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NEWTON CLASSES

JEE (MAIN & ADV.), MEDICAL
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PHYSICS

For Joint Entrance Examination (JEE)

Electrostatics and Current Electricity

B.M. Sharma

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Preface

While the paper-setting pattern and assessment methodology have been revised many times over and newer criteria devised to help develop more aspirant-friendly engineering entrance tests, the need to standardize the selection processes and their outcomes at the national level has always been felt. A combined national-level engineering entrance examination has finally been proposed by the Ministry of Human Resource Development, Government of India. The Joint Entrance Examination (JEE) to India's prestigious engineering institutions (IITs, IIITs, NITs, ISM, IISERs, and other engineering colleges) aims to serve as a common national-level engineering entrance test, thereby eliminating the need for aspiring engineers to sit through multiple entrance tests.

While the methodology and scope of an engineering entrance test are prone to change, there are two basic objectives that any test needs to serve:

1. The objective to test an aspirant's caliber, aptitude, and attitude for the engineering field and profession.
2. The need to test an aspirant's grasp and understanding of the concepts of the subjects of study and their applicability at the grassroot level.

Students appearing for various engineering entrance examinations cannot bank solely on conventional shortcut measures to crack the entrance examination. Conventional techniques alone are not enough as most of the questions asked in the examination are based on concepts rather than on just formulae. Hence, it is necessary for students appearing for joint entrance examination to not only gain a thorough knowledge and understanding of the concepts but also develop problem-solving skills to be able to relate their understanding of the subject to real-life applications based on these concepts.

This series of books is designed to help students to get an all-round grasp of the subject so as to be able to make its useful application in all its contexts. It uses a right mix of fundamental principles and concepts, illustrations which highlight the application of these concepts, and exercises for practice. The objective of each book in this series is to help students develop their problem-solving skills/accuracy, the ability to reach the crux of the matter, and the speed to get answers in limited time. These books feature all types of problems asked in the examination—be it MCQs (one or more than one correct), assertion-reason type, matching column type, comprehension type, or integer type questions. These problems have skillfully been set to help students develop a sound problem-solving methodology.

Not discounting the need for skilled and guided practice, the material in the books has been enriched with a number of fully solved concept application exercises so that every step in learning is ensured for the understanding and application of the subject. This whole series of books adopts a multi-faceted approach to mastering concepts by including a variety of exercises asked in the examination. A mix of questions helps stimulate and strengthen multi-dimensional problem-solving skills in an aspirant.

It is imperative to note that this book would be as profound and useful as you want it to be. Therefore, in order to get maximum benefit from this book, we recommend the following study plan for each chapter.

Step 1: Go through the entire opening discussion about the fundamentals and concepts.

Step 2: After learning the theory/concept, follow the illustrative examples to get an understanding of the theory/concept.

Overall the whole content of the book is an amalgamation of the theme of physics with ahead-of-time problems, which equips the students with the knowledge of the field and paves a confident path for them to accomplish success in the JEE.

With best wishes!

B.M. Sharma

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CHAPTER

1

Coulomb's Laws and Electric Field

- Electric Charge
- Charging of a Body
- Work Function of a Body
- Properties of Electric Charge
- Coulomb's Law
- Coulomb's Law in Vector Form
- Electric Field
- Different Patterns of Electric Field Lines
- Field of Ring Charge
- Field of Uniformly Charged Disk
- Field of Two Oppositely Charged Sheets
- Electric Dipole
- Electric Field Due to a Dipole
- Electric Field Intensity Due to a Short Dipole at Some General Point
- Dipole in a Uniform Electric Field

ELECTRIC CHARGE

Electric charge, like mass, is one of the fundamental attributes of the particle of which the matter is made of. Charge is the physical property of certain fundamental particles (such as, electron, proton) by virtue of which they interact with the other similar fundamental particles.

- Charge is an intrinsic property of some fundamental particles which accompanies these particles wherever they exist.
- Charge is that property of a body/particle which is responsible for 'electrical force' between them.

To distinguish the nature of interaction, charges are divided into two parts: (i) positive (ii) negative

Figure 1.1 shows an experiment to demonstrate that there are two types of charges.

We know that matter consists of atoms. An atom consists of a central core (called nucleus) and electrons. Electrons orbit around the nucleus. Nucleus consists of neutrons and protons. Neutrons do not contain any net charge. Protons and electrons have equal charges, but of opposite nature. Protons are positively charged, whereas electrons are negatively charged. Protons, however, are very heavy when compared with electrons, about 1836 times. Protons are imprisoned in the nucleus along with neutrons due to the strongest binding force existing in nature called 'strong or nuclear force'. Thus, protons do not travel from atom to atom. The outermost electrons may travel from atom to atom. Hence, we say that electrons are the basis of electricity.

Charge on a proton or on an electron is of indivisible nature. We designate this charge as $+e$ and $-e$, respectively. Hence, charge in or on any object is always an integral multiple of the electronic charge.

In a normal atom:

i. Number of protons are equal to number of electrons.
ii. Protons have the basic $+e$ charge and electrons have the basic $-e$ charge.
iii. Hence, a normal atom is electrically neutral.

Electrons can travel from one atom to another and from one body to another. If a body loses one electron, it becomes positively charged with $+e$ charge and vice versa.

A body, however, cannot lose or gain any proton, which is heavy and remains imprisoned in the nucleus, by ordinary methods.

Note: ***Basic unit of charge = e, whose magnitude is equal to the magnitude of charge on an electron or proton, i.e., $e = 1.6 \times 10^{-19}$ C.***

S.I. unit of charge: As mentioned, $e = 1.6 \times 10^{-19}$ C. Here, e stands for one electronic charge which is the basic unit of charge; C stands for coulomb (note the small c in coulomb). coulomb is the S.I. unit of charge.

CHARGING OF A BODY

Ordinarily, matter contains equal number of protons and electrons. A body can be charged by the transfer of electrons or redistribution of electrons.

A body can be charged by the transfer of electrons and by the transfer of protons. Why?

It is because protons are inside the nucleus and it is very difficult to remove them from there. Electrons lie in the outer shells and it is easier to remove them.

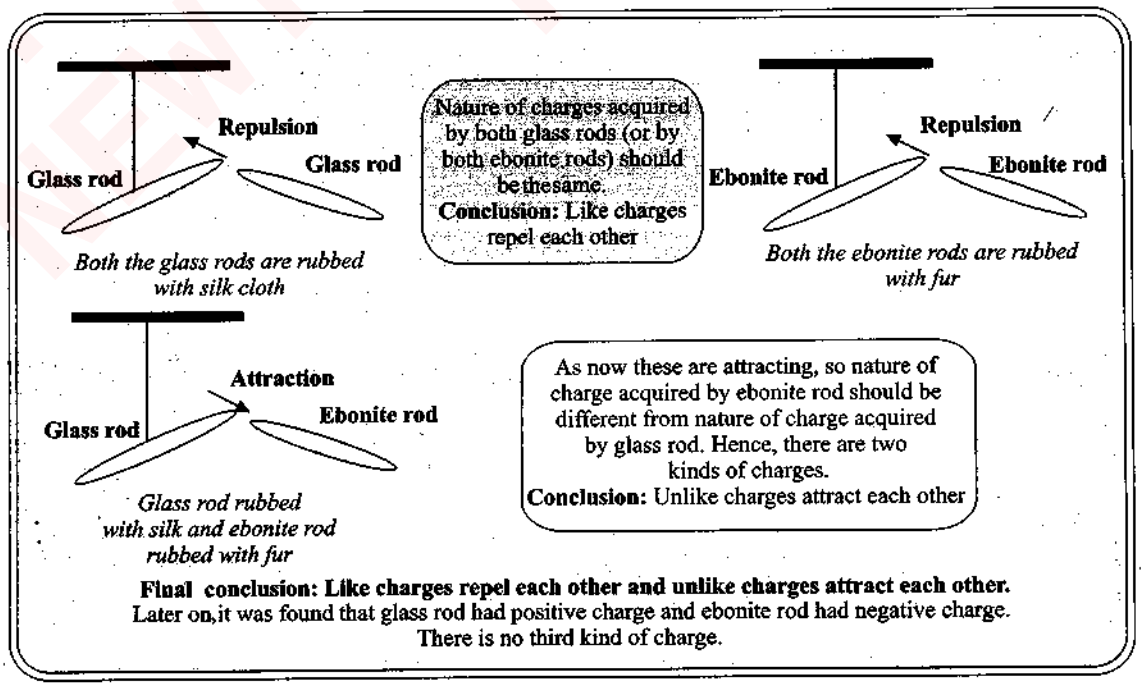


Fig. 1.1

To charge a body negatively: some electrons are given to it.

To charge a body positively: some electrons are taken from it.

WORK FUNCTION OF A BODY

It is the amount of work to be done on a body in order to remove an electron from its surface. Obviously, it is easier to remove an electron from a body whose work function is lower.

Let us see how bodies get charged due to friction:

As shown in Fig. 1.2, let $W_2 > W_1$. Now, suppose A and B are rubbed together.

Net transfer of electrons will take place from A to B.

It is because electrons in A are loosely bound as work function of A is less than B.

It is to be noted that mass is also affected during charging.

(Mass of negatively charged body increases and that of positively charged body decreases.)

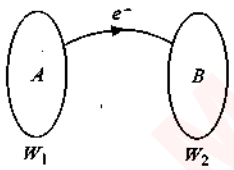


Fig. 1.2

Basically charging can be done by three methods:

1. Friction, **2.** Conduction, and **3.** Induction

Charging by Friction

When two bodies are rubbed together, electrons are transferred from one body to the other making one body positively charged and the other negatively charged.

Example: When a glass rod is rubbed with silk, the rod becomes positively charged, whereas silk gets negatively charged. However, ebonite on rubbing with wool becomes negatively charged making the wool positively charged.

Charging by Conduction

The process of charging from an already charged body can happen by either conduction or induction. Conduction from a charged body involves transfer of like charges. A positively charged body can cause more bodies to get positively charged, but the sum of the total charge on all positively charged bodies will be the same as charge on initially considered charged body.

Charging by Induction

Induction is a process by which a charged body can be used to charge neutral bodies without touching them or losing its own charge. If a charged body is brought near a neutral body, the charged body attracts opposite charge and repels similar charge present on the neutral body. If the neutral body is now earthed, the like charge is neutralized by the flow of charge from earth, leaving unlike charge on the body. Now, the earthing and the charging body is removed leaving the initially neutral body charged. The whole process is as shown in Fig. 1.3.

PROPERTIES OF ELECTRIC CHARGE

Quantization of Charge

Charge exists in discrete packets rather than in continuous amount, i.e., charge on any body is the integral multiple of the charge on an electron or proton.

$$Q = \pm ne, \text{ where } n = 0, 1, 2, \ldots$$

Conservation of Charge

Charge is conserved, i.e., total charge on an isolated system is constant. By isolated system, we here mean a system through the boundary of which no charge is allowed to escape or enter. This

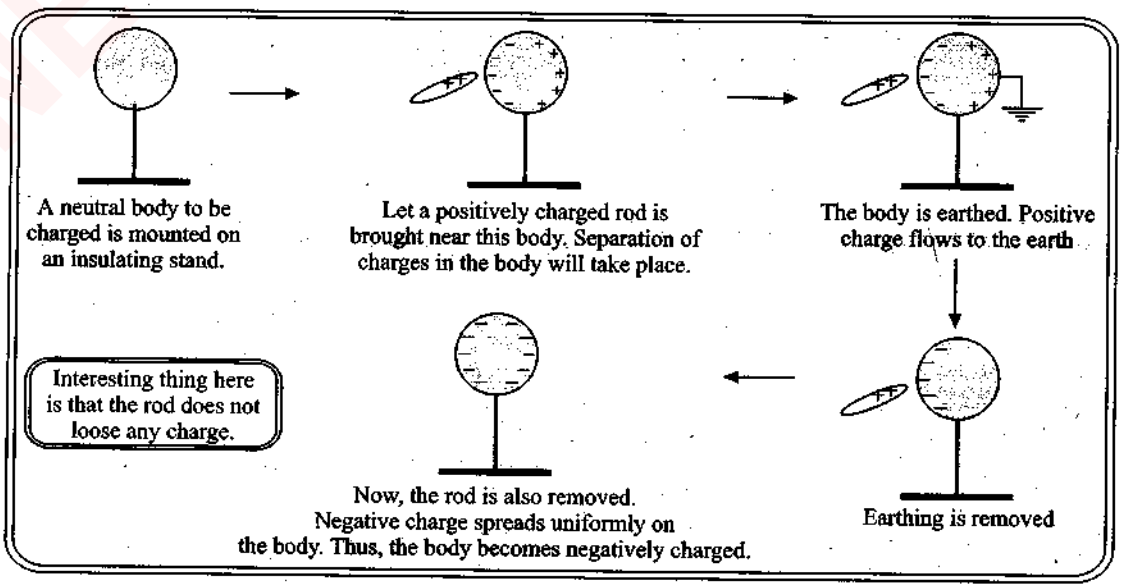


Fig. 1.3

does not require that the amounts of positive and negative charges are separately conserved.

Additivity of Charge

Total charge on a body is the algebraic sum of all the charges located anywhere on the body. While adding the charges, their sign must be taken into consideration.

For example, if a body has charges 2 C, −5 C, 4 C and 6 C (Fig. 1.4), then the total charge on the body is $2-5+4+6=7$ C.

Note that charges are added like real numbers. They have no direction. *So, charge is a scalar quantity.*

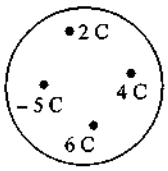


Fig. 1.4

Charge is Invariant

Charge does not depend on the speed of body.

Points to Remember

- There are two types of forces which act between two charges. If the charges are stationary, there is only one type of force between them. It is called electric or electrostatic force. It is given by Coulomb's law for point charges. If the charges are moving, then two types of forces act between them. The first one is electric force. The other force which emerges due to motion is called magnetic force. We shall study magnetic force the following chapter.
- **Charge produces electric and magnetic fields and radiates energy:** A stationary charged particle produces only electric field in the space surrounding it. A charged particle moving without acceleration produces electric as well as magnetic field. A charged particle in accelerated motion radiates energy as well, in the form of electromagnetic waves.

Illustration 1.1 **A glass rod is rubbed with a silk cloth. The glass rod acquires a charge of $+19.2\times10^{-19}$C.**

1. **Find the number of electrons lost by glass rod.**
2. **Find the negative charge acquired by silk.**
3. **Is there transfer of mass from glass to silk?**

Given, $m_e = 9\times10^{-31}$ kg.

Sol.

1. Number of electrons lost by glass rod is

$$n=\frac{q}{e}=\frac{19.2\times10^{-19}}{1.6\times10^{-19}}=12$$

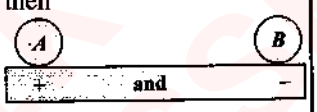
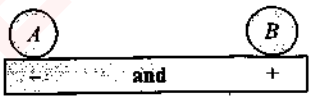
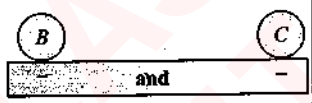
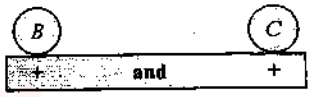
2. Charge on silk $= -19.2\times10^{-19}$ C
3. Since an electron has a finite mass ($m_e = 9\times10^{-31}$ kg), there will be transfer of mass from glass rod to silk cloth. Mass transferred $= 12\times(9\times10^{-31}) = 1.08\times10^{-29}$ kg

Note that mass transferred is negligibly small. This is expected because the mass of an electron is extremely small.

Illustration 1.2 **Electric charges *A* and *B* attract each other. Electric charges *B* and *C* repel each other. If *A* and *C* are held close together, they will**

a. attract b. repel. c. not affect each other
d. more information is needed to answer.

Sol. a.

Case 1	Case 2
If *A* and *B* attract each other, then 	If *A* and *B* repel each other, then 
If *B* and *C* attract each other, then 	If *B* and *C* repel each other, then 

From both cases, we see that *A* and *C* will be having unlike charges. Hence, if the charges *A* and *C* are held together, they will attract each other.

Illustration 1.3 **If an object made of substance *A* is rubbed with an object made of substance *B*, then *A* becomes positively charged and *B* becomes negatively charged. If, however, an object made of substance *A* is rubbed against an object made of substance *C*, then *A* becomes negatively charged. What will happen if an object made of substance *B* is rubbed against an object made of substance *C*?**

a. *B* becomes positively charged and *C* becomes positively charged.
b. *B* becomes positively charged and *C* becomes negatively charged.
c. *B* becomes negatively charged and *C* becomes positively charged.
d. *B* becomes negatively charged and *C* becomes negatively charged.

Sol. c. When *A* and *B* are rubbed, *A* becomes positively charged and *B* becomes negatively charged. It means Electrons are loosely bound with *A* in comparison to *B*. When *A* and *C* are rubbed together, *A* becomes negatively charged and *C* positively charged. It means Electrons are loosely bound with *C* in comparison to *A*. Hence, in *C*, electrons are most loosely bound. So, if *B* and *C* are rubbed together, *C* will loose electrons and *B* will receive electrons. Hence, *C* will become positively charged and *B* will become negatively charged.

Illustration 1.4 **Objects *A*, *B* and *C* are three identical, insulated, spherical conductors. Originally, *A* and *B* have**

charges of +3 mC, whereas C has a charge of −6 mC. Objects A and C are allowed to touch, then they are moved apart. Objects B and C are allowed to touch before they are moved apart.

i. If objects A and B are now held near each other, they will

a. attract b. repel c. have no effect on each other.

ii. If instead objects A and C are held near each other, they will

a. attract b. repel c. have no effect on each other.

Sol.

Initially, A (+3 mC), B (+3 mC), C (−6 mC)

- When the objects A and C are allowed to touch and then moved apart:

A C $\quad A \Longleftrightarrow C$

$[(+3\text{ mC}+(-6\text{ mC})=-3\text{ mC})]$ $\quad -\frac{3}{2}$ mC $\quad -\frac{3}{2}$ mC

- When the objects B and C are allowed to touch and then moved apart:

B C $\quad B \Longleftrightarrow C$

$\left[(+3\text{ mC})+\left(-\frac{3}{2}\text{ mC}\right)=+\frac{3}{2}\text{ mC}\right]$ $\quad +\frac{3}{4}$ mC $\quad +\frac{3}{4}$ mC

i.a. Hence, if A and B are now held near each other, they will attract each other.

ii.a. If A and C are now held near each other, they will also attract each other.

Illustration 1.5 **Figure 1.5 shows that a positively charged rod is brought near two uncharged metal spheres A and B clam red on insulated stands and placed in contact with each other.**

i. What would happen if the rod is removed before the spheres are separated?

ii. Would the induced charges be equal in magnitude even if the spheres had different sizes or different conductors?

iii. What will happen if the spheres are separated first and then the rod is removed far away?

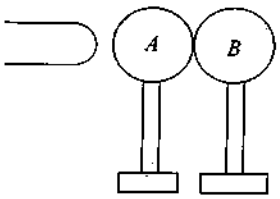


Fig. 1.5

Sol.

i. When a positively charged rod is brought near A, the free electrons in the sphere A are attracted towards the rod and moved on the left side of A. This movement leaves unbalanced positive charge on B. If the rod is removed before the spheres are separated, the excess electrons on sphere A would flow back to B. Both the spheres will become uncharged.

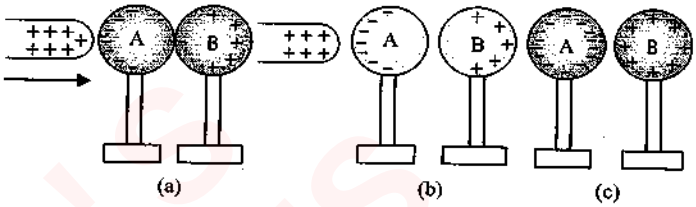


Fig. 1.6

ii. Yes, net charge is conserved. Before the rod is brought near A, both A and B were neutral. They will remain so even if they have different sizes or materials.

iii. If the rod is removed after the spheres are separated, then sphere A will have net negative charge and sphere B will have net positive charge of same magnitude as shown in Fig. 1.6.

Concept Application Exercise 1.1

1. **a.** How many electrons are in 1 coulomb of negative charge?
   b. Which is the true test of electrification, attraction or repulsion?
   c. Can a body have charge of 0.8×10^{-19} C?
2. If only one charge is available, can itself be used to obtain a charge many times greater than itself in magnitude?
3. **a.** Can two bodies having like charges attract each other? (Yes/No)
   b. Can a charged body attract an uncharged body? (Yes/No)
   c. Two identical metallic spheres of exactly equal masses are taken, one is given a positive charge q and the other an equal negative charge. Their masses after charging are different. Comment on the statement.
4. A particle has charge of $+10^{-12}$ C.
   a. Does it contain more or less number of electrons as compared with the neutral state?
   b. Calculate the number of electrons transferred to provide this charge.
5. An ebonite rod is rubbed with fur. The ebonite rod is found to have a charge of -3.2×10^{-8} C on it.
   a. Calculate the number of electrons transferred.
   b. What is the charge on fur after rubbing?
6. The electric charge of macroscopic bodies is actually a surplus or deficiency of electrons. Why not protons?
7. A charged rod attracts bits of dry paper which after touching the rod often jump away from it violently. Explain.
8. A person standing on an insulating stool touches a charged insulated conductor. Will the conductor get completely discharged?
9. An electron moves along a metal tube with variable cross section. How will its velocity change when it approaches the neck of the tube (Fig. 1.7)?

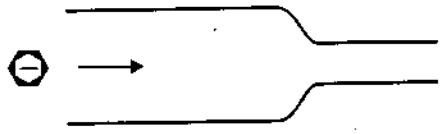


Fig. 1.7

10. Define the following statement "If there were only one electrically charged particle in the entire universe, the concept of electric charge would be meaningless".

COULOMB'S LAW

The force of interaction between two point charges is proportional to the product of magnitudes of the two charges and inversely proportional to the square of distance between them.

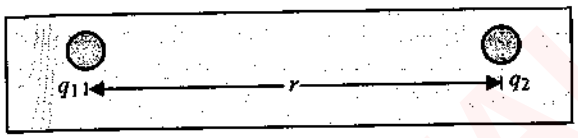


Fig. 1.8

Let two point electric charges q_1 and q_2 be at rest, separated by a distance r, then they exert a force on each other which is given by

$$F = k\frac{q_1 q_2}{r^2}$$

where k is a proportionality constant known as *electrostatic force constant*

If there is free space (or vacuum) between the two charges, then $k = \frac{1}{4\pi\varepsilon_0} = 9\times10^9\ \text{Nm}^2\ \text{C}^{-2}$ (in SI units)

where $\varepsilon_0 = \frac{1}{4\pi k} = 8.85\times10^{-12}\ \text{C}^2\text{N}^{-1}\text{m}^{-2}$ is the absolute electric permittivity of the free space.

So, force between two charges is given as

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \quad \text{(i)}$$

Equation (i) is applicable only for point charges placed in vacuum. Now, what happens if the two charges are placed in some medium? In a medium, the force is given as

$$F' = k'\frac{q_1 q_2}{r^2} \quad \text{(ii)}$$

where $k' = \frac{1}{4\pi\varepsilon}$ and in this ε is known as absolute electrical permittivity of medium. Then,

$$F' = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2} \quad \text{(iii)}$$

The ratio $\varepsilon/\varepsilon_0 = \varepsilon_r$ is known as relative electrical permittivity of medium. It is also known as *dielectric constant* and denoted by K.

So, $\varepsilon/\varepsilon_0 = \varepsilon_r = K$

The value of K for different materials: vacuum = 1, air = 1.006, glass = 3 to 4, water = 81, conductor = ∞.

In general, $K \geq 1$.

Now, from Eqs. (i) and (iii), $\frac{F'}{F} = \frac{\varepsilon_0}{\varepsilon} = \frac{1}{K} \Rightarrow F' = \frac{F}{K}$. It means when the charges are placed in a medium, the force decreases K times. Also, $K = F/F'$.

So, *the dielectric constant of a medium may be defined as the ratio of force between two charges when they are placed in vacuum to that when they are placed in that medium at same separation.*

Note:

- ***Coulomb's law is not valid for distances < 10^{-15} m.***
- ***Electrostatic forces are comparatively stronger than gravitational forces. Can you show this?***

(As an example—when we hold a book in our hand, electric force between hand and the book is sufficient to balance the gravitational force of earth on the book due to entire earth.)

Some Important Points

- Coulomb's law is applicable only for point charges.
- Coulomb's law is similar to Newton's gravitational law and both obey inverse square law.
- Coulomb's law obeys Newton's third law, i.e., the forces exerted by the two charges on each other are equal and opposite.
- This force acts along the line joining the two particles (called central force).
- Electrostatic force is a conservative force.

COULOMB'S LAW IN VECTOR FORM

Let q_1 and q_2 be two like charges placed at points A and B, respectively, in vacuum.

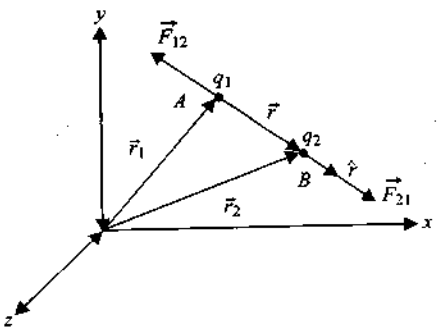


Fig. 1.9

$\vec{r}_1$ is the position vector of point A and $\vec{r}_2$ is the position vector of point B.

Let $\vec{r}$ is vector from A to B, then $\vec{r} = \vec{r}_2 - \vec{r}_1$ and $r = |\vec{r}_2 - \vec{r}_1|$.

$$\Rightarrow \quad \hat{r} = \frac{\vec{r}}{r} = \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|}$$

Let $\vec{F}_{21}$ be the force on charge q_2 due to q_1; and $\vec{F}_{12}$ be the force on charge q_1 due to q_2.

From Fig. 1.9, it is clear that $\vec{F}_{21}$ and $\hat{r}$ are in the same direction, so

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}\frac{\vec{r}}{r} = \frac{q_1q_2}{4\pi\varepsilon_0}\frac{\vec{r}}{r^3}$$

$$\Rightarrow \qquad F_{21} = \frac{q_1q_2}{4\pi\varepsilon_0}\frac{\vec{r}_2-\vec{r}_1}{|\vec{r}_2-\vec{r}_1|^3}$$

The above equations give the Coulomb's law in vector form.

As we know that charges apply equal and opposite forces on each other, so we have

$$\vec{F}_{12} = -\vec{F}_{21} \Rightarrow \vec{F}_{12} = \frac{q_1q_2}{4\pi\varepsilon_0}\frac{\vec{r}_1-\vec{r}_2}{|\vec{r}_1-\vec{r}_2|^3}$$

Superposition Principle

It enables us to calculate the force acting on a charge due to more than one charge.

According to superposition principle, the total force on a given charge is vector sum of all the individual forces exerted by each of the other charges.

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots + \vec{F}_n$$

Another important point is that the force between two charges remains unaffected due to the presence of a third charge.

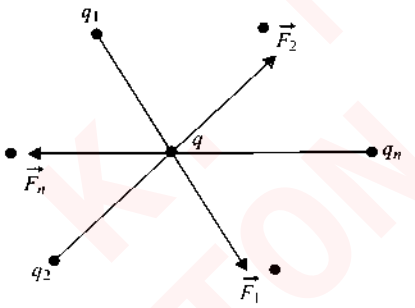
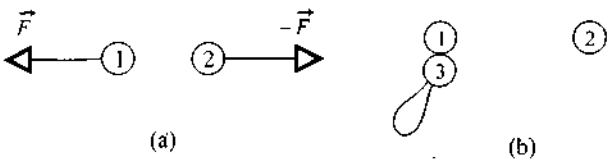


Fig. 1.10

Note:

- ***Coulomb's law and principle of superposition together can explain whole of the electrostatics.***
- ***Both Coulomb's law and Gravitational law describe inverse square law that involve a property of interacting particles—the charge in one case and mass in the other case.***

Illustration 1.6 **Two identical conducting spheres 1 and 2 carry equal amounts of charge and are fixed at a certain distance apart that is large compared with their diameters. The spheres repel each other with an electrical force of 88 mN. Suppose now that a third identical sphere 3, having an insulating handle and initially uncharged, is touched first to sphere 1 and then to sphere 2 and finally removed. Find the force between spheres 1 and 2 as shown in Fig. 1.11(d).**



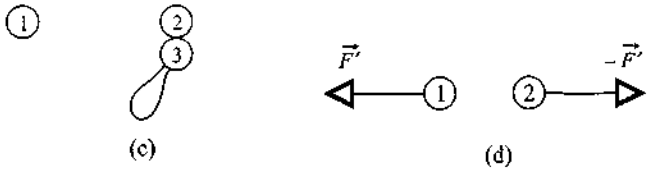


Fig. 1.11

Sol. Initial force between '1' and '2' is $F = kq^2/r^2 = 88$ mN

Charge on '1' after sphere '3' is touched with '1' is $q/2$. Same charge will be on sphere '3' also.

Charge on '2' after sphere '3' is touched with '2' is

$$\frac{q+(q/2)}{2} = \frac{3q}{4}$$

Now, force between '1' and '2' in situation (d) is

$$F' = \frac{K(q/2)(3q/4)}{r^2} = \frac{3}{8}\frac{kq^2}{r^2} = \frac{3}{8}\times 88 = 33 \text{ mN}$$

Illustration 1.7 **Two identical He-filled spherical balloons each carrying a charge q are tied to a weight W with strings and float in equilibrium as shown in Fig. 1.12(a). Find**

i. the magnitude of q, assuming that the charge on each balloon acts as if it were concentrated at the centre.

ii. the volume of each balloon.

Take density of He as ρ_{He} and density of air as ρ_a. Ignore the weight of the unfilled balloons.

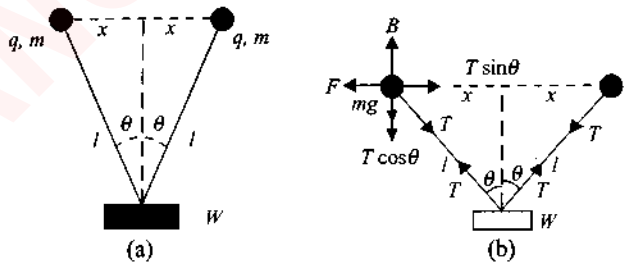


Fig. 1.12

Sol. i. $2T\cos\theta = W, T\sin\theta = F$ [Fig.1.12(b)]

$$\Rightarrow \qquad \frac{\tan\theta}{2} = \frac{F}{W} \Rightarrow F = W\frac{\tan\theta}{2}$$

$$\Rightarrow \qquad \frac{q^2}{4\pi\varepsilon_0(2x)^2} = \frac{W\tan\theta}{2} \Rightarrow q = \sqrt{8W\tan\theta\pi\varepsilon_0 x^2}$$

ii. $T\cos\theta + mg = B \Rightarrow \frac{W}{2} + V\rho_{He}g = V\rho_a g$

$$\Rightarrow \qquad V = \frac{W}{2(\rho_a - \rho_{He})g}$$

Illustration 1.8 **Two particles, each having a mass of 5 g and charge 10^{-7} C, stay in limiting equilibrium on a horizontal table with a separation of 10 cm between them. Find the coefficient of friction between each particle and the table, which is the same between each particle and table.**

Sol. Friction force f will balance the electrostatic repulsion, i.e.,

$$f = F \Rightarrow \mu mg = \frac{q^2}{4\pi\varepsilon_0 r^2}$$

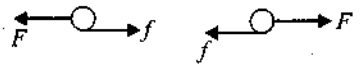


Fig. 1.13

$$\Rightarrow \quad \mu \times \frac{5}{1000} \times 10 = \frac{9\times10^9\times(10^{-7})^2}{(0.10)^2} \Rightarrow \mu = 0.18$$

Illustration 1.9 **A particle of mass m carrying a charge $-q_1$ starts moving around a fixed charge $+q_2$ along a circular path of radius r. Prove that period of revolution T of charge $-q_1$ is given by $T = \sqrt{\frac{16\pi^3\varepsilon_0 m r^3}{q_1 q_2}}$.**

Sol. Electrostatic force on $-q_1$ due to $+q_2$ will provide the necessary centripetal force, hence

$$\frac{kq_1q_2}{r^2} = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{kq_1q_2}{mr}}$$

Now, $$T = \frac{2\pi r}{v} = \sqrt{\frac{16\pi^3\varepsilon_0 m r^3}{q_1q_2}}$$

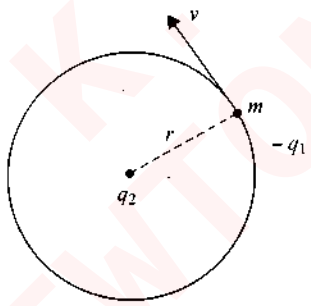


Fig. 1.14

Illustration 1.10 **Consider three charges q_1, q_2 and q_3, each equal to q, at the vertices of an equilateral triangle of side l. What is the force on a charge Q placed at the centroid of the triangle?**

Sol. Method 1. The resultant of three equal coplanar vectors acting at a point will be zero if these vectors form a closed polygon (Fig. 1.15). Hence, the vector sum of the forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ is zero.

Method 2. The forces acting on the charge Q are

$$\vec{F}_1 = \text{force on } Q \text{ due to } q_1 = \frac{1}{4\pi\varepsilon_0}\frac{Qq_1}{AO^2}\widehat{AO}$$

$$\vec{F}_2 = \text{force on } Q \text{ due to } q_2 = \frac{1}{4\pi\varepsilon_0}\frac{Qq_2}{BO^2}\widehat{BO}$$

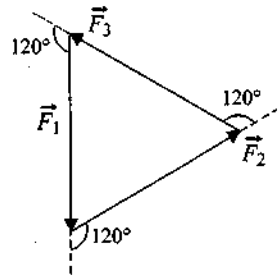


Fig. 1.15

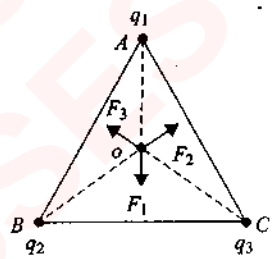


Fig. 1.16

$$\vec{F}_3 = \text{force on } Q \text{ due to } q_3 = \frac{1}{4\pi\varepsilon_0}\frac{Qq_3}{CO^2}\widehat{CO}$$

The resultant force is $\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Qq}{AO^2}(\widehat{AO} + \widehat{BO} + \widehat{CO}) = 0$$

(as $|q_1| = |q_2| = |q_3|$ and $|\overrightarrow{AO}| = |\overrightarrow{BO}| = |\overrightarrow{CO}|$)

Also, $\overrightarrow{AO} + \overrightarrow{BO} + \overrightarrow{CO} = 0$ because these are three equal vectors in a plane making angles of 120° with each other.

Method 3. The resultant force $\sum \vec{F}$ is the vector sum of individual forces, i.e.,

$$\sum \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 \quad \text{or}$$

$$\sum F_x = F_{1x} + F_{2x} + F_{3x}$$

$$= 0 + F_2\cos 30° - F_3\cos 30° \qquad \text{(i)}$$

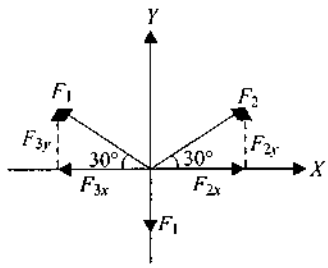


Fig. 1.17

And $\sum F_y = F_{1y} + F_{2y} + F_{3y}$

$$= -F_1 + F_2\sin 30° + F_3\sin 30° \qquad \text{(ii)}$$

As $|F_1| = |F_2| = |F_3| = |F|$ (say), Eqs. (i) and (ii) become $\sum F_x = 0$ and $\sum F_y = 0$. Hence, resultant force $\sum \vec{F} = 0$.

Illustration 1.11 **Point charges are placed at the vertices of a square of side a as shown in Fig. 1.18. What should be sign of charge q and magnitude of the ratio $|q/Q|$ so that**

i. net force on each Q is zero?

ii. net force on each q is zero?

iii. Is it possible that the entire system could be in electrostatic equilibrium?

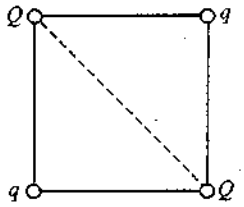
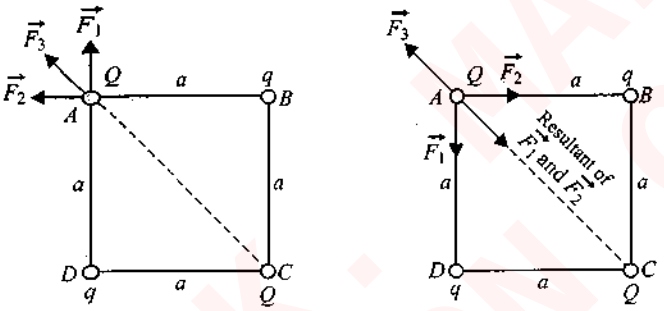


Fig. 1.18

Sol.

i. Consider the forces acting on charge Q placed at A (shown in Fig. 1.19(a) and (b))

Case 1. Let the charges q and Q are of same sign.



(a) (q and Q are of same nature) Here, net force cannot be zero.

(b) (q and Q are of opposite nature) Here, net force can be zero.

Fig. 1.19

Here, $F_1 \frac{1}{4\pi\varepsilon_0}\frac{qQ}{a^2}$ (force of q at D on Q at A)

$F_2 \frac{1}{4\pi\varepsilon_0}\frac{qQ}{a^2}$ (force of q at B on Q at A)

$F_3 \frac{1}{4\pi\varepsilon_0}\frac{QQ}{2a^2}$ (force of Q at C on Q at A)

In Fig. 1.19(a), resultant of forces $\vec{F}_1$ and $\vec{F}_2$ will lie along $\vec{F}_3$ so that net force on Q cannot be zero. Hence, q and Q have to be of opposite signs.

Case II. Let the charges q and Q are of opposite sign.

In this case, as shown in Fig. 1.19(b), resultant of $\vec{F}_1$ and $\vec{F}_2$ will be opposite to $\vec{F}_3$ so that it becomes possible to obtain a condition of zero net force.

Let us write $\vec{F}_R = \vec{F}_1 + \vec{F}_2$

$$\therefore \quad \sqrt{F_1^2 + F_2^2} = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{a^2}\sqrt{2}$$

Direction of $\vec{F}_R$ will be along AC ($\vec{F}_R$, being resultant of forces of equal magnitude, bisects the angle between the two) $\vec{F}_R$ and $\vec{F}_3$ are in opposite directions. Net force on Q can be zero if their magnitudes are also equal, i.e.,

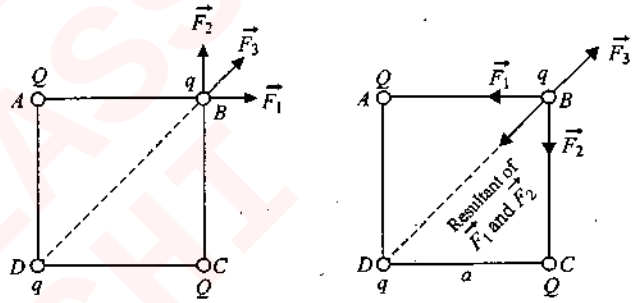
$$\frac{1}{4\pi\varepsilon_0}\frac{qQ}{a^2}\sqrt{2} = \frac{1}{4\pi\varepsilon_0}\frac{QQ}{2a^2} \text{ or } \frac{Q}{4\pi\varepsilon_0 a^2}\left(\sqrt{2}q - \frac{Q}{2}\right) = 0$$

$$\Rightarrow q = \frac{Q}{2\sqrt{2}} \Rightarrow \left|\frac{q}{Q}\right| = \frac{1}{2\sqrt{2}} \quad \{Q \neq 0\}$$

Therefore, the sign of q should be negative of Q.

ii. Consider now the forces acting on charge q placed at B (see Fig. 1.20(a) and (b)).

In a similar manner, as discussed in (i), for net force on q to be zero, q and Q have to be of opposite signs. This is also shown in the given figures.



(a) (q and Q are of same sign) Here, net force cannot be zero.

(b) (q and Q are of opposite sign) Here, net force can be zero.

Fig. 1.20

Now, $F_1 = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{a^2}$ (force of Q at A on q at B)

$F_2 = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{a^2}$ (force of Q at C on q at B)

$F_3 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{2a^2}$ (force of q at D on q at B)

Referring to Fig. 1.20(b), let us write $\vec{F}_R = \vec{F}_1 + \vec{F}_2$

$$\therefore \quad F_R = \sqrt{F_1^2 + F_2^2} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{a^2}\sqrt{2}$$

Resultant of $\vec{F}_1$ and $\vec{F}_2$, i.e., $\vec{F}_R$, is opposite to $\vec{F}_3$. Net force can become zero if their magnitudes are also equal, i.e.,

$$\frac{1}{4\pi\varepsilon_0}\frac{Qq}{a^2}\sqrt{2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{2a^2} \Rightarrow \frac{q}{4\pi\varepsilon_0 a^2}\left(\sqrt{2}Q - \frac{q}{2}\right) = 0$$

$$\Rightarrow \quad Q = \frac{q}{2\sqrt{2}} \Rightarrow \left|\frac{q}{Q}\right| = 2\sqrt{2} \quad \{q \neq 0\}$$

Therefore, the sign of q should be negative of Q.

In this case, we need not to repeat the calculation as the present situation is same as the previous one; we can directly write $|q/Q| = 2\sqrt{2}$.

iii. The entire system cannot be in equilibrium since both conditions, i.e., $q = -\frac{Q}{2\sqrt{2}}$ and $Q = -\frac{q}{2\sqrt{2}}$ cannot be satisfied together.

Illustration 1.12 **Two identical small charged spheres, each having a mass m, hang in equilibrium as shown in Fig.1.21(a). The length of each string is l and the angle made by any string with vertical is θ. Find the magnitude of the charge on each sphere.**

Sol. The forces acting on the sphere are tension in the string T; force of gravity, mg; repulsive electric force, F_e, as shown in the free-body diagram of the sphere (Fig. 1.21(b)). The sphere is in equilibrium. The forces in the horizontal and vertical directions must separately add up to zero.

$$\sum F_x = T\sin\theta - F_e = 0 \quad \text{(i)}$$

$$\sum F_y = T\cos\theta - mg = 0 \quad \text{(ii)}$$

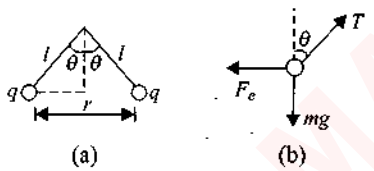


Fig. 1.21

From Eq. (ii), $T = \frac{mg}{\cos\theta}$. Thus, we can eliminate T from Eq. (i) to obtain

$$F_e = mg\tan\theta \text{ or } \frac{kq^2}{r^2} = mg\tan\theta \quad \text{(iii)}$$

where $k = \frac{1}{4\pi\varepsilon_0}$ and $r = 2l\sin\theta$.

Equation (iii) now reduces to $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2l\sin\theta)^2} = mg\tan\theta$

or $q = \sqrt{16\pi\varepsilon_0 l^2 mg\tan\theta\sin^2\theta}$

Illustration 1.13 **Two identical balls each having a density ρ are suspended from a common point by two insulating strings of equal length. Both the balls have equal mass and charge. In equilibrium, each string makes an angle θ with the vertical. Now, both the balls are immersed in a liquid. As a result, the angle θ does not change. The density of the liquid is σ. Find the dielectric constant of the liquid.**

Sol. Let V is the volume of each ball, then mass of each ball is

$$m = \rho V$$

When the balls are in air, from the previous problem,

$$F = mg\tan\theta = \rho Vg\tan\theta \quad \text{(i)}$$

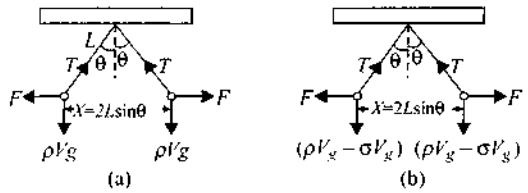


Fig. 1.22

When the balls are suspended in liquid, the Coulombic force is reduced to $F' = F/K$ and apparent weight = weight − upthrust, i.e.,

$$W' = (\rho Vg - \sigma Vg)$$

According to the problem, angle θ is unchanged. Therefore,

$$F' = W'\tan\theta = (\rho Vg - \sigma Vg)\tan\theta \quad \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{F}{F'} = K = \frac{\rho Vg}{\rho Vg - \sigma Vg} = \frac{\rho}{\rho - \sigma}$$

Illustration 1.14 **Three particles, each of mass 'm' and carrying a charge q each, are suspended from a common point by insulating massless strings, each of length 'L'. If the particles are in equilibrium and are located at the corners of an equilateral triangle of side 'a', calculate the charge q on each particle. Assume $L >> a$.**

Sol. From Fig. 1.23(a), for equilibrium of a particle along a vertical line, we get

$$T\cos\theta = mg \quad \text{(i)}$$

While for equilibrium in the plane of equilateral triangle, we get

$$T\sin\theta = 2F\cos 30° \quad \text{(ii)}$$

So, from Eqs. (i) and (ii), we have

$$\tan\theta = \frac{\sqrt{3}F}{mg} \quad \text{(iii)}$$

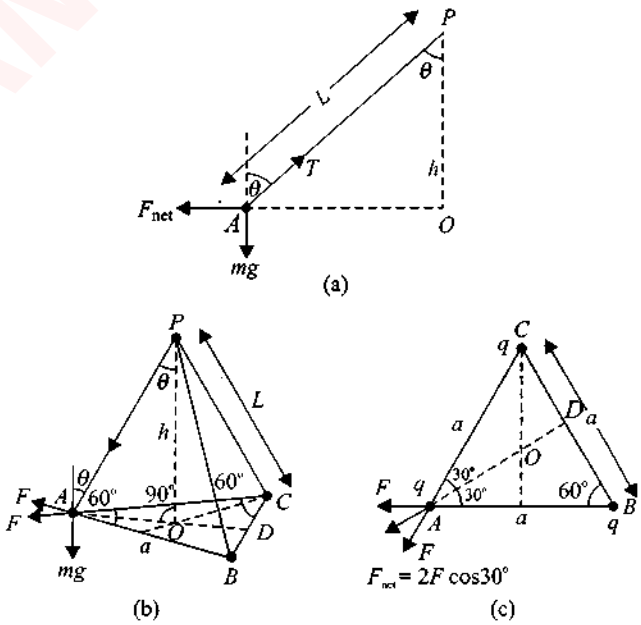


Fig. 1.23

Here, $F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2}$ and $\tan\theta = \frac{OA}{OP} = \frac{OA}{\sqrt{L^2 - OA^2}}$

Also, from Fig. 1.23(c), we get

$$OA = \frac{2}{3}AD = \frac{2}{3}a\sin 60° = \frac{a}{\sqrt{3}}$$

So, $$\tan\theta = \frac{(a/\sqrt{3})}{\sqrt{L^2(a^2/3)}} = \frac{a}{(\sqrt{3})L} \quad (\text{as } L >> a)$$

On substituting the above values of F and $\tan\theta$ in Eq. (iii), we get

$$\frac{a}{(\sqrt{3})L} = \frac{\sqrt{3}}{mg}\frac{q^2}{4\pi\varepsilon_0 a^2}, \text{i.e.}, q = \left[\frac{4\pi\varepsilon_0 a^3 mg}{3L}\right]^{1/2}$$

Illustration 1.15 **A thin fixed ring of radius 'a' has a positive charge 'q' uniformly distributed over it. A particle of mass 'm', having a negative charge 'Q', is placed on the axis at a distance of x ($x << a$) from the centre of the ring. Show that the motion of the negatively charged particle is approximately simple harmonic. Calculate the time period of oscillation.**

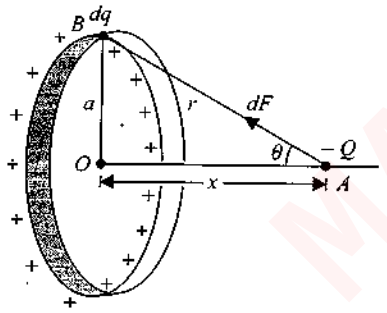


Fig. 1.24

Sol. The force on the point charge Q due to the element dq of the ring is

$$dF = \frac{1}{4\pi\varepsilon_0}\frac{dqQ}{r^2} \text{ along } AB$$

For every element of the ring, there is symmetrically situated diametrically opposite element, the components of forces along the axis will add up while those perpendicular to it will cancel each other. Hence, net force on the charge $-Q$ is $-$ve sign shows that this force will be towards the centre of ring.

$$F = \int dF\cos\theta = \cos\theta\int dF$$

$$= \frac{x}{r}\int\frac{1}{4\pi\varepsilon_0}\left[-\frac{Qdq}{r^2}\right]$$

So, $$F = -\frac{1}{4\pi\varepsilon_0}\frac{Qx}{r^3}\int dq = -\frac{1}{4\pi\varepsilon_0}\frac{Qqx}{(a^2+x^2)^{3/2}} \quad \text{(i)}$$

(as $r = (a^2+x^2)^{1/2}$ and $\int dq = q$)

As the restoring force is not linear, the motion will be oscillatory. However, if $x \ll a$ so that $x^2 \ll a^2$, then

$$F = -\frac{1}{4\pi\varepsilon_0}\frac{Qq}{a^3}x = -kx \text{ with } k = \frac{Qq}{4\pi\varepsilon_0 a^3}$$

i.e., the restoring force will become linear and so the motion is simple harmonic with time period

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{4\pi\varepsilon_0 ma^3}{qQ}}$$

Concept Application Exercise 1.2

1. A negatively charged particle is placed exactly midway between two fixed particles having equal positive charges. What will happen to the charge
   i. if it is displaced at right angle to the line joining the positive charges?
   ii. if it is displaced along the line joining the positive charges?
2. Does an electric charge experience a force due to the field produced by itself ? (Yes/No)
3. Two negative charges of a unit magnitude and a positive charge 'q' are placed along a straight line. At what position and value of q will the system be in equilibrium? (Negative charges are fixed.)
4. Figure 1.25 shows three arrangements of an electron e and two protons p (where $D > d$).
   a. Rank the arrangements according to the magnitude of the net electrostatic force on the electron due to the protons, largest first.

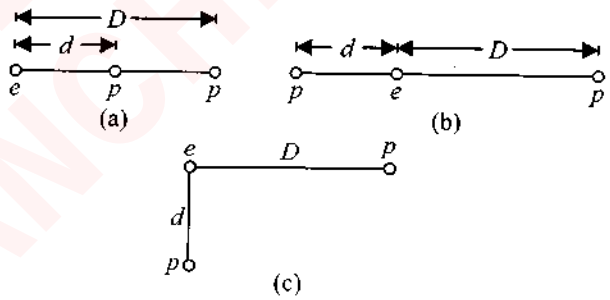


Fig. 1.25

   b. In situation (c), is the angle between the net force on the electron and the line labeled horizontal less than or more than 45°?
5. Figure 1.26 shows two charge particles on an axis. The charges are free to move. At one point, however, a third charged particle can be placed such that all three particles are in equilibrium.

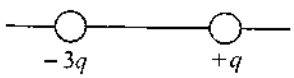


Fig. 1.26

   a. Is that point to the left of the first two particles, to their right, or between them?
   b. Should the third particle be positively or negatively charged?
   c. Is the equilibrium stable or unstable?
6. In Fig. 1.27, a central particle of charge $-q$ is surrounded by two circular rings of charged particles of radii r and R, with $R > r$. What is the magnitude and direction of the net electrostatic force on the central particle due to the other particles?

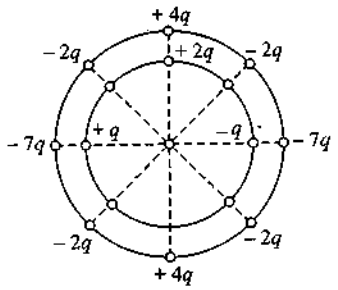


Fig. 1.27

7. Figure 1.28 shows four situations in which particles of charge $+q$ or $-q$ are fixed in place. In each, the particles on the x-axis are equidistant from the y-axis. The particle on the y-axis experiences an electrostatics force F from each of these two particles.

a. Are the magnitudes F of those forces the same or different?

b. Is the magnitude of the net force on the particle on the y-axis equal to, greater than or less than $2F$?

c. Do the x components of the two forces add or cancel?

d. Do the y components of the forces add or cancel?

e. Is the direction of the net force on the middle particle that of the canceling components or the adding components?

f. What is the direction of the net force on the middle particle?

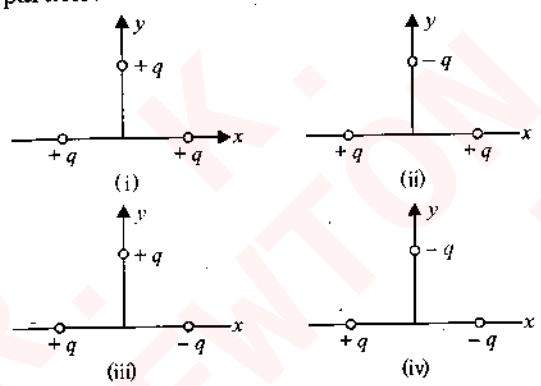


Fig. 1.28

8. Force between two point electric charges kept at a distance 'd' apart in air is F. If these charges are kept at the same distance in water, the force between the charges is F'. The ratio F'/F is equal to _____.

9. Two small balls each having charge q are suspended by two insulating threads of equal length L from a hook in an elevator. The elevator is freely falling. Calculate the angle between the two threads and tension in each thread.

10. Suppose we have a large number of identical particles, very small in size. Any two of them at 10 cm separation repel with a force of 3×10^{-10} N.

a. If one of them is at 10 cm from a group (of very small size) of n others, how strongly do you expect it to be repelled?

b. Suppose you measure the repulsion and find it 6×10^{-6} N How many particles were there in the group?

ELECTRIC FIELD

If we place a single charge q at some point in space, it will experience no force. But if some other charge (say Q) is placed near it, q will start experiencing a force given by

$$F = \frac{kQq}{r^2}$$

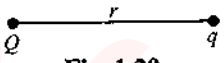


Fig. 1.29

Now, question arises, how does Q apply a force on q or how does q know the presence of Q when there is no direct contact between them.

Basically, the force between two charges can be seen as a two-step process:

1. Firstly, charge Q creates something around itself known as electric field.
2. Secondly, any other charge particle like q if placed at some point in that field experiences a force, or we can say that charges interact with each other through electric field.

So, we can define electric field as the space around a charge in which its influence can be felt by any other charged particle.

How to Measure Electric Field

Strength of electric field at a point in space can be measured in terms of two measureable quantities:

1. Electric field intensity is denoted by E. It is a vector quantity.
2. Electric field potential is denoted by V. It is a scalar quantity.

First, we will discuss them separately and then we will see what is the relation between them and how to obtain one from the other.

Electric Field Intensity E

How to find electric field intensity E at a point?

General method: Electric field intensity, E, is a vector quantity. At a point in a given space, it has both magnitude and direction. Let us calculate E at some point P created due to some charges around P. Bring a small charge q_0 [test charge, generally positive] at point P. Let this charge experiences a force $\vec{F}$ due to charges placed in the vicinity of P. Then we define electric field intensity at P as force experienced per unit test charge (Fig. 1.30).

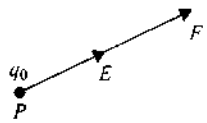


Fig. 1.30

$\vec{E} = \lim_{q_0\to0}\frac{\vec{F}}{q_0}$. The direction $\vec{E}$ will be same as that of $\vec{F}$.

Note: ***Q. Why the magnitude of test charge is kept small?***
Ans. Otherwise, large magnitude may disturb the original charge distribution and then we will get electric field due to disturbed configuration and not original.
Q. What is the minimum possible value of q_0?
Ans. 1.6×10^{-19} C

Unit of E: N/C (newton per coulomb)

$$\text{Dimensional formula of } E = \frac{\text{Force}}{\text{Charge}} = \frac{MLT^{-2}}{\text{ampere} \times \text{time}} = \frac{MLT^{-2}}{AT} = [MLT^{-3}A^{-1}]$$

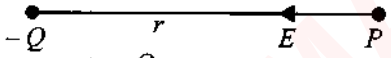
Note: ***If a test charge experiences no force at a point, the electric field at that point must be zero.***

Electric field due to a point charge is illustrated in Fig. 1.31.

(i) Positive point charge

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \text{ away from the charge}$$

(ii) Negative point charge



$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \text{ towards the charge}$$

Fig. 1.31

Electric field due to a point charge is spherically symmetric.

Point Charge in an Electric Field

What happens if a point charge q is placed at any point in an electric field which is produced by some other stationary charges. Let this electric field is $\vec{E}$. Charge q will experience a force at this point, let this force is $\vec{F}$. Then, value of electric field at that point must be

$$\vec{E} = \frac{\vec{F}}{q} \Rightarrow \vec{F} = q\vec{E}. \text{ This is the force on } q \text{ by } E.$$

Direction of $\vec{F}$: The direction of $\vec{F}$ will be same as that of $\vec{E}$ if q is +ve and opposite if q is −ve (Fig. 1.32).

(a) Positive charge q_0 placed in an electric field: force on q_0 is in same direction as $\vec{E}$.

(b) Negative charge q_0 placed in an electric field: force on q_0 is in opposite direction as $\vec{E}$.

Fig. 1.32

Note: ***q has no contribution in $\vec{E}$. A charge particle is not affected due to its own field. It means a charge particle can experience force due to field produced by other charge particles, but not due to field produced by itself.***

Electric Field Intensity due to a Point Charge in Position Vector Form

Electric field at P due to charge Q is

$$\vec{E} = \frac{Q(\vec{r} - \vec{r}_0)}{4\pi\varepsilon_0 |\vec{r} - \vec{r}_0|^3}$$

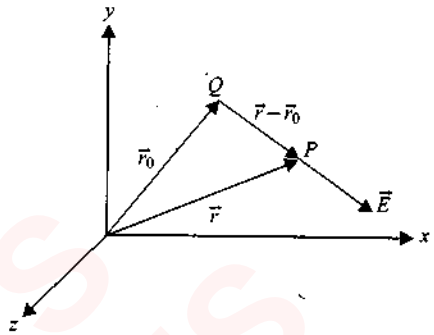


Fig. 1.33

If a charge q is placed at P, then force on this charge by Q is

$$\vec{F} = q\vec{E}$$

$$\Rightarrow \quad \vec{F} = \frac{qQ(\vec{r} - \vec{r}_0)}{4\pi\varepsilon_0 |\vec{r} - \vec{r}_0|^3}$$

Electric Field Intensity due to a Group of Charges

Using the principle of superposition, net field at point P (see Fig. 1.34) is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \cdots + \vec{E}_n$$

$$\Rightarrow \quad \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1^2}\hat{r}_1 + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2^2}\hat{r}_2 + \cdots + \frac{1}{4\pi\varepsilon_0}\frac{q_n}{r_n^2}\hat{r}_n$$

$$\Rightarrow \quad \vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{r}_i$$

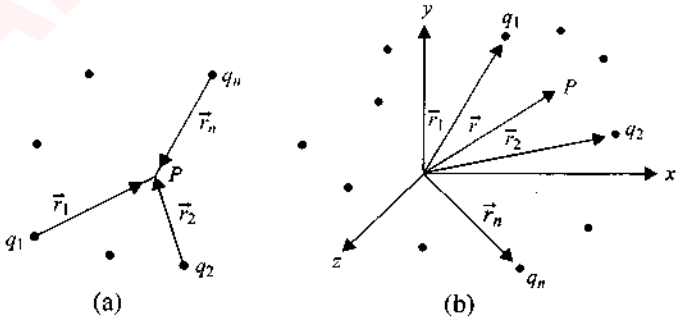


Fig. 1.34

In terms of position vectors:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i(\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|^3}$$

Illustration 1.10 **Two point-like charges a and b whose strengths are equal in absolute value are positioned at a certain distance from each other. Assuming the field strength is positive in the direction coinciding with the positive direction of the r axis, determine the signs of the charges for each distribution of the field strength between charges shown in Figs. 1.35(a), (b), (c) and (d).**

Sol.

a. As electric field tends away at a and towards at b, hence there should be + charge at a and negative charge at b, i.e., q_a is '+' and q_b is '−'.

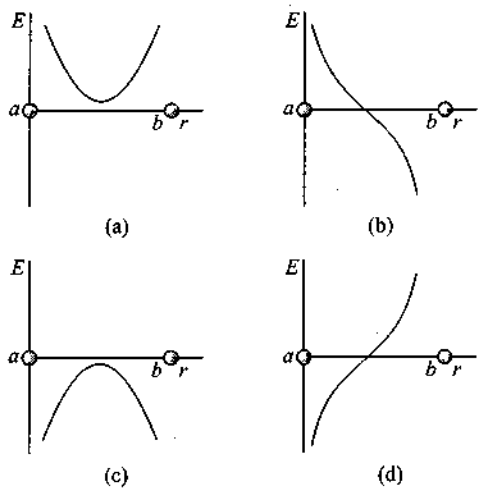


Fig. 1.35

b. The neutral point exists between a and b only when both q_a and q_b are of same sign. As direction of electric field is away from both, both charges are positive, i.e., q_a is '+' and q_b is '+'.

Similarly, for (c) and (d) in Fig. 1.36:

c. q_a is '–' and q_b is '+'

d. q_a is '–' and q_b is '–'.

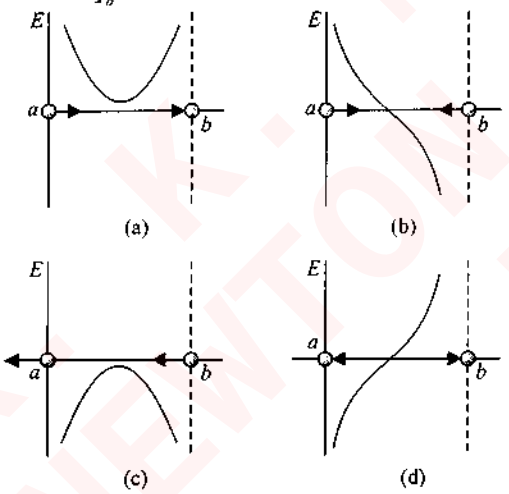


Fig. 1.36

Illustration 1.17 **Two point charges $\pm q$ are placed on the axis at $x = -a$ and $x = +a$, as shown in Fig. 1.37.**

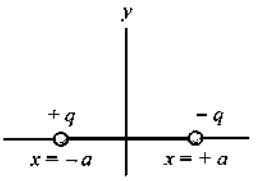


Fig. 1.37

i. Plot the variation of E along the x-axis.

ii. Plot the variation of E along the y-axis

Sol.

i. Variation of E along the x-axis as shown in Fig. 1.38(a).

ii. Variation of E along the y-axis as shown in Fig. 1.38(b). In this case, field will be maximum at origin because at origin, field due to both charges is directly added.

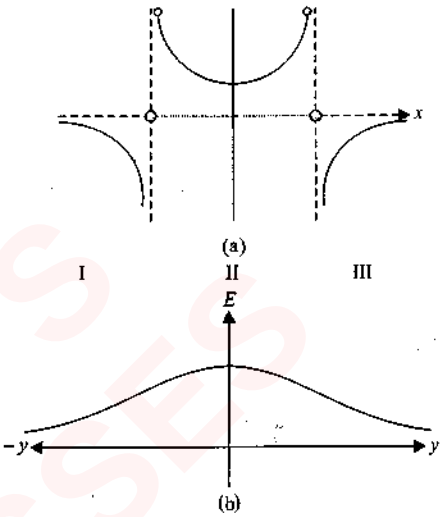


Fig. 1.38

Illustration 1.18 **In Fig. 1.39, determine the point (other than infinity) at which the electric field is zero.**

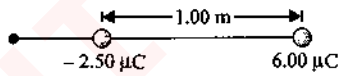


Fig. 1.39

Sol. Electric field will be zero at a point closer to the charge smaller in magnitude. Let electric field is zero at P (see Fig. 1.40).

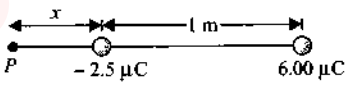


Fig. 1.40

Then, we have

$$\frac{k(2.5\times10^{-6})}{x^2} = \frac{k(6\times10^{-6})}{(1+x)^2}$$

$$\Rightarrow \qquad x = 1.82 \text{ m}$$

Illustration 1.19 **Four charges are arranged as shown in Fig. 1.41. A point P is located at distance r from the centre of the configuration. Assuming $r >> l$, find**

i. the magnitude of the field at point P.

ii. the angle of its vector with the x-axis.

Sol. i. Electric field due to charges placed on the y-axis [Fig. 1.41(a)].

$$E_y = 2E_1 \sin\theta = 2\frac{1}{4\pi\varepsilon_0}\frac{q}{\left(r^2+\left(\frac{l}{2}\right)^2\right)}\frac{(l/2)}{\left(r^2+\frac{l^2}{4}\right)^{1/2}}$$

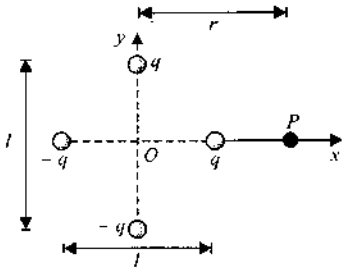


Fig. 1.41

$$E_y = \frac{1}{4\pi\varepsilon_0}\frac{ql}{\left(r^2+\frac{l^2}{4}\right)^{3/2}} = \frac{1}{4\pi\varepsilon_0}\frac{ql}{r^3} \text{ (as } r >> l)$$

Electric field due to charges placed on x-axis [Fig. 1.42(b)]

$$E_x = E_3 - E_4 = \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(r-\frac{l}{2}\right)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(r+\frac{l}{2}\right)^2}$$

$$= \frac{1}{2\pi\varepsilon_0}\frac{ql}{r^3}$$

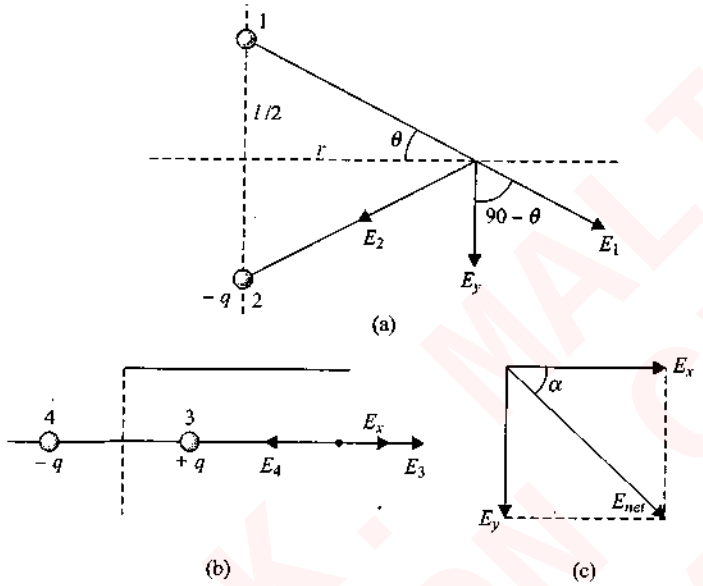


Fig. 1.42

$$\Rightarrow \quad E_{net} = \sqrt{E_x^2 + E_y^2} = \sqrt{5}\frac{ql}{4\pi\varepsilon_0 r^3}$$

ii. The angle that E_{net} makes with x-axis [Fig.1.42(c)] is

$$\alpha = \tan^{-1}\left(\frac{E_y}{E_x}\right) = \tan^{-1}\left(\frac{1}{2}\right) \text{ below } x\text{-axis}$$

Illustration 1.20 **A uniform electric field E exists between two metal plates one –ve and other +ve. The plate length is l and the separation of the plates is d.**

i. An electron and a proton start from the negative plate and positive plate, respectively, and go to opposite plates. Which one of them wins this race?

ii. An electron and a proton start moving parallel to the plates towards the other end from the midpoint of the separation of plates at one end of the plates. Which of the two will have greater deviation when they come out of the plates if they start with the

a. same initial velocity,

b. same initial kinetic energy and

c. same initial momentum.

Sol.

i. $a_e = \frac{eE}{m_e}, a_p = \frac{eE}{m_p}; d = \frac{1}{2}at^2$

$$t = \sqrt{\frac{2d}{a}} \text{ or } t = \sqrt{\frac{2md}{eE}}; \text{ therefore, we have } \frac{t_e}{t_p} = \sqrt{\frac{m_e}{m_p}}.$$

As $m_e < m_p$, therefore $t_e < t_p$. Hence, electron will take less time, i.e., the electron wins the race.

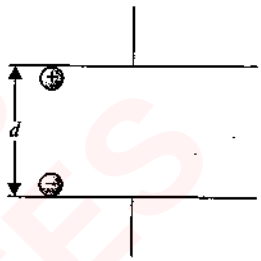


Fig. 1.43

ii. Time to cross the plates $t = \frac{l}{u}$

Deviation: $y = \frac{1}{2}at^2 = \frac{1}{2}\frac{eE}{m}\left(\frac{l}{u}\right)^2$

$$\frac{y_e}{y_p} = \frac{m_p}{m_e}\left(\frac{u_p}{u_e}\right)^2 \quad \text{(i)}$$

a. If $u_p = u_e$, then $\frac{y_e}{y_p} = \frac{m_p}{m_e}$.

As $m_p > m_e$, therefore $y_e > y_p$.

Hence, deviation of electron will be more.

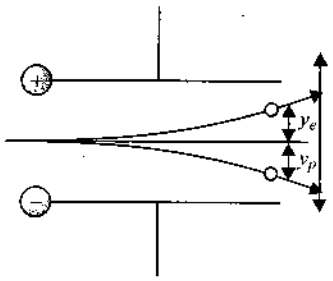


Fig. 1.44

b. From Eq. (i), $\frac{y_e}{y_p} = \left(\frac{m_p u_p^2}{m_e u_e^2}\right) = 1$ (as given)

Hence, deviation of both electron and proton will be the same.

c. From Eq. (i), $\frac{y_e}{y_p} = \left(\frac{m_p u_p}{m_e u_e}\right)^2 \frac{m_e}{m_p} = \frac{m_e}{m_p}$

As $m_e < m_p, y_e < y_p$.

Hence, the deviation of proton will be more.

Illustration 1.21 **A charge 10^{-9} coulomb is located at origin in free space and another charge Q at (2, 0, 0). If the x-component of the electric field at (3, 1, 1) is zero, calculate the value of Q. Is the y-component zero at (3, 1, 1)?**

Sol. The electric field due to a point charge q_i at position $\vec{r}_i$ in vector form is given by

$$\vec{E}_i = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_i^3}\vec{r}_i$$

Here $\vec{r}_1 = (3-0)\hat{i} + (1-0)\hat{j} + (1-0)\hat{k} = 3\hat{i} + \hat{j} + \hat{k}$

$$= \sqrt{(3^2+1^2+1^2)} = \sqrt{11}\text{ m}$$

$$\vec{r}_2 = (3-2)\hat{i} + (1-0)\hat{j} + (1-0)\hat{k} = \hat{i} + \hat{j} + \hat{k}$$

$$= \sqrt{(1^2+1^2+1^2)} = \sqrt{3}\text{ m}$$

So,

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{10^{-9}}{(11)^{3/2}}[3\hat{i} + \hat{j} + \hat{k}] \text{ and}$$

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(3)^{3/2}}[\hat{i} + \hat{j} + \hat{k}]$$

Hence, net field is $\vec{E} = \vec{E}_1 + \vec{E}_2$

$$= \frac{1}{4\pi\varepsilon_0}\left[\left(\frac{3\times10^{-9}}{11\sqrt{11}} + \frac{Q}{3\sqrt{3}}\right)\hat{i} + \left(\frac{10^{-9}}{11\sqrt{11}} + \frac{Q}{3\sqrt{3}}\right)\hat{j} + \left(\frac{10^{-9}}{11\sqrt{11}} + \frac{Q}{3\sqrt{3}}\right)\hat{k}\right]$$

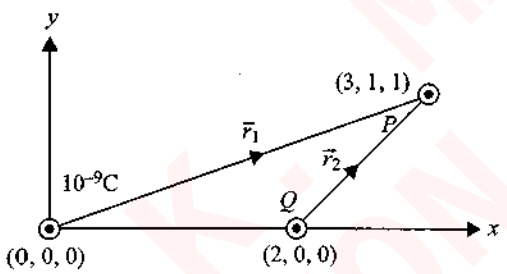


Fig. 1.45

According to the given problem

$$E_x = 0, \text{ i.e., } \frac{1}{4\pi\varepsilon_0}\left(\frac{3\times10^{-9}}{11\sqrt{11}} + \frac{Q}{3\sqrt{3}}\right) = 0$$

So, $$Q = -\left[\frac{3}{11}\right]^{3/2} \times 3\times10^{-9}\text{coulomb}$$

And for this value of Q

$$E_y = \frac{1}{4\pi\varepsilon_0}\left[\frac{10^{-9}}{11\sqrt{11}} - \frac{(3/11)^{3/2}\times3\times10^{-9}}{3\sqrt{3}}\right]$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{2\times10^{-9}}{11\sqrt{11}} \neq 0 \text{ i.e., } E_y \text{ is not zero.}$$

Lines of Force

This idea was given by Michael Faraday. The lines of force provide a nice idea to visualise the pattern of electric field in a given space. We assume that space around a charged body is filled with some lines known as electric lines of force. *These lines of force are drawn in space in such a way that tangent to the line at any point gives the direction of electric field at that point.* It has been found quite convenient to visualize the electric field in terms of lines of force.

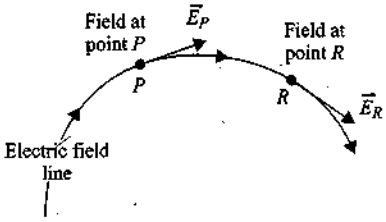


Fig. 1.46

Properties of Electric Lines of Force

- Electric lines of force start (or diverge out) from a positive charge and end (or converge) on a negative charge.
- The tangent drawn at any point on the line of force gives the direction of force acting on a positive charge at that point (see Fig. 1.47).
- In S.I. system of units, the number of electric lines of force originating or terminating on a charge of q coulomb is equal to q/ε_0.

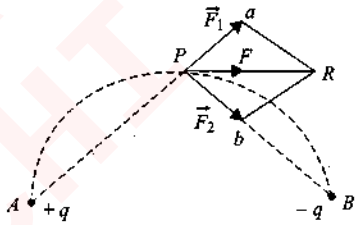


Fig. 1.47

- Two electric lines of force never cross each other, because if they do so then at the point of intersection, intensity will have two directions which is absurd.
- Electric lines of force can never be closed loops, as a line can never start and end on the same charge.
- The electric lines of force do not pass through a conductor as electric field inside a conductor is always zero.
- Lines of force have a tendency to contract longitudinally like a stretched elastic string producing attraction between opposite charges and repel each other laterally resulting in repulsion between similar charges and edge effect (curving of lines of force near the edges of a charged conductor).
- Electric lines of force end or start normally on the surface of a conductor.
- Tangent to the line of force at a point in an electric field gives the direction of intensity or force or acceleration which a positive charge will experience there but not the direction of motion always, therefore a positive point charge free to move in an electric field may or may not follow the line of force. It will follow the line of force if it is a straight line (as direction of velocity and acceleration will be same) and will not follow the line if it is curved as the direction of motion will be different from that of acceleration and the particle will move in the direction of neither motion nor acceleration (line of force).

The use of the electric lines of force is that we can compare the intensities at two points just by looking at the distribution of lines of force. Where the field lines are close together, E is large and where they are far apart, E is small.

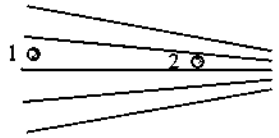


Fig. 1.48

In Fig. 1.48, electric lines of forces are shown. The electric field intensity at point 2 will be greater in comparison to that at point 1.

DIFFERENT PATTERNS OF ELECTRIC FIELD LINES

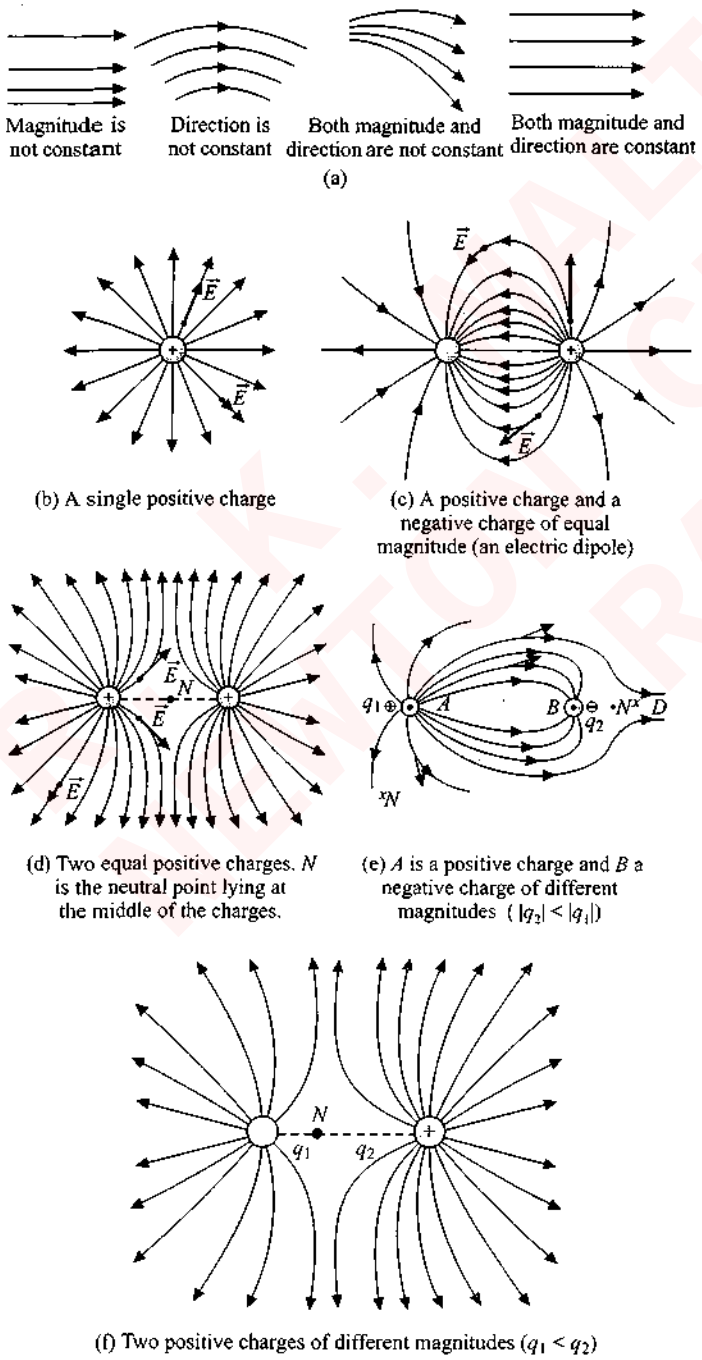


Fig. 1.49

Note: ***Neutral point (N) is the location where the net electric field due to charges is zero. It lies near the charge of smaller magnitude.***

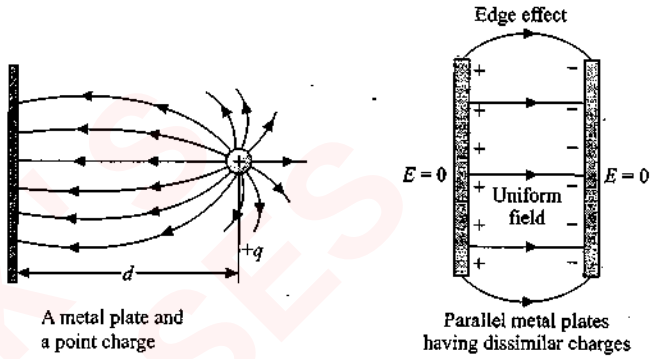


Fig. 1.50

Illustration 1.22 **Figure 1.51 shows the sketch of field lines for two point charges $2Q$ and $-Q$. The pattern of field lines can be deduced by considering the following points:**

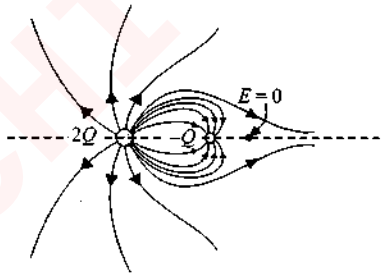


Fig. 1.51

Sol.

i. Symmetry: For every point above the line joining the two charges, there is an equivalent point below it. Therefore, the pattern must be symmetrical about the line joining the two charges.

ii. Near field: Very close to a charge, its field predominates. Therefore, the lines are radial and spherically symmetric.

iii. Far field: Far from the system of charges, the pattern should look like that of a single point charge of value $(2Q - Q) = +Q$, i.e., the lines should be radially outward.

iv. Null point or neutral point: There is one point at which $E = 0$. No lines should pass through this point. Neutral point lies near the position of charge of smaller magnitude.

v. Number of lines: Twice as many lines leave $+2Q$ as enter $-Q$.

Note: ***Excess lines from 2Q charge will meet at infinity.***

Illustration 1.23 **Charges $+q$ and $-2q$ are fixed at distance d apart as shown in the figure.**

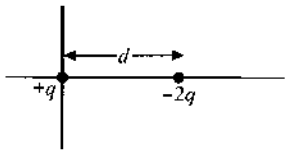


Fig. 1.52

i. Sketch roughly the pattern of electric field lines, showing position of neutral point.
ii. Where should a charge particle q be placed so that it experiences no force?

Sol. Let net force on q at P is zero, then

$$\frac{kq^2}{x^2} = \frac{kq\,2q}{(d+x)^2} \Rightarrow x = \frac{d}{\sqrt{2}-1}$$

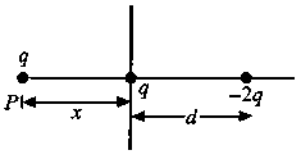


Fig. 1.53

P is the neutral potential where electric field will be zero.

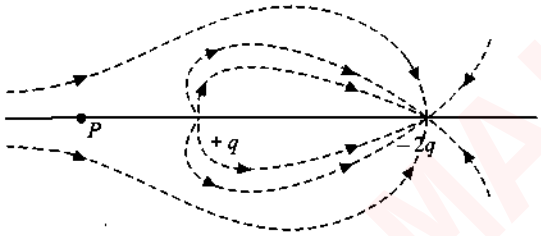


Fig. 1.54

Illustration 1.24 **The field lines for two point charges are shown in Fig. 1.55.**

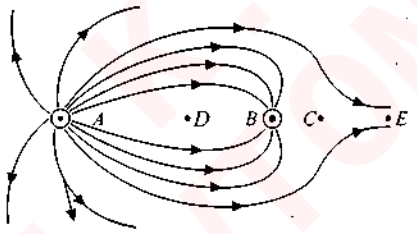


Fig. 1.55

i. Is the field uniform?
ii. Determine the ratio q_A/q_B.
iii. What are the sign of q_A and q_B?
iv. Apart from infinity, where is the neutral point?
v. If q_A and q_B are separated by a distance $10(\sqrt{2}-1)$ cm, find the position of neutral point.
vi. Where will the lines meet which are coming from A and are not meeting at q_B?
vii. Will a positive charge follow the line of force if free to move?

Sol.

i. No

ii. Number of lines coming from or coming to a charge is proportional to magnitude of charge, so

$$\frac{q_A}{q_B} = \frac{12}{6} = 2$$

iii. q_A is positive and q_B is negative.

iv. C is the other neutral point.

v. For neutral point $E_A = E_B$

$$\frac{1}{4\pi\varepsilon_0}\frac{q_A}{(l+x)^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_B}{x^2}$$

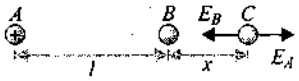


Fig. 1.56

$$\left(\frac{l+x}{x}\right)^2 = \frac{q_A}{q_B} = 2 \Rightarrow x = 10 \text{ cm}$$

vi. At infinity.

vii. No. As lines of force are curved, the direction of velocity and acceleration will be different. Hence, a charge cannot follow strictly the line of force. Also to move on some curved path, centripetal force is required, whereas lines of force will provide only tangential force.

FIELD OF RING CHARGE

A ring-shaped conductor with radius a carries a total charge Q uniformly distributed around it. Let us calculate the electric field at a point P that lies on the axis of the ring at a distance x from its centre.

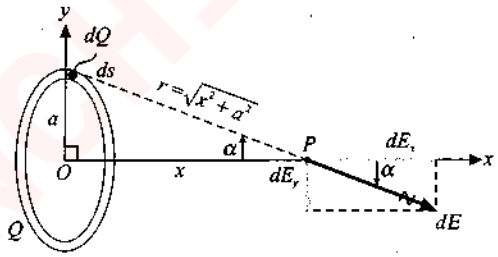


Fig. 1.57

As shown in the figure, the ring is divided into infinitesimal segments each of length ds. Each segment has charge dQ and acts as a point charge source of electric field. Let $d\vec{E}$ be the electric field from one such segment; the net electric field at P is then the sum of all contributions $d\vec{E}$ from all the segments that make up the ring. (The same technique works for any situation in which charge is distributed along a line or a curve.) The calculation of $\vec{E}$ is greatly simplified because the field point P is on the symmetry axis of the ring. If we consider two ring segments at the top and bottom of the ring, we see that the contributions $d\vec{E}$ to the field at P from these segments have the same x-component but opposite y-components. Hence, the total y-component of field due to this pair of segments is zero. When we add up the contributions from all such pairs of segments, the total field $\vec{E}$ will have only a component along the ring's symmetry axis (the x-axis), with no component perpendicular to that axis (that is, no y-component or z-component). So, the field at P is described completely by its x-component E_x.

To calculate E_x, note that the square of the distance r from a ring segment to the point P is $r^2 = x^2 + a^2$. Hence, the magnitude of this segment's contribution to the electric field at P is

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{x^2+a^2}$$

Using $\cos\alpha = \frac{x}{r} = \frac{x}{(x^2+a^2)^{1/2}}$ the component dE_x of this field along the x-axis is

$$dE_x = dE\cos\alpha = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{x^2+a^2}\frac{x}{\sqrt{x^2+a^2}} = \frac{1}{4\pi\varepsilon_0}\frac{xdQ}{(x^2+a^2)^{3/2}}$$

To find the total x-component E_x of the field at P, we integrate this expression over all segments of the ring, i.e.,

$$E_x = \int \frac{1}{4\pi\varepsilon_0}\frac{xdQ}{(x^2+a^2)^{3/2}}$$

Since x does not vary as we move from point to point around the ring, all the factors on the right side except dQ are constant and can be taken outside the integral. The integral of dQ is just the total charge Q and we finally get

$$\vec{E} = E_x\hat{i} = \frac{1}{4\pi\varepsilon_0}\frac{xQ}{(x^2+a^2)^{3/2}}\hat{i} \qquad \text{(i)}$$

- Electric field is directed away from positively charged ring.
- For $x = 0$, $E = 0$. This conclusion may be arrived at by the symmetry consideration.
- At a large distance from the ring, the electric field will be zero. Hence, it should have certain maximum value between $x = 0$ and $x = \infty$ (or $x = -\infty$).
- If we maximize Eq. (i), we can get the value of x_m as well as E_{max}.

For the maximum value of E_x, we get

$$\frac{d}{dx}\left\{\frac{1}{4\pi\varepsilon_0}Q\frac{x}{(x^2+a^2)^{3/2}}\right\} = 0$$

$$\frac{(x^2+a^2)^{3/2}(1-x)\frac{3}{2}(x^2+a^2)^{1/2}(2x)}{(x^2+a^2)^3} = 0$$

$$(x^2+a^2)-3x^2 = 0 \Rightarrow x = \pm\frac{a}{\sqrt{2}}$$

and the maximum value of the electric field is

$$E_{x(max)} = \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q}{3\sqrt{3}\,a^2}\right)$$

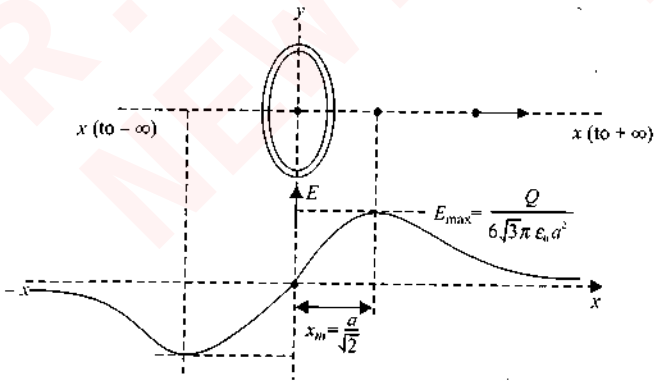


Fig. 1.58

Illustration 1.25 **Two identical point charges having magnitude q each are placed as shown in the figure.**

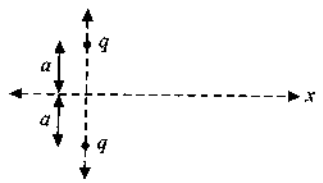


Fig. 1.59

i. Plot the variation of electric field on x-axis.

ii. Where will the magnitude of electric field be maximum on x-axis? Find the maximum value of electric field on x-axis.

iii. If we place a negative charge (of magnitude $-q$ and mass m) at the mid point of charges and displaced along x-axis, examine whether it will perform simple harmonic motion. If yes, then find the time period of oscillation of the particle.

Sol. i.

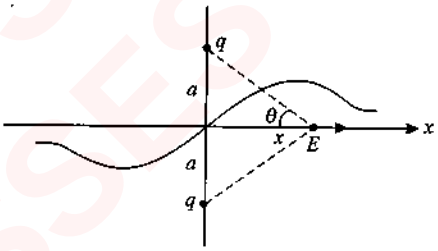


Fig. 1.60

ii. Field at $x = x$: $E = 2\left[\frac{1}{4\pi\varepsilon_0}\frac{q}{(a^2+x^2)}\right]\cos\theta$

$$\Rightarrow \quad E = \frac{q}{2\pi\varepsilon_0}\frac{x}{(a^2+x^2)^{3/2}}$$

For E to be maximum, $\frac{dE}{dx} = 0$

Solving, we get $x = \pm\frac{a}{\sqrt{2}} \Rightarrow E_{max} = \frac{q}{3\sqrt{3}\pi\varepsilon_0 a^2}$

iii. Force on particle: $F = -qE = \frac{-q^2}{2\pi\varepsilon_0}\frac{x}{(a^2+x^2)^{3/2}}$

For $x << a$, particle will execute S.H.M. with time period

$$T = 2\pi\sqrt{\frac{2\pi\varepsilon_0 ma^3}{q^2}}$$

Electric Field due to an Infinite Line Charge

Positive electric charge Q is distributed uniformly along a line, lying along the y-axis. Let us find the electric field at point D on the x-axis at a distance r_0 from the origin.

We divide the line charge into infinitesimal segments, each of which acts as a point charge; let the length of a typical segment at height l be dl. If the charge is distributed uniformly with the linear charge density λ, then the charge dQ in a segment of length dl is $dQ = \lambda dl$. At point D, the differential electric field dE created by this element is

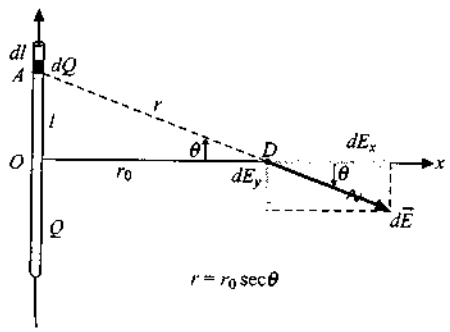


Fig. 1.61

$$dE = \frac{dQ}{4\pi\varepsilon_0 r^2} = \frac{\lambda dl}{4\pi\varepsilon_0 r^2} = \frac{\lambda dl}{4\pi\varepsilon_0 r_0^2 \sec^2\theta} \quad \text{(i)}$$

In triangle AOD; $OA = OD \tan\theta$, i.e., $l = r_0 \tan\theta$.

Differentiating this equation with respect to θ, we get $dl = r_0 \sec^2\theta d\theta$.

Substituting the value of dl in Eq. (i), we get

$$dE = \frac{\lambda d\theta}{4\pi\varepsilon_0 r_0}$$

Field dE has components dE_x dE_y given by

$$dE_x = \frac{\lambda\cos\theta d\theta}{4\pi\varepsilon_0 r_0} \text{ and } dE_y = \frac{\lambda\sin\theta d\theta}{4\pi\varepsilon_0 r_0}$$

On integrating expression for dE_x and dE_y in limits $\theta = -\pi/2$ to $\theta = +\pi/2$, we obtain E_x and E_y. Note that as the length of wire increases, the angle θ also increases. For a very long wire (infinitely long wire), θ approaches $\pi/2$.

$$E_x = \int_{-\pi/2}^{+\pi/2} \frac{\lambda\cos\theta\, d\theta}{4\pi\varepsilon_0 r_0} = \frac{\lambda}{2\pi\varepsilon_0 r_0} \text{ and}$$

$$E_y = \int_{-\pi/2}^{+\pi/2} \frac{\lambda\sin\theta\, d\theta}{4\pi\varepsilon_0 r_0} = 0$$

Thus, $E = E_x = \dfrac{\lambda}{4\pi\varepsilon_0 r_0}$

Note: ***Using a symmetry argument, we could have guessed that E_y would be zero; if we place a positive test charge at D, the upper half of the line of charge pushes downward on it, and the lower half pushes upward with equal magnitude.***

- If the wire has finite length and the angles subtended by ends of wire at a point are θ_1 and θ_2, the limits of integration would change.

$$E_x = \int_{-\theta_1}^{+\theta_2} \frac{\lambda\cos\theta\, d\theta}{4\pi\varepsilon_0 r_0}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 r_0}(\sin\theta_1 + \sin\theta_2)$$

$$E_y = \int_{-\theta_1}^{+\theta_2} \frac{\lambda\sin\theta d\theta}{4\pi\varepsilon_0 r_0}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 r_0}(\cos\theta_1 - \cos\theta_2)$$

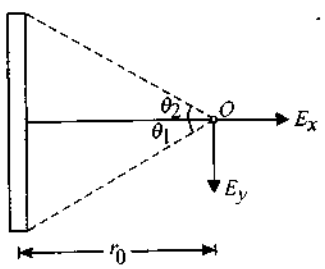


Fig. 1.62

- If we wish to determine field at the end of a long wire, we may substitute $\theta_1 = 0$ and $\theta_2 = \pi/2$ in the expressions for E_x and E_y, i.e.,

$$E_x = \frac{\lambda}{4\pi\varepsilon_0 r_0}\left[\sin(0) + \sin\left(\frac{\pi}{2}\right)\right] = \frac{\lambda}{4\pi\varepsilon_0 r_0} \text{ and}$$

$$E_y = \frac{\lambda}{4\pi\varepsilon_0 r_0}\left[\cos(0) - \cos\left(\frac{\pi}{2}\right)\right] = \frac{\lambda}{4\pi\varepsilon_0 r_0}$$

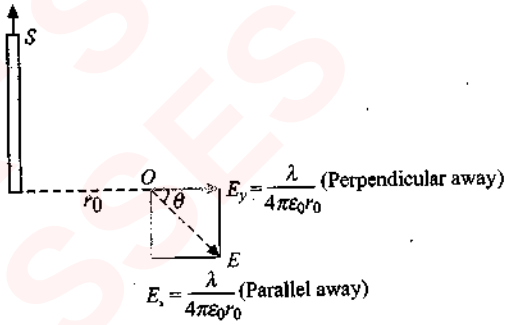


Fig. 1.63

Magnitude of resultant field $\vec{E}$ is

$$|\vec{E}| = \sqrt{E_x^2 + E_y^2} = \frac{\sqrt{2}\lambda}{4\pi\varepsilon_0 r_0}$$

$\vec{E}$ makes an angle θ with the x-axis, where $\tan\theta = |E_y|/|E_x| = 1; \theta = 45°$

FIELD OF UNIFORMLY CHARGED DISK

Let us find the electric field caused by a disk of radius R with a uniform positive surface charge density (charge per unit area) σ at a point on the axis of the disk at a distance x from its centre.

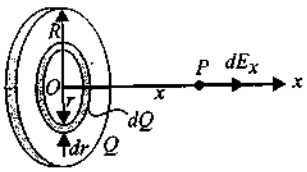


Fig. 1.64

The situation is shown in Fig.1.64. We can represent this charge distribution as a collection of concentric rings of charge. We already know how to find the field of a single ring on its axis of symmetry, therefore we will add the contribution of all the rings. As shown in the figure, a typical ring has charge dQ, inner radius r and outer radius $r + dr$. Its area dA is approximately equal to its width dr times its circumference $2\pi r$, or $dA = 2\pi r\, dr$. The charge per unit area is $\sigma = dQ/dA$, so the charge of ring is $dQ = \sigma(2\pi r dr)$, or $dQ = 2\pi\sigma r dr$. The field component dE_x at point P due to charge dQ of a ring of radius r is

$$dE_x = \frac{1}{4\pi\varepsilon_0}\frac{(2\pi\sigma r dr)x}{(x^2 + r^2)^{3/2}}$$

To find the total field due to all the rings, we integrate dE_x over r. To include the whole disk, we must integrate from 0 to R (not from $-R$ to R), i.e.,

$$E_x = \int dE_x = \int_0^R dE_x = \int_0^R \frac{1}{4\pi\varepsilon_0}\frac{(2\pi\sigma r dr)x}{(x^2+r^2)^{3/2}}$$

Remember that x is a constant during the integration and that the integration variable is r. The integral can be evaluated by the use of the substitution $z = x^2 + r^2$. We will let you work out the details; the result is

$$E_x = \frac{\sigma x}{2\varepsilon_0}\left[-\frac{1}{\sqrt{x^2+R^2}}+\frac{1}{x}\right] = \frac{\sigma}{2\varepsilon_0}\left[1-\frac{x}{\sqrt{x^2+R^2}}\right] \qquad \text{(i)}$$

In this figure, the charge is assumed to be positive. At a point on the symmetry axis of a uniformly charged ring, the electric field due to the ring has no components perpendicular to the axis. Hence, at point P in the figure, $dE_y = dE_z = 0$ for each ring, and thus the total field has $E_y = E_z = 0$.

Again, we can ask what happens if the charge distribution gets very large. Suppose we keep increasing the radius R of the disk, simultaneously adding charge so that the surface charge density σ (charge per unit area) is constant. In the limit that R is much larger than the distance x of the field point from the disk ($R >> x$), i.e., the situation becomes the electric field near infinite plane sheet of charge.

From Eq. (i), we get

$$E_x = \frac{\sigma}{2\varepsilon_0}\left[1-\frac{x}{x\sqrt{1+\frac{R^2}{x^2}}}\right] = \frac{\sigma}{2\varepsilon_0}\left[1-\frac{1}{\sqrt{1+\frac{R^2}{x^2}}}\right];$$

As $R \gg x$, then the term $\frac{1}{\sqrt{1+\frac{R^2}{x^2}}} \to 0$

And we get $E_x = \frac{\sigma}{2\varepsilon_0}$

Our final result does not contain the distance x from the plane. This is a correct but rather surprising result.

We observe the following.

- The electric field produced by an infinite plane sheet of charge is independent of the distance from the sheet.
- The electric field is uniform; everywhere its direction is perpendicular to the sheet and away from it.
- Infinite plane sheet of charge is a hypothetical case. In real practice, there is no such infinite plane sheet of charge. Again, there is no such thing as an infinite sheet of charge, but if the dimensions of the sheet are much larger than the distance x of the observation point P from the sheet, the field is very nearly the same as for an infinite sheet.

FIELD OF TWO OPPOSITELY CHARGED SHEETS

Two infinite plane sheets are placed parallel to each other, separated by a distance d (as shown in Fig. 1.65). The lower sheet has a uniform positive surface charge density σ, and the upper sheet has a uniform negative surface charge density $-\sigma$ with the same magnitude. Let us find the electric field between the two sheets, above the upper sheet and below the lower sheet.

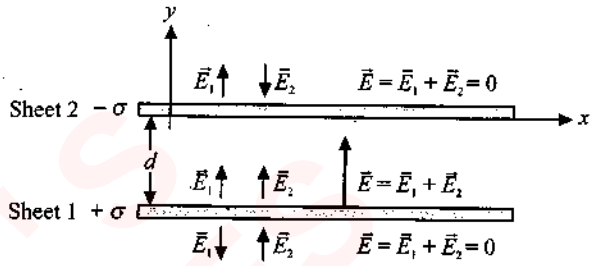


Fig. 1.65

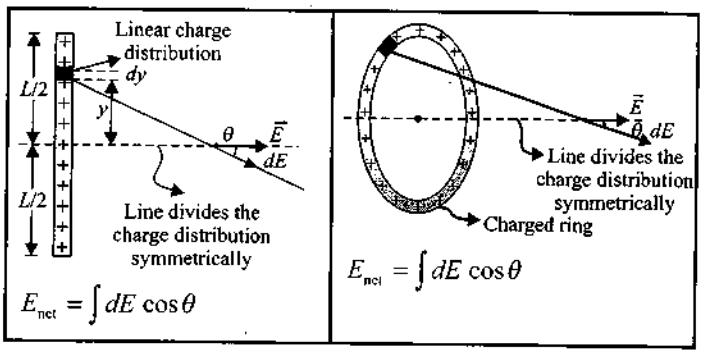
The situation described in this example in an idealization of two finite, oppositely charged sheets, like the plates shown in the figures. If the dimensions of the sheets are large in comparison to the separation d, then to good approximation we can consider the sheets to be infinite in extent. We know the field due to a single infinite plane sheet of charge. We can then find the total field by using the principle of superposition of electric fields. Let sheet 1 be the lower sheet of positive charge, and let sheet 2 be the upper sheet of negative charge; the fields due to each sheet are $\vec{E}_1$ and $\vec{E}_2$, respectively, and both have the same magnitude at all points, no matter how far from either sheet, i.e., $E_1 = E_2 = \sigma/2\varepsilon_0$.

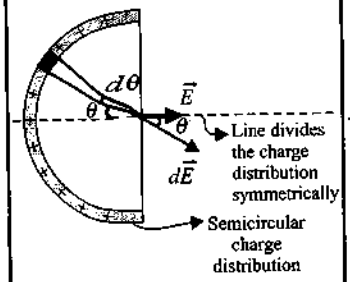
At all points, the direction of $\vec{E}_1$ is away from the positive charge of sheet 1, and the direction of $\vec{E}_2$ is towards the negative charge of sheet 2. These fields, as well as the x- and y-axes, are shown in the figure. At points between the sheets, the fields add each other and at points above the upper sheet or below the lower sheet cancel each other. Thus, the total field is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \begin{cases} 0 & \text{above the upper sheet} \\ \frac{\sigma}{\varepsilon_0}\hat{j} & \text{between the sheets} \\ 0 & \text{above the upper sheet} \end{cases}$$

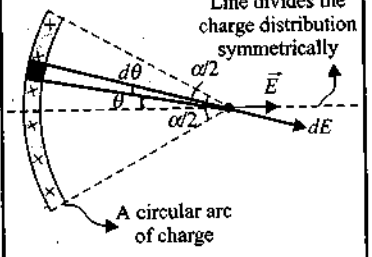
Because we considered the sheets to be infinite, our result does not depend on the separation d.

Symmetry plays very important role in problem solving. Electric field is in the direction along the line which divides the charge distribution symmetrically.



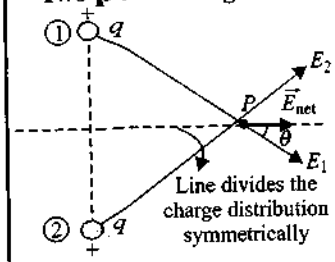


$E_{net} = \int dE \cos\theta$



$E_{net} = \int dE \cos\theta$

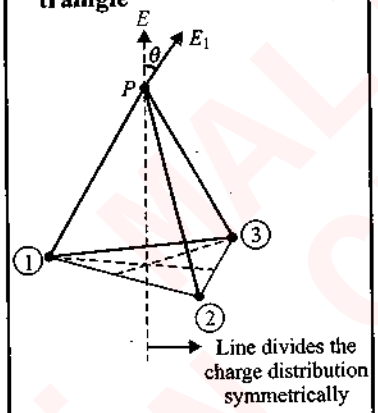
Two point charge



Here, $|\vec{E}_1| = |\vec{E}_2|$

$E_{net} = 2|E_1|\cos\theta$

Three point charge at the corner of an equilateral traingle

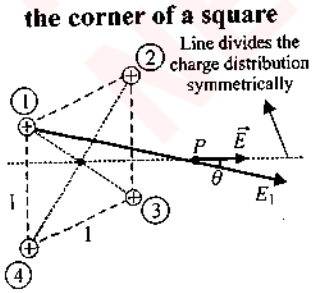


Here, electric fields at P due to charges (1), (2) and (3) are equal,

i.e., $|\vec{E}_1| = |\vec{E}_2| = |\vec{E}_3|$.

Hence, $E_{net} = 3|\vec{E}_1|\cos\theta$

Four point charges at the corner of a square

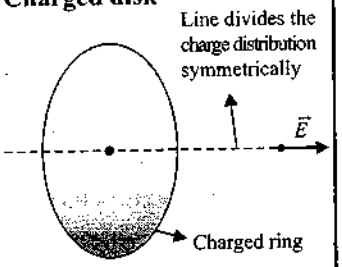


The electric field at point P due to charges (1), (2), (3) and (4) is $|\vec{E}_1| = |\vec{E}_2| = |\vec{E}_3| = |\vec{E}_4|$

Hence, net electric field at P is

$|\vec{E}_{net}| = 4|\vec{E}_1|\cos\theta$

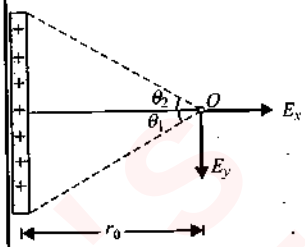
Charged disk



Net field at a point on the axis is along the axis of disk.

Some Useful Results

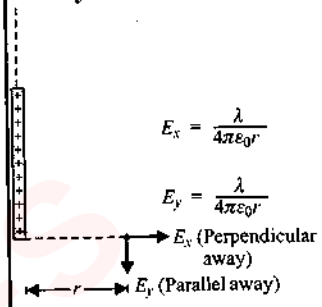
A charged rod of fixed length having charge density λ



$E_x = \frac{\lambda}{4\pi\varepsilon_0 r_0}(\sin\theta_1 + \sin\theta_2)$

$E_y = \frac{\lambda}{4\pi\varepsilon_0 r_0}(\cos\theta_1 - \cos\theta_2)$

Semi-infinite rod having charge density λ



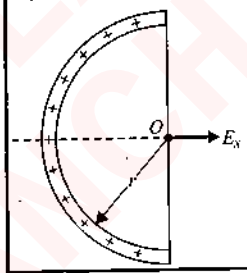
$E_x = \frac{\lambda}{4\pi\varepsilon_0 r}$

$E_y = \frac{\lambda}{4\pi\varepsilon_0 r}$

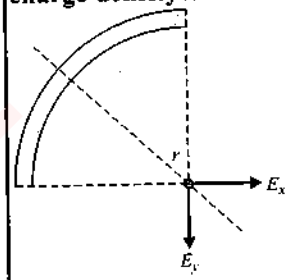
Semicircular ring having charge density λ

$E_x = \frac{\lambda}{2\pi\varepsilon_0 r}$

$E_y = 0$

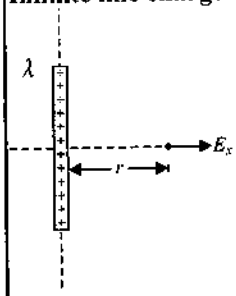


Quarter circular ring having charge density λ



$E_x = \frac{\lambda}{4\pi\varepsilon_0 r}, E_y = \frac{\lambda}{4\pi\varepsilon_0 r}$

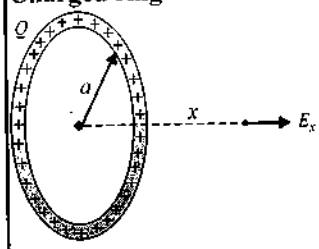
Infinite line charge



$E_x = \frac{\lambda}{2\pi\varepsilon_0 r}$

$E_y = 0$

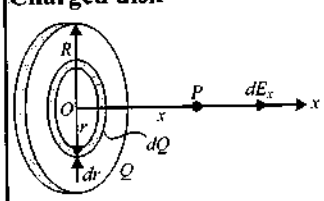
Charged ring



$E_x = \frac{kQx}{(x^2 + a^2)^{3/2}}$

$E_y = 0$

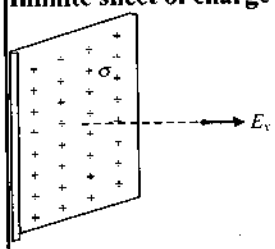
Charged disk



$E_x = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{(x^2 + R^2)}}\right]$

$E_y = 0$

Infinite sheet of charge



$E_x = \frac{\sigma}{2\varepsilon_0}$

$E_y = 0$

R. K. MALIK'S NEWTON CLASSES RANCHI

Concept Application Exercise 1.3

1. a. A particle with positive charge Q is held fixed at the origin (Fig. 1.66). A second particle with positive charge q is fired at the first particle, and follows the trajectory as shown in the figure. Is the angular momentum of second particle constant about some axis? Why or why not? Give reason to support your answer.

   b. Does the Coulomb force that one charge exerts on other charges change if the other charges are brought nearby? (Yes/No)

   a. points A and B?

   b. At which point, A or B, will the acceleration of the test charge be greater if the charge is released?

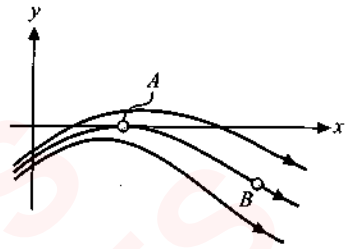


Fig. 1.69

Direction of magnetic moment $\vec{M}$ is perpendicular to the plane of orbit.

b $$\vec{\tau} = \vec{M} \times \vec{B}$$

$$|\vec{\tau}| = MB \sin\theta$$

where θ is the angle between $\vec{M}$ and $\vec{B}$

$$\theta = 30^0$$

$$\therefore \quad \tau = \left(\frac{eh}{4\pi m}\right)(B)\sin 30^0 = \frac{ehB}{8\pi m}$$

The direction of $\vec{\tau}$ is perpendicular to both $\vec{M}$ and $\vec{B}$

Example 57 **A particle of mass m and charge q is moving in a region where uniform, constant electric and magnetic fields $\vec{E}$ and $\vec{B}$ are present. $\vec{E}$ and $\vec{B}$ are parallel to each other. At time $t = 0$, the velocity $\vec{v}_0$ of the particle is perpendicular to $\vec{E}$**

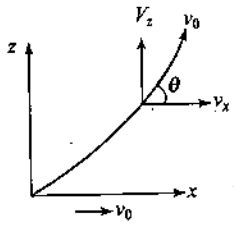


Fig. 1.77

$$\therefore \quad v_x = v_0 \cos \omega t = v_0 \cos\left(\frac{qB}{m}t\right) \quad \text{(ii)}$$

$$v_z = v_0 \sin \omega t = v_0 \sin\left(\frac{qB}{m}t\right) \quad \text{(iii)}$$

From eqs (i), (ii) and (iii)

$$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

$$\left(\frac{qB}{m}\right)(\vec{v}_0) = \frac{qE}{m}(\vec{E}) \qquad \left(\frac{qB}{m}\right)(\vec{v}_0 \times \vec{B})$$

20. A droplet of ink in an industrial ink-jet printer carries a charge of 1.6×10^{-10} C and is deflected onto the paper by a force of 3.2×10^{-4} N. Find the strength of the electric field to produce this force.

ELECTRIC DIPOLE

- An electric dipole is a system of two equal and opposite point charges separated by a very small and finite distance.
- Figure 1.77 shows an electric dipole consisting of two equal and opposite point charges $-q$ and $+q$ separated by a small distance $2l$. The strength of an electric dipole is measured by a vector quantity known as *electric dipole moment*. Its magnitude is equal to the product of the magnitude of either charge and the distance between the two charges, i.e.,

$$p = 2ql$$

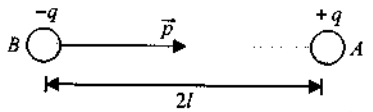


Fig. 1.77

The direction of p is from negative charge to positive charge.

- In S.I. system of units, p is measured in coulomb-metre.

ELECTRIC FIELD DUE TO A DIPOLE

Electric Field Intensity due to an Electric Dipole at a Point on the Axial Line

- A line passing through the negative and positive charges of the electric dipole is called the axial line of the electric dipole.

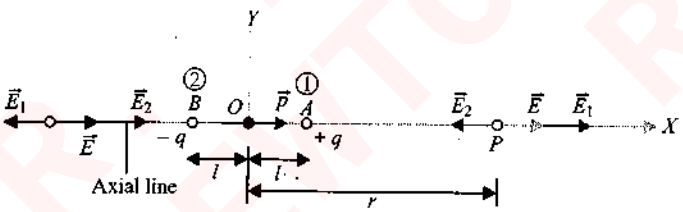


Fig. 1.78

- Suppose an electric dipole AB is located in a medium of dielectric constant K (as shown in Fig.1.78). Let the dipole consists of two point charges $-q$ and $+q$ separated by a short distance $2l$ metre. Let P be an observation point on the axial line such that its distance from the midpoint O of the electric dipole is r. We are interested to calculate the intensity of electric field at P.
- $E_1 = \dfrac{1}{4\pi\varepsilon_0 K}\dfrac{q}{(r-l)^2}$ due to q at P (along the direction OX)

and $E_2 = \dfrac{1}{4\pi\varepsilon_0 K}\dfrac{q}{(r+l)^2}$ due to $-q$ at P (along the direction OB)

The intensities E_1 and E_2 are along the same line but in opposite directions. Since $E_1 > E_2$, the resultant intensity E at the point P will be equal to their differences and in the direction $\vec{A}P$. Thus,

$$E = E_1 - E_2 = \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r-l)^2} - \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r+l)^2}$$

$$= \frac{q}{4\pi\varepsilon_0 K}\left[\frac{4lr}{(r^2-l^2)^2}\right] = \frac{1}{4\pi\varepsilon_0 K}\left[\frac{2(2ql)r}{(r^2-l^2)^2}\right]$$

But $2ql = p =$ electric dipole moment;

$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0 K}\frac{2pr}{(r^2-l^2)^2}$$

- If l is very small as compared to $r (l \ll r)$, then l^2 can be neglected in comparison to r^2. Then, the electric field intensity at the point P due to a short dipole is given by

$$E = \frac{1}{4\pi\varepsilon_0 K}\frac{2pr}{r^4} = \frac{1}{4\pi\varepsilon_0 K}\frac{2p}{r^3}$$

$$= \frac{1}{4\pi\varepsilon_0 K}\frac{2p}{r^3}$$

- If dipole is placed in air or vacuum, then $K = 1$ and

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$$

Note: ***The direction of electric field E is in the direction of $\vec{p}$, i.e., parallel to the axis of dipole from the negative charge towards the positive charge.***

In vector form, we can write

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{r^3}$$

Electric Field Intensity due to an Electric Dipole at a Point on the Equatorial Line

An equatorial line of the electric dipole is a line perpendicular to the axial line and it passes through a point midway between charges.

- Let us now suppose that the observation point P is situated on the equatorial line of dipole such that its distance from midpoint O of the electric dipole is r (as shown in Fig.1.79). Let us assume again that the medium between the electric dipole and the observation point has dielectric constant K.
- $E_1 = \dfrac{1}{4\pi\varepsilon_0 K}\dfrac{q}{(r^2+l^2)}$ (along the direction PD)

and $E_2 = \dfrac{1}{4\pi\varepsilon_0 K}\dfrac{q}{(r^2+l^2)}$ (along the direction PC)

The magnitude of E_1 and E_2 is equal but directions are different.

Net intensity: $E = E_1\cos\theta + E_2\cos\theta$

[since components cancel out]

$$= \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r^2+l^2)}\cos\theta + \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r^2+l^2)}\cos\theta$$

$$= \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r^2+l^2)} \times 2\cos\theta \text{ along } PR$$

But from the figure,

$$\cos\theta = \frac{OA}{PA} = \frac{OA}{(OP^2+OA^2)^{1/2}} = \frac{l}{(r^2+l^2)^{1/2}}$$

$$\therefore E = \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r^2+l^2)} \times \frac{2l}{(r^2+l^2)^{1/2}} = \frac{1}{4\pi\varepsilon_0 K} \times \frac{2ql}{(r^2+l^2)^{3/2}}$$

But $2ql = p$ = electric dipole moment

$$\therefore \quad E = \frac{1}{4\pi\varepsilon_0 K} \times \frac{p}{(r^2+l^2)^{3/2}}$$

- If l is very small as compared to $r (l \ll r)$, then l^2 can be neglected in comparison to r^2.

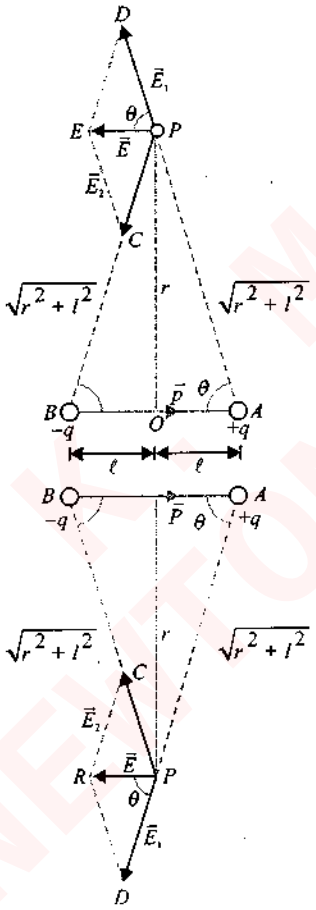


Fig. 1.79

Then, the electric field intensity at the point P due to a short dipole is given by

$$E = \frac{1}{4\pi\varepsilon_0 K} \times \frac{p}{(r^2)^{3/2}} = \frac{1}{4\pi\varepsilon_0 K}\frac{p}{r^3}$$

- If dipole is placed in air or vacuum, then K=1 and $E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$.

The direction of resultant electric field is opposite to the direction of dipole moment, so we can write

$$\vec{E} = -\frac{1}{4\pi\varepsilon_0} \times \frac{\vec{p}}{r^3}$$

Note: ***The direction of electric field E is opposite to the direction of $\vec{p}$, i.e., antiparallel to the axis of dipole from the positive charge towards the negative charge.***

ELECTRIC FIELD INTENSITY DUE TO A SHORT DIPOLE AT SOME GENERAL POINT

- Let AB be a short electric dipole of dipole moment $\vec{p}$ (directed from B to A). We are interested to find the electric field at some general point P. The distance of observation point P w.r.t. midpoint O of the dipole is r and the angle made by the line OP w.r.t. axis of dipole is θ.
- We know that dipole moment of a dipole is a vector quantity. It can be resolved into two rectangular components $\vec{p}_1$ and $\vec{p}_2$ as shown in Fig. 1.80, so that $\vec{p}_1 = \vec{p}_1 + \vec{p}_2$.
  The magnitudes of $\vec{p}_1$ and $\vec{p}_2$ are $p_1 = p\cos\theta$ and $p_2 = p\sin\theta$.
- It is clear from figure that point P lies on the axial line of dipole with moment $\vec{p}_1$. Hence, the magnitude of the electric field intensity $\vec{E}_1$ at P due to $\vec{p}_1$ is

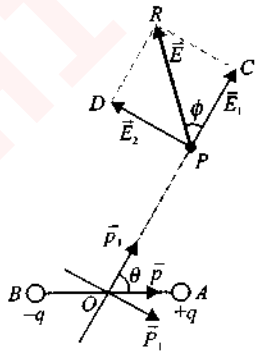


Fig. 1.80

$$E_1 = \frac{1}{4\pi\varepsilon_0} \times \frac{2p\cos\theta}{r^3} \text{ (along } OC) \qquad \text{(i)}$$

Similarly, P lies on the equatorial line of dipole with moment $\vec{p}_2$. Hence, the magnitude of electric field intensity $\vec{E}_2$ at P due to $\vec{p}_2$ is

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^3} \text{ (opposite to } p_2) \qquad \text{(ii)}$$

Hence, the resultant intensity at P is $\vec{E} = \vec{E}_1 + \vec{E}_2$.

The magnitude of $\vec{E}$ is $E = \sqrt{(E_1^2 + E_2^2)}$ (as $\vec{E}_1$ and $\vec{E}_2$ are mutually perpendicular).

$$\text{or} \quad E = \sqrt{\left(\frac{2p\cos\theta}{4\pi\varepsilon_0 r^3}\right)^2 + \left(\frac{p\sin\theta}{4\pi\varepsilon_0 r^3}\right)^2}$$

$$= \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{4\cos^2\theta + \sin^2\theta} = \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{1+3\cos^2\theta}$$

- If the resultant field intensity vector $\vec{E}$ makes an angle ϕ with the direction of $\vec{E}_1$, then

$$\tan\phi = \frac{E_2}{E_1} = \frac{(p\sin\theta/4\pi\varepsilon_0 r^3)}{(2p\cos\theta/4\pi\varepsilon_0 r^3)} = \frac{1}{2}\tan\theta$$

Illustration 1.26 **Three charges $-q, +2q$ and $-q$ are arranged on a line as shown in Fig.1.81. Calculate the field at a distance $r \gg a$ on the line.**

Sol. The field at point P is superposition of fields $\vec{E}_1, \vec{E}_2, \vec{E}_3$ due to each charge.

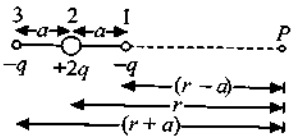


Fig. 1.81

$$\vec{E}_1 = -\frac{q}{4\pi\varepsilon_0(r-a)^2}\hat{i};\ \vec{E}_2 = +\frac{2q}{4\pi\varepsilon_0 r^2}\hat{i};\ \vec{E}_3 = -\frac{q}{4\pi\varepsilon_0(r+a)^2}\hat{i}$$

Now,

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{(r-a)^2} + \frac{2}{r^2} - \frac{1}{(r+a)^2}\right]\hat{i}$$

$$E = \frac{q}{4\pi\varepsilon_0 r^2}\left[-\left\{1-\left(\frac{a}{r}\right)\right\}^{-2} + 2 - \left\{1+\left(\frac{a}{r}\right)\right\}^{-2}\right]$$

If $r \gg a$, we can use binomial approximation:

$$(1+\alpha)^n \simeq 1 + n\alpha + \frac{n(n-1)}{\lfloor 2}\alpha^2 + \cdots \text{ for } \alpha \ll 1$$

Therefore,

$$E = \frac{q}{4\pi\varepsilon_0 r^2}\left[-\left\{\left(1-2\left(-\frac{a}{r}\right)\right) + \frac{-2(-2-1)}{2}\left(-\frac{a}{r}\right)^2\right\}\right.$$

$$\left. +2 - \left\{1 - 2\frac{a}{r} + \frac{-2(-2-1)}{2}\left(\frac{a}{r}\right)^2\right\}\right] = \frac{6a^2 q}{4\pi\varepsilon_0 r^4}$$

The charge in this problem may be considered as two dipoles placed close together. Such an arrangement of charge is called an electric quadrupole.

Illustration 1.27 **What is the force on a dipole of dipole moment p placed as shown in Fig.1.82.**

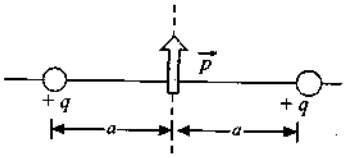


Fig. 1.82

Sol. Force on any q by dipole is

$$F = qE_{\text{dipole}} = \frac{q}{4\pi\varepsilon_0}\frac{p}{a^3} \text{ downwards}$$

So from the third law, force on dipole due to both charges is

$$2F = \frac{qp}{2\pi\varepsilon_0 a^3} \text{ upwards}$$

Net Force on a Dipole in a Non-Uniform Field

Suppose an electric dipole with dipole moment $\vec{P}$ is placed in a non-uniform electric field $\vec{E} = E\hat{i}$ that points along the x-axis (Fig.1.83). Let E depends only on x. The electric field at the position of negative charge is E and at the position of positive charge $(E + \Delta E)$. Then, the net force acting on the dipole is

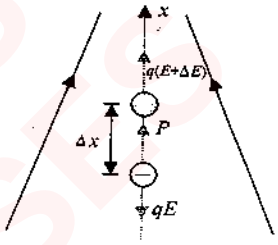


Fig. 1.83

$$F = q(E+\Delta E) - qE = q\Delta E = q\left[\frac{\Delta E}{\Delta x}\Delta x\right]$$

$$\left[\text{as } \frac{\Delta E}{\Delta x} = \frac{dE}{dx}\right]$$

$$F = q\Delta x\frac{dE}{dx} = p\frac{dE}{dx}$$

$$|\vec{F}| = \left|P\frac{d\vec{E}}{dx}\right|$$

where dE/dx is the gradient of the field in the x-direction.

Illustration 1.28 **Find the force on a small electric dipole of dipole moment $\vec{P}$ due to a point charge Q placed at a distance r.**

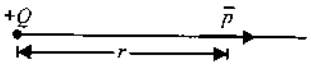


Fig. 1.84

Sol. Electric field of a point charge is a non-uniform electric field. Electric field at a distance x from the point charge is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{x^2} \Rightarrow \frac{dE}{dx} = -\frac{1}{4\pi\varepsilon_0}\frac{2Q}{x^3}$$

The magnitude of force on the dipole is

$F = \left|p\frac{dE}{dx}\right|_{x=r} = \frac{-1}{4\pi\varepsilon_0}\frac{2pQ}{r^3}$, negative sign indicates that force is towards Q or it is attractive.

Alternatively: The same result can be calculated as force on the point charge due to dipole which is the same as the force on dipole due to the point charge (Newton's third law). The electric field of small dipole at a distance r is

$E = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$ Hence, force on the point charge Q is

$$F = EQ = \frac{1}{4\pi\varepsilon_0}\frac{2pQ}{r^3}$$

DIPOLE IN A UNIFORM ELECTRIC FIELD

Torque: When a dipole is placed in a uniform field as shown in Fig.1.85, the net force on it is $F_R = \left[q\vec{E} + (-q)\vec{E}\right] = 0$

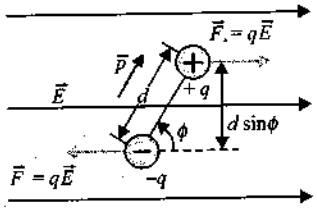


Fig. 1.85

Hence, net force on a dipole is zero in a uniform electric field.

While the torque $\tau = qE \times d\sin\phi$

i.e., $\tau = pE \times \sin\phi$ {as $p = qd$}

or $\vec{\tau} = \vec{p} \times \vec{E}$ (by electric field)

and $\vec{\tau} = \vec{E} \times \vec{p}$ (by us if the dipole is in equilibrium)

From the expression, it is clear that couple acting on a dipole is maximum $(= pE)$ when dipole is perpendicular $(\phi = 90°)$ to the field and minimum $(= 0)$ when dipole is parallel $(\phi = 0°)$ or antiparallel $(\phi = 180°)$ to the field.

By applying a torque, electric field tends to align a dipole in its own direction.

Illustration 1.29 **An electric dipole consists of two charges of 0.1 μC separated by a distance of 2.0 cm. The dipole is placed in an external field of 10^5 NC^{-1}. What maximum torque does the field exert on the dipole?**

Sol. $\tau = pE\sin\theta = q \times 2a \times E\sin\theta$. The value of τ will be maximum when $\sin\theta = 1$

$$\therefore \tau_{max} = 10^{-7} \times 2 \times 10^{-2} \times 10^5 \times 1 = 2 \times 10^{-4} \text{ N m}$$

Concept Application Exercise 1.4

1. State the following statements as true/false:
   a. An electric dipole is kept in a uniform electric field at some angle with it. It experiences a force but no torque.
   b. An electric dipole may experience a net force when it is placed in a non-uniform electric field.
   c. An electric dipole is kept in a non-uniform electric field. It can experience a force and a torque.
2. Electric intensity due to an electric dipole varies with distance as $E \propto r^n$, where n is _____.
3. An electric dipole of moment $\vec{p}$ is placed at the origin along the x-axis. The electric field E at a point P, whose position vector makes an angle θ with the x-axis, will make an angle with x-axis which is _____.

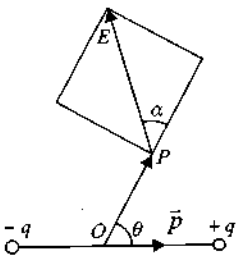


Fig. 1.86

4. Two point charges of +1 μC and −1 μC are separated by a distance of 100 Å. A point P is at a distance of 10 cm from the midpoint and on the perpendicular bisector of the line joining the two charges. Find the electric field at P.
5. An electric dipole consists of two opposite charges of magnitude 2×10^{-6} C each and separated by a distance of 3 cm. It is placed in an electric field of 2×10^5 NC^{-1}. Determine the maximum torque on the dipole.
6. Three charges are arranged on the vertices of an equilateral triangle as shown in Fig.1.87. Find the dipole moment of the combination.

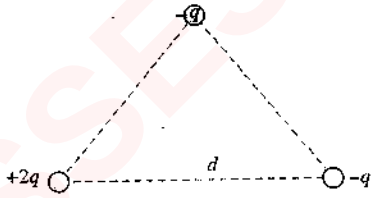


Fig. 1.87

7. The electric field at A due to dipole p is perpendicular to p. the angle θ is _____.

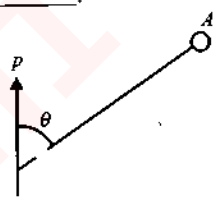


Fig. 1.88

8. An electric dipole is formed by two particles fixed at the end of a light rod of length l. The mass of each particle is m and the charges are $-q$ and $+q$. The system is placed in such a way that the dipole axis is parallel to a uniform electric field E that exists in region. The dipole is slightly rotated about its centre and released. Show that for small angular displacement, the motion is S.H.M. Evaluate its time period.

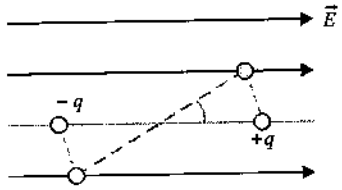


Fig. 1.89

9. A dipole consists of two particles carrying charges +2 μC and −2 μC and masses 1 and 2 kg, respectively, separated by a distance of 6 m. It is placed in a uniform electric field of 8×10^4 Vm^{-1}. For small oscillations about its equilibrium position, find the angular frequency.
10. A small electric dipole of dipole moment P is placed near a point charge $+Q$ as shown. Then, the net force on the dipole is towards _____.

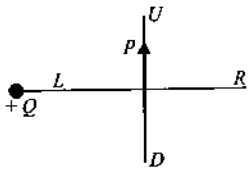


Fig. 1.90

Solved Examples

Example 1.1 **A uniformly charged wire with linear charge density λ is laid in the form of a semicircle of radius R. Find the electric field generated by the semicircle at the centre.**

Sol. We consider a differential element dl on the ring that subtends an angle $d\theta$ at the centre of the ring, i.e., $dl = Rd\theta$.

Charge on this element $= dQ = \lambda Rd\theta$.

This element creates a field dE which makes an angle θ at the centre as shown in Fig.1.91.

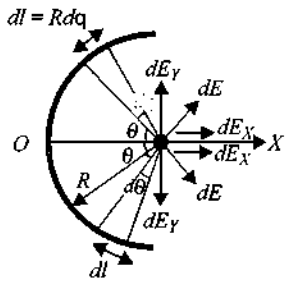


Fig. 1.91

For each differential element in the upper half of the ring, there corresponds a symmetrically placed charge element in the lower half. The y-components of the field due to these symmetric elements cancel out and the x-components remain. We get

$$dE_x = dE\cos\theta = \frac{dQ}{4\pi\varepsilon_0 R^2}\cos\theta = \frac{\lambda(Rd\theta)\cos\theta}{4\pi\varepsilon_0 R^2}$$

On integrating the expression for dE_x w.r.t. angle θ in limits $\theta = -\pi/2$ to $\theta = +\pi/2$, we obtain

$$E = \int_{-\pi/2}^{+\pi/2} \frac{\lambda R}{4\pi\varepsilon_0 R^2}\cos\theta\, d\theta = \frac{\lambda}{2\pi\varepsilon_0 R}$$

In terms of the total charge, say Q, on the ring, $\lambda = Q/\pi R$ and we get $E = Q/(2\pi^2\varepsilon_0 R^2)$.

If we consider the wire in the form of an arc as shown in Fig. 1.92, the symmetry consideration is not useful in cancelling out x- and y-components of the fields, if θ_1 and θ_2 are different. We will integrate dE_x as well as dE_y in limits $\theta = -\theta_1$ to $\theta = +\theta_2$.

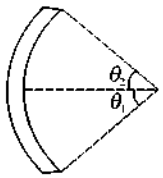


Fig. 1.92

$$E_x = \int_{-\theta_1}^{+\theta_2} \frac{\lambda R}{4\pi\varepsilon_0 R^2}\cos\theta\, d\theta = \frac{\lambda}{4\pi\varepsilon_0 R}(\sin\theta_1 + \sin\theta_2)$$

$$E_y = -\int_{-\theta_1}^{+\theta_2} \frac{\lambda R}{4\pi\varepsilon_0 R^2}\sin\theta\, d\theta = \frac{\lambda}{4\pi\varepsilon_0 R}(\cos\theta_2 - \cos\theta_1)$$

For a symmetrical arc, $\theta_1 = \theta_2$. Thus, E_y vanishes and

$$E_x = \frac{\lambda\sin\theta}{2\pi\varepsilon_0 R}$$

Example 1.2 **A long wire with a uniform charge density λ is bent in two configurations shown in Fig. 1.93 (a) and (b). Determine the electric field intensity at point O.**

Sol. In Fig. 1.93(a),

a. Field due to segment (1):

$$\vec{E}_1 = \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat{i} + \left(-\frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat{j}$$

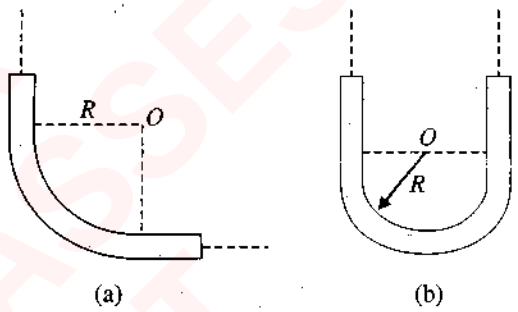


Fig. 1.93

Field due to segment (2):

$$\vec{E}_2 = \left(-\frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat{i} + \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat{j}$$

Field due to quarter shape wire segment (3):

$$\vec{E}_3 = \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat{i} + \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat{j} \qquad (\because \theta_1 = 90°, \theta_2 = 0°)$$

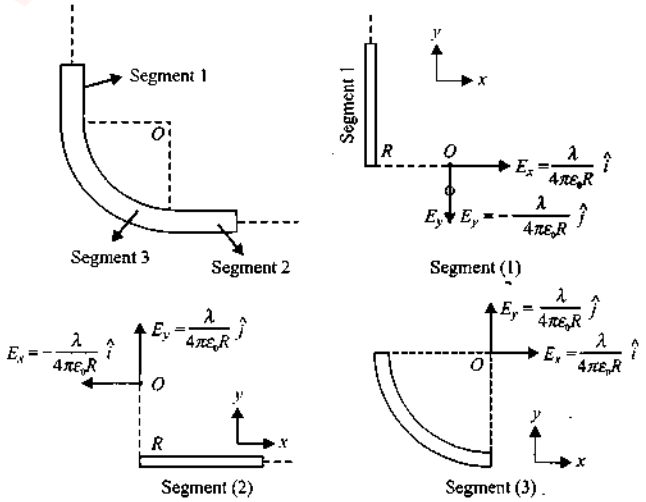


Fig. 1.94

Resultant field is superposition of the fields due to each part, i.e.,

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 \qquad \text{(i)}$$

Substituting the values of $\vec{E}_1$, $\vec{E}_2$ and $\vec{E}_3$ in Eq. (i), we get

$$\vec{E} = \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat{i} + \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat{j}$$

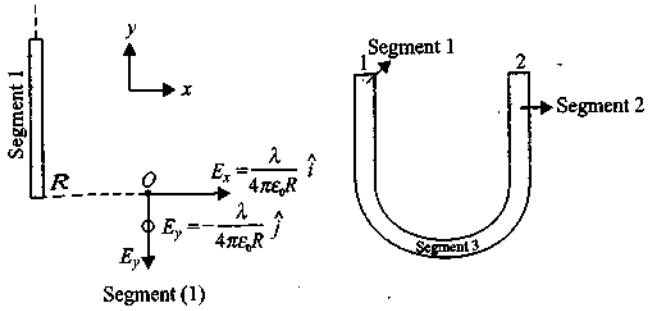


Fig. 1.95

$$|\vec{E}| = \left[\left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)^2 + \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)^2\right]^{1/2} = \frac{\sqrt{2}\lambda}{4\pi\varepsilon_0 R}$$

Here, $E_x = E_y = \frac{\lambda}{4\pi\varepsilon_0 R}$. Hence, the resultant field will make an angle of 45° with the axis.

b. Field due to segment 1

$$\vec{E}_1 = \frac{\lambda}{4\pi\varepsilon_0 R}[\hat{i} - \hat{j}]$$

Field due to segment 2, $\vec{E}_{x_2} = -\frac{\lambda}{4\pi\varepsilon_0 R}\hat{i}$

$$\vec{E}_{y_2} = -\frac{\lambda}{4\pi\varepsilon_0 R}\hat{j}$$

$$\vec{E}_2 = -\frac{\lambda}{4\pi\varepsilon_0 R}[\hat{i} + \hat{j}]$$

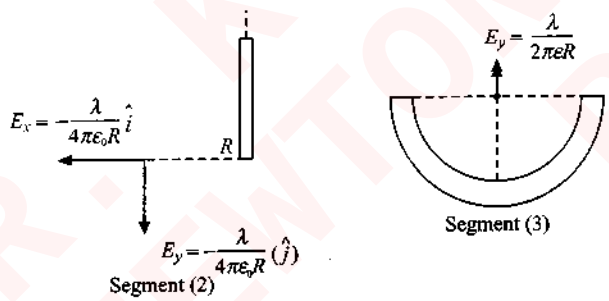


Fig. 1.96

Field due to segment 3,

$$\vec{E}_3 = \frac{\lambda}{2\pi\varepsilon_0 R}\hat{j}$$

From the principle of superposition of electric fields,

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \frac{\lambda}{4\pi\varepsilon_0 R}[\hat{i} - \hat{j}] - \frac{\lambda}{4\pi\varepsilon_0 R}[\hat{i} + \hat{j}]$$

$$+\frac{\lambda}{2\pi\varepsilon_0 R}\hat{j} = 0$$

Hence, the net field is zero.

Example 1.3 **A particle having charge that of an electron and mass 1.6×10^{-30} kg is projected with an initial speed u at an angle 45° to the horizontal from the lower plate of a parallel-plate capacitor as shown in the figure. The plates are sufficiently long and have separation of 2 cm. Find the maximum value of the velocity of the particle so that it does not hit the upper plate. Take electric field between the plates = 10^3 Vm^{-1} directed upwards.**

Sol. Resolving the velocity of the particle parallel and perpendicular to the plate, we get

$$u_{\parallel} = u\cos 45° = \frac{u}{\sqrt{2}} \text{ and } u_{\perp} = u\sin 45° = \frac{u}{\sqrt{2}}$$

Force on the charged particle in downward direction normal to the plate = eE.

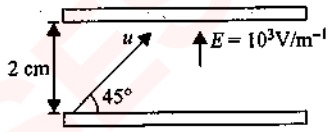


Fig. 1.97

Therefore, acceleration $a = eE/m$, where m is the mass of the charged particle.

The particle will not hit the upper plate, if the velocity component normal to the plate becomes zero before reaching it, i.e., $0 = -u^2_{\perp} - 2ay$ with $y \le d$, where d is the distance between the plates.

Therefore, the maximum velocity for the particle not to hit the upper plate (for this $y = d = 2$ cm) is

$$u_{\perp} = \sqrt{2ay} = \sqrt{\frac{2\times 1.6\times 10^{-19}\times 10^3\times 2\times 10^{-2}}{1.6\times 10^{-30}}}$$

$$= 2\times 10^6 \text{ ms}^{-1}$$

$$\Rightarrow \quad u_{max} = u_{\perp}/\cos 45° = 2\sqrt{2}\times 10^6 \text{ ms}^{-1}$$

Example 1.4 **A particle of mass m and charge q is released from rest in a uniform field of magnitude E. The uniform field is created between two parallel-plates of charge densities $+\sigma$ and $-\sigma$, respectively. The particle accelerates towards the other plate a distance d apart. Determine the speed at which it strikes the opposite plate.**

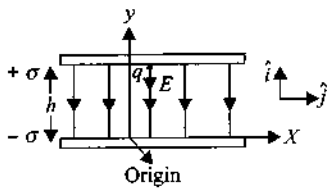


Fig. 1.98

Sol. The applied electric field is $\vec{E} = -E_0\hat{J}$.

The force experienced by the charge q is, $\vec{F} = q\vec{E} = -qE_0\hat{J}$.

The force is constant, and so the acceleration is constant as well, i.e.,

$$\vec{a} = \frac{\vec{F}}{m} = -\frac{qE_0}{m}\hat{j}$$

Because of constant acceleration, the particle moves in the – ve y-direction; the problem is analogous to motion of a mass released from rest in a gravitational field.

From equations of motion, we get

$$v_y = v_{y0} + a_y t = 0 - \frac{qE_0}{m} t \qquad \text{(i)}$$

And $$y = y_0 + v_0 t + \frac{1}{2} a_y t^2; 0 = d + 0 - \frac{1}{2}\frac{qE_0}{m} t^2 \qquad \text{(ii)}$$

Particle starts at $y_0 = d$ and impact occurs at $y = 0$

From Eq. (ii), we get $t = \left(\frac{2dm}{qE_0}\right)^{1/2}$

From Eq. (i), we get $v_y = -\frac{qE_0}{m}\left(\frac{2dm}{qE_0}\right)^{1/2} = -\sqrt{\frac{2qE_0 d}{m}}$

Example 1.5 **Two balls of charges q_1 and q_2 initially have a velocity of the same magnitude and direction. After a uniform electric field has been applied for a certain time interval, the direction of first ball changes by 60° and the velocity magnitude is reduced by half. The direction of velocity of the second ball changes by 90°. In what ratio will the velocity of the second ball change? Determine the magnitude of the charge-to-mass ratio of the second ball if it is equal to α_1 for the first ball. Ignore the electrostatic interaction between the balls.**

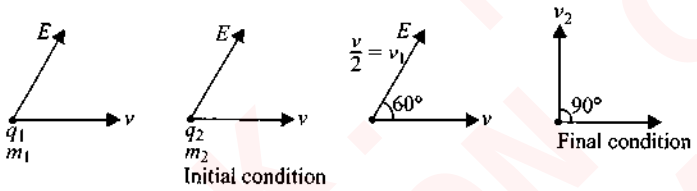


Fig. 1.99

Sol. Let the electric field on each ball be given by

$$E = E_x\hat{i} + E_y\hat{j}$$

From impulse–momentum equation, we have

Impulse = Change in momentum

Let the final velocities of the balls be v_1 and v_2. Noting that $v_1 = v/2$, we have

$$q_1(E_x\hat{i} + E_y\hat{j})\Delta t = m_1\left(\frac{v}{2}\cos 60°\hat{i} + \frac{v}{2}\sin 60°\hat{j}\right) - m_1 v\hat{i} \qquad \text{(i)}$$

$$q_2(E_x\hat{i} + E_y\hat{j})\Delta t = m_2\left(v_2\cos 90°\hat{i} + v_2\sin 90°\hat{j}\right) - m_2 v\hat{i} \qquad \text{(ii)}$$

On comparing the x- and y-components on both sides of Eq. (i), we get

$$\frac{q_1}{m_1}E_x\Delta t = -\frac{3}{4}v \text{ and } \frac{q_1}{m_1}E_y\Delta t = \frac{\sqrt{3}}{4}v \qquad \text{(iii)}$$

Similarly, for Eq. (ii), we get

$$\frac{q_2}{m_2}E_x\Delta t = -v \text{ and } \frac{q_2}{m_2}E_y\Delta t = v_2 \qquad \text{(iv)}$$

From Eq. (iii) and (iv), by dividing the equations expression for x-components, we get

$$\frac{q_1/m_1}{q_2/m_2} = \frac{3}{4} \qquad \text{(v)}$$

or $$\frac{q_2}{m_2} = \frac{4}{3}\frac{q_1}{m_1} = \frac{4}{3}\alpha_1$$

Also $$\frac{q_1/m_1}{q_2/m_2} = \frac{\sqrt{3}v}{4v_2} \Rightarrow \frac{\sqrt{3}v}{4v_2} = \frac{3}{4} \Rightarrow v_2 = \frac{v}{\sqrt{3}}$$

Example 1.6 **A rigid insulated wire frame in the form of a right-angled triangle ABC is set in a vertical plane as shown in the figure. Two beads of equal masses m each and carrying charges q_1 and q_2 are connected by a cord of length l and can slide without friction on the wires. (IIT-JEE, 1978)**

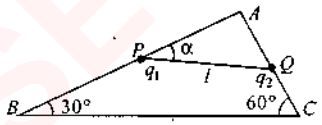


Fig. 1.100

Considering the case when the beads are stationary determine

(i) the angle α

(ii) the tension in the cord

(iii) the normal reaction on the beads

If the cord is now cut what are the value of the charges for which the beads continue to remain stationary?

Sol. Tension and electrostatic force are in opposite direction and along the string. Now each bead is in equilibrium under the concurrent forces: Normal reaction (N), weight (mg) and the resultant of tension and electrostatic force, i.e., $T - F_e$

where $$F_e = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{\ell^2}$$

Applying Lami's theorem for both the beads, we get

$$\frac{N_1}{\sin(120 - \alpha)} = \frac{mg}{\cos\alpha} = \frac{T - F_e}{\cos 60°} \qquad \text{(i)}$$

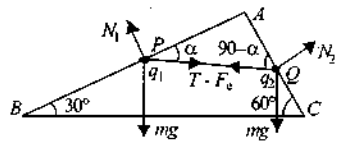


Fig. 1.101

$$\frac{N_2}{\sin(60° + \alpha)} = \frac{mg}{\sin\alpha} = \frac{T - F_e}{\cos 30°} \qquad \text{(ii)}$$

Dividing Eq. (i) by Eq. (ii), we have

(i) $$\tan\alpha = \frac{\cos 30°}{\cos 60°} = \sqrt{3}, \text{ therefore } \alpha = 60°$$

(ii) $$T = F_e + mg = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q_1 q_2}{\ell^2} + mg \qquad \text{(iii)}$$

(iii) $$N_1 = \sqrt{3}\,mg \quad \text{and} \quad N_2 = mg$$

From Eq. (iii), $T = 0$ when string is cut

or $$q_1 q_2 = -(4\pi\varepsilon_0)mg\,\ell^2$$

EXERCISES

Subjective Type

Solutions on page 1.41

1. Calculate the number of electrons in a small, electrically neutral silver pin that has a mass of 10.0 g. Silver has 47 electrons per atom, and its molar mass is 107.87 $gmol^{-1}$.
2. A charged particle of radius 5×10^{-7} m is located in a horizontal electric field of intensity 6.28×10^5 Vm^{-1}. The surrounding medium has coefficient of viscosity $\eta = 1.6 \times 10^5$ Nsm^{-2}. The particle starts moving under the effect of electric field and finally attains a uniform horizontal speed of 0.02 ms^{-1}. Find the number of electrons on it. Assume gravity free space.
3. Suppose that 1.00 g of hydrogen is separated into electrons and protons. Suppose also that the protons are placed at the Earth's north pole and the electrons are placed at the south pole. What is the resulting compression force on the Earth? (Given that radius of earth is 6400 km).
4. Two identical conducting small spheres are placed with their centres 0.300 m apart. One is given a charge of 12.0 nC and the other a charge of −18.0 nC.
   a. Find the electric force exerted by one sphere on the other?
   b. If the spheres are connected by a conducting wire, find the electric force between the two after they attain equilibrium.
5. Four equal point charges, each of magnitude $+Q$, are to be placed in equilibrium at the corners of a square. What should be the magnitude and sign of the point charge that should be placed at the centre of square to do this job?
6. Two point electric charges of values q and $2q$ are kept at a distance d apart from each other in air. A third charge Q is to be kept along the same line in such a way that the net force acting on q and $2q$ is zero. Find the location of the third charge from charge 'q'.
7. Two fixed point charges $+4e$ and $+e$ unit are separated by a distance 'a'. Where the third point charge should be placed from $+4e$ charge for it to be in equilibrium?
8. Two identical particles are charged and held at a distance of 1 m from each other. They are found to be attracting each other with a force of 0.027 N. Now, they are connected by a conducting wire, so that charge flows between them. When the charge flow stops, they are found to be repelling each other with a force of 0.009 N. Find the initial charge on each particle.
9. Two similarly and equally charged identical metal spheres A and B repel each other with a force of 2×10^{-5} N. A third identical uncharged sphere C is touched with A and then placed at the mid-point between A and B. Find the net electric force on C.
10. Three point charges of +2 μC, −3 μC and −3 μC are kept at the vertices A, B and C, respectively, of an equilateral triangle of side 20 cm as shown in the figure. What should be the sign and magnitude of the charge (q) to be placed at the midpoint (M) of side BC so that the charge at A remains in equilibrium?

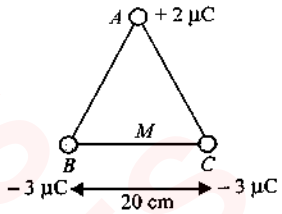


Fig. 1.102

11. Two small beads having positive charges $3q$ and q are fixed at the opposite ends of a horizontal, insulating rod, extending from the origin to the point $x = d$. As shown in the figure, a third small charged bead is free to slide on the rod. At what position is the third bead in equilibrium? Can it be in stable equilibrium?

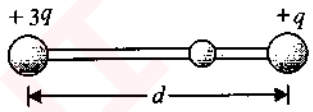


Fig. 1.103

12. A copper atom consists of copper nucleus surrounded by 29 electrons. The atomic weight of copper is 63.5 $gmol^{-1}$. Let us now take two pieces of copper each weighing 10 g. Let us consider one electron from one piece is transferred to another for every 1000 atoms in a piece.
   a. Find the magnitude of charge appearing on each piece.
   b. What will be the Coulomb force between the two pieces after the transfer of electrons if they are 10 cm apart?
   [Avogradro's number = 6×10^{23} mol^{-1}]
13. A flat square sheet of charge of side 50 cm carries a uniform surface charge density. An electron 0.5 cm from a point near the centre of the sheet experiences a force of 1.8×10^{-12} N directed away from the sheet. Determine the total charge on the sheet.
14. A particle of mass 9×10^{-31} kg having a negative charge of 1.6×10^{-19} C is projected horizontally with a velocity of 10^6 ms^{-1} into a region between two infinite horizontal parallel plates of metal. The distance between the plates is $d = 0.3$ cm and the particle enters 0.1 cm below the top plate. The top and bottom plates are connected, respectively, to the positive and negative terminals of a 30 V battery. Find the components of the velocity of the particle just before it hits one of the plates.

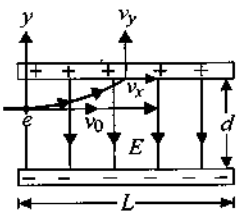


Fig. 1.104

15. Point charges q and $-q$ are located at the vertices of a square with diagonals $2l$ as shown in the figure. Evaluate the magnitude of the electric field strength at a point located symmetrically with respect to the vertices of the square at a distance x from the centre.

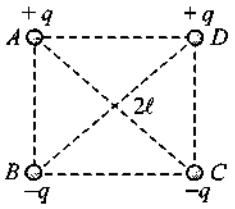


Fig. 1.105

16. Two mutually perpendicular long straight conductors carrying uniformly distributed charges of linear charge densities λ_1 and λ_2 are positioned at a distance a from each other. How does the interaction between the rods depend on a?

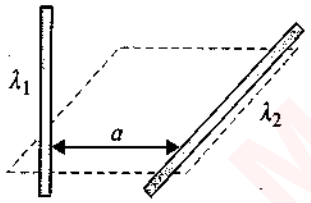


Fig. 1.106

17. A ring of radius 0.1 m is made out of a thin metallic wire of area of cross section 10^{-6} m^2. The ring has a uniform charge of π coulombs. Find the change in the radius of the ring when a charge of 10^{-8} C is placed at the centre of the ring. Young's modulus of the metal is 2×10^{11} Nm^{-2}.

18. A charged cork ball of mass m is suspended on a light string in the presence of a uniform electric field as shown in the figure. When $E = (A\hat{i} + B\hat{j})$ NC^{-1}, where A and B are positive numbers, the ball is in equilibrium at the θ. Find (a) the charge on the ball and (b) the tension in the string.

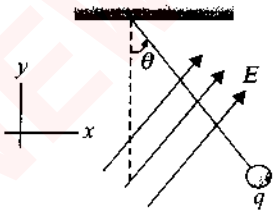


Fig. 1.107

19. A ring of radius R has charge $-Q$ distributed uniformly over it. Calculate the charge that should be placed at the centre of the ring such that the electric field becomes zero at a point on the axis of the ring distant 'R' from the centre of the ring.

20. Two identical small equally charged conducting balls are suspended from long threads secured at one point. The charges and masses of the balls are such that they are in equilibrium when the distance between them is a (the length of thread $L \gg a$). Then one of the balls is discharged. What will be the distance b ($b \ll L$) between the balls when equilibrium is restored?

21. Two point charges Q_a and Q_b are positioned at points A and B. The field strength to the right of charge Q_b on the line that passes through the two charges varies according to a law that is represented schematically in the figure accompanying the problem (without employing a definite scale). The field strength is assumed to be positive if its direction coincides with the positive direction of the x-axis. The distance between the charges is $l = 21$ cm (Fig. 1.108). Find

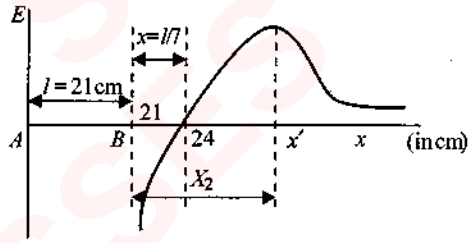


Fig. 1.108

a. the signs of the charges
b. the ratio between the absolute values of charges Q_a and Q_b
c. the coordinate x of the point where the field strength is maximum

22. Two semicircular wires ABC and ADC, each of radius 'R', are lying on $x - y$ and $x - z$ plane, respectively, as shown in Fig.1.109.
If the linear charge density of the semicircular parts and straight parts is λ, find the electric field intensity $\vec{E}$ at the origin.

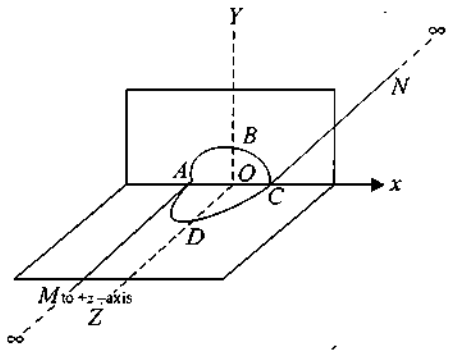


Fig. 1.109

23. An infinite wire having linear charge density λ is arranged as shown in Fig 1.110. A charge particle of mass m and charge q is released from point P. Find the initial acceleration of the particle (at $t = 0$) just after the particle is released.

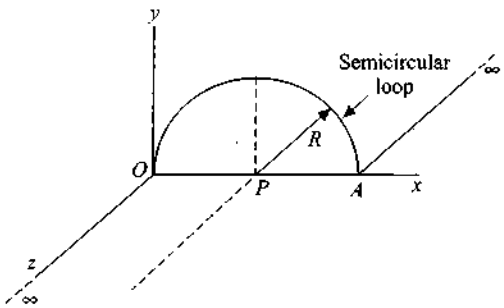


Fig. 1.110

24. Two similar balls, each of mass m and charge q, are hung from a common point by two silk threads, each of length l (Fig.1.111). Prove that separation between the balls is $x = \left[\frac{q^2 l}{2\pi\varepsilon_0 mg}\right]^{1/3}$, if θ is small.

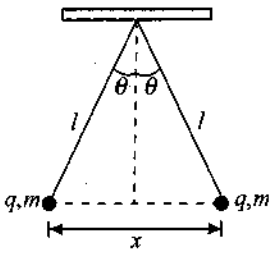


Fig. 1.111

Find the rate dq/dt with which the charge should leak off each sphere if the velocity of approach varies as $v = a/\sqrt{x}$, where a is a constant.

25. Three equal negative charges, $-q_1$ each, form the vertices of an equilateral triangle. A particle of mass m and a positive charge q_2 is constrained to move along a line perpendicular to the plane of triangle and through its centre which is at a distance r from each of the negative charges as shown in the figure. The whole system is kept in gravity free space. Find the time period of vibration of the particle for small displacement from equilibrium position.

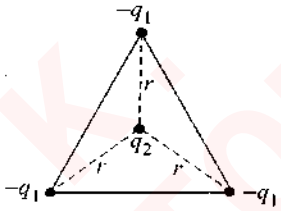


Fig. 1.112

26. A ball of radius R carries a positive charge whose volume density at a point is given as $\rho = \rho_0(1 - r/R)$, where ρ_0 is a constant and r is the distance of the point from the centre. Assuming the permittivities of the ball and the environment to be equal to unity, find

a. the magnitude of the electric field strength as a function of the distance r both inside and outside the ball

b. the maximum intensity E_{max} and the corresponding distance r_m.

Objective Type

Solutions on page 1.46

1. If a body is charged by rubbing it, its weight
a. always decreases slightly
b. always increases slightly
c. may increase slightly or may decrease slightly
d. remains precisely the same

2. In S.I. system, the value of ε_0 is
a. $C^2 N^{-1} m^{-2}$ **b.** $9 \times 10^9 C^2 N^{-1} m^{-2}$
c. $\frac{1}{9\times10^9} C^2 N^{-1} m^{-2}$ **d.** $\frac{1}{4\pi\times9\times10^9} C^2 N^{-1} m^{-2}$

3. Dimensions of ε_0 are
a. $M^{-1}L^{-3}T^4A^2$ **b.** $M^0L^{-3}T^3A^3$
c. $M^{-1}L^{-3}T^3A$ **d.** $M^{-1}L^{-3}TA^2$

4. The dimensional formula of electric intensity is
a. $MLT^{-2}A^{-1}$ **b.** $MLT^{-3}A^{-1}$
c. $ML^2T^{-3}A^{-1}$ **d.** $ML^2T^{-3}A^{-2}$

5. The dielectric constant K of an insulator can be
a. –1 **b.** 0 **c.** 0.5 **d.** 5

6. Choose the correct statement:
a. The total charge of the universe is constant.
b. The total number of the charged particles is constant.
c. The total positive charge of the universe remains constant.
d. The total negative charge of the universe remains constant.

7. Two neutrons are placed at some distance apart from each other. They will
a. attract each other
b. repel each other
c. neither attract nor repel each other
d. cannot say

8. When a soap bubble is charged, its size
a. increases
b. decreases
c. remains the same
d. increases if it is given positive charge and decreases if it is given negative charge

9. Two point charges at certain distance apart in air repel each other with a force F. A glass plate is introduced between the charges. The force becomes F_1, where
a. $F_1 < F$ **b.** $F_1 = F$
c. $F_1 > F$ **d.** data is insufficient

10. There are two charges +1 μC and +5 μC. The ratio of the forces (force on one due to other) acting on them will be
a. 1:1 **b.** 1:2 **c.** 1:3 **d.** 1:4

11. Two point charges Q_1 and Q_2 are 3 m apart, and their sum of charges is 10 μC. If force of attraction between them is 0.075 N, then the values of Q_1 and Q_2, respectively, are
a. 5 μC, 5 μC **b.** 15 μC, –5μC
c. 5μC, 15μC **d.** –15 μC, 5 μC

12. A certain charge 'Q' is to between divided into two parts q and $Q - q$. What is the relationship 'Q' and 'q' if the two parts, placed at a given distance 'r' apart, are to have the maximum Coulomb repulsion?
a. $q = Q/2$ **b.** $q = Q/3$ **c.** $q = 2Q/2$ **d.** $q = Q/4$

13. Three charged particles are placed on a straight line as shown in the figure. q_1 and q_2 are fixed but q_3 can be moved. Under the action of the forces from q_1 and q_2, q_3 is in equilibrium. What is the relation between q_1 and q_2?

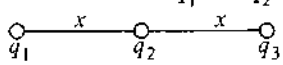


Fig. 1.113

a. $q_1 = 4q_2$ **b.** $q_1 = -q_2$ **c.** $q_1 = -4q_2$ **d.** $q_1 = q_2$

14. Two particles A and B (B is right of A) having charges 8×10^{-6} C and -2×10^{-6} C, respectively, are held fixed with separation of 20 cm. Where should a third charged particle be placed so that it does not experience a net electric force.
a. 5 cm right of B **b.** 5 cm left of A
c. 20 cm left of A **d.** 20 cm right of B

15. Five balls numbered 1, 2, 3, 4, 5 are suspended using separate threads. The balls (1, 2), (2, 4) and (4, 1) show electrostatic attraction, while balls (2, 3) and (4, 5) show repulsion. Therefore, ball 1 must be
a. negatively charged **b.** positively charge
c. neutral **d.** made of metal

16. Electric charges A and B repel each other. Electric charges B and C also repel other. If A and C are held close together, they will
a. attract **b.** repel
c. not affect each other **d.** none of these

17. Two point charges repel each other with a force of 100 N. One of the charges is increased by 10% and the other is reduced by 10%. The new force of repulsion at the same distance would be
a. 100 N **b.** 121 N
c. 99 N **d.** none of these

18. Three charges $+Q_1$, $+Q_2$ and q are placed on a straight line such that q is somewhere in between $+Q_1$ and $+Q_2$. If this system of charges is in equilibrium, what should be the magnitude and sign of charge q?
a. $\frac{Q_1Q_2}{(\sqrt{Q_1}+\sqrt{Q_2})^2}$, +ve **b.** $\frac{Q_1+Q_2}{2}$, +ve
c. $\frac{Q_1Q_2}{(\sqrt{Q_1}+\sqrt{Q_2})^2}$, −ve **d.** $\frac{Q_1+Q_2}{2}$, −ve

19. Two positive and equal charges are fixed at a certain distance. A third small charge is placed in between the two charges and it experiences zero net force due to the other two.
a. The equilibrium is stable if small charge is positive
b. The equilibrium is stable if small charge is negative
c. The equilibrium is always stable
d. The equilibrium is not stable

20. An isolated charge q_1 of mass m is suspended freely by a thread of length l. Another charge q_2 is brought near it ($r >> l$). When q_1 is in equilibrium, tension in thread will be

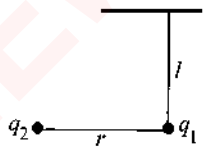


Fig. 1.114

a. mg **b.** $>mg$
c. $<mg$ **d.** none of these

21. Three equal charges, each $+q$, are placed on the corners of an equilateral triangle of side a. Then, the coulomb force experienced by one charge due to the rest of the two is
a. kq^2/a^2 **b.** $2kq^2/a^2$ **c.** $\sqrt{3}\,kq^2/a^2$ **d.** zero

22. A positively charged ball hangs from a long silk thread. Electric field at a certain point (at the same horizontal level of ball) due to this charge is E. Let us put a positive test charge q_0 at this point and measure F/q_0 on this charge. Then, E
a. $> F/q_0$ **b.** $< F/q_0$
c. $= F/q_0$ **d.** none of these

23. Electric field near a straight wire carrying a steady current is
a. proportional to the distance from the wire
b. proportional to inverse square of the distance from the wire
c. inversely proportional to the distance from the wire
d. zero

24. A force of 2.25 N acts on a charge of 15×10^{-4} C. Calculate the intensity of electric field at the point.
a. 1500 NC^{-1} **b.** 150 NC^{-1}
c. 15000 NC^{-1} **d.** none of these

25. An α particle is situated in an electric field of strength 15×10^4 NC^{-1}. Force acting on it is
a. 4.8×10^{-12} N **b.** 4.8×10^{-14} N
c. 48×10^{-14} N **d.** none of these

26. Two particles of masses in the ratio 1 : 2, with charges in the ratio 1 : 1, are placed at rest in a uniform electric field. They are released and allowed to move for the same time. The ratio of their kinetic energies will be finally
a. 2 : 1 **b.** 8 : 1 **c.** 4 : 1 **d.** 1 : 4

27. Three equal charges, each $+q$, are placed on the corners of an equilateral triangle. The electric field intensity at the centroid of the triangle is
a. kq/r^2 **b.** $3kq/r^2$ **c.** $\sqrt{3}\,kq/r^2$ **d.** zero

28. A point charge of 100 μC is placed at $3\hat{i}+4\hat{j}$ m. Find the electric field intensity due to this charge at a point located at $9\hat{i}+12\hat{j}$ m.
a. 8000 Vm^{-1} **b.** 9000 Vm^{-1}
c. 2250 Vm^{-1} **d.** 4500 Vm^{-1}

29. Electric lines of force
a. exist everywhere
b. exist only in the immediate vicinity of electric charges
c. exist only when both positive and negative charges are near one another
d. are imaginary

30. Two charges $Q_1 = 18$ μC and $Q_2 = -2$ μC are separated by a distance R and Q_1 is to the left of Q_2. The distance of the point where the net electric field is zero is
a. between Q_1 and Q_2 **b.** left of Q_1 at $R/2$
c. right of Q_2 at R **d.** right of Q_2 at $R/2$

31. An oil drop, carrying six electronic charges and having a mass of 1.6×10^{-12} g, falls with some terminal velocity in a medium. What magnitude of vertical electric field is required to make the drop move upwards with the same speed as it was formerly moving downwards with? Ignore buoyancy.
a. 10^5 NC^{-1} **b.** 10^4 NC^{-1}
c. 3.3×10^4 NC^{-1} **d.** 3.3×10^5 NC^{-1}

32. What is the largest charge a metal ball of 1 mm radius can hold? Dielectric strength of air is 3×10^6 Vm^{-1}.
a. 3 nC **b.** 1/3 nC **c.** 2 nC **d.** 1/2 nC

33. Five point charges, $+q$ each, are placed at the five vertices of a regular hexagon. The distance of centre of the hexagon from any of the vertices is a. The electric field at the centre of the hexagon is

a. $\frac{q}{4\pi\varepsilon_0 a^2}$ b. $\frac{q}{8\pi\varepsilon_0 a^2}$ c. $\frac{q}{16\pi\varepsilon_0 a^2}$ d. zero

34. A ring of charge with radius 0.5 m has 0.002 π m gap. If the ring carries a charge of +1 C, the electric field at the centre is

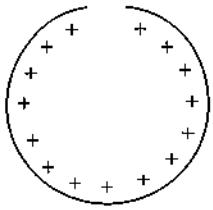


Fig. 1.115

a. 7.5×10^7 NC^{-1} b. 7.2×10^7 NC^{-1}
c. 6.2×10^7 NC^{-1} d. 6.5×10^7 NC^{-1}

35. A block of mass m containing a net negative charge $-q$ is placed on a frictionless horizontal table and is connected to a wall through an unstretched spring of spring constant k as shown. If horizontal electric field E parallel to the spring is switched on, then the maximum compression of the spring is

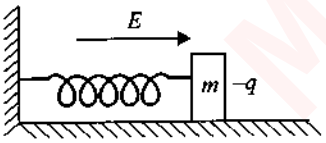


Fig. 1.116

a. $\sqrt{qE/k}$ b. $2qE/k$ c. qE/k d. zero

36. Figure 1.117 shows the electric lines of force emerging from a charged body. If the electric fields at A and B are E_A and E_B, respectively, and if the distance between A and B is r, then

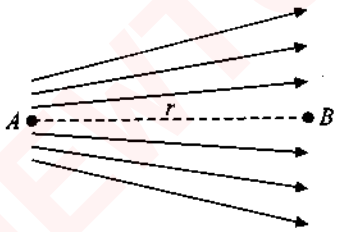


Fig. 1.117

a. $E_A > E_B$ b. $E_A < E_B$ c. $E_A = E_B/r$ d. $E_A = E_B/r^2$

37. If an electron has an initial velocity in a direction different from that of a uniform electric field, the path of the electron is

a. a straight line b. a circle
c. an ellipse d. a parabola

38. A point charge q_1 is moved along a circular path of radius r in the electric field of another point charge q_2 at the centre of the path. The work done by the electric field on the charge q_1 in half revolution is

a. zero b. positive
c. negative d. none of these

39. A spherical conducting ball is suspended by a grounded conducting thread. A positive point charge is moved near the ball. The ball will

a. be attracted to the point charge and swing toward it.
b. be repelled from the point charge and swing away from it.
c. not be affected by the point charge
d. none of these

40. Three +ve charges of equal magnitude 'q' are placed at the vertices of an equilateral triangle of side 'l'. How can the system of charges be placed in equilibrium?

a. By placing a charge $Q = (-q/\sqrt{3})$ at the centroid of the triangle
b. By placing a charge $Q = (q/\sqrt{3})$ at the centroid of the triangle
c. By placing a charge $Q = q$ at a distance l from all the three charges
d. By placing a charge $Q = -q$ above the plane of the triangle at a distance l from all the three charges

41. In the figure, two equal positive point charges $q_1 = q_2 = 2.0$ μC interact with a third point charge $Q = 4.0$ μC. The magnitude as well as direction of the net force on Q is

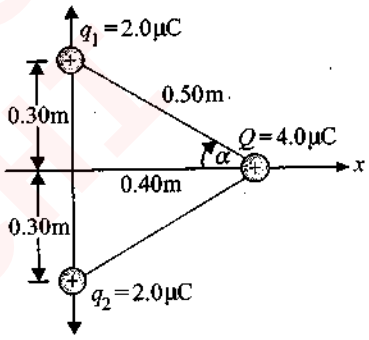


Fig. 1.118

a. 0.23 N in +x direction b. 0.46 N in +x direction
c. 0.23 N in −x direction d. 0.46 N in −x direction

42. Three identical spheres, each having a charge q and radius R, are kept in such a way that each touches the other two. The magnitude of the electric force on any sphere due to other two is

a. $\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{R}\right)^2$ b. $\frac{\sqrt{3}}{4\pi\varepsilon_0}\left(\frac{q}{R}\right)^2$

c. $\frac{\sqrt{3}}{16\pi\varepsilon_0}\left(\frac{q}{R}\right)^2$ d. $\frac{\sqrt{5}}{16\pi\varepsilon_0}\left(\frac{q}{R}\right)^2$

43. Five point charges, each of value +q, are placed on five vertices of a regular hexagon of side L. The magnitude of the force on a point charge of value −q coulomb placed at the centre of the hexagon is

a. $\frac{1}{\pi\varepsilon_0}\left(\frac{q}{L}\right)^2$ b. $\frac{2}{\pi\varepsilon_0}\left(\frac{q}{L}\right)^2$

c. $\frac{1}{2\pi\varepsilon_0}\left(\frac{q}{L}\right)^2$ d. $\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{L}\right)^2$

44. It is required to hold equal charges q in equilibrium at the corners of a square. What charge when placed at the center of the square will do this?

a. $-\frac{q}{2}(1+2\sqrt{2})$ b. $\frac{q}{2}(1+2\sqrt{2})$

c. $\frac{q}{4}(1+2\sqrt{2})$ d. $-\frac{q}{4}(1+2\sqrt{2})$

45. A point charge $q = -8.0$ nC is located at the origin. The electric field (in NC^{-1}) vector at the point $x = 1.2$ m, $y = -1.6$ m as shown in Fig.1.119 is

a. $-14.4\hat{i}+10.8\hat{j}$ b. $-14.4\hat{i}-10.8\hat{j}$
c. $-10.8\hat{i}+14.4\hat{j}$ d. $-10.8\hat{i}-14.4\hat{j}$

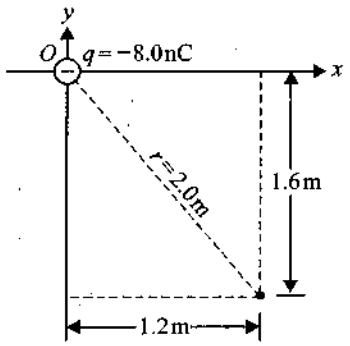


Fig. 1.119

46. A positive point charge 50 μC is located in the plane xy at a point with radius vector $\vec{r}_0 = 2\hat{i}+3\hat{j}$. The electric field vector $\vec{E}$ at a point with radius vector $\vec{r} = 8\hat{i}-5\hat{j}$, where r_0 and r are expressed in metre, is

a. $(1.4\hat{i}-2.6\hat{j})$ kNC^{-1} b. $(1.4\hat{i}+2.6\hat{j})$ kNC^{-1}
c. $(2.7\hat{i}-3.6\hat{j})$ kNC^{-1} d. $(2.7\hat{i}+3.6\hat{j})$ kNC^{-1}

47. Four identical charges Q are fixed at the four corners of a square of side a. The electric field at a point P located symmetrically at a distance $a/\sqrt{2}$ from the centre of the square is

a. $\frac{Q}{2\sqrt{2}\pi\varepsilon_0 a^2}$ b. $\frac{Q}{\sqrt{2}\pi\varepsilon_0 a^2}$ c. $\frac{2\sqrt{2}Q}{\pi\varepsilon_0 a^2}$ d. $\frac{\sqrt{2}Q}{\pi\varepsilon_0 a^2}$

48. A thin glass rod is bent into a semicircle of radius r. A charge $+Q$ is uniformly distributed along the upper half and a charge $-Q$ is uniformly distributed along the lower half, as shown in Fig.1.120. The electric field E at P, the centre of the semicircle, is

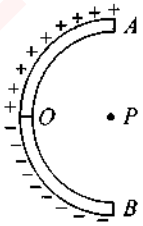


Fig. 1.120

a. $\frac{Q}{\pi^2\varepsilon_0 r^2}$ b. $\frac{2Q}{\pi^2\varepsilon_0 r^2}$ c. $\frac{4Q}{\pi^2\varepsilon_0 r^2}$ d. $\frac{Q}{4\pi^2\varepsilon_0 r^2}$

49. A system consists of a thin charged wire ring of radius r and a very long uniformly charged wire oriented along the axis of the ring, with one of its ends coinciding with the centre of the ring. The total charge on the ring is q and the linear charge density on the straight wire is λ. The interaction force between the ring and the wire is

a. $\frac{\lambda q}{4\pi\varepsilon_0 r}$ b. $\frac{\lambda q}{2\sqrt{2}\pi\varepsilon_0 r}$ c. $\frac{2\sqrt{2}\lambda q}{\pi\varepsilon_0 r}$ d. $\frac{4\lambda q}{\pi\varepsilon_0 r}$

50. Find the electric field vector at P (a, a, a) due to three infinitely long lines of charges along the x-, y- and z-axes, respectively. The charge density, i.e., charge per unit length of each wire is λ.

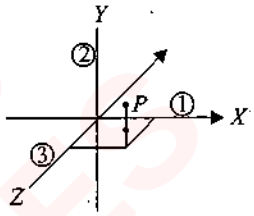


Fig. 1.121

a. $\frac{\lambda}{3\pi\varepsilon_0 a}(\hat{i}+\hat{j}+\hat{k})$ b. $\frac{\lambda}{2\pi\varepsilon_0 a}(\hat{i}+\hat{j}+\hat{k})$
c. $\frac{\lambda}{2\sqrt{2}\pi\varepsilon_0 a}(\hat{i}+\hat{j}+\hat{k})$ d. $\frac{\sqrt{2}\lambda}{\pi\varepsilon_0 a}(\hat{i}+\hat{j}+\hat{k})$

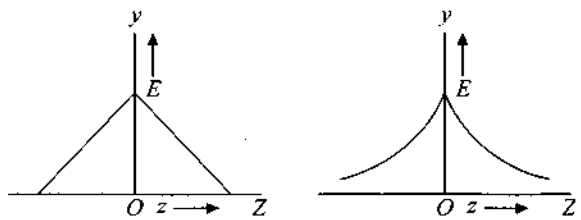
51. A particle of mass m and charge $-q$ moves diametrically through a uniformly charged sphere of radius R with total charge Q. The angular frequency of the particle's simple harmonic motion, if its amplitude $< R$, is given by

a. $\sqrt{\frac{1}{4\pi\varepsilon_0}\frac{qQ}{mR}}$ b. $\sqrt{\frac{1}{4\pi\varepsilon_0}\frac{qQ}{mR^2}}$
c. $\sqrt{\frac{1}{4\pi\varepsilon_0}\frac{qQ}{mR^3}}$ d. $\sqrt{\frac{1}{4\pi\varepsilon_0}\frac{m}{qQ}}$

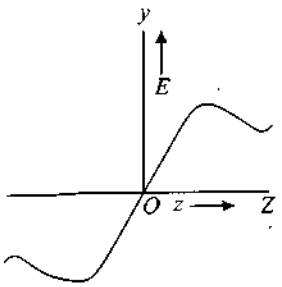
52. A particle of mass m carrying a positive charge q moves simple harmonically along x-axis under the action of a varying electric field E directed along x-axis. The motion of the particle is confined between $x = 0$ and $x = 2l$. The angular frequency of the motion is ω. Then, which of the following is correct?

a. $qE = -m\omega^2(x-l)$
b. $qE = m\omega^2(x-l)$
c. Electric field to the right of origin is directed along +ve x-axis for all values of x.
d. Electric field to the right of origin is directed along −ve x-axis for all values of x.

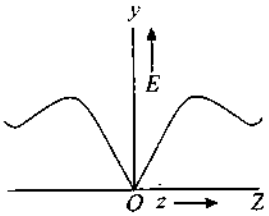
53. A circular ring carries a uniformly distributed positive charge and lies in $X - Y$ plane with centre at origin of coordinate system. If at a point $(0, 0, z)$ the electric field is E, then which of the following graphs is correct?



a. b.



c.



d.

Multiple Correct Answers Type

Solutions on page 1.50

1. For the arrangement shown in Fig.1.122, the two positive charges, $+Q$ each, are fixed. Mark out the correct statement(s) regarding a third charged particle q placed at midpoint P that can be displaced along or perpendicular to the line connecting the charges.

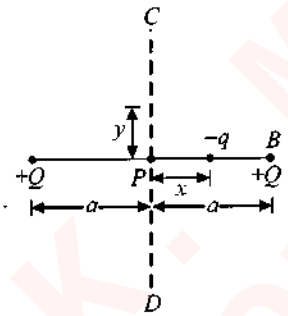


Fig. 1.122

a. The particle will perform S.H.M. for $x << a$.
b. The particle will oscillate about P but not harmonically for any x.
c. The particle will perform S.H.M. for $y << a$.
d. The particle will oscillate about P but not harmonically for y comparable to a.

2. A particle of mass m and charge $-q$ has been projected from ground as shown in Fig.1.123. Mark out the correct statement(s). XY plane is vertical.

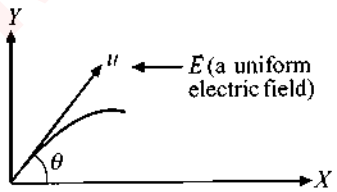


Fig. 1.123

a. The path of motion of the particle may be parabolic.
b. The path of motion of the particle may be a straight line.
c. Time of flight of the particle is $(2u\sin\theta)/g$.
d. Range of motion of the particle can be less than, greater than or equal to $(u^2\sin 2\theta)/g$.

3. For the arrangement shown in Fig.1.124, the two point charges are in equilibrium. The infinite wire is fixed in the horizontal plane and the two point charges are placed one above and the other below the wire.

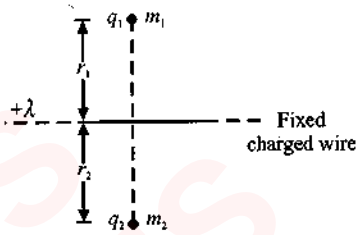


Fig. 1.124

Considering the gravitational effect of the earth, the nature of q_1 and q_2 can be

a. $q_1 \to +ve, q_2 \to +ve$
b. $q_1 \to +ve, q_2 \to -ve$
c. $q_1 \to -ve, q_2 \to -ve$
d. $q_1 \to -ve, q_2 \to +ve$

Assertion-Reasoning Type

Solutions on page 1.51

In the following questions, each question contains Statement I (Assertion) and Statement II (Reason). Each question has four choices (a), (b), (c) and (d). out of which **ONLY ONE** is correct.

a. Statement I is True, Statement II is True; Statement II is a correct explanation for Statement I.
b. Statement I is True, Statement II is True; Statement II is **NOT** a correct explanation for Statement I.
c. Statement 1 is True, Statement II is False.
d. Statement 1 is False, Statement II is True.

1. **Statement I:** If a point charge is rotated in a circle around another charge at the centre of circle, the work done by electric field will be zero.
Statement II: Work done is equal to dot product of force and displacement.
2. **Statement I:** A positive point charge initially at rest in a uniform electric field starts moving along electric lines of forces. (Neglect all other forces except electric forces.)
Statement II: A point charge released from rest in an electric field always moves along the lines of force.
3. **Statement I:** When a neutral body acquires +ve charge, its mass decreases.
Statement II: A body acquires +ve charge when it loses electrons.
4. **Statement I:** Two similarly charged bodies may attract each other.
Statement II: When charge on one body (Q) is much greater than that on another (q) and they are close enough to each other, then force of attraction between Q and induced charges exceeds the force of repulsion between Q and q.
5. **Statement I:** Charge is quantized because only integral number of electrons can be transferred.
Statement II: There is no possibility of transfer of some fraction of electron.

Comprehension Type

Solutions on page 1.51

For Problems 1–2

Two small identical conducting balls A and B of charges of + 10 μC and +30 μC, respectively, are kept at a separation of 50 cm. These balls have been connected by a wire for a short time.

1. The final charge on each of the balls A and B will be
   a. 10 μC and 30 μC, respectively
   b. 20 μC on each ball
   c. 30 μC and 10 μC, respectively
   d. −40 μC and 80 μC, respectively
2. The force of interaction between the balls is
   a. 28.8 N b. 32.6 N c. 14.4 N d. 72 N

For Problems 3–4

Two free point charges A and B having charges $+q$ and $+4q$, respectively, are a distance l apart. A third charge is so placed that the entire system is in equilibrium.

3. The third charge should be placed
   a. left of A at a distance $l/3$ from A
   b. right of A at a distance $l/3$ from B
   c. between A and B at a distance $2l/3$ from A
   d. between A and B at a distance $l/3$ from A
4. The third charge has magnitude and sign
   a. $Q = \left(-\frac{4}{9}\right) q$ b. $Q = \left(\frac{4}{9}\right) q$
   c. $Q = \left(\frac{3}{5}\right) q$ d. $Q = \left(-\frac{3}{5}\right) q$

For Problems 5–7

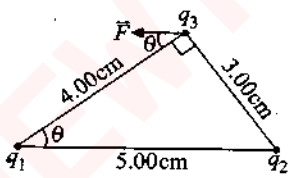


Fig. 1.125

Three charges are placed as shown in Fig.1.125. The magnitude of $q1$ is 2.00 μC, but its sign and the value of the charge q_2 are not known. Charge q_3 is +4.00 μC, and the net force on q_3 is entirely in the negative x-direction.

5. As per the condition given in the problem, the sign of q_1 and q_2 will be
   a. +, + b. +, − c. −, + d. −, −
6. The magnitude of q_2 is
   a. $\frac{27}{64}$ μC b. $\frac{27}{32}$ μC c. $\frac{13}{32}$ μC d. $\frac{13}{64}$ μC
7. The magnitude of force acting on q_3 is
   a. 25.25 N b. 32.5 N c. 56.25 N d. 13.5 N

For Problems 8–10

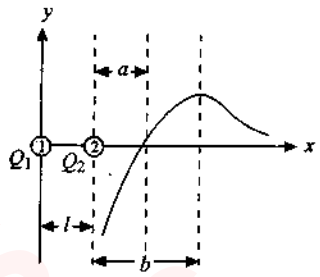


Fig. 1.126

Two point like charges Q_1 and Q_2 are positioned at points 1 and 2. The field intensity to the right of the charge Q_2 on the line that passes through the two charges varies according to a law that is represented schematically in Fig.1.126. The field intensity is assumed to be positive if its direction coincides with the positive direction on the x-axis. The distance between the charges is l.

8. The sign of each charge Q_1 and Q_2 is
   a. +, − b. −, + c. +, + d. −, −
9. The ratio of the absolute values of the charges $|Q_1/Q_2|$ is
   a. $\left(\frac{a+l}{a}\right)^2$ b. $\left(\frac{l}{a}\right)^2$ c. $\left(\frac{a}{a+l}\right)^2$ d. $\left(\frac{a}{l}\right)^2$
10. The value of b, where the field intensity is maximum, is
   a. $\frac{l}{(Q_1/Q_2)^{1/3}+1}$ b. $\frac{l}{(Q_1/Q_2)^{1/3}-1}$
   c. $\frac{l}{(Q_2/Q_1)^{1/3}+1}$ d. $\frac{l}{(Q_2/Q_1)^{1/3}-1}$

For Problems 11–12

Four equal positive charges, each of value Q, are arranged at the four corners of a square of diagonal $2a$. A small body of mass m carrying a unit positive charge is placed at a height h above the centre of the square.

11. What should be the value of Q in order that this body may be in equilibrium ?
   a. $\pi\varepsilon_0 \frac{mg}{2h}(h^2+2a^2)^{3/2}$ b. $\pi\varepsilon_0 \frac{mg}{h}(h^2+a^2)^{3/2}$
   c. $\pi\varepsilon_0 \frac{2mg}{h}(h^2+2a^2)^{3/2}$ d. $\pi\varepsilon_0 \frac{mg}{2h}(h^2-a^2)^{3/2}$
12. The type of equilibrium of the point mass is (consider only vertical displacement)
   a. stable equilibrium b. unstable equilibrium
   c. neutral equilibrium d. cannot be determined

For Problems 13–16

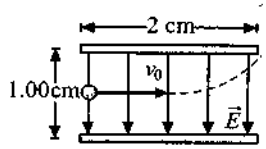


Fig. 1.127

An electron is projected with an initial speed $v_0 = 1.60 \times 10^6$ ms^{-1} into the uniform field between the parallel plates as shown in Fig. 1.127. Assume that the field between the plates is uniform and directed vertically downwards, and that the field outside the

plates is zero. The electron enters the field at a point midway between the plates. Mass of electron = 9.1×10^{-31} kg.

13. If the electron just misses the upper plate, the time of flight of electron up to this instant is
a. 1.25×10^{-9} s **b.** 32.5×10^{-6} s
c. 1.25×10^{-8} s **d.** 32.5×10^{-8} s

14. For condition of previous situation, the magnitude of electric field is
a. 124 NC^{-1} **b.** 364 NC^{-1}
c. 224 NC^{-1} **d.** 520 NC^{-1}

15. If instead of electron, a proton were projected with the same speed, then compare the paths travelled by the electron and the proton.
a. The proton will hit the upper plate.
b. The proton will hit the lower plate.
c. The proton will not hit either plate.
d. None of these.

16. The vertical displacement travelled by the proton as it exits the region between the plates is. Mass of proton = 1.67×10^{-27} kg.
a. 1.6×10^{-8} m **b.** 3.25×10^{-8} m
c. 5.25×10^{-6} m **d.** 2.73×10^{-6} m

For Problems 17–18

An electron is projected as shown in Fig. 1.128, with kinetic energy K, at an angle $\theta = 45°$ between two charged plates. Ignore the gravity.

17. The magnitude of the electric field, so that the electron just fails to strike the upper plate is
a. K/qd **b.** $2K/qd$ **c.** $K/2qd$ **d.** infinite

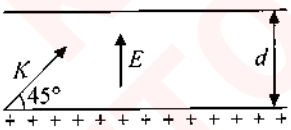


Fig. 1.128

18. At what distance from the starting point will the electron strike the lower plate?
a. d **b.** $2d$ **c.** $3d$ **d.** $4d$

For Problems 19–20

In 1909, Robert Millikan was the first to find the charge of an electron in his now-famous oil-drop experiment. In that experiment, tiny oil drops were sprayed into a uniform electric field between a horizontal pair of oppositely charged plates. The drops were observed with a magnifying eyepiece, and the electric field was adjusted so that the upward force on some negatively charged oil drops was just sufficient to balance the downward force of gravity. That is, when suspended, upward force qE just equalled mg. Millikan accurately measured the charges on many oil drops and found the values to be whole number multiples of 1.6×10^{-19} C— the charge of the electron. For this, he won the Nobel prize.

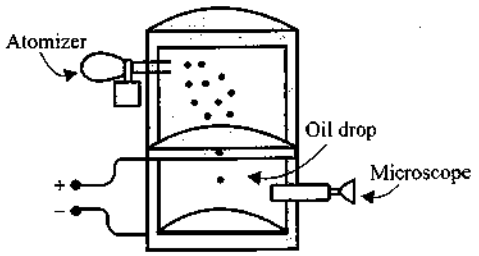


Fig. 1.129

19. If a drop of mass 1.08×10^{-14} kg remains stationary in an electric field of 1.68×10^{5} NC^{-1}, then the charge of this drop is
a. 6.40×10^{-19} C **b.** 3.2×10^{-19} C
c. 1.6×10^{-19} C **d.** 4.8×10^{-19} C

20. Extra electrons on this particular oil drop (given the presently known charge of the electron) are
a. 4 **b.** 3 **c.** 5 **d.** 8

Matching Column Type

Solutions on page 1.52

1. In Fig.1.130, charges, each $+q$, are fixed at L and M. O is the midpoint of distance LM. X- and Y- axes are as shown. Consider the situations given in column I and match them with the information in Column II:

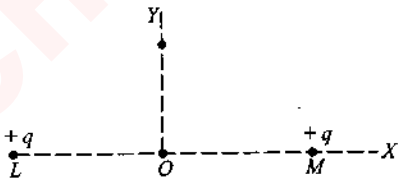


Fig. 1.130

Column I	Column II
i. Let us place a charge $+q$ at O, displace it slightly along X-axis and release. Assume that it is allowed to move only along X-axis. At position O,	**a.** force on the charge is zero
ii. Place a charge $-q$ at O. Displace it slightly along X-axis and release. Assume that it is allowed to move only along X-axis. At position O,	**b.** potential energy of the system is maximum
iii. Place a charge $+q$ at O. Displace it slightly along Y-axis and release. Assume that it is allowed to move only along Y-axis. At position O,	**c.** potential energy of the system is minimum
iv. Place a charge $-q$ at O. Displace it slightly along Y-axis and release. Assume that it is allowed to move only along Y-axis. At position O,	**d.** the charge is in equilibrium

2. Match the forces given in Column I with the properties given in Column II:

Column I	Column II
i. Electric force of nucleus on electron	a. Conservative
ii. Your weight, i.e., the force that earth exerts on you	b. Action reaction force
iii. Force between earth and sun	c. Depends on the nature of medium between the interacting objects.
iv. Repulsive force between two protons	d. Principle of superposition applies provided that other forces, which are acting on the object, are also of this nature

3. Match Column I with Column II:

Column I	Column II
i. Force on an electron in an atom	a. Gravitational force
ii. Force between a proton and a neutron inside nucleus	b. Strong force
iii. Force between a proton and a proton inside nucleus	c. Coulomb force
iv. Conservative force	d. Electric force

4. Match the facts given in Column I with the systems given in Column II:

Column I	Column II
i. Charge cannot exist	a. without charge
ii. Mass can exist	b. without mass
iii. Charge is	c. not conserved
iv. Mass is	d. conserved

5.

Column I	Column II
i. ε_0	a. 1
ii. K	b. $[M^1L^1T^{-2}]$
iii. E	c. $[M^{-1}L^{-3}T^4A^2]$
iv. F	d. $[M^1L^1T^{-3}A^{-1}]$

Integer Answer Type

Solutions on page 1.53

1. Two concentric rings, one of radius 'a' and the other of radius 'b', have the charges $+q$, and $-(2/5)^{-3/2}q$, respectively, as shown in the figure.

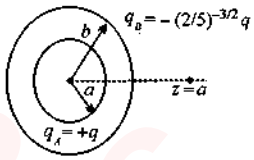


Fig. 1.131

Find the ratio b/a if a charge particle placed on the axis at $z = a$ is in equilibrium.

2. A rod AB of length L and mass m is uniformly charged with a charge Q and it is suspended from end A as shown in the figure. The rod can freely rotate about A in the plane of figure.

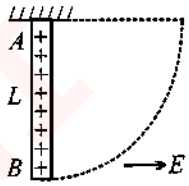


Fig. 1.132

An electric field E is suddenly switched on in the horizontal direction due to which rod gets turned by a maximum angle 90°. The magnitude of E is equal to nMg/Q. Find the value of n.

3. Two particles of masses m and $2m$ having same charges q each are placed in a uniform electric field E and allowed to move for the same time. Find the ratio of their kinetic energies.

4. There is an electric field E in $+x$ direction. If work done by electric field in moving a charge 0.2 C through a distance of 2 m along a line making an angle of 60 degree with x-axis is 1.0 J, what is the value of E in N/C?

5. An electric field is given by $\vec{E} = (y\hat{i} + x\hat{j})N/C$. Find the work done (in J) by electric field in moving a 1 C charge from $\vec{r}_A = (2\hat{i} + 2\hat{j})$ m to $\vec{r}_B = (4\hat{i} + \hat{j})$ m.

ANSWERS AND SOLUTIONS

Subjective Type

1. No. of atoms in 10 g of silver,

$$n = \frac{6.023 \times 10^{23} \times 10}{107.87} = 5.58 \times 10^{22}$$

No. of electron = $47n = 2.62 \times 10^{24}$

2. $qE = 6\pi\eta rv$

$\Rightarrow \quad NeE = 6\pi\eta rv \Rightarrow N = 6\pi\eta rv / eE$

$$\Rightarrow \quad N = \frac{6\pi \times 1.6 \times 10^5 \times 5 \times 10^{-7} \times 0.02}{1.6 \times 10^{-19} \times 6.28 \times 10^5} = 3 \times 10^{11}$$

3. No. of electrons or protons in 1.00 g of hydrogen,

$$n = 2\left[\frac{1}{2} \times 6.023 \times 10^{23}\right] = 6.023 \times 10^{23}$$

Magnitude of charge on north or south pole:

$$q = ne = 6.023 \times 10^{23} \times 1.6 \times 10^{-19} = 96368 \text{ C}$$

$$F = \frac{kq^2}{r^2} = \frac{9\times10^9\times(96368)^2}{(2\times6400\times10^3)^2} = 5.10\times10^5 \text{ N}$$
$$= 5.10 \text{ kN}$$

4. a. $F = \dfrac{9\times10^9\times12\times10^{-9}\times18\times10^{-9}}{(0.30)^2}$

$= 2.16\times10^{-5}$ N attractive

b. When they are connected by a conducting wire, finally charge on each will be half of total charge on both. Let q be the final charge on each, then $q = (12-18)/2 = -3$ nC.

$$F' = \frac{9\times10^9\times(3\times10^{-9})^2}{(0.30)^2} = 9\times10^{-7}\text{ N repulsive}$$

5. $F_3 = F_2 + 2F_1\cos 45°$

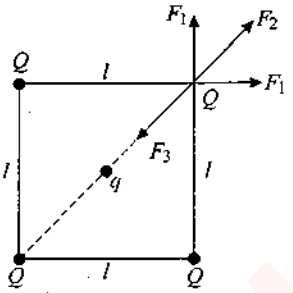


Fig. 1.133

$$\Rightarrow \quad \frac{kqQ}{(l/\sqrt{2})^2} = \frac{kQQ}{(\sqrt{2}l)^2} + 2\frac{kQQ}{l^2}\frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad q = \frac{Q(2\sqrt{2}+1)}{4}$$

Q should be negative as there is attraction between Q and q.

So, $q = \dfrac{-Q(2\sqrt{2}+1)}{4}$

6. Q should be negative and it should be placed somewhere between q and $2q$.

Fig. 1.134

For net force on q to be zero: $\dfrac{kqQ}{x^2} = \dfrac{kq2q}{d^2}$ (i)

For net force on $2q$ to be zero: $\dfrac{kQ2q}{(d-x)^2} = \dfrac{kq2q}{d^2}$ (ii)

From Eqs. (i) and (ii), we get

$$\frac{1}{x^2} = \frac{2}{(d-x)^2} \Rightarrow \quad x = \frac{d}{1+\sqrt{2}}$$

7. The third charge (q) is in equilibrium only when

$$\frac{k(4e)q}{x^2} = \frac{k(e)q}{(a-x)^2}$$

Fig. 1.135

On solving, we get $x = \dfrac{2}{3}a$.

8. As there is attraction (here we have assumed Q_1 and Q_2 in μC), so we get

$$\frac{9\times10^9\times Q_1Q_2\times10^{-12}}{1^2} = -0.027 \Rightarrow Q_1Q_2 = -3$$

– ve sign is due to attraction

For repulsion: $\dfrac{9\times10^9\times\left(\dfrac{Q_1+Q_2}{2}\right)^2\times10^{-12}}{1^2} = 0.009$

$\Rightarrow \quad (Q_1+Q_2)^2 = 4$

$\Rightarrow \quad Q_1+Q_2 = \pm2$

Here, we have two sets of equations.

(i) $\left\{\begin{matrix}\text{from } Q_1Q_2 = -3\\ \text{and } Q_1+Q_2 = 2\end{matrix}\right\} \rightarrow$ we get 3 μC and −1 μC

(ii) $\left\{\begin{matrix}\text{from } Q_1Q_2 = -3\\ \text{and } Q_1+Q_2 = -2\end{matrix}\right\} \rightarrow$ we get −3 μC and 1 μC

Both are valid.

9. Let charge on each sphere be q, then

$$F = k\frac{q^2}{r^2} = 2\times10^{-5} \Rightarrow \frac{q^2}{r^2} = \frac{2\times10^{-5}}{9\times10^9}$$

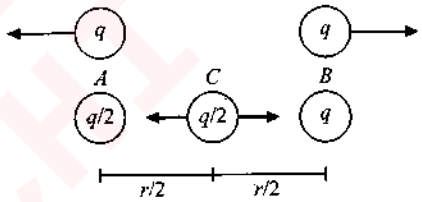


Fig. 1.136

When sphere C is touched by sphere A, charge gets equally divided between the two.

$$F_{CA} = k\frac{q^2/4}{r^2/4} = k\frac{q^2}{4}\times\frac{4}{r^2} = \frac{kq^2}{r^2}$$

$$F_{CB} = k\frac{q^2/2}{r^2/4} = k\frac{q^2}{2}\times\frac{4}{r^2} = 2k\frac{q^2}{r^2}$$

$$F_C = F_{CB} - F_{CA} = k\frac{q^2}{r^2} = 2\times10^{-5}\text{ N towards } A.$$

10. Net electric field at A due to charges at B and C is

$$E_A = 2E_{AC}\sin 60° = 2\times9\times10^9\times\frac{3\times10^{-6}}{(0.20)^2}\times\frac{\sqrt{3}}{2}$$
$$= \sqrt{3}\times6.75\times10^5$$

$$AM = \sqrt{(20)^2-(10)^2} = \sqrt{400-100} = 10\sqrt{3}\text{ cm}$$

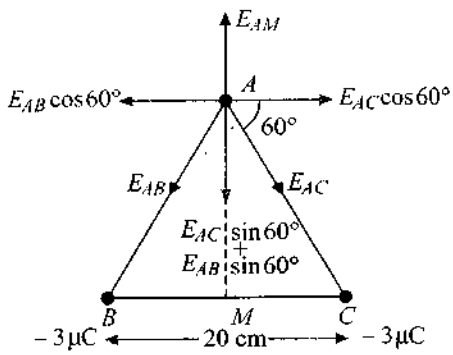


Fig. 1.137

Let the charge at M be q. Charge q should be positive so that there can be repulsion between the charges at A and M.

$$E_{AM} = 9\times10^9 \times \frac{q}{(10\sqrt{3}/100)^2} = 3q\times10^{11}$$

For A to be in equilibrium:

$$E_A = E_{AM} \Rightarrow \sqrt{3}\times6.75\times10^5 = 3q\times10^{11}$$

$$\Rightarrow \quad q = \frac{9\sqrt{3}}{4}\times10^{-6} = \frac{9\sqrt{3}}{4}\,\mu C$$

11. For Q to be in equilibrium:

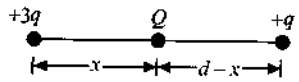


Fig. 1.138

$$\frac{k3qQ}{x^2} = \frac{kQq}{(d-x)^2}$$

$$\Rightarrow \quad x = \frac{\sqrt{3}d}{\sqrt{3}+1}$$

The bead can be in stable equilibrium, if it has positive charge.

12. No. of atoms in 10 g of copper,

$$n = \frac{10}{63.5}\times6\times10^{23} = 9.448\times10^{22}$$

Number of electrons transferred,

$$N = \frac{n\times1}{1000} = 9.448\times10^{19}$$

a. Magnitude of charge appearing on either piece,

$$q = Ne = 9.448\times10^{19}\times1.6\times10^{-19} = 15.12\ C$$

b. $F = \frac{kq^2}{r^2} = \frac{9\times10^9\times(15.12)^2}{(0.10)^2} = 2.05\times10^{14}$ N

13. Electron is 0.5 cm from the sheet of charge and is far away from the edges of the sheet. Therefore, the sheet looks effectively infinite.

$$E = \frac{\sigma}{2\varepsilon_0} = \frac{q}{2\varepsilon_0 A} \Rightarrow \quad F = -eE = -e\left(\frac{q}{2\varepsilon_0 A}\right)$$

$$q = -\frac{2\varepsilon_0 AF}{e}$$

$$= -\frac{2(8.854\times10^{-12})\times(0.5)^2\times(1.8\times10^{-12})}{1.6\times10^{-19}}$$

$$= -50\ \mu C$$

14. An electron is a negatively charged particle, it will be attracted by the positive plate with force $F = eE$. Hence, acceleration of electron along y-axis will be

$$a = \frac{F}{m} = \frac{eE}{m} = \frac{eV}{md} \quad \text{(i)} \left\{\text{as } E = \frac{V}{d}\right\}$$

So, from equation of motion, $v^2 = u^2 + 2as$ along the x-axis,

$v_x = v_0 = 10^6\ (ms^{-1})$ $\{\text{as } a_x = 0\}$ (ii)

And along the y-axis, $v_y^2 = 2ay_0$ $\{\text{as } u = 0 \text{ and } s = y_0\}$

Now, as $y_0 = 1$ cm (given) and is given by Eq. (i), the electron will hit the top plate with the velocity,

$$v_y = \sqrt{\frac{2y_0 eV}{md}} = \sqrt{\frac{2\times1.6\times10^{-19}\times30\times1\times10^{-3}}{9\times10^{-31}\times3\times10^{-3}}}$$

$$v_y = (4\sqrt{2}/3)\times10^6 = 1.885\times10^6\ ms^{-1} \quad \text{(iii)}$$

So, the electron will hit the upper plate with the velocity,

$v_x = 10^6\ (ms^{-1})$ and $v_y = 1.885\times10^6\ ms^{-1}$

Note: ***In this problem, time taken by the electron to hit the plate,***

$$t = \sqrt{\frac{2y_0}{a}} = \sqrt{\frac{2y_0 m}{eE}} = \sqrt{\frac{2y_0 md}{eV}}$$

$$t = \sqrt{\frac{2\times(2\times10^{-3})\times(9\times10^{-31})\times(3\times10^{-3})}{1.6\times10^{-19}\times30}}$$

$$= \frac{3}{2\sqrt{2}}\times10^{-9}s = 1.06\times10^{-9}s$$

And in this time, electron will travel a horizontal distance,

$s = v_0 t = 10^6\times1.06\times10^{-9} = 1.06\times10^{-3}$ **m**

15. $E_1 = 2E\cos\theta = \frac{2kq(l)}{[x^2+l^2]^{3/2}}$ along AC

$$E_2 = 2E\cos\theta = \frac{2kql}{[x^2+l^2]^{3/2}} \text{ along } DB$$

So $E_1 \perp^r E_2$. Net field is

$$E_0 = \sqrt{E_1^2+E_2^2} = \frac{2\sqrt{2}kql}{(x^2+l^2)^{3/2}} = \frac{ql}{\sqrt{2}\pi\varepsilon_0(x^2+l^2)^{3/2}}$$

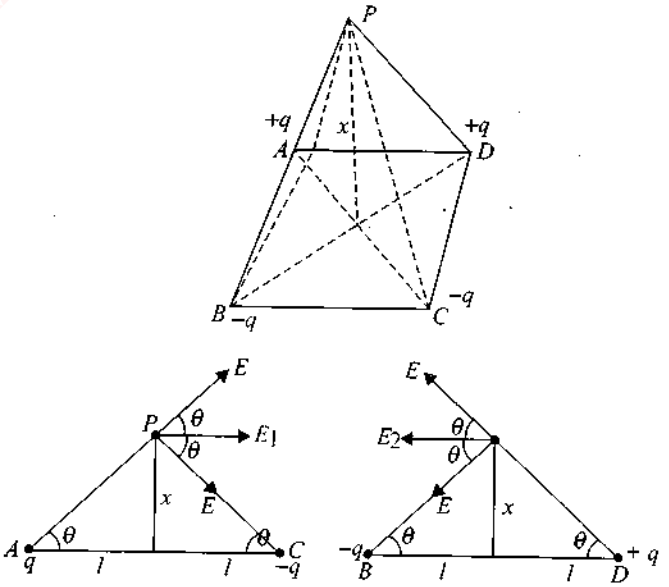


Fig. 1.139

16. Let us make two-dimensional view of the situation. Because of symmetry we can say the force on wire 2 (λ_2) will be along the negative y-direction.

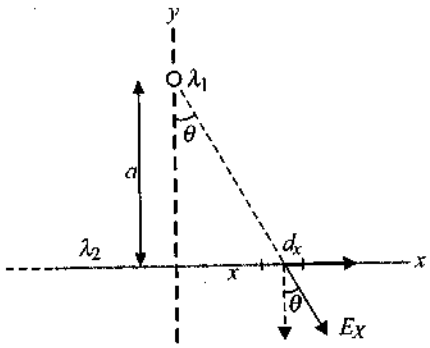


Fig. 1.140

$$dF = E_x\, dq \cos\theta$$

$$F = \int dF = \int \frac{\lambda_1}{2\pi\varepsilon_0\sqrt{a^2+x^2}}(\lambda_2 dx)\cos\theta$$

where $\cos\theta = \dfrac{a}{\sqrt{a^2+x^2}}$

$$F = \frac{\lambda_1\lambda_2}{2\pi\varepsilon_0} a\int_{-\infty}^{\infty}\frac{dx}{a^2+x^2} = \frac{\lambda_1\lambda_2 a}{2\pi\varepsilon_0}\times\frac{1}{a}\left[\tan^{-1}\frac{x}{a}\right]_{-\infty}^{\infty}$$

$F = \dfrac{\lambda_1\lambda_2}{2\pi\varepsilon_0}$. It is independent of a.

17. $q = 10^{-8}$ C, $\lambda = \dfrac{Q}{2\pi r} = \dfrac{\pi}{2\pi(0.1)} = 5\ \text{Cm}^{-1}$

Take an arc of ring subtending small angle θ at the centre (Fig. 1.141). Charge on this arc: $dq = \lambda r\theta$.

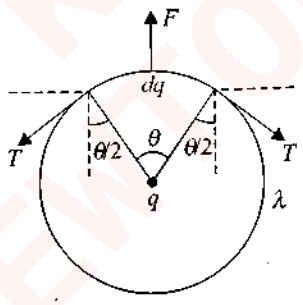


Fig. 1.141

For net force to be zero on this arc,

$$2T\sin\left(\frac{\theta}{2}\right) = F$$

$$\Rightarrow \quad 2T\sin\left(\frac{\theta}{2}\right) = \frac{kq(\lambda r\theta)}{r^2}$$

$$\Rightarrow \quad T = \frac{kq\lambda}{r}$$

$$= \frac{9\times10^9\times10^{-8}\times5}{0.1} = 4500\ \text{N}$$

$$\Delta r = \frac{Tr}{AY} = \frac{4500\times0.1}{10^{-6}\times2\times10^{11}}$$

$$= 225\times10^{-5}\ \text{m} = 2.25\ \text{mm}$$

18. $T\cos\theta + qB = mg$

$$\Rightarrow \quad T\cos\theta = mg - qB \qquad \text{(i)}$$

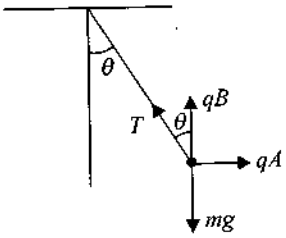


Fig. 1.142

$$T\sin\theta = qA \qquad \text{(ii)}$$

Dividing Eqs. (i) and (ii), we get

$$\tan\theta = \frac{qA}{mg - qB}$$

a. $q = \dfrac{mg\tan\theta}{A + B\tan\theta}$

Putting the value of q in Eq. (ii).

b. $T = \dfrac{A\,mg\sec\theta}{A + B\tan\theta}$

19. Electric field at point P due to charge of ring is

$$E = \frac{kQx}{(R^2+x^2)^{3/2}}$$

At $x = R$: $E = \dfrac{kQ}{2\sqrt{2}R^2}$ directed towards the centre.

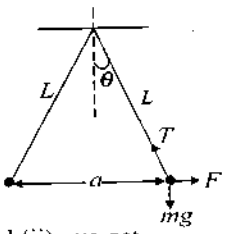
Electric field at P due to charge at centre: $\dfrac{kq}{R^2}$

For net field to be zero at P:

$$\frac{kq}{R^2} = \frac{kQ}{2\sqrt{2}R^2} \Rightarrow q = \frac{Q}{2\sqrt{2}}$$

20. As we see $\quad T\sin\theta = F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{a^2} \qquad$ (i)

$$T\cos\theta = mg \qquad \text{(ii)}$$



From Eqs. (i) and (ii), we get

$$\tan\theta = \frac{q^2}{4\pi\varepsilon_0 a^2 mg}$$

$$\frac{a/2}{L} = \frac{q^2}{4\pi\varepsilon_0 a^2 mg} \quad (\because \text{for small } \theta, \tan\theta \approx a/2L)$$

$$\Rightarrow \quad \frac{a^3}{L} = \frac{q^2}{2\pi\varepsilon_0 mg} \qquad \text{(iii)}$$

When one of the balls is discharged, the balls come closer and touch each other and again separate due to repulsion. The charge on each ball after touching each other is $q/2$. Replacing q with $q/2$ in Eq. (iii), we get

$$\frac{b^3}{L} = \frac{(q/2)^2}{2\pi\varepsilon_0 mg} \qquad \text{(iv)}$$

From Eqs. (iii) and (iv), $\frac{b^3}{a^3} = \frac{1}{4} \Rightarrow b = \frac{a}{2^{2/3}}$

21. **a.** Electric field near B and to the right of B is along −ve x-axis, so the sign of Q_b should be −ve. There is a neutral point at x = 24 cm, so the sign of Q_a should be opposite of Q_b.
Hence, Q_a should be +ve.

b. At neutral point (at x = 24 cm),

$$k\frac{Q_a}{24^2} = k\frac{Q_b}{3^2} \Rightarrow \left|\frac{Q_a}{Q_b}\right| = \left(\frac{24}{3}\right)^2 = 64$$

c. Electric field to the right of charges (at $x = x$),

$$E = k\frac{Q_a}{x^2} - k\frac{Q_b}{(x-21)^2}$$

For field to be maximum: $\frac{dE}{dx} = 0$

$$\Rightarrow \qquad \Rightarrow \frac{kQ_a(-2)}{x^3} - \frac{kQ_b(-2)}{(x-21)^3} = 0 \Rightarrow x = 28 \text{ cm}$$

22. Electric field due to MA: $\vec{E}_1 = \frac{\lambda}{4\pi\varepsilon_0 R}(\hat{i} - \hat{k})$

Electric field due to ADC: $\vec{E}_2 = \frac{\lambda}{2\pi\varepsilon_0 R}(-\hat{k})$

Electric field due to ABC: $\vec{E}_3 = \frac{\lambda}{2\pi\varepsilon_0 R}(-\hat{j})$

Electric field due to NC: $\vec{E}_4 = \frac{\lambda}{4\pi\varepsilon_0 R}(-\hat{i} + \hat{k})$

Net electric field,

$$\vec{E}_0 = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4 = -\frac{\lambda}{2\pi\varepsilon_0 R}(\hat{j} + \hat{k})$$

23. Electric field due to straight wires will cancel out. The net electric field will be due to semicircular wire only.

Hence, $\vec{E}_{net} = \vec{E}_{circular} = \frac{\lambda}{2\pi\varepsilon_0 R}(-\hat{j})$

So, acceleration: $\vec{a} = \frac{q\vec{E}_{net}}{m} = \frac{q\lambda}{2\pi\varepsilon_0 mR}(-\hat{j})$

24. $\tan\theta = \frac{F}{mg} = \frac{kq^2}{x^2 mg}$

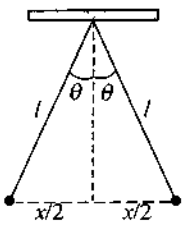


Fig. 1.143

$$\frac{x}{2l} = \frac{q^2}{4\pi\varepsilon_0 mg}$$

$$x = \left[\frac{q^2 l}{2\pi\varepsilon_0 mg}\right]^{1/3}. \text{ Hence proved.}$$

Now $q^2 = \frac{2\pi\varepsilon_0 mgx^3}{l}$

$$q = \sqrt{\frac{2\pi\varepsilon_0 mg}{l}}x^{3/2}$$

$$\frac{dq}{dt} = \sqrt{\frac{2\pi\varepsilon_0 mg}{l}}\frac{3}{2}\sqrt{x}\frac{dx}{dt}$$

$$(\sqrt{x}\frac{dx}{dt} = \sqrt{x}(-v)$$

$$= -\sqrt{x}a/\sqrt{x} = -a)$$

$$\frac{dq}{dt} = -\frac{3}{2}a\sqrt{\frac{2\pi\varepsilon_0 mg}{l}}$$

25. $F_{net} = 3F\cos\theta = \frac{3kq_1q_2}{R^2}\frac{x}{R} = ma$

$$\Rightarrow \quad a = \left[\frac{3q_1q_2}{4\pi\varepsilon_0 mr^3}\right]x \quad \text{(as for small } x, R = r)$$

$$T = 2\pi\sqrt{\frac{4\pi\varepsilon_0 mr^3}{3q_1q_2}}$$

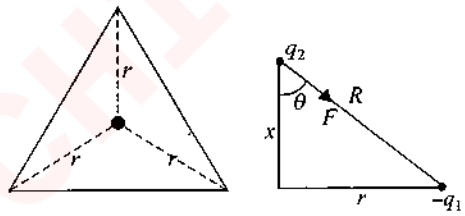


Fig. 1.144

26. Charge within radius r,

$$q = \int_0^r \rho_0\left[1 - \frac{r}{R}\right]4\pi r^2 dr$$

$$= 4\pi\rho_0\int_0^r\left(r^2 - \frac{r^3}{R}\right)dr$$

$$= 4\pi\rho_0\left(\frac{r^3}{3} - \frac{r^4}{4R}\right) \qquad \text{(i)}$$

For $r < R$,

$$E = \frac{kq}{r^2} = \frac{\rho_0 r}{3\varepsilon_0}\left[1 - \frac{3r}{4R}\right] \qquad \text{(ii)}$$

Total charge: Put $r = R$ in Eq. (i)

$$Q = 4\pi\rho_0\left[\frac{R^3}{3} - \frac{R^3}{4}\right] = \frac{4\pi\rho_0 R^3}{12}$$

For $r > R$,

$$E = \frac{kQ}{r^2} = \frac{\rho_0 R^3}{12\varepsilon_0 r^2} \qquad \text{(iii)}$$

At surface, from both Eqs. (ii) and (iii), put $r = R$.

$$E_0 = \frac{\rho_0 R}{12\varepsilon_0}$$

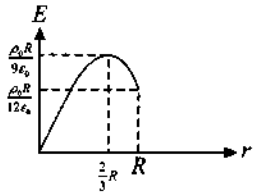


Fig. 1.145

For maximum $\frac{dE}{dr} = 0$ from Eq. (ii)

$$r = \frac{2}{3}R$$

$$E = \frac{\rho_0 R}{9\varepsilon_0}$$

Also, at $r = R/3$

$$E = \frac{\rho_0 R}{12\varepsilon_0}$$

Objective Type

1. **c.** The body may slightly get charged either negatively or positively.
2. **d.** $K = \frac{1}{4\pi\varepsilon_0} = 9\times10^9, \varepsilon_0 = \frac{1}{4\pi\times9\times10^9}$ $C^2N^{-1}m^{-2}$
3. **a.** $F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}$

$$[\varepsilon_0] = \frac{[q^2]}{[Fr^2]} = \frac{(\text{current}\times\text{time})^2}{MLT^{-2}L^2}$$

$$= \frac{A^2T^2}{MLT^{-2}L^2} = M^{-1}L^{-3}T^4A^2$$

4. **b.** $E = \frac{F}{q} = \frac{MLT^{-2}}{AT} = MLA^{-1}T^{-3}$
5. **d.** Dielectric constant of a material is $K \geq 1$.
6. **a.** Net charge of universe is constant. Positive and negative charges may separately be created or destroyed. But net sum of charge remains constant.
7. **a.** There will be gravitational attraction between the neutrons.
8. **a.** When we charge a soap bubble, there will be either net positive or net negative charge on the bubble. Due to the repulsion between like charges, size of the bubble will tend to increase.
9. **a.** Glass plate will act like a dielectric.
10. **a.** Newton's third law.
11. **b.** Given $Q_1 + Q_2 = 10$ (i)

And $\frac{kQ_1Q_2}{r^2} = -0.075$

$$\Rightarrow \quad \frac{9\times10^9 Q_1Q_2\times10^{-12}}{(3)^2} = -0.075$$

(Here, we have multiplied by 10^{-12} because we have considered Q_1 and Q_2 to be in μC)

$$\Rightarrow \quad Q_1Q_2 = -75 \quad \text{(ii)}$$

Solving Eqs. (i) and (ii), we get $Q_1 = 15$ μC, $Q_2 = -5$ μC

12. **a.** $F = \frac{1}{4\pi\varepsilon_0}\frac{q(Q-q)}{r^2}$

For the force to be maximum:

$$\frac{dF}{dq} = \frac{1}{4\pi\varepsilon_0 r^2}\frac{d}{dq}[Qq - q^2] = 0 \Rightarrow q = \frac{Q}{2}$$

And $\frac{d^2F}{dq^2}$ is negative at $q = \frac{Q}{2}$.

Hence, force will be maximum if $q = Q/2$.

13. **c.** Net force on q_3: $F_1 + F_2 = 0$

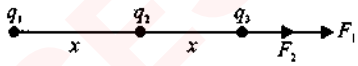


Fig. 1.146

$$\Rightarrow \quad \frac{kq_1q_3}{(2x)^2} + \frac{kq_2q_3}{x^2} = 0 \Rightarrow q_1 = -4q_2$$

14. **d.**

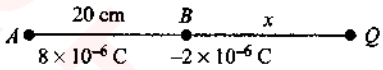


Fig. 1.147

Let the third charge Q be placed at a distance x to the right of B. Then

$$\frac{kQ\times8\times10^{-6}}{(20+x)^2} = \frac{kQ(2\times10^{-6})}{x^2}$$

$$\Rightarrow \quad x = 20 \text{ cm}$$

15. **c.** 1 2 3 4 5
+ − +

If 1 is positively charged, then 2 should be negatively charged, then 4 should be positively charged. Now, 1 and 4 cannot attract. It means ball 1 should be neutral (2 and 4 cannot be neutral because they are showing repulsion with 3 and 5, respectively).

16. **b.** A and B have like charges, and B and C have like charges. So, A and C also have like charges. Hence, they also repel each other.
17. **c.** $100 = k\,q_1q_2/r^2$ and $F = k\,(1.1q_1)(0.9q_2)/r^2$

$$\Rightarrow \quad \frac{F}{100} = 0.99 \Rightarrow F = 99 \text{ N}$$

18. **c.** Force on Q_2 is zero (q should be −ve)

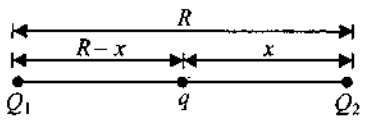


Fig. 1.148

$$\Rightarrow \quad \frac{kQ_1Q_2}{R^2} = \frac{kqQ_2}{x^2} \Rightarrow \frac{x}{R} = \sqrt{\frac{q}{Q_1}}$$

Force on q is zero: $\frac{kQ_1q}{(R-x)^2} = \frac{kqQ_2}{x^2}$

$$\Rightarrow \quad \frac{R-x}{x} = \frac{\sqrt{Q_1}}{\sqrt{Q_2}}$$

$$\Rightarrow \quad \frac{R}{x} = \frac{\sqrt{Q_1}+\sqrt{Q_2}}{\sqrt{Q_2}} \Rightarrow \frac{\sqrt{Q_1}}{\sqrt{q}} = \frac{\sqrt{Q_1}+\sqrt{Q_2}}{\sqrt{Q_2}}$$

$$\Rightarrow \quad q = \frac{Q_1Q_2}{(\sqrt{Q_1}+\sqrt{Q_2})^2}$$

19. d. The equilibrium can be stable only if on displacing the small charge slightly in any direction, the forces act on it in such a way so as to bring back the charge to its equilibrium position.

20. b. Initial tension: $T_1 = mg$

Final tension: $T_2 \cos\theta = mg$

$$\Rightarrow \quad T_2 = \frac{mg}{\cos\theta}$$

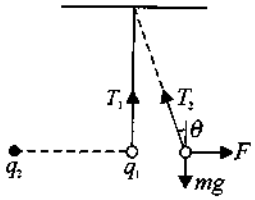


Fig. 1.149

Obviously, $T_2 > mg$.

Here we have assumed θ to be small so that F is almost horizontal.

21. c. $F_{net} = 2F\cos 30°$

$$= \frac{2kq^2}{a^2}\frac{\sqrt{3}}{2} = \frac{\sqrt{3}kq^2}{a^2}$$

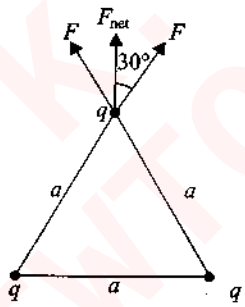


Fig. 1.150

22. a. $E = \frac{kQ}{r^2}$ **(i)**

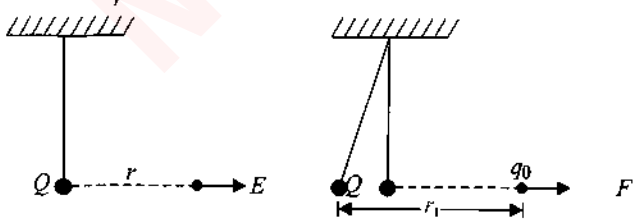


Fig. 1.151

$$F = \frac{kQq_0}{r_1^2} \Rightarrow \frac{F}{q_0} = \frac{kQ}{r_1^2} \quad \textbf{(ii)}$$

As $r_1 > r$, so from Eqs. (i) and (ii), we get $E > F/q_0$

23. d. Net charge on a current carrying wire will be zero at any time. Because when current flows through a wire, the amount of charge entering from one end is equal to the amount of charge leaving the other end.

24. a. $E = \frac{F}{q} = \frac{2.25}{15\times10^{-4}} = 1500\ \text{NC}^{-1}$

25. b. $F = Eq = 15\times10^4\times2\times1.6\times10^{-19} = 4.8\times10^{-14}$ N

26. a. As charge on both particles is same, so electric force acting on them will be same. Since the particles are allowed to move for the same time, their final momentum will be same. Because

Change in momentum = impulse = force × time

So, from KE = $\frac{P^2}{2m}$, we have KE $\propto \frac{1}{m}$

$$\frac{\text{KE}_1}{\text{KE}_2} = \frac{m_2}{m_1} = 2:1$$

27. d. Each charge will produce the same magnitude of intensity, say E, at the centroid. These are directed at angles of 120° with each other. So, their vector sum will be zero.

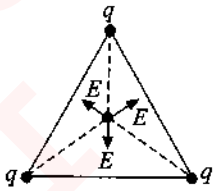


Fig. 1.152

28. b. $\vec{r} = (9-3)\hat{i} + (12-4)\hat{j} = 6\hat{i} + 8\hat{j}$

$$\Rightarrow \quad r = \sqrt{6^2+8^2} = 10\ \text{m}$$

$$E = \frac{9\times10^9\times100\times10^{-6}}{10^2} = 9000\ \text{Vm}^{-1}$$

29. d. Electric lines of force is an imaginary concept, They do not exist in reality.

30. d. $\frac{kQ_2}{x^2} = \frac{kQ_1}{(x+R)^2} \Rightarrow x = \frac{R}{2}$

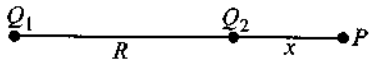


Fig. 1.153

31. c. For the first case: $F = mg$

For the second case: $F + mg = 6eE \Rightarrow 2mg = 6eE$

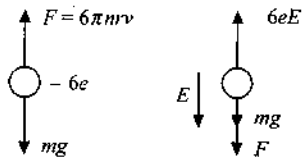


Fig. 1.154

$$\Rightarrow \quad E = \frac{mg}{3e} = \frac{1.6\times10^{-15}\times10}{3\times1.6\times10^{-19}} = 3.3\times10^4\ \text{NC}^{-1}$$

32. b. Dielectric strength means the maximum electric field which a medium can bear. Here, if field becomes more than this, then the charge will start leaking from the metal ball. So,

$$E = \frac{kQ}{R^2} \Rightarrow Q = \frac{ER^2}{k} = \frac{3\times10^6\times(0.001)^2}{9\times10^9} = 1/3\ \text{nC}$$

33. a. $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}$

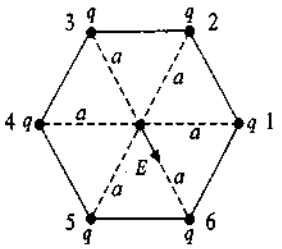


Fig. 1.155

Suppose the charge is present at the sixth vertex also, then electric field at centre would be zero. Now, if charge is not present at this vertex, the electric field at centre would be because of other five charges, which should be equal and opposite to the field produced due to single charge at the sixth vertex.

34. b. Charge on the element opposite to the gap is

$$dq = \frac{Q}{2\pi r}(0.002\pi)$$

$$= \frac{1}{2\pi(0.5)} \times \frac{2\pi}{1000} = 2\times10^{-3}\text{ C}$$

$$E = \frac{9\times10^9 \times 2\times10^{-3}}{(0.5)^2} = 7.2\times10^7\text{ NC}^{-1}$$

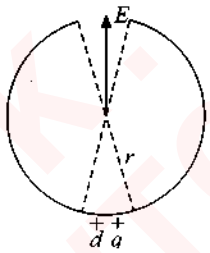


Fig. 1.156

35. b. Force on the block: $F = qE$ towards left.
Let spring be compressed maximum by x. Then

$$Fx = \frac{1}{2}kx^2 \Rightarrow qEx = \frac{1}{2}kx^2 \Rightarrow x = \frac{2qE}{k}$$

36. a. Strength of electric intensity is more if field lines are closer.

37. d. We can compare this situation with that of a mass moving as a projectile.

38. a. Work done is zero, because force applied by q_2 on q_1 is always perpendicular to the velocity of q_1.

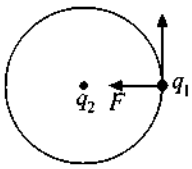


Fig. 1.157

39. a. Due to induced negative charge, there will be attraction.

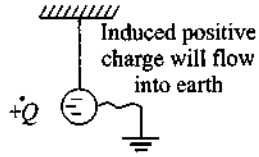


Fig. 1.158

40. a. To keep the system in equilibrium, net force experienced by charges at 'A', 'B' and 'C' should be zero. For this, another charge of opposite sign should be placed at the centroid of the triangle. Let this charge be '$-Q$'.

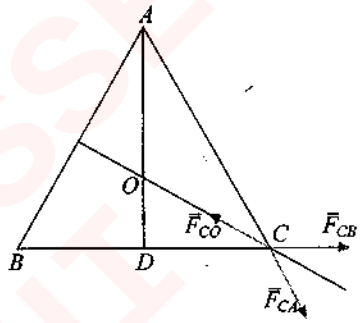


Fig. 1.159

$$AD = l\cos 30° = \frac{l\sqrt{3}}{2},\ AO = \frac{2}{3}AD = \frac{1}{\sqrt{3}}$$

$$2|\vec{F}_{CA}|\cos 30° = |\vec{F}_{CO}|$$

$$2\times\frac{1}{4\pi\varepsilon_0}\times\frac{q^2}{l^2}\times\frac{\sqrt{3}}{2} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{(l/\sqrt{3})^2} \Rightarrow Q = -\frac{q}{\sqrt{3}}$$

41. b. $F_{net} = 2|F_{31}|\cos\alpha$

$$= 2\times\frac{1}{4\pi\varepsilon_0}\times\frac{2\times4\times10^{-12}}{(0.5)^2}\times\frac{4}{5} = 0.46\text{ N}$$

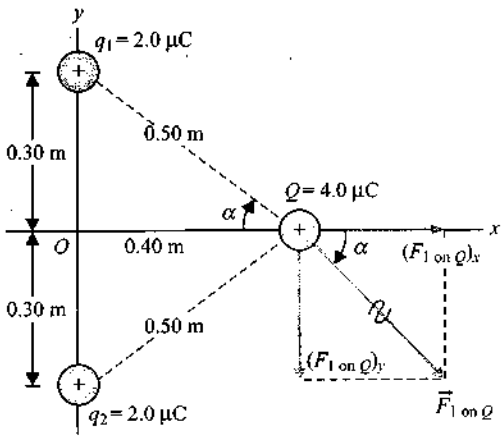


Fig. 1.160

42. c. For external points, a charged sphere behaves as if the whole of its charge is concentrated at its centre.

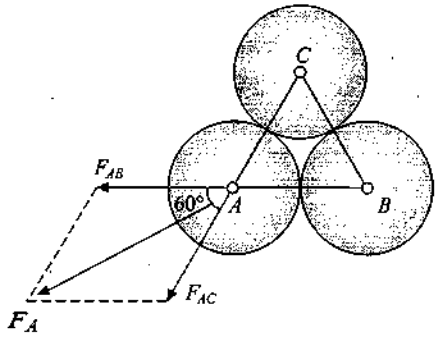


Fig. 1.161

Force on A due to B,

$$F_{AB} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2R)^2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{4R^2} \text{ along } \overline{BA}$$

And force on A due to C,

$$F_{AC} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2R)^2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{4R^2} \text{ along } \overline{CA}$$

Now as angle between BA and CA is 60° and

$$|F_{AB}| = |F_{AC}| = F$$

$$\therefore \quad F_A = \sqrt{F^2 + F^2 + 2F.F.\cos 60} = \sqrt{3}F$$

$$F_A = \frac{1}{4\pi\varepsilon_0}\frac{\sqrt{3}}{4}\left(\frac{q}{R}\right)^2$$

43. d. If there had been a sixth charge $+q$ at the remaining vertex of hexagon, force due to all the six charges on $-q$ at O will be zero.

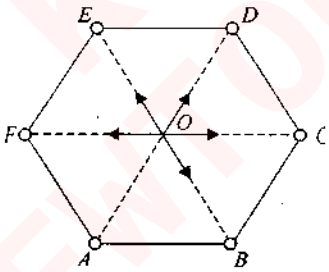


Fig. 1.162

Now if f is the force due to the sixth charge and F due to the remaining five charges, then

$$\vec{F} + \vec{f} = 0, \quad \text{i.e., } \vec{F} = -\vec{f}$$

$$|\vec{F}| = |\vec{f}| = \frac{1}{4\pi\varepsilon_0}\frac{q \times q}{L^2} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{L}\right)^2$$

44. d. $AC = \sqrt{2}l = BD$

$$\Rightarrow \quad BO = \frac{1}{\sqrt{2}}$$

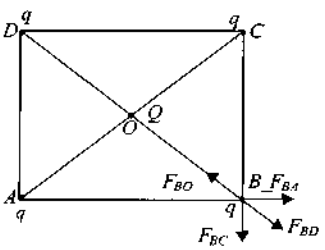


Fig. 1.163

$$F_{BO} = F_{BD} + (F_{BA} + F_{BC})\cos 45°$$

Solving, we get

$$Q = \frac{q}{4}(1 + 2\sqrt{2}), \; Q \text{ should be negative of } q.$$

45. c. $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$

$$\hat{r} = \frac{\vec{r}}{r} = \frac{(1.2-0)\hat{i} + (-1.6-0)\hat{j}}{\sqrt{(1.2)^2 + (1.6)^2}}$$

$$= \frac{1}{2}(1.2\hat{i} - 1.6\hat{j})$$

$$\vec{E} = 9 \times 10^9 \times \frac{(-8 \times 10^{-9})}{2^2} \times \left[\frac{1}{2}(1.2\hat{i} - 1.6\hat{j})\right]$$

$$= -10.8\hat{i} + 14.4\hat{j}$$

46. c. As $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{|\vec{r} - \vec{r}_0|^3}(\vec{r} - \vec{r}_0)$

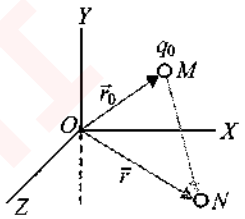


Fig. 1.164

$$\overline{MN} = \vec{r} - \vec{r}_0 = (8\hat{i} - 5\hat{j}) - (2\hat{i} + 3\hat{j})$$

$$= (6\hat{i} - 8\hat{j})$$

$$|\vec{r} - \vec{r}_0| = \sqrt{6^2 + 8^2} = 10 \text{ m}$$

$$\vec{E} = 9 \times 10^9 \times \frac{50 \times 10^{-6}}{(10)^3}(6\hat{i} - 8\hat{j})$$

$$= (2.7\hat{i} - 3.6\hat{j}) \text{ kNC}^{-1}$$

47. b.

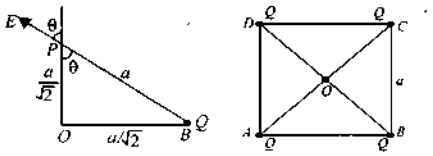


Fig. 1.165

$$E_{net} = 4E\cos\theta = \frac{4}{4\pi\varepsilon_0}\frac{Q}{a^2}\frac{1}{\sqrt{2}} = \frac{Q}{\sqrt{2}\pi\varepsilon_0 a^2}$$

48. a. Take PO as the x-axis and PA as the y-axis. Consider two elements EF and $E'F'$ of width $d\theta$ at angular distance θ above and below PO, respectively.

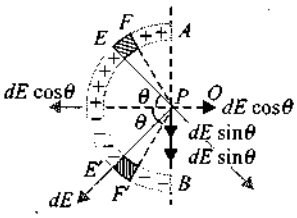


Fig. 1.166

The magnitude of the field at P due to either element is

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{rd\theta \times Q/(\pi r/2)}{r^2} = \frac{Q}{2\pi^2\varepsilon_0 r^2}d\theta$$

Resolving the fields, we find that the components along PO sum up to zero and hence, the resultant field is along PB.

Therefore, field at P due to pair of elements $= 2dE\sin\theta$

$$E = \int_0^{\pi/2} 2dE\sin\theta$$

$$= 2\int_0^{\pi/2}\frac{Q}{2\pi\varepsilon_0 r^2}\sin\theta d\theta = \frac{Q}{\pi^2\varepsilon_0 r^2}$$

49. a. Net force $F_{net} = qE_x$

$$F = q\frac{\lambda}{4\pi\varepsilon_0} = \frac{\lambda q}{4\pi\varepsilon_0 r}$$

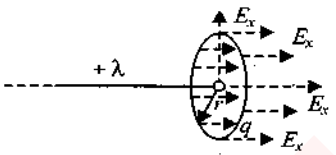


Fig. 1.167

50. b. Let us consider the electric field due to wire (3) only.

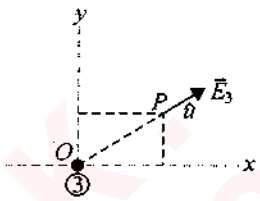


Fig. 1.168

$$\vec{E}_3 = E\hat{u}$$

$$= \frac{\lambda}{2\pi\varepsilon_0(a^2+a^2)^{1/2}}(\hat{i}\cos 45° + \hat{j}\cos 45°)$$

$$= \frac{\lambda}{2\sqrt{2}\pi\varepsilon_0 a}\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$$

$$= \frac{\lambda}{4\pi\varepsilon_0 a}(\hat{i}+\hat{j})$$

Similarly, electric field due to wires (1) and (2)

$$\vec{E}_1 = \frac{\lambda}{4\pi\varepsilon_0 a}(\hat{j}+\hat{k}) \text{ and } \vec{E}_2 = \frac{\lambda}{4\pi\varepsilon_0 a}(\hat{i}+\hat{k})$$

$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \frac{\lambda}{2\pi\varepsilon_0 a}(\hat{i}+\hat{j}+\hat{k})$$

51. c. $F = \frac{kQrq}{R^3} = -ma$

$$a = -\left(\frac{Qq}{4\pi\varepsilon_0 mR^3}\right)r \Rightarrow a = -\omega^2 r$$

$$\omega = \sqrt{\frac{Qq}{4\pi\varepsilon_0 mR^3}}$$

52. a. Mean position: $x = l$

$$F = -k(x-\ell) \Rightarrow \text{ where } k = m\omega^2$$

$$\text{or } qE = -m\omega^2(x-\ell) \Rightarrow \text{At } x = \ell, E = 0$$

To the right of $x = \ell$, E is $-$ve, so towards left and to the left of $x = \ell$, E is +ve.

53. c. The following two arguments shall lead us to the right choice.

i. Electric field at the centre of the ring is zero.

ii. Electric field is directed away from the ring.

Multiple Correct Answers Type

1. c.,d. The particle will oscillate or perform SHM if equilibrium is a stable one. For negative charge, equilibrium is stable if the particles are displaced along CD and unstable for displacement along AB.

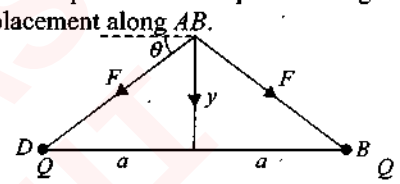


Fig. 1.169

For $y \ll a$, in displaced position, net resultant force towards equilibrium position is

$$F_{res} = 2F\sin\theta = \frac{2\times Q}{4\pi\varepsilon_0(a^2+y^2)}\times\frac{y}{(a^2+y^2)^{1/2}} = \frac{2Qy}{4\pi\varepsilon_0 a^3}\ [a >> y]$$

$F_{res} \propto y$, so SHM for $y << a$.

For $y \simeq a$, there would be restoring force, but F is not proportional to y.

So, there is oscillatory motion but not SHM.

2. a., b., c.

Along X-axis, $u_x = u\cos\theta \Rightarrow a_x = -\frac{qE}{m}$

Along Y-axis, $u_y = u\sin\theta \Rightarrow a_y = -g$

Equation of motion along X- and Y-axes would be as follows: a is resultant aceleration. If a is opposite to u, i.e., $\alpha = \theta$, then the path will be a straight line. For $\alpha \neq \theta$, the path will be parabolic.

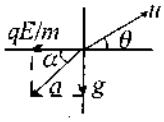


Fig. 1.170

Time of flight remains unchanged as vertical motion is not affected by $\vec{E}$. Range of the particle in the present case is always less than $= \frac{u^2\sin 2\theta}{g}$ whatever may be the value of E.

$$R = x_t = T = u\cos\theta \times \frac{2u\sin\theta}{g} - \frac{1}{2}$$

$$\times\frac{qE}{m}\left[\frac{2u\sin\theta}{g}\right]^2 < \frac{u^2\sin 2\theta}{g}$$

3. b., c. Just draw the FBD of both charge particles and see whether force can be zero or not on the particle. At least, one force should be opposite to others.

Assertion-Reasoning Type

1. a. Force by electric field will be perpendicular to the displacements.
2. c. If the field lines are curved, then the charge particle does not exactly follow the curved path.
3. a. A body can be charged by the transfer of electrons only.
4. a. Reason truly explains the assertion. However, if charges are point, no induction will take place and they will never attract.
5. a. Both the assertion and the reason are true, and the reason is the correct explanation of the assertion.

Comprehension Type

For Problems 1–2

1. b., 2, c.

Sol. Charge on each ball $= \frac{10+30}{2} = 20\ \mu C$

$$F = 9\times10^9 \times \frac{(20\times10^{-6})^2}{(0.50)^2} = 14.4\ N$$

For Problems 3–4

3. d., 4. a.

Sol.

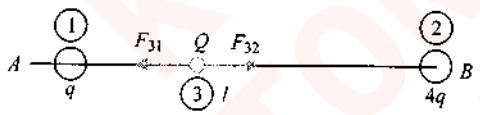


Fig. 1.171

For system to be in equilibrium net force on each charge should be zero, hence the third charge should be negative and it should be placed near q between (1) and (2). For equilibrium of (3), we have

$$\frac{1}{4\pi\varepsilon_0}\frac{qQ}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q4q}{(l-x)^2} \Rightarrow x = \frac{l}{3}$$

For equilibrium of (1), we have

$$\frac{1}{4\pi\varepsilon_0}\frac{q4q}{l^2} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{x^2}$$

$$Q = 4q\left(\frac{x}{l}\right)^2 = 4q\left(\frac{1}{3}\right)^2 = \frac{4q}{9} \Rightarrow Q = \frac{4q}{9}$$

Q should be negative of q, otherwise the resultant force on q cannot be zero.

For Problems 5–7

5. c., 6. a., 7. c.

Sol. The four possible force diagrams are as follows:

Only the last picture can result in an electric field in the $-$ve x-direction.

$q_1 = -2.00\ \mu C$, $q_3 = +4.00\ \mu C$ and $q_2 > 0$.

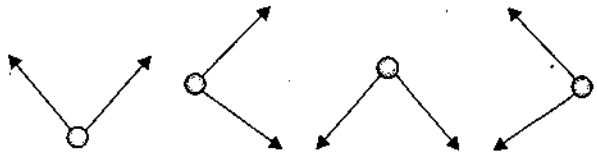


Fig. 1.172

$$E_y = 0 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{(0.0400\,m)^2}\sin\theta - \frac{1}{4\pi\varepsilon_0}\frac{q_2}{(0.0300\,m)^2}\cos\theta$$

$$\Rightarrow q_2 = \frac{9}{16}q_1\frac{\sin\theta}{\cos\theta} = \frac{9}{16}q_1\frac{3/5}{4/5} = \frac{27}{64}q_1 = 0.843\ \mu C$$

$$F_3 = q_3E_x = q_3\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{0.0016}\frac{4}{5} + \frac{q_2}{0.0009}\frac{3}{5}\right) = 56.25\ N$$

For Problems 8–10

8. a., 9. a., 10. b.

Sol. Electric field at (2) tends to $-\infty$, hence the charge at (2) should be negative. There is a neutral point to the right of charges. This is possible only when the charge at (1) should be positive.

Hence, Q_1 is positive and Q_2 is negative.

At neutral point, $\vec{E}_1 = \vec{E}_2$

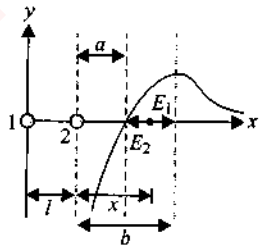


Fig. 1.173

$$\frac{1}{4\pi\varepsilon_0}\frac{Q_1}{(a+l)^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{a^2} \Rightarrow \frac{Q_1}{Q_2} = \left(\frac{a+l}{a}\right)^2$$

Electric field at any position to the right of charges

$$E = E_1 - E_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{(l+x)^2} - \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{x^2}$$

For the maximum value of E

$$\frac{dE}{dx} = 0 \Rightarrow \frac{d}{dx}\left[\frac{1}{4\pi\varepsilon_0}\frac{Q_1}{(l+x)^2} - \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{x^2}\right] = 0$$

$$Q_1\frac{(-2)}{(l+x)^3} - Q_2\frac{(-2)}{x^3} = 0$$

$$\frac{Q_1}{(l+x)^3} = \frac{Q_2}{x^3} \Rightarrow \left(\frac{l+x}{x}\right)^3 = \frac{Q_1}{Q_2}$$

$$\frac{l+x}{x} = \left(\frac{Q_1}{Q_2}\right)^{1/3} \Rightarrow \frac{l}{x} = \left(\frac{Q_1}{Q_2}\right)^{1/3} - 1$$

$$x = \frac{l}{\left(\frac{Q_1}{Q_2}\right)^{1/3} - 1}$$

For Problems 11–12

11. b., 12. a.

Sol. $OA = OB = OC = OD = a$

The magnitude of the electric force due to each charge is $\frac{1}{4\pi\varepsilon_0}\frac{Q}{a^2+h^2}$.

The components of the force perpendicular to OP sum up to zero because of the symmetrical distribution of charges about OP. Hence, the resultant force at P is upward along OP. The magnitude of the force is given by

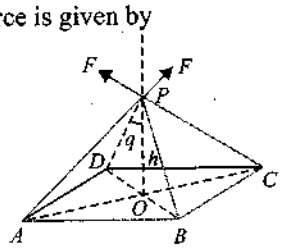


Fig. 1.174

$$F_{up} = 4\times\frac{1}{4\pi\varepsilon_0}\frac{Q}{h^2+a^2}\cos\theta$$

$$= \frac{Q}{\pi\varepsilon_0(h^2+a^2)}\frac{h}{\sqrt{h^2+a^2}} = \frac{Qh}{\pi\varepsilon_0(h^2+a^2)^{3/2}}$$

$$F_{down} = \text{weight} = mg$$

For equilibrium, $mg = \frac{Qh}{\pi\varepsilon_0(h^2+a^2)^{3/2}}$

$$\Rightarrow \quad Q = \pi\varepsilon_0\frac{mg}{h}(h^2+a^2)^{3/2}$$

Yes, this is a stable equilibrium as it will regain its position if displaced a little.

For Problems 13–16

13. c., 14. b., 15. c., 16. d.

Sol. Passing between the charged plates, the electron feels a force upward and just misses the top plate. The distance it travels in the y-direction is 0.005 m.

$$\text{Time of flight} = t = \frac{0.0200\text{ m}}{1.60\times10^6\text{ m/s}}$$

$$= 1.25\times10^{-8}\text{s}$$

and the initial y-velocity is zero.

Now, $y = v_{0y}t + \frac{1}{2}at^2$

So, $0.005\text{ m} = \frac{1}{2}a(1.25\times10^{-8}\text{ s})^2$

$$\Rightarrow \quad a = 6.40\times10^{13}\text{ ms}^{-2}$$

But also, $a = \frac{F}{m} = \frac{eE}{m_e}$

$$E = \frac{(9.1\times10^{-31}\text{ kg})(6.40\times10^{13}\text{ ms}^{-2})}{1.60\times10^{-19}\text{C}}$$

$$= 364\text{ NC}^{-1}$$

Since the proton is more massive, it will accelerate less, and NOT hit the plates. To find the vertical displacement when it exits the plates, we use the kinematic equations again:

$$y = \frac{1}{2}at^2 = \frac{1}{2}\frac{eE}{m_p}(1.25\times10^{-8}\text{s})^2$$

$$= 2.73\times10^{-6}\text{m}$$

As mentioned in (b), the proton will not hit one of the plates because although the electric force felt by the proton is the same as the electron felt, a smaller acceleration results for the more massive proton.

The acceleration produced by the electric force is much greater than g; it is reasonable to ignore gravity.

For Problems 17–18

17. c., 18. d.

Sol. $d = \frac{u^2\sin^2\theta}{2a}$, $\quad a = \frac{qE}{m}$

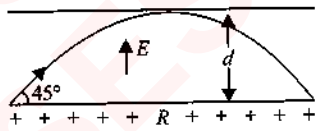


Fig. 1.175

$$d = \frac{mu^2\sin^2 45°}{2qE}$$

$$d = \frac{K}{2qE} \Rightarrow E = \frac{K}{2dq}$$

$$R = \frac{u^2}{a} = \frac{2d}{\sin^2\theta} = 4d$$

For Problems 19–20

19. a., 20. a.

Sol. $qE = mg \Rightarrow q\times1.68\times10^5 = 1.08\times10^{-14}g$

$$\Rightarrow \quad q = 6.40\times10^{-19}\text{ C}$$

$$q = ne \Rightarrow 6.4\times10^{-19} = n\times1.6\times10^{-19} \Rightarrow n = 4$$

Matching Column Type

1. i. → a., c., d. ii. → a., b., d. iii. → a., b., d. iv. → a., c., d.

Sol. In case of stable equilibrium, the potential energy is minimum and in case of unstable equilibrium, potential energy is maximum.

Cases (i) and (iv) are the cases of stable equilibrium, hence answers are (a), (c) and (d). Cases (ii) and (iii) are the cases of unstable equilibrium. Hence, the answers are (a), (b) and (d).

2. i. → a., b., c. ii. → a., b. iii. → a., b. iv. → a., b., c.

Electrostatic force and gravitational force are conservative, action–reaction forces. Also, electrostatic force depends upon the nature of the medium between interacting objects. The principle of superposition is applicable, irrespective of the nature of force.

3. i. → a., c., d. ii. → a., b. iii. → a., b., c., d. iv. → a., c., d.

Electrostatic forces or the Coulombic force exists between the charged bodies. Gravitational force exists between all bodies. Strong force exists between particles inside the nucleus.

4. i. → b. ii. → a. iii. → d. iv. → c.
Charge always needs mass to reside. Mass may or may not be charged. Charge is a conserved quantity but mass is not.

5. i. → c. ii. → a. iii. → d. iv. → b.

$$\varepsilon_0 \propto \frac{q_1 q_2}{Fr^2} \Rightarrow [\varepsilon_0] = [M^{-1} L^{-3} T^4 A^2]$$

$k =$ Dimensionless

$$E = \frac{F}{q} \Rightarrow [E] = [M^1 L^1 T^{-2} Q^{-1}]$$
$$= [M^1 L^1 T^{-2} A^{-1} T^{-1}]$$
$$= [M^1 L^1 T^{-3} A^{-1}]\text{s}$$
$$[F] = [M^1 L^1 T^{-2}]$$

Integer Answer Type

1. (2) For the equilibrium of charge at $z = a$, the net electric field at this point due to both the ring should be zero.

$$E_A + E_B = 0 \Rightarrow \frac{kq_A a}{(a^2 + a^2)^{3/2}} + \frac{kq_B a}{(b^2 + a^2)^{3/2}} = 0$$

Put the values and solve to get $b/a = 2$.

2. (1) After turning through 90°, the rod comes at rest momentarily. So there is no change in KE. Net work done by all the forces should be zero.

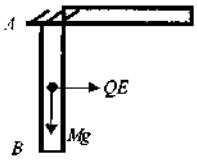


Fig. 1.176

$$QE\frac{L}{2} - Mg\frac{L}{2} = 0 \quad \Rightarrow \quad E = \frac{Mg}{Q}$$

3. (2) $a_1 = \frac{qE}{m}$ and $a_2 = \frac{qE}{2m} \Rightarrow a_1 = 2a_2$

$$K_1 = \frac{1}{2}m_1 v_1^2 = \frac{1}{2}m_1 (a_1 t)^2, \; K_2 = \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_2 (a_2 t)^2$$

$$\frac{K_1}{K_2} = \frac{m_1 a_1^2}{m_2 a_2^2} = \frac{m}{2m} \times \frac{4}{1} = \frac{2}{1}$$

4. (5) $W = qE \times s \cos\theta \Rightarrow 1 = 0.2E \times 2 \cos 60°$

$$\Rightarrow \quad E = 5 \text{ NC}^{-1}$$

5. (0) $W = q\int_{\vec{r}_A}^{\vec{r}_B} \vec{E} \cdot d\vec{r} = 1 \times \int_{(2,2)}^{(4,1)} (E_x dx + E_y dy)$

$$= 1 \times \int_{(2,2)}^{(4,1)} (y\,dx + x\,dy) = \int_{(2,2)}^{(4,1)} d(xy)$$

$$= [xy]_{(2,2)}^{(4,1)} = 4 \times 1 - 2 \times 2 = 0$$

R. K. MALIK'S
NEWTON CLASSES
RANCHI

CHAPTER
2

Electric Flux and Gauss's Law

- Electric Flux
- Gauss's Law
- Field of a Charged Conducting Sphere
- Field of a Line Charge
- Field of an Infinite Plane Sheet of Charge
- Field at the Surface of a Conductor
- Field of a Uniformly Charged Sphere
- Electric Field Due to a Long Uniformly Charged Cylinder
- Electric Field Near Uniformly Volume Charged Plane
- Appendix

R. K. MALIK'S NEWTON CLASSES RANCHI

ELECTRIC FLUX

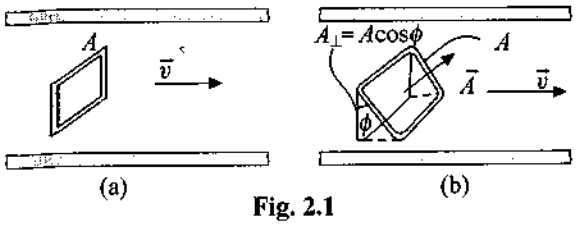
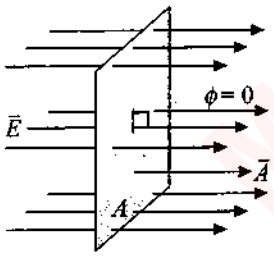


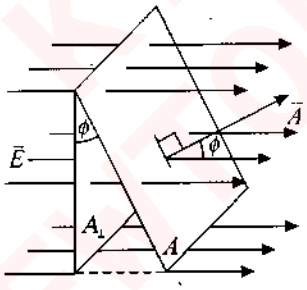
Fig. 2.1

Figure 2.1 shows a fluid flowing steadily from left to right. Let us examine the volume flow rate dV/dt (in, say, cubic meters per second) through the rectangular wire with area A. When the area is perpendicular to the flow velocity [as shown in Fig. 2.1(a)] and the flow velocity is the same at all points in the fluid, the volume flow rate dV/dt is the area A multiplied by the flow speed v

$$\frac{dV}{dt} = vA$$



(a) Surface face on to electric field $\vec{E}$ and $\vec{A}$; parallel angle between $\vec{E}$ and $\vec{A}$ is $\phi = 0$; flux $\Phi_E = \vec{E}\cdot\vec{A} = EA$



(b) Surface tilted from face-on orientation by an angle ϕ; angle between $\vec{E}$ and $\vec{A}$ is ϕ; flux $\Phi_E = \vec{E}\cdot\vec{A} = EA\cos\phi$

Fig. 2.2

When the rectangle is tilted at an angle ϕ (as shown in Fig. 2.1(b)) so that its face is not perpendicular to $\vec{v}$, the area that counts is the silhouette area that we see when we look in the direction of $\vec{v}$. This area, which is labelled $A_\perp$ in Fig. 2.1(b), is the projection of area A onto a surface perpendicular to $\vec{v}$. Two sides of the projected rectangle have the same length as the original one, but the other two are foreshortened by a factor of $\cos\phi$, so the projected area $A_\perp$ is equal to $A\cos\phi$. Thus, the volume flow rate through A is

$$\frac{dV}{dt} vA\cos\phi$$

If $\phi = 90°$, $dV/dt = 0$; the rectangle is edge-on to the flow, and no fluid passes through the rectangle.

We can also write $dV/dt = A(v\cos\phi)$.

Also, $v\cos\phi$ is the component of the vector $\vec{v}$ perpendicular to the plane of area A. Calling this component $v_\perp$, we can rewrite the volume flow rate as $dV/dt = v_\perp A$.

We can express the volume flow rate more compactly by using the concept of vector area $\vec{A}$, a vector quantity with magnitude A and direction perpendicular to the plane of the area we are describing. The vector area $\vec{A}$ describes both the size of an area and its orientation in space. In terms of $\vec{A}$, we can write the volume flow rate of fluid through the rectangle in Fig. 2.1(b) as a scalar (dot) product, i.e.,

$$\frac{dV}{dt} = \vec{v}\cdot\vec{A}$$

Using the analogy between electric field and fluid flow, we now define electric flux (Φ_E) in the same way as we have just defined the volume flow rate of a fluid.

Roughly speaking, we can picture Φ_E in terms of the field lines passing through A (Fig. 2.2).

We generalize our definition of electric flux for a uniform electric field as

$\Phi_E = EA\cos\phi$ (electric flux for uniform, flat surface) (i)

Since $E\cos\phi$ is the component of E perpendicular to the area, we can rewrite Eq. (i) as

$\Phi_E = E_\perp A$ (electric flux for uniform, flat surface) (ii)

In terms of the vector area $\vec{A}$ perpendicular to the area, we can write the electric flux as the scalar product of $\vec{E}$ and $\vec{A}$, i.e.,

$\Phi_E = \vec{E}\cdot\vec{A}$ (electric flux for uniform, flat surface) (iii)

A surface has two sides, so there are two possible directions for $\vec{A}$. We must always specify which direction of $\vec{A}$ we choose. For a closed surface, we will always choose the direction of $\vec{A}$ to be outward, and we will speak of the flux out of a closed surface. Thus, we call 'outward electric flux' corresponds to a positive value of Φ_E and we call 'inward electric flux' corresponds to a negative value of Φ_E.

Illustration 2.1 **Find the flux of the electric field through each of the five surfaces of the inclined plane as shown in Fig. 2.3. What is the total flux through the entire closed surface?**

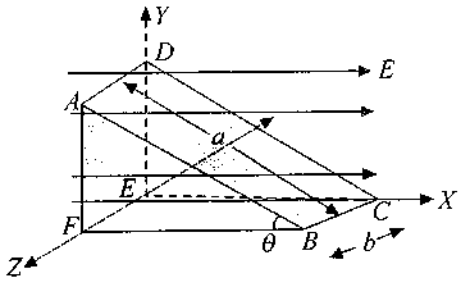


Fig. 2.3

Sol. Note that flux through the faces ABF, CDE and $BCEF$ is zero. Area vector of face ABF points in the positive z-direction, area vector of CDF points in the negative z-direction and area vector of $BCEF$ points in the negative y-direction. In all the three cases, field E is normal to area vector.

Flux through face $ABCD$ [Fig. 2.4(b)]:

Magnitude of area vector of face $ABCD = ab$

$$\phi_E = \vec{E}\cdot\vec{A} = E(ab)\cos(90°-\theta) = Eab\sin\theta$$

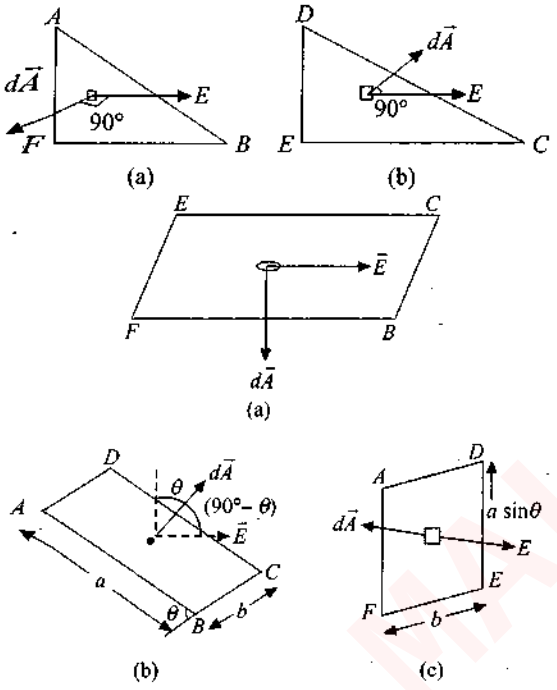


Fig. 2.4

Flux through face $ADEF$ [Fig. 2.4(c)]:
Magnitude of area vector of face

$$ADEF = (a\sin\theta)b = ab\sin\theta$$

$$\phi_E = E\cos 180°\,(ab\sin\theta) = -Eab\sin\theta$$

Thus, we obtain

$$(\phi_E)_{ABF} = 0,\ (\phi_E)_{CDE} = 0,\quad (\phi_E)_{BCEF} = 0,$$

$$(\phi_E)_{ABCD} = +Eab\sin\theta \text{ and } (\phi_E)_{ADEF} = -Eab\sin\theta.$$

Flux is a scalar quantity, therefore the total flux is algebraic sum of flux through each surface.

$$\phi_{total} = (\phi_E)_{ABF} + (\phi_E)_{CDE} + (\phi_E)_{BCEF} + (\phi_E)_{ABCD} + (\phi_E)_{ADEF}$$

$$= 0 + 0 + 0 + Eab\sin\theta - Eab\sin\theta = 0$$

- Note that the contribution to the flux for a closed surface is positive for the surface where the field is directed out ($ABCD$) and negative for the surface where the field is directed into the surface ($ADEF$).
- The net flux for this closed surface can also be seen to be zero from the examination of the field lines. If the field is uniform, the number of lines that enter the closed surface equals the number of lines that come out.
- The flux of a constant vector through any closed surface is zero.

Illustration 2.2 **Consider a cylindrical surface of radius R, length l, in a uniform electric field E. Compute the electric flux if the axis of the cylinder is parallel to the field direction.**

Sol. We can divide the entire surface into three parts, right and left plane faces and curved portion of its surface. Hence, the surface integral consists of the sum of the three terms:

$$\phi_E = \oint E\cdot dA = \int_{\text{left end}} E\cdot dA + \int_{\text{right end}} E\cdot dA + \int_{\text{curved}} E\cdot dA$$

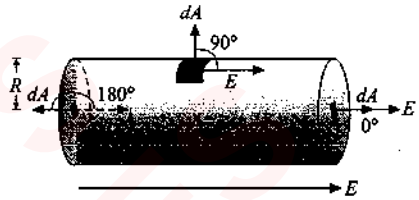


Fig. 2.5

All the area elements on the left end and electric field E are at an angle of 180°

$$(\phi_E)_{\text{left end}} = \oint_{\text{left end}} E\cdot dA = \oint_{\text{left end}} E\,dA\cos 180°$$

$$= -E\oint_{\text{left end}} dA = -E\pi R^2$$

Note that E is constant over the entire plane surface of left end; therefore, we take it out from the integral.

Similarly, all the area elements on the right end are parallel to electric field E, i.e., angle is 0°.

$$(\phi_E)_{\text{right end}} = \oint_{\text{right end}} E\cdot dA = \oint_{\text{right end}} E\,dA\cos 0°$$

$$= +E\oint_{\text{right end}} dA = E\pi R^2$$

Finally, at every point on the curved surface the area vectors are perpendicular to the direction of the electric field. Thus,

$$(\phi_E)_{\text{curved}} = \oint_{\text{curved surface}} E\cdot dA = \oint_{\text{curved surface}} E\,dA(\cos 90°) = 0$$

$$\text{Total flux} = (\phi_E)_{\text{right end}} + (\phi_E)_{\text{left end}} + (\phi_E)_{\text{curved surface}}$$

$$= (+E\pi R^2) + (-E\pi R^2) + 0 = 0$$

Hence, we see that in a uniform field the flux through a closed surface is zero. This is true for any shape of closed surface.

Illustration 2.3 **A point charge q is placed at a distance $a/2$ from the centre of a square of side a as shown in Fig. 2.6. Calculate the electric flux passing through the square.**

Sol.

- This problem can solved by symmetry consideration and Gauss's law.

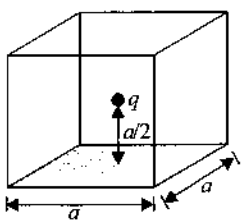


Fig. 2.6

- We can enclose the charged particle by a cube of side 'a' and keep the particle at the centre of the cube.
- The total flux passing through the close cube cube $\phi = q/\varepsilon_0$.

- All the six surfaces are symmetrical with respect to charge, hence they will have equal contribution of the flux. So, flux through any one face is $\phi' = \phi/6 = q/6\varepsilon_0$.

Illustration 2.4 **In Fig. 2.7, a charge q is placed at a distance $\delta \to 0$ near one of the corners of a cube of edge l on a line of symmetry along diagonal.**

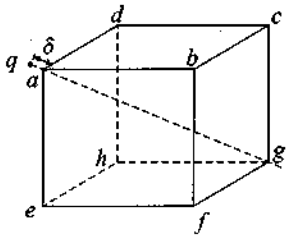


Fig. 2.7

1. **What is flux through each of the faces containing the point a?**
2. **What is the flux through the other three faces?**

Sol. Use of symmetry consideration may be useful in problems of flux calculation.

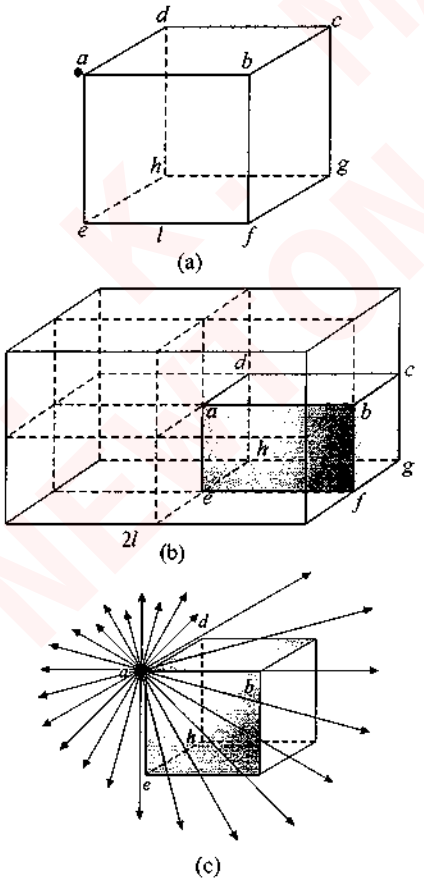


Fig. 2.8

- We can imagine a charged particle placed at the centre of a cube of side $2l$.
- The flux enclosed with the cube $\phi = q/\varepsilon_0$.
- The flux passing through one of the faces of the cube $\phi' = \phi/6 = q/6\varepsilon_0$.
- Hence, the flux passing through the face $bcgf = \phi'/4 = q/24\varepsilon_0$.
- Each of the faces $(efgh)$, $(bcgf)$ and $(dcgh)$ symmetrical with respect to charge. Hence, the flux passing through each of the face is $q/24\varepsilon_0$.
- The electric field lines for the faces $(efgh)$, $(bcgf)$ and $(dcgh)$ is away from the faces. Hence, the flux associated with each of the faces will be positive $(\text{i.e.}, +q/24\varepsilon_0)$.

  Hence, the total flux through these sides $= \dfrac{3\times q}{24\varepsilon_0} = \dfrac{q}{8\varepsilon_0}$.
- As $\delta \to 0$, we can say the faces $(abcd)$, $(abfe)$ and $(adhe)$ are also symmetrical about charge. Charge is slightly outside the cube.
- The number of electric field lines which are passing through the faces which do not contain the point a are same as the number of electric field lines passing through the faces containing the point a.
- Hence, the same amount of flux will pass through the faces containing the point a, i.e., $q/8\varepsilon_0$.
- The electric field lines are towards the faces containing the point a. Hence, the flux will be negative, i.e., $\phi'' = -q/8\varepsilon_0$.
- Hence, the flux through each of the faces containing the point 'a' will be $\phi''/3 = q/24\varepsilon_0$.

Your Task: Repeat Illustration 2.4 if the charge is exactly at the corner of the cube given.

Concept Application Exercise 2.1

1. A thin metallic spherical shell contains a charge Q on it. A point charge q is placed at the centre of the shell and another charge q_1 is placed outside it as shown in the figure. All the three charges are positive. Find the force on the charge

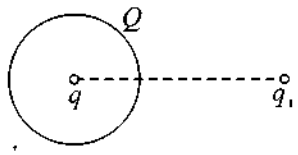


Fig. 2.9

   a. at centre due to all charges.
   b. at centre due to shell.
2. If we introduce a large thin metal plate between two point charges, what will happen to the force between the charges?
3. A dipole lies on the x-axis, with the positive charge $+q$ at $x = +d/2$ and the negative charge at $x = -d/2$. Find the electric flux ϕ_E through the yz plane midway between the charges.

4. A charge Q is distributed uniformly on a ring of radius r. A sphere of equal radius r is constructed with its centre at the periphery of the ring. Find the flux of the electric field through the surface of the sphere.

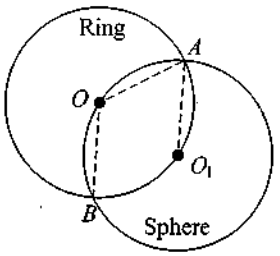


Fig. 2.10

5. Figure 2.11(a) shows an imaginary cube of edge $L/2$. A uniformly charged rod of lenght L moves towards left at a small but constant speed v. At $t = 0$, the left end of the rod just touches the right face of the cube. Which of the graphs in Fig. 2.11(b) represents the flux of the electric field through the cube as the rod goes through it?

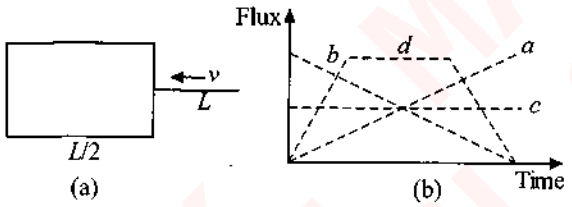


Fig. 2.11

6. A hemispherical body is placed in a uniform electric field E. What is the flux linked with the curved surface, if the field is (a) parallel to base of the body [Fig. 2.12(a)]; (b) perpendicular to base of the body [Fig. 2.12(b)] and (c) perpendicular to the curved surface at every points as in Fig. 2.12(c).

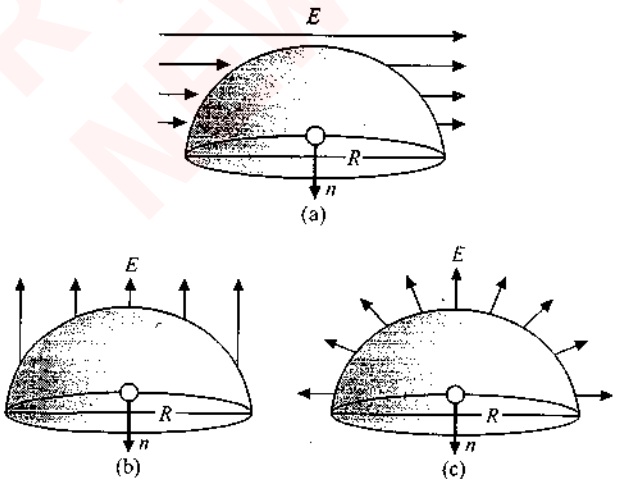


Fig. 2.12

7. What is the field in the cavity, if a conductor having a cavity is charged? Does the result depend on the shape and size of cavity or conductor?

8. Figure 2.13 shows a closed surface which intersects a conducting sphere. If a positive charge is placed at point P, find the sign of flux passing through the curved surface S.

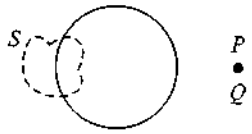


Fig. 2.13

9. In which position (A, B, C or D) of second charge, the flux of the electric field through the hemisphere remains unchanged?

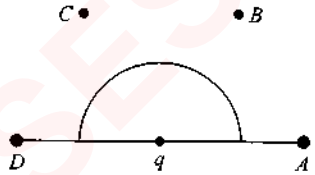


Fig. 2.14

10. A point charge Q is located just above the centre of the flat face of a hemisphere of radius R as in Fig. 2.15. What is the flux

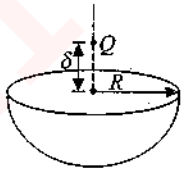


Fig. 2.15

a. through the curved surface, and

b. through the flat face?

c. Repeat parts (a) and (b) if the charge is exactly at the centre.

11. In Fig. 2.16, a cone lies in a uniform electric field E. Determine the electric flux entering the cone.

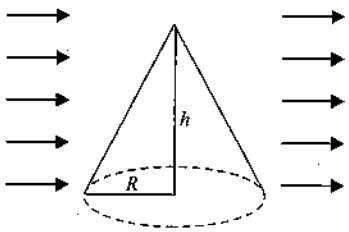


Fig. 2.16

12. A uniform electric field $a\hat{i}+b\hat{j}$ intersects a surface of area A. What is the flux through this area if the surface lies: (a) in the yz plane? (b) in the xz plane? (c) in the xy plane?

13. a. A point charge q is located at distance d from an infinite plane. Determine the electric flux through the plane due to the point charge.

b. A point charge q is located at a very small distance from the centre of a very large square on the line perpendicular to the square passing through its centre. Determine the approximate electric flux through the square due to the point charge.

14. Calculate the total electric flux through the paraboloidal surface due to a uniform electric field of magnitude E_0 in the direction shown in Fig. 2.17.

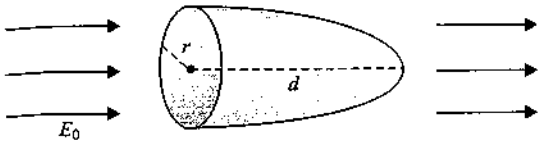


Fig. 2.17

15. Consider a closed surface of arbitatry shape as shown in Fig. 2.18. Suppose a single charge Q_1 is located at some point within the surface and second charge Q_2 is located outside the surface.

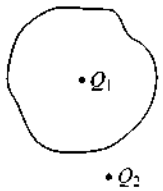


Fig. 2.18

a. What is the total flux passing through the surface due to charge Q_1?

b. What is the total flux passing through the surface due to charge Q_2?

16. If Coulomb's law involved $1/r^3$ (instead of $1/r^2$) would Gauss's law still be true?

GAUSS'S LAW

Gauss's law is an alternative to Coulomb's law. Although completely equivalent to Coulomb's law, Gauss' law provides a different way to express the relationship between electric charge and electric field.

Gauss's law states that the total electric flux through a closed surface is proportional to the total electric charge enclosed within the surface. This law is useful in calculating field caused by charge distributions that have various symmetry properties.

Mathematically, Gauss's law can be written as

$$\phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

One thing is to note that the electric field appearing in the Gauss's law is the resultant electric field due to all the charges present inside as well as outside the given closed surface. On the other hand, the charges q_{in} appearing in the law are only the charges contained within the closed surface. The contribution of the charges outside the closed surface in producing the flux is zero. A surface on which Gauss's law is applied is sometimes called the Gaussian surface (Fig. 2.19).

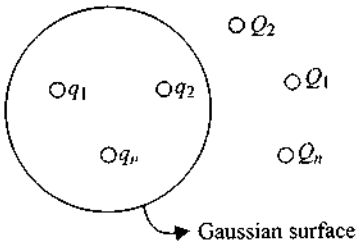
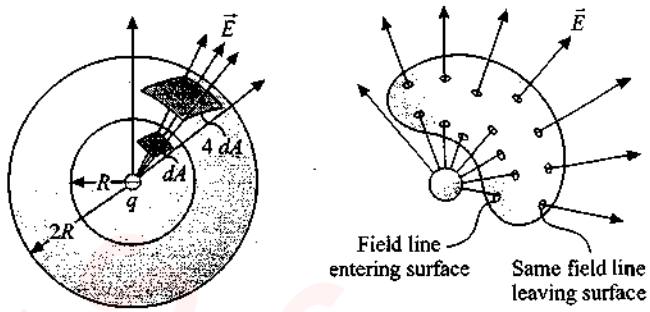


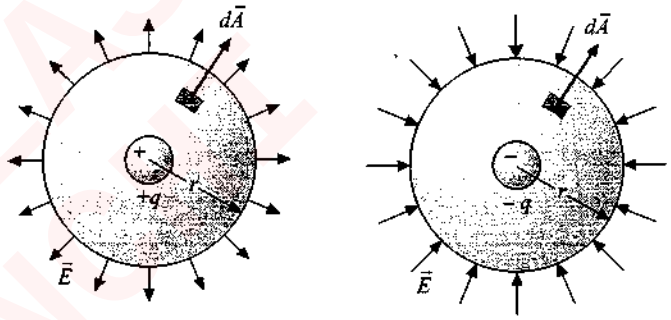
Fig. 2.19

Caution: Remember that the closed surface in Gauss's law is imaginary; there need not be any material object at the position of the surface.



Projection of an element of area dA of a sphere of radius R onto a concentric sphere of radius $2R$. The projection multiplies each linear dimension by 2, so the area element on the larger sphere is 4 dA. The same number of field lines and the same flux pass through these two area elements.

A point charge outside a closed surface that encloses no charge. If an electric field line from the external charge enters the surface at one point, it must leave at another. Figure illustrates this point. Electric field lines can begin or end inside a region of space only when there is charge in that region.



(a) Spherical Gaussian surface around positive charge; positive (outward) flux.

(b) Spherical Gaussian surface around negative charge; negative (inward) flux.

Fig. 2.20

Note that the electric field in the expression $\oint \vec{E} \cdot d\vec{A} = q_{in}/\varepsilon_0$ is the resultant field at any point on the Gaussian surface, whereas q_{in} is the charge enclosed by the Gaussian surface. Consider the two Gaussian surfaces A_1 and A_2 as shown in Fig. 2.21.

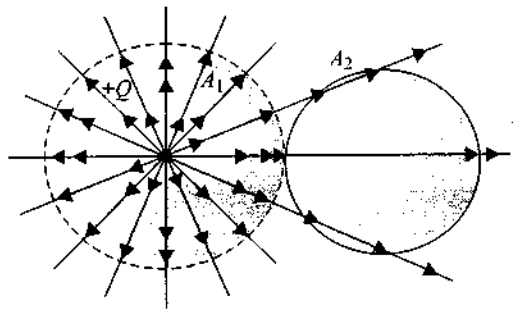


Fig. 2.21

Charge Q lies at the centre of the Gaussian surface A_1. For surface A_1, the net flux through A_1 is Q/ε_0. For surface A_2, charge Q is outside A_2 so that the net flux through A_2 is zero. Note that the field lines that enter the Gaussian surface A_2 (net flux in) also leave it (net flux out).

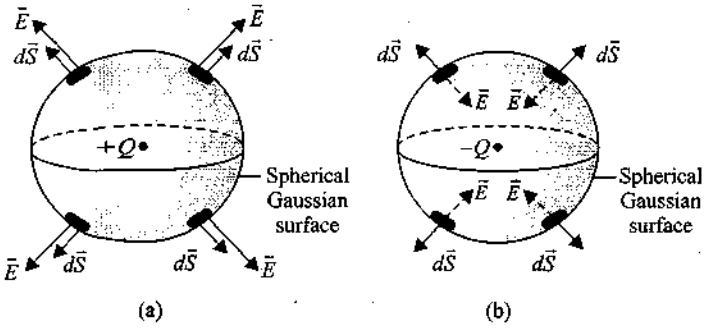


Fig. 2.22

Problem-Solving Strategy

Identify the relevant concepts: Gauss's law is most useful in situations where the charge distribution has spherical or cylindrical symmetry or is distributed uniformly over a plane. In these situation, we determine the direction of $\vec{E}$ from the symmetry of the charge distribution. If we are given the charge distribution, we can use Gauss' law to find the magnitude of $\vec{E}$. Alternatively, if we are given the field, we can use Gauss's law to determine the details of the charge distribution. In either case, begin your analysis by asking the question, 'What is the symmetry?'

The problem uses the following steps:

1. Select the surface that you will use with Gauss's law. We often call it a Gaussian surface. If you are trying to find the field at a particular point, then that point must lie on the Gaussian surface.

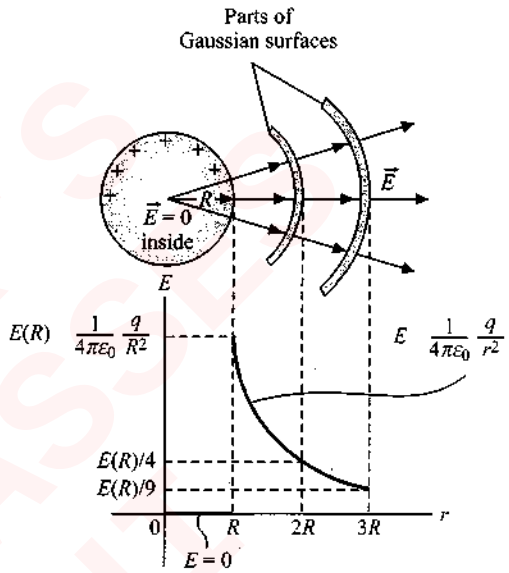
Charge Distribution	Gaussian Surface	Electric Field
Point charge	Spherical	Radial
Spherical charge distribution	Spherical	Radial
Line of charge	Cylindrical	Radial
Planer charge	Plane parallel to charge distribution	Normal to surface

2. The Gaussian surface does not have to be a real physical surface, such as a surface of a solid body. Often the appropriate surface is an imaginary geometric surface; it may be in empty space, embedded in a solid body or both.

3. Usually, you can evaluate the integral in Gauss's law (without using a computer) only if the Gaussian surface and the charge distribution have some symmetry property. If the charge distribution has cylindrical or spherical symmetry, choose the Gaussian surface to be a coaxial cylinder or a concentric sphere, respectively.

FIELD OF A CHARGED CONDUCTING SPHERE

We place positive charge q on a solid conducting sphere with radius R (as shown in Fig. 2.23). All the charges must be on the surface of the sphere.



Under electrostatic conditions, the electric field inside a solid conducting sphere is zero. Outside the sphere, the electric field drops off as $1/r^2$ as though all the excess charge on the sphere is concentrated at its centre.

Fig. 2.23

Selection of Gaussian Surface

The system has spherical symmetry. To take advantage of the symmetry, we take an imaginary sphere as our Gaussian surface of radius r centred on the conductor. To calculate the field outside the conductor, we take r to be greater than the radius of the conductor R; to calculate the field inside, we take r to be less than R. In either case, the point where we want to calculate $\vec{E}$ lies on Gaussian surface.

Electric Field Outside the Sphere

We first consider the field outside the conductor, so we choose $r > R$. The entire conductor is within the Gaussian surface, so the enclosed charge is q. The area of the Gaussian surface is $4\pi r^2$; $\vec{E}$ is uniform over the surface and perpendicular to it at each point. The flux integral $\oint E_\perp dA$ in Gauss's law is therefore $E(4\pi r^2)$ which gives

$$E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

or $$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \quad \text{(outside a charged conducting sphere)}$$

This expression for the field at any point outside the sphere ($r > R$) is the same as for a point charge; the field due to the charged sphere is the same as if the entire charge were concentrated at its centre. Just outside the surface of the sphere, where $r = R$, we get

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$$

(at the surface of a charged conducting sphere)

Electric Field Inside the Sphere

We know that extra charge on a conductor lies on its outer surface. So there is no charge inside the Gaussian surface, i.e., $q_{in} = 0$ (Fig. 2.24)

$$\therefore \oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0} = 0 \Rightarrow E4\pi r^2 = 0 \Rightarrow E = 0$$

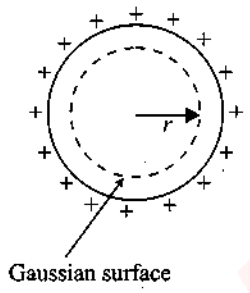


Fig. 2.24

Hence, at a point inside the sphere, electric field is zero.

FIELD OF A LINE CHARGE

Electric charge is distributed uniformly along an infinitely long, thin wire. The charge per unit length is λ (assumed positive) known as linear charge density.

Selection of Gaussian Surface

The system has cylindrical symmetry. This property suggest that we use a cylinder as a Gaussian surface with arbitrary radius r and arbitrary length l, with its ends perpendicular to the wire (Fig. 2.25). We break the surface integral for the flux Φ_E into an integral over each flat end and one over the curved side wall.

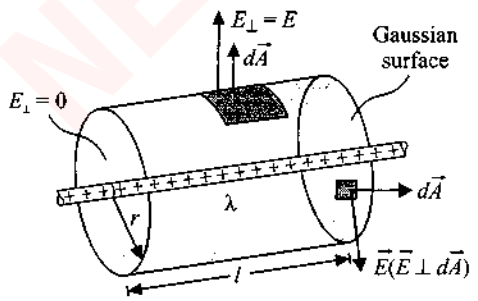


Fig. 2.25

There is no flux through the ends because $\vec{E}$ lies in the plane of the surface. To find the flux through the side walls, note that $\vec{E}$ is perpendicular to the surface at each point; by symmetry, E has the same value everywhere on the walls (curved surface). The area of the side wall is $2\pi rl$. Hence, the total flux Φ_E through the entire cylinder is the sum of the flux through the side wall, which is $(E)(2\pi rl)$, and the zero flux through the two ends. Finally, we need the total enclosed charge, which is the charge per unit length multiplied by the length of wire inside the Gaussian surface, or $Q_{encl} = \lambda l$. From Gauss's law, we get

$$\Phi_E = (E)(2\pi rl) = \frac{\lambda \ell}{\varepsilon_0} \text{ and } E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}$$

(field of an infinite line of charge)

We have assumed that λ is positive. If it is negative, $\vec{E}$ is directed radially inward towards the line of charge, and in the above expression for the field magnitude E, we must interpret λ as the magnitude (absolute value) of the charge per unit length.

FIELD OF AN INFINITE PLANE SHEET OF CHARGE

Let us consider a thin, flat, infinite sheet on which there is a uniform positive charge per unit area σ.

Selection of Gaussian Surface

To take advantage of these symmetry properties, we use a cylinder as our Gaussian surface with its axis perpendicular to the sheet of charge, with ends of area A (Fig. 2.26).

The charged sheet passes through the middle of the cylinder's length, so the cylinder is perpendicular to the surface; hence, the flux through each end is EA. Because $\vec{E}$ is perpendicular to the charged sheet, it is parallel to the curved side wall of the cylinder, and there is no flux through these wall.

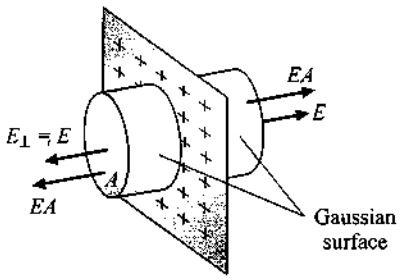


Fig. 2.26

The total flux integral in Gauss's law is then $2EA$ (EA from each end and zero from the side wall). The net charge within the Gaussian surface is the charge per unit area multiplied by the sheet area enclosed by the surface, or $Q_{encl} = \sigma A$. Hence, according to the Gauss's law

$$2EA = \frac{\sigma A}{\varepsilon_0}$$

or $$E = \frac{\sigma}{2\varepsilon_0} \text{ (field of an infinite sheet of charge)}$$

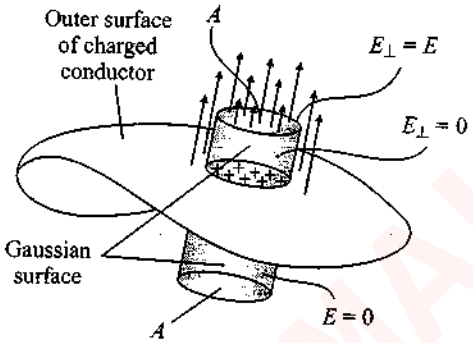
If the charge density is negative, $\vec{E}$ is directed towards the sheet, the flux through the Gaussian surface in figure is negative, and in the expression $E = \sigma/2\varepsilon_0$, σ denotes the magnitude (absolute value) of the charge density.

The assumption that the sheet is infinitely large is an idealization; nothing in nature is really infinitely large. But the

result $E = \sigma/2\varepsilon_0$ is a good approximation for points that are close to the sheet (compared to the sheet's dimensions) and not too near its edges. At such points, field is very nearly uniform and perpendicular to plane.

FIELD AT THE SURFACE OF A CONDUCTOR

To find a relation between σ at any point on the surface and the perpendicular component of the electric field at that point, we construct a Gaussian surface in the form of a small cylinder (as shown in Fig. 2.27).



The field just outside a charged conductor is perpendicular to the surface and its perpendicular component $E_\perp$ is equal to σ/ε_0.

Fig. 2.27

One end face, with area A, lies within the conductor and the other lies just outside. The electric field is zero at all points within the conductor. Outside the conductor, the component of $\vec{E}$ perpendicular to the side walls of the cylinder is zero, and over the end face the perpendicular component is equal to $E_\perp$. (If σ is positive, the electric field points out of the conductor and $E_\perp$ is positive; if σ is negative, the field points inward and $E_\perp$ is negative.) Hence, the total flux through the surface is $E_\perp A$. The charge enclosed within the Gaussian surface is σA. So from the Gauss's law, we get

$$E_\perp A = \frac{\sigma A}{\varepsilon_0}$$

or $$E_\perp = \frac{\sigma}{\varepsilon_0} \quad \text{(field at the surface of a conductor)}$$

We can check this with the results we have obtained for spherical, cylindrical and plane surfaces.

FIELD OF A UNIFORMLY CHARGED SPHERE

Positive electric charge Q is distributed uniformly throughout the volume of an insulating sphere with radius R.

Selection of Gaussian Surface

The system is spherically symmetric. To make use of this symmetry, we choose a sphere as our Gaussian surface with radius r, concentric with the charge distribution.

Electric Field Inside the Sphere

From symmetry, the magnitude E of the electric field has the same value at every point on the Gaussian surface, and the direction of $\vec{E}$ is radial at every point on the surface. Hence, the total electric flux through the Gaussian surface is the product of E and the total area of the surface $A = 4\pi r^2$, that is, $\Phi_E = 4\pi r^2 E$.

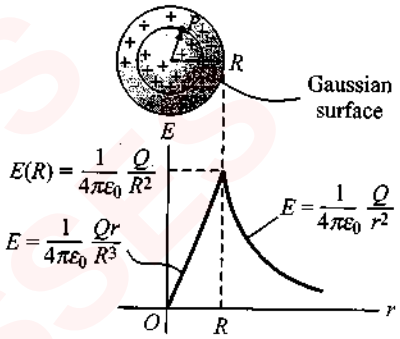


Fig. 2.28

The amount of charge enclosed within the Gaussian surface depends on the radius r. Let us first find the field magnitude inside the charged sphere of radius R; so we choose $r < R$.

The volume charge density ρ is the charge Q divided by volume of the entire charged sphere of radius R, i.e.,

$$\rho = \frac{Q}{4\pi R^3/3}$$

$$Q_{encl} = \rho V_{encl} = \left(\frac{Q}{4\pi R^3/3}\right)\left(\frac{4}{3}\pi r^3\right) = Q\frac{r^3}{R^3}$$

Then using Gauss's law, we get $\phi = \int E dA = \frac{Q_{encl}}{\varepsilon_0}$

$$E\int dA = E4\pi r^2 = \frac{Q}{\varepsilon_0}\frac{r^3}{R^3}$$

or $$E = \frac{1}{4\pi\varepsilon_0}\frac{Qr}{R^3} \quad \text{(field inside a uniformly charged sphere)}$$

The field magnitude is proportional to the distance r of the field point from the centre of the sphere (Fig. 2.28).

At the centre ($r = 0$), $E = 0$.

Electric field in terms of charge density (at inside point) is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{\left(\rho\frac{4}{3}\pi R^3\right)r}{R^3} = \frac{\rho r}{3\varepsilon_0}$$

$\Rightarrow$ $$E = \frac{\rho r}{3\varepsilon_0} \quad \text{(field inside a uniformly charged sphere)}$$

Electric Field Outside the Charged Sphere

We use a spherical Gaussian surface of radius $r > R$. This surface encloses the entire charged sphere, so $Q_{encl} = Q$ and Gauss's law gives

$$E4\pi r^2 = \frac{Q}{\varepsilon_0}$$

or $$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \quad \text{(field outside a uniformly charged sphere)}$$

For any spherically symmetric charged body, the electric field outside the body is the same; however, the entire charge is concentrated at the centre.

ELECTRIC FIELD DUE TO A LONG UNIFORMLY CHARGED CYLINDER

Consider a long uniformly charged cylinder of volumetric charge density ρ and radius R.

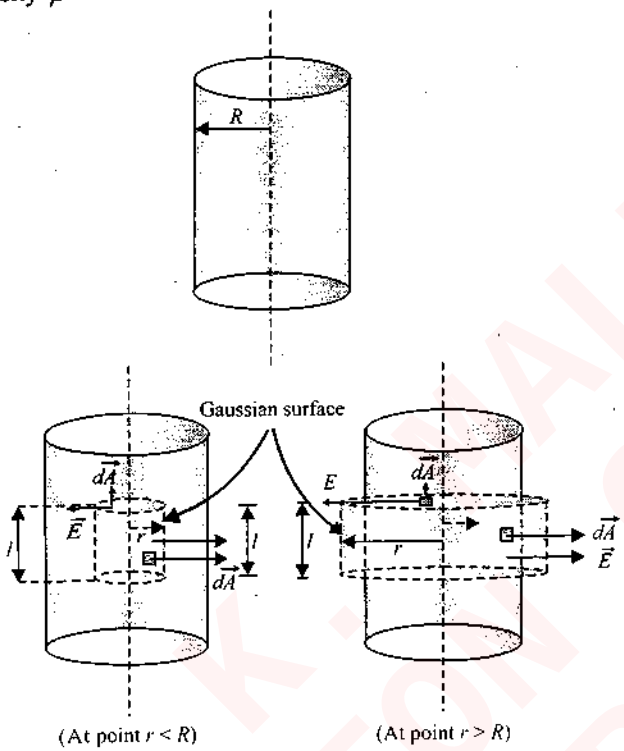


Fig. 2.29

For any point $r < R$ or $r > R$, the Gaussian surface will be cylindrical as shown in Fig. 2.29. For any point inside the cylinder ($r < R$), we get

$$E(2\pi rl) = \frac{q_{in}}{\varepsilon_0} = \frac{(\rho\pi r^2 l)}{\varepsilon_0}$$

$$E = \frac{\rho r}{2\varepsilon_0} \Rightarrow E \propto r$$

For any point outside the cylinder ($r > R$), we have

$$E(2\pi rl) = \frac{q_{in}}{\varepsilon_0} = \frac{(\rho\pi R^2 l)}{\varepsilon_0}$$

$$E = \frac{\rho R^2}{2\varepsilon_0 r} \Rightarrow E \propto \frac{1}{r}$$

Electric field inside the long uniformly charged cylinder varies lineary, i.e., $E \propto r$ and outside the cylinder the electric field varies inversely to the distance from the axis, i.e., $E \propto 1/r$ (see Fig. 2.30).

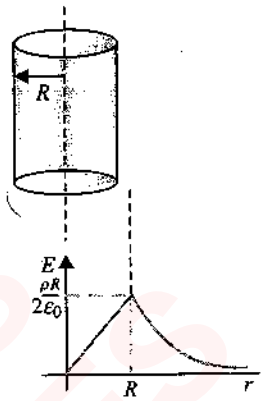


Fig. 2.30

ELECTRIC FIELD NEAR UNIFORMLY VOLUME CHARGED PLANE

Let there be charge distributed uniformly in an infinite plane of thickness d with the volume charge density ρ. Due to symmetry, the electric field will be normally away and same in magnitude at same distances from the plane of symmetry.

Field Inside the Plane

Consider the Gaussian surface of the form of a cylinder of area S and thickness $2r(<d)$ placed symmetrically (Fig. 2.31). On the curved surface, flux of electric field will be zero as the area vector is perpendicular to the field vector. On the left and right surfaces, flux is positive (outcoming flux). Hence,

$$\oint_S \vec{E}d\vec{S} = \frac{q_{enclosed}}{\varepsilon_0},$$

where $\oint_S \vec{E}d\vec{S} = \oint_{Left} \vec{E}d\vec{S} + \oint_{Right} \vec{E}d\vec{S} + 0$, $q_{enclosed} = V\rho = S2r\rho$

$$\Rightarrow \quad \oint_S \vec{E}d\vec{S} = 2ES = \frac{S(2r)\rho}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{\rho r}{\varepsilon_0} \quad \text{(field inside a uniformly charged plane)}$$

Hence, the electric field inside the plane sheet ($r < d/2$) is directly proportional to the distance of point r from the central plane.

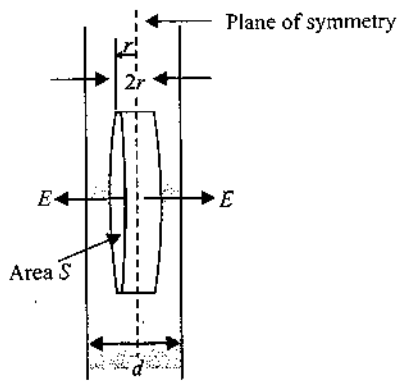


Fig. 2.31

Now, consider the Gaussian surface of the form of a cylinder of area S and thickness $2r$ ($> d$) placed symmetrically. On the curved surface, the flux of electric field will be zero as the area vector is perpendicular to the field vector. On left and right surfaces, flux is positive (outcoming flux). Hence,

$$\oint_S \vec{E}d\vec{S} = \frac{q_{\text{inclosed}}}{\varepsilon_0}$$

$$\Rightarrow \quad 2ES = \frac{Sd\rho}{\varepsilon_0}$$

$$\Rightarrow \quad E = \left(\frac{\rho}{\varepsilon_0}\right)\frac{d}{2} \text{ (field outside uniformly charged plane)}$$

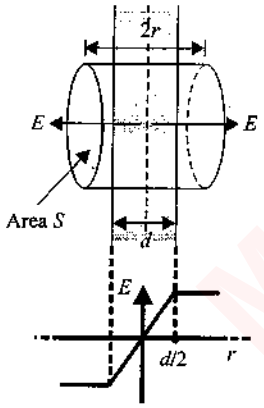


Fig. 2.32

Hence, the electric field outside the plane sheet ($r < d/2$) is constant and does not depend on the distance of point r from the central plane (see Fig. 2.32).

APPENDIX

Solid Angle

It is the cone subtended by an area at the point of interest (See Fig. 2.33). The magnitude of solid angle subtended by an area S at a point is defined as

$$\Omega = \int_S \frac{\cos\theta}{r^2} ds$$

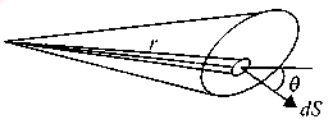


Fig. 2.33

at its centre. Therefore, total solid angle around a point in space is the solid angle subtended by entire spherical surface on its centre.

$$\Omega_0 = \frac{4\pi R^2}{R^2} = 4\pi \text{ steradian}$$

Solid Angle Subtended by a Disk at a Point on its Axis

Consider a coaxial area element of radius x and thickness dx (see Fig. 2.34).

$$dS = 2\pi x dx$$

Solid angle subtended by this element at point P is

$$d\Omega = \frac{dS\cos\theta}{(x^2+a^2)} \Rightarrow d\Omega = \frac{2\pi x dx a}{(x^2+a^2)\sqrt{a^2+x^2}}$$

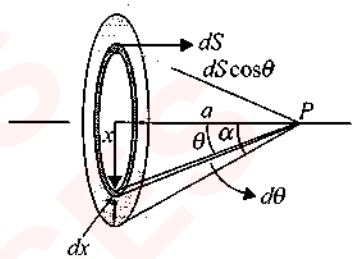


Fig. 2.34

Hence, the total solid angle subtended by the disk is

$$\Omega = \pi a \int_0^R \frac{2x dx}{(x^2+a^2)^{3/2}} \quad \Rightarrow \Omega = 2\pi a\left(-\frac{1}{\sqrt{a^2+x^2}}\right)_0^R$$

$$\Rightarrow \quad \Omega = 2\pi\left(1 - \frac{a}{\sqrt{a^2+R^2}}\right)$$

$\Omega = 2\pi(1-\cos\alpha)$; where α is the semi-vertical angle of the cone subtended by the disk at P.

Concept Application Exercise 2.2

1. A ball of charge q is placed in a hollow conducting uncharged sphere. After this, the sphere is connected with earth for a short time and the ball is then removed from the sphere. The ball has not been brought into contact with the sphere.
   a. What charge will the sphere have after these operations? Where and how will this charge be distributed?
   b. What will be the electric field inside as well as outside the sphere?
2. Figure 2.35 shows the field produced by two point charges $+q$ and $-q$ of equal magnitude but opposite signs (an electric dipole). Find the electric flux through each of the closed surfaces A, B, C and D.

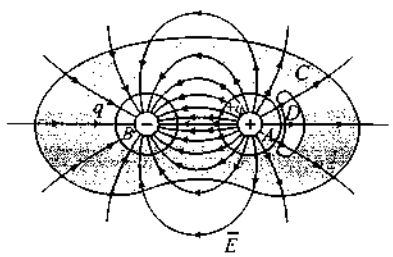
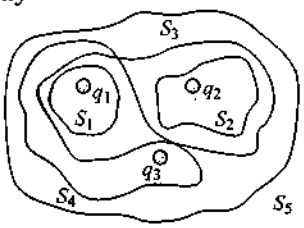


Fig. 2.35

3. The three small spheres as shown in Fig. 2.36 carry charges $q_1 = 4$ nC, $q_2 = -7.8$ nC and $q_3 = 2.4$ nC. Find the net electric flux through each of the following closed surfaces shown in cross section in the figure.
   a. S_1 b. S_2
   c. S_3 d. S_4
   e. S_5

Do your answers to parts from (a) to (e) depend on how the charge is distributed over each small sphere? Why or Why not?



Surface	What it encloses
S_1	q_1
S_2	q_2
S_3	q_1 and q_2
S_4	q_1 and q_3
S_5	q_1, q_2 and q_3

Fig. 2.36

4. A conducting sphere carrying charge Q is surrounded by a spherical conducting shell.

a. What is the net charge on the inner surface of the shell?

b. Another charge q is placed outside the shell. Now, what is the net charge on the inner surface of the shell?

c. If q is moved to a position between the shell and the sphere, what is the net charge on the inner surface of the shell?

d. Are your answers valid if the sphere and shell are not concentric?

5. A solid insulating sphere of radius a carries a net positive charge $3Q$, uniformly distributed throughout its volume. Concentric with this sphere is a conducting spherical shell with inner radius b and outer radius c and having a net charge $-Q$, as shown in Fig. 2.37.

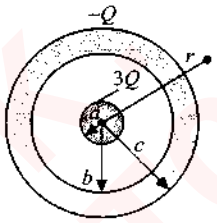


Fig. 2.37

a. Consider a spherical Gaussian surface of radius $r > c$, the net charge enclosed by this surface is _____.

b. The direction of the electric field $r > c$ is _____.

c. The electric field at $r > c$ is _____.

d. The electric field in the region with radius r, where $c > r > b$, is _____.

e. Consider a spherical Gaussian surface of radius r, where $c > r > b$, the net charge enclosed by this surface is _____.

f. Consider a spherical Gaussian surface of radius r, where $b > r > a$, the net charge enclosed by this surface is _____.

g. The electric field in the region $b > r > a$ is _____.

h. Consider a spherical Gaussian surface of radius $r < a$. Find an expression for the net charge $Q(r)$ enclosed by this surface as a function of r. Note that the charge inside this surface is less than $3Q$.

i. The electric field in the region $r < a$ is _____.

j. The charge on the inner surface of the conducting shell is _____.

k. The charge on the outer surface of the conducting shell is _____.

l. Make a plot of the magnitude of the electric field versus r.

6. A small conducting spherical shell with inner radius a and outer radius b is concentric with a larger conducting sphereical shell with inner radius c and outer radius d. The inner shell has total charge $+2q$ and the outer shell has charge $+4q$.

a. Make a plot of the magnitude of the electric field versus r.

b. Calculate the electric field (magnitude and direction in terms of q) and the distance r from the common centre of the two shell for (i) $r < a$; (ii) $a < r < b$; (iii) $b < r < c$; (iv) $c < r < d$; (v) $r > d$. Show your result in a graph with radial component of $\vec{E}$ as function of r.

c. What is the total charge on the

i. inner surface of the small shell;

ii. outer surface of the small shell;

iii. inner surface of the large shell;

iv. outer surface of the large shell?

7. Which of the following staements is/are correct?

a. Electric field calculated by Gauss law is the field due to only those charges which are enclosed inside the Gaussian surface.

b. Gauss law is applicable only when there is a symmetrical distribution of charge.

c. Electric flux through a closed surface is equal to total flux due to all the charges enclosed within that surface only.

8. Which of the following statements is correct? If $E = 0$, at all points of a closed surface

a. the electric flux through the surface is zero.

b. the total charge enclosed by the surface is zero.

9. A hollow dielectric sphere, as shown in Fig. 2.38, has inner and outer radii of R_1 and R_2, respectively. The total charge carried by the sphere is $+Q$, this charge is uniformly distributed R_1 and R_2.

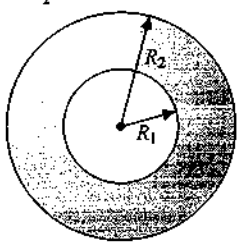


Fig. 2.38

Then,

a. the electric field for $r < R_1$ is zero. (Yes/No)

b. the electric field for $R_1 < r < R_2$ is given by _____.

c. the electric field for $r > R_2$ is given by _____.

10. A ring of diameter d is rotated in a uniform electric field until the position of maximum electric flux is found. The flux is found to be ϕ. What is the electric field strength?

11. Two infinite, non-conducting sheets of charge are parallel to each other, as shown in Fig. 2.39. The sheet on the left has a uniform surface charge density σ, and the one on the right has a uniform charge density $-\sigma$. Calculate the electric field at points (a) to the left of, (b) in between and (c) to the right of the two sheets.

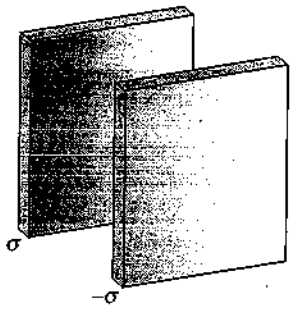


Fig. 2.39

12. S_1 and S_2 are two hollow consentric spheres enclosing charge Q and $2Q$, respectively, as shown in Fig. 2.40. What is the ratio of the electric flux leaving through the surface of S_1 and S_2?

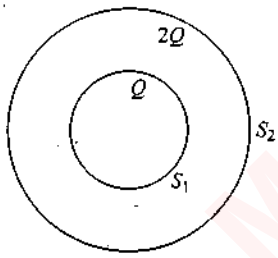


Fig. 2.40

13. A hollow half cylinder surface of radius R and lenght l is placed in a uniform electric field $\vec{E}$. Electric field is acting perpendicular on the plane $ABCD$. Find the flux through the curved surface of the hollow cylindrical surface.

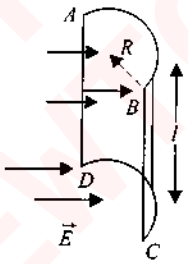


Fig. 2.41

14. Consider two concentric conducting spheres. The outer sphere is hollow and initially has a charge $-7Q$ on it. The inner sphere is solid and has a charge $+2Q$ on it.
   a. How much charge is on the outer surface and inner surface of the outer sphere.
   b. If a wire is connected between the inner and outer sphere, after electrostatic equilibrium is established how much total charge is on the outer sphere? How much charge is no the outer surface and inner surface of outer sphere? Does the electric field at the surface of the inside sphere change when the wire is connected?
   c. We return to original condition in (a). We now connect the outer sphere to ground with a wire and then disconnect it. How much total charge will be on the outer sphere? How much charge will be on the inner surface and outer surface of the outer sphere?

Solved Examples

Example 2.1 **A long, straight wire is surrounded by a hollow metal cylinder whose axis coincides with that of the wire. The wire has a charge per unit length of λ, and the cylinder has a net charge per unit length of 2λ. From this information, use Gauss's law to find (a) the charge per unit length on the inner and outer surfaces of the cylinder and (b) the electric field outside the cylinder, a distance r from the axis.**

Sol.

a. Inside surface: Consider a cylindrical surface within the metal. Since E inside the conducting shell is zero, the total charge inside the Gaussian surface must be zero, so the charge/length on the inner surface of the cylinder is $-\lambda$.

Outside surface: The total charge on the metal/cylinder is

$$2\lambda l = q_{in} + q_{out} \Rightarrow q_{out} = 2\lambda l - q_{in}, \text{ where } q_{in} = -\lambda l$$

$q_{out} = 2\lambda l + \lambda l = 3\lambda l$ so the outside charge/length is $q_{out}/l = 3\lambda$.

b. $E = \dfrac{2k(3\lambda)}{r} = \dfrac{6k\lambda}{r} = \dfrac{3\lambda}{2\pi\varepsilon_0 r}$ radially outward.

Example 2.2 **A point charge q is placed on the apex of a cone of semi-vertex angle θ. Show that the electric flux through the base of the cone is $q(1-\cos\theta)/2\varepsilon_0$.**

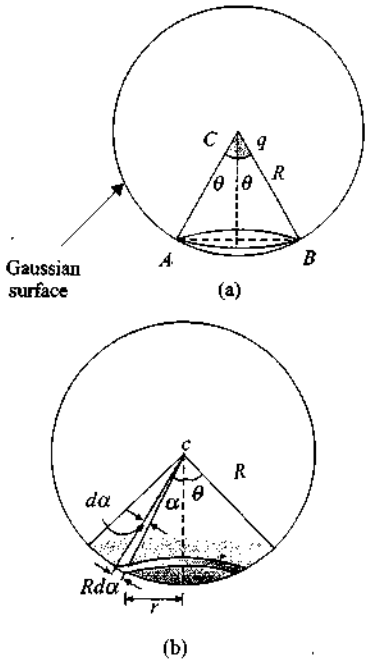


Fig. 2.42

Sol. Method 1: For point charge, Gaussian surface should be spherical. Consider a Gaussian sphere with its centre at the apex and radius the slant length of the cone. The flux through the whole sphere is q/ε_0. Therefore, the flux through the base of the cone is

$\phi_E = (A/A_0)\, q/\varepsilon_0$

Here, A_0 = area of the whole sphere $4\pi R^2$, and
A = area of the sphere below the base of the cone.

Consider a differential ring of radius r and thickness dr, then

$$dA = (2\pi r)Rd\alpha = (2\pi R\sin\alpha)R\,d\alpha \quad [\text{as } r = R\sin\alpha]$$
$$= (2\pi R^2)\sin\alpha\,d\alpha$$

$$A = \int_0^\theta (2\pi R^2)\sin\alpha\,d\alpha; \quad A = 2\pi R^2(1-\cos\theta)$$

The desired flux is $\phi_E = \left(\frac{A}{A_0}\right)\frac{q}{\varepsilon_0}$

$$= \frac{(2\pi R^2)(1-\cos\theta)}{(4\pi R^2)}\frac{q}{\varepsilon_0} = \frac{(1-\cos\theta)q}{2\varepsilon_0}$$

Method 2: Using the concept of solid angle.

The total solid angle around a point in space is 4π steradian. Solid angle subtended by the base of the cone at the apex of cone is $2\pi(1-\cos\theta)$

As the flux associated with solid angle 4π is q/ε_0, the flux associated with solid angle $2\pi = (1-\cos\theta)$ is

$$\phi = \frac{q}{\varepsilon_0}\frac{2\pi(1-\cos\theta)}{4\pi} = \frac{q(1-\cos\theta)}{2\varepsilon_0}$$

Example 2.2 **A cube of side l has one corner at the origin of coordinates and extends along the positive x-, y- and z-axes. Suppose the electric field in this region is given by $\vec{E} = (a+by)\hat{j}$. Determine the charge inside the cube. a and b are some constants.**

Sol. The faces *adhe*, *bcgf*, *cdhg*, *abfe* will contribute zero flux because the area vector is normal to electric field for these faces.

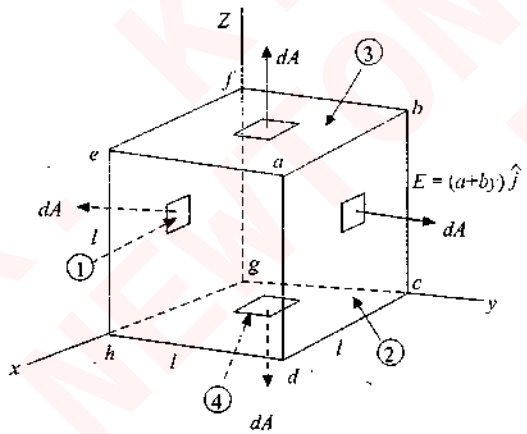


Fig. 2.43

Flux through face *efgh* is

$$\phi_1 = \int \vec{E}\cdot d\vec{A} \quad = a(\hat{j})\cdot l^2(-\hat{j}) = -al^2$$

The field at the face *efgh* (that lies in the *yz* plane, $y = 0$) is $\vec{E} = a\hat{j}$ and area vector is $l^2(-\hat{j})$ (direction outward normal)

Flux through face *abcd*: $\phi_2 = (a+bl)\hat{j}\cdot l^2\hat{j} = (al^2 + bl^3)$, for this $y = l$.

Net flux through the cube $= \phi_1 + \phi_2 = bl^3$

From Gauss's law, $\phi = \frac{Q_{\text{enclosed}}}{\varepsilon_0}$, $Q_{\text{enclosed}} = \varepsilon_0\phi_E = \varepsilon_0 bl^3$.

Example 2.3 **The electric field in a cubical volume is**

$$\vec{E} = E_0\left(1+\frac{z}{a}\right)\hat{i} + E_0\left(\frac{z}{a}\right)\hat{j}$$

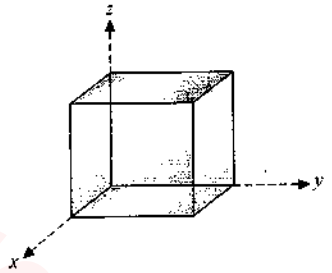


Fig. 2.44

Each edge of the cube measures d and one of the corners lies at the origin of coordinates. Determine the net charge within the cube.

Sol.

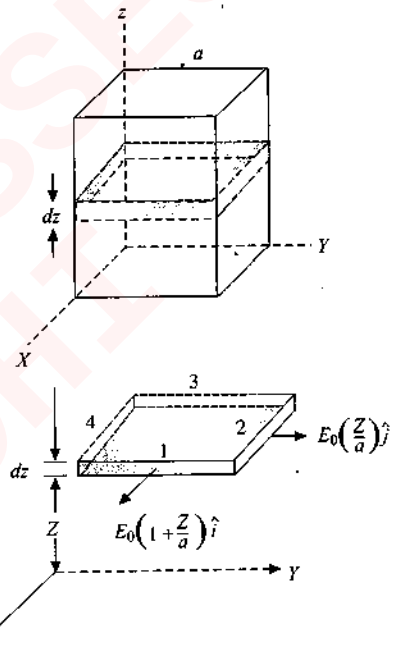


Fig. 2.45

We choose a differential slab of thickness dz at a distance z from y-axis. The electric field varies with z-coordinate only. The field components at this position have constant magnitude. Consider faces 1 and 3. Net flux due to y-component of field is zero (area vector and field vector are perpendicular) and net flux due to x-component is also zero because net flux in through face 3 is equal to net flux out through face 1. Similarly, net flux through faces 2 and 4 is also zero. Flux through each differential slab in the cube is zero. Therefore, from Gauss's law, net charge enclosed by cubical volume is zero.

Example 2.4 **Two identical metal plates, each having surface area 'A' and charge 'q_1' and 'q_2' are placed facing each other at a separation 'd'. Find the charge appearing on surface (1), (2), (3) and (4). Assume the size of the plate is much larger than the separation between the plates.**

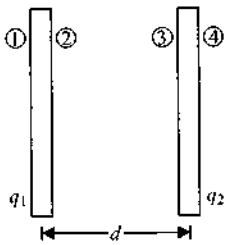


Fig. 2.46

Sol. The surfaces each other have equal and opposite charge (by Gauss theorem). Let these surfaces have the charge x and $-x$ [surface (2) and (3), respectively].

Then, the charge on the surfaces (1) and (2) should be $(q_1 - x)$ and x, respectively (by conservation of charge).

The metallic surfaces facing other always have equal and opposite charge, hence charge appearing on surfaces (3) and (4) will be $-x$ and $(q_2 + x)$, respectively.

Let us consider point P inside the left plate. Net electric field at P should be zero.

Net electric field at P will be due to the resultant of electric field due to charge appearing on all four surfaces. We get

$$E_1 + E_3 = E_2 + E_4$$

$$\Rightarrow \quad \frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_3}{2\varepsilon_0} + \frac{\sigma_2}{2\varepsilon_0} + \frac{\sigma_4}{2\varepsilon_0}$$

$$\Rightarrow \quad q_1 - x + x = q_2 + x + x$$

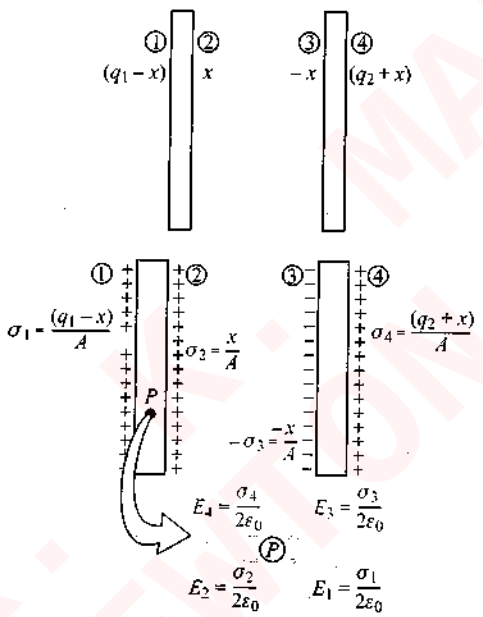


Fig. 2.47

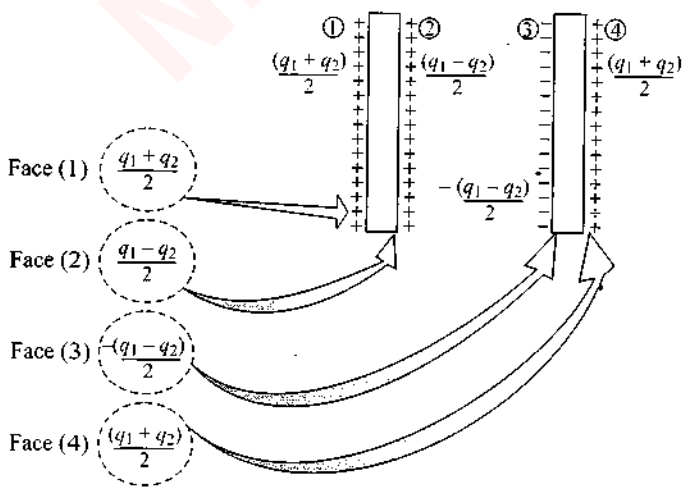


Fig. 2.48

$$\Rightarrow \quad x = \frac{q_1 - q_2}{2}$$

Hence, charges appearing on different surfaces are as shown in Fig. 2.49.

Important Note:

- Facing surfaces have the equal and opposite nature of charge with magnitude 'half the difference of the charge on different plates', i.e., $\left(\frac{q_1 - q_2}{2}\right)$ in surface (2) and $\frac{(q_2 - q_1)}{2}$ or $-\left(\frac{q_1 - q_2}{2}\right)$ in surface (3).
- Outer surfaces always have equal charges of magnitude 'half the summation of charges', i.e., $\frac{(q_1 + q_2)}{2}$ in each surface (1) and (4).
- If we have this type of charge distribution, then the electric field inside any metal plate will be zero.
- The charge appearing on the surfaces (2) and (3) is called bounded charge and the charge appearing on the surfaces (1) and (4) is called free charge.
- If we connect second plate (right plate) with ground (Fig. 2.49), the charge appearing on the surface (4) will go to the earth and charge distribution will be as shown in the figure.

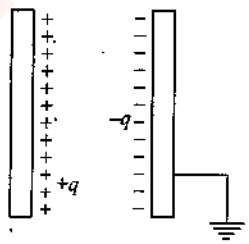


Fig. 2.49

- Any metal plate or object connected to the earth need not have zero charge. If the conducting body is isolated and connected to earth, then it will have no charge. If the conducting body connected to earth has any charged object near to it, then the body may not have zero charge.

Example 24 **A point charge $+Q$ is placed at the centre of an uncharged spherical conducting shell of inner radius a and outer radius b as shown in Fig. 2.50.**

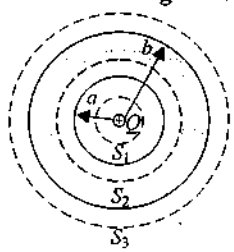


Fig. 2.50

i. Find the electric field for $r < a$.
ii. What is the magnitude and sign of the induced charge q' on the inner shell surface?
iii. What is the field for $a < r < b$?

iv. What is the electric field at points $r > b$?

v. What is the surface charge on the outer surface of the conductor?

Sol.

i. Consider a Gaussian surface S_1 of radius $r < R$ inside the cavity, centred on the charge Q. From Gauss's law, we get

$$\phi_E = \oint E dA = E(4\pi r^2) = \frac{+Q}{\varepsilon_0}$$

From which we find the electric field to be

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

This result is the same as that of a point charge in vacuum.

ii. Consider a Gaussian surface inside the conducting material, s_2. We do not know if there is a charge on the inside surface of the conductor or not. We assume that the charge is q'. Because the Gaussian surface is inside the conductor, the electric field is zero.

From Gauss's law, we get

$$\oint \vec{E} d\vec{A} = E(4\pi r^2) = \frac{Q_{enclosed}}{\varepsilon_0} = 0$$

$$\Rightarrow \quad \frac{Q_{enclosed}}{\varepsilon_0} = \frac{Q+q'}{\varepsilon_0} = 0$$

which implies $q' = -Q$

There is a charge on the inside surface of the conductor. The total charge induced on the inside surface of the cavity is the negative of the charge placed at its centre.

iii. The field inside a conductor in electrostatic equilibrium is always zero.

iv. For E $(r > b)$, consider a Gaussian surface S_3. From Gauss's law, we have

$$\phi_E = \oint E dA = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{+Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

It was stated in the problem that the conducting sphere has no net charge. Consequently, the total charge inside our Gaussian surface S_3 is sum of charge $+Q$ and induced charges $-Q$ on the inner surface of conductor and $+Q$ on the outer surface. Once more we can see that field outside the sphere is the same as for a point charge. The conducting sphere has no shielding effect at all. However, such a conducting shield does prevent electrostatic fields from charges outside the shell from entering it.

v. The conducting shell has no net charge, yet there is a surface charge $-Q$ on its inner surface. Because the net charge on the shell is zero and no charge can reside inside a conductor, there must be $+Q$ charge on the outer surface of the conductor (see Fig. 2.51).

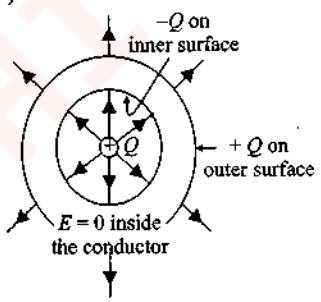


Fig. 2.51

EXERCISES

Subjective Type

Solutions on page 1.41

1. A solid spherical region, having a spherical cavity whose diameter 'R' is equal to the radius of the spherical region, has a total charge 'Q'. Find the electric field at a point P as shown.

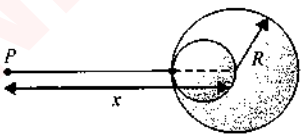


Fig. 2.52

2. A sphere of radius R has a uniform volume charge density ρ. A spherical cavity of radius b whose centre lies at $\vec{r} = \vec{a}$ is removed from the sphere.

a. Find the electric field at any point inside the spherical cavity.

b. Find the electric field outside the cavity.

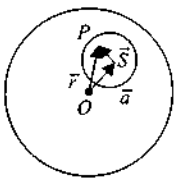


Fig. 2.53

3. A very long, solid insulating cylinder with radius R has a cylindrical hole with radius a bored along its entire length. The axis of the hole is a distance b from the axis of the cylinder, where $a < b < R$ (as shown in the figure). The solid material of the cylinder has a uniform volume charge density ρ. Find the magnitude and direction of the electric field inside the hole, and show that this is uniform over the entire hole.

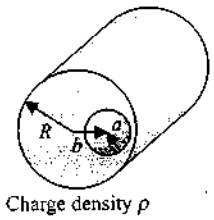


Fig. 2.54

4. An infinite wire having charge density λ passes through one of the edges of a cube having edge length l. Find the

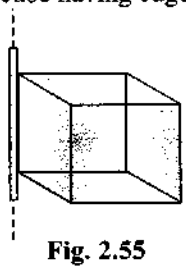


Fig. 2.55

a. total flux passing through the cube.
b. flux passing through the surfaces which are in contact with the wire.
c. flux passing through the surfaces which are not in contact with the wire.

5. It has been experimentally observed that the electric field in a large region of Earth's atmosphere is directed vertically down. At an altitude of 300 m, the electric field is 60 Vm^{-1}. At an altitude of 200 m, the field is 100 Vm^{-1}. Calculate the net amount of charge contained in the cube of 100 m edge, located between 200 and 300 m altitude.

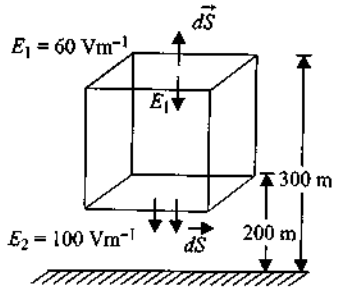


Fig. 2.56

6. A point charge Q is located on the axis of a disk of radius R at a distance b from the plane of the disk (Fig. 2.57). Show that if one-fourth of the electric flux from the charge passes through the disk, then $R = \sqrt{3}b$.

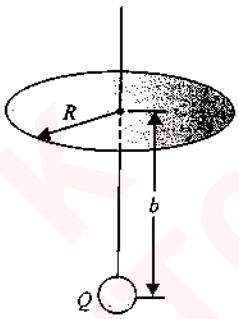


Fig. 2.57

7. A very long uniformly charged wire oriented along the axis of a circular ring of radius R rests on its centre with one of the ends (as shown in Fig. 2.58). The linear charge density on the wire is λ. Evaluate the flux of the vector $\vec{E}$ across the circle area.

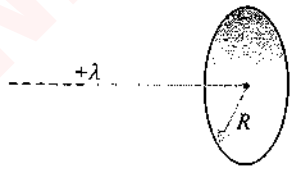


Fig. 2.58

8. Two point charges q and $-q$ are separated by a distance $2a$ (Fig. 2.59). Evaluate the flux of electric field strength vector across a circle of radius R

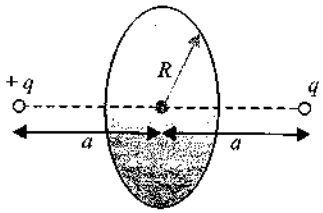


Fig. 2.59

9. An infinitely long line charge having a uniform charge per unit length λ lies at a distance d from point O as shown in Fig. 2.60. Determine the total electric flux through the surface of a sphere of radius R centred at O resulting from this line charge. Consider both cases where $R < d$ and $R > d$.

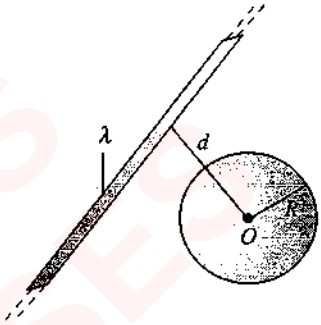


Fig. 2.60

10. Find the electric flux crossing the wire frame $ABCD$ of length l, width b and whose centre is at a distance $OP = d$ from an infinite line of charge with linear charge density λ. Consider that the plane of frame is perpendicular to the line OP (Fig. 2.61).

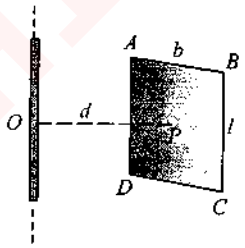


Fig. 2.61

11. A solid insulating sphere of radius R has a non-uniform charge density that varies with r according to the expression $\rho = Ar^2$, where A is a constant and $r < R$ is measured from the centre of the sphere. Show that (a) the magnitude of the electric field outside ($r > R$) the sphere is $E = AR^5/5\varepsilon_0 r^2$ and (b) the magnitude of the electric field inside ($r < R$), the sphere is $E = Ar^3/5\varepsilon_0$.

12. The electric field in a region is radially outwards with magnitude $E = \alpha r$. Calculate the charge contained in a sphere of radius R centred at the origin. Calculate the value of charge if α = 100 V/m^2 and R = 0.30 m.

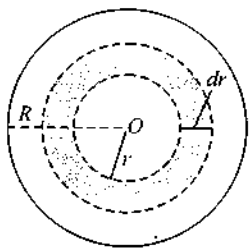


Fig. 2.62

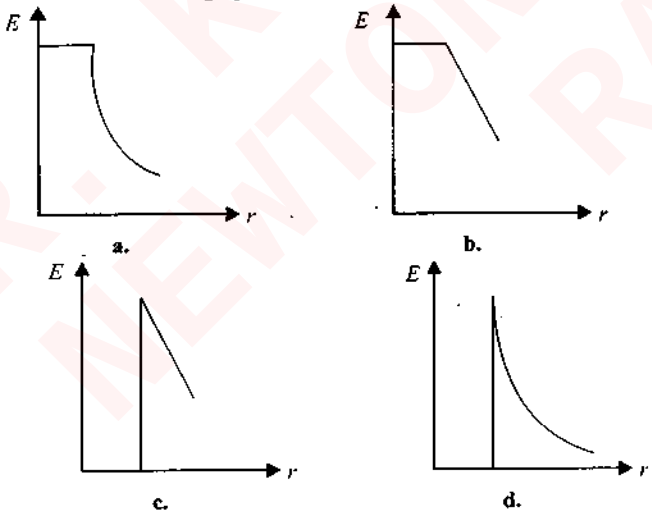
13. A system consists of a ball of radius R carrying a uniformly distributed charge q and surrounding space filled with a charge of volume density $\rho = \alpha/r$ where α is a constant and r is the distance from the centre of the ball. Find the charge on the ball for which the magnitude of electric field strength

outside the ball is constant ? The dielectric constant of ball and surrounding may be taken equal to unity.

Objective Type

Solutions on page 2.28

1. Units of electric flux are
   a. $NC^{-1}m^2$ b. JC^{-1} c. V d. Vm
2. Positive electric flux from a closed surface indicates that electric lines of force are directed
   a. outwards b. inwards
   c. outwards or inwards d. none of these
3. A surface encloses an electric dipole. The net flux through the surface is
   a. zero b. positive c. negative d. infinite
4. In a region with a uniform electric field, the number of lines of force per unit area is E. If a spherical metallic conductor is placed in the area, the field inside the conductor will be
   a. zero b. E
   c. more than E d. less than E
5. An insulated sphere of radius R has a uniform volume charge density ρ. The electric field at a point P inside the sphere at a distance r from the centre is
   a. $\frac{R\rho}{3\varepsilon_0}$ b. $\frac{r\rho}{3\varepsilon_0}$ c. zero d. $\frac{2}{3}\left(\frac{r\rho}{\varepsilon_0}\right)$
6. Which one of the following graphs shows the variation of electric field strength E with distance r from the centre of a hollow conducting sphere?



7. A cylinder of length L and radius b has its axis coincident with the x-axis. The electric field in this region is $\vec{E} = 200\hat{i}$. Find the flux through the left end of cylinder.
   a. 0 b. $200\pi b^2$ c. $100\pi b^2$ d. $-200\pi b^2$
8. Consider the Gaussian surface that surrounds part of the charge distribution shown in Fig. 2.63. Then, the contribution to the electric field at point P arises from charges
   a. q_1 and q_2 only b. q_3 and q_4 only
   c. q_1, q_2, q_3 and q_4 d. none of the above

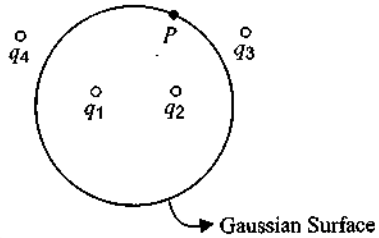


Fig. 2.63

9. Charge on an originally uncharged conductor is separated by holding a positively charged rod very closely nearby, as in Fig. 2.64. Assume that the induced negative charge on the conductor is equal to the positive charge q on the rod. Then, flux through surface S_1 is

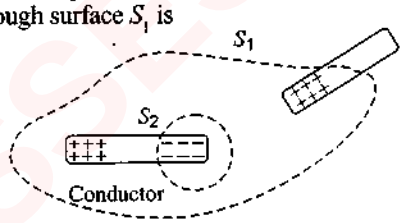


Fig. 2.64

   a. zero b. q/ε_0
   c. $-q/\varepsilon_0$ d. none of these
10. A thin metallic spherical shell contains a charge Q on its surface. A point charge q_1 is placed at the centre of the shell and another charge q_2 is placed outside the shell. All the three charges are positive. Then, the force on charge q_1 is

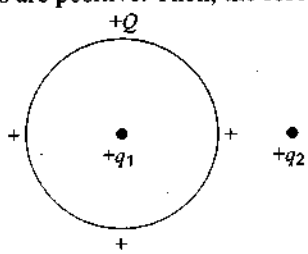
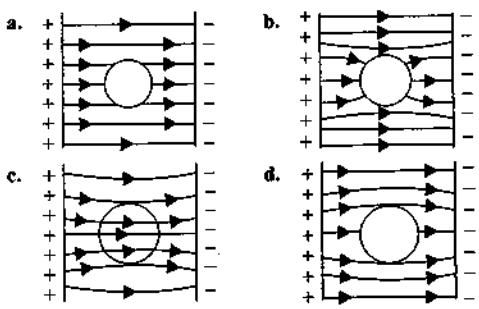


Fig. 2.65

   a. towards right b. towards left
   c. zero d. none of these
11. If one penetrates a uniformly charged spherical cloud, electric field strength
   a. decreases directly as the distance from the centre
   b. increases directly as the distance from the centre
   c. remains constant
   d. none of the above
12. An uncharged metal sphere is placed between two equal and oppositely charged metal plates. The nature of lines of force will be



13. A hollow metallic sphere of radius 10 cm is given a charge of 3.2×10^{-9}C. The electric intensity at a point 4 cm from the centre is
a. 9×10^{-9} NC^{-1} **b.** 288 NC^{-1} **c.** 2.88 NC^{-1} **d.** zero

14. The surface density on a copper sphere is σ. The electric field strength on the surface of the sphere is
a. σ **b.** $\sigma/2$ **c.** $\sigma/2\varepsilon_0$ **d.** σ/ε_0

15. A cylinder of radius R and length l is placed in a uniform electric field E parallel to the axis of the cylinder. The total flux over the curved surface of the cylinder is
a. zero **b.** πR^2E **c.** $2\pi R^2E$ **d.** $E/\pi R^2$

16. A cube of side 10 cm encloses a charge of 0.1μC at its centre. Calculate the number of lines of force through each face of the cube.
a. 1.113×10^{11} **b.** 1.13×10^4 **c.** 1.13×10^9 **d.** 1883

17. Number of electric lines of force from 0.5 C of positive charge in a dielectric medium of dielectric constant 10 is
a. 5.65×10^9 **b.** 1.13×10^{11} **c.** 9×10^9 **d.** 8.85×10^{-12}

18. The electric flux from a cube of edge l is ϕ. If edge of the cube is made $2l$ and charge enclosed is halved, its value will be
a. 4ϕ **b.** 2ϕ **c.** $\phi/2$ **d.** ϕ

19. In a certain region of space, there exists a uniform electric field of $2\times10^2\hat{k}$ Vm^{-1}. A rectangular coil of dimension 10 cm $\times$20 cm is placed in $x-y$ plane. The electric flux through the coil is
a. zero **b.** 30 Vm **c.** 40 Vm **d.** 50 Vm

20. Consider two concentric spherical surfaces S_1 with radius a and S_2 with radius $2a$, both centred on the origin. There is a charge $+q$ at the origin, and no other charges. Compare the flux ϕ_1 through S_1 with the flux ϕ_2 through S_2
a. $\phi_1=4\phi_2$ **b.** $\phi_1=2\phi_2$ **c.** $\phi_1=\phi_2$ **d.** $\phi_1=\phi_2/2$

21. Under what conditions can the electric flux ϕ_E be found through a closed surface?
a. If the magnitude of electric field is known everywhere on the surface.
b. If the total charge inside the surface is specified.
c. If the total charge outside the surface is specified.
d. Only if the location of each point charge inside the surface is specified.

22. Fig. 2.66 shows four charges q_1, q_2, q_3, q_4 fixed in space. Then, the total flux of electric field through a closed surface S, due to all charges q_1, q_2, q_3 and q_4, is

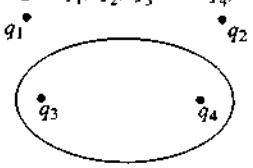


Fig. 2.66

a. not equal to the total flux through S due to q_3 and q_4
b. equal to the total flux through S due to q_3 and q_4
c. zero if $q_1+q_2=q_3+q_4$
d. twice the total flux through S due to q_3 and q_4 if $q_1+q_2=q_3+q_4$

23. If the flux of the electric field through a closed surface is zero, then
a. the electric field must be zero everywhere on the surface
b. the total charge inside the surface must be zero
c. the electric field must be uniform throughout the closed surface
d. the charge outside the surface must be zero

24. Eight charges, 1 μC, −7 μC, −4 μC, 10 μC, 2 μC, −5 μC, −3 μC and 6 μC, are situated at the eight corners of a cube of side 20 cm. A spherical surface of radius 80 cm encloses this cube. The centre of the sphere coincides with the centre of the cube. Then, the total outgoing flux from the spherical surface (in units of Vm) is
a. $36\pi\times10^3$ **b.** $684\pi\times10^3$ **c.** zero **d.** none of these

25. Three charges of $q_1=1\times10^{-6}$ C, $q_2=2\times10^{-6}$ C and $q_3=-3\times10^{-6}$ C have been placed as shown. Then, the net electric flux will be maximum for the surface

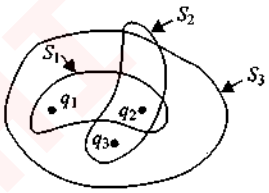


Fig. 2.67

a. S_1 **b.** S_2
c. S_3 **d.** same for all three

26. In a region of space, the electric field is given by $\vec{E}=8\hat{j}+4\hat{j}+3\hat{k}$. The electric flux through a surface of area of 100 units in $x-y$ plane is
a. 800 units **b.** 300 units **c.** 400 units **d.** 1500 units

27. A spherical shell of radius R = 1.5 cm has a charge q = 20 μC uniformly distributed over it. The force exerted by one half over the other half is
a. zero **b.** 10^{-2} N **c.** 500 N **d.** 2000 N

28. A flat, square surface with sides of length L is described by the equations

$$x=L,\ 0\le y\le L, 0\le z\le L$$

The electric flux through the square due to a positive point charge q located at the origin ($x=0, y=0, z=0$) is
a. $\frac{q}{4\varepsilon_0}$ **b.** $\frac{q}{6\varepsilon_0}$ **c.** $\frac{q}{24\varepsilon_0}$ **d.** $\frac{q}{48\varepsilon_0}$

29. The electric field $\vec{E}_1$ at one face of a parallelopiped is uniform over the entire face and is directed out of the face. At the opposite face, the electric field $\vec{E}_2$ is also uniform over the entire face and is directed into that face (as shown in Fig. 2.68). The two faces in question are inclined at 30° from the horizontal, $\vec{E}_1$ and $\vec{E}_2$ (both horizontal) have magnitudes of 2.50×10^4 N/C and 7.00×10^4 N/C, respectively. Assuming that no other electric field lines cross the surfaces of the parallelopiped, the net charge contained within is
a. $-67.5\varepsilon_0$ C **b.** $37.5\,\varepsilon_0$ C **c.** $105\,\varepsilon_0$ C **d.** $-105\,\varepsilon_0$ C

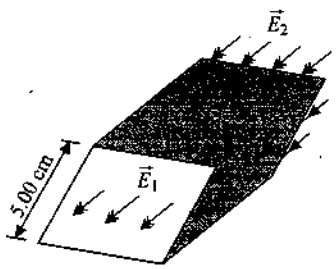


Fig. 2.68

30. A dielectric in the form of a sphere is introduced into a homogeneous electric field. A, B and C are three points as shown in Fig. 2.69.

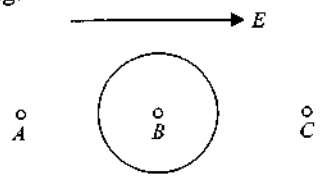


Fig. 2.69

Then,

a. intensity at A increases while that at B and C decreases

b. intensity at A and B decreases, whereas intensity at C increases

c. intensity at A and C increases and that at B decreases

d. intensity at A, B and C decreases

31. The electric field on two sides of a large charged plate is shown in Fig. 2.70. The charge density on the plate in S.I. units is given by (ε_0 is the permittivity of free space in S.I. units)

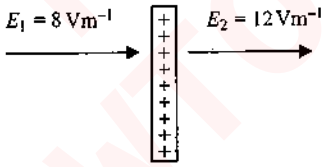


Fig. 2.70

a. $2\varepsilon_0$ **b.** $4\varepsilon_0$ **c.** $10\varepsilon_0$ **d.** zero

Multiple Correct Answers Type

Solutions on page 2.30

1. 10 C charge is given to a conducting spherical shell and a −3 C point charge is placed inside the shell. For this arrangement, find the correct statement(s).

a. The charge on the inner surface of the shell will be +3 C and it can be distributed uniformly or non-uniformly.

b. The charge on the inner surface of the shell will be +3 C and its distribution would be uniform.

c. The net charge on the outer surface of the shell will be +7 C and its distribution can be uniform or non-uniform.

d. The net charge on the outer surface of the shell will be +7 C and its distribution would be uniform.

2. Consider Gauss's law $\oint \vec{E} \cdot \vec{ds} = \frac{q}{\varepsilon_0}$.

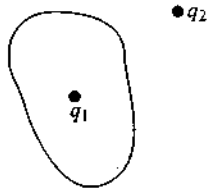


Fig. 2.71

Then, for the situation shown in Fig. 2.71 at the Gaussian surface

a. $\vec{E}$ due to q_2 would be zero.

b. $\vec{E}$ due to both q_1 and q_2 would be non-zero.

c. ϕ due to both q_1 and q_2 would be non-zero.

d. ϕ due to q_2 would be zero.

3. For Gauss's law, mark out the correct statement(s).

a. If we displace the enclosed charges (within a Gaussian surface) without crossing the boundary, then both $\vec{E}$ and ϕ remain same.

b. If we displace the enclosed charges without crossing the boundary, then $\vec{E}$ changes but ϕ remains the same.

c. If charge crosses the boundary, then both $\vec{E}$ and ϕ would change.

d. If charge crosses the boundary, then ϕ changes but $\vec{E}$ remains the same.

Assertion-Reasoning Type

Solutions on page 2.30

In the following questions, each question contains Statement I (Assertion) and Statement II (Reason). Each question has 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

a. Statement I is True, Statement II is True; Statement II is a correct explanation for Statement I.

b. Statement I is True, Statement II is True; Statement II is **NOT** a correct explanation for Statement I.

c. Statement I is True, Statement II is False.

d. Statement I is False, Statement II is True.

1. Statement I: $\vec{E}$ in outside vicinity of a conductor depends only on the local charge density σ and it is independent of the other charges present anywhere on the conductor.
Statement II: $\vec{E}$ in outside vicinity of a conductor is given by σ/ε_0.

2. Statement I: Upon displacement of charges within a closed surface, $\vec{E}$ at any point on the surface does not change.
Statement II: The flux crossing through a closed surface is independent of the location of charge within the surface.

3. Statement I: If Gaussian surface does not enclose any charge, then $\vec{E}$ at any point on the Gaussian surface must be zero.
Statement II: No net charge is enclosed by Gaussian surface, so net flux passing through the surface is zero.

4. Statement I: For the situation shown in Fig. 2.73, if we displace the charge q within the conducting shell, then nature of distribution of charge on the outer surface of the shell does not change.
Statement II: Any conducting shell divides the entire space

into two regions (inside and outside the shell), which are independent to each other in terms of electric field.

5. **Statement I:** Electric field on the surface of a conductor is more at the sharp corners.
Statement II: Surface charge density on conductor's surface is inversely proportional to the radius of curvature.

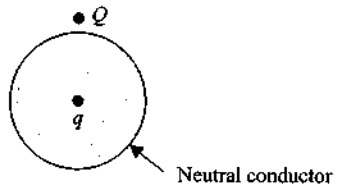


Fig. 2.72

Comprehension Type

Solutions on page 2.30

For Problems 1–4

The cube as shown in Fig. 2.73 has sides of length $L = 10.0$ cm. The electric field is uniform, has a magnitude $E = 4.00 \times 10^3$ NC^{-1}, and is parallel to the xy-plane at an angle of 37° measured from the $+x$-axis towards the $+y$-axis.

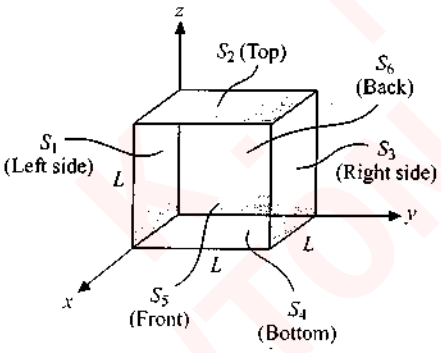


Fig. 2.73

1. The surfaces that have zero flux are
   a. S_1 and S_3 b. S_5 and S_6
   c. S_2 and S_4 d. S_1 and S_1
2. Electric flux passing through the surface S_1 is
   a. -24 Nm^{2}C^{-1} b. 24 Nm^{2}C^{-1}
   c. 32 Nm^{2}C^{-1} d. -32 Nm^{2}C^{-1}
3. Electric flux passing through the surface S_6 is
   a. -24 Nm^{2}C^{-1} b. -24 Nm^{2}C^{-1}
   c. 32 Nm^{2}C^{-1} d. -32 Nm^{2}C^{-1}
4. The total net electric flux through all faces of the cube is
   a. 8 Nm^{2}C^{-1} b. -8 Nm^{2}C^{-1}
   c. 24 Nm^{2}C^{-1} d. zero

For Problems 5–8

A cube has sides of length $L = 0.300$ m. It is placed with one corner at the origin as shown in Fig. 2.74. The electric field is not uniform, but is given by $\vec{E} = (-5.00 \text{ NC}^{-1})x\hat{i} + (3.00 \text{ NC}^{-1})z\hat{k}$.

5. The surfaces that have zero flux are
   a. S_2, S_4 and S_5 b. S_1, S_3, S_4 and S_6
   c. S_1, S_2 and S_3 d. S_2, S_3 and S_4
6. The flux passing through the surface S_5 will be
   a. -0.135 Nm^{2}C^{-1} b. -0.054 Nm^{2}C^{-1}
   c. 0.081 Nm^{2}C^{-1} d. 0.054 Nm^{2}C^{-1}
7. The total flux passing through the cube is
   a. -0.135 Nm^{2}C^{-1} b. -0.054 Nm^{2}C^{-1}
   c. 0.081 Nm^{2}C^{-1} d. zero
8. The total electric charge inside the cube is
   a. $-0.054\ \varepsilon_0$ C b. $0.081\ \varepsilon_0$ C
   c. $0.135\ \varepsilon_0$ C d. $0.054\ \varepsilon_0$ C

For Problems 9–10

A cube has sides of length L. It is placed with one corner at the origin as shown in Fig. 2.74. The electric field is uniform and given by $\vec{E} = -B\hat{i} + C\hat{j} - D\hat{k}$, where B, C and D are positive constants.

9. The flux passing through the different surfaces (match the table) is

Surface	Flux
(i) S_1	(m) BL^2
(ii) S_2	(n) $-BL^2$
(iii) S_3	(o) CL^2
(iv) S_4	(p) $-CL^2$
(v) S_5	(q) DL^2
(vi) S_6	(r) $-DL^2$

   a. (i, p), (ii, r), (iii, o), (iv, q), (v, n), (vi, m)
   b. (i, r), (ii, p), (iii, q), (iv, n), (v, m), (vi, o)
   c. (i, m), (ii, n), (iii, o), (iv, p), (v, q), (vi, r)
   d. (i, r), (ii, q), (iii, p), (iv, o), (v, n), (vi, m)
10. The total flux passing through the cube is
   a. $(B + C + D)\,L^2$ b. $2(B + C + D)\,L^2$
   c. $6(B + C + D)\,L^2$ d. zero

For Problems 11–12

A cube of side a is placed such that the nearest face which is parallel to the $y-z$ plane is at a distance 'a' from the origin. The electric field components are $E_x = \alpha x^{1/2}, E_y = E_z = 0$.

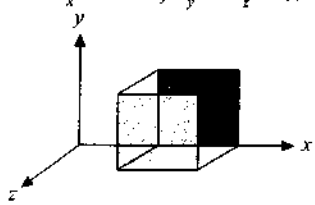


Fig. 2.74

11. The flux ϕ_E through the cube is
   a. $\sqrt{2}\alpha a^{5/2}$ b. $-\alpha a^{5/2}$
   c. $(\sqrt{2} - 1)\,\alpha a^{5/2}$ d. zero
12. The charge within the cube is
   a. $\sqrt{2}\alpha\varepsilon_0 a^{5/2}$ b. $-\alpha\varepsilon_0 a^{5/2}$
   c. $(\sqrt{2} - 1)\,\alpha\varepsilon_0 a^{5/2}$ d. zero

For Problems 13–14

A hollow conducting ball has a single positive charge $+q$ fixed at the centre. The ball has no net charge.

13. The charge on the inner surface of the ball is
   a. $+2q$ b. $+q$ c. $-q$ d. 0

14. The charge on the outer surface of the ball is
a. $+2q$ b. $+q$ c. $-q$ d. 0

For Problems 15–16

Suppose that a net charge $+q$ is placed on the ball in the previous question; the point charge remains at its centre.

15. The charge on the inner surface of the ball is
a. $+2q$ b. $+q$ c. $-q$ d. 0

16. The charge on the outer surface of the ball is
a. $+2q$ b. $+q$ c. $-q$ d. 0

For Problems 17–18

The positive charge at the centre of the ball in Problems 13–14 is moved off centre and closer to the inner surface, but it does not touch the inner surface.

17. The total charge on the inner surface of the ball will
a. increase
b. decrease
c. remain the same
d. change, depending on how close the ball gets to the inner surface

18. The total charge on the outer surface of the ball will
a. increase
b. decrease
c. remain the same
d. change, depending on how close the ball gets to the inner surface

For Problems 19–24

A small conducting spherical shell with inner radius a and outer radius b is concentric with a larger conducting spherical shell with inner radius c and outer radius d (as shown in Fig. 2.75). The inner shell has total charge $+2q$ and the outer shell has charge $+4q$. Calculate the electric field in terms of q and the distance r from the common centre of the two shells for:

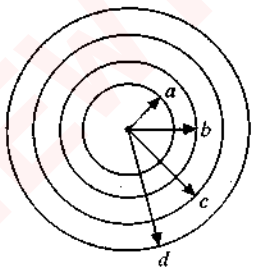


Fig. 2.75

19. $r < a$
a. zero
b. $\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$
c. $-\frac{1}{4\pi\varepsilon_0}\frac{6q}{r^2}$
d. $\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

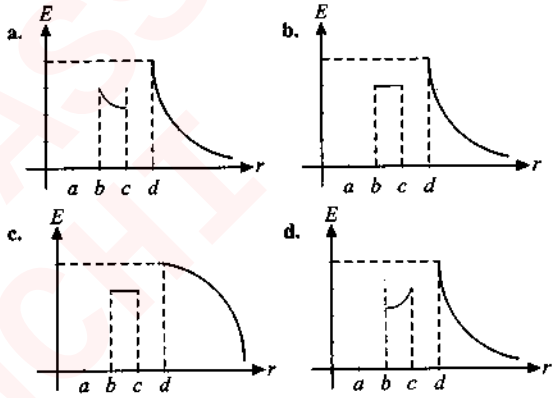
20. $a < r < b$
a. zero
b. $\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$
c. $-\frac{1}{4\pi\varepsilon_0}\frac{6q}{r^2}$
d. $\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

21. $b < r < c$
a. zero
b. $\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$
c. $-\frac{1}{4\pi\varepsilon_0}\frac{6q}{r^2}$
d. $\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

22. $c < r < d$
a. zero
b. $\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$
c. $\frac{1}{4\pi\varepsilon_0}\frac{6q}{r^2}$
d. $\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

23. $r > d$
a. zero
b. $\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$
c. $\frac{1}{4\pi\varepsilon_0}\frac{6q}{r^2}$
d. $\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

24. The graph of the radial component of E as a function of r will be



For Problems 25–28

According to problems the total charge on the

25. inner surface of the small shell is
a. zero b. $2q$ c. $-2q$ d. $6q$

26. outer surface of the small shell is
a. zero b. $2q$ c. $-2q$ d. $6q$

27. inner surface of the large shell is
a. zero b. $2q$ c. $-2q$ d. $6q$

28. outer surface of the large shell is
a. zero b. $2q$ c. $-2q$ d. $6q$

For Problems 29–34

Consider the previous problem, let the outer shell have charge $-4q$. As in the above problem, the inner shell has charge $+2q$. Calculate the electric field in terms of q and the distance r from the common centre of the two shells for:

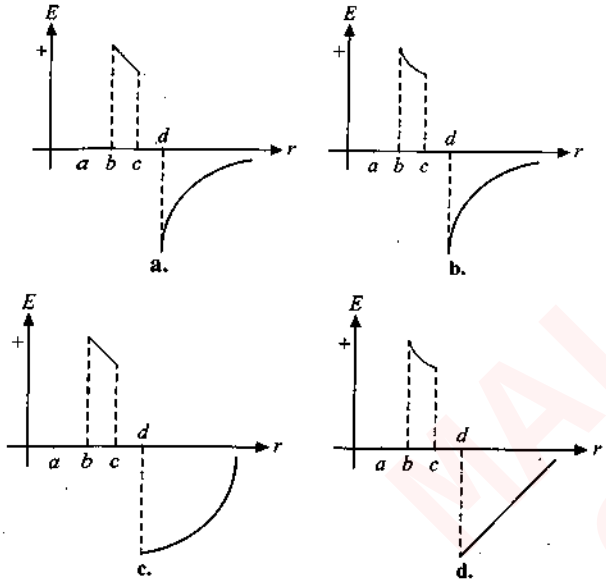
29. $r < a$
a. zero
b. $\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$
c. $-\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$
d. $\frac{1}{4\pi\varepsilon_0}\frac{6q}{r^2}$

30. $a < r < b$
a. zero
b. $\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$
c. $-\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$
d. $\frac{1}{4\pi\varepsilon_0}\frac{6q}{r^2}$

31. $b < r < c$

a. zero
b. $\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$
c. $-\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$
d. $\frac{1}{4\pi\varepsilon_0}\frac{6q}{r^2}$

32. The graph of the radial component of E as a function of r will be



33. $c < r < d$

a. zero
b. $\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$
c. $-\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$
d. $\frac{1}{4\pi\varepsilon_0}\frac{6q}{r^2}$

34. $r > d$

a. zero
b. $\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$
c. $-\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$
d. $\frac{1}{4\pi\varepsilon_0}\frac{6q}{r^2}$

For Problems 35–38

According to problems what is the total charge on the

35. inner surface of the small shell?
a. zero b. $2q$ c. $-2q$ d. $6q$

36. outer surface of the small shell?
a. zero b. $2q$ c. $-2q$ d. $6q$

37. inner surface of the large shell?
a. zero b. $2q$ c. $-2q$ d. $6q$

38. outer surface of the large shell?
a. zero b. $2q$ c. $-2q$ d. $6q$

For Problems 39–44

Consider the above problem, let the outer shell have charge $-2q$. As in the above problem, the inner shell has charge $+2q$. Calculate the electric field in terms of q and the distance r from the common centre of the two shells for:

39. $r < a$

a. zero
b. $\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$
c. $-\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$
d. $\frac{1}{4\pi\varepsilon_0}\frac{6q}{r^2}$

40. $a < r < b$

a. zero
b. $\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$
c. $-\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$
d. $\frac{1}{4\pi\varepsilon_0}\frac{6q}{r^2}$

41. $b < r < c$

a. zero
b. $\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$
c. $-\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$
d. $\frac{1}{4\pi\varepsilon_0}\frac{6q}{r^2}$

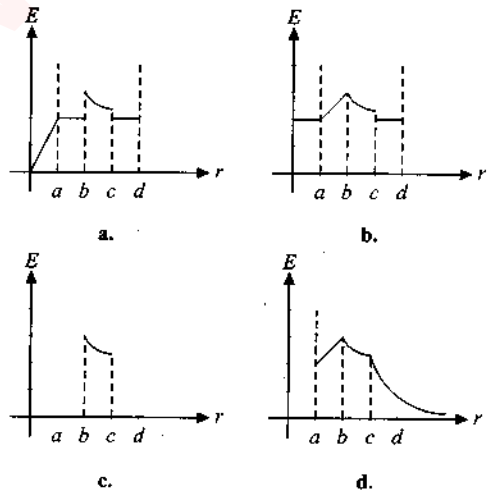
42. $c < r < d$

a. zero
b. $\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$
c. $-\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$
d. $\frac{1}{4\pi\varepsilon_0}\frac{6q}{r^2}$

43. $r > d$

a. zero
b. $\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$
c. $-\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$
d. $\frac{1}{4\pi\varepsilon_0}\frac{6q}{r^2}$

44. The graph of the radial component of E as a function of r will be



For Problems 45–48

In the previous problem, the total charge on the

45. inner surface of the small shell is
a. zero b. $2q$ c. $-2q$ d. $4q$

46. outer surface of the small shell is
a. zero b. $2q$ c. $-2q$ d. $4q$

47. inner surface of the large shell is
a. zero b. $2q$ c. $-2q$ d. $4q$

48. outer surface of the large shell is
a. zero b. $2q$ c. $-2q$ d. $4q$

For Problems 49–51

Two spherical cavities of radii a and b are hollowed out from the interior of a neutral conducting sphere of radius R. At the centre

of each cavity, a point charge is placed. Call these charges q_a and q_b.

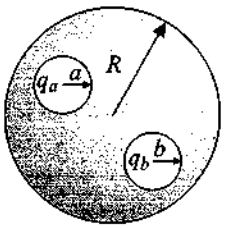


Fig. 2.76

49. Match the table

(i) σ_a	(m) $\frac{q_a+q_b}{4\pi R^2}$
(ii) σ_b	(n) $\frac{-q_a}{4\pi a^2}$
(iii) σ_R	(o) $\frac{-q_b}{4\pi b^2}$

a. (i, o), (ii, n), (iii, m) **b.** (i, n), (ii, o), (iii, m)
c. (i, m), (ii, o), (iii, n) **d.** (i, n), (ii, m), (iii, o)

50. The electric field at a distance r outside the conductor is

a. $\frac{1}{4\pi\varepsilon_0}\frac{q_b}{r^2}$ **b.** zero

c. $\frac{1}{4\pi\varepsilon_0}\frac{q_a+q_b}{r^2}$ **d.** $\frac{1}{4\pi\varepsilon_0}\frac{q_a}{r^2}$

51. The electric field inside the cavity of radius a at a distance r from the centre of cavity is

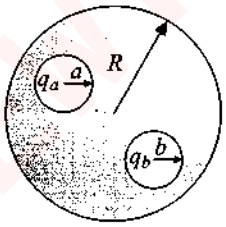


Fig. 2.77

a. $\frac{1}{4\pi\varepsilon_0}\frac{q_a}{r^2}$ **b.** $-\frac{1}{4\pi\varepsilon_0}\frac{q_a}{r^2}$

c. $\frac{1}{4\pi\varepsilon_0}\frac{q_a+q_b}{r^2}$ **d.** zero

Matching Column Type

Solutions on page 2.32

1. Column I specifies a point at distance r from the centre/axis of a symmetrical distribution of charge. Column II gives the variation of electric intensity at P as a function of r. Match the columns.

Column I	Column II
i. P lies outside a cylinder having uniform volume charge density.	**a.** $E \propto \frac{1}{r^2}$
ii. P lies inside a spherical charged conductor.	**b.** $E \propto \frac{1}{r}$
iii. P lies inside a spherical body having uniform volume charge density.	**c.** $E \propto r$
iv. P lies inside a solid cylinder having uniform volume charge density.	**d.** $E \propto r^0$
v. P lies inside a plane infinite sheet of some thickness. The sheet is charged uniformly throughout its volume.	
vi. P lies outside the plane sheet mentioned in part(v).	

2. Three identical metal plates with large surface areas are kept plate parallel to each other as shown in Fig. 2.78. The leftmost plate is given charge Q, the rightmost given charge $-2Q$ and the middle one remains neutral. Then

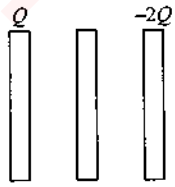


Fig. 2.78

Column I	Column II
i. The charge appearing on outer surface of the rightmost plate	**a.** $+\frac{Q}{2}$
ii. The charge appearing on outer surface of the leftmost plate	**b.** $-\frac{Q}{2}$
iii. The charge appearing on left surface of the middle plate	**c.** $-\frac{3Q}{2}$
iv. The charge appearing on right surface of the middle plate	**d.** $\frac{3Q}{2}$

3. Electric field due to

Column I	Column II
i. Infinite plane sheet of charge	**a.** 0
ii. Infinite conducting plane sheet of uniform thickness	**b.** $\frac{\sigma}{2\varepsilon_0}$
iii. Non-conducting charged solid sphere at its surface	**c.** $\frac{R\rho}{3\varepsilon_0}$
iv. Non-conducting charged solid sphere at its centre	**d.** $\frac{\sigma}{\varepsilon_0}$

where symbols have their usual meaning.

Integer Answer Type

Solutions on page 2.33

1. The figure shows three infinite non-conducting plates of charge perpendicular to the plane of the paper with charge per unit area $+\sigma$, $+2\sigma$ and $-\sigma$.

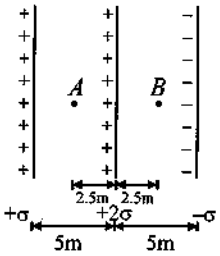


Fig. 2.79

Find the ratio of the net electric field at point A to that at point B.

2. The dielectric strength of air is 3.0×10^6 N/C. Find the largest charge (in nC) that a metallic sphere of 3 mm radius can hold.

3. A particle having a charge of $q = 8.85\ \mu$C is placed on the axis of a circular ring of radius $R = 30$ cm. Distance of the particle from centre of the ring is $a = 40$ cm. Calculate the electric flux passing through the ring [in 10^5 N/C].

4. A solid sphere of radius R has a charge Q distributed in its volume with a charge density $\rho = kr^a$, where k and a are constants and r is the distance from its centre. If the electric field at $r = R/2$ is 1/8 times that at $r = R$, find the value of a.

5 A cube of side $l = 1$ m has one corner at the origin of coordinates and extends along the positive x, y and z-axes. Suppose the electric field in this region is given by $\vec{E} = (2+3y)\hat{j}$. The charge inside the cube is $W\varepsilon_0$. Find W.

ANSWERS AND SOLUTIONS

Subjective Type

1. Volumetric charge density of given structure,

$$\rho = \frac{Q}{\frac{4}{3}\pi R^3 - \frac{4}{3}\pi\left(\frac{R}{2}\right)^3} \Rightarrow \rho = \frac{8}{7}\frac{Q}{\frac{4}{3}\pi R^3}$$

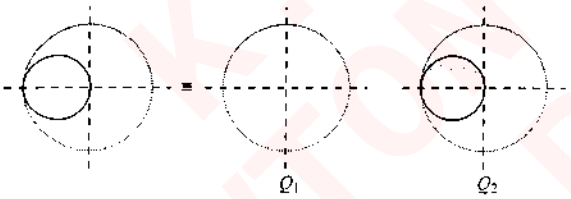


Fig. 2.80

$$Q_1 = \rho\frac{4}{3}\pi R^3 = \frac{8}{7}Q,\quad Q_2 = \rho\frac{4}{3}\pi\left(\frac{R}{2}\right)^3 = \frac{1}{7}Q$$

$$E_p = E_1 - E_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{x^2} - \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{\left(x-\frac{R}{2}\right)^2}$$

$$\Rightarrow \qquad E_p = \frac{Q}{28\pi\varepsilon_0}\left[\frac{8}{x^2} - \frac{1}{\left(x-\frac{R}{2}\right)^2}\right]$$

2. a. The field within the cavity or outside is the superposition of the field due to the original uncut sphere, plus the field due to a sphere of the size of the cavity but with a uniform negative charge density. The effective charge distribution is composed of a uniformly charged sphere of radius r, charge density ρ, superposed on it, a charge density $-\rho$ filling the cavity. Electric field $\vec{E}_1$ caused by the charge distribution $+\rho$ at a point $\vec{r}$; inside the spherical cavity.

$\vec{E}_1 = \frac{\rho r}{3\varepsilon_0}\hat{r} = \frac{\rho\vec{r}}{3\varepsilon_0}$; where $\hat{r}$ is a unit vector in radial direction.

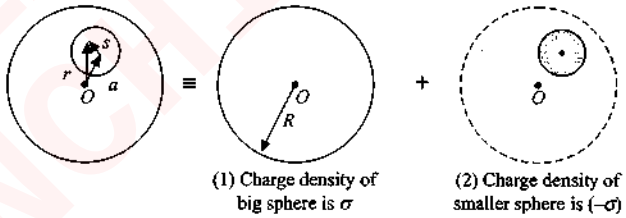


Fig. 2.81

Similarly, the electric field $\vec{E}_2$ formed by the charge density $-\rho$ inside the cavity is

$$\vec{E}_2 = \frac{(-\rho)\vec{s}}{3\varepsilon_0};\ \vec{s} = \vec{r} - \vec{a}$$

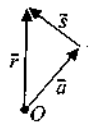


Fig. 2.82

$\vec{s}$ is the radius vector from cavity centre to the point P.

$$\therefore \qquad \vec{E}_2 = \frac{-\rho(\vec{r}-\vec{a})}{3\varepsilon_0}$$

The resultant electric field inside the cavity is, therefore, given by the superposition of $\vec{E}_1$ and $\vec{E}_2$, i.e.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\rho\vec{r}}{3\varepsilon_0} + \left[\frac{-\rho(\vec{r}-\vec{a})}{3\varepsilon_0}\right]$$

$$= +\frac{\rho\vec{a}}{3\varepsilon_0} = \text{constant}$$

$$\Rightarrow \qquad \vec{E} = \frac{\rho\vec{a}}{3\varepsilon_0}$$

b. (i) Electric field at points inside the large sphere but outside the cavity,

$$\vec{E} = \frac{\rho \vec{r}}{3\varepsilon_0}$$

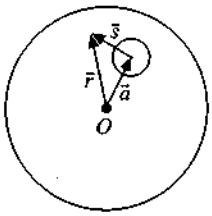


Fig. 2.83

and $\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q\vec{s}}{s^3} = \frac{\left(-\frac{4}{3}\pi\rho b^3\right)(\vec{r}-\vec{a})}{4\pi\varepsilon_0 |\vec{r}-\vec{a}|^3}$

The resultant electric field is, therefore,

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\rho}{3\varepsilon_0}\left[\vec{r} - \left(\frac{b}{|\vec{r}-\vec{a}|}\right)(\vec{r}-\vec{a})\right]$$

(ii)

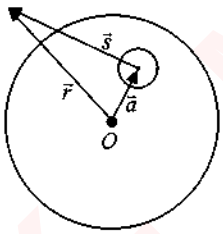


Fig. 2.84

Electric field at points outside the large sphere,

$$\vec{E}_1 = \frac{Q_{total}}{4\pi\varepsilon_0 r^3}\vec{r} = \frac{\left(\frac{4}{3}\pi R^3\rho\right)}{4\pi\varepsilon_0 r^3}\vec{r} = \frac{R^3\rho}{3\varepsilon_0 r^3}\vec{r}$$

$$\vec{E}_2 = \frac{q}{4\pi\varepsilon_0 r^3}\vec{r} = \frac{\left(-\frac{4}{3}\pi b^3\rho\right)}{4\pi\varepsilon_0(|\vec{r}-\vec{a}|)^3}(\vec{r}-\vec{a})$$

$$= \frac{-\rho b^3}{3\varepsilon_0(|\vec{r}-\vec{a}|)^3}(\vec{r}-\vec{a})$$

The resultant electric field,

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\rho}{3\varepsilon_0}\left[\left(\frac{R}{r}\right)^3\vec{r} - \left(\frac{b}{|\vec{r}-\vec{a}|}\right)^3(\vec{r}-\vec{a})\right]$$

$$E(\vec{r}) = \begin{cases} \frac{\rho\vec{a}}{3\varepsilon_0} \text{ Electric field inside the cavity} \\ \frac{\rho}{3\varepsilon_0}\left[\vec{r} - \left(\frac{b}{|\vec{r}-\vec{a}|}\right)^3(\vec{r}-\vec{a})\right] \\ \text{Electric field outside the cavity but inside the large sphere} \\ \frac{\rho}{3\varepsilon_0}\left[\left(\frac{R}{r}\right)^3\vec{r} - \left(\frac{b}{|\vec{r}-\vec{a}|}\right)^3(\vec{r}-\vec{a})\right] \\ \text{Electric field outside the large sphere} \end{cases}$$

3. $\vec{E}_{whole} = \vec{E}_{cylinder} - \vec{E}_{hole}$

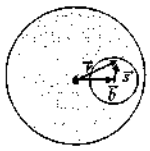


Fig. 2.85

$$\vec{E}_{whole} = \frac{\rho\vec{r}}{2\varepsilon_0} - \frac{\rho\vec{s}}{2\varepsilon_0} = \frac{\rho}{2\varepsilon_0}(\vec{r}-\vec{s}) = \frac{\rho}{2\varepsilon_0}\vec{b}$$

Note that $\vec{E}_{whole}$ is uniform within the hole.

4. a. If we construct three more cubes as shown in Fig. 2.86 (by dotted line), then

the net flux through all the four cubes is $\frac{q_{in}}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0}$

So, flux through any one cube $= \frac{\lambda l}{4\varepsilon_0}$

b. Field lines will be parallel to the surfaces in contact with the wire. Hence, the flux through these surfaces will be zero. There are four such surfaces.

c. There are two surfaces which are not in contact with the wire. So, the flux $\lambda l/4\varepsilon_0$ will be divided among these two surfaces. So, the flux through each surface is $\lambda l/8\varepsilon_0$.

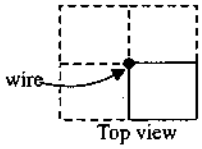


Fig. 2.86

5. $\phi_E = \int_{Bottom} \vec{E}_1 d\vec{S} + \int_{Top} \vec{E}_2 d\vec{S} + 4\int_{Vertical} \vec{E} d\vec{S}$

$$= \int_{Bottom} E_1 dS\cos 0° + \int_{Top} E_2 dS\cos 180° + 4\int_{Vertical} E dS\cos 90°$$

$$= \int_{Bottom} E_1 dS - \int_{Top} E_2 dS = E_1 S - E_2 S = (E_1 - E_2)S$$

Also, $\phi_E = \frac{q}{\varepsilon_0}$

$$q = \varepsilon_0(E_1 - E_2)S$$

$$= (8.85\times10^{-12}\ C^2N^{-1}m^{-2})$$

$$\times(100\ Vm^{-1} - 60\ Vm^{-1})(100\ m)^2$$

$$= 3.54\times10^{-6}\ C$$

6. Solid angle subtended by disk at Q,

$$\Omega = 2\pi(1-\cos\alpha)$$

$$= 2\pi\left[1 - \frac{b}{\sqrt{b^2+R^2}}\right]$$

Flux through disk $= \dfrac{Q\Omega}{\varepsilon_0 4\pi}$

It is given to be $\dfrac{Q}{4\varepsilon_0}$

So, $\dfrac{Q}{4\varepsilon_0} = \dfrac{Q\Omega}{\varepsilon_0 4\pi}$.

Solve to get $R = \sqrt{3}b$.

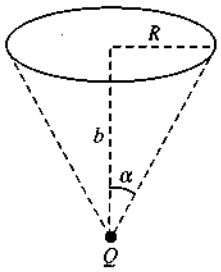


Fig. 2.87

7. The component of electric field E_y will not contribute to flux as angle between E_y and $\vec{A}$ is 90°. Hence, flux due to E_x:

$d\phi = E_x 2\pi y dy$

$$\phi = \int d\phi = \int_0^R \frac{\lambda}{4\pi\varepsilon y} 2\pi y dy$$

$$= \frac{\lambda}{2\varepsilon_0}\int_0^R dy = \frac{\lambda R}{2\varepsilon_0}$$

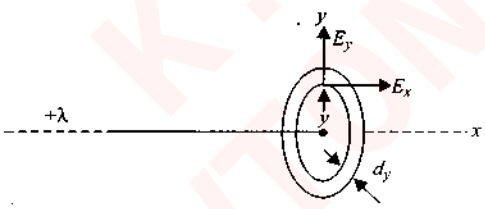


Fig. 2.88

8. $d\phi = 2E\cos\theta 2\pi y dy$

$$= 2\frac{1}{4\pi\varepsilon_0}\frac{q}{(a^2+y^2)}\frac{a\,2\pi y dy}{(a^2+y^2)^{1/2}}$$

$$\phi = \int d\phi = \frac{qa}{\varepsilon_0}\int_0^R \frac{y dy}{(a^2+y^2)^{3/2}}$$

$$= \frac{q}{\varepsilon_0}\left[1 - \frac{1}{\sqrt{1+(R/a)^2}}\right]$$

9. For $R < d$, the flux will be zero. It is because there is no change line inside the sphere (Fig. 2.89).

For $R > d$, the length inside the sphere $= 2(\sqrt{R^2 - d^2})$ (Fig. 2.90).

So, the charge inside $= 2\lambda\sqrt{R^2 - d^2}$

So, $\phi = \dfrac{2\lambda\sqrt{R^2 - d^2}}{\varepsilon_0}$

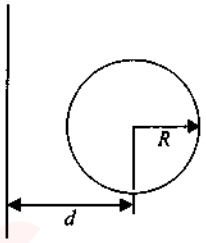


Fig. 2.89

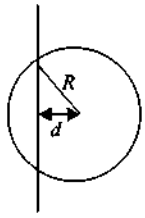


Fig. 2.90

10. The flux passing through the strip of area dA is

$$d\phi = EdA\cos\theta$$

$$= \frac{\lambda}{2\pi\varepsilon_0\sqrt{d^2+x^2}}(l\,dx)\frac{d}{\sqrt{d^2+x^2}}$$

$$= \frac{\lambda dl}{2\pi\varepsilon_0}\frac{dx}{(d^2+x^2)}$$

$$\therefore \quad \phi = \int d\phi = \frac{\lambda dl}{2\pi\varepsilon_0}\int_{-b/2}^{b/2}\frac{dx}{d^2+x^2}$$

$$= \frac{\lambda dl}{2\pi\varepsilon_0}\frac{1}{d}\left[\tan^{-1}\frac{x}{d}\right]_{-b/2}^{b/2}$$

$$= \frac{\lambda l}{2\pi\varepsilon_0}\times 2\tan^{-1}\left(\frac{b}{2d}\right)$$

$$= \frac{\lambda l}{\pi\varepsilon_0}\tan^{-1}\left(\frac{b}{2d}\right)$$

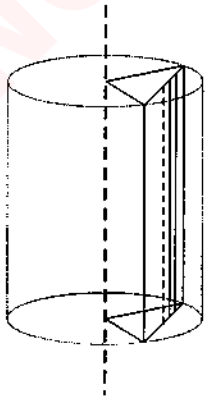


Fig. 2.91

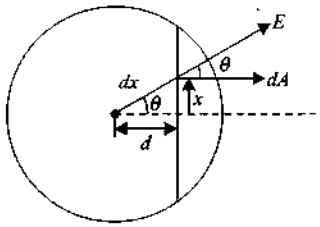


Fig. 2.92

Alternative Method:

The number of lines passing through a surface is proportional to the flux passing through the surface. The number of lines passing through the plane surface $ABCD$ is equal to the number of lines passing through the curved surface.

The amount of flux passing through the plane surface is equal to the line flux passing through the curved surface.

From Fig. 2.93, $\tan\dfrac{\theta}{2} = \dfrac{b}{2d} \Rightarrow \theta = 2\tan^{-1}\left(\dfrac{b}{2d}\right)$

2π angle corresponds to flux $\left(\dfrac{\lambda l}{\varepsilon_0}\right)$.

Hence, θ angle corresponds to the flux, i.e.

$$\phi = \frac{\theta}{2\pi}\left(\frac{\lambda l}{\varepsilon_0}\right) = \frac{\lambda l}{\pi\varepsilon_0}\tan^{-1}\left(\frac{b}{2d}\right)$$

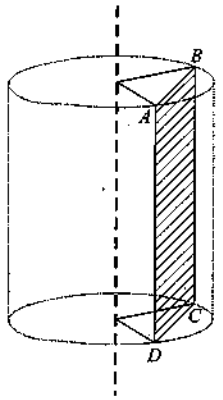


Fig. 2.93

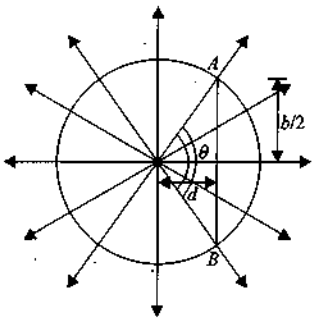


Fig. 2.94

11. $\int E\,dA = E(4\pi r^2) = \frac{q_{in}}{\varepsilon_0}$

(a) For $r > R$, $q_{in} = \int_0^R Ar^2(4\pi r^2)dr = 4\pi\frac{AR^5}{5}$

So $$E = \frac{AR^5}{5\varepsilon_0 r^2}$$

(b) For $r < R$, $q_{in} = \int_0^r Ar^2(4\pi r^2)dr = \frac{4\pi Ar^5}{5}$

So $$E = \frac{Ar^3}{5\varepsilon_0}$$

12. Consider a spherical shell of radius x. The electric flux through this surface is

$$\phi = \int_s \vec{E}\cdot d\vec{S} = E_r 4\pi r^2$$

Therefore, the electric flux through spherical surface of radius R will be

$$\phi = E_R 4\pi R^2$$

When $r = R$, $E_R = \alpha R$, we have $\phi = \alpha R 4\pi R^2$.
By the Gauss theorem, net electric flux is

$$\frac{1}{\varepsilon_0}\times\text{change enclosed}$$

$$\alpha R 4\pi R^2 = \frac{1}{\varepsilon_0}Q_{enclosed}$$

$$\therefore \quad Q_{enclosed} = (4\pi\varepsilon_0)\alpha R^3$$

Given $R = 0.30$ m, $\alpha = 100$ V/m^2

$$Q_{enclosed} = \frac{1}{9\times10^9}\times100\times(0.30)^3 = 3\times10^{-10}\text{ C}$$

13. Consider a spherical surface of radius $r(< R)$ having centre at the centre of the ball. If E is the magnitude of electric field strength at the surface, then the electric flux through this surface is

$$\int_s \vec{E}\cdot d\vec{S} = \int_s E dS\cos 0°$$

$$= E\int_s dS = E4\pi r^2$$

By the Gauss theorem,

$$\int_s \vec{E}\cdot d\vec{S} = \frac{1}{\varepsilon_0}(Q_{enclosed}) \quad \text{(i)}$$

If q is charge on ball, then

$$Q_{enclosed} = q + \int_R^r \rho dv = q + \int_R^r \frac{\alpha}{x}\cdot 4\pi x^2 dx$$

$$= q + 2\pi\alpha(r^2 - R^2)$$

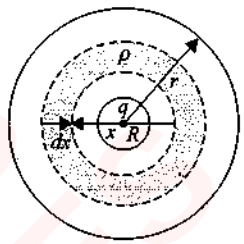


Fig. 2.95

From Eq. (i), we get

$$E4\pi r^2 = \frac{1}{\varepsilon_0}[q + 2\pi\alpha(r^2 - R^2)]$$

$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0 r^2}[q + 2\pi\alpha(r^2 - R^2)]$$

or $$E = \frac{\alpha}{2\varepsilon_0} + \frac{1}{\varepsilon_0}\left(\frac{q}{4\pi r^2} - \frac{\alpha}{2}\frac{R^2}{r^2}\right) \quad \text{(ii)}$$

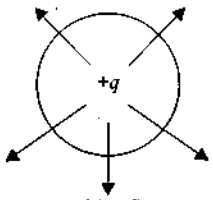
This will be independent of r if the second term on R.H.S. is zero, i.e., $\frac{q}{4\pi r^2} - \frac{\alpha R^2}{2r^2} = 0 \Rightarrow q = 2\pi\alpha R^2$ (iii)

Then the electric field strength E will be $\alpha/2\varepsilon_0$.

Objective Type

1. a. $\phi = E = \text{NC}^{-1}\text{m}^2$

2. a. Outgoing flux is taken as +ve.



Positive flux

Fig. 2.96

3. a. Because net charge enclosed by the surface is zero.

4. a. Inside a conductor, electric field is zero. Because applied field is cancelled by field produced due to induced charges.

5. b. $E = \frac{kQr}{R^3} = \frac{1}{4\pi\varepsilon_0}\frac{(4/3)\pi R^3\rho r}{R^3} = \frac{\rho r}{3\varepsilon_0}$

6. d. Inside a hollow conductor, electric field is zero. At surface, it is maximum and then decreases.

7. d. $\phi = EA\cos\theta = E\pi b^2\cos 180° = -200\pi b^2$

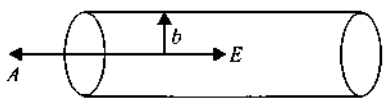


Fig. 2.97

8. c. Electric field at any point on Gaussian surface is due to all charges present inside or outside the surface.

9. b. Net charge on the conductor will be zero. So, net charge inside S_1 will be the charge on the rod. Hence, flux through S_1 is q/ε_0.

10. c. Electric field is zero inside the shell due to the outside changes.

11. a. Inside a uniformly charged solid sphere: $E \propto r$

12. b. There is no electric field inside a conductor, and at surface lines of forces are perpendicular to the conductor at any point.

13. d. At any point inside the conductor, net electric field is zero.

14. d. Electric field at any point near the surface of an arbitrary conductor is $E = \sigma/\varepsilon_0$, where σ is the surface change density at that point.

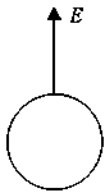


Fig. 2.98

15. a. At any point on the curved surface, E and area vector are perpendicular to each other.

16. d. $\frac{q}{6\varepsilon_0} = \frac{0.1\times10^{-6}}{6\times8.85\times10^{-12}} = 1883$

17. a. $\frac{Q}{K\varepsilon_0} = \frac{0.5}{10\times8.85\times10^{-12}} = 5.65\times10^{9}$

18. c. $\phi = \frac{q}{\varepsilon_0} \rightarrow$ Independent of dimensions

19. c. $\vec{E} = 2000\,\hat{k}, \vec{A} = 10\times20\times10^{-4}\hat{k}$

$\phi = \vec{E}\cdot\vec{A} = 40$ Vm

20. c. Flux through both will be the same as net charge enclosed by both is same.

21. b. We should know the total charge inside. There is no contribution in the flux due to outside charges.
Also, we should know both magnitude and direction of electric field at any point on the surface.

22. b. Net flux is due to charges inside S only.

23. b. $\phi = \frac{q_{in}}{\varepsilon_0} \Rightarrow 0 = \frac{q_{in}}{\varepsilon_0} \Rightarrow q_{in} = 0$

24. c. $1 - 7 - 4 + 10 + 2 - 5 - 3 + 6 = 0$
Sum of all the charges is zero, so net flux is zero.

25. a. Charge inside $S_1 = q_1 + q_2 = 3\times10^{-6}$ C
Charge inside $S_2 = q_2 + q_3 = -1\times10^{-6}$ C
Charge inside $S_3 = 0$
Charge inside S_1 is greatest. So, flux through S_1 is maximum.

26. b. $\vec{E} = 8\hat{i} + 4\hat{j} + 3\hat{k}, \vec{A} = 100\,\hat{k}$

27. d. $\phi = \vec{E}\cdot\vec{A} = 300$ units

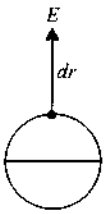


Fig. 2.99

$$E = \frac{\sigma}{2\varepsilon_0}$$

$$F = E\,dr = \frac{\sigma dA\sigma}{2\varepsilon_0}$$

$$P = \frac{F}{dA} = \frac{\sigma^2}{2\varepsilon_0}$$

$$\text{Force} = P\pi R^2 = \frac{\sigma^2}{2\varepsilon_0}\pi R^2$$

$$= \frac{Q^2}{16\pi^2R^4}\frac{\pi R^2}{2\varepsilon_0} = \frac{Q^2}{32\pi\varepsilon_0 R^2}$$

$$= \frac{(20\times10^{-6})^2\times9\times10^{9}}{8(1.5\times10^{-2})^2} = 2000\text{ N}$$

28. c.

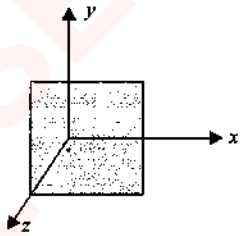


Fig. 2.100

Imagine a charge q at the centre of a cube of edge length $2L$ (Fig. 2.100). Then

$$\phi = \frac{q}{\varepsilon_0}.$$

Here, the square is one 24th of the surface area of the imaginary cube, so it intercepts 1/24 of the flux. That is,

$$\Phi = \frac{q}{24\varepsilon_0}$$

29. a. To find the charge enclosed, we need the flux through the parallelopiped:

$\Phi_1 = AE_1\cos60°$

$= (0.0500\text{ m})(0.0600\text{ m})(2.50\times10^4\text{ NC}^{-1})\cos60°$

$= 37.5\text{ Nm}^2\text{C}^{-1}$

$\Phi_2 = AE_2\cos120°$

$= (0.0500\text{ m})(0.0600\text{ m})(7.00\times10^4\text{ NC}^{-1})\cos120°$

$= -105\text{ Nm}^2\text{C}^{-1}$

So, the total flux is

$\Phi = \Phi_1 + \Phi_2 = (37.5 - 105)\text{Nm}^2\text{C}^{-1} = -67.5\text{ Nm}^2\text{C}^{-1}$

$q = \Phi\varepsilon_0 = (-67.5\text{Nm}^2/\text{C})\,\varepsilon_0 = -5.97\times10^{-10}$ C.

There must be a net charge (negative) in the parallelopiped since there is a net flux flowing into the surface. Also, there must be an external field, otherwise all lines would point towards the slab.

30. c. The dielectric gets polarized as shown. Induced charges will also contribute in electric field.

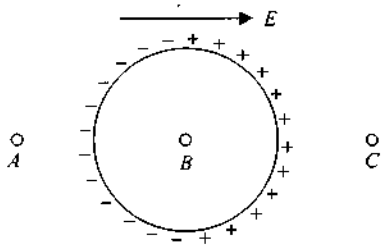


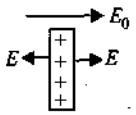
Fig. 2.101

So, intensity at points A and C will increase and at B intensity will decrease.

31. b. From the figure, it is clear that the plate is placed in an external electric field. Let the electric field due to plate be E, and E_0 is the external electric field.

$\therefore$ $E_0 + E = 12\ \text{Vm}^{-1}$

$E_0 - E = 8\ \text{Vm}^{-1}$



Solving equations, $E = 2\ \text{Vm}^{-1}$ and $E_0 = 10\ \text{Vm}^{-1}$

Now, electric field due to plate $= \dfrac{\sigma}{2\varepsilon_0} = 2$

$\therefore \sigma = 4\varepsilon_0$

Multiple Correct Answers Type

1. a., d. Due to induction, charges on various faces are as shown in Fig. 2.102.

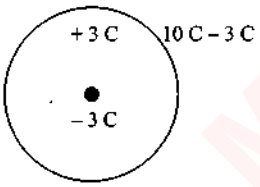


Fig. 2.102

Charge on the inner surface of the shell = +3 C
Net charge on the outer surface of the shell = −3 C + 10 C = +7 C.
Distribution of charge on the inner surface would be uniform if charge is placed at the centre, otherwise non-uniform. On outer surface, charge would be always uniformly distributed as the displacement of inside charges does not affect the distribution of the outer charge.

2. b., d. In L.H.S. of Gauss's law, $\vec{E}$ is due to all point charges present in space and ϕ depends only on the enclosed charges.

3. b., c. ϕ crossing through Gaussian surface does not depend on the location of charge, while $\vec{E}$ depends on it.
If q crosses the boundary, then q_{enclosed} changes and hence the flux and $\vec{E}$ also change.

Assertion-Reasoning Type

1. d. $\vec{E}$ in outside vicinity of the conductor's surface depends on all the charges present in the space, but expression $\vec{E} = \sigma/\varepsilon_0$.

2. d. Due to displacement of charges within closed surface, $\vec{E}$ at any point may change. But net flux crossing the surface will not change.

3. d. $\vec{E}$ at any point on the Gaussian surface may be due to outside charges also.

4. a. Statement II is true and according to that distribution of charge on the outer surface is not affected by the inside location of the charges.

5. a. From $\sigma \propto 1/R$ and electric field at the conductor's surface, $E = \sigma/\varepsilon_0$. We can say that Statement I is correct and Statement II is a correct explanation for it.

Comprehension Type

For Problems 1–4

1. c., 2. a., 3, d., 4. d.

Sol. $\Phi = \vec{E}\cdot\vec{A} = EA\cos\theta$, where, $\vec{A} = A\hat{n}$

$\hat{n}_{S_1} = -\hat{j}$(left), $\Phi_{S_1} = -(4\times10^3\ \text{NC}^{-1})(0.1\ \text{m})^2\cos(90°-37°)$
$= -24\ \text{Nm}^2\text{C}^{-1}$

$\hat{n}_{S_2} = +\hat{k}$(top),

$\Phi_{S_2} = -(4\times10^3\ \text{NC}^{-1})(0.1\ \text{m})^2\cos 90° = 0$

$\hat{n}_{S_3} = +\hat{j}$(right),

$\Phi_{S_3} = +(4\times10^3\ \text{NC}^{-1})(0.1\ \text{m})^2\cos(90°-37°)$
$= +24\text{Nm}^2\text{C}^{-1}$

$\hat{n}_{S_4} = -\hat{k}$(bottom),

$\Phi_{S_4} = (4\times10^3\ \text{NC}^{-1})(0.1\ \text{m})^2\cos 90° = 0$

$\hat{n}_{S_5} = +\hat{i}$(front),

$\Phi_{S_5} = +(4\times10^3\ \text{NC}^{-1})(0.1\ \text{m})^2\cos 37° = 32\ \text{Nm}^2\text{C}^{-1}$

$\hat{n}_{S_6} = -\hat{i}$(back),

$\Phi_{S_6} = -(4\times10^3\ \text{NC}^{-1})(0.1\ \text{m})^2\cos 37° = -32\ \text{Nm}^2\text{C}^{-1}$

As the field is uniform, so the total flux through the cube must be zero; any flux entering the cube must also leave it.

For Problems 5–8

5. b, 6. a, 7. b, 8. a.

For S_1: $\vec{A} = L^2(-\hat{j})$, $\vec{E} = -5x\hat{i} + 3z\hat{k}$

Here $\vec{E} \perp \vec{A}$, so $\phi_{S_1} = 0$

For S_2: $\vec{A} = L^2\hat{k}$, $\vec{E} = -5x\hat{i} + 3L\hat{k}$

Here $-5x\hat{i}$ is perpendicular to $\vec{A}$, so no flux due to this component.

$\phi_{S_2} = (3L\hat{k})\cdot L^2\hat{k} = 3L^3 = 81\times10^{-3}\ \text{Nm}^2\ \text{C}^{-1}$

For S_3: $\vec{A} = L^2\hat{j}$, $\vec{E} = -5x\hat{i} + 3z\hat{k}$

Here $\vec{E} \perp \vec{A}$, so $\phi_{S_3} = 0$

For S_4: $\vec{A} = -L^2\hat{k}$, $\vec{E} = -5x\hat{i} + 0\hat{k}$ $(\because z = 0)$

Here $\vec{E} \perp \vec{A}$, so $\phi_{S_4} = 0$

For S_5: $\vec{A} = L^2\hat{i}$, $\vec{E} = -5L\hat{i} + 3z\hat{k}$

No flux due to $3z\hat{k}$ component.

$\phi_{S_5} = (-5L\hat{i})\cdot L^2\hat{i} = -5L^3 = -135\times10^{-3}\ \text{Nm}^2\ \text{C}^{-1}$

For S_6: $\vec{A} = -L^2\hat{i}$, $\vec{E} = 0\hat{i} + 3z\hat{k}$ $(\because x = 0)$

Here $\vec{E} \perp \vec{A}$, so $\phi_{S_6} = 0$

Total flux through the cube is

$$\phi = \phi(S_1 + S_2 + S_3 + S_5 + S_6)$$

$$= 81\times10^{-3} - 135\times10^{-3} = -54\times10^{-2}\ \text{Nm}^2\ \text{C}^{-1}$$

Now $\phi = \dfrac{q_{in}}{\varepsilon_0} \Rightarrow q_{in} = \phi\varepsilon_0 = -0.054\varepsilon_0\ \text{C}$

For Problems 9–10

9. a., 10. d.

Sol. Given that $\vec{E} = -B\hat{i} + C\hat{j} - D\hat{k}$, $\Phi = \vec{E}\cdot\vec{A}$, Edge length L, and

$$\hat{n}_{S_1} = -\hat{j} \Rightarrow \Phi_1 = \vec{E}\cdot A\hat{n}_{S_1} = -CL^2$$

$$\hat{n}_{S_2} = +\hat{k} \Rightarrow \Phi_2 = \vec{E}\cdot A\hat{n}_{S_2} = -DL^2$$

$$\hat{n}_{S_3} = +\hat{j} \Rightarrow \Phi_3 = \vec{E}\cdot A\hat{n}_{S_3} = +CL^2$$

$$\hat{n}_{S_4} = -\hat{k} \Rightarrow \Phi_4 = \vec{E}\cdot A\hat{n}_{S_4} = +DL^2$$

$$\hat{n}_{S_5} = +\hat{i} \Rightarrow \Phi_5 = \vec{E}\cdot A\hat{n}_{S_5} = -BL^2$$

$$\hat{n}_{S_6} = -\hat{i} \Rightarrow \Phi_6 = \vec{E}\cdot A\hat{n}_{S_6} = +BL^2$$

Total flux is $\sum_{i=1}^{6}\Phi_i = 0$

For Problems 11–12

11. c., 12. c.

Sol. $\phi_1 = -E_1a^2 = -\alpha a^{1/2}a^2 = -\alpha a^{5/2}$

and $\phi_2 = E_2a^2 = \alpha(2a)^{1/2}a^2 = \sqrt{2}\alpha a^{5/2}$

$\phi_{net} = (\sqrt{2}-1)\alpha a^{5/2}$

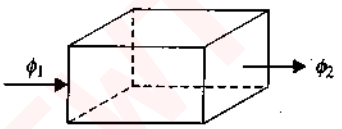


Fig. 2.103

Using the Gauss theorem, we get

$$\phi = \frac{q_{in}}{\varepsilon_0}$$

$$\Rightarrow \quad q_{in} = \phi\varepsilon_0 = (\sqrt{2}-1)\varepsilon_0\alpha a^{5/2}$$

For Problems 13–14

13. c., 14. b.

Sol. Induced charge on the inner surface will be equal and opposite to the charge placed at the centre.

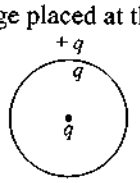


Fig. 2.104

Equal amount of opposite charge as that of the inner surface will be induced on the outer surface while net charge is zero on the ball.

For Problems 15–16

15 c., 16. a.

Sol. The extra given charge will reside on the outer surface.

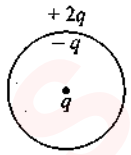


Fig. 2.105

For Problems 17–18

17. c., 18. c.

Sol. As far as the net charge induced on the inner and outer surfaces is concerned, it does not depend upon the location of point charge within the ball.

For Problems 19–24

19. a., 20. a., 21. b., 22. a., 23. c., 24. a.

Sol.

i. $r < a$. $E = 0$, since $Q = 0$. (Fig.2.106)

ii. $a < r < b$. $E = 0$, since $Q = 0$.

iii. $b < r < c$. Since $Q = +2q$, $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2q}{r^2}$,

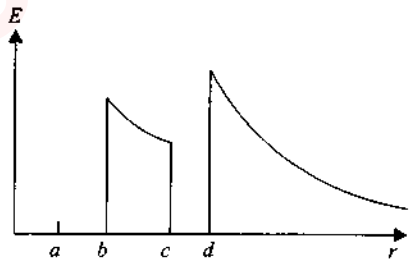


Fig. 2.106

iv. $c < r < d$. $E = 0$, since $Q = 0$.

v. $r > d$. $\dfrac{1}{4\pi\varepsilon_0}\dfrac{6q}{r^2}$, since $Q = +6q$.

For Problems 25–28

25. a., 26. b., 27. c., 28. d.

Sol.

i. Small shell inner surface: $Q = 0$

ii. Small shell outer surface: $Q = +2q$

iii. Large shell inner surface: $Q = -2q$

iv. Large shell outer surface: $Q = +6q$

For Problems 29–34

29. a., 30. a., 31. b., 32. b., 33. a., 34. c.

Sol.

i. $r < a$. $E = 0$, since charge enclosed is zero.

ii. $a < r < b$. $E = 0$, since charge enclosed is zero.

iii. $b < r < c$. $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2q}{r^2}$, since charge enclosed is $+2q$.

iv. $c < r < d$. $E = 0$, since charge enclosed is zero (Fig. 2.107).

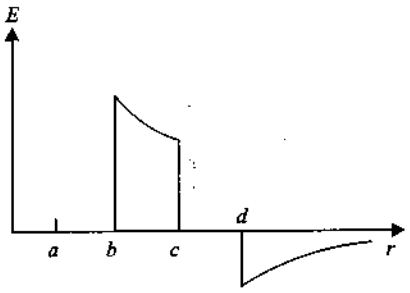


Fig. 2.107

v. $r > d$. $E = -\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$, since charge enclosed is $-2q$.

For Problems 35–38

35. a., 36. b., 37. c., 38. c.

Sol.

i. Small shell inner surface: $Q = 0$

ii. Small shell outer surface: $Q = +2q$

iii. Large shell inner surface: $Q = -2q$

iv. Large shell outer surface: $Q = -2q$

For Problems 39–44

39. a., 40. a., 41. b., 42. a., 43. a., 44. c.

Sol.

i. $r < a$. $E = 0$, since the charge enclosed is zero (Fig. 2.108).

ii. $a < r < b$. $E = 0$, since the charge enclosed is zero.

iii. $b < r < c$. $E = \frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$, since the charge enclosed is $2q$.

iv. $c < r < d$. $E = 0$, since the net charge enclosed is zero.

v. $r > d$. $E = 0$, since the net charge enclosed is zero.

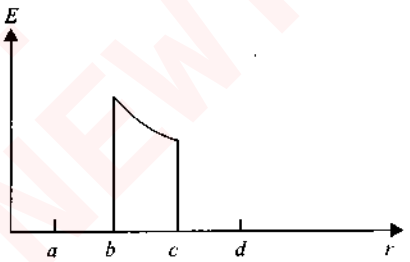


Fig. 2.108

For Problems 45–48

45. a. 46. b., 47. c., 48. a.

i. Small shell inner surface: $Q = 0$

ii. Small shell outer surface: $Q = +2q$

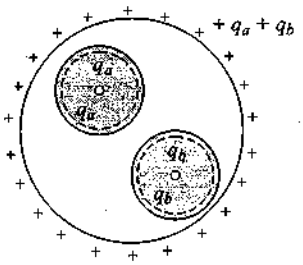
iii. Large shell inner surface: $Q = -2q$

iv. Large shell outer surface: $Q = 0$

For Problems 49–51

49. b. 50. c. 51. a.

Sol. i. $\sigma_A = \frac{-q_a}{4\pi a^2}, \sigma_B = \frac{-q_b}{4\pi b^2}, \sigma_R = \frac{q_a + q_b}{4\pi R^2}$



(a)

Fig. 2.109

ii. $E = \frac{1}{4\pi\varepsilon}\frac{q_a + q_b}{r^2}$ (Fig. 2.109)

iii. $E_a = \frac{1}{4\pi\varepsilon}\frac{q_b}{r^2}$

Matching Column Type

1. i. → b., ii. → d., iii. → c.
iv. → c., v. → c., vi. → d.

i. $E = \frac{\lambda}{2\pi\varepsilon_0 r}$.

ii. Inside electric field, $E = 0$.

iii. $E = \frac{\rho r}{3\varepsilon_0}$.

iv. $E = \frac{\lambda r}{2\pi\varepsilon_0 R^2}$.

v. Inside a plane sheet, $E \propto r$.

vi. E is independent of r outside the plane sheet.

2. i. → b., ii. → b., iii. → c., iv. → d.

Sol. Distribution of charge on different surfaces of the plates has been shown.

Take a point P on the leftmost plate. The electric field at P is

$$E = \frac{Q-q}{2A\varepsilon_0} - \frac{q}{2A\varepsilon_0} + \frac{q}{2A\varepsilon_0}$$

$$-\frac{q}{2A\varepsilon_0} + \frac{q}{2A\varepsilon_0} - \frac{(q-2Q)}{2A\varepsilon_0} = 0$$

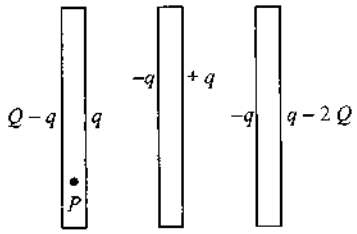


Fig. 2.110

(inside a conductor, $E = 0$).

So, $Q - q - q + 2Q = 0$

or $2q = 3Q \Rightarrow q = \frac{3Q}{2}$

The charge appearing on the outer surface of the rightmost plate is

$$q - 2Q = \frac{3Q}{2} - 2Q = -\frac{Q}{2}$$

The charge on the outer surface of the leftmost plate is

$$Q - q = Q - \frac{3Q}{2} = -\frac{Q}{2}$$

Charge appearing on the middle plate is $-q$ and $+q$ ·

i.e., $-\frac{3Q}{2}$ and $+\frac{3Q}{2}$.

Note: We see that charge appearing on each of the outer surfaces is half of the total sum of charge on all three plates.

3. i. → b., ii. → d., iii. → c., iv. → a.

Sol. i. Infinite conducting plane sheet of charge is $\sigma/2\varepsilon_0$.

ii. Infinite conducting plane sheet of uniform thickness is σ/ε_0.

iii. Non-conducting charged solid sphere at its surface is $R\rho/3\varepsilon_0$.

iv. Non-conducting charged solid sphere at its centre is zero.

Integer Answer Type

1. (0) Electric field at point A will be zero and at B will be non-zero. So the required ratio is 0.

2. (3) Electric field on the surface of the sphere should not exceed 3.0×10^6 N/C. Therefore,

$$9\times10^9\times\frac{q}{(3\times10^{-3})^2} = 3.0\times10^6 \quad \Rightarrow q = 3\times10^{-9}\text{ C}.$$

3. (1) Consider a coplanar and concentric ring of radius x and radial thickness dx.

Its area $ds = 2\,xdx$

Distance of every point of this ring from the point charge is

$$r = \sqrt{a^2 + x^2}$$

Electric field at circumference of this ring is

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

Electric flux passing through this ring is

$$d\phi = \vec{E}\cdot\vec{ds} = (Eds\cos\theta)$$

$$d\phi = \left(\frac{1}{4\pi\varepsilon_0}\frac{q}{(a^2+x^2)}\right)2\pi xdx\left(\frac{a}{\sqrt{(a^2+x^2)}}\right)$$

Hence, total flux passing through this ring is

$$\phi = \int_{x=0}^{x=R}\frac{aq}{2\epsilon_0}\frac{xdx}{(a^2+x^2)^{3/2}} = \frac{qa}{2\epsilon_0}\left[\frac{1}{a} - \frac{1}{\sqrt{a^2+R^2}}\right]$$

Putting the values, we get

$$\phi = 1\times10^5\text{ N/C}$$

4. (2) $\rho = kr^a$

$$E\left(r = \frac{R}{2}\right) = \frac{1}{8}E(r = R)$$

$$\frac{q_{\text{enclosed}}}{4\pi\varepsilon_0(R/2)^2} = \frac{1}{8}\frac{Q}{4\pi\varepsilon_0R^2}$$

$$32\,q_{\text{enclosed}} = Q$$

$$q_{\text{enclosed}} = \int_0^{R/2} kr^a 4\pi r^2 dr = \frac{4\pi k}{(a+3)}\left(\frac{R}{2}\right)^{(a+3)}$$

$$Q = \frac{4\pi k}{(a+3)}R^{(a+3)}$$

$$\frac{Q}{q_{\text{enclosed}}} = 2^{a+3} \Rightarrow 2^{a+3} = 32 \Rightarrow a = 2$$

5. (3) The faces $adhe$, $bcgf$, $cdhg$ and $abfe$ will contribute zero flux because the area vector is normal to the electric field for these faces.

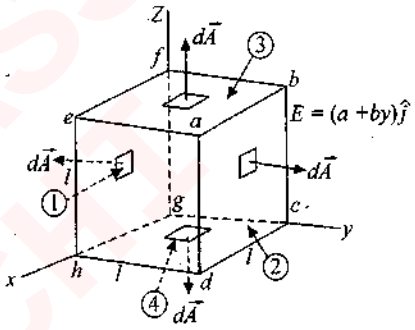


Fig. 2.111

Flux through face $efgh$, $\phi_1 = \int\vec{E}\cdot d\vec{A}$

$$= a(\hat{j})\cdot l^2(-\hat{j}) = -al^2$$

As the field at the face $efgh$ (that lies in the yz plane, $y = 0$) is $E = a\hat{j}$ and area vector is $l^2(-\hat{j})$ (direction outward normal)

Flux through face $abcd$:

$$\phi_2 = (a + bl)\hat{j}\cdot l^2\hat{j} = (al^2 + bl^3), \text{ for this } y = l$$

Net flux through the cube $= \phi_1 + \phi_2 = bl^3$

From Gauss's law, $\phi = \frac{Q_{\text{enclosed}}}{\varepsilon_0}$

$$Q_{\text{enclosed}} = \varepsilon_0\phi_E = \varepsilon_0 bl^3$$

After substituting the values $l = 1$ m and $b = 3$, we get

$$Q_{\text{enclosed}} = \varepsilon_0\phi_E = 3\varepsilon_0$$

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CHAPTER
3

Electric Potential

- Electric Potential and Energy
- Electric Potential Energy of Two Point Charges
- Electric-Volt
- Electric Potential
- Equipotential Surface
- Relation Between Electric Field and Potential
- Finding Electric Field from Electic Potential
- Electric Potential of Some Continuous Charge Distributions
- A Uniform Line of Charge
- A Ring of Charge
- A Charged Disk
- Potential Due to an Electric Dipole
- Work Done in Rotating an Electric Dipole in a Uniform Electric Field
- Potential Energy of an Electric Dipole in a Uniform Electric Field

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The concept of potential energy was introduced in mechanics in connection with such conservative forces as the gravitational force and the elastic force exerted by a spring. By using the law of conservation of energy, we were able to avoid working directly with forces when solving various problems in mechanics. The concept of potential energy is also of great value in the study of electricity. Because the electrostatic force is conservative, electrostatic phenomena can be conveniently described in terms of an electric potential energy. This idea enables us to define a scalar quantity known as *electric potential*. Because the electric potential at any point in an electric field is a scalar quantity, we can use it to describe electrostatic phenomena more simply than if we were to rely only on the electric field and electric forces.

The potential is characteristic of the field only, independent of a charged test particle that may be placed in the field. Potential energy is characteristic of the charge–field system due to an interaction between the field and a charged particle placed in the field.

ELECTRIC POTENTIAL AND ENERGY

Electrostatic force is a conservative force. Thus, when an electrostatic force acts between two or more charged particles within a system of particles, we can assign an electric potential energy U to the system.

We have a relation between change in potential energy and work done by conservative force

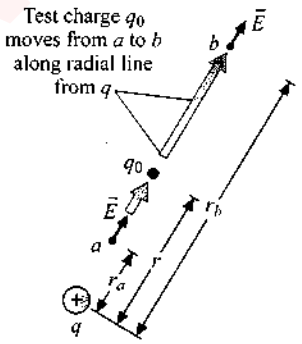
$$\Delta U = -W \Rightarrow U_b - U_a = -W \qquad \text{(i)}$$

ELECTRIC POTENTIAL ENERGY OF TWO POINT CHARGES

Let us calculate the work done on a test charge q_0 moving in the electric field caused by a single, stationary point charge q.

We will consider first a displacement along the radial line as shown in Fig. 3.1, from point a to point b. The force on q_0 is given by Coulomb's law, i.e.,

$$F_r = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2} \qquad \text{(i)}$$



Test charge q_0 moves along a straight line extending radially from charge q. As it moves from a to b, the distance varies from r_a to r_b.

Fig. 3.1

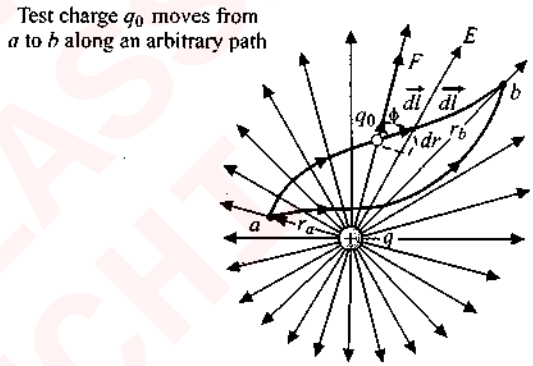
The force is not constant during the displacement, and we have to integrate to calculate the work $W_{a\to b}$ done on q_0 by this force as q_0 moves from a to b. We find

$$W_{a\to b} = \int_{r_a}^{r_b} F_r\, dr = \int_{r_a}^{r_b} \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\, dr = \frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right) \qquad \text{(ii)}$$

The work done by the electric force for this particular path depends only on the end points.

In fact, the work is the same for all possible paths from a to b. To prove this, we consider a more general displacement (as shown in Fig. 3.2). The work done on q_0 during this displacement is given by

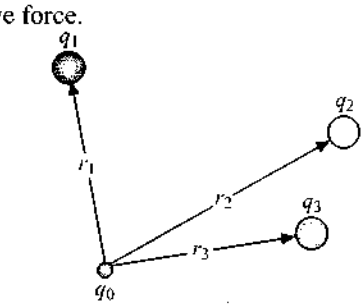
$$W_{a\to b} = \int_{r_a}^{r_b} F\cos\phi\, dl = \int_{r_a}^{r_b} \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\cos\phi\, dl \qquad \text{(iii)}$$



The work done on charge q_0 by the electric field of charge q does not depend on the path taken, but only on the distances r_a and r_b.

Fig. 3.2

But Fig. 3.2 shows that $\cos\phi\, dl = dr$. That is, work done during a small displacement $d\vec{l}$ depends only on the change dr in the distance r between the charges, which is the radial component of the displacement. Thus, Eq. (iii) is valid even for this more general displacement; the work done on q_0 by the electric field produced by q depends only on r_a and r_b not on the details of the path (Fig. 3.3). These are the needed characteristics for a conservative force, as we defined it in the section. Thus, the force on q_0 is a conservative force.



The potential energy associated with a charge q_0 at point a depends on the other charges q_1, q_2 and q_3 and on their distance r_1, r_2 and r_3 from point a.

Fig. 3.3

We see that Eqs. (i) and (iii) are consistent if we define $qq_0/4\pi\varepsilon_0 r_a$ to be the potential energy U_a when q_0 is at point a,

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a distance r_a from q, and we define $qq_0/4\pi\varepsilon_0 r_b$ to be the potential energy U_b when q_0 is at point b, a distance r_b from q. Thus, the potential energy U when the test charge q_0 is at any distance r from charge q is

$$U = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r}$$

(electric potential energy of two point charges q and q_0)
In case of discrete distribution charges,

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}} + \frac{q_2q_3}{r_{23}} + \cdots\right] = \frac{1}{2}\frac{1}{(4\pi\varepsilon_0)}\sum_{i\neq j}\frac{q_iq_j}{r_{ij}}$$

[1/2 is used as each term in summation will appear twice]

ELECTRON-VOLT

It is the smallest practical unit of energy used in atomic and nuclear physics. An electron-volt is defined as the energy acquired by a particle having one quantum of charge (i.e., e) when accelerated by 1 volt, i.e.,

$$1\text{ eV} = (1.6\times10^{-19}\text{ C})\times(1\text{ JC}^{-1}) = 1.6\times10^{-19}\text{ J}$$

(as $U = qV$)

ELECTRIC POTENTIAL

The work done by the electric force during a displacement from a to b: $W_{a\to b} = -\Delta U = -(U_b - U_a)$. On a "work per unit charge" basis, we divide this equation by q_0, obtaining

$$\frac{W_{a\to b}}{q_0} = -\frac{\Delta U}{q_0} = -\left(\frac{U_b}{q_0} - \frac{U_a}{q_0}\right)$$
$$= -(V_b - V_a) = V_a - V_b \qquad \text{(i)}$$

where $V_a = U_a/q_0$ is the potential energy per unit charge at point a and similarly for V_b. We call V_a and V_b the potential at point a and potential at point b, respectively. *Thus, the work done per unit charge by the electric force when a charged body moves from a to b is equal to the potential at 'a' minus the potential at 'b'.*

The difference $V_a - V_b$ is called the potential of a with respect to b. We sometimes abbreviate this difference as $V_{ab} = V_a - V_b$ (note the order of the subscripts). In electric circuits which we will analyze in later chapters, the potential difference between two points is often called *voltage*. Equation (i) then states that V_{ab}, *the potential of 'a' with respect to 'b', equals the work done by the electric force when a unit charge moves from 'a' to 'b'.*

Also, V_{ab}, the potential of a with respect to b, equals the work that must be done to move a unit charge slowly from b to a against the electric force.

To find the potential V due to a single point charge q, we divide U by q_0, i.e.,

$$V = \frac{U}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \qquad \text{(ii)}$$

(potential due to a point charge q at a distance r from it) where r is the distance from the point charge q to the point at which the potential is evaluated.

Regarding potential, it is worth noting that

- It is a scalar quantity having dimensions

$$[V] = \left[\frac{W}{q_0}\right]$$
$$= \left[\frac{ML^2T^{-2}}{AT}\right] = [ML^2T^{-3}A^{-1}]$$

- And S.I. or practical unit (JC^{-1}) is called volt (V). Also, as in electric field *principle of superposition* is valid for discrete distribution of charge, the electrical potential also follows the same.

$$V = V_1 + V_2 + V_3 + \cdots = \frac{1}{4\pi\varepsilon_0}\sum\frac{q_i}{r_i}$$

- The potential due to a continuous charge distribution is the sum of potentials of all the infinitesimal charge elements in which the distribution may be divided.

$$V = \int dV \Rightarrow V = \int\frac{dq}{4\pi\varepsilon_0 r}$$

- When a positive charge moves from low potential to high potential, its potential energy increases and if negative charge does so, its potential energy decreases.

Illustration 3.1 **Two electric charges q and $-2q$ are placed at a distance 6 m apart on a horizontal plane. Find the locus of point on this plane where the potential has a value zero.**

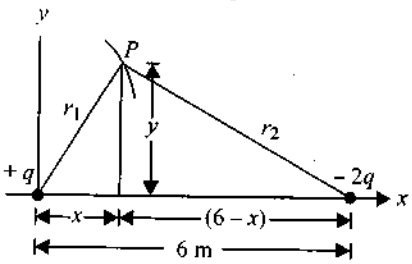

Fig. 3.4

Sol. If point P is at a distance r_1 from point charge $+q$ and r_2 from charge $-2q$ (Fig. 3.4), then

$$V_P = V_1 + V_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{+q}{r_1} + \frac{-2q}{r_2}\right]$$

According to the problem, $V_P = 0$, i.e., $\frac{q}{r_1} - \frac{2q}{r_2} = 0$

or $\qquad r_2 = 2r_1 \qquad$ (i)

But if the charge q is assumed to be situated at the origin, then

$$r_1^2 = x^2 + y^2 \text{ and } r_2^2 = (6-x)^2 + y^2$$

So, substituting these values of r_1 and r_2 in Eq. (i), we get

$$(x+2)^2 + y^2 = 16$$

So, the locus of the point P is a circle with radius 4 m and centre (−2, 0).

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Illustration 3.2 **An alpha particle with kinetic energy 10 MeV is heading towards a stationary tin nucleus of atomic number 50. Calculate the distance of closest approach (Fig. 3.5).**

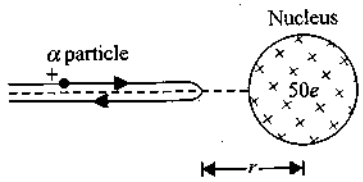


Fig. 3.5

Sol. In approaching the nucleus, kinetic energy of alpha particle is converted into electrical potential energy. So, if r is the distance of the closest approach, then

$$\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$$

or,
$$r = \frac{1}{4\pi\varepsilon_0}\frac{(2e)(50e)}{(\text{K.E.})}$$

or,
$$r = (9\times10^9)\frac{(2\times1.6\times10^{-19})(1.6\times10^{-19}\times50)}{(10\times10^6\times1.6\times10^{-19})}$$

$$= 14.4\times10^{-15}\,\text{m} = 14.4\text{ fm}$$

Illustration 3.3 **A proton moves with a speed of u directly towards a free proton originally at rest. Find the distance of closest approach for the two protons. Given that mass of the proton is 'm' and charge of the proton is $+e$.**

Sol. As here the particle at rest is free to move, when one particle approaches the other, due to electrostatic repulsion the other particle will also start moving and so the velocity of first particle will decrease while that of the other particle will increase and at the closest approach both will move with the same velocity. So, if v is the common velocity of each particle at the closest approach, as no external forces are acting on them (the system of particles), the linear momentum of the system of particles will be conserved. By the conservation of momentum, we get

$$mu = mv + mv, \text{ i.e., } v = \frac{1}{2}u$$

And by the conservation of energy, we have

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

So,
$$r = \frac{e^2}{\pi\varepsilon_0 mu^2} \quad (\text{as } v = u/2)$$

EQUIPOTENTIAL SURFACE

For a given charge distribution, loci of all points or regions for which the electric potential has a constant value are called *equipotential regions*. Such equipotential regions can be surfaces, volumes or lines.

1. Equipotential surfaces can never cross each other, otherwise potential at a point will have two values which is not possible (Fig. 3.6).

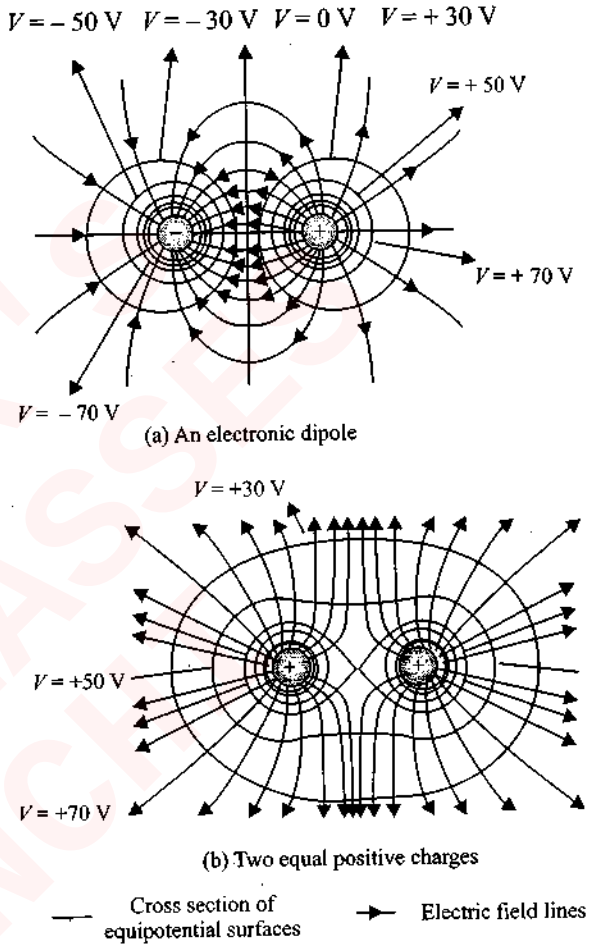
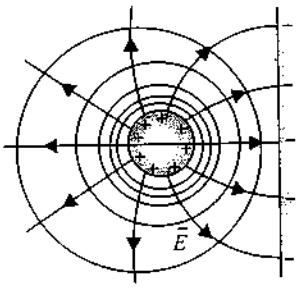
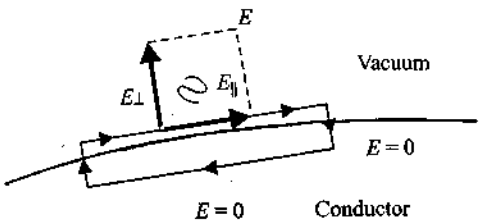


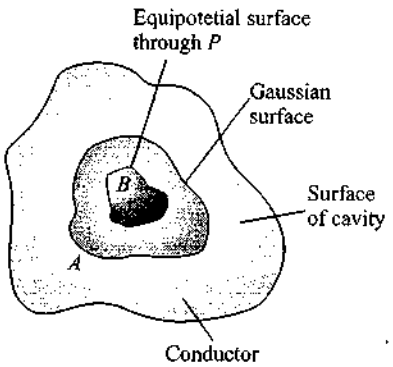
Fig. 3.6



When charges are at rest, a conducting surface is always an equipotential surface. Field lines are perpendicular to a conducting surface.



Inside the conductor, $E = 0$. If, just outside the conductor field, E had a component $E_{\parallel}$ parallel to the conductor surface, then a test charge moving around the rectangular loop and returning to the starting point would have non-zero total work done on it by the field. Because the field E is conservative, this is impossible. Therefore, E just outside the surface must be perpendicular to it.



A cavity in a conductor. If the cavity contains no charge, every point in the cavity is at the same potential, the electric field is zero everywhere in the cavity, and there is no charge everywhere on the surface of the cavity.

Fig. 3.7

2. Equipotential surface is always perpendicular to electric lines of force.
3. If a charge is moved from one point to the other over an equipotential surface, work done will be zero (Fig. 3.7).

$$W_{AB(by\ us)} = U_{BA} = q(V_B - V_A) = 0 \quad (\text{as } V_B = V_A)$$

RELATION BETWEEN ELECTRIC FIELD AND POTENTIAL

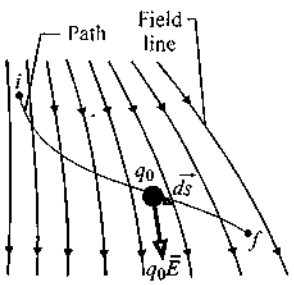
Consider an arbitrary electric field (represented by the field lines in Fig. 3.8) and a positive test charge q_0 that moves along the path shown from point i to point f. At any point on the path, an electrostatic force q_0E acts on the charge as it moves through a differential path element ds.

$$U_f + U_i = -W = -\int_i^f q_0\vec{E}\cdot\overrightarrow{ds} \Rightarrow \frac{U_f - U_i}{q_0} = -\int_i^f \vec{E}\cdot\overrightarrow{ds}$$

$$V_f - V_i = -\int_i^f \vec{E}\cdot\overrightarrow{ds} \qquad \text{(i)}$$

If we take $V_i = 0, i \to \infty, f \to r$, then

$$V = -\int_\infty^r \vec{E}\cdot\overrightarrow{ds} \qquad \text{(ii)}$$



A test charge q_0 moves from point i to point f along the path shown in a non-uniform electric field. During a displacement ds, an electrostatic force q_0E acts on the test charge. This force points in the direction of the field line at the location of the test charge.

Fig. 3.8

FINDING ELECTRIC FIELD FROM ELECTRIC POTENTIAL

We know in an electric field $\Delta U = -W$, where W is the work done by electric field.

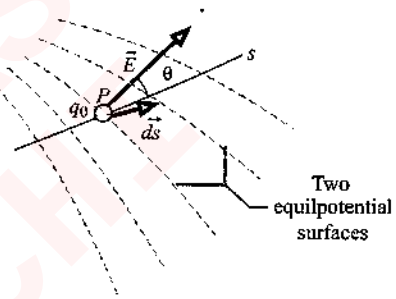
$$U_f - U_i = -\int_i^f q\vec{E}\cdot\overrightarrow{ds} \Rightarrow \frac{U_f - U_i}{q} = -\int_i^f \vec{E}\cdot\overrightarrow{ds}$$

$$\Rightarrow \qquad V_f - V_i = -\int_i^f \vec{E}\cdot\overrightarrow{ds}$$

We can write $dV = -\vec{E}\cdot\overrightarrow{ds} = E\cos\theta\, ds$ (iii)

$$E\cos\theta = -\frac{dV}{ds}$$

$E\cos\theta$ is the component of E in the direction of ds (see Fig. 3.9).



A test charge q_0 moves a distance ds from one equipotential surface to another. (The separation between the surfaces has been exaggerated for clarity.) The displacement $\overrightarrow{ds}$ makes an angle θ with the direction of the electric field $\vec{E}$.

Fig. 3.9

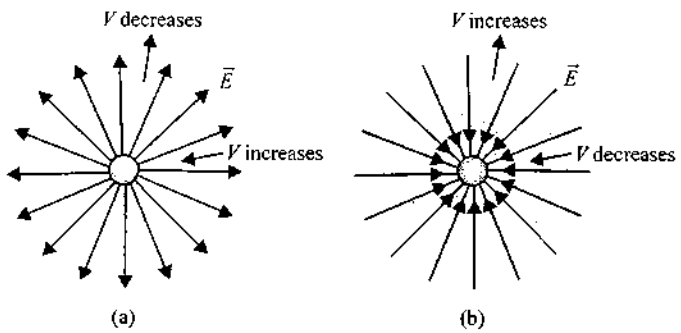
$$\Rightarrow \qquad E_S = -\frac{dV}{ds} \qquad \text{(iv)}$$

We write $\vec{E}$ and $\overrightarrow{ds}$ in terms of their components, i.e.,

$$\vec{E} = \hat{i}E_x + \hat{j}E_y + \hat{k}E_z \text{ and } \overrightarrow{ds} = \hat{i}dx + \hat{j}dy + \hat{k}dz.$$

Then, we have

$$-dV = E_x dx + E_y dy + E_z dz$$



(a) A positive point charge. (b) A negative point charge.

In both cases, if you move in the direction of $\vec{E}$, electric potential V decreases; if you move in the direction opposite to $\vec{E}$, V increases.

Fig. 3.10

Suppose the displacement is parallel to the x-axis, so $dy = dz = 0$. Then, $-dV = E_x dx$ or $E_x = -\left(\frac{dV}{dx}\right)_{y,z \text{ constant}}$,

where the subscript reminds us that only x varies in the derivative; recall that V is, in general, a function of x, y and z. But this is just what is meant by the partial derivative $\delta V/\delta x$. The y- and z-components of E are related to the corresponding derivatives of V in the same way, so we have

$$E_x = -\frac{\partial V}{\partial x};\ E_y = -\frac{\partial V}{\partial y};\ E_z = -\frac{\partial V}{\partial z}$$

(components of E in terms of V)

The equation states the negative of the rate of change of potential with position in any direction is the component of E in that direction. The minus sign implies that E points in the direction of decreasing V (Fig. 3.10).

Concept Application Exercise 3.1

1. The electric field inside a hollow charged conductor is zero. Is this true or false?
2. How can you check whether the electric potential in a given region of space has constant value?
3. Are we free to call the potential of earth +100 V instead of zero? What effect would such an assumption have on measured values of
   a. potentials, b. potential difference?
4. If you know E at a given point, can you calculate V at that point? If not, what further information do you need?
5. Can there be a potential difference between two adjacent conductors that carry same amount of positive charge?

Illustration 3.4 **The potential at any point is given by $V = x(y^2 - 4x^2)$. Calculate the cartesian components of the electric field at the point.**

Sol. Here, $V = x(y^2 - 4x^2)$

$$E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}[xy^2 - 4x^3]$$

$$= -[y^2 - 12x^2] \quad \text{or} \quad E_x = [12x^2 - y^2]$$

Similarly, $E_y = -\frac{\partial V}{\partial y} \Rightarrow E_y = -\frac{\partial}{\partial y}[x(y^2 - 4x^2)]$

or $$E_y = -\frac{\partial}{\partial y}[xy^2 - 4x^3] = -2xy$$

$E_z = -0$ because V does not depend upon the z-coordinate.

Illustration 3.5 **Two point charges Q_1 and Q_2 lie along a line at a distance from each other. Figure 3.11 shows the potential variation along the line of charges. At which of the points 1, 2 and 3 is the electric field zero? What are the signs of the charges Q_1 and Q_2 and which of the two charges is greater in magnitude?**

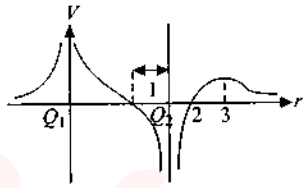


Fig. 3.11

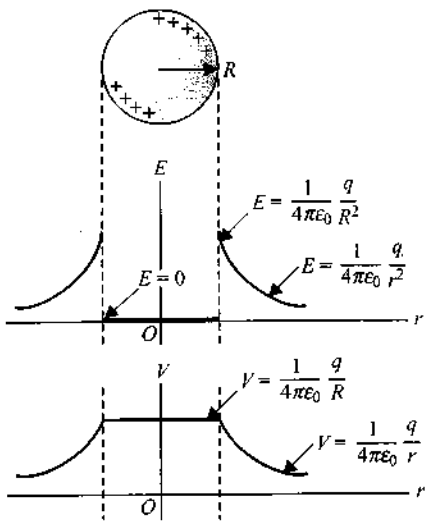
Sol. The electric field vector is zero at point 3. As $-dV/dr = E_r$, the negative of slope of V verus r curve represents component of electric field along r. Slope of curve is zero only at 3.

Near positive charge, net potential is positive and near a negative charge net potential is negative. Thus, charge Q_1 is positive and Q_2 negative. From the graph, it can be seen that net potential due to the two charges is positive everywhere in the region left of charge Q_1. Therefore, the magnitude of potential due to charge Q_1 is greater than that due to Q_2. Therefore, the absolute value of charge Q_1 is greater than that of Q_2. Secondly, the point 1, where potential due to two charges is zero, is nearer to charge Q_2 thereby implying that Q_1 has greater absolute value. Also, potential is zero at 2 which is towards right of Q_2, as we know that potential is zero at an outside point towards the side of charge smaller in magnitude.

ELECTRIC POTENTIAL OF SOME CONTINUOUS CHARGE DISTRIBUTIONS

A Charged Conducting Sphere

A solid conducting sphere of radius R has a total charge q. At all points outside the sphere, the field is the same at equal distance from centre of sphere, as if the sphere were removed and replaced by a point charge q. We take $V = 0$ at infinity, as we did for a point charge (Fig. 3.12).



Electric field magnitude E and potential V at points inside and outside a positively charged spherical conductor.

Fig. 3.12

Then, the potential at a point outside the sphere at a distance r from its centre is the same as the potential due to a point charge q at the centre i.e., $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

The potential at the surface of the sphere is

$$V_{\text{surface}} = \frac{q}{4\pi\varepsilon_0 R}$$

Inside the sphere, $\vec{E}$ is zero everywhere; otherwise, charge would move within the sphere. Hence, if a test charge moves from one point to any other point inside the sphere, no work is done on that charge. This means that the potential is the same at every point inside the sphere and is equal to its value $\left(\frac{q}{4\pi\varepsilon_0 R}\right)$ at the surface.

A Non-Conducting Solid Sphere

Outside the Sphere

The field intensity outside the sphere (Fig. 3.13) is

$$E_{\text{outside}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

$$\frac{dV_{\text{outside}}}{dr} = -E_{\text{outside}} \Rightarrow dV_{\text{outside}} = -E_{\text{outside}}dr$$

$$\int_{\infty}^{V} dV_{\text{outside}} = -\int_{\infty}^{V}\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}dr$$

$$\Rightarrow \quad V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \text{ as } V_{\infty} = 0$$

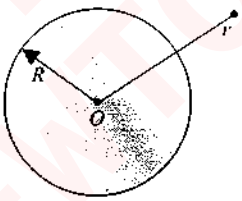


Fig. 3.13

At $r = R$, $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$, i.e., at the surface potential is

$$V_S = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

Inside the Sphere

Inside the sphere, $E_{\text{inside}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^3}r$

$$\Rightarrow \quad \frac{dV_{\text{inside}}}{dr} = -E_{\text{inside}}$$

$$\Rightarrow \quad dV_{\text{inside}} = -E_{\text{inside}}dr$$

$$\int_{V_S}^{V} dV_{\text{inside}} = -\frac{1}{4\pi\varepsilon_0}\frac{q}{R^3}\int_R^r r\,dr$$

$$\Rightarrow \quad V - V_S = -\frac{1}{4\pi\varepsilon_0}\frac{q}{R^3}\left[\frac{r^2}{2}\right]_R^r$$

Substituting $V_S = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$, we get

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}\left[\frac{3}{2} - \frac{1}{2}\frac{r^2}{R^2}\right] = \frac{V_S}{2}\left[3 - \frac{r^2}{R^2}\right]$$

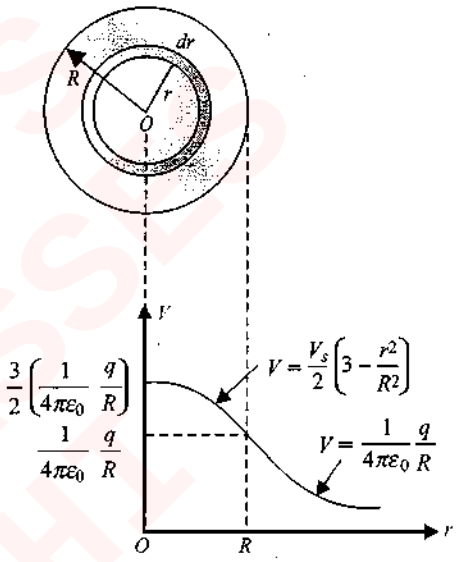


Fig. 3.14

At the centre $r = 0$ and $V_C = \frac{3}{2}\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{R}\right) = \frac{3}{2}V_S$, i.e., potential at the centre is 1.5 times the potential at the surface (Fig. 3.14). Thus, for a uniformly charged non-conducting sphere, we have the following formula for potential.

$$V_{\text{outside}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}, \quad V_{\text{surface}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}, \text{ and}$$

$$V_{\text{inside}} = \frac{q}{8\pi\varepsilon_0}\left(3 - \frac{r^2}{R^2}\right)$$

A UNIFORM LINE OF CHARGE

We can use the geometry as shown in Fig. 3.15 to find the potential due to a uniform line of positive charge at point P, a distance y from the rod on its perpendicular bisector. Using the charge element $dq = \lambda dz$ (where λ is the linear charge density), we have

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dz}{\sqrt{z^2 + y^2}}$$

Carrying out the integration over the length L and noting that y is a constant, we obtain

$$V = \frac{1}{4\pi\varepsilon_0}\int_{-L/2}^{+L/2}\frac{\lambda dz}{\sqrt{z^2 + y^2}}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left[\ln\left(z + \sqrt{z^2 + y^2}\right)\right]_{-L/2}^{+L/2}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\ln\left[\frac{L/2 + \sqrt{L^2/4 + y^2}}{-L/2 + \sqrt{L^2/4 + y^2}}\right]$$

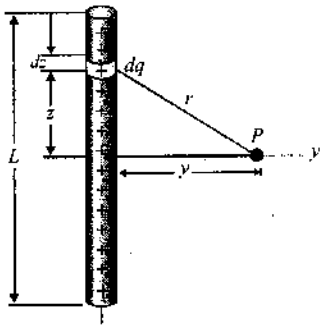


Fig. 3.15

For an infinite rod, the potential at any point near the rod is infinite (Fig. 3.16). But the potential difference between two points situated at the distances a and b can easily be calculated as

$$V_b - V_a = \int_a^b -E_r dr \Rightarrow V_b - V_a = \int_a^b \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{r}dr$$

$$V_a - V_b = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{b}{a}\right)$$

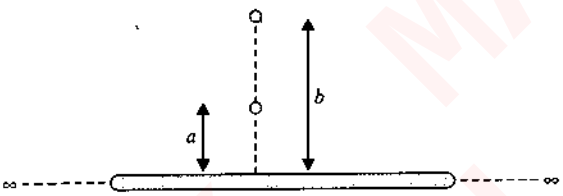


Fig. 3.16

For an infinite rod, $L \to \infty \Rightarrow V \to \infty$. Hence, the absolute potential of an infinite rod is not defined. However, we can define potential difference between two points near infinite line of charge.

Points to Remember

Absolute potential of an infinite rod or an infinite sheet of charge is not defined but we can define potential difference near the infinite charged distribution.

A RING OF CHARGE

Figure 3.17 shows a uniform ring of positive charge. The contribution to the potential at point P on its axis due to the charge element

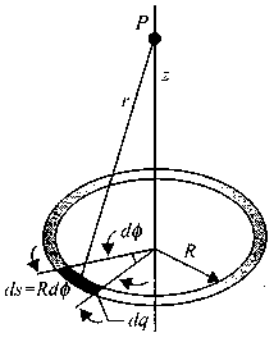


Fig. 3.17

$dq = \lambda\, ds = \lambda R d\phi$ is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda R d\phi}{\sqrt{R^2+z^2}}$$

Integrating around the ring as we note that both R and z remain constant. The variable of integration is ϕ, which ranges from 0 to 2π.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{\lambda R}{\sqrt{R^2+z^2}}\int_0^{2\pi} d\phi = \frac{1}{4\pi\varepsilon_0}\frac{2\pi\lambda R}{\sqrt{R^2+z^2}}$$

The total charge on the ring $q = 2\pi\lambda R$. The distance of the point under consideration from the ring $r = \sqrt{R^2+z^2}$.

$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$, where r is the distance from the ring to the point under consideration (Fig. 3.18).

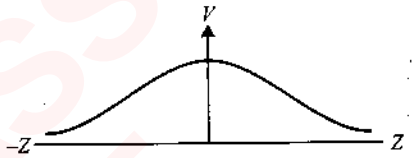


Fig. 3.18

At $z = 0, V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$ and at $z \to \infty$, $V = 0$. The maximum potential is at the centre of the ring.

A CHARGED DISK

With the geometry of figure, due to the ring of radius w and charge $dq = \sigma\, dA$ with area element (Fig. 3.19), we get

$$dA = 2\pi w\, dw$$

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\sqrt{w^2+z^2}}$$

$$= \frac{1}{4\varepsilon_0\pi}\frac{2\pi\sigma\, w\, dw}{\sqrt{w^2+z^2}}$$

To sum the contributions from all the rings on the disk, we integrate as w ranges from 0 to R, i.e.,

$$V = \frac{\sigma}{2\varepsilon_0}\int_0^R \frac{w\, dw}{\sqrt{w^2+z^2}}$$

$$= \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2+z^2} - |z|\right)$$

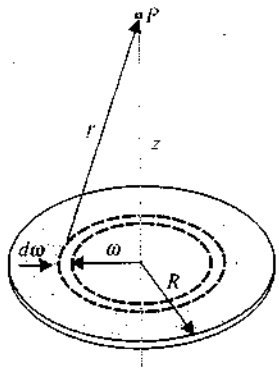


Fig. 3.19

Note: *As $R \to \infty$ (i.e., for an infinite sheet of charge), $V \to \infty$. Hence, we cannot define absolute potential of an infinite sheet of charge; but we can define potential difference between two points near the infinite sheet of charge.*

Concept Application Exercise 3.2

1. An electron moves from the positive to negative terminal of a battery (9 V). How much potential energy did it gain or lose?
2. We want to produce a proton with kinetic energy of 4.3×10^{-15} J. Through what difference of potential should we accelerate the proton in order to obtain this kinetic energy assuming that it starts from rest and no other forces are present?
3. The potential produced by a point charge is $V = kQ/r$. Use this information to determine the shape of equipotential surface?
4. **a.** A circle is drawn with centre as a charge $+q$. What is the work done in moving a charge $+q$ from B to C along the circumference of the circle?

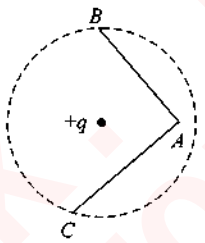


Fig. 3.20

b. In the above question, if the charge $+q$ is first taken from B to A and then from A to C, on which path, the magnitude of work is greater?

5. A uniform electric field E exists between two oppositely charged plates (Fig. 3.21). What will be the work done in moving a charge q along a closed rectangular path?

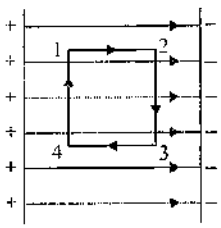


Fig. 3.21

6. Show that if at some part of a field the lines of force have the form of concentric circles whose centres are at point O (as given in Fig. 3.22), the field intensity at each point in this part of the field should be inversely proportional to the distance from the point O.

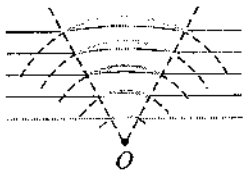


Fig. 3.22

7. If you carry out the integral of the electric field $\int \vec{E} \cdot \vec{dl}$ for a closed path like that as shown in Fig. 3.23, the integral will always be equal to zero, independent of the shape of the path and independent of where charges may be located relative to the path. Explain why.

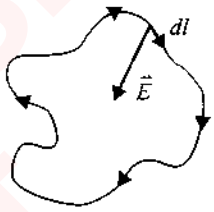


Fig. 3.23

POTENTIAL DUE TO AN ELECTRIC DIPOLE

Suppose an electric dipole consists of two charges $-q$ and $+q$ at B and A, respectively, separated by a distance $2l$ (as shown in Fig. 3.24).

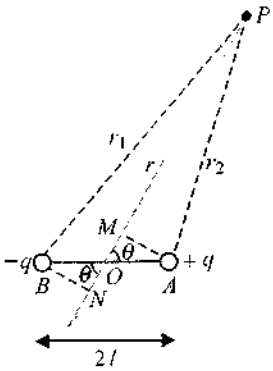


Fig. 3.24

We have to calculate the electric potential at any point P where

$$OP = r, \angle AOP = \theta$$

Let $BP = r_1$ and $AP = r_2$. Draw $AM \perp PO$ and $BN \perp PO$ produced backwards.

Then,

$$OM = ON$$

$$\text{In } \Delta OMA, \frac{OM}{OA} = \cos\theta$$

$$\therefore OM = OA\cos\theta = l\cos\theta = ON$$

Hence, $r_1 = BP \simeq NP = OP + ON = r + l\cos\theta$ (i)

and $r_2 = AP \simeq MP = OP - OM = r - l\cos\theta$ (ii)

Therefore, the potential at point P due to both the charges $-q$ (at B) and $+q$ (at A) can be written as

$$V = \frac{-q}{4\pi\varepsilon_0}\frac{1}{r_1} + \frac{q}{4\pi\varepsilon_0}\frac{1}{r_2} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r - l\cos\theta} - \frac{1}{r + l\cos\theta}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{r + l\cos\theta - r + l\cos\theta}{r^2 - l^2\cos^2\theta}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\cdot\frac{2l\cos\theta}{r^2 - l^2\cos^2\theta}$$

i.e., $$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p\cos\theta}{r^2 - l^2\cos^2\theta} \quad (\because p = q(2l))$$

Special Cases

1. If $l << r$, then electric dipole is very short. So, potential is given by $V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$.
2. If P lies on the axial line, we have $\theta = 0°$ so that
$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos 0°}{r^2 - l^2\cos^2 0°}$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2 - l^2} \quad (\because \cos 0° = 1)$$
3. If $l << r$, then the electric dipole is very short and $V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$ (for a point on the axial line).
4. If P lies on the equatorial line, then $\theta = 90°$ so that $V = 0$.

WORK DONE IN ROTATING AN ELECTRIC DIPOLE IN A UNIFORM ELECTRIC FIELD

Suppose an electric dipole of dipole moment p (= $q2l$) is rotated in a uniform electric field E through an angle θ from its stable equilibrium position.

The work done by an external agent in rotating the dipole further from this position through a very small angle $d\theta$ is

$$dW = \text{couple} \times \text{angular displacement} = (pE\sin\theta)\, d\theta$$

This work will be done on the dipole by external agent as it is being rotated against its natural tendency (which is to align itself along the direction of electric field). Hence, the work done in rotating the dipole through the angle θ from its equilibrium position is

$$W = \int_0^\theta pE\sin\theta\, d\theta = pE\,[-\cos\theta]_0^\theta$$

$$= pE\,[-\cos\theta + \cos 0] = pE\,[1 - \cos\theta]$$

$$\Rightarrow \quad W_{\text{external}} = pE\,(1 - \cos\theta)$$

POTENTIAL ENERGY OF AN ELECTRIC DIPOLE IN A UNIFORM ELECTRIC FIELD

Work done in bringing an electric dipole from infinity to some point inside the field is equal to the potential energy of an electric dipole in an electric field.

Let us bring an electric dipole from infinity into a uniform electric field E in such a way that the dipole moment P is always along the direction of the field.

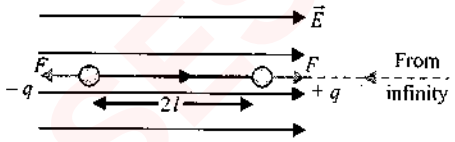


Fig. 3.25

It is clear from Fig. 3.25, as the dipole is brought from infinity into the field, the charge $-q$ covers $2l$ distances more than the charge $+q$. Therefore, the work done on $-q$ will be greater. Hence, net work done by the system in bringing the dipole from infinity into the field

$$W_{\text{by us}} = \text{Force on charge } (-q) \times \text{extra distance moved}$$

$$= -qE \times 2l = -pE \quad \text{(i)}$$

where $p(= q \times 2l)$ is the dipole moment of the electric dipole.

According to the law of conservation of energy, this work will be stored in the form of potential energy of the electric dipole. It is represented by U. Hence,

$$U = -pE \quad \text{(ii)}$$

In this position, the electric dipole will be in stable equilibrium inside the field.

If we now rotate the dipole in the field through an angle θ, then the work done on the dipole is given by

$$W = pE\,(1 - \cos\theta) \quad \text{(iii)}$$

This will result in an increase in the potential energy of the dipole. Hence, the potential energy of an electric dipole in the position θ is given by

$$U_0 = U + W = -pE + pE(1 - \cos\theta)$$

$$= -pE\cos\theta = -\vec{p}\cdot\vec{E} \quad \text{(iv)}$$

Note: ***When a dipole rotates from an initial orientation θ_i to another orientation θ_f, the work W done on the dipole by the electric field is $W = -\Delta U = -(U_f - U_i) = -pE(\cos\theta_i - \cos\theta_f)$, where U_f and U_i are calculated with Eq. (iv). If the change in orientation is caused by an applied torque (commonly said to be due to an external agent), then the work W_a done on the dipole by the applied torque is the negative of the work done on the dipole by the field; that is,***

$$W_a = -W = (U_f - U_i) = -pE(\cos\theta_f - \cos\theta_i)$$

Illustration 3.6 **Two point charges of 3.2×10^{-19} C and -3.2×10^{-19} C are separated from each other by 2.4×10^{-10} m. The dipole is situated in a uniform electric field of intensity 4×10^5 Vm^{-1}. Calculate the work done in rotating the dipole by 180°.**

Sol. Work done in rotating the dipole by angle θ is

$$W = pE(1-\cos\theta), \text{ here } \theta = 180°$$
$$= pE(1-[-1]) = 2pE = 2qd\,E$$
$$= 2\times3.2\times10^{-19}\times2.4\times10^{-10}\times4\times10^5\,\text{J}$$
$$= 61.44\times10^{-24}\,\text{J}$$

Illustration 3.7 **What is the potential energy of the charge and dipole system shown in Fig. 3.26?**

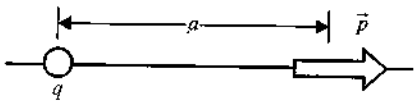


Fig. 3.26

Sol. $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{a^2}$ (Fig. 3.27)

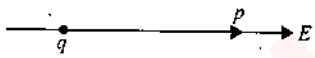


Fig. 3.27

$$U = -pE\cos\theta = -\frac{pq}{4\pi\varepsilon_0 a^2} \quad (\theta = 0°)$$

Concept Application Exercise 3.3

1. An electric dipole of length 4 cm, when placed with its axis making an angle of 60° with a uniform electric field, experiences a torque of $4\sqrt{3}$ N m. Calculate the (a) magnitude of the electric field and (b) potential energy of the dipole, if the dipole has charges of ±8 nC.
2. An electric dipole consists of two opposite charges each of magnitude 1 μC separated by 2 cm. The dipole is placed in an external uniform field of 10^5 NC^{-1} intensity. Find the
   a. maximum torque exerted by the field on the dipole and
   b. work done in rotating the dipole through 180° starting from the position $\theta = 0°$.
3. Figure 3.28 shows four orientations of an electric dipole in an external electric field. Rank the orientations according to the
   a. magnitude of the torque on the dipole, and
   b. potential energy of the dipole, greatest first.

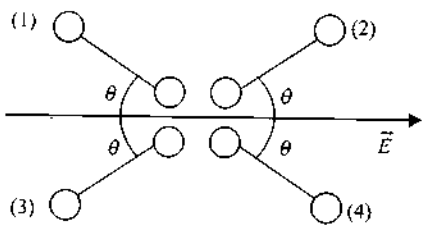


Fig. 3.28

4. **a.** In question 3, if the dipole rotates from orientation 1 to orientation 2, is the work done on the dipole by the field positive, negative, or zero?
   b. If, instead, the dipole rotates from the orientation 1 to orientation 4, is the work done by the field more than, less than or the same as in (a)?
5. The potential energies associated with four orientations of an electric dipole in an electric field are (1) $-5U_0$, (2) $-7U_0$, (3) $3U_0$ and (4) $5U_0$, where U_0 is positive. Rank the orientations according to the
   a. angle between the electric dipole moment $\vec{P}$ and the electric field $\vec{E}$, and
   b. magnitude of the torque on the electric dipole, greatest first.
6. A neutral water molecule (H_2O) in its vapour state has an electric dipole moment of magnitude 6.2×10^{-30} Cm.
   a. How far apart are the molecule's centres of positive and negative charge?
   b. If a molecule is placed in an electric field of 1.5×10^4 NC^{-1}, what maximum torque can the field exert on it? (Such a field can easily be set up in the laboratory)
   c. How much work must an external agent do to turn this molecule end for end in this field, starting from its fully aligned position, for which $\theta = 0$?
7. What is the potential energy of dipole with charge particles system as shown in Fig. 3.29?

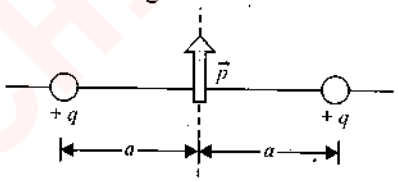


Fig. 3.29

8. State the following statements as True or False.
   a. Electric potential at any point on the bisector of a dipole is zero.
   b. A dipole experiences maximum torque at the position where potential energy is zero.
9. A charge $+q$ is carried from point $A(r, 135°)$ to point $B(r, 45°)$ following a path which is a quadrant of circle of radius r (Fig. 3.30). If the dipole moment is P, find the work done by the external agent (assume short dipole).

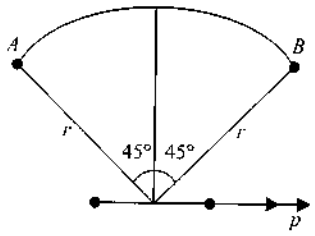


Fig. 3.30

Solved Examples

Example 3.8 **A hollow uncharged spherical conductor has inner radius a and outer radius b. A positive point charge $+q$ is in the cavity at the centre of the sphere (Fig. 3.31). Find the potential $V(r)$ everywhere, assuming that $V = 0$ at $r \to \infty$.**

Sol. The conductor is an equipotential volume, so V is constant for $a \le r \le b$. The field lines inside the cavity must end on the inner surface of the cavity, so this surface has an induced charge of $-q$. Since the shell is uncharged, a positive charge $+q$ is on the outer surface.

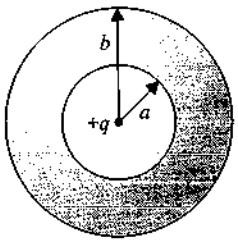


Fig. 3.31

The three charges q at the centre, $-q$ on the inner surface, and $+q$ on the outer surface produce a field $E_r = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$ for $r > b$, so the potential for $r > b$ is $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$.

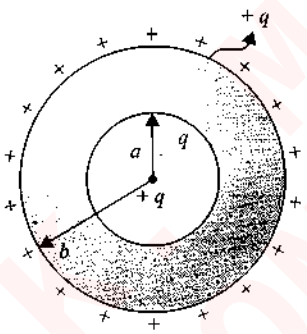


Fig. 3.32

Outside the shell, $V(r)$ is the same as that due to a point charge q at the origin. Choosing $V = 0$ at $r = \infty$, we have

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}, \quad r \ge b$$

At $r = b$, the potential is $\frac{1}{4\pi\varepsilon_0}\frac{q}{b}$. V remains at this constant value throughout the spherical shell from $r = b$ to $r = a$, so

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{b}, \quad a \le r \le b$$

For any point inside the cavity $(r < a)$

$V(r)$ = Electric potential due to 'q' at centre + Electric potential due charge distributed on spherical surface of radius 'a' + Electric potential due to charge distributed on spherical surface of radius 'b'.

$$V(r) = V_q + V_{a,\text{ inside}} + V_{b,\text{ outside}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{(q)}{r} + \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{a} + \frac{1}{4\pi\varepsilon_0}\frac{(q)}{b}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r} + \frac{1}{4\pi\varepsilon_0}\frac{q}{b} - \frac{1}{4\pi\varepsilon_0}\frac{q}{a}, \quad r \le a$$

Figure 3.33 shows the electric potential as a function of distance from the centre of the cavity.

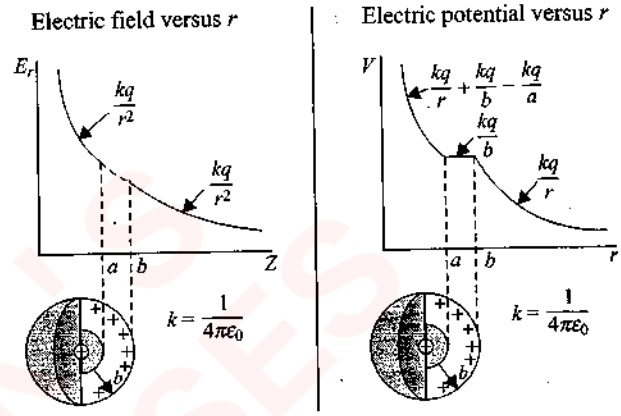


Fig. 3.33

Example 3.2 **Figure 3.34 shows two concentric conducting shells of radii r_1 and r_2 carrying uniformly distributed charges q_1 and q_2, respectively. Find out an expression for the potential of each shell.**

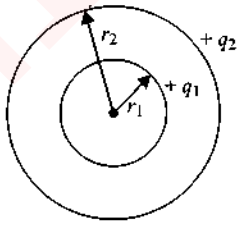


Fig. 3.34

Sol. The potential of each sphere consists of two points:

- one due to its own charge, and
- second due to the charge on the other sphere.

Using the principle of superposition, we have

$$V_1 = V_{r_1,\text{ surface}} + V_{r_2,\text{ inside}} \text{ and } V_2 = V_{r_1,\text{ outside}} + V_{r_2,\text{ surface}}$$

Hence, $$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2}$$

and $$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_2} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2}$$

Example 3.3 **In the previous example, if the charge $q_1 = +q_0$ and the outer shell is earthed, then**

a. determine the charge on the outer shell, and

b. find the potential of the inner shell.

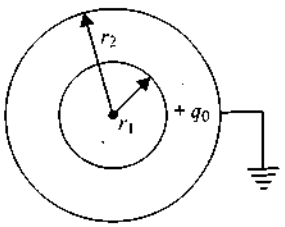


Fig. 3.35

Sol.

a. We know that charge on facing surfaces is equal and opposite. So, if charge on inner sphere is q_0, then charge on inner surface of shell should be $-q_0$. Now, let charge on outer surface of shell be q_2.

As the shell is earthed, so its potential should be zero. So,

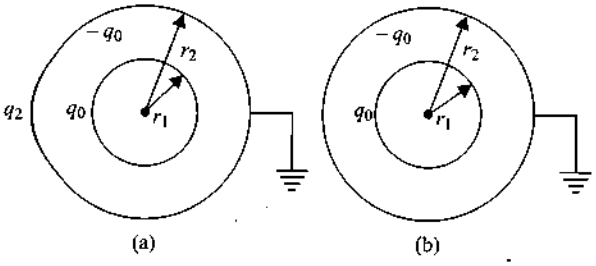


Fig. 3.36

$$V_{\text{shell}} = \frac{k\,q_0}{r_2} + \frac{k\,(-q_0)}{r_2} + \frac{k\,q_2}{r_2} = 0 \Rightarrow q_2 = 0$$

Hence, charge on outer surface of shell is zero. Final charges appearing are shown in Fig. 3.36(b).

b. Potential of inner sphere:

$$V_i = \frac{k\,q_0}{r_1} + \frac{k\,(-q_0)}{r_2} = \frac{q_0}{4\pi\varepsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

Example 3.4 **Consider two concentric spherical metal shells of radii a and b, where $b > a$. The outer shell has charge Q, but the inner shell has no charge. Now, the inner shell is grounded. This means that the inner shell will come at zero potential and that electric field lines leave the outer shell and go to infinity but other electric field lines leave the outer shell and end on the inner shell.**

a. Find the charge on the inner shell.

b. Find the potential of outer sphere.

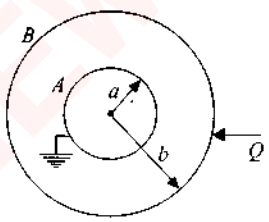


Fig. 3.37

Sol. a. When an object is connected to earth (grounded), its potential is reduced to zero. Let q' be the charge on A after it is earthed as shown in Fig. 3.37.

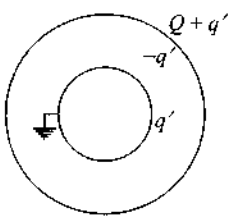


Fig. 3.38

The charge q' on A induces $-q'$ on inner surface of B and $+q'$ on outer surface of B. In equilibrium, the charge distribution is as shown in Fig. 3.38.

Potential of inner sphere = potential due to charge on A + potential due to charge on $B = 0$

$$V_A = \frac{q'}{4\pi\varepsilon_0 a} - \frac{q'}{4\pi\varepsilon_0 b} + \frac{Q+q'}{4\pi\varepsilon_0 b} = 0 \text{ or } q' = -Q\left(\frac{a}{b}\right)$$

This implies that a charge $+Q(a/b)$ has been transferred to the earth leaving negative charge on A.

Final charge distribution will be as shown in Fig. 3.39.

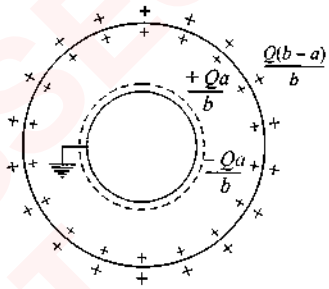


Fig. 3.39

As $b > a$, so charge on the outer surface of outer shell will be $\frac{Q(b-a)}{b} > 0$.

b. Potential of outer surface V_B = Potential due to charge on A + Potential due to charge on B.

$$V_B = V_{a,\text{ out}} + V_{b,\text{ both surface}} = \frac{1}{4\pi\varepsilon_0}\frac{q'}{b} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{b}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\left(-Q\frac{a}{b}\right)}{b} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{b} = \frac{Q(b-a)}{4\pi\varepsilon_0 b^2}$$

Example 3.5 **Two circular loops of radii 0.05 and 0.09 m, respectively, are put such that their axes coincide and their centres are 0.12 m apart. Charge of 10^{-6} coulomb is spread uniformly on each loop. Find the potential difference between the centres of loops.**

Sol. The potential at the centre of a ring will be due to charge on both the rings and as every element of a ring is at a constant distance from the centre, so

$$V_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{R_1} + \frac{q_2}{\sqrt{R_2^2 + x^2}}\right]$$

$$= 9\times10^9\left[\frac{10^{-4}}{5} + \frac{10^{-4}}{\sqrt{9^2+12^2}}\right]$$

$$= 9\times10^5\left[\frac{1}{5} + \frac{1}{15}\right] = 2.40\times10^5\text{ V}$$

Similarly, $$V_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_2}{R_2} + \frac{q_1}{\sqrt{R_1^2 + x^2}}\right]$$

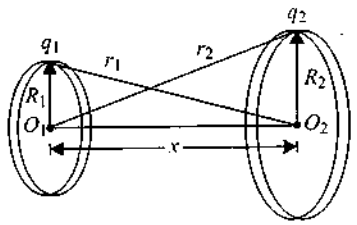


Fig. 3.40

or, $$V_2 = 9\times10^5\left[\frac{1}{9}+\frac{1}{13}\right] = \frac{198}{117}\times10^5$$

$$V_2 = 1.69\times10^5 \text{ V}$$

So, $$V_1 - V_2 = (2.40-1.69)\times10^5 = 71 \text{ kV}$$

Example 3.6 **A very small sphere of mass 80 g having a charge q is held at a height of 9 m vertically above the centre of a fixed conducting sphere of radius 1 m, carrying an equal charge q. When released, it falls until it is repelled back just before it comes in contact with the sphere as shown in Fig. 3.41. Calculate the charge q. [g = 10 m/s²]**

Sol. Keeping in mind that here both electric and gravitational potential energies are changing and for an external point, a charged sphere behaves as if whole of its charge were concentrated at its centre. Applying conservation of energy between initial and final positions, we have

$$\frac{1}{4\pi\varepsilon_0}\frac{qq}{9} + mg\times9 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{1} + mg\times1$$

as K. E. is zero at both locations

or $$q^2 = \frac{80\times10^{-3}\times10}{10^9} \text{ or } q = 20\sqrt{2}\ \mu\text{C}$$

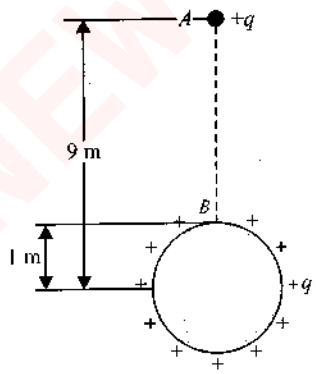


Fig. 3.41

Example 3.7 **A circular ring of radius R with uniform positive charge density λ per unit length is located in the y–z plane with its centre at the origin O. A particle of mass m and positive charge q is projected from the point $p[-\sqrt{3}R, 0, 0]$ on the negative x-axis directly towards O, with initial speed v. Find the smallest (non-zero) value of the speed such that the particle does not return to P? (IIT-JEE, 1993)**

Sol. As the electric field at the centre of a ring is zero, the particle will not come back due to repulsion if it crosses the centre (Fig. 3.42), i.e.,

$$\frac{1}{2}mv^2 + \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r} > \frac{1}{4\pi\varepsilon_0}\frac{qQ}{R}$$

But here, $Q = 2\pi R\lambda$ and $r = \sqrt{(\sqrt{3}R)^2 + R^2} = 2R$

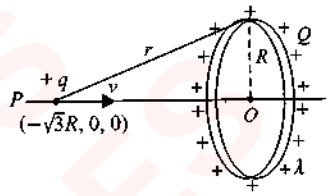


Fig. 3.42

So, $$\frac{1}{2}mv^2 > \frac{1}{4\pi\varepsilon_0}\frac{2\pi R\lambda q}{R}\left[1-\frac{1}{2}\right] \text{ or } v > \sqrt{\left(\frac{\lambda q}{2\varepsilon_0 m}\right)},$$

So, $$v_{min} = \sqrt{\left(\frac{\lambda q}{2\varepsilon_0 m}\right)}$$

Example 3.8 **A non-conducting disk of radius a and uniform positive surface charge density σ is placed on the ground, with its axis vertical. A particle of mass m and positive charge q is dropped, along the axis of the disk, from a height H with zero initial velocity. The particle has $q/m = 4\varepsilon_0 g/\sigma$.**

a. Find the value of H if the particle just reaches the disk.
b. Sketch the potential energy of the particle as a function of its height and find its equilibrium position.

(IIT-JEE, 1999)

Sol.

a. Given that a = radius of disk, σ = surface charge density, $q/m = 4\varepsilon_0 g/\sigma$

The K.E. of the particle, when it reaches the disk can be taken as zero.

Potential due to a charged disk at any axial point situated at a distance x from O is

$$V(x) = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{a^2+x^2} - x\right]$$

Hence, $$V(H) = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{a^2+H^2} - H\right] \text{ and } V(O) = \frac{\sigma a}{2\varepsilon_0}$$

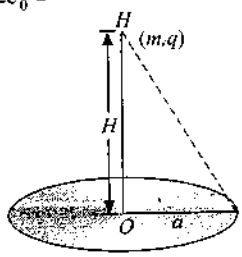


Fig. 3.43

According to the law of conservation of energy, we have loss of gravitation potential energy = gain in electric potential energy

$$mg\,H = q\Delta V = q[V(0) - V(H)]$$

$$= q[a - \{\sqrt{(a^2+H^2)} - H\}]\frac{\sigma}{2\varepsilon_0} \qquad \text{(i)}$$

From the given relation $\frac{\sigma}{2\varepsilon_0} = 2\,mg$ (given)

Putting this in Eq. (i), we get

$$mg\,H = 2mg[a - \{\sqrt{(a^2+H^2)} - H\}]$$

or $H = 2[a + H - \sqrt{(a^2+H^2)}]$

$= 2a + 2H - 2\sqrt{(a^2+H^2)}$ or $2\sqrt{(a^2+H^2)} = H + 2a$

$4a^2 + 4H^2 = H^2 + 4a^2 + 4aH$ or $3H^2 = 4aH$ or $H = \frac{4a}{3}$

[$\because H = 0$ is not valid]

b. The total potential energy of the particle at height x is

$$U(x) = mgx + qV(x) = mgx + \frac{q\sigma}{2\varepsilon_0}(\sqrt{a^2+x^2} - x)$$

$$= mgx + 2mg + \left[\sqrt{(a^2+x^2)} - x\right]$$

$$= mg\,[2\sqrt{(a^2+x^2)} - x] \qquad \text{(ii)}$$

For equilibrium: $\frac{dU}{dx} = 0$

This gives $x = \frac{a}{\sqrt{3}}$

From Eq. (ii), graph between $U(x)$ and x will be as shown in Fig. 3.44.

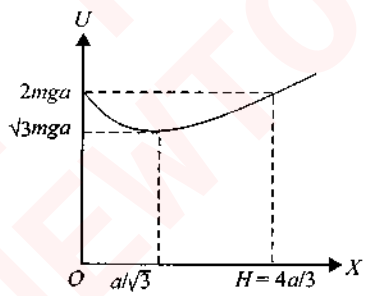


Fig. 3.44

Example 3.9 **Three concentric conducting shells of radii a, b and c are shown in Fig. 3.45. Charge on the shell of radius b is Q. If the key K is closed, find the charges on the innermost and outermost shells and ratio of charge densities of the shells. Given that $a : b : c = 1 : 2 : 3$.**

Sol. After closing the key, the innermost and outermost shells will be at the same potential. Let the charge on the outer shell be q and that on the inner shell be $-q$, the total charge on inner and outer shells is zero.

Potential on innermost shell,

V_a = sum of potentials due to $-q$, Q and q

$$= -\frac{q}{a} + \frac{Q}{b} + \frac{q}{c}$$

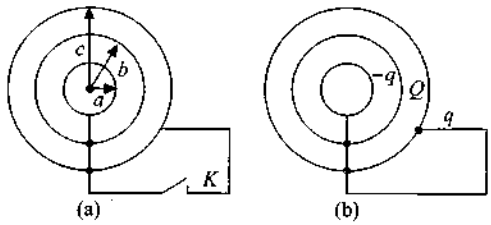


Fig. 3.45

Similarly, the potential on the outermost shell,

$$V_c = -\frac{q}{c} + \frac{Q}{c} + \frac{q}{c}$$

As $V_a = V_c$, we have $-\frac{q}{a} + \frac{Q}{b} + \frac{q}{c} = -\frac{q}{c} + \frac{Q}{c} + \frac{q}{c}$

From the given conditions, $c = 3a$, $b = 2a$.

Equation (i) now becomes

$$-\frac{q}{a} + \frac{Q}{2a} = -\frac{q}{3a} + \frac{Q}{3a}$$

or $q = \frac{Q}{4}$

Thus, the charge on outermost shell $= \frac{Q}{4}$

The charge on innermost shell $= -\frac{Q}{4}$

$$\sigma_a = \frac{1}{4\pi a^2}\left(-\frac{Q}{4}\right); \quad \sigma_b = \frac{+Q}{4\pi b^2} = \frac{Q}{4\pi(4a^2)};$$

$$\sigma_c = \frac{1}{4\pi c^2}\left(\frac{Q}{4}\right) = \frac{+Q}{16\pi(9a^2)}$$

Example 3.10 **A conducting sphere S_1 of radius r is attached to an insulating handle. Another conducting sphere S_2 of radius R is mounted on an insulating stand. S_2 is initially uncharged.**

S_1 is given a charge Q, brought into contact with S_2 and removed. S_1 is recharged such that the charge on it is again Q; and it is again brought into contact with S_2 and removed. This procedure is repeated n times.

a. Find the electrostatic energy of S_2 after n such contacts with S_1.

b. What is the limiting value of this energy as $n \to \infty$?

(IIT-JEE, 1998)

Sol.

a. When the spheres S_1 and S_2 come in contact, there is transfer of charges till the potentials of the two spheres become equal.

During first contact,

$V_1 = V_2$ [q_1 charge shifts from S_1 to S_2]

$$\frac{K(Q-q_1)}{r} = \frac{Kq_1}{R} \Rightarrow q_1 = Q\left(\frac{R}{R+r}\right)$$

During second contact, again $V_1 = V_2$

$$\frac{K[Q-(q_2-q_1)]}{r} = \frac{Kq_2}{R}$$

[$(q_2 - q_1)$ charge shifts from S_1 to S_2]

$$q_2 = Q\left[\frac{R}{R+r} + \left(\frac{R}{R+r}\right)^2\right]$$

On third contact, again $V_1 = V_2$

$$\frac{K[Q-(q_3-q_2)]}{r} = \frac{Kq_3}{R}$$

[$(q_3 - q_2)$ charge shifts from S_1 to S_2]

$$q_3 = Q\left[\frac{R}{R+r} + \left(\frac{R}{R+r}\right)^2 + \left(\frac{R}{R+r}\right)^3\right]$$

On nth contact, by symmetry $V_1 = V_2$

$$\frac{K[Q-(q_n-q_{n-1})]}{r} = \frac{Kq_n}{R}$$

[$(q_n - q_{n-1})$ charge shift from S_1 to S_2]

$$q_n = Q\left[\frac{R}{R+r} + \left(\frac{R}{R+r}\right)^2 + \cdots + \left(\frac{R}{R+r}\right)^n\right]$$

$$= \frac{QR}{r}\left[1-\left(\frac{R}{R+r}\right)^n\right]$$

The electrostatic energy of S_2 after n contacts is

$$U_n = \frac{1}{2}\frac{q_n^2}{C} = \frac{1}{2}\times\frac{1}{4\pi\varepsilon_0 R}\times\left\{\frac{QR}{r}\left[1-\left(\frac{R}{R+r}\right)^n\right]\right\}^2$$

b. The limiting value is

$$\lim_{n\to\infty} U_n = \lim_{n\to\infty}\left[\frac{1}{2}\times\frac{1}{4\pi\varepsilon_0 R}\left\{\frac{QR}{r}\left[1-\left(\frac{R}{R+r}\right)^n\right]\right\}^2\right]$$

$$= \frac{Q^2 R}{2(4\pi\varepsilon_0)r^2}$$

Example 3.11 **Two isolated metallic solid spheres of radii R and $2R$ are charged such that both of these have same charge density σ. The spheres are located far away from each other, and connected by a thin conducting wire. Find the new charge density on the bigger sphere.**

(IIT-JEE, 1996)

Sol. For sphere of radius R (Fig. 3.46), $\sigma = \dfrac{q_1}{4\pi R^2}$

$$q_1 = \sigma \times 4\pi R^2$$

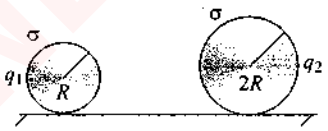


Fig. 3.46

For sphere of radius $2R$ (Fig. 3.47), $\sigma = \dfrac{q_2}{4\pi(2R)^2}$

$$q_2 = \sigma \times 16\pi R^2$$

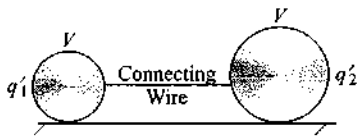


Fig. 3.47

When the two spheres are connected, the potential on the two spheres will be the same. There will be a rearrangement of charge for this to happen.

Let q_1' and q_2' be the new charges on the two spheres. Since the total charge remains the same, we get

$$q_1' + q_2' = q_1 + q_2 = \sigma \times 20\pi R^2 \quad \text{(i)}$$

Also, since $V_1 = V_2$

$$\frac{1}{4\pi\varepsilon_0}\frac{q_1'}{R} = \frac{1}{4\pi\varepsilon_0}\frac{q_2'}{2R}; \quad q_1' = \frac{q_2'}{2} \quad \text{(ii)}$$

Substituting the value of q_1' from Eq. (ii) in Eq. (i)

$$\frac{q_2'}{2} + q_2' = \sigma \times 20\pi R^2$$

$$\Rightarrow \quad \frac{3q_2'}{2} = \sigma \times 20\pi R^2$$

$$\Rightarrow \quad \frac{q_2'}{4\pi(2R)^2} = \frac{\sigma}{3}\times\frac{5}{2}$$

New charge density on the bigger sphere $= \dfrac{q_2'}{4\pi(2R)^2} = \dfrac{5\sigma}{6}$

Example 3.12

a. A charge of Q coulomb is uniformly distributed over a spherical volume of radius R metre. Obtain an expression for the energy of the system.

b. What will be the corresponding expression for the energy needed to completely disassemble the planet earth against the gravitational pull among its constituent particles? Assume the earth to be a sphere of uniform mass density. Calculate this energy, given the product of the mass and the radius of the earth to be 2.5×10^{31} kg m.

c. If the same charge of Q coulomb as in part (a) is given to a spherical conductor of the same radius R, what will be the energy of the system? **(IIT-JEE, 1992)**

Sol.

a. Let us consider a shell of the thickness dx at a distance x from the centre of a sphere (Fig. 3.48).

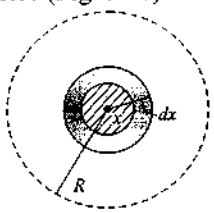


Fig. 3.48

The volume of the shell $= \left[\dfrac{4}{3}\pi(x+dx)^3 - \dfrac{4}{3}\pi x^3\right]$

$$= \frac{4}{3}\pi[(x+dx)^3 - x^3]$$

$$= \frac{4}{3}\pi x^3\left[\left(1+\frac{dx}{x}\right)^3 - 1\right]$$

$$= \frac{4}{3}\pi x^3\left[1+\frac{3dx}{x} - 1\right]$$

$$= \frac{4}{3}\pi x^3 \times \frac{3dx}{x} = 4\pi x^2 dx$$

Let ρ be the charge per unit volume of the sphere.

Charge of the shell $= dq = 4\pi x^2 \rho dx$

Potential at the surface of the sphere of radius x is

$$V = \frac{1}{4\pi\varepsilon_0} \times \frac{\rho \times \frac{4}{3}\pi x^3}{x} = \frac{\rho x^2}{3\varepsilon_0} \left[\because V = k\frac{q}{r}\right]$$

Work done in bringing the charge dq on the sphere of radius x is

$$dW = \frac{\rho x^2}{3\varepsilon_0} \times dq$$

i.e., $$dW = \frac{\rho x^2}{3\varepsilon_0} \times 4\pi x^2 \rho\, dx$$

Therefore, the work done in accumulating the charge Q over a spherical volume of radius R metre is

$$W = \int_0^R \frac{4\pi\rho^2}{3\varepsilon_0} x^4 dx = \frac{4\pi\rho^2}{3\varepsilon_0}\left[\frac{x^5}{5}\right]_0^R = \frac{4\pi\rho^2}{3\varepsilon_0}\frac{R^5}{5}$$

$$= \frac{4\pi}{3\varepsilon_0}\left(\frac{Q}{[4/3]\pi R^3}\right)^2 \frac{R^5}{5} = \frac{3Q^2}{20\pi\varepsilon_0 R}$$

This is also the energy stored in the system.

b. The above energy calculated is

$$E = \frac{3Q^2}{5\times(4\pi\varepsilon_0)R} = \frac{3KQ^2}{5R}, \text{ where } K = \frac{1}{4\pi\varepsilon_0}$$

In case of the Earth and gravitational pull, K may be replaced by G. Therefore, the energy required to disassemble the planet earth again the gravitational pull among its constituent particles is the work required to make the Earth from its constituent particles, i.e.,

$$E = \frac{3GM^2}{5R} \quad [\because Q \text{ is replaced by } M]$$

$$\left(\text{Using } F = \frac{Kq_1q_2}{r^2} \text{ and } F = \frac{Gm_1m_2}{r^2}\right)$$

But $$g = \frac{GM}{R^2} \Rightarrow gMR = \frac{GM^2}{R}$$

$$\therefore \quad E = \frac{3}{5}gMR = \frac{3}{5}\times 10\times 2.5\times 10^{31} = 1.5\times 10^{32} \text{ J}$$

c. During the charging process, let at any instant the spherical conductor has a charge q on its surface (Fig. 3.49). The potential at the surface $= \frac{1}{4\pi\varepsilon_0}\times\frac{q}{R}$

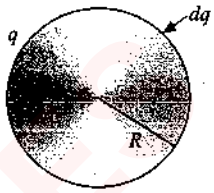


Fig. 3.49

The small amount of work done in bringing a charge dq move on the surface will be $dW = \frac{1}{4\pi\varepsilon_0}\times\frac{q}{R}\times dq$

Therefore, the total amount of work done in bringing charge Q on the surface of spherical conductor is

$$W = \frac{1}{4\pi\varepsilon_0 R}\int_0^Q q\, dq = \frac{1}{4\pi\varepsilon_0 R}\left[\frac{q^2}{2}\right]_0^Q = \frac{Q^2}{(8\pi\varepsilon_0 R)}$$

Alternative solution of part (c):

If we consider the charged spherical capacitor as an isolated capacitor, then the energy stored in the capacitor is given by

$$E = \frac{1}{2}\frac{Q^2}{C} \quad \text{(where } C = \text{capacitance of the capacitor)}$$

For an isolated capacitor, $C = 4\pi\varepsilon_0 R$.

$$\therefore \quad E = \frac{Q^2}{8\pi\varepsilon_0 R} = \text{energy of the system}$$

EXERCISES

Subjective Type

Solutions on page 3.30

1. The variation of electric potential for an electric field directed parallel to the x-axis is shown in the given graph (Fig. 3.50). Draw the variation of electric field strength with x-axis.

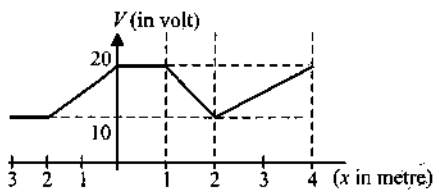


Fig. 3.50

2. Identical charges $-q$ each are placed at eight corners of a cube of each side b, find the electrostatic potential energy of a charge $+q$ which is placed at the centre of the cube.
3. At a point due to a point charge, the values of electric field intensity and potential are 32 NC^{-1} and 16 JC^{-1}, respectively. Calculate the
   a. magnitude of the charge, and
   b. distance of the charge from the point of observation.
4. Four charges $+q$, $-q$, $+q$ and $-q$ are placed in order on the four consecutive corners of a square of side a. Find the work done in interchanging the positions of any two neighbouring charges of opposite sign.
5. Water from a tap maintained at a constant potential of V is allowed to fall by drops of radius r through a small hole into a hollow conducting sphere of radius R standing on an insulating stand until it fills the entire sphere. Find the potential of the hollow conductor, after it is completely filled with water.
6. Three point charges of 0.1 C each are placed at the corners of an equilateral triangle with side L = 1 m. If this system is supplied energy at the rate of 1 kW, how much time will be

required to move one of the charges on to the mid-point of the line joining the other two?

7. Figure 3.51 shows a large conducting ceiling having uniform charge density σ below which a charge particle of charge q_0 and mass m is hung from point O, through a small string of length l. Calculate the minimum horizontal velocity v required for the string to become horizontal.

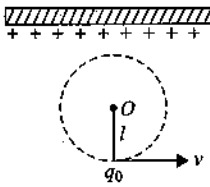


Fig. 3.51

8. A non-conducting sphere of radius $R = 5$ cm has its centre at the origin O of coordinate system as shown in Fig. 3.52. Its has two spherical cavities of radius $r = 1$ cm, whose centres are at (0, 3 cm), (0, –3 cm), respectively, and solid material of the sphere has uniform positive charge density $\rho = 1/\pi$ μCm^{-3}. Calculate electric potential at point P (4 cm, 0).

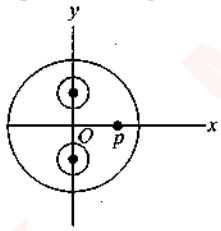


Fig. 3.52

9. Two identical thin rings, each of radius R metre, are coaxially placed at a distance of R metre from each other. If Q_1 coulomb and Q_2 coulomb are the charges uniformly spread on the two rings, find the work done in moving a charge q from the centre of one ring to that of the other.

10. Three conducting spherical shells have radii a, b and c such that $a < b < c$ (Fig. 3.53). Initially, the inner shell is uncharged, the middle shell has a positive charge Q and the outer shell has a negative charge $-Q$.
   a. Find the electric potential of the three shells.
   b. If the inner and outer shells are now connected by a wire that is insulated as it passes through the middle shell, what is the electric potential of each of the three shells? Also, what is the final charge on each shell?

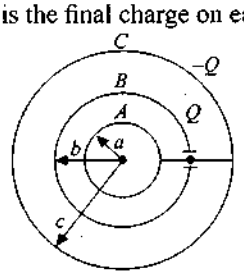


Fig. 3.53

11. Figure 3.54 shows three concentric spherical conductors A, B and C with radii R, $2R$ and $4R$, respectively. A and C are connected by a conducting wire and B is uniformly charged (charge = $+Q$). Find
   a. charges on conductors A and C,
   b. potentials of A and B.

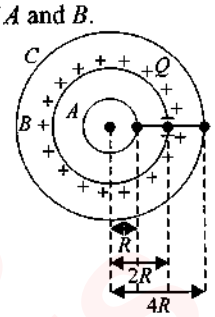


Fig. 3.54

12. Two concentric shells of radii R and $2R$ are shown in Fig. 3.55. Initially, a charge q is imparted to the inner shells. Now, key K_1 is closed and opened and then key K_2 is closed and opened. After the keys K_1 and K_2 are alternately closed n times each, find the potential difference between the shells. Note that finally the key K_2 remains closed.

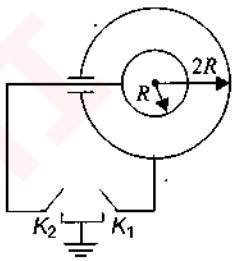


Fig. 3.55

13. Three charges each of value q are placed at the corners of an equilateral triangle. A fourth charge Q is placed at the centre of the triangle.
   a. Find the net force on charge 'q'.
   b. If $Q = -q$, will the charges at the corners move towards the centre or fly away from it?
   c. For what value of Q at O will charges remain stationary?
   d. In situation (c), how much work is done in removing the charges to infinity? **(IIT-JEE, 1978)**

14. A small ball of mass 2×10^{-3} kg having a charge of 1 μC is suspended by a string of length 0.8 m. Another identical ball having the same charge is kept at the point of suspension. Determine the minimum horizontal velocity which should be imparted to the lower ball so that it can make a complete revolution.

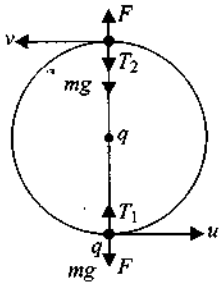


Fig. 3.56

15. Two fixed charges $-2Q$ and Q are located at the points with coordinates $(-3a, 0)$ and $(+3a, 0)$, respectively, in the x–y plane.

a. Show that all points in the x–y plane where the electric potential due to the two charges is zero lie on a circle. Find its radius and the location of its centre.

b. Give the expression $V(x)$ at a general point on the x-axis and sketch the function $V(x)$ on the whole x-axis.

c. If a particle of charge $+q$ starts from rest at the centre of the circle, show by a short quantitative argument that the particle eventually crosses the circle. Find its speed when it does so.

(IIT-JEE, 1991)

16. A point charge q is located at the centre O of an uncharged spherical capacitor provided with a small orifice. The inside and outside radii of the capacitor are a and b, respectively (Fig. 3.57). What amount of work has to be performed to slowly transfer the charge q bit by bit from the point O through the orifice to infinity ?

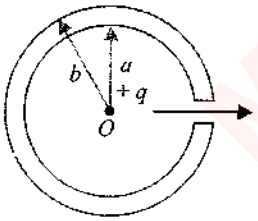


Fig. 3.57

17. Four point charges +8 μC, −1 μC, −1 μC and +8 μC are fixed at the points $-\sqrt{\frac{27}{2}}$ m, $-\sqrt{\frac{3}{2}}$ m, $+\sqrt{\frac{3}{2}}$ m and $+\sqrt{\frac{27}{2}}$ m, respectively, on the y-axis. A particle of mass 6×10^{-4} kg and charge +0.1 μC moves along the x-direction. Its speed at $x = +\infty$ is V_0. Find the least value of V_0 for which the particle will cross the origin. Find also the kinetic energy of the particle at the origin. Assume that space is gravity free. Given $\frac{1}{4\pi\varepsilon_0} = 9\times10^9 \text{Nm}^2\text{C}^{-2}$. **(IIT-JEE, 2000)**

18. Charges $+q$ and $-q$ are located at the corners of a cube of side as shown in Fig. 3.58. Find the work done to separate the charges to infinite distance. **(IIT-JEE, 2003)**

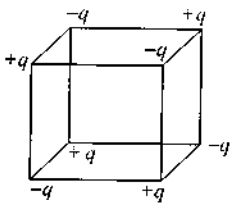


Fig. 3.58

19. Two uniformly charged large plane sheets S_1 and S_2 having charge densities σ_1 and $\sigma_2 (\sigma_1 > \sigma_2)$ are placed at a distance d parallel to each other. A charge q_0 is moved along a line of length a $(a < d)$ at an angle 45° with the normal to S_1. Calculate the work done by the electric field. **(IIT-JEE, 2004)**

20. Figure 3.59 shows two dipole moments parallel to each other and placed at a distance x apart. What is the magnitude of force of interaction? What is the nature of force, attractive or repulsive?

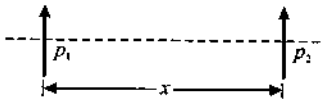


Fig. 3.59

21. Two dipoles p_1 and p_2 are placed along the same axis at a distance x apart, as shown in Fig. 3.60. What is the magnitude of force of interaction? What is the nature of force, attractive or repulsive?

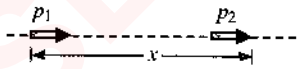


Fig. 3.60

22. A short dipole is placed along x-axis at $x = x$ (Fig. 3.61).

a. Find the force acting on the dipole due to a point charge q placed at origin.

b. Find the force on dipole if the dipole is rotated by 180° about z-axis.

Fig. 3.61

c. Find the force on dipole if the dipole is rotated by 90° anticlockwise about z-axis, i.e., it becomes parallel to y-axis.

23. For the electrostatic charge system as shown in Fig. 3.62,

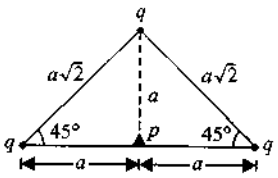


Fig. 3.62

a. find the net force on electric dipole.

b. Also, find electrostatic energy of the system.

24. Four charge particles each having charge Q are fixed at corners of base (at A, B, C and D) of a square pyramid with slant length 'a' ($AP = BP = DP = PC = a$). A charge $-Q$ is fixed at point P. A dipole with dipole moment P is placed at the centre of base and perpendicular to its plane as shown in Fig. 3.63.

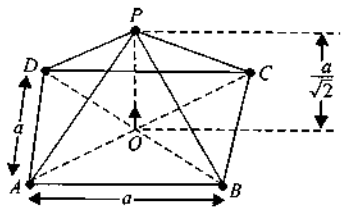


Fig. 3.63

Find
a. force on dipole due to charge particles.
b. potential energy of the system.

25. Three identical dipoles with charges q and $-q$ and separation a between the charges are placed on the corners of an equilateral Δ of side d as shown in Fig. 3.64. Find the interaction energy of the system ($a << d$).

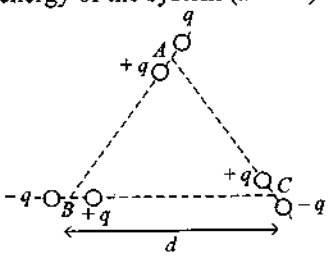


Fig. 3.64

Objective Type

Solutions on page 3.36

1. If a conductor is electrically neutral, then
   a. net charge on it should be zero
   b. potential on it should be zero
   c. both charge and potential should be zero
   d. none of them may not be zero

2. An electron is released from rest in a region of space with a non-zero electric field. Which of the following statements is true?
   a. The electron will begin moving towards a region of higher potential.
   b. The electron will begin moving towards a region of lower potential.
   c. The electron will begin moving along a line of constant potential.
   d. Nothing can be concluded unless the direction of the electric field is known.

3. Figure 3.65 shows two parallel surfaces A and B at the same potential, kept at a small distance r from each other. A point charge q is taken from the surface A to B.

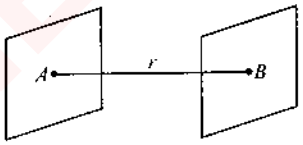


Fig. 3.65

The amount of work done is

a. $\frac{q^2}{2\pi\varepsilon_0 r}$ b. $\frac{q^2}{8\pi\varepsilon_0 r}$ c. $\frac{q^2}{4\pi\varepsilon_0 r}$ d. zero

4. Inside a hollow charged spherical conductor, the potential
   a. is constant
   b. varies directly as the distance from the centre
   c. varies inversely as the distance from the centre
   d. varies inversely as the square of the distance from the centre

5. A hollow metal sphere of radius 5 cm is charged such that the potential on its surface is 10 V. The potential at the centre of the sphere is
   a. 0 V b. 10 V
   c. same as at point 5 cm away from the surface
   d. same as at a point 20 cm away from the surface

(IIT-JEE, 1983)

6. The electric potential at the surface of an atomic nucleus (Z = 50) of radius of 9×10^{-15} m is
   a. 80 V b. 8×10^6 V c. 9 V d. 9×10^5 V

7. A ball of mass 1 g carrying a charge 10^{-8} C moves from a point A at potential 600 V to a point B at zero potential. The change in its K.E. is
   a. -6×10^{-6} erg b. -6×10^{-6} J
   c. 6×10^{-6} J d. 6×10^{-6} erg

8. A large insulated sphere of radius r charged with Q units of electricity is placed in contact with a small insulated uncharged sphere of radius r' and is then separated. The charge on the smaller sphere will now be
   a. $\frac{Q(r'+r)}{r'}$ b. $\frac{Q(r'+r)}{r}$ c. $\frac{Qr}{r'+r}$ d. $\frac{Qr'}{r'+r}$

9. Two point charges 4 μC and −2 μC are separated by a distance of 1 m in air. At what point in between the charges and on the line joining the charges, is the electric potential zero?
   a. In the middle of the two charges
   b. 1/3 m from 4 μC
   c. 1/3 m from −2 μC
   d. Nowhere the potential is zero

10. Two conducting spheres of radii r_1 and r_2 have same electric field near their surfaces. The ratio of their electrical potentials is
   a. r_1^2/r_2^2 b. r_2^2/r_1^2 c. r_1/r_2 d. r_2/r_1

11. Three charges $2q$, $-q$ and $-q$ are located at the vertices of an equilateral triangle. At the centre of the triangle
   a. the field is zero but potential is non-zero
   b. the field is non-zero but potential is zero
   c. both field and potential are zero
   d. both field and potential are non-zero

12. The variation of potential with distance R from fixed point is shown in Fig. 3.66.

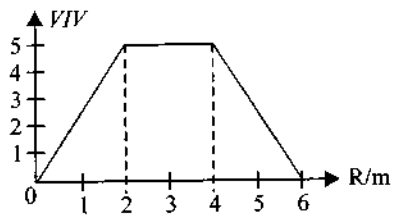


Fig. 3.66

The electric field at $R = 5$ m is
a. 2.5 Vm^{-1} b. −2.5 Vm^{-1} c. 0.4 Vm^{-1} d. −0.4 Vm^{-1}

13. When a 2 μC of charge is carried from point A to point B, the amount of work done by electric field is 50 μJ. What is the potential difference and which point is at a higher potential?
   a. 25 V, B b. 25 V, A

c. 20 V, B d. Both are at same potential

14. The work done in taking a unit positive charge from P to A is W_A and from P to B is W_B.

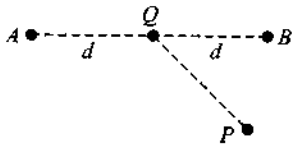


Fig. 3.67

Then

a. $W_A > W_B$ b. $W_A < W_B$ c. $W_A = W_B$ d. $W_A + W_B = 0$

15. Four charges $+q$, $-q$, $+q$ and $-q$ are put together at four corners of a square as shown in Fig. 3.68. The work done by external agent in assembling this configuration is

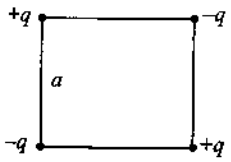


Fig. 3.68

a. zero b. $-2.59kq^2/a$
c. $+2.59kq^2/a$ d. none of these

16. For an isolated charged conductor shown in Fig. 3.69, the potentials at points A, B, C and D are V_A, V_B, V_C, and V_D, respectively. Then

a. $V_A = V_B > V_C > V_D$ b. $V_D > V_C > V_B = V_A$
c. $V_D > V_C > V_B > V_A$ d. $V_D = V_C = V_B = V_A$

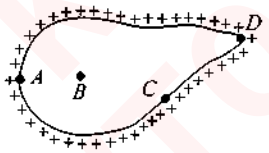


Fig. 3.69

17. The electric field in a region surrounding the origin is uniform and along the x-axis. A small circle is drawn with the centre at the origin cutting the axes at points A, B, C and D having coordinates $(a, 0)$, $(0, a)$, $(-a, 0)$ and $(0, -a)$, respectively, as shown in Fig. 3.70. Then, the potential is minimum at

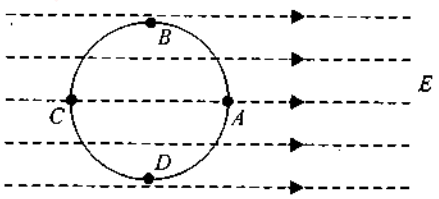


Fig. 3.70

a. A b. B c. C d. D

18. A small conducting sphere of radius a, carrying a charge of $+Q$, is placed inside an equal and oppositely charged conducting shell of radius b such that their centres coincide. Determine the potential at a point which is at a distance of c from centre such that $a < c < b$.

a. $k(Q/c + Q/b)$ b. $k(Q/a + Q/b)$
c. $k(Q/a - Q/b)$ d. $k(Q/c - Q/b)$

19. Two metal spheres (radii r_1, r_2 with $r_1 < r_2$) are very far apart but are connected by a thin wire. If their combined charge is Q, then what is their common potential?

a. $kQ/(r_1 + r_2)$ b. $kQ/(r_1 - r_2)$ c. $-kQ/(r_1 + r_2)$ d. $-kQ/r_1r_2$

20. Mark the correct statement:

a. If E is zero at certain point, then V should be zero at that point.
b. If E is not zero at certain point, then V should not be zero at that point.
c. If V is zero at certain point, then E should be zero at that point.
d. If V is zero at certain point, then E may or may not be zero.

21. Variation in potential is maximum if one goes

a. along the line of force
b. perpendicular to the line of force
c. in any direction
d. none of these

22. An uncharged conductor A is brought near a positively charged conductor B. Then

a. the charge on B will increase but the potential of B will not change
b. the charge on B will not change but the potential of B will decrease
c. the charge on B will decrease but the potential of B will not change
d. the charge on B will not change but the potential of B will increase

23. Two spherical conductors of radii R_1 and R_2 are separated by a distance much larger than the radius of the either sphere. The spheres are connected by a conducting wire as shown in Fig. 3.71. If the charges on the spheres in equilibrium are q_1 and q_2, respectively, what is the ratio of the field strength at the surfaces of the spheres?

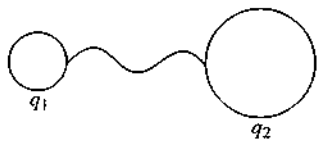


Fig 3.71

a. R_2/R_1 b. R_2^2/R_1^2 c. R_1/R_2 d. R_1^2/R_2^2

24. There is an electric field E in the x-direction. If the work done by electric field in moving a charge of 0.2 C through a distance of 2 m along a line making an angle 60° with the x-axis is 4 J, then what is the value of E?

a. $\sqrt{3}$ NC^{-1} b. 4 NC^{-1} c. 5 NC^{-1} d. 20 NC^{-1}

25. Some equipotential surfaces are shown in Fig. 3.72. The magnitude and direction of the electric field is

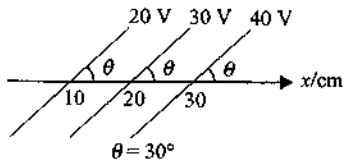


Fig. 3.72

a. 100 Vm^{-1} making angle 120° with the x-axis
b. 200 Vm^{-1} making angle 60° with the x-axis
c. 200 Vm^{-1} making angle 120° with the x-axis
d. none of the above

26. In moving from A to B along an electric field line, the work done by the electric field on an electron is 6.4×10^{-19} J. If ϕ_1 and ϕ_2 are equipotential surfaces, then the potential difference $V_C - V_A$ is

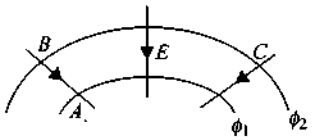


Fig. 3.73

a. −4 V b. 4 V c. zero d. 6.4 V

27. An electron is taken from point A to point B along the path AB in a uniform electric field of intensity $E = 10$ Vm^{-1}. Side $AB = 5$ m and side $BC = 3$ m. Then, the amount of work done on electron by us is

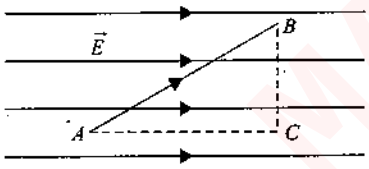


Fig 3.74

a. 50 eV b. 40 eV c. −50 eV d. −40 eV

28. Two identical rings P and Q of radius 0.1 m are mounted coaxially at a distance 0.5 m apart. The charges on the two rings are 2 and 4 μC, respectively. The work done in transferring a charge of 5 μC from the centre of P to that of Q is

a. 1.28 J b. 0.72 J c. 0.144 J d. 2.24 J

29. n charged drops, each of radius r and charge q, coalesce to form a big drop of radius R and charge Q. If V is the electric potential and E is the electric field at the surface of a drop, then

a. $E_{big} = n^{2/3}E_{small}$ b. $V_{big} = n^{1/3}V_{small}$
c. $E_{small} = n^{2/3}E_{big}$ d. $V_{big} = n^{2/3}V_{small}$

30. A small positively charged sphere is placed inside a positively charged spherical shell. What happens if the inner sphere is connected with the outer shell by a conducting wire?

a. The entire charge of inner sphere will be transferred to outer shell and then both will be at same potential.
b. The entire charge of inner sphere will be transferred to outer shell and then both will be at different potential.
c. The entire charge of outer shell will be transferred to inner sphere and then both will be at same potential.
d. Nothing can be predicted.

31. At a point in space, the electric field points towards north. In the region surrounding this point, the rate of change of potential will be zero along

a. north b. south c. north-south d. east-west

32. A spherical charged conductor has surface density of charge as σ. The electric field intensity on its surface is E. If the radius of surface is doubled, keeping σ unchanged, what will be the electric field intensity on the new sphere?

a. $E/2$ b. $E/4$ c. $2E$ d. E

33. In the previous question, if V is the electric potential of the first sphere, what would be the electric potential of the second sphere?

a. $2V$ b. $V/2$ c. $V/4$ d. V

34. Which of the following is discontinuous across a charged conducting surface?

a. Electric potential
b. Electric intensity
c. Both electric potential and intensity
d. None of the above

35. The electric field lines are closer together near object A than they are near object B. We can conclude

a. the potential near A is greater than the potential near B
b. the potential near A is less than the potential near B
c. the potential near A is equal to the potential near B
d. nothing about the relative potentials near A and B

36. As shown in Fig. 3.75, a dust particle with mass $m = 5.0 \times 10^{-9}$ kg and charge $q_0 = 2.0$ nC starts from rest at point a and moves in a straight line to point b. What is its speed v at point b?

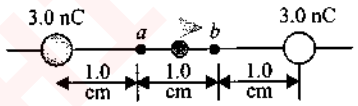


Fig. 3.75

a. 26 ms^{-1} b. 34 ms^{-1} c. 46 ms^{-1} d. 14 ms^{-1}

37. Charges $-q$, Q and $-q$ are placed at an equal distance on a straight line. If the total potential energy of the system of three charges is zero, then find the ratio Q/q.

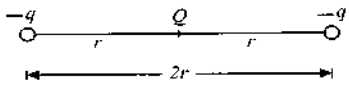


Fig. 3.76

a. 1/2 b. 1/4 c. 2/3 d. 3/4

38. ABC is a right-angled triangle, where AB and BC are 25 and 60 cm, respectively. A metal sphere of 2 cm radius charged to a potential of 9×10^5 volt is placed at B as in Fig. 3.77. Find the amount of work done in carrying a positive charge of 1 coulomb from C to A.

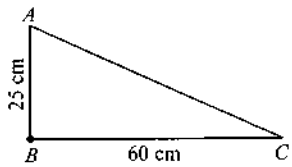


Fig. 3.77

a. 21 kJ b. 42 kJ c. 14 kJ d. 52 kJ

39. Two charged particles having charges 1 and −1 μC and of mass 50 g each are held at rest while their separation is 2 m. Now the charges are released. Find the speed of the particles when their separation is 1 m.

a. $\frac{1}{5}$ m/s b. $\frac{3}{5}$ m/s c. $\frac{3}{10}$ m/s d. $\frac{2}{7}$ m/s

40. A 100 eV electron is projected directly towards a large metal plate that has surface charge density of -2.0×10^{-6} Cm^{-2}. From what distance must the electron be projected, if it is to just fail to strike that plate?

a. 0.44 mm b. 0.20 mm c. 1 mm d. 0.30 mm

41. A solid sphere of radius R is charged uniformly. At what distance from its surface is the electrostatic potential half of the potential at the centre?

a. R b. $R/2$ c. $R/3$ d. $2R$

42. Four identical charges are placed at the points (1, 0, 0), (0, 1, 0), (−1, 0, 0) and (0, −1, 0). Then,

a. the potential at the origin is zero
b. the electric field at the origin is not zero
c. the potential at all points on the z-axis, other than the origin, is zero
d. the field at all points on the z-axis, other than the origin, acts along the z-axis

43. When the separation between two charges is increased, the electric potential energy of the charges

a. increases b. decreases
c. remains the same d. may increase or decrease

44. A positive charge is moved from a low potential point A to a high potential point B. Then, the electric potential energy of the system

a. increases b. decreases
c. will remain the same
d. nothing definite can be predicted

45. If a charge is moved against the coulomb force of an electric field, then the

a. positive work is done by the electric field
b. energy is used from some outside source which does positive work
c. strength of the field is decreased
d. energy of the system is decreased

46. Mark the correct statement:

a. An electron and a proton when released from rest in a uniform electric field experience the same force and the same acceleration.
b. Two equipotential surfaces may intersect.
c. A solid conducting sphere holds more charge than a hollow conducting sphere of the same radius.
d. No work is done in taking a positive charge from one point to another inside a negatively charged metallic sphere.

47. Two point charges Q and $-Q/4$ placed along the x-axis are separated by a distance r. Take $-Q/4$ as origin and it is placed on right of Q. Then, the potential is zero

a. at $x = r/3$ only
b. at $x = -r/5$ only
c. both at $x = r/3$ and at $x = -r/5$
d. there exist two points on the axis where the electric field is zero

48. The electric potential decreases uniformly from 120 V to 80 V as one moves on the X-axis from $x = -1$ cm to $x = +1$ cm. The electric field at the origin

a. must be equal to 20 Vcm^{-1}
b. must be equal to 20 Vm^{-1}
c. may be greater than 20 Vcm^{-1}
d. may be less than 20 Vcm^{-1}

49. A small sphere with mass 1.2 g hangs by a thread between two parallel vertical plates 5.00 cm apart. The plates are insulating and have uniform surface charge densities $+\sigma$ and $-\sigma$. The charge on the sphere is $q = 9 \times 10^{-6}$ C. What potential difference between the plates will cause the thread to assume an angle of 37° with the vertical as shown in Fig. 3.78.?

a. 30 V b. 12 V c. 50 V d. 25 V

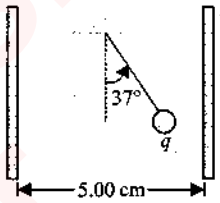


Fig. 3.78

50. A particle of mass m carrying charge 'q' is projected with velocity 'v' from point 'P' towards an infinite line of charge from a distance 'a'. Its speed reduces to zero momentarily at point Q which is at a distance $a/2$ from the line of charge. If another particle with mass m and charge '$-q$' is projected with the same velocity 'v' from P towards the line of charge, what will be its speed at Q?

a. $v_1 = \sqrt{2}v$ b. $v_1 = 2v$ c. $v_1 = \sqrt{v}$ d. $v_1 = \sqrt{2}$

51. Charge Q is given a displacement $\vec{r} = a\hat{i} + b\hat{j}$ = in an electric field $\vec{E} = E_1\hat{i} + E_2\hat{j}$. The work done is

a. $Q(E_1a + E_2b)$ b. $Q\sqrt{(E_1a)^2 + (E_2b)^2}$
c. $Q(E_1 + E_2)\sqrt{a^2 + b^2}$ d. $Q\sqrt{(E_1^2 + E_2^2)^2}\sqrt{a^2 + b^2}$

52. A point charge q is placed inside a conducting spherical shell of inner radius $2R$ and outer radius $3R$ at a distance of R from the centre of the shell. Find the electric potential at the centre of the shell.

a. $\frac{1}{4\pi\varepsilon_0}\frac{q}{2R}$ b. $\frac{1}{4\pi\varepsilon_0}\frac{4q}{3R}$ c. $\frac{1}{4\pi\varepsilon_0}\frac{5q}{6R}$ d. $\frac{1}{4\pi\varepsilon_0}\frac{2q}{3R}$

53. An electric field is expressed as $\vec{E} = 2\hat{i} + 3\hat{j}$. Find the potential difference $(V_A - V_B)$ between two points A and B whose position vectors are given by $r_A = \hat{i} + 2\hat{j}$ and $r_B = 2\hat{i} + \hat{j} + 3\hat{k}$.

a. −1 V b. 1 V c. 2 V d. 3 V

54. The potential function of an electrostatic field is given by $V = 2x^2$. Determine the electric field strength at the point (2 m, 0, 3 m).

a. $\vec{E} = 4\hat{i}$ (NC^{-1}) b. $\vec{E} = -4\hat{i}$ (NC^{-1})
c. $\vec{E} = 8\hat{i}$ (NC^{-1}) d. $\vec{E} = -8\hat{i}$ (NC^{-1})

55. Figure 3.79 shows equipotential surfaces concentric at O, the magnitude of electric field at a distance r measured from O is

a. $\frac{9}{r^2}(\text{Vm}^{-1})$ b. $\frac{6}{r^2}(\text{Vm}^{-1})$

c. $\frac{2}{r^2}(\text{Vm}^{-2})$ d. $\frac{16}{r^2}(\text{Vm}^{-2})$

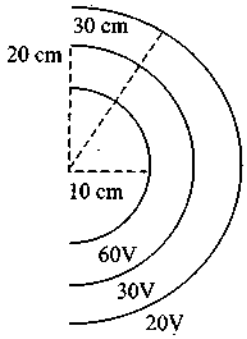


Fig. 3.79

56. A conducting sphere A of radius a, with charge Q, is placed concentrically inside a conducting shell B of radius b. B is earthed. C is the common centre of A and B. Study the following statements.

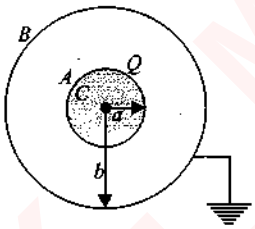


Fig. 3.80

i. The potential at a distance r from C, where $a \le r \le b$, is $\frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{r}\right)$

ii. The potential difference between A and B is

$$\frac{1}{4\pi\varepsilon_0}Q\left(\frac{1}{a}-\frac{1}{b}\right)$$

iii. The potential at a distance r from C, where $a \le r \le b$, is $\frac{1}{4\pi\varepsilon_0}Q\left(\frac{1}{r}-\frac{1}{b}\right)$

Which of the following statements are correct?

a. Only (i) and (ii) b. Only (ii) and (iii)
c. Only (i) and (iii) d. All

57. An electron having charge e and mass m starts from the lower plate of two metallic plates separated by a distance d. If potential difference between the plates is V, the time taken by the electron to reach the upper plate is given by

a. $\sqrt{\frac{2md^2}{eV}}$ b. $\sqrt{\frac{md^2}{eV}}$

c. $\sqrt{\frac{md^2}{2eV}}$ d. $\frac{2md^2}{eV}$

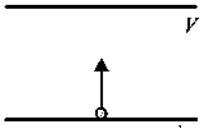


Fig. 3.81

58. There is an infinite straight chain of alternating charges q and $-q$. The distance between the two neighbouring charges is equal to a. Find the interaction energy of any charge with all the other charges.

a. $-\frac{2q^2}{4\pi\varepsilon_0 a}$ b. $\frac{2q^2\log_e 2}{4\pi\varepsilon_0 a}$

c. $-\frac{2q^2\log_e 2}{4\pi\varepsilon_0 a}$ d. None of these

59. The electric field in a certain region is A/x^3. Then, the potential at a point (x, y, z), assuming the potential at infinity to be zero, is

a. zero b. $A/2x^2$ c. $3A/x^4$ d. A/x^2

60. Three identical metallic uncharged spheres A, B and C each of radius a, are kept at the corners of an equilateral triangle of side d ($d >> a$) as shown in Fig. 3.82. The fourth sphere (of radius a), which has a charge q, touches A and is then removed to a position far away. B is earthed and then the earth connection is removed. C is then earthed. The charge on C is

a. $\frac{qa}{2d}\left(\frac{2d-a}{2d}\right)$ b. $\frac{qa}{2d}\left(\frac{2d-a}{d}\right)$

c. $-\frac{qa}{2d}\left(\frac{d-a}{d}\right)$ d. $\frac{2qa}{d}\left(\frac{d-a}{2d}\right)$

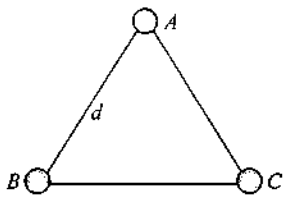
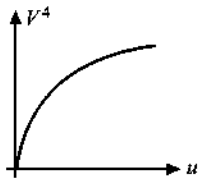


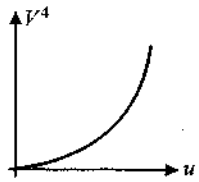
Fig. 3.82

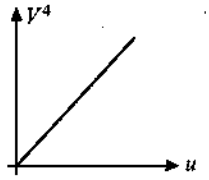
61. A solid conducting sphere of radius 10 cm is enclosed by a thin metallic shell of radius 20 cm. A charge $q = 20\ \mu\text{C}$ is given to the inner sphere. Find the heat generated in the process when the inner sphere is connected to the shell by a conducting wire.

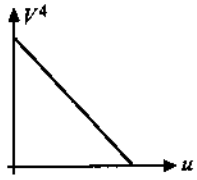
a. 12 J b. 9 J c. 24 J d. zero

62. If V and u are electric potential and energy density, respectively, at a distance r from a positive point charge, then which of the following graphs is correct?

a.


b.


c. 

d. 

63. Find the potential V of an electrostatic field $\vec{E} = a(y\hat{i} + x\hat{j})$, where a is a constant.

a. $axy + C$ **b.** $-axy + C$ **c.** axy **d.** $-axy$

64. We have three identical metallic spheres A, B and C, A is given a charge Q and B and C are uncharged. The following processes of touching of two spheres one carried out in succession. Each process is carried out with sufficient time:

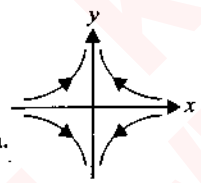
i. A and B ii. B and C
iii. C and A iv. A and B v. B and C

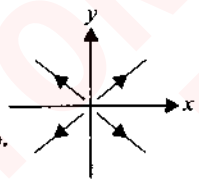
The final charges on the spheres are

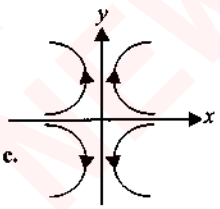
a. $\frac{11Q}{32}, \frac{5Q}{16}, \frac{11Q}{32}$ **b.** $\frac{11Q}{32}, \frac{11Q}{32}, \frac{5Q}{16}$

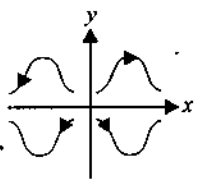
c. $\frac{8Q}{8}, \frac{5Q}{16}, \frac{5Q}{16}$ **d.** $\frac{5Q}{16}, \frac{11Q}{32}, \frac{11Q}{32}$

65. The potential field depends on the x- and y-coordinates as $V = x^2 - y^2$. The corresponding electric field lines in $x - y$ plane are as

a. 

b. 

c. 

d. 

66. A solid conducting sphere having a charge Q is surrounded by an uncharged concentric conducting hollow spherical shell. The potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell is V. If the shell is now given a charge of $-3Q$, the new potential difference between the same two surfaces is **(IIT-JEE, 1989)**

a. V **b.** $2V$ **c.** $4V$ **d.** $-2V$

Multiple Correct Answers Type

Solutions on page 3.39

1. A wire having a uniform linear charge density λ, is bent in the form of a ring of radius R. Point A as shown in Fig. 3.83 is in the plane of the ring but not at the centre. Two elements of the ring of lengths a_1 and a_2 subtend very small same angle at the point A. They are at distances r_1 and r_2 from the point A, respectively. ($r_2 > r_1$)

a. The ratio of charges of element a_1 to that of element a_2 is r_1/r_5.
b. The element a_1 produced greater magnitude of electric field at A than element a_2.
c. The elements a_1 and a_2 produce same potential at A.

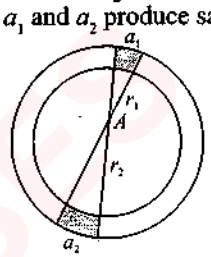


Fig. 3.83

d. The direction of net electric field produced by elements only at A is towards element a_2.

2. Two point charges of different magnitudes and of opposite signs are separated by some distance. There can be

a. only one point in space where net electric field intensity is zero
b. only two points in space where net electric potential is zero
c. infinite number of points in space where net electric field intensity is 0
d. infinite number of points in space where net electric potential is zero

3. Two infinite, parallel, non-conducting sheets carry equal positive charge density σ. One is placed in the y–z plane and the other at distance $x = a$. Take potential $V = 0$ at $x = 0$. Then

a. for $0 \le x \le a$, potential $V = 0$
b. for $x \ge a$, potential $V = \frac{\sigma}{\varepsilon_0}(x - a)$
c. for $x \ge a$, potential $V = -\frac{\sigma}{\varepsilon_0}(x - a)$
d. for $x \le 0$, potential $V = \frac{\sigma}{\varepsilon_0}x$

4. A negative charge is moved by an external agent in the direction of electric field.

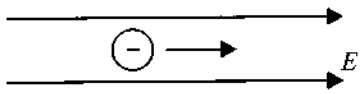


Fig. 3.84

Then

a. potential energy of the charge increases
b. potential energy of the charge decreases
c. positive work is done by the electric field
d. negative work is done by the electric field

5. If a charged conductor is enclosed by a hollow charged conducting shell (assumed concentric and spherical in shape), and they are connected by a conducting wire, then which of the following statement(s) would be correct?

a. Potential difference between two conductors becomes zero.
b. If charge on inner conductor is q and on outer conductor it is $2q$, then finally charge on outer conductor will be $3q$.
c. The charge on the inner conductor is totally transferred to the outer conductor.
d. If charge on the inner conductor is q and charge on the outer conductor is zero, then finally charge on each conductor will be $q/2$.

6. For the situation shown in Fig. 3.85, mark out the correct statement(s).

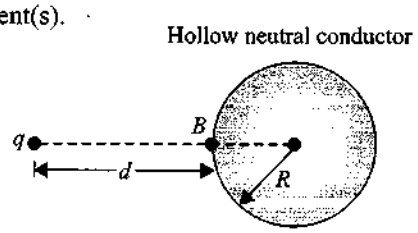


Fig. 3.85

a. Potential of the conductor is $q/[4\pi\varepsilon_0(d+R)]$.
b. Potential of the conductor is $q/4\pi\varepsilon_0 d$.
c. Potential of the conductor cannot be determined as nature of distribution of induced charges is not known.
d. Potential at point B due to induced charges is $-qR/[4\pi\varepsilon_0(d+R)d]$.

Assertion-Reasoning Type

Solutions on page 3.40

In the following questions, each question contains Statement I (Assertion) and Statement II (Reason). Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

a. Statement I is True, Statement II is True; Statement II is a correct explanation for Statement I.
b. Statement I is True, Statement II is True; Statement II is **NOT** a correct explanation for Statement I.
c. Statement I is True, Statement II is False.
d. Statement I is False, Statement II is True.

1. **Statement I:** Positive charge always moves from a higher potential point to a lower potential point if left free in electric field.
   Statement II: Electric potential is a vector quantity.
2. **Statement I:** The surface of a conductor is an equipotential surface.
   Statement II: Conductors allow the free flow of charge within themselves.
3. **Statement I:** No work is done in taking a small positive charge from one point to other inside a positively charged metallic sphere while outside the sphere work is done in taking the charge towards the sphere. Neglect induction due to small charge.
   Statement II: Inside the sphere, electric potential is same at each point, but outside it is different for different points.
4. **Statement I:** Electric potential of Earth is taken to be zero as a reference.
   Statement II: The electric field produced by Earth in surrounding space is zero.

Comprehension Type

Solutions on page 3.40

For Problems 1–2

Two point charges are located on the x-axis, $q_1 = -e$ at $x = 0$ and $q_2 = +e$ at $x = a$.

1. The work done by an external force to bring a third point charge $q_3 = +e$ from infinity to $x = 2a$ is
   a. $\frac{e^2}{4\pi\varepsilon_0 a}$ b. $\frac{e^2}{8\pi\varepsilon_0 a}$ c. $\frac{-e^2}{8\pi\varepsilon_0 a}$ d. $\frac{-e^2}{4\pi\varepsilon_0 a}$
2. The total potential energy of the system of three charges is
   a. $\frac{e^2}{4\pi\varepsilon_0 a}$ b. $\frac{e^2}{8\pi\varepsilon_0 a}$ c. $\frac{-e^2}{8\pi\varepsilon_0 a}$ d. $\frac{-e^2}{4\pi\varepsilon_0 a}$

For Problems 3–4

A small metal sphere, carrying a net charge of $q_1 = -2\ \mu C$, is held in a stationary position by insulating supports. A second small metal sphere, with a net charge of $q_2 = -8\ \mu C$ and mass 1.50 g, is projected towards q_1. When the two spheres are 0.800 m apart, q_2 is moving towards q_1 with speed 20 ms^{-1} as shown in Fig. 3.86. Assume that the two spheres can be treated as point charges. You can ignore the force of gravity.

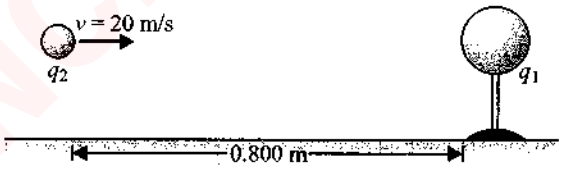


Fig. 3.86

3. The speed of q_2 when the spheres are 0.400 m apart is
   a. $2\sqrt{10}$ ms^{-1} b. $2\sqrt{6}$ ms^{-1} c. $4\sqrt{10}$ ms^{-1} d. $4\sqrt{6}$ ms^{-1}
4. How close does q_2 get to q_1?
   a. 0.20 m b. 0.30 m c. 0.10 m d. 0.15 m

For Problems 5–7

Two point charges $q_1 = +2.40$ nC and $q_2 = -6.50$ nC are 0.100 m apart. Point A is midway between them; point B is 0.080 m from q_1 and 0.060 m from q_2 as shown in Fig. 3.87. Take the electric potential to be zero at infinity.

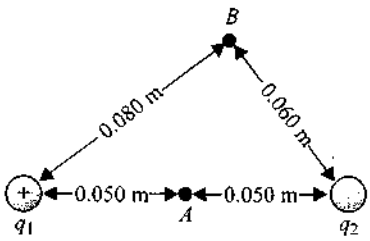


Fig. 3.87

5. The potential at point A is
   a. −738 V b. −323 V c. −705 V d. −120 V
6. The potential at point B is
   a. −738 V b. −323 V c. −705 V d. −120 V

7. The work done by the electric field on a charge of 2.50 nC that travels from point B to point A is
   a. -8.25×10^{-8} J b. 8.25×10^{-8} J
   c. 1.25×10^{8} J d. -1.25×10^{-8} J

For Problems 8–11

Four charges $+q$, $+q$, $-q$ and $-q$ are placed, respectively, at the corners A, B, C and D of a square of side a, arranged in the given order. E and F are the midpoints of sides BC and CD, respectively. O is the centre of square.

8. The electric field at O is
   a. $\frac{q}{\sqrt{2}\pi\varepsilon_0 a^2}$ b. $\frac{q}{\sqrt{3}\pi\varepsilon_0 a^2}$
   c. $\frac{\sqrt{3}q}{\pi\varepsilon_0 a^2}$ d. $\frac{\sqrt{2}q}{\pi\varepsilon_0 a^2}$

9. The electric potential at O is
   a. $\frac{\sqrt{2}q}{\pi\varepsilon_0 a}$ b. $\frac{\sqrt{3}q}{\pi\varepsilon_0 a}$
   c. $\frac{q}{\pi\varepsilon_0 a}$ d. zero

10. The work done in carrying a charge e from O to E is
   a. $\frac{\sqrt{2}qe}{\pi\varepsilon_0 a}$ b. $\frac{qe}{\pi\varepsilon_0 a}\left[\frac{1}{\sqrt{5}}-1\right]$
   c. $\frac{qe}{\pi\varepsilon_0 a}\left[\frac{1}{\sqrt{5}}+1\right]$ d. zero

11. The work done in carrying a charge e from O to F is
   a. $\frac{\sqrt{2}qe}{\pi\varepsilon_0 a}$ b. $\frac{qe}{\pi\varepsilon_0 a}\left[\frac{1}{\sqrt{5}}-1\right]$
   c. $\frac{qe}{\pi\varepsilon_0 a}\left[\frac{1}{\sqrt{5}}+1\right]$ d. zero

For Problems 12–13

A charge Q is distributed over two concentric hollow spheres of radii r and R ($R > r$) such that their surface densities are equal.

12. The charges on smaller and bigger shells are
   a. $\frac{Qr^2}{r^2+R^2}$ and $\frac{QR^2}{r^2+R^2}$, respectively
   b. $Q\left(1+\frac{r^2}{R^2}\right)$ and $Q\left(1+\frac{R^2}{r^2}\right)$, respectively
   c. $Q\left(1-\frac{r^2}{R^2}\right)$ and $Q\left(1-\frac{R^2}{r^2}\right)$, respectively
   d. $\frac{QR^2}{r^2+R^2}$ and $Q\frac{Qr^2}{r^2+R^2}$, respectively

13. The potential at the common centre is
   a. $\frac{\sqrt{2}}{\pi\varepsilon_0}\frac{Q(R+r)}{(R^2+r^2)}$ b. $\frac{1}{2\pi\varepsilon_0}\frac{Q(R+r)}{(R^2+r^2)}$
   c. $\frac{1}{4\pi\varepsilon_0}\frac{Q(R+r)}{(R^2+r^2)}$ d. $\frac{1}{\pi\varepsilon_0}\frac{Q(R-r)}{(R^2+r^2)}$

For Problems 14–15

Refer to the uniform electric field shown in Fig. 3.88.

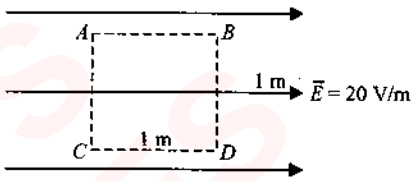


Fig. 3.88

14. The potential difference $V_A - V_C$ is
   a. zero b. 20 V c. −20 V d. $20\sqrt{2}$ V

15. The potential difference $V_A - V_D$ is
   a. zero b. 20 V c. −20 V d. $20\sqrt{2}$ V

For Problems 16–18

The electrical potential function for an electrical field directed parallel to the x-axis is shown in the given graph in Fig. 3.89:

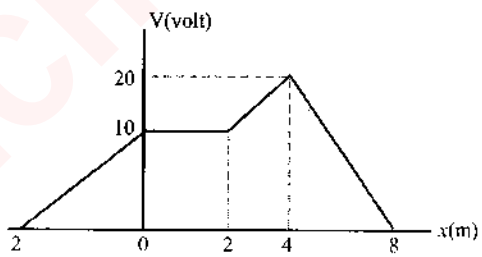
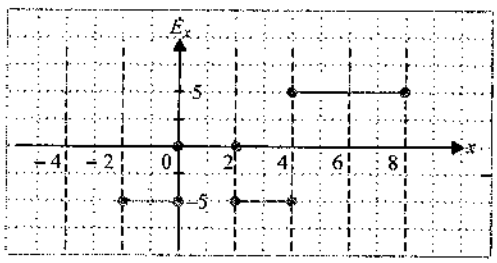


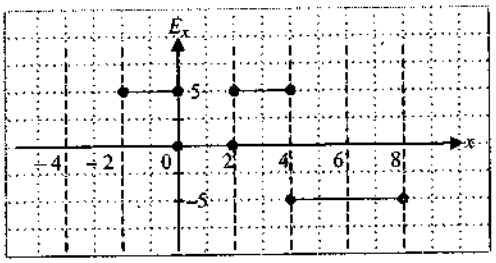
Fig. 3.89

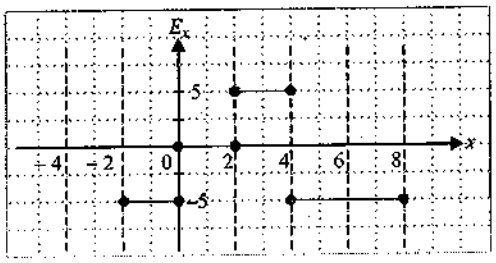
16. The intervals in which the magnitude of electric field in x-direction is maximum are
   a. $-2 \le x \le 0$, $4 \le x \le 8$ and $0 \le x \le 2$
   b. $-2 \le x \le 0$ and $0 \le x \le 2$
   c. $-2 \le x \le 0$, $2 \le x \le 4$ and $4 \le x \le 8$
   d. $0 \le x \le 2$ and $4 \le x \le 8$

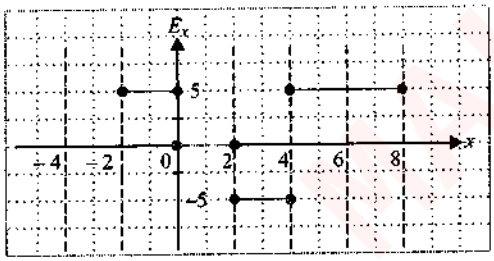
17. The magnitude of electric field in x-direction in the interval $4 \le x \le 8$ is
   a. 2.5 NC^{-1} b. 5 NC^{-1}
   c. −2.5 NC^{-1} d. −5 NC^{-1}

18. The graph E_x versus x will be

a. 

b.


c.


d.


For Problems 19–20

A uniform electric field of 100 Vm^{-1} is directed at 30° with the positive x-axis as shown in Fig. 3.90. $OA = 2$ m and $OB = 4$ m.

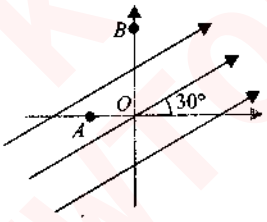


Fig. 3.90

19. The potential difference $V_O - V_A$ is

a. $100\sqrt{3}$ V b. $200\sqrt{3}$ V

c. $-100\sqrt{3}$ V d. $-200\sqrt{3}$ V

20. The potential difference $V_B - V_A$ is

a. $-100\left[2+\sqrt{3}\right]$ V b. $100\left[2+\sqrt{3}\right]$ V

c. $100\left[2-\sqrt{3}\right]$ V d. $-100\left[2-\sqrt{3}\right]$ V

For Problems 21–24

The electric potential varies in space according to the relation $V = 3x + 4y$. A particle of mass 0.1 kg starts from rest from point (2, 3.2) under the influence of this field. The charge on the particle is +1 μC. Assume V and (x, y) are in S.I. units.

21. The component of electric field in the x-direction (E_x) is

a. -3 Vm^{-1} b. 4 Vm^{-1} c. 5 Vm^{-1} d. 8 Vm^{-1}

22. The component of electric field in the y-direction (E_y) is

a. 3 Vm^{-1} b. -4 Vm^{-1} c. 5 Vm^{-1} d. 8 Vm^{-1}

23. The time taken to cross the x-axis is

a. 20 s b. 40 s c. 200 s d. 400 s

24. The velocity of the particle when it crosses the x-axis is

a. 20×10^{-3} ms^{-1} b. 40×10^{-3} ms^{-1}

c. 30×10^{-3} ms^{-1} d. 50×10^{-3} ms^{-1}

For Problems 25–26

Three concentric spherical metallic shells A, B and C of radii a, b and c ($a < b < c$) have surface charge densities $\sigma, -\sigma$ and σ, respectively.

25. If V_A, V_B and V_C are potential of shells A, B and C, respectively, match the columns

Column A	Column B
a. V_A	i. $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2-b^2+c^2}{c}\right]$
b. V_B	ii. $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2}{b}-b+c\right]$
c. V_C	iii. $\frac{\sigma}{\varepsilon_0}[a-b+c]$

26. If shells A and C are at the same potential, the relation between the radii a, b and c is

a. $a = b + c$ b. $c = a + b$ c. $b = a + c$ d. $2a = b - c$

(IIT-JEE, 1990)

For Problems 27–30

We have an isolated conducting spherical shell of radius 10 cm. Some positive charge is given to it so that resulting electric field has a maximum intensity of 1.8×10^6 NC^{-1}. The same amount of negative charge is given to another isolated conducting spherical shell of radius 20 cm. Now, first shell is placed inside the second so that both are concentric as shown in Fig. 3.91.

27. The electric potential at any point inside the first shell is

a. 18×10^4 V b. 9×10^4 V

c. 4.5×10^4 V d. 1.8×10^4 V

28. The electric field intensity just inside the outer sphere is

a. 4.5×10^5 N/C b. 9×10^5 N/C

c. 4.5×10^6 N/C d. 5×10^4 N/C

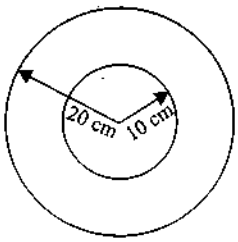


Fig. 3.91

29. The electrostatic energy stored in the system is

a. 1.0 J b. 0.045 J c. 0.09 J d. 1.8 J

30. If both the spheres are connected by a conducting wire, then

a. Nothing will happen.

b. Some part of the energy stored in the system will convert into heat.

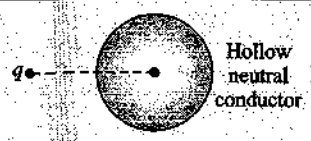
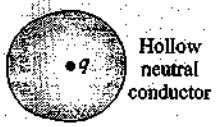
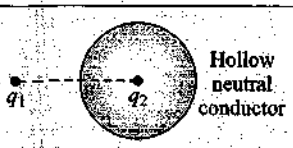
c. charge on both spheres will be positive.

d. entire amount of the energy stored in the system will convert into heat.

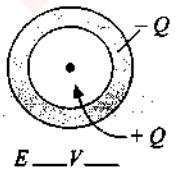
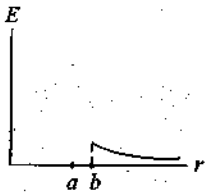
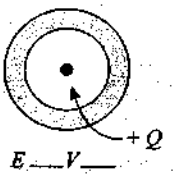
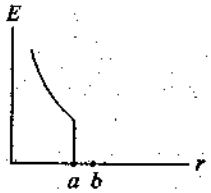
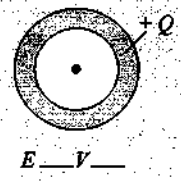
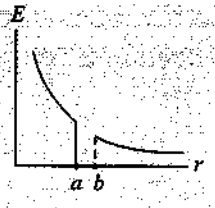
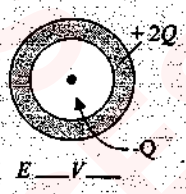
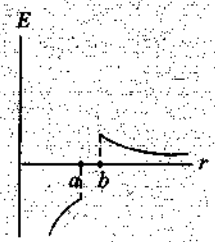
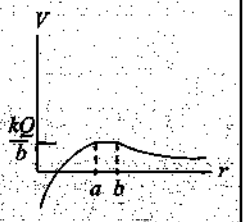
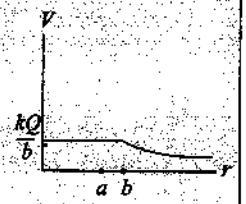
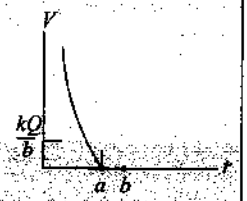
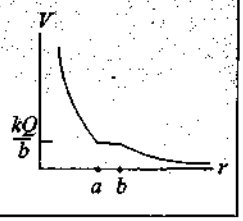
Matching Column Type

Solutions on page 3.42

1. Match the entries of Column I with the entries of Column II.

Column I	Column II
i. 	a. $\vec{E}$ inside the conductor is zero
ii. 	b. $\lvert\vec{E}\rvert$ inside the conductor is constant but not zero
iii. 	c. $\lvert\vec{E}\rvert$ inside the conductor is varying
	d. Potential inside the conductor is same as that of conductor
	e. Potential inside the conductor is varies

2. Column I shows four hollow metal spheres each with internal radius a and external radius b. Match these charge distributions with its corresponding E-graph and V-graph in Column II. A point charge of $+Q$ or $-Q$ is present at the centre of the spheres [in figure (iii), there is no charge at the centre of sphere]. The charge indicated on the spherical shell itself is the net charge on the shell, that is, any induced charge distribution is not shown. Also, the label for the net charge on a conducting sphere does not necessarily indicate the actual position of the charge on or within the conductor.

Column I	Column II
i. 	a. 
ii. 	b. 
iii. 	c. 
iv. 	d. 
	e. 
	f. 
	g. 
	h. 

3. Figure 3.92 shows three concentric thin spherical shells A, B and C of radii R, $2R$ and $3R$ respectively. The shell B is earthed and A and C are given charges q and $2q$, respectively. If the charge on surfaces 1, 2, 3 and 4 are q_1, q_2, q_3 and q_4, respectively, then match the following columns.

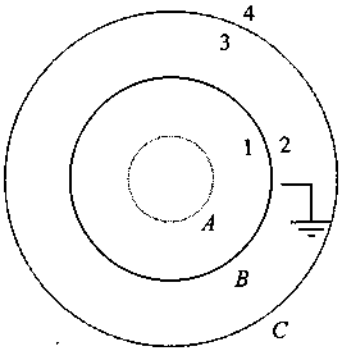


Fig. 3.92

Column I	Column II
i. q_1	a. $\frac{2}{3}q$
ii. q_2	b. $\frac{4}{3}q$
iii. q_3	c. $-\frac{4}{3}q$
iv. q_4	d. $-q$

Integer Answer Type

Solutions on page 3.43

1. There are 27 drops of a conducting fluid. Each drop has radius r and each of them is charged to same potential V_1. They are then combined to form a bigger drop. The potential of the bigger drop is V_2. Find the ratio V_2/V_1. Ignore the change in density of fluid on combining the drops.
2. Figure 3.93 shows two conducting thin concentric shells of radii r and $3r$.

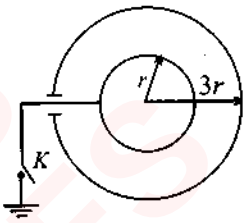


Fig. 3.93

The outer shell carries charge q and inner shell is neutral. The amount of charge which flows from inner shell to the earth after the key K is closed is equal to $(1/n)^{th}$ of the charge on outer shell. What is the value of n?

3. A charge of 5 μC is placed at the centre of a square $ABCD$ of side 10 cm. Find the work done (in μJ) in moving a charge of 1 μC from A to B.
4. The linear charge density on a dielectric ring of radius R vanes with θ as $\lambda = \lambda_0 \cos\theta/2$, where λ_0 is constant. Find the potential at the centre O of ring. [in volt]
5. The point charges $-2q$, $-2q$ and $+q$ are put on the vertices of an equilateral triangle of side a. Find the work done by some external force in increasing the separation to $2a$ (in J).

ANSWERS AND SOLUTIONS

Subjective Type

1.

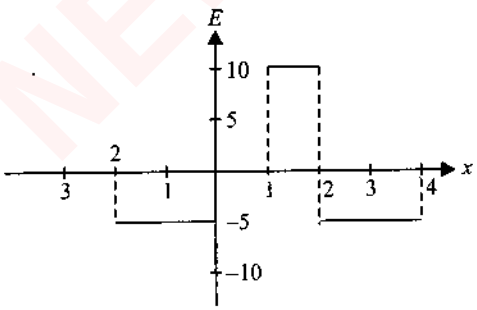


Fig. 3.94

2. Length of the cube diagonal $= \sqrt{3}\,b$
Distance of centre of the cube from its each corner,

$$r = \frac{\sqrt{3}}{2}b$$

Total P.E. of charge q at the centre is

$$\frac{-8q(q)}{4\pi\varepsilon_0 r} = \frac{-8q^2}{4\pi\varepsilon_0\sqrt{3}(b/2)} = \frac{-4q^2}{\pi\varepsilon_0\sqrt{3}b}$$

3. If r is the distance of the charge Q from the point of observation, then

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = 32 \quad \text{(i)}$$

and

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = 16 \quad \text{(ii)}$$

Dividing Eq. (ii) by Eq. (i), we get $r = 0.5$ m and putting the value of r in Eq. (ii), we get

$$Q = \frac{16\times 0.5}{9\times 10^9} = (8/9)\times 10^{-9}\,\text{C}$$

4. Let U_I be the potential energy of system I and U_{II} be the potential energy of system II.

Work done = Change in P.E. = $U_{II} - U_I$.

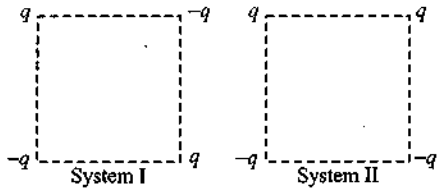


Fig. 3.95

$$U_1 = -\frac{4q^2}{4\pi\varepsilon_0 a} + \frac{2q^2}{4\pi\varepsilon_0\sqrt{2}a}$$

$$U_{II} = \frac{-2q^2}{4\pi\varepsilon_0\sqrt{2}a}$$

$$\therefore \quad W = \frac{-2q^2}{4\pi\varepsilon_0\sqrt{2}a} + \frac{4q^2}{4\pi\varepsilon_0 a} - \frac{2q^2}{4\pi\varepsilon_0\sqrt{2}a}$$

$$= \frac{q^2}{4\pi\varepsilon_0 a}(4 - 2\sqrt{2})$$

5. Potential of a drop, $V = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q}{r}$

$$q = 4\pi\varepsilon_0\varepsilon_r r V$$

Therefore, total charge $Q = nq = 4\pi\varepsilon_0\varepsilon_r r Vn$

(n = total no. of drops)

Also, $\frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3 \Rightarrow n = \frac{R^3}{r^3}$

Hence, potential of the hollow conductor,

$$V' = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{Q}{R} = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \times \frac{4\pi\varepsilon_0\varepsilon_r r V n}{R}$$

$$= \frac{r}{R} \times V \times \left(\frac{R^3}{r^3}\right)$$

$$= \left(\frac{R}{r}\right)^2 V$$

6. As potential energy of two point charges separated by a distance r is given by $U = (q_1q_2/4\pi\varepsilon_0 r)$, the initial and final potential energies of the system will be

$$(Us)_I = 3 \times \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r} = 3 \times 9 \times 10^9 \times \frac{(0.1)^2}{1} = 2.7 \times 10^8 \text{ J}$$

$$(Us)_F = 2 \times \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(r/2)} + \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r} = 5 \times \frac{q^2}{4\pi\varepsilon_0 r} = 4.5 \times 10^8 \text{ J}$$

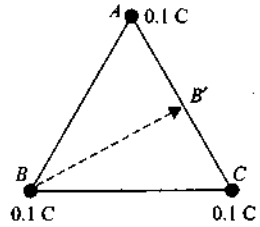


Fig. 3.96

So, work done in changing the configuration of the system,

$$W = (Us)_F - (Us)_I = 1.8 \times 10^8 \text{ J}$$

Now, as energy is supplied at the rate of 1 kW, i.e., 10^3 Js^{-1}, time required to do work W is given by

$$t = \frac{W}{P} = \frac{1.8 \times 10^8}{10^3} = 1.8 \times 10^5 \text{ s} = 50 \text{ h}$$

7. $W_{net} = \Delta KE \Rightarrow -mgl - q_0El = 0 - \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2mgl + 2q_0El}{m}}$,

when $E = \frac{\sigma}{\varepsilon_0}$

8. Charge on the sphere (including cavities),

$$Q = \frac{4}{3}\pi R^3\rho = \frac{4}{3}\pi(5 \times 10^{-2})^3 \frac{1}{\pi} \times 10^{-6} = \frac{500}{3} \times 10^{-12} \text{ C}$$

Charge in a volume equal to that of cavity,

$$q = \frac{4}{3}\pi R^3\rho = \frac{4}{3}\pi(1 \times 10^{-2})^3 \frac{1}{\pi} \times 10^{-6} = \frac{4}{3} \times 10^{-12} \text{ C}$$

Potential at P,

$$V_P = V_{\text{whole sphere}} - 2V_{\text{cavity}}$$

$$= \frac{kQ}{2R}\left[3 - \frac{x^2}{R^2}\right] - 2\frac{kq}{5 \times 10^{-2}}$$

$$= \frac{9 \times 10^9 \times (500/3) \times 10^{-12}}{2 \times 5 \times 10^{-2}}\left[3 - \left(\frac{4}{5}\right)^2\right]$$

$$- \frac{2 \times 9 \times 10^9 \times (4/3) \times 10^{-12}}{5 \times 10^{-2}}$$

$$= \frac{873}{25} = 34.92 \text{ V}$$

9. Work done = charge × difference in potential

Let the particle move from O to O' (Fig. 3.97)

Potential at point O is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{R} + \frac{Q_2}{\sqrt{2}R}$$

and potential at point O' is

$$V' = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_2}{R} + \frac{Q_1}{\sqrt{2}R}\right]$$

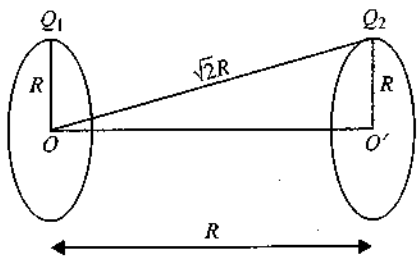


Fig. 3.97

Difference in potential,

$$\Delta V = V - V'$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1}{R} + \frac{Q_2}{\sqrt{2}R}\right] - \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_2}{R} + \frac{Q_1}{\sqrt{2}R}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{1}{R}(Q_1 - Q_2) + \frac{1}{\sqrt{2}R}(Q_2 - Q_1)\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1 - Q_2}{R}\left(1 - \frac{1}{\sqrt{2}}\right)\right]$$

$$\therefore \quad W.d. = q\Delta V = q\frac{1}{4\pi\varepsilon_0}\left[\frac{(Q_1 - Q_2)}{\sqrt{2}R}(\sqrt{2} - 1)\right]$$

10. a. Potential of shell A,

$$V_A = V_{\text{due to charge on } A} + V_{\text{due to charge on } B} + V_{\text{due to charge on } C}$$

$$= 0 + \frac{Q}{4\pi\varepsilon_0 b} - \frac{Q}{4\pi\varepsilon_0 c}$$

Potential of shell B,

$$V_B = V_{\text{due to charge on } A} + V_{\text{due to charge on } B} + V_{\text{due to charge on } C}$$

$$= 0 + \frac{Q}{4\pi\varepsilon_0 b} - \frac{Q}{4\pi\varepsilon_0 c}$$

Potential of shell C,

$$V_C = V_{\text{due to charge on } A} + V_{\text{due to charge on } B} + V_{\text{due to charge on } C}$$

$$= 0 + \frac{Q}{4\pi\varepsilon_0 c} - \frac{Q}{4\pi\varepsilon_0 c} = 0$$

b. After connecting inner and outer shells:

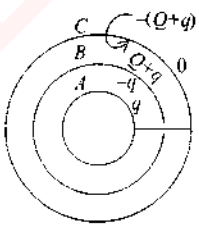


Fig. 3.98

$$V_A = V_C$$

$$\Rightarrow \quad \frac{q}{a} - \frac{q}{b} + \frac{Q+q}{b} - \frac{Q+q}{c} = \frac{q - q + Q + q - (Q+q)}{c}$$

$$\Rightarrow \quad q = \frac{Qa}{b}\frac{(b-c)}{(c-a)}$$

Final potential of A and C is zero.

$$V_B = k\left[\frac{q}{a} - \frac{q}{b} + \frac{Q+q}{b} - \frac{(Q+q)}{c}\right] = k(Q+q)\left[\frac{c-b}{bc}\right]$$

$$= k\left[Q + \frac{Qa(b-c)}{b(c-a)}\right]\left[\frac{c-b}{bc}\right] = kQ\left[\frac{(b-a)(c-b)}{b^2(c-a)}\right]$$

11. a. Let the charges on A and C be q_1 and q_2, respectively (Fig. 3.99). From conservation of charge, we have

$$q_1 + q_2 = 0$$

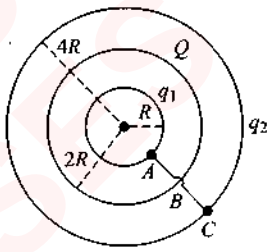


Fig. 3.99

Hence, $q_1 = -q_2$.

Since A and C are connected by a conducting wire, so they have the same potential.

V_A = potential of A

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1}{R} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{2R} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{4R}$$

V_C = potential of C

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1}{4R} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{4R} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{4R}$$

Equalizing the potentials of A and C, i.e., $V_A = V_C$, we get

$$\frac{q_1}{R} + \frac{Q}{2R} + \frac{q_2}{4R} = \frac{q_1}{4R} + \frac{Q}{4R} + \frac{q_2}{4R}$$

$$\text{or } 4q_1 + 2Q = q_1 + Q$$

$$q_1 = -Q/3$$

Hence, $q_2 = Q/3$.

b. $$V_A = \frac{1}{4\pi\varepsilon_0 R}\left[\frac{-Q}{3} + \frac{Q}{2} + \frac{Q}{12}\right] = \frac{Q}{16\pi\varepsilon_0 R}$$

$$V_B = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{2R} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{2R} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{4R}$$

$$= \frac{1}{8\pi\varepsilon_0 R}\left[\frac{-Q}{3} + Q + \frac{Q}{6}\right] = \frac{5Q}{48\pi\varepsilon_0 R}$$

12. When K_1 is closed first time, the outer sphere is earthed and the potential on it becomes zero. Let the charge on it be q_1'.

V_1' = Potential due to charge on the inner sphere and that due to charge on the outer sphere is

$$V_1' = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{2R} + \frac{q_1'}{2R}\right] = 0 \text{ or } q_1' = -q$$

When K_2 is closed first time, the potential V_2' on the inner sphere becomes zero as it is earthed. Let the new charge on inner sphere be q_2'.

$$0=\frac{1}{4\pi\varepsilon_0}\frac{q_2'}{R}+\frac{1}{4\pi\varepsilon_0}\frac{(-q)}{(2R)} \Rightarrow q_2'=\frac{q}{2}$$

Now, when K_1 will be closed second time, charge on the outer sphere will be $-q_2'$, i.e., $-q/2$.

After one event involving closure and opening of K_1 and K_2, charge is reduced to half of its initial value.

Similarly, when K_1 will be closed n^{th} time, charge on the outer sphere will be $-q/(2^{n-1})$ as each time charge will be reduced to half of the previous value.

After closing K_2 n^{th} time, charge on the inner shell will be negative of half the charge on the outer shell, i.e., $(+q/2^n)$ and potential on it will be zero.

For potential of the outer shell,

$$V_0=\frac{1}{4\pi\varepsilon_0}\frac{(+q/2^n)}{2R}+\frac{1}{4\pi\varepsilon_0}\frac{(-q/2^{n-1})}{2R}$$

$$=\frac{-q[-1+2]}{4\pi\varepsilon_0 2^{n+1}R}=\frac{-q}{4\pi\varepsilon_0 2^{n+1}R}$$

Potential difference $=V_0-V_i=\frac{-q}{4\pi\varepsilon_0 2^{n+1}R}-0=\frac{-q}{4\pi\varepsilon_0 2^{n+1}R}$

13. a. Consider the situation Q is a positive charge. The resultant force on q at A will be

$$\vec{F}_R=\vec{F}_1+\vec{F}_2+\vec{F}_3$$

Here, $$F_1=F_2=\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2}$$

So, $$(\vec{F}_1+\vec{F}_2)=2\times\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2}\cos 30^\circ=\frac{\sqrt{3}q^2}{4\pi\varepsilon_0 a^2}\text{ along }\overline{OA}$$

While $$\vec{F}_3=\frac{1}{4\pi\varepsilon_0}\frac{qQ}{(a/\sqrt{3})^2}=\frac{3q\,Q}{4\pi\varepsilon_0 a^2}\text{ along }\overline{OA}$$

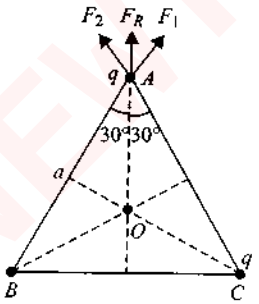


Fig. 3.100

So, $$F_R=\frac{q}{4\pi\varepsilon_0 a^2}\left[\sqrt{3}q+3Q\right]\text{ along }\overline{OA} \quad \text{(i)}$$

b. For $Q=-q$,

$$F_R=\frac{q}{4\pi\varepsilon_0 a^2}\left[\sqrt{3}q-3q\right]\text{ along }\overline{OA}$$

$$=\frac{\sqrt{3}q}{4\pi\varepsilon_0 a^2}\left[\sqrt{3}-1\right]\text{ along }\overline{AO}$$

i.e., the charges q at corners A, B and C will move towards the centre O.

c. Charge will remain stationary if $F_R=0$, which in Eq. (i) is possible only if

$$\left[(\sqrt{3})q+3Q\right]=0,\text{ i.e., }Q=-\left(\frac{q}{\sqrt{3}}\right)$$

d. Potential energy of the system,

$$U=\frac{1}{2}\times\frac{1}{4\pi\varepsilon_0}\sum_{i\neq j}\frac{q_iq_j}{r_{ij}}=\frac{1}{4\pi\varepsilon_0}\left[3\frac{q\times q}{a}+\frac{3q(-q/\sqrt{3})}{(a/\sqrt{3})}\right]=0$$

And for finite charge distribution potential energy at infinity is always zero.

$$\therefore \quad W=U_F-U_i=0-0=0$$

14. If the ball has to just complete the circle, then the tension must vanish at the topmost point, i.e., A

From Newton's second law

$$T_2+mg-\frac{q^2}{4\pi\varepsilon_0 l^2}=\frac{mv^2}{l} \quad \text{(i)}$$

At the topmost point, $T_2=0$

$$\therefore \quad mg-\frac{q^2}{4\pi\varepsilon_0 l^2}=\frac{mv^2}{l} \quad \text{(ii)}$$

From energy conservation,

Energy at lowest point = Energy at topmost point

$$\frac{1}{2}mu^2=\frac{1}{2}mv^2+mg2l \quad \text{(iii)}$$

$$v^2-u^2=-4gl \quad \text{(iv)}$$

From Eq. (ii), $v^2=gl-\frac{q^2}{4\pi\varepsilon_0 ml}$ (v)

From Eqs. (iv) and (v),

$$u=\sqrt{5gl-\frac{q^2}{4\pi\varepsilon_0 ml}}=\left(\frac{275}{8}\right)^{1/2}=5.86\text{ ms}^{-1}$$

15. a. Let P be a point in the XY plane with coordinates (x, y) at which the potential due to charges $-2Q$ and $+Q$ placed at A and B, respectively, be zero (Figure 3.101).

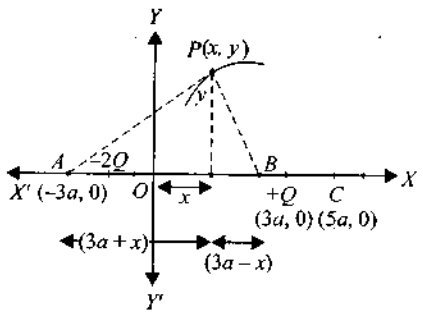


Fig. 3.101

$$V_P=\frac{K(-2Q)}{\sqrt{(3a+x)^2+y^2}}+\frac{K(+Q)}{\sqrt{(3a-x)^2+y^2}}=0$$

$$2\sqrt{(3a-x)^2+y^2}=\sqrt{(3a+x)^2+y^2}$$

$$\Rightarrow \quad 4\left[(3a-x)^2+y^2\right]=\left[(3a+x)^2+y^2\right]$$

$$\Rightarrow \quad (x-5a)^2+(y-0)^2=(4a)^2$$

This is the equation of a circle with centre at $(5a, 0)$ and radius $4a$. Thus, $c\,(5a, 0)$ is the centre of the circle, where potential is zero.

b. Let us find potential at various points on the x-axis:

For $x > 3a$,

$$V=\frac{kQ}{x-3a}-\frac{k2Q}{x+3a}=\frac{kQ(9a-x)}{x^2-9a^2}.$$

We see that at $x = 9a$, $V = 0$.

For $-3a < x < 3a$,

$$V=\frac{kQ}{3a-x}-\frac{k2Q}{x+3a}=\frac{kQ3(x-a)}{9a^2-x^2}$$

We see that at $x = a$, $V = 0$.

For $x < -3a$,

$$V=\frac{kQ}{3a-x}+\frac{k(-2Q)}{-3a-x}=\frac{-kQ(9a-x)}{x^2-9a^2}$$

We see that $V \neq 0$ for $x < -3a$ and V is always negative for $x < -3a$.

c. Applying energy conservation, we get

$$(\text{K.E.}+\text{P.E.})_{\text{centre}}=(\text{K.E.}+\text{P.E.})_{\text{circumference}}$$

$$0+K\left[\frac{Qq}{2a}-\frac{2Qq}{8a}\right]=\frac{1}{2}mv^2+K\left[\frac{Qq}{6a}-\frac{2Qq}{12a}\right]$$

$$\frac{1}{2}mv^2=\frac{KQq}{4a} \Rightarrow v=\sqrt{\frac{KQq}{2ma}}=\sqrt{\frac{1}{4\pi\varepsilon_0}\left(\frac{Qq}{2ma}\right)}$$

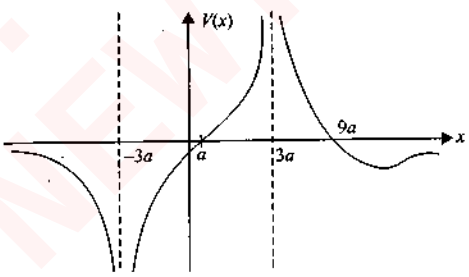


Fig. 3.102

16. Potential at centre O due to induced charges is

$$V_1=-\frac{q}{4\pi\varepsilon_0 a}+\frac{q}{4\pi\varepsilon_0 b}$$

$$V_\infty=0$$

Small work done to transfer a small charge dq from centre to infinity is

$$dW=dq(V_\infty-V_1)=\frac{1}{4\pi\varepsilon_0}\left[\frac{1}{a}-\frac{1}{b}\right]q\,dq$$

$$\Rightarrow \quad W=\int dW=\frac{q^2}{8\pi\varepsilon_0}\left[\frac{1}{a}-\frac{1}{b}\right]$$

17. Let electric field at P due to all four charges be zero.

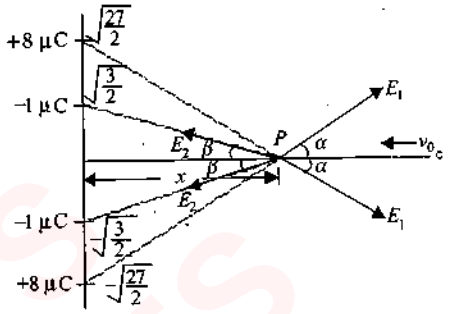


Fig. 3.103

$$2E_1\cos\alpha=2E_2\cos\beta$$

$$\Rightarrow \quad \frac{2k\times8\times10^{-6}x}{(x^2+27/2)^{3/2}}=\frac{2k\times1\times10^{-6}x}{(x^2+3/2)^{3/2}} \Rightarrow x=\sqrt{\frac{5}{2}}\text{ m}$$

Beyond P, E is towards right. To the left of P, E is towards left. Once the charge crosses point P, after this attractive forces will pull the charge to origin.

Applying conservation of energy between ∞ and P, we get

$$\frac{1}{2}mv_0^2=2k\left[\frac{8\times10^6}{\sqrt{\frac{27}{2}+\frac{5}{2}}}-\frac{1\times10^6}{\sqrt{\frac{3}{2}+\frac{5}{2}}}\right]\times0.1\times1\times10^{-6}$$

$$\Rightarrow \quad v_0=3\text{ m/s}$$

To find K.E. at origin, apply conservation of energy between ∞ and origin.

$$\text{K.E.}_{x=0}+2k\times0.1\times10^{-12}\left[\frac{8}{\sqrt{27/2}}-\frac{1}{\sqrt{3/2}}\right]=\frac{1}{2}mv_0^2$$

$$\Rightarrow \quad \text{K.E.}_{x=0}=2.5\times10^{-4}\text{ J}$$

18. $$W_{\text{external}}=\Delta\text{P.E.}=\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\left[-\frac{3}{1}+\frac{3}{\sqrt{2}}-\frac{1}{\sqrt{3}}\right]\times4$$

$$=\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\times\frac{4}{\sqrt{6}}[3\sqrt{3}-3\sqrt{6}-\sqrt{2}]$$

19. As we see $E_1=\frac{\sigma_1}{2\varepsilon_0}$, $E_2=\frac{\sigma_2}{2\varepsilon_0}$

$$E=E_1-E_2=\frac{\sigma_1-\sigma_2}{2\varepsilon_0}$$

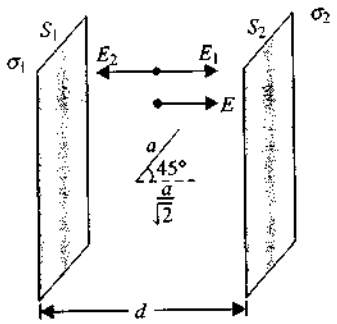


Fig. 3.104

$$W=q_0E\times\frac{a}{\sqrt{2}}=\frac{q_0a}{2\sqrt{2}\varepsilon_0}(\sigma_1-\sigma_2)$$

20. Potential energy of dipole system

$$U = -\vec{p}_2 \cdot \vec{E}_{21} = -p_2 \frac{1}{4\pi\varepsilon_0}\frac{p_1}{x^3}\cos\pi$$

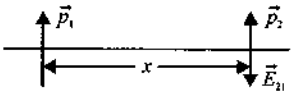


Fig. 3.105

E_{21} is the field due to p_1 at p_2 as shown.

$$U = \frac{1}{4\pi\varepsilon_0}\frac{p_1 p_2}{x^3}$$

$$F = -\frac{\partial U}{\partial x} = \frac{3}{4\pi\varepsilon_0}\frac{p_1 p_2}{x^4}$$

F comes out to be positive, so it is a repulsive force.

21. Potential energy of system is

$$U = -\vec{p}_2 \cdot \vec{E}_{21} = -p_2 \frac{1}{4\pi\varepsilon_0}\frac{2p_1}{x^3}\cos 0 = -\frac{1}{4\pi\varepsilon_0}\frac{2p_1 p_2}{x^3}$$

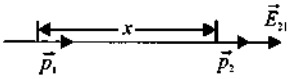


Fig. 3.106

$$F = -\frac{\partial U}{\partial x} = -\frac{6}{4\pi\varepsilon_0}\frac{p_1 p_2}{x^4} = -\frac{3}{2\pi\varepsilon_0}\frac{p_1 p_2}{x^4}$$

F comes out to be negative, so it is an attractive force.

22. a. $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}$

$$U = -pE\cos 0° = -\frac{qp}{4\pi\varepsilon_0 x^2}$$

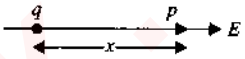


Fig. 3.107

$$F = -\frac{\partial U}{\partial x} = \frac{-pq}{2\pi\varepsilon_0 x^3}$$

–ve sign indicates that force on dipole is towards the –ve x-direction or the force is attractive.

b. $U = -pE\cos 180° = \frac{qp}{4\pi\varepsilon_0 x^2}$

$$F = -\frac{\partial U}{\partial x} = \frac{pq}{2\pi\varepsilon_0 x^3}$$

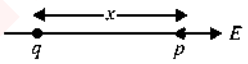


Fig. 3.108

+ve sign indicates that force on dipole is towards the +ve x-direction or the force is repulsive.

c. $E = \frac{1}{4\pi\varepsilon_0}\frac{p}{x^3}$

Let us first find force on q due to p

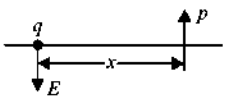


Fig. 3.109

$$F = qE = \frac{qp}{4\pi\varepsilon_0 x^3}$$

Charge q will also apply the same force on dipole but in an opposite direction, so the force on dipole is

$F = \frac{qp}{4\pi\varepsilon_0 x^3}$ along $\vec{p}$ or parallel to y-axis

23. a. Force exerted by the upper charge on dipole,

$$F_1 = \frac{1}{2\pi\varepsilon_0}\frac{pq}{a^3}\text{ (down)}$$

Force exerted by the left charge on dipole,

$$F_2 = \frac{1}{4\pi\varepsilon_0}\frac{pq}{a^3}\text{ (up)}$$

Force exerted by the right charge on dipole,

$$F_3 = \frac{1}{4\pi\varepsilon_0}\frac{pq}{a^3}\text{ (up)}$$

Net force on the dipole due to all charges,

$$F = F_1 + F_2 + F_3 = 0$$

Hence, net force on the dipole is zero.

The total electric potential energy consists of interaction of all the three charges among themselves and interaction of these three charges with dipole. So,

$$U = 2\left(\frac{1}{4\pi\varepsilon_0}\frac{q^2}{\sqrt{2}a}\right) + \frac{1}{4\pi\varepsilon_0}\frac{q^2}{2a}$$

$$-\vec{P}\cdot\vec{E}_{up} - \vec{P}\cdot\vec{E}_{left} - \vec{P}\cdot\vec{E}_{right}$$

$$\vec{P}\cdot\vec{E}_{left} = \vec{P}\cdot\vec{E}_{right} = 0$$

(Because electric fields produced by left and right charges are perpendicular to P.)

$$-\vec{P}\cdot\vec{E}_{up} = -P\cdot\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}\right)\cos\pi = \frac{1}{4\pi\varepsilon_0}\frac{qp}{a^2}$$

b. Putting the values, we get

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{2a}\left[2\sqrt{2}+1\right] + \frac{1}{4\pi\varepsilon_0}\frac{pq}{a^2}$$

24. a. Charges at A, B, C and D are placed at an equilateral position of dipole. Hence, the force on each of them due to dipole is

$$F_1 = \frac{Qp}{4\pi\varepsilon_0 (a/\sqrt{2})^3}.$$

This force is downward on charges. Hence, force due to these charges on dipole is $4F_1$ (upwards)

Force on dipole due to charge at P is

$$F_2 = \frac{2pQ}{4\pi\varepsilon_0 (a/\sqrt{2})^3}\text{ (upward)}$$

Net force on dipole: $F = 4F_1 + F_2 = \frac{3\sqrt{2}\,Qp}{\pi\varepsilon_0 a^3}$ (upward)

b. P.E. of the system is

U = (10 pairs of charged particles) + (5 pairs of dipole and charged particles)

As potential energy of the dipole with four charges at A, B, C and D will be zero,

$$\therefore \quad U = 4\left[\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{a}\right] + 2\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{\sqrt{2}a} - 4\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{a}$$

$$-\frac{1}{4\pi\varepsilon_0}\frac{pQ}{(a/\sqrt{2})^2}$$

$$U = \frac{Q^2}{2\sqrt{2}\pi\varepsilon_0 a} - \frac{pQ}{2\pi\varepsilon_0 a^2}$$

25.

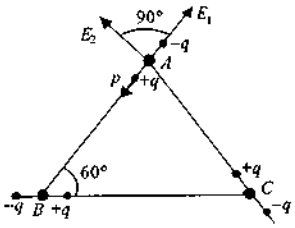


Fig. 3.110

Electric field E_1 due to dipole at B is

$$E_1 = \frac{k\,2p\cos 60°}{d^3} \text{ and } E_2 = \frac{k\,p\sin 60°}{d^3}$$

Potential energy between A and B is

$$-\vec{p}\cdot\vec{E} = -\vec{p}\cdot(\vec{E}_1 + \vec{E}_2) = -\vec{p}\cdot\vec{E}_1 \quad (\because \vec{p} \perp \vec{E}_2)$$

$$\Rightarrow \quad -pE_1\cos 180° = pE_1 = \frac{k2p^2\cos 60°}{d^3} = \frac{q^2a^2}{4\pi\varepsilon_0 d^3}$$

Net energy of system $= \dfrac{3q^2a^2}{4\pi\varepsilon_0 d^3}$

Objective Type

1. a. Electrically neutral means net charge zero. Potential of a neutral conductor may or may not be zero. It depends upon the other charges present in the surrounding.

2. a. Negative charge itself goes from low to high potential.

3. d. $W = q(V_f - V_i) = 0$, since $V_f = V_i$.

4. a. Because electric field intensity is zero inside a spherical conductor.

5. b. The potential at the surface of a hollow or conducting sphere is same as the potential at the centre of the sphere and any point inside the sphere.

6. b. Apply $V = \dfrac{kQ}{R}$, where $Q = Ze$

7. c. $\text{K.E.}_i + \text{P.E.}_i = \text{K.E.}_f + \text{P.E.}_f$

$$0 + q \times 600 = \text{K.E.}_f + q \times 0$$

$$\text{K.E.}_f = 600 \times 10^{-8} = 6 \times 10^{-6}\ \text{J}$$

8. d. Let the charge on the smaller sphere be q. As the potential of both will be the same finally, i.e.,

$$\frac{q}{r'} = \frac{Q-q}{r} \Rightarrow q = \frac{Qr'}{r+r'}$$

9. c. $\dfrac{k \times 4}{1-x} = \dfrac{k \times 2}{x} \quad \Rightarrow x = \dfrac{1}{3}\text{ m}$

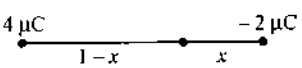


Fig. 3.111

10. c. $E_1 = E_2 \quad \Rightarrow \quad \dfrac{kQ_1}{r_1^2} = \dfrac{kQ_2}{r_2^2}$

$$\frac{Q_1}{Q_2} = \left(\frac{r_1}{r_2}\right)^2$$

$$\frac{V_1}{V_2} = \frac{Q_1}{r_1} \times \frac{r_2}{Q_2} = \left(\frac{r_1}{r_2}\right)^2 \frac{r_2}{r_1} = \frac{r_1}{r_2}$$

11. b. E is a vector quantity, V is a scalar quantity.

12. a. $E = -\dfrac{dV}{dr} =$ negative of the slope of V–r graph.

13. b. $W_{eq} = q(V_A - V_B)$

$$50 \times 10^{-6} = 2 \times 10^{-6}(V_A - V_B)$$

$$V_A - V_B = 25\text{ V}$$

14. c. As potential at A and B is the same, $V_A = V_B = kQ/d$. So, work done in both the cases will be the same.

15. b. $U = \sum \dfrac{kq_1q_2}{r}$. There will be six pairs, four on a side of square and two as diagonal.

16. d. A conductor is an equipotential body. Potential on it or within it is the same everywhere.

17. a. Potential decreases in the direction of electric field.

18. d. $V = \dfrac{kQ}{c} - \dfrac{kQ}{b}$

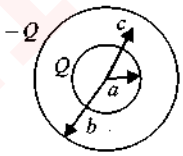


Fig. 3.112

19. a. $V_C = V_1 = V_2$

$$\frac{kq_1}{r_1} = \frac{kq_2}{r_2}$$

$$\frac{q_1}{q_2} = \frac{r_1}{r_2} \qquad \text{(i)}$$

$$q_1 + q_2 = Q \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$q_1 = \frac{Qr_1}{r_1 + r_2}$$

Putting V_1, we get $V_C = \dfrac{kQ}{r_1 + r_2}$

20. d. V is a scalar quantity, E is a vector quantity.

21. a. $dV = -E\,dr\cos\theta$

Along the line of force, θ is 0°, hence dV is maximum. So, the variation of potential is maximum along the line of force.

22. b. Charge will induce on A but total charge on A will remain zero. Negative charge of A will be more closer to B than positive charge on A. So potential of B will decrease.

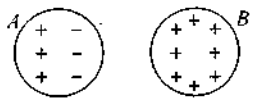


Fig. 3.113

23. a. Their potential will be same, i.e., $V_1 = V_2$

$$\frac{kq_1}{R_1} = \frac{kq_2}{R_2} \Rightarrow \frac{q_1}{q_2} = \frac{R_1}{R_2}$$

$$\frac{E_1}{E_2} = \frac{kq_1/R_1^2}{kq_2/R_2^2} = \frac{q_1}{q_2}\left(\frac{R_2}{R_1}\right)^2$$

$$= \frac{R_1}{R_2}\left(\frac{R_2}{R_1}\right)^2 = \frac{R_2}{R_1}$$

24. d. $F = qE, W = qE \times 2\cos 60°$

$\Rightarrow \quad 4 = 0.2E \times 2 \times \frac{1}{2} \Rightarrow E = 20\ \text{NC}^{-1}$

25. c. Perpendicular distance between equipotential surfaces: $d = 10 \sin 30° = 5$ cm

$$E = \frac{\Delta V}{d} = \frac{30-20}{5/100} = 200\ \text{Vm}^{-1}$$

Direction of electric field will be in the direction of decreasing potential.

26. b. $W_{\text{el.}} = q(V_i - V_f)$

$\Rightarrow \quad 6.4 \times 10^{-19} = -1.6 \times 10^{-19}(V_A - V_B)$

$\Rightarrow \quad V_A - V_B = -4\ \text{V}$

$\Rightarrow \quad V_A - V_C = -4\ \text{V}(\because V_B = V_C)$

$\Rightarrow \quad V_C - V_A = 4\ \text{V}$

27. b. $W_{AB} = W_{AC} + W_{CB}$

W_{CB} should be zero, because in moving from C to B, we always move perpendicular to field. Hence, force applied by field and displacement will be at 90°.

$$W_{AC} = -e(V_C - V_A)$$

$$V_C - V_A = -E \times AC$$

$$= -10 \times 4 = -40$$

$$W_{AC} = -e \times (-40) = 40e$$

So $W_{AB} = 40e\ \text{J} = 40\ \text{eV}$

28. b. $V_1 = \frac{k \times 2 \times 10^{-6}}{0.1} + \frac{k \times 4 \times 10^{-6}}{\sqrt{(0.1)^2 + (0.5)^2}}$

$$V_2 = \frac{k \times 4 \times 10^{-6}}{0.1} + \frac{k \times 2 \times 10^{-6}}{\sqrt{(0.1)^2 + (0.5)^2}}$$

Work done $= q(V_2 - V_1) = 0.72$ J

29. d. $Q = nq, R = n^{1/3}r$

$$E_{\text{small}} = \frac{kq}{r^2}, E_{\text{big}} = \frac{kQ}{R^2}$$

$$E_{\text{big}} = \frac{k\,n\,q}{n^{2/3}r^2} = (n^{1/3})\frac{kq}{r^2} = n^{1/3}E_{\text{small}}$$

$$V_{\text{small}} = \frac{kq}{r}, V_{\text{big}} = \frac{kQ}{R}$$

$$V_{\text{big}} = \frac{k\,n\,q}{n^{1/3}r} = n^{2/3}\frac{kq}{r} = n^{2/3}V_{\text{small}}$$

30. a. We know that any extra charge resides on the outer surface. Finally, both will be at the same potential. It is because both are connected.

31. d. Equipotential surfaces are perpendicular to the electric lines of forces.

32. d. $E = \frac{\sigma}{\varepsilon_0}$ does not depend upon radius if σ is constant.

33. a. $V = ER$; If R is doubled, V also gets doubled.

34. b. Electric intensity inside is zero, whereas outside it is not zero.

35. d. Potential decreases in the direction of electric field. So it depends whether the lines of forces are from A to B or from B to A.

36. c. Apply conservation of mechanical energy between point a and b: $(\text{K.E.} + \text{P.E.})_a = (\text{K.E.} + \text{P.E.})_a$

$$\Rightarrow \quad 0 + \frac{k(3\times10^{-9})q_0}{0.01} - \frac{k(3\times10^{-9})q_0}{0.02}$$

$$\Rightarrow \quad \frac{1}{2}mv^2 + \frac{k(3\times10^{-9})q_0}{0.02} - \frac{k(3\times10^{-9})q_0}{0.01}$$

Put the values and get: $v = 12\sqrt{15} = 46\ \text{ms}^{-1}$

37. b. $U = -\frac{kqQ}{r} - \frac{kqQ}{r} + \frac{kq^2}{2r} = 0 \Rightarrow Q/q = 1/4$

38. b. Find potential at A and C due to charge at B, then the required work done is $W = q(V_A - V_C)$.

39. c. Apply conservation of mechanical energy,

$$\frac{-9\times10^9\times10^{-12}}{2} = \frac{-9\times10^9\times10^{-12}}{1} + \frac{1}{2}(2m)v^2$$

$\Rightarrow \quad v = 3/10$ m/s

40. a. Electric field due to large metal plate $= \sigma/\varepsilon_0$.

Since the electron has energy of 100 eV, so it can cross a potential difference of 100 V only. So

$$100 = \frac{\sigma}{\varepsilon_0}d\text{, solve to get } d.$$

41. c. $\frac{kQ}{x} = \frac{1}{2}\left(\frac{3}{2}\frac{kQ}{R}\right) \Rightarrow x = 4R/3$

Distance from surface: $x - R = R/3$

42. d. All the charges are placed in the x-y plane such that they form a square with origin as its centre. So electric field at the origin will be zero but at other points on z-axis, field will be along z-axis.

43. d. It depends whether both charges are of the same or opposite sign.

44. a. Because work is to be done by an external agent in moving a positive charge from low potential to high potential and this work gets stored in the form of potential energy of the system. Hence, it increases.

45. b. Electric field will do the negative work, because the force of electric field is opposite to the displacement. So external agent has to do positive work. So its energy will be used.

46. d. Option (a) is wrong, because force will be the same, but acceleration will be different because masses of electrons and protons are different.

Option (b) is wrong, because at a point there can be only one potential.

Option (c) is wrong, because charge always lies on the outer surface of a conductor.

Option (d) is correct, because the whole conductor will be an equipotential body.

47. c. $\frac{kQ}{4x_1} = \frac{kQ}{r+x_1} \Rightarrow r + x_1 = 4x_1 \Rightarrow x_1 = r/3$

$\frac{kQ}{4x_2} = \frac{kQ}{r-x_2} \Rightarrow x_2 = \frac{r}{5}$

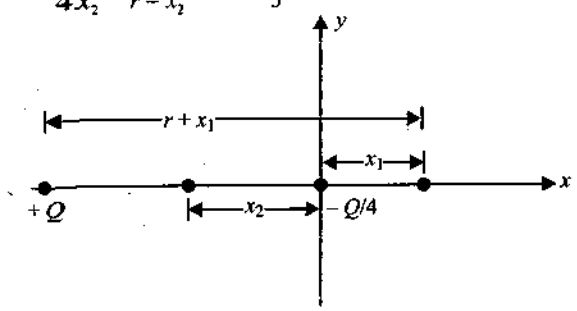


Fig. 3.114

48. c. $E_x = \frac{dV}{dx} = \frac{120-80}{2} = 20\ \text{Vcm}^{-1}$

There may be the y- and z-components of field also.

49. c. $F_e = mg\tan\theta$

$= (1.20\times10^{-3}\ \text{kg})(10\ \text{ms}^{-2})\tan(37°) = 0.0090\ \text{N}$

$$F_e = Eq = \frac{Vq}{d}$$

$$\therefore \quad V = \frac{F_e d}{q} = \frac{(0.009\ \text{N})(0.0500\ \text{m})}{9.0\times10^{-6}\ \text{C}} = 50.0\ \text{V}$$

50. a. $\left(0 - \frac{1}{2}mv^2\right) = q(V_P - V_Q)$ (i)

$\left(\frac{1}{2}mv_1^2 - \frac{1}{2}mv^2\right) = -q(V_P - V_Q)$ (ii)

From Eqs. (i) and (ii), $v_1 = \sqrt{2}v$

51.a. $W = \vec{F}\cdot\vec{r} = q\vec{E}\cdot\vec{r}$

52.c. $V = k\frac{q}{R} - \frac{kq}{2R} + \frac{kq}{3R} = \frac{kq}{R}\left[1 - \frac{1}{2} + \frac{1}{3}\right]$

$= \frac{k\,5q}{6\,R}$

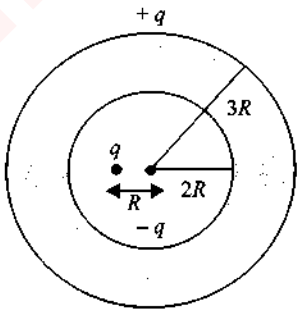


Fig. 3.115

53. a. $V_B - V_A = -\left[\int_1^2 2dx + \int_2^1 3dy\right]$

$= -[2(2-1) + 3(1-2)]$

$= -[2-3] = 1\ \text{V}$

Hence, $V_A - V_B = -1\ \text{V}$

54. d. $E_x = -\frac{dV}{dx} = -4x = -4\times2 = -8,$

$E_y = 0, E_z = 0$

Hence, $\vec{E} = -8\hat{i}\ \text{NC}^{-1}$

55. b. Equipotential lines are concentric circles,

These pattern of equipotential lines should be due to point charge at the centre of circles.

Let us consider few equipotential lines,

$$20 = \frac{1}{4\pi\varepsilon_0}\frac{q}{30\times10^{-2}} \Rightarrow \frac{q}{4\pi\varepsilon_0} = 6$$

$$60 = \frac{1}{4\pi\varepsilon_0}\frac{q}{10\times10^{-2}} \Rightarrow \frac{q}{4\pi\varepsilon_0} = 6$$

For point charge, electric field should be in radial direction.

The value is $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{6}{r^2}\ (\text{Vm}^{-1})$

56. b. Charge $+Q$ will flow into earth.

$$V_C = \frac{kQ}{r} - \frac{kQ}{b}$$

$$V_A = \frac{kQ}{a} - \frac{kQ}{b}$$

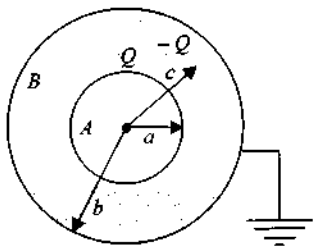


Fig. 3.116

Potential at B is zero.

57. a. $E = \frac{V}{d}, F = eE = eV/d$

$$a = \frac{F}{m} = \frac{eV}{md}$$

$$d = \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2d}{a}}$$

$$t = \sqrt{\frac{2d\ md}{eV}} = \sqrt{\frac{2\ md^2}{eV}}$$

58. c. $U = 2kq^2\left[-\frac{1}{a} + \frac{1}{2a} - \frac{1}{3a} + \frac{1}{4a} + \cdots\right]$

$= -\frac{2q^2}{4\pi\varepsilon_0 a}\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots\right] = -\frac{2q^2\log_e 2}{4\pi\varepsilon_0 a}$

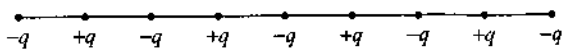


Fig. 3.117

59. b. $V_x - V_\infty = -\int_\infty^x E_x dx \Rightarrow V_x - 0 = -\int_\infty^x \frac{A}{x^3} dx$

$\Rightarrow \quad V_x = \frac{A}{2x^2}$

60. c. $\frac{k\,q_1}{a} + \frac{k\,q}{2d} = 0 \Rightarrow q_1 = \frac{-q\,a}{2d}$

$$\frac{kq_2}{a} + \frac{kq}{2d} + \frac{kq_1}{d} = 0 \Rightarrow q_2 = \frac{-qa}{2d}\left(\frac{d-a}{d}\right)$$

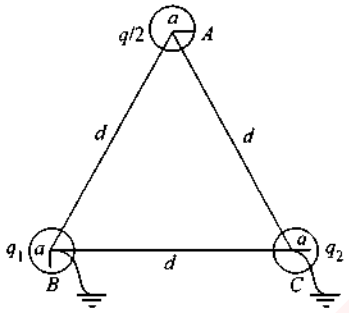


Fig.3.118

61. b. On connecting, the entire amount of charge will shift to the outer sphere.
Heat generated

$$U_i - U_f = \frac{q^2}{8\pi\varepsilon_0 R_1} - \frac{q^2}{8\pi\varepsilon_0 R_2}$$

$$= \frac{(20\times10^{-6})\times9\times10^9}{2}\left[\frac{1}{0.10} - \frac{1}{0.20}\right] = 9\text{ J}$$

62. c. $V = \frac{kQ}{r} \Rightarrow u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\frac{\varepsilon_0 k^2 Q^2}{r^4}$

$$V^4 \propto u$$

63. b. $\vec{E} = a(y\hat{i} + x\hat{j})$

$$V_2 - V_1 = -\int(ay\,dx + ax\,dy)$$

Take $V_1 = C$ and $V_2 = V$

$$V = -a\int(ydx + xdy) + C$$
$$= -a\int d(yx) + C = -axy + C$$

64. d. (i) $Q/2, Q/2, 0$
(ii) $Q/2, Q/4, Q/4$
(iii) $3Q/8, Q/4, 3Q/8$
(iv) $5Q/16, 5Q/16, 3Q/8$
(v) $5Q/16, 11Q/32, 11Q/32$

65. a. $V = x^2 - y^2$

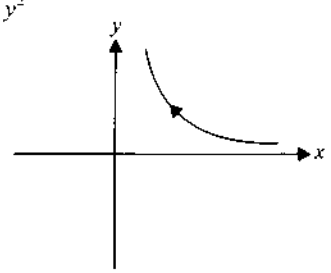


Fig. 3.119

$$E_x = -\frac{dV}{dx} = -2x$$

$$E_y = 2y$$

66. a. As we add $-3Q$ charge on the surface, the potential on the surface changes by the same amount as that inside. Therefore, the potential difference remains the same.

Multiple Correct Answers Type

1. a., b., c., d.

Charge on $a_1 = (r_1\theta)\lambda$

Charge on $a_2 = (r_2\theta)\lambda$

Ratio of charges $= \frac{r_1}{r_2}$

E_1 (Field produced by a_1) $= \frac{K[(r_1\theta)\lambda]}{r_1^2} = \frac{K\theta\lambda}{r_1}$

E_2 (Field produced by a_2) $= \frac{K\theta\lambda}{r_2}$

As $r_2 > r_1$, therefore $E_1 > E_2$

i.e., Net field at A is towards a_2.

$$V_1 = \frac{K(r_1\theta)}{r_1} = K\theta\lambda$$

$$V_2 = \frac{K(r_2\theta)}{r_2} = K\theta\lambda$$

$$V_1 = V_2$$

2. a., d. E is a vector quantity; V is a scalar quantity.

3. a., c., d. $0 \le x \le a; V_x = \left[-\int_0^x E_x dx\right] + V_{(0)} = 0$

(as $E_x = 0$)

$$x \ge a; V = -\int_a^x E_x dx + V_{(a)}$$

$$V = \left[-\int_a^x \frac{\sigma}{\varepsilon_0} dx\right] + V_{(a)} = -\frac{\sigma}{\varepsilon_0}(x-a) = \frac{-\sigma}{\varepsilon_0}(x-a)$$

$$x \le 0;\ V = -\int_0^x E_x dx + V_{(0)}\left[\because E_x = \frac{-\sigma}{\varepsilon_0}\right]$$

$$= -\left(-\frac{\sigma}{\varepsilon_0}x\right) + V_{(0)} = \frac{\sigma}{\varepsilon_0}x$$

4. a., d. When a negative charge moves from high potential to low potential, its potential energy increases.

$$W_{el} = q(V_1 - V_2)$$

As $V_1 > V_2$ and q is negative, hence

$$W_{el} = \text{negative.}$$

5. a., b., c. This question is based on the working principle of a generator.

$$V_1 - V_{II} = \left(\frac{q_1}{4\pi\varepsilon_0 R_1} + \frac{q_2}{4\pi\varepsilon_0 R_2}\right) - \left(\frac{q_1 + q_2}{4\pi\varepsilon_0 R_2}\right)$$

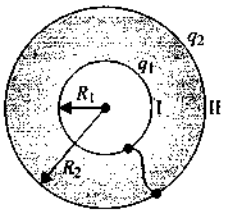


Fig. 3.120

$$V_I - V_{II} = \frac{q_1}{4\pi\varepsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

As two conductors are connected, transfer of charge takes place from one conductor to other till both acquire same potential, i.e., here $V_I - V_{II} = 0$.

Potential difference depends only on q_1 (charge of inner conductor), so $V_I - V_{II} = 0$, where $q_1 = 0$, i.e., total charge is transferred to the outer conductor.

6. a., d. As no charge is present inside the conductor, potential at any point inside the conductor is same as that of the potential of conductor.

So, potential of the conductor = Potential at the centre

$$= V_q + V_{\text{induced charges}}$$

i.e., $$V_{\text{conductor}} = \frac{q}{4\pi\varepsilon_0(d+R)} + 0$$

As the total induced charge at conductor's surface is equal to zero, hence the potential at centre due to the induced charges is zero.

For point B

$$V_{\text{conductor}} = V_{\text{at point B}} = V_q + V_{\text{induced charges}}$$

$$V_{\text{induced charges}} = \frac{q}{4\pi\varepsilon_0(d+R)} - \frac{q}{4\pi\varepsilon_0 d} = \frac{-qR}{4\pi\varepsilon_0 d(d+R)}$$

Assertion-Reasoning Type

1. c. Electric field is directed from high potential to low potential.

2. a. The surface of a conductor is an equipotential surface as electric field inside the conductor is zero. If there is some potential difference between two points of conductor, then charge will flow within conductor to make the potential same.

3. a. Inside electric field is zero but not outside.

4. c. Earth also has some surface charge density due to which it produces electric field in the surrounding space.

Comprehension Type

For Problems 1–2

1. b., 2. c.

Sol. The work that must be done on q_3 by an external force $\vec{F}_{ext}$ is equal to the difference between two quantities: the potential energy U associated with q_3 when it is at $x = 2a$ and the potential energy when it is infinitely far away.

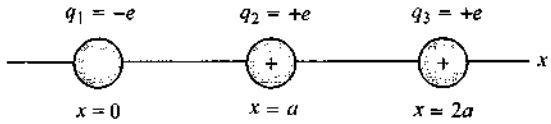


Fig. 3.121

The second of these is zero, so the work that must be done is equal to U. The distances between the charges are $r_{13} = 2a$ and $r_{23} = a$, so

$$W = U = \frac{q_3}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right)$$

$$= \frac{+e}{4\pi\varepsilon_0}\left(\frac{-e}{2a} + \frac{+e}{a}\right) = \frac{+e^2}{8\pi\varepsilon_0 a}$$

If q_3 is brought in from infinity along the $+x$-axis, it is attracted by q_1 but is repelled more strongly by q_2; hence, positive work must be done to push q_3 to the position at $x = 2a$.

The total potential energy of the assemblage of three charges is

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{(-e)(e)}{a} + \frac{(-e)(e)}{2a} + \frac{(e)(e)}{a}\right) = \frac{-e^2}{8\pi\varepsilon_0 a}$$

For Problems 3–4

3. c., 4. b.

Sol. $$E_i = K_i + U_i = \frac{1}{2}(0.0015\text{ kg})(20.0\text{ ms}^{-1})^2 + \frac{k(2.00\times10^{-6}\text{ C})(8.00\times10^{-6}\text{ C})}{0.800\text{ m}}$$

$$= 0.48\text{ J}$$

$$E_i = E_f = \frac{1}{2}mv_f^2 + \frac{kq_1q_2}{r_f}$$

$$v_f = \sqrt{\frac{2(0.48\text{ J} - 0.36\text{ J})}{0.0015\text{ kg}}} = 4\sqrt{10}\text{ ms}^{-1}$$

At the closest point, the velocity is zero:

$$0.48\text{ J} = \frac{kq_1q_2}{r}$$

$$r = \frac{k(2.00\times10^{-6}\text{C})(8.00\times10^{-6}\text{C})}{0.48\text{ J}} = 0.30\text{ m}$$

For Problems 5–7

5. a., 6. c., 7. b.

Sol. At $A: V_A = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$

$$= k\left(\frac{2.40\times10^{-9}\text{ C}}{0.05\text{ m}} + \frac{-6.50\times10^{-9}\text{ C}}{0.05\text{ m}}\right) = -738\text{ V}$$

At $B: V_B = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$

$$= k\left(\frac{2.40\times10^{-9}\text{ C}}{0.08\text{ m}} + \frac{-6.50\times10^{-9}\text{ C}}{0.06\text{ m}}\right) = -705\text{ V}$$

$$W_{e_1} = q(V_i - V_f)$$

$$= 2.5 \times 10^{-9}[-705 + 738]$$

$$= 8.25 \times 10^{-8} \text{ J}$$

For Problems 8–11

8. d., 9. d., 10. d., 11. b.

Sol. Electric field at O (Fig. 3.122)

$$E_0 = 2(2|\bar{E}_A|\cos 45°)$$

$$= 4\frac{1}{4\pi\varepsilon_0}\frac{q}{(a/\sqrt{2})^2} \times \frac{1}{\sqrt{2}}$$

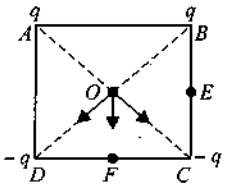


Fig. 3.122

$$E_0 = \frac{\sqrt{2}q}{\pi\varepsilon_0 a^2}$$

Electric potential at O will be zero.

Also $V_E = 0$, so work done in carrying a charge from O to E will be zero.

$$V_F = 2\left[\frac{1}{4\pi\varepsilon_0}\frac{q}{(a^2 + a^2/4)^{1/2}} - \frac{1}{4\pi\varepsilon_0}\frac{q}{a/2}\right]$$

$$= \frac{q}{\pi\varepsilon_0 a}\left[\frac{1}{\sqrt{5}} - 1\right]$$

Now $W_{OF} = e(V_F - V_O)$

For Problems 12–13

12. a., 13. c.

Sol. If q_1 and q_2 are the charges on the spheres of radii r and R, respectively, then, by conservation of charge

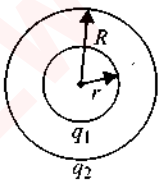


Fig. 3.123

$$q_1 + q_2 = Q \qquad \text{(i)}$$

But $\sigma_1 = \sigma_2$

$$\frac{q_1}{4\pi r^2} = \frac{q_2}{4\pi R^2}$$

$$\frac{q_1}{q_2} = \frac{r^2}{R^2} \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$q_1 = \frac{Qr^2}{r^2 + R^2} \text{ and } q_2 = \frac{QR^2}{r^2 + R^2}$$

Hence, potential at common centre,

$$V = V_1 + V_2$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r} + \frac{q_2}{R}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{Qr}{(R^2 + r^2)} + \frac{QR}{(R^2 + r^2)}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q(R + r)}{(R^2 + r^2)}$$

For Problems 14–15

14. a., 15. b.

Sol. The points A and C lie on equipotential line

$$\therefore \quad V_A = V_C$$

The points B and D lie on equipotential line

$$\therefore \quad V_B = V_D$$

$$V_A - V_D = V_A - V_B = E\Delta x = 20 \times 1 = 20 \text{ V}$$

For Problems 16–18

16. c., 17. b., 18. a.

Sol.

$$E_r = -\frac{\Delta V}{\Delta r}; \; E_x = -\frac{\Delta V}{\Delta x}$$

For $-2 \le x \le 0$; $E_x = -\frac{(10 - 0)}{[0 - (-2)]} = -5 \text{ NC}^{-1}$

For $0 \le x \le 2$; $E_x = 0$

For $2 \le x \le 4$; $E_x = -\frac{[20 - 10]}{[4 - 2]} = -5 \text{ NC}^{-1}$

For $4 \le x \le 8$; $E_x = -\frac{[0 - 20]}{[8 - 4]} = 5 \text{ NC}^{-1}$

For Problem 19–20

19. c., 20 a.

Sol. $d_1 = AO\cos 30° = 2\sqrt{3}/2 = \sqrt{3}$ m

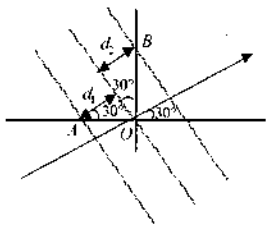


Fig. 3.124

$$d_2 = BO\sin 30° = 4 \times \frac{1}{2} = 2 \text{ m}$$

$$V_A - V_O = Ed_1 = 100\sqrt{3} \text{ V}$$

$$\Rightarrow \quad \Rightarrow V_O - V_A = -100\sqrt{3} \text{ V}$$

$$V_A - V_B = E(d_1 + d_2) = 100(\sqrt{3} + 2) \text{ V}$$

$$\Rightarrow \quad \Rightarrow V_B - V_A = -100(\sqrt{3} + 2) \text{ V}$$

For Problems 21–24

21. a., 22. b., 23. d., 24. a.

Sol. $E_x = -\frac{\delta V}{\delta x} = -3\ \text{Vm}^{-1}; E_y = -\frac{\delta V}{\delta y} = -4\ \text{Vm}^{-1}$

$$a_x = \frac{qE_x}{m} = -\frac{1\times10^{-6}\times3}{0.1} = -3\times10^{-5}\,\text{ms}^{-2}$$

$$a_y = \frac{qE_y}{m} = -\frac{1\times10^{-6}\times4}{0.1} = -4\times10^{-5}\,\text{ms}^{-2}$$

Time taken to cross the x-axis:

Using $s = ut + \frac{1}{2}at^2$

$$3.2 = \frac{1}{2}\times4\times10^{-5}\times t^2$$

$$t = 400\ \text{s}$$

$$v_x = a_x t = -3\times10^{-5}\times400$$

$$= 12\times10^{-3}\,\text{ms}^{-1}$$

$$v_y = a_y t = -4\times10^{-5}\times400$$

$$= 16\times10^{-3}\,\text{ms}^{-1}$$

$$v = \sqrt{v_x^2 + v_y^2}$$

$$= 20\times10^{-3}\,\text{ms}^{-1}$$

For Problems 25–26

25. a. → iii., b. → ii., c. → i., 26. b.

Sol. $q_1 = \sigma 4\pi a^2, q_2 = -\sigma 4\pi b^2, q_3 = \sigma 4\pi c^2$

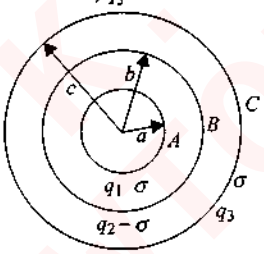


Fig. 3.125

$$V_A = \frac{kq_1}{a} + \frac{kq_2}{b} + \frac{kq_3}{c}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{\sigma 4\pi a^2}{a} - \frac{\sigma 4\pi b^2}{b} + \frac{\sigma 4\pi c^2}{c}\right]$$

$$\frac{\sigma}{\varepsilon_0}[a - b + c]$$

Similarly, $V_B = \frac{kq_1}{b} + \frac{kq_2}{b} + \frac{kq_3}{c}$,

$$V_C = \frac{kq_1}{c} + \frac{kq_2}{c} + \frac{kq_3}{c}$$

For problem 26, put $V_A = V_c$

For Problems 27–30

27. b., 28. a., 29. c., 30. d.

Sol. $1.8\times10^6 = \frac{9\times10^9\times q_1}{(0.1)^2} \Rightarrow q_1 = 2\times10^{-6}$ C

$$V = 9\times10^9\times2\times10^{-6}\left[\frac{1}{0.1} - \frac{1}{0.2}\right] = 9\times10^4\ \text{V}$$

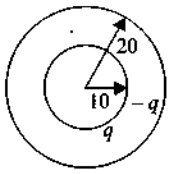


Fig. 3.126

28. $E = \frac{kq_1}{r^2} = \frac{9\times10^9\times2\times10^{-6}}{(0.20)^2} = 4.5\times10^5$ N/C

29. Interaction energy,

$$U = \frac{kq}{0.2}(-q) = \frac{9\times10^9\times(2\times10^{-6})^2}{0.2} = -0.18\ \text{J}$$

Self-energy,

$$U_1 = \frac{q_2}{8\pi\varepsilon_0 r_1} = \frac{9\times10^9\times(2\times10^{-6})^2}{2\times0.1} = 0.18\ \text{J}$$

$$U_2 = \frac{q_2}{8\pi\varepsilon_0 r_2} = \frac{9\times10^9\times(2\times10^{-6})^2}{2\times0.2} = 0.09\ \text{J}$$

Total energy: $U + U_1 + U_2 = 0.09$ J

30. Both the charges will get neutralized and there will be no charge left on any sphere. So, no energy will be left in the system. It means whole amount of energy will convert into heat.

Matching Column Type

1. i. → a., d., ii. → c., e., iii. → c., e.

Sol. i. Due to q, charge will be induced on the conductor, such that net field due to q and induced charge becomes zero at any point inside the conductor.

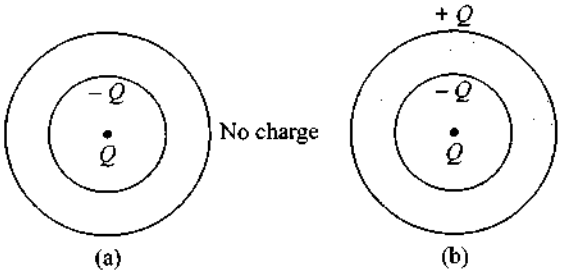
Since $E = 0$ everywhere inside the conductor, so potential is constant inside and same as that of surface of conductor.

ii. Due to q, field and potential both will vary inside.

iii. Due to q_2, field and potential both will vary inside. Because inside charge system has nothing to do with outside system.

2. i. → b., g., ii. → c., h., iii. → a., f., iv. → d., e.

Sol. i. There is no charge on the outer surface, so no electric field at $r > a$ [Fig. 3.127(a)]. E and V exist only inside $r > a$ as shown by (b) and (g).



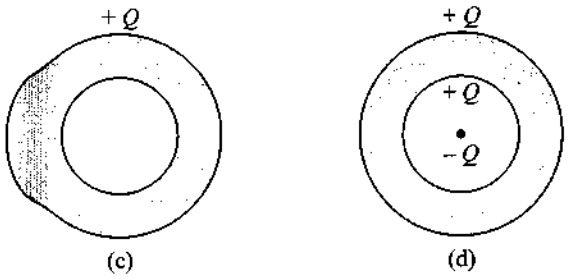


Fig. 3.127

ii. No field from $r = a$ to b, potential at outside point will be same as that produced by a point charge Q at centre [Fig.3.127(b)].

iii. No charge inside, hence no electric field inside $r < b$. For $r \geq b$, field will be same as that produced by a potential charge at centre [Fig. 3.127(c)].

iv. For $r < a$, field will be that of $-Q$ at centre [Fig. 3.127(d)].
For $r > b$, field will be that of $+Q$ at centre.
For $a < r < b$, no field.

3. i → d., ii. → c., iii. → b., iv. → a.

Sol. Charge induced at the surface of '1'

$$q_1 = -q$$

Any point located on the surface of sphere 2 will have zero potential

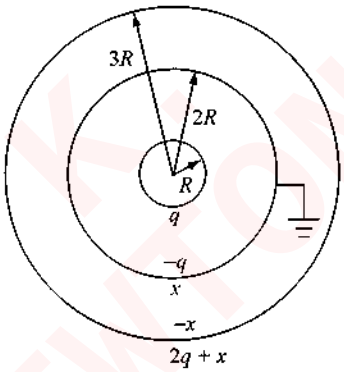


Fig. 3.128

$$V_P = 0 = V_{A,\text{out}} + V_{B,\text{surface}} + V_{C,\text{in}}$$

$$0 = k\frac{(q)}{2R} + k\frac{(-q+x)}{2R} + k\frac{(2q)}{3R}$$

Hence, $x = -\frac{4}{3}q$.

Hence, charge appearing on surface 2 is $-(4/3)q$ and charge appearing on surface 3 is $(4/3)q$ and charge appearing on surface 4 is $q_4 = 2q - (4/3)q = (2q/3)$.

Integer Answer Type

1. (9) Let q be the charge on each drop: $V_1 = \frac{kq}{r}$

Let R be radius of single drop. Then

$$\frac{4}{3}\pi R^3 = 27\frac{4}{3}\pi r^3 \Rightarrow R = 3r$$

$$V_2 = \frac{k(27q)}{R} = \frac{k27q}{3r} = 9V_1 \Rightarrow \frac{V_2}{V_1} = 9$$

2. (3) Let charge appearing on inner sphere be x,

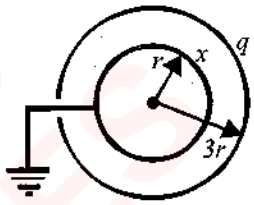


Fig. 3.129

Now potential of inner sphere is zero, i.e.

$$\frac{kq}{3r} + \frac{kx}{r} = 0 \Rightarrow x = -q/3$$

Now charge which flows from the inner shell to the earth shall be $-x = q/3$.

3. (0) All the four corners will be at the same potential.

4. (0) The charge on the infinitesimal elements of arc which subtend an angle $d\theta$ at the centre of the ring.

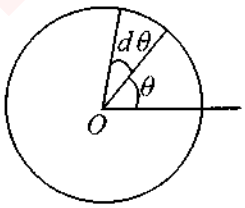


Fig. 3.130

$$dQ = \lambda R d\theta = \lambda_0 \cos\frac{\theta}{2} R d\theta$$

Potential at the centre of ring due to charge dQ

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{R} = \frac{\lambda_0 \cos\frac{\theta}{2} R d\theta}{4\pi\varepsilon_0 R}$$

$$V = \int dV \Rightarrow V = \frac{\lambda_0}{4\pi\varepsilon_0}\int_0^{2\pi} \cos\frac{\theta}{2} d\theta$$

$$= \frac{\lambda_0}{4\pi\varepsilon_0}\left[\frac{\sin\frac{\theta}{2}}{\frac{1}{2}}\right]_0^{2\pi} = 0 \text{ V}$$

5. (0) $W = U_f - U_i$

where $U_i = \frac{1}{4\pi\varepsilon_0 a} \times [(-2q)(-2q) + (-2q)(q) + (q)(-2q)] = 0$

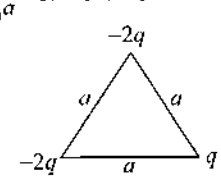


Fig. 3.131

Also, $U_f = 0$ So, $W = 0$

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CHAPTER
4

Capacitor and Capacitance

- Capacitor
- Units of Capacitance
- Combination of Capacitors
- Kirchhoff's Rules for Capacitors
- Sign Convention
- Dielectric
- Force on a Dielectric Slab
- Spherical Capacitor
- Cylindrical Capacitor

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CAPACITOR

Any two conductors separated by an insulator (or a vacuum) form a capacitor (see Fig. 4.1). An electrical capacitor is not a device to store electric charge, but it stores electric energy in the form of electric field. Basic elements of a capacitor are two isolated conductors of arbitrary shape. No matter what their geometry is, these conductors are called *plates*. If these two isolated conductor plates are kept parallel to each other, the combination is called a parallel-plate capacitor as shown in Fig. 4.2.

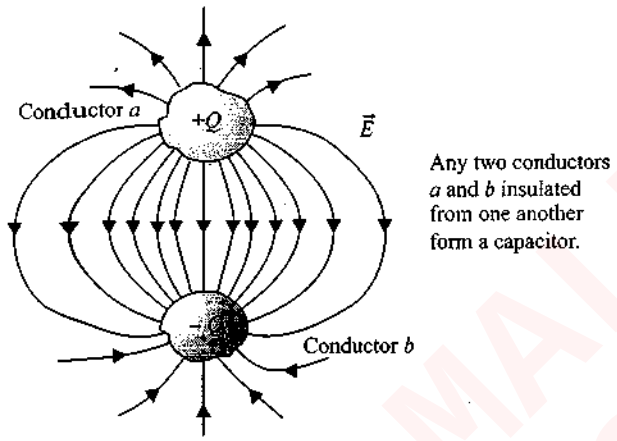
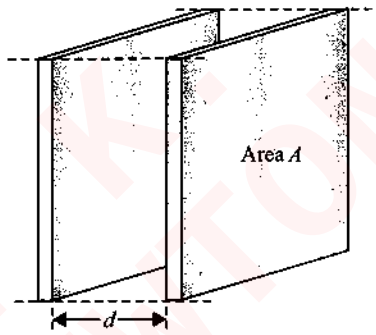


Fig. 4.1



Parallel-plate capacitor

Fig. 4.2

The symbol for a capacitor is ┤├. Although this symbol is based upon the geometry of a parallel-plate capacitor, in fact, it is used for any type of capacitor, e.g., a parallel-plate capacitor, a spherical capacitor or a cylindrical capacitor etc.

The region between the two conductor plates is generally filled by an insulating material called *dielectric*.

Practically, almost always, when a capacitor is charged, its plates have equal but opposite charges $+q$ and $-q$. However, we refer to the charge on a capacitor as being q, the absolute value of the charges on the plates. Please note it consciously that q is not the net charge on the capacitor, in fact the net charge on the capacitor is zero.

Or simply, when we say that a capacitor has charge q, or that a charge q is stored on the capacitor, it means that the conductor at higher potential has charge $+q$ and the conductor at lower potential has charge $-q$ (assuming that q is positive).

Because the plates are conductors, they are equipotential, i.e., all the points on a plate are at the same electric potential. Further, there is a potential difference between the two plates, which is denoted by V for historical reasons, rather than by ΔV.

The charge q and potential difference V for a capacitor are proportional to each other, i.e., $q = CV$.

The constant of proportionality C is called the capacitance of the capacitor. To determine the capacitance of a capacitor, electrical techniques can also be used.

The electric field at any point in the region between the conductors is proportional to the magnitude q of charge on each conductor. It follows that the potential difference V between the conductors is also proportional to q. If we double the magnitude of charge on each conductor, the charge density at each point doubles, the electric field at each point doubles and the potential difference between conductors doubles; however, the ratio of charge to potential difference does not change. This ratio is called the capacitance C of the capacitor as mentioned earlier, $C=q/V$ (definition of capacitance).

The greater the capacitance C of a capacitor, the greater the magnitude of charge q on either conductor for a given potential difference V and hence the greater the amount of energy stored.

The value of C is independent of the charge q or potential difference V. It purely depends on the geometry and shape of the conductors forming the capacitor. It also depends upon the material medium placed between the conductors.

Note: ***A single conductor can also act as a capacitor by assuming that the other conductor is placed at infinity.***

UNITS OF CAPACITANCE

The S.I. unit of capacitance is coulomb per volt which is frequently written as farad (F). The symbol F is used for this. This is a very large unit. Some smaller units such as microfarad (μF), nanofarad (nF), picofarad (pF), etc. are also used frequently.

Parallel-Plate Capacitor

It consists of two large plates placed parallel to each other with a separation d small in comparison to the two dimensions (length and breadth) of the plates (Fig. 4.3).

In an ideal capacitor, electric field resides in the region within the plates. No electric field is outside the plates (neglecting fringing effect for ideal case). So, the entire energy resides within the capacitor and no energy is, therefore, outside the capacitor. Electric field is directed from positive plate to negative plate in such a way that the lines emerge perpendicularly from positive plate and terminate perpendicularly to the negative plate.

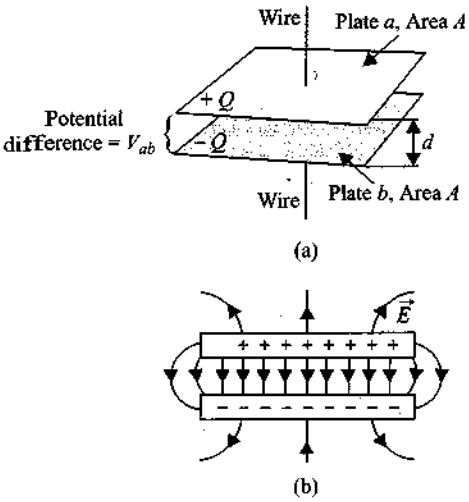
Electric field between the plates is

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

Therefore, potential difference between the plates is

$$V = Ed = \frac{Qd}{A\varepsilon_0}$$

and therefore capacitance $C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}$



(a) A charged parallel plate capacitor.
(b) When the separation of the plates is small as compared to their size, there is a slight fringing of the electric field $\vec{E}$ at the edges.

Fig. 4.3

Capacitance of a Spherical Conductor or Capacitor

As we have already said that a single conductor can also act as capacitor, here we will find the capacitance of a single isolated sphere (Fig. 4.4). For this, let a charge q is given to a spherical conductor of radius R, then potential on it is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

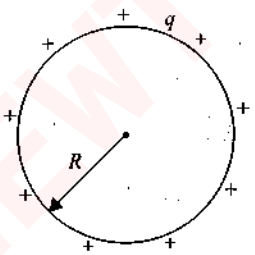


Fig. 4.4

The other conductor is supposed to be at infinity whose potential will be taken as zero.

So, the potential difference between the sphere and the conductor at infinity $= V - 0 = V$.

Then, capacitance $C = q/V = 4\pi\varepsilon_0 R$.

Thus, the capacitance of a spherical conductor is $C = 4\pi\varepsilon_0 R$.

Energy Stored in a Charged Conductor or Capacitor

Work has to be done in charging a conductor or capacitor against the force of repulsion by the already existing charges on it. The work is stored as a potential energy in the form of electric field of the conductor. Suppose a conductor of capacity C is to be charged up to a potential V_0 and let q_0 be the final charge on the conductor. Let the potential of the conductor when (during charging) the charge on it was q ($< q_0$) be $V = q/C$.

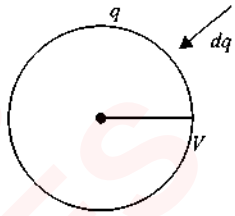


Fig. 4.5

Now, work done in bringing a small charge dq at this potential is $dW = Vdq = (q/C)dq$.

Therefore, the total work done in charging it from 0 to q_0 is

$$W = \int_0^{q_0} dW = \int_0^{q_0} \frac{q}{C}dq = \frac{1}{2}\frac{q_0^2}{C}$$

This work is stored as the potential energy, i.e.

$$U = \frac{1}{2}\frac{q_0^2}{C}$$

Further, by using $q_0 = CV_0$, we can write this expression as

$$U = \frac{1}{2}CV_0^2 = \frac{1}{2}q_0V_0$$

In general, if a conductor (or capacitor) of capacity C is charged to a potential V by giving it a charge q, then

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}qV$$

Potential energy of a spherical capacitor made of a single sphere is

$$U = \frac{q^2}{2C} = \frac{q^2}{8\pi\varepsilon_0 R} \quad (\because C = 4\pi\varepsilon_0 R)$$

Note: ***It is a common misconception that electric field energy is a new kind of energy, different from the electric potential energy described before. This is not the case; it is simply a different way of interpreting electric potential energy. We can regard the energy of a given system of charges as being a shared property of all the charges, or we can think of the energy as being a property of the electric field that the charges create. Either interpretation leads to the same value of the potential energy.***

Force Between the Plates of a Parallel-Plate Capacitor

Consider a parallel-plate capacitor with plate area A (Fig 4.6). Suppose a positive charge $+Q$ is given to one plate and a negative charge $-Q$ to the other plate. The electric field due to only the positive plate is $E = \sigma/2\varepsilon_0 = Q/2A\varepsilon_0$ at all points if the plate is large. The negative charge $-Q$ on the other plate finds itself in the field of this positive charge. Therefore, force on this plate is $F = EQ = Q^2/2A\varepsilon_0$. This force will be attractive.

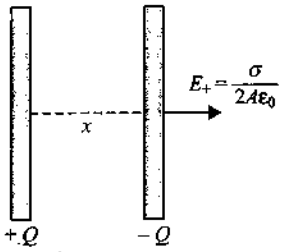


Fig. 4.6

Energy Density (Energy Per Unit Volume) in Electric Field

Consider a parallel-plate capacitor of plate area A and plate separation d. Let the charge on the capacitor be Q, then the electric field in the region between the plates is $E = \sigma/\varepsilon_0 = Q/A\varepsilon_0$.

Volume of capacitor is $V = Ad$.

Energy stored, $U = \frac{Q^2}{2C} = \frac{Q^2}{2\varepsilon_0 A} = \left(\frac{Q}{A\varepsilon_0}\right)^2 \frac{1}{2}\varepsilon_0 Ad$

Energy per unit volume, $u = \frac{U}{V} = \left(\frac{Q}{A\varepsilon_0}\right)^2 \frac{1}{2}\frac{\varepsilon_0 Ad}{Ad}$

$$= u\frac{1}{2}\varepsilon_0 E^2$$

Although we have proved the above result for a parallel-plate capacitor, in general this is true for any kind of capacitor or any other kind of electric field.

Potential energy of a spherical capacitor made of a single sphere using concept of energy density:

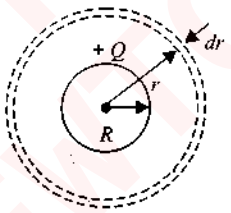


Fig. 4.7

Energy density, $u = \frac{1}{2}\varepsilon_0 E^2$

$$dU = udV = \frac{1}{2}\varepsilon_0\left(\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\right)^2 4\pi r^2 dr$$

$$U = \frac{1}{8\pi\varepsilon_0}Q^2\int_R^\infty \frac{dr}{r^2} = \frac{Q^2}{8\pi\varepsilon_0 R}$$

Loss of Energy During Redistribution of Charge

Let two capacitors of capacitances C_1 and C_2 be charged separately up to potential differences of V_1 and V_2, respectively, as shown in Fig. 4.8(a). Now, suppose these two capacitors are connected (positive plate of one capacitor to the positive plate of other and negative plate to negative plate) with each other as shown in Fig.4.8(b). If $V_1 \neq V_2$, then redistribution of charge will occur until the potentials of both capacitors become equal. Let V be the final common potential difference across each capacitor.

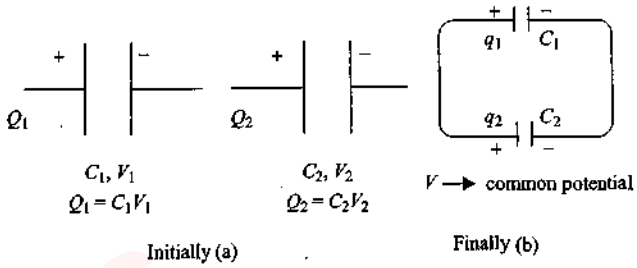


Fig. 4.8

Initial charges: $Q_1 = C_1V_1$, $Q_2 = C_2V_2$. Let final charges be q_1 and q_2 as shown in Fig. 4.8(b).

From conservation of charge, we get $q_1 + q_2 = Q_1 + Q_2$ (i)

And $V = \frac{q_1}{C_1} = \frac{q_2}{C_2}$ (ii)

Solving Eqs. (i) and (ii), we get

$$V = \frac{Q_1+Q_2}{C_1+C_2}, q_1 = \frac{C_1(Q_1+Q_2)}{C_1+C_2}, q_2 = \frac{C_2(Q_1+Q_2)}{C_1+C_2}$$

We can show that in redistribution of charge, energy is always lost.

Initial potential energy, $U_i = \frac{1}{2}\frac{Q_1^2}{C_1} + \frac{1}{2}\frac{Q_2^2}{C_2}$

Final potential energy, $U_f = \frac{1}{2}(C_1+C_2)V^2 = \frac{1}{2}\frac{(Q_1+Q_2)^2}{C_1+C_2}$

Loss of energy is

$$\Delta U = U_i - U_f = \frac{1}{2}\left[\frac{Q_1^2}{C_1} + \frac{Q_2^2}{C_2} - \frac{(Q_1+Q_2)^2}{C_1+C_2}\right]$$

or $$\Delta U = \frac{1}{2C_1C_2(C_1+C_2)}[Q_1^2C_1C_2 + Q_1^2C_2^2 + Q_2^2C_1^2 + Q_2^2C_1C_2 - Q_1^2C_1C_2 - Q_2^2C_1C_2 - 2Q_1Q_2C_1C_2]$$

or $$\Delta U = \frac{C_1^2C_2^2}{2C_1C_2(C_1+C_2)}\left[\frac{Q_1^2}{C_1^2} + \frac{Q_2^2}{C_2^2} - \frac{2Q_1Q_2}{C_1C_2}\right]$$

$$= \frac{C_1C_2}{2(C_1+C_2)}(V_1 - V_2)^2$$

Illustration 4.1 **Two capacitors C_1 and C_2 (where $C_1 > C_2$) are charged to the same initial potential difference. The charged capacitors are removed from the battery and their plates are connected as shown in Fig. 4.9(a). The switches S_1 and S_2 are then closed, as shown in Fig. 4.9(b).**

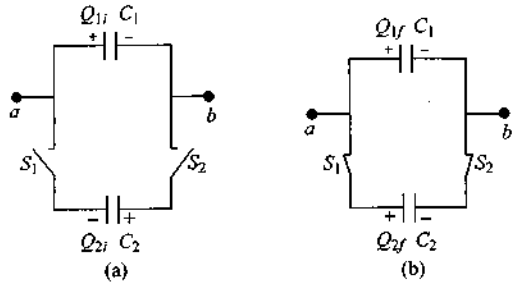


Fig. 4.9

a. Find the final potential difference between *a* and *b* after the switches are closed.

b. Find the total energy stored in the capacitors before and after the switches are closed and the ratio of the final energy to the initial energy.

Sol. a. The left-hand plates of the capacitors in Fig 4.9(b) act as an isolated system because they are not connected to the right-hand plates by conductors. The charges on the left-hand plates before the switches are closed are

$$Q_{1i} = C_1 \Delta V_i \text{ and } Q_{2i} = C_2 \Delta V_i$$

The total charge Q in the system is

$$Q = Q_{1i} + Q_{2i} = (C_1 - C_2)\Delta V_1 \qquad \text{(i)}$$

After the switches are closed, the total charge in the system remains the same.

$$Q = Q_{1f} + Q_{2f} \qquad \text{(ii)}$$

The charges redistribute until the entire system is at the same potential ΔV_f. Thus, the final potential difference across C_1 must be the same as the final potential difference across C_2. The charges on the capacitors after the switches are closed are

$$Q_{1f} = C_1 \Delta V_f \text{ and } Q_{2f} = C_2 \Delta V_f$$

Dividing Eq. (i) by Eq. (ii), we have

$$\frac{Q_{1f}}{Q_{2f}} = \frac{C_1 \Delta V_f}{C_2 \Delta V_f} = \frac{C_1}{C_2} \Rightarrow Q_{1f} = \frac{C_1}{C_2} Q_{2f}$$

Combining Eqs. (ii) and (iii), we obtain

$$Q = Q_{1f} + Q_{2f} = \frac{C_1}{C_2} Q_{2f} + Q_{2f} = Q_{2f}\left(1 + \frac{C_1}{C_2}\right)$$

$$\Rightarrow Q_{2f} = Q\left(\frac{C_2}{C_1 + C_2}\right)$$

Using Eq. (iii) to find Q_{1f} in terms of Q, we have

$$Q_{1f} = \frac{C_1}{C_2} Q_{2f} = \frac{C_1}{C_2} Q\left(\frac{C_2}{C_1 + C_2}\right) = Q\left(\frac{C_1}{C_1 + C_2}\right)$$

Finally, voltage across each capacitor is

$$\Delta V_f = \frac{Q_{1f}}{C_1} = \frac{Q\left(\frac{C_1}{C_1 + C_2}\right)}{C_1} = \frac{Q}{C_1 + C_2}$$

To express ΔV_f in terms of the given quantities C_1, C_2 and ΔV_i, we substitute the value of Q from Eq. (i) to obtain

$$\Delta V_f = \left(\frac{C_1 - C_2}{C_1 + C_2}\right)\Delta V_i$$

b. Before the switches are closed, the total energy stored in the capacitors is

$$U_i = \frac{1}{2} C_1 (\Delta V_i)^2 + \frac{1}{2} C_2 (\Delta V_i)^2 = \frac{1}{2}(C_1 + C_2)(\Delta V_i)^2$$

After the switches are closed, the total energy stored in the capacitors is

$$U_f = \frac{1}{2} C_1 (\Delta V_f)^2 + \frac{1}{2} C_2 (\Delta V_f)^2 = \frac{1}{2}(C_1 + C_2)(\Delta V_f)^2$$

$$= \frac{1}{2}(C_1 + C_2)\left(\frac{Q}{C_1 + C_2}\right)^2 = \frac{1}{2}\frac{Q^2}{C_1 + C_2}$$

Using Eq. (i), we can express this as

$$U_f = \frac{1}{2}\frac{Q^2}{(C_1 + C_2)} = \frac{1}{2}\frac{(C_1 - C_2)^2 (\Delta V_i)^2}{(C_1 + C_2)}$$

Therefore, the ratio of the final energy stored to the initial energy stored is

$$\frac{U_f}{U_i} = \frac{\frac{1}{2}\frac{(C_1 - C_2)^2 (\Delta V_i)^2}{(C_1 + C_2)}}{\frac{1}{2}(C_1 + C_2)(\Delta V_i)^2} = \left(\frac{C_1 - C_2}{C_1 + C_2}\right)^2$$

Illustration 4.2 **Four identical metal plates are arranged in air at equal distance *d* from each other. The area of each plate is *A*. A battery of e.m.f. *V* is connected across the plates 1 and 2. Discuss the charge distribution and find the capacitance of the system between points 1 and 2 if the other two plates are connected by a conducting wire as shown in Fig. 4.10.**

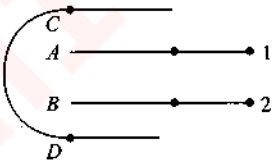


Fig. 4.10

Sol. Method 1: Imagine a battery connected to points 1 and 2, so that there is a total charge $+Q$ on plate A and $-Q$ on plate B. Charge on A and B would induce a charge $-Q'$ on C while $+Q'$ on D, plates C and D together remain neutral.

Consequently, charge Q on A is divided into two parts: $+Q'$ on the upper side of plate A and $(Q - Q')$ on lower side. Similarly, charge $-Q$ on B is also divided, $-Q'$ on lower side and $-(Q - Q')$ on upper side of plate B.

Due to charges on the plates, electric fields will appear in the gaps between the plates. Fields E' between AC and DB would be same, while between AB, field E would be different.

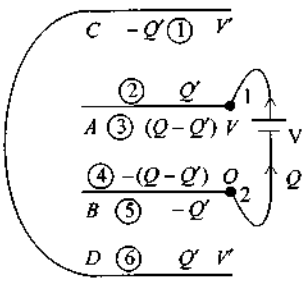


Fig. 4.11

Let the potential of plate A be V and that of plate B be zero. As the plates C and D are connected together, they will be at common potential, say V'.

The capacity of the system can be written as

$$C = \frac{Q}{V_+ - V_-} = \frac{Q}{V} \qquad \text{(i)}$$

We can write equation for facing surfaces (1) and (2) as

$$Q' = C_0(V - V') \quad \text{(ii)}$$

For facing surfaces (3) and (4), the equation is

$$(Q - Q') = C_0(V - 0) \quad \text{(iii)}$$

For facing surfaces (5) and (6), the equation is

$$Q' = C_0(V' - 0) \quad \text{(iv)}$$

From Eqs. (ii), (iii) and (iv), we have

$$Q = \frac{3}{2}C_0V = \frac{3}{2}\frac{\varepsilon_0 AV}{d} \text{ and } Q' = \frac{C_0V}{2} = \frac{\varepsilon_0 AV}{2d}$$

Hence, the equivalent capacitance is $C_{eq} = \frac{Q}{V} = \frac{3}{2}\frac{\varepsilon_0 A}{d}$.

And charges on different surfaces are

Surfaces	Charges
(1), (5)	$-\frac{\varepsilon_0 AV}{2d}$
(2), (6)	$+\frac{\varepsilon_0 AV}{2d}$
(3)	$\frac{\varepsilon_0 AV}{d}$
(4)	$-\frac{\varepsilon_0 AV}{d}$

Method 2: The equivalent circuit can be drawn as shown in Fig. 4.12.

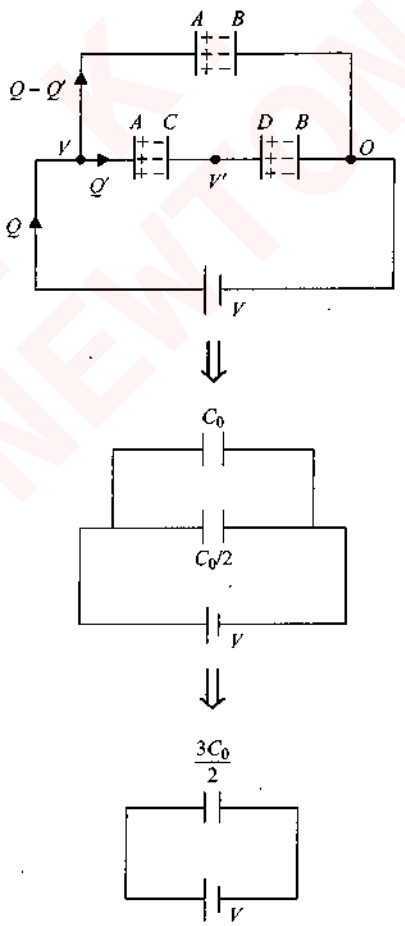


Fig. 4.12

Hence, the equivalent capacitance $C_{eq} = \frac{3}{2}C_0 = \frac{3}{2}\frac{\varepsilon_0 A}{d}$

Charge supplied by battery $Q = \frac{3}{2}\frac{\varepsilon_0 A}{d}V$ and $Q' = C_0V = \frac{\varepsilon_0 A}{d}$.

Concept Application Exercise 4.1

1. Suppose the two plates of a capacitor have different areas. When the capacitor is charged by connecting it to a battery, do the charges on the two plates have equal magnitude, or can they be different? Explain your reasoning.
2. The charged plates of a capacitor attract each other, so to pull the plates farther apart requires work by some external force. What becomes of the energy added by this work? Explain your reasoning.
3. The two plates of a capacitor are given charges $\pm Q$. The capacitor is then disconnected from the charging device so that the charges on the plates cannot change, and the capacitor is immersed in a tank of oil. Does the electric field between the plates increase, decrease or remain the same? Explain your reasoning. How can this field be measured?
4. **a.** How many excess electrons must be added to one plate and removed from the other to give a 5.00 nF parallel-plate capacitor 25.0 J of stored energy?
   b. How could you modify the geometry of this capacitor to get it to store 50.0 J of energy without changing the charge on its plates?
5. A capacitor of capacitance C is charged to a potential difference V from a cell and then disconnected from it. A charge $+Q$ is now given to its positive plate. Find the potential difference across the capacitor.
6. A capacitor is connected across a battery.
   a. Why does each plate receive a charge of exactly the same magnitude?
   b. Is this true even if the plates are of different size? (Yes/No)
7. Three identical large metallic plates are placed parallel to each other at very small separation as shown in Fig. 4.13. The central plate is given a charge Q. What amount of charge will flow to earth when key is pressed?

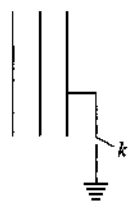


Fig. 4.13

8. The plates of a plane capacitor are drawn apart keeping them connected to a battery. Next, the same plates are drawn apart from the same initial condition keeping battery disconnected. In which case is more work done?
9. If a small charge q is moved along a closed path in the field between the plates of a parallel-plate capacitor, will

any work be done by the agent which moves the charge? (Yes/No)

10. At which of the two points, 1 or 2, of a charged capacitor with non-parallel plates is the surface charge density greater?

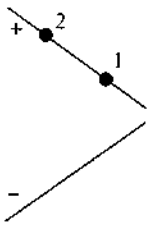


Fig. 4.14

11. A parallel-plate air capacitor is connected to a battery. If plates of the capacitor are pulled farther apart, then state whether the following statements are true or false.

a. Strength of electric field inside the capacitor remains unchanged, if battery is disconnected before pulling the plates.

b. During the process, work is done by external force applied to pull the plates irrespective of whether the battery is disconnected or not.

c. Strain energy in the capacitor decreases if the battery remains connected.

COMBINATION OF CAPACITORS

Sometimes to obtain a desired value of capacitance, we group two or more than two capacitors together. In this section, we will learn how to find the equivalent capacitance when two or more than two capacitors are grouped together?

Equivalent capacitance of a group of capacitors is that value of capacitance of a single capacitor which will allow to flow (or store) the same amount of charge for a given potential difference as done by the combination.

In other words, that single capacitor will perform the same task as done by the combination in all respects.

Capacitors can be grouped in many ways, but the two common types of combinations are: (i) Series combination and (ii) Parallel combination.

Capacitors Connected in Series

Let n capacitors are connected in series as shown in Fig. 4.15. C_{eq} is the equivalent capacitance of this system.

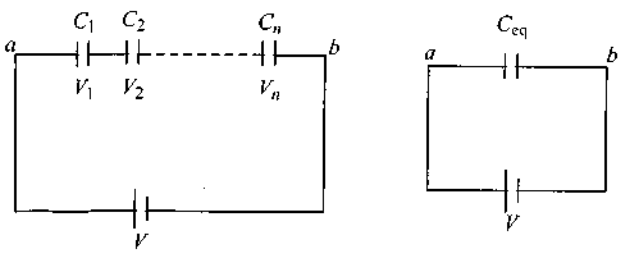


Fig. 4.15

Let a battery V be applied across the combination, then

$$V = V_1 + V_2 + \cdots + V_n$$

$$\Rightarrow \quad \frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \cdots + \frac{Q}{C_n}$$

$$\Rightarrow \quad \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$$

Thus, the reciprocal of the equivalent capacitance of a series combination equals the sum of the reciprocals of the individual capacitances.

Note:

- ***In series combination, the equivalent capacitance is always less than any of the individual capacitance.***
- ***In series combination, charge on each capacitor is same but potential is different. From $V = q/C$, the larger the capacitance, the lesser the potential is***

Energy in Series Combination

$$U = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} + \cdots + \frac{Q^2}{2C_n}$$

$$= \frac{Q^2}{2}\left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots \frac{1}{C_n}\right) = \frac{Q^2}{2C_{eq}}$$

Thus, a single capacitor of capacitance C_{eq} will store the same amount of energy as stored by $C_1, C_2, \ldots, C_n$ when connected in series.

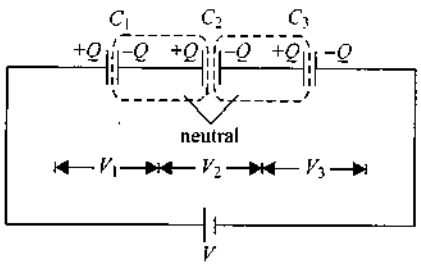


Fig. 4.16

As an example, let us take the case of a circuit consisting of three capacitors in series as shown in Fig. 4.16. Charge on each capacitor is same, but potential can be different.

$$Q = C_1V_1 = C_2V_2 = C_3V_3 \quad \text{(i)}$$

$$V = V_1 + V_2 + V_3 \quad \text{(ii)}$$

$$= \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \quad \text{(iii)}$$

or $$Q = \frac{V}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} = \frac{C_1C_2C_3V}{C_1C_2 + C_2C_3 + C_1C_3}$$

Hence, $$V_1 = \frac{Q}{C_1} = \frac{C_2C_3V}{C_1C_2 + C_2C_3 + C_3C_1}$$

$$V_2 = \frac{Q}{C_2} = \frac{C_3C_1V}{C_1C_2 + C_2C_3 + C_3C_1}$$

and
$$V_3 = \frac{Q}{C_3} = \frac{C_1C_2V}{C_1C_2 + C_2C_3 + C_3C_1}$$

Special Case of Two Capacitors in Series

Let two capacitors C_1 and C_2 be connected in series and a potential difference V is applied across them as in Fig. 4.17.

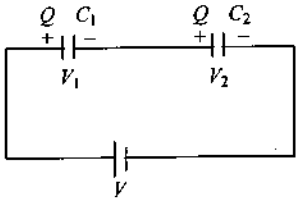


Fig. 4.17

Potentials appearing on capacitors are V_1 and V_2, respectively. Q is the net charge that flows in circuit, then

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C_{eq} = \frac{C_1C_2}{C_1 + C_2}$$

$$V_1 + V_2 = V \quad \text{(i)}$$

$$Q = C_1V_1 = C_2V_2 \quad \text{(ii)}$$

From Eq. (ii), $\frac{V_1}{V_2} = \frac{C_2}{C_1}$ (iii)

It means potential will be divided on the capacitors in the inverse ratio of their capacitances.

From (i) and (iii), $V_1 = \frac{C_2V}{C_1 + C_2}, V_2 = \frac{C_1V}{C_1 + C_2}$

In the above discussion, take $C_1 = 4\ \mu F$, $C_2 = 8\ \mu F$ and $V = 12$ V, then

$$C_{eq} = \frac{C_1C_2}{C_1 + C_2} = \frac{4 \times 8}{4 + 8} = \frac{8}{3}\ \mu F$$

$$V_1 = \frac{C_2V}{C_1 + C_2} = \frac{8 \times 12}{4 + 8} = 8\ V$$

$$V_2 = \frac{C_1V}{C_1 + C_2} = \frac{4 \times 12}{4 + 8} = 4\ V$$

Energy in $C_1 : U_1 = \frac{1}{2}C_1V_1^2 = \frac{1}{2} \times 4 \times (8)^2 = 128\ \mu J$

Energy in $C_2 : U_2 = \frac{1}{2}C_2V_2^2 = \frac{1}{2} \times 8 \times (4)^2 = 64\ \mu J$

Total energy: $U = U_1 + U_2 = 128 + 64 = 192\ \mu J$

We can find the total energy by using equivalent capacitance. Equivalent capacitance will store the same amount of energy as stored by the combination on applying the same potential, i.e.,

$$U = \frac{1}{2}C_{eq}V^2 = \frac{1}{2} \times \frac{8}{3} \times (12)^2 = 192\ \mu J$$

Capacitors Connected in Parallel

Let n capacitors be connected in parallel as shown in Fig. 4.18. C_{eq} is the equivalent capacitance of this system.

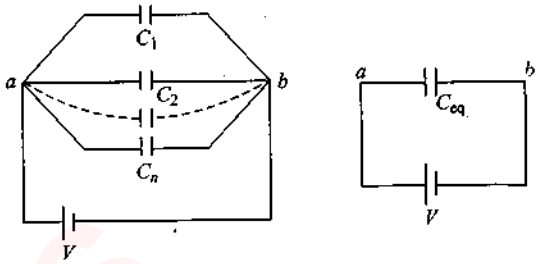


Fig. 4.18

Let total charge q flows through the battery, then

$$q = q_1 + q_2 + \cdots + q_n$$

$$\Rightarrow \quad C_{eq}V = C_1V + C_2V + \cdots + C_nV$$

$$= C_1 + C_2 + \cdots + C_n$$

The equivalent capacitance of a parallel combination equals the sum of the individual capacitances.

Note:

- ***In a parallel combination, the equivalent capacitance is always greater than any individual capacitance.***
- ***In a parallel combination, the potential on each capacitor is the same, but charge may be different. From $q = CV$, the greater the capacitance, the greater the charge is.***
- ***Charge is distributed on the capacitors in the ratio of their capacitances, i.e., $q_1 : q_2 : \ldots : q_n = C_1 : C_2 : \ldots : C_n$.***

Energy in Parallel Combination

$$U = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 + \cdots + \frac{1}{2}C_nV^2$$

$$= \frac{1}{2}(C_1 + C_2 + \cdots + C_n)V^2 = \frac{1}{2}C_{eq}V^2$$

Thus a single capacitor of capacitance C_{eq} will store the same amount of energy as stored by $C_1, C_2, \ldots, C_n$ when connected in parallel.

Special Case of Two Capacitors in Parallel

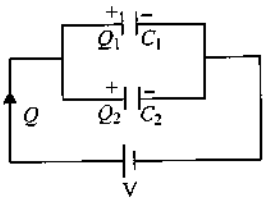


Fig. 4.19

Let two capacitors C_1 and C_2 be connected in parallel and a potential difference V be applied across them as shown in Fig. 4.19. Then

$$C_{eq} = C_1 + C_2$$

$$Q_1 + Q_2 = Q \quad \text{(i)}$$

$$V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} \quad \text{(ii)}$$

From Eq. (ii), we get $\frac{Q_1}{Q_2} = \frac{C_1}{C_2}$ (iii)

It means charge will be divided in the direct ratio of the capacitances.

From Eqs. (i) and (iii), $Q_1 = \frac{C_1 Q}{C_1 + C_2}, Q_2 = \frac{C_2 Q}{C_1 + C_2}$

In the above discussion, take $C_1 = 4\ \mu F$, $C_2 = 8\ \mu F$ and $V = 12$ V

$$C_{eq} = C_1 + C_2 = 4 + 8 = 12\ \mu F$$
$$Q_1 = C_1 V = 4 \times 12 = 48\ \mu C$$
$$Q_2 = C_2 V = 8 \times 12 = 96\ \mu C$$

The total charge that flows through the battery is

$$Q = Q_1 + Q_2 = 48 + 96 = 144\ \mu C$$

Also: $Q = C_{eq} V = 12 \times 12 = 144\ \mu C$

$$Q_1 = \frac{C_1 Q}{C_1 + C_2} = \frac{4 \times 144}{4 + 8} = 48\ \mu C$$
$$Q_2 = \frac{C_2 Q}{C_1 + C_2} = \frac{8 \times 144}{4 + 8} = 96\ \mu C$$

Energy stored in $C_1 : U_1 = \frac{1}{2} C_1 V^2 = \frac{1}{2} \times 4 \times 12^2 = 288\ \mu J$

Energy stored in $C_2 : U_2 = \frac{1}{2} C_2 V^2 = \frac{1}{2} \times 8 \times 12^2 = 576\ \mu J$

Total energy: $U = U_1 + U_2 = 288 + 576 = 864\ \mu J$

Also: $U = \frac{1}{2} C_{eq} V^2 = \frac{1}{2} \times 12 \times (12)^2 = 864\ \mu J$

Illustration 4.3 **A capacitor is made of a flat plate of area A and a second plate having a stair-like structure as shown in Fig. 4.20. The area of each stair is $A/3$ and the height is d. Find capacitance of this arrangement.**

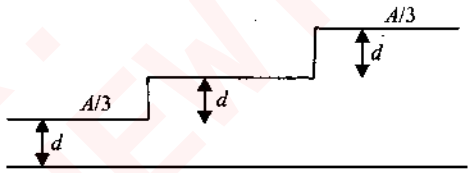


Fig. 4.20

Sol. The arrangement is a parallel combination of three capacitors. Each capacitor has a plate area $A/3$ and the separation between the plates is d, $2d$ and $3d$, respectively.

$$C_1 = \frac{\varepsilon_0 A/3}{3d} = \frac{\varepsilon_0 A}{3d}, C_2 = \frac{\varepsilon_0 A/3}{2d} = \frac{\varepsilon_0 A}{6d} \text{ and}$$
$$C_3 = \frac{\varepsilon_0 A/3}{3d} = \frac{\varepsilon_0 A}{9d}$$

As these three capacitances are in parallel, their equivalent capacitance is given by

$$C = C_1 + C_2 + C_3 = \frac{\varepsilon_0 A}{3d} + \frac{\varepsilon_0 A}{6d} + \frac{\varepsilon_0 A}{9d} = \frac{11\varepsilon_0 A}{18d}$$

Illustration 4.4 **Three capacitors of capacitances 3 μF, 6 μF and 4 μF are connected as shown across a battery of e.m.f. 6 V.**

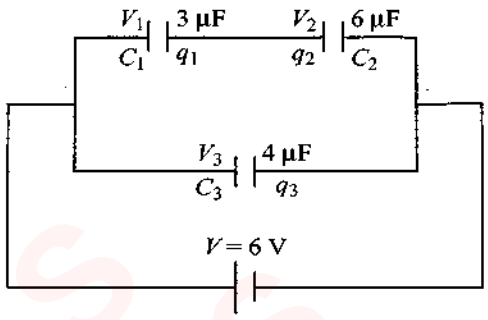


Fig. 4.21

i. Find equivalent capacitance.
ii. Find potential difference and charge on each capacitor.
iii. Find energy stored in each capacitor and total energy stored in the system of capacitors.

Sol.

i. C_1 and C_2 are in series, so their equivalent capacitance is

$$C' = \frac{C_1 C_2}{C_1 + C_2} = \frac{3 \times 6}{3 + 6} = 2\ \mu F$$

This C' will be in parallel with C_3, so $C_{eq} = C' + C_3 = 2 + 4 = 6\ \mu F$

ii. 6 V will be divided across C_1 and C_2 in the inverse ratio of the capacitances. So

$$\text{p.d. across } C_1 : V_1 = \frac{C_2 V}{C_1 + C_2} = \frac{6 \times 6}{3 + 6} = 4 \text{ V}$$
$$\text{p.d. across } C_2 : V_2 = \frac{C_1 V}{C_1 + C_2} = \frac{3 \times 6}{3 + 6} = 2 \text{ V}$$

Because C_3 is connected directly across the battery without any other capacitor in between, so

p.d. across C_3: $V_3 = V = 6$ V.

Charge on C_1: $q_1 = C_1 V_1 = 3 \times 4 = 12\ \mu C$, charge on C_2: $q_2 = C_2 V_2 = 6 \times 2 = 12\ \mu C$

We see that $q_1 = q_2$, and it should have been, because in series charge is the same.

Charge on C_3: $q_3 = C_3 V_3 = 4 \times 6 = 24\ \mu C$

Alternative method to find charge: We can find charge in this way also.

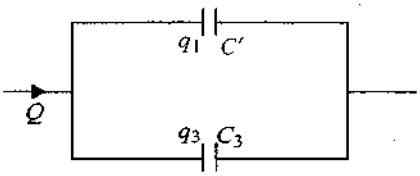


Fig. 4.22

Total charge that will flow through the battery is

$$Q = C_{eq} V = 6 \times 6 = 36\ \mu C$$

It will be divided in direct ratio of C' and C_3, so

$$q_1 = \frac{C' Q}{C' + C_3} = \frac{2 \times 36}{2 + 4} = 12\ \mu C$$
$$q_3 = \frac{C_3 Q}{C' + C_3} = \frac{4 \times 36}{2 + 4} = 24\ \mu C$$

iii. Energy in $C_1 : U_1 = \frac{1}{2}C_1V_1^2 = \frac{1}{2}\times 3\times 4^2 = 24\ \mu J$

Energy in $C_2 : U_2 = \frac{1}{2}C_2V_2^2 = \frac{1}{2}\times 6\times 2^2 = 12\ \mu J$

Energy in $C_3 : U_3 = \frac{1}{2}C_3V^2 = \frac{1}{2}\times 4\times 6^2 = 72\ \mu J$

Total energy: $U = U_1 + U_2 + U_3 = 24 + 12 + 72 = 108\ \mu J$

Alternatively: $U = \frac{1}{2}C_{eq}V^2 = \frac{1}{2}\times 6\times 6^2 = 108\ \mu J$

Illustration 4.5 **Three capacitors of capacitances 4 μF, 4 μF and 8 μF are connected as shown across a battery of e.m.f. 12 V.**

i. Find equivalent capacitance.
ii. Find potential difference and charge on each capacitor.
iii. Find energy stored in each capacitor and total energy stored in the system of capacitors.

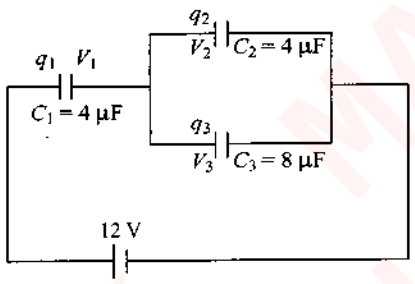


Fig. 4.23

Sol.

i. C_2 and C_3 are in parallel, so their equivalent capacitance is

$$C' = C_2 + C_3 = 4 + 8 = 12\ \mu F$$

This C' will be in series with C_1, so the net equivalent capacitance is

$$C_{eq} = \frac{C_1C'}{C_1 + C'} = \frac{4\times 12}{4+12} = 3\ \mu F$$

ii. $q_1 = C_{eq}V = 3\times 12 = 36\ \mu C$. q_1 will be divided into q_2 and q_3, so

$$q_2 = \frac{C_2q_1}{C_2 + C_3} = \frac{4\times 36}{4+8} = 12\ \mu C$$

$$q_3 = \frac{C_3q_1}{C_2 + C_3} = \frac{8\times 36}{4+8} = 24\ \mu C$$

$$V_1 = \frac{q_1}{C_1} = \frac{36}{4} = 9\ V,\ V_2 = V_3 = \frac{q_2}{C_2} = \frac{q_3}{C_3} = \frac{36}{4} = 3\ V$$

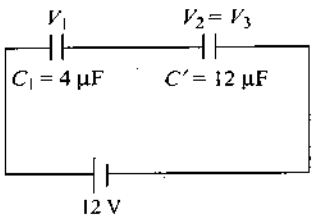


Fig. 4.24

Alternatively, potentials can also be calculated by dividing 12 V in inverse ratio of C_1 and C'.

$$V_1 = \frac{C'V}{C_1 + C'} = \frac{12\times 12}{4+12} = 9\ V$$

$$V_2 = V_3 = \frac{C_1V}{C_1 + C'} = \frac{4\times 12}{4+12} = 3\ V$$

iii. $U_1 = \frac{1}{2}C_1V_1^2 = \frac{1}{2}\times 4\times 9^2 = 162\ \mu J$

$U_2 = \frac{1}{2}C_2V_2^2 = \frac{1}{2}\times 4\times 3^2 = 18\ \mu J$

$U_3 = \frac{1}{2}C_3V_3^2 = \frac{1}{2}\times 8\times 3^2 = 36\ \mu J$

Total energy $= U_1 + U_2 + U_3 = 216\ \mu J$

Alternatively: $U = \frac{1}{2}C_{eq}V^2 = \frac{1}{2}\times 3\times (12)^2 = 216\ \mu J$

KIRCHHOFF'S RULES FOR CAPACITORS

Kirchhoff's rules can be used to determine the p.d. and charge on the plates of a capacitor in any electric circuit. It is explained with the help of Illustration 4.6.

Illustration 4.6 **Find charges Q_1 and Q_2 on the capacitors C_1 and C_2 in the circuit shown in Fig. 4.25.**

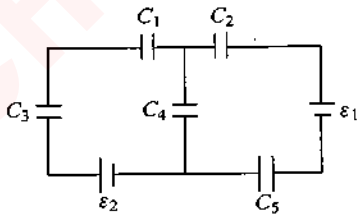


Fig. 4.25

Sol. **Step I.** Identify isolated regions (IRs), considering cells as continuous circuit elements and capacitances as discontinuous elements.

An IR is a continuous region or group of circuit branches in which you can travel from any one point to any other point in that region. There are four such regions in the given circuit, depicted by lines having different thickness, namely,

(1) C_3 and C_1, (2) C_1, C_2 and C_4, (3) C_3, ε_2 C_4 and C_5, (4) C_2, ε_1 and C_5.

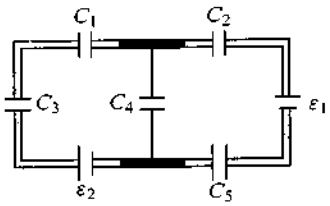


Fig. 4.26

Conservation of Charge: Electric charge will remain conserved in every isolated region.

Step II. Allocate charges on various plates of each capacitor rationally on the basis of the following points:

a. Please note that cells do not supply any extra charge. They only cause charge to flow from one point to another.

b. The algebraic sum of all the plate charges in each IR must be zero (conservation of charge).

c. The two plates of each capacitor must be shown to have equal and opposite charges.

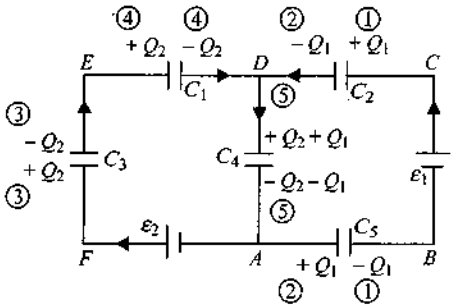


Fig. 4.27

This is done in five steps depicted by 1, 2, 3, 4 and 5 in the given circuit (Fig. 4.27).

Now, calculate and show p.d. across the plates of each capacitor, using the formula $V = Q/C$.

The algebraic sum of potential changes in each closed loop must be zero.

1. For determining a closed loop, consider each capacitor as well as cell as a continuous circuit element. Please note that capacitors were considered as discontinuous elements while identifying IRs in step 1.
2. A closed loop is the path travelled by you to start from any point and reach the same point again.
3. Sign convention is same as in the case of resistances.

SIGN CONVENTION

If you travel from (+)vely charged plate of a capacitor or positive terminal of a cell to (−)vely charged plate of a capacitor or negative terminal of a cell, then the sign of V or ε is (−) and vice versa.

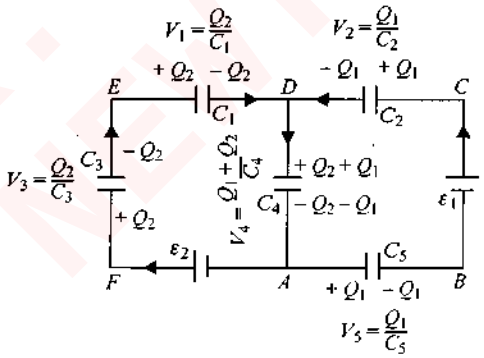


Fig. 4.28

Applying rule (2) in closed loop $ABCDA$, we get

$$-\frac{Q_1}{C_5}+\varepsilon_1-\frac{Q_1}{C_2}-\frac{Q_1+Q_2}{C_4}=0$$

$$\Rightarrow \quad Q_1\left(\frac{1}{C_1}+\frac{1}{C_4}+\frac{1}{C_5}\right)+\frac{Q_2}{C_4}=\varepsilon_1 \qquad \text{(i)}$$

Applying rule (2) in closed loop $ADEFA$, we get

$$\frac{Q_1+Q_2}{C_4}+\frac{Q_2}{C_1}+\frac{Q_2}{C_3}-\varepsilon_2=0$$

$$\Rightarrow \quad \frac{Q_1}{C_4}+Q_2\left[\frac{1}{C_1}+\frac{1}{C_3}+\frac{1}{C_4}\right]=\varepsilon_2 \qquad \text{(ii)}$$

Now, you have got Eqs. (i) and (ii) and two unknown Q_1 and Q_2 to solve.

Illustration 4.7 **Consider the following circuit. Find the charge and potential difference across capacitor C_1 of capacitance 1 μF.**

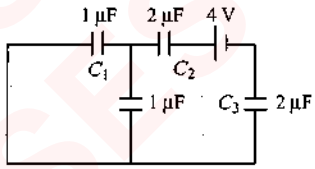


Fig. 4.29

Sol. Applying Kirchhoff's rule to the closed loop $ABFEA$ (Fig. 4.30), we get

$$+\frac{Q_2-Q_1}{C_4}-\frac{Q_1}{C_1}=0 \Rightarrow \frac{Q_2-Q_1}{1\times10^{-6}}-\frac{Q_1}{1\times10^{-6}}=0$$

$$Q_2 = 2Q_1 \qquad \text{(i)}$$

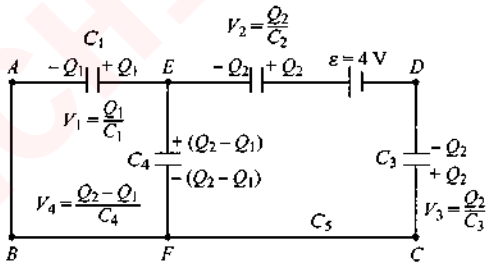


Fig. 4.30

Applying Kirchhoff's rule to closed loop $ABCDA$, we get

$$-\frac{Q_2}{C_3}+4-\frac{Q_2}{C_2}-\frac{Q_1}{C_1}=0$$

$$\Rightarrow \quad \frac{Q_1}{C_1}+Q_2\left(\frac{1}{C_2}+\frac{1}{C_3}\right)=4$$

$$\frac{Q_1}{1\times10^{-6}}+Q_2\left(\frac{1}{2\times10^{-6}}+\frac{1}{2\times10^{-6}}\right)=4$$

$$\Rightarrow \quad Q_1+Q_2=4\times10^{-6} \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$Q_1=\frac{4}{3}\ \mu\text{C and } V_1=\frac{4}{3}\ \text{V}$$

Illustration 4.8 **In the circuit shown in Fig. 4.31, find the charge on each capacitor.**

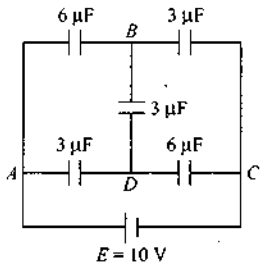


Fig. 4.31

Sol. Let total charge q flow through the battery, which gets divided at point A in q_1 and q_2. The final charge on each capacitor is as shown in Fig. 4.32.

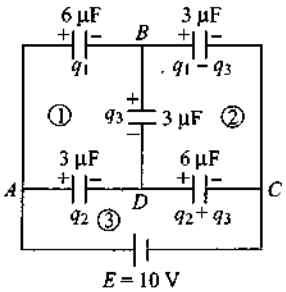


Fig. 4.32

Applying KVL in the loop (1), traversing clockwise, we get

$$-\frac{q_1}{6} - \frac{q_3}{3} + \frac{q_2}{3} = 0 \quad \text{or} \; -q_1 - 2q_3 + 2q_2 = 0 \qquad \text{(i)}$$

For loop (2), traversing clockwise, we have

$$-\left(\frac{q_1 - q_3}{3}\right) + \frac{q_2 + q_3}{6} + \frac{q_3}{3} = 0$$

$$\Rightarrow \qquad -2q_1 + 5q_3 + q_2 = 0 \qquad \text{(ii)}$$

For loop $ABCEA$, traversing clockwise, we have

$$-\frac{q_1}{6} - \frac{(q_1 - q_3)}{3} + 10 = 0 \Rightarrow 3q_1 - 2q_3 = 60 \qquad \text{(iii)}$$

Solving Eqs. (i), (ii) and (iii), we get

$$q_1 = 24\ \mu\text{C}, q_2 = 18\ \mu\text{C}, q_3 = 6\ \mu\text{C}$$

DIELECTRIC

These are non-conductors up to a certain value of field depending upon their nature. If the field exceeds the limiting value, called dielectric strength, dielectric loses its insulating property and begins to conduct.

Dielectric Constant

When a dielectric material is placed between the plates of a capacitor, the capacitance of the capacitor increases. The ratio of the capacitance of a given capacitor with the material filling the entire space between its plates to the capacitance of the same capacitor in vacuum is called dielectric constant of that material.

Dielectric constant $K = \dfrac{C}{C_0}$

Dielectric in an Electric Field

Suppose a slab of dielectric material is placed in a uniform electric field $\vec{E}_0$ set up between the parallel plates of a charged capacitor. The slab becomes electrically polarized. That is, its molecules become electric dipoles oriented in the direction of the field. The net effect is the appearance of negative charge on one face of the slab and an equal positive charge on the opposite face. The net charge in the interior of the slab remains zero. The polarization charges induced on the two faces of the slab produce their own electric field $\vec{E}_0'$, which opposes the external field $\vec{E}_0$.

Hence, the resultant field $\vec{E}$ within the dielectric is smaller than E_0, but it points in the same direction as $\vec{E}_0$.

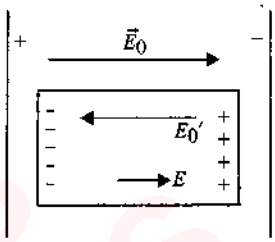


Fig. 4.33

The field in the rest of the (free) space is still $\vec{E}_0$. Hence, we conclude that when a dielectric is placed in an electric field, the field 'within' the dielectric is weakened (but not reduced to zero).

The weakening of electric field within the dielectric is illustrated in Fig. 4.33. Here, the dielectric fully fills the space between the plates. Figure 4.34 (a) shows the original field. In other figures, some of the lines of force leaving the positive plate of the capacitor penetrate the dielectric; others terminate on the charges induced on the dielectric.

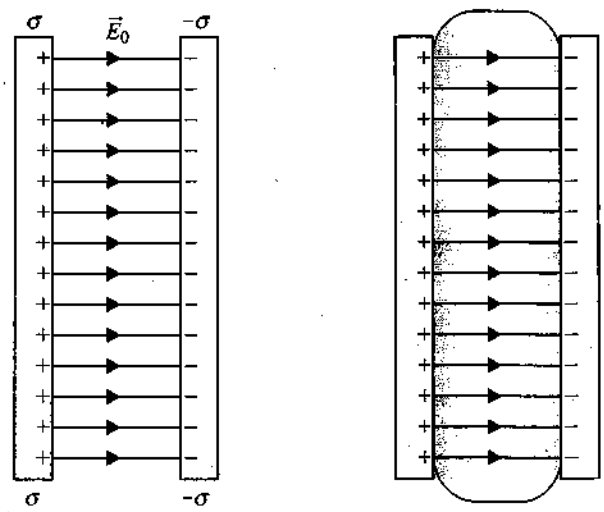
The reduction in the magnitude of the electric field from E_0 to E causes a reduction in the potential difference between the plates of the capacitor. Let V_0 and V be the potential difference with and without the plates, then we have

$$\frac{E_0}{E} = \frac{V_0}{V} \qquad (\because E = V/d)$$

But $\dfrac{V_0}{V} = \dfrac{C}{C_0} = K$, where K is the dielectric constant of the slab. So, $\dfrac{E_0}{E} = K \Rightarrow E = \dfrac{E_0}{K}$.

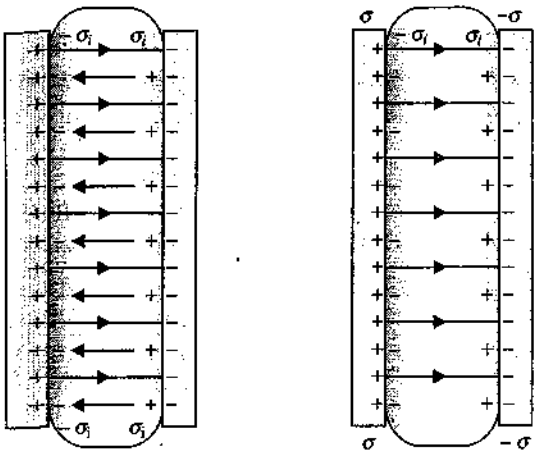
Thus, the electric field within the dielectric is reduced by a factor K.

Induced Charge on the Surface of Dielectric



(a) Electric field of magnitude E_0 between two charged plates.

(b) Introduction of a dielectric constant K.



(c) The induced surface charges and their fields (thinner lines).

(d) Resultant field of magnitude E_0/K when a dielectric is placed between charged plates.

Fig. 4.34

Let K = dielectric constant,

E_0 = original field in vacuum if dielectric slab is not there,

E_i = electric field induced in the dielectric slab,

E = net electric field in the dielectric slab.

$$\therefore \quad E = E_0 - E_i \qquad \text{(i)}$$

$$\text{and } \frac{E_0}{E} = K \text{ (by definition of } K \text{ of } \varepsilon_r) \text{ or } E = \frac{E_0}{K} \qquad \text{(ii)}$$

From Eqs. (i) and (ii), $E_0 - E_i = \dfrac{E_0}{K}$

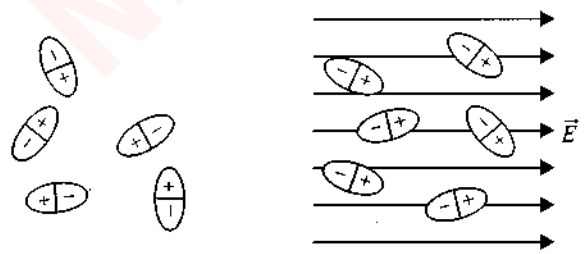
$$\Rightarrow \quad E_0K - E_iK = E_0 \Rightarrow E_0K - E_0 = E_iK$$

$$\Rightarrow \quad E_i = \frac{K-1}{K}E_0 \qquad \text{(iii)}$$

$$\Rightarrow \quad \frac{\sigma_i}{\varepsilon_0} = \frac{K-1}{K}\frac{\sigma}{\varepsilon_0}$$

$$\Rightarrow \quad \sigma_i = \frac{K-1}{K}\sigma$$

$$\Rightarrow \quad \frac{Q_i}{A} = \frac{K-1}{K}\frac{Q}{A} \Rightarrow Q_i = Q\left(1-\frac{1}{K}\right) \qquad \text{(iv)}$$

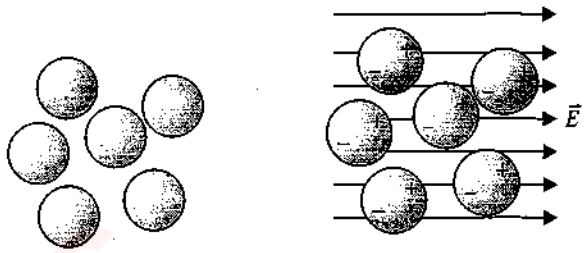


(a) Polar molecules with no applied electric field

(b) Polar molecules with applied electric field

(a) Polar molecules have random orientations when there is no applied electric field.

(b) The molecules tend to line up with an applied electric field $\vec{E}$.



(a) Nonpolar molecules with no applied electric field

(b) Nonpolar molecules with applied electric field

(a) Nonpolar molecules have centres of their positive and negative charge at the same point.

(b) These centres becomes slightly separated by an applied electric field $\vec{E}$.

Fig. 4.35

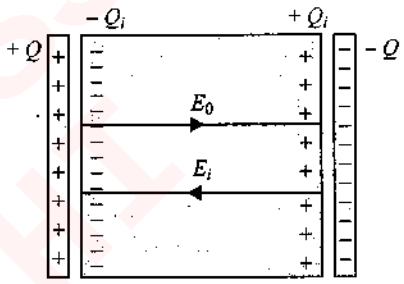


Fig. 4.36

This is irrespective of the thickness of the dielectric slab, i.e., whether it fills up the entire space between the charged plates or it fills only part of it.

Dielectric Breakdown

If a dielectric material is subjected to a sufficiently strong electric field, dielectric breakdown takes place and the dielectric becomes a conductor (a partial ionization that permits conduction through it). This occurs when the electric field is so strong that electrons are ripped loose from their molecules and crashed into other molecules, liberating even more electrons. This avalanche of moving charge, forming a spark or arc discharge, often starts quite suddenly.

Because of dielectric breakdown, capacitors always have maximum voltage ratings. When a capacitor is subjected to excessive voltage, an arc may form through a layer of dielectric, burning or melting a hole in it. This arc creates a conducting path (a short circuit) between the conductors. If a conducting path remains after the arc is extinguished, the device is rendered permanently useless and cannot be used as a capacitor.

The maximum electric field that a material can withstand without the occurrence of breakdown is called its *dielectric strength*.

Capacity of Parallel-Plate Capacitor with Dielectric

Suppose a parallel-plate capacitor has a plate area A and a separation d and a dielectric slab of thickness t and area A is inserted between the plates.

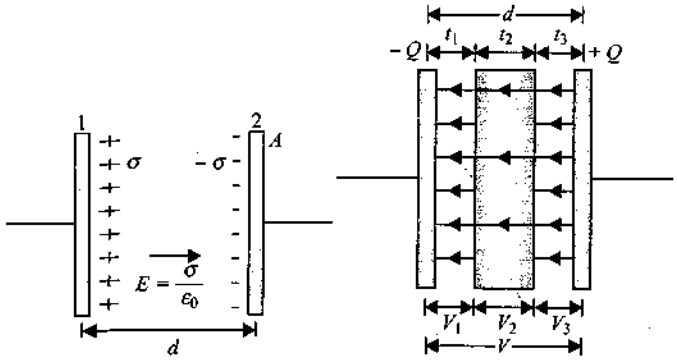


Fig. 4.37

Let Q be the charge given to the capacitor, plates. The electric field between the plates of the capacitor is given by

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

The electric field in the region of dielectric slab is

$$E' = \frac{E}{K} = \frac{Q}{K\varepsilon_0 A}$$

We know that if the electric field is constant, then potential difference between two points separated by distance d along the field line is Ed.

Therefore, the potential difference between two plates is,

$$V = V_1 + V_2 + V_3 = Et_1 + E't_2 + Et_3$$

$$= \frac{Q}{\varepsilon_0 A}t_1 + \frac{Q}{K\varepsilon_0 A}t_2 + \frac{Q}{\varepsilon_0 A}t_3$$

$$= \frac{Q}{\varepsilon_0 A}(t_1 + t_3) + \frac{Q}{K\varepsilon_0 A}t_2 = \frac{Q}{\varepsilon_0 A}(d - t) + \frac{Q}{K\varepsilon_0 A}t$$

The capacity $C = \frac{Q}{V_+ - V_-} = \frac{1}{\frac{(d-t)}{\varepsilon_0 A} + \frac{t}{K\varepsilon_0 A}}$

$$= \frac{\varepsilon_0 A}{(d-t) + \frac{t}{K}}$$

Note:

- ***The capacitance in the above situation is independent of the position of dielectric slab with respect to the plates. The capacitance depends on the thickness of the dielectric slab and dielectric constant.***
- ***The dielectric constant of conducting slab (metal plate) is infinity (∞), therefore term t/K reduces to zero. If we insert a metal plate of thickness t between the plates of capacitor having area A and separation d, the capacitance will become $C = \varepsilon_0 A/d - t$. Also, the capacitance will be independent of position of metal plate between the plates of the capacitor.***
- ***If $t << d$, then $C = \varepsilon_0 A/d$. Hence, if we place a thin metal plate parallel to the plates of a capacitor, the capacitance of the capacitor remains unchanged.***

If we place many dielectric slabs parallel to the plates of a capacitor as shown in Fig. 4.38, then capacitance is given by

$$C = \frac{\varepsilon_0 A}{d - (t_1 + t_2 + \cdots) + \left(\frac{t_1}{k_1} + \frac{t_2}{k_2} + \cdots\right)}$$

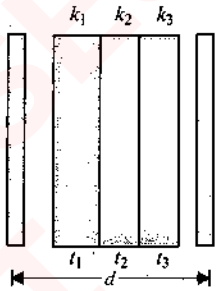


Fig. 4.38

If we introduce a number of dielectric slabs which completely fill the space between the plates, i.e., $d = t_1 + t_2 + t_3 + \cdots + t_n$, then the capacity of the capacitor will be

$$C = \frac{\varepsilon_0 A}{\left(\frac{t_1}{k_1} + \frac{t_2}{k_2} + \cdots\right)}$$

Illustration 4.9 **A parallel-plate capacitor with plates of area A and plate separation d is partially filled with a dielectric slab of constant K, as shown in Fig. 4.39. The thickness of the dielectric slab is $d/4$. What is the equivalent capacitance of this arrangement?**

Sol. (a) If we introduce a thin metal plate between the plates of the capacitor and parallel to it (Fig. b), the capacitance of the system will remain unchanged. The given capacitor arrangement is equivalent to the series combination of the two capacitors as shown in Fig. (c) because the electric potential is the same at all points on the lower surface of the dielectric. All the points on the surface of a conductor are equipotential, therefore all points on the I-shaped conductor connecting the two capacitors are equipotential. Thus, we can divide the entire region of capacitors in two parts.

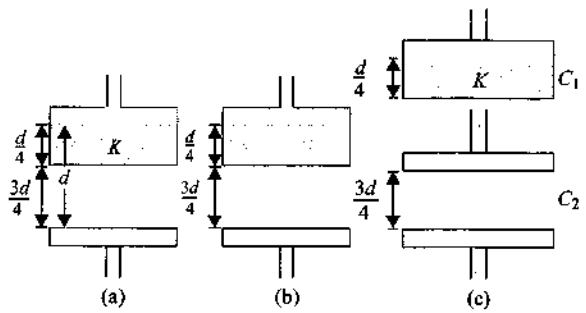


Fig. 4.39

The capacitance of these two parts is

$$C_1 = \frac{AK\varepsilon_0}{d/4} \text{ and } C_2 = \frac{A\varepsilon_0}{(3d/4)}$$

Equivalent capacity, $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d/4}{AK\varepsilon_0} + \frac{3d/4}{A\varepsilon_0}$

$$\Rightarrow \quad C_{eq} = \frac{4\varepsilon_0 A}{d}\left(\frac{K}{3K+1}\right)$$

Alternative method: We can use direct formula;

$$C_{eq} = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}} \quad \text{(i)}$$

Here, $t = d/4$. Thus, from Eq. (i), we get

$$C_{eq} = \frac{\varepsilon_0 A}{d - \frac{d}{4} + \frac{d/4}{K}} = \frac{\varepsilon_0 A}{\left(\frac{3d}{4} + \frac{d}{4K}\right)} = \frac{4\varepsilon_0 A}{d}\left(\frac{K}{3K+1}\right)$$

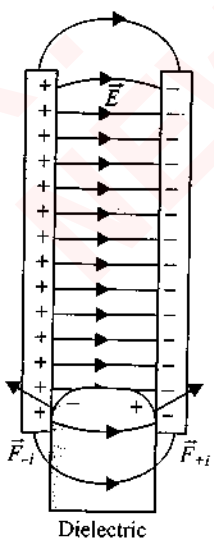
FORCE ON A DIELECTRIC SLAB

When a dielectric slab is placed near a charged capacitor, due to fringing effect the slab experiences a net force towards inside the capacitor.

We will find the force on a dielectric under two conditions: (i) first, when the battery remains connected and (ii) second, after the battery is disconnected.

(i) When battery remains connected: Here, V remains constant.

Let us consider a parallel-plate capacitor with plates of width b and length l. The distance between the plates is d.



The fringing field at the edges of the capacitor exerts forces $\vec{F}_{-i}$ and $\vec{F}_{+i}$ on the negative and positive induced surface charges of a dielectric, pulling the dielectric into the capacitor.

Fig. 4.40

The capacitor is connected to a battery of e.m.f. V. If a dielectric is inserted up to distance x, then

$$C_1 = \frac{\varepsilon_0 b(l-x)}{d}; C_2 = \frac{K\varepsilon_0 bx}{d}$$

$$C = C_1 + C_2 = \frac{\varepsilon_0 b}{d}[l + x(K-1)]$$

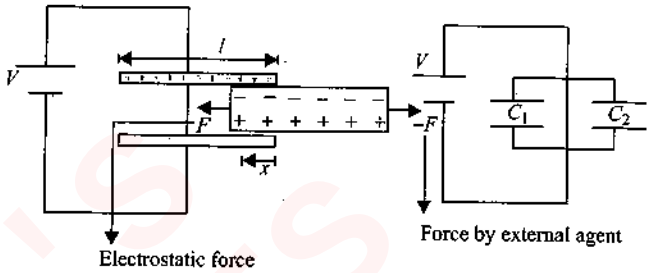


Fig. 4.41

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{\varepsilon_0 b}{d}[l + x(K-1)]V^2$$

Now we have

$$W_{battery} + W_{ext\text{-}agent} = dU + \text{Heat}$$

Here heat will not be generated, as the potential difference across the capacitor remains the same as that of the battery.

$$\Rightarrow \quad V(dQ) - Fdx = \frac{1}{2}dCV^2$$

$$\Rightarrow \quad V[(dC)V] - Fdx = \frac{1}{2}dCV^2$$

$$\Rightarrow \quad Fdx = \frac{1}{2}dCV^2$$

$$\Rightarrow \quad F = \frac{V^2}{2}\frac{dC}{dx}$$

$$= \frac{\varepsilon_0 bV^2}{2d}(K-1)$$

(ii) When battery is disconnected after charging the capacitor: Here, q remains constant.

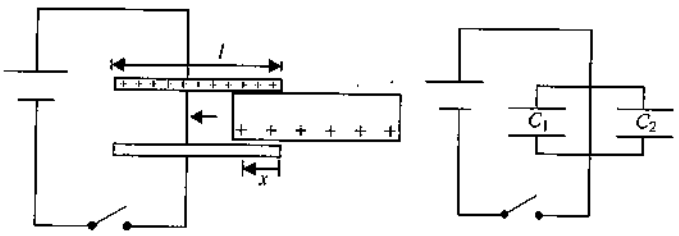


Fig. 4.42

$$U = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}\frac{q^2 d}{\varepsilon_0[l + x(K-1)]b}$$

$$F = -\frac{dU}{dx} = -\frac{q^2 d}{2\varepsilon_0 b}\frac{(K-1)(-1)}{[l + x(K-1)]^2}$$

Therefore, force on dielectric is $F = \frac{q^2 d(K-1)}{2\varepsilon_0 b[l + x(K-1)]^2}$

Effect of Dielectric on Different Parameters

Let the entire space between the plates of a capacitor is filled with a dielectric of dielectric constant K under two conditions:

(a) first, when the battery remains connected and (b) second, after the battery is disconnected.

(a) When battery remains connected (see Fig. 4.43): In this case, potential difference across the plates will remain same, i.e., $V = V_0$, as battery is a source of constant potential difference.

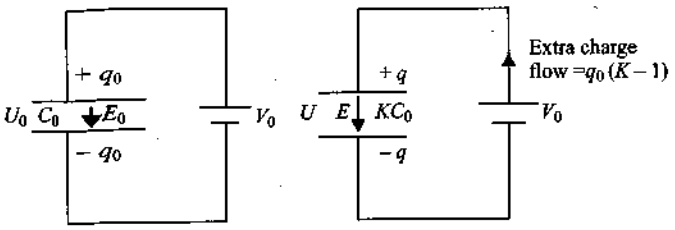


Fig. 4.43

- Capacitance increases, i.e., $C = KC_0$, as capacitance depends upon geometrical factors only.
- Charge on capacitor: $q = CV = KC_0 V_0 = Kq_0$
  ($\therefore$ initially $q_0 = C_0 V_0$)
  Thus, charge increases and becomes K times of previous charge.
- Electric field is
  $$E = \left[\frac{V}{d}\right] = \frac{V_0}{d} = E_0 \{\text{as } V = V_0 \text{ and } V_0/d = E_0\}$$
  Thus, the electric field remains same.
- Energy stored in the capacitor:
  $$U = \frac{1}{2}CV^2 = \frac{1}{2}(KC_0)(V_0)^2 = K\frac{1}{2}C_0V_0^2 = KU_0$$
  $$\left\{\text{as } C = KC_0 \text{ and } U_0 = \frac{1}{2}C_0V_0^2\right\}$$
  Thus, energy increases and becomes K times of the previous energy.

Note: ***On insertion of dielectric, an extra charge of $q - q_0 = q_0(K-1)$ will flow in the circuit through the battery from the negative plate to the positive plate of the capacitor.***

(b) When battery is disconnected (see Fig. 4.44): In this case, charge on the plates will remain same, i.e., $q = q_0$, as in an isolated system, charge remains conserved.

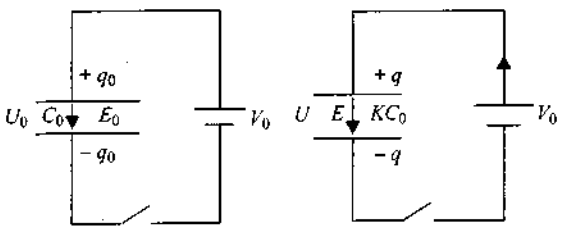


Fig. 4.44

- Capacitance increases, i.e., $C = KC_0$, as capacitance depends upon geometrical factors only.
- Potential difference between the plates is
  $$V = \frac{q}{C} = \frac{q_0}{KC_0} = \frac{V_0}{K}$$
  So, the potential difference decreases.
- Field between the plates is
  $$E = \frac{V}{d} = \frac{V_0}{Kd} = \frac{E_0}{K} \left\{\text{as } V = \frac{V_0}{K} \text{ and } E_0 = \frac{V_0}{d}\right\}$$
  So, the electric field decreases.
- Energy stored in the capacitor is
  $$U = \frac{q^2}{2C} = \frac{q_0^2}{2KC_0} = \frac{U_0}{K} \{\text{as } q = q_0 \text{ and } C = KC_0\}$$
  So, energy decreases.

SPHERICAL CAPACITOR

It consists of two concentric spherical conducing shells of radii a and b, say $b > a$ (Fig. 4.45). The outer shell is earthed. Place a charge $+Q$ on the inner shell. It will reside on the outer surface of the shell. A charge $-Q$ will be induced on inner surface of outer shell. A charge $+Q$ will flow from outer shell to the earth.

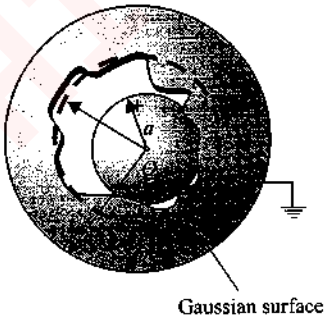


Fig. 4.45

Consider a Gaussian spherical surface of radius r such that $a < r < b$.

From Gauss's law, electric field at distance $r > a$ is
$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

Potential difference is
$$V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{r} = -\int_a^b \frac{Q}{4\pi\varepsilon_0 r^2}dr$$

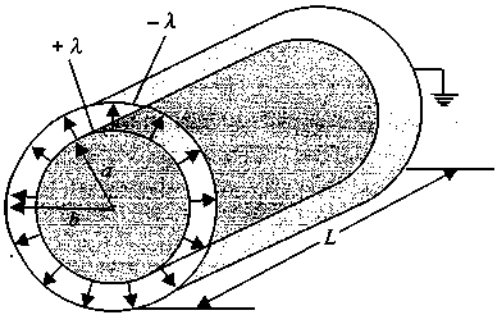
Since $V_b = 0$, we have
$$V_a = \int_a^b \frac{Q}{4\pi\varepsilon_0 r^2}dr$$
$$= \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{Q(b-a)}{4\pi\varepsilon_0 ab}$$

Therefore, capacitance, $C = \dfrac{Q}{V_a - V_b} = \dfrac{Q}{V_a} = \dfrac{4\pi\varepsilon_0 ab}{b-a}$

CYLINDRICAL CAPACITOR

It consists of two coaxial cylinders of radii a and b, say $b > a$ (Fig. 4.46). The outer one is earthed. The cylinders are long enough so that we can neglect fringing of electric field at the

ends. Electric field at a point between the cylinders will be radial and its magnitude will depend on the distance from the central axis. Consider a Gaussian surface of length y and radius r such that $a < r < b$. Flux through the plane surface is zero because electric field and area vector are perpendicular to each other.



A long cylindrical capacitor. The linear charge density λ is assumed to be positive in this figure. The magnitude of charge in a length L of either cylinder is λL.

Fig. 4.46

For curved part, $\phi = \int \vec{E} \cdot d\vec{s} = \int E ds$

$$= E \int ds = E \cdot 2\pi r y$$

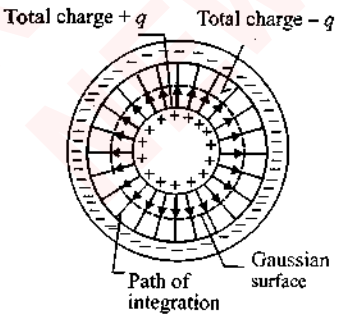
Charge inside the Gaussian surface, $q = \dfrac{Qy}{L}$

From Gauss's law, $\phi = E 2\pi r y = \dfrac{Qy}{L\varepsilon_0} \Rightarrow E = \dfrac{Q}{2\pi\varepsilon_0 L r}$

Potential difference:

$$V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{r} = -\int_a^b \frac{Q}{2\pi\varepsilon_0 L r} dr = -\frac{Q}{2\pi\varepsilon_0 L}\int_a^b \frac{1}{r} dr$$

$$\Rightarrow \qquad V_a = \frac{Q}{2\pi\varepsilon_0 L} \ln \frac{b}{a} \qquad (\text{since } V_b = 0)$$



A cross section of a long cylindrical capacitor, showing a cylindrical Gaussian surface of radius r (that encloses the positive plate). This figure also serves to illustrate a spherical capacitor in a cross section through its centre.

Fig. 4.47

Capacitance, is $C = \dfrac{Q}{V_a - V_a} = \dfrac{Q}{V_a} = \dfrac{2\pi\varepsilon_0 L}{\ln\left(\dfrac{b}{a}\right)}$

Concept Application Exercise 4.2

1. A conductor is an extreme case of dielectric, since if an electric field is applied to a conductor, charges are free to move within the conductor to set up "induced charges". What is the dielectric constant of a perfect conductor? Is it $K = 0$, $K \to \infty$ or something in between? Explain your reasoning.
2. A capacitor of capacitance C is charged to a potential difference V_0. The terminals of the charged capacitor are then connected to those of an uncharged capacitor of capacitance $C/2$. Compute
   a. the original charge of the system;
   b. the final potential difference across each capacitor;
   c. the final energy of the system;
   d. the decrease in energy when the capacitors are connected.
   e. where did the "lost" energy go?
3. A parallel plate vacuum capacitor with plate area A and separation x has charges $+Q$ and $-Q$ on its plates. The capacitor is disconnected from the source of charge, so the charge on each plate remains fixed.
   a. What is the total energy stored in the capacitor?
   b. The plates are pulled apart an additional distance dx. What is the change in the stored energy?
   c. If F is the force with which the plates attract each other, then the change in the stored energy must equal the work $dW = Fdx$ done in pulling the plates apart. Find an expression for F.
   d. Explain why F is not equal to QE, where E is the electric field between the plates?
4. You have two identical capacitors and an external potential source.
   a. Compare the total energy stored in the capacitors when they are connected to the applied potential in series and in parallel.
   b. Compare the maximum amount of charge stored in each case.
   c. Energy storage in a capacitor can be limited by the maximum electric field between the plates. What is the ratio of the electric field for the series that for the parallel combinations?
5. A circuit has section AB as shown in Fig. 4.48.

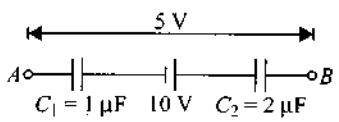


Fig. 4.48

The e.m.f. of the cell is 10 V and the capacitors have capacitances $C_1 = 1\ \mu F$ and $C_2 = 2\ \mu F$. The potential difference $V_{AB} = 5$ V. Find the charges on the capacitors.

6. While a capacitor remains connected to a battery, a dielectic slab is slipped between the plates. Describe qualitatively what happens to the charge, the capacitance, the potential difference, the electric field strength and the stored energy. Is work required to insert the slab?
7. If you have several 2.0 μF capacitors, each capable of withstanding 200 V without breakdown, how would

you assemble a combination which has an equivalent capacitance of

a. 0.4 μF, and

b. 1.2 μF, each withstanding 1000 V?

8. N identical capacitors are connected in parallel and then a potential difference of V is applied to them. Find the potential difference when these capacitors are reconnected in series, their charges being left undisturbed?

9. In the arrangement shown in Fig. 4.49, plate B is given a charge equal to 60 μC. The ratio $d_1/d_2 = 2$. Then

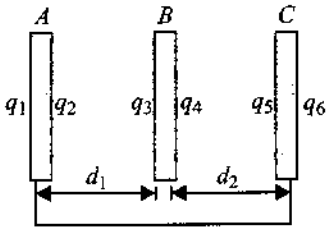


Fig. 4.49

$q_1 =$ _____
$q_2 =$ _____
$q_3 =$ _____
$q_4 =$ _____
$q_5 =$ _____
$q_6 =$ _____

10. In Fig.4.50, the plate A has 100 μC charge, while the plate B has 60 μC charge, while the plate B has 60 μC charge.

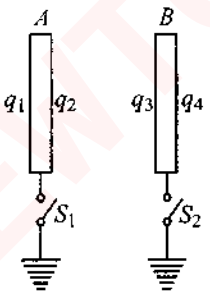


Fig. 4.50

a. When both switches are open, then

$q_1 =$ _____ $q_2 =$ _____
$q_3 =$ _____ $q_4 =$ _____

b. When only switch S_1 is closed, then

$q_1 =$ _____ $q_2 =$ _____
$q_3 =$ _____ $q_4 =$ _____

c. When switch S_2 is also closed, then

$q_1 =$ _____ $q_2 =$ _____
$q_3 =$ _____ $q_4 =$ _____

11. For the network of capacitors as shown in Fig. 4.51.

a. Find the potential of junction B,

b. Find the potential of junction D,

c. Find the charge on 2 μF capacitor.

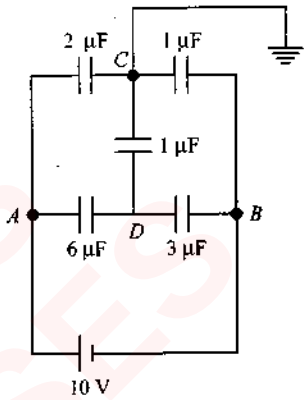


Fig. 4.51

12. In Fig. 4.52, the system is in steady state.

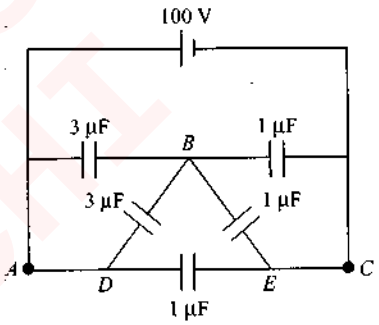


Fig. 4.52

Find

a. $V_A - V_B =$ _____

b. $V_B - V_C =$ _____

c. $V_D - V_E =$ _____

d. The energy stored in the circuit = _____

13. State whether the following statements are true or false.

a. If a battery is connected across a circuit consisting of two identical capacitors, it is found that, in steady state, the two capacitors have equal charge, then the two capacitors must be in series with each other.

b. If a battery is connected across a circuit consisting of two capacitors having different capacitances and it is found that, in steady state, the two capacitors have equal charge, then the two capacitors must be in series with each other.

c. If a battery is connected across a circuit consisting of two identical capacitors and it is found that, in steady state, the two capacitors have equal charge, then the two capacitors may be in series with each other.

14. Find the equivalent capacitance between points A and B shown in Fig. 4.53.

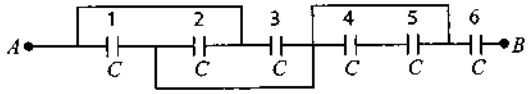


Fig. 4.53

Solved Examples

Example 4.1 **Figure 4.54 shows two identical parallel-plate capacitors connected to a battery with the switch S closed. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant (or relative permittivity) $K = 3$. Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric. (IIT-JEE, 1983)**

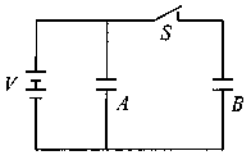


Fig. 4.54

Sol. Initially, when the switch is closed, both the capacitors A and B are in parallel and therefore, the energy stored in the capacitors is

$$U_i = 2\times\frac{1}{2}CV^2 = CV^2 \qquad \text{(i)}$$

When switch S is opened, B gets disconnected from the battery. The capacitor B is now isolated, the charge on an isolated capacitor remains constant, often referred to as bound charge. On the other hand, A remains connected to battery. Hence, potential V remains constant on it.

When the capacitors are filled with dielectric, their capacitance increases to KC.

Therefore, energy stored in B changes to $\frac{1}{2}\frac{Q^2}{KC}$, where $Q = CV$ is the charge on B which remains constant, and energy stored in A changes to $1/2KCV^2$, where V is the potential on A which remains constant. Thus, the final total energy stored in the capacitors is

$$U_f = \frac{1}{2}\frac{(CV)^2}{KC} + \frac{1}{2}KCV^2 = \frac{1}{2}CV^2\left(K+\frac{1}{K}\right) \qquad \text{(ii)}$$

From Eq. (i) and (ii), we find $\frac{U_i}{U_f} = \frac{2K}{K^2+1}$

It is given that $K = 3$. Therefore, we have $\frac{U_i}{U_f} = \frac{3}{5}$

Example 4.2 **Two parallel-plate capacitors of capacitance C each are connected in series with a battery of e.m.f. ε. Then, one of the capacitors is filled with a dielectric of dielectric constant K.**

i. Find the change in electric field in the two capacitors, if any.

ii. What amount of charge flows through the battery?

iii. Find the change in energy stored in the circuit, if any.

Sol.

i. Two capacitors A and B initially have same charge Q and potential $V = Q/C$. The electric field between the capacitor plates is given by $E = V/d$. Since the two capacitors are connected in series with the battery, the sum of potentials across the capacitors must be equal to ε, i.e;

$$\varepsilon = 2V = \frac{2Q}{C} \qquad \text{(i)}$$

ii. When one of the capacitors, say A, if filled with a dielectric, the capacity of A increases to $C' = KC$ while that of B remains unchanged, i.e., C.

Suppose, now charge on the capacitors becomes Q' and potentials across A and B are V_A' and V_B', respectively, with $V_A' + V_B' = \varepsilon$. Hence, we have

$$V_A' = \frac{Q'}{C'} = \frac{Q'}{KC} \text{ and } V_B' = \frac{Q'}{C'}$$

Hence, $\varepsilon = \frac{Q'}{C}\left(1+\frac{1}{K}\right)$ (ii)

From Eq. (i) and (ii), we get $Q' = \frac{2K}{1+K}Q$

Since $K > 1$, therefore $Q' > Q$. Also, $V_A' = \frac{2K}{1+K}$ V and $V_B' = \frac{2K}{1+K}$ V.

Thus, the electric field (or potential difference) in capacitor A increases by a factor of $\left(\frac{2}{1+K}\right)$ while that in B increases by a factor of $\left(\frac{2K}{1+K}\right)$.

The amount of charge that flows into the circuit is given by

$$\Delta Q = Q' - Q = \left(\frac{2K}{1+K} - 1\right)Q = \frac{K-1}{K+1}Q$$
$$= \frac{1}{2}\frac{K-1}{K+1}C\varepsilon$$

iii. Initially, the energy is given by $U_i = 2\times\frac{1}{2}CV^2 = CV^2$

Finally, the energy is

$$U_f = \frac{1}{2}C'V_A'^2 + \frac{1}{2}CV_B'^2$$
$$= \frac{1}{2}KC\left(\frac{2}{1+K}\right)^2 V^2 + \frac{1}{2}C\left(\frac{2K}{1+K}\right)^2 V^2$$
$$= \frac{4K}{1+K}\left(\frac{1}{2}CV^2\right) = \frac{2K}{1+K}U_i$$

Example 4.3 **Five identical conducting plates 1, 2, 3, 4 and 5 are fixed parallel and equidistant from each other as shown in Fig. 4.55. Plates 2 and 5 are connected by a conductor while plates 1 and 3 are joined by another conductor. The junction of plates 1 and 3 and the plate 4 are connected to a source of constant e.m.f. V_0. Find**

i. the effective capacity of the system between the terminals of the source,

ii. the charges on plates 3 and 5.

Given d = distance between any two successive plates and A = area of either face of each plate.

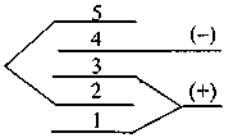


Fig. 4.55

Sol.

i. The equivalent circuit is shown in Fig. 4.56. The system consists of four capacitors, i.e., C_{12}, C_{32}, C_{34} and C_{54}. The capacity of each capacitor is $(K\varepsilon_0 A/d) = C_0$.

The capacitors C_{12} and C_{32} are in parallel, their capacity is $C_0 + C_0 = 2C_0$. The capacitor C_{54} is in series with the parallel combination of C_{12} and C_{32}. Hence, the resultant capacity will be $C_1 = \dfrac{C_0 \times 2C_0}{C_0 + 2C_0} = \dfrac{2C_0}{3}$

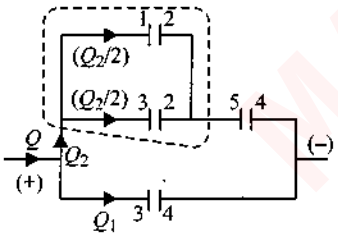


Fig. 4.56

Further, C_{34} is again in parallel with the combination of C_{12}, C_{32}, C_{54}. Hence, the effective capacity is

$$C_{\text{eff.}} = C_0 + \frac{C_0 \times 2C_0}{C_0 + 2C_0} = \frac{5}{3}C_0 \frac{5}{3}K\varepsilon_0 \frac{A}{d}$$

ii. Charge on the plate 5 = charge on the upper half of parallel combination, $Q_5 = V_0\left(\dfrac{2}{3}C_0\right) = \dfrac{2}{3}\dfrac{K\varepsilon_0 A V_0}{d}$

Charge on plate 3 on the surface facing 4 is

$$Q_1 = V_0 C_0 = \frac{K\varepsilon_0 A V_0}{d}$$

Charge on plate 3 on the surface facing 2

= [potential difference across (3 – 2)] C_0

$$= V_0 \frac{C_0}{C_0 + 2C_0} C_0 = K\varepsilon_0 \frac{AV_0}{3d}$$

Net charge on plate 3 is

$$Q_3 = \frac{K\varepsilon_0 A V_0}{d} + K\varepsilon_0 \frac{AV_0}{3d} = \frac{K\varepsilon_0 A V_0}{d}\left[1 + \frac{1}{3}\right]$$

$$= \frac{4}{3}K\varepsilon_0 \frac{A}{d} V_0$$

Example 4.4 **Two capacitors of capacity 3.00 μF and 2.00 μF are separately charged with a 24.0 V battery. After they are fully charged, they are connected as shown in Fig. 4.57 (a) and (b).**

In each of the two arrangements, find the energy stored on each capacitor (a) before switch S is closed and (b) after switch S is closed.

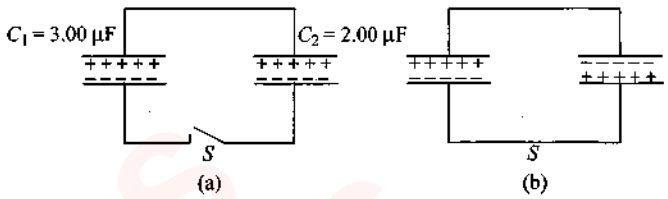


Fig. 4.57

Sol. a. When the switch S is open, the charges on the unconnected plates are bound, they have no place to go. Charges cannot move in open wires, a closed circuit is a must. The charges on the connected plates are held in place due to Coulomb attraction of bound charges on the unconnected plates, which prevents them from combining.

Each capacitor retains its charges and the potential difference it had when originally connected to the battery. Initially, the energy stored in the capacitors,

$$U_1 = \frac{1}{2}C_1(\Delta V)^2 = \frac{1}{2}(3.00\times10^{-6})(24.0)^2 = 8.64\times10^{-4}\ \text{J}$$

$$U_2 = \frac{1}{2}C_2(\Delta V)^2 = \frac{1}{2}(2.00\times10^{-6})(24.0)^2 = 5.76\times10^{-4}\ \text{J}$$

The total energy of this system of capacitors is

$$U = U_1 + U_2 = 1.44\times10^{-3}\ \text{J}$$

b. When the switch is closed, the charges can combine such that net charge is conserved. The magnitude of initial charge on each capacitor is

$$Q_1 = C_1\Delta V = (3.00\times10^{-6})(24) = 72\times10^{-6}\ \text{C} = 72\ \mu\text{J}$$

$$Q_2 = C_2\Delta V = (2\times10^{-6})(24) = 48\times10^{-6}\ \text{C} = 48\ \mu\text{J}$$

Now the positive plate of C_1 is connected to negative plate of C_2; therefore, net charge on both top plates is $(Q_1 - Q_2)$ and on the lower plates is $-(Q_1 - Q_2)$.

$$Q_{\text{net}} = Q_1 - Q_2 = (72 - 48)\times10^{-6}$$

$$= 24 \times 10^{-6}\ \text{C} = 24\ \mu\text{C} \qquad \text{(i)}$$

After the switch is closed, the two capacitors are in parallel. Consequently, potential differences across the two capacitors are equal, i.e.,

$$\frac{Q_1'}{C_1} = \frac{Q_2'}{C_2} \qquad \text{(iii)}$$

On solving Eqs. (i) and (iii), we obtain final charges on the two capacitors i.e.,

$$Q_1' = 14.4\ \mu\text{C and } Q_2' = 9.60\ \mu\text{C}$$

Now the potential difference across each capacitor is

$$\Delta V = \frac{Q_1'}{C_1} = \frac{Q_2'}{C_2} = \frac{14.4}{3.0} = \frac{9.60}{2} = 4.80\ \text{V}$$

Final energy on the two capacitors,

$$U_1' \frac{Q_1'^2}{2C_1} = \frac{(14.4\times10^{-6})^2}{2(3\times10^{-6})} = 3.46\times10^{-5}\ \text{J}$$

and $U_2' \frac{Q_2'^2}{2C_1} = \frac{(9.60\times10^{-6})^2}{2(2\times10^{-6})} = 2.30\times10^{-5}$ J

Now the total energy is

$$U' = U_1' + U_2' = 5.76\times10^{-5} \text{ J}$$

Example 4.45 **Two parallel-plate capacitors A and B have the same separation $d = 8.85 \times 10^{-4}$ m between the plates. The plate areas of A and B are 0.04 m² and 0.02 m², respectively. A slab of dielectric constant (relative permittivity) $K = 9$ has dimensions such that it can exactly fill the space between the plates of capacitor B.**

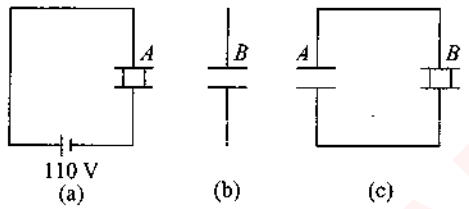


Fig. 4.58

i. The dielectric slab is placed inside A as shown in Fig. 4.58 (a). A is then charged to a potential difference of 110 V. Calculate the capacitance of A and the energy stored in it.

ii. The battery is disconnected and then the dielectric slab is removed from A. Find the work done by the external agency in removing the slab from A.

iii. The same dielectric slab is now placed inside B, filling it completely. The two capacitors A and B are then connected as shown in Fig. 4.58 (c). Calculate the energy stored in the system. (IIT-JEE, 1993)

Sol. i. The capacitor A with dielectric can be considered as two capacitors in parallel, one having dielectric and the other having no dielectric state. Such capacitor has an area of $A/2$.

Fig. 4.59

The combined capacitance is

$$C = C_1 + C_2 = \frac{(A/2)\varepsilon_0}{d} + \frac{(A/2)\varepsilon_0 K}{d} = \frac{A}{2}\frac{\varepsilon_0}{d}[1+K]$$

$$= \frac{0.04\times8.85\times10^{-12}}{2\times8.85\times10^{-4}}[1+9] = 2\times10^{-9} \text{ F}$$

$$\text{Energy stored} = \frac{1}{2}CV^2 = \frac{1}{2}\times2\times10^{-9}\times(110)^2$$

$$= 1.21\times10^{-5} \text{ J}$$

ii. Work done in removing the dielectric state = (Energy stored in capacitor without dielectric) − (Energy stored in capacitor with dielectric).

It may be noted that while taking out the dielectric the charge on the capacitor plate remains the same.

$$W = \frac{q^2}{2C'} - \frac{q^2}{2C}$$

Here, $C = 2\times10^{-9}$ F, $C' = \frac{A\varepsilon_0}{d} = \frac{0.04\times8.85\times10^{-12}}{8.85\times10^{-4}}$

$$= 0.4\times10^{-9} \text{ F}$$

$$q = CV = 2\times10^{-9}\times110 = 2.2\times10^{-7} \text{ C}$$

$$\therefore W = \frac{(2.2\times10^{-7})^2}{2}\left[\frac{1}{0.4\times10^{-9}} - \frac{1}{2\times10^{-9}}\right]$$

$$= 4.84\times10^{-5} \text{ J}$$

iii. The capacitance of $B = \frac{\varepsilon_0 K A_B}{d}$

$$C_B = 1.8\times10^{-9} \text{ F}$$

The charge on A, $q_A = 2.2 \times 10^{-7}$ C, gets distributed into two parts.

$$q_1 + q_2 = 2.2 \times 10^{-7} \text{ C}$$

Also, the p. d. across A = p.d. across B

$$\frac{q_1}{C_A} = \frac{q_2}{C_B} \Rightarrow q_1 = \frac{C_A}{C_B}q_2 = \frac{0.4\times10^{-9}}{1.8\times10^{-9}}q_2 = 0.22q_2$$

$$0.22q_2 + q_2 = 2.2\times10^{-7}$$

$$\Rightarrow \quad q_2 = \frac{2.2}{1.22}\times10^{-7} = 1.8\times10^{-7} \text{ C and}$$

$$q_1 = 0.4\times10^{-7} \text{ C}$$

$$\text{Total energy stored} = \frac{q_1^2}{2C_A} + \frac{q_2^2}{2C_B}$$

$$= 0.2\times10^{-5} + 0.9\times10^{-5} = 1.1\times10^{-5} \text{ J}$$

Alternatively, the combined capacitance of the two capacitors can be found. The total charge on the two capacitors is known. The energy can be found using the formula $Q^2/2C_{eq}$.

Example 4.46 **Two capacitors A and B with capacities 3 and 2 μF are charged to a potential difference of 100 and 180 V, respectively. The plates of the capacitors are connected as shown in Fig. 4.60 with one wire free from each capacitor. The upper plate of A is positive and that of B is negative. An uncharged 2 μF capacitor C with lead wires falls on the free ends to complete the circuit. Calculate**

i. the final charge on the three capacitors,

ii. the amount of electrostatic energy stored in the system before and after the completion of the circuit.

(IIT-JEE, 1997)

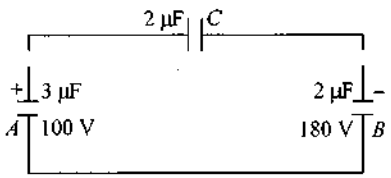


Fig. 4.60

i. Method 1: Initial charge on capacitor A: $q_A = 3 \times 100 = 300\ \mu C$

Initial charge on capacitor B: $q_B = 2 \times 180 = 360\ \mu C$

After completing the circuit, let charge on capacitor A be q_1, on B is q_2 and on C is q_3 with polarities as shown in Fig. 4.61.

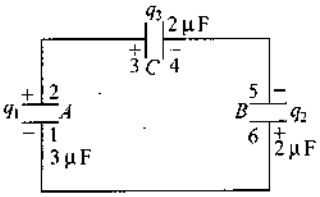


Fig. 4.61

Applying conservation of charge for plates 2 and 3, we get

$$q_1 + q_3 = 300 + 0 = 300 \quad \text{(i)}$$

Applying conservation of charge for plates 4 and 5, we get

$$-q_2 - q_3 = -360 + 0 \Rightarrow q_2 + q_3 = 360 \quad \text{(ii)}$$

Applying Kirchhoff's law in the whole circuit, we get

$$\frac{q_1}{3} - \frac{q_3}{2} + \frac{q_2}{2} = 0 \quad \text{(iii)}$$

On solving, we get $q_1 = 90\ \mu C$, $q_2 = 150\ \mu C$, $q_3 = 210\ \mu C$.

Method 2: After completing the circuit, let charge q flow in the circuit in clockwise direction. Then the final charges on capacitors will be as shown.

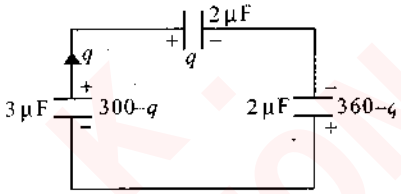


Fig. 4.62

Applying Kirchhoff's law, we get

$$\frac{300-q}{3} - \frac{q}{2} + \frac{360-q}{2} = 0 \Rightarrow q = 210\ \mu C$$

Now charges on all three capacitors can be found.

ii. Initially, energy stored is

$$U_i = \frac{1}{2} \times 3 \times 10^{-6} \times (100)^2 + \frac{1}{2} \times 2 \times 10^{-6} (180)^2 = 4.74 \times 10^{-2}\ J$$

Finally, the energy stored is

$$U_f = \frac{(90 \times 10^{-6})^2}{2 \times 3 \times 10^{-6}} + \frac{(150 \times 10^{-6})^2}{2 \times 2 \times 10^{-6}} + \frac{(210 \times 10^{-6})^2}{2 \times 2 \times 10^{-6}} = 1.8 \times 10^{-2}\ J$$

Example 4.7 **A capacitor has rectangular plates of length a and width b. The top plate is inclined at a small angle as shown in Fig. 4.63. The plate separation varies from $d = y_0$ at the left to $d = 2y_0$ at the right, where y_0 is much less than a or b. Calculate the capacitance of the system.**

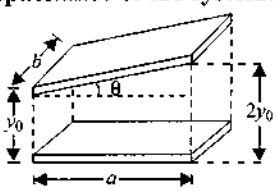


Fig. 4.63

Sol. We consider a differential strip of width dx and length b to approximate a differential capacitor of area $bd\,x$ and separation $d = y_0 + \left(\frac{y_0}{a}\right)x$. All such differential capacitors are in parallel arrangement.

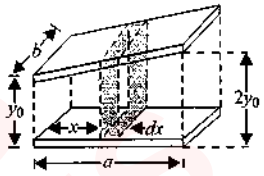


Fig. 4.64

$$dC = \frac{\varepsilon_0 (b\,dx)}{y_0 + \left(\frac{y_0}{a}\right)x} \Rightarrow C = \int dC$$

$$C = \varepsilon_0 b \int_0^a \frac{dx}{\left(y_0 + \frac{y_0}{a}x\right)}$$

$$= \frac{\varepsilon_0 b}{(y_0/a)} \left[\ln \left(\frac{y_0 + \frac{y_0}{a} \times a}{y_0} \right) \right] = \frac{\varepsilon_0 ab}{y_0} \ln 2$$

We can determine the expression for capacity in terms of θ as

$$d = (y_0 + x \tan\theta)$$

$$C = \int dC = \int_b^a \frac{\varepsilon_0 b\,dx}{(y_0 + x \tan\theta)}$$

$$= \frac{\varepsilon_0 b}{\tan\theta} \ln \frac{(y_0 + x \tan\theta)}{y_0}$$

For small θ, $\tan\theta = \theta \Rightarrow C = \frac{\varepsilon_0 b}{\theta} \ln \left(1 + \frac{a\theta}{y_0}\right)$

Now, we can use the expansion

$$\log(1+x) = x - \frac{1}{2}x^2 + \cdots$$

For $x < 1$, we can neglect higher powers. Thus,

$$C = \frac{\varepsilon_0 b}{\theta} \left[\frac{a\theta}{y_0} - \frac{1}{2}\left(\frac{a\theta}{y_0}\right)^2 \right] = \frac{\varepsilon_0 ab}{y_0} \left[1 - \frac{a\theta}{2y_0} \right]$$

Example 4.8 **In the circuit shown (Fig. 4.65), the e.m.f. of each battery is 60 V and $C_1 = 2\ \mu F$ and $C_2 = 3\ \mu F$. Find the charges that will flow through the sections 1, 2 and 3 after the key is closed.**

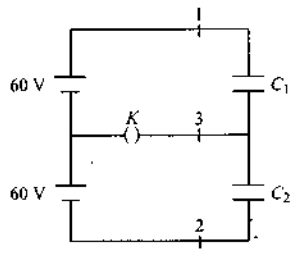


Fig. 4.65

Sol. Before closing the switch [Fig. 4.66(a)], both capacitors are in series. Their equivalent capacitance is

$$C_{eq} = \frac{2\times 3}{2+3} = \frac{6}{5}\ \mu F$$

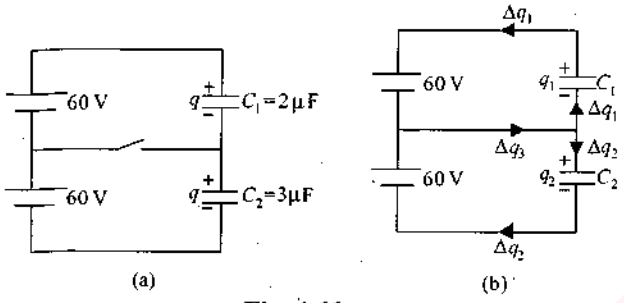


Fig. 4.66

So charge on each capacitor is

$$q = C_{eq}\times 120 = \frac{6}{5}\times 120 = 144\ \mu C$$

After closing the switch [Fig. 4.66 (b)], potential difference across each capacitor is 60 V. So charge on them is

$$q_1 = C_1\times 60 = 2\times 60 = 120\ \mu C$$

$$q_2 = C_2\times 60 = 3\times 60 = 180\ \mu C$$

Charge flown through section 1:

$$\Delta q_1 = q - q_1 = 144 - 120 = 24\ \mu C$$

Charge flown through section 2:

$$\Delta q_2 = q_2 - q = 180 - 144 = 36\ \mu C$$

Charge flown through section 3:

$$\Delta q_3 = \Delta q_1 + \Delta q_2 = 24 + 36 = 60\ \mu C$$

Example 4.9 **Figure 4.67 shows that a conducting sphere *A* of radius '*a*' is surrounded by a neutral conducting spherical shell *B* of radius *b* (> *a*). Initially, switches S_1, S_2 and S_3 are open and the sphere *A* carries a charge *Q*. First, switch S_1 is closed to connect shell *B* with ground, and then S_1 is opened. Now, switch S_2 is closed so that sphere *A* is grounded and then S_2 is opened. Finally, switch S_3 is closed to connect the spheres together. Find the heat produced on closing switch S_3.**

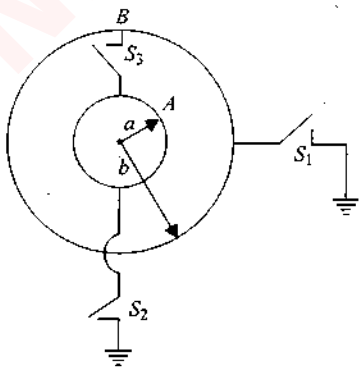


Fig. 4.67

Sol. When the outer sphere is connected to ground, charge $-Q$ resides on the inner surface of sphere *B*, with no charge on its outer surface. When it is disconnected from the ground, $-Q$ remains on its inner surface. Now, the sphere *A* is connected to the earth, so potential on its surface becomes zero. Let the charge on the sphere *A* become *q*, then

$$V_A = \frac{1}{4\pi\varepsilon_0}\frac{q}{a} - \frac{1}{4\pi\varepsilon_0}\frac{Q}{b} = 0 \qquad q = \frac{a}{b}Q$$

The resulting charges on various surfaces are shown in Fig. 4.68.

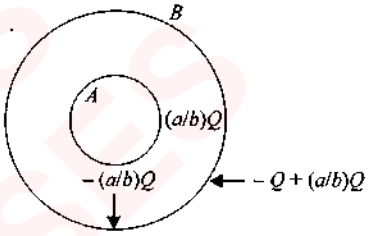


Fig. 4.68

In this position, the energy stored is

$$U_1 = \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{1}{a}\left(\frac{1}{b}Q\right)^2 + \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{1}{b}Q^2 + \frac{1}{4\pi\varepsilon_0}\frac{1}{b}\left(\frac{a}{b}Q\right)(-Q) \qquad \text{(i)}$$

As shown in Eq. (i), the above energy is sum of the energies of each of the individual spheres and their interaction energy (represented by the third term).

$$+\frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{1}{b}\left(\frac{a}{b}-1\right)^2 Q^2$$

When switch S_3 is closed, the total charge will appear on the outer surface of shell *B*. In this position, energy stored is

$$U_2 = \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{1}{b}\left(\frac{a}{b}-1\right)^2 Q^2$$

This energy is corresponding to the outer sphere only.

$$\text{Heat produced} = U_1 - U_2 = \frac{Q^2 a(b-a)}{8\pi\varepsilon_0 b^3}$$

Example 4.10 **In the circuit shown in Fig. 4.69, the P.D. between the points *a* and *b* is 4 V. Find the e.m.f. *E* of the battery. Assume that before connecting the battery in the circuit, all the capacitors were uncharged.**

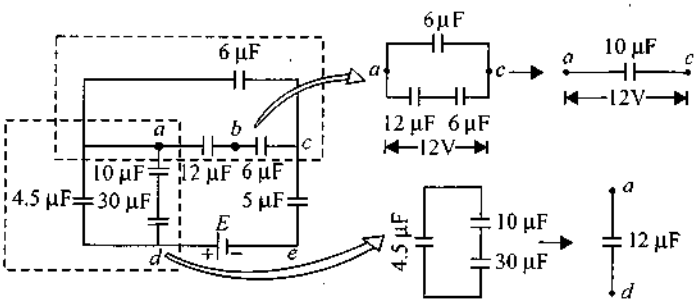
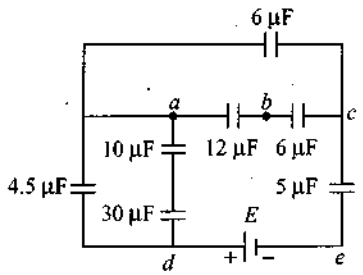


Fig. 4.69

Sol. The capacitors 12 μF and 6 μF are in series arrangement. The charge in each will be equal $q_1 = CV = 12 \times 4 = 48\ \mu C$



(a)

Fig. 4.70

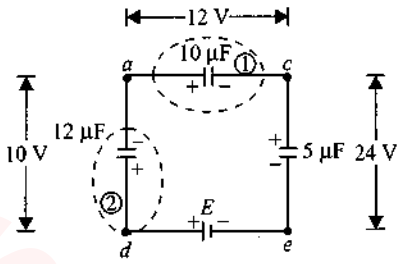
The P.D. across 6 μF capacitors $= \frac{12\times 4}{6}$ V = 8 V

The P.D. between points a and $c = V_a - V_c = 4 + 8 = 12$ V

The equivalent capacity in the portion (1) is 10 μF.

The charge on it =(10 × 12) μC =120 μC

The portions (1) and (2) of the circuit are in series combination.



(a)

Fig. 4.71

Hence, $12\times(V_d - V_a) = 120\ \mu C \Rightarrow V_d - V_a = 10$ V

The capacitor 5 μF is also in series with 10 μF, (see Fig. 4.71) Hence, charge on it is 120 μC.

Thus, $V_c - V_e = 120/5 = 24$ V

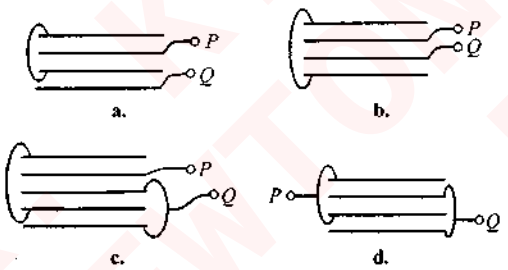
Now $E = (12 + 10 + 24)$ V = 46 V

EXERCISES

Subjective Type

Solutions on page 4.36

1. Identical metal plates are located in air at equal distance d from one another. The area of each plate is equal to A. Evaluate the capacitance of the system between P and Q if the plates are interconnected as shown in Fig. 4.72.



2. The capacitors in Fig. 4.72 are initially uncharged and are connected as in the diagram with switch S open. The applied potential difference is V_{ab} = +360 V.

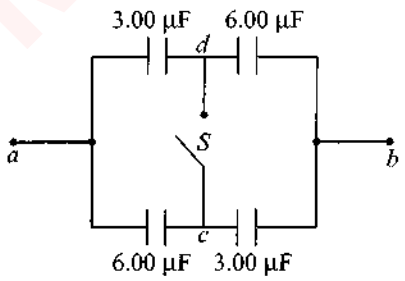


Fig. 4.72

a. What is the potential difference V_{cd}?

b. What is the potential difference across each capacitor after switch S is closed?

c. How much charge will flow through the switch after it is closed?

3. If the area of each plate is A and the successive separations are d, $2d$ and $3d$, then find the equivalent capacitance across A and B.

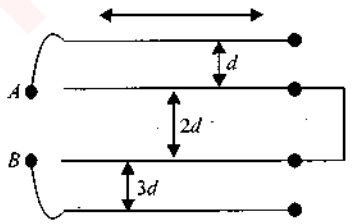


Fig. 4.73

4. Condensers with capacities C, 2 C, 3 C and 4 C are charged to the voltage V, 2 V, 3 V and 4 V, respectively, (Fig. 4.74). The circuit is closed. Find the voltage on all the condensers in equilibrium.

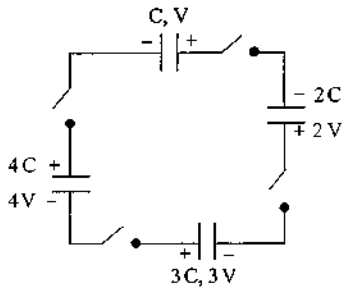


Fig. 4.74

5. In Fig. 4.75, when switch is swapped from 1 to 2, find the heat produced in the circuit.

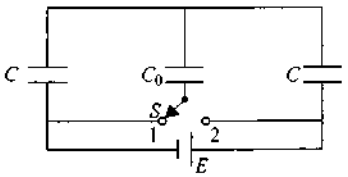


Fig. 4.75

6. Consider the network shown in Fig. 4.76. Find the effective capacity between A and B. Assume C = 25 μF.

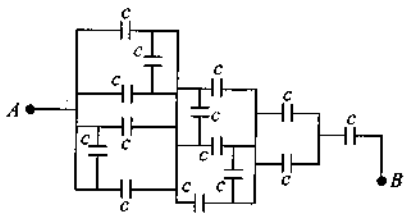


Fig. 4.76

7. A capacitor of capacitance $C_1 = 1$ μF can withstand a maximum voltage of $V_1 = 6$ kV and another capacitor of capacitance $C_2 = 2$ μF can withstand a maximum voltage of $V_2 = 4$ kV. If they are connected in series, what maximum voltage will the system withstand?
8. What charges (in μC) will flow through section B of the circuit in the direction shown when switch S is closed?

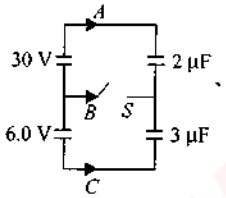


Fig. 4.77

9. A parallel-plate capacitor contains a mica sheet (thickness 10^{-3} m) and a sheet of fibre (thickness 0.5×10^{-3} m). The dielectric constant of mica is 8 and that of fibre is 2.5. Assuming that the fibre breaks down when subjected to an electric field of 6.4×10^6 Vm^{-1}, find the maximum safe voltage that can be applied to the capacitor.
10. Find the potential difference between the points M and N of system shown in Fig. 4.78 if the e.m.f. is equal to $E = 110$ V and the capacitance ratio $C_2/C_1 = 23$.

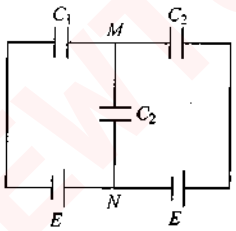


Fig. 4.78

11. Figure 4.79 shows a network of seven capacitors. If charge on 5 μF capacitor is 10 μC, find the potential difference between points A and C.

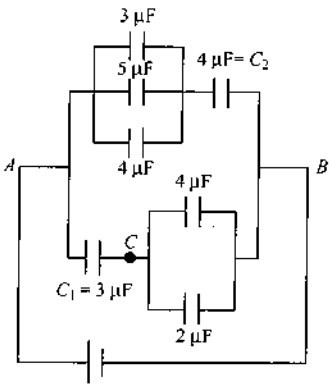


Fig. 4.79

12. Find the potential difference between the points A and B and that between E and F of the circuit shown in Fig. 4.80.

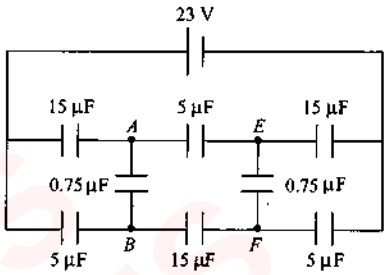


Fig. 4.80

13. Some capacitors each of capacitance 30 μF are connected as shown in Fig. 4.81. Calculate equivalent capacitance between terminals A and B.

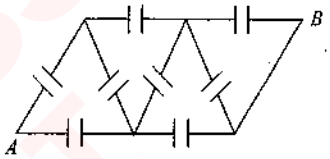


Fig. 4.81

Objective Type

Solutions on page 4.40

1. Two copper spheres of same radii, one hollow and the other solid, are charged to same potential. Then, which, if any, of the two will have more charge?
   a. Hollow
   b. Solid
   c. Both will have the same charge
   d. Nothing can be predicted
2. The distance between the plates of a parallel-plate capacitor is d. A metal plate of thickness $d/2$ is placed between the plates. What will be its effect on the capacitance?
   a. Capacitance will be halved
   b. Capacitance will be doubled
   c. Capacitance will not change
   d. Capacitance will become 1.5 times original
3. Two metallic charged spheres of radii R_1 and R_2 having charges Q_1 and Q_2, respectively, are connected to each other. There is
   a. no change in the energy of the system
   b. an increase in the energy of the system
   c. always a decrease in the energy of the system
   d. a decrease in energy of the system unless $Q_1R_2 = Q_2R_1$
4. In the circuit of Fig. 4.82, find the charge of the condenser having capacity 5 μF.

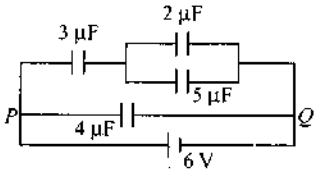


Fig. 4.82

a. 4.5 μC b. 9 μC c. 7 μC d. 30 μC

5. In the accompanying diagram, if $C_1 = 3$ μF, $C_2 = 6$ μF, $C_3 = 9$ μF, $C_4 = 12$ μF, $C_5 = 15$ μF and $C_6 = 18$ μF, then the equivalent capacitance between the ends A and B is

a. 1.22 μF b. 5.16 μF c. 2.25 μF d. 2.51 μF

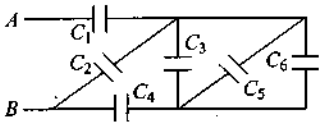


Fig. 4.83

6. Three capacitors of capacitances 2, 3 and 4 pF are connected in parallel. What is the charge (in pC) on each capacitor if the combination is connected to a 100 V supply?

a. 200, 300, 400 b. 300, 200, 400
c. 400, 300, 200 d. 400, 200, 300

7. In the above question, if the capacitors were connected in series, find the potential difference (in V) across each capacitor.

a. $\frac{300}{13}, \frac{600}{13}, \frac{400}{13}$ b. $\frac{600}{13}, \frac{300}{13}, \frac{400}{13}$
c. $\frac{300}{13}, \frac{400}{13}, \frac{600}{13}$ d. $\frac{600}{13}, \frac{400}{13}, \frac{300}{13}$

8. Four identical metal plates, each with a surface area A (on one side), are placed at a distance d from each other as shown in Fig. 4.84. The two inner plates are connected to point B and the other two plates to another point A. Then, the capacitance of the system is

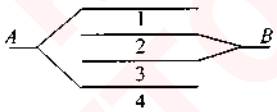


Fig. 4.84

a. $\varepsilon_0 A/d$ b. $2\varepsilon_0 A/d$
c. $3\varepsilon_0 A/d$ d. $2\varepsilon_0 A/3d$

9. We wish to obtain a capacitance of 5 μF, by using some capacitors, each of 2 μF. Then, the minimum number of capacitors required is

a. 3 b. 4
c. 5 d. not possible

10. A number of capacitors, each of equal capacitance C, are arranged as shown in Fig. 4.85. The equivalent capacitance between A and B is

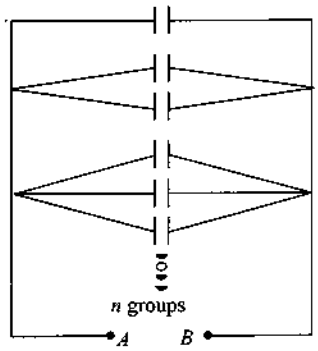


Fig. 4.85

a. n^2C b. $(2n+1)C$
c. $\frac{(n-1)n}{2}C$ d. $\frac{(n+1)n}{2}C$

11. The plates of a parallel-plate capacitor are charged up to 100 V. Now, after removing the battery, a 2 mm thick plate is inserted between the plates. Then, to maintain the same potential difference, the distance between the capacitor plates is increased by 1.6 mm. Dielectric constant of the plate is

a. 5 b. 1.25 c. 4 d. 2.5

12. Three plates A, B, C each of area 50 cm² have separation 3 mm between A and B and 3 mm between B and C. The energy stored when the plates are fully charged is

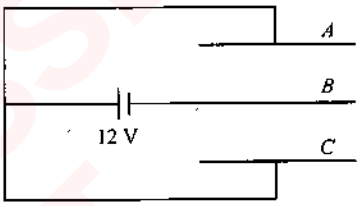


Fig. 4.86

a. 6×10^{-9} J b. 3.12×10^{-9} J
c. 2.12×10^{-9} J d. none of these

13. Four metallic plates, each with a surface area of one side A, are placed at a distance d from each other. The alternate plates are connected to points A and B as shown in Fig. 4.87. Then the capacitance of the system is

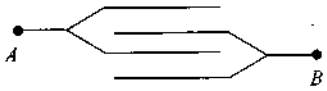


Fig. 4.87

a. $\frac{\varepsilon_0 A}{d}$ b. $\frac{2\varepsilon_0 A}{d}$ c. $\frac{3\varepsilon_0 A}{d}$ d. $\frac{4\varepsilon_0 A}{d}$

14. The capacitance of an infinite circuit formed by the repetition of the same link consisting of two identical capacitors, each with capacitance C (Fig. 4.88), is

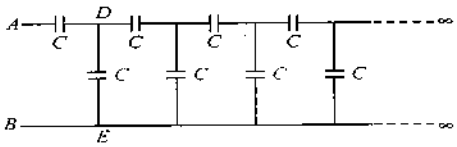


Fig. 4.88

a. zero b. $\frac{\sqrt{5}-1}{2}$ C c. $\frac{\sqrt{5}+1}{2}$ C d. infinite

15. Two parallel-plate capacitors, each of capacitance 40 μF, are connected in series. The space between the plates of one capacitor is filled with a dielectric of dielectric constant $K = 3$, then the equivalent capacitance of the combination is

a. 30 μF b. 120 μF c. 40 μF d. 160 μF

16. For making a parallel-plate capacitor you have two plates of copper, a sheet of mica (thickness = 0.10 mm, $K = 5.4$), a sheet of glass (thickness = 0.20 mm, $K = 7$) and a slab of paraffin (thickness = 1.0 cm, $K = 2$). To obtain the largest capacitance, which sheet should you place between the copper plates?

a. Mica b. Copper
c. Glass d. Information is insufficient

17. For configuration of media of permittivity ε_0, ε, ε_0 between parallel plates each of area A, as shown in Fig.4.89, the equivalent capacitance is

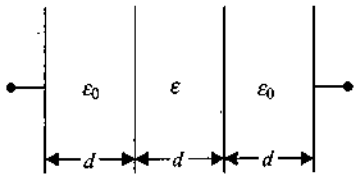


Fig. 4.89

a. $\varepsilon_0 A/d$ b. $\varepsilon\varepsilon_0 A/d$
c. $\dfrac{\varepsilon\varepsilon_0 A}{d(\varepsilon+\varepsilon_0)}$ d. $\dfrac{\varepsilon\varepsilon_0 A}{(2\varepsilon+\varepsilon_0)d}$

18. A parallel-plate capacitor is connected across a battery. Now, keeping the battery connected, a dielectric slab is inserted between the plates. In this process,
a. no work is done
b. work is done by the battery and the stored energy increases
c. work is done by the external agent and the stored energy decreases
d. work is done by the battery as well as external agent but the stored energy does not change

19. When a dielectric slab is introduced between the plates of an isolated charged capacitor, it
a. increases the capacitance of the capacitor
b. decreases the electric field between the plates
c. decreases the amount of energy stored in the capacitor
d. all of the above

20. Seven capacitors, each of capacitance 2 μF, are to be combined to obtain a capacitance of 10/11 μF. Which of the following combination is possible?
a. 2 in parallel, 5 in series
b. 3 in parallel, 4 in series
c. 4 in parallel, 3 in series
d. 5 in parallel, 2 in series

21. A spherical capacitor has an inner sphere of radius 12 cm and an outer sphere of radius 13 cm. The outer sphere is earthed and the inner sphere is given a charge of 2.5 μC. The space between the concentric spheres is filled with a liquid of dielectric constant 32. Determine the potential of the inner sphere.
a. 400 V b. 450 V c. 500 V d. 300 V

22. A parallel-plate capacitor has plates of area A and separation d and is charged to a potential difference V. The charging battery is then disconnected and the plates are pulled apart until their separation is $2d$. What is the work required to separate the plates?
a. $2\varepsilon_0 AV^2/d$ b. $\varepsilon_0 AV^2/d$
c. $3\varepsilon_0 AV^2/2d$ d. $\varepsilon_0 AV^2/2d$

23. A parallel-plate capacitor is charged and then disconnected from the source of potential difference. If the plates of the condenser are then moved farther apart by the use of insulated handle, which one of the following is true?
a. The charge on the capacitor increases
b. The charge on the capacitor decreases
c. The capacitance of the capacitor increases
d. The potential difference across the plates increases

24. For section AB of a circuit shown in Fig. 4.90, $C_1 = 1$ μF, $C_2 = 2$ μF, $E = 10$ V and the potential difference $V_A - V_B = -10$ V. Charge on capacitor C_1 is

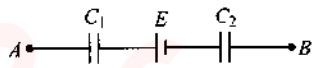


Fig. 4.90

a. 0 μC b. 20/3 μC
c. 40/3 μC d. none of these

25. A 600 pF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. What is the common potential (in V) and energy lost (in J) after reconnection?
a. 100, 6×10^{-6} b. 200, 6×10^{-5}
c. 200, 5×10^{-6} d. 100, 6×10^{-5}

26. Two parallel-plate capacitors of capacitances C and $2C$ are connected in parallel and charged to potential difference V. The battery is then disconnected and the region between the plates of C is filled completely with a material of dielectric constant K. The common potential difference across the combination becomes
a. $\dfrac{2V}{K+2}$ b. $\dfrac{V}{K+2}$
c. $\dfrac{3V}{K+3}$ d. $\dfrac{3V}{K+2}$

27. Three capacitors A, B and C are connected in a circuit as shown in Fig. 4.91. What is the charge in μC on the capacitor B ?

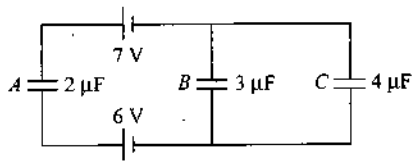


Fig. 4.91

a. 1/3 b. 2/3 c. 1 d. 4/3

28. A parallel-plate capacitor is made by stacking n equally spaced plates connected alternatively. If the capacitance between any two adjacent plates is C, then the resultant capacitance is
a. nC b. C c. $(n+1)C$ d. $(n-1)C$

29. Three capacitors are connected as shown in Fig. 4.92. Then, the charge on capacitor C_1 is

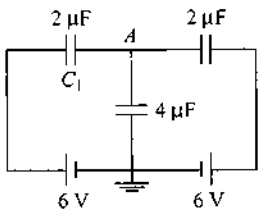


Fig. 4.92

a. 6 μC b. 12 μC c. 18 μC d. 24 μC

30. In the above question, the potential of point A is

a. 3 V b. 6 V c. 9 V d. zero

31. In Fig. 4.93, if the potential at point B is taken as zero, then the potential at point A will be

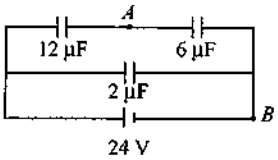


Fig. 4.93

a. 8 V b. 16 V
c. 24 V d. none of the above

32. A capacitor of capacitance $C_1 = 1$ μF charged up to a voltage $V = 110$ V is connected in parallel to the terminals of a circuit consisting of two uncharged capacitors connected in series and possessing capacitances $C_2 = 2$ μF and $C_3 = 3$ μF. Then, the amount of charge that will flow through the connecting wires is

a. 40 μC b. 50 μC c. 60 μC d. 110 μC

33. Ten capacitors are joined in parallel and charged with a battery up to a potential V. They are then disconnected from battery and joined in series. Then, the potential of this combination will be

a. 1 V b. 10 V c. 5 V d. 2 V

34. In Fig. 4.94, three capacitors C_1, C_2 and C_3 are joined to a battery. With symbols having their usual meaning, the correct conditions will be

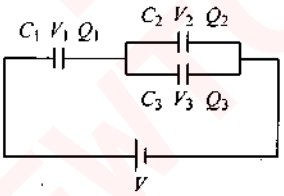


Fig. 4.94

a. $Q_1 = Q_2 = Q_3$ and $V_1 = V_2 = V_3 + V$
b. $Q_1 = Q_2 + Q_3$ and $V = V_1 + V_2 + V_3$
c. $Q_1 = Q_2 + Q_3$ and $V = V_1 + V_2$
d. $Q_2 = Q_3$ and $V_2 = V_3$

35. The cross section of a cable is shown in Fig. 4.95. The inner conductor has a radius of 10 mm and the dielectric has a thickness of 5 mm. The cable is 8 km long. Then, the capacitance of the cable is (given $\log_e 1.5 = 0.4$)

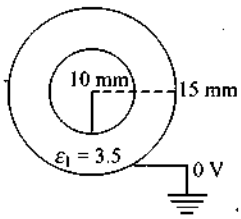


Fig. 4.95

a. 3.8 μF b. 1.1 μF
c. 4.8×10^{-10} F d. none of these

36. An uncharged parallel-plate capacitor having a dielectric of dielectric constant K is connected to a similar air cored parallel-plate capacitor charged to a potential V_0. The two share the charge and the common potential becomes V. The dielectric constant K is

a. $\frac{V_0}{V} - 1$ b. $\frac{V_0}{V} + 1$ c. $\frac{V}{V_0} - 1$ d. $\frac{V}{V_0} + 1$

37. Two identical parallel-plate capacitors are connected in series and then joined in series with a battery of 100 V. A slab of dielectric constant $K = 3$ is inserted between the plates of the first capacitor. Then, the potential difference across the capacitors will be, respectively,

a. 25 V, 75 V b. 75 V, 25 V
c. 20 V, 80 V d. 50 V, 50 V

38. In the given network of capacitors as shown in Fig. 4.96, given that $C_1 = C_2 = C_3 = 400$ pF and $C_4 = C_5 = C_6 = 200$ pF. The effective capacitance of the circuit between X and Y is

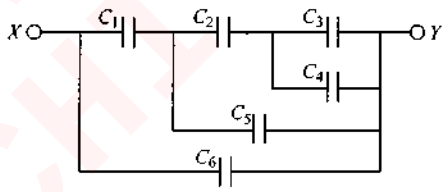


Fig. 4.96

a. 810 pF b. 205 pF c. 600 pF d. 410 pF

39. The work done in increasing the potential of a capacitor from V volt to $2V$ volt is W. Then, the work done in increasing the potential of the same capacitor from $2V$ volt to $4V$ volt will be

a. W b. $2W$ c. $4W$ d. $8W$

40. The plates of a parallel-plate capacitor have an area of 90 cm² each and are separated by 2 mm. The capacitor is charged by connecting it to a 400 V supply. Then the energy density of the energy stored (in Jm⁻³) in the capacitor is (Take $\varepsilon_0 = 8.8 \times 10^{-12}$ Fm⁻¹)

a. 0.113 b. 0.117
c. 0.152 d. none of these

41. Three identical capacitors, each of capacitance C, are connected in series with a battery of e.m.f. V and get fully charged. Now, the battery is removed and the capacitors are connected in parallel with positive terminals at one point and negative terminals at other point. Then, the common potential will be

a. V b. $3V$ c. $V/3$ d. zero

42. In Fig. 4.97, given $C_1 = 3$ μF, $C_2 = 5$ μF, $C_3 = 9$ μF and $C_4 = 13$ μF. What is the potential difference between points A and B?

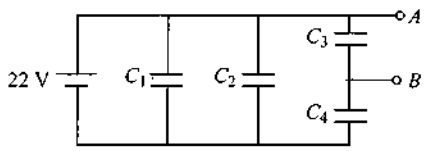


Fig. 4.97

a. 13 V b. 9 V c. 0 V d. 11 V

43. When a metal plate is introduced between the two plates of a charged capacitor and insulated from them, then
   a. the metal plate divides the capacitor into two capacitors connected in parallel to each other
   b. the metal plate divides the capacitor into two capacitors connected in series with each other
   c. the metal plate is equivalent to a dielectric of zero dielectric constant
   d. capacitance of the capacitor decreases

44. The potential gradient at which dielectric of the condenser just gets punctured is called
   a. dielectric constant
   b. dielectric strength
   c. dielectric resistance
   d. dielectric number

45. A parallel-plate capacitor is charged and then isolated. What is the effect of increasing the plate separation on charge, potential and capacitance, respectively?
   a. Constant, decreases, decreases
   b. Increases, decreases, decreases
   c. Constant, decreases, increases
   d. Constant, increases, decreases

46. The effective capacitance between points X and Y in Fig. 4.98, assuming $C_2 = 10\ \mu F$ and that outer capacitors are all 4 μF each, is

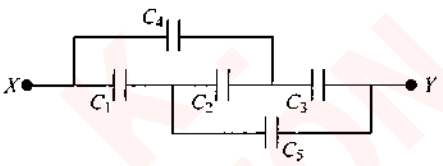


Fig. 4.98

a. 1 μF b. 3 μF c. 4 μF d. 5 μF

47. The resultant capacitance between the points A and B in Fig. 4.99 is

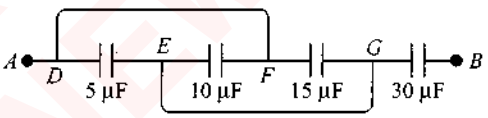


Fig. 4.99

a. 15 μF b. 30 μF c. 60 μF d. 45 μF

48. Two condensers C_1 and C_2 in a circuit are joined as shown in Fig. 4.100. The potential of point A is V_1 and that of B is V_2. The potential of point D will be

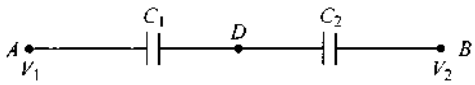


Fig. 4.100

a. $\frac{1}{2}(V_1 + V_2)$ b. $\frac{C_1V_2 + C_2V_1}{C_1 + C_2}$

c. $\frac{C_1V_1 + C_2V_2}{C_1 + C_2}$ d. $\frac{C_2V_1 - C_1V_2}{C_1 + C_2}$

49. A capacitor is charged to store an energy U. The charging battery is disconnected. An identical capacitor is now connected to the first capacitor in parallel. The energy in each of the capacitor is now
   a. $3U/2$ b. U c. $U/4$ d. $U/2$

50. Consider a parallel-plate capacitor of capacity 10 μF with air filled in the gap between the plates. Now, one half of the space between the plates is filled with a dielectric of dielectric constant 4 as shown in Fig. 4.101. The capacity of the capacitor changes to

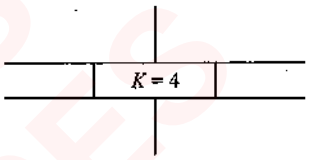


Fig. 4.101

a. 25 μF b. 20 μF c. 40 μF d. 5 μF

51. A 2 μF capacitor is charged to 100 V and then its plates are connected by a conducting wire. The heat produced is
   a. 0.001 J b. 0.01 J c. 0.1 J d. 1 J

52. In Fig. 4.102, the initial status of capacitance and their connection is shown. Which of the following is incorrect about this circuit?

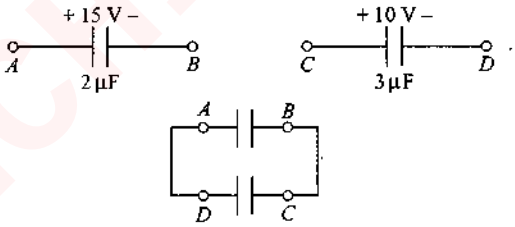


Fig. 4.102

   a. Final charge on each capacitor will be zero
   b. Final total electrical energy of the capacitance will be zero
   c. Total charge flown from A to D is 30 μC
   d. Total charge flown from A to D is –30 μC

53. A parallel-plate capacitor with no dielectric has a capacitance of 0.5 μF. The space between the plates is filled with equal amounts of two dielectric materials of dielectric constants 2 and 3 as shown in Fig. 4.103. Find the capacitance of the system now.

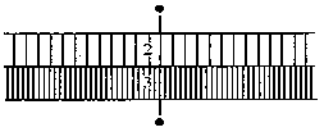


Fig. 4.103

a. 1.2 μF b. 1.8 μF
c. 1.25 μF d. none of these

54. Solve the above question if the dielectric materials were filled as shown in Fig. 4.104.

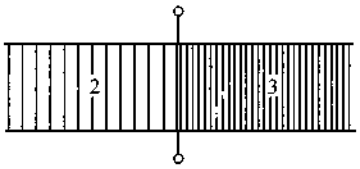


Fig. 4.104

a. 1.2 μF b. 1.25 μF
c. 1.80 μF d. none of these

55. Two metallic spheres of radii A and B are separated by a distance d as shown in Fig. 4.105. The capacity of the system is

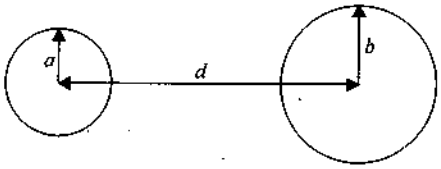


Fig. 4.105

a. $\dfrac{4\pi\varepsilon_0}{\frac{1}{a}+\frac{1}{b}-\frac{2}{d}}$ b. $\dfrac{4\pi\varepsilon_0}{\frac{1}{a}+\frac{1}{b}+\frac{2}{d}}$

c. $\dfrac{4\pi\varepsilon_0}{\frac{1}{a}-\frac{1}{b}+\frac{2}{d}}$ d. $\dfrac{4\pi\varepsilon_0}{\frac{1}{a}-\frac{1}{b}-\frac{2}{d}}$

56. A capacitor of capacitance C_0 is charged to a potential V_0 and then isolated. A small capacitor C is then charged from C_0, discharged and charged again; the process being repeated n times. Due to this, potential of the larger capacitor is decreased to V. The value of C is

a. $C_0\left(\frac{V_0}{V}\right)^{1/n}$ b. $C_0\left[\left(\frac{V_0}{V}\right)^{1/n}-1\right]$

c. $C_0\left[\left(\frac{V}{V_0}\right)-1\right]^n$ d. $C_0\left[\left(\frac{V}{V_0}\right)^n+1\right]$

57. The distance between plates of a parallel-plate capacitor is $5d$. The positively charged plate is at $x = 0$ and negatively charged plate is at $x = 5d$.

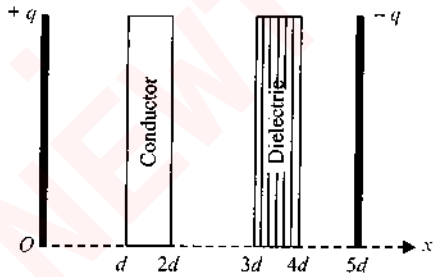
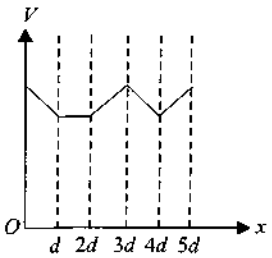
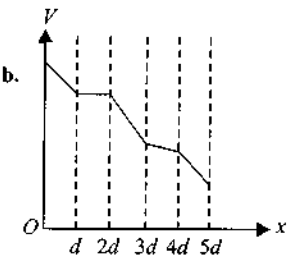
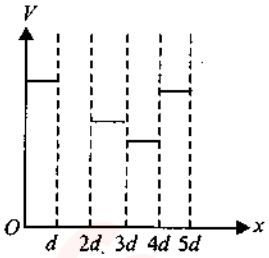


Fig. 4.106

Two slabs, one of conductor and the other of a dielectric of same thickness d, are inserted between the plates as shown in Fig. Potential V versus distance x graph will be

a.


b.


c.


d. None of these

58. In the circuit shown in Fig. 4.107, $C = 6$ μF. The charge stored in the capacitor of capacity C is

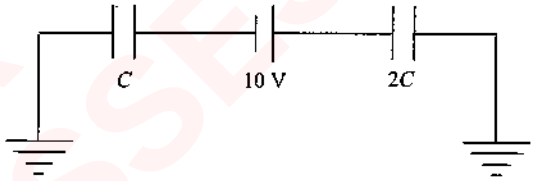


Fig. 4.107

a. zero b. 90 μC c. 40 μC d. 60 μC

59. One plate of a capacitor is fixed and the other is connected to a spring as shown in Fig. 4.108. Area of both the plates is A. In steady state (equilibrium), separation between the plates is $0.8d$ (spring was unstretched and the distance between the plates was d when the capacitor was uncharged). The force constant of the spring is approximately

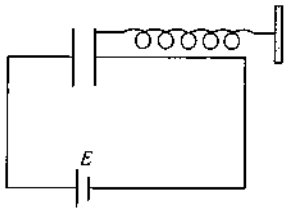


Fig. 4.108

a. $\frac{125}{32}\frac{\varepsilon_0 AE^2}{d^3}$ b. $\frac{2\varepsilon_0 AE^2}{d^3}$ c. $\frac{6\varepsilon_0 E^2}{Ad^2}$ d. $\frac{\varepsilon_0 AE^3}{2d^3}$

60. A dielectric slab of area A and thickness d is inserted between the plates of a capacitor of area $2A$ with constant speed υ as shown in Fig. 4.109. Distance between the plates is d.

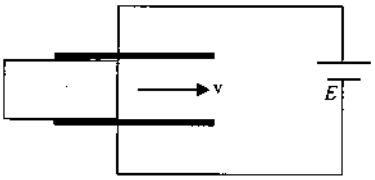
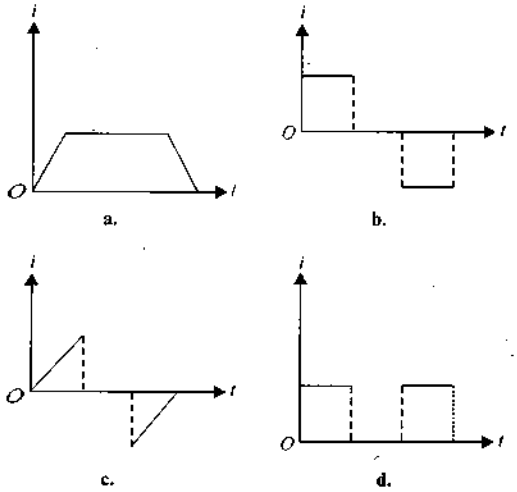


Fig. 4.109

The capacitor is connected to a battery of e.m.f. E. The current in the circuit varies with time as



61. A photographic flash unit consists of a xenon-filled tube. It gives a flash of average power 2000 W for 0.04 s. The flash is due to discharge of a fully charged capacitor of 40 μF. The voltage to which it is charged before a flash is given by the unit is

a. 1500 V **b.** 2000 V **c.** 2500 V **d.** 3000 V

62. A parallel-plate capacitor is constructed using three different dielectric materials as shown in Fig. 4.110. What is the capacitance across P and Q?

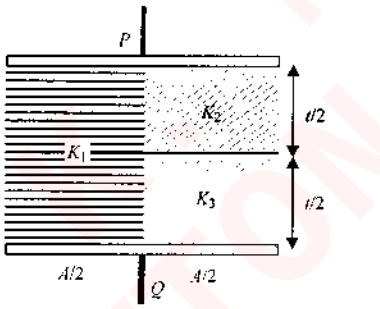


Fig. 4.110

a. $\left(\frac{K_1}{2}+\frac{K_2K_3}{K_2+K_3}\right)\frac{\varepsilon_0 A}{t}$ **b.** $\left(K_1+\frac{K_2K_3}{K_2+K_3}\right)\frac{\varepsilon_0 A}{t}$

c. $\left(K_1+\frac{2K_2K_3}{K_2+K_3}\right)\frac{\varepsilon_0 A}{t}$ **d.** $\left(K_1+\frac{K_2K_3}{2(K_2+K_3)}\frac{\varepsilon_0 A}{t}\right)$

63. Two square plates ($l \times l$) and dielectric $\left(\frac{l}{2}\times\frac{t}{2}\times l\right)$ are arranged as shown in Fig. 4.111. Find the equivalent capacitance of the structure.

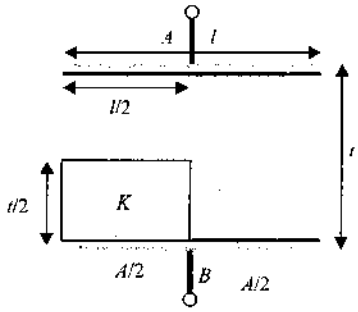


Fig. 4.111

a. $\frac{\varepsilon_0 A}{t}\left(\frac{3K+1}{2(K+1)}\right)$ **b.** $\frac{2\varepsilon_0 A}{t}\left(\frac{K+1}{K+3}\right)$

c. $\frac{\varepsilon_0 A}{t}\left(\frac{K+1}{K+3}\right)$ **d.** $\frac{\varepsilon_0 A}{t}\left(\frac{2K+1}{2K+3}\right)$

64. The equivalent capacitance across AB (Fig. 4.112) is

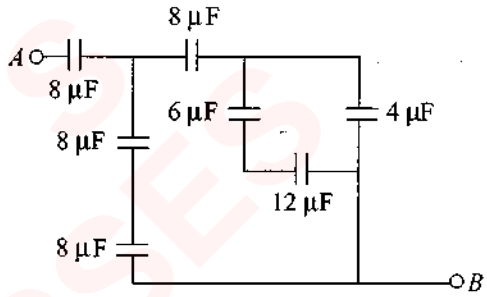


Fig. 4.112

a. 8 μF **b.** 12 μF **c.** 4 μF **d.** 24 μF

65. Find the capacitance between P and O (Fig. 4.113). Each capacitor has capacitance C.

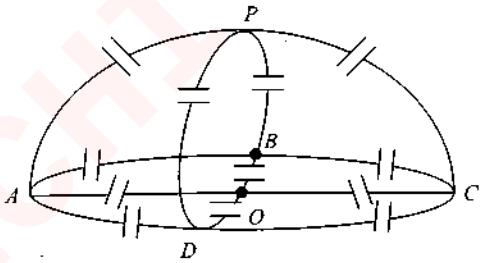


Fig. 4.113

a. $2C$ **b.** $3C$ **c.** $8C$ **d.** $6C$

66. In the circuit shown in Fig. 4.114, $C_1 = 6$ μF, $C_2 = 3$ μF and battery $B = 20$ V. The switch S_1 is first closed. It is then opened and afterwards S_2 is closed. What is the final charge on C_2?

a. 120 μC **b.** 80 μC
c. 40 μC **d.** 20 μC

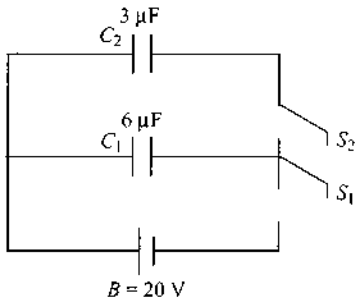


Fig. 4.114

Multiple Correct Answers Type

Solutions on page 4.44

1. To two plates of a parallel-plate capacitors, charges Q_1 and Q_2 are given. The capacity of the capacitor is C. When the switch is closed, mark the correct statement(s). [Assume both Q_1, Q_2 to be +ve]

a. The charge flown through switch is zero
b. The charge flown through switch is $Q_1 + Q_2$

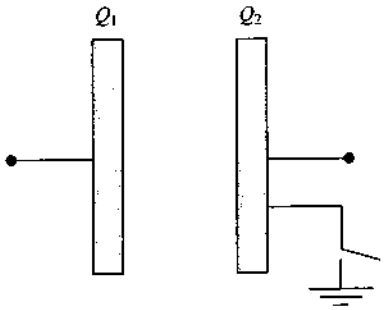


Fig. 4.115

c. Potential difference across the capacitor plate is Q_1/C
d. The charge of the capacitor is Q_1

2. A dielectric slab fills the lower half region of parallel-plate capacitor as shown in Fig. 4.116. [Take plate area as A]

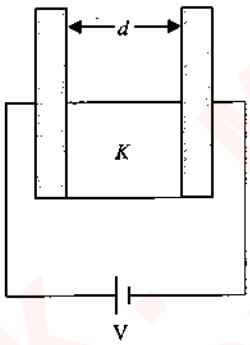


Fig. 4.116

a. Equivalent capacity of the system is $\frac{\varepsilon_0 A}{2d}(1+K)$
b. The net charge of lower half of the left hand plate is $1/K$ times the charge on upper half of the plate
c. Net charges on lower and upper halves of left hand plate are different
d. Net charge on lower half of left hand plate is $\frac{K\varepsilon_0 A}{2d} \times V$

3. A parallel-plate air capacitor has initial capacitance C. If plate separation is slowly increased from d_1 to d_2, then mark the correct statement(s). [Take potential of the capacitor to be constant, i.e., throughout the process it remains connected to battery.]
a. Work done by electric force = – work done by external agent.
b. Work done by external force = $-\int \vec{F}\cdot\vec{dx}$, where $\vec{F}$ is the electric force of attraction between the plates at plate separation x.
c. Work done by electric force ≠ – ve of work done by external agent.
d. Work done by battery = 2 times the change in electric potential energy stored in capacitor.

4. A capacitor of 5 μF is charged to a potential of 100 V. Now, this charged capacitor is connected to a battery of 100 V with positive terminal of battery connected to negative plate of the capacitor. For the given situation, mark the correct statement(s).
a. The charge flown through 100 V battery is 500 μC.
b. The charge flown through 100 V battery is 1000 μC.
c. Heat dissipated in the circuit is 0.1 J.
d. Work done on the battery is 0.1 J.

5. Two identical capacitors with identical dielectric slabs in between them are connected in series as shown in Fig. 4.117. Now, the slab of one capacitor is pulled out slowly with the help of an external force F at steady state as shown. Mark out the correct statement(s).

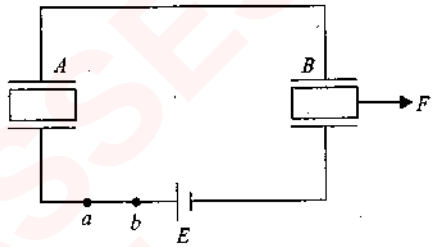


Fig. 4.117

a. During the process, charge (positive) flows from b to a.
b. During the process, charge of capacitor B is equal to charge on A at all instants.
c. Work done by F is positive.
d. During the process, the battery has been charged.

Assertion-Reasoning Type

Solutions on page 4.44

In the following questions, each question contains Statement I (Assertion) and Statement II (Reason). Each question has four choices a., b., c. and d. out of which **ONLY ONE** is correct.

a. Statement I is True, Statement II is True; Statement II is a correct explanation for Statement I.
b. Statement I is True, Statement II is True; Statement II is **NOT** a correct explanation for Statement I.
c. Statement I is True, Statement II is False.
d. Statement I is False, Statement II is True.

1. **Statement I:** A capacitor can be given only a limited quantity of charge.
Statement II: Charge stored by a capacitor depends upon shape and size of the plates of capacitor and the surrounding medium.
2. **Statement I:** Capacity of a parallel-plate capacitor increases when distance between the plates is decreased.
Statement II: Capacitance of a capacitor is directly proportional to distance between them.
3. **Statement I:** The capacity of a conductor, under given circumstances, remains constant irrespective of the charge present on it.
Statement II: Capacity depends on size and shape of conductor and also on the medium.
4. **Statement I:** A charged plane parallel-plate capacitor has half interplanar region (I) filled with dielectric slab. The other half region II has air.

Then, the magnitude of net electric field in region I is less than that in region II.

Statement II: In a dielectric medium, induced (or polarized) charges tend to reduce the electric field.

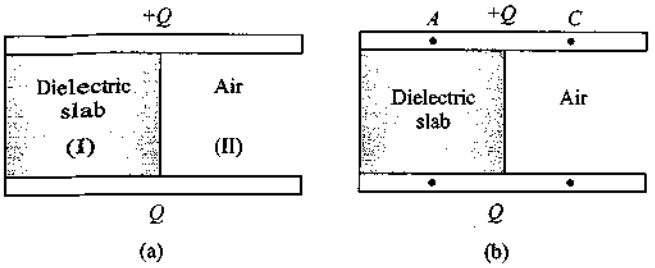


Fig. 4.118

5. **Statement I:** A dielectric is inserted between the plates of an isolated fully charged capacitor. The dielectric completely fills the space between the plates. The magnitude of electrostatic force on either metal plate decreases, as it was before the insertion of dielectric medium.

Statement II: Due to insertion of dielectric slab in an isolated parallel-plate capacitor (the dielectric completely fills the space between the plates), the electrostatic potential energy of the capacitor decreases.

Comprehension Type

Solutions on page 4.45

For Problems 1–2

An inflated balloon is covered with a conducting surface that carries charge q. The balloon develops a leak and the radius starts to decrease, but no charge is lost from surface.

1. How does the capacitance of the balloon change as the balloon leaks?
   a. C increases
   b. C decreases
   c. C remains the same
   d. There is not enough information to answer the question
2. How does the stored electrical energy change as the balloon leaks?
   a. U increases
   b. U decreases
   c. U remains the same
   d. There is not enough information to answer the question

For Problems 3–4

Figure 4.119 shows two capacitors in series, the rigid centre conducting section of length b being movable vertically.

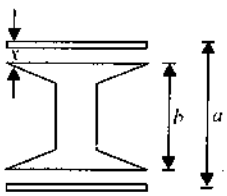


Fig. 4.119

3. The equivalent capacitance of given structure is
   a. $\frac{\varepsilon_0 A}{(a-b-x)}$ b. $\frac{\varepsilon_0 A}{(a-b)}$ c. $\frac{\varepsilon_0 A}{(b-x)}$ d. $\frac{\varepsilon_0 A}{(a-x)}$
4. If potentials of upper and lower plates are V_1 and V_2, respectively, then the potential of the rigid section is
   a. $V_1 - \frac{(V_1-V_2)x}{(a-b)}$ b. $V_1 - \frac{(V_2-V_1)x}{(a-b)}$
   c. $V_1 - \frac{(V_1-V_2)x}{(a+b)}$ d. $V_1 - \frac{(V_2-V_1)x}{(a+b)}$

For Problems 5–6

A 1 μF and a 2 μF capacitor are connected in series across a 1200 V supply.

5. The charged capacitors are disconnected from the line and from each other, and are now reconnected with terminals of like sign together. Find the final charge on each capacitor and voltage across each capacitor.
   a. Charge on capacitors: $\left(\frac{1400}{3}\right)$ μC and $\left(\frac{3200}{3}\right)$ μC; potential difference across each capacitor: $\left(\frac{1600}{3}\right)$ V
   b. Charge on capacitors: $\left(\frac{1600}{3}\right)$ μC and $\left(\frac{3200}{3}\right)$ μC; potential difference across each capacitor: $\left(\frac{1600}{3}\right)$ V
   c. Charge on each capacitor is 1600 μC and potential difference across each capacitor is 800 V
   d. Charge and potential difference across each capacitor are zero
6. If the charged capacitors are reconnected with terminals of opposite sign together, find the final charge and voltage across each capacitor.
   a. Charge on capacitors: $\left(\frac{1400}{3}\right)$ μC and $\left(\frac{3200}{3}\right)$ μC, potential difference across each capacitor: $\left(\frac{1600}{3}\right)$ V
   b. Charge on capacitors: $\left(\frac{1600}{3}\right)$ μC and $\left(\frac{3200}{3}\right)$ μC; potential difference across each capacitor: $\left(\frac{1600}{3}\right)$ V
   c. Charge on each capacitor is 1600 μC and potential difference across each capacitor is 800 V
   d. Charge and potential difference across each capacitor are zero

For Problems 7–9

For the arrangement shown in Fig. 4.120, when the switch S is closed, then find

7. the final charge on 6 μF capacitor is
   a. 12 μC b. 24 μC c. 32 μC d. 48 μC
8. the final potential difference across the 4 μF capacitor is
   a. 12 V b. 8 V c. 20 V d. 32 V

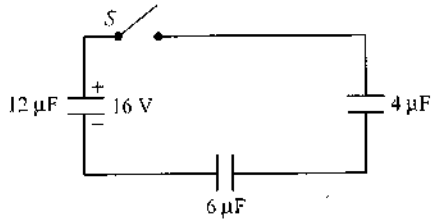


Fig. 4.120

9. the final potential difference across the 12 μF capacitor is

a. $\frac{40}{3}$ V **b.** $\frac{20}{3}$ V **c.** 12 V **d.** 24 V

For Problems 10–11

Each plate of a parallel-plate air capacitor has area $S = 5 \times 10^{-3}$ m² and distance between the plates $d = 8.80$ mm. Plate A has positive charge $q_1 = +10^{-10}$ C and plate B has charge $q_2 = +2 \times 10^{-10}$ C. A battery of e.m.f. $E = 10$ V has its positive terminal connected to plate A and negative terminal to plate B. (Given $\varepsilon_0 = 8.8 \times 10^{-12}$ Nm² C⁻²)

10. Charge supplied by the battery is

a. 120 pC **b.** 100 pC **c.** 60 pC **b.** 50 pC

11. Energy supplied by the battery is

a. 10^{-9} J **b.** 5×10^{-9} J **c.** 50×10^{-9} J **d.** 25×10^{-9} J

For Problems 12–13

Consider Fig. 4.121. Each capacitor has capacitance C.

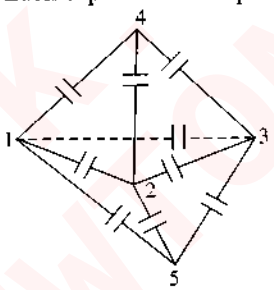


Fig. 4.121

12. The equivalent capacitance between 4 and 5 is

a. $\left(\frac{3C}{4}\right)$ **b.** $\left(\frac{3C}{2}\right)$ **c.** $\left(\frac{3C}{5}\right)$ **d.** $\left(\frac{5C}{4}\right)$

13. The capacitance between 1 and 3 is

a. $\left(\frac{3C}{4}\right)$ **b.** $\left(\frac{3C}{2}\right)$ **c.** $\left(\frac{5C}{2}\right)$ **d.** $\left(\frac{5C}{4}\right)$

For Problems 14–16

Consider Fig. 4.122.

14. The charge appearing on C_2 is

a. $E\left(\frac{C_3C_4}{C_1+C_2}\right)$ **b.** $E\left(\frac{C_1C_2}{C_1+C_2}\right)$

c. $E\left(\frac{C_1C_2}{C_3+C_4}\right)$ **d.** $E\left(\frac{C_3C_4}{C_3+C_4}\right)$

15. The potential difference $V_A - V_B$ is

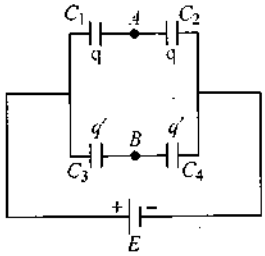


Fig. 4.122

a. $E\left[\frac{C_1C_4 - C_2C_3}{(C_1+C_2)(C_3+C_4)}\right]$ **b.** $E\left[\frac{C_1C_4 + C_2C_3}{(C_1+C_3)(C_2+C_4)}\right]$

c. $E\left[\frac{C_1C_3 - C_2C_4}{(C_1+C_2)(C_3+C_4)}\right]$ **d.** $E\left[\frac{C_1C_3 - C_2C_4}{(C_1+C_3)(C_2+C_4)}\right]$

16. The condition for which the potential difference between A and B is zero is

a. $C_1 C_2 = C_3 C_4$ **b.** $C_1 C_4 = C_2 C_3$

c. $C_1 C_3 = C_2 C_4$ **d.** none of these

For Problems 17–18

C_1, C_2, C_3 and C_4 are four capacitors connected to a battery of constant e.m.f. equal to 12 V, as shown in Fig. 4.123. Given $C_1 = 1$ μF, $C_2 = 2$ μF, $C_3 = 3$ μF and $C_4 = 4$ μF.

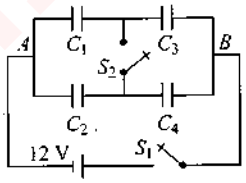


Fig. 4.123

17. Find the charge on C_1 when only S_1 is closed.

a. 16 μC **b.** 9 μC **c.** 6 μC **d.** 8 μC

18. If the switch S_2 is also closed. Match the table.

a. (m, q) (n, r) (o, s) (p, t) **b.** (m, t) (n, s) (o, r) (p, q)

c. (m, t) (n, q) (o, s) (p, r) **d.** (m, r) (n, s) (o, t) (p, q)

Capacitor	Charge
(m) C_1	(q) $\left(\frac{84}{5}\right)$ μC
(n) C_2	(r) $\left(\frac{72}{5}\right)$ μC
(o) C_3	(s) $\left(\frac{54}{5}\right)$ μC
(p) C_4	(t) $\left(\frac{42}{5}\right)$ μC

For Problems 19–21

In Fig. 4.124, each capacitance C_1 is 6.0 μF, and each capacitance C_2 is 4.0 μF.

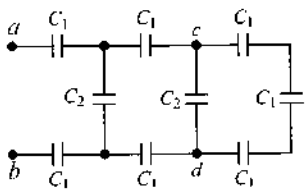


Fig. 4.124

19. The equivalent capacitance of the network between points a and b is

a. 2 μF **b.** 4 μF **c.** 6 μF **d.** 8 μF

20. The charge on C_1 nearest to a when V_{ab} =420 V is

a. 840 μC **b.** 560 μC **c.** 600 μC **d.** 320 μC

21. With 420 V across a and b, the value of $(V_c - V_d)$ is

a. 24.6 V **b.** 46.7 V **c.** 18 V **d.** 72 V

For Problems 22–24

Consider Fig. 4.125.

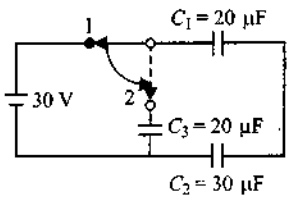


Fig. 4.125

22. In the circuit shown in the figure, the switch can be shifted to positions '1' and '2'. The charge on capacitor C_1 when the switch is at position '1' is

a. 120 μC **b.** 240 μC **c.** 360 μC **d.** 80 μC

23. Now, the switch is shifted to position '2'. The charge appearing on capacitor C_3 is

a. 225 μC **b.** 135 μC **c.** 270 μC **d.** 75 μC

24. The charge on capacitor C_1 is

a. 225 μC **b.** 135 μC **c.** 270 μC **d.** 360 μC

For Problems 25–26

For the system shown in Fig. 4.126, capacitance is C. Left plate is given a charge Q_1 and right plate is uncharged. Now, switch is closed.

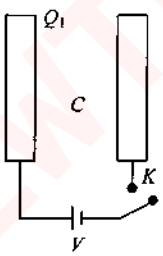


Fig. 4.126

25. Find the amount of charge that will flow through the battery before the steady state is achieved.

a. CV **b.** $CV - Q_1$

c. $CV + \frac{Q_1}{2}$ **d.** $CV - \frac{Q_1}{2}$

26. Find the charge appearing on the inner face of the left plate.

a. $CV - \frac{Q_1}{2}$ **b.** $CV + Q_1$

c. $CV + \frac{Q_1}{2}$ **d.** CV

For Problems 27–28

Consider the circuit shown in Fig. 4.127, after switch S is closed.

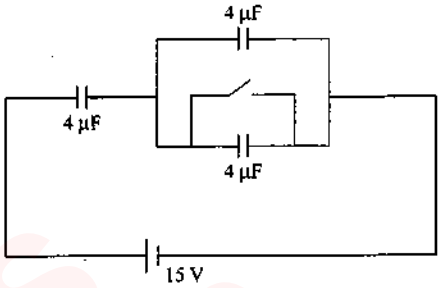


Fig. 4.127

27. What amount of charge will flow through the battery?

a. 20 μC **b.** 60 μC

c. 40 μC **d.** No charge will flow

28. What amount of charge will flow through the switch?

a. 20 μC **b.** 60 μC

c. 40 μC **d.** No charge will flow

Matching Column Type

Solutions on page 4.48

1. Two identical capacitors are connected in series and the combination is connected with a battery, as shown. Some changes in capacitor 1 are now made independently after the steady state is achieved as listed in column I. Some effects which may occur in new steady state due to these changes on the capacitor 2 are listed in column II. Match the changes on capacitor 1 in column I with corresponding effect on capacitor 2 in column II.

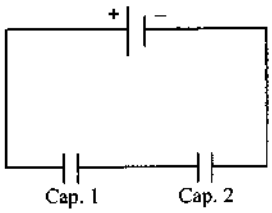


Fig. 4.128

Column I	Column II
i. A dielectric slab is inserted	a. Charge on the capacitor increases
ii. Separation between plates increased	b. Charge on the capacitor decreases
iii. A metal plate is inserted connecting both plates	c. Energy stored in the capacitor increases
iv. The left plate is grounded	d. No change occurrs

2. Observe the circuit in Fig. 4.129 and match the following (assume q_1, q_2 and q_3 be the charges on three capacitors).

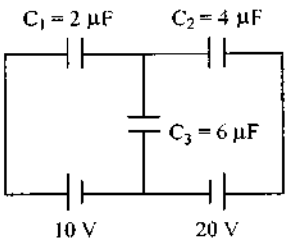


Fig. 4.129

Column I	Column II
i. q_1 (in μC)	a. 50
ii. q_2(in μC)	b. $\frac{10}{3}$
iii. q_3(in μC)	c. $\frac{140}{3}$
iv. Potential difference across 6 μF capacitor is (in volt)	d. $\frac{25}{3}$

3. Five identical capacitor place, each of area A, are arranged that adjacent plates are at d distance apart. Plates are connected to a source of e.m.f. V as shown in Fig. 4.130. Match the following:

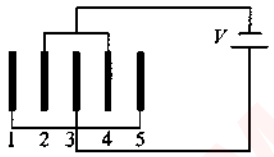


Fig. 4.130

Column I	Column II
i. Charge on plate 1	a. $-2\varepsilon_0 AV/d$
ii. Charge on plate 4	b. $+\varepsilon_0 AV/d$
iii. Potential difference between plates 2and 3	c. zero
iv. Potential difference between plates 1 and 5	d. V

Integer Answer Type

Solutions on page 4.48

1. Each capacitance shown in the figure is in μF. Find the charge on 6 μF in μC.

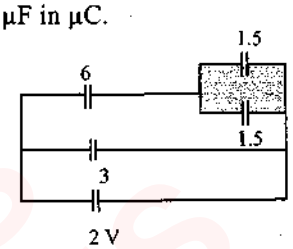


Fig. 4.131

2. In the figure, a potential of +12 V is given to point A and point B is earthed, what is the potential at the point P in V?

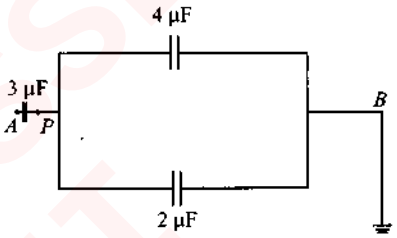


Fig. 4.132

3. Capacitance of a capacitor becomes 4/3 times its original value if a dielectric slab of thickness $t = d/2$ is inserted between the plates (d = separation between the plates). What is the dielectric constant of the slab?
4. A spherical drop of capacitance 12 μF is broken into eight drops of equal radius. Then, what is the capacitance of each small drop in μF?
5. A parallel-plate capacitor of capacity C_0 is charged to a potential V_0. (i) The energy stored in the capacitor when the battery is disconnected and the plate separation is doubled is E_1. (ii) The energy stored in the capacitor when the charging battery is kept connected and the separation between the capacitor plates is doubled is E_2. Find the ratio E_1/E_2.

ANSWERS AND SOLUTIONS

Subjective Type

1. a.

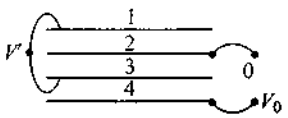
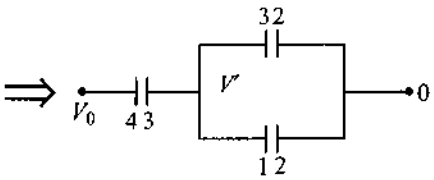


Fig. 4.133

Equivalent circuit



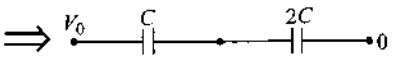


Fig. 4.134

$$C_{eq} = \frac{2C}{3} = \frac{2\varepsilon_0 A}{3d}$$

b.

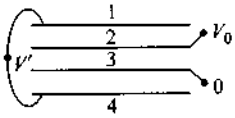


Fig. 4.135

Equivalent circuit

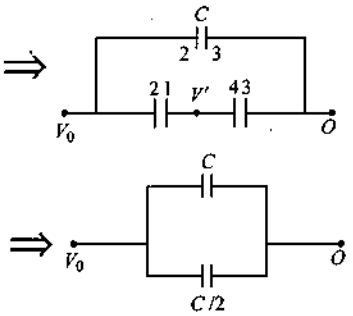


Fig. 4.136

$$C_{eq} = \frac{3}{2}C = \frac{3\varepsilon_0 A}{2d}$$

c.

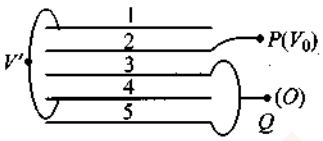


Fig. 4.137

Equivalent circuit

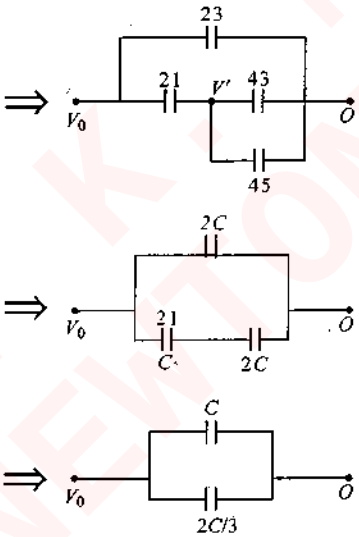


Fig. 4.138

d.

$$C_{eq} = \frac{5C}{2} = \frac{5}{3}\frac{\varepsilon_0 A}{d}$$

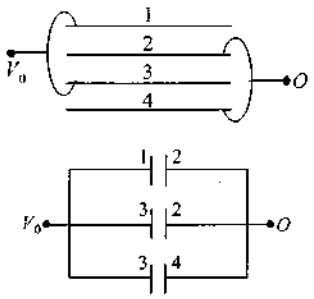


Fig. 4.139

$$C_{eq} = 3C = \frac{3\varepsilon_0 A}{d}$$

2. a. Refer to Fig. 4.140.

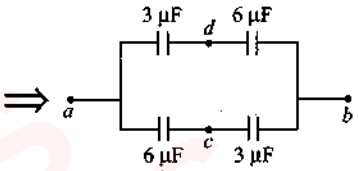


Fig. 4.140

$$V_a - V_d = \frac{6}{6+3} \times 360 = 240 \text{ V}$$

and $V_d - V_c = \frac{3}{6+3} \times 360 = 120 \text{ V}$

$\therefore \quad V_c - V_d = V_{cd} = 120 \text{ V}$

b. Refer to Fig. 4.141.

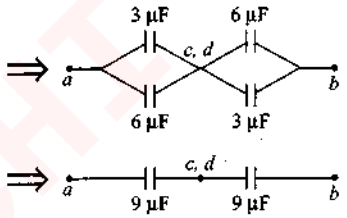


Fig. 4.141

$$V_a - V_c = V_a - V_d = V_d - V_b = V_c - V_b$$

$$= \frac{360}{2} = 180 \text{ V}$$

i.e., the potential across each capacitor is 180 V

c. Charge flown after the switch is closed:

$$\Delta q_1 = 1080 - 720 = 360 \ \mu\text{C}$$

$$\Delta q_2 = 720 - 540 = 180 \ \mu\text{C}$$

Now,

$$\Delta q = \Delta q_1 + \Delta q_2 = 360 + 180 = 540 \ \mu\text{C}$$

Hence, charge of 540 μC flows from c to d after closing the switch.

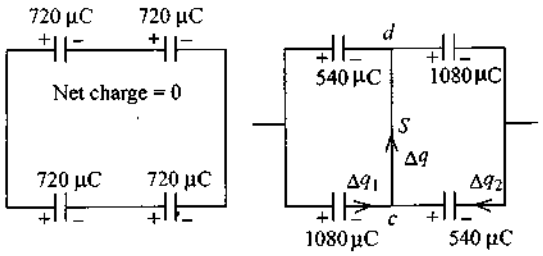


Fig. 4.142

3.

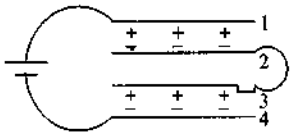


Fig. 4.143

Plates can be rearranged as shown in Fig. 4.144.
Plates 1 and 2 and plates 3 and 4 form two capacitors which are in series between A and B. Plates 2 and 3 do not form any capacitor as they are at the same potential.

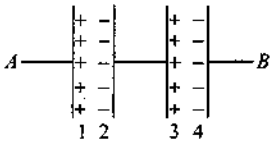


Fig. 4.144

$$\text{So, } C_{eq} = \frac{C_1C_2}{C_1+C_2} = \frac{\left(\frac{\varepsilon_0 A}{d}\right)\times\left(\frac{\varepsilon_0 A}{3d}\right)}{\frac{\varepsilon_0 A}{d}+\frac{\varepsilon_0 A}{3d}} = \frac{\varepsilon_0}{4d}$$

4. Let a charge q flow in clockwise direction after closing the switch.

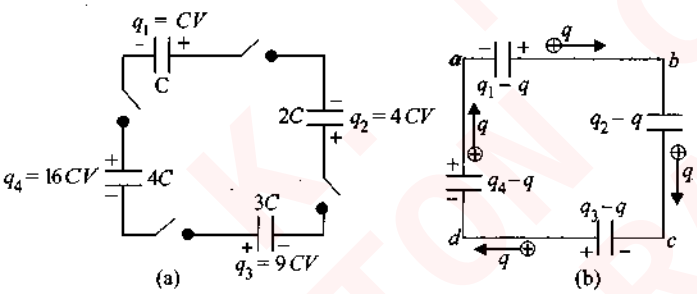


Fig. 4.145

Applying loop law in *abcda*, we get

$$\frac{q_1-q}{C}+\frac{q_2-q}{2C}+\frac{q_3-q}{3C}+\frac{q_4-q}{4C}=0$$

Substituting values of q_1, q_2, q_3 and q_4, we get

$$q = \frac{24}{5}CV$$

$$|\Delta V|_C = \left|\frac{q_1-q}{C}\right| = \frac{19}{5}\text{V}$$

$$|\Delta V|_{2C} = \left|\frac{q_2-q}{2C}\right| = \frac{2}{5}\text{V}$$

$$|\Delta V|_{3C} = \left|\frac{q_3-q}{3C}\right| = \frac{7}{5}\text{V}$$

and $$|\Delta V|_{4C} = \left|\frac{q_4-q}{4C}\right| = \frac{14}{5}\text{V}$$

5. For Fig. 4.146(a) $C_{eq} = \dfrac{(C+C_0)C}{2C+C_0}$ and $Q = C_{eq}E$

$$\therefore \quad Q_1 = \frac{C_0Q}{C+C_0} \text{ and } Q_2 = \frac{CQ}{C+C_0}$$

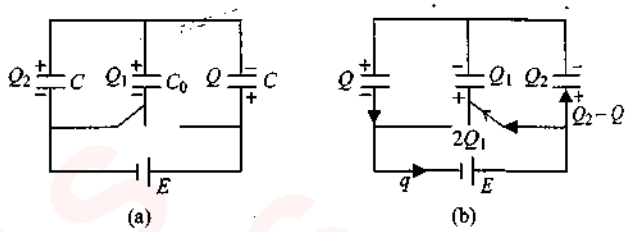


Fig. 4.146

Charge flown through battery on reconnection, as in Fig. 4.146(b)

$$q = Q - Q_2 = Q - \frac{CQ}{C+C_0} = \frac{C_0Q}{C+C_0} = \frac{C_0E}{C+C_0}C_{eq}$$

Total energy on capacitors before and after reconnection is the same. Hence, whole work done by battery will go in the form of heat. So, heat generated is

$$W_b = qE = \frac{C_0E^2}{C+C_0}\times\frac{(C+C_0)C}{(2C+C_0)} = \frac{CC_0E^2}{2C+C_0}$$

Alternatively: Heat $= \sum\dfrac{(\Delta Q)^2}{2C} = 2\left[\dfrac{(Q-Q_2)^2}{2C}\right]$

$$+\frac{(2Q_1)^2}{2C_0}$$

(Charge flown through the switch is $2Q_1$)

$$= \frac{C_0^2Q^2}{(C+C_0)^2C}+\frac{2C_0^2Q^2}{C_0[C+C_0]^2} = \frac{C_0^2Q^2}{(C+C_0)^2}\left[\frac{1}{C}+\frac{2}{C_0}\right]$$

$$= \frac{E^2CC_0}{2(C+C_0)}$$

6. Vertically cross-connected capacitors in the diagram are ineffective as they all are short-circuited. So, circuits can be redrawn as

Hence, $\dfrac{1}{C_{eq}} = \dfrac{1}{4C}+\dfrac{1}{3C}+\dfrac{1}{2C}+\dfrac{1}{C}$

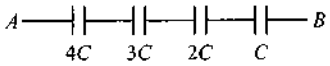


Fig. 4.147

or $C_{eq} = \dfrac{12}{25}C = \dfrac{12}{25}\times 25\,\mu\text{F} = 12\,\mu\text{F}$

7. Maximum charge C_1 can hold

$$Q_1 = C_1V_1 = 1\times10^{-6}\times6\times10^3 = 6\times10^{-3}\text{ C}$$

and maximum charge C_2 can hold

$$Q_2 = C_2V_2 = 2\times10^{-6}\times4\times10^3 = 8\times10^{-3}\text{ C}$$

When connected in series, both will have equal charges and so each can have charge Q_1 which is smaller of the two.
In this case:
Voltage across $C_1 = 6$ kV

And voltage across

$$C_2 = \frac{Q_1}{C_2} = \frac{6\times10^{-3}}{2\times10^{-6}} = 3\text{ kV}$$

Therefore, the maximum voltage across the system = 6 + 3 = 9 kV

8. When switch is opened, 2 and 3μF capaitors are in series.

So, $$C_{eq} = \frac{2\times3}{5} = \frac{6}{5}\ \mu\text{F}$$

Hence, charge on each capacitor is

$$q = CV = \frac{6}{5}\times 90 = 108\ \mu\text{C}$$

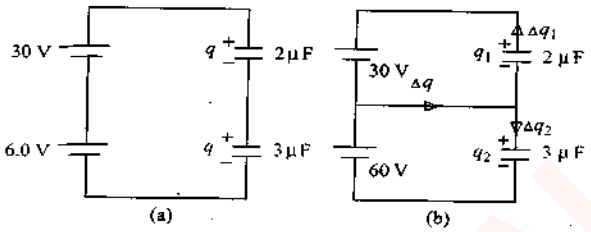


Fig. 4.148

When switch S is closed, let q_1 and q_2 be charges on the two capacitors as in Fig 4.148 (b). So,

$$q_1 = 2\times30 = 60\ \mu\text{C}$$
$$q_2 = 3\times60 = 180\ \mu\text{C}$$

Let charge q_B goes to the upper plate of 3μF capacitor and lower plate of 2μF capacitor. Initially, both the plates have charge $+q - q = 0$

Finally, they have charges $q_2 - q_1$. So,

$$q_2 - q_1 = q_B + 0$$

or $$q_B = q_2 - q_1 = 180 - 60 = 120\ \mu\text{C}$$

Alternatively: After closing the switch, charge flown out of 2 μF capacitor:

$$\Delta q_1 = 108 - 60 = 48\ \mu\text{C}$$

Charge flown into 3 μF capacitor:

$$\Delta q_2 = 180 - 108 = 72\ \mu\text{C}$$

So the net charge flown through the switch:

$$\Delta q = \Delta q_1 + \Delta q_2 = 48 + 72 = 120\ \mu\text{C}$$

which is same as q_B obtained earlier.

9. Let E_0 be the electric field due to charges on plates (not due to induced charges). Given $E_0 = 6.4\times10^6$ V/m

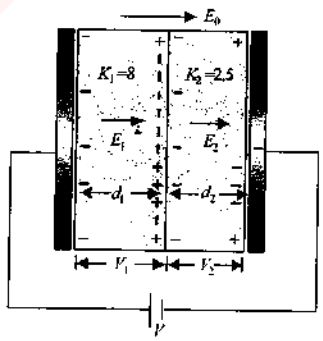


Fig. 4.149

$$V = V_1 + V_2 = E_1d_1 + E_2d_2 \quad\Rightarrow\quad V = \frac{E_0}{K_1}d_1 + \frac{E_0}{K_2}d_2$$

$$\Rightarrow V = \frac{6.4\times10^6\times10^{-3}}{8} + \frac{6.4\times10^6\times0.5\times10^{-3}}{2.5}$$

$$= 2.08\times10^3\text{ V} = 2.08\text{ kV}$$

10. Let $C_1 = C$ and $C_2 = 2C$ and charges on different capacitors have been shown. Net charge on isolated system should be zero. Hence,

$$q_1 - q_2 - q_3 = 0 \qquad \text{(i)}$$

For left loop, $E - \frac{q_3}{2C} - \frac{q_1}{C} = 0$ (ii)

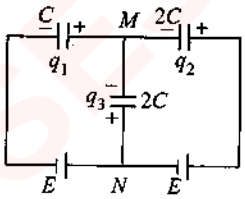


Fig. 4.150

$$E - \frac{q_3}{2C} - \left(\frac{q_2+q_3}{C}\right) = 0 \quad (\text{using } q_1 = q_2 + q_3)$$

or $$E - \frac{q_3}{2C} - \frac{q_2}{C} - \frac{q_3}{C} = 0$$

$$E - \frac{q_2}{C} - \frac{3}{2}\frac{q_3}{C} = 0 \qquad \text{(iii)}$$

For right loop, $E - \frac{q_2}{2C} - \frac{q_3}{2C} = 0$

$$\Rightarrow \quad 2E - \frac{q_2}{C} + \frac{q_3}{C} = 0 \qquad \text{(iv)}$$

Solving Eqs. (iii) and (iv), we get

$$q_3 = \frac{-2CE}{5}$$

So $$V_{MN} = \frac{-q_3}{2C} = \frac{2CE}{5\times2C} = \frac{E}{5} = \frac{110}{5} = 22\text{ V}$$

So $$V_{MN} = 22\text{ V}$$

11. Charge across 5 μF capacitor = 10 μC.

Therefore, potential difference across 5 μF capacitor is

$$= \frac{q}{C} = \frac{10\times10^{-6}}{5\times10^{-6}} = 2\text{ V}$$

As 3, 4 and 5 μF capacitors are in parallel, so potential difference across each of the them equals 2 V.

Charge on 3 μF capacitor = $CV = 3\times10^{-6}\times2 = 6\ \mu\text{C}$

Charge on 4 μF capacitor = $4\times10^{-6}\times2 = 8\ \mu\text{C}$

Total charge flowing in upper branch of circuit is

$$10\ \mu\text{C} + 6\ \mu\text{C} + 8\ \mu\text{C} = 24\mu\text{C}$$

Potential difference across $C_2 = \frac{24\ \mu\text{C}}{4\ \mu\text{F}} = 6\text{ V}$

Total potential difference across $AB = 6 + 2 = 8$ V

Equivalent capacitance of lower branch of circuit is

$$\frac{6\times3}{6+3}=2\,\mu F$$

So, charge flowing $= 2\times10^{-6}\times8 = 16\,\mu C$

Therefore, potential difference between A and C is

$$\frac{16\,\mu C}{3\,\mu F}=\frac{16}{3}=5.33\text{ V}$$

12. The distribution of charge is shown in Fig. 4.151 in compliance with the point rule. Applying loop rules,

Left loop: $-\frac{q_2}{5}+\frac{q_3}{0.75}+\frac{q_1}{15}=0$

$\Rightarrow$ $q_1-3q_2+20\,q_3=0$ (i)

Middle loop: $-\frac{q_2+q_3}{15}-\frac{q_3}{0.75}+\frac{q_1-q_3}{5}-\frac{q_3}{0.75}=0$

$\Rightarrow$ $3q_1-q_2-44\,q_3=0$ (ii)

Outermost loop: $23-\frac{q_2}{5}-\frac{q_2+q_3}{15}-\frac{q_2}{5}=0$

$\Rightarrow$ $345=7q_2+q_3$ (iii)

Solving for q_1, q_2 and q_3, we get

$$q_1=\frac{19\times345}{92},\ q_2=\frac{13\times345}{92},\ q_3=\frac{345}{92}$$

Therefore, P.D. between A and $B=\frac{q_3}{0.75}=\frac{345}{92}\times\frac{4}{3}=5$ V

P.D. between E and F is also 5 V but in the opposite direction.

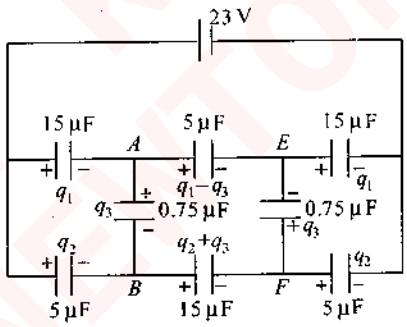


Fig. 4.151

13. $Q=Q_1+Q_2$

Loop $ACDA$: $Q_1=Q_2+Q_3$

Loop $DCED$: $Q_3+Q_1-Q_2+2Q_3=Q_2-Q_3$

$\Rightarrow$ $Q_1-2Q_2+4Q_3=0$

$\Rightarrow$ $Q_1-2Q_2+4(Q_1-Q_2)=0$

$\Rightarrow$ $5Q_1=6Q_2 \Rightarrow 6Q=11Q_1$

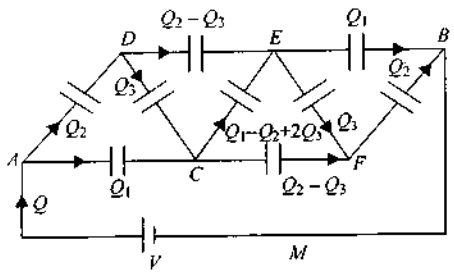


Fig. 4.152

Loop $ADEBMA$:

$CV=Q_2+Q_2-Q_3+Q_1 \Rightarrow CV=Q+Q_2-Q_3$

$CV=Q+Q_2-(Q_1-Q_2)$

$=Q-Q_1+2Q_2$

$=Q-Q_1+2(Q-Q_1)$

$=3Q-3Q_1$

$=3Q-3\times\frac{6Q}{11}=\frac{15}{11}Q$

$\Rightarrow$ $C_{eq}=\frac{Q}{V}=\frac{11}{15}C$

$=\frac{11}{15}\times30=22\,\mu F$

Objective Type

1. c. Charge resides only on the outer surfaces.

2. b. $C'=\frac{\varepsilon_0 A}{d-t}=\frac{\varepsilon_0 A}{d-d/2}=\frac{2\varepsilon_0 A}{d}$

Hence, capacitance is doubled.

3. d. Let their potentials be the same.

$V_1=V_2 \Rightarrow \frac{kQ_1}{R_1}=\frac{kQ_2}{R_2}\Rightarrow Q_1R_2=Q_2R_1$

If potentials are the same, then no flow of charge will occur, otherwise charge will flow and there will be loss of energy.

4. b. Potential on 5 μF capacitor is

$$\frac{3\times6}{3+(2+5)}=1.8\text{ V}$$

So, charge on this capacitor $=CV=5\times1.8=9\,\mu C$

5. d. C_3, C_5 and C_6 are in parallel and C_4 is in series with it. Then C_2 is in parallel and C_1 is in series.

6. a. In parallel, potential is same on each capacitor.

7. d. In series, charge is the same on each capacitor.

8. b. Plate 2 and 3 will be at the same potential. So, there will be two capacitors in parallel.

9. b. Minimum number of capacitors is 4.

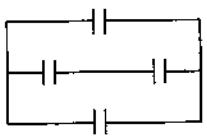


Fig. 4.153

10. d. $C_{eq}=C+2C+3C+\cdots+nC$

$=\frac{(n+1)n}{2}C$

11. a. As battery is disconnected, so charge will remain the same. It is given that final potential is the same. So, final capacitance should be equal to initial capacitance

$$\frac{\varepsilon_0 A}{d}=\frac{\varepsilon_0 A}{(1.6+d)-t(1-1/K)}$$

$\Rightarrow$ $K=5$

12. c. There are two capacitors in parallel.

$$U=\frac{1}{2}CV^2,\ C=\frac{2\varepsilon_0 A}{d}.$$

13. c. The four plates are alternately connected. They form three capacitors in parallel. Capacity of each capacitor is

$\varepsilon_0 A/d$. So, the net capacity is $3\varepsilon_0 A/d$.

14. b. Let capacitance between A and B be x. If we remove the first two capacitors, then capacitance across D and E will also be x. So the circuit can be redrawn as follows:

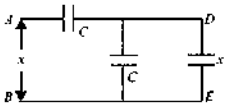


Fig. 4.154

$$x = \frac{C(C+x)}{C+(C+x)}, \text{ Solve to get } x = \frac{\sqrt{5}-1}{2}C$$

15. a. $C_1 = 40\ \mu F$, $C_2 = K \times 40 = 3 \times 40 = 120\ \mu F$

$$C_{eq} = \frac{C_1C_2}{C_1+C_2} = \frac{40\times 120}{40+120} = 30\ \mu F$$

16. a. $C = \frac{K\varepsilon_0 A}{d}$, find K/d for each sheet. Capacitance will be largest for which K/d is largest.

17. d. $C_{eq} = \dfrac{\varepsilon_0}{\frac{d}{K_1}+\frac{d}{K_2}+\frac{d}{K_3}}$

Here, $K_1 = K_3 = 1$, $K_2 = \varepsilon/\varepsilon_0$

18. b. Extra charge will flow through battery, so work is done by battery. External agent will do negative work.

19. d. Here, battery is disconnected, so charge remains the same.

20. d. Check each option separately.

21. b. $V - 0 = \frac{Q}{C} = \frac{Q(b-a)}{K4\pi\varepsilon_0 ab}$

$$V = \frac{9\times 10^9 \times 2.5\times 10^{-6}(0.13-0.12)}{32\times 0.13\times 0.12}$$

$= 450$ V

22. d. $W = U_2 - U_1 = \frac{q^2}{2}\left[\frac{1}{C_2}-\frac{1}{C_1}\right]$

$$C_1 = \frac{\varepsilon_0 A}{d}, C_2 = \frac{C_1}{2} = \frac{\varepsilon_0 A}{2d}$$

$$q = C_1V = \frac{\varepsilon_0 AV}{d}$$

Solve to get, $W = \frac{1}{2}\frac{\varepsilon_0 AV^2}{d}$

Alternatively: $W = Fd = \frac{Q^2}{2A\varepsilon_0}d = \frac{C_1^2V^2}{2\varepsilon_0 A}d = \frac{1}{2}\frac{\varepsilon_0 AV^2}{d}$

23. d. $V = Ed$. As d increases, V also increases. Note that E remains the same.

24. c. $V_A - \frac{q}{C_1} - E - \frac{q}{C_2} = V_B \Rightarrow V_A - V_B - E = q\left(\frac{1}{C_1}+\frac{1}{C_2}\right)$

$\Rightarrow \quad q = -\frac{40}{3}\mu C$

25. a. $V_C = \frac{C_1V_1}{C_1+C_2} = 100$ V

Energy lost $= U_i - U_f$

$$= \frac{1}{2}C_1V_1^2 - \frac{1}{2}(C_1+C_2)V_c^2$$

$$= 6\times 10^{-6}\text{ J}$$

26. d. $V_C = \frac{Q_1+Q_2}{C_1+C_2} = \frac{CV+2CV}{KC+2C} = \frac{3V}{K+2}$

27. b. Capacitors B and C are in parallel, then A is in series.

$$C_{eq} = \frac{2\times(3+4)}{2+(3+4)} = \frac{14}{9}\mu F$$

$$Q = C_{eq}V = \frac{14}{9}(7-6) = \frac{14}{9}\mu C$$

Q will be divided between B and C. So, charge on B is

$$q = \frac{3\times(14/9)}{3+4} = \frac{2}{3}\mu C$$

28. d. There will be $n-1$ capacitors, all connected in parallel.

29. a. $6 = \frac{q}{2}+\frac{2q}{4} \Rightarrow q = 6\ \mu C$

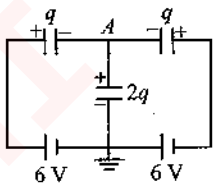


Fig. 4.155

30. a. $V_A - 0 = \frac{2q}{4} = 3$ V

31. b. Potential across 6 μF capacitor:

$$V_A - V_B = \frac{12\times 24}{12+6} = 16$$

$\Rightarrow \quad V_A - 0 = 16$

$\Rightarrow \quad V_A = 16$ V

32. c. Initial charge on C_1: $Q_1 = C_1V = 110\ \mu C$

Let x charge flow through wires $\frac{Q_1 - x}{C_1} = \frac{x}{C_{eq}}$

where $C_{eq} = \frac{C_2\times C_3}{C_2+C_3}$. Solve to get $x = 60\ \mu C$.

33. b. In series, all the potentials will be added.

34. c. At junction A, Q_1 will divide into Q_2 and Q_3. Hence, $Q_1 = Q_2 + Q_3$.

C_2 and C_3 are in parallel, so potential on them will be the same, i.e. $V_2 = V_3$.

V will divide into V_1 and V_2 (or V_3)

Hence, $V = V_1 + V_2$ or $V = V_1 + V_3$

35. a. $C = \frac{K\varepsilon_0 2\pi l}{\ln(b/a)}$

$$= \frac{3.5\times 2\pi\times 8.85\times 10^{-12}\times 8\times 10^3}{\ln(15/10)}$$

$= 3.84 \times 10^{-6}$ F

36. a. $C_1 = C$, $V_1 = V_0$, $C_2 = KC$, $V_2 = 0$ and $V_{common} = V$

We know that $V_{common} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$

$$V = \frac{CV_0 + KC \times O}{C + KC}$$

$$K = \frac{V_0}{V} - 1$$

37. a. $V_1 = \frac{C \times 100}{3C + C} = 25$ V, $V_2 = 100 - 25 = 75$ V

38. d. Start with C_3 and C_4 in parallel, then C_2 in series, then C_5 in parallel, then C_1 in series and finally C_6 in parallel.

39. c. $W = \frac{1}{2}C[(2V)^2 - V^2] = \frac{3}{2}CV^2$

$$W' = \frac{1}{2}C[(2V)^2 - (2V)^2] = 6CV^2$$

$$= \frac{6 \times 2W}{3} = 4W$$

40. b. $A = 90$ cm², $d = 2$ mm

$$E = \frac{V}{d}, V = 400V$$

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2$$

$$= \frac{1}{2}(8.85 \times 10^{-12})\left(\frac{400}{2 \times 10^{-3}}\right)^2 = 0.177 \text{ Jm}^{-3}$$

41. c. In series, potential is divided.

42. a. 22 V get will divide into series combination of C_3 and C_4.

43. b. Knowledge based.

44. b. Dielectric strength, it is the maximum electric field which a material can bear.

45. d. As the capacitor is isolated, so charge will remain the same. Now, as the separation between the plates is increased, capacitance ($\varepsilon_0 A/d$) will decrease.

$V = \frac{Q}{C}$, if C decreases, V increases.

46. c. Wheatstone bridge will be formed.

47. a. 5 μF, 10 μF, 15 μF will be in parallel, then 30 μF will be in series.

48. c. $V_1 - V_2$ will be divided between C_1 and C_2 in series.

Potential difference across C_1:

$$V_1 - V_D = \frac{C_2(V_1 - V_2)}{C_1 + C_2}$$

$$V_D = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

49. c. $U = \frac{1}{2}CV^2$

Potential of each capacitor now = $V/2$

$$U' = \frac{1}{2}C\left(\frac{V}{2}\right)^2 = \frac{U}{4}$$

50. a. Given $\frac{\varepsilon_0 A}{d} = 10$

$$C' = \frac{\varepsilon_0 A/2}{d} + \frac{4\varepsilon_0(A/2)}{d} = 10\left[\frac{1}{2} + 2\right] = 25\ \mu\text{F}$$

51. b. The whole amount of energy stored in capacitor will convert into heat.

Heat $= \frac{1}{2}CV^2 = \frac{1}{2} \times 2 \times 10^{-6} \times (100)^2 = 0.01$ J

52. d. $V = \frac{Q_1 + Q_2}{C_1 + C_2} = 0$, where $Q_1 = 30\ \mu$C, $Q_2 = -30\ \mu$C

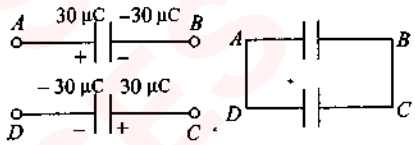


Fig. 4.156

Final potential difference = zero

Final energy = zero

Charge flow 30 μC from A to d.

53. a. $C = \frac{\varepsilon_0 A}{d} = 0.5$

New capacitance

$$C' = \frac{\varepsilon_0 A}{\frac{d/2}{2} + \frac{d/2}{3}} = \frac{\varepsilon_0 A}{d}\left[\frac{1}{\frac{1}{4} + \frac{1}{6}}\right]$$

$$= C \times 2.4 = 1.2\ \mu\text{F}$$

54. b. $C' = \frac{2\varepsilon_0 A/2}{d} + \frac{3\varepsilon_0 A/2}{d}$

$$C\left[1 + \frac{3}{2}\right] = \frac{5}{2} \times 0.5 = 1.25\ \mu\text{F}$$

55. a. $V_1 = \frac{kQ}{a} - \frac{kQ}{d}, V_2 = \frac{-kQ}{b} + \frac{kQ}{d}$

$$C = \frac{Q}{V_1 - V_2} = \frac{4\pi\varepsilon_0}{\frac{1}{a} + \frac{1}{b} - \frac{2}{d}}$$

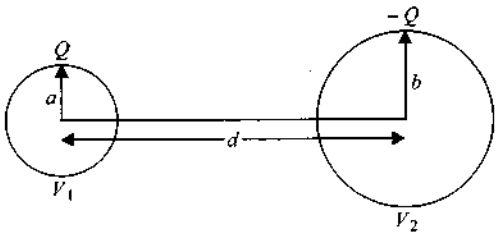


Fig. 4.157

56. b. Potential of larger capacitor after the first charging,

$$V_1 = \frac{C_0V_0}{(C + C_0)}$$

After the second charging

$$V_2 = \frac{C_0V_1}{(C + C_0)} \Rightarrow V_2 = \left(\frac{C_0}{C + C_0}\right)^2 V_0$$

After the nth charging,

$$V_n = \left(\frac{C_0}{C + C_0}\right)^n V_0, \text{ but } V_n = V$$

$$\Rightarrow \qquad C = C_0\left[\left(\frac{V_0}{V}\right)^{1/n} - 1\right]$$

57. b. Electric field is the –ve of slope of $V - x$ graph. Inside the conductor, electric field is zero, so slope of $V - x$ graph is zero. Inside dielectric, field decreases, so slope also decreases.

58. c. $0 - \frac{Q}{C} + 10 - \frac{Q}{2C} = 0$

$$Q = \frac{20\,C}{3} = \frac{20 \times 6}{3} = 40\ \mu C$$

59. a. $F = Kx$

$$\frac{Q^2}{2A\varepsilon_0} = K(0.2d)$$

$$\frac{\left(\frac{\varepsilon_0 A}{0.8d}E\right)^2}{2A\varepsilon_0} = 0.2\,Kd$$

$$K = 3.9\ \varepsilon_0 AE^2/d^3$$

$$K = 4\varepsilon_0 AE^2/d^3$$

60. b. $C = \frac{K\varepsilon_0 vtb}{d} + \frac{\varepsilon_0(l - vt)b}{d}$; $q = CV$

$$I = \frac{dq}{dt} = V\frac{dC}{dt} = V\left[\frac{K\varepsilon_0 bv}{d} - \frac{\varepsilon_0 bv}{d}\right] = \text{constant}$$

So, till the dielectric is fully inserted, a constant current will flow. When the dielectric is fully inside, C will be constant, so $I = 0$. And when dielectric starts coming out, again a constant current will flow but in opposite direction.

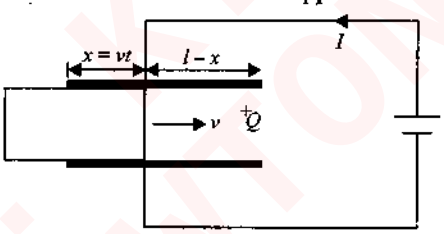


Fig. 4.158

61. b. $2000 \times 0.04 = \frac{1}{2} \times 40 \times 10^{-6} V^2$

$$V^2 = 4 \times 10^6 \Rightarrow V = 2000\ \text{V}$$

62. a.

$$C_1 = \frac{K_1\varepsilon_0 A/2}{t}$$

$$C_2 = \frac{K_2\varepsilon_0 A/2}{t/2}$$

$$C_3 = \frac{K_3\varepsilon_0 A/2}{t/2}$$

Now, $C_{eq} = C_1 + \frac{C_2C_3}{C_2 + C_3}$

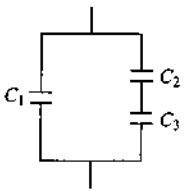


Fig. 4.159

63. a. We can make equivalent circuit of given system in two ways as in Fig. 4.160 (a) and (b). But (b) is wrong and (a) is correct.

Here $C_1 = \frac{\varepsilon_0 A/2}{t/2} = \frac{\varepsilon_0 A}{t}$

$$C_2 = \frac{K\varepsilon_0 A/2}{t/2} = \frac{K\varepsilon_0 A}{t} \text{ and } C_3 = \frac{\varepsilon_0 A/2}{t} = \frac{\varepsilon_0 A}{2t},$$

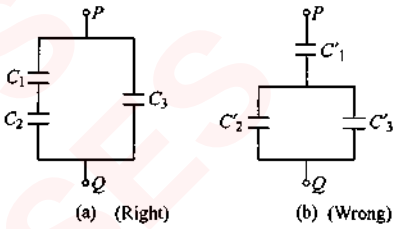
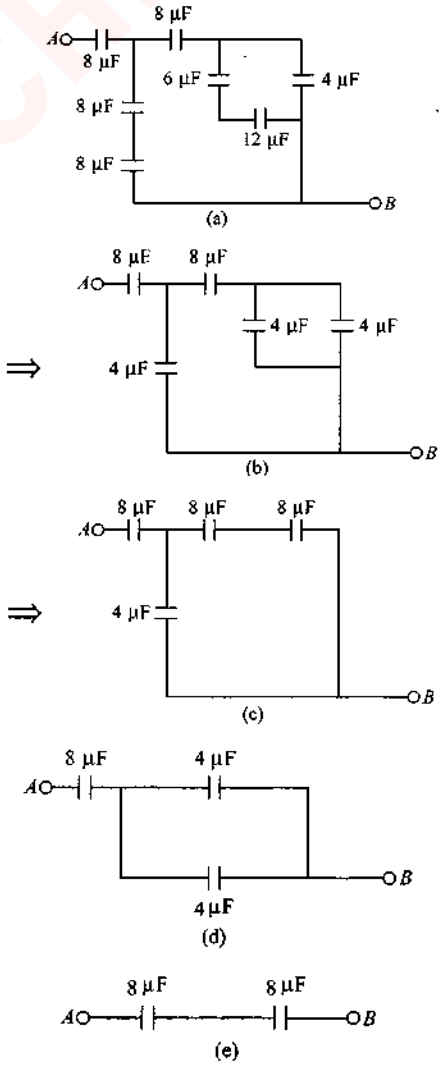


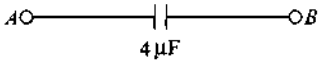
Fig. 4.160

Now, $C_{eq} = \frac{C_1C_2}{C_1 + C_2} + C_3$

$$= \frac{\varepsilon_0 A}{t}\left[\frac{3K+1}{2(K+1)}\right]$$

64. c. Using the method of successive reduction (see Fig. 4.161).





(f)

Fig. 4.161

65. a. A, B, C and D are equipotential points (see Fig.4.162)

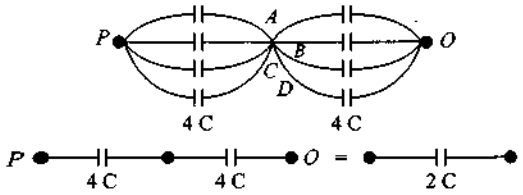


Fig. 4.162

66. c. After closing S_1, charge on C_1: $q = 6 \times 20 = 120\ \mu C$. Now, S_1 is opened. On closing S_2, charge q will be distributed between C_1 and C_2 according to their capacitances.

So, charge on C_2: $q_2 = \dfrac{C_2 q}{C_1 + C_2} = \dfrac{3 \times 120}{3+6} = 40\ \mu C$

Multiple Correct Answers Type

1. b., c., d. As the switch is closed, the charge on the outer surface of the second plate becomes zero. From the concept in electrostatics that electric field inside the bulk of the material of conductor is zero, we can find the charges on various faces.

So, it is clear that $Q_1 + Q_2$ charge goes from the second plate to the earth. Charge on capacitor is Q_1 and hence its potential is Q_1/C.

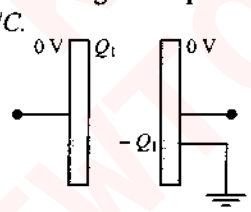


Fig. 4.163

2. a., c., d. This system can be considered as two capacitors in parallel, with $C_1 = \dfrac{\varepsilon_0 A}{2d}$ and $C_2 = \dfrac{K\varepsilon_0 A}{2d}$.

C_1 is due to upper half of two plates while C_2 is due to lower half.

As potential difference across two capacitors is the same, charge would be different as capacitors are different.

3. a., b., d. $C = \dfrac{\varepsilon_0 A}{d_1}, C' = \dfrac{\varepsilon_0 A}{d_2}$

Extra charge flown $= Q' - Q = (C' - C)V$

$$= \varepsilon_0 A V\left[\frac{1}{d_2} - \frac{1}{d_1}\right]$$

Work done by battery:

$$W_b = V \times \text{charge flown} = \varepsilon_0 A V^2\left[\frac{1}{d_2} - \frac{1}{d_1}\right]$$

Change in P.E. of capacitor $= \Delta U = \dfrac{1}{2}(C' - C)V^2$

$$= \frac{1}{2}\varepsilon_0 A V^2\left[\frac{1}{d_2} - \frac{1}{d_1}\right]$$

4. b., c. Initial condition just after the connection of battery:

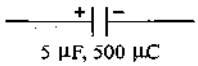


Fig. 4.164

Condition after a long time:

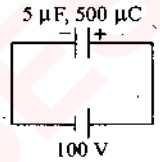


Fig. 4.165

It means battery supplies 1000 μC charge from its positive terminal and an equal and opposite charge enters from its negative terminal, i.e., charge flow through battery is 1000 μC.

Work done by the battery, $W_{battery} = 100\ V \times 10^3\ \mu C = 0.1\ J$

From energy conservation law, $U_i + W_{battery} = U_f + \Delta H$

$U_i = U_f$ so $\Delta H = 0.1$ J.

5. b., c., d. As the dielectric slab is pulled out, the equivalent capacity of the system decreases and hence charge supplied by the battery decreases as potential of the system remains constant. It means charging of battery takes place and a positive charge flows from a to b. As the two capacitors are connected in series, so charge on both capacitors remains the same at all instants.

From energy conservation law,

$U_i + W_{ext} = U_f$ + work done on battery + ΔH

As dielectric slab is attracted by the plates of capacitors, to pull it out F has to perform some work, i.e., $W_{ext}(F) > 0$.

Assertion-Reasoning Type

1. a. The maximum amount of charge given to a capacitor depends upon the geometrical factors.

2. c. Capacitance is inversely proportional to the distance between the plates.

3. a. Capacitance of a capacitor does not depend upon charge but it depends upon the geometrical factors.

4. d. Let the electric field in regions I and II be E_1 and E_2, respectively. The potential difference across left half capacitor and half capacitor is the same. Therefore, $E_1 d = E_2 d$, where d = inner planar gap.

Hence, statement 1 is false, statement 2 is correct by definition.

$\therefore \quad E_1 = E_2$

5. d. The electrostatic force on metal of capacitor is

$$\text{Pressure} \times \text{Area of plate} = \frac{\sigma^2}{2\varepsilon_0}A$$

Since charge on the metal plate of an "isolated" capacitor does not change, force on metal plate remains the same.

Electric field decreases due to induced charge in dielectric, but this does not affect the charge distribution on isolated metal plate.

Capacitance increases, Q remains the same.

$$U = \frac{Q^2}{2C}$$

Hence, U decreases.

Comprehension Type

For Problems 1–2

1. b., 2. a.

Sol. As radius of the balloon decreases, so capacitance also decreases.

But charge remains the same, so energy increases.

For Problems 3–4

3. b., 4. a.

Sol. The circuit is equivalent to two capacitors in series.

$$C_1 = \frac{\varepsilon_0 A}{x}$$

$$C_2 = \frac{\varepsilon_0 A}{(a-b-x)}$$

Fig. 4.166

$$\frac{1}{C_{eq}} = \frac{1}{\varepsilon_0 A}[a-b-x+x] = \frac{(a-b)}{\varepsilon_0 A}$$

$$= \frac{\varepsilon_0 A}{(a-b)} \text{ independent of } x.$$

We can reduce this structure in the following circuit (Fig.4.167).

$$(V_1 - V) = (V_1 - V_2)\frac{C_2}{C_1 + C_2}$$

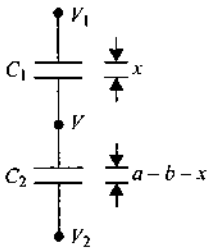


Fig. 4.167

$$V_1 - V = (V_1 - V_2)\frac{\dfrac{\varepsilon_0 A}{(a-b-x)}}{\dfrac{\varepsilon_0 A}{x} + \dfrac{\varepsilon_0 A}{a-b-x}}$$

$$= (V_1 - V_2)\frac{x}{(a-b)}$$

$$-(V_1 - V_2)\frac{x}{(a-b)}$$

For Problems 5–6

5. b., 6. d.

Sol.

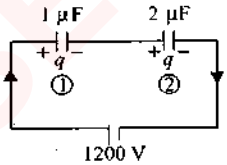


Fig. 4.168

$$1200 - \frac{q}{1} - \frac{q}{2} = 0 \Rightarrow q = 800\ \mu C$$

Charge on each capacitor = 800 μC

$$V_1 = \frac{q}{C_1} = \frac{800}{1} = 800\text{ V}$$

$$V_2 = \frac{q}{C_2} = \frac{800}{2} = 400\text{ V}$$

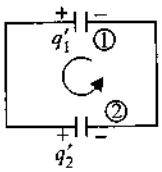


Fig. 4.169

(i) $q_1' + q_2' = 1600$ (i)

Also, $\dfrac{q_1'}{1} - \dfrac{q_2'}{2} = 0$

$\Rightarrow \quad q_2' = 2q_1'$

From Eq. (i), $3q_1' = 1600 \Rightarrow q_1' = \dfrac{1600}{3}\ \mu C$

$$3q_2' = \frac{3200}{3}\ \mu C$$

$$V = \frac{q_1 + q_2}{C_1 + C_2} = \frac{1600}{3}\text{ V}$$

(ii) $q_1' + q_2' = 800 - 800 = 0$ (ii)

Also, $\dfrac{q_1'}{1} - \dfrac{q_2'}{2} = 0$

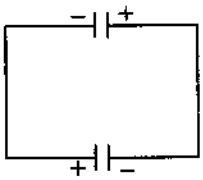


Fig. 4.170

From Eq. (ii), $q_1' = q_2' = 0$.

Therefore, potential difference across each capacitor is zero.

For Problems 7–9

7. c., 8. b., 9. a.

Sol.

In the close loop,

$$\frac{q_0 - \Delta q}{C_1} - \frac{\Delta q}{C_2} - \frac{\Delta q}{C_2} = 0$$

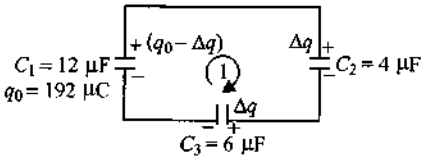


Fig. 4.171

$$\frac{q_0}{C_1} = \Delta q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]$$

$$\Delta q = \frac{q_0}{C_1\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]} = \frac{192}{[1+3+2]} = 32\ \mu C$$

P.D. across $C_2 = \frac{\Delta q}{C_2} = 8$ V

P.D. across $C_1 = \frac{(192-32)}{12} = \frac{160}{12} = \frac{40}{3}$ V

For Problems 10–11

10. b., 11. a.

Sol.

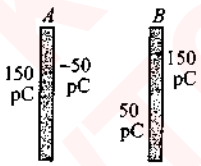


Fig. 4.172

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.8\times10^{-12}\times5\times10^{-3}}{8.8\times10^{-3}} = 5\times10^{-12}\ F$$

Charge on the plate after connection with the battery is

$$q = CV = 5\times10^{-12}\times10 = 50\ pC$$

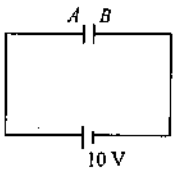


Fig. 4.173

Charge supplied by the battery is

$$\Delta q = (50 + 50)\ pC = 100\ pC$$

Energy supplied by the battery is

$$U_{battery} = \Delta qV = 100\times10 = 1000\ pJ$$
$$= 10^{-9}\ J$$

For Problems 12–13

12. b., 13. c.

4, 2, 5 are equipotential points (Fig 4.174).

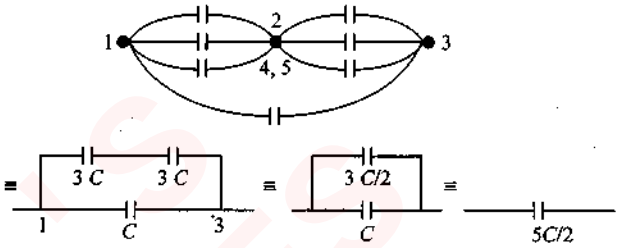


Fig. 4.174

1, 2, 3 are equipotential points (Fig 4.175).

For Problems 14–16

14. b., 15. a., 16. b.

Sol. Let charge be as shown. (Capacitors in series have the same the charge.)

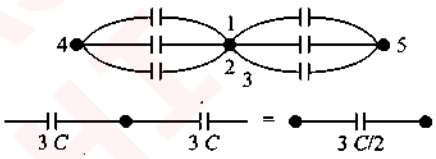


Fig. 4.175

Take loop containing C_1, C_2 and E, we get

$$\frac{q}{C_1} + \frac{q}{C_2} - E = 0 \Rightarrow q = E\left[\frac{C_1C_2}{C_1+C_2}\right]$$

Similarly, from loop containing C_3, C_4 and E, we get

$$\frac{q'}{C_3} + \frac{q'}{C_4} - E = 0 \Rightarrow q' = E\left[\frac{C_3C_4}{C_3+C_4}\right]$$

Now, $V_A - V_B = \frac{q}{C_2} - \frac{q'}{C_4}$

$$= E\left[\frac{C_1}{C_1+C_2} - \frac{C_3}{C_3+C_4}\right]$$

$$\Rightarrow \quad V_A - V_B = E\left[\frac{C_1C_4 - C_3C_2}{(C_1+C_2)(C_3+C_4)}\right]$$

Now if $V_A - V_B = E \Rightarrow C_1C_4 - C_2C_3 = 0$

or $\frac{C_1}{C_2} = \frac{C_3}{C_4}$

For Problems 17–18

17. b., 18. c.

Sol. When S_1 is closed: $-\frac{q_1}{3} - \frac{q_1}{1} + 12 = 0$

$4q_1 = 36 \Rightarrow q_1 = 9\ \mu C$

When S_2 is also closed:

$$C_{eq} = \frac{3\times7}{10} = \frac{21}{10}\mu F$$

$$q = C_{eq}V = \frac{21}{10}\times12 = \frac{126}{5}\ \mu C$$

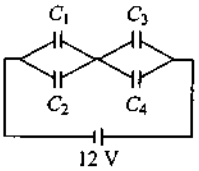


Fig. 4.176

Charge on C_3, $q_3 = q\left(\frac{C_3}{C_3+C_4}\right)$

$$\Rightarrow \quad q_3 = \frac{126}{5}\times\frac{3}{7} = \frac{54}{5}\ \mu C$$

$$q_4 = \frac{72}{5}\ \mu C$$

$$q_1 = q\left(\frac{C_1}{C_1+C_2}\right) = \frac{126}{5}\left(\frac{1}{1+2}\right)$$

$$\Rightarrow \quad \Rightarrow q_1 = \frac{42}{5}\ \mu C$$

$$q_2 = \frac{126}{5}\left(\frac{2}{3}\right) = \frac{84}{5}\ \mu C$$

For Problems 19–21

19. a., 20. a., 21. b.

Sol. Reduction of the farthest right leg yields

$$C = \left(\frac{1}{6.0\,\mu F}+\frac{1}{6.0\,\mu F}+\frac{1}{6.0\,\mu F}\right)^{-1} = 2.0\,\mu F = \frac{C_1}{3}$$

It combines in parallel with C_2, i.e.,

$$C = 4.0\ \mu F + 2.0\ \mu F = 6.0\ \mu F = C_1$$

So, the next reduction is the same as the first: $C = 2.0\ \mu F = C_1/3$. And the next is the same as the second, leaving $3C_1$'s in series. So, $C_{eq} = 2.0\ \mu F = C_1/3$.

For the three capacitors nearest to points a and b:

$$Q_{C_1} = C_{eq}V = (2.0\times10^{-6}\ F)\ (420\ V) = 8.4\times10^{-4}\ C$$

$$V_{cd} = \frac{1}{3}\left(\frac{420}{3}\ V\right) = 46.7\ V$$

Since by symmetry, the total voltage drop over the equivalent capacitance of the part of the circuit from the junctions between c and d is (420/3) V and the equivalent capacitance is that of three equal capacitors C_1 in series. V_{cd} is the voltage over just one of those capacitors, i.e., 1/3 of 420/3 V.

For Problems 22–24

22. c., 23. a., 24. b.

Sol. When switch is in position 1,

$$30 - \frac{q}{20} - \frac{q}{30} = 0 \Rightarrow q = 360\ \mu C$$

Now, the switch is in position 2,

$$q_1' + q_3' = 360 \qquad \text{(i)}$$

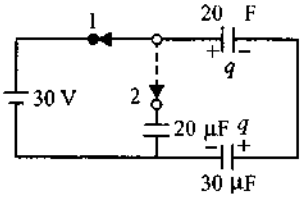


Fig. 4.177

Loop $ABCD$,

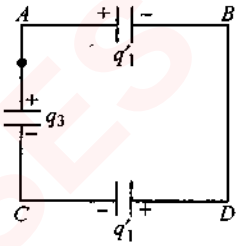


Fig. 4.178

$$-\frac{q_1'}{20} - \frac{q_1'}{30} + \frac{q_3}{20} = 0$$

$$-3q_1' - 2q_1' + 3q_3 = 0$$

$$5q_1' = 3q_3 \Rightarrow q_1' = \frac{3}{5}q_3$$

$$\frac{8q_3}{5} = 360$$

$$q_3 = \frac{360\times5}{8} = \frac{450}{2}\mu C = 225\ \mu C \qquad \text{(ii)}$$

$$q_1' = \frac{3}{5}\left(\frac{450}{2}\right) = 135\ \mu C$$

For Problems 25–26

25. d., 26. d.

Sol. $Q = CV$

Charge flow: $\Delta q = \left(Q + \frac{Q_1}{2}\right) - Q_1$

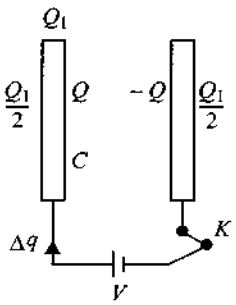


Fig. 4.179

$$Q - \frac{Q_1}{2} = CV - \frac{Q_1}{2}$$

and charge on the inner surface of the left plate: $Q = CV$.

For Problems 27–28

27. a., 28. b

Figure 4.180 (a) corresponds to when switch is open.

Figure 4.180 (b) corresponds to when switch is closed

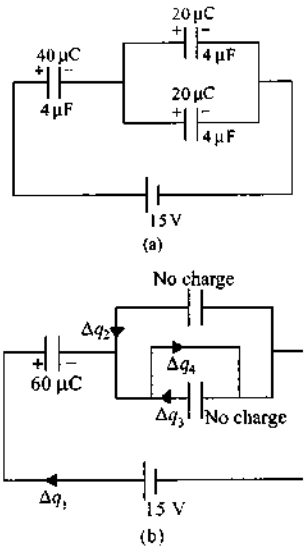


Fig. 4.180

Various charges flown after closing the switch are given as:

$$\Delta q_1 = 60 - 40 = 20\ \mu C$$

$$\Delta q_2 = 20 - 0 = 20\ \mu C$$

$$\Delta q_3 = 20 - 0 = 20\ \mu C$$

$$\Delta q_4 = \Delta q_1 + \Delta q_2 + \Delta q_3 = 60\ \mu C$$

Matching Column Type

1. i. → a., c., ii. → b., iii., → a., c., iv.→ d.

Sol. a. By inserting dielectric slab, capacitance of 1 increases thereby increasing charge on capacitor. Thus, more charge is flown through the battery. Energy stored in capacitor also increases.

b. By increasing separation between the plates, capacitance of C_1 decreases. Charge on C_2 also decreases.

c. By shorting capacitor 1, only capacitor 2 remains in the circuit. Potential difference across C_2 increases, thereby increasing charge on 2 as well as energy stored.

d. By earthing plate of capacitor 1, potentials will change but there will be no change in potential difference, making no overall change in the circuit.

2. i. → b., ii. → c., iii. → a., iv. → d.

Sol. The charges on three plates which are in contact add to zero, because these plates taken together form an isolated system which cannot receive charges from the batteries.

Thus, $q_3 - q_1 - q_2 = 0$

Applying Kirchhoff's law in loop *ABCFA* and *CDEFC*, we get

$$-\frac{q_1}{2} - \frac{q_3}{6} + 10 = 0$$

or $$q_3 + 3q_1 = 60$$

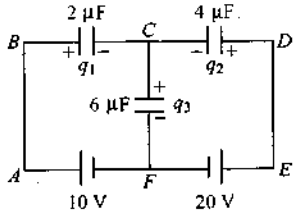


Fig. 4.181

and $$20 - \frac{q_2}{4} - \frac{q_3}{6} = 0$$

or $$3q_2 + 2q_3 = 240 \quad \text{(ii)}$$

Solving the above three equations, we have

$$q_1 = \frac{10}{3}\mu C,\ q_2 = \frac{140}{3}\mu C,\ q_3 = 50\mu C$$

Potential difference across 6 μF capacitor is

$$\frac{q_3}{6} = \frac{50\ \mu C}{6\ \mu F} = \frac{25}{3}\ V$$

3. i. → b., ii. → a., iii. → d., iv. → c.

Sol.

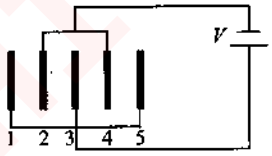


Fig. 4.182

These five plates constitute four identical capacitors in parallel, each of capacity $\varepsilon_0 A/d$. Now, as plate 1 is connected to the positive terminal of the battery and is a part of one capacitor only, so charge on it is

$$q_1 = +\left(\frac{\varepsilon_0 AV}{d}\right)$$

So, **i. → b.**

However, the plate 4 is connected to the negative terminal of the battery and is common to the two identical capacitors in parallel.

So, $$q_4 = -\frac{2\varepsilon_0 AV}{d}$$

Hence, **ii. → a.**

From the circuit, it is obvious that between the plates 2 and 3, the battery is connected, so the potential difference will be V. Plates 1 and 5 get connected through connecting wire, so the potential difference between 1 and 5 is zero.

Hence, **iii → d. iv. → c.**

Integer Answer Type

1. (4) To determine charge on 6 μF, we can remove 3 μF from the circuit.

After this, $$C_{eq} = 2\ \mu F$$

$$Q = C_{eq}V = 2 \times 2 = 4\ \mu C$$

2. (4) $\Delta V_{AP} = \frac{Q}{3}$, $\Delta V_{PB} = \frac{Q}{6}$ $[\because (4+2)\mu F = 6\mu F]$

Also, $$\frac{Q}{3} + \frac{Q}{6} = 12 \Rightarrow Q = 24\mu C$$

$$\Delta V_{AP} = \frac{24}{3} = 8 \Rightarrow V_A - V_P = 8$$

$$\Rightarrow \quad 12 - V_P = 8 \Rightarrow V_P = 4\,V$$

3. (2) $\dfrac{\varepsilon_0 A}{d - \dfrac{d}{2} + \dfrac{d}{2k}} = \dfrac{4}{3}\left(\dfrac{\varepsilon_0 A}{d}\right)$. Solve to get $k = 2$.

4. (6) Volume of eight drops = Volume of big drop

$$8 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \Rightarrow r = \frac{R}{2}$$

$$C = 4\pi\varepsilon_0 R = 12\ \mu\text{F}$$

Thus, capacitance of smaller drop

$$C = 4\pi\varepsilon_0 r = \frac{4\pi\varepsilon_0 R}{2} = \frac{12}{2} = 6\ \mu\text{F}$$

5. (4) Initial energy: $E = \frac{1}{2}C_0V_0^2$

(i) $E_1 = 2E$, because charge remains the same and the capacitance becomes half.

(ii) $E_2 = E/2$, because potential remains the same and the capacitance becomes half.

$$\Rightarrow \quad \frac{E_1}{E_2} = 4$$

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APPENDIX

A1

Miscellaneous Assignments and Archives on Chapters 1–4

R. K. MALIK'S NEWTON CLASSES RANCHI

EXERCISES

Objective Type

Solutions on page A1.40

1. A metal sphere having a radius r_1 charged to a potential V_1 is enveloped by a thin-walled conducting spherical shell of radius r_2. Determine the potential V_1 acquired by the sphere after it has been connected for a short time to the shell by a conductor.

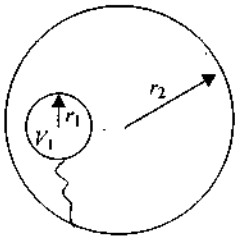


Fig. A1.1

a. $\frac{V_1 r_2}{r_1}$ b. $\frac{V_1 r_1}{r_2}$

c. V_1 d. $\frac{V_1(r_1+r_2)}{r_2}$

2. An insulated conductor initially free from charge is charged by repeated contacts with a plate which after each contact is replenished to a charge Q from an electrophorus. If q is the charge on the conductor after the first operation, find the maximum charge which can be given to the conductor in this way.

a. $\frac{Qq}{Q+q}$ b. $\frac{3Qq}{2Q+q}$

c. $\frac{Qq}{Q-q}$ d. $\frac{2Qq}{Q+q}$

3. In the given circuit diagram (Fig. A1.2), switch S_w is shifted from position 1 to position 2. Then

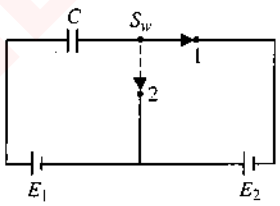


Fig. A1.2

a. a charge of amount CE_2 will be supplied to battery E_1

b. heat generated in the circuit is $\frac{1}{2}CE_2^2$

c. a charge of amount CE_2 will be supplied by battery E_1

d. heat generated in the circuit is $\frac{1}{2}CE_1E_2$

4. Find the electric dipole moment of a non-conducting ring of radius a, made of two semicircular rings having linear charge densities $-\lambda$ and $+\lambda$ as shown in Fig. A1.3.

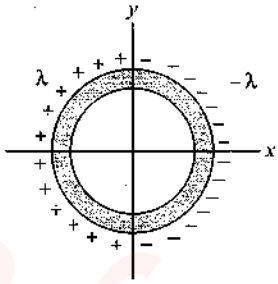


Fig. A1.3

a. $2\lambda a^2$ b. $4\lambda a^2$ c. λa^2 d. $2\sqrt{2}\lambda a^2$

5. The electric potential in a certain region of space is given by $V = -3x^2 + 4x$, where x is in metres and V is in volts. In this region, the equipotential surfaces are

a. planes parallel to XY plane

b. planes parallel to YZ plane

c. concentric cylinders with axis as x-axis

d. concentric spheres centred at origin

6. Mark out the correct statement about the electric lines of force.

a. The number of lines of force leaving a +ve charge or entering a −ve charge is proportional to the magnitude of charge.

b. The path of a charge particle in an electric field is not always along the field lines.

c. The direction of electric lines of force is along the normal to the equipotential surface.

d. All of the above.

7. A point charge q is placed at the centre of a closed cylindrical conductor of length l and radius R. The flux crossing through the conducting cylinder would be

a. $\frac{ql}{2\varepsilon_0[R^2+l^2/4]^{1/2}}$ b. zero

c. $\frac{ql}{\varepsilon_0[R^2+l^2/4]^{1/2}}$ d. $\frac{q}{\varepsilon_0}$

8. A point charge q is placed at the centre of an imaginary Gaussian surface as shown in Fig. A1.4.
Find the flux crossing through the curved surface.

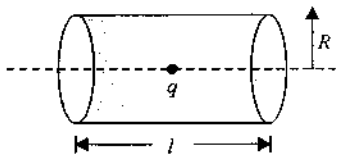


Fig. A1.4

a. zero b. $\frac{q}{\varepsilon_0}\left[1-\frac{l}{\sqrt{R^2+(l^2/4)}}\right]$

c. $\frac{q}{2\varepsilon_0}\left[1-\frac{l}{2\sqrt{(l^2/4)+R^2}}\right]$ d. $\frac{ql}{2\varepsilon_0\sqrt{(l^2/4)+R^2}}$

9. Consider a capacitor as shown in Fig. A1.5(a). If we pull the plates of capacitor apart to a final position as shown in

Fig. A1.5 (b), then we must perform some work against the electric force. For this situation, mark out the correct statement(s). [Take areas of plates as A]

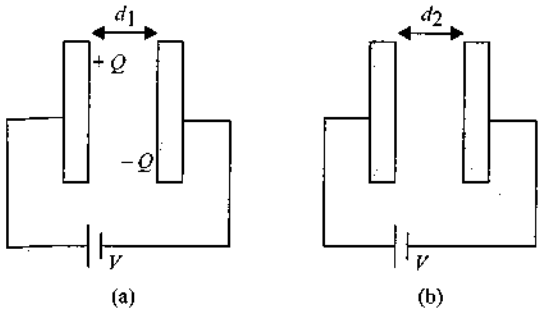


Fig. A1.5

a. Work done is $\frac{Q^2}{2\varepsilon_0 A}(d_2 - d_1)$ and is stored in volume $A(d_2 - d_1)$.

b. Work done is $+\frac{Q^2}{2\varepsilon_0 A}(d_2 - d_1)$ and is stored in volume Ad_2.

c. Work done is $+\frac{\varepsilon_0 A V^2}{2}\frac{(d_2 - d_1)}{d_1 d_2}$ and is stored in volume $A(d_2 - d_1)$.

d. Work done is $\frac{\varepsilon_0 A V^2}{2}\frac{(d_2 - d_1)}{d_1 d_2}$ and is stored in volume Ad_2.

10. An isolated capacitor of capacitance C is charged to a potential V. Then, a dielectric slab of dielectric constant K is inserted as shown in Fig. A1.6. The net charge on four surfaces 1, 2, 3 and 4 would be respectively,

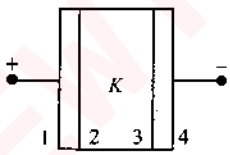


Fig. A1.6

a. $0, CV, -CV, 0$ **b.** $0, \frac{CV}{K}, \frac{-CV}{K}, 0$

c. $CV, 0, 0, -CV$ **d.** $CV, \frac{-CV}{K}, \frac{CV}{K}, -CV$

11. Mark out the correct statement(s).

a. Capacitance of an isolated sphere depends on its charge.

b. Capacitance of a non-isolated sphere depends only on its shape and size.

c. Capacitance of a non-isolated sphere is increased because of the presence of other conducting bodies nearby it.

d. Capacitance of a conductor cannot be affected because of the presence of other conducting bodies nearby it.

12. Three concentric spherical conductors are as shown in Fig. A1.7. Determine the equivalent capacitance of the system between A and B.

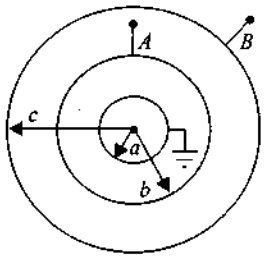


Fig. A1.7

a. $\frac{4\pi\varepsilon_0 ac}{c-a}$ **b.** $\frac{4\pi\varepsilon_0 bc}{c-b} + \frac{4\pi\varepsilon_0 ab}{b-a}$

c. $\frac{4\pi\varepsilon_0 bc}{c-b} + \frac{4\pi\varepsilon_0 abc}{ab + c(b-a)}$ **d.** $\frac{4\pi\varepsilon_0 ac}{c-a} + 4\pi\varepsilon_0 c$

13. Capacitor C_1 is connected to a battery and charged till the magnitude of the charge on each plate is q_0. Then, the battery is disconnected and C_1 is connected to two other uncharged capacitors C_2 and C_3 as shown (Fig. A1.8). Final charges on the capacitors (q_1, q_2 and q_3) are related by

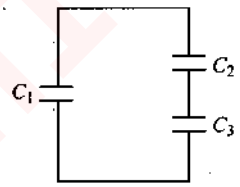


Fig. A1.8

a. $q_0 = q_1 + q_2 + q_3$ **b.** $q_1 + q_2 + q_3 = 0$

c. $q_0 = q_3 + q_2, q_1 = 0$ **d.** $q_0 = q_1 + q_2, q_2 = q_3$

14. Three concentric conducting spherical shells have radii r, $2r$ and $3r$ and charges q_1, q_2 and q_3 respectively. Innermost and outermost shells are earthed as shown in Fig. A1.9. The charges shown are after earthing. Select the correct alternative.

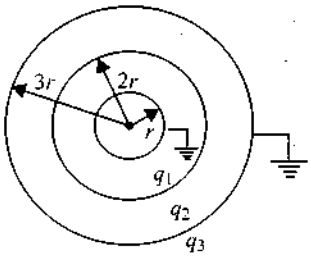


Fig. A1.9

a. $q_1 + q_3 = -q_2$ **b.** $q_1 = -q_2$

c. $\frac{q_3}{q_2} = -\frac{1}{3}$ **d.** None of these

15. Two infinitely long parallel wires having linear charge densities λ_1 and λ_2, respectively, are placed at a distance R. The force per length on either wire will be

a. $k\frac{2\lambda_1\lambda_2}{R^2}$ **b.** $k\frac{2\lambda_1\lambda_2}{R}$ **c.** $k\frac{\lambda_1\lambda_2}{R^2}$ **d.** $k\frac{\lambda_1\lambda_2}{R}$

16. If a positively charged pendulum is oscillating in a uniform electric field as shown in Fig. A1.10. Its time period as compared with that when it was uncharged

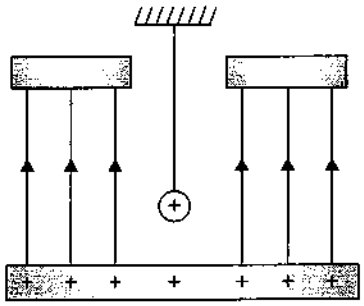


Fig. A1.10

a. will increase
b. will decrease
c. will not change
d. will first increase and then decrease

17. A uniformly charged and infinitely long line having a linear charge density λ is placed at a normal distance y from a point O. Consider a sphere of radius R with O as centre and $R > y$. Electric flux through the surface of the sphere is

a. zero b. $\frac{2\lambda R}{\varepsilon_0}$

c. $\frac{2\lambda\sqrt{R^2 - y^2}}{\varepsilon_0}$ d. $\frac{\lambda\sqrt{R^2 + y^2}}{\varepsilon_0}$

18. A point charge $+Q$ is placed at the centroid of an equilateral triangle. When a second charge $+Q$ is placed at a vertex of the triangle, the magnitude of the electrostatic force on the central charge is 8 N. The magnitude of the net force on the central charge when a third charge $+Q$ is placed at another vertex of the triangle is

a. zero b. 4 N c. $4\sqrt{2}$ N d. 8 N

19. Five Styrofoam balls are suspended from insulating threads. Several experiments are performed on the balls and the following observations are made:

(i) Ball A repels C and attracts B.
(ii) Ball D attracts B and has no effect on E.
(iii) A negatively charged rod attracts both A and E.

An electrically neutral Styrofoam ball gets attracted if placed nearby a charged body due to induced charge. What are the charges, if any, on each ball A, B, C, D and E?

a. + + − + O + b. + − + + O
c. + − + O O d. − + − O O

20. A charge Q is placed at a distance of $4R$ above the centre of a disk of radius R. The magnitude of flux through the disk is ϕ. Now, a hemispherical shell of radius R is placed over the disk such that it forms a closed surface. The flux through the curved surface, taking direction of area vector along outward normal as positive, is

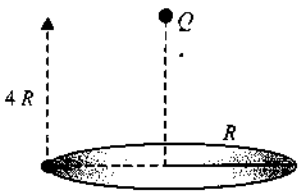


Fig. A1.11

a. zero b. ϕ c. $-\phi$ d. 2ϕ

21. Two very large thin conducting plates having same cross-sectional area are placed as shown in Fig. A1.12. They are carrying charges Q and $3Q$ respectively.

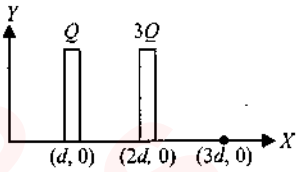
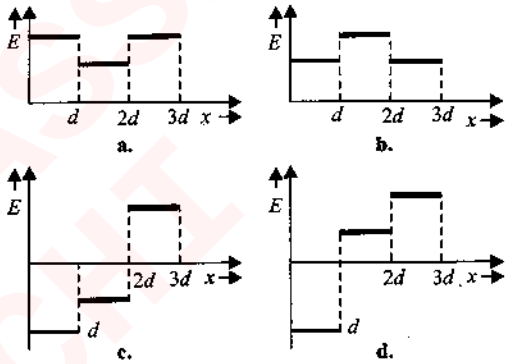


Fig. A1.12

The variation of electric field as a function of x (for $x = 0$ to $x = 3d$) will be best represented by



22. The negative charge $-q_2$ is fixed, while positive charge q_1 as well as the conducting sphere 'S' is free to move (Fig. A1.13). If the system is released from rest,

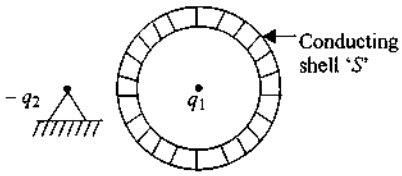


Fig. A1.13

a. both S and q_1 move towards left
b. q_1 moves towards right while S moves towards left
c. q_1 remains at rest, S moves towards left
d. both q_1 and S remain at rest

23. A charged particle of mass $m = 2$ kg and charge 1 μC is projected from a horizontal ground at an angle $\theta = 45°$ with speed 10 ms^{-1}. In space, a horizontal electric field towards the direction of projection $E = 2 \times 10^7$ NC^{-1} exists. The range of the projectile is

a. 20 m b. 60 m c. 200 m d. 180 m

24. A wire of linear charge density λ passes through a cuboid of length l, breadth b and height h ($l > b > h$) in such a manner that the flux through the cuboid is maximum. The position of the wire is now changed, so that the flux through the cuboid is minimum. The ratio of maximum flux to minimum flux will be

a. $\sqrt{\frac{l^2 + b^2}{h}}$ b. $\sqrt{\frac{l^2 + b^2 + h^2}{h}}$

c. $\dfrac{h}{\sqrt{l^2+b^2}}$ d. $\dfrac{h}{\sqrt{l^2+b^2+h^2}}$

25. A positively charged particle P enters the region between two parallel plates with a velocity u, in a direction parallel to the plates. There is a uniform electric field in this region. P emerges from this region with a velocity v.

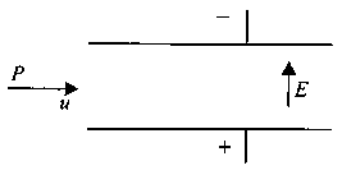


Fig. A1.14

Taking C as a constant, v will depend on u as

a. $v = Cu$ b. $v = \sqrt{u^2+Cu}$

c. $v = \sqrt{u^2+\dfrac{C}{u}}$ d. $v = \sqrt{u^2+\dfrac{C}{u^2}}$

26. Consider the situation shown in Fig. A1.15. We find electric field E at point P using Gauss's law and it comes out to be $E = \sigma/\varepsilon_0$. This electric field is due to

a. all the amount of charges present on both the plates
b. all the charges present on positive plate only
c. positive charge present only inside the Gaussian surface
d. positive charge present inside the Gaussian surface plus equal magnitude of negative charge present on negative plate in front of Gaussian surface

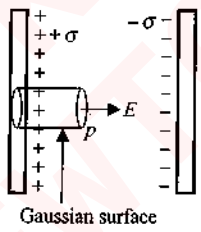


Fig. A1.15

27. Figure A1.16 shows three circular arcs, each of radius R and total charge as indicated. The net electric potential at the centre of curvature is

a. $\dfrac{Q}{2\pi\varepsilon_0 R}$ b. $\dfrac{Q}{4\pi\varepsilon_0 R}$ c. $\dfrac{2Q}{\pi\varepsilon_0 R}$ d. $\dfrac{Q}{\pi\varepsilon_0 R}$

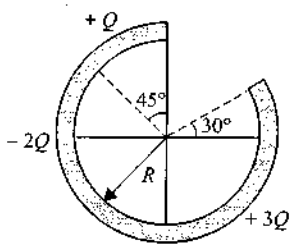


Fig. A1.16

28. Four charges are placed at four corners of a square as shown (Fig. A1.17). The side of the square is a. Two charges are positive and two are negative but their magnitudes are the same. Now, an external agent starts decreasing all the sides of the square slowly and at the same rate. What happens to the electrical potential energy of the system and what will be the nature of work done by the agent?

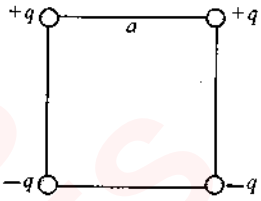


Fig. A1.17

a. Increases, positive b. Increases, negative
c. Decreases, negative d. Increases, positive

29. Given a metallic uniformly charged sphere. The radius of the sphere is increased keeping its potential same. What is the effect on the value of electric field intensity at its surface?

a. Increases b. Decreases
c. Remains constant d. Cannot say

30. One-fourth of a sphere of radius R is removed as shown (Fig. A1.18). An electric field E exists parallel to x-y plane. Find the flux through the remaining curved part.

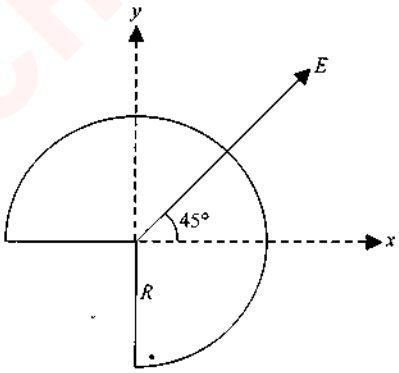


Fig. A1.18

a. $\pi R^2 E$ b. $\sqrt{2}\pi R^2 E$
c. $\pi R^2 E/\sqrt{2}$ d. none of these

31. At a distance r from a point located at origin in space, the electric potential varies as $V = 10r$. Find the electric field at $\vec{r} = 3\hat{i}+4\hat{j}-5\hat{k}$.

a. $\sqrt{2}(3\hat{i}+4\hat{j}-5\hat{k})$ b. $-\sqrt{2}(3\hat{i}+4\hat{j}-5\hat{k})$
c. $-\sqrt{3}(3\hat{i}+4\hat{j}-5\hat{k})$ d. None of the above

32. A conducting spherical shell is earthed. A positive charge $+q_1$ is placed at the centre and another small positive charge $+q_2$ is placed at a distance r from q_1 (Fig. A1.19). Ignore the effect of induced charge due to q_2 on the sphere. Then the coulomb force on q_2 is

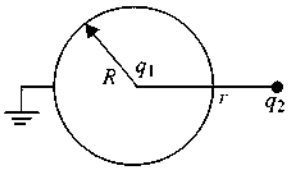


Fig. A1.19

a. zero
b. $\frac{q_1q_2}{4\pi\varepsilon_0 r^2}$
c. $\frac{q_1q_2}{4\pi\varepsilon_0 (r-R)^2}$
d. $\frac{q_1q_2}{4\pi\varepsilon_0 (r^2-R^2)}$

33. In question 32, if the conducting spherical shell is not earthed but is neutral, then the force on q_2 is

a. zero
b. $\frac{q_1q_2}{4\pi\varepsilon_0 r^2}$
c. $\frac{q_1q_2}{4\pi\varepsilon_0 (r-R)^2}$
d. none of these

34. Two parallel conducting plates, each of area A, are separated by a distance d. Now, the left plate is given a positive charge Q. A positive charge q of mass m is released from a point near the left plate. Find the time taken by the charge to reach the right plate.

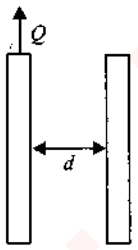


Fig. A1.20

a. $\sqrt{\frac{3dm\varepsilon_0 A}{qQ}}$
b. $\sqrt{\frac{4dm\varepsilon_0 A}{qQ}}$
c. $\sqrt{\frac{2dm\varepsilon_0 A}{qQ}}$
d. None of these

35. A uniform electric field of 100 Vm^{-1} is directed at 60° with the positive x-axis as shown in Fig. A1.21. If $OA = 2$ m, the potential difference $V_O - V_A$ is

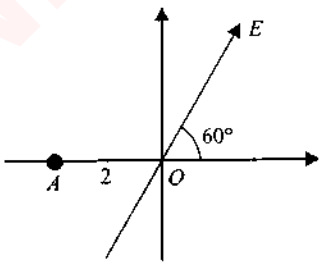


Fig. A1.21

a. 50 V
b. 50 V
c. 100 V
d. −100 V

36. A graph of the x-component of the electric field as a function of x in a region of space is shown in Fig. A1.22. The y- and z-components of the electric field are zero in this region. If the electric potential is 10 V at the origin, then the potential at $x = 2.0$ m is

a. 10 V
b. 40 V
c. −10 V
d. 30 V

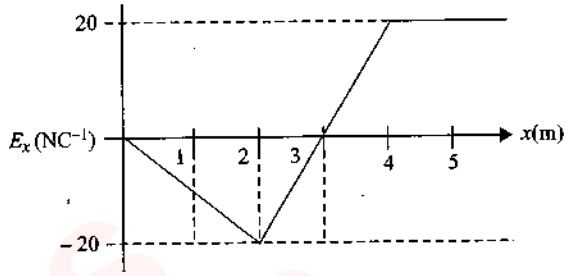


Fig. A1.22

37. An uncharged conductor A is brought near a positively charged conductor B. The size of the conductor A is much greater than the size of conductor B. Then,

a. the charge on B will increase but the potential of B will not change
b. the charge on B will not change but the potential of B will decrease
c. the charge on B will decrease but the potential of B will not change
d. the charge on B will not change but the potential of B will increase

38. Inside a hollow conducting sphere, which is uncharged, a charge q is placed at its centre. Let electric field at a distance x from centre at point p be E and potential at this point be V. Now, some positive charge Q is given to this sphere, then

a. E will remain the same
b. E will increase
c. V will decrease
d. V will remain the same

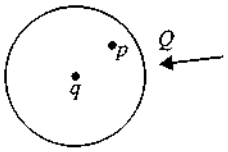


Fig. A1.23

39 A charge particle moves in a circle around an infinite line charge with the centre of circle at the line and line being perpendicular to the plane of circle. Let r be the radius of circle. The velocity of the particle depends upon which power of r.

a. 1
b. 2
c. −1
d. None of these

40. Two large plates are given the charges as shown in Fig. A1.24. Now, the left plate is earthed. Find the amount of charge that will flow from the earth to the plate.

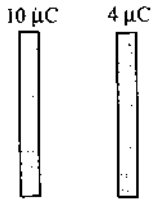


Fig. A1.24

a. 14 μC
b. −14 μC
c. 7 μC
d. −7 μC

41. Three identical metallic plates are kept parallel to one another at separations a and b. The outer plates are connected to the

ground and the middle plate is given charge Q (Fig. A1.25). Then, the charge on the right side of middle plate is

a. $Q/2$ b. $-\frac{Qb}{a+b}$ c. $\frac{Qb}{a+b}$ d. $\frac{Qa}{a+b}$

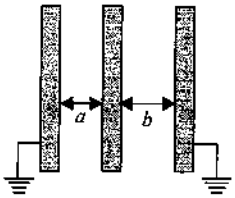


Fig. A1.25

42. An electric dipole consists of charges $\pm 2.0 \times 10^{-8}$ C separated by distance of $\pm 2.0 \times 10^{-3}$ m. It is placed at a distance of 6 cm from a line charge of linear charge density 4.0×10^{-4} Cm^{-1} as shown in Fig. A1.26 such that the dipole makes an angle of $\theta = 60°$ with line AB. Find the force acting on the dipole.

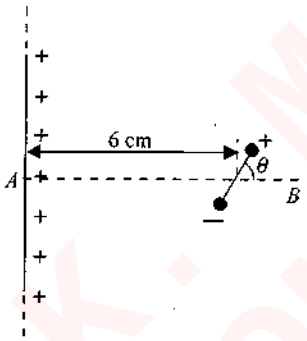


Fig. A1.26

a. 0.012 N b. 1.8 N
c. 0.04 N d. None of these

43. A 16 μF capacitor, initially charged to 5 V, is started charging at $t = 0$ by a source at the rate of $40t$ μCs^{-1}. How long will it take to raise its potential to 10 V?

a. 1 s b. 2 s
c. 3 s d. None of these

44. Find the equivalent capacitance between C and B.

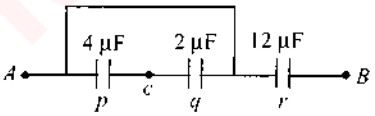


Fig. A1.27

a. 6/5 μF b. 5/6 μF
c. 4 μF d. None of these

45. A parallel plate capacitor of capacitance 10 μF is connected across a battery of e.m.f. 5 mV. Now, the space between the plates of the capacitor is filled with a dielectric material of dielectric constant $K = 5$. Then, the charge that will flow through the battery till equilibrium is reached is

a. 250 μC b. 250 nC
c. 200 nC d. 200 μC

46. Six plates of equal area A and plate separation as shown (Fig. A1.28) are arranged. The equivalent capacitance between A and B is

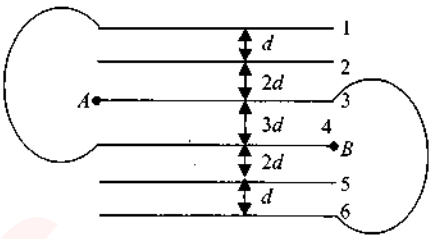


Fig. A1.28

a. $\frac{\varepsilon_0 A}{d}$ b. $\frac{2\varepsilon_0 A}{d}$ c. $\frac{3\varepsilon_0 A}{d}$ d. $\frac{\varepsilon_0 A}{4d}$

47. If area of each plate is A and the successive separations are d, $2d$ and $3d$, then the equivalent capacitance across A and B is

a. $\frac{\varepsilon_0 A}{6d}$ b. $\frac{\varepsilon_0 A}{4d}$ c. $\frac{3\varepsilon_0 A}{4d}$ d. $\frac{\varepsilon_0 A}{3d}$

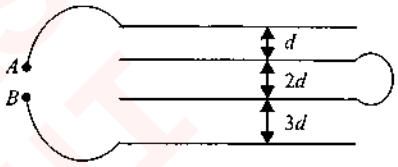


Fig. A1.29

48. A circuit has a section AB (Fig. A1.30). Potential difference between points A and B (i.e., $V_A - V_B$) equals 5 V. The voltage across 2 μF capacitor is

a. 5 V b. 10 V c. 15 V d. zero

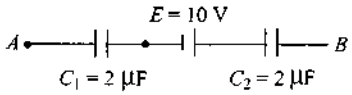


Fig. A1.30

49. In the circuit shown (Fig. A1.31), the equivalent capacitance between the points X and Y is

a. 2 μF b. 3 μF c. 4 μF d. 5 μF

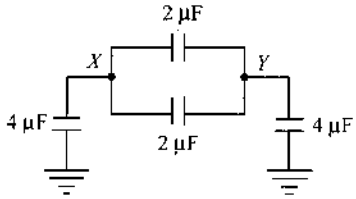


Fig. A1.31

50. An electric dipole is placed at origin in the x-y plane with its orientation along the positive x-axis. The direction of electric field

a. at $(-x_0, 0, 0)$ is along the positive x-axis.
b. at $(0, y_0, 0)$ is along the negative x-axis.
c. at $(0, 0, z_0)$ is along the negative x-axis.
d. all the above.

51. For the arrangement shown in Fig. A1.32, identify the correct statement.

a. The charge on the 12 μF capacitor is zero.
b. The charge on the 12 μF capacitor is 30 μC.
c. The charge on the 4 μF capacitor is 30 μC.
d. None of these

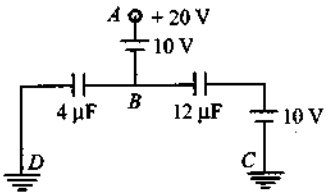


Fig. A1.32

52. The electric field in a region is given by

$$\vec{E} = \frac{E_0 x}{l}\hat{i}$$

Find the charge contained inside a cubical volume bounded by the surfaces $x = 0, x = a, y = 0, y = a, z = 0$ and $z = a$. Take $E_0 = 5 \times 10^3$ N/C, $l = 0.02$ m and $a = 0.01$ m.

a. 1.1×10^{-12}C **b.** 2.2×10^{-12}C
c. 4.4×10^{-12}C **d.** 5.5×10^{-12}C

53. Three identical metal plates with large surface areas are kept parallel to each other as shown in Fig. A1.33. The leftmost plate is given a charge Q, the rightmost a charge $-2Q$ and the middle one remains neutral. Find the charge appearing on the outer surface of the rightmost plate.

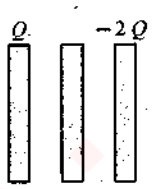


Fig. A1.33

a. $-Q/4$ **b.** $-2Q$ **c.** $-Q$ **d.** $-Q/2$

54. The particle P shown in the figure has a mass of 10 mg and a charge of -0.01 μC. Each plate has a surface area 10^{-2} m^2 on one side. What potential difference V should be applied to the combination to hold the particle P in equilibrium?

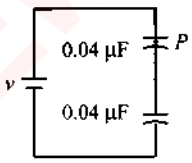


Fig. A1.34

a. 43 mV **b.** 35 mV **c.** 50 mV **d.** 55 mV

55. Figure A1.35 shows two identical parallel plate capacitors connected to a battery through a switch S. Initially, the switch is closed so that the capacitors are completely charged. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant 3. Find the ratio of the initial total energy stored in the capacitors to the final total energy stored.

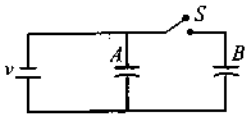


Fig. A1.35

a. 9 : 16 **b.** 5 : 9 **c.** 2 : 3 **d.** 3 : 5

56. A point charge q is situated at X between two parallel plates which have a potential difference V and carry charges $+Q$ and $-Q$. What is the electric field strength at X?

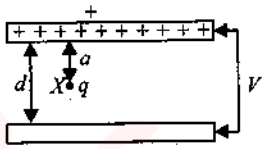


Fig. A1.36

a. $\frac{V}{d}$ **b.** $\frac{Vq}{d}$
c. $\frac{Q}{4\pi\varepsilon_0 a^2}$ **d.** $\frac{qQ}{4\pi\varepsilon_0 a^2}$

57. In the circuit shown, a capacitor of capacitance 3 μF is charged form a battery of e.m.f. 6 V with switch connected to terminal P. The switch is now connected to Q. This charges the 6 μF capacitor from the 3 μF one. What is the new potential difference across combination?

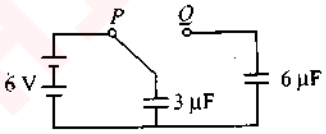


Fig. A1.37

a. 1 V **b.** 2 V **c.** 4 V **d.** 6 V

58. In the given arrangement of capacitors, 6 μC charge is added to point a. Find the charge on upper capacitor.

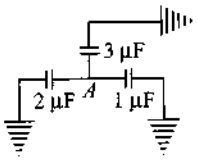


Fig. A1.38

a. 3 μC **b.** 1 μC **c.** 2 μC **d.** 6 μC

59. Two capacitors A and B with capacities C_1 and C_2 are charged to potential difference of V_1 and V_2, respectively. The plates of the capacitors are connected as shown in the figure with one wire free from each capacitor. The upper plate of A is positive and that of B is negative. An uncharged capacitor of capacitance C_3 and lead wires falls on the free ends to complete circuit, then

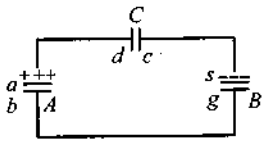


Fig. A1.39

a. the final charge on each capacitor are same to each other
b. the final sum of charge on plates a and d is C_1V_1
c. the final sum of charge on plates b and g is $C_2V_2 - C_1V_1$
d. both (b) and (c) are correct

60. A parallel plate capacitor C is equally filled with parallel layers of materials of dielectric constants K_1 and K_2. Then the ratio of new capacitance to the previous capacitor is

a. $\frac{2K_1K_2}{K_1+K_2}$ b. K_1+K_2

c. $\frac{K_1K_2}{K_1+K_2}$ d. none of the above

61. The time period of simple pendulum of charged bob is T as shown in Fig. A1.40. Now a massless charge q is placed at point B and time period of oscillation is T', then

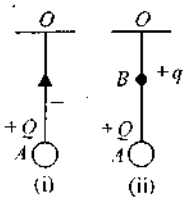


Fig. A1.40

a. $T' > T$ b. $T' < T$

c. $T' = T$ d. cannot say

62. In the given electric field $\bar{E} = \left[\alpha(d+x)\hat{i} + E_0\hat{j}\right]$ N/C; where $\alpha = 1$ N/C hypothetical closed surface is taken as shown in Fig. A1.41.

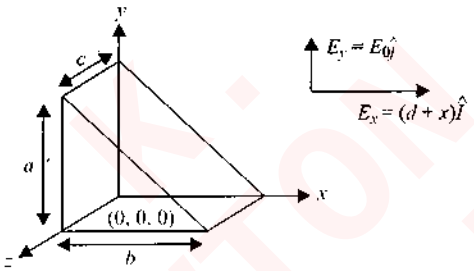


Fig. A1.41

The total charge enclosed within the close surface is

a. $\frac{abc\varepsilon_0}{2}$ b. $\frac{acd\varepsilon_0}{2}$

c. $\frac{abd\varepsilon_0}{2}$ d. none of above

63. A spherical conductor A contains two spherical cavities. The total charge on the conductor itself is zero. However, there is a point charge q_b at the centre of one cavity and q_c at the centre of the other. At considerable distance r away from the centre of the spherical conductor, there is another charge q_d. Forces acting on q_b, q_c and q_d are F_1, F_2 and F_3, respectively. [Assume all charges are positive]

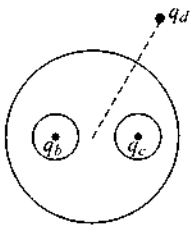


Fig. A1.42

a. $F_1 < F_2 < F_3$ b. $F_1 = F_2 < F_3$

c. $F_1 = F_2 > F_3$ d. $F_1 > F_2 > F_3$

64. Three infinite planes have a uniform surface charge distribution σ on its surface. All charges are fixed. On each of the three infinite planes, parallel to the $y-z$ plane placed at $x = -a$, $x = 0$ and $x = a$, there is a uniform surface charge of the same density, σ.

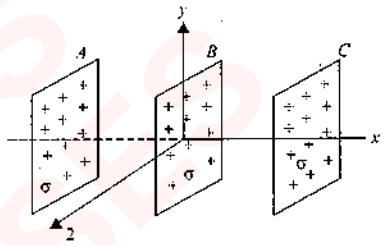


Fig. A1.43

The potential difference between A and C is

a. $\frac{\sigma}{2\varepsilon_0}a$ b. $\frac{\sigma}{\varepsilon_0}a$

c. $\frac{\sigma a}{2\varepsilon_0}$ d. none of the above

65. In the circuit shown in Fig. A1.44, the charge stored on a capacitor of capacitance 3 μF is

a. zero b. 40 μC c. 60 μC d. 90 μC

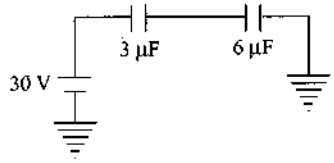


Fig. A1.44

66. A parallel-plate capacitor is connected to a battery. The plates are pulled apart with uniform speed. If x is the separation between the plates, then the rate of change of electrostatic energy of the capacitor is proportional to

a. x^2 b. x c. $\frac{1}{x}$ d. $\frac{1}{x^2}$

67. Find the equivalent capacitance between A and B. [Assume each conducting plate is having same dimensions and neglect the thickness of the plate, $\varepsilon_0 A/d = 7$ μF [where A is area of plates, $A >> d$].

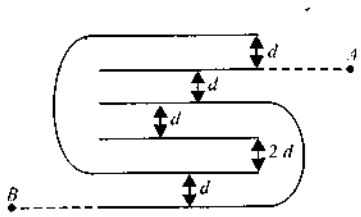


Fig. A1.45

a. 7 μF b. 11 μF c. 12 μF d. 15 μF

68. If the plates of a parallel-plate capacitor are not equal in area, then

a. quantity of charge on the plates will be the same, but nature of charge will differ
b. quantity of charge as well as nature of charge on the plates will be different
c. quantity of charge on the plates will be different, but nature of charge will be the same
d. quantity of charge as well as nature of charge will be the same

69. Electric charges q, q and $-2q$ are placed at the corners of an equilateral triangle ABC of side l. The magnitude of electric dipole moment of the system is
a. ql b. $2ql$ c. $\sqrt{3}ql$ d. ql

70. How does the electric field strength vary when we enter a uniformly charged spherical cloud?
a. Decreases inversely as the square of the distance from the surface
b. Decreases directly as the square of the distance from the surface
c. Decreases directly as the square of the distance from the centre
d. Decreases directly as the distance from the centre or $E \propto r$

71. A ring of radius R carries a charge $+q$. A test charge $-q_0$ is released on its axis at a distance from its centre. How much kinetic energy will be acquired by the test charge when it reaches the centre of the ring?
a. $\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{R}$ b. $\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{2R}$
c. $\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{\sqrt{3}R}$ d. $\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{3R}$

72. Two points are at distances a and b ($a < b$) from a long string of charge per unit length λ. The potential difference between the points is proportional to
a. b/a b. b^2/a^2 c. $\sqrt{b/a}$ d. $\log b/a$

73. In a certain charge distribution, all points having zero potential can be joined by a circle S. Points inside S have positive potential, and points outside S have negative potential. A positive charge, which is free to move, is placed inside S
a. it will remain in equilibrium
b. it can move inside S, but it cannot cross S
c. it must cross S at some time
d. it may move but will ultimately return to its starting point

74. A charge $+q$ is placed at each of the points $x = x_0$, $x = 3x_0$, $x = 5x_0$ at infinitum on the x-axis and a charge $-q$ is placed at each of the points $x = 2x_0$, $x = 4x_0$, $x = 6x_0$ at infinitum. Here, x_0 is a positive constant. Take the electric potential at a point due to a charge Q at a distance r from it to be $Q/4\pi\varepsilon_0 r$. Then the potential at the origin due to the above system of charges is
a. 0 b. $\frac{q}{8\pi\varepsilon_0 x_0 \log 2}$
c. ∞ d. $\frac{q\log 2}{4\pi\varepsilon_0 x_0}$

75. A spring block system undergoes vertical oscillations above a large horizontal metal sheet with uniform positive charge. The time period of the oscillation is T. If the block is given a charge Q, its time period of oscillation will be
a. T
b. $> T$
c. $< T$
d. $> T$ if Q is +ve and $< T$ if Q is −ve

76. A thin aluminium sheet is placed between the plates of a parallel-plate capacitor. Its capacitance will
a. increases b. decreases
c. remain same d. become infinite

77. A fully charged capacitor has a capacitance C. It is discharged through a small coil of resistance wire embedded in a thermally insulated block of specific heat capacity s and mass m. If the temperature of the block is raised by T, the potential difference V across the capacitance is
a. $\frac{msT}{C}$ b. $\sqrt{\frac{2msT}{C}}$
c. $\sqrt{\frac{2msCT}{s}}$ d. $\frac{mCT}{s}$

78. The work done in placing a charge of 8×10^{-18} C on a condenser of capacity 100 mF is
a. 32×10^{-32} J b. 16×10^{-32} J
c. 3.1×10^{-26} J d. 4×10^{-10} J

79. A 40 μF capacitor in a defibrillator is charged to 3000 V. The energy stored in the capacitor is sent through the patient during a pulse of duration 2 ms. The power delivered to the patient is
a. 45 kW b. 90 kW c. 180 kW d. 360 kW

80. Three charges q, q and $-2q$ are fixed on the vertices of an equilateral triangular plate of edge length a. This plate is in equilibrium between two very large plates having surface charge density σ_1 and σ_2, respectively. Find the time period of small angular oscillation about an axis passing through its centroid and perpendicular to the plane. Moment of inertia of the system about this axis is I.
a. $2\pi\sqrt{\frac{\varepsilon_0 I}{qa|\sigma_1-\sigma_2|}}$ b. $2\pi\sqrt{\frac{\varepsilon_0 I}{2qa|\sigma_1-\sigma_2|}}$
c. $2\pi\sqrt{\frac{2\varepsilon_0 I}{\sqrt{3}qa|\sigma_1-\sigma_2|}}$ d. $2\pi\sqrt{\frac{2\varepsilon_0 I}{qa|\sigma_1-\sigma_2|}}$

81. The capacitor plates are fixed on an inclined plane and connected to a battery of e.m.f. E. The capacitor plates have plate area a, length l and the distance between them is d. A dielectric slab of mass m and dielectric constant K is inserted into the capacitor and tied to mass M by a massless string as shown in the figure. Find the value of M for which the slab will stay in equilibrium. There is no friction between slab and plates.
a. $\frac{m}{2}+\frac{E^2\varepsilon_0 A(k-1)}{2lgd}$ b. $\frac{m}{2}-\frac{E^2\varepsilon_0 A(k-1)}{2lgd}$
c. $\frac{m}{2}+\frac{E^2\varepsilon_0 A(k-1)}{lgd}$ d. $\frac{m}{2}-\frac{E^2\varepsilon_0 A(k-1)}{lgd}$

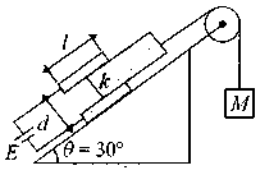


Fig. A1.46

82. Two plates, each of area A, are placed parallel to each other at a distance d. One plate is connected to a battery of e.m.f. E and its negative is earthed. The other plate is also earthed.

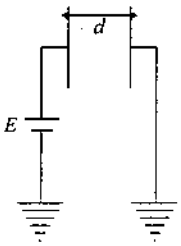


Fig. A1.47

The charge drawn by plate is

a. $\frac{2\varepsilon_0 AE}{d}$ **b.** $\frac{\varepsilon_0 AE}{d}$ **c.** $\frac{\varepsilon_0 AE}{2d}$ **d.** $\frac{3\varepsilon_0 AE}{d}$

83. A non-conducting sphere with a cavity has volume charge density ρ. O_1 and O_2 represent the two centres as shown. The electric field inside the cavity is E_0. Now, an equal and opposite charge is given uniformly to the sphere on its outer surface. The magnitude of electric field inside the cavity becomes

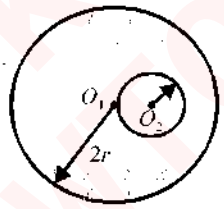


Fig. A1.48

a. zero **b.** E_0
c. $2E_0$ **d.** $3E_0$

84. Five Styrofoam balls are suspended from insulating threads. Several experiments are performed on the balls and the following observations are made

(i) Ball A repels C and attracts B.

(ii) Ball D attracts B and has no effect on E

(iii) A negatively charged rod attracts both A and E

An electrically neutral Styrofoam ball gets attracted if placed nearby a charged body due to induced charge. What are the charges, if any, on each ball?

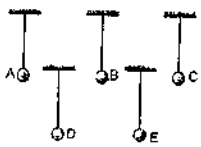


Fig. A1.49

	A	B	C	D	E
a.	+	–	+	0	+
b.	+	–	+	+	0
c.	–	+	–	0	0
d.	–	–	+	0	0

85. There exists a uniform electric field in the space as shown. Four points A, B, C and D are marked which are equidistant from the origin. If V_A, V_B, V_C and V_D are their potentials, respectively,

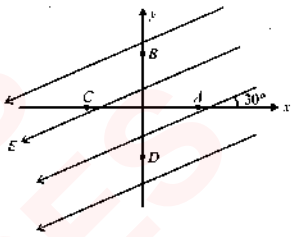


Fig. A1.50

a. $V_B > V_A > V_C > V_D$ **b.** $V_A > V_B > V_D > V_C$
c. $V_A = V_B > V_C = V_D$ **d.** $V_B > V_C > V_A > V_D$

86. The figure shows two large, closely placed, parallel, non-conducting sheets with identical (positive) uniform surface charge densities, and a sphere with a uniform (positive) volume charge density. Four points marked as 1, 2, 3 and 4 are shown in the space in between. If E_1, E_2, E_3 and E_4 are magnitude of net electric fields at these points, respectively, then

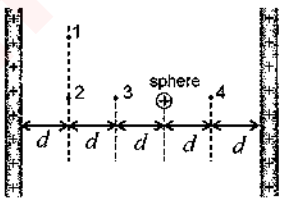


Fig. A1.51

a. $E_1 > E_2 > E_3 > E_4$ **b.** $E_1 > E_2 > E_3 = E_4$
c. $E_3 = E_4 > E_2 > E_1$ **d.** $E_1 = E_2 = E_3 = E_4$

87. Three positive point charges q_1, q_2 and q_3 form an isolated system. Suppose the charges have generated a property due to which like charges also attract. The charges are moving along a circle with same speed maintaining angles as shown in the figure. The charge q_1 experiences a force f_1 due to other two charges. Similarly, q_2 experiences a force f_2 and q_3, a force f_3. The ratio $f_1 : f_2 : f_3$ is

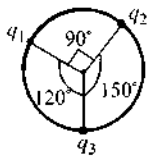


Fig. A1.52

a. 1 : 1 : 1 **b.** $q_1 : q_2 : q_3$
c. $1 : \sqrt{3} : 2$
d. this ratio cannot be calculated.

88. In the above question, the ratio of masses of the charge particles q_1, q_2 and q_3 is

a. 1 : 1 : 1 **b.** $q_1 : q_2 : q_3$
c. $1 : \sqrt{3} : 2$
d. this ratio cannot be calculated.

89. In a conducting hollow sphere of inner and outer radii 5 cm and 10 cm, respectively, a point charge 1 C is placed at

point A, that is 3 cm from the centre C of the hollow sphere. An external uniform electric field of magnitude 20 N/C is also applied. Net electric force on this charge is 15 N, away from the centre of the sphere as shown. Find the magnitude of net force exerted on the charge placed at point A by the induced charges on the sphere.

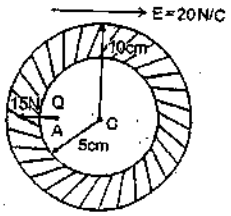


Fig. A1.53

a. 15 N **b.** 20 N **c.** 5 N **d.** 35 N

90. Consider two concentric metal spheres. The outer sphere is given a charge $Q > 0$, then

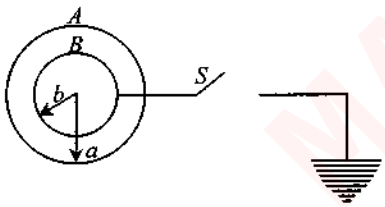


Fig. A1.54

a. the electrons will flow from earth to inner sphere if S is shorted
b. the electrons will flow from inner sphere to the earth if S is shorted
c. the shorting of S will produce a charge of $-Q$ on the inner sphere
d. none of the above

91. A conic surface is placed in a uniform electric field E as shown such that field is perpendicular to the surface on the side AB. The base of the cone is of radius R and height of the cone is h. The angle of cone is θ as shown. Find the magnitude of that flux which only enters the cone curved surface. Do not count the outgoing flux. Take $\theta < 45°$.

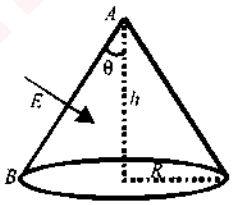


Fig. A1.55

a. $ER[h\cos\theta + \pi(R/2)\sin\theta]$
b. $ER(h\sin\theta + \pi R\cos\theta)$
c. $ER(h\cos\theta + \pi R\sin\theta)$
d. None of these

92. A wheel having mass μ has charges $+q$ and $-q$ on diametrically opposite points. It remains in equilibrium on a rough inclined plane in the presence of uniform vertical electric field $E =$

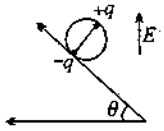


Fig. A1.56

a. $\mu g/q$ **b.** $\mu g/2q$ **c.** $\mu g \tan\theta/2q$ **d.** none

93. A, B, C, D, P and Q are points in a uniform electric field. The potentials at these points are $V(A) = 2$ V. $V(P) = V(B) = V(D) = 5$ V. $V(C) = 8$ V. The electric field at P is

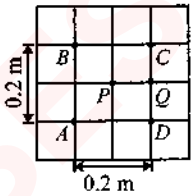


Fig. A1.57

a. 10 V/m along PQ **b.** $15\sqrt{2}$ V/m along PA
c. 5 V/m along PC **d.** 5 V/m along PA

94. Find the charge that will flow through the wire A to B if the switch S is closed. The capacitance of each capacitor shown in the figure is C.

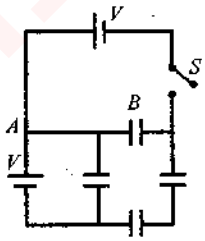


Fig. A1.58

a. CV **b.** $CV/3$ **c.** zero **d.** $2CV/3$

95. An infinite non-conducting sheet has surface charge density s. There is a small hole in the sheet as shown in the figure. A uniform rod of length ℓ having linear charge density λ is hinged in the hole as shown. If the mass of the rod is m, then the time period of oscillation for small angular displacement is

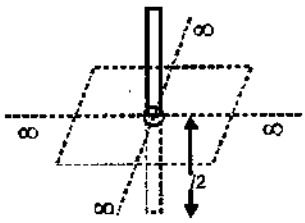


Fig. A1.59

a. $\pi\sqrt{\dfrac{m\varepsilon_0}{3\sigma\lambda}}$ **b.** $2\pi\sqrt{\dfrac{2m\varepsilon_0}{3\sigma\lambda}}$

c. $\dfrac{\pi}{2}\sqrt{\dfrac{m\varepsilon_0}{3\sigma\lambda}}$ **d.** $4\pi\sqrt{\dfrac{m\varepsilon_0}{3\sigma\lambda}}$

96. The flux passing through the shaded surface of a sphere when a point charge q is placed at the centre is

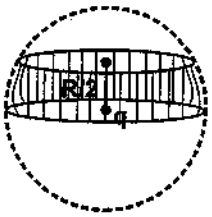


Fig. A1.60

a. q/ε_0 b. $q/2\varepsilon_0$ c. $q/4\varepsilon_0$ d. zero

97. The left plate of the capacitor shown in the figure carries a charge $+Q$ while the right plate is uncharged. The total final charge on the right plate after closing the switch S will be

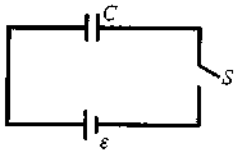


Fig. A1.61

a. $\frac{Q}{2}+C\varepsilon$ b. $\frac{Q}{2}-C\varepsilon$

c. $-\frac{Q}{2}$ d. none of these

98. Two point charges of $+Q$ each have been placed at the positions $(-a/2, 0, 0)$ and $(a/2, 0, 0)$. The locus of the points where $-Q$ charge can be placed such that total electrostatic potential energy of the system can become equal to zero, can be represented by which of the following equations?

a. $z^2 + (y-a)^2 = 2a$ b. $z^2 + (y-a)^2 = 27a^2/4$

c. $z^2 + y^2 = 15a^2/4$ d. None

99. Figure shows equipotential surfaces for a two charges system. At which of the labelled points will an electron have the highest potential energy?

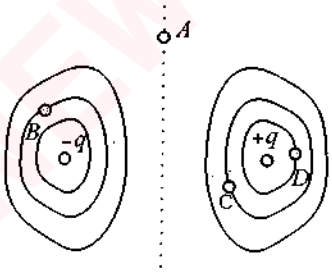


Fig. A1.62

a. Point A b. Point B c. Point C d. Point D

100. The effective capacitance between points A and B is (the capacitance of each of the capacitors is C)

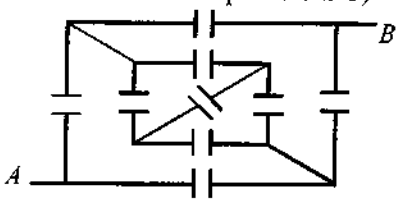


Fig. A1.63

a. C b. $C/2$ c. $36C/17$ d. $42C/17$

101. An electric dipole is placed in a uniform electric field $\vec{E}$ of magnitude 40 N/C. Graph shows the magnitude of the torque on the dipole versus the angle θ between the field $\vec{E}$ and the dipole moment $\vec{P}$. The magnitude of dipole moment $\vec{P}$ is equal to

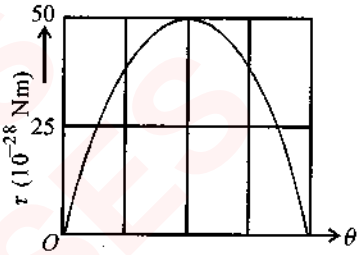


Fig. A1.64

a. 2.5×10^{-28} Cm b. 2.0×10^{-25} Cm

c. 1.25×10^{-28} Cm d. 5.0×10^{-28} Cm

102. Three large plates are arranged as shown. How much charge will flow through the key k if it is closed?

a. $\frac{5Q}{6}$ b. $\frac{4Q}{3}$

c. $\frac{3Q}{2}$ d. none

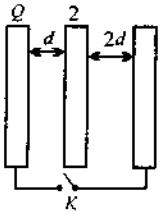
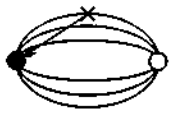
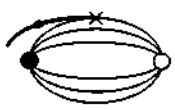


Fig. A1.65

103. The following diagrams show the electric field lines between two opposite charges. The positive charge is indicated by the black circle, whereas the negative charge by the white circle. An electron starting from rest at the indicated position (X), and accelerated to high speed by the electric field will follow most likely which trajectory?

a. 

b.

c. 

d.

104. Two identical conducting very large plates P_1 and P_2 having charges $+4Q$ and $+6Q$ are placed very closed to each other at separation d. The plate area of either face of the plate is A. The potential difference between plates P_1 and P_2 is

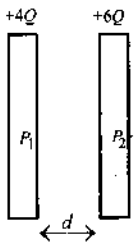


Fig. A1.66

a. $V_{P_1} - V_{P_2} = \frac{Qd}{A\varepsilon_0}$ b. $V_{P_1} - V_{P_2} = \frac{-Qd}{A\varepsilon_0}$

c. $V_{P_1} - V_{P_2} = \frac{5Qd}{A\varepsilon_0}$ d. $V_{P_1} - V_{P_2} = \frac{-5Qd}{A\varepsilon_0}$

105. In the above question, if plates P_1 and P_2 are connected by a thin conducting wire, then the amount of heat produced will be

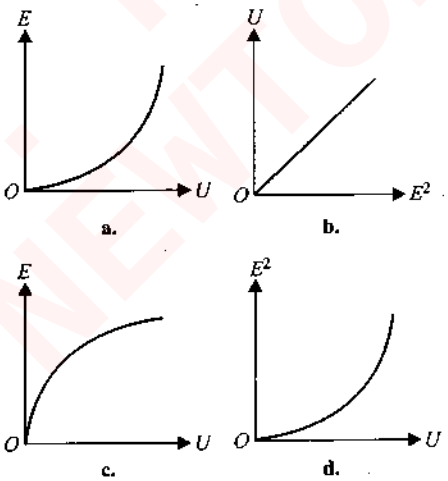
a. $\frac{Q^2}{A\varepsilon_0}d$ b. $\frac{5Q^2}{A\varepsilon_0}d$

c. $\frac{2Q^2}{A\varepsilon_0}d$ d. none of these

Multiple Correct Answers Type

Solutions on page A1.51

1. If at distance r from a positively charged particle, electric field strength and energy density are E and U, respectively, which of the following graphs is/are correct?



2. Mark out the incorrect statement(s).

a. A proton tends to go from a region of low electric potential to a region of high electric potential.

b. The electric potential of a negatively charged conductor must be negative.

c. If $\vec{E} = 0$ at a point P, then V must be zero at P.

d. If $V = 0$ at a point P, then $\vec{E}$ must be zero at P.

3. In a uniformly charged dielectric sphere, a very thin tunnel has been made along the diameter as shown in Fig. A1.67 below. A charge particle $-q$ having mass m is released from rest at one end of the tunnel.

For the situation described, mark out the correct statement(s). [Neglect gravity.]

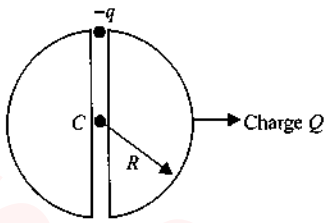


Fig. A1.67

a. Charge particle will perform S.H.M. about centre of the sphere as mean position.

b. Time period of the particle is $2\pi\sqrt{\frac{2\pi\varepsilon_0 mR^3}{qQ}}$.

c. Particle will perform oscillation but not S.H.M.

d. Speed of the particle while crossing the mean position is $\sqrt{\frac{qQ}{4\pi\varepsilon_0 mR}}$.

4. A charge particle q is projected in an electric field produced by a fixed point charge Q as shown in Fig. A1.68. Mark out the correct statements.

a. The path taken by q is a straight line.

b. The path taken by q is not a straight line.

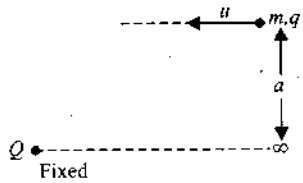


Fig. A1.68

c. The minimum distance between the two particles is $\frac{\frac{qQ}{2\pi\varepsilon_0} + \sqrt{\left(\frac{qQ}{2\pi\varepsilon_0}\right)^2 + 4m^2u^2a^2}}{2mu^2}$

d. Velocity of the particle q is changing in both magnitude and direction.

5. Four identical particles, each having mass m and charge q, are placed at the vertices of a square of side l. All the particles are free to move without any friction and released simultaneously from rest. Then,

a. at all instants, the particles remain at vertices of square whose edge length is changing

b. the configuration is changing (not remaining square) as the time passes

c. the speed of the particles when one of the particles gets displaced by $\frac{1}{\sqrt{2}}$ is $\sqrt{\frac{q^2}{8\pi\varepsilon_0 ml}\left(2+\frac{1}{\sqrt{2}}\right)}$

d. the speed of the particles cannot be found

6. A small sphere is charged uniformly and placed at some point $A(x_0, y_0)$ so that at point B(9 m, 4 m) electric field strength is $\vec{E} = (54\hat{i} + 72\hat{j})$ NC^{-1} and potential is 1800 V. Then

a. the magnitude of charge on the sphere is 4 μC
b. the magnitude of charge on the sphere is 2 μC
c. coordinates of A are $x_0 = -3$ m, $y_0 = -12$ m
d. coordinates of A are $x_0 = 4$ m, $y_0 = -1$ m

7. Two conducting plates M and N, each having large surface area A (on one side), are placed parallel to each other (Fig. A1.69). The plate M is given charge Q_1 and N, charge Q_2 ($<Q_1$). Then,
   a. electric field at point A is $\frac{Q_1 - Q_2}{2A\varepsilon_0}$ towards right
   b. electric field at point B is $\frac{Q_1 + Q_2}{2A\varepsilon_0}$ towards right
   c. electric potential of N is greater than M
   d. all of the above

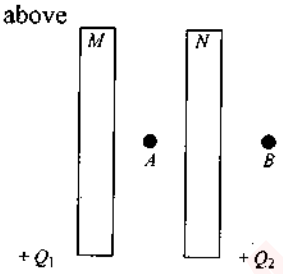


Fig. A1.69

8. A point charge q is placed within the cavity of an electrically neutral conducting shell whose outer surface has spherical shape (Fig. A1.70). Then,
   a. the potential V at point P lying outside the shell at a distance r from the centre O of the outer surface depends upon the value of x
   b. potential at P does not depend upon the value of x
   c. a total charge q will be induced on the outer surface of the shell which will be distributed uniformly on the outer surface
   d. a total charge $-q$ will be induced on the inner surface of the shell which will be distributed non-uniformly on the inner surface

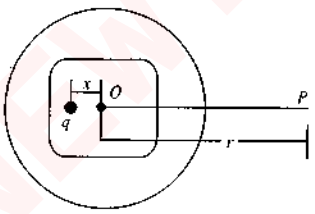


Fig. A1.70

9. In Fig. A1.71 the plates of a parallel plate capacitor have unequal charges. Its capacitance is C. P is a point outside the capacitor and close to the plate of charge $-Q$. The distance between the plates is d.

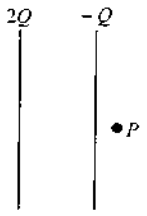


Fig. A1.71

Then,
   a. a point charge at point P will experience electric force due to the capacitor
   b. the potential difference between the plates will be $3Q/2C$
   c. the energy stored in the electric field in the region between the plates is $9Q^2/8C$
   d. the force on one plate due to the other plate is $Q^2/2\pi\varepsilon_0 d^2$

10. A particle of mass 2 kg and charge 1 mC is projected vertically with a velocity 10 ms^{-1}. There is a uniform horizontal electric field of 10^4 NC^{-1}. Then,
   a. the horizontal range of the particle is 10 m
   b. the time of flight of the particle is 2 s
   c. the maximum height reached is 5 m
   d. the horizontal range of the particle is 0

11. A thin-walled spherical conducting shell S of radius R is given charge Q. The same amount of charge is also placed at its centre C. Which of the following statements are correct?
   a. On the outer surface of S, the charge density is $\frac{1}{2}\frac{Q}{\pi R^2}$.
   b. The electric field is zero at all points inside S.
   c. At a point just outside S, the electric field is double the field at a point just inside S in the cavity.
   d. At any point inside S (i.e. within its cavity), the electric field is inversely proportional to the square of its distance from C.

12. The electric potential at a certain distance from a point charge is 600 V and the electric field is 200 NC^{-1}. Which of the following statements will be true?
   a. The work done in moving a point charge of 1 μC from the given point to a point at a distance of 9 m will be 4×10^{-4} J.
   b. The distance of the given point from the charge is 3 m.
   c. The potential at a distance of 9 m will be 200 V.
   d. The magnitude of charge is 0.2×10^{-3} C.

13. An electric charge 2×10^{-8} C is placed at the point (1, 2, 4). At the point (3, 2, 1), the electric
   a. field will increase by a factor K if the space between the points is filled with a dielectric of dielectric constant K
   b. field will be along the y-axis
   c. potential will be 49.9 V
   d. field will have no y-component

14. The electric potential in a region along the x-axis varies with x according to the relation $V(x) = 4 + 5 \times 2$. Then,
   a. potential difference between the points $x = 1$ and $x = -2$ is 15 V
   b. the force experienced by the above charge will be towards the $+x$-axis
   c. a uniform electric field exists in this region along the $+x$-axis
   d. force experienced by a 1 C charge at $x = -1$ m will be 10 N

15. When a positively charged sphere is brought near a metallic sphere, it is observed that a force of attraction exists between the two. It means
   a. the metallic sphere may be electrically neutral
   b. the metallic sphere is necessarily negatively charged
   c. nothing can be said about the charge of the metallic sphere
   d. the metallic sphere may be negatively charged

16. A conducting sphere of radius R has a charge. Then,
- **a.** the charge is uniformly distributed over its surface, if there is no external electric field
- **b.** the distribution of charge over its surface will be non-uniform if an external electric field exists in the space
- **c.** the potential at every point of the sphere must be the same
- **d.** the electric field strength inside the sphere will be equal to zero only when no external electric field exists

17. A, B and C are three large, parallel conducting plates, placed horizontally. A and C are rightly fixed and earthed (Fig. A1.72). B is given some charge. Under electrostatic and gravitational forces, B may be
- **a.** in equilibrium if it is closer to A than to C
- **b.** in equilibrium midway between A and C
- **c.** B can never be in stable equilibrium
- **d.** in equilibrium if it is closer to C than to A

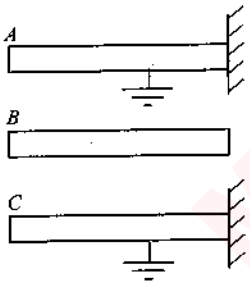


Fig. A1.72

18. A small conducting sphere of radius a mounted on an insulated handle and a positive charge q is inserted through a hole in the wall of a hollow conducting sphere of inner radius b and outer radius c. The hollow sphere is supported on an insulating stand and is initially uncharged. The small sphere is placed at the centre of the hollow sphere. Neglect any effect of the hole. Which of the following statements will be true for this system?
- **a.** No work will be done in carrying a small charge from the inner conductor to the outer conductor.
- **b.** The electric field at a point in the region between the spheres at a distance r from the centre is equal to $q/4\pi\varepsilon_0 r^2$.
- **c.** The electric field at a point outside the hollow sphere at a distance r from the centre is $q/4\pi\varepsilon_0 r^2$.
- **d.** The potential of the inner sphere with respect to the outer sphere is given by $V_{ab} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$.

19. Consider two identical charges placed distance $2d$ apart, along the x-axis (Fig. A1.73). The equilibrium of a positive test charge placed at point O midway between them is
- **a.** stable for displacements along the x-axis
- **b.** neutral
- **c.** unstable for displacement along the y-axis
- **d.** stable for displacements along the y-axis

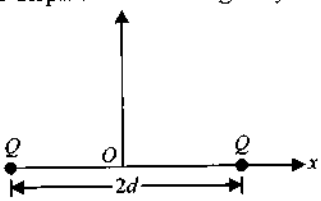


Fig. A1.73

20. Mark out the correct statements.
- **a.** A given conducting sphere cannot be charged to a potential greater than a certain value.
- **b.** A given conducting sphere can be charged to a potential less than a certain minimum value.
- **c.** A given conducting sphere can be charged to any extent.
- **d.** None of the above

21. For the situation shown in Fig. A1.74 (assume $r >>$ length of dipole) mark out the correct statement(s).

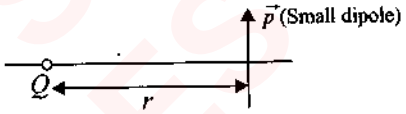


Fig. A1.74
- **a.** Force acting on the dipole is zero.
- **b.** Force acting on the dipole is approximately $\frac{pQ}{4\pi\varepsilon_0 r^3}$ and is acting upward.
- **c.** Torque acting on the dipole is $\frac{pQ}{4\pi\varepsilon_0 r^2}$ in clockwise direction.
- **d.** Torque acting on the dipole is $\frac{pQ}{4\pi\varepsilon_0 r^2}$ in anti-clockwise direction.

22. A dipole is placed in $x-y$ plane parallel to the line $y = 2x$. There exists a uniform electric field along z-axis. Net force acting on the dipole will be zero. But it can experience some torque. We can show that the direction of this torque will be parallel to the line
- **a.** $y = 2x + 1$
- **b.** $y = -2x$
- **c.** $y = -\frac{1}{2}x$
- **d.** $y = -\frac{1}{2}x + 2$

23. A hollow conducting sphere of inner radius r and outer radius $2R$ is given charge Q as shown in the figure, then the

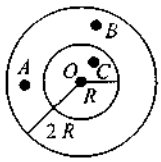


Fig. A1.75
- **a.** potential at A and B is same
- **b.** potential at O and B is same
- **c.** potential at O and C is same
- **d.** potential at A, B, C and O is same

24. An insulating spherical shell of uniform surface charge density is cut into two parts and placed at a distance d apart as shown in Fig. A1.76.

$\vec{E}_P$ and $\vec{E}_Q$ denote the electric fields at P and Q, respectively. As d (i.e. PQ) $\to \infty$

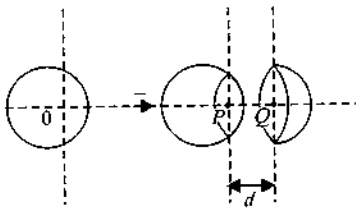


Fig. A1.76

a. $|\vec{E}_P| > |\vec{E}_Q|$ b. $|\vec{E}_P| = |\vec{E}_Q|$

c. $|\vec{E}_P| < |\vec{E}_Q|$ d. $\vec{E}_P + \vec{E}_Q = 0$

25. A point charge μ is placed at origin. Let $\vec{E}_A, \vec{E}_B$ and $\vec{E}_C$ be the electric field at three points A (1, 2, 3), B (1, 1, –1) and C (2, 2, 2) due to charge μ. Then

a. $\vec{E}_A \perp \vec{E}_B$ b. $\vec{E}_A \| \vec{E}_B$

c. $|\vec{E}_B| = 4|\vec{E}_C|$ d. $\vec{E}_B = 16|\vec{E}_C|$

26. Two identical parallel-plate capacitors are connected in one case in parallel and in the other in series. In each case the plates of one capacitor are brought closer by a distance a and the plates of the other are moved apart by the same distance a. Then

a. total capacitance of first system increases
b. total capacitance of first system decreases
c. total capacitance of second system decreases
d. total capacitance of second system remains constant

27. A charge Q is imparted to two identical capacitors in parallel. Separation of the plates in each capacitor is d_0. Suddenly, the first plate of the first capacitor and the second plate of the second capacitor start moving to the left with speed v, then

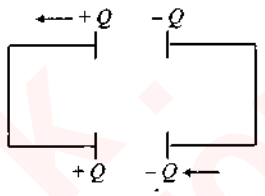


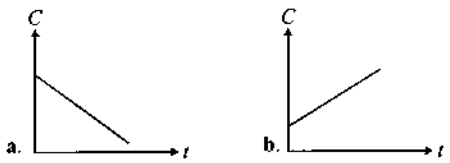
Fig. A1.77

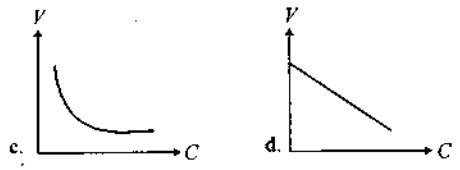
a. charges on the two capacitors as a function of time are $\frac{Q(d_0 - vt)}{2d_0}, \frac{Q(d_0 + vt)}{2d_0}$

b. charges on the two capacitors as a function of time are $\frac{Qd_0}{2(d_0 - vt)}, \frac{Qd_0}{2(d_0 + vt)}$

c. current in the circuit will increase as time passes on
d. current in the circuit will be constant

28. A parallel-plate capacitor has a dielectric slab in it. The slab just fills the space inside the capacitor. The capacitor is charged by a battery and then battery is disconnected. Now the slab is started to pull out slowly at $t = 0$. If at time t the capacitance of the capacitor is C and potential difference between the plates capacitor is V, then which of the following graphs is/are correct





29. Two plates of a parallel-plate capacitors carry charges q and $-q$ and are separated by a distance a from each other. The capacitor is connected to a constant voltage source V_0. The distance between the plates is changed to $x + dx$.

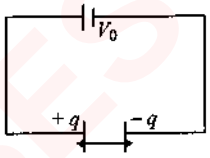


Fig. A1.78

Then in steady state

a. change in electrostatic energy stored in the capacitor is $-Udx/x$
b. change in electrostatic energy in the capacitor is $-Udx/dx$
c. attraction force between the plates is $1/2qE$
d. attraction force between the plates is qE (where E is electric field between the plates)

30. A charged particle having a positive charge q approaches a grounded metallic spheres of radius R with a constant small speed v as shown in Fig. A1.79.

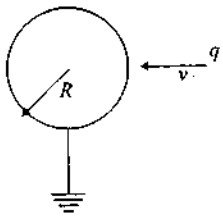


Fig. A1.79

In this situation,

a. as the charge draws nearer to the surface of the sphere, a current flows into the ground
b. as the charge draws nearer to the surface of the sphere, a current flows out of the ground into the sphere
c. as the charged particle draws nearer, the magnitude of the current flowing in the connector joining the shell to the ground increases
d. as the charged particle draws nearer, the magnitude of the current flowing in the connector joining the sphere to the ground decreases

31. A particle with a specific charge s starts from rest in a region where the electric field has a constant direction, but whose magnitude increases linearly with time. The particle acquires velocity v in time t.

a. $v \propto s$ b. $v \propto \sqrt{s}$

c. $v \propto t$ d. $v \propto t^2$

32. The electric potential at a point certain distance from a point charge is 600 V and the electric field is 200 NC^{-1}. Which of the following statements will be true?

a. The magnitude of the charge is 20×10^{-3} C.
b. The distance of the given point from the charge is 3 m.
c. The potential at a distance of 9 m will be 200 V.
d. The work done by an external agent in moving a point charge of 1 mC from the given point to a point at a distance of 9 m will be 4×10^{-4} J.

33. Three capacitors, each having capacitance $C = 2\ \mu F$, are connected with a battery of e.m.f. 30 V as shown in the figure.

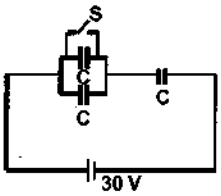


Fig. A1.80

When the switch S is closed,
a. the amount of charge flown through the battery is $20\ \mu C$
b. the heat generated in the circuit is 0.6 mJ
c. the energy supplied by the battery is 0.6 mJ
d. the amount of charge flown through the switch S is $60\ \mu C$

34. Three identical parallel conducting plates A, B and C are placed as shown. Switches S_1 and S_2 are open, and can connect A and C to earth when closed. $+Q$ charge is given to B.

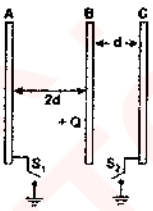


Fig. A1.81

a. If S_1 is closed with S_2 open, a charge of amount Q will pass through S_1.
b. If instead S_2 were closed with S_1 open, a charge of amount Q will pass through S_2.
c. If S_1 and S_2 are closed together, a charge of amount $Q/3$ will pass through S_1, and a charge of amount $2Q/3$ will pass through S_2.
d. If S_1 and S_2 are closed together, a charge of amount $2Q/3$ will pass through S_1, and a charge of amount $Q/3$ will pass through S_2.

35. A parallel-plate capacitor is charged from a cell and then isolated from it. The separation between the plates is now increased.
a. The force of attraction between the plates will decrease.
b. The field in the region between the plates will not change.
c. The energy stored in the capacitor will increase.
d. The potential difference between the plates will decrease.

36. The figure shows two point charges $2Q$ (> 0) and $-Q$. The charges divide the line joining them in three parts I, II and III.

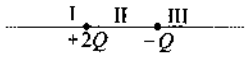


Fig. A1.82

a. Region III has a local maxima of electric field.
b. Region I has a local minima of electric field.
c. Equilibrium position for a test charge lies in region II.
d. The equilibrium (if there is any) is stable for a negative test charge.

Assertion-Reasoning Type

Solutions on page A1.55

In the following questions, each question contains Statement I (Assertion) and Statement II (Reason). Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.
a. Statement I is True, Statement II is True; Statement II is a correct explanation for Statement I.
b. Statement I is True, Statement II is True; Statement II is **NOT** a correct explanation for Statement I.
c. Statement I is True, Statement II is False.
d. Statement I is False, Statement II is True.

1. Statement I: When air between the plates of a parallel-plate condenser is replaced by an insulating dielectric medium, its capacity decreases.
Statement II: Electric field intensity between the plates with dielectric in between is reduced.

2. Statement I: If electric potential in certain region is constant, then the electric field must be zero in this region.
Statement II: $\vec{E} = -\dfrac{dV}{dr}\hat{r}$

3. Statement I: A conducting body is connected to the earth, hence it is electrically neutral.
Statement II: The potential of a conducting body connected to the earth is zero.

4. Statement I: A uniformly charged disk has a pin hole at its centre. The electric field at the centre of the disk is zero.
Statement II: Disk can be supposed to be made up of many rings. Also, electric field at the centre of a uniformly charged ring is zero.

5. Statement I: In a region where uniform electric field exists, the net charge within volume of any size is zero.
Statement II: The electric flux within any closed surface in region of uniform electric field is zero.

6. Statement I: Though large number of free electrons are present in a metal, yet there is no current in the absence of electric field.
Statement II: In the absence of electric field, electrons move randomly in all directions.

7. Statement I: The lightning conductor at the top of a high building has sharp pointed ends.
Statement II: The surface density of charge at sharp ends is very high resulting in setting up of electric wind.

8. Statement I: Electric field is always directed perpendicular to an equipotential surface.
Statement II: Equipotential surface is a surface on which at each point potential is same.

9. **Statement I:** Two positively charged conductors are put in contact, the common potential will be less than the initial potential of one of the conductors.
Statement II: Both conductors will attract each other.
10. **Statement I:** The circuits containing capacitor are handled cautiously, even when there is no current.
Statement II: A dielectric differs from an insulator.
11. **Statement I:** If distance between the parallel plates of a capacitor is halved, then its capacitance is doubled.
Statement II: The capacitance depends upon the introduced dielectric constant.
12. **Statement I:** If the plates of a parallel capacitor are not same in cross-sectional area, then the quantity of charge on the plates will be same but nature of charge will be different.
Statement II: They will have the same nature of charge.
13. **Statement I:** It is not possible to make a spherical capacitor of capacity 1 F.
Statement II: It is possible for the earth as its radius is 6.4×10^6 m.
14. **Statement I:** A dipole always tends to align in the direction of electric field.
Statement II: In this direction, torque acting on the dipole is zero.
15. **Statement I:** The electric field due to a dipole on its axial line at a distance r is E. Then electric field due to the same dipole on the equatorial line and at the same distance will be $E/2$.
Statement II: Electric field due to dipole varies inversely as the square of distance.

Comprehension Type

Solutions on page A1.56

For Problems 1–3

Gauss's law relates the net flux ϕ of an electric field through a closed surface to the net charge q_{in} that is enclosed by that surface.

$$\varepsilon_0 \phi = q_{in} \text{ or } \varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{in}$$

Charge outside the surface no matter how larger or how close it may be is not included in the term q_{in} in Gauss's law. The exact form or location of the charge inside the Gaussian surface is also of no concern. The electric field on the left-hand side of equation, however, is the net electric field resulting from all charges, both inside and outside the Gaussian surface. This may seem to be inconsistent, but keep in mind that the electric field due to a charge outside the Gaussian surface contributes zero net flux through the surface, because as many field lines due to that charge enter the surface as leave it.

1. Figure A1.83 shows five charged lumps of plastic and an electrically neutral coin. The cross section of Gaussian surface S is indicated. Assuming $q_1 = q_4 = +3.1$ nC, $q_2 = q_5 = -5.9$ nC and $q_3 = -3.1$ nC, the net electric flux through the surface is

a. $-666\ \text{Nm}^2\text{C}^{-1}$ **b.** $+666\ \text{Nm}^2\text{C}^{-1}$
c. $-360\ \text{Nm}^2\text{C}^{-1}$ **d.** $+360\ \text{Nm}^2\text{C}^{-1}$

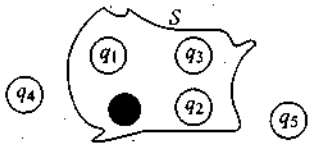


Fig. A1.83

2. A charge Q is uniformly distributed over a large plastic plate. The electric field at point P close to the centre of plate is 20 Vm^{-1}. If the plastic plate is replaced by aluminium plate of same geometrical dimensions, carrying the same charge Q, the electric field at the point P will become

a. zero **b.** 10 Vm^{-1} **c.** 20 Vm^{-1} **d.** 40 Vm^{-1}

3. In an electric field due to a point charge $+Q$, a spherical closed surface is drawn as shown by dotted circle (Fig. A1.84). The electric flux through the surface drawn is zero by Gauss's law. A conducting sphere is inserted intersecting the previously drawn Gaussian surface. The electric flux through the surface

Fig. A1.84

a. still remains zero **b.** is non-zero but positive
c. is non-zero but negative **d.** becomes infinite

For Problems 4–7

In many systems, equilibrium is possible due to the balancing of electric forces by other forces. We shall here consider a system which also involves force of buoyancy. A body placed in a fluid displaces fluid and it acts in the upward direction. Consequently, effective weight of the body is reduced. In Fig. A1.85 two identical helium-filled balloons of small size are tied to an object of mass 1.1 g with threads of equal lengths, each 1 m. Each balloon carries a charge q that can be assumed as if it were concentrated at the centre of balloon. The system floats in equilibrium as shown. Volume of helium in each balloon, in this situation, is V. The figure also shows that equilibrium separation between the balloons is 1.2 m. Helium is now replaced by another gas 'A' of a density twice that of helium. In this new situation, it is observed that volume of gas 'A' in each balloon has to be V' so as to result in equilibrium, under the condition that charge on each balloon retains its earlier value.

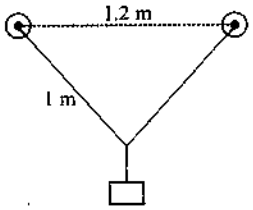


Fig. A1.85

Given density of air = 1.3 kgm^{-3}, density of He = 0.2 kgm^{-3}, $g = 10\ \text{ms}^{-2}$ and weight of unfilled balloon $\simeq 0$.
Now answer the following questions.

4. Volume of He in each balloon so that the system floats in equilibrium is
   a. 500 cc b. 800 cc c. 1125 cc d. 375 cc
5. Tension in each thread when the system is in equilibrium, with He filled in each balloon, will be
   a. 3.44×10^{-3} N b. 6.88×10^{-3} N
   c. 8.25×10^{-3} N d. 6.84×10^{-5} N
6. Charge on each balloon is nearly
   a. 1.5 μC b. 1.2 μC c. 0.8 μC d. 0.6 μC
7. Which of the following is correct?
   a. In both situations, i.e., with He in each balloon and then with gas A in each balloon, vertical component of tension has the same value in the equilibrium condition. This value is 5.5×10^{-3} N
   b. In case of He, vertical component of tension is 5.5×10^{-3} N but with gas A, it is 11×10^{-3} N
   c. In case of He, vertical component of tension is 11×10^{-3} N but with gas A, it is 5.5×10^{-3} N
   d. In both cases, vertical component of tension has the same value and is equal to 2.75×10^{-3} N

For Problems 8–10

In a certain region, electric field E exists along the x-axis which is uniform. Given $AB = 2\sqrt{3}$ m, $BC = 4$ m. Points A, B, C are in the x, y plane.

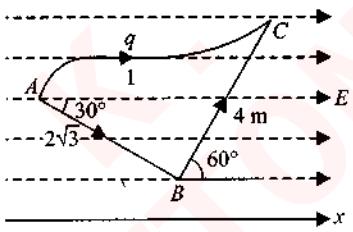


Fig. A1.86

8. Find the potential difference $V_A - V_B$ between the points A and B.
   a. E b. $2E$ c. $3E$ d. $2E$
9. Find potential difference $V_C - V_B$ between the points C and B.
   a. $-E$ b. $-2E$ c. $2E$ d. $3E$
10. A charged particle q is moved from A to C as shown in path 1. What is the work done by electric field in this process?
   a. qE b. $4qE$ c. $2qE$ d. $5qE$

For Problems 11–13

We know that electric field (E) at any point in space can be calculated using the relation

$$\bar{E} = -\frac{\delta V}{\delta x}\hat{i} - \frac{\delta V}{\delta y}\hat{j} - \frac{\delta V}{\delta z}\hat{k},$$

if we know the variation of potential (V) at that point. Now let the electric potential in volt along the x-axis vary as $V = 2x^2$, where x is in m. Its variation is as shown in Fig. A1.87.

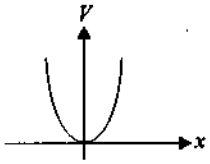
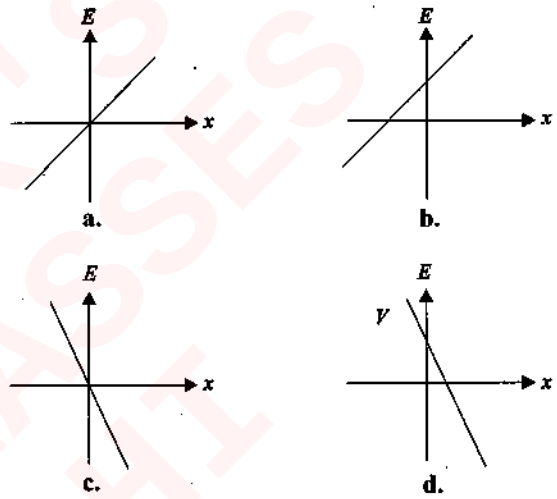


Fig. A1.87

11. Draw the variation of electric field (E) along the x-axis.



12. A charge particle of mass 10 mg and charge 2.5 μC is released from rest at $x = 2$ m. Find its velocity when it crosses origin.
   a. 0.5 ms^{-1} b. 1 ms^{-1} c. 2 ms^{-1} d. 4 ms^{-1}
13. Will the particle perform a simple harmonic motion? Also, find the time period of its oscillations.
   a. Yes, time period = 2π s
   b. No, time period = 2π s
   c. Yes, time period = 4π s
   d. The particle will perform S.H.M., but time period cannot be found from the given data.

For Problems 14–17

Consider a system of two equal point charges, each Q=8 μC, which are fixed at points (2 m, 0) and (−2 m, 0). Another charge μ is held at a point (0, 0.1 m) on the y-axis. Mass of the charge μ is 91 mg. At $t = 0$, μ is released from rest and it is observed to oscillate along y-axis in a simple harmonic manner. It is also observed that at $t = 0$, the force experienced by it is 9×10^{-3} N.

14. Charge q is
   a. −8 μC b. −6.5 μC c. −5 μC d. +6.5 μC
15. Amplitude of motion is
   a. 10 cm b. 20 cm c. 30 cm d. 40 cm
16. Frequency of oscillation is
   a. 8 b. 10 c. 5 d. 2
17. Equation of S.H.M. (displacement from mean position) can be expressed as
   a. $y = 0.1\sin(10\pi t)$ b. $y = 0.1\sin(10\pi t + \pi/2)$
   c. $y = 0.1\sin(5\pi t + \pi/2)$ d. $y = 0.2\sin(5\pi t)$

For Problems 18–21

In a certain experiment to measure the ratio of charge to mass of elementary particles, a surprising result was obtained in which

two particles moved in such a way that the distance between them always remained constant. It was also noticed that this two-particle system was isolated from all other particles and no force was acting on this system except the force between these two masses. After careful observation followed by intensive calculation, it was deduced that velocity of these two particles was always opposite in direction and magnitude of velocity was $10^3\ ms^{-1}$ and $2\times10^3\ ms^{-1}$ for first and second particle, respectively, and masses of these particles were 2×10^{-30} kg and 10^{-30} kg, respectively. Distance between them were 12Å (1Å=10^{-10}m).

18. Acceleration of the first particle was

a. zero b. $4\times10^{16}\ ms^{-2}$
c. $2\times10^{16}\ ms^{-2}$ d. $2.5\times10^{15}\ ms^{-2}$

19. Acceleration of the second particle was

a. $5\times10^{15}\ ms^{-2}$ b. $4\times10^{16}\ ms^{-2}$
c. $2\times10^{16}\ ms^{-2}$ d. zero

20. If the first particle is stopped for a moment and then released, the velocity of centre of mass of the system just after the release will be

a. $\frac{1}{3}\times10^{-30}\ ms^{-1}$ b. $\frac{1}{3}\times10^{3}\ ms^{-1}$
c. $\frac{2}{3}\times10^{3}\ ms^{-1}$ d. none of these

21. Paths of the two particles were

a. intersecting straight lines
b. parabolic
c. circular
d. straight line w.r.t. each other

For Problems 22–23

In the circuit shown in Fig. A1.88, initially the switch is opened. The switch is closed now.

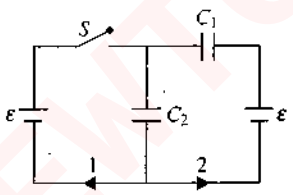


Fig. A1.88

22. The charge that will flow in direction '1' is

a. $-\frac{C_2^2\varepsilon}{C_1+C_2}$ b. $-\left(\frac{C_1C_2}{C_1+C_2}\right)\varepsilon$
c. $\frac{C_1^2\varepsilon}{C_1+C_2}$ d. $C_2\varepsilon$

23. The charge that will flow in direction '2' is

a. $-\frac{C_2^2\varepsilon}{C_1+C_2}$ b. $\left(\frac{C_1C_2}{C_1+C_2}\right)\varepsilon$
c. $\frac{C_1^2\varepsilon}{C_1+C_2}$ d. $C_2\varepsilon$

For Problems 24–26

Capacitors $C_1 = 2\ \mu F$ and $C_2 = 3\ \mu F$ are connected in series to a battery of e.m.f. $\varepsilon = 120$ V whose midpoint is earthed. The wire connecting the capacitors can be earthed through a key K. Now, key K is closed. Determine the charge flowing through the sections 1, 2 and 3 in the directions indicated in Fig. A1.89.

24. In section 1,

a. $-24\ \mu C$ b. $-36\ \mu C$ c. $-60\ \mu C$ d. $60\ \mu C$

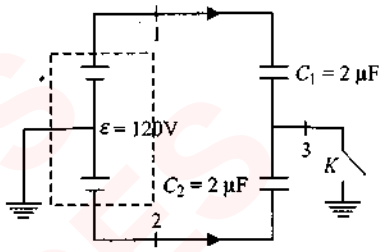


Fig. A1.89

25. In section 2,

a. $-24\ \mu C$ b. $-36\ \mu C$ c. $-60\ \mu C$ d. $60\ \mu C$

26. In section 3,

a. $-24\ \mu C$ b. $-36\ \mu C$ c. $-60\ \mu C$ d. $60\ \mu C$

For Problems 27–28

A researcher studying the properties of ions in the upper atmosphere wishes to construct an apparatus with the following characteristics: Using an electric field, a beam of ions, each having charge q, mass m, and initial velocity $v\hat{i}$, is turned through an angle of 90° as each ion undergoes displacement $R\hat{i}+R\hat{j}$. The ions enter a chamber as shown in Fig. A1.90 and leave through the exit port with the same speed they had when they entered the chamber. The electric field acting on the ions is to have constant magnitude.

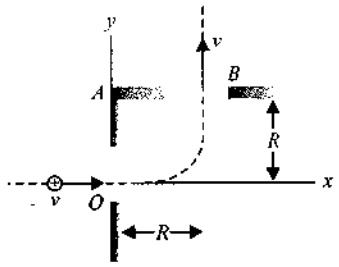


Fig. A1.90

27. Suppose the electric field is produced by two concentric cylindrical electrodes not shown in the diagram, and hence is radial. What magnitude should the field have?

a. $\frac{mv^2}{2qR}$ centred at A b. $\frac{mv^2}{2qR}$ centred at O
c. $\frac{mv^2}{qR}$ centred at A d. $\frac{mv^2}{qR}$ centred at O

28. If the field is produced by two flat plates and is uniform in direction, what value should the field have in this case?

a. $\frac{mv^2}{2qR}(\hat{i}+\hat{j})$ b. $\frac{mv^2}{2qR}(-\hat{i}+\hat{j})$
c. $\frac{2mv^2}{qR}(\hat{i}-\hat{j})$ d. $\frac{2mv^2}{qR}(-\hat{i}+\hat{j})$

For Problems 29–32

A and B are two capacitors having air as the dielectric medium between the plates. They have the same separation between the plates but the area of plates of A is twice that of B. A and B are connected in series with a 12 V dc supply as shown in Fig. A1.91.

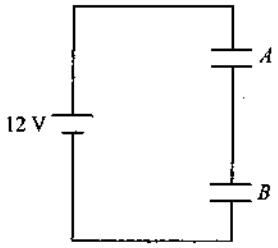


Fig. A1.91

Without removing the battery, a dielectric material ($K=6$) is uniformly filled between the plates of capacitor B. The dimensions of dielectric are just enough to completely fill the space between the plates of B. As a result, electric energy stored in the two capacitors changes by 1.2×10^{-4} J. The capacitors are now disconnected from the battery and also from each other and the dielectric ($K = 6$) that had been filled between the plates of B is also removed.

Consider now the following two situations:

i. The capacitors are combined such that positive plate of A is connected to negative plate of B and vice versa.

ii. The capacitors are combined such that positive plate of A is connected to the positive plate of B and negative plate of A is connected to negative plate of B.

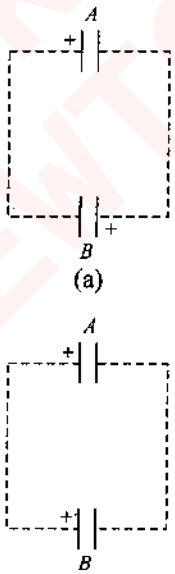


Fig. A1.92

Now answer the following questions.

29. Initially, A and B, with air as the dielectric between the plates of both the capacitors, are connected in series with 12 V supply. As described above, dielectric ($K = 6$) is then inserted between the plates of B. Consequently, electric energy stored in capacitor B

a. reduces by 10^{-5} J **b.** increases by 2×10^{-5} J
c. reduces by 2×10^{-5} J **d.** increases by 10^{-5} J

30. Work done in removing the dielectric from capacitor B is

a. 3.4×10^{-4} J **b.** 2.7×10^{-4} J
c. 1.8×10^{-4} J **d.** zero

31. In situation (i), the final value of electric energy stored by the capacitor is

a. 4.3×10^{-4} J **b.** 5.6×10^{-4} J
c. 3.2×10^{-4} J **d.** zero

32. In situation (ii), the final value of electric energy stored by the capacitors is

a. 4.3×10^{-4} J **b.** 7.3×10^{-4} J **c.** 2.6×10^{-4} J **d.** zero

For Problems 33–35

An isolated parallel-plate capacitor consists of two metal plates of area A and separation d. A slab of thickness t and dielectric constant $K = 2$ is inserted between the plates with its faces parallel to the plates and having the same surface area as that of the plates as shown (Fig. A1.93)

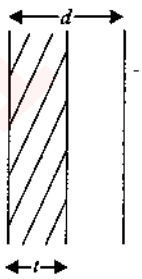


Fig. A1.93

33. The capacitance of the system is

a. $\dfrac{\varepsilon_0 A}{\left(d-\frac{t}{2}\right)}$ **b.** $\dfrac{\varepsilon_0 A}{\left(d+\frac{t}{2}\right)}$ **c.** $\dfrac{\varepsilon_0 A}{d-t}$ **d.** $\dfrac{\varepsilon_0 A}{d+t}$

34. For what value of t/d, will the capacitance of system be (3/2) times that of the capacitance with air filling the full space?

a. $\dfrac{2}{3}$ **b.** $\dfrac{3}{2}$ **c.** 1 **d.** $\dfrac{1}{3}$

35. The ratio of energy in first case (with air) to that of second case (with dielectric) will be

a. $\dfrac{2}{3}$ **b.** $\dfrac{3}{2}$ **c.** 1 **d.** $\dfrac{1}{3}$

For Problems 36–40

A point charge q_1 is placed inside cavity 1 and another point charge q_2 is inside cavity 2. A point charge q is placed outside the conductor. For the situation described above, answer the following questions.

36. The charge on outer surface of the conductor would be

a. $Q+q_1+q_2$ and non-uniformly distributed

b. $Q+q_1+q_2$ and its uniform or non-uniform distribution depends upon locations of q_1 and q_2

c. $Q+q_1+q_2$ and would be distributed uniformly

d. $Q+q_1+q_2$ and the distribution depends upon the locations of q_1, q_2 and q

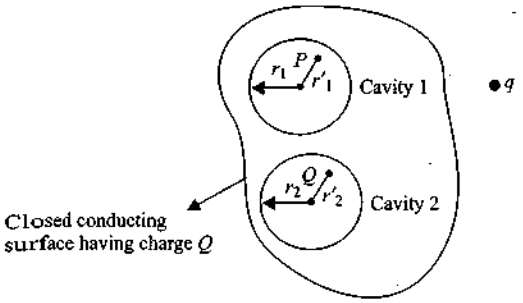


Fig. A1.94

37. If q_1 is at the centre of cavity 1, then $\vec{E}$ at point S, at a distant r from the centre of cavity 1($r > r_1$), due to the induced charge on the surface of cavity 1 is

a. $\frac{q_1}{4\pi\varepsilon_0 r^2}$ away from the centre of cavity 1

b. $\frac{q_1}{4\pi\varepsilon_0 r_1^2}$ away from the centre of cavity 1

c. zero

d. $\frac{q_1}{4\pi\varepsilon_0 r^2}$ towards the centre of cavity 1

38. $\vec{E}$ inside the conductor at point S distant r from point charge q, due to charge on outer surface of the conductor, would be

a. $\frac{Q+q_1+q_2}{4\pi\varepsilon_0 r^2}$ away from charge q

b. $\frac{q}{4\pi\varepsilon_0 r^2}$ towards charge q

c. zero

d. cannot be determined

39. If charge q_2 is at point Q (inside cavity 2), then $\vec{E}$ at the centre of cavity 2 due to induced charge on the surface of cavity 2 would be

a. $\frac{q_2}{4\pi\varepsilon_0 r_2'^2}$ towards q_2

b. $\frac{q_2}{4\pi\varepsilon_0 r_2'^2}$ away from q_2

c. zero

d. cannot be determined

40. If the potential of the conductor is V_0 and charge q_2 is placed at the centre of cavity 2, then potential at point Q is

a. $\frac{q_2}{4\pi\varepsilon_0 r_2'}+V_0$

b. $\frac{q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_2'}+\frac{1}{r_2}\right)+V_0$

c. $\frac{q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_2'}-\frac{1}{r_2}\right)+V_0$

d. V_0

For Problems 41–44

Four concentric hollow spheres of radii R, $2R$, $3R$ and $4R$ are given the charges as shown in Fig. A1.95. Then the conductors 1 and 3, 2 and 4 are connected by conducting wires (both the connections are made at the same time).

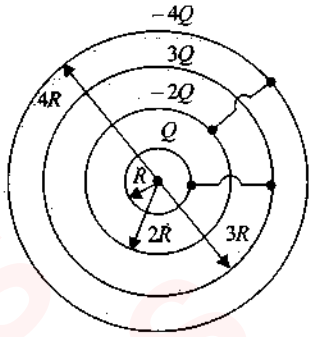


Fig. A1.95

41. The charge on the inner surface of the third conductor is

a. $-6Q/5$ b. $6Q/5$. c. $-2Q$ d. $+2Q$

42. The charge on the fourth conductor is

a. $\frac{22Q}{5}$ b. $+\frac{11Q}{3}$ c. $-\frac{11Q}{3}$ d. $-\frac{22Q}{5}$

43. The potential of conductor 1 is

a. $\frac{3Q}{40\pi\varepsilon_0 R}$

b. $-\frac{19Q}{40\pi\varepsilon_0 R}$

c. $-\frac{3Q}{40\pi\varepsilon_0 R}$

d. $-\frac{19Q}{40\pi\varepsilon_0 R}$

44. The potential of conductor 2 is

a. $-\frac{Q}{8\pi\varepsilon_0 R}$

b. $\frac{Q}{8\pi\varepsilon_0 R}$

c. $\frac{Q}{32\pi\varepsilon_0 R}$

d. $-\frac{Q}{32\pi\varepsilon_0 R}$

For Problems 45–48

Some cell walls in the human body have a layer of negative charge on the inside surface and a layer of positive charge of equal magnitude on the outside surface. Consider one such cell having the cell wall of thickness 10^{-10} m. The charge densities on the walls are $-\sigma$ and $+\sigma Cm^{-2}$, respectively, and the relative permittivity of the cell wall material is 5. Taking the volume based on the above information, answer the following questions.

45. Which wall is at a higher potential?

a. Inner

b. Outer

c. Both are at the same potential

d. Cannot be determined from the given information

46. Determine the electric field intensity in between the cell walls.

a. $\frac{\sigma}{5\varepsilon_0}$ NC^{-1}

b. $\frac{\sigma}{2\varepsilon_0}$ C NC^{-1}

c. $\frac{\sigma}{5\varepsilon_0}\times10^{-5}$ NC^{-1}

d. $\frac{\sigma}{2\varepsilon_0}\times10^{-5}$ NC^{-1}

47. The potential difference between inside and outside walls of the cell is

a. $\frac{\sigma}{5\varepsilon_0}\times10^{-5}$ V b. $\frac{\sigma}{2\varepsilon_0}\times10^{-5}$ V

c. $\frac{\sigma}{5\varepsilon_0}\times10^{-10}$ V d. $\frac{\sigma}{2\varepsilon_0}\times10^{-10}$ V

48. The energy stored in the cell wall is

a. $\frac{\sigma^2}{5\varepsilon_0}\times10^{-5}$ J b. $\frac{\sigma^2}{2\varepsilon_0}\times10^{-15}$ J

c. $\frac{\sigma^2}{2\varepsilon_0}\times10^{-16}$ J d. $\frac{\sigma^2}{\varepsilon_0}\times10^{-16}$ J

For Problems 49–50

A parallel-plate capacitor is connected to a battery as shown in Fig. A1.96. A thin conducting plate is inserted mid-way between the two plates. Takes plate area as A.

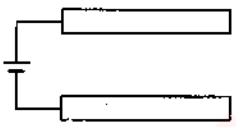


Fig. A1.96

49. What is the capacitance of the system after the plate is inserted?

a. $\frac{\varepsilon_0 A}{d}$ b. $\frac{\varepsilon_0 A}{2d}$

c. $\frac{2\varepsilon_0 A}{d}$ d. Cannot say anything

50. If the thin and upper plates are shortened, the extra charge flown through the battery is

a. $\frac{2\varepsilon_0 AV}{d}$ b. zero c. $\frac{\varepsilon_0 AV}{d}$ d. $\frac{\varepsilon_0 AV}{2d}$

For Problems 51–53

Two plates of a parallel-plate capacitor are connected to a battery as shown in Fig. A1.97. The separation between the plates is increased from x to $x + dx$ (very slowly). Take plate area as A.

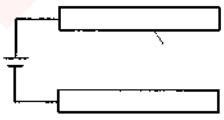


Fig. A1.97

51. What is the change in electrostatic energy stored in the capacitor?

a. $-\frac{\varepsilon_0 AV^2}{2}\times\frac{dx}{x^2}$ b. $\frac{\varepsilon_0 AV^2 dx}{2x^2}$

c. $\frac{\varepsilon_0 AV^2}{2x}$ d. Remains constant

52. The charge flown out from positive terminal of battery and work done by battery is [during the process]

a. $-\frac{\varepsilon_0 AVdx}{x^2}, \frac{\varepsilon_0 AV^2 dx}{x^2}$ b. $\frac{\varepsilon_0 AVdx}{x^2}, \frac{\varepsilon_0 AV^2 dx}{x^2}$

c. $\frac{\varepsilon_0 AVdx}{x^2}, -\frac{\varepsilon_0 AV^2 dx}{x^2}$ d. $-\frac{\varepsilon_0 AVdx}{x^2}, -\frac{\varepsilon_0 AV^2 dx}{x^2}$

53. Work done by external agent is

a. $\frac{\varepsilon_0 AV^2 dx}{x^2}$

b. $-\frac{\varepsilon_0 AV^2}{2}\times\frac{dx}{x^2}$

c. $\frac{\varepsilon_0 AV^2}{2}\times\frac{dx}{x^2}$

d. cannot be calculated, as force with which the plates have been pulled is not given

For Problems 54–56

A simple electric dipole consists of a positive and negative charge of equal magnitude held very close to one another. The components of the electric field pointing away from a dipole has magnitude $E = 2kp\cos\theta/d^3$ where d is the distance from the centre of the dipole to the point in question, $K = 9.0\times10^9$ Nm^{2}C^{-2} is a universal constant and q is the magnitude of the dipole moment vector, which specifies the strength and direction of the dipole. Here θ denotes the angle between the dipole moment vector and d, the displacement vector (from the dipole to the point in question).

A student performs an experiment to determine whether a mystery object is an electric dipole. (The mystery object is only a

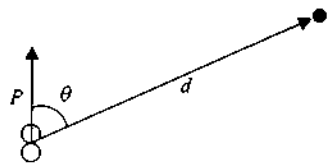


Fig. A1.98

few millimeters long.) Using a sophisticated instrument, the student measures the component of the electric field pointing away from the object, at various distances from the centre of the object. By taking each measurement along an imaginary line emanating outward from the centre of the mystery object, he ensures that θ remains the same throughout the experiment. Table A1.1 shows the electric field he found at various distances.

Table A1.1

Trial	Distance (m)	Field (NC^{-1})
1	0.010	3.6×10^{-10}
2	0.020	9.0×10^{-11}
3	0.030	4.0×10^{-11}

54. From the given information, can we calculate the electrostatic force that would act on a point charge $q = 25\times10^{-7}$ held at the location where the student measured the electric field in trial 2?

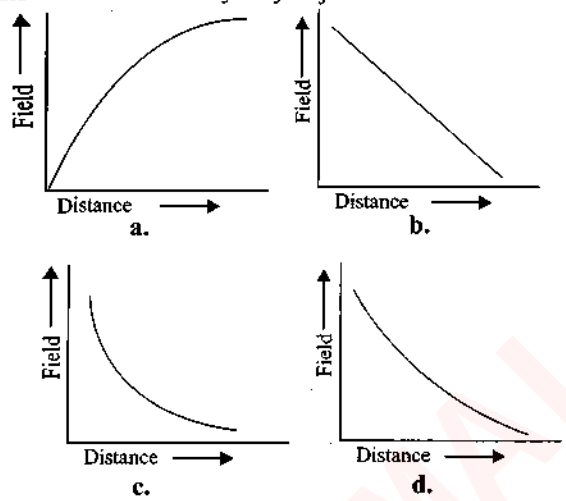
a. Yes, because we can use $E = 2kp\cos\theta/r^3$ with $r = 0.02$ m.

b. Yes, because we know the electric field at the relevant point.

c. No, because the formula $F = kq_1q_2/r_2$ does not apply and we do not know the dipole moment.

d. No, because even though $F = kq_1q_2/r_2$ applies, we do not know the dipole moment.

55. Which of the following graphs best determines how the electric field measured by the student varies with the distance from the mystery object?



56. Consider the electric field produced by a dipole. If both the dipole moment and the distance from the dipole are doubled, while θ is kept the same, them the electric field component pointing away from the dipole decreases by a factor of

a. 2 b. 4 c. 8 d. 1

For Problems 57–58

The electric field in a certain region of space obeys $E_y \neq 0$,

$$E_x = E_z = 0 \text{ and } \frac{\partial \vec{E}}{\partial x} \neq 0, \frac{\partial \vec{E}}{\partial y} = \frac{\partial \vec{E}}{\partial z} = 0$$

57. The net force on an electric dipole oriented parallel to the x-axis in this field is

a. directed along the x-axis
b. directed along the y-axis
c. directed along the z-axis
d. none of the above

58. The net torque on an electric dipole parallel to the x-axis in this field is

a. directed along the x-axis
b. directed along the y-axis
c. directed along the z-axis
d. none of the above

For Problems 59–60

A large sheet carries uniform surface charge density σ. A rod of length $2l$ has a linear charge density λ on one half and $-\lambda$ on the other half. The rod is hinged at midpoint O and makes an angle θ with the normal to the sheet.

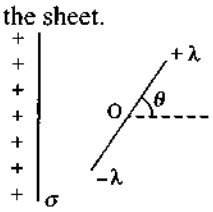


Fig. A1.99

59. The net force experienced by the rod is

a. $\dfrac{\sigma\lambda l}{2\varepsilon_0}$ b. $\dfrac{\sigma\lambda l}{\varepsilon_0}$

c. zero d. none of these

60. The torque experienced by the rod is

a. $\dfrac{\sigma\lambda l^2}{2\varepsilon_0}\cos\theta$ b. $\dfrac{\sigma\lambda l^2 \sin\theta}{2\varepsilon_0}$

c. $\dfrac{\sigma\lambda l}{2\varepsilon_0}\cos^2\theta$ d. $\dfrac{\sigma\lambda l \sin^2\theta}{2\varepsilon_0}$

For Problems 61–63

Three metallic plates out of which the middle is given charge Q as shown in the figure. The outer plates can be earthed with the help of switches S_1 and S_2. The area of each plates is the same.

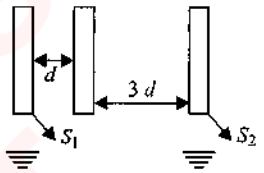


Fig. A1.100

Answer the following question based on the following passage.

61. The charge appearing on the outer surface of extreme left plate is

a. $-(Q/2)$ b. $(Q/2)$ c. Q d. $-Q$

62. The charge that will flow to the earth when only switch S_1 is connected to the earth is

a. $-(Q/2)$ b. $(Q/2)$ c. Q d. $-Q$

63. The charge that will flow to the earth through S_2 when both the switches S_1 and S_2 are grounded simultaneously is

a. Q b. $-Q$ c. $\frac{3}{4}Q$ d. $\frac{Q}{4}$

For Problems 64–66

Three concentric spherical conductors A, B and C of radii r $2R$ and $4R$, respectively. A and C is shorted and B is uniformly charged (charge $+Q$).

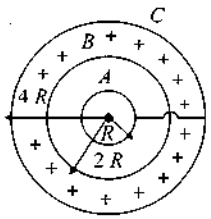


Fig. A1.101

64. Charge on conductor A is

a. $Q/3$ b. $-Q/3$ c. $2Q/3$ d. $-2Q/3$

65. Potential at A is

a. $\dfrac{Q}{4\pi\varepsilon_0 R}$ b. $\dfrac{Q}{16\pi\varepsilon_0 R}$ c. $\dfrac{Q}{20\pi\varepsilon_0 R}$ d. $\dfrac{5Q}{48\pi\varepsilon_0 R}$

66. Potential at B is

a. $\frac{Q}{4\pi\varepsilon_0 R}$ b. $\frac{Q}{16\pi\varepsilon_0 R}$ c. $\frac{Q}{48\pi\varepsilon_0 R}$ d. $\frac{5Q}{48\pi\varepsilon_0 R}$

For Problems 67–69

Two capacitors of capacity 6 μF and 3 μF are charged to 100 V and 50 V separately and connected as shown in Fig. A1.102. Now all the three switches S_1, S_2 and S_3 are closed.

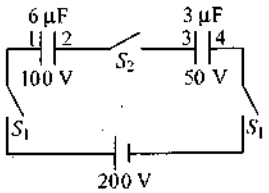


Fig. A1.102

67. Which plate(s) form an isolated system?
a. Plate 1 and plate 4 separately
b. Plate 2 and plate 3 separately
c. Plate 1 and plate 4 jointly
d. Plate 2 and plate 3 jointly

68. Charges on 6 μF and 3 μF capacitors in steady state will be
a. 400 μC, 400 μC b. 700 μC, 700 μC
c. 800 μC, 350 μC d. 300 μC, 450 μC

69. Let q_1, q_2 and q_3 be the magnitudes of charges flown from switches S_1, S_2 and S_3 after they are closed. Then
a. $q_1 = q_3$ and $q_2 = 0$ b. $q_1 = q_3 = q_2/2$
c. $q_1 = q_3 = 3q_2$ d. $q_1 = q_2 = q_3$

For Problems 70–72

Most capacitors have a non-conducting material between their conducting plates. A solid dielectric placed between the plates of capacitor serves the following three functions:

i. It solves the problems of maintaining two large metal sheets at a very small separation without actual contact.
ii. It increases the maximum possible potential difference which can be applied between the plates of capacitor without the dielectric breakdown.
iii. It increases the capacity of the capacitor. When *a* dielectric material is inserted between the plates (keeping the charge to be constant) the electric field and hence potential difference decreases by *a* factor K. Electric field decreases because an induced charge of the opposite sign appears on each surface of dielectric.

70. Two metal plates having charges Q and $-Q$ face each other at some separation and are dipped into an oil tank. If all oil is pumped out, the electric field between the plates will be
a. increased b. decreased
c. remain same d. become zero

71. A dielectric slab is inserted between the plates of an isolated charged capacitor. Which of the following quantities will remain same?
a. The electric field in the capacitor
b. The charge on capacitor
c. The potential difference between the plates
d. Stored energy in the capacitor

72. In a parallel-plate capacitor, the region between the plates is filled by a dielectric slab. The capacitor is connected to a cell and the slab is taken out then,
a. some charge is drawn from cell
b. some charge is returned to the cell
c. the potential difference across the capacitor is reduced
d. no work is done by an external agent in taking the slab out

For Problems 73–75

As shown in Fig. A1.103 $m_A = m_B = 1$ kg, block A is neutral while block B carries charge -1 C. Sizes of A and B are negligible. Block B is released from rest at a distance 1.8 m from A. Initially, spring is neither compressed nor stretched.

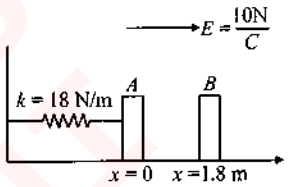


Fig. A1.103

73. If collision between A and B is perfectly inelastic, what is the velocity of combined mass just after collision?
a. 6 m/s b. 3 m/s c. 9 m/s d. 12 m/s

74. Equilibrium position of the combined mass is at $x =$ _____ m.
a. $-\frac{2}{9}$ b. $-\frac{1}{3}$
c. $-\frac{5}{9}$ d. $-\frac{7}{9}$

75. The amplitude of oscillation of the combined mass will be
a. $\frac{2}{3}$ m b. $\frac{\sqrt{124}}{3}$ m
c. $\frac{\sqrt{72}}{9}$ m d. $\frac{\sqrt{106}}{9}$ m

For Problems 76–78

A charged particle is suspended at the centre of two thin concentric spherical charged shells, made of non-conducting material. Figure A shows cross section of the arrangement. Figure B gives the net flux ϕ through a Gaussian sphere centred on the particle, as a function of the radius r of the sphere.

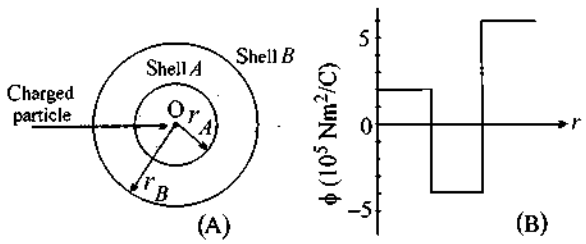


Fig. A1.104

76. What is the charge on the central particle?
a. 0.2 μC b. 2 μC
c. 1.77 μC d. 3.4 μC

77. What is the charge on shell A?
a. 5.31 μC b. −5.31 μC
c. −3.54 μC d. −1.77 μC

78. In which range of the values of r is the electric field zero?
a. 0 to r_A b. r_A to r_B
c. for $r > r_B$
d. for no range of r, electric field is zero

For Problems 79–81

Capacitor C_3 in the circuit is a variable capacitor (its capacitance can be varied). C_1 and C_2 are of fixed values. Graph is plotted between potential difference V_1 (across capacitor C_1) versus C_3. Electric potential V_1 approaches on asymptote of 10 V as $C_3 \to \infty$.

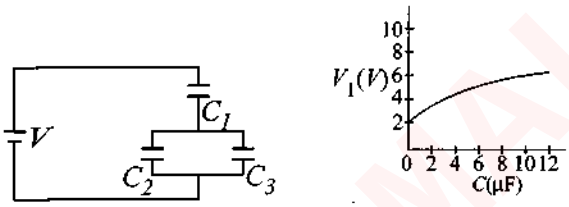


Fig. A1.105

79. The electric potential V across the battery is equal to
a. 10 V b. 12 V
c. 16 V d. 20 V

80. Relation between C_1 and C_2 is
a. $C_1 = C_2$ b. $C_1 = 4C_2$
c. $4C_1 = C_2$ d. any relation

81. When $V_1 = 4$ V, then C_3 is equal to
a. $5C_2/2$ b. $5C_1/3$
c. $5C_2/3$ d. none of these

Matching Column Type

Solutions on page A1.65

1. A conducting shell of inner radius R_1 and outer radius R_2 is given a charge $+Q$. A point charge q_1 is placed inside the shell and q_2 is placed outside the shell. Then for various locations of q_1 and q_2, match the entries of Column I with the entries of Column II.

Column I	Column II
i. If q_1 is at centre and $q_2 = 0$, then $\vec{E}$ at centre of shell due to charge on outer surface of shell is	a. $\dfrac{q_1}{4\pi\varepsilon_0 r^2}$
ii. If q_1 is not at centre and q_2 is at distance r from the centre, then $\vec{E}$ at the inner surface of shell (at a point closest to q_2) due to charge on outer surface of the shell is	b. $\dfrac{q_2}{4\pi\varepsilon_0 (r-R_1)^2}$
iii. If q_1 is at centre and q_2 is at distance r from the centre, then $\vec{E}$ at a point distant r_2 ($> r$) from the centre of the shell due to outer surface charge is	c. zero
iv. If q_1 is not at centre and $q_2 = 0$, then $\vec{E}$ at point P (P is at distance r from q_1 due to charge of the inner surface of shell is	d. cannot be determined

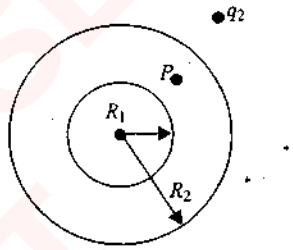


Fig. A1.106

2. For the situation shown in Fig. A1.107, match the entries of Column I with the entries of Column II.

Column I	Column II
i. If we displace the inside charge,	a. distribution of charge on the inner surface of the conductor is uniform
ii. If we displace the outside charge keeping the inside charge at centre	b. distribution of charge on the inner surface of the conductor is non-uniform
iii. If both the charges are displaced	c. distribution of charge on the outer surface of the conductor is uniform
iv. If outside charge is not present and inside charge is at centre,	d. distribution of charge on the outer surface of the conductor is non-uniform

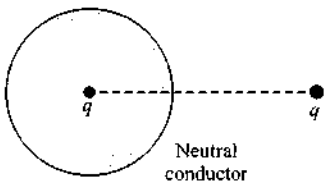


Fig. A1.107

3.

Column I	Column II
i. When a dielectric slab is gradually inserted between the plates of an isolated parallel-plate capacitor,	a. the electric potential energy of the system decreases
ii. When a dielectric slab is gradually inserted between the plates of a parallel-plate capacitor and its potential is kept constant,	b. work done by external agent is positive
iii. When the plates of a parallel-plate capacitor are pulled apart keeping its potential constant,	c. work done by battery is positive
iv. When the plates of a parallel-plate capacitor are pulled apart, keeping its charge constant,	d. work done by external agent is negative

4. A capacitor of capacitance C is charged to a potential V. Now it is connected to a battery of e.m.f. E as shown in Fig. A1.108. Based on this information, match the entries of Column I with the entries of Column II in the following table.

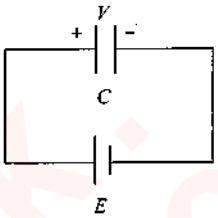


Fig. A1.108

Column I	Column II
i. If $V = E$, then	a. charge flows in the circuit
ii. If $V > E$, then	b. no charge flows in the circuit
iii. If $V < E$, then	c. non-zero thermal energy will be dissipated in the circuit
iv. If the plates of capacitor are shorted, then	d. outer surfaces of the plates of capacitor have zero charge

5. In Fig. A1.109, the separation between the plates of C_1 is slowly increased to double of its initial value. Now match the entries in columns I and II.

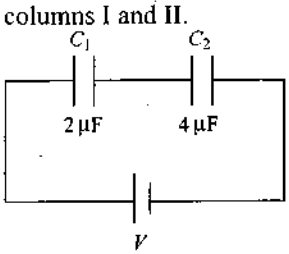


Fig. A1.109

Column I	Column II
i. The potential difference across C_1	a. increases
ii. The potential difference across C_2	b. decreases
iii. The energy stored in C_1	c. increases by a factor of 6/5
iv. The energy stored in C_2	d. decreases by a factor of 18/25

6.

Column I	Column II
i. Spherical charged conductor	a. at the surface, electric field is continuous and maximum
ii. Spherical non-conductor having uniform volume distribution of charge	b. at the surface, electric field is discontinuous
iii. Charged ring	c. electric field is uniform
iv. Infinite sheet of charge	d. at the centre, electric field is zero

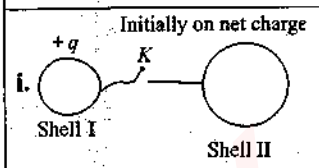
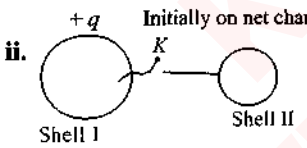
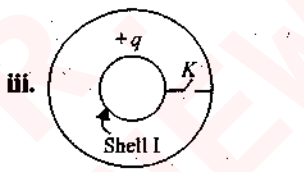
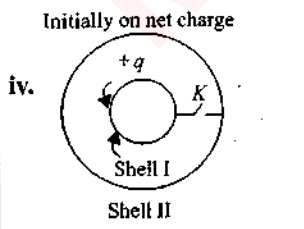
7.

Column I	Column II
i. Infinite sheet of charge	a. uniform non-zero electric field intensity
ii. At the centre of uniformly charged solid sphere	b. uniform decreasing potential
iii. Due to infinite line charge assuming potential to be zero at infinity	c. infinite potential everywhere
iv. Inside a charged conducting sphere	d. maximum magnitude of potential

8. A solid conducting sphere of radius a is placed inside a conducting shell of radius b so that both are concentric. Now, the shell is given a charge Q. Match the following:

Column I	Column II
i. Charge appearing on the inner sphere	a. zero
ii. Charge appearing on the inner sphere after it is earthed	b. Q
iii. Electric field intensity (E) inside the inner sphere (before earthing)	c. Qa/b
iv. Electric field intensity (E) inside the inner sphere after it is earthed	d. $\frac{1}{4\pi\varepsilon_0}\frac{Q}{ab}$

9. In the following table, Column I gives certain situations involving two thin conducting shells connected by a conducting wire via key K. In all situations, one sphere has net charge $+q$ and other sphere has no net charge. After key K is pressed, Column II gives some resulting effect. Match the figures in Column I with the statements in Column II.

Column I	Column II
i. 	a. Charge flows through the connecting wire
ii. 	b. Potential energy of system of spheres decreases
iii. 	c. No heat is produced
iv. 	d. Sphere I has no charge after equilibrium is reached

10. Figure A1.110(a) shows a uniformly charged ring of radius r. Its axis is along the X-axis and the ring is in the YZ plane. Point P can be anywhere on the X-axis and p' in the XY plane.
Figure A1.110(b) shows a uniformly charged disk of radius r. Its axis is along the X-axis and the disk is in the YZ plane. Point P can be anywhere on the X-axis and p' in the XY plane.

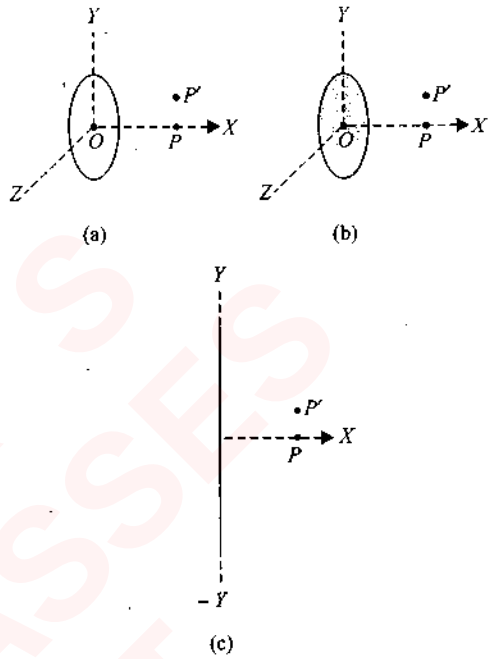


Fig. A1.110

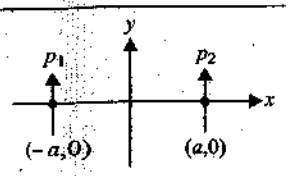
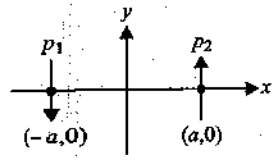
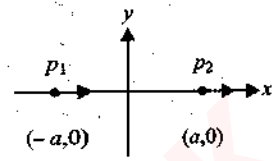
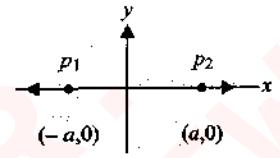
Figure A1.110(c) shows an infinite line charged uniformly placed along y-axis. P is a point on X-axis and P' in the XY plane. E_x, E_y and E_z are the components of electric field along X-,y-, and z-axis and m and $E_{p'}$ are net electric fields at P and P', respectively.
Now match the following columns

Column I	Column II
i. $E_x = 0,\ E_y = 0,\ E_z = 0$	a. Point P' in part (c)
ii. $E_x \neq 0, E_y = 0,\ E_z = 0$	b. Point P' in part (b)
iii. $E_x \neq 0, E_y \neq 0,\ E_z = 0$	c. Point P in part (c)
iv. $E_P = E_{P'}$	d. Point P in part (a) and (b)

11. An electric dipole is placed in a uniform external electric field. θ is the angle between the dipole moment and the field direction. In general, the dipole rotates under a torque. With reference to the behaviour of the dipole in an electric field, match Column I with Column II.

Column I	Column II
i. Potential energy of the dipole is maximum	a. conserved
ii. Angular acceleration of the dipole is maximum	b. not conserved
iii. Angular momentum of the dipole	c. $\theta = 180°$
iv. Kinetic energy of the dipole	d. $\theta = 90°$

12. In each situation of Column I, two electric dipoles having dipole moments $\vec{p}_1$ and $\vec{p}_2$ of same magnitude (that is $p_1 = p_2$) are placed on the x-axis symmetrically about origin in different orientations as shown. In Column II, certain inferences are drawn for these two dipoles. Then, match the different orientations of dipoles in Column I with the corresponding results in Column II.

Column I	Column II
 i. ($\vec{p}_1$ and $\vec{p}_2$ are perpendicular to the x-axis as shown)	a. The torque on one dipole due to other is zero
 ii. ($\vec{p}_1$ and $\vec{p}_2$ are perpendicular to the x-axis as shown)	b. The potential energy of one dipole in electric field of other dipole is negative
 iii. ($\vec{p}_1$ and $\vec{p}_2$ are parallel to the x-axis as shown)	c. There is one straight line in $x-y$ plane (not at infinity) which is equipotential
 iii. ($\vec{p}_1$ and $\vec{p}_2$ are parallel to the x-axis as shown)	d. Electric field at origin is zero.

13.

Column I	Column II
i. $\lvert\vec{E}_{axial}\rvert$ for a dipole	a. $\dfrac{\lvert\vec{P}\rvert}{4\pi\varepsilon_0 r^3}$
ii. $\lvert\vec{E}_{equipotential}\rvert$ for a dipole	b. $\dfrac{2\lvert\vec{P}\rvert}{4\pi\varepsilon_0 r^3}$
iii. When dipole is in uniform electric field when $\theta = 0°$	c. $F_{net} = 0,\ \tau_{net} = 0$
iv. When dipole is in non-uniform electric field when $\theta = 0°$	d. $F_{net} \neq 0,\ \tau_{net} = 0$

14. A certain electric field is given as

$$E = -[(2xy + z^2)\hat{i} + (2yz + x^2)\hat{j} + (2zx + y^2)\hat{k}]$$

Column I	Column II
i. Work done by electric field in taking a unit charge from (0, 0, 0) to (3 4 0) along a straight line is	a. –36 units
ii. Work done by electric field in taking a charge from (3, 4, 0) to (0, 0, 0) along a straight line is	b. +36 units
iii. Work done by external agent in taking a charge from (3, 4, 0) to (6, 8, 0) without change in K.E. is	c. zero unit
iv. Net charge enclosed in a sphere of radius 5 units centred at the origin is	d. +252 units

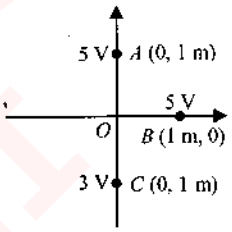


Fig. A1.111

15. An uniform electric field exists in straight line in the $X-Y$ plane. The potentials at different points in region are shown in the figure. Now match the following table where charge particles P and Q (mass = 10^{-6} kg), charge = $-2\sqrt{2}$ μC are released from the origin.

Column I	Column II
i. Coordinate of position of particle P at time $t = 2\sqrt{2}$ sec is	a. $(4\sqrt{2}\text{ m}, 4\sqrt{2}\text{ m})$
ii. Coordinate of position of particle Q at time $t = 2$ sec is	b. $(-4\sqrt{2}\text{ m}, -4\sqrt{2}\text{ m})$
iii. Distance travelled by particle Q in 2 sec is	c. 8 m
iv. Distance travelled by particle P in 2 sec is	d. 4 m

Integer Answer Type

Solutions on page A1.67

1. Two points charges $q_1 = q_2 = 2$ μC are fixed at $x_1 = +3$ m and $x_2 = -3$ m as shown. A third particle of mass 1 g and charge $q_3 = -4$ μC is released from rest at $y = 4$ m. Find the speed (in terms of the nearest integer in m/s) of the particle as it reaches the origin.

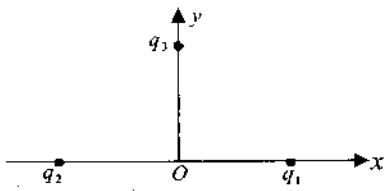


Fig. A1.112

2. Two concentric spherical conducting shells of radii R and $2R$ are carrying charges q and $2q$, respectively. Both are now connected by a conducting wire. Find the change in electric potential (in V) on the outer shell.

3. When a dielectric slab of thickness $d/2$ is inserted between the plates of a capacitor (with d as the separation between the plates), the capacitance of a capacitor becomes 3/4 times the original value. Find the dielectric constant of the slab.

4. A particle having charge that of an electron and mass 1.6×10^{-30} kg is projected with an initial speed u at an angle 45° to the horizontal from the lower plate of a parallel-plate capacitor as shown in the figure. The plates are sufficiently long and have separation 2 cm. The maximum value of velocity of particle not to hit the upper plate is $\sqrt{W} \times 10^6$ m/s. Take electric field between the plates as 10^3 V/m directed upward. Find W.

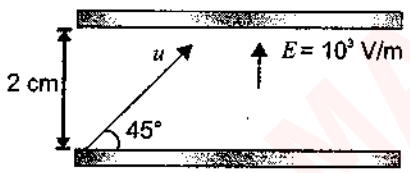


Fig. A1.113

5. The electric field in a region is radially outward with magnitude $E = ar$. If $a = 100$ V/m² and $R = 0.30$ m, then the value of charge contained in a sphere of radius R centred at the origin is $W \times 10^{-10}$ coulomb. Find W.

6. A wire is bent in the form of a regular hexagon and a total charge q is distributed uniformly on it. What is the electric field at the centre? You may answer this part without making any numerical calculations.

7. A uniform electric field of 10 N/C exists in the vertically downward direction. Find the increase in the electric potential (in V) as one goes up through a height of 50 cm.

8. The kinetic energy of a charged particle decreases by 10 J as it moves from a point at potential 100 V to a point at potential 200 V. Find the charge on the particle. (in $\times 10^{-1}$ C)

9. A charge of 20 μC is placed on the positive plate of an isolated parallel-plate capacitor of capacitance 10 μF. Calculate the potential difference (in V) developed between the plates.

10. A capacitor with stored energy 4.0 J is connected with an identical capacitor with no electric field in between. Find the total energy stored (in J) in the two capacitors.

11. Four point charges, each of $+q$, are rigidly fixed at the four corners of a square planar soap film of side 'a'. The surface tension of the soap film is γ. The system of charges and planar film is in equilibrium and $a = k[q^2/\gamma]^{1/N}$, where 'k' is a constant. Then N is _____. **(IIT-JEE, 2011)**

12. Excess electrons are placed on a small lead sphere of mass 6.90 g so that its net charge is -3.20×10^{-9} C. Find the number of excess electrons (in terms of $\times 10^{10}$) on the sphere.

13. The angles of inclination of the suspended charged pendulum (having same charge in each case) are 30° and 60° due to electric field of the infinite sheets of surface charge densities σ_1 and σ_2, respectively. Then find σ_2/σ_1.

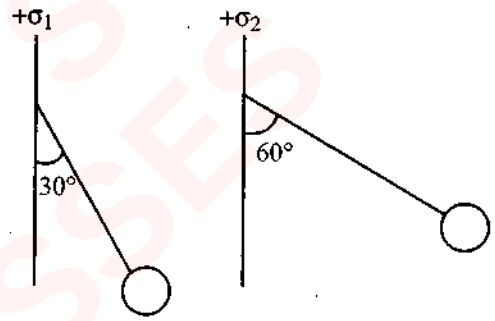


Fig. A1.114

14. The magnitude of the resultant force on a charge of 1 μC held at P due to two charges of $+2.0 \times 10^{-8}$ C and -10^{-8} C at A and B, respectively is found to be $18\sqrt{a} \times 10^{-3}$ N. Find a.

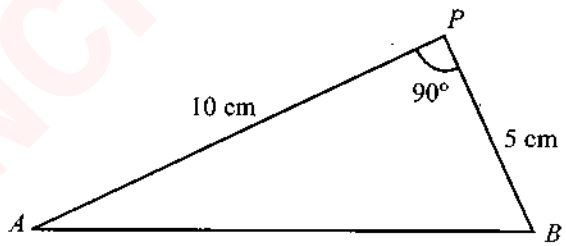


Fig. A1.115

15. The earth has a net electric charge that causes a field at points near its surface equal to 150 N/C and directed in toward the centre of the earth. What magnitude of charge (in C) would a 60 kg man have to acquire to overcome his weight by the force exerted by the earth's electric field?

16. In the figure shown, find the ratio of the linear charge densities λ_1 (on semi-infinite straight wire) and λ_2 (on semi-circular part), that is, λ_1/λ_2 so that the field O is along y direction.

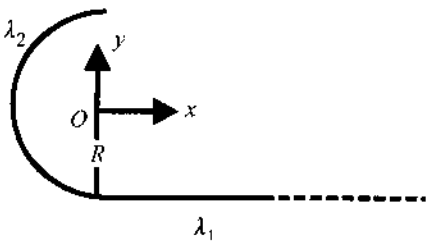


Fig. A1.116

17. A point charge q is placed at the origin. The net flux of electric field passing through the curved surface S is found to be $Nq/64\varepsilon_0$. Find the value of N.

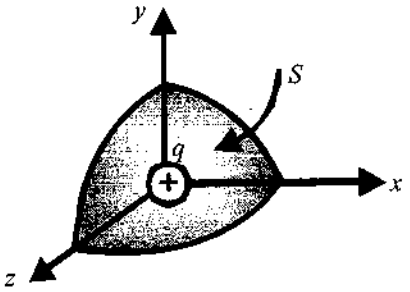


Fig. A1.117

Archives

Solutions on page A1.69

Fill in the Blanks Type

1. Five identical capacitor plates, each of area A, are arranged such that adjacent plates are at distance d apart. The plates are connected to a source of e.m.f. V as shown in Fig. A1.118.

(IIT-JEE, 1984)

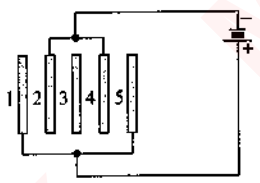


Fig. A1.118

2. Figure A1.119 shows line of constant potentials in a region in which an electric field is present. The values of the potentials are written in brackets. Of the points A, B and C, the magnitude of the electric field is greater at the point _____.

(IIT-JEE, 1984)

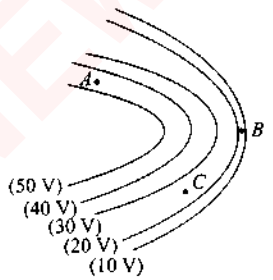


Fig. A1.119

3. Two small balls, having equal positive charge Q (coulomb) on each, are suspended by two insulating strings of equal length L (metre) from a hook fixed to a stand. The whole setup is taken in a satellite into space where there is no gravity (state of weightlessness). The angle between the two strings is _____ and the tension in each string is _____ newton.

(IIT-JEE, 1986)

4. Two parallel-plate capacitors of capacitances C and $2C$ are connected in parallel and charged to potential difference V. The battery is then disconnected and the region between the plates of the capacitor C is completely filled with a material of dielectric constant K. The potential difference across the capacitors now becomes _____

(IIT-JEE, 1988)

5. A point charge q moves from point P to point S along the path $PQRS$ in a uniform electric field E pointing parallel to the positive direction of the x-axis (Fig. A1.120). The coordinates of the points P, Q, R and S are $(a\ b\ 0)$ $(2a\ 0\ 0)(a^{-b}\ 0)$ and $(0, 0, 0)$, respectively. The work done by the field in the above process is given by the expression _____.

(IIT-JEE, 1989)

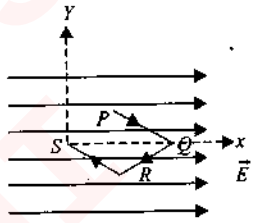


Fig. A1.120

6. The electric potential V at any point x, y, z (all in metres) in space is given by $V = 4x^2$ V. The electric field at the point (1 m, 0, 2 m) is _____ Vm^{-1}.

(IIT-JEE, 1992)

7. Five point charges, each of value $+q$ coulomb, are placed on five vertices of a regular hexagon of side L metres. The magnitude of the force on the point charge of value $-q$ coulomb placed at the centre of the hexagon is _____ newton.

(IIT-JEE, 1992)

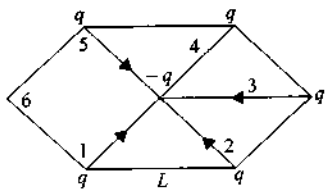


Fig. A1.121

True or False Type

1. The work done in carrying a point charge from one point to another in an electrostatic field depends on the path along which the point charge is carried.

(IIT-JEE, 1981)

2. Two identical metallic spheres of exactly equal masses are taken. One is given a positive charge Q coulomb and the other an equal negative charge. Their masses after charging are different.

(IIT-JEE, 1983)

3. A small metal ball is suspended in a uniform electric field with the help of an insulated thread. If a high-energy X-ray beam falls on the ball, then the ball will be deflected in the direction of the field.

(IIT-JEE, 1983)

4. Two protons A and B are placed in between the two plates of a parallel-plate capacitor charged to a potential difference V as shown in the figure. The forces on the two protons are identical.

(IIT-JEE, 1986)

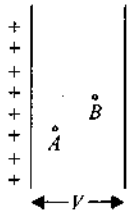


Fig. A1.122

5. A ring of radius r carries a uniformly distributed charge $+Q$. A point charge $-q$ is placed on the axis of the ring at a distance $2R$ from the centre of the ring and released from rest. The particle executes a simple harmonic motion along the axis of the ring.

(IIT-JEE, 1988)

Multiple Choice Questions with One Correct Answer Type

1. A hollow metal sphere of radius 5 cm is charged such that the potential on its surface is 10 V. The potential at the centre of the sphere is
   a. zero
   b. 10 V
   c. same as at a point 5 cm away from the surface
   d. same as at a point 25 cm away from the surface

(IIT-JEE, 1983)

2. Two point charges $+q$ and $-q$ are held fixed at $(-d, 0)$ and $(d, 0)$, respectively, of a $x-y$ coordinate system. Then
   a. electric field E at all points on the x-axis has the same direction.
   b. electric field at all points on the y-axis is along the x-axis.
   c. work has to be done in bringing a test charge from ∞ to the origin.
   d. the dipole moment is $2qd$ along the x-axis.

(IIT-JEE, 1995)

3. A parallel-plate capacitor of capacitance C is connected to a battery and is charged to a potential difference V. Another capacitor of capacitance $2C$ is similarly charged to potential difference $2V$. The charging battery is now disconnected and the capacitors are connected in parallel to each other in such a way that the positive terminal of one is connected to the negative terminal of the other. The final energy of the conuration is
   a. zero
   b. $\frac{3}{2}CV^2$
   c. $\frac{25}{6}CV^2$
   d. $\frac{9}{2}CV^2$

(IIT-JEE, 1995)

4. An electron of mass m_e initially at rest moves through a certain distance in a uniform electric field in time t_1. A proton of mass m_p also initially at rest takes time t_2 to move through an equal distance in this uniform electric field. Neglecting the effect of gravity, the ratio t_2/t_1 is nearly equal to
   a. 1
   b. $(m_p/m_e)^{1/2}$
   c. $(m_e/m_p)^{1/2}$
   d. 1836

(IIT-JEE, 1997)

5. A non-conducting ring of radius 0.5 m carries a total charge of 1.11×10^{-10} C distributed non-uniformly on its circumference producing an electric field E everywhere in space. The value of the integral $\int_{l=\infty}^{l=0} -\vec{E}.d\vec{l}$ ($l = 0$ being centre of the ring) in volts is
   a. +2
   b. −1
   c. −2
   d. zero

(IIT-JEE, 1997)

6. Two identical metal plates are given positive charges Q_1 and $Q_2(< Q_1)$, respectively. If they are brought close together to form a parallel-plate capacitor with capacitance C, the potential difference between them will be
   a. $(Q_1+Q_2)/(2C)$
   b. $(Q_1+Q_2)/C$
   c. $(Q_1-Q_2)/C$
   d. $(Q_1-Q_2)/(2C)$

(IIT-JEE, 1999)

7. For the circuit shown in Fig. A1.123 which of the following statements is true?

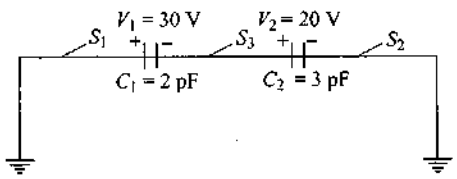


Fig. A1.123

   a. With S_1 closed, $V_1 = 15$ V, $V_2 = 20$ V
   b. With S_3 closed, $V_1 = V_2 = 25$ V
   c. With S_1 and S_2 closed, $V_1 = V_2 = 0$
   d. With S_1 and S_2 closed, $V_1 = 30$ V, $V_2 = 20$ V

(IIT-JEE, 1999)

8. Three charges q, $+q$ and $+q$ are placed at the vertices of a right-angled isosceles triangle as shown (Fig. A1.124). The net electrostatic energy of the configuration is zero if Q is equal to
   a. $\frac{-q}{1+\sqrt{2}}$
   b. $\frac{-2q}{2+\sqrt{2}}$
   c. $-2q$
   d. $+q$

(IIT-JEE, 2000)

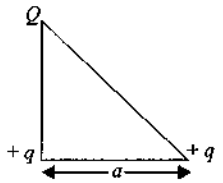


Fig. A1.124

9. A parallel-plate capacitor of area A, plate separation d and capacitance C is filled with three different dielectric materials having dielectric constants k_1, k_2 and k_3 as shown (Fig. A1.125). If a single dielectric material is to be used to have the same capacitance C in this capacitor, then its dielectric constant k is given by

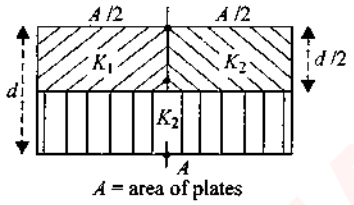


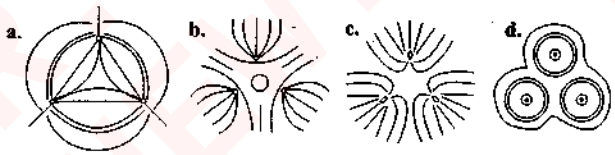
Fig. A1.125

a. $\frac{1}{k}=\frac{1}{k_1}+\frac{1}{k_2}+\frac{1}{2k_3}$ b. $\frac{1}{k}=\frac{1}{k_1+k_2}+\frac{1}{2k_3}$

c. $k=\frac{k_1k_2}{k_1+k_2}+2k_3$ d. $k=k_1+k_2+2k_3$

(IIT-JEE, 2000)

10. Three positive charges of equal value q are placed at the vertices of an equilateral triangle. The resulting lines of force should be sketched as in



(IIT-JEE, 2001)

11. Consider the situation shown in Fig. A1.126. The capacitor A has a charge q on it, whereas B is uncharged. The charge appearing on the capacitor B a long time after the switch is closed is

(IIT-JEE, 2001)

a. zero b. $q/2$
c. q d. $2q$

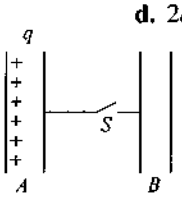


Fig. A1.126

12. A uniform electric field pointing in the positive x-direction exists in a region. Let A be the origin, B be the point on the x-axis at $x = +1$ cm and C be the point on the y-axis at $y = +1$ cm. Then, the potentials at the points A, B and C satisfy

(IIT-JEE, 2001)

a. $V_A < V_B$ b. $V_A > V_B$
c. $V_A < V_C$ d. $V_A > V_C$

13. Two equal point charges are fixed at $x = -a$ and $x = +a$ on the x-axis. Another point charge Q is placed at the origin. The change in the electrical potential energy of Q, when it is displaced by a small distance x along the x-axis, is approximately proportional to

(IIT-JEE, 2002)

a. x b. x^2
c. x^3 d. $1/x$

14. Two identical capacitors have the same capacitance C. One of them is charged to potential V_1 and the other to V_2. The negative ends of the capacitors are connected together. When the positive ends are also connected, the decrease in energy of the combined system is

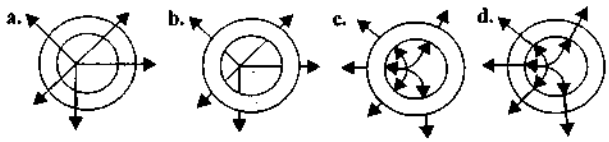
a. $\frac{1}{4}C(V_1^2-V_2^2)$ b. $\frac{1}{4}C(V_1^2+V_2^2)$

c. $\frac{1}{4}C(V_1-V_2)^2$ d. $\frac{1}{4}C(V_1+V_2)^2$

(IIT-JEE, 2002)

15. A metallic shell has a point charge 'q' kept inside its cavity. Which one of the following diagrams correctly represents the electric lines of force?

(IIT-JEE, 2003)



16. Six charges of equal magnitude, three positive and three negative, are to be placed on $PQRSTU$ corners of a regular hexagon, such that field at the centre is double that of what it would have been if only one +ve charge is placed at R.

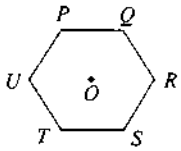


Fig. A1.127

a. +,+,+,−,−,−
b. −,+,+,+,−,−
c. −,+,+,−,+,−
d. +,−,+,−,+,− **(IIT-JEE, 2004)**

17. A Gaussian surface in Fig. A1.128 is shown by dotted line.

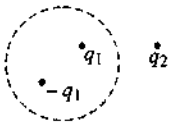


Fig. A1.128

The electric field on the surface will be

a. due to q_1 and q_2 only
b. due to q_2 only
c. zero
d. due to all **(IIT-JEE, 2004)**

18. Three infinitely long charge sheets are placed as shown in Fig. A1.129.

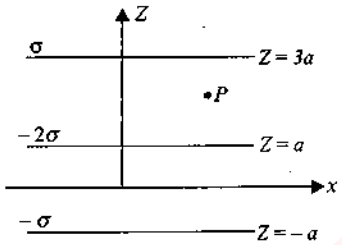


Fig. A1.129

The electric field at point P is

a. $\frac{2\sigma}{\varepsilon_0}\hat{k}$
b. $\frac{4\sigma}{\varepsilon_0}\hat{k}$
c. $-\frac{2\sigma}{\varepsilon_0}\hat{k}$
d. $-\frac{4\sigma}{\varepsilon_0}\hat{k}$

(IIT-JEE, 2005)

19. A long hollow conducting cylinder is kept coaxially inside another long hollow conducting cylinder of larger radius. Both the cylinders are initially electrically neutral. Then

a. a potential difference appears between the two cylinders when a charge density is given to the inner cylinder.
b. a potential difference appears between the two cylinders when a charge density is given to the outer cylinder.
c. no potential difference appears between the two cylinders when a uniform line charge is kept along the axis of the cylinders.
d. no potential difference appears between the two cylinders when same charge density is given to both the cylinders.

(IIT-JEE, 2007)

20. Consider a neutral conducting sphere. A positive point charge is placed outside the sphere. The net charge on the sphere is then **(IIT-JEE, 2007)**

a. negative and distributed uniformly over the surface of the sphere
b. negative and appears only at the point on the sphere closest to the point charge
c. negative and distributed non-uniformly over the entire surface of the sphere
d. zero

21. A spherical portion has been removed from a solid sphere having a charge distributed uniformly in its volume as shown in Fig. A1.130. The electric field inside the empty space is **(IIT-JEE, 2007)**

a. zero everywhere
b. non-zero and uniform
c. non-uniform
d. zero only at its centre

Fig. A1.130

22. Positive and negative point charges of equal magnitude are kept at $(0, 0, a/2)$ and $(0, 0, -a/2)$, respectively. The work done by the electric field when another positive point charge is moved from $(-a, 0, 0)$ to $(0, a, 0)$ is

a. positive
b. negative
c. zero
d. depends on the path connecting the initial and final positions

(IIT-JEE, 2007)

23. Consider a system of three charges $q/3$, $q/3$ and $-2q/3$ placed at points A, B and C, respectively, as shown in Fig. A1.131. Take O to be the centre of the circle of radius R and angle $CAB = 60°$. Then **(IIT-JEE, 2008)**

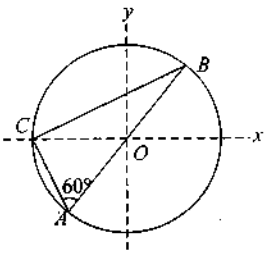


Fig. A1.131

a. the electric field at point O is $q/8\pi\varepsilon_0 R^2$ directed along the negative x-axis.
b. the potential energy of the system is zero.
c. the magnitude of the force between the charges at C and B is $q^2/54\pi\varepsilon_0 R^2$.
d. the potential at point O is $q/12\pi\varepsilon_0 R$.

24. A disk of radius $a/4$ having a uniformly distributed charge $6C$ is placed in the x-y plane with its centre at $(-a/2, 0, 0)$. A rod of length carrying a uniformly distributed charge $8C$ is placed on the x-axis from $x = a/4$ to $x = 5a/4$. Two point charges $-7C$ and $3C$ are placed at $(a/4, -a/4, 0)$ and $(-3a/4, 3a/4, 0)$, respectively. Consider a cubical surface formed by six surfaces $x = \pm a/2$, $y = \pm a/2$, $z = \pm a/2$.

The electric flux through this cubical surface is **(IIT-JEE, 2009)**

a. $\frac{-2C}{\varepsilon_0}$ b. $\frac{2C}{\varepsilon_0}$

c. $\frac{10C}{\varepsilon_0}$ d. $\frac{12C}{\varepsilon_0}$

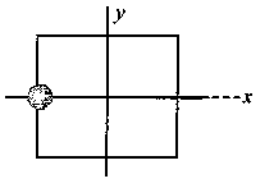


Fig. A1:132

25. Three concentric metallic spherical shells of radii R, $2R$, $3R$ are given charges Q_1, Q_2, Q_3, respectively. It is found that the surface charge densities on the outer surfaces of the shells are equal. Then, the ratio of the charges given to the shells $Q_1 : Q_2 : Q_3$ is **(IIT-JEE, 2009)**

a. 1:2:3 b. 1:3:5

c. 1:4:9 d. 1:8:18

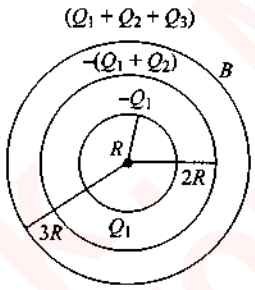


Fig. A1.133

26. A uniformly charged thin spherical shell of radius R carries uniform surface charge density of σ per unit area. It is made of two hemispherical shells, held together by pressing them with force F. F is proportional to **(IIT-JEE, 2010)**

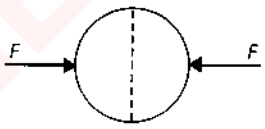


Fig. A1.134

a. $\frac{1}{\varepsilon_0}\sigma^2 R^2$ b. $\frac{1}{\varepsilon_0}\sigma^2 R$

c. $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R}$ d. $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R^2}$

27. A tiny spherical oil drop carrying a net charge q is balanced in still air with a vertical uniform electric field of strength $(81\pi/7) \times 105$ Vm^{-1}. When the field is switched off, the drop is observed to fall with terminal velocity 2×10^{-3} ms^{-1}. Given $g = 9.8$ ms^{-2}, viscosity of the air $= 1.8 \times 10^{-5}$ Ns m^{-2} and the density of oil = 900 kg m^{-3}, the magnitude of q is **(IIT-JEE, 2010)**

a. 1.6×10^{-19} C b. 3.2×10^{-19} C

c. 4.8×10^{-19} C d. 8.0×10^{-10} C

28. Consider an electric field $\vec{E} = E_0\hat{x}$, where E_0 is a constant. The flux through the shaded area (as shown in the figure) due to this field is

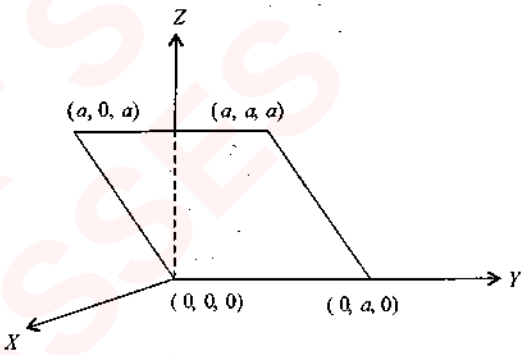


Fig. A1.135

a. $2E_0a^2$ b. $\sqrt{2}E_0a^2$

c. E_0a^2 d. $\frac{E_0a^2}{\sqrt{2}}$

(IIT-JEE, 2011)

29. A 2 μF capacitor is charged as shown in the figure. The percentage of its stored energy dissipated after switch S is turned to position 2 is

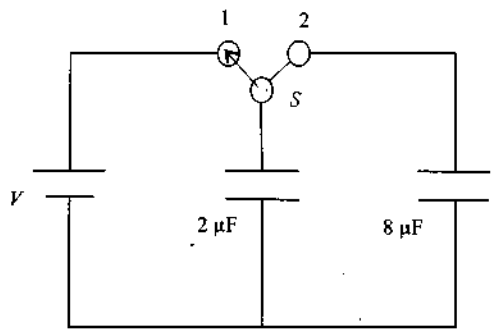


Fig. A1.136

a. 0% b. 20% c. 75% d. 80%

(IIT-JEE, 2011)

30. A wooden block performs S.H.M. on a frictionless surface with frequency ν_0. The block carries a charge $+Q$ on its surface. If a uniform electric field **E** is switched on as shown, then the S.H.M. of the block will be

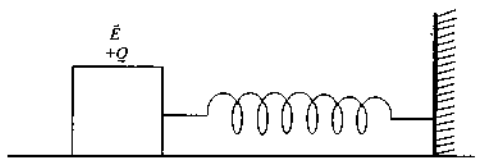


Fig. A1.137

a. of the same frequency and with shifted mean position
b. of the same frequency and with the same mean position
c. of changed frequency and with shifted mean position
d. of changed frequency and with the same mean position

Assertion and Reasoning Type

a. Statement I is True, Statement II is True; Statement II is a correct explanation for Statement I.
b. Statement I is True, Statement II is True; Statement II is **NOT** a correct explanation for Statement I.
c. Statement I is True, Statement II is False.
d. Statement I is False, Statement II is True.

1. **Statement I:** For practical purposes, the earth is used as a reference at zero potential in electrical circuits.
Statement II: The electrical potential of a sphere of radius R with charge Q uniformly distributed on the surface is given by $Q/4\pi\varepsilon_0 R$.

Comprehension Type

For Problems 1–3

The nuclear charge (Ze) is non-uniformly distributed within a nucleus of radius R. The charge density $\rho(r)$ [charge per unit volume] is dependent only on the radial distance r from the centre of the nucleus as shown in Fig.A1.138. The electric field is only along the radial direction.

(IIT-JEE, 2008)

1. The electric field at $r = R$ is
a. independent of a
b. directly proportional to a
c. directly proportional to a^2
d. inversely proportional to a

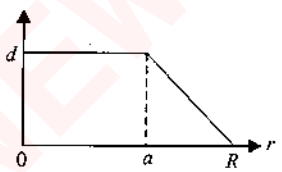


Fig. A1.138

2. For $a = 0$, the value of d (maximum value of ρ as shown in Fig.A1.13?) is

a. $\dfrac{3Ze}{4\pi R^3}$ b. $\dfrac{3Ze}{\pi R^3}$
c. $\dfrac{4Ze}{3\pi R^3}$ d. $\dfrac{Ze}{3\pi R^3}$

3. The electric field within the nucleus is generally observed to be linearly dependent on r. This implies

a. $a = 0$ b. $a = \dfrac{R}{2}$
c. $a = R$ d. $a = \dfrac{2R}{3}$

For Problems 4–5

A positive point charge q is fixed at origin. A dipole with a dipole moment $\vec{p}$ is placed along the x-axis far away from the origin with $\vec{p}$ pointing along the positive x-axis.

(IIT-JEE, 2003)

4. Find the kinetic energy of the dipole when it reaches a distance d from the origin.

a. $\dfrac{3}{4\pi\varepsilon_0}\dfrac{pq}{d^2}$ b. $\dfrac{3}{2\pi\varepsilon_0}\dfrac{pq}{d^2}$
c. $\dfrac{1}{2\pi\varepsilon_0}\dfrac{pq}{d^2}$ d. $\dfrac{1}{4\pi\varepsilon_0}\dfrac{pq}{d^2}$

5. Find the force experienced by the charge q at this moment.

a. $\dfrac{1}{4\pi\varepsilon_0}\dfrac{pq}{d^2}$ b. $\dfrac{1}{2\pi\varepsilon_0}\dfrac{pq}{d^2}$
c. $\dfrac{1}{2\sqrt{2}\pi\varepsilon_0}\dfrac{pq}{d^2}$ d. $\dfrac{\sqrt{2}}{\pi\varepsilon_0}\dfrac{pq}{d^2}$

Multiple Choice Questions with One or More than One Correct Answer Type

1. Two equal negative charges $-q$ are fixed at points $(0, \pm a)$ on the y-axis. A positive charge Q is released from rest at the point $(2a, 0)$ on the x-axis. The charge Q will
a. execute simple harmonic motion about the origin
b. move to the origin and remain at rest
c. move to infinity
d. execute oscillatory but not simple harmonic motion

(IIT-JEE, 1984)

2. A parallel-plate air capacitor is connected to a battery. The quantities charge, voltage, electric field and energy associated with the capacitor are given by Q_0, V_0, E_0 and U_0, respectively. A dielectric slab is now introduced to fill the space between the plates with battery still in connection. The corresponding quantities now given by Q, V, E and U are related to the previous one as

(IIT-JEE, 1985)

a. $Q > Q_0$ b. $V > V_0$
c. $E > E_0$ d. $U > U_0$

3. A charge Q is placed at the centre of the line joining two equal charges Q. The system of the three charges will be in equilibrium if q is equal to **(IIT-JEE, 1987)**

a. $-\dfrac{Q}{2}$ b. $-\dfrac{Q}{4}$
c. $+\dfrac{Q}{4}$ d. $+\dfrac{Q}{2}$

4. A parallel-plate capacitor is charged and the charging battery is then disconnected. If the plates of the capacitor

are moved farther apart by means of insulating handles
a. the charge on the capacitor increases
b. the voltage across the plates increases
c. the capacitance increases
d. the electrostatic energy stored in the capacitor increases

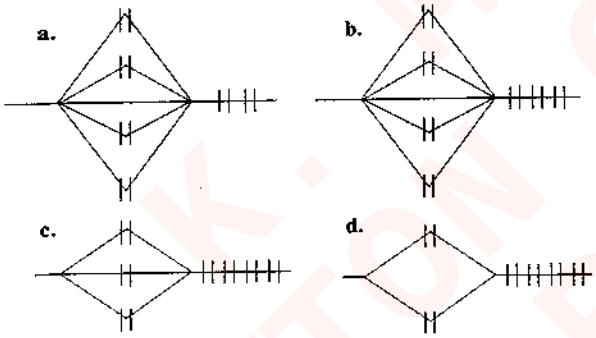
(IIT-JEE, 1987)

5. A solid conducting sphere having charge Q is surrounded by an uncharged concentric conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and the outer surface of the hollow shell be V. If the shell is now given a charge of $-3Q$, the new potential difference between the same two surfaces is

a. V b. $2V$
c. $4V$ d. $-2V$

(IIT-JEE, 1989)

6. Seven capacitors, each of capacitance 2 μF, are to be connected in a configuration to obtain an effective capacitance of 10/11 μF. Which of the combination (s) shown in the given graphs will achieve the desired result?



(IIT-JEE, 1990)

7. A parallel-plate capacitor of plate area a and plate separation d is charged to potential difference V and then the battery is disconnected. A slab of dielectric constant K is then inserted between the plates of the capacitor so as to fill the space between the plates. If Q, E and W denote, respectively, the magnitude of charge on each plate, the electric field between the plates (after the slab is inserted) and work done on the system, in question, in the process of inserting the slab, then

(IIT-JEE, 1991)

a. $Q = \frac{\varepsilon_0 AV}{d}$ b. $Q = \frac{\varepsilon_0 KAV}{d}$

c. $E = \frac{V}{Kd}$ d. $W = \frac{\varepsilon_0 AV^2}{2d}\left[1 - \frac{1}{K}\right]$

8. Two identical thin rings, each of radius R metres, are coaxially placed a distance r metres apart. If Q_1 coulomb and Q_2 coulomb are, respectively, the charges uniformly spread on the two rings, the work done in moving a charge q from the centre of one ring to that of the other is

a. zero b. $\frac{q(Q_1 - Q_2)(\sqrt{2}-1)}{(4\sqrt{2}\pi\varepsilon_0 R)}$

c. $\frac{q\sqrt{2}(Q_1 + Q_2)}{(4\pi\varepsilon_0 R)}$ d. $\frac{q(Q_1 + Q_2)(\sqrt{2}+1)}{(4\sqrt{2}\pi\varepsilon_0 R)}$

(IIT-JEE, 1992)

9. The magnitude of electric field $\vec{E}$ in the annular region of a charged cylindrical capacitor **(IIT-JEE, 1996)**
a. is same throughout
b. is higher near the outer cylinder than near the inner cylinder
c. varies as $1/r$, where r is the distance from axis
d. varies as $1/r^2$ where r is the distance from the axis

10. A metallic solid sphere is placed in a uniform electric field. The lines of force follow the path(s) shown in Fig. A1.139 as **(IIT-JEE, 1996)**

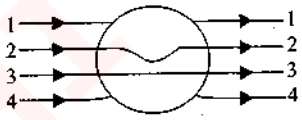


Fig. A1.139

a. 1 b. 2
c. 3 d. 4

11. A dielectric slab of thickness d is inserted in a parallel-plate capacitor whose negative plate is at $x = 0$ and positive plate is $x = 3d$. The slab is equidistant from the plates. The capacitor is given some charge. As x goes from 0 to $3d$,
a. the magnitude of the electric field remains the same
b. the direction of the electric field remains the same
c. the electric potential increases continuously
d. the electric potential increases at first, then decreases and again increases

(IIT-JEE, 1998)

12. A charge $+q$ is fixed at each of the points $x = x_0$, $x = 3x_0$, $x = 5x_0$ and so on, on the x-axis, and a charge $-q$ is fixed at each of the points $x = 2x_0, x = 4x_0, x = 6x_0$ and so on. Here, x_0 is a positive constant. Take the electric potential at a point due to charge Q at a distance r from it to be $Q/(4\pi\varepsilon_0 r)$. Then, the potential at the origin due to the above system of charges is

(IIT-JEE, 1998)

a. 0 b. $\frac{q}{8\pi\varepsilon_0 x_0 \ln 2}$

c. ∞ d. $\frac{q \ln 2}{4\pi\varepsilon_0 x_0}$

13. A positively charged thin metal ring of radius R is fixed in the xy plane with its centre at the origin O. A negatively charged

particle P is released from rest at the point $(0, 0, z_0)$, where $z_0 > 0$. Then, the motion of P is

a. periodic, for all values of z_0 satisfying $0 < z_0 < \infty$
b. simple harmonic, for all values of z_0 satisfying $0 < z_0 \le R$
c. approximately simple harmonic, provided $z_0 << R$
d. such that P crosses O and continues to move along the negative z-axis towards $z = -\infty$.

(IIT-JEE, 1998)

14. A non-conducting solid sphere of radius R is uniformly charged. The magnitude of the electric field due to the sphere at a distance r from its centre

(IIT-JEE, 1998)

a. increases as r increases, for $r < R$
b. decreases as r increases, for $0 < r < \infty$
c. decreases as r increases, for $R < r < \infty$
d. is discontinuous at $r = R$

15. An elliptical cavity is carved within a perfect conductor (Fig. A1.140). A positive charge q is placed at the centre of the cavity. The points A and B are on the cavity surface as shown in the figure.

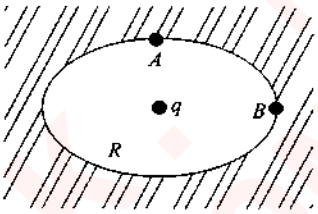


Fig. A1.140

Then

a. electric field near A in the cavity = electric field near B in the cavity.
b. charge density at A = charge density at B
c. potential at A = potential at B
d. total electric field flux through the surface of the cavity is q/ε_0

(IIT-JEE, 1999)

16. The electrostatic potential (ϕ_r) of a spherical symmetric system, kept at origin, is shown in Fig. A1.141, and given as **(IIT-JEE, 2006)**

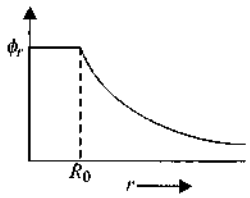


Fig. A1.141

$$\phi_r = \frac{q}{4\pi\varepsilon_0 r}\ (r \ge R_0),\ \phi_r = \frac{q}{4\pi\varepsilon_0 R_0}\ (r \le R_0)$$

Which of the following option(s) is/are correct?

a. For spherical region $r \le R_0$, total electrostatic energy stored is zero.
b. Within $r = 2R_0$, total charge is Q.
c. There will be no charge anywhere except at $r = R_0$.
d. Electric field is discontinuous at $r = R_0$.

17. A few electric field lines for a system of two charges Q_1 and Q_2 fixed at two different points on the x-axis are shown in the figure. These lines suggest that **(IIT-JEE, 2010)**

a. $|Q_1| > |Q_2|$
b. $|Q_1| < |Q_2|$
c. at a finite distance to the left of Q_1, the electric field is zero
d. at a finite distance to the right of Q_2, the electric field is zero

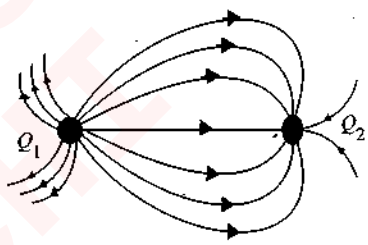


Fig. A1.142

18. Which of the following statement(s) is/are correct?

a. If the electric field due to a point charge varies as $r^{-2.5}$ instead of r^{-2}, then the Gauss's law will still be valid.
b. The Gauss's law can be used to calculate the field distribution around an electric dipole.
c. If the electric field between two point charges is zero somewhere, then the sign of the two charges is the same.
d. The work done by the external force in moving a unit positive charge from point A at potential V_A to point B at potential V_B is $(V_B - V_A)$.

(IIT-JEE, 2011)

19. A spherical metal shell A of radius R_A and a solid metal sphere B of radius R_B $(<R_A)$ are kept far apart and each is given charge 'Q'. Now they are connected by a thin metal wire. Then

a. $E_A^{\text{inside}} = 0$
b. $Q_A > Q_B$
c. $\dfrac{\sigma_A}{\sigma_B} = \dfrac{R_B}{R_A}$
(d) $E_A^{\text{on surface}} < E_B^{\text{on surface}}$

(IIT-JEE, 2011)

20. A solid sphere of radius r has a charge Q distributed in its volume with a charge density $\rho = kr^a$ where k and a are constants and r is the distance from its centre. If the electric field at $r = R/2$ is 1/8 times that at $r = R$, find the value of a.

(IIT-JEE, 2009)

ANSWERS AND SOLUTIONS

Objective Type

1. b. Charge on the metal sphere: $V_1 = \frac{kq}{r_1} \Rightarrow q = \frac{V_1 r_1}{k}$

On connecting the sphere with shell, entire charge will be transferred to the shell. So, potential of sphere now is

$$V_2 = \frac{kq}{r_2} = \frac{V_1 r_1}{r_2}$$

2. c. Let capacitance of the conductor be C_1 and that of plate be C_2 (Fig.A1.143). After first operation, we get

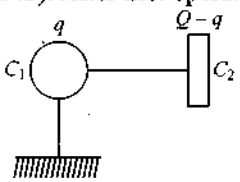


Fig. A1.143

$$\frac{q}{C_1} = \frac{Q-q}{C_2} \quad \text{(i)}$$

Let q_0 be the maximum charge that can be transferred to the conductor (Fig.A1.144). Then,

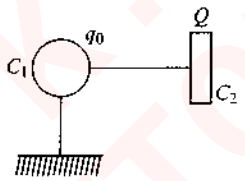


Fig. A1.144

$$\frac{q_0}{C_1} = \frac{Q}{C_2} \quad \text{(ii)}$$

From Eqs. (i) and (ii), $q_0 = \frac{Qq}{Q-q}$

3. b. Initially, when the switch is closed on position 1, the capacitor C is connected in series with batteries E_1 and E_2. From KVL, we have

$$\frac{Q_i}{C} - E_2 + E_1 = 0$$

or $$Q_i = (E_2 - E_1)C \quad \text{(i)}$$

Depending upon the sign of $(E_2 - E_1)$, charge Q_i on the left plate may be positive (if $E_2 > E_1$), or negative (if $E_2 < E_1$); charge on right plate would be equal and opposite.

When the switch is moved to position 2, the left plate (earlier having charge $+Q_i$) will now have charge

$$Q_f = -E_1 C \quad \text{(ii)}$$

The net charge flow through the circuit is

$$\Delta Q = Q_f - Q_i = [-E_1 - (E_2 - E_1)]C = -E_2 C$$

We can say that a net positive charge equal to $E_2 C$ is pulled by the battery of e.m.f. E_1 from the left plate of the capacitor, which flows through battery E_1 and is transferred to the right plate of the capacitor. Work done by battery E_1 in the process of charge transfer is

$$\Delta W = E_1 E_2 C \quad \text{(iii)}$$

A part of this work changes the energy of the capacitor:

$$\Delta W_c = \frac{Q_f^2}{2C} - \frac{Q_i^2}{2c} = \frac{1}{2}E_1^2 C - \frac{1}{2}(E_2 - E_1)^2 C$$

$$= \frac{1}{2}(2E_1E_2 - E_2^2)C$$

and the remaining part is lost as Joule heating:

$$H = \Delta W - \Delta W_C = \frac{1}{2}E_2^2 C$$

4. b. From Fig. A1.145, we get

$$dp = (\lambda a d\theta)2a\cos\theta$$

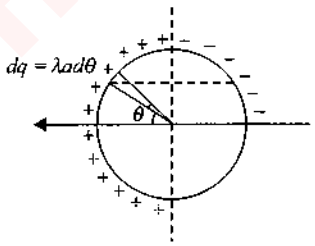


Fig. A1.145

$$p = \int dp = 2\lambda a^2 \int_{-\pi/2}^{\pi/2} \cos\theta\, d\theta$$

$$= 2\lambda a^2 [\sin\theta]_{-\pi/2}^{\pi/2}$$

$$|p| = 4\lambda a^2$$

5. b. $\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$

$$= (-6x + 4)\hat{i}$$

So, the equipotential surfaces would be planes parallel to YZ plane, as $\vec{E}$ is perpendicular to the equivalent surface.

6. a. All the statements are general properties of electric lines of force.

7. b. Equal and opposite charges get induced on the inner surface of conductor, so the net charge enclosed by the surface is zero and hence flux crossing through the surface is zero.

8. c. $\phi = \phi_{ps_1} + \phi_{cs} + \phi_{ps_2}$

$$= 2\phi_{ps} + \phi_{cs} \qquad [\because \phi_{ps_1} = \phi_{ps_2}]$$

$$\phi_{cs} = \phi - 2\phi_{ps} = \frac{q}{\varepsilon_0} - 2\phi_{ps}$$

To find ϕ_{ps}, use integration technique.

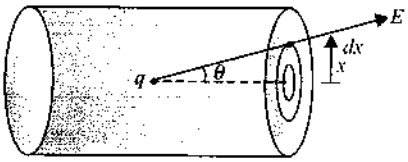


Fig. A1.146

$$\vec{E} = \frac{q}{4\pi\varepsilon_0} \times \frac{1}{(x^2 + l^2/4)}$$

$$d\phi_{ps} = \vec{E} \cdot d\vec{s} = E \times 2\pi x\, dx \times \cos\theta$$

$$= \frac{q}{2\varepsilon_0} \times \frac{x\,dx}{(x^2 + l^2/4)} \times \frac{l/2}{\sqrt{x^2 + l^2/4}}$$

$$= \frac{ql}{4\varepsilon_0} \times \frac{x\,dx}{(x^2 + l^2/4)^{3/2}}$$

$$\phi_{ps} = \frac{ql}{4\varepsilon_0}\int_0^R \frac{x\,dx}{(x^2 + l^2/4)^{3/2}} = \frac{q}{2\varepsilon_0} - \frac{ql}{4\varepsilon_0\sqrt{R^2 + l^2/4}}$$

9. c. This work done by us is stored in the capacitor in the volume $A(d_2 - d_1)$ where new electric field is created.

If you calculate the work done by using the expression $W_{ext} - W_{el} = dU = \frac{\varepsilon_0 A V^2}{2}\left[\frac{1}{d_2} - \frac{1}{d_1}\right] < 0$, you may get confused. Here, battery is also doing work, so from energy conservation principle, we get

$$W_{ext} + W_{el} + W_{battery} = 0$$

10. b. Due to polarization, charge on dielectric slab would be $CV\left(1 - \frac{1}{K}\right)$.

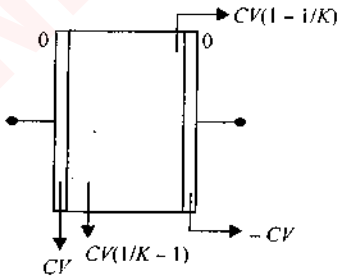


Fig. A1.147

So, charge on 1 = 0

Charge on $2 = CV - CV\left(1 - \frac{1}{K}\right) = \frac{CV}{K}$

Charge on $3 = CV\left(1 - \frac{1}{K}\right) - CV = -\frac{CV}{K}$

Charge on 4 = 0

11. c. Refer concepts and formulate isolated sphere and non-isolated sphere.

12. c. The innermost conductor is at zero potential (the same potential as we assumed for infinity).

The conductors with radius a and b make one capacitor between b and c; other conductor and c make a capacitor with its other plate at infinity. So, equivalent diagram can be drawn as

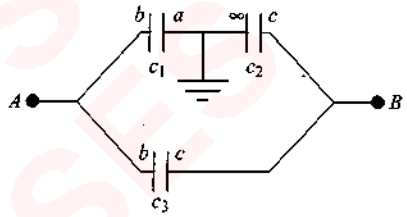


Fig. A1.148

Here, $C_1 = \frac{4\pi\varepsilon_0 ab}{b - a}$

$C_2 = 4\pi\varepsilon_0 c$

$C_3 = \frac{4\pi\varepsilon_0 bc}{c - b}$

$$\therefore \quad C_{eq} = C_3 + \frac{C_1 C_2}{C_1 + C_2} = 4\pi\varepsilon_0\left[\frac{bc}{c - b} + \frac{abc}{ab + c(b - a)}\right]$$

13. d. Initial situation after the reconnection is shown in Fig. A1.149(a) and the final situation in Fig.A1.149(b).

The charge transferred by C_1 is $q_0 - q_1$ and the capacitors C_2 and C_3 are in series, so $q_2 = q_3$.

Other way to solve the questions is to equalize the potentials across C_1 and series combination of C_2 and C_3.

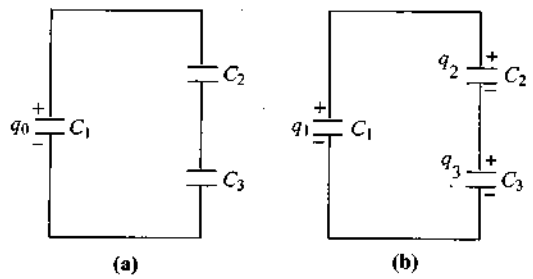


Fig. A1.149

14. a. Potential of outer sphere $V = \frac{kq_1}{3r} + \frac{kq_2}{3r} + \frac{kq_3}{3r} = 0$

$$\Rightarrow \quad q_1 + q_3 = -q_2$$

15. b. Electric field due to one wire $= \frac{1}{4\pi\varepsilon_0}\frac{2\lambda_1}{R}$

Charge per unit length on other wire: $q_2 = \lambda_2 l$

$$\Rightarrow \quad q_2 = \lambda_2 \times 1 = \lambda_2$$

Force per metre length on other wire

$$F = q_2 E = \lambda_2 E = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda_1\lambda_2}{R} = K \times \frac{2\lambda_1\lambda_2}{R}$$

16. a. $T = 2\pi\sqrt{\dfrac{l}{g_{\text{eff}}}}$

Here, $g_{\text{eff}} = g - \dfrac{qE}{m}$

$\Rightarrow$ g_{eff} will decrease.

Hence, T will increase.

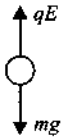


Fig. A1.150

17. c. Electric flux $\oint_S \vec{E}\cdot\vec{dS} = \dfrac{q_{\text{in}}}{\varepsilon_0}$

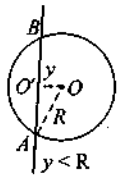


Fig. A1.151

q_{in} is the charge enclosed by the Gaussian surface which, in the present case, is the surface of given sphere. As shown, length AB of the line lies inside the sphere.

In $\Delta OO'A, R^2 = y^2 + (O'A)^2$

$O'A = \sqrt{R^2 - y^2}$ and $AB = 2\sqrt{R^2 - y^2}$

Charge on length $AB = 2\sqrt{R^2 - y^2} \times \lambda$

Therefore, electric flux $\oint_S \vec{E}\cdot\vec{dS} = \dfrac{2\lambda\sqrt{R^2 - y^2}}{\varepsilon_0}$

18. d. $R = \sqrt{8^2 + 8^2 + 2\times 8\times 8\cos 120°} = 8$ N

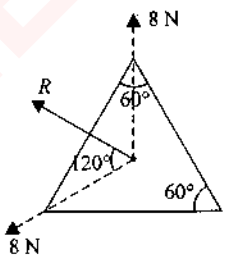


Fig. A1.152

19. c. From Eq. (i), A and C both are charged, either positively or negatively. From Eq. (ii), B is charged and D and E have no charge. From Eq. (iii), A is positively charged. Therefore, from Eq. (i), B is negatively charged.

20. c. After covering with a hemispherical shell;

$$\phi_{\text{shell}} + \phi_{\text{disc}} = 0 \text{ (from Gauss's law)}$$
$$\phi_{\text{shell}} = -\phi_{\text{disc}} = -\phi$$

21. c. Using the formula for electric field produced by large sheet, $E = Q/2A\varepsilon_0$ and we get

$$E_A = \frac{4Q}{2A\varepsilon_0}(-\hat{i});\ E_B = \frac{2Q}{2A\varepsilon_0}(-\hat{i})$$
$$E_C = \frac{4Q}{2A\varepsilon_0}(+\hat{i})$$

22. c. Net force on q_1 is zero, while that on the conducting sphere is towards the left due to attraction of $-q_2$ (Fig. A1.153).

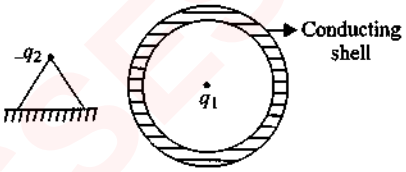


Fig. A1.153

23. a. $a_x = \dfrac{qE}{m} = \dfrac{10^{-6}\times 2\times 10^7}{2} = 10 \text{ ms}^{-2}$

Time of flight $T = \dfrac{2u\sin\theta}{g} = \dfrac{2\times 10\times\frac{1}{\sqrt{2}}}{10}$

$T = \sqrt{2}$ sec

Hence, range $R = u_x T + \dfrac{1}{2}a_x T^2$

$R = 10\cos 45° \times T + \dfrac{1}{2}a_x T^2$

$= \dfrac{10}{\sqrt{2}}\times\sqrt{2} + \dfrac{1}{2}\times 10\times 2$

$\Rightarrow$ $R = 20$ m

24. b. From Gauss's law, $\phi = \dfrac{q}{\varepsilon_0}$

So, $\dfrac{\phi_{\max}}{\phi_{\min}} = \dfrac{Q_{\max}}{Q_{\min}} = \dfrac{\lambda(l^2 + b^2 + h^2)^{1/2}}{\lambda h}$

$= \dfrac{\sqrt{l^2 + b^2 + h^2}}{h}$

25. d. l = length of the each tube

$u_x = u$ = constant

Time of travel between the plates $t = l/u$

Let a = constant acceleration in y-direction

So, $v_y = at$ when the particle emerges from the plates

So, $v^2 = u_x^2 + v_y^2 = u^2 + a^2t^2$

$= u^2 + a^2\dfrac{l^2}{u^2} = u^2 + \dfrac{C}{u^2}$ (where $a^2l^2 = C$)

So, $v = \sqrt{u^2 + \dfrac{C}{u^2}}$

26. a. According to Gauss's law, electric field at any point on Gaussian surface is due to all the charges present inside or outside.

27. a. $V = V_1 + V_2 + V_3$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} + \frac{1}{4\pi\varepsilon_0}\left(\frac{-2Q}{R}\right) + \frac{1}{4\pi\varepsilon_0}\left(\frac{3Q}{R}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q}{R}\right) = \frac{Q}{2\pi\varepsilon_0 R}$$

28. c. Electrical potential energy of system:

$U = -2\left[\frac{kq^2}{\sqrt{2}a}\right] = \frac{-\sqrt{2}kq^2}{a}$ (four pairs will cancel each other)

If a decreases, U also decreases and if U decreases, the agent will do negative work.

29. b. $V = \frac{kQ}{R}, E = \frac{kQ}{R^2}$

$$\frac{V}{E} = R \Rightarrow E = \frac{V}{R}$$

If R increases, E decreases.

30. c.

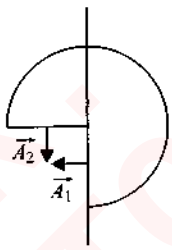


Fig. A1.154

From Fig. A1.154, we have

$$\vec{A}_1 = -\frac{\pi R^2}{2}\hat{i}$$

$$\vec{A}_2 = -\frac{\pi R^2}{2}\hat{j}$$

$$\vec{E} = E\cos 45°\,\hat{i} + E\sin 45°\,\hat{j} = \frac{E}{\sqrt{2}}\hat{i} + \frac{E}{\sqrt{2}}\hat{j}$$

$$\phi = \vec{E}\cdot(\vec{A}_1 + \vec{A}_2) = \frac{-E}{\sqrt{2}}\frac{\pi R^2}{2} - \frac{E}{\sqrt{2}}\frac{\pi R^2}{2}$$

$$= \frac{-\pi R^2 E}{\sqrt{2}}$$

This is the flux entering. So, flux leaving $= \frac{\pi R^2 E}{\sqrt{2}}$

31. b. $V = 10r = 10\sqrt{x^2 + y^2 + z^2}$

$$E_x = -\frac{dv}{dx} = -\frac{10(2x)}{2\sqrt{x^2 + y^2 + z^2}}$$

$$= \frac{-10x}{\sqrt{x^2 + y^2 + z^2}} = \frac{-10\times 3}{\sqrt{3^2 + 4^2 + 5^2}} = -3\sqrt{2}$$

Similarly, $E_y = -4\sqrt{2}$

$$E_z = 5\sqrt{2}\,;\ \vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$$

32. a. There is no charge on the outer surface. Hence, no force on q_2 (Fig. A1.155).

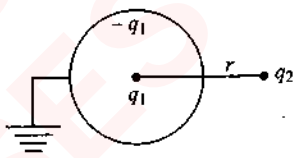


Fig. A1.155

33. b. The charge on outer surface of shell will apply a force on q_2

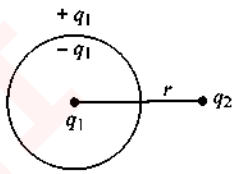


Fig. A1.156

$$F = \frac{kq_1q_2}{r^2}$$

34. b. $E = \frac{Q}{2A\varepsilon_0}$

$$a = \frac{qE}{m} = \frac{qQ}{2Am\varepsilon_0}$$

$$d = \frac{1}{2}at^2; t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2d\,2Am\varepsilon_0}{qQ}}$$

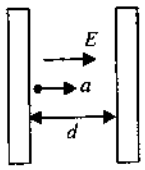


Fig. A1.157

35. d. $OB = OA\cos 60° = 2\times\frac{1}{2} = 1\,\text{m}$

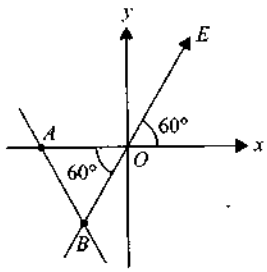


Fig. A1.158

$$V_B - V_O = E(OB) = 100 \times 1 = 100 \text{ V}$$
$$V_A - V_O = 100 \text{ V } [\because V_B = V_A]$$
$$V_O - V_A = -100 \text{ V}$$

36. d. $V_B - V_A = -\int E_X dx = -[\text{Area under } E_x - x \text{ curve}]$

$$V_B - 10 = -\frac{1}{2} \times 2 \times (-20) = 20$$
$$V_B = 30 \text{ V}$$

37. b. Charge on B will remain the same as it is not touched with any other body. Charge induced on A will decrease the potential of B (Fig. A1.159).

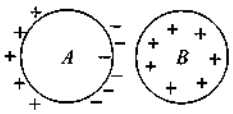


Fig. A1.159

38. a. Electric field produced due to charge Q will be zero at P, but potential produced by Q at P will not be zero.

39. d. $\frac{\lambda q}{2\pi\varepsilon_0 r} = \frac{mv^2}{r}$ or $v = \sqrt{\frac{\lambda}{2\pi\varepsilon_0 mq}}$ independent of r

40. b. Final charges will be as shown in Fig. A1.160.

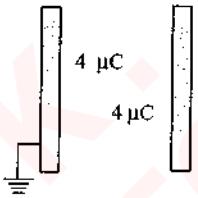


Fig. A1.160

So, charge flowing from earth to plate = final charge − initial charge = $-4 - 10 = -14$ μC.

41. d. $V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$

$$Q_1 + Q_2 = Q$$

where $C_1 = \varepsilon_0 A/a$ and $C_2 = \varepsilon_0 A/b$

Solve to find $Q_2 = \frac{Qa}{a+b}$

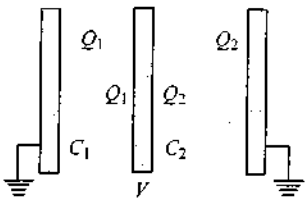


Fig. A1.161

42. c. $P = 2 \times 10^{-8} \times 2 \times 10^{-3} = 4 \times 10^{-11}$ Cm

$$E = \frac{\lambda}{2\pi\varepsilon_0 x}, F = p\cos\theta \frac{dE}{dx}$$

$$F = -\frac{p\cos\theta\lambda}{2\pi\varepsilon_0 x^2} = \frac{-4 \times 10^{-11} \times \cos 60° \times 4 \times 10^{-4}}{\frac{1}{2 \times 9 \times 10^9} \times \left(\frac{6}{100}\right)^2}$$
$$= 0.04 \text{ N}$$

43. b. $q_i = CV_i = 16 \times 5 = 80$ μC, $q_f = CV_f = 16 \times 10 = 160$ μC

Given $\frac{dq}{dt} = 40t \Rightarrow \int_{q_i}^{q_f} dq = \int_0^t 40t$

$$\Rightarrow \quad q_f - q_i = \frac{40t^2}{2}$$
$$\Rightarrow \quad 160 - 80 = 20t^2 \Rightarrow T = 2 \text{ s}$$

44. c. p and q are in parallel and then r in series.

45. c. The charge on capacitor before dielectric is

$$q_1 = CV = 50 \times 10^{-9} \text{ C}$$

Final charge on capacitor after dielectric is

$$q_2 = (KC)V = 5 \times 50 \times 10^{-9}$$
$$= 250 \times 10^{-9} \text{ C}$$

Charge flow from battery, $\Delta q = q_2 - q_1 = 200$ nC

46. a. Given plates can be rearranged as shown:

$$C_1 = \frac{\varepsilon_0 A}{d}; C_2 = \frac{\varepsilon_0 A}{2d}$$
$$C_3 = \frac{\varepsilon_0 A}{3d}; C_4 = \frac{\varepsilon_0 A}{2d}; C_5 = \frac{\varepsilon_0 A}{d}$$

C_1 and C_2 are in series and its effective capacity is

$$\frac{\frac{\varepsilon_0 A}{d} \times \frac{\varepsilon_0 A}{2d}}{\frac{\varepsilon_0 A}{d} + \frac{\varepsilon_0 A}{2d}} = \frac{\varepsilon_0 A}{3d}$$

Effective capacitance of C_4 and $C_5 = \frac{\varepsilon_0 A}{3d}$

$$\therefore \quad C_{AB} = \frac{\varepsilon_0 A}{3d} + \frac{\varepsilon_0 A}{3d} + \frac{\varepsilon_0 A}{3d} = \frac{\varepsilon_0 A}{d}$$

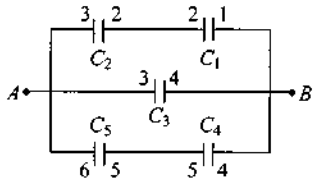


Fig. A1.162

47. b. Plates can be rearranged as shown in Fig. A1.163. Plates 1 and 2 and plates 3 and 4 form two capacitors which are in series between A and B. Plates 2 and 3 do not form any capacitor as they are at same potential.

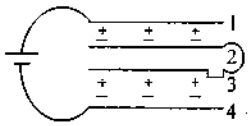


Fig. A1.163

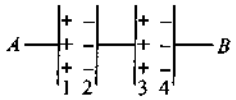


Fig. A1.164

$$\text{So, } C_{eq} = \frac{C_1C_2}{C_1+C_2} = \frac{\left(\frac{\varepsilon_0 A}{d}\right)\times\left(\frac{\varepsilon_0 A}{3d}\right)}{\frac{\varepsilon_0 A}{d}+\frac{\varepsilon_0 A}{3d}}$$

$$= \frac{\varepsilon_0 A}{4d}$$

48. a. Let the charge following through section AB be Q (Fig. A1.165).

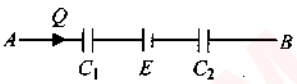


Fig. A1.165

Applying Kirchhoff's law, we get

$$V_A - \frac{Q}{C_1} + E - \frac{Q}{C_2} = V_B$$

$$V_A - V_B + E = Q\left(\frac{1}{C_1}+\frac{1}{C_2}\right)$$

$$5+10 = Q\left(\frac{C_1+C_2}{C_1C_2}\right) = Q\left(\frac{1+2}{2}\right)$$

$$Q = \frac{15\times 2}{3} = 10\ \mu C$$

So, potential difference across 2 μF capacitor

$$V = \frac{Q}{C_2} = \frac{10\ \mu C}{2\ \mu F} = 5\ V$$

49. d. Circuit is redrawn as shown in Fig.A1.166.

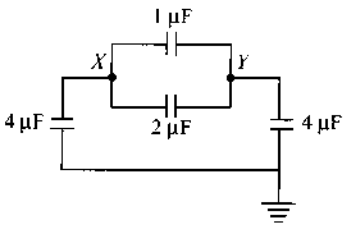


Fig. A1.166

In the circuit, capacitors 4 μF and 4 μF are connected in series. Combination of this is in parallel with 1 μF and 2 μF capacitors.

50. d. Field on the axis of the dipole lies in the direction of dipole.

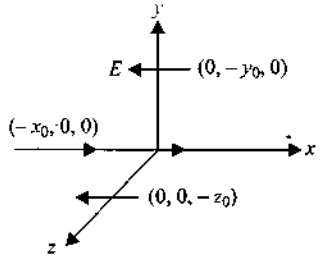


Fig. A1.167

Field on the bisector of the dipole lies opposite to the direction of dipole.

51 a. The potential of point B is + 10 V, therefore no potential difference exists across 12 μF capacitor, hence $q_{12} = 0$. Charge on the 4 μF capacitor is $q_4 = (4)(10) = 40\ \mu C$.

52. b. Given:

$$E = \frac{E_0}{l}\hat{i},\ l = 2\text{ cm}, a = 1\text{ cm},\ E_0 = 5\times 10^3\text{ N/c}$$

We see that flux passes mainly through surface area $ABDC$ and $EFGH$. As the $AEFB$ and $CHGD$ are parallel to the flux again in $ABDC = 0$. Hence, the flux only passes through the surface area $EFGHE = 0$.

$$\text{Flux} = E_0\frac{a}{l}\times\text{area} = 5\times 10^3\times\frac{a}{l}\times a^2$$

$$= 5\times 10^3\times\frac{a^3}{l}$$

$$= 5\times 10^3\times\frac{(0.01)^3}{2\times 10^{-2}} = 2.5\times 10^{-1}$$

So, $q = \varepsilon_0$ flux

$$= 8.85\times 10^{-12}\times 2.5\times 10^{-1}$$

$$= 22.125\times 10^{-13} = 2.2125\times 10^{-12}\ C$$

53. d. Consider the Gaussian surface where the induced charge is as shown in the figure

The net field at P due to all the charges is zero, i.e.,

$$-2Q + \frac{q}{2A\varepsilon_0}(\text{left}) + \frac{q}{2A\varepsilon_0}(\text{right}) + q - \frac{q}{2A\varepsilon_0}(\text{right}) = 0$$

$$\Rightarrow \quad -2q + q - Q + q = 0$$

$$\Rightarrow \quad q = \frac{3}{2}Q$$

Charge on the right side of the rightmost plate is

$$-2Q + Q = -2Q + \frac{3}{2}Q = -\frac{Q}{2}$$

54 a. Electrical force is balanced by the weight of the mass

$$mg = qE, \text{ where } E = \text{electric field}$$

$$\text{or } mg = q\frac{V}{d} \text{ (where } d = \text{separation at the plates)}$$

$$= q\cdot\frac{V}{2}\left(\therefore \frac{\varepsilon_0 A}{d} = C\ \frac{1}{d} = \frac{C}{\varepsilon_0 A}\right)$$

or $V = \dfrac{2mg\varepsilon_0 A}{qC}$

$$= \frac{10^{-6} \times 10 \times 2.2 \times 10^{-12} \times 10^{-2}}{10^{-5} \times 4 \times 10^{-8}}$$

$$= \frac{10^{-6} \times 9.8 \times 8.88 \times 10^{-12} \times 100 \times 10^{-4}}{10^{-8} \times 0.04 \times 10^{-6}}$$

$= 43$ mV

55. d. Initial total energy $= \frac{1}{2}CV^2 + \frac{1}{2}CV^2 = CV^2$

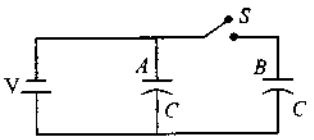


Fig. A1.168

When the switch is open, dielectric is introduced
Then capacitance $C = KC = 3C$

Energy stored in $C = \frac{1}{2}3CV^2 = \frac{3}{2}CV^2$

Since the switch is open, charge will be the same in B so energy $B = \frac{1}{2}\frac{C}{3}V^2$.

So, total final energy $= \frac{3}{2}CV^2 + \frac{1}{6}CV^2$

$$= \frac{9CV^2 + 1CV^2}{6} = \frac{10}{6}CV^2$$

So, required ratio $= \dfrac{CV^2}{\frac{10}{6}CV^2} = \frac{3}{5} = 3:5$

56 a. The electric field strength in a parallel-plate capacitor with applied voltage of V and separation d is given by $E = V/d$.

57. b. Initially at switch position Q, by conservation of charge, the voltage V across the two capacitors is thus given by

$$q = q_1 + q_2 = C_1V + C_2V = (C_1 + C_2)V$$

$\Rightarrow$ $18 = (3 + 6)$ V

$\Rightarrow$ $V = (18/9) = 2$ V

58. a. $\frac{q_1}{1} = \frac{q_2}{2} = \frac{q_3}{3} = K$

$q_1 + q_2 + q_3 = 6$

Solving, we get

$K = 1$

$\Rightarrow q_3 = 3\ \mu C$

59. d.

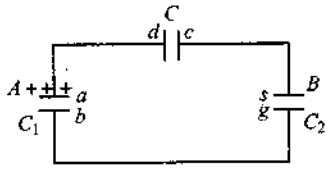


Fig. A1.169

Charge on positive plate of $A = C_1V_1$

Charge on negative plate of $B = -C_2V_2$

When d plate of capacitor C is connected with the plate of A, then the total charge of (d, a) plate system will be C_1V_1 (conservation of charge).

Similarly on (C, S) plates, total charge will be $-C_2V_2$.

The total charge on (b, g) plate system will be $+C_2V_2 - C_1V_1$.

60. a.

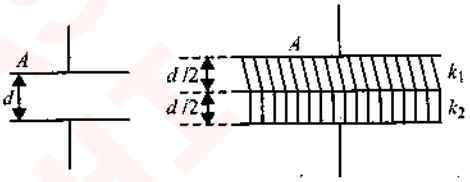


Fig. A1.170

$$C_0 = \frac{\varepsilon_0 A}{d}$$

$$C_1 = \frac{k_1\varepsilon_0 A}{d/2} \Rightarrow C_2 = \frac{k_2\varepsilon_0 A}{d/2}$$

C_1 and C_2 are in series

Equivalent dielectric constant $K = \dfrac{2k_1k_2}{k_1 + k_2}$

which is the ratio of capacitor.

61. c. $+q$ will not provide restoring torque.

62. a. $\phi_{ABCD} = -acd$ unit

$\phi_{CDEF} = -bcE_0$ unit

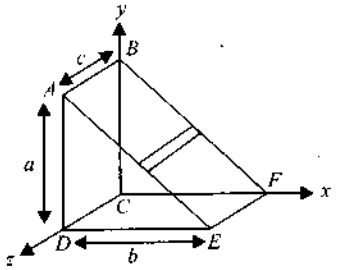


Fig. A1.171

$$\phi_{ABEF} = bcE_0 + c\int_0^a (d + x)\,dy$$

$$= +bcE_0 + acd + c\int_0^a x\,dy$$

$$= +bcE_0 + acd + \frac{ca}{b}\int_0^b x\,dx$$

[Since $\frac{x}{b} + \frac{y}{a} = 1 \Rightarrow \frac{dx}{b} = \frac{-dy}{a}$]

$$= \left[+bcE_0 + acd + \frac{acb}{2}\right] \text{ unit}$$

Using Gauss's law, we get

$$\phi_{net} = \frac{q_{in}}{\varepsilon_0} \Rightarrow q_{in} = \frac{abc\varepsilon_0}{2}$$

63. b. $F_3 = \frac{k(q_b + q_d)}{r^2}$

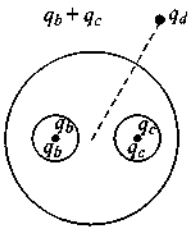


Fig. A1.172

$F_1 = F_2 =$ zero

64. d. $\Delta V = -\int_A^B \vec{E}\cdot dl = \text{zero}$

65. c. Circuit is redrawn as shown in Fig. A1.173.

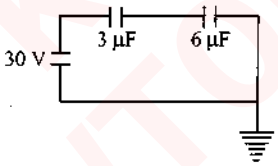


Fig. A1.173

Equivalent capacity of combination

$$C_{eq} = \frac{3\times 6}{9} = 2\,\mu\text{F}$$

Hence, charge $q = C_{eq}V$

$$q = 2\mu\text{F}\times 30\text{ V} = 60\,\mu\text{C}$$

66. d. The capacitance of capacitor $C = \frac{\varepsilon_0 A}{x}$

Charge on capacitor

$$q = CV = \frac{\varepsilon_0 AV}{x};\ U = \frac{1}{2}qV = \frac{1}{2}\frac{\varepsilon_0 AV^2}{x}$$

So, $$\frac{dU}{dt} = \frac{1}{2}\varepsilon_0 AV^2\left(-\frac{1}{x^2}\right)\frac{dx}{dt}$$

$$= \frac{1}{2}\varepsilon_0 AV^2 v\left(-\frac{1}{x^2}\right)$$

From the earlier expression, $\frac{dU}{dt} \propto \frac{1}{x^2}$

67. b.

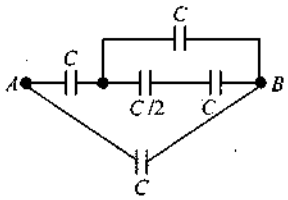


Fig. A1.174

$$C_{eq} = \frac{\frac{4C}{3}\times C}{\frac{7C}{3}} + C = \frac{11C}{7}$$

68. a. The total charge on each plate will have to be same. They should also carry opposite charges.

69. c. There will be two dipoles inclined to each other at an angle of 60°. The dipole moment of each dipole will be $q\lambda$. The resultant dipole moment is

$$\sqrt{(ql)^2 + (ql)^2 + 2(ql)(ql)\cos 60°} = \sqrt{3}ql$$

70. d. $E = \frac{1}{4\pi\varepsilon_0}\left(\frac{\frac{4}{3}\pi r^3\rho}{r^2}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{4}{3}\pi r\rho\right)$ or $E \propto r$

71. b. Potential V_1 due to a ring is given by:

$$V_1 = \frac{1}{4\pi\varepsilon_0}\times\frac{q}{\sqrt{R^2 + r^2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\times\frac{q}{\sqrt{R^2 + 3R^2}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{2R}$$

At the centre, $V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$

$$\text{K.E.} = q_0(V_2 - V_1) = \frac{1}{4\pi\varepsilon_0}\left(\frac{qq_0}{2R}\right)$$

72. d. $E = \frac{\lambda}{2\pi\varepsilon_0 r} = -\frac{\partial V}{\partial r}$ or $\int_{V_a}^{V_b} dV = -\int_a^b \frac{\lambda}{2\pi\varepsilon_0}\frac{dr}{r}$

$$V_a - V_b = \frac{\lambda}{2\pi\varepsilon_0}\log\left(\frac{b}{a}\right)$$

Hence, $V_a - V_b \propto \log\frac{b}{a}$.

73. c. A positive charge, which is free to move, will always move from higher to lower potential.

74. d. $V = K\left[\frac{q}{x_0} - \frac{q}{2x_0} + \frac{q}{3x_0} - \frac{q}{4x_0} + \cdots\right]$

$$= \left(\frac{q}{x_0}\right)K\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{x_0}\log 2$$

75. a. Though effective value of g will change, as the time period of spring block system does not depend on g, so time period will remain the same, i.e., T.

76. c. On placing this aluminium sheet, it will form two capacitors in series, say initial separation is d.

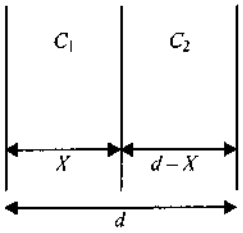


Fig. A1.175

$$C_1 = \frac{\varepsilon_0 A}{x}, C_2 = \frac{\varepsilon_0 A}{d-x}$$

$$C_{eq} = \frac{C_1C_2}{C_1+C_2} = \frac{\frac{\varepsilon_0 A}{x}\cdot\frac{\varepsilon_0 A}{d-x}}{\frac{\varepsilon_0 A}{x}+\frac{\varepsilon_0 A}{d-x}}$$

$$= \frac{\varepsilon_0 A}{x(d-x)\left[\frac{1}{x}+\frac{1}{d-x}\right]} = \frac{\varepsilon_0 A}{x+d-x}$$

$$= \frac{\varepsilon_0 A}{d}$$

Hence, capacitance remains unchanged.

77. b. $\frac{1}{2}CV^2$ = mass × specific heat × change in temperature

$$V^2 = \frac{2\times m\times s\times T}{C}; V = \sqrt{\frac{2msT}{C}}$$

78. a. $u = \frac{q^2}{2C} = \frac{(8\times10^{-18})^2}{2\times100\times10^{-6}} = \frac{64\times10^{-36}}{2\times10^{-4}} = 32 \times 10^{-32}$ J

79. b. Power $= \frac{\text{Energy}}{\text{Time}} = \frac{(1/2)CV^2}{\text{Time}}$

$$= \frac{40\times10^{-6}\times(3000)^2}{2\times2\times10^{-3}} = \frac{40\times10^{-6}\times9\times10^6}{4\times10^{-3}}$$

$$= 9 \times 10 = 90 \text{ kW}$$

80. c. Net dipole moment $= \sqrt{3}qa$

$$\tau = -\sqrt{3}\,qa\frac{|\sigma_1-\sigma_2|\theta}{2\varepsilon_0} = I\alpha$$

$$\frac{\sqrt{3}qa|\sigma_1-\sigma_2|}{2\varepsilon_0} = I\left(\frac{2\pi}{T}\right)^2$$

$$T = 2\pi\sqrt{\frac{2\varepsilon_0 I}{\sqrt{3}qa|\sigma_1-\sigma_2|}}$$

81. a. For equilibrium, we have

$$T = Mg$$

And $T = F + \sin 30°$

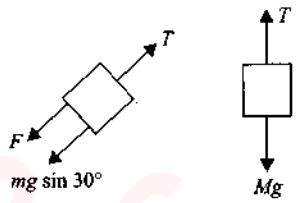


Fig. A1.176

$$\Rightarrow \quad Mg = F + \frac{mg}{2}$$

$$\Rightarrow \quad m = \frac{m}{2} + \frac{F}{g} = \frac{m}{2} + \frac{E^2\varepsilon_0 A(k-1)}{2lgd}$$

82. b. The circuit can be redrawn as shown in Fig. A1.177.

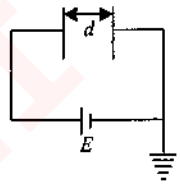


Fig. A1.177

Charge on capacitor is

$$Q = CE = \frac{\varepsilon_0 AE}{d}$$

Options (a), (c) and (d) are incorrect.

83. b. Electric field will remain the same, because electric field due to surface charge distributed uniformly will be zero at any point inside the sphere.

84. d. Ball D has no effect on E, it means both D and E should be neutral.

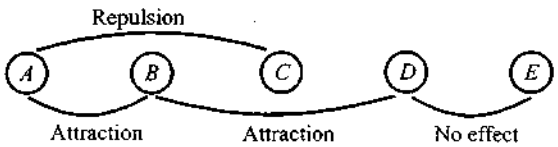


Fig. A1.178

A repels C, so both should have similar type of charge. Negatively charged rod attracts A, so A should have positive charge. So C should also have positive charge.

A attracts B, so B is either negatively charged or has no charge. But B attracts D which is uncharged, so B should have negative charge.

85. b. Four lines perpendicular to lines of electric field and passing through A, B, C and D are drawn. These are equipotential lines. As potential decreases in the direction of electric field, $VA > VB > VD > VC$.

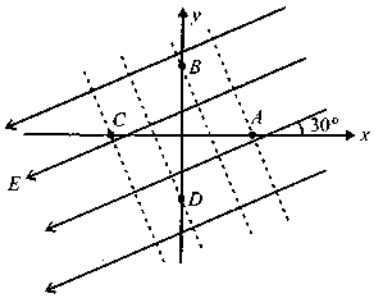


Fig. A1.179

86. c. Electric field due to both the plates will be cancelled out for all the points. So the net electric field at the points will be governed only by the sphere. The farther the point from the sphere, the lesser the magnitude of the electric field.

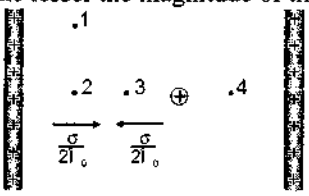


Fig. A1.180

Since the charges exert force on one another that are in action–reaction pair, sum of forces $\vec{f}_1, \vec{f}_2$ and $\vec{f}_3$ is zero. So force vectors form a triangle, as shown in the figure.

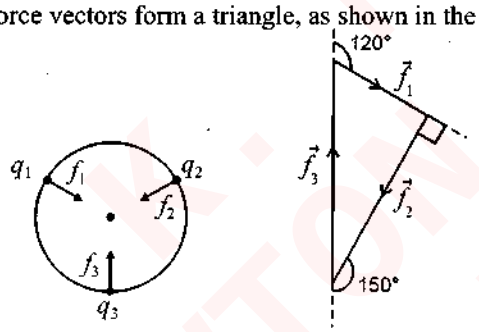


Fig. A1.181

Applying sine rule, we have

$$\frac{f_1}{\sin 30} = \frac{f_2}{\sin 60} = \frac{f_3}{\sin 90}$$

or $$2f_1 = \frac{2f_2}{\sqrt{3}} = f_3 = K \text{ (suppose)}$$

$$f_1 = \frac{k}{2},\ f_2 = \frac{\sqrt{3}}{2}K,\ f_3 = K$$

So $f_1 : f_2 : f_3 = \frac{1}{2} : \frac{\sqrt{3}}{2} : 1 = 1 : \sqrt{3} : 2$

88. c. Since magnitude of forces is proportional to m as $f = mv2/r$, $m_1 : m_2 : m_3 = 1 : \sqrt{3} : 2$.

89. d. External applied field's effect is neutralized by charge induced on outer surface, i.e., their combined effect is zero. Hence, force applied by induced charges on outer surface should be equal and opposite to that applied by electric field. This is equal to $qE = 1 \times 20 = 20$ N towards left. 15 N force is only due to induced charge on the inside surface of the hollow sphere. Hence, net force by induced charges is 15 + 20 = 35 N.

90. a. When S is open, spheres A and B have same, positive potential. When S is closed, potential of B will become zero. So, the potential of B will decrease. This decrease of potential takes place due to flow of electrons from earth to inner sphere B. But magnitude of charge acquired by inner sphere will be less than Q.

91. a. Flux entering the cone from side AB will ultimately also pass through area A_1 and A_2.

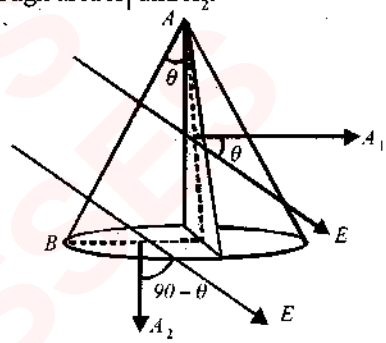


Fig. A1.182

Hence, $\phi = EA_1 \cos\theta + EA_2 \cos(90-\theta)$

$$= E\left(\frac{1}{2}2Rh\cos\theta + \frac{\pi}{2}R^2\sin\theta\right)$$

$$= ER\left(h\cos\theta + \pi(R/2)\sin\theta\right)$$

92. b.

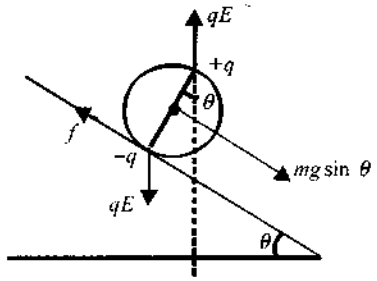


Fig. A1.183

Balancing torque about $-q$ is

$$qE\ 2R\sin\theta = mg\sin\theta\ R \Rightarrow E = \frac{mg}{2q}$$

93. b. Potential decreases in the direction of electric field. Hence, electric field should be along PA.

$$E = \frac{\Delta V}{\Delta r} = \frac{5-2}{0.1\times\sqrt{2}} = 15\sqrt{2}\ \text{V/m}$$

94. d. Charge flown from A to B:

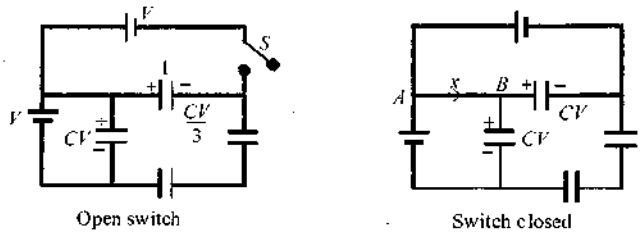


Fig. A1.184

$x =$ change in charge on capacitor '1'

$$= CV - \frac{CV}{3} = \frac{2CV}{3}$$

95. b.

$$\tau = 2\left(\frac{\sigma}{2\varepsilon_0}\lambda dx\right)x\sin\theta = \frac{\sigma}{\varepsilon_0}\lambda\int_0^{\ell/2} x.dx\sin\theta = \frac{\sigma}{2\varepsilon_0}\lambda\frac{\ell^2}{4}\sin\theta$$

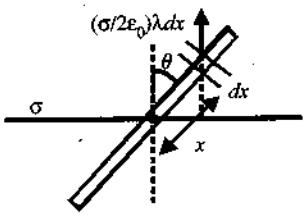


Fig. A1.185

$$\tau = -I\alpha \Rightarrow \alpha = -\tau/I = \frac{\sigma}{2\varepsilon_0}\lambda\frac{\ell^2}{4}\frac{12}{m\ell^2}\sin\theta$$

$$\alpha = -\left(\frac{3\sigma\lambda}{2m\varepsilon_0}\theta\right) \text{ (for small angle)}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2m\varepsilon_0}{3\sigma\lambda}}$$

96. c. $\alpha = 60°$, solid angle subtended by BCD:
$\omega = 2\pi\,(1 - \cos\alpha) = \pi$

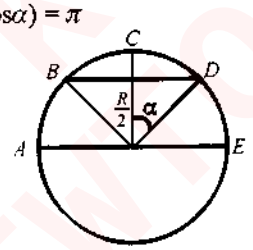


Fig. A1.186

Solid angle subtended by $ABDE$ is
$\omega_{(ABCDE)} - \omega_{(BCD)} = 2\pi - \pi = \pi$

Hence, flux through $ABDE$: $\phi = \frac{q}{\varepsilon_0}\frac{\pi}{4\pi} = \frac{q}{4\varepsilon_0}$

97. b. Electric field between the capacitor plates:

$$E = \frac{Q+x}{2A\varepsilon_0} + \frac{x}{2A\varepsilon_0} = \frac{1}{2A\varepsilon_0}[Q+2x]$$

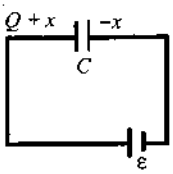


Fig. A1.187

Potential difference $= Ed = \frac{d}{2A\varepsilon_0}[Q+2x] = \varepsilon$

$$\varepsilon = \frac{Q+2x}{2C} - x = \frac{Q}{2} - C\varepsilon$$

98. c. As in none of the options, the x-coordinate appears, so we have to find the locus in the y-z plane.

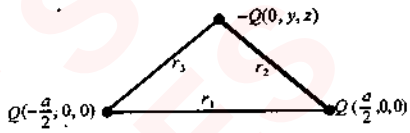


Fig. A1.188

$$U = \frac{kQ^2}{r_1} - \frac{kQ^2}{r_2} - \frac{kQ^2}{r_3} = 0$$

$$kQ^2\left[\frac{1}{a} - \frac{1}{\sqrt{(a/2)^2 + y^2 + z^3}} - \frac{1}{\sqrt{(a/2)^2 + y^2 + z^2}}\right] = 0$$

$$\frac{1}{a} = \frac{2}{\sqrt{a^2/4 + y^2 + z^2}} \Rightarrow y^2 + z^2 = 15a^2/4$$

99. b. Potential energy of an electron at any point is $U = -eV$, where V is the potential at that point.
V is negative maximum at point B, hence U is maximum for this point.

100. a. This is the case of a balanced Wheatstone bridge. The middle five capacitors will have no charge and will be useless.

101. c. $\vec{\tau} = \vec{p} \times \vec{E}$

$\tau_{max} = pE$

$50 \times 10^{-28} = p40 \Rightarrow p = 1.25 \times 10^{-28}$ column

102.a. Before closing the switch After closing the switch

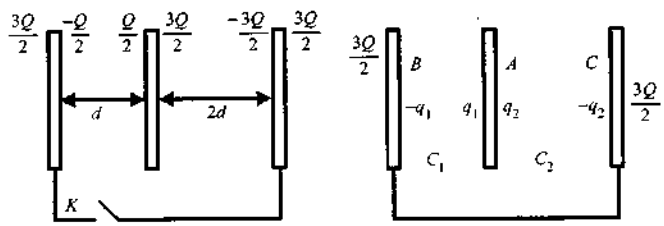


Fig. A1.189

$q_1 + q_2 = 2Q$

$$V_{AB} = V_{AC} \Rightarrow \frac{q_1}{C_1} = \frac{q_2}{C_2}$$

$$\frac{q_1}{q_2} = \frac{C_1}{C_2} = 2 \Rightarrow q_1 = 2q_2$$

Solving, we get $q_1 = \frac{4Q}{3}, q_2 = \frac{2Q}{3}$

Charge flown through K:

$$= -\frac{Q}{2} - (-q_1) = -\frac{Q}{2} + \frac{4Q}{3} = 5Q/6$$

103 c. Trajectory will be tangent to the electric field.

104 b. Potential difference between the plates

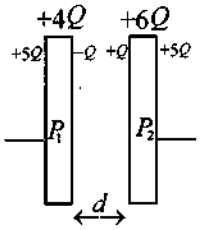


Fig. A1.190

$$V_{P_1} - V_{P_2} = \frac{-Q}{(\varepsilon_0 A/d)} = \frac{-Qd}{\varepsilon_0 A}$$

105. d. Final charges are as shown. Energy stored in the capacitor will be zero finally.

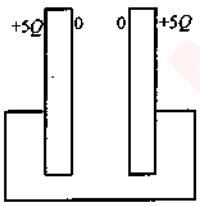


Fig. A1.191

Heat produced $= \Delta H = U_i - U_f = \frac{1}{2}\frac{Q^2}{\varepsilon_0 A/d} - 0$

Multiple Correct Answers Type

1. b., c. Energy density, $u = \frac{1}{2}\varepsilon_0 E^2$

2. a., b., c., d. Positive charge moves from a region of high electric potential to a region of low electric potential, while for negative charge it reverses. If some charge is present near the conductor, then its potential gets affected.

For Fig. A1.192, $\vec{E} = 0$ at point P but V is not.

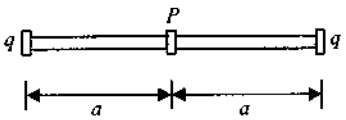


Fig. A1.192

For Fig. A1.193, $V = 0$ at point P but E is not.

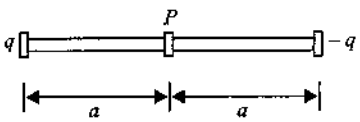


Fig. A1.193

3. a., b., d. At a centre, the force experienced by the charge particle is zero, so it is a position of equilibrium. As we displace the charge from the equilibrium position, electric force starts acting on it towards equilibrium position and hence equilibrium is a stable one.

At a distance x from the centre of sphere (equilibrium position), force experienced by the charge particle is

$F = \frac{qQx}{4\pi\varepsilon_0 R^3}$ [for $x < R$].

As $F \propto x$, it performs S.H.M. about the centre.

Time period can be calculated by using $F = m\omega^2 x$.

4. b., c., d. The path traced by q is shown in Fig. A1.194, the path is curvilinear and acceleration is due to the force exerted by Q on q.

The separation between them is minimum if relative velocity of the particle along the line joining them is zero. Let d be the minimum separation between them. As torque about Q is zero, angular momentum remains conserved.

$mua = mvd \Rightarrow v = \frac{ua}{d}$

From energy conservation law, $\frac{mu^2}{2} = \frac{mv^2}{2} + \frac{1}{4\pi\varepsilon_0}\frac{qQ}{d}$

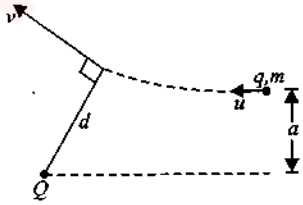


Fig. A1.194

5. a., c. Since the situation is having symmetry, the particles move symmetrically along the diagonal as shown in Fig. A1.195.

The energy conservation law is to be used.

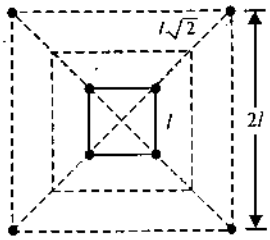


Fig. A1.195

$$U_i = \frac{1}{4\pi\varepsilon_0}\left[\frac{4q^2}{l} + \frac{2q^2}{\sqrt{2}l}\right]$$

$$U_f = \frac{1}{4\pi\varepsilon_0}\left[\frac{4q^2}{2l} + \frac{2q^2}{\sqrt{2}\times\sqrt{2}l}\right]$$

$$U_i = U_f + \frac{4mv^2}{2}$$

$$2mv^2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{2q^2}{l} + \frac{q^2}{l\sqrt{2}}\right]$$

$$v = \left[\frac{q^2}{8\pi\varepsilon_0 ml}\left(2 + \frac{1}{\sqrt{2}}\right)\right]^{1/2}$$

6. a.,c. $\vec{E} = 54\hat{i} + 74\hat{j},\ E = 90\,\text{NC}^{-1}$

$$90 = \frac{9\times10^9 Q}{r^2} \quad \text{(i)}$$

$$V = 1800 = \frac{9\times10^9 Q}{r} \quad \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$r = 20M,\ q = 4\ \mu\text{C}$$

Now, $\dfrac{(9-x_0)\hat{i} + (4-y_0)\hat{j}}{20} = \dfrac{54\hat{i} + 72\hat{j}}{90}$

$\Rightarrow$ $x_0 = -3,\ y_0 = -12$

7. a., b. Electric field lines will be from M to N, so potential of N will be less than that of M.

8. b., c., d. $V_P = \dfrac{kq}{r}$ (independent of x)

9. a., b., c. i. $E = \dfrac{2Q}{2A\varepsilon_0} + \dfrac{Q}{2A\varepsilon_0} \Rightarrow E = \dfrac{3Q}{2A\varepsilon_0}$

$$E = \frac{3}{2}\frac{Q}{Cd} \Rightarrow Ed = \frac{3Q}{2C} = V$$

ii. $F = E(-Q);\ F = \left(\dfrac{2Q}{2A\varepsilon_0}\right)\times\dfrac{(-Q)}{1} = -\dfrac{Q^2}{A\varepsilon_0}$

$\Rightarrow$ $F = \dfrac{Q^2}{A\varepsilon_0}$

iii. Energy $= \dfrac{1}{2}\varepsilon_0 E^2 Ad = \dfrac{1}{2}\varepsilon_0\left(\dfrac{3Q}{2Cd}\right)^2 Ad = \dfrac{9}{8}\dfrac{Q^2}{C}$

10. a., b., c. Time of flight $(t) = \dfrac{2u}{g} = \dfrac{2\times10}{10} = 2\,\text{sec}$

$$H = \frac{u^2}{2g} = \frac{10^2}{2\times10} = 5\,\text{m}$$

$$R = 0 + \frac{1}{2}\left(\frac{qE}{m}\right)t^2 = \frac{1}{2}\times\frac{10^{-3}\times10^4\times2\times2}{2} = 10\,\text{m}$$

11. a., c., d.

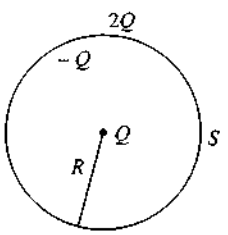


Fig. A1.196

Charge density at the outer surface (Fig. A1.196) is

$$\frac{2Q}{4\pi R^2} = \frac{Q}{2\pi R^2}$$

$$E_{\text{inside}} = \frac{Q}{4\pi\varepsilon_0 R^2},\ E_{\text{outside}} = \frac{2Q}{4\pi\varepsilon_0 R^2}$$

$\therefore$ $E_{\text{outside}} = 2E_{\text{inside}}$

12. a., b., c. $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r} = 600$ V

And $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2} = 200\ \text{NC}^{-1}$

Hence, $\dfrac{V}{E} = r = \dfrac{600}{200} = 3$ m

Now, $\dfrac{q}{4\pi\varepsilon_0 r} = 600;\ \dfrac{q}{4\pi\varepsilon_0 r'} = V'$

$\therefore$ $\dfrac{v'}{600} = \dfrac{r}{r'} = \dfrac{3}{9}$

or $V' = 200$ V

$$W = q(V - V') = 10^{-6}(600 - 200)$$
$$= 4\times10^{-4}\ \text{J}$$

13. c., d. $\vec{r}_P = \hat{i} + 2\hat{j} + 4\hat{k};\ \vec{r}_q = 3\hat{i} + 2\hat{j} + \hat{k}$

$$|\vec{r}_q - \vec{r}_P| = \sqrt{[(2)^2 + (-3)^2]} = \sqrt{13}$$

As $q = 2\times10^{-8}$ C, hence

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{|\vec{r}_q - \vec{r}_p|}$$

$$= \frac{9\times10^9\times2\times10^{-8}}{\sqrt{13}} = 49.9\ \text{V}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{|\vec{r}_q - \vec{r}_P|^2}\frac{(\vec{r}_q - \vec{r}_P)}{|\vec{r}_q - \vec{r}_P|}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{|\vec{r}_q - \vec{r}_P|^3}(2\hat{i} - 3\hat{k})$$

(By filling a dielectric of dielectric constant K, electric field gets decreased as $E = E_0/K$ and $K > 1$.)

14. a., c., d.

$$V(x) = 4 + 5x^2$$
$$V(x = 1) = 9\ \text{V}$$

And $V(X = -2) = 24$ V

Hence, $\Delta V = 15$ V

$$E = -(\Delta V/\Delta x) = -10x$$

$\therefore$ $E(\text{at } x = -1\text{ m}) = 10\ \text{NC}^{-1}$

So, $F = qE = +10$ N (along +ve x-axis).

15. a., d. Suppose a positively charged sphere is brought near an uncharged metallic sphere, then on nearer surface of

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the uncharged sphere, negative charge is induced and on farther surface, positive charge is induced. Hence, a force of attraction will be observed between these two spheres. Therefore, if a force of attraction is observed between a positively charged sphere and a metallic sphere, it cannot be concluded that the metallic sphere is necessarily negatively charged. Therefore, option (b) is wrong and options (a) and (d) are correct.

16. a., b., c. If there is no external electric field, then the charge given to a conducting sphere gets uniformly distributed over its surface. Therefore, option (a) is correct.
If an external electric field exists, then the charge gets distributed over the surface of the sphere in such a way that the electric field inside the sphere can become equal to zero. Hence, distribution of the charge on the surface of sphere will be non-uniform.
Therefore, option (b) is correct. Obviously, option (d) is wrong.
Since electric field inside the conducting sphere is equal to zero, therefore, potential difference between two points in the sphere is equal to zero. It means, the potential is same at every point of the sphere. Therefore, option (c) is correct.

17. a., c. As A and C are earthed, they are connected to each other. Hence, '$A+B$' and '$B+C$' are two capacitors with the same potential difference. If B is closer to A than to C, then the capacitance $C_{AB} > C_{BC}$. The upper surface of B will have greater charge than the lower surface. As the force of attraction between the plates of a capacitor is proportional to Q^2, there will be a net upward force on B. This can balance its weight.

18. b., c., d. Charge on inner sphere can be supposed to be concentrated as a point charge at the centre, hence electric field at a point in the region between the spheres at a distance r from the centre $=(q/4\pi\varepsilon_0 r^2)$. Due to induction, equal and opposite charges will appear on the inner and outer surfaces of the outer sphere. Hence, net charge which can be supposed to be placed at the centre $= q$ and electric field due to it at a point outside the hollow sphere at a distance r from centre $=(q/4\pi\varepsilon_0 r^2)$.

Potential of inner sphere $= \dfrac{q}{4\pi\varepsilon_0}\left(\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}\right)$

And potential of outer sphere $= \dfrac{q}{4\pi\varepsilon_0 c}$

Potential of inner sphere with respect to the outer sphere

$$\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$$

19. a., c. If positive test charge is displaced along x-axis, then net force will always act in a direction opposite to that of displacement and the test charge will always come back to its original position. But if test charge is displaced along y-axis, it will never come back to its original position and will fly away along y-axis.

20. a., b. If the charge is given to a conducting sphere, then an electric field is established in the surrounding space. Magnitude of electric field is maximum just outside the sphere. This maximum electric field may be increased to the dielectric strength of the surrounding medium. Therefore, there is a limiting value of maximum charge which can be given to the conducting sphere. Hence, option (c) is wrong. Obviously, the conducting sphere cannot be charged to a potential greater than a certain value. Hence, option (a) is correct. It can be easily said that option (b) is also correct.

21. b., c. The situation is shown in Fig. A1.197.

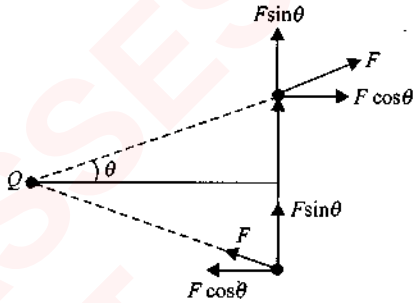

Fig. A1.197

$$\vec{F}_{net} = 2F\sin\theta \uparrow = \frac{PQ}{4\pi\varepsilon_0 r^3}$$

$$\vec{\tau} = F\cos\theta \times 2a \text{ in clockwise direction}$$

$$= \frac{PQ}{4\pi\varepsilon_0 r^2}$$

22. c., d. Torque will be perpendicular to the line $y = 2x$ and it should be in x–y plane, because electric field is in z-direction. The lines in options (c) and (d) are perpendicular to $y = 2x$.

23. a., b., c., d. Points A and B lie within the same metal hence $V_A = V_B$. The potential inside a hollow sphere is same as potential at the surface, hence $V_A = V_B = V_C = V_0$.

24. b., d. The electric field inside any point of the sphere is zero.

25. a., c. $\vec{E}_A$ is along $\overrightarrow{OA}$ and $\overrightarrow{OA} = \hat{i}+2\hat{j}+3\hat{k}$

$\vec{E}_B$ is along $\rightarrow \overrightarrow{OB} = \hat{i}+\hat{j}-\hat{k}$

Since $\overrightarrow{OA}\cdot\overrightarrow{OB} = (\hat{i}+2\hat{j}+3\hat{k})\cdot(\hat{i}+\hat{j}-\hat{k}) = 0$

So $\overrightarrow{OA} \perp \overrightarrow{OB} \Rightarrow \vec{E}_A \perp \vec{E}_B$

So, option (a) is correct.

Since $E_B = \dfrac{kq}{|OB|^2} = \dfrac{kq}{3}$

$E_C = \dfrac{kq}{|OC|^2} = \dfrac{kq}{12}$

So $\dfrac{E_B}{E_C} = 4$ or $|\vec{E}_B| = 4|\vec{E}_C|$

So, option (c) is correct.
Options (b) and (d) are wrong from the explanation.

26. a., d. When capacitors are connected in parallel, initial capacitance is $C = 2\varepsilon_0 A/d$. After the distance between the

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plates is changed, the capacitance becomes

$$C = \frac{\varepsilon_0 A}{d+a} + \frac{\varepsilon_0 A}{d-a} \text{ or}$$

$$C = \frac{2\varepsilon_0 A}{d-(a^2/d)}$$

which is greater than initial one. Hence, option (a) is correct and option (b) is incorrect.

When capacitors are connected in series, then

$$\frac{1}{C} = \frac{2d}{\varepsilon_0 A}$$

After the distance between the plates is changed,

$$\frac{1}{C} = \frac{d+a}{\varepsilon_0 A} + \frac{d-a}{\varepsilon_0 A} = \frac{2d}{\varepsilon_0 A}$$

That is, the capacitance remains unchanged. Hence, option (d) is correct and option (c) is incorrect.

27. a., d. Let q_1 and q_2 be the instantaneous charges on capacitors. Since they are in parallel, then

$$\frac{q_1}{C_1} = \frac{q_2}{C_2} \text{ and } q_1 + q_2 = Q$$

$$C_1 = \frac{\varepsilon_0 A}{d_0 + vt},\ C_2 = \frac{\varepsilon_0 A}{d_0 - vt}$$

So $$\frac{q_1}{q_2} = \frac{C_1}{C_2} = \frac{d_0 - vt}{d_0 + vt} \Rightarrow q_2\left(\frac{d_0 - vt}{d_0 + vt}\right) + q_2 = 0$$

So $$q_2 = \frac{Q(d_0 + vt)}{2d_0} \text{ and } q_1 = \frac{Q(d_0 - vt)}{2d_0}$$

Hence, option (a) is correct and option (b) is incorrect.

$$i = \frac{-dq_1}{dt} \text{ or } \frac{dq_2}{dt} \text{ or } i = \frac{Qv}{2d_0}$$

which does not depend on time. So option (d) is correct and option (c) is incorrect.

28. a., c. Let the length, width and thickness of slab be l, b and d respectively. K be the dielectric constant of dielectric.

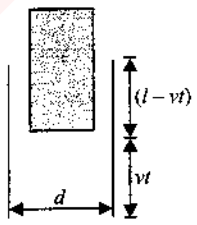


Fig. A1.198

At time, t length t of the slab has been pulled out, so at this instant the capacitor may be considered as a parallel combination of two capacitors as shown. The capacitance of combination is

$$C = \frac{\varepsilon_0 vtb}{d} + \frac{\varepsilon_0 Kb(l - vt)}{d};$$

$$C = \frac{\varepsilon_0 b}{d}[vt + K(l - vt)]$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 b}{d}[Kl - (K-1)vt]$$

This equation is of straight line, having positive intercept and negative slope. Hence, option (a) is correct and option (b) is incorrect.

Since battery is disconnected, so charge on capacitor remains constant.

$$V = \frac{q}{C} \text{ or } V \propto \frac{1}{C}$$

Hence, current between V and C will be rectangular hyperbola.

So, option (c) is correct and option (d) is incorrect.

29. a., c. Initial stored energy $U_i = \frac{1}{2}\frac{\varepsilon_0 A V_1^2}{x}$

Final stored energy $U_f = \frac{1}{2}\frac{\varepsilon_0 A V_0^2}{2(x+dx)}$

So $$\Delta U = U_f - U_i = \frac{1}{2}\varepsilon_0 A V_0^2\left[\frac{1}{x+dx} - \frac{1}{x}\right]$$

$$= \frac{1}{2}\varepsilon_0 A V_0^2\left[\frac{x - x - dx}{x(x+dx)}\right]$$

$$= -\frac{1}{2}\frac{\varepsilon_0 A V_0^2}{x^2}dx = -\frac{1}{2}\frac{\varepsilon_0 A V_0^2}{x}\cdot\frac{dx}{x} = -\frac{U\,dx}{x}$$

So, option (a) is correct and option (b) is incorrect.

$$F = -\frac{dU}{dx} = -\frac{U}{x}$$

$$= -\frac{1}{2}\frac{\varepsilon_0 A}{x}\cdot\frac{V_0^2}{x} = -\frac{1}{2}\left(\frac{\varepsilon_0 A}{x}V_0\right)\frac{V_0}{x} = -\frac{1}{2}qE$$

So magnitude of attractive force is $\frac{1}{2}qE$.

So, option (c) is correct and option (d) is incorrect.

30. a., c. $$\frac{1}{4\pi\varepsilon_0}\frac{Q}{R} + \frac{1}{4\pi\varepsilon_0}\frac{q}{(R_0 - vt)} = 0$$

where R_0 is the initial distance of the charged particle.

$$Q = \frac{Rq}{R_0 - vt} \Rightarrow \frac{dQ}{dt} = i = \frac{Rqv}{(R_0 - vt)^2}$$

31. a., d.

$$E = at (a = \text{constant})\ F = QE$$

$$a = F/m = QE/m = Es = ast$$

$$= \frac{dv}{dt} = ast$$

$$\text{or } v = \frac{1}{2}ast^2 \Rightarrow v \propto s \text{ and } t^2$$

32. b., c.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{x} = 600 \quad \text{(i)}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2} = 200 \quad \text{(ii)}$$

From Eq. (i) and Eq. (ii), $x = 3$ m

From Eq. (i), $q = \frac{600\times 3}{9\times 10^9} = 2\times 10^{-7}$ C

$$V' = \frac{1}{4\pi\varepsilon_0}\frac{q}{x'} = 9\times 10^9 \times \frac{2\times 10^{-7}}{9} = 200 \text{ V}$$

$$W = q(V'-V) = 10^{-6}(200-600) = -4\times 10^{-4} \text{ J}$$

33. a., c., d.

The amount of charge flown through the battery is q = 20 μC. Because charge on the right capacitor changes by 20 μC. Charge flown through the switch is 60 mC. It is because the sum of charges of both the left capacitors and the charge which flows through the battery will pass through the switch.

Therefore, energy supplied by the battery is

$$U = qV = (20 \times 10\text{-}6)\ (30) \text{ J} = 0.6 \text{ mJ}$$

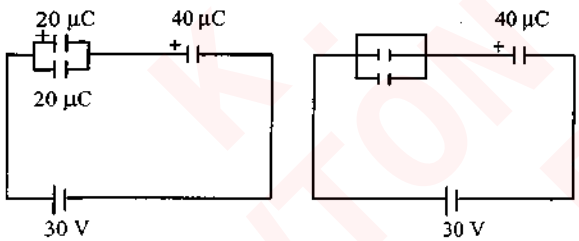


Fig. A1.199

Energy stored in all the capacitors before closing the switch S is

$$U_i = \frac{1}{2}C_{net}V^2 = \frac{1}{2}\left(\frac{4}{3}\times 10^{-6}\right)(30)^2 = 0.6 \text{ mJ}$$

and after closing the switch

$$U_f = \frac{1}{2}C_{net}V^2 = \frac{1}{2}(2\times 10^{-6})\ (30)^2 = 0.9 \text{ mJ}$$

Therefore, heat generated $H = U - (U_f - U_i) = 0.3$ mJ

34. .a., b., c.

When either A or C is earthed (but not both together), a parallel-plate capacitor is formed with B, with $\pm Q$ charges on the inner surfaces. (The other plate, which is not earthed, plays no role.) Hence, charge of amount $+Q$ flows to the earth.

When both are earthed together, A and C effectively become connected. The plates now form two capacitors in parallel, with capacitances in the ratio 1 : 2, and hence share charge Q in the same ratio.

$$C_1 = \frac{\varepsilon_0 A}{2d}, C_2 = \frac{\varepsilon_0 A}{d}\ \frac{C_1}{C_2} = \frac{1}{2}$$

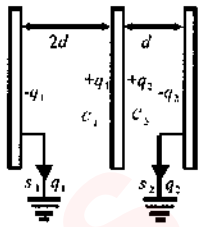


Fig. A1.200

Now we can write $\frac{q_1}{q_2} = \frac{C_1}{C_2} = \frac{1}{2}$, because potential difference of both the capacitors is same, and $q_1 + q_2 = Q$. Solving these, we get

$$q_1 = \frac{Q}{3}, q_2 = \frac{2Q}{3}$$

35. b., c.

$$C = \frac{Q}{V} \quad \text{Also, } C = \frac{\varepsilon_0 A}{d}$$

As d increases, C will decrease.

Therefore, V should increase, as Q remains the same.

$$E = \frac{V}{d} = \frac{Q}{A\varepsilon_0} \text{ will be constant.}$$

Energy, $U = \frac{Q^2}{2C}$. As C decreases, U will increase.

36. a., d.

Electric field is zero only at a point in region III. Equilibrium at this point will be stable for a negative charge.

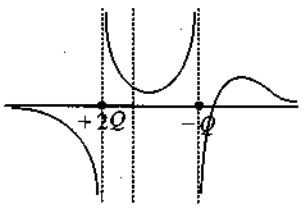


Fig. A1.201

Assertion-Reasoning Type

1. **d.** Capacitance increases and electric field decreases.
2. **a.** If $E = 0$ in some region, then potential is same everywhere in that region.
3. **d.** An electrically neutral body means charge on the body is zero. A body connected to earth may possess some charge.
4. **a.** The electric field due to disk is superposition of electric fields due to its constituent rings as given in reason. Assertion is true, reason is true; reason is a correct explanation for assertion.
5. **a.** (Moderate) Electric flux within any closed surface in region of uniform field is zero because the total number of

electric lines of force entering the closed surface equals that leaving the surface. Hence, from Gauss's theorem the net charge enclosed within such a closed surface is zero.

6. **a.** Free electrons present in the metal are moving randomly in all directions in absence of the electric field. Hence, the average velocity of electrons is zero. Because of it the current does not flow in the metal in the absence of electric field.

7. **a.** We know that surface density of a charge is very large on the sharp ends of a conductor. Hence, the lightning conductor conducts charges of lighting to earth and there is no harm to the building.

8. **b.** On an equipotential surface, the potential at each point is same throughout.

Let $\vec{E}$ be electric field at a point on an equipotential surface.

Then, small work done in moving a test charge q_0 through a small displacement along the surface

$$dW = \vec{E}\cdot d\vec{r} = (q_0\vec{E})\cdot d\vec{r}$$

Since work in moving a test charge along an equipotential surface is always zero,

$$\therefore \quad (-q_0\vec{E})\cdot d\vec{r} = 0 \text{ or } \vec{E}\cdot d\vec{r} = 0$$

Hence, electric field is directed perpendicular to the surface.

9. **c.** We know that common potential will be between two initial values, so it will be less than the potential of one of the conductors. Since both have positive charge, they will repel each other.

10. **b.** A capacitor does not discharge itself. In case the capacitor is connected in a circuit containing a source of high voltage, the capacitor charges itself to a very high potential. So, if a person handles it without discharging, he/she may get a severe shock. Dielectrics and insulators are the same.

11. **b.** The capacitance of a parallel-plate capacitor is given by

$$C = \frac{\varepsilon_0 A}{d}$$

where A is area of the plates and d is the distance between plates.

Hence,
$$\frac{C'}{C} = \frac{\dfrac{\varepsilon_0 A}{d}}{\left(\dfrac{\varepsilon_0 A}{\frac{d}{2}}\right)} = 2 \Rightarrow C' = 2C$$

When a dielectric of dielectric constant K is introduced in between the plates, then the capacitance

$$C = \frac{K\varepsilon_0}{d} \Rightarrow C \propto K$$

i.e., C depends on induced dielectric constant.

12. **c.** The total charge on each plate will have the charge in same quantity, as charge is independent of the area. But the charge will be of opposite nature.

13. **c.** The capacitance of a spherical conductor of radius r is given by $C = 4\pi\varepsilon_0 r$.

The radius of the spherical conductor having capacitance 1 F is given by

$$r = \frac{C}{4\pi\varepsilon_0} = 1\times 9\times 10^9 = 9\times 10^9 \text{ m} = 9\times 10^6 \text{ km}$$

Since one cannot have a spherical conductor of such a big radius, even greater than earth, so it is not possible to make a spherical capacitance of 1 F and earth itself cannot have capacitance of 1 F.

14. **b.** Torque acting on the dipole is also zero when it is oppositely directed w.r.t. electric field.

15. **c.** We know that for an electric dipole

$$E_{\text{axial}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{2p}{r^3}\right) \text{ and } E_{\text{equatorial}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{p}{r^3}\right)$$

Hence, $$\frac{E_{\text{axial}}}{2} = E_{\text{equatorial}}$$

or $$E_{\text{equatorial}} = \frac{E}{2}$$

Reason is false as electric field due to dipole varies inversely as cube of distance, i.e., $E \propto 1/r_3$.

Comprehension Type

For Problems 1–3

1. a., 2. c., 3. b.

Sol.

1 a. $$\phi = \frac{q_{\text{en}}}{\varepsilon_0} = \frac{q_1 + q_2 + q_3}{\varepsilon_0}$$

$$= \frac{(+3.1\times 10^{-9} - 5.9\times 10^{-9} - 3.1\times 10^{-9}\ 8.85\times 10^{-12}}{8.85\times 10^{-12}}$$

$$= -666 \text{ Nm}^2\text{C}^{-1}$$

2. **c.** By replacing the plastic plate with aluminium plate of same dimensions, the charge density (i.e., σ) does not get affected. Hence, electric field will remain constant.

3. **b.** Due to induction, some positive charge will lie within the Gaussian surface and hence flux becomes positive.

For Problems 4–7

4. a., 5. b., 6. c., 7. a.

Sol. $2[m_1 g - B] + m_2 g = 0$

$$2[V\rho_{\text{He}} g - V\rho_{\text{air}} g] = -m_2 g$$

$$V = \frac{-m_2}{2[\rho_{\text{He}} - \rho_{\text{air}}]} = \frac{1.1\times 10^{-3}}{2[1.3 - 0.2]} = \frac{10^{-3}}{2}\text{ m}^3$$

$$= \frac{10^3}{2}\text{cm}^3 = 500\text{cm}^3$$

$$T\cos\theta = \frac{kq^2}{r^2};\ T\,(0.6) = \frac{9\times 10^9\times q^2}{(1.2)^2} \quad \text{(i)}$$

$$T\sin\theta = \frac{m_2 g}{2};\ T(0.8) = \frac{1.1\times10^{-3}\times10}{2} \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{0.6}{0.8} = \frac{9\times10^9\times q^2}{(1.2)^2}\times\frac{2}{1.1\times10^{-3}\times10}$$

$$\Rightarrow \quad q = 8.12\times10^{-7}\ C = 0.8\ \mu C$$

and $$T = \frac{9\times10^9\times q^2}{(1.2)^2\times(0.6)} = 6.875\times10^{-3}\ N$$

Also, $$T\sin\theta = 5.5\times10^{-3}\ N$$

For Problems 8–10

8. c, 9. b, 10. d.

Sol.

8. c. From Fig. A1.202

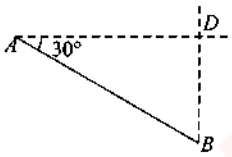


Fig. A1.202

$$V_A - V_B = V_A - V_D = EAD$$

$$= E\,AB\cos30° = E2\sqrt{3}\frac{\sqrt{3}}{2} = 3E$$

9. b. $V_C - V_B = V_F - V_B = E(BF)$ (Fig. A1.203)

$$= -E(BC)\cos60° = -E\times4\times\frac{1}{2} = -2E$$

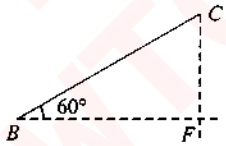


Fig. A1.203

10. d. $W_{el} = q(V_A - V_C)$

$$= q(V_A - V_B) + (V_B - V_C) = q\ (3E + 2E) = 5qE$$

For Problems 11–13

11. c., 12. c., 13. a.

Sol.

11. c. $E = -\dfrac{dV}{dx} = -4x$

12. c. $F = qE = 2.5\times10^{-6}(-4x) = -10^{-5}x$

$$W = \int_2^0 F\,dx = -10^{-5}\int_2^0 x\,dx$$

$$\frac{1}{2}mv^2 = 10^{-5}\left[\frac{x^2}{2}\right]_0^2 \quad \Rightarrow V = 2\,ms^{-1}$$

13. a. $F = -10^{-5}x$

$$ma = -10^{-5}x \quad \Rightarrow \quad 10\times10^{-6}a = -10^{-5}x$$

$$\Rightarrow a = -x$$

$$\omega^2 = 1,\ \omega = 1\ \text{rad.} \Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{1} = 2\pi s$$

For Problems 14–17

14. c., 15. a., 16. c., 17. b.

Sol. Since $Q = +8\ \mu C$, if q is a positive charge, resultant force on it due to Q at A and Q at B will be along positive Y-axis and it would move away along Y-axis. But the charge q here is observed to oscillate. This is possible only if q is a negative charge so that resultant force on it due to Q at A and Q at B is towards O. Under the action of this force, q moves towards O, crosses O and as it is moving along negative Y direction, resultant force on it will again be towards O. This force retards the motion of q along negative Y-axis. It comes to rest at some point and then moves back towards O and so on (Fig. A1.204).

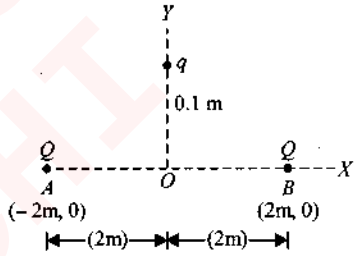


Fig. A1.204

Force applied by Q on q has a magnitude

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{y^2+a^2}$$

Force applied by Q at A on q can be resolved into rectangular components: $F\cos\theta$ and $F\sin\theta$. Similarly, force applied by Q at B on q can be resolved into components: $F\cos\theta$ and $F\sin\theta$. $F\sin\theta$ components of the two forces balance each other so that the net force on q is $2F\cos\theta$ towards O.

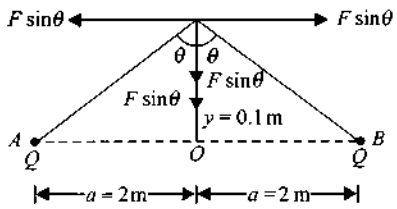


Fig. A1.205

Therefore, net force on q, $F_n = 2\dfrac{1}{4\pi\varepsilon_0}\dfrac{Qq}{y^2+a^2}\cos\theta$

$$F_n = \frac{1}{4\pi\varepsilon_0}\frac{2Qq}{y^2+a^2}\frac{y}{(y^2+a^2)^{1/2}} = \frac{1}{4\pi\varepsilon_0}\frac{2Qqy}{(y^2+a^2)^{3/2}} \qquad \text{(i)}$$

For $y << a$, net force on q, $F_n = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2Qqy}{a^3}$ (ii)

Here, $Q = 8\ \mu C = 8\times10^{-6}$ C

At $t = 0$, q is at $y = 0.1$ m. Obviously, $y < a$ (= 2 m).

Since the motion is simple harmonic, we can use the approximation $y << a$ so that net force, from Eq. (ii), will be proportional to displacement (y). Initially, i.e., at $y = 0$ m, force on q is 9×10^{-3} N.

Using Eq. (i), we get

$$9 \times 10^{-3} = (9 \times 10^9) \frac{2\,(8 \times 10^{-6})\, q\,(0.1)}{(2)^3}$$

$$\Rightarrow \qquad q = 5 \times 10^{-6}\,\text{C} = 5\ \mu\text{C}$$

This, in fact, is the magnitude of q. We know that q, as explained earlier, is a negative charge. Hence, $q = -5\ \mu$C. So, correct option is (c).

At $t = 0$, q is released at a point 0.1 m from O on Y-axis. As it oscillates, its other extreme position will be 0.1 m from O on the negative Y-axis, assuming undamped simple harmonic motion. Hence, amplitude of oscillations is 0.1 m or 10 cm. So, correct option is (a).

From Fig. A1.205, we get

$$F_n = \frac{1}{4\pi\varepsilon_0}\frac{2Qqy}{a^3} = ky$$

where $$k = \frac{1}{4\pi\varepsilon_0}\frac{2Qq}{a^3}$$

Thus, $F_n \propto y$

We also know that F_n always acts towards O (mean position).

Time period of resulting S.H.M. will be $T = 2\pi\sqrt{\frac{m}{k}}$

or frequency $$= \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$\therefore \quad f = \frac{1}{2\pi}\sqrt{\frac{1}{4\pi\varepsilon_0}\frac{2Qq}{ma^3}}$$

$$= \frac{1}{(2 \times 3.14)}\sqrt{(9 \times 10^9)\frac{(2)(8 \times 10^{-6})(5 \times 10^{-6})}{(91 \times 10^{-6})(2)^3}} = 5$$

$[m = 91\ \text{mg} = 91 \times 10^{-6}\ \text{kg}]$

Thus, the correct option is (c).

In S.H.M., equation of displacement from mean position can be expressed as $y = a\sin(\omega t + \phi)$

Here, $a = 0.1\ \mu$; $\omega = 2\pi f = 2\pi \times 5 = 10\pi$

$$y = 0.1\sin(10\pi t + \phi)$$

But at $t = 0$, $y = 0.1$ (given)

Hence, $0.1 = 0.1\sin\phi$ or $\sin\phi = 1$

$$\Rightarrow \qquad \phi = \frac{\pi}{2};\ y = 0.1\sin(10\pi t + \pi/2)$$

Thus, the correct option is (b).

For Problems 18–21

18.d., 19. a., 20. c., 21. c.

Sol. The two particles move in different circles.

The mutual interaction force provides the required centripetal force to the particle. As magnitude of the interaction force is same, we get

$$F_{12} = \frac{m_1 v_1^2}{r_1} \text{ and } F_{21} = \frac{m_2 v_2^2}{r_2}$$

$$\left|\vec{F}_1\right| = \left|\vec{F}_2\right| \Rightarrow \frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2}$$

Putting values, we get $r_2 = 2r_1$

Also, $r_1 + r_2 = 12 \times 10^{-12}$ m (given)

$$r_1 = 4 \times 10^{-12}\ \text{m};\ r_2 = 8 \times 10^{-12}\ \text{m}$$

Acceleration of first particle $= \frac{v_1^2}{r_1} = \frac{(10^3)^2}{(4 \times 10^{-12})}$

$$= 2.5 \times 10^{15}\ \text{ms}^{-2}$$

19. a. Acceleration of second particle is

$$\frac{v_2^2}{r_2} = \frac{(2 \times 10^3)^2}{(8 \times 10^{-12})} = 5 \times 10^{15}\ \text{ms}^{-2}$$

20. c. Just after release,

$$V_{CM} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$= \frac{(2 \times 10^{-30})(0) + (10^{-30})(2 \times 10^3)}{3 \times 10^{-30}} = \frac{2}{3} \times 10^3\ \text{ms}^{-1}$$

21. c. From Fig. A1.206,

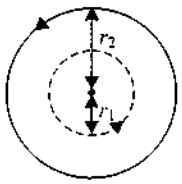


Fig. A1.206

Since the distance between them always remains constant but they move with different velocities, therefore they must move in different circles with common centre as shown in Fig. A1.206.

For Problems 22–23

22. b., 23. d.

Sol. Charge on capacitors C_1 and C_2 before closing the switch S (Fig. A1.88) is

$$q_0 = \left(\frac{C_1 C_2}{C_1 + C_2}\right)\varepsilon$$

After closing S, charge in C_2 (final charge) is

$$q_2 = C_2\varepsilon$$

In loop $ABDEFGA$, $\varepsilon - \frac{q_1}{\varepsilon} - \varepsilon = 0 \quad \Rightarrow q_1 = 0$

Final charge on C_1, $q_1 = 0$

To make final charge in C_1, the charge q_0 will flow towards battery.

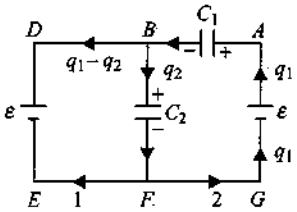


Fig. A1.207

Hence, charge flown towards direction 2 is

$$\Delta q_2 = -\left(\frac{C_1C_2}{C_1+C_2}\right)\varepsilon$$

To make charge on capacitor C_2 to final value q_2, the charge flow into capacitor is

$$\Delta q = C_2\varepsilon - \frac{C_1C_2}{C_1+C_2}\varepsilon = C_2\varepsilon\left[1-\frac{C_1}{C_1+C_2}\right]$$

$$\Delta q_B = \frac{C_2^2\varepsilon}{C_1+C_2}$$

At junction F,

$$\Delta q_1 = \Delta q - \Delta q_2 = \frac{C_2\varepsilon}{C_1+C_2}[C_2+C_1] = C_2\varepsilon$$

Hence, charge flown in the direction of $1 = \Delta q = C_2\varepsilon$

Alternative Method: (Fig. A1.208)

In loop $ABDEG$, $-\dfrac{(q_0+\Delta q_1)}{C_1} - \varepsilon + \varepsilon = 0$

$$\Delta q_1 = -q_0 = -\frac{C_1C_2\varepsilon}{C_1+C_2}$$

In loop $BFEDB$, $-\dfrac{(q_0+\Delta q_1)}{C_2} + \varepsilon = 0$

$$\Delta q = C_2\varepsilon - q_0$$

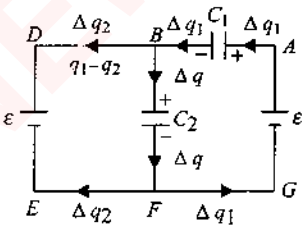


Fig. A1.208

At junction F, $\Delta q_2 = \Delta q_1 - \Delta q_1 = -q_0 - (C_2\varepsilon - q_0)$

$$= -C_2\varepsilon$$

Hence charge flow, in the direction of:

$$(1) = -\frac{C_1C_2\varepsilon}{C_1+C_2}$$

$$(2) = C_2\varepsilon$$

For Problems 24–26

24. a., 25. b., 26. d.

Sol. Before earthing charge on each capacitor (Fig. A1.209),

$$q_0 = 120 \times \frac{6}{5}$$

$$q_0 = 24 \times 6 = 144\ \mu C$$

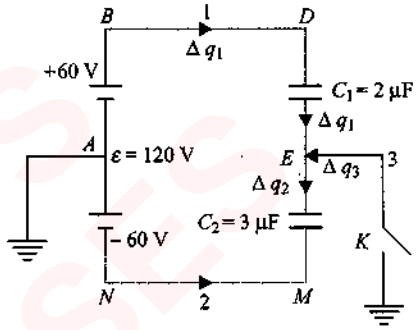


Fig. A1.209

After earthing, we get

Path $ABDE$, $0 + 60 - \dfrac{(q_0+\Delta q_1)}{2} = 0$

$$\Delta q_1 = 120 - q_0 = 24\ \mu C$$

Path $ANME$, $0 - 60 + \dfrac{(q_0+\Delta q_2)}{3} = 0$

$$\Delta q_2 = 180 - q_0 = 36\ \mu C$$

At junction E, $\Delta q_1 + \Delta q_3 = \Delta q_2$

$$\Delta q_3 = \Delta q_2 - \Delta q_1 = 36 + 24 = 60\ \mu C$$

Hence, the charge flown in the direction

(1) $= -24\ \mu C$

(2) $= -36\ \mu C$

(3) $= +60\ \mu C$

For Problems 27–28

27. c., 28. b.

Sol. Concentric cylindrical electrodes will produce radial electric field. As ion is entering at O and leaving at B, hence the path followed by ion should be circular and centred at A. Required centripetal force should be provided by force on the ion due to electric field.

Hence, $qE = \dfrac{mv^2}{R} \Rightarrow E = \dfrac{mv^2}{qR}$

As final velocity along x-axis becomes zero and finally the ion starts moving towards y direction, hence the electric field should have component towards x- and y-directions respectively.

For x-component of electric field (using $v_x^2 = u_x^2 + 2a_x\,\Delta x$):

$$0 = v^2 - 2\left(\frac{qE_x}{m}\right)R \Rightarrow E_x = \frac{mv^2}{2qR}$$

For y-component of electric field (again using $v_y^2 = u_y^2 + 2a\,\Delta y$)

$$v^2 = 0 + 2\left(\frac{qE_y}{m}\right)R \Rightarrow E_y = \frac{mv^2}{2qR}$$

Hence, net electric field is

$$\vec{E} = -\frac{mv^2}{2qR}\hat{i} + \frac{mv^2}{2qR}\hat{j} = \frac{mv^2}{2qR}(-\hat{i} + \hat{j})$$

For Problems 29–32

29. a., 30. b., 31. d., 32. d.

Sol

29. a. Let capacitance of A is C_1 and that of B is C_2. Both are with air.

Given $C_1 = 2C_2$

Before insertion of dielectric:

$$C_{eq_1} = \frac{2}{3}C_2$$

After insertion: $C_{eq1} = \frac{6C_2}{C_1 + 6C_2} = \frac{3}{2}C_2$

$$\Delta U = \frac{1}{2}(C_{eq2} - C_{eq1})V^2$$

$$1.2 \times 10^{-4} = \frac{1}{2}\left[\frac{3}{2}C_2 - \frac{2}{3}C_2\right](12)^2$$

$$C_2 = 2 \times 20^{-6}, F = 2\ \mu F$$

Now before insertion of dielectric (Fig. A1.210):

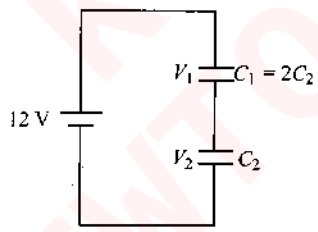


Fig. A1.210

$$V_2 = \frac{2 \times 12}{2+1} = 8\ V$$

Energy in $B = U_1 = \frac{1}{2}C_2V_2^2 = 64 \times 10^{-6}$ J

After insertion of dielectric (Fig. A1.211):

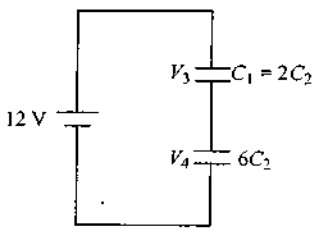


Fig. A1.211

$$V_4 = \frac{2 \times 12}{2+6} = 3\ V$$

Energy in $B = U_2 = \frac{1}{2}6C_2V^2 = 54 \times 10^{-6}$ J

Decrease in energy $= U_1 - U_2 = 10^{-5}$ J

30. b. Since dielectric is removed after disconnecting the battery, charge on it will remain same.

$$Q = C_{eq2}V = \frac{3}{2}C_2V = 36 \times 10^{-6}\ C$$

Work done in removing the dielectric = change in energy of B

$$= \frac{Q^2}{2C_2} - \frac{Q^2}{2 \times 6C_2}$$

$$= 2.7 \times 10^{-4}\ J$$

31. d. Before connecting as in situation (ii), both capacitors will carry equal charge. They will get neutralized on connecting as in situation (i). So, net charge on any capacitor will become zero. Hence, final electrical energy stored on them will be zero.

32. d. Charges acquired by capacitors after the whole process described is completed (Fig. A1.212).

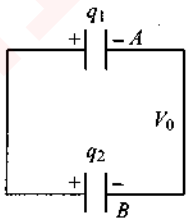


Fig. A1.212

$$Q = \frac{3}{2}C_2 \times 12 = 36 \times 10^{-6}\ C$$

For situation (ii), let q_1 and q_2 be the final charges. V_0 is common potential.

$$q_1 + q_2 = 36 + 36 = 72\ \mu C$$

$$V_0 = \frac{q_1}{C_1} = \frac{q_2}{C_2}, q_1 = 48\ \mu C, \quad q_2 = 24\ \mu C$$

Final electric energy is

$$\frac{1}{2}\left[\frac{q_1^2}{C_1} + \frac{q_2^2}{C_2}\right] = 432 \times 10^{-6}\ J = 4.3 \times 10^{-4}\ J$$

For Problems 33–35

33. a., 34. a., 35. b.

Sol.

33. a. System is equivalent to two capacitors, i.e.,

$$C_1 = \frac{K\varepsilon_0 A}{t}, C_2 = \frac{\varepsilon_0 A}{d-t}$$

Since both are connected in series, so

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{t}{K\varepsilon_0 A} + \frac{(d-t)}{\varepsilon_0 A}$$

$$= \frac{t + Kd - Kt}{K\varepsilon_0 A} = \frac{K(d-t)+t}{K\varepsilon_0 A}$$

or $$C = \frac{K\varepsilon_0 A}{K(d-t)+t}$$

$$= \frac{\varepsilon_0 A}{(d-t)+\frac{t}{K}} = \frac{\varepsilon_0 A}{(d-t)+\frac{t}{2}}$$

$$= \frac{\varepsilon_0 A}{d-\frac{t}{2}}$$

34. a. According to the question, $\frac{\varepsilon_0 A}{d-\frac{t}{2}} = \frac{3}{2}\frac{\varepsilon_0 A}{d}$

Solving, we get $\frac{t}{d} = \frac{2}{3}$

35. b. If charge q on the capacitor remains unchanged, then

$$U_i = \frac{1}{2}\frac{q^2}{C_a}, U_f = \frac{q^2}{2C_{eq}}$$

or $$\frac{U_i}{U_f} = \frac{C_{eq}}{C_a} = \frac{d}{t} = \frac{3}{2}$$

For Problems 36–40

36. a., 37. d., 38. b., 39. d., 40. c.

Sol.

36. a. The charge distribution on various surfaces is as shown in the following figure. $-q_1$ on the surface of cavity 1 will spread uniformly if q_1 is at the centre, otherwise the distribution would be non-uniform. Same is the case with $-q_2$.

The charge appearing on outer surface of the conductor is $q_1 + q_2 + Q$ which would be non-uniformly distributed as radius of curvature at various points of conductor's surface is different. The presence of q_1 and q_2 (and their location) has no effect on the distribution of charge on the outer surface. The presence of q will change the distribution of charge on outer surface, but still it remains non-uniform.

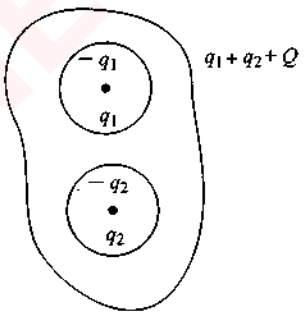


Fig. A1.213

37. d. Electric field intensity outside the cavity due to q_1 and $-q_1$ would be zero.

$$\bar{E}_{-q_1} + \bar{E}_{-q_1} = 0$$

$$\bar{E}_{-q_1} = -\bar{E}_{q_1} = \frac{q_1}{4\pi\varepsilon_0 r^2} \text{ towards centre of cavity 1}$$

38. b. $\bar{E}$ inside the conductor due to outside charges = 0

$$\bar{E}_q + \bar{E}_{q_1+q_2+Q} = 0$$

$$\Rightarrow \quad \bar{E}_{q_1+q_2+Q} = -\bar{E}_q$$

$$= \frac{q}{4\pi\varepsilon_0 r^2} \text{ towards } q.$$

39. d. If q_2 is at point Q, then induced charge $-q_2$ would be non-uniformly distributed. So, we cannot determine $\vec{E}$ due to $-q_2$ at any inside point.

40. c. Potential at point Q = Potential of conductor + Potential due to q_2 + Potential due to $-q_2$

$$= V_0 + \frac{q_2}{4\pi\varepsilon_0 r_2'} + \frac{-q_2}{4\pi\varepsilon_0 r_2} = \frac{q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_2'} - \frac{1}{r_2}\right) + V_0$$

For Problems 41–44

41. b., 42. d., 43. c., 44. b.

Sol. Let the charge distribution be as shown in Fig. A1.214.

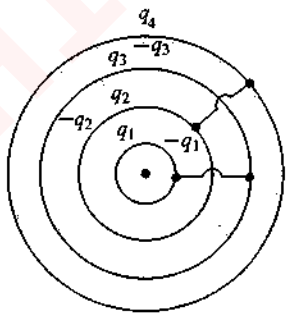


Fig. A1.214

From Gauss's theorem, we know that facing surfaces of the conductor acquire equal and opposite charges.

i.e., $V_1 = V_3$ and $V_2 = V_4$

$$q_1 + q_3 - q_2 = +4Q \quad \text{(i)}$$

Now, $$q_2 - q_1 + q_4 - q_3 = -6Q \quad \text{(ii)}$$

$$V_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{R} + \frac{q_2-q_1}{2R} + \frac{q_3-q_2}{3R} + \frac{q_4-q_3}{4R}\right]$$

$$V_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{2R} + \frac{q_2-q_1}{2R} + \frac{q_3-q_2}{3R} + \frac{q_4-q_3}{4R}\right]$$

$$V_3 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{3R} + \frac{q_2-q_1}{3R} + \frac{q_3-q_2}{3R} + \frac{q_4-q_3}{4R}\right]$$

$$V_4 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{4R} + \frac{q_2-q_1}{4R} + \frac{q_3-q_2}{3R} + \frac{q_4-q_3}{4R}\right]$$

From $$V_1 = V_3, q_1 = -\frac{q_2}{3}$$

From $$V_2 = V_4, q_2 = -\frac{q_3}{2}$$

On solving Eq. (ii), $q_1 = \frac{2Q}{5}, q_2 = -\frac{6Q}{5}$ and $q_3 = \frac{12Q}{5}$

Substituting these values in Eq. (ii), we get $q_4 = -2Q$.

41. b. Charge on the inner surface of the third conductor $= -q_2 = 6Q/5$.

42. d. Charge on the fourth conductor $= q_4 - q_3 = -2Q - \frac{12Q}{5} = \frac{-22Q}{5}$.

43. c. Potential of conductor 1, $V_1 = \frac{-3Q}{40\pi\varepsilon_0 R} = V_3$

44. b. Potential of conductor 2, $V_2 = \frac{Q}{8\pi\varepsilon_0 R} = V_4$

For Problems 45–48

45. b., 46. a., 47. c., 48. d.

Sol.

46. a. Inner wall is negatively charged, so it is at a lower potential (Fig. A1.215).

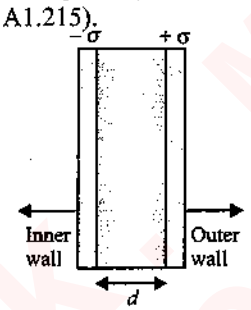


Fig. A1.215

46. a. Area of cell wall is $A = V/d \Rightarrow \quad A = 10^{-5}\ \text{m}^2$

$$E = \frac{\sigma}{5\varepsilon_0}\ \text{NC}^{-1}$$

47. c. $dV = -\vec{E}\cdot\vec{dr}$

$$V_{\text{outer}} - V_{\text{inner}} = \frac{\sigma}{5\varepsilon_0} \times 10^{-10}\ \text{V}$$

48. d. $U = \frac{1}{2}\varepsilon_0 E^2 \times \text{volume}$

$$= \frac{1}{2}(5\varepsilon_0)\left[\frac{\sigma}{5\varepsilon_0}\right]^2 \times 10^{-15}\ \text{J} = \frac{\sigma^2}{\varepsilon_0} \times 10^{-16}\ \text{J}$$

For Problems 49–50

49. c., 50. c.

Sol.

49. c. This can be treated as a system of two capacitors in series (Fig. A1.216).

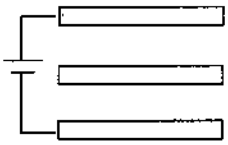


Fig. A1.216

$$C_{\text{eq}} = \frac{C_1 C_1}{C_1 + C_2}$$

where $\quad C_1 = C_2 = \frac{2\varepsilon_0 A}{d}$

50. c. Initially, charge supplied by battery, $q_i = \frac{\varepsilon_0 A}{d} \times V$

After shorting, $C_{\text{eq}} = \frac{2\varepsilon_0 A}{d}$, so $q_f = \frac{2\varepsilon_0 AV}{d}$

So, extra charge flown $= q_f - q_i = \frac{\varepsilon_0 AV}{d}$

For Problems 51--53

51. a., 52. d., 53. c.

Sol.

51. a. $C_i = \frac{\varepsilon_0 d}{x}, \quad C_f = \frac{\varepsilon_0 A}{x + dx}$

$$U_i = \frac{C_i V^2}{2}, \quad U_f = \frac{C_f V^2}{2}$$

$$dU = U_f - U_i = \frac{\varepsilon_0 AV^2}{2}\left[\frac{1}{x+dx} - \frac{1}{x}\right]$$

$$= -\frac{\varepsilon_0 AV^2}{2} \times \frac{dx}{x^2}\ [x >> dx]$$

52. d. $q_i = C_i V, q_f = C_f V$

So, the required charge,

$$q = q_f - q_i$$

$$= \varepsilon_0 AV\left[\frac{1}{x+dx} - \frac{1}{x}\right]$$

$$= -\frac{\varepsilon_0 AV\,dx}{x^2}$$

Negative sign implies that charge is entering into the battery from its positive terminal.

Work done by battery $= qv = -\frac{\varepsilon_0 AV^2 dx}{x^2}$

Negative sign means that work is done on the battery.

53. c. From the work–energy theorem

$$dK = 0 = W_{\text{el}} + W_{\text{ext}} + W_{\text{battery}}$$

$$W_{\text{el}} = \frac{\varepsilon_0 AV^2}{2} \times \frac{dx}{x^2}$$

For Problems 54–56

54. b., 55. b., 56. b.

Sol.

54. b. The electrostatic force generated by an electric field E on a point charge q is always $F = qE$.
No matter what object generates that electric field. Think of the mystery object as creating an electric field, which then pushes on q. If we know the field, we need not know what kind of object created the field.

55. b. As discussed in previous problem, the data indicate an inverse square relationship, doubling the distance decreases

the field by a factor of 4. Only graph (d) captures this insight. Graph (c) represents a regular inverse proportionality, in which doubling the distance cuts the field in half.

56. b. According to the formula $E = 2kp\cos\theta/d^3$, doubling the distance from the dipole decreases the field by a factor $2^3 = 8$. (Mathematically, that is because doubling d increases the denominator by 8.) By similar reasoning, doubling the dipole moment P increases the field by a factor of 2. So, when we "turn on" both of these effects at once, the field decreases by a factor of 4.

If this reasoning does not make sense, you can reason in steps as follows. Suppose the electric field has strength E_0. First, we double P. This increases the field strength to $2E_0$. Next, we double d. This decreases the field by a factor of 8 to $2E_0/8 = E_0/4$. The order of the steps makes no difference, you get the same answer either way. Therefore, this step-by-step reasoning works even, in real life, the dipole moment and the distance get doubled simultaneously.

For Problems 57–58

57. a., 58. d.

Sol. $\partial E/\partial x$ is not zero, it means E_y may change, as we move along x-axis. So, $-q$ and $+q$ may find different forces along y-axis, hence net force on dipole may be along y-axis (Fig. A1.217).

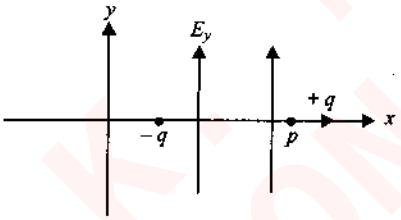


Fig. A1.217

Obviously, forces along y-axis will rotate the dipole in x–y plane, producing torque along z-axis.

For Problems 59–60

59. c., 60. b.

Sol.

59. c. Net charge on the rod is zero, so the net force will also be zero. Location of charge will not matter as the electric field produced by a large sheet is uniform.

60. b. $\tau = 2\int_0^l (\lambda dx E)\, x \sin\theta$

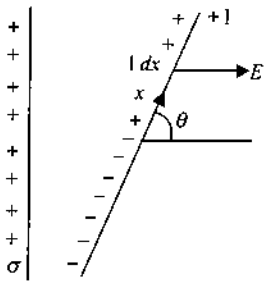


Fig. A1.218

$$\Rightarrow \quad \tau = \frac{2\lambda\sigma\sin\theta}{2\varepsilon_0}\int_0^l x\,dx = \frac{\sigma\lambda l^2\sin\theta}{2\varepsilon_0}$$

For Problems 61–63

61. b., 62. c., 63. d.

Sol.

61. b. Field inside the conductor is zero.

62. c. Charge distribution is shown in Fig. A1.219.

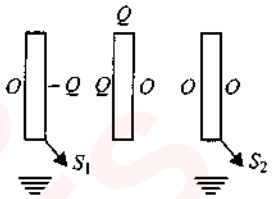


Fig. A1.219

63. d. $\frac{(Q-q)}{\varepsilon_0 A}3d = \frac{qd}{\varepsilon_0 A}$

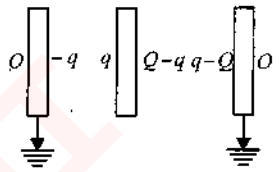


Fig. A1.220

$$\Rightarrow \quad q = \frac{3Q}{4}$$

For Problems 64–66

64. b., 65. b., 66. d.

Sol.

64. b.

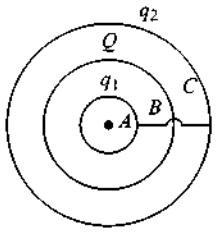


Fig. A1.221

$$q_1 + q_2 = 0$$

$$v_A = \frac{kq_1}{R} + \frac{kQ}{2R} + \frac{kq_2}{4R}$$

$$v_C = \frac{kq_1}{4R} + \frac{kQ}{4R} + \frac{kq_2}{4R}$$

$$v_A = v_C$$

$$q_1 = -\frac{Q}{3} \text{ and } q_2 = \frac{Q}{3}$$

65. b.

$$v_A = k\left[\frac{-Q}{3R} + \frac{Q}{2R} + \frac{Q}{12R}\right] = \frac{Q}{16\pi\varepsilon_0 R}$$

66. d. $v_B = k\left[\frac{-Q}{6R}+\frac{Q}{2R}+\frac{Q}{12R}\right]=\frac{5Q}{48\pi\varepsilon_0 R}$

For Problems 67–69

67. d., 68. b., 69. d.

Sol.

67. d. Plate 2 and 3 are joined together and they are neither connected to any of the terminals of the battery nor to any other source of charge. So, they jointly form an isolated system.

68. b. Before closing the switches, charges are shown in Fig. A1.222. After closing the switch, let q charge goes from battery. Then charge on each will increase by q.

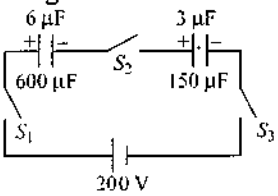


Fig. A1.222

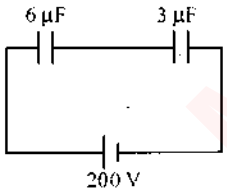


Fig. A1.223

Applying Kirchhoff's voltage law, we get

$$200-\left(\frac{600+q}{6}\right)-\left(\frac{150+q}{3}\right)=0$$

Or q = 100 μC.

So, charge on 6 μF capacitor will be 70 μC and 3 μF capacitor will be 250 μC.

69. d. From all the switches 100 μC of charge will flow.

For Problems 70–72

70. a., 71. b., 72. b.

Sol.

70. a. When oil is pumped out then plates act as air capacitor between which electric field is more than electric field in dielectric capacitor. So choice (a) is correct and other choices are wrong.

71. b. Since charged capacitor is isolated, hence, charge is conserved. So, choice (b) is correct and other choices are wrong.

72. b. Since capacitor is connected to cell so potential difference across capacitor remains constant. When slab is pulled out capacitance decreases.

Charge on capacitor $Q = CV$

So charge on capacitor decreases and some charge will be returned to cell.

Choices (a), (c) and (d) are wrong.

For Problems 73–75

73. b., 74. c., 75. d.

Sol. 73. b. Electrostatic force on block A is zero, while on block B it is

$$F = qE = 1\times 10 = 10\text{N}$$

This force acts along negative direction of X-axis.

Acceleration of block B is

$$a=\frac{F}{q}=\frac{10}{1}=10\text{ m/s}^2$$

But before collision, velocity of block B,

$$v=\sqrt{2aS}$$

or $$v=\sqrt{2\times 10\times 1.8}=6\text{ m/s}$$

Now, from conservation of linear momentum

$$(m_A+m_B)v=m_Av_A+m_Bv_B$$

$$(1+1)\,v=0+1\times 6 \text{ or } v=\frac{6}{2}=3\text{ m/s}$$

74. c. At equilibrium, net force on system is zero. Let x_0 be the compression in the spring in equilibrium, then

Kx_0 = electrostatic force

$18x_0 = 10$

or $x_0=\frac{10}{18}=\frac{5}{9}$

So, equilibrium position will be $x=-\frac{5}{9}$ m.

75. d. Angular frequency of S.H.M. will be

$$\omega=\sqrt{\frac{k}{m_A+m_B}}=\sqrt{\frac{18}{2}}=3\text{ rad/sec}$$

At $x=\frac{5}{9}$ m, speed is 3 m/s

Therefore from $v=\omega\sqrt{A^2-x^2}$

or $$3=3\sqrt{A^2-x^2} \text{ or } 1=1\sqrt{A^2-x^2}$$

or $$A^2-x^2=1 \text{ or } A^2=1+x^2$$

$$=1+\left(\frac{5}{9}\right)^2=1+\frac{25}{81}=\frac{106}{81} \text{ or } A=\frac{\sqrt{106}}{9}\text{ m}$$

Hence, option (d) is correct.

For Problems 76–78

76. b., 77. c., 78. d.

Sol.

76. b. We see that ϕ remains constant up to r_1. So $r_1 < r_A$. Let q be the charge at centre then

$$\phi=\frac{q_{in}}{\varepsilon_0}\Rightarrow 2\times 10^5=\frac{q}{8.85\times 10^{-12}}\Rightarrow q = 1.77\ \mu\text{C}$$

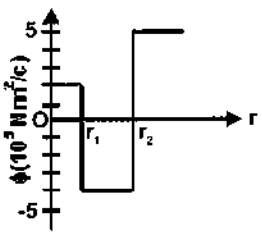


Fig. A1.224

Similarly: $\phi = -4\times10^5 = \dfrac{q+q_A}{\varepsilon_0} \Rightarrow q_A = -5.31\ \mu C$

Where q_A is the charge on A.

Electric field is zero nowhere as ϕ is not zero anywhere.

For Problems 79–81

79. b., 80. c., 81. d.

Sol.

79. b. When $C_3 \to \infty$, it means distance between the plates of C_3 is zero or both plates will be at the same potential. Then C_2 will be shorted. Entire potential V will be across C_1. Hence, $V = V_1 = 10$ V.

80. c. From graph, when $C_3 = 0$, $V_1 = 2$ V. So, C_3 will act like open switch. C_1 and C_2 will be in series. Potential different across C_2 is $V_2 = 10 - V_1 = 8$ V.

$q = C_1V_1 = C_2V_2 \Rightarrow C_{12} = C_{28} \Rightarrow C_1 = 4C_2$

81. d. $V_1 = 4$ V, $V_2 = 10 - 4 = 6$ V

$q = C_1V_1 = (C_2 + C_3)\,V_2$

$\Rightarrow \quad C_{14} = (C_2 + C_3)6 \Rightarrow 16C_2 = 6C_2 + 6C_3$

$\Rightarrow \quad C_3 = 5\,C_{2/3}$

Matching Column Type

1. i. → c.; ii. → b.; iii. → d.; iv → a.

Sol. i. Since $q_2 = 0$, so charge $Q+q_1$ on outer surface will be distributed uniformly. Hence, electric field of $Q+q_1$ at centre will be zero.

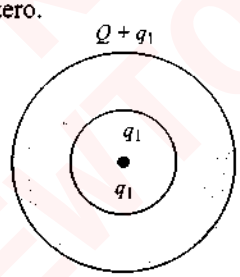


Fig. A1.225

ii. Since q_2 is not zero, so $Q+q_1$ will be induced non-uniformly on outer surface. But due to q_2 and $Q+q_1$, net field at any point inside the outer surface should be zero.

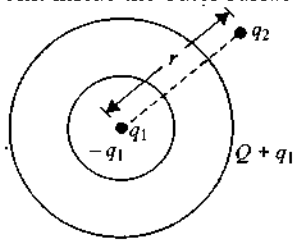


Fig. A1.226

$$E_{q_2} + E_{Q+q_1} = 0$$

$$\frac{1}{4\pi\varepsilon_0}\frac{q_2}{(r-R_1)^2} + E_{Q+q_1} = 0$$

$$E_{Q+q_1} = -\frac{1}{4\pi\varepsilon_0}\frac{q_2}{(r-R_1)^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_2}{(r-R_1)^2}, \text{ radially outward}$$

iii. Since $Q+q_1$ will be induced non-uniformly, so it is difficult to determine its electric field at an outside point.

iv.

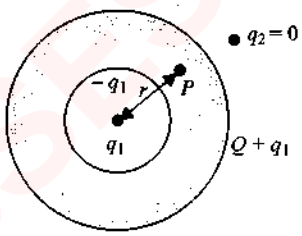


Fig. A1.227

Net field at P due to q_1 and $-q_1$ should be zero.

$$E_{q_1} + E_{(-q_1)} = 0$$

$$E_{(-q_1)} = -E_{-q_1} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2}$$

2. i. → b., d.; ii. → a., d.; iii. → b., d.; iv → a., c.

Sol. If any charge is present outside, then charge on outer surface will be distributed non-uniformly. But if there is no charge present outside, then charge on outer surface will be distributed uniformly. This is irrespective of location of the charge inside.

If charge is at the centre, then charge on inner surface will be distributed uniformly, but if charge is displaced from centre, then charge on inner surface will be distributed non-uniformly. This is irrespective of whether the charge is present outside or not.

3. i. → a., d.; ii. → c., d.; iii. → a., b.; iv. → b.

Sol. i. When we insert the dielectric slowly, we have to apply the force on dielectric in opposite direction, so we have to do negative work. Because of this negative work done, energy of system decreases.

ii. Here, again work done by external agent is negative, but here battery will supply some energy (or the battery will do positive work) due to which there is the overall increase in energy of the system.

iii. $U = \frac{1}{2}CV^2$, V remains same and C decreases so U also decreases. Work done by external agent will be positive in pulling apart the plates against the attractive force between plates.

iv. $U = \dfrac{q^2}{2C}$, q remains same and C decreases so U increases. Work done by external agent will be positive in pulling apart the plates against the attractive force between plates.

4. **i. → b., d.; ii. → a., c., d.; iii. → a., c., d.; iv. → a., c., d.**

Sol. Charge on the outer surfaces of the plates of capacitor will be always zero. If $V = E$, then no charge will flow in the circuit and hence no thermal energy will be dissipated. But if $V \neq E$, then charge will flow in the circuit and thermal energy will be dissipated.

5. **i. → a., c.; ii. → b.; iii. → b., d.; iv. → b.**

Sol. i. Initial potential difference across C_1:

$$V_1 = \frac{4V}{2+4} = \frac{2V}{3}$$

On doubling the distance between plates, C_1 becomes half. So, final potential difference across C_1:

$$V_1' = \frac{4V}{1+4} = \frac{2V}{5}$$

This increases by a factor $\frac{V_1'}{V_1} = \frac{4V/5}{2V/3} = \frac{6}{5}$

ii. Across C_2:

Initially: $V_2 = \frac{2V}{2+4} = \frac{V}{3}$

Finally: $V_2' = \frac{1V}{1+4} = \frac{V}{5}$

This decreases by a factor of

$$\frac{V_2'}{V_2} = \frac{V/5}{V/3} = \frac{3}{5}$$

iii. Energy in C_1

Initially: $U_1 = \frac{1}{2} \times 2\left(\frac{2V}{3}\right)^2 = \frac{4V^2}{9}$

Finally: $U_1' = \frac{1}{2} \times 1 \times \left(\frac{4V}{5}\right)^2 = \frac{8V^2}{25}$

This decreases by a factor

$$\frac{U_1'}{U_1} = \frac{8V^2/25}{4V^2/9} = \frac{18}{25}$$

iv. Energy in C_2

Initially: $U_2 = \frac{1}{2} 4\left(\frac{V}{3}\right)^2 = \frac{2V^2}{9}$

Finally: $U_2' = \frac{1}{2} 4\left(\frac{V}{3}\right)^2 = \frac{2V^2}{25}$

This decreases by a factor $\frac{U_2'}{U_2} = \frac{9}{25}$

6. **i. → b., d.; ii. → a., d.; iii. → d.; iv. → c.**

Sol. In case of hollow or solid conducting sphere of radius r, for an internal point (i.e., $r < R$) electric field is zero. At the surface, E can be either minimum (= 0) or maximum ($= Q/4\pi\varepsilon_0 R^2$). It is worth mentioning here that one should take $E = 0$ at the surface for dealing internal behaviour and ($= Q/4\pi\varepsilon_0 R^2$) for dealing external behaviour. So, electric field is discontinuous.

Hence, **i. → b., d.**

In case of spherical volume distribution of charge, inside the sphere electric field $E_{in} = (1/4\pi\varepsilon_0)(Qr/R^3)$. From this expression, it is clear that electric field is zero at the centre. At the surface, E is continuous and maximum ($= Q/4\pi\varepsilon_0 R^2$).

Hence, **ii. → a., d.**

In case of charged ring, electric field is zero at the centre.

In case of infinite sheet of charge, electric field $E = \sigma/2\varepsilon_0$, which is constant.

Hence, **iii. → d. iv. → c.**

7. **i. → a., b.; ii. → d.; iii. → c.; iv. → d.**

Sol. Potential difference to a line charge

$$V_2 - V_1 = -\frac{\lambda}{2\pi\varepsilon_0}(\ln r_2 - \ln r_1)$$

$$V_2 = V_1 - \frac{\lambda}{2\pi\varepsilon_0}\ln r_2 + \frac{\lambda}{2\pi\varepsilon_0}\ln r_1$$

At $r_1 = \infty, V_1 = 0$

Hence, at any value of r_2, V_2 is infinite.

8. **i. → a.; ii. → c.; iii. → a.; iv. → a.**

Sol. $\frac{kq}{a} + \frac{kQ}{b} = 0 \Rightarrow q = -\frac{Qa}{b}$

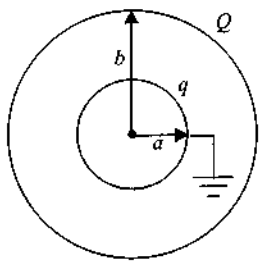


Fig. A1.228

9. **i. → a., b.; ii. → a., b.; iii. → a., b., d.; iv. → c., d.**

Sol. In situation (i), (ii) and (iii), shells I and II are not at same potential. Hence, charge shall flow from Sphere I to Sphere II till both acquire same potential.

If charge flows, the potential energy of system decreases and heat is produced.

In situations (i) and (ii) charges shall divide in some fixed ratio, but in situation (iii) complete charge shall be transferred to Shell II for potential of Shells I and II to be same.

In situation (iv) both the shells are at same potential, hence no charge flows through connecting wire.

10. **i. → d.; ii. → a., c., d.; iii. → b.; iv. → a., c.**

In this case, electric field is totally zero because in (a) and (b) particle P can be at origin.

For points P, P' in Fig. A1.110 (c) electric field will be along x-axis only. Also, for point P in Fig. A1.110 (a) and

(b) E_x will remain non-zero while other two components of electric field will be zero.

iii. → b. Only in the case of disk x–y components of electric field will be present as per given statements.

iv. → a., c. For the two points P and P' in Fig. A1.112(c), the field will be same as these points are at equal distances from infinite line of charge.

11. i. → c.; ii. → d.; iii. → b.; iv. → b.

Sol. i. $U = -\vec{p}\cdot\vec{E}$. So, when $\theta = 180°$, $U = pE$.

ii. Angular acceleration is maximum when torque is maximum. $\vec{\tau} = \vec{p}\times\vec{E}$ is maximum for $\theta = 90°$.

iii. Angular momentum will not be conserved as there is no external force on the system but torque will act on the system.

iv. During movement of dipole, total energy remains conserved. Only K.E. cannot be conserved.

12. i. → a., c.; ii. → a., b., c., d.; iii. → a., b., c.; iv. → a., d.

Sol. The electric field due to one dipole at centre of other dipole is parallel to the dipole in all cases. Hence, torque on dipole is zero in all cases.

In cases (ii) and (iii), the electic field at second dipole due to first is along the second dipole, hence electrostatic potential energy of second dipole is negative.

In cases (i) and (ii), x-axis is the line of zero potential. In case (iii), y-axis is the line of zero potential. In cases (ii) and (iii), electric field at origin is zero.

13. i. → b.; ii. → a.; iii. → c.; iv. → d.

Sol. i. $\vec{E}_{\text{axial}}$ for a short dipole $= \dfrac{2|\vec{P}|}{4\pi\varepsilon_0 r^3}$

ii. $\vec{E}_{\text{potential}}$ for a dipole $= \dfrac{|\vec{P}|}{4\pi\varepsilon_0 r^3}$

iii. Net force $= qE - qE = 0$.

Torque $= PE\sin 0° = 0$

iv. Net force $\neq 0$. as the value of E is different at both ends.

But as the angle $\theta = 0°$, hence the torque is zero.

14. i. → a.; ii. → b.; iii. → d.; iv. → c.

Sol. From question: $\dfrac{\partial V}{\partial x} = -(2xy + z^2)$

$$\frac{\partial V}{\partial y} = -(2yz + x^2)$$

$$\frac{\partial V}{\partial z} = -(2zx + y^2)$$

$$V = x^2y + y^2z + z^2x + C$$

Using this, we can find out the works

Further, $E(x, y, z) = E(-x, -y, -z)$

Direction of field at x, y, z is same as direction of field at $(-x, -y, -z)$ and direction of area vector is just opposite. That net flux will turn out to be zero.

15. i. → b.; ii. → a.; iii. → c.; iv. → d.

Sol. $\vec{E} = -\hat{i} - \hat{j} \Rightarrow \vec{E} = \sqrt{2}$ N/C

$$a_P = 2\text{ m/s}^2 \quad a_\theta = 4\text{ m/s}^2$$

$$S_P = \frac{1}{2}\times 2\times 4\times 2 = 8\text{ m}$$

$$S_Q = \frac{1}{2}\times 4\times 4 = 8\text{ m}$$

Integer Answer Type

1. (0) When shells are connected, charge on inner shell will be transferred to the outer shell.

$$V_i = \frac{KQ}{2R} + \frac{K(2Q)}{2R} = \frac{3KQ}{2R}$$

$$V_f = \frac{K(3Q)}{2R} \rightarrow \Delta V = V_f - V_i = 0$$

2. (2) $\dfrac{\varepsilon_0 A}{\left[d - \dfrac{d}{2} + \dfrac{d}{2k}\right]} = \dfrac{4}{3}\dfrac{\varepsilon_0 A}{d}$

Or $\dfrac{1}{\dfrac{d}{2}\left(1 + \dfrac{1}{K}\right)} = \dfrac{4}{3d}$

On solving, we get $K = 2$.

3. (8) Resolving the velocity of particle parallel and perpendicular to the plate.

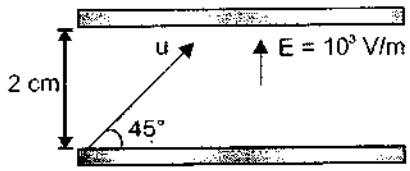


Fig. A1.229

$$u_{\parallel} = u\cos 45° = \frac{u}{\sqrt{2}} \text{ and } u_{\perp} = u\sin 45° = \frac{u}{\sqrt{2}}$$

Force on the charged particle in the downward direction normal to the plate $= eE$.

Acceleration is $a = eE/m$, where m, mass of charged particle. The particle will not hit the upper plate, if the velocity component normal to plate becomes zero before reaching it, i.e., $0 = u_{\perp}^2 - 2ay$, with $y \le d$, where d, distance between the plates.

Therefore, the maximum velocity for the particle not to hit the upper plate, (for this $y = d = 2$ cm) is

$$u_{\perp} = \sqrt{2ay} = \sqrt{\frac{2\times 1.6\times 10^{-19}\times 10^3\times 2\times 10^{-2}}{1.6\times 10^{-30}}}$$

$$= 2\times 10^6 \text{ m/s}$$

$$\Rightarrow \quad u_{\max} = u_{\perp}/\cos 45° = 2\sqrt{2}\times 10^6 \text{ m/s}$$

4. (3)

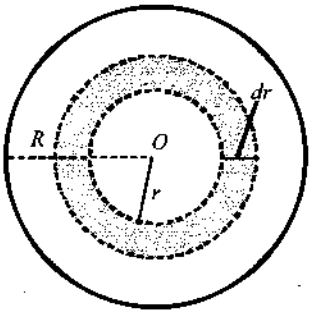


Fig. A1.230

Given $E = \alpha r$

When $r = R$, $ER = \alpha R$

$\therefore \phi = E_R(\text{area}) = \alpha R\, 4\pi R^2$

By the Gauss's theorem, the net electric flux is

$$\frac{1}{\varepsilon_0}\text{ charge enclosed}$$

$$\alpha R 4\pi R^2 = \frac{1}{\varepsilon_0} Q_{\text{enclosed}}$$

$$\therefore \qquad Q_{\text{enclosed}} = (4\pi\varepsilon_0)\,\alpha R^3$$

Given $R = 0.30$ m, $\alpha = 100$ V/m^2

$$Q_{\text{enclosed}} = \frac{1}{9\times10^9}\times100\times(0.30)^3 = 3\times10^{-10}\text{ C}$$

5. (0) Electric field at the centre will be cancelled due to opposite sides.

6. (5) $E = 10$ N/C, $S = 50$ cm $= \frac{1}{2}$ m

$$\Delta V = E.S = 10\times\frac{1}{2} = 5\text{ V}$$

7. (1) Decrease in K.E. = increase in P.E.

$10\text{ J} = (200 - 100)\times q_0$

$100\times q_0 = 10 \Rightarrow q_0 = \frac{10}{100} = 0.1$ C

8. (1) Given that capacitance C = 10 μF, charge = 20 μC. But the effective charge = q = 10 μC.

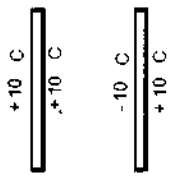


Fig. A1.231

Potential difference = q/C = 1 V

9. (2) On joining two capacitors, charge on each will be half of the previous charge on the first capacitor. So energy stored on each capacitor will be one-fourth of energy stored in the previously in the first capacitor, that is 1 J. Hence, the net energy stored in both will be 1 + 1 = 2 J.

10. (6) Here the charge q_3 is attracted towards q_1 and q_2 both, so the net force on q_3 is towards the origin. As only force acting is conservative, we can use conservation of mechanical energy.

Let v = speed of particle at origin.

From energy of conservation principle, we get

$Ui + Ki = U + K$

$$\text{or} \qquad \frac{1}{4\pi\varepsilon_0}\left[\frac{q_3q_2}{(r_{32})_i}+\frac{q_3q_1}{(r_{31})_i}+\frac{q_2q_1}{(r_{21})_i}\right]+0$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_3q_2}{(r_{32})_f}+\frac{q_3q_1}{(r_{31})_f}+\frac{q_2q_1}{(r_{21})_f}\right]+\frac{1}{2}mv^2$$

Here, $(r_{21})_i = (r_{21})_f$

Substituting the proper values,

$$9\times10^9\left[\frac{(-4)(2)}{5}+\frac{(-4)(2)}{5}\right]\times10^{-12}$$

$$= 9\times10^9\left[\frac{(-4)(2)}{3}+\frac{(-4)(2)}{3}\right]\times10^{-12}+\frac{1}{2}\times10^{-3}v^2$$

$$\text{Or} \quad (9\times10^{-3})\left(\frac{-16}{5}\right) = (9\times10^{-3})\left(\frac{-16}{3}\right)+\frac{1}{2}\times10^{-3}v^2$$

$$\text{Or} \quad 9\times10^{-3}(16)\left(\frac{2}{15}\right) = \frac{1}{2}\times10^{-3}v^2$$

Or v = 6.2 m/s = 6 m/s

11. (3) Electrostatic force will balance the surface tension force,

$$\frac{q^2}{a^2}\propto\gamma a \quad\Rightarrow\quad a^3\propto\frac{q^2}{\gamma}$$

$$\Rightarrow \quad a\propto\left(\frac{q^2}{\gamma}\right)^{1/3} \Rightarrow a = k\left(\frac{q^2}{\gamma}\right)^{1/3} \Rightarrow \quad N = 3$$

12. (2) $q = n_e e \Rightarrow n_e = \frac{-3.20\times10^{-9}}{-1.6\times10^{-9}} = 2.0\times10^{10}$

13. (3) $\tan\theta = \frac{qE}{mg} = \frac{q\sigma}{2\varepsilon_0 mg}$

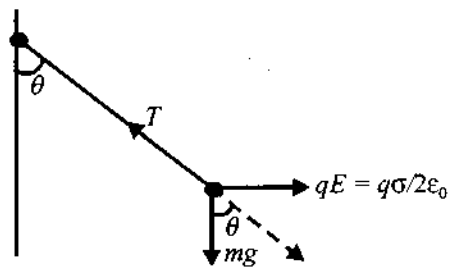


Fig. A1.232

$$\frac{\sigma_2}{\sigma_1} = \frac{\tan\theta_2}{\tan\theta_1} = \frac{\tan 60^\circ}{\tan 30^\circ} = 3$$

14. (5) $F_1 = \dfrac{9\times10^9\times2\times10^{-8}\times10^{-6}}{(0.1)^2} = 0.018$ N

$$F_2 = \frac{9\times10^9\times10^{-6}\times10^{-8}}{(0.05)^2} = 0.036 \text{ N}$$

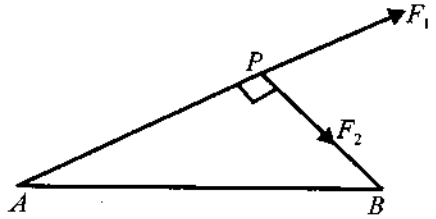


Fig. A1.233

Net force: $F = \sqrt{F_1^2 + F_2^2} = 18\sqrt{5}\times10^{-3}$ N

Hence, $a = 5$.

15. (4) The electric field of the earth points towards the ground, so a negative charge will have to acquire.

$$mg = qE \Rightarrow q = \frac{(60.0 \text{ kg})\,(10 \text{ m/s}^2)}{150 \text{ N/C}} = 4 \text{ C}.$$

16. (2) Electric field due to semicircular part:

$$E_2 = \frac{\lambda_2}{2\pi\varepsilon_0 R}$$

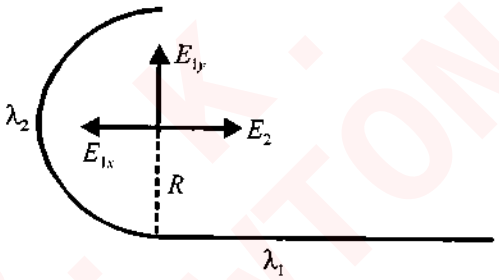


Fig. A1.234

Electric field due to straight part:

$$E_{1x} = E_{1y} = \frac{\lambda_1}{4\pi\varepsilon_0 R}$$

For the net field to be along the y-axis:

$$E_2 = E_{1x} \Rightarrow \frac{\lambda_2}{2\pi\varepsilon_0 R} = \frac{\lambda_1}{4\pi\varepsilon_0 R} \Rightarrow \frac{\lambda_1}{\lambda_2} = 2$$

17. (8) The given curved surface subtends solid angle of $\Omega = \dfrac{4\pi}{8} = \dfrac{\pi}{2}$ steradian at origin. Hence, the flux through the surface:

$$\phi = \frac{q}{\varepsilon_0 4\pi}\Omega = \frac{q}{\varepsilon_0 4\pi}\frac{\pi}{2} = \frac{q}{8\varepsilon_0} = \frac{8q}{64\varepsilon_0}. \text{ Hence, } N = 8.$$

Archives

Fill in the Blanks Type

1. For plate 1

$$q = CV = \frac{\varepsilon_0 A}{d}\times V$$

For plate 2:

$$2q = \frac{2\,\varepsilon_0 A}{d}\times V$$

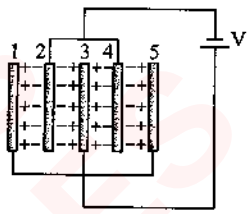


Fig. A1.235

2. It is greatest at point B since at B, the equipotential surfaces are closest.

3. Where there is no gravitational force, then in this case only electrostatic force of repulsion is acting which will take the two balls as far as possible. The angle between the two strings will be 180°. The tension in the string will be equal to the electrostatic force of repulsion, i.e.,

$$T = \frac{1}{4\pi\varepsilon_0}\times\frac{Q\times Q}{(2L)^2} = \frac{1}{4\pi\varepsilon_0}\times\frac{Q^2}{4L^2}.$$

4. Initially, charge on capacitance $C = q_1 = CV$.

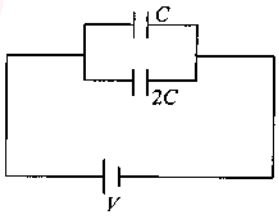


Fig. A1.237

Charge on capacitance $2C = q_2 = 2CV$.

Finally, charge on capacitance $C = q_1' = KCV'$.

Charge on capacitance $2C = q_2' = 2CV'$.

Since charge will not change, we get

$CV + 2CV = KCV' + 2CV' \Rightarrow V' = 3V/(k+2) - 4pt$

5. Since electric field is conservative in nature, the work done by the field along $PQRS$ will be the same as along P to M to S:

Work done from P to $M = \vec{F}\cdot\vec{PM}$

$= F\,(PM)\cos 90° = 0$

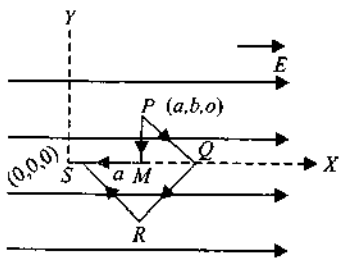


Fig. A1.237

Work done from M to $S = \vec{F}\cdot\overrightarrow{MS} = F(MS)\cos 180°$
$= -qEa$

6. $V = 4x^2$ V
The electric potential changes only along x-axis
We know that
$$E_x = \frac{-dV}{dx} \Rightarrow E_x = -\frac{d}{dx}(4x^2) = -8x$$
The electric field at point (1, 0, 2) will be (here $x = 1$)
$$E_x = -8\text{V}$$

7. If we place a charge q at the sixth vertex of the regular hexagon, then the net force on the charge $-q$ placed at the centre of hexagon will be zero due to symmetry.

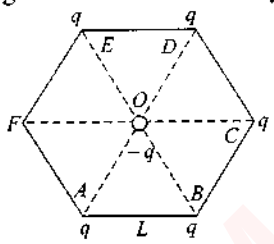


Fig. A1.238

The force on charge $-q$ due to the charge q placed on the sixth vertex balances the net force on charge $-q$ due to the other five charges placed at the five vertices. The force on charge $-q$ due to charge q placed at the sixth vertex will be
$$F = \frac{1}{4\pi\varepsilon_0}\frac{q\times q}{L^2}$$
where l is the distance of the centre of hexagon from any vertex (directed from O to C)
The magnitude of force on the point charge of value $-q$ coulomb placed at the centre of the hexagon is $(1/4\pi\varepsilon_0)(q^2/L^2)$ directed from O to F.

True or False Type

1. Let us consider two points A and B in an electric field. Let the potentials at A and B be V_A and V_B, respectively.
Now, by the definition of potential difference, the potential difference between two points B and A is the amount of work done in carrying a unit positive charge from A to B.

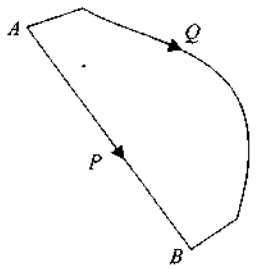


Fig. A1.239

Mathematically, for path $A \to P \to B$,
$$\frac{W_{APB}}{q} = V_B - V_A$$
$$W_{APB} = q(V_B - V_A) \quad \text{(i)}$$
For path $A \to Q \to B$,
$$\frac{W_{AQB}}{q} = V_B - V_A$$
$$W_{AQB} = q(V_B - V_A) \quad \text{(ii)}$$
Since the R.H.S. of Eqs. (i) and (ii) is the same,
$$\Rightarrow \quad W_{APB} = W_{AQB} \quad \text{(iii)}$$

2. The statement is true. The metallic sphere which gets negatively charged gains electrons and hence its mass increases. The metallic sphere which gets positively charged loses electrons and hence its mass decreases.
3. When a high-energy x-ray beam falls, it will knock out electrons from the small metal ball making it positively charged. Therefore, the ball will be deflected in the direction of electric field.
4. The electric field produced between the parallel-plate capacitor is uniform. The force acing on a charged particle placed in an electric field is given by $F = qE$.
In the case of two protons, q and E are equal and hence force will be equal.
The statement is true.
5. Force on charge $-q$ due to small charge dq situated at length dl is
$$dF = k\frac{qdq}{5R^2}$$
Resolving this force into two parts $dF\cos\theta$ and $dF\sin\theta$ as shown

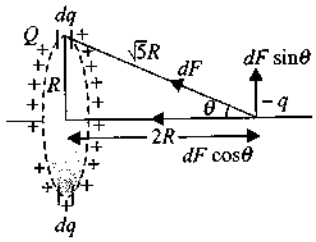


Fig. A1.240

If we take another diametrically opposite length dl, the charge on it being dq, then the force on charge $-q$ by this small charge dq will be
$$dF = k\frac{qdq}{5R^2}$$
Again, resolving this force, we find $dF\sin\theta$ cancels out with $dF\sin\theta$ of the previous force and $dF\cos\theta$ components add up.
$$F = \int_0^{2\pi R} dF\cos\theta = \int_0^{2\pi R}\frac{kqdq}{5R^2}\times\frac{2R}{\sqrt{5}R}$$
Charge on length $2\pi R = Q$

Charge on length $dl = \frac{Qdl}{2\pi R} = dq$

$$F = \int_0^{2\pi R} \frac{2kq}{5\sqrt{5}R^2} \times \frac{Qdl}{2\pi R}$$

$$= \frac{2kQq}{5\sqrt{5}\times 2\pi R^3} \times 2\pi R = \frac{2kQq}{5\sqrt{5}R^2}$$

This is not an equation of simple harmonic motion.
Therefore, the statement is false.

Multiple Choice Questions with One Correct Answer Type

1. b. Concept: The potential at the surface of a sphere is the same as the potential at the centre of the sphere.

2. b. If we take a point M on the x-axis as shown in Fig. A1.241, then the net electric field is in the x-direction.

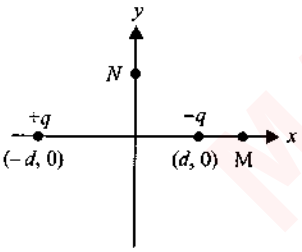


Fig. A1.241

Therefore, option (a) is incorrect.
If we take a point N on y-axis, we find net electric field along $+x$-direction. The same will be true for any point on Y-axis.
Option (c) is incorrect. The direction of dipole moment is from −ve to +ve. Therefore, (d) is incorrect.

3. b. Energy stored, $U = \frac{1}{2}C_{eq}V_{net}^2 = \frac{1}{2}(3C)V^2$ (Fig. A1.242)

$$= \frac{3}{2}CV^2$$

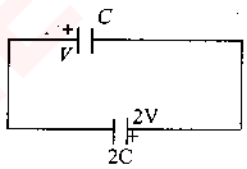


Fig. A1.242

4. b. Let the distance to be travelled be x. Let the strength of uniform electric field be E.
For the electron:

$$u = 0, S = x, a = \frac{eF}{m_e}, t = t_1$$

$$S = ut + \frac{1}{2}at^2 \Rightarrow x = \frac{1}{2}\frac{eE}{m_e} \times t_1^2 \quad \text{(i)}$$

For the proton:

$$u = 0, s = x, a = \frac{eE}{mp}, t = t_1$$

$$S = ut + \frac{1}{2}at^2 \Rightarrow x = \frac{1}{2}\frac{eE}{m_p} \times t_2^2 \quad \text{(ii)}$$

From Eqs. (i) and (ii),

$$\frac{t_2^2}{t_1^2} = \frac{m_p}{m_e} \Rightarrow \frac{t_2}{t_1} = \left[\frac{m_p}{m_e}\right]^{1/2}$$

5. a. $V_0 = K\frac{q}{R}, V_\infty = 0$

$$\int_{t=a}^{t=0} -\vec{E}\cdot\vec{dl} = V_0 - V_\infty$$

$$= \frac{Kq}{r} - 0$$

$$= \frac{9\times 10^9 \times 1.11\times 10^{-10}}{0.5}$$

$$= 2 \text{ V}$$

6. d. For the capacitor to get charged upto 0.75 V, the charge on the plates should be
$q = CV = 10^{-5} \times 0.75 = 0.75 \times 10^{-5}$ C

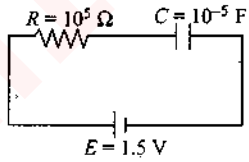


Fig. A1.243

Using the equation of charging of capacitor
$q = CE\,[1 - e^{-t/RC}]$

$$\Rightarrow \quad 0.75\times 10^{-5} = 10^{-5}\times 1.5\left[1 - e^{\frac{t}{10^5\times 10^{-5}}}\right]$$

$$\Rightarrow \quad \frac{1}{2} = [1 - e^{-t}] \Rightarrow e^{-t} = \frac{1}{2}$$

Taking log on both sides, we get

$$-t = \ln 2$$

$$\Rightarrow \quad t = 0.693 \text{ sec}$$

7. b. With the closing of switch S3, the potential across C_1 and C_2 would become identical to the average of V_1 and V_2, i.e.,
(30 V + 20 V)/2 = 25 V

8. b. Here, we have $= \frac{Qq}{a} + \frac{q^2}{a} + \frac{Qq}{a\sqrt{2}} = 0$

or $\quad Q\sqrt{2} + q\sqrt{2} + Q = 0$ or $Q(\sqrt{2}+1) = -q\sqrt{2}$

$$\Rightarrow \quad Q = -\frac{q\sqrt{2}}{\sqrt{2}+1} = -\frac{2q}{2+\sqrt{2}}$$

9. b. The effective capacitance is given by

$$\frac{\varepsilon_0 A}{d}\left[\frac{1}{(k_1+k_2)} + \frac{1}{2k_3}\right]^{-1}$$

The capacitance of a single capacitor will be $\frac{\varepsilon_0 A}{d}k$

$$k = \left[\frac{1}{(k_1+k_2)} + \frac{1}{2k_3}\right]^{-1} \text{ or } \frac{1}{k} = \frac{1}{(k_1+k_2)} + \frac{1}{2k_3}$$

10. c. Option (a) is not possible because all the three charges are positive and the electric lines of force will expand laterally and not contract longitudinally.
Option (b) is not possible as electric lines of force are continuous lines but there are three lines which end up abruptly somewhere in between the electric field which is not possible. Option (d) makes no sense.

11. a. When S is closed, there will be no shifting of negative charge from plate A to B as the charge $-q$ is held by the charge $+q$. Neither there will be any shifting of charge from B to A.

12. b. As we move along the direction of electric field, potential decreases.

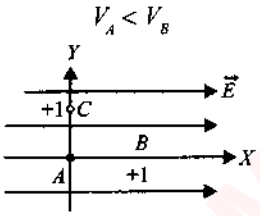


Fig. A1.244

13. b. $U_i = \frac{2Qq}{4\pi\varepsilon_0(a)}; U_f = \frac{Qq}{4\pi\varepsilon_0}\left[\frac{1}{a+x} + \frac{1}{a-x}\right];$

$U_i - U_f = \frac{(Qqx^2)}{2\pi\varepsilon_0 a^3}$ [for $x << a$, x^2 can be neglected in comparison to a^2 in denominator]

14. c. Initially,

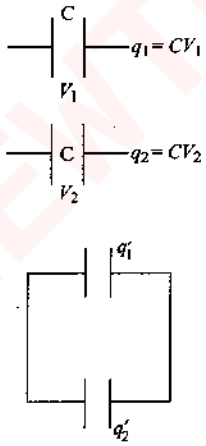


Fig. A1.245

Initial energy $= \frac{1}{2}C(V_1^2 + V_2^2)q_1' + q_2' = CV_1 + CV_2$

$\frac{q_1'}{C} = \frac{q_2'}{C} \Rightarrow q_1' = q_2'$ (Charge conservation)

$$q_1' = \frac{C(V_1+V_2)}{2}$$

Final energy $= \frac{C(V_1+V_2)^2}{4}$

Therefore, change in energy = Initial energy – Final energy

$$= \frac{1}{2}C(V_1^2+V_2^2) - \frac{C}{4}(V_1^2+V_2^2+2V_1V_2)$$

$$= \frac{C}{4}[2V_1^2+2V_2^2-V_1^2-V_2^2-2V_1V_2] = \frac{C}{4}(V_1-V_2)^2$$

15. c. Electric field is perpendicular to the equipotential surfaces and is zero everywhere inside the metal.

16. c. $|\vec{E}| = \frac{kq}{r^2}$

Electric field due to P on O is cancelled by electric field due to S on O (Fig. A1.246).
Similarly, electric field due to Q to O is cancelled by electric field due to T and O. The electric field due to R on O in the same direction as that of U and O. Therefore, the net electric field is $2\vec{E}$.

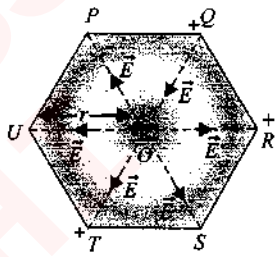


Fig. A1.246

17. d. The flux through the Gaussian surface is due to the charges inside the Gaussian surface. But the electric field on the Gaussian surface will be due to the charges present in the Gaussian surface and outside it. It will be due to all the charges.

18. c. Figure A1.247 shows the electric fields due to the sheets 1, 2 and 3 at point P. The direction of electric fields is according to the charge on the sheets (away from positively charged sheet and towards the negatively charged sheet and perpendicular).

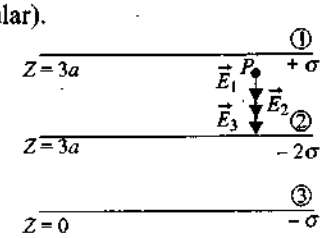


Fig. A1.247

The total electric field is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$= E_1(-\hat{k}) + E_2(-\hat{k}) + E_3(-\hat{k})$$

$$= \left[\frac{\sigma}{2\varepsilon_0} + \frac{2\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}\right](-\hat{k})$$

$$= -\frac{2\sigma}{\varepsilon_0}\hat{k}$$

19. a. When a charge density is given to the inner cylinder, the potential developed at its surface is different from that on the outer cylinder. This is because the potential decreases

with distance from a charged conducting cylinder when the point of consideration is outside the cylinder.
But when a charge density is given to the outer cylinder, it will change its potential by the same amount as that of the inner cylinder. Therefore, no potential difference will be produced between the cylinders in this case.

20. d. When a positive point charge is placed outside a conducting sphere, a rearrangement of charge takes place on the surface. But the total charge on the sphere is zero as no charge has left or entered the sphere.

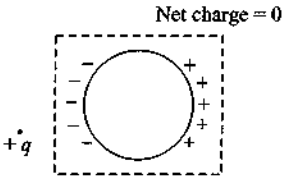


Fig. A1.248

21. b. Let us consider a uniformly charged solid sphere without any cavity. Let the charge per unit volume be σ and O be the centre of the sphere.
Let us consider a uniformly charged sphere of negative charged density σ having its centre at O'. Also, let OO' be equal to a (Fig. A1.249).
Let us consider an arbitrary point P in the small sphere. The electric field due to charge on big sphere $\vec{E} = (\sigma/3\varepsilon_0)\overrightarrow{OP}$.

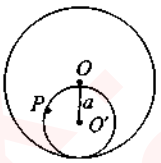


Fig. A1.249

Also, the electric field due to small sphere is

$$\vec{E}_2 = \frac{\sigma}{3\varepsilon_0}\overrightarrow{PO}$$

Therefore, the total electric field is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\sigma}{3\varepsilon_0}[\overrightarrow{OP} + \overrightarrow{PO}] = \frac{\sigma}{3\varepsilon_0}\overrightarrow{OO'}$$

This will have a finite value which will be uniform.

22. c. The charges make an electric dipole. Points A and B lie on the equatorial plane of the dipole. Therefore, potential at A = potential at B = 0
$W = q(V_A - V_B) = q \times 0 = 0$

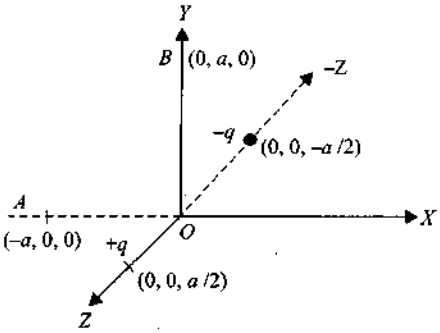


Fig. A1.250

23. c. $F_{BC} = \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{q}{3}\right)\left(\frac{2q}{3}\right)}{(R/\sqrt{3})^2} = \frac{q^2}{54\pi\varepsilon_0 R^2}$

24. a. Total charge enclosed by cube is $-2C$. Hence, electric flux through the cube is $-2C/\varepsilon_0$.

25. b. $\frac{Q_1}{4\pi R^2} = \frac{Q_1 + Q_2}{4\pi(2R)^2} = \frac{Q_1 + Q_2 + Q_3}{4\pi(3R)^2}$

$\Rightarrow \quad Q_1 : Q_2 : Q_3 :: 1 : 3 : 5$

26. a. Pressure $= \frac{\sigma^2}{2\varepsilon_0}$ and force $\frac{\sigma^2}{2\varepsilon_0} \times \pi R^2$

27. b. $\frac{4}{5}\pi R^3 \rho g = qE = 6\rho\eta R v_T$

$\therefore q = 8.0 \times 10^{-19}$ C

28. c. $\vec{A} = A\cos 45°\hat{i} - A\sin 45°\hat{k}$

$$= \sqrt{2}\,a^2\frac{1}{\sqrt{2}}\hat{i} - \sqrt{2}\,a^2\frac{1}{\sqrt{2}}\hat{k} = a^2\hat{i} - a^2\hat{k}$$

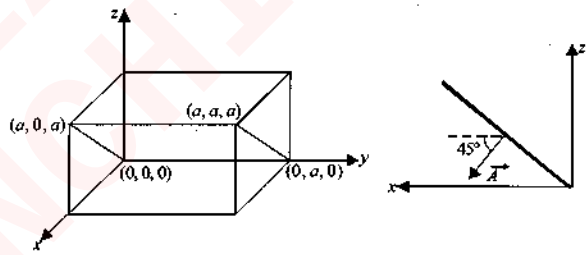


Fig. A1.251

$$\phi = \vec{E}\cdot\vec{A} = E_0\hat{i}\cdot(a^2\hat{i} - a^2\hat{k}) = E_0 a^2$$

29. d. $U_i = \frac{1}{2}(2)V^2 = V^2$. After connecting S to 2:

$$V_{\text{common}} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{2V + 8\times 0}{2+8} = \frac{V}{5}$$

$$U_f = \frac{1}{2}(2+8)\left(\frac{V}{5}\right)^2 = \frac{V^2}{5}$$

$$\%\text{ energy dissipated} = \frac{U_i - U_f}{U_i}\times 100$$

$$= \frac{V^2 - \frac{V^2}{5}}{V^2}\times 100 = 80\%$$

30. a. The frequency will be the same $\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$, frequency is independent of any external constant force. But due to the constant force qE towards right, the equilibrium position gets shifted by qE/K in forward direction.

Assertion and Reasoning Type

1. **b.** Both are basic facts and have no relation with each other

Comprehension Type

For Problems 1–3

1. a., 2. b., 3. c.

Sol. Net charge with in $r < R$ is constant; hence, electric field is independent of a.

$$q = \int_0^R \frac{d}{R}(R-x)4\pi x^2 dx = Ze$$

$$d = \frac{3Ze}{\pi R^3}$$

If within a sphere ρ is constant, then $E \propto r$.

For Problems 4–5

4. d., 5. b.

Sol. Conserving energy (Fig. A1.252)

$$(-\vec{p}\vec{E})_{\text{initial}} + (\text{K.E.})_{\text{initial}} = \text{constant}$$

$$0 + 0 = -p \times \frac{1}{4\pi\varepsilon_0}\frac{q}{q^2} + \text{K.E.}$$

$$\text{K.E.} = \frac{pq}{4\pi\varepsilon_0 d^2}$$

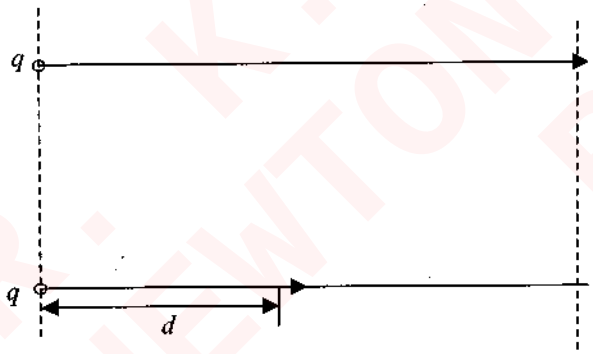


Fig. A1.252

$$|F| = \left|\frac{dU}{dx}\right| = \frac{pq}{4\pi\varepsilon_0}\frac{d}{dx}\left(\frac{1}{x^2}\right)$$

$$= \frac{pq}{4\pi\varepsilon_0 0}(-2x^{-3}) = \frac{pq}{2\pi\varepsilon_0}\frac{1}{x^3}$$

Force on dipole $|F| = \frac{1}{2\pi\varepsilon_0}\frac{pq}{d^3}$

Multiple Choice Questions with One or More than One Correct Answer Type

1. **d.** Let us consider the positive charge Q at any instant of time t at a distance x from the origin. It is under the influence of two forces, F_1 (= I) and f_2(= F). On resolving these two forces, we find that $F \sin\theta$ cancels out. The resultant force is

$$F_R = 2F\cos\theta = 2 \times \frac{kQq}{\sqrt{x^2+a^2}} \times \frac{x}{\sqrt{x^2+a^2}}$$

$$= \frac{2kQqx}{(x^2+a^2)^{3/2}}$$

Since F_R is not proportional to x, the motion is not simple harmonic. The charge Q will accelerate till the origin and gain velocity. At the origin, the net force is zero but due to momentum it will cross the origin and move towards left. As it comes on the negative x-axis, the force is again towards the origin.

2. **a., d.** (Fig. A1.253)

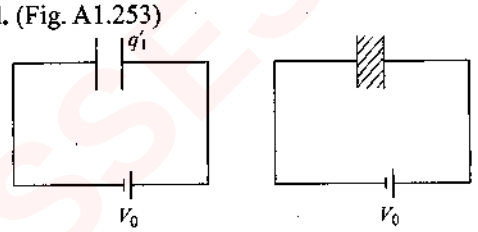


Fig. A1.253

Before introducing dielectric slab:
P.d. = V_0
Capacitance = C
Charge, $q_0 = CV_0$

$$\text{P.E., } U_0 = \frac{1}{2}CV_0^2$$

After introduction of dielectric slab
P.d. = V_0
Capacitance = KC [K is the dielectric constant of slab; $K > 1$]
New charge, $Q = KCV_0$

$$\text{New P.E., } U = \frac{1}{2}KCV_0^2$$

3. **b.** q has to be negative for equilibrium.
Considering equilibrium of 1

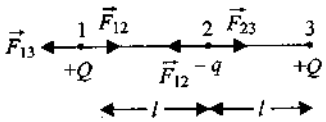


Fig. A1.254

$$F_{13} = F_{12}$$

$$\frac{KQ \times Q}{(2l)^2} = \frac{kQq}{l^2} \text{ or } q = \frac{Q}{4}$$

4. **b., d.** From Fig. A1.255,

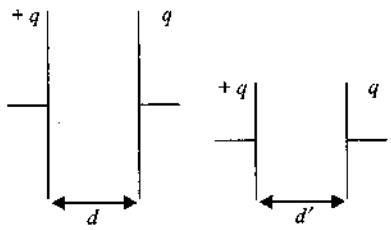


Fig. A1.255

Charge on plate is q

$$C = \frac{\varepsilon_0 A}{d}; \quad C' = \frac{\varepsilon_0 A}{d'} \Rightarrow C' < C$$

$$q = CV \Rightarrow V = \frac{q}{C}; \quad V' = \frac{q}{C'} \Rightarrow V' > V$$

$$U = \frac{1}{2} q \times V; \quad U' = \frac{1}{2} qV' \Rightarrow U' > U$$

5. a. The potential inside the shell will be the same everywhere as on its surface. As we add $3Q$ charge on the surface, the potential on the surface changes by the same amount as that inside. Therefore, the potential difference remains the same.

6. a. The equivalent capacitance (Fig. A1.256) is

$$\frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2 \times 5} = \frac{11}{10}$$

$$C_{eq} = \frac{11}{10} \lambda F$$

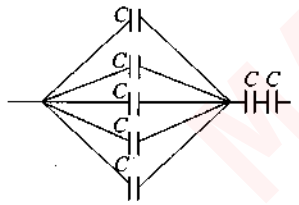


Fig. A1.256

7. a., c., d. $C = \frac{\varepsilon_0}{d} A, \quad C' = \frac{K\varepsilon_0 A}{d}$

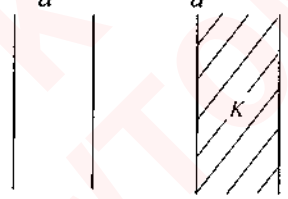


Fig. A1.257

$$Q = CV = \frac{\varepsilon_0 A}{d} \times V = \frac{\varepsilon_0 AV}{d}$$

[Q will remain the same as no charge is leaving or entering the plates during the process of slab insertion]

$$Q = C'V' = C'E'd$$

$$E' = \frac{Q}{C'd} = \frac{(\varepsilon_0 AV/d)}{(K\varepsilon_0 A/d)} \times d = \frac{V}{Kd}$$

Work done is the change in energy stored, i.e.,

$$W = \frac{1}{2} C'V'^2 - \frac{1}{2} CV^2$$

$$= \frac{1}{2} \frac{K\varepsilon_0 A}{d} \times \frac{V^2}{K^2} - \frac{1}{2} \frac{\varepsilon_0 A}{d} \times V^2 \quad \left[\because V' = E'd = \frac{V}{K}\right]$$

$$= \frac{1}{2} \frac{\varepsilon_0 A}{d} V^2 \left[\frac{1}{K} - 1\right]$$

8. b. The work done in moving a charge from A to B

$W = (\text{T.P.E.})_A - (\text{T.P.E.})_B$

T.P.E = Total potential energy

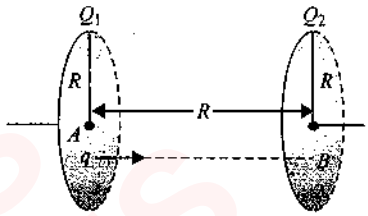


Fig. A1.258

$$(\text{T.P.E.})_A = [\text{P.E. due to } Q_1 + \text{P.E. due to } Q_2]$$

$$= \left[\left(\frac{Q_1}{4\pi\varepsilon_0 R}\right) \times q + \left(\frac{Q_2}{4\pi\varepsilon_0 \sqrt{R^2 + R^2}}\right) q\right]$$

$$= \frac{q}{4\pi\varepsilon_0 R}\left[Q_1 + \frac{Q_2}{\sqrt{2}}\right]$$

$$(\text{T.P.E.})_B = [\text{P.E. due to } Q_2 + \text{P.E. due to } Q_1]$$

$$= \left[\left(\frac{Q_2}{4\pi\varepsilon_0 R}\right) q + \left(\frac{Q_1}{4\pi\varepsilon_0 \sqrt{R^2 + R^2}}\right) q\right]$$

$$= \frac{q}{4\pi\varepsilon_0 R}\left[Q_2 + \frac{Q_1}{\sqrt{2}}\right]$$

$$\therefore \quad W = \frac{q}{4\pi\varepsilon_0 R}\left[Q_1 + \frac{Q_2}{\sqrt{2}} - Q_2 - \frac{Q_1}{\sqrt{2}}\right]$$

$$= \frac{q}{4\pi\varepsilon_0 R}\left[Q_1\left(1 - \frac{1}{\sqrt{2}}\right) - Q_2\left(1 - \frac{1}{\sqrt{2}}\right)\right]$$

$$= \frac{q(Q_1 - Q_2)}{4\pi\varepsilon_0 R}\left(\frac{\sqrt{2} - 1}{\sqrt{2}}\right)$$

9. c. Let λ be the charge per unit length. Let us consider a Gaussian surface (dotted cylinder) (Fig. A1.259).

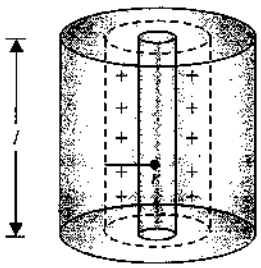


Fig. A1.259

Applying Gauss's law, we get

$$\phi = \oint \vec{E} \cdot \vec{ds} = \frac{\lambda l}{\varepsilon_0}$$

For the flat portions of the Gaussian surface, the angle between electric field and surface is 90°. Hence, flux through flat portions is zero. By symmetry, the electric field on the curved surface is the same throughout.

The angle between $\vec{E}$ and $\vec{ds}$ is 0° (for curved surface)

$$E\int ds = \frac{\lambda l}{\varepsilon_0} \Rightarrow E \times 2\pi r l = \frac{\lambda l}{\varepsilon_0} \Rightarrow E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$E \propto \frac{1}{r}$$

10. d. The electric lines of force cannot enter the metallic sphere as electric field inside the solid metallic sphere is zero. Also, the origination and termination of the electric lines of force from the metallic surface is normal (directed towards the centre)

11 b., c. In regions I and III, there will be electric field $\vec{E}_0$ directed from +ve to −ve. In region II, due to orientation of dipoles, there is an electric field $\vec{E}_0$ present in the opposite direction of $\vec{E}_0$. But since $\vec{E}_0$ is also present, the net electric field is $E_0 - E_k$ in the direction of $\vec{E}_0$ as shown in Fig. A1.260 ($\because E_0 > E_k$).

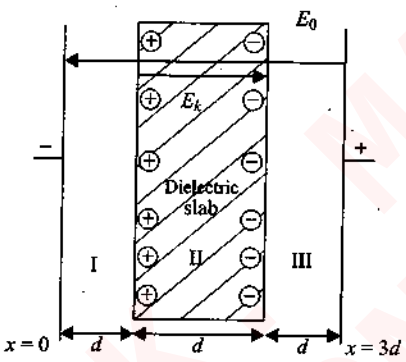


Fig. A1.260

Also, please note that when one moves opposite to the direction of electric field, the potential always increases. The stronger the electric field, the more is the increase in potential. Since in region II, the electric field is less as compared to regions I and III, therefore the increase in potential will be less but there has to be an increase in potential in all the regions from $x = 0$ to $x = 3d$.

12. d. Potential at origin will be given by

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{x_0} - \frac{1}{2x_0} + \frac{1}{3x_0} - \frac{1}{4x_0} + \cdots\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{1}{x_0}\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots\right]$$

$$= \frac{q}{4\pi\varepsilon_0 x_0}\ln(2)$$

13. a., c. Let Q be the charge on the ring, the negative charge $-q$ is released from point $P(0, 0, Z_0)$. The electric field at P due to the charged ring will be along positive z-axis and its magnitude will be

$$E = \frac{1}{4\pi\varepsilon_0}\frac{QZ_0}{(R^2 + Z_0^2 - 4pt)^{3/2}}$$

Therefore, force on charge P will be towards the centre as shown, and its magnitude is

$$F_e = qE = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{(R^2 + Z_0^2)^{3/2}}Z_0 \quad \text{(i)}$$

Similarly, when it crosses the origin, the force is again towards centre O.

Thus, the motion of the particle is periodic for all values of Z_0 lying between 0 and ∞.

Secondly, if $Z_0 \ll R$, $(R^2 + Z_0^2)^{3/2} \to R^3$

$$F_e = \frac{1}{4p\varepsilon_0} \times \frac{Qq}{R^3} \times Z_0 \text{ [from Eq. (i)]}$$

i.e., the restoring force $F_e \propto -Z_0$. Hence, the motion of the particle will be simple harmonic. (Here, negative sign implies that the force is towards its mean position.)

14. a., c. The expressions of the electrical field are

Inside the sphere $(r < R)$, $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}r$;

Outside the sphere $(R < r < \infty)$, $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

Hence, E increases for $r < R$ and decreases for $R < r < \infty$.

15. c., d. When two points are connected with a conducting path in electrostatic condition, then the potential of the two points is equal. Therefore, option (c) is the correct option. Option (d) follows from Gauss's law.
Options (a) and (b) are dependent on the curvature which are different at points A and B.

16. a., b., c., d.

For $r > R_0$, $E = \frac{d\phi}{dr} = \frac{Q}{4\pi\varepsilon_0 r^2}$

Therefore, charge enclosed by concentric spherical surface of r is

$$2R_0 = \varepsilon_0\phi_E 4\pi r^2 = \varepsilon_0\frac{Q}{4\pi\varepsilon_0 r^2}4\pi r^2 = Q$$

For $r < R_0$, $E = -\frac{dV}{dr} = 0$ and for $r > R_0$

$$E = -\frac{dV}{dr} = 4\pi\varepsilon_0 r^2 \text{ (Here, } V = \phi)$$

17. a., d. A number of electric field lines of forces emerging from Q_1 are larger than terminating at Q_2.

18. c.

a. This option is wrong, because otherwise flux due a charge through a Gaussian surface (in which the charge is present) will depend upon the shape and size of the surface.

b. It is almost impossible to calculate the field distribution around an electric dipole using Gauss's law because proper symmetry will not be formed.

c. This is obvious.

d. This option is not correct because it is not given that charge is moving slowly.

19. a., b., c., d. From conservation of charge: $Q_A + Q_B = 2Q$ (i)

Potential of both should be the same: $\frac{KQ_A}{R_A} = \frac{KQ_B}{R_B}$ (ii)

Fig. A1.261

From Eqs. (i) and (ii) $\frac{Q_A}{Q_B} = \frac{R_A}{R_B}$ (iii)

Solving Eqs. (i) and (iii), we get

$$Q_A = \frac{2QR_A}{R_A + R_B} \text{ and } Q_B = \frac{2QR_B}{R_A + R_B} \Rightarrow Q_A > Q_B$$

$$\frac{\sigma_A}{\sigma_B} = \frac{Q_A / 4\pi R_A^2}{Q_B / 4\pi R_B^2} = \frac{R_B}{R_A} \quad \text{using Eq. (iii)}$$

$\sigma_B > \sigma_A$ (as $R_A > R_B$)

On surface, $E_A = \frac{\sigma_A}{\varepsilon_0}$ and $E_B = \frac{\sigma_B}{\varepsilon_0}$

$\Rightarrow \quad E_A < E_B \quad (\because \sigma_A < \sigma_B)$

20. (2) $\rho = kr^a$

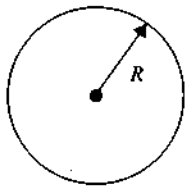


Fig. A1.262

$$E\left(r - \frac{R}{2}\right) = \frac{1}{8}E(r - R)$$

$$\frac{q_{\text{enclosed}}}{4\pi\varepsilon_0 (R/2)^2} = \frac{1}{8}\frac{Q}{4\pi\varepsilon_0 R^2}$$

$$32q_{\text{enclosed}} = Q$$

$$q_{\text{enclosed}} = Q$$

$$q_{\text{enclosed}} = \int_0^{R/2} kr^a 4\pi r^2 dr = \frac{4\pi k}{(a+3)}\left(\frac{R}{2}\right)^{(a+3)} \quad Q = \frac{4\pi k}{(a+3)}R^{(a+3)}$$

$$\frac{Q}{q_{\text{enclosed}}} = 2^{(a+3)}$$

$$2^{a+3} = 32$$

$$a = 2$$

R. K. MALIK'S
NEWTON CLASSES
RANCHI

CHAPTER
5

Electric Current and Circuits

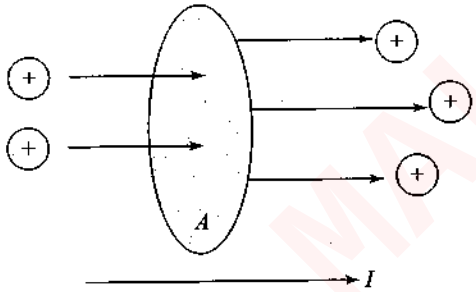
- Electric Current
- Current Density
- Drift Velocity
- Structural Model for Electrical Conductor
- Mobility
- Temperature Coefficient of Resistivity
- Validity and Failure of Ohm's Law
- Electromotive Force and Potential Difference
- Internal Resistance of a Cell
- Combination of Resistances
- Kirchhoff's Law: Kirchhoff's Laws For Electrical Networks
- Wheatstone Bridge: Balanced Wheatstone Bridge
- Combination of Cells
- Superposition Principle
- Charging and Discharging of a Capacitor through a Resistance

R. K. MALIK'S NEWTON CLASSES RANCHI

In electrostatics, our discussion of electric phenomena has been focused on charges at rest. In the previous chapter, we treated the concept of electric potential, which is measured in volt. Now we will see that this voltage acts like an "electrical pressure" that can produce a flow of charge or current, which is measured in ampere (or simply, amp and abbreviated as A) and that the resistance that restrains this flow is measured in ohm (Ω).

ELECTRIC CURRENT

An isolated metallic conductor, say a wire, contains a few electrons which are moving at random with high speeds. These are called conduction electrons. The rate at which these electrons pass from left to right through an area in a wire is the same as the rate at which they pass from right to left through the same area, i.e., net rate is zero.



Charges in motion through an area A. The time rate at which charge flows through the area is defined as the current I. The direction of the current is the direction in which positive charges flow when free to do so.

Fig. 5.1

To define the current mathematically, suppose charged particles are moving perpendicular to a surface of area A as in Fig. 5.1 (this area could be the cross-sectional area of a wire, for example). The current is defined as the rate at which electric charge flows through this surface. If ΔQ is the amount of charge that passes through this area in time interval Δt, then average current, I_{avg}, over this time interval through this area is the ratio of the charge to the time interval, i.e.,

$$I_{avg} = \frac{\Delta Q}{\Delta t} \qquad \text{(i)}$$

It is possible for the rate at which the charge flows to vary with time. We define the instantaneous current I as the limit of the preceding expression as Δt tends to zero;

$$I \equiv \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

Also we can write, $dQ = Idt \Rightarrow Q = \int Idt$ (ii)

From above equation, total charge flown in a time interval can be found.

Points to Remember

- The particles flowing through a surface can be charged positively or negatively, or we can have two or more types of particles moving, with charges of both the signs in the flow. Conventionally, we define the direction of the current as the direction of flow of positive charge.
- In a common conductor such as copper, the current is in physical state due to the motion of the negatively charged electrons. Therefore, when we speak of current in such a conductor, the direction of the current is opposite to the direction of flow of electrons.
- On the other hand, if one considers a beam of positively charged protons in a particle accelerator, then the current is in the direction of the motion of the protons.
- In some cases—gases and electrolytes, for example, the current is the result of the flow of both positive and negative charged particles.
- It is common to refer to a moving charged particle (whether it is positive or negative) as a mobile charge carrier. For example, the charge carriers in a metal are electrons.

When a wire is connected to a battery, an electric field is set up at every point within the wire. This field exerts a force on each conduction electron. Although the electrons are continuously accelerated by the field, due to their frequent collisions with the atoms of the wire, they on an average simply drift at a small constant speed in the direction opposite to the field. Thus, there is a net flow of charge in the wire at a small rate. The total charge passing through any cross section per second is the electric current in the wire.

In the steady state, current through each section of the conducting loop would be same—no matter what is the location or orientation of area of that section see Fig. 5.2 This is because of the fact that charge is conserved.

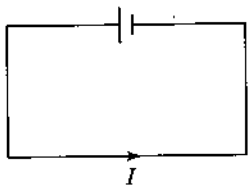


Fig. 5.2

Unit of Electric Current

In SI system, it is ampere (A).

"The current is said to be one ampere when one coulomb of charge flows past any cross section of a conductor every second."

Statement of Ohm's Law

"The electrical current in any conductor is proportional to the potential difference between its ends, other factors remaining constant."

The ratio of the potential difference to current is termed as the resistance of the conductor. Accordingly,

$$R = \frac{V}{I} \Rightarrow V = IR$$

where, I = current, V = p.d., and R = resistance

The resistance of the conductor is the opposition offered by the conductor to the flow of electric current passing through it.

The resistance R not only depends on the material of the conductor but also on the dimensions of the conductor.

The resistance of an ohmic conducting wire is found to be proportional to its length ℓ and inversely proportional to its cross-sectional area A, i.e.,

$$R = \rho \frac{\ell}{A} \qquad \text{(iii)}$$

where the constant of proportionality ρ is called the *resistivity* of the material, which has the unit ohm metre (Ωm). To understand the relationship between resistance and resistivity, we should know that ρ depends on the properties of the material and on temperature. On the other hand, the resistance R of a particular conductor depends on its size and shape as well as on the resistivity of the material.

The inverse of resistivity is defined as *conductivity* σ. Hence, the resistance of an ohmic conductor can be expressed in terms of its conductivity as

$$R = \frac{\ell}{\sigma A}, \quad \text{where } \sigma = \frac{1}{\rho}$$

Resistance and Resistivity: Resistivity is a property of a substance, whereas resistance is a property of an object. We have seen similar pairs of variables before. For example, density is a property of a substance, whereas mass is a property of an object. Equation (iii) relates resistance to resistivity.

SI Unit of Resistance

The unit of resistance is ohm (Ω).

"*The resistance of a conductor is said to be 1 ohm if a current of 1 ampere flows through it when the p.d. across its ends is 1 volt.*"

CURRENT DENSITY

Current density at a point is defined as the amount of current flowing per unit area around that point provided area is normal to the direction of current. It is denoted by J.

So we can write: $J = I/A$, where I is current and A is area.

Unit of current denisty: A/m²

Current density is a vector quantity. So it must have direction also. Its direction at a point is the direction of motion of the positive charge or direction of current at that point.

Average Current Density: Let through an area ΔA around a point P, a current Δi is passing normal to the area. Then the average current density over the area is given by $J_{av} = \Delta i/\Delta A$.

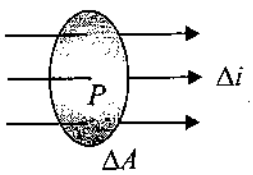


Fig. 5.3

Current Density at a Point: Now suppose we want to find current density at point P in the preceding figure. Take an infinitesimally small area dA around point P. Let the current through this small area is di. Then, the current density at point P is given by $J = di/dA$.

If Current is not Perpendicular to Area: Here we divide the current by the component of area in the direction of the current. Hence, the average current density is given by

$$J_{av} = \frac{\Delta i}{\Delta A \cos\theta}$$

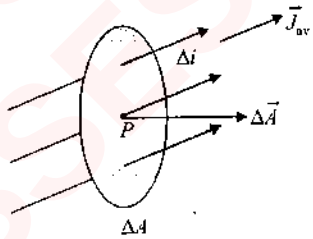


Fig. 5.4

We can also write, $\Delta i = J_{av}\Delta A\cos\theta \Rightarrow \Delta i = \vec{J}_{av}.\Delta\vec{A}$

Curent Density at *P*: Take an infinitesimally small area dA around point P. Let the current through this small area is di. Then the current density at point P is given by $J = di/(dA\cos\theta)$

we can also write, $di = JdA\cos\theta \Rightarrow di = \vec{J}.\vec{dA} \Rightarrow i = \int \vec{J}.\vec{dA}$.

Note: ***While electric current is a scalar quantity, electric current density is a vector quantity.***

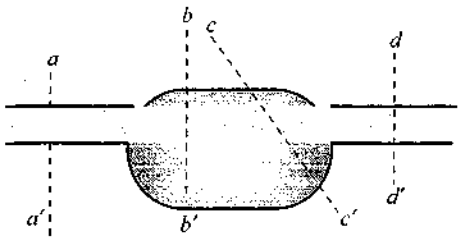
In case of conductors:

$$J = \frac{I}{A} = \frac{V}{RA} = \frac{EL}{(\rho L/A)A} = \frac{E}{\rho} = \sigma E \quad \text{or} \quad \vec{J} = \sigma\vec{E}$$

a. Current density is proportional to electric field and direction of current density is the same as that of electric field.
b. If electric field is uniform (i.e., is constant), current density will be constant
c. If electric field is zero (as in electrostatics inside a conductor), current density and hence current will be zero.

Points to Remember

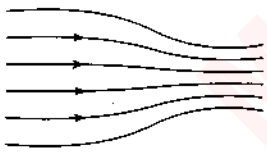
- If the current has not reached a steady state, i.e., the flow of charge is not constant, then the current through different cross sections at a particular instant may have different values.
- Electric current may be distributed non-uniformly over the surface through which it passes. Hence, to characterize the current in greater detail, current density vector $\vec{J}$ is introduced.
- Current density, $\vec{J}$, tells us how charge flows at a certain point and its direction tells us about the direction of the flow of charge at that point, while the current describes how charge flows through an extended object.



Current through all sections would be same in steady state

Fig. 5.5

- The direction of current density is the same as that of the velocity of +ve charge or opposite to the direction of the velocity of −ve charge.
- Current density can be represented by a similar set of lines known as stream lines. The spacing of the stream lines suggests the value of current density. Narrower stream lines mean more current density, spaced stream lines mean less current density (Fig. 5.6).



A conductor having non uniform cross section

Fig. 5.6

Illustration 5.1 **In a hydrogen discharge tube, the number of protons drifting across a cross section per second is 1.0×10^{18}, while the number of electrons drifting in the opposite direction across the same cross section is 2.7×10^{18} per second. Find the current flowing in the tube.**

Sol. As electrons and protons are moving in the opposite directions, they will effectively produce current in the same direction and the total current in the tube is

$$I = (n_p + n_e)e/t$$
$$= (1.0 \times 10^{18} + 2.7 \times 10^{18}) \times 1.6 \times 10^{-19}/1$$
$$= 3.7 \times 1.6 \times 10^{-1}\ \text{A} = 0.592\ \text{A}$$

Illustration 5.2 **You need to produce a set of cylindrical copper wires 2.5 m long that will have a resistance of 0.125 Ω each. What will be the mass of each of these wires? (Density of copper is 8.9×10^3 kg/m³, resistivity of copper is 1.72×10^{-8} Ωm).**

Sol. Given that $L = 2.5$ m, $R = 0.125\ \Omega$. To find the mass of each wire, we need to calculate the volume of one of the wires as

Mass = Density × Volume

Volume = Area × Length $\left[\because R = \dfrac{\rho L}{A} \Rightarrow A = \dfrac{\rho L}{R}\right]$

$$\text{Volume} = AL = \frac{\rho L^2}{R} = \frac{1.72 \times 10^{-8} \times (2.5)^2}{0.125} = 8.6 \times 10^{-7}\ \text{m}^3$$
$$m = d \times V = 8.9 \times 10^3 \times 8.6 \times 10^{-7}$$
$$= 7.654 \times 10^{-3}\ \text{kg} = 7.654\ \text{g}$$

Illustration 5.3 **Consider a wire of length l, area of cross section A and resistivity ρ with resistance 10 Ω. Its length is increased by applying a force and it becomes four times of its original value. Find the changed resistance of the wire.**

Sol. Here $l_1 = l$, $A_1 = A$, and $R = 10\ \Omega$. Similarly, $l_2 = 4l$ and $R_2 = ?$ Resistivity is same in each case as the material is same. The volume of the wire will remain the same even after the increase in the length. $A_1 l_1 = A_2 l_2 \Rightarrow A_2 = \dfrac{A_1 l_1}{l_2} = \dfrac{Al}{4l} = \dfrac{A}{4}$

The formula used for measuring resistance of wire is $R = \rho\,(l/A)$.

Using this formula in both cases, we have

$$R_1 = \rho \frac{l_1}{A_1} = \frac{\rho l}{A} \quad \text{(i)}$$

and
$$R_2 = \rho \frac{l_2}{A_2} = \rho \frac{4l}{A/4} = 16\,\rho\,\frac{l}{A} \quad \text{(ii)}$$

Dividing Eq. (ii) by Eq. (i), $\dfrac{R_2}{R_1} = 16 \Rightarrow R_2 = 160\ \Omega$

Illustration 5.4 **A wire of mass m, length l, density d, and area of cross section A is stretched in such a way that its length increases by 10% of its original value. Express the changed resistance in percentage.**

Sol. Given mass m, length $l_1 = l$, density d, and area of cross section $A_1 = A$. Let ρ be the resistivity and R_1 be the resistance of the wire. Mass of wire m = volume × density = $Al \times d = Al\,d$.

Therefore, area of cross section $A_1 = m/ld$, the resistance of the wire is

$$R_1 = \rho \frac{l}{A_1} = \rho \frac{l^2 d}{m} = \left(\frac{\rho d}{m}\right) l^2 = k\,l^2 \quad \text{(i)}$$

Let l_2 be the new length, then $l_2 = l + \dfrac{10}{100}\,l = l + 0.1\,l = 1.1\,l$

Let R_2 be the resistance of the wire after stretching, then

$$R_2 = k l_2^2 \quad \text{(ii)}$$

Dividing Eq. (ii) by Eq. (i), we have $\dfrac{R_2}{R_1} = \dfrac{l_2^2}{l_1^2} = \dfrac{(1.1)^2 l_1^2}{l_1^2} = 1.21$

$$\Rightarrow \quad R_2 = 1.21\,R_1 = R_1 + 0.21\,R_1 \Rightarrow R_2 - R_1 = 0.21\,R_1$$

Hence, the percentage change in the resistance is

$$\frac{R_2 - R_1}{R_1} \times 100 = 21\%$$

Illustration 5.5 **A uniform copper wire of mass 2.23×10^{-3} kg carries a current of 1 A when 1.7 V is applied across it. Calculate the length and the area of cross section. If the wire is uniformly stretched to double its length; calculate the new resistance. Density of copper is 8.92×10^3 kg/m³ and resistivity is 1.7×10^{-8} Ω m.**

Sol. As m = volume × density = $(L \times S) \times d$

[as volume = $L \times S$]

So
$$L \times S = \frac{m}{d} = \frac{2.23 \times 10^{-3}}{8.92 \times 10^3} = \frac{1}{4} \times 10^{-6} \quad \text{(i)}$$

And as, $V = IR$, i.e.,
$$R = \frac{V}{I} = \frac{1.7}{1} = 1.7\ \Omega \quad \text{(ii)}$$

But by definition, $R = \rho\,(L/S)$

$$\Rightarrow \qquad \frac{L}{S} = \frac{R}{\rho} = \frac{1.7}{1.7\times10^{-8}} = 10^8 \qquad \text{(iii)}$$

Solving Eqs. (i) and (iii) for L and S, we get $L = 5$ m and $S = 5 \times 10^{-8}$ m^2

When the wire is stretched uniformly to double its length, the volume will remain unchanged, i.e.,

$$SL = S'(2L) \quad \text{so} \quad S' = S/2$$

and hence the new resistance will be

$$R' = \rho\frac{(2L)}{(S/2)} = 4\rho\frac{L}{S} = 4R = 4\times1.7 = 6.8\ \Omega$$

DRIFT VELOCITY

Under the normal conditions of temperature and pressure and without the influence of any external electrostatic field, the motion of free electrons in a conductor is due to the thermal energy.

Now, when this conductor is placed in an external field, these free electrons start experiencing electric force and start moving under the influence of this force (see Fig. 5.7). However, although they are free to move, they are not able to move in a straight line because they encounter other electrons, ions, atoms, or molecules in their way (see Fig. 5.8). Hence, they experience collisions after collisions, but are able to drift in a particular direction because of this external field. The drifting of these free electrons over some period of time is called drift velocity.

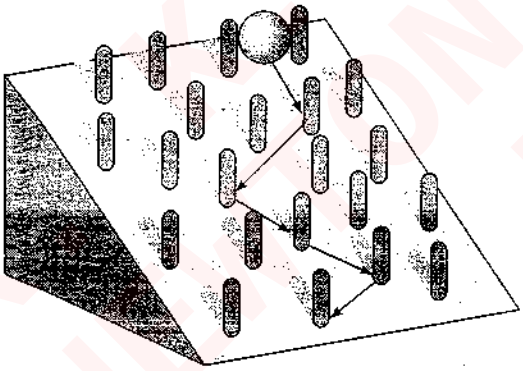


Fig. 5.7

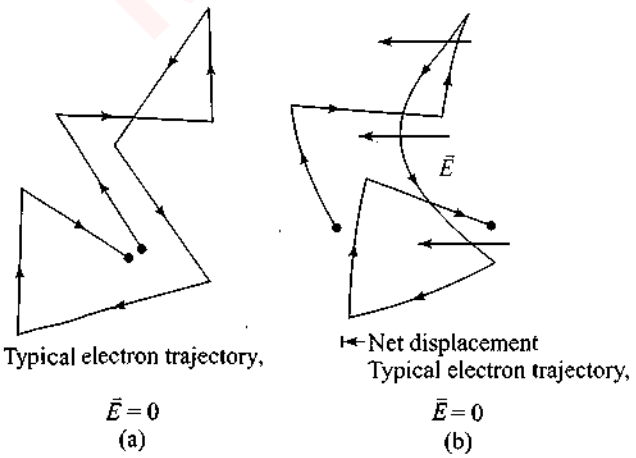


Fig. 5.8

Relation Between Drift Velocity and Current

Let A = area of cross section of the conductor, e = charge on each electron, v_d = drift velocity, n = number of free electrons per unit volume, and I = current, then the total number of electrons between cross sections P and Q which are v_d distance apart = (volume between P and Q) $\times\, n = Av_d n = nAv_d$

Therefore, the total charge in this volume is $nAv_d e = neAv_d$.

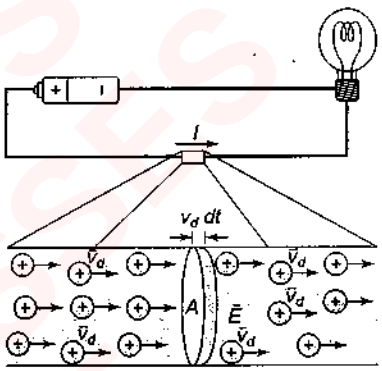


Fig. 5.9

Now, the electron which is present at cross section Q will reach cross section P after 1/sec because P and Q are so selected that the distance between them is v_d which is the drift speed of the electron. Therefore, $neAv_d$ is the charge which will pass through cross section at P (where P can be any point on the conductor). Hence, this is the electric current which flows through the conductor (see Fig. 5.10).

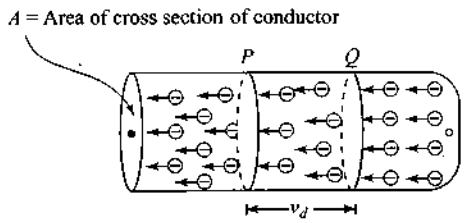


Fig. 5.10

$$\therefore \qquad I = neAv_d$$

Also, accordingly current density, $J = I/A = nev_d$.

The equation $J = nev_d$ can be written in vector form as follows: $\vec{J} = nq\vec{v}_d$ where q is the charge of the charge carrier and v_d is the average drift velocity. This equation is correct for both the signs of q. If $q > 0$, $\vec{v}_d$ is in the direction of electric field $\vec{E}$ and $\vec{J}$ is in the direction of $\vec{E}$. If $q < 0$ ($q = -e$ for electrons), as it is in metallic conductor, $\vec{v}_d$ is opposite to $\vec{E}$, and $\vec{J} = -ne\vec{v}_d$ continues to be in the direction of $\vec{E}$.

Illustration 5.6 **A copper wire has a square cross section of 6 mm on a side. The wire is 10 m long and carries a current of 3.6 A. The density of free electrons is 8.5×10^{28}/m^3. Find the magnitude of (a) the current density in the wire; (b) the electric field in the wire. (c) How much time is required for an electron to travel the length of the wire? (ρ, electrical resistivity, is $1.72 \times 10^{-8}\ \Omega$m)**

Sol. Given $r = 6$ mm, $l = 10$ m, $I = 3.6$ A, $n = 8.5 \times 10^{28}/\text{m}^3$.

(a) To find the current density, formula used should be $J = I/A$

$$\Rightarrow \quad J = \frac{3.6}{(6\times10^{-3})^2} = \frac{3.6}{36\times10^{-6}} = 10^5 \text{ A/m}^2$$

(b) To find electric field,

$$E = \rho J = 1.72 \times 10^{-8} \times 10^5 = 1.72 \times 10^{-3} \text{ V/m}$$

(c) Time taken, $t = \dfrac{l}{v_d} = \dfrac{l\,ne\,A}{I}$

$$= \frac{10 \times 8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times (6\times10^{-3})^2}{3.6}$$

$$= 1.36 \times 10^6 \text{ s}$$

Illustration 5.7 **Consider a wire of length 0.1 m with an area of cross section 1 mm² connected to 5 V. Find the current flowing through the metallic wire where $\mu = 5 \times 10^{-6}$ m² V⁻¹ s⁻¹, $e = 1.6 \times 10^{-19}$ C and $n = 8 \times 10^{28}$ m⁻³.**

Sol. Given $A = 1$ mm² $= 10^{-6}$ m², $l = 0.1$ m, $V = 5$ V.

Due to the applied potential difference across the wire, an electric field is set up in the conductor.

Formula used, $E = \dfrac{V}{l}, = \dfrac{5}{0.1} = 50 \text{ V m}^{-1}$

The current flowing through the wire is given by

$$I = n\,Aev_d = n\,Ae\mu E \qquad [\text{where, } \mu = v_d/E]$$

$$= 8 \times 10^{28} \times 10^{-6} \times 1.6 \times 10^{-19} \times 5 \times 10^{-6} \times 50 = 3.2 \text{ A}$$

Illustration 5.8 **A current 16 A is made to pass through a conductor where the number density of free electrons is 4×10^{28} m⁻³ and its area of cross section is 10^{-5} m². Find out the value of the drift velocity of free electrons.**

Sol. Given $I = 16$ A, $A = 10^{-5}$ m², $n = 4 \times 10^{28}$ m⁻³.

Drift velocity $v_d = \dfrac{I}{ne\,A} = \dfrac{16}{4\times10^{28}\times1.6\times10^{-19}\times10^{-5}}$

$$= 2.5 \times 10^{-4} \text{ ms}^{-1}$$

Illustration 5.9 **(a) Estimate the average drift speed of conduction electrons in a copper wire of cross-sectional area 1.0×10^{-7} m² carrying a current of 1.5 A. Assume that each copper atom contributes roughly one conduction electron. The density of copper is 9.0×10^3 kg/m³, and its atomic mass is 63.5 u. (b) Compare the drift speed obtained with the speed of propagation of electric field along the conductor which causes the drift motion.**

Sol.

(a) The direction of drift velocity of conduction electrons is opposite to the electric field direction, i.e., the electrons drift in the direction of increasing potential. The drift speed v_d is given by $v_d = (I/neA)$.

Now, $e = 1.6 \times 10^{-19}$ C, $A = 1.0 \times 10^{-7}$ m², $I = 1.5$A, the density of n conduction electrons is equal to the number of atoms per cubic metre (assuming one conduction electron per Cu atom is reasonable from its valence electron count of one). A cubic metre of copper has a mass of 9.0×10^3 kg. Since 6.0×10^{23} copper atoms have a mass of 63.5 g, Therefore,

$$n = \frac{6.0\times10^{23}}{63.5} \times 9.0 \times 10^6 = 8.5 \times 10^{28}$$

which gives

$$v_d = \frac{1.5}{8.5\times10^{28}\times1.6\times10^{-19}\times1.0\times10^{-7}}$$

$$= 1.1 \times 10^{-3} \text{ ms}^{-1} = 1.1 \text{ mms}^{-1}$$

(b) An electric field travelling along the conductor has a speed of an electromagnetic wave, i.e., equal to 3.0×10^8 ms⁻¹. The drift speed is, in comparison, extremely small; smaller by a factor of 10^{-11}.

STRUCTURAL MODEL FOR ELECTRICAL CONDUCTOR

Consider a conductor as a regular array of atoms containing free electrons (sometimes called conduction electrons). Such electrons are free to move throughout the conductor. In the absence of an electric field, the free electrons move in random directions with average speeds of the order of 10^6 m/s. The situation is similar to the motion of the gas molecules confined in a vessel that we studied in kinetic theory of gases. In fact, the conduction electrons in a metal are often called electron gas.

Conduction electrons are not totally free because they are confined to the interior of the conductor and undergo frequent collisions with the array of atoms. The collisions are the predominant mechanism contributing to the resistivity of a metal at normal temperatures. Note that there is no current in a conductor in the absence of an electric field because the average velocity of the free electrons is zero. On an average, just as many electrons move in one direction as in the opposite direction, so there is no net flow of charge.

However, the situation is modified, when an electric field is applied to the metal. In addition to the random thermal motion, the free electrons drift slowly in a direction opposite to that of the electric field, with an average drift speed of v_d, which is much less (typically 10^{-4} m/s) than the average speed between collisions (typically 10^6 m/s).

In our structural model, we shall assume that the excess kinetic energy acquired by the electrons in the electric field is lost to the conductor in the collision process. The energy given up to the atoms in the collisions increases the total vibrational energy of the atoms, causing the conductor to warm up. The model also assumes that an electron's motion after a collision is independent of its motion before the collision.

On the basis of our model, we now take the first step towards obtaining an expression of the drift speed. Let a mobile charged particle of mass m and charge q is subjected to an electric field $\vec{E}$. For electrons in a metal, $\vec{F}_e = e\vec{E}$. The motion of the electron can be determined from Newton's second law, $\sum \vec{F} = m_e\vec{a}$. The acceleration of the electron is

$$\vec{a} = \frac{\sum \vec{F}}{m_e} = \frac{\vec{F}_e}{m_e} = \frac{-e\vec{E}}{m_e} \qquad \text{(i)}$$

The acceleration, which occurs for only a short time interval between collisions, changes the velocity of the electron. Because the force is constant, the acceleration is constant, and we can

model the electron as a particle under constant acceleration. If $\vec{v}_0$ is the velocity of the electron just after a collision, at which we define the time as $t = 0$, the velocity of the electron at time t is

$$\vec{v} = \vec{v}_0 + \vec{a}t = \vec{v}_0 - \frac{e\vec{E}}{m_e}t \qquad \text{(ii)}$$

The motion of the electron through the metal is characterized by a very large number of collisions per second. Consequently, we consider the average value of $\vec{v}$ over a time interval compared with the time interval between collisions, which gives us the drift velocity $\vec{v}_d$. Because the velocity of the electron after a collision is assumed to be independent of its velocity before the collision, the initial velocities are randomly distributed in direction, so that the average value of $\vec{v}_0$ is zero. In the second term on the right of Eq. (ii), the charge, electric field and the mass are all constant. Therefore, the only factor affected by the averaging process is the time t. The average value of this term is $(-e\vec{E}/m_e)\tau$, where τ is the average time interval between collisions. Therefore, after the averaging process, Eq. (ii) becomes,

$$\vec{v}_d = \frac{-e\vec{E}}{m_e}\tau \qquad \text{(iii)}$$

Substituting the magnitude of this drift velocity (the drift speed) in equation $I_{avg} = \Delta Q/\Delta t = nev_d A$, we have

$$I = nev_d A = ne\left(\frac{eE}{m_e}\tau\right)A = \frac{ne^2E}{m_e}\tau A \qquad \text{(iv)}$$

According to Ohm's law, the current is related to the macroscopic variables of potential difference and resistance, i.e.,

$$I = \frac{\Delta V}{R}$$

Substituting $R = \rho(\ell/A)$, we can write this expression as

$$I = \frac{\Delta V}{\left(\rho\frac{\ell}{A}\right)} = \frac{\Delta V}{\rho\ell}A$$

In the conductor, the electric field is uniform, therefore we use $\Delta V = E\ell$, for the magnitude of the potential difference across the conductor, i.e.,

$$I = \frac{E\ell}{\rho\ell}A = \frac{E}{\rho}A \qquad \text{(v)}$$

Equating Eqs.(iv) and (v) for the current, we solve for the resistivity, i.e.,

$$I = \frac{ne^2E}{m_e}\tau A = \frac{E}{\rho}A \rightarrow \rho = \frac{m_e}{ne^2\tau} \qquad \text{(vi)}$$

According to this structural model, resistivity does not depend on the electric field or, equivalently, on the potential difference, but depends only on fixed parameters associated with the material and the electron. This feature is characteristic of a conductor obeying Ohm's law. The model shows that the resistivity can be calculated from the knowledge of the density of the electrons, their charge and mass and the average time interval τ between collisions.

We can also write current density as

$$J = nev_d = (ne)\left(\frac{eE}{m}\tau\right) = \frac{ne^2}{m}\tau E$$

Illustration 5.10 **A copper wire of cross-sectional area 3.00×10^{-6} m² carries a current 10.0 A. Find**

a. the drift speed of the electrons in the wire. Assume that each copper atom contributes one free electron to the body of material.

b. the average time between collisions for electrons in the copper at 20°C. The density of copper is 8.95 g/cm³, molar mass of copper = 63.5 g/mol, Avagadro number 6.02×10^{23} electrons/mol and resistivity of copper = 1.7×10^{-8} Ωm.

Sol.

a. The volume occupied by 63.5 g of copper is

$$V = \frac{M}{\rho} = \frac{63.5}{8.95} = 7.09 \text{ cm}^3/\text{mol}$$

As each copper atom contributes one free electron to the body of the material, the density of free electrons is

$$n = \frac{6.02\times10^{23}}{7.09\times10^{-6}} = 8.48\times10^{28} \text{ electron/m}^3$$

The drift speed $v_d = \frac{I}{neA}$;

$$v_d = \frac{10.0}{8.48\times10^{28}\times1.60\times10^{-19}\times3\times10^{-6}} = 2.46\times10^{-4} \text{ m/s}$$

b. Average time between collision for electrons is

$$\tau = \frac{m_e}{ne^2\rho} = \frac{9.10\times10^{-31}}{8.48\times10^{28}\times(1.6\times10^{-19})^2\times1.7\times10^{-8}}$$

$$= 2.5\times10^{-14} \text{ s}$$

MOBILITY

As we have seen that conductivity arises from mobile charge carriers. In metals, these mobile charge carriers are electrons; in an ionized gas, they are electron and positive charged ions; in an electrolyte, there can be both positive and negative ions. In a semi-conductor material such as germanium or silicon, conduction is partly due to electrons and partly due to electron vacancies called holes. Holes are sites of missing electrons which act like positive charges.

An important quantity is the mobility μ defined as the magnitude of the drift velocity per unit electric field

$$\mu = \frac{v_d}{E}$$

Illustration 5.11 **Consider a conductor of length 40 cm where a potential difference of 10 V is maintained between the ends of conductor. Find out the mobility of the electrons provided the drift velocity of the electrons is 5×10^{-6} ms⁻¹.**

Sol. Given $L = 40$ cm, $V = 10$ V, $v_d = 5 \times 10^{-6}$ ms⁻¹.

To find the electron mobility, we need the value of the electric field which can be obtained using the following formula

$$E = \frac{V}{l} \Rightarrow E = \frac{10}{0.4} = 25 \text{ V/m}$$

Also the formula used for electron mobility:

$$\mu = \frac{v_d}{E} = \frac{5\times10^{-6}}{25} = \frac{1}{5}\times10^{-6}$$

$$= 0.2\times10^{-6} = 2\times10^{-7}\ \text{m}^2\text{V}^{-1}\text{s}^{-1}$$

Effect of Temperature on Resistivity and Resistance: As long as the temperature of the material is constant, the resistivity of material also remains constant. As the temperature of material increases, the relaxation time (t) decreases, the resistivity increases and hence resistance also increases.

As the number of electrons per unit volume goes up, the resistance decreases. This can be easily visualized, since if the density of electrons increases, more electrons can flow in response to the potential difference and hence the current will increase. Therefore, the resistance will decrease.

TEMPERATURE COEFFICIENT OF RESISTIVITY

Let us study the equation

$$R_T = R_{T_0}[1 + \alpha(T - T_0) + \beta(T - T_0)^2]$$

where both α and β are called temperature coefficients of resistance, though different in magnitude.

Temperature Coefficients of Resistance

In case of pure metals, β is negligibly small, so the resistance varies linearly with the rise of temperature (see Fig. 5.11).

$$R_T = R_{T_0}[1 + \alpha(T - T_0)]$$

where R_T = resistance at temperature T

R_{T_0} = resistance at temperature T_0 (called reference temperature)

α = a constant for a given metal and for a given reference temperature and is called temperature coefficient of resistance. Its unit is per degree temperature (°C^{-1}).

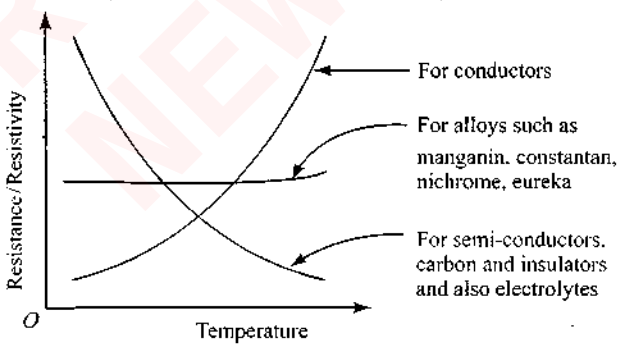


Fig. 5.11

T_0 is some reference temperature generally either 0°C or 20°C. α is (+) for metals, 0.004 per degree for Cu, (+) but very small (almost zero) for alloys such as manganin ($\alpha = 0.00001$ per degree), nichrome, constantan, eureka and (–) for semiconductors and insulators.

Let $\rho_T = \rho_0[1 + \alpha(T - T_0)]$ (see Fig. 5.11)

$\Rightarrow$ $\rho_T - \rho_0 = \alpha\rho_0(T - T_0)$

$$\Rightarrow \quad \alpha = \frac{1}{\rho_0}\times\frac{(\rho_T - \rho_0)}{(T - T_0)} \Rightarrow \rho = \frac{1}{\rho}\times\frac{d\rho}{dT}$$

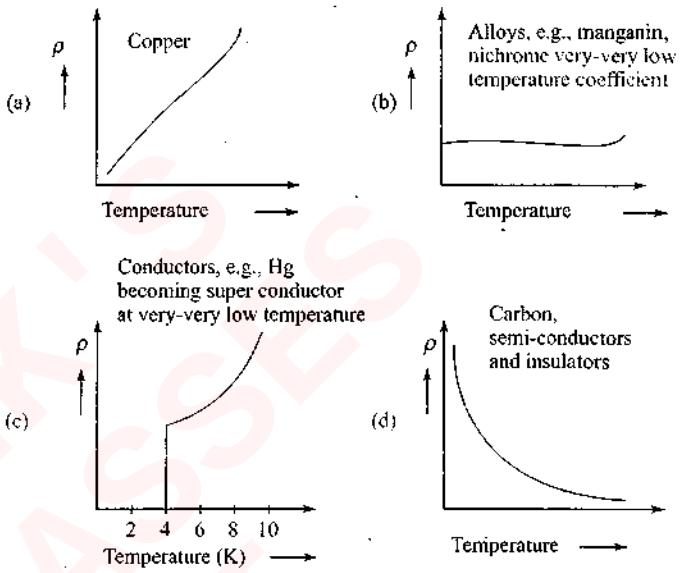


Fig. 5.12

These observations can be understood qualitatively using the relation for resistivity, i.e., $\rho = m/ne^2\tau$.

For metals, the number of free electrons is fixed. As temperature increases, the amplitude of vibration of atoms/ions increases and collisions of electrons with them become more effective and frequent, resulting in the decrease in τ and hence, increase in ρ. Thus, for metals, ρ increases with temperature.

Finding Coefficient of Resistance

i. $\alpha = \dfrac{R_2 - R_1}{R_1(t_2 - t_1)}$, where R_2 = resistance at t_2°C and R_1 = resistance at t_1°C.

Derivation:

$R_1 = R_0(1 + \alpha t_1)$ (i)

and $R_2 = R_0(1 + \alpha t_2)$ (ii)

Dividing Eq. (ii) by Eq. (i), we have

$$\frac{R_2}{R_1} = \frac{1+\alpha t_2}{1+\alpha t_1} \Rightarrow \frac{R_2}{R_1} = (1+\alpha t_2)(1+\alpha t_1)^{-1}$$

$$\frac{R_2}{R_1} = (1+\alpha t_2)(1+\alpha t_1)$$

(Using binomial theorem and thus neglecting higher terms)

$$\frac{R_2}{R_1} = (1+\alpha)(t_2 - t_1)$$

(since α^2 is very small and hence neglected)

$$R_2 = R_1 + R_1\alpha(t_2 - t_1) \Rightarrow \alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)}$$

ii. $\alpha = \dfrac{R_2 - R_1}{R_1 t_2 - R_2 t_1}$; where R_2 = resistance at t_2°C

Derivation:

Dividing Eq. (ii) by Eq. (i), we have

$$\frac{R_2}{R_1} = \frac{1+\alpha t_2}{1+\alpha t_1} \Rightarrow R_2 + R_2\alpha t_1 = R_1 + R_1\alpha t_2$$

$$R_2 - R_1 = \alpha(R_1 t_2 - R_2 t_1) \Rightarrow \alpha = \frac{R_2 - R_1}{R_1 t_2 - R_2 t_1}$$

iii. $\alpha = \dfrac{R_t - R_0}{R_0 t}$; where R_t = resistance at t °C and R_0 = resistance at 0°C

Derivation: $R_t = R_0(1+\alpha t) \Rightarrow R_t - R_0 = R_0\alpha t$

$$\Rightarrow \quad \alpha = \frac{R_t - R_0}{R_0 t}$$

Illustration 5.12 **A copper coil has a resistance of 20.0 Ω at 0°C and a resistance of 26.4 Ω at 80°C. Find out the temperature coefficient of resistance of copper.**

Sol. $R_{80°C} = R_{0°C}[1 + \alpha\,\Delta T]$

$$\Rightarrow \quad 26.4 = 20.0[1 + \alpha \times (80-0)] \Rightarrow \frac{26.4}{20} = 1 + 80\,\alpha$$

On solving, we get $\alpha = 4 \times 10^{-3}$°C^{-1}.

Illustration 5.13 **A metallic wire has a resistance of 120 Ω at 20° C. Find the temperature at which the resistance of same metallic wire rises to 240 Ω where the temperature coefficient of wire is 2×10^{-4} °C^{-1}.**

Sol. Given $R_1 = R_{20} = 120\,\Omega$, $R_2 = R_T = 240\,\Omega$, $t = ?$, $\alpha = 2 \times 10^{-4}$°C^{-1}, $t_1 = 20$°C, $t_2 = T = ?$

Formula used to find the temperature coefficient of resistance is

$$\alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)} \Rightarrow \alpha = \frac{R_T - R_{20}}{R_{20}(T-20)}$$

$$2 \times 10^{-4} = \frac{240-120}{120(T-20)} \Rightarrow T - 20 = \frac{120}{120 \times 2 \times 10^{-4}}$$

$$\Rightarrow \quad T = \frac{1}{2 \times 10^{-4}} + 20 = 0.5 \times 10^4 + 20 = 5020°\text{C}$$

VALIDITY AND FAILURE OF OHM'S LAW

Ohm's law is not a law of nature i.e., it is not a universal law which would apply everywhere under all conditions. Ohm's law is obeyed by metallic conductors, which accordingly are called ohmic conductors, that too at about normal working temperatures. At very high currents/voltages, even ohmic conductors do not follow this law as shown in the following figures. Semi-conductors also do not follow Ohm's law as shown in Figs. 5.13 and 5.14.

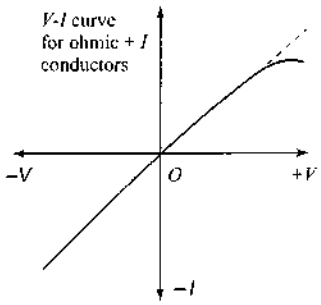


Fig. 5.13

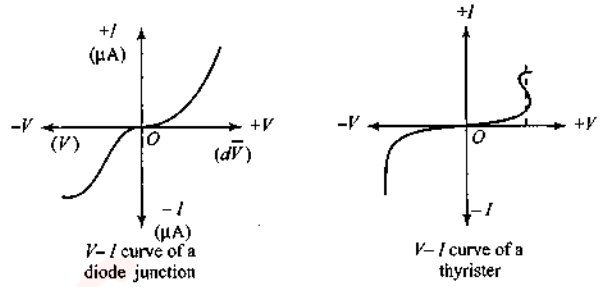


Fig. 5.14

Thus, Ohm's law is not followed in the following cases:

Materials: (i) Vacuum tubes; (ii) crystal rectifiers; (iii) transistors; (iv) thermistors, thyristors; (v) superconductors.

Conditions: (i) At very high temperatures; (ii) at very low temperatures (superconductivity); (iii) at very high potential differences.

Concept Application Exercise 5.1

1. How many electrons per second pass through a section of wire carrying a current of 0.7 A?
2. A current of 7.5 A is maintained in a wire for 45 s. In this time (a) how much charge and (b) how many electrons flow through the wire?
3. If 0.6 mol of electrons flows through a wire in 45 min, what are (a) the total charge that passes through the wire and (b) the magnitude of the current?
4. A typical copper wire might have 2×10^{21} free electrons in 1 cm of its length. Suppose that the drift speed of the electrons along the wire is 0.05 cm/s. How many electrons would pass through a given cross section of the wire each second. How large would a current be flowing in the wire?
5. A coil of wire has a resistance of 25.00 Ω at 20°C and a resistance of 25.17 Ω at 35°C. What is its temperature coefficient of resistance?
6. A metal wire of diameter 2 mm and of length 300 m has a resistance of 1.6424 Ω at 20°C and 2.415 Ω at 150°C. Find the values of α, R_0 and ρ_0, where the zero subscript refers to 0°C, and $\rho_{20°C}$. Identify the metal.
7. It is desired to make a 20 Ω coil of wire which has a zero thermal coefficient of resistance. To do this, a carbon resistor of resistance R_1 is placed in series with an iron resistor of resistance R_2. The proportions of iron and carbon are so chosen that $R_1 + R_2 = 20$ Ω for all temperatures near 20°C. How long are R_1 and R_2?
   $\alpha_{carbon} = -0.5 \times 10^{-3}$/°C, $\alpha_{iron} = 5 \times 10^{-3}$/°C
8. A resistance thermometer measures temperature with the increase in resistance of a wire of high temperature. If the wire is platinum and has a resistance of 10 Ω at 20°C and a resistance of 35 Ω in a hot furnace, what is the temperature of the furnace? ($\alpha_{platinum} = 0.0036$°C^{-1}.)
9. Though the drift velocity of electrons responsible for current in a conductor under ordinary circumstances is very small, lights in a room turn on immediately after the switch is closed. Explain why and how?

10. **a.** A steady current flows in a metallic conductor of non-uniform cross section. State which of the quantities i.e., current, current density, electric field and drift velocity remains constant?

b. A steady current passes through a cylindrical conductor. Is there an electric field inside the conductor?

(IIT-JEE, 1982)

11. A potential difference V is applied to copper wire of diameter d and length L. What will be the effect on the electron drift speed by doubling (a) voltage V; (b) length L and (c) diameter d?

12. A wire has a resistance R. What will be its resistance if (a) it is double on itself and (b) it is stretched so that (i) its length is doubled, (ii) its radius is halved.

13. The current in a wire varies with time according to the equation $i = 4 + 2t$, where i is in ampere and t is in second. Calculate the quantity of charge which has passed through a cross section of the wire during the time $t = 2$ s to $t = 6$ s.

14. The following table gives the length of three copper rods, their diameters, and the potential differences between their ends. Rank the rods according to (a) the magnitude of the electric field within them, (b) the current density within them, and (c) the drift speed of electrons through them, greatest first.

Rod	Length	Diameter	P.D.
1	L	$3d$	V
2	$2L$	d	$2V$
3	$3L$	$2d$	$2V$

15. V-I graph for a metallic wire at two different temperatures T_1 and T_2 is shown in Fig. 5.15. Which of the two temperatures T_1 and T_2 is higher and why?

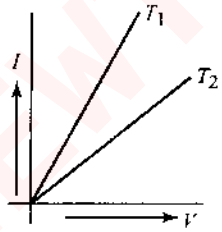


Fig. 5.15

16. The voltage–current variation of two metallic wires X and Y at constant temperature is shown in Fig. 5.16. Assuming that the wires have the same length and the same diameter, explain which of the two wires will have larger resistivity.

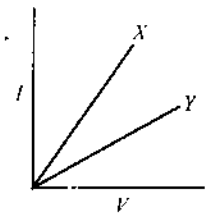


Fig. 5.16

17. A beam contains 2.0×10^8 doubly charged positive ions per cubic centimeter, all of which are moving toward north with a speed of 1.0×10^5 m/s. (a) What are the magnitude and direction of the current density $\vec{J}$? (b) Can you calculate the total current i in this ion beam? If not, what additional information is needed.

18. The current density across a cylindrical conductor of radius R varies in magnitude according to the equation $J = J_0\left(1 - \frac{r}{R}\right)$ where r is the distance from the central axis. Thus, the current density is a maximum J_0 at that axis ($r = 0$) and decreases linearly to zero at the surface ($r = R$). Calculate the current in terms of J_0 and the conductor's cross-sectional area $A = \pi R^2$.

19. Figure 5.17 shows a conductor of length l having a circular cross section.

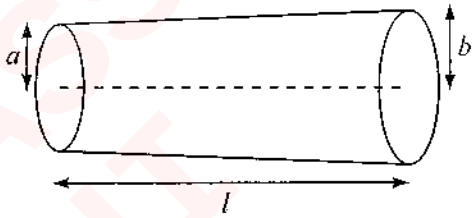


Fig. 5.17

The radius of cross section varies linearly from a to b. The resistivity of the material is ρ. Assuming that $b - a \leftrightarrow l$, find the resistance of the conductor.

20. If a copper wire is stretched to make it 0.1% longer, what is the percentage change in its resistance?

(IIT-JEE, 1978)

21. A copper wire having cross-sectional area of 0.5 mm² and a length of 0.1 m is initially at 25°C and is thermally insulated from the surrounding. If a current of 10 A is set up in this wire, (a) find the time in which the wire will start melting. The change of resistance with the temperature of the wire may be neglected. (b) What will this time be, if the length of the wire is doubled? Given for copper: resistivity = 1.6×10^{-8} Ωm, density = 9,000 kg/m³ and specific heat capacity = 0.09 cal/gm°C. Melting temperature of copper is 1075°C. **(IIT-JEE, 1979)**

ELECTROMOTIVE FORCE AND POTENTIAL DIFFERENCE

A device which can supply energy to charge carriers and thereby maintain their flow in a circuit is called a set of electromotive force (abbreviated as an e.m.f). A voltaic cell, a thermocouple and a coil rotating in a magnetic field are some common examples of the seats of e.m.f. Thus, a seat of e.m.f. is a device in which energy is converted from non-electrical to electrical form and in which charge carriers are passed on from lower potential to higher potential. The e.m.f. of such a device is measured by the rate at which the energy is converted from non-electrical to electrical form during the passage of unit charge. If ΔW = energy converted into

the electrical form when Δq amount of electricity passes through it, then e.m.f. (ε) of the device is given by $\varepsilon = (\Delta W/\Delta q)$ joule per coulomb or volt. Thus, charge carriers carrying Δq amount of charge receive $\Delta W = \varepsilon \Delta q$ joules of energy while passing through a seat of e.m.f. from lower (negative) potential plate to the higher (positive) potential plate (see Fig. 5.18).

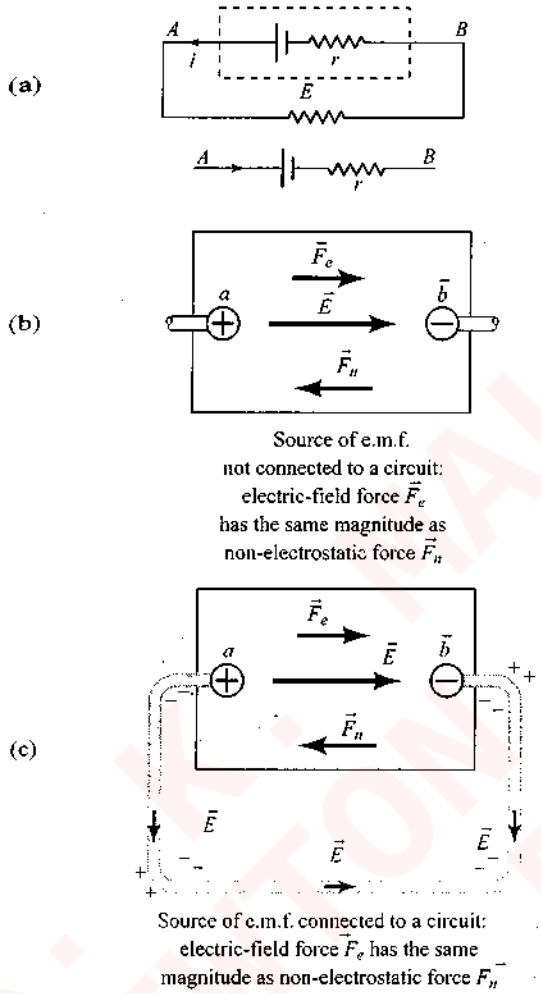


Fig. 5.18

The potential difference between any two points is defined as the energy given up by charge carriers carrying unit charge and it is, therefore, equal to the work done by the charge carriers carrying unit charge from one point to the other.

If ΔW is the energy given up by charge carriers carrying Δq charge then the potential difference between the ends of a current carrying conductor is $V = (\Delta W)/(\Delta q)$ joule per coulomb or volt, i.e.,

$$V_{ab} = \varepsilon \qquad \text{(i)}$$

Now let us make a complete circuit by connecting a wire path resistance R to the terminals of a source [as shown in Fig. 5.18(c)]. The potential difference between terminals a and b sets up an electric field within the wire; this causes current to flow around the loop from a to b, from higher to lower potential. Notice that where the wire bends, equal amounts of positive and negative charge persist on the "inside" and "outside" of the bend. These charges exert the forces that cause the current to follow the bends in the wire.

From $V = IR$ (equation of relationship between voltage, current and resistance), the potential difference between the ends of the wire in Fig. 5.18(a) is given by $V_{ab} = IR$. Combining with Eq. (i), we have

$$\varepsilon = V_{ab} = IR \qquad \text{(ii)}$$

That is, when a positive charge q flows around the circuit, the potential rise ε as it passes through the ideal source is numerically equal to the potential drop $V_{ab} = IR$ as it passes through the remainder of the circuit. Once ε and R are known, this relation determines the current in the circuit.

Difference Between e.m.f. (E) and Potential Difference (V)

As the name suggests potential difference (p.d.) between two points is the difference in their potentials. Terminal p.d. of a cell, however, has a special meaning. It means p.d. between the terminals of a cell in closed circuit, i.e., when the current is flowing through the cell. E.m.f. is the p.d. between the terminals of a cell in an open circuit, i.e., when no current flows through the cell.

INTERNAL RESISTANCE OF A CELL

Real sources in a circuit do not behave in exactly the way we have described. The potential difference across a real source in a circuit is not equal to the e.m.f. The reason is that charge moving through the material of any real source encounters resistance. We call this the internal resistance of the source, denoted by r. If this resistance behaves according to Ohm's law, r is constant, and is independent of the current I. As the current moves through r, it experiences an associated drop in the potential which is equal to Ir. Thus, when a current is flowing through a source from the negative terminal b to the positive terminal a, the potential difference V_{ab} between the terminals is $V_{ab} = \varepsilon - Ir$.

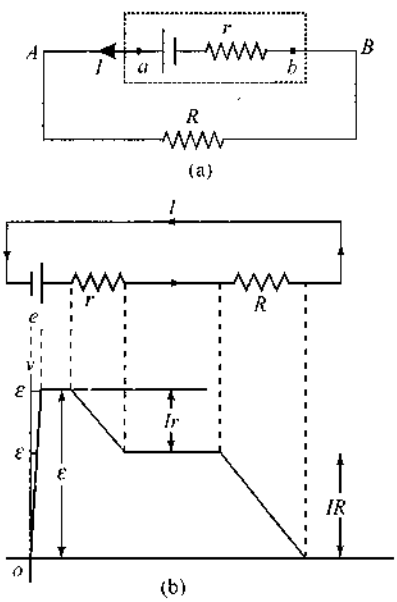


Fig. 5.19

A cell of e.m.f. ε always has some internal resistance r.

If current I is drawn from the cell, a potential drop Ir occurs inside the cell and, therefore, if the potential difference between the terminals of the battery is V, then

$$\varepsilon - V = Ir \Rightarrow V_A - V_B = \varepsilon - Ir$$

If the external resistance is R, then

$$V = IR \Rightarrow I = \left(\frac{\varepsilon}{R+r}\right)$$

Variation of the potential difference in different parts of the circuit is shown in Fig. 5.19.

COMBINATION OF RESISTANCES

Resistances in Series

If resistances are connected as shown in Fig. 5.20 such that the current flowing through them is the same, the resistances are said to be in series.

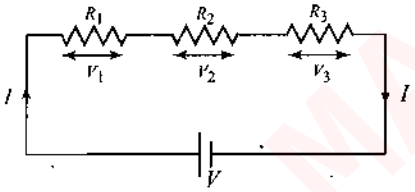


Fig. 5.20

If I = current flowing through the resistances then potential drop across each is $V_1 = IR_1$, $V_2 = IR_2$, $V_3 = IR_3$. Adding, we get

$$V_1 + V_2 + V_3 = I(R_1 + R_2 + R_3) \quad \text{(i)}$$

but $V_1 + V_2 + V_3 = V$, so we get from Eq. (i) as

$$V = I(R_1 + R_2 + R_3)$$

where V = potential difference across the combinations. Also from Ohm's law

$$V = IR \quad \text{(ii)}$$

From Eqs. (i) and (ii), we get $R = R_1 + R_2 + R_3$ where R is known as *equivalent resistance.*

Resistances in Parallel

If the resistances are connected between the same two points such that the potential drop across each resistance is the same, then the resistances are said to be in parallel (Fig. 5.21).

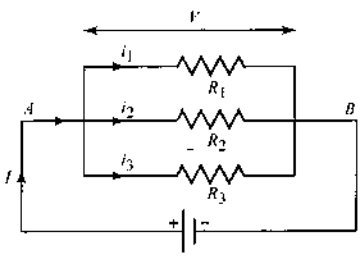


Fig. 5.21

In this case, if V is the potential difference between the points A and B, then $V = i_1R_1$, $V = i_2R_2$, and $V = i_3R_3$.

$$\Rightarrow \quad i_1 = \frac{V}{R_1}, \quad i_2 = \frac{V}{R_2}, \quad \text{and} \quad i_3 = \frac{V}{R_3}$$

Therefore, the total current flowing through the battery is

$$i = i_1 + i_2 + i_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

i.e.,

$$i = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

Also by Ohm's law, $i = V/R$, where R is the equivalent resistance between A and B. So we get $\frac{1}{R} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$.

Voltage Divider

In a series circuit, current through each resistor is the same (see Fig. 5.22).

$$V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3$$

$$I = \frac{V}{R_1 + R_2 + R_3}$$

$$V_1 = \frac{VR_1}{R_1 + R_2 + R_3}, \quad V_2 = \frac{VR_2}{R_1 + R_2 + R_3},$$

$$V_3 = \frac{VR_3}{R_1 + R_2 + R_3}$$

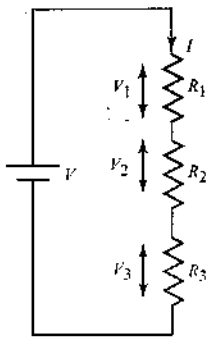


Fig. 5.22

Current Divider for Two Resistances

From Fig. 5.23, we have

$$I = I_1 + I_2 \quad \text{(i)}$$

$$I_1R_1 = I_2R_2 \quad \text{(ii)}$$

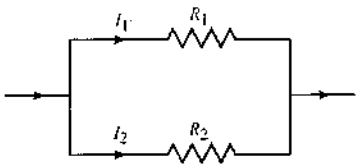


Fig. 5.23

On solving Eq. (i) and (ii), we get

$$I_1 = \left(\frac{R_2}{R_1 + R_2}\right)I; \quad I_2 = \left(\frac{R_1}{R_1 + R_2}\right)I$$

The division of current in the branches of a parallel circuit is inversely proportional to their resistances.

Current Divider for Three Resistances

The division of current in the branches of a parallel circuit is inversely proportional to their resistances (see Fig. 5.24).

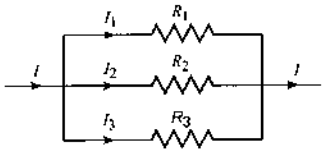


Fig. 5.24

$$I = I_1 + I_2 + I_3$$

$$I_1R_1 = I_2R_2 = I_3R_3$$

and
$$R_{eq} = \frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}$$

$$I_1 = I\left[\frac{R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}\right]$$

$$I_2 = I\left[\frac{R_1R_2}{R_1R_2 + R_2R_3 + R_3R_1}\right]$$

$$I_3 = I\left[\frac{R_1R_2}{R_1R_2 + R_2R_3 + R_3R_1}\right]$$

It is easy to remember the expressions for I_1, I_2 and I_3. Notice which resistance is missing in the numerator.

In all parts in Fig. 5.25, the resistances R_1, R_2 and R_3 are connected in parallel between points M and N.

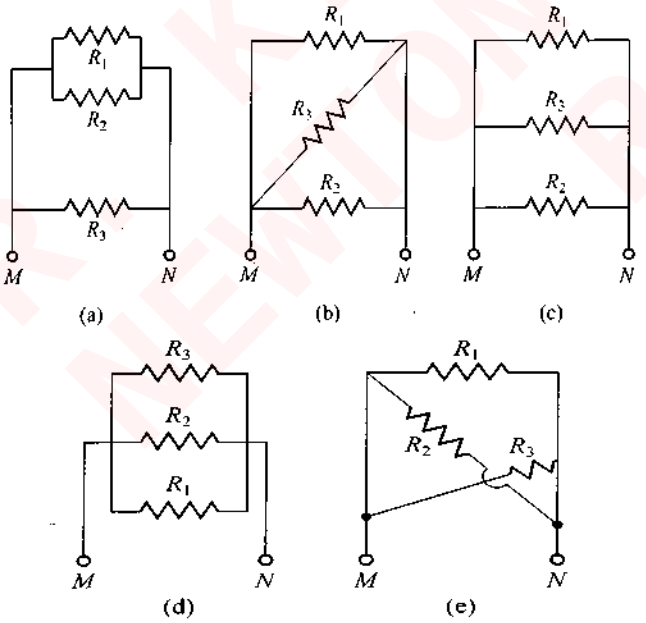


Fig. 5.25

Calculation of Effective Resistance

Method of Successive Reduction

It is the most common method to determine the equivalent resistance. This method is applicable only when we are able to identify resistances in series or in parallel. The method is based on the simplification of the circuit by successive reduction of the series and parallel combinations.

To calculate the equivalent resistance between points M and N, the network shown in Fig. 5.26 may be successively reduced as described step by step.

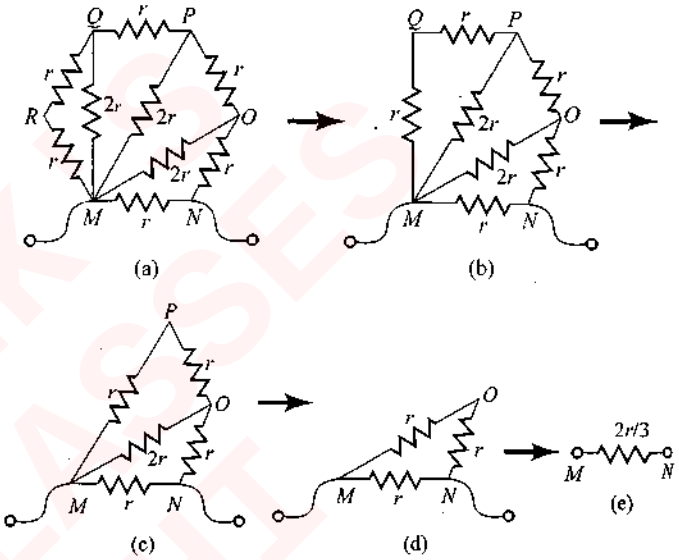


Fig. 5.26

Method of Equipotential Reduction

This method is based on identifying the points of equal potential and connecting them. By doing so the electric resistance network reduces to an arrangement of series and parallel combinations that can be easily solved by the successive reduction method.

Now the question arises: how to identify the points of same potential?

In this section, we will discuss the method to calculate the equivalent resistance and capacitance using symmetry techniques.

There are various kinds of symmetry considerations. The most common are:

- parallel axis of symmetry,
- perpendicular axis of symmetry,
- shifted symmetry or shifted asymmetry, and
- path symmetry.

We will discuss each one of these in the following sections:

Parallel Axis of Symmetry: It is along the direction of current flow.

Let us discuss this concept by an example.

In the circuit shown in Fig. 5.27 (a), even though the resistors 1 and 2 do not appear to be connected in parallel, they can be treated as parallel, why? To explain this, we have to use the concept of symmetry. Note that the circuit is symmetric about the line MN. Therefore, all characteristics such as potential and current should also be symmetrical.

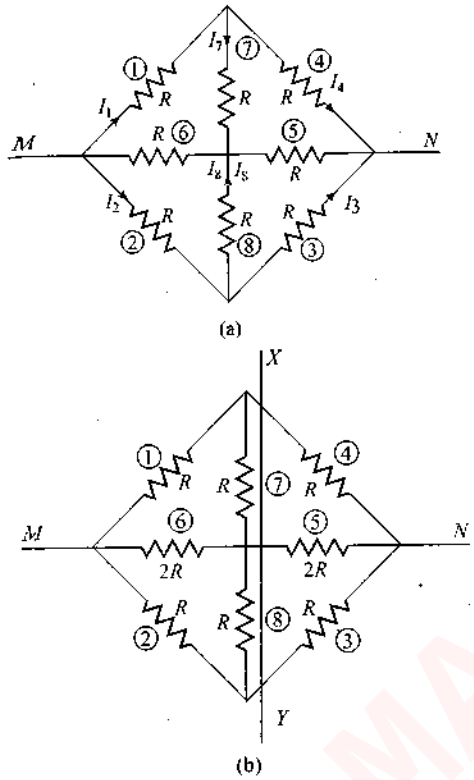


Fig. 5.27

From this, it means that current in 1 (I_1) = current in 2 (I_2), current in 3 (I_3) = current in 4 (I_4), current in 7 (I_7) = current in 8 (I_8), hence potential difference across 1 is equal to the potential difference across 2.

Note that the conditions for 1 and 2 to be in parallel are satisfied. Hence, we can consider them to be in parallel; however, if one of the resistors had a different value, we cannot use this method.

Perpendicular Axis of Symmetry: It is perpendicular to the direction of flow of current.

Consider the circuit diagram given in Fig. 5.27 (b). The circuit has perpendicular axis of symmetry about *XY*.

The perpendicular axis of symmetry means that the circuit diagram is symmetric except for the fact that the input and output are reversed. That is only the flow of current will not be a mirror image about this particular axis. For example, in the above case, element 1 and 4 are symmetric about *XY*, but the current flow condition is not a mirror image. The current flow condition is in the same direction. This implies that current into 6 = current out of 5, current into 1 = current out of 4, etc.

Perpendicular axis of symmetry is a very powerful principle. In fact, by just looking at the circuit, we can easily say that since the circuit has perpendicular axis of symmetry about *XY*, no current will flow in elements 7 and 8.

Therefore, we can ignore these two elements completely. In some cases, we may not be able to use symmetry to simplify the circuit, but we can find out some of the characteristics of current /potential based on symmetry. We should always look out for these characteristics and use them as much as possible.

Illustration 5.14 **In the network shown in Fig. 5.28 find the equivalent resistance across the points *M* and *M′*.**

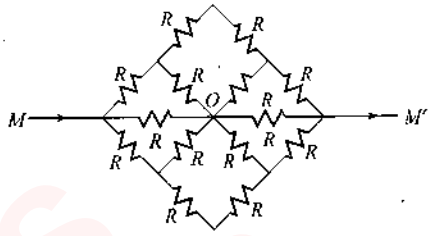


Fig. 5.28

Sol.

i. The axis *MM′* is the parallel axis of symmetry, and the axis *NN′* is the perpendicular axis of symmetry.

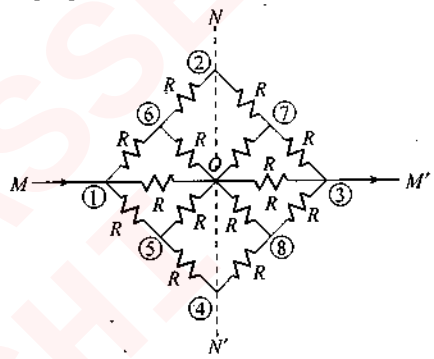


Fig. 5.29

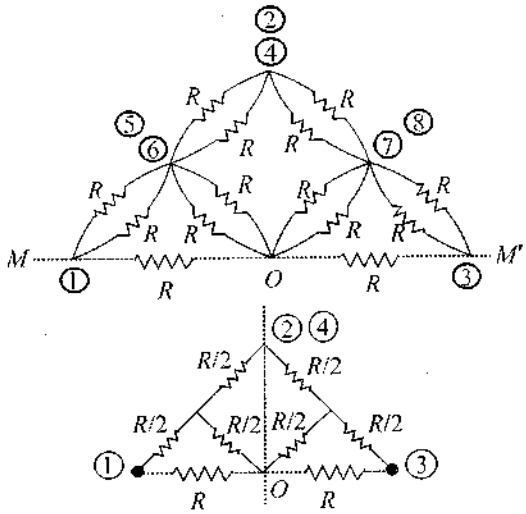
ii. Points lying on the perpendicular axis of symmetry may have same potential. In the given network, points 2, 0 and 4 are at the same potential.

iii. Points lying on the parallel axis of symmetry can never have the same potential.

iv. The network can be folded about the parallel axis of symmetry, and the overlapping nodes have same potential.

Thus, as shown in Fig. 5.29, the following points have same potential, (5 and 6), (2, 0, 4), (7 and 8).

After folding the network about the axis *MM′*, the circuit may be simplified by using the method of successive reduction (Fig. 5.30).



(*Contd.*)

Derivation:

Dividing Eq. (ii) by Eq. (i), we have

$$\frac{R_2}{R_1} = \frac{1+\alpha t_2}{1+\alpha t_1} \Rightarrow R_2 + R_2\alpha t_1 = R_1 + R_1\alpha t_2$$

$$R_2 - R_1 = \alpha(R_1 t_2 - R_2 t_1) \Rightarrow \alpha = \frac{R_2 - R_1}{R_1 t_2 - R_2 t_1}$$

iii. $\alpha = \dfrac{R_t - R_0}{R_0 t}$; where R_t = resistance at t °C and R_0 = resistance at 0°C

Derivation: $R_t = R_0(1+\alpha t) \Rightarrow R_t - R_0 = R_0\alpha t$

$$\Rightarrow \quad \alpha = \frac{R_t - R_0}{R_0 t}$$

Illustration 5.12 **A copper coil has a resistance of 20.0 Ω at 0°C and a resistance of 26.4 Ω at 80°C. Find out the temperature coefficient of resistance of copper.**

Sol. $R_{80°C} = R_{0°C}[1 + \alpha\,\Delta T]$

$$\Rightarrow \quad 26.4 = 20.0[1 + \alpha \times (80-0)] \Rightarrow \frac{26.4}{20} = 1 + 80\,\alpha$$

On solving, we get $\alpha = 4 \times 10^{-3}$°C^{-1}.

Illustration 5.13 **A metallic wire has a resistance of 120 Ω at 20° C. Find the temperature at which the resistance of same metallic wire rises to 240 Ω where the temperature coefficient of wire is 2×10^{-4} °C^{-1}.**

Sol. Given $R_t = R_{20} = 120\,\Omega$, $R_2 = R_T = 240\,\Omega$, $t = ?$, $\alpha = 2 \times 10^{-4}$°C^{-1}, $t_1 = 20$°C, $t_2 = T = ?$

Formula used to find the temperature coefficient of resistance is

$$\alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)} \Rightarrow \alpha = \frac{R_T - R_{20}}{R_{20}(T-20)}$$

$$2 \times 10^{-4} = \frac{240-120}{120(T-20)} \Rightarrow T - 20 = \frac{120}{120 \times 2 \times 10^{-4}}$$

$$\Rightarrow \quad T = \frac{1}{2 \times 10^{-4}} + 20 = 0.5 \times 10^4 + 20 = 5020°\,C$$

VALIDITY AND FAILURE OF OHM'S LAW

Ohm's law is not a law of nature i.e., it is not a universal law which would apply everywhere under all conditions. Ohm's law is obeyed by metallic conductors, which accordingly are called ohmic conductors, that too at about normal working temperatures. At very high currents/voltages, even ohmic conductors do not follow this law as shown in the following figures. Semi-conductors also do not follow Ohm's law as shown in Figs. 5.13 and 5.14.

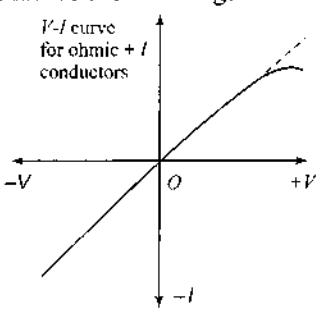


Fig. 5.13

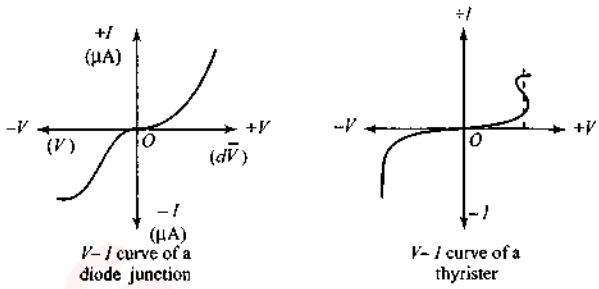


Fig. 5.14

Thus, Ohm's law is not followed in the following cases:

Materials: (i) Vacuum tubes; (ii) crystal rectifiers; (iii) transistors; (iv) thermistors, thyristors; (v) superconductors.

Conditions: (i) At very high temperatures; (ii) at very low temperatures (superconductivity); (iii) at very high potential differences.

Concept Application Exercise 5.1

1. How many electrons per second pass through a section of wire carrying a current of 0.7 A?
2. A current of 7.5 A is maintained in a wire for 45 s. In this time (a) how much charge and (b) how many electrons flow through the wire?
3. If 0.6 mol of electrons flows through a wire in 45 min, what are (a) the total charge that passes through the wire and (b) the magnitude of the current?
4. A typical copper wire might have 2×10^{21} free electrons in 1 cm of its length. Suppose that the drift speed of the electrons along the wire is 0.05 cm/s. How many electrons would pass through a given cross section of the wire each second. How large would a current be flowing in the wire?
5. A coil of wire has a resistance of 25.00 Ω at 20°C and a resistance of 25.17 Ω at 35°C. What is its temperature coefficient of resistance?
6. A metal wire of diameter 2 mm and of length 300 m has a resistance of 1.6424 Ω at 20°C and 2.415 Ω at 150°C. Find the values of α, R_0 and ρ_0, where the zero subscript refers to 0°C, and $\rho_{20°C}$. Identify the metal.
7. It is desired to make a 20 Ω coil of wire which has a zero thermal coefficient of resistance. To do this, a carbon resistor of resistance R_1 is placed in series with an iron resistor of resistance R_2. The proportions of iron and carbon are so chosen that $R_1 + R_2 = 20$ Ω for all temperatures near 20°C. How long are R_1 and R_2?
   $\alpha_{carbon} = -0.5 \times 10^{-3}$/°C, $\alpha_{iron} = 5 \times 10^{-3}$/°C
8. A resistance thermometer measures temperature with the increase in resistance of a wire of high temperature. If the wire is platinum and has a resistance of 10 Ω at 20°C and a resistance of 35 Ω in a hot furnace, what is the temperature of the furnace? ($\alpha_{platinum} = 0.0036$°C^{-1}.)
9. Though the drift velocity of electrons responsible for current in a conductor under ordinary circumstances is very small, lights in a room turn on immediately after the switch is closed. Explain why and how?

10. **a.** A steady current flows in a metallic conductor of non-uniform cross section. State which of the quantities i.e., current, current density, electric field and drift velocity remains constant?

b. A steady current passes through a cylindrical conductor. Is there an electric field inside the conductor?

(IIT-JEE, 1982)

11. A potential difference V is applied to copper wire of diameter d and length L. What will be the effect on the electron drift speed by doubling (a) voltage V; (b) length L and (c) diameter d?

12. A wire has a resistance R. What will be its resistance if (a) it is double on itself and (b) it is stretched so that (i) its length is doubled, (ii) its radius is halved.

13. The current in a wire varies with time according to the equation $i = 4 + 2t$, where i is in ampere and t is in second. Calculate the quantity of charge which has passed through a cross section of the wire during the time $t = 2$ s to $t = 6$ s.

14. The following table gives the length of three copper rods, their diameters, and the potential differences between their ends. Rank the rods according to (a) the magnitude of the electric field within them, (b) the current density within them, and (c) the drift speed of electrons through them, greatest first.

Rod	Length	Diameter	P.D.
1	L	$3d$	V
2	$2L$	d	$2V$
3	$3L$	$2d$	$2V$

15. V-I graph for a metallic wire at two different temperatures T_1 and T_2 is shown in Fig. 5.15. Which of the two temperatures T_1 and T_2 is higher and why?

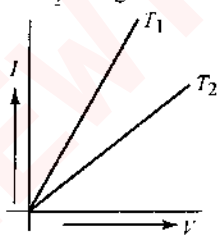


Fig. 5.15

16. The voltage–current variation of two metallic wires X and Y at constant temperature is shown in Fig. 5.16. Assuming that the wires have the same length and the same diameter, explain which of the two wires will have larger resistivity.

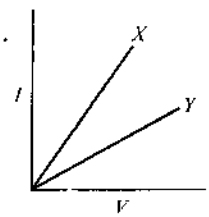


Fig. 5.16

17. A beam contains 2.0×10^8 doubly charged positive ions per cubic centimeter, all of which are moving toward north with a speed of 1.0×10^5 m/s. (a) What are the magnitude and direction of the current density $\vec{J}$? (b) Can you calculate the total current i in this ion beam? If not, what additional information is needed.

18. The current density across a cylindrical conductor of radius R varies in magnitude according to the equation $J = J_0\left(1-\frac{r}{R}\right)$ where r is the distance from the central axis. Thus, the current density is a maximum J_0 at that axis ($r = 0$) and decreases linearly to zero at the surface ($r = R$). Calculate the current in terms of J_0 and the conductor's cross-sectional area $A = \pi R^2$.

19. Figure 5.17 shows a conductor of length l having a circular cross section.

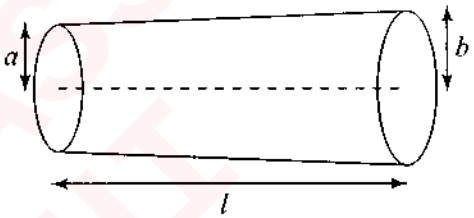


Fig. 5.17

The radius of cross section varies linearly from a to b. The resistivity of the material is ρ. Assuming that $b-a \leftrightarrow l$, find the resistance of the conductor.

20. If a copper wire is stretched to make it 0.1% longer, what is the percentage change in its resistance?

(IIT-JEE, 1978)

21. A copper wire having cross-sectional area of 0.5 mm² and a length of 0.1 m is initially at 25°C and is thermally insulated from the surrounding. If a current of 10 A is set up in this wire, (a) find the time in which the wire will start melting. The change of resistance with the temperature of the wire may be neglected. (b) What will this time be, if the length of the wire is doubled? Given for copper: resistivity = 1.6×10^{-8} Ωm, density = 9,000 kg/m³ and specific heat capacity = 0.09 cal/gm°C. Melting temperature of copper is 1075°C. **(IIT-JEE, 1979)**

ELECTROMOTIVE FORCE AND POTENTIAL DIFFERENCE

A device which can supply energy to charge carriers and thereby maintain their flow in a circuit is called a set of electromotive force (abbreviated as an e.m.f). A voltaic cell, a thermocouple and a coil rotating in a magnetic field are some common examples of the seats of e.m.f. Thus, a seat of e.m.f. is a device in which energy is converted from non-electrical to electrical form and in which charge carriers are passed on from lower potential to higher potential. The e.m.f. of such a device is measured by the rate at which the energy is converted from non-electrical to electrical form during the passage of unit charge. If ΔW = energy converted into

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the electrical form when Δq amount of electricity passes through it, then e.m.f. (ε) of the device is given by $\varepsilon = (\Delta W/\Delta q)$ joule per coulomb or volt. Thus, charge carriers carrying Δq amount of charge receive $\Delta W = \varepsilon \Delta q$ joules of energy while passing through a seat of e.m.f. from lower (negative) potential plate to the higher (positive) potential plate (see Fig. 5.18).

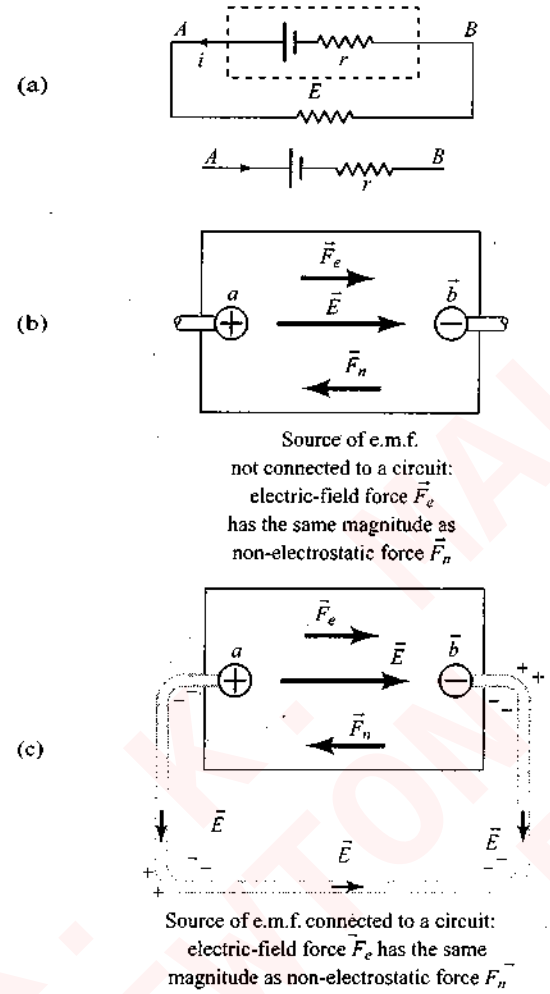

Fig. 5.18

The potential difference between any two points is defined as the energy given up by charge carriers carrying unit charge and it is, therefore, equal to the work done by the charge carriers carrying unit charge from one point to the other.

If ΔW is the energy given up by charge carriers carrying Δq charge then the potential difference between the ends of a current carrying conductor is $V = (\Delta W)/(\Delta q)$ joule per coulomb or volt, i.e.,

$$V_{ab} = \varepsilon \qquad \text{(i)}$$

Now let us make a complete circuit by connecting a wire path resistance R to the terminals of a source [as shown in Fig. 5.18(c)]. The potential difference between terminals a and b sets up an electric field within the wire; this causes current to flow around the loop from a to b, from higher to lower potential. Notice that where the wire bends, equal amounts of positive and negative charge persist on the "inside" and "outside" of the bend. These charges exert the forces that cause the current to follow the bends in the wire.

From $V = IR$ (equation of relationship between voltage, current and resistance), the potential difference between the ends of the wire in Fig. 5.18(a) is given by $V_{ab} = IR$. Combining with Eq. (i), we have

$$\varepsilon = V_{ab} = IR \qquad \text{(ii)}$$

That is, when a positive charge q flows around the circuit, the potential rise ε as it passes through the ideal source is numerically equal to the potential drop $V_{ab} = IR$ as it passes through the remainder of the circuit. Once ε and R are known, this relation determines the current in the circuit.

Difference Between e.m.f. (E) and Potential Difference (V)

As the name suggests potential difference (p.d.) between two points is the difference in their potentials. Terminal p.d. of a cell, however, has a special meaning. It means p.d. between the terminals of a cell in closed circuit, i.e., when the current is flowing through the cell. E.m.f. is the p.d. between the terminals of a cell in an open circuit, i.e., when no current flows through the cell.

INTERNAL RESISTANCE OF A CELL

Real sources in a circuit do not behave in exactly the way we have described. The potential difference across a real source in a circuit is not equal to the e.m.f. The reason is that charge moving through the material of any real source encounters resistance. We call this the internal resistance of the source, denoted by r. If this resistance behaves according to Ohm's law, r is constant, and is independent of the current I. As the current moves through r, it experiences an associated drop in the potential which is equal to Ir. Thus, when a current is flowing through a source from the negative terminal b to the positive terminal a, the potential difference V_{ab} between the terminals is $V_{ab} = \varepsilon - Ir$.

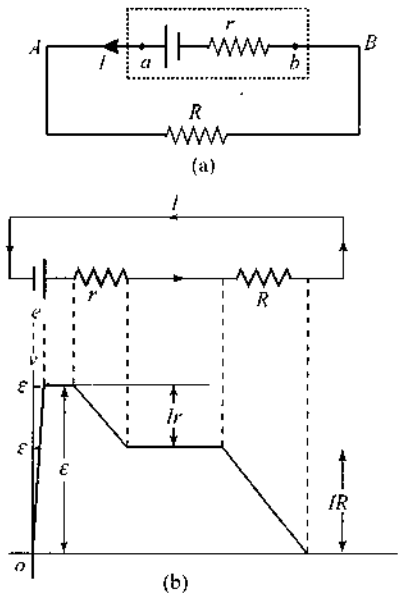

Fig. 5.19

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A cell of e.m.f. ε always has some internal resistance r.

If current I is drawn from the cell, a potential drop Ir occurs inside the cell and, therefore, if the potential difference between the terminals of the battery is V, then

$$\varepsilon - V = Ir \Rightarrow V_A - V_B = \varepsilon - Ir$$

If the external resistance is R, then

$$V = IR \Rightarrow I = \left(\frac{\varepsilon}{R+r}\right)$$

Variation of the potential difference in different parts of the circuit is shown in Fig. 5.19.

COMBINATION OF RESISTANCES

Resistances in Series

If resistances are connected as shown in Fig. 5.20 such that the current flowing through them is the same, the resistances are said to be in series.

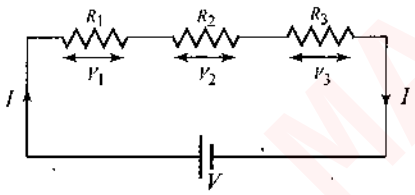


Fig. 5.20

If I = current flowing through the resistances then potential drop across each is $V_1 = IR_1$, $V_2 = IR_2$, $V_3 = IR_3$. Adding, we get

$$V_1 + V_2 + V_3 = I(R_1 + R_2 + R_3) \quad \text{(i)}$$

but $V_1 + V_2 + V_3 = V$, so we get from Eq. (i) as

$$V = I(R_1 + R_2 + R_3)$$

where V = potential difference across the combinations. Also from Ohm's law

$$V = IR \quad \text{(ii)}$$

From Eqs. (i) and (ii), we get $R = R_1 + R_2 + R_3$ where R is known as *equivalent resistance*.

Resistances in Parallel

If the resistances are connected between the same two points such that the potential drop across each resistance is the same, then the resistances are said to be in parallel (Fig. 5.21).

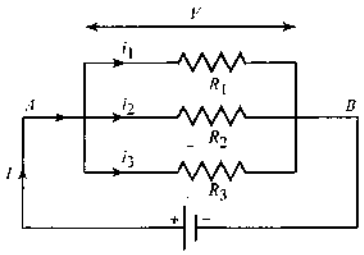


Fig. 5.21

In this case, if V is the potential difference between the points A and B, then $V = i_1R_1$, $V = i_2R_2$, and $V = i_3R_3$.

$$\Rightarrow \quad i_1 = \frac{V}{R_1}, \quad i_2 = \frac{V}{R_2}, \quad \text{and} \quad i_3 = \frac{V}{R_3}$$

Therefore, the total current flowing through the battery is

$$i = i_1 + i_2 + i_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

i.e.,

$$i = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

Also by Ohm's law, $i = V/R$, where R is the equivalent resistance between A and B. So we get $\frac{1}{R} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$.

Voltage Divider

In a series circuit, current through each resistor is the same (see Fig. 5.22).

$$V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3$$

$$I = \frac{V}{R_1 + R_2 + R_3}$$

$$V_1 = \frac{VR_1}{R_1 + R_2 + R_3}, \quad V_2 = \frac{VR_2}{R_1 + R_2 + R_3},$$

$$V_3 = \frac{VR_3}{R_1 + R_2 + R_3}$$

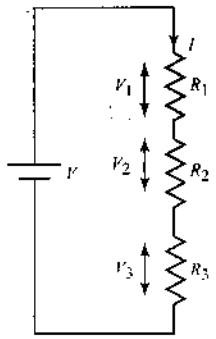


Fig. 5.22

Current Divider for Two Resistances

From Fig. 5.23, we have

$$I = I_1 + I_2 \quad \text{(i)}$$

$$I_1R_1 = I_2R_2 \quad \text{(ii)}$$

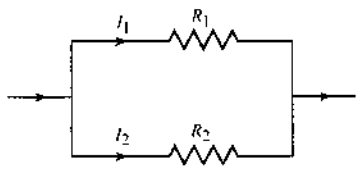


Fig. 5.23

On solving Eq. (i) and (ii), we get

$$I_1 = \left(\frac{R_2}{R_1 + R_2}\right)I; \quad I_2 = \left(\frac{R_1}{R_1 + R_2}\right)I$$

The division of current in the branches of a parallel circuit is inversely proportional to their resistances.

Current Divider for Three Resistances

The division of current in the branches of a parallel circuit is inversely proportional to their resistances (see Fig. 5.24).

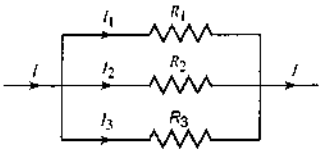


Fig. 5.24

$$I = I_1 + I_2 + I_3$$

$$I_1R_1 = I_2R_2 = I_3R_3$$

and
$$R_{eq} = \frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}$$

$$I_1 = I\left[\frac{R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}\right]$$

$$I_2 = I\left[\frac{R_1R_2}{R_1R_2 + R_2R_3 + R_3R_1}\right]$$

$$I_3 = I\left[\frac{R_1R_2}{R_1R_2 + R_2R_3 + R_3R_1}\right]$$

It is easy to remember the expressions for I_1, I_2 and I_3. Notice which resistance is missing in the numerator.

In all parts in Fig. 5.25, the resistances R_1, R_2 and R_3 are connected in parallel between points M and N.

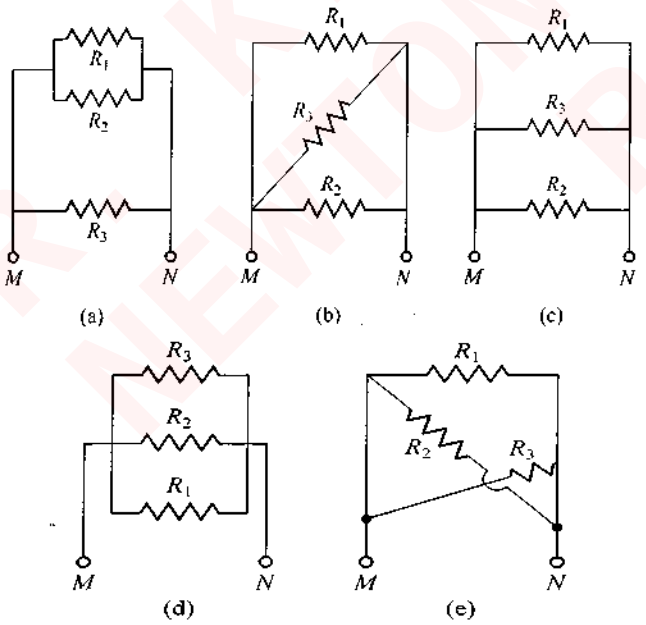


Fig. 5.25

Calculation of Effective Resistance

Method of Successive Reduction

It is the most common method to determine the equivalent resistance. This method is applicable only when we are able to identify resistances in series or in parallel. The method is based on the simplification of the circuit by successive reduction of the series and parallel combinations.

To calculate the equivalent resistance between points M and N, the network shown in Fig. 5.26 may be successively reduced as described step by step.

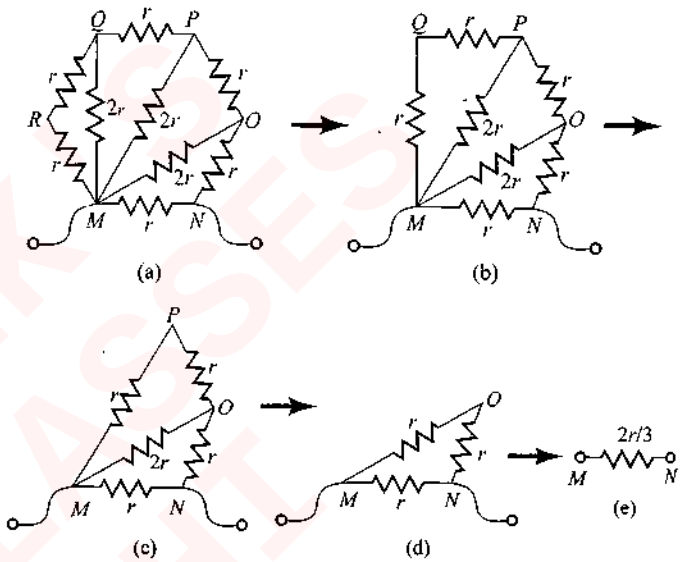


Fig. 5.26

Method of Equipotential Reduction

This method is based on identifying the points of equal potential and connecting them. By doing so the electric resistance network reduces to an arrangement of series and parallel combinations that can be easily solved by the successive reduction method.

Now the question arises: how to identify the points of same potential?

In this section, we will discuss the method to calculate the equivalent resistance and capacitance using symmetry techniques.

There are various kinds of symmetry considerations. The most common are:

- parallel axis of symmetry,
- perpendicular axis of symmetry,
- shifted symmetry or shifted asymmetry, and
- path symmetry.

We will discuss each one of these in the following sections:

Parallel Axis of Symmetry: It is along the direction of current flow.

Let us discuss this concept by an example.

In the circuit shown in Fig. 5.27 (a), even though the resistors 1 and 2 do not appear to be connected in parallel, they can be treated as parallel, why? To explain this, we have to use the concept of symmetry. Note that the circuit is symmetric about the line MN. Therefore, all characteristics such as potential and current should also be symmetrical.

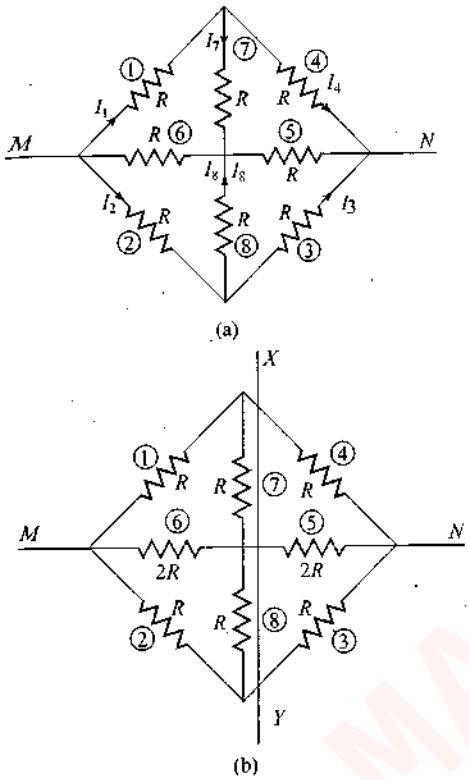


Fig. 5.27

From this, it means that current in 1 (I_1) = current in 2 (I_2), current in 3 (I_3) = current in 4 (I_4), current in 7 (I_7) = current in 8 (I_8), hence potential difference across 1 is equal to the potential difference across 2.

Note that the conditions for 1 and 2 to be in parallel are satisfied. Hence, we can consider them to be in parallel; however, if one of the resistors had a different value, we cannot use this method.

Perpendicular Axis of Symmetry: It is perpendicular to the direction of flow of current.

Consider the circuit diagram given in Fig. 5.27 (b). The circuit has perpendicular axis of symmetry about XY.

The perpendicular axis of symmetry means that the circuit diagram is symmetric except for the fact that the input and output are reversed. That is only the flow of current will not be a mirror image about this particular axis. For example, in the above case, element 1 and 4 are symmetric about XY, but the current flow condition is not a mirror image. The current flow condition is in the same direction. This implies that current into 6 = current out of 5, current into 1 = current out of 4, etc.

Perpendicular axis of symmetry is a very powerful principle. In fact, by just looking at the circuit, we can easily say that since the circuit has perpendicular axis of symmetry about XY, no current will flow in elements 7 and 8.

Therefore, we can ignore these two elements completely. In some cases, we may not be able to use symmetry to simplify the circuit, but we can find out some of the characteristics of current /potential based on symmetry. We should always look out for these characteristics and use them as much as possible.

Illustration 5.14 **In the network shown in Fig. 5.28 find the equivalent resistance across the points M and M'.**

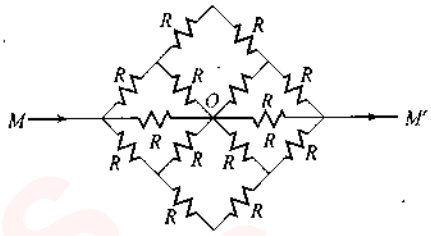


Fig. 5.28

Sol.

i. The axis MM' is the parallel axis of symmetry, and the axis NN' is the perpendicular axis of symmetry.

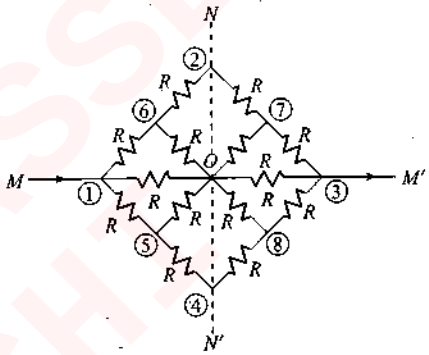


Fig. 5.29

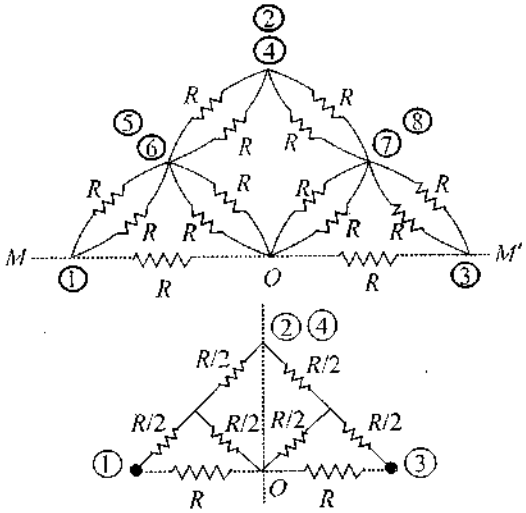
ii. Points lying on the perpendicular axis of symmetry may have same potential. In the given network, points 2, 0 and 4 are at the same potential.

iii. Points lying on the parallel axis of symmetry can never have the same potential.

iv. The network can be folded about the parallel axis of symmetry, and the overlapping nodes have same potential.

Thus, as shown in Fig. 5.29, the following points have same potential, (5 and 6), (2, 0, 4), (7 and 8).

After folding the network about the axis MM', the circuit may be simplified by using the method of successive reduction (Fig. 5.30).



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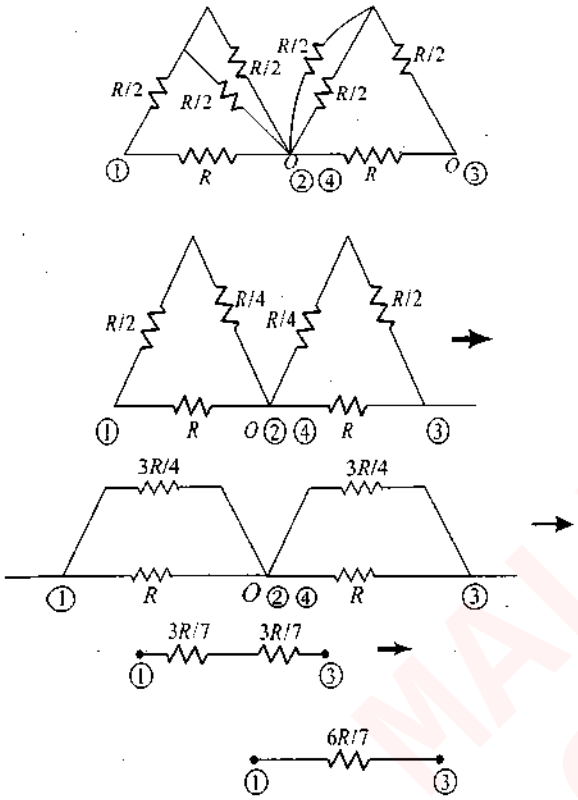
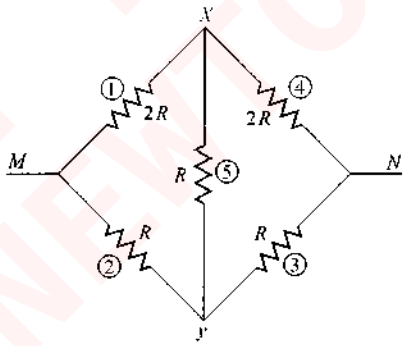


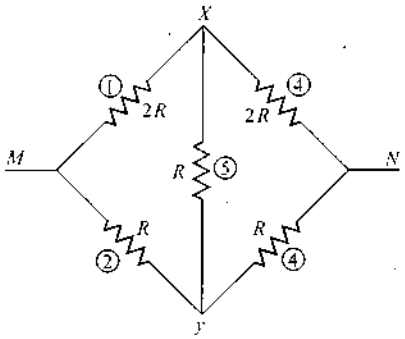
Fig. 5.30

Shifted Symmetry

Shifted symmetry is the same as the parallel axis of symmetry and the perpendicular axis of symmetry principles, except that the symmetry is shifted. [Figs. 5.31(a) and (b)]



(a)



(b)

Fig. 5.31

Consider the following situation:

In this case, the system has a perpendicular axis of symmetry about MN, therefore, we can say that current in 1 must be equal to the current out of 4.

In Fig. 5.31 (a), if we interchange the positions of 3 and 4 the diagram still has perpendicular axis of symmetry of xy, but the positions have been interchanged. Now we can say that the current in 1 = current in 4.

Path Symmetry

Path symmetry is also very powerful method that one can use. In case of path symmetry: *"If all paths from one point to another point have the same configuration of resistance or capacitance, then the charge or current into the beginning of the path must be the same"*.

Illustration 5.15 **Consider a more complex example where you have resistors on all edges of a cube. The resistors are all the same. Then find the equivalent resistance between the corners A and B as shown in Fig. 5.32.**

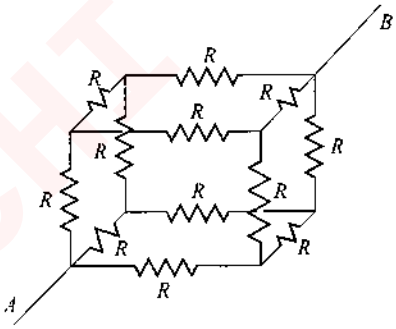


Fig. 5.32

Sol. Let us number the corners as 1, 2, 3, … and so on as shown in Fig. 5.33.

a (1 – 8 – 7 – 6), b(1 – 2 – 7 – 6), c(1 – 8 – 5 – 6), d(1 – 2 –3 – 6), e(1 – 4 – 5 – 6), f (1 – 4 – 3 – 6)

Now for each of these paths, we have identical resistances. Let the current entering at point A be I. The current will be distributed equally among all the three parts. Therefore, we can say that the current is

$$I_{12} = I_{14} = I_{18} = I/3$$

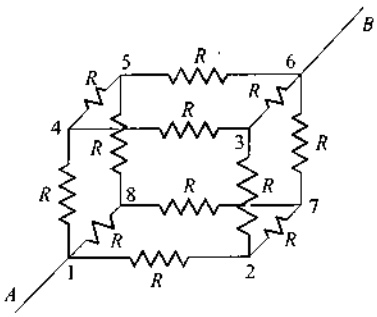


Fig. 5.33

Now look at all paths from 2 to 6. We have the following paths g(2 – 7 – 6) and h(2 – 3 – 6).

Since they also have the same resistances, we can assume that current in 2—3 and current in 2—7 must be same. Therefore,

$$I_{27} = I_{23} = \frac{1}{2}\left(\frac{I}{3}\right) = \frac{I}{6}$$

Similarly, we can say that $I_{43} = I_{45} = I_{85} = I_{87} = I/6$

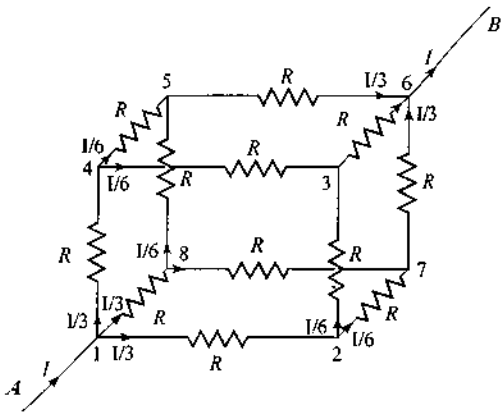


Fig. 5.34

Therefore, to calculate the potential difference across 1—6 (see Fig. 5.34) the path A_{1276}, we have

$$V_1 - \frac{I}{3}R - \frac{I}{6}R - \frac{I}{3}R = V_6 \Rightarrow V_1 - V_6 = I\left[\frac{R}{3} + \frac{R}{6} + \frac{R}{3}\right] = \frac{5}{6}IR$$

Hence, $\dfrac{(V_1 - V_6)}{I} = \dfrac{5}{6}R$

Therefore, the equivalent resistance is $\dfrac{5}{6}R$.

Illustration 5.16 **In the given circuits [Figs. 5.35 (a) and (b)], calculate the resistance between the points MM'.**

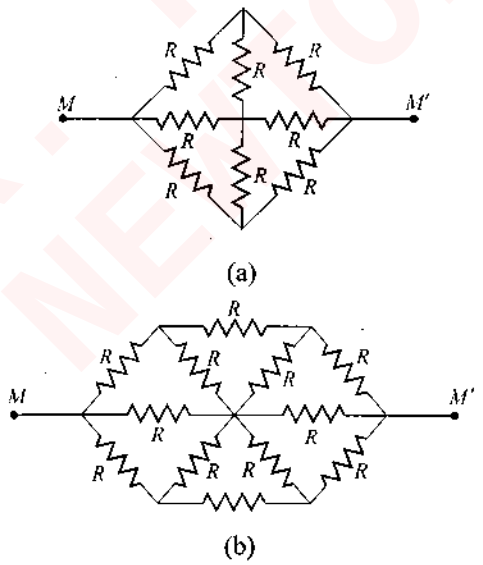


Fig. 5.35

Sol.

1. When a circuit is symmetrical about a line ac (by symmetry we mean that the two parts are mirror images of each others), then the potential and current must also be symmetrical [see Fig. 5.36 (a)]. Therefore, current in ab and ad is the same. Current in dc and bc is also the same. Potentials of the points b, e and d are same. The equivalent circuit is redrawn, the equivalent resistance is (2/3) R. Note that there is no current in branches be and ed.

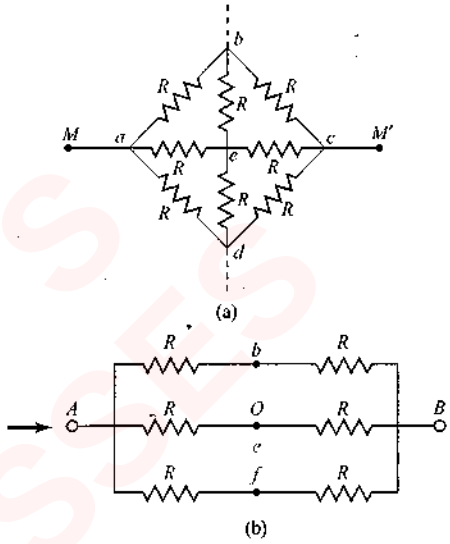


Fig. 5.36

Another symmetry is visible along the line bd. The current flow is not a mirror image in branches ab and bc because the flow is in the same direction. This is called asymmetric condition. The special thing about this asymmetry is that incoming current at b is equal to outgoing current, similar situation exists at e and d also. Thus, resistors in branches be and de are ineffective.

2. In Fig. 5.37, there is asymmetry along line xy. The current reaching O (i_2) is equal to the outgoing current that means there is no mingling of current from upper branch and lower branch into the middle branch.

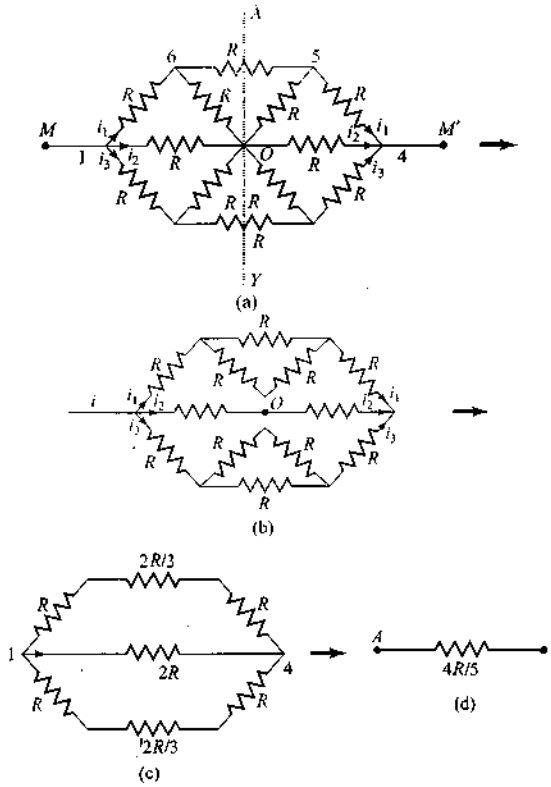


Fig. 5.37

The resulting circuit is simple enough, the equivalent resistance is $4R/5$.

Illustration 5.17 **In Fig. 5.38 (a), the resistances are connected as shown. Determine the equivalent resistance between points A and D.**

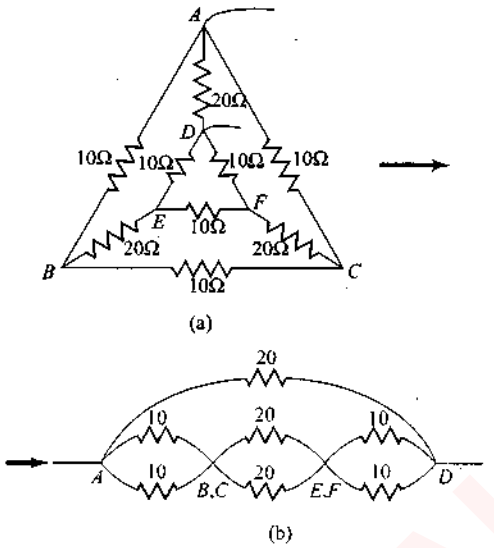


Fig. 5.38

Sol. Points B and C, and E and F are at the same potential, so the circuit can be redrawn as shown in the figure. Thus, the equivalent resistance is 10 Ω. There exists parallel axis of symmetry. The points across the parallel axis of symmetry can be treated as equipotenital points.

KIRCHHOFF'S LAW: KIRCHHOFF'S LAWS FOR ELECTRICAL NETWORKS

As indicated in the preceding section, some simple circuits can be analyzed using $\Delta V = IR$ and the rules for series and parallel combinations of resistors. Resistors, however, can be connected so that the circuits formed cannot be reduced to a single equivalent resistor. Consider the circuit in Fig. 5.39, for example. If either battery were removed from this circuit, the resistors could be combined with the simple, either series or parallel combination. With both the batteries present, however, that cannot be done.

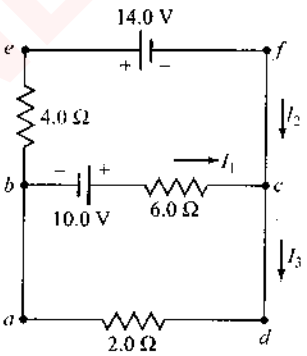


Fig. 5.39

The procedure for analyzing such circuits is greatly simplified by the use of two simple rules called Kirchhoff's rules:

- At any junction, the algebraic sum of the currents must be equal to zero. This rule is often referred to as the junction rule. In Fig. 5.39, there are junctions at b and c.
- The sum of the potential differences across each element around any closed circuit loop must be zero. This rule is usually called the loop rule. In Fig. 5.39, we can identify three loops: *abcda*, *aefda* and befcb.

Kirchhoff's rules are generally used to determine the current in each element in the circuit. By using these rules, we first draw the circuit diagram and assume a direction for the current in each device of the circuit. We draw an arrow representing that direction next to the device and assign a symbol to each independent current, such as I_1, I_2, and so on. Figure 5.39 shows the three different currents that exist in this circuit. Keep in mind that currents in devices connected in series are the same, so the currents in these devices will have the same assigned symbol.

The junction rule is a statement of conservation of charge. The amount of charge that enters a given point in a circuit in a time interval must also leave that point in the same time interval because the charge cannot build up or disappear at a point. Current with a direction into the junction is entered into the junction rule as $+I$, whereas current with a direction out of a junction is entered as $-I$. If we apply the rule to the junction, we have $I_1 + I_2 - I_3 = 0$.

The loop rule is equivalent to the law of conservation of energy. Suppose a charge moves around any closed loop in a circuit (the charge starts and ends at the same point), in this case, the circuit must gain as much energy as it loses. In this isolated system model for the system of the circuit, no energy is transferred across the boundary of the system (ignoring energy transfer by radiation and heat into the air from warm circuit elements), but energy transformations do occur within the system. The energy of the circuit may decrease due to a potential drop $-IR$ as a charge moves through a resistor or as a result of having the charge move in the reverse direction through an emf. In the latter case, electric potential energy is converted to chemical energy as the battery is charged. The potential energy increases when the charge moves through a battery in the same direction as the e.m.f.

Some Useful Conventions to be Followed in Circuit Analysis:

While discharging, current is drawn from the battery, the current comes out from positive terminal and enters negative terminal.

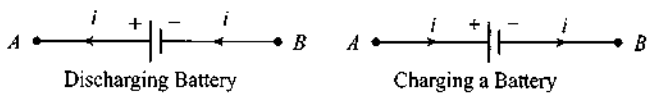


Fig. 5.40

While charging the battery, current is forced from positive terminal of the battery to negative terminal.

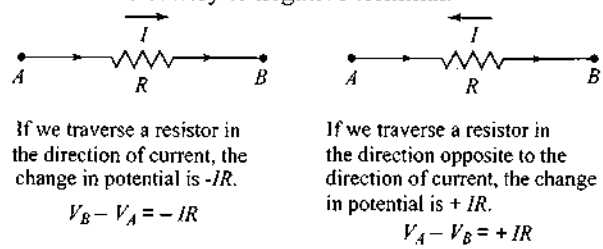


Fig. 5.41

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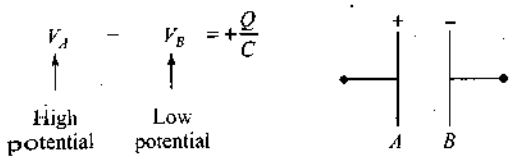

Fig. 5.42

If a capacitor is traversed from negative plate to positive plate, the change in potential is $+Q/C$.

Illustration 5.18 **Calculate the current through each resistance in the given circuit (see Fig. 5.43). Also calculate the potential difference between the points *a* and *b*.**

$E_1 = 6$ V, $E_2 = 8$ V, $E_3 = 10$ V,
$R_1 = 5\ \Omega$, $R_2 = 10\ \Omega$, $R_3 = 4\ \Omega$

Assume that all the cells have no internal resistance.

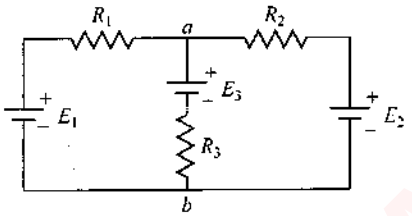

Fig. 5.43

Sol. The process of solving a circuit involves three steps:

i. Assume unknowns $(x, y, \ldots)$ for currents in different branches of the circuit. Use the Kirchhoff's current law at the junctions so that the number of unknowns introduced is minimum.
Let x be the current through R_1 and y be the current through R_3 as shown in Fig. 5.44. Kirchhoff's current law at the junction a gives a current $(x - y)$ through R_2.

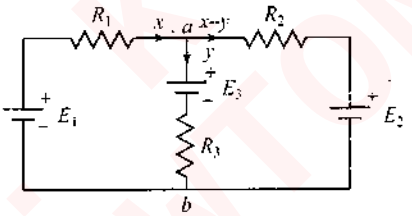

Fig. 5.44

ii. Select as many loops as the number of unknowns introduced for currents. Apply Kirchhoff's voltage law through every loop.
Going anticlockwise through the loop containing R_1 and R_3 (starting from junction a)

$$+xR_1 - E_1 + yR_3 + E_3 = 0$$

$$\Rightarrow \quad 5x + 4y = -4 \qquad \text{(i)}$$

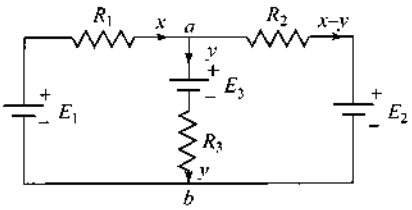

Fig. 5.45

Going clockwise through the loop containing R_2 and R_3 (starting from junction a)

$$-R_2(x - y) - E_2 + yR_3 + E_3 = 0$$

$$\Rightarrow \quad 5x - 7y = 1 \qquad \text{(ii)}$$

Some currents may come out to be negative. This simply means that their directions was incorrectly assumed. So the signs of the currents will give us the correct direction of each current.
Solving Eqs. (i) and (ii), we get

$$x = \frac{-24}{55}\text{ A} \quad \text{and} \quad y = \frac{-5}{11}\text{ A}, x - y = \frac{1}{55}\text{ A}$$

The signs indicate that the direction of x and y was assumed incorrectly while the direction of $(x - y)$ was correct.

The current i_1 (through R_1) $= \frac{24}{55}$ A towards left

The current i_2 (through R_2) $= \frac{1}{55}$ A towards right

The current i_3 (through R_3) $= \frac{5}{11}$ A upwards

The current directions are shown in the circuit diagram Fig. 5.46.

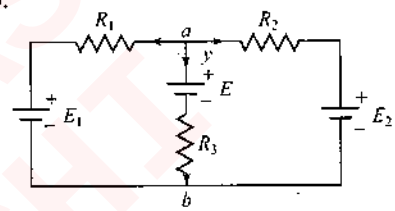

Fig. 5.46

iii. Potential difference between a and b
The p.d. between any two points in a circuit so calculated by adding changes in potential while going through any path from one point to the other point.

Hence, let us go from b to a through R_3.

$$V_a - V_b = +yR_3 + E_3 = \left(\frac{-5}{11}\right) \times 4 + 10 = \frac{90}{11}\text{ V}$$

Illustration 5.19 **In the circuit shown in Fig. 5.47, find the current through the branch *BD*.**

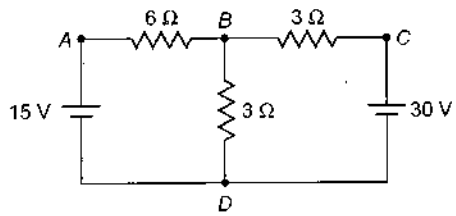

Fig. 5.47

Sol. To find the current in a particular branch in a circuit where resistances and voltages are given, lower loop rule is applicable. In order to find out the solution, the given circuit is directed into parts or close loops which are analyzed using Kirchhoff's second law and corresponding sign conventions.

Assume the currents in the circuit as shown in Fig. 5.48.

Applying KVL along the loop *ABDA* and moving in clockwise direction, we get

$$-6I_1 - 3I_2 + 15 = 0 \Rightarrow 2I_1 + I_2 = 5 \qquad \text{(i)}$$

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While moving in the direction of current corresponding IR, products are taken as negative and if the negative terminal of the battery comes first, then e.m.f. is taken as +ve (Fig. 5.48).

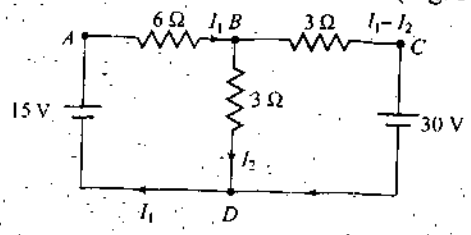


Fig. 5.48

Applying KVL along the loop $BCDB$, we get

$$-3(I_1 - I_2) - 30 + 3I_2 = 0 \Rightarrow -I_1 + 2I_2 = 10 \quad \text{(ii)}$$

Solving Eqs. (i) and (ii) for I_2, we get $I_2 = 5$ A.

Illustration 5.20 **In the circuit shown in Fig. 5.49, determine the voltage drop between A and D.**

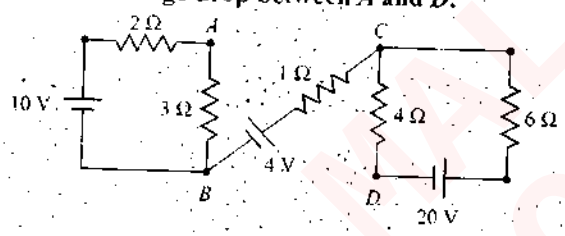


Fig. 5.49

Sol. We need to divide this circuit into three parts. We have left loop, right loop, and the central part. To find out voltage drop between points A and D we have to apply Kirchhoff's second law to these circuits one by one.

Also let us assume that current I_1 flows in the left circuit in the clockwise direction and current I_2 flows in the right circuit in the anticlockwise direction.

Direction of both the currents is decided by the battery present in the circuit as current flows from positive to negative.

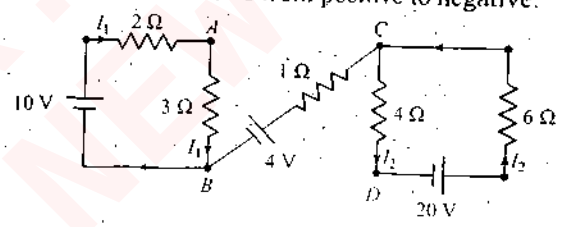


Fig. 5.50

Applying Kirchhoff's second law to left loop, we get

$$-2I_1 - 3I_1 + 10 = 0 \Rightarrow -5I_1 = -10 \Rightarrow I_1 = 2 \text{ A}$$

Now applying Kirchhoff's second law to right loop, we get

$$4I_2 + 6I_2 - 20 = 0 \Rightarrow 10I_2 = 20 \Rightarrow I_2 = 2 \text{ A}$$

Applying between the points A and D, we get

$$V_A - 3I_1 + 4 - 4I_2 = V_D \Rightarrow V_A - V_D = 3I_1 + 4I_2 - 4$$

Putting values of I_1 and I_2 in the above expression, we get

$$V_A - V_D = 3 \times 2 + 4 \times 2 - 4 = 6 + 8 - 4 = 10 \text{ V}$$

Illustration 5.21 **Find the current through 12 Ω resistor in Fig. 5.51.**

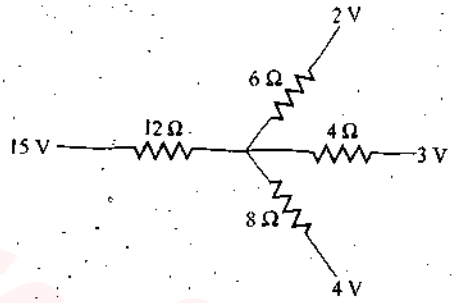


Fig. 5.51

Sol. Let V be the potential at P, then applying KCL at junction P, we get

$$I = I_1 + I_2 + I_3 \Rightarrow \frac{15 - V}{12} = \frac{V - 2}{6} + \frac{V - 3}{4} + \frac{V - 4}{8}$$

$$\text{or } V = \frac{68}{15} \text{ V} \quad \text{and} \quad I = \frac{15 - (68/15)}{12} = \frac{157}{180} \text{ A}$$

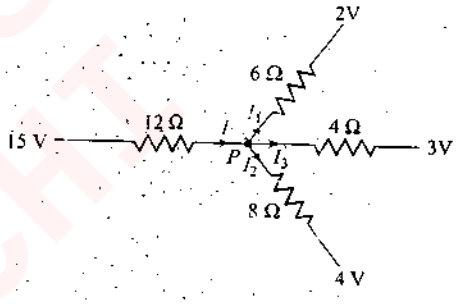
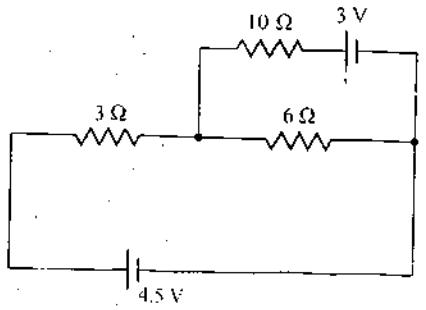


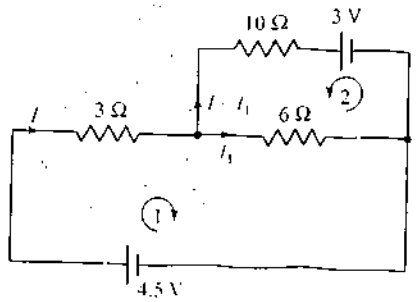
Fig. 5.52

Illustration 5.22 **Find the current in each part of the circuit.**

Sol. Apply loop law in Fig. 5.53 (b).



(a)



(b)

Fig. 5.53

(1) $-3I - 6I_1 + 4.5 = 0$ or $I + 2I_1 = 1.5$ (i)

(2) $10(I - I_1) + 3 - 6I_1 = 0$ or $10I - 16I_1 = -3$ (ii)

Solving Eqs. (i) and (ii), we get

$$I = \frac{1}{2}\text{ A}, I_1 = \frac{1}{2}\text{ A and } I - I_1 = \frac{1}{2} - \frac{1}{2} = 0$$

WHEATSTONE BRIDGE: BALANCED WHEATSTONE BRIDGE

Wheatstone bridge is also known as a metre bridge or slide wire bridge.

If four resistances P, Q, R and S are joined as shown in Fig. 5.54, both the keys (K_1 and K_2) are on and no current flows through the galvanometer (i.e., $I_g = 0$), then the combination of resistances is called a balanced Wheatstone bridge. Then

$$\frac{P}{Q} = \frac{R}{S}$$

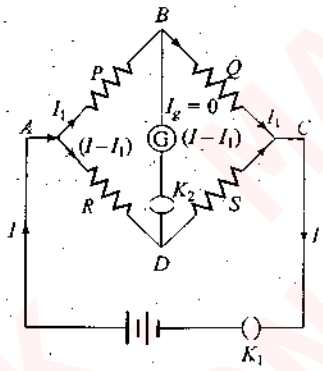


Fig. 5.54

If, say, S is an unknown resistance, then $S = R \times (Q/P)$.

Now if we are given R, P and Q or R and ratio Q/P, then we can calculate the value of S easily.

Proof: Applying loop rule to loop $ABDA$ (moving in clockwise direction).

$$-I_1P + (I - I_1)R = 0 \quad (\because I_g = 0)$$

or $$I_1P = (I - I_1)R \quad \text{(i)}$$

Applying loop rule to loop $BCDB$, we get

$$-I_1Q + (I - I_1)S = 0 \text{ or } I_1Q = (I - I_1)S \quad \text{(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get $P/Q = R/S$

The balanced Wheatstone bridge method is an accurate method because

i. We do not have to read the deflection, but we only have to see that needle remains at dead zero.
ii. It is not affected by internal resistance of cells, resistances of galvanometers, etc.

This is the principle used in the metre bridge or in the slide-wire bridge.

Other circuits which can form Wheatstone bridge are as follows

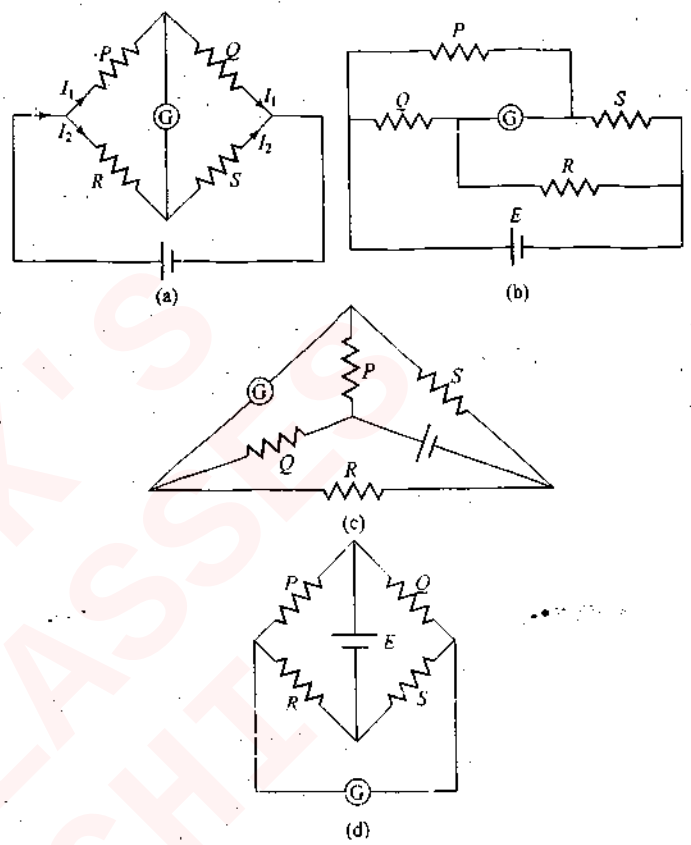


Fig. 5.55

All the four circuits, (a), (b), (c), and (d) represent a Wheatstone bridge network.

COMBINATION OF CELLS

Series Grouping

Suppose n cells each of e.m.f. E and internal resistance r are connected in series as shown in Fig. 5.56. Then

Net e.m.f. = nE; total resistance = $nr + R$,

Therefore, current in the circuit $i = \dfrac{\text{net e.m.f.}}{\text{total resistance}}$ or $i = \dfrac{nE}{nr + R}$

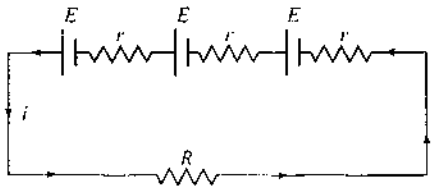


Fig. 5.56

Note: ***If polarity of m cells is reversed, then the equivalent e.m.f. = (n – 2m)E. While the total resistance is still nr + R***

$$i = \frac{(n - 2m)E}{nr + R}$$

If the same current passes through every resistor in a given branch, irrespective of the presence of sources in that branch, the resistors are in series even though they are not directly connected to each other. Same is true about capacitors.

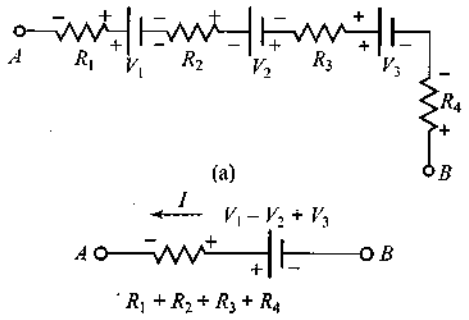


Fig. 5.57

Parallel Grouping

Although E and r of each cell are different, still the positive terminals of all cells are connected at one junction while negative terminals at the other.

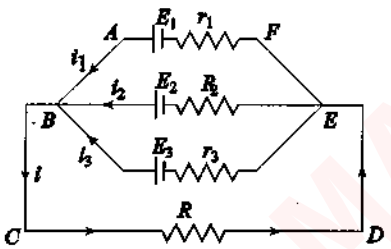


Fig. 5.58

Applying Kirchhoff's second law in loop *ABCDEFA*, we get

$$E_1 - iR - i_1 r_1 = 0 \text{ or } i_1 = -\frac{iR}{r_1} + \frac{E_1}{r_1} \quad \text{(i)}$$

Similarly, we can write

$$i_2 = -i\frac{R}{r_2} + \frac{E_2}{r_2} \quad \text{(ii)}$$

Adding all above equations, we have

$$(i_1 + i_2 + \cdots + i_n) = -iR\sum\left(\frac{1}{r}\right) + \sum\left(\frac{E}{r}\right)$$

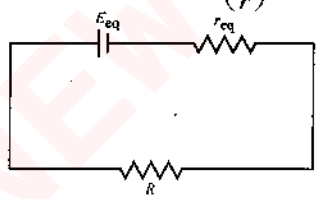


Fig. 5.59

But $\quad i_2 + i_2 + \cdots + i_n = i$

$$\therefore \quad i = -iR\sum\left(\frac{1}{r}\right) + \sum\left(\frac{E}{r}\right)$$

$$= \frac{\sum\left(\frac{E}{r}\right)}{1 + R\sum\left(\frac{1}{r}\right)} = \frac{E_{eq.}}{R_{eq.}}$$

$$\text{where } E_{eq} = \frac{\sum\left(\frac{E}{r}\right)}{\sum\left(\frac{1}{r}\right)} \quad \text{and} \quad R_{eq} = R + \frac{1}{\sum\left(\frac{1}{r}\right)}$$

From the expression, we can see that $i = \dfrac{E}{R + r/n}$ if n cells of same e.m.f. E and internal resistance r are connected in parallel. This is because $\sum\left(\dfrac{E}{r}\right) = \dfrac{nE}{r}$ and $\sum\left(\dfrac{1}{r}\right) = \dfrac{n}{r}$

$$\therefore \quad i = \frac{nE/r}{1 + nR/r} = \frac{E}{R + r/n}$$

We can also write $i = \dfrac{(E_1/r_1) + (E_2/r_2) + (E_3/r_3)}{1 + R\left(\dfrac{1}{r_1} + \dfrac{1}{r_2} + \dfrac{1}{r_3}\right)}$

Mixed Grouping

The situation is shown in

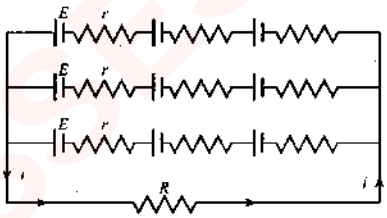


Fig. 5.60

There are n identical cells in a row and number of rows are m. E.m.f. of each cell is E are internal resistance is r. Treating each row as a single cell of e.m.f. nE and internal resistance nr, we have

Net e.m.f. = nE, total internal resistance = nr/m

Total external resistance = R

Therefore, current through the external resistance R is

$$i = \frac{nE}{R + \frac{nr}{m}}$$

This expression after some rearrangement can also be written as

$$i = \frac{mnE}{(\sqrt{mR} - \sqrt{nr})^2 + 2\sqrt{mnrR}}$$

From this expression, we see that i is maximum when

$$\sqrt{mR} = \sqrt{nR} \text{ or } R = nr/m$$

or total external resistance = total internal resistance.

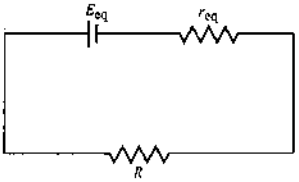


Fig. 5.61

Thus, we can say that the current and hence the power transferred to the load are maximum when the load resistance is equal to internal resistance. This is known as *maximum power transfer theorem.*

Illustration 5.23 **In a battery, 12 cells each having the same e.m.f. are connected in series and are kept in a closed box. Some of the cells are wrongly connected. This battery is connected in series with an ammeter and two cells identical with the others. The current is 3 A when the cells and the battery aid each other and 2 A when the cells and battery oppose each other. How many cells in the battery are wrongly connected?**

Sol. Let n be the cells in the battery that are wrongly connected, then $E_B = (12-n)E - nE = (12-2n)E$ and $r_B = 12r$

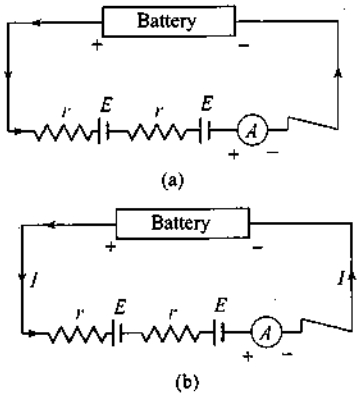


Fig. 5.62

So according to the given problem as shown in Figs. 5.62 (a) and (b), we have

$$\frac{(12-2n)E+2E}{12r+2r} = 3 \quad \text{(i)}$$

and

$$\frac{(12-2n)E-2E}{12r+2r} = 2 \quad \text{(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get $\frac{14-2n}{10-2n} = \frac{3}{2}$, i.e., $n = 1$

This means that in the battery only one cell is wrongly connected.

SUPERPOSITION PRINCIPLE

Concepts

Whenever a circuit has more than one cell or battery. The superposition principle may be used to find current and voltages. This principle is based on the fact that every cell or battery acts independently of the presence of others.

According to this principle, the total current I in the circuit equals the algebraic sum of currents ($I_1, I_2, \ldots, I_n$) produced by each source (cell or battery), taken one at a time.

Mathematically,

$$I = I_1 + I_2 + \cdots + I_n$$

The superposition splits the original two source problem into two one source problems. Instead of solving a difficult two source problem, we can solve two simple problems.

Determine the current I in the 2 Ω resistor.

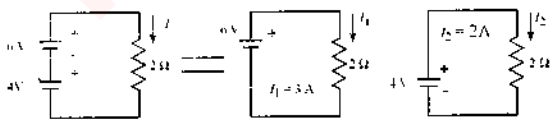
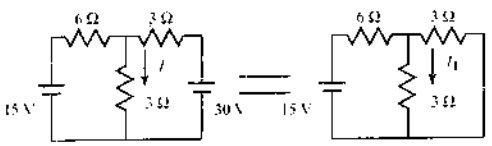


Fig. 5.63

$$I = I_1 + I_2 = \frac{6}{2} + \frac{4}{2} = 3 + 2 = 5 \text{ A}$$

Find the value of I in the circuit shown in Fig. 5.64.



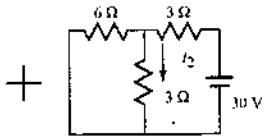


Fig. 5.64

$$I_1 = 1 \text{ A}, \quad I_2 = 4 \text{ A} \Rightarrow I = I_1 + I_2 = 1 + 4 = 5 \text{ A}$$

Situation Where Superposition Principle is not Valid:

In the circuit shown in Fig. 5.65, find the current I flowing through the 2 Ω resistor.

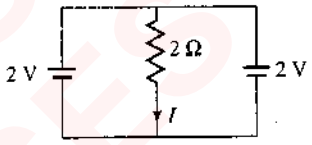


Fig. 5.65

Many students come out with an answer $I = 2$ A with a wrong reason.

Each battery contributes a current of 1 A, as there are two batteries so the total current in the 2 Ω resistor is 2 A.

The circuit may be split into two parts as shown in Fig. 5.66.

With each battery, the current in the 2 Ω resistor is zero. It happens so because when we remove one battery from a branch, then that particular branch gets short-circuited, as there is no other resistance present in that branch.

Note that in such a situation the principle of superposition is not applicable.

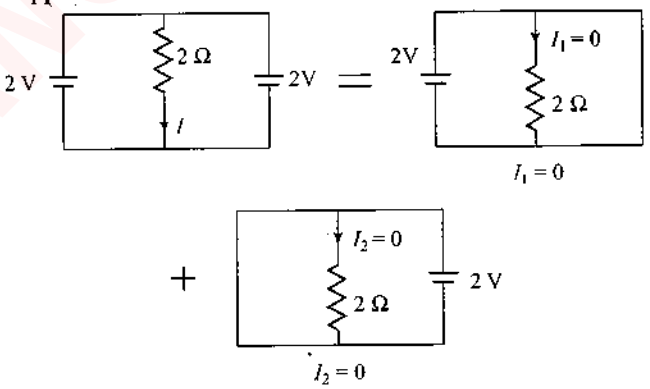


Fig. 5.66

Conditions for the Applicability of Principal of Superposition:

Whenever a cell or a battery is present in a branch there must be some resistance (internal or external or both) present in that branch in order to apply the superposition principle.

In practical situations, it always happens because we can never have an ideal cell or a battery with zero resistance.

In this case, the current flowing through the 2 Ω resistor is 1A, since voltage drop across it is 2V (correct).

Concept Application Exercise 5.2

1. Calculate the value of the electric currents I_1, I_2 and I_3 in the given electrical network.

   a. $I_1 =$ —

b. I_2 = —

c. I_3 = —

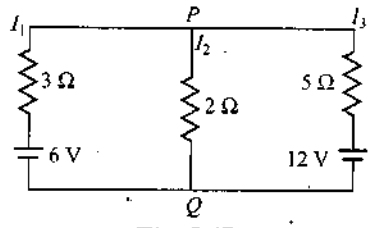


Fig. 5.67

2. Determine the voltage drop across the resistor R_1 in the circuit given below with $E = 65$ V, $R_1 = 50\ \Omega$, $R_2 = 100\ \Omega$, $R_3 = 100\ \Omega$ and $R_4 = 300\ \Omega$.

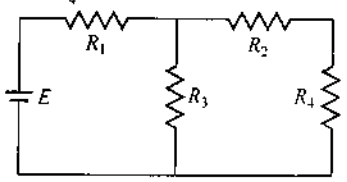


Fig. 5.68

3. A 20 V battery of internal resistance 1 Ω is connected to three coils of 12 Ω, 6 Ω and 4 Ω in parallel, a resistor of 5 Ω and a reversed battery (e.m.f. 8 V and internal resistance 2 Ω) as shown in Fig. 5.69.

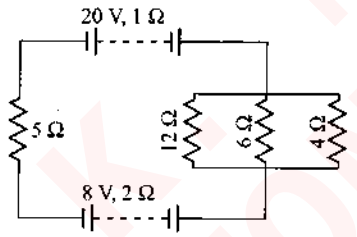


Fig. 5.69

Calculate

a. the current in the circuit,

b. current in resistor of 12 Ω coil and

c. p.d. across each battery.

4. Potential difference across terminals of a cell were measured (in volts) against different currents (in ampere) flowing through the cell. A graph was drawn which was a straight line *ABC*. Using the data given in the graph (see Fig. 5.70), determine

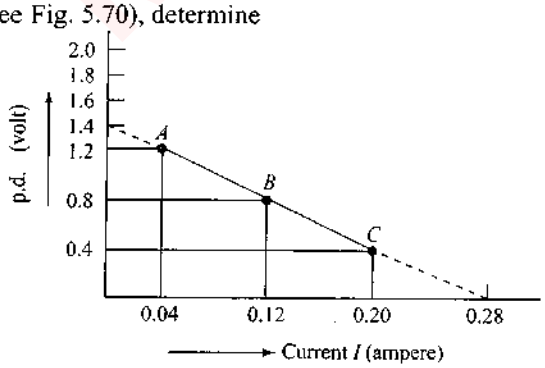


Fig. 5.70

a. the e.m.f. and

b. the internal resistance of the cell.

5. Find the current in each resistor in the circuit as shown in Fig. 5.71.

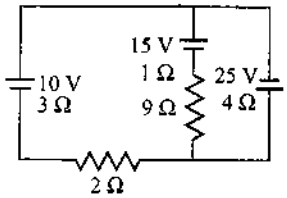


Fig. 5.71

6. The given Wheatstone bridge is showing no deflection in the galvanometer joined between the points *B* and *D* in Fig. 5.72. Calculate the value of *R*.

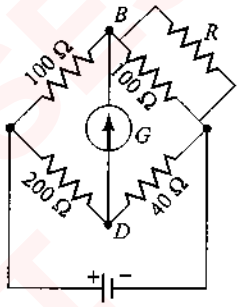


Fig. 5.72

7. When 10 identical cells in series are connected to the ends of a resistance of 59 Ω, the current is found to be 0.25 A. But when the same cells, being connected in parallel, are joined to the ends of a resistance of 0.05 Ω, the current is 25 A. Calculate the internal resistance and the e.m.f. of each cell.

8. Find the minimum number of cells required to produce a current of 1.5 A through a resistance of 30 Ω. Given that the e.m.f. of each cell is 1.5 V and the internal resistance is 1 Ω.

9. The current in a simple series circuit is 5 A. When an additional resistance of 2 Ω is introduced, the current is reduced to 4 A. Calculate the resistance of the original circuit. Assume that the applied potential difference is the same in both the cases.

10. Calculate the resistance between the terminals *A* and *B* of the network shown in Fig. 5.73.

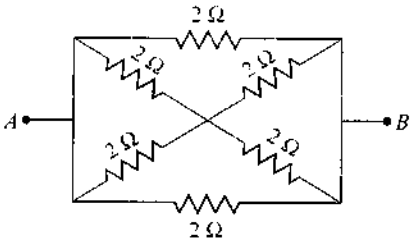


Fig. 5.73

11. In the circuit (Fig. 5.74), the cells E_1 and E_2 have e.m.f.'s of 4 V and 8 V and internal resistances 0.5 Ω and 1.0 Ω, respectively. Calculate the current through 6 Ω resistance.

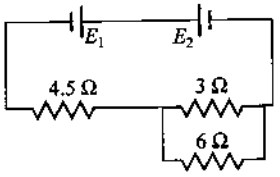


Fig. 5.74

12. Determine the currents I_1, I_2 and I_3 for the network shown below (Fig. 5.75).

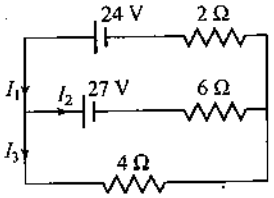


Fig. 5.75

a. $I_1 = —$, **b.** $I_2 = —$, **c.** $I_3 = —$.

13. Find the current supplied by the source in Fig. 5.76. The resistors are mounted around in a cylindrical form.

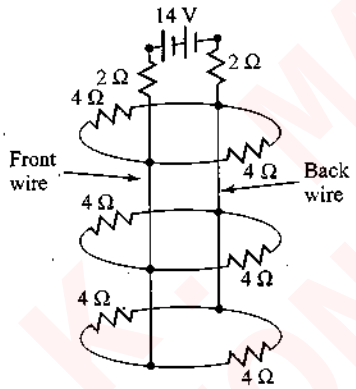


Fig. 5.76

14. A parallel combination of an 8 Ω resistor and an unknown resistor R is connected in series with a 16 Ω resistor and a battery. This circuit is then disassembled and the three resistors are then connected in series with each other and the same battery. In both arrangements, the current through the 8 Ω resistor is the same. What is the unknown resistance R?

15. For the resistor network shown in Fig. 5.77, the potential drop between a and b is 12 V. Find the

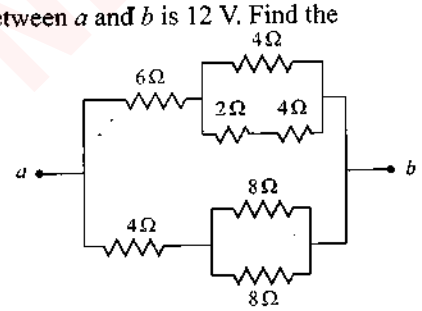


Fig. 5.77

a. Current through resistance of 6 Ω is _____.
b. Current through resistance of 2 Ω is _____.
c. Current through resistance of 8 Ω is _____.

16. For the circuit in Fig. 5.78, find the potential difference between points a and b.

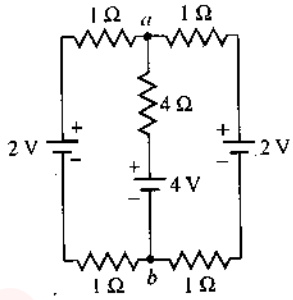


Fig. 5.78

17. Find the current in each resistor of the circuit shown in Fig. 5.79.

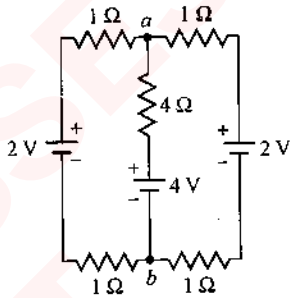


Fig. 5.79

a. Current through resistance of 6 Ω is _____.
b. Current through resistance of 3 Ω is _____.
c. Current through resistance of 2 Ω is _____.
d. Current through resistance of 5 Ω is _____.

18. Given that 5.0 A passes along the branch from C to B in Fig. 5.80. What is the voltage of points A, D and G?

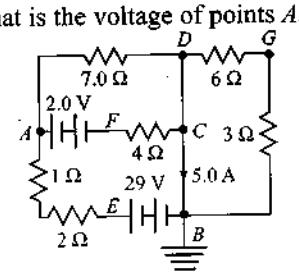


Fig. 5.80

a. $V_A = —$ **b.** $V_D = —$ **c.** $V_G = —$

19. In an experiment, a graph (Fig. 5.81) was plotted of the potential difference V between the terminals of a cell against circuit current I by varying load rheostat. Find the internal conductance of the cell.

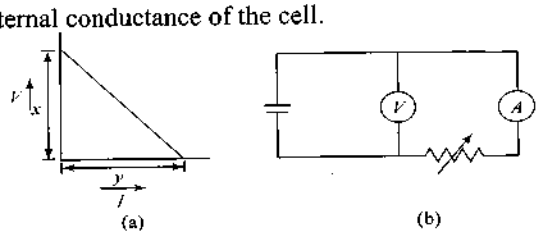


Fig. 5.81

20. Each resistance in the circuit (Fig. 5.82) is of 2,000 Ω.

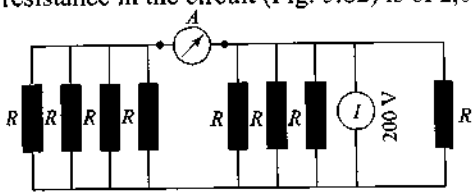


Fig. 5.82

The combination is put across a supply of 200 V. Find the reading of ammeter.

21. Consider the circuits shown in Fig. 5.83. Both the circuits are taking same current from battery, but current through R in the second circuit is (1/10) th of current through R in the first circuit. If R is 11 Ω, then find the value of R_1 and R_2.

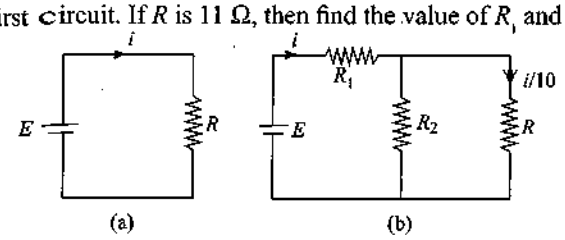


Fig. 5.83

22. Calculate the current through the resistance connected across MN and the current supplied by each of the batteries in the circuit diagram shown in Fig. 5.84.

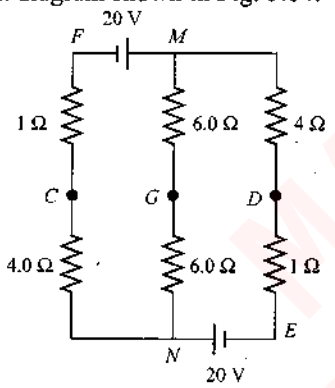


Fig. 5.84

23. A hemispherical network of radius a is made by using a conducting wire of resistance per unit length r. Find the equivalent resistance across OP.

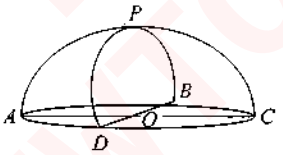


Fig. 5.85

24. If each of the resistances in the shown network is R, then what is the resistance between A and B. **(IIT-JEE, 1978)**

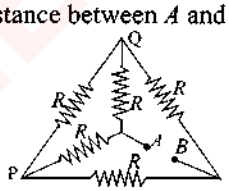


Fig. 5.86

25. A battery of e.m.f. 2 V and internal resistance 0.1 Ω is being charged with a current of 5 A. In what direction will the current flow inside the battery? What is the potential difference between the two terminals of the battery? **(IIT-JEE, 1980)**

26. How a battery is to be connected so that the shown rheostat will behave like a potential divide?

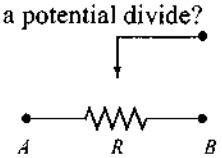


Fig. 5.87

Also indicate the points about which output can be taken. **(IIT-JEE, 2003)**

CHARGING AND DISCHARGING OF A CAPACITOR THROUGH A RESISTANCE

(a) Charging: Consider an uncharged capacitor C is connected to a resistor R through a battery of e.m.f. E. Let switch K be closed at $t = 0$.

Let at any time t, charge on the capacitor be q and current in the circuit be I. We want to find the values of q and I.

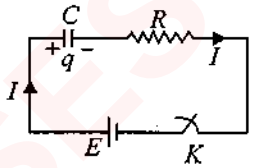


Fig. 5.88

Initial conditions:

at $t = 0$, $q = 0$ because capacitor is uncharged at $t = 0$

at any time: $I = \dfrac{dq}{dt}$

Applying Kirchhoff's law at any time, we get

$$E = \frac{q}{C} + IR \qquad \text{(i)}$$

$$\Rightarrow \quad E = \frac{q}{C} + \frac{dq}{dt}R \Rightarrow \frac{dq}{dt}R = -\frac{q}{C} + E$$

$$\Rightarrow \quad \int_0^q \frac{dq}{EC - q} = \int_0^t \frac{dt}{RC} \Rightarrow \left(\frac{\ln(EC - q)}{-1}\right)_0^q = \frac{t}{RC}$$

$$\Rightarrow \quad \ln(EC - q) - \ln EC = -\frac{t}{RC}$$

$$\Rightarrow \quad \ln\left(\frac{EC - q}{EC}\right) = -\frac{t}{RC} \Rightarrow \frac{EC - q}{EC} = e^{-t/RC}$$

$$\Rightarrow \quad q = EC[1 - e^{-t/RC}] \Rightarrow q = Q(1 - e^{-t/\tau}) \qquad \text{(ii)}$$

where $Q = EC \rightarrow$ maximum charge on capacitor and

$\tau = RC \rightarrow$ time constant of the circuit.

Let us find the charge at $t = \tau$.

Putting $t = \tau$ in Eq. (ii), we get

$$q = Q(1 - e^{-\tau/\tau}) = Q(1 - e^{-1})$$

$$= Q\left(1 - \frac{1}{e}\right) = \frac{Q(e-1)}{e} = 0.632Q$$

So one time constant (τ) *is the time in which charge on the capacitor becomes 63.2 % of maximum charge.*

The variation of charge q with time t is as shown in the given figure.

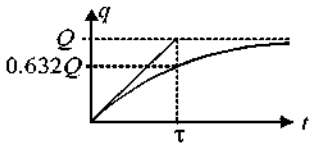


Fig. 5.89

Current or Rate of Charging:

Differentiating Eq. (ii), we get

$$I = \frac{dq}{dt} = Q\left(0 - e^{-t/\tau}\left(-\frac{1}{\tau}\right)\right) \; I = \frac{Q}{\tau}e^{-t/\tau} \Rightarrow I = \frac{EC}{RC}e^{-t/\tau}$$

$$\Rightarrow \quad I = \frac{E}{R}e^{-t/\tau} \Rightarrow I = I_0 e^{-t/\tau} \quad \text{(iii)}$$

where $I_0 = E/R \rightarrow$ Maximum current in the circuit

Let us find the current at $t = \tau$, putting $t = \tau$ in Eq. (iii), we get $I = I_0 e^{-\tau/\tau} = \frac{I_0}{e} = 0.368 I_0$

So time constant (τ) may also be defined as the time in which current decreases to 36.8% of its initial maximum value.

The variation of current I with time t is as shown in the given figure.

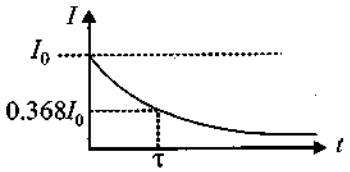


Fig. 5.90

Note:
- ***Putting $t = 0$ in Eqs. (ii) and (iii), we get $q = 0$, $I = I_0$. Initially, capacitor is uncharged and current is maximum. It means when capacitor is uncharged, it offers no resistance to the flow of current or capacitor behaves like a simple connecting wire (or short-circuited).***
- ***Putting $t = \infty$ in Eqs. (ii) and (iii), we get $q = Q$, $I = 0$. Now the capacitor is fully charged and no current flows through the circuit. The capacitor here offers infinite resistance (or behaves like open circuit). This is known as steady state. In steady state, no current flows through the branch containing capacitor.***
- ***Initial rate of charging i.,e., initial current (at $t = 0$) is $I = I_0 = E/R$. Let us see how much time it will take to charge the capacitor if it were to charge with a constant rate of initial rate of charging.***

  Here total time taken is

  $$t = \frac{Q}{I_0} = \frac{EC}{E/R} = RC \Rightarrow t = \tau$$

  This comes out to be equal to one time constant. So time constant may also be defined as the time during which the charging would have been completed, had the growth rate been as it began initially.
- ***Remember this result of integration:***

  If $\int_0^x \frac{dx}{a-bx} = \int_0^t c\,dt$, then $x = \frac{a}{b}\left(1 - e^{-bct}\right)$
- ***From Eq. (i) also, we can see that at $t = 0$, $q = 0$, then $I = E/R$ and at $t = \infty$, $q = Q = EC$, then $I = 0$***
- ***Dimensional formula of $\tau = RC$ is [T].***

Charging of the Capacitor—Other Approach

Differentiating Eq. (i) with respect to time on both the sides,

we get $0 = \frac{1}{C}\frac{dq}{dt} + R\frac{dI}{dt} \Rightarrow \frac{1}{C}I + R\frac{dI}{dt} = 0$

$$\Rightarrow \quad \frac{dI}{I} = -\frac{1}{RC}dt$$

Integrating both sides, we get

$$\int_{I_0}^{I}\frac{dI}{I} = -\frac{1}{RC}\int_0^t dt \Rightarrow [\ln I]_{I_0}^{I} = -\frac{1}{RC}[t]_0^t$$

$$\ln\frac{I}{I_0} = -\frac{t}{RC} \Rightarrow \frac{I}{I_0} = e^{-\frac{t}{RC}} \Rightarrow I = I_0 e^{-\frac{t}{RC}}$$

This is same as Eq. (iii) obtained earlier.

Illustration 5.24 **A resistor with resistance 10 MΩ is connected in series with a capacitor of capacitance 1.0 μF and a battery with e.m.f. 12.0 V. Before the switch is closed at time $t = 0$, the capacitor is uncharged.**

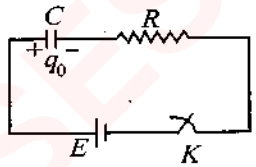


Fig. 5.91

a. What is the time constant?
b. What fraction of the final charge is on the plates at time = 46 s?
c. What fraction of the initial current remains at $t = 46$ s?

Sol.

a. The time constant is $\tau = RC = 10 \times 10^6 \times 1 \times 10^{-6} = 10$ s

b. The fraction f of the final charge on capacitor is q/Q

$$\frac{q}{Q} = 1 - e^{-t/RC} = 1 - e^{-46/10} = 0.99$$

The capacitor is 99% charged after a time equal to 4.6 RC, or 4.6 time constants.

c. $$\frac{I}{I_0} = e^{-4.6} = 0.010$$

After 4.6 time constants, the current has decreased to 1.0% of its initial value.

Illustration 5.25 **An uncharged capacitor is connected to a 15 V battery through a resistance of 10 Ω. It is found that in a time of 2 μs, potential difference across the capacitor becomes 5 V. Find the capacitance of the capacitor. Take ln(1.5) = 0.4.**

Sol. We know that charge on the capacitor at any time is given by $q = Q(1 - e^{-t/\tau})$ where $Q = EC = 15C$. Here charge q at any time is given by $q = VC$ where V is potential difference across capacitor at that time. Here $V = 5$ V, so $q = 5C$.

Putting the values: $5C = 15C(1 - e^{-t/\tau}) \Rightarrow e^{-t/\tau} = 2/3$

$$\Rightarrow \quad \frac{t}{\tau} = \ln\left(\frac{3}{2}\right) \Rightarrow \frac{t}{RC} = \ln\left(\frac{3}{2}\right)$$

$$\Rightarrow \quad C = \frac{t}{R\ln(3/2)} = \frac{2 \times 10^{-6}}{10\ln(3/2)} = 0.5\ \mu\text{F}$$

Illustration 5.26 **Consider the circuit shown in the figure Find out the steady-state current in the 4 Ω resistor. Assume the internal resistance of 8 V battery to be negligible.**

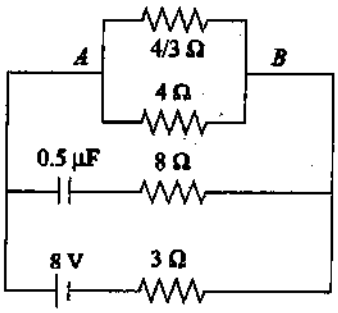


Fig. 5.92

Sol. The equivalent resistance between points A and B is

$$\frac{1}{R_p}=\frac{3}{4}+\frac{1}{4}=1 \Rightarrow R_p=1\,\Omega$$

In steady state, the current will not pass through 0.5 μF capacitor as it offers infinite resistance to steady-state current or direct current. So the total resistance offered by the circuit $= 1 + 3 = 4\ \Omega$.

The current from the battery $I = 8/4 = 2$ A. Hence, the potential difference between points A and $B = I \times R_p = 2$ V.

Therefore, the current through 4 Ω resistor = 2/4 = 0.5A.

Illustration 5.27 **Find out the potential difference between the points x and y in Fig. 5.93.**

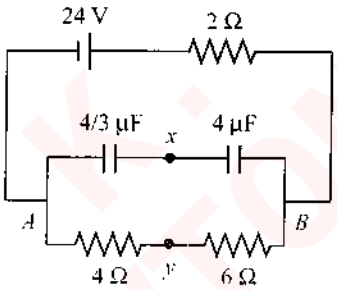


Fig. 5.93

Sol. Given $E = 24$ V, $R = 2\ \Omega$, $R_1 = 4\ \Omega$, $R_2 = 6\ \Omega$, $C_1 = (4/3)$ μF, $C_2 = 4$ μF.

As the capacitor offers a very high resistance to the current in the steady state, so the current is prevented to pass through the capacitors.

Now, the total resistance in the circuit is

$$R_{eq} = R + R_1 + R_2 = 2 + 4 + 6 = 12\ \Omega$$

Hence, the net current in the circuit $= \dfrac{E}{R_{eq}} = \dfrac{24}{12} = 2$ A

Therefore, the terminal potential difference across is

$$AB = 24 - 2 \times 2 = 20 \text{ V}$$

Note: ***As the capacitors and resistances are in parallel, the potential difference of 20 V is available to both the capacitors and the resistors.***

$$V_B - V_x = 20 \times \frac{6}{10} = 12 \text{ V} \qquad \text{(i)}$$

$$V_B - V_x = \frac{20 \times 4/3}{4/3 + 4} = 5 \text{ V} \qquad \text{(ii)}$$

From Eqs. (i) and (ii), $V_x - V_y = 12 - 5 = 7$ V

Illustration 5.28 **Consider the circuit shown in the figure where a battery of e.m.f. 4 V and a capacitor of capacitance 1 μF is connected to a combination of resistances. Find out the steady-state current in the battery.**

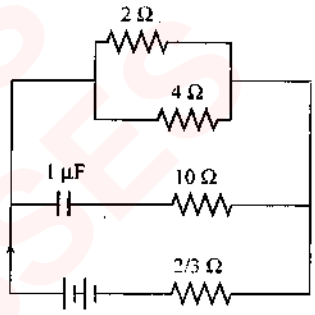


Fig. 5.94

Sol. Let us consider I to be the steady state current through the circuit. In the steady-state condition, current I is constant in the circuit and the capacitor offers infinite resistance. So the resistance 10 Ω becomes ineffective in the circuit. So in this case, the equivalent resistance of resistors 2 Ω and 4 Ω that are connected in parallel is

$$\frac{1}{R_p}=\frac{1}{2}+\frac{1}{4}=\frac{3}{4} \Rightarrow R_p=\frac{4}{3}\ \Omega$$

Therefore, the total resistance of the circuit $= \dfrac{4}{3}+\dfrac{2}{3}=\dfrac{6}{3}=2\ \Omega$

Hence, the steady-state current in the circuit = 4/2 = 2 A

Energy Considerations:

Total work done by battery in fully charging the capacitor is $W_b = EQ = E^2C$.

Final energy stored in the capacitor $U = \dfrac{1}{2}CE^2$

We see that final energy stored in the capacitor is half of the work done by battery. Now question arises, where the other half has gone. It might have been dissipated in the resistor in the form of heat. Let us check how much energy is dissipated in the resistor. For this

$$H = \int_0^\infty I^2 R dt = \int_0^\infty I_0^2 e^{-2t/\tau} R dt = I_0^2 R \left(\frac{e^{-2t/\tau}}{-2/\tau} \right)_0^\infty$$

$$= \frac{-I_0^2 R\tau}{2}(0-1) = \left(\frac{E}{R}\right)^2 \frac{R}{2} RC = \frac{1}{2}CE^2$$

Hence, the other half is dissipated in the resistor in the form of heat. One point to notice here is that final energy dissipated in the resistor is independent of its value R.

Note: ***Multiplying Eq. (i) with I, we get***

$$EI = \frac{q}{C}I + I^2R \Rightarrow EI = \frac{q}{C}\frac{dq}{dt} + I^2R$$

The term on the left-hand side is the power supplied by battery which is used in two ways as indicated on R.H.S. The first term on the R.H.S. $\left(\frac{q}{C}\frac{dq}{dt} = \frac{d}{dt}\left(\frac{q^2}{2C}\right)\right)$ indicates the rate at which energy is being stored in the capacitor and second term (I^2R) is the rate of dissipation of energy in the resistor.

Illustration 5.29 **A capacitor of capacitance 10 μF is connected to a resistance 2 Ω and a battrery of e.m.f. 5 V of negligible internal resistance. After 20 μs of completing the circuit, find**

(a) power delivered by the battery

(b) power dissipated as heat

(c) rate of energy stored in the capacitor

Sol. Here $C = 10\ \mu F$, $R = 2\ \Omega$, $E = 5$ V, $\tau = RC = 2 \times 10 \times 10^{-6}$ s = 20 μs

(a) Current at $t = 20$ μs, $I = \frac{E}{R}e^{-t/\tau} = \frac{5}{2}e^{-20/20} = \frac{5}{2e}$

Power delivered by battery: $P_1 = EI = 5 \times \frac{5}{2e} = \frac{25}{2e} = 4.6$ W

(b) Power dissipated as heat:

$$P_2 = I^2R = \left(\frac{5}{2e}\right)^2 2 = \frac{25}{2e^2} = 1.7 \text{ W}$$

(c) Energy on the capacitor: $U = \frac{q^2}{2C}$

Rate of energy stored in the capacitor:

$$P_3 = \frac{dU}{dt} = \frac{2q}{2C}\frac{dq}{dt} = \frac{q}{C}\frac{dq}{dt} \Rightarrow P_3 = \frac{Q(1-e^{-t/\tau})}{C}I$$

$$\Rightarrow \quad P_3 = \frac{EC}{C}\left(1-e^{-t/\tau}\right)I = EI\left(1-e^{-t/\tau}\right)$$

$$= \frac{5\times 5}{2e}\left(1-e^{-20/20}\right) \text{ W} = \frac{25}{2e} - \frac{25}{2e^2} = 2.9$$

Note: ***We see that $P_1 = P_2 + P_3$.***

(b) Discharing: Consider a capacitor is given a charge Q and then it is connected to a resistor R. Let at $t = 0$, switch is closed.

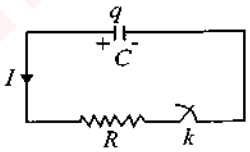


Fig. 5.95

At $t = 0$, $q = Q$. Let at any time t charge on the capacitor is q

$$\Rightarrow \quad q = Qe^{-t/\tau}$$

At $t = \tau$, we get $q = Qe^{-\tau/\tau} \Rightarrow q = Q/e = 0.368Q$

So time constant (τ) may also be defined as the time in which charge decreases to 36.8% of its initial maximum value in a discharging circuit.

The variation of charge q with time t is shown in the figure.

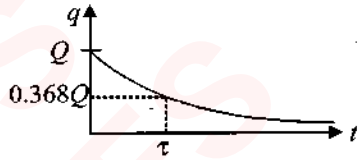


Fig. 5.96

Current or rate of discharging:

$$I = \frac{-dq}{dt} = -Qe^{-t/\tau}\left(-\frac{1}{\tau}\right)$$

$$\Rightarrow \quad I = \frac{Q}{\tau}e^{-t/\tau} \Rightarrow I = \frac{Q}{RC}e^{-t/\tau} = \frac{V}{R}e^{-t/\tau}$$

(where $V = Q/C$ is initial potential of capacitor)

$\Rightarrow \quad I = I_0e^{-t/\tau}$ (where $I_0 = V/R$ is the maximum current)

At $t = \tau$, we get $I = I_0e^{-\tau/\tau} = I_0/e = 0.368I_0$

So time constant (τ) may also be defined as the time in which current decreases to 36.8% of its initial maximum value in a discharging circuit.

The variation of charge I with time t is shown in the figure:

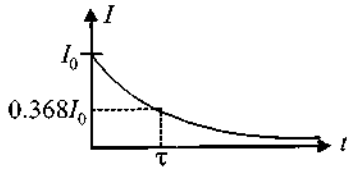


Fig. 5.97

At $t = 0$, the capacitor is acting as a battery of e.m.f. = Q/C. As the time passes, the charge on the capacitor decreases and the potential difference across the capacitor decreases. Hence the capacitor acts as a battery of decreasing e.m.f.

Energy Considerations:

Let us see how much energy is dissipated in the resistor:

For this

$$H = \int_0^\infty I^2Rdt = \int_0^\infty \left(\frac{Q}{RC}\right)^2 e^{-2t/\tau}Rdt = \frac{Q^2}{RC^2}\left(\frac{e^{-2t/\tau}}{-2/\tau}\right)^\infty$$

2. Find the average value of current (in A) shown graphically in Fig. 7.64, from $t = 0$ to $t = 2$ s.

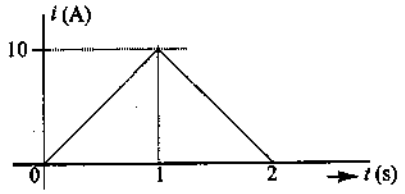


Fig. 7.64

3. The average value of current $i = I_m \sin\omega t$ from $t = \frac{\pi}{2\omega}$ to $t = \frac{3\pi}{2\omega}$ is how many times of I_m?

4. An ac ammeter is used to measure current in a circuit. When a given direct current passes through the circuit, the ac ammeter reads 3 A. When another alternating current passes through the circuit, the ac ammeter reads 4 A. Then find the reading of this ammeter (in A), if dc and ac flow through the circuit simultaneously.

Archives

Solutions on page 7.41

Subjective Type

1. In a series L-R circuit ($L = 35$ mH and $R = 11\ \Omega$), a variable emf source ($V = V_0 \sin \omega t$) of $V_{rms} = 220$ V and frequency 50 Hz is applied. Find the current amplitude in the circuit and phase of current with respect to voltage. Draw current–time graph on given graph. **(IIT-JEE, 2004)**

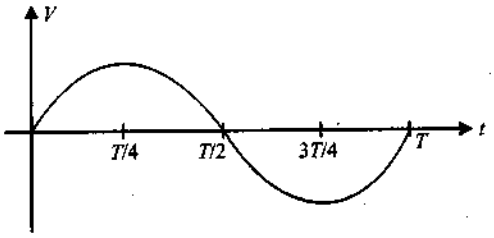


Fig. 7.65

Single Correct Answer Type

1. When an ac source of emf $e = E_0 \sin(100\ t)$ is connected across a circuit, the phase difference between emf e and current i in the circuit is observed to be $\pi/4$, as shown in Fig. 7.66. If the circuit consists possibly only of R-C or R-L or L-C in series, find the relationship between the two elements. **(IIT-JEE, 2003)**

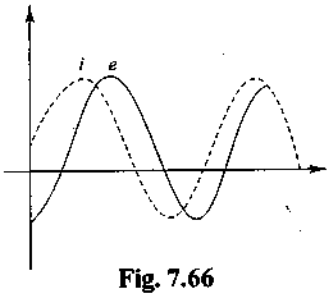


Fig. 7.66

a. $R = 1\ k\Omega, C = 10\ \mu F$
b. $R = 1\ k\Omega, C = 1\ \mu F$
c. $R = 1\ k\Omega, L = 10$ H
d. $R = 1\ k\Omega, L = 1$ H

2. An ac voltage source of variable angular frequency ω and fixed amplitude V_0 is connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When ω is increased
   a. the bulb glows dimmer
   b. the bulb glows brighter
   c. total impedance of the circuit is unchanged
   d. total impedance of the circuit increases

(IIT-JEE, 2010)

Multiple Correct Answers Type

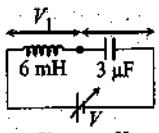
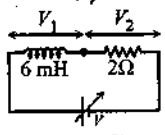
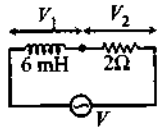
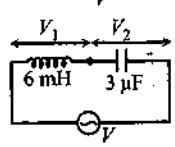
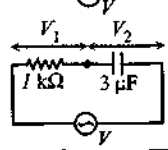
1. A series R-C circuit is connected to an ac voltage source. Consider two cases: (A) when C is without a dielectric medium and (B) when C is filled with a dielectric of constant 4. The current I_R through the resistor and voltage V_C across the capacitor are compared in the two cases. Which of the following is/are true?

a. $I_R^A > I_R^B$
b. $I_R^A < I_R^B$
c. $V_C^A > V_C^B$
d. $V_C^A < V_C^B$

(IIT-JEE, 2011)

Matching Column Type

1. You are given many resistors, capacitors and inductors. These are connected to a variable dc voltage source (the first two circuits) or an ac voltage source of 50 Hz frequency (the next three circuits) in different ways as shown in Column II. When a current I (steady state for dc or rms for ac) flows through the circuit, the corresponding voltages V_1 and V_2 (indicated in circuits) are related as shown in Column I. Match the two.

Column I	Column II
i. $I \neq 0$, V_1 is proportional to I	a. 
ii. $I \neq 0$, $V_2 > V_1$	b. 
iii. $V_1 = 0$, $V_2 = V$	c. 
iv. $I \neq 0$, V_2 is proportional to I	d. 
	e. 

(IIT-JEE, 2010)

Integer Answer Type

1. A series R-C combination is connected to an ac voltage of angular frequency $\omega = 500$ rad/s. If the impedance of the R-C circuit is $R\sqrt{1.25}$, the time constant (in millisecond) of the circuit is _____.

(IIT-JEE, 2011)

ANSWERS AND SOLUTIONS

Subjective Type

1. Let us solve the question for time interval $t = 0$ to $t = 1$ s. The rms and average value calculated for this time interval will be same as for overall graph for long time, because the graph is repeating again and again.

Analytical Method: For $t = 0$ to $t = 1$ s:

$$f(t) = 4t - 2$$

Average value:

$$f(t)_{av} = \frac{\int_0^1 f(t)dt}{1-0} = \frac{\int_0^1 (4t-2)\,dt}{1} = \left[\frac{4t^2}{2} - 2t\right]_0^1 = 0$$

Hence average value of waveform is zero.

$$\text{rms value} = f(t)_{rms} = \sqrt{\frac{\int_0^1 f^2(t)\,dt}{1-0}} = \sqrt{\frac{\int_0^1 (4t-2)^2\,dt}{1}}$$

$$= \sqrt{\left[\frac{(4t-2)^3}{4\times 3}\right]_0^1} = \sqrt{\frac{4}{3}}$$

Alternate method (Graphically):

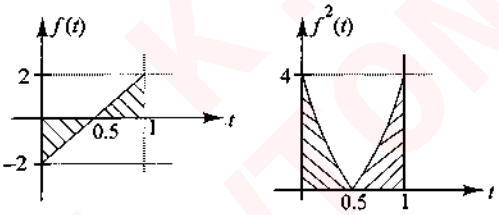


Fig. 7.67

$$f(t)_{av} = \frac{\int_0^1 f(t)dt}{1-0}$$

$$= \frac{\text{area of } f(t)\text{–}t \text{ graph from } t = 0 \text{ to } t = 1 \text{ s}}{1}$$

$$= \frac{\frac{1}{2}(0.5)(-2) + \frac{1}{2}(0.5)2}{1} = 0$$

$$f(t)_{rms} = \sqrt{\frac{\int_0^1 f^2(t)dt}{1-0}}$$

$$= \sqrt{\frac{\text{area under } f^2(t) - t \text{ graph from } t = 0 \text{ to } t = 1\text{s}}{1}}$$

$$= \sqrt{\frac{\frac{1}{3}\times 0.5\times 4 + \frac{1}{3}\times 0.5\times 4}{1}} = \sqrt{\frac{4}{3}}$$

2. Here, time of 1 cycle $T = 1/50$ s. So, we have to calculate the average energy as time >> T.

Energy consumed in time t

$$= V_{rms} I_{rms} \cos\phi t = \frac{I_M}{\sqrt{2}} \times \frac{V_M}{\sqrt{2}} \times \frac{R}{Z} t$$

$$V = \frac{V_M^2 R}{2Z^2} t \qquad \left(\because I_M = \frac{V_M}{Z}\right)$$

$$\text{Now, } Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$= \sqrt{(50)^2 + \left(314\times 20\times 10^{-3} - \frac{1}{314\times 100\times 10^{-6}}\right)^2}$$

$$= \sqrt{3153.6} \approx 56\ \Omega$$

$$\text{Energy consumed} = \frac{10^2 \times 50 \times 20 \times 60}{2 \times 3153.6} = 951\text{ J}$$

When resistance is removed,

$$\cos\phi = \frac{R}{Z'} = 0 \quad \text{or,} \quad \phi = \frac{\pi}{2} \text{ or } -\frac{\pi}{2}$$

$$Z' = \frac{1}{\omega C} - \omega L' = \frac{1}{314\times 10^{-4}} - 314\times 40\times 10^{-3} = 19.3$$

[here $X_C > X_L$, hence $\phi = \pi/2$]

$$I = \frac{V_M}{Z'}\sin(\omega t + \phi) = \frac{10}{19.3}\sin\left(314t + \frac{\pi}{2}\right)$$

$$= 0.52\cos 314t$$

3.

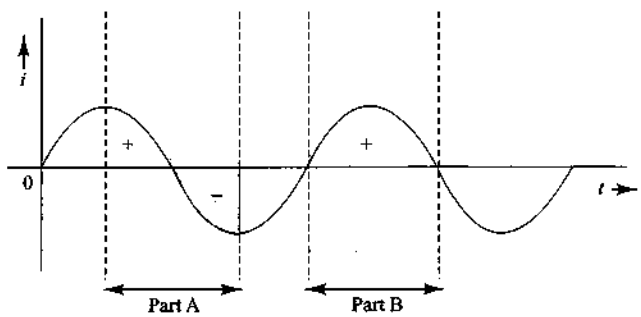


Fig. 7.68

Figure 7.68 shows two parts, A and B, each half cycle. In part A we can see that the net area is zero.
$\therefore$ $<i>$ in part A = 0.
In part B, area is positive hence in this part $<i> \neq 0$.

4. $$<i> = \frac{\int_0^1 i\,dt}{1} = 2\sqrt{2}\int_0^1 \sin\left(\pi t + \frac{\pi}{4}\right) = \frac{4}{\pi}$$

5. **Analytically:**

$$i = 2\sin 100\pi t + 2\cos(100\pi t + 30°)$$

$$\Rightarrow \quad i = 2\sin 100\pi t + 2\cos 100\pi t \cos 30° - 2\sin 100\pi t \sin 30°$$

$$\Rightarrow \quad i = \sin 100\pi t + \sqrt{3}\cos 100\pi t$$

$$\Rightarrow \quad i = I_m \sin(100\pi t + \phi)$$

where $I_m = \sqrt{1 + (\sqrt{3})^2} = 2$ A

$$I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}\text{ A}$$

Graphically:

The given equation can be written as:

$$i = 2\sin 100\pi t + 2\sin(100\pi t + 120°)$$

It is the sum of two phasors having phase difference 120°.

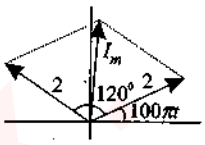


Fig. 7.69

$$I_m = \sqrt{2^2 + 2^2 + 2\times 2\times 2\cos 120°} = 2\text{ A}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}\text{ A}$$

6. $$P_{av} = v_{rms} I_{rms}\cos\phi$$

$$= \frac{200\sqrt{2}}{\sqrt{2}}\frac{2}{\sqrt{2}}\cos(30°) = 100\sqrt{6}\text{ W}$$

7. $$X_C = \frac{1}{\omega C} = \frac{1}{100\times 10^{-6}} = 10^4\ \Omega$$

As ac instruments read rms value, the reading of ammeter will be,

$$I_{rms} = \frac{E_{rms}}{X_C} = \frac{E_0}{\sqrt{2}X_C} \qquad \left[\text{as } E_{rms} = \frac{E_0}{\sqrt{2}}\right]$$

i.e., $$I_{rms} = \frac{200\sqrt{2}}{\sqrt{2}\times 10^4} = 0.02\text{ A} = 20\text{ mA}$$

8. $$X_C = \frac{10^6}{\frac{100}{\pi}(2\pi 50)} = 100\ \Omega$$

i. $Z = \sqrt{R^2 + X_C^2} = \sqrt{100^2 + (100)^2} = 100\sqrt{2}\ \Omega$

ii. $\tan\phi = \frac{X_C}{R} = 1 \quad \therefore\ \phi = 45°$

iii. Power factor $= \cos\phi = \frac{1}{\sqrt{2}}$

iv. Current $I_{rms} = \frac{V_{rms}}{Z} = \frac{200}{100\sqrt{2}} = \sqrt{2}$ A

v. Maximum current $= I_{rms}\sqrt{2} = 2$ A

vi. Voltage across $R = V_{R,rms} = I_{rms}R = \sqrt{2}\times 100$ V

vii. Voltage across $C = V_{C,rms} = I_{rms}X_C = \sqrt{2}\times 100$ V

viii. Max voltage across $R = \sqrt{2}\,V_{R,rms} = 200$ V

ix. Max voltage across $C = \sqrt{2}\,V_{C,rms} = 200$ V

x. $<P> = V_{rms}I_{rms}\cos\phi = 200\times\sqrt{2}\times\frac{1}{\sqrt{2}} = 200$ W

xi. $<P_R> = I_{rms}^2 R = 200$ W

xii. $<P_C> = 0$

xiii. Applied voltage can be written as:

$$V_s(t) = V_0\sin\omega t \qquad (\omega = 2\pi f = 2\pi 50 = 100\pi\text{ rad/s})$$

$$= 200\sqrt{2}\sin(100\pi t)$$

$$i(t) = I_0\sin(100\pi t + \phi)$$

$$= \frac{V_0}{Z}\sin\left(100\pi t + \frac{\pi}{4}\right) = \frac{200\sqrt{2}}{100\sqrt{2}}\sin\left(100\pi t + \frac{\pi}{4}\right)$$

$$= 2\sin\left(100\pi t + \frac{\pi}{4}\right)$$

$$V_R(t) = V_{R0}\sin\left(100\pi t + \frac{\pi}{4}\right) = I_0 R\sin\left(100\pi t + \frac{\pi}{4}\right)$$

$$= 2\times 100\sin\left(100\pi t + \frac{\pi}{4}\right) = 200\sin\left(100\pi t + \frac{\pi}{4}\right)$$

$$V_C(t) = V_{C0}\sin\left(100\pi t + \frac{\pi}{4} - \frac{\pi}{2}\right)$$

$$= I_0 X_C\sin\left(100\pi t - \frac{\pi}{4}\right) = 200\sin\left(100\pi t - \frac{\pi}{4}\right)$$

9. According to the given problem,

$$I = \frac{V}{Z} = \frac{V}{[R^2 + (1/C\omega)^2]^{1/2}} \qquad (1)$$

and, $$\frac{I}{2} = \frac{V}{[R^2 + (3/C\omega)^2]^{1/2}} \qquad (2)$$

From (1) in (2), $$\frac{1}{C^2\omega^2} = \frac{3}{5}R^2$$

So that, $$\frac{X}{R} = \frac{(1/C\omega)}{R} = \frac{\left(\frac{3}{5}R^2\right)^{1/2}}{R} = \sqrt{\frac{3}{5}}$$

10. Here $X_L = \omega L = 2\pi f L = 2\pi \times 50 \times \frac{9}{100\pi} = 9\,\Omega$

So, $Z = \sqrt{R^2 + X_L^2} = \sqrt{12^2 + 9^2} = 15\,\Omega$

So (a) $I = \frac{V}{Z} = \frac{225}{15} = 15\text{ A}$

and (b) $\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{9}{12}\right) = \tan^{-1}(3/4) = 37°$

i.e., the current will lag the applied voltage by 37° in phase.

11. When the coil is connected to a dc source, the final current is decided by the resistance of the coil.

$\therefore \quad r = \frac{10}{2.5} = 4\,\Omega$

When the coil is connected to an ac source, the final current is decided by the impedance of the coil.

$\therefore \quad Z = \frac{10}{2} = 5\,\Omega$

But $\quad Z = \sqrt{(r)^2 + (X_L)^2} \Rightarrow X_L = 3\,\Omega$

$\therefore \quad \omega L = 2\pi f L = 3 \Rightarrow 2\pi 50 L = 3 \Rightarrow L = 3/100\pi$ H

12. From the rating of the bulb, the resistance of the bulb is

$$R = \frac{V^2}{P} = 100\,\Omega$$

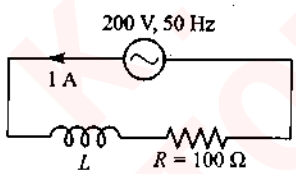


Fig. 7.70

For the bulb to be operated at its rated value, the rms current through it should be 1A

Also, $I_{rms} = \frac{V_{rms}}{Z}$

$\therefore \quad 1 = \frac{200}{\sqrt{100^2 + (2\pi 50 L)^2}} \Rightarrow L = \frac{\sqrt{3}}{\pi}$ H

13. As for the arc lamp $V_R = IR = 10 \times 5 = 50$ V, so when it is connected to 160 V ac source through a choke in series,

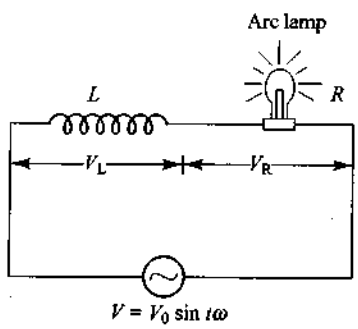


Fig. 7.71

$V^2 = V_R^2 + V_L^2$, $V_L = \sqrt{160^2 - 50^2} = 152$ V

and as, $V_L = IX_L = I\omega L = 2\pi f L I$

So, $\quad L = \frac{V_L}{2\pi f I} = \frac{152}{2 \times \pi \times 50 \times 10} = 4.84 \times 10^{-2}$ H

Now the lamp is to be operated at 160 V dc; instead of choke if additional resistance r is put in series with it,

$V = I(R + r)$, i.e., $160 = 10(5 + r)$

i.e., $\quad r = 11\,\Omega$

In case of ac, as choke has no resistance, power loss in the choke will be zero while the bulb will consume,

$P = I^2 R = 10^2 \times 5 = 500$ W

However, in case of dc as resistance r is to be used instead of choke, the power loss in resistance r will be.

$PL = 10^2 \times 11 = 1100$ W

while the bulb will still consume 500 W, i.e., when the lamp is run on resistance r instead of choke more than double the power consumed by the lamp is wasted by the resistance r.

14. Impedance of the coil:

$$Z = \sqrt{R^2 + (2\pi f L)^2} = \sqrt{100^2 + 4\pi^2 (50)^2 (0.5)^2} = 186.2\,\Omega$$

Maximum current: $I_0 = \frac{E_0}{Z} = \frac{240\sqrt{2}}{186.2} = 1.82$ A

$$\tan\phi = \frac{X_C - X_L}{R} = \frac{0 - 2\pi(50)(0.5)}{100} = -\frac{\pi}{2}$$

$$\Rightarrow \phi = -\tan^{-1}\left(\frac{\pi}{2}\right) = -57.51° = -1.003 \text{ rad}$$

Let emf is maximum at t_1, then $\omega t_1 = \frac{\pi}{2}$

[we have $E = E_0 \sin \omega t$ and $I = I_0 \sin(\omega t + \phi)$]

Let current is maximum at t_2, then $\quad \omega t_2 + \phi = \frac{\pi}{2}$

From above we have $\omega(t_2 - t_1) = -\phi$

Time lag between voltage maximum and current maximum:

$$t_2 - t_1 = -\frac{\phi}{\omega} = \frac{1.003}{2\pi f} = 3.2 \times 10^{-3} \text{ s}$$

15. Impedance of circuit is

$$Z = \sqrt{(40)^2 + \frac{1}{(120\pi \times 100 \times 10^{-6})^2}} = 48\,\Omega$$

Maximum current: $I_0 = \frac{E_0}{Z} = \frac{110\sqrt{2}}{48} = 3.24$ A

[we have $E = E_0 \sin \omega t$ and $I = I_0 \sin(\omega t + \phi)$]

$$\tan\phi = \frac{X_C}{R} = \frac{1}{120\pi(100 \times 10^{-6})40}$$

$$\Rightarrow \phi = 33.5° = 0.186\pi$$

Required time lag: $t_0 = \frac{\phi}{\omega} = \frac{0.186\pi}{2\pi 60} = 1.55 \times 10^{-3}$ s

Objective Type

1. b. $E_v = \sqrt{120^2 + 160^2} = 200\text{ V}$

2. b. The period of sinusoidal voltage, $T = 0.4$ s.

$$f = \frac{1}{T} = \frac{1}{0.4} = 2.5\text{ Hz}$$

N lags M by 0.1 s which is equivalent to $\left(\frac{0.1}{0.4}\right)2\pi$ or $\frac{\pi}{2}$ rad.

Thus, the lead of N over M is $-\frac{\pi}{2}$ rad.

3. a. The equivalent primary load is

$$R_1 = \left(\frac{N_1}{N_2}\right)^2 R_2 = \left(\frac{20}{1}\right)^2 (6.0) = 2400\ \Omega$$

Current in the primary coil

$$= \frac{240}{R_1} = \frac{240}{2400} = 0.1\text{ A}$$

4. a. Here, $V_L = V_C$. They are in opposite phase. Hence, they will cancel each other. Now the resultant potential difference is equal to the applied potential difference = 100 V

$Z = R$ $\quad (\because\ X_L = X_C)$

$$\therefore \quad I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{R} = \frac{100}{50} = 2\text{ A}$$

5. c. According to the given question,

$$\tan 60° = \frac{\omega L}{R} \text{ and } \tan 60° = \frac{1/\omega C}{R}$$

$\therefore \quad \omega L = (1/\omega C)$(case of resonance)

$$\text{Now} \quad Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = 100\ \Omega$$

$$\therefore \quad I_{rms} = \frac{E_{rms}}{Z} = \frac{200\text{ V}}{100\ \Omega} = 2\text{ A}$$

6. d. Time for reaching maximum or peak value from 0

$$= \frac{T}{4} = \frac{1}{4} \times \frac{1}{50}\text{ s} = \frac{1}{200}\text{ s} = 5 \times 10^{-3}\text{ s}$$

$$I_0 = 10\sqrt{2}\text{ A} = 14.14\text{ A}$$

7. b. Given that $E_0 = 10$ V, $t = \frac{1}{600}$ s

$$\therefore \quad E = E_0 \cos 2\pi f t$$

$$= 10 \cos\left[2\pi \times 50 \times \frac{1}{600}\right]$$

$$= 10 \cos(\pi/6) = 10(\sqrt{3}/2) = 5\sqrt{3}\text{ V}$$

8. b. Wattless component of ac

$$= I_V \sin\phi = \frac{E_v}{Z}\frac{X_L}{Z} = \frac{E_v X_L}{Z^2} = \frac{220 \times \omega L}{(R^2 + \omega^2 L^2)}$$

As $\quad \omega L = 0.7 \times 2\pi \times 50 = 220\ \Omega$

Hence, wattless component of ac

$$= \frac{200 \times (220)}{(220^2 + 220^2)} = 0.5\text{ A}$$

9. a. $R = \frac{E}{I} = \frac{100}{1} = 100\ \Omega$

for ac: $\quad Z = [R^2 + (2\pi f L)^2]^{1/2}$

$$Z = \frac{E_v}{I_v} = \frac{100}{0.5} = 200\ \Omega$$

or $\quad 200 = [(100)^2 + (100\pi L)^2]^{1/2}$

Solving, we get $L = 0.55$ H

10. c. Clearly, $X_L = R$

or $L \times 2 \times 3.14 \times 1000 = 100$

$$\text{or} \quad L = \frac{100}{2 \times 3.14 \times 1000}\text{ H}$$

$$= 15.9 \times 10^{-3}\text{ H} = 15.9\text{ mH} \approx 16\text{ mH}$$

11. a. Current remains unchanged in R. However, it becomes half in L, because reactance is doubled on doubling the frequency.

12. c. The current of 1.6 A lags emf in phase by $\pi/2$. The current of 0.4 A leads emf in phase by $\pi/2$. So, these two currents are 180° out of phase with each other.

$\therefore$ Net current, $I_1 = (1.6 - 0.4)\text{ A} = 1.2\text{ A}$

$$1.6 = \frac{E_v}{X_L} \text{ and } 0.4 = \frac{E_v}{X_C}$$

$$\Rightarrow \frac{X_C}{X_L} = 4 \Rightarrow \frac{1}{\omega C \omega L} = 4$$

$$\Rightarrow \omega = \frac{1}{2\sqrt{LC}}$$

$$\Rightarrow f = \frac{\omega}{2\pi} = \frac{1}{4\pi\sqrt{LC}}$$

13. d. Reactance of the inductor O is given by $X_L = 2\pi f L$. Therefore, rms current through inductor L is I_2

$= \frac{V}{2\pi f L} \propto \frac{1}{f}$ where V is the rms of the supply voltage.

Reactance of the capacitor C is given by $X_C = \frac{1}{2\pi f C}$

Therefore, rms current through the capacitor is

$$I_3 = \frac{V}{X_C} = 2\pi f C V \propto f$$

The total current I_1 is given by $I_1 = I_2 + I_3$

$$= \left(\frac{V}{2\pi f L} - 2\pi f C V\right)$$

Thus, I_2 is best represented by curve R.
I_3 is best represented by curve P.
I_1 is best represented by curve Q.

14. d. $V_{6\Omega} = 3 = 6I_V \quad \therefore I_V = 0.5\,A$

$$I_V = \frac{1}{2} = \frac{5}{\sqrt{6^2 + X_L^2}},\ X_L = 8\,\Omega$$

Now, $V_L = I_V \cdot X_L = \frac{1}{2} \times 8 = 4\,V$

15. b. $X_C = \frac{1}{2\pi fC} = \frac{10^6}{2\pi 50 \times 500} = \frac{20}{11}\Omega$

$$X_L = 2\pi fL = 2\pi \times 50 \times 10 \times 10^{-3} = \pi\Omega$$

Since $X_L < X_C$, so inductive branch has less impedance, and so more current. Hence B_2 will be brighter.

16. a. Since the capacitor is connected in series to the resistor, current I_C from the supply and I_R through the resistor is in phase as represented by choice (a).

17. b. Root mean square current of the sinusoidal waveform,

$$I = \frac{I_0}{\sqrt{2}}.$$

Power output of the heater,

$$P = I^2R = \left(\frac{I_0}{\sqrt{2}}\right)^2 R = \frac{I_0^2 R}{2}$$

18. a. Concept based

19. d. Both V and I are in the same phase. So, let us calculate the time taken by the voltage to change from peak value to rms value. Now, $220 = 220 \sin 100\pi t_1$

or $100\pi t_1 = \frac{\pi}{2}$ or $t_1 = \frac{1}{200}$ s

Again, $\frac{220}{\sqrt{2}} = 220 \sin 100\pi t_2$

or $\frac{1}{\sqrt{2}} = \sin 100\pi t_2$ or $100\pi t_2 = \frac{3\pi}{4}$

or $t_2 = \frac{3}{400}$ s

Required time $= t_2 - t_1 = \frac{1}{400}\,s = 2.5 \times 10^{-3}$ s

20. c. Quality factor

$$= \frac{f_0}{f_2 - f_1} = \frac{600}{650 - 550} = \frac{600}{100} = 6$$

21. a. $E = E_0 \cos\omega t$

$\therefore \quad \omega = 50\pi$

$2\pi f = 50\pi \Rightarrow f = 25$ Hz

In one cycle ac current becomes zero twice.
Therefore, 50 times the current becomes zero in 1 s.

22. a. In an ac circuit, a pure inductor does not consume any power. Therefore, power is consumed by the resistor only.

$\therefore \quad P = I_v^2 R$

or $108 = (3)^2 R$ or $R = 12\,\Omega$

23. c. $\lambda = 300$ m, $c = 3 \times 10^8$ m/s

Frequency $f = \frac{c}{\lambda} = \frac{3 \times 10^8}{300} = 10^6$ Hz

Resonance frequency

$$f = \frac{1}{2\pi\sqrt{LC}} \quad \text{or} \quad L = \frac{1}{C4\pi^2 f^2}$$

$$\therefore \quad L = \frac{1}{4\pi^2 (10^6)^2 \times 2.4 \times 10^{-6}} \cong 10^{-8}\,H$$

24. a. Change on the capacitor,

$q_0 = CV = 1 \times 10^{-6} \times 1 = 10^{-6}$ C

or $I_0 = \omega q_0 =$ Maximum current

Now $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-9}}} = (10^9)^{1/2}$

$\therefore \quad I_0 = (10^9)^{1/2} \times (1 \times 10^{-6}) = \sqrt{10^{-3}}$ A $= \sqrt{1000}$ mA

25. b. $E = E_0 \sin\omega t$

Voltage read is rms value.

$\therefore \quad E_0 = \sqrt{2} \times 234\,V = 331\,V$

and $\omega t = 2\pi f t = 2\pi \times 50 \times t = 100\pi t$

Thus, the equation of the line voltage is given by

$$E = 331 \sin(100\pi t)$$

26. b. Here inductance and resistance are connected in series. We know that in case of resistance, both current and potential difference are in the same phase. In inductor, voltage leads current by $\pi/2$.

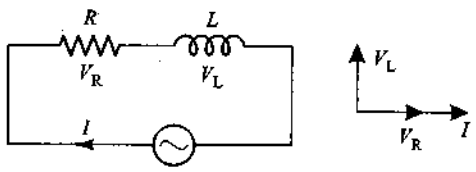


Fig. 7.72

27. b. In an ac circuit, capacitor does not consume any power. Therefore, power is consumed by the resistor only.

$\therefore \quad P = I_v^2 R$ or $100 = (2)^2 R$ or $R = 25\,\Omega$

28. c. Impedance $Z = \frac{E_v}{I_v}$ or $Z = \frac{200}{2} = 100\,\Omega$

But $Z^2 = R^2 + \left(\frac{1}{\omega C}\right)^2$

or $\left(\frac{1}{\omega C}\right)^2 = Z^2 - R^2 = (100)^2 - (25)^2 = 125 \times 75$

or $X_C = \frac{1}{\omega C} = \sqrt{125 \times 75}\ \Omega$

29. d. $\frac{1}{\omega C} = \sqrt{125 \times 75}$

Here $\omega = 2\pi f = 2\pi \times 50 = 100\pi$ radian/sec

$\therefore$ $C = \frac{1}{100\pi\sqrt{125 \times 75}}$ F

30. b. Here $R = X_L = X_C$ ($\therefore$ voltage across them is same)
When capacitor is short circuited,

$$I = \frac{10}{\left(R^2 + X_L^2\right)^{1/2}} = \frac{10}{\sqrt{2}R}$$

$\therefore$ Potential drop across inductance $= IX_L = IR = 10/\sqrt{2}$ V

31. a. $R = \frac{125}{12.5} = 10\ \Omega$

$$X_L = \omega L = 2\pi f L = \frac{V}{I} = \frac{125}{10} = 12.5$$

$\therefore$ $X'_L = 2\pi L \times f' = 0.25 \times 40 = 10\ \Omega$

Impedance of the circuit

$$Z = \sqrt{R^2 + X_L^2} = 10\sqrt{2}\ \Omega$$

$\therefore$ Current $= \frac{100}{10\sqrt{2}} = 10/\sqrt{2}$ A

32. b. Given $I = 5 + 10\sin\omega t$

$$I_{eff} = \left[\frac{\int_0^T I^2 dt}{\int_0^T dt}\right]^{1/2} = \left[\frac{1}{T}\int_0^T (5 + 10\sin\omega t)^2\, dt\right]^{1/2}$$

$$= \left[\frac{1}{T}\int_0^T \left(25 + 100\sin\omega t + 100\sin^2\omega t\right)\right]^{1/2}$$

But as $\frac{1}{T}\int_0^T \sin\omega t\, dt = 0$

and $\frac{1}{T}\int_0^T \sin^2\omega t\, dt = \frac{1}{2}$

so $I_{eff} = \left[25 + \frac{1}{2} \times 100\right]^{1/2} = 5\sqrt{3}$ A

33. a. $X_L = L\omega = 5 \times 10^{-3} \times 2000 = 10\ \Omega$

$$X_C = \frac{1}{C\omega} = \frac{1}{50 \times 10^{-6} \times 2000} = \frac{100}{10}\Omega = 10\ \Omega$$

Since $X_L = X_C$, therefore

$$Z = R = 5.9 + 0.10 + 4 = 10\ \Omega$$

$$I_v = \frac{E_v}{Z} = \frac{20}{10}\text{ A} = 2\text{ A}$$

34. a. $P = E_v I_v \cos\phi;\ P = E_v \frac{E_v}{Z}\frac{R}{Z}$

or $P = \frac{E_v^2 R}{Z^2} = \frac{110 \times 110 \times 11}{22 \times 22} = 275$ W

35. b. $V_C^2 + V_R^2 = V^2$

$$50^2 + V_R^2 = 110^2 \Rightarrow V_R = \sqrt{160 \times 60} = 98\text{ V}$$

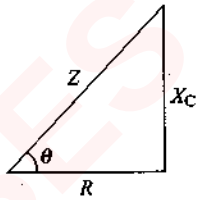


Fig. 7.73

Then, $I_V = \frac{\sqrt{160 \times 60}}{50}$ ($\therefore R = 50\ \Omega$)

Also, $I_V = \frac{110}{\sqrt{R^2 + X_C^2}}$ $\therefore$ $\frac{98}{50} = \frac{110}{\sqrt{50^2 + X_C^2}}$

Flux X_C and now using $X_C = \frac{1}{\omega C}$, we get $C = 104\ \mu$F

36. c. $V_R = I_v R = 1 \times 50 = 50\ \Omega$

$$V_L = \sqrt{E_v^2 - V_R^2} = \sqrt{220^2 - 50^2} = 214\text{ V}$$

37. a. $q_0 = CE_0 = 8 \times 10^{-6} \times 220\sqrt{2} = 2.5 \times 10^{-3}$ C

38. a. Resultant current is superposition of two currents, i.e., I (instantaneous total current) $= 6 + I_0 \sin\omega t$
dc ammeter will read average value

$$= \overline{6 + I_0 \sin\omega t} = 6 \qquad (\therefore\ \overline{I_0 \sin\omega t = 0})$$

ac ammeter will read

$$= \sqrt{\overline{(6 + I_0 \sin\omega t)^2}}$$

$$= \sqrt{\overline{36 + 12 I_0 \sin\omega t + I_0^2 \sin^2\omega t}}$$

$$(\therefore\ \overline{I_0 \sin\omega t = 0})$$

Since $\overline{\sin^2\omega t} = \frac{1}{2}$ and $I_{rms} = 8 = \frac{I_0}{\sqrt{2}} \Rightarrow I_0 = 8\sqrt{2}$ A

Therefore, ac reading

$$= \sqrt{36 + \frac{I_0^2}{2}} = \sqrt{36 + 64} = 10\text{ A}$$

39. c. $I = I_1 \cos\omega t + I_2 \sin\omega t$

$$(I^2)_{mean} = I_1^2\ \overline{\cos^2\omega t} + I_2^2\ \overline{\sin^2\omega t} + 2I_1 I_2\ \overline{\cos\omega t \sin\omega t}$$

$$= I_1^2 \times \frac{1}{2} + I_2^2 \times \frac{1}{2} + 2I_1 I_2 \times 0$$

$$I_{rms} = \sqrt{(I^2)_{mean}} = \sqrt{\frac{I_1^2 + I_2^2}{2}}$$

40. b. Resistance of bulb: $R = \frac{V_0^2}{P_0} = \frac{(220)^2}{1100} = 44\,\Omega$

when L is maximum, power consumed will be minimum

which is $P = \frac{1100}{5} = 220$ W

Now $P = I_v^2 R \Rightarrow 220 = I_v^2 \times 44 \Rightarrow I_v = \sqrt{5}$ A

$$I_v = \frac{E_v}{Z} \Rightarrow \sqrt{5} = \frac{220}{\sqrt{44^2 + X_L^2}} \Rightarrow X_L = 88\,\Omega$$

$$\Rightarrow 2\pi f\, L_{max} = 88 \Rightarrow 2 \times \frac{22}{7} \times 50\, L_{max} = 88 \Rightarrow L_{max} = 0.28 \text{ H}$$

41. c. $E_1 = E_0 \sin \omega t$; $E_2 = E_0 \sin[\omega t + (\pi/3)]$

$$E = E_2 + E_1$$
$$= E_0 \sin[\omega t + (\pi/3)] + E_0 \sin \omega t$$
$$= E_0 [2 \sin\{\omega t + (\pi/6)\} \cos(\pi/6)]$$
$$= \sqrt{3}\, E_0 \sin[\omega t + (\pi/6)]$$

42. d. If an ac source $E = E_0 \sin \omega t$ is applied across an inductance and capacitance in parallel, the current in inductance will lag the applied voltage while that across the capacitor will lead, and so,

$$I_L = \frac{E_0}{X_L} \sin\left(\omega t - \frac{\pi}{2}\right) = -0.8\sqrt{2} \cos \omega t$$

$$I_C = \frac{V}{X_C} \sin\left(\omega t + \frac{\pi}{2}\right) = 0.6\sqrt{2} \cos \omega t$$

So the current drawn from the source

$$I = I_L + I_C = -0.2\sqrt{2} \cos \omega t$$

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{0.2\sqrt{2}}{\sqrt{2}} = 0.2 \text{ A}$$

43. d. As current at any instant in the circuit will be

$$I = I_{dc} + I_{ac} = a + b \sin \omega t$$

so, $$I_{eff} = \left[\frac{\int_0^T I^2 dt}{\int_0^T dt}\right]^{1/2} = \left[\frac{1}{T}\int_0^T (a + b \sin \omega t)^2 dt\right]^{1/2}$$

i.e., $$I_{eff} = \left[\frac{1}{T}\int_0^T (a^2 + 2ab \sin \omega t + b^2 \sin^2 \omega t)\, dt\right]^{1/2}$$

but as $\frac{1}{T}\int_0^T \sin \omega t\, dt = 0$ and $\frac{1}{T}\int_0^T \sin^2 \omega t\, dt = \frac{1}{2}$

So, $$I_{eff} = \left[a^2 + \frac{1}{2}b^2\right]^{1/2}$$

44. c. The equation of a semi-circular wave is

$$x^2 + y^2 = a^2 \quad \text{or} \quad y^2 = a^2 - x^2$$

$$I_{rms} = \sqrt{\frac{1}{2a}\int_{-a}^{+a} y^2 dx}$$

$$I_{rms}^2 = \frac{1}{2a}\int_{-a}^{+a} (a^2 - x^2)\, dx$$

$$= \frac{1}{2a}\int_{-a}^{+a} (a^2 - x^2)\, dx = \frac{1}{2a}\left|a^2 x - \frac{x^3}{3}\right|_{-a}^{+a}$$

$$= \frac{1}{2a}\left(a^3 - \frac{a^3}{3} + a^3 - \frac{a^3}{3}\right) = \frac{2a^2}{3}$$

$$I_{rms} = \sqrt{\frac{2a^2}{3}} = \sqrt{\frac{2}{3}}\, a$$

45. b. $X_C = \frac{1}{\omega C} = \frac{1}{100 \times 10^{-6}} = 10^4\,\Omega$

So, $$I_{rms} = \frac{E_{rms}}{Z} = \frac{50\sqrt{2}}{\sqrt{2} \times 10^4} = 5 \text{ mA}$$

46. b. At resonance, the series combination of L and C gives zero impedance.

At resonance, the voltages across L and C are equal but opposite in phase.

47. b. As Resistance of the lamp

$$R = \frac{V_S^2}{P_0} = \frac{100^2}{50} = 200\,\Omega$$

The rms current $I = \frac{V}{R} = \frac{100}{200} = \frac{1}{2}$ A.

So when the lamp is put in series with a capacitance and run at 200 V ac, from $V = IZ$, we have

$$Z = \frac{V}{I} = \frac{200}{(1/2)} = 400\,\Omega$$

Now as in case of C–R circuit,

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

i.e., $$R^2 + \left(\frac{1}{\omega C}\right)^2 = 160000$$

or, $$\left(\frac{1}{\omega C}\right)^2 = 16 \times 10^4 - (200)^2 = 12 \times 10^4$$

$$\frac{1}{\omega C} = \sqrt{12} \times 10^2$$

$$C = \frac{1}{100\pi \times \sqrt{12} \times 10^2} \text{ F}$$

$$C = \frac{100}{\pi\sqrt{12}}\,\mu\text{F} = \frac{50}{\pi\sqrt{3}} = 9.2\,\mu\text{F}$$

48. b. Here, $X_L = \omega L = 2\pi f L = 2\pi \times 50 \times 1 = 100\,\pi\Omega$

$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 10 \times 10^{-6}} = \frac{10^3}{\pi}\Omega$$

So, $X = |X_L - X_C| = \left|100\pi - \frac{10^3}{\pi}\right| = \left|10^2\left[\frac{\pi^2 - 10}{\pi}\right]\right|\Omega$

49. a. $i_3 = i_1 + i_2 = 3\sin\omega t + 4\cos\omega t$

$= 3\sin\omega t + 4\sin(\omega t + 90°)$

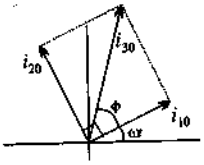


Fig. 7.74

$$i_{30} = \sqrt{i_{10}^2 + i_{20}^2} = \sqrt{3^2 + 4^2} = 5$$

$$\tan\phi = \frac{4}{3} \Rightarrow \phi = 53°$$

So $i_3 = i_{30}\sin(\omega t + \phi) = 5\sin(\omega t + 53°)$

50. a. Current leads emf so the circuit is $R - C$.

$\tan\phi = X_C / R$, $\phi = 45°$, $R = 1000\,\Omega$, $\omega = 100$

$C = ?$

Since $\tan 45° = \frac{1}{\omega CR}$, So $C = 10\,\mu F$

51. b. $V_{rms} = \sqrt{16^2 + 20^2} = 25.6$ V

52. c. $i = 3\sin\omega t + 4\cos\omega t$

$$= 5\left[\frac{3}{5}\sin\omega t + \frac{4}{5}\cos\omega t\right] = 5[\sin(\omega t + \delta)]$$

rms value $= \frac{5}{\sqrt{2}}$ A

If voltage applied is $V = V_m \sin\omega t$ then I, given by Eq. (i), indicates that it is ahead of V by δ where $0 < \delta < 90$ which indicates that the circuit contains R and C.

If we find average value for first half period (0 to $T/2$), then it will be $6/\pi$ A. But for different time interval, it will be different.

53. a. $v = v_0 \sin[\omega t + (\pi/4)]$

$= v_0 \cos[\omega t - (\pi/4)]$

Since V lags current, an inductor can bring it in phase with current.

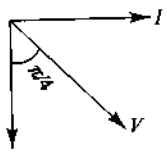
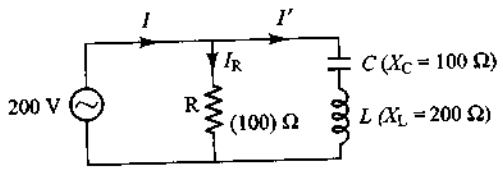


Fig. 7.75

54. b.



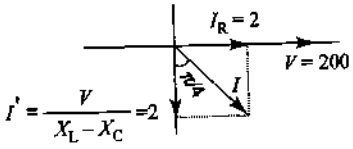


Fig. 7.76

$$I_R = \frac{V}{R} = \frac{200}{100} = 2\text{ A}$$

$$I' = \frac{V}{X_L - X_C} = \frac{200}{100} = 2\text{ A}$$

$$I = \sqrt{I_R^2 + I'^2} = 2\sqrt{2}\text{ A}$$

55. c. The circuit will have inductive nature if

$$\omega > \frac{1}{\sqrt{LC}}\left(\omega L > \frac{1}{\sqrt{LC}}\right)$$

Hence (a) is false. Also, if circuit has inductive nature the current will lag behind voltage. Hence (d) is also false.

If $\omega = \frac{1}{\sqrt{LC}}\left(\omega L = \frac{1}{\omega C}\right)$ the circuit will have resistance nature. Hence (b) is false.

Power factor, $\cos\phi = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = 1$

If $\omega L = \frac{1}{\omega C}$. Hence (c) is true.

56. c. $X_L = X_C$ at resonance

$\frac{X_L}{X_C} = 1$ for both circuits.

Impedance may be different if applied voltage is different.

57. d. Since, $\cos\phi = \frac{R}{Z} = \frac{IR}{IZ} = \frac{8}{10} = \frac{4}{5}$

(Also $\cos\phi$ can never be greater than 1)

Hence (c) is wrong.

Also $IX_C > IX_L \Rightarrow X_C > X_L$

$\therefore$ Current will be leading.

In an LCR circuit,

$V = \sqrt{(v_L - v_C)^2 + v_R^2} = \sqrt{(6-12)^2 + 8^2}$

$V = 10$; which is less than voltage drop across capacitor.

58. c. Since current is leading emf, capacitor must be present in the circuit. Along with capacitor there can be either (i) only resistor or (ii) resistor and inductor if $X_C > X_L$.

In both of the above conditions, we can obtain a phase difference of $\pi/4$ with current leading emf.

59. a. $V = \frac{V_0}{T/4}t$; $V = \frac{4V_0}{T}t$

$$V_{rms} = \sqrt{\langle V^2 \rangle} = \frac{4V_0}{T}\sqrt{\langle t \rangle}$$

$$= \frac{4V_0}{T}\left\{\frac{\int_0^{T/4} t^2 dt}{\int_0^{T/4} dt}\right\}^{1/2} = \frac{V_0}{\sqrt{3}}$$

60. d. $< V > = \dfrac{\int_0^T V dt}{\int_0^T dt} = 0$

61. b. $(0.5)^2 R\left(\frac{T}{2}\right) + (0.5)^2 R\left(\frac{T}{2}\right) = I^2_{rms} RT$

or $I_{rms} = \frac{1}{2}\text{A} = 0.5\text{ A}$

62.c. $\cos\phi = \dfrac{R}{\sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}}$

Putting the values, $C = 500\ \mu\text{F}$

63. a. Given:
Voltage in primary coil, $V_P = 200$ V
Current in primary coil, $i_p = 2$ A
Voltage in secondary coil, $V_s = 2000$ V
The relation for the current in the secondary coil is

$$\frac{V_s}{V_p} = \frac{i_p}{i_s} \Rightarrow \frac{2000}{200} = \frac{2}{i_s} = \frac{2\times 200}{2000} = 0.2\text{ A}$$

Multiple Correct Ansers Type

1. a., c.

$$f_{res} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{\frac{1}{\pi}\times\frac{1}{\pi}\times 10^{-6}}}$$

At resonance, $Z = R$,

So current $I = \frac{V}{Z} = \frac{V}{R}$

When L and C are in series, voltage across capacitor and inductor is 180° out of phase.

2. a., b. (f = 900 Hz) < (f_{res} = 1000 Hz), so $X_C > X_L$, circuit becomes capacitive and current leads emf.
At resistance, impedance is minimum.
Not only at resonance, but at all conditions voltage across L and C differs in phase by 180° when connected in series.

3. a., b., c.
$H = I^2Rt/4.2$ (Heat produced in a conductor is independent of the nature of current. If the current flowing in a conductor changes its direction, then also heat being produced in it will not change). Hence, (a), (b) and (c) are true.

4. a., b., c.
The frequency

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \frac{1}{2\pi}\sqrt{\frac{1}{0.6\times 10\times 10^{-6}}} = 65\text{ Hz}$$

The current at resonance is

$$I_0 = \frac{200\text{ V}}{5\Omega} = 40\text{ A}$$

The quantity $V_0 = I_0\sqrt{R^2 + (\omega L)^2}$

$$= 40\sqrt{25^2 + (0.6\times 2\pi\times 65)^2} = 9.8\text{ kV}$$

$$V_1 = \frac{I_0}{\omega C} = \frac{40\times 10^5}{2\pi\times 65} = 9.8\text{ kV}$$

5. a., c., d. $V_R^2 + V_L^2 = 100^2$, $|V_C - V_L| = 120$

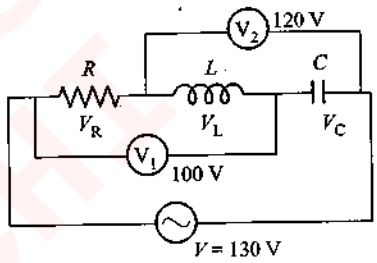


Fig. 7.77

$$130^2 = V_R^2 + (V_C - V_L)^2 \quad \Rightarrow \quad V_R = 50\text{ V}$$

$$\Rightarrow V_L = \sqrt{100^2 - 50^2} = 50\sqrt{3}\text{ V}$$

$$V_C = 120 + 50\sqrt{3}\text{ V}$$

$$\cos\phi = \frac{V_R}{V} = \frac{50}{130} = \frac{5}{13}$$

Since $V_C < V_L$, so circuit is capacitive.

Assertion-Reasoning Type

1. d. The maximum value of rms current $= \dfrac{\varepsilon_{rms}}{Z} = \dfrac{\varepsilon_{rms}}{R}$. It does not depend upon ω.

2. c. $P_{av} = \dfrac{VI\cos\phi}{2}$
At resonance condition, $\cos\phi = 1$
But $Z = R$
Which is minimum.

3. c. Voltage will be added vectorially.

4. b. Average power consumed by capacitor or inductor is zero.

5. b. Like direct current, alternating current also produces magnetic field. But the magnitude and direction of the field goes on changing continuously with time.

6. c. Both ac and dc produce heat, which is proportional to the square of current. The reversal of direction of current in ac is immaterial as far as production of heat is concerned.
7. d. Statement 1 is false because the given relation is true if all voltages are instantaneous.

Comprehension Type

For Problems 1 – 3

1. a., 2. b., 3. a.

Sol. 1. The peak value $V_0 = 220\sqrt{2} = 311\text{V}$

$$V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{220\sqrt{2}}{\sqrt{2}} = 200 \text{ V}$$

2. $V_{av} = \frac{2}{\pi}V_0 = \frac{2}{\pi}\times 311 = \frac{622}{\pi}\text{V}$

3. As $\omega = 2\pi f, 2\pi f = 314$, i.e., $f = \frac{314}{2\times\pi} = 50\text{Hz}$

For Problems 4 – 5

4. a., 5. b.

Sol. For finding the average value, we would find the total area of the trapezium and divide it by π (as shown in Fig. 7.57).

$$\text{Area} = 2\times\Delta OAE + \text{rectangle } ABDE$$

$$= 2\times(1/2)\times F_m\alpha + (\pi - 2\alpha)F_m = (\pi-\alpha)F_m$$

$$\text{Average value} = (\pi-\alpha)F_m/\pi$$

rms value : From similar triangles, we get

$$\frac{y}{\theta} = \frac{F_m}{\alpha} \text{ or, } y^2 = \frac{F_m^2}{\alpha^2}\theta^2$$

This gives the equation of the signal over the two triangles OAE and DBC. The signal remains constant over the angle α to $(\pi - \alpha)$, i.e., over an angular distance of $(\pi - \alpha) - \alpha = (\pi - 2\alpha)$.

Sum of the square

$$= \frac{2F_m^2}{\alpha^2}\int_0^\alpha \theta^2 d\theta + F_m^2(\pi - 2\alpha) = F_m^2\left(\pi - \frac{4\alpha}{3}\right)$$

The mean value of the squares

$$= \frac{1}{\pi}F_m^2\left(\pi - \frac{4\alpha}{3}\right) = F_m^2\left(1 - \frac{4\alpha}{3\pi}\right)$$

$$\text{rms valus} = F_m\sqrt{\left(1 - \frac{4\alpha}{3\pi}\right)}$$

For Problems 6 – 7

6. d., 7. a.

Sol. 6. Equation of line AB is given by $y = 10 + \left(\frac{10}{T}\right)t$

$$y_{av} = \frac{1}{T}\int_0^T y\,dt = \frac{1}{T}\int_0^T\left(10 + \frac{10}{T}t\right)dt = \frac{1}{T}\left|10t + \frac{5t^2}{T}\right|_0^T = 15$$

7. Mean square value

$$= \frac{1}{T}\int_0^T y^2 dt = \int_0^T\left(10 + \frac{10}{T}t\right)^2 dt$$

$$= \frac{1}{T}\int_0^T\left(100 + \frac{100}{T^2}t^2 + \frac{200}{T}t\right)$$

$$= \frac{1}{T}\left|100t + \frac{100t^3}{3T^2} + \frac{100t^2}{T}\right|_0^T = \frac{700}{3}$$

rms value = $10\sqrt{7/3}$

For Problems 8 – 9

8. b., 9. a.

Sol. Here,

$$X_L = \omega L = 2\pi fL = 2\pi\times 50\times 0.21 = 21\pi\,\Omega$$

So, $Z = \sqrt{R^2 + X_L^2} = \sqrt{12^2 + (21\pi)^2}$

$$= \sqrt{12^2 + (21\times 22/7)^2} = \sqrt{4500} = 30\sqrt{5}\ \Omega$$

So, (a) $I = \frac{V}{Z} = \frac{220}{30\sqrt{5}} = \frac{22}{3\sqrt{5}}\text{A}$

and (b) $\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{21\pi}{12}\right) = \tan^{-1}\left(\frac{7\pi}{4}\right)$

i.e., the current will lag the applied voltage.

For Problems 10 – 11

10. a., 11. c.

Sol. When dc is applied, $R = \frac{V}{I} = \frac{100}{1} = 100\,\Omega$

and when ac of 50 Hz is applied

$$I = \frac{V}{Z} \quad \text{i.e.,} \quad Z = \frac{V}{I} = \frac{100}{0.5} = 200\,\Omega$$

but, $Z = \sqrt{R^2 + \omega^2 L^2}$

i.e., $\omega^2 L^2 = Z^2 - R^2$

i.e., $(2\pi f L)^2 = 200^2 - 100^2 = 3\times 10^4$ (as $\omega = 2\pi f$)

So, $L = \frac{\sqrt{3}\times 10^2}{2\pi\times 50} = \frac{\sqrt{3}}{\pi}\text{H} = 0.55\text{H}$

For Problems 12 – 15

12. a., 13. d., 14. b., 15. d.,

Sol. 12. As this circuit is a series *LCR* circuit, current will be maximum at resonance, i.e.,

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(4.9\times10^{-3})(10^{-6})}} = \frac{10^5}{7} \text{ rad/s}$$

with, $I = \frac{V}{R} = \frac{10}{(32+68)} = \frac{1}{10}$ A

So the impedance, $Z_P = [R_1^2 + (1/\omega C)^2]^{1/2} = \sqrt{5924} = 77\,\Omega$

13. $Z_Q = [R_2^2 + (\omega L)^2]^{1/2}$

$$= [(68)^2 + (4.9\times10^{-3}\times10^5/7)^2]^{1/2} = \sqrt{9524} = 97.6\,\Omega$$

14. $V_P = IZ_P = \frac{1}{10}\times(77) = 7.7$ V

15. $V_Q = IZ_Q = \frac{1}{10}\times(97.6) = 9.76$ V

For Problems 16 – 18

16. b., 17. a., 18. c.

Sol. At resonance as $X = 0$, $I = \frac{V}{R} = \frac{60}{120} = \frac{1}{2}$ A

as $V_L = IX_L = I\omega L$, $L = \frac{V_L}{I\omega}$

So, $L = \frac{40}{(1/2)\times4\times10^5} = 0.2$ mH

$$\omega = \frac{1}{\sqrt{LC}} \Rightarrow C = \frac{1}{L\omega^2}$$

i.e., $C = \frac{1}{0.2\times10^{-3}\times(4\times10^5)^2} = \frac{1}{32}\,\mu$F

Now in case of series *LCR* circuit,

$$\tan\phi = \frac{X_L - X_C}{R}$$

So current will lag the applied voltage by 45° if,

$$\tan 45° = \frac{\omega L - \frac{1}{\omega C}}{R}$$

$$1\times120 = \omega\times2\times10^{-4} - \frac{1}{\omega(1/32)\times10^{-6}}$$

$$\omega^2 - 6\times10^5\omega - 16\times10^{10} = 0$$

i.e., $\omega = \frac{6\times10^5 \pm \sqrt{(6\times10^5)^2 + 64\times10^{10}}}{2}$

i.e., $\omega = \frac{6\times10^5 + 10\times10^5}{2} = 8\times10^5$ rad/s

For Problems 19 – 20

19. c., 20. a.

Sol. 19. When dc is applied, $I = \frac{V}{R}$, i.e., $R = \frac{12}{4} = 3\,\Omega$

and when ac is applied, $Z = \left(\frac{V}{I}\right) = \left(\frac{12}{2.4}\right) = 5\,\Omega$

$R^2 + X_L^2 = 5^2$ (as $Z = \sqrt{R^2 + X_L^2}$)

So, $X_L^2 = 5^2 - R^2 = 5^2 - 3^2 = 4^2$, i.e., $X_L = 4\Omega$

but as, $X_L = \omega L$, $L = \frac{X_L}{\omega} = \frac{4}{50} = 0.08$ H

20. Now when the capacitor is connected to the above circuit in series,

$$X_C = \frac{1}{\omega C} = \frac{1}{50\times2500\times10^{-6}} = \frac{10^3}{125} = 8\,\Omega$$

so, $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{3^2 + (4-8)^2} = 5\,\Omega$

and hence, $I_{rms} = \frac{V}{Z} = \frac{12}{5} = 2.4$ A

So, $P_{av} = V_{rms}I_{rms}\cos\phi = (I_{rms}\times Z)\times I_{rms}\times\left(\frac{R}{Z}\right)$

i.e., $P_{av} = I_{rms}^2 R = (2.4)^2\times3 = 17.28$ W

For Problems 21 – 22

21. a., 22. a.

Sol. $Z^2 = (X_C - X_L)^2 + R^2 = (31.85 - 6.28)^2 + (50)^2 = 3154$

$$P = \left(\frac{E_{rms}^2}{Z^2}\right)R = \frac{(10/\sqrt{2})^2}{3154}\times50 = 0.8 \text{ W}$$

Heat produced in 20 min $= (0.8)(20\times60) = 960$ J

$X_C - X_L = 31.85 - 2(6.28) = 19.29$

$$I_m = \frac{10}{19.29} = 0.52$$

Hence, $I = 0.52\sin(314t + \pi/2) = 0.52\cos 314t$

For Problems 23–25

23. a., 24. a., 25. d.

Sol. $d\phi = BdA$

$$d\phi = \left[\frac{\mu_0 I}{2\pi x} + \frac{\mu_0 I}{2\pi(3a - x)}\right]adx$$

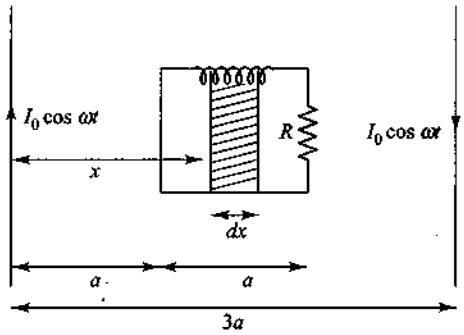


Fig. 7.78

$$\phi = \frac{\mu_0 I}{2\pi}\left[\int_a^{2a}\frac{dx}{x} + \int_a^{2a}\frac{dx}{(3a-x)}\right]a; \quad \phi = \frac{\mu_0 I a}{\pi}\ln 2$$

Magnitude of emf in this circuit:

$$\varepsilon = \left|\frac{d\phi}{dt}\right| = \frac{\mu_0 a(\ln 2)}{\pi}\left|\frac{dI}{dt}\right|$$

$$\varepsilon = \frac{\mu_0 a \ln 2}{\pi} I_0 \omega \sin \omega t$$

ac current, $I = \dfrac{\mu_0 a \ln 2 I_0 \omega}{\pi\sqrt{R^2+\omega^2 L^2}}\sin(\omega t - \phi)$

Matching Column Type

1. i. → b.,c., ii. → b.,c., iii. → a.,b.,c.,d., iv. → b.,c.,d.

i. $L = \dfrac{\mu N^2 A}{\ell}$

So, L depends upon shape, size (l, A) and medium (μ) inserted.

ii. $C = \dfrac{\varepsilon A}{d}$

So, C depends upon shape, size (A, d) and medium (e) inserted.

iii. $Z = \sqrt{R^2 + x_L^2}$

When $x_L = \omega L$, ω depends upon the external voltage source. So, Z depends upon all the factors in column II.

iv. $X_C = \dfrac{1}{\omega C}$ (independent of resistivity)

2. i. → d., ii. → a., d., iii. → b.,d., iv. → b.

i. For sinusoidal curve $i_{rms} = \dfrac{i_0}{\sqrt{2}}$ and average current for positive half cycle is $\dfrac{2i_0}{\pi}$

ii. Let us find the average current (or rms current) for $t = 0$ to $T/4$. Then it will also be average current (or rms current) for half cycle, i.e., from $t = 0$ to $T/2$

For $t = 0$ to $T/4$: $i = \dfrac{4i_0}{T}t$

then $i_{av} = \dfrac{\int_0^{T/4} i\,dt}{T/4} = \dfrac{\int_0^{T/4}\frac{4i_0}{T}t\,dt}{T/4} = \dfrac{i_0}{2}$

and $i_{rms} = \sqrt{\dfrac{\int_0^{T/4} i^2 dt}{T/4}} = \sqrt{\dfrac{64 i_0^2}{T^3}\int_0^{T/4} t^2 dt} = \dfrac{i_0}{\sqrt{3}}$

Full cycle average current is zero.

iii. For positive half cycle, average current

$$= \frac{\int i\,dt}{\int dt} = \frac{i_0(T/2)}{T/2} = i_0$$

Full cycle average current is zero.

iv. Average current for positive half cycle is i_0.
For full cycle, average current

$$= \frac{\int i\,dt}{\int dt} = \frac{i_0(T/2)+0}{T} = \frac{i_0}{2}$$

and $i_{rms} = \sqrt{\dfrac{\int_0^{T/2} i_0^2 dt}{T}} = \dfrac{i_0}{\sqrt{2}}$

3. i → b., c., ii. → a., d., iii → a., c., iv. → b., d.

i. $\tan\phi = \dfrac{1/\omega C}{R} = 1 \Rightarrow \phi = \dfrac{\pi}{4}$, current leads source voltage because reactance is capacitive.

ii. Pure inductive circuit $\phi = \dfrac{\pi}{2}$, current lags behind source voltage because reactance is inductive

iii. as $R = 0$, $\tan\phi = \infty$ $(X_C > X_L)$
$\phi = \pi/2$, current leads source voltage because reactance is capacitive

iv. $\tan\phi = \dfrac{\omega L}{R} = 1 \Rightarrow \phi = \dfrac{\pi}{4}$, current lags behind source voltage because reactance is inductive

4. i. → b., d., ii. → a., iii. → a., iv. → b., c.

$$X_L = \omega L = 2\pi \times 50 \times \frac{100}{\pi}\times 10^{-3} = 10\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi \times 50 \times \frac{100}{\pi}\times 10^{-6}}\ \Omega = 100\ \Omega$$

Impedence of the circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = 134.5\ \Omega$$

rms value of the current through the circuit is

$$I_{rms} = \frac{220}{Z} = 1.63\ \text{A}$$

rms value of voltage drop across the capacitor is
$V_{Cv} = I_{rms} X_C = 163$ V

Average power dissipated in the resistor is

$$P_{av} = I_{rms}^2 R = 1.63^2 \times 100 = 265.7\ \text{W}$$

Average power dissipated in inductor and capacitor would be zero.

5. i. → a., b., ii. → c. iii. → a., b., c., d., iv. → d.

i. For LR series circuit, the phasor diagram is as shown below

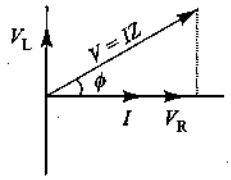


Fig. 7.79

I lags voltage by an angle $\phi (< \pi/2)$.

I lags V_L by an angle $\pi/2$.

ii. For RC series ac circuit, I leads V by an angle less than $\pi/2$.

iii. Depending on the value of L, C and R, circuit would be either capacitive, inductive or purely resistive. All possibilities can be there.

iv. In purely resistive circuit, current and voltage are in same phase.

Integer Answers Type

1. (i. 4, ii. 1) In such cases it is difficult to develop a single equation. Hence, it is usual to consider two equations, one applicable from 0 to 1 ms and another from 1 to 2 ms.
For t lying between 0 and 1 ms, $V_1 = 4$. For t lying between 1 and 2 ms, $V_2 = -4t + 4$.

i. $$V_{rms} = \sqrt{\frac{1}{2}\left(\int_0^1 V_1^2 dt + \int_1^2 V_2^2 dt\right)}$$

$$V_{rms}^2 = \frac{1}{2}\left[\int_0^1 4^2 dt + \int_1^2 (-4t+4)^2 dt\right]$$

$$= \frac{1}{2}\left[\left|16t\right|_0^1 + \left|\frac{16t^3}{3}\right|_1^2 + \left|16t\right|_1^2 - \left|\frac{32t^2}{2}\right|_1^2\right] = \frac{32}{3}$$

$$V_{rms} = \sqrt{\frac{32}{3}} = 4\sqrt{\frac{2}{3}}\text{ V}, \text{ Hence } N = 4$$

ii. $$V_{av} = \frac{1}{2}\left[\int_0^1 V_1 dt + \int_1^2 V_2 dt\right] = \frac{1}{2}\left[\int_0^1 4\,dt + \int_1^2 (-4t+4)\,dt\right]$$

$$= \frac{1}{2}\left[\left|4t\right|_0^1 + \left|-\frac{4t^2}{2} + 4t\right|_1^2\right] = 1\text{ V}$$

2. (5) From the $i - t$ graph (Fig. 7.63), area from $t = 0$ to $t = 2$ s

$$\frac{1}{2} \times 2 \times 10 = 10\text{ A s}$$

$$\therefore \text{Average current} = \frac{10}{2} = 5\text{ A}$$

3. (0) $$\langle i \rangle = \frac{\int_{\pi/2\omega}^{3\pi/2\omega} I_m \sin\omega t\, dt}{\frac{3\pi}{2\omega} - \frac{\pi}{2\omega}} = \frac{I_m\left(-\frac{\cos\omega t}{\omega}\right)_{\pi/2\omega}^{3\pi/2\omega}}{\frac{\pi}{\omega}} = 0$$

4. (5) Net current: $I = 3 + 4\sqrt{2}\sin\omega t$

Now find $$I_v = \sqrt{\frac{\int_0^T (3 + 4\sqrt{2}\sin\omega t)^2 dt}{T}} = 5\text{ A}$$

Archives

Subjective Type

1. Inductive reactance

$$X_L = \omega L = (50)(2\pi)(35 \times 10^{-3}) \approx 11\,\Omega$$

Impedance $Z = \sqrt{R^2 + X_L^2} = \sqrt{(11)^2 + (11)^2} = 11\sqrt{2}\,\Omega$

Given, $V_{rms} = 220$V

Hence, amplitude of voltage

$$V_0 = \sqrt{2}V_{rms} = 220\sqrt{2}\,V$$

Amplitude of current $i_0 = \frac{V_0}{Z} = \frac{220\sqrt{2}}{11\sqrt{2}} = 20$ A

Phase difference $\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{11}{11}\right) = \frac{\pi}{4}$

In L-R circuit voltage leads the current. Hence, instantaneous current in the circuit is,

$$i = (20A)\sin\left(\omega t - \frac{\pi}{4}\right)$$

Corresponding i–t graph is shown in the following figure.

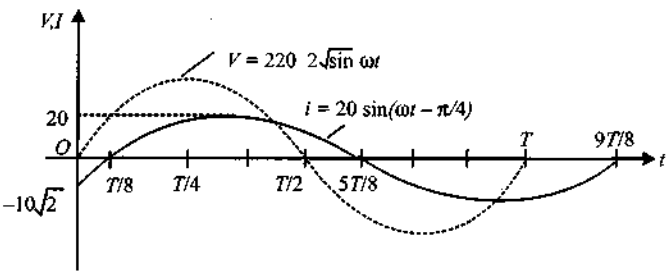


Fig. 7.80

Single Correct Answer Type

1. a. Since current leads emf (as seen from the graph), therefore this is an R–C circuit.

$$\tan\phi = \frac{X_C - X_L}{R} \quad \text{Here, } \phi = 45°$$

$\therefore \quad X_C = R \qquad [X_L = 0$ as there is no inductor$]$

$$\frac{1}{\omega C} = R$$

$$\text{L.H.S.} = \frac{1}{100\times10\times10^{-6}} = R = 10^3\ \Omega$$

2. b.

$$\text{Impedance } Z = \sqrt{\left(\frac{1}{\omega C}\right)^2 + R^2}$$

As ω increases, Z decreases. Hence, bulb will glow brighter.

Multiple Correct Answers Type

1. b., c. Case (A): $Z_A = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

Case (B): $Z_B = \sqrt{R^2 + \left(\frac{1}{\omega KC}\right)^2}$

So $Z_B < Z_A$

$I_R^A = \frac{V}{Z_A}$ and $I_R^B = \frac{V}{Z_B}$ clearly $I_R^A < I_R^B$

Since current in case (B) is greater, so p.d. across R will increase in case (B) and thus across capacitor will decrease.

Hence $V_C^A > V_C^B \qquad (\because V_R^2 + V_C^2 = V_0^2)$

Matching Column Type

1. **i → c., d., e.,** **ii → b., c., d., e.,**
ii → a., b., **iv → b., c., d., e.**

a. In this case, steady state current is zero ($I = 0$) because of the presence of capacitor. Entire potential will be across capacitor in steady state. Hence $V_1 = 0$ and $V_2 = V$.

b. In steady state $V_1 = 0$, so $V_2 = 0$. Also V_2 is proportional to I.

c. $X_L = \omega L = 2\pi fL = 2\pi\times50\times6\times10^{-3} \simeq 1.885\,\Omega$

$R = 2\,\Omega$

Since $R > X_L$, so $V_2 > V_1$

Here $I \neq 0$

V_1 and V_2 both are proportional to I.

d. $X_L = 1.885\,\Omega$,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC} = \frac{1}{2\pi\times50\times3\times10^{-6}} = 1061\ \Omega$$

Hence $X_C > X_L$, so $V_2 > V_1$

e. $X_C = 1.061\ \text{k}\Omega,\ R = 1\ \text{k}\Omega$

$X_C > R$, so $V_2 > V_1$

Integer Answer Type

1. (4) $\omega = 500$ rad/s

$$Z = \sqrt{\left(\frac{1}{\omega C}\right)^2 + R^2} = R\sqrt{1.25}$$

$$\left(\frac{1}{\omega C}\right)^2 + R^2 = R^2(1.25) \Rightarrow \left(\frac{1}{\omega C}\right)^2 = \frac{R^2}{4}$$

$$\Rightarrow CR = \frac{2}{\omega} = \frac{2}{500}\ \text{s} = \frac{2}{500}\times10^3\ \text{ms} = 4\ \text{ms}$$

CHAPTER
8

Miscellaneous Assignments, Appendix A1

R. K. MALIK'S NEWTON CLASSES RANCHI

EXERCISES

Single Correct Answer Type

1. A positive charge particle of mass m and charge q is projected with velocity v as shown in Fig. A1.1. If radius of curvature of charge particle in magnetic field region is greater than d, then find the time spent by the charge particle in magnetic field.

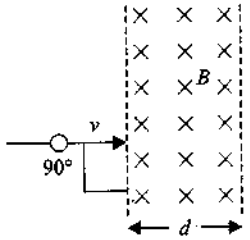


Fig. A1.1

a. $\frac{2m}{qB}\sin^{-1}\left(\frac{dqB}{mv}\right)$ b. $\frac{2m}{qB}\sin^{-1}\left(\frac{mv}{dqB}\right)$

c. $\frac{m}{qB}\sin^{-1}\left(\frac{dqB}{mv}\right)$ d. $\frac{m}{qB}\sin^{-1}\left(\frac{mv}{dqB}\right)$

2. A current carrying wire is placed in the grooves of an insulating semicircular disc of radius 'R', as shown in Fig. A1.2. The current enters at point A and leaves from point B. Determine the magnetic field at point D.

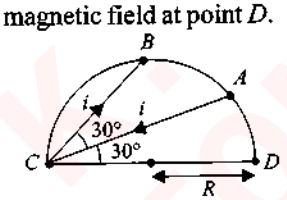


Fig. A1.2

a. $\frac{\mu_0 i}{8\pi R\sqrt{3}}$ b. $\frac{\mu_0 i}{4\pi R\sqrt{3}}$

c. $\frac{\sqrt{3}\mu_0 i}{4\pi R}$ d. none of these

3. Determine the magnetic field at the centre of the current carrying wire arrangement shown in Fig. A1.3. The arrangement extends to infinity. (The wires joining the successive squares are along the line passing through the centre.)

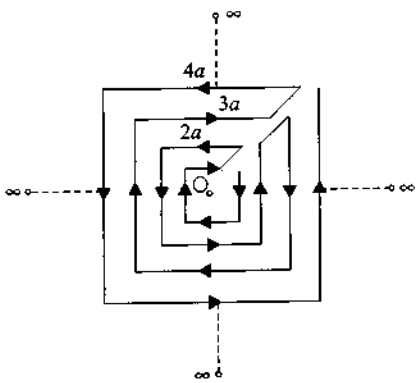


Fig. A1.3

a. $\frac{\mu_0 i}{\sqrt{2}\pi a}$ b. 0

c. $\frac{2\sqrt{2}\mu_0 i}{\pi a}\ln 2$ d. none of these

4. A uniform magnetic field $\vec{B} = 3\hat{i} + 4\hat{j} + \hat{k}$ exists in region of space. A semicircular wire of radius 1 m carrying current 1 A having its centre at (2, 2, 0) is placed in x-y plane as shown in Fig. A1.4. The force on semicircular wire will be

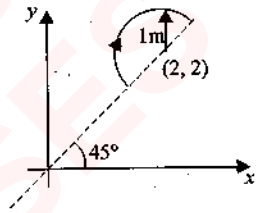


Fig. A1.4

a. $\sqrt{2}(\hat{i} + \hat{j} + \hat{k})$ b. $\sqrt{2}(\hat{i} - \hat{j} + \hat{k})$

c. $\sqrt{2}(\hat{i} + \hat{j} - \hat{k})$ d. $\sqrt{2}(-\hat{i} + \hat{j} + \hat{k})$

5. An electron gun ejects electrons at an angle of 45° with magnetic field boundary as shown below. Find the angular deviation of electrons as it comes out of field.

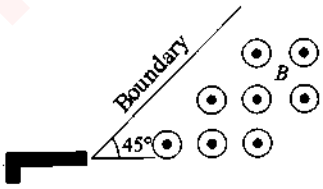


Fig. A1.5

a. 45° b. 90°

c. 60° d. None of these

6. In a certain region of space, there exists a uniform and constant electric field of strength E along x-axis and uniform constant magnetic field of induction B along z-axis. A charged particle having charge q and mass m is projected with speed v parallel to x-axis from a point $(a, b, 0)$. When the particle reaches a point $2a, b/2, 0$ its speed becomes $2v$. Find the value of electric field strength in terms of m, v and co-ordinates.

a. $\frac{3}{2}\frac{mv^2}{qa}$ b. $\frac{mv^2}{qB}$

c. $\frac{2mv^2}{qBa}$ d. $\frac{3}{2}vB$

7. A particle of specific charge $q/m = \pi$ C/kg is projected from the origin towards positive x-axis with a velocity of 10 m/s in a uniform magnetic field $\vec{B} = -2\hat{k}$ Tesla. The velocity $\vec{V}$ of the particle after time $t = 1/6$ s will be

a. $(5\hat{i} + 5\sqrt{3}\hat{j})$ m/s b. $10\hat{j}$ m/s

c. $(5\sqrt{3}\hat{i} - 5\hat{j})$ m/s d. $-10\hat{j}$ m/s

8. An α-particle and a proton are both simultaneously projected in opposite directions into a region of constant magnetic field perpendicular to the direction of the field. After some time it is found that the velocity of the α-particle has changed in a direction by 45°. Then at this time, the angle between velocity vectors of α-particle and proton is
   a. 90° b. 45° c. 135° d. none

9. The torque experienced by a given current carrying loop in a uniform magnetic field $\vec{B}$ given by $B_0(\vec{i}-\vec{j})$ would have magnitude

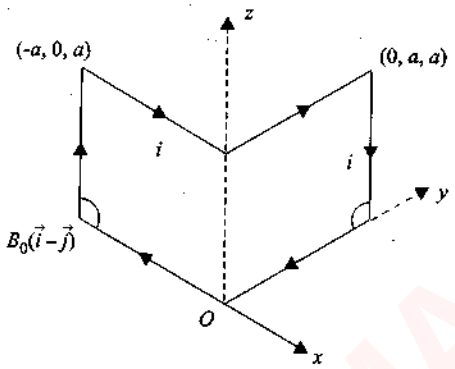


Fig. A1.6

   a. $\sqrt{2}B_0a^2I$ b. zero
   c. $2B_0a^2I$ d. B_0a^2I

10. A wire is wound on a long rod of material of relative permeability $\mu_r = 4000$ to make a solenoid. If the current through the wire is 5 A and number of turns per unit length is 1000 per metre, then the magnetic field inside the solenoid is
   a. 4πmT b. 8πmT c. 4πT d. 8πT

11. A uniform magnetic field B and electric field E exist along y and negative z axis respectively. Under the influence of these fields a charge particle moves along OA undeflected. If electric field is swithed off, find the pitch of helical trajectory in which the particle will move.

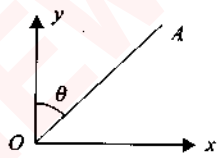
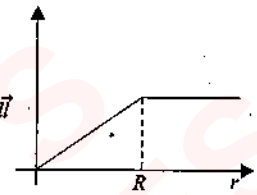


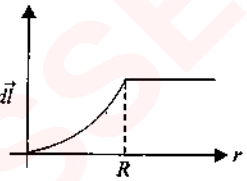
Fig. A1.7

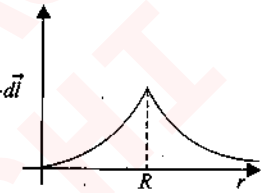
   a. $\dfrac{2\pi mE}{qB^2\cot\theta}$ b. $\dfrac{4\pi mE}{qB^2\tan\theta}$
   c. $\dfrac{4\pi mE}{qB^2\cot\theta}$ d. $\dfrac{2\pi mE}{qB^2\tan\theta}$

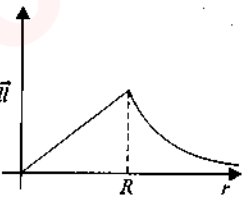
12. Axis of a solid cylinder of infinite length and radius R lies along y-axis. It carries a uniformly distributed current i along $+y$ direction. Magnetic field at a point $(R/2, y, R/2)$ is
   a. $\dfrac{\mu_0 I}{4\pi R}(\hat{i}-\hat{k})$ b. $\dfrac{\mu_0 I}{2\pi R}(\hat{j}-\hat{k})$
   c. $\dfrac{\mu_0 I}{4\pi R}\hat{j}$ d. $\dfrac{\mu_0 I}{4\pi R}(\hat{i}+\hat{k})$

13. A cylindrical wire of radius R is carrying uniformly distributed current i over its cross-section. If a circular loop of radius r is taken as amperian loop, then the variation value of $\oint \vec{B}\cdot\vec{d\ell}$ over this loop with radius 'r' of loop will be best represented by

   a.


   b.


   c.


   d.


14. A parabolic section of wire OA is located in the x-y plane and carries current $I = 12$ A. A uniform magnetic field $B = 4.0$ T making an angle 60° with x axis exists in x-y plane. Calculate the magnetic force on the wire OA. Co-ordinates of A are (0.25m, 1 m)

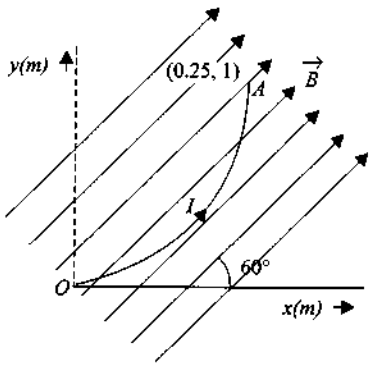


Fig. A1.8

   a. $6(\sqrt{3}-4)\hat{k}$ N b. $6(4-\sqrt{3})\hat{k}$ N
   c. $3(\sqrt{3}-4)\hat{k}$ N d. $3(4-\sqrt{3})\hat{k}$ N

15. Two coils of self inductance 100 mH and 400 mH are placed very close to each other. Find the maximum mutual inductance between the two when 4 A current passes through them.
a. 200 mH b. 300 mH
c. $100\sqrt{2}$ mH d. none of these

16. In a region at a distance r from z-axis, magnetic field $\vec{B} = B_0 rt\hat{k}$ is present where B_0 is constant and t is time. Then the magnitude of induced electric field at a distance r from z-axis is given by
a. $\frac{r}{2}B_0$ b. $\frac{r^2}{2}B_0$ c. $\frac{r^2}{3}B_0$ d. none

17. A small square loop of edge length a and resistance R is moved with velocity v_0 away from an infinitely long current carrying conductor carrying current i so that the conductor and side of square are always in same plane. Find induced current in loop at a separation of r.
a. $\frac{\mu_0 i a^2 v}{\pi r^2 R}$ b. $\frac{\mu_0 i a^2 v}{4\pi r^2 R}$
c. $\frac{\mu_0 i a^2 v}{2\pi r^2 R}$ d. $\frac{\mu_0 i a v}{2\pi r R}$

18. In Fig. A1.9 (a) and (b), two air-cored solenoids P and Q have been shown. They are placed near each other. In Fig. A1.9 (a), when I_P the current in P, changes at the rate of 5 A/s, an emf of 2 mV is induced in Q. The current in P is then switched off, and a current changing at 2 A/s is fed through Q as shown in diagram. What emf will be induced in P?

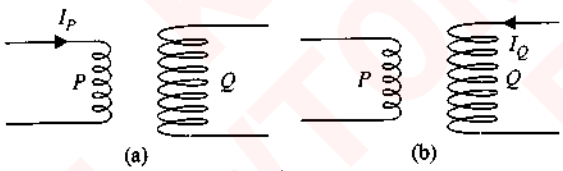


Fig. A1.9

a. 8×10^{-4} V b. 2×10^{-3} V
c. 5×10^{-3} V d. 8×10^{-2} V

19. Radius of a circular ring is changing with time and the coil is placed in uniform magnetic field perpendicular to its plane. The variation of 'r' with time 't' is shown in Fig. A1.10.

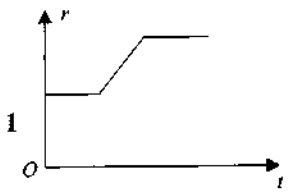
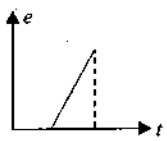
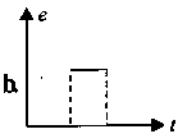
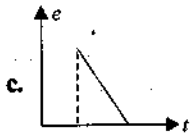


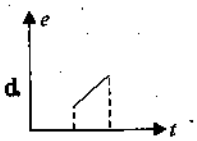
Fig. A1.10

Then induced emf e with time t will be best represented by

a. 

b. 

c. 

d. 

20. A triangular wire frame (each side = 2 m) is placed in a region of time variant magnetic field $dB/dt = \sqrt{3}$ T/s. The magnetic field is perpendicular to the plane of the triangle and its centre coincides with the centre of triangle. The base of the triangle AB has a resistance 1 Ω while the other two sides have resistance 2 Ω each. The magnitude of potential difference between the points A and B will be

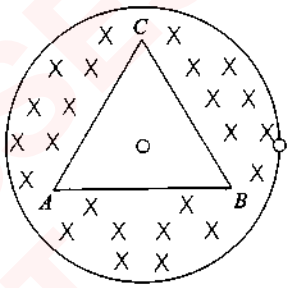


Fig. A1.11

a. 0.4 V b. 0.6 V c. 1.2 V d. None

21. A metallic square loop $PQRS$ is moving in its own plane with velocity v in a uniform magnetic field perpendicular to its plane as shown in Fig. A1.12. If V_P, V_Q, V_R and V_S are the potentials of points P, Q, R and S then which of the following is an incorrect statement?

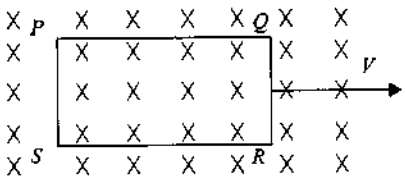


Fig. A1.12

a. $V_P = V_Q$ b. $V_P > V_S$
c. $V_P > V_R$ d. $V_S > V_R$

22. Switch S is closed at $t = 0$, in the circuit shown. The change in flux in the inductor (L = 500 mH) from $t = 0$ to an instant when it reaches steady state is

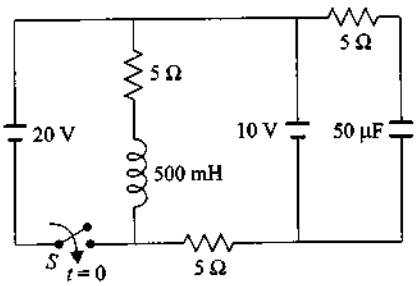


Fig. A1.13

a. 2 Wb b. 1.5 Wb c. 0 Wb d. none

23. An AC source rated 100 V (rms) supplies a current of 10 A (rms) to a circuit. The average power delivered by the source:
a. must be 1000 W

b. may be greater than 1000 W
c. may be less than 1000 W
d. all of the above three are possible

24. The voltage of an AC source varies with time according to the relation: $E = 120 \sin 100\pi t \cos 100\pi t$ V. What is the peak voltage of the source?

a. 60 V b. 120 V c. 30 V d. $\sqrt{2}$ V

25. The figure below shows a battery with emf 15 V in a circuit with $R_1 = 30\,\Omega$, $R_2 = 10\,\Omega$, $R_3 = 20\,\Omega$ and $L = 3.0$ H. The switch S is initially in the open position and is then closed at time $t = 0$. Then the graph which shows the correct variation of current through battery after switch S is closed

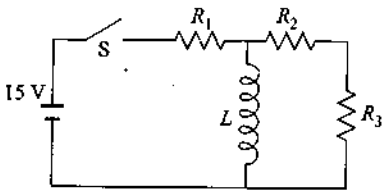
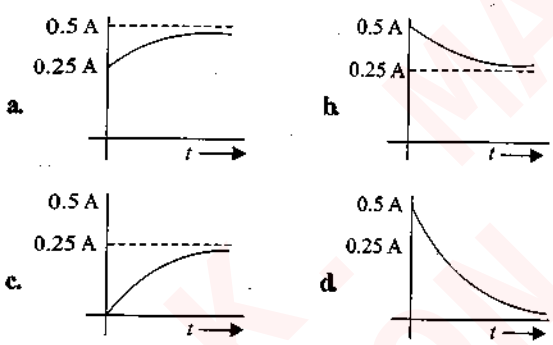


Fig. A1.14



26. A plane loop is shaped as two squares (as shown) and placed in a uniform magnetic field at right angle to the loop's plane. The magnetic induction varies with time as $B = B_0 \sin \omega t$, where $B_0 = 10$ mT and $\omega = 100$ rads^{-1}. The wires do not touch at point A. If resistance per unit length of the loop is 50 mΩ/m, then amplitude of current induced in the loop is

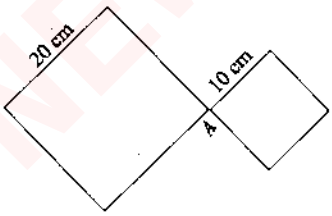


Fig. A1.15

a. 1.5 A b. 1.0 A c. 0.5 A d. 2.0 A

27. Two infinitely long, thin, insulated, straight wires lie in the x–y plane along the x- and y-axes respectively. Each wire carries a current I, respectively in the positive x-direction and positive y-direction. The magnetic field will be zero at all points on the straight line:

a. $y = x$ b. $y = -x$ c. $y = x - 1$ d. $y = -x + 1$

28. Two infinitely long straight parallel wires carry equal currents 'i' in the same direction and are 2 m apart. Magnetic induction at a point which is at same normal distance $\sqrt{2}$ m from each wire has magnitude

a. $\frac{\mu_0 i}{2\pi}\sqrt{2}$ b. $\frac{\mu_0 i}{4\pi}\sqrt{2}$

c. $\frac{\mu_0 i}{2\sqrt{2}\pi}$ d. $\frac{\mu_0 i}{2\pi}$

29. Two infinitely long straight parallel wires are 5 m apart, perpendicular to the plane of paper. One of the wires, as it passes perpendicular to the plane of paper, intersects it at A and carries current 'i' in the downward direction. The other wire intersects the plane of paper at point B and carries current 'i' in the outward direction O in the plane of paper as shown in Fig. A1.16. With x and y axes as shown, magnetic induction at O in the component form can be expressed as

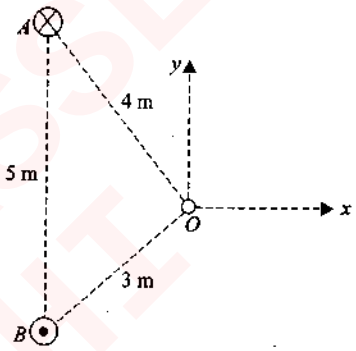


Fig. A1.16

a. $\vec{B} = \frac{\mu_0 i}{2\pi}\left[-\hat{i} + \frac{3\hat{j}}{5}\right]$ b. $\vec{B} = \frac{\mu_0 i}{5\pi}\left[-\hat{i} + \frac{7\hat{j}}{24}\right]$

c. $\vec{B} = \frac{\mu_0 i}{4\pi}\left[-3\hat{i} + \frac{7\hat{j}}{18}\right]$ d. $\vec{B} = \frac{\mu_0 i}{7\pi}\left[-2\hat{i} + \frac{3\hat{j}}{5}\right]$

30. In Fig. A1.17, AB is a non-conducting rod. Equal charges of magnitude q are fixed at various points on the rod as shown. The rod is rotated uniformly about an axis passing through O and perpendicular to its length such that linear speed at the end A or B of the rod is 3 m/s. Magnetic field at O is

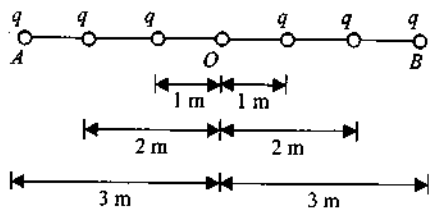


Fig. A1.17

a. $\frac{11\mu_0 q}{12\pi}$ b. $\frac{3\mu_0 q}{7\pi}$

c. $\frac{\mu_0 q}{2\pi}$ d. $\frac{6\mu_0 q}{13\pi}$

31. Infinite wires, each carrying a current 0.5 A, are kept parallel to y-axis and intersecting x-axis at $x = \pm 1$ m, ± 2 m, ± 4 m, ± 8 m etc. Wires kept at positive values of x carry current in the same direction while wires kept at negative values of x carry current successively in opposite directions as shown

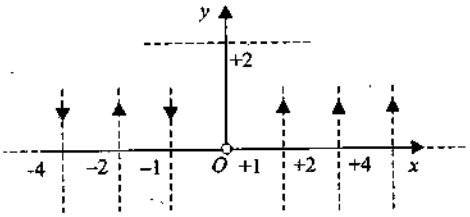


Fig. A1.18

A wire is kept parallel to x-axis and intersecting y-axis at $y = 2$ m. It carries a current 'i'. Assuming this wire to be insulated from others then the current 'i' in it such that magnetic induction at O is zero is:
(assume each wire to be infinitely long)

a. $\frac{8}{3}$A along positive x-axis

b. $\frac{4}{3}$A along negative x-axis

c. $\frac{5}{3}$A along negative x-axis

d. $\frac{7}{3}$A along positive x-axis

32. Three infinitely long wires, each carrying a current 1A, are placed such that one end of each wire is at the origin, and, one of these wires is along x-axis, the other along y-axis and the third along z-axis. Magnetic induction at point (–2 m, 0, 0) due to the system of these wires can be expressed as

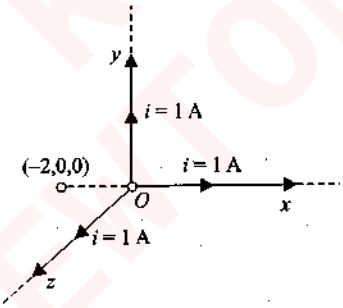


Fig. A1.19

a. $\frac{\mu_0}{4\pi}(\hat{j}+\hat{k})$ b. $\frac{\mu_0}{4\pi}(\hat{j}-\hat{k})$

c. $\frac{\mu_0}{8\pi}(-\hat{j}+\hat{k})$ d. $\frac{\mu_0}{8\pi}(\hat{j}+\hat{k})$

33. Two parallel wires P and Q placed at a separation $d = 6$ cm on x-axis carry electric current $i_1 = 5$ A and $i_2 = 2$ A in opposite directions as shown in Fig.A1.20. Find the point on the line PQ where the resultant magnetic field is zero.

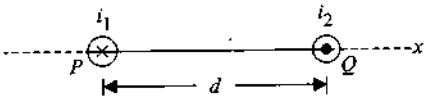


Fig. A1.20

a. 4 cm left of P b. 4 cm right of Q

c. middle of PQ d. At no position on x-axis

34. In Fig. A1.21, the conductors carry equal currents i. All straight segments are very long, and the two circular loops have equal radii. However, the currents around the loops have opposite senses. The ratio of the magnetic fields at a and b, at the centres of the two loops, is

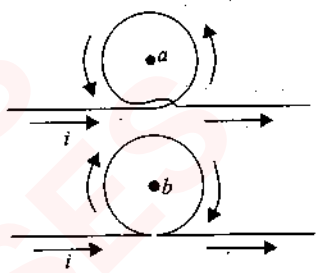


Fig. A1.21

a. $\frac{B_a}{B_b} = \frac{\pi+1}{-\pi+1}$ b. $\frac{B_a}{B_b} = \frac{\pi-1}{\pi+1}$

c. $\frac{B_a}{B_b} = \frac{\pi-2}{\pi+2}$ d. $\frac{B_a}{B_b} = \frac{\pi+2}{\pi-2}$

35. What is the magnitude of magnetic field at the centre 'O' of loop of radius $\sqrt{2}$ m made of uniform wire when a current of 1 A enters in the loop and is taken out of it by two long wires as shown in Fig. A1.22.

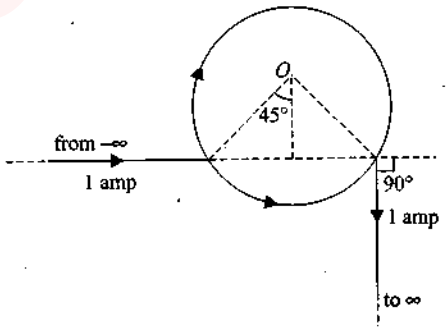


Fig. A1.22

a. $\frac{\mu_0}{\pi}\left[1-\frac{1}{\sqrt{2}}\right]T$ b. $\frac{\mu_0}{\sqrt{2}\pi}\left[1-\frac{1}{\sqrt{2}}\right]T$

c. zero d. None of these

36. Find the magnetic field at the origin in the figure shown below.

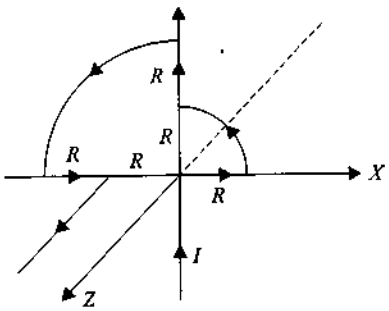


Fig. A1.23

a. $\frac{\mu_0 I}{4R}\left(\frac{3}{4}\hat{k}+\frac{1}{\pi}\hat{j}\right)$ b. $\frac{\mu_0 I}{4R}\left(\frac{3}{2}\hat{k}+\frac{1}{\pi}\hat{j}\right)$

c. $\frac{\mu_0 I}{4R}\left(\frac{3}{4}\hat{k}+\frac{1}{2\pi}\hat{j}\right)$ d. None of these

37. Find the magnitude of the magnetic induction B of a magnetic field generated by a system of thin conductors along which a current i is flowing at a point A (O, R, O), that is the centre of a circular conductor of radius R. The ring is in yz plane.

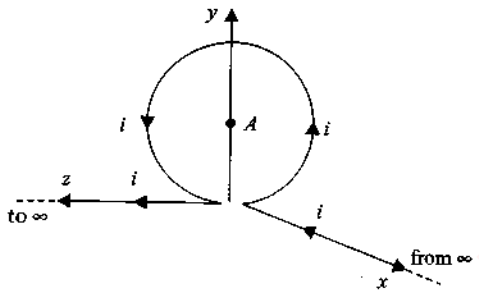


Fig. A1.24

a. $B=\frac{\mu_0 i}{4\pi R}\sqrt{(2\pi^2-2\pi+1)}$

b. $B=\frac{\mu_0 i}{4\pi R}\sqrt{2(2\pi^2-2\pi+1)}$

c. $B=\frac{\mu_0 i}{2\pi R}\sqrt{(2\pi^2-2\pi+1)}$

d. None of these

38. A charged particle (charge q, mass m) has velocity v_0 at origin in $+x$ direction. In space there is a uniform magnetic field B in $-z$ direction. Find the y coordinate of particle when is crosses y axis.

a. $\frac{mv_0}{qB}$ b. $\frac{2mv_0}{qB}$

c. $\frac{mv_0}{2qB}$ d. None of these

39. AB and AC are the boundary lines within which a magnetic field B exists. If the magnetic field is absent, a charged particle of mass m and charge q must have passed through a point P on angle bisector of $\angle BAC$, at a distance $r\sqrt{2}$ from A, if it has fallen on AB normally at a point Q such that $QP = r$. If the magnetic field is present, how much time the charged particle will take to come out of the magnetic field

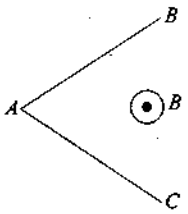


Fig. A1.25

a. $\frac{m\pi}{Bq}$ b. $\frac{2m\pi}{Bq}$

c. $\frac{4m\pi}{Bq}$ d. $\frac{m\pi}{2Bq}$

40. Two rectangular plates A and B placed at a distance '$2a$' apart, are connected to a battery to produce an electric field. There are insulators between plates C and other two plates. A magnetic field exists along z-axis. A charged particle of mass m and charge q passes through a hole at the middle of the plate A with velocity v and strikes at Q which is the middle of the bottom edge of plate B after passing through a hole in plate C. If $E=\frac{mv^2}{qa}$, what will be the speed of the particle at Q?

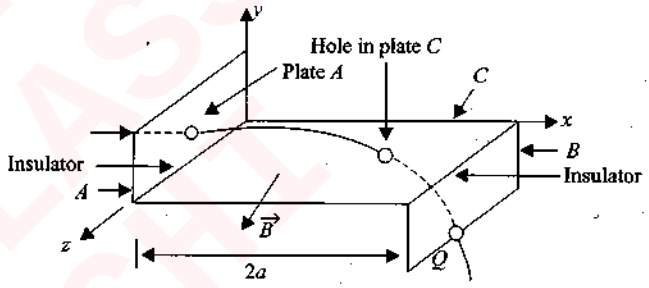


Fig. A1.26

a. $v\sqrt{2}$ b. $2v$

c. $v\sqrt{5}$ d. $v\sqrt{3}$

41. A magnetic field B exists between OA and OB. Inclined at an angle θ, a charged particle strikes at point A on surface OA, at a distance $(2K\cos q)/(qvB)$ from O, where K is kinetic energy, q is the charge and v is the velocity of the particle. At what angle with horizontal (measured from end B) will the charged particle emerge from OB?

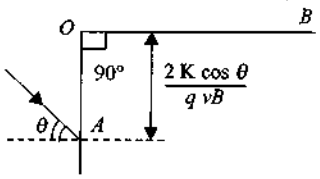


Fig. A1.27

a. θ b. $90°-\theta$

c. $90°+\theta$ d. $90°$

42. To the right of line PQ is a uniform magnetic field $\vec{B}$. B_1A is the line of incidence of a charged particle, which comes out of the field along DE, CA and DF are the normals at A and D. $\angle B_1AC = \theta$. Angle measured from CA in clockwise direction is taken as positive. What will be the value of θ so that angle subtended by the part of the circle (along which charged particle moves in the field) at its centre and facing the circle is less than π?

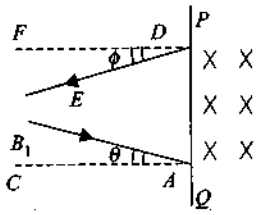


Fig. A1.28

a. θ is positive
b. $\theta = 0$
c. θ is negative
d. θ depends upon $\vec{B}$ and charge

43. In the previous problem, if O is the point on AD and OO_1 is the perpendicular from the centre of the circle, O_{-1}, then OA/OD will be:

a. 1 b. >1
c. <1 d. depend upon $\vec{B}$ and charge

44. In the previous problem, if ϕ is the angle between line of emergence DE and normal DF at point D, ratio of ϕ/θ for positive value of θ will be

a. 1 b. >1
c. <1 d. depends upon $\vec{B}$ and charge

45. In the previous problem, if time period, $T = 2\pi\dfrac{m}{BQ}$ where Q is the charge of the particle and m is its mass, the ratio of time spent by the particle in the field when θ is positive to when θ is negative is given by

a. $\left(\dfrac{\pi/2+\theta}{\pi/2-\theta}\right)$ b. $\left(\dfrac{\pi+\theta}{\pi-\theta}\right)$
c. $\left(\dfrac{\pi-\theta}{\pi+\theta}\right)$ d. $\left(\dfrac{\pi/2-\theta}{\pi/2+\theta}\right)$

46. In the previous problem, the maximum range of movement of the centre of the part of the circle from line AD in which charged particle of charge Q moves with a velocity v when θ is positive to when θ is negative is given by

a. $\pm\dfrac{mv}{2QB}$ b. $\pm\dfrac{mv}{QB}$
c. $\pm\dfrac{2mv}{QB}$ d. $\pm\dfrac{2mv}{3QB}$

47. A triangular system of mass 100 gm consisting of 3 wires of lengths, as shown in Fig. A1.29 and length of AO as 4 units, are placed in the magnetic field of 1T. The current of 1A flows through wire AO. The wires are of same material and cross-sectional area. In which direction will the system move?

a. at $\tan^{-1}(3/4)$ with $(-x)$ axis.
b. along x-axis
c. along y-axis
d. along $(-x)$ axis

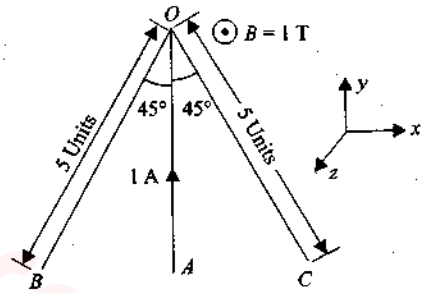


Fig. A1.29

48. In the previous problem, what will be force with which system moves? (Use $\sqrt{2} = 1.4$)

a. 3.05 N b. 4.0 N c. 0.5 N d. 1.0 N

49. In the previous problem, instead of being stationary, the system enters the magnetic field with a velocity 0.5 m/sec along y-direction. Along which direction the system will move now?

a. along $a\hat{i} + b\hat{j}$
b. along a circular path
c. system will become stationary
d. along a direction in x-y plane

50. In the previous problem, what will be acceleration of the system (in m/sec^2)?

a. $5\sqrt{2}$ b. $0.5\sqrt{2}$
c. zero d. 2.5

51. A conducting loop of radius R is present in a uniform magnetic field B perpendicular to the plane of the ring. If radius R varies as a function of time 't', as $R = R_0 + t$. The emf induced in the loop is

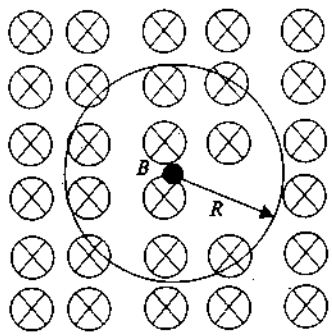


Fig. A1.30

a. $2\pi(R_0 + t)$B clockwise
b. $\pi(R_0 + t)$B clockwise
c. $2\pi(R_0 + t)$B anticlockwise
d. zero.

52. A wire loop is placed in a region of time varying magnetic field which is oriented orthogonally to the plane of the loop as shown in Fig. A1.31. The graph shows the magnetic field variation as the function of time. Assume the positive emf is the one which drives a current in the clockwise direction and seen by the observer in the direction of B. Which of the following graphs best represents the induced emf as a function of time ?

But $\Delta H = \Delta H_1 + \Delta H_2 = \frac{\Delta H_2}{2} + \Delta H_2 = \frac{3}{2}\Delta H_2$

$\therefore \quad \Delta H_2 = \frac{2}{3}\Delta H = \frac{2}{3} \times \frac{1}{2} C_0 E_0^2 = \frac{1}{3} C_0 E_0^2$

91. b. In the steady state, no current is passing through the capacitor. Let the charge on each capacitor be q.
Since the current through galvanometer is zero.

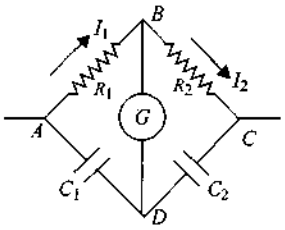


Fig. 5.298

$\therefore \quad I_1 = I_2$

The potential difference between the ends of the galvanometer will be zero.

$\therefore \quad V_A - V_B = V_A - V_D$

$$I_1 R_1 = \frac{q}{C_1} \quad \text{(i)}$$

Similarly, $V_B - V_C = V_D - V_C$

$$I_2 R_2 = \frac{q}{C_2} \quad \text{(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{I_1 R_1}{I_2 R_2} = \frac{q/C_1}{q/C_2} = \frac{C_2}{C_1} \Rightarrow \frac{C_1}{C_2} = \frac{R_2}{R_1}$$

92. b. As the capacitors are identical, each of them finally have charge $Q/2$.

Initial energy of the system $= E_i = \frac{Q^2}{2C}$

Final energy of the system $= E_f = 2\left[\frac{(Q/2)^2}{2C}\right] = \frac{Q^2}{4C}$

Heat produced = loss in energy $= E_i - E_f = \frac{Q^2}{4C}$

93. c. In the steady state, no current flow through C

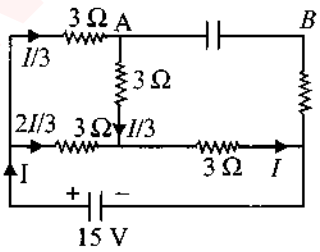


Fig. 5.299

$15 = 3 \times 2I/3 + 3I \Rightarrow I = 3\text{A}$

$V_A - 3I/3 - 3I = V_B$

$\Rightarrow \quad V_A - V_B = 4I = 4 \times 3 = 12\text{ V}$

94. c. $\alpha(T) = \frac{1}{R_0}\frac{dR}{dT} \Rightarrow (3T^2 + 2T) = \frac{1}{R_0}\frac{dR}{dT}$

$\Rightarrow \quad dR = R_0(3T^2 + 2T)\,dT$

$\Rightarrow \quad \int_{R_0}^{R} dR = R_0\left[3\int_0^T T^2 dT + 2\int_0^T T\,dT\right]$

$\Rightarrow \quad R = R_0[1 + T^2 + T^3]$

95. c. Momentum of each charge carrier moving with a drift velocity v is mv.

Total number of charge carriers in the sample is $N = n(A\ell)$, where n is the number of charge carriers per unit volume and A is area of cross section of the conductor.

Total momentum $= p = N(mv) = nA\ell mv$

Further, we have $v = I/neA$

$\Rightarrow \quad p = nA\ell m\left(\frac{I}{neA}\right) \Rightarrow p = \ell\left(\frac{m}{e}\right)I$

Since $F = \frac{dp}{dt}$

$\Rightarrow F = \frac{\ell}{S}\dot{I} \Rightarrow \frac{F}{\ell} = \frac{\dot{I}}{s} \quad [\because s = \text{specific charge} = e/m]$

96. c.

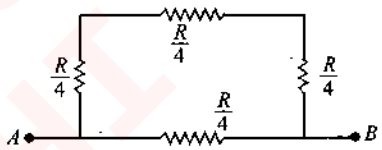


Fig. 5.300

$$R_{AB} = \frac{3R}{16} = \frac{3(16)}{16} = 3\Omega$$

97. c. When each element of the circuit is multiplied by a factor k, then the equivalent resistance also becomes k times.
Let the equivalent resistance between A and B be x.

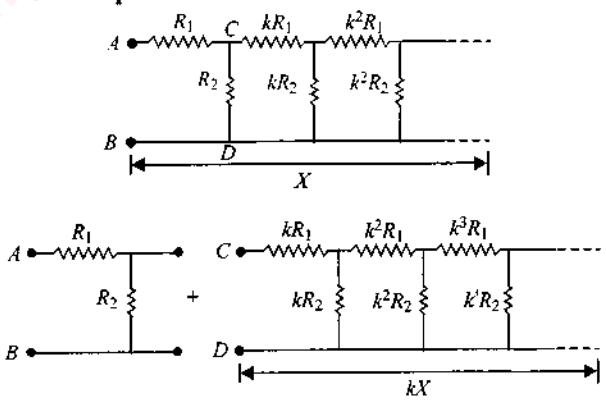


Fig. 5.301

So the equivalent circuit becomes

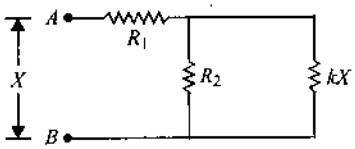


Fig. 5.302

$$X = R_1 + \frac{kXR_2}{kX + R_2}$$

For $k = \frac{1}{2} \Rightarrow X = \frac{(R_1 - R_2) + \sqrt{R_1^2 + R_2^2 + 6R_1R_2}}{2}$

98. b. $R_{AB} = \frac{7}{6}R$

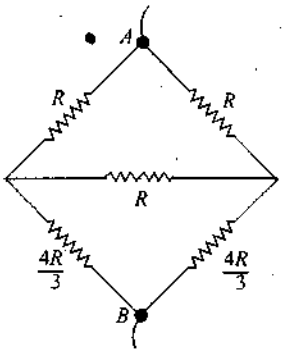


Fig. 5.303

99. c. We will solve this problem by the superposition principle. Let current I enters at A, then current $I/6$ will flow through AB. Now, in the second case, let the current is extracted from B, again current $I/6$ will flow through AB. Now, if simultaneously current enters at A and leaves at B, then the current $I/6 + I/6 = I/3$ should flow through AB and current $I - I/3 = 2I/3$ will flow through the remaining part. Let the resistance of the remaining part be R_1.

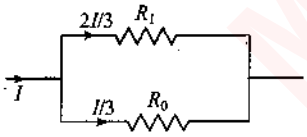


Fig. 5.304

Now $\frac{R_0 I}{3} = \frac{R_1 2I}{3} \Rightarrow R_1 = \frac{R_0}{2}$

$$R_{eq} = \frac{R_1 R_0}{R_1 + R_0} = \frac{(R_0/2)R_0}{(R_0/2) + R_0} = \frac{R_0}{3}$$

100. a. Resistors 7 Ω and 3 Ω are in parallel; 6 Ω and 4 Ω are in parallel and both in series.

So $R_{eq} = \frac{7 \times 3}{7+3} + \frac{4 \times 6}{4+6} = 4.5\ \Omega$

101. d. At junction E or F:

$I_1 + I_3 + I_4 = I_2 - I_3 \Rightarrow I_1 - I_2 + 2I_3 + I_4 = 0$

Loop $AFBA$ or loop $ECDE$

$rI_1 = rI_2 + rI_3 \Rightarrow I_1 = I_2 + I_3$

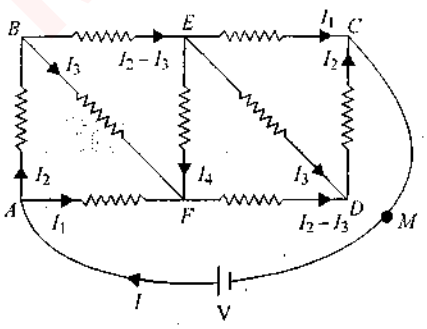


Fig. 5.305

Loop $BEFB$ or loop $EDFE$:

$$r(I_2 - I_3) + rI_4 = rI_3 \Rightarrow I_2 - 2I_3 + I_4 = 0$$

Loop $AFDCMA$ or $ABECMA$:

$$V = rI_1 + r(I_2 - I_3) + rI_2 \Rightarrow V = rI_1 + 2rI_2 - rI_3$$

Solve to get $I_1 = 2V/5r$, $I_2 = V/3r$

$$R_{eq} = \frac{V}{I} = \frac{V}{I_1 + I_2} = \frac{V}{2V/5r + V/3r} = \frac{15r}{11}$$

102. d. Equivalent resistance between A and E is

$$y = \frac{x+1}{x+2}$$

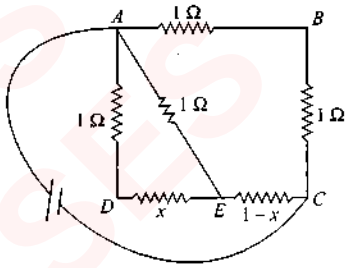


Fig. 5.306

For B and E to be equipotent, we get

$$\frac{R_{AE}}{R_{AB}} = \frac{R_{EC}}{R_{BC}} \Rightarrow \frac{x+1}{(x+2)\times 1} = \frac{1-x}{1}$$

Solve to get $x = \sqrt{2} - 1\ \Omega$

Now $\frac{CE}{ED} = \frac{1-x}{x} = \sqrt{2}\ \Omega$

103. b. Charge on capacitor at any time

$$q = Qe^{-t/\tau} \quad \text{(i)}$$

Differentiating Eq. (i) w.r.t. time, we get the current

$$I = -\frac{dq}{dt} = \frac{Q}{\tau}e^{-t/\tau} \quad \text{(ii)}$$

Required ratio: $q/I = \tau$ [by dividing Eq. (i) by Eq. (ii)]

104. c. Initial rate of discharging means initial current which is equal to V/R, where V is initial potential. Here V and R are the same for both.

105. b. The given problem may be approached in a simplified manner by the following considerations. Since the resistances R and R are in series with the battery in the steady state, the potential difference across R (which charges the capacitor) will have the maximum value of $E/2$. Also, if the battery is absent, and the (charged) capacitor were to discharge, the two resistors R and R will be connected in parallel, i.e., the effective resistance across the capacitor is $R/2$ and hence the time constant in the circuit is $\tau = RC/2$. Thus, the growth of the potential in the capacitor will be given by

$$V(t) = \frac{E}{2}\left[1 - \exp\left(-\frac{t}{\tau}\right)\right] = \frac{E}{2}\left[1 - \exp\left(-\frac{2t}{RC}\right)\right]$$

Multiple Correct Answers Type

1. a., b., d. Let V = potential at D

$70 - V = 10i_1$, $V - 0 = 20i_2$, $V - 10 = 30(i_1 - i_2)$

Solve for i_1, i_2, and V.

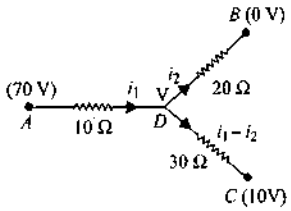
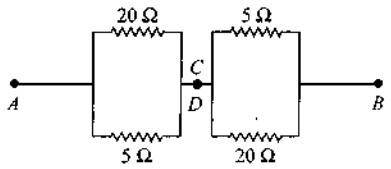


Fig. 5.307

2. a., b., d. As C and D are joined, they must be at the same potential, and may be treated as the same point. This gives the equivalent resistance as 8 Ω. If we distribute current in the network, using symmetry.



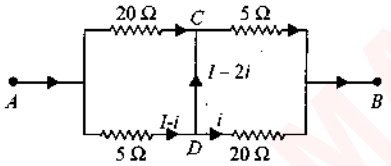


Fig. 5.308

$$V_A - V_D = V_A - V_C$$

or $$20i = 5(I - i)$$

or $$i = I/5$$

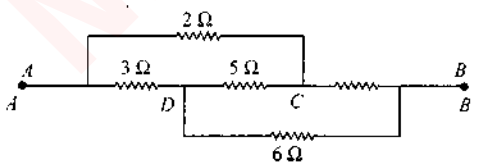
$$I - 2i = I - \frac{2I}{5} = \frac{3I}{5} = \text{current flowing from } D \text{ to } C.$$

3. a., d. The last three resistors 2 Ω, 4 Ω and 2 Ω are in series having the equivalent resistance as 2 + 4 + 2 = 8 Ω. This will be in series with the 8 Ω next to them. So, their equivalent resistance becomes 4 Ω. In this way, net equivalent resistance of the circuit becomes R_{eq} = 9 Ω. This will be in series with r = 1 Ω. So, the current through 3 Ω is

$$I = e/(R + r) = 10/(9 + 1) = 1 \text{ A}.$$

Further, current will get divided at C and E into half at each point. So, finally the current reaching in 4 Ω will be 0.25 A.

4. a, d.



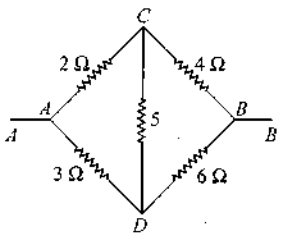


Fig. 5.309

Rearrangement of the circuit as shown in Fig. 5.309 gives a balanced Wheatstone bridge, and no current flows through the 5 Ω resistor. It can thus be removed from the circuit.

5. a., b., c. Potential difference across resistance = Potential difference across the terminals of the battery

So $$V = E - Ir$$

This is an equation of a straight line. Comparing this with the given graph, we can see that E = 10 V and r = 5 Ω.

Also $I_{max} = \frac{E}{r} = \frac{10}{5} = 2\text{A}$

When external resistance R = 0.

6. a., d. Resistors 2 Ω and 2 Ω are in series, 2 Ω and 4 Ω are in series, then their resultant are in parallel. Producing net resistance R = 2.4 Ω

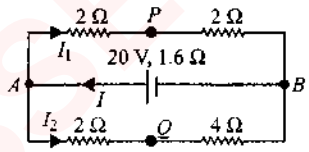


Fig. 5.310

$$I = \frac{20}{r + R} = \frac{20}{1.6 + 2.4} = 5 \text{ A}$$

$$I_1 + I_2 = 5 \text{ A} \quad \text{(i)}$$

$$V_{AB} = 4I_1 = 6I_2 \Rightarrow 2I_1 = 3I_2 \quad \text{(ii)}$$

From Eqs. (i) and (ii), I_1 = 3A, I_2 = 2 A

$$V_A - V_P = 6 \text{ V}, V_A - V_Q = 2I_2 = 4 \text{ V}$$

Thus, $$V_Q - V_P = 2 \text{ V}$$

7. a., b., d. We know that the equivalent internal resistance is

$$R_{eq} = \frac{1}{\sum \frac{1}{R}} \Rightarrow R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

$$= \frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}$$

and the equivalent e.m.f is

$$E_{eq} = \frac{\sum \frac{E}{R}}{\sum \frac{1}{R}} = \frac{\frac{E_1}{R_1} + \frac{E_2}{R_2} + \frac{E_3}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Use $$E_3 = \frac{E_1R_2 + E_2R_1}{R_1 + R_2} \text{ to get}$$

$$E_{eq} = E_3 = \frac{E_1R_2 + E_2R_1}{R_1 + R_2} \Rightarrow E_{eq} = E_3$$

8. b., c., d. 7 Ω resistor is short-circuited, so no current will flow through it. Potential difference across each of 3 Ω and 6 Ω is 12 V, so we can find current in them.

9. a., c., d.

$$I = \frac{dQ}{dt} = a - 2bt$$

$$I = 0 \text{ for } t = (a/2b) \text{ and } (dI/dt) = -2b$$

10. a., c., d. For loop $CEFD$: $-2(I - I_1) + 1I_1 + 9 = 0$

$\Rightarrow$ $$3I_1 - 2I = -9$$

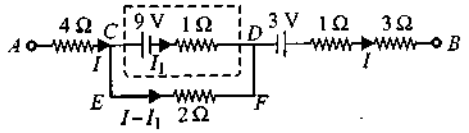


Fig. 5.311

From A to B via CD: $V_A - 4I - 9 + 3 - 1I_1 - 1I - 3I = V_B$

But $V_A - V_B = 16$ V, so $8I + I_1 = 10$

$\Rightarrow$ $I_1 = -2$A, $I = 1.5$A

$I - I_1 = 1.5 + 2 = 3.5$ A

$V_C - 9 - 1I_1 = V_D \Rightarrow V_C - V_D = 7$ V

11. c., d. Note that points a, h, g and f have the same potential. They are connected by conducting wires without any circuit elements between them. Similarly, points b, c, d and e have the same potential. Hence, the potential drop across branch e and f, and a and b is the same. The two resistors (6 Ω and 4 Ω in series) are directly connected across the terminals of 12 V battery.

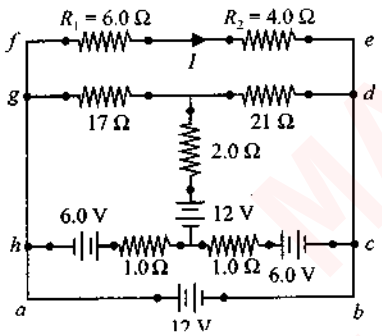


Fig. 5.312

The current $I = \dfrac{V}{R_1 + R_2} = \dfrac{12}{10} = 1.2$ A

Hence, $V_1 = (1.2)(6) = 7.2$ V, $V_2 = (1.2)(4) = 4.8$ V

12. a., b., d. The circuit can be redrawn as shown in the figure

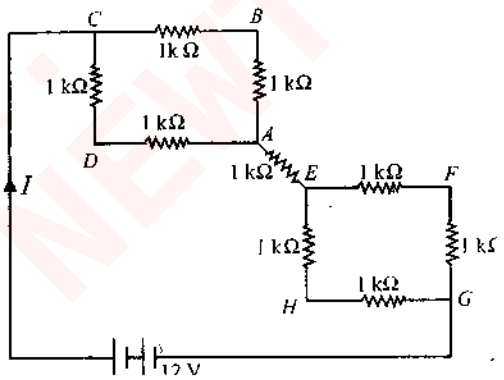


Fig. 5.313

$R_{eq} = 3$ kΩ, $I = \dfrac{12}{3 \times 10^3} = 4$ mA

$V_E - V_G = I(1\text{ k}\Omega) = 4 \times 10^{-3} \times 1 \times 10^3 = 4$ V

13. a., c. When S is opened: $V_c - V_a = \dfrac{18 \times 6}{6+3} = 12$ V

$V_c - V_a = \dfrac{18 \times 6}{6+3} = 6$ V $\Rightarrow V_b - V_a = 12 - 6 = 6$ V

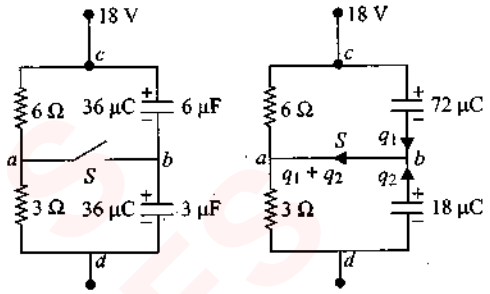


Fig. 5.314

Charges flown after S is closed:

$q_1 = 72 - 36 = 36$ μC, $q_2 = 36 - 18 = 18$ μC

Charges flown through S after it is closed:

$36 + 18 = 54$ μC

Final potential of b is 6 V

14. a., d. When X is joined to Y for a long time (charging), energy stored in the capacitor = heat produced in $R = H_1$. When X is joined to Z (discharging), the energy stored in $C(=H_1)$ reappears as heat (H_2) in R. Thus, $H_1 = H_2$.

15. b., c., d. Disregard the capacitors and find the current through G. The potential difference across each capacitor is then found from the potential differences across the resistances in parallel with them.

16. b., c., d. Just after closing the switch, capacitors will act like conducting wires. Find the equivalent resistance and calculate the current.

After a long time, the switch is closed, no current flows in the circuit as the capacitors act like infinite resistance.

Heat developed $= (1/2)C_{eq}V^2$.

17. a., b., c., d.

Circuit *A*: $x = \dfrac{R(2R + x)}{3R + x} \Rightarrow 3Rx + x^2 = 2R^2 + Rx$

$\Rightarrow x^2 + 2Rx - 2R^2 = 0 \Rightarrow x = \dfrac{-2R + \sqrt{4R^2 + 8R^2}}{2}$

$\Rightarrow x = -\dfrac{2R + 2\sqrt{3}R}{2} \Rightarrow x = (\sqrt{3} - 1)R$

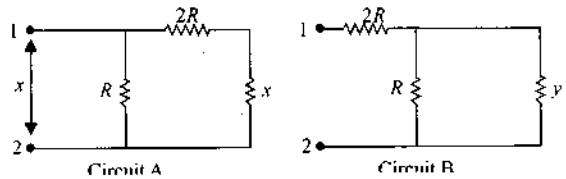


Fig. 5.315

Circuit *B*: $y = \dfrac{yR}{y+R} + 2R \Rightarrow y^2 + Ry = yR + 2Ry + 2R^2$

$\Rightarrow y^2 - 2YR - 2R^2 = 0 \Rightarrow y = (\sqrt{3} + 1)R$

18.a., c. As the cross sections are in series, so the current in them will be the same. From $I = neAv_d$, we have $v_d \propto 1/A$. At one cross section is more, hence the drift velocity is less.

Assertion–Reasoning Type

1. **c.** On increasing the temperature of metals, the resistance of metal increases. Therefore, the temperature coefficient of resistance of metals is positive.
   On increasing the temperature of the insulators, the resistance decreases. Therefore, the temperature coefficient of the resistance of insulators is negative.
2. **a.** $v_d = I/neA$, If radius is doubled, A becomes four times and hence V_d becomes one-fourth.
3. **a.** Both $R/2$ are in parallel, so their equivalent resistance is $R/4$.

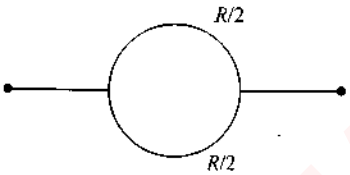
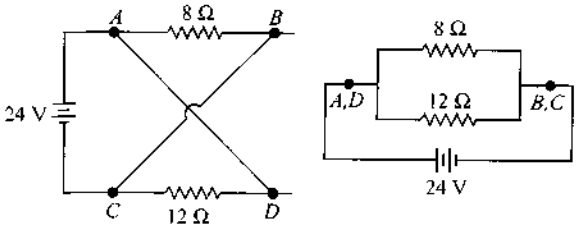


Fig. 5.316

4. **b.** Both the statements are correct, but independent of each other.
5. **d.** It is true that resistance of a wire is directly proportional to its length, but here when length is doubled, area of cross-section decreases as the volume remains constant. Finally, resistance becomes n^2 times.
6. **c.** The smaller is the resistance, the more is the current in parallel.
7. **a.** In series, current in both resistances will be the same. For the same current, the more is the resistance, the more is the potential drop.
8. **a.** Copper is a conductor and germanium is a semi-conductor.
9. **c.** Drift velocity is directly proportional to electric field. If there is no electric field, then there is no drifting of electrons in a particular direction, hence no current in the conductor.
10. **(a)** $V = IR \Rightarrow V = neAv_d \dfrac{\rho\ell}{A} \Rightarrow v_d = \dfrac{V}{ne\rho\ell} \Rightarrow v_d \propto \dfrac{1}{\ell}$

Comprehension Type

For Problems 1–3
1. d., 2. a., 3. b.
Sol. The current are as shown.



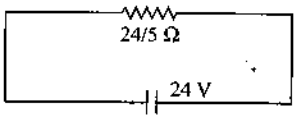


Fig. 5.317

$$I = \frac{24}{24/5} = 5\text{ A}$$

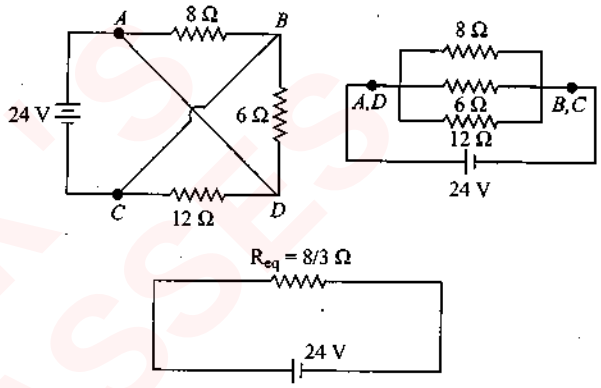


Fig. 5.318

$$\frac{1}{R_{eq}} = \frac{1}{8} + \frac{1}{6} + \frac{1}{12} \Rightarrow R_{eq} = \frac{8}{3}\,\Omega \quad I = \frac{24}{8/3} = 9\text{ A}$$

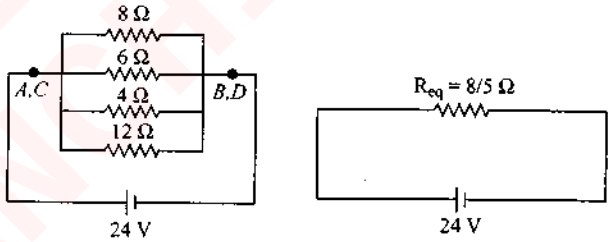


Fig. 5.319

$$\frac{1}{R_{eq}} = \frac{1}{8} + \frac{1}{6} + \frac{1}{4} + \frac{1}{12} \quad R_{eq} = \frac{8}{5}\,\Omega, \quad I = \frac{24}{8/5} = 15\text{ A}$$

For Problems 4–6
4. c., 5. a., 6. b.
Sol.

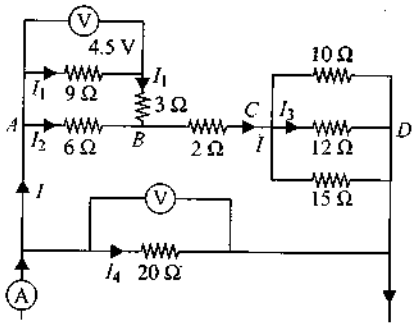


Fig. 5.320

$$I_1 = \frac{4.5}{9} = \frac{1}{2}\text{ A}$$

4. Potential difference across AB:

$$4.5 + 3I_1 = 6I_2 \Rightarrow I_2 = 1\text{ A}, I = I_1 + I_2 = 1.5\text{ A}$$

Equivalent resistance between C and D: $R_1 = 4\,\Omega$

Potential difference across $CD = R_1 I = 4 \times 1.5 = 6$ V

Current through 12 Ω, $I_3 = \dfrac{6}{12} = \dfrac{1}{2}$ A

5. Potential difference across AD is

$$6I_2 + 2I + 12I_3 = 6\times 1 + 2\times 1.5 + 12\times 0.5 = 15\text{ V}$$

This will be equal to the reading of voltmeter across 20 Ω

6. $I_4 = \frac{15}{20} = 0.75$ A

Reading of the ammeter $= I + I_4 = 1.5 + 0.75 = 2.25$ A

For Problems 7–9

7. d., 8. c., 9. d.

Sol.

7. Given $V_1 = \frac{V_0}{k}, V_2 = \frac{V_1}{k}, V_3 = \frac{V_2}{k}, \quad I = I_1 + I_2$

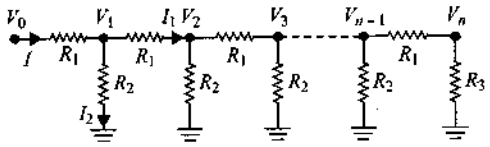


Fig. 5.321

$$\frac{V_0 - V_1}{R_1} = \frac{V_1 - V_2}{R_1} + \frac{V_1 - 0}{R_2}$$

$$\frac{V_0 - V_0/k}{R_1} = \frac{V_0/k - V_0/k^2}{R_1} + \frac{V_0/k}{R_2}$$

$$\frac{R_1}{R_2} = \frac{(k-1)^2}{k}$$

8. Current in R_1 and R_3 will be the same: $\frac{V_{n-1} - V_n}{R_1} = \frac{V_n}{R_3}$

$$\frac{V_{n-1} - \frac{V_{n-1}}{k}}{R_1} = \frac{V_{n-1}}{kR_3}$$

$$\Rightarrow \quad R_1 = R_3(k-1)$$

Put the value of R_1 in the above expression, we get:

$$\frac{R_2}{R_3} = \frac{k}{k-1}$$

9. Current in R_2 nearest to V_0:

$$I_2 = \frac{V_1}{R_2} = \frac{V_0/k}{R_3\left(\frac{k}{k-1}\right)} = \left(\frac{k-1}{k^2}\right)\frac{V_0}{R_3}$$

For Problems 10–12

10. a., 11. d., 12. c.

Sol. 10. $q = \int i\,dt$ = area of given curve $= \frac{1}{2} i_0 t_0$

11. $\frac{i}{i_0} + \frac{t}{t_0} = 1 \Rightarrow i = i_0\left(1 - \frac{t}{t_0}\right)$

12. Heat $= \int i^2 R\,dt = \int_0^{t_0} i_0^2 R\left(1 - \frac{t}{t_0}\right)^2 dt$

$$H = \left.\frac{i_0^2 R\left(1 - \frac{t}{t_0}\right)^3}{3\left(-\frac{1}{t_0}\right)}\right|_0^{t_0} \Rightarrow H = \frac{Rt_0 i_0^2}{3}$$

For Problems 13–15

13. b., 14. b., 15. d.

Sol. In the steady state, there will be no current in capacitor branch.

Loop $EABE$: $6 = i_1 + (1+1)(i_1 + i_2) + 2i_1 \Rightarrow 5i_1 + 2i_2 = 6$ (i)

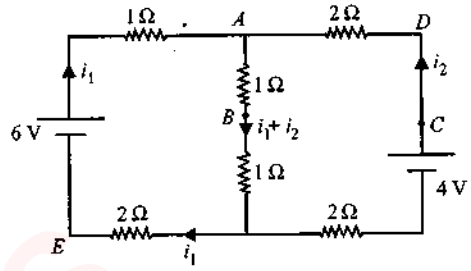


Fig. 5.322

loop $ADCBA$: $4 = 2i_2 + (1+1)(i_1 + i_2) + 2i_1$

$\Rightarrow \quad 6i_2 + 2i_1 = 4$ (ii)

Solving Eqs. (i) and (ii), $i_1 = 14/13$ A, $i_2 = 4/13$ A

Given $V_E = 0$, $\quad V_E + 6 - 1i_1 - 1(i_1 + i_2) = V_B$

$$\Rightarrow \quad V_B = 6 - 2i_1 - i_2 = 6 - 2\times\frac{14}{13} - \frac{4}{13} = \frac{46}{13}\text{ V}$$

Now along $BADC$: $V_B + 1(i_1 + i_2) + 2i_2 = V_C$

$$V_C - V_B = i_1 + 3i_2 = \frac{14}{13} + 3\times\frac{4}{13} = 2\text{ V}$$

Charge on capacitor: $q = C(V_C - V_B) = 4\times 2 = 8\ \mu\text{C}$

For Problems 16–18

16. c., 17. b., 18. a.

Sol.16. c. If we connect a battery across A and B, then E and D will be at the same potential and F and C will be at the same potential.

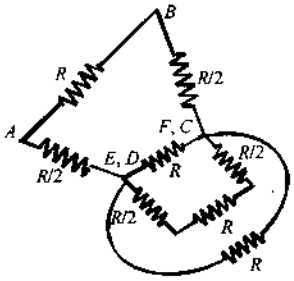


Fig. 5.323

Find $R_{eq(AB)} = \frac{7R}{12}$

17. b. If we connect the battery across A and G, then B, E and D will be at the same potential. C, F and H will be at the same potential.

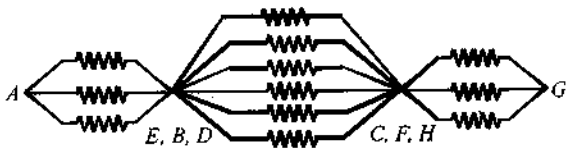


Fig. 5.324

$R_{eq} = \frac{5R}{6}$

18. a. If we connect the battery across A and C, then no current will flow in the resistances between B and F and between H and D. So they can be removed.

Now get $R_{eq} = \frac{3R}{4}$

For Problems 19–21

19. d., 20. d., 21. a.

Sol. 19.d. D, B and E are symmetrical to A, hence potential at these points is same. Similarly, C, F and H are symmetrically to G. So potential at these points is the same.

20.d. Resulting diagram will be as shown: C, D, F and E will be at the same potential.

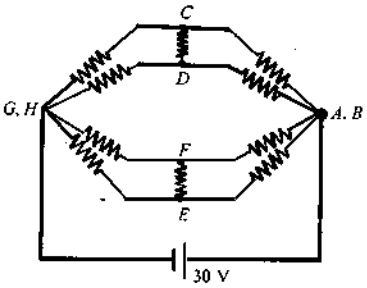


Fig. 5.325

21.a. P.d. between C and G will be half of the applied potential, i.e.,

$$V_{CG} = 30/2 = 15 \text{ V}$$

For Problems 22–24

22. a., 23. a., 24. c.

22. a. If we connect battery between E and F, then A, B, C and D will be all at the same potential.

Fig. 5.326

$$R_{eq} = \frac{R}{4} + \frac{R}{4} = \frac{R}{2}$$

23. a. If we connect a battery between A and C, then E and F will be at the same potential.

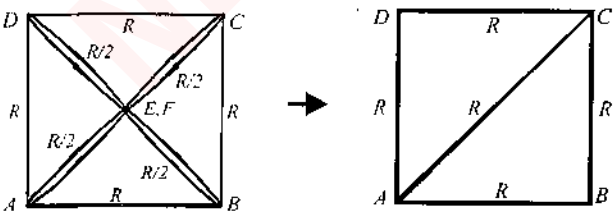


Fig. 5.327

Now resistances between DE and EB will be useless:

$\Rightarrow$ $R_{eq} = R/2$

24. c. Let us remove the resistance between A and B, then E and F will be at the same potential, we can draw the remaining as:

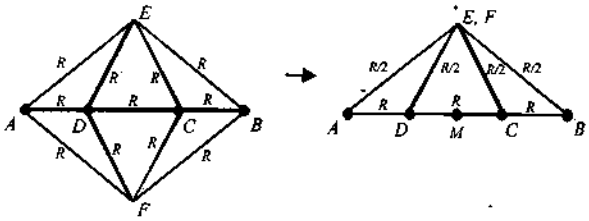


Fig. 5.328

M is the midpoint of DC, M will be at the same pot as E and F.

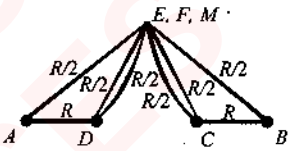


Fig. 5.329

Now find $R_{AB} = 5/7R$. The resistance which we had removed will be in parallel to it so

$$R_{eq} = \frac{R_{AB}R}{R_{AB} + R} = \frac{(5/7)RR}{(5/7)R + R} = \frac{5}{12}R$$

Matching Column Type

1. i. → a., ii. → c, iii. → b., iv. → d.

Sol. The equivalent circuit is as shown in Fig. 5.330 (b).

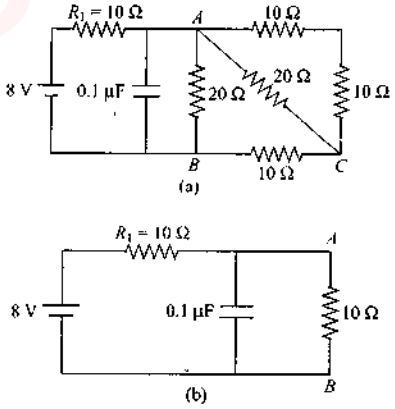


Fig. 5.330

The current through $R_1 = \frac{8}{20} = 0.4$ A

In the steady state, the potential difference across AB is 4 V.

Charge on capacitor in steady state is

$$q = CV = 0.4 \ \mu\text{C}$$

Current through resistor R is $I = \frac{V}{R} = \frac{4}{20} = 0.2$ A

2. i. → c., ii. → b., iii. → b., iv. → a.

Sol. When a steady state is reached, no current passes through the capacitor or the branch CE.

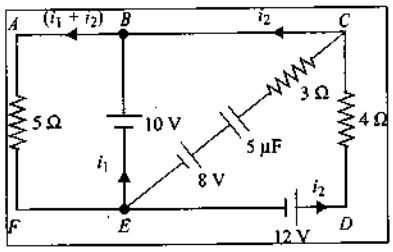


Fig. 5.331

Considering the loop *ABEFA*,

$$5 \times (i_1 + i_2) = 10 \text{ or } i_1 + i_2 = 2 \text{ A} \quad \text{(i)}$$

Considering the loop *BCDEB*

$$4i_2 = 12 - 10 = 2 \Rightarrow i_2 = 0.5 \text{ A}$$

So, $i_1 = 2 - 0.5 = 1.5$ A

To find the charge on capacitors, we must know potential difference across the plates.

Consider the loop *CEDC*:

$$-12 + 4i_2 + 3 \times 0 - V_C + 8 = 0$$

Or $V_C = -2$ V. So charge on capacitor $Q = CV = 10\ \mu$C

3. i. → b., ii. → a.,b.,c., iii. → b., iv. → a.,b.,c.,d.

Sol.

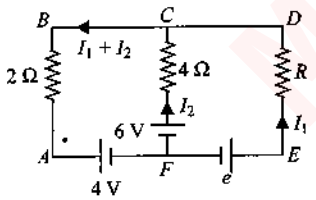


Fig. 5.332

Loop *FEDCF*: $e - 6 = RI_1 - 4I_2$ (i)

Loop *AFCBA*: $6 - 4 = 4I_2 + 2(I_1 + I_2)$

$$2 = 2I_1 + 6I_2 \quad \text{(ii)}$$

Solving them we get:

$$I_1 = \frac{3e - 14}{4 + 3R}, I_2 = \frac{R + 6 - e}{4 + 3R}$$

i. $I_2 = 0 \Rightarrow e = R + 6$

$e > 6$ V ($\therefore R \neq 0$)

ii. For current from *F* to *C* direction

$$I_2 > 0 \Rightarrow R + 6 > e \Rightarrow e < R + 6$$

Possible for any finite value of *e*, because *R* is finite.

iii. For current from *C* to *F* direction

$$I_2 < 0 \Rightarrow e > R + 6$$

iv. For current in 2 Ω from *B* to *A* direction

$$I_1 + I_2 = \frac{R - 8 + 2e}{4 + 3R} > 0$$

$$R - 8 + 2e > 0 \Rightarrow e > 4 - \frac{R}{2}$$

Depending upon the value of *R*, *e* can take any value from zero to infinity.

4. i. → d., ii. → d., iii. → a., iv. → b.

Sol. Let current through various branches is as shown in the figure.

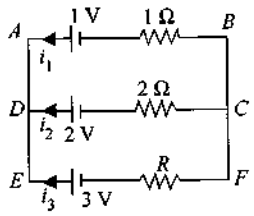


Fig. 5.333

$$i_1 + i_2 + i_3 = 0 \quad \text{(i)}$$

$$V_A - V_B = V_D - V_C = V_E - V_F$$

$$1 - 1i_1 = 2 - 2i_2 = 3 - Ri_3 \quad \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$i_1 + \frac{2 - 1 + i_1}{2} + \frac{3 - 1 + i_1}{R} = 0 \Rightarrow i_1 = \frac{-(R+4)}{3R+2}$$

Again from Eqs. (i) and (ii), we get

$$\frac{2i_2 - 1}{1} + i_2 + \frac{1 + 2i_2}{R} = 0 \Rightarrow i_2 = \frac{R-1}{3R+2}$$

Now $i_3 = -i_1 - i_2 = \frac{5}{3R+2}$

i. Current i_3 cannot be zero.

ii. Current i_1 cannot be zero.

iii. $i_2 = 0 \Rightarrow R = 1\ \Omega$

iv. $i_3 = \frac{5}{8} \Rightarrow \frac{5}{3R+2} = \frac{5}{8} \Rightarrow R = 2\Omega$

5 .i. → b,c., ii. → a,b,c., iii. → d., iv. → a,b,c.

i. Electrical conductivity of a conductor depends on temperature of the material. It decreases with the increase in temperature. It also depends upon the nature of material.

ii. Conductance (the reciprocal of resistance) also depends upon dimensions of material apart from temperature and nature of material.

iii. $J = \sigma E$, so current density depends upon electric field. For a given conductor, σ is fixed.

iv. Current will depend upon resistance which depends upon dimensions, temperature and nature of material.

Integer Answer Type

1. (3) By symmetry, points *D* and *E* have the same potential. Similarly points *C* and *F* have the same potential.

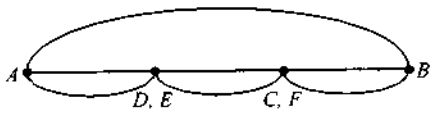


Fig. 5.334

$$r_{eq} \frac{\frac{(3r)}{2} r}{\frac{3r}{2} + r} = \frac{3r}{5} = \frac{3}{5} \times 5 = 3\ \Omega$$

2. (6) 9 V and 16 Ω can be ignored because potential difference between A and C is fixed which is 4 V. The circuit can be drawn as shown below.

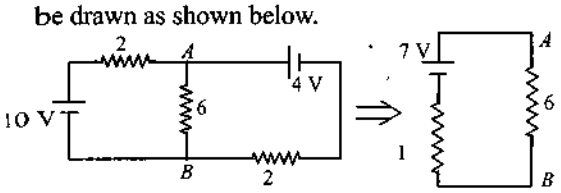


Fig. 5.335

Equivalent e.m.f.: $e = \frac{\Sigma(E/r)}{\Sigma(1/r)} = \frac{10/2 + 4/2}{1/2 + 1/2} = 7$ V

Equivalent internal resistance: $r_0 = \frac{1}{\Sigma(1/r)} = \frac{1}{1/2 + 1/2} = 1\ \Omega$

Now $V_{AB} = \frac{7}{(6+1)} \times 6 = 6$ V

3. (1) Current through the battery will be minimum when all the three switches are open. The resulting circuit is shown in Fig. 5.336.

Hence, $V_{AB} = (0.5)V_{AB} \times \frac{24}{12} = 1$ V

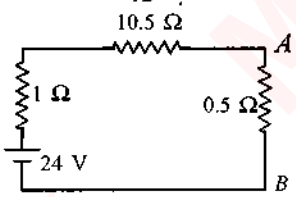


Fig. 5.336

4. (3) Let a battery of e.m.f. E is applied between points A and B. Let a current I enter through point A. The current distribution is shown in the figure.

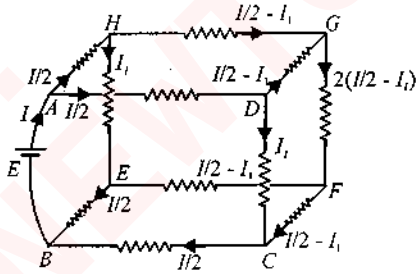


Fig. 5.337

If R_{AB} is a equivalent resistance between points A and B, then from Ohm's law

$$R_{AB}I = E \qquad \text{(i)}$$

Applying Kirchhoff's second law to mesh $DGFC$, we get

$$\left(\frac{I}{2} - I_1\right)R + (I - 2I_1)R + \left(\frac{I}{2} - I_1\right)R - I_1R = 0 \Rightarrow I_1 = \frac{2}{5}I \qquad \text{(ii)}$$

Applying Kirchhoff's second law to external circuit $AHEBA$, we get

$$E = \frac{I}{2}R + I_1R + \frac{I}{2}R \qquad \text{(iii)}$$

Solving Eqs. (i), (ii) and (iii), we get $R_{AB} = \frac{7}{5}R = \frac{7}{5} \times \frac{15}{7} = 3\ \Omega$

Alternative Method: The figure can be redrawn as shown below.

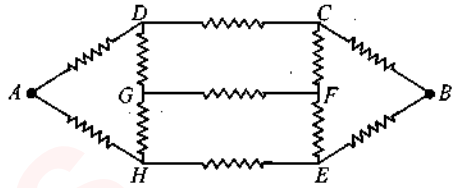


Fig. 5.338

If we connect a battery between A and B, then D and H will be at the same potential; similarly C and E will be at the same potential. Resulting figure will be as shown.

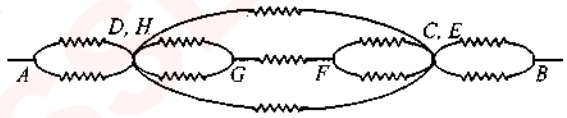


Fig. 5.339

Now simplify to get $R_{AB} = \frac{7}{5}R$.

5. (6) If we connect battery between A and B, then different points will be at the same potentials.
(1,2,3,4); (5,6,7,8); (9,10,11,12,13,14,15,16) and (17,18,19,20)

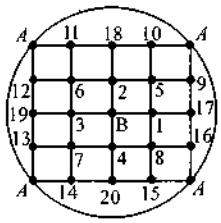


Fig. 5.340

The resulting circuit can be drawn as shown below.

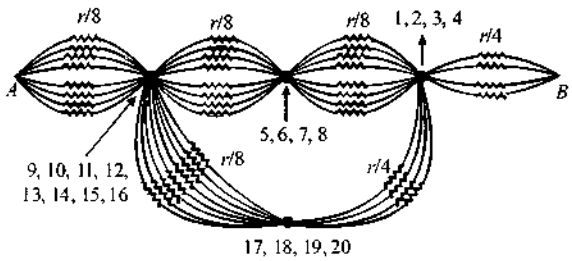


Fig. 5.341

Solve to get $R_{AB} = \frac{21}{40}r \Rightarrow R_{AB} = \frac{21}{40} \times \frac{80}{7} = 6\ \Omega$

6. (8) Due to symmetry, the line of resistors passing through b will be useless.

$$r_{eq} = 2r + \frac{2rr_{eq}}{2r + r_{eq}} \Rightarrow r_{eq} = (1 + \sqrt{5})r$$

$$= (\sqrt{5} + 1)2(\sqrt{5} - 1) = 8\ \Omega$$

7. (9) At junction C: $i_1 + i_2 = i$,

where $i_1 = \frac{0-x}{1}$, $i_2 = \frac{1-x}{1}$

At D: $4i + i_1 = 0$, solve to get $x = \frac{4}{9}$

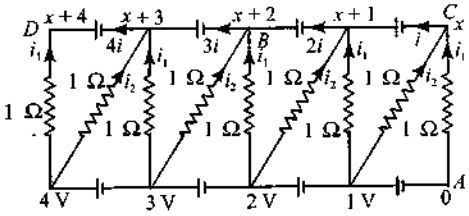


Fig. 5.342

Now $V_A - i_1 + 1 + 1 = V_B \Rightarrow V_A - V_B = -\left(2 + \frac{4}{9}\right) = -\frac{22}{9}$ V

8. (9) The resulting circuit can be drawn as shown. No current will flow into the earth.

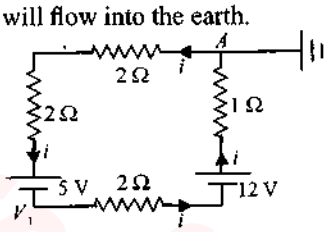


Fig. 5.343

$i = (12 - 5)/7 = 1$ A, $V_A - 2i - 2i - 5 = V_1$ but $V_A = 0$

$\Rightarrow$ $V_1 = -9$ V

CHAPTER
6

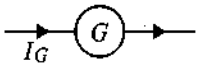
Electrical Measuring Instruments

- Galvanometer
- Ammeter
- Voltmeter
- Potentiometer
- Meter Bridge or Slide Wire Bridge

R. K. MALIK'S NEWTON CLASSES RANCHI

GALVANOMETER

It is a device which is used to measure the magnitude and direction of current in a circuit. It is a very sensitive device and can be used to measure only small currents, say in μA. If we pass a current through galvanometer more than a specified current, then it may get damaged. The maximum current which can be passed through a galvanometer is known as *full-scale deflection current*. Let it be I_G. The resistance of a galvanometer is very small.



In a galvanometer, there is a needle attached with a coil, which is subject to magnetic field. Attached to the coil is a spring, similar to the hairspring on the balance wheel of a watch.

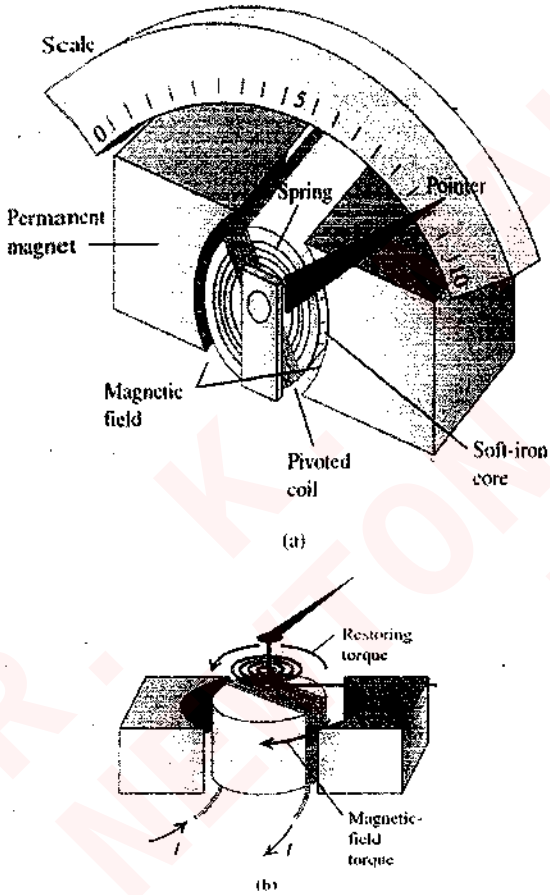


Fig. 6.1

In equilibrium position, with no current in the coil, the needle is at zero. When there is current in the coil, the magnetic field exerts a torque on it that is proportional to the current. This torque is responsible for the rotation of the coil and hence the needle deflects. As the coil turns, the spring exerts a restoring torque that is proportional to the angular displacement.

AMMETER

A galvanometer cannot be used directly to measure higher currents. To measure high currents, we have to carry out some modifications in the galvanometer. After modifications what we get is known as *ammeter*.

Suppose we want to make an ammeter which can measure the current up to I. Such an ammeter is said to have a range of $0-I$. For this we have to connect a small resistance (known as *shunt*) in parallel with the galvanometer as shown in the figure.

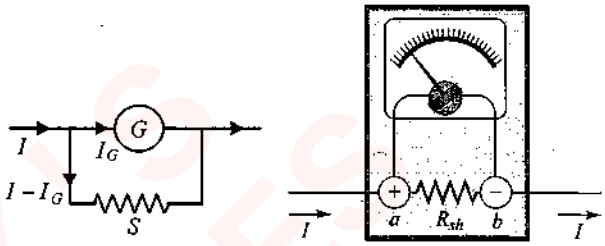


Fig. 6.2

The value of S is so selected that only I_G current passes through galvanometer and remaining $I-I_G$ through shunt. Let resistance of the galvanometer be G. Since G and S are in parallel, potential difference across them should be same, i.e.,

$$I_G G = S(I-I_G) \Rightarrow S = \frac{GI_G}{I-I_G}$$

The above equation gives the value of S to be connected in parallel with galvanometer to convert it into an ammeter of range $0-I$.

Note:

1. ***Resistance of ammeter: G and S are in parallel, so their equivalent resistance is given by*** $R_A = \frac{SG}{S+G}$. ***Since S is a very small resistance (it has to be small if we want most of the current to pass through it), R_A is even smaller than S. In general, resistance of an ammeter is very small. For an ideal ammeter, resistance is zero.***
2. ***An ammeter is used in series to measure the current.***
3. ***The reading of an ammeter is generally lesser than the actual current in the circuit. It is because when we connect ammeter in the circuit to measure the current, ammeter introduces its own resistance in the circuit which results into an increase in the resistance of the circuit and decrease in the current.***
4. ***The lower the value of shunt resistance, the higher the range of ammeter.***
5. ***How an ammeter reads the current: The current through the galvanometer is responsible for the deflection of the needle of the galvanometer. We have to use this fraction of current (I_G) in measuring the actual current I.***
   As the potential difference across G is the same as that across S, from the equation $I_G G = (I-I_G)S$, we get

$$I_G = \left(\frac{S}{G+S}\right) I \Rightarrow \boxed{I_G \propto I}$$

Hence, I_G is proportional to I or the deflection of the needle is proportional to the current I. If the value of current I is changed, then the deflection of the needle also changes. The scale can be graduated to read the value of I directly.

Illustration 6.1 **A galvanometer has a resistance of 50 Ω and its full-scale deflection current is 50 μA. What shunt resistance should be added so that the ammeter can have a range of 0 – 5 mA?**

Sol. Given $I_G = 50$ μA. The upper limit gives the maximum current to be measured which is $I = 5$ mA. The galvanometer resistance, $G = 50\ \Omega$. Now

$$S = \frac{I_G G}{I - I_G} = \frac{50\times10^{-6}\times50}{5\times10^{-3} - 50\times10^{-6}} = \frac{50\times10^{-6}\times50}{5\times10^{-3}} \approx 0.5\ \Omega$$

Illustration 6.2 **What is the value of shunt which passes 10% of the main current through a galvanometer of 99 Ω?**

Sol. Given $G = 99\ \Omega$ and $I_G = (10/100)\,I = 0.1I$

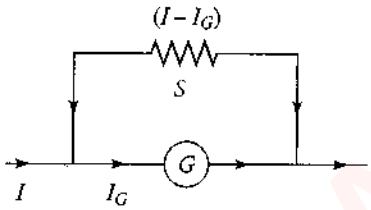


Fig. 6.3

Now $$S = \frac{I_G G}{(I - I_G)} = \frac{0.1I\times99}{(I - 0.1I)} = \frac{0.1}{0.9}\times99 = 11\ \Omega$$

Illustration 6.3 **The deflection in a moving coil galvanometer falls from 50 divisions to 10 divisions when a shunt of 12 Ω is applied. What is the resistance of the galvanometer? Assume main current to remain same.**

Sol. In case of a galvanometer, $I \propto \theta$

Given $$\frac{I_G}{I} = \frac{10}{50} = \frac{1}{5}, \quad \text{i.e.,} \quad I_G = \frac{1}{5}I$$

Now as in case of a shunted galvanometer as shown in the figure,

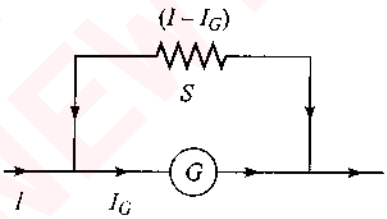


Fig. 6.4

$$(I - I_G)\,S = I_G G, \quad \text{i.e.,} \quad \left(I - \frac{1}{5}I\right)\times12 = \frac{1}{5}IG$$

or $G = 4\times12 = 48\ \Omega$

Illustration 6.4 **Consider a circuit as shown in figure (i). We want to measure the current *i* flowing in the circuit.**

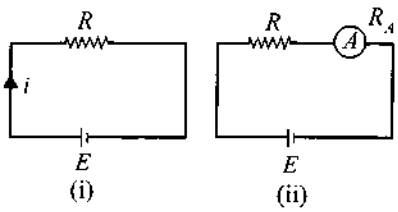


Fig. 6.5

For this, we connect an ammeter of resistance R_A as show in the Fig. 6.5(ii). Find the percentage error in the current.

Sol. Actual current, $i = E/R$. Let current measured by ammeter be i_1, then $i_1 = E/(R + R_A)$, clearly L_1 is less than i.

Percentage error is

$$\frac{i - i_1}{i}\times100 = \left(\frac{E/R - E/(R+R_A)}{E/R}\right)\times100\left(\frac{R_A}{R+R_A}\right)\times100$$

VOLTMETER

It is an instrument used to find the potential difference across any two points in a circuit. A galvanometer directly can measure small potential differences, only. To measure high potential differences, we have to do some modifications in galvanometer. After modifications what we get is known as *voltmeter*.

Conversion of a Galvanometer into Voltmeter:

Suppose we want to make a voltmeter which can measure the potential difference up to V and range of the voltmeter is 0–V. For this, a suitable high resistance is connected in series with galvanometer such that when a potential difference of V is applied, only a current I_G passes through the galvanometer as shown in the figure.

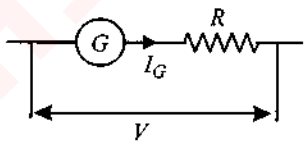


Fig. 6.6

We can write $$V = I_G(G + R) \Rightarrow R = \frac{V}{I_G} - G$$

The above equation gives the value of R in terms of V.

Note:

- ***Resistance of voltmeter: $R_V = G + R$. Generally, R_V is very high and for an ideal voltmeter, R_V is infinite.***
- ***A voltmeter is used in parallel to measure the potential difference.***
- ***The higher the value of R, the higher the range of voltmeter.***
- ***How a voltmeter reads the potential difference:***
  Let V be the potential difference across a resistor which is to be measured. We have the relation,

$$I_G = \frac{V}{R + G} \Rightarrow \boxed{I_G \propto V}$$

We know that deflection of needle is proportional to the current I_G and hence to V. The scale can be graduated to read the potential difference directly.

Illustration 6.5 **A galvanometer has a resistance of 50 Ω and its full-scale deflection current is 50 μA. What resistance should be added to it so that it can have a range of 0–5 V?**

Sol. Given $I_G = 50$ μA. Maximum voltage to be measured is $V = 5$ V. The galvanometer resistance, $G = 50\ \Omega$. Now,

$$R = \frac{V}{I_G} - G = \frac{5}{50\times10^{-6}} - 50 = 100\ \text{K}\Omega$$

Illustration 6.6 **A galvanometer has a resistance of 30 Ω and a current of 2 mA is needed for a full-scale deflection. What is the resistance and how is it to be connected to convert the galvanometer (a) into an ammeter of 0.3 A range and (b) into a voltmeter of 0.2 V range?**

Sol. Given $G = 30\ \Omega$ and $I_g = 2$ mA

(a) To convert the galvanometer into an ammeter of range 0.3 A,

$$(I - I_G)\,S = I_G\,G \Rightarrow (0.3 - 0.002)\,S = 0.002\times30$$

$$\Rightarrow \quad S = \frac{0.002\times30}{0.298} = 0.2013\ \Omega$$

(b) To convert the galvanometer into a voltmeter of range 0.2 V,

$$V = I_G\,(R+G) \Rightarrow 0.2 = 2\times10^{-3}(30+R) \Rightarrow R = 70\ \Omega$$

Illustration 6.7 **The scale of a galvanometer is divided into 150 equal divisions. The galvanometer has current sensitivity of 10 divisions per mA and a voltage sensitivity of 2 divisions per mV. How can the galvanometer be designed to read (i) 6 A per division and (ii) 1 V per division?**

Sol. Full-scale voltage = 150 divisions/2 divisions per mV = 75 mV

Full-scale current = 150 divisions/10 divisions per mA = 15 mA

So resistance of galvanometer,

$$G = \frac{\text{Full-scale voltage}}{\text{Full-scale current}} = \frac{75\times10^{-3}}{15\times10^{-3}} = 5\ \Omega$$

(i) Range of ammeter, $I = 150\times6 = 900$ A. So,

$$S = \frac{I_G G}{I - I_G} = \frac{15\times10^{-3}\times5}{(900 - 15\times10^{-3})} = 8.3\times10^{-5}\ \Omega$$

(ii) Range of voltmeter $V = 150\times1 = 150$ V. So,

$$R = \frac{V}{I_G} - G = \frac{150}{15\times10^{-3}} - 5 = 9995\ \Omega$$

Illustration 6.8 **(i) In the figure, find the potential difference between the points *A* and *B*.**

(ii) Now we wish to measure this potential difference by using a voltmeter of resistance 2 kΩ. Find the reading of the voltmeter and percentage error.

(iii) Solve part (ii) if the voltmeter were of resistance 20 kΩ.

What conclusion do you draw from the results you get in the above parts?

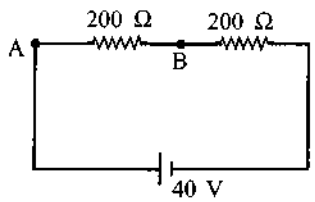


Fig. 6.7

Sol. (i) As both the resistances are same, 40 V will be divided equally among both the resistances. Hence, potential difference across *A* and *B* is 20 V.

(ii) Equivalent resistance of 200 Ω and 2 kΩ is

$$R_1 = \frac{200\times2000}{200+2000} = \frac{2000}{11}\ \Omega$$

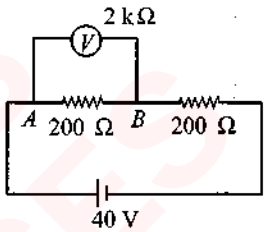


Fig. 6.8

Reading of voltmeter = potential difference across *AB*

$$= V_1 = \frac{2000/11\times40}{2000/11+200} = 19.05\ \text{V}$$

$$\%\ \text{Error} = \frac{20-19.05}{20}\times100 = 4.75\%$$

(iii) In this case: $R_1 = \dfrac{200\times20{,}000}{200+20{,}000} = \dfrac{20{,}000}{101}\ \Omega$

Reading of voltmeter = potential difference across *AB*

$$= V_2 = \frac{20{,}000/101\times40}{20{,}000/101+200} = 19.90\ \text{V}$$

$$\%\ \text{Error} = \frac{20-19.9}{20}\times100 = 0.5\%$$

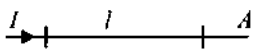
In case (iii), percentage error is less than that in case (ii). It means the more the resistance of voltmeter, the more accurate it will give the reading.

POTENTIOMETER

When we measure the current or potential difference using ammeter or voltmeter, we do not get exact values. Because when we connect ammeter or voltmeter in circuit they disturb the original circuit. But potentiometer is an instrument that can measure current or potential difference accurately. Basically, potentiometer does not draw any current from the original circuit and hence does not disturb the original circuit.

It is based on the principle that if a constant current is passed through a wire of uniform cross section, the potential difference across any segment of the wire is proportional to its length.

Let a current *I* pass through a wire of uniform cross section area *A* as shown in the figure.



Consider a segment of length *l* of the wire. Resistance of this segment is given by $R = \rho l/A$.

Potential difference across this segment is given by

$$V = IR = I\rho l/A \Rightarrow V = kl \Rightarrow k = V/l$$

where $k = I\rho/A$ is the potential difference per unit length known as *potential gradient*.

Construction of Potentiometer

Potentiometer consists of a wire AB of uniform cross section, generally 1 to 10 m long fixed on a wooden board. Let the resistance of AB be r.

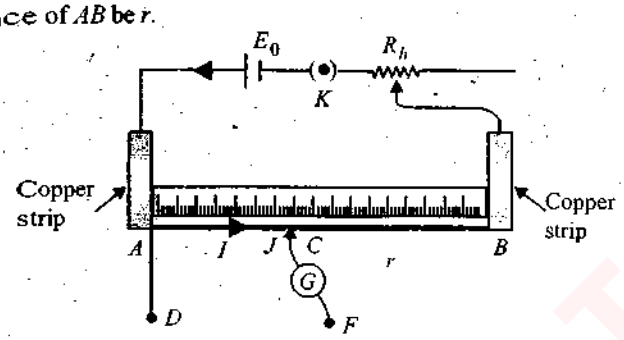


Fig. 6.9

The ends A and B are connected to a battery E_0 (known as driving battery), a switch K and a rheostat R_h. On closing the switch, current I is established in the wire AB, i.e.,

$$I = \frac{E_0}{R_h + r}$$

Current I can be adjusted by varying the value of R_h. A jockey J can slide freely on wire AB. Jockey touches the wire at C. A scale is fitted on the wooden board parallel to wire AB so that length AC can be read. Between points D and F, we can insert any cell, resistance or any other device of which we need the current or voltage information. After inserting such a device, jockey is slid on the wire and location of C is so selected that finally there is no current through the galvanometer. In this condition, the applied potential difference across D and F will be equal to the potential difference across the segment AC.

Uses of Potentiometer

1. **Comparison of e.m.f. of Two Cells**: Consider two cells of e.m.f.s. E_1 and E_2 are connected across points D and F along with switches K_1 and K_2 as shown in the figure. Initially, both the switches are open.

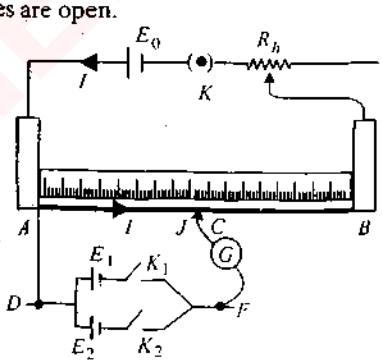


Fig. 6.10

First close switch K_1 and move the jockey on the wire AB until the galvanometer shows no deflection. When $AC = \ell_1$, deflection in the galvanometer becomes zero. Then from the principle of potentiometer, we get

$$E_1 = k\ell_1 \quad \text{(i)}$$

where k is the potential gradient of wire AB. We can say that E_1 is balanced at length ℓ_1.

Similarly, on opening switch K_1 and closing K_2, let E_2 be balanced at length ℓ_2, then

$$E_2 = k\ell_2 \quad \text{(ii)}$$

From Eqs. (i) and (ii), $\dfrac{E_2}{E_1} = \dfrac{\ell_2}{\ell_1}$

Thus we can compare the e.m.f.s. of the cells.

2. **Determination of Internal Resistance of a Cell:** Suppose we have a cell of e.m.f. E whose internal resistance r is to be measured.

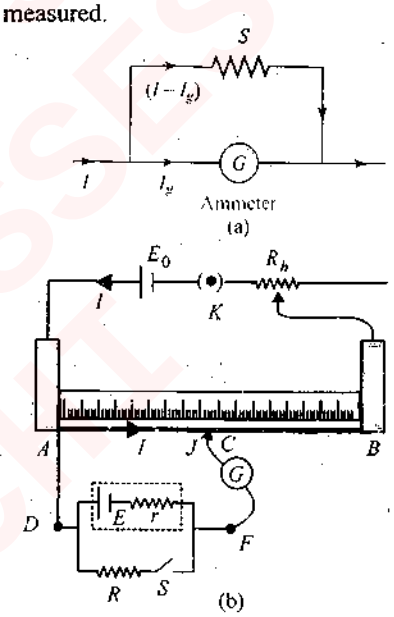


Fig. 6.11

Connect the cell in the circuit of potentiometer as shown in the figure. First keep switch S open and slide the jockey so that there is no deflection. Let the balancing length be ℓ_1. Then

$$E = k\ell_1 \quad \text{(i)}$$

Now close switch S and again find the balancing length. Let the balancing length be ℓ_2. A separate current say I_1 is established in the lower circuit. I_1 is given by $I_1 = \dfrac{E}{R+r}$.

Let the terminal potential difference across cell be V, then

$$V = k\ell_2 \Rightarrow I_1 R = k\ell_2 \Rightarrow \frac{E}{R+r}R = k\ell_2 \quad \text{(ii)}$$

From Eqs. (i) and (ii), $\dfrac{R+r}{R} = \dfrac{\ell_1}{\ell_2} \Rightarrow r = \left(\dfrac{\ell_1 - \ell_2}{\ell_2}\right)R$

Thus, we can measure the internal resistance of a cell.

Note: ***we can also write:***

$$r = \left(\frac{\ell_1 - \ell_2}{\ell_2}\right)R = \left(\frac{k\ell_1 - k\ell_2}{k\ell_2}\right)R \Rightarrow r = \left(\frac{E - V}{V}\right)R$$

Sensitivity of Potentiometer: The sensitiveness of a potentiometer means the smallest potential difference that can be measured with its help. The sensitiveness of a potentiometer can be increased by

decreasing its potential gradient. The same can be achieved
(i) by increasing the length of potentiometer wire.
(ii) by reducing the current in the potentiometer wire circuit with the help of rheostat if the potentiometer wire is of fixed length.

Illustration 6.9 **Figure 6.12 shows a potentiometer circuit for comparison of two resistances. The balance point with a standard resistor $R = 10.0\ \Omega$ is found to be 58.3 cm, while that with the unknown resistance X is 68.5 cm. Determine the value of X. What would you do if you fail to find a balance point with the given cell E?**

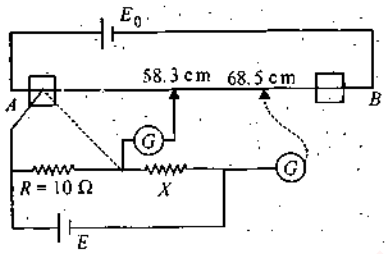


Fig. 6.12

Sol. Here, $l_1 = 58.3$ cm, $l_2 = 68.5$ cm, $R = 10\ \Omega$, $X = ?$

Let I be the current in the potentiometer wire and E_1 and E_2 be the potential drops across R and X, respectively. Then

$$\frac{E_2}{E_1} = \frac{IX}{IR} = \frac{X}{R} \quad \text{or} \quad X = \frac{E_2}{E_1}R \qquad \text{(i)}$$

But
$$\frac{E_2}{E_1} = \frac{l_2}{l_1}$$

From Eq. (i),
$$X = \frac{l_2}{l_1}R = \frac{68.5}{58.3} \times 10.0 \doteq 11.75\ \Omega$$

If there is no balance point with the given cell of e.m.f. E, it means potential drop across R or X is greater than the potential drop across the potentiometer wire AB. In order to obtain the balance point, the potential drops across R and X are to be reduced, which is possible by reducing the current. For that, either a suitable resistance should be put in series with R and X or a cell of smaller e.m.f. should be used. Another possible way is to increase the potential drop across the potentiometer wire by increasing the voltage of driver cell.

Illustration 6.10 **Figure 6.13 shows a 2.0 V potentiometer used for the determination of internal resistance of a 1.5 V cell. The balance point of the cell in open circuit is 76.3 cm. When a resistor of 9.5 Ω is used in the external circuit of the cell, the balance point shifts to 64.8 cm, length of the potentiometer. Determine the internal resistance of the cell.**

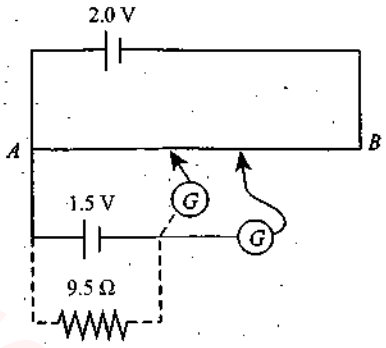


Fig. 6.13

Sol. Here, $l_1 = 76.3$ cm, $l_2 = 64.8$ cm, $r = ?$, $R = 9.5\ \Omega$

Now, $r = \left(\frac{l_1 - l_2}{l_2}\right)R = \left(\frac{76.3 - 64.8}{64.8}\right) \times 9.5 = 1.68\ \Omega$

Illustration 6.11 **A voltage V_0 is applied to a potentiometer whose sliding contact is exactly in the middle. A voltmeter V is connected between the sliding contact and one fixed end of the potentiometer. It is assumed that the resistance of the voltmeter is not very high is in comparison with the resistance of the potentiometer wire. What voltage will the voltmeter show, higher than, less than or equal to $V_0/2$?**

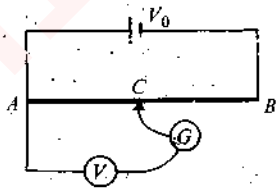


Fig. 6.14

Sol. Let R be the resistance of the whole potentiometer wire and R_V be the resistance of the voltmeter. Then the total resistance across section AC is

$$R_{AC} = \frac{R_V(R/2)}{R_V + R/2} = \frac{R}{2(1 + R/2R_V)} < \frac{R}{2}$$

The resistance of section CB is equal to $R/2$. So the reading of voltmeter will be less than $V_0/2$.

Illustration 6.12 **Potentiometer wire PQ of 1 m length is connected to a standard cell E_1. Another cell E_2 of e.m.f. 1.02 V is connected with a resistance r and a switch S as shown in the circuit diagram. With switch S open, null position is obtained at a distance of 51 cm from P. Calculate**
(a) potential gradient of the potentiometer wire.
(b) e.m.f. of cell E_1.
(c) when switch S is closed, will null point move towards P or towards Q? Give reason for your answer.

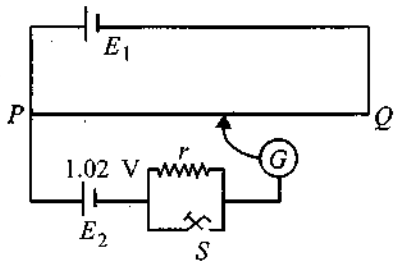


Fig. 6.15

Sol. **(a)** Potential gradient, $k = \frac{V}{l} = \frac{1.02}{0.51} = 2$ V/m

(b) e.m.f. of the cell $E_1 = k\ell = 2 \times 1 = 2$ V

(c) When switch S is closed, there is no shift in the position of null point as the position of null point depends upon the potential gradient along the potentiometer wire (which depends upon the e.m.f. of battery E_1 and resistance of potentiometer wire circuit and length of potentiometer) and e.m.f. of the cell E_2 which does not change when switch S is closed.

Illustration 6.13 **In Fig. 6.16, AB is a 1 m long uniform wire of 10 Ω resistance. Other data are shown in the diagram. Calculate (i) potential gradient along AB, (ii) length AO when galvanometer shows no deflection.**

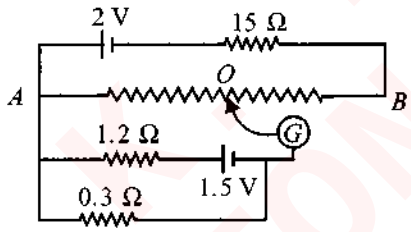


Fig. 6.16

Sol. **i.** Current in wire AB is $I = 2/(15 + 10) = 2/25$ A
Potential difference across AB, $V = IR = 2/25 \times 10 = 0.8$ V
Potential gradient along AB, $k = V/l = 0.8/1 = 0.8$ V/m

ii. Current through 0.3 Ω $= \frac{1.5}{1.2 + 0.3} = 1$ A

Potential difference across 0.3 Ω $= 1 \times 0.3 = 0.3$ V
Let l_1 be the length AO, then $0.3 = 0.8 \times l_1$
or $l_1 = 0.3/0.8 = 0.375$ m

Illustration 6.14 **Cells A and B and a galvanometer G are connected to a slide wire OS by two sliding contacts C and D as shown in Fig. 6.17. The slide wire is 100 cm long and has a resistance of 12 Ω. With OD = 75 cm, the galvanometer gives no deflection when OC is 50 cm. If D is moved to touch the end of wire S, the value of OC for which the galvanometer shows no deflection is 62.5 cm. The e.m.f. of cell B is 1.0 V. Calculate**

a. the potential difference across O and D when D is at 75 cm mark from O
b. the potential difference across OS when D touches S
c. internal resistance of cell A
d. the e.m.f. of cell A

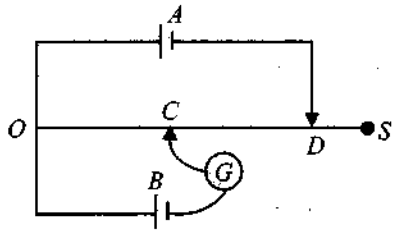


Fig. 6.17

Sol. Resistance of wire $OD = \frac{12}{100} \times 75 = 9$ Ω. Let E and r be the e.m.f. and internal resistance of cell E, respectively

a. Since 1 V is balanced across 50 cm, so potential gradient of wire = 1/50 V/cm. Therefore, voltage drop across the wire OD of length 75 cm = (1/50) × 75 = 1.5 V.

b. Now potential gradient of wire = 1/62.5 V/cm. Therefore, voltage drop across the wire OS of length 100 cm = (1/62.5) × 100 = 1.6 V

c. For first case: $\left(\frac{E}{9+r}\right) \times 9 = 1.5$ (i)

For second case: $\left(\frac{E}{12+r}\right) \times 12 = 1.6$ (ii)

d. On solving Eqs. (i) and (ii), we get $r = 3$ Ω and $E = 2$ V.

METER BRIDGE OR SLIDE WIRE BRIDGE

Slide wire bridge is a particle application of Wheatstone bridge and is used for (a) measuring an unknown resistance and (b) comparing two unknown resistances. Slide wire bridge works on the principle of Wheatstone bridge.

Construction

It consists of a uniform wire AC, usually of eureka or manganin of 1 m length. It is stretched on a wooden board between two copper strips. A metre scale is fitted on the board parallel to the length of wire. Another copper strip is fitted on the wooden board in order to provide two gaps.

In one of the gaps (say left gap) a resistance box R is connected, while in other gap (right gap) an unknown resistance S is connected. A cell E and a key K are connected across the ends A and C as shown in the figure.

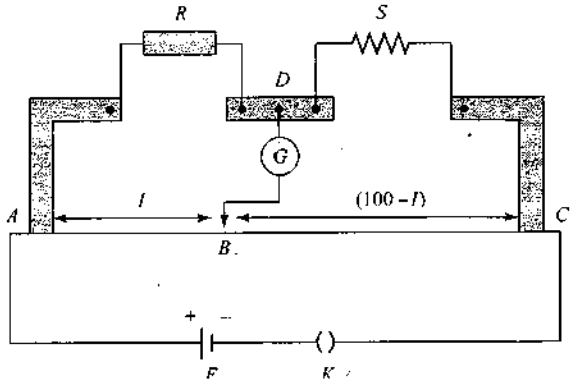


Fig. 6.18

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Checking of Connections

Close the key K and put the jockey at the end A of the wire and see the direction of deflection in the galvanometer. Now remove the jockey from A and put it at the end B of the wire and note the direction of deflection in the galvanometer. If the direction of deflection reverses, the connections are correct.

Working

Close the key K and take out some suitable low resistance R from the resistance box. Now move the jockey gently on wire AC till the galvanometer shows no deflection. Let this point be B on the wire.

Let $AB = l$

$\therefore$ $BC = (100 - l)$

Let the resistance of the wire between A and $B = P$ and the resistance of the wire between B and $C = Q$.

If r = resistance of wire of unit length, then

$P = lr$ and $Q = (100 - l)\,r$

According to the principle of Wheatstone bridge,

$$\frac{P}{Q} = \frac{R}{S} \Rightarrow \frac{\ell r}{(100-\ell)r} = \frac{R}{S} \Rightarrow S = \left(\frac{100-\ell}{\ell}\right)R$$

knowing λ and R, S can be calculated.

Illustration 6.15 **In the simple potentiometer circuit, where the length AB of the potentiometer wire is 1 m, the resistors X and Y have values of 5Ω and 2Ω, respectively. When X is shunted by a wire, the balance point is found to be 0.625 m from A. What is the resistance of the shunt?**

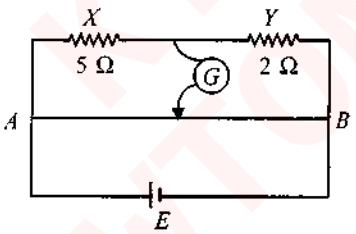

Fig. 6.19

Sol. Let R be the resistance of the shunted wire, the effective resistance of R and 5Ω in parallel = $5R/(5 + R)$.

At balance point, $\dfrac{5R/(5+R)}{2} = \dfrac{0.625}{1-0.625} = \dfrac{0.625}{0.0375} = \dfrac{5}{3}$

On solving, we get $R = 10\ \Omega$.

Concept Application Exercise 6.1

1. An ammeter is always connected in series and a voltmeter is connected in parallel to the circuit element in any electrical circuit. Why?
2. By mistake a voltmeter is connected in series and an ammeter in parallel with a resistance in an electrical circuit, what will happen to the instrument?
3. A 100 V voltmeter having an internal resistance of 20 kΩ is connected in series with a large resistance R across a 110 V line. What is the magnitude of resistance R if the voltmeter reads 5 V?
4. What will be the effect on the accuracy of the result if we replace a single wire potentiometer by a potentiometer having 12 wires, the length of each wire being 1 m?
5. In the circuit shown in Fig. 6.20, a metre bridge is in its balance state. The metre bridge wire has a resistance of 1 ohm cm^{-1}. Calculate the value of unknown resistance X and the current drawn from the battery of negligible internal resistance.

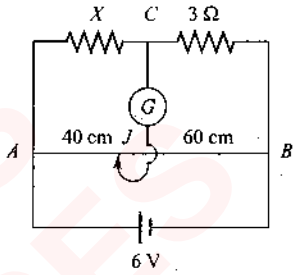

Fig. 6.20

6. The variation of potential difference V with length ℓ in case of two potentiometers X and Y is as shown in the given diagram. Which one of these two will you prefer for comparing e.m.f.s of two cells and why?

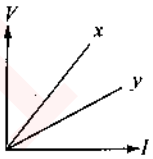

Fig. 6.21

7. Two unknown resistances X and Y are placed on the left and right gaps of a meter bridge. The null point in the galvanometer is obtained at a distance of 80 cm from left. A resistance of 100 Ω is now connected in parallel across X. The null point is then found by shifting the sliding contact towards left by 20 cm. Calculate X and Y.
8. A galvanometer with a coil of resistance 12.0 Ω shows full-scale deflection for a current of 2.5 mA. How will you convert the meter into
   i. an ammeter of range 0 to 7.5 A?
   ii. a voltmeter of range 0 to 1.0 V?
9. What shunt resistance is required to make a 1.00 mA, 20.0 Ω meter into an ammeter with a range of 0 A to 50.0 mA?
10. How can we make a galvanometer with R_G = 20.0 Ω and I_g = 0.00100 A into a voltmeter with a maximum range of 10.0 V?
11. In an experiment with a potentiometer, the null point is obtained at a distance of 60 cm along the wire from the common terminal with a Leclanche cell. When a shunt resistance of 1 Ω is connected across the cell, the null point shifts to a distance of 30 cm from the common terminal. What is the internal resistance of the cell?
12. In the experiment of calibration of voltmeter, a standard cell of e.m.f. 1.1 V is balanced against 440 cm of potentiometer wire. The potential difference across the ends of a resistance is found to balance against 220 cm of the wire. The corresponding reading of voltmeter is 0.5 V. Find the error in the reading of voltmeter.
13. It is required to measure the resistance of a circuit operating at 120 V. There is only one galvanometer of current sensitivity 10^{-6} A per division. How should the galvano-

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meter be connected in the circuit to operate an ohmmeter? What minimum resistance can be measured with such a galvanometer if its full-scale has 40 divisions?

14. In the circuit shown in Fig. 6.22, a voltmeter reads 30 V when it is connected across a 400 ohm resistance. Calculate what the same voltmeter would read when it is connected across the 300 ohm resistance.

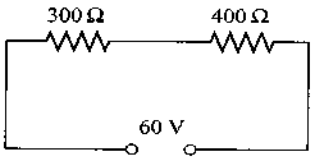


Fig. 6.22

(IIT-JEE, 1980)

15. Draw a circuit diagram to verify Ohm's law with the help of a main resistance of 100 Ω, a variable dc source, two galvanometers and two resistances of values 10^6 Ω and 10^{-3} Ω. Show the correct positions of voltmeter and ammeter.

(IIT-JEE, 2004)

Solved Examples

Example 6.1 **A galvanometer of resistance 95 Ω, shunted by a resistance of 5 Ω, gives a deflection of 50 divisions when joined in series with a resistance of 20 Ω and a 2 V accumulator. What is the current sensitivity of the galvanometer (in div/μA)?**

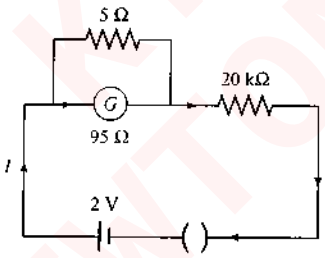


Fig. 6.23

Sol. In accordance with the given problem, the situation is depicted by the circuit diagram in the figure. As 20 kΩ is much greater than the resistance of the shunted galvanometer (<5 Ω), the current in the circuit will be

$$I = \frac{2}{20 \times 10^3} = 10^{-4}\ \text{A} = 100\ \mu\text{A}$$

And as this current produces a deflection of 50 divisions in the galvanometer, $CS = \frac{\theta}{I} = \frac{50\,\text{div}}{100\,\mu\text{A}} = \frac{1}{2}\frac{\text{div}}{\mu\text{A}}$

Example 6.2 **An electrical circuit is shown in Fig. 6.24. Calculate the potential difference across the resistor of 400 Ω, as will be measured by the voltmeter V of resistance 400 Ω, either by applying Kirchhoff's rule or otherwise.**

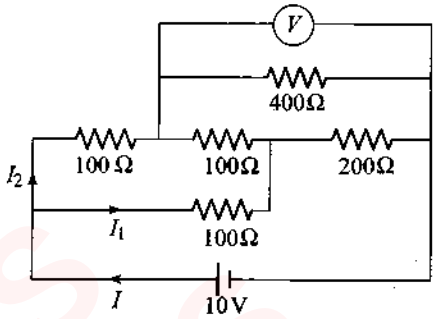


Fig. 6.24

(IIT-JEE, 1996)

Sol. Applying Kirchhoff's law in loop *JMGDJ*, we get

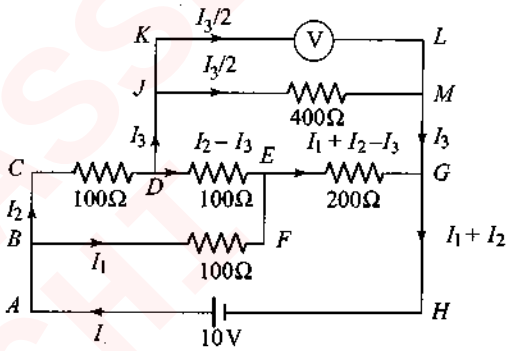


Fig. 6.25

$$-\frac{I_3}{2} \times 400 + (I_1 + I_2 - I_3)\,200 + (I_2 - I_3)\,100 = 0$$

$$\Rightarrow \quad -500\,I_3 + 200I_1 + 300I_2 = 0 \Rightarrow 2I_1 + 3I_2 - 5I_3 = 0 \qquad \text{(i)}$$

Applying Kirchhoff's law in *CDEFBC*, we get

$$-100I_2 - 100(I_2 - I_3) + 100I_1 = 0$$

$$\Rightarrow \quad I_1 - 2I_2 + I_3 = 0 \qquad \text{(ii)}$$

Multiplying Eq.(ii) by 2 and subtracting from, Eq. (i), we have

$$2I_1 + 3I_2 - 5I_3 - 2I_1 + 4I_2 - 2I_3 = 0 \Rightarrow I_2 = I_3 \qquad \text{(iii)}$$

Applying Kirchhoff's law in *ABFEGHA*, we get

$$-3I_1 - 2I_2 + 2I_3 + 0.1 = 0 \qquad \text{(iv)}$$

Multiplying Eq. (ii) by 3 and adding it with Eq. (iv), we get

$$3I_1 - 6I_2 + 3I_3 - 3I_1 - 2I_2 + 2I_3 + 0.1 = 0$$

$$\Rightarrow \quad -8I_2 + 5I_3 + 0.1 = 0 \Rightarrow -8I_3 + 5I_3 + 0.1 = 0$$

$$\Rightarrow \quad 3I_3 = 0.1 \Rightarrow I_3 = \frac{0.1}{3}$$

Therefore, potential difference across *JM* is

$$\frac{0.1/3}{2} \times 400 = \frac{0.1}{2 \times 3} \times 400$$

$$= \frac{20}{3} = 6.67\ \text{V}$$

Alternatively: We can redraw the circuit as shown in Fig. 6.26.

The equivalent resistance between G and D is

$$R_{GD} = \frac{400 \times 400}{400 + 400} = 200\,\Omega, \text{ Since } \frac{R_{GE}}{R_{GD}} = \frac{R_{EB}}{R_{DB}}$$

Therefore, it is a case of balanced Wheatstone bridge.

The equivalent resistance across G and B is

$$R_{GB} = \frac{300 \times 300}{300 + 300} = 150\,\Omega$$

Therefore, current $I = \frac{V}{R_{GB}} = \frac{10}{150} = \frac{1}{15}$ A

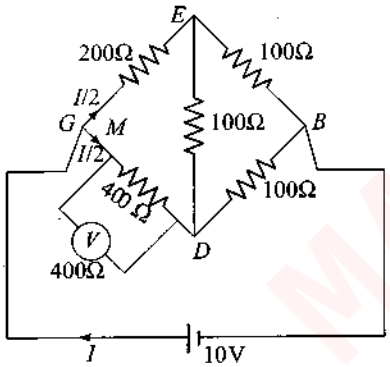


Fig. 6.26

Since $RGEB = RGDB$, the current is divided at G into two equal parts. The current $I/2$ further divides into two equal parts at M. Therefore, the potential difference across the voltmeter is $\frac{I}{4} \times 400 = \frac{1}{15} \times 100 = \frac{20}{3}$ V.

Example 6.3 **A thin uniform wire AB of length 1 m, an unknown resistance X and a resistance of 12 Ω are connected by thick conducting strips, as shown in Fig. 6.27. A battery and a galvanometer (with a sliding jockey connected to it) are also available. Connections are to be made to measure the unknown resistance X using the principle of Wheatstone bridge. Answer the following questions.**

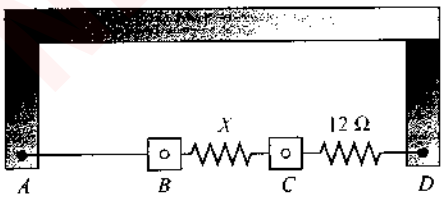


Fig. 6.27

(a) Are there positive and negative terminals on the galvanometer?

(b) Copy the figure in your answer book and show the battery and the galvanometer (with jockey) connected at appropriate points.

(c) After appropriate connections are made, it is found that no deflection takes place in the galvanometer when the sliding jockey touches the wire at a distance of 60 cm from A. Obtain the value of the resistance X.

(IIT-JEE, 2002)

Sol.

(a) No. There are no positive and negative terminals on the galvanometer.

(b)

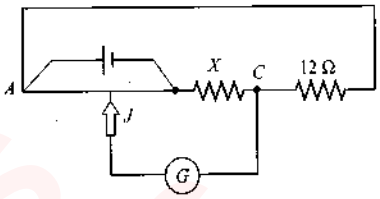


Fig. 6.28

(c) Since bridge is balanced, $\frac{R_{Al}}{R_{lB}} = \frac{0.6\rho}{0.4\rho} = \frac{12\Omega}{X} \Rightarrow X = 8\,\Omega$

where ρ is the resistance per unit length.

Example 6.4 **An unknown resistance is to be determined using resistances R_1, R_2 or R_3. Their corresponding null points are A, B and C. Find which of the above will give the most accurate reading and why? $R = R_1$ or R_2 or R_3**

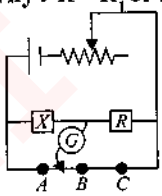


Fig. 6.29 **(IIT-JEE, 2005)**

Sol. All null point, the Wheatstone bridge will be balanced

$$\therefore \frac{X}{r_1} = \frac{R}{r_2} \Rightarrow X = R\frac{r_1}{r_2}$$

where R is a constant and r_1 and r_2 are variables. The maximum fraction error is $\frac{\Delta X}{X} = \frac{\Delta r_1}{r_1} + \frac{\Delta r_2}{r_2}$

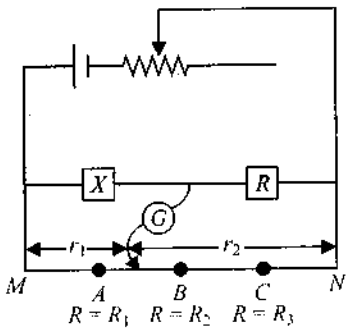


Fig. 6.30

Here $\Delta r_1 = \Delta r_2 = y$ (say) then

$$\frac{\Delta X}{X} = y\left[\frac{r_2 + r_1}{r_1 r_2}\right]$$

For $\frac{\Delta X}{X}$ to be minimum, $r_1 \times r_2$ should be maximum

$[\because r_1 + r_2 = c \text{ (constant)}]$

Let $E = r_1 \times r_2 \Rightarrow E = r_1 \times (c - r_1) = r_1 c - r_1^2$

$$\therefore \frac{dE}{dr_1} = c - 2r_1 = 0 \Rightarrow r_1 = \frac{c}{2} \Rightarrow r_2 = \frac{c}{2} \Rightarrow r_1 = r_2$$

$\Rightarrow R_2$ gives the most accurate value

EXERCISES

Subjective Type

Solutions on page 6.20

1. Consider the potentiometer circuit arranged as in the figure. The potentiometer wire is 600 cm long and its resistance is $15r$.

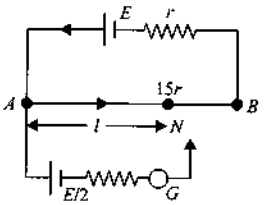


Fig. 6.31

a. At what distance from point A should the jockey touch the wire to get zero deflection in the galvanometer?

b. If the jockey touches the wire at a distance of 560 cm from A, what will be the current in the galvanometer?

2. Figure 6.32 shows a metre bridge (which is nothing but a practical Wheatstone bridge), consisting of two resistors X and Y together in parallel with a metre long constantan wire of uniform cross section.

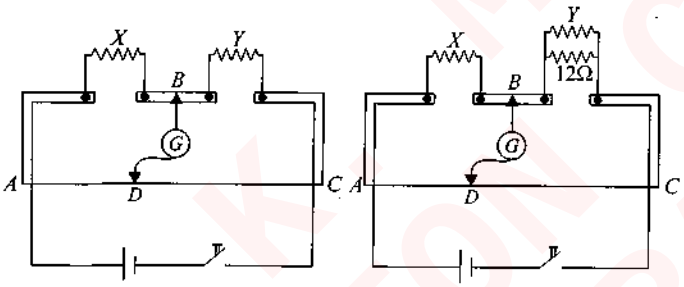


Fig. 6.32

With the help of a movable contact D, one can change the ratio of resistance of the two segments of the wire until a sensitive galvanometer G connected across B and D shows no deflection. The null point is found to be at a distance of 33.7 cm. The resistor Y is shunted by a resistance of 12 Ω and the null point is found to shift by a distance of 18.2 cm. Determine the resistance of X and Y.

3. The circuit shown in Fig. 6.33 shows the use of potentiometer to measure the internal resistance of a cell.

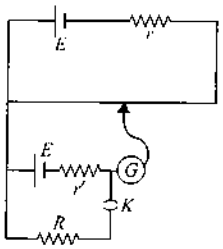


Fig. 6.33

a. When the key is open, how does the balance point change, if the current from the driver cell decreases?

b. When key is closed, how does the balance point change, if R is increased, keeping the current from the driver cell constant?

4. Let V and I represent, respectively, the readings of the voltmeter and ammeter shown in Fig. 6.34, and let R_V and R_A be their equivalent resistances. Because of the resistances of the meters, the resistance R is not simply equal to V/I.

a. When the circuit is connected as shown in Fig.6.34 (a), show that $R = \dfrac{V}{I} - R_A$.

Explain why the true resistance R is always less than V/I.

b. When the connections are as shown in Fig. 6.34 (b)

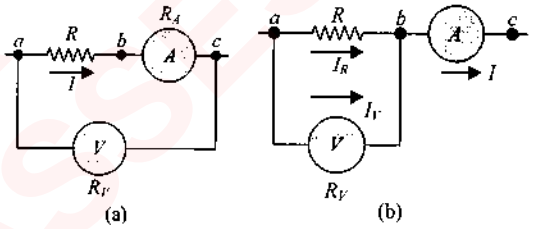


Fig. 6.34

show that $R = \dfrac{V}{I - (V/R_V)}$

Explain why the true resistance R is always greater than V/I.

c. Show that the power delivered to the resistor in part (i) is $IV - I^2R_A$ and that in part (ii) is $IV - (V^2/R_V)$.

5. You are given two resistors X and Y whose resistances are to be determined using an ammeter of resistance 0.5 Ω and a voltmeter of resistance 20 kΩ. It is known that X is in the range of a few ohms, while Y is in the range of several thousand ohms. In each case, which of the following two connections (Fig. 6.35) would you choose for resistance measurement? Justify your answer quantitatively. [**Hint:** For each connection, determine the error in resistance measurement. The connection that corresponds to a smaller error (for a given range of resistance) is to be preferred.]

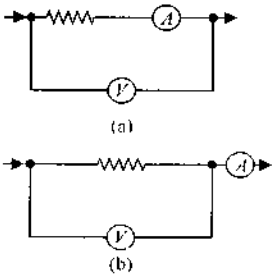


Fig. 6.35

6. Figure 6.36 shows a potentiometer with a cell of e.m.f. 2.0 V and internal resistance 0.4 Ω maintaining a potential drop across the resistor wire AB. A standard cell which maintains a constant e.m.f. of 1.02 V (for very moderate currents upto a few μA) gives a balance point at 67.3 cm length of the wire. To ensure very low current is drawn from the standard cell, a very high resistance of 600 kΩ is put in series with it, which is shorted close to the balance point. The standard cell is then replaced by a cell of unknown e.m.f. ε and the balance point found, similarly, turns out to be at 82.3 cm length of the wire.

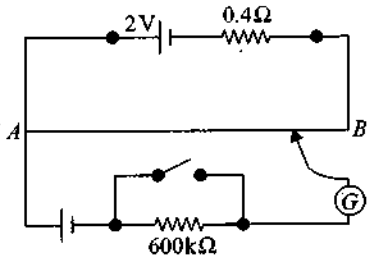


Fig. 6.36

a. What is the value of ε?
b. What purpose does the high resistance of 600 kΩ have?
c. Is the balance point affected by this high resistance?
d. Is the balance point affected by internal resistance of the driver cell?
e. Would the method work in the above situation if the driver cell of the potentiometer had an e.m.f. of 1.0 V instead of 2.0 V?
f. Would the circuit work well for determining an extremely small e.m.f., say of the order of a few mV (such as the typical e.m.f. of a thermocouple)? If not, how will you modify the circuit?

7. In a meter bridge circuit, the two resistances in the gap are 5 Ω and 10 Ω. The wire resistance is 4 Ω. The e.m.f. of the cell connected at the ends of wire is 5 V and its internal resistance is 1 Ω. What current will flow through the galvanometer of resistance 30 Ω if the contact is made at the mid-point of the wire ?

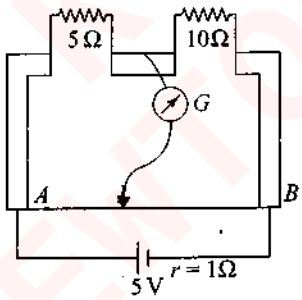


Fig. 6.37

8. In the given circuit, a meter bridge is shown in a balanced state. The bridge wire has a resistance of 1 Ω/cm. Find the value of the unknown resistance X and the current drawn from the battery of negligible internal resistance.

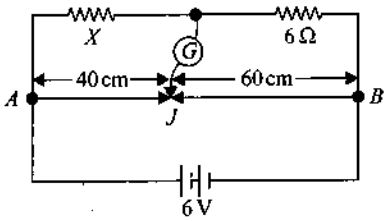


Fig. 6.38

9. An experiment with a post office box, the ratio of arms are 1000:10. If the value of the third resistance is 999 Ω, find the unknown resistance.

10. A voltmeter reads 5.0 V at full-scale deflection and is graded according to its resistance per volt at full-scale deflection as 5000 Ω/V. How will you convert it into a voltmeter that reads 20 V at full-scale deflection? Will it still be graded as 5000 Ω/V? Will you prefer this voltmeter to one that is graded as 2000 Ω/V?

11. A battery of e.m.f. 1.4 V and internal resistance 2 Ω is connectd to a resistor of 100 Ω resistance through an ammeter. The resistance of the ammeter is 4/3 Ω. A voltmeter has also been connected to find the potential difference across the resistor.
(i) Draw the circuit diagram.
(ii) The ammeter reads 0.02 A. What is the resistance of the voltmeter?
(iii) The voltmeter reads 1.1 V. What is the error in the reading?

12. A potentiometer wire has a length of 10 m and resistance 4 Ω/m. An accumulator of e.m.f. 2 V and a resistance box are connected in series with it. Calculate the resistance to be introduced in the box so as to get a potential gradient of (i) 0.1 V/m and (ii) 0.1 mV/m.

Objective Type

Solutions on page 6.23

1. How will the reading in the ammeter A of Fig. 6.39 be affected if another identical bulb Q is connected in parallel to P as shown. The voltage in the mains is maintained at a constant value.

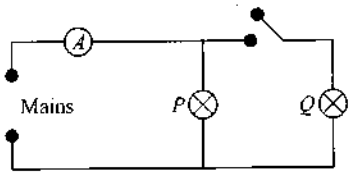


Fig. 6.39

a. The reading will be reduced to one-half
b. The reading will not be affected
c. The reading will be double of the previous one
d. The reading will be increased four-fold

2. A potentiometer is connected across A and B and a balance is obtained at 64.0 cm. When potentiometer lead to B is moved to C, a balance is found at 8.0 cm. If the potentiometer is now connected across B and C, a balance will be found at

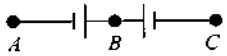


Fig. 6.40

a. 8.0 cm b. 56.0 cm c. 64.0 cm d. 72.0 cm

3. In the circuit shown in Fig. 6.41, the reading of the ammeter is (assume internal resistance of the battery be to zero)

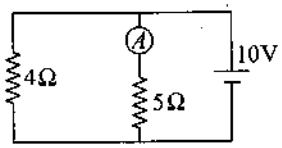


Fig. 6.41

a. $\frac{40}{29}$ A b. $\frac{10}{9}$ A c. $\frac{5}{3}$ A d. 2 A

4. In the circuit shown in Fig. 6.42, resistors X and Y, each with resistance R, are connected to a 6 V battery of negligible internal resistance. A voltmeter, also of resistance R, is connected across Y.

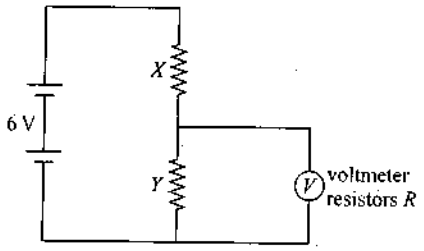


Fig. 6.42

What is the reading of the voltmeter?

a. Zero b. Between zero and 3 V
c. 3 V d. Between 3 V and 6 V

5. In the shown arrangement of the experiment of a meter bridge, if AC corresponding to null deflection of galvanometer is x, what would be its value if the radius of the wire AB is doubled?

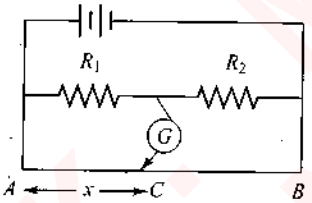


Fig. 6.43

a. x b. $x/4$ c. $4x$ d. $2x$

6. The length of a wire of a potentiometer is 100 cm, and the e.m.f. of its standard cell is E volt. It is employed to measure the e.m.f. of a battery whose internal resistance is 0.5 Ω. If the balance point is obtained at $l = 30$ cm from the positive end, e.m.f. of the battery is **(AIEEE, 2003)**

a. $\frac{30E}{100}$ b. $\frac{30E}{100.5}$

c. $\frac{30E}{(100-0.5)}$ d. $\frac{30(E-0.5i)}{100}$

where i is the current in the potentiometer wire.

7. In a metre bridge experiment, null point is obtained at 20 cm from one end of the wire when resistance X is balanced against another resistance Y. If $X < Y$, then where will be the new position of the null point from the same end, if one decides to balance a resistance of $4X$ against Y? **(AIEEE, 2004)**

a. 50 cm b. 80 cm c. 40 cm d. 70 cm

8. In the circuit shown in Fig. 6.44, the galvanometer G shows zero deflection. If the batteries A and B have negligible internal resistance, the value of the resistor R will be **(AIEEE, 2005)**

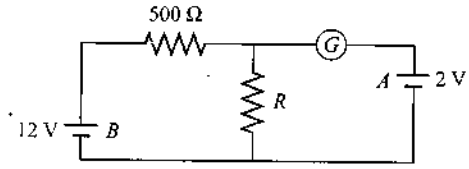


Fig. 6.44

a. 1000 Ω b. 500 Ω c. 100 Ω d. 200 Ω

9. In a potentiometer experiment, the balancing with a cell is at length 240 cm. On shunting the cell with a resistance of 2 Ω, the balancing length becomes 120 cm. The internal resistance of the cell is **(AIEEE, 2005)**

a. 2 Ω b. 4 Ω c. 0.5 d. 1 Ω.

10. If in the experiment of Wheatstone bridge, the positions of cells and galvanometer are interchanged, then the balance points will

a. change
b. remain unchanged
c. depend on the internal resistance of the cell and resistance of the galvanometer
d. none of these

11. Sensitivity of the potentiometer can be increased by

a. increasing the e.m.f. of the cell
b. increasing the length of the potentiometer
c. decreasing the length of the potentiometer wire
d. none of the above

12. The resistance of a galvanometer is 10 Ω. It gives full-scale deflection when 1 mA current is passed. The resistance connected in series for converting it into a voltmeter of 2.5 V will be

a. 24.9 Ω b. 249 Ω c. 2490 Ω d. 24900 Ω

13. A milliammeter of range 10 mA has a coil of resistance 1 Ω. To use it as an ammeter of range 1A, the required shunt must have a resistance of

a. $\frac{1}{101}$ Ω b. $\frac{1}{100}$ Ω c. $\frac{1}{99}$ Ω d. $\frac{1}{9}$ Ω

14. Two cells of e.m.f.s E_1 and E_2 ($E_1 > E_2$) are connected as shown in Fig. 6.45

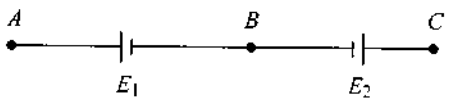


Fig. 6.45

When a potentiometer is connected between A and B, the balancing length of the potentiometer wire is 300 cm. On connecting the same potentiometer between A and C, the balancing length is 100 cm. The ratio E_1/E_2 is

a. 3:1 b. 1:3 c. 2:3 d. 3:2

15. Figure 6.46 shows a Wheatstone net, with $P = 1000$ Ω, $Q = 10.0$ Ω, R (unknown), S variable and near 150 Ω for balance. If the connections across A, C and B, D are interchanged, the error range in R determination would

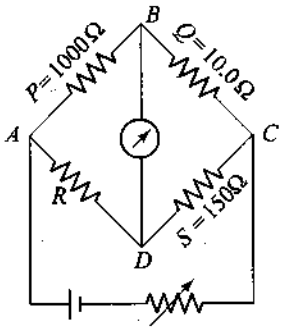


Fig. 6.46

a. remain unaffected b. increase substantially
c. increase marginally d. decrease substantially

16. An ideal ammeter (zero resistance) and an ideal voltmeter (infinite resistance) are connected as shown in Fig. 6.47. The ammeter and voltmeter readings are

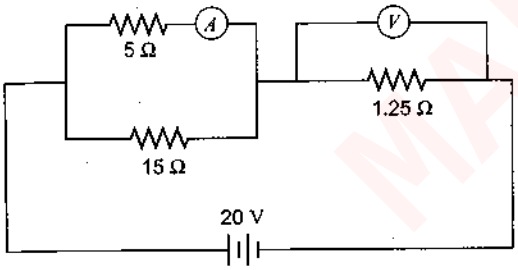


Fig. 6.47

a. 6.25 A, 3.75 V b. 3.00 A, 5 V
c. 3.00 A, 3.75 V d. 6.00 A, 6.25 V

17. A constant 60 V d.c. supply is connected across two resistors of resistance 400 kΩ and 200 kΩ. What is the reading of the voltmeter, also of resistance 200 kΩ, when connected across the second resistor as shown in Fig. 6.48?

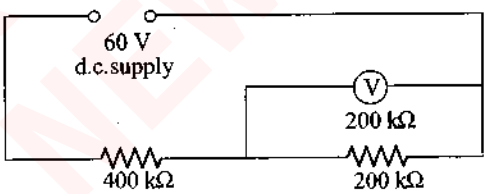


Fig. 6.48

a. 12 V b. 15 V c. 20 V d. 30 V

18. Figure 6.49 shows a circuit which may be used to compare the resistance R of an unknown resistor with a 100 Ω standard. The distances l from one end of the potentiometer slider wire to the balance point are 400 mm and 588 mm when X is connected to Y and Z, respectively. The length of the slide wire is 1.00 m. What is the value of resistance R?

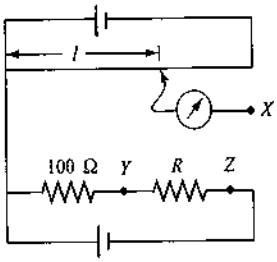


Fig. 6.49

a. 32 Ω b. 47 Ω c. 68 Ω d. 147 Ω

19. In the circuit shown in Fig. 6.50, an ideal ammeter and an ideal voltammeter are used. When key is open, the voltmeter reads 1.53 V. When the key is closed, the ammeter reads 1.0 A and the voltmeter reads 1.03 V. The resistance R is

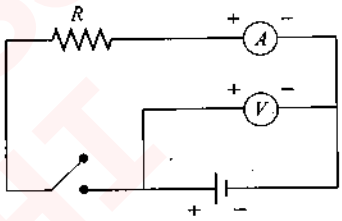
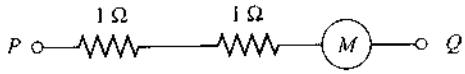
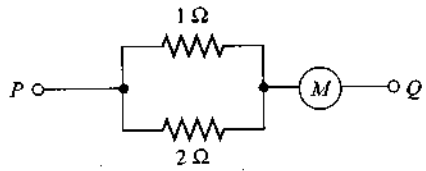


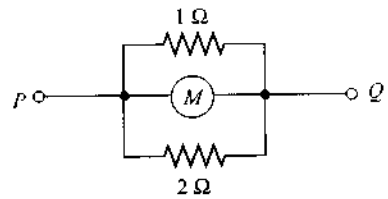
Fig. 6.50

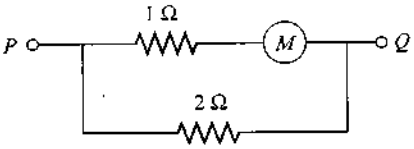
a. 0.5 Ω b. 1.03 Ω c. 1.53 Ω d. 0.53 Ω

20. In which one of the following arrangements of resistors does the meter M, which has a resistance of 2 Ω, give the largest reading when the same potential difference is applied between points P and Q?

a.


b.


c.


d.


21. Figure 6.51 shows a simple potentiometer circuit for measuring a small e.m.f. produced by a thermocouple.

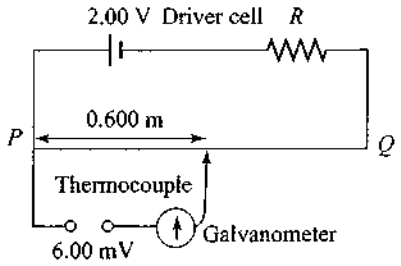


Fig. 6.51

The metre wire PQ has a resistance of 5 Ω and the driver cell has an e.m.f. of 2.00 V. If a balance point is obtained 0.600 m along PQ when measuring an e.m.f. of 6.00 mV, what is the value of resistance R?

a. 95 Ω **b.** 995 Ω **c.** 195 Ω **d.** 1995 Ω

22. Figure 6.52 shows a balanced Wheatstone net. Now, it is disturbed by changing P to 11 Ω. Which of the following steps will not bring the bridge to balance again?

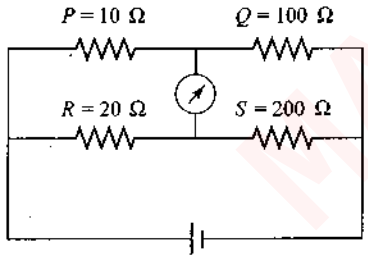


Fig. 6.52

a. Increasing R by 2 Ω
b. Increasing S by 20 Ω
c. Increasing Q by 10 Ω
d. Making product $RQ = 2200\ (\Omega)^2$

23. In an experiment to measure the internal resistance of a cell by a potentiometer, it is found that the balance point is at a length of 2 m when the cell is shunted by a 5 Ω resistance and is at a length of 3 m when the cell is shunted by a 10 Ω resistance, the internal resistance of the cell is then

a. 1.5 Ω **b.** 10 Ω **c.** 15 Ω **d.** 1 Ω

24. When an ammeter of negligible internal resistance is inserted in series with circuit, it reads 1 A. When the voltmeter of very large resistance is connected across R_1, it reads 3 V. But when the points A and B are short-circuited by a conducting wire, then the voltmeter measures 10.5 V across the battery. The internal resistance of the battery is equal to

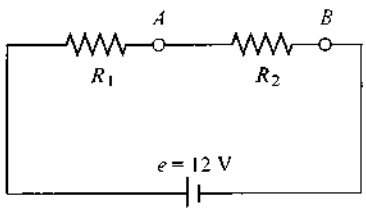


Fig. 6.53

a. $\frac{3}{7}\ \Omega$ **b.** 5 Ω
c. 3 Ω **d.** none of these

25. An 80 Ω galvanometer deflects full-scale for a potential of 20 mV. A voltmeter deflecting full-scale of 5 V is to be made using this galvanometer. We must connect

a. a resistance of 19.92 kΩ parallel to the galvanometer
b. a resistance of 19.92 kΩ in series with the galvanometer
c. a resistance of 20 kΩ parallel to the galvanometer
d. a resistance of 20 kΩ in series with the galvanometer

26. A voltmeter having a resistance of 1800 Ω is employed to measure the potential difference across 200 Ω resistance which is connected to dc power supply of 50 V and internal resistance 20 Ω. What is the approximate percentage change in the p.d. across 200 Ω resistance as a result of connecting the voltmeter across it?

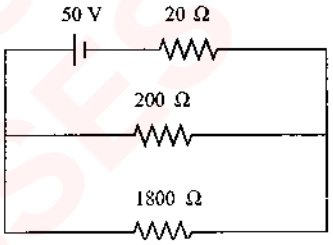


Fig. 6.54

a. 2.2% **b.** 5% **c.** 10% **d.** 20%

27. In the given circuit, the voltmeter and the electric cell are ideal. Find the reading of the voltmeter

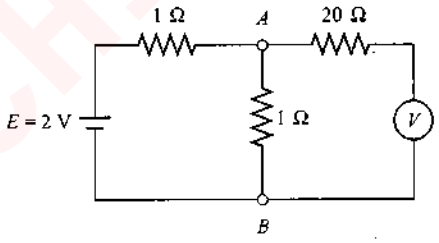


Fig. 6.55

a. 1 V **b.** 2 V
c. 3 V **d.** None of these

28. The e.m.f. of the driver cell of a potentiometer is 2 V and its internal resistance is negligible. The length of the potentiometer wire is 100 cm and resistance is 5 Ω. How much resistance is to be connected in series with the potentiometer wire to have a potential gradient of 0.05 mV/cm?

a. 1990 Ω **b.** 2000 Ω
c. 1995 Ω **d.** None of these

29. In the above question, if the balancing length for a cell of e.m.f. E is 60 cm, the value of E will be

a. 3 mV **b.** 5 mV **c.** 6 mV **d.** 2000 Ω

30. A, B and C are voltmeters of resistance R, $1.5R$ and $3R$, respectively. When some potential difference is applied between X and Y, the voltmeter readings are V_A, V_B and V_C, respectively. Then

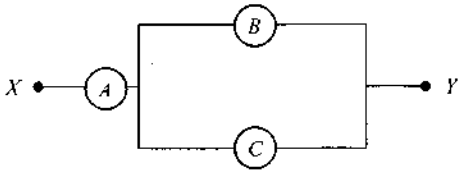


Fig. 6.56

a. $V_A = V_B = V_C$ b. $V_A \neq V_B = V_C$
c. $V_A = V_B \neq V_C$ d. $V_B \neq V_A = V_C$

31. A milliammeter of range 10 mA and resistance 9 Ω is joined in a circuit as shown in Fig. 6.57. The meter gives full-scale deflection for current I when A and B are used as its terminals, if current enters at A and leaves at B (C is left isolated), the value of I is

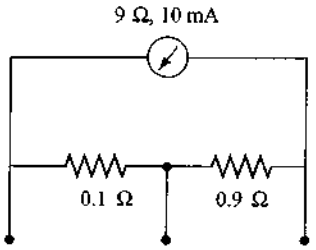


Fig. 6.57

a. 100 mA b. 900 mA c. 1 A d. 1.1 A

32. A candidate connects a moving coil voltmeter V, a moving coil ammeter A and a resistor R as shown in Fig. 6.58. If the voltmeter reads 20 V and the ammeter reads 4 A, R is

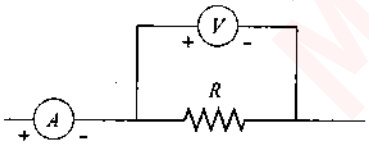


Fig. 6.58

a. equal to 5 Ω
b. greater than 5 Ω
c. less than 5 Ω
d. greater or less than 5 Ω depending upon its material

33. If a shunt of 1/10th of the coil resistance is applied to a moving coil galvanometer, its sensitivity becomes

a. 10-fold b. 11-fold c. $\frac{1}{10}$-fold d. $\frac{1}{11}$-fold

34. In Fig. 6.59, when an ideal voltmeter is connected across 4000 Ω resistance, it reads 30 V. If the voltmeter is connected across 3000 Ω resistance, it will read

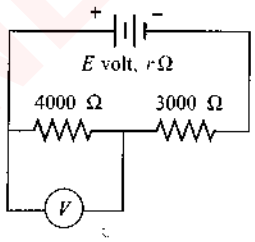


Fig. 6.59

a. 20 V b. 22.5 V c. 35 V d. 40 V

35. A voltmeter has a resistance of G ohm and range V volt. The value of resistance used in series to convert it into voltmeter of range nV volt is

a. nG b. $(n-1)G$ c. G/n d. $G/(n-1)$

36. A galvanometer has a resistance of 3663 Ω. A shunt S is connected across it such that (1/34) of the total current passes through the galvanometer. The value of the shunt is

a. 3663 Ω b. 111 Ω c. 107.7 Ω d. 3555.3 Ω

37. In Q.36 the combined resistance of the shunt and the galvanometer is

a. 3665 Ω b. 111 Ω c. 107.7 Ω d. 3555.3 Ω

38. In Q.36, the external resistance which must be connected in series with the main circuit so that the total current in the main circuit remains unaltered even when the galvanometer is shunted is

a. 3663 Ω b. 111 Ω c. 107.7 Ω d. 3555.3 Ω

39. Two moving coil galvanometers 1 and 2 are with identical field magnets and suspension torque constants, but with coil of different number of turns, N_1 and N_2, area per turn A_1 and A_2 and resistance R_1 and R_2. When they are connected in series in the same circuit, they show deflections θ_1 and θ_2. Then (θ_1/θ_2) is

a. (A_1N_1/A_2N_2) b. (A_1N_2/A_2N_1)
c. $(A_1R_2N_1/A^2R^2N^2)$ d. $(A_1R_1N_1/A_2R_2N^2)$

40. An ammeter is obtained by shunting a 30 Ω galvanometer with a 30 Ω resistance. What additional shunt should be connected across it to double the range?

a. 15 Ω b. 10 Ω
c. 5 Ω d. None of these

41. Three voltmeters are connected as shown.

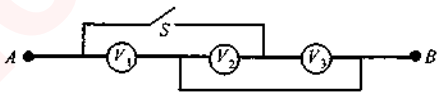
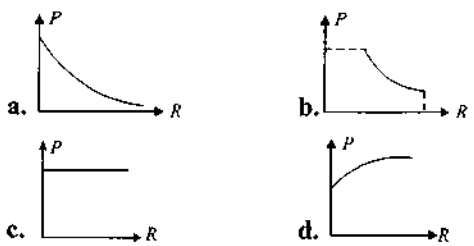


Fig. 6.60

A potential difference has been applied between A and B. On closing the switch S, what will be the effect on the readings of voltmeters?

a. V_1 increases
b. V_1 decreases
c. V_2 and V_3 both increase
d. One of V_2 and V_3 increases and other decreases.

42. A constant p.d. is applied across a resistance. Consider variation of resistance with temperature. Which graph represents best the variation of power produced in resistance versus resistance?



43. In the figure, voltmeter and ammeter shown are ideal. Then voltmeter and ammeter readings, respectively, are

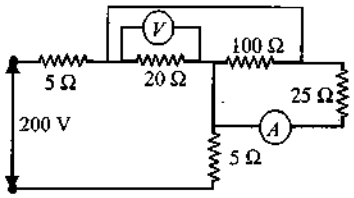


Fig. 6.61

a. 125 V, 3 A **b.** 100 V, 4 A
c. 120 V, 4 A **d.** 120 V, 3 A

44. A potentiometer arrangement is shown in the figure. Deriving cell has e.m.f. e and internal resistance r.

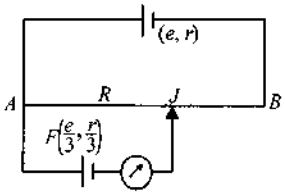


Fig. 6.62

Resistance of potentiometer wire AB is R. F is the cell of e.m.f. $e/3$ and internal resistance $r/3$. Balance point (J) can be obtained for all finite values of

a. $R > r/2$ **b.** $R < r/2$
c. $R > r/3$ **d.** $R < r/3$

Multiple Correct Answers Type

Solutions on page 6.25

1. A voltmeter reads the potential difference across the terminals of an old battery as 1.40 V while a potentiometer reads its voltage to be 1.55 V. The voltmeter resistance is 280 Ω. Then

a. the e.m.f. of the battery is 1.4 V
b. the e.m.f. of the battery is 1.55 V
c. the internal resistance r of the battery is 30 Ω
d. the internal resistance r of the battery is 5 Ω

2. In the circuit shown in Fig. 6.63, the cell is ideal with e.m.f. 9 V. If the resistance of the coil of galvanometer is 1 Ω, then

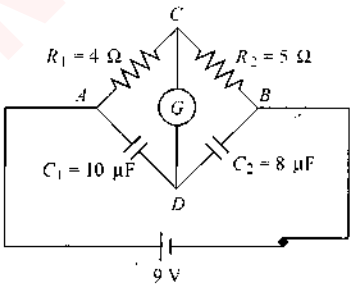


Fig. 6.63

a. No current flows in the galvanometer
b. Charge flowing through 8 μF is 40 μC
c. Potential difference across 10 μF is 5 V
d. Potential difference across 10 μF is 4 V

3. Figure 6.64 shows a balanced Wheatstone bridge

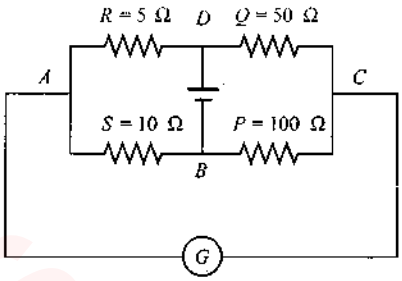


Fig. 6.64

a. If P is slightly increased, the current in the galvanometer flows from C to A
b. If P is slightly increased, the current in the galvanometer flows from A to C
c. If Q is slightly increased, the current in the galvanometer flows from C to A
d. If Q is slightly increased, the current in the galvanometer flows from A to C

4. Two voltmetes and two resistances are connected as shown in the figure. On closing the switch S, what will be the effect on the readings of the voltemters ?

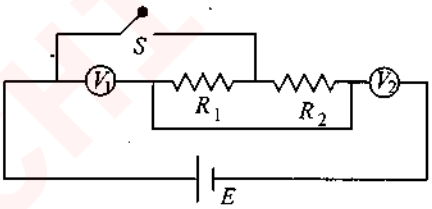


Fig. 6.65

a. V_1 increases **b.** V_1 decreases
c. V_2 increases **d.** V_2 decreases

5. In the figure shown, voltmeter is not ideal. If voltmeter is removed from R_1 and then put across R_2, what will be the effect on current I? Given $R_1 > R_2$.

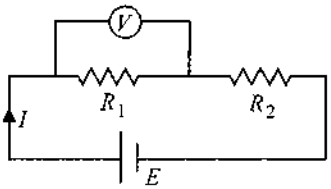


Fig. 6.66

a. Decreases
b. remains same
c. increases
d. I would have been same if voltmeters were ideal.

6. Two ideal batteries and two ammeters are arranged as shown in the figure.

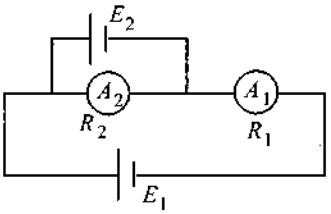


Fig. 6.67

a. Readings of both ammeters can be same if $E_1 > E_2$
b. Readings of both ammeters can be same if $E_2 > E_1$ provided $R_2 > R_1$
c. Readings of both ammeters can be same if $E_2 > E_1$ provided $R_2 < R_1$
d. If $E_2 > E_1$, then current in ammeters will flow in opposite directions

7. In the above question, if polarity of E_2 is reversed. Then
a. current in both ammeters will flow in same direction
b. current in both ammeters will flow in opposite directions
c. current in both ammeters can be same if $R_1 > R_2$
d. current in both can be same if $R_1 < R_2$

Assertion-Reasoning Type

Solutions on page 6.26

In the following questions, each question contains STATEMENT 1 (Assertion) and STATEMENT 2 (Reason). Each question has 4 choices **a.**, **b.**, **c.** and **d.** out of which **ONLY ONE** is correct.

a. Statement 1 is True, Statement 2 is True; Statement 2 is a correct explanation for Statement 1.
b. Statement 1 is True, Statement 2 is True; Statement 2 is **NOT** a correct explanation for Statement 1.
c. Statement 1 is True, Statement 2 is False.
d. Statement 1 is False, Statement 2 is True.

1. **Statement I:** A potentiometer is preferred over a voltmeter for the measurement of e.m.f. of a cell.
Statement II: A potentiometer is preferred, as it does not draw any current from the cell.
2. **Statement I:** The wire of a potentiometer should be of uniform area of cross section.
Statement II: It satisfies the requirement of the principle of a potentiometer.
3. **Statement I:** The higher the range, the greater is the resistance of an ammeter.
Statement II: To increase the range of an ammeter, additional shunt is needed to be used across it.
4. **Statement I:** The resistance of an ideal voltmeter should be infinite.
Statement II: Lower resistance of voltmeters gives a reading lower than the actual potential difference across the terminals.
5. **Statement I:** Voltmeter always gives e.m.f. of a cell if it is connected across the terminals of a cell.
Statement II: Terminal potential of a cell is given by $V = E - ir$
6. **Statement I:** The e.m.f. of the driver cell in the potentiometer experiment should be greater than the e.m.f. of the cell to be determined.
Statement II: The fall of potential across the potentiometer wire should not be less than the e.m.f. of the cell to be determined.
7. **Statement I:** In a metre bridge experiment, a high resistance is always connected in series with a galvanometer.
Statement II: As resistance increases, current through the circuit increases.

Comprehension Type

Solutions on page 6.26

For Problems 1–2

A battery is connected to a potentiometer and a balance point is obtained at 84 cm along the wire. When its terminals are connected by a 5 Ω resistor, the balance point changes to 70 cm.

1. Calculate the internal resistance of the cell.
a. 4 Ω **b.** 2 Ω **c.** 5 Ω **d.** 1 Ω
2. Find the new position of the balance point when 5 Ω resistor is changed by 4 Ω resistor.
a. 26.5 cm **b.** 52 cm **c.** 67.2 cm **d.** 83.3 cm

For Problems 3–4

A cell of e.m.f. 3.4 V and internal resistance 3 Ω is connected to an ammeter having resistance 2 Ω and an external resistance of 100 Ω. When a voltmeter is connected across the 100 Ω resistance, the ammeter reading is 0.04 A.

3. Find the resistance of the voltmeter.
a. 400 Ω **b.** 200 Ω
c. 300 Ω **d.** 500 Ω
4. Had the voltmeter been an ideal one, what would have been its reading ?
a. 7.2 V **b.** 1.8 V
c. 0.5 V **d.** 3.24 V

For Problems 5–7

In the connection shown in Fig. 6.68, initially switch K is open and the capacitor is uncharged. Then the switch is closed and the capacitor is charged up to the steady state and the switch is opened again. Determine the values indicated by the ammeter. [Given: $V_0 = 30$ V, $R_1 = 10$ kΩ, $R_2 = 5$ kΩ]

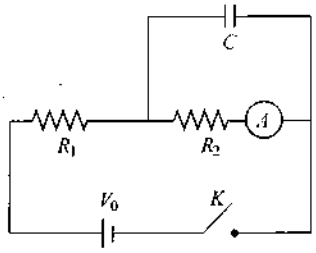


Fig. 6.68

5. Just after closing the switch
a. 2 mA **b.** 3 mA
c. 0 mA **d.** none of these
6. A long time after the switch was closed
a. 2 mA **b.** 3 mA
c. 6 mA **d.** none of these
7. Just after reopening the switch
a. 2 mA **b.** 3 mA
c. 6 mA **d.** none of these

For Problems 8–10

Potentiometer is an ideal voltmeter as voltmeter draws some current through the circuit while potentiometer needs no current to work. Potentiometer works on the principle of e.m.f. comparison. In working condition, a constant current flows throughout the wire of potentiometer using standard cell of e. m. f. e_1. The wire of potentiometer is made of uniform material and cross-sectional area and it has uniform resistance per unit length. The potential gradient depends upon the current in the wire.

A potentiometer with a cell of e. m. f. 2 V and internal resistance 0.4 Ω is used across the wire *AB*. A standard cadmium cell of e.m.f. 1.02 V gives a balance point at 66 cm length of wire. The standard cell is then replaced by a cell of unknown e.m.f. *e* (internal resistance *r*) and the balance point found similarly turns out to be 88 cm length of the wire. The length of potentiometer wire *AB* is 1 m.

8. The value of *e* is
a. 1.36 V **b.** 2.63 V **c.** 1.83 V **d.** none

9. The reading of potentiometer, if 4 V battery is used instead of *e*, is
a. 88.3 cm **b.** 47.3 column
c. 95 cm **d.** cannot be calculated

10. If the resistance is connected across the cell *e*, the balancing length will
a. increase **b.** decrease
c. remain same **d.** none

For Problems 11–13

The length of a potentiometer wire *AB* is 600 cm and it carries a constant current of 40 mA in direction from *A* to *B*. For a cell of e.m.f. 2 V and internal resistance 10 Ω the null point is found at 500 cm from *A*. When a voltmeter is connected across the cell, the balancing length of the wire is decreased by 10 cm.

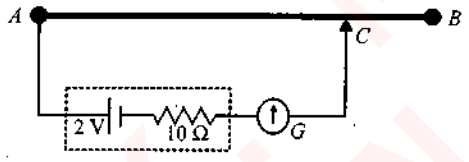


Fig. 6.69

11. Potential gradient along *AB* is
a. 1/5 V/m **b.** 2/5 V/m **c.** 3/5 V/m **d.** 4/5 V/m

12. Reading of the voltmeter is
a. 2 V **b.** 2.04 V **c.** 1.96 V **d.** 1.0 V

13. Resistance of the voltmeter is
a. 400 Ω **b.** 500 Ω **c.** 510 Ω **d.** 490 Ω

For Problems 14–15

AB is a potentiometer wire of length 100 cm. When a cell E_2 is connected across *AC*, where *AC* = 75 cm, no current flows from E_2. The internal resistance of the cell E_1 is negligible.

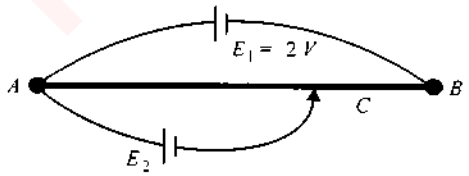


Fig. 6.70

14. Find the potential gradient along *AB*.
a. 0.01 V/cm **b.** 0.03 V/cm
c. 0.04 V/m **d.** 0.02 V/cm

15. Find e.m.f. of the cell E_2.
a. 2 V **b.** 1.5 V **c.** 1 V **d.** 1.75 V

Matching Column Type

Solutions on page 6.27

1. In the circuit shown in Fig. 6.71, battery, ammeter and voltmeter are ideal and the switch *S* is initially closed as shown. When switch *S* is opened, match the parameters of column I with the effects in column II

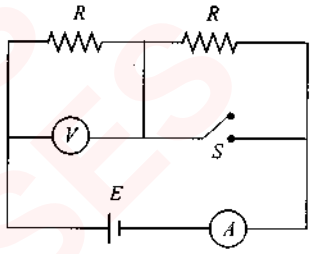


Fig. 6.71

Column I	Column II
i. Equivalent resistance across the battery	a. remains same
ii. Power dissipated by left resistance *R*	b. increases
iii. Voltmeter reading	c. decreases
iv. Ammeter reading	d. becomes zero

2. In a potentiometer experiment:

Column I	Column II
i. Deflection of galvanometer is in the same direction at the two ends of the wire	a. Accuracy in measurement increases
ii. A protective resistance added in series to the galvanometer	b. Accuracy in measurement decreases
iii. A short wire is used as a potentiometer	c. e.m.f. of the battery in the primary circuit is less than the e.m.f. of the cell to be measured
iv. More length of potentiometer up to null point	d. Uncertainty in the location of balance

3. Figure shows the internal wiring of a three-range voltmeter whose binding posts are marked +, 2V, 10 V and 100 V. When the meter is connected to the circuit being measured, one connection is made to the post marked + and the other to the post marked with the desired voltage range. The resistance of the moving coil R_G is 40 Ω and a current of 1 mA in the coil causes it to deflect full-scale. Then match the following:

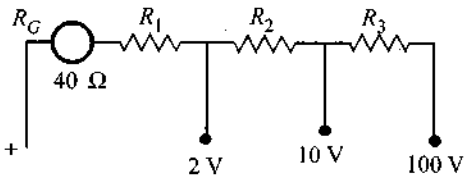


Fig. 6.72

Column I	Column II
i. Value of resistance R_1 in kΩ	a. 100
ii. Value of resistance R_2 in kΩ	b. 2
iii. Overall resistance of the meter in 100 V range in kΩ	c. 1.96
iv. Overall resistance of the meter in 2 V range in kΩ	d. 90

Integer Answer Type

Solutions on page 6.28

1. A potentiometer wire of length 10 m and resistance 30 Ω is connected in series with a battery of e.m.f. 2.5 V, internal resistance 5 Ω and an external resistance R. If the fall of potential along the potentiometer wire is 50 μ V/mm, then the value of R is found to be $23n$ Ω. What is n?
2. The ammeter shown in the figure. consists of 480 Ω coil connected in parallel to 20 Ω shunt. The reading of the ammeter comes out to be $\frac{1}{*}$ A. What is '*'?

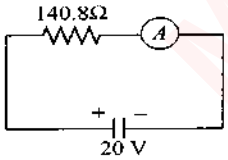


Fig. 6.73

3. A 5 m potentiometer wire having 3 Ω resistance per metre is connected to a storage cell of steady e.m.f. 2 V and internal resistance 1 Ω. A primary cell is balanced against 3.5 m of it. When a resistance of $32/n$ Ω is put in series with the storage cell, the null point shifts to the centre of the last wire, i.e., 4.5 m. What is 'n'?
4. In the circuit shown in the figure, the internal resistance of the cell is negligible. For value of $R = 40/x$ Ω, no current flows through the galvanometer. What is x?

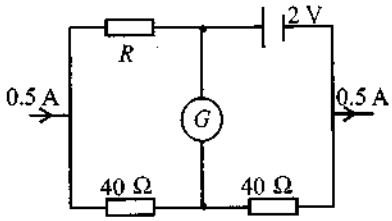


Fig. 6.74

5. Find the reading of the ammeters A_1 (in A) connected as shown in the network.

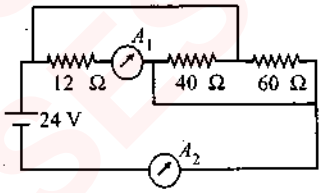


Fig. 6.75

6. A galvanometer together with an unknown resistance in series is connected across two identical batteries of each 1.5 V. When the batteries are connected in series, the galvanometer records a current of 1 A and when the batteries are connected in parallel, the current is 0.6 A. In this case, internal resistance of the battery is $\frac{1}{*}$ Ω. What is the value of '*'?
7. In the figure, ammeter (I) reads a current of 10 mA, while the voltmeter reads a potential difference of 3 V. The ammeters are identical, the internal resistance of the battery is negligible. (consider all ammeters and voltmeters as non-ideal)

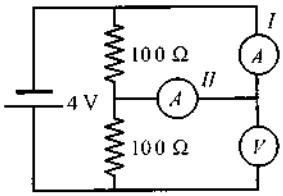


Fig. 6.76

The resistance of ammeter is $m \times 10^2$ Ω. What is the value of m?

8. In the above question, the reading of ammeter is $200/x$ mA. What is the value of x?

ANSWERS AND SOLUTIONS

Subjective Type

1. a. Current in AB, $I = \frac{E}{r+15r} = \frac{E}{16r}$

Potential gradient, $k = \frac{15rI}{600} \Rightarrow k = \frac{15r}{600} \times \frac{E}{16r} = \frac{E}{640}$

Let $E/2$ be balanced at ℓ, then

$$\frac{E}{2} = k\ell \Rightarrow \frac{E}{2} = \frac{E}{640}\ell \Rightarrow \ell = 320 \text{ cm}$$

b. Let potential at C be zero and at A be x.

Then applying Kirchhoff's first law at C, we get

$$\frac{x-0}{14r} + \frac{(x-E)/(2-0)}{r} + \frac{(x-E-0)}{2r} = 0 \Rightarrow x = \frac{14E}{22}$$

$$I_g = \frac{x - \frac{E}{2}}{r} = \frac{\left(\frac{14E}{22}\right) - \frac{E}{2}}{r} = \frac{3E}{22r}$$

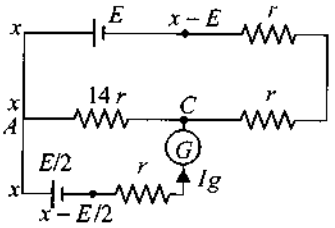


Fig. 6.77

2. As the wire is of uniform cross section, hence the resistances of the two segments AD and DC of the wire are in the ratio of the lengths of AD and DC.

According to the condition of balance of Wheatstone bridge, $\frac{X}{Y} = \frac{\ell_1}{\ell_2}$

Here $\ell_1 = 33.7$ cm and $\ell_2 = 100 - 33.7 = 66.3$ cm

$$\frac{X}{Y} = \frac{33.7}{66.3} \qquad \text{(i)}$$

As resistance Y' is due to a parallel combination of resistance Y and a resistance of 12 Ω, hence

$$\frac{1}{Y'} = \frac{1}{Y} + \frac{1}{12} = \frac{12+Y}{12Y} \text{ or } Y' = \frac{12Y}{12+Y}$$

Since Y' is less than Y, hence the ratio X/Y' will be greater than X/Y and the null point should shift towards end C.

$$\frac{X}{Y'} = \frac{33.7+18.2}{66.3-18.2} = \frac{51.9}{48.1}$$

$$\frac{X(12+Y)}{12Y} = \frac{51.9}{48.1}$$

$$12 + Y = \frac{51.9}{48.1} \times 12 \times \frac{Y}{X} = \frac{51.9}{48.1} \times 12 \times \frac{66.3}{33.7} = 25.47\ \Omega$$

$Y = 25.47 - 12 = 13.47\ \Omega$

Putting this value in Eq. (i), we get

$$X = \frac{33.7}{66.3} \times Y = \frac{33.7}{66.3} \times 13.47 = 6.85\Omega$$

3. a. If current from driver cell decreases, then potential gradient decreases. So the balancing length should increase. Hence, balance point shifts towards right.

b. If R is increased, current in the lower circuit decreases due to which terminal potential difference of battery increases. To balance increased terminal potential difference, balancing length should increase, hence balance point shifts towards right.

4. a. $V = IR + IR_A \Rightarrow R = \frac{V}{I} - R_A$.

The true resistance R is always less than the reading (V/I) because in the circuit the ammeter's resistance causes the current to be less than the actual. Thus the smaller current requires the resistance R to be calculated larger than what it should be.

b. $I = \frac{V}{R} + \frac{V}{R_V} \Rightarrow R = \frac{VR_V}{IR_V - V} = \frac{V}{I - V/R_V}$

Now the current measured is greater than that through the resistor, so $R = V/I_R$ is always greater than V/I.

c. (i) $P = I^2R = I^2(V/I - R_A) = IV - I^2R_A$.

(ii) $P = V^2/R = V(I - V/R_V) = IV - V^2/R_V$.

5. let us measure a resistance R using circuit (a)

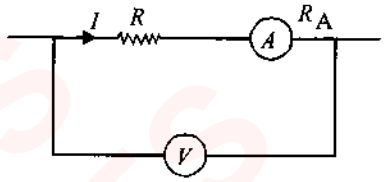


Fig. 6.78

True value = R, measured value = $\frac{V}{I} = R + R_A$

$$\%\text{ Error} = \frac{R + R_A - R}{R} \times 100 = \frac{R_A \times 100}{R} = \frac{0.5 \times 100}{R} = \frac{50}{R}$$

So the more the value of R, the lesser the percentage error. Hence, circuit (a) should be used to measure Y.

Now, let us measure R using in circuit (b).

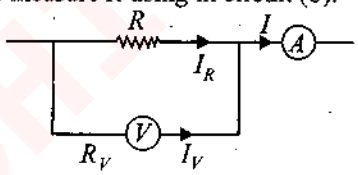


Fig. 6.79

True value = R

$$\text{Measured value} = \frac{V}{I} = \frac{V}{I_R + I_V} = \frac{V}{\frac{V}{R} + \frac{V}{R_V}} = \frac{RR_V}{R + R_V}$$

$$\%\text{ Error} = \frac{R - \frac{RR_V}{R + R_V}}{R} \times 100 = \frac{1}{1 + R_V/R} \times 100$$

The lesser the value of R, the lesser the percentage error. Hence, circuit (b) should be used to measure X.

6. a. By formula, $\frac{\varepsilon}{\varepsilon_{\text{standard}}} = \frac{\ell}{\ell_{\text{standard}}}$

$$\varepsilon = \frac{\ell \times \varepsilon_{\text{standard}}}{\ell_{\text{standard}}} = \frac{82.3 \times 1.02}{67.3} = 1.2474\text{ V}$$

b. To reduce the current through the galvanometer when the movable contact is far from the balance point.

c. No.

d. Yes.

e. No. If ε is greater than the e.m.f. of the driver cell of the potentiometer, there will be no balance point on the wire AB.

f. The circuit, as it is, would be unsuitable, because the balance point (for e of the order of a few mV) will be very close to the end A and the percentage error in measurement will be very large. The circuit is modified by putting a suitable resistor R in series with the wire AB so that potential drop across AB is only slightly greater than the e.m.f. to be measured. Then the balance point will be at larger length of the wire and the percentage error will be much smaller.

7. The distribution of currents is shown in Fig. 6.80. Applying Kirchhoff's law to closed mesh (1), (2) and (3), we have

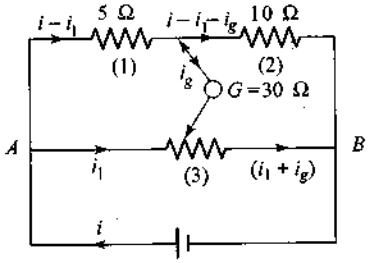


Fig. 6.80

$5(i-i_1)+30i_g-2i_1=0$ or $5i-7i_1+30i_g=0$ (i)

$10(i-i_1-i_g)-2(i_1+i_g)-30i_g=0$

$10i-12i_1-42i_g=0$ (ii)

$2i_1+2(i_1+i_g)+i=5$ or $4i_1+2i_g+i=5$ (iii)

From Eqs. (i) and (ii), $i_1=51i_g$ (iv)

From Eqs. (ii) and (iii), $13i=36i_g+15$ (v)

Substituting Eqs (iv) and (v) in Eq. (iii), we get

$$i_g=\frac{25}{1357}\text{A}$$

8. For the balanced bridge, the ratio of the two resistances is equal to the ratio of the lengths of the two parts AJ and JB of the wire, i.e.,

$$\frac{X}{6\,\Omega}=\frac{40\text{ cm}}{60\text{ cm}} \quad \text{or} \quad X=4\,\Omega$$

No current flows through the galvanometer G, the resistance of the parts AJ and JB is 40 Ω and 60 Ω, respectively. If R be the equivalent resistance between the points A and B, then we have

$$\frac{1}{R}=\frac{1}{(X+6)\,\Omega}+\frac{1}{(40+60)\,\Omega}$$

or $$R=\frac{100}{11}\Omega,\ i=\frac{V}{R}=\frac{6\text{ V}}{(100/11)\Omega}=0.66\text{ A}$$

9. In the given case, the ratio arms are 1000:10

$$\frac{P}{Q}=\frac{1000}{10}=100$$

Third resistance, R = 999 V

Let X be the unknown resistance. Then,

$$\frac{P}{Q}=\frac{R}{X} \quad \text{or} \quad X=\frac{Q}{P}\times R=\frac{1}{100}\times 999=9.99\,\Omega$$

10. Resistance per volt at full-scale deflection = 5000 ΩV^{-1}

Reading of voltmeter at full-scale deflection = 5 V

Therefore, resistance of voltmeter $G=5000\times 5=25000\ \Omega$

Also current for maximum deflection,

$$I_g=\frac{1\text{ V}}{5000\,\Omega}=0.0002\text{ A}$$

Range of voltmeter to be changed to V = 20 V

Now, $$R=\frac{V}{I_g}-G=\frac{20}{0.0002}-25000$$

$$=100000-25000=75000\ \Omega$$

Thus, 7500 Ω resistor is to be connected in series.

Resistance of 20 V voltmeter = 75000 + 25000 = 1,00,000 Ω

Its grading becomes $=\frac{100000}{20}=5000\ \Omega V^{-1}$ which is same as in the earlier case. A voltmeter with grading 2000 ΩV^{-1} will have less resistance and is, therefore, not preferred.

11. (i) The circuit diagram is shown.

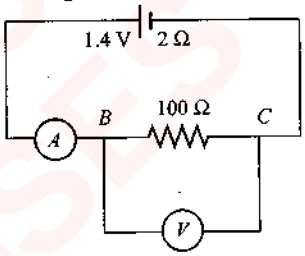


Fig. 6.81

(ii) Let resistance of the voltmeter be R ohm. The equivalent resistance of voltmeter (R ohm) and 100 Ω in parallel is

$$\frac{100\times R}{100+R}=\frac{100R}{100+R}$$

Resistance of the ammeter $=\frac{4}{3}\Omega$.

Total resistance of the circuit $=\frac{100R}{100+R}+\frac{4}{3}+2\Omega$

Current in the circuit as read by the ammeter = 0.02 A.

Now, $$0.02=\frac{1.4}{\frac{100R}{100+R}+\frac{4}{3}+2} \quad \left(\because I=\frac{V}{R}\right)$$

or $R=200\ \Omega$

Therefore, resistance of the voltmeter = 200 Ω.

(iii) Effective resistance between B and $C=\frac{100\times 200}{100+200}$

$$=\frac{200}{3}\Omega$$

The potential drop across this resistance is

$$\text{circuit current}\times\frac{200}{3}=0.02\times\frac{200}{3}=\frac{4}{3}\text{ V}=1.333\text{ V}$$

$$=\frac{4}{3}\text{ V}=1.333\text{ V}$$

Reading of the voltmeter = 1.1 V

Error in the reading of the voltmeter is

$$1.1-1.333=-0.233\text{ V}$$

12. Let R be the resistance to be introduced in the box. Then current in the potentiometer wire is given by

$$1=\frac{E}{R+\ell\rho}$$

where ρ is the resistance per unit length of the wire and ℓ is the length of the wire. Now, potential gradient is

$$k = I\rho = \frac{E\rho}{R + \ell\rho}$$

Here, $\ell = 10$ m, $\rho = 4\ \Omega/\text{m}$

(i) For $k = 0.1$ V/m, we have

$$0.1 = \frac{2 \times 4}{R + 10 \times 4} = \frac{8}{R + 40} \quad \text{or} \quad R = \frac{4}{0.1} = 40\ \Omega$$

(ii) For $k = 0.1$ mV/m $= 0.1 \times 10^{-3}$ V/m, we have

$$0.1 \times 10^{-3} = \frac{2 \times 4}{R + 10 \times 4}$$

$$\text{or} \quad 10^{-4} = \frac{8}{R + 40} \quad \text{or} \quad R = 79960\ \Omega$$

Objective Type

1. c. Resistance is halved. Current is doubled.

2. b. $E_1 \propto 64$

$$E_1 - E_2 \propto 8$$

$$E_2 \propto l$$

$$\therefore \quad 64 - l = 8 \quad \text{or} \quad l = 64 - 8 = 56 \text{ cm}$$

3. d. Voltage across $5\ \Omega = 10$ V

$$\therefore \quad I = \frac{10}{5}\text{ A} = 2\text{ A}$$

4. b. The circuit may be redrawn as

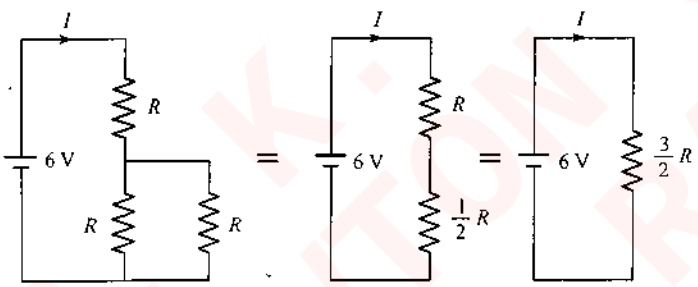


Fig. 6.82

Current is given by $I = \dfrac{6}{\frac{3}{2}R} = \dfrac{4}{R}$ A

Therefore, current through the voltmeter is $I/2$ or $2/R$ A.

Hence, the reading of the voltmeter is $(2/R)(R)$ or 2 V.

5. a. At null point, $R_1/R_2 = R_3/R_4 = x/(100 - x)$. If radius of the wire is doubled, then the resistance of AC will change and the resistance of CB will also change. But since R_1/R_2 does not change, so R_3/R_4 should also not change at null point. Therefore, point C does not change.

6. a. Using the principle of potentiometer, $V \propto l$

$$\therefore \quad \frac{V}{E} = \frac{l}{L} \quad \text{or} \quad V = \frac{l}{L}E = \frac{30}{100}E = \frac{30E}{100}$$

7. a. $\dfrac{X}{Y} = \dfrac{20}{80} = \dfrac{1}{4}$ or $Y = 4X$

$$\frac{4X}{Y} = \frac{l}{100 - l} \quad \text{or} \quad \frac{4X}{4X} = \frac{l}{100 - l} \quad \text{or} \quad l = 50 \text{ cm}$$

8. c. Current through $R = \dfrac{12}{500 + R}$

Voltage across $R = \dfrac{12R}{500 + R}$

Since galvanometer shows zero deflection, so

$$\frac{12R}{500 + R} = 2 \Rightarrow R = 100\ \Omega$$

9. a. $r = \dfrac{l_1 - l_2}{l_2}R = \dfrac{240 - 120}{120} \times 2\Omega = 2\Omega.$

10. b. When Wheatstone bridge is balanced, then

$$\frac{P}{Q} = \frac{R}{S} \quad \text{or} \quad \frac{P}{R} = \frac{Q}{S}$$

If the galvanometer is replaced with a cell in balanced Wheatstone bridge, the condition for balanced bridge will be $P/R = Q/S$, which is there.

Hence, balance point will remain unchanged; where galvanometer shows no current.

11. b. Sensitivity of potentiometer means the smallest potential difference it can measure. It can be increased by reducing the potential gradient. The same is possible by increasing the length of the potentiometer.

12. c. $i_g = \dfrac{V}{G + R} \Rightarrow 10^{-3} = \dfrac{2.5}{10 + R} \Rightarrow R = 2490\ \Omega$

13. c. $S = \dfrac{i_g G}{(I - i_g)} = \dfrac{0.01 \times 1}{1 - 0.01} = \dfrac{1}{99}\ \Omega$

14. d. $E_1 \propto 300, \quad E_1 - E_2 \propto 100$

$$\frac{E_1}{E_1 - E_2} = 3 \quad \text{or} \quad E_1 = 3E_1 - 3E_2$$

$$\text{or} \quad 3E_2 = 2E_1 \quad \text{or} \quad \frac{E_1}{E_2} = \frac{3}{2}$$

15. d. R is the order of 15,000 Ω. The junctions of the highest two and the lowest two resistances are A and C, and for better sensitivity, the galvanometer will be between these. So, error will decrease with the suggested interchange.

Alternatively: Let R be not exactly 15,000 Ω. Then some potential difference will be created between points B and D. Let it be ΔV_1. Now suppose the connections are interchanged, then the same type of potential difference will be created between points A and C. Let it be ΔV_2. On calculations, we can find that $\Delta V_2 > \Delta V_1$. So larger current will flow through galvanometer in the second case telling us that value of R needs to be changed. And we will try to put more accurate value of R. In this way, error will decrease.

16. b. Total resistance is

$$\left(\frac{5 \times 15}{5 + 15} + 1.25\right) = \left(\frac{75}{20} + 1.25\right)\Omega = (3.75 + 1.25)\Omega = 5\ \Omega$$

$$I = \frac{20}{5}\text{ A} = 4\text{ A}$$

Current through 5 Ω = $\frac{15}{20} \times 4\text{ A} = 3\text{ A}$

Voltmeter reading = Potential drop across 1.25 Ω

$= 4 \times 1.25\text{ V} = 5\text{ V}$

17. a. Effective resistance across the voltmeter = 200/2 = kΩ

Total resistance across d.c. supply = 400 + 100 = 500 kΩ.

Thus, the voltage across the voltmeter = 100/500(60 V) = 12 V.

18. b. When X is connected to Y, the balance length l is proportional to the p.d. across the 100 Ω resistor. When X is connected to Z, the balance length is proportional to the p.d. across the 100 Ω resistor and resistor R.

$$\frac{100+R}{100} = \frac{588}{400} \Rightarrow R = 47\,\Omega$$

19. b. A careful analysis would show that the voltage along R is 1.03 V.

$$1.03 = 1 \times R \quad \text{or} \quad R = 1.03\ \Omega$$

20. c. If V is the potential difference applied across P and Q, the current through M is determined by

Circuit	Current
(a)	$V/4$
(b)	$3V/8$
(c)	$V/2$
(d)	$V/3$

Hence, circuit arrangement (c) gives the largest reading in ammeter M.

21. b. The voltage per unit length on the metre wire PQ is

$$\frac{6.00\text{ mV}}{0.60\text{ m}} \text{ or } 10\text{ mV/m}$$

Hence, potential across the metre wire PQ is 10 mV/m (1 m) = 10 mV. Current drawn from the driver cell is

$$I = \frac{10\text{ mV}}{5\,\Omega} = 2\text{ mA}$$

Resistance of the resistor R is

$$R = \frac{2\text{ V} - 10\text{ mV}}{2\text{ mA}} = \frac{1990\text{ mV}}{2\text{ mV}} = 995\ \Omega$$

22. b. $P/Q = R/S$. If P is increased then either R or Q should be increased or S should be decreased.

23. b. In case of internal resistance measurement by potentiometer,

$$\frac{V_1}{V_2} = \frac{\ell_1}{\ell_2} = \frac{[ER_1/(R_1+r)]}{[ER_2/(R_2+r)]} = \frac{R_1(R_2+r)}{R_2(R_1+r)}$$

Here $\ell_1 = 2$ m, $\ell_2 = 3$ m, $R_1 = 5\ \Omega$ and $R_2 = 10\ \Omega$

$$\therefore \quad \frac{2}{3} = \frac{5(10+r)}{10(5+r)} \quad \text{or} \quad r = 10\ \Omega$$

24. a. $3 = IR_1$ or $3 = 1 \times R_1$ or $R_1 = 3\ \Omega$

When points A and B are connected by a conducting wire R_2 is short-circuited.

$$\therefore \quad 10.5 = I'R_1 \quad \text{or} \quad 10.5 = I' \times 3$$

$$\therefore \quad I' = \frac{10.5}{3} = 3.5\text{ A}$$

But $\quad 10.5 = E - I'r \quad \text{or} \quad 10.5 = 12 - 3.5r$

$$\therefore \quad r = \frac{1.5}{3.5} = \frac{3}{7}\ \Omega$$

25. b. The current through the galvanometer producing full scale deflection is

$$I = \frac{V}{R} = \frac{20 \times 10^{-3}}{80} = 2.5 \times 10^{-4}\text{ A}$$

To convert the galvanometer into a voltmeter, a high resistance is connected in series with the galvanometer

$$\therefore \quad 5\text{ V} = (2.5 \times 10^{-4})(R + 80) \quad \text{or} \quad R = 19.92\ \text{k}\Omega$$

26. a. $V_1 = E - ir = 50 - \frac{50}{220} \times 20$

$= 50 - 4.6 = 45.4$ V

Now, $\quad V_2 = 50 - \frac{50}{180} \times 20 = 44.4$ V

Percentage change $= \frac{V_1 - V_2}{V_1} \times 100 = 2.27$ (also see the question)

27. a. The electric current through ideal voltmeter is zero. According to the loop rule,

$$E - 1 \times I - 1 \times I = 0 \Rightarrow I = \frac{E}{2} = \frac{2}{2} = 1\text{ A}$$

Reading of the voltmeter $= V_A - V_B$

$= [1 \times I] = [1 \times 1] = 1$ V

28. c. $k = 0.05$ mV/cm = 5 mV/m

$$I = \frac{kl}{r} = \frac{5 \times 10^{-3} \times 1}{5} = 10^{-3}\text{ A}$$

Now, $2 = I(R + r) \Rightarrow 2 = 10^{-3}(R + 5) \Rightarrow R = 1995\ \Omega$

29. a. The value of E = Potential gradient × Length

(5 mV/m)(60 cm) = 3 mV

30. a. $V_A = iR$, $V_B = \frac{2i}{3} \times 1.5R = iR$, $V_C = (i/3)(3R) = iR$

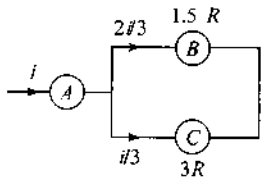


Fig. 6.83

31. c. $\quad i_g = 10$ mA = 0.01 A

$$V_A - V_B = (I - i_g)\,0.1 = i_g(9 + 0.9) \quad \text{or} \quad I = \frac{10 \times 0.01}{0.1} = 1\text{ A}$$

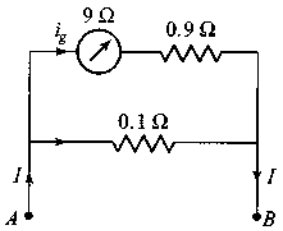


Fig. 6.84

32. b. Let a current of x ampere pass through the voltmeter; then $(4-x)$ ampere passes through the resistance R. Therefore, voltmeter reading is

$$20 = (4-x)R, \text{ i.e., } R = \frac{20}{4-x}, \text{ i.e., } R > 5\ \Omega$$

33. d. $\frac{I_g}{I} = \frac{S}{S+G} \Rightarrow \frac{I_g}{I} = \frac{(G/10)}{(G/10)+G} = \frac{1}{11}$

Initially, $\alpha_i = \theta/I_g$ (i)

Finally, after the shunt is used,

$\alpha_f = \theta/I$ (ii)

$$\therefore \quad \frac{\alpha_f}{\alpha_i} = \frac{\theta/I}{\theta/I_g} = \frac{I_g}{I} = \frac{1}{11}$$

So, current sensitivity becomes 1/11-fold.

34. b. Let I be the current in the circuit, then $I \times 4000 = 30$ V

If voltmeter is put across 3000 Ω resistance, then reading of voltmeter $= I \times 3000 = \frac{30}{4000} \times 3000 = 22.5$ V

35. b. We know that $R = \frac{V}{I_g} - G$

The voltmeter gives the full-scale deflection for potential difference V. Its resistance is G. Hence, $I_g = (V/G)$.

Given that $V = nV$

$$\therefore \quad R = \frac{nV}{(V/G)} - G = (n-1)G$$

36. b. $\frac{I_g}{I} = \frac{S}{S+G}$ or $\frac{1}{34} = \frac{S}{S+G}$

$$\therefore \quad S = (G/33) = (3663/33) = 111\ \Omega$$

37. c. $R_s = \frac{SG}{S+G} = \frac{111 \times 3663}{111+3663} = 107.7\ \Omega$

38. d. Compensation external resistance

$$G - \frac{SG}{S+G} = 3663 - 107.7 = 3555.3\ \Omega$$

39. a. $I = \frac{k}{NBA}\theta$. Given that $I_1 = I_2$

$$\therefore \quad \frac{K\theta_1}{N_1BA_1} = \frac{K\theta_2}{N_2BA_2}. \text{ So } \frac{\theta_1}{\theta_2} = \frac{A_1N_1}{A_2N_2}$$

40. a. Total range is doubled, i.e., $4I_g$

Now shunt required, $S = \frac{I_g}{4I_g - I_g} \times G = 10\ \Omega$

$$\frac{1}{30} + \frac{1}{x} = \frac{1}{10} \Rightarrow x = 15\ \Omega$$

41. c. Initially, when switch is open, V_2 and V_3 will be out of circuit or they are short-circuited. So no p.d. across V_2 and V_3. On closing the switch, all three voltmeters will be in parallel. Now, there is some p.d. across V_2 and V_3. So p.d. across V_2 and V_3 increases and p.d. across V_1 remains the same.

42. b. $P = \frac{V^2}{R} \Rightarrow P \propto \frac{1}{R}$

43. b. Resistors 20 Ω, 100 Ω and 25 Ω will be in parallel. Their equivalent is 10 Ω.

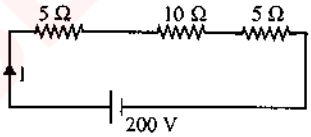


Fig. 6.85

$$I = \frac{200}{5+10+5} = 10\text{A}$$

p.d. across 10 Ω, $10I = 10 \times 10 = 100$ V

This will be the voltmeter reading. Also, this will be the p.d. across each of 20 Ω, 100 Ω and 25 Ω resistors.

Ammeter reading = current through 25 Ω $= \frac{100}{25} = 4$ A.

44. a. Current in $AB = I = \frac{e}{R+r}$

p.d. across $AB = IR = \frac{eR}{R+r}$

To obtain balanced point: $\frac{eR}{R+r} > \frac{e}{3} \Rightarrow R > r/2$

Multiple Correct Answers Type

1. b., c. The potential measures the exact value of e.m.f. of a battery. Therefore, $E = 1.55$ V.

Also, $1.4 = I(280)$ $\therefore$ $I = 0.005$ A

Also, $V = E - Ir$ $\therefore$ $r = \frac{(E-V)}{I} = \frac{1.55 - 1.40}{0.005} = 30\ \Omega$

2. a., b., d.

Since $R_1/R_2 = C_2/C_1$, the Wheatstone bridge is balanced. Hence, $V_C = V_D$. No current passes through the galvanometer.

Hence, option **(a)** is correct.

Potential difference across R_1 = potential difference across $C_1 = 4$ V

Potential difference across R_2 = potential difference across $C_2 = 5$ V

Potential difference across 5 Ω is the potential difference across 8 μF capacitor. So, charge across 8 μF capacitor $Q = CV = 8\ \mu\text{F} \times 5\text{ V} = 40\ \mu\text{C}$

Hence, option **(b)** is correct and so is option **(d)**.

3. **b., c.** If P is slightly increased, potential of C will decrease.

Hence, current will form A to C.

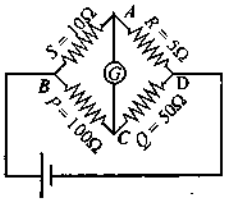


Fig. 6.86

If Q is slightly increased, potential of C will increase, hence current will flow from C to A.

4. **b.,c.** When switch S is open, R_1 and R_2 are short-circuited. E will be divided among V_1 and V_2 only. After closing the switch, V_1, R_1 and R_2 will be in parallel. Their effective resistance will be less than the resistance of V_1. Hence, p.d. across V_2 decreases and so across V increases.

5. **a., d.** Let resistance of voltmeter be R_V. Then the equivalent resistance of circuit is

$$R_{eq} = R_2 + \frac{R_1 R_V}{R_1 + R_V} = \frac{R_1R_2 + R_2R_V + R_VR_1}{R_1 + R_V}$$

$$I = \frac{E}{R_{eq}} = \frac{E(R_1 + R_V)}{R_1R_2 + R_2R_V + R_VR_1}$$

On putting voltmeter across R_2, we can get current

$$I' = \frac{E(R_2 + R_V)}{R_1R_2 + R_2R_V + R_VR_1} \quad \text{as} \quad R_1 > R_2 \Rightarrow I > I'$$

If voltmeter is ideal, then in both cases R_1 and R_2 will be in series and current in both cases will be $I = E/(R_1 + R_2)$.

6. **a., b., d.** By applying Kirchhoff's law, we can obtain

$$I_2 = \frac{E_2}{R_2} \text{ and } I_1 = \frac{E_1 - E_2}{R_1}$$

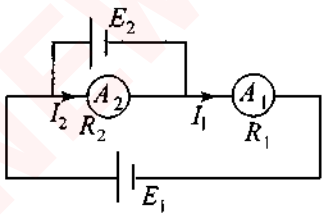


Fig. 6.87

If $E_1 > E_2$, I_1 and I_2 can be the same for some value of R_1 and R_2, hence (a) is correct.

For (b): if $E_2 > E_1$, then I_1 will be in the direction opposite to that shown in the figure and magnitude of I_1 will be $I_1 = (E_2 - E_1)/R_1$.

Now I_1 and I_2 can be the same only if $R_1 < R_2$.

7. **b., c.**

If polarity of E_2 is reversed, then

$$I_2 = \frac{E_2}{R_2} \text{ and } I_1 = \frac{E_1 + E_2}{R_1}$$

for $I_1 = I_2$, $R_1 > R_2$

Assertion-Reasoning Type

1. **a.** In a balanced condition, the potentiometer does not draw any current, and hence does not disturb the circuit.
2. **a.** Principle of a potentiometer states that drop of potential across any segment of the potentiometer wire is directly proportional to its length. This can be satisfied if the wire of the potentiometer has a uniform area of cross section.
3. **d.** The lower the resistance of an ammeter, the higher the range.
4. **a.** If resistance of a voltmeter is not infinite, it will draw some current from the circuit and finally the reading will be less than actual.
5. **d.** Voltmeter gives terminal potential (V) though it can give e.m.f. if internal resistance of the cell is zero.
6. **a.** If either the e.m.f. of the driver cell or the potential difference across the whole potentiometer wire is lesser than the e.m.f. of the experimental cell, the balance point will not be obtained.
7. **c.** The resistance of a galvanometer is fixed. In metre bridge experiments, to protect the galvanometer from a high current, high resistance is connected to the galvanometer in order to protect it from any damage.

Comprehension Type

For Problems 1–2

1. d., 2. c.

Sol.

1. d. $r = \frac{l_1 - l_2}{l_2} R = \frac{84 - 70}{70} \times 5 = 1\ \Omega$

2. c. $1 = \frac{l_1 - l_3}{l_3} \times 4$, put $l_1 = 84$ cm and get $l_3 = 67.2$ cm

For Problems 3–4

3. a., 4. d.

Sol.

Let R_1 and R_2 be the resistances of the ammeter and the voltmeter, respectively. Let the external resistance be denoted by R and the internal resistance of battery be r. The equivalent resistance of the parallel combination of R and R_2 is given by

$$R' = \frac{RR_2}{R + R_2}$$

The total resistance RT of circuit then becomes

$$RT = R_1 + r + \frac{RR_2}{R + R_2}$$

The current in the circuit is given by

$$I = \frac{E}{R_1 + r + \frac{RR_2}{R + R_2}}$$

This must be equal to 0.04 A, the reading indicated by the ammeter.

$$\frac{3.4}{2 + 3 + \frac{100R_2}{100 + R_2}} = 0.04 \Rightarrow R_2 = 400\ \Omega$$

In case of an ideal voltmeter, no current flows through it. In that case, current in the circuit is

$$I' = \frac{3.4}{2 + 3 + 100} = \frac{3.4}{105} = 0.0324\ \text{A}$$

Potential drop across the resistance R would be $100 \times 0.0324 = 3.24$ V. This should be the reading indicated by an ideal voltmeter.

For Problems 5–7

5. c., 6. a., 7. a.

Sol.

5. c. Just after closing, capacitor behaves as a short circuit and all current flows through it, hence ammeter reads zero.

6. a. After a long time, capacitor behaves like an open circuit and no current flows through it.

Therefore, $i = \frac{V_0}{R_1 + R_2} = \frac{30}{10 + 5} = 2$ mA

7. a. Just after reopening, potential difference across R_2 remains the same as charge on the capacitor does not change initially, hence current remains the same.

For Problems 8–10

8. a., 9. d., 10. b.

Sol.

8. a. $\frac{e}{1.02} = \frac{88}{66} \Rightarrow e = 1.36$ V

9. d. 4 V is greater than applied e.m.f. 2 V, hence no balance point is obtained.

10. b. On connecting the resistance across e, current will flow in e due to which terminal potential difference will be less than e.m.f. and the balancing length will decrease.

For Problems 11–13

11. b., 12. c., 13. d.

Sol.

11. b. Since 2 V is balanced across a length of 500 cm = 5 m. So the potential gradient is $k = 2/5$ V/m

12. c. Reading of voltmeter = p.d. across voltmeter

$$= V_A - V_D = k(4.9)$$

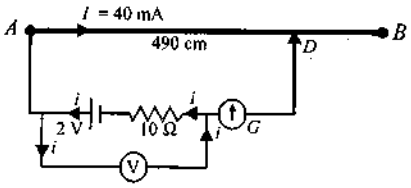


Fig. 6.88

$\Rightarrow$ Reading of the voltmeter $= \frac{2}{5} \times 4.9 = 1.96$ V

13. d. Terminal p.d. of the battery will also be 1.96 V.

So $1.96 = 2 - i \times 10 \Rightarrow i = 4 \times 10^{-3}$ A

So, resistance of the voltmeter is $R_v = 1.96/(4 \times 10^{-3}) = 490\ \Omega$.

For Problems 14–15

14. d., 15. b.

Sol.

14.d. $k = \frac{E_1}{100} = \frac{2}{100}$ 0.02 V/cm

15.b. $E_2 = k(75) = 0.02 \times 75 = 1.5$ V

Matching Column Type

1. i. → b., ii. → c., iii. → c., iv. → c.

When the switch is closed, the equivalent resistance is R. After opening the switch, the equivalent resistance becomes $2R$. Hence, the equivalent resistance increases.

Also, current through the battery decreases, hence ammeter reading decreases. Current through the left R also decreases. So voltmeter reading decreases and power dissipated through the left R also decreases.

2. i. → c., ii. → d., iii. → b,d., iv. → a.

i. If deflection in a galvanometer is in the same direction for the position of jockey on one side of the null point, then for the position of jockey on the other side of the null point, the deflection in the galvanometer should be in the opposite direction. But if e.m.f. of the battery in the primary circuit is less than the e.m.f. of the cell to be measured, then for all positions of jockey on wire, deflection in the galvanometer will be in one direction only.

ii. Due to protective resistance, the galvanometer will show less deflection when away from the null point. Hence, uncertainty in the location of the null point increases.

iii. For a short potentiometer wire, accuracy is less.

iv. For a long potentiometer wire, accuracy is more.

3. i. → c., ii. → d., iii. → a., iv. → b.

For 2 V range: $I_g (R_G + R_1) = 2$ V

$$\Rightarrow \quad R_1 = \frac{2}{10^{-3}} - 40 = 1960\ \Omega = 1.96\ \text{k}\Omega$$

For 10 V range: $I_G(R_G + R_1 + R_2) = 10$ V

$$\Rightarrow \quad R_2 = \frac{10}{1\times10^{-3}} - 40 - 1960 = 8000\ \Omega = 8\ \text{k}\Omega$$

For 100 V range: $I_G(R_G + R_1 + R_2 + R_3) = 100$ V

$$\Rightarrow R_3 = \frac{100}{10^{-3}} - 40 - 1960 - 8000 = 90000\ \Omega = 90\text{k}\Omega$$

The overall resistance of the meter on 100 V range is

$$R_G + R_1 + R_2 + R_3 = 100\ \text{k}\Omega$$

The overall resistance of the meter on 2 V range is

$$R_G + R_1 = 2\ \text{k}\Omega$$

Integer Answer Type

1. (5) Potential gradient, $k = \frac{IR_P}{L} = \frac{ER_P}{(R + R_P + r)L}$

(where R_P is resistance of the wire)

$$\Rightarrow 50\times10^{-3} = \frac{2.5\times30}{(R+30+5)\times10} \Rightarrow R = 115\ \Omega$$

2. (8) Equivalent resistance of the ammeter is

$$\frac{(480\ \Omega)(20\ \Omega)}{480\ \Omega + 20\ \Omega} = 19.2\ \Omega$$

The equivalent resistance of the circuit is 140.8 Ω + 19.2 Ω = 160 Ω. Therefore, current, i = 20 V/160 Ω = (1/8)A

3. (7) This means you will not get the balancing point.

$$i = \frac{2}{1+15} = \frac{1}{8},\ E_P = \frac{1}{8}\times3\times\frac{7}{2} = \frac{21}{16}$$

$$\frac{21}{16} = i\times3\times\frac{9}{2} \Rightarrow i = \frac{7}{72}\ \text{A, now}\ \frac{7}{72} = \frac{2}{16+R} \Rightarrow R = \frac{32}{7}\ \Omega$$

4. (9) $\frac{V-2}{40} = \frac{2-0}{40} \Rightarrow V = 4$

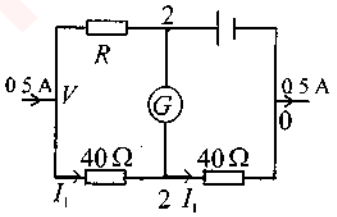


Fig. 6.89

$$I_1 = \frac{2}{40} = 0.05\ \text{A}$$

$$\therefore \quad \frac{V-2}{R} = 0.45$$

$$R = \frac{4-2}{0.45} = \frac{2}{0.45} = \frac{40}{9}\ \Omega$$

5. (2) Here all the resistances will be in parallel.
Current through 12 Ω: $I_1 = 24/12 = 2$ A
Hence, reading of $A_1 = I_1 = 2$ A

6. (3) Let R be the combined resistance of galvanometer and an unknown resistance and r the internal resistance of each battery. When the batteries, each of e.m.f. E are connected in series, the net e.m.f. = $2E$ and net internal resistance = $2r$.

Current $i_1 = \frac{2E}{R+2r} \Rightarrow 1.0 = \frac{2\times1.5}{R+2r}$

$$\Rightarrow \quad R + 2r = 3 \qquad \text{(i)}$$

When the batteries are connected in parallel, the e.m.f. remains E and net internal resistance become $r/2$. Therefore, current is $i_2 = \frac{E}{R+\frac{r}{2}} = \frac{2E}{2R+r}$

$$\Rightarrow \quad 2R + r = \frac{2E}{i_2} = \frac{2\times1.5}{0.6} = 5.0 \qquad \text{(ii)}$$

Solving Eqs. (i) and (ii), we get $r = \frac{1}{3}\ \Omega$

7. (1) R_A = resistance of ammeter

$1\ \text{V} - 0\ \text{V} = (10\ \text{mA})\ R_A \Rightarrow R_A = 100\ \Omega$

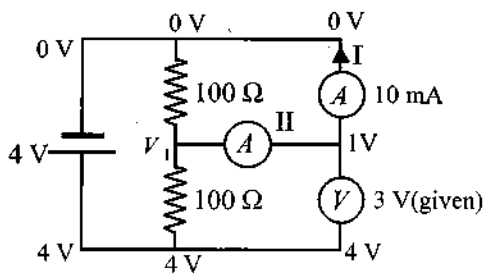


Fig. 6.90

8. (3) $\frac{4-V_1}{100} = \frac{V_1-1}{100} + \frac{V_1-0}{100} \Rightarrow V_1 = \frac{5}{3}$ V

Current in ammeter (II) = $\frac{V_1-1}{R_A} = \frac{(5/3)-1}{200} = \frac{2}{300}\ \text{A} = \frac{20}{3}$ mA

CHAPTER
7

Heating Effects of Current

- Heating Effects of Current
- Some Applications

R. K. MALIK'S NEWTON CLASSES RANCHI

HEATING EFFECTS OF CURRENT

When an electric current is passed through a conductor, it becomes hot and its temperature starts rising. This is known as *heating effect of current* or *Joule's heating effect*. Here electric energy converts into heat energy.

Various appliances, such as geyser, iron, heater, fuse wire, etc. work on this basis.

Cause of Heating

When current is passed through a conductor, the electrons start drifting towards the +ve end. They gain additional K.E. apart from thermal K.E. These electrons suffer collisions with atoms/ions more violently and transfer their K.E. to atoms/ions. It increases the amplitude of vibrations of ions/atoms. Thus, average K.E of vibrations of atoms/ions increases which shows up in the form of increased temperature. Here, the electric energy supplied by the source of e.m.f. is converted into heat.

Heat Produced by an Electric Current

Suppose a current I is flowing in a resistor of resistance R (Fig. 7.1). Amount of charge passed through resistor in time t: $q = It$.

Decreases in the potential of this charge: $V = IR$

Decreases in the potential energy of charge $= qV = I^2Rt$

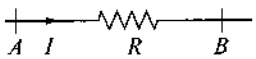


Fig. 7.1

This decrease in the energy will appear in the form of heat energy.

So electric energy produced in a resistor of resistance R in time t in which a current I is flowing is given by

$H = I^2RT$ (This is Joule's law of heating.)

Joule's law of heating: It states that amount of heat produced in a conductor is directly proportional to (i) square of current, (ii) resistance of conductor and (iii) time.

Other forms of H: $H = I^2Rt = \dfrac{V^2}{R} = VIt$

Joule's heating effect is irreversible. The resistor will become hot (and not cool down), if current is sent in any direction.

As $H \propto I^2$, heating effect of current is common to both d.c. and a.c. This is why instruments and appliances such as filament bulb, heater, geyser, press, toaster, etc. work on both d.c. and a.c.

Electric Power Produced in the Circuit

It is the energy produced in the resistor per unit time.

$$P = \frac{H}{t} \Rightarrow P = I^2R = \frac{V^2}{R} = VI$$

Units of Electric Energy and Electric Power

Unit of electric energy: J, cal, kWh, etc.

$$1 \text{ cal} = 4.18 \text{ J} \simeq 4.2 \text{ J}$$

Relation between kWh and joule: 1 kWh = 1000 W × hour = 1000 W × 3600 s = 3.6×10^6 Ws = 3.6×10^6 J.

1 kWh is the energy consumed by an appliance of power 1 kW when it runs for 1 h.

Commercial unit: 1 kWh is one unit of electricity. To calculate number of units, we can use

$$\text{Number of units} = \frac{\text{watt} \times \text{hour}}{1000}$$

The energy dissipated in kWh can be calculated using the following relation:

$$E = \frac{V \text{ (in volt)} \times I \text{ (in ampere)} \times t \text{ (in hour)}}{1000}$$

Unit of electric power: W, kW, MW, hp; 1 kW = 10^3 W, 1 MW = 10^6 W, 1 hp = 746 W.

Some Important Points

1. If the resistances are connected in series, then using $P = I^2R$, power developed will be higher in the resistor of higher value as current will be same in all resistors.
2. If the resistances are connected in parallel, then using $P = V^2/R$, the power developed will be higher in the resistor of lower value as potential will be same across all resistors.

Illustration 7.1 **Two wires of same mass, having ratio of the lengths 1:2, density 1:3 and resistivity 2:1. They are connected one by one to the same voltage supply. The rate of heat dissipation in the first wire is found to be 10 W. Find the rate of heat dissipation in the second wire.**

Sol. Given $\dfrac{\ell_1}{\ell_2} = \dfrac{1}{2}, \dfrac{d_1}{d_2} = \dfrac{1}{3}, \dfrac{\rho_1}{\rho_2} = \dfrac{2}{1}$; $m = A_1\ell_1d_1 = A_2\ell_2d_2$

$$\frac{P_2}{P_1} = \frac{V^2/R_2}{V^2/R_1}$$

$$\Rightarrow \frac{P_2}{P_1} = \frac{R_1}{R_2} = \frac{\rho_1\ell_1A_2}{\rho_2\ell_2A_1} = \frac{\rho_1\ell_1^2d_1}{\rho_2\ell_2^2d_2}$$

$$\Rightarrow \frac{P_2}{P_1} = \frac{2}{1} \times \left(\frac{1}{2}\right)^2 \times \frac{1}{3} = \frac{1}{6}$$

$$\Rightarrow P_2 = \frac{P_1}{6} = \frac{10}{6} = \frac{5}{3} \text{ W}$$

3. **To determine the resistance of a bulb (or other appliances):** Let a bulb be designed to operate on a voltage V_0 and its power indicated on it be P_0 (see Fig. 7.2). The resistance of the bulb is given by $R = V_0^2/P_0$.

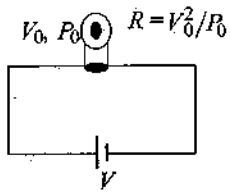


Fig. 7.2

Now let a potential difference of V is applied across this bulb, then power consumed is given by $P = \frac{V^2}{R} = \left(\frac{V}{V_0}\right)^2 P_0$.

If $V = V_0$, then $P = P_0$.

The above formula is very the convenient to calculate the power consumption when the applied voltage is different from the specified one.

An electric appliance consumes the specified power P_0 only if it runs at the specified voltage V_0. If the applied voltage V_A is greater than the specified voltage the appliance may get damaged as in this situation $I = (V_A/R)$ will exceed its current capacity $I_C = (V_0/R)$. Further, if an appliance is made to run at a voltage lower than the specified, then true power consumption will be less than the specified value.

Illustration 7.2 **A 100 W bulb is designed to operate on a potential difference of 230 V.**

i. Find the resistance of the bulb.
ii. Find the current drawn by the bulb if it is operated at a potential difference for which it is designed.
iii. Find the current drawn and power consumed by the bulb if it is connected to 200 V supply.

Sol. Power rating of the bulb: $P_0 = 100$ W, voltage rating of the bulb: $V_0 = 230$ V.

i. Resistance of the bulb: $R = \frac{V_0^2}{P_0} = \frac{(230)^2}{100} = 529\ \Omega$

ii. Current drawn: $I = \frac{V_0}{R} = \frac{230}{529} = \frac{10}{23}$ A

iii. $I = \frac{200}{529}$ A, $P = I^2R = \left(\frac{200}{529}\right)^2 \times 529 = 75.6$ W

Illustration 7.3 **A 500 W heating unit is designed to operate from a 200 V line. By what percentage will its heat output drop if the line voltage drops to 160 V? Find the heat produced by it in 10 min.**

Sol. Actual power consumed: $P = \left(\frac{V}{V_0}\right)^2 P_0 = \left(\frac{160}{200}\right)^2 500$

$= 320$ W. Heat output drop $= 500 - 320 = 180$ W.

% Heat drop $= \frac{180}{500} \times 100 = 36\%$.

Heat produced in 10 min (600 s) is given by $H = 320 \times 600 = 192\ 000$ J $= 192$ kJ

4. Two bulbs connected in series: Suppose two bulbs of same voltage rating V_0 and power ratings P_{01} and P_{02} are connected in series. Here $P_{01} > P_{02}$. Suppose potential V is applied across them as shown in Fig. 7.3.

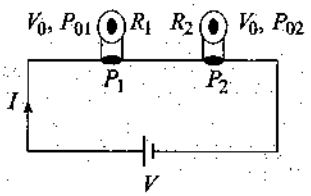


Fig. 7.3

Resistances of the bulbs:

$$R_1 = \frac{V_0^2}{P_{01}}, R_2 = \frac{V_0^2}{P_{02}}$$

Suppose power produced in them are P_1 and P_2, respectively. Then $P_1 = I^2R_1$ and $P_2 = I^2R_2$.

Now given: $P_{01} > P_{02} \Rightarrow R_1 < R_2 \Rightarrow P_1 < P_2$

It means the bulb having more power rating will consume less power when connected in series.

Total power produced:

$$P = P_1 + P_2 = \frac{V^2}{R_1 + R_2}$$

$$= \frac{V^2}{\frac{V_0^2}{P_{01}} + \frac{V_0^2}{P_{02}}} = \left(\frac{V}{V_0}\right)^2 \left(\frac{P_{01}P_{02}}{P_{01} + P_{02}}\right)$$

If $V = V_0$, then $P = \frac{P_{01}P_{02}}{P_{01} + P_{02}} \Rightarrow \frac{1}{P} = \frac{1}{P_{01}} + \frac{1}{P_{02}}$

Note: ***In series if any one bulb gets fused, then others will not glow.***

5. Two bulbs are connected in parallel: Suppose two bulbs of same voltage rating V_0 and power ratings P_{01} and P_{02} are connected in parallel. Here $P_{01} > P_{02}$. Suppose potential V is applied across them as shown in Fig. 7.4.

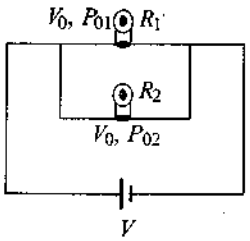


Fig. 7.4

Resistances of the bulbs:

$$R_1 = \frac{V_0^2}{P_{01}} R_2 = \frac{V_0^2}{P_{02}}$$

Let power produced in them be P_1 and P_2, respectively.

Then $P_1 = \frac{V^2}{R_1}$ and $P_2 = \frac{V^2}{R_2}$.

Now given: $P_{01} > P_{02} \Rightarrow R_1 < R_2 \Rightarrow P_1 > P_2$

It means that the bulb having more power rating will consume more power when connected in parallel.
Total power produced:

$$P = P_1 + P_2 = \frac{V^2}{R_1} + \frac{V^2}{R_2} = \frac{V^2}{V_0^2}P_{01} + \frac{V^2}{V_0^2}P_{02}$$

$$= \left(\frac{V}{V_0}\right)^2 (P_{01} + P_{02})$$

If $V = V_0$, then $P = P_{01} + P_{02}$

Note: ***In parallel if any one bulb gets fused, then others will continue to glow.***

Illustration 7.4 **Two bulbs are marked 220 V–100 W and 220 V–50 W.**

i. Which bulb will produce more illumination if they are connected in parallel to a 220 V supply?

ii. Which bulb will produce more illumination if they are connected in series to a 220 V supply?

iii. Also find the total power consumed by both the bulbs in each of the two parts above.

Sol.

i. First bulb. In parallel, the more the power rating, the more the power produced.

ii. Second bulb. In series, the more the power rating, the lesser the power produced.

iii. In first part: $P = P_{01} + P_{02} = 100 + 50 = 150$ W. In second part:

$$P = \frac{P_{01}P_{02}}{P_{01} + P_{02}} = \frac{100 \times 50}{150} = \frac{100}{3} \text{ W}$$

Illustration 7.5 **Two bulbs are rated 30 W–200 V and 60 W–200 V. They are connected with a 400 V power supply. Find which bulb will get fused if they are connected in (i) series and (ii) parallel.**

Sol.

i. $R_1 = \frac{(200)^2}{30}\Omega, R_2 = \frac{(200)^2}{60}\Omega \Rightarrow R_1 > R_2$

Hence, voltage across first bulb will be greater than 200 V. So, it will get fused.

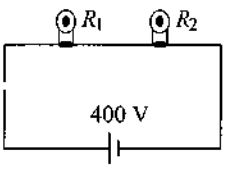


Fig. 7.5

ii. In parallel, potential across both the bulbs will be same and equal to 400 V. So both will get fused.

Illustration 7.6 **An electric tea kettle has two heating coils. When one of the coils is switched on, boiling begins in 6 min. When the other coil is switched on, the boiling begins in 8 min. In what time, will the boiling begin if both coils are switched on simultaneously (i) in series and (ii) in parallel.**

Sol. Let power of first coil be P_1 and that of second coil is P_2. Let H be the amount of heat required to boil water. Then $H = P_1t_1 = P_2t_2$ where $t_1 = 6$ min, $t_2 = 8$ min.

i. When the coils are connected in series:

$$P = \frac{P_1P_2}{P_1 + P_2}$$

$$t = \frac{H}{P} = H\left[\frac{1}{P_1} + \frac{1}{P_2}\right] = H\left[\frac{t_1}{H} + \frac{t_2}{H}\right]$$

$$= t_1 + t_2 = 6 + 8 = 14 \text{ min}$$

ii. When the coils are connected in parallel:

$$P = P_1 + P_2$$

$$t = \frac{H}{P} = \frac{H}{P_1 + P_2} = \frac{H}{\frac{H}{t_1} + \frac{H}{t_2}}$$

$$= \frac{t_1t_2}{t_1 + t_2} = \frac{6 \times 8}{6 + 8} = 3.43 \text{ min}$$

6. Let a resistance R under a potential difference V dissipate power:

$$P = \frac{V^2}{R}$$

So, if the resistance is changed from R to (R/n) keeping V same, the power consumed will be

$$P' = \frac{V^2}{(R/n)} = n\frac{V^2}{R} = nP$$

that is, if for a given voltage, resistance is changed from R to (R/n), power consumed changes from P to nP.

7. If n equal resistances are connected in series with a voltage source, then power dissipated will be

$$P_s = \frac{V^2}{nR} \qquad [\text{as } R_s = nR]$$

And if the same resistances are connected in parallel with the same voltage source,

$$P_P = \frac{V^2}{(R/n)} = \frac{nV^2}{R} \qquad [\text{as } R_p = (R/n)]$$

So, $\frac{P_P}{P_S} = n^2$ or $P_P = n^2P_S$

that is, power consumed by n equal resistors in parallel is n^2 times that of the power consumed in series, if V remains same.

Maximum Power Transfer Theorem

Suppose we want to find that for what value of the external resistance the maximum power will be drawn from a battery? For this, in the shown network (Fig. 7.6), let power developed in resistance R be

$$P = I^2R = \frac{E^2}{(R+r)^2}R \qquad \left(\text{as } I = \frac{E}{R+r}\right)$$

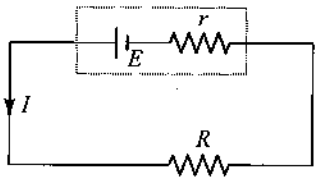


Fig. 7.6

Now, for $dP/dR = 0$ (for P to be maximum $dP/dR = 0$)

$$\Rightarrow \quad E^2\frac{(R+r)^2 - 2(R)(R+r)}{(R+r)^4} = 0$$

$$\Rightarrow \quad (r+R) = 2R \quad \Rightarrow \quad r = R$$

It means power output is maximum, when the external resistance equals the internal resistance $R = r$.

Illustration 7.7 **How will you connect (series and parallel) 24 cells each of internal resistance 1 Ω so as to get maximum power output across a load of 10 Ω?**

Sol. Suppose there are m rows and each row has n cells. Total number of cells is $mn = 24$. Here $I = \dfrac{ne}{R + nr/m}$

For maximum power:

$$\frac{nr}{m} = R \Rightarrow n = 10m \Rightarrow 10m^2 = 24 \Rightarrow m = \sqrt{2.4} = 1.55$$

i. If $m = 1, n = 24$; $I = \dfrac{24e}{34}$, then $P_1 = \left(\dfrac{24e}{34}\right)^2 \times 10 = 4.98e^2$

ii. If $m = 2, n = 12$; $I = \left(\dfrac{12e}{16}\right)$, then $P_2 = \left(\dfrac{12e}{16}\right)^2 \times 10 = 5.625e^2$

So, we have two rows ($m = 2$) each containing 12 cells ($n = 12$) in series.

Illustration 7.8 **A dry cell of e.m.f. of 1.5 V and internal resistance 0.10 Ω is connected across a resistor in series with a very low resistance ammeter. When the circuit is switched on, the ammeter reading settles to a steady value of 2.0 A.**

a. What is the steady rate of chemical energy consumption of the cell?
b. What is the steady rate of energy dissipation inside the cell?
c. What is the steady rate of energy dissipation inside the resistor?
d. What is the steady power output of the source?

Sol. a. Rate of chemical energy consumption of the cell is

$EI = 1.5 \times 2 = 3$ W.

b. Rate of energy dissipation inside the cell $= I^2r = (2)^2 \times 0.1 = 0.4$ W.

c. Rate of energy dissipation inside the resistor $= I^2R = EI - I^2r = 3 - 0.4 = 2.6$ W.

d. Power out $= I^2R = 2.6$ W.

SOME APPLICATIONS

(a) Fusing of bulb when it is switched on:

Usually filament bulbs get fused when they are switched on. This is because with the rise in temperature, the resistance of the bulb increases and becomes constant in steady state. So the power consumed by the bulb (V^2/R) initially is more than that in steady state and hence the bulb glows more brightly in the beginning and may get fused.

Illustration 7.9 **Two wires made of tinned copper having identical cross section ($=10^{-6}$ m²) and lengths 10 and 15 cm are to be used as fuses. Show that the fuses will melt at the same value of current in each case.**

Sol. The temperature of the wire rises to a certain steady temperature when the heat produced per second by the current just becomes equal to the rate of loss of heat from its surface.

Heat produced per second by the current $= I^2R = I^2\dfrac{\rho l}{\pi r^2}$,

where l is the length, r is radius of the wire and ρ is the specific resistance. Let H be heat lost per second per unit surface area of the wire. If we neglect the loss of heat from the end faces of the wire, then heat lost per second by the wire is $H \times$ surface area of wire $= H \times 2\pi rl$.

At steady state temperature,

$$H \times 2\pi rl = \frac{I^2\rho l}{\pi r^2} \text{ or } H = \frac{I^2\rho}{2\pi^2 r^3} \qquad \text{(i)}$$

From Eq. (i) we note that the rate of loss of heat (H) which, in turn, depends upon the temperature of the wire is independent of length of the wire. Hence, the fuses of two wires of the same values of r and ρ but of different lengths will melt for the same value of current in each case.

(b) Decrease in the brightness of bulb after long use:

Also due to evaporation of metal from the filament (which deposits as black substance on the inner side of glass wall), the filament of the bulb becomes thinner and thinner with use. This increases the resistance [$R = \rho L/\pi r^2$] of the bulb and as $P = V^2/R$ the brightness of light emitted by a bulb decreases gradually with time.

(c) Decrease in brightness of a bulb in a room when a heavy current appliance is switched on:

As shown in Fig. 7.7, if the bulb draws a current I_1 from the source, then terminal voltage of source $= V = (E - I_1 r)$

and so power consumed by the bulb is

$$P = \frac{V^2}{R} = \frac{(E - I_1 r)^2}{R}.$$

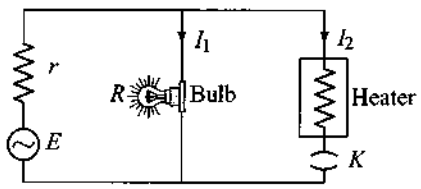


Fig. 7.7

Now, when a heavy current appliance such as motor, heater or geyser is switched on, it will draw a heavy current, say I_2, from the source so that terminal voltage will become

$$V' = [E - (I_1 + I_2)r] = (V - I_2 r) \qquad (< v)$$

and hence power consumed by the bulb will now be

$$p' = \frac{V'^2}{R} = \frac{(V - I_2 r)^2}{R} < P$$

So, the brightness of the bulb decreases.

Note: ***If the source is ideal, i.e., $r = 0$, $V' = V = E$ then $P' = P$, i.e., there will be no change in the brightness of the bulb, if the source is ideal.***

(d) Fuse and its action:

It is a metallic conducting wire of 75% Pb and 25% Sn with low melting point and higher resistance and is in series with an appliance (Fig. 7.8).

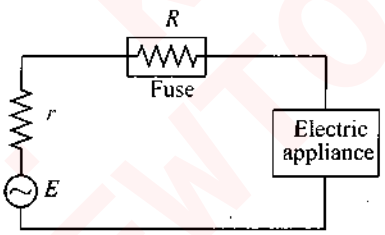


Fig. 7.8

It is a safety device which protects the appliance from getting damaged, by melting and opening the circuit, if the current in the circuit exceeds a specific predetermined value, called 'current capacity'.

(e) Long distance power transmission:

When power is transmitted through a power line of resistance R, power loss will be

Power loss $= I^2R$

Now, let power P_0 is transmitted at voltage V, then $P_0 = VI$,

i.e., $I = \frac{P_0}{V}$

So, power loss $= \frac{P_0^2}{V^2} \times R$

Now, as for a given power and line, P_0 and R are constant.

So, power loss $\propto \frac{1}{V^2}$

So, if power is transmitted at high voltage, power loss will be small and vice versa, e.g., power loss at 22 kV is 10^{-4} times than that at 220 V. This is why long distance power transmission is carried out at high voltage.

(f) The wires supplying current to a bulb are not heated while the filament of the bulb becomes hot. It is because resistance of the wires is very small in comparison to the resistance of bulb. If alone the wires are connected, then whole of the potential differences will be across the wires and because their resistance is very small, a large amount of heat will be generated from $P = V^2/R$. This happens when wires are short-circuited.

(g) The resistance of high electric power instrument will be smaller than that of low electric power instrument. Because for a given voltage: $P = V^2/R$. For example, iron, heater, geyser.

The heating element of these appliances is made of nichrome. It is an alloy of Ni and Cr. Its resistivity is higher in comparison to platinum, tungsten and copper, etc. Nichrome is used because

i. it has high resistivity and high melting point,
ii. it is not oxidized when heated,
iii. it can be easily drawn into wires.

Resistivity is kept higher so that smaller length can be used, as $H = \frac{V^2}{R}t = \frac{V^2 At}{\rho \ell}$.

(h) Incandescent electric lamp:

It consists of a metal filament generally made of tungsten. It is enclosed in a glass bulb with some inert gas and at suitable pressure. The filament gets heated, then it becomes white hot (known as incandescent stage) and starts emitting white light. The filament should have high melting point.

Illustration 7.10 **An electric kettle taking 3 A at 210 V brings 1 ℓ of water from 20°C to 80°C in 10 min. Find its efficiency.**

Sol. $\eta = \frac{OP}{IP} \times 100 = \frac{10^3 \times 4.2 \times 60}{210 \times 3 \times 600} \times 100 = 66.67\%$

Illustration 7.11 **A line having a total resistance of 0.2 Ω delivers 10 kW at 220 V to a small factory. Calculate the efficiency of transmission.**

Sol. $I = \frac{10000}{220} = \frac{500}{11}$ A. Loss $= I^2 r = \left(\frac{500}{11}\right)^2 \times 0.2 = 413.22$

Efficiency: $\eta = \frac{10000}{10000 + 413.22} \times 100 = 96\%$

Concept Application Exercise 7.1

1. a. What is the rate at which energy being delivered to a light bulb higher; just after it is turned on, the glow of the filament is increasing, or after it has been on for a few seconds and the glow is steady?

b. If a piece of wire were used to connect points b and c in Fig. 7.9, does the brightness of bulb R_1 increase, decrease or stay constant? What happens to the brightness of bulb R_2?

$I_1 = I_2 = I$

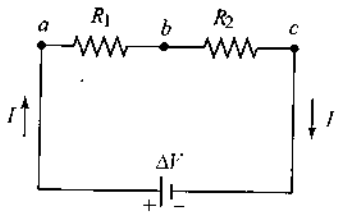


Fig. 7.9

c. Compare the brightness of four identical light bulbs in Fig. 7.10. What happens if bulb A fails, so that it cannot conduct? What if C fails? What if D fails?

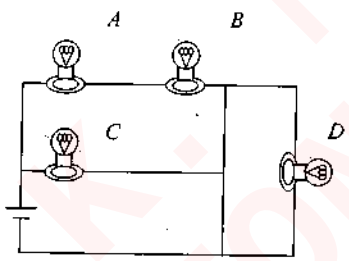


Fig. 7.10

d. If electric power is transmitted over long distances, the resistance of the wires becomes significant. Why? Which mode of transmission would result in less energy loss–high current and low voltage or low current and high voltage? Discuss.

e. In Fig. 7.11, describe what happens to the light bulb after the switch is closed. Assume the capacitor has a large capacitance and is initially uncharged.

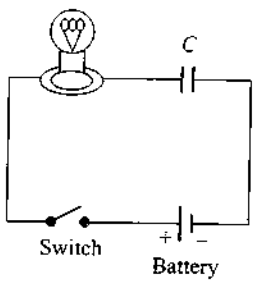


Fig. 7.11

f. A student claims that a second light bulb in series is less bright than the first, because the first bulb uses up some of the current. How would you respond to this statement?

g. If you were to design an electric heater using Nichrome wire as the heating element, what parameters of the wire could you vary to meet a specific power output, such as 1000 W?

2. A heater joined in series with a 50 W bulb is connected to the mains. If the 50 W bulb is replaced by a 100 W bulb, then will the heater now give more heat, less heat or same heat ? Why?

3. Each of the three resistors in Fig. 7.12 has a resistance of 2 Ω and can dissipate a maximum of 18 W without becoming excessively heated.

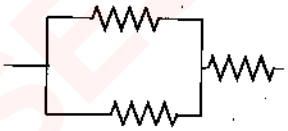


Fig. 7.12

Find the maximum power the circuit can dissipate.

4. An electric bulb rated 220 V and 60 W is connected in series with another electric bulb rated 220 V and 40 W. The combination is connected across 220 V source of e.m.f. Which bulb will glow more?

5. We have a 30 W, 6 V bulb which we want to glow by a supply of 120 V. What can be done for it?

6. Two heater coils made of the same material are connected in parallel across the mains, the length and the diameter of one coil is double that of the other. Which of them will produce more heat?

7. A series circuit consists of three identical lamps connected to a battery as shown in Fig. 7.13.

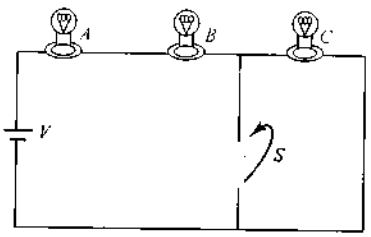


Fig. 7.13

When the switch S is closed, what happens

a. to the intensities of lamp A and B

b. to the intensity of lamp C

c. to the current in the circuit and

d. to the voltage drop across the three lamps? Does the power dissipated in the circuit increase, decrease or remain the same?

8. Two wires of the same material and having the same uniform area of cross section are connected in an electric circuit. The masses of the wires are m and $2m$, respectively. When a current I flows through both of them connected in series, then find the ratio of heat produced in them in a given time.

9. Water boils in an electric kettle in 15 min after being switched on. Using the same main supply, should the

length of the heating element be increased or decreased if the water is to be boiled in 10 min? Why?

10. An electric motor operating on a 50 V d.c. supply draws a current of 12 A. If the efficiency of the motor is 30%, estimate the resistance of the windings of the motor.

11. A fuse with a circular cross-sectional radius of 0.15 mm blows at 15 A. What should be the radius of cross section of a fuse made of the same material which blows at 30 A?

12. A motor operating on 120 V draws a current of 2 A. If the heat is developed in the motor at the rate of 9 cal s^{-1}, what is its efficiency?

13. The walls of a closed cubical box of edge 40 cm are made of a material of thickness 1 mm and thermal conductivity 4×10^{-4} $cals^{-1}cm^{-1}$ $°C^{-1}$. The interior of the box is maintained at 100°C above the outside temperature by a heater placed inside the box and connected across 400 V d.c. Calculate the resistance of the heater.

14. Two tungsten lamps with resistances R_1 and R_2, respectively, are connected first in parallel and then in series in a lighting circuit of negligible internal resistance. Given that $R_1 > R_2$.

a. Which lamp will glow more brightly when they are connected in parallel?

b. If the lamp of resistance R_1 now burns out, how will the net illumination produced change?

c. Which lamp will glow more brightly when they are connected in series?

d. If the lamp of resistance R_2 now burns out and the lamp R_1 alone is plugged in, will the net illumination increase or decrease?

15. n identical bulbs are connected in series and illuminated by a power supply. One of the bulbs gets fused. The fused bulb is removed and the remaining bulbs are again illuminated by the same power supply. Find fractional change in the illumination of **(a)** all the bulbs, and **(b)** one bulb.

16. An electric motor is designed to work at 100 V and draws a current of 6 A. The output power supplied by the motor is 150 W and remaining goes to heat. What is the resistance of the windings of the motor and its percentage efficiency?

17. A house is fitted with certain numbers of 100 W, 230 V incandescent lamps. The power to the house is fed by a generator producing the power at 240 V. The resistance of the wires from generator to the house is 2 Ω. Find the maximum number of lamps that can be illuminated so that voltage across none of the lamps drops below 230 V.

18. A house is fitted with seven tubelights of rating 220 V – 40 W each, two bulbs of rating 220 V – 60 W each, five fans each drawing a current of 0.4 A at 220 V and a heater of resistance 48.4 Ω. The main line power supplied to the house is at 220 V. Calculate the bill for the month of January if tubelights and bulbs are used for 6 h daily, fans for 1 h daily and heater for 10 h daily. The electricity is to cost Rs. 2 per unit.

19. Two bulbs are marked 200 V, 300 W and 200 V, 600 W, respectively. The bulbs are connected in series and the combination is connected to a 200 V supply.

a. Which bulb will produce more illumination?

b. Find the total power consumed by both the bulbs.

c. Find the total power consumed if both the bulbs were connected in parallel.

20. A voltage stabilizer restricts the voltage output to 220 V ± 1%. If the electric bulb rated at 220 V, 100 W is connected to it, what will be the minimum and maximum power consumed by it?

21. The efficiency of a cell when connected to a resistance R is 60%. What will be its efficiency if the external resistance is increased by six times?

22. A 25 W and a 100 W bulb are joined in series and connected to the mains. Which bulb will glow brighter?

(IIT-JEE, 1979)

Solved Examples

Example 7.1 **A series battery of six cells each of e.m.f. 2 V and internal resistance 0.5 Ω is charged by a 100 V d.c. supply. What resistance should be used in the charging circuit in order to limit the charging current to 8 A. Using this relation, obtain (a) the power supplied by the d.c. source, (b) the power dissipated as heat and (c) the chemical energy stored in the battery in 15 min.**

Sol. Given: number of cells $n = 6$; e.m.f. of each cell, $E = 2$V; internal resistance of each cell, $r = 0.5$ Ω; charging voltage $V = 100$ V. Let R be the resistance used in the series of the circuit while charging the cells. Then current in the circuit will be

$$i = \frac{V - nE}{nr + R} \text{ or } R \frac{V - nE}{i} - nr = \frac{100 - 6 \times 2}{8} - 6 \times 0.5$$

$$= 11 - 3 = 8\ \Omega$$

a. Power supplied by d.c. source $= V \times i = 100 \times 8 = 800$ W

b. Power dissipated as heat $= i^2(R + nr) = 8^2\,(8 + 6 \times 0.5)$
$= 704$ W

c. Rate at which the chemical energy is stored $= 800 - 704$
$= 96$ W

Therefore, chemical energy stored in 15 min $= 96 \times 15 \times 60$
$= 86400$ J

Example 7.2 **Determine the current through the battery of internal resistance 0.5 Ω for the circuit shown in Fig. 7.14. How much power is dissipated in 6 Ω resistance?**

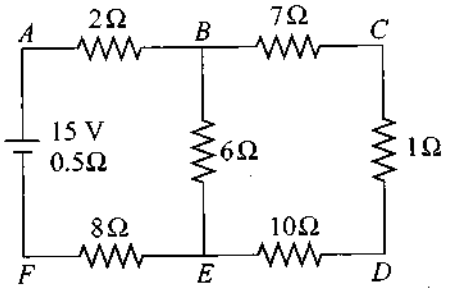


Fig. 7.14

Sol. Resistance of arm $BCDE = 7 + 1 + 10 = 18\ \Omega$. Here 18 and $6\ \Omega$ are in parallel. Their effective resistance is

$$R_p = \frac{18 \times 6}{18 + 6} = \frac{18 \times 6}{24} = 4.5\ \Omega$$

Total resistance of the circuit $= 2 + 4.5 + 8 + 0.5 = 15\ \Omega$

Therefore, current through the circuit, $i = \frac{15}{15} = 1\text{A}$

Potential difference across B and $E = i \times R_p = 1 \times 4.5 = 4.5$ V

Therefore, power dissipated as heat due to resistance $6\ \Omega$ is

$$\frac{(4.5)^2}{6} = 3.375\text{ W}$$

Example 7.3 **Two uniform wires of same material each weighing 1 g but one having double the length of the other are connected in series, carrying a current of 10 A. The length of the longer wire is 20 cm. Calculate the rate of consumption of energy in each of the two wires. Which wire gets hotter? Density of the material of wire = 11 gcm^{-3}, specific resistance of the material is $20 \times 10^{-5}\ \Omega$ cm.**

Sol. Let a_1 and a_2 be the area of cross section of shorter and longer wires, respectively.

As mass = volume × density = $al\rho$

$\therefore \quad 1 = a_1 \times 10 \times 11 = a_2 \times 20 \times 11$

or $\quad a_1 = \frac{1}{10 \times 11}\text{ cm}^2 \quad$ and $\quad a_2 = \frac{1}{20 \times 11}\text{ cm}^2$

$$\therefore \quad R_1 = 20 \times 10^{-5} \times \frac{10}{1/(10 \times 11)}$$

$$= 20 \times 10^{-5} \times 10 \times 10 \times 11 = 22 \times 10^{-2}\ \Omega$$

$$R_2 = 20 \times 10^{-5} \times \frac{20}{1/(20 \times 11)} = 88 \times 10^{-2}\ \Omega$$

And rate of heat produced

$$H_1 = I^2 R_1 = (10)^2 \times 22 \times 10^{-2} = 22\text{ W}$$

and $\quad H_2 = I^2 R_2 = (10)^2 \times 88 \times 10^{-2} = 88$ W

Thus, the wire of longer length gets hotter.

Example 7.4 **In a house having 220 V line, the following appliances are operating:**

(i) a 60 W bulb, (ii) a 1000 W heater and (iii) a 40 W radio. Calculate (a) the current drawn by heater and (b) the current passing through fuse in the main line.

Sol. Here, $V = 220$ V; $P_1 = 60$ W; $P_2 = 1000$ W; $P_3 = 40$ W.

a. Current drawn by heater $= \frac{P_2}{V} = \frac{1000}{220} = \frac{50}{11}$ A

b. Current drawn by bulb $= \frac{P_1}{V} = \frac{60}{220} = \frac{3}{11}$ A

Current drawn by radio $= \frac{P_3}{V} = \frac{40}{220} = \frac{2}{11}$ A

Current passing through fuse for the line $= \frac{50}{11} + \frac{3}{11} + \frac{2}{11} = 5$ A

Example 7.5 **A heater is designed to operate with a power of 1000 W in a 100 V line. It is connected, in combination with a resistance R, to a 100 V mains as shown in Fig. 7.15. What should be the value of R such that heater may operate with a power of 62.5 W?**

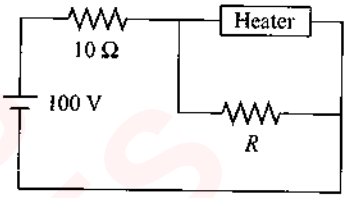


Fig. 7.15

(IIT-JEE, 1978)

Sol. The resistance of the heater is

$$R = \frac{V^2}{P} = \frac{100 \times 100}{1000} = 10\ \Omega$$

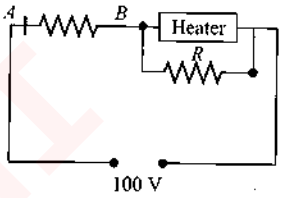


Fig. 7.16

The power on which it operates is 62.5 W.

$\therefore \quad V = \sqrt{R \times P'} = \sqrt{10 \times 62.5} = \sqrt{625} = 25$ V

$\Rightarrow$ The potential drop across $AB = 75$ V

$\therefore$ The current in $AB = I = \frac{V}{R} = \frac{75}{10} = 7.5$ A

This current divides into two parts. Let I_1 be the current that passes through the heater. Therefore

$$25 = I_1 \times 10 \quad \Rightarrow \quad I_1 = 2.5\text{ A}$$

$\Rightarrow$ current through R is 5 A

Applying Ohm's law across R, we get:

$$25 = 5 \times R \quad \Rightarrow \quad R = 5\ \Omega$$

Example 7.6 **(i) Find the time taken by a filament of 200 W to heat 500 ml of water from 25°C to 75°C. Specific heat of water = 1 calg^{-1}°C^{-1}. Take 1 cal = 4.2 J.**

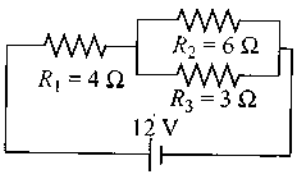


Fig. 7.17

(ii) Find the power produced by each resistor shown in Fig. 7.17. If R_1 is dipped in 1000 ml of water at 30°C, find the time taken by it to boil the water.

Sol.

(i) $200\,t = 0.5 \times 4200 \times 50 \quad \Rightarrow \quad t = 525$ s

(ii) $R_{eq} = 6\Omega, I_1 = 12/6 = 2$ A in $4\ \Omega, I_2 = (2/3)$ A in $6\ \Omega, I_3 = (4/3)$ A in 3Ω

$$P_1 = I_1^2 R_1 = 2^2 \times 4 = 16 \text{ W}, P_2 = \left(\frac{2}{3}\right)^2 \times 6 = \frac{8}{3} \text{ W}$$

$$P_3 = \left(\frac{4}{3}\right)^2 \times 3 = \frac{16}{3} \text{ W}$$

Now, $P_1 t = 1 \times 4200 \times 70 \Rightarrow t = \dfrac{4200 \times 70}{16} = 18375 \text{ s}$

Example 7.7 **A heating coil of 2000 W is immersed in water. How much time will it take in raising the temperature of 1 l of water from 4°C to 100°C? Only 80% of the thermal energy produced is used in raising the temperature of water.**

Sol. Here, $P = 2000$ W, $t = ?$ (in seconds)

Volume of water = 1 l = 1000 cm³

Mass of water, m = volume × density = 1000 × 1 = 1000 g

Rise in temperature $= \theta_2 - \theta_1 = 100 - 4 = 96°C$

We know specific heat of water, $c = 1$ cal g⁻¹ °C⁻¹

Therefore, heat taken by water is $mc(\theta_2 - \theta_1) = 1000 \times 1 \times 96 = 96000$ cal

Energy spent in heating coil $= Pt = 2000 \times t$

Useful energy produced = 80% = $2000 \times t \times 80/100$ J

Useful heat produced $= \dfrac{2000 \times t \times 80}{100 \times 4.2}$ cal

As this heat is taken by water, $\dfrac{2000 \times t \times 80}{100 \times 4.2} = 96000$

or $$t = \frac{96000 \times 100 \times 4.2}{2000 \times 80} = 252 \text{ s}.$$

Example 7.8 **Consider the following circuit (Fig. 7.18) where some resistances have been arranged in a definite order. With the given condition that heat produced by 6 Ω resistance is 60 cal s⁻¹ due to the current flowing through it, find out the heat produced across 2 Ω resistance in calorie per second.**

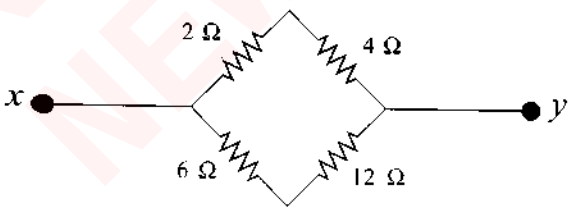


Fig. 7.18

Sol. Same current flows through resistances connected in series.

Heat produced in 6 Ω resistance $= \dfrac{I^2 R}{J}$

$$\Rightarrow \quad 60 = \frac{I^2 \times 6}{4.2} \Rightarrow I^2 = 42 \Rightarrow I = \sqrt{42} \text{ A}$$

Now the voltage drop across x and y is

$$(6 + 12)\sqrt{42} = 18\sqrt{42} \text{ V}$$

As this potential drop is same in every area of a parallel circuit, the potential drop across upper part of the circuit is the same.

Therefore, current through 2 and 4 Ω resistances $= (18\sqrt{42})/6 = 3\sqrt{42}$ A

Hence, the heat produced across the 2 Ω resistance is

$$\frac{I^2 R}{J} = \frac{9 \times 42 \times 2}{4.2} = 180 \text{ cal}$$

Example 7.9 **Consider a Wheatstone bridge *PQRS* as shown in Fig. 7.19 where current *I* is in the circuit of four resistances 10, 20, 30 and 40 Ω. Find the ratio of the heat generated in the four arms *PQ*, *QR*, *PS* and *SR*.**

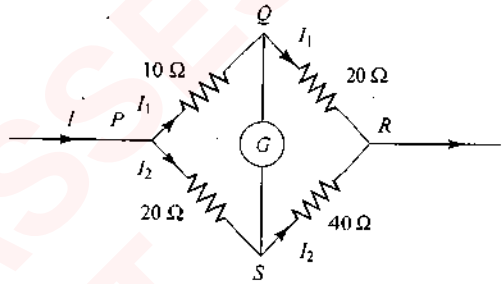


Fig. 7.19

Sol. Given: $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$, $R_3 = 20\ \Omega$ and $R_4 = 40\ \Omega$.

Now, $$\frac{R_1}{R_2} = \frac{10}{20} = \frac{1}{2} \text{ and } \frac{R_3}{R_4} = \frac{20}{40} = \frac{1}{2}$$

$$\therefore \quad \frac{R_1}{R_2} = \frac{R_3}{R_4}$$

Hence, Wheatstone bridge is balanced. Now as the bridge is balanced, no current will flow through arm *QS*.

Let I_1 and I_2 be the currents flowing in arms *PQ* and *PS*, respectively.

Then potential difference across *P* and *Q* is equal to potential difference across *P* and *S*.

$$\Rightarrow \quad I_1 \times 10 = I_2 \times 20 \Rightarrow I_1 = 2I_2$$

Therefore, heat produced in arm *PQ* is

$$H_1 = I_1^2 \times 10 = 40 I_2^2 \text{ J}$$

Also heat produced in arm *QR* is

$$H_2 = I_1^2 \times 20 = 80 I_2^2 \text{ J}$$

Similarly, heat produced in arm *PS* is

$$H_3 = I_2^2 \times 20 = 20 I_2^2 \text{ J}$$

And heat produced in arm *SR* is

$$H_4 = I_2^2 \times 40 = 40 I_2^2 \text{ J}$$

$\therefore H_1 : H_2 : H_3 : H_4 = 40 I_2^2 : 80 I_2^2 : 20 I_2^2 : 40 I_2^2 = 2 : 4 : 1 : 2$, which is the required ratio.

Example 7.10 **A person with body resistance between his hands of 10 kΩ accidentally grasps the terminals of a 18 kV power supply. (a) If the internal resistance of the power supply is 2000 Ω, what is the current through the person's body? (b) What is the power dissipated in his body? (c) If the power supply is to be made safe by increasing its internal resistance, what should the internal resistance be**

for the maximum current in the above situation to be 1.00 mA or less?

Sol. Given: $R = 10\ k\Omega$, $V = 18\ kV$

a. To find out current flowing through the body, we need to sum up the resistances present in the circuit and divide the voltage by it.

$$I = \frac{V}{R+r} = \frac{18\times10^3}{10\times10^3 + 2\times10^3} = \frac{18\times10^3}{12\times10^3}$$

$$= \frac{3}{2} = 1.5\ A$$

b. We know power dissipated is

$$V_1 = I^2R = (1.5)^2\,(10000) = 2.25\times10000$$
$$= 22500 = 22.5\ kW$$

c. To find the internal resistance for the safe limit of power, we can use the formula as in part (a). The only difference here is I is given and r is to be calculated.

$$R + r = \frac{V}{I} = \frac{18\times10^3}{1\times10^{-3}} \quad\Rightarrow\quad R + r = 18\times10^6$$

$$\Rightarrow \qquad r = 18\times10^6 - 10\times10^3 = (18\ M\Omega - 10\ k\Omega)$$
$$\cong 18\ M\Omega$$

Example 7.11 **An electric kettle has two coils of same power. When one coil is switched on, it takes 15 min to boil water and when the second coil is switched on it takes 30 min. How long will it take to boil water when both the coils are used in (a) series and (b) parallel ?**

Sol. Heat produced in resistance R in time t is $H = Pt = \frac{V^2}{R}t$

For coil 1: $H_1 = \frac{V^2}{R_2}(15\times60)$ (i)

For coil 2: $H_2 = \frac{V^2}{R_2}(15\times60)$ (ii)

But according to the given problem

$$H_1 = H_2,\ \text{i.e.,}\ \frac{15}{R_1} = \frac{30}{R_2}\ \text{or}\ R_2 = 2R_1 \qquad \text{(iii)}$$

a. Both the coils are used in series:

$$H_s = \frac{V^2}{(R_1+R_2)}t_S = \frac{V^2}{3R_1}\times t_S \qquad [\text{as } R_2 = 2R_1]$$

But as here, $H_S = H_1\ (= H_2)$

So, $\frac{V^2}{R_1}(15\times60) = \frac{V^2}{3R_1}t_S$ or $t_S = (45\times60)\,s = 45\,min$

b. Both the coils are used in parallel:

$$H_P = \left[\frac{V^2}{R_1} + \frac{V^2}{R_2}\right]\times t_P = \frac{3V^2}{2R_1}\times t_P \qquad [\text{as } R_2 = 2R_1]$$

According to the given problem, $H_P = H_1$

$$\frac{3V^2}{2R_1}\times t_P = \frac{V^2}{R_1}\times(15\times60) \quad\text{or}\quad t_P = (10\times60)\ s = 10\ min$$

Example 7.12 **What amount of heat will be generated in a coil of resistance R due to a total charge q passing through it if the current in the coil:**

(a) decreases down to zero uniformly during a time interval t_0?

(b) decreases down to zero halving its value every t_0 seconds?

Sol.

a. The current decreases uniformly with time, therefore i vs. t curve is a straight line as shown in Fig. 7.20 with slope $m = -i_0/t_0$. Current as a function of time can be written as

$$i = i_0 - \left(\frac{i_0}{t_0}\right)t \qquad \text{(i)}$$

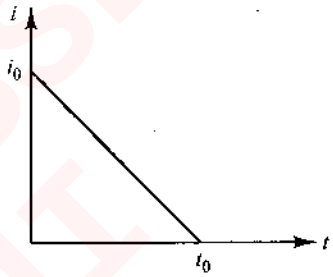


Fig. 7.20

Area under the $i-t$ graph gives the flow of charge q, therefore

$$q = \frac{1}{2}(t_0)(i_0) \Rightarrow i_0 = \frac{2q}{t_0}$$

Substituting in Eq. (i), we get

$$i = \frac{2q}{t_0}\left(1 - \frac{t}{t_0}\right) \Rightarrow i = \left(\frac{2q}{t_0} - \frac{2qt}{t_0^2}\right)$$

Heat produced in a time interval t_0 is

$$\int dH = \int i^2R\,dt \text{ or } H = \int_0^{t_0}\left(\frac{2q}{t_0} - \frac{2qt}{t_0^2}\right)^2 R\,dt = \frac{4}{3}\frac{q^2R}{t_0}$$

b. Here, current decreases from i_0 to zero exponentially with half-life of t_0. The $i-t$ equation in this case is an exponential function like the radioactive decay law:

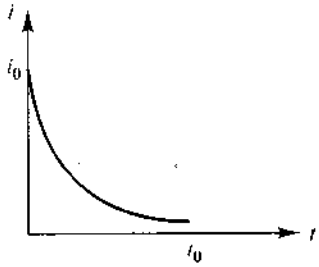


Fig. 7.21

$$i = i_0e^{-\lambda t} \quad\text{where}\quad \lambda = \frac{\ln(2)}{t_0}$$

Total charge, $q = \int_0^\infty i\,dt = \int_0^\infty i_0e^{-\lambda t}\,dt = \left(\frac{i_0}{\lambda}\right)$

or $\qquad i_0 = \lambda q \quad$ or $\quad i = (\lambda q)e^{-\lambda t}$

Heat produced in time interval dt is

$$dH = i^2R\,dt = \lambda^2q^2e^{-2\lambda t}R\,dt$$

or $$H = \lambda^2 q^2 R \int_0^\infty e^{-2\lambda t}\, dt = \frac{q^2 \lambda R}{2} = \frac{q^2 R \ln(2)}{2 t_0}$$

Example 7.12 **A variable capacitor is adjusted in position of its lowest capacitance C_0 and is connected with a source of constant voltage V for a long time. Resistance of connecting wires is R. At $t = 0$, its capacitance starts to increase so that a constant current I starts to flow through the circuit. Calculate at time t:**

i. power supplied by the source
ii. thermal power generated in the connecting wire
iii. rate of increase of electrostatic energy stored in capacitor
iv. What do you infer from the above three results?

Sol.

i. Since voltage V of the source is constant and the circuit draws constant current I from it, therefore, power supplied by the source is $P = VI$.

ii. Thermal power generated in connecting wires is $H = I^2R$.

iii. Since initial capacitance of the capacitor was equal to C_0 and it was connected with the source for long time, therefore initial charge on capacitor was $q_0 = C_0V$.

Since a constant current I starts to flow at $t = 0$, therefore at time t, charge on capacitor becomes $q = (C_0V + It)$.

At time t, the circuit will be as shown in Fig. 7.21. Potential difference across the capacitor is

$$V_C = V_A - V_B = (V - IR) \quad \rightarrow \quad \text{constant}$$

Therefore, electrostatic energy in capacitor at this instant is

$$U = \frac{1}{2} qV_C$$

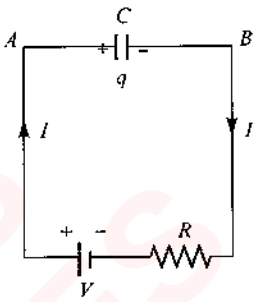


Fig. 7.22

Rate of increase of electrostatic energy is

$$\frac{dU}{dt} = \frac{1}{2} V_C \frac{dq}{dt} = \frac{1}{2}(V - IR)I$$

$$= \frac{1}{2}(VI - I^2R)$$

But power acting across the capacitor at this instant is

$$P_0 = P - H = (VI - I^2R)$$

while the rate of increase of electrostatic energy in the capacitor is half of it.

iv. In fact, a force of attraction exists between the plates of the capacitor. When these surfaces move towards each other, capacitance increases. Hence, the remaining part of the power acting across the capacitor is used to increase kinetic energy of surface (plate) of the capacitor.

EXERCISES

Subjective Type

Solutions on page 7.20

1. A circuit shown in Fig. 7.23 has resistances $R_1 = 20\,\Omega$ and $R_2 = 30\,\Omega$. At what value of the resistance R_x will the thermal power generated in it be practically independent of small variations of that resistance. The voltage between the points A and B is supposed to be constant in this case.

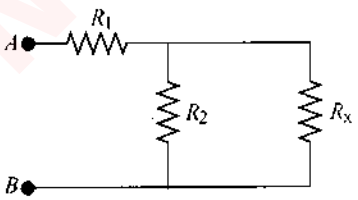


Fig. 7.23

2. A 1 kW heater is meant to operate at 200 V.
   a. What is the resistance?
   b. How much power will it consume if the line voltage drops to 100 V?
   c. How many units of electrical energy will it consume in a month (of 30 days) if it operates 10 h daily at the specified voltage (200 V)?

3. A resistor R_1 consumes electrical power P_1 when connected to an e.m.f. ε. When resistor R_2 is connected to the same e.m.f., it consumes electrical power P_2. In terms of P_1 and P_2, what is the total electrical power consumed when they are both connected to this e.m.f. source
   a. in parallel? **b.** in series?

4. 200 identical electrical bulbs, each having resistance 400 Ω, are connected in parallel to a d.c. source of e.m.f. 100 V and internal resistance 0.1 Ω. What is the power consumed by each bulb. Also find the percentage change is power consumed by each bulb if one bulb turns out.

5. In the circuit shown in Fig. 7.24, all the resistors are rated at a maximum power of 1.00 W. What is the maximum e.m.f. e that the battery can have without burning up any of the resistors?

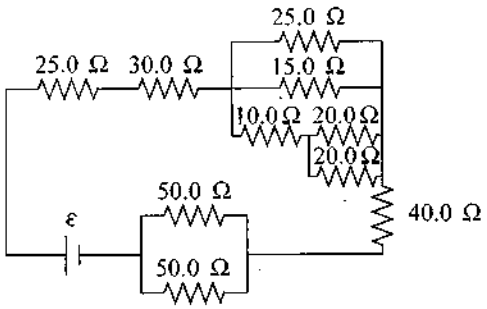


Fig. 7.24

6. In the circuit shown in Fig. 7.25,

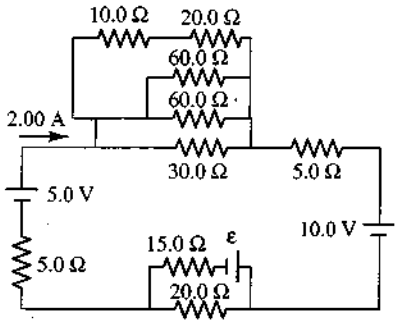


Fig. 7.25

a. What must the e.m.f. e of the battery be in order for a current of 2.00 A to flow through the 5.00 V battery, as shown? Is the polarity of the battery correct as shown?
b. How long does it take for 60.0 J of thermal energy to be produced in the 10.0 Ω resistor?

7. If two bulbs of 25 and 100 W rated at 220 V are connected in series across a 440 V supply, will both the bulbs fuse? If not which one?

8. Three 60 W, 120 V light bulbs are connected across a 120 V power line as shown in Fig. 7.26. Find (a) the voltage across each bulb and (b) the total power dissipated in the three bulbs.

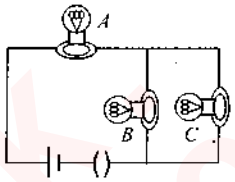


Fig. 7.26

Objective Type

Solutions on page 7.21

1. The operating temperature of the filament of lamp is 2000°C. The temperature coefficient of the material of filament is 0.005 °C⁻¹. If the atmospheric temperature is 0°C, then the current in the 100 W–200 V lamp when it is switched on is nearest to
   a. 2.5 A b. 3.5 A
   c. 4.5 A d. 5.5 A

2. In the circuit below (Fig. 7.27), bulb B does not light although ammeter A indicates that the current is flowing. Why does the bulb not light?

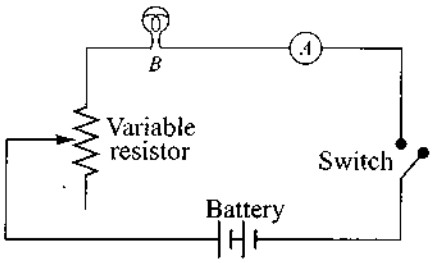


Fig. 7.27

a. The bulb is fused
b. There is break in the circuit between bulb and ammeter
c. The variable resistor has too large resistance
d. There is a break in the circuit between the bulb and the variable resistance

3. Three bulbs B_1, B_2 and B_3 are connected to the mains as shown in Fig. 7.28. How will the brightness of bulb B_1 be affected if B_2 or B_3 are disconnected from the circuit?

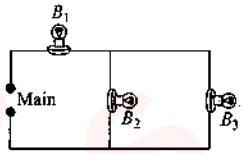


Fig. 7.28

a. Bulb B_1 becomes brighter
b. Bulb B_1 becomes dimmer
c. No change occurs in the brightness
d. Bulb B_1 becomes brighter if bulb B_2 is disconnected and dimmer if bulb B_3 is disconnected

4. Three identical cells, each having an e.m.f. of 1.5 V and a constant internal resistance of 2.0 Ω, are connected in series with a 4.0 Ω resistor R, first as in circuit (i), and secondly as in circuit (ii).

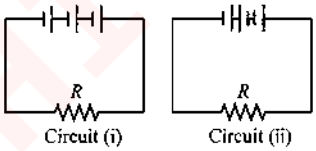


Fig. 7.29

What is the ratio $\dfrac{\text{Power in } R \text{ in circuit (i)}}{\text{Power in } R \text{ in circuit (ii)}}$?

a. 9.0 b. 7.2 c. 1.8 d. 3.0

5. All bulbs in Fig. 7.30 are indentical. Which bulb lights more brightly?

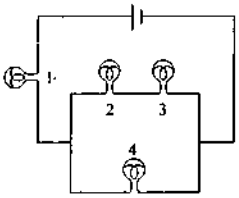


Fig. 7.30

a. 1 b. 2 c. 3 d. 4

6. Which of the two switches S_1 and S_2 shown in Fig. 7.31 will produce short-circuiting?

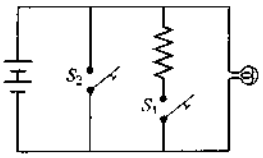


Fig. 7.31

a. S_1 b. S_2
c. Both S_1 and S_2 d. Neither S_1 nor S_2

7. Three similar light bulbs are connected to a constant voltage d.c. supply as shown in Fig. 7.32. Each bulb operates at normal brightness and the ammeter (of negligible resistance) registers a steady current.

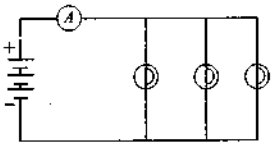


Fig. 7.32

The filament of one of the bulbs breaks. What happens to the ammeter reading and to the brightness of the remaining bulbs?

	Ammeter reading	*Bulb brightness*
a.	increases	increases
b.	increases	unchanged
c.	unchanged	unchanged
d.	decreases	unchanged

8. The circuit shown in Fig. 7.33 contains a battery, a rheostat and two identical lamps. What will happen to the brightness of the lamps if the resistance of the rheostat is increased?

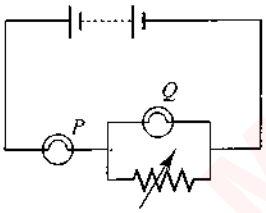


Fig. 7.33

	Lamp P	*Lamp Q*
a.	Less bright	Brighter
b.	Less brighter	Less brighter
c.	Brighter	Less brighter
d.	No change	Brighter

9. A cell of internal resistances r is concocted to a load of resistance R. Energy is dissipated in the load, but some thermal energy is also wasted in the cell. The efficiency of such an arrangement is found from the expression

$$\frac{\text{Energy dissipated in the load}}{\text{Energy dissipated in the complete circuit}}$$

Which of the following gives the efficiency in this case?

a. $\frac{r}{R}$ **b.** $\frac{R}{r}$

c. $\frac{r}{R+r}$ **d.** $\frac{R}{R+r}$

10. Two identical batteries each of e.m.f. $E = 2$ V and internal resistance $r = 1\ \Omega$ are available to produce heat in an external circuit. What is the maximum rate of production of heat that can be obtained in the external circuit?

a. 1 W **b.** 2 W **c.** 4 W **d.** 8 W

11. Two similar head light lamps are connected in parallel to each other. Together, they consume 48 W from a 6 V battery. What is the resistance of each filament?

a. 6 Ω **b.** 4 Ω
c. 3.0 Ω **d.** 1.5 Ω

12. Two electric bulbs, rated for the same voltage, have powers of 200 and 100 W, respectively. If their resistances are r_1 and r_2, respectively, then

a. $r_1 = 2r_2$ **b.** $r_2 = 2r_1$
c. $r_2 = 4r_1$ **d.** $r_1 = 4r_2$

13. If the current in electric bulb decreases by 0.5% the power in the bulb decreases by approximately

a. 1% **b.** 2%
c. 0.5% **d.** 0.25%

14. An electric bulb rated for 500 W at 100 V is used in a circuit having a 200 V supply. The resistance R that must be put in series with the bulb, so that the bulb draws 500 W, is

a. 18 Ω **b.** 20 Ω
c. 40 Ω **d.** 700 Ω

15. A 1°C rise in temperature is observed in a conductor by passing a certain current. If the current is doubled, then the rise in temperature is approximately

a. 2.5°C **b.** 4°C
c. 2°C **d.** 1°C

16. Two electric bulbs have tungsten filament of same length. If one of them gives 60 W and the other 100 W, then

a. 100 W bulb has thicker filament
b. 60 W bulb has thicker filament
c. both filaments are of same thickness
d. it is not possible to get different wattages unless the lengths are different

17. n identical light bulbs, each designed to draw p power from a certain voltage supply, are joined in series across that supply. The total power which they will draw is

a. nP **b.** P
c. P/n **d.** P/n^2

18. How many calories of heat will be approximately developed in a 210 W electric bulb in 5 min?

a. 15,000 **b.** 1,050
c. 63,000 **d.** 80,000

19. A constant voltage is applied between the two ends of a metallic wire. If both the length and the radius of the wire are doubled, the rate of heat developed in the wire will

a. be halved **b.** be doubled
c. remain the same **d.** be quadrupled

20. The power rating of an electric motor which draws a current of 3.75 A, when operated at 200 V, is nearly

a. 54 W **b.** 1 hp
c. 500 W **d.** 750 hp

21. A cable of resistance 10 Ω carries electric power from a generator producing 250 kW at 10,000 V. The current in the cable is

a. 1,000 A **b.** 250 A
c. 100 A **d.** 25 A

22. In the previous problem, the power lost in the cable during transmission is

a. 3.15 kW **b.** 12.5 kW
c. 6.25 kW **d.** 25 kW

23. The heat generated through 4 and 9 Ω resistances separately, when a capacitor of 100 μF capacity charged to 200 V is discharged one by one, will be

a. 2 and 8 J, respectively
b. 8 and 2 J, respectively
c. 2 and 4 J, respectively
d. 2 and 2 J, respectively

24. If the length of the filament of a heater is reduced by 10%, the power of the heater will
a. increase by about 9% b. increase by about 11%
c. increase by about 19% d. decrease by about 10%

25. A 2 kW heater used for 1 h every day consumes the following electrical energy in 30 days
a. 60 units b. 120 units
c. 15 units d. none of the above

26. Two cells, each of e.m.f. E and internal resistance r, are connected in parallel across a resistor R. The power delivered to the resistor is maximum if R is equal to
a. $r/2$ b. r
c. $2r$ d. 0

27. A constant voltage is applied between the two ends of a uniform metallic wire. Some heat is developed in it. The heat developed is doubled if
a. both the length and radius of the wire are halved
b. both the length and radius of the wire are doubled
c. the radius of the wire is doubled
d. the length of the wire is doubled

28. A given resistor cannot carry currents exceeding 20 A, without exceeding its maximum power dissipation ratings. By forced air cooling suppose that we increase the rate at which heat can be carried by a factor of 2. Now the maximum current that the resistor can carry is
a. 10 A b. $20\sqrt{2}$ A
c. $30\sqrt{2}$ A d. 40 A

29. A wire when connected to 220 V mains supply has power dissipation P_1. Now, the wire is cut into two equal pieces which are connected in parallel to the same supply. Power dissipation in this case is P_2. Then $P_2 : P_1$ is **(AIEEE, 2002)**
a. 1 b. 4
c. 2 d. 3

30. A 220 V, 1,000 W bulb is connected across a 110 V main supply. The power consumed will be **(AIEEE, 2003)**
a. 1000 W b. 750 W
c. 500 W d. 250 W

31. A heater coil is cut into two equal parts and only one part is now used in the heater. The heat generated will now be **(AIEEE, 2005)**
a. halved b. one-fourth
c. four times d. doubled

32. Three 10 Ω, 2 W resistors are connected as in Fig. 7.34. The maximum possible voltage between points A and B without exceeding the power dissipation limits of any of the resistors is

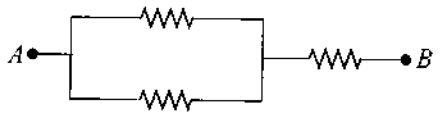


Fig. 7.34

a. $5\sqrt{3}$ V b. $3\sqrt{5}$ V
c. 15 V d. $\frac{5}{3}$ V

33. A torch bulb rated 4.5 W, 1.5 V is connected as shown in Fig. 7.35. The e.m.f. of the cell needed to make the bulb glow at full intensity is

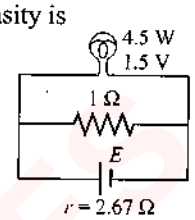


Fig. 7.35

a. 4.5 V b. 1.5 V c. 2.67 V d. 13.5 V

34. A heater is designed to operate with a power of 1000 W on a line of 100 V. It is connected in combination with resistance of 10 Ω and a resistance R to line of 100 V. The value of R so that the entire circuit operates with a power of 625 W is

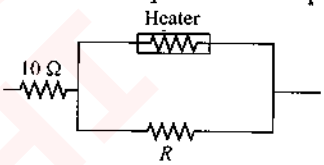


Fig. 7.36

a. 5 Ω b. 10 Ω c. 15 Ω d. 20 Ω

35. The main supply voltage to a room is 120 V. The resistance of the lead wires is 6 Ω. A 60 W bulb is already giving light. What is the decrease in voltage across the bulb when a 240 W heater is switched on?
a. No change b. 10 V
c. 20 V d. More than 10 V

36. Figure 7.37 shows a network of three resistances. When some potential difference is applied across the network, thermal powers dissipated by A, B and C are in the ratio

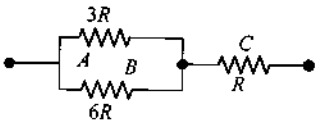


Fig. 7.37

a. 2 : 3 : 4 b. 2 : 4 : 3
c. 4 : 2 : 3 d. 3 : 2 : 4

37. Resistors P, Q and R in the circuit have equal resistances. If the battery is supplying a total power of 12 W, what is the power dissipated as heat in resistor R?

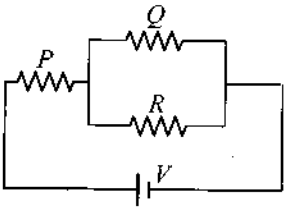


Fig. 7.38

a. 2 W b. 6 W
c. 3 W d. 8 W

38. Three bulbs of 40, 60 and 100 W are connected in series with a 240 V source.

a. The potential difference will be maximum across the 40 W bulb.
b. The current will be maximum in 100 W bulb.
c. The resistance of the 40 W bulb is minimum.
d. The current through the 60 W bulb will be 0.1 A.

39. In the circuit shown in Fig. 7.39, the heat produced in the 5 Ω resistor due to the current flowing through it is 10 cal s^{-1}. The heat generated in the 4 Ω resistor is

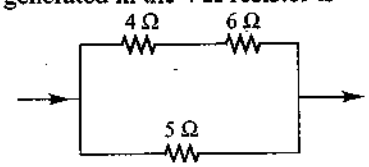


Fig. 7.39

a. 1 cal s^{-1} b. 2 cal s^{-1}
c. 3 cal s^{-1} d. 4 cal s^{-1}

40. A battery of internal resistance 4 Ω is connected to the network of resistances as shown in Fig. 7.40. In order that the maximum power can be delivered to the network, the value of R in Ω should be

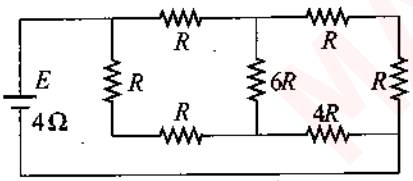


Fig. 7.40

a. $\frac{4}{9}$ b. 2 c. $\frac{8}{3}$ d. 18

41. Four resistances carrying a current shown in Fig. 7.41 are immersed in a box containing ice at 0°C. How much ice must be put in the box every 10 min to keep the average quantity of ice in the box constant? Latent heat of ice is 80 cal g^{-1}.

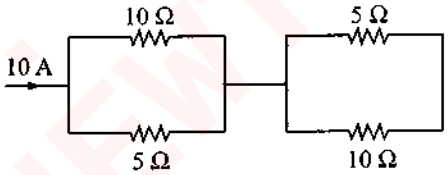


Fig. 7.41

a. 1.190 kg b. 3.20 kg
c. 4.2 kg d. 0.25 kg

42. The three resistances of equal values are arranged in the different combinations shown below. Arrange them in increasing order of power dissipation.

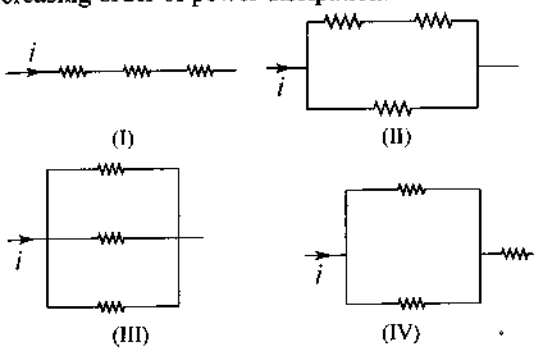


Fig. 7.42

a. III < II < IV < I b. II < III < IV < I
c. I < IV < III < II d. I < III < II < IV

43. An ideal gas is filled in a closed rigid and thermally insulated container. A coil of 100 Ω resistor carrying current 1 A for 5 min supplies heat to the gas. The change in internal energy of the gas is

a. 10 kJ b. 30 kJ c. 20 kJ d. 0 kJ

44. The resistance in which the maximum heat is produced is given by (Fig. 7.43)

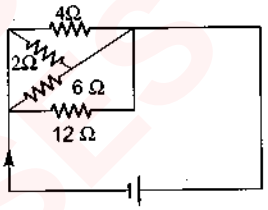


Fig. 7.43

a. 2 Ω b. 6 Ω c. 4 Ω d. 12 Ω

45. The resistance of hot tungsten filament is about 10 times the cold resistance. What will be the resistance of 100 W and 200 V lamp when not in use?

a. 14000 Ω b. 400 Ω c. 40 Ω d. 4 Ω

46. The resistance of the filament of a lamp increases with the increase in temperature. A lamp rated 100 W and 220 V is connected across 220 V power supply. If the voltage drops by 10%, then the power of the lamp will be

a. 90 W b. 81 W
c. between 90 and 100 W d. between 81 and 90 W

47. A wire of length L and three identical cells of negligible internal resistance are connected in series. Due to the current, the temperature of the wire is raised by ΔT in time t. A number N of similar cells is now connected in series with a wire of same material and cross section but of length $2L$. The temperature of the wire is raised by the same amount ΔT in the same time. The value of N is

a. 4 b. 6 c. 8 d. 9

48. An electric immersion heater of 1.08 kW is immersed in water. After it has reached a temperature of 100°C, how much time will be required to produce 100 g of steam?

a. 50 s b. 420 s c. 105 s d. 210 s

49. Two electric bulbs A and B are rated 60 and 100 W, respectively. If they are connected in parallel to the same source, then

a. both the bulbs draw the same current
b. bulb A draws more current than bulb B
c. bulb B draws more current than bulb A
d. currents drawn in the bulbs are in the ratio of their resistances

50. A 25 W–220 V bulb and a 100 W–220 V bulb are connected in series across a 220 V line; which electric bulb will glow more brightly?

a. 25 W bulb
b. 100 W bulb
c. Both will have equal incandescence
d. Neither will give light

51. Two identical heaters rated 220 V–1000 W are placed in series with each other across 220 V line; then the combined power is

a. 1000 W b. 2000 W
c. 500 W d. 4000 W

52. Figure 7.44 shows three similar lamps L_1, L_2 and L_3 connected across a power supply. If the lamp L_3 fuses, how will the light emitted by L_1 and L_2 change?

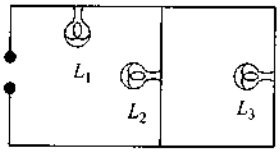


Fig. 7.44

a. No change
b. Brilliance of L_1 decreases and that of L_2 increases
c. Brilliance of both L_1 and L_2 increases
d. Brilliance of both L_1 and L_2 decreases

53. If a wire of resistance 20 Ω is covered with ice and a voltage of 210 V is applied across the wire, then the rate of melting of ice is
a. 8.85 gs^{-1} b. 1.92 gs^{-1}
c. 6.56 gs^{-1} d. none of these

54. A factory is served by a 220 V supply line. In a circuit protected by a fuse marked 10 A, the maximum number of 100 W lamps in parallel that can be turned on is
a. 11 b. 22
c. 33 d. 66

55. It takes 16 min to boil some water in an electric kettle. Due to some defect it becomes necessary to remove 10% turns of the heating coil of the kettle. After repairs, how much time will it take to boil the same mass of water?
a. 17.7 min b. 14.4 min
c. 20.9 min d. 13.7 min

56. An electric kettle (rated accurately at 2.5 kW) is used to heat 3 kg of water from 15°C to boiling point. It takes 9.5 min. Then the amount of heat that has been lost is
a. 3.5×10^5 J b. 7×10^5 J
c. 3.5×10^4 J d. 7×10^6 J

57. How many 60 W lamps may be safely run on a 230 V circuit fitted with a 5 A fuse?
a. 2 b. 19
c. 20 d. 4

58. If a given volume of water in a 220 V heater is boiled in 5 min, then how much time will it take for the same volume of water in a 110 V heater to be boiled?
a. 20 min b. 30 min
c. 25 min d. 40 min

59. The charge flowing through a resistance R varies with time t as $Q = at - bt^2$. The total heat produced in R is
a. $\frac{a^3R}{6b}$ b. $\frac{a^3R}{3b}$ c. $\frac{a^3R}{2b}$ d. $\frac{a^3R}{b}$

60. A 100 W bulb designed to operate on 100 V is to be connected across a 500 V source. Find the resistance to be put in sereis so that bulb consumes 100 W only.
a. 100 Ω b. 500 Ω
c. 400 Ω d. 300 Ω

Multiple Correct Answers Type

Solutions on page 7.25

1. Two electric bulbs rated 25 W, 220 V and 100 W, 220 V are connected in series across a 220 V voltage source. The 25 and 100 W bulbs now draw P_1 and P_2 powers, respectively.
a. $P_1 = 16$ W b. $P_1 = 4$ W
c. $P_2 = 16$ W d. $P_2 = 4$ W

2. A voltmeter and an ammeter are connected in series to an ideal cell of e.m.f. E. The voltmeter reading is V and the ammeter reading is I. Then
i. $V < E$
ii. the voltmeter resistance is V/I
iii. the potential difference across the ammeter is $E - V$
iv. voltmeter resistance plus ammeter resistance = E/I
Correct statements are
a. i and ii b. ii and iii
c. iii and iv d. all

3. In the circuit shown in Fig. 7.45,

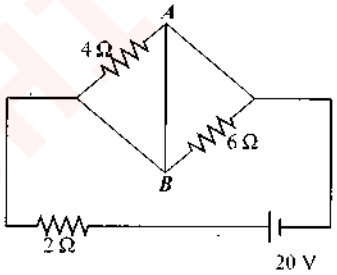


Fig. 7.45

a. power supplied by the battery is 200 W
b. current flowing in the circuit is 5 A
c. potential difference across the 4 Ω resistance is equal to the potential difference across the 6 Ω resistance
d. current in wire AB is zero

4. Two bulbs consume same energy when operated at 200 V and 300 V, respectively. When these bulbs are connected in series across a d.c. source of 500 V, then
a. ratio of potential difference across them is 3/2
b. ratio of potential difference across them is 4/9
c. ratio of power produced in them is 4/9
d. ratio of power produced in them is 2/3

5. An element with e.m.f. ε and internal resistance r is connected across an external resistance R. The maxium power in externl circuit is 9 W. The current flowing through the circuit in these conditions is 3 A. Then which of the following is/are correct ?
a. ε = 6 V b. $r = R$
c. $r = 1$ Ω d. $r = 3$ Ω

Assertion-Reasoning Type

Solutions on page 7.26

In the following questions, each question contains STATEMENT I (Assertion) and STATEMENT II (Reason). Each question has four choices a., b., c. and d. out of which **ONLY ONE** is correct.

a. Statement I is True, Statement II is True; Statement II is a correct explanation for Statement I.
b. Statement I is True, Statement II is True; Statement II is NOT a correct explanation for Statement I.
c. Statement I is True, Statement II is False.
d. Statement I is False, Statement II is True.

1. **Statement I:** The wires supplying current to an electric heater are not heated appreciably.
Statement II: Resistance of connecting wires is very small and $H \propto R$.
2. **Statement I:** If the current of a lamp increases by 20%, the percentage increase in the illumination of the lamp is 40%.
Statement II: Illumination of the lamp is directly proportional to the square of the current through the lamp.
3. **Statement I:** Heater wire must have high resistance than connecting wires and high metallic point.
Statement II: If resistance is high, the electrical conductivity will be less.
4. **Statement I:** In the circuit in Fig. 7.46, both cells are ideal and of fixed e.m.f., the resistor R_1 has fixed resistance and the resistance of resistor R_2 can be varied (but R_2 is always non-zero). Then the electric power delivered to resistor of the resistance R_1 is independent of the value of resistance R_2.

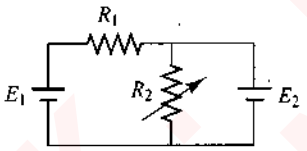


Fig. 7.46

Statement II: If potential difference across a fixed resistance is unchanged, the power delivered to the resistor remains constant.
5. **Statement I:** Since all the current coming to our house returns to powerhouse (as current travels in a closed loop), there is no need to pay the electricity bill.
Statement II: The electricity bill is paid for the power used, not for the current used.
6. **Statement I:** Internal resistance of a battery is drawn parallel to a battery in electrical circuit.
Statement II: Heat generated in a battery is due to internal resistance.

Comprehension Type

Solutions on page 7.26

For Problems 1–2

In Fig. 7.47 circuit section, AB absorbs energy at a rate of 50 W when a current i = 1.0 A passes through it in the indicated direction.

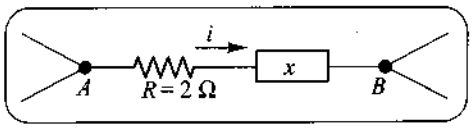


Fig. 7.47

1. What is the potential difference between A and B?
a. 10 V b. 50 V
c. 20 V d. 30 V
2. E.m.f. device X does not have internal resistance. What is its e.m.f.?
a. 24 V b. 32 V
c. 48 V d. 12 V

For Problems 3–4

A three-way light bulb has three brightness settings (low, medium, high) but only two filaments. The two filaments are arranged in three settings, when connected across a 120 V line and can dissipate 60, 120 and 180 W. Answer the following questions:

3. i. Higher resistance filament only working for 60 W
ii. Low resistance filament working for 120 W
iii. Low resistance filament working for 60 W
iv. High resistance filament working for 120 W
v. Low and high resistance filaments in parallel for 180 W
vi. Low and high resistance filament in series for 180 W
a. i, ii and v are correct
b. i, ii and vi are correct
c. iii, iv and v are correct
d. iii, iv and vi are correct
4. When the filament of higher resistance burns out, then intensity in
a. all three settings is 120 W
b. all three settings is 60 W
c. two settings is 60 W
d. two settings is 120 W

For Problems 5–7

In Fig. 7.48, each of the segments (e.g., AE, GM, etc.) has resistance r. A battery of e.m.f. V is connected between A and C. Internal resistance of the battery is negligible.

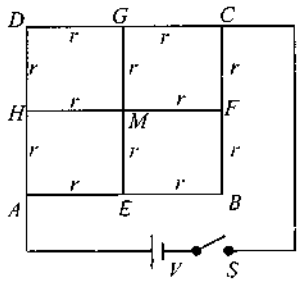


Fig. 7.48

5. What is the equivalent resistance of the system about A and C?
a. r b. $\frac{r}{2}$ c. $\frac{3r}{2}$ d. $2r$
6. Find the ratio of the power developed in segment AE to that in segment HM.
a. 1 b. 2 c. 3 d. 4
7. If a potentiometer circuit having potential gradient k is connected across the points H and C, find the balancing length shown by the potentiometer is

a. $\frac{v}{k}$ b. $\frac{2v}{3k}$

c. $\frac{3v}{2k}$ d. none of these

For Problems 8–10

Refer to Fig. 7.49.

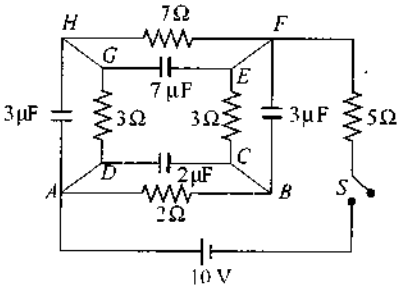


Fig. 7.49

8. At $t = 0$, the switch is closed. Just after closing the switch, find the current through the 5 Ω resistor.

a. $\frac{4}{5}$ A b. $\frac{2}{5}$ A c. $\frac{6}{5}$ A d. 2 A

9. Long time after closing the switch, find the current through the 5 Ω resistor.

a. $\frac{4}{5}$ A b. $\frac{2}{5}$ A c. $\frac{6}{5}$ A d. $\frac{8}{5}$ A

10. Now, the switch is opened after closing it for a long time. Find the total energy dissipated in the system.

a. 40.8 μJ b. 50.8 μJ

c. 40 μJ d. None of these

For Problems 11–13

All bulbs consume same power. The resistance of bulb 1 is 36 Ω. Answer the following questions:

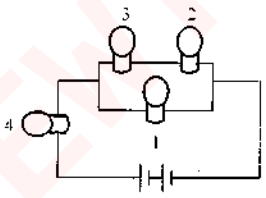


Fig. 7.50

11. What is the resistance of bulb 3?

a. 4 Ω b. 9 Ω c. 12 Ω d. 18 Ω

12. What is the resistance of bulb 4?

a. 4 Ω b. 9 Ω c. 12 Ω d. 18 Ω

13. What is the voltage output of the battery if the power of each bulb is 4 W?

a. 12 V b. 16 V

c. 24 V d. none of these

Matching Column Type

Solutions on page 7.28

Column I and Column II contain four entries each. Entries of column I are to be matched with some entries of column II. One or more than one entries of column I may have the matching with the same entries of column II and one entry of column I may have one or more than one matching with entries of column II.

1. In Fig. 7.51, the resistance R is variable, r is the internal resistance of battery of e.m.f. E.

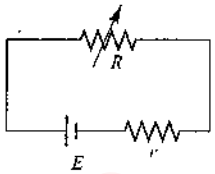


Fig. 7.51

Column I	Column II
i. Terminal potential difference across the cell to be maximum	a. $R > r$
ii. Power transferred to R is less than the maximum possible	b. $R < r$
iii. Power dissipated in the cell is maximum	c. $R = \infty$
iv. Fastest drift of ions in the electrolyte in the cell will be for	d. $R = 0$

2. Figure 7.52 shows a charging circuit of a capacitor. At $t = 0$, S is closed.

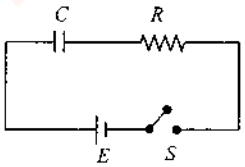


Fig. 7.52

Column I	Column II
i. When the charging rate of the capacitor is maximum, the current through R is	a. maximum
ii. When charge on the capacitor is maximum, then the current through R is	b. minimum but not zero
iii. When the power supplied by the battery is maximum, then charge on the capacitor is	c. zero
iv. The difference in the power supplied by battery and power consumed in R at $t = 0$ is	d. not equal to zero

3. For the circuit shown in Fig. 7.53, four cells are arranged. In Column I, the cell number is given, while in Column II, some statement related to cells are given. Match the entries of Column I with the entries of Column II.

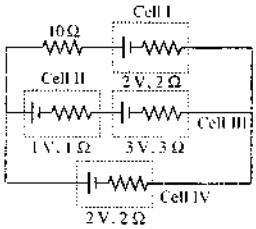


Fig. 7.53

Column I	Column II
i. Cell I	a. Chemical energy of cell is decreasing
ii. Cell II	b. Chemical energy of cell is increasing
iii. Cell III	c. Work done by cell is + ve
iv. Cell IV	d. Thermal energy developed in cell is +ve

Integer Answer Type

Solutions on page 7.28

1. Two circular rings of identical radii and resistance of 36 Ω each are placed in such a way that they cross each other's centre C_1 and C_2 as shown in the figure. Conducting joints are made at intersection points A and B of the rings. An ideal cell of e.m.f. 20 V is connected across A and B. Find the power delivered by cell (in 10^2 W).

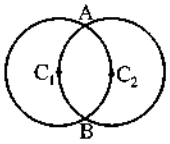


Fig. 7.54

2. Three identical resistors are connected in series. When a certain potential difference is applied across the combination, the total power dissipated is 27 W. How many times the power would be dissipated if the three resistors were connected in parallel across the same potential difference?
3. A heating coil is rated 100 W, 220 V. The coil is cut in half and two pieces are joined in parallel to the same source. Now what is the energy (in $\times 10^2$ J) liberated per second?
4. A dynamo develops 0.5 A at 6 V. Find the energy which it generates in 1 sec.
5. If in the circuit shown in the figure, power dissipation is 150 W, then find the value of R (in Ω).

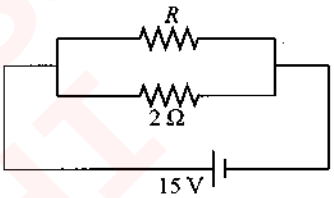


Fig. 7.55

ANSWERS AND SOLUTIONS

Subjective Type

1. The equivalent resistance between the points is

$$R_0 = R_1 + \frac{R_2 R_x}{R_2 + R_x}$$

Power generated by R_x is $I_x^2 R_x$

From Fig. 7.56, $$I = \frac{V}{R_1 + \dfrac{R_2 R_x}{R_2 + R_x}}$$

$$I_x = \left(\frac{V}{R_1 + \dfrac{R_2 R_x}{R_2 + R_x}}\right)\left(\frac{R_2}{R_2 + R_x}\right)$$

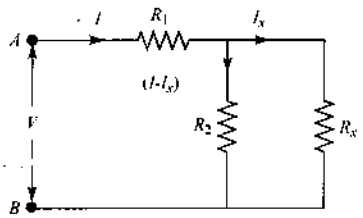


Fig. 7.56

$$P_x = I_x^2 R_x = \left[\frac{VR_2}{R_1R_2 + R_1R_x + R_2R_x}\right]^2 R_x$$

For the maximum value, $\dfrac{dP_x}{dR_x} = 0$

$$\Rightarrow \quad R_1R_2 + R_1R_x + R_2R_x = 2(R_1 + R_2)R_x$$

$$\Rightarrow \quad R_x = \frac{R_1R_2}{R_1 + R_2} = \frac{20 \times 30}{20 + 30} = 12\ \Omega$$

2. **a.** The resistance of an electric appliance is given by

$$R = \frac{V_S^2}{P_0} = \frac{(200)^2}{1000} = 40\ \Omega$$

b. The 'actual power' consumed by an electric appliance is given by

$$P = \left(\frac{V_A}{V_S}\right)^2 \times P_0 = \left(\frac{100}{200}\right)^2 \times 1000 = 250\text{ W}$$

c. The total electrical energy consumed by an electric appliance in a specified time is given by

$$E = \frac{\Sigma W_i h_i}{1000}\text{ kWh} = \frac{1000 \times (10 \times 30)}{1000} = 300\text{ kWh}$$

3. $P_1 = \varepsilon^2/R_1$, so $R_1 = \varepsilon^2/P_1$

$P_2 = \varepsilon^2/R_1$, so $R_2 = \varepsilon^2/P_2$

a. When the resistors are connected in parallel to the e.m.f., the voltage across each resistor and the power dissipated by each resistor are the same as if only one resistor was connected. $P_{tot} = P_1 + P_2$

b. When the resistors are connected in series, the equivalent resistance is $R_{eq} = R_1 + R_2$.

$$p_{tot} = \frac{\varepsilon^2}{R_1 + R_2} = \frac{\varepsilon^2}{\varepsilon^2/P_1 + \varepsilon^2/P_2} = \frac{P_1 P_2}{P_1 + P_2}$$

4. If R is the resistance of each bulb, then the equivalent resistance of N bulbs in parallel, $R_{eq} = R/N$. Therefore, current supplied by battery, $i = E/(R/N + r)$.

This current is equally divided among all the N bulbs, as potential drop across each bulb is the same.

So, power consumed by each bulb,

$$P = \left(\frac{i}{N}\right)^2 R = \frac{E^2 R}{N^2\left(\frac{R}{N} + r\right)^2} = \frac{E^2 R}{(Nr + R)^2} \qquad \text{(i)}$$

With $(N-1)$ bulbs, the power consumed by each bulb can be obtained by replacing N by $(N-1)$ in Eq. (i).

So, $$P' = \frac{E^2 R}{[(N-1)r + R]^2} \qquad \text{(ii)}$$

Now, percentage change in power consumption of each bulb is

$$\frac{P' - P}{P} \times 100 = \left[\frac{(rN + R)^2}{[r(N-1) + R]^2} - 1\right] \times 100$$

$$= \left[\frac{1}{\left[1 - \frac{r}{Nr + R}\right]^2} - 1\right] \times 100$$

As $r << (Nr + R)$, $\left(1 - \frac{r}{Nr + R}\right)^{-2} - 1 + \frac{2r}{Nr + R}$

Therefore, % change in power is

$$\frac{2r}{Nr + R} \times 100 = \frac{2 \times 0.1 \times 100}{200 \times 0.1 + 400} = 0.048\%$$

5. First recognize that if the 40 Ω resistor is safe, all the other resistors are also safe.

$$I^2 R = P \Rightarrow I^2(40\ \Omega) = 1\ \text{W} \Rightarrow I = 0.158\ \text{A}$$

Now, use series/parallel reduction to simplify the circuit. The upper parallel branch is 6.38 Ω and the lower one is 25 Ω. The series sum is now 126.38 Ω. Ohm's law gives

$$\varepsilon = (126.38\ \Omega)(0.158\ \text{A}) = 20\ \text{V}$$

6. a. First, do series/parallel reduction:

Now, apply Kirchhoff's laws and solve for e:

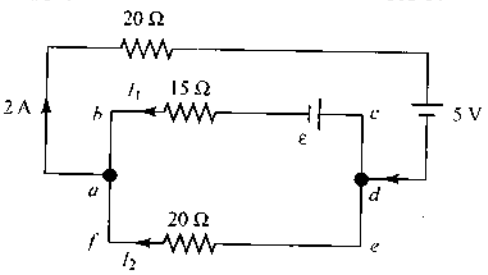


Fig. 7.57

$$\Delta V_{adefa} = 0: -(20\ \Omega)(2\ \text{A}) - 5\text{V} - (20\ \Omega) I_2 = 0$$

$$I_2 = -2.25\ \text{A}$$

$$I_1 + I_2 = 2\ \text{A} \Rightarrow I_1 = 2\ \text{A} - (-2.25\ \text{A}) = 4.25\ \text{A}$$

$$\Delta V_{adefa} = 0: \ (15\ \Omega)(4.25\ \text{A}) + \varepsilon - (20\ \Omega)(-2.25\ \text{A}) = 0$$

$\varepsilon = -108.75$ V; polarity should be reversed.

b. Parallel branch has a 10 Ω resistance.

$$\Delta V_{par} = RI = (10\ \Omega)(2\ \text{A}) = 20\ \text{V}$$

Current in upper part: $I = \frac{\Delta V}{R} = \frac{20\text{V}}{30\Omega} = \frac{2}{3}\ \text{A}$

$$Pt = U \Rightarrow I^2 Rt = U$$

$$\left(\frac{2}{3}\ \text{A}\right)^2 (10\ \Omega)t = 60\ \text{J} \quad \Rightarrow \quad t = 13.5\text{s}$$

7. As for an electric appliance, $R = V_S^2/W$, so for same specified voltage V_S

$$\frac{R_{25}}{R_{100}} = \frac{100}{25} = 4, \text{ i.e., } R_{25} = 4R \text{ with } R_{100} = R$$

Now, in series, potential divides in proportion to resistance.

So, $V_1 = \frac{R_1}{(R_1 + R_2)}\text{V}$, i.e., $V_{25} = \frac{4}{5} \times 440 = 352\text{V}$

And $V_2 = \frac{R_1}{(R_1 + R_2)}\text{V}$, i.e., $V_{100} = \frac{1}{5} \times 440 = 88\text{V}$

From this, it is clear that voltage across the 100 W bulb (= 88 V) is lesser than specified (220 V), while across the 25 W bulb (= 352 V) is greater than specified (220 V), so the 25 W bulb will fuse.

8. a. As bulbs B and C are in parallel voltage across B and C will be the same, i.e., $V_B = V_C$. Further, if R is the resistance of each bulb (as bulbs are identical), the resistance of bulbs B and C together $(=R/2)$ is in series with resistance R of bulb A and as in series, potential divides in proportion to resistance,

$$V_A = \frac{R}{R - 0.5R} = \frac{2}{3} V_S = \frac{2}{3} \times 120 = 80\text{V}$$

$$V_B = V_C = \frac{0.5R}{R + 0.5R}, V = \frac{1}{3} V_S = \frac{1}{3} \times 120 = 40\text{V}$$

[or $V_B = V_C = V - V_A = 120 - 80 = 40$ V]

b. As actual power consumed by a bulb is

$$P = \frac{V_A^2}{R} = \left[\frac{V_A}{V_S}\right]^2 \times W \qquad \left[\because \ R = \frac{V_S^2}{W}\right]$$

So, the total power consumption is

$$P = P_A + P_B + P_C = P_A + 2P_B = \frac{(4+2)60}{9} = 40\ \text{W}$$

Objective Type

1. d. $$R_{200} = \frac{200 \times 200}{100} = 400\ \Omega$$

So, $400 = R_0\,[1 + 0.005 \times 2000]$

$\therefore R_0 = \frac{400}{11}\,\Omega \approx 36\,\Omega$, Hence, current $I = \frac{200}{R_0} = 5.5$ A.

2. c. It is a case of weak current.

3. a. After disconnecting B_2 and B_3, the whole of potential difference will be across B_1. Hence, potential difference across B_1 increases, so its brightness increases.

4. a. Power $P = I^2R = \left(\frac{V}{R+3r}\right)^2 R \Rightarrow P \propto V^2$

Power ratio $= \left(\frac{V_1}{V_2}\right)^2 = \left(\frac{4.5}{1.5}\right)^2 = 3^2 = 9$

5. a. Maximum current flows through bulb 1.

6. b. When switch S_2 is closed, the whole of current shall flow through the connecting wire only which is supposed to have zero resistance.

7. d. Suppose V is the voltage of the supply and R is the resistance of each bulb.

Now, $R_p = R/3$ and current in ammeter, $I = V/R_p = 3V/R$, provided all three bulbs are working properly.

If one bulb has broken down, then

$$R_p = \frac{R}{2} \quad \text{and} \quad I = 2\frac{V}{R}$$

Therefore, current decreases and since current through each bulb is V/R the same as before, brightness of bulbs is not affected.

8. a. Consider two extreme cases: (i) When the resistance of the rheostat is zero, the current through Q is zero since Q is short-circuited. The circuit is then essentially a battery in series with lamp P. (ii) When the resistance of the rheostat is very large, almost no current flows through it. So, the currents through P and Q are almost equal. The circuit is essentially a battery in series with lamps P and Q.

9. d. Let current I flow through the circuit.

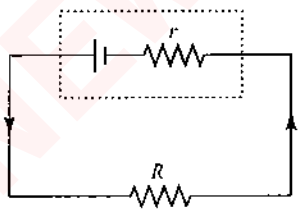


Fig. 7.58

Energy dissipated in load $= I^2R$.

Energy dissipated in the complete circuit $= I^2(r + R)$.

Therefore, the efficiency $= \frac{I^2R}{I^2(R+r)} = \frac{R}{R+r}$

10. b. Let both the batteries are connected in series. Power output is maximum when external resistance is 2 Ω. Current in the circuit = 4 V/4 Ω = 1 A and power in the external circuit $= (1)^2 \times 2 = 2$ W.

11. d. Power consumed by each lamp = 24 W.

Hence using $R = (V^2/P)$ we find

$$R = (36/24) = 1.5\ \Omega$$

12. b. $P = \frac{V^2}{R}$, $\frac{P_1}{P_2} = \frac{R_2}{R_1} \Rightarrow r_2 = 2r_1$

13. a. $P = i^2R$, $\frac{dP}{P} = 2\frac{dI}{I} = 2 \times 0.5\% = 1\%$

14. b. $P = VI$, $I = \frac{P}{V}$ or $I = \frac{500\text{ W}}{100\text{ V}} = 5$ A

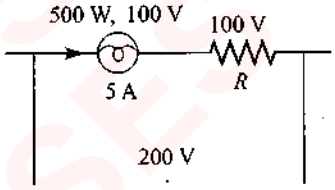


Fig. 7.59

Now, $5R = 100$ or $R = 20\ \Omega$

15. b. When current is doubled, heating effect increases four times. Now, $Q \propto \Delta T$. So, ΔT increases four times.

16. a. $P = V^2/R$. If P is more, R is less. $R = \rho\,\ell/a$. For less R, 'a' is more. So, the 100 W bulb has thicker element.

17. c. Total resistance $= \frac{nV^2}{P}$. Power $= \frac{V^2P}{nV^2} = \frac{P}{n}$.

18. a. Heat in calories $= \frac{210 \times 5 \times 60}{4.2} = 15{,}000$

19. b. $P = \frac{V^2}{R}$ or $P = \frac{V^2a}{\rho\ell} = \frac{V^2\pi r^2}{\rho\ell}$

20. b. $P = 200 \times 3.75$ W $= 750$ W ≈ 1 hp

21. d. $P = VI$; $I = \frac{P}{V} = \frac{250 \times 1000}{10000}$ A $= 25$ A

22. c. Power lost $= 25 \times 25 \times 10$ W $= 6250$ W $= 6.25$ kW

23. d. $U = \frac{1}{2}CV^2 = \frac{1}{2} \times 100 \times 10^{-6} \times 200 \times 200$ J $= 2$ J

24. b. $P = \frac{V^2}{\rho\frac{\ell}{a}} \Rightarrow P \propto \frac{1}{\ell}$, $P' \propto \frac{1}{\ell - \frac{10}{100}\ell}$

$$\frac{P'}{P} = \frac{10}{9} \Rightarrow \left(\frac{P'}{P} - 1\right) \times 100 = \left(\frac{10}{9} - 1\right) \times 100$$

$$\frac{P' - P}{P} \times 100 = \frac{100}{9} \approx 11$$

25. a. 2 kW × 30 h = 60 kWh = 60 units

26. a. The condition for delivering maximum power is that the external resistance is equal to the internal resistance.

27. b. Heat developed, $H = \frac{V^2}{R}t$

Heat developed will be doubled when R is halved.

Further, $R = \rho\,\ell/(\pi r^2)$

$H = V^2\pi r^2 t/\rho\ell$. So, heat produced will be doubled when both the length and radius of the wire are doubled.

28. b. Heat produced $\propto I^2$, initial heat produced $= k(20)^2$, final heat produced $= 2k(20)^2$. If final current is I', then

$$kI'^2 = 2\times k\times(20)^2 \Rightarrow I' = 20\sqrt{2}\text{ A}$$

29. b. $P = V^2/R$. R is reduced by a factor of 4. So, P is increased by a factor of 4.

30. d. $P = V^2/R$. If V is halved, P is reduced by a factor of 4. So, new power is (1000/4) W, i.e., 250 W.

31. d. $Q = \frac{V^2}{R}t$. When R is halved, then Q is doubled.

32. b. Power $= \frac{V^2}{R}$

or $$V = \sqrt{PR} = \sqrt{2\times 10} = \sqrt{20} = 2\sqrt{5}\text{ V}$$

Clearly, voltage across single resistor of 10 Ω cannot exceed $2\sqrt{5}$ V. Note that the resistance of parallel combination is half of 10 Ω. Thus, the maximum possible voltage between A and B is $3\sqrt{5}$ V.

33. d. Resistance of bulb $= \frac{1.5\times 1.5}{4.5}\,\Omega = 0.5\ \Omega$

Resistance of parallel combination, $R = \frac{1\times\frac{1}{2}}{1+\frac{1}{2}}\Omega = \frac{1}{3}\Omega$

Now, $r = \frac{E-V}{V}R \Rightarrow \frac{8}{3} = \frac{E-1.5}{1.5}\times\frac{1}{3} \Rightarrow E = 13.5\text{ V}$

34. c. Power of the heater $P = 1000$ W

Potential difference $V = 100$ V

Therefore, resistance $R_1 = \frac{V^2}{P} = \frac{100\times 100}{1000} = 10\ \Omega$

Now resistance of the circuit is

$$R_2 = 10 + \frac{10R}{10+R} = \frac{100+20R}{10+R}$$

Therefore, power $= \frac{V^2}{R_2} = 625$ W

$$R_2 = \frac{V^2}{625} \Rightarrow \frac{100+20R}{10+R} = \frac{625}{100\times 100} \Rightarrow R_2 = 15\,\Omega$$

35. d. $$R_{60} = \frac{120\times 120}{60}\,\Omega = 240\ \Omega$$

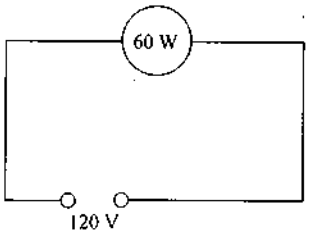


Fig. 7.60

$$\text{Current} = \frac{120}{240+6}\text{ A} = \frac{120}{246}\text{ A}$$

$$\text{Voltage across bulb} = \frac{120}{246}\times 240\text{ V} = 117.1\text{ V}$$

$$R_{240} = \frac{120\times 120}{240}\,\Omega = 60\ \Omega$$

$$\text{Resistance of parallel combination} = \frac{60\times 240}{60+240}\,\Omega = 48\ \Omega$$

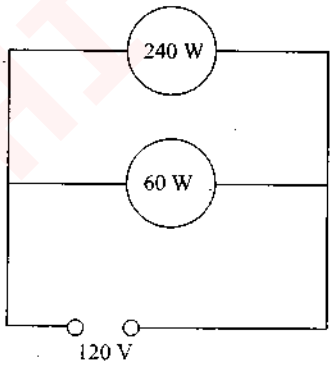


Fig. 7.61

Total resistance = (48 + 6) Ω = 54 Ω

Current $I = \frac{120}{54}$ A

Voltage across parallel combination is

$$\frac{120}{54}\times 48\text{ V} = 106.7\text{ V}$$

Change in voltage = (117.1 – 106.7) V = 10.4 V.

36. c. Let current flow from b to a as shown (Fig. 7.62).

Ratio is $\left(\frac{2}{3}I\right)^2 3R : \left(\frac{1}{3}I\right)^2 6R : I^2R$

or $\frac{4}{3} : \frac{2}{3} : 1$ or $4:2:3$.

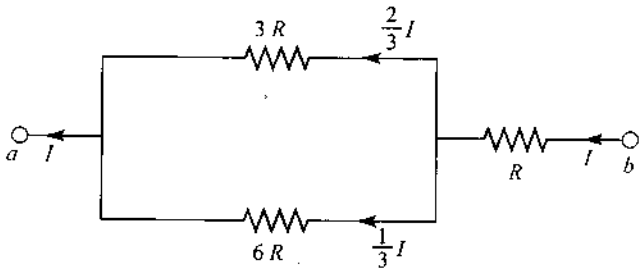


Fig. 7.62

37. a. Let resistors P, Q and R have resistance r. The effective resistance across the source is

$$R_{eff} = r + r \| r = r + \frac{(r)(r)}{r+r} = r + \frac{r}{2} = \frac{3r}{2}.$$

Current drawn from source is

$$I_s^2 R_{eff} = 12$$

$$\Rightarrow \quad I_s = \sqrt{\frac{12}{R_{eff}}} = \sqrt{\frac{8}{r}} \text{ A}.$$

Since Q and R have equal resistance r, each draws a current of I which is given by

$$I = \frac{1}{2} I_s = \sqrt{\frac{2}{r}} \text{ A}$$

Heat dissipation in R can now be determined and is given by

$$I^2 r = \left(\frac{2}{r}\right) r = 2 \text{ W}.$$

38. a. Since all the three resistors are in series the same current will flow through them. Their resistances are given by $R = V^2/P$.

$$R_{40} = \frac{240 \times 240}{40} = 1440\ \Omega$$

$$R_{60} = 960\ \Omega \quad \text{and} \quad R_{100} = 576\ \Omega$$

Total resistance $R = 2976\ \Omega$

$$I = \frac{240}{2976} = 0.0806 \text{ A}$$

potential difference across the 40 W bulb = $1440 \times 0.0806 = 116$ V

39. b. Let current in 5 Ω resistor be I_1 and in 4 Ω resistors be I_2, then $I_2/I_1 = 1/2$. The heat generated in the 5 Ω resistor is 10 cal s^{-1} = given $I_1^2 \times 5 = 10 \Rightarrow (2I_2)^2 \times 5 = 10 \Rightarrow I_2^2 = 1/2$

Heat in the 4 Ω resistor will be $(I_2)^2 \times 4 = 2$ cal s^{-1}.

40. b. For the maximum power, external resistance is equal to internal resistance. Therefore, $2R = 4$ or $R = 2\ \Omega$.

41. a. Net resistance of circuit, $R = \frac{10 \times 5}{10+5} + \frac{10 \times 5}{10+5} = \frac{20}{3}\Omega$

Heat generated in circuit per minute is

$$Q = I^2 Rt = (10)^2 \times \frac{20}{3} \times (10 \times 60) = 4 \times 10^5 \text{ J} = \frac{4 \times 10^5}{4.2 \times 10^3} \text{ kcal}$$

$$m = \frac{Q}{L} = \frac{4 \times 10^5}{4.2 \times 10^3 \times 80} = 1.19 \text{ kg}$$

42. a. $P_I = I^2(3R)$, $P_{II} = I^2\left(\frac{2R}{3}\right)$, $P_{III} = I^2\left(\frac{R}{3}\right)$

$$P_{IV} = I^2\left(\frac{3R}{2}\right)$$

$\therefore$ III < II < IV < I

43. b. The heat supplied under these conditions is the change in internal energy $Q = \Delta U$. The heat supplied $Q = I^2Rt$ $1 \times 1 \times 100 \times 5 \times 60 = 30000$ J = 30 kJ.

44. a. All resistances are in parallel.

So, for parallel combination $H = \frac{V^2}{R} t \Rightarrow H \propto \frac{1}{R}$

45. c. When hot, $R = \frac{V^2}{P} = \frac{200 \times 200}{100} = 400\ \Omega$

Hence, when cold, the resistance is 40 Ω.

46. d. Let the resistance of the lamp filament be R. Then $100 = (220)^2/R$. When the voltage drops, expected power is $P = (220 \times 0.9)^2/R'$.

Here, R' will be less than R, because now the rise in temperature will be less. Therefore, P is more than $(220 \times 0.9)^2/R = 81$ W. But it will not be 90% of the earlier value, because fall in temperature is small. Hence, option (d) is correct.

47. b. In the first case,

$$\frac{(3E)^2}{R} t = (m) s \Delta T \quad \text{(i)}$$

When the length of the wire is doubled, resistance and mass both are doubled. Therefore, in the second case,

$$\frac{(NE)^2}{2R} t = (2m) s \Delta T \quad \text{(ii)}$$

Dividing Eq. (ii) by Eq. (i), we get $N = 6$.

48. d. L is the latent heat of vaporization of water, the heat required for producing 1 g of steam,

$L = 540$ cal $= 540 \times 4.2 = 2268$ J.

Energy supplied = 1080 Js^{-1}.

Time required to boil 100 g of water

$$t = 540 \times 4.2 \times 100/1080 = 210 \text{ s}$$

49. c. Given: $P_A = 60$ W; $P_B = 100$ W

We know that current flowing through the bulb,

$$I = \frac{P}{V}$$

We also know that as both the bulbs are connected in parallel, therefore potential difference (V) across both the bulbs is the same. Thus, $I \propto P$.

Since the power of bulb B is greater than that of bulb A, therefore bulb B draws more current than bulb A.

50. a. As both bulbs are in series, current through them will be the same. But resistance of 25 W bulb is more. Hence from the relation $P = I^2R$, the 25 W bulb will glow more brightly.

51. c. Let the resistance of the two heaters be denoted by R_1 and R_2, respectively. Then

$$R_1 = \left(\frac{V^2}{P_1}\right) \text{ and } R_2 = \left(\frac{V^2}{P_2}\right)$$

If the resistance of the series combination is denoted by R_s and the corresponding power by R_s, then $R_s = R_1 + R_2$.

or $$\frac{V^2}{P_S} = \frac{V^2}{P_1} + \frac{V^2}{P_2}$$

or $$P_s = \frac{P_1 P_2}{P_1 + P_2} = \frac{1000 \times 1000}{2000} = 500 \text{ W}$$

52. b. Let R be the resistance of each lamp. If E is the applied e.m.f., then the current in the circuit I_1 is given by

$$I_1 = \frac{E}{R + (R/2)} = 2E/3R$$

Current flowing through L_2 or L_3 is

$$\frac{1}{2}\left[\frac{2E}{3R}\right] = \frac{E}{3R}$$

When L_3 is fused, the whole current flows through L_1 and L_2. Thus $I_2 = \left(\frac{E}{2R}\right)$.

So, current through L_1 decreases and the current through L_2 increases.

53. c. The rate of heat developed is given by

$$\frac{V^2}{R} = \frac{(210)^2}{20} = 2205 \text{ Js}^{-1} = \frac{2205}{4.2} \text{ cal s}^{-1}$$

Let m be the amount of ice melting per sec and, then

$$m \times 80 = \frac{2205}{4.2} \quad \text{or} \quad m = 6.56 \text{ gs}^{-1}$$

54. b. Current required by each bulb,

$$I = \frac{P}{V} = \frac{100}{220} \text{ A}$$

If n bulbs are joined in parallel, then

$$nI = I_{fuse} \quad \text{or} \quad n \times \frac{100}{220} = 10 \quad \text{or} \quad n = 22$$

55. b. If N be the initial number of turns in the coil and r be the radius of coil, then its resistance,

$$R = \rho\frac{L}{A} = \rho\frac{N2\pi r}{A} \quad \text{or} \quad \frac{V^2 tA}{4.2\,\rho N 2\pi r} = Q = msd\theta$$

or $$\frac{t}{n} = \frac{ms\,d\theta \times 4.2\,\rho 2\pi r}{V^2 A} = \text{constant}$$

or $$\frac{t_1/N_1}{t_2/N_2} = 1 \text{ or } t_2 = \frac{N_2}{N_1}\,t_1 = \left(\frac{\frac{9}{10}N_1}{N_1}\right)t_1 = \frac{9}{10} \times 16 \text{ min}$$

56. a. Energy consumed in 9.5 min is

$$2.5 \times 1000 \times 9.5 \times 60 \text{ J} = 1425000 \text{ J}$$

Heat usefully consumed is

$$3 \times 4.2 \times 1000 \times (100 - 15) \text{ J} = 1071000 \text{ J}$$

Loss $= 3.54 \times 10^5$ J

57. b. Watt = volt × ampere

$\Rightarrow$ $60 = 230 \times I$ or $I = (6/23)$ A

If n lamps are used in parallel, each allowing 6/23 A, then total current, $n \times (6/23) \le 5$

or $n \le 19.1$ or $n = 19$

58. a. $H = V^2t/R$. When voltage is halved, the heat becomes one-fourth. Hence, time taken to heat the water becomes four times.

59. a. $Q = at - bt^2 \Rightarrow i = dQ/dt = a - 2bt$

$i = 0$ for $t = t_0 = a/2b$, i.e., current flows from $t = 0$ to $t = t_0$.

The heat produced $= \int_0^{t_0} i^2 R\,dt = a^3R/6b$

60. c. Resistance of bulb: $R_b = \frac{V^2}{P} = \frac{(100)^2}{100} = 100\ \Omega$

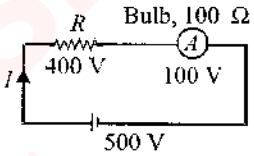


Fig. 7.63

If bulb consumes 100 W, then p.d. across the bulb should be 100 V. So $I = 100/100 = 1$ A. And p.d. across $R = 500 - 100 = 400$ V.

So $R = \frac{400}{I} = \frac{400}{1} = 400\ \Omega$

Multiple Correct Answers Type

1. a., d. Let $V = 200$ V; R_1 and R_2 = resistance of the 25 W and 100 W bulbs.

$P_1 = 25 = V^2/R_1$ or $R_1 = V^2/25$ and $R_2 = V^2/100$

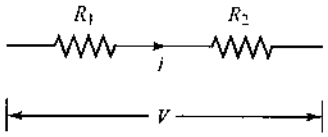


Fig. 7.64

When the bulbs are joined in series, the current is

$$I = \frac{V}{R_1 + R_2}$$

Power in the 25 W bulb $= R_1I^2$ and in the 100 W bulb $= R_2I^2$.

2. a., b., c., d. Treat all voltmeters and ammeters as resistances. Draw the circuit and find the currents and potential differences for each section.

3. a., c. Resistors 4 Ω and 6 Ω are short-circuited. Therefore, no current will flow through these resistances. Current passing through the battery is $I = (20/2) = 10$ A.
This is also the current passing in wire AB from B to A.
Power supplied by the battery is

$$P = EI = (20)(10) = 200 \text{ W}$$

Potential difference across the 4 Ω resistance
= potential difference across the 6 Ω resistance

4. b., c.

$P = \frac{V^2}{R}$, therefore $R = \frac{V^2}{P}$ or $R \propto V^2$, i.e., $\frac{R_1}{R_2} = \left(\frac{200}{300}\right)^2 = \frac{4}{9}$

When connected in series, potential drop is in the ratio of their resistances. So,

$$\frac{V_1}{V_2} = \frac{R_1}{R_2} = \frac{4}{9}. \text{ Now, } P = I^2 R$$

or $P \propto R$ (in series I is the same)

or $$\frac{P_1}{P_2} = \frac{R_1}{R_2} = \frac{4}{9}$$

5. a.,b.,c. Current through the circuit $I = \varepsilon/(r + R)$

Power dissipated in external resistance is

$$P = I^2 R = \frac{\varepsilon^2}{(r+R)^2} R$$

We know that power dissipated is maximum when $r = R$

$$P_{max} = \frac{\varepsilon^2 r}{(2r)^2} = \frac{\varepsilon^2}{4r} = 9 \text{ W}, \quad i = \frac{\varepsilon}{2r} = 3 \text{ A}$$

Solve to get, $\varepsilon = 6$ V and $r = 1\ \Omega$.

Assertion-Reasoning Type

1. **a.** Resistance of the connecting wires is much smaller than the electric appliances to which current is supplied by the wires.
2. **d.** $H \propto I^2 \Rightarrow \frac{H_2}{H_1} = \left(\frac{I_2}{I_1}\right)^2 = \left(\frac{1.2 I_1}{I_1}\right)^2 = 1.44$

   There is 44% increase in illumination.
3. **b.** Heater wire must have high resistance than connecting wires so that most of the potential drops across the heater wire. Because in series the more the resistances, the more the potential difference. Obviously, it should have high melting point.
4. **a.** The potential difference across resistance R_1 is always $|E_1 - E_2|$. Hence, assertion and reason are true and the reason is the correct explanation of the assertion.
5. **d.** Power used $= i^2 R$

   Hence, power is consumed, not the current.
6. **d.** Internal resistance is drawn in series with battery.

Comprehension Type

For Problems 1–2

1. b., 2. c.

Sol. As section AB consumes 50 W at 1 A and $P = VI$,

$$V = V_A - V_B = \frac{P}{I} = \frac{50\ W}{1\ A} = 50 \text{ V}$$

As R and C are in series

$$V = V_R + V_C, \quad \text{i.e.,} \quad V_C = V - V_R$$

But here $V = 50$ V and $V_R = IR = 1 \times 2 = 2$ V

So, $V_C = 50 - 2 = 48$ V

Now as the element C absorbs energy and has no resistance, it is a source of e.m.f. with zero internal resistance (i.e., ideal battery).

So e.m.f., $E = V + Ir = 48 + 1 \times 0 = 48$ V

and as in charging, positive and negative terminals of the charger are connected to the positive and negative terminals of the battery, respectively, B is connected to the negative terminal of element C.

For Problems 3–4

3. a., 4. d.

Sol. 60 W bulb will be of higher resistance. If only switch '1' is closed, 120 W bulb will glow. If only switch '2' is closed, 60 W bulb will glow. If both the switches are closed, both bulbs will glow with total illumination of 180 W.

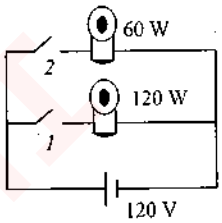


Fig. 7.65

When the filament of higher resistance (of 60 W bulb) burns out, then for two settings (either '1' is closed or both are closed) power dissipated will be 120 W.

For Problems 5–7

5. c., 6. d., 7. b.

Sol. In loop $EBFME$:

$$V_E - ri_1 - ri_1 + r\left(\frac{i}{2} - i_1\right) + r\left(\frac{i}{2} - i_1\right) = V_E$$

$$\Rightarrow \quad 2i_1 = 2\left(\frac{i}{2} - i_1\right) \Rightarrow i_1 = \frac{i}{4}$$

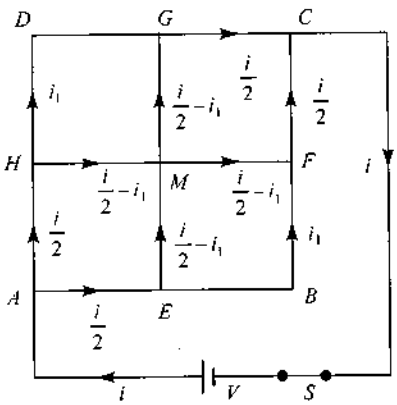


Fig. 7.66

Loop $AEBCSA$: $V_A - r\frac{i}{2} - ri_1 - ri_1 - r\frac{i}{2} + V = V_A$

$$\Rightarrow \quad V = ri + 2ri_1 \quad \Rightarrow \quad V = ri + 2r\frac{i}{4}$$

$$\Rightarrow \quad V = \frac{3ri}{2},\ R_{eq} = \frac{V}{i} = \frac{3r}{2}$$

$$\frac{P_1}{P_2} = \frac{\left(\frac{i}{2}\right)^2 r}{\left(\frac{i}{2} - i_1\right)^2 r} = \frac{i^2}{(i - 2i_1)^2} = 4$$

Let ℓ be the balancing length, then

$$k\ell = V_B - V_C$$

$$\Rightarrow \quad k\ell = \left(\frac{i}{2} - i_1\right)r + \left(\frac{i}{2} - i_1\right)r + \frac{i}{2}r$$

$$\Rightarrow \quad k\ell = \left(\frac{3i}{2} - 2i_1\right)r \quad \Rightarrow \quad k\ell = ir$$

$$\Rightarrow \quad \ell = \frac{2V}{3k} \quad \Rightarrow \quad \ell = \frac{2V}{3k}$$

For Problems 8–10

8. d., 9. c., 10. a.

Sol. Initially, when all the capacitors are uncharged, they will act as conducting wires. Hence, all the resistances (except 5 Ω) will be short-circuited. So, no resistance occurs between A and F. So, current through the 5 Ω resistor will be (10/5) A = 2 A.

In steady state, distribution of current is shown.

Loop $ABCEFMA$: $10 = 2I_1 + 3I_1 + 5(I_1 + I_2)$

or $10I_1 + 5I_2 = 10$ (i)

Loop $ADGHFMA$: $10 = 3I_2 + 7I_2 + 5(I_1 + I_2)$

or $5I_1 + 15I_2 = 10$ (ii)

From Eqs. (i) and (ii), $I_1 = \frac{4}{5}$ A, $I_2 = \frac{2}{5}$ A

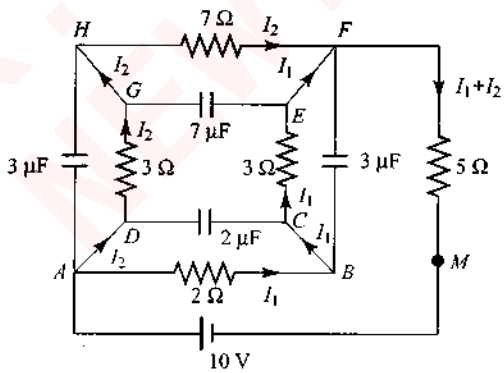


Fig. 7.67

Hence, current through the 5 Ω resistor is $I_1 + I_2 = (6/5)$ A.

Potential difference across the 2 μF capacitor is

$$2I_1 = 2 \times \frac{4}{5} = \frac{8}{5}\text{ V}$$

Energy in it is given by $U_1 = \frac{1}{2} \times 2\left(\frac{8}{5}\right)^2 = (64/25)$ μJ.

Potential difference across the 3 μF capacitor is

$$3I_1 = 3 \times \frac{4}{5} = \frac{12}{5}\text{ V}$$

Energy in it is given by $U_2 = \frac{1}{2} \times 3\left(\frac{12}{5}\right)^2 = (216/25)$ μJ.

Potential difference across the 3 μF capacitor is

$$3I_2 = 3 \times \frac{2}{5} = \frac{6}{5}\text{ V}$$

Energy in it is given by $U_3 = \frac{1}{2} \times 3\left(\frac{6}{5}\right)^2 = (54/25)$ μJ.

Potential difference across the 7 μF capacitor is

$$7I_2 = 7 \times \frac{2}{5} = \frac{14}{5}\text{ V}$$

Energy in it is given by $U_4 = \frac{1}{2} \times 7\left(\frac{14}{5}\right)^2 = (686/25)$ μJ.

This whole of the energy stored in all capacitors will be dissipated after the switch is opened. So the energy dissipated is $U_1 + U_2 + U_3 + U_4 = 40.8$ μJ.

For Problems 11–13

11. b., 12. a., 13. b.

Sol. $i_2 = i_3 = I_b$, $V_1 = (V_2 + V_3) = V$, and $P_2 = P_3$

$\Rightarrow$ $R_2 = R_3$ and $V_2 = V_3 = V/2$

$P_1 = P_2 = P_3 = P_4$ (given)

$$P_1 = \frac{V^2}{36};\quad P_3 = \frac{(V/2)^2}{R_3}$$

As $P_1 = P_3$, so $R_3 = 9\ \Omega$

Also $R_3 = R_2 = 9\ \Omega$

$I_a = \frac{I}{3}$ and $I_4 = I$, $P_1 = P_4 = 4$ W

$$\left(\frac{I}{3}\right)^2 R_1 = (I)^2 R_4 = 4\text{ W} \Rightarrow \frac{R_1}{9} = R_4 \Rightarrow R_4 \Rightarrow 4\ \Omega$$

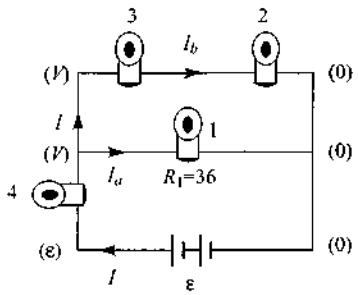


Fig. 7.68

Also $I = 1$ A

$R_{eq} = 16\ \Omega$ and $I = 1$ A, $\varepsilon = 16$ V

Matching Column Type

1. i. → c., ii. → a.,b.,c.,d., iii. → d., iv. → d.

Sol. $I = \dfrac{E}{R+R}$

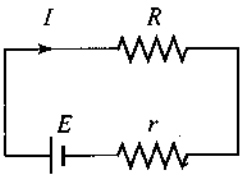


Fig. 7.69

Terminal potential difference across the cell is $V = E - Ir$. This is maximum if $I = 0$, and this is possible if $R = \infty$. Hence, i. → c.

Power transferred to R is maximum if $R = r$. In all other cases it will be somewhat less. Hence, ii. → a.,b.,c.,d.

Power dissipated in cell = EI. This is maximum if I is maximum, for this $R = 0$. Hence, iii. → d.

Also if I is maximum, there will be the fastest drift of ions in electrolyte in the cell. Hence, iv. → d.

2. i. → a.,d., ii. → c., iii. → c., iv. → c.

Sol. Charging rate means current $\left(\dfrac{dq}{dt} = I\right)$. So, if charging rate is maximum, then current is also maximum. Hence, i. → a, d.

Charge on the capacitor is maximum in steady state. In steady state current becomes zero. Hence, ii. → c.

Power supplied by the battery is maximum initially because current is maximum initially. At this time, charge on the capacitor is zero. Hence, iii. → c.

Difference in the power supplied by battery and power consumed in R is $EI - I^2R = EI - I(IR) = EI - IE = 0$

Hence iv. → c.

3. i. → b.,d., ii. → a.,c.,d., iii. → a.,c.,d., iv. → b.,d.

Sol. We have $I_1 = -\dfrac{1}{20}$ A, $I_2 = \dfrac{7}{20}$ A, $I_3 = -\dfrac{6}{20}$ A.

In each cell, thermal energy will be dissipated due to internal resistance whether the chemical energy of the cell is increasing or decreasing.

- **i.** Cell I is getting charged, hence its chemical energy increases.
- **ii.** Cells II and III both are getting discharged, hence their chemical energies are decreasing. So, work done by both of them is positive.
- **iv.** Cell IV is getting charged, hence its chemical energy increases.

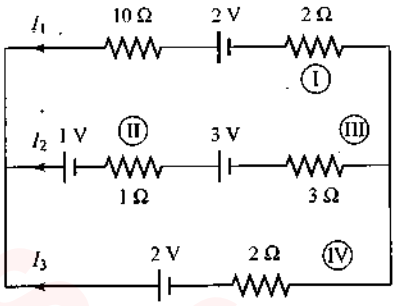


Fig. 7.70

Integer Answer Type

1. (1) The resistance of four sections are shown in the figure.

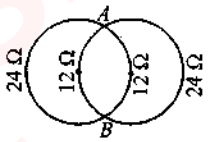


Fig. 7.71

Hence, the equivalent resistance R across AB is

$$\frac{1}{R} = \frac{1}{24} + \frac{1}{12} + \frac{1}{12} + \frac{1}{24} \Rightarrow R = 4\ \Omega$$

Power = $V^2/R = 20^2/4 = 100$ W

2. (9) For three identical resistors in series, $P_s = V^2/3R$

If they are now in parallel over the same voltage,

$$P_p = \frac{V^2}{R_{eq}} = \frac{V^2}{R/3} = \frac{9V^2}{3R} = 9P_s = 9 \times (27\ \text{W})$$

3. (4) Let resistance of heating coil be R, then

$$100 = \frac{220^2}{R} \Rightarrow R = \frac{(220)^2}{100}\ \Omega$$

Resistance of each cut part = $R/2$

Now, power dissipated $= P = 2\left(\dfrac{(220)^2}{R/2}\right)$

$$\Rightarrow \quad P = 4\frac{(220)^2}{R} = 4 \times 100 = 400\ \text{W}$$

4. (3) $R = \dfrac{6}{0.5} = 12\ \Omega$

$$H = I^2Rt = (0.5)^2 \times 12 \times 1 = 3\ \text{J}$$

5. (6) $150 = \dfrac{15 \times 15}{R_{eq}}$ or $R_{eq} = \dfrac{15 \times 15}{150}\ \Omega = \dfrac{3}{2}\ \Omega$

Now, $\dfrac{2R}{2+R} = \dfrac{3}{2} \Rightarrow R = 6\ \Omega$

APPENDIX
A2

Miscellaneous Assignments and Archives on Chapters 5–7

R. K. MALIK'S NEWTON CLASSES RANCHI

EXERCISES

Objective Type

Solutions on page A2.32

1. A source of constant potential difference is connected across a conductor having irregular cross section as shown in Fig. A2.1.

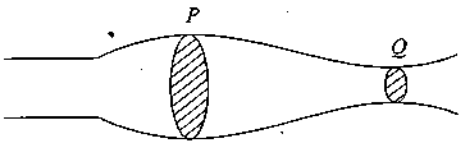


Fig. A2.1

a. Electric field intensity at P is greater than that at Q
b. Rate of electrons crossing per unit area of cross section at P is less than that at Q
c. The rate of generation of heat per unit length at P is greater than that at Q
d. Mean kinetic energy of free electrons at P is greater than that at Q

2. Current–voltage characteristics of two elements A and B are as shown in Fig. A2.2 (a) and (b)

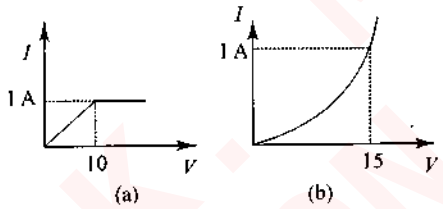
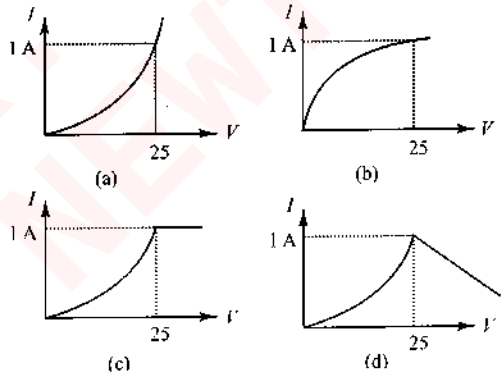


Fig. A2.2

Which of the following graphs represents current–voltage characteristics for their series combination?



3. In the given network (Fig. A2.3) the batteries getting charged are

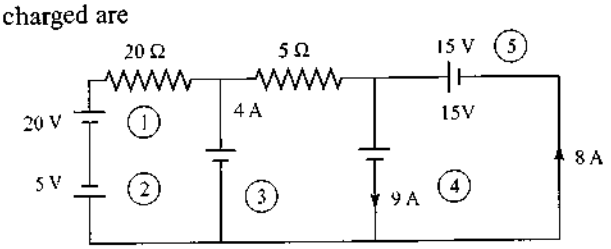


Fig. A2.3

a. 1 and 3
b. 1, 3 and 5
c. 1 and 4
d. 1, 2 and 5

4. When a galvanometer is shunted with a 4 Ω resistance, the deflection is reduced to 1/5. If the galvanometer is further shunted with a 2 Ω wire, the new deflection will be (assuming the main current remains the same)

a. 5/13 of the deflection when shunted with 4 Ω only
b. 8/13 of the deflection when shunted with 4 Ω only
c. 3/4 of the deflection when shunted with 4 Ω only
d. 3/13 of the deflection when shunted with 4 Ω only

5. Two cells A and B of electromotive forces 1.3 V and 1.5 V, respectively, are arranged as shown in Fig. A2.4. The voltmeter (assumed ideal) reads 1.45 V, the internal resistances of cells A and B are r_A and r_B, respectively. Which of the following is correct ?

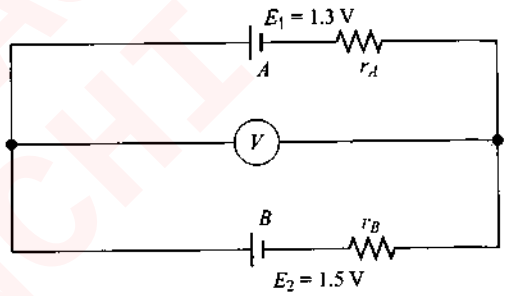


Fig. A2.4

a. $r_A = 2\,r_B$
b. $r_A = 3\,r_B$
c. $r_B = 2\,r_A$
d. $r_B = 3\,r_A$

6. In the circuit shown in Fig. A2.5, the battery E_1 has an e.m.f. of 12 V and zero internal resistance, while the battery E_2 has an e.m.f. of 2 V. If the galvanometer G reads zero, then the value of the resistance Y is

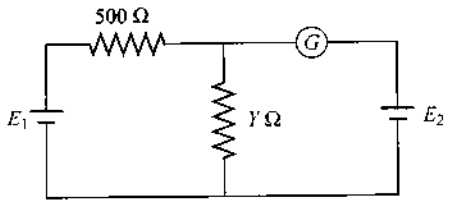


Fig. A2.5

a. 10 Ω
b. 100 Ω
c. 500 Ω
d. 200 Ω

7. Two cells of e.m.f.s E_1 and E_2 and of negligible internal resistances are connected with two variable resistors as shown in Fig. A2.6. When the galvanometer shows no deflection, the values of the resistances are P and Q.

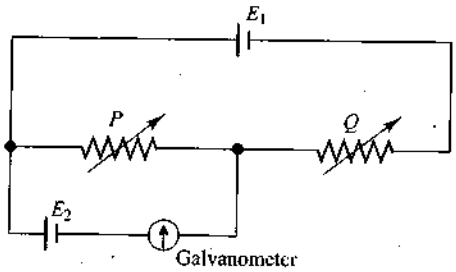


Fig. A2.6

What is the value of the ratio E_2/E_1?

a. $\frac{P}{Q}$ **b.** $\frac{P}{P+Q}$ **c.** $\frac{Q}{P+Q}$ **d.** $\frac{P+Q}{P}$

8. The Wheatstone's bridge shown in the Fig. A2.7 is balanced. If the positions of the cell C and the galvanometer G are now interchanged, G will show zero deflection

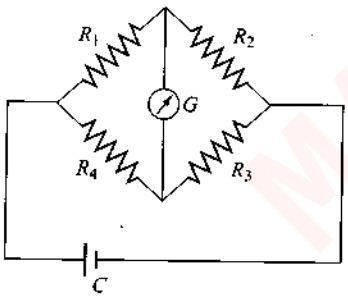


Fig. A2.7

a. in all cases
b. only if all the resistances are equal
c. only if $R_1 = R_3$ and $R_2 = R_4$
d. only if $R_1/R_3 = R_2/R_4$

9. In Fig. A2.8, two cells have equal e.m.f. E but internal resistances are r_1 and r_2. If the reading of the voltmeter is zero, then relation between R, r_1 and r_2 is

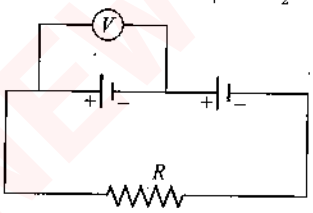


Fig. A2.8

a. $R = r_1 - r_2$ **b.** $R = r_1 + r_2$
c. $2r_1 - r_2$ **d.** $r_1 r_2$

10. A constant voltage dc source is connected, as shown in Fig. A2.9, across two resistors of resistances 400 kΩ and 100 kΩ.

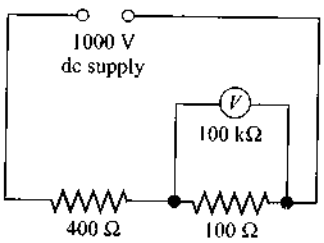


Fig. A2.9

What is the reading of the voltmeter, also of resistance 100 kΩ, when connected across the second resistor as shown?

a. 111 V **b.** 250 V
c. 125 V **d.** 333 V

11. In the given circuit (Fig. A2.10) in which case will the ammeter reading not change when R_2 is varied?

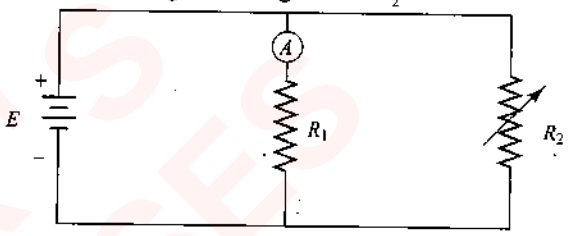


Fig. A2.10

a. $R_1 = r$ **b.** $R_1 = 2r$
c. $R_1 > R_2$ **d.** $r = 0$

12. An ammeter and a voltmeter are joined in series to a cell. Their readings are A and V, respectively. If a resistance is now joined in parallel with the voltmeter, then

a. both A and V will increase
b. both A and V will decrease
c. A will decrease, V will increase
d. A will increase, V will decrease

13. A capacitor of capacitance C is connected to two voltmeters A and B (Fig. A2.11). A is ideal, having infinite resistance, while B has resistance R. The capacitor is charged and then the switch S is closed. The readings of A and B will be equal

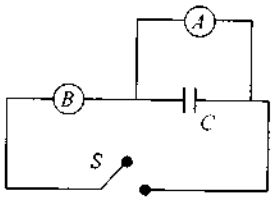


Fig. A2.11

a. at all times
b. after time RC
c. after time RC in 2
d. only after a very long time

14. The equivalent resistance of the combination across AB (Fig. A2.12) is

a. $\left(\frac{3+\sqrt{17}}{4}\right)$ **b.** $3+\sqrt{17}$

c. $\frac{3+\sqrt{17}}{2}$ **d.** $2(3+\sqrt{17})$

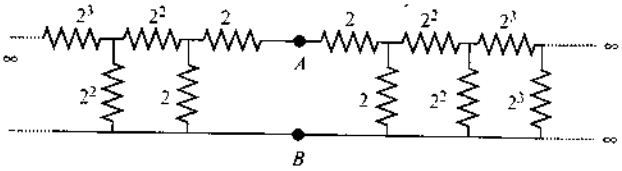


Fig. A2.12

15. The capacitor shown in Fig. A2.13 is in steady state. The energy stored in the capacitor is

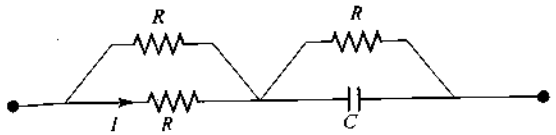


Fig. A2.13

a. CI^2R^2 **b.** $2CI^2R^2$
c. $4CI^2R^2$ **d.** None of the above

16. Charge on the capacitor having capacitance C_2 in steady state (Fig. A2.14) is

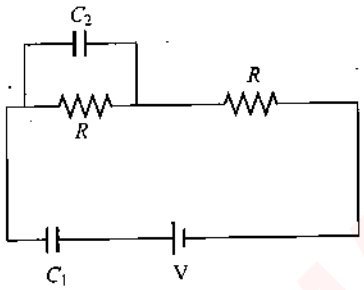


Fig. A2.14

a. Zero **b.** $(C_1 + C_2)V$
c. C_2V **d.** C_1V

17. An 80 μC charge is given to the 4 μF capacitor in the circuit shown in Fig. A2.15 so that the upper plate A is positively charged. An unknown resistance R is connected in the left limb. As soon as the switch S in the central limb is closed, a current of 2 A flows through the 2 Ω resistor in the central limb. The capacitive time constant for the circuit is

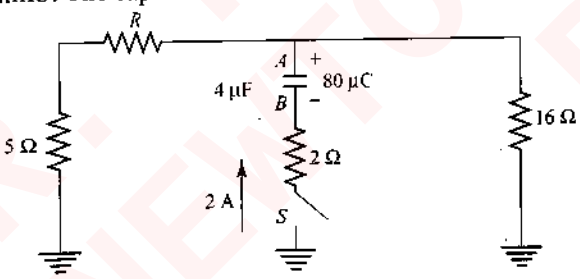


Fig. A2.15

a. 56 μs **b.** 8 μs
c. 200 μs **d.** 40 μs

18. In the given circuit (Fig. A2.16), when key K is open, reading of ammeter is I. Now key K is closed then the correct statement is

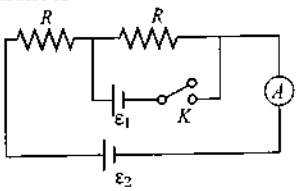


Fig. A2.16

a. If $\varepsilon_1 < IR$, reading of the ammeter is less than I
b. If $IR < \varepsilon_1$, reading of the ammeter is greater than I
c. If $\varepsilon_1 < 2IR$, reading of the ammeter will be zero
d. Reading of ammeter will not change

19. In the circuit shown (Fig. A2.17), the batteries have e.m.f. $E_1 = E_2 = 1$ V, $E_3 = 2.5$ V and the resistance $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$, Capacitance $C = 10$ μF. The charge on the left plate of the capacitor C at steady state is

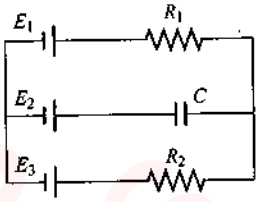


Fig. A2.17

a. + 2 μC **b.** – 4 μC
c. – 5 μC **d.** + 12 μC

20. A voltmeter with resistance $R_V = 2500\ \Omega$ indicates a voltage of 125 V in the circuit shown in Fig. A2.18. What is the series resistance (R) to be connected with voltmeter in this circuit so that it indicates 100 V?

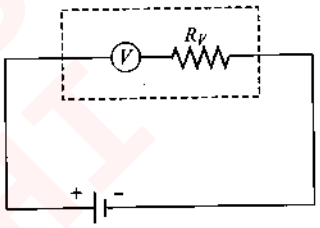


Fig. A2.18

a. 625 Ω **b.** 120 Ω
c. 550 Ω **d.** Data are insufficient

21. Figure A2.19 shows a battery with e.m.f. 15 V in a circuit with $R_1 = 30\ \Omega$, $R_2 = 10\ \Omega$, $R_3 = 20\ \Omega$ and capacitance $C = 10$ μF. The switch S is initially in the open position and is then closed at time $t = 0$. What will be the final steady-state charge on capacitor?

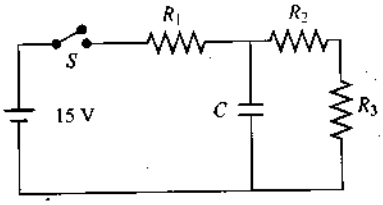


Fig. A2.19

a. 75 μC **b.** 50 μC
c. 10 μC **d.** None of these

22. A conductor of resistivity ρ and resistance R, as shown in the figure, is connected across a battery of e.m.f. V. Its radius varies from a at left end to b at right end. The electric field at a point P at distance x from left end of it is

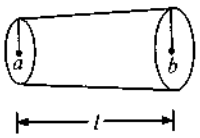


Fig. A2.20

a. $\dfrac{VI^2\rho}{\pi R(Ia+(b-a)x)^2}$ **b.** $\dfrac{2VI^2\rho}{\pi R(Ia+(b+a)x)^2}$

c. $\dfrac{VI^2\rho}{2\pi R(Ia+(b-a)x)^2}$ **d.** None of these

23. A piece of conducting wire of resistance R is cut into $2n$ equal parts. Half the parts are connected in series to form a bundle and remaining half in parallel to form another bundle. These bundles are then connected to give the maximum resistance. The maximum resistance of the combination is

a. $\frac{R}{2}\left(1+\frac{1}{n^2}\right)$ b. $\frac{R}{2}(1+n^2)$

c. $\frac{R}{2(1+n^2)}$ d. $R\left(n+\frac{1}{n}\right)$

24. In the circuit shown in Fig. A2.21, if the switch S is closed at $t = 0$, the capacitor charges with a time constant

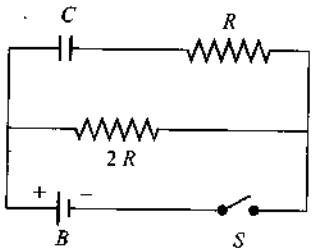


Fig. A2.21

a. RC b. $3RC$

c. $\frac{2}{3}RC$ d. $RC\ln\left(\frac{2}{3}\right)$

25. A metal wire of length L and radius r is made of copper and aluminium contributing equal lengths (Fig. A2.22). This wire is now coated by nickel till the radius of wire becomes R. If specific resistances of these materials are ρ_{Cu}, ρ_{Al} and ρ_{Ni} the equivalent conductance of the system across the length will be

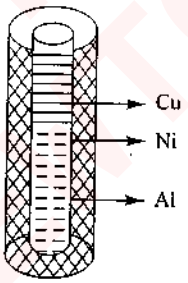


Fig. A2.22

a. $\frac{\pi r^2}{L}\left[\frac{1}{\rho_{Cu}}+\frac{1}{\rho_{Al}}+\frac{R^2}{r^2\rho_{Ni}}\right]$

b. $\frac{\pi}{L}\left[\frac{r^2}{\rho_{Cu}}+\frac{r^2}{\rho_{Al}}+\frac{2(R^2-r^2)}{r\rho_{Ni}}\right]$

c. $\frac{\pi}{L}\left[\frac{2(R^2-r^2)}{\rho_{Cu}+\rho_{Al}}-\frac{r^2}{\rho_{Ni}}\right]$

d. $\frac{\pi}{L}\left[\frac{2r^2}{\rho_{Cu}+\rho_{Al}}+\frac{R^2-r^2}{\rho_{Ni}}\right]$

26. In the given circuit (Fig. A2.23), the potential difference across the capacitor is 12 V. Each resistance is of 3 Ω. The cell is ideal. The e.m.f. of the cell is

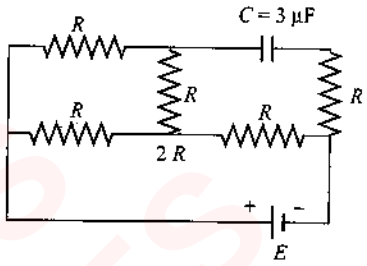


Fig. A2.23

a. 15 V b. 9 V

c. 12 V d. 24 V

27. A battery of internal resistance 4 Ω is connected to the network of the resistance as shown in Fig. A2.24. If the maximum power can be delivered to the network, the magnitude of resistance in Ω should be

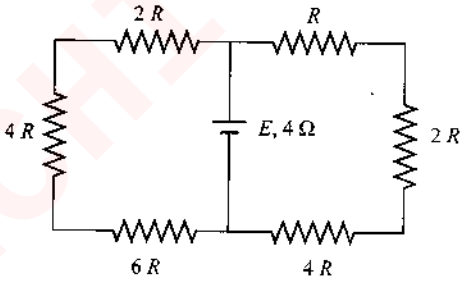


Fig. A2.24

a. $\frac{19}{21}\Omega$ b. $\frac{84}{19}\Omega$

c. 12 Ω d. 7 Ω

28. Find the effective resistance between A and $A2$.

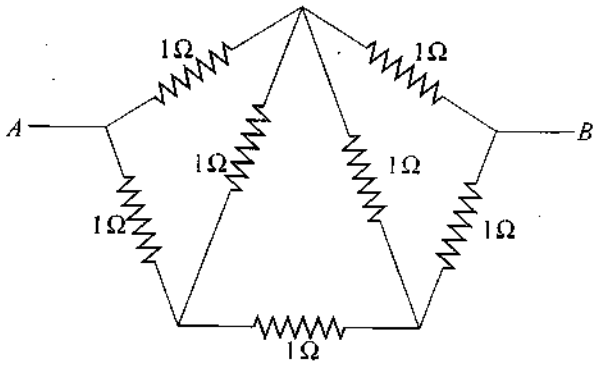


Fig. A2.25

a. 2 Ω b. 1 Ω

c. 8/7 Ω d. 7 Ω

29. A milli-ammeter of range 10 mA and resistance 9 Ω are joined in a circuit as shown in the Fig. A2.26. The meter gives full scale deflection, when current in the main circuit is

I and A and D are used as terminals. The value of I is

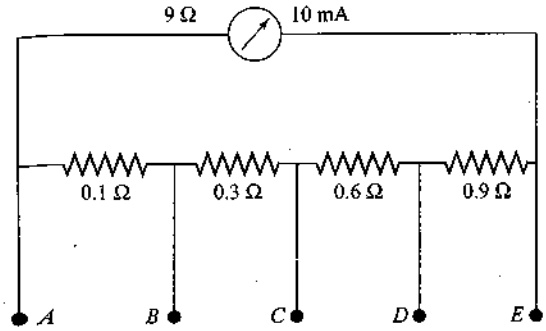


Fig. A2.26

a. 1.09 A **b.** 10.9 A **c.** zero **d.** 0.109 A

30. A potential of 400 V is applied at the point A. The resistances $R_1 = 1000\ \Omega$, $R_2 = 2000\ \Omega$ and $R_3 = 1000\ \Omega$ are connected between points A and G as shown in Fig. A2.27. Point G is earthed. The measured potential difference by an ideal voltmeter connected across R_2 is

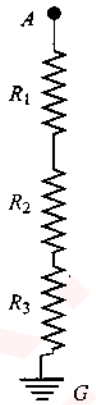


Fig. A2.27

a. 100 V **b.** 200 V **c.** 300 V **d.** 400 V

31. In the arrangement shown in Fig. A2.28, when the switch S_2 is open, the galvanometer shows no deflection for $\ell = L/2$. When the switch S_2 is closed, the galvanometer shows no deflection for $\ell = 5/12L$. The internal resistance (r) of 6 V cell, and the e.m.f. E of the other battery are respectively

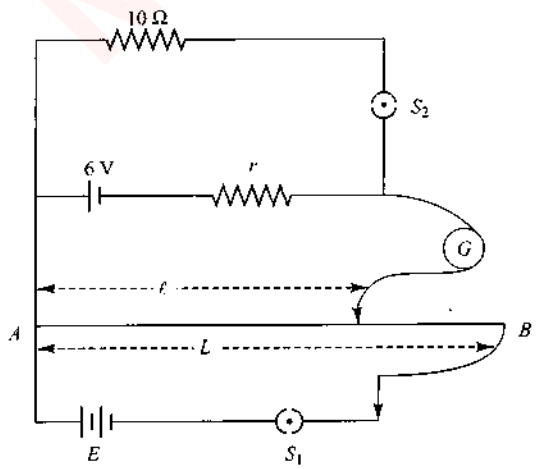


Fig. A2.28

a. 3 Ω, 8 V **b.** 2 Ω, 12 V **c.** 2 Ω, 24 V **d.** 3 Ω, 12 V

32. Calculate the energy stored in the capacitor of capacitance 2 μF. The voltmeter gives a reading of 15 V and the ammeter A reads 15 mA.

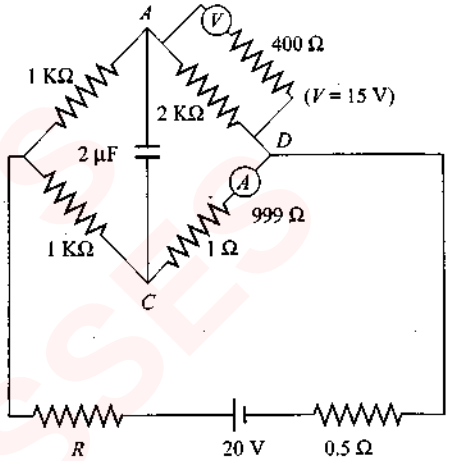


Fig. A2.29

a. 5 μJ **b.** 10 μJ
c. 0.5 μJ **d.** zero

33. In the given circuit (Fig. A2.30), $R_1 \neq R_2$ and the reading of the voltmeter is the same, irrespective of whether the switch S is open or closed. Then, which of the following is correct?

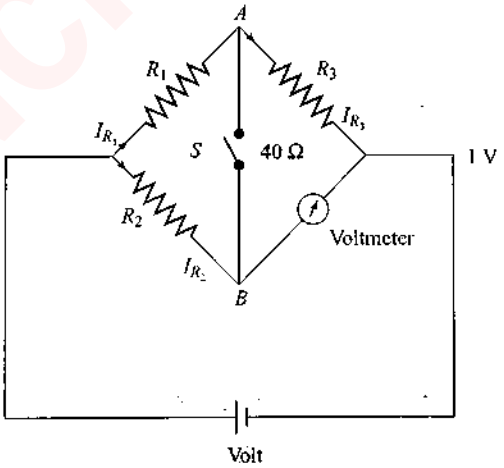


Fig. A2.30

a. $I_{R_2} = I_V$ **b.** $I_{R_1} = I_{R_2}$

c. $I_{R_3} = I_V$ **d.** None of the above

34. In Fig. A2.31, the charge that flows from P to Q when the switch S is closed is

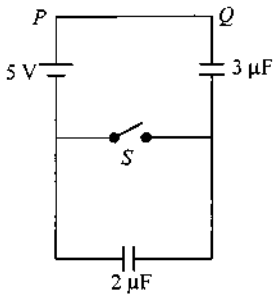


Fig. A2.31

a. 3 μC b. 6 μC
c. 9 μC d. 15 μC

35. In the circuit shown (Fig. A2.32), the value of R in ohm that will result in no current through the 30 V battery is

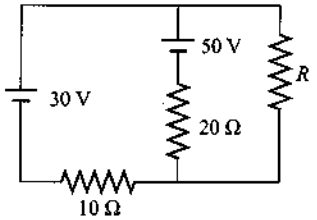


Fig. A2.32

a. 10 Ω b. 25 Ω
c. 30 Ω d. 40 Ω

36. In Fig. A2.33, the current flowing through $2R$ is

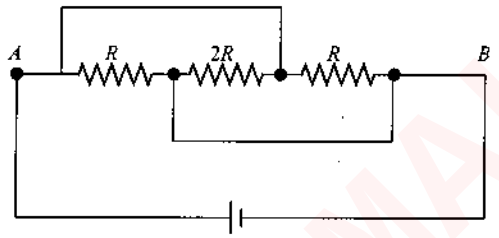


Fig. A2.33

a. from left to right b. from right to left
c. no current d. none of these

37. In the circuit shown (Fig. A2.34), switch S_2 is closed first and is kept closed for a long time. Now S_1 is closed. Just after that instant, the current through S_1 is

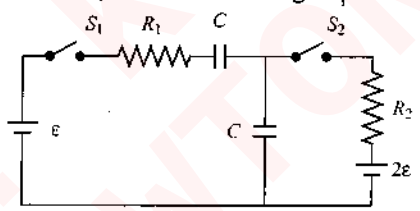


Fig. A2.34

a. $\frac{\varepsilon}{R_1}$ towards right b. $\frac{\varepsilon}{R_1}$ towards left
c. zero d. $\frac{2\varepsilon}{R_1}$

38. Each resistor in the following circuit (Fig. A2.35) has a resistance of 2 MΩ and the capacitors have capacitances of 1 μF. The battery voltage is 3 V. The voltage across the resistor A in the following circuit in steady state is

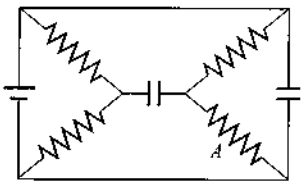


Fig. A2.35

a. 0 V b. 0.5 V
c. 0.75 V d. 1.5 V

39. Initially, switch S is connected to position 1 for a long time (Fig. A2.36). The net amount of heat generated in the circuit after it is shifted to position 2 is

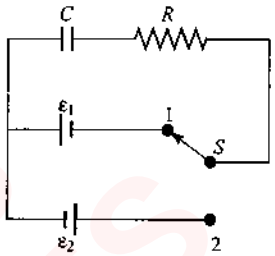


Fig. A2.36

a. $\frac{C}{2}(\varepsilon_1+\varepsilon_2)\varepsilon_2$ b. $C(\varepsilon_1+\varepsilon_2)\varepsilon_2$
c. $\frac{C}{2}(\varepsilon_1+\varepsilon_2)^2$ d. $C(\varepsilon_1+\varepsilon_2)^2$

40. The switch S in the circuit diagram (Fig. A2.37) is closed at $t = 0$. The charge on capacitors at any time t is

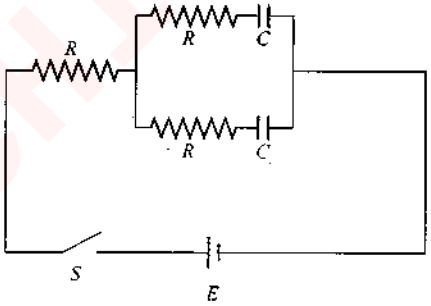


Fig. A2.37

a. $q_{(t)} = EC(1-e^{-2t/3RC})$ b. $q_{(t)} = EC(1-e^{-t/2RC})$
c. $q_{(t)} = EC(1-e^{-t/3RC})$ d. $q_{(t)} = EC(1-e^{-3t/2RC})$

41. The charge on the capacitor in steady state in the circuit shown (Fig. A2.38) is

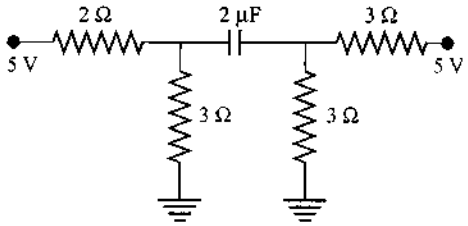


Fig. A2.38

a. 0.5 μC b. 1 μC c. 2 μC d. 4 μC

42. In the circuit shown in Fig. A2.39, XY is a potentiometer wire 100 cm long. The circuit is connected up as shown. With switches S_2 and S_3 open, a balance point is found at Z. After switch S_1 has remained closed for some time, it is found that contact at Z must be moved towards Y to maintain a balance. Which of the following is the most likely reason for this?

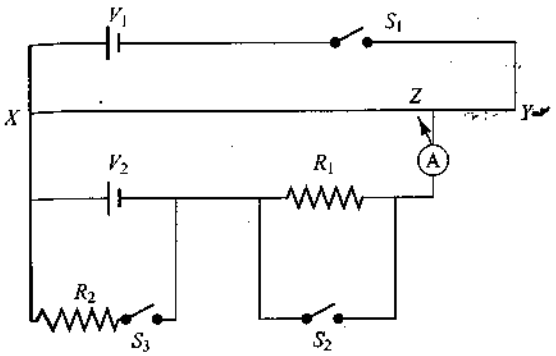


Fig. A2.39

a. The cell V_1 is running down
b. The cell V_2 is running down
c. The wire XZ is getting warm and its resistance is increasing
d. The resistor R_1 is getting warm and increasing in value

43. In the circuit shown (Fig. A2.40), if switches S_1 and S_2 have been closed for a long time, then the charge on the capacitor

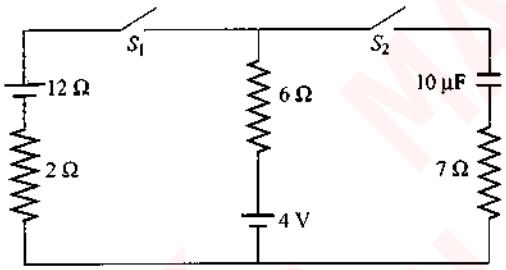


Fig. A2.40

a. is 100 μC
b. increases to 120 μC if one-third of the gap of the capacitor's plates is filled with a dielectric ($K = 2$) of same area
c. both a. and b.
d. charge on the capacitor remains unchanged if one-third of the gap of the capacitor's plates is filled with a dielectric ($K = 2$) of same area

44. The circuit shown in Fig. A2.41 consists of a battery of e.m.f. $\varepsilon = 10$ V; a capacitor of capacitance $C = 1.0$ μF and three resistors of values $R_1 = 2\,\Omega$, $R_2 = 2\,\Omega$ and $R_3 = 1\,\Omega$. Initially, the capacitor is completely uncharged and the switch S is open. The switch S is closed at $t = 0$. Then

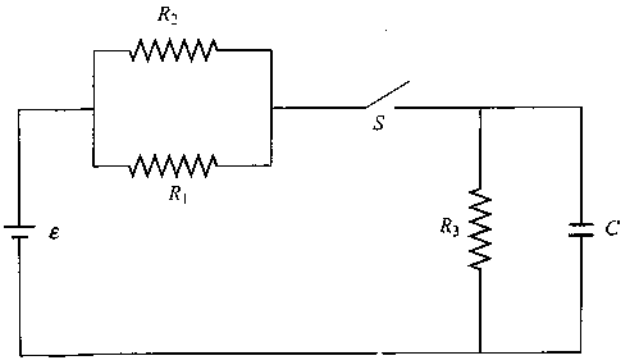


Fig. A2.41

a. the current through resistor R_3 at the moment the switch closed is zero
b. the current through resistor R_3 a long time after the switch closed is 5 A
c. the ratio of current through R_1 and R_2 is always constant
d. all of these

45. Figures A2.42 (a) and (b) show a circuit used in an experiment to determine the e.m.f. and internal resistance of the battery C. A graph was plotted of the potential difference V between the terminals of the battery against the current I, which was varied by adjusting the rheostat. The graph is shown in Fig. A2.42 (b); x and y are the intercepts of the graph with the axes as shown. What is the internal resistance of the battery?

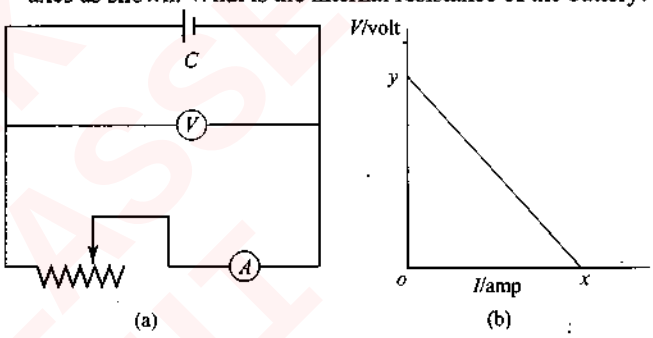


Fig. A2.42

a. x b. y c. x/y d. y/x

46. In the circuit shown (Fig. A2.43), what is the change of total electrical energy stored in the capacitors when the key is pressed?

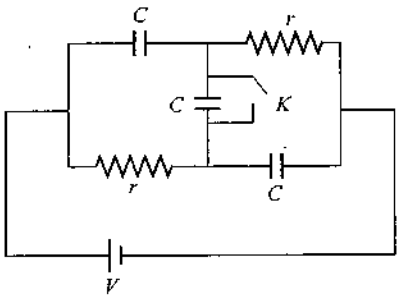


Fig. A2.43

a. $\dfrac{CV^2}{12}$ b. $\dfrac{7CV^2}{8}$
c. $\dfrac{5CV^2}{4}$ d. $\dfrac{3CV^2}{8}$

47. In the circuit shown (Fig. A2.44), calculate the current through 6 V battery.

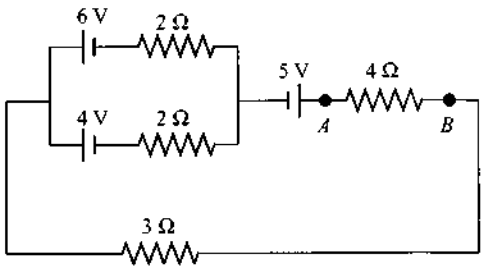


Fig. A2.44

a. (1/4) A b. (1/8) A
c. (1/2) A d. none of these

48. In Fig. A2.45, points A and B are connected by a perfectly conducting wire. Calculate the current through AB

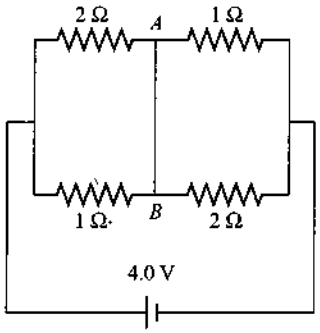


Fig. A2.45

a. 2 A b. 1 A
c. 1.5 A d. 2.5 A

49. In the potentiometer arrangement (Fig.A2.46), the driving cell A has e.m.f. ε and internal resistance r. The e.m.f. of the cell B is to be rechecked has e.m.f. $\varepsilon/2$ and internal resistance $2r$. The potentiometer wire CD is 100 cm long. If balance is obtained, the length $CJ = l$ is

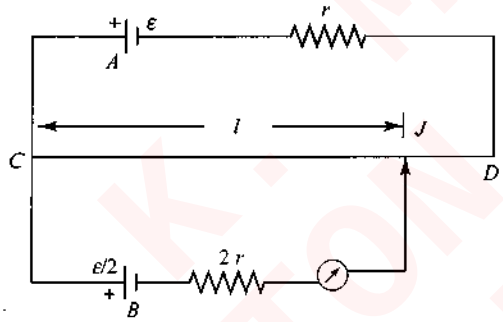


Fig. A2.46

a. $l = 50$ cm b. $l > 50$ cm
c. $l < 50$ cm d. Balance cannot be obtained

50. In a practical Wheatstone bridge circuit (Fig. A2.47), when one more resistance of 100 Ω is connected in parallel with unknown resistance x, then the ratio l_1 / l_2 becomes 2. l_1 is the balance length. AB is a uniform wire. Then the value of x must be

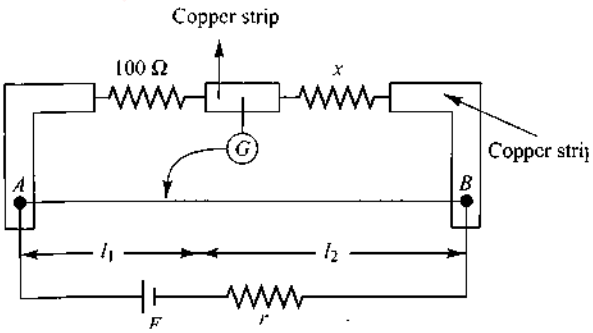


Fig. A2.47

a. 50 Ω b. 100 Ω c. 200 Ω d. 400 Ω

51. To get the maximum current through a resistance of 2.5 Ω, one can use m rows of cells, each row having n cells. The internal resistance of each cell is 0.5 Ω. What are the values of n and m, if the total number of cells is 45?

a. 3, 15 b. 5, 9 c. 9, 5 d. 15, 3

52. Two circular rings of identical radii and resistance of 36 Ω each are placed in such a way that they cross each other's centre C_1 and C_2 as shown in Fig. A2.48. Conducting joints are made at intersection points A and B of the rings. An ideal cell of e.m.f. 20 V is connected across AB. The power delivered by cell is

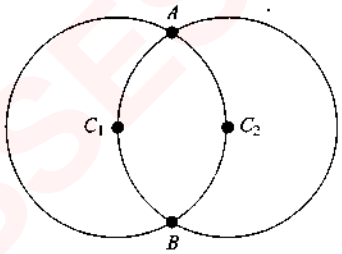


Fig. A2.48

a. 80 W b. 100 W c. 120 W d. 200 W

53. Circuit for the measurement of resistance by potentiometer is shown in Fig. A2.49. The galvanometer is first connected at point A and zero deflection is observed at length $PJ = 10$ cm. In second case, it is connected at point C and zero deflection is observed at a length 30 cm from P, then the unknown resistance X is

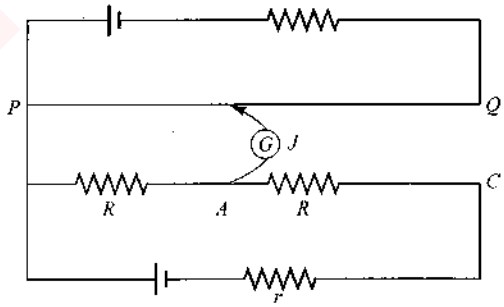


Fig. A2.49

a. 2 R b. $R/2$ c. $R/3$ d. 3 R

54. Which of the following circuits (Fig. A2.50) gives the correct value of resistance, when computed by using $R = V/I$ where V and I are voltmeter and ammeter readings, respectively? The meters are not ideal.

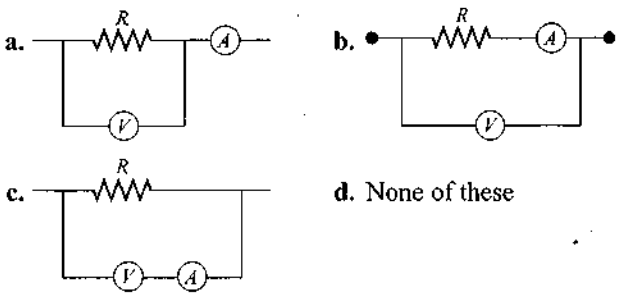


Fig. A2.50

55. Two ideal batteries having e.m.f. E_1 and E_2 are connected as shown in Fig. A2.51. The values of resistances are chosen in such a way that ammeter reading is zero. The reading of voltmeter will be (consider the meters to be ideal)

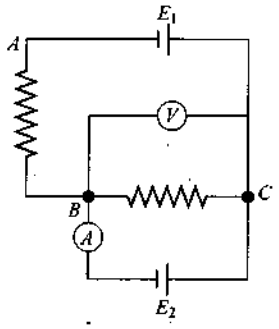


Fig. A2.51

a. E_1 **b.** E_2

c. In between E_1 and E_2

d. Nothing can be predicted about voltmeter reading from the given information

56. A cell of e.m.f. ε and internal resistance r is charged by a current i, then

a. the cell stores chemical energy at the rate of εi

b. the cell stores chemical energy at the rate of $i^2 r$

c. the cell stores chemical energy at the rate of $\varepsilon i - i^2 r$

d. the storage of chemical energy rate cannot be calculated

57. Two long coaxial and conducting cylinders of radius a and b are separated by a material of conductivity σ and a constant potential difference V is maintained between them by a battery. Then the current per unit length of the cylinder flowing from one cylinder to the other is

a. $\dfrac{4\pi\sigma}{\ell n(b/a)}V$ **b.** $\dfrac{4\pi\sigma}{(b/a)}V$

c. $\dfrac{2\pi\sigma}{\ell n(b/a)}V$ **d.** $\dfrac{2\pi\sigma}{(b+a)}V$

58. 50 V battery is supplying current of 10 A when connected to a resistor. If the efficiency of the battery at this current is 25%, then the internal resistance of the battery is

a. 2.5 Ω **b.** 3.75 Ω

c. 1.25 Ω **d.** 5 Ω

59. A battery is supplying power to a tape recorder by cable of resistance of 0.02 Ω. If the battery is generating 50 W power at 5 V, then the power received by the tape recorder is

a. 50 W **b.** 45 W

c. 30 W **d.** 48 W

60. In a Wheatstone bridge, resistance P, Q and R are connected in the three arms and the fourth arm is formed by two resistances S_1 and S_2 connected in parallel. The condition for the bridge to be balanced will be **[AIEEE, 2006]**

a. $\dfrac{P}{Q} = \dfrac{R(S_1+S_2)}{2S_1S_2}$ **b.** $\dfrac{P}{Q} = \dfrac{R}{S_1+S_2}$

c. $\dfrac{P}{Q} = \dfrac{2R}{S_1+S_2}$ **d.** $\dfrac{P}{Q} = \dfrac{R(S_1+S_2)}{S_1S_2}$

61. Figure A2.52 shows a meter-bridge set up with null deflection in the galvanometer. The value of the unknown resistor R is **[AIEEE, 2008]**

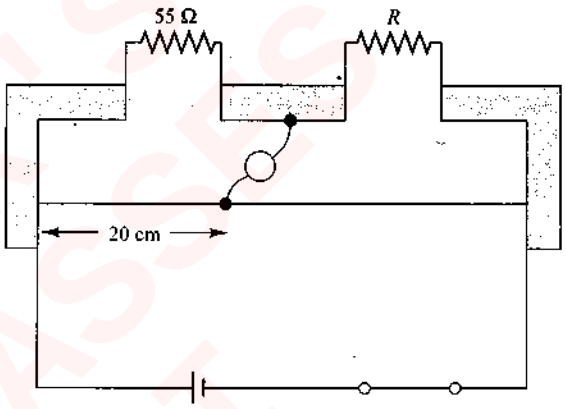


Fig. A2.52

a. 110 Ω **b.** 55 Ω

c. 13.75 Ω **d.** 220 Ω

62. A number of resistors are connected as shown in the figure. The equivalent resistance between A and B is

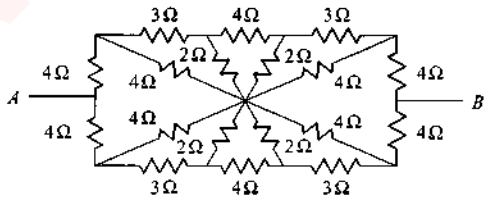


Fig. A2.53

a. 6 Ω **b.** 12 Ω

c. 9 Ω **d.** 15 Ω

63. In the given arrangement, the reading of ammeter is same in each case when either K_1 or K_2 is closed. The reading of the ammeter is

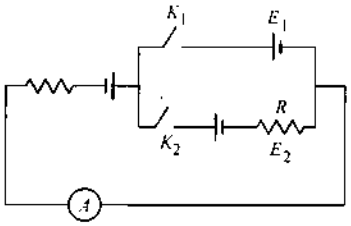


Fig. A2.54

a. $\dfrac{E_1+E_2}{R}$ **b.** $\dfrac{E_1-E_2}{R}$

c. data given is not sufficient **d.** none of these

64. The charge on the capacitor as in figure is

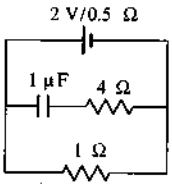


Fig. A2.55

a. 2 μC **b.** $\frac{2}{3}$ μC
c. $\frac{4}{3}$ μC **d.** zero

65. Find the potential drop across the capacitor in the given circuit.

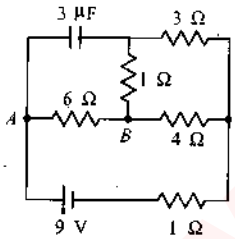


Fig. A2.56

a. 6 V **b.** 6.5 V
c. 7 V **d.** None of these

66. A galvanometer may be converted into ammeter or voltmeter. In which of the following cases the resistance of the device will be largest ? (Assume the maximum range of galvanometer = 1 mA)
a. An ammeter of range 10 A
b. A voltmeter of range 5 V
c. An ammeter of range 5 A
d. A voltmeter of range 10 V

67. In the diagrams, all right bulbs are identical, all cells are ideal and identical. In which circuit (a, b, c, d) will the bulbs be dimmest?

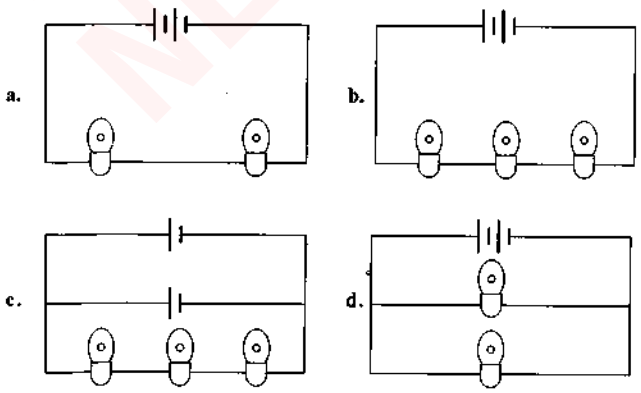


Fig. A2.57

68. For the circuit shown, a shorting wire of negligible resistance is added to the circuit between points A and B. When this shorting wire is added, bulb 3 goes out. Which bulbs (all identical) in the circuit brighten?

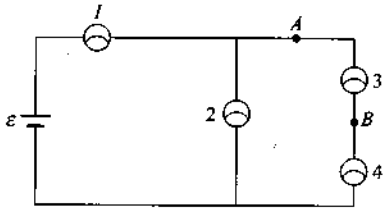


Fig. A2.58

a. Only bulb 2 **b.** Only bulb 4
c. Only bulbs 1 and 4 **d.** Only bulbs 2 and 4

69. A wire has linear resistance ρ (in Ω/m). Find the resistance R between points A and B if the side of the big square is d.

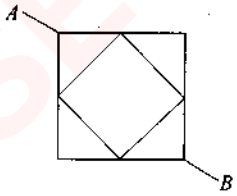


Fig. A2.59

a. $\frac{\rho d}{\sqrt{2}}$ **b.** $\sqrt{2}\rho d$
c. $2\rho d$ **d.** None of these

70. The equivalent resistance between A and B in the arrangement of resistances as shown is

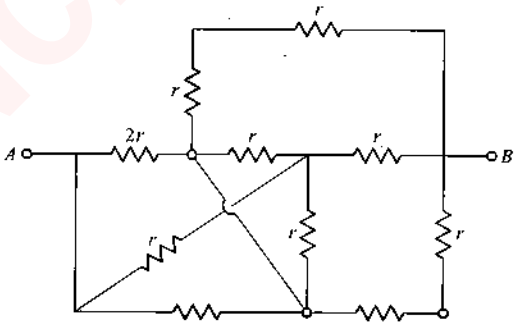


Fig. A2.60

a. $4r$ **b.** $3r$
c. $2.5r$ **d.** r

71. A capacitor of capacitance 10 μF is charged up to a potential difference of 2 V and then the cell is removed. Now it is connected to a cell of e.m.f. 4 V and is charged fully. In both cases the polarities of the two cells are in the same directions. Total heat produced in the complete charging process is
a. 10 mJ **b.** 20 μJ
c. 40 μJ **d.** 80 mJ

72. In the shown wire frame, each side of a square (the smallest square) has a resistance R. The equivalent resistance of the circuit between the points A and B is

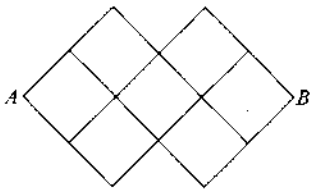


Fig. A2.61

a. R b. $2R$
c. $4R$ d. $8R$

73. A spherical shell, made of material of electrical conductivity $10^9/\pi(\Omega\,\text{m})^{-1}$, has thickness $t = 2$ mm and radius $R = 10$ cm. In an arrangement, its inside surface is kept at a lower potential than its outside surface.

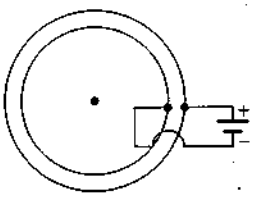


Fig. A2.62

The resistance offered by the shell is equal to
a. $5\pi \times 10^{-12}\ \Omega$ b. $2.5 \times 10^{-11}\ \Omega$
c. $5 \times 10^{-12}\ \Omega$ d. $5 \times 10^{-11}\ \Omega$

74. Two cylindrical rods of uniform cross-sectional area A and $2A$, having free electrons per unit volume $2n$ and n, respectively, are joined in series. A current I flows through them in steady state. Then the ratio of drift velocity of free electron in left rod to drift velocity of electron in the right rod (V_L/V_R) is

a. 1/2 b. 1
c. 2 d. 4

75. A charge passing through a resistor varies with time as shown in the figure. The amount of heat generated in time 't' is best represented (as a function of time) by

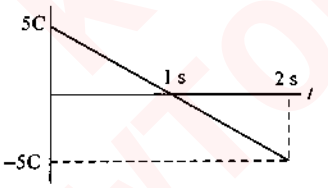
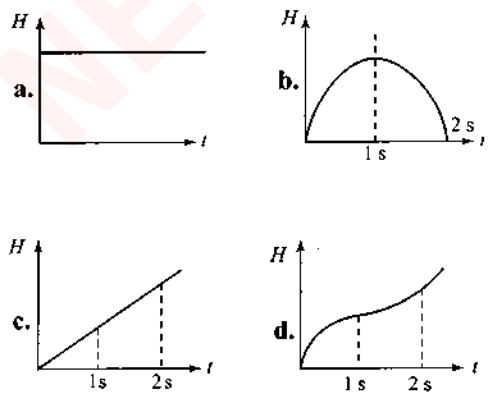


Fig. A2.63



76. Two cylindrical rods of same cross-sectional area and same length are connected in series to an ideal cell as shown. The resistivity of left rod is ρ and that of right rod is 2ρ. Then the variation of potential at any point P distant x from the left end of combined rod system is given by

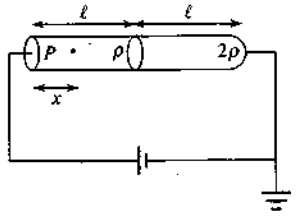
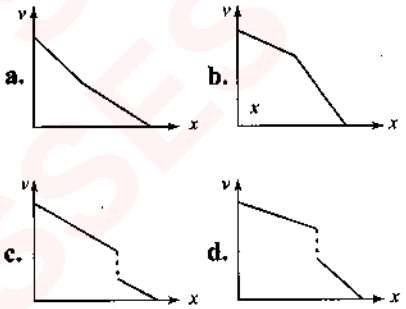


Fig. A2.64



77. The effective resistance between A and B of the shown network, where resistance of each resistor is R, is

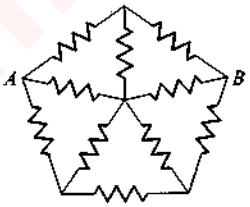


Fig. A2.65

a. $\frac{8R}{11}$ b. $\frac{6R}{11}$
c. $\frac{6R}{5}$ d. none of these

78. A potentiometer wire AB as shown is 40 cm long of resistance 50 Ω/m free end of an ideal voltmeter is touching the potentiometer wire. What should be the velocity of the jockey as a function of time so that reading in voltmeter varies with time as (2 sin πt)?

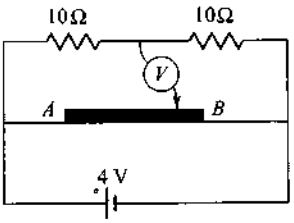


Fig. A2.66

a. $10\pi \sin \pi t$ cm/s b. $10\pi \cos \pi t$ cm/s
c. $20\pi \sin \pi t$ cm/s d. $20\pi \cos \pi t$ cm/s

79. The relation between R and r (internal resistance of the battery) for which the power consumed in the external part of the circuit is maximum.

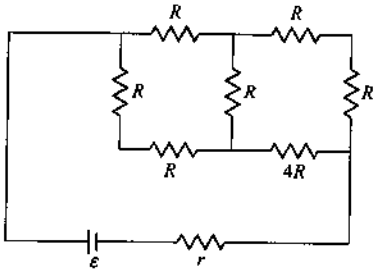


Fig. A2.67

a. $R = r$ **b.** $R = r/2$

c. $R = 2r$ **d.** $R = 1.5r$

80. For the potentiometer arrangement shown in the figure, length of wire AB is 100 cm and its resistance is 9 Ω. Find the length AC for which the galvanometer G will show zero deflection.

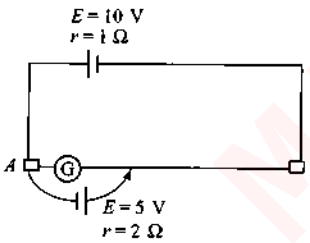


Fig. A2.68

a. 66.7 cm **b.** 60 cm
c. 50 cm **d.** 33.3 cm

81. A resistance R = 12 Ω is connected across a source of e.m.f. as shown in the figure. Its e.m.f. changes with time as shown in the graph. What is the heat developed in the resistance in the first 4 s ?

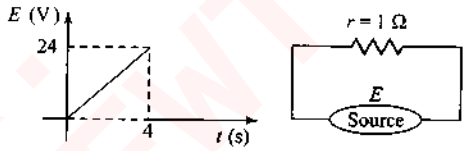


Fig. A2.69

a. 72 J **b.** 64 J
c. 108 J **d.** 100 J

82. If the switch at point P is opened (shown in the figure), hoose the correct option.

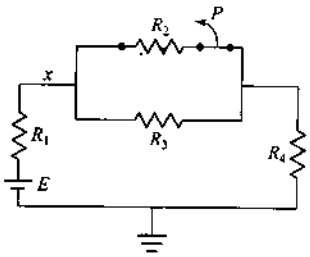


Fig. A2.70

a. The current in R_1 would not change
b. The potential difference between point X and the ground would increase
c. The current provided by the battery would increase
d. The e.m.f. produced by the battery (assumed to have no internal resistance) would change

83. Figure shows a circuit model for the transmission of an electrical signal, such as cable TV, to a large number of subscribers. Each subscriber connects a load resistance R_L between the transmission line and the ground. Assume the ground to be at zero potential and to have negligible resistance. The resistance of the transmission line between the connection points of different subscribers is modelled as the constant resistance R_T. The equivalent resistance across the signal source is

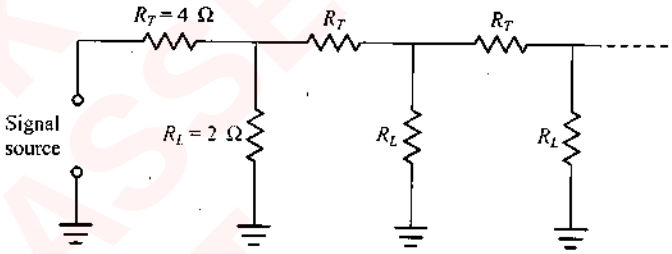


Fig. A2.71

a. 10 Ω **b.** 5 Ω
c. $\sqrt{55}\,\Omega$ **d.** $\sqrt{65}\,\Omega$

84. In the circuit shown in the figure, the switch S is in position '1' for a long time. Now the switch is thrown to position '2'. Find the total heat developed in the circuit till the steady state is reached again.

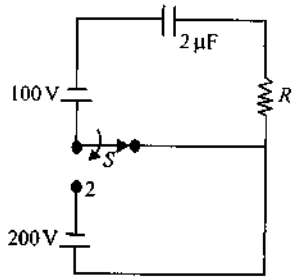


Fig. A2.72

a. 0.04 J **b.** 0.4 J
c. 0.02 J **d.** 0.2 J

85. A total charge Q flows across a resistor R during a time interval = T in such a way that the current vs. time graph for 0 to T is like the loop of a sine curve in the range 0 to π. The total heat generated in the resistor is

a. $Q^2\pi^2R/8T$ **b.** $2Q^2\pi^2R/T$
c. $Q^2\pi^2R/T$ **d.** $Q^2\pi^2R/2T$

86. An ammeter A of finite resistance and a resistor R are joined in series to an ideal cell C. A potentiometer P is joined in parallel to R. The ammeter reading is I_0 and the potentiometer reading is V_0. P is now replaced by a voltmeter of finite resistance. The ammeter reading now is I and the voltmeter reading is V. Then

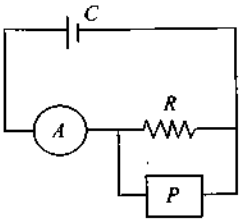


Fig. A2.73

a. $I > I_0, V < V_0$ b. $I > I_0, V > V_0$
c. $I = I_0, V < V_0$ d. $I < I_0, V > V_0$

87. In the given potentiometer circuit length of the wire AB is 3 m and resistance is $R = 4.5\ \Omega$. The length AC for no deflection in galvanometer is

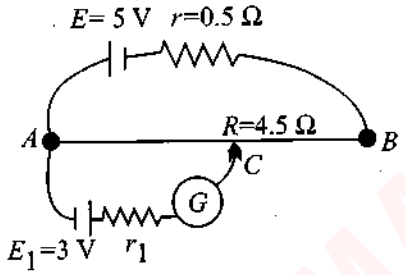


Fig. A2.74

a. 2 m b. 1.8 m
c. dependent on r_1 d. none of these

88. A battery of e.m.f. $\varepsilon_0 = 12$ V is connected across a 4 m long uniform wire having resistance 4 Ω/m. The cells of small e.m.f. $\varepsilon_1 = 2$ V and $\varepsilon_2 = 4$ V having internal resistances $r_1 = 2\ \Omega$ and $r_2 = 6\ \Omega$, respectively, are connected as shown in the figure. If galvanometer shows no deflection at point N, the distance of point N from point A is equal to

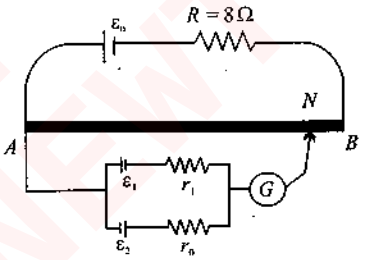


Fig. A2.75

a. 1/6 m b. 1/3 m
c. 25 cm d. 50 cm

89. A capacitor having initial charge $q_0 = CE/2$ is connected to a cell of e.m.f. E through a resistor R as shown. Find the total heat generated in the circuit after the switch S is closed.

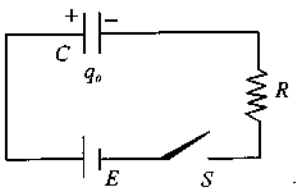


Fig. A2.76

a. $(1/12)CE^2$ b. $(1/8)CE^2$
c. $(1/4)CE^2$ d. None of these

90. Given that current through $CA = 1$ A, current through $C'A' = 2$ A. Now if A is connected to A' and B is connected B'. Find currents through CA and $C'A'$, respectively,

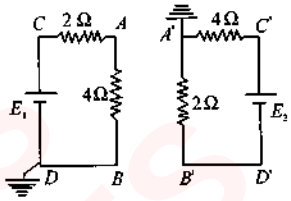


Fig. A2.77

a. 1 A, 3 A b. 3 A, 1 A
c. 1 A, 1 A d. 3 A, 3 A

91. In the circuit shown, the capacitor C_1 is initially charged with charge q_0. The switch S is closed at time $t = 0$. The charge on C_2 after time t is

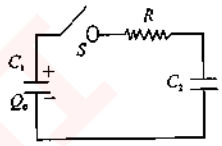


Fig. A2.78

a. $\frac{q_0C_1}{C_1+C_2}\left(1-e^{-\frac{t(C_1+C_2)}{C_1C_2R}}\right)$ b. $\frac{q_0C_2}{C_1+C_2}\left(1-e^{-\frac{t(C_1+C_2)}{C_1C_2R}}\right)$

c. $\frac{q_0C_1}{C_1+C_2}\left(1-e^{-\frac{t}{C_2R}}\right)$ d. $\frac{q_0C_2}{C_1+C_2}\left(1-e^{-\frac{t}{C_2R}}\right)$

92. Consider the circuit shown in the figure. Find the charge on capacitor C between A and D in steady state.

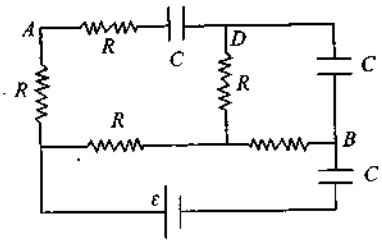


Fig. A2.79

a. $C\varepsilon$ b. $C\varepsilon/2$
c. $C\varepsilon/3$ d. zero

93. The variation of current (I) and voltage (V) is as shown in the figure.

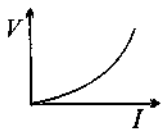
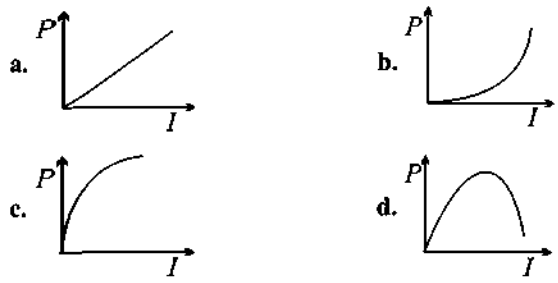


Fig. A2.80

The variation of power P with current I is best shown by which of the following graphs?



94. What amount of heat will be generated in a coil of resistance R due to a charge q passing through it if the current in the coil decreases to zero uniformly during a time interval T?

a. $\frac{4}{3}\frac{q^2R}{T}$ b. $\ln\left(\frac{q^2R}{2T}\right)$

c. $\frac{2q^2R}{3T}$ d. $\ln\left(\frac{2T}{q^2R}\right)$

95. A wire of length L and three identical cells of negligible internal resistances are connected in series. Due to the current, the temperature of the wire is raised by ΔT in time t. N number of similar cells are now connected in series with a wire of the same material and cross section but of length $2L$. The temperature of the wire is raised by the same amount ΔT in the same time t. Assume no loss of heat from the wire to the surrounding. The value of N is

a. 4 b. 5
c. 8 d. 6

96. In the given circuit, the capacitor of capacitance C is charged by closing key K at $t = 0$. Find the time required to charge the capacitor up to the maximum charge for the given circuit, if it were to be charged with the constant initial charging rate at $t = 0$ in the given circuit.

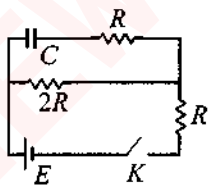


Fig. A2.81

a. $\frac{RC}{3}$ b. $\frac{2RC}{5}$

c. $\frac{2RC}{3}$ d. $\frac{5RC}{3}$

97. V_c is the ideal voltmeter in the figure. Resistance of the resistor shown is R. Initially the switch is in position 2 when charging of the capacitor starts. Initially the capacitor was uncharged. The switch in circuit shifts automatically from 2 to 1 when $V_c > 2V/3$ and goes back to 2 from 1 when $V_c < V/3$. The ideal voltmeter reads voltage across capacitor as plotted. What is the period T of the wave form in terms of R and C?

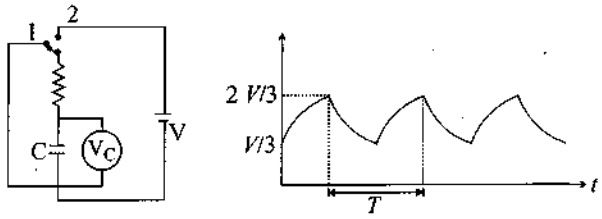


Fig. A2.82

a. $3RC \ln 3$ b. $2RC \ln 2$
c. $(RC/2) \ln 2$ d. none of these

98. In the circuit shown in the figure, take the potential of point Q to be zero, and potential of P to be V.

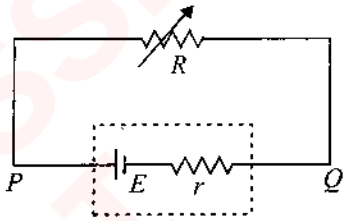
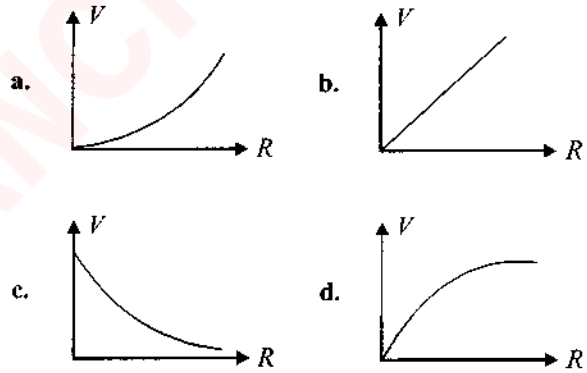


Fig. A2.83

The variation of V w.r.t R is correctly shown by



99. Consider a cell of e.m.f. ε and internal resistance r. Its terminals are directly connected. Some current I is obtained in the circuit. Now take two such cells.

a. Current more than I can be obtained if both cells are connected in series and shorting the two ends.
b. Current more than I can be obtained if both cells are connected in parallel and then shorting the two ends of cells.
c. Both of the above
d. None of the above

Multiple Correct Answers Type

Solutions on page A2.43

1. The electron in a hydrogen atom moves in a circular orbit of radius 5×10^{-11} m with a speed of $0.6\pi \times 10^6$ m/s, then

a. the frequency of the electron is 6×10^{15} rev/s
b. the electron carries -1.6×10^{-19} C around the loop
c. the current in the orbit is 0.96 mA
d. the current flow is in the opposite direction to the direction of the motion of electron

2. For the circuit shown in Fig. A2.84, select the correct statements from the following options.

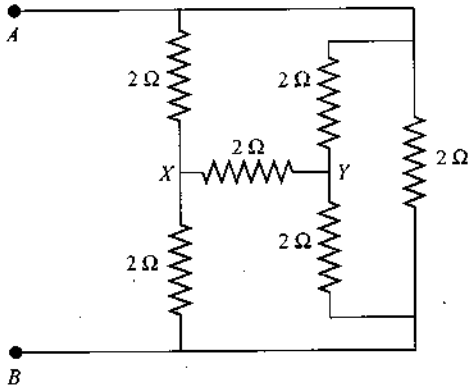


Fig. A2.84

a. x and y are equipotential points
b. Effective resistance between A and B is 2 Ω
c. Effective resistance between A and B is 1 Ω
d. None of the above

3. Two bulbs 25 W, 100 V (upper bulb in figure) and 100 W, 200 V (lower bulb in figure) are connected in the circuit as shown in Fig. A2.85. Choose the correct answer(s).

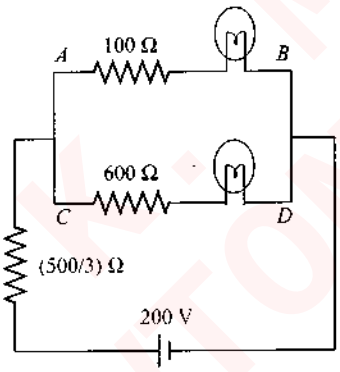


Fig. A2.85

a. Heat lost per second in the circuit will be 80 J
b. Ratio of heat produced per second in bulb will be 1:1
c. Ratio of heat produced in branch AB to that produced in branch CD will be 1:2
d. Current drawn from the cell is 0.4 A

4. In Fig. A2.86, battery of e.m.f. E has internal resistance r and a variable resistor. At an instant, current flowing through the circuit is i, potential difference between the terminals of cells is V, thermal power developed in external circuit is P and thermal power developed in the cell is equal to fraction η of total electrical generated in it. Which of the following graphs is/are correct?

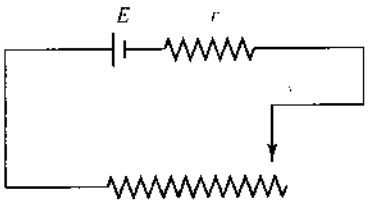
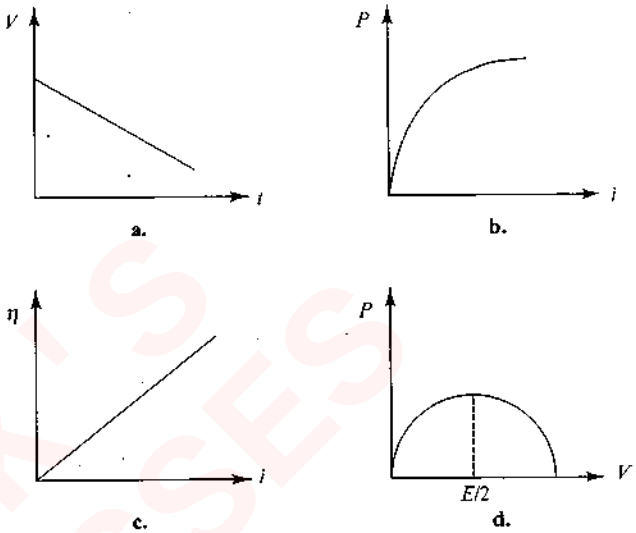


Fig. A2.86



5. In the given circuit (Fig. A2.87),

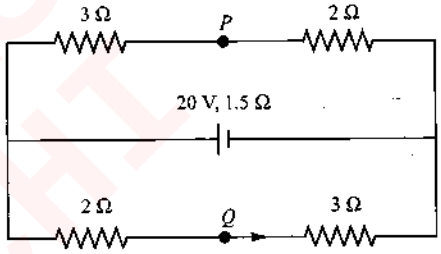


Fig. A2.87

a. the current through the battery is 5.0 A
b. P and Q are at the same potential
c. P is 2.5 V higher than Q
d. Q is 2.5 V higher than P

6. In the circuit shown (Fig. A2.88), the cell is ideal with e.m.f. = 2 V. The resistance of the coil of the galvanometer G is 1Ω. Then

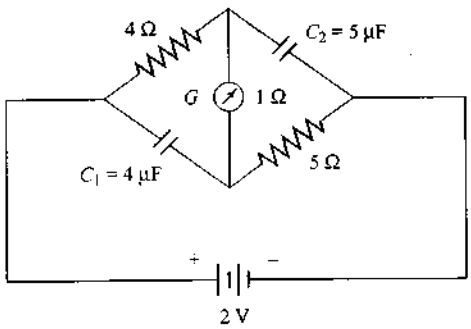


Fig. A2.88

a. no current flows in G
b. 0.2 A current flows in G
c. potential difference across C_1 is 1 V
d. potential difference across C_2 is 1.2 V

7. Which of the following statements are correct?
a. If bulbs of difference wattages are joined in parallel, then the lowest wattage bulb glows with the maximum brightness
b. If bulbs of difference wattages are joined in parallel,

then the highest wattage bulb glows with the maximum brightness

c. If bulbs of difference wattages are joined in series, then the lowest wattage bulb glows with maximum brightness

d. If bulbs of difference wattages are joined in series, then the highest wattage bulb glows with the maximum brightness

8. Three ammeters A, B and C of resistances R_A, R_B and R_C, respectively, are joined as shown in Fig. A2.89. When some potential difference is applied across the terminals T_1 and T_2, their readings are I_A, I_B and I_C, respectively, then

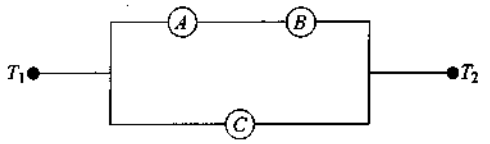


Fig. A2.89

a. $I_A = I_B$

b. $I_A R_A + I_B R_B = I_C R_C$

c. $\frac{I_A}{I_C} = \frac{R_C}{R_A}$

d. $\frac{I_B}{I_C} = \frac{R_C}{R_A + R_B}$

9. A battery of e.m.f. 2 V and initial resistance 1 Ω is connected across terminals A and B of the circuit shown in Fig. A2.90.

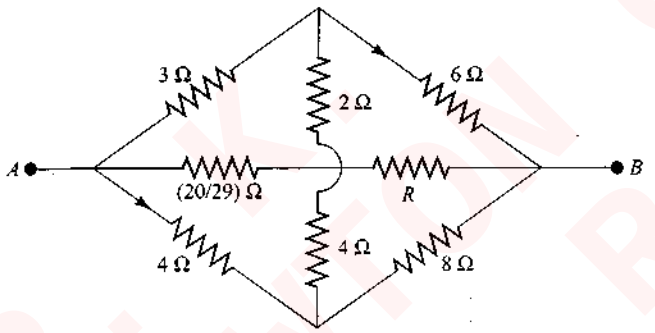


Fig. A2.90

a. Thermal power generated in the external circuit will be maximum possible when $R = \frac{16}{25}\,\Omega$

b. Maximum possible thermal power generated in the external circuit is equal to 4 W

c. Ratio of current through 3Ω to that through 8Ω is independent of R

d. None of above

10. Two electric bulbs rated at 25 W–220 V and 100 W–220 V are connected in series across a 220 V voltage source. The 25 W and 100 W bulbs now draw P_1 and P_2 powers, respectively, therefore

a. $P_1 = 16$ W

b. $P_1 = 4$ W

c. $P_2 = 16$ W

d. $P_2 = 4$ W

11. In the circuit shown in Fig. A2.91,

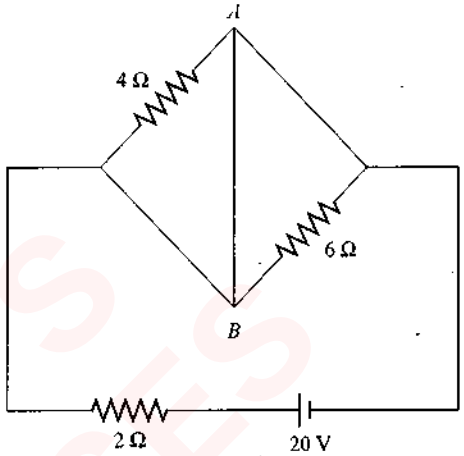


Fig. A2.91

a. power supplied by the circuit is 200 W

b. current flowing in the circuit is 5 A

c. potential difference across 4 Ω resistance is equal to the potential difference across 6 Ω resistance

d. current in wire AB is zero

12. Two cells of unequal e.m.f.s E_1 and E_2 and internal resistances r_1 and r_2 are joined as shown in Fig. A2.92. V_P and V_Q are identical at P and Q, respectively, then

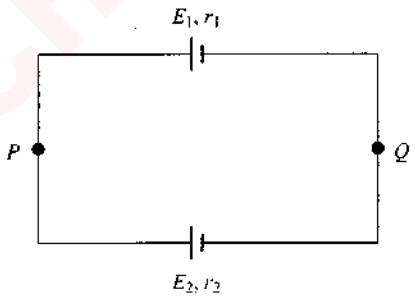


Fig. A2.92

a. the potential difference across both the cells will be equal

b. one of the cells will supply energy to the other cell

c. the potential difference across one of the cells will be greater than its e.m.f.

d. $V_P - V_Q = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$

13. A battery of e.m.f. $\varepsilon_0 = 5$ V and internal resistance 5 Ω is connected across a long uniform wire AB of length 1 m and resistance per unit length 5 Ωm^{-1}. Two cells of $\varepsilon_1 = 1$ V and $\varepsilon_2 = 2$ V are connected as shown in the figure.

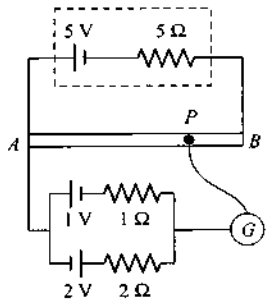


Fig. A2.93

a. The null point is at A
b. If the jockey is touched to point B, the current in the galvanometer will be going towards B
c. When jockey is connected to point A, no current flows through 1 V battery
d. The null point is at distance of 8/15 m from A_2

14. The diagram shows a modified meter bridge, which is used for measuring two unknown resistances at the same time. When only the first galvanometer is used, for obtaining the balance point, it is found at point C. Now the first galvanometer is removed and the second galvanometer is used, which gives balance point at D. Using the details given in the diagram, find out the value of R_1 and R_2.

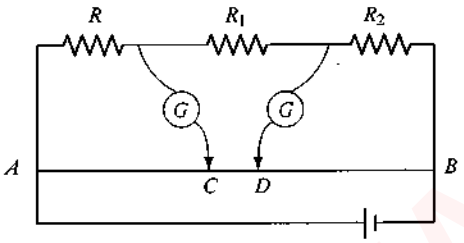


Fig. A2.94

a. $R_1 = 5R/3$ b. $R_2 = 4R/3$
c. $R_1 = 4R/3$ d. $R_2 = 5R/3$

15. A parallel-plate capacitor of capacitance 10 μF is connected to a cell of e.m.f 10 V and is fully charged. Now a dielectric slab ($k = 3$) of thickness equal to the gap between the plates is completely filled in the gap, keeping the cell connected. During the filling process,
a. the increase in charge on the capacitor is 200 μC
b. the heat produced is zero
c. energy supplied by the cell = increase in stored potential energy + work done on the person who is filling the dielectric slab
d. energy supplied by the cell = increase in stored potential energy + work done on the person who is filling the dielectric slab + heat produced

16. The galvanometer shown in the figure has resistance 10 Ω. It is shunted by a series combination of a resistance $S = 1\ \Omega$ and an ideal cell of e.m.f. 2 V. A current 2 A passes as shown.

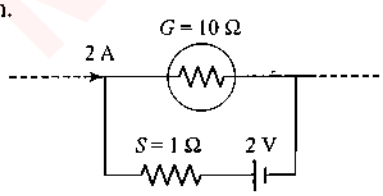


Fig. A2.95

a. The reading of the galvanometer is 1 A
b. The reading of the galvanometer is zero
c. The potential difference across the resistance S is 1.5 V
d. The potential difference across the resistance S is 2 V

17. In the circuit shown in figure, E_1 and E_2 are two ideal sources of unknown e.m.f.s. Some currents are shown. Potential difference appearing across 6 Ω resistance is $V_A - V_B = 10$ V.

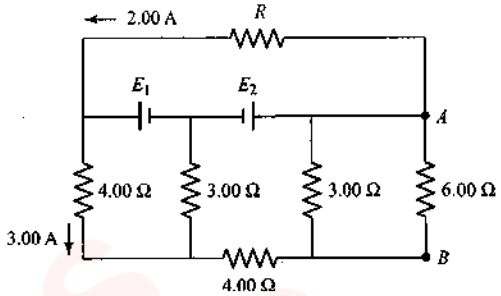


Fig. A2.96

a. The current in the 4.00 Ω resistor is 5 A
b. The unknown e.m.f. E_1 is 36 V
c. The unknown e.m.f. E_2 is 54 V
d. The resistance R is equal to 9 Ω

18. Consider a resistor of uniform cross-sectional area connected to a battery of internal resistance zero, if the length of the resistor is doubled by stretching it, then
a. the current will become four times
b. the electric field in the wire will become half
c. the thermal power produced by the resistor will become one fourth
d. the product of the current density and conductance will become half

19. A variable current flows through a 1 Ω resistor for 2 s. Time dependence of the current is shown in the graph.

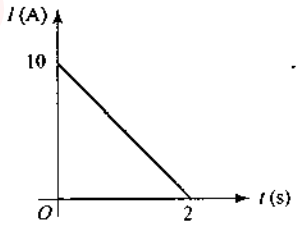


Fig. A2.97

a. Total charge flown through the resistor is 10 C
b. Average current through the resistor is 5 A
c. Total heat produced in the resistor is 50 J
d. Maximum power during the flow of current is 100 W

20. A conductor made of an isotropic material has shape of a truncated cone. A battery of constant e.m.f. is connected across it and its left end is earthed as shown in the figure.

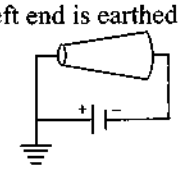
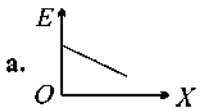
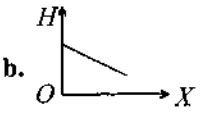
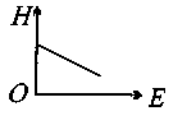


Fig. A2.98

If at a section distant x from left end, electric field intensity, potential and time rate of generation of heat per unit length are E, V and H, respectively, which of the following graphs is/are correct?

a. 

b. 

c. 

d.

Assertion-Reasoning Type

Solutions on page A2.46

a. Statement I is true, Statement II is true; Statement II is a correct explanation for Statement I.
b. Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I.
c. Statement I is true, Statement II is false.
d. Statement I is false, Statement II is true.

1. **Statement I:** If an electric field is applied to a metallic conductor, the free electrons experience a force but do not accelerate; they only drift at a constant speed.
Statement II: The force exerted by the electric field is completely balanced by the Coulomb force between electrons and protons.

2. **Statement I:** In the meter bridge experiment shown in Fig. A2.99, the balance length AC corresponding to null deflection of the galvanometer is x. If the radius of the wire AB is doubled, the balanced length becomes $4x$.

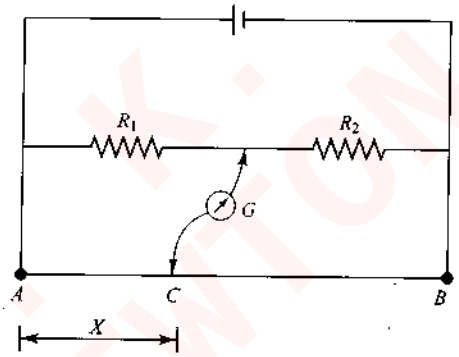


Fig. A2.99

Statement II: The resistance of a wire is inversely proportional to the square of its radius.

3. **Statement I:** In the potentiometer circuit shown in Fig. A2.100, E_1 and E_2 are the e.m.f. of cells C_1 and C_2, respectively, with $E_1 > E_2$. Cell C_1 has negligible internal resistance. For a given resistor R, the balance length is x. If the diameter of the potentiometer wire AB is increased, the balance length x will decrease.

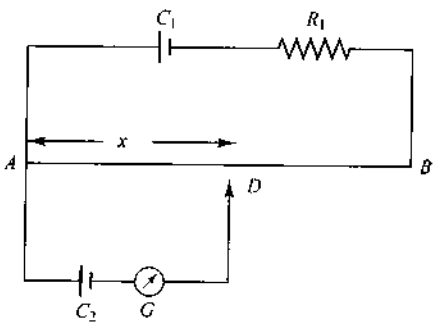


Fig. A2.100

Statement II: At the balance point, the potential difference between AD due to cell C_1 is E_2, the e.m.f. of cell C_2.

4. **Statement I:** When an external resistor of resistance R (connected across a cell to internal resistance r) is varied, power consumed by resistance R is maximum when $R = r$.
Statement II: Power consumed by a resistor of constant resistance R is maximum when current through it is maximum.

5. **Statement I:** The current density $\vec{J}$ at any point in ohmic resistor is in the direction of electric field $\vec{E}$ at that point.
Statement II: A point charge when released from rest in a region having only electrostatic field always moves along electric lines of force.

6. **Statement I:** A wire of uniform cross section and uniform resistivity is connected across an ideal cell. Now the length of the wire is doubled keeping volume of the wire constant. The drift velocity of electrons after stretching the wire becomes one-fourth of what it was before stretching the wire.
Statement II: If a wire (of uniform resistivity and uniform cross section) of length l_0 is stretched to length nl_0, then its resistance becomes n^2 times of what it was before stretching the wire (the volume of wire is kept constant in stretching process). Further at constant potential difference, current is inversely proportional to resistance. Finally, drift velocity of free electron is directly proportional to current and inversely proportional to cross-sectional area of current carrying wire.

7. **Statement I:** The switch S shown in Fig. A2.101 is closed at $t = 0$. Initial current flowing through battery is $E/(R + r)$.

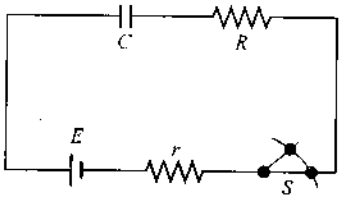


Fig. A2.101

Statement II: Initially, capacitor was uncharged, so the resistance offered by capacitor at $t = 0$ is zero.

8. **Statement I:** Consider the two situations shown in Fig. A2.102. Potential difference between points A and B in Case I is more as compared to Case II.
Statement II: In Case I, $V_A - V_B = E + Ir$
In Case II, $V_A - V_B = E - Ir$

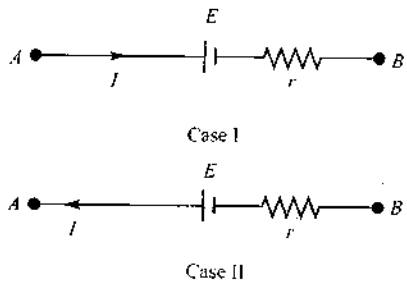


Fig. A2.102

9. **Statement I:** Electric field outside the conducting wire which carries a constant current is zero.

Statement II: Net charge on conducting wire is zero.

10. **Statement I:** A conductor carrying electric current becomes electrically charged.
Statement II: A conductor carrying electric current contains same number of positive and negative charges and thus conductor is electrically neutral.

11. **Statement I:** In the following circuit, e.m.f. is 2 V, internal resistance of the cell is 1 Ω and R = 1 Ω, the reading of the voltmeter is 1 V.
Statement II: $V = E - ir$, where E = 2V, i = 1A and R = 1 Ω.

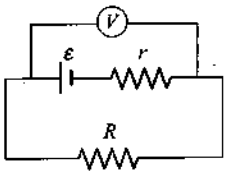


Fig. A2.103

12. **Statement I:** Direction of current cannot be from negative potential.
Statement II: Direction of current is opposite to the flow of electrons.

13. **Statement I:** When a cell is charged by connecting its positive electrode with positive terminal of the charger battery, then the potential difference across the electrodes of cell will be smaller to the e.m.f. of cell.
Statement II: Potential difference across electrodes in a cell providing electric current is $E - Ir$, where E is e.m.f. and r internal resistance.

14. **Statement I:** The drift velocity of electrons in a metallic wire will decrease, if the temperature of the wire is increased.
Statement II: On increasing temperature, the conductivity of the metallic wire decreases.

15. **Statement I:** In a simple battery circuit, the point at the lowest potential is the positive terminal of the battery.
Statement II: The current flows towards the point of lowest potential for battery, as it does in a circuit from the positive to negative terminal.

Comprehension Type

Solutions on page A2.46

For Problems 1–5

Ram and Shyam purchased two electric tea kettles A and B of same size, same thickness and same volume of 0.4 litre. They studied the specification of kettles as under

Kettle A:
Specific heat capacity = 1680 J/kgK
Mass = 200 g
Cost = Rs. 400

Kettle B:
Specific heat capacity = 2450 J/kgK
Mass = 400 g
Cost = Rs. 400

When kettle A is switched on with constant potential source, the tea begins to boil in 6 min. When kettle B is switched on with the same source separately, then tea begins to boil in 8 min. The efficiency of kettle is defined as

$$\frac{\text{Energy used for liquid heating}}{\text{Total energy supplied}}$$

They made discussion on specification and efficiency of kettles and subsequently prepared a list of questions to draw the conclusions. Some of them are as under (Assume specific heat of tea liquid as 4200 J/kgK and density 1000 kg/m³.)

1. Efficiency of kettle A is
a. 63.34 % **b.** 83.34 % **c.** 93.34 % **d.** 73.34 %

2. Efficiency of kettle B is
a. 82.5 % **b.** 72.5 % **c.** 92.5 % **d.** 62.5 %

3. Ratio of efficiency consumed charges for one time boiling of tea in kettle A to that in kettle B
a. 3 : 5 **b.** 2 : 3 **c.** 3 : 4 **d.** 1 : 1

4. If resistances of coil of kettles A and B are R_A and R_B, respectively, then we can say
a. $R_A > R_B$ **b.** $R_A = R_B$ **c.** $R_A < R_B$
d. cannot be ascertained by above date

5. If both the kettles are joined with the same source in series one after the other. Then boiling starts in kettle A and kettle B after
a. 4 times of their original time
b. equal to their original time
c. 2 times of their original time
d. cannot be ascertained by the above data

For Problems 6–8

A car battery with a 12 V e.m.f. and an internal resistance of 0.04 Ω is being charged with a current of 50 A.

6. The potential difference V across the terminals of the battery is
a. 10 V **b.** 12 V **c.** 14 V **d.** 16 V

7. The rate at which energy is being dissipated as heat inside the battery is
a. 100 W **b.** 500 W **c.** 600 W **d.** 700 W

8. The rate of energy conservation from electrical to chemical is
a. 100 W **b.** 500 W **c.** 600 W **d.** 700 W

For Problems 9–12

By ammeters and voltmeters in combination

A voltmeter and an ammeter can be used together to measure resistance and power. The resistance R of a resistor equals the potential difference V_{ab} between its terminals divided by the current I; that is, $R = V_{ab}/I$. The power input P to any circuit element is the product of the potential difference across it and the current through it; that is, $P = V_{ab} I$. In principle, the most straightward way to measure R or P is to measure V_{ab} and I simultaneously.

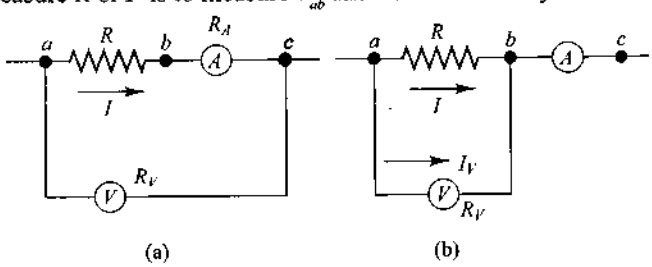


Fig. A2.104

By Ohmmeters

An alternative method for measuring resistance is to use a d' Arsonval meter in an arrangement called an ohmmeter. It consists of a meter, a resistor and a source (often a flashlight battery) connected in series (Fig. A2.105). The resistance R to be measured is connected between terminals X and Y.

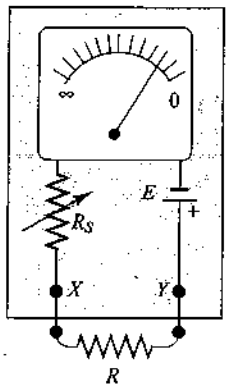
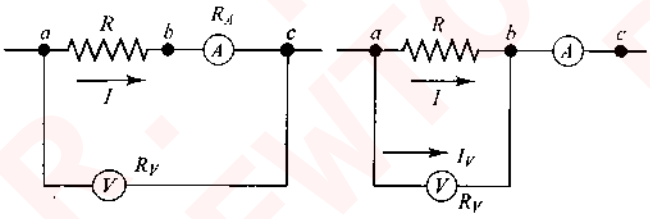
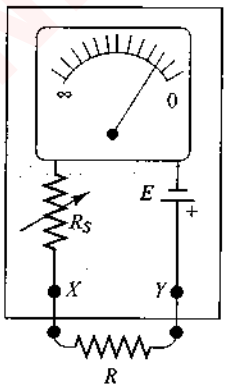


Fig. A2.105

The series resistance R_s is variable; it is adjusted so that when terminals X and Y are short-circuited (that is, when $R = 0$), the meter shows full-scale deflection. When nothing is connected to terminals X and Y, so that the circuit between X and Y is open (that is, when $x \to \infty$), there is no current and no deflection. For any intermediate value of R the meter deflection depends on the value of R, and the meter scale can be calibrated to read the resistance R directly. Larger currents correspond to smaller resistance, so this scale reads backward compared to the scale showing the current.

Measurement of Resistances





(c)

Fig. A2.106

9. Suppose we want to measure an unknown resistance R, using the circuit in Fig. A2.106 (a). The voltmeter resistance and ammeter resistance are, respectively, $R_v = 10{,}000\ \Omega$ and $R_A = 2.00\ \Omega$. If the voltmeter reads 12.0 V and the ammeter reads 0.100 A, then resistance R is

a. 120 Ω **b.** 118 Ω **c.** 121 Ω **d.** 108 Ω

10. Suppose the meters in the above problem are connected to a different resistor in the circuit of Fig. A2.106 (b), and the readings obtained on the meters are the same as in the above problem. The value of this new resistance is

a. 120 Ω **b.** 118 Ω **c.** 121 Ω **d.** 108 Ω

11. In the ohmmeter in Fig. A2.106 (c), the coil of the meter has resistance $R_C = 50\ \Omega$, and the current required for full-scale deflection is 1.50 mA, the source is a flash light battery with $E = 0.75$ V having negligible internal resistance. The ohmmeter is to show a meter deflection of 1/2 of full scale when it is connected to a resistor with $R = 500$ W. The series resistance R_s required is equal to

a. 50 Ω **b.** 500 Ω **c.** 450 Ω **d.** 45 Ω

12. In Fig. A2.107, 1.50 mA ohmmeter having a resistance of 100 Ω (at current of 1.50 mA, meter shows full-scale deflection). The battery has an e.m.f. of 0.9 V and negligible internal resistance R is chosen so that when the terminals a and b are shorted, the meter reads 3/4 of full scale. When a and b are open, the meter reads zero. The corresponding value of R is

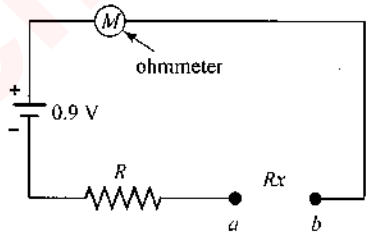


Fig. A2.107

a. 100 Ω **b.** 800 Ω **c.** 700 Ω **d.** 900 Ω

For Problems 13–15

Important Aspect of Fuse Wire Nd Battery

Electric fuse is a protective device used in series with an electric circuit or an electric appliance to save it from damage due to overheating produced by strong current in the circuit or application. Fuse wire is generally made from an alloy of lead and tin which has high resistance and low melting point. It is connected in series in an electric installation. If a circuit gets accidentally short-circuited, a large current flows, then fuse wire melts away which causes a break in the circuit. The power through fuse (F') is equal to heat energy lost per unit area per unit time (h) (neglecting heat loses from ends of the wire).

$$P = I^2 R = h \times 2\pi r l \quad \left[R = \frac{\rho l}{\pi r^2}\right]$$

where r and l are the length and radius of fuse wire, respectively.

A battery is described by its e.m.f. (E) and internal resistance (r). Efficiency of a battery (η) is defined as the ratio of the output power to the input power

$$\eta = \frac{\text{Output power}}{\text{Input power}} \times 100\%$$

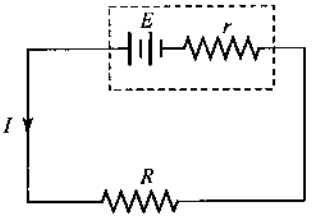


Fig. A2.108

but $I = \dfrac{E}{R+r}$, input power $= EI$

Output power $= EI - I^2r$

Then
$$\eta = \left(\frac{EI - I^2r}{EI}\right)\times 100\left(1-\frac{Ir}{E}\right)\times 100$$
$$= 1-\left(\frac{E}{R+r}\right)\left(\frac{r}{E}\right)\times 100$$
$$\eta = \left(\frac{R}{R+r}\right)\times 100$$

We know that output power of a source is maximum when the external resistance is equal to internal resistance, i.e., $R = r$.

13. Two fuse wires of same potential material are having length ratio 1 : 2 and ratio 4:1. Then respective ratio of their current rating will be
a. 8 : 1 **b.** 2 : 1 **c.** 1 : 8 **d.** 4 : 1

14. The maximum power rating of a 20.0 Ω fuse wire is 2.0 kW, then this fuse wire can be connected safely to a D.C. source (negligible internal resistance) of
a. 300 volt **b.** 190 volt **c.** 250 volt **d.** 220 volt

15. Efficiency of a battery (non-ideal) when delivering the maximum power is
a. 100% **b.** 50% **c.** 90% **d.** 40%

For Problems 16–18

In the connection shown in the figure, initially the switch K is open and the capacitor is uncharged. Then the switch is closed and the capacitor is charged up to the steady state and the switch is opened again. Determine the values indicated by the ammeter. [Given: $V_0 = 30$ V, $R_1 = 10$ kΩ, $R_2 = 5$ kΩ]

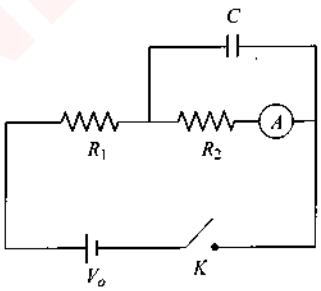


Fig. A2.109

16. Just after closing the switch
a. 2 mA **b.** 3 mA **c.** 0 mA **d.** None of these

17. A long time after the switch was closed
a. 2 mA **b.** 3 mA **c.** 6 mA **d.** none of these

18. Just after reopening the switch
a. 2 mA **b.** 3 mA **c.** 6 mA **d.** None of these

For Problems 19–21

Resistance value of an unknown resistor is calculated using the formula $R = V/I$ where V and I are the readings of the voltmeter and the ammeter, respectively. Consider the circuits below. The internal resistances of the voltmeter and the ammeter (R_V and R_G respectively) are finite and non-zero.

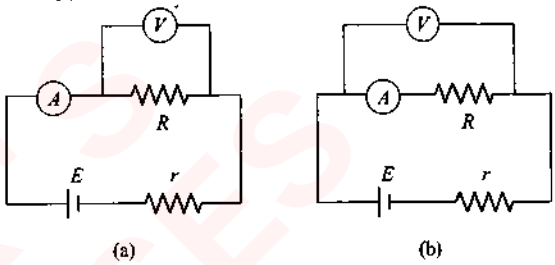


Fig. A2.110

Let R_A and R_B be the calculated values in the two cases A and B, respectively.

19. The relation between R_A and the actual value of R is
a. $R > R_A$ **b.** $R < R_A$
c. $R = R_A$ **d.** dependent upon E and r

20. The relation between R_B and the actual value of R is
a. $R < R_B$ **b.** $R > R_B$
c. $R = R_B$ **d.** dependent upon E and r

21. If the resistance of voltmeter is $R_V = 1$ kΩ and that of ammeter is $R_G = 1$ Ω, the magnitude of the percentage error in the measurement of R (the value of R is nearly 10 Ω) is
a. zero in both cases
b. non-zero but equal in both cases
c. more in circuit A
d. more in circuit B

For Problems 22–24

In the circuit given in the figure, both batteries are ideal. e.m.f. E_1 of battery 1 has a fixed value, but e.m.f. E_2 of battery 2 can be varied between 1.0 V and 10.0 V. The graph gives the currents through the two batteries as a function of E_2, but are not marked as which plot corresponds to which battery. But for both plots, current is assumed to be negative when the direction of the current through the battery is opposite to the direction of that battery's e.m.f. (direction of e.m.f. is from negative to positive).

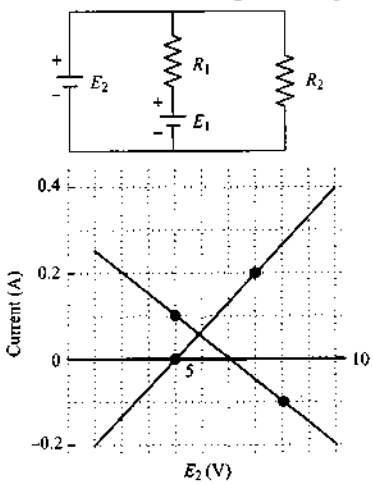


Fig. A2.111

22. The value of e.m.f. E_1 is
a. 8 V b. 6 V
c. 4 V d. 2 V

23. The resistance R_1 has value
a. 10 Ω b. 20 Ω
c. 30 Ω d. 40 Ω

24. The resistance R_2 is equal to
a. 10 Ω b. 20 Ω
c. 30 Ω d. 40 Ω

For Problems 25–27

The circuit shown in a steady state:

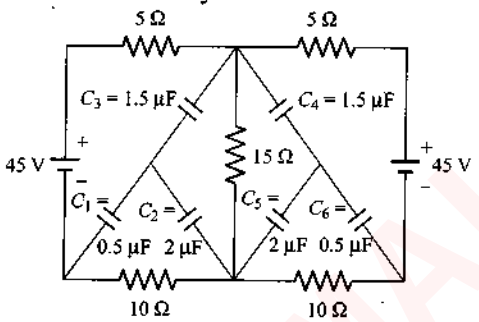


Fig. A2.112

25. The charge in capacitor C_1 is
a. 20 μC b. 30 μC
c. 40 μC d. 10 μC

26. The charge in capacitor C_2 is
a. 30 μC b. 10 μC
c. 20 μC d. 40 μC

27. The charge in capacitor C_3 is
a. 10 μC b. 30 μC
c. 20 μC d. 40 μC

For Problems 28–30

Most of the times we connect remote speakers to play music in another room along with the buit-in speakers. These speakers are connected in parallel with the music system.

At the instant represented in the picture, the voltage across the speakers is 6.00 V. The resistance of the main speaker is 8 Ω and the remote speaker has resistance 4 Ω.

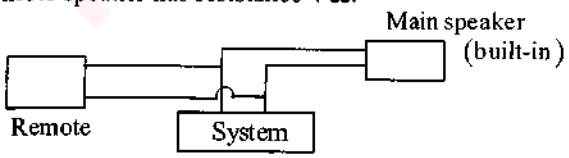


Fig. A2.113

28. The equivalent resistance of the speakers is
a. 12 Ω b. 7/3 Ω
c. 8/3 Ω d. 0.375 Ω

29. The total current supplied by the music system is
a. 2.25 A b. 1 A
c. 16 A d. 1.5 A

30. The power dissipated in the speaker of 4 Ω resistance is
a. 9 W b. 4.5 W
c. 13.5 W d. 0

Matching Column Type

Solutions on page A2.48

Column I and column II contains four entries each. Entries of column I are to be matched with some entries of column II. One or more than one entries of column I may have the matching with the same entries of column II and one entry of column I may have one or more than one matching with entries of column II.

1. A battery of e.m.f. E is connected across a conductor as shown in Fig. A2.114. As one observes from A to B, match the following:

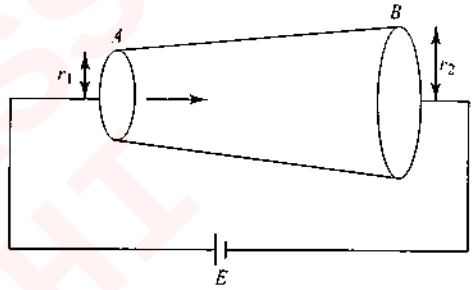


Fig. A2.114

Column I	Column II
a. Current	p. increases
b. Drift velocity of electron	q. decreases
c. Electric field	r. remains same
d. Potential drop across the length	s. cannot be determined

2. Column I gives physical quantities based on a situation in which an ideal cell of e.m.f. V is connected across a cylindrical rod of uniform cross-sectional area and conductivity (s) as shown in the figure. E, J, ϕ and i are electric field at, current density through, electric flux through and current through the shaded cross section, respectively, as shown in Fig. A2.115 Physical quantities in column II are related to those in column I. Match the expressions in column I with the statements in columns II.

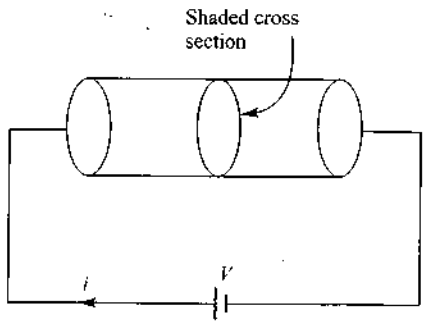


Fig. A2.115

Column I	Column II
a. $\frac{\phi}{i}$	p. Conductivity of the rod
b. $\frac{E}{J}$	q. Resistance of the rod
c. $\sigma\phi V$	r. Resistivity of the rod
d. $\frac{V}{\sigma\phi}$	s. Power delivered to the rod

3. In the circuit shown in Fig. A2.116, battery, ammeter and voltmeter are ideal and the switch S is initially closed as shown. When switch S is opened, match the parameter in column I with the effects in column II and indicate your answer by darkening appropriate bubbles in the 4×4 matrix given in the OMR.

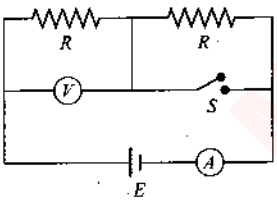


Fig. A2.116

Column I	Column II
a. Equivalent resistance across the battery	p. remains same
b. Power dissipated by left resistance R	q. increases
c. Voltmeter reading	r. decreases
d. Ammeter reading	s. becomes zero

4. Column I gives physical quantities of a situation in which current i passes through two rods I and II of equal length that are joined in series. The ratio of free electron density (η), resistivity (ρ) and cross-sectional area (A) of both rods are in ratio $n:n_2 = 2:1$; $r_2 = 2:1$ and $A_1:A_2 = 1:2$, respectively. Column II gives corresponding results. Match the ratios in column I with the values in column II and indicate your answer by darkening appropriate bubbles in the 4×4 matrix given in the OMR.

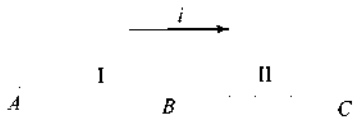


Fig. A2.117

Column I	Column II
a. $\frac{\text{Drift velocity of free electron in rod I}}{\text{Drift velocity of free electron in rod II}}$	p. 0.5
b. $\frac{\text{Electric field in rod I}}{\text{Electric field in rod II}}$	q. 1
c. $\frac{\text{Potential difference across rod I}}{\text{Potential difference across rod II}}$	r. 2
d. $\frac{\text{Average time taken by free electron to move from } A \text{ to } B}{\text{Average time taken by free electron to move from } B \text{ to } C}$	s. 4

5. Match the statements in column I with the results in column II

Column I	Column II
a. A variable resistor is connected across a non-ideal cell. As the resistance of the variable resistor is continuously increased from zero to a very large value, the electric power consumed by the variable resistor	p. first increases for some time and then decreases
b. A circular ring lies in space having uniform and constant magnetic field. Initially, the direction of magnetic field is parallel to the plane of the ring. Keeping the centre of ring fixed, the ring is rotated by 180° about one of its diameters with constant angular speed. For the duration the ring rotates, the magnitude of the induced e.m.f. in the ring	q. first decreases for some time and then increases
c. A thin rod of length 1 cm lies along the principal axis of a convex lens of focal length 5 cm. One end of rod is at a distance 1 cm from optical centre of the lens. The convex lens is moved (without rotation) perpendicular to the initial principal axis by 5 mm and brought back to its initial position. The length of the image of the rod	r. is always constant
d. A bulb (of negligible inductance) and a capacitor in series are connected across an ideal ac source of constant peak voltage and variable frequency. As frequency of ac source is continuously increased, the brightness of bulb	s. increases or may increase over some time interval

6. In the circuit shown, all the ammeters are ideal. Match the following based on the circuit

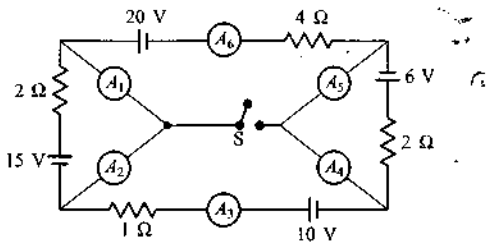


Fig. A2.118

Column I	Column II
a. If switch S is open, the ammeter(s) that read(s) less than 10 A	p. A_1 and A_2
b. If switch S is open, the ammeter(s) that read(s) equal current	q. A_4 and A_5
c. If switch S is closed, the ammeter(s) that read(s) more than 5 A	r. A_3 and A_4
d. If switch S is closed, the ammeter(s) that show(s) increase in the reading	s. A_6 and A_5

7. Consider two identical cells each of e.m.f. E and internal resistance r connected to a load resistance R.

Column I	Column II
a. Maximum power transferred to load if cells are connected in series	p. $\frac{4E^2}{9r}$
b. Maximum power transferred to load if cells are connected in parallel	q. $\frac{E^2}{2r}$
c. Power transferred to load if cells are connected in series and $R = r$	r. $E_{eq} = E$, $r_{eq} = \frac{r}{2}$
d. Power transferred to load if cells are connected in parallel and $R = r$	s. $E_{eq} = 2E$, $r_{eq} = 2r$

8. In an experiment for comparing e.m.f.s of two primary cells using potentiometer, some observations are given in column I.

Column I	Column II
a. Deflection of galvanometer is in same direction at two ends of the wire	p. Accuracy in measurement increase
b. A series protective resistance added in series to the galvanometer	q. The positive terminals of all batteries/cells are not connected at a point
c. A short wire is used in potentiometer	r. e.m.f. of battery in primary circuit is less than the e.m.f. of cell to be measured
d More length of wire up to null point	s. Uncertainity in the location of balance point increases

Integer Answer Type

Solutions on page A2.50

1. A 10 m long nichrome wire having 80 Ω resistance has current carrying capacity of 5 A. This wire can be cut into equal parts and equal parts can be connected in series or parallel. What is the maximum power which can be obtained as heat by the wire from a 200 V mains supply? (in kW)
2. The maximum current in a galvanometer can be 10 mA. Its resistance is 10 Ω. To convert it into an ammeter of 1 A, what resistance should be connected in parallel with galvanometer (in 10^{-1} Ω)?
3. Find current in the branch CD of the circuit (in ampere).

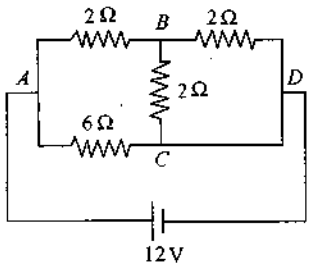


Fig. A2.119

4. Find the current (in A) in the rightmost resistor shown in figure.

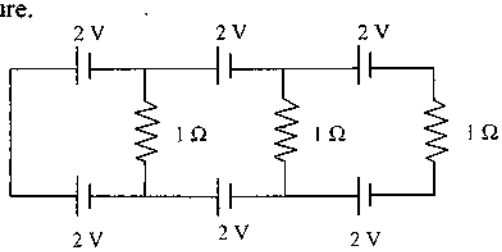


Fig. A2.120

5. The current in a conductor and the potential difference across its ends are measured by an ammeter and a voltmeter, respectively. The voltmeter draws negligible currents. The ammeter is accurate but the voltmeter has a zero error (that is, it does not read zero when no potential difference is applied). Calculate the zero error if the readings for two different conditions are 1.75 A, 14.4 V and 2.75 A, 22.4 V. (in × 10^{-1} V)

6. An electric current of 2.0 A passes through a wire of resistance 25 Ω. How much heat will be developed in 1 min ? (in $\times 10^3$ J)
7. At what value of the resistance in the circuit shown in the figure will the total resistance between points A and B be independent of the number of cells? If $R = (\sqrt{3}+1)\,\Omega$, then the value of x will be "....." Ω.

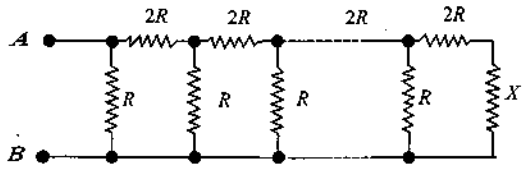


Fig. A2.121

8. In the given circuit, each resistance is r = 18.75 Ω. The current (in A) in the resistance connected across A and B is '....' A.

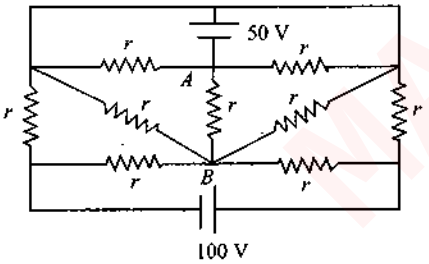


Fig. A2.122

Archives

Solutions on page A2.50

Fill in the Blanks Type

1. An electric bulb rated for 500 watts at 100 volts is used in a circuit having a 200 volts supply. The resistance R that must be put in series with the bulb, so that the bulb delivers 500 watts is_____ohms. **(IIT-JEE, 1987)**
2. The equivalent resistance between points A and B in the circuit (Fig. A2.123) is _____ Ω.

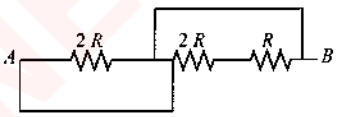


Fig. A2.123

(IIT-JEE, 1997)

3. In the circuit shown (Fig. A2.124), each battery is 5 V and has an internal resistance of 0.2 ohm.

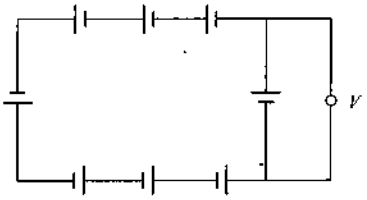


Fig. A2.124

The reading in the ideal voltmeter V is _____ V.

True or False Type

1. In an electrolyte solution, the electric current is mainly due to movement of free electrons. **(IIT-JEE, 1980)**
2. Electrons in a conductor have no motion in the absence of a potential difference across it. **(IIT-JEE, 1981)**
3. The current–voltage graphs for a given metallic wire at two different temperature T_1 and T_2 are shown in Fig. A2.125

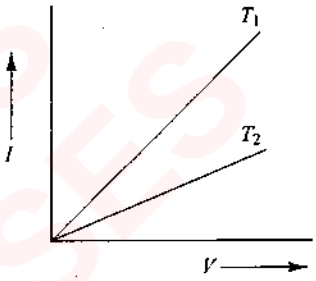


Fig. A2.125

The temperature T_1 is greater than T_2. **(IIT-JEE, 1993)**

Multiple Choice Questions with One Correct Answer Type

1. The temperature coefficient of resistance of a wire is 0.00125 per °C. At 300 K, its resistance is 1 ohm. This resistance of the wire will be 2 ohm at
   a. 1154 K b. 1100 K
   c. 1400 K d. 1127 K **(IIT-JEE, 1980)**
2. A constant voltage is applied between the two ends of a uniform metallic wire. Some heat is developed in it. The heat developed is doubled if
   a. both the length and the radius of the wire are halved
   b. both the length and the radius of the wire are doubled
   c. the radius of the wire is doubled
   d. the length of the wire is doubled **(IIT-JEE, 1980)**
3. The electrostatic field due to a point charge depends on the distance r as $1/r^2$. Indicate which of the following quantities shows same dependence on r.
   a. Intensity of light from a point source.
   b. Electrostatic potential due to a point charge.
   c. Electrostatic potential at a distance r from the centre of a charged metallic sphere. Given r < radius of the sphere.
   d. None of these **(IIT-JEE, 1980)**
4. In the circuit shown in the figure the heat produced in the 5 ohm resistor due to the current flowing through it is 10 calories per second.
   The heat generated in the 4 ohm resistor is

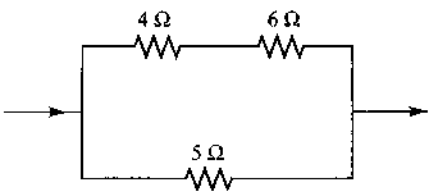


Fig. A2.126

a. 1 calorie/sec
b. 2 calories/sec
c. 3 calories/sec
d. 4 calories/sec

(IIT-JEE, 1981)

5. The current i in the circuit is

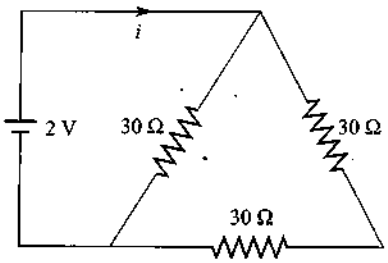


Fig. A2.127

a. 1/45 A
b. 1/15 A
c. 1/10 A
d. 1/5 A **(IIT-JEE, 1983)**

6. A piece of copper and another of germanium are cooled from room temperature to 80 K. The resistance of
a. each of them increases
b. each of them decreases
c. copper increases and germanium decreases
d. copper decreases and germanium increases

(IIT-JEE, 1981)

7. A battery of internal resistance 4 Ω is connected to the network of resistances as shown. In order that the maximum power can be delivered to the network, the value of R in Ω should be

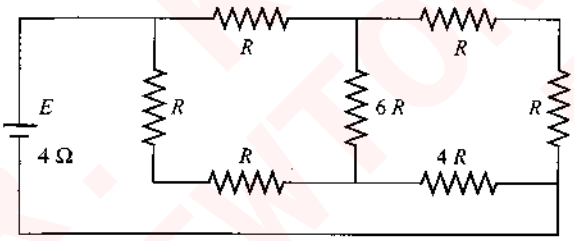


Fig. A2.128

a. 4/9
b. 2
c. 8/3
d. 18 **(IIT-JEE, 1995)**

8. A steady current flows in a metallic conductor of non-uniform cross section. The quantity/quantities constant along the length of the conductor is/are
a. current, electric field and drift speed
b. drift speed only
c. current and drift speed
d. current only **(IIT-JEE, 1997)**

9. A parallel combination of 0.1 MΩ resistor and a 10 μF capacitor is connected across a 1.5 V source of negligible resistance. The time required for the capacitor to get charged up to 0.75 V is approximately (in seconds)
a. ∞
b. $\log_e 2$
c. $\log_{10} 2$
d. Zero

(IIT-JEE, 1997)

10. In the circuit $P \neq R$, the reading of the galvanometer is same with switch S open or closed. Then

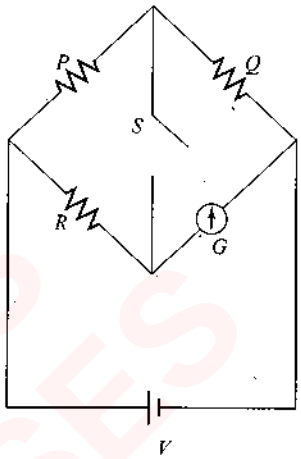


Fig. A2.129

a. $I_R = I_G$
b. $I_P = I_G$
c. $I_Q = I_G$
d. $I_Q = I_R$ **(IIT-JEE, 1999)**

11. In the given circuit, with steady current, the potential drop across the capacitor must be

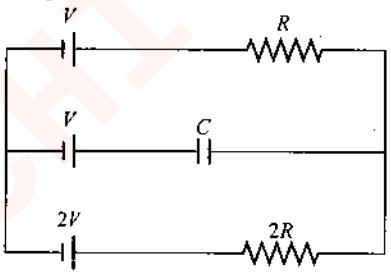


Fig. A2.130

a. V
b. $V/2$
c. $V/3$
d. $2V/3$ **(IIT-JEE, 2001)**

12. A wire of length L and three identical cells of negligible internal resistances are connected in series. Due to the current, the temperature of the wire is raised by ΔT in a time t. A number N of similar cells is now connected in series with a wire of the same material and cross section but of length $2L$. The temperature of the wire is raised by the same amount ΔT in the same time t, the value of N is
a. 4
b. 6
c. 8
d. 9 **(IIT-JEE, 2001)**

13. In the given circuit (Fig. A2.131), it is observed that the current I is independent of the value of the resistance R_6. Then the resistance values must satisfy

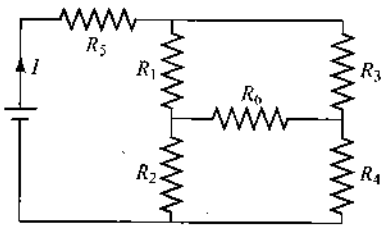


Fig. A2.131

a. $R_1 R_2 R_5 = R_3 R_4 R_6$

b. $\dfrac{1}{R_5} + \dfrac{1}{R_6} = \dfrac{1}{R_1 + R_2} + \dfrac{1}{R_3 + R_4}$

c. $R_1R_4 = R_2R_3$

d. $R_1R_3 = R_2R_4 = R_5R_6$ **(IIT-JEE, 2001)**

14. The effective resistance between point P and Q of the electrical circuit shown in Fig. A2.132 is

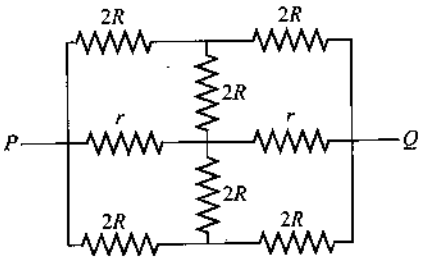


Fig. A2.132

a. $\frac{2Rr}{R+r}$ b. $\frac{8R(R+r)}{3R+r}$

c. $2r+4R$ d. $\frac{5R}{2}+2r$

(IIT-JEE, 2002)

15. A 100 W bulb B_1 and two 60 W bulbs B_2 and B_3 are connected to a 250 V source, as shown in Fig. A2.133. Now W_1, W_2 and W_3 are the output powers of the bulbs B_1, B_2 and B_3, respectively. Then

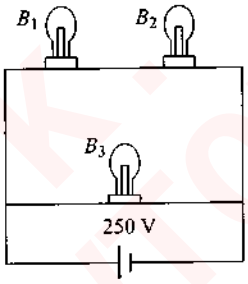


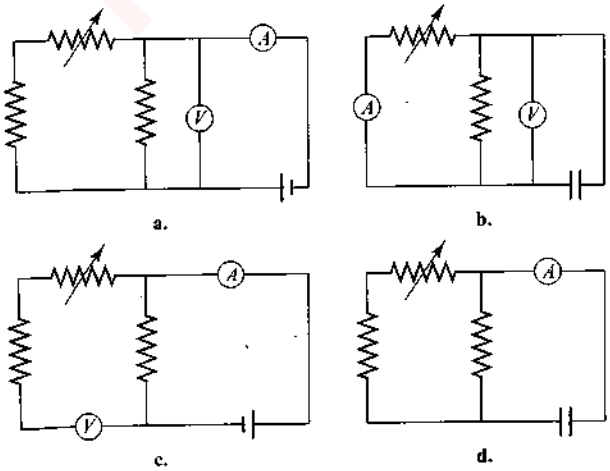
Fig. A2.133

a. $W_1 > W_2 = W_3$ b. $W_1 > W_2 > W_3$

c. $W_1 < W_2 = W_3$ d. $W_1 < W_2 < W_3$

(IIT-JEE, 2002)

16. Express which of the following setups can be used to verify Ohm's law?



(IIT-JEE, 2003)

17. In the shown arrangement of the experiment of the meter bridge (Fig. A2.134) if AC corresponding to null deflection of galvanometer is x, what would be its value if the radius of the wire AB is doubled?

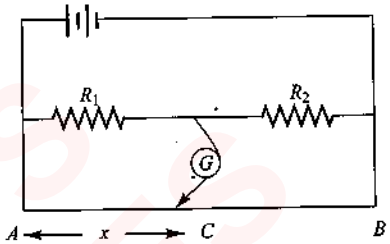


Fig. A2.134

a. x b. $x/4$

c. $4x$ d. $2x$ **(IIT-JEE, 2003)**

18. The three resistances of equal value are arranged in the different combinations (Fig. A2.135). Arrange them in the increasing order of power dissipation.

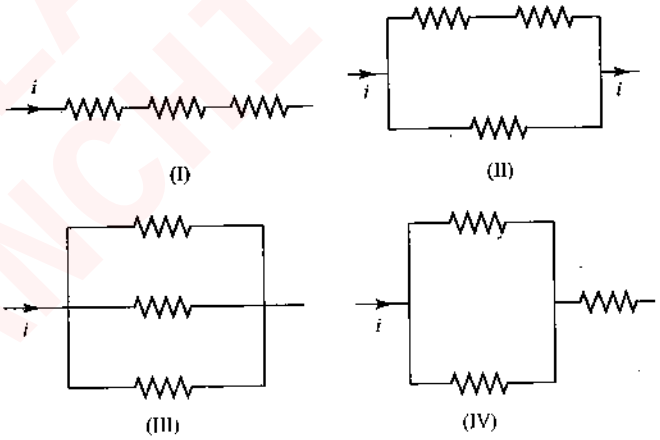


Fig. A2.135

a. III < II < IV < I b. II < III < IV < I

c. I < IV < III < II d. I < III < II < IV

(IIT-JEE, 2003)

19. Figure A2.136 shows a post office box. In order to calculate the value of external resistance, it should be connected between

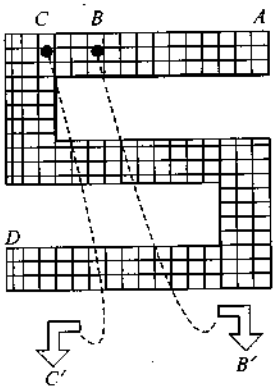


Fig. A2.136

a. B' and C' b. A and D

c. C and D d. B and D **(IIT-JEE, 2004)**

20. Six identical resistors are connected as shown in Fig. A2.137. The equivalent resistance will be

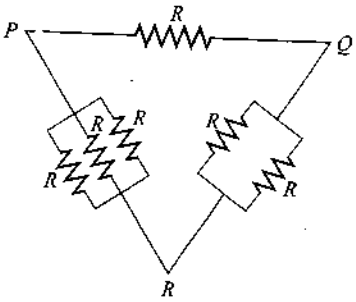


Fig. A2.137

a. Maximum between P and R
b. Maximum between Q and R
c. Maximum between P and Q
d. All are equal

(IIT-JEE, 2004)

21. A capacitor is charged using an external battery with a resistance x in series. The dashed line shows the variation of ln 1 with respect to time. If the resistance is changed to $2x$, the new graph will be

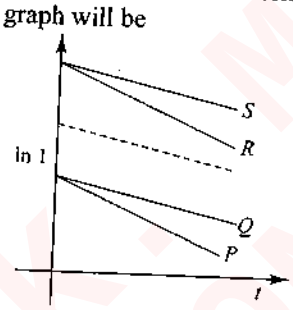


Fig. A2.138

a. P **b.** Q
c. R **d.** S

(IIT-JEE, 2004)

22. Find out the value of current through 2 Ω resistance for the given circuit (Fig. A2.139).

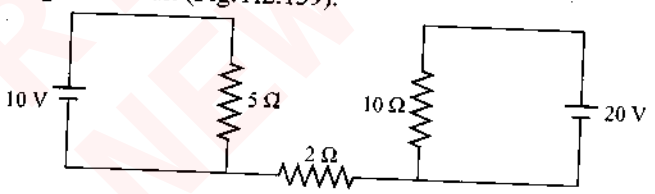


Fig. A2.139

a. 0 **b.** 2 A
c. 5 A **d.** 4 A

(IIT-JEE, 2005)

23. A 4 μF capacitor and a resistance of 2.5 MΩ are in series with 12 V battery. Find the time after which the potential difference across the capacitor is 3 times the potential difference across the resistor. [ln2 = 0.693]

a. 13.86 s **b.** 6.93 s
c. 7 s **d.** 14 s

(IIT JEE, 2005)

24. A moving coil galvanometer of resistance 100 Ω is used as an ammeter using a resistance 0.1 Ω. The maximum deflection current in the galvanometer is 100 μA. Find the minimum current in the circuit so that the ammeter shows maximum deflection.

a. 100.1 mA **b.** 1000.1 mA
c. 10.01 mA **d.** 1.01 mA

(IIT-JEE, 2005)

25. An ideal gas is filled in a closed rigid and thermally insulated container. A coil of 100 Ω resistor carrying current 1 A for 5 min supplies heat to the gas. The change in internal energy of the gas is

a. 10 kJ **b.** 30 kJ
c. 20 kJ **d.** 0 kJ

(IIT-JEE, 2005)

26. In Fig. A2.140, $R_1 = 1\ \Omega$, $R_2 = 2\ \Omega$, $C_1 = 4\ \mu F$, $C_2 = 2\ \mu F$, the time constants (in μs) for the circuits I, II, III, respectively, are

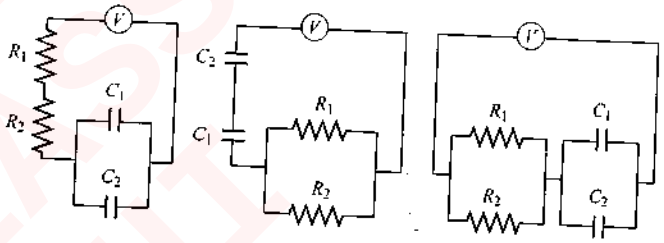


Fig. A2.140

a. 18, 4, 8/9 **b.** 18, 8/9, 4
c. 4, 18, 8/9 **d.** 4, 8/9, 18

(IIT-JEE, 2006)

27. Two bars of radius r and $2r$ are kept in contact as shown in Fig. A2.141. An electric current I is passed through the bars. Which of the following is correct?

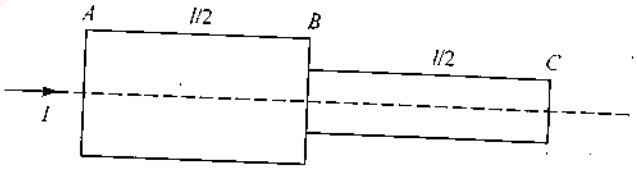


Fig. A2.141

a. $V_{AB} = 2V_{BC}$
b. Power across BC is 4 times the power across AB
c. Current density in AB and BC is equal.
d. Electric field due to current inside AB and BC is equal.

(IIT-JEE, 2006)

28. A resistance of 2 Ω is connected across one gap of a metre-bridge (the length of the wire is 100 cm) and an unknown resistance, greater than 2 Ω, is connected across the other gap. When these resistances are interchanged, the balance point shifts by 20 cm. Neglecting any corrections, the unknown resistance is

a. 3 Ω **b.** 4 Ω
c. 5 Ω **d.** 6 Ω

(IIT-JEE, 2007)

29. A circuit is connected as shown in Fig. A2.142 with the switch S open. When the switch is closed, the total amount of charge that flows from Y to X is

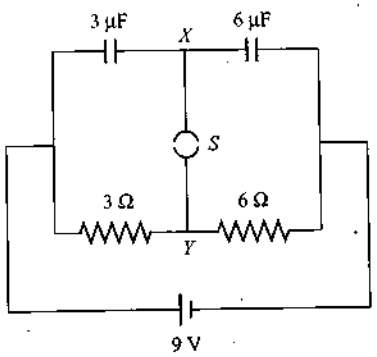


Fig. A2.142

a. 0 b. 54 μC
c. 27 μC d. 81 μC **(IIT-JEE, 2007)**

30. A parallel-plate capacitor C with plates of unit area and separation d is filled with a liquid of dielectric constant $K = 2$. The level of liquid is $d/3$ initially. Suppose the liquid level decreases at a constant speed V, the time constant as a function of time t is

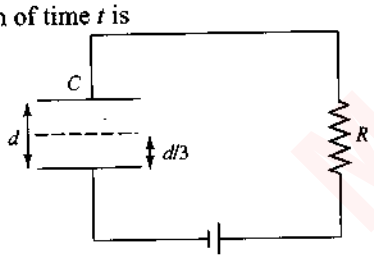


Fig. A2.143

a. $\dfrac{6\varepsilon_0 R}{5d+3Vt}$ b. $\dfrac{(15d+9Vt)\varepsilon_0 R}{2d^2-3dVt-9V^2t^2}$

c. $\dfrac{6\varepsilon_0 R}{5d-3Vt}$ d. $\dfrac{(15d-9Vt)\varepsilon_0 R}{2d^2+3dVt-9V^2t^2}$

(IIT-JEE, 2008)

31. Figure A2.144 shows three-resistor configuration R_1, R_2, R_3 connected to 3 V battery. If the power dissipated by the configuration R_1, R_2 and R_3 is P_1, P_2 and P_3 respectively, then

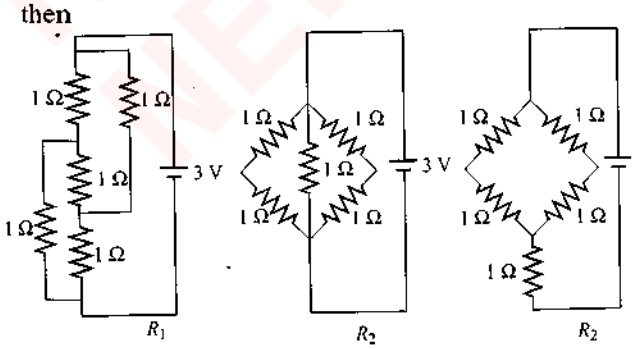


Fig. A2.144

a. $P_1 < P_2 < P_3$ b. $P_1 > P_2 > P_3$
c. $P_2 > P_1 > P_3$ d. $P_3 > P_2 > P_1$

(IIT-JEE, 2008)

32. Statement I: In a meter-bridge experiment, null point for an unknown resistance is measured. Now, the unknown resistance is put inside an enclosure maintained at a higher temperature. The null point can be obtained at the same point as before by decreasing the value of the standard resistance.

Statement II: Resistance of a metal increases with the increase in temperature.

a. Statement I is true, Statement II is true; Statement II is a correct explanation for Statement I.
b. Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I.
c. Statement I is true, Statement II is false.
d. Statement I is false, Statement II is true. **(IIT-JEE, 2008)**

33. The equivalent resistance of the circuit across points A and B is equal to

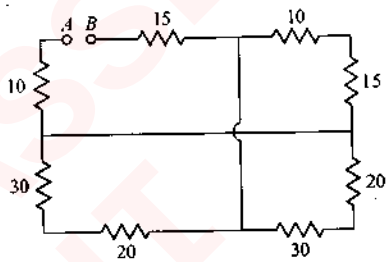


Fig. A2.145

a. 22.5 Ω b. 25 Ω
c. 37.5 Ω d. 75 Ω **(IIT-JEE, 2007)**

34. Incandescent bulbs are designed by keeping in mind that the resistance of their filament increases with the increase in temperature. If at room temperature, 100 W, 60 W and 40 W bulbs have filament resistances R_{100}, R_{60} and R_{40}, respectively, the relation between these resistances is

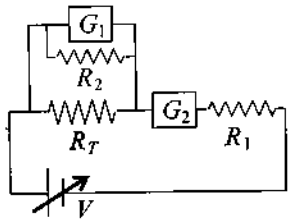
a. $\dfrac{1}{R_{100}} = \dfrac{1}{R_{40}} + \dfrac{1}{R_{60}}$ b. $R_{100} = R_{40} + R_{60}$

c. $R_{100} > R_{60} > R_{40}$ d. $\dfrac{1}{R_{100}} > \dfrac{1}{R_{60}} > \dfrac{1}{R_{40}}$

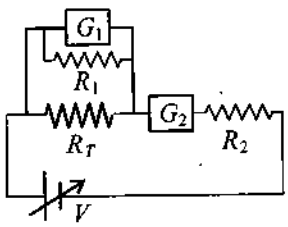
(IIT-JEE, 2010)

35. To verify Ohm's law, a student is provided with a test resistor R_T, a high resistance R_1, a small resistance R_2, two identical galvanometers G_1 and G_2, and a variable voltage source V. The correct circuit to carry out the experiment is

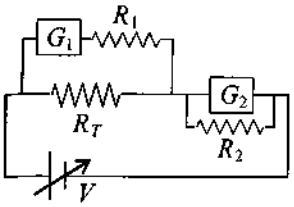
a.



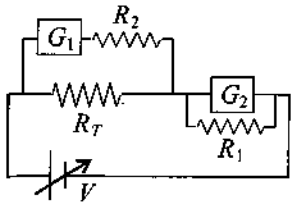
b.



c.



d.



(IIT-JEE, 2010)

36. Consider a thin square sheet of side L and thickness t, made of a material of resistivity ρ. The resistance between two opposite faces, shown by the shaded areas in the figure, is

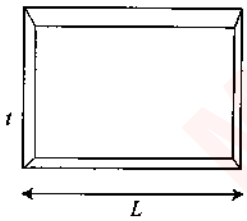


Fig. A2.146

a. directly proportional to L
b. directly proportional to t
c. independent of L
d. independent of t **(IIT-JEE, 2010)**

Multiple Choice Questions with or More than One Correct Answer Type

1. Capacitor C_1 of capacitance 1 micro-farad and capacitor C_2 of capacitance 2 micro-farad are separately charged fully by a common battery. The two capacitors are then separately allowed to discharge through equal resistors at time $t = 0$.
a. The current in each of the two discharging circuits is zero at $t = 0$
b. The currents in the two discharging circuits at $t = 0$ are equal but not zero.
c. The currents in the two discharging circuits at $t = 0$ are unequal
d. Capacitor C_1 loses 50% of its initial charge sooner than C_2 loses 50% of its initial charge. **(IIT-JEE, 1989)**

2. Read the following statements carefully:
Y: The resistivity of a semiconductor decreases with the increase in temperature.
Z: In a conducting solid, the rate of collisions between free electrons and ions increases with the increase in temperature
Select the correct statement(s) from the following:
a. Y is true but Z is false
b. Y is false but Z is true
c. Both Y and Z are true
d. Y is true and Z is the correct reason for Y
(IIT-JEE, 1993)

3. In the circuit shown in Fig. A2.147, the current through the

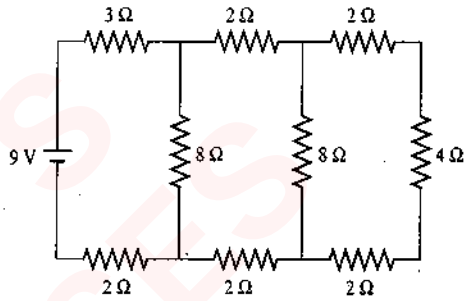


Fig. A2.147

a. 3 W resistor is 0.50 A **b.** 3 W resistor is 0.25 A
c. 4 W resistor is 0.50 A **d.** 4 W resistor is 0.25 A
(IIT-JEE, 1998)

4. When a potential difference is applied across the current passing through
a. an insulator at 0 K is zero
b. a semiconductor at 0 K is zero
c. a metal at 0 K is finite
d. a p–n dipole at 300 K is finite, if it is reverse biased
(IIT-JEE, 1999)

5. For the circuit shown in the figure,

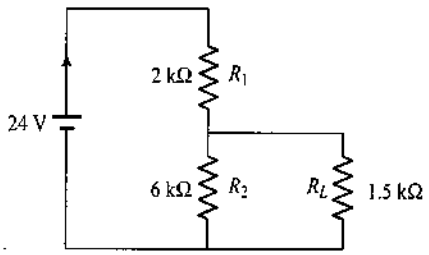


Fig. A2.148

a. the current I through the battery is 7.5 mA
b. the potential difference across R_L is 18 V
c. ratio of powers dissipated in R_1 to that in R_2 is 3
d. If R_1 and R_2 are interchanged, the magnitude of the power dissipated in R_L will decrease by a factor of 9
(IIT-JEE, 2009)

Integer Answer Type

1. When two identical batteries of internal resistance 1 Ω each are connected in series across a resistor R, the rate of heat produced in R is J_1. When the same batteries are connected in parallel across R, the rate is J_2. If $J_1 = 2.25\ J_2$, then the value of R in Ω is _____. **(IIT-JEE, 2010)**

2. At time $t = 0$, a battery of 10 V is connected across points A and B in the given circuit. If the capacitors have no charge initially, at what time (in seconds) does the voltage across them becomes 4 volt? [take ln 5 = 1.6, ln 3 = 1.1]
(IIT-JEE, 2010)

ANSWERS AND SOLUTIONS

Objective Type

1. b. When a source is connected, a current starts to flow through the conductor. Let it be I. Then current density at a section is equal to I/A, where A = cross-sectional area. Since cross-sectional area at P is maximum, current density at P is minimum.

Since electric field is $\frac{J'}{\sigma}\left(-\frac{1}{A\sigma}=\frac{I\rho}{A}\right)$, at P electric field is minimum while that at Q is maximum.

Rate of generation of heat per unit length at a section will be equal to $I^2\rho/A$. It is minimum at P and maximum at Q.

The mean kinetic energy of free electrons $=\frac{1}{2}mv_d^2$ which is minimum at P and maximum at Q.

2. c. In element A, the resistance remains constant up to the potential drop of 10 V. Further increase in the voltage does not increase this current (which is constant at 1 A). This means that the ratio V/R_A = constant and this resistance R_A increases linearly with voltage.

In element B, the resistance decreases gradually up to 15 V and afterwards the resistance R_B increases linearly with voltage. When both A and B are in series, the current in the circuit will increase nonlinearly up to 1 A when the total voltage drop across A and B becomes 10 + 15 = 25 V. Further increase in this voltage does not bring about any change in the current as shown in option (c). The voltage drop across A will go on increasing while that across B remains fixed at 15 V.

3. c. Applying Kirchhoff's law at A, C, D, the direction of the currents in each branch will be as shown in Fig. A2.149. Now, it is clear from the figure that the batteries 1 and 4 are getting charged.

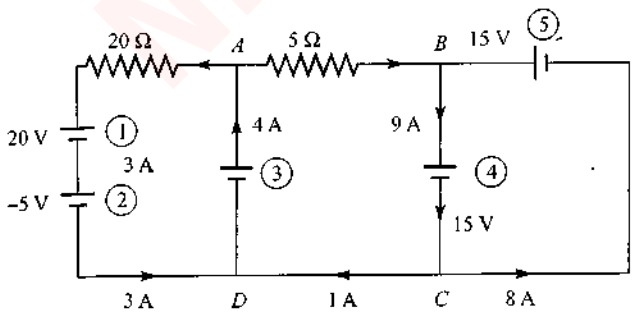


Fig. A2.149

4. a. When only 4 Ω resistance is shunted $(i_g) = i/5$

$G \times i/5 = 4 \times (4/5) \Rightarrow G = 16\ \Omega$

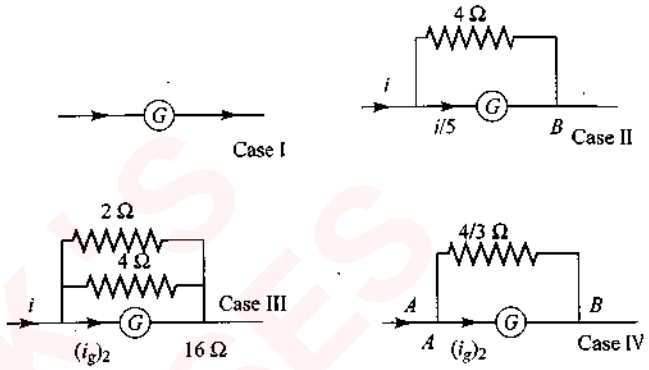


Fig. A2.150

5. b. $1.45 = 1.3 + Ir_A$

$1.45 = 1.5 - Ir_B$

$Ir_B = 0.05$

$Ir_A = 0.15$

Dividing, we get $\frac{r_B}{r_A}=\frac{1}{3}$

or $r_A = 3r_B$

6. b. Clearly, $E_2 = IY$

$2=\frac{12}{500+Y}Y$ or $500+Y=6Y$

or $5Y = 500$ or $Y = 100\ \Omega$

7. b. Potential difference V_P across P as determined from E_1 is given by $V_P=\left(\frac{E_1}{P+Q}\right)P$.

Potential V_P across P as determined from E_2 is same as E_2 because no current is drawn, i.e., $V_P = E_2$.

Therefore,

$E_2=E_1\left(\frac{P}{P+Q}\right) \Rightarrow \frac{E_2}{E_1}=\frac{P}{P+Q}$

8. a. In a Wheatstone bridge, the deflection in the galvanometer does not change if the battery and galvanometer are interchanged.

9. a. $V = E - Ir_1$, $V = 0$

$\therefore$ $E = Ir_1$

Total e.m.f. $= Ir_1 = Ir_1 = 2Ir_1$

Total resistance $= R + r_1 + r_2$

Now, $I=\frac{2Ir_1}{R+r_1+r_2}$

or $R + r_1 + r_2 = 2r_1$

or $R = 2r_1 - r_1 - r_2$ or $R = r_1 - r_2$

10. a. Effective resistance across voltmeter = 50 kΩ. Total resistance across the dc supply = 450 kΩ.

Current drawn from supply $\frac{1000\text{ V}}{450\text{ k}\Omega} = \frac{1}{450}$ A. Potential difference across voltmeter $= \frac{50 \times 1000}{450}\text{ V} = \frac{1000}{9}\text{ V} = 111\text{ V}$

11. d. When $r = 0$ the terminal potential difference in the arm containing R_1 remains unchanged, so the ammeter reading does not change.

12. d. When a resistance is joined in parallel with the voltmeter, the total resistance of the circuit decreases. Current will increase, so ammeter reading will increase. The equivalent resistance across the voltmeter decreases and hence its reading will decrease.

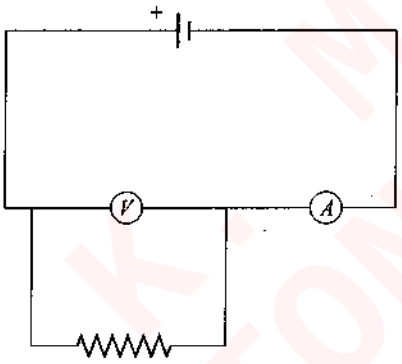


Fig. A2.151

13. a. A and B are effectively in parallel and hence give the same reading every time.

14. a. The equivalent circuit can be redrawn as shown in Fig. A2.152.

Let $R = \frac{2R \times 2}{2R + 2} + 2$

or $R = \frac{2R}{R+1} + 2$

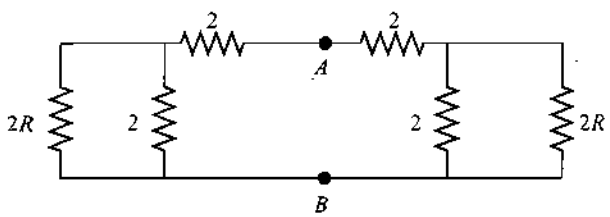


Fig. A2.152

or $R = \frac{2R + 2R + 2}{R+1} \Rightarrow R = \frac{2 + 4R}{R+1}$

$R^2 - 3R - 2 \Rightarrow R = \frac{3 + \sqrt{17}}{2}$

$R_{AB} = \frac{R}{2} = \frac{3 + \sqrt{17}}{4}$

15. b. $U = \frac{1}{2}CV^2 = \frac{1}{2}C(2IR)^2 = 2I^2R^2C$

16. a. Potential difference = 0. Therefore, charge = 0.

17. d. Potential difference across the central limb = 16 V = potential difference across 16 Ω = potential difference across the left limb.

Current through 16 Ω = 1 A

Current through the left limb = 1 A and R = 11 Ω

$\therefore \quad \tau_C = \left(\frac{16}{2} + 2\right)\Omega \times (4 \times 10^{-6})\text{ F} = 40\,\mu\text{s}$

18. b. $R(i_1 + i_2) = E_1$

$i_2R + (i_1 + i_2)R = E_2$

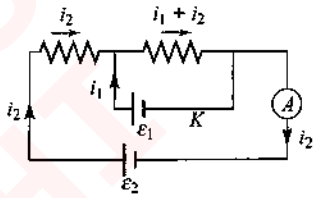


Fig. A2.153

$i_2 = \frac{E_2 - E_1}{R}$

[Initially, $E = E_2/R$]

19. c. $i = \frac{2.5 - 1}{30} = 0.05$ A

$-20i + 2.5 = V_{\text{capacitor}} + 1$

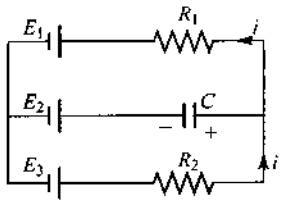


Fig. A2.154

$V_{\text{capacitor}} = 0.5$ V

$q^{(-)} = CV = 10 \times 0.5 = 5$

20. a. $R_0 = 2500\ \Omega$

$E = 125$ V

When r is connected in series

$i = \frac{E}{R_0 + r}$

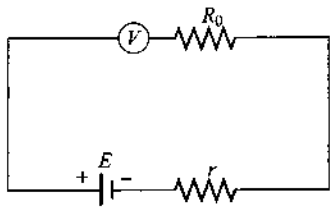


Fig. A2.155

Reading in voltmeter $= E - ir = 100$

$\therefore \quad 100 = 125 - \dfrac{125}{2500 + r} \times r$

$\Rightarrow \quad r = 625\ \Omega$

21. a. $i = \dfrac{15}{60} = \dfrac{1}{4}$ A

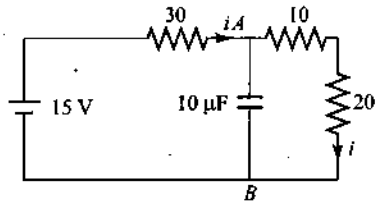


Fig. A2.156

$$V_{AB} = \frac{1}{4} \times 30 = 7.5 \text{ V}$$

$$q = eV = 10 \times 7.5 = 75\ \mu\text{C}$$

22. a. $E = j\rho$ [j = current density]

$j = \dfrac{I}{\pi r^2}$ [r = radius of c.s. at distance 'x' from left end]

$$r = \left[a + \frac{(b-a)}{l}x\right]$$

Hence, $E = \dfrac{Vl^2\rho}{\pi R[al + (b-a)x]^2}$

23. a. Resistance of each part $= R/2n$.

For 'n' such parts connected in series, equivalent resistances, say

$$R_1 = n = \left[\frac{R}{2n}\right] = \frac{R}{2}$$

Similarly, equivalent resistance, say R_2, for another set of n identical, respectively, in parallel resistances would be

$$\frac{1}{n}\left(\frac{R}{2n}\right) = \frac{R}{2n^2}.$$

For getting maximum of R_1 and R_2, the resistances should be connected in series and hence,

$$R_{eq} = R_1 + R_2 = \frac{R}{2}\left(1 + \frac{1}{n^2}\right)$$

24. a. Either apply Kirchhoff's law or solve analytically.

25. d. Use resistance in parallel and series combination.

26. a. In the steady state, no current flows through the capacitor.

27. a. Recall the condition for maximum power flow through a circuit.

28. c. $R_{equal} = \dfrac{\frac{8}{3} \times 2}{\frac{8}{3} + 2} = \dfrac{16}{14} = \dfrac{8}{7}\ \Omega$

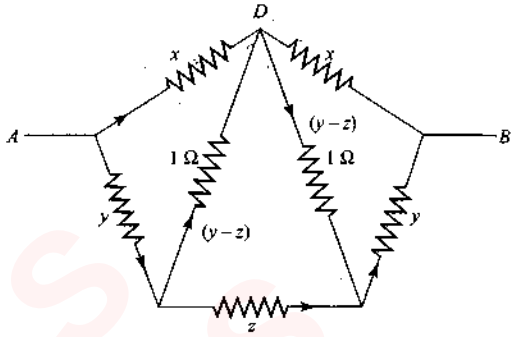


Fig. A2.157

The charge x passing between A and D will entirely pass to $DA2$. So, we can detach the circuit at point D and shown in Fig. A2.158.

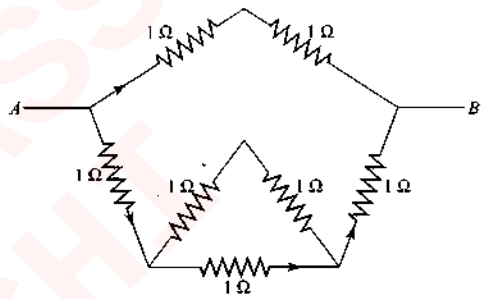


Fig. A2.158

29. d. $(0.1 + 0.3 + 0.6)i_1 = (9 + 0.9) \times 10$

$i = 99$ mA

$I = (99 + 10)$ mA $= 0.109$ A

30. b. The potential of point G should be kept zero.

31. b. When switch S_1 is open

$\dfrac{6}{E} = \dfrac{L}{2} \Big/ L = \dfrac{1}{2}$

$\therefore \quad E = 12$ V

When switch S_2 is closed, we get

$$\frac{6 \times 10}{10 + r} = \frac{5L}{12L} E = \frac{5}{12} \times 12 = 5$$

$\therefore\ 10 + r = 12$ or $r = 2\ \Omega$

32. d. From Fig. A2.159

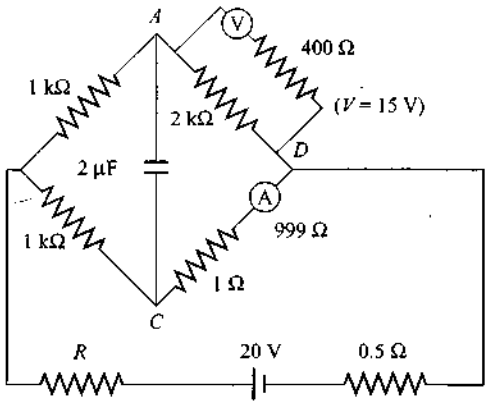


Fig. A2.159

$V_A - V_D = 15$ V

and $V_C - V_D = iR$

$= 15 \times 10^{-3} \times (1 + 999) = 15$ V

$V_A - V_C = 15 - 15 = 0$

Energy stored = 0

33. a. This is a case of balanced Wheatstone bridge.

34. c. The equivalent capacitance C_{eq} is

$$\frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

$$C_{eq} = \frac{6}{5}\mu F$$

The charge that flows from P to Q is

$$Q = C_{eq}V$$
$$= \frac{6}{5} \times 5$$
$$= 6\mu C$$

35. c. $i = \dfrac{50}{20 + R}$

Potential drop across R = potential drop across AB

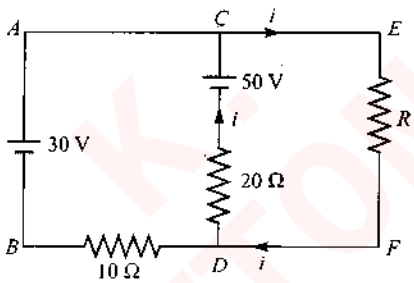


Fig. A2.160

$$\frac{50}{20 + R} R = 30 \quad \Rightarrow \quad R = 30\ \Omega$$

36. b. In Fig. A2.161, all resistances are connected in parallel.

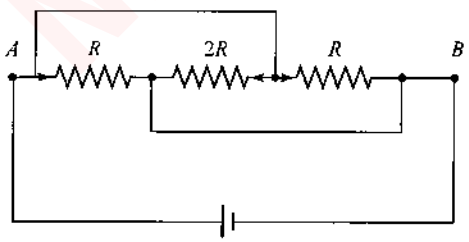


Fig. A2.161

So, $R_{eq} = \dfrac{2R \times R/2}{2R + R/2}$ and current in all resistances flows from positive terminal of battery (means A end) to negative terminal of battery (means B end).

37. b. Just before S_1 is closed, the potential difference across capacitor 2 is 2ε.

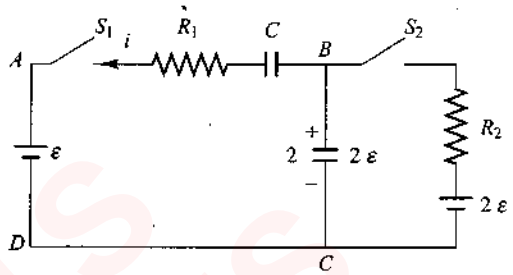


Fig. A2.162

Just after S_1 is closed, the potential differences across capacitors 1 and 2 are 0 and 2ε, respectively. Applying KVL to loop $ABCD$ immediately after S_1 is closed,

$$\varepsilon = -iR_1 + 0 + 2\varepsilon, i = \frac{E}{R_1} \text{ towards left}$$

38. d. This is a dc circuit because the battery is the only source of voltage. Hence, the capacitors behave like open circuits. An equivalent circuit is then two parallel sets of two identical series resistors (see Fig. A2.163). The voltage drop across each parallel branch must be the battery voltage of 3 V. Since the resistors are identical, there is an equal voltage drop of 1.5 V across each resistor. In particular, there is a drop of 1.5 V across resistor A.

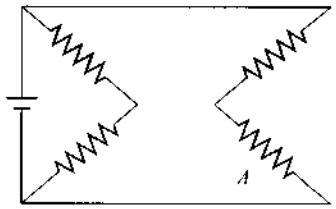


Fig. A2.163

39. c. $U_i = \dfrac{1}{2}C\varepsilon_1^2$

$$U_f = \frac{1}{2}C\varepsilon_2^2, \quad \Delta U = \frac{1}{2}C(\varepsilon_2^2 - \varepsilon_1^2)$$

$$Q_{in} = +C\varepsilon_1, Q_f = -C\varepsilon_2, \Delta Q = |Q_f - Q_i|$$

$$= C(\varepsilon_2 + \varepsilon_1)$$

Work done by battery $W_b = \varepsilon_2 \Delta Q = C(\varepsilon_2 + \varepsilon_1)\varepsilon_2$

Heat generated $= W_b - \Delta U$

$$= C\varepsilon_2^2 + C\varepsilon_1\varepsilon_2 - \frac{1}{2}C(\varepsilon_2^2 - \varepsilon_1^2)$$

$$= \frac{1}{2}C(\varepsilon_2^2 + \varepsilon_1^2 + 2\varepsilon_1\varepsilon_2) = \frac{1}{2}C(\varepsilon_1 + \varepsilon_2)^2$$

40. c. Applying KVL, we get

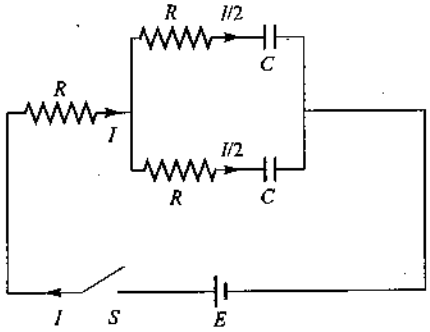


Fig. A2.164

$$\frac{I}{2}=\frac{dq}{dt}\Rightarrow I=2\frac{dq}{dt}$$

$$E-IR-\frac{I}{2}R-\frac{q}{C}=0$$

$$E-\frac{3}{2}IR-\frac{q}{C}=0$$

$$E-\frac{3}{2}\times 2\frac{dq}{dt}R-\frac{q}{C}=0$$

$$q_{(t)}=EC\left(1-e^{-t/3RC}\right)$$

41. b. $I_1=\frac{5}{2+3}=1$ A

$$I_2=\frac{5}{3+3}=\frac{5}{6}\text{ A}$$

$V_A-0=3I_1 \Rightarrow V_A=3$ V

$V_B-0=3I_2 \Rightarrow V_B=2.5$ V

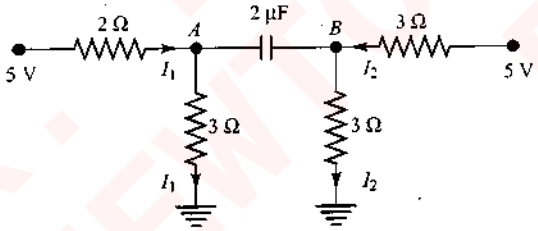


Fig. A2.165

Potential difference across capacitor is

$$3\text{ V}-2.5\text{ V}=0.5\text{ V}=1/2\text{ V}$$

$$q=CV=2\times 1/2=1\ \mu\text{C}$$

42. a. If V_1 decreases, then potential difference across xz will decrease. So, to have same potential difference, i.e., V_2 across xz, length xz has to be increased or z should be moved towards y.

b. is incorrect because for this, z should have been moved towards x.

c. will be incorrect because xz can get warm up if current through it increases which is not possible.

d. is incorrect because there is no current in R_1.

43. c. After long time, current through the capacitor = 0.

Therefore, current through the 6 Ω resistor,

$$i=\frac{12-4}{8}=1\text{ A}$$

Voltage across capacitor, $V=4+(6)(1)=10$ V

Charge on the capacitor, $Q=(10)(10)=100\ \mu$C

After the insertion of the dielectric, we get

$$C'=\frac{A\varepsilon_0}{\frac{d}{3}+\frac{d}{3}+\frac{d}{3(2)}}=\frac{A\varepsilon_0}{d}\left[\frac{1}{\left(\frac{1}{3}+\frac{1}{3}+\frac{1}{6}\right)}\right]$$

$$=(10)\left(\frac{1}{(5/6)}\right)=\frac{5}{6}(10)=12\ \mu\text{F}$$

Hence, voltage across the capacitor remains the same.

Charge on the capacitor, $Q'=(12)(10)=120\ \mu$C.

44. d. At $t=\infty$, the equivalent circuit is

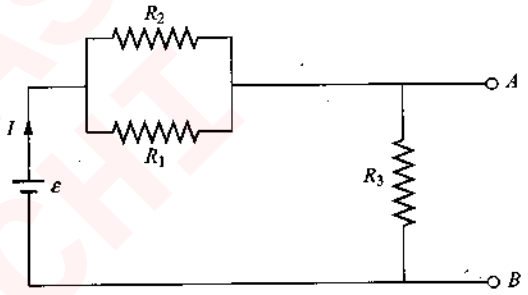


Fig. A2.166

$$\therefore\quad I=\frac{\varepsilon}{R_3+\frac{R_1R_2}{R_1+R_2}}=\frac{10}{1+\frac{(2)(2)}{2+2}}=\frac{10}{1+1}=5\text{A}$$

Also, $I_{R_1}R_1=I_{R_2}R_2 \Rightarrow \frac{I_{R_1}}{I_{R_2}}=\frac{R_2}{R_1}$

$\therefore\quad V_{AB}=IR_3=(5)(1)=5$ V

$\therefore\quad Q_{max}=CV_{AB}=5\ \mu$C

At $t=0$, the equivalent circuit is

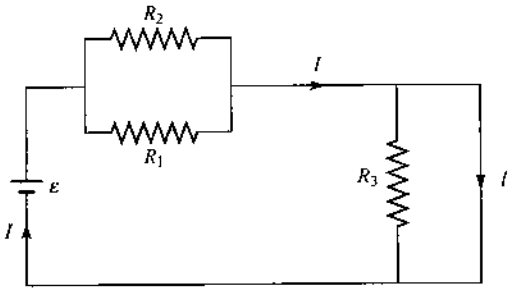


Fig. A2.167

$\therefore\quad I_{R_3}=0$

and $I_{R_1}R_1=I_{R_2}R_2 \Rightarrow \frac{I_{R_1}}{I_{R_2}}=\frac{R_2}{R_1}$

45. d. As $V = \varepsilon - Ir$

From graph it is clear that slope $= -r = -y/x$

or $r = y/x$

46. c. Initially, all three capacitors are in parallel

$$E_i = \frac{1}{2} \times 3CV^2 = \frac{3}{2}CV^2$$

When key is closed, two capacitors are in series.

$$E_f = \frac{1}{2}C\left(\frac{V}{2}\right)^2 \times 2 = \frac{CV^2}{4}$$

$$\Delta E = E_i - E_f = \frac{5}{4}CV^2$$

47. c. Equivalent circuit:

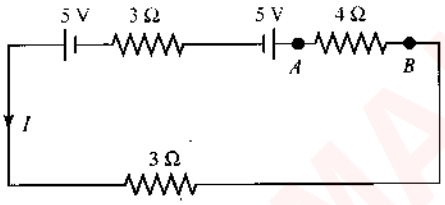


Fig. A2.168

$$I = \frac{5-5}{8} = 0 \Rightarrow V_A - V_B = 0$$

So, current will be only in the smaller loop containing the 6 V and 4 V battery. So, current through the 6 V battery is

$$I' = \frac{6-4}{4} = \frac{1}{2}\text{ A}$$

48. b.

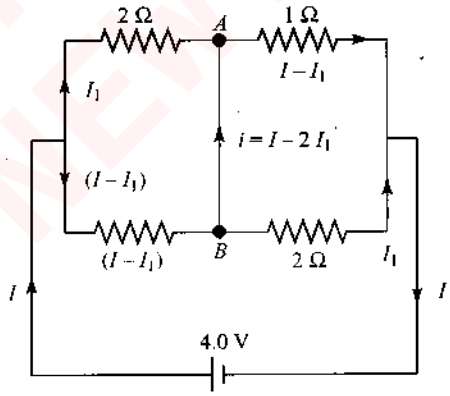


Fig. A2.169

Using KVL, we get

$2I_1 - (I - I_1) = 0 \Rightarrow I = 3I_1$

and $(I - I_1) + 2I_1 = 4$

Solving, we get $I_1 = 1$ A and $I = 3$ A

Therefore, current through $AB = I - 2I_1 = (3 - 2)$ A $= 1$ A

49. b. As potential gradient k in the potentiometer wire will be less than $(\varepsilon\varepsilon/100)$ (V cm^{-1}) because of the presence of r. So for the battery of e.m.f. $\varepsilon/2$, the balance point l will be greater than 50 cm as shown.

$$l = \frac{e/2}{k(< e/100)} \Rightarrow l > 50\text{ cm}$$

50. b.

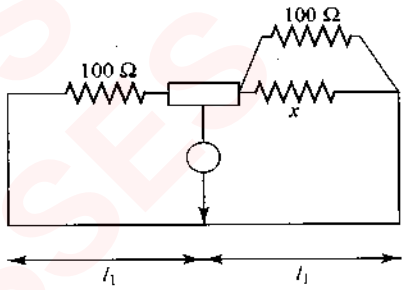


Fig. A2.170

The Wheatstone bridge is in balanced condition, so

$$\frac{100}{l_1} = \frac{\frac{100x}{100+x}}{l_2}$$

$$\therefore \quad \frac{l_1}{l_2} = 2 \Rightarrow x = 100\ \Omega$$

51. d. For maximum current, net resistance of cells must be equal to 2.5 Ω.

i.e., $\frac{n(0.5)}{m} = 25$ (i)

and $m \times n = 45$ (ii)

Solving, we get $n = 15$, $m = 3$

52. b.

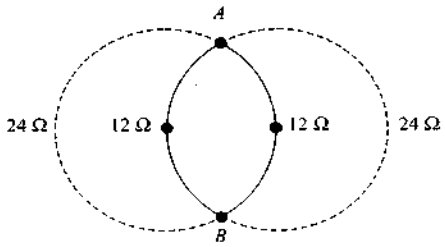


Fig. A2.171

From Fig. A2.171, we get

$AC_1 = AC_2 = C_1C_2 =$ radius

$\angle AC_1B = 120°$

The resistances of the four sections are 24, 12, 12 and 24 Ω. Hence, equivalent resistance R across AB is

$$\frac{1}{R} = \frac{1}{24} + \frac{1}{12} + \frac{1}{12} + \frac{1}{24} \text{ or } R = 4\ \Omega$$

Therefore, power $= \frac{v^2}{R} = \frac{(20)^2}{4} = 100$ W

53. a. In potentiometer wire, potential difference is directly proportional to length. Let the potential drop of unit length of a potentiometer wire be K.

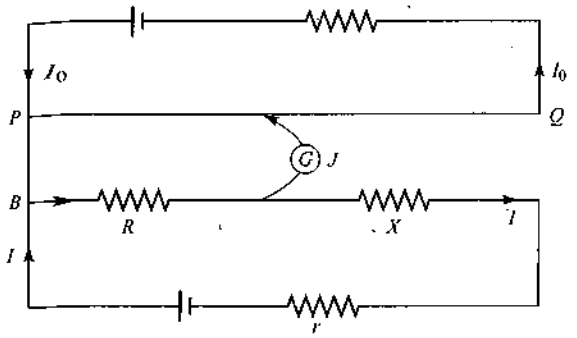


Fig. A2.172

For zero deflection, the current will flow independently in two circles:

$$IR = K \times 10 \quad \text{(i)}$$

$$IR + IX = K \times 30 \quad \text{(ii)}$$

Subtracting Eq. (ii) from Eq. (i), we get

$$IX = k \times 20 \quad \text{(iii)}$$

Dividing Eq. (i) by Eq. (ii), we have

$$\frac{R}{X} = \frac{1}{2}$$

54. d. In first two circuit diagrams, corrections have to be made while third one is absolutely wrong. So, none of the circuit diagrams is giving correct value of R.

55. b. The ammeter is not showing any reading. It means the potential drop across CB due to E_1 is equal to that of the e.m.f. E_2. And this is the reading of the voltmeter.

56. c. From work–energy theorem, we get $\Delta = \varepsilon i - i^2 r$.

57. c. $E = \dfrac{\lambda}{2\pi\varepsilon_0 r'}$, where λ is the linear charge density of the inner cylinder.

And $$V = \int_a^b E d\ell = \frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{b}{a}\right) \quad \text{(i)}$$

Now, $$I = \int \vec{J} \cdot d\vec{A} = \sigma \int \vec{E} \cdot d\vec{A}$$

$$= \sigma \int \frac{\lambda}{2\pi\varepsilon_0 r} 2\pi r\, dr$$

Current per unit length will be

$$I = \frac{\sigma\lambda}{\varepsilon_0} \quad \text{(ii)}$$

From Eq. (i), we get

$$I = \frac{2\sigma\pi\varepsilon_0}{\varepsilon_0 \ln(b/a)} v = \frac{2\pi\sigma}{\ln(b/a)} v$$

Alternatively,

$$I = \frac{V}{R} \Rightarrow R = \int_{x=a}^{b} \frac{1}{\sigma} \frac{dx}{2\pi x 1} = \frac{1}{2\pi r} \ln\left(\frac{b}{a}\right)$$

$$\therefore \quad I = \frac{2\pi\sigma V}{\ln(b/a)}$$

58. b. $50 = 10[R + r]$

$R + r = 5\ \Omega$

$$\eta = \frac{R}{R+r} \Rightarrow 0.25 = \frac{R}{R+r}$$

$R + r = 4R$

$\Rightarrow \quad r = 3R$

Then $R = 5/4 = 1.2\ \Omega$ and $r = 3.75\ \Omega$

59. d. $P = VI \Rightarrow 50 = 5 \times I \Rightarrow I = 10$ A

Power lost in cable $= I^2 R = 10 \times 10 \times 0.2 = 2$ W

Power supplied to the tape recorder = 50 W – 2 W = 48 W

60. d. Using the condition for Wheatstone bridge to be balanced, we get

$$\frac{P}{Q} = \frac{R}{\dfrac{S_1 S_2}{S_1 + S_2}} \quad \text{or} \quad \frac{P}{Q} = \frac{R(S_1 + S_2)}{S_1 S_2}$$

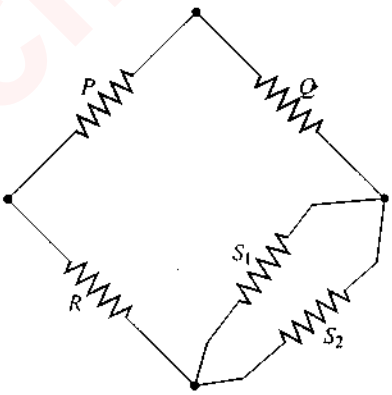


Fig. A2.173

61. d. For balanced meter bridge (null deflection), we get

$$\frac{55}{R} = \frac{20}{80} \quad \text{or} \quad R = 220\ \Omega$$

62. a. Across the dotted line, the circuit is symmetric.

$R_{eq} = 6\Omega$

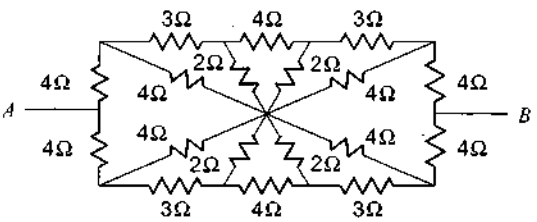


Fig. A2.174

63. b. $I = \dfrac{E - E_1}{R'} = \dfrac{E - E_2}{R + R'}$

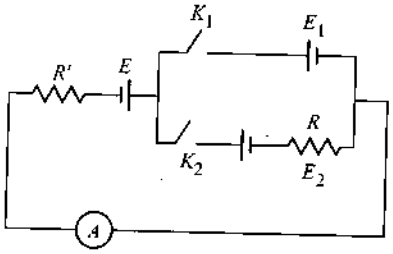


Fig. A2.175

$$\frac{E - E_2}{E - E_2} = \frac{R'}{R + R'}$$

$$\frac{E - E_2}{E - E_2} = \frac{R}{R'}$$

$$I = \frac{E_1 - E_2}{R}$$

64. c. Potential drop across $1\,\Omega = \dfrac{2 \times 1}{1.5}$ V. This potential drop exists across capacitor.

$\therefore \quad Q = CV = \dfrac{4}{3}\ \mu C$

65. b. The current will pass through only resistance part of the circuit. Hence, $I = \dfrac{9}{1 + 6 + 2}$ 1 A.

Potential drop across 3 μF capacitance is

$V_{AB} = V_{AB} + V_{Bc} = 1 \times 6 + \dfrac{1}{2} \times 1 = 6.5$ V

(Apply current division for current in branch BC which is 1/2 A.)

66. d. With these diagrams, we can easily find that the maximum equivalent resistance will be across voltmeter of range 10 V.

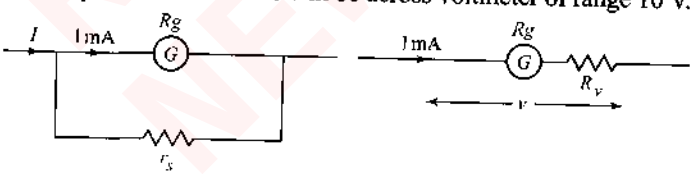


Fig. A2.176

67. c. In this option, e.m.f. is least and resistance is maximum, so bulbs will be dimmest in this option.

68. c. Initially, $R_{eq} = 5R/3$. Finally, $R_{eq} = 3R/2$.

Equivalent resistance decreases, so current increases in circuit and in '1' also. Hence, brightness of '1' increases. It means p.d. across '1' increases, so across '2' p.d. decreases, hence brightness of 2 decreases.

Initially, p.d. across '4' is $V_{4i} = \dfrac{1}{2}\left[\dfrac{(2R/3)\varepsilon}{2R/3 + R}\right] = \dfrac{\varepsilon}{5}$

Finally, $V_{4f} = \dfrac{(R/2)\varepsilon}{R/2 + R} = \dfrac{\varepsilon}{3}$

Since $V_{4f} > V_{4i}$; hence, brightness of 4 increases.

69. a. The circuit is equivalent to

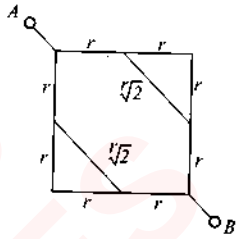


Fig. A2.177

Let each half side has resistance $\rho rd/2$

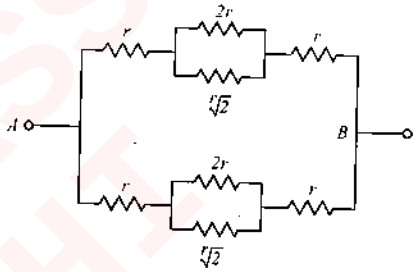


Fig. A2.178

On solving, we get

$$R = \frac{1}{2}\left[2r + \frac{(2r)(r\sqrt{2})}{(2 + \sqrt{2})r}\right] = r\sqrt{2}$$

$$= \rho d/\sqrt{2}$$

70. d.

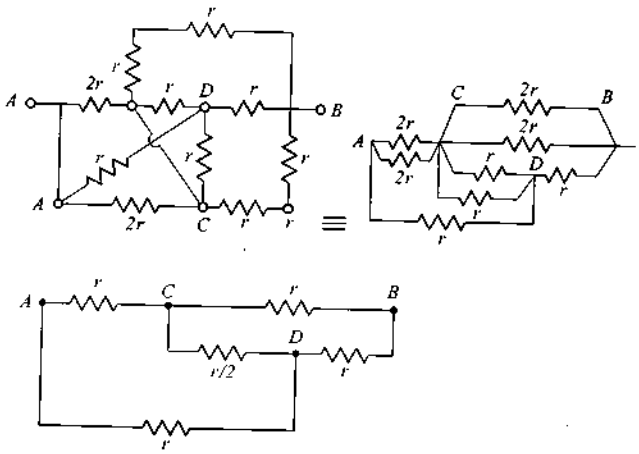


Fig. A2.179

$\equiv r$ (By Wheatstone balanced bridge concept)

71. b. Energy stored in capacitor when it is charged up to 2 V is

$\dfrac{1}{2} \times 10 \times 2^2 = 20\ \mu J = u_1$ (suppose)

Energy stored in capacitor when it is charged up to 4 V is

$\dfrac{1}{2} \times 10 \times 4^2 = 80\ \mu J = u_2$ (suppose)

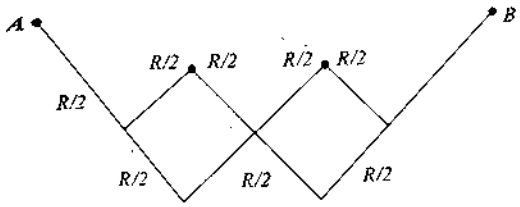
Increase in charge = 40 – 20 = 20 μC

Energy drawn from cell = $20 \times 4 = 80\ \mu J = u$ (suppose)

Heat produced = $u_1 + u - u_2$

$= 20 + 80 - 80 = 20\ \mu J$

72. b. The circuit can be folded about B and redrawn as



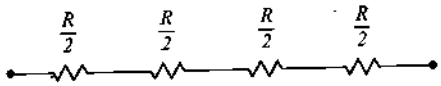


Fig. A2.180

Hence, the equivalent resistance between A and B is $2R$.

73. d. $R = \frac{1}{\sigma} \times \frac{t}{4\pi R^2}$

Using values, $R = 5 \times 10^{-11}\ \Omega$

74. b. Since current $IneAv_d$ through both the rods is the same, i.e., $2(n)eAv_L = ne(2A)v_R$.

$$\Rightarrow \frac{v_L}{v_R} = 1$$

75. c. $i = \frac{dq}{dt}$ = slope of q–t graph

$= -5$ (which is constant)

Amount of heat generated in time $t = i^2RT \propto t$

76. b. From relation $E = \rho J$, the magnitude of electric field is greater in right rod as compared to left rod. Therefore, magnitude of potential gradient in the right rod is greater (remember potential is continuous).

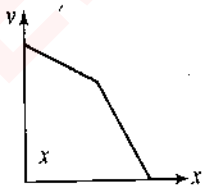


Fig. A2.181

Therefore, the variation is shown by the figure.

77. a. Equivalent circuit, $R_{net} = \frac{8R}{11}$

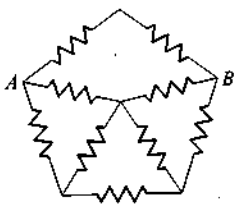


Fig. A2.182

78. d. $v_P = 2\left[\frac{20+x}{40}\right]$

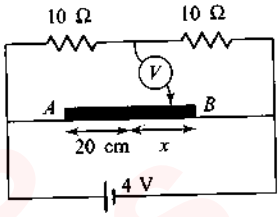


Fig. A2.183

$v_Q = 2$

$v_P - v_Q = \frac{x}{10}$

$2 \sin \pi t = \frac{x}{10}$

$x = 20 \sin \pi t$

$\frac{dx}{dt} = (20\ \pi \cos \pi t)$ cm/s

79. b. Circuit is forming a Wheatstone bridge, so $R = 2R$. For the maximum power transfer, $2R = r$.

80. a. Potential difference across AC is zero.

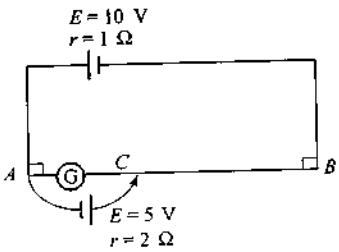


Fig. A2.184

$5 - 2I = 0$

$I = 2.5$ A

Let the resistance of part BC be r

Applying KVL, we get

$10 + 5 - 2I - Ir - I = 0$

$2.5r = 7.5 \Rightarrow r = 3\ \Omega$

As resistance of part $AB = 9\ \Omega$,

length $AC = 66.7$ cm

81. b. $H = \int_0^4 \frac{E^2}{R} dt = \int_0^4 \frac{(6t)^2\ dt}{12} = 64$ J

82. b. If P is disconnected, R_{eq} of circuit increases hence less current is drawn. Therefore,

$0 + 2\ iR_1 - x$

Therefore, as i decreases, x increases

83. b. From the hint, the equivalent resistance of the circuit is

$$R_T + \frac{1}{1/R_L + 1/R_{eq}} = R_{eq}$$

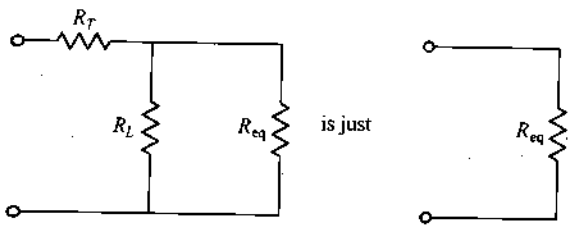


Fig. A2.185

$$R_T + \frac{R_L\, R_{eq}}{R_L + R_{eq}} = R_{eq}$$

$$R_T R_L + R_T\, R_{eq} + R_L\, R_{eq} = R_L\, R_{eq} + R_{eq}^2$$

$$R_{eq}^2 - R_T R_{eq} - R_T\, R_L = 0$$

$$R_{eq} = \frac{R \pm \sqrt{R_T^2\ 4(1)(-R_T R_L)}}{2(1)}$$

Only the positive sign is physical.

$$R_{eq} = \frac{1}{2}\left(\sqrt{4R_L\, R_T + R_T^2} + R_T\right)$$

For example, if $R_T = 1\ \Omega$ and $R_L = 20\ \Omega$, $R_{eq} = 5\ \Omega$

84. a. Initial charge on capacitor: $Q_1 = 2 \times 100 = 200\ \mu C$

Final charge: $Q_2 = 2 \times 300 = 600\ \mu C$

$W_b = 300\,[Q_2 - Q_1] = 12{,}0000\ \mu J$

$\Delta U = \frac{1}{2}C\left[300^2 - 100^2\right] = 8{,}0000\ \mu J$

Heat $= W_b - \Delta U = 4{,}0000\ \mu J = 0.04$ J

85. a. Time period: $T_0 = 2T$, $\omega = \frac{2\pi}{T_0} = \frac{\pi}{T}$

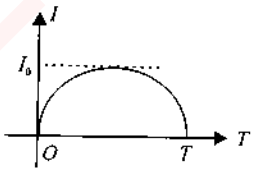


Fig. A2.186

$$I = I_0 \sin\omega t = I_0 \sin\left(\frac{\pi}{T}t\right)$$

$$Q = \int_0^T I\,dt \Rightarrow Q = I_0\int_0^T \sin\left(\frac{\pi}{T}t\right)dt$$

$$= \frac{-I_0}{\pi/T}\left[\cos\left(\frac{\pi}{T}t\right)\right]_0 = \frac{2TI_0}{\pi}$$

$$\Rightarrow \qquad I_0 = Q\pi/2T$$

$$\text{Heat} = \int_0^T I^2 R\,dt = \int_0^T I_0^2 \sin^2\left(\frac{\pi}{T}t\right)R\,dt = Q^2\pi^2 R/8T$$

86. a. Effective resistance of R and V is less than effective resistance of R and P, hence p.d. across voltmeter (or R) is less than previous value.

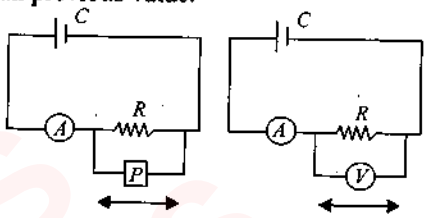


Fig. A2.187

Also overall resistance decreases, so overall current increases, hence reading of ammeter increases.

87. d. E_1 could be balanced if positive terminal of E_1 would be connected to A.

88. c. $R_{AB} = 16\ \Omega$, $V_{AB} = \frac{R_{AB}\,\varepsilon_0}{R + R_{AB}} = \frac{16 \times 12}{8 + 16} = 8$ V

Current in the loop of ε_1 and ε_2:

$$I = \frac{\varepsilon_2 + \varepsilon_1}{r_1 + r_2} = \frac{4+2}{2+6} = \frac{3}{4}\text{ A}$$

$$\varepsilon_1 - Ir_1 = V_{AB} = \frac{AN}{AB}(V_{AB}) \quad \Rightarrow 2 - \frac{3}{4} \times 2 = \frac{AN}{4} \times 8$$

$$\Rightarrow AN = \frac{1}{4}\text{ m} = 25\text{ cm}$$

89. b. Final charge on capacitor $q_f = CE$

Charge flown $= CE - CE/2 = CE/2$

Work done by battery: $W_b = E\,(CE/2) = \frac{1}{2}\,CE^2$

Change in energy of capacitor is

$$\Delta U = \frac{1}{2C}\left[q_f^2 - q_0^2\right]$$

$$= \frac{1}{2C}\left[(CE)^2 - (CE/2)^2\right] = \frac{3CE^2}{8}$$

$$\text{Heat} = W_b - \Delta U = \frac{1}{8}CE^2$$

90. d. $E_1 = (2 + 4) \times \ = 6$ V, $E_2 = (2 + 4) \times 2 = 12$ V

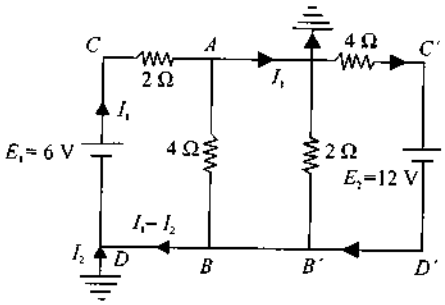


Fig. A2.188

There will be no current in AB and $A'B'$, because all these points are at the same potential.

From D to A': $V_D + 6 - 2I_1 = V_{A'}$, but $V_D = V_{A'} = 0$
$\Rightarrow I_1 = 3$ A
For outermost loop: $6 - 12 = 2I_1 + 4(I_1 - I_2) \Rightarrow I_2 = 6$ A
Current through $C'A' = I_1 - I_2 = 3 - 6 = -3$ A

91. b. $I = \dfrac{dq}{dt}$; apply Kirchhoff's law: $\dfrac{q_0 - q}{C_1} = IR + \dfrac{q}{C_2}$

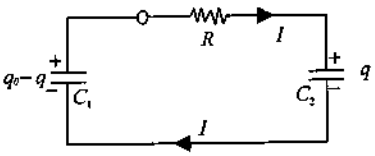


Fig. A2.189

$$\frac{q_0}{C_1} - q\left[\frac{C_1 + C_2}{C_1C_2}\right] = \frac{dq}{dt}R$$

$$\int_0^q \frac{dq}{\dfrac{q_0}{C_1} - q\left(\dfrac{C_1 + C_2}{C_1C_2}\right)} = \int_0^t \frac{dt}{R}$$

Solve to get: $q = \dfrac{q_0C_2}{C_1 + C_2}\left[1 - e^{-\frac{t(C_1+C_2)}{C_1C_2R}}\right]$

92. d. In steady state, there will be no current in any branch. So A and D will be at the same potential. Hence, charge on capacitor between A and D will be zero.

93. b. $V \propto I^n, n > 1 \quad V = kI^n$
$P = VI = kI^{n+1}$. Here $n+1 > 0$.

94. a. $I = I_0 - kt$, at $t = T, I = 0 \Rightarrow k = I_0/T$

So $I = I_0 - I_0t/T$, $q = \int_0^T I dt = I_0\int_0^T\left(1 - \dfrac{t}{T}\right)dt$

$\Rightarrow q = \dfrac{I_0T}{2} \Rightarrow I_0 = \dfrac{2q}{T}$

Heat $= \int_0^T I^2 R dt = I_0^2R\int_0^T\left(1 - \dfrac{t}{T}\right)^2 dt = \dfrac{4q^2R}{3T}$

95. d. $\dfrac{V^2}{R}t = ms\Delta T$

For first case: $\dfrac{(3E)^2}{(\rho L/A)}t = ms\Delta T$

For second case: $\dfrac{(NE)^2}{\rho 2L/A}t = 2ms\Delta T$

Dividing, we get $N = 6$.

96. d. Initial charging rate = initial current in the line of capacitor
$= i = 2E/5R$.

Steady state p.d. across capacitor is

$$V_0 = \frac{2}{3}E \Rightarrow q_0 = CV_0 = \frac{2}{3}EC$$

$$t = \frac{q_0}{i} = \frac{(2/3)EC}{2E/5R} = \frac{5}{3}RC$$

97. b. t_1 is the time of capacitor to discharge from $2V/3$ to $V/3$.

For this, $\dfrac{V}{3} = \dfrac{2V}{3}e^{-t_1/\tau} \Rightarrow t_1 = \tau\ln 2 = RC\ln 2$

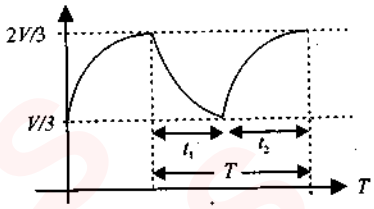


Fig. A2.190

t_2 is time for charging of capacitor from $V/3$ to $2V/3$.

So $\displaystyle\int_{V/3}^{2V/3}\frac{dq}{VC - q} = \int_0^{t_2}\frac{dt}{RC} \Rightarrow t_2 = RC\ln 2$

$T = t_1 + t_2 = 2RC\ln 2$

98. d.

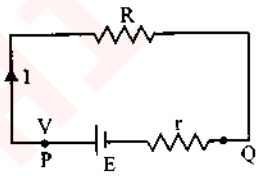


Fig. A2.191

$V = IR$ (i)
$V = E - Ir$ (ii)

Eliminate I from Eqs. (i) and (ii), we get

$$V = \frac{ER}{R + r} = \frac{E}{1 + r/R}$$

Now at $R = 0$, $V = 0$; at $R = \infty$, $V = E$

For this (d) is the suitable graph.

99. b. When the terminals of a single cell are directly connected, current obtained is $I = \varepsilon/r$.
If both are connected in series, then $I = 2\varepsilon/2r = \varepsilon/r$.

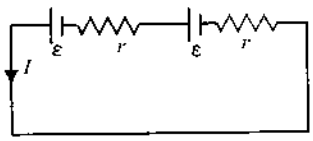


Fig. A2.192

If both are connected in parallel then, apply Kirchhoff's law for loop $ABCD$: $\varepsilon - (I/2)r = 0 \Rightarrow I = 2\varepsilon/r$.

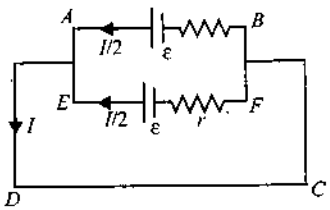


Fig. A2.193

Multiple Correct Answers Type

1. a., b., c., d. Electric current at a point on the circle

$i = fe$, where f = frequency

$$= \frac{\omega}{2\pi} = \frac{V}{2\pi r} = \frac{0.6\times10^6\times\pi}{2\pi\times5\times10^{-11}} = 6\times10^{15}\text{ rev/s}$$

$$= 6\times10^{15}\times1.6\times10^{-19}$$

$$= 0.96\times10^{-3}\text{ A} = 0.96\text{ mA}$$

2. a., c. Use symmetry.

3. a., d. Resistance of upper bulb, $R_1 = 100^2/25 = 400\ \Omega$

Resistance of lower bulb, $R_2 = 200^2/100 = 400\ \Omega$

Equivalent resistance of the circuit: $R_{eq} = 500\ \Omega$.

Heat lost per second in the circuit is

$$e^2/R_{eq} = 200^2/500 = 80\text{ J s}^{-1}$$

Potential difference across branches AB and CD will be same. Hence, ratio of heat generated in them is

$$\frac{\text{Heat}_{AB}}{\text{Heat}_{CD}} = \frac{R_{CD}}{R_{AB}} = \frac{1000}{500} = 2:1$$

Hence, (c) is incorrect.

Since potential difference across branches AB and CD will be the same but their resistances are different, so current in them will be different. As the resistances of the bulbs are same, so heat generated in them will be different. Hence, (b) is incorrect. Current drawn from the cell is $e/R_{eq} = 200/500 = 0.4$ A.

4. a., b., c., d. Potential difference across the terminals of the cell is given by $V = E - ir$. Hence, the graph between V and i will be a straight line having negative slope and positive intercept. Hence, (a) is correct.

Total electrical power generated is Ei and thermal power generated in the cell is i^2r. Hence, thermal power generated in the external circuit is $P = Ei - i^2r$. So, the graph between P and i is a parabola passing through the origin. Hence, choice (b) is correct.

When terminal potential difference across the cell is V, current flowing through the circuit will be $i = E-V/R$ and thermal power generated in the external circuit will be equal to $P = V_i = V\left(\frac{E-V}{r}\right) = \frac{EV}{r} - \frac{V^2}{r}$.

Hence, the graph between P and V will be a parabola passing through the origin and have a maximum value of P at $V = E/R$. Hence, choice (d) is correct.

At an instant, thermal power generated in the cell is equal to i^2r and total electrical power generated in the cell is equal to Ei. Hence, the fraction $\eta = i^2r/Ei = (r/E)i$. Hence, the graph between η and i will be a straight line passing through the origin and have a positive slope. Hence, (c) is correct.

5. a., d.

Equivalent resistance of the circuit = (2.5 + 1.5) = 4 Ω.

Current through battery is $I = 20/4 = 5$ A

Current through P = Current through Q = 5/2 = 2.5 A

$$V_A - V_Q = 3\times2.5 = 7.5\text{ V} \quad \text{(i)}$$

$$V_A - V_Q = 2\times2.5 = 5\text{ V} \quad \text{(ii)}$$

From Eqs. (i) and (ii), we get $V_Q - V_P = 2.5$ V.

6. b., c., d. Disregard capacitors and find the current through G. The potential difference across each capacitor is then found from the potential difference across the resistances in parallel with them.

7. b., c. As $R \propto 1/W$ and actual power consumed $P = V^2/R$, the lesser the resistance (i.e., higher the wattage), the greater the power consumed. If the bulbs are connected in series, actual power $P = i^2R \Rightarrow P \propto R$. Hence, in series connection, low wattage bulbs glow more.

8. a., b., d. As A and B are connected in series, hence $IA = IB$. The potential difference across both the branches will be the same.

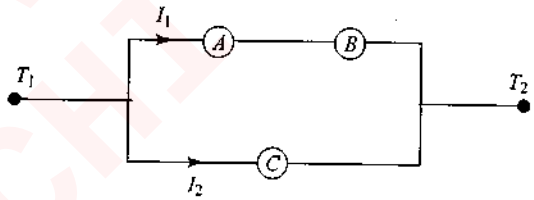


Fig. A2.194

$$I_AR_B + I_BR_B = I_CR_C \text{ or } \frac{I_B}{I_C} = \frac{R_C}{R_A + R_B}$$

9. a., b., c. In the simplified circuit, the circuit is a balanced Wheatstone bridge and a branch of 20/29 Ω and R is parallel with this balanced bridge for maximum power.

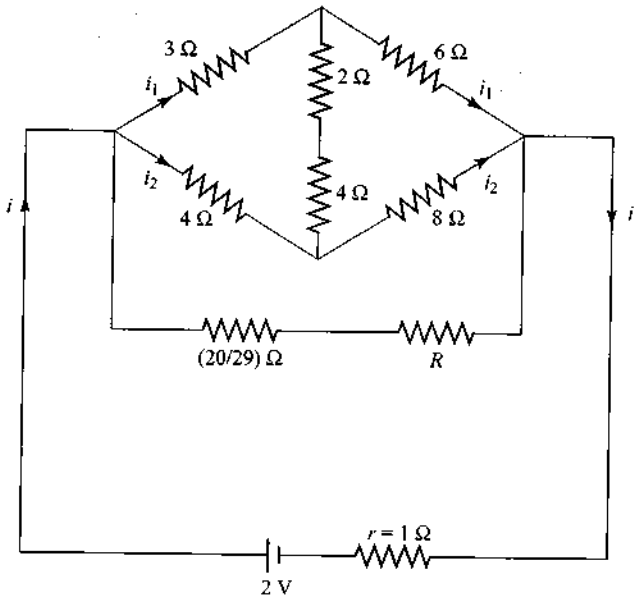


Fig. A2.195

$$r = R_{external}$$

$$1 = \frac{1}{\frac{1}{(3+6)} + \frac{1}{(20/29+R)} + \frac{1}{(4+8)}}$$

or $$R = \frac{16}{25}\ \Omega$$

Maximum power developed in the external circuit is

$$P_{max} = i^2 R = \left(\frac{2}{1+1}\right)^2 \times 1 = 1\ \text{W}$$

Current through the upper branch

$$i_1 = i\left[\frac{\left(\frac{20}{29}+R\right)(4+8)}{9+\left(\frac{20}{29}+R\right)+12}\right]$$

$$i_2 = i\left[\frac{\left(\frac{20}{29}+R\right)(3+6)}{9+\left(\frac{20}{29}+R\right)+12}\right]$$

Therefore, i_1/i_2 is independent of R.

10. a., d.

$$R_1 = \frac{V^2}{W_1} = \frac{(220)^2}{25} = (22)^2 \times 4\ \Omega$$

$$R_2 = \frac{V^2}{W_2} = \frac{(220)^2}{100} = (22)^2\ \Omega$$

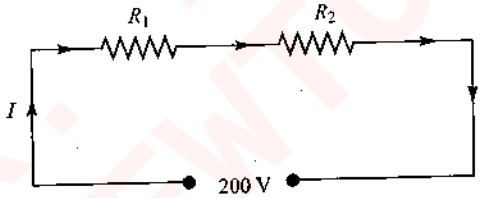


Fig. A2.196

Current in the circuit when the bulbs are connected in series,

$$I = \frac{220}{(22)^2 \times 4 + (22)^2} = \frac{10}{22 \times 5} = \frac{1}{11}\ \text{A}$$

Hence, $$P_1 = I^2 R_1 = \left(\frac{1}{11}\right)^2 \times (22)^2 \times 4 = 16\ \text{W}.$$

And $$P_2 = I^2 R_2 = \left(\frac{1}{11}\right)^2 \times (22)^2 = 4\ \text{W}.$$

11. a., c. Resistances 4 and 6 Ω are short-circuited. Therefore, no current will flow through these two resistances. Current passing through the battery is $I = (20/2) = 10$ A.
This is also the current passing in wire AB from B to A.
Power supplied by the battery is

$$P = EI = (20)(10) = 200\ \text{W}$$

Potential difference across the 4 Ω resistance = potential difference across the 6 Ω resistance = 0

12. a., b., c., d.

(a) Potential difference across each cell = $V_P - V_Q$.

(b) If it is clockwise, then E_2 supplies energy and for the reverse case, E_1 supplies the energy.

(c) Potential difference = $E = ir$, when battery supplies energy. It is $E + ir$, when battery consumes energy.

By KVL, $i = \frac{E_1 - E_2}{r_1 + r_2}$ (anticlockwise)

$$V_P - V_Q = E_1 - ir_1 = \frac{E_1 r - E_2 r}{r_1 + r_2}$$

13. a., b. For null point, current flows in the loop CD only.

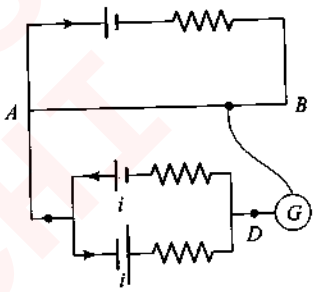


Fig. A2.197

$$i = \frac{3\ \text{V}}{2\ \Omega + 1\ \Omega} = 1\ \text{A}$$

$$V_{CD} = 1\text{V} - 1(1) = 0$$

Therefore, option (a) is correct. That is, $V_A > V_B$.

When jockey touches B, current from A to B increases the P.D. across the secondary circuit.

Therefore, option (b) is correct.

14. a., b.

$$\frac{R}{(R_1 + R_2)} = \frac{L/4}{3L/4}$$

$$3R = (R_1 + R_2)$$

$$\frac{R_1 + R_2}{R} = \frac{2/3}{1/3}$$

$$3R = R_2 + R_1$$

$$R = 2R_2 - R_1$$

$$4R = 3R_2$$

$$R_1 = \frac{5R}{3}$$

$$R_2 = \frac{4R}{3}$$

15. a., b., c. Initial charge (before filling the dielectric slab) is $10 \times 10 = 100\ \mu C$.

Final charge (after filling the dielectric slab) $= 10 \times 30 = 300\ \mu C$.

Increase in charge $= 20\ \mu C$

16. a., b., d. Let the currents be as shown in the figure

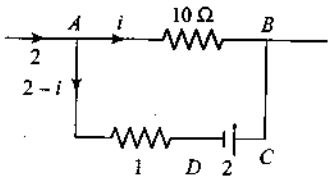


Fig. A2.198

Applying KVL along ABCDA, we get

$\Rightarrow \quad -10i - 2 + (2 - i)1 = 0$

$\therefore \quad i = 0$

Potential difference acrosss $= (2 - i)1 = 2 \times 1 = 2$ V

17. a., b., c., d.

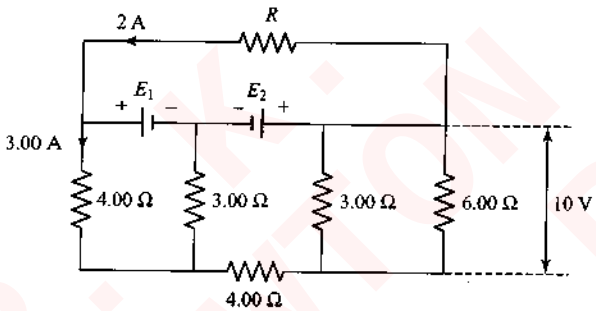


Fig. A2.199

After redrawing the circuit

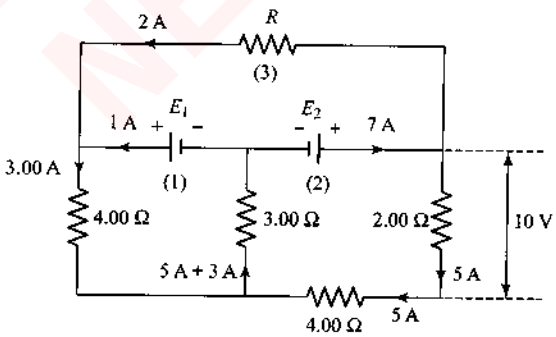


Fig. A2.200

a. $I_4 = 5$ A

b. From loop (1)

$-8(3) + E_1 - 4(3) = 0 \quad \Rightarrow \quad E_1 = 36$ V

From loop (2)

$+ 4(5) + 5(2) - E_2 + 8(3) = 0$

$E_2 = 54$ V

c. From loop (3)

$-2R - E_1 + E_2 = 0$

$$R = \frac{E_2 - E_1}{2} = -36 = 9\ \text{W}$$

18. b., c. (Moderates) As the length is doubled, the cross-sectional area of the wire becomes half, thus the resistance of the wire $R = \rho\frac{L}{A}$ becomes four times the previous value. Hence after the wire is elongated, the current becomes one-fourth. Electric field is potential difference per unit length and hence becomes half the initial value. The power delivered to resistance is $P = \frac{V^2}{R}$ and hence becomes one-fourth.

19. a., b., d.

Total charge $= \int I\,dt =$ Area under the curve $= 10$ C

$$\text{Average current} = \frac{\int I\,dt}{\int dt} = 5\text{A}$$

Total heat produced $= \int I^2\,dt$

$$= \int_0^2 (-5t + 10)^2 1\,dt = \frac{200}{3}\ \text{J}$$

Maximum power $= I^2R$ when I is maximum current $= 100 \times 1 = 100$W.

20. b., c. $V - (V + dV) = IdR \Rightarrow -dV = IdR$

$$E = -\frac{dV}{dx} = I\frac{dR}{dx} \quad \Rightarrow E = \frac{I\rho dx}{Adx} = \frac{I\rho}{A}$$

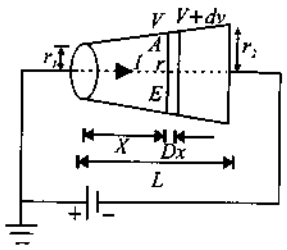


Fig. A2.201

We can find $r = \frac{r_1\ell + x(r_2 - r_1)}{\ell}$, $A = \pi r^2$

$$E = \frac{I\rho}{A} = \frac{I\rho}{\pi r^2} = \frac{I\rho\ \ell^2}{\pi\left[r_1\ell + x(r_2 - r_1)\right]^2};$$ option (a) is incorrect.

Rate of heat generation in length dx

$$dQ = I^2 dR = \frac{I^2\rho dx}{A} \quad \frac{dQ}{dx} = H = \frac{I^2\rho}{A}$$

Hence, option (b) is correct.

$H = I\left(\frac{I\rho}{A}\right) = IE$, hence option (c) is correct.

At $x = 0$, V is zero, hence option (d) is incorrect.

Assertion-Reasoning Type

1. **c.** The electrons suffer a large number of collisions with the positive ions of the conductor. Although the electric field accelerates an electron between two collisions, it is decelerated by collision. The net acceleration averages out to zero and the electron acquires a constant average speed. The gain in speed between collisions is lost in the next collision.
2. **d.** The condition for no deflection of the galvanometer is $\frac{R_1}{R_2} = \frac{R_{AC}}{R_{CB}}$

   where R_{AC} and R_{CB} are the resistances of the bridge wire of length AC and CB, respectively. If the radius of the wire AB is doubled, the ratio R_{AC}/R_{CB} will remain unchanged. Hence, the balance length will remain the same.
3. **d.** If the diameter of wire AB is increased, its resistance will decrease. Hence, the potential difference between A and B due to cell C_1 will decrease. Therefore, the null point will be obtained at a higher value of x.
4. **b.** Both statements I and II are true. In Statement I, R is varied while in Statement II, R is kept constant. Hence, both statements are independent.
5. **c.** From relation $\vec{J} = \sigma\vec{E}$, the current density $\vec{J}$ at any point in ohmic resistor is in direction of electric field $\vec{E}$ at that point. In space having non-uniform electric field, charges released from rest may not move along EL. Hence, Statement I is true while Statement II is false.
6. **d.** As the length of the wire is doubled, the cross-sectional area of the wire becomes half. Therefore, resistance of the wire becomes four times and the current becomes one-fourth of the initial value.

   Also, $V_d = \frac{I}{neA}$

   Since current becomes one-fourth and cross-sectional area of the wire becomes half, therefore, from the above equation the drift velocity of electron becomes half. Hence, Statement I is false.
7. **a.** Charge on capacitor $q = CE(1 - e^{-t/CR_{eq}})$

   $\therefore \quad I = \frac{dq}{dt} = \frac{E}{R_{eq}} e^{-t/CR}$

   at $\quad t = 0,\ I = \frac{E}{R_{eq}} = \frac{E}{R + r}$

   Therefore, resistance offered by the capacitor is zero.
8. **a.** $V_A - E - Ir + V_B \Rightarrow V_A - V_B = E + Ir$

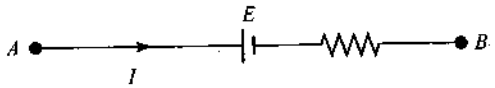


Fig. A2.202

Similarly, $V_A - V_B = E - Ir$.

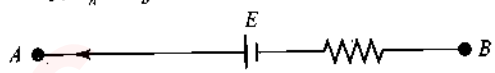


Fig. A2.203

9. **a.** When current flows through a conductor it always remains uncharged. Hence, no electric field is produced outside it.
10. **d.** The electrons are in motion which constitute electric current in a conductor but numbers of positive and negative charges are same.
11. **a.** Here $E = 2$ V, $I = \frac{2}{2} = 1$ A and $r = 1\ \Omega$

    $V = E - ir = 2 - (1)(1) = 1$ V
12. **d.** Direction of flow of current is from higher potential to lower potential.
13. **d.** $V = E + ir$ when charging of cell takes place.
14. **b.** On increasing temperature of the wire the K.E. of electrons increases and so they collide more rapidly with each other and hence their drift velocities decrease. Resistivity also increases and resistivity is inversely proportional to conductivity of material.
15. **d.** In a simple battery circuit, the point at the lowest potential is the negative terminal of battery. The current flows in the circuit from positive terminal to negative terminal.

Comprehension Type

For Problems 1–5

1.b., 2. b., 3. c., 4. b, 5. a

Sol.

1. **b.** $\eta = \frac{0.4 \times 4200(\theta - \theta_1)}{420 \times 4.8(\theta - \theta_1)} \times 100 = 250/3 = 83.34\%$

2. **d.** $\eta = \frac{0.4 \times 4200(\theta - \theta_1)}{420 \times 6.4(\theta - \theta_1)} \times 100 = 62.5\%$

3. **c.** $H_A = 0.2 \times 1680(\theta - \theta_1) + 0.4 \times 4200(\theta - \theta_1)$
   $= 420 \times [0.8 + 4](\theta - \theta_1)$
   $H_B = 0.4 \times 2450(\theta - \theta_1) + 0.4 \times 4200(\theta - \theta_1)$
   $= 420 \times (2.4 + 4)(\theta - \theta_1)$
   $\frac{H_A}{H_B} = \frac{4.8}{6.4} = \frac{3}{4}$.

4. **b.** $H_A = \frac{V^2}{R_A} \times 6$ (i)

   $H_B = \frac{V^2}{R_B} \times 8$ (ii)

   $\frac{H_A}{H_B} = \frac{R_B}{R_A} \times \frac{3}{4}$ but $\frac{H_A}{H_B} = \frac{3}{4}$

Then $R_B = R_A$.

5. a. (i) $\frac{V^2}{R_A}\times 6 = \frac{V^2}{(R_A+R_B)^2}R_A \times t_A$

But $R_A = R_B \Rightarrow \frac{V^2}{R_A}\times 6 = \frac{V^2}{4R_A^2}R_A \times t_A$

$t_A = 24$ min

(ii) $\frac{V^2}{R_B}\times 8 = \frac{V^2}{(R_A+R_B)^2}R_B \times t_B$

$\frac{8}{R_B} = \frac{R_B}{4R_{B^2}}t_B \Rightarrow t_B = 32$ min

For Problems 6–8
6.c., 7. a., 8. c.
Sol.

6. c. $V = E + ir$ (during charging) = 14 V.

7. a. $P = I^2 r$ (due to internal resistance) = $50^2 \times 4 \times 10^{-2}$ = 100 W.

8. c. Rate of charging = EI = 12 V × 50 A = 600 W.

For Problems 9–12
9.b., 10. c., 11. c., 12. c.
Sol.

9. b. As per the figure, $V_{bc} = 0.10 \times 2 = 0.2$ V

So actual drop = 12 – 0.2 = 11.8 V

Then $R = \frac{11.8}{0.1} = 118\ \Omega$

10. c. $I_V = \frac{V}{R_V} = \frac{12}{10000} = 1.2$ mA

Then the actual current I in the resistor is I = 0.100 – 0012 = 0.0988 A.

$$R = \frac{V_{ab}}{I} = \frac{12.0}{0.0988} = 121\ \Omega$$

11. c. $\frac{1.50\times 10^{-3}}{2} = \frac{0.75}{50+R_S+500}$

$R_S = 450\ \Omega$

12. c. $\frac{3}{4}\times 1.50\times 10^{-3} = \frac{0.9}{R+100}$

$R + 100 = 800$

$R = 700\ \Omega$.

For Problems 13–15
13. a. 14.b., 15. b.
Sol.

13.a. $I \propto r^{3/2}$

$\frac{I_1}{I_2} = \frac{(4r)^{3/2}}{r^{3/2}} = \frac{8}{1}$

14. b. $P = \frac{V^2}{R} \Rightarrow 2\times 10^3 = \frac{V^2}{20}$

$V = 200$ V

15. b. $\eta = \frac{R}{R+r}\times 100$, where $R = r$, efficiency will be maximum.

For Problems 16–18
16.c., 17. a., 18. a.
Sol.

16. c. Just after closing, capacitor behaves as short circuit and all current flows through it; hence ammeter reads zero.

17. a. After a long time, capacitor behaves as an open circuit and no current flows through it.

Therefore, $i = \frac{v_0}{R_1+R_2} = \frac{30}{10+5} = 2$ mA

18. a. Just after reopening, potential difference across R_2 remains the same initially as charge on capacitor does not change initially. Hence, current remains the same.

For Problems 19–21
19.a., 20. a., 21. d.
Sol.

19. a. $R_A = R_A = \frac{R\times R_V}{R+R_V} < R$

20 a. $R_B = R + R_0 > R$

21 d. (Tough)% error in case A.

$$\frac{R_A - R}{R}\times 100 = \left(\frac{R_V}{R+R_V} - 1\right)\times 100$$

$$\frac{R}{R+R_V}\times 100 = -1\%$$

% Error in case B

$$\frac{R_B - R}{R}\times 100 = \frac{R_G}{R}\times 100 = 10\%$$

Hence, percentage error in circuit B is more than that in A.

For Problems 22–24
22.b., 23. b., 24. d.
Sol.

$i_0 = 0.1$ A, $E_2 = 4$ V, $i_2 = 0$

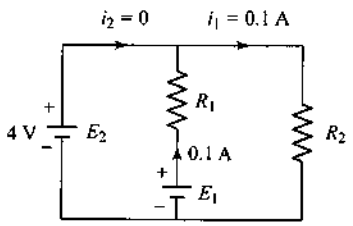


Fig. A2.204

As $0.1\,R_1 + 0.1\,R_2 - E_1 = 0$

$0.1\,R_2 - 4$ V $= 0$

$R_2 = 40\ \Omega$

Now $i_2 = 0.3$ A, $i_1 = 0.1$ A, $E_2 = 8$ A

Now $0.1\,R_1 + E_1 - 8 = 0$

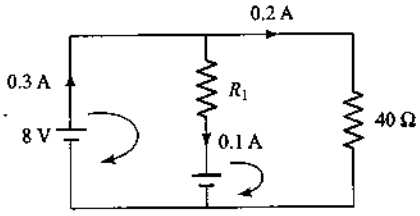


Fig. A2.205

$0.1 + 4 - E_1 = 0$

$0.2\,R_1 - 4 = 0$

$R_1 = \dfrac{4}{0.2} = 20\ \Omega$

$0.2\,E_1 = 2 + 4 = 6$ V

For Problems 25–27

Sol.

25.d., 26. c., 27. b.

Sol. At steady state, the circuit will be simplified to the following circuit:

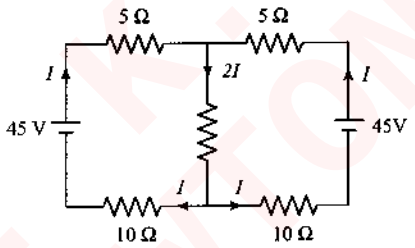


Fig. A2.206

Using Kirchhoff's law, we can calculate $I = 1$ A.
Further using Kirchhoff's law in original circuit, we can calculate charges on different capacitors.

For Problems 28–30

28. c., 29. a., 30. a.

Sol.

28.c. 8 Ω and 4 Ω will be in parallel: $R_{eq} = \dfrac{8\times 4}{8+4} = \dfrac{8}{3}\ \Omega$

29.a. $I = \dfrac{V}{R_{eq}} = \dfrac{6}{8/3} = 2.25$A

30.a. $P = \dfrac{V^2}{R} = \dfrac{6^2}{4} = 9$ W

Matching Column Type

1. a. → r.; b. → q.; c. → q.; d. → q.

Current through any cross section should remain the same.

$I = neAv_d$, as area increases, drift velocity decreases.
E is directly proportional to v_d, so E also decreases.
If E decreases, then p.d. across a segment of fixed length also decreases.

2. a. → r.; b. → r.; c. → s.; d. → q.

From relation $J = \sigma E$
Multiplying both sides by cross-sectional area of rod A, we get $JA = \sigma EA$ or $i = \sigma\phi$

$\dfrac{\phi}{i} = \dfrac{1}{\sigma}$ = resistivity of the rod

$\dfrac{E}{J} = \dfrac{1}{\sigma}$ = resistivity of the rod

$\sigma V = iV$ = power delivered to the rod

$\dfrac{V}{\sigma\phi} = \dfrac{V}{i}$ = resistance of the rod.

3. a. → q.; b. → r.; c. → r.; d. → r.

When the switch is closed, equivalent resistance is R. After opening the switch, the equivalent resistance becomes $2R$. Hence, equivalent resistance increases.
Also current through battery decreases, hence ammeter reading decreases. Current through left R also decreases. So voltmeter reading decreases and power dissipated through left R also decreases.

4. a. → p.; b. → r.; c. → s.; d. → r.

a. Since current in both rods is the same,

$\therefore\ n_1ev_1A_1 = n_2ev_2A_2$

$\therefore\ \dfrac{v_1}{v_2} = \dfrac{n_2A_2}{n_1A_1} = \dfrac{1}{2}\times\dfrac{2}{1} = 1$

b. $\therefore\ E = \rho J = \rho\dfrac{1}{A}$

$\therefore\ \dfrac{E_1}{E_2} = \dfrac{\rho_2}{\rho_1} = \dfrac{A_2}{A_1} = \dfrac{2}{1}\times\dfrac{2}{1} = 4$

c. $\dfrac{\text{p.d. across rod I}}{\text{p.d. across rod II}} = \dfrac{E_1\times AB}{E_2\times BC} = 4$

d. $\dfrac{\text{Average time taken by free electron to move from } A \text{ to } B}{\text{Average time taken by free electron to move from } B \text{ to } C}$

$= \dfrac{AB}{V_1}\times\dfrac{V_2}{BC} = 1$

5. a. → p.; b. → q.; c. → p.,s.; d. → s.

a. We know that power developed by R is maximum for $R = r$.

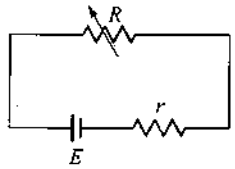


Fig. A2.207

b. Initially, angle between area vector of loop and magnetic field is 90°, it becomes zero and then again 90°after 180° rotation.

$\phi = BA\cos\theta$

$$e = -\frac{d\phi}{dt} = BA\sin\theta\left(\frac{d\phi}{dt}\right) = BA\,\omega\sin\theta$$

c. AB is the angle of the rod before the lens is displaced. CD is the image of the rod after displacing the lens perpendicular to the principle axis.

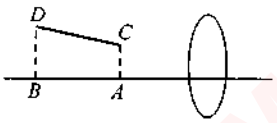


Fig. A2.208

d. $Z = \sqrt{R_2 + X_C^2} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

As ω is increased, z is decreased, so current through bulb will increase thus increasing the brightness.

6. a. → p.,q.,r.,s.; b. → p.,q.; c. → q.,r.; d. → q.,r.,s.

When S is open:

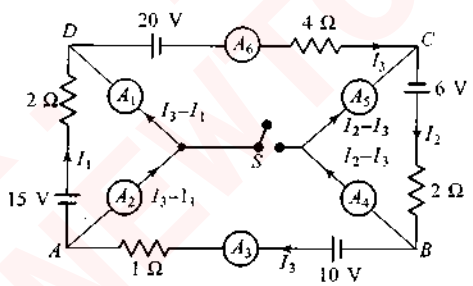


Fig. A2.209

$I_1 = \frac{15}{2} = 7.5$ A

$I_2 = \frac{6}{2} = 3$ A

Loop $ADCBA$:

$15 - 20 + 6 + 10 = 2I_1 + 4I_3 + 2I_2 + 1I_3$

$I_3 = -2$ A

Current in A_1 and A_2:

$I_3 - I_1 = -2 - 7.5 = -9.5$A

Current in A_4 and A_5:

$I_2 - I_3 = 3 - (-2) = 5$A

After closing S:

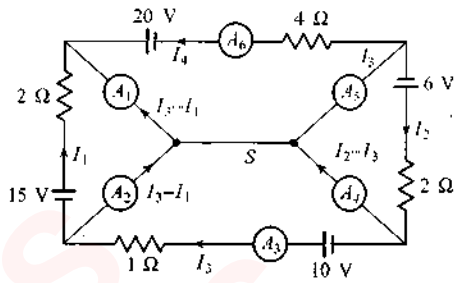


Fig. A2.210

$I_1 = \frac{15}{2} = 7.5$ A

$I_2 = \frac{6}{2} = 3$ A

$I_3 = \frac{10}{1} = 10$ A

$I_4 = \frac{20}{4} = 5$ A

Current in A_1:

$I_1 + I_4 = 7.5 + 5 = 12.5$ A

Current in A_2:

$I_3 - I_1 = 10 - 7.5 = 2.5$ A

Current in A_4:

$I_2 - I_3 = 3 - 10 = -7$ A

Current in A_5:

$I_4 - I_2 = 5 + 3 = 8$ A

7.a. → q.,s.; b. → q.,r.; c. → p.,s.; d. → p.,r.

a. For $P_{max} = R = 2r$

$$P_{max} = \left(\frac{2E}{R+2r}\right)^2 \quad R = \frac{E^2}{2r}$$

b. For P_{max} : $R = \frac{r}{2}$

$$P_{max} = \left(\frac{E}{R+R/2}\right)^2 \quad R = \frac{E^2}{2r}$$

c. $P = \left(\frac{2E}{r+2r}\right)^2 \quad r = \frac{4E^2}{9r}$

d. $P = \left(\frac{2E}{r+r/2}\right)^2 \quad r = \frac{4E^2}{9r}$

8.a. → q.,r.; b. → s.; c. → s.; d. → p.

a. It is possible that small length of potentiometer wire is used for positive terminal of e.m.f. to be measured may not be connected at extreme ends of the potentiometer wire.

b. Sensitivity of the potentiometer decreases.

c. Sensitivity of the potentiometer decreases

d. Accuracy of the potentiometer increases.

Integer Answer Type

1. (2) If the wire is connected as such across the battery, then current in wire,

$$i = \frac{V}{R} = \frac{200}{80} = 2.5 \text{ A}$$

and power obtained,

$$P = \frac{V^2}{R} = \frac{200 \times 200}{80} = 500 \text{ W}$$

The wire can carry maximum current of 5 A, therefore to double the current, the resistance should be halved. Thus if we divide the wire in two parts and the two parts are connected in parallel across 200 V mains supply, then the resistance of each part = 40 W; therefore, current in each wire = 200/40 = 5 A.

Net resistance, $R' = \frac{R_1 R_1}{R_1 + R_2} = \frac{40 \times 40}{40 + 40} = 20\ \Omega$

and new power obtained is

$$P_{max} = \frac{V^2}{R} = \frac{200 \times 200}{20} = 2000 \text{ watts}$$

Thus, the maximum power is 2000 W and this is obtained when wire is cut in two halves and they are connected in parallel across the given supply.

2. (1) $I_G = 10$ mA, $G = 10\ \Omega$

$S(I - I_G) = I_G G$ where S is shunt in parallel, solve to get $S = 0.1\ \Omega$.

3. (4) $R_{eq} = 2\ \Omega$; $i = 12/2 = 6$ A

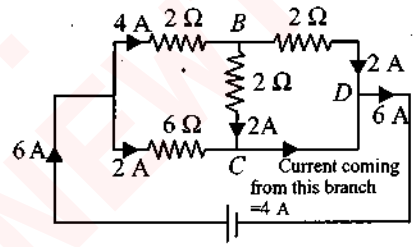


Fig. A2.211

Current through branch CD = 4 A.

4. (0) Potential difference across each of the resistors will be zero; hence, no current in any resistor.

5. (4) Let the voltmeter reading when the voltmeter has zero error be V.

$$\frac{I_1 R}{I_2 R} = \frac{V_1}{V_2} \Rightarrow \frac{1.75}{2.75} = \frac{14.4 - V}{22.4 - V} \Rightarrow \frac{7}{11} = \frac{14.4 - V}{22.4 - V}$$

$$\Rightarrow 7 \times (22.4 - V) = 11(14.4 - V)$$

$$\Rightarrow 156.8 - 7V = 158.4 - 11V \Rightarrow V = 0.4 \text{ V}$$

6. (6) $i = 2$ A, $R = 25$ W, $t = 1$ min $= 60$ sec

Heat developed $= i^2 Rt = (2 \times 2 \times 25 \times 60) = 100 \times 60$ J

$= 6000$ J

7. (2) $x = \frac{(2R + x)R}{3R + x}$

$3Rx + x^2 = 2R^2 + Rx$

$x^2 + 2Rx - 2R^2 = 0$

$$x = \frac{-2R \pm \sqrt{4R^2 + 8R^2}}{2} = R(\sqrt{3} - 1)$$

$$= (\sqrt{3} + 1)(\sqrt{3} - 1) = 3 - 1 = 2\ \Omega$$

8. (2) At y according to Kirchhoff's junction law,

$$\frac{y - x}{r} + \frac{y - x - 100}{r} + \frac{y - 50}{r} + \frac{y}{r} + \frac{y - 50}{r} = 0$$

$$5y - 2x = 200 \quad \text{(i)}$$

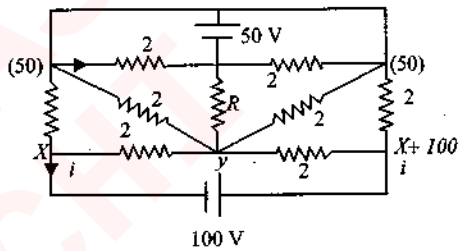


Fig. A2.212

Similarly at x,

$$i = \frac{50 - x}{r} + \frac{y - x}{r} \quad \text{(ii)}$$

at $x + 100$,

$$i = \frac{x + 100 - 50}{r} + \frac{x + 100 - y}{r} \quad \text{(iii)}$$

From Eqs. (ii) and (iii), we get $y - 2x = 50$ (iv)

From Eqs. (i) and (iv), $y = 37.5$ V

So, current through R is 37.5/18.75 = 2 A

Archives

Fill in the Blanks Type

1. We know that $P = \frac{V^2}{R}$

$\therefore\ R = \frac{V^2}{P} = \frac{100 \times 100}{500} = 20\Omega$. For the bulb to deliver 500 W, it should have a potential difference of 100 V across it. This would be possible only when $R = 20\ \Omega$. Because in that case both resistances will share equal potential difference and the potential difference across the bulb will be 200 V. Hence, the answer is 20 Ω.

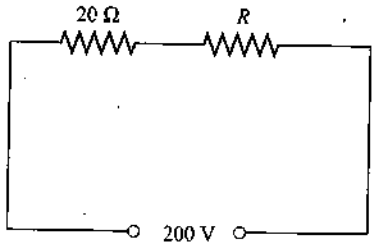


Fig. A2.213

2. The given circuit may be redrawn as shown in Fig. A2.214.
Thus, the resistances $2R$, $2R$ and R are in parallel.
Hence, $\frac{1}{R_{AB}} = \frac{1}{2R} + \frac{1}{2R} + \frac{1}{R} = \frac{2}{R}$. Hence, $R_{AB} = \frac{R}{2}$.

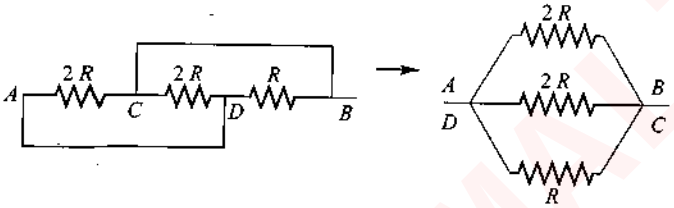


Fig. A2.214

3. Let a current I flow through the circuit. Net e.m.f. of the circuit = 8 (5 V) = 40 V. Net resistance in the circuit = 8(0.2 Ω) = 1.6 Ω.
Current flowing through the circuit, $I = 40$ V/1.6 Ω = 25 A.
The voltmeter reading would be $V = E - IR$ = (5 V) – (25 A) (0.2 Ω) = 5 V – 5 V = 0.
Alternatively: We can apply Kirchhoff's law in the loop and find the current.

True or False Type

1. **False:** An electrolyte solution is formed by mixing an electrolyte in a solvent. The electrolyte on dissolution furnishes ions. The preferred movement of ions under the influence of electric field is responsible for electric current.
2. **False:** The electrons in a conductor are free and have thermal velocities. Thus, electrons will be in motion even in the absence of potential difference.
3. **True:** For a given voltage, current is more in case of T_1 (Fig. A2.215)

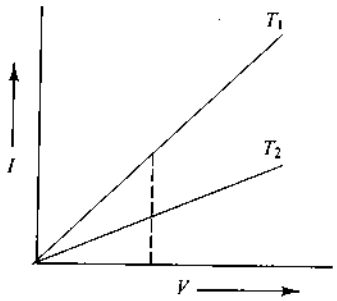


Fig. A2.215

Since, $V = IR$; $R = V/I$. Resistance is less in case of T_1. For a metallic wire, resistance increases with temperature, therefore $T_2 > T_1$.

Multiple Choice Questions with One Correct Answer Type

1. **b.** $R_2 = R_1(1 + \alpha\Delta T)$

$$2 = 1(1 + 00125 \times \Delta T) \Rightarrow \Delta T = \frac{1}{0.00125} = 800$$

$$\Rightarrow T_2 - T_1 = 800 \Rightarrow T_2 = 800 + T_1 = 800 + 300 = 1100 \text{ K}.$$

2. **d.** $H = I^2Rt = \frac{l}{\pi r^2}t$

3. **a.** $I \propto \frac{1}{r^2}; V \propto \frac{1}{r}$ for metallic sphere, at inside, at inside point

4. **b.** Let I_1 be the current flowing in 5 Ω and $(I - I_1)$ in 4 Ω and 6 Ω.

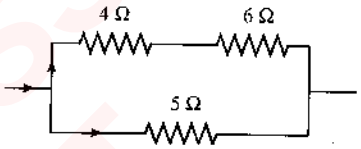


Fig. A2.216

The heat generated in the 5 Ω resistor is 10 cal/s = 4.2 × 10 J/s ∴ $4.2 \times 10 = I_1^2 R$

$$\therefore \quad I_1 = \sqrt{\frac{4.2 \times 10}{5}} = \sqrt{8.4} = 2.9 \text{ A}. \qquad \text{(i)}$$

Since AB and CD are in parallel, therefore the potential difference remains the same between C and D, and between A and B.

∴ $(I - I_1)(4 + 6) = I_1 \times 5$ on solving using I_1 from Eq. (i) we get $(I - 2.9)10 = 2.9 \times 5$

∴ $I - 2.9 = 1.45$

∴ $I = 4.35$. Heat released per second in 4 Ω will be $(4.35 - 2.9)^2 \times 4$ = A2.4 J/s = 2 cal/s.

5. **c.** $\frac{1}{R_{eq}} = \frac{1}{30} + \frac{1}{60} = \frac{90}{30 \times 60}, R_{eq} = 20\ \Omega\ V = IR$

$$I = \frac{2}{20} = 0.1 \text{ A}$$

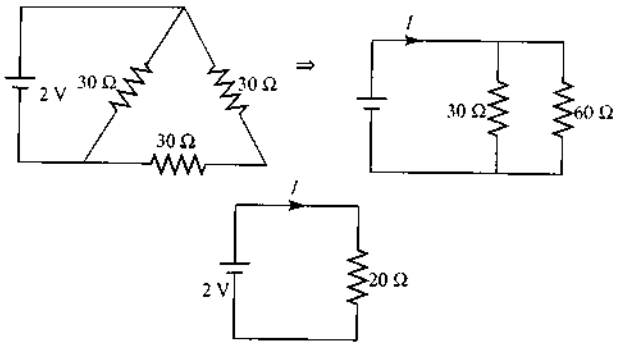


Fig. A2.217

6. **d.** Copper is a metal, whereas germanium is semi conductor.
7. **b.** For the maximum power,

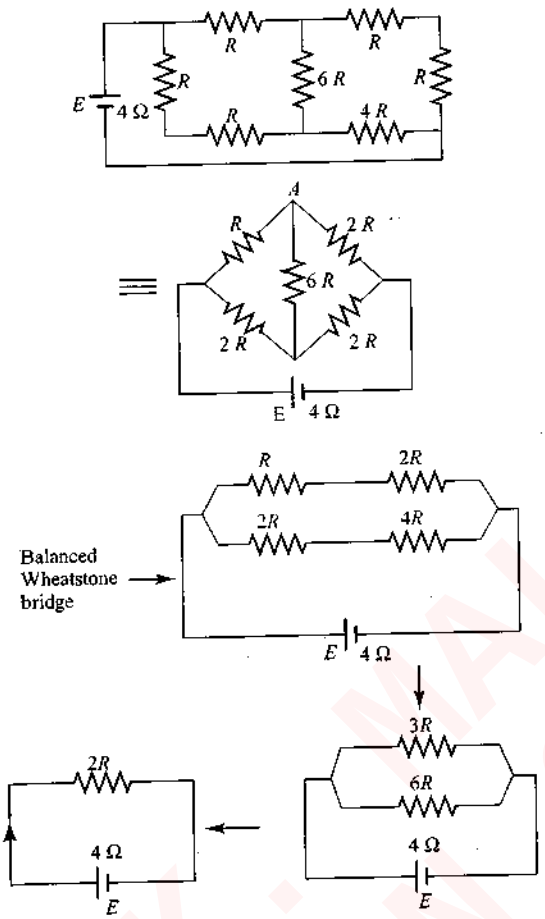


Fig. A2.218

External resistance = Internal resistance

$\therefore\ 2R = 4, R = 2.$

8. d. When a steady current flows in a metallic conductor of non-uniform cross section, then drift speed $V_d = I/n_e A$ and electric field $E = I/\sigma A \Rightarrow V_d \propto 1/A$ and $E \propto 1/A$. It implies only current remains is constant.

9. b. For the capacitor to get charged up to 0.75 V, the charge on the plates should be as follows.

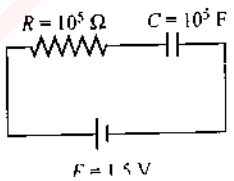


Fig. A2.219

$$q = CE[1 - e^{-t/RC}]$$

$$0.75\times10^{-5} = 10^{-5}\times1.5\left[1 - e^{\frac{-t}{10^5\times10^{-5}}}\right]$$

$$\frac{1}{2} = [1 - e^{-t}] \Rightarrow e^{-t} = \frac{1}{2}$$

Taking log on both sides, we get

$$-t = -\ln 2 \Rightarrow t = 0.693 \text{ s.}$$

10. a. Since opening or closing the switch does not affect the current through G, it means that in both the cases there is no current passing through S. This means that potential at A is equal to potential at B and it is the case of balanced Wheatstone bridge.

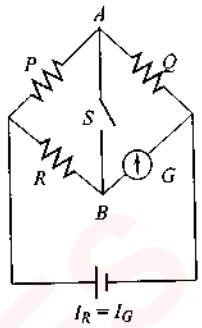


Fig. A2.220

$I_P = I_Q I_R = I_G$ and (a) is the correct option.

11. c. Method I: In this, there will be no current flowing in branch BE in steady condition. Let I be the current flowing in the loop $ABCDEFA$. Applying Kirchhoff's law in the loop moving in anticlockwise direction starting from C + 2V $- I(2R) - I(R) - V = 0 \Rightarrow V = 3IR \Rightarrow I = V/3R$. Applying Kirchhoff's law in the circuit $ABEF/A$ we get on moving in anticlockwise direction starting from B

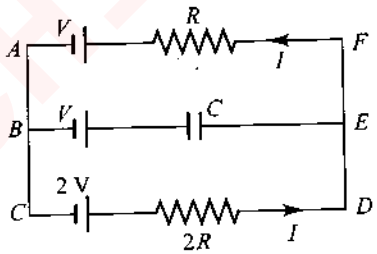


Fig. A2.221

$+ V + V_{cap} - IR - V = 0$ where V_{cap} is the potential difference across capacitor

$$\therefore\ V_{cap} IR = \left(\frac{V}{3R}\right)\times R = \frac{V}{3}.$$

Method II:

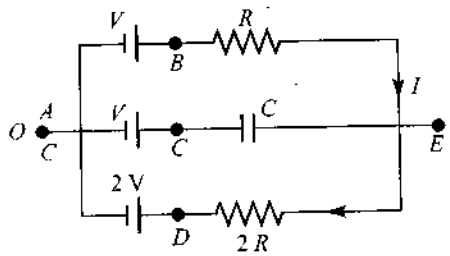


Fig. A2.222

Let us consider A to be at OV. Then points B, C and D will be at V, V and 2 V, respectively. Let the current be flowing in the clockwise direction. Applying Kirchhoff's law in the outer loop, we get $V - IR - I(2R) - 2V = 0$

$\therefore I = -V/3R$. The minus sign here indicates that the current is in the opposite direction to what we have assumed. Applying Kirchhoff's law from A to E via B, we get $V_A + V + IR = V_E$

$$\therefore\ 0 + V + \frac{V}{3R}\times R = V_E = \frac{4V}{3}$$

Again applying Kirchhoff's law from A to E via C, we get

$V_A + V + V_{cap} = V_c \therefore V_{cap} = \frac{V}{3}$.

2. b. Let R be the resistance of rod

Energy released in t seconds $= \frac{(3V^2)}{R} \times t$

$\therefore \quad Q = \frac{(9V^2)}{R} \times t$

But $Q = mc\Delta T = \frac{(9V^2)}{R} \times t$ (i)

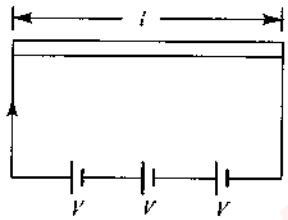


Fig. A2.223

Let R' be the resistance of rod

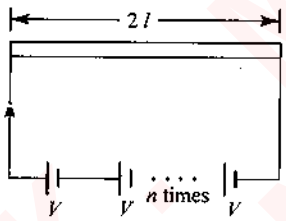


Fig. A2.224

$\Rightarrow \quad R' = 2R$ (as length is twice)

Therefore, energy released in t seconds $= \frac{(NV^2)}{2R} \times t$

$\therefore \quad Q' = \frac{(N^2V^2)}{2R} \times t$

Applying $Q' = m'c\Delta T$, we get

$2mc\Delta T = \frac{(N^2V^2)}{2R} \times t$ (ii)

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{mc\Delta T}{2mc\Delta T} = \frac{9V^2 \times t/R}{N^2V^2\, t/2R}$$

$\therefore \frac{1}{2} = \frac{9 \times 2}{N^2} \Rightarrow N^2 = 18 \times 2$

$\therefore N = 6$

13. c. Since current I is independent of R_6, it follows that the resistances R_1, R_2, R_3 and R_5 must form a balanced Wheatstone bridge.

14. a. The circuit is symmetrical about the axis PQ. Therefore, the equivalent circuit is

$$\therefore \quad \frac{1}{R_{PQ}} = \frac{1}{4R} + \frac{1}{4R} + \frac{1}{2r} = \frac{1}{2R} + \frac{1}{2r} \frac{R+r}{2Rr}$$

$$\Rightarrow \quad R_{PQ} = \frac{2Rr}{R+r}$$

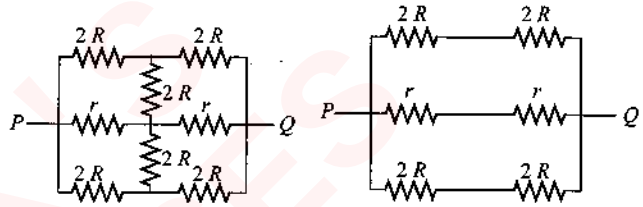


Fig. A2.225

15. d. We know that $R = \frac{V^2}{P}$

$$\therefore R_1 = \frac{V^2}{100}, R_2 = \frac{V^2}{60} = R_3$$

$$W_1 = \frac{V_1^2}{R_1} = \frac{V^2 R_1}{(R_1 + R_2)} W_2 = \frac{V_2^2}{R_2}, W_2 = \frac{V_2^2}{R_2}$$

$$W_3 : W_2 : W_1 : \frac{(250)^2}{R_3} : \frac{(250)^2}{(R_1 + R_2)^2} R_2 \frac{(250)^2}{(R_1 + R_2)}$$

$$= 60 : \frac{(250)^2 V^2 / 60}{\left(\frac{V^2}{100} + \frac{V^2}{60}\right)} : \frac{(250)^2 V^2 / 100}{\left(\frac{V^2}{100} + \frac{V^2}{60}\right)}$$

$= 60:25:15$

Hence, $W_3 > W_2 > W_1$

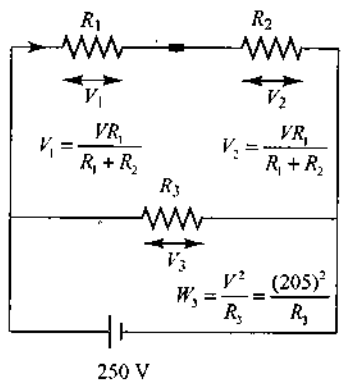


Fig. A2.226

16. a. In Ohm's law, we check $V = IR$ where I is the current flowing through a resistor and V is the potential difference across that resistor. Only option (a) fits the criteria. Remember that ammeter is connected in series with resistance and voltmeter in parallel with the resistance.

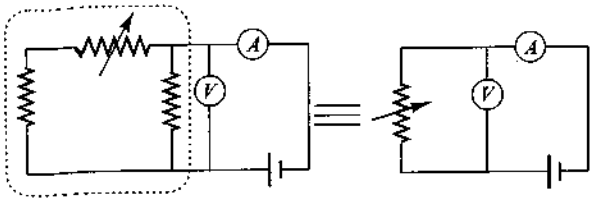


Fig. A2.227

17. a. At null point $\frac{R_1}{R_2}=\frac{R_3}{R_4}=\frac{x}{100-x}$. If radius of the wire is doubled, then the resistance of AC will change and also the resistance of CB will change. But since R_1/R_2 does not change, so R_3/R_4 should also not change at null point. Therefore, the point C does not change.

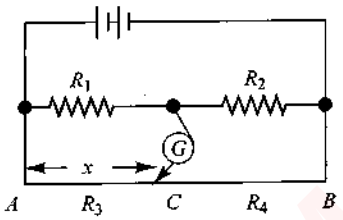
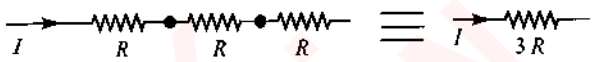


Fig. A2.228

$R \propto \frac{1}{A}, \frac{R_3}{R_4}=\frac{A_4}{A_3}=1$, Ratio does not change.

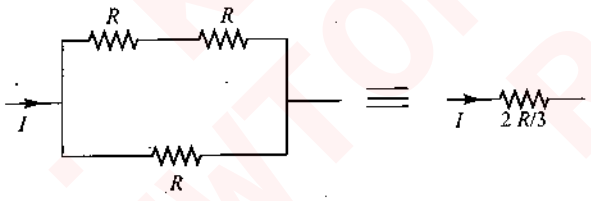
18. a.

(I)



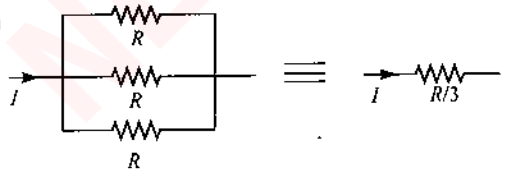
$P_t = I^2(3R)$

(II)



$P_{II} = I^2\left(\frac{2R}{3}\right)$

(III)



$P_{III} = I^2\left(\frac{R}{3}\right)$

(IV)

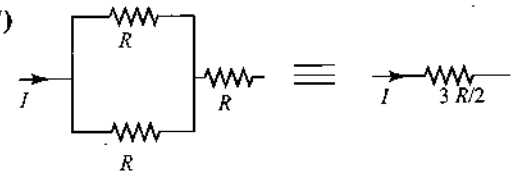


Fig. A2.229

$P_{IV} = I^2\left(\frac{3R}{2}\right)$

III < II < IV < I.

19. b. Total external resistance will be the total resistance of whole length of the box. It should be connected between A and D.

20. c. For various combinations, equivalent resistance is maximum between P and Q.

21. b. The current in RC circuit is given by $I = I_0 e^{-URC}$

$\Rightarrow \ln I = \ln I_0 - \frac{t}{RC}$ or $\ln I = \left(\frac{-t}{RC}\right) + 1NI_0$

$\ln I = \left(-\frac{t}{RC}\right) + \ln\left(\frac{E_0}{R}\right)$

On comparing with $y = mx + C$, we get

Intercept $= \ln\left(\frac{E_0}{R}\right)$ and slope $= -\frac{1}{RC}$

When R is changed to $2R$, then slope decreases in magnitude. New graph is Q.

22. a. The current in 2 Ω resistor will be zero because it is not a part of any closed loop.

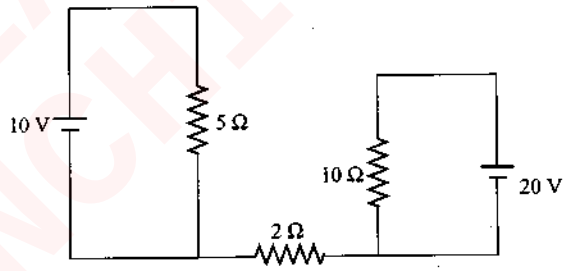


Fig. A2.230

23. a. Let at any instant of time t during charging process, the transient current in the circuit be I. $I = (V_0/R)e^{-t/RC}$. Therefore, potential difference across resistor R is $\left[\frac{V_0}{R}e^{-t/RC}\right]\times R = V_0 e^{-t/RC}$

Therefore, potential difference across

$C = V_0 - V_0 e^{-t/RC} = V_0(1-e^{-t/RC})$

$\Rightarrow 1 - e^{-t/RC} = 3e^{-t/RC} \Rightarrow 1 = 4e^{-t/RC}$

Taking log on both sides, we get

$$\log_e 1 = 2\log_e 2 + \left(-\frac{t}{RC}\right)$$

$\Rightarrow 0 = 2\times 2.303\log_{10} 2 - \frac{t}{RC}$

$\Rightarrow t = [2\times 2.303\log_{10} 2]\times 2.5\times 10^6 \times 4\times 10 = 13.86$ s

24. a. $I_g G = (I - I_g)S$. Here, $I_g = 100\times 10^{-6}$ A

$G = 100\ \Omega, S = 0.1\ \Omega$

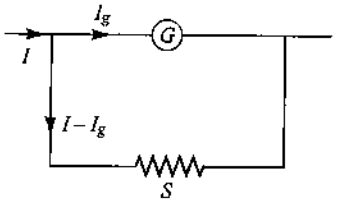


Fig. A2.231

$$\therefore\ I = I_g\left(\frac{G}{S}+1\right) = 100\times10^{-6}\left(\frac{100}{0.1}+1\right)$$

$$= 100\times10^{-6}\times1000.1 = 100.01 \text{ mA}$$

25. b. The heat supplied under these conditions is the change in internal energy $Q = \Delta U$. The heat supplied is
$Q = i^2Rt = 1\times1\times100\times5\times60 = 30{,}000 \text{ J} = 30 \text{ kJ}$

26. b. We know that time constant of an RC circuit is RC. Find equivalent resistance and capacitance in each case and then determine the time constant.

27. b.

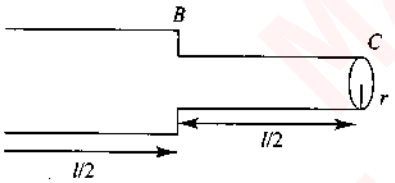


Fig. A2.232

(a) $\dfrac{V_{AB}}{V_{BC}} = \dfrac{I_{AB}R_{AB}}{I_{BC}R_{BC}} = \dfrac{R_{AB}}{R_{BC}} = \dfrac{\rho\dfrac{l}{2[\pi\times4r^2]}}{\rho\dfrac{l}{2[\pi r^2]}} = \dfrac{1}{4}$

(since $I_{AB} = I_{BC}$, wire is of the same material)
Therefore, option (a) is incorrect.

(b) $\dfrac{P_{BC}}{P_{AB}} = \dfrac{I_2R_{BC}}{I_2R_{AB}} = \dfrac{\rho\dfrac{l}{2[\pi\times4r^2]}}{\rho\dfrac{l}{2[\pi r^2]}} = \dfrac{1}{4}$

$\Rightarrow P_{AB} = 4P_{BC}$. Therefore, option (b) is correct.

(c) $\dfrac{J_{AB}}{J_{BC}} = \dfrac{\dfrac{I}{\pi\times4r^2}}{\dfrac{I}{\pi\times r^2}} = \dfrac{1}{4}$. Therefore, option (c) is incorrect.

(d) $\dfrac{E_{AB}}{E_{BC}} = \dfrac{\left[\dfrac{V_{AB}}{l/2}\right]}{\left[\dfrac{V_{BC}}{l/2}\right]} = \dfrac{1}{4}$. Therefore, option (d) is incorrect.

28. a. Given X is greater than 2 Ω when the bridge is balanced.

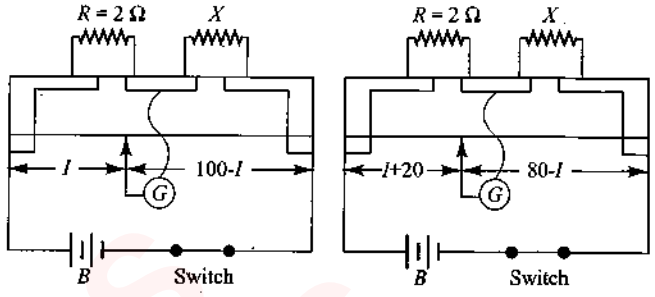


Fig. A2.233

$$\frac{R}{l} = \frac{X}{100-l} \Rightarrow 100R - Rl = lX$$

$$\Rightarrow\ 200 - 2l = lX \Rightarrow l = \frac{200}{X+2}$$

When the resistances are interchanged, the jockey shift 20 cm, therefore $\dfrac{X}{l+20} = \dfrac{2}{80-l}$

$$\Rightarrow\ 80X = \frac{200}{(x+2)} + 40 \Rightarrow X = \frac{240}{80} = 3\ \Omega$$

29. c. When steady state is reached, the current I from the battery is

$$9 = I(3+6) \Rightarrow I = 1\text{A}$$

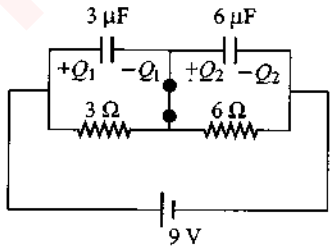


Fig. A2.234

Potential difference across 3 Ω resistance = 3 V
and potential difference across 6 Ω resistance = 6 V
p.d. across 3 μF capacitor = 3 V
and p.d. across 6 μF capacitor = 6 V
Therefore, charge on 3 μF capacitor

$$Q_1 = 3\times3 = 9\ \mu\text{C}$$

Charge on 6 μF capacitor

$$Q_2 = 6\times6 = 36\ \mu\text{C}$$

Charge $(-Q_1)$ is shifted from the positive plate of 6 μF capacitor. The remaining charge on the positive plate of 6 μF capacitor is shifted through the switch.
Therefore, charge passing through the switch $= 36 - 9 = 27\ \mu\text{C}$.

30. a. $C_{eq} = \dfrac{\dfrac{2\varepsilon_0}{d/3-Vt}\times\dfrac{\varepsilon_0}{2d/3+Vt}}{\dfrac{2\varepsilon_0}{d/3-Vt}+\dfrac{\varepsilon_0}{2d/3+Vt}} = \dfrac{6\varepsilon_0}{5d+3Vt}$

Now $\tau = C_{eq}R = \dfrac{6\varepsilon_0 R}{5d+3Vt}$.

31. c. Here, $R_1 = 1\ \Omega, R_2 = 1/2\ \Omega, R_3 = 3\ \Omega$

As $P = \frac{V^2}{R}$, so $P_2 > P_1 > P_3$.

32. d. Unknown resistance is directly proportional to standard resistance. On increasing the temperature, the value of unknown resistance will increase, hence standard resistance is also to be increased.

33. c. Equivalent circuit is as follows:

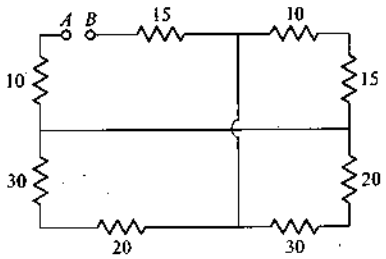


Fig. A2.235

Equivalent resistance = 37.5 Ω

34. d. Power $\propto 1/R$

35. c. G_1 is acting as voltmeter and G_2 is acting as ammeter.

36. We see that $R = \frac{\rho L}{Lt}$

Multiple Choice Questions with or More than One Correct Answer Type

1. b., d. During the decay of charge in RC circuit $I = I_0 e^{-t/RC}$ where $I_0 = q_0/RC$

When $t = 0$, $I = I_0 = q_0/RC$

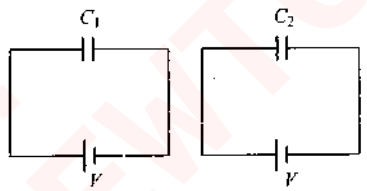


Fig. A2.236

Since potential difference between the plates is initially the same, therefore I is the same in both the cases at $t = 0$ and is equal to $I = \frac{q_0}{RC} = \frac{V}{R}$. Also $q = q_0 e^{-t/RC}$.

When $q_0 = \frac{q_0}{2}$, then $\frac{q_0}{2} = q_0 e^{-t/RC}$

$\Rightarrow e^{+t/RC} = 2 \frac{t}{RC} = \log_e 2 \Rightarrow t = RC \log_e \Rightarrow t \propto C$.

Therefore, time taken for the first capacitor (1 μF) for discharging 50% of initial charge will be less.

2. c. Both are basic facts

3. d.) The net resistance of the circuit is 9 Ω as shown in Fig. A2.237.

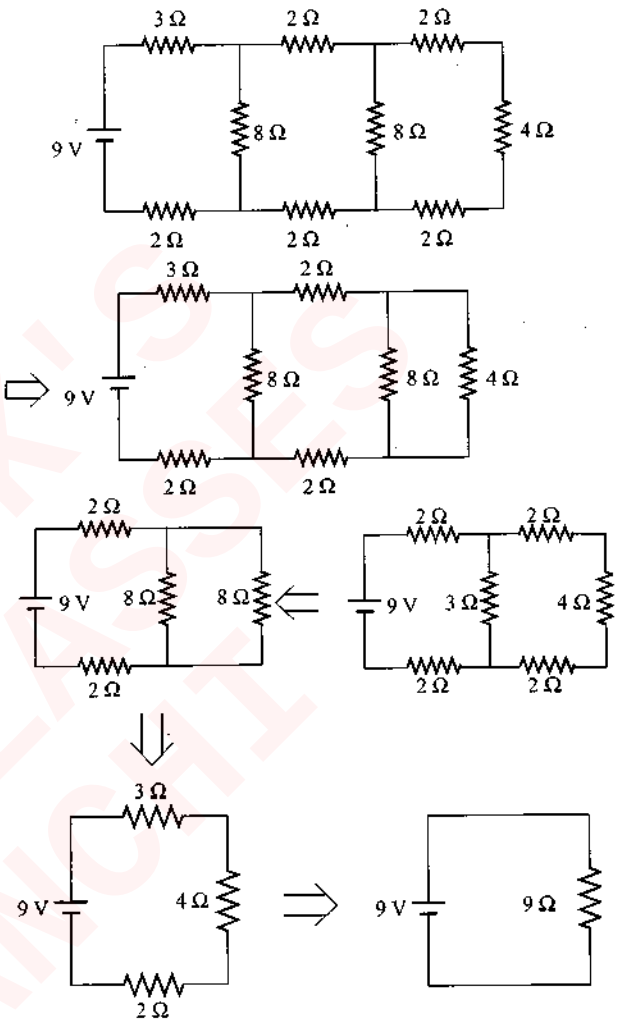
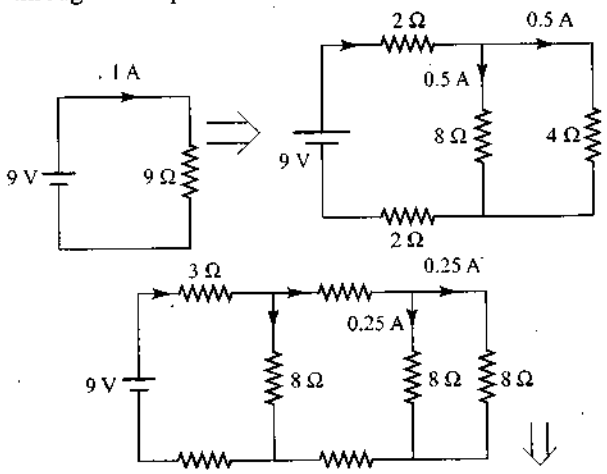


Fig. A2.237

The current flowing in the circuit is $I = V/R = 9\ V/9\ \Omega = 1.0$ A.

The flow of current in the circuit is as follows. Please note that the current gets divided into two equal parts if it passes through two equal resistances.



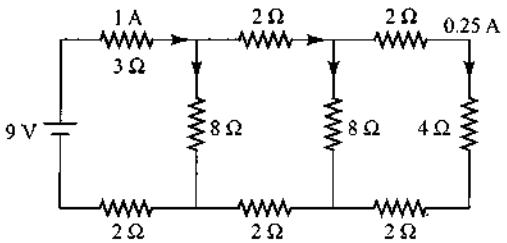


Fig. A2.238

4. a., b., d.

In case of metal, the current flow will be very-very high.

5. a., d.

$24 - 2 \times 10^3 I - 6 \times 10^3 (I - i) = 0$

$24 - 2 \times 10^3 I - 1.5 \times 10^3 i = 0$

Hence, $I = 7.5$ mA

$i = 6$ mA

$24 - 6 \times 10^3 I' - 2 \times 10^3 (I' - i') = 0$

$24 - 6 \times 10^3 I' - 1.5 \times 10^3 i' = 0$

$I' = 3.5$ mA

$i' = 2$ mA

$$\frac{P_1}{P_2} = \frac{6^2}{2^2} = 9$$

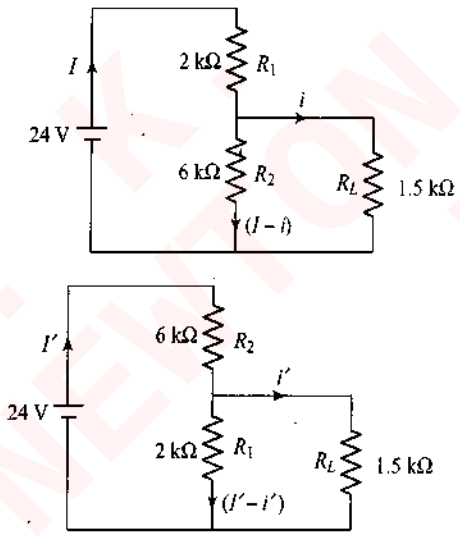


Fig. A2.239

Integer Answer Type

1. $J_1 = \left(\frac{2E}{R+2}\right)^2 R$

$J_2 = \left(\frac{E}{R+1/2}\right) R$ since $J_1 / J_2 = 2.25$

$R = 4\ \Omega$

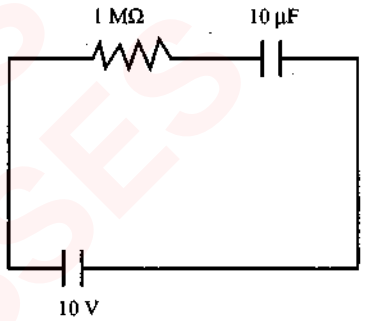


Fig. A2.240

2. $4 = 10\,(1 - e^{-t/4})$

$\therefore t = 2$ sec

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NEWTON CLASSES
RANCHI

Appendix 3

Solutions to Concept Application Exercises

CHAPTER 1

Exercise 1.1

1. a. $q = ne$

$$n = \frac{q}{e} = \frac{1}{1.6\times10^{-19}} = 6.25\times10^{18}$$

b. Repulsion. As a charged body can attract uncharged body.

c. No, as charge is quantized.

2. By induction.
3. a. Yes. Two bodies are placed close to each other where one has much more charge than the other. Then due to induction, force of attraction becomes more than force of repulsion.

b. Yes, due to induction effect.

c. Yes, mass of negatively charged sphere will be more.

4. a. Since the particle has positive charge, it has deficiency of electrons. So it has less number of electrons compared to the neutral state.

b. $q = ne$

$$10^{-12} = n \times 1.6 \times 10^{-19}$$
$$n = 6.25 \times 10^{6}$$

5. a. $q = ne$

$$3.2 \times 10^{-8} = n \times 1.6 \times 10^{-19}$$
$$n = 2 \times 10^{11}$$

b. Charge on fur should be equal and opposite to that on ebonite, i.e., $+3.2 \times 10^{-8}$ C.

6. Protons cannot be transferred, as they reside in nucleus. To remove protons, a great amount of energy is required.
7. When the paper comes in contact with the rod, the paper also gets similarly charged. Hence, repulsion takes place.
8. Conductor will not get completely discharged, because the person is standing on the insulating stool. Some charge of the rod may transfer to the person to make the potential of the both same.
9.

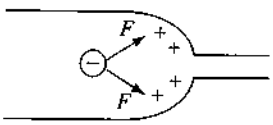


Fig. A3-1.1

Velocity increases due to induced charges on the neck.

10. There should be other charged particles to feel the effect of a charge particle, otherwise the concept of electric charge would be meaningless.

Exercise 1.2

1. i. There is always a restoring force towards its original position.The particle (negative charge) will perform oscillation. (The case of stable equilibrium.)

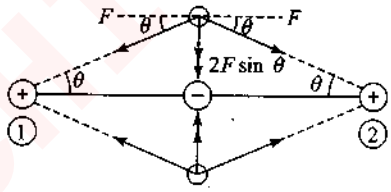


Fig. A3-1.2

ii. Let the negative charge be displaced towards charge (1). Then $\vec{F}_1$ will be more than $\vec{F}_2$. The negative charge will experience net force towards charge (1) or in the direction of its displacement. The negative charge is under unstable equilibrium.

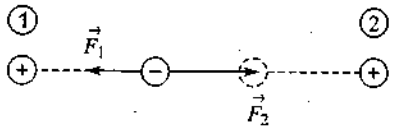
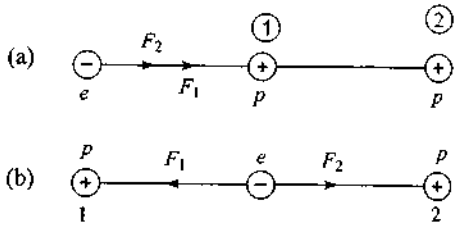


Fig. A3-1.3

2. No. A charged particle cannot apply force on itself.
3. Since negative charges are fixed, net force on them will always be zero and they will be in equilibrium. q will be in equilibrium if it is placed at the midpoint of negative charges.
4.



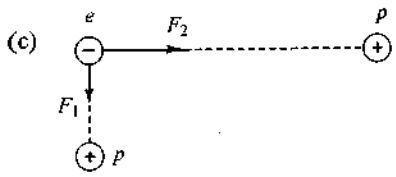


Fig. A3-1.4

a. It is clear from the diagram

$$F_a > F_c > F_b$$

b. Since $F_1 > F_2$; $\tan\theta = \dfrac{F_1}{F_2}$

$$\theta > 45°$$

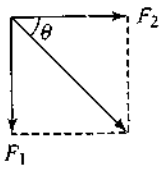


Fig. A3-1.5

5. a. Right, because third particle should be placed near the charge smaller in magnitude and not between the charges.

b. The third particle should be negatively charged, only then net force on any charge due to other two charges can be zero.

c. Equilibrium is unstable, because if we displace any of the charged particles from its equilibrium position, it may not return to its initial position for all directions of displacement.

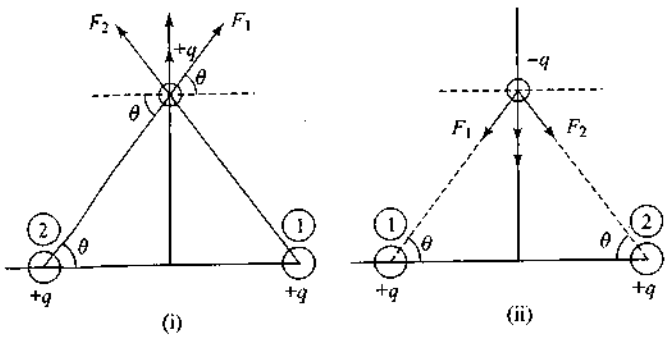
6. Net force on charge at centre due to outer ring will be zero. Due to inner ring: Magnitude of force on $-q$ at centre

$$F_n = \sqrt{F^2 + F^2} = \sqrt{2}F = \sqrt{2} \times \frac{1}{4\pi\varepsilon_0} \times \frac{2q \times q}{r^2}$$

$$= \frac{q^2}{\sqrt{2}\pi\varepsilon_0 r^2}$$

The direction is at angle 45° from the x-axis in IV-quadrant.

7.



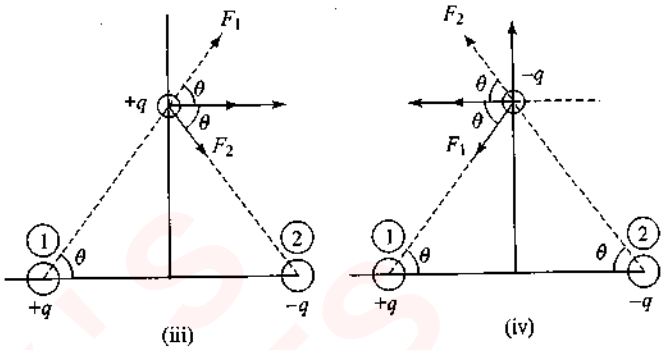


Fig. A3-1.6

a. It is clear from the diagrams that the forces $F(F_1$ or $F_2)$ in case (i), (ii), (iii) and (iv) will be same.

b. Less than $2F$

c. Cancel in situations (i) and (ii), and add in situations (iii) and (iv)

d. Add in situations (i) and (ii), and cancel in situations (iii) and (iv)

e. Obviously, the direction of net force will be that of adding components.

f. i. $+y$ **ii.** $-y$ **iii.** $+x$ **iv.** $-x$

8. When charges are brought in the medium, force between them will decrease by a factor of K, where K is known as dielectric constant of that medium. Hence,

$$\frac{F'}{F} = \frac{1}{\text{Dielectric constant}} = \frac{1}{81}$$

9. Because of the free fall, there will be weightlessness in the elevator. The balls will go maximum away from each other and finally angle between them will be 180°.

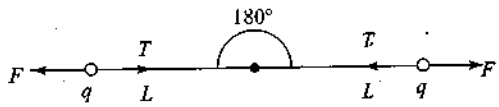


Fig. A3-1.7

Repulsion force F between them will be balanced by tension T. So

$$T = F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2L)^2} = \frac{q^2}{16\pi\varepsilon_0 L^2}$$

10. a. Since the size of group is small, the group of n small charged particles must behave as single point charge, so that it can have a separation of 10 cm from the particle in question. Obviously, force on this particle due to the group of n particles is n times the force due to a single particle. Hence, force due to group of n particles, $F = n \times 3 \times 10^{-10}$ N.

b. Here, $F = 6 \times 10^{-6}$ N

$$n = \frac{F}{3\times10^{-10}} = \frac{6\times10^{-6}}{3\times10^{-10}} = 2\times10^4$$

Exercise 1.3

1. a. Angular momentum of q will be constant about Q, because force acting on q due to Q will pass through Q and this force produces no torque about Q.
   b. No
2. a. Top (+); middle (–) bottom (+)
   Electric field lines are away from the positive charge and towards the negative charge.

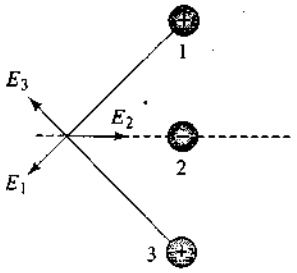


Fig. A3-1.8

   b. We can get two neutral points in the plane of paper, left and right side of middle charge, i.e., electric field will be zero at these locations.
   c. On perpendicular bisector of line joining the charges.
3.

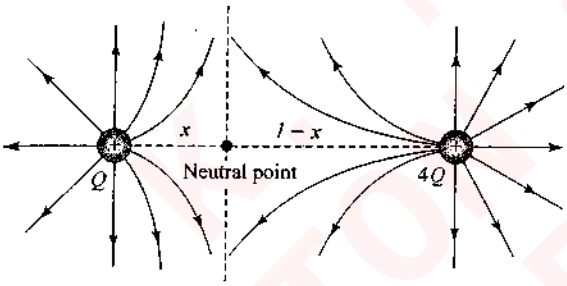


Fig. A3-1.9

For its location,

$$\frac{1}{4\pi\varepsilon_0}\frac{Q}{x^2}=\frac{1}{4\pi\varepsilon_0}\frac{4Q}{(\ell-x)^2}$$

$$\left(\frac{\ell-x}{x}\right)^2=4 \quad \Rightarrow x=\frac{\ell}{3}=\frac{12}{3}=4\,\text{cm}$$

Hence, neutral point will be at 4 cm from charge Q.

4. No, in given hypothetical field lines electric field is strong at bottom and weak at top. If we move a charge in a close path some work will be done by electric field, but in an electric field no work is done in close path.

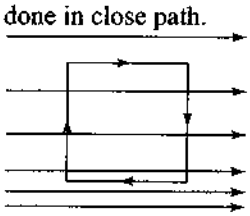


Fig. A3-1.10

5. a. Along the tangents at points A and B.
   b. At A, as intensity of electric field is more at A.
6. a. Yes
   b. Towards
   c. No, it is less than $2E$
   d. Cancel
   e. Add
   f. Adding components
   g. Negative y
7. Make use of symmetry property.

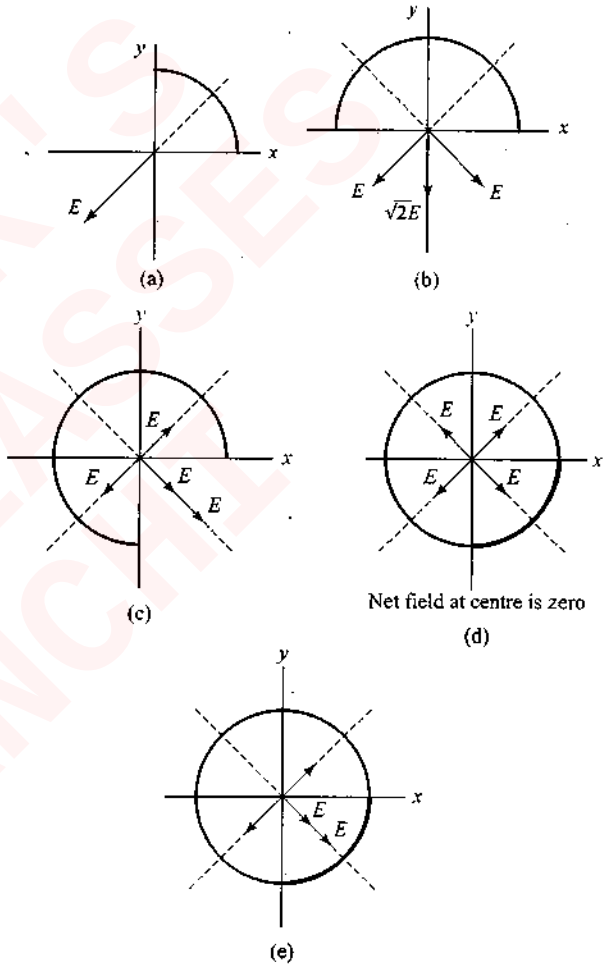


Fig. A3-1.11

It is clear from diagrams that order of fields will be (greatest first): (e), (b), (a,c), (d) [$2E$, $\sqrt{2}E$ and zero respectively]

8. It is clear from the diagram each other the components perpendicular to normal to plate will cancel and along the normal $dE\cos\theta$ add will. As the distance decreases, dE increases but $\cos\theta$ also decreases in such a way that $dE\cos\theta$ remains constant for all positions.

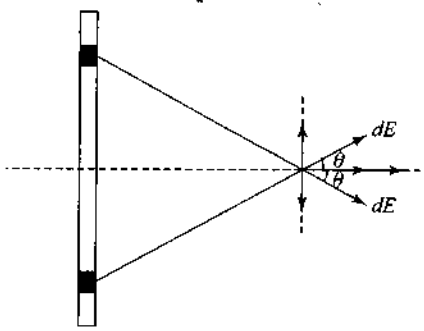


Fig. A3-1.12

It can be observed that near an infinite sheet, electric field is constant everywhere.

9. (1) and (2) are negative while (3) is positive.
Acceleration of each particle,

$$a = \frac{qE}{m}$$

Deflection suffered by a particle,

$$y = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}\frac{qE}{m}t^2 \Rightarrow y \propto \frac{q}{m}$$

Hence, (3) will have the highest q/m ratio.

10. **a.** Along C.

b. Path D. As x starts moving the magnitude of electric field due to charge on y decreases while the magnitude of electric field due to charge on z increases. The resultant electric field changes the direction as shown in the figure. Hence, we can say the charge particle follows path D.

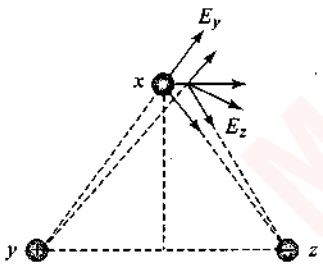


Fig. A3-1.13

11. Here we have assumed that the particle has started from the positive side of the x-axis.

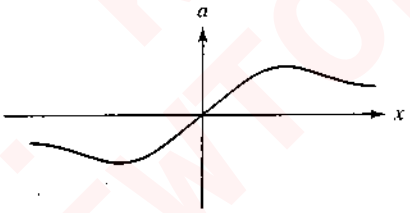


Fig. A3-1.14

Acceleration at infinity will be zero. When the particle comes towards the origin, first its acceleration will increase, becomes maximum and again becomes zero at origin and similarly on the negative side of the x-axis.

12. **b.** Magnitude of electric field will remain same but direction will be opposite.

13. We have to find intensity at P.

$$E = \frac{kQ}{a^2}, \frac{a/2}{x} = \cos 30° \quad \Rightarrow x = \frac{a}{\sqrt{3}}$$

$$y = \sqrt{a^2 - x^2} = \sqrt{a^2 - \frac{a^2}{3}} = \left(\sqrt{\frac{2}{3}}\right)a$$

$$\cos\theta = \frac{y}{a} = \sqrt{\frac{2}{3}}$$

Net intensity at P :

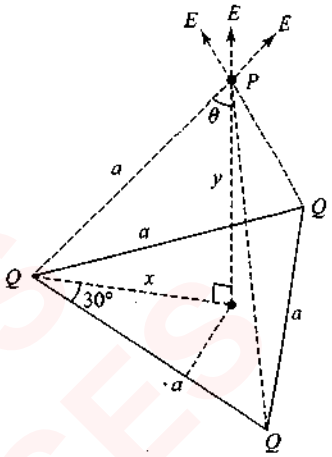


Fig. A3-1.15

$$E_0 = 3E\cos\theta = \frac{3kQ}{a^2}\sqrt{\frac{2}{3}}$$

$$= \frac{3}{4\pi\varepsilon_0}\frac{Q}{a^2}\frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{3}Q}{2\sqrt{2}\pi\varepsilon_0 a^2}$$

14. Let electric field at point P is zero, then

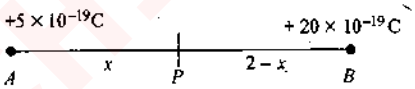


Fig. A3-1.16

$$\frac{1}{4\pi\varepsilon_0}\frac{5\times10^{-19}}{X^2} = \frac{1}{4\pi\varepsilon_0}\frac{20\times10^{-19}}{(2-X)^2}$$

or $$\frac{2-X}{X} = 2 \text{ or } X = \frac{2}{3}\text{m}$$

15. The acceleration experienced by a charged particle under the action of an electric field E is

$$a = \frac{eE}{m}$$

If it falls through a distance h, starting from rest,

then $$h = \frac{1}{2}at^2 \qquad (\text{as } u = 0)$$

or $$t^2 = \frac{2h}{a} = \frac{2h}{eE/m} = \frac{2mh}{eE}$$

or $$t \propto \sqrt{m} \qquad \text{(i)}$$

From Eq. (i) the time of fall will be

$\frac{t_{\text{electron}}}{t_{\text{proton}}} = \sqrt{\frac{m_e}{m_p}}$. Now since $m_p > m_e$, so $t_p > t_e$, i.e., time of fall through the same distance is greater for a heavier particle (i.e., proton), which is in contrast with the situation of free fall under gravity where the time of fall is independent of the mass of the body.

16. **a.** The force F experienced by the electron towards the positive plate is given by

$$F = qE = 1.6 \times 10^{-19} \times (45/16) \times 10^3$$
$$= 4.5 \times 10^{-16}\ \text{N}$$

Hence, the acceleration experienced by the electron is given by

$$a = \frac{F}{m} = \frac{4.5\times10^{-16}\ \text{N}}{9.0\times10^{-31}\ \text{kg}} = 5.0\times10^{14}\ \text{m/s}^2$$

Now, the electron is released from the negative plate and is travelling towards the positive plate. Hence we have

$$u = 0,\ x = 0.10\ \text{m and}\ a = 5.0 \times 10^{14}\ \text{m/s}^2$$

According to equation $x = ut + \frac{1}{2}at^2$,

$$t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2\times0.10}{5.0\times10^{14}}} = 2\times10^{-8}\ \text{s}$$

b. $v = u + at = 0 + 5.0 \times 10^{14} \times 2.0 \times 10^{-8} = 10^7$ m/s

17. Since q is in equilibrium with Q and Q,

$$k\frac{Qq}{x^2} = k\frac{Qq}{(r-x^2)} \quad \Rightarrow x = \frac{r}{2}$$

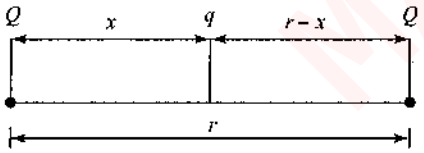


Fig. A3-1.17

Since Q is also in equilibrium with Q and q

$$k\frac{QQ}{r^2} = k\frac{Qq}{x^2} \Rightarrow q = \frac{Q}{4}$$

The sign of q should be opposite to that of Q.

18. **i.** Both the charges cannot be of same sign and magnitudes of charge should be different.

ii. The observation point (where electric field intensity is zero) has to be closer to the charge smaller in magnitude.

19. Electric field at the centre of the full ring will be zero, because field at centre due to some element will be cancelled by the opposite element of same length.

But in half ring, there will be a net field at the centre of the ring. So field due to half ring has greater magnitude.

20. $E = F/q = 3.2 \times 10^{-4}/1.6 \times 10^{-10} = 2 \times 10^6$ N/C

Exercise 1.4

1. **a.** False. Net force will be zero because equal and opposite forces will act on positive and negative charges. Torque $= pE \sin\theta$.

b. True. In a non-uniform electric field, forces on both charges may not be equal and opposite.

c. True. In a non-uniform electric field, a dipole can experience a force as well as a torque.

2. $E \propto \frac{1}{r^3}$, so $n = -3$.

3. As p is directed from negative charge to the positive charge, so angle made by E with p is $= \theta + \alpha$.

4. $E = \frac{kp}{r^3} = \frac{9\times10^9\times1\times10^{-6}\times100\times10^{-10}}{(0.1)^3} = 0.09$ N/C

5. Maximum torque $= pE = 1.2\times10^{-2}$ Nm

6. $p = qd$

$$P_{net} = 2p\cos 30°$$
$$= 2qd\frac{\sqrt{3}}{2} = \sqrt{3}qd$$

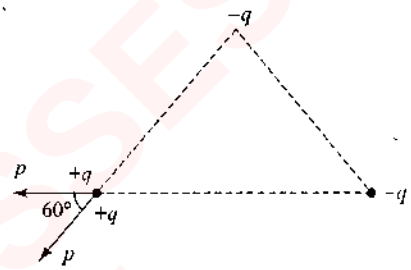


Fig. A3-1.18

7. $\alpha = 90 - \theta$

We know that $\tan\alpha = \frac{\tan\theta}{2}$

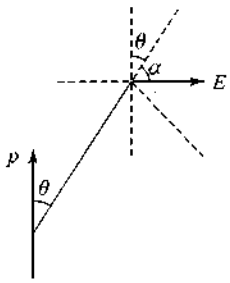


Fig. A3-1.19

$$\Rightarrow \quad \tan(90-\theta) = \frac{\tan\theta}{2} \quad \Rightarrow \tan\theta = \sqrt{2}$$

$$\Rightarrow \quad \theta = \tan^{-1}(\sqrt{2})$$

8. Suppose that the dipole axis makes an angle θ with the electric field at an instant. The magnitude of the torque on it is $\tau = pE \sin\theta$. This torque will tend to rotate the dipole back towards the electric field. Also, for small angular displacement, $\sin\theta \approx \theta$ so that

$$\tau = -q\ell E\theta$$

Negative sign is because torque applied by field will be opposite to angular displacement θ.

The moment of inertia of the system about the axis of rotation is

$$I = 2\times m\left(\frac{\ell}{2}\right)^2 = \frac{m\ell^2}{2}$$

Thus, the angular acceleration is

$$\alpha = \frac{\tau}{I} = -\frac{2qE}{m\ell}\theta = -\omega^2\theta$$

where $\omega^2 = \dfrac{2qE}{m\ell}$

Thus, the motion is angular simple harmonic and the time period is

$$T = 2\pi\sqrt{\frac{m\ell}{2qE}}$$

9. Distances of centre of mass from 2 kg, and 1 kg, respectively, are $x_1 = 2$ m, $x_2 = 4$ m

$$\tau = I\alpha; \quad \alpha = -\frac{pE}{I}\theta$$

$$T = 2\pi\sqrt{\frac{I}{pE}} = 2\pi\sqrt{\frac{2(2)^2 + 1(4)^2}{8\times10^4\times2\times10^{-6}\times6}}$$

$$\omega = \frac{2\pi}{T} = \sqrt{\frac{16\times6\times10^{-2}}{24}} = 0.2 \text{ rad/s}$$

10. x components will be cancelled.

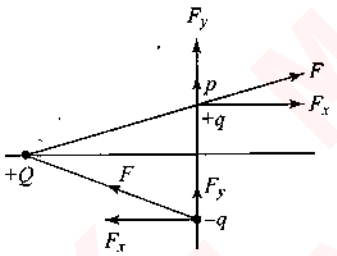


Fig. A3-1.20

y components will be added, giving net force along U.

CHAPTER 2

Exercise 2.1

1. a. No force will be experienced by the charge at the centre as it is electrostatically shielded.

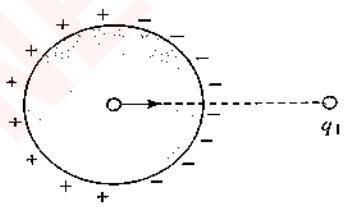


Fig. A3-2.1

b. There will not be any field at centre due to the charges Q and q. The charge induced at the surface of spherical shell due to q will produce the field towards right. This field will be responsible for producing force at centre towards right.

2. Becomes zero, thin metal plate will provide the shielding between the charges due to which electric field of one charge will not be able to reach the other charge.

3. Net flux though yz plane is $2\left(\dfrac{q}{2\varepsilon_0}\right) = \dfrac{q}{\varepsilon_0}$.

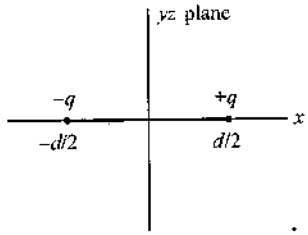


Fig. A3-2.2

So flux depends upon q but is independent of d.

4. As $\theta = 120°$, hence charge enclosed within sphere = $Q/3$.

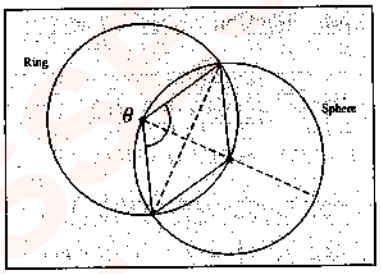


Fig. A3-2.3

$$\phi = \frac{Q_{\text{enclosed}}}{\varepsilon_0} = \frac{Q}{3\varepsilon_0}$$

5. As the rod enters the box, the charge increases; hence, flux increases linearly. When rod enters the cube completely, the charge within the cube becomes constant for sometime and hence the flux also becomes constant.
When the rod starts moving out of the cube, the charge inside the cube starts decreasing and hence the flux will also start decreasing linearly. Hence, graph d is the current graph.

6. a. Number of field lines coming to the curved surface is equal to the field lines moving away from the surface. Hence, the flux linked with curved surface is zero.
b. Flux passing through the plane surface = the flux passing through the curved surface.

$$\phi = E\pi R^2$$

(positive as field lines are away from the surface)

c. $\phi = E2\pi R^2$

7. Zero. No, as the charge resides on the outer surface of a conductor.

8. The charge induced on left side is positive.

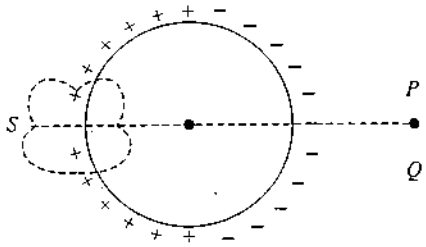


Fig. A3-2.4

Hence, charge enclosed by Gaussian surface is positive. Thus, the sign of the flux is '+ve'.

9. If the charge is placed at A or D, any electric field line will cross the curved surface twice.

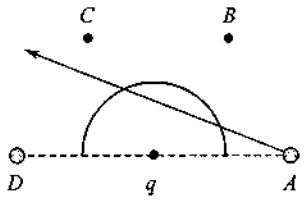


Fig. A3-2.5

Hence, the flux passing through the curved surface should be unchanged. If the charge is placed at B, some of the electric field lines may cut the surface once thereby changing the flux through the surface.

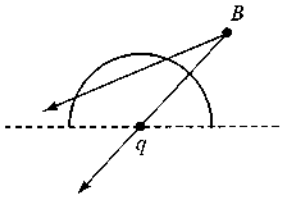


Fig. A3-2.6

10. **a.** and **b.**: Since the point charge Q is located just above the centre of the flate face, then almost half the lines emitting from Q will pass through the flate surface. Any line passing through the flat surface will also pass through the curved surface below it. Hence the required flux is $\phi = q/2\varepsilon_0$. Here flux through curved surface will be positive and through flat surface will be negative having same magnitude.

c. If the charge is placed exactly at the centre, then any line emitting from the charge will be parallel to the flat surface. Hence, there is no flux through the flat surface. But the flux through the curved surface will remain the same as $\phi = q/2\varepsilon_0$.

11. Effective area of the cone through which flux passes is

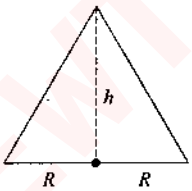


Fig. A3-2.7

$$A = 2\left[\frac{1}{2}Rh\right] = Rh$$

Flux $\qquad \phi = EA = ERh$

12. $\vec{E} = a\hat{i} + b\hat{j}$

a. $\vec{A} = A\hat{i} \quad \Rightarrow \phi = \vec{E}\cdot\vec{A} = Aa$

b. $\vec{A} = A\hat{j} \quad \Rightarrow \phi = \vec{E}\cdot\vec{A} = Ab$

c. $\vec{A} = A\hat{k} \quad \Rightarrow \phi = \vec{E}\cdot\vec{A} = 0$

13. **a.** Any line emitting from charge q to the right of line AB will pass through the infinite plane. It means half flux will pass through the plane. So the flux passing through the plane is $q/2\varepsilon_0$.

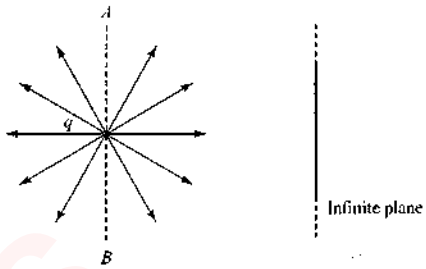


Fig. A3-2.8

b. q is above the centre at a small distance. So this becomes similar to the case (a).

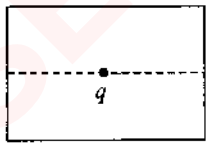


Fig. A3-2.9

14. Effective area $= A = \pi r^2$

$\therefore \qquad \phi = E_0 A = E_0 \pi r^2$

15. **a.** Since Q_1 is inside the surface, so flux due to Q_1 is Q_1/ε_0.

b. Since Q_2 is outside, so there is no contribution to the flux due to Q_2.

16. Let Coulomb's law involves $1/r^3$. Now, let us find the flux due to a point charge.

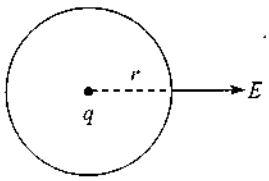


Fig. A3-2.10

$$E = \frac{kq}{r^3}$$

$$\phi = \int \vec{E}\cdot d\vec{s} = \int \frac{kq}{r^3} ds$$

$$\frac{kq}{r^3}\int ds = \frac{kq}{r^3} 4\pi r^2$$

$$\phi = \frac{4\pi kq}{r}$$

It means flux depends upon r, which is not true. So Gauss's law will not be true. Because in Gauss's law, flux through a closed surface depends upon the charge present inside and not on the shape or size of the Gaussian surface.

Exercise 2.2

1. **a.i.** When we bring a charge q at the centre of a sphere, charges $-q$ and $+q$ will be induced on inner surface and outer surface, respectively, as shown in Fig. A3-2.11(a).

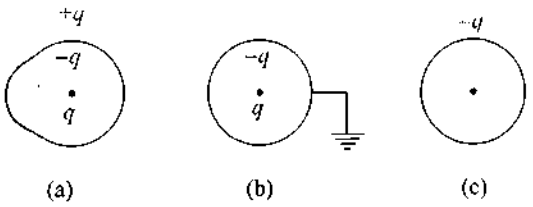


Fig. A3-2.11

ii. After earthing, $+q$ will be transferred to earth as shown in Fig. A3-2.11(b).

iii. After removing the earthing and charge q from centre, $-q$ will be distributed uniformly on the external surface of the sphere as shown in Fig. A3-2.11(c). So finally the sphere will have charge $-q$ distributed uniformly on the external surface.

b. The intensity of the electric field inside the sphere will be zero. An electric field similar to the field of the point charge q situated at the centre of the sphere will be set up outside the sphere. Direction of the field will be radially inward.

2. Flux through closed surface A,

$$\phi_A = \frac{Q_{in}}{\varepsilon_0} = \frac{q}{\varepsilon_0}$$

Flux through closed surface B,

$$Q_B = \frac{-q}{\varepsilon_0}$$

Flux through surface C,

$$Q_C = \frac{q-q}{\varepsilon_0} = 0 \text{ and } Q_D = \frac{Q_{in}}{\varepsilon_0} = 0$$

3. a. $\Phi_{S_1} = q_1/\varepsilon_0 = (4.00\times10^{-9}\text{ C})/\varepsilon_0 = 452\text{ Nm}^2/\text{C}$

b. $\Phi_{S_2} = q_2/\varepsilon_0 = (-7.80\times10^{-9}\text{ C})/\varepsilon_0 = -881\text{ Nm}^2/\text{C}$

c. $\Phi_{S_3} = (q_1+q_2)/\varepsilon_0 = ((4.00-7.80)\times10^{-9}\text{ C})/\varepsilon_0$

$= -429\text{ Nm}^2/\text{C}$

d. $\Phi_{S_4} = (q_1+q_3)/\varepsilon_0 = ((4.00+2.40)\times10^{-9}\text{ C})/\varepsilon_0$

$= 723\text{ Nm}^2/\text{C}$

e. $\Phi_{S_5} = \dfrac{(q_1+q_2+q_3)}{\varepsilon_0} = \dfrac{(4.00-7.80+2.40)\times10^{-9}\text{ C}}{\varepsilon_0}$

$= -158\text{ Nm}^2/\text{C}$

All that matters for Gauss's law is the total amount of charge enclosed by the surface, not its distribution within the surface.

4. We know that charge on facing surface is equal and opposite.

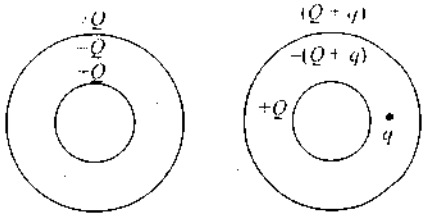


Fig. A3-2.12 **Fig. A3-2.13**

a. Net charge on inner surface is $-Q$.
b. Net charge on the inner surface will remain $-Q$.
c. Net charge on inner surface is $-(Q+q)$.
d. Yes, as location of charge does not matter.

5. The distribution of charge is as shown in Fig. A3-2.14.

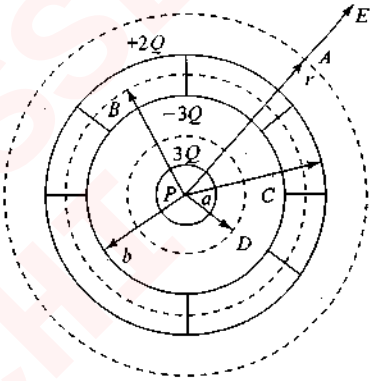


Fig. A3-2.14

a. Net charge enclosed by Gaussian surface ($r > c$) is $3Q - 3Q + 2Q = 2Q$.

b. For $r > c$, the direction of electric field is radially outward as shown at point A.

c. $E = \dfrac{kq_{in}}{r^2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2Q}{r^2}$

d. For $c < r < b$, we have point B. If we draw a Gaussian surface through B, net charge enclosed in it will be zero. So electric field at B will be zero.

e. As explained above, net charge enclosed is zero.

f. For $b > r > a$, we have point D. If we draw a Gaussian surface through D, it will have net charge enclosed $3Q$.

g. $E = \dfrac{kq_{in}}{r^2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{3Q}{r^2}$

h. $\rho = \dfrac{3Q}{\frac{4}{3}\pi a^3}$ (volume charge density)

So, charge within radius $r\,(< a)$ is $q = \rho\dfrac{4}{3}\pi r^3 = \dfrac{3Qr^3}{a^3}$

i. $E = \dfrac{kq_{in}}{r^2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{3Qr^3}{a^3 r^2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{3Qr}{a^3}$

j. As shown in the figure, the charge on the inner surface of shell is $-3Q$.

k. As shown in the figure, the charge on outer surface of shell is $+2Q$.

1.

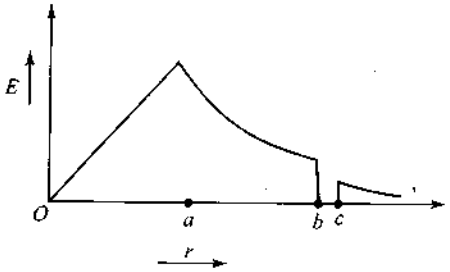


Fig. A3-2.15

6.

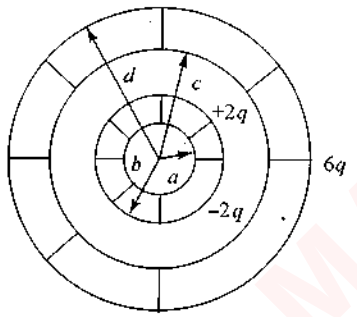


Fig. A3-2.16

b. i, ii. For $r < b$, electric field is zero, because there is no charge inside.

iii. For $c > r > b$, electric field will be similar to that as if a point charge $2q$ is placed at the centre.

iv. For $d > r > c$, electric field will be zero, as there is no charge inside.

v. For $r > d$, electric field will be similar to that as if a point charge $6q$ is placed at the centre.

The resulting plot is as shown in Fig. A3-2.17

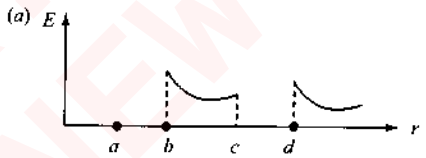


Fig. A3-2.17

c. i. 0

ii. $+2q$

iii. $-2q$

iv. $+6q$

7. a. False. Electric field is due to all charges present inside or outside.

b. False. Gauss's law is applicable for any distribution of charge. But it is more useful when the charge is distributed symmetrically.

c. True. Net flux is due to the charges inside only.

8. a. True. If E is zero at each point, then net flux will be zero.

b. True. If net flux is zero, then net charge enclosed will be zero.

9. a. Yes, as there is no charge for $r < R_1$.

b. Volume charge density $\rho = \dfrac{Q}{\frac{4}{3}\pi(R_2^3 - R_1^3)}$.

Charge within radius r is $q = \rho \dfrac{4}{3}\pi(r^3 - R_1^3)$

$$= \frac{Q(r^3 - R_1^3)}{R_2^3 - R_1^3}$$

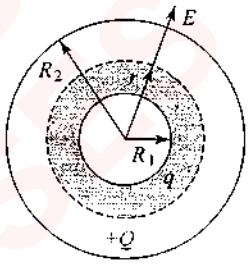


Fig. A3-2.18

$$E = \frac{kq}{r^2} = \frac{Q}{4\pi\varepsilon_0 r^2}\left[\frac{r^3 - R_1^3}{R_2^3 - R_1^3}\right]$$

c. For $r > R_2$,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

10. $\phi = EA;\ \phi = E\dfrac{\pi d^2}{4} \Rightarrow E = \dfrac{4\phi}{\pi d^2}$

11. a. $E = 0$, electric field will be cancelled due to both.

b. σ/ε_0 to the right, field of both will be added.

c. $E = 0$, electric field will be cancelled due to both.

12. According to Gauss' law,

$$\phi_1 = \frac{Q}{\varepsilon_0},\ \phi_2 = \frac{(Q + 2Q)}{\varepsilon_0}$$

$$\therefore \quad \frac{\phi_1}{\phi_2} = \frac{Q/\varepsilon_0}{3Q/\varepsilon_0} = \frac{1}{3}$$

13. The flux passing through curved surface

= the flux passing through the plane $ABCD$

$= E\,2R\,l$

14. a. The charge on the inner sphere induces equal magnitude of charge, but opposite in sign, on the inner surface of the outer sphere. Sum of all the induced charges is always zero. Therefore, an equal amount of charge must appear on the outer surface. Thus outer and inner surface of the outer sphere have charges $-5Q$ and $-2Q$, respectively.

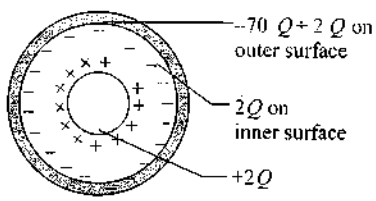


Fig. A3-2.19

b. When outer and inner spheres are connected by a wire, the entire charge is transferred to the outer surface from the inner sphere. In electrostatic equilibrium, charge does not reside inside a conductor. Total charge on the outer surface of the outer sphere is $-5Q$.
Total charge on the inner surface is 0.
The electric field at the surface of the inside sphere goes to zero after connection. Consider a Gaussian surface just on the surface of the inner sphere.

$$\phi_E = E(4\pi r^2) = \frac{Q_{enclosed}}{\varepsilon_0} = 0, \text{ as } Q_{enclosed} = 0$$

Thus, we have $E = 0$

c. When the outer sphere is grounded the charge on the outer surface is transferred to ground, thus charge is reduced to zero on the outer surface. The final charge distribution is shown in Fig. A3-2.20.

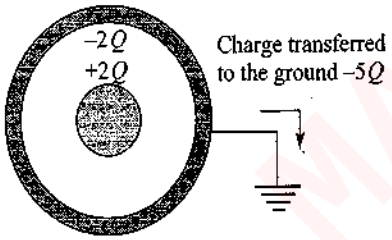


Fig. A3-2.20

CHAPTER 3

Exercise 3.1

1. True, because induced charges cancel the external electric field.
2. Electric field intensity E in that region should be zero, if potential is constant in a given region. This we can check by putting a charged particle in that region. If it experiences no force, E should be zero.
3. Yes, **a.** Potential will increase by 100 V. **b.** No effect on potential difference.
4. We know E in certain region, then we can find potential difference between any two points in that region. To find potential at one point, we should know the potential at some other points.
5. Yes, if shape and size of the conductors are different.

Exercise 3.2

1. It will loose the energy of $eV = 1.6 \times 10^{-19} \times 9 = 14.4 \times 10^{-19}$ J.
2. $eV = 4.3 \times 10^{-15} \Rightarrow 1.6 \times 10^{-19}\,V = 4.3 \times 10^{-15}\,V = 26875\,V$
3. $V = kQ/q$, for same distance r from a point, potential is the same. This is possible only for a spherical surface. So shape of equipotential surface should be spherical.
4. **a.** Points B and C lie on equipotential surface.
   So work done is zero.
   b. $V_B - V_A = V_C - V_A$, so magnitude of change in potential is same; hence, magnitude of work done will be the same.
5. Electric field is a conservative field and in any conservative field, work done in a close path must be zero.
6. For closed path, net work is zero.

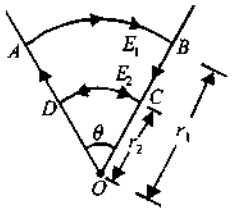


Fig. A3-3.1

$$W_{AB} + W_{BC} + W_{CD} + W_{DA} = 0$$
$$E_1 r_1 \theta + 0 - E_2 r_2 \theta + 0 = 0$$
$$\Rightarrow \quad E_1 r_1 = E_2 r_2 \Rightarrow E \propto \frac{1}{r}$$

7. $V_f - V_i = \int_i^f \vec{E}.d\ell$

   In a closed path, $V_i = V_f$. Hence, $\int_i^f \vec{E}.d\ell$ in a close path will be zero.

Exercise 3.3

1. Here, $q = \pm 8 \times 10^{-9}$ C; $2a = 4$ cm $= 0.04$ m
   $\tau = 4\sqrt{3}$ Nm and $\theta = 60°$

   a. Now, $p = q(2a) = 8 \times 10^{-9} \times 0.04 = 3.2 \times 10^{-10}$ Cm
   Torque on the electric dipole, $\tau = pE \sin\theta$

   or $$E = \frac{\tau}{p \sin\theta} = \frac{4\sqrt{3}}{3.2 \times 10^{-10} \times \sin 60°}$$
   $$= \frac{4\sqrt{3} \times 2}{3.2 \times 10^{-10} \times \sqrt{3}} = 2.5 \times 10^{10}\ \text{NC}^{-1}$$

   b. Potential energy of the electric dipole,
   $$U = -pE\cos\theta$$
   $$= -3.2 \times 10^{-10} \times 2.5 \times 10^{10} \times \cos 60°$$
   $$= -3.2 \times 2.5 \times 0.5 = -4\ \text{J}$$

2. Here, $E = 10^5\ \text{NC}^{-1}$; $q = 1\ \mu\text{C} = 10^{-6}$ C
   $2a = 2$ cm $= 0.02$ m
   $p = q(2a) = 10^{-6} \times 0.02 = 2 \times 10^{-8}$ Cm

   a. The torque acting on a dipole is given by
   $$\tau = pE\sin\theta$$
   Torque is maximum when $\sin\theta = 1$
   $$\tau_{max} = 2 \times 10^{-8} \times 10^5 \times 1 = 0.002\ \text{Nm}$$

   b. Now, work done in rotating the dipole from $\theta = \theta_1$ to $\theta = \theta_2$ is given by
   $$W = pE(\cos\theta_1 - \cos\theta_2)$$
   Here, $\theta_1 = 0°, \theta_2 = 180°$
   $$\therefore \quad W = 2 \times 10^{-8} \times 10^5 (\cos 0° - \cos 180°)$$
   $$= 0.002(1 - (-1)) = 0.004\ \text{J}$$

3. For 2 and 4, $\tau = pE\sin\theta$, $U = -pE\cos\theta$
   For 1 and 3, $\tau = pE\sin(180 - \theta) = pE\sin\theta$
   $U = -pE\cos(180 - \theta) = pE\cos\theta$

   Hence, torque is same for all and U is greater for 1 and 3 than 2 and 4.

4. a. Electric field will do positive work, if dipole rotates such that its angle decreases with E.
   b. 2 and 4 orientations are identical. Hence, work done will be same.
5. a. $U = -pE\cos\theta = pE\cos(180-\theta)$
   The more is U, the more is θ. So $\angle 4 > \angle 3 > \angle 1 > \angle 2$
   b. If angle between p and E is more closer to 90°, then more is torque. If angle is closer to 0° or 180°, then torque is less. It the means the lesser is the magnitude of potential energy, the more is torque and vice versa. So $3 > 1 = 4 > 2$.
6. a. As $p = 2l \times q$

$$l = \frac{p}{2q} = \frac{6.2\times10^{-30}}{2q}$$

   b. $\tau_{max} = pE = 6.2 \times 10^{-30} \times 1.5 \times 10^4 = 9.3 \times 10^{-26}$ Nm
   c. $W = 2pE = 2 \times 9.3 \times 10^{-26} = 1.86 \times 10^{-25}$ J
7. Potential energy of dipole will be zero because electric field due to point charges at dipole will cancel out. Also electric field due to any point charge is $\perp^r$ to the dipole moment.
8. a. True b. True
9. $V = \dfrac{kp\cos\theta}{r^2}$

For A, $\theta = 135°$; $V_A = \dfrac{kp\cos135°}{r^2} = \dfrac{-kp}{\sqrt{2}\,r^2}$

For B, $\theta = 45°$, $V_B = \dfrac{kp\cos45°}{r^2} = \dfrac{Kp}{\sqrt{2}\,r^2}$

$$W = q(V_B - V_A) = \frac{q\sqrt{2}\,k\,p}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\sqrt{2}\,qp}{r^2}$$

CHAPTER 4

Exercise 4.1

1. Yes, magnitude of charge on both plates will be equal. Because all the electrons lost from positive plate will go to negative plate.

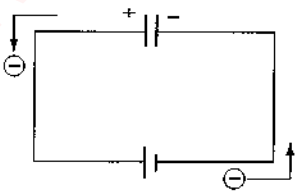


Fig. A3-4.1

2. This energy is stored as electrostatic potential energy between the plates.
3. Electric field between the plates of a capacitor is $E = \sigma/\varepsilon_0$. When the capacitor is submerged in a tank of oil,

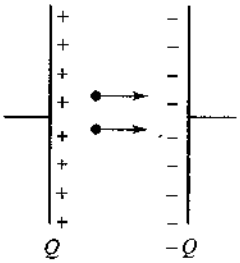


Fig. A3-4.2

the electric field between the plates is $E = \sigma/\varepsilon = \sigma/\varepsilon_0\varepsilon_r$. Hence, electric field decreases.

The field can be measured by measuring potential difference betwen the plates (V) and then it can be calculated using $E = V/d$.

4. a. $U = Q^2/2C$

$$Q = \sqrt{2UC} = \sqrt{2(25.0\text{ J})(5.00\times10^{-9}\text{ F})} = 5.00\times10^{-4}\text{ C}$$

The number of electrons N that must be removed from one plate and added to the other is

$N = Q/e = (5.00 \times 10^{-4}\text{ C})/(1.6 \times 10^{-19}\text{ C})$

$= 3.125 \times 10^{15}$ electrons

   b. To double U while keeping Q constant, decrease C by a factor of 2, $C = \varepsilon_0 A/d$, halve the plate area or double the plate separation.
5. Positive plate is given a charge Q, the charge distribution on different surface are shown in Fig. A3-4.3.

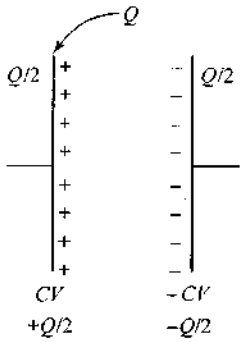


Fig. A3-4.3

Hence, new charge on capacitor is $\left(CV + \dfrac{Q}{2}\right)$; the potential difference between the plates will be

$$V' = \frac{CV + \frac{Q}{2}}{C} = V + \frac{Q}{2C}$$

6. a. The charge from one plate gets transferred to another plate through the battery. The battery pumps the charge from one plate to another.
   b. Yes, size of plates does not matter.
7. On connecting with earth, the charge on central plate will shift to its right face. $-Q$ charge will be induced on the left face of the earthed plate. Thus, $+Q$ charge will flow to the earth.

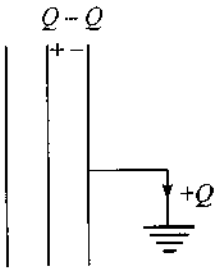


Fig. A3-4.4

8. $q = CV$, C decreases in both the cases.
   i. In the first case, V remains the same; hence, q decreases due to which force of attraction will decrease.
   ii. But in the second case, q will remain the same; hence, force of attraction remains the same. So more work will be done in the second case.
9. No, because electric field is conservative, so net work done in a closed path should be zero.
10. At point 1, distance between the plates is less, so attraction between the charges here can be more due to which concentration of charge at point 1 will be more.
11. a. True
    b. True
    c. True

Exercise 4.2

1. Electric field in a dielectric is $E = E_0/K$, in a conductor $E = 0$
   $\Rightarrow K = \infty$
2. a. $Q = CV_0$
   b. They must have equal potential difference, and their combined charge must add up to the original charge. Therefore,

$$V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} \text{ and also } Q_1 + Q_2 = CV_0$$

$$C_1 = C \quad \text{and} \quad C_2 = \frac{C}{\ }. \text{ so } \frac{Q_1}{C} = \frac{Q_2}{(C/2)}$$

$$\Rightarrow \quad Q_2 = \frac{Q_1}{2}$$

$$Q = \frac{3}{2}Q_1 \Rightarrow Q_1 = \frac{2}{3}Q$$

So $$V = \frac{Q_1}{C_1} = \frac{2}{3}\frac{Q}{C} = \frac{2}{3}V_0$$

c. $$U = \frac{1}{2}\left(\frac{Q_1^2}{C_1} + \frac{Q_2^2}{C_2}\right) = \frac{1}{2}\left[\frac{(\frac{2}{3}Q)^2}{C} + \frac{2(\frac{1}{3}Q)^2}{C}\right]$$

$$= \frac{1}{3}\frac{Q^2}{C} = \frac{1}{3}CV_0^2.$$

d. The original U was $U = \frac{1}{2}CV_0^2 \quad \Rightarrow \Delta U = \frac{-1}{6}CV_0^2$.

e. Thermal energy of capacitor, wires, etc. and electromagnetic radiation.

3. a. $U_0 = \dfrac{Q^2}{2C} = \dfrac{xQ^2}{2\varepsilon_0 A}$.

   b. Increase the separation by dx

$$\Rightarrow \qquad U = \frac{(x+dx)Q^2}{2\varepsilon_0 A} = U_0(1 + dx/x)$$

The change is then $(Q^2/2\varepsilon_0 A)dx$.

   c. The work done in increasing the separation is given by

$$dW = U - U_0 = \frac{dxQ^2}{2\varepsilon_0 A} = Fdx \Rightarrow F = \frac{Q^2}{2\varepsilon_0 A}.$$

   d. The reason for the difference is that E is the field due to both plates. The force is QE if E is the field due to one plate and Q is the charge on the other plate.
4. Let the applied voltage be V. Let each capacitor have capacitance C.

$$U = \frac{1}{2}CV^2$$

for a single capacitor with voltage V.

   a. Series: Voltage across each capacitor is $V/2$. The total energy stored is

$$U_s = 2\left(\frac{1}{2}C[V/2]^2\right) = \frac{1}{4}CV^2$$

   Parallel: Voltage across each capacitor is V. The total energy stored is

$$U_p = 2\left(\frac{1}{2}CV^2\right) = CV^2 \text{ or } U_p = 4U_s$$

   b. $Q = CV$ for a single capacitor with voltage V.

$$Q_s = 2(C[V/2]) = CV;$$

$$Q_p = 2(CV) = 2CV;$$

   c. $E = V/d$ for a capacitor with voltage V.

$$E_s = V/2d; \quad E_p = V/d; \quad E_p = 2E_s$$

5.

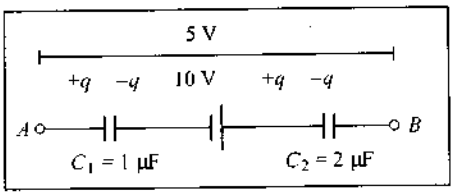


Fig. A3-4.5

$$V_A - \frac{q}{1} + 10 - \frac{q}{2} = V_B \quad \Rightarrow V_A - V_B = q + \frac{q}{2} - 10$$

$$5 = \frac{3}{2}q - 10 \quad \Rightarrow q = 10\ \mu\text{C}$$

6. Charge increases, capacitance increases, potential difference remains the same, electric field remains the same and energy stored increases. External agent will have to do negative work.

7. a. Five capacitors in series

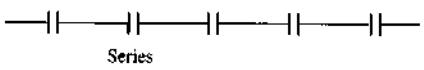
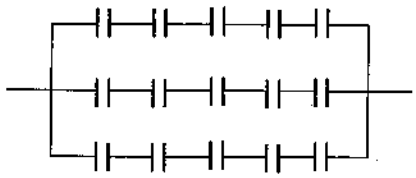


Fig. A3-4.6

b. Three rows in parallel with each row containing five capacitors



Series and parallel common grouping

Fig. A3-4.7

8. In series, potentials of all capacitors will be added, so net potential will be NV.

9. Since charges on outer surfaces will be half of the total charge of all the three plates: so $q_1 = q_6 = 60/2 = 30\mu C$

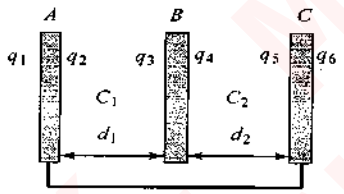


Fig. A3-4.8

$$q_3 = -q_2,\ q_4 = -q_5 \quad \Rightarrow V_{AB} = V_{CB}$$

$$\Rightarrow \quad \frac{q_2}{C_1} = \frac{q_5}{C_2}$$

$$q_2 d_1 = q_5 d_2 \quad \Rightarrow q_2 2d_2 = q_5 d_2$$

$$q_5 = 2q_2 \quad \Rightarrow q_4 = 2q_3$$

And $\quad q_3 + q_4 = 60$

$$q_3 = 20\ \mu C \quad \Rightarrow q_4 = 40\ \mu C$$

And $\quad q_2 = -20\ \mu C \quad \Rightarrow q_5 = -40\ \mu C$

10. a. $q_1 = 80\ \mu C,\ q_2 = 20\ \mu C$

$q_3 = -20\ \mu C,\ q_4 = 80\ \mu C$

b. $q_1 = 0,\ q_2 = -60\ \mu C$

$q_3 = 60\ \mu C,\ q_4 = 0$

c. $q_1 = q_2 = q_3 = q_4 = 0$

11. $V_C = 0$

Loop $ADBMA \rightarrow \quad \dfrac{q_1}{6} + \dfrac{q_1 - q_3}{3} = 10 \qquad$ (i)

Loop $ACBMA \rightarrow \quad \dfrac{q_2}{2} + \dfrac{q_2 + q_3}{1} = 10 \qquad$ (ii)

Loop $CDAC \rightarrow \quad -\dfrac{q_2}{2} + \dfrac{q_3}{1} + \dfrac{q_1}{6} = 0 \qquad$ (iii)

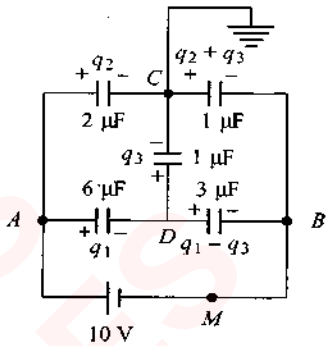


Fig. A3-4.9

Solving, we get $\quad q_1 = 20\ \mu C \quad \Rightarrow q_2 = 20/3\ \mu C,\ q_3 = 0$

a. $V_C - V_B = \dfrac{q_2 + q_3}{1} \quad \Rightarrow 0 - V_B = \dfrac{20}{3} + 0$

$$V_B = -\frac{20}{3}\text{V}$$

b. $V_C - V_D = \dfrac{-q_3}{1} \quad \Rightarrow V_D = 0$

c. $q_2 = \dfrac{20}{3}\ \mu C$

12. Both 3 μF capacitors will be in parallel, and similarly upper two 1 μF capacitors will be in parallel. The circuit may be redrawn as follows:

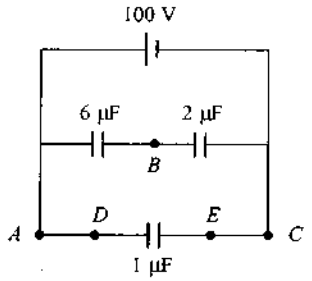


Fig. A3-4.10

a. $V_A - V_B = \dfrac{100 \times 2}{6 + 2} = 25\ \text{V}$

b. $V_A - V_C = \dfrac{100 \times 6}{6 + 2} = 75\ \text{V}$

c. $V_D - V_E = 100\ \text{V}$

d. $C_{eq} = \dfrac{6 \times 2}{6 + 2} + 1 = \dfrac{5}{2}\ \mu\text{F};$

$$U = \frac{1}{2} C_{eq} V^2 = \frac{1}{2}\left(\frac{5}{2} \times 10^{-6}\right) \times (100)^2$$

$$= 125 \times 10^{-4}\ \text{J}$$

13. a. False

b. True

c. True

14. Capacitors 4 and 5 are short-circuited. So, they are useless. '1', '2' and '3' will be in parallel and then 6 will be in series with them. $C_{eq} = 3C/4$

CHAPTER 5

Exercise 5.1

1. We know that $I = \frac{q}{t} = \frac{ne}{t}$, therefore $n = \frac{It}{e} = \frac{0.7 \times 1}{1.6 \times 10^{-19}}$

So number of electrons $n = 0.44 \times 10^{19}$

2. a. $q = it = (7.5\text{ A})(45\text{ s}) = 337.5\text{ C}$

b. The number of electrons N is given by

$$N = \frac{q}{e} = \frac{337.5\text{C}}{1.6 \times 10^{-19}\text{C}} = 2.1 \times 10^{21}$$

where $e = 1.6 \times 10^{-19}$ C is the charge of an electron.

3. a. The number N of electrons in 0.6 mol is
$N = (0.6\text{ mol})(6.02 \times 10^{23}\text{ electrons/mol}) = 3.6 \times 10^{23}$ electrons
$q = Ne = (3.6 \times 10^{23})(1.6 \times 10^{-19}) = 5.78 \times 10^4$ C

b. $t = (45\text{ min})(60\text{ s/min}) = 2.7 \times 10^3$ s

$$I = \frac{q}{t} = \frac{5.78 \times 10^4}{2.7 \times 10^3\text{ Cs}} = 21.4\text{ A}$$

4. Number per second = (number/length) (velocity)
$= (2 \times 10^{21})(0.05)$
$= 1 \times 10^{20}$ electrons/s
$I = Q/t = (1 \times 10^{20})(1.6 \times 10^{-19}) = 16$ A

5. $R = R_0\,[1 + \alpha(T - T_0)]$ or $\alpha = \Delta R/(R_0 \Delta T)$, with $\Delta R = R - R_0 = 0.17\,\Omega$ and $\Delta T = T - T_0 = 15°\text{C}$. Then $\alpha = (0.17)/(25.00 \times 15) = 4.5 \times 10^{-4}\text{ C}^{-1}$.

6. $R_{150°C} = 2.415 = R_0(1 + 150\alpha)$, $R_{20°C} = 1.6424 = R_0(1 + 20\alpha)$
Solving these relations simultaneously, $\alpha = 3.9 \times 10^{-3}\text{ C}^{-1}$ and $R_0 = 1.5236\,\Omega$. From $R_0 = \rho_0(L/A)$, we get

$$1.5236 = \frac{\rho_0(300)}{\pi(2 \times 10^{-3})^2/4} \quad \text{or} \quad \rho_0 = 1.596 \times 10^{-8}\text{ Wm}$$

Then, $\rho_{20°C} = \rho_0(1 + 20\alpha) = (1.596 \times 10^{-8})[1 + (3.9 \times 10^{-3})(20)] = 1.720 \times 10^{-8}\,\Omega\text{m}$

This indicates that the material is copper.

7. We need $R_1(1 + \alpha_1\Delta t) + R_2(1 + \alpha_2\Delta t) = 20$. Because $R_1 + R_2 = 20$ when $\Delta t = 0$, we must have $R_1\alpha_1 = -R_2\alpha_2$ with $\alpha_1 = -0.5 \times 10^{-3}$ and $\alpha_2 = 5 \times 10^{-3}$. Solving the two equations $R_1 + R_2 = 20$ and $R_1 = 10R_2$ simultaneously leads to $R_1 = 18.18\,\Omega$ and $R_2 = 1.82\,\Omega$.

8. We assume that α is constant over the needed temperature range. Then $\Delta R = \alpha R \Delta t$ leads to $(35 - 10) = 0.0036(10)\,\Delta t$. Solving, we get $\Delta t = 25/0.036 = 694°\text{C}$. Finally, $694 + 20 = 714°\text{C}$ (furnace temperature).

9. When switch is made on, the electric field $\vec{E}$ responsible for setting current is transmitted through the wire from switch to bulb at speed of light $c = 3 \times 10^8$ m/s. So in time L/c, i.e., immediately after the field is set up in the circuit and electrons start drifting producing current and hence light. However, the electron at the switch will take time L/v_d, i.e., hours to reach the bulb if it reaches the bulb at all.

10. a. As by definition, current and current density are related to each other through the relation $J = I/S$, so for steady current (i.e., I = constant) but nonuniform cross section J will not be constant but will vary with cross-sectional area S

as $J \propto \frac{I}{S}$ [I being constant]

Now, as J is related to electric field E and drift velocity v_d through the relations $J = \sigma E$ and $J = nev_d$

$$E = \frac{I}{\sigma S} \text{ and } v_d = \frac{I}{neS} \qquad \left[\text{as } J = \frac{I}{S}\right]$$

But as for a given metal σ and n are constants,

so $E \propto \frac{1}{S}$ and $v_d \propto \frac{I}{S}$ [as I = constant]

i.e., in case of steady current flow through a metallic conductor of non-uniform cross section, current is constant while current density, electric field and drift velocity are not constant and all vary inversely with area of cross section.

b. Yes, current flows in a conductor only when electric field established within the conductor exerts force on the electrons.

11. According to 'electron theory of metals' the drift speed of an electron inside a metal in the presence of an electric field E is given by $v_d = \left(\frac{e\tau}{m}\right)E = \left(\frac{e\tau}{m}\right)\left(\frac{V}{L}\right)$ $\left[\text{as } E = \frac{V}{L}\right]$

a. As $vd \propto V$, on doubling V, drift velocity will be doubled.

b. As $v_d \propto (1/L)$, on doubling L, drift velocity will be halved.

c. As drift velocity is independent of diameter d, it will not change on doubling the diameter.

12. In case of a resistance wire, $R = \rho\frac{L}{S} = \rho\frac{L}{\pi r^2}$ [as $S = \pi r^2$].

And in changing dimensions without any change in mass, volume remains constant, i.e., $V = V'$ or $SL = S'L'$.

a. When the wire is doubled on itself (as shown in Fig. A3-5.1(b)), its length will be halved, i.e., $L' = (L/2)$.

$$S \times L = S' \times (L/2), \quad \text{i.e., } S' = 2S$$

$$\therefore \quad R' = \rho\left[\frac{L'}{S'}\right] = \rho\frac{(L/2)}{(2S)} = \frac{1}{4}\rho\frac{L}{S} = \frac{1}{4}R$$

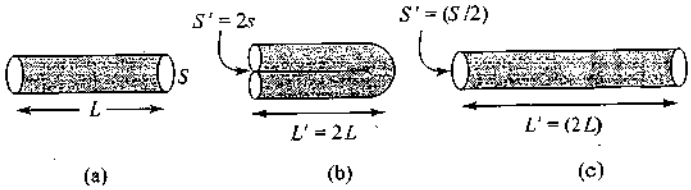


Fig. A3-5.1

b. **i.** When the length of the wire is doubled, i.e., $L' = 2L$ (as shown in Fig. A3-5.1(c))

$$SL = 2LS', \text{ i.e., } S' = S/2$$

$$R' = \rho\left[\frac{L'}{S'}\right] = \rho\frac{(2L)}{(S/2)} = 4\rho\left[\frac{L}{S}\right] = 4R$$

ii. When the radius of the wire is halved, i.e., $r' = r/2$

$$S' = \pi r'^2 = \pi(r/2)^2 = \pi r^2/4 = S/4$$

And hence from $SL = S'L'$, i.e., $SL = (S/4)L'$

we have $L' = 4L$, $R' = \rho\frac{L'}{S'} = \rho\frac{(4L)}{(S/4)} = 16\rho\frac{L}{S} = 16R$

13. Let dq be the charge which has passed in a small interval of time dt, then $dq = i\,dt = (4 + 2t)\,dt$.
Hence, the total charge passed between $t = 2$ and $t = 6$ is

$$q = \int_2^6 (4+2t)dt = 48 \text{ C}$$

14. **a.** $E_1 = \frac{V}{L}$, $E_2 = \frac{2}{2L} = \frac{V}{L}$, $E_3 = \frac{2V}{3L}$. Hence, $E_1 = E_2 > E_3$

c. $V_d \propto E$. Hence, $V_{d_1} = V_{d_2} > V_{d_3}$

b. $I = neAv_d = \frac{ne\pi d^2}{4}v_d = \frac{ne\pi}{4}v_d d^2$

$\Rightarrow I \propto v_d d^2 \Rightarrow I \propto Ed^2 \Rightarrow I = KEd^2$

$$I_1 = \frac{KV}{L}(3d)^2 = \frac{9KVd^2}{L},\ I_2 = K\frac{V}{L}d^2A$$

$$I_3 = K\frac{2V}{3L}(2d)^2 = \frac{8KVd^2}{3L}. \quad \text{Hence, } I_1 > I_3 > I_2.$$

15. We know $R = \frac{V}{I}$ and slope of IV curve $= \frac{I}{R}$

or $R = \frac{1}{\text{slope}}$

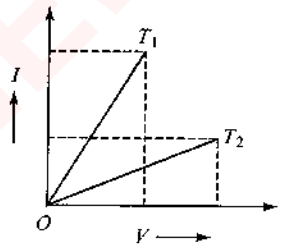


Fig. A3-5.2

Since the slope of OT_2 is smaller than OT_1, hence the resistance of wire at T_2 is greater than that at T_1 as resistance increases with temperature, so temperature T_2 is greater than T_1.

16. $R = \frac{\rho\ell}{A}$, therefore $\rho \propto R$. But $R_Y > R_X$

Because for same V, current in Y is less So, $\rho_Y > \rho_X$.

17. **a.** $J = nqv_d = (2 \times 10^8 \times 10^6) \times (2 \times 1.6 \times 10^{-19}) \times 10^5 = 6.4$ A/m^2.

As the velocity of positive ions is towards north, the direction of $\vec{J}$ will also be towards north.

b. No, we need area of cross section of the wire.

18. $dI = J2\pi r\,dr = J_0\left(1-\frac{r}{R}\right)2\pi r\,dr = 2\pi J_0\left(1-\frac{r}{R}\right)r\,dr$

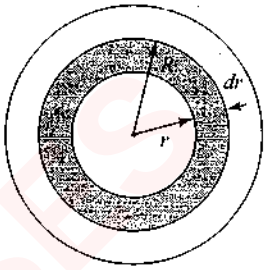


Fig. A3-5.3

$$I = \int dI = 2\pi J_0\int_0^R\left(1-\frac{r}{R}\right)r\,dr = 2\pi J_0\left[\int_0^R r\,dr - \frac{1}{R}\int_0^R r^2 dr\right]$$

$$= 2\pi J_0\left[\frac{R^2}{2} - \frac{R^2}{3}\right] = 2\pi J_0\frac{R^2}{6} = \frac{J_0 A}{3}$$

19. Let at distance x, the radius is r. Then,

$$r = a + \Delta r = a + \left(\frac{b-a}{\ell}\right)x$$

$$R = \int dR = \int\frac{\rho dx}{\pi r^2} = \int_0^\ell \frac{\rho dx}{\pi\left[a+\left(\frac{b-a}{\ell}\right)x\right]^2}$$

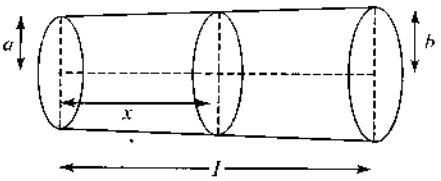


Fig. A3-5.4

Let $a + \left(\frac{b-a}{\ell}\right)x = z \Rightarrow \left(\frac{b-a}{\ell}\right)dx = dz$

z goes from a to b, we get

$$R = \int dR = \int_a^b \frac{\rho\ell}{\pi(b-a)}\frac{dz}{z^2} = \frac{\rho\ell}{\pi(b-a)}\int_a^b\frac{dz}{z^2}$$

$$= \frac{\rho\ell}{\pi(b-a)}\left[\frac{1}{a} - \frac{1}{b}\right] = \frac{\rho\ell}{\pi ab}$$

20. $$\frac{R_f - R_i}{R_i} = \frac{\rho\frac{l_f}{A_f} - \rho\frac{l_i}{A_i}}{\rho\frac{l_i}{A_i}} = \frac{\frac{l_f}{A_f} - \frac{l_i}{A_i}}{\frac{l_i}{A_i}} \quad \text{(i)}$$

Let the initial length of the wire be 100 cm, then the new length is $100 + \frac{0.1}{100}\times 100$, $l_f = 100.1$ cm (ii)

Let A_i and A_f be the initial and final area of cross section. Then $100 \times A_i = 100.1\, A_f$

$$\Rightarrow \quad A_f = \frac{100}{100.1} A_i \qquad \text{(iii)}$$

From Eqs. (i), (ii) and (iii), we get

$$\frac{R_f - R_i}{R_i} \times 100 = \frac{\dfrac{(100.1)^2}{100 A_i} - \dfrac{100}{A_i}}{\dfrac{100}{A_i}} \times 100$$

$$= \frac{(100.1)^2 - (100)^2}{(100)^2} \times 100$$

$$= \frac{200.1 \times 0.1}{100 \times 100} \times 100 = 0.2\%$$

21. $a = 0.5\ \text{mm}^2 = 0.5 \times 10^{-6}\ \text{m}^2$, $l = 0.1$ m, $T_1 = 25°\text{C}$, $I = 10$ A, $T_2 = 1075\ °\text{C}$

a. $I^2 Rt = ms\Delta T \Rightarrow I^2 \rho \dfrac{\ell}{a} \times t = ms\Delta T$

$$\Rightarrow \quad t = \frac{ms\Delta T \times a}{I^2 \rho l} = \frac{(d \times a \times l) s \Delta T \times a}{I^2 \rho l} = \frac{da^2 s \Delta T}{\rho I^2}$$

$$= \frac{9 \times 10^3 \times (0.5 \times 10^{-6})^2 \times 9 \times 10^{-2} \times 10^3 \times 4.18 \times 1050}{10 \times 10 \times 1.6 \times 10^{-8}}$$

$= 555.5$ s $= 9$ min 16 s.

b. Since length does not occur in the expression of time, the melting does not depend on the length. So time taken will be the same.

Exercise 5.2

1. $\varepsilon_{eq} = \dfrac{\dfrac{6}{3} + \dfrac{12}{5}}{\dfrac{1}{3} + \dfrac{1}{5}} = \dfrac{33}{4}$ V, $r_{eq} = \dfrac{1}{\dfrac{1}{3} + \dfrac{1}{5}} = \dfrac{15}{8}\ \Omega$

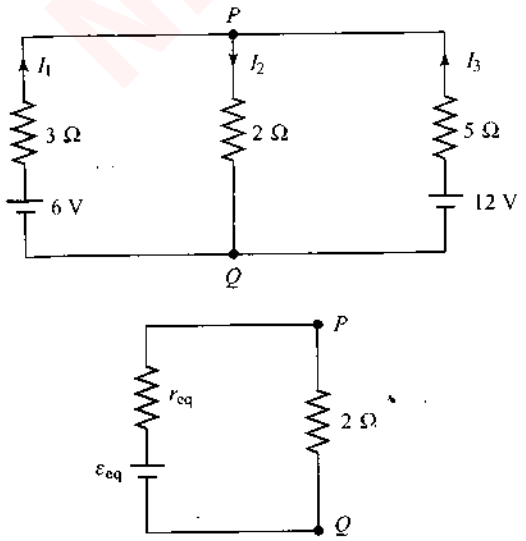


Fig. A3-5.5

Hence, $I_2 = \dfrac{\dfrac{33}{4}}{2 + \dfrac{15}{8}} = \dfrac{66}{31}$ A

Potential difference across PQ is

$$V_{PQ} = I_2 R = \frac{66}{31} \times 2 = \frac{132}{31}\ \text{V}$$

Current through 6 V battery: $\dfrac{132}{31} = 6 - 3I_1 \Rightarrow I_1 = \dfrac{18}{31}$ A

Hence, $I_3 = I_2 - I_1 = \dfrac{66}{31} - \dfrac{18}{31} = \dfrac{48}{31}$ A

2. R_3 and R_4 are in series. The combined resistance is 400 Ω. Now $R_2 = 100\ \Omega$ and $400\ \Omega$ resistances are in parallel. The combined resistance is $(100 \times 400)/500 = 80\ \Omega$. Total resistance $R = 80 + 50 = 130\ \Omega$

$I = 65/130 = 1/2$ A. So, $V = IR_1 = 1/2 \times 50 = 25$ V

3. Equivalent resistance R' of the parallel combination of 12Ω, 6 Ω and 4 Ω resistances is given by

$$\frac{1}{R'} = \frac{1}{12} + \frac{1}{6} + \frac{1}{4} = \frac{1}{2} \quad \text{or } R' = 2\ \Omega$$

Total resistance of the circuit is $R = 1 + 5 + 2 + 2 = 10\ \Omega$

$E = 20 - 8 = 12$ V

a. Current in the circuit, $I = \dfrac{E}{R} = \dfrac{12}{10} = 1.2$ A

b. Combined resistance of 6 Ω and 4 Ω in parallel is

$$\frac{6 \times 4}{6 + 4} = 2.4\ \Omega$$

Current in resistor of 12 Ω coil $= \dfrac{2.4}{12 + 2.4} \times 1.2 = 0.2$ A

c. p.d. across the 20 V battery $= (20 - 1.2 \times 1) = 18.8$ V

p.d. across the 8 V battery $= (8 + 1.2 \times 2) = 10.4$ V

4. a. When no current is drawn from a cell, potential difference across the terminals of the cell is equal to its e.m.f. It is clear that e.m.f. = 1.4 V

b. Further, $V = \varepsilon - Ir \Rightarrow r = \dfrac{\varepsilon - V}{I}$

Here, $\varepsilon = 1.4$ V. Consider any given value of potential difference from graph, say $V = 1.2$ V.

Current corresponding to this p.d. is 0.04 A

$$\therefore \quad r = \frac{1.4 - 1.2}{0.04} = 5\ \Omega$$

5. The circuit can be redrawn as shown:

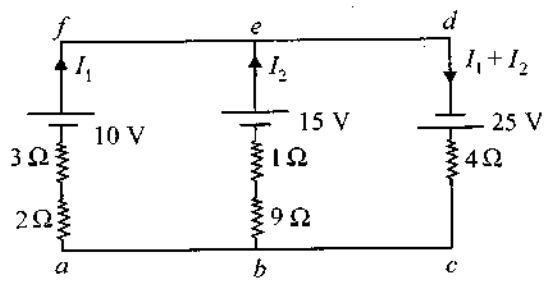


Fig. A3-5.6

Loop *afdca*: $(2 + 3)I_1 + 4(I_1 + I_2) = 10 + 25$

$\Rightarrow \quad 9I_1 + 4I_2 = 35$ (i)

Loop *edcbe* : $(1+9)I_2 + 4(I_1 + I_2) = 15 + 25$

$\Rightarrow \quad 4I_1 + 14I_2 = 40$ (ii)

From Eqs. (i) and (ii), we get $I_1 = 3\text{A}, I_2 = 2\text{A}$.

6. Since the galvanometer shows no deflection, therefore Wheatstone bridge is balanced. Applying condition for balanced Wheatstone bridge, $\frac{100(100+R)}{100R} = \frac{200}{40} = 5$

or $5R = 100 + R$ or $R = 25\ \Omega$

7. $0.25 = \frac{10E}{59+10r}$ and $25 = \frac{E}{0.05 + \frac{r}{10}}$. On solving, we get

$E = 1.5$ V and $r = 0.1\ \Omega$

8. $nm = ?$, $30 = \frac{n \times 1}{m}$ or $n = 30$ m, $1.5 = \frac{n \times 1.5}{30 + \frac{n}{m}}$

On solving, we get $n = 60$, $m = 2$, total number of batteries $= nm = 120$.

9. If V is the applied potential difference and R is the original resistance, then $V/R = 5$ and $V/(R+2) = 4$.

Dividing, we get $\frac{R+2}{R} = \frac{5}{4}$ or $R = 8\Omega$

10. An equivalent of the given network is shown in Fig. A3-5.8. It is clear from the network of the figure that the resistances of 2 Ω, 4 Ω, 4 Ω and 2 Ω are connected in parallel with each other. If R_p is the total resistance, then

$$\frac{1}{R_p} = \frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{1}{2} \Rightarrow \frac{1}{R_p} = \frac{2+1+1+2}{4} = \frac{3}{2}$$

or $R_p = \frac{2}{3}\Omega$

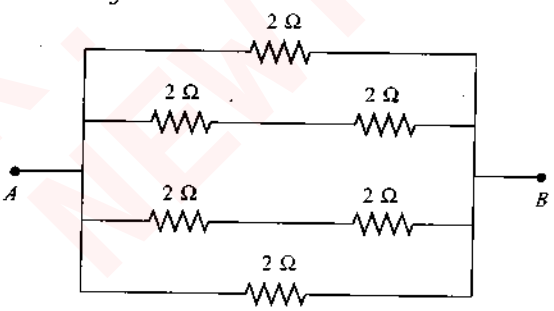


Fig. A3-5.7

11. Net e.m.f. $= 8 - 4 = 4$ V, $R_{AB} = 6 \times 3/9 = 2\ \Omega$

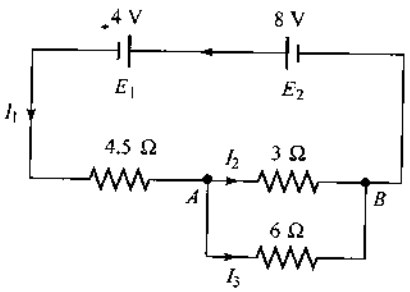


Fig. A3-5.8

Total resistance of the circuit $= 8\ \Omega$

So, $I_1 = 4/8 = 0.5$ A, $V_{AB} = 0.5 \times 2 = 1$ V

So, $I_2 = 1/3$ A and $I_3 = 1/6$ A

12. Traversing the upper and lower loops anticlockwise, we get

$2I_1 + 6I_2 = 24 - 27 \Rightarrow 2I_1 + 6I_2 = -3$ (i)

and $4I_3 - 6I_2 = 27 \Rightarrow 4(I_1 - I_2) - 6I_2 = 27$

$\Rightarrow \quad 4I_1 - 10I_2 = 27$ (ii)

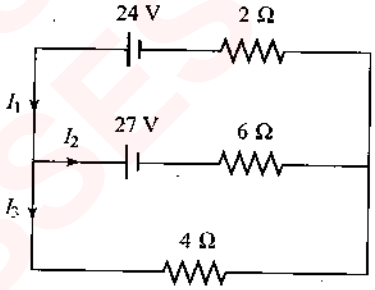


Fig. A3-5.9

Solving Eqs. (i) and (ii), we get $I_1 = 3$ A, $I_2 = -1.5$ A

$I_3 = I_1 - I_2 = 3 + 1.5 = 4.5$ A

13. The given circuit can be redrawn as follows:

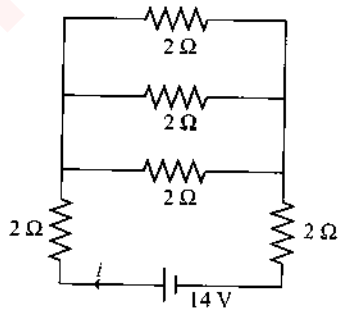


Fig. A3-5.10

$$\frac{1}{R'} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \Rightarrow R' = \frac{2}{3}\Omega$$

$\therefore R_{\text{Total}} = 2 + \frac{2}{3} + 2 = \frac{14}{3}\ \Omega$. Hence, $i = \frac{V}{R} = \frac{14 \times 3}{14} = 3$ A

14. Let current through 8 Ω be i in both cases.

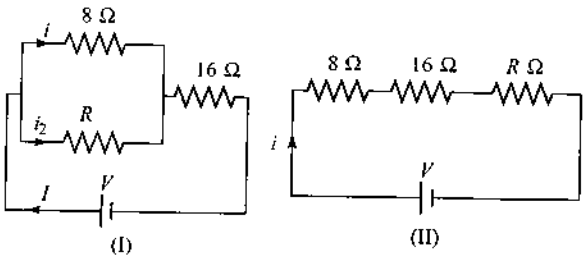


Fig. A3-5.11

In case (I), $V_1 = V_2 \Rightarrow i \times 8 = R \times i_2 \Rightarrow i_2 = \frac{8i}{R}$

$I = i + i_2 = i + \frac{8i}{R}$, $V = I(R_{\text{Total}})$

$$V=\left(i+\frac{8i}{R}\right)\left(\frac{8R}{R+8}+16\right)=i\left(\frac{8+R}{R}\right)\left(\frac{8R}{R+8}+16\right) \quad \text{(i)}$$

In case (II), $V=i(24+R)$ (ii)

Now, from Eqs. (i) and (ii), we get

$$\frac{24R+128}{R}=24+R \Rightarrow R=\sqrt{128}\ \Omega$$

15. $V_{ab}=I_1\left(6+\frac{4\times 6}{4+6}\right)\Rightarrow V_{ab}=I_1(8.4)$ (i)

$V_{ab}=I_2\left(4+\frac{8\times 8}{8+8}\right)\Rightarrow V_{ab}=I_2(8)$ (ii)

a. From Eq. (i), we get $I_1=\frac{12}{8.4}=\frac{10}{7}$ A.

I_1 is the current in the 6 Ω resistor.

b. $I_{2,4}=\frac{4}{6+4}\times\frac{10}{7}=\frac{4}{7}$ A

c. From Eq. (ii), we get $I_2=\frac{3}{2}$ A, so current in the 4 Ω resistor is $\frac{3}{2}$ A.

It will equally divide into the 8 Ω and 8 Ω resistors which one in parallel. Therefore, $I_8=\frac{3}{2\times 2}=\frac{3}{4}$ A

16. Applying Kirchhoff's law in any branch, we get

$4-4(2i)-i-2-i=0 \Rightarrow i=0.2$ A

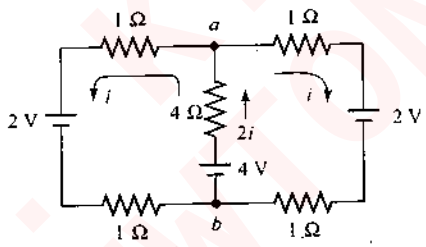


Fig. A3-5.12

$V_a-V_b=4-(2i)=4-4(0.4)=2.4$ V

17. Applying Kirchhoff's current law,

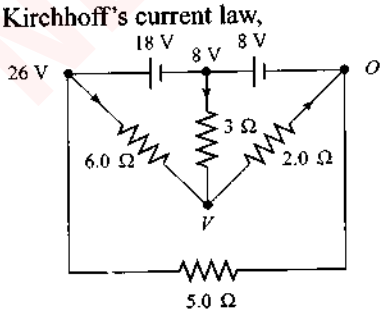


Fig. A3-5.13

At node V, $\frac{26-V}{6}+\frac{8-V}{3}=\frac{V-0}{2}\Rightarrow V=7$ V

Current through the 6 Ω resistor is $I_6=\frac{26-7}{6}=\frac{19}{6}$ A

Current through the 3 Ω resistor is $I_3=\frac{8-7}{3}=\frac{1}{3}$ A

Current through the 2 Ω resistor is $I_2=\frac{7}{2}$ A

Current through the 5 Ω resistor is $I_6=\frac{26}{5}$ A

18. Given $i_2-i_3=5$ (A)

In ADC, $-7i_1-4i_3+2=0$ (i)

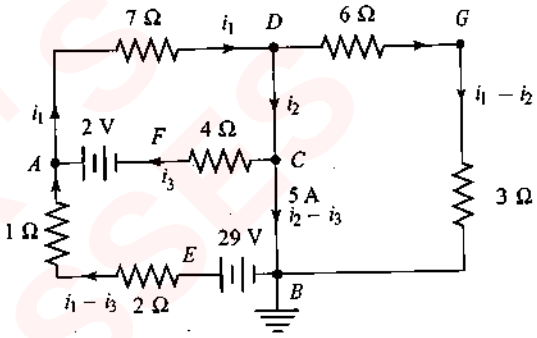


Fig. A3-5.14

In $FAEBC$, $2+3(i_1-i_3)-29-4i_3=0$

$\Rightarrow -27+3i_1-7i_3=0$ (ii)

(i) × 3 + (ii) × 7 ⇒ $i_3=-3$ A

Puting in Eq. (i), we get $i_1=2$ A

Puting in Eq. (A), we get $i_2=2$A

In ACB, $V_A-V_B=2-(-3\times 4)\Rightarrow V_A-0=14$ V

a. $|V_A|=14$ V, $V_A-V_D=7\times 2=14$

b. $V_D=14-14=0$

$V_D-V_G=6\times(i_1-i_2)=6(2-2)\Rightarrow V_G=0+V_D$

c. $V_G=0$ V

19. Here internal resistance is given by the slope of graph, i.e., x/y but conductance = 1/resistance = x/y.

20. In parallel combination, each resistor has same potential difference, i.e., 200 V. The circuit, therefore, reduces to

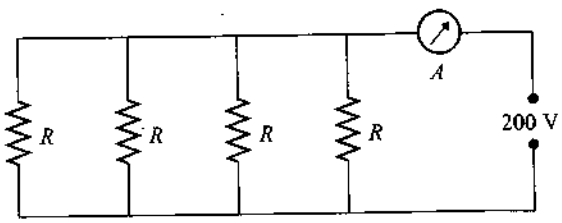


Fig. A3-5.15

$\therefore R_{eq}=\frac{2000}{4}=500\ \Omega; I=\frac{200}{500}=0.4$A

21. From the circuit given in question,

current through $R_2=i-\frac{i}{10}=\frac{9i}{10}$

Potential difference across R_2 = potential difference across R

$\therefore R_2\times\frac{9}{10}i=R\times\frac{i}{10}$, i.e., $R_2=\frac{R}{9}=\frac{11}{9}\ \Omega$

$$\therefore R_{eq} = \frac{R_2 \times R}{R_2 + R} = \frac{\frac{11}{9} \times \frac{11}{1}}{\frac{11}{9} + \frac{11}{1}} = \frac{11}{10}\,\Omega$$

Total circuit resistance $= \frac{11}{10} + R_1 = R = 11$

$$\therefore \quad R_1 = 11 - \frac{11}{10} = 9.9\,\Omega$$

22. Loop $AFMN$: $20 = 5I_1 + 12(I_1 - I_2)$

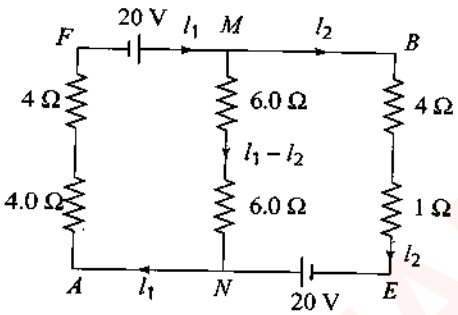


Fig. A3-5.16

$$20 = 17I_1 - 12I_2 \qquad \text{(i)}$$

Loop $AFBEA$: $20 + 20 = 5I_1 + 5I_2 \Rightarrow 8 = I_1 + I_2$ (ii)

Solving, we get $I_1 = 4$ A, $I_2 = 4$ A

So current through $MN = I_1 - I_2 = 4 - 4 = 0$

And current through each of the batteries is 4 A.

23. In the given circuit, A, B, C and D are at same potential by symmetry.

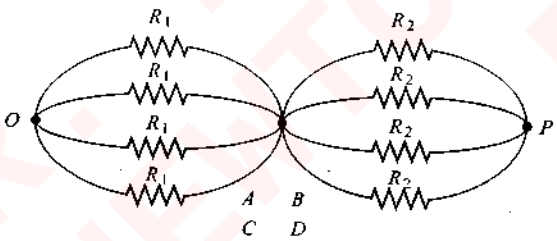


Fig. A3-5.17

$$R_{eq} = \frac{R_1 + R_2}{4}, \quad R_1 = ar, R_2 = \frac{\pi a}{2} r.$$

$$R_{eq} = \frac{ar}{4}\left(1 + \frac{\pi}{2}\right) = \frac{ar}{8}(2 + \pi)$$

24. The given circuit can be redrawn as shown below. It is a balanced Wheatstone bridge. The resistance between P and Q can be removed. So $R_{AB} = R$.

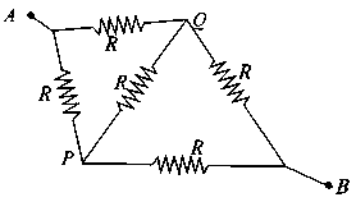


Fig. A3-5.18

25. The current will flow from the positive terminal to the negative terminal in the battery. During charging, the potential difference $= V + Ir = 2 + 5 \times 0.1 = 2.5$ V.

26. Battery should be connected across A and B. Output can be taken across the terminals A and C or B and C,

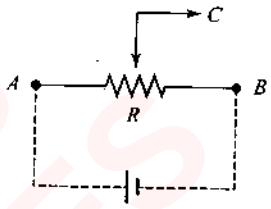


Fig. A3-5.19

Exercise 5.3

1. a. At $t = 0$, C will behave as a short circuit, so no current passes through R_2. And $I = I_2 = \varepsilon/R_1$.

b. At $t = \infty$, C will block the current. So $I_2 = 0$.

And $$I = I_1 = \frac{\varepsilon}{R_1 + R_2}$$

2. a. In each case, in steady state (equilibrium), potential difference across capacitor will be E. So charge in each case will be finally $q = CE$.

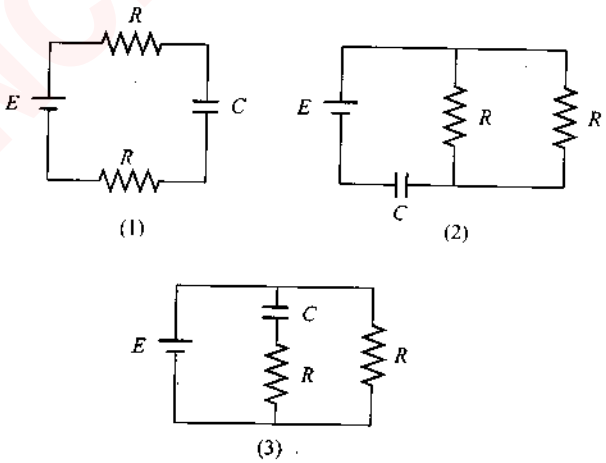


Fig. A3-5.20

b. $\tau_1 = 2RC, \tau_2 = \frac{R}{2}C, \tau_3 = RC$

For (1), time constant is the largest so it will take maximum time. For (2), it is the smallest. So time taken will be in the order; (1) > (3) > (2).

3. a. Immediately after the switch is closed, C_1 will act as a simple wire due to which R_2 and R_3 will be short-circuited.

So, $I = E/R_1$

b. After a long time, C_1 and C_2 will block the current. Current will pass through only R_1 and R_3. $I = \frac{E}{R_1 + R_3}$

4. For $t = t_0$ to $t = 2t_0$, $V = at + b$

Charge on capacitor $q = CV = C[at + b]$

Current, $I = \frac{dq}{dt} = Ca \rightarrow$ constant

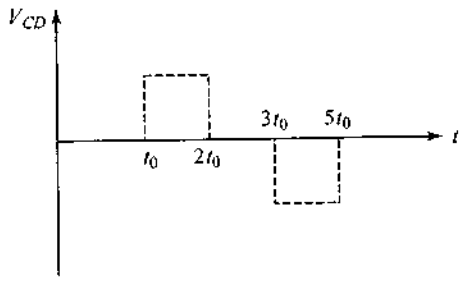


Fig. A3-5.21

Potential across $CD = IR = CaR \rightarrow$ constant
From $2t_0$ to $3t_0$, V is constant, so no current flows through the capacitor or resistor. Hence, V_{CD} is zero.

5. a. S_1 is closed, S_2 is open.

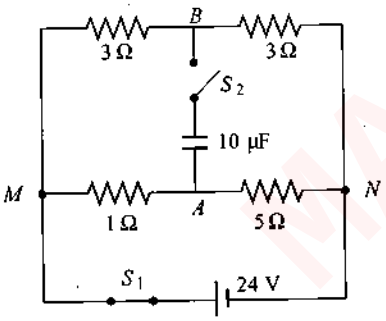


Fig. A3-5.22

$$V_M - V_A = \frac{1\times 24}{1+5} = 4\text{ V},\ V_M - V_B = \frac{3\times 24}{3+3} = 12\text{ V}$$

$V_A - V_B = 8$ V

b. (i) Just after closing S_2, A and B will come to the same potential, so $V_A - V_B = 0$.
(ii) After a long time, no current will flow through AB, so $V_A - V_B = 8$ V

6. $I = \dfrac{E}{R_1 + R_2} = \dfrac{6}{10+20} = \dfrac{1}{5}$ A

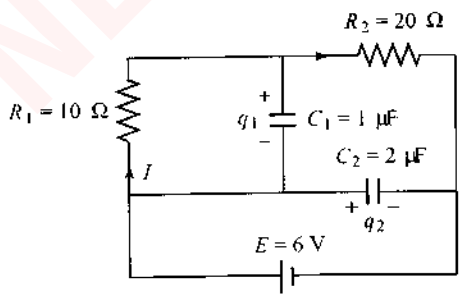


Fig. A3-5.23

$$q_1 = C_1V_1 = C_1IR_1 = 1\times\frac{1}{5}\times 10 = 2\ \mu\text{C}$$

$$q_2 = C_2V_2 = 2\times 6 = 12\ \mu\text{C}$$

7. a. At $t = 0$ capacitor acts as short-circuit.
There will not be any current in the 40 Ω resistance.

$$\therefore \quad I_0 = \frac{10}{20} = 0.5\text{ A}$$

b. Capacitor acts as open circuit, $I = \dfrac{10}{60} = \dfrac{1}{6}$ A

c. Voltage across capacitor is

$$V_{40} = IR = \frac{1}{6}\times 40, Q = CV = 0.50\times\frac{1}{6}\times 40 = \frac{10}{3}\ \mu\text{C}$$

d. $q = q_0(e^{-t/RC})$, capacitor will discharge through the 40 Ω resistor when S is open, i.e.,

$$\frac{20}{100}q_0 = q_0\left(e^{-\frac{t}{40\times 0.5\times 10^{-6}}}\right) \Rightarrow \frac{1}{5} = \frac{1}{e^{\frac{t}{20\times 10^{-6}}}}$$

or $\quad t = 20\times 10^{-6}\ln(5)$

8. $q_1 = 400\ \mu$C, $C = 50\ \mu$F
$R = 1000\ \Omega$, $\tau = RC = 50\times 10^{-6}\times 1000 = 5\times 10^{-2}$ s

$$q = q_0e^{-t/\tau} = 400e^{-\frac{1}{5\times 10^{-2}}} = \frac{400}{e^{+20}}\ \mu\text{C}$$

9. $q = q_0/3$, from $q = q_0e^{-t/\tau}$ we have $q_0/3 = q_0e^{-t/\tau}$

$$\Rightarrow t = \tau\ln 3 = RC\ln 3 \Rightarrow R = \frac{t}{C\ln 3} = \frac{4.4\times 10^{-3}}{2\times 10^{-6}\ln 3} = \frac{2200}{\ln 3}\ \Omega$$

10. $q = q_0(1 - e^{-t/\tau})$, Putting $t = \tau$, we get $\dfrac{q}{q_0} = 1 - e^{-1}$

$$\Rightarrow \frac{q}{q_0}\times 100 = \frac{e-1}{e}\times 100 = 63.2\%$$

11. $\dfrac{q^2}{2C} = \dfrac{1}{2}\dfrac{q_0^2}{2C} \Rightarrow q = \dfrac{q_0}{\sqrt{2}}$

So, $\quad \dfrac{q_0}{\sqrt{2}} = q_0[1 - e^{-t/\tau}] \Rightarrow t = \tau\ln\left(\dfrac{\sqrt{2}}{\sqrt{2}-1}\right) = 1.23\tau$

12. Power deliverd by a battery is given by $P = EI$.
$\Rightarrow P \propto I$, so if power becomes half, current becomes half.

$$I = I_0e^{-t/\tau} \Rightarrow \frac{I_0}{2} = I_0e^{-t/\tau} \Rightarrow e^{t/\tau} = 2 \Rightarrow t = \tau\ln 2 = 0.69\tau$$

13. Let charge on the second capacitor at any time t be q, then the remaining charge on first capacitor will be $Q - q$.
The current in the circuit at any time is $I = dq/dt$.
Applying Kirchhoff's law, we get

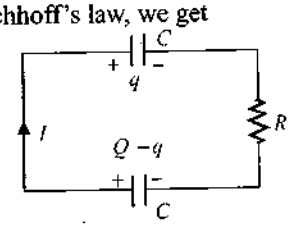


Fig. A3-5.24

$$\frac{Q-q}{C} = \frac{q}{C} + IR \Rightarrow \frac{Q-2q}{C} = \frac{dq}{dt}R \Rightarrow \int_0^q \frac{dq}{Q-2q} = \int_0^t \frac{dt}{RC}$$

Solving, we get $q = \dfrac{Q}{2}(1 - e^{-2t/RC})$.

CHAPTER 6

Exercise 6.1

1. In series, I is same for all elements, so if an ammeter is connected in series with a resistance R, its reading will be equal to the current through R (as shown in Fig. A3-6.1).

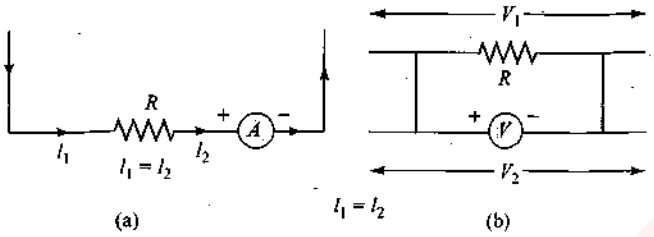


Fig. A3-6.1

Similarly, in parallel, V is same for all elements, so if a voltmeter is connected in parallel with a resistance R, potential difference across it will be same as that across R. This is why an ammeter is always connected in series, while a voltmeter is connected in parallel with the element to measure I (through it) and V (across it).

2. With the presence of a voltmeter in series with resistance R in a circuit, due to voltmeter's high resistance, the current I in the circuit will decrease appreciably to I' and as this I' will further divide in resistance R and ammeter, the current through ammeter I_2 will be even lesser than I'. So instruments will not be damaged.

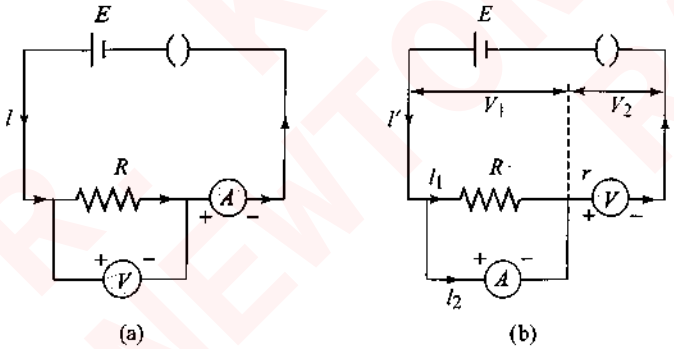


Fig. A3-6.2

3. As shown in Fig. A3-6.3, the voltmeter is in series with R, so $V = V_1 + V_R$, i.e., $110 = 5 + V_R$

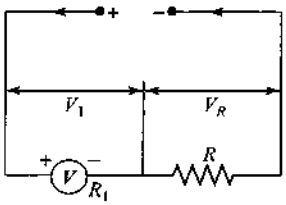


Fig. A3-6.3

i.e., $V_R = 110 - 5 = 105$ V

And as in series, potential divides in proportion to resistance, i.e.,

$$\frac{V_R}{V_1} = \frac{R}{R_1}, \text{ i.e., } \frac{105}{5} = \frac{R}{20\ \text{k}\Omega} \text{ or } R = 420\ \text{k}\Omega$$

4. Let the length of the potentiometer wire be ℓ, and V be the potential difference across it, then potential gradient $K = V/\ell$.

Let balance point is obtained at l_1, then measured potential difference $= K\ell_1 = V\ell_1/\ell$.

Let us make error Δl in measuring l_1, then the percentage error $= \frac{k\Delta\ell}{k\ell_1} \times 100 = \frac{\Delta\ell}{\ell_1} \times 100$.

If l is increased, then l_1 also increases and hence the percentage error will decrease. So accuracy increases.

5. For balanced Wheatstone bridge, $\frac{X}{40} = \frac{3}{60} \Rightarrow X = 2\,\Omega$

6. A potentiometer is said to be sensitive if fall of potential per unit length, i.e, potential gradient (dV/dl) is small. The slope of V-I graph gives potential gradient, which is smaller for potentiometer with Y than for potentiometer wire X. Therefore, potentiometer Y will be preferred for comparing e.m.f.s of the two cells.

7. From the first null point: $\frac{X}{Y} = \frac{80}{20}$ (i)

From the second null point: $\frac{\left(\frac{100X}{100+X}\right)}{Y} = \frac{60}{40}$ (ii)

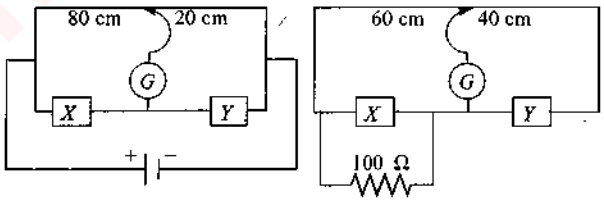


Fig. A3-6.4

From Eqs. (i) and (ii), we get

$$X = \frac{500}{3}\Omega \text{ and } Y = \frac{125}{3}\Omega$$

8. Given, $G = 12.0\ \Omega$, $I_g = 2.5$ mA $= 2.5 \times 10^{-3}$ A

i. $I = 7.5$ A, $S = ?$

$$S = G \times \frac{I_g}{I - I_g} = 12 \times \frac{2.5 \times 10^{-3}}{7.5 - 2.5 \times 10^{-3}}\ \Omega \approx 4.0 \times 10^{-3}\ \Omega$$

ii. $V = 10.0$ V, $R = ?$

$$\therefore R = \frac{V}{I_g} - G = \frac{10.0}{2.5 \times 10^{-3}} - 12.0 = 3988\ \Omega$$

9. We have $I_2 = 1.00$ mA $= 1.00 \times 10^{-3}$ A and $R_2 = 20.0\ \Omega$, and the ammeter should be able to handle maximum current, $I = 50.0 \times 10^{-3}$ A.

Solving equation $I_g R_g = (I - I_g)R_s$ for R_s, we get

$$R_s = \frac{I_g R_g}{I - I_g} = \frac{(1.00\times10^{-3}\,\text{A})(20.0\,\Omega)}{50.0\times10^{-3}\,\text{A} - 1.00\times10^{-3}\,\text{A}} = 0.408\,\Omega$$

10. Solving equation for R_S, we have

$$R_s = \frac{V}{I_g} - R_G = \frac{10.0\text{ V}}{0.00100\text{ A}} - 20.0\,\Omega = 9980\,\Omega$$

At full-scale deflection, $V_{ab} = 10.0$ V, the voltage across the meter is 0.0200 V, the voltage across R_S is 9.98 V and the current through the voltmeter is 0.00100 A. In this case, most of the voltage appears across the series resistor. The equivalent meter resistance is $R_{eq} = 20.0 + 9980 = 10000\,\Omega$. Such a meter is described as a "1000 ohms per volt", referring to the ratio of resistance to full-scale deflection voltage. The voltmeter draws off only a small fraction of the current and disturbs only slightly the circuit being measured.

11. Internal resistance of the cell is given by $r = R\left(\frac{\ell_1 - \ell_2}{\ell_2}\right)$

Hence, $r = 1\times\frac{60-30}{30}\,\Omega = 1\,\Omega$

12. $k = \frac{E_B}{L} = \frac{1.1}{440} = 0.0025$ V/cm

Potential difference across R is $0.0025 \times 220 = 0.550$ V
Error in the reading of voltmeter is
reading of voltmeter – reading of potentiometer
$= 0.5 - 0.55 = -0.05$ V

13. We should connect this galvanometer in series with the given resistor. In general, resistance of galvanometer is small, so it can be neglected. If I is the current in circuit, then $R = 120/I$

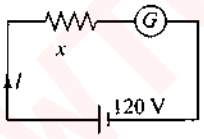


Fig. A3-6.5

$$I_{max} = 40\times10^{-6}\text{ A}, R_{min} = \frac{120}{40\times10^{-6}} = 3\text{ M}\Omega$$

14. Let the resistance of the voltmeter be R_V and the current flowing through it be I_V.

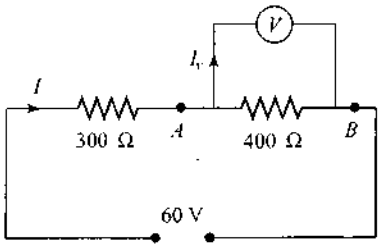


Fig. A3-6.6

Then $I_V R_V = 30$ and $(I - I_V) \times 400 = 30$

Also, $300 \times I = 30 \Rightarrow I = 0.1$ A

Therefore, $0.1 - I_V = 3/40$.

$$\Rightarrow I_V = 0.1 - \frac{3}{40} \Rightarrow I_V = \frac{4-3}{40} = \frac{1}{40}\text{ A}$$

$$\therefore \quad \frac{1}{40}\times R_V = 30 \quad \Rightarrow R_V = 1200\,\Omega$$

For the circuit in Fig. A3-6.7:

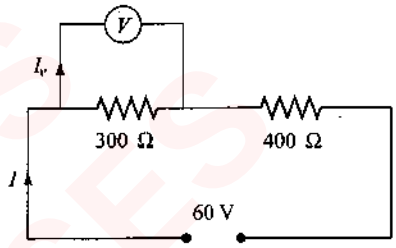


Fig. A3-6.7

$I_V \times R_V = (I - I_V)\,300 \Rightarrow I_V \times 1200 = (I - I_V)\,300$

$\Rightarrow 4I_V = I - I_V \quad \Rightarrow I = 5I_V$ (i)

Also, $300(I - I_V) + 400\,I = 60 \quad \Rightarrow 700\,I - 300\,I_V = 60$

$3500\,I_V - 300\,I_V = 60 \quad \Rightarrow I_V = \frac{60}{3200} = \frac{3}{160}$ A

Therefore, reading of voltmeter $= \frac{3}{160}\times1200 = 22.5$ V

15. For the experimental verification of Ohm's law, ammeter and voltmeter should be connected as shown in Fig. A3-6.8.

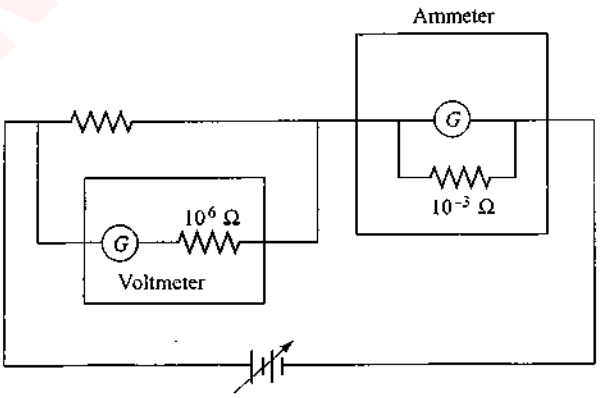


Fig. A3-6.8

CHAPTER 7

Exercise 7.1

1. a. Just after it is turned on, the energy supplied to the bulb is higher. Because initially resistance is low and it increases with the increase in temperature.

b. After connecting b and c, R_2 will be short-circuited. Hence, it will not glow. Now, entire potential difference will be on R_1. So brightness of R_1 increases.

c. D is short-circuited, so D will not glow. Potential difference across C is V. Potential difference across the series combination of A and B is V, so potential difference across

each of A and B is $V/2$. So, brightness of C is the greatest and that of A and B are equal.
If A fails: B will not glow. No effect on C and D.
If C fails: C will not glow. No effect on A, B and D.
If D fails: No effect anywhere.

d. Because resistance is directly proportional to length. At low current and high voltage energy, loss will be less. Because if current is less, then loss of energy $= I^2R$ will be less.

e. Initially, current will be high in the circuit, so bulb will glow with maximum possible brightness. But as time increases, current will decrease and brightness of the bulb falls and finally the bulb goes out.

f. This is not true. Because in series, current through all the bulbs is the same. The current is simultaneously set up in series through all the bulbs.

g. $P = \dfrac{V^2}{R} = \dfrac{V^2 A}{P\ell}$

Here, P and V are fixed. We can vary A or ℓ.

2. Resistance of a bulb is given by: $R = V^2/P$. Now, 100 W bulb will have less resistance. So we are connecting less resistance in series with the heater. It means potential difference across the heater will increase. So the heater will give more heat.

3. $P_{max} = (i_{max})^2 R \Rightarrow 18 = i_{max}^2 \times 2 \Rightarrow i_{max} = 3$ A
Here $R_{eq} = 3\ \Omega$.
So the maximum power of circuit $= i_{max}^2\ R_{eq} = 27$ W

4. We know $R = V^2/P$. Therefore, resistance of the first bulb is $R_1 = V^2/P_1$. And resistance of the second bulb is $R_2 = V^2/P_2$.
In series, same current will pass through each bulb.
Therefore, power developed across the first bulb is $P_1' = I^2(V^2/P_1)$ and that across the second bulb is $P_2' = I^2(V^2/P_2)$.

$$\frac{\text{Power in bulb 1}}{\text{Power in bulb 2}} = \frac{P_2}{P_1} \Rightarrow \frac{\text{Power in bulb 1}}{\text{Power in bulb 2}} < 1 (\text{as } P_2 < P_1)$$

Hence, the bulb rated 220 V and 40 W will glow more.

5. Here, $P = 30$ W, $V = 6$ V
Therefore, resistance of the bulb,
$R_1 = V^2/P = (6)^2/30 = 1.2\ \Omega$

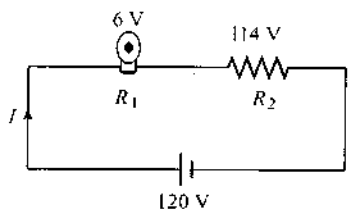


Fig. A3-7.1

Current capacity of the bulb, $I = \dfrac{P}{V} = \dfrac{30}{6} = 5$ A

Supply voltage, $V' = 120$ V
Let R_2 be the resistance used in series with the bulb to have a current of 5 A in the circuit.

Total resistance $= R_2 + R_1 = R_2 + 1.2$
Therefore, current $I = V'/(R_2 + 1.2)$

or $5 = \dfrac{120}{R_2 + 1.2}$ or $R_2 = \dfrac{120}{5} - 1.2 = 22.8\ \Omega$ in series

Alternatively:
Potential difference across the bulb should not exceed 6 V.
For this: $\dfrac{6}{114} = \dfrac{R_1}{R_2} \Rightarrow R_2 = \dfrac{114}{6} R_1 = \dfrac{114}{6} \times 1.2 = 22.8\ \Omega$

6. According to Joule heating, the rate at which heat is produced in a resistance is given by $P = I^2R = V^2/R$. And in parallel V is same, i.e., $V_1 = V_2$.

So, $\dfrac{P_2}{P_1} = \dfrac{R_1}{R_2} = \dfrac{L_1}{L_2}\left[\dfrac{r_2}{r_1}\right]^2$ $\left[\text{as } R = \rho\dfrac{L}{\pi r^2}\right]$

$\dfrac{P_2}{P_1} = \dfrac{L_1}{2L_1}\left[\dfrac{2r_1}{r_1}\right]^2$ [as $L_2 = 2L_1$ and $r_2 = 2r_1$]

i.e., $P_2 = 2P_1$

i.e., heat produced in the thicker coil is more (i.e., double) than that produced in the other coil.

7. As the three bulbs are in series and identical, initially when switch S is open, $V_A = V_B = V_C = \dfrac{1}{3}$ V

Also $P_A = P_B = P_C = \dfrac{(V/3)^2}{R} = \dfrac{V^2}{9R} = P$

Now, the switch S is closed,

a. The bulb C is short-circuited and hence potential difference across it $V_C' = 0$ and so $V_A' = V_B' = V/2$ with $V_C' = 0$. And hence, $P_A' = P_B' = \dfrac{(V/2)^2}{R} = \dfrac{V^2}{4R}$

i.e., $P_A' = P_B' = \dfrac{9}{4}P$, i.e., intensities of bulbs A and B will increase and become 2.25 of their initial values.

b. As $V_C' = 0$, no current will pass through bulb C, so it will give no light, i.e., $P_C' = 0$.

c. As R_T changes from $3R$ to $2R$, so the current in the circuit will change from

$$I = \frac{V}{3R} \text{ to } I' = \frac{V}{2R}, \quad \text{i.e.,} \quad I' = \frac{3}{2}I$$

i.e., current in the circuit will increase and will become 1.5 times its initial value.

d. As explained earlier, initially the voltage V divides equally across A, B and C, i.e., $V_A = V_B = V_C = V/3$. Now when the switch S is closed, bulb C is short-circuited, i.e., $V_C' = 0$, so voltage V now will divide equally across A and B, i.e., $V_A' = V_B' = V/2$. So voltage across bulbs A and B will change from $V/3$ to $V/2$ while across C from $V/3$ to zero, i.e., voltage drop across A and B will increase while across C it will decrease and become zero.

Further as R_T changes from $3R$ to $2R$, so power consumed in the circuit changes from

$$P_T = \frac{V^2}{3R} \text{ to } P'_T = \frac{V^2}{2R}, \quad \text{i.e.,} \quad P'_T = \frac{3}{2}P_T$$

i.e., power dissipated in the circuit increases and becomes 1.5 times its initial value.

8. Since $m = \rho A l$, the lengths and hence the resistances of the wires are in the ratio 1:2. Also when they are connected in series, heat produced will be in the ratio 1:2, because $H = I^2R$.

9. $P = \frac{V^2}{R} = \frac{V^2 A}{\rho \ell}$. To boil water in 10 min, power should be increased. To increase power, length should be decreased.

10. Power drawn by motor: $P = VI = 50 \times 12 = 600$ W

Power loss $= \frac{70}{100} \times 600 = 420$ W.

This power is lost in the form of heat in the resistance.

$$I^2R = 420 \Rightarrow (12)^2R = 420 \Rightarrow R = 2.9\ \Omega$$

11. Heat produced = Heat loss through surface of wire

$$I^2R = H\,2\pi r\ell$$

where H is the heat loss per unit surface area of the wire.

$$I^2 \frac{\rho\ell}{a} = H 2\pi r l$$

Put $a = \pi r^3$, therefore $I^2 \propto r^2$

$$\Rightarrow \left(\frac{15}{30}\right)^2 = \left(\frac{0.15}{r}\right)^3 \Rightarrow r = 0.238 \text{ mm}$$

12. $P_{in} = 12 \times 2 = 240$ W, heat loss $= 9$ cal/s $= 9 \times 4.20 = 378$ J/s

$$P_{out} = 240 - 37.8 = 202.2 \text{ W}$$

Efficiency $\eta = \frac{P_{out}}{P_{in}} \times 100 = 84.25\%$

13. Heat produced = Heat loss $\Rightarrow \frac{V^2}{R} = \frac{KA\Delta T}{\ell}$

Here $V = 400$ V, $K = 4 \times 10^{-4}$ cal/cm s°C $= 16.8 \times 10^{-4}$ J/cm s°C

$\Delta T = 100$, $\ell = 1$ mm $= 0.1$ cm, $A = 6(40)^2$ cm^2

$$\frac{400^2}{R} = \frac{16.8 \times 10^{-4} \times 6 \times (40)^2 \times 100}{0.1} \Rightarrow R = 9.92\ \Omega$$

14. **a.** When the lamps are connected in parallel, then potential difference V across each lamp will be the same and will be equal to potential difference necessary for full brightness of each bulb. As $P = V^2/R$, illumination produced by the second bulb will be higher than that produced by the first bulb, i.e., the bulb having lower resistance will shine more brightly.

b. When R_1 burns out, then power is dissipated in R_2 only. Because internal resistance is quite low in lighting circuit, potential difference is still equal to V, hence power dissipated in the second lamp, i.e.,

$\frac{V^2}{R_2} < \left(\frac{V^2}{R_1} + \frac{V^2}{R_2}\right)$, net power consumed initially.

In other words, net illumination will now decrease.

c. When the two lamps are connected in series, the potential difference across each lamp will be different but current I flowing through each lamp will be the same, as $P = I^2R$. Hence, illumination produced by the first lamp will be higher as compared to that produced by the second lamp, i.e., the lamp having higher resistance will glow more brightly.

d. When lamp of resistance R_2 burns out and only lamp of resistance R_1 is connected in the circuit, then current flowing through the circuit will change. Let new current be i'. Because potential difference still remains the same (due to low internal resistance), hence

$$i' R_1 = i(R_1 + R_2) \Rightarrow i' = [i(R_1 + R_2)]/R_1$$

If P' is the power consumed, then

$$P' = i'^2 R_1 = [i^2(R_1 + R_2)(R_1 + R_2)]/R_1$$

When both the lamps were present, then total power consumed was given by

$P_S = P_1 + P_2 = i^2(R_1 + R_2)$, i.e., illumination gets increased when one bulb is used.

15. Let the voltage supply be V. Let resistance of each bulb be R. Total resistance of circuit $= nR$

Power illuminated by all bulbs, $P_1 = \frac{V^2}{nR}$

Power illuminated by one bulb, $P_2 = \frac{P_1}{n} = \frac{V^2}{n^2R}$

After one bulb is fused, the powers are

$$P'_1 = \frac{V^2}{(n-1)R}, \quad P'_2 = \frac{V^2}{(n-1)^2 R}$$

a. Fractional change in the illumination of all the bulbs:

$$f_1 = \frac{P'_1 - P_1}{P_1} = \frac{P'_1}{P_1} - 1 = \frac{n}{n-1} - 1 = \frac{1}{n-1}$$

b. Fractional change in the illumination of one bulb:

$$f_2 = \frac{P'_2 - P_2}{P_2} = \frac{n^2}{(n-1)^2} - 1 = \frac{2n-1}{(n-1)^2}$$

16. Power input $= P_i = VI = 100 \times 6 = 600$ W

Power output $= 150$ W

Efficiency: $n = \frac{150}{600} \times 100 = 25\%$

Heat produced is $600 - 150 = 450$ W

$\Rightarrow I^2 R = 450$ where R is the resistance of windings

$$\Rightarrow \quad R = \frac{450}{6^2} = \frac{450}{36} = 12.5\ \Omega$$

17. Voltage drop across line $= 10$ V, current $= \frac{10}{2} = 5$ A.

Current drawn by one lamp $= \frac{100}{230}$ A

So number of lamps = $\frac{5\times 230}{100} = 11.5$.

Hence, maximum number of lamps = 11.

18. Total wattage consumed (units) is

$$\frac{7\times 40\times 6}{1000}+\frac{2\times 60\times 6}{1000}+\frac{5\times(220\times 0.4)\times 1}{1000}+\frac{(220)^2}{48.4}\times\frac{10}{1000}$$

= 12.84 per day

No. of units consumed in one month (Jan) = 12.84 × 31 = 398.04. Total bill = 2 × 398.04 = ₹ 796.08.

19. **a.** $R_1 = \frac{200^2}{300}, R_2 = \frac{200^2}{600}$

The resistance of the first bulb is more. Hence, in series, the first bulb will produce more illumination.

b. $P = \frac{P_1P_2}{P_1+P_2} = \frac{300\times 600}{300+600} = 200$ W

c. $P = P_1 + P_2 = 300 + 600 = 900$ W

20. Let stabilizer give voltage V, then power produced by the bulb is

$$P = \left(\frac{V}{220}\right)^2 100 \text{ or } \frac{dP}{P} = 2\frac{dV}{V}$$

If $\frac{dV}{V} = \pm 1\%$, then $\frac{dP}{P} = \pm 2\%$

Hence, the maximum power produced is

$P_{max} = 100\% + 2\% = 102$ W

Minimum power produced is

$P_{min} = 100\% - 2\% = 98$ W

21. Efficiency $= \eta = \frac{\text{Output power}}{\text{Input power}}$

$\eta = \frac{i^2R}{\varepsilon i}$ where $i = \frac{\varepsilon}{R+r}$

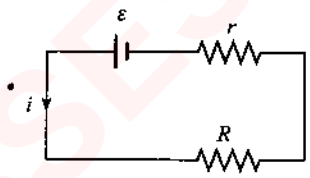


Fig. A3-7.2

$$\eta = \frac{R}{R+r} \Rightarrow 0.6 = \frac{R}{R+r} \Rightarrow 2R = 3r$$

Therefore, new efficiency $\eta_1 = \frac{6R}{6R+r} = 0.9 = 90\%$

22. 25 W bulb will glow brighter. This is because $P = I^2R$ where I is the current flowing and R is the resistance of the appliance. When I is same, $P \propto R$. The resistance of 25 W bulb is more and therefore P is more.

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