Modern C++ Programming

2. Basic Concepts I

Fundamental Types

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Table of Context

Preparation

What Compiler Should I Use?

Most popular compilers:

- Microsoft Visual Code (**MSVC**) is the compiler offered by Microsoft
- The GNU Compiler Collection (**GCC**) contains the most popular C++ Linux compiler
- **Clang** is a C++ compiler based on LLVM Infrastructure available for Linux/Windows/Apple (default) platforms

Suggested compiler on Linux for beginner: **Clang**

- Comparable performance with GCC/MSVC and low memory usage
- Expressive diagnostics (examples and propose corrections)
- Strict C++ compliance. GCC/MSVC compatibility (inverse direction is not ensured)
- Includes very useful tools: memory sanitizer, static code analyzer, automatic formatting, linter, etc.

Install the Compiler on Linux

Install the last gcc/g++ (v11) (v12 on Ubuntu 22.04)

```
$ sudo add-apt-repository ppa:ubuntu-toolchain-r/test
$ sudo apt update
$ sudo apt install gcc-11 g++-11
$ gcc-11 --version
```

Install the last clang/clang++ (v16)

```
$ bash -c "$(wget -O - https://apt.llvm.org/llvm.sh)"
$ wget https://apt.llvm.org/llvm.sh
$ chmod +x llvm.sh
$ sudo ./llvm.sh 16
$ clang++ --version
```

Install the Compiler on Windows

Microsoft Visual Studio

- Direct Installer: `Visual Studio Community 2022`

Clang on Windows

Two ways:

- Windows Subsystem for Linux (WSL)
  - `Run` → `optionalfeatures`
  - Select `Windows Subsystem for Linux`, `Hyper-V`, `Virtual Machine Platform`
  - `Run` → `ms-windows-store:` → Search and install `Ubuntu 22.04 LTS`
- Clang + MSVC Build Tools
  - Download Build Tools per Visual Studio
  - Install `Desktop development with C++`

What Editor/IDE Compiler Should I Use?

Popular C++ IDE (Integrated Development Environment):

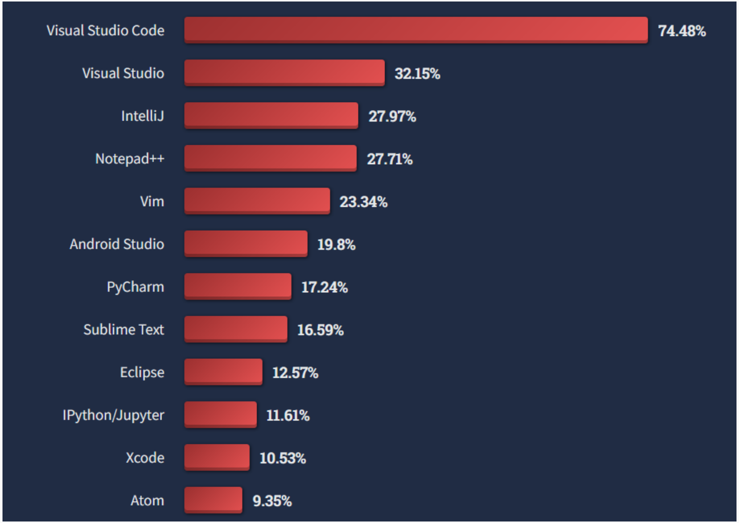
- **Microsoft Visual Studio** (MSVC) (link). Most popular IDE for Windows
- **Clion** (link). (free for student). Powerful IDE with a lot of options
- **QT-Creator** (link). Fast (written in C++), simple
- **XCode**. Default on Mac OS
- **Cevelop** (Eclipse) (link)

Standalone editors for coding (multi-platform):

- **Microsoft Visual Studio Code** (VSCode) (link)
- **Sublime Text editor** (link), written in C++
- **Vim**. Powerful, but needs expertise

Not suggested: `Notepad, Gedit, and other similar editors` (lack of support for programming)

What Editor/IDE Compiler Should I Use?



StackOverflow Developer Survey 2022

How to Compile?

Compile C++11, C++14, C++17, C++20 programs:

```
g++  -std=c++11 <program.cpp> -o program
g++  -std=c++14 <program.cpp> -o program
g++  -std=c++17 <program.cpp> -o program
g++  -std=c++20 <program.cpp> -o program
```

Any C++ standard is backward compatible

C++ is also backward compatible with C, even for very old code, except if it contains C++ keywords (`new, template, class, typename, etc.`)

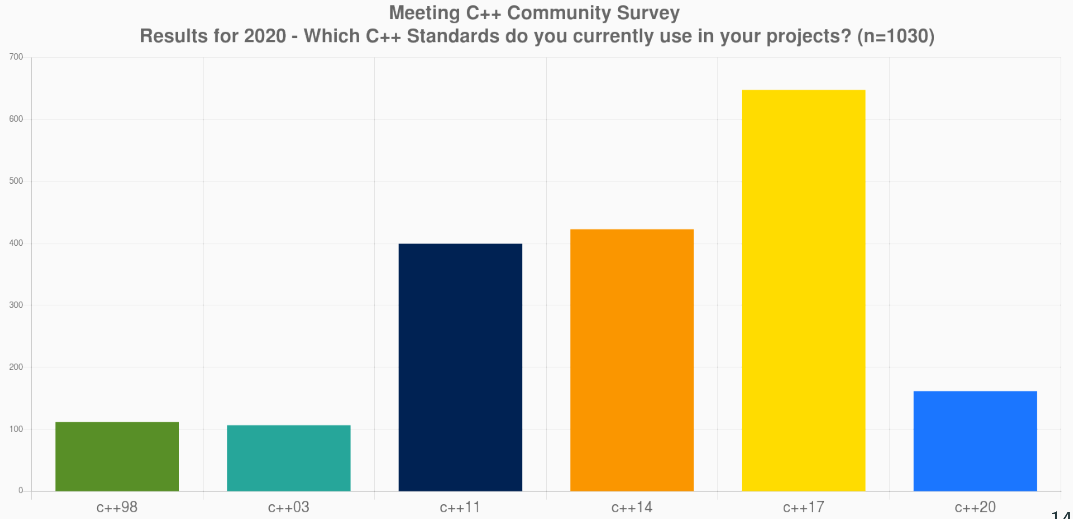
We can potentially compiles a pure C program in C++20

C++ Standard

Compiler	C++11		C++14		C++17		C++20	
	Core	Library	Core	Library	Core	Library	Core	Library
`g++`	`4.8.1`	`5.1`	`5.1`	`5.1`	`7.1`	`9.0`	`11+`	`11+`
`clang++`	`3.3`	`3.3`	`3.4`	`3.5`	`5.0`	`11.0`	`12+`	`14+`
`MSVC`	`19.0`	`19.0`	`19.10`	`19.0`	`19.15`	`19.15`	`19.29+`	`19.29`

`en.cppreference.com/w/cpp/compiler_support`

C++ Standard


Meeting C++ Community Survey
Results for 2020 - Which C++ Standards do you currently use in your projects? (n=1030)
700
600
500
400
300
200
100
0
c++98
c++03
c++11
c++14
c++17
c++20

Hello World

Hello World 1/2

C code with `printf`:

```
#include <stdio.h>

int main() {
    printf("Hello World!\n");
}
```

`printf`

prints on standard output

C++ code with `streams`:

```
#include <iostream>

int main() {
    std::cout << "Hello World!\n";
}
```

`cout`

represent the standard output stream

Hello World 2/2

The previous example can be written with the global `std` namespace:

```cpp
#include <iostream>
using namespace std;

int main() {
    cout << "Hello World!\n";
}
```

Note: For sake of space and for improving the readability, we intentionally omit the `std` namespace in the next slides

I/O Stream (std::cout) 1/3

`std::cout` is an example of *output* stream. Data is redirected to a destination, in this case the destination is the standard output

C:

```c
#include <stdio.h>
int main() {
    int    a   = 4;
    double b   = 3.0;
    char   c[] = "hello";
    printf("%d %f %s\n", a, b, c);
}
```

C++:

```cpp
#include <iostream>
int main() {
    int    a   = 4;
    double b   = 3.0;
    char   c[] = "hello";
    std::cout << a << " " << b << " " << c << "\n";
}
```

I/O Stream (Why should we prefer I/O stream?) 2/3

- **Type-safe**: The type of object provided to the I/O stream is known statically by the compiler. In contrast, `printf` uses `%` fields to figure out the types dynamically

- **Less error prone**: With IO Stream, there are no redundant `%` tokens that have to be consistent with the actual objects pass to I/O stream. Removing redundancy removes a class of errors

- **Extensible**: The C++ IO Stream mechanism allows new user-defined types to be pass to I/O stream without breaking existing code

- **Comparable performance**: If used correctly may be faster than C I/O (`printf` , `scanf` , etc.) .

I/O Stream (Common C errors) 3/3

- Forget the number of parameters:

```
printf("long phrase %d long phrase %d", 3);
```

- Use the wrong format:

```
int a = 3;
...many lines of code...
printf(" %f", a);
```

- The `%c` conversion specifier does not automatically skip any leading white space:

```
scanf("%d", &var1);
scanf(" %c", &var2);
```

Fundamental Types Overview

Arithmetic Types - Integral

Native Type	Bytes	Range	Fixed width types `<cstdint>`
`bool`	1	true, false	
`char` †	1	implementation defined	
`signed char`	1	-128 to 127	int8_t
`unsigned char`	1	0 to 255	uint8_t
`short`	2	-2^{15} to 2^{15}-1	int16_t
`unsigned short`	2	0 to 2^{16}-1	uint16_t
`int`	4	-2^{31} to 2^{31}-1	int32_t
`unsigned int`	4	0 to 2^{32}-1	uint32_t
`long int`	4/8*		int32_t/int64_t
`long unsigned int`	4/8*		uint32_t/uint64_t
`long long int`	8	-2^{63} to 2^{63}-1	int64_t
`long long unsigned int`	8	0 to 2^{64}-1	uint64_t

* 4 bytes on Windows64 systems, † `signed/unsigned`, two-complement from C++11

Arithmetic Types - Floating-Point

Native Type	IEEE	Bytes	Range	Fixed width types C++23 `<stdfloat>`
(bfloat16)	N	2	$\pm 1.18 \times 10^{-38}$ to $\pm 3.4 \times 10^{+38}$	std::bfloat16_t
(float16)	Y	2	0.00006 to $65,536$	std::float16_t
`float`	Y	4	$\pm 1.18 \times 10^{-38}$ to $\pm 3.4 \times 10^{+38}$	std::float32_t
`double`	Y	8	$\pm 2.23 \times 10^{-308}$ to $\pm 1.8 \times 10^{+308}$	std::float64_t

Arithmetic Types - Short Name

Signed Type	short name
`signed char`	/
`signed short int`	`short`
`signed int`	`int`
`signed long int`	`long`
`signed long long int`	`long long`

Unsigned Type	short name
`unsigned char`	/
`unsigned short int`	`unsigned short`
`unsigned int`	`unsigned`
`unsigned long int`	`unsigned long`
`unsigned long long int`	`unsigned long long`

Arithmetic Types - Suffix (Literals)

Type	SUFFIX	Example	Notes
`int`	/	2	
`unsigned int`	u, U	3u	
`long int`	l, L	8L	
`long unsigned`	ul, UL	2ul	
`long long int`	ll, LL	4ll	
`long long unsigned int`	ull, ULL	7ULL	
`float`	f, F	`3.0f`	only decimal numbers
`double`		`3.0`	only decimal numbers

C++23 Type	SUFFIX	Example	Notes
`std::bfloat16_t`	bf16, BF16	`3.0bf16`	only decimal numbers
`std::float16_t`	f16, F16	`3.0f16`	only decimal numbers
`std::float32_t`	f32, F32	`3.0f32`	only decimal numbers
`std::float64_t`	f64, F64	`3.0f64`	only decimal numbers
`std::float128_t`	f128, F128	`3.0f128`	only decimal numbers

Arithmetic Types - Prefix (Literals)

Representation	PREFIX	Example
Binary C++14	0b	0b010101
Octal	0	0308
Hexadecimal	0x or 0X	0xFFA010

C++14 also allows *digit separators* for improving the readability `1'000'000`

Other Arithmetic Types

- C++ also provides `long double` (no IEEE-754) of size 8/12/16 bytes depending on the implementation

- Reduced precision floating-point supports before C++23:
  - Some compilers provide support for *half* (16-bit floating-point) (GCC for ARM: `__fp16`, LLVM compiler: `half`)
  - Some modern CPUs and GPUs provide *half* instructions
  - Software support: OpenGL, Photoshop, Lightroom, `half.sourceforge.net`

- C++ does not provide **128-bit integers** even if some architectures support it. `clang` and `gcc` allow 128-bit integers as compiler extension (`__int128`)

`void` Type

`void` is an incomplete type (not defined) without a value

- `void` indicates also a function with no return type or no parameters e.g. `void f()`, `f(void)`
- In C `sizeof(void) == 1` (GCC), while in C++ `sizeof(void)` does not compile!!

```
int main() {
//  sizeof(void); // compile error
}
```

nullptr Keyword

C++11 introduces the new keyword `nullptr` to represent a null pointer (`0x0`) and replacing the `NULL` macro

```
int* p1 = NULL;      // ok, equal to int* p1 = 0l
int* p2 = nullptr;   // ok, nullptr is a pointer not a number

int n1 = NULL;       // ok, we are assigning 0 to n1
// int n2 = nullptr; // compile error we are assigning
//                      a null pointer to an integer variable

// int* p2 = true ? 0 : nullptr; // compile error
//                               // incompatible types
```

Remember: `nullptr` is not a pointer, but an object of type `nullptr_t` → safer

Fundamental Types Summary

The *fundamental types*, also called *primitive* or *built-in*, are organized into three main categories:

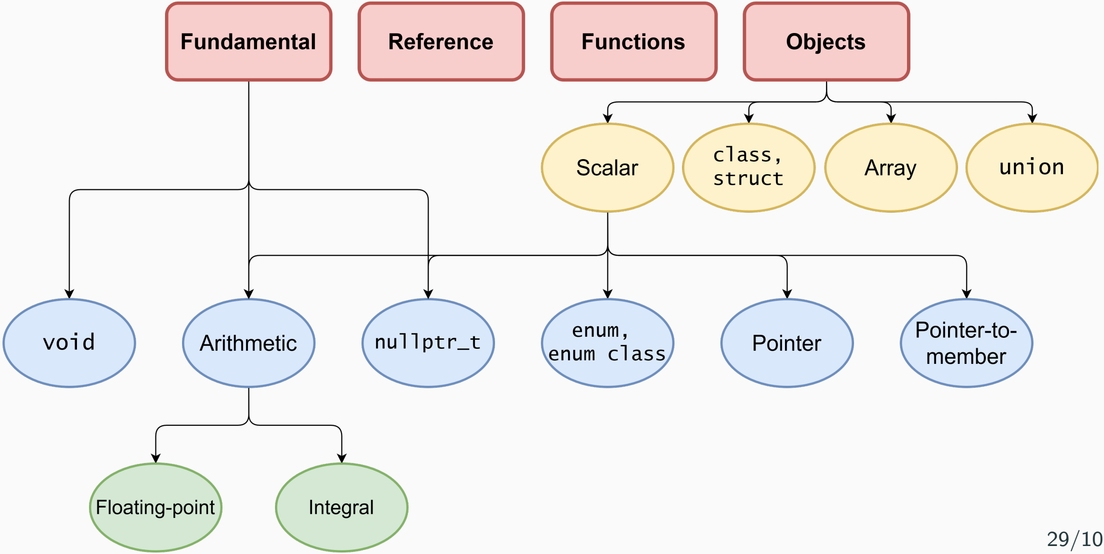
- Integers
- Floating-points
- `void`, `nullptr`

Any other entity in C++ is

- an *alias* to the correct type depending to the context and the architectures
- a *composition* of builtin types: `struct/class`, `array`, `union`

en.cppreference.com/w/cpp/language/types
en.cppreference.com/w/cpp/types/integer

C++ Types Summary



Conversion Rules

Conversion Rules

Implicit type conversion rules, applied in order, before any operation:

⊗: any operation (*, +, /, -, %, etc.)

(A) Floating point promotion

`floating_type` ⊗ `integer_type` → `floating_type`

(B) Implicit integer promotion

`small_integral_type` := any signed/unsigned integral type smaller than `int`

`small_integral_type` ⊗ `small_integral_type` → `int`

(C) Size promotion

`small_type` ⊗ `large_type` → `large_type`

(D) Sign promotion

`signed_type` ⊗ `unsigned_type` → `unsigned_type`

Examples and Common Errors

```
float    f = 1.0f;
unsigned u = 2;
int      i = 3;
short    s = 4;
uint8_t  c = 5; // unsigned char

f * u; // float × unsigned → float: 2.0f
s * c; // short × unsigned char → int: 20
u * i; // unsigned × int → unsigned: 6u
+c;    // unsigned char → int: 5
```

Integers are not floating points!

```
int   b = 7;
float a = b / 2;    // a = 3 not 3.5!!
int   c = b / 2.0; // again c = 3 not 3.5!!
```

Implicit Promotion

Integral data types smaller than 32-bit are *implicitly* promoted to `int`, independently if they are *signed* or *unsigned*

- Unary `+, -, ∼` and Binary `+, -, &, etc.` promotion:

```
char a = 48;       // '0'
cout << a;         // print '0'
cout << +a;        // print '48'
cout << (a + 0); // print '48'

uint8_t a1 = 255;
uint8_t b1 = 255;
cout << (a1 + b1);  // print '510' (no overflow)
```

`auto` Declaration

`auto` Keyword 1/3

C++11 The `auto` keyword specifies that the type of the variable will be automatically deduced by the compiler (from its initializer)

```
auto a = 1 + 2;   // 1 is int, 2 is int, 1 + 2 is int!
//    -> 'a' is "int"
auto b = 1 + 2.0; // 1 is int, 2.0 is double. 1 + 2.0 is double
//    -> 'b' is "double"
```

`auto` can be very useful for maintainability and for hiding complex type definitions

```
for (auto i = k; i < size; i++)
    ...
```

On the other hand, it may make the code less readable if excessively used because of type hiding

Example: `auto x = 0;` in general makes no sense (`x` is `int`)

auto Keyword - Functions ★ 2/3

In C++11/C++14, `auto` (as well as `decltype`) can be used to define function output types

```
auto g(int x) -> int { return x * 2; } // C++11
// "-> int" is the deduction type
// a better way to express it is:

auto g2(int x) -> decltype(x * 2) { return x * 2; }  // C++11

auto h(int x) { return x * 2; }        // C++14

//--------------------------------------------------------------

int x = g(3); // C++11
```

auto Keyword - Functions ★ 3/3

In C++20, `auto` can be also used to define function input

```cpp
void f(auto x) {}
// equivalent to templates but less expensive at compile-time

//--------------------------------------------------------------

f(3);    // 'x' is int
f(3.0);  // 'x' is double
```

C++ Operators

Operators Precedence 1/3

Precedence	Operator	Description	Associativity
1	a++ a--	Suffix/postfix increment and decrement	Left-to-right
2	+a -a ++a --a ! ∼	Prefix increment/decrement, Logical/Bitwise Not	Right-to-left
3	a*b a/b a%b	Multiplication, division, and remainder	Left-to-right
4	a+b a-b	Addition and subtraction	Left-to-right
5	≪ ≫	Bitwise left shift and right shift	Left-to-right
6	< <= > >=	Relational operators	Left-to-right
7	== !=	Equality operators	Left-to-right
8	&	Bitwise AND	Left-to-right
9	ˆ	Bitwise XOR	Left-to-right
10	|	Bitwise OR	Left-to-right
11	&&	Logical AND	Left-to-right
12	||	Logical OR	Left-to-right
13	+= -= *= /= %= <<= >>= &= ˆ= |=	Compound	Right-to-left

Operators Precedence 2/3

- **Unary** operators have higher precedence than **binary operators**

- **Standard math operators** (`+, *, etc.`) have higher precedence than **comparison**, **bitwise**, and **logic** operators

- **Comparison** operators have higher precedence than **bitwise** and **logic operators**

- **Bitwise** operators have higher precedence than **logic** operators

- **Compound assignment** operators `+=`, `-=`, `*=`, `/=`, `%=`, `^=`, `!=`, `&=`, `>>=`, `<<=` have lower priority

- The **comma** operator has the lowest precedence (see next slides)

en.cppreference.com/w/cpp/language/operator_precedence

Operators Precedence 3/3

Examples:

```
a + b * 4;           // a + (b * 4)

a * b / c % d;       // ((a * b) / c) % d

a + b < 3 >> 4;      // (a + b) < (3 >> 4)

a && b && c || d;    // (a && b && c) || d

a | b & c || e && d; // ((a | (b & c)) || (e && d)
```

Important: sometimes parenthesis can make expression worldly... but they can help!

Prefix/Postfix Increment Semantic

Prefix Increment/Decrement `++i`, `--i`

(1) Update the value

(2) Return the new (updated) value

Postfix Increment/Decrement `i++`, `i--`

(1) Save the old value (temporary)

(2) Update the value

(3) Return the old (original) value

Prefix/Postfix increment/decrement semantic applies not only to built-in types but also to objects

Operation Ordering Undefined Behavior ★

Expressions with undefined (implementation-defined) behavior:

```
int i = 0;
i = ++i + 2;       // until C++11: undefined behavior
                   // since C++11:  i = 3
i = 0;
i = i++ + 2;       // until C++17: undefined behavior
                   // since C++17: i = 3

f(i = 2, i = 1);   // until C++17: undefined behavior
                   // since C++17:  i = 2
i = 0;
a[i] = i++;        // until C++17: undefined behavior
                   // since C++17: a[1] = 1

f(++i, ++i);       // undefined behavior
i = ++i + i++;     // undefined behavior
```

Assignment, Compound, and Comma Operators

Assignment and **compound assignment** operators have *right-to-left associativity* and their expressions return the assigned value

```
int y = 2;
int x = y = 3; // y=3, then x=3
               // the same of x = (y = 3)
if (x = 4)     // assign x=4 and evaluate to true
```

The **comma** operator has *left-to-right associativity*. It evaluates the left expression, discards its result, and returns the right expression

```
int a = 5, b = 7;
int x = (3, 4); // discards 3, then x=4
int y = 0;
int z;
z = y, x;       // z=y (0), then returns x (4)
```

Spaceship Operator <=>

C++20 provides the **three-way comparison operator** `<=>`, also called *spaceship operator*, which allows comparing two objects in a similar way of `strcmp`. The operator returns an object that can be directly compared with a positive, 0, or negative integer value

```
(3 <=> 5)     == 0; // false
('a' <=> 'a') == 0; // true

(3 <=> 5)     < 0;  // true
(7 <=> 5)     < 0;  // false
```

The semantic of the *spaceship operator* can be extended to any object (see next lectures) and can greatly simplify the comparison operators overloading

Safe Comparison Operators

C++20 introduces a set of functions `<utility>` to safely compare integers of different types (`signed`, `unsigned`)

```
bool cmp_equal(T1 a, T2 b)
bool cmp_not_equal(T1 a, T2 b)
bool cmp_less(T1 a, T2 b)
bool cmp_greater(T1 a, T2 b)
bool cmp_less_equal(T1 a, T2 b)
bool cmp_greater_equal(T1 a, T2 b)
```

example:

```
#include <utility>
unsigned a  = 4;
int      b  = -3;
bool     v1 = (a > b);           // false!!!, see next slides
bool     v2 = cmp_greater(a, b); // true
```

Integral Data Types

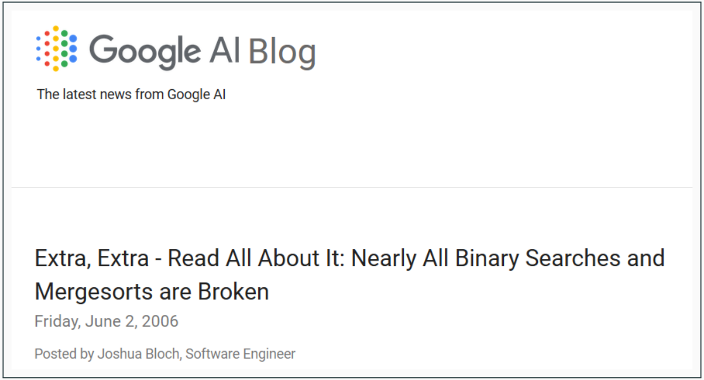
A Firmware Bug

> *"Certain SSDs have a firmware bug causing them to irrecoverably fail after exactly 32,768 hours of operation. SSDs that were put into service at the same time will fail simultaneously, so RAID won't help"*

`HPE SAS Solid State Drives - Critical Firmware Upgrade`

Via twitter.com/martinkl/status/1202235877520482306?s=19

Overflow Implementations



Google AI Blog

The latest news from Google AI

Extra, Extra - Read All About It: Nearly All Binary Searches and Mergesorts are Broken

Friday, June 2, 2006

Posted by Joshua Bloch, Software Engineer

Note: Computing the average in the right way is not trivial, see `On finding the average of two unsigned integers without overflow`

related operations: ceiling division, rounding division

`ai.googleblog.com/2006/06/extra-extra-read-all-about-it-nearly.html`

Potentially Catastrophic Failure

$$51\ days = 51 \cdot 24 \cdot 60 \cdot 60 \cdot 1000 = 4\,406\,400\,000\ ms$$

Boeing 787s must be turned off and on every 51 days to prevent 'misleading data' being shown to pilots

C++ Data Model

LP32 Windows 16-bit APIs (no more used)

ILP32 Windows 32-bit APIs, Unix 32-bit (Linux, Mac OS)

LLP64 Windows 64-bit APIs

LP64 Linux 64-bit APIs

Model/Bits	`short`	`int`	`long`	`long long`	`pointer`
ILP32	16	32	32	64	32
LLP64	16	32	32	64	64
LP64	16	32	64	64	64

`char` is always 1 byte

Fixed Width Integers

int*_t <cstdint>

C++ provides fixed width integer types.

They have the same size on any architecture:

int8_t, uint8_t

int16_t, uint16_t

int32_t, uint32_t

int64_t, uint64_t

Good practice: Prefer fixed-width integers instead of native types. `int` and `unsigned` can be directly used as they are widely accepted by C++ data models

Fixed Width Integers

`int*_t` types are not "real" types, they are merely *typedefs* to appropriate fundamental types

C++ standard does not ensure a one-to-one mapping:

- There are **five** distinct *fundamental types* (`char`, `short`, `int`, `long`, `long long`)
- There are **four** `int*_t` *overloads* (`int8_t`, `int16_t`, `int32_t`, and `int64_t`)

ithare.com/c-on-using-int_t-as-overload-and-template-parameters

Fixed Width Integers 3/3

Warning: I/O Stream interprets `uint8_t` and `int8_t` as `char` and not as integer values

```
int8_t var;
cin >> var;  // read '2'
cout << var; // print '2'
int a = var * 2;
cout << a;   // print '100' !!
```

size_t and ptrdiff_t

size_t ptrdiff_t <cstddef>

size_t and **ptrdiff_t** are *aliases* data types capable of storing the biggest representable value on the current architecture

- size_t is an unsigned integer type (of at least 16-bit)
- ptrdiff_t is the signed version of size_t commonly used for computing pointer differences
- size_t is commonly used to represent size measures
- size_t / ptrdiff_t are 4 bytes on 32-bit architectures, and 8 bytes on 64-bit architectures
- C++23 adds uz / UZ literal for size_t
- C++23 adds z / Z literal for ptrdiff_t

When Use Signed/Unsigned Integer? 1/3

`Signed` and **`unsigned`** integers use the same hardware for their operations, but they have very different semantic:

`signed` integers

- Represent positive, negative, and zero values ($\mathbb{Z}$)
- More negative values ($2^{31}-1$) than positive ($2^{31}-2$)
- Overflow/underflow is undefined behavior
  Possible behavior:
  overflow: $(2^{31}-1)+1 \rightarrow min$
  underflow: $-2^{31}-1 \rightarrow max$
- Bit-wise operations are implementation-defined
- Commutative, reflexive, not associative (overflow/underflow)

When Use Signed/Unsigned Integer? 2/3

`unsigned` integers

- Represent only *non-negative* values ($\mathbb{N}$)
- Overflow/underflow is well-defined (modulo 2^{32})
- Discontinuity in 0, $2^{32} - 1$
- Bit-wise operations are well-defined
- Commutative, reflexive, associative

When Use Signed/Unsigned Integer? 3/3

Google Style Guide

> Because of historical accident, the C++ standard also uses unsigned integers to represent the size of containers - many members of the standards body believe this to be a mistake, but it is effectively impossible to fix at this point

Solution: use `int64_t`

max value: $2^{63} - 1 =$ 9,223,372,036,854,775,807 or
9 quintillion (9 billion of billion),
about 292 years (nanoseconds),
9 million terabytes

Subscripts and sizes should be signed, WG21 P1428R0, *Bjarne Stroustrup*
Don't add to the signed/unsigned mess, WG21 P1491R0, *Bjarne Stroustrup*

Arithmetic Type Limits

Query properties of arithmetic types in C++11:

```
#include <limits>

std::numeric_limits<int>::max();      // 2^31 − 1
std::numeric_limits<uint16_t>::max(); // 65,535

std::numeric_limits<int>::min();      // −2^31
std::numeric_limits<unsigned>::min(); // 0
```

* this syntax will be explained in the next lectures

Promotion and Truncation

Promotion to a larger type keeps the sign

```
int16_t x = -1;
int     y = x; // sign extend
cout << y;     // print -1
```

Truncation to a smaller type is implemented as a modulo operation with respect to the number of bits of the smaller type

```
int     x = 65537; // 2^16 + 1
int16_t y = x;     // x % 2^16
cout << y;         // print 1

int     z = 32769; // 2^15 + 1 (does not fit in a int16_t)
int16_t w = z;     // (int16_t) (x % 2^16 = 32769)
cout << w;         // print -32767
```

Mixing Signed/Unsigned Errors 1/2

```
unsigned a = 10;  // array is small
int      b = -1;
array[10ull + a * b] = 0; // ?
```

☠ Segmentation fault!

```
int f(int a, unsigned b, int* array) {  // array is small
    if (a > b)
        return array[a - b]; // ?
    return 0;
}
```

☠ Segmentation fault for ”a” ¡ 0!

```
// v.size() return unsigned
for (size_t i = 0; i < v.size() - 1; i++)
    array[i] = 3; // ?
```

☠ Segmentation fault for v.size() = 0!

Mixing Signed/Unsigned Errors 2/2

Easy case:

```
unsigned x = 32;     // x can be also a pointer
x         += 2u - 4; // 2u - 4 = 2 + (2^32 - 4)
                     //        = 2^32 - 2
                     // (32 + (2^32 - 2)) % 2^32
cout << x;           // print 30 (as expected)
```

What about the following code?

```
uint64_t x = 32;     // x can be also a pointer
x         += 2u - 4;
cout << x;
```

Undefined Behavior 1/4

More negative values than positive

```
int x = std::numeric_limits<int>::max() * -1; //  (2^31 -1) * -1
cout << x;                                    //  -2^31 +1 ok

int y = std::numeric_limits<int>::min() * -1; // -2^31 * -1
cout << y; // hard to see in complex examples // 2^31 overflow!!
```

A pratical example:

```
void f(int* ptr, int pos) {
    pos++;
    if (pos < 0)
        return;     // <-- the compiler assumes that
    ptr[pos] = 0;   //      signed overflow never happen
}                   //      and removes the if statement
int main() {        // compiled with optimizations
    int tmp[10];    // leads to segmentation faults
    f(tmp, INT_MAX);
}
```

Undefined Behavior 2/4

Initialize an integer with a value larger then its range is undefined behavior

```
int z = 3000000000; // undefined behavior!!
```

Bitwise operations on signed integer types is undefined behavior

```
int y = 1 << 12;    // undefined behavior!!
```

Shift larger than #bits of the data type is undefined behavior even for **unsigned**

```
unsigned y = 1u << 32u; // undefined behavior!!
```

Undefined behavior in implicit conversion

```
uint16_t a = 65535; // 0xFFFF
uint16_t b = 65535; // 0xFFFF                    expected: 4'294'836'225
cout << (a * b);    // print '-131071' undefined behavior!! (int overflow)
```

Undefined Behavior 3/4

Even worse example:

```
#include <iostream>

int main() {
    for (int i = 0; i < 4; ++i)
        std::cout << i * 1000000000 << std::endl;
}
// with optimizations, it is an infinite loop
// --> 1000000000 * i > INT_MAX
// undefined behavior!!

// the compiler translates the multiplication constant into an addition
```

Why does this loop produce undefined behavior?

Undefined Behavior

Is the following loop safe?

```
void f(int size) {
    for (int i = 1; i < size; i += 2)
        ...
}
```

- What happens if `size` is equal to `INT_MAX`?
- How to make the previous loop safe?
- `i >= 0 && i < size` is not the solution because of *undefined behavior* of signed overflow
- Can we generalize the solution when the increment is `i += step`?

Overflow / Underflow

Detecting overflow/underflow for unsigned integral types is **not trivial**

```
// some examples
bool is_add_overflow(unsigned a, unsigned b) {
    return (a + b) < a || (a + b) < b;
}

bool is_mul_overflow(unsigned a, unsigned b) {
    unsigned x = a * b;
    return a != 0 && (x / a) != b;
}
```

Overflow/underflow for signed integral types is **not defined** !! *Undefined behavior* must be checked before performing the operation

Floating-point Types and Arithmetic

IEEE Floating-Point Standard

IEEE754 is the technical standard for floating-point arithmetic

The standard defines the binary format, operations behavior, rounding rules, exception handling, etc.

- First Release: 1985
- Second Release: 2008. Add 16-bit floating point
- Third Release: 2019. Specify `min/max` behavior

see `The IEEE Standard 754: One for the History Books`

IEEE764 technical document:

`754-2019 - IEEE Standard for Floating-Point Arithmetic`

In general, **C/C++ adopts IEEE754 floating-point standard**:

`en.cppreference.com/w/cpp/types/numeric_limits/is_iec559`

32/64-bit Floating-Point

- **IEEE764 Single precision** (32-bit) `float`

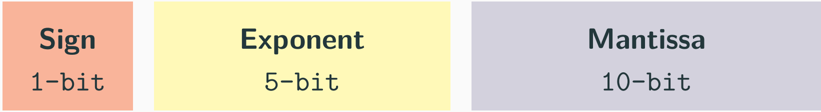
Sign	**Exponent** (or base)	**Mantissa** (or significant)
`1-bit`	`8-bit`	`23-bit`

- **IEEE764 Double precision** (64-bit) `double`

Sign	**Exponent** (or base)	**Mantissa** (or significant)
`1-bit`	`11-bit`	`52-bit`

16-bit Floating-Point (Standardized in C++23)

- **IEEE754 16-bit Floating-point** (`std::binary16`) →, GPU, Arm7

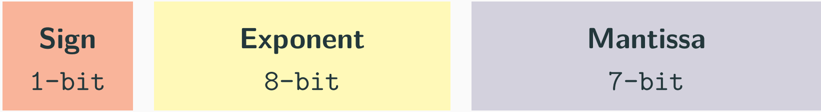


Sign 1-bit

Exponent 5-bit

Mantissa 10-bit

- **Google 16-bit Floating-point** (`std::bfloat16`) →, TPU, GPU, Arm8



Sign 1-bit

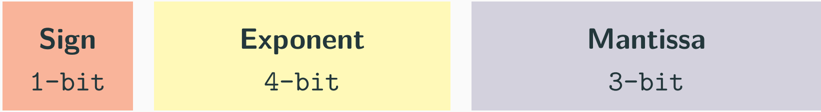
Exponent 8-bit

Mantissa 7-bit

`half-precision-arithmetic-fp16-versus-bfloat16`

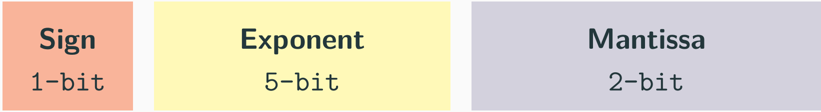
8-bit Floating-Point (Non-Standardized in C++/IEEE)

- `E4M3`



Sign 1-bit | Exponent 4-bit | Mantissa 3-bit

- `E5M2`



Sign 1-bit | Exponent 5-bit | Mantissa 2-bit

- `Floating Point Formats for Machine Learning`, *IEEE draft*
- `FP8 Formats for Deep Learning`, *Intel, Nvidia, Arm*

Other Real Value Representations (Non-standardized in C++/IEEE)

- **TensorFloat-32 (TF32)** Specialized floating-point format for deep learning applications

- **Posit** (John Gustafson, 2017), also called *unum III* (*universal number*), represents floating-point values with *variable-width* of exponent and mantissa.
  It is implemented in experimental platforms

- **Fixed-point** representation has a fixed number of digits after the radix point (decimal point). The gaps between adjacent numbers are always equal. The range of their values is significantly limited compared to floating-point numbers.
  It is widely used on embedded systems

- NVIDIA Hopper Architecture In-Depth
- Beating Floating Point at its Own Game: Posit Arithmetic
- Comparing posit and IEEE-754 hardware cost

Floating-point Representation 1/2

Floating-point number:

- *Radix* (or base): β
- *Precision* (or digits): p
- *Exponent* (magnitude): e
- *Mantissa*: M

$$n = \underbrace{M}_{p} \times \beta^{e} \quad \rightarrow \quad \text{IEEE754: } 1.M \times 2^{e}$$

```
float  f1 = 1.3f;   // 1.3
float  f2 = 1.1e2f; // 1.1 · 10^2
float  f3 = 3.7E4f; // 3.7 · 10^4
float  f4 = .3f;    // 0.3
double d1 = 1.3;    // without "f"
double d2 = 5E3;    // 5 · 10^3
```

Floating-point Representation 2/2

Exponent Bias

In IEEE754 floating point numbers, the exponent value is offset from the actual value by the **exponent bias**

- The exponent is stored as an unsigned value suitable for comparison
- Floating point values are lexicographic ordered
- For a single-precision number, the exponent is stored in the range $[1, 254]$ (0 and 255 have special meanings), and is biased by subtracting 127 to get an exponent value in the range $[-126, +127]$

0	10000111	11000000000000000000000
+	$2^{(135-127)} = 2^8$	$\frac{1}{2^1} + \frac{1}{2^2} = 0.5 + 0.25 = 0.75 \overset{normal}{\rightarrow} 1.75$

$$+1.75 * 2^8 = 448.0$$

Floating-point - Normal/Denormal 1/2

Normal number

A **normal** number is a floating point value that can be represented with *at least one bit set in the exponent* or the mantissa has all 0s

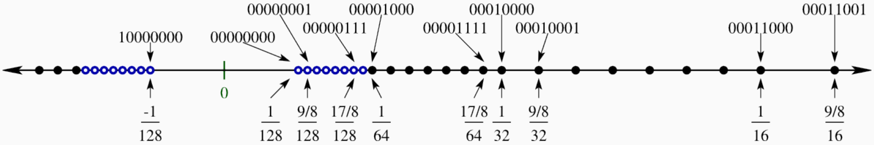
Denormal number

Denormal (or subnormal) numbers fill the underflow gap around zero in floating-point arithmetic. Any non-zero number with magnitude smaller than the smallest normal number is denormal

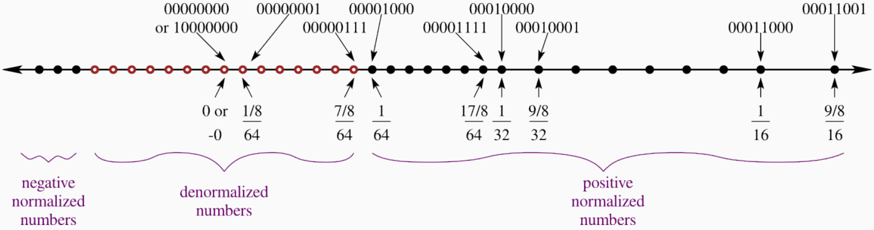
A **denormal** number is a floating point value that can be represented with *all 0s in the exponent*, but the mantissa is non-zero

Floating-point - Normal/Denormal 2/2

Why denormal numbers make sense: (↓ normal numbers)



The problem: distance values from zero (↓ denormal numbers)



www.toves.org/books/float/

Infinity 1/2

Infinity

In the IEEE754 standard, **`inf`** (infinity value) is a numeric data type value that exceeds the maximum (or minimum) representable value

Operations generating **`inf`**:

- $\pm\infty \cdot \pm\infty$
- $\pm\infty \cdot \pm\texttt{finite\_value}$
- $\texttt{finite\_value}$ *op* $\texttt{finite\_value} > \texttt{max\_value}$
- $\texttt{non-NaN} \,/ \pm 0$

There is a single representation for `+inf` and `-inf`

Comparison: $\texttt{(inf == finite\_value)} \rightarrow \texttt{false}$

$\texttt{(}\pm\texttt{inf == }\pm\texttt{inf)} \rightarrow \texttt{true}$

Infinity 2/2

```cpp
cout << 0 / 0;         // undefined behavior
cout << 0.0 / 0.0;     // print "nan"
cout << 5.0 / 0.0;     // print "inf"
cout << -5.0 / 0.0;    // print "-inf"

auto inf = std::numeric_limits<float>::infinity;
cout << (-0.0 == 0.0);                     // true, 0 == 0
cout << ((5.0f / inf) == ((-5.0f / inf));  // true, 0 == 0
cout << (10e40f) == (10e40f + 9999999.0f); // true, inf == inf
cout << (10e40)  == (10e40f + 9999999.0f); // false, 10e40 != inf
```

Not a Number (NaN)

NaN

In the IEEE754 standard, `NaN` (not a number) is a numeric data type value representing an undefined or unrepresentable value

Operations generating `NaN`:

- Operations with a `NaN` as at least one operand
- $\pm\infty \cdot \mp\infty$, $0 \cdot \infty$
- $0/0, \infty/\infty$
- $\sqrt{x}$, $\log(x)$ for $x < 0$
- $\sin^{-1}(x), \cos^{-1}(x)$ for $x < -1$ *or* $x > 1$

There are many representations for `NaN` (e.g. $2^{24} - 2$ for `float`)

Comparison: `(NaN == x)` $\rightarrow$ `false`, for every `x`

`(NaN == NaN)` $\rightarrow$ `false`

Machine Epsilon

Machine epsilon

Machine epsilon ε (or *machine accuracy*) is defined to be the smallest number that can be added to 1.0 to give a number other than one

IEEE 754 Single precision : $\varepsilon = 2^{-23} \approx 1.19209 * 10^{-7}$

IEEE 754 Double precision : $\varepsilon = 2^{-52} \approx 2.22045 * 10^{-16}$

Units at the Last Place (ULP)

ULP

Units at the Last Place is the gap between consecutive floating-point numbers

$$ULP(p, e) = \beta^{e-(p-1)} \rightarrow 2^{e-(p-1)}$$

Example:

$$\beta = 10,\ p = 3$$
$$\pi = 3.1415926... \rightarrow x = 3.14 \times 10^0$$
$$ULP(3, 0) = 10^{-2} = 0.01$$

Relation with ε:

- $\varepsilon = ULP(p, 0)$
- $ULP_x = \varepsilon * \beta^{e(x)}$

Floating-Point Representation of a Real Number

The machine floating-point representation $\mathbf{fl}(x)$ of a *real number* x is expressed as

$fl\,(x) = x\,(1 + \delta)$, where δ is a small constant

The approximation of a *real number* x has the following properties:

Absolute Error: $|fl(x) - x| \leq \frac{1}{2} \cdot ULP_x$

Relative Error: $\left|\frac{fl\,(x) - x}{x}\right| \leq \frac{1}{2} \cdot \varepsilon$

Floating-point - Cheatsheet

- NaN (mantissa $\neq 0$)

*	11111111	***********************

- ± infinity

*	11111111	00000000000000000000000

- Lowest/Largest ($\pm 3.40282 * 10^{+38}$)

*	11111110	11111111111111111111111

- Minimum (normal) ($\pm 1.17549 * 10^{-38}$)

*	00000001	00000000000000000000000

- Denormal number ($< 2^{-126}$)(minimum: $1.4 * 10^{-45}$)

*	00000000	***********************

- ± 0

*	00000000	00000000000000000000000

Floating-point - Cheatsheet 2/3

	`E4M3`	`E5M2`	`half`
Exponent	4 [0*-14] (no inf)	5-bit [0*-30]	
Bias	7	15	
Mantissa	4-bit	2-bit	10-bit
Largest ($\pm$)	$1.75 * 2^8$ 448	$1.75 * 2^{15}$ $57,344$	2^{16} $65,536$
Smallest ($\pm$)	2^{-6} 0.015625	2^{-14} 0.00006	
Smallest (denormal*)	2^{-9} 0.001953125	2^{-16} $1.5258 * 10^{-5}$	2^{-24} $6.0 \cdot 10^{-8}$
Epsilon	2^{-4} 0.0625	2^{-2} 0.25	2^{-10} 0.00098

Floating-point - Cheatsheet 3/3

	`bfloat16`	`float`	`double`
Exponent	8-bit [0*-254]		11-bit [0*-2046]
Bias	127		1023
Mantissa	7-bit	23-bit	52-bit
Largest ($\pm$)	2^{128} $3.4 \cdot 10^{38}$		2^{1024} $1.8 \cdot 10^{308}$
Smallest ($\pm$)	2^{-126} $1.2 \cdot 10^{-38}$		2^{-1022} $2.2 \cdot 10^{-308}$
Smallest (denormal*)	/	2^{-149} $1.4 \cdot 10^{-45}$	2^{-1074} $4.9 \cdot 10^{-324}$
Epsilon	2^{-7} 0.0078	2^{-23} $1.2 \cdot 10^{-7}$	2^{-52} $2.2 \cdot 10^{-16}$

Floating-point - Limits

```cpp
#include <limits>
// T: float or double

std::numeric_limits<T>::max();        // largest value

std::numeric_limits<T>::lowest();     // lowest value (C++11)

std::numeric_limits<T>::min();        // smallest value

std::numeric_limits<T>::denorm_min() // smallest (denormal) value

std::numeric_limits<T>::epsilon();    // epsilon value

std::numeric_limits<T>::infinity()    // infinity

std::numeric_limits<T>::quiet_NaN()  // NaN
```

Floating-point - Useful Functions

```
#include <cmath> // C++11

bool std::isnan(T value)     // check if value is NaN
bool std::isinf(T value)     // check if value is ±infinity
bool std::isfinite(T value)  // check if value is not NaN
                             // and not ±infinity

bool std::isnormal(T value); // check if value is Normal

T    std::ldexp(T x, p)      // exponent shift x * 2^p
int  std::ilogb(T value)     // extracts the exponent of value
```

Floating-point Arithmetic Properties 1/2

Floating-point operations are written

- $\oplus$ addition
- $\ominus$ subtraction
- $\otimes$ multiplication
- $\oslash$ division

$\odot \in \{\oplus, \ominus, \otimes, \oslash\}$

$op \in \{+, -, *, \backslash\}$ denotes exact precision operations

Floating-point Arithmetic Properties

(P1) In general, $a \ op \ b \neq a \odot b$

(P2) **Not Reflexive** $a \neq a$

- *Reflexive* without `NaN`

(P3) **Not Commutative** $a \odot b \neq b \odot a$

- *Commutative* without `NaN` ($\texttt{NaN} \neq \texttt{NaN}$)

(P4) In general, **Not Associative** $(a \odot b) \odot c \neq a \odot (b \odot c)$

(P5) In general, **Not Distributive** $(a \oplus b) \otimes c \neq (a \cdot c) \oplus (b \cdot c)$

(P6) **Identity on operations is not ensured** $(k \oslash a) \otimes a \neq k$

(P7) **No overflow/underflow** Floating-point has *“saturation”* values `inf`, `-inf`

- Adding (or subtracting) can “saturate” before `inf`, `-inf`

Detect Floating-point Errors ★ 1/2

C++11 allows determining if a floating-point exceptional condition has occurred by using floating-point exception facilities provided in `<cfenv>`

```
#include <cfenv>
// MACRO
FE_DIVBYZERO  // division by zero
FE_INEXACT    // rounding error
FE_INVALID    // invalid operation, i.e. NaN
FE_OVERFLOW   // overflow (reach saturation value +inf)
FE_UNDERFLOW  // underflow  (reach saturation value -inf)
FE_ALL_EXCEPT // all exceptions

// functions
std::feclearexcept(FE_ALL_EXCEPT); // clear exception status
std::fetestexcept(<macro>);        // returns a value != 0 if an
                                   // exception has been detected
```

Detect Floating-point Errors ★

```
#include <cfenv>    // floating point exceptions
#include <iostream>
#pragma STDC FENV_ACCESS ON // tell the compiler to manipulate the floating-point
                            // environment (not supported by all compilers)
                            // gcc: yes, clang: no
int main() {
    std::feclearexcept(FE_ALL_EXCEPT); // clear
    auto x = 1.0 / 0.0;                // all compilers
    std::cout << (bool) std::fetestexcept(FE_DIVBYZERO); // print true

    std::feclearexcept(FE_ALL_EXCEPT); // clear
    auto x2 = 0.0 / 0.0;               // all compilers
    std::cout << (bool)  std::fetestexcept(FE_INVALID);  // print true

    std::feclearexcept(FE_ALL_EXCEPT); // clear
    auto x4 = 1e38f * 10;              // gcc: ok
    std::cout << std::fetestexcept(FE_OVERFLOW);         // print true
}
```

see What is the difference between quiet NaN and signaling NaN?

Floating-point Issues

Some Examples... 1/4

Ariene 5: data conversion from 64-bit floating point value to 16-bit signed integer → *$137 million*

Patriot Missile: small chopping error at each operation, 100 hours activity → *28 deaths*

Some Examples... 2/4

Integer type is more accurate than floating type for large numbers

```
cout << 16777217;          // print 16777217
cout << (int) 16777217.0f; // print 16777216!!
cout << (int) 16777217.0;  // print 16777217, double ok
```

`float` numbers are different from `double` numbers

```
cout << (1.1 != 1.1f); // print true !!!
```

Some Examples... 3/4

The floating point precision is finite!

```
cout << setprecision(20);
cout << 3.33333333f; // print 3.333333254!!
cout << 3.33333333;  // print 3.333333333
cout << (0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1); // print 0.59999999999999998
```

Floating point arithmetic is not associative

```
cout << 0.1 + (0.2 + 0.3) == (0.1 + 0.2) + 0.3; // print false
```

IEEE764 Floating-point computation guarantees to produce **deterministic** output, namely the exact bitwise value for each run, if and only if the **order of the operations is always the same**

→ *same result on any machine and for all runs*

Some Examples... 4/4

"Using a double-precision floating-point value, we can represent easily the number of atoms in the universe.
If your software ever produces a number so large that it will not fit in a double-precision floating-point value, chances are good that you have a bug"

***Daniel Lemire**, Prof. at the University of Quebec*

" NASA uses just 15 digits of π to calculate interplanetary travel.
With 40 digits, you could calculate the circumference of a circle the size of the visible universe with an accuracy that would fall by less than the diameter of a single hydrogen atom"

***Latest in space**, Twitter*

`Number of atoms in the universe versus floating-point values`

Floating-point Algorithms

- `addition` **algorithm** (simplified):

(1) Compare the exponents of the two numbers. Shift the smaller number to the right until its exponent would match the larger exponent

(2) Add the mantissa

(3) Normalize the sum if needed (shift right/left the exponent by 1)

- `multiplication` **algorithm** (simplified):

(1) Multiplication of mantissas. The number of bits of the result is twice the size of the operands ($46 + 2$ bits, with $+2$ for implicit normalization)

(2) Normalize the product if needed (shift right/left the exponent by 1)

(3) Addition of the exponents

- `fused multiply-add` (`fma`):
  - Recent architectures (also GPUs) provide `fma` to compute addition and multiplication in a single instruction (performed by the compiler in most cases)
  - The rounding error of $fma(x, y, z)$ is less than $(x \otimes y) \oplus z$

Catastrophic Cancellation

Catastrophic Cancellation

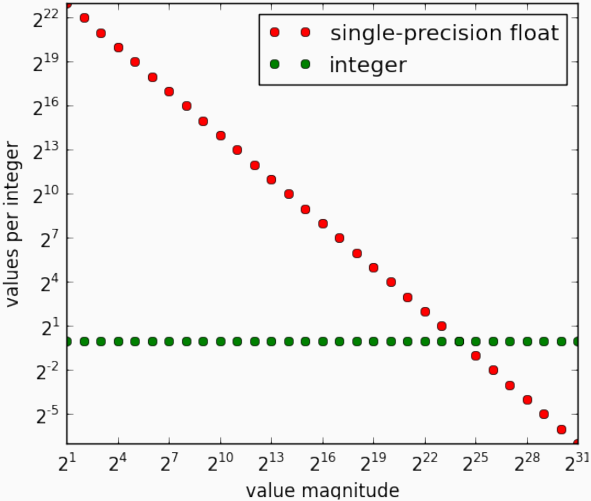
Catastrophic cancellation (or *loss of significance*) refers to loss of relevant information in a floating-point computation that cannot be revered

Two cases:

(C1) $\mathbf{a} \pm \mathbf{b}$, where $\mathbf{a} \gg \mathbf{b}$ or $\mathbf{b} \gg \mathbf{a}$. The value (or part of the value) of the smaller number is lost

(C2) $\mathbf{a} - \mathbf{b}$, where $\mathbf{a}, \mathbf{b}$ are approximation of exact values and $\mathbf{a} \approx \mathbf{b}$, namely a loss of precision in both $\mathbf{a}$ and $\mathbf{b}$. $\mathbf{a} - \mathbf{b}$ cancels most of the relevant part of the result because $\mathbf{a} \approx \mathbf{b}$. It implies a *small absolute error* but a *large relative error*

Catastrophic Cancellation (case 1) - Granularity



Intersection $= 16,777,216 = 2^{24}$

Catastrophic Cancellation (case 1) 3/5

How many iterations performs the following code?

```
while (x > 0)
    x = x - y;
```

How many iterations?

```
float:  x = 10,000,000   y = 1     -> 10,000,000
float:  x = 30,000,000   y = 1     -> does not terminate
float:  x =    200,000   y = 0.001 -> does not terminate
bfloat: x =        256   y = 1     -> does not terminate !!
```

Catastrophic Cancellation (case 1) 4/5

Floating-point increment

```
float x = 0.0f;
for (int i = 0; i < 20000000; i++)
x += 1.0f;
```

What is the value of `x` at the end of the loop?

Ceiling division $\left\lceil \frac{a}{b} \right\rceil$

```
//        std::ceil((float) 101 / 2.0f) -> 50.5f -> 51
float x = std::ceil((float) 20000001 / 2.0f);
```

What is the value of `x`?

Catastrophic Cancellation (case 2) 5/5

Let's solve a quadratic equation:

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$x^2 + 5000x + 0.25$

```
(-5000 + std::sqrt(5000.0f * 5000.0f - 4.0f * 1.0f * 0.25f)) / 2 // x2
(-5000 + std::sqrt(25000000.0f - 1.0f)) / 2 // catastrophic cancellation (C1)
(-5000 + std::sqrt(25000000.0f)) / 2
(-5000 + 5000) / 2 = 0                      // catastrophic cancellation (C2)
// correct result: 0.00005!!
```

$$\textit{relative error}: \frac{|0 - 0.00005|}{0.00005} = 100\%$$

Floating-point Comparison 1/3

The problem

```
cout << (0.11f + 0.11f < 0.22f); // print true!!
cout << (0.1f + 0.1f > 0.2f);    // print true!!
```

Do not use absolute error margins!!

```
bool areFloatNearlyEqual(float a, float b) {
    if (std::abs(a - b) < epsilon); // epsilon is fixed by the user
        return true;
    return false;
}
```

Problems:

- Fixed `epsilon` "looks small" but it could be too large when the numbers being compared are very small
- If the compared numbers are very large, the `epsilon` could end up being smaller than the smallest rounding error, so that the comparison always returns `false`

Floating-point Comparison 2/3

Solution: Use relative error $\frac{|a-b|}{b} < \varepsilon$

```
bool areFloatNearlyEqual(float a, float b) {
    if (std::abs(a - b) / b < epsilon); // epsilon is fixed
        return true;
    return false;
}
```

Problems:

- `a=0, b=0` The division is evaluated as `0.0/0.0` and the whole `if` statement is `(nan < espilon)` which always returns `false`
- `b=0` The division is evaluated as `abs(a)/0.0` and the whole `if` statement is `(+inf < espilon)` which always returns `false`
- **`a and b very small.`** The result should be `true` but the division by `b` may produces wrong results
- **`It is not commutative.`** We always divide by `b`

Floating-point Comparison 3/3

Possible solution: $\frac{|a-b|}{\max(|a|,|b|)} < \varepsilon$

```cpp
bool areFloatNearlyEqual(float a, float b) {
    constexpr float normal_min     = std::numeric_limits<float>::min();
    constexpr float relative_error = <user_defined>

    if (!std::isfinite(a) || !isfinite(b)) // a = ±∞, NaN  or  b = ±∞, NaN
        return false;
    float diff  = std::abs(a - b);
    // if "a" and "b" are near to zero, the relative error is less effective
    if (diff <= normal_min) // or also: user_epsilon *  normal_min
        return true;

    float abs_a = std::abs(a);
    float abs_b = std::abs(b);
    return (diff / std::max(abs_a, abs_b)) <= relative_error;
}
```

Minimize Error Propagation - Summary

- Prefer **multiplication/division** rather than addition/subtraction

- Try to reorganize the computation to **keep near** numbers with the same scale (e.g. sorting numbers)

- Consider to **put a zero** very small number (under a threshold). Common application: iterative algorithms

- Scale by a **power of two** is safe

- **Switch to log scale**. Multiplication becomes Add, and Division becomes Subtraction

- Use a **compensation algorithm** like Kahan summation, Dekker's FastTwoSum, Rump's AccSum

References

Suggest readings:

- What Every Computer Scientist Should Know About Floating-Point Arithmetic
- Do Developers Understand IEEE Floating Point?
- Yet another floating point tutorial
- Unavoidable Errors in Computing

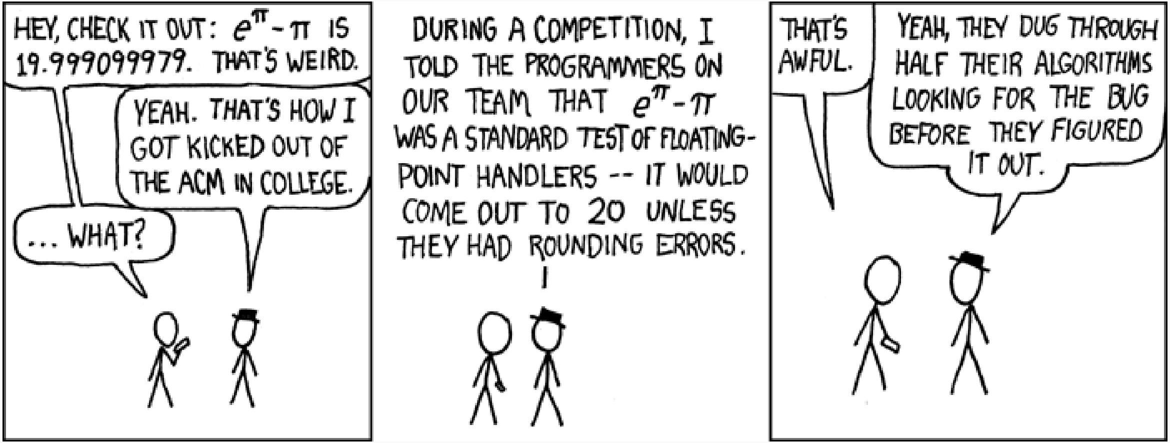
Floating-point Comparison readings:

- The Floating-Point Guide - Comparison
- Comparing Floating Point Numbers, 2012 Edition
- Some comments on approximately equal FP comparisons
- Comparing Floating-Point Numbers Is Tricky

Floating point tools:

- IEEE754 visualization/converter
- Find and fix floating-point problems

On Floating-Point


HEY, CHECK IT OUT: $e^{\pi} - \pi$ IS 19.999099979. THAT'S WEIRD.
YEAH. THAT'S HOW I GOT KICKED OUT OF THE ACM IN COLLEGE.
... WHAT?
DURING A COMPETITION, I TOLD THE PROGRAMMERS ON OUR TEAM THAT $e^{\pi} - \pi$ WAS A STANDARD TEST OF FLOATING-POINT HANDLERS -- IT WOULD COME OUT TO 20 UNLESS THEY HAD ROUNDING ERRORS.
THAT'S AWFUL.
YEAH, THEY DUG THROUGH HALF THEIR ALGORITHMS LOOKING FOR THE BUG BEFORE THEY FIGURED IT OUT.