Problem Set #7

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Elaboration

A major part of financial crisis literature is concerned with the causes of starting a debt crises. When the investment of a country is higher than its savings, the country needs to decline the consumption to service the debt. But if the production of the country (GDP) does not cover the consumption, investment and debt services, debt will rise Stein (2005). In this practice, I want to see the optimum amount of a debt for a country. Hence, I will define my problem as follows:

There are a continuum of measure one of countries uniformly distributed on $[0, 1]$ in the infinite time horizon. The preferences as a function of consumption are defined as:

$$u^i(c^i) = \log c^i$$

where $i = l, b$ is type of the country because in each period some countries are lenders (l) and some others are borrowers (b) of external debt. This economy has the technology of a neoclassical growth model as:

$$y^i_t = Ak^{i\alpha}_t$$

In this economy, the information is determined through

$$\varepsilon_t \sim i.i.d.N(0, \sigma)$$

$$\pi_{ij} : Pr(\varepsilon_{t+1} = j|\varepsilon_t = i), \quad i, j = l, b$$

Transition equations in this problem are defined as:

$$k^i_{t+1} = (1 - \delta)k^i_t + I_t$$

$$D^i_{t+1} = y^i_t - c^i_t - I^i_t - (1 + r^i_t\varepsilon_t)D^i_t$$

The countries problem is defined as:

$$\begin{aligned}
\max_{c,k',D'} \quad & \sum_{t=0}^{\infty} \beta^t u^i(c^i_t) \\
\text{s.t.} \quad & c^i_t + I^i_t + (1 + r^i_t\varepsilon_t)D^i_t \leq y^i_t + D^i_{t+1} \\
& c^i_t \geq 0 \\
& \lim_{t\to\infty} (\prod_{s=0}^{t} \frac{1}{1 + r^i_s})D^i_{t+1} = 0 \\
& k^i_{t+1} = (1 - \delta)k^i_t + I^i_t
\end{aligned} \tag{1}$$

Markets of goods and external debt are cleared as

$$c^i_t = y^i_t - k^i_{t+1} + (1 - \delta)k^i_t$$

$$\sum_i D^i_t = 0$$

To solve this problem for a country, due to Inada conditions, the country has an interior solution and $c_t^i > 0$. The Bellman equation for the planner is determined as:

$$V(\varepsilon, k, D) = \max_{k', D'} \quad u(Ak^\alpha - k' + (1-\delta)k + D' - (1+r\varepsilon)D) + \beta E_{\varepsilon'|\varepsilon} V(\varepsilon', k', D') \tag{2}$$

The state variables are k, D, ε and the control variables are c, k', D'. First order necessary conditions (FONCs):

$$\begin{aligned}\frac{\partial V(\varepsilon, k', D')}{\partial D'} &= \frac{1}{Ak^\alpha - k' + (1-\delta)k + D' - (1+r\varepsilon)D} \\ &+ \beta E(V_{k'}(\varepsilon', Ak^\alpha - k' + (1-\delta)k + D' - (1+r\varepsilon)D)) \\ &= 0\end{aligned} \tag{3}$$

$$\begin{aligned}\frac{\partial V(\varepsilon, k', D')}{\partial k'} &= \frac{-1}{Ak^\alpha - k' + (1-\delta)k + D' - (1+r\varepsilon)D} \\ &+ \beta E(\frac{[A\alpha k'^{\alpha-1} + (1-\delta)]^2}{Ak'^\alpha - k'' + (1-\delta)k' + D'' - (1+r'\varepsilon')D'}) \\ &= 0\end{aligned} \tag{4}$$

$$\begin{aligned}\frac{\partial V(\varepsilon, k, D)}{\partial k} &= \frac{A\alpha k^{\alpha-1} - (1-\delta)}{Ak^\alpha - k' + (1-\delta)k + D' - (1+r\varepsilon)D} \\ &= 0\end{aligned} \tag{5}$$

Equations 3, 4, 5 along with the market clearing conditions will solve the problem for the country.

References

Stein, Jerome L. 2005. Applications of stochastic optimal control/dynamic programming to international finance and debt crises. *Nonlinear Analysis: Theory, Methods & Applications*, **63**(5-7), e2033–e2041.