Research
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RESEARCH NOTES

Primes (part 04): Playing around

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Abstract

Some playing around with different ways of intersection solutions.

Content

$\underline{x_2 > x_1}$: $x_2 = x_1 + \Delta x_{1,2}$

$\Leftrightarrow x_1 = x_2 - \Delta x_{1,2}$

$$\tilde{n}_1^2 = (2x_1+1)(2u_1+1) - 2\Delta x_{1,2}\Delta_1$$

$$\tilde{n}_2^2 = (2x_2+1)(2u_2+1) - 2\Delta x_{1,2}\Delta_2$$

$\Rightarrow$

$$0 = +(2x_2+1)(2u_2+1) - (2x_1+1)(2u_1+1) - 2\Delta x_{1,2}\Delta_2 + 2\Delta x_{1,2}\Delta_1$$

$$0 = (2x_1+1)(2u_2+1) + 2\Delta x_{1,2}(2u_2+1) - (2x_1+1)(2u_1+1) + 2\Delta x_{1,2}(-\Delta_2+\Delta_1)$$

$$(2x_1+1)(2u_1+1) = (2x_1+1)(2u_2+1) + 2\Delta x_{1,2}(2u_2+1) + 2\Delta x_{1,2}(-\Delta_2+\Delta_1)$$

$$2u_1+1 = 2u_2+1 + \frac{2\Delta x_{1,2}(2u_2+1) + 2\Delta x_{1,2}(-\Delta_2+\Delta_1)}{2x_1+1}$$

$$2u_1+1 = 2u_2+1 + 2\Delta x_{1,2}\underbrace{\frac{(2u_2+1)+(-\Delta_2+\Delta_1)}{2x_1+1}}_{*}$$

$\Rightarrow$ $\underline{\text{Se}}$ $u_2 = (-\Delta_2+\Delta_1+\alpha)\beta+\gamma$

$$* = \frac{2\left((-\Delta_2+\Delta_1+\alpha)\beta+\gamma\right)+(-\Delta_2+\Delta_1)}{2x_1+1}$$

$$= \frac{2(-\Delta_2+\Delta_1)\beta + 2\alpha\beta + 2\gamma + (-\Delta_2+\Delta_1)}{2x_1+1}$$

$$= (-\Delta_2+\Delta_1)\frac{2\beta+1}{2x_1+1} + \underbrace{\frac{2\overset{1}{\alpha}\beta+2\overset{1/2}{\gamma}}{2x_1+1}}$$

$$\boxed{\Rightarrow \gamma = \tfrac{1}{2} \quad \Rightarrow \alpha = 1}$$

$\Rightarrow$

$$\boxed{u_2 = (-\Delta_2+\Delta_1+1)\cdot(2x_1y+x_1+y) + \tfrac{1}{2}}$$

$$= (-\Delta_2+\Delta_1)\frac{2\beta+1}{2x_1+1} + \frac{2\cdot\beta+1}{2x_1+1}$$

$$\boxed{\beta = 2x_1y + x_1 + y}$$

$$= \frac{(2\beta+1)(-\Delta_2+\Delta_1+1)}{2x_1+1} = \frac{\overbrace{\left(2\cdot(2x_1y+x_1+y)+1\right)}^{(2x_1+1)(2y+1)}(-\Delta_2+\Delta_1+1)}{2x_1+1}$$

$$= (2y+1)(-\Delta_2+\Delta_1+1)$$

$$\left\| \begin{array}{l} \bar{n}_1' = (2x_1+1)(2u_1+1) - 2\Delta x_{1/2}\Delta_1 \\ \bar{n}_2' = (2x_2+1)(2u_2+1) - 2\Delta x_{1/2}\Delta_2 \end{array} \right.$$

$$\Rightarrow (2u_1+1) = (-\Delta_2+\Delta_1+1)\, x_2 \xrightarrow{=} (-\Delta_2+\Delta_1+1)\,(2x_2y_2+x_2+y_2)$$

$$(2u_2+1) = (-\Delta_2+\Delta_1+1)\, x_1 \xrightarrow{=} (-\Delta_2+\Delta_1+1)\,\underbrace{(2x_1y_1+x_1+y_1)}_{((2x_1+1)(2y_1+1)-1)\frac{1}{2}}$$

Be: $x_2 > x_1$, $2x_2+1 = (2x_1+1)(2x_1'+1)$

$$0 = (2x_1+1)(2x_1'+1)(2u_2+1) - 2\Delta x_{1/2}\Delta_2$$
$$-(2x_1+1)(2u_1+1) + 2\Delta x_{1/2}\Delta_1$$

$$(2x_1+1)(2u_1+1) = (2x_1+1)(2x_1'+1)(2u_2+1) - 2\Delta x_{1/2}\Delta_2$$
$$+ 2\Delta x_{1/2}\Delta_1$$

$$(2u_1+1) = (2x_1'+1)(2u_2+1) + 2\Delta x_{1/2}\frac{(-\Delta_2+\Delta_1)}{2x_1+1}$$
$$= (2x_1'+1)(-\Delta_2+\Delta_1+1)\left[(2x_1+1)(2y_1+1)-1\right)\frac{1}{2}$$
$$+ 2\Delta x_{1/2}\frac{-\Delta_2+\Delta_1}{2x_1+1}$$

$$= (2x_1'+1)(2x_1+1)(2y_1+1)(-\Delta_2+\Delta_1+1)\cdot\frac{1}{2}$$
$$-(2x_1'+1)(-\Delta_2+\Delta_1+1)\frac{1}{2} + 2\Delta x_{1/2}\frac{-\Delta_2+\Delta_1}{2x_1+1}$$

$$0 = (2x_2+1)(2u_2+1) - 2\Delta x_{1/2}\Delta_2$$
$$- (2x_1+1)(2u_1+1) + 2\Delta x_{1/2}\Delta_1$$

$x_2 = x_1 + \Delta x_{1/2}$

$$0 = (2x_1+1)(2u_2+1) + 2\Delta x_{1/2}(2u_2+1) - 2\Delta x_{1/2}\Delta_2$$
$$- (2x_1+1)(2u_1+1) + 2\Delta x_{1/2}\Delta_1$$

$x_1 = x_2 - \Delta x_{1/2}$

$$0 = (2x_2+1)(2u_2+1) - 2\Delta x_{1/2}\Delta_2$$
$$- (2x_2+1)(2u_1+1) + 2\Delta x_{1/2}(2u_2+1) + 2\Delta x_{1/2}\Delta_1$$

Fehler !!!

Be $(2x_2+1) = (2x_1+1)(2x_1'+1)$

$$\Rightarrow \quad 0 = (2x_1+1)(2x_1'+1)(2u_2+1) - 2\Delta x_{1/2}\Delta_2$$
$$- (2x_1+1)(2x_1'+1)(2u_1+1) - 2\Delta x_{1/2}(2u_1+1) + 2\Delta x_{1/2}\Delta_1$$

$$(2x_1+1)(2x_1'+1)(2u_1+1)$$
$$= (2x_1+1)(2x_1'+1)(2u_2+1) - 2\Delta x_{1/2}\Delta_2$$
$$- 2\Delta x_{1/2}(2u_1+1) + 2\Delta x_{1/2}\Delta_1$$

$$(2u_1+1) = (2u_2+1) + \frac{1}{(2x_1+1)(2x_1'+1)}\left(-2\Delta x_{1/2}\Delta_2 - 2\Delta x_{1/2}(2u_1+1) + 2\Delta x_{1/2}\Delta_1\right)$$

$$= (2u_2+1) + 2\Delta x_{1/2}\frac{(-\Delta_2+\Delta_1) - (2u_1+1)}{\underbrace{(2x_1+1)(2x_1'+1)}_{=2\cdot(2x_1x_1'+x_1+x_1')+1}}$$

Be $u_1 = (-\Delta_2+\Delta_1+\alpha)(2x_1x_1'+x_1+x_1') + \gamma$

$$= (2u_2+1) + 2\Delta x_{1/2}\frac{(-\Delta_2+\Delta_1) - 2(-\Delta_2+\Delta_1+\alpha)(2x_1x_1'+x_1+x_1') - 2\gamma - 1}{(2x_1+1)(2x_1'+1)}$$

$\cancel{\gamma = -\tfrac{1}{2}}$

$\cancel{-2(-\tfrac{1}{2})-1}$
$\cancel{= 1-1=0}$

$$= (2u_2+1) + \cancel{2\Delta x_{1/2}(-\Delta_2+\Delta_1)\frac{1-2\cdot(2x_1x_1'+x_1+x_1')}{(2x_1+1)(2x_1'+1)}}$$
$$= \cancel{(2u_2+1) - 2\Delta x_{1/2}(-\Delta_2+\Delta_1)\frac{2\cdot(2x_1x_1'+x_1+x_1')-1}{(2x_1+1)(2x_1'+1)}}$$

$$= (2u_2+1) + (2\Delta x_{1,2})\frac{(-\Delta_2+\Delta_1) - 2(-\Delta_2+\Delta_1)(2x_1x_1' + x_1 + x_1') - 2\alpha(2x_1x_1'+x_1+x_1')}{(2x_1+1)(2x_1'+1)}$$

$$+ \frac{-2\gamma-1}{(2x_1+1)(2x_1'+1)}$$

$$= (2u_2+1)(2\Delta x_{1,2}) + \underbrace{2\Delta x_{1,2}\cdot(-\Delta_2+\Delta_1)\frac{1-2(2x_1x_1'+x_1+x_1')}{(2x_1+1)(2x_1'+1)}}_{1}$$

$$+ \underbrace{\frac{-2\alpha(2x_1x_1'+x_1+x_1') - 2\gamma - 1}{(2x_1+1)(2x_1'+1)}}_{2}$$

$$(2x_1+1)(2x_1'+1) = 2\cdot(2x_1x_1'+x_1+x_1') + 1$$

$$2 = -\frac{2\alpha(2x_1x_1'+x_1+x_1') + 2\gamma + 1}{(2x_1+1)(2x_1'+1)}$$

→ Be $\gamma = 0$ and $\alpha = 1$

$$= -\frac{2\cdot 1(2x_1x_1'+x_1+x_1') + 2\cdot 0 + 1}{(2x_1+1)(2x_1'+1)}$$

$$= -1$$

$$1 = +2\Delta x_{1,2}(-\Delta_2+\Delta_1)\frac{-2(2x_1x_1'+x_1+x_1')+1}{(2x_1+1)(2x_1'+1)}$$

$$= -2\Delta x_{1,2}(-\Delta_2+\Delta_1)\frac{2(2x_1x_1'+x_1+x_1')+1-2}{(2x_1+1)(2x_1'+1)}$$

$$= -2\Delta x_{1,2}(-\Delta_2+\Delta_1)\left[1 + \frac{-2}{(2x_1+1)(2x_1'+1)}\right]$$

$$= -2\Delta x_{1,2}(-\Delta_2+\Delta_1)$$

?

$$- \frac{2\cdot 2\Delta x_{1,2}(-\Delta_2+\Delta_1)}{(2x_1+1)(2x_1'+1)} \quad \text{(eq 1)}$$

We have: $x_2 = x_1 + \Delta x_{1,2} \Rightarrow \Delta x_{1,2} = x_2 - x_1$

and $(2x_2+1) \stackrel{!}{=} (2x_1+1)(2x_1'+1)$

$$0 = (2x_2+1)(2u_2+1) - 2\Delta x_{1,2}\Delta_2 - (2x_1+1)(2u_1+1) + 2\Delta x_{1,2}\Delta_1$$

Be $x_2 > x_1$, $x_1 = x_2 - \Delta x_{1,2}$, and $(2x_2+1) = (2x_1+1)(2x_1'+1)$

$(\Delta x_{1,2} = x_2 - x_1)$

$$(2x_1+1)(2u_1+1) = (2x_2+1)(2u_2+1) + 2\Delta x_{1,2}(-\Delta_2+\Delta_1)$$

$$(2x_1+1)(2u_1+1) = (2x_1+1)(2x_1'+1)(2u_2+1) + 2\Delta x_{1,2}(-\Delta_2+\Delta_1)$$

$$(2u_1+1) = (2x_1'+1)(2u_2+1) + \underbrace{\frac{2\Delta x_{1,2}(-\Delta_2+\Delta_1)}{2x_1+1}}_{1}$$

$$1 = \frac{2\cdot(x_2-x_1)(-\Delta_2+\Delta_1)}{2x_1+1}$$

$$= \frac{2\cdot\left(((2x_1+1)(2x_1'+1)-1)\frac{1}{2} - x_1\right)(-\Delta_2+\Delta_1)}{2x_1+1}$$

$$= \frac{((2x_1+1)(2x_1'+1) - 1 - 2x_1)(-\Delta_2+\Delta_1)}{(2x_1+1)}$$

$$= \frac{\cancel{(2x_1+1)}(2x_1'+1)(-\Delta_2+\Delta_1)}{\cancel{(2x_1+1)}} - \frac{(2x_1+1)(-\Delta_2+\Delta_1)}{(2x_1+1)}$$

$$= (2x_1'+1)(-\Delta_2+\Delta_1) - (-\Delta_2+\Delta_1)$$

$$= (-\Delta_2+\Delta_1)(2x_1'+1-1)$$

$$= (-\Delta_2+\Delta_1)\,2x_1'$$

$$\Rightarrow \boxed{2u_1+1 = (2x_1'+1)(2u_2+1) + (-\Delta_2+\Delta_1)\cdot 2x_1'}$$

$$= 2x_1'u_2\cdot 2 + 2x_1' + 2u_2 + \cancel{1} + (-\Delta_2+\Delta_1)\cdot 2x_1'$$

$$2u_1 = 2^2x_1'u_2 + 2x_1' + 2u_2 + (-\Delta_2+\Delta_1)\cdot 2x_1'$$

$$\boxed{u_1 = 2x_1'u_2 + x_1' + u_2 + (-\Delta_2+\Delta_1)x_1'}$$

$$\boxed{u_1 = \underbrace{2x_1'u_2 + u_2}_{(2x_1'+1)u_2} + (-\Delta_2+\Delta_1+1)x_1'}$$

$$0 = (2x_2+1)(2u_2+1) - 2\Delta x_{1,2}\Delta_2$$
$$-(2x_1+1)(2u_1+1) + 2\Delta x_{1,2}\Delta_1$$

Be

$\underline{x_1 = x_2 - \Delta x_{1,2}}$

$$0 = (2x_2+1)(2u_2+1) - 2\Delta x_{1,2}\Delta_2$$
$$-(2(x_2-\Delta x_{1,2})+1)(2u_1+1) + 2\Delta x_{1,2}\Delta_1$$

$$0 = (2x_2+1)(2u_2+1) - 2\Delta x_{1,2}\Delta_2$$
$$-(2x_2+1)(2u_1+1) + 2\Delta x_{1,2}(2u_1+1)$$

Or $\underline{x_2 = x_1 + \Delta x_{1,2}}$

$$0 = (2(x_1+\Delta x_{1,2})+1)(2u_2+1) - 2\Delta x_{1,2}\Delta_2$$
$$-(2x_1+1)(2u_1+1) + 2\Delta x_{1,2}\Delta_1$$

$$0 = (2x_1+1)(2u_2+1) + 2\Delta x_{1,2}(2u_2+1) - 2\Delta x_{1,2}\Delta_2$$
$$-\underline{(2x_1+1)(2u_1+1)} + 2\Delta x_{1,2}\Delta_1$$

$$(\cancel{2x_1+1})(2u_1+1) = (\cancel{2x_1+1})(2u_2+1) + 2\Delta x_{1,2}(2u_2+1) + 2\Delta x_{1,2}(-\Delta_2+\Delta_1)$$
$$(2u_1+1) = (2u_2+1) + \frac{2\Delta x_{1,2}(2u_2+1) + 2\Delta x_{1,2}(-\Delta_2+\Delta_1)}{2x_1+1}$$

$$= (2u_2+1) + \underset{x_2-x_1}{\underset{\uparrow}{2\Delta x_{1,2}}}\,\frac{(2u_2+1)+(-\Delta_2+\Delta_1)}{2x_1+1}$$

$\underline{\text{Be } u_2 = (-\Delta_2+\Delta_1+1)(2x_1y_1 + x_1 + y_1):}$

$$\cancel{= (2u_2+1) + 2\Delta x_{1,2}}$$

$$\left(= (2u_2+1) + \underbrace{\frac{2(x_2-x_1)(2u_2+1) + (-\Delta_2+\Delta_1)}{2x_1+1}}_{\frac{2x_2 2u_2 + 2x_2 - 2u_2 x_1 - x_1 + (-\Delta_2+\Delta_1)}{2x_1+1}} \right)$$

$$= (2u_2+1) + 2\Delta x_{1,2}\,\frac{\cancel{2}\cdot(-\Delta_2+\Delta_1+1)\overbrace{(2x_1y_1+x_1+y_1)}^{((2x_1+1)(2y_1+1)-1)\cancel{\frac{1}{2}}} + 1 + (-\Delta_2+\Delta_1)}{2x_1+1}$$

$$= (2u_2+1) + 2\Delta x_{1,2}\cdot\frac{(-\Delta_2+\Delta_1+1)(\cancel{2x_1+1})(2y_1+1) - \cancel{(-\Delta_2+\Delta_1+1)} + \overset{(-\Delta_2+\Delta_1+1)}{\cancel{(1+\ldots)}}}{(\cancel{2x_1+1})}$$

$$= (2u_2+1) + 2\Delta x_{1,2}\cdot(-\Delta_2+\Delta_1+1)(2y_1+1)$$

✓

~~Be~~

$$\cancel{x_1 = x_2 + \Delta x_{1,2} \quad , \quad \Delta x_{1,2} \in \mathbb{Z}^- \quad , \quad x_2 > x_1}$$

$$\cancel{0 = (2x_2+1)(2u_2+1) + 2\Delta x_{1,2}\Delta_2}$$

$$\cancel{-(2x_2+1)(2u_1+1)}$$

$$\boxed{x_1 = x_2 - \Delta x_{1,2}}$$

$$0 = (2x_2+1)(2u_2+1) - 2\Delta x_{1,2}\Delta_2$$

$$-(2x_1+1)(2u_1+1) + 2\Delta x_{1,2}(2u_1+1) + 2\Delta x_{1,2}\Delta_1$$

$$\boxed{\text{Be } (2x_2+1) = (2x_1+1)(2x_1'+1)}$$

$$0 = \overset{(2x_1+1)(2x_1'+1)}{\cancel{(2x_2+1)}}(2u_2+1) - 2\Delta x_{1,2}\Delta_2$$

$$-(2x_1+1)(2x_1'+1)(2u_1+1) + 2\Delta x_{1,2}(2u_1+1) + 2\Delta x_{1,2}\Delta_1$$

$$(2x_1+1)(2x_1'+1)(2u_1+1)$$

$$\cancel{= (2x_2+1)(2u_2+1) - 2\Delta x_{1,2}}$$

$$= (2x_1+1)(2x_1'+1)(2u_2+1) - 2\Delta x_{1,2}\Delta_2$$

$$+ 2\Delta x_{1,2}(2u_1+1) + 2\Delta x_{1,2}\Delta_1$$

$$2u_1+1 = (2u_2+1) + 2\Delta x_{1,2}\frac{(-\Delta_2+\Delta_1) + (2u_1+1)}{(2x_1+1)(2x_1'+1)}$$

$$\boxed{\text{Be } u_1 = (-\Delta_2+\Delta_1+\alpha)\underbrace{(2x_1x_1' + x_1 + x_1')}_{x_2} + \gamma}$$

$$\Rightarrow 2u_1+1 = (2u_2+1) + 2\Delta x_{1,2}\frac{(-\Delta_2+\Delta_1) + 2\cdot(-\Delta_2+\Delta_1+\alpha)(2x_1x_1'+x_1+x_1') + 2\gamma+1}{(2x_1+1)(2x_1'+1)}$$

$$= (2u_2+1) + 2\Delta x_{1,2}\frac{(-\Delta_2+\Delta_1) + 2\cdot(-\Delta_2+\Delta_1)(2x_1x_1'+x_1+x_1') + 2\alpha(2x_1x_1'+x_1+x_1')}{(2x_1+1)(2x_1'+1)}$$

$$+ \frac{2\gamma+1}{(2x_1+1)(2x_1'+1)}$$

$$= (2u_2+1) + 2\Delta x_{1,2}(-\Delta_2+\Delta_1)\frac{1+2(2x_1x_1'+x_1+x_1')}{(2x_1+1)(2x_1'+1)}$$

$$+ 2\Delta x_{1,2}\frac{2\alpha(2x_1x_1'+x_1+x_1') + 2\gamma+1}{(2x_1+1)(2x_1'+1)}$$

$(2x_1+1)(2x_1'+1)$

$= 2\cdot(2x_1x_1'+x_1+x_1')+1$

$$= (2u_2+1) + 2\Delta x_{1/2}(-\Delta_2+\Delta_1)\cdot 1$$

$$+ 2\Delta x_{1/2}\,\frac{2\cdot\alpha(2x_1x_1'+x_1+x_1')+2\gamma+1}{(2x_1+1)(2x_1'+1)} \quad \Rightarrow \alpha=1 \quad \Rightarrow \gamma=0$$

$$= (2u_2+1) + 2\Delta x_{1/2}(-\Delta_2+\Delta_1) + 2\Delta x_{1/2}\cdot 1$$

$$= (2u_2+1) + 2\Delta x_{1/2}(-\Delta_2+\Delta_1+1)$$

$$\Rightarrow \boxed{u_1 = (-\Delta_2+\Delta_1+1)\underbrace{(2x_1x_1'+x_1+x_1')}_{=\left[(2x_1+1)(2x_1'+1)\frac{1}{2}-1\right]}}$$

$$\boxed{\text{Be } u_1 = (-\Delta_2+\Delta_1+\alpha)(2x_1y_1+x_1+y_1)+\gamma}$$

$$\Rightarrow 2u_1+1 = (2u_2+1)+2\Delta x_{1/2}\,\frac{(-\Delta_2+\Delta_1)+2\cdot(-\Delta_2+\Delta_1+\alpha)(2x_1y_1+x_1+y_1)+2\gamma+1}{(2x_1+1)(2x_1'+1)}$$

$$= (2u_2+1)+2\Delta x_{1/2}\,\frac{(-\Delta_2+\Delta_1)+2\cdot(-\Delta_2+\Delta_1)(2x_1y_1+x_1+y_1)+2\alpha(2x_1y_1+x_1+y_1)+2\gamma+1}{(2x_1+1)(2x_1'+1)}$$

$$= (2u_2+1)+2\Delta x_{1/2}(-\Delta_2+\Delta_1)\,\frac{1+2\cdot(2x_1y_1+x_1+y_1)}{(2x_1+1)(2x_1'+1)}$$

$$+2\Delta x_{1/2}\,\frac{2\alpha(2x_1y_1+x_1+y_1)+2\gamma+1}{(2x_1+1)(2x_1'+1)}$$

$$= (2u_2+1)+2\Delta x_{1/2}(-\Delta_2+\Delta_1)\,\frac{2\cdot((2x_1+1)(2y_1+1)-1)\frac{1}{2}+1}{(2x_1+1)(2x_1'+1)}$$

$$+2\Delta x_{1/2}\,\frac{2\alpha((2x_1+1)(2y_1+1)-1)\frac{1}{2}+2\gamma+1}{(2x_1+1)(2x_1'+1)}$$

$$= (2u_2+1) + 2\Delta x_{1,2}\,(-\Delta_2+\Delta_1)\,\frac{(2x_1+1)(2y_1+1)-1+1}{(2x_1+1)(2x_1'+1)}$$

$$+\,2\Delta x_{1,2}\,\frac{\alpha\,(2x_1+1)(2y_1+1)+2\gamma+1-\alpha}{(2x_1+1)(2x_1'+1)}$$

$\boxed{\text{Be } \gamma=0 \text{ and } \alpha=1}$

$$= (2u_2+1)+2\Delta x_{1,2}(-\Delta_2+\Delta_1)\,\frac{\cancel{(2x_1+1)}(2y_1+1)}{\cancel{(2x_1+1)}(2x_1'+1)}$$

$$+\,2\Delta x_{1,2}\,\frac{\cancel{(2x_1+1)}(2y_1+1)}{\cancel{(2x_1+1)}(2x_1'+1)}$$

$$= (2u_2+1)+2\Delta x_{1,2}\,(-\Delta_2+\Delta_1)\,\frac{2y_1+1}{2x_1'+1}+2\Delta x_{1,2}\,\frac{2y_1+1}{2x_1'+1}$$

$$= (2u_2+1)+2\Delta x_{1,2}\,\frac{2y_1+1}{2x_1'+1}\,(-\Delta_2+\Delta_1+1)$$

$x_1 = x_2-\Delta x_{1,2}$ $\qquad x_1+\Delta x_{1,2}=x_2=((2x_1+1)(2x_1'+1)-1)\frac{1}{2}$

$\Delta x_{1,2}=x_2-x_1$

$$= (2u_2+1)+2\cdot(x_2-x_1)\,\frac{2y_1+1}{2x_1'+1}\,(-\Delta_2+\Delta_1+1)$$

$$= (2u_2+1)+2\cdot\left[((2x_1+1)(2x_1'+1)-1)\frac{1}{2}-x_1\right]\frac{2y_1+1}{2x_1'+1}\,(-\Delta_2+\Delta_1+1)$$

$$= (2u_2+1)+\left[(2x_1+1)(2x_1'+1)-1-2x_1\right]\frac{2y_1+1}{2x_1'+1}\,(-\Delta_2+\Delta_1+1)$$

$$= (2u_2+1)+\frac{(2x_1+1)(2x_1'+1)(2y_1+1)-(2x_1+1)(2y_1+1)}{2x_1'+1}\,(-\Delta_2+\Delta_1+1)$$

$$= (2u_2+1)+\left[(2x_1+1)(2y_1+1)-\frac{(2x_1+1)(2y_1+1)}{2x_1'+1}\right]\cdot(-\Delta_2+\Delta_1+1)$$

$$= (2u_2+1)+(2x_1+1)(2y_1+1)\cdot(-\Delta_2+\Delta_1+1)$$

$$\underbrace{-\frac{(2x_1+1)(2y_1+1)}{2x_1'+1}}$$

$$-\frac{(2x_1+1)\cancel{(2x_1'+1)}(2\hat{y}_1+1)}{\cancel{2x_1'+1}}$$

$$= -(2x_1+1)(2\hat{y}_1+1)$$

Be $y_1 = 2x_1'\hat{y}_1+x_1'+\hat{y}_1$

$= ((2x_1'+1)(2\hat{y}_1+1)-1)\frac{1}{2}$

$2y_1+1 = (2x_1'+1)(2\hat{y}_1+1)$

$= 2(2x_1'\hat{y}_1'+x_1'+\hat{y}_1)+1$

$$\Rightarrow K_2 = (-\Delta_2+\Delta_1+1)\big(2x_1(2x_1'\tilde{y}_1 + x_1' + \tilde{y}_1) + x_1 + (2x_1'\tilde{y}_1 + x_1' + \tilde{y}_1)\big)$$

$\tilde{y}_1 \in \mathbb{N}$

$$= (-\Delta_2+\Delta_1+1)\big(2^2 x_1 x_1'\tilde{y}_1 + 2x_1x_1' + 2\tilde{y}_1 + x_1 + 2x_1'\tilde{y}_1 + x_1' + \cancel{\tilde{y}_1}\big)$$

$$K_2 = (-\Delta_2+\Delta_1+1)\big(2^2 x_1 x_1'\tilde{y}_1 + 2x_1x_1' + 3\tilde{y}_1 + 2x_1'\tilde{y}_1 + x_1 + x_1'\big)$$

$$= (-\Delta_2+\Delta_1+1)(2x_1y_1 + x_1 + y_1)$$

$$= (-\Delta_2+\Delta_1+1)\big((2x_1+1)(2y_1+1)-1\big)\tfrac{1}{2}$$

$$= (-\Delta_2+\Delta_1+1)\Big[(2x_1+1)\big(\cancel{2}((2x_1'+1)(2\tilde{y}_1+1)-1)\cancel{\tfrac{1}{2}}+1\big)-1\Big]\tfrac{1}{2}$$

$$= (-\Delta_2+\Delta_1+1)\Big[(2x_1+1)\big((2x_1'+1)(2\tilde{y}_1+1)\cancel{-1}\cancel{+1}\big)-1\Big]\tfrac{1}{2}$$

$$K_2 = (-\Delta_2+\Delta_1+1)\big[(2x_1+1)(2x_1'+1)(2\tilde{y}_1+1)-1\big]\tfrac{1}{2}$$

$\cancel{= (-\Delta_2+\Delta_1+1)}$

$$\begin{cases} \text{I} & \overline{n}_1 = (2x_1+1)(2u_1+1) - 2\Delta x_{1/2}\Delta_1 \\ \text{II} & \overline{n}_2 = (2x_2+1)(2u_2+1) - 2\Delta x_{1/2}\Delta_2 \end{cases} \quad \text{(eq.1)}$$

Be $x_2 > x_1$ $\quad (x_2 = x_1 + \Delta x_{1/2})$

$x_1 = x_2 - \Delta x_{1/2}$

$\Delta x_{1/2} = x_2 - x_1$

1. Substitute: $x_2 = x_1 + \Delta x_{1/2}$

$$\text{(eq2)} \quad 0 = (2x_2+1)(2u_2+1) - (2x_1+1)(2u_1+1) + 2\Delta x_{1/2}(-\Delta_2+\Delta_1)$$

$$0 = (2(x_1+\Delta x_{1/2})+1)(2u_2+1) - (2x_1+1)(2u_1+1) + 2\Delta x_{1/2}(-\Delta_2+\Delta_1)$$

$$= (2x_1+1)(2u_2+1) + 2\Delta x_{1/2}(2u_2+1) - (2x_1+1)(2u_1+1) + 2\Delta x_{1/2}(-\Delta_2+\Delta_1)$$

$$(2x_1+1)(2u_1+1) = (2x_1+1)(2u_2+1) + 2\Delta x_{1/2}(2u_2+1) + 2\Delta x_{1/2}(-\Delta_2+\Delta_1)$$

$$\text{(eq3.)} \quad (2u_1+1) = (2u_2+1) + 2\Delta x_{1/2} \frac{\cancel{(2u_2+1) + 2\Delta x_{1/2}}\ (2u_2+1) + (-\Delta_2+\Delta_1)}{(2x_1+1)}$$

Case: $\Delta x_{1/2} = n \cdot (2x_1+1)$ $\quad (n \in \mathbb{N})$ $\quad$ (That is equal to $(2x_2+1) = (2x_1+1)(2n+1)$)

$$(2u_1+1) = (2u_2+1) + 2 \cdot n \cdot \cancel{(2x_1+1)} \frac{(2u_2+1) + (-\Delta_2+\Delta_1)}{\cancel{2x_1+1}}$$

$$(2u_1+1) = (2u_2+1) + 2 \cdot n \cdot (2u_2+1) + (-\Delta_2+\Delta_1)$$

Solve this for u_2: $(2u_2+1)$

$$(2u_1+1) = (2u_2+1)(2n+1) + (-\Delta_2+\Delta_1)$$

$$(2u_2+1) = \frac{(2u_1+1) - (-\Delta_2+\Delta_1)}{\cancel{2u_1+\Delta_1}\ 2n+1}$$

We need integer solutions for this!

$\Rightarrow$ Be $u_1 = (-\Delta_2+\Delta_1-1)(2ny_1+n+y_1)$

$$(2u_2+1) = \frac{2 \cdot (-\Delta_2+\Delta_1-1)(2ny_1+n+y_1) + 1 - (-\Delta_2+\Delta_1)}{2n+1}$$

$$= \frac{2 \cdot (-\Delta_2+\Delta_1-1)((2n+1)(2y_1+1)-1)\frac{1}{2} + 1 - (-\Delta_2+\Delta_1)}{2n+1}$$

③ 2/7 ×2

$$(2u_2+1) = \frac{\cancel{2}\cdot(-\Delta_2+\Delta_1-1)(2n+1)(2y_1+1)\cdot\cancel{\tfrac{1}{2}} - \cancel{2}\cdot(-\Delta_2+\Delta_1-1)\cdot\cancel{\tfrac{1}{2}}+1-(-\Delta_2+\Delta_1)}{2n+1}$$

$$= -(-\Delta_2+\Delta_1-1) - (-\Delta_2+\Delta_1-1)$$

$$= (-\Delta_2+\Delta_1-1)(2y_1+1) + \frac{-(-\Delta_2+\Delta_1-1)+1-(-\Delta_2+\Delta_1)}{2n+1}$$

$$= (-\Delta_2+\Delta_1-1)(2y_1+1) - 2\,\frac{(-\Delta_2+\Delta_1-1)}{2n+1} \quad ↯$$

Or Be: $u_1 = -(-\Delta_2+\Delta_1-1)(2ny_1+n+y_1)$

$$(2u_2+1) = \frac{-\cancel{2}(-\Delta_2+\Delta_1-1)(2n+1)(2y_1+1)\cancel{\tfrac{1}{2}} + 2(-\Delta_2+\Delta_1-1)\tfrac{1}{2}+1-(\Delta_2+\Delta_1)}{2n+1}$$

$$= -(-\Delta_2+\Delta_1-1)(2y_1+1) + \frac{\overbrace{(-\Delta_2+\Delta_1-1) - (-\Delta_2+\Delta_1-1)}^{=0}}{2n+1}$$

$$(2u_2+1) = -(-\Delta_2+\Delta_1-1)(2y_1+1)$$

$$u_2 = \left(-(-\Delta_2+\Delta_1-1)(2y_1+1) - 1\right)\tfrac{1}{2}$$

$$= -(-\Delta_2+\Delta_1-1)\left((2y_1+1)-1\right)\tfrac{1}{2}$$

$$u_2 = -(-\Delta_2+\Delta_1-1)\, y_1 \quad \text{arbitrary!!}$$

(can e.g. be ~~$y_1 = 2x_1y_1 + x_1+y_1$~~ $y_1 = 2x_1\alpha_1 + x_1 + \alpha_1$)

Case: $\Delta x_{1/2} \neq n\cdot(2x_1+1)$:

$$\text{(eq.3)} \quad (2u_1+1) = (2u_2+1) + 2\Delta x_{1/2}\,\frac{(2u_2+1)+(-\Delta_2+\Delta_1)}{(2x_1+1)}$$

Be $u_2 = (-\Delta_2+\Delta_1+1)(2x_1y_1+x_1+y_1)$

$$= (-\Delta_2+\Delta_1+1)\left((2x_1+1)(2y_1+1)-1\right)\tfrac{1}{2}$$

$$\Rightarrow (2u_1+1) = (2u_2+1) + 2\Delta x_{1/2}\,\frac{2\cdot(-\Delta_2+\Delta_1+1)\cancel{((2x_1+1)(2y_1+1)-1)}^{2x_1y_1+x_1+y_1}\tfrac{1}{2}+1+(-\Delta_2+\Delta_1)}{(2x_1+1)}$$

$$\cancel{= (2u_2+1) + 2\Delta x_{1/2}\,\frac{(-\Delta_2+\Delta_1+1)(2x_1+1)(2y_1+1) - (-\Delta_2+\Delta_1+1) + 1 + (-\Delta_2+\Delta_1)}{2x_1+1}}$$

$$\cancel{= (2u_2+1) + 2\Delta x_{1/2}\left[(-\Delta_2+\Delta_1+1)(2y_1+1) + \frac{-(-\Delta_2+\Delta_1+1)}{}\right.}$$

③ 3/7 x3

$$\Rightarrow (2u_1+1) = (2u_2+1) + 2\Delta x_{1/2} \frac{2\cdot(-\Delta_2+\Delta_1+1)(2x_1y_1+x_1+y_1)+(1+(-\Delta_2+\Delta_1)\cdot)}{2x_1+1}$$

$$(2u_1+1) = (2u_2+1) + 2\Delta x_{1/2} \cdot \frac{2\cdot(-\Delta_2+\Delta_1)(2x_1y_1+x_1+y_1)+2(2x_1y_1+x_1+y_1)}{2x_1+1}$$

$$+ 2\Delta x_{1/2} \cdot \frac{(1+1-\Delta_2+\Delta_1)}{2x_1+1}$$

$$= (2u_2+1) + 2\Delta x_{1/2}\left[\cancel{(-\Delta_2+1)}(-\Delta_2+\Delta_1+1)\frac{2\cdot(2x_1y_1+x_1+y_1)+1}{2x_1+1}\right]$$

$$= (2u_2+1) + 2\Delta x_{1/2}(-\Delta_2+\Delta_1+1)\frac{2\cdot[(2x_1+1)(2y_1+1)-1]\cdot\frac{1}{2}+1}{2x_1+1}$$

$$= (2u_2+1) + 2\Delta x_{1/2}(-\Delta_2+\Delta_1+1)\frac{(2x_1+1)(2y_1+1) \cancel{-1+1}}{2x_1+1}$$

$$\boxed{(2u_1+1) = (2u_2+1) + 2\Delta x_{1/2}(-\Delta_2+\Delta_1+1)(2y_1+1)}$$

$$(2u_1+1) = 2\cdot[(2x_1+1)(2y_1+1)-1]\frac{1}{2}+1 + 2(x_2-x_1)(-\Delta_2+\Delta_1+1)(2y_1+1)$$

$$2u_1+1 = (2x_1+1)(2y_1+1) \cancel{-1+1} + 2(x_2-x_1)(-\Delta_2+\Delta_1+1)(2y_1+1)$$

$$(2u_1+1) = \cancel{(2x_1+1)(2y_1+1) + 2x_2(-\Delta_2+\Delta_1+1)(2y_1+1)}$$

$$= (2y_1+1)[(2x_1+1) + 2(x_2-x_1)(-\Delta_2+\Delta_1+1)]$$

$$\left(u_1 = \{(2y_1+1)[(2x_1+1) + 2(x_2-x_1)(-\Delta_2+\Delta_1+1)]-1\}\tfrac{1}{2}\right)$$

$$(2u_1+1) = (2y_1+1)2x_1 + (2y_1+1) + (2x_2-2x_1)(-\Delta_2+\Delta_1+1)$$

$$= (2y_1+1)[\underbrace{(2x_1+1) + 2(x_2-x_1)(-\Delta_2+\Delta_1+1)}]$$

$$= 2x_1+1 + 2x_2(-\Delta_2+\Delta_1+1) - 2x_1(-\Delta_2+\Delta_1+1)$$

$$=$$

$$(2u_1+1) = 2\cdot(-\Delta_2+\Delta_1+1)(2x_1y_1+x_1+y_1)+1+\cancel{2\Delta x_{1/2}}\ 2(x_2-x_1)(-\Delta_2+\Delta_1+1)(2y_1+1)$$

$$= (-\Delta_2+\Delta_1+1)[2\underbrace{(2x_1y_1+x_1+y_1)}_{=[(2x_1+1)(2y_1+1)-1]\frac{1}{2}} + 2(x_2-x_1)(2y_1+1)]+1$$

$$= (-\Delta_2+\Delta_1+1)[(2x_1+1)(2y_1+1)-1 + 2(x_2-x_1)(2y$$

③ 4/7 ×4

$$(2u_1+1) = (2u_2+1) + 2\Delta x_{1,2}(-\Delta_2+\Delta_1+1)(2y_1+1)$$

$$= \left\{2\left((-\Delta_2+\Delta_1+1)\left((2x_1+1)(2y_1+1)-1\right)\tfrac{1}{2}\right)+1\right\}$$

$$+ 2(x_2-x_1)(-\Delta_2+\Delta_1+1)(2y_1+1)$$

$$= (-\Delta_2+\Delta_1+1)(2x_1+1)(2y_1+1) - (-\Delta_2+\Delta_1+1) + 1$$

$$+ 2x_2(-\Delta_2+\Delta_1+1)(2y_1+1) - 2x_1(-\Delta_2+\Delta_1+1)$$

$$= (-\Delta_2+\Delta_1+1)\left[(2x_1+1)(2y_1+1) - 1 + 2x_2 - 2x_1\right] + 1$$

$$(2u_1+1) = (-\Delta_2+\Delta_1+1)\left[2(2x_1y_1+x_1+y_1)\cancel{+1}\cancel{-1} + 2x_2 - 2x_1\right] + 1$$

$$= (-\Delta_2+\Delta_1+1)\left[2^2x_1y_1 + \cancel{2x_1} + 2y_1 + 2x_2 - \cancel{2x_1}\right] + 1$$

$$= (-\Delta_2+\Delta_1+1)\left[2^2x_1y_1 + 2y_1 + 2x_2\right] + 1$$

$$= (-\Delta_2+\Delta_1+1)\left[2\cdot(2x_1y_1+y_1+x_2)\right] + 1$$

?

$$\boxed{\text{Be } (2u_2+1) = (-\Delta_2+\Delta_1)\cdot 2\cdot(2x_1y_1+x_1+y_1)} + Z\cdot(2x_1+1)$$

$$\Rightarrow (\text{eq.}3)\quad (2u_1+1) = (2u_2+1) + 2\Delta x_{1,2}\,\frac{(2u_2+1)+(-\Delta_2+\Delta_1)+Z\cdot(2x_1+1)}{2x_1+1}$$

$$= (2u_2+1) + 2\Delta x_{1,2}\,\frac{(-\Delta_2+\Delta_1)\cdot 2\cdot(2x_1y_1+x_1+y_1)+(-\Delta_2+\Delta_1) + Z(2x_1+1)}{2x_1+1}$$

$$= (2u_2+1) + 2\Delta x_{1,2}\cdot(-\Delta_2+\Delta_1)\,\frac{2\cdot\left((2x_1+1)(2y_1+1)-1\right)\tfrac{1}{2}+1 + Z(2x_1+1)}{2x_1+1}$$

$$= (2u_2+1) + 2\Delta x_{1,2}(-\Delta_2+\Delta_1)\,\frac{(2x_1+1)(2y_1+1) + Z(2x_1+1)}{(2x_1+1)}$$

$$\boxed{(2u_1+1) = (2u_2+1) + 2\Delta x_{1,2}(-\Delta_2+\Delta_1)\left((2y_1+1) + Z\right)} + Z2\Delta x_{1,2}$$

$Z(2x_1+1)$

$$\cancel{(2u_1+1) = 2\cdot(-\Delta_2+\Delta_1)\cdot 2(2x_1y_1+x_1+y_1)+1+2(x_2-x_1)(-\Delta_2+\Delta_1)\cdot(2y_1+1)}$$

③ x^5 5/7

$$(2u_1+1) = (-\Delta_2+\Delta_1)\cdot 2\left[\cancel{2}\left((2x_1+1)(2y_1+1)-1\right)\cancel{\tfrac{1}{2}} + (x_2-x_1)(2y_1+1)\right] + 1$$

$$= (-\Delta_2+\Delta_1)\cdot 2\left[(2x_1+1)(2y_1+1) - 1 + (x_2-x_1)(2y_1+1)\right]+1$$

$$= (-\Delta_2+\Delta_1)\cdot 2\left[(2y_1+1)(2x_1+1+x_2-x_1)-1\right]+1$$

$$= (-\Delta_2+\Delta_1)\cdot 2\cdot\left[(2y_1+1)(x_1+x_2+1)-1\right]+1$$

$= x_2 - \Delta x_{1,2}$

$$(2u_1+1) = (-\Delta_2+\Delta_1)\cdot 2\cdot(2x_2y_2+x_2+y_2)$$

$x_2 = x_1 + \Delta x_{1,2}$

$$(2u_1+1) = (2u_2+1) + 2\underset{(x_2-x_1)}{\Delta x_{1,2}}(-\Delta_2+\Delta_1)(2y_1+1)$$

$$= (-\Delta_2+\Delta_1)\cdot 2(2x_1y_1+x_1+y_1)$$

$$+ 2\cdot(x_2-x_1)(-\Delta_2+\Delta_1)(2y_1+1)$$

$$= (-\Delta_2+\Delta_1)\left[2\cdot\underbrace{(2x_1y_1+x_1+y_1)}_{=((2x_1+1)(2y_1+1)-1)\frac{1}{2}} + 2(x_2-x_1)(2y_1+1)\right]$$

be $(2x_1+1)\cdot\beta$

$$= (-\Delta_2+\Delta_1)\left[(2x_1+1)(2y_1+1)-1+2\widehat{(x_2-x_1)}(2y_1+1)\right]$$

$2\cdot 2x_1y_1 + 2x_1 + 2y_1 + 1 - 1 + 2\cdot(2x_1+1)\beta(2y_1+1)$

$$= (-\Delta_2+\Delta_1)\left[\underline{2\cdot 2x_1y_1} + \cancel{2x_1} + \underline{2y_1} + \cancel{1} - \cancel{1} + \underline{2x_2 2y_1} + \underline{2x_2} - 2^2x_1y_1 - \cancel{2x_1}\right]$$

$$= (-\Delta_2+\Delta_1)\left[2^2\cancel{x_1y_1} + 2y_1 + 2^2x_2y_1 + 2x_2 - \cancel{2^2x_1y_1}\right]$$

$$= (-\Delta_2+\Delta_1)\left[2y_1 + 2^2x_2y_1 + 2x_2\right]$$

$$\boxed{(2u_1+1) = (-\Delta_2+\Delta_1)\cdot 2(2x_2y_1+x_2+y_1)}$$ ✓✓✓

$+\cancel{1}\cdot(2x_2+1)$

$$= (-\Delta_2+\Delta_1)\left[\underline{2\cdot 2x_1y_1}+\underline{2x_1}+\underline{2y_1}+\cancel{1}-\cancel{1}+ 2^3x_1\beta y_1+2^2x_1\beta + 2^2\beta y_1+2\beta\right]$$

$$= (-\Delta_2+\Delta_1)\left[2\cdot 2x_1y_1 + x_1(2+2^2\beta) + y_1(2+2^2\beta)+2^3x_1\beta y_1+2\beta\right]$$

$$= (-\Delta_2+\Delta_1)\cdot 2\left[2x_1y_1 + \underbrace{x_1(1+2\beta)}_{=x_1+2x_1\beta} + \underbrace{y_1(1+2\beta)}_{=y_1+2y_1\beta}+2^2x_1\beta y_1+\beta\right]$$

NR:

$$2\cdot x_2y_1+x_2+y_1$$
$$= 2\cdot(2x_1+1)\beta y_1 + (2x_1+1)\beta + y_1$$
$$= 2^2x_1\beta y_1 + 2\beta y_1 + 2x_1\beta+\beta+y_1$$

$$(2x_1+1)y_1 = 2x_1y_1+y_1$$

$$= (-\Delta_2+\Delta_1)\cdot 2\left[(2x_2y_1+x_2+y_1) + 2x_1y_1+x_1\right]$$

$$= (-\Delta_2+\Delta_1)\cdot 2\left[(2x_2y_1+x_2+y_1) + (2y_1+1)x_1\right]$$

For $\Delta x_{1/2} = (2x_1+1)\beta$ follows

$$(2u_1+1) = (-\Delta_2+\Delta_2)\cdot 2\,\underbrace{(2x_2y_1+x_2+y_1)}_{((2x_2+1)(2y_1+1)-1)\frac{1}{2}} + \underbrace{2\cdot(2y_1+1)x_1}_{2\cdot(2y_1x_1+x_1)}$$

$$= 2\cdot((2x_1+1)y_1-y_1)$$
$$= 2\cdot(2x_1+1)y_1-2y_1$$

$$= (-\Delta_2+\Delta_1)\left[(2x_2+1)(2y_1+1)-1+2\cdot(2y_1+1)x_1\right]$$

$$= (-\Delta_2+\Delta_1)\left[(2y_1+1)\left[\underbrace{(2x_2+1)+2x_1}_{2\cdot(x_2+x_1)+1}\right]-1\right]$$

$$\cancel{= (-\Delta_2+\Delta_1)\left[(2y_1+1)(2(x_1+1)+1)-1\right]}$$

$$\cancel{= (-\Delta_2+\Delta_1)\left[2^2y_1(x_1+1)+2y_1+2(x_1+1)+1-1\right]}$$

$$\cancel{= (-\Delta_2+\Delta_1)\cdot 2\cdot(2y_1(x_1+1)+y_1+(x_1+1))}$$

=

$$(2u_1+1) = (-\Delta_2+\Delta_1)\cdot 2\cdot(2x_2y_1+\underset{\substack{(x_1+\Delta x_{1/2})\\ (2x_1+1)\beta}}{\overset{\substack{(2x_1+1)\beta\\ x_1+\Delta x_{1/2}}}{x_2}}+y_1)$$

$$(2x_2+1)y_1+(2x_1+1)y_1$$
$$= (2x_2+1+2x_1+1)y_1$$
$$= (2(x_2+x_1)+2)y_1$$
$$= 2((x_2+x_1)+1)+y_1$$

$$(2u_1+1) = (-\Delta_2+\Delta_1)\cdot 2\,(\underbrace{2x_2y_1+x_2}_{(2x_2+1)y_1}+y_1-y_1) + 2\cdot(2x_1+1)y_1$$

$$(2u_1+1)=(2u_2+1)+2\cdot \underset{u}{\Delta x_{1,2}}\;\frac{(2u_2+1)+(-\Delta_2+\Delta_1)}{2x_1+1}$$

$$\Delta x_{1,2}=n(2x_1+1)$$

$$=(2u_2+1)+2\cdot n\,(2u_2+1)+(-\Delta_2+\Delta_1)$$

$$=(2u_2+1)+2\cdot n\cdot(-\Delta_2+\Delta_1)\cdot 2\,(2x_1y_1+x_1+y_1)+(-\Delta_2+\Delta_1)$$

$$=(2u_2+1)+2\cdot(-\Delta_2+\Delta_1)\,\big(2n\,(2x_1y_1+x_1+y_1)+1\big)$$

$$=(2u_2+1)+2\cdot(-\Delta_2+\Delta_1)\,\big(\cancel{2}n\,((2x_1+1)(2y_1+1)-1)\,\cancel{\tfrac{1}{2}}+1\big)$$

$$=(2u_2+1)+2\cdot(-\Delta_2+\Delta_1)\cdot\big(n\cdot(2x_1+1)(2y_1+1)\,\cancel{-1+1}\big)$$

$$=(2u_2+1)+2\cdot(-\Delta_2+\Delta_1)\cdot n\cdot(2x_1+1)(2y_1+1)$$

$$(2u_1+1)=(2u_2+1)+2\cdot\big(n\cdot(2\underbrace{x_1}_{x_2-\Delta x_{1,2}}+1)\big)\,(-\Delta_2+\Delta_1)\cdot(2y_1+1)$$

Be $(2u_2+1)=(-\Delta_2+\Delta_1)\cdot 2\cdot(2x_1y_1+x_1+y_1)+z(2x_1+1)$

$$(2u_1+1)=(2u_2+1)+2\Delta x_{1,2}\;\frac{(-\Delta_2+\Delta_1)\cdot 2\,\overbrace{(2x_1y_1+x_1+y_1)}^{((2x_1+1)(2y_1+1)-1)\frac{1}{2}}+z(2x_1+1)\;\cancel{+(-\Delta_2+\Delta_1)}}{2x_1+1}$$

$$=(2u_2+1)+2\Delta x_{1,2}\;\cancel{(-\Delta_2+\Delta_1)(2x_1+1)(2y_1+1)}$$

$$(-\Delta_2+\Delta_1)\;\frac{\cancel{2\cdot((2x_1+1)(2y_1+1)-1)\tfrac{1}{2}+1+\dots}}{2x_1+1}$$

$$+\,2\Delta x_{1,2}\;\frac{z(2x_1+1)}{2x_1+1}$$

$$=(2u_2+1)+2\Delta x_{1,2}\big((-\Delta_2+\Delta_1)(2y_1+1)+z\big)$$

NR: $2x_1+1+2\Delta x_{1,2}$
$=2\cdot(x_1+\Delta x_{1,2})+1$
$=2x_2+1$ ✓

$$(2u_1+1)=(2u_2+1)+2\Delta x_{1,2}\,(-\Delta_2+\Delta_1)(2y_1+1)+2\Delta x_{1,2}\,z$$

$$=(-\Delta_2+\Delta_1)\cdot 2\,(2x_1y_1+x_1+y_1)+z(2x_1+1)+2\Delta x_{1,2}\,(-\Delta_2+\Delta_1)(2y_1+1)+2\Delta x_{1,2}\,z$$

$=z(2x_2+1)$

$$(2u_1+1)=(-\Delta_2+\Delta_1)\cdot 2\,(2x_2y_1+x_2+y_1)+z\big((2x_1+1)+2\Delta x_{1,2}\big)$$