$x_2 > x_1$: $x_2 = x_1 + \Delta x_{1,2}$

$\Leftrightarrow \; x_1 = x_2 - \Delta x_{1,2}$

$$\bar{n}_1^2 = (2x_1+1)(2u_1+1) - 2\Delta x_{1,2}\Delta_1$$

$$\bar{n}_2^2 = (2x_2+1)(2u_2+1) - 2\Delta x_{1,2}\Delta_2$$

$\Rightarrow$

$$0 = +(2x_2+1)(2u_2+1) - (2x_1+1)(2u_1+1) - 2\Delta x_{1,2}\Delta_2 + 2\Delta x_{1,2}\Delta_1$$

$$0 = (2x_1+1)(2u_2+1) + 2\Delta x_{1,2}(2u_2+1) - (2x_1+1)(2u_1+1) + 2\Delta x_{1,2}(-\Delta_2+\Delta_1)$$

$$(2x_1+1)(2u_1+1) = (2x_1+1)(2u_2+1) + 2\Delta x_{1,2}(2u_2+1) + 2\Delta x_{1,2}(-\Delta_2+\Delta_1)$$

$$2u_1+1 = 2u_2+1 + \frac{2\Delta x_{1,2}(2u_2+1) + 2\Delta x_{1,2}(-\Delta_2+\Delta_1)}{2x_1+1}$$

$$2u_1+1 = 2u_2+1 + 2\Delta x_{1,2}\underbrace{\frac{(2u_2+1)+(-\Delta_2+\Delta_1)}{2x_1+1}}_{(*)}$$

$\Rightarrow$ Be $u_2 = (-\Delta_2+\Delta_1+\alpha)\beta+\gamma$

$$(*) = \frac{2((-\Delta_2+\Delta_1+\alpha)\beta+\gamma) + (-\Delta_2+\Delta_1)}{2x_1+1}$$

$$= \frac{2(-\Delta_2+\Delta_1)\beta + 2\alpha\beta + 2\gamma + (-\Delta_2+\Delta_1)}{2x_1+1}$$

$$= (-\Delta_2+\Delta_1)\frac{2\beta+1}{2x_1+1} + \underbrace{\frac{2\overset{1}{\alpha}\beta+2\overset{1/2}{\gamma}}{2x_1+1}}$$

$$\boxed{\Rightarrow \gamma = \tfrac{1}{2} \quad \Rightarrow \alpha = 1}$$

$$= (-\Delta_2+\Delta_1)\frac{2\beta+1}{2x_1+1} + \frac{2\cdot\beta+1}{2x_1+1}$$

$$\boxed{\beta = 2x_1y + x_1 + y}$$

$$= \frac{(2\beta+1)(-\Delta_2+\Delta_1+1)}{2x_1+1} = \frac{(\overbrace{2\cdot(2x_1y+x_1+y)+1}^{(2x_1+1)(2y+1)})(-\Delta_2+\Delta_1+1)}{2x_1+1}$$

$$= (2y+1)(-\Delta_2+\Delta_1+1)$$

$\Rightarrow$

$$\boxed{u_2 = (-\Delta_2+\Delta_1+1)\cdot(2x_1y+x_1+y) + \tfrac{1}{2}}$$

$$\bar{n}_1' = (2x_1+1)(2u_1+1) - 2\Delta x_{1/2}\Delta_1$$
$$\bar{n}_2' = (2x_2+1)(2u_2+1) - 2\Delta x_{1/2}\Delta_2$$

$$\Rightarrow (2u_1+1) = (-\Delta_2+\Delta_1+1)\,x_2 \overset{\rightarrow}{=} (-\Delta_2+\Delta_1+1)\,(2x_2y_2+x_2+y_2)$$
$$(2u_2+1) = (-\Delta_2+\Delta_1+1)\,x_1 \overset{\rightarrow}{=} (-\Delta_2+\Delta_1+1)\,\underbrace{(2x_1y_1+x_1+y_1)}_{((2x_1+1)(2y_1+1)-1)\frac{1}{2}}$$

<u>Be:</u> $x_2 > x_1$, $2x_2+1 = (2x_1+1)(2x_1'+1)$

$$0 = (2x_1+1)(2x_1'+1)(2u_2+1) - 2\Delta x_{1/2}\Delta_2 - (2x_1+1)(2u_1+1) + 2\Delta x_{1/2}\Delta_1$$

$$(2x_1+1)(2u_1+1) = (2x_1+1)(2x_1'+1)(2u_2+1) - 2\Delta x_{1/2}\Delta_2 + 2\Delta x_{1/2}\Delta_1$$

$$(2u_1+1) = (2x_1'+1)(2u_2+1) + 2\Delta x_{1/2}\frac{(-\Delta_2+\Delta_1)}{2x_1+1}$$
$$= (2x_1'+1)(-\Delta_2+\Delta_1+1)\left((2x_1+1)(2y_1+1)-1\right)\frac{1}{2} + 2\Delta x_{1/2}\frac{-\Delta_2+\Delta_1}{2x_1+1}$$
$$= (2x_1'+1)(2x_1+1)(2y_1+1)(-\Delta_2+\Delta_1+1)\cdot\frac{1}{2} - (2x_1'+1)(-\Delta_2+\Delta_1+1)\frac{1}{2} + 2\Delta x_{1/2}\frac{-\Delta_2+\Delta_1}{2x_1+1}$$

$$0 = (2x_2+1)(2u_2+1) - 2\Delta x_{1,2}\Delta_2$$
$$- (2x_1+1)(2u_1+1) + 2\Delta x_{1,2}\Delta_1$$

$x_2 = x_1 + \Delta x_{1,2}$

$$0 = (2x_1+1)(2u_2+1) + 2\Delta x_{1,2}(2u_2+1) - 2\Delta x_{1,2}\Delta_2$$
$$- (2x_1+1)(2u_1+1) + 2\Delta x_{1,2}\Delta_1$$

$x_1 = x_2 - \Delta x_{1,2}$

$$0 = (2x_2+1)(2u_2+1) - 2\Delta x_{1,2}\Delta_2$$
$$- (2x_2+1)(2u_1+1) + 2\Delta x_{1,2}(2u_2+1) + 2\Delta x_{1,2}\Delta_1$$

Fehler !!!

Be $(2x_2+1) = (2x_1+1)(2x_1'+1)$

$$\Rightarrow \quad 0 = (2x_1+1)(2x_1'+1)(2u_2+1) - 2\Delta x_{1,2}\Delta_2$$
$$- (2x_1+1)(2x_1'+1)(2u_1+1) - 2\Delta x_{1,2}(2u_1+1) + 2\Delta x_{1,2}\Delta_1$$

$$(2x_1+1)(2x_1'+1)(2u_1+1)$$
$$= (2x_1+1)(2x_1'+1)(2u_2+1) - 2\Delta x_{1,2}\Delta_2$$
$$- 2\Delta x_{1,2}(2u_1+1) + 2\Delta x_{1,2}\Delta_1$$

$$(2u_1+1) = (2u_2+1) + \frac{1}{(2x_1+1)(2x_1'+1)}\left(-2\Delta x_{1,2}\Delta_2 - 2\Delta x_{1,2}(2u_1+1) + 2\Delta x_{1,2}\Delta_1\right)$$

$$= (2u_2+1) + 2\Delta x_{1,2}\frac{(-\Delta_2+\Delta_1) - (2u_1+1)}{\underbrace{(2x_1+1)(2x_1'+1)}_{=2\cdot(2x_1x_1'+x_1+x_1')+1}}$$

Be $u_1 = (-\Delta_2+\Delta_1+\alpha)(2x_1x_1'+x_1+x_1') + \gamma$

$$= (2u_2+1) + 2\Delta x_{1,2}\frac{(-\Delta_2+\Delta_1) - 2(-\Delta_2+\Delta_1+\alpha)(2x_1x_1'+x_1+x_1') - 2\gamma - 1}{(2x_1+1)(2x_1'+1)}$$

$\gamma = \frac{1}{2}$

$$\cancel{-2\cdot\tfrac{1}{2}-1}$$
$$\cancel{= -1-1 = 0}$$

$$= (2u_2+1) + 2\Delta x_{1,2}(-\Delta_2+\Delta_1)\frac{1 - 2\cdot(2x_1x_1'+x_1+x_1')}{(2x_1+1)(2x_1'+1)}$$

$$= \cancel{(2u_2+1) - 2\Delta x_{1,2}(-\Delta_2+\Delta_1)\frac{2\cdot(2x_1x_1'+x_1+x_1')-1}{(2x_1+1)(2x_1'+1)}}$$

$$= (2u_2+1) + (2\Delta x_{1,2}) \frac{(-\Delta_2+\Delta_1) - 2(-\Delta_2+\Delta_1)(2x_1x_1' + x_1 + x_1') - 2\alpha(2x_1x_1'+x_1+x_1')}{(2x_1+1)(2x_1'+1)}$$

$$+ \frac{-2\gamma-1}{(2x_1+1)(2x_1'+1)}$$

$$= (2u_2+1)(2\Delta x_{1,2}) + \underbrace{2\Delta x_{1,2}\cdot(-\Delta_2+\Delta_1)\frac{1-2(2x_1x_1'+x_1+x_1')}{(2x_1+1)(2x_1'+1)}}_{①}$$

$$+ \underbrace{\frac{-2\alpha(2x_1x_1'+x_1+x_1')-2\gamma-1}{(2x_1+1)(2x_1'+1)}}_{②}$$

$(2x_1+1)(2x_1'+1)$
$= 2\cdot(2x_1x_1'+x_1+x_1')$
$+1$

$$② = -\frac{2\alpha(2x_1x_1'+x_1+x_1')+2\gamma+1}{(2x_1+1)(2x_1'+1)}$$

→ Be $\gamma=0$ and $\alpha=1$

$$= -\frac{2\cdot 1(2x_1x_1'+x_1+x_1')+2\cdot 0+1}{(2x_1+1)(2x_1'+1)}$$

$$= -1$$

$$① = +2\Delta x_{1,2}(-\Delta_2+\Delta_1)\frac{-2(2x_1x_1'+x_1+x_1')+1}{(2x_1+1)(2x_1'+1)}$$

$$= -2\Delta x_{1,2}(-\Delta_2+\Delta_1)\frac{2(2x_1x_1'+x_1+x_1')+1-2}{(2x_1+1)(2x_1'+1)}$$

$$= -2\Delta x_{1,2}(-\Delta_2+\Delta_1)\left[1+\frac{-2}{(2x_1+1)(2x_1'+1)}\right]$$

$$= -2\Delta x_{1,2}(-\Delta_2+\Delta_1)$$

?

$$- \frac{2\cdot 2\Delta x_{1,2}(-\Delta_2+\Delta_1)}{(2x_1+1)(2x_1'+1)} \quad \text{(eq 1)}$$

We have: $x_2 = x_1 + \Delta x_{1,2} \Rightarrow \Delta x_{1,2} = x_2 - x_1$

and $(2x_2+1) \doteq (2x_1+1)(2x_1'+1)$

$$\boxed{O = (2x_2+1)(2u_2+1) - 2\Delta x_{1,2}\Delta_2 - (2x_1+1)(2u_1+1) + 2\Delta x_{1,2}\Delta_1}$$

Be $x_2 > x_1$, $x_1 = x_2 - \Delta x_{1,2}$, and $(2x_2+1) = (2x_1+1)(2x_1'+1)$

$(\Delta x_{1,2} = x_2 - x_1)$

$$(2x_1+1)(2u_1+1) = (2x_2+1)(2u_2+1) + 2\Delta x_{1,2}(-\Delta_2+\Delta_1)$$

$$(2x_1+1)(2u_1+1) = (2x_1+1)(2x_1'+1)(2u_2+1) + 2\Delta x_{1,2}(-\Delta_2+\Delta_1)$$

$$(2u_1+1) = (2x_1'+1)(2u_2+1) + \underbrace{\frac{2\Delta x_{1,2}(-\Delta_2+\Delta_1)}{2x_1+1}}_{①}$$

$$① = \frac{2\cdot(x_2-x_1)(-\Delta_2+\Delta_1)}{2x_1+1}$$

$$= \frac{2\cdot\left(((2x_1+1)(2x_1'+1)-1)\frac{1}{2} - x_1\right)(-\Delta_2+\Delta_1)}{2x_1+1}$$

$$= \frac{((2x_1+1)(2x_1'+1) - 1 - 2x_1)(-\Delta_2+\Delta_1)}{(2x_1+1)}$$

$$= \frac{(2x_1+1)(2x_1'+1)(-\Delta_2+\Delta_1)}{(2x_1+1)} - \frac{(2x_1+1)(-\Delta_2+\Delta_1)}{(2x_1+1)}$$

$$= (2x_1'+1)(-\Delta_2+\Delta_1) - (-\Delta_2+\Delta_1)$$

$$= (-\Delta_2+\Delta_1)(2x_1'+1-1)$$

$$= (-\Delta_2+\Delta_1)\,2x_1'$$

$$\Rightarrow \boxed{2u_1+1 = (2x_1'+1)(2u_2+1) + (-\Delta_2+\Delta_1)\cdot 2x_1'}$$

$$= 2\cancel{x_1'}u_2\cdot 2 + \cancel{2x_1'} + \cancel{2u_2} + \cancel{1} + (-\Delta_2+\Delta_1)\cdot 2x_1'$$

$$2u_1 = 2^2 x_1' u_2 + 2x_1' + 2u_2 + (-\Delta_2+\Delta_1)\cdot 2x_1'$$

$$\boxed{u_1 = 2x_1'u_2 + x_1' + u_2 + (-\Delta_2+\Delta_1)x_1'}$$

$$\boxed{u_1 = \underbrace{2x_1'u_2 + u_2}_{(2x_1'+1)u_2} + (-\Delta_2+\Delta_1+1)x_1'}$$

$$0 = (2x_2+1)(2u_2+1) - 2\Delta x_{1,2}\Delta_2$$
$$\quad - (2x_1+1)(2u_1+1) + 2\Delta x_{1,2}\Delta_1$$

Be

~~[illegible]~~ $x_1 = x_2 - \Delta x_{1,2}$

$$0 = (2x_2+1)(2u_2+1) - 2\Delta x_{1,2}\Delta_2$$
$$\quad - (2(x_2-\Delta x_{1,2})+1)(2u_1+1) + 2\Delta x_{1,2}\Delta_1$$

$$0 = (2x_2+1)(2u_2+1) - 2\Delta x_{1,2}\Delta_2$$
$$\quad - (2x_2+1)(2u_1+1) + 2\Delta x_{1,2}(2u_1+1)$$

Bx $x_2 = x_1 + \Delta x_{1,2}$

$$0 = (2(x_1+\Delta x_{1,2})+1)(2u_2+1) - 2\Delta x_{1,2}\Delta_2$$
$$\quad - (2x_1+1)(2u_1+1) + 2\Delta x_{1,2}\Delta_1$$

$$0 = (2x_1+1)(2u_2+1) + 2\Delta x_{1,2}(2u_2+1) - 2\Delta x_{1,2}\Delta_2$$
$$\quad - \underline{(2x_1+1)(2u_1+1)} + 2\Delta x_{1,2}\Delta_1$$

$$(2\cancel{x_1}+1)(2u_1+1) = (2\cancel{x_1}+1)(2u_2+1) + 2\Delta x_{1,2}(2u_2+1) + 2\Delta x_{1,2}(-\Delta_2+\Delta_1)$$

$$(2u_1+1) = (2u_2+1) + \frac{2\Delta x_{1,2}(2u_2+1) + 2\Delta x_{1,2}(-\Delta_2+\Delta_1)}{2x_1+1}$$

$$= (2u_2+1) + 2\underbrace{\Delta x_{1,2}}_{x_2-x_1}\frac{(2u_2+1)+(-\Delta_2+\Delta_1)}{2x_1+1}$$

Be $u_2 = (-\Delta_2+\Delta_1+1)(2x_1y_1+x_1+y_1)$:

$$\cancel{= (2u_2+1) + 2\Delta x_{1,2}}$$

$$\left(= (2u_2+1) + \underbrace{\frac{2(x_2-x_1)(2u_2+1) + (-\Delta_2+\Delta_1)}{2x_1+1}}_{\frac{2x_2 2u_2 + 2x_2 - 2u_2 x_1 - x_1 + (-\Delta_2+\Delta_1)}{2x_1+1}} \right)$$

$$= (2u_2+1) + 2\Delta x_{1,2}\frac{\cancel{2}\cdot(-\Delta_2+\Delta_1+1)\overbrace{(2x_1y_1+x_1+y_1)}^{((2x_1+1)(2y_1+1)-1)\cancel{\frac{1}{2}}} + 1 + (-\Delta_2+\Delta_1)}{2x_1+1}$$

$$= (2u_2+1) + 2\Delta x_{1,2}\cdot\frac{(-\Delta_2+\Delta_1+1)(2x_1+1)(2y_1+1) - \cancel{(-\Delta_2+\Delta_1+1)} + \cancel{(-\Delta_2+\Delta_1+1)}}{(2x_1+1)}$$

$$(2u_2+1) + 2\Delta x_{1,2}\cdot(-\Delta_2+\Delta_1+1)(2y_1+1)$$

✓