_l} - 1\right) \times 10$$

$$\Rightarrow \quad \frac{-1}{5} = \frac{1.5}{n_l} - 1$$

$$\Rightarrow \quad \frac{1.5}{n_l} = \frac{4}{5} \Rightarrow \frac{5 \times 1.5}{4} = n_l \Rightarrow n_l = \frac{15}{8}$$

Hence correct option is (b).

13.



We have, $\frac{\sin i}{\sin r_1} = \frac{\mu_1}{\mu}$ ...(i)

$$\frac{\sin r_1}{\sin r_2} = \frac{\mu_2}{\mu_1} \quad \text{...(ii)}$$

and $\frac{\sin r_2}{\sin r_3} = \frac{\mu_3}{\mu_2}$ ...(iii)

$$\frac{\sin r_3}{\sin x} = \frac{\mu_4}{\mu_3} \quad \text{...(iv)}$$

Multiplying (i), (ii), (iii) and (iv), we get

$$\frac{\sin i}{\sin x} = \frac{\mu_4}{\mu} \Rightarrow \sin x = \frac{\mu}{\mu_4}\sin i$$

Hence correct option is (b)

14. Let radius of curvature of the lens is R

then $\frac{1}{f} = (n-1)\left(\frac{1}{R} + \frac{1}{R}\right) \Rightarrow f = \frac{R}{2(x-1)}$

Let focal length of one part is f'

then $\frac{1}{f'} = (n-1)\left[\frac{1}{R} - \frac{1}{\infty}\right]$

$$\Rightarrow \quad f' = \frac{R}{(n-1)} = 2f'$$

The focal length of the combination is

$$\frac{1}{F} = \frac{1}{2f} + \frac{1}{2f} + \frac{1}{2f} + \frac{1}{2f} \Rightarrow F = \frac{f}{2}$$

Hence correct option is (b).

15. Here $P = +5\text{D} \Rightarrow f + = 20\,\text{cm}$

$$\Rightarrow \quad \frac{1}{20} = (1.5-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad \text{...(i)}$$

and $-\frac{1}{100} = \left(\frac{1.5}{n_e} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$ ...(ii)

Dividing Eq. (ii) by (i) $-\frac{1}{5} = \frac{\left(\frac{1.5}{n_l} - 1\right)}{0.5}$

$$\Rightarrow \quad \frac{1}{10} + 1 = \frac{1.5}{n_l}$$

$$\Rightarrow \quad n_l = \frac{5}{3}$$

Hence correct option is (b).

16. We know that

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1f_2}$$

$\because$ whole system is concave $\Rightarrow F<0$

$$\Rightarrow \quad \frac{d}{f_1f_2}>\frac{1}{f_1}+\frac{1}{f_2}\Rightarrow d>f_1+f_2$$

$$\Rightarrow \quad d>(10+20)\text{ cm}\Rightarrow d>30\text{ cm}$$

Hence only possible value of the given values is 40 cm.

Hence correct option is (d).

17. Since when $A>2\theta_c$ total light is reflect (TIR) take place hence maximum value of A is $2\theta_c$.

Hence correct option is (c).

18. $\because$ emergent ray is $\perp$ to the surface

$$\Rightarrow \quad i_2=r_2=0$$

$$\because \quad r_1+r_2=A\Rightarrow r_1=A$$

Now $\quad \mu=\dfrac{\sin i}{\sin A}$

Since i and A are small

$$\Rightarrow \quad \sin i\simeq i \text{ and } \sin A\simeq A$$

$$\Rightarrow \quad \mu=\frac{i}{A}\Rightarrow i=\mu A$$

Hence correct option is (c).

19. $\mu=\dfrac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin\dfrac{A}{2}}$

$$\Rightarrow \quad \cot+\frac{A}{2}=\frac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin\dfrac{A}{2}}$$

$$\Rightarrow \quad \frac{A+\delta_m}{2}=90^\circ-\frac{A}{2}$$

$$\Rightarrow \quad A+\delta_m=180^\circ-A$$

$$\Rightarrow \quad \delta_m=180^\circ-2A$$

Hence correct option is (d).

20. Minimum deviation condition

$$r_1=r_2=r\Rightarrow 90^\circ-r+90^\circ-r+A=180^\circ$$

$$\Rightarrow \quad A=2r$$

$$\Rightarrow \quad r=\frac{A}{2}=30^\circ$$

Now $\quad \mu=\dfrac{\sin i}{\sin r}$

$$\Rightarrow \quad \sqrt{2}=\frac{\sin i}{\sin 30^\circ}$$

$$\Rightarrow \quad \sin i=\frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad i=45^\circ$$

Hence correct option is (a).

21. $\dfrac{1}{F}=\dfrac{1}{f_1}+\dfrac{1}{f_1}-\dfrac{d}{f_1f_2}$ ...(i)

$$\Rightarrow \quad \frac{1}{2F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{2d}{f_1f_2} \quad \text{...(ii)}$$

$$\frac{1}{F'}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{4d}{f_1f_2} \quad \text{...(iii)}$$

On solving (i) , (ii) and (iii) we get $F'=2F$.

Correct option is (a).

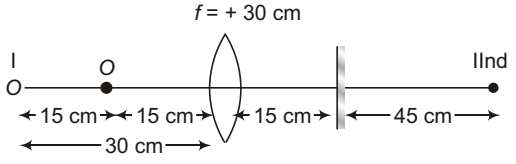
22. $\left|\dfrac{1}{f}\right|=(n-1)\left|\dfrac{1}{R_1}-\dfrac{1}{R_2}\right|$

$$\Rightarrow \quad \frac{1}{24}=(1.5-1)\left|\frac{1}{R}-\frac{1}{2R}\right|$$

$$\Rightarrow \quad \frac{1}{24}=\frac{0.5}{2R}\Rightarrow R=6 \text{ and } 2R=12$$

Hence correct option is (a).

23. The system is shown in figure.



Using $\quad \dfrac{1}{v}-\dfrac{1}{u}=\dfrac{1}{f}$

$$\Rightarrow \quad \frac{1}{v}-\frac{1}{(-15)}=\frac{1}{30}\Rightarrow v=-30\text{ cm}$$

This image I_1 act as a virtual object for mirror since plane mirror form image at same distance as object. Hence the distance between object and image is $(30 + 45)$ cm $= 75$ cm.

Hence correct option is (c).

24. $\dfrac{\sin 45°}{\sin r_1} = \sqrt{3}$ ...(i)

$$\frac{\sin r_1}{\sin r_2} = \sqrt{\frac{2}{3}}$$

Multiplying Eq. (i) and (ii)

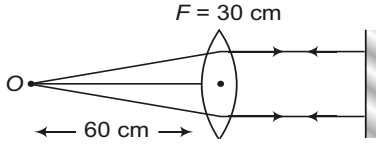
$$\frac{\sin 45°}{\sin r_2} = \sqrt{2}$$

$\Rightarrow \quad \sin r_2 = \dfrac{1}{2}$

$\Rightarrow \quad r_2 = 30°$

Hence correct option is (a).

25. For small angle prism



$\delta_{\min} = (\mu - 1)\,A$ if μ increases

$\Rightarrow \quad \delta_{\min} \rightarrow$ increases

Hence correct option is (a)

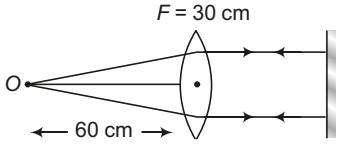
26. $\dfrac{\sin i}{\sin r} = \dfrac{n_2}{n_1} \Rightarrow \dfrac{\sin i}{\sin\left(\dfrac{i}{2}\right)} = n$

$\because r = \dfrac{i}{2}$

$\Rightarrow \quad 2\cos\dfrac{i}{2} = n \;\Rightarrow i = 2\cos^{-1}\left(\dfrac{n}{2}\right)$

Hence correct option is (c).

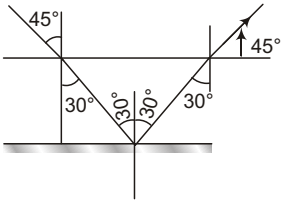
27. Situation is shown in figure.



Since image after reflection form on object itself hence the object must be placed at focus of the lens. The rays after refraction by lens becomes parallel to optic axis. Hence reflection rays follow the same path and final image form on x itself. Hence $x = 30$ cm.

Correct option is (b).

28. $\dfrac{\sin 45°}{\sin r} = \sqrt{2} \Rightarrow \sin r = \dfrac{1}{2} \Rightarrow r = 30°$



hence total deflection $= 45° - (-45°) = 90°$

Hence correct option is (a).

29. $\mu = \dfrac{\sin\left(\dfrac{A + \delta_m}{2}\right)}{\sin\dfrac{A}{2}}$

$\Rightarrow \quad \sqrt{\dfrac{3}{2}} = \dfrac{\sin\left(\dfrac{90° + \delta_m,}{2}\right)}{\sin 45°}$

$\Rightarrow \quad \sin\left(\dfrac{90° + \delta_m}{2}\right) = \dfrac{\sqrt{3}}{2} = \sin 60°$

$\Rightarrow \quad \delta_m = 30°$

Hence correct option is (c)

30. $\dfrac{1}{f} = (n - 1)\left[\dfrac{1}{R_1} - \dfrac{1}{R_2}\right]$

here $\quad n = \dfrac{n_{\text{air}}}{n_{\text{glass}}} = \dfrac{1}{1.5}$

$\Rightarrow \dfrac{1}{f} = \left(\dfrac{1}{1.5} - 1\right)\left[\dfrac{1}{10} + \dfrac{1}{10}\right]$

$\Rightarrow f = -15$ cm. Hence lens is concave

Correct option is (a).

31. For lens $u = -12$ cm and $F = +10$ cm

We have $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\Rightarrow \quad \frac{1}{v} + \frac{1}{12} = \frac{1}{10}$

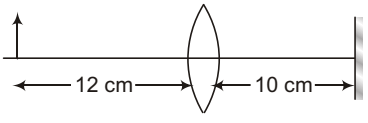
$\Rightarrow \quad v = 60$ cm

Since the distance between lens and mirror is 10 cm. Hence the image formed 50 cm from convex mirror. The rays retrace its path if image is formed at the centre of curvature of the mirror *i.e.,*

$R_{\text{convex}} = 50 \text{ cm} \Rightarrow F = \frac{R}{2} = 25 \text{ cm}$

Hence the correct option is (b)

32.



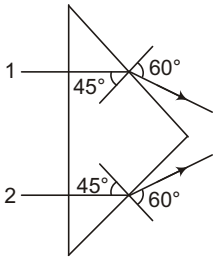
Using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{(-12)} = \frac{1}{20}$

$\Rightarrow \quad v = -30$ cm

Hence the distance between mirror and this image is 40 cm. Therefore second image formed 40 cm behind the mirror.

Hence correct option is (c).

33. We have $\mu = \frac{\sin i}{\sin r}$



$\Rightarrow \quad \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sin 45°}{\sin r}$

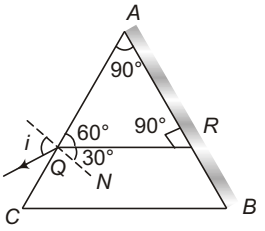
$\Rightarrow \sin r = \frac{\sqrt{3}}{2} \Rightarrow r = 60°$ (on both faces)

Hence the angle between emergent ray

$= 90° - (30 + 30°) = 30°$

Hence correct option is (b).

34.



Since the ray retrace its path hence $\angle ARQ = 90° \Rightarrow \angle RQN = \angle r = 30°$

$\mu = \frac{\sin i}{\sin r} \Rightarrow \sin i = \mu \sin r = \sqrt{2} \times \sin 30°$

$\Rightarrow \quad \sin i = \frac{1}{\sqrt{2}} \Rightarrow i = 45°$

Hence correct option is (c).

35. $f = 10$ cm

the focal length of $= \frac{1}{F} = \frac{1}{10} + \frac{1}{10}$

$\Rightarrow \quad F = 5$ cm

Here $u = -7.5$ cm, $F = 5$ cm

$\Rightarrow \quad \frac{1}{v} + \frac{1}{7.5} - \frac{1}{5} \Rightarrow \frac{1}{v} = \frac{7.5 - 5}{7.5 \times 5}$

$\Rightarrow \quad v = +15$ cm

Hence $|m| = \left|\frac{v}{u}\right| \Rightarrow \frac{\text{height of image}}{\text{height of object}} = \frac{15}{7.5}$

Height of image $= 2 \times 1$ cm $= 2$ cm

Hence correct option is (a).

36. $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$

$\Rightarrow \quad \frac{1}{v} - \frac{3}{2}\frac{1}{(\infty)} = \frac{(3/2 - 1)}{-20}$

$\Rightarrow \quad v = +40$ cm

Hence correct option is (a).

37. $\frac{1}{f} = \left(\frac{n_g}{n_L} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Here $R_1 = -30$ cm and $R_2 = -50$ cm

$$\Rightarrow \quad \frac{1}{f} = \left(\frac{1.5}{1.4} - 1\right)\left(-\frac{1}{30} + \frac{1}{50}\right)$$

On solving

$$\Rightarrow \quad f = -1050 \text{ cm}$$

Hence correct option is (d).

38. From the figure, it is clear that $\mu_1 = \mu_3 < \mu_2$

Hence correct option is (b).

39. Here $A = 60°$, $\delta_m = 60°$

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\frac{A}{2}} = \frac{\sin 60°}{\sin 30°}$$

$$\mu = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$$

Hence correct option is (a).

JEE Corner

Assertion and Reason

1. Due to shifting of image on refraction Shayam appear nearer to Ram and light suffer two refraction. Hence, both (a) and (b) are correct but reason does not explain the assertion.

Correct option is (b).

2. Applying lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Let u = object distance from lens

$v = d - u$ distance of image from lens.

$$\because \quad v + u = d$$

$$\Rightarrow \quad \frac{1}{d-u} - \frac{1}{-u} = \frac{1}{f}$$

$$\Rightarrow \quad u^2 - du + df = 0$$

$$\Rightarrow \quad u = \frac{d \pm \sqrt{d - 4f}}{2}$$

$\because u$ is real hence $d \geq 4f$

Thus mean distance $v = 4f$, if $u = -2f, v = 2f$. Hence both assertion and reason are true, and reason explain or may not explain assertion. Hence correct option is (a, b)

3. Correct option is (b).

4. Correct option is (c).

5. Since both assertion and reason are true built reason is not explain assertion Hence correct option is (b).

6. Using mirror formula.

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Here $\quad u = -\infty$

$f = -f$ (concave lens)

$$\frac{1}{v} + \frac{1}{\infty} = \frac{1}{-f}$$

$$\Rightarrow \quad v = -f$$

Hence image is formed at principle focus thus assertion is false but reason is true. Hence correct option is (d).

7. $$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin A/2}$$

Here $\quad A = 60°, \ \mu = \sqrt{2}$

$$\Rightarrow \quad \sqrt{2} = \frac{\sin\left(\frac{60 + \delta_m}{2}\right)}{\sin 30°}$$

$$\Rightarrow \quad \frac{60 + \delta_m}{2} = 450 \Rightarrow \delta_m = 30°$$

Hence both assertion and reason are true and reason explain assertion correctly.

Correct option is (a).

8. Focal length of combination

$$\frac{1}{F}=\frac{1}{F_{\text{convex}}}+\frac{1}{F_{\text{convex}}}=\frac{1}{f_1}-\frac{1}{f_2}$$

if $\quad f_1 > f_2$

$\Rightarrow \quad \frac{1}{f_1}<\frac{1}{f_2}$

$\Rightarrow F = -$ negative

Hence assertion is true. Since power is a measure of converging or divergence of a lens. Hence reason is not true. Correct option is (c).

9. Since glass slab produced a net shift. Hence v is increased. Thus magnified image is obtained but image may be real or virtual depending on the position of slab.

Correct option is (b)

10. In this case image distance of O_1 and O_2 are same from the lens.

$\because \quad \frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ and reason is trure.

Hence correct option is (d).

11. Assertion is false since only ray energe if refractve index of the colour less than the prism and angle of incidence is less than critical angle but reason is true. Correct option is (a).

12. If two object is placed between pole and focus image is real hence assertion is true. Also reason is correct.

Hence correct option is (b)

13. Since both assertion and reason are true and reason explanation is correct.

Hence correct option is (a).

Objective Questions (Level 2)

■ Single option correct

1. We have $\mu=\frac{\text{Real depth}}{\text{App. depth}}$

$\Rightarrow \quad \frac{4}{3}=\frac{1}{\text{App. depth}}$

$\Rightarrow$ App. depth $=\frac{3}{4}$

Hence the distance between bird and mirror

$$=2+\frac{3}{4}=\frac{11}{4}\text{m}$$

Since plane mirror form image behind the mirror (for real object) at same distance as object hence the distance between bird and its image

$$=\frac{11}{4}+\frac{11}{4}=\frac{11}{2}\text{ m}$$

Correct option is (d).

2. $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$

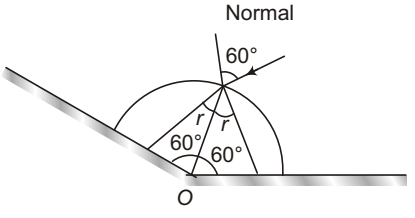
Here $\quad \mu=-\infty \; u_1=1$

and $\quad \mu_2=1.5$

$\Rightarrow \quad \frac{1.5}{v}+\frac{1}{\infty}=\frac{0.5}{R}\Rightarrow v=3R$

Hence correct option is (b).

3. From figure, $r=30°$



$\because \quad r=\theta+\alpha-90=120°-90°$

Hence $\mu = \dfrac{\sin 60°}{\sin 30°} = \sqrt{3}$

The correct option is (d).

4. The lens become diverging if

$$\mu_1 - \mu_2 > \mu_2 - \mu_3$$

$$\Rightarrow \quad \mu_1 + \mu_3 > 2\mu_2$$

or $\quad 2\mu_2 < \mu_1 + \mu_3$

Hence correct option is (b).

5. $\dfrac{1}{v_1} + \dfrac{1}{16} = \dfrac{1}{f} \Rightarrow v_1 = \dfrac{16f}{16-f}$

$$m_1 = \frac{-v_1}{16} = \frac{-16f}{16(16-f)} = \frac{-f}{(16-f)} \quad \ldots(i)$$

and $\quad \dfrac{1}{v_2} + \dfrac{1}{6} = \dfrac{1}{f} \Rightarrow v_2 = \dfrac{6f}{6-f}$

$$\mu_2 = \frac{v_2}{6} \quad (\because \text{image is virtual})$$

$$\Rightarrow \quad m_2 = \frac{6f}{6(6-f)} = \frac{f}{6-f} \quad \ldots(ii)$$

But $\quad m_1 = m_2 \Rightarrow \dfrac{-f}{16-f} = \dfrac{f}{6-f}$

$$\Rightarrow \quad -6 + f = 16 - f \Rightarrow 2f = 22$$

$$\Rightarrow \quad f = 11 \text{ cm}$$

Hence correct option is (d).

6. Let real depth at any instant t of the water is h then volume of water $V = \pi R^2 h$

$$\Rightarrow \quad \frac{dV}{dt} = \pi R^2 \frac{dh}{dt} \quad \ldots(i)$$

Let apparent depth at this instant is h'

$$\because \quad \mu = \frac{\text{Real depth}}{\text{Apparent depth}}$$

$$\Rightarrow \quad \frac{n_2}{n_1} = \frac{h}{h_1} \Rightarrow h_1 = \frac{n_1}{n_2} h$$

Now $\quad \dfrac{dh'}{dt} = x = \dfrac{n_1}{n_2}\dfrac{dh}{dt}$

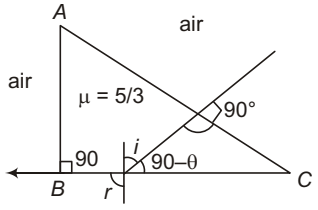
$$\Rightarrow \quad \frac{dh}{dt} = \frac{xn_2}{n_1} \quad \ldots(ii)$$

From Eq. (i) and (ii), we get

$$\frac{dV}{dt} = \frac{\pi R^2 x n_2}{n_1}$$

Hence correct option is (b).

7.



$\because \theta = 37° \Rightarrow i = [90 - (90 - \theta)] = \theta = 37°$

Applying Snell's law on face BC.

$$\frac{\sin 37°}{\sin r} = \frac{\mu_2}{\mu_1} = \frac{3}{5}$$

$$\Rightarrow \quad \sin r = \frac{4}{3}\sin 37° = \frac{5}{3} \times \frac{3}{5} = 1$$

$$\Rightarrow \quad r = 90°$$

Hence deviation $\delta = 90° + 37° = 127°$

Correct option is (b).

8. Let refractive index of liquid is μ

For position of fish w.r.t. bird is

$$\mu = \frac{\text{Real depth}}{\text{App. depth}} \Rightarrow \mu = \frac{x}{h_1} \quad \ldots(i)$$

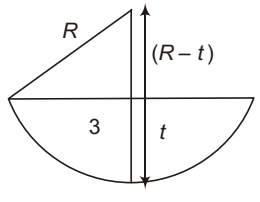
For position of bird w.r.t. fish is

$$\frac{1}{\mu} = \frac{y}{h_2} \quad \ldots(2)$$

From Eq. (i) and (ii) we get $u = \dfrac{h_2}{h_1}$

Hence correct option is (a).

9.



$$\Rightarrow (R - t)^2 + 3^2 = R^2$$

$$\Rightarrow \quad 2Rt \simeq 9 \quad \because R >> T$$

$$\Rightarrow \qquad 2R = \frac{900\text{ mm}}{3\text{ mm}} = 300\text{ mm}$$

$$\Rightarrow \qquad R = 150\text{ mm} = 15\text{ cm}$$

Hence correct option is (a).

10. $\frac{1}{v_1} + \frac{1}{v_1} = \frac{1}{f} \Rightarrow v_1 = \frac{fv_1}{v_1 - f}$

$$m_1 = \frac{-v_1}{v_1} = \frac{-f}{(v_1 - f)} \qquad \ldots(i)$$

$$\frac{1}{v_2} + \frac{1}{v_2} = \frac{1}{f} \Rightarrow v_2 = \frac{fv_2}{v_2 - f}$$

$$m_2 = \frac{v_2}{v_2} = \frac{f}{\mu_2 - f} \qquad \ldots(ii)$$

$\because \quad m_1 = m_2$

$$\Rightarrow \frac{-f}{v_1 - f} = \frac{f}{v_2 f} \Rightarrow f = \frac{v_1 + v_2}{2}$$

Hence the correct option is (d).

11. $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$

$$\Rightarrow \qquad F = \frac{f_1 f_2}{f_1 + f_2 - d}$$

as d increases $f_1 + f_2 - d$ decreases hence F increases. Hence image move to right.

Correct option is (b).

12. In this case, minimum deviation of ray 1 is same as ray 2.

Hence correct option is (c).

13. For critical angle at glass air surface

$$\sin\theta_c = \frac{1}{\mu_g} = \frac{2}{3} \qquad \ldots(i)$$

Now for glass water surface.

$$\frac{\mu_w}{\mu_g} = \frac{\sin\theta_c}{\sin r} \Rightarrow \frac{4/3}{3/2} = \frac{2}{3 \times \sin r}$$

$$\Rightarrow \qquad \sin r = \frac{4}{3}$$

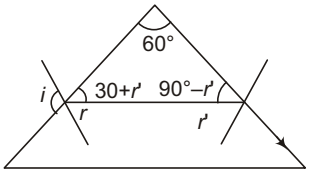
Now for water air surface

$$\frac{\mu_w}{\mu_a} = \frac{\sin r}{\sin r'} \Rightarrow \frac{4}{3} = \frac{4}{3} \times \frac{1}{\sin r'}$$

$$\Rightarrow \qquad \sin r' = 1 \Rightarrow r' = 90^\circ$$

Hence correct option is (d).

14. For limiting angle of incident emergent ray become parallel to the 2nd face



$$\because \qquad \frac{\sin r'}{\sin 90^\circ} = \sqrt{\frac{3}{7}} \Rightarrow r' = \sin^{-1}\sqrt{\frac{3}{7}}$$

Now $r + 30^\circ + r' = 90^\circ$

$$\Rightarrow \qquad r = 60^\circ - r'$$

$$\Rightarrow \qquad r = (60^\circ - \sin^{-1}\sqrt{3/7})$$

Now $\qquad \mu = \frac{\sin i}{\sin r}$

$$\Rightarrow \qquad \sqrt{\frac{7}{3}} = \frac{\sin i}{\sin[60 - \sin^{-1}\sqrt{3/7}]}$$

$$\Rightarrow \quad i = \sin^{-1}\{\sqrt{7/3} \times \sin(60 - \sin^{-1}\sqrt{3/7}\}$$

$$= \sin^{-1}\left\{\sqrt{\frac{7}{3}} \times \sin(60^\circ - 21^\circ)\right\}$$

$$= \sin^{-1}\{\sqrt{7/3} \times \sin 19^\circ\}$$

$$= \sin^{-1}\{0.49\} \simeq 30^\circ$$

Hence correct option is (a).

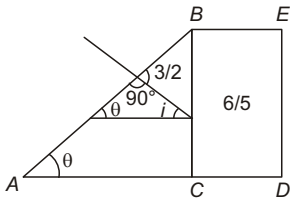
15. The image form the object itself if the rays incident parallel to optical axis on the mirror *i.e.,* image of refraction is formed at ∞. It is possible when O is placed at focus *i.e.*, $d = 10\text{ cm}$

Hence correct option is (c).

16. The dot will appear at c for all values of μ. Since position does not in same medium.

Hence correct option is (b).

17. We have for total internal reflection



$$\frac{\sin i}{\sin 90} = \frac{6/5}{3/2}$$

$$\Rightarrow \quad \sin i = \frac{4}{5} \Rightarrow i = \sin^{-1}\left(\frac{4}{5}\right) = 53°$$

Hence the ray will not cross BC if $i > 53°$

$$\Rightarrow \quad 90 + \theta + i = 180°$$

$$\theta = 90 - i \because i > 53°$$

$$\Rightarrow \quad \theta < 37°$$

Hence correct option is (a).

18. For reflection at curved surface

$$\frac{1}{v} - \frac{1}{(-x)} = \left(\frac{3}{2} - 1\right) \times \frac{1}{10}$$

$$\Rightarrow \quad v = \frac{20x}{x - 20}$$

This image act as virtual object for plane-glass-water surface

$$\Rightarrow \quad \frac{\mu_g}{\infty} - \frac{\mu_w(x-20)}{20x} = \frac{\mu_g - \mu_w}{\infty}$$

$$\Rightarrow \quad x = 20 \text{ cm}$$

Hence answer is (c).

19. The ratio of focal length in the situation II and III is 1 : 1.

Hence correct option is (c).

20. We have $\frac{1}{OB} - \frac{1}{(-OA)} = \frac{1}{f}$

$$\Rightarrow \quad \frac{1}{OB} + \frac{1}{OA} = \frac{1}{f}$$

$$\Rightarrow \quad f = \frac{OB.OA}{OA + OB} \quad \because OB + OA = AB$$

$$\Rightarrow \quad f = \frac{OB.OA}{AB} \quad \text{...(i)}$$

Now $AB^2 = AC^2 + BC^2$

$$(OA + OB)^2 = OC^2 + OA^2 + OB^2 + OC^2$$

$$\Rightarrow$$

$$OA^2 + OB^2 + 2OAOB = 20\,C^2 + OA^2 + OB^2$$

$$\Rightarrow \quad OC^2 = OA\ OB \quad \text{...(ii)}$$

Putting this value in Eq. (i), we get

$$\Rightarrow \quad f = \frac{OC^2}{AB}$$

Hence correct option is (c).

21. The shift produce

$$\Delta t = t\left[1 - \frac{2}{{}_w\mu_g}\right]$$

$$= 36\left[1 - \frac{1}{9/8}\right] \because {}_w\mu_g = \frac{3/2}{4/3} = \frac{9}{8}$$

$$= \frac{36 \times 1}{9} = 4 \text{ cm}$$

Hence correct option is (b).

22. $\mu = \frac{\text{Real depth}}{\text{App. depth}}$

$$\Rightarrow \quad \frac{4}{3} = \frac{\text{real depth}}{10.5 \text{ cm}}$$

$$\Rightarrow \text{ Real depth} = \frac{4}{3} \times 10.5 \text{ cm} = 14 \text{ cm}$$

Hence correct option is (d).

23. $\because y_0 = +1$ cm and $y_i = -2$ cm

$$m = -\frac{v}{u} \Rightarrow 2 = \frac{v}{u}$$

Now let x be the position of lens then $v = 50 - x$ and $v = (40 + x)$.

$$\Rightarrow \quad 2 = \frac{50 - x}{40 + x}$$

$$\Rightarrow \quad 80 + 2x = 50 - x$$

$$\Rightarrow \quad -3x = 30 \Rightarrow x = -10 \text{ cm}$$

Hence correct option is (c).

24. If the plane surface of plano-convex lens is silvered it behave the concave mirror of focal length $f_m/2$

$\because \quad f_m = 10\,\text{cm}$

$\Rightarrow \quad f_e = 5\,\text{cm}$ hence $R = 10\,\text{cm}$

Correct option is (c).

25. Since lens made real and magnified image, hence it is a convex lens when lens dipped in water its focal length.

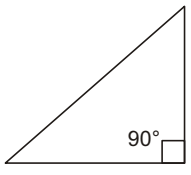
$$\frac{1}{f} = \left(\frac{\mu_w}{\mu_g} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$= \left(\frac{4/3}{3/2} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{-1}{9}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$\because f$ is – ve lens behave as concave, hence the image is virtual and magnified.

Correct option is (c).

26. The prism transmit the light for which angle of incidence(c), $2c \le 90° \Rightarrow c \le 45°$



Hence $\mu = \dfrac{1}{\sin C} = \dfrac{1}{\sin 45°} = \sqrt{2} = 1.414$

Correct option is (b).

27. $\dfrac{1}{v} - \dfrac{1}{(-16)} = \dfrac{1}{f} \because |m| = \left|\dfrac{v}{u}\right| = 3$

For convex lens $v = 3u$

$\dfrac{1}{48} + \dfrac{1}{16} = \dfrac{1}{f} \Rightarrow f = 12\,\text{cm}$ (for real image)

Similarly when distance is 6 cm, 3 times virtual image is formed hence mirror is convex with focal length 12 cm.

Correct option is (c).

28. $v_g = n\lambda_g \Rightarrow \dfrac{c}{\mu_g} = n\lambda_g$ ...(i)

and $\dfrac{c}{\mu_w} = n\lambda_w$ ...(ii)

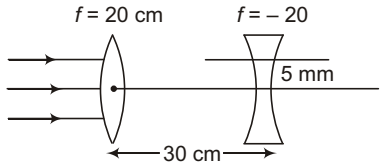
Dividing (i) and (ii)

$$\frac{\mu_w}{\mu_g} = \frac{\lambda_g}{\lambda_w} \Rightarrow \frac{4}{3\mu g} = \frac{4}{5}$$

$\Rightarrow \quad \mu_g = \dfrac{5}{3}$

Hence correct option is (a).

29. $x = \dfrac{f_1 f_2 + d(f_1 - d)}{f_1 + f_2 - d}$ and $y = \dfrac{(f_1 - d)\,\Delta}{f_1 + f_2 - d}$



Here $f_1 = f_2 = 20\,\text{cm}$, $d = 30\,\text{cm}$

and $\Delta = 55\text{m} = 0.5\,\text{cm}$

Putting these values we get

$x = 25\,\text{cm}$ and $y = 0.25\,\text{cm}$

Correct option is (b).

30. Since for each θ angle of incidence at glass-air boundry remains 0° hence there will never be total internal reflection.

Correct option is (d).

31. Diameter $= \mu \times$ Original diameter

$$= \frac{4}{3} \times 1\,\text{cm} = -\frac{4}{3}\,\text{cm}$$

Hence correct option is (a).

32. $\dfrac{1}{f_1} = (1.5 - 1)\left[\dfrac{1}{20} + \dfrac{1}{20}\right]$

$\Rightarrow \quad f_1 = 20\,\text{cm}$

Here $u_1 = -30\,\text{cm}$

$$\frac{1}{v} - \frac{1}{u_1} = \frac{1}{f_1} \Rightarrow \frac{1}{v_1} = \frac{1}{20} - \frac{1}{30} \Rightarrow v_1 = 60\,\text{cm}$$

Magnification $|m_1| = \left|-\dfrac{v}{u}\right| = \dfrac{60}{30} = +2$

(Inverted image)

For second lens. $\frac{1}{v_2}+\frac{1}{60}=\frac{1}{20}\Rightarrow v_2=+30\text{ cm}$

Magnification $|m_2|=\left|\frac{30}{60}\right|=\frac{1}{2}$

Total magnification $=m_1\times m_2=1$

Hence object size remains 3mm and it is formed at $(120+30)\text{ cm}=50\text{ cm}$ from first lens.

Hence correct option is (b).

33. From left hand side refraction occur from $n_2=2$ to $n_1=1$.

$$n=\frac{n_1}{n_2}=\frac{1}{2}=0.5$$

$$\frac{n}{v}-\frac{1}{u}=\frac{n-1}{R}$$

$$\frac{0.5}{v}-\frac{1}{10}=\frac{-0.5}{10}+\frac{1}{10}=\frac{0.5}{10}$$

$\Rightarrow$ $v=10\text{ cm}$

Hence correct option is (a).

34. For lens $v=-20\text{ cm}$, $f=+10\text{ cm}$

Using $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ we get $v=20\text{ cm}$

This image acts virtual object for convex mirror. For mirror $v=-(x-20)$

and $v=-(20+x)$ and $f=+60\text{ cm}$

Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$$\frac{1}{-(20+x)}-\frac{1}{(x-20)}=\frac{1}{60}$$

After solving we get $x=20\text{ cm}$.

Hence correct option is (c).

35. In this case system behave as concave mirror or focal length $=\mu\times f_e$

$$=1.5\times 20=30\text{ cm}$$

Now using mirror formula

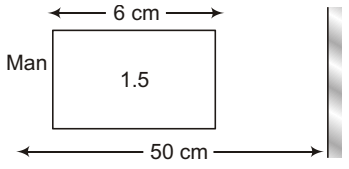
$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}\text{ we get}$$

$$\frac{1}{v}-\frac{1}{20}=-\frac{1}{30}$$

$\Rightarrow$ $\frac{1}{v}=\frac{1}{20}-\frac{1}{30}\Rightarrow v=+60\text{ cm}$

Hence correct option is (d).

36. Shift in mirror $=6\left(1-\frac{1}{1.5}\right)=2\text{ cm}$



Hence man lie at 48 cm from mirror. The distance of image from observer $=2\times 48$

$=96\text{ cm}$

Hence correct option is (b).

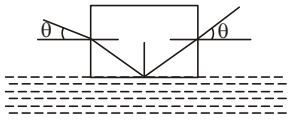
37. Using lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$\Rightarrow$ $\frac{1}{(f+40)}+\frac{1}{(f+10)}=\frac{1}{f}$

On solving we get $f=20\text{ cm}$

Hence correct option is (c).

38.



$$\frac{\sin\theta}{\sin(90-\theta_c)}=n_1$$

$$\frac{\sin\theta}{\cos\theta_c}=n_1$$

Now $\cos\theta_c<\sqrt{\frac{n_1^2-n_2^2}{n_1}}$

$\Rightarrow$ $\sin\theta<\sqrt{\frac{n_1^2-n_2^2}{n_1}}\times n_1$

$\Rightarrow$ $\sin\theta<\sqrt{n_1^2-n_2^2}$

Hence correct option is (a).

39. For mirror $u = -1$ cm (taking upward direction + ve)

$$f = -2 \text{ cm}$$

$$\frac{1}{v} = -\frac{1}{f} - \frac{1}{u}$$

$$\Rightarrow \quad \frac{1}{v} = -\frac{1}{2} + 1 = \frac{1}{2}$$

$$\Rightarrow \quad v = 2 \text{ cm}$$

Hence mirror form virtual image behind mirror at 2 cm from pole. This image acts as virtual object for slab on see below the slab the shift is

$$\Delta t = \left(1 - \frac{1}{3/2}\right) \times 9 = 3 \text{ cm}$$

Hence virtual image form on object thus correct option is (a).

40. We have $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

For real object u is + ve

$$\Rightarrow \quad \frac{\mu_2}{v} = \frac{(\mu_2 - \mu_1)}{R} + \frac{\mu_1}{u}$$

$\because$ R is + ve and $\mu_1 > \mu_2$

$$\Rightarrow \quad \frac{\mu_2}{v} = -\text{ve} + \frac{\mu_1}{u}$$

$$\Rightarrow \quad v = -\text{ve}$$

Hence, if $\mu_1 > \mu_2$ then these cannot be real image of real object.

Hence, correct option is (a).

41. At oil-concave surface

$$\frac{1}{f_1} = (1.6 - 1)\left[\frac{1}{\infty} + \frac{1}{10}\right] \Rightarrow \frac{1}{f_1} = \frac{0.6}{10}$$

At other surface light goes from oil to glass

$$\frac{1}{f_2} = \left(\frac{1.5}{1.6} - 1\right)\left[-\frac{1}{10} - \frac{1}{20}\right]$$

$$\frac{1}{f_2} = \frac{0.3}{20 \times 1.6}$$

Let focal length of combination is F

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{0.6}{10} + \frac{0.3}{20 \times 1.6}$$

$$\Rightarrow \quad F = 28.57 \text{ cm}$$

Hence correct option is (d).

42. Using formula

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

For real image μ is –ve

$$\Rightarrow \quad \frac{\mu_2}{v} + \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\Rightarrow \quad \frac{1.5}{v} = \frac{0.5}{R} - \frac{1}{u}$$

$$\Rightarrow \quad \frac{1.5}{v} = \frac{1}{2R} - \frac{1}{u} \quad v \text{ is + ve if } u > 2R$$

Hence correct option is (b).

43. $v = \frac{fu}{f - u}$ when $u < f$

u lend to lens

$\Rightarrow$ if $\quad u \to 0, v \to 0$

Hence correct option is (d).

44. Since image formed by diverging lens is always virtual.

Hence correct option is (a)

45. If the object place at first focus the image forms at ∞ *ie*, rays incident the plane surface normally and retrace its path.

Now $x = f_1 = \frac{-\mu_1 R}{\mu_2 - \mu_1} - \frac{-60}{0.5} = -120$ cm

Hence correct option is (a).

46. $\frac{\mu_2}{v_1} - \frac{\mu_1}{u_1} = \frac{\mu_2 - \mu_1}{R}$ for $\mu_2 = 1.6$

$$\frac{1.6}{v_1} - \frac{1}{(-2)} = \frac{1.6 - 1}{1} = \frac{6}{10}$$

$$\Rightarrow \quad \frac{1.6}{v_1} = \frac{6}{10} - \frac{1}{2} = \frac{1}{10}$$

$$\Rightarrow \quad v_1 = 16 \text{ m}$$

For $\quad \mu_2 = 2$

$$\frac{2}{v_2} - \frac{1}{(-2)} = \frac{2-1}{1} = 1$$

$$\frac{2}{v_2} = 1 - \frac{1}{2} = \frac{1}{2} \Rightarrow v_2 = 4 \text{ m}$$

Hence separation between images $= v_1 - v_2$

$= (16 - 4)$ cm

$= 12$ cm

Hence correct option is (a).

47. System behave a a concave mirror

Here $u = -10, v = -40$ cm

Using mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \text{ we get}$$

$$\frac{-1}{40} - \frac{1}{10} = \frac{1}{f}$$

$\Rightarrow \quad f = -8$ cm

Hence correct option is (a).

48. The focal length of lens combination is $2f$

Hence $\frac{1}{2f} = \frac{(1.8 - 1.2)}{(1.2 - 1)} - \left[\frac{1}{\infty} + \frac{1}{R}\right]$

$$\frac{1}{16} = \frac{0.6}{0.2R}$$

$\Rightarrow \quad R = 48$ cm

Hence correct option is (a).

49. If plane surface is silvered the system acts a concave mirror having focal length

$$= \frac{R}{2} = 24 \text{ cm}$$

Hence correct option is (c).

■ More than one options correct

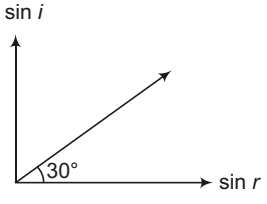
1. Since prism are identical hence if a right prism produce deviation δ inverted prism produce deviation $-\delta$.

if $n = 2m$ deviation becomes zero.

if $n = 2m + 1$ deviation produce is δ

Hence correct options are (a) and (b).

2.



$\because \sin i = \mu \sin r$

$\Rightarrow \quad \mu = \tan 30° = \sqrt{3}$

if speed of light in medium x is v

then speed of light; in medium $y = \frac{v}{\mu} = \frac{v}{\sqrt{3}}$

Since y is denser w.r.t. x hence total internal reflection take place when incidence in y.

The correct options are (b) and (d).

3. The correct options are (b) and (c).

4. The lens form real image if $D \geq 4F$ (displacement method)

and $\quad f = \frac{D^2 - x^2}{4D}$

and the magnification $m_1 m_2 = 1$

Hence correct options are (b), (c) and (d).

5. Deviation produced by prism

$\delta = (\mu - 1) A = (1.5 - 1) 4° = 2°$

if the mirror is rotated $\theta = 2°$ ray become horizontal after reflection from mirror.

Again if mirror is rotated by 1° reflection ray deviated by 2° from horizontal and after passing through prism again ray become horizontal.

Hence correct option are (a) and (b).

■ Match the Columns

1. Correct match is

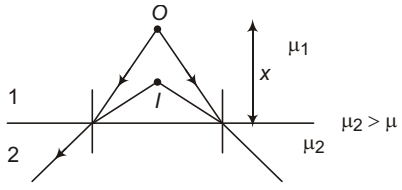
(a) $\rightarrow q, r$ (b) $\rightarrow p, s$

(c) $\rightarrow p, r$ (d) $\rightarrow p, r$

2. (a) $\rightarrow p, r$ (b) $\rightarrow q, s$

(c) $\rightarrow q, r$ (d) $\rightarrow q, r$

3.



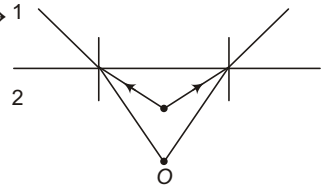
Hence image distance is less than x and virtual.

$\Rightarrow$ (a) $\rightarrow q, s$

Similarly as in (a) the correct match for

(b) $\rightarrow q, r$

(c) $\rightarrow$



(c) $\rightarrow p, s$

(d) p, r

4. (a) $\rightarrow q$,

(b) $\rightarrow r$

(c) $\rightarrow r$

(d) $\rightarrow p$

5. Since $\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

here $n = 1.5$ for (a) $n = 1.4$

$$\Rightarrow \quad \frac{1}{f} = \left(\frac{1.5}{1.4} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$\Rightarrow$ f is + ve

hence

(a) > $p, s \because f$ increases, power decreases

(b) $\rightarrow q, s \because f$ is – ve and increase in magnification

Similarly

(c) $\rightarrow q, s$ and

(d) $\rightarrow p, s$

6. For real object at $2c$ convex lens form image at $2c$ similarly for virtual object concave lens does.

(a) $\rightarrow q, s$ (b) $\rightarrow q, r$

(c) $\rightarrow q, r$ (d) $\rightarrow q, r$

28 Interference and Diffraction of Light

Introductory Exercise 28.1

1. Because they are incoherent *ie*, $\Delta\phi$ does not remain constant.

2. Since laser is highly coherent and monochromatic source of light

3. $I = I_0 \cos^2 \theta/2$

$$\Rightarrow \quad \frac{3I_0}{4} = I_0 \cos^2 \theta/2$$

$$\Rightarrow \quad \cos\theta/2 = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \frac{\theta}{2} = \frac{\pi}{6}$$

$$\Rightarrow \quad \theta = \frac{\pi}{3}$$

$$\Rightarrow \quad \Delta x = \frac{\lambda}{2\lambda} \times \phi = \frac{\lambda}{2\pi} \times \frac{\pi}{3} = \frac{\lambda}{6}$$

But $\quad \Delta x = \dfrac{yd}{D}$

$$\Rightarrow \quad y = \frac{D\,\Delta x}{d} = \frac{D}{d} \times \frac{\lambda}{6}$$

$$\Rightarrow y = \frac{1.2 \times 600 \times 10^{-9}}{0.25 \times 10^{-2} \times 6} = 48\ \mu\text{m}$$

4. $2\mu t = \left(n - \dfrac{1}{2}\right)\lambda$ for minimum thickness $n = 1$

$$\Rightarrow \quad t = \frac{\lambda}{4\mu} = \frac{3}{4 \times 1.5} = 0.5\ \text{cm}$$

5. Here $a_1 = 3\,a$ and $a_2 = a$

$$R^2 = (3a)^2 + (a)^2 + 2 \times 3a \times a \times \cos\theta$$

$$\Rightarrow I = 9I_0 + I_0 + 6I_0 \cos\theta$$

$$\Rightarrow I = [10 + 6\cos\theta]I_0$$

$$I = \left[10 + 6 \times \left[2\cos^2\frac{\theta}{2} - 1\right]\right]I_0$$

$$= \left[10 + 12\cos^2\frac{\theta}{2} - 6\right]I_0$$

$$= \left[4 + 12\cos^2\frac{\theta}{2}\right]I_0$$

$$= 4I_0\left[1 + 3\cos^2\frac{\theta}{2}\right]$$

Now, $I_{\max} = \dfrac{I_0}{9}$

$$\Rightarrow \quad I = \frac{4}{9}I_{\max}\left[1 + 3\cos^2\frac{\theta}{2}\right]$$

6. $\Delta x = \dfrac{yd}{D} - (\mu - 1)\,t$

if $\quad t = \dfrac{\lambda}{2(\mu - 1)}$

$$\Rightarrow \quad \Delta x = \frac{yd}{D} - \frac{\lambda}{2}$$

For maxima $\Delta x = n\lambda$

$$\Rightarrow \quad \frac{yd}{D} = \left(2n + \frac{1}{2}\right)\lambda$$

This become minima.

For minima $\Delta x = \left(n - \dfrac{1}{2}\right)\lambda$

$$\frac{yd}{\Delta} - \frac{\lambda}{2} = n\lambda - \frac{\lambda}{2}$$

$\Rightarrow \dfrac{yd}{D} - n\lambda$ this become maxima.

Hence maxima and minima are interchanged.

7. For two slit experiment

$$d \sin\theta = n\lambda$$

$$\Rightarrow \quad \sin\theta = \frac{n\lambda}{d}$$

But $\quad \sin\theta \le 1 \Rightarrow \frac{n\lambda}{\delta} \le 1$

$$\Rightarrow \quad n \le \frac{d}{\lambda} \Rightarrow n \le \frac{4\times10^{-6}}{6\times10^{-7}} = 6.67$$

$$\Rightarrow \quad n = 6$$

8. Since amplitude of each wave is equal. The amplitude of resultant wave is zero if waves are equally displaced in phase

ie, $\quad \theta = \frac{360°}{8} = 45°$

Hence phase difference must be 45°

AIEEE Corner

■ Subjective Question (Level-1)

1. $R^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\phi$

(i) $R = 2a,\ a_1 = a_2 = a$

$4a^2 = a^2 + a^2 + 2a^2\cos\phi$

$\Rightarrow \cos\phi = 1 \Rightarrow \phi = 0°$

(ii) $2a^2 = 2a^2 + 2a^2\cos\phi$

$\Rightarrow \phi = 90°$

(iii) $a^2 = 2a^2 + 2a^2\cos\phi$

$\Rightarrow \cos\phi = \frac{-1}{2} \Rightarrow \phi = 120°$

(iv) $0 = 2a^2 + 2a^2\cos\phi \Rightarrow \phi = 180°$

2. $$\frac{I_{max}}{I_{min}} = \frac{(a_1+a_2)^2}{(a_1-a_2)^2} = \frac{\left(\frac{a_1}{a_2}+1\right)^2}{\left(\frac{a_1}{a_2}-1\right)^2}$$

$$\Rightarrow \quad \frac{I_{max}}{I_{min}} = \frac{\left(\frac{5}{3+1}\right)^2}{\left(\frac{5}{3-1}\right)^2} = \frac{8^2}{2^2} = 16$$

$$I_{max} : I_{min} = 16:1$$

3. $I = I_{max}\cos^2\frac{\theta}{2}$

$$\Rightarrow \quad \frac{I_{max}}{2} = I_{max}\cos^2\frac{\theta}{2}$$

$$\Rightarrow \quad \cos\frac{\theta}{2} = \cos\frac{\pi}{4}$$

$$\Rightarrow \quad \theta = \frac{\pi}{2}$$

Now, $\quad \phi = \frac{2\pi}{\lambda}\times\Delta x$

$$\frac{\pi}{2} = \frac{2\pi}{\lambda}\times\Delta x$$

$$\Rightarrow \quad \Delta x = \frac{\lambda}{4}$$

But $\quad \Delta x = \frac{yd}{D}$

$$\Rightarrow \quad y = \frac{D\,\Delta x}{d}$$

$$\Rightarrow \quad y = \frac{1\text{m}\times\lambda}{4\times1\text{mm}} = \frac{1\times500\times10^{-9}}{4\times10^{-3}}$$

$$\Rightarrow \quad y = 1.25\times10^{-4}\ \text{m}$$

4. $I = I_{max}\cos^2\frac{\theta}{2}$

(a) $\frac{I_{max}}{2} = I_{max}\cos^2\frac{\theta}{2} \Rightarrow \theta = \frac{\pi}{2}$

Now $\theta = \frac{2\pi}{\lambda}\times\Delta x$

$\Rightarrow \Delta x = \frac{\lambda}{4}$ and $\Delta x = \frac{yd}{D} \Rightarrow \ = \frac{D\lambda}{4d}$

(b) $\frac{1}{4}I_{max} = I_{max}\cos^2\frac{\theta}{2}$

$\Rightarrow \cos\frac{\theta}{2} = \cos\frac{\pi}{3} \Rightarrow \quad \theta = \frac{2\pi}{3}$

$$\phi = \frac{2\pi}{\lambda}\times\Delta x$$

$$\Rightarrow \frac{2\pi}{3} = \frac{2\pi}{\lambda} \times \Delta x \Rightarrow \Delta x = \frac{\lambda}{3}$$

Now $\Delta x = \frac{yd}{D} \Rightarrow y = \frac{D\lambda}{3d}$

5. (a) $I = I_{\max} \cos^2 \frac{\theta}{2}$

$I_{\max} = I_0$ and $\theta = 60°$

$$\Rightarrow \quad I = I_0 \cos^2 30° = \frac{3}{4} I_0 = 0.75 I_0$$

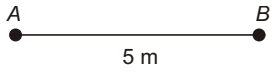
(b) $\theta = \frac{2\pi}{\lambda} \times \Delta x$

$$\Rightarrow \quad \frac{\pi}{3} = \frac{2\pi}{\lambda} \times \Delta x$$

$$\Rightarrow \quad \Delta x = \frac{\lambda}{6} = \frac{480}{6} \text{ nm}$$

$$\Rightarrow \quad \Delta x = 80 \text{ nm}$$

6. $\lambda_A = \lambda_B = 6\text{m}$



For constructive interference difference wavelength $= 0, \lambda, 2\lambda$

$\because \quad \lambda = 6 > d_{AB} = 5$

Hence only constructive interference occur at $\Delta\lambda = 0$

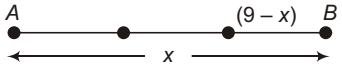
$$\Rightarrow \quad x = \frac{5}{2}\text{m} = 2.5 \text{ m}$$

For destructive interference $\Delta\lambda = \lambda, \frac{3\lambda}{2}$

Only possibility at $\Delta\lambda = 6$

Which occur at $x = 1$ m and $x = 4$ m from A.

7. The wavelength $\lambda = \frac{c}{n} = \frac{3 \times 10^8}{120 \times 10^6} = 2.5$ m



For constructive interference

$x - (9 - x) = n\lambda$, where $n = 0, 1, 2 \ldots$

$x = 4.5, 5.75, 7, 8.25$

The other points are 3.25, 2, .75

8. In Young's double slit experiment

$$w = \frac{D\lambda}{d}$$

$$\Rightarrow \quad 2.82 \times 10^{-3}\text{m} = \frac{2.2 \times \lambda}{.460 \times 10^{-3}}$$

$$\Rightarrow \quad \lambda = 590 \text{ nm}$$

9. x_n for bright fringe is given by

$$x_n = \frac{nD\lambda}{d}$$

$$\Rightarrow \quad x_1 = \frac{D\lambda}{d} \text{ and } x_2 = \frac{2D\lambda}{d}$$

$\Delta x = \frac{D\lambda}{d}$ the angular separation

$$\sin(\Delta\theta) = \frac{\Delta x}{D} = \frac{\lambda}{d} = \frac{5 \times 10^{-7}}{2 \times 10^{-3}} = 2.5 \times 10^{-4}$$

$$(\Delta\theta) = \sin^{-1}[0.00025]$$

$$= 0.014°$$

10. When whole appratus is immersed in water

$$\lambda = n_w \lambda = \frac{4}{3} \times 700 \times 10^{-9}\text{m}$$

$$w = \frac{D\lambda}{d} = \frac{48 \times 10^{-2} \times 4 \times 7 \times 10^{-7}}{3 \times 25 \times 10^{-5}} = 0.90 \text{ mm}$$

11. $w_1 = \frac{D_1\lambda}{d}$ and $w_2 = \frac{D_2\lambda}{d}$

$$\Rightarrow \quad w_1 - w_2 = \frac{(D_1 - D_2)\lambda}{d}$$

$$\Rightarrow \quad 3 \times 10^{-5} = \frac{1.5 \times 10^{-2}}{10^{-3}} \times \lambda$$

$$\Rightarrow \quad \lambda = \frac{3}{1.5} \times \frac{10^{-8}}{10^{-2}} = 2 \times 10^{-6}\text{m} = 2\mu\text{m}$$

12. For bright fringe $x_n = \frac{Dn\lambda}{d}$

For first light ($\lambda = 480$ nm) the third order Bright fringe is $x_3 = \frac{1 \times 3 \times 480 \times 10^{-9}}{5 \times 10^{-3}}$

For second light ($\lambda = 600$ nm)

$$x_3' = \frac{1 \times 3 \times 600 \times 10^{-9}}{5 \times 10^{-3}}$$

$$\Delta x = x_3' - x_3 = \frac{3 \times 10^{-9} \times (600 - 480)}{5 \times 10^{-3}}$$

$$= \frac{3 \times 120}{5} \times 10^{-6}$$

$$\Delta x = 72 \times 10^{-6}, \Delta x = 72\,\mu\text{m}$$

13. Fringe width :

$$\omega = \frac{\lambda D}{d}$$

$$= \frac{(500 \times 10^{-9})(75 \times 10^{-2})}{(0.45 \times 10^{-3})}\text{ m}$$

$$= 0.83 \times 10^{-3}\text{ m} = 0.83\text{ mm}$$

Distance between second and third dark line = one fringe width = 0.83 mm.

14. For first order bright fringe

$$x = \frac{D\lambda}{d}$$

$$\Rightarrow \quad 4.94 \times 10^{-3} = \frac{3 \times 600 \times 10^{-9}}{d}$$

$$\Rightarrow \quad d = \frac{18 \times 10^{-7}}{4.94 \times 10^{-3}} = \frac{18}{5.94} \times 10^{-4}\text{m}$$

Let for wavelength λ first dark fringe is obtained at this point for first dark fringe

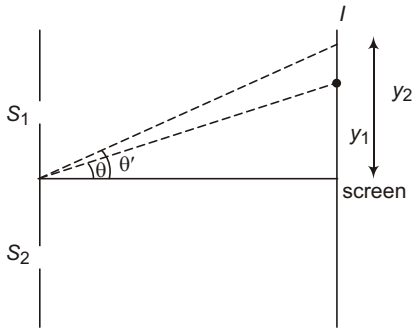
$$x = \frac{D\lambda}{2d}$$

$$\Rightarrow 4.94 \times 10^{-3} = \frac{3 \times \lambda \times 4.94}{2 \times 18 \times 10^{-4}}$$

$$\Rightarrow 2 \times 18 \times 10^{-7} = 3\lambda$$

$$\Rightarrow \quad \lambda = 12 \times 10^{-7}\text{m}\ \lambda = 1200\text{ nm}$$

15.



For dark fringe

$$d \sin\theta = (2n - 1)\frac{\lambda}{2}$$

For first dark fringe $n = 1$ and for 2nd, $n = 2$

$$d \sin\theta = \frac{\lambda}{2}$$

$$d \sin\theta' = \frac{3\lambda}{2} \Rightarrow \sin\theta = \frac{\lambda}{2d}$$

$$\Rightarrow \quad \frac{y_1}{\sqrt{D^2 + y_1^2}} = \frac{\lambda}{2d}$$

and $$\frac{y_2}{\sqrt{D^2 + y_1^2}} = \frac{3\lambda}{2d}$$

$$\Rightarrow \quad \frac{y_1}{\sqrt{(35 \times 10^{-2})^2 + y_1^2}} = \frac{550 \times 10^{-9}}{2 \times 1.8 \times 10^{-6}} \quad \ldots\text{(i)}$$

and $$\frac{y_2}{\sqrt{(35 \times 10^{-2})^2 + y_2^2}} = \frac{3 \times 550 \times 10^{-9}}{2 \times 1.8 \times 10^{-6}}$$

On solving $y_2 - y_1 = 12.6$ cm

16. Wavelength of source $\lambda = \dfrac{\lambda}{\sqrt{2meV}}$

$$\Rightarrow \lambda = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 100 \times 1.6 \times 10^{-19}}}$$

$$\Rightarrow \lambda = 1.24 \times 10^{-10}\text{m}$$

$$w = \frac{D\lambda}{d} = \frac{3 \times 1.24 \times 10^{-10}}{10 \times 10^{-10}}\text{m} = 36.6\text{ cm}$$

17. Given $\lambda = 546$ nm $= 5.46 \times 10^{-7}$m, $D = 1$m

$d = 0.3$ mm $= 0.3 \times 10^{-3}$m

(a) At distance $y = 10$ mm $= 10 \times 10^{-3}$m from the central fringe path difference will be

$$\Delta y = \frac{y.d}{D} = \frac{10 \times 10^{-3} \times 0.3 \times 10^{-3}}{1}$$

$$= 3 \times 10^{-6}\text{ m}$$

Corresponding phase difference

$$\phi = \frac{2\lambda}{\lambda} \times \Delta x = \frac{2\pi}{5.46 \times 10^{-7}} \times 3 \times 10^{-6}\text{ rad}$$

$$= 1978°$$

$$I = I_0 \cos^2 \frac{\theta}{2}$$

$$= I_0 \cos^2 \left(\frac{1978°}{2}\right) = I_0 \cos^2 989°$$

$$I = 3 \times 10^{-4} I_0$$

(b) Fringe width $w = \frac{\lambda D}{d}$

$$w = \frac{5.46 \times 10^{-7} \times 1}{0.3 \times 10^{-3}} = 1.82 \text{ mm}$$

$$\text{Number of fringes} = \frac{10 \text{ mm}}{1.82 \text{ mm}} = 5.49$$

Hence the number of fringe is five.

18. Shift due to sheet of thickness 10μ and refractive index 1.6 is

$$\Delta x_1 = \frac{(\mu - 1)\, tD}{d}$$

$$\Rightarrow \Delta x_1 = (1.6 - 1) \times \frac{10 \times 10^{-6} \times 1.5}{1.5 \times 10^{-3}}$$

$$\Rightarrow \Delta x_1 = 0.6 \times 10 \times 10^{-3} = 6 \times 10^{-3} \text{m}$$

Shift due to sheet of thickness 15μ and refractive index 1.2 is

$$\Delta x_2 = \frac{(1.2 - 1) \times 15 \times 10^{-6} \times 1.5}{1.5 \times 10^{-3}}$$

$$\Delta x_2 = 3 \times 10^{-3} \text{m}$$

Since these shifts are in opposite direction of central maxima hence net shift

$$\Delta x = \Delta x_1 - \Delta x_2 = 6 \times 10^{-3} \text{m} - 3 \times 10^{-3} \text{m}$$

$$= 3 \times 10^{-3} \text{m} = 3 \text{ mm}$$

19. Let λ is the wavelength of light D is screen distance from source and d is the separation between slits (all are in metres)

$$\text{Shift} = \Delta x = \frac{(\mu - 1)\, tD}{d}$$

$$\Rightarrow \quad \Delta x = \frac{(1.6 - 1) \times 1.964 \times 10^{-6} \times D}{d}$$

$$\Rightarrow \quad \Delta x = \frac{1.1784\, D}{d} \times 10^{-6} \text{m}$$

Now when t is removed and D is doubled the distance between successive maximum (or minima) *i.e.*, fringe width

$$w = \frac{2D\lambda}{d}$$

but according to question $\Delta x = w$

$$\Rightarrow \quad \frac{1.178 \times 10^{-6} D}{d} = \frac{2D\lambda}{d}$$

$$\Rightarrow \quad \lambda = 0.589 \times 10^{-6} \text{m} = 589 \text{ nm}$$

20. Let n bright fringe ($\lambda = 5500$ Å) concide with 10th

bright fringe of 6000 Å

$$\Rightarrow \quad n \times 5500 \text{ Å} = 6000 \text{ Å} \times 10$$

$$\Rightarrow \quad n \simeq 11$$

Similarly first bright fringe concide with 1st fringe. Now fringe width

$$w = \frac{14.74 - 12.5}{10} = 0.224 \text{ mm}$$

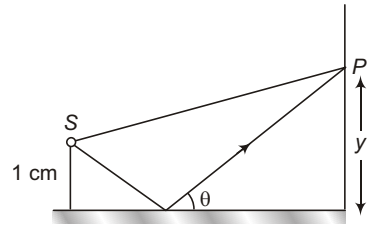
Hence position of 10th bright fringe

$$= 14.74 - 0.224 \simeq 14.55 \text{ mm}$$

Position of zero order bright fringe

$$= 12.75 - 0.224 \simeq 12.25 \text{ mm}$$

21. Here $d \simeq 1$ cm



$$D = 100 \text{ m}$$

$$\lambda = 500 \text{ nm}$$

For first dark fringe

$$y = \frac{\Delta\lambda}{2d} = \frac{100 \times 500 \times 10^{-9}}{2 \times 10^{-2}}$$

$\Rightarrow \quad y = 2.5\,\text{mm}$

22. The destructive interference will be $2\mu t = n\lambda$ for thinnest $n = 1$

$\Rightarrow \quad 2\mu t = \lambda$

$\Rightarrow \quad t = \frac{\lambda}{2\mu} = \frac{650 \times 10^{-9}}{2 \times 1.42}$

$\Rightarrow \quad t = 114$ nm

23. Here $t = 0.485\,\mu\text{m}$, $t = 485$ nm, $n = 1.53$

(a) Condition for constructive interference in refractive system

$$2\mu t = \left(n - \frac{1}{2}\right)\lambda \quad \text{where, } n = 1, 2$$

For $\quad n = 1, \lambda_1 = 4\mu t = 4 \times 1.53 \times 485$

$= 2968.2$ mm

which does not lie in visible region put

$n = 2, 3, \ldots\ldots$

we get $\lambda = 424$ nm, 594 nm, …

(b) For structive interference in transmitted system

$2\mu t = n\lambda$, putting $n = 1, 2,$

only $\lambda = 495$ nm is lie in visible region.

24. For constructive interference

$$2\mu t = \left(n - \frac{1}{2}\right)\lambda \text{ for } t \text{ to be minimum } n = 1$$

$\Rightarrow \quad t = \frac{\lambda}{4\mu} = \frac{6000}{4 \times 1.3} = 1154$ Å

25. For destructive interference

$2\mu t = n\lambda$ for t to be minimum

$n = 1$

$\Rightarrow \quad t = \frac{\lambda}{2\mu} = \frac{800}{2\mu} \quad \ldots(i)$

For constructive interference

$2\mu t = (2n - 1)\frac{\lambda}{2}$

$2\mu \times \frac{800}{2\mu} = (2n - 1)\frac{\lambda}{2}$

$1600 = (2n - 1)\lambda$

For $n = 1$, $\lambda = 600$ nm which does not lie in visible region

For $\quad n = 2$

$1600 = 3\lambda$

$\Rightarrow \quad \frac{1600}{3} = \lambda \Rightarrow \lambda = 533$ nm

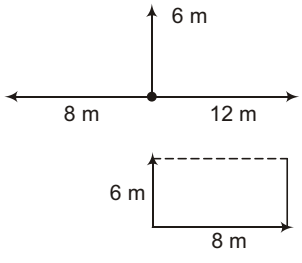
for $n = 3$, $\lambda = \frac{1600}{5} = 320$ nm which does lie in visible

Hence $\lambda = 533$ nm

Objective Questions (Level-1)

■ (Single option correct)

1. Using phasor method



$$R = \sqrt{8^2 + 6^2} = 10\,\text{m}$$

Hence correct option is (b).

2. Here $\frac{I_1}{I_2} = B^2$

$I_{max} = [\sqrt{I_1} + \sqrt{I_1}]^2$

$I_{min} = [\sqrt{I_1} - \sqrt{I_2}]^2$

$$\frac{I_{min}}{I_{max}} = \left[\frac{\sqrt{\frac{I_1}{I_2}} - 1}{\sqrt{\frac{I_1}{I_2}} + 1}\right]^2 = \left[\frac{\beta - 1}{\beta + 1}\right]^2$$

$$\Rightarrow \frac{(I_{max} - I_{min})}{(I_{max} + I_{min})} = \left[\frac{\beta + 1 - \beta + 1}{2\beta}\right]^2$$

$$\Rightarrow \frac{I_{max} - I_{min}}{I_{max} - I_{min}} = \frac{1}{\beta^2}$$

Hence, correct option is (d).

3. For nth dark fringe

$$x_n = (2n - 1)\frac{D\lambda}{2d}$$

For 1st dark fringe $n = 1$

$$\Rightarrow \quad x_1 = \frac{D\lambda}{2d}$$

Angular position $\theta = \sin^{-1}\left(\frac{x_1}{D}\right)$

$$\Rightarrow \quad \theta = \sin^{-1}\left[\frac{\lambda}{2a}\right]$$

$$\Rightarrow \quad \theta = \sin^{-1}\left[\frac{5460 \times 10^{-10}}{2 \times 0.1 \times 10^{-3}}\right]$$

$$\Rightarrow \quad \theta = \sin^{-1}[273 \times 10^{-5}]$$

$$\Rightarrow \quad \theta = \sin^{-1}[0.00273]$$

$$\Rightarrow \quad \theta = 0.16°$$

Hence correct option is (b)

4. For 10th bright fringe $x_{10} = \frac{10D\lambda}{d}$

For 6th dark fringe $x_6 = (2 \times 6 - 1)\frac{D\lambda'}{2d}$

But $\quad x_6 = x_{10}$

$$\frac{11}{2a}D\lambda' = \frac{10D\lambda}{d}$$

$$\Rightarrow \quad \frac{\lambda'}{\lambda} = \frac{20}{11}$$

But $\quad \mu = \frac{\lambda'}{\lambda} = \frac{20}{11} = 1.8$

Hence correct option is (a).

5. Shift $= \frac{(\mu - 1)tD}{d}$

$$= \frac{(1.5 - 1) \times 10 \times 10^{-6} \times 100 \times 10^{-2}}{2.5 \times 10^{-3}}$$

$$= \frac{5 \times 10^{-6}}{2.5 \times 10^{-3}} = 2 \times 10^{-3}\text{m} = 2\text{mm}$$

Hence correct option is (a).

6. $x_n = \frac{Dn\lambda}{d}$ for nth bright fringe

$x_n' = \frac{D\lambda}{2d}(2n - 1)$ for nth dark fringe

$$\Delta x = x_n - x_n' = \frac{D\lambda}{2d}[2n - 2n + 1]$$

$$\Delta x = \frac{D\lambda}{2d}$$

Hence correct option is (c).

7. Since at centre path difference for all colour is always zero hence centre will be white.

Hence correct option is (a).

8. $n_1 \times \lambda_1 = n_2 \times \lambda_2$

$$60 \times 4000 = n_2 \times 6000 \Rightarrow n_2 = 40$$

Hence correct option is (a).

9. Initial fringe width $w_1 = \frac{D\lambda}{d}$

Final fringe width $w_2 = \frac{(D - 5 \times 10^{-2})\lambda}{d}$

$$|\Delta w| = |w_2 - w_1| = \frac{\lambda}{d} \times 5 \times 10^{-2}$$

$$\Rightarrow \quad 3 \times 10^{-5} = \frac{\lambda}{10^{-3}} \times 5 \times 10^{-2}$$

$$\Rightarrow \quad \frac{3 \times 10^{-6}}{5} = \lambda \Rightarrow 6000 \text{ Å} = \lambda$$

Hence correct option is (a).

10. $\frac{I_{max}}{I_{min}} = \frac{49}{9}$

Now $\frac{I_{max}}{I_{min}} = \left[\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right]^2 = \left[\frac{\sqrt{\frac{I_1}{I_2}}+1}{\sqrt{\frac{I_1}{I_2}}-1}\right]^2$

$$\frac{I_{max}}{I_{min}} = \left[\frac{\sqrt{\frac{49}{9}}+1}{\sqrt{\frac{49}{9}}-1}\right]^2 = \left[\frac{\frac{7}{3}+1}{\frac{7}{3}-1}\right]^2 = \frac{100}{16}$$

$\Rightarrow \frac{I_{max}}{I_{min}} = \frac{25}{4}$

Hence correct option is (a)

11. $x_n = \frac{Dn\lambda}{d}$

$\Rightarrow \quad x_3 = \frac{3D\lambda}{d} = 7.5 \times 10^{-3}\text{m}$

$\Rightarrow \quad \lambda = \frac{7.5 \times 10^{-3} \times 0.2 \times 10^{-3}}{3 \times 1}$

$\Rightarrow \quad \lambda = 0.5 \times 10^{-6} = 500 \text{ nm}$

Hence correct option is (b).

12. Let nth fringe of 6500 Å concide with n'th fringe of 5200 Å.

$\Rightarrow x_n = \frac{Dn \times 6500 \text{ Å}}{d} = \frac{Dn'}{d} \times 5200 \text{ Å}$

$\Rightarrow \frac{n}{n'} = \frac{5200}{6500} = \frac{4}{5}$

It means 4th fringe of 6500 Å coincide with 5th fringe of 5200 Å hence the distance

$$x = \frac{4 \times 120 \times 10^{-2} \times 6500 \times 10^{-10}}{2 \times 10^{-3}}$$

$x = 0.156 \text{ cm}$

Hence correct option is (a).

13. Since number of minima does not depends on orientation hence $n_1 = n_2$

Hence correct option is (a).

■ Assertion and Reason

1. We have $I = 4I_0 \cos^2 \frac{\phi}{2}$ if $\phi = \frac{2\pi}{3}$

we get $I = I_0$

Hence assertion is true.

Now path difference $= \frac{\lambda}{2\pi} \times$phase difference

Path difference $= \frac{\lambda}{2\pi} \times \frac{2\pi}{3} = \frac{\lambda}{3}$

Hence reason is true.

But reason is not the explanation of assertion. Hence correct option is (b).

2. Here assertion and reason are both true but reason is not correct explanation of assertion. Correct option is (b).

3. Assertion is wrong since fringes are symmetrical *ie*, fringes obtained both above and below point O. Reason is true.

Correct option is (a).

4. Here both assertion and reason are true and reason correctly explain assertion. Hence correct option is (a).

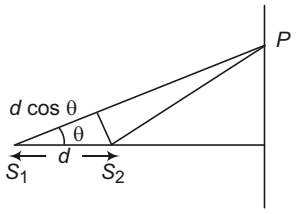
5. Both assertion and reason are true and reason correctly explain the assertion.

Hence correct option is (a).

6. Assertion is true since locus of all fringes is circle. But reason is wrong since fringes may have any shape.

Correct option is (c).

7.



Path difference at point P is $\Delta x = d\cos\theta$

The path difference decreases as θ increases.

$\because$ as $\theta \to$ increases, $\cos\theta \to$ decreases

Hence order of fringe $n = \dfrac{d\cos\theta}{\lambda}$ decreases as we go above P. Hence assertion is wrong (false).

For 11th order maxima path differenc is more hence reason is true but assertion is false correct option is (d).

8. Here assertion is true and reason is false and reason does not correctly explain assertion. Correct option is (c).

9. $\because\ d\sin\theta = n\lambda$

$\Rightarrow \sin\theta = \dfrac{n\lambda}{d}$ but $\dfrac{d}{\lambda} = 4$

$\Rightarrow\ \sin\theta = \dfrac{n}{4}$

Now if $\theta = 30°$

$\Rightarrow \qquad n = 4\sin 30° = 2$

Hence assertion is true.

Also reason is true and does not correctly explain assertion correct option is (b).

10. Here assertion is false. Since shift $= \dfrac{(\mu - 1)tD}{d}$ is independent of λ.

Hence shift of red colour = shift of violet colour.

and reason is true

$\because \qquad \mu_V > \mu_R$

Hence correct option is (d)

Objective Questions (Level 2)

1. $I = 4I_0\cos^2\dfrac{\pi x}{\alpha}$

$\Rightarrow \qquad I_0 = 4I_0\cos^2\dfrac{\lambda x_1}{\alpha}$

$\Rightarrow \qquad x_1 = \dfrac{\alpha}{3}$ ...(i)

and $\qquad 2I_0 = 4I_0\cos^2\dfrac{\pi x_2}{\alpha}$

$\Rightarrow \qquad x_2 = \dfrac{\alpha}{4}$ ...(ii)

$\Delta x = x_1 - x_2 = \dfrac{\alpha}{3} - \dfrac{\alpha}{4} = \dfrac{\alpha}{12}$

Hence correct option is (c).

2. $I = 4I_0\cos^2\left(\dfrac{\pi x}{\lambda}\right) = k\cos^2\left(\dfrac{\pi x}{\lambda}\right)$

$I = k\cos^2\left(\dfrac{\pi}{\lambda}\times\lambda\right) = k$ ...(i)

$\dfrac{I}{4} = k\cos^2\left(\dfrac{\pi x}{\lambda}\right)$

$\Rightarrow \dfrac{k}{4} = k\cos^2\left(\dfrac{\pi x}{\lambda}\right)$

$\Rightarrow \qquad \cos\dfrac{\pi x}{\lambda} = \dfrac{1}{2}$

$\dfrac{\pi x}{\lambda} = \cos^{-1}\left(\dfrac{1}{2}\right)$

$\Rightarrow \qquad \dfrac{\pi x}{\lambda} = \dfrac{\pi}{3}$ or $\dfrac{\pi x}{\lambda} = \dfrac{2\pi}{3}$

$\Rightarrow \qquad x = \dfrac{\lambda}{3}$ or $x = \dfrac{2\lambda}{3}$

Hence correct option is (b).

3. Light of wavelength λ is strongly reflected if

$2ut = \left(n + \dfrac{1}{2}\right)\lambda \quad n = 0, 1, 2$ ...(i)

$\Rightarrow 2ut = 2\times 1.5\times 5\times 10^{-7}\,\text{m} = 1.5\times 10^{-6}\,\text{m}$

Putting $\lambda = 400$ nm in eq. (1) and using eq. (ii)

$$1.5 \times 10^{-6}\ \text{m} = \left(n + \frac{1}{2}\right) \times 4 \times 10^{-7}\ \text{m}$$

$\Rightarrow \quad n = 3.25$

Putting $\quad \lambda = 700$

$$1.5 \times 10^{-6}\ \text{m} = \left(n + \frac{1}{2}\right) \times 7 \times 10^{-7}$$

$$n = 1.66$$

Hence n can take values 2 and 3.

From (i) if $n = 2$, $\lambda = \dfrac{4ut}{2 \times 2 + 1} = 600$ nm

if $\quad n = 3\ \lambda = 429$ nm

only $\lambda = 600$ is given in the options.

Hence correct option is (b).

4. $x_n = \dfrac{D\lambda n}{d} = \dfrac{100 \times 10^{-2}}{0.01 \times 10^{-3}} \lambda \times n$

$\Rightarrow \quad x_n = \dfrac{n\lambda}{10^{-5}} \qquad$ For $\lambda = 4000$ Å

$$x_n = \frac{n \times 4000 \times 10^{-10}\text{m}}{10^{-5}} = 4 \times 10^{-5} \times n$$

$$= 0.04 \times n$$

$\Rightarrow x_n = 4n$ mm

Similarly for $\lambda = 7000$ Å

$x_n = 7n$ mm $n = 5, 6$

hence only $x = 5$

Passes through hole $\lambda = 5000$ Å

Hence correct option is (b).

5. $I = 4I_0 \cos^2\left(\dfrac{\pi x}{\alpha}\right)$ where $\alpha = \dfrac{\Delta\lambda}{d}$

$\Rightarrow \alpha = \dfrac{1 \times 6000 \times 10^{-10}}{1 \times 10^{-3}} = (6 \times 10^{-4})$

$$75\%\ I_0 = I_0 \cos^2\left(\frac{\pi x}{\alpha}\right)$$

$$\frac{3}{16} = \cos^2 \frac{\pi x}{\alpha}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{4} = \cos \frac{\pi x}{\alpha}$$

$$\pi x = \alpha \cos^{-1}\left(\sqrt{\frac{3}{4}}\right) \Rightarrow x = \frac{\alpha}{\pi} \cos^{-1}\left(\sqrt{\frac{3}{4}}\right)$$

$\Rightarrow \quad x = 0.20$ mm

Hence correct option is (d).

6. Number of fringes shifted $= \dfrac{(\mu - 1)\, t}{\lambda}$

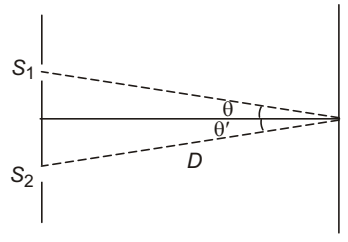
$$4 = \frac{(1.5 - 1) \times t}{6000\ \text{Å}}$$

$$\Rightarrow \quad \frac{4}{0.5} \times 6 \times 10^{-7}\text{m} = t$$

$\Rightarrow \quad t = 4.8\ \mu$m

Hence correct option is (a).

7. For nth order minima



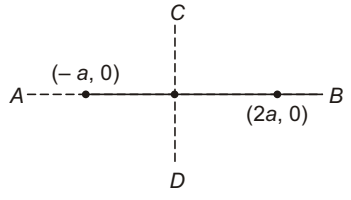
$y_n = \dfrac{(2n - 1)\, D\lambda}{2d}$ for 3rd minima $n = 3$

$$y_3 = \frac{5D\lambda}{2d} = \frac{5\lambda}{2\theta}$$

$\because \quad \theta \simeq \tan\theta = \dfrac{d}{D}$

Hence correct option is (b).

8. $AB = d = 3a = \dfrac{3a \times 5}{5}$



$AB = 15\lambda$

Hence total maxima $= 14 \times 4 + 4$

$= 60°$

Hence correct option is (a).

9. $I = 4I_0 \cos^2\left[\frac{\pi x}{\alpha}\right]$

$\Rightarrow I = 4I_0 \cos^2\left[\frac{\pi}{\alpha} \times \frac{\alpha}{4}\right]$

$I = 4I_0 \cos^2 \frac{\pi}{4}$

$= \frac{4I_0}{2} = 2I_0$

$\Rightarrow \frac{I_0}{I} = \frac{1}{2}$

Hence correct option is (b).

10. $I = I_0 \cos^2\left[\frac{\pi}{\alpha}(\mu - 1)t\right]$

at $\mu = I \Rightarrow I = I_0$

Hence correct variation is (c)

11. $I = I_0 \cos^2\left(\frac{\pi x}{\lambda}\right)$

$\frac{3}{4} I_0 = I_0 \cos^2\left(\frac{\pi x}{\lambda}\right)$

$\Rightarrow \frac{\pi x}{\lambda} = \frac{\pi}{6}$

$\Rightarrow x = \frac{\lambda}{6}$ but $x = (\mu - 1)t$

$\Rightarrow (1.5 - 1) \times t = \frac{\lambda}{6}$

$\Rightarrow t = \frac{\lambda}{3} = \frac{6000}{3}$ Å

$\Rightarrow t = 2000$ Å $= 0.2\,\mu$m

Hence correct option is (a).

12. Net shift $= (\mu_1 - 1)t - (\mu_2 - 1)t$

$= (\mu_1 - \mu_2)t = (1.52 - 1.40) \times t$

Net shift $= 0.12 \times 10.4\ \mu$m

$= 12 \times 104 \times 10^{-9}$m

$= 1248 \times 10^{-9}$m

$= 1248$ nm

Net shift $= n\lambda$

where n is + ve integer

$\Rightarrow n\lambda = 1248$

Now, for $416 = \lambda$, $n = 3$

For $624 = \lambda$, $n = 2$

Hence correct option is (c).

13. $w = \frac{\lambda D}{d \times n} = \frac{6300\text{ Å} \times 1.33\text{ m}}{1\text{ mm} \times 1.33}$

$= \frac{63 \times 10^{-8} \times 1.33\text{ m}}{10^{-3} \times 1.33}$

$= 0.63$ mm

Hence correct option is (a).

14. $\Delta x = \frac{7D\lambda}{nd} - \frac{3D\lambda}{nd} = 4\left(\frac{D\lambda}{nd}\right)$

$\Delta x = 4 \times 0.63$ mm $= 2.52$ mm

Correct option is (a).

15. $\Delta x = 2$ of fridge width

$(\mu - 1)t = \frac{2 \times 0.63\text{ mm}}{2}$

$t = \frac{2 \times 0.63}{0.53} = 1.57$ mm

Hence correct option is (b).

16. Since on introducing thin glass sheet fringe width does not change hence fringe width $= 0.63$ mm.

The correct option is (a).

More than one options correct

1. At centre path difference between all colours is zero hence cnetre is white since violet has least wavelength hence next to central will be violet and since intensity is different for different for colours hence there will be not a completely dark fringe

   Hence correct options, are (b), (c) and (d).

2. Correct options are (a), (c) and (d).

3. $\because I = 4I_0 \cos^2 \frac{\theta}{2}$ at centre $\theta = 0 \Rightarrow I = 4I_0$

   and at distance 4 mm above o is again maxima hence its intensity is also $4I_0$.

   Hence correct options are (a) and (c).

4. The correct options, are (a), (c) and (d)

   Since phase difference = constant

   Light should be monochromatic.

5. Since in this case fringe pattern shift upward hence the correct options are (a), (b) and (c).

6. $\because n_1\lambda_1 = n_2\lambda_2$ for maxima

   Using this option (a) is satisfied and $(2n_1 - 1)\lambda_1 = (2n_2 - 1)\lambda_2$ for minima

   Using this 3rd option is satified

   Hence correct options are (a) and (c).

Match the Columns

1. $\because I = 4I_0 \cos^2 \frac{\theta}{2}$

   if $\theta = 60°$ $I = 3I_0$

   if $\theta = 90°$ $I = 2I_0$

   if $\theta = 0°$ $I = 4I_0$

   if $\theta = 120°$ $I = I_0$

   Hence correct match is

   (a) $\rightarrow s$; (b) $\rightarrow q$; (c) $\rightarrow p$; (d) $\rightarrow r$

2. $\Delta x = \frac{\lambda}{2\pi} \times \Delta\phi$

   $\Rightarrow \Delta\phi = \frac{2\pi}{\lambda}\Delta x$ if $\Delta x = \frac{\lambda}{3}$

   $\Rightarrow \Delta\phi = \frac{2\pi}{3} = 120°$

   if $\Delta x = \frac{\lambda}{6}$

   $\Delta\phi = 60°$ if $\Delta x = \frac{\lambda}{4}$, $\Delta\phi = 90°$

   Using $I = 4I_0 \cos^2\left(\frac{\Delta\theta}{2}\right)$

   The correct match are

   (a) $\rightarrow q$; (b) $\rightarrow p$; (c) $\rightarrow r$; (d) $\rightarrow s$

3. Distance between third order maxima and central maxima $= \frac{3D\lambda}{d} = 3w$

   Distance between 3rd order minima and central

   Maxima $= \left(3 - \frac{1}{2}\right)\frac{D\lambda}{d} = 2.5\frac{D\lambda}{d} = 2.5\,w$

   Distance between first minima and forth order

   maxima $= \frac{4D\lambda}{d} = \frac{D\lambda}{2d} = 2.5\,w$

   Distance between first minima and forth order

   maxima $= \frac{4D\lambda}{d} - \frac{D\lambda}{2d} = 3.5\,w$

   Distance between 2nd order maxima and fifth order minima $= 4.5\,w - 2w = 2.5\,w$

   hence correct match is

   (a) $\rightarrow q$; (b) $\rightarrow p$; (c) $\rightarrow r$; (d) $\rightarrow p$

4. Since fringe shift in the division of sheet placed soure hence

(a) $\rightarrow p$

Similarly for other the correct match

(b) $\rightarrow r, s$; (c) $\rightarrow p$; (d) $\rightarrow p$

5. When $y = \frac{\lambda D}{2d}$ there will a dark fringe at 0.

hence (a) $\rightarrow q$

when $y = \frac{\lambda D}{6d} = 3\left(\frac{\lambda D}{2d}\right)$

The intensity becomes $3I$

(b) $\rightarrow p$

when $y = \frac{\lambda D}{4d}$ Intensity $= 2I$

(c)$\rightarrow s$

when $y = \frac{\lambda D}{3d}$ Intensity $= I$

(d) $\rightarrow r$

6. When a thin plate (transparent) is placed in front of S_1 zero order fringe shift above from O hence

(a) $\rightarrow r$

When S_1 is closed interference disappear and uniform illuminance is obtained on screen hence

(b) $\rightarrow p, q$

Similarly (c) $\rightarrow r, s$ and

When s is removed and two real sources s_1 and s_2 emitting light of same wavelength are placed interference disappear. Since sources become non-cohrrent hence (d) $\rightarrow p, q$

29. Modern Physics I

Introductory Exercise 29.1

1. The positron has same mass m as the electron. The reduced mass of electron positron atom is

$$\mu = \frac{m \times m}{m + m} = \frac{1}{2}\text{m}$$

$$R_H = \frac{me^4}{8\varepsilon_0^2 ch^3}$$

$$\Rightarrow \quad R_P = \frac{R_H}{2}$$

$$\frac{1}{\lambda_H} = R_H\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$\frac{1}{\lambda_P} = R_P\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$\Rightarrow \quad \frac{\lambda_P}{\lambda_H} = \frac{R_H}{R_P} = 2$$

$$\Rightarrow \lambda_p = 2\lambda_H = 2 \times 6563 \text{ Å} = 13126 \text{ Å}$$

$$= 1.31\,\mu\text{m}$$

$$\frac{1}{\lambda_{\text{He}}} = R_H\left(\frac{1}{2^2} - \frac{1}{3^2}\right) \cdot z^2$$

$$\Rightarrow \quad \frac{1}{\lambda_{\text{He}}} = \frac{1}{\lambda_{\text{H}}} \cdot z^2 \Rightarrow \quad \lambda_{\text{He}} = \frac{\lambda_{\text{H}}}{2^2} = \frac{6563 \text{ Å}}{2^2}$$

$$\Rightarrow \quad \lambda_{\text{He}} = 164 \text{ nm}$$

2. $\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n^2}\right]$ for largest wavelength $n = 3$

$$\Rightarrow \quad \frac{1}{\lambda} = R\left[\frac{1}{4} - \frac{1}{9}\right]$$

$$\Rightarrow \quad \lambda = \frac{36}{5R} = \frac{36}{5 \times 1.097 \times 10^7}$$

$$\Rightarrow \quad \lambda = 656 \text{ nm}$$

3. For H-atom $r_n = \frac{\varepsilon_0 n^2 h^2}{\pi me^2}$

$$u_n = \frac{e^2}{2\varepsilon_0 nh}$$

$$T_n = \frac{2\pi r_n}{u_n} = 2\pi \frac{\varepsilon_0 n^2 h^2 \times 2\varepsilon_0 nh}{\pi me^2 \times e^2}$$

$$= \frac{4\varepsilon_0 n^3 h^3}{me^4}$$

$$r_n = \frac{1}{T_n} = \frac{me^4}{4\varepsilon_0^2 n^3 h^3}$$

$$\Rightarrow \quad r_1 = \frac{me^4}{4\varepsilon_0^2 h^3}$$

$$\Rightarrow \quad \nu_1 = \frac{9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^4}{4 \times (8.85 \times 10^{-12})^2 \times (6.6 \times 10^{-34})^3}$$

$$\Rightarrow \quad \nu_1 = 6.58 \times 10^{15} \text{ Hz}$$

$$\nu_2 = \frac{\nu_1}{2^3} = \frac{\nu_1}{8} = \frac{6.58 \times 10^{15}}{8}$$

$$= 0.823 \times 10^{15} \text{ Hz}$$

(b) $\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right]$

$$\Rightarrow \quad \nu = \frac{c}{\lambda} = 3 \times 10^8 \times R\left[\frac{3}{4}\right]$$

$$\Rightarrow \nu = \frac{9 \times 10^8 \times 1.097 \times 10^7}{4}$$

$$= 2.46 \times 10^{15} \text{ Hz}$$

(c) Number of revolutions

$$= \upsilon_2 \times T = 0.823 \times 10^{15} \times 1 \times 10^{-8}$$

$$= 8.23 \times 10^6 \text{ revolution}$$

4. Reduce mass

$$= \frac{m_\mu m_p}{m_\mu + m_p} = \frac{207\,\text{m} \times 1836\,\text{m}}{(207\,\text{m} + 1836\,\text{m})} = 186\,\text{m}$$

$$r_1 = 4\pi\varepsilon_0\left(\frac{h^2}{4\pi^2\mu e^2}\right) = 4\pi\varepsilon_0\left(\frac{h^2}{4\pi^2(186m)\,e^2}\right)$$

Putting the value we get

$$r_1 = 2.55 \times 10^{-13}\text{m}$$

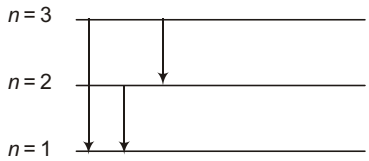
$$E_1 = \frac{-\mu e^4}{8\varepsilon_0^2 h^2} = -2810\,\text{eV}$$

Ionization energy $= -E_1 = 2.81$ keV

5. (a) $\lambda = \dfrac{h}{mv} = \dfrac{6.6 \times 10^{-34}}{46 \times 10^{-3} \times 30} = 4.8 \times 10^{-34}\text{m}$

(b) $\lambda = \dfrac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^7} = 7.3 \times 10^{-11}\text{m}$

6. (a) After absorbing 12.3 eV the atom excited to $n = 3$ state



$$\frac{1}{\lambda_L} = R\left[1 - \frac{1}{n^2}\right]$$

$$\frac{1}{\lambda_{L_1}} = R\left[1 - \frac{1}{9}\right] = \frac{8R}{9}$$

$$\Rightarrow \lambda_{L_1} = \frac{9}{8R} = \frac{9}{8 \times 1.097 \times 10^7} = 102\text{ nm}$$

$$\frac{1}{\lambda_{L_2}} = R\left[1 - \frac{1}{4}\right]$$

$$\Rightarrow \lambda_{L_2} = \frac{4}{3R} = \frac{4}{3 \times 1.097 \times 10^7} = 122\text{ nm}$$

$$\frac{1}{\lambda_B} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5R}{36}$$

$$\Rightarrow \lambda_B = \frac{36}{5R}$$

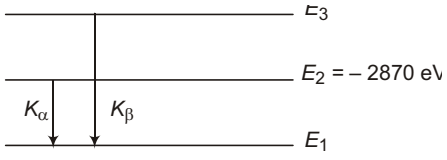
$$\Rightarrow \lambda_B = \frac{36}{5 \times 1.097 \times 10^7} = 653\text{ nm}$$

7. $\nu_{K_\beta} = \nu_{K_\alpha} + \nu_{L_\alpha} \Rightarrow \lambda_{L_\alpha} = \dfrac{\lambda_{K_\alpha} \times \lambda_{K_\beta}}{\lambda_{K_\alpha} - \lambda_{K_\beta}}$

$$\frac{c}{\lambda_{K_\beta}} = \frac{c}{\lambda_{K_\alpha}} + \frac{c}{\lambda_{L_\alpha}} \Rightarrow \lambda_{L_\alpha} = \frac{0.71 \times 0.63}{0.71 - 0.63}$$

$$\Rightarrow \frac{1}{\lambda_{L_\alpha}} = \frac{1}{\lambda_{K_\beta}} - \frac{1}{\lambda_{K_\alpha}} \quad \lambda_{L_\alpha} = 5.59\text{ nm}$$

8. $\lambda = \dfrac{hc}{\Delta E}$



$$\Rightarrow \quad \lambda_{K_\alpha} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{(E_1 - 2870) \times 1.6 \times 10^{-19}}$$

$$\Rightarrow 0.71 \times 10^{-9} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{(E_1 - 2870) \times 1.6 \times 10^{-19}}$$

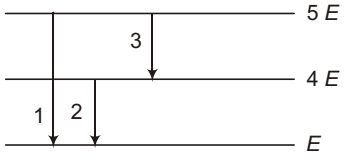
Solving this we get

$$E_1 = -4613\text{ eV}$$

$$\lambda_{K_B} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{(4613 - E_3) \times 1.6 \times 10^{-19}} = 0.63$$

Solving this we get $E_3 = -2650$ eV

9. $\nu_{31} = \dfrac{4E}{h} = f$ ...(i)



$$\nu_{21} = \frac{3E}{h} = \frac{3}{4}\left(\frac{4E}{h}\right) = \frac{3f}{4}$$

$$\nu_{32} = \frac{E}{h} = \frac{f}{4}$$

Introductory Exercise 29.2

1. $eV_0 = \frac{hc}{\lambda} - W$

$\Rightarrow eV_0 = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{2 \times 10^{-7} \times 1.6 \times 10^{-19}} - 4.3 \text{ eV}$

$\Rightarrow eV_0 = 6.2\text{eV} - 4.3 \text{ eV} = 1.9 \text{ eV}$

$\Rightarrow V_0 = 1.9 \text{ volt}$

2. $P = 1.5\text{mW} = 1.5 \times 10^{-3} \text{ W}$

Energy of each photon

$$= \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{4 \times 10^{-7}}$$

$$= 4.96 \times 10^{-19} \text{ J}$$

Number of photons incident per second

$$= \frac{P}{\text{Energy of each photon}} = \frac{1.5 \times 10^{-3}}{4.96 \times 10^{-19}}$$

$$\simeq 3 \times 10^{15}$$

The number of photoelectrons produce

$$= 0.1\% \times 3 \times 10^{15} = 3 \times 10^{12}$$

Current $i = 3 \times 10^{12} \times 1.6 \times 10^{-19}$ A

$$= 4.8 \times 10^{-7} \text{ A} = 0.48 \ \mu\text{A}$$

3. $K_{max} = hf - W = hf - hf_0$

$\Rightarrow K_{max} \propto (f - f_0)$

4. $K_{max} = \frac{hc}{\lambda} - W$

$$= \left[\frac{6.62 \times 10^{-34} \times 3 \times 10^8}{2 \times 10^{-7} \times 1.6 \times 10^{-19}} - 3\right] \text{eV}$$

$$= [6.20 - 3]\text{eV} = 3.20 \text{ eV}$$

The minimum kinetic energy = 0.

5. $K_{max} = h[f - f_0]$

$1.2 \text{ eV} = h[f - f_0]$ ...(i)

$4.2 \text{ eV} = h[1.5f - f_0]$ ...(ii)

Dividing Eq. (i) and Eq. (ii)

$$\frac{124}{42} = \frac{f - f_0}{1.5f - f_0}$$

$\Rightarrow 3f - 2f_0 = 7f - 7f_0$

$\Rightarrow 5f_0 = 4f$

$\Rightarrow f_0 = \frac{4}{5} f = 0.8f$

$\Rightarrow 1.2 \times 1.6 \times 10^{-19} = 6.62 \times 10^{-34} \left[\frac{f_0}{0.8} - f_0\right]$

$\Rightarrow \frac{1.2 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34}} = \frac{2f_0}{8} = \frac{f_0}{4}$

$\Rightarrow f_0 = 1.16 \times 10^{15} \text{Hz}$

Subjective Questions (Level I)

1. Here $\lambda = 280 \text{ nm} = 28 \times 10^{-8} \text{ m}$

$E = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{28 \times 10^{-8}} \text{ J}$

$E = \frac{19.8 \times 10^{-34} \times 10^{16}}{28} \text{ J} = \frac{198 \times 10^{-19}}{28} \text{ J}$

$E = \frac{198 \times 10^{-19}}{28 \times 1.6 \times 10^{-19}} \text{ eV} = \frac{198}{28 \times 1.6} \simeq 4.6 \text{ eV}$

We have $E = mc^2 \Rightarrow m = \frac{E}{c^2} = \frac{198 \times 10^{-19}}{28 \times 9 \times 10^{16}}$

$\Rightarrow m = 8.2 \times 10^{-36} \text{ kg}$

Momentum $p = mc = 8.2 \times 10^{-36} \times 3 \times 10^8$

$$= 2.46 \times 10^{-27} \text{ kg-m/s}$$

2. Intensity of light at a distance 2 m

From the source $= \frac{1}{4\pi \times (2)^2} = \frac{1}{16\pi} \text{ W/m}^2$

Let plate area is A

Energy incident on unit time is

$$E_1 = \frac{1}{16\pi} A\omega$$

Energy of each photon

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{4.8 \times 10^{-7}}$$

Number of photons striking per unit area

$$n = \frac{\frac{1}{16\pi} \times A \times 4.8 \times 10^{-7}}{A \times 6.6 \times 10^{-34} \times 3 \times 10^8 \times A}$$

$$= 4.82 \times 10^{16} \text{ per m}^2 \text{ s}$$

3. Here $p = 8.24 \times 10^{-28}$ kg-m/s

(a) Energy of photon $E = pc$

$\Rightarrow E = 8.24 \times 10^{-28} \times 3 \times 10^8$

$\Rightarrow E = 2.47 \times 10^{-19}$ J

$$\text{Energy in eV} = \frac{E \text{ in joule}}{1.6 \times 10^{-19}}$$

$$= \frac{2.47 \times 10^{-19}}{1.6 \times 10^{-19}}$$

Energy (in eV) = 1.54 eV

(b) Wavelength

$$\lambda = \frac{h}{p} = \frac{6.6 \times 10^{-34}}{8.24 \times 10^{-28}} = 804 \text{ nm}$$

This wavelength in Infrared region.

4. We have $c = f\lambda \Rightarrow f = \frac{c}{\lambda} = \frac{3 \times 10^8 \text{ m/s}}{6 \times 10^{-7} \text{ m}}$

$\Rightarrow \quad f = 5 \times 10^{14}$ Hz

We have $p = \frac{E}{t} \Rightarrow E = pt$ power per sec = energy

$\Rightarrow \quad P = E$

$\Rightarrow \quad 75 = (h \times v) \times n$

$\Rightarrow \quad = \frac{75}{6.6 \times 10^{-34} \times 5 \times 10^{14}}$

$\Rightarrow \quad n = 2.3 \times 10^{20}$ photons/sec

5. (a) $E = 2.45 \text{ MeV} = 2.45 \times 1.6 \times 10^{-19} \times 10^6$ J

$$E = h\nu \Rightarrow \nu = \frac{E}{h} = \frac{3.92 \times 10^{-13}}{6.6 \times 10^{-34}}$$

$\Rightarrow \nu = 5.92 \times 10^{20}$ Hz

(b) We have $c = \nu\lambda \Rightarrow \lambda = \frac{c}{\nu}$

$$\Rightarrow \lambda = \frac{3 \times 10^8}{5.92 \times 10^{20}} = 5.06 \times 10^{-13} \text{ m}$$

6. We have $\quad p = \sqrt{2mK}$

$\Rightarrow \quad p_1 = \sqrt{2mK_1}$ and $p_2 = \sqrt{2mK_2}$

$\Rightarrow \quad \frac{p_1}{p_2} = \sqrt{\frac{K_1}{K_2}}$

$\Rightarrow \quad \frac{1}{2} = \sqrt{\frac{K_1}{K_2}} \qquad [\because p_2 = 2p_1]$

$\Rightarrow \quad K_2 = 4K_1$

(b) $E_1 = p_1 c$ and $E_2 = p_2 c \Rightarrow E_2 = 2p_1 c$

$\Rightarrow E_2 = 2E_1$

7. (a) Since power = energy per unit line

let n be the number of photons

$\Rightarrow P = nE \Rightarrow 10 = n \times \frac{nc}{\lambda}$

$$\Rightarrow n = \frac{10 \times \lambda}{hc} = \frac{10 \times 500 \times 10^{-9}}{6.6 \times 10^{-34} \times 3 \times 10^8}$$

$\Rightarrow n = 2.52 \times 10^{19}$

(b) Force exerted on that surface

$$F = \frac{P}{c} = \frac{10}{3 \times 10^8} = 3.33 \times 10^{-8} \text{ N}$$

8. Absorbing (power) light = 70% of incident light

$\Rightarrow P_a = 70\%$ of 10 W = 7 W

Refractive power = 30% of 10 W

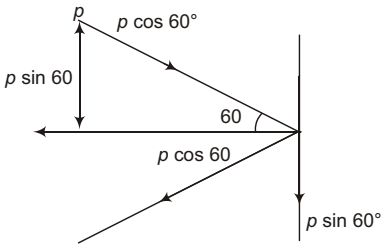
$\Rightarrow P_R = 3$ W

$$\text{The force exerted} = \frac{P_a}{c} + \frac{2P_R}{c}$$

$$= \frac{7 + 2 \times 3}{c} = \frac{13}{3 \times 10^8}$$

$$= 4.3 \times 10^{-8} \text{ N}$$

9. Force = rate of change of momentum



$$F = \frac{\Delta p}{\Delta t} \text{ here } \Delta t = 1 \text{ s}$$

$$F = 2p\cos 60°$$

$$F = p$$

$$\Rightarrow \quad F = \frac{nh}{\lambda}$$

where n is number of photons striking per second

$$\Rightarrow F = \frac{1 \times 10^{19} \times 6.63 \times 10^{-34}}{663 \times 10^{-9}} = 10^{-8} \text{ N}$$

10. Here output energy = 60 W/s

$$\text{Pressure } p = \frac{2 \times 60}{3 \times 10^8} = 4 \times 10^{-7} \text{ N}$$

de-Broglie wavelength

11. Here $m = 5$ g $\Rightarrow$ m $= 5 \times 10^{-3}$kg, $v = 340$ m/s

by de-Broglie hypothesis wavelength

$$\lambda = \frac{h}{mv} = \frac{6.62 \times 10^{-34}}{5 \times 10^{-3} \times 340}$$

$$\Rightarrow \quad \lambda = 3.9 \times 10^{-34} \text{ m}$$

Since λ is too small. No wave like property is exhibit.

12.(a) $\lambda_e = \dfrac{h}{m_e v} = \dfrac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 4.7 \times 10^6}$

$$= 1.55 \times 10^{-10} \text{ m}$$

(b) $\lambda_p = \dfrac{6.6 \times 10^{-34}}{1836 \times 9.1 \times 10^{-31} \times 4.7 \times 10^{-6}}$

$$= 8.44 \times 10^{-14} \text{m}$$

13. (a) $\lambda = \dfrac{h}{p} \Rightarrow p = \dfrac{h}{\lambda} = \dfrac{6.6 \times 10^{-34}}{2.8 \times 10^{-10}}$

$$\Rightarrow p = 2.37 \times 10^{-24} \text{ kg-m/s}$$

(b) $\because p^2 = 2m_e K \Rightarrow K = \dfrac{p^2}{2m_e}$

$$\Rightarrow K = \frac{(2.37 \times 10^{-24})^2}{2 \times 9.1 \times 10^{-31}}$$

$$\Rightarrow K = 3.07 \times 10^{-18} \text{ J}$$

$$K \text{ (in eV)} = \frac{K \text{ in J}}{1.6 \times 10^{-19}} = \frac{3.07 \times 10^{-18}}{1.6 \times 10^{-19}}$$

$$\Rightarrow \quad K = 19.2 \text{ eV}$$

14. Here $T = 273° + 20° = 293$ K

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} \Rightarrow \lambda = \frac{h}{Mv_{\text{rms}}}$$

$$\Rightarrow \lambda = \frac{h}{\sqrt{3\,MRT}}$$

$$= \frac{6.6 \times 10^{-34}}{\sqrt{3 \times 1836 \times 9.1 \times 10^{-31} \times 8.31 \times 293}}$$

$$\Rightarrow \lambda = 1.04 \text{ Å}$$

15. For hydrogen like atom

$$E = -K \text{ Here } E = -3.4 \text{ eV}$$

$$\Rightarrow \quad K = 3.4 \text{ eV} = 3.4 \times 1.6 \times 10^{-19} \text{ J}$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_e K}}$$

$$= \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 3.4 \times 1.6 \times 10^{-19}}}$$

$$\Rightarrow \lambda = 6.663 \text{ Å}$$

16. In Bohr model the velocity of electron in nth orbit is given by

$$U_n = \frac{e^2}{2\varepsilon_0 nh}$$

Putting the values of e, ε_0, h and $n = 1$, we get

$$U_1 = 2.19 \times 10^7 \text{ m/s and } U_4 = \frac{2.19 \times 10^6}{4} \text{ m/s}$$

$$\Rightarrow \lambda_1 = \frac{h}{m_e v_1} \text{ and } \lambda_4 = \frac{h}{m_e u_4} = \frac{h}{m_e \frac{u_1}{4}} = 4\,\lambda_1$$

$$\Rightarrow \quad \lambda_1 = \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 2.19 \times 10^{6}}$$

$$= 3.32 \times 10^{-10}\text{m}$$

$$\Rightarrow \quad \lambda_4 = 4\lambda_1 = 4 \times 3.332 \times 10^{-10}\text{m}$$

$$= 13.28 \times 10^{-10}\ \text{m}$$

$$= 1.33 \times 10^{-9}\text{m}$$

The radius of first Bohr orbit

$$r_1 = 0.529 \times 10^{-10}$$

The radius of fourth Bohr orbit

$$r_4 = 16 \times 0.529 \times 10^{-10}$$

$$\Rightarrow \quad 2\pi r_1 = 2 \times 3.14 \times 0.529 \times 10^{-10}$$

$$\approx 3.32 \times 10^{-10}\text{m} = \lambda_1$$

Bohr Atomic Model and Emission Spectrum

17. For hydrogen like atom we can write

$$e_n = \frac{-z^2}{n^2}(13.6\,\text{eV})$$

For lithium atom $z = 3$ we get

$$E_n = \frac{-9}{n^2}(13.6\,\text{eV}) = \frac{-122.4}{n^2}\text{eV}$$

The ground state energy is for $n = 1$

$$\Rightarrow \quad E_1 = \frac{-122.4}{1^2}\text{eV} = -122.4\ \text{eV}$$

Ionization potential $= -E_1 = 122.4$ eV

18. For hydrogen atom we can write

(a) $E = -K \Rightarrow K = 3.4$ eV

(b) PE $= -2K = -2 \times 3.4 = -6.8$ eV

Since potential energy depends upon refrence hence it will changed.

19. Binding energy of an electron in He-atom is $E_0 = 24.6$ eV. *ie*, the energy required to remove one electron from He-atom $= 24.6$ eV

Now, He-atom becomes He^+ and energy of He^+ ion is given by

$$E_n = \frac{-z^2(13.6)}{n^2} \text{ for } He^+\ z = 2, \text{ we get}$$

$$E_1 = -4 \times 13.6 = -54.4\ \text{eV}.$$

Hence energy required to remove this electron $= 54.4$ eV, thus total energy $= 24.6 + 54.4 = 79$ eV

20. For hydrogen atom $E_n = \dfrac{-13.6\,\text{eV}}{n^2}$

Putting $n = 3$, we get

$$E_3 = \frac{-13.6\,\text{eV}}{9} = -1.51\,\text{eV}$$

Hence hydrogen atom is in third excited state the angular momentum

$$L = \frac{nh}{2\pi} = \frac{3h}{2\pi} = \frac{3 \times 6.62 \times 10^{-34}}{2 \times 3.14}$$

$$\Rightarrow \ L = 3.16 \times 10^{-34}\,\text{kg-m}^2/\text{s}$$

21. We have $\lambda = \dfrac{hc}{\Delta E} = \dfrac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{\Delta E}$

$$\Rightarrow \Delta E = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{1023 \times 10^{-10}} \text{ (in Joule)}$$

$$\Rightarrow \Delta E = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{1023 \times 10^{-10} \times 1.6 \times 10^{-19}} \text{ (in eV)}$$

$$\simeq 12.1\,\text{eV}$$

$\Rightarrow E_n - E_1 = 12.1$ eV But $E_1 = -13.6$ eV

$\Rightarrow E_n \simeq -1.51$ eV

For H-atom $E_n = \dfrac{-13.6}{n^2} \ \Rightarrow\ -1.51 = -\dfrac{13.6}{n^2}$

$\Rightarrow \ n = 3$

Hence atom goes to 3rd excited state. The possible transition are ($3 \to 2, 3 \to 1$, and $2 \to 1$) *ie*, 3 transitions are possible and the largest wavelength $= 1023$ Å

(From $3 \to 1$)

22. For hydrogen like atom

$$E_n = \frac{-z^2}{n^2}(13.6)\text{ eV}$$

For Li^{++} $\quad z = 3$

$$\Rightarrow E_n = \frac{-122.4}{n^2}\text{eV} \Rightarrow E_1 = -122.4\text{ eV}$$

$$E_3 = \frac{-122.4}{9}\text{eV} = -13.6\text{ eV}$$

$$\Delta E = E_3 - E_1 = (122.4 - 13.6)\text{ eV} = 108.8\text{ eV}$$

$$\lambda = \frac{hc}{E} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{108.8 \times 1.6 \times 10^{-19}} = 113.74\text{ Å}$$

23. The excited state energy He^+ atom will be equal to the sum of energies of the photons having wavelength 108.5 nm and 30.4 nm.

$$\Rightarrow E_n - E_1 = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2}$$

$$\frac{6.63 \times 10^{-34} \times 3 \times 10^8}{10^{-10}}\left[\frac{1}{1085} + \frac{1}{304}\right]$$

$$\Rightarrow \quad E_n - E_1 = 83.7 \times 10^{-19}\text{ J}$$

$$= \frac{83.7 \times 10^{-19}\text{ J}}{1.6 \times 10^{-19}\text{ J/eV}} = 52.3\text{ eV} \quad \ldots\text{(i)}$$

For He^+ atom $E_n = \frac{-z^2(13.6)}{n^2}\text{eV}$

For He^+ $\quad z = 2$

$$E_n = \frac{54.4}{n^2}\text{ eV}$$

$$\Rightarrow \quad E_1 = -54.4\text{ eV}$$

$$\Rightarrow \quad E_n - E_1 = 54.4\left(1 - \frac{1}{n^2}\right)\text{eV} \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$52.3 = 54.4\left(1 - \frac{1}{n^2}\right)$$

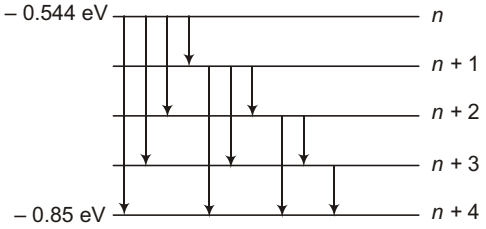
$$\Rightarrow \quad 1 - \frac{1}{n^2} = 0.96$$

$$\Rightarrow \quad n = 5$$

24. For hydrogen like atom

$$E_n = \frac{-z^2}{n^2}(13.6)\text{eV}$$

Let for ten transitions quantum numbers of energy levels are n, $n + 1$, $n + 2$, $n + 3$ and $n + 4$



$$\frac{-z^2(13.6)\text{ eV}}{n^2} = -0.85\text{ eV} \quad \ldots\text{(i)}$$

$$\frac{-z^2(13.6)\text{ eV}}{(n+4)^2} = -0.544\text{ eV} \quad \ldots\text{(ii)}$$

Dividing Eq. (i) and Eq. (ii)

$$\frac{(n+4)^2}{n^2} = \frac{0.85}{0.544} = 1.5625$$

$$\Rightarrow \quad \frac{n+4}{n} = 1.25$$

$$\Rightarrow \quad 1 + \frac{4}{n} = 1.25$$

$$\Rightarrow \quad \frac{4}{n} = 0.25$$

$$\Rightarrow \quad n = \frac{4}{0.25} = 16$$

Putting this value of n in Eq. (i)

$$\frac{-z^2(13.6)}{(16)^2} = -0.85$$

$$\Rightarrow \quad z^2 = \frac{256 \times 0.85}{13.6}$$

$$\Rightarrow \quad z^2 = 16 \Rightarrow z = 4$$

Hence atom no. of atom is = 4

We know that $\Delta E = \frac{hc}{\lambda}$

$\Rightarrow \lambda = \frac{hc}{\Delta E}$ for smallest wavelength ΔE is maximum

$$\lambda = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{[-0.544 - (-0.85)] \times 1.6 \times 10^{-19}}$$

$$\lambda = 40441\text{ Å}$$

25. Here $E_1 = -15.6$ eV

(a) Hence ionization potential

$= -E_1 = 15.6$ eV

(b) We have $\lambda = \dfrac{hc}{\Delta E}$ for short wavelength

ΔE is maximum

$$\Rightarrow \lambda = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{0 - (-5.3) \times 1.6 \times 10^{-19}} \simeq 2335 \text{ Å}$$

(c) Excitation potential for $n = 3$ state is

$= E_3 - E_1 = -3.08 + 15.6 = 12.52$ V

(d) From $n = 3$ to $n = 1$

$\Delta E = E_3 - E_1 = -3.08 + 15.6 = 12.52$ eV

We know $\lambda = \dfrac{hc}{\Delta E}$

$$\Rightarrow \frac{1}{\lambda} = \frac{\Delta E}{hc} = \frac{12.52 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34} \times 3 \times 10^8}$$

$$= 1.01 \times 10^7 \text{ (m}^{-1}\text{)}$$

26. (a) $E_1 = -6.52$ eV

$\lambda = 860$ nm $= 8600$ Å

Energy of this photon

$$= \frac{12375 \text{ (eV)}}{8600} = 1.44 \text{ eV}$$

hence internal energy of atom after absorbing this photon is given by

$E_i = E_1 + 1.44$ eV $= -6.52 + 1.44$

$= -5.08$ eV

(b) $\lambda_2 = \dfrac{12375 \text{ (eV)}}{4200} = 2.95$ eV

hence internal energy of the atom after emission of this photon is given by

$E_i = E_1 - 2.95$ eV $= (-2.68 - 2.95)$ eV

$\Rightarrow \quad E_i = -5.63$ eV

27. Heve $U = \dfrac{1}{2} m\omega^2 r^2 \Rightarrow F = \left| \dfrac{dU}{dr} \right| = m^2\omega^2 r$

But $\quad \dfrac{mv^2}{r} = m^2\omega^2 r$

$\Rightarrow \quad v^2 = m\omega^2 r^2$

But by Bohr's postulate $mvr = \dfrac{nh}{2\pi}$

$$\Rightarrow \quad m^2 v^2 r^2 = \frac{n^2 h^2}{4\pi^2}$$

$$\Rightarrow \quad m^3 \omega^2 r^4 = \frac{n^2 h^2}{4\pi^2}$$

$$\Rightarrow \quad r^4 = \frac{n^2 h^2}{4\pi^2 m^3 \omega^2}$$

$$\Rightarrow \quad r \propto \sqrt{n}$$

28. $\dfrac{1}{\lambda_{K_\alpha}} = R(z-1)^2 \left[1 - \dfrac{1}{2^2}\right] = \dfrac{1}{\lambda_0}$

$$\frac{1}{\lambda_{K_\beta}} = R(z-1)^2 \left[1 - \frac{1}{3^2}\right]$$

$$\Rightarrow \quad \frac{\lambda_{K_\beta}}{\lambda_{K_\alpha}} = \frac{3 \times 9}{4 \times 8} = \frac{27}{32}$$

$$\Rightarrow \quad \lambda_{K_\beta} = \frac{27}{32}\lambda_{K_\alpha} = \frac{27}{32}\lambda_0$$

29. $\lambda_0 = \dfrac{hc}{eV}$

$$\Rightarrow \lambda_{0_1} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19} \times 25 \times 10^3}$$

$\Rightarrow \lambda_{0_1} = 49.5$ pm

$\Rightarrow \lambda_{0_2} = 2\lambda_{0_1} = 2 \times 49.5$ pm $= 99$ pm

[1 pm $= 10^{-12}$ m]

30. $f_{K\alpha} = (2.48 \times 10^{15})$ Hz $(z-1)^2$

$$\Rightarrow \frac{3 \times 10^8}{\lambda_{K\alpha}} = 2.48 \times 10^{15}(2-1)^2$$

$$\Rightarrow \frac{3 \times 10^8}{0.76 \times 10^{-10} \times 2.58 \times 10^{15}} = (z-1)^2$$

$(z-1) \simeq 40 \Rightarrow z = 41$

31. $\lambda_i = \dfrac{hc}{eV}$ $\quad \Delta\lambda = 26$ pm when $V_f = 1.5$ V

$\Rightarrow \quad \lambda_f = (\lambda_i - 26)$ pm

$$\lambda_f = \frac{hc}{e \times 1.5\ V} = \frac{1}{1.5}\frac{hc}{eV} = \frac{12}{1.5} = \frac{1}{1.5}\lambda_i$$

$$\Rightarrow \quad (\lambda_i - 26) = \frac{2}{3}\lambda_i$$

$$\Rightarrow \quad 3\lambda_i - 26 \times 3 = 2\lambda_i$$

$$\Rightarrow \quad \lambda_i = 78 \text{ pm}$$

$$\Rightarrow \quad 78 \times 10^{-12} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19} \times V}$$

$$\Rightarrow \quad V = \frac{6.6 \times 3 \times 10^{-26}}{1.6 \times 78 \times 10^{-12} \times 10^{-19}}$$

$$\Rightarrow \quad V = 15865 \text{ volt}$$

32. $\sqrt{V} = a(z - b)$

$$\Rightarrow \quad \frac{c}{\lambda} = a^2(z-b)^2$$

$$\Rightarrow \quad \frac{1}{\lambda} = \frac{a^2}{c}(z-b)^2$$

$$\Rightarrow \quad \frac{1}{887 \text{ pm}} = \frac{a^2}{c}(13-b)^2 \quad \ldots\text{(i)}$$

and $$\frac{1}{146 \text{ pm}} = \frac{a^2}{z}(30-b)^2 \quad \ldots\text{(ii)}$$

Dividing Eq. (i) and Eq. (ii)

$$\frac{146}{887} = \left[\frac{(13-b)}{(30-b)}\right]^2$$

$$\Rightarrow \quad 2.5 = \frac{30-b}{13-b}$$

$$\Rightarrow \quad 32.5 - 2.5b = 30 - b$$

$$2.5 = 1.5b$$

$$b = \frac{5}{3}$$

$$\frac{1}{\lambda_{26}} = \frac{a^2}{c}\left(26 - \frac{5}{3}\right)^2$$

$$\frac{1}{\lambda_{26}} = \frac{\phi\left(26 - \frac{5}{3}\right)^2}{887 \text{ pm} \times \left(13 - \frac{5}{3}\right)^2}$$

$$\Rightarrow \quad \lambda_{26} = \frac{887 \text{ pm} \times \left(13 - \frac{5}{3}\right)^2}{\left(26 - \frac{5}{3}\right)^2}$$

$$= 887 \text{ pm} \times \frac{34^2}{(73)^2}$$

$$\simeq 198 \text{ pm}$$

33. $\sqrt{f} = \sqrt{\frac{3RC}{4}}(z-1)$

$$\sqrt{4.2 \times 10^{18}} = \sqrt{\frac{3 \times 1.1 \times 10^7 \times 3 \times 10^8}{4}}(z-1)^2$$

$$\Rightarrow \quad \frac{4.2 \times 10^{18} \times 4}{9 \times 1.1 \times 10^{15}} = (z-1)^2$$

$$\Rightarrow \quad (z-1) = 41 \Rightarrow z = 42$$

34. $P = Vi = 40 \text{ kW} \times 10 \text{ mA} = 400 \text{ W}$

$$\% \text{ of } P = \frac{400 \times 1}{100} = 4 \text{ W}$$

(a) Total power of X-rays = 4 W

(b) Heat produced per second

$$= 400 - 4 = 396 \text{ J/s}$$

Photoelectric effect

35. Einstein photo electric equation is

$$K_{\max} = h\nu - W$$

$$\Rightarrow \quad eV_0 = \frac{hc}{\lambda} - W \quad \because K_{\max} = eV_0$$

$$\Rightarrow \quad 10.4 \text{ eV} = \frac{12375}{\lambda(\text{Å})} - 1.7 \text{ eV}$$

$$\Rightarrow \quad \lambda(\text{Å}) = \frac{12375}{12.1} = 1022 \text{ Å}$$

For H-atom $\lambda = \frac{hc}{\Delta E} \Rightarrow \Delta E = \frac{12375}{1022} = 12.1 \text{ eV}$

This difference equal to $n = 3 \rightarrow n = 1$ transition.

36. $K_{max} = h\nu - W$

$$\Rightarrow K_{max} = \frac{6.6 \times 10^{-34} \times 1.5 \times 10^{15}}{1.6 \times 10^{-19}} - 3.7$$

$$\Rightarrow K_{max} = 6.18 - 3.7 = 2.48 \text{ eV}$$

37. Here work function

$$W(\text{in eV}) = \frac{12375}{5000 \text{ Å}} = 2.475 \text{ eV}$$

$$K_{max} = eV_0 = 3 \text{ eV}$$

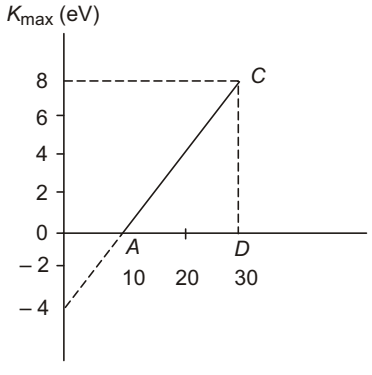
$$K_{max} = \frac{hc}{\lambda} - W$$

$$\Rightarrow \quad 3 = \frac{12375}{\lambda(\text{in Å})} - 2.475$$

$$\Rightarrow \quad \lambda = \frac{12375}{5.475} = 2260 \text{ Å}$$

38. Comparing the given graph with

$$K_{max} = h\nu - W$$



$f = 1 \times 10^{14}$ Hz

(a) ν_o = threshold frequency $\theta = 10 \times 10^{14}$ Hz

$= 10^{15}$ Hz

(b) $W = 4$ eV

(c) h = slope of the graph

$$= \frac{CD}{AD} = \frac{8 \text{ eV}}{20 \times 10^{14} \text{Hz}}$$

$$\Rightarrow h = \frac{8 \times 1.6 \times 10^{-19}}{2 \times 10^{15}} = 6.4 \times 10^{-34} \text{ J-s}$$

39. Here $\frac{v_{1(max)}}{u_{2(max)}} = \frac{3}{1}$

Using Einstein equation, $K_{max} = \frac{hc}{\lambda} - W$,

we get

$$\frac{1}{2} m v_{max}^2 = \frac{hc}{\lambda} - W$$

where m is the mass of photoelectron

$$\Rightarrow \quad \frac{1}{2} m [v_{1(max)}]^2 = \frac{hc}{\lambda_1} - W \quad \ldots(i)$$

$$\text{and} \quad \frac{1}{2} m [v_{2(max)}]^2 = \frac{hc}{\lambda_2} - W \quad \ldots(ii)$$

Dividing Eq. (i) and Eq. (ii), we get

$$\left(\frac{v_{1\,max}}{v_{2\,max}}\right)^2 = \frac{\frac{hc}{\lambda_1} - W}{\frac{hc}{\lambda_2} - W} \Rightarrow (3)^2 = \frac{\frac{hc}{\lambda_1} - W}{\frac{hc}{\lambda_2} - W}$$

$$\Rightarrow \quad \frac{9hc}{\lambda_2} - \frac{hc}{\lambda_1} = 8W$$

$$\Rightarrow \quad hc\left[\frac{9}{6000} - \frac{1}{3000}\right] = 8W$$

$$\Rightarrow \quad \frac{6.6 \times 10^{-34} \times 3 \times 10^8 \times 7}{6000 \times 10^{-10} \times 1.6 \times 10^{-19}} = 8W$$

$$\Rightarrow \quad W = 1.81 \text{ eV}$$

Putting the value of W in Eq. (i)

$$\frac{1}{2} m (u_{1\,max})^2 = \frac{hc}{3000 \times 10^{-10}} - 1.81 \times 1.6 \times 10^{-19}$$

$$\frac{1}{2} m (u_{1\,max})^2 = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{3 \times 10^7} - 2.896 \times 10^{-19}$$

$$\frac{1}{2} m (u_{1\,max})^2 = 6.62 \times 10^{-19} - 2.896 \times 10^{-19}$$

$$\frac{1}{2} m (u_{1\,max})^2 = 3.724 \times 10^{-19}$$

$$\Rightarrow (u_{1\,\max})^2 = \frac{3.724 \times 10^{-19} \times 2}{9.1 \times 10^{-31}}$$

$$-1.81 \times 1.6 \times 10^{-19} \quad \because \quad m_e = 9.1 \times 10^{-31}$$

$$\Rightarrow u_{1\,\max} = 9 \times 10^5 \text{ m/s}$$

and $v_{2\,\max} = \frac{1}{3} v_{1\,\max} = 3 \times 10^5$ m/s

40. Here intensity $I = 2$ W/m^2 and

Area $A = 1 \times 10^{-4}$ m^2

Energy incident per unit time on the metal surface

$$E = IA = 2 \times 10^{-4} \text{ W}$$

$$= 2 \times 10^{-4} \text{ J/s} = \frac{2 \times 10^{-4}}{1.6 \times 10^{-19}} \frac{\text{eV}}{\text{s}}$$

$$= \frac{2 \times 10^{15}}{1.6} \text{ eV/s}$$

Energy of each photon $= 10.6$ eV

Number of photons incident on surface

$$= \frac{2 \times 10^{15}}{1.6 \times 10.6}$$

Number of photoelectrons emitted

$$= \frac{0.53}{100} \times \frac{2 \times 10^{15}}{1.6 \times 10.6}$$

$$= 6.25 \times 10^{11} \text{ per second}$$

Minimum KE $= 0$

Maximum KE $= (10.6 - 5.6)$ eV $= 5$ eV

41. $K_{\max} = \frac{hc}{\lambda} - W$

$\Rightarrow$

$$K_{\max} = \frac{6.6 \times 10^{34} \times 3 \times 10^8}{180 \times 10^{-9}} - 2 \times 1.6 \times 10^{-19}$$

$$\frac{1}{2} m_e v_{\max}^2 = \frac{6.6 \times 3 \times 10^{-18}}{18} - 2 \times 1.6 \times 10^{-19}$$

$$\frac{1}{2} m_e v_{\max}^2 = 11 \times 10^{-19} - 3.2 \times 10^{-19}$$

$$= 7.2 \times 10^{-19} \text{ J}$$

$$v_{\max} = \sqrt{\frac{2 \times 7.2 \times 10^{-19}}{9.1 \times 10^{-31}}} = 1.25 \times 10^6 \text{ m/s}$$

$$r = \frac{mv_{\max}}{eB} = \frac{9.1 \times 10^{-31} \times 1.25 \times 10^6}{1.6 \times 10^{-19} \times 5 \times 10^{-5}}$$

$r = 0.148$ m

42. The given equation is

$$E = (100 \text{ v / m}) [\sin((5 \times 10^{15}) t + \sin(8 \times 10^{15}) t]$$

Light consist of two different frequencies.

Maximum frequency

$$= \frac{8 \times 10^{15}}{2\pi} = 1.27 \times 10^{15} \text{ Hz}$$

For maximum KE we will use Einstein's equation

$$(\text{KE})_{\max} = h\nu - W$$

$$= \frac{1.27 \times 10^{15} \times 6.62 \times 10^{-34}}{1.6 \times 10^{-19}} - 2$$

$$\Rightarrow (\text{KE})_{\max} = 3.27 \text{ eV}$$

43. Here $E = E_0 \sin 1.57 \times 10^7 (x - ct)$ frequency of the wave

$$\nu = \frac{1.57 \times 3 \times 10^{15}}{2 \times 3.14} = 0.75 \times 10^{15}$$

We have

$$eV_0 = \left(\frac{6.6 \times 10^{-34} \times 0.75 \times 10^{15}}{1.8 \times 10^{-19}} - 1.9 \right) \text{eV}$$

$$\Rightarrow \quad V_0 = 1.2 \text{ V}$$

■ Objective Questions (Level-1)

1. Einstein photo electric equation is

$$K_{\max} = h\nu - W$$

its slope $= h =$ planck constant which is same for all metals and independent of intensity of radiation.

Hence correct option is (d).

2. Since current is directly proportional to intensity therefore as current is increased

intensity is increased since $\lambda_{min} \propto \frac{1}{V}$, if V is decreased λ_{min} is increased.

Hence correct option is (c).

3. For hydrogen atom (Bohr's model) nth orbital speed $v_n = \frac{e^2}{2\varepsilon_0 nh}$

For first orbit $n = 1$

$$\Rightarrow \quad v_1 = \frac{e^2}{2\varepsilon_0 \times h}$$

$$= \frac{(1.6 \times 10^{-19})^2}{2 \times 8.85 \times 10^{-12} \times 6.62 \times 10^{-34}}$$

$$\Rightarrow \quad v \simeq \left(\frac{1}{137}\right) \times 3 \times 10^8 = \frac{c}{137}$$

Hence correct option is (c).

4. ${}_{86}A^{22} \xrightarrow[3\alpha]{} {}_{80}X^{210} \xrightarrow{4\beta} {}_{84}B^{210}$

Hence correct option is (b)

5. $\lambda_{min}(\text{in Å}) = \frac{12375}{V(\text{in volt})} = \frac{12375}{20 \times 1000} \simeq 0.62$ Å

Hence correct option is (c).

6. We have $\frac{1}{2} m_e v_{max}^2 = \text{eV}$

$$\Rightarrow \quad v_{max} = \sqrt{\frac{2eV}{m}}$$

$$= \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 18 \times 1000}{9.1 \times 10^{-31}}}$$

$$\Rightarrow \quad v_{max} \simeq 8 \times 10^7 \text{ m/s}$$

Hence correct option is (a).

7. For hydrogen atom $v_n = \frac{e^2}{2\varepsilon_0 nh}$

$$\Rightarrow \quad v_2 = \frac{e^2}{2\varepsilon_0 \times 2h}, v_3 = \frac{e^2}{2\varepsilon_0 \times 3h}$$

$$\Rightarrow \quad \frac{v_2}{v_3} = \frac{3}{2}$$

Let λ_2 and λ_3 are the de-Broglie wavelengths

$$\Rightarrow \quad \frac{\lambda_2}{\lambda_3} = \frac{\frac{h}{mv_2}}{\frac{n}{mv_3}} = \frac{v_3}{v_2}$$

$$\Rightarrow \quad \frac{\lambda_2}{\lambda_3} = \frac{2}{3}$$

Hence correct option is (a).

8. For hydrogen like atom

$$E_n = \frac{-z^2}{n^2}(13.6 \text{ eV})$$

For ground state $n = 1$

$$\Rightarrow \quad E_1 = -z^2 \times 13.6 \text{ eV}$$

But $\quad E_1 = -122.4$ eV

$$\Rightarrow \quad -122.4 \text{ eV} = -z^2 \times 13.6 \text{ eV}$$

$$\Rightarrow \quad z^2 = 9$$

$$\Rightarrow \quad z = 3$$

Hence it is Li^{2+}

The correct option is (c).

9. $\lambda_{min} = \frac{hc}{eV} \Rightarrow \frac{\Delta\lambda_{min}}{\lambda_{min}} \times 100 = -\frac{\Delta V}{V} \times 100$

Percentage change in $\lambda_{min} = -2\%$

Hence λ_{min} is decreased by 2%

correct option is (c)

10. $E_n = \frac{-z^2}{n^2}(13.6 \text{ eV})$ for first excited state $n = 2$

$$\Rightarrow \quad E_2 = \frac{-z^2}{4}(13.6) \text{ eV}$$

$$\Rightarrow \quad -13.6 \text{ eV} = \frac{-z^2}{4} \times 13.6 \text{ eV}$$

$$\Rightarrow \quad z = 2$$

Hence it is He^+

Correct option is (a).

11. $\lambda_{min}(\text{in Å}) = \frac{12375}{V(\text{in volt})}$

$$\Rightarrow \quad V = \frac{12375}{1} = 12.375 \times 10^3 \text{ V}$$

$$\Rightarrow \quad V = 12.4 \text{ eV}$$

Hence correct option is (c).

12. We have $\lambda = \frac{h}{p}$

$$\Rightarrow \quad p = \frac{h}{\lambda} = \frac{6.62 \times 10^{-34}}{0.5 \times 10^{-10}}$$

$$\Rightarrow \lambda = 13.26 \times 10^{-24} \text{ kg-m/s}$$

Hence correct option is (c).

13. $W = \frac{hc}{\lambda_0} \Rightarrow \lambda_0 = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19} \times 1.6}$

$$\Rightarrow \quad \lambda_0 = 7750 \text{ Å}$$

Hence correct option is (a).

14. For H-like atom Balmer series is $\frac{1}{\lambda} = z^2 R\left[\frac{1}{2^2} - \frac{1}{n^2}\right]$

For third Balmer line $n = 5$

$$\Rightarrow \frac{1}{1085 \times 10^{-10}} = z^2 \times 1.09 \times 10^7 \left[\frac{1}{4} - \frac{1}{5^2}\right]$$

$$\Rightarrow \quad z^2 = \frac{100 \times 1000}{1085 \times 1.097 \times 21} = 4$$

$$\Rightarrow \quad z = 2$$

Binding energy $= (13.6) \times z^2 \text{eV} = 13.6 \times 4 \text{ eV}$

$= 54.4 \text{ eV}$

Hence correct option is (a).

15. If $V_1 = 0$ then total energy $=$ KE

$$\Rightarrow \quad \text{KE} = 13.6 \text{ eV}$$

and the energy difference between two states $= 10.2$ eV

Hence total energy in this state

$= 13.6 + 10.2$

$= 23.8$ eV

The correct option is (c).

16. $eV_0 = \frac{hc}{3300} - W$ ...(i)

$2eV_0 = \frac{hc}{2200}$ ...(ii)

Substracted Eq. (i) from Eq. (ii), we get

$$eV_0 = hc \times \frac{3300 - 2200}{3300 \times 2200}$$

$$= \frac{hc}{3 \times 2200 \times 10^{-10}}$$

$$V_0 = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{3 \times 2200 \times 1.6 \times 10^{-19} \times 10^{-10}}$$

$$= \frac{30}{16} = \frac{15}{8}$$

$$\Rightarrow \quad V_0 = \frac{15}{8} \text{V}$$

Hence the correct option is (c).

17. $E = \frac{hc}{\lambda} - W$ ...(i)

$$4E = \frac{hc}{\lambda/3} - W$$

$$\Rightarrow \quad 4E = \frac{3hc}{\lambda} - W \quad \text{...(ii)}$$

From Eqs. (i) and (ii)

$$4\left(\frac{hc}{\lambda} - W\right) - \frac{3hc}{\lambda} - W$$

$$\Rightarrow \quad \frac{hc}{\lambda} = 3W$$

$$\Rightarrow \quad W = \frac{hc}{3\lambda}$$

Hence correct option is (b).

18. By Moseley's law

$\sqrt{f} = a(z - b)$ for K_α line $b = 1$

$\sqrt{f} = a(z - 1)$

$\Rightarrow \sqrt{f} = a(31 - 1) = a \times 30$ ...(i)

and $\sqrt{f'} = a(51 - 1) = 9 \times 50$

$$\Rightarrow \quad \sqrt{\frac{f}{f'}} = \frac{3}{5}$$

$$\Rightarrow \quad f' = \frac{25f}{9}$$

Hence correct option is (a).

19. $\sqrt{f} = \sqrt{(RC)}\,(z - 1)\sqrt{\left[\frac{1}{12} - \frac{1}{n^2}\right]}$

for K_α, $n = 2$

K_β, $n = 3$

$$\sqrt{f_\alpha} = \sqrt{RC}\,(z-1)\sqrt{\left[1-\frac{1}{2^2}\right]} = \sqrt{RC}\,(z-1)\times\sqrt{\frac{3}{4}}$$

$$\sqrt{f_\beta} = \sqrt{RC}\,(z-1)\sqrt{1-\frac{1}{9}} = \sqrt{RC}\,(z-1)\times\sqrt{\frac{8}{9}}$$

$$\frac{\sqrt{f_\beta}}{\sqrt{f_\alpha}} = \sqrt{\frac{8\times 4}{9\times 3}} = \sqrt{\frac{32}{27}},$$

Correct option is (a).

20. $\frac{1}{\lambda} = R\left[\frac{1}{2^2}-\frac{1}{n^2}\right]$ here $n = 3$

$$\Rightarrow \frac{1}{\lambda} = R\left[\frac{1}{2^2}-\frac{1}{3^2}\right] = \left(\frac{9-4}{9\times 4}\right)R = \frac{5R}{36}$$

$$\Rightarrow \lambda = \frac{36}{5R}$$

Hence correct option is (c)

21. $\lambda_D = \frac{h}{\sqrt{2\,meV}}$ and $\lambda_{\min} = \frac{hc}{eV}$

$$\Rightarrow \frac{\lambda_D}{\lambda_{\min}} = \frac{1}{c}\sqrt{\frac{eV}{2m}}$$

$$= \frac{1}{3\times 10^8}\sqrt{\frac{1.8\times 10^{11}\times 10000}{2}}$$

$$\Rightarrow \frac{\lambda_D}{\lambda_{\min}} = \frac{3\times 10^7}{3\times 10^8} = \frac{1}{10}$$

Hence correct option is (c).

22. $5eV_0 = \frac{hc}{\lambda} - W$ ...(i)

$eV_0 = \frac{hc}{3\lambda} - W$ ...(ii)

From Eqs. (i) and (ii)

$$\frac{5hc}{3\lambda} - 5W = \frac{hc}{\lambda} - W$$

$$\Rightarrow \frac{5hc}{3\lambda} - \frac{hc}{\lambda} = 4\,W$$

$$\Rightarrow \frac{hc}{\lambda}\frac{(5-3)}{3} = 4\,W$$

$$\Rightarrow \frac{2hc}{3\lambda} = 4\,W$$

$$\Rightarrow W = \frac{hc}{6\lambda}$$

Hence correct option is (a).

23. $eV_0 = h[2V_0 - V_0] = hV_0$ ...(i)

$eV = h[3V_0 - V_0] = h\times 2V_0 = 2hV_0$ ...(ii)

From Eqs. (i) and (ii)

$$eV = 2\times eV_0 \Rightarrow V = 2V_0$$

Hence correct option is (b).

24. For H-atom Lyman series is

$$\frac{1}{\lambda} = R\left[\frac{1}{1^2}-\frac{1}{n^2}\right]$$

$$\Rightarrow v = \frac{c}{\lambda} = RC\left[\frac{1}{1^2}-\frac{1}{n^2}\right]$$

For H-like atom

$$\nu' = RC\left[\frac{1}{12}-\frac{1}{n^2}\right]\times z^2$$

For Li^{++}, $n = 3 \Rightarrow \nu' = \nu\times 3^2 = 9\,v$

Hence correct option is (c).

25. Ground state energy of H-atom $= -13.6$ eV

For Li^{++} atom $E_n = \frac{(-13.6)\,eV}{n^2}\times z^2$

$$\Rightarrow -13.6\text{ eV} = \frac{-13.6\text{ eV}\times 9}{n^2}$$

$$\Rightarrow n = 3$$

Hence correct option is (c).

26. $\frac{1}{2}mv_1^2 = h\nu_1 - W$ ...(i)

$\frac{1}{2}mv_2^2 = h\nu_2 - W$ ...(ii)

$$\Rightarrow \frac{1}{2}m[v_1^2 - v_2^2] = h[\nu_1 - \nu_2]$$

$$\Rightarrow v_1^2 - v_2^2 = \frac{2h}{m}[\nu_1 - \nu_2]$$

Hence correct option is (b).

27. For Lyman series

$$\frac{1}{\lambda} = R\left[\frac{1}{1^2}-\frac{1}{n^2}\right]$$

For largest wavelength $n = 2$

$$\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right]$$

$$\Rightarrow \quad \frac{1}{\lambda} = R\left[1 - \frac{1}{4}\right]$$

$$\Rightarrow \quad \lambda = \frac{4}{3R}$$

For He^+ atom

$$\frac{1}{\lambda_{He}} = R(z)^2\left[\frac{1}{2^2} - \frac{1}{n^2}\right] = 4R\left[\frac{1}{4} - \frac{1}{n^2}\right]$$

$$\Rightarrow \quad \frac{3R}{4} = 4R\left[\frac{1}{4} - \frac{1}{n^2}\right]$$

$$\Rightarrow \quad \frac{3}{16} = \frac{1}{4} - \frac{1}{n^2}$$

$$\Rightarrow \quad \frac{1}{n^2} = \frac{1}{4} - \frac{3}{16} = \frac{4-3}{16}$$

$$\Rightarrow \quad \frac{1}{n^2} = \frac{1}{16}$$

$$\Rightarrow \quad n = 4$$

Hence correct option is (b).

28. We have $\sqrt{\frac{1}{\lambda}} = \sqrt{\frac{3R}{4}} \times (z-1)$

$$\Rightarrow \sqrt{\frac{1}{\lambda}} = \sqrt{\frac{3 \times 1.0973 \times 10^7}{4}} \times (92-1)$$

$$\Rightarrow \lambda = \frac{4}{3 \times 91 \times 91 \times 1.0973 \times 10^7} = 0.15 \text{ Å}$$

Hence correct option is (c).

29. For K_α, K_β and L_α of X-rays

$$\nu_{K_\beta} = \nu_{K_\alpha} + \nu_{L_\alpha}$$

$$\Rightarrow \quad Y_2 = Y_1 + Y_3$$

Hence correct option is (b).

30. We have $\lambda = \frac{h}{\sqrt{2mqV}}$

$$\lambda_p = \frac{h}{\sqrt{2m_p eV}} \text{ and } \lambda_\alpha = \frac{h}{\sqrt{2m_{\alpha\,(2e)}V}}$$

$$\Rightarrow \frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{2m_\alpha}{m_p}} = \sqrt{\frac{2 \times 4m_p}{m_p}} = \sqrt{8} = 2\sqrt{2}$$

Hence correct option is (c).

31. We have, $\lambda_e = \frac{h}{\sqrt{2m_e E_1}}$, $\lambda_\alpha = \frac{h}{\sqrt{2m_\alpha E_2}}$

$$\lambda_p = \frac{h}{\sqrt{2m_p E_3}} \quad \because m_\alpha > m_p > m_e$$

and $\quad \lambda_e = \lambda_\alpha = \lambda_p \quad \because E_1 > E_3 > E_2$

Hence correct option is (a).

32. KE in ground state $= 13.6$ eV

Total energy in $n = \infty = \text{KE} + \text{Energy}$ difference between $n = \infty$ to $n = 1$

$\Rightarrow$ Total energy

$$= 13.6 \text{ eV} + 13.6 \text{ eV}$$

$$= 27.2 \text{ eV}$$

Hence correct option is (b).

33. Here $P = 1000$ W, $\nu = 880$ kHz $= 880 \times 10^3$ Hz

Let n is the number of photon p emitted per second

$$\Rightarrow \quad n = \frac{P}{h\nu} = \frac{1000}{6.62 \times 10^{-34} \times 880 \times 10^3}$$

$$= 1.7 \times 10^{30}$$

Correct option is (b).

34. Here $\lambda = 3000$ Å $= 3 \times 10^{-7}$ m

Energy of incident radiation $E = \frac{hc}{\lambda}$ joule

$$\Rightarrow \quad E = \frac{hc}{\lambda \times 1.6 \times 10^{-19}} \text{ (in eV)}$$

$$E = \frac{6.62 \times 3 \times 10^{-26}}{3 \times 1.6 \times 10^{-26}} = 4.125 \text{ eV}$$

$\because E <$ work function hence no emission of electrons it means sphere remain natured.

Hence correct option is (c).

35. $\because E_n = \frac{-13.6 \text{ eV}}{n^2}$ for $n = 5$

$$E_5 = \frac{-13.6 \text{ eV}}{5^2} = -0.54 \text{ eV}$$

Hence correct option is (a).

36. $K_{max} = E - W = (6.2 - 4.2)\,\text{eV} = 2\,\text{eV}$

$\Rightarrow K_{max} = 2 \times 1.6 \times 10^{-19}\ \text{J}$

$= 3.2 \times 10^{-19}\,\text{J}$

Hence correct option is (b).

37. $\lambda = \dfrac{6.62 \times 10^{-34}}{5200 \times 10^{-10}} = 9.1 \times 10^{-31} \times v$

$\Rightarrow \quad v = \dfrac{6.625 \times 10^{-27}}{5.2 \times 9.1 \times 10^{-31}} \simeq 1400\,\text{m/s}$

Hence correct option is (c).

38. $K_{max} = \left[\dfrac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{3000 \times 10^{-10} \times 1.6 \times 10^{-19}} - 1\right]\text{eV}$

$K_{max} = 3.14\ \text{eV} = 3.14 \times 1.6 \times 10^{-19}\,\text{J}$

$\Rightarrow \dfrac{1}{2} m_e v_{max}^2 = 3.14 \times 1.6 \times 10^{-19}$

$\Rightarrow v_{max} = \sqrt{\dfrac{3.14 \times 3.2 \times 10^{-19}}{9.1 \times 10^{-31}}} \simeq 10^6\,\text{m/s}$

Hence correct option is (d).

JEE Corner

1. Here both assertion and reason are true and reason explain correctly assertion. Correct option is (a).

2. For photon $E = \dfrac{hc}{\lambda}$ and $p = \dfrac{E}{c}$

$\Rightarrow$ If λ is doubled, E and p are reduced to half. Hence assertion is true. Since speed of photon is always c. Hence reason is false. Hence correct option is (c).

3. If frequency is increased keeping intensity constant photoelectron emitted the plate reach other plate in less time hence saturation current can be increased. Reason can be true or not hence correct option is (a, b).

4. Here both assertion and reason is true and reason correctly explain assertion. Hence correct option is (a).

5. Here assertion is true since possible transition are $6 \to 3, 6 \to 4, 6 \to 5, 5 \to 3, 5 \to 4$, and $4 \to 3$. According to reason total $\dfrac{n(n-1)}{2}$ transition has $n = 3 \Rightarrow \dfrac{6 \times (3-1)}{2} = 6$ it may explain or may not explain assertion

Hence correct option is (a, b)

6. We have

$$eV_0 = h[\nu - \nu_0]$$

$$\Rightarrow \quad V_0 = \frac{h}{e}\nu_0 - \frac{h}{e}\nu_0 \qquad \ldots(i)$$

if $\nu \to 2\nu_0$ ν_0 does not become double hence assertion is false but reason is true. Hence correct option is (d).

7. Here both assertion and reason are true and reason may or may not explain assertion correct option is (a, b).

8. Here assertion and reason are both true.

$\because \quad \lambda_{min} = \dfrac{hc}{eV}$ if $V \to$ increases

$\lambda_{min} \to$ decreases

but reason is not correct explanation of assertion hence correct option is (b).

9. $\because \quad E_n = \dfrac{-13.6}{n_2}$

$\Rightarrow \quad E_2 > E_1$

Hence assertion is true and $E = -K = +\dfrac{v}{2}$

$\Rightarrow v$ is more in $n = 2$

Here reason is also true but it is not correct explanation of assertion hence correct option is (b).

10. Here assertion is false but reason is true. Hence correct option is (a).

Objective Questions (Level 2)

1. $F_a = F_c \Rightarrow \frac{GmM}{r^2} = \frac{mv^2}{r}$ ...(i)

and $mvr = \frac{nh}{2\pi}$ ...(ii)

From Eq. (ii)

$\Rightarrow v = \frac{nh}{2\pi mr}$

Putting this value in Eq. (i)

$\frac{GM}{r} = \frac{h^2h^2}{4\pi^2m^2r^2}$

$\Rightarrow \quad r = \frac{n^2h^2}{4\pi^2m^2GM}$

$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}\frac{m \times GM}{r} = \frac{GMm}{2r}$

$\text{PE} = \frac{-GMm}{r}$

$\Rightarrow E = \text{KE} + \text{PE}$

$\Rightarrow E = \frac{-GMm}{2r} = \frac{-GMm \times 4\pi^2m^2GM}{2n^2h^2}$

$\Rightarrow E = \frac{-2\pi^2G^2M^2m^3}{n^2h^2}$ for ground state $n = 1$

$\Rightarrow E = \frac{-2\pi^2G^2M^2m^3}{n^2}$

Hence correct option is (b).

2. We have $\mu_n = i_nA_n = \frac{e}{T_n} \times \pi r_n^2$

$\Rightarrow \mu_n = \frac{e \times \pi \times r_n^2 \times u_n}{2\pi r_n} = \frac{eu_nr_n}{2}$

$\Rightarrow \mu_n = e \times \frac{v_1}{n} \times \frac{r_1 \times n^2}{2} = \frac{ev_1r_1}{2} \times n$

$\Rightarrow \quad \mu_2 = \frac{ev_1r_1 \times 2}{2}$

and $\mu_1 = \frac{ev_1r_1}{2} \times 1$

$\Rightarrow \quad \mu_1 = \frac{\mu_2}{2}$

Hence magnetic moment decreases two times correct option is (b).

3. For H-like atom $E_n = \frac{-(13.6)Z^2\text{eV}}{n^2}$

Here $E_2 = \frac{-(13.6)z^2}{4}$ and $E_1 = -13.6\,z^2$

$E_2 - E_1 = 40.8$ eV

$\Rightarrow \quad 13.6Z^2 \times \left[1 - \frac{1}{4}\right] = 40.8$

$\Rightarrow \quad Z^2 = \frac{40.8 \times 4}{13.6 \times 3} \Rightarrow Z^2 = 4$

$\Rightarrow \quad Z = 2$

Energy needed to remove the electron from ground state is

$-E_1 = +(13.6) \times Z^2 = +13.6 \times 4 = 54.4$ eV

Hence correct option is (a).

$i_n = \frac{e}{T_n} = \frac{e}{\frac{2\pi r_n}{u_n}} = \frac{ev_n}{2\pi r_n}$

$\because \quad u_n \propto \frac{1}{n}$ and $r_n \propto \frac{1}{n^2}$

$\Rightarrow \quad i_n \propto \frac{1}{n^3} \Rightarrow \frac{i_2}{i_1} = \frac{1}{2^3}$

$\Rightarrow \quad i_1 = 8i_2$

Hence current increases 8 times correct option is (c).

5. Since five dark lines are possible hence atom is excited to $n = 6$ state.

The number of transition in emission line $= \frac{n(n-1)}{2}$

Number of emission transition $= \frac{6 \times 5}{2} = 15$

Hence correct option is (c).

6. $A_n = \pi r_n^2$ for hydrozen atom $r_n = kn^2$

where k is constant.

$\Rightarrow \quad A_n = \pi k^2 n^4$

$\Rightarrow \quad A_1 = \pi k^2 \propto \frac{A_n}{A_1} = n^4$

Taking log both sides $\log\left(\frac{A_n}{A_1}\right) = 4\log_n$

Hence it is a straight line with slope = 4

Correct option is (b)

7. For hydrogen atom $i_n \propto \frac{1}{n^3}$ and

$$B_n \propto \frac{i_n}{r_n} \propto B_n = k\frac{1}{n^5} \qquad [\because r_n \times n^2]$$

$$\Rightarrow \qquad B_2 = \frac{k}{2^5} \text{ and } B_1 = k$$

$$\Rightarrow \qquad \frac{B_2}{B_1} = \frac{1}{2^5} = \frac{1}{32}$$

$$\Rightarrow \qquad B_1 = 32\,B_2$$

Hence magnatic field increases 32 times.

The correct option is (d).

8. For H-atom Lyman series is given by

$\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{n^2}\right]$ for first line $n = 2$

$$\Rightarrow \qquad \frac{1}{\lambda} = R\left[1 - \frac{1}{2^2}\right]$$

$$\Rightarrow \qquad \frac{1}{\lambda} = \frac{3R}{4}$$

Momentum of photon $P_p = \frac{h}{\lambda}$

Let momentum of atom p_A

$\because$ Initial momentum was zero. Hence using momentum conservation law, we get

$$p_A = p_B \Rightarrow Mv = \frac{h}{\lambda} = \frac{3hR}{4}$$

$$\Rightarrow \qquad v = \frac{3hR}{4M}$$

Hence the correct option is (a).

9. Light wave equation is

$$200\text{ V/m}\sin(1.5\times10^{15}\text{sec}^{-1})\,t \times \cos(0.5\times10^{15}\text{sec}^{-1})\,t$$

Here maximum frequency $= \frac{1.5\times10^{15}}{2\pi}$

Maxmum incident energy

$$= \frac{1.5\times10^{15}}{2\pi} \times \frac{6.6\times10^{-34}}{1.6\times10^{-19}}$$

$$= 0.98\text{ eV}$$

Since work function $= 2\text{ eV} >$ maximum energy hence no emission of electrons.

Thus correct option is (d).

10. Since in Balmer series of H-like atom wavelengths (in visible region) are found same or smaller hence the gas was initially in second excited state.

Correct option is (c).

11. For H-atom $T_H = 2\pi n^3$

and for H-like atom

$$T_x = \frac{2\pi n^3}{z^2}$$

For H-atom in ground state $T_H = 2\pi$

For H-like atom in first excited state

$$T_x = \frac{(2\pi)\times 2^3}{z^2} = \frac{2\pi\times 8}{z^2}$$

But $T_H = 2T_x \Rightarrow 2\pi = \frac{2\times2\pi\times8}{z^2}$

$$\Rightarrow \qquad z^2 = 16$$

$$\Rightarrow \qquad z = 4$$

Hence correct option is (c).

12. For K_α line of X-ray

$$\frac{1}{\lambda} = \frac{a^2}{c}(z-1)^2$$

$\because z$ (atomic No.) for Pb^{204}, Pb^{206}, Pb^{208} are same hence $\lambda_1 = \lambda_2 = \lambda_3$.

Hence correct option is (c).

13. The correct option is (d).

14. Since $E_n = \frac{-13.6\text{ eV}}{n^2}$

$$\Rightarrow E_1 = -13.6\text{ eV}$$

and first excited state $E_2 = \frac{-13.6}{4}\text{eV}$

$\Rightarrow \quad E_2 = -3.4 \text{ eV}$

$\Delta E = E_2 - E_1 = 10.2 \text{ if } K < 10.2 \text{ eV}$

The electron collide elastically with H-atom in ground state.

The correct option is (c).

15. For Lyman series $\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{n^2}\right]$ here $n = 3$

$\Rightarrow \quad \frac{1}{\lambda} = R\left[1 - \frac{1}{9}\right]$

$\Rightarrow \quad \frac{1}{\lambda} = \frac{8R}{9}$

$P_{\text{photon}} = \frac{h}{\lambda} = \frac{8Rh}{9}$

But $\quad P_{\text{Photon}} = P_{\text{H-atom}}$

$\Rightarrow \quad \frac{8Rh}{9} = M_p \times v$

$\Rightarrow v = \frac{8 \times 1.097 \times 10^7 \times 6.6 \times 10^{-34}}{9 \times 1837 \times 9.1 \times 10^{-31}}$

$\Rightarrow v = 4 \text{ m/s}$

Hence correct option is (a).

16. Power $= VI = 150 \times 10^3 \times 10 \times 10^{-3} = 1500$ W

The 99% power heated the target hence

Heating power $= \frac{99}{100} \times 1500 = 15 \times 99$ W

The rate at which target is heated per sec. (in cal)

$= \frac{15 \times 99}{4.2} \simeq 355 \qquad \left[\because 1\text{J} = \frac{1}{4.2}\text{cal}\right]$

Hence correct option is (c).

17. $E_n = -\frac{13.6 \text{ eV} \times z^2}{n^2}$

$E_3 = -\frac{13.6 \text{ eV}}{9} z^2$ and $E_4 = \frac{-13.6 \text{ eV}}{16} z^2$

$\Delta E = E_4 - E_3 = (13.6) \text{ eV} \times z^2 \left[\frac{1}{9} - \frac{1}{16}\right]$

$\Rightarrow \Delta E = \frac{13.6(\text{eV}) \times 2^2 \times 7}{16 \times 9} = 32.4 \text{ eV}$

$\Rightarrow z^2 = \frac{16 \times 32.4 \times 9}{13.6 \times 7} = 49$

$\Rightarrow z = 7$

Hence correct option is (d).

18. $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_p K}} = \frac{h}{\sqrt{2m_p eV}}$

$\Rightarrow$

$10^{-13} = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 1836 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times V}}$

$\Rightarrow \quad V = 8.15 \times 10^4$ volt

Hence correct option is (b).

19. Since $E_n \propto \frac{1}{n^2}$ and $L_n \propto n$

Hence $\quad E_n \propto \frac{1}{L_{n^2}}$

The correct option is (d).

20. Since $\mu_n = \frac{e u_n r_n}{2\pi} \because u_n \propto \frac{1}{n}$ and $r_n \propto n^2$

$\Rightarrow \mu_n = kn$

Where k is constant for H-atom

For ground state $\quad \mu_1 = k \times 1 = k \qquad$ ...(i)

For third excited state $n = 4$

$\mu_2 = k \times 4 = 4k \qquad$ ...(ii)

From Eqs. (i) and (ii) we get $\mu_2 = 4\mu_1$

Hence correct option is (d).

21. By conservation of momentum

$M_H v = (M_H + M_H) v' \Rightarrow v' = \frac{v}{2}$

Let initial KE of H-atom $= K$

Final KE of each-H-atom $= \frac{K}{2}$

For excitation $\frac{K}{2} = E_2 - E_1 = \frac{-13.6}{4} + 13.6$

$\Rightarrow \quad \frac{K}{2} = 10.2 \text{ eV}$

$\Rightarrow \quad K = 20.4 \text{ eV}$

Hence correct option is (a).

22. We know that for H-like atom

$$E_n = -K_n \Rightarrow K_n = 3.4 \text{ eV}$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_e K}}$$

$$= \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 3.4 \times 1.6 \times 10^{-19}}}$$

$$\Rightarrow \lambda = 6.6 \text{ Å}$$

Hence both options (a) and (b) are correct.

Hence answer is (c) both are correct.

23-25. $$0.6e = \frac{hc}{4950 \times 10^{-10}} - W \quad \text{...(i)}$$

$$1.1\,e = \frac{hc}{\lambda_2} - W \quad \text{...(ii)}$$

Subtracting Eq. (i) from Eq. (ii) we get

$$0.5\,e = hc\left[\frac{1}{\lambda_2} - \frac{1}{4950 \times 10^{-10}}\right]$$

$$\Rightarrow \frac{0.5 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34} \times 3 \times 10^8} + \frac{1}{4950 \times 10^{-10}} = \frac{1}{\lambda_2}$$

$$\Rightarrow \lambda_2 \simeq 4111 \text{ Å}$$

From Eq. (i)

$$W = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{4.95 \times 10^{-7} \times 1.6 \times 10^{-19}} \simeq 1.9 \text{ eV}$$

23. $W = 1.9$ eV

Hence correct option is (c).

24. $\lambda = 4111$ Å

Hence correct option is (c)

25. Since magnatic field does not change the KE of electrons hence retarding potential remain same.

Hence correct option is (c).

26. $(\text{KE})_{\max} = 5 \text{ eV} - 3 \text{ eV} = 2 \text{ eV}$

$$= 2 \times 1.6 \times -19 \text{ J}$$

$$\lambda_{\min} = \frac{h}{p} = \frac{h}{\sqrt{2m_e K}}$$

$$\Rightarrow \lambda_{\min} = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 2 \times 1.6 \times 10^{-19}}}$$

$$= 8.69 \text{ Å}$$

Hence correct option is (b).

27. Photo emission will stop when potential of sphere becomes stopping potential

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{q}{r} = 2 \text{ V}$$

Since $(\text{KE})_{\max} = eV_0$

hence $V_0 = 2$

$$\Rightarrow \quad q = 8\pi\varepsilon_0 r \text{ coulomb}$$

Hence correct option is (b)

28. Let t be the time for photo emission

$$\frac{1}{4\pi\varepsilon_0} \times \frac{q}{r} \times t = 2$$

$$t = \frac{8\pi\varepsilon_0 r}{q}$$

Intensity of light at 0.8 from source

$$I = \frac{3.2 \times 10^{-3}}{4\pi \times (0.8)^2} \simeq 4 \times 10^{-4} \text{ W/m}^2$$

Energy incident on the sphere in unit time

$$E_1 = \pi \times (8 \times 10^{-3})^2 \times 4 \times 10^{-4} = 8.04 \times 10^{-8} \text{ W}$$

Energy of each photon

$$E_2 = 5 \times 1.6 \times 10^{-19} = 8 \times 10^{-19} \text{ J}$$

Total number of photons incident on the sphere per second

$$\eta = \frac{E_1}{E_2} = \frac{8.04 \times 10^{-8}}{8 \times 10^{-19}} = 10^{11}$$

Since 10^6 photons emit one electron.

Hence the total number of photoelectron per sec is $$n_2 = \frac{n_1}{10^1} = \frac{10^{11}}{10^6} = 10^5$$

Therefore,

$$q = n_2 \times e \times t = 10^5 \times 1.6 \times 10^{-19} \times t$$

$$\Rightarrow \quad \frac{9 \times 10^9 \times 1.6 \times 10^{-14} \times t}{r} = 2$$

$$t = \frac{2 \times 8 \times 10^{-3}}{9 \times 1.6 \times 10^{-5}} \Rightarrow t = 111 \text{ s}$$

Hence correct option is (c).

■ More than one options are correct

1. Since $\lambda_0 = \frac{hc}{eV}$ if v increases λ_0 decreases hence the interval between $\lambda_{K\alpha}$ and λ_0 as well as $\lambda_{K\beta}$ and λ_0 increases.

The correct options are (b) and (c).

2. $R \propto n^2, V \propto \frac{1}{n}$ and $E \propto \frac{1}{n^2}$ for Bohr model of H-atom

$$\Rightarrow \qquad VR \propto n \text{ and } \frac{V}{E} \propto n$$

Hence, the correct options are (a) and (c).

3. For Bohr model of H-atom

$$L \propto n, \quad r \propto n^2 \quad \text{and} \quad T \propto n^3$$

Hence $\frac{rL}{T}$ is independent of n

$$\frac{L}{T} \propto \frac{1}{n^2} \text{ and } \frac{T}{r} \propto n, L \propto n^3$$

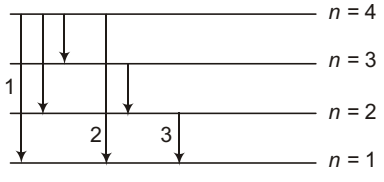
Hence correct options are (a), (b) and (c).

4. $\because \lambda = \frac{h}{mv}$ and $\lambda = \frac{h}{\sqrt{2mK}}$

Hence heavy particle has smallest wavelength when speed and KE both particle are same.

The correct options are (a) and (c).

5. Since there are six different wavelength



Hence, final state will be $n = 4$.

Since two wavelengths are longer than λ_0 [(From $n = 4 \to 3$ and $n = 3 \to 2$)]

Hence initial state was $n = 2$

and there are three transitions shown as (1), (2) and (3) belonging to Lymen series.

Hence correct options are (a), (b) and (d).

6. $\because \sqrt{f} = a(z - b)$

$\sqrt{f}$ *versus* z is a straight line

$$\because \qquad f = \frac{c}{\lambda} \Rightarrow \sqrt{\frac{1}{\lambda}} = \frac{a}{\sqrt{c}}(z - b)$$

hence $\sqrt{\frac{1}{\lambda}}$ *versus* z is a straight line

$$f = a^2(z - b)$$

$\Rightarrow \log f = \log a^2 + \log(z - b)$ which is a straight line

Hence correct options are (a), (b) and (c).

■ Match the Columns :

1. Lymen series lies in UV region, Balmer series lies in visible region and Paschen and Brackelt series lie in infrared region. Hence

(a)	r
(b)	q
(c)	p
(d)	p

2. For H-atom $E_n = \frac{-13.6}{n^2}$ eV

$$\Rightarrow \qquad E_2 = \frac{-13.6}{2^2} \text{eV} = \frac{-13.6}{4} \text{eV}$$

Ionization energy from first excited state of H-atom

$$E = -E_2 = \frac{13.6}{4} \text{eV} \qquad \ldots\text{(i)}$$

For He^+ ion

$$E_{H(\text{He})} = \frac{-(13.6) \text{ eV}}{n^2} \times Z^2 \text{ for } He^+ \ z = 2$$

$$\Rightarrow E_n(\text{He}) = \frac{-13.6 \text{ eV}}{n^2} \times 4$$

Ionization energy of He^+ atom from ground state $= (13.6)\text{eV} \times 4 = 4E \times 4$ from Eq. (i)

$$= 16E$$

$$E_2(\text{He}) = \frac{-(13.6)\,\text{eV} \times 4}{4} = -(13.6)\,\text{eV}$$

But $E_2 = -K_2 \Rightarrow K_2 = (13.6)$

and $\overline{U} = -2K = -2 \times (13.6) = -2 \times 4E = -8E$

From Eq. (i)

KE in ground state of He^+ ion

$$= (13.6)\,\text{eV} \times 4$$

$$= 4E \times 4 = 16E$$

Ionisation energy from Ist executive state

$$= -\left(\frac{-13.6\,\text{eV}}{4}\right) \times 4 = 13.6\,\text{eV} = 4E$$

Hence correct match are

(a) $\longrightarrow$	s
(b) $\longrightarrow$	r
(c) $\longrightarrow$	s
(d) $\longrightarrow$	p

3. $K_{\max} = h\nu - W$ and $V_0 = \frac{h}{e}\nu - \frac{W}{e}$

Slope of line –1 is h, $Y_1 = W$

Slope of line –2 is $\frac{h}{e}$, $Y_2 = \frac{W}{e}$

Hence correct match are

(a) $\longrightarrow$	q
(b) $\longrightarrow$	p
(c) $\longrightarrow$	r
(d) $\longrightarrow$	s

4. $T = \frac{2\pi r}{v} \because r \propto \frac{n^2}{z} \Rightarrow U \propto \frac{z}{n}$

$$\Rightarrow \quad T \propto \frac{n^3}{z^2}$$

$\Rightarrow (a) \rightarrow r$

$\because \quad L \propto n \Rightarrow (b) \rightarrow s$

$\because \quad V \propto \frac{z}{n} \Rightarrow (c) \rightarrow s$

$\because \quad R \propto \frac{n^2}{z} \Rightarrow (d) \rightarrow q$

5. Balmer series is

$$\frac{1}{\lambda_B} = R\left[\frac{1}{2^2} - \frac{1}{n^2}\right] n = 3, 4, 5 \ldots$$

For 2nd line $n = 4$ and $\lambda_B = \lambda$

$$\Rightarrow \quad \frac{1}{\lambda} = \left(\frac{1}{4} - \frac{1}{16}\right)R$$

$$\Rightarrow \quad \lambda = \frac{16}{3R}$$

$$\frac{1}{\lambda_{B_1}} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$$

$$\Rightarrow \quad \lambda_{B_1} = \frac{36}{5R} = \frac{36}{5\left(\frac{16}{3\lambda}\right)} = \frac{36 \times 3\lambda}{16 \times 5}$$

$$\Rightarrow \quad \lambda_{B_1} = \left(\frac{27}{20}\right)\lambda$$

$\Rightarrow (a) \rightarrow p$

$$\frac{1}{\lambda_{B_3}} = R\left[\frac{1}{2^2} - \frac{1}{5^2}\right] = \frac{R \times (25-4)}{25 \times 4} = \frac{21R}{100}$$

$$\Rightarrow \lambda_{B_3} = \frac{100}{21R} = \frac{100}{21 \times \left(\frac{16}{3\lambda}\right)}$$

$$= \frac{100 \times 3\lambda}{21 \times 16} = \frac{25\lambda}{28}$$

$\Rightarrow (b) \rightarrow (s)$

For Lyman series

$$\frac{1}{\lambda_L} = R\left[\frac{1}{1^2} - \frac{1}{n^2}\right] \quad n = 2, 3, 4$$

$$\Rightarrow \frac{1}{\lambda_{L_1}} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right]$$

$$\Rightarrow \lambda_{L_1} = \frac{4}{3R}$$

$$\Rightarrow \lambda_{L_1} = \frac{5}{\left(\frac{16}{\lambda}\right)} = \frac{\lambda}{4} \Rightarrow (c) \rightarrow q$$

$$\frac{1}{\lambda_{L_2}} = R\left[1 - \frac{1}{9}\right] = \frac{8R}{9}$$

$$\Rightarrow \lambda_{L_2} = \frac{9}{8R} = \frac{9}{8 \times \frac{16}{3\lambda}} = \frac{9 \times 3\lambda}{16 \times 8} = \frac{27\lambda}{128}$$

$\Rightarrow (d) \rightarrow s$

6. The proper match are

(a)$\rightarrow s$

$\because$ X-ray is inverse process of photoelectric effect [high energy electrons convert in electromagnetic radiation]

$\Rightarrow$ (b) $\rightarrow p$

$\because \quad \lambda_c \propto \frac{1}{V} \Rightarrow$ (c)$\rightarrow q$

$\because$ Wavelength of continuos X-ray depends on voltage

$\Rightarrow$ (d) $\rightarrow q$

7. $\because (\text{KE})_{\max} \propto f$ and stopping potential $\propto f$

$\Rightarrow$ (a) $\rightarrow p, r$

$\because$ stopping potental $\propto f$ hence it remains same

$\Rightarrow$ (b)$\rightarrow s$

$\because$ Current is directly proportional to Intensity. Hence sat current increases but stopping potential does not chase

$\Rightarrow$ (c) $\rightarrow q, s$

$\because (\text{KE})_{\max} = hf - W$ and stopping potential

$$= \left(\frac{hf}{e} - \frac{W}{e}\right)$$

If W is decreased $(\text{KE})_{\max}$ and stopping potential increased

$\Rightarrow$ (d) $\rightarrow p, r$

Modern Physics II

Introductory Exercise 30.1

1. $R_0 = \lambda N_0 = 8000\,\text{Bq}$

$$R_1 = \lambda N = 1000\,\text{Bq}$$

$$\Rightarrow \quad \frac{R_1}{R_0} = \frac{1}{8} = \frac{N}{N_0} = e^{-\lambda t} \Rightarrow \log 8 = \lambda t$$

$$\Rightarrow \quad 3\log 2 = \frac{\log 2}{T_{1/2}} \times 9$$

$$\Rightarrow \quad T_{1/2} = \frac{9}{3} = 3 \text{ days}$$

Average, life $= 1.44 \times T_{1/2} = 1.44 \times 3 = 4.33$ days

2. $R_0 = \lambda N_0$

$$\Rightarrow \quad 40 \times 3.7 \times 10^{10} \times 10^{-6} = \frac{0.693}{64.8} \times N_0$$

$$\Rightarrow \quad N_0 = \frac{40 \times 64.8 \times 3.7 \times 10^4}{0.693}$$

$$= 13.83 \times 10^7$$

Now, $\quad N = N_0 e^{-\lambda t}$

$$N_{10} = N_0 e^{\frac{-0.693 \times 10}{64.8}}$$

$$= 13.83 \times 10^7 \times e^{\frac{-0.693 \times 10}{64.8}}$$

$$N_{12} = 13.83 \times 10^7 \times e^{\frac{-0.693 \times 12}{64.8}}$$

$$\Rightarrow \quad N_{10} - N_{12} = 13.83 \times 10^7 \left[e^{\frac{-0.693 \times 10}{64.8}} - e^{\frac{-0.693 \times 12}{64.8}} \right]$$

$$\approx 9.47 \times 10^9 \text{ nuclei}$$

3. (a) $\quad R_0 = 10\,\text{mCi}, R = 8\,\text{mCi} \Rightarrow \frac{R_0}{R} = e^{\lambda t}$

$$\Rightarrow \quad \log \frac{10}{8} = \lambda \times 4 \times 3600$$

$$\Rightarrow \quad \lambda = \frac{0.223}{4 \times 3600}$$

$$\Rightarrow \quad \lambda = 1.55 \times 10^{-5}/\text{s}$$

$$T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{1.55 \times 10^{-5}} = 12.4 \text{ h}$$

(b) $R_0 = 10\,\text{mci} = 10 \times 3.7 \times 10^{10} \times 10^{-3}\,\text{Bq}$

$$= 3.7 \times 10^8 \text{ Bq}$$

$$R_0 = \lambda N_0$$

$$\Rightarrow N_0 = \frac{R_0}{\lambda} = \frac{3.7 \times 10^8}{1.55 \times 10^{-5}}$$

$$= 2.39 \times 10^{13} \text{ (atoms)}$$

(c) $R = R_0 e^{-\lambda t}$

$$= 10\,\text{mCi} \times e^{-\frac{0.693}{12.4} \times 30} = 1.87 \text{ mCi}$$

4. $R_0 = \lambda N_0$

$$\Rightarrow \lambda = \frac{R_0}{N_0} = \frac{6 \times 10^{11}\text{Bq}}{10^{15}}$$

$$= 6 \times 10^{-4}/\text{s}$$

$$T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{6 \times 10^{-4}} = 1.16 \times 10^3 \text{s}$$

5. $N_x = N_y = N_0$

$T_{1/2x} = 50\,\text{min}$ and $T_{1/2y} = 100\,\text{min}$

$$N_x = (N_0)\left(\frac{1}{2}\right)^n = N_0\left(\frac{1}{2}\right)^4 = \frac{N_0}{16}$$

$$N_y = N_0\left(\frac{1}{2}\right)^n = N_0\left(\frac{1}{2}\right)^2 = \frac{N_0}{4}$$

$$\frac{N_x}{N_y} = \frac{\frac{N_0}{16}}{\frac{N_0}{4}} = \frac{1}{4}$$

Introductory Exercise 30.2

1. $\Delta E = \Delta Mc^2$

Here, $P = 10^9$ J/s $= 10^9 \times 24 \times 60$ J/day

$$\Rightarrow \Delta M = \frac{10^9 \times 24 \times 60}{(3 \times 10^8)^2} = 9.6 \times 10^{-4} \text{ kg}$$

2. Number of fission $= \dfrac{10^9 \text{ J/s}}{200 \times 10^6 \times 1.6 \times 10^{-19}}$

$$= 3.125 \times 10^{19}$$

3. Given reaction is

$${}_{92}U^{238} \longrightarrow {}_{90}Th^{234} + {}_2He^4$$

$\Delta M = (238.050784 - 234.043593 - 4.002602)$ u

$\Delta M = 0.0004589$ u

$\Delta E = \Delta M \times 931.5$ MeV

$= 0.0004589 \times 931.5$

$= 4.27$ MeV

4. Complete reactions are

(a) ${}_3Li^6 + {}_1H^2 \rightarrow {}_4Be^7 + {}_0n^1$

(b) ${}_{17}Cl^{35} + {}_1H^1 \rightarrow {}_{16}S^{32} + {}_2He^4$

(c) ${}_4Be^9 + {}_2He^4 \rightarrow 3({}_2He^4) + {}_0n^1$

(d) ${}_{35}Br^{79} + {}_1H^2 \rightarrow {}_{36}Kr^{79} + 2({}_0n^1)$

AIEEE Corner

■ Subjective Questions (Level 1)

Radioactivity

1. (a) Initially the rate of distingration is

$$-\left(\frac{dN}{dt}\right)_0 = \lambda N_0$$

After 5 min $\dfrac{-dN}{dt} = \lambda N$

$$\Rightarrow \frac{N_0}{N} = \frac{\left(\frac{dN}{dt}\right)_0}{\frac{dN}{dt}} = \frac{4750}{2700} = 1.76$$

Now $N = N_0 e^{-\lambda t}$ or $\lambda = -\dfrac{\log\left(\frac{N}{N_0}\right)}{t}$

$$\Rightarrow \lambda = \frac{2.3026}{t}\log_{10}\frac{N_0}{N}$$

$$= \frac{2.3026}{5}\log_{10}(1.76)$$

$\Rightarrow \lambda = 0.113$ /min

(b) Half-life $= \dfrac{0.693}{\lambda} = \dfrac{0.693}{0.113} = 6.132$ min

2. We have $A = \lambda N$

$$6 \times 10^{11} = \lambda \times 1 \times 10^{15}$$

$$\Rightarrow \lambda = 6 \times 10^{-4} \text{ s}^{-1}$$

$$T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{6 \times 10^{-4}} = 1155 \text{ s}$$

$$= 19.25 \text{ min}$$

3. $A = \lambda N$

$A = 8$ Ci $= 8 \times 3.7 \times 10^{10}$ decay/s

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{5.3 \text{ yr}}$$

$$= \frac{0.693}{5.3 \times 365 \times 24 \times 60 \times 60 \text{ s}}$$

$$\Rightarrow N = \frac{A}{\lambda}$$

$$= \frac{8 \times 3.7 \times 10^{10} \times 5.3 \times 365 \times 24 \times 3600}{0.693}$$

$$\Rightarrow N = 7.2 \times 10^{19}$$

$$6.023 \times 10^{23} \text{ nuclei} = 60 \text{ g}$$

$$1 = \frac{60 \text{ g}}{6.023 \times 10^{23}}$$

Hence 7.2×10^{19} nuclei

$$= \frac{60 \times 7.2 \times 10^{19}}{6.023 \times 10^{23}} = 7.11 \times 10^{-3} \text{ g}$$

4. Number of decay per second

$$\frac{m}{M} \times N_A \times \lambda = \frac{1}{238} \times 6 \times 10^{23} \times \frac{0.693}{4.5 \times 10^9 \text{ yr}}$$

$$= \frac{6 \times 10^{23}}{238} \times \frac{0.693}{4.5 \times 10^9 \times 365 \times 24 \times 60 \times 60 \text{s}}$$

$$= 1.23 \times 10^4 \text{ decay/s}$$

5. Probability of decay

$$P = (1 - e^{-\lambda t}) = (1 - e^{-t/T_{\text{mean}}})$$

$$\Rightarrow P = (1 - e^{-5/10}) = 1 - e^{-2} = 0.39$$

6. Since initially no ^{206}Pb nuclei is present and after time t the ratio of $\frac{N_v}{N_{\text{Pb}}} = 3$

It means $\frac{1}{4}$ of original ^{238}U nuclei decays.

Hence $$N = N_0 e^{-\lambda t}$$

$$\left(N_0 - \frac{N_0}{4}\right) = N_0 e^{-\lambda}$$

$$\frac{3}{4} = e^{-\lambda t}$$

$$\Rightarrow \quad t = \frac{\log 4 - \log 3}{\lambda}$$

$$\Rightarrow \quad t = \frac{(\log 4 - \log 3)}{0.693} \times 4.5 \times 10^9 \text{ yr}$$

$$\Rightarrow \quad t = 1.88 \times 10^9 \text{ yr}$$

7. $(R_1)_{0\text{P}_{32}} = \lambda_1 4 N_0$

$$(R_2)_{0\text{P}_{33}} = \lambda_2 N_0$$

$$R_0 = (R_1)_{0\text{P}_{32}} + (R_2)_{0\text{P}_{33}}$$

$$= N_0 (4\lambda_1 + \lambda_2) = 3 \text{ mCi}$$

$$(R_1)_{t\text{P}_{32}} = \lambda_1 N = \lambda_1 4 N_0 e^{-\lambda_1 t}$$

$$(R_1)_{t\text{P}_{32}} = \lambda_1 \times 4 N_0 e^{-\frac{\log 2}{14} \times 60 \times 365}$$

$$(R_2)_{t\text{P}_{33}} = \lambda_2 \times N_0 e^{-\frac{\log 2}{25} \times 60 \times 365}$$

$$R = (R_1)_{t\text{P}_{32}} + (R_2)_{t\text{P}_{33}}$$

$$= N_0 \left[4\lambda_1 e^{-\frac{\log 2}{14} \times 60 \times 365} + \lambda_2 e^{-\frac{\log 2}{25} \times 60 \times 365} \right]$$

$$= \frac{3 \text{ mCi}}{(4\lambda_1 + \lambda_2)} \left[4\lambda_1 e^{-\frac{\log 2 \times 60 \times 365}{14}} + \lambda_2 e^{-\frac{\log 2}{25} \times 60 \times 365} \right]$$

$$= \frac{3 \text{ mCi}}{\left[\frac{4 \times \log 2}{14} + \frac{\log 2}{25}\right]} \left[\frac{4 \log 2}{14} e^{-\frac{\log 2 \times 60 \times 365}{14}} + \frac{\log 2}{25} \times e^{-\frac{\log 2}{25} \times 60 \times 365} \right]$$

$$\approx 0.205 \text{ mCi}$$

8. Complete reactions are

(a) $_{88}\text{Ra}^{226} \longrightarrow \alpha + {}_{86}\text{RN}^{222}$

(b) $_{8}\text{O}^{19} \longrightarrow {}_{9}\text{F}^{19} + e + \bar{\nu}$

(c) $_{13}\text{Al}^{25} \longrightarrow {}_{12}\text{Mg}^{25} + e^{+} + \nu$

9. Only reaction (b) is possible.

10. $\Delta E = (7 \times 1.000783 + 7 \times 1.00867 - 14.00307) \times 931.5 \text{ MeV}$

$$\Rightarrow \Delta E = 104.72 \text{ MeV}$$

11. $\Delta E = [8m_p + 8m_n - m(_8O^{16})] \times 931.5$

$$= (8 \times 1.007825 + 8 \times 1.008665 - 15.994915) \times 931.5 = 127.6 \text{ MeV}$$

12. (a) Number of nuclei in kg

$$= \frac{6.023 \times 10^{23}}{235} \times 1$$

Energy

$$= \frac{6.023 \times 10^{23}}{235} \times 200 \times 10^6 \times 1.6 \times 10^{-19}$$

$$= 8.09 \times 10^{13} \text{J}$$

(b) Mass $= \dfrac{8.09 \times 10^{13}\,\text{J}}{30 \times 10^{3}\,\text{J/g}}$

$= \dfrac{8.09 \times 10^{13}}{30 \times 10^{3}}\,\text{g}$

$= \dfrac{8.09}{3} \times 10^{9} \times \dfrac{1}{10^{3}}\,\text{kg}$

$= 2.7 \times 10^{6}\,\text{kg}$

13. Applying conservation of momentum

$$M_\alpha v_\alpha = M_{\text{Ti}} v_{\text{Ti}}$$

$$\Rightarrow \quad v_{\text{Ti}} = \frac{M_\alpha v_\alpha}{M_{\text{Ti}}}$$

$$K_\alpha = \frac{1}{2} M_\alpha v_\alpha^2 = 6.802\,\text{MeV}$$

$$K_{\text{Ti}} = \frac{1}{2} M_{\text{Ti}}\, v_{\text{Ti}}^2 = \frac{1}{2} M_{\text{Ti}} \frac{M_\alpha^2 v_\alpha^L}{M_{\text{Ti}}^2}$$

$$\Rightarrow \quad K_{\text{Ti}} = \frac{M_\alpha}{M_{\text{Ti}}}\left(\frac{1}{2} M_\alpha v_\alpha^2\right)$$

$$= \frac{4}{208} \times 6.802\,\text{MeV}$$

$$= \frac{1}{52} \times 6.802 = 0.1308\,\text{MeV}$$

14. Power $= 100\,\text{MW} = 10^8\,\text{W} = 10^8\,\text{J/s}$

$$= \frac{10^8}{1.6 \times 10^{-13}\,\text{J/MeV}} = \frac{10^{21}}{1.6}\,\frac{\text{MeV}}{\text{s}}$$

Energy per fission $= 185\,\text{MeV}$

Hence number of fissions $= \dfrac{10^{21}}{1.6 \times 185}$ / s.

Number of nuclei in 1 kg

$U^{235} = \dfrac{6.023 \times 10^{26}}{235}$

Hence $t = \dfrac{6.023 \times 10^{26} \times 1.6 \times 185}{235 \times 10^{21}}$

$= 8.78$ days

15. (a) The given reaction is

$${}_1H^2 + {}_1H^2 \longrightarrow {}_1H^3 + {}_1H^1$$

$$\Delta M = [2m({}_1H^2) - m({}_1H^3) - m({}_1H^1)]$$

$$= [2(2.014102) - 3.016049 - 1.007825]u$$

$$\Delta M = 0.000433\ \text{u}$$

$$Q = \Delta M \times 931.5 = 0.000433 \times 931.5$$

$$\approx 4.05\,\text{MeV}$$

(b) $\Delta M = (2 \times 2.014102 - 3.016049 - 1.008665) \times 931.5$

$\Delta M = 3.25\,\text{MeV}$

(c) $\Delta M = (2.014102 + 3.016049 - 4.002603 - 1.008665) \times 931.5$

$\Delta M = 17.57\ \text{MeV}$

16. The given reaction is

$$He^4 + He^4 \longrightarrow Be^8$$

$$\Delta M = (2 \times 4.0026 - 8.0053)\ \text{u}$$

$$= (8.0052 - 8.0053)\ \text{u}$$

$$\Delta M = -(0.0001)\ \text{u}$$

$\because \Delta M$ is negative this reaction is not energetically favourable

$$\Delta E = \Delta M \times 931.5$$

$$= -1 \times 10^{-4} \times 931.5\,\text{MeV}$$

$$= -93.15\ \text{keV}$$

17. Number of nuclei in 1 kg water

$$= \frac{6.023 \times 10^{26}}{18}$$

Heavy water $= \dfrac{6.023 \times 10^{26}}{18} \times \dfrac{1.5 \times 10^{-2}}{100}$

$= \dfrac{6.023 \times 1.5}{18} \times 10^{22}$

Energy realesed per fission

$= (2 \times 2.014102 - 3.016049 - 1.007825) \times 931.5$

$= 4.33 \times 10^{-3} \times 931.5 \times 10^{6} \times 1.6 \times 10^{-19}\,\text{J}$

Hence total energy $= \dfrac{6.023 \times 1.5 \times 10^{22}}{18}$

$\times\ 4.33 \times 931.5 \times 1.6 \times 10^{-16}$

$= 3200\,\text{MJ}$

■ Objective Questions (Level-1)

1. Since during β^- decay a neutron in the nucleus is transformed into a proton, an electron and an antineutrino as $n \to P + e^- + \bar{\gamma}$.

Hence Correct option is (c).

2. Since nuclear force is same for all nucleons. Hence $F_1 = F_2 = F_3$

Correct option is (a)..

3. Given reaction is ${}_{90}X^{200} \longrightarrow {}_{80}Y^{168}$

Difference in mass number $= 200 - 168 = 32$

Hence Number of α-particles $= \frac{32}{4} = 8$

Difference in atomic number $= 10$

hence number of β-particles $= 6$

Hence correct option is (d).

4. The reaction is

${}_{92}U^{235} + {}_0n^1 \longrightarrow {}_{54}Xe^{138} + {}_{38}Sr^{94} + 3({}_0n^1)$

The correct option is (b) three neutrons.

5. The reactions are $A \to \beta + \alpha$ and $\beta \to C + 2\beta$

After one α atomic number reduced by 2 and after 2β atomic number increased by.

Hence A and C are isotopes Correct option is (d).

6. Here $m_p = 1.00785$ u, $m_n = 1.00866$ u

and $m_\alpha = 4.00274$ u

$$\Delta m = 2(m_p + m_n) - m_\alpha$$

$$\Rightarrow \Delta m = [2(1.00785 + 1.00866) - 4.00274]\text{ u}$$

$$\Rightarrow \Delta m = 0.03028 \text{ u}$$

$$\Delta E = \Delta m \times 931.5 \text{ MeV}$$

$$= 0.3028 \times 931.5 = 28.21 \text{ MeV}$$

Hence correct option is (c).

7. $N = N_0 - \frac{7}{8}N_0 = \frac{1}{8}N_0$

But $N = N_0\left(\frac{1}{2}\right)^n \Rightarrow \frac{1}{8}N_0 = N_0\left(\frac{1}{2}\right)^n$

$\Rightarrow \quad n = 3$

$\Rightarrow \quad 3 \times T_{1/2} = 8$ s

$\Rightarrow \quad T_{1/2} = \frac{8}{3}$ s

Hence correct option is (d).

8. $N = N_0 e^{-\lambda t}$ for mean life $t = \frac{1}{\lambda}$

$\Rightarrow N = N_0 e^{-\lambda \times \frac{1}{\lambda}} = \frac{N_0}{e}$

Hence the fraction disintegrated

$$= \frac{N_0 - N}{N_0} = \left(1 - \frac{1}{e}\right)$$

Correct option is (b).

9. $N = N_0 - \frac{7}{8}N_0 = \frac{1}{8}N_0$

But $N = N_0\left(\frac{1}{2}\right)^n$

$\Rightarrow \quad \frac{1}{8} = \left(\frac{1}{2}\right)^n \Rightarrow n = 3$

Hence half life $T_{1/2} = \frac{15 \text{ min}}{3} = 5$ min

Correct option is (a).

10. Since radioactive substance loses half of its activity in 4 days it means its half life

$$T_{1/2} = 4 \text{ days}$$

Now $A = 5\%$ of A_0

$\Rightarrow \quad A = \frac{1}{20}A_0$

$\Rightarrow \quad \frac{A}{A_0} = \frac{1}{20}$

But $\quad A = A_0 e^{-\lambda t} \Rightarrow \frac{1}{20} = e^{-\lambda t}$

$\Rightarrow \quad \log 20 = \lambda t \Rightarrow t = \frac{\log 20}{\lambda}$

But $\lambda = \dfrac{\log_e 2}{T_{1/2}}$

$\Rightarrow t = T_{1/2} \dfrac{\log_e 20}{\log_e 2} = T_{1/2} \dfrac{\log_{10} 20}{\log_{10} 2} = 4.32 \times 4$

$\Rightarrow t = 17.3$ days

Hence correct option is (c).

11. Total energy released per sec

$$= 1.6\,\text{MW} = 1.6 \times 10^6\,\text{J/s}$$

Energy released per fission $= 200$ MeV

$= 200 \times 10^6 \times 1.6 \times 10^{-19}$ J

$= 2 \times 1.6 \times 10^{-11}$ J

Number of fission per second

$$= \frac{1.6 \times 10^6}{2 \times 1.6 \times 10^{-11}} = 5 \times 10^{16}/\text{s}$$

Hence correct option is (a).

12. $\because R = R_0 A^{1/3}$

Volume $= \dfrac{4}{3}\pi R^3 = \dfrac{4}{3}\pi R_0^3 A$

Mass of nucleus $= A \times 1.67 \times 10^{-27}$ kg

Density $\rho = \dfrac{\text{mass}}{\text{volume}} = \dfrac{A \times 1.67 \times 10^{-27}\text{ kg}}{\dfrac{4}{3}\pi R_0^3 A}$

$\Rightarrow \quad \rho = \dfrac{1.67 \times 10^{-27}\text{ kg}}{\dfrac{4}{3}\pi R_0^3}$

$\because \rho$ is independent of A, hence ratio of densities $\dfrac{\rho_1}{\rho_2} = 1$.

Correct option is (d).

13. $N = N_0 e^{-\lambda t} \Rightarrow \dfrac{N}{N_0} = e^{-\lambda t}$

$\Rightarrow \quad \dfrac{N}{N_0} = e^{-\frac{0.693}{6.93} \times 10} = e^{-1} = \dfrac{1}{e}$

Fractional change

$$= \frac{N_0 - N}{N_0} = \left(1 - \frac{1}{e}\right) \simeq 0.63$$

Hence correct option is (b).

14. Since radioactive substance reduce to about 6% it means $N = \dfrac{N_0}{16}$

We have $\quad N = (N_0)\left(\dfrac{1}{2}\right)^n$

$\Rightarrow \quad \dfrac{1}{16} = \left(\dfrac{1}{2}\right)^n \Rightarrow n = 4$

$4 \times T_{1/2} = 2\,h \Rightarrow T_{1/2} = 30$ min

Hence correct option is (a).

15. Probability of a nucleus for survival of time t.

$$p_{(\text{survival})} = \frac{N}{N_0} = \frac{N_0\, e^{-\lambda t}}{N_0} = e^{-\lambda t}$$

For one mean life $t = \dfrac{1}{\lambda}$

$\Rightarrow P_{\text{survival}} = e^{-\lambda \times \frac{1}{\lambda}} = e^{-1} = \dfrac{1}{e}$

Hence Correct option is (a).

■ Assertion and Reason

1. Here both assertion and reason are true but reason does not explain assertion. Hence correct option is (b).

2. Here assertion is false but reason is true since for heavier nucleus binding energy per nucleon is least.

Correct option is (d).

3. Here both assertion and reason are true but reason is not correct explanation of assertion. Hence correct option is (b).

4. Here a ssertion is true but reason is false since electromagnetic waves are produced by accelerating charge particles.

Correct option is (c).

5. Here assertion is wrong since β-decay process is $n \rightarrow p + e^- + \bar{v}$

but reason is true hence correct option is (d).

6. Here assertion is true but reason is false. Correct option is (c).

7. Here both assertion and reason are true and reason may or may not be true. Correct option is (a, b)

8. Both assertion and reason are true but reason is not correct explanation of assertion. Hence correct option is (b).

9. Here reason is true but assertion is false

$\because$ 1 amu = 931.5 MeV

Correct option is (d)

10. Both assertion and reason are true but reason does not correctly explain assertion. Hence correct option is (b).

11. Here both assertion and reason are true and reason may or may not be correct explanation of assertion .

Hence correct options are (a, b).

Objective Questions (Level 2)

■ Single option correct

1. Let initially substance have N_i nuclei then

$$N = N_i\, e^{-\lambda t}$$

$$\frac{dN}{dt} = -\lambda N_i\, e^{-\lambda t}$$

At $t = t$

we get

$$\left(\frac{dN}{dt}\right)_{t=t} = -\lambda N_i\, e^{-\lambda t} = N_0 \quad \ldots\text{(i)}$$

At $t = 4t$

$$\left(\frac{dN}{dt}\right)_{t=4t} = -\lambda N_i\, e^{-4\lambda t} = \frac{N_0}{16} \quad \ldots\text{(ii)}$$

Dividing Eq. (i) and Eq. (ii) we get

$$e^{3\lambda t} = 16 \quad \ldots\text{(iii)}$$

Now at $t = \left(\frac{11}{2}\right)t$

$$\left(\frac{dN}{dt}\right)_{t=\frac{11t}{2}} = -\lambda N_i\, e^{\frac{-11\lambda}{2}t}$$

$$= -\lambda N_i\, e^{\frac{-8\lambda t}{2}} \times e^{\frac{-3\lambda t}{2}}$$

$$= \frac{-\lambda N_i\, e^{-4\lambda t}}{\sqrt{e^{3\lambda t}}} = \frac{N_0}{16 \times \sqrt{16}}$$

From Eqs. (ii) and (iii)

$$= \frac{N_0}{64}$$

Hence, correct option is (b).

2. We have $\lambda = \lambda_1 + \lambda_2 = \frac{\log 2}{30} + \frac{\log 2}{60}$

$$\Rightarrow \quad \lambda = \frac{\log 2}{20}$$

Now $\quad N = N_0\, e^{-\lambda t}$

$$\Rightarrow \quad \frac{N_0}{4} = N_0\, e^{\frac{-t \log 2}{20}}$$

$$\Rightarrow \quad \log 4 = \frac{t}{20}\log 2$$

$$\Rightarrow \quad 2\log 2 = \frac{t}{20}\log 2$$

$$\Rightarrow \quad t = 40\text{ yr}$$

Hence correct option is (c).

3. From graph it is clear that number of nucleons in X is N_3 and binding energy per nucleon is E_3 for Y nucleon is N_2 and BE per nucleon is E_2.

Hence $X + Y = E_3N_3 + E_2N_2$

Similerly $W = E_1N_1$

The reaction is $W \rightarrow X + Y$

The energy released is
$(E_3N_3 + E_2N_2 - E_1N_1)$

Hence Correct option is (b).

4. Energy $= (110 \times 8.2 + 90 \times 8.2 - 200 \times 7.4)$

$$= 200 \times (8.2 - 7.4)$$

$$= 200 \times 0.8 = 160 \text{ MeV}$$

Hence correct option is (d).

5. The reaction is

$_1H^2 + {}_1H^2 \rightarrow {}_2He^4 + \text{energy}$

Energy $= (4 \times 7 - 2 \times 1.1)$ MeV

$= (28 - 4.4) = 23.6$ MeV

Hence Correct option is (b).

6. Total energy released per second

$$= 16 \times 10^6 \text{ W}$$

$$= 16 \times 10^6 \text{ J/s}$$

Energy per fission $= 200$ MeV

$$= 200 \times 10^6 \times 1.6 \times 10^{-19}$$

$$= 2 \times 1.6 \times 10^{-11} \text{ J}$$

$\because$ Efficiency $= 50\%$

Hence power (energy converted per second)

$$= 2 \times 1.6 \times 10^{-11} \times \frac{50}{100} = 1.6 \times 10^{-11} \text{ J}$$

Number of fission $= \dfrac{16 \times 10^6}{1.6 \times 10^{-11}} = 10^{18}$ /s

Hence correct option is (d).

7. $\dfrac{dN}{dt} = A - \lambda N$

$\because$ After time N become

conservation $\Rightarrow \dfrac{dN}{dt} = 0$

$$\Rightarrow \quad N = \frac{A}{\lambda} = \frac{A}{\frac{\log 2}{T}} = \frac{AT}{\log 2}$$

Hence correct option is (d).

8. By conservation of momentum

$$M_H v = (M_H + M_H) v'$$

$$\Rightarrow \quad v' = \frac{v}{2}$$

Let initial KE of H-atom $= K$

Final KE of each H-atom $= \dfrac{K}{2}$

For excitation

$$\frac{K}{2} = E_2 - E_1 = \left(\frac{-13.6}{4} + 13.6\right) \text{eV}$$

$$\Rightarrow \quad \frac{K}{2} = 10.2 \text{ eV}$$

$$\Rightarrow \quad K = 2 \times 10.2 \times 1.6 \times 10^{-19}$$

$$\Rightarrow \quad \frac{1}{2} M_H u^2 = 2 \times 10.2 \times 1.2 \times 1.6 \times 10^{-19}$$

$$\Rightarrow \quad u_H = \sqrt{\frac{2 \times 1.2 \times 10.2 \times 2 \times 1.6 \times 10^{-19}}{1.673 \times 10^{27}}}$$

$$= 6.25 \times 10^4 \text{ m/s}$$

Hence correct option is (c).

9. Let us suppose just before the death no radioactive atoms were present hence original activity A_0 is given as

$$A_0 = \lambda N_0 \qquad \ldots\text{(i)}$$

After death the radioactivity decreases exponentially *ie*,

$$A = \frac{dN}{dt} = \lambda N = \lambda N_0 e^{-\lambda t} \qquad \ldots\text{(ii)}$$

Dividing eq. (ii) by eq. (i) we get

$$\frac{A}{A_0} = e^{-\lambda t}$$

or $\lambda t = \log \dfrac{A_0}{A}$ or $t = \dfrac{1}{\lambda} \log \dfrac{A_0}{A}$

Now $A_0 = 15$ decay/min/gram

$$A = \frac{375}{200} \text{ decay/min/g}$$

but $\quad \lambda = \dfrac{0.693}{5730 \text{ yr}}$

$$t = \frac{5730}{0.693} \times \log \frac{15 \times 200}{375} = \frac{5730}{0.693} \times \log \frac{200}{25}$$

$$\Rightarrow \quad t = \frac{5730}{0.693} \times \log 8$$

$$= \frac{5730}{0.693} \times 3 \log 2 = 5730 \times 3$$

$$\Rightarrow \quad t = 17190 \text{ yr}$$

Hence correct option is (c).

10. $N_P = N_0 \, e^{-\lambda(t_1 + t)}$ ...(i)

and $N_Q = N_0 \, e^{-\lambda t}$

Now $A_P = \lambda N_P$ and $A_Q = \lambda N_Q$

$$\Rightarrow \quad \frac{A_P}{A_Q} = \frac{N_P}{N_Q} = e^{-\lambda t_1}$$

$$\Rightarrow \quad \lambda t_1 = \log\left(\frac{A_Q}{A_P}\right)$$

$$\Rightarrow \quad t_1 = \frac{1}{\lambda} \log\left(\frac{A_Q}{A_P}\right) = T \log\left(\frac{A_Q}{A_P}\right)$$

Hence correct option is (b).

11. The given reactions are

$${}_1H^2 + {}_1H^2 \rightarrow {}_1H^3 + p$$

$${}_1H^2 + {}_1H^3 \rightarrow {}_2He^4 + n$$

$$\Rightarrow \; 3\,{}_1H^2 \rightarrow {}_2He^4 + n + p$$

Mass defect

$$\Delta m = (3 \times 2.014 - 4.001 - 1.007 - 1.008)$$

$$\Delta m = 0.026 \text{ amu}$$

Energy Released

$$= 0.026 \times 931 \text{ MeV} = 3.87 \times 10^{-12} \text{ J}$$

This energy produced by the three deutronatoms. Total energy released by 10^{40} deutrons

$$= \frac{10^{40}}{3} \times 3.87 \times 10^{-12} \text{ J} = 1.29 \times 10^{28} \text{ J}$$

The average power $P = 10^{16}$ W $= 10^{16}$ J/s

Therefore total time to exhaust all deutrons of the star will be

$$t = \frac{1.29 \times 10^{28}}{10^{16}} \text{ s} \approx 10^{12} \text{ s}.$$

Hence correct option is (c).

12. $N_1 = N_0 \, e^{-\lambda_1 t} = N_0 \, e^{\frac{-\log 2}{t_1} t}$ ...(i)

$N_2 = N_0 \, e^{-\lambda_2 t} = N_0 \, e^{\frac{-\log 2}{t_2} t}$ ...(ii)

$R_1 = \lambda_1 N_1$ ...(iii)

and $R_2 = \lambda_2 = \lambda_2 N_2$ ...(iv)

Let after time t, $R_1 = R_2$ then

$$\Rightarrow \quad \frac{R_1}{R_1} = 1$$

$$\Rightarrow \quad \frac{\lambda_1}{\lambda_2} \times \frac{N_1}{N_2} = 1$$

$$\Rightarrow \quad \frac{\lambda_1}{\lambda_2} = \frac{N_2}{N_1} = e^{\log 2 \left[\frac{t_2 - t_1}{t_1 t_2}\right] \times t}$$

$$\Rightarrow \quad \log \frac{t_2}{t_1} = t \times \log 2 \times \left(\frac{t_2 - t_1}{t_1 t_2}\right)$$

$$\Rightarrow \quad t_1 = \frac{t_1 t_2}{0.693\,(t_2 - t_1)} \times \log \frac{t_2}{t_1}$$

Hence Correct option is (a).

13. The given reaction is ${}_Z X^{232} \longrightarrow {}_{90} Y^A + \alpha$

$$\Rightarrow \quad {}_Z X^{232} \longrightarrow {}_{90} Y^A + {}_2He^4$$

$$\Rightarrow \quad Z = 92 \text{ and } A = 228$$

$\because$ Initially X is in rest hence momentum of α-particle after decay will be equal and opposite of Y.

$$\Rightarrow \quad M_Y v_Y = M_\alpha v_\alpha$$

$$\Rightarrow \quad v_Y = \frac{M_\alpha}{M_Y} v_\alpha$$

Total kinetic energy

$$K_T = \frac{1}{2}(M_\alpha v_\alpha^2 + M_Y v_Y^2)$$

$$\Rightarrow \quad K_T = \frac{1}{2}\left[M_\alpha v_\alpha^2 + M_y \frac{M_\alpha^2 v_\alpha^2}{M_Y^2}\right]$$

$$\Rightarrow \quad K_T = \frac{1}{2} M_\alpha v_\alpha^2 \left[1 + \frac{M_\alpha}{M_Y}\right]$$

$$\Rightarrow \quad K_T = K_\alpha \left[1 + \frac{4}{228}\right]$$

$$\Rightarrow \quad K_\alpha = \frac{232}{228} K_T$$

Hence Correct option is (b).

14. Energy of emitted photon $= 7$ MeV

$$= 7 \times 10^6 \times 1.6 \times 10^{-19} \text{ J}$$

$$= 11.2 \times 10^{-13} \text{J}$$

Momentum of photon $= \dfrac{11.2 \times 10^{-13} \text{J}}{3 \times 10^8 \text{ m/s}}$

$$= \frac{11.2}{3} \times 10^{-21} \text{ kg-m/s}$$

$\because$ Initial nucleus is stationary

Applying conservation of momentum principle

$$O = \vec{\mathbf{P}}_{\text{nuc}} + \vec{\mathbf{P}}_{\text{photon}} \Rightarrow \vec{\mathbf{P}}_{\text{nuc}} = -\vec{\mathbf{P}}_{\text{photon}}$$

$$\Rightarrow |\vec{\mathbf{P}}_{\text{nuc}}| = |-\vec{\mathbf{P}}_{\text{photon}}|$$

$$\Rightarrow P_{\text{nuc}} = \frac{11.2}{3} \times 10^{-21} \text{ kg-m/s}$$

Mass of nucleus $= 24$ amu

$$= 24 \times 1.66 \times 10^{-27} \text{ kg}$$

But $\quad P_{\text{nuc}}^2 = 2mK_{\text{nuc}}$

$\Rightarrow$

$$K_{\text{nuc}} = \frac{P_{\text{nuc}}^2}{2m} = \frac{11.2 \times 11.2 \times 10^{-42}}{9 \times 2 \times 24 \times 1.66 \times 10^{-27}} \text{ Joule}$$

$\Rightarrow$

$$K_{\text{nucc}} = \frac{11.2 \times 11.2 \times 10^{-42}}{18 \times 24 \times 1.66 \times 10^{-27} \times 1.6 \times 10^{-19}} \text{ eV}$$

$$\approx 1.1 \text{ keV}$$

Hence correct option is (b).

15. Let time interval between two instants is t_1 then

$$N_1 = N_0 \, e^{-\lambda(t + t_1)}$$

and $\quad N_2 = 2N_0 \, e^{-\lambda t}$

$$A_1 = N_1\lambda = \lambda N_0 \, e^{-\lambda(t + t_1)}$$

$$A_2 = N_2\lambda = \lambda(2N_0) \, e^{-\lambda t}$$

$$\Rightarrow \quad \frac{A_1}{A_2} = \frac{1}{2} \times e^{-\lambda t_1}$$

$$\Rightarrow \quad \frac{2A_1}{A_2} = e^{-\lambda t_1}$$

$$\Rightarrow \quad \log\left(\frac{A_2}{2A_1}\right) = \lambda t_1$$

$$\Rightarrow \quad t_1 = \frac{1}{\lambda} \log\left(\frac{A_2}{2A_1}\right)$$

$$\Rightarrow \quad t_1 = \frac{T}{\log 2} \log\left(\frac{A_2}{2A_1}\right)$$

Hence Correct option is (c).

16. The given reaction is

$${}_1\text{H}^2 + {}_1\text{H}^2 \longrightarrow {}_1\text{H}^3 + {}_1\text{H}^1$$

$$\Delta M = [2m({}_1\text{H}^2) - m({}_1\text{H}^3) - m({}_1\text{H}^1)]$$

$$= (2 \times 2.014102 - 3.016049 - 1.007825) \text{ amu}$$

$$= 4.33 \times 10^{-3} \text{amu}$$

$$\Delta E = 4.33 \times 10^{-3} \times 931.5 \text{ MeV}$$

$$= 4 \text{ MeV}$$

Hence correct option is (c).

17. Number of fusion required to generate 1 kWh

$$= \frac{1 \times 10^3 \times 3600}{4 \times 10^6 \times 1.6 \times 10^{-19}}$$

$$= \frac{36 \times 10^{18}}{6.4} = 5.6 \times 10^{18} \approx 10^{18}$$

Hence correct option is (b).

18. The energy released $= 4$ MeV

This energy produced by two atoms.

Hence energy produced per atom

$$= 2 \text{ MeV} = 2 \times 1.6 \times 10^{-13} \text{J}$$

Hence number of atom fused to produced 1 kWJ

$$= \frac{36 \times 10^5}{2 \times 1.6 \times 10^{-13}} = \frac{18}{1.6} \times 10^{18}$$

Mass of deutrium which contain

$$\frac{18}{1.6} \times 10^{18} \text{ atom}$$

$$= \frac{18 \times 10^{18}}{1.6 \times 6.02 \times 10^{23}} = 3.7 \times 10^{-5} \text{ kg.}$$

Hence correct option is (c).

■ More Than one Option is Correct

1. $x = N_0,\ y = \lambda N_0$

$$\Rightarrow \quad \frac{x}{y} = \frac{N_0}{\lambda N_0} = \frac{1}{\lambda}$$

where λ is decay constant

Hence $\frac{x}{y}$ is constant throught.

$$\because \quad \frac{x}{y} = \frac{1}{\lambda} = \frac{1}{\frac{0.693}{T}} = \frac{T}{0.693}$$

$$\Rightarrow \quad \frac{x}{y} > T,\ xy = \lambda (N_0)^2$$

For one half life $N = \frac{N_0}{2}$

$$\Rightarrow (xy)_T = \lambda \left(\frac{N_0}{2}\right)^2 = \frac{\lambda N_0^2}{4} = \frac{xy}{4}$$

Hence correct options are (a), (b) and (d).

2. The correct options are (a), (b), (c) and (d).

3. A nucleus in excited state emits a high energy photon called as γ-ray. The reaction is

$$X^* \longrightarrow X + \gamma$$

Hence by gamma radiation atomic number and mass number are not changed. Since after emission of one α atomic number reduced by $2({}_2\alpha^4)$ and after 2β atomic number is increased by (2). Hence correct options are (a), (b) and (c).

4. Here half lives are T and $2T$ and $N_x = N_0$, $N_y = N_0$ after $4T$ for first substance = 4 half lives and after $4T$ the second substance = 2 Half lines.

$$\Rightarrow \quad N_x = N_0 \left(\frac{1}{2}\right)^4 = \frac{N_0}{16}$$

$$N_y = N_0 \left(\frac{1}{2}\right)^2 = \frac{N_0}{4}$$

$$\Rightarrow \quad x = \frac{N_x}{N_y} = \frac{N_0/16}{N_0/4} = \frac{1}{4}$$

Let their activity are R_x and R_y.

$$\Rightarrow \quad R_x = \lambda_x N_x \text{ and } R_y = \lambda_y N_y$$

$$\Rightarrow \quad y = \frac{R_x}{R_y} = \frac{\lambda_x}{\lambda_y} \times \frac{N_x}{N_y} = \frac{\frac{0.693}{T}}{\frac{0.693}{2T}} \times \frac{1}{4}$$

$$\Rightarrow \quad \frac{R_x}{R_y} = y = \frac{1}{2}$$

Hence correct options are (b) and (c).

5. Since nuclear forces are vary short range charge independent, no electromagnetic and they exchange ($n \to p$ or $p \to n$). Hence the correct options are (a), (b), (c) and (d).

6. $\because R = R_0 A^{1/3}$

$$\rho = \frac{M}{4/3\ \pi R^3} = \frac{A \times 1.67 \times 10^{-27} \text{ kg}}{\frac{4}{3} \pi R_0^3 A}$$

$\Rightarrow$ ρ is independent of A.

$$\text{But } \rho = \frac{1.67 \times 10^{-27} \text{ kg}}{\frac{4}{3} \times 3.14 \times (1.3 \times 10^{-15})^3}$$

$$= 1.8 \times 10^{17} \text{ kg/m}^3$$

Hence correct options are (b) and (c).

Match the Columns

1. Here $N_0 = x$, let λ is the decay constant.

$\Rightarrow \; N = N_0 e^{-\lambda t} = x e^{-\lambda t}$

$\left|\frac{dN}{dt}\right| = \lambda x e^{-\lambda t}$ but $\left|\frac{dN}{dt}\right|_{t=0} = y$

$\Rightarrow \qquad y = \lambda x e^0 = \lambda x$

$\Rightarrow \qquad \lambda = \frac{y}{x}$

Hence

(a) $\longrightarrow$ s

Half life $T_{1/2} = \frac{\log 2}{\lambda} = \frac{\log 2}{\frac{y}{x}} = \frac{x}{y}\log 2$

Hence

(b) $\longrightarrow$ p

We have activity $R = \lambda N = \lambda x e^{-\lambda t}$

at $t = \frac{1}{\lambda}$

$R = \lambda x e^{-\lambda \times \frac{1}{\lambda}} = \lambda x e^{-1} = \frac{\lambda x}{e}$

but $\lambda x = y$

$\Rightarrow \qquad R = \frac{y}{e}$

Hence

(c) $\longrightarrow$ r

Number of nuclei after time $t = \frac{1}{\lambda}$

$N = x e^{-\lambda \times \frac{1}{\lambda}} = \frac{x}{e}$

Hence

(d) $\longrightarrow$ s

Thus correct match is

(a) $\longrightarrow$ s

(b) $\longrightarrow$ r

(c) $\longrightarrow$ r

(d) $\longrightarrow$ s

2. In reaction $P + P \rightarrow Q$ energy is released

$\because$ Binding energy increases when two or more lighter nucleus combine to form haviour nucleus.

Hence correct match is

(a) $\longrightarrow$ p

Similerly for reaction

$P + P + R \rightarrow R$

Correct match is

(b) $\longrightarrow$ p

for reaction

$P + R = 2Q$ from graph BE per nucleon increases.

Hence energy is released.

Correct match is

(c) $\longrightarrow$ p

For the reaction

$P + Q = R$

We will check energy process if BE per nucleon is given. Hence data is not sufficient correct match is

(d) $\longrightarrow$ s

3. Since A and B are radioactive nuclei of $(A + B)$ decreases with time. Hence correct match is

(a) $\longrightarrow$ q

$\because A$ is converted into B and B is converted into C and decay rate of $A \rightarrow B$ and $B \rightarrow C$ are not known. Hence correct match is

(b) $\longrightarrow$ s

Since at time passes A is converted to B and B is converted to C. Hence nuclei of $(B + C)$ increases. Correct match is

(c) $\longrightarrow$ p

Similerly the correct match for (d) is

(d) $\longrightarrow$ s

4. After emission of 1α particle mass no decreased by 4 but after emission of 1β particle atomic number will increase or decrease by 1. Hence for (a)

(a) $\longrightarrow$ p, s

(b) $\longrightarrow$ p, r

(c) $\longrightarrow$ s

(d) $\longrightarrow$ q, r

5. For

(a) $\longrightarrow$ p

Since BE per nucleon of heavy nuclei is about 7.2 MeV. Hence

(b) $\longrightarrow$ s

X-ray photon have wavelength about 1 Å the energy of this wavelength is of order of 10 keV.

(c) $\longrightarrow$ r

$\because$ Visible light energy of order of ≈ 2 eV. Hence

(d) $\longrightarrow$ q