Research
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RESEARCH NOTES

Primes (part 01): Recursion basics

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Abstract

This notes give an overview over all necessary items for calculating prime numbers (only odds, we always ignore number 2) recursively. We start, with the assumption that we know that 3 and 5 are prime numbers. We define equations which gave us all numbers which not belong to their time tables, do an intersection of this, and hence determine the next prime numbers from this. So we can do the same thing again. Taking all our knowing prime numbers, 3, 5 and the new ones, can define equations for the numbers which not belong to this time tables, do an intersection of this, and hence get the new prime numbers.
This notes talks about all aspects which we have to consider to do so, and shows that we always have all necessary information to do this recursion without limitations. Since, we are also able to determine a closed analytical equation for the intersection of an arbitrary number of intersection, we are also able to do an recursive considering of it.
In future work, it is still necessary to think about a nicer describing equation of this intersection, to find a better presentation of the final recursion formula.

Content

I. Given be $n \in \mathbb{N}$ with $\boxed{n_i = (2x_i+1)y_i \quad , x_i, y_i \in \mathbb{N},}$ (eq. 1)
the set of the odd times tables. (without 1)
n_i

With this, we can write:

$$\boxed{\bar{n}_i = (2x_i+1)y_i - \Delta_i \quad , \Delta_i \in \{1,\dots,2x_i\}} \quad \text{(eq. 2)}$$
$\Delta_i \in \mathbb{N}$

the set of all numbers, which does not belong to the times table n_i.

Now, we assume that we have two different $\bar{n}_i$ equations.

$$\left.\begin{array}{ll} \text{I.} \quad \bar{n}_1 = (2x_1+1)y_1 - \Delta_1 & , \Delta_1 \in \{1,\dots,2x_1\} \\ \text{II.} \quad \bar{n}_2 = (2x_2+1)y_2 - \Delta_2 & , \Delta_2 \in \{1,\dots,2x_2\} \end{array}\right\} \text{(eq. 3)}$$

with $x_2 > x_1$: $x_2 = x_1 + \Delta x_{1,2}$.

Let's do the intersection:

$$\bar{n}_1 = \bar{n}_2$$
$$(2x_1+1)y_1 - \Delta_1 = (2x_2+1)y_2 - \Delta_2$$
$$0 = (2x_2+1)y_2 - (2x_1+1)y_1 - \Delta_2 + \Delta_1$$

with $x_2 = x_1 + \Delta x_{1,2}$

$\Rightarrow$
$$0 = (2(x_1+\Delta x_{1,2})+1)y_2 - (2x_1+1)y_1 - \Delta_2 + \Delta_1$$
$$0 = (2x_1+1)y_2 + 2\Delta x_{1,2} y_2 - (2x_1+1)y_1 - \Delta_2 + \Delta_1$$
$$0 = (2x_1+1)(y_2 - y_1) + 2\Delta x_{1,2} y_2 - \Delta_2 + \Delta_1$$

$\Rightarrow$
$$(2x_1+1)y_1 = (2x_1+1)y_2 + 2\Delta x_{1,2} y_2 - \Delta_2 + \Delta_1$$
$$y_1 = (2x_1+1)^{-1}\left((2x_1+1)y_2 + 2\Delta x_{1,2} y_2 - \Delta_2 + \Delta_1\right)$$
$$y_1 = y_2 + \frac{2\Delta x_{1,2} y_2 - \Delta_2 + \Delta_1}{2x_1+1}$$

Be $y_2 = x_1 \cdot (-\Delta_2 + \Delta_1) + (2x_1+1) z_{1,2}$ / $y_1 = x_2 \cdot (-\Delta_2 + \Delta_1) + (2x_2+1) z_{1,2}$

Test: $$\frac{2\Delta x_{1,2} x_1(-\Delta_2+\Delta_1) + (-\Delta_2+\Delta_1)}{2x_1+1} \quad \text{(eq. 4)}$$

Case: $\Delta x_{1,2} = 1$:

$$\Rightarrow \frac{2 \cdot 1 \cdot x_1(-\Delta_2+\Delta_1) + (-\Delta_2+\Delta_1)}{2x_1+1}$$

$$= \frac{2x_1(-\Delta_2+\Delta_1) + (-\Delta_2+\Delta_1)}{2x_1+1}$$

$$= \frac{(-\Delta_2+\Delta_1)\cdot(2x_1+1)}{2x_1+1}$$

$$= (-\Delta_2+\Delta_1) \quad ✓$$

Case: $\Delta x_{1,2} > 1$:

$$\Rightarrow \frac{2 \cdot \Delta x_{1,2} \cdot x_1(-\Delta_2+\Delta_1) + (-\Delta_2+\Delta_1)}{2x_1+1}$$

$= ?$, we see, in this case, we have a problem to find integer solutions.

let's Assume instead of (eq. 4), we have

New!

$$\frac{2\Delta x_{1,2}\, x_1(-\Delta_2+\Delta_1) + \Delta x_{1,2}(-\Delta_2+\Delta_1)}{2x_1+1} \qquad \text{(eq. 5)}$$

Now, we can solve our intersection:

$$y_1 = y_2 + \frac{2\Delta x_{1,2}\, y_2 + (-\Delta_2+\Delta_1)\cdot \Delta x_{1,2}}{2x_1+1}$$

with $y_2 := x_1(-\Delta_2+\Delta_1) + (2x_1+1)z_{1,2}$, $z_{1,2} \in \mathbb{Z}$

$y_1 = x_2 \cdot (-\Delta_2+\Delta_1) + (2x_2+1)z_{1,2}$

Test: $y_1 = x_1(-\Delta_2+\Delta_1) + (2x_1+1)z_{1,2} + \frac{2\Delta x_{1,2}\, x_1(-\Delta_2+\Delta_1)}{2x_1+1}$

$$+ \frac{2\Delta x_{1,2}(2x_1+1)z_{1,2}}{(2x_1+1)} + \frac{(-\Delta_2+\Delta_1)\Delta x_{1,2}}{2x_1+1}$$

$$= x_1(-\Delta_2+\Delta_1) + (2x_1+1)z_{1,2} + 2\Delta x_{1,2} z_{1,2} + \frac{(-\Delta_2+\Delta_1)\Delta x_{1,2}(2x_1+1)}{2x_1+1}$$

because of $x_2 = x_1 + \Delta x_{1,2}$

$$= x_2(-\Delta_2+\Delta_1) + (2x_2+1)z_{1,2} \quad ✓$$

So, we have our new equations:

New!

$$\boxed{\begin{aligned} &\mathrm{I}': \quad \bar{n}_1' = (2x_1+1)y_1' - \Delta x_{1,2}\Delta_1 \quad , \Delta_1 \in \{1,..,2x_1\} \\ &\mathrm{II}': \quad \bar{n}_2' = (2x_2+1)y_2' - \Delta x_{1,2}\Delta_2 \quad , \Delta_2 \in \{1,..,2x_2\} \end{aligned}} \quad \text{(eq. 6)}$$

For the intersection $\bar{n}_1' = \bar{n}_2$,

we have the solutions:

$$\boxed{\begin{aligned} &\mathrm{III}'. \quad y_1' = x_2\cdot(-\Delta_2+\Delta_1) + (2x_2+1)z_{1,2} \quad , z_{1,2} \in \mathbb{Z} \\ &\mathrm{IV}'. \quad y_2' = x_1\cdot(-\Delta_2+\Delta_1) + (2x_1+1)z_{1,2} \end{aligned}} \quad \text{(eq. 7)}$$

and hence finally:

$$\begin{aligned} \bar{n}_{1,2}' &= (2x_1+1)y_1' - \Delta x_{1,2}\Delta_1 \\ &= (2x_1+1)\left[x_2(-\Delta_2+\Delta_1) + (2x_2+1)z_{1,2}\right] - \Delta x_{1,2}\Delta_1 \end{aligned}$$

$$\boxed{\bar{n}_{1,2}' = (2x_1+1)(2x_2+1)z_{1,2} + \underbrace{(2x_1+1)(-\Delta_2+\Delta_1)x_2 - \Delta x_{1,2}\Delta_1}_{(*)}} \quad \text{(eq. 8}^1\text{)}$$

$$\begin{aligned} \bar{n}_{1,2}' &= (2x_2+1)y_2' - \Delta x_{1,2}\Delta_2 \\ &= (2x_2+1)\left[x_1\cdot(-\Delta_2+\Delta_1) + (2x_1+1)z_{1,2}\right] - \Delta x_{1,2}\Delta_2 \end{aligned}$$

$$\boxed{\bar{n}_{1,2}' = (2x_1+1)(2x_2+1)z_{1,2} + \underbrace{(2x_2+1)(-\Delta_2+\Delta_1)x_1 - \Delta x_{1,2}\Delta_2}_{(**)}} \quad \text{(eq. 8}^2\text{)}$$

$$\cancel{(*) = (2x_1x_2 + x_2)(-\Delta_2+\Delta_1) - \Delta x_{1,2}\Delta_1}$$

$*\,(x_1 = x_2 - \Delta x_{1,2})$

With $x_2 = x_1 + \Delta x_{1,2}$, it follows:

$$(**) = (2(x_1+\Delta x_{1,2})+1)(-\Delta_2+\Delta_1)\overset{(x_2-\Delta x_{1,2})}{\cancel{x_1}} - \Delta x_{1,2}\Delta_2$$

$$\left(= (2x_1+1)(-\Delta_2+\Delta_1)x_1 + 2\Delta x_{1,2}(-\Delta_2+\Delta_1)(x_2-\Delta x_{1,2}) - \Delta x_{1,2}\Delta_2\right),$$

$$\begin{aligned} &= (2x_1+1)(-\Delta_2+\Delta_1)(x_2-\Delta x_{1,2}) \\ &\quad + 2\Delta x_{1,2}(-\Delta_2+\Delta_1)(x_2-\Delta x_{1,2}) \\ &\quad - \Delta x_{1,2}\Delta_2 \\ &= (2x_1+1)(-\Delta_2+\Delta_1)x_2 \\ &\quad (2x_1+1)(-\Delta_2+\Delta_1)(-\Delta x_{1,2}) + 2\Delta x_{1,2}(-\Delta_2+\Delta_1)\underbrace{(x_2-\Delta x_{1,2})}^{x_1} \\ &\quad - \Delta x_{1,2}\Delta_2 \end{aligned}$$

$$= (2x_1+1)(-\Delta_2+\Delta_1)x_2$$
$$+ (-2x_1\Delta x_{1,2} - \Delta x_{1,2})(-\Delta_2+\Delta_1)$$
$$+ 2\Delta x_{1,2}\, x_1 (-\Delta_2+\Delta_1)$$
$$- \Delta x_{1,2}\Delta_2$$

$$= (2x_1+1)(-\Delta_2+\Delta_1)x_2$$
$$+ \overset{1}{\cancel{2x_1\Delta x_{1,2}\Delta_2}} - \overset{3}{\cancel{2x_1\Delta x_{1,2}\Delta_1}} + \overset{2}{\cancel{\Delta x_{1,2}\Delta_2}} - \Delta x_{1,2}\Delta_1$$
$$- \overset{1}{\cancel{2\Delta x_{1,2}\Delta_2 x_1}} + \overset{3}{\cancel{2\Delta x_{1,2}x_1\Delta_1}}$$
$$- \overset{2}{\cancel{\Delta x_{1,2}\Delta_2}}$$

$$= (2x_1+1)(-\Delta_2+\Delta_1)x_2 \quad - \Delta x_{1,2}\Delta_1$$
$$= (*) \quad \checkmark$$

II Be $\overline{n}_1 = (2x_1+1)y_1 - \Delta_1$

and $\overline{n}_1' = (2x_1+1)\,y_1' - \Delta x_{1,2}\,\Delta_1$

<u>Relationship between old and new equation.</u>

$$\overline{n}_1' = \overline{n}_1$$

$$(2x_1+1)y_1' - \Delta x_{1,2}\Delta_1 = (2x_1+1)\,y_1 - \Delta_1$$

$$0 = (2x_1+1)\,y_1 - (2x_1+1)y_1' - \Delta_1 + \Delta x_{1,2}\Delta_1$$

$$(2x_1+1)y_1' = (2x_1+1)\,y_1 - \Delta_1 + \Delta x_{1,2}\,\Delta_1$$

$$y_1' = y_1 + \frac{-\Delta_1 + \Delta x_{1,2}\Delta_1}{2x_1+1}$$

$$\boxed{\begin{aligned} y_1' &= y_1 + \frac{\Delta_1(-1+\Delta x_{1,2})}{2x_1+1} && \text{(eq. 9}^1\text{)} \\ \Leftrightarrow\quad y_1 &= y_1' - \frac{\Delta_1(-1+\Delta x_{1,2})}{2x_1+1} && \text{(eq. 9}^2\text{)} \end{aligned}}$$

Be y_1^{m} given.

$$y_1' = y_1^{m} + \frac{\Delta_1(-1+\Delta x_{1,2})}{2x_1+1}$$

and

$$y_1' = x_2\cdot(-\Delta_2+\Delta_1) + (2x_2+1)\,z_{1,2}$$

$$\Rightarrow\quad x_2\cdot(-\Delta_2+\Delta_1) + (2x_2+1)\,z_{1,2} = y_1^{m} + \frac{\Delta_1(-1+\Delta x_{1,2})}{2x_1+1}$$

$$\Leftrightarrow\quad (2x_2+1)\,z_{1,2} = y_1^{m} + \frac{\Delta_1(-1+\Delta x_{1,2})}{2x_1+1} - x_2(-\Delta_2+\Delta_1)$$

$$\Leftrightarrow\quad \text{(eq. 10)}\quad \boxed{z_{1,2} = \frac{y_1^{m}}{2x_2+1} + \frac{\Delta_1(-1+\Delta x_{1,2})}{(2x_1+1)(2x_2+1)} - \frac{x_2(-\Delta_2+\Delta_1)}{2x_2+1}}$$

Warning W1 still $\lfloor x \rfloor$ and $\lceil x \rceil$ neccessary !!! for $z_{1,2}$?

$$\Rightarrow\quad \overline{n}_1' = (2x_1+1)y_1' - \Delta x_{1,2}\Delta_1$$

and $y_1' = x_2(-\Delta_2+\Delta_1) + (2x_2+1)\,z_{1,2}$

$$\boxed{\begin{aligned} \Rightarrow\ \overline{n}_{1,2} &= (2x_1+1)\left[x_2(-\Delta_2+\Delta_1) + (2x_2+1)\,z_{1,2}\right] - \Delta x_{1,2}\,\Delta_1 \\ &= (2x_1+1)(2x_2+1)\,z_{1,2} + (2x_1+1)(-\Delta_2+\Delta_1)\,x_2 - \Delta x_{1,2}\Delta_1 \end{aligned}}$$

III A few words about our changing of the equations and its consequences. ((eq.3) → (eq.6)) (see Warning W1)

(eq.3¹) I. $\bar{n}_1 = (2x_1+1)y_1 - \Delta_1$, $\Delta_1 \in \{1,\dots,2x_1\}$

(eq.3²) II. $\bar{n}_2 = (2x_2+1)y_2 - \Delta_2$, $\Delta_2 \in \{1,\dots,2x_1\}$

→ In this equations are: $x_i, y_i \in \mathbb{N}$, $\bar{n}_i \in \mathbb{N}$

→ Then we used our Ansatz: $\bar{n}_1 = \bar{n}_2$

$$y_1 = y_2 + \frac{2\Delta x_{1,2} y_2 - \Delta_2 + \Delta_1}{2x_1+1}$$

and, of course, we searched for integer solution for y_1 and y_2 for this.

→ Since, we had problems to solve this, we changed to (for $\Delta x_{1,2} > 1$)

$$y_1' = y_2' + \frac{2\Delta x_{1,2} y_2 + \Delta x_{1,2}(-\Delta_2 + \Delta_1)}{2x_1+1}$$

and we also looked here for integer solutions y_1' and y_2' for

(eq.6¹) I. $\bar{n}_1' = (2x_1+1)y_1' - \Delta x_{1,2}\,\Delta_1$, $\Delta_1 \in \{1,\dots,2x_1\}$

(eq.6²) II. $\bar{n}_2' = (2x_2+1)y_2' - \Delta x_{1,2}\,\Delta_2$, $\Delta_2 \in \{1,\dots,2x_1\}$

→ But...

... if we look at the relationship between this two versions (eq.3) and (eq.6), we get

(eq.9¹) I. $y_1' = y_1 + \frac{\Delta_1(-1+\Delta x_{1,2})}{2x_1+1}$

⇔ (eq.9²) II. $y_1 = y_1' - \frac{\Delta_1(-1+\Delta x_{1,2})}{2x_1+1}$

, we see that in general in (eq.9) y_i and y_i' are not integers at the same time.

→ So the change ~~betw~~ between (eq.3) ⇔ (eq.6) is a ~~change~~ transition between integer ⇔ non-integer values.

→ Since (eq.6) ist also defined ~~at inte~~ for integer values like (eq.3), we have to take care that this is always ~~fullfield~~ fulfield.

→ We do this by using the floor function for (eq.9)

$$\text{(eq. }9^1\text{v2)}\quad \boxed{\text{I. } y_1' = y_1 + \left\lfloor \frac{\Delta_1(-1+\Delta x_{1,2})}{2x_1+1} \right\rfloor}$$

$$\Leftrightarrow\quad \text{(eq. }9^2\text{v2)}\quad \boxed{\text{II. } y_1 = y_1' - \left\lfloor \frac{\Delta_1(-1+\Delta x_{1,2})}{2x_1+1} \right\rfloor}$$

of cours y_1 and y_1' will not generate the same $\bar{n}$ anymore, but we will see, that this is not a problem for our calculations, in the next steps.

→ With this, we also have to change the solution for $z_{1,2}$.

$$y_1' = y_1^m + \left\lfloor \frac{\Delta_1(-1+\Delta x_{1,2})}{2x_1+1} \right\rfloor$$

and $\quad y_1' = x_2(-\Delta_2+\Delta_1) + (2x_2+1) z_{1,2}$

$$x_2(-\Delta_2+\Delta_1) + (2x_2+1) z_{1,2} = y_1^m + \left\lfloor \frac{\Delta_1(-1+\Delta x_{1,2})}{2x_1+1} \right\rfloor$$

$$\Leftrightarrow\quad (2x_2+1) z_{1,2} = y_1^m + \left\lfloor \frac{\Delta_1(-1+\Delta x_{1,2})}{2x_1+1} \right\rfloor - x_2(-\Delta_2+\Delta_1)$$

$$\Leftrightarrow\quad \text{(eq. 10 v2)}\quad \boxed{z_{1,2} = \left\lfloor \frac{1}{2x_2+1} \left\{ y_1^m + \left\lfloor \frac{\Delta_1(-1+\Delta x_{1,2})}{2x_1+1} \right\rfloor - x_2(-\Delta_2+\Delta_1) \right\} \right\rfloor}$$

$2_L \quad 1_L \quad 1_R \quad 2_R$

Have attention, that we add additonally second floor brackets to this equation, for getting an integer $z_{1,2}$.

Notation Info !

From now on, we will do a small notation changing.

Until now, we have had: $x_2 > x_1 \Leftrightarrow x_1 < x_2$

Since now, we will have: $x_2 < x_1 \Leftrightarrow x_1 > x_2$

Or, more general: $x_i > x_j$, with $i < j$, $i, j \in \mathbb{N}: i \neq j$

We will see, that this will lead to less confusion because of notation, in the next steps.

$\Rightarrow$ <u>Now, we will do the intersection of several equations.</u>

<u>Given be:</u> $\bar{n}_i' = (2x_i + 1)y_i' - \Delta x_{ij} \Delta_i$

<u>Let's start!</u>

$\Rightarrow$ **<u>Step 1:</u>**

$$\bar{n}_1' = (2x_1 + 1)y_1' - \Delta x_{1,2} \Delta_1 \qquad \Delta_1 \in \{1, \dots, 2x_1\}$$

$$\bar{n}_2' = (2x_2 + 1)y_2' - \Delta x_{1,2} \Delta_2 \qquad \Delta_2 \in \{1, \dots, 2x_2\}$$

(S1S) $\Rightarrow$

$$\boxed{\bar{n}_{1,2}' = (2x_1+1)(2x_2+1)z_{1,2} + \underbrace{(2x_1+1)(-\Delta_2+\Delta_1)x_2 - \Delta x_{1,2}\Delta_1}_{=:-\Delta_{12}}}$$

$\Rightarrow$ **<u>Step 2:</u>**

$$\Rightarrow \bar{n}_{1,2}' = (2x_1+1)(2x_2+1)z_{1,2} + \Delta x_{12,3}\{\underbrace{(2x_1+1)(-\Delta_2+\Delta_1)x_2 - \Delta x_{1,2}\Delta_1}_{= -\Delta_{12}}\}$$

$$\bar{n}_3' = (2x_3+1)y_3' - \Delta x_{12,3}\Delta_3 \qquad \Delta_3 \in \{1, \dots, 2x_3\}$$

(S2S) $\Rightarrow$

$$\boxed{\begin{aligned} \bar{n}_{12,3}' = {} & (2x_1+1)(2x_2+1)(2x_3+1) z_{12,3} \\ & \underbrace{+ (2x_1+1)(2x_2+1)\{-\Delta_3 - (2x_1+1)(-\Delta_2+\Delta_1)x_2 + \Delta x_{1,2}\Delta_1\}x_3 - \Delta x_{12,3}\{(2x_1+1)(-\Delta_2+\Delta_1)x_2 + \Delta x_{1,2}\Delta_1\}}_{=: -\Delta_{123}} \end{aligned}}$$

$\Rightarrow$ **<u>Step 3:</u>**

$$\bar{n}_{12,3}' = (2x_1+1)(2x_2+1)(2x_3+1)z_{12,3} + \Delta x_{123,4} \cdot (-\Delta_{123})$$

$$\bar{n}_4' = (2x_4+1)y_4' - \Delta x_{123,4}\Delta_4 \qquad \Delta_4 \in \{1, \dots, 2x_4\}$$

(S3S)

$$\Rightarrow \bar{n}'_{123,4} = (2x_1+1)(2x_2+1)(2x_3+1)(2x_4+1)\,z_{123,4}$$
$$+ (2x_1+1)(2x_2+1)(2x_3+1)$$
$$\cdot \Big\{ -\Delta_4 - (2x_1+1)(2x_2+1)\big\{-\Delta_3 - (2x_1+1)(-\Delta_2+\Delta_1)x_2 + \Delta x_{1,2}\Delta_1\big\}x_3$$
$$+ \Delta x_{12,3}\big\{-(2x_1+1)(-\Delta_2+\Delta_1)x_2 + \Delta x_{1,2}\Delta_1\big\}\Big\}$$
$$\cdot\, x_4$$

$$- \Delta x_{123,4} \cdot \Big\{ -(2x_1+1)(2x_2+1)\big\{-\Delta_3 - (2x_1+1)(-\Delta_2+\Delta_1)x_2 + \Delta x_{1,2}\Delta_1\big\}x_3$$
$$+ \Delta x_{12,3}\big\{-(2x_1+1)(-\Delta_2+\Delta_1)x_2 + \Delta x_{1,2}\Delta_1\big\}\Big\}$$

....and so on.

Let's try to bring this in a more general solution....

We want a general ~~sol~~ form of $\bar{n}'_{123\ldots,n}$, $n \geq 2$, $n \in \mathbb{N}$

for the intersection of n-equations.

Part 1:

That's easy:

(1) $\boxed{\text{We have } \bar{n}'_{123,\ldots,n} = \prod_{k=1}^{n} (2x_k+1)\,z_{12\ldots n-1,n} + A}$

Part 2: A? That's much harder.

(2) **S1S:** ($\bar{n}'_{1,2}$) ($n=2$)

$$A_{(2)} = (2x_1+1)(-\Delta_2+\Delta_1)x_2 - \Delta x_{1,2}\Delta_1$$
$$= \prod_{e_1=1}^{n-1}(2x_{e_1}+1)\cdot\{-\Delta_n+\Delta_{n-1}\}\,x_n - \Delta x_{12\ldots n-1,n}\cdot\Delta_{n-1}$$

S2S ($\bar{n}'_{12,3}$) ($n=3$)

$$A_{(3)} = \prod_{e_1=1}^{n-1}(2x_{e_1}+1)\Big\{-\Delta_n - \prod_{e_2=1}^{n-2}(2x_{e_2}+1)(-\Delta_{n-1}+\Delta_{n-2})\,x_{n-1}$$
$$+ \Delta x_{1\ldots n-2,n-1}\Delta_{n-2}\Big\}x_n$$
$$- \Delta x_{1\ldots n-1,n}\Big\{-\prod_{e_3=1}^{n-2}(2x_{e_3}+1)(-\Delta_{n-1}+\Delta_{n-2})\,x_{n-1}$$
$$+ \Delta x_{1\ldots n-2,n-1}\Delta_{n-2}$$

S3S: $\overline{(n)}_{123,4}$ $(n=4)$

$$A_{(4)} = \prod_{e_1=1}^{n-1}(2x_{e_1}+1)$$

$$\cdot \Big\{ -\Delta_{n-0} - \prod_{e_2=1}^{n-2}(2x_{e_2}+1)\Big\{-\Delta_{n-1} - \prod_{e_3=1}^{n-3}(2x_{e_3}+1)(-\Delta_{n-2}+\Delta_{n-3})x_{n-2}$$

$$+ \Delta x_{1..n-3,n-2}\,\Delta_{n-3}\Big\} x_{n-1}$$

$$+ \Delta x_{1..n-2,n-1}\Big\{-\prod_{e_4=1}^{n-3}(2x_{e_4}+1)(-\Delta_{n-2}+\Delta_{n-3})x_{n-2}$$

$$+ \Delta x_{1..n-3,n-2}\,\Delta_{n-3}\Big\}\Big\}$$

$$-\Delta x_{1...n-1,n}\Big\{-\prod_{e_5=1}^{n-2}(2x_{e_5}+1)\Big\{-\Delta_{n-1} - \prod_{e_6=1}^{n-3}(2x_{e_6}+1)(-\Delta_{n-2}+\Delta_{n-3})\cdot x_{n-2}$$

$$+ \Delta_{1..n-3,n-2}\,\Delta_{n-3}\Big\}x_{n-1}$$

$$+ \Delta x_{1..n-2,n-1}\Big\{-\prod_{e_7=1}^{n-3}(2x_{e_7}+1)(-\Delta_{n-2}+\Delta_{n-3})\cdot x_{n-2}$$

$$+ \Delta x_{1..n-3,n-2}\,\Delta_{n-3}\Big\}\Big\}$$

$$\cancel{A_{(n)} = \prod_{e_1=1}^{n-1}(2x_{e_1}+1)\cdot\Big\{-\Delta_{n-0} - \sum_{i=2}^{n-1}\prod_{e_i=1}^{n-i}(2x_{e_i}+1)\Big(-\sum_{k=i-1}^{2}\Delta_{n-k}}$$

$$\cancel{-\Delta_n - \sum \prod\prod (2x+1)}$$

S1S: $$A_{(2)} = \prod_{e_1=1}^{n-1}(2x_{e_1}+1)\left(\sum_{k_1=1}^{n}(-1)^{k_1+1}\Delta_{k_1}\right)x_n - \underbrace{\Delta x_{1,2}\,\Delta_1}$$

$$= \Delta x_{1,n}\,\Delta_1$$

$$= \Delta x_{1..n-1,n}\,\Delta_1$$

S2S:

$$A_{(3)} = \Big\{(2x_1+1)(2x_2+1)(-\Delta_3) - \big((2x_1+1)(2x_2+1)\big)(2x_1+1)(-\Delta_2+\Delta_1)x_2$$

$$+ (2x_1+1)(2x_2+1)\cdot\Delta x_{1,2}\,\Delta_1\Big\}x_3 - \Delta x_{12,3}\Big\{-(2x_1+1)(-\Delta_2+\Delta_1)x_2$$

$$+ \Delta x_{1,2}\,\Delta_1\Big\}$$

$$\mathcal{A}_{(3)} = \prod_{e_1=1}^{n-1}(2x_{e_1}+1)\left(\sum_{u_1=1}^{n}(-1)^{u_1}\Delta_{u_1}\right)x_n - \Delta x_{1\dots n-1,n}\,\Delta_{1\dots n}$$

$$= \cancel{\prod_{e_1=1}^{n-1}(2x_{e_1}+1)\left\{\sum_{u_1=1}^{n}(-1)^{u_1}\Delta_{u_1}\cdot\prod_{e_2=1}^{n-u_1}(2x_{e_2}+1)\right.}$$

$$= \prod_{e_1=1}^{n-1}(2x_{e_1}+1)\left\{\underbrace{-\Delta_n}_{1} - \underbrace{\prod_{e_2=1}^{n-2}(2x_{e_2}+1)\left(\sum_{u_1=1}^{n-1}(-1)^{u_1+1}\Delta_{u_1}\right)x_{n-1}}_{2} - \Delta x_{1\dots n-2,n-1}\,\Delta_1\right\}x_n$$

$$- \Delta x_{1\dots n-1,n}\left\{-\prod_{e_2=1}^{n-2}(2x_{e_2}+1)\left(\sum_{u_1=1}^{n-1}(-1)^{u_1+1}\Delta_{u_1}\right)x_{n-1} - \Delta x_{1\dots n-2,n-1}\,\Delta_1\right\}$$

$$= \cancel{\prod_{e_1=1}^{n-1}(2x_{e}+1)\left\{\sum_{u_2=1}^{n}(-1)^{u_2}\Delta_{u_2}\right.}$$

$$\cancel{\prod_{e_2=1}^{n-1}(2x_{e_2}+1)}$$

① $\prod_{e=1}^{n-3}(2x_e+1)\left(\sum_{u=n}^{n}(-1)^{u}\Delta_u\right)$

$$\Rightarrow \left\{\underbrace{\prod_{e_2=1}^{n-3}(2x_{e_2}+1)\left(\sum_{u_1=n}^{n}(-1)^{u_1}\Delta_{u_1}\right)}_{1} + \underbrace{\prod_{e_3=1}^{n-2}(2x_{e_3}+1)\left(\sum_{u_2=1}^{n-1}(-1)^{u_2}\Delta_{u_2}\right)\cdot x_{n-1}}_{2}\right.$$

$$= \sum_{i=2}^{n}\prod_{e=1}^{n-i}(2x_e+1)\left(\sum_{\substack{u=\cancel{n-i}\\ u=i-1}}^{n-1}(-1)^{u}\Delta_u\right)$$

$n-i+1$ Elemente

$$\sum \cancel{\prod (2x_e+1)(\sum(-1)^u \Delta u)}$$

$=$

$=$

$$\bar{n}'_{1,2} = (2x_1+1)(2x_2+1)z_{1,2} + (2x_1+1)(-\Delta_2+\Delta_1)x_2 - \underset{(x_1-x_2)}{\underset{\uparrow}{\Delta x_{1,2}}}\Delta_1$$

We have: $x_1 = x_2 + \Delta x_{1,2}$

$\Leftrightarrow \Delta x_{1,2} = x_1 - x_2$

$$\Rightarrow \bar{n}'_{1,2} = (2x_1+1)(2x_2+1)z_{1,2} + (2x_1+1)(-\Delta_2)x_2 + (2x_1+1)\Delta_1 x_2 - x_1\Delta_1 + x_2\Delta_1$$

$$= (2x_1+1)(2x_2+1)z_{1,2} + (2x_1+1)(-\Delta_2)x_2 - x_1\Delta_1 + (2x_1+1)\Delta_1 x_2 + x_2\Delta_1$$

$$= (2x_1+1)(2x_2+1)z_{1,2} - 2x_1x_2\Delta_2 - \Delta_2 x_2 + 2x_1x_2\Delta_1 + \Delta_1 x_2 - x_1\Delta_1 + x_2\Delta_1$$

$$= (2x_1+1)(2x_2+1)z_{1,2} - \Delta_2\cdot(2x_1x_2 + x_2) + \Delta_1(\overbrace{2x_1x_2 + x_2 - x_1 + x_2}^{2x_1x_2+2x_2-x_1})$$

$$= (2x_1+1)(2x_2+1)z_{1,2} - \Delta_2(2x_1+1)x_2 + \Delta_1((2x_2+1)x_1 - 2x_1 + 2x_2)$$

$$= (2x_1+1)(2x_2+1)z_{1,2} - \Delta_2(2x_1+1)x_2 + \Delta_1(2x_2+1)x_1 + \underbrace{2(-x_1+x_2)}_{=-2(x_1-x_2) = -2\cdot\Delta x_{1,2}}\cdot\Delta_1$$

$$\boxed{\bar{n}'_{1,2} = (2x_1+1)(2x_2+1)z_{1,2} + \underbrace{(-\Delta_2(2x_1+1)x_2 + \Delta_1(2x_2+1)x_1)}_{=:-\Delta_{12}} - 2\underbrace{(x_1-x_2)}_{\Delta x_{1,2}}\cdot\Delta_1}$$

$$\bar{h}'_{12,3} = (2x_1+1)(2x_2+1)(2x_3+1)\, z_{12,3}$$

$$+ \Big\{ -\Delta_3 \,(2x_1+1)(2x_2+1)\, x_3$$

$$+ \Big(\big(+\Delta_2 (2x_1+1)\, x_2 \cdot -\Delta_1 (2x_2+1)\, x_1\big) + 2(x_1-x_2)\Delta_1 \Big)$$

① $$\cdot (2x_3+1) \Big((2x_1+1)(2x_2+1) - 1 \Big) \frac{1}{2} \Big\}$$

$$-2 \cdot \Big\{ \big((2x_1+1)(2x_2+1)-1\big)\frac{1}{2} - x_3 \Big\} \Delta_1$$ ②

$$① = \Big\{ -\Delta_3 (2x_1+1)(2x_2+1)\, x_3$$

$$+\Delta_2 (2x_1+1)\, x_2\, (2x_3+1) \Big((2x_1+1)(2x_2+1) - 1 \Big) \frac{1}{2}$$

$$-\Delta_1 (2x_2+1)\, x_1 (2x_3+1) \Big((2x_1+1)(2x_2+1) - 1 \Big) \cdot \frac{1}{2}$$

$$+ 2(x_1 - x_2)\Delta_1 \cdot (2x_3+1) \Big((2x_1+1)(2x_2+1) - 1 \Big) \frac{1}{2} \Big\}$$ $*$

$$② = \Big(-2 \big((2x_1+1)(2x_2+1) - 1 \big) \frac{1}{2} + 2x_3 \Big) \cdot \Delta_1$$

$$= -\Delta_1 \big((2x_1+1)(2x_2+1) - 1 \big) \frac{1}{2} \cdot 2 \;-\; 2^2 x_3 \Delta_1$$

$$\Rightarrow -\Delta_1 \big((2x_1+1)(2x_2+1) - 1 \big) \frac{1}{2} \cdot \Big\{ 2 + (2x_2+1)\, x_1 (2x_3+1) \Big\}$$

$$-\Delta_1 2^2 x_3$$

$$\Rightarrow -\Delta_1 \Big\{ \big((2x_1+1)(2x_2+1) - 1 \big) \frac{1}{2} \cdot \big\{ 2 + (2x_2+1)\, x_1 (2x_3+1) \big\} + 2^2 x_3 \Big\}$$

$$\cancel{m^2_{1\ldots n-1,n} = \prod_{k=1}^{n}(2x_k+1)\, z_{1\ldots n-1,n}}$$

$$\circledast = \Big\{ 2(x_1-x_2)\cdot(2x_3+1)\big((2x_1+1)(2x_2+1)-1\big)\tfrac{1}{2} - 2\big((2x_1+1)(2x_2+1)-1\big)\tfrac{1}{2} + 2^2 x_3 \Big\}\Delta_1$$

$$= 2\Big\{(x_1-x_2)(2x_3+1)\big((2x_1+1)(2x_2+1)-1\big)\tfrac{1}{2} - \big((2x_1+1)(2x_2+1)-1\big)\tfrac{1}{2} + 2x_3\Big\}\Delta_1$$

$$\boxed{= -2\Big\{-(x_1-x_2)(2x_3+1)\big((2x_1+1)(2x_2+1)-1\big)\tfrac{1}{2} + \big((2x_1+1)(2x_2+1)-1\big)\tfrac{1}{2} - 2x_3\Big\}\Delta_1}$$

$$= -2\Big\{\big((2x_1+1)(2x_2+1)-1\big)\tfrac{1}{2}\big(-(x_1-x_2)(2x_3+1)+1\big) - 2x_3\Big\}\Delta_1$$

$$= -2\Big\{\big((2x_1+1)(2x_2+1)-1\big)\tfrac{1}{2}\underbrace{\big((-x_1+x_2)(2x_3+1)+1\big)-2x_3}\Big\}\Delta_1$$

$$= -2x_3x_1 - x_1 + 2x_2x_3 + x_2 + 1 - 2x_3$$

$$= (2x_2x_3 + x_2 + x_3) - 3x_3 - (2x_1x_3 + x_1 + x_3) + x_3$$

$$\boxed{= -2\Big\{\big((2x_1+1)(2x_2+1)-1\big)\tfrac{1}{2}\Big(\big((2x_2+1)(2x_3+1)-1\big)\tfrac{1}{2} - \big((2x_1+1)(2x_3+1)-1\big)\tfrac{1}{2} - 2x_3\Big)\Big\}\Delta_1}$$

$$= -2\Big\{\big((2x_1+1)(2x_2+1)-1\big)\tfrac{1}{2}\big((2x_2+1)(2x_3+1)-1\big)\tfrac{1}{2}$$

$$- \big((2x_1+1)(2x_2+1)-1\big)\tfrac{1}{2}\big((2x_1+1)(2x_3+1)-1\big)\tfrac{1}{2}$$

$$- \big((2x_1+1)(2x_2+1)-1\big)\tfrac{1}{2}(-2x_3)$$

Given be $\bar{n}_i' = (2x_i+1)y_i' - \Delta x_{i,j}\Delta_i$ with $\Delta_i \in \{1,\dots,2x_i\}$

from $\bar{n}_i = (2x_i+1)y_i - \Delta_i$, $x_i, y_i \in \mathbb{N}$

Now since this function shall start with $y_i \geq 2$ –

But, we would still like to have $\mathbb{N}$ as definition set for x_i and y_i, we do a substitution.

$$\boxed{y_i = \tilde{y}_i + 1 \ , \ \tilde{y}_i \in \mathbb{N}} \Leftrightarrow \boxed{\tilde{y}_i = y_i - 1}$$

$$\Rightarrow \bar{n}_i = (2x_i+1)(\tilde{y}_i+1) - \Delta_i$$

$$\Rightarrow \boxed{\bar{n}_i = (2x_i+1)\tilde{y}_i + (2x_i+1) - \Delta_i} \qquad \text{(eq. 11)}$$

and for $\bar{n}_i'$

$$y_i' = y_i + \frac{\Delta_i(-1+\Delta x_{i,j})}{2x_i+1}$$

$$\tilde{y}_i' + 1 = \tilde{y}_i + 1 + \text{" " "}$$

$$\Leftrightarrow \tilde{y}_i' = \tilde{y}_i + \frac{\Delta_i(-1+\Delta x_{i,j})}{2x_i+1}$$

$\Rightarrow$ The same relationship like before

$$\Rightarrow \bar{n}_i' = (2x_i+1)(\tilde{y}_i'+1) - \Delta x_{i,j}\Delta_i$$

$$\Rightarrow \boxed{\bar{n}_i' = (2x_i+1)\tilde{y}_i' + (2x_i+1) - \Delta x_{i,j}\Delta_i} \qquad \text{(eq. 12)}$$

Intersection:

$$\bar{n}_1' = (2x_1+1)\tilde{y}_1' + (2x_1+1) - \Delta x_{1,2}\Delta_1$$

$$\bar{n}_2' = (2x_2+1)\tilde{y}_2' + (2x_2+1) - \Delta x_{1,2}\Delta_2$$

$$(2x_1+1)\tilde{y}_1' + (2x_1+1) - \Delta x_{1,2}\Delta_1 = (2x_2+1)\tilde{y}_2' + (2x_2+1) - \Delta x_{1,2}\Delta_2$$

$$0 = (2x_2+1)\tilde{y}_2' - (2x_1+1)\tilde{y}_1' + \overbrace{(2x_2+1) - (2x_1+1)}^{=2x_1+1+2\Delta x_{1,2}-2x_1-1} - \Delta x_{1,2}\Delta_2 + \Delta x_{1,2}\Delta_1$$

Be $x_2 = x_1 + \Delta x_{1,2}$

$$0 = (2x_1+1)\tilde{y}_2' + 2\Delta x_{1,2}\tilde{y}_2' - (2x_1+1)\tilde{y}_1' + 2\Delta x_{1,2} - \Delta x_{1,2}\Delta_2 + \Delta x_{1,2}\Delta_1$$

$\cancel{(2x_1+1)\tilde{y}_2}$

$$(2x_1+1)\tilde{y}_1' = (2x_1+1)\tilde{y}_2' + (2\Delta x_{1,2})(\tilde{y}_2'+1) + \Delta x_{1,2}\Delta_2 + \Delta x_{1,2}\Delta_1$$

$$\Rightarrow \tilde{y}_1' = \tilde{y}_2' + \frac{2\Delta x_{1,2}(\tilde{y}_2'+1) + \Delta x_{1,2}(-\Delta_2+\Delta_1)}{2x_1+1} \checkmark$$

$$\Rightarrow \boxed{\begin{aligned} \tilde{y}_1' &= x_2(-\Delta_2+\Delta_1) + (2x_2+1)z_{1,2} - 1 \\ \tilde{y}_2' &= x_1(-\Delta_2+\Delta_1) + (2x_1+1)z_{1,2} - 1 \end{aligned}} \quad \text{(eq.13)}$$

$$\begin{aligned}
\bar{n}_{1,2}' &= (2x_1+1)\tilde{y}_1' + (2x_1+1) - \Delta x_{1,2}\Delta_1 \\
&= (2x_1+1)\left[x_2(-\Delta_2+\Delta_1) + (2x_2+1)z_{1,2} - 1\right] \\
&\quad + (2x_1+1) - \Delta x_{1,2}\Delta_1 \\
&= (2x_1+1)(2x_2+1)z_{1,2} \\
&\quad + (2x_1+1)(-\Delta_2+\Delta_1)x_2 - \cancel{(2x_1+1)} + \cancel{(2x_1+1)} \\
&\quad - \Delta x_{1,2}\Delta_1 \\
&= (2x_1+1)(2x_2+1)z_{1,2} + (2x_1+1)(-\Delta_2+\Delta_1)x_2 - \Delta x_{1,2}\Delta_1
\end{aligned}$$

the ~~same~~ same like before. ✓

$$\boxed{\overline{n}_1 = (2x_1+1)y_1 - \Delta_1}$$

→ We only need odd $\overline{n}$. $, m \in \mathbb{N}$ (eq. 14)

$$\Rightarrow \quad \boxed{2m+1 = \overline{n}_1 = (2x_1+1)y_1 - \Delta_1} \quad \text{with } \Delta_1 \in \{1,\ldots,2x_1\}$$

We have different possibilities to rewrite the right side, to get the necessary numbers. We will choose the following:

$$2m+1 = \overline{n}_1 = (2x_1+1)\underbrace{(2u_1+1)}_{=:y_1,\ u_1 \in \mathbb{N}} - \Delta_1$$

→ we only have odd y_1 now.

$\Rightarrow \Delta_1$ has to be 2ℓ, $\ell \in \mathbb{N}$

$\Rightarrow$ So, be $\Delta_1^0 \in \{1,\ldots,x_i\}$ but now (with only odd numbers) we have a doubbled intervall, so we have in real $\Delta_1^0 \in \{1,\ldots,2x_1\}$ again

It follows:

$$\boxed{2m+1 = \overline{n}_1^0 = (2x_1+1)\underbrace{(2u_1+1)}_{=:y_1'} - 2\Delta_1^0}, \text{ with } \boxed{\Delta_1^0 \in \{1,\ldots,2x_1\}}$$

(eq. 15)

$$\Rightarrow \overline{n}_1^0 = (2x_1+1)(2u_1+1) - 2\Delta_1^0 \cdot \Delta x_{1,2}$$

$$\overline{n}_2^0 = (2x_2+1)\underbrace{(2u_2+1)}_{=:y_2'} - 2\Delta_2^0 \cdot \Delta x_{1,2}$$

$$\Rightarrow y_1' = (2u_1+1) = x_2 \cdot (-\Delta_2 + \Delta_1) + (2x_2+1) z_{1,2}$$

$$u_1 = \frac{1}{2}\left(x_2(-\Delta_2+\Delta_1) + (2x_2+1) z_{1,2} - 1 \right)$$

$$(2x_1+1)(2u_1+1) - 2\Delta_1^0 \cdot \Delta x_{1,2} = (2x_2+1)(2u_2+1) - 2\Delta_2^0 \cdot \Delta x_{1,2}$$

$\underline{x_2 = x_1 + \Delta x_{1,2}}$

$$0 = (\underbrace{2(x_1+\Delta x_{1,2})+1}_{(2x_1+1)+2\Delta x_{1,2}})(2u_2+1) - (2x_1+1)(2u_1+1) - 2\Delta_2^0 \cdot \Delta x_{1,2} + 2\Delta_1^0 \cdot \Delta x_{1,2}$$

$$\cdot\tfrac{1}{2} \Big/ = (2x_1+1)(\underbrace{2u_2+1-2u_1+1}_{2(u_2-u_1)+2}) + 2\Delta x_{1,2}(2u_2+1) - 2\Delta_2^0 \Delta x_{1,2} + 2\Delta_1^0 \Delta x_{1,2}$$

$$= (2x_1+1)(u_2-u_1+1) + \Delta x_{1,2}(2u_2+1) + \Delta x_{1,2}(-\Delta_2+\Delta_1)$$

$$(2x_1+1)(2u_1+1) - 2\Delta_1^\circ \cdot \Delta x_{1,2} = (2x_2+1)(2u_2+1) - 2\Delta_2^\circ \cdot \Delta x_{1,2}$$

$$0 = (2x_2+1)(2u_2+1) - (2x_1+1)(2u_1+1) - 2\Delta_2^\circ \Delta x_{1,2} + 2\Delta_1^\circ \Delta x_{1,2}$$

$$\underline{x_2 = x_1 + \Delta x_{1,2}}$$

$$0 = (2x_1+1)(2u_2+1) + 2\Delta x_{1,2}(2u_2+1) - (2x_1+1)(2u_1+1)$$
$$+ 2\Delta x_{1,2}(-\Delta_2^\circ + \Delta_1^\circ)$$

$$\cdot\tfrac{1}{2} \; / \; 0 = (2x_1+1)(\underbrace{2u_2+1-2u_1-1}_{2(u_2-u_1)}) + 2\Delta x_{1,2}(2u_2+1) + 2\Delta x_{1,2}(-\Delta_2^\circ+\Delta_1^\circ)$$

$$0 = (2x_1+1)(u_2-u_1) + \Delta x_{1,2}(2u_2+1) + \Delta x_{1,2}(-\Delta_2^\circ+\Delta_1^\circ)$$

$$(2x_1+1)u_1 = (2x_1+1)u_2 + 1\Delta x_{1,2}(2u_2+1) + \Delta x_{1,2}(-\Delta_2^\circ+\Delta_1^\circ)$$

$$\boxed{u_1 = u_2 + \frac{\Delta x_{1,2}(2u_2+1) + \Delta x_{1,2}(-\Delta_2^\circ+\Delta_1^\circ)}{2x_1+1}} \quad \text{(eq. 16)}$$

Be $\underline{u_2 = (-\Delta_2 + \Delta_1^\circ + \alpha)x_1}$ $\qquad \underline{2\cdot(-\Delta_2^\circ+\Delta_1^\circ)x_1} + 2\alpha x_1 + 1 + \underline{(-\Delta_2^\circ+\Delta_1^\circ)}$

$$\Rightarrow \quad u_1 = (-\Delta_2^\circ+\Delta_1^\circ+1)x_1 + \Delta x_{1,2} \, \frac{2(-\Delta_2^\circ+\Delta_1^\circ+\alpha)x_1 + 1 + (-\Delta_2^\circ+\Delta_1^\circ)}{2x_1+1}$$

$$= (-\Delta_2^\circ+\Delta_1^\circ+1)x_1 + \Delta x_{1,2} \cdot \frac{(-\Delta_2^\circ+\Delta_1^\circ)\cdot(2x_1+1) + 2\alpha x_1 + 1}{2x_1+1}$$

$$= (-\Delta_2^\circ+\Delta_1^\circ+1)x_1 + \Delta x_{1,2}(-\Delta_2^\circ+\Delta_1^\circ) + \frac{2\alpha x_1+1}{2x_1+1}\cdot\Delta x_{1,2}$$

be $\alpha = 1$

$$= (-\Delta_2^\circ+\Delta_1^\circ+1)x_1 + \Delta x_{1,2}(-\Delta_2^\circ+\Delta_1^\circ) + 1\Delta x_{1,2}$$

$$= (-\Delta_2^\circ+\Delta_1^\circ+1)x_1 + \Delta x_{1,2}(-\Delta_2^\circ+\Delta_1^\circ+1)$$

$$= (-\Delta_2^\circ+\Delta_1^\circ+1)(x_1+\Delta x_{1,2})$$

$$\underline{u_1 = (-\Delta_2^\circ+\Delta_1^\circ+1)\,x_2}$$

$$\boxed{2m_1+1 = \bar{n}_1^\circ = (2x_1+1)(2k_1+1) - 2\Delta_1^\circ}$$

$$\Rightarrow \quad 2m_1+1 = (2x_1+1)(2k_1+1) - 2\Delta_1^\circ$$

$$2m_1 = (2x_1+1)(2k_1+1) - 2\Delta_1^\circ - 1$$

$$m_1 = \frac{1}{2}\left\{(2x_1+1)(2k_1) + (2x_1+1) - 2\Delta_1^\circ - 1\right\}$$

$$= \frac{1}{2}\left\{(2x_1+1)\,2k_1 + 2x_1 - 2\Delta_1^\circ\right\}$$

$$\boxed{m_1 = (2x_1+1)k_1 + x_1 - \Delta_1^\circ} \quad (\text{eq.}17)$$

'$\Delta x_{1,2}$ for our $\bar{n}_1^\circ$' version

$\Rightarrow$ That's the equation which we already know from my work „The recursively calculation of prime numbers"

Available on:

[1] https://github.com/Samdney/primescalc

For the intersection of an arbitrary number n of equation $\bar{n}_i^\circ$, $(n>1)$, $n \in \mathbb{N}$, we have from [1]:

(eq.I1)

(eq.18)

$$\begin{aligned}
m_{\underbrace{1,\dots,n-1}_{i},\underbrace{n}_{j}} = {} & \prod_{k=1}^{n}(2x_k+1)\,z_{1,\dots,n-1,n} \\
& + \frac{1}{2}\left(\prod_{k=1}^{n}(2x_k+1) - 1\right) \\
& - \left((-1)^{n+1}\,\Delta_1 x_1 \prod_{l\neq 1}^{n}(2x_l+1)\prod_{f=1}^{n-2,\,n>2}\left(\frac{1}{2}\left(\prod_{m=1}^{f+1}(2x_m+1)-1\right)\right)\right) \\
& - \left(\sum_{k=2}^{n}(-1)^{n+k}\,\Delta_k x_k \prod_{l\neq 1}^{n}(2x_l+1)\prod_{f=1}^{n-k,\,n>2}\left(\frac{1}{2}\left(\prod_{m=1}^{f+1}(2x_m+1)-1\right)\right)\right)
\end{aligned}$$

With the definition $\prod_{f=1}^{0} A_f := 1$.

$$\Rightarrow \bar{n}'_{1,\dots,n-1,n} = 2m_{1\dots n-1,n} + 1$$

(eq. I2)

(eq. 19)

$$= 2 \cdot \Bigg\{ \prod_{u=1}^{n} (2x_u + 1)_{z_{1\dots n-1,n}}$$

$$+ \frac{1}{2} \left(\prod_{u=1}^{n} (2x_u + 1) - 1 \right)$$

$$- \left((-1)^{n+1} \Delta_1 x_1 \prod_{e \neq 1}^{n} (2x_e + 1) \prod_{f=1}^{n-2,\, n>2} \left(\frac{1}{2} \left(\prod_{m=1}^{f+1} (2x_m + 1) - 1 \right) \right) \right)$$

$$- \left(\sum_{u=2}^{n} (-1)^{n+u} \Delta_u x_u \prod_{e \neq 1}^{n} (2x_e + 1) \prod_{f=1}^{n-u,\, n>2} \left(\frac{1}{2} \left(\prod_{m=1}^{f+1} (2x_m + 1) - 1 \right) \right) \right) \Bigg\}$$

$$+ 1$$

VII → Be given

$$n_1 = (2x_1+1)y_1 + \alpha \qquad x_i, y_i \in \mathbb{N}$$

and $n_2 = (2x_2+1)y_2 + \beta$

with $x_2 > x_1$.

→ So, we we can write $x_2 = x_1 + \Delta x_{1,2}$, $\Delta x_{1,2} \in \mathbb{N}$

$$\Rightarrow \quad n_1 = (2x_1+1)y_1 + \alpha$$

$$n_2 = (2\cdot(x_1+\Delta x_{1,2})+1)y_2 + \beta$$

→ $\boxed{\text{Now, let's assume: } \Delta x_{1,2} = (2x_1+1)}$

$$\begin{aligned}
\Rightarrow n_2 &= (2\cdot(x_1+(2x_1+1))+1)y_2 + \beta \\
&= (2\cdot(x_1+2x_1+1)+1)y_2 + \beta \\
&= (2\cdot(3x_1+1)+1)y_2 + \beta \\
&= (6x_1+2+1)y_2 + \beta \\
&= (6x_1+3)y_2 + \beta \\
&= 3\cdot(2x_1+1)y_2 + \beta \\
n_2 &= (2x_1+1)(\underbrace{3y_2}_{=:\tilde{y}_2}) + \beta \\
&= (2x_1+1)\tilde{y}_2 + \beta
\end{aligned}$$

→ Now, look at

$$(2\tilde{x}+1) = \tilde{n}_{1,2} = \underbrace{(2x_1+1)(2x_2+1)}z_{1,2} + \underbrace{(2x_1+1)(-\Delta_2+\Delta_1)x_2 - \Delta x_{1,2}\Delta_1}_{=:\gamma}$$

$$= 2\cdot 2x_1x_2 + 2x_1 + 2x_2 + 1$$

$$= 2\cdot(2x_1x_2 + x_1 + x_2) + 1$$

Be $\tilde{x} = 2x_1x_2 + x_1 + x_2 - \tilde{\Delta}x_{1,2}$

$$\Rightarrow 2\cdot(2x_1x_2+x_1+x_2-\tilde{\Delta}x_{1,2})+1 = (2(2x_1x_2+x_1+x_2)+1)z_{1,2} + \gamma$$

$$\Leftrightarrow 2(2x_1x_2+x_1+x_2) - 2\tilde{\Delta}x_{1,2} + 1 = (2(2x_1x_2+x_1+x_2)+1)z_{1,2} + \gamma$$

$$\Leftrightarrow 2(2x_1x_2+x_1+x_2) - 2(2x_1x_2+x_1+x_2)z_{1,2} = 2z_{1,2} + 2\tilde{\Delta}x_{1,2} - 1 + \gamma$$

$$\Leftrightarrow 2(2x_1x_2+x_1+x_2)\cdot(1-z_{1,2}) = 2(z_{1,2}+\tilde{\Delta}_{1,2}) - 1 + \gamma$$

$$\Leftrightarrow 2(2x_1x_2+x_1+x_2)(1-z_{1,2}) = 2\cdot(z_{1,2}-1) + 1 + 2\tilde{\Delta}_{1,2} + \gamma$$

$$\Leftrightarrow\ 2(2x_1x_2+x_1+x_2)(1-z_{1,2}) = -2(1-z_{1,2})+1+2\hat{\Delta}_{1,2}+\gamma$$

$$\Leftrightarrow\ 2(2x_1x_2+x_1+x_2)(1-z_{1,2})+(1-z_{1,2}) = -(1-z_{1,2})+1+2\hat{\Delta}_{1,2}+\gamma$$

$$\Leftrightarrow\ \left(2(2x_1x_2+x_1+x_2)+1\right)(1-z_{1,2}) = -1+z_{1,2}+1+2\hat{\Delta}x_{1,2}+\gamma$$

$$\Leftrightarrow\ (2x_1+1)(2x_2+1)(1-z_{1,2}) = z_{1,2}+2\hat{\Delta}x_{1,2}+\gamma$$

$$\Leftrightarrow\ 2(2x_1x_2+x_1+x_2)-2\hat{\Delta}_{1,2}+1 = \left(2(2x_1x_2+x_1+x_2)+1\right)z_{1,2}+\gamma$$

$$\Leftrightarrow\ (2x_1+1)(2x_2+1)-2\hat{\Delta}_{1,2} = (2x_1+1)(2x_2+1)z_{1,2}+\gamma$$

$$\Leftrightarrow\ (2x_1+1)(2x_2+1)-(2x_1+1)(2x_2+1)z_{1,2} = 2\hat{\Delta}_{1,2}+\gamma$$

$$\Leftrightarrow\ (2x_1+1)(2x_2+1)(1-z_{1,2}) = 2\hat{\Delta}_{1,2}+\gamma$$

$$\Leftrightarrow\ (2x_1+1)(2x_2+1)(1-z_{1,2})-\gamma = 2\hat{\Delta}_{1,2}$$

$\rightarrow$ We know that we have for odd solutions $\tilde{n} = 2\tilde{x}+1$, always $2\Delta_i^\circ$. $\Delta_i^\circ \in \frac{1}{2}\{1,..,2x_i\}$

$\Delta_i \in$

$$\Leftrightarrow\ \tilde{\Delta}x_{1,2} = \underbrace{\underbrace{(2x_1+1)(2x_2+1)(1-z_{1,2})\cdot\tfrac{1}{2}}_{①}}_{\text{this part is always: } =(2x_1+1)\cdot k \text{ respectively } =(2x_2+1)k'} - \underbrace{\tfrac{1}{2}\gamma}_{②} \qquad (*)$$

(eq. 20)

we can resubstitute this, with $\frac{(1-z_{1,2})}{2} = z^{new}_{1,2}$ for all $z_{1,2}$ odd

$(**)$ $\forall k_1,k_2 \in \mathbb{N}$
$\tilde{\Delta}x_{1,2} \neq (2x_1+1)k_1$
and $\tilde{\Delta}x_{1,2} \neq (2x_2+1)k_2$
$\Rightarrow$ so we can always use our $\Delta\tilde{x}_{1,2}$ from recursion for the next step, in the valid range

<u>To ②:</u> $\rightarrow$ From the deffinition of our original equation $\tilde{n}_i$, we know that all results are between $(2x_i+1)k$ and $(2x_i+1)(k+1)$, $\forall k \in \mathbb{N}$. excluded $(2x_i+1)k$ $\forall k \in \mathbb{N}$

<u>and</u> $\rightarrow$ From this follows, that also all values from $\tilde{n}_{ij}$ are between $(2x_1+1)(2x_2+1)k$ and $(2x_1+1)(2x_2+1)(k+1)$ $\forall k \in \mathbb{N}$, $(2x_i+1)(2x_2+1)k$ excluded $\forall k \in \mathbb{N}$

$\Longrightarrow$ all values of $(*)$ are not element of $(2x_1+1)$ times table <u>and</u> not element of $(2x_2+1)$ times table <u>and</u> not element of $(2x_1+1)(2x_2+1)$ times table $\Rightarrow (**)$

Let's have a look at the differences between

$$\boxed{\begin{array}{ll} & \bar{n}_1 = (2x_1+1)y_1 - \Delta_1 \\ \text{and} & \bar{n}_1' = (2x_1+1)y_1' - \Delta x_{1/2}\Delta_1 \end{array}}$$

Example: $x_1 = 1$, $\Delta x_{1/2} = 1$

$$\Rightarrow \quad \bar{n}_1 = 3y_1 - \Delta_1$$
$$\bar{n}_1' = 3y_1' - \Delta_1$$

Be $\Delta_1 = 1$: $\bar{n}_1 = 3\cdot y_1 - 1$, $y_1 = 1 \Rightarrow \bar{n}_1 = 3-1 = 2$

$$y_1' = y_1 + \frac{\Delta_1(-1+\Delta x_{1/2})}{2x_1+1}$$
$$= 1 + \frac{1\cdot(-1+1)}{2\cdot 1+1}$$
$$= 1+0$$
$$y' = 1$$

Example: $x_1 = 1$ $\Delta x_{1/2} = 2$

$$\Rightarrow \quad \bar{n}_1 = 3y_1 - \Delta_1$$
$$\bar{n}_1' = 3y_1' - 2\Delta_1$$

Be $\Delta_1 = 1$: $y_1 = 2 \Rightarrow \bar{n}_1 = 3\cdot 2 - 1 = 6-1 = 5$

$$y_1' = y_1 + \frac{\Delta_1(-1+\Delta x_{1/2})}{2\cdot x_1+1}$$
$$= 2 + \frac{1\cdot(-1+2)}{3}$$
$$y_1' = 2 + \frac{1}{3}$$

$\Rightarrow$ So, we see, if we use the same Δ_1 in both equations, we get a not integer y_1'.

Let's do $y_1' = \lfloor 2 + 1/3 \rfloor = 2$, $\Delta_1 = 1$

$$\Rightarrow \quad \bar{n}_1' = 3\cdot 2 - 2\cdot 1 = 6-2 = 4$$

For which values do we get $\bar{n}_1' = 5$?

$$5 = 3y_1' - 2\Delta_1 = \left\{ \begin{array}{l} y_1' = 2,\ \Delta_1 = 1 \\ \Rightarrow 3\cdot 2 - 2\cdot 1 \\ \quad = 6-2 = 4 \end{array} \right.$$

$\Rightarrow$ we get all values of $\bar{n}_1$ also by $\bar{n}_1'$! but with different (y_1, Δ_1) pairs, if we want to fulfield the constraint $y_i \in \mathbb{N}$!

(I already showed (in other notes) tha this is fulfield. I will not repead it here, but you can also easy check this by yourself)

Assume we have given:

$$\circledast \quad \boxed{\bar{n}_1 = (2x_1+1)y_1 - \Delta_1} \qquad \Delta_1 \in \{1, .., 2x_1\}$$

$\rightarrow$ Have, fore example, a look at: $(x_1 = 1)$:

$$\bar{n}_1 = 3y_1 - \Delta_1 \quad , \Delta_1 \in \{1, 2\}$$

$y_1 = 1$: $\quad \bar{n}_1 = \begin{cases} \Delta_1 = 1 : 2 \\ \Delta_1 = 2 : 1 \end{cases}$

$\leftarrow$ We see: The number „3" is missing!

$y_1 = 2$: $\quad \bar{n}_1 = \begin{cases} \Delta_1 = 1 : 5 \\ \Delta_1 = 2 : 4 \end{cases}$

$\rightarrow$ That's not what, we want, since we want to find all prime numbers.

$y_1 = 3$: $\quad \bar{n}_1 = \begin{cases} \Delta_1 = 1 : 8 \\ \Delta_1 = 2 : 7 \end{cases}$

$\rightarrow$ with this we will not get „3"

$\vdots \qquad \vdots \qquad \vdots$

$\Rightarrow$ Our equation $\circledast$ is not usable (gives us not a valid range) for values ≤ 3.

$\rightarrow$ But..., we already have to know „3" to generate this $\circledast$ equation, so the given number „3" comes from an earlier calculation step.

$\Rightarrow$ So it's just valid for $y_1 \geq 2$.!

Now let's switch to (only odd solutions)

$$\circledast\circledast \quad \boxed{\bar{n}_1^o = (2x_1+1)(2u_1+1) - 2\Delta_1^o \quad , \Delta_1^o \in \{1, .., 2x_1\}}$$

$$u_1 \in \mathbb{N}$$

→ Let's look again at our example ($x_1 = 1$):

$$\overline{n}_1^o = 3 \cdot (2u_1+1) - 2\Delta_1^o \; , \; \Delta_1^o \in \{1,2\}$$

(+) →

$u_1 = 1$: $\overline{n}_1^o = \begin{cases} \Delta_1^o = 1: & 3 \cdot 3 - 2 \cdot 1 = 9 - 1 \cdot 2 = 9 - 2 = 7 \\ \Delta_1^o = 2: & 3 \cdot 3 - 2 \cdot 2 = 9 - 2 \cdot 2 = 9 - 4 = 5 \end{cases}$

$u_1 = 2$: $\overline{n}_1^o = \begin{cases} \Delta_1^o = 1: & 3 \cdot 5 - 2 \cdot 1 = 15 - 1 \cdot 2 = 15 - 2 = 13 \\ \Delta_1^o = 2: & 3 \cdot 5 - 2 \cdot 2 = 15 - 2 \cdot 2 = 15 - 4 = 11 \end{cases}$

$u_1 = 3$: $\overline{n}_1^o = \begin{cases} \Delta_1^o = 1: & 3 \cdot 7 - 2 \cdot 1 = 21 - 2 \cdot 1 = 21 - 2 = 19 \\ \Delta_1^o = 2: & 3 \cdot 7 - 2 \cdot 2 = 21 - 2 \cdot 2 = 21 - 4 = 17 \end{cases}$

⋮

(+) → Here we see, that we don't generate numbers $\overline{n}_1^o \leq 6 = 3 \cdot 2$

→ So, if we use this equation (**), we only generate ~~a valid~~ valid numbers > 6.

⇒ ~~Consequences for intersection of two equations of the form (**):~~

~~$$\overline{n}_1^o = (2x_1+1)(2u_1+1) - 2\Delta_1^o \; , \; \Delta_1^o \in \{1,..,2x_1\}$$~~

~~$$\overline{n}_2^o = (2x_2+1)(2u_2+1) - 2\Delta_2^o \; , \; \Delta_2^o \in \{1,..,2x_2\}$$~~

~~$$\Rightarrow \overline{n}_1^o = \overline{n}_2^o$$~~

~~Assume: $x_2 > x_1$~~

~~Then our solution $\overline{n}_{12}^o$ of $\overline{n}_1^o = \overline{n}_2^o$ gives us only valid results in the range $\overline{n}_{12}^o \in](2x_2+1), \infty[$ and not-valid results in the range $\overline{n}_{12}^o \in [1, (2x_2+1)]$~~

(***)

$(2x_2+1)(2 \cdot 0+1)$

~~So, what is "1"?~~

$\Rightarrow$ Consequences for intersection of two equations of the form (**)

Example: $\bar{n}_1^o = 3(2u_1+1) - 2\Delta_1^o \quad , \Delta_1^o \in \{1, 2\}$

$\bar{n}_2^o = 5(2u_2+1) - 2\Delta_2^o \quad , \Delta_2^o \in \{1, .., 4\}$

and $\bar{n}_3^o = 7(\underbrace{2u_3+1}_{=3,5,7,9,11}) - 2\Delta_3^o \quad , \Delta_3^o \in \{1, .., 6\}$

$\Rightarrow n_1$: $\underline{\underline{3}}$ 4 5 6 7 8 $\boxed{9}$ 11 12 13 14 $\boxed{15}$

n_2: ... 4 $\underline{\underline{5}}$ 6 ... 14 $\boxed{15}$ 16 ...... 24 $\boxed{25}$

n_3: ... 6 $\underline{\underline{7}}$ 8 20 $\boxed{21}$ 22 ...... 34 $\boxed{35}$

$n^o_{i(u=0)}$ $\quad \bar{n}^o_i$ $\quad n^o_{i(u \in \mathbb{N})}$

even / odd

Minimum ($\bar{n}_1^o = \bar{n}_2^o$)

Assume: $x_2 > x_1$

$\Rightarrow$ We see, our smallest values for n_1 and n_2 are 9 and 15 for $u=1$.

$\Rightarrow$ So, for our range we have our Minimum at

$$\Rightarrow \boxed{\bar{n}^{o\,min}_{1,2} = \underbrace{(2x_2+1)}_{x_2 > x_1} \cdot (2\underset{\overset{\shortparallel}{1}}{u_2}+1) - 2 \cdot \underset{\overset{\shortparallel}{2 \cdot x_2}}{\Delta_2^o}} \quad \text{(eq. 21)}$$

(here: $\bar{n}^{min}_{1,2} = (2 \cdot 2+1) \cdot (2 \cdot 1+1) - 2 \cdot 2 \cdot 2$

$= 5 \cdot 3 - 8 = 15 - 8 = \underline{\underline{7}}$)

$\Rightarrow$ Let's look for our Maximum

If we look at our numbers, we see that we can ~~use~~ use ~~$(2x_2+1) \cdot (2u_2+1)$~~ here 21 safely as maximum range since it's the first number of the next larger number.

$$\Rightarrow \boxed{\bar{n}^{max}_{1,2} = \underbrace{(2(x_2+1)+1)}_{x_2 > x_1} \cdot (2\underset{\overset{\shortparallel}{1}}{u_2}+1) - 2} \quad \text{(eq. 22)}$$

(here: $\bar{n}^{max}_{1,2} = (2 \cdot (2+1)+1) \cdot (2 \cdot 1+1) - 2$

$= (2 \cdot 3+1) \cdot (2+1) - 2$

$= 7 \cdot 3 - 2$

$= 19$)

$(x_2 > x_1)$

$$\boxed{\Rightarrow \underline{\text{valid } \bar{n}^o_{1,2} \text{ range}}: \bar{n}^o_{1,2} \in [(2x_2+1) \cdot 3 - 2 \cdot (2 \cdot x_2), (2(x_2+1)+1) \cdot 3 - 2]} \quad \text{(eq. 23)}$$

Since, we now know our valid $\bar{n}_i^0$ range, we can finally also have a look at z_{ij}.

For the intersection of two equations, we have given

$$\text{(1)} \quad z_{ij} = \left\lfloor \frac{1}{2x_j+1} \left\{ y_i + \left\lfloor \frac{\Delta_i(-1+\Delta x_{ij})}{2x_i+1} \right\rfloor - x_j(-\Delta_j + \Delta_i) \right\} \right\rfloor$$

respectively

$$\text{(2)} \quad z_{ij} = \left\lfloor \frac{1}{2x_i+1} \left\{ y_j + \left\lfloor \frac{\Delta_j(-1+\Delta x_{ij})}{2x_j+1} \right\rfloor - x_i(-\Delta_j + \Delta_i) \right\} \right\rfloor$$

→ assume „i" be the intersection of all $(2x_i+1)$ apart from the Maximum ($\Rightarrow$ „j" $\hat{=}$ Maximum) and do the intersection with this Maximum as the last intersection, at all.

→ So, if we take eq(2), we can take y_j „Maximum" and determine z_{ij} from this.

→ If we also do the same for the Minimum in the intersection step before, we can determine our valid z_{ij} range from this.

Approximations for floor function: (from different Internet sources)

1. $$\lfloor x \rfloor = -\frac{1}{2} + x + \frac{\arctan(\cot(\pi x))}{\pi} = -\frac{1}{2} + x - \frac{\arctan(\tan(\pi(x-\frac{1}{2})))}{\pi}$$

(eq.24)

2. Fourier series of the Floor function
$$\lfloor x \rfloor = -\frac{1}{2} + x + \frac{1}{\pi}\sum_{k=1}^{\infty}\frac{\sin(2\pi k x)}{k}$$

(eq.25) which converges to
$$= \underbrace{-\frac{1}{2} + x - \frac{i\left(\ln(1-e^{-2\pi i x}) - \ln(1-e^{2\pi i x})\right)}{2\pi}}_{\text{real} \qquad (\mathbb{C})}$$

3. $$\lfloor x \rfloor = \lim_{n\to\infty}\left(\sum_{k=-n}^{n}\mu(x-k)\right) - n - 1$$

(eq.26) μ step function: $\mu(x) = \lim_{n\to\infty} f(nx)$
with $f(x) = e^{-x^2} + \frac{2}{\pi}\operatorname{Si}(x)$.
So
$$f(x) = 1 + \frac{2}{\pi}x + \sum_{k=1}^{\infty}\frac{(-1)^k}{\prod_{j=1}^{k} j}x^{2k} + \frac{2}{\pi}\frac{\left(\prod_{j=1}^{2k} j\right)(-1)^k}{\left(\prod_{j=1}^{2k+1} j\right)^2}x^{2k+1}$$

Taylor series converges for every $x \in \mathbb{R}$.

4. $$\lfloor x \rfloor = \sum_{n=-\infty}^{\infty} n\Theta(x-n)\Theta(n+1-x), \quad x \notin \mathbb{Z}$$

(eq.27)
$$\lfloor x \rfloor = \lim_{z\to x^+}\lfloor z \rfloor$$

Let's put it all together:

1. We will ignore the prime number 2.

2. We assume, we already know the first two prime numbers

$$p_1 = \overline{n}_1 = 2 \cdot 1 + 1 = 3 \qquad \Rightarrow \text{We know all primes } \underbrace{p \in \underbrace{[1,5]}_{=: I_1}}_{\overline{n}'_{0,1}}$$

$$p_2 = \overline{n}_2 = 2 \cdot 2 + 1 = 5$$

$$\Rightarrow \left\| \begin{array}{lll} \text{I.} & \overline{n}_1 = 3\overbrace{(2k_1+1)}^{=: y_1} - 2\Delta_1 & , \Delta_1 \in \{1, 2\} \\ \text{II.} & \overline{n}_2 = 5\underbrace{(2k_2+1)}_{=: y_2} - 2\Delta_2 & , \Delta_2 \in \{1, .., 4\} \end{array} \right.$$

3. We have also:

$$\Rightarrow \left\| \begin{array}{lll} \text{I.} & \overline{n}'_1 = 3\overbrace{(2k'_1+1)}^{= y'_1} - 2\overset{1}{\overset{\shortparallel}{\Delta x_{1,2}}}\Delta_1 & , \Delta_1 \in \{1, 2\} \\ \text{II.} & \overline{n}'_2 = 5\underbrace{(2k'_2+1)}_{=: y'_2} - 2\underset{1}{\underset{\shortparallel}{\Delta x_{1,2}}}\Delta_2 & , \Delta_2 \in \{1, .., 4\} \end{array} \right.$$

4. Do the intersection: we receive $\overline{n}'_{1,2}$

5. Determine: $y_1^{min}, y_1^{max}, y_2^{min}, y_2^{max}$

6. Calculate with this $y_1^{min'}, y_1^{max'}, y_2^{min'}, y_2^{max'}$

7. Determine $z_{1,2}^{min}$ and $z_{1,2}^{max}$ from Step 6. Make an good approximation for $z_{1,2}$.

   $\Rightarrow$ Now we have $\overline{n}'_{1,2}$ which generate all prime numers, in the allowed next range I_2.

8. Now, we can do $\overline{n}'_{1,2,3}(\overline{n}'_{1,2}, \overline{n}'_{0,1})$ ~~$\overline{n}'_{1,2,3}(\overline{n}'_{1,2,1})$ to generate the primes in the~~

   and determine $z'_{1,2,3}(\overline{n}'_{1,2}, \underbrace{y_1^{min'}, y_1^{max'}, y_2^{min'}, y_2^{max'}}_{z'_{1,2}}) = z'_{1,2,3}(z'_{1,2})$

9. Now, we can do $\overline{n}'_{1,2,3,4}(\overline{n}'_{1,2,3}(\overline{n}'_{1,2}, \overline{n}'_{0,1}), \overline{n}'_{1,2}, \overline{n}'_{0,1})$

   and determine $z'_{1,2,3,4}(z'_{1,2,3}(z'_{1,2}))$

10. ... and so on.

!! $\Rightarrow$ All what we need are the equations for z, $\overline{n}'_i$ and m'_i:
(eq. 10 v2), (eq. I1) and (eq. I2)

! ⇒ We ~~know~~ know the whole time, during every step, everything what we need to calculate the next step!
determine

ToDo:

→ Find a better way of writing for equations (eq. I 1) and (eq. I 2)

→ Think about possible approximations for z (eq. 10 v2)

→ Think about other useful $y^{min/max}$ ranges / ~~choices~~ choices

~~→ Determine the simplified recursion equation~~

→ Determine the simplified recursion equation.