Given be $n \in \mathbb{N}$ with $\boxed{n_i = (2x_i+1)y_i \quad , x_i, y_i \in \mathbb{N},}$ (eq. 1)

the set of the odd times tables. (without 1)
n_i

With this, we can write:

$$\boxed{\bar{n}_i = (2x_i+1)y_i - \Delta_i \quad , \Delta_i \in \{1, \ldots, 2x_i\}} \quad \text{(eq. 2)}$$

$\Delta_i \in \mathbb{N}$

the set of all numbers, which does not belong to the times table n_i.

Now, we assume that we have two different $\bar{n}_i$ equations.

$$\left.\begin{array}{lll} \text{I.} & \bar{n}_1 = (2x_1+1)y_1 - \Delta_1 & , \Delta_1 \in \{1, \ldots, 2x_1\} \\ \text{II.} & \bar{n}_2 = (2x_2+1)y_2 - \Delta_2 & , \Delta_2 \in \{1, \ldots, 2x_2\} \end{array}\right\} \text{(eq. 3)}$$

with $x_2 > x_1$: $x_2 = x_1 + \Delta x_{1,2}$.

Let's do the intersection:

$$\bar{n}_1 = \bar{n}_2$$

$$(2x_1+1)y_1 - \Delta_1 = (2x_2+1)y_2 - \Delta_2$$

$$0 = (2x_2+1)y_2 - (2x_1+1)y_1 - \Delta_2 + \Delta_1$$

with $x_2 = x_1 + \Delta x_{1,2}$

$$\Rightarrow \quad 0 = (2(x_1+\Delta x_{1,2})+1)y_2 - (2x_1+1)y_1 - \Delta_2 + \Delta_1$$

$$0 = (2x_1+1)y_2 + 2\Delta x_{1,2}y_2 - (2x_1+1)y_1 - \Delta_2 + \Delta_1$$

$$0 = (2x_1+1)(y_2 - y_1) + 2\Delta x_{1,2}y_2 - \Delta_2 + \Delta_1$$

$$\Rightarrow \quad (2x_1+1)y_1 = (2x_1+1)y_2 + 2\Delta x_{1,2}y_2 - \Delta_2 + \Delta_1$$

$$y_1 = (2x_1+1)^{-1}\left((2x_1+1)y_2 + 2\Delta x_{1,2}y_2 - \Delta_2 + \Delta_1\right)$$

$$y_1 = y_2 + \frac{2\Delta x_{1,2}y_2 - \Delta_2 + \Delta_1}{2x_1+1}$$

Be $y_2 = x_1 \cdot (-\Delta_2 + \Delta_1) + (2x_1+1)z_{1,2}$ / $y_1 = x_2 \cdot (-\Delta_2 + \Delta_1) + (2x_2+1)z_{1,2}$

Test: $$\frac{2\Delta x_{1,2}x_1(-\Delta_2+\Delta_1) + (-\Delta_2+\Delta_1)}{2x_1+1} \quad \text{(eq. 4)}$$

Case: $\Delta x_{1,2} = 1$:

$$\Rightarrow \frac{2 \cdot 1 \cdot x_1(-\Delta_2+\Delta_1) + (-\Delta_2+\Delta_1)}{2x_1+1}$$

$$= \frac{2x_1(-\Delta_2+\Delta_1) + (-\Delta_2+\Delta_1)}{2x_1+1}$$

$$= \frac{(-\Delta_2+\Delta_1)\cdot(2x_1+1)}{2x_1+1}$$

$$= (-\Delta_2+\Delta_1) \quad \checkmark$$

Case: $\Delta x_{1,2} > 1$:

$$\Rightarrow \frac{2 \cdot \Delta x_{1,2} \cdot x_1(-\Delta_2+\Delta_1) + (-\Delta_2+\Delta_1)}{2x_1+1}$$

$= ?$, we see, in this case, we have a problem to find integer solutions.

Let's Assume instead of (eq. 4), we have

New!

$$\frac{2\Delta x_{1,2}\, x_1(-\Delta_2+\Delta_1) + \Delta x_{1,2}(-\Delta_2+\Delta_1)}{2x_1+1} \qquad \text{(eq. 5)}$$

Now, we can solve our intersection:

$$y_1 = y_2 + \frac{2\Delta x_{1,2}\, y_2 + (-\Delta_2+\Delta_1)\cdot\Delta x_{1,2}}{2x_1+1}$$

with $y_2 := x_1(-\Delta_2+\Delta_1) + (2x_1+1)z_{1,2}$, $z_{1,2} \in \mathbb{Z}$

$y_1 = x_2\cdot(-\Delta_2+\Delta_1) + (2x_2+1)z_{1,2}$

Test:

$$y_1 = x_1(-\Delta_2+\Delta_1) + (2x_1+1)z_{1,2} + \frac{2\Delta x_{1,2}\, x_1(-\Delta_2+\Delta_1)}{2x_1+1} + \frac{2\Delta x_{1,2}(2x_1+1)z_{1,2}}{(2x_1+1)} + \frac{(-\Delta_2+\Delta_1)\Delta x_{1,2}}{2x_1+1}$$

$$= x_1(-\Delta_2+\Delta_1) + (2x_1+1)z_{1,2} + 2\Delta x_{1,2} z_{1,2} + \frac{(-\Delta_2+\Delta_1)\Delta x_{1,2}(2x_1+1)}{2x_1+1}$$

because of $x_2 = x_1 + \Delta x_{1,2}$

$$= x_2(-\Delta_2+\Delta_1) + (2x_2+1)z_{1,2} \quad \checkmark$$

So, we have our new equations:

New!

$$\boxed{\begin{aligned} &\mathrm{I}': \quad \bar{n}_1' = (2x_1+1)y_1' - \Delta x_{1,2}\Delta_1 \quad, \Delta_1 \in \{1,\dots,2x_1\} \\ &\mathrm{II}': \quad \bar{n}_2' = (2x_2+1)y_2' - \Delta x_{1,2}\Delta_2 \quad, \Delta_2 \in \{1,\dots,2x_2\} \end{aligned}} \quad \text{(eq. 6)}$$

For the intersection $\bar{n}_1' = \bar{n}_2$,

we have the solutions:

$$\boxed{\begin{aligned} &\mathrm{III}': \quad y_1' = x_2\cdot(-\Delta_2+\Delta_1) + (2x_2+1)z_{1,2} \quad, z_{1,2}\in\mathbb{Z} \\ &\mathrm{IV}': \quad y_2' = x_1\cdot(-\Delta_2+\Delta_1) + (2x_1+1)z_{1,2} \end{aligned}} \quad \text{(eq. 7)}$$

and hence finally:

$$\begin{aligned} \bar{n}_{1,2}' &= (2x_1+1)y_1' - \Delta x_{1,2}\Delta_1 \\ &= (2x_1+1)\left[x_2(-\Delta_2+\Delta_1) + (2x_2+1)z_{1,2}\right] - \Delta x_{1,2}\Delta_1 \end{aligned}$$

$$\boxed{\bar{n}_{1,2}' = (2x_1+1)(2x_2+1)z_{1,2} + \underbrace{(2x_1+1)(-\Delta_2+\Delta_1)x_2 - \Delta x_{1,2}\Delta_1}_{(*)}} \quad \text{(eq. } 8^1\text{)}$$

$$\begin{aligned} \bar{n}_{1,2}' &= (2x_2+1)y_2' - \Delta x_{1,2}\Delta_2 \\ &= (2x_2+1)\left[x_1\cdot(-\Delta_2+\Delta_1) + (2x_1+1)z_{1,2}\right] - \Delta x_{1,2}\Delta_2 \end{aligned}$$

$$\boxed{\bar{n}_{1,2}' = (2x_1+1)(2x_2+1)z_{1,2} + \underbrace{(2x_2+1)(-\Delta_2+\Delta_1)x_1 - \Delta x_{1,2}\Delta_2}_{(**)}} \quad \text{(eq. } 8^2\text{)}$$

$$\cancel{(*) = (2x_1x_2 + x_2)(-\Delta_2+\Delta_1) - \Delta x_{1,2}\Delta_1}$$

$*(x_1 = x_2 - \Delta x_{12})$

With $x_2 = x_1 + \Delta x_{1,2}$, it follows:

$$(**) = (2(x_1+\Delta x_{1,2})+1)(-\Delta_2+\Delta_1)\overset{(x_2-\Delta x_{1,2})}{\cancel{x_1}} - \Delta x_{1,2}\Delta_2$$

$$\left(= (2x_1+1)(-\Delta_2+\Delta_1)x_1 + 2\Delta x_{1,2}(-\Delta_2+\Delta_1)(x_2-\Delta x_{1,2}) - \Delta x_{1,2}\Delta_2\right),$$

$$\begin{aligned} &= (2x_1+1)(-\Delta_2+\Delta_1)(x_2-\Delta x_{1,2}) \\ &\quad + 2\Delta x_{1,2}(-\Delta_2+\Delta_1)(x_2-\Delta x_{1,2}) \\ &\quad - \Delta x_{1,2}\Delta_2 \\ &= (2x_1+1)(-\Delta_2+\Delta_1)x_2 \\ &\quad (2x_1+1)(-\Delta_2+\Delta_1)(-\Delta x_{1,2}) + 2\Delta x_{1,2}(-\Delta_2+\Delta_1)(\overbrace{x_2-\Delta x_{1,2}}^{x_1}) \\ &\quad - \Delta x_{1,2}\Delta_2 \end{aligned}$$

$$= (2x_1+1)(-\Delta_2+\Delta_1)x_2$$
$$+ (-2x_1\Delta x_{1,2} - \Delta x_{1,2})(-\Delta_2+\Delta_1)$$
$$+ 2\Delta x_{1,2}\, x_1 (-\Delta_2+\Delta_1)$$
$$- \Delta x_{1,2}\Delta_2$$

$$= (2x_1+1)(-\Delta_2+\Delta_1)x_2$$
$$+ \overset{1}{\cancel{2x_1\Delta x_{1,2}\Delta_2}} - \overset{3}{\cancel{2x_1\Delta x_{1,2}\Delta_1}} + \overset{2}{\cancel{\Delta x_{1,2}\Delta_2}} - \Delta x_{1,2}\Delta_1$$
$$- \overset{1}{\cancel{2\Delta x_{1,2}\Delta_2 x_1}} + \overset{3}{\cancel{2\Delta x_{1,2}x_1\Delta_1}}$$
$$- \overset{2}{\cancel{\Delta x_{1,2}\Delta_2}}$$

$$= (2x_1+1)(-\Delta_2+\Delta_1)x_2 \quad - \Delta x_{1,2}\Delta_1$$
$$= (*) \quad \checkmark$$

II Be $\overline{n}_1 = (2x_1+1)y_1 - \Delta_1$

and $\overline{n}_1' = (2x_1+1)\,y_1' - \Delta x_{1,2}\,\Delta_1$

<u>Relationship between old and new equation.</u>

$$\overline{n}_1' = \overline{n}_1$$

$$(2x_1+1)y_1' - \Delta x_{1,2}\Delta_1 = (2x_1+1)\,y_1 - \Delta_1$$

$$0 = (2x_1+1)\,y_1 - (2x_1+1)y_1' - \Delta_1 + \Delta x_{1,2}\Delta_1$$

$$(2x_1+1)y_1' = (2x_1+1)\,y_1 - \Delta_1 + \Delta x_{1,2}\,\Delta_1$$

$$y_1' = y_1 + \frac{-\Delta_1 + \Delta x_{1,2}\Delta_1}{2x_1+1}$$

$$\boxed{\begin{aligned} y_1' &= y_1 + \frac{\Delta_1(-1+\Delta x_{1,2})}{2x_1+1} && \text{(eq. 9}^1\text{)}\\ \Leftrightarrow\quad y_1 &= y_1' - \frac{\Delta_1(-1+\Delta x_{1,2})}{2x_1+1} && \text{(eq. 9}^2\text{)}\end{aligned}}$$

Be $y_1^{m\text{[redacted]}}$ given.

$$y_1' = y_1^{m\text{[redacted]}} + \frac{\Delta_1(-1+\Delta x_{1,2})}{2x_1+1}$$

and

$$y_1' = x_2\cdot(-\Delta_2+\Delta_1) + (2x_2+1)\,z_{1,2}$$

$$\Rightarrow \quad x_2\cdot(-\Delta_2+\Delta_1) + (2x_2+1)\,z_{1,2} = y_1^{m\text{[redacted]}} + \frac{\Delta_1(-1+\Delta x_{1,2})}{2x_1+1}$$

$$\Leftrightarrow \quad (2x_2+1)\,z_{1,2} = y_1^{m\text{[redacted]}} + \frac{\Delta_1(-1+\Delta x_{1,2})}{2x_1+1} - x_2(-\Delta_2+\Delta_1)$$

$$\Leftrightarrow \quad \text{(eq. 10)} \quad \boxed{z_{1,2} = \frac{y_1^{m\text{[redacted]}}}{2x_2+1} + \frac{\Delta_1(-1+\Delta x_{1,2})}{(2x_1+1)(2x_2+1)} - \frac{x_2(-\Delta_2+\Delta_1)}{2x_2+1}}$$

<u>**Warning W1**</u> still $\lfloor x \rfloor$ [redacted] and $\lceil x \rceil$ neccessary !!! for $z_{1,2}$?

$$\Rightarrow \quad \overline{n}_1' = (2x_1+1)y_1' - \Delta x_{1,2}\Delta_1$$

and $y_1' = x_2(-\Delta_2+\Delta_1) + (2x_2+1)\,z_{1,2}$

$$\boxed{\begin{aligned}\Rightarrow \overline{n}_{1,2} &= (2x_1+1)\,[\,x_2(-\Delta_2+\Delta_1) + (2x_2+1)\,z_{1,2}\,] - \Delta x_{1,2}\,\Delta_1\\ \text{[redacted]} &= (2x_1+1)(2x_2+1)\,z_{1,2} + (2x_1+1)(-\Delta_2+\Delta_1)\,x_2 - \Delta x_{1,2}\Delta_1\end{aligned}}$$

III A few words about our changing of the equations and its consequences. ((eq.3) → (eq.6)) (see Warning W1)

(eq.3^1) I. $\bar{n}_1 = (2x_1+1)\,y_1 - \Delta_1$, $\Delta_1 \in \{1,\dots,2x_1\}$

(eq.3^2) II. $\bar{n}_2 = (2x_2+1)\,y_2 - \Delta_2$, $\Delta_2 \in \{1,\dots,2x_1\}$

→ In this equations are: $x_i, y_i \in \mathbb{N}$, $\bar{n}_i \in \mathbb{N}$

→ Then we used our Ansatz: $\bar{n}_1 = \bar{n}_2$

$$\Downarrow$$

$$y_1 = y_2 + \frac{2\Delta x_{1,2}\, y_2 - \Delta_2 + \Delta_1}{2x_1+1}$$

and, of course, we searched for <u>integer</u> solution for <u>y_1</u> and <u>y_2</u> for this.

→ Since, we had problems to solve this, we changed to (for $\Delta x_{1,2} > 1$)

$$y_1' = y_2' + \frac{2\Delta x_{1,2}\, y_2 + \Delta x_{1,2}(-\Delta_2+\Delta_1)}{2x_1+1}$$

and we <u>also</u> looked here for <u>integer</u> solutions <u>y_1'</u> and <u>y_2'</u> for

(eq.6^1) I. $\bar{n}_1' = (2x_1+1)\,y_1' - \Delta x_{1,2}\,\Delta_1$, $\Delta_1 \in \{1,\dots,2x_1\}$

(eq.6^2) II. $\bar{n}_2' = (2x_2+1)\,y_2' - \Delta x_{1,2}\,\Delta_2$, $\Delta_2 \in \{1,\dots,2x_1\}$

→ <u>But</u>...

...if we look at the relationship between this two versions (eq.3) and (eq.6), we get

(eq.9^1) I. $y_1' = y_1 + \frac{\Delta_1(-1+\Delta x_{1,2})}{2x_1+1}$

⇔ (eq.9^2) II. $y_1 = y_1' - \frac{\Delta_1(-1+\Delta x_{1,2})}{2x_1+1}$

, we see that in general in (eq.9) y_i and y_i' <u>are not</u> <u>integers</u> at the same time.

→ So the change ~~betw~~ between (eq.3) ⇔ (eq.6) is a ~~change~~ transition between integer ⇔ non-integer values.

→ Since (eq.6) ist also defined ~~at inte~~ for integer values like (eq.3), we have to take care that this is always ~~fullfield~~ fulfield.

→ We do this by using the floor function for (eq.9)

$$\text{(eq. 9}^1\text{v2)}\quad \boxed{\text{I. } y_1' = y_1 + \left\lfloor \frac{\Delta_1(-1+\Delta x_{1,2})}{2x_1+1} \right\rfloor}$$

$$\Leftrightarrow \quad \text{(eq.9}^2\text{v2)}\quad \boxed{\text{II. } y_1 = y_1' - \left\lfloor \frac{\Delta_1(-1+\Delta x_{1,2})}{2x_1+1} \right\rfloor}$$

Of cours y_1 and y_1' will not generate the same $\overline{n}$ anymore, but we will see, that this is not a problem for our calculations, in the next steps.

→ With this, we also have to change the solution for $z_{1,2}$.

$$y_1' = y_1^m + \left\lfloor \frac{\Delta_1(-1+\Delta x_{1,2})}{2x_1+1} \right\rfloor$$

and $\quad y_1' = x_2(-\Delta_2+\Delta_1) + (2x_2+1)z_{1,2}$

$$x_2(-\Delta_2+\Delta_1) + (2x_2+1)z_{1,2} = y_1^m + \left\lfloor \frac{\Delta_1(-1+\Delta x_{1,2})}{2x_1+1} \right\rfloor$$

$$\Leftrightarrow \quad (2x_2+1)z_{1,2} = y_1^m + \left\lfloor \frac{\Delta_1(-1+\Delta x_{1,2})}{2x_1+1} \right\rfloor - x_2(-\Delta_2+\Delta_1)$$

$$\Leftrightarrow \quad \text{(eq.10 v2)}\quad \boxed{z_{1,2} = \left\lfloor \frac{1}{2x_2+1} \left\{ y_1^m + \left\lfloor \frac{\Delta_1(-1+\Delta x_{1,2})}{2x_1+1} \right\rfloor - x_2(-\Delta_2+\Delta_1) \right\} \right\rfloor}$$

$2_L \quad 1_L \quad 1_R \quad 2_R$

Have attention, that we add additonally second floor brackets to this equation, for getting an integer $z_{1,2}$.

IV

Notation Info !

From now on, we will do a small notation changing.

Until now, we have had: $x_2 > x_1 \Leftrightarrow x_1 < x_2$

Since now, we will have: $x_2 < x_1 \Leftrightarrow x_1 > x_2$

Or, more general: $x_i > x_j$, with $i < j$, $i,j \in \mathbb{N}: i \neq j$

We will see, that this will lead to less confusion because of notation, in the next steps.

⇒ Now, we will do the intersection of several equations.

Given be: $\bar{n}_i' = (2x_i + 1)y_i' - \Delta x_{ij}\Delta_i$

Let's start!

⇒ **Step 1:**

$$\bar{n}_1' = (2x_1+1)y_1' - \Delta x_{12}\Delta_1 \qquad \Delta_1 \in \{1,\dots,2x_1\}$$

$$\bar{n}_2' = (2x_2+1)y_2' - \Delta x_{12}\Delta_2 \qquad \Delta_2 \in \{1,\dots,2x_2\}$$

$$(S1S) \Rightarrow \boxed{\bar{n}_{12}' = (2x_1+1)(2x_2+1)z_{12} + \underbrace{(2x_1+1)(-\Delta_2+\Delta_1)x_2 - \Delta x_{12}\Delta_1}_{=:\Delta_{12}}}$$

⇒ **Step 2:**

$$\Rightarrow \bar{n}_{12}' = (2x_1+1)(2x_2+1)z_{12} + \Delta x_{12,3}\{\underbrace{(2x_1+1)(-\Delta_2+\Delta_1)x_2 - \Delta x_{12}\Delta_1}_{= -\Delta_{12}}\}$$

$$\bar{n}_3' = (2x_3+1)y_3' - \Delta x_{12,3}\Delta_3 \qquad \Delta_3 \in \{1,\dots,2x_3\}$$

$$(S2S) \Rightarrow \boxed{\begin{aligned}\bar{n}_{12,3}' = {} & (2x_1+1)(2x_2+1)(2x_3+1)z_{12,3} \\ & \underbrace{+ (2x_1+1)(2x_2+1)\{-\Delta_3 + (2x_1+1)(-\Delta_2+\Delta_1)x_2 + \Delta x_{12}\Delta_1\}x_3 - \Delta x_{12,3}\{(2x_1+1)(-\Delta_2+\Delta_1)x_2 + \Delta x_{12}\Delta_1\}}_{=: -\Delta_{123}}\end{aligned}}$$

⇒ **Step 3:**

$$\bar{n}_{12,3}' = (2x_1+1)(2x_2+1)(2x_3+1)z_{12,3} + \Delta x_{123,4} \cdot (-\Delta_{123})$$

$$\bar{n}_4' = (2x_4+1)y_4' - \Delta x_{123,4}\Delta_4 \qquad \Delta_4 \in \{1,\dots,2x_4\}$$

$$\Rightarrow \bar{n}'_{123,4} = (2x_1+1)(2x_2+1)(2x_3+1)(2x_4+1)\, z_{123,4}$$
$$+ (2x_1+1)(2x_2+1)(2x_3+1)$$
$$\cdot \Big\{ -\Delta_4 - (2x_1+1)(2x_2+1)\big\{ -\Delta_3 - (2x_1+1)(-\Delta_2+\Delta_1)x_2 + \Delta x_{1,2}\Delta_1 \big\} x_3$$
$$+ \Delta x_{12,3}\big\{ -(2x_1+1)(-\Delta_2+\Delta_1)x_2 + \Delta x_{1,2}\Delta_1 \big\} \Big\}$$
$$\cdot\, x_4$$
$$- \Delta x_{123,4} \cdot \Big\{ -(2x_1+1)(2x_2+1)\big\{ -\Delta_3 - (2x_1+1)(-\Delta_2+\Delta_1)x_2 + \Delta x_{1,2}\Delta_1 \big\} x_3$$
$$+ \Delta x_{12,3}\big\{ -(2x_1+1)(-\Delta_2+\Delta_1)x_2 + \Delta x_{1,2}\Delta_1 \big\} \Big\}$$

....and so on.

<u>Let's try to bring this in a more general solution....</u>

We want a general ~~sol~~ form of $\bar{n}'_{123\ldots,n}$, $n \geq 2$, $n \in \mathbb{N}$

for the intersection of n-equations.

<u>Part 1</u>:

That's easy:

1 $$\boxed{\text{We have } \bar{n}'_{123,\ldots,n} = \prod_{k=1}^{n} (2x_k+1)\, z_{12\ldots n-1,n} + \mathcal{A}}$$

<u>Part 2</u>: $\mathcal{A}$? That's much harder.

2 <u>S1S:</u> ($\bar{n}'_{1,2}$) ($n=2$)
$$\mathcal{A}_{(2)} = (2x_1+1)(-\Delta_2+\Delta_1)x_2 - \Delta x_{1,2}\Delta_1$$
$$= \prod_{e_1=1}^{n-1}(2x_{e_1}+1)\cdot\{-\Delta_n+\Delta_{n-1}\}\, \overset{x_2}{x_n} - \Delta x_{12\ldots n-1,n}\cdot\Delta_{n-1}$$

<u>S2S</u> ($\bar{n}'_{12,3}$) ($n=3$)
$$\mathcal{A}_{(3)} = \prod_{e_1=1}^{n-1}(2x_{e_1}+1)\Big\{-\Delta_n - \prod_{e_2=1}^{n-2}(2x_{e_2}+1)(-\Delta_{n-1}+\Delta_{n-2})\,\overset{x_2}{x_{n-1}}$$
$$+ \Delta x_{1\ldots n-2,n-1}\Delta_{n-2}\Big\}\, x_n$$
$$- \Delta x_{1\ldots n-1,n}\Big\{-\prod_{e_3=1}^{n-2}(2x_{e_3}+1)(-\Delta_{n-1}+\Delta_{n-2})x_{n-1}$$
$$+ \Delta x_{1\ldots n-2,n-1}\Delta_{n-2}$$

S3S: $(\bar{n})_{1,2,3,4}$ $(n=4)$

$$\vartheta_{(4)} = \prod_{e_1=1}^{n-1}(2x_{e_1}+1)$$

$$\cdot\Big\{-\Delta_{n-0} - \prod_{e_2=1}^{n-2}(2x_{e_2}+1)\Big\{-\Delta_{n-1} - \prod_{e_3=1}^{n-3}(2x_{e_3}+1)(-\Delta_{n-2}+\Delta_{n-3})\overset{x_2}{\downarrow}x_{n-2}$$

$$+\Delta x_{1..n-3,n-2}\Delta_{n-3}\Big\}x_{n-1}$$

$$+\Delta x_{1..n-2,n-1}\Big\{-\prod_{e_4=1}^{n-3}(2x_{e_4}+1)(-\Delta_{n-2}+\Delta_{n-3})x_{n-2}$$

$$+\Delta x_{1..n-3,n-2}\Delta_{n-3}\Big\}\Big\}$$

$$-\Delta x_{1...n-1,n}\Big\{-\prod_{e_5=1}^{n-2}(2x_{e_5}+1)\Big\{-\Delta_{n-1} - \prod_{e_6=1}^{n-3}(2x_{e_6}+1)(-\Delta_{n-2}+\Delta_{n-3})\cdot x_{n-2}$$

$$+\Delta_{1..n-3,n-2}\Delta_{n-3}\Big\}x_{n-1}$$

$$+\Delta x_{1..n-2,n-1}\Big\{-\prod_{e_7=1}^{n-3}(2x_{e_7}+1)(-\Delta_{n-2}+\Delta_{n-3})\cdot x_{n-2}$$

$$+\Delta x_{1..n-3,n-2}\Delta_{n-3}\Big\}\Big\}$$

$$\cancel{\vartheta_{(n)} = \prod_{e_1=1}^{n-1}(2x_{e_1}+1)\cdot\Big\{-\Delta_{n-0} - \sum_{i=2}^{n-1}\prod_{e_i=1}^{n-i}(2x_{e_i}+1)\Big(-\sum_{k=i-1}^{n-i}\Delta_{n-k}}$$

$$\cancel{-\Delta_n - \sum\prod\prod(2x+1)}$$

S1S: $\vartheta_{(2)} = \prod_{e_1=1}^{n-1}(2x_{e_1}+1)\Big(\underbrace{\sum_{k_1=1}^{n}(-1)^{k_1+1}\Delta_{k_1}}\Big)x_n - \underbrace{\Delta x_{1,2}\Delta_1}$

$= \Delta x_{1,n}\Delta_1$

$= \Delta x_{1..n-1,n}\Delta_1$

S2S:

$$\vartheta_{(3)} = \Big\{(2x_1+1)(2x_2+1)(-\Delta_3) - \big((2x_1+1)(2x_2+1)\big)(2x_1+1)(-\Delta_2+\Delta_1)x_2$$

$$+(2x_1+1)(2x_2+1)\cdot\Delta x_{1,2}\Delta_1\Big\}x_3 - \Delta x_{1,2,3}\Big\{-(2x_1+1)(-\Delta_2+\Delta_1)x_2$$

$$+\Delta x_{1,2}\Delta_1\Big\}$$

$$\mathcal{A}_{(3)} = \prod_{e_1=1}^{n-1}(2x_{e_1}+1)\left(\sum_{u_1=1}^{n}(-1)^{u_1}\Delta_{u_1}\right)x_n - \Delta x_{1\ldots n-1,n}\,\Delta_{1\ldots n}$$

$$\cancel{= \prod_{e_1=1}^{n-1}(2x_{e_1}+1)\left\{\sum_{u_1=1}^{n}(-1)^{u_1}\Delta_{u_1}\cdot\prod_{e_2=1}^{n-1-u_1}(2x_{e_2}+1)\right.}$$

$$= \prod_{e_1=1}^{n-1}(2x_{e_1}+1)\left\{\underbrace{-\Delta_n}_{1}\underbrace{-\prod_{e_2=1}^{n-2}(2x_{e_2}+1)\left(\sum_{u_1=1}^{n-1}(-1)^{u_1+1}\Delta_{u_1}\right)x_{n-1} - \Delta x_{1\ldots n-2,n-1}\Delta_1}_{2}\right\}x_n$$

$$- \Delta x_{1\ldots n-1,n}\left\{-\prod_{e_2=1}^{n-2}(2x_{e_2}+1)\left(\sum_{u=1}^{n-1}(-1)^{u_1+1}\Delta_{u_1}\right)x_{n-1} - \Delta x_{1\ldots n-2,n-1}\Delta_1\right\}$$

$$\cancel{= \prod_{e_1=1}^{n-1}(2x_{e}+1)\left\{\sum_{u_2=1}^{n}(-1)^{u_2}\Delta_{u_2}\right.}$$

$$\cancel{\prod_{e_2=1}^{n-1}(2x_{e_2}+1)}$$

$$1 \quad \prod_{e=1}^{n-3}(2x_e+1)\left(\sum_{u=n}^{n}(-1)^{u}\Delta_u\right)$$

$$\Rightarrow \left\{\underbrace{\prod_{e_2=1}^{n-3}(2x_{e_2}+1)\left(\sum_{u_1=n}^{n}(-1)^{u_1}\Delta_{u_1}\right)}_{1} + \underbrace{\prod_{e_3=1}^{n-2}(2x_{e_3}+1)\left(\sum_{u_2=1}^{n-1}(-1)^{u_2}\Delta_{u_2}\right)\cdot x_{n-1}}_{2}\right.$$

$$= \sum_{i=2}^{n}\prod_{e=1}^{n-i}(2x_e+1)\left(\sum_{\substack{k=\cancel{n-i}\\ k=i-1}}^{\cancel{n-1}\; i}(-1)^{k}\Delta_k\right)$$

$n-i+1$ Elemente

$$\sum \cancel{\prod (2x_e+1)\left(\sum (-1)^u \Delta u\right)}$$

$$=$$

[illegible]

$$= \bar{n}'_{1,2} = (2x_1+1)(2x_2+1)z_{1,2} + (2x_1+1)(-\Delta_2+\Delta_1)x_2 - \underset{\uparrow \, (x_1-x_2)}{\Delta x_{1,2}}\Delta_1$$

We have: $x_1 = x_2 + \Delta x_{1,2}$

$$\Leftrightarrow \Delta x_{1,2} = x_1 - x_2$$

$$\Rightarrow \bar{n}'_{1,2} = (2x_1+1)(2x_2+1)z_{1,2} + (2x_1+1)(-\Delta_2)x_2 + (2x_1+1)\Delta_1 x_2 - x_1\Delta_1 + x_2\Delta_1$$

$$= (2x_1+1)(2x_2+1)z_{1,2} + (2x_1+1)(-\Delta_2)x_2 - x_1\Delta_1 + (2x_1+1)\Delta_1 x_2 + x_2\Delta_1$$

$$= (2x_1+1)(2x_2+1)z_{1,2} - 2x_1x_2\Delta_2 - \Delta_2 x_2 + 2x_1x_2\Delta_1 + \Delta_1 x_2 - x_1\Delta_1 + x_2\Delta_1$$

$$= (2x_1+1)(2x_2+1)z_{1,2} - \Delta_2\cdot(2x_1x_2 + x_2) + \Delta_1\big(\overbrace{2x_1x_2 + x_2 - x_1}^{2x_1x_2+2x_2-x_1} + x_2\big)$$

$$= (2x_1+1)(2x_2+1)z_{1,2} - \Delta_2(2x_1+1)x_2 + \Delta_1\left((2x_2+1)x_1 - 2x_1 + 2x_2\right)$$

$$= (2x_1+1)(2x_2+1)z_{1,2} - \Delta_2(2x_1+1)x_2 + \Delta_1(2x_2+1)x_1 + \underbrace{2(-x_1+x_2)}_{= -2(x_1-x_2) = -2\cdot\Delta x_{1,2}}\cdot\Delta_1$$

$$\boxed{\bar{n}'_{1,2} = (2x_1+1)(2x_2+1)z_{1,2} + \underbrace{\left(-\Delta_2(2x_1+1)x_2 + \Delta_1(2x_2+1)x_1\right)}_{=: -\Delta_{12}} - 2(\underbrace{x_1-x_2}_{\Delta x_{1,2}})\cdot\Delta_1}$$

$$\bar{n}'_{12,3} = (2x_1+1)(2x_2+1)(2x_3+1)\,z_{12,3}$$

$$+\Big\{ -\Delta_3\,(2x_1+1)(2x_2+1)\,x_3$$

$$+\Big(\big(+\Delta_2(2x_1+1)\,x_2 - \Delta_1(2x_2+1)\,x_1\big) + 2(x_1-x_2)\Delta_1\Big)$$

$$① \quad \cdot(2x_3+1)\Big((2x_1+1)(2x_2+1)-1\Big)\tfrac{1}{2}\Big\}$$

$$-2\cdot\Big\{\big((2x_1+1)(2x_2+1)-1\big)\tfrac{1}{2} - x_3\Big\}\Delta_1 \quad ②$$

$$① = \Big\{ -\Delta_3\,(2x_1+1)(2x_2+1)\,x_3$$

$$+\Delta_2\,(2x_1+1)\,x_2\,(2x_3+1)\Big((2x_1+1)(2x_2+1)-1\Big)\tfrac{1}{2}$$

$$-\Delta_1\,(2x_2+1)\,x_1\,(2x_3+1)\underline{\Big((2x_1+1)(2x_2+1)-1\Big)\cdot\tfrac{1}{2}}$$

$$+2(x_1-x_2)\Delta_1\cdot(2x_3+1)\Big((2x_1+1)(2x_2+1)-1\Big)\tfrac{1}{2}\Big\} \quad ⊛$$

$$② = \Big(-2\big((2x_1+1)(2x_2+1)-1\big)\tfrac{1}{2} + 2x_3\Big)\cdot\Delta_1$$

$$= -\Delta_1\big((2x_1+1)(2x_2+1)-1\big)\tfrac{1}{2}\cdot 2 \;-\; 2^2 x_3\Delta_1$$

$$\Rightarrow -\Delta_1\big((2x_1+1)(2x_2+1)-1\big)\tfrac{1}{2}\cdot\big\{2 + (2x_2+1)\,x_1(2x_3+1)\big\}$$

$$-\Delta_1\,2^2 x_3$$

$$\Rightarrow -\Delta_1\Big\{\big((2x_1+1)(2x_2+1)-1\big)\tfrac{1}{2}\cdot\big\{2 + (2x_2+1)\,x_1(2x_3+1)\big\} + 2^2 x_3\Big\}$$

$$\cancel{m_{1,\dots,n-1,n} = \prod_{k=1}^{n} (2x_k+1)\, z_{1,\dots,n-1,n}}$$

$$\circledast = \Big\{ 2(x_1-x_2)\cdot(2x_3+1)\big((2x_1+1)(2x_2+1)-1\big)\tfrac{1}{2} - 2\big((2x_1+1)(2x_2+1)-1\big)\tfrac{1}{2} + 2^2 x_3 \Big\}\Delta_1$$

$$= 2\Big\{ (x_1-x_2)(2x_3+1)\big((2x_1+1)(2x_2+1)-1\big)\tfrac{1}{2} - \big((2x_1+1)(2x_2+1)-1\big)\tfrac{1}{2} + 2x_3 \Big\}\Delta_1$$

$$\boxed{= -2\Big\{ -(x_1-x_2)(2x_3+1)\big((2x_1+1)(2x_2+1)-1\big)\tfrac{1}{2} + \big((2x_1+1)(2x_2+1)-1\big)\tfrac{1}{2} - 2x_3 \Big\}\Delta_1}$$

$$= -2\Big\{ \big((2x_1+1)(2x_2+1)-1\big)\tfrac{1}{2}\big(-(x_1-x_2)(2x_3+1)+1\big) - 2x_3 \Big\}\Delta_1$$

$$= -2\Big\{ \big((2x_1+1)(2x_2+1)-1\big)\tfrac{1}{2}\underbrace{\big((-x_1+x_2)(2x_3+1)+1\big)} - 2x_3 \Big\}\Delta_1$$

$$= -2x_3x_1 - x_1 + 2x_2x_3 + x_2 + 1 - 2x_3$$

$$= (2x_2x_3 + x_2 + x_3) - 3x_3 - (2x_1x_3 + x_1 + x_3) + x_3$$

$$\boxed{= -2\Big\{ \big((2x_1+1)(2x_2+1)-1\big)\tfrac{1}{2}\Big(\big((2x_2+1)(2x_3+1)-1\big)\tfrac{1}{2} - \big((2x_1+1)(2x_3+1)-1\big)\tfrac{1}{2} \Big) - 2x_3 \Big\}\Delta_1}$$

$$= -2\Big\{ \big((2x_1+1)(2x_2+1)-1\big)\tfrac{1}{2}\big((2x_2+1)(2x_3+1)-1\big)\tfrac{1}{2}$$

$$- \big((2x_1+1)(2x_2+1)-1\big)\tfrac{1}{2}\big((2x_1+1)(2x_3+1)-1\big)\tfrac{1}{2}$$

$$- \big((2x_1+1)(2x_2+1)-1\big)\tfrac{1}{2}(-2x_3)$$

Given be $\bar{n}_i' = (2x_i+1)y_i' - \Delta x_{i,j}\Delta_i$ with $\Delta_i \in \{1,\dots,2x_i\}$

from $\bar{n}_i = (2x_i+1)y_i - \Delta_i$, $x_i, y_i \in \mathbb{N}$

Now since this function shall start with $y_i \geq 2$ —

But, we would still like to have $\mathbb{N}$ as definition set for x_i and y_i, we do a substitution.

$$\boxed{y_i = \tilde{y}_i + 1, \quad \tilde{y}_i \in \mathbb{N}} \Leftrightarrow \boxed{\tilde{y}_i = y_i - 1}$$

$$\Rightarrow \bar{n}_i = (2x_i+1)(\tilde{y}_i+1) - \Delta_i$$

$$\Rightarrow \boxed{\bar{n}_i = (2x_i+1)\tilde{y}_i + (2x_i+1) - \Delta_i} \quad \text{(eq. 11)}$$

and for

$\underline{\bar{n}_i'}$ $\cancel{\tilde{y}_i' = \tilde{y}_i + \Delta}$

$$y_i' = y_i + \frac{\Delta_i(-1+\Delta x_{i,j})}{2x_i+1}$$

$$\tilde{y}_i' + 1 = \tilde{y}_i + 1 + \text{〃 〃 〃}$$

$$\Leftrightarrow \tilde{y}_i' = \tilde{y}_i + \frac{\Delta_i(-1+\Delta x_{i,j})}{2x_i+1}$$

$\Rightarrow$ The same relationship like before

$$\Rightarrow \bar{n}_i' = (2x_i+1)(\tilde{y}_i'+1) - \Delta x_{i,j}\Delta_i$$

$$\Rightarrow \boxed{\bar{n}_i' = (2x_i+1)\tilde{y}_i' + (2x_i+1) - \Delta x_{i,j}\Delta_i} \quad \text{(eq. 12)}$$

<u>Intersection</u>:

$$\bar{n}_1' = (2x_1+1)\tilde{y}_1' + (2x_1+1) - \Delta x_{1,2}\Delta_1$$

$$\bar{n}_2' = (2x_2+1)\tilde{y}_2' + (2x_2+1) - \Delta x_{1,2}\Delta_2$$

$$(2x_1+1)\tilde{y}_1' + (2x_1+1) - \Delta x_{1,2}\Delta_1 = (2x_2+1)\tilde{y}_2' + (2x_2+1) - \Delta x_{1,2}\Delta_2$$

$$= 2x_1+1 + 2\Delta x_{1,2} - 2x_1 - 1$$

$$0 = (2x_2+1)\tilde{y}_2' - (2x_1+1)\tilde{y}_1' + \overbrace{(2x_2+1) - (2x_1+1)} - \Delta x_{1,2}\Delta_2 + \Delta x_{1,2}\Delta_1$$

Be $x_2 = x_1 + \Delta x_{1,2}$

$$0 = (2x_1+1)\tilde{y}_2' + 2\Delta x_{1,2}\tilde{y}_2' - (2x_1+1)\tilde{y}_1' + 2\Delta x_{1,2} - \Delta x_{1,2}\Delta_2 + \Delta x_{1,2}\Delta_1$$

$\cancel{(2x_1+1)\tilde{y}_2}$

$$(2x_1+1)\tilde{y}_1' = (2x_1+1)\tilde{y}_2' + (2\Delta x_{1,2})\cdot(\tilde{y}_2'+1) - \Delta x_{1,2}\Delta_2 + \Delta x_{1,2}\Delta_1$$

$$\Rightarrow \tilde{y}_1' = \tilde{y}_2' + \frac{2\Delta x_{1,2}(\tilde{y}_2'+1) + \Delta x_{1,2}(-\Delta_2+\Delta_1)}{2x_1+1}$$ ✓

$$\Rightarrow \boxed{\begin{aligned} \tilde{y}_1' &= x_2(-\Delta_2+\Delta_1) + (2x_2+1)z_{1,2} - 1 \\ \tilde{y}_2' &= x_1(-\Delta_2+\Delta_1) + (2x_1+1)z_{1,2} - 1 \end{aligned}} \qquad \text{(eq.13)}$$

$$\begin{aligned} \overline{n}_{1,2}' &= (2x_1+1)\tilde{y}_1' + (2x_1+1) - \Delta x_{1,2}\Delta_1 \\ &= (2x_1+1)\left[x_2(-\Delta_2+\Delta_1) + (2x_2+1)z_{1,2} - 1\right] \\ &\quad + (2x_1+1) - \Delta x_{1,2}\Delta_1 \\ &= (2x_1+1)(2x_2+1)z_{1,2} \\ &\quad + (2x_1+1)(-\Delta_2+\Delta_1)x_2 - \cancel{(2x_1+1)} + \cancel{(2x_1+1)} \\ &\quad - \Delta x_{1,2}\Delta_1 \\ &= (2x_1+1)(2x_2+1)z_{1,2} + (2x_1+1)(-\Delta_2+\Delta_1)x_2 - \Delta x_{1,2}\Delta_1 \end{aligned}$$

the ~~same~~ same like before. ✓

$$\boxed{\bar{n}_1 = (2x_1+1)y_1 - \Delta_1}$$

$\rightarrow$ We only need odd $\bar{n}$. $\quad , m \in \mathbb{N} \quad$ (eq. 14)

$$\Rightarrow \boxed{2m+1 = \bar{n}_1 = (2x_1+1)y_1 - \Delta_1} \text{ with } \Delta_1 \in \{1,\dots,2x_1\}$$

We have different possibilities to rewrite the right side, to get the necessary numbers. We will choose the following:

$$2m+1 = \bar{n}_1 = (2x_1+1)\underbrace{(2u_1+1)}_{=:y_1,\ u_1 \in \mathbb{N}} - \Delta_1$$

$\rightarrow$ we only have odd y_1 now.

$\Rightarrow \Delta_1$ has to be 2ℓ, $\ell \in \mathbb{N}$

$\Rightarrow$ So, be $\Delta_1^0 \in \{1,\dots,x_i\}$ but now (with only odd numbers) we have a doubbled intervall, so we have in real $\Delta_1^0 \in \{1,\dots,2x_i\}$ again

It follows:

$$\boxed{2m+1 = \bar{n}_1^0 = (2x_1+1)\underbrace{(2u_1+1)}_{=:y_1'} - 2\Delta_1^0}, \text{ with } \boxed{\Delta_1^0 \in \{1,\dots,2x_1\}} \quad \text{(eq. 15)}$$

$$\Rightarrow \bar{n}_1^0 = (2x_1+1)(2u_1+1) - 2\Delta_1^0 \cdot \Delta x_{1,2}$$

$$\bar{n}_2^0 = (2x_2+1)\underbrace{(2u_2+1)}_{=:y_2'} - 2\Delta_2^0 \cdot \Delta x_{1,2}$$

$$\Rightarrow y_1' = (2u_1+1) = x_2 \cdot (-\Delta_2+\Delta_1) + (2x_2+1) z_{1,2}$$

$$u_1 = \frac{1}{2}\left(x_2(-\Delta_2+\Delta_1) + (2x_2+1) z_{1,2} - 1 \right)$$

$$(2x_1+1)(2u_1+1) - 2\Delta_1^0 \cdot \Delta x_{1,2} = (2x_2+1)(2u_2+1) - 2\Delta_2^0 \cdot \Delta x_{1,2}$$

$x_2 = x_1 + \Delta x_{1,2}$

$$0 = \overbrace{(2(x_1+\Delta x_{1,2})+1)}^{(2x_1+1)+2\Delta x_{1,2}}(2u_2+1) - (2x_1+1)(2u_1+1) - 2\Delta_2^0 \cdot \Delta x_{1,2} + 2\Delta_1^0 \cdot \Delta x_{1,2}$$

$$\cdot\tfrac{1}{2} \Big| = (2x_1+1)\underbrace{(2u_2+1-2u_1+1)}_{2(u_2-u_1)+2} + 2\Delta x_{1,2}(2u_2+1) - 2\Delta_2^0 \Delta x_{1,2} + 2\Delta_1^0 \Delta x_{1,2}$$

$$= (2x_1+1)(u_2-u_1+1) + \Delta x_{1,2}(2u_2+1) + \Delta x_{1,2}(-\Delta_2+\Delta_1)$$

$$(2x_1+1)(2u_1+1) - 2\Delta_1^\circ \cdot \Delta x_{1,2} = (2x_2+1)(2u_2+1) - 2\Delta_2^\circ \cdot \Delta x_{1,2}$$

$$0 = (2x_2+1)(2u_2+1) - (2x_1+1)(2u_1+1) - 2\Delta_2^\circ \Delta x_{1,2} + 2\Delta_1^\circ \Delta x_{1,2}$$

$\underline{x_2 = x_1 + \Delta x_{1,2}}$

$$0 = (2x_1+1)(2u_2+1) + 2\Delta x_{1,2}(2u_2+1) - (2x_1+1)(2u_1+1) + 2\Delta x_{1,2}(-\Delta_2^\circ + \Delta_1^\circ)$$

$$\cdot\tfrac{1}{2} \;/\; 0 = (2x_1+1)\underbrace{(2u_2+1-2u_1-1)}_{2(u_2-u_1)} + 2\Delta x_{1,2}(2u_2+1) + 2\Delta x_{1,2}(-\Delta_2^\circ+\Delta_1^\circ)$$

$$0 = (2x_1+1)(u_2-u_1) + \Delta x_{1,2}(2u_2+1) + \Delta x_{1,2}(-\Delta_2^\circ + \Delta_1^\circ)$$

$$(2x_1+1)u_1 = (2x_1+1)u_2 + \Delta x_{1,2}(2u_2+1) + \Delta x_{1,2}(-\Delta_2^\circ+\Delta_1^\circ)$$

$$\boxed{u_1 = u_2 + \frac{\Delta x_{1,2}(2u_2+1) + \Delta x_{1,2}(-\Delta_2^\circ+\Delta_1^\circ)}{2x_1+1}} \quad (\text{eq. } 16)$$

$\underline{\text{Be } u_2 = (-\Delta_2 + \Delta_1^\circ + \alpha)x_1}$

$$\Rightarrow \quad u_1 = (-\Delta_2^\circ + \Delta_1^\circ + 1)x_1 + \Delta x_{1,2} \frac{\overbrace{2(-\Delta_2^\circ+\Delta_1^\circ+\alpha)x_1+1}^{2\cdot(-\Delta_2^\circ+\Delta_1^\circ)x_1 + 2\alpha x_1 + 1} + (-\Delta_2^\circ+\Delta_1^\circ)}{2x_1+1}$$

$$= (-\Delta_2^\circ + \Delta_1^\circ + 1)x_1 + \Delta x_{1,2} \cdot \frac{(-\Delta_2^\circ+\Delta_1^\circ)\cdot(2x_1+1) + 2\alpha x_1 + 1}{2x_1+1}$$

$$= (-\Delta_2^\circ + \Delta_1^\circ + 1)x_1 + \Delta x_{1,2}(-\Delta_2^\circ+\Delta_1^\circ) + \frac{2\alpha x_1+1}{2x_1+1}\cdot \Delta x_{1,2}$$

be $\alpha = 1$

$$= (-\Delta_2^\circ + \Delta_1^\circ + 1)x_1 + \Delta x_{1,2}(-\Delta_2^\circ+\Delta_1^\circ) + \Delta x_{1,2}$$

$$= (-\Delta_2^\circ + \Delta_1^\circ + 1)x_1 + \Delta x_{1,2}(-\Delta_2^\circ + \Delta_1^\circ + 1)$$

$$= (-\Delta_2^\circ + \Delta_1^\circ + 1)(x_1 + \Delta x_{1,2})$$

$$\underline{u_1 = (-\Delta_2^\circ + \Delta_1^\circ + 1)\, x_2}$$

$$\boxed{2m_1+1 = \bar{n}_1^{\circ} = (2x_1+1)(2k_1+1) - 2\Delta_1^{\circ}}$$

$$\Rightarrow \quad 2m_1+1 = (2x_1+1)(2k_1+1) - 2\Delta_1^{\circ}$$

$$2m_1 = (2x_1+1)(2k_1+1) - 2\Delta_1^{\circ} - 1$$

$$m_1 = \frac{1}{2}\left\{(2x_1+1)(2k_1) + (2x_1+1) - 2\Delta_1^{\circ} - 1\right\}$$

$$= \frac{1}{2}\left\{(2x_1+1)\,2k_1 + 2x_1 - 2\Delta_1^{\circ}\right\}$$

$$\boxed{m_1 = (2x_1+1)\,k_1 + x_1 - \Delta_1^{\circ}} \quad \text{(eq.17)}$$

'$\Delta x_{1,2}$ for our $\bar{n}_1^{\circ}$' version

$\Rightarrow$ That's the equation which we already know from my work „The recursively calculation of prime numbers“

Available on:

[1] https://github.com/Samdney/primescalc

For the intersection of an arbitrary number n of equation $\bar{n}_i^{\circ}$, $(n>1)$, $n \in \mathbb{N}$ we have from [1]:

(eq.I1)
(eq.18)

$$m_{\underbrace{1,\dots,n-1}_{i},\underbrace{n}_{j}} = \prod_{k=1}^{n}(2x_k+1)\,z_{1\dots n-1,n}$$
$$+ \frac{1}{2}\left(\prod_{k=1}^{n}(2x_k+1) - 1\right)$$
$$- \left((-1)^{n+1}\,\Delta_1\, x_1 \prod_{\ell \neq 1}^{n}(2x_\ell+1)\prod_{f=1}^{n-2,\,n>2}\left(\frac{1}{2}\left(\prod_{m=1}^{f+1}(2x_m+1)-1\right)\right)\right)$$
$$- \left(\sum_{k=2}^{n}(-1)^{n+k}\,\Delta_k\, x_k \prod_{\ell \neq 1}^{n}(2x_\ell+1)\prod_{f=1}^{n-k,\,n>2}\left(\frac{1}{2}\left(\prod_{m=1}^{f+1}(2x_m+1)-1\right)\right)\right)$$

With the definition $\prod_{f=1}^{0} A_f := 1$.

(eq.I2)

(eq.19)

$$\Rightarrow \bar{n}'_{1,\ldots,n-1,n} = 2m_{1\ldots n-1,n} + 1$$

$$= 2 \cdot \Bigg\{ \prod_{k=1}^{n} (2x_k+1)_{z_{1\ldots n-1,n}} + \frac{1}{2}\left(\prod_{k=1}^{n} (2x_k+1) - 1 \right) - \left((-1)^{n+1} \Delta_1 x_1 \prod_{e\neq 1}^{n} (2x_e+1) \prod_{f=1}^{n-2,\, n>2} \left(\frac{1}{2}\left(\prod_{m=1}^{f+1} (2x_m+1) - 1 \right)\right)\right) - \left(\sum_{k=2}^{n} (-1)^{n+k} \Delta_k x_k \prod_{e\neq 1}^{n} (2x_e+1) \prod_{f=1}^{n-k,\, n>2} \left(\frac{1}{2}\left(\prod_{m=1}^{f+1} (2x_m+1) - 1 \right)\right)\right) \Bigg\}$$

$$+1$$

VII → Be given

$$n_1 = (2x_1 + 1)y_1 + \alpha \qquad x_i, y_i \in \mathbb{N}$$

and $$n_2 = (2x_2 + 1)y_2 + \beta$$

with $x_2 > x_1$.

→ So, we we can write $x_2 = x_1 + \Delta x_{1,2}$, $\Delta x_{1,2} \in \mathbb{N}$

$$\Rightarrow \quad n_1 = (2x_1 + 1)y_1 + \alpha$$

$$n_2 = (2\cdot(x_1 + \Delta x_{1,2}) + 1)y_2 + \beta$$

→ $\boxed{\text{Now, let's assume :} \quad \Delta x_{1,2} = (2x_1 + 1)}$

$$\Rightarrow n_2 = (2\cdot(x_1 + (2x_1 + 1)) + 1)y_2 + \beta$$
$$= (2\cdot(x_1 + 2x_1 + 1) + 1)y_2 + \beta$$
$$= (2\cdot(3x_1 + 1) + 1)y_2 + \beta$$
$$= (6x_1 + 2 + 1)y_2 + \beta$$
$$= (6x_1 + 3)y_2 + \beta$$
$$= 3\cdot(2x_1 + 1)y_2 + \beta$$
$$n_2 = (2x_1 + 1)\underbrace{(3y_2)}_{=: \tilde{y}_2} + \beta$$
$$= (2x_1 + 1)\tilde{y}_2 + \beta$$

→ Now, look at

$$* \quad (2\tilde{x} + 1) = \tilde{n}_{1,2} = \underbrace{(2x_1 + 1)(2x_2 + 1)}z_{1,2} + \underbrace{(2x_1 + 1)(-\Delta_2 + \Delta_1)x_2 - \Delta x_{1,2}\Delta_1}_{=: \gamma}$$

$$= 2\cdot 2x_1x_2 + 2x_1 + 2x_2 + 1$$
$$= 2\cdot(2x_1x_2 + x_1 + x_2) + 1$$

Be $$\tilde{x} = 2x_1x_2 + x_1 + x_2 - \tilde{\Delta}x_{1,2}$$

$$\Rightarrow \quad 2\cdot(2x_1x_2 + x_1 + x_2 - \tilde{\Delta}x_{1,2}) + 1 = (2(2x_1x_2 + x_1 + x_2) + 1)z_{1,2} + \gamma$$

$$\Leftrightarrow \quad 2(2x_1x_2 + x_1 + x_2) - 2\tilde{\Delta}x_{1,2} + 1 = (2(2x_1x_2 + x_1 + x_2) + 1)z_{1,2} + \gamma$$

$$\Leftrightarrow \quad 2(2x_1x_2 + x_1 + x_2) - 2(2x_1x_2 + x_1 + x_2)z_{1,2} = 2z_{1,2} + 2\tilde{\Delta}x_{1,2} - 1 + \gamma$$

$$\Leftrightarrow \quad 2(2x_1x_2 + x_1 + x_2)\cdot(1 - z_{1,2}) = 2(z_{1,2} + \tilde{\Delta}_{1,2}) - 1 + \gamma$$

$$\Leftrightarrow \quad 2(2x_1x_2 + x_1 + x_2)(1 - z_{1,2}) = 2\cdot(z_{1,2} - 1) + 1 + 2\tilde{\Delta}_{1,2} + \gamma$$

$$\Leftrightarrow \quad 2(2x_1x_2+x_1+x_2)(1-z_{1,2}) = -2(1-z_{1,2})+1+2\hat{\Delta}_{1,2}+\gamma$$

$$\Leftrightarrow \quad 2(2x_1x_2+x_1+x_2)(1-z_{1,2}) + (1-z_{1,2}) = -(1-z_{1,2})+1+2\hat{\Delta}x_{1,2}+\gamma$$

$$\Leftrightarrow \quad \left(2(2x_1x_2+x_1+x_2)+1\right)(1-z_{1,2}) = -1+z_{1,2}+1+2\hat{\Delta}x_{1,2}+\gamma$$

$$\Leftrightarrow \quad (2x_1+1)(2x_2+1)(1-z_{1,2}) = z_{1,2}+2\hat{\Delta}x_{1,2}+\gamma$$

$$\Leftrightarrow \quad 2(2x_1x_2+x_1+x_2)-2\hat{\Delta}x_{1,2}+1 = \left(2(2x_1x_2+x_1+x_2)+1\right)z_{1,2}+\gamma$$

$$\Leftrightarrow \quad (2x_1+1)(2x_2+1)-2\hat{\Delta}x_{1,2} = (2x_1+1)(2x_2+1)z_{1,2}+\gamma$$

$$\Leftrightarrow \quad (2x_1+1)(2x_2+1)-(2x_1+1)(2x_2+1)z_{1,2} = 2\hat{\Delta}x_{1,2}+\gamma$$

$$\Leftrightarrow \quad (2x_1+1)(2x_2+1)(1-z_{1,2}) = 2\hat{\Delta}x_{1,2}+\gamma$$

$$\Leftrightarrow \quad (2x_1+1)(2x_2+1)(1-z_{1,2}) - \gamma = 2\tilde{\Delta}x_{1,2}$$

→ We know that we have for odd solutions $\tilde{n} = 2\tilde{x}+1$, always $2\Delta_i$. $\Delta^\circ_i \in \frac{1}{2}\{1,..,2x_i\}$

$\Delta_i \in$

$$\Leftrightarrow \quad \tilde{\Delta}x_{1,2} = \underbrace{\underbrace{(2x_1+1)(2x_2+1)\frac{(1-z_{1,2})}{2}}_{①}}_{\text{this part is always}} - \underbrace{\frac{1}{2}\gamma}_{②} \qquad (*)$$

(eq. 20)

① we can resubstitute this, with $\frac{(1-z_{1,2})}{2} = z^{new}_{1,2}$ for all $z_{1,2}$ odd

this part is always: $= (2x_1+1)\cdot u$ respectively $= (2x_2+1)\,u'$

(**) $\forall u_1,u_2 \in \mathbb{N}$

$\tilde{\Delta}x_{1,2} \neq (2x_1+1)u_1$

and $\tilde{\Delta}x_{1,2} \neq (2x_2+1)u_2$

⟹ so we can always use our $\Delta\tilde{x}_{1,2}$ from recursion for the next step, in the valid range

To ②: → From the deffinition of our original equation $\bar{n}_i$, we know that all results are between $(2x_i+1)u$ and $(2x_i+1)(u+1)$, $\forall u \in \mathbb{N}$. excluded $(2x_i+1)u$ $\forall u \in \mathbb{N}$

and → From this follows, that also all values from $\bar{n}_{ij}$ are between $(2x_1+1)(2x_2+1)u$ and $(2x_1+1)(2x_2+1)(u+1)$ $\forall u \in \mathbb{N}$, $(2x_i+1)(2x_2+1)u$ excluded $\forall u \in \mathbb{N}$

⟹ all values of (*) are not element of $(2x_1+1)$ times table and not element of $(2x_2+1)$ times table and not element of $(2x_1+1)(2x_2+1)$ times table ⟹ (**)

Let's have a look at the differences between

$$\boxed{\begin{aligned} &\bar{n}_1 = (2x_1+1)y_1 - \Delta_1 \\ \text{and} \quad &\bar{n}_1' = (2x_1+1)y_1' - \Delta x_{1/2}\Delta_1 \end{aligned}}$$

Example: $x_1 = 1$, $\Delta x_{1/2} = 1$

$$\Rightarrow \quad \bar{n}_1 = 3y_1 - \Delta_1$$
$$\bar{n}_1' = 3y_1' - \Delta_1$$

Be $\Delta_1 = 1$: $\bar{n}_1 = 3\cdot y_1 - 1$, $y_1 = 1 \Rightarrow \bar{n}_1 = 3 - 1 = 2$

$$y_1' = y_1 + \frac{\Delta_1(-1+\Delta x_{1/2})}{2x_1+1}$$
$$= 1 + \frac{1\cdot(-1+1)}{2\cdot 1+1}$$
$$= 1 + 0$$
$$y' = 1$$

Example: $x_1 = 1$ $\Delta x_{1/2} = 2$

$$\Rightarrow \quad \bar{n}_1 = 3y_1 - \Delta_1$$
$$\bar{n}_1' = 3y_1' - 2\Delta_1$$

Be $\Delta_1 = 1$: $y_1 = 2 \Rightarrow \bar{n}_1 = 3\cdot 2 - 1 = 6 - 1 = 5$

$$y_1' = y_1 + \frac{\Delta_1(-1+\Delta x_{1/2})}{2\cdot x_1+1}$$
$$= 2 + \frac{1\cdot(-1+2)}{3}$$
$$y_1' = 2 + \frac{1}{3}$$

$\Rightarrow$ So, we see, if we use the same Δ_1 in both equations, we get a not integer y_1'.

Let's do $y_1' = \lfloor 2 + 1/3 \rfloor = 2$, $\Delta_1 = 1$

$$\Rightarrow \quad \bar{n}_1' = 3\cdot 2 - 2\cdot 1 = 6 - 2 = 4$$

For which values do we get $\bar{n}_1' = 5$?

$$5 = 3y_1' - 2\Delta_1 = \begin{cases} y_1' = 2, \Delta_1 = 1 \\ \Rightarrow 3\cdot 2 - 2\cdot 1 = 6 - 2 = 4 \end{cases}$$

⇒ we get all values of $\bar{n}_1$ also by $\bar{n}_1'$! but with different (y_1, Δ_1) pairs, if we want to fulfield the constraint $y_i \in \mathbb{N}$!

(I already showed (in other notes) tha this is fulfield. I will not repead it here, but you can also easy check this by yourself)

Assume we have given:

$$(*) \quad \boxed{\bar{n}_1 = (2x_1+1)y_1 - \Delta_1} \qquad \Delta_1 \in \{1, \dots, 2x_1\}$$

→ Have, fore example, a look at: $(x_1 = 1)$:

$$\bar{n}_1 = 3y_1 - \Delta_1 \quad , \Delta_1 \in \{1, 2\}$$

$y_1 = 1$: $\quad \bar{n}_1 = \begin{cases} \Delta_1 = 1 : 2 \\ \Delta_1 = 2 : 1 \end{cases}$

← We see: The number „3" is missing !

$y_1 = 2$: $\quad \bar{n}_1 = \begin{cases} \Delta_1 = 1 : 5 \\ \Delta_1 = 2 : 4 \end{cases}$

→ That's not what we want, since we want to find all prime numbers.

$y_1 = 3$: $\quad \bar{n}_1 = \begin{cases} \Delta_1 = 1 : 8 \\ \Delta_1 = 2 : 7 \end{cases}$

→ with this we will not get „3"

$\vdots \qquad \vdots \qquad \vdots$

⇒ Our equation (*) is not usable (gives us not a valid range) for values ≤ 3.

⇒ So it's just valid for $y_1 \geq 2$. !

→ But..., we already have to know „3" to generate this (*) equation, so the given number „3" comes from an earlier calculation step.

Now let's switch to (only odd solutions)

$$(**) \quad \boxed{\bar{n}_1^o = (2x_1+1)(2u_1+1) - 2\Delta_1^o \quad , \Delta_1^o \in \{1, \dots, 2x_1\}}$$

$$u_1 \in \mathbb{N}$$

→ Let's look again at our example ($x_1 = 1$):

$$\bar{n}_1^0 = 3\cdot(2u_1+1) - 2\Delta_1^0 \,,\; \Delta_1^0 \in \{1,2\}$$

(+) →

$u_1 = 1$:

$$\bar{n}_1^0 = \begin{cases} \Delta_1^0 = 1: & 3\cdot 3 - 2\cdot 1 = 9 - 1\cdot 2 = 9 - 2 = 7 \\ \Delta_1^0 = 2: & 3\cdot 3 - 2\cdot 2 = 9 - 2\cdot 2 = 9 - 4 = 5 \end{cases}$$

$u_1 = 2$:

$$\bar{n}_1^0 = \begin{cases} \Delta_1^0 = 1: & 3\cdot 5 - 2\cdot 1 = 15 - 1\cdot 2 = 15 - 2 = 13 \\ \Delta_1^0 = 2: & 3\cdot 5 - 2\cdot 2 = 15 - 2\cdot 2 = 15 - 4 = 11 \end{cases}$$

$u_1 = 3$:

$$\bar{n}_1^0 = \begin{cases} \Delta_1^0 = 1: & 3\cdot 7 - 2\cdot 1 = 21 - 2\cdot 1 = 21 - 2 = 19 \\ \Delta_1^0 = 2: & 3\cdot 7 - 2\cdot 2 = 21 - 2\cdot 2 = 21 - 4 = 17 \end{cases}$$

$\vdots$

(+) → Here we see, that we don't generate numbers $\bar{n}_1^0 \le 6 = 3\cdot 2$

→ So, if we use this equation (**), we only generate ~~a valid~~ valid numbers > 6.

⇒ Consequences for intersection of two equations of the form (**):

$$\bar{n}_1^0 = (2x_1+1)(2u_1+1) - 2\Delta_1^0 \,,\; \Delta_1^0 \in \{1,..,2x_1\}$$

$$\bar{n}_2^0 = (2x_2+1)(2u_2+1) - 2\Delta_2^0 \,,\; \Delta_2^0 \in \{1,..,2x_2\}$$

$$\Rightarrow \bar{n}_1^0 = \bar{n}_2^0$$

Assume: $x_2 > x_1$

Then our solution $\bar{n}_{12}^0$ of $\bar{n}_1^0 = \bar{n}_2^0$ gives us only valid results in the range $\bar{n}_{12}^0 \in](2x_2+1), \infty[$ (***)

and not-valid results in the range $\bar{n}_{12}^0 \in [1, (2x_2+1)]$

$(2x_2+1)(2\cdot 0+1)$

So, what is [illegible]?

⇒ Consequences for intersection of two equations of the form (**)

Example:

$$\bar{n}_1^o = 3(2u_1+1) - 2\Delta_1^o \,, \quad \Delta_1^o \in \{1,2\}$$

$$\bar{n}_2^o = 5(2u_2+1) - 2\Delta_2^o \,, \quad \Delta_2^o \in \{1,..,4\}$$

and

$$\bar{n}_3^o = 7(\underbrace{2u_3+1}_{=3,5,7,9,11}) - 2\Delta_3^o \,, \quad \Delta_3^o \in \{1,..,6\}$$

⇒ n_1: 3 | 4 5 | 6 | 7 8 $\boxed{9}$ 11 12 13 14 $\boxed{15}$

n_2: ... 4 5 | 6 ... 14 $\boxed{15}$ 16 ...... 24 $\boxed{25}$

n_3: ... 6 | 7 8 20 $\boxed{21}$ 22 ...... 34 $\boxed{35}$

$n^o_{i(u=0)}$ $\quad$ $\bar{n}^o_i$ ~~even~~ odd $\quad$ $n^o_{i(u\in\mathbb{N})}$

Minimum ($\bar{n}_1^o = \bar{n}_2^o$)

Assume: $x_2 > x_1$

⇒ We see, our smallest values for n_1 and n_2 are 9 and 15 for $u=1$.

⇒ So, for our range we have our Minimum at

$$\Rightarrow \boxed{\bar{n}^{o\,min}_{1,2} = \underbrace{(2x_2+1)}_{x_2>x_1}\cdot(2\underbrace{u_2}_{=1}+1) - 2\cdot\underbrace{\Delta_2^o}_{=2\cdot x_2}} \quad \text{(eq.21)}$$

(here: $\bar{n}^{min}_{1,2} = (2\cdot2+1)\cdot(2\cdot1+1) - 2\cdot2\cdot2$
$= 5\cdot3 - 8 = 15-8 = 7$)

⇒ Let's look for our Maximum

If we look at our numbers, we see that we can ~~use~~ use ~~$(2x_2+1)\cdot(2u_2+1)$~~ here 21 safely as maximum range since it's the first number of the next larger number.

$$\Rightarrow \boxed{\bar{n}^{max}_{1,2} = \underbrace{(2(x_2+1)+1)}_{x_2>x_1}\cdot(2\underbrace{u_2}_{=1}+1) - 2} \quad \text{(eq.22)}$$

(here: $\bar{n}^{max}_{1,2} = (2\cdot(2+1)+1)\cdot(2\cdot1+1) - 2$
$= (2\cdot3+1)\cdot(2+1) - 2$
$= 7\cdot3 - 2$
$= 19$)

$(x_2 > x_1)$

⇒ valid $\bar{n}^o_{1,2}$ range: $\bar{n}^o_{1,2} \in [(2x_2+1)\cdot3 - 2\cdot(2\cdot x_2),\ (2(x_2+1)+1)\cdot3 - 2]$ (eq.23)

Since, we now know our valid $\bar{n}_i^0$ range, we can finally also have a look at z_{ij}.

For the intersection of two equations, we have given

$$(1) \quad z_{ij} = \left\lfloor \frac{1}{2x_j+1} \left\{ y_i + \left\lfloor \frac{\Delta_i(-1+\Delta x_{ij})}{2x_i+1} \right\rfloor - x_j(-\Delta_j+\Delta_i) \right\} \right\rfloor$$

respectively

$$(2) \quad z_{ij} = \left\lfloor \frac{1}{2x_i+1} \left\{ y_j + \left\lfloor \frac{\Delta_j(-1+\Delta x_{ij})}{2x_j+1} \right\rfloor - x_i(-\Delta_j+\Delta_i) \right\} \right\rfloor$$

→ assume „i" be the intersection of all $(2x_i+1)$ apart from the Maximum ($\Rightarrow$ „j" $\hat{=}$ Maximum) and do the intersection with this Maximum as the last intersection, at all.

→ So, if we take Eq(2), we can take y_j „Maximum" and determine z_{ij} from this.

→ If we also do the same for the Minimum in the intersection step before, we can determine our valid z_{ij} range from this.

Approximations for floor function: (from different Internet sources)

1. $$\lfloor x \rfloor = -\frac{1}{2} + x + \frac{\arctan(\cot(\pi x))}{\pi} = -\frac{1}{2} + x - \frac{\arctan(\tan(\pi(x-\frac{1}{2})))}{\pi}$$

(eq.24)

2. Fourier series of the Floor function

$$\lfloor x \rfloor = -\frac{1}{2} + x + \frac{1}{\pi}\sum_{k=1}^{\infty}\frac{\sin(2\pi k x)}{k}$$

(eq.25) which converges to

$$= \underbrace{-\frac{1}{2} + x - \frac{i\left(\ln(1-e^{-2\pi i x}) - \ln(1-e^{2\pi i x})\right)}{2\pi}}_{\text{real}} \quad (\mathbb{C})$$

3. $$\lfloor x \rfloor = \lim_{n\to\infty}\left(\sum_{k=-n}^{n}\mu(x-k)\right) - n - 1$$

(eq.26) μ step function: $\mu(x) = \lim_{n\to\infty} f(nx)$

with $f(x) = e^{-x^2} + \frac{2}{\pi}Si(x)$.

So

$$f(x) = 1 + \frac{2}{\pi}x + \sum_{k=1}^{\infty}\frac{(-1)^k}{\prod_{j=1}^{k} j}x^{2k} + \frac{2}{\pi}\frac{\left(\prod_{j=1}^{2k} j\right)(-1)^k}{\left(\prod_{j=1}^{2k+1} j\right)^2}x^{2k+1}$$

Taylor series converges for every $x \in \mathbb{R}$.

4. $$\lfloor x \rfloor = \sum_{n=-\infty}^{\infty} n\,\Theta(x-n)\,\Theta(n+1-x), \quad x \notin \mathbb{Z}$$

(eq.27)

$$\lfloor x \rfloor = \lim_{x\to x^+}\lfloor z \rfloor$$

Let's put it all together:

1. We will ignore the prime number 2.

2. We assume, we already know the first two prime numbers

$p_1 = \bar{n}_1 = 2 \cdot 1 + 1 = 3$ $\Rightarrow$ We know all primes $p \in \underbrace{[1,5]}_{=: I_1}$

$p_2 = \bar{n}_2 = 2 \cdot 2 + 1 = 5$ $\underbrace{}_{\bar{n}'_{0,1}}$

$$\Rightarrow \left\| \begin{array}{ll} \text{I.} & \bar{n}_1 = 3\overbrace{(2u_1+1)}^{=: y_1} - 2\Delta_1 \quad , \Delta_1 \in \{1,2\} \\ \text{II.} & \bar{n}_2 = 5\underbrace{(2u_2+1)}_{=: y_2} - 2\Delta_2 \quad , \Delta_2 \in \{1,..,4\} \end{array} \right.$$

3. We have also:

$$\Rightarrow \left\| \begin{array}{ll} \text{I.} & \bar{n}'_1 = 3\overbrace{(2u'_1+1)}^{= y'_1} - 2\overset{1}{\overset{\shortparallel}{\Delta x_{1,2}}}\Delta_1 \quad , \Delta_1 \in \{1,2\} \\ \text{II.} & \bar{n}'_2 = 5\underbrace{(2u'_2+1)}_{=: y'_2} - 2\underset{1}{\underset{\shortparallel}{\Delta x_{1,2}}}\Delta_2 \quad , \Delta_2 \in \{1,..,4\} \end{array} \right.$$

4. Do the intersection: we receive $\bar{n}'_{1,2}$

5. Determine: $y_1^{min}, y_1^{max}, y_2^{min}, y_2^{max}$

6. Calculate with this $y_1^{min'}, y_1^{max'}, y_2^{min'}, y_2^{max'}$

7. Determine $z_{1,2}^{min}$ and $z_{1,2}^{max}$ from Step 6. Make an good approximation for $z_{1,2}$.

$\Rightarrow$ Now, we have $\bar{n}'_{1,2}$ which generate all prime numers, in the allowed next range I_2.

8. Now, we can do $\bar{n}'_{1,2,3}(\bar{n}'_{1,2}, \bar{n}'_{0,1})$ to generate the primes in the

and determine $z'_{1,2,3}(\bar{n}'_{1,2}, \underbrace{y_1^{min'}, y_1^{max'}, y_2^{min'}, y_2^{max'}}_{z'_{1,2}}) = z'_{1,2,3}(z'_{1,2})$

9. Now, we can do $\bar{n}'_{1,2,3,4}(\bar{n}'_{1,2,3}(\bar{n}'_{1,2}, \bar{n}'_{0,1}), \bar{n}'_{1,2}, \bar{n}'_{0,1})$

and determine $z'_{1,2,3,4}(z'_{1,2,3}(z'_{1,2}))$

10. ... and so on.

!! $\Rightarrow$ All what we need are the equations for z, $\bar{n}'_i$ and $\bar{m}'_i$:
(eq. 10 v2), (eq. I1) and (eq. I2)

! ⇒ We ~~know~~ know the whole time, during every step, everything what we need to calculate the next step!
determine

ToDo:

→ Find a better way of writing for equations (eq. I 1) and (eq. I 2)

→ Think about possible approximations for z (eq. 10 v2)

→ Think about other useful $Y^{min/max}$ ranges / ~~choices~~ choices

~~→ Determine the simplified recursion equation~~

→ Determine the simplified recursion equation.