3 LEVELLING

3.1 LEVELLING

Levelling is an operation in surveying performed to determine the difference in levels of two points. By this operation the height of a point from a *datum*, known as *elevation*, is determined.

3.2 LEVEL SURFACE

A *level surface* is the equipotential surface of the earth's gravity field. It is a curved surface and every element of which is normal to the plumb line.

3.3 DATUM

A *datum* is a reference surface of constant potential, called as a level surface of the earth's gravity field, for measuring the elevations of the points. One of such surfaces is the mean sea level surface and is considered as a standard datum. Also an arbitrary surface may be adopted as a datum.

3.4 LEVEL LINE

A line lying in a level surface is a *level line*. It is thus a curved line.

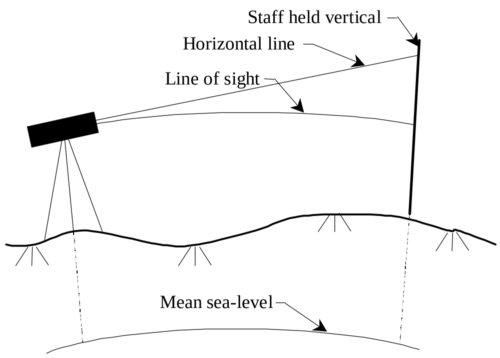

Fig. 3.1

A level in proper adjustment, and correctly set up, produces a horizontal line of sight which is at right angles to the direction of gravity and tangential to the level line at the instrument height. It follows a constant height above mean sea level and hence is a curved line, as shown in Fig. 3.1.

Over short distances, such as those met in civil engineering works, the two lines can be taken to coincide. Over long distances a correction is required to reduce the staff readings given by the horizontal line of sight to the level line equivalent. Refraction of the line of sight is also to be taken into account. The corrections for the curvature of the level line C_c and refraction C_r are shown in

Fig. 3.2. The combined correction is given by

$$C_{cr} = -\frac{3}{7}\frac{d^2}{R} \qquad ...(3.1)$$

where

C_{cr} = the correction for the curvature and refraction,

d = the distance of the staff from the point of tangency, and

R = the mean earth's radius.

For the value of R = 6370 km and d in kilometre, the value of C_{cr} in metre is given as

$$C_{cr} = -0.067d^2 \qquad ...(3.2)$$

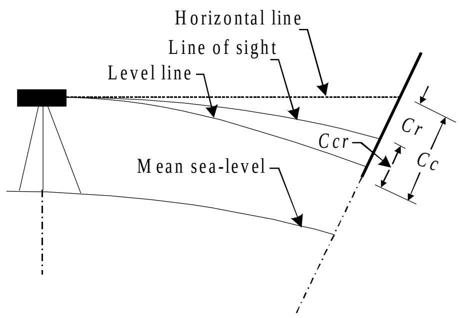

Fig. 3.2

3.5 DIRECT DIFFERENTIAL OR SPIRIT LEVELLING

Differential levelling or spirit levelling is the most accurate simple direct method of determining the difference of level between two points using an instrument known as *level* with a *levelling staff*. A level establishes a horizontal line of sight and the difference in the level of the line of sight and the point over which the levelling staff is held, is measured through the levelling staff.

Fig. 3.3 shows the principle of determining the difference in level Δh between two points A and B, and thus the elevation of one of them can be determined if the elevation of the other one is known. S_A and S_B are the staff readings at A and B, respectively, and h_A and h_B are their respective elevations.

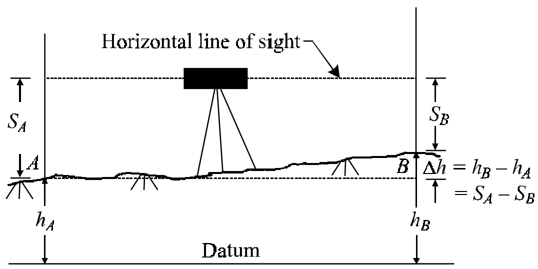

Fig. 3.3

From the figure, we find that

(*i*) if $S_B < S_A$, the point B is higher than point A.

(*ii*) if $S_B > S_A$, the point B is lower than point A.

(*iii*) to determine the difference of level, the elevation of ground point at which the level is set up, is not required.

Booking and Reducing the Levels

Before discussing the booking and methods of reducing levels, the following terms associated with differential levelling must be understood.

Station: A station is the point where the levelling staff is held. (Points A, a, b, B, c, and C in Fig. 3.4).

Height of instrument (H.I.) or height of collimation: For any set up of the level, the elevation of the line of sight is the height of instrument. (H.I. = $h_A + S_A$ in Fig. 3.3).

Back sight (B.S.): It is the first reading taken on the staff after setting up the level usually to determine the height of instrument. It is usually made to some form of a bench mark (B.M.) or to the points whose elevations have already been determined. When the instrument position has to be changed, the first sight taken in the next section is also a back sight. (Staff readings S_1 and S_5 in Fig. 3.4).

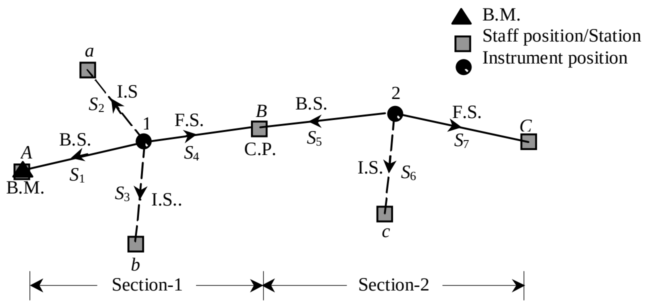

Fig. 3.4

Fore sight (F.S.): It is the last reading from an instrument position on to a staff held at a point. It is thus the last reading taken within a section of levels before shifting the instrument to the next section, and also the last reading taken over the whole series of levels. (Staff readings S_4 and S_7 in Fig. 3.4).

Change point (C.P.) or turning point: A change point or turning point is the point where both the fore sight and back sight are made on a staff held at that point. A change point is required before moving the level from one section to another section. By taking the fore sight the elevation of the change point is determined and by taking the back sight the height of instrument is determined. The change points relate the various sections by making fore sight and back sight at the same point. (Point B in Fig. 3.4).

Intermediate sight (I.S.): The term ‘intermediate sight’ covers all sightings and consequent staff readings made between back sight and fore sight within each section. Thus, intermediate sight station is neither the change point nor the last point. (Points a, b, and c in Fig. 3.4).

Balancing of sights: When the distances of the stations where back sight and fore sight are taken from the instrument station, are kept approximately equal, it is known as balancing of sights. Balancing of sights minimizes the effect of instrumental and other errors.

Reduced level (R.L.): Reduced level of a point is its height or depth above or below the assumed datum. It is the elevation of the point.

Rise and fall: The difference of level between two consecutive points indicates a rise or a fall between the two points. In Fig. 3.3, if $(S_A - S_B)$ is positive, it is a rise and if negative, it is a fall. Rise and fall are determined for the points lying within a section.

Section: A section comprises of one back sight, one fore sight and all the intermediate sights taken from one instrument set up within that section. Thus the number of sections is equal to the number of set ups of the instrument. (From *A* to *B* for instrument position 1 is section-1 and from *B* to *C* for instrument position 2 is section-2 in Fig. 3.4).

For booking and reducing the levels of points, there are two systems, namely the *height of instrument* or *height of collimation method* and *rise and fall method*. The columns for booking the readings in a level book are same for both the methods but for reducing the levels, the number of additional columns depends upon the method of reducing the levels. Note that except for the change point, each staff reading is written on a separate line so that each staff position has its unique reduced level. This remains true at the change point since the staff does not move and the back sight from a forward instrument station is taken at the same staff position where the fore sight has been taken from the backward instrument station. To explain the booking and reducing levels, the levelling operation from stations *A* to *C* shown in Fig. 3.4, has been presented in Tables 3.1 and 3.2 for both the methods. These tables may have additional columns for showing chainage, embankment, cutting, etc., if required.

Table 3.1 Height of instrument method

Station	B.S.	I.S.	F.S.	H.I.	R.L.	Remarks	
A	S_1			H.I.$_A$ = $h_A + S_1$	h_A	B.M. = *ha*	Section-1
a		S_2			h_a = H.I.$_A - S_2$		
b		S_3			h_b = H.I.$_A - S_3$		
B	S_5		S_4	H.I.$_B$ = $h_B + S_5$	h_B = H.I.$_A - S_4$	C.P.	–
c		S_6			h_c = H.I.$_B - S_6$		Section-2
C			S_7		H_C = H.I.$_B - S_7$		
	Σ B.S.		Σ F.S.				
Check: Σ B.S. – Σ F.S. = Last R.L. – First R.L.							

In reducing the levels for various points by the height of instrument method, the height of instrument (H.I.) for the each section highlighted by different shades, is determined by adding the elevation of the point to the back sight reading taken at that point. The H.I. remains unchanged for all the staff readings taken within that section and therefore, the levels of all the points lying in that section are reduced by subtracting the corresponding staff readings, i.e., I.S. or F.S., from the H.I. of that section.

In the rise and fall method, the rises and the falls are found out for the points lying within each section. Adding or subtracting the rise or fall to or from the reduced level of the backward station obtains the level for a forward station. In Table 3.2, r and f indicate the rise and the fall, respectively, assumed between the consecutive points.

Table 3.2 Rise and fall method

Station	B.S.	I.S.	F.S.	Rise	Fall	R.L.	Remarks	
A	S_1					h_A	B.M. = h_a	Section-1
a		S_2		$r_1 = S_1 - S_2$		$h_a = h_A + r_1$		
b		S_3			$f_1 = S_2 - S_3$	$h_b = h_a - f_1$		
B	S_5		S_4		$f_2 = S_3 - S_4$	$h_B = h_b - f_2$	C.P.	
c		S_6			$f_3 = S_5 - S_6$	$h_c = h_B - f_3$		
C			S_7	$r_2 = S_6 - S_7$		$H_C = h_c + r_2$		Section-2
	Σ B.S.		Σ F.S.	Σ Rise	Σ Fall			
Check:	Σ B.S. – Σ F.S. = Σ Rise – Σ Fall = Last R.L. – First R.L.							

The arithmetic involved in reduction of the levels is used as check on the computations. The following rules are used in the two methods of reduction of levels.

(*a*) For the height of instrument method

(*i*) Σ B.S. – Σ F.S. = Last R.L. – First R.L.

(*ii*) Σ [H.I. × (No. of I.S.'s + 1)] – Σ I.S. – Σ F.S. = Σ R.L. – First R.L.

(*b*) For the rise and fall method

Σ B.S. – Σ F.S. = Σ Rise – Σ Fall = Last R.L. – First R.L.

3.6 COMPARISON OF METHODS AND THEIR USES

Less arithmetic is involved in the reduction of levels with the height of instrument method than with the rise and fall method, in particular when large numbers of intermediate sights is involved. Moreover, the rise and fall method gives an arithmetic check on all the levels reduced, i.e., including the points where the intermediate sights have been taken, whereas in the height of instrument method, the check is on the levels reduced at the change points only. In the height of instrument method the check on all the sights is available only using the second formula that is not as simple as the first one.

The height of instrument method involves less computation in reducing the levels when there are large numbers of intermediate sights and thus it is faster than the rise and fall method. The rise and fall method, therefore, should be employed only when a very few or no intermediate sights are taken in the whole levelling operation. In such case, frequent change of instrument position requires determination of the height of instrument for the each setting of the instrument and, therefore, computations involved in the height of instrument method may be more or less equal to that required in the rise and fall method. On the other hand, it has a disadvantage of not having check on the intermediate sights, if any, unless the second check is applied.

3.7 LOOP CLOSURE AND ITS APPORTIONING

A *loop closure* or *misclosure* is the amount by which a level circuit fails to close. It is the difference of elevation of the measured or computed elevation and known or established elevation of the same point. Thus loop closure is given by

$$e = \text{computed value of R.L.} - \text{known value of R.L.}$$

If the length of the loop or circuit is L and the distance of a station to which the correction c is computed, is l, then

$$c = -e\frac{l}{L} \quad \text{...(3.3)}$$

Alternatively, the correction is applied to the elevations of each change point and the closing point of known elevation. If there are n_1 change points then the total number points at which the corrections are to be applied is

$$n = n_1 + 1$$

and the correction at each point is

$$= -\frac{e}{n} \quad \text{...(3.4)}$$

The corrections at the intermediate points are taken as same as that for the change points to which they are related.

Another approach could be to apply total of $-e/2$ correction equally to all the back sights and total of $+e/2$ correction equally to all the fore sights. Thus if there are n_B back sights and n_F fore sights then

$$\text{correction to each back sight} = -\frac{e}{2n_B}$$

$$\text{correction to each fore sight} = +\frac{e}{2n_F} \quad \text{...(3.5)}$$

3.8 RECIPROCAL LEVELLING

Reciprocal levelling is employed to determine the correct difference of level between two points which are quite apart and where it is not possible to set up the instrument between the two points for balancing the sights. It eliminates the errors due to the curvature of the earth, atmospheric refraction and collimation.

If the two points between which the difference of level is required to be determined are A and B then in reciprocal levelling, the first set of staff readings (a_1 and b_1) is taken by placing the staff on A and B, and instrument close to A. The second set of readings (a_2 and b_2) is taken again on A and B by placing the instrument close to B. The difference of level between A and B is given by

$$\Delta h = \frac{(a_1 - b_1) + (a_2 - b_2)}{2} \quad \text{...(3.6)}$$

and the combined error is given by

$$e = \frac{(b_1 - a_1) - (b_2 - a_2)}{2} \qquad ...(3.7)$$

where

$$e = e_l + e_c - e_r \qquad ...(3.8)$$

e_l = the collimation error assumed positive for the line of sight inclined upward,

e_c = the error due to the earth's curvature, and

e_r = the error due to the atmospheric refraction.

We have

$$e_c - e_r = \text{the combined curvature and refraction error} = 0.067d^2.$$

The collimation error is thus given by

$$e_l = e - 0.067d^2 \text{ in metre} \qquad ...(3.9)$$

where d is the distance between A and B in kilometre.

3.9 TRIGONOMETRIC LEVELLING

Trigonometric levelling involves measurement of vertical angle and either the horizontal or slope distance between the two points between which the difference of level is to be determined. Fig. 3.5 shows station A and station B whose height is to be established by reciprocal observations from A on to signal at B and from B on to signal at A. Vertical angles α (angle of elevation) and β (angle of depression), are measured at A and B, respectively. The refracted line of sight will be inclined to the direct line AB and therefore, the tangent to the refracted line of sight makes an angle υ with AB. The vertical angle α is measured with respect to this tangent and to the horizontal at A.

Similarly, from B the angle of depression β is measured from the horizontal to the tangent to the line of sight. The point C lies on the arc through A, which is parallel to the mean sea level surface. d is the geodetic or spheroidal distance between A and B, and could be deduced from the geodetic coordinates of the two points. Angle BAC between AB and chord AC is related to angle θ between the two verticals at A and B which meet at the earth's centre, since AC makes an angle of $\theta/2$ with the horizontal at A.

If the elevations of A and B are h_A and h_B, respectively, then the difference in elevation Δh between A and B, is found out by solving the triangle ABC for BC. Thus in triangle ABC

$$BC = AC \frac{\sin\left(\alpha + \frac{\theta}{2} - \upsilon\right)}{\sin\left[180^\circ - \left(90^\circ + \frac{\theta}{2}\right) - \left(\alpha + \frac{\theta}{2} - \upsilon\right)\right]}$$

$$= AC \frac{\sin(\alpha + \frac{\theta}{2} - \upsilon)}{\cos(\alpha + \theta - \upsilon)} \qquad ...(3.10)$$

The correction for curvature and refraction at A and B is $(\theta/2 - \upsilon)$ and this refers angles a and β to chords AC and BD, respectively.

Thus angle of elevation $BAC = \alpha + \theta/2 - \upsilon$ and angle of depression $DBA = \beta - \theta/2 + \upsilon$.

Since AC is parallel to BD, $\angle BAC = \angle DBA$,

Therefore, $\angle BAC + \angle DBA = (\alpha + \theta/2 - \upsilon) + (\beta - \theta/2 + \upsilon)$

or $$2\angle BAC = \alpha + \beta$$

or $$\angle BAC = \frac{\alpha + \beta}{2} = \angle DBA.$$

This is the correct angle of elevation at A or angle of depression at B and is mean of the two angles α and β. In addition, we can write Eq. (3.10) as

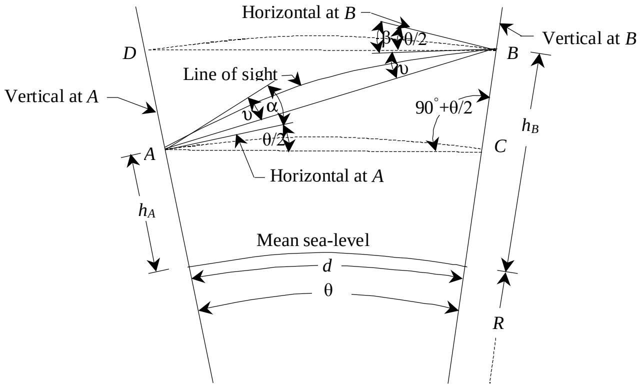

Fig. 3.5

$$BC = AC \tan(\alpha + \theta/2 - \upsilon)$$

or $$\Delta h = AC \tan\left(\frac{\alpha + \beta}{2}\right).$$

Therefore, $$h_B = h_A + \Delta h$$

$$= h_A + AC \tan\left(\frac{\alpha + \beta}{2}\right) \quad \ldots (3.11)$$

Note that in Eq. (3.11) it is assumed that α is the angle of elevation and β is angle of depression. In practice, only magnitudes need to be considered, not signs provided one angle is elevation and the other is depression.

The coefficient of refraction K in terms of the angle of refraction υ and the angle θ subtended at the centre of the spheroid by the arc joining the stations, is given by

$$K = \frac{\upsilon}{\theta} \quad \ldots (3.12)$$

3.10 SENSITIVITY OF A LEVEL TUBE

The *sensitivity of a level tube* is expressed in terms of angle in seconds subtended at the centre by the arc of one division length of the level tube. The radius of curvature of the inner surface of the upper portion of the level tube is also a measure of the sensitivity. The sensitivity of a level tube depends upon radius of curvature of the inner surface of the level tube and its diameter. It also depends upon the length of the vapour bubble, viscosity and surface tension of the liquid and smoothness of the inner surface of the tube.

If α' is the sensitivity of the level tube it is given by

$$\alpha' = \frac{s}{nD\sin 1''} \text{ seconds} \quad \ldots(3.13)$$

where

s = the change in the staff reading for movement of the bubble by n divisions (Fig.3.6), and

D = the distance of the staff from the instrument.

The radius of curvature of the level tube is expressed as

$$R = \frac{nlD}{s} \quad \ldots(3.14)$$

where l is the length of one division of the level tube.

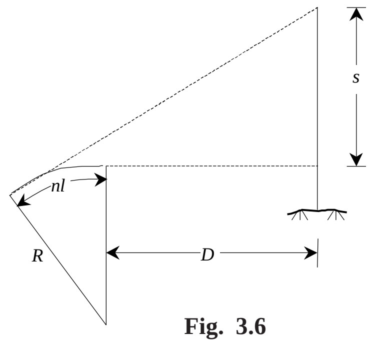

Fig. 3.6

3.11 TWO-PEG TEST

Two-peg test is conducted for checking the adjustment of a level. Fig. 3.7 shows the method of conducting the test. Two rigid points A and B are marked on the ground with two pegs and the instrument is set up exactly between them at point C. Readings are taken on the staff held at A and B, and the difference between them gives the correct difference in level of the pegs. The equality in length of back sight and fore sight ensures that any instrumental error, e, is equal on both sights and is cancelled out in the difference of the two readings. The instrument is then moved to D so that it is outside the line AB and it is near to one of the pegs. Readings are again taken on the staff held at A and B. The difference in the second set of the staff readings is equal to the difference in level of the points A and B, and it will be equal to that determined with the first set of readings if the instrument is in adjustment. If the two values of the difference in level differ from each other, the instrument is out of adjustment.

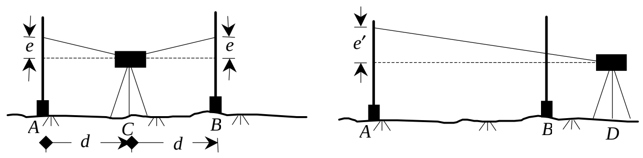

Fig. 3.7

The adjustment of the instrument can also be tested by determining the difference in level of the points A and B by placing the instrument at C and D as shown in Fig. 3.8.

Fig. 3.8

3.12 EYE AND OBJECT CORRECTION

The heights of the instrument and signal at which the observation is made, are generally not same and thus the observed angle of elevation θ' as shown in Fig. 3.9, does not refer to the ground levels at A and B. This difference in height causes the observed vertical angle θ' to be larger than that θ which would have been observed directly from those points. A correction (ε) termed as *eye and object correction,* is applied to the observed vertical angle to reduce it to the required value.

The value of the eye and object correction is given by

$$\varepsilon = \frac{h_s - h_i}{d} \text{ radians} \qquad \ldots(3.15)$$

$$= 206265 \frac{h_s - h_i}{d} \text{ seconds} \qquad \ldots(3.16)$$

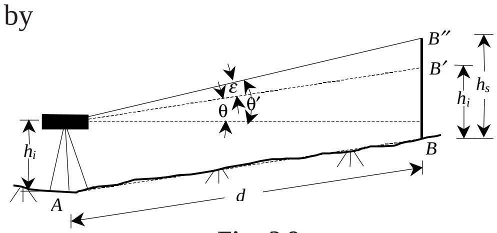

Fig. 3.8

Example 3.1. The following readings were taken with a level and 4 m staff. Draw up a level book page and reduce the levels by the height of instrument method.

0.578 B.M.(= 58.250 m), 0.933, 1.768, 2.450, (2.005 and 0.567) C.P., 1.888, 1.181, (3.679 and 0.612) C.P., 0.705, 1.810.

Solution:

The first reading being on a B.M., is a back sight. As the fifth station is a change point, 2.005 is fore sight reading and 0.567 is back sight reading. All the readings between the first and fifth readings are intermediate sight-readings. Similarly, the eighth station being a change point, 3.679 is fore sight reading, 0.612 is back sight reading, and 1.888, 1.181 are intermediate sight readings. The last reading 1.810 is fore sight and 0.705 is intermediate sight-readings. All the readings have been entered in their respective columns in the following table and the levels have been reduced by height of instrument method. In the following computations, the values of B.S., I.S., H.I., etc., for a particular station have been indicated by its number or name.

Section-1:

$$\text{H.I.}_1 = h_1 + \text{B.S.}_1 = 58.250 + 0.578 = 58.828 \text{ m}$$
$$h_2 = \text{H.I.}_1 - \text{I.S.}_2 = 58.828 - 0.933 = 57.895 \text{ m}$$
$$h_3 = \text{H.I.}_1 - \text{I.S.}_3 = 58.828 - 1.768 = 57.060 \text{ m}$$
$$h_4 = \text{H.I.}_1 - \text{I.S.}_4 = 58.828 - 2.450 = 56.378 \text{ m}$$
$$h_5 = \text{H.I.}_1 - \text{F.S.}_5 = 58.828 - 2.005 = 56.823 \text{ m}$$

Section-2:

$$\text{H.I.}_5 = h_5 + \text{B.S.}_5 = 56.823 + 0.567 = 57.390 \text{ m}$$
$$h_6 = \text{H.I.}_2 - \text{I.S.}_6 = 57.390 - 1.888 = 55.502 \text{ m}$$
$$h_7 = \text{H.I.}_2 - \text{I.S.}_7 = 57.390 - 1.181 = 56.209 \text{ m}$$
$$h_8 = \text{H.I.}_2 - \text{F.S.}_8 = 57.390 - 3.679 = 53.711 \text{ m}$$

Section-3:

$$\text{H.I.}_8 = h_8 + \text{B.S.}_8 = 53.711 + 0.612 = 54.323 \text{ m}$$
$$h_9 = \text{H.I.}_8 - \text{I.S.}_9 = 54.323 - 0.705 = 53.618 \text{ m}$$
$$h_{10} = \text{H.I.}_8 - \text{F.S.}_{10} = 54.323 - 1.810 = 52.513 \text{ m}$$

Additional check for H.I. method: Σ [H.I. $\times$ (No. of I.S.s + 1)] – Σ I.S. – Σ F.S. = Σ R.L. – First R.L.

[58.828 × 4 + 57.390 × 3 + 54.323 × 2] – 8.925 – 7.494 = 557.959 – 58.250 = 499.709 (*O.K.*)

Table 3.3

Station	B.S.	I.S.	F.S.	H.I.	R.L.	Remarks
1	0.578			58.828	58.250	B.M.=58.250 m
2		0.933			57.895	
3		1.768			57.060	
4		2.450			56.378	
5	0.567		2.005	57.390	56.823	C.P.
6		1.888			55.502	
7		1.181			56.209	
8	0.612		3.679	54.323	53.711	C.P.
9		0.705			53.618	
10			1.810		52.513	
Σ	1.757	8.925	7.494		557.956	
Check: 1.757 – 7.494 = 52.513 – 58.250 = – 5.737 (*O.K.*)						

Example 3.2. Reduce the levels of the stations from the readings given in the Example 3.1 by the rise and fall method.

Solution:

Booking of the readings for reducing the levels by rise and fall method is same as explained in Example 3.1. The computations of the reduced levels by rise and fall method is given below and the results are tabulated in the table. In the following computations, the values of B.S., I.S., Rise (r), Fall (f), etc., for a particular station have been indicated by its number or name.

(i) Calculation of rise and fall

Section-1:

$$f_2 = \text{B.S.}_1 - \text{I.S.}_2 = 0.578 - 0.933 = 0.355$$
$$f_3 = \text{I.S.}_2 - \text{I.S.}_3 = 0.933 - 1.768 = 0.835$$
$$f_4 = \text{I.S.}_3 - \text{I.S.}_4 = 1.768 - 2.450 = 0.682$$
$$r_5 = \text{I.S.}_4 - \text{F.S.}_5 = 2.450 - 2.005 = 0.445$$

Section-2:
$$f_6 = \text{B.S.}_5 - \text{I.S.}_6 = 0.567 - 1.888 = 1.321$$
$$f_7 = \text{I.S.}_6 - \text{I.S.}_7 = 1.888 - 1.181 = 0.707$$
$$f_8 = \text{I.S.}_7 - \text{F.S.}_8 = 1.181 - 3.679 = 2.498$$

Section-3:
$$f_9 = \text{B.S.}_8 - \text{I.S.}_9 = 0.612 - 0.705 = 0.093$$
$$f_{10} = \text{I.S.}_9 - \text{F.S.}_{10} = 0.705 - 1.810 = 1.105$$

(*ii*) Calculation of reduced levels

$$h_2 = h_1 - f_2 = 58.250 - 0.355 = 57.895 \text{ m}$$
$$h_3 = h_2 - f_3 = 57.895 - 0.835 = 57.060 \text{ m}$$
$$h_4 = h_3 - f_4 = 57.060 - 0.682 = 56.378 \text{ m}$$
$$h_5 = h_4 + r_5 = 56.378 + 0.445 = 56.823 \text{ m}$$
$$h_6 = h_5 - f_6 = 56.823 - 1.321 = 55.502 \text{ m}$$
$$h_7 = h_6 + r_7 = 55.502 + 0.707 = 56.209 \text{ m}$$
$$h_8 = h_7 - f_8 = 56.209 - 2.498 = 53.711 \text{ m}$$
$$h_9 = h_8 - f_9 = 53.711 - 0.093 = 53.618 \text{ m}$$
$$h_{10} = h_9 - f_{10} = 53.618 - 1.105 = 52.513 \text{ m}$$

Table 3.4

Station	B.S.	I.S.	F.S.	Rise	Fall	R.L.	Remarks
1	0.578					58.250	B.M.=58.250 m
2		0.933			0.355	57.895	
3		1.768			0.835	57.060	
4		2.450			0.682	56.378	
5	0.567		2.005	0.445		56.823	C.P.
6		1.888			1.321	55.502	
7		1.181		0.707		56.209	
8	0.612		3.679		2.498	53.711	C.P.
9		0.705			0.093	53.618	
10			1.810		1.105	52.513	
Σ	1.757		7.494	1.152	6.889		
Check: 1.757 – 7.494 = 1.152 – 6.889 = 52.513 – 58.250 = – 5.737 (*O.K.*)							

Example 3.3. The following consecutive readings were taken with a level on continuously sloping ground at a common interval of 20 m. The last station has an elevation of 155.272 m. Rule out a page of level book and enter the readings. Calculate

(*i*) the reduced levels of the points by rise and fall method, and (*ii*) the gradient of the line joining the first and last points.

0.420, 1.115, 2.265, 2.900, 3.615, 0.535, 1.470, 2.815, 3.505, 4.445, 0.605, 1.925, 2.885.

Solution:

Since the readings have been taken along a line on a continuously sloping ground, any sudden large change in the reading such as in the sixth reading compared to the fifth reading and in the eleventh reading compared to the tenth reading, indicates the change in the instrument position. Therefore, the sixth and eleventh readings are the back sights and fifth and tenth readings are the fore sights. The first and the last readings are the back sight and fore sight, respectively, and all remaining readings are intermediate sights.

The last point being of known elevation, the computation of the levels is to be done from last point to the first point. The falls are added to and the rises are subtracted from the known elevations. The computation of levels is explained below and the results have been presented in the following table.

(i) Calculation of rise and fall

Section-1:

$$f_2 = \text{B.S.}_1 - \text{I.S.}_2 = 0.420 - 1.115 = 0.695$$
$$f_3 = \text{I.S.}_2 - \text{I.S.}_3 = 1.115 - 2.265 = 1.150$$
$$f_4 = \text{I.S.}_3 - \text{I.S.}_4 = 2.265 - 2.900 = 0.635$$
$$f_5 = \text{I.S.}_4 - \text{F.S.}_5 = 2.900 - 3.615 = 0.715$$

Section-2:

$$f_6 = \text{B.S.}_5 - \text{I.S.}_6 = 0.535 - 1.470 = 0.935$$
$$f_7 = \text{I.S.}_6 - \text{I.S.}_7 = 1.470 - 2.815 = 1.345$$
$$f_8 = \text{I.S.}_7 - \text{I.S.}_8 = 2.815 - 3.505 = 0.690$$
$$f_9 = \text{I.S.}_8 - \text{F.S.}_9 = 3.505 - 4.445 = 0.940$$

Section-3:

$$f_{10} = \text{B.S.}_9 - \text{I.S.}_{10} = 0.605 - 1.925 = 1.320$$
$$f_{11} = \text{I.S.}_{10} - \text{F.S.}_{11} = 1.925 - 2.885 = 0.960$$

(ii) Calculation of reduced levels

$$h_{10} = h_{11} + f_{11} = 155.272 + 0.960 = 156.232 \text{ m}$$
$$h_9 = h_{10} + f_{10} = 156.232 + 1.320 = 157.552 \text{ m}$$
$$h_8 = h_9 + f_9 = 157.552 + 0.940 = 158.492 \text{ m}$$
$$h_7 = h_8 + f_8 = 158.492 + 0.690 = 159.182 \text{ m}$$
$$h_6 = h_7 + f_7 = 159.182 + 1.345 = 160.527 \text{ m}$$
$$h_5 = h_6 + f_6 = 160.527 + 0.935 = 161.462 \text{ m}$$
$$h_4 = h_5 + f_5 = 161.462 + 0.715 = 162.177 \text{ m}$$
$$h_3 = h_4 + f_4 = 162.177 + 0.635 = 162.812 \text{ m}$$
$$h_2 = h_3 + f_3 = 162.812 + 1.150 = 163.962 \text{ m}$$
$$h_1 = h_2 + f_2 = 163.962 + 0.695 = 164.657 \text{ m}$$

Table 3.5

Station	Chainage (m)	B.S.	I.S.	F.S.	Rise	Fall	R.L.	Remarks
1	0	0.420					164.657	
2	20		1.115			0.695	163.962	
3	40		2.265			1.150	162.812	
4	60		2.900			0.635	162.177	
5	80	0.535		3.615		0.715	161.462	C.P.
6	100		1.470			0.935	160.527	
7	120		2.815			1.345	159.182	
8	140		3.505			0.690	158.492	
9	160	0.605		4.445		0.940	157.552	C.P.
10	180		1.925			1.320	156.232	
11	200			2.885		0.960	155.272	Elevation = 155.272 m
Σ		1.560		10.945	0.000	9.385		
Check: 1.560 – 10.945 = 0.000 – 9.385 = 155.272 – 164.657 = – 9.385 (*O.K.*)								

(*iii*) Calculation of gradient

The gradient of the line 1-11 is

$$= \frac{\text{difference of level between points } 1\text{-}11}{\text{distance between points } 1\text{-}11}$$

$$= \frac{155.272 - 164.657}{200} = \frac{-9.385}{200}$$

= 1 in 21.3 (*falling*)

Example 3.4. A page of level book is reproduced below in which some readings marked as (×), are missing. Complete the page with all arithmetic checks.

Solution:

The computations of the missing values are explained below.

$$\text{B.S.}_4 - \text{I.S.}_5 = f_5, \quad \text{B.S.}_4 = f_5 + \text{I.S.}_5 = -0.010 + 2.440 = \mathbf{2.430}$$

$$\text{B.S.}_9 - \text{F.S.}_{10} = f_{10}, \quad \text{B.S.}_9 = f_{10} + \text{F.S.}_{10} = -0.805 + 1.525 = \mathbf{0.720}$$

$$\text{B.S.}_1 + \text{B.S.}_2 + \text{B.S.}_4 + \text{B.S.}_6 + \text{B.S.}_7 + \text{B.S.}_9 = \Sigma\text{B.S.}$$

$$3.150 + 1.770 + 2.430 + \text{B.S.}_6 + 1.185 + 0.720 = 12.055$$

$$\text{B.S.}_6 = 12.055 - 9.255 = \mathbf{2.800}$$

Table 3.6

Station	B.S.	I.S.	F.S.	Rise	Fall	R.L.	Remarks
1	3.150					×	
2	1.770		×		0.700	×	C.P.
3		2.200			×	×	
4	×		1.850	×		×	C.P.
5		2.440			0.010	×	
6	×		×	1.100		×	C.P.
7	1.185		2.010	×		222.200	C.P.
8		–2.735		×		×	Staff held inverted
9	×		1.685		4.420	×	C.P.
10			1.525		0.805	×	
Σ	12.055		×	×	×		

$$\text{B.S.}_1 - \text{F.S.}_2 = f_2, \quad \text{F.S.}_2 = \text{B.S.}_1 - f_2 = 3.150 - (-0.700) = \mathbf{3.850}$$
$$\text{I.S.}_5 - \text{F.S.}_6 = r_6, \quad \text{F.S.}_6 = \text{I.S.}_5 - r_6 = 2.440 - 1.100 = \mathbf{1.340}$$
$$\text{B.S.}_2 - \text{I.S.}_3 = 1.770 - 2.200 = -0.430 = \mathbf{0.430}\ (fall) = f_3$$
$$\text{I.S.}_3 - \text{F.S.}_4 = 2.200 - 1.850 = \mathbf{0.350} = r_4$$
$$\text{B.S.}_6 - \text{F.S.}_7 = 2.800 - 2.010 = \mathbf{0.790} = r_7$$
$$\text{B.S.}_7 - \text{I.S.}_8 = 1.185 - (-2.735) = \mathbf{3.920} = r_8$$

For the computation of reduced levels the given reduced level of point 7 is to be used. For the points 1 to 6, the computations are done from points 6 to 1, upwards in the table and for points 8 to 10, downwards in the table.

$$h_6 = h_7 - r_7 = 222.200 - 0.790 = 221.410 \text{ m}$$
$$h_5 = h_6 - r_6 = 221.410 - 1.100 = 220.310 \text{ m}$$
$$h_4 = h_5 + f_5 = 220.310 + 0.010 = 220.320 \text{ m}$$
$$h_3 = h_4 - r_4 = 220.320 - 0.350 = 219.970 \text{ m}$$
$$h_2 = h_3 + f_3 = 219.970 + 0.430 = 220.400 \text{ m}$$
$$h_1 = h_2 + f_2 = 220.400 + 0.700 = 221.100 \text{ m}$$
$$h_8 = h_7 + r_8 = 222.200 + 3.920 = 226.120 \text{ m}$$
$$h_9 = h_8 - f_9 = 226.120 - 4.420 = 221.700 \text{ m}$$
$$h_{10} = h_9 - f_{10} = 221.700 - 0.805 = 220.895 \text{ m}$$

The computed missing values and the arithmetic check are given Table 3.7.

Table 3.7.

Station	B.S.	I.S.	F.S.	Rise	Fall	R.L.	Remarks
1	3.150					221.100	
2	1.770		3.850		0.700	220.400	C.P.
3		2.200			0.430	219.970	
4	2.430		1.850	0.350		220.320	C.P.
5		2.440			0.010	220.310	
6	2.800		1.340	1.100		221.410	C.P.
7	1.185		2.010	0.790		222.200	C.P.
8		-2.735		3.920		226.120	Staff held inverted
9	0.720		1.685		4.420	221.700	C.P.
10			1.525		0.805	220.895	
Σ	12.055		12.266	6.610	6.365		
Check: 12.055 – 12.266 = 6.610 – 6.365 = 220.895 – 221.100 = – 0.205 (*O.K.*)							

Example 3.5. Given the following data in Table 3.8, determine the R.L.s of the points 1 to 6. If an uniform upward gradient of 1 in 20 starts at point 1, having elevation of 150 m, calculate the height of embankment and depth of cutting at all the points from 1 to 6.

Table 3.8

Station	Chainage (m)	B.S.	I.S.	F.S.	Remarks
B.M.	–	10.11			153.46 m
1	0		3.25		
2	100		1.10		
3	200	6.89		0.35	
4	300		3.14		
5	400	11.87		3.65	
6	500			5.98	

Solution:

Reduced levels of the points by height of instrument method

$$\text{H.I.}_{\text{B.M.}} = \text{R.L.}_{\text{B.M.}} + \text{B.S.}_{\text{B.M.}} = 153.46 + 10.11 = 163.57 \text{ m}$$

$$h_1 = \text{H.I.}_{\text{B.M.}} - \text{I.S.}_1 = 163.57 - 3.25 = 160.32 \text{ m}$$

$$h_2 = \text{H.I.}_{\text{B.M.}} - \text{I.S.}_2 = 163.57 - 1.10 = 162.47 \text{ m}$$

$$h_3 = \text{H.I.}_{\text{B.M.}} - \text{F.S.}_3 = 163.57 - 0.35 = 163.22 \text{ m}$$

$$\text{H.I.}_3 = h_3 + \text{B.S.}_3 = 163.22 + 6.89 = 170.11 \text{ m}$$

$$h_4 = \text{H.I.}_3 - \text{I.S.}_4 = 170.11 - 3.14 = 166.97 \text{ m}$$
$$h_5 = \text{H.I.}_3 - \text{F.S.}_5 = 170.11 - 3.65 = 166.46 \text{ m}$$
$$\text{H.I.}_5 = h_5 + \text{B.S.}_5 = 166.46 + 11.87 = 178.33 \text{ m}$$
$$h_6 = \text{H.I.}_5 - \text{F.S.}_6 = 178.33 - 5.98 = 172.35 \text{ m}$$

Levels of the points from gradient

Since the gradient is 1 in 20, for every 100 m the rise is 5 m.

Level of point 1, $h'_1 = 150$ m (given)

Level of point 2, $h'_2 = 150 + 5 = 155$ m

Level of point 3, $h'_3 = 155 + 5 = 160$ m

Level of point 4, $h'_4 = 160 + 5 = 165$ m

Level of point 5, $h'_5 = 165 + 5 = 170$ m

Level of point 6, $h'_6 = 170 + 5 = 175$ m

Height of embankment and depth of cutting

At point 1 $h_1 - h'_1 = 160.32 - 150.00 = +\ 10.32$ m (embankment)

At point 2 $h_2 - h'_2 = 162.47 - 155.00 = +\ 7.47$ m (embankment)

At point 3 $h_3 - h'_3 = 163.22 - 160.00 = +\ 3.22$ m (embankment)

At point 4 $h_4 - h'_4 = 166.97 - 165.00 = +\ 1.97$ m (embankment)

At point 5 $h_5 - h'_5 = 166.46 - 170.00 = -\ 3.54$ m (cutting)

At point 6 $h_6 - h'_6 = 172.35 - 175.00 = -\ 2.65$ m (cutting)

The computed values of the height of embankment and depth of cutting are tabulated below.

Table 3.9

Station	Chainage (m)	B.S.	I.S.	F.S.	H.I.	R.L.	Gradient level	Embankment/cutting	
								Height (m)	Depth (m)
B.M.	–	10.11			163.57	153.46	–		
1	0		3.25			160.32	150	10.32	
2	100		1.10			162.47	155	7.47	
3	200	6.89		0.35	170.11	163.22	160	3.22	
4	300		3.14			166.97	165	1.97	
5	400	11.87		3.65	178.33	166.46	170		3.54
6	500			5.98		172.35	175		2.65

Example 3.6. The readings given in Table 3.10, were recorded in a levelling operation from points 1 to 10. Reduce the levels by the height of instrument method and apply appropriate checks. The point 10 is a bench mark having elevation of 66.374 m. Determine the loop closure and adjust the calculated values of the levels by applying necessary corrections. Also determine the mean gradient between the points 1 to 10.

Table 3.10

Station	Chainage (m)	B.S.	I.S.	F.S.	Remarks
1	0	0.597			B.M.= 68.233 m
2	20	2.587		3.132	C.P
3	40		1.565		
4	60		1.911		
5	80		0.376		
6	100	2.244		1.522	C.P
7	120		3.771		
8	140	1.334		1.985	C.P
9	160		0.601		
10	180			2.002	

Solution:

Reduced levels of the points

$$\text{H.I.}_1 = h_1 + \text{B.S.}_1 = 68.233 + 0.597 = 68.830 \text{ m}$$
$$h_2 = \text{H.I.}_1 - \text{F.S.}_2 = 68.233 + 3.132 = 65.698 \text{ m}$$
$$\text{H.I.}_2 = h_2 + \text{B.S.}_2 = 65.698 + 2.587 = 68.285 \text{ m}$$
$$h_3 = \text{H.I.}_2 - \text{I.S.}_3 = 68.285 - 1.565 = 66.720 \text{ m}$$
$$h_4 = \text{H.I.}_2 - \text{I.S.}_4 = 68.285 - 1.911 = 66.374 \text{ m}$$
$$h_5 = \text{H.I.}_2 - \text{I.S.}_5 = 68.285 - 0.376 = 67.909 \text{ m}$$
$$h_6 = \text{H.I.}_2 - \text{F.S.}_6 = 68.285 - 1.522 = 66.763 \text{ m}$$
$$\text{H.I.}_6 = h_6 + \text{B.S.}_6 = 66.763 + 2.244 = 69.007 \text{ m}$$
$$h_7 = \text{H.I.}_6 - \text{I.S.}_7 = 69.007 - 3.771 = 65.236 \text{ m}$$
$$h_8 = \text{H.I.}_6 - \text{F.S.}_8 = 69.007 - 1.985 = 67.022 \text{ m}$$
$$\text{H.I.}_8 = h_8 + \text{B.S.}_8 = 67.022 + 1.334 = 68.356 \text{ m}$$
$$h_9 = \text{H.I.}_8 - \text{I.S.}_9 = 68.356 - 0.601 = 67.755 \text{ m}$$
$$h_{10} = \text{H.I.}_8 - \text{F.S.}_{10} = 68.356 - 2.002 = 66.354 \text{ m}$$

Loop closure and loop adjustment

The error at point 10 = computed R.L. – known R.L.

= 66.354 – 66.374 = –0.020 m

Therefore correction = +0.020 m

Since there are three change points, there will be four instrument positions. Thus the total number of points at which the corrections are to be applied is four, i.e., three C.P.s and one last F.S. It is reasonable to assume that similar errors have occurred at each station. Therefore, the correction for each instrument setting which has to be applied progressively, is

$$= +\frac{0.020}{4} = 0.005 \text{ m}$$

i.e., the correction at station 1 0.0 m
the correction at station 2 + 0.005 m
the correction at station 6 + 0.010 m
the correction at station 8 + 0.015 m
the correction at station 10 + 0.020 m

The corrections for the intermediate sights will be same as the corrections for that instrument stations to which they are related. Therefore,

$$\text{correction for I.S.}_3,\ \text{I.S.}_4,\ \text{and I.S.}_5 = +0.010 \text{ m}$$
$$\text{correction for I.S.}_7 = +0.015 \text{ m}$$
$$\text{correction for I.S.}_9 = +0.020 \text{ m}$$

Applying the above corrections to the respective reduced levels, the corrected reduced levels are obtained. The results have been presented in Table 3.11.

Table 3.11

Station	Chainage (m)	B.S.	I.S.	F.S.	H.I.	R.L.	Correction	Corrected R.L.
1	0	0.597			68.830	68.233	–	68.233
2	20	2.587		3.132	68.285	65.698	+ 0.005	65.703
3	40		1.565			66.720	+ 0.010	66.730
4	60		1.911			66.374	+ 0.010	66.384
5	80		0.376			67.909	+ 0.010	67.919
6	100	2.244		1.522	69.007	66.763	+ 0.010	66.773
7	120		3.771			65.236	+ 0.015	65.251
8	140	1.334		1.985	68.356	67.022	+ 0.015	67.037
9	160		0.601			67.755	+ 0.020	67.775
10	180			2.002		66.354	+ 0.020	66.374
Σ		6.762		8.641				
Check: 6.762 – 8.641 = 66.354 – 68.233 = – 1.879 (*O.K.*)								

Gradient of the line 1-10

The difference in the level between points 1 and 10, $\Delta h = 66.324 - 68.233 = -1.909$ m

The distance between points 1-10, $D = 180$ m

$$\text{Gradient} = -\frac{1.909}{180} = -0.0106$$

$$= \mathbf{1\ in\ 94.3}\quad (\textit{falling})$$

Example 3.7. Determine the corrected reduced levels of the points given in Example 3.6 by two alternative methods.

Solution: Method-1

From Eq. (3.3), the correction $c = -e\dfrac{l}{L}$

The total correction at point 10 (from Example 3.6) = + 0.020 m

The distance between the points 1 and 10 = 180 m

$$\text{Correction at point 2} = +\frac{0.020}{180}\times 20 = +\ 0.002 \text{ m}$$

$$\text{Correction at point 6} = +\frac{0.020}{180}\times 100 = +\ 0.011 \text{ m}$$

$$\text{Correction at point 8} = +\frac{0.020}{180}\times 140 = +\ 0.016 \text{ m}$$

$$\text{Correction at point 10} = +\frac{0.020}{180}\times 180 = +\ 0.020 \text{ m}$$

Corrections at points 3, 4, and 5 = + 0.011 m

Correction at point 7 = + 0.016 m

Correction at point 9 = + 0.020 m

The corrections and the corrected reduced levels of the points are given in Table 3.12.

Table 3.12

Station	R.L.	Correction	Corrected R.L.
1	68.233	–	68.233
2	65.698	+ 0.002	65.700
3	66.720	+ 0.011	66.731
4	66.374	+ 0.011	66.385
5	67.909	+ 0.011	67.920
6	66.763	+ 0.011	66.774
7	65.236	+ 0.016	65.252
8	67.022	+ 0.016	67.038
9	67.755	+ 0.020	67.775
10	66.354	+ 0.020	66.374

Method-2

In this method half of the total correction is applied negatively to all the back sights and half of the total correction is applied positively to all the fore sights.

Total number of back sights = 4

Total number of fore sights = 4

$$\text{Correction to each back sight} = -\left(\frac{-0.020}{2\times 4}\right) = +\ 0.0025 \text{ m}$$

$$\text{Correction to each fore sight} = +\left(\frac{-0.020}{2\times 4}\right) = -0.0025 \text{ m}$$

The correction to each intermediate sight is also the same as for the fore sights, i.e., - 0.0025 m. The correction and the corrected values of the reduced levels are tabulated in Table 3.13.

Table 3.13

Station	Observed		Correction	Corrected			F.S.	H.I.	Corrected
	B.S.	I.S.		F.S.	B.S.	I.S.			R.L.
1	0.597	+ 0.0025	0.5995			68.8325	68.233		
2	2.587	3.132	+ 0.0025	– 0.0025	2.5895		3.1295	68.2925	65.703
3		1.565		– 0.0025		1.5625			66.730
4		1.911		– 0.0025		1.9085			66.384
5		0.376		– 0.0025		0.3735			67.919
6	2.244	1.522	+ 0.0025	– 0.0025	0.2465		1.5195	69.0195	66.773
7		3.771		– 0.0025		3.7685			65.251
8	1.334	1.985	+ 0.0025	– 0.0025	1.3365		1.9825	68.3735	67.037
9		0.601		– 0.0025		0.5985			67.775
10			2.002	– 0.0025			1.9995		66.374

Example 3.8. Reciprocal levelling was conducted across a wide river to determine the difference in level of points A and B, A situated on one bank of the river and B situated on the other. The following results on the staff held vertically at A and B from level stations 1 and 2, respectively, were obtained. The level station 1 was near to A and station 2 was near to B.

Instrument at	Staff reading on	
	A	*B*
1	1.485	1.725
2	1.190	1.415

(a) If the reduced level of B is 55.18 m above the datum, what is the reduced level of A?

(b) Assuming that the atmospheric conditions remain unchanged during the two sets of the observations, calculate (*i*) the combined curvature and refraction correction if the distance AB is 315 m, and (*ii*) the collimation error.

Solution:

To eliminate the errors due to collimation, curvature of the earth and atmospheric refraction over long sights, the reciprocal levelling is performed.

From the given data, we have

$$a_1 = 1.485 \text{ m}, \quad a_2 = 1.725 \text{ m}$$
$$b_1 = 1.190 \text{ m}, \quad b_2 = 1.415 \text{ m}$$

The difference in level between A and B is given by

$$\Delta h = \frac{(a_1 - b_1) + (a_2 - b_2)}{2}$$

$$= \frac{(1.485 - 1.190) + (1.725 - 1.415)}{2} = 0.303 \text{ m}$$

R.L. of B = R.L. of $A + \Delta h$

R.L. of A = R.L. of $B - \Delta h$

= 55.18 – 0.303 = **54.88 m**.

The total error $e = e_l + e_c - e_r$

where

$$e = \frac{(b_1 - a_1) - (b_2 - a_2)}{2}$$

$$= \frac{(1.190 - 1.485) - (1.415 - 1.725)}{2} = 0.008 \text{ m}$$

and $e_c - e_r = 0.067\ d^2$

$= 0.067 \times 0.315^2 = 0.007$ m.

Therefore collimation error $e_l = e - (e_c - e_r)$

= 0.008 – 0.007 = **0.001 m**.

Example 3.9. To determine difference in level between two stations A and B, reciprocal vertical angles have been observed as + 6°32′58.3″ from A to B and – 6°33′36.7″ from B to A, the horizontal distance AB being 1411.402 m.

Compute

(*i*) the corrected vertical angle,

(*ii*) the coefficient of refraction,

(*iii*) the correction for the earth's curvature and atmospheric refraction, and

(*iv*) the elevation of B if the elevation of A is 116.73 m.

Take the mean radius of the earth equal to 6383.393 km.

Solution: (Fig. 3.10)

In Fig. 3.10, from Δ AEO, we have

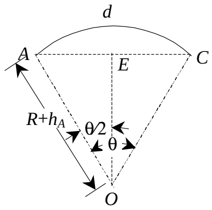

Fig. 3.10

$$\text{Chord } AC = 2\ (R + h_A) \sin \frac{\theta}{2}$$

But for all practical purposes we can take

$$\text{Chord } AC = \text{arc } AC = d\,\frac{(R + h_A)}{R}$$

Unless h_A is appreciable, chord $AC = d$ since h_A becomes negligible compared to R. From Fig. 3.10, we get

$$\sin\frac{\theta}{2} = \frac{AE}{AO}\ \frac{\text{chord } AC/2}{R} = \frac{d}{2R}$$

$$= \frac{1411.402}{2 \times 6383.393 \times 1000}$$

$$\frac{\theta}{2} = 22.8''$$

The observed angle of elevation $\alpha = 6°32'58.3''$
and the observed angle of depression $\beta = 6°33'36.7''$

(*i*) Correct vertical angle $= \dfrac{\alpha + \beta}{2}$

$$= \frac{6°32'58.3'' + 6°33'36.7''}{2} = 6°33'17.5''$$

We know that

$$\alpha + \theta/2 - \upsilon = \frac{\alpha + \beta}{2}$$

$$\upsilon = \alpha + \theta/2 - \frac{\alpha + \beta}{2}$$

$$= 6°32'58.3'' + 22.8'' - 6°33'17.5''$$

$$= \mathbf{3.6''}.$$

(*ii*) Coefficient of refraction $K = \dfrac{\upsilon}{\theta}$

$$= \frac{3.6}{2 \times 22.8} = \mathbf{0.079}.$$

Combined correction for curvature and refraction

$$C_{cr} = -\frac{3}{7}\frac{d^2}{R}$$

$$= -\frac{3}{7} \times \frac{1.411402^2 \times 1000}{6383.393} = \mathbf{-\ 0.134\ m}.$$

(*iii*) The difference of level in A and B is given by

$$\Delta h = AC \tan\left(\frac{\alpha + \beta}{2}\right)$$

$$= d \tan\left(\frac{\alpha + \beta}{2}\right)$$

$$= 1411.402 \times \tan 6°33'17.5'' = 162.178 \text{ m}$$

Elevation of B $h_B = h_A + \Delta_h$

$$= 116.73 + 162.178 = \mathbf{278.91\ m.}$$

Example 3.10. The following observations were made to determine the sensitivity of two bubble tubes. Determine which bubble tube is more sensitive.The distance of the staff from the instrument was 80 m and the length of one division of both the bubble tubes is 2 mm.

Bubble tube	Bubble reading			Staff reading
		L.H.S.	R.H.S.	
A	(*i*)	13	5	1.618
	(*ii*)	18	12	1.767
B	(*i*)	15	3	1.635
	(*ii*)	6	14	1.788

Solution:

Bubble tube *A*

The distance of the bubble from the centre of its run

(*i*) $n_1 = \frac{1}{2} \times (13 - 5) = 4$ divisions

(*ii*) $n_2 = \frac{1}{2} \times (12 - 8) = 2$ divisions

The total number of divisions n through which bubble has moved $= n_1 + n_2 = 6$

The staff intercept $s = 1.767 - 1.618 = 0.149$ m

The sensitivity of the bubble tube

$$\alpha'_A = 206265 \times \frac{s}{nD} \text{ seconds}$$

$$= 206265 \times \frac{0.149}{6 \times 80} \text{ seconds} = 1'4''$$

Bubble tube *B*

The distance of the bubble from the centre of its run

(*i*) $n_1 = \frac{1}{2} \times (15 - 3) = 6$ divisions

(*ii*) $n_2 = \frac{1}{2} \times (14 - 6) = 4$ divisions

The total number of divisions n through which bubble has moved $= n_1 + n_2 = 10$

The staff intercept $s = 1.788 - 1.635 = 0.153$ m

The sensitivity of the bubble tube

$$\alpha'_B = 206265 \times \frac{0.153}{10 \times 80} \text{ seconds} = 40''$$

Since $\alpha'_A > \alpha'_B$, the bubble A is more sensitive than B.

Example 3.11. If sensitivity of a bubble tube is 30″ per 2 mm division what would be the error in staff reading on a vertically held staff at a distance of 200 m when the bubble is out of centre by 2.5 divisions?

Solution:

The sensitivity of a bubble tube is given by

$$\alpha' = 206265 \frac{s}{nD} \text{ seconds}$$

where s can be taken as the error in staff reading for the error in the bubble tube.

Therefore $$s = \frac{nD\alpha'}{206265}$$

$$= \frac{2.5 \times 200 \times 30}{206265} = \mathbf{0.073\ m}.$$

Example 3.12. Four stations C, A, B, and D were set out in a straight line such that $CA = AB = BD = 30$ m. A level was set up at C and readings of 2.135 and 1.823 were observed on vertically held staff at A and B, respectively, when bubble was at the centre of its run. The level was then set up at D and readings of 2.026 and 1.768 were again observed at A and B, respectively. Determine the collimation error of the level and correct difference in level of A and B.

Solution: (Fig. 3.8)

Apparent difference in level of A and B when instrument at C

$$\Delta h_1 = 2.135 - 1.823 = 0.312 \text{ m}$$

Apparent difference in level of A and B when instrument at D

$$\Delta h_2 = 2.026 - 1.768 = 0.258 \text{ m}$$

Since the two differences in level do not agree, the line of collimation is inclined to the horizontal and not parallel to the axis of the bubble tube. Let the inclination of the line of collimation with the horizontal be θ, directed upwards. The distance d between consecutive stations is 30 m. If the errors in the staff readings at A and B for the instrument position at C are e_{A1} and e_{B1} and that for the instrument position D are e_{A2} and e_{B2}, respectively, then

$$e_{A1} = d\theta$$

$$e_{B1} = 2\ d\theta$$

$$e_{A2} = 2\ d\theta$$

$$e_{B2} = d\theta$$

The correct staff readings for the instrument position at C are $(2.135 - d\theta)$ and $(1.823 - 2\ d\theta)$ and that for the instrument position at D are $(2.026 - 2\ d\theta)$ and $(1.768 - d\theta)$

Substituting $d = 30$ m, the correct difference in level are

$$\Delta h_1 = (2.135 - 30\theta) - (1.823 - 60\theta) = 0.312 + 30\theta$$

$$\Delta h_2 = (2.026 - 60\theta) - (1.768 - 30\theta) = 0.258 - 30\theta$$

Since both $\Delta h_1$and Δh_2 are the correct differences in level, they must be equal.

Therefore $$\Delta h_1 = \Delta h_2$$

$$0.312 + 30\theta = 0.258 - 30\theta$$

$$\theta = -0.0009 \text{ radians}$$

$$= -0.0009 \times 206265 \text{ seconds}$$

$$= -3'5.64''$$

The negative sign shows the line of collimation is inclined downwards rather upwards as assumed.

The correct difference of level between A and B

$$\Delta h = 0.312 + 30 \times (-0.0009)$$

$$= \mathbf{0.285\ m}.$$

The correct difference in level can also be obtained from Eq. (3.6) by reciprocal levelling

$$\Delta h = \frac{(a_1 - b_1) + (a_2 - b_2)}{2}$$

= half the sum of apparent difference in level

$$= \frac{0.312 + 0.258}{2} = 0.285 \text{ m}.$$

Example 3.13. Fig. 3.11 shows a rectangle $ABCD$, in which A, B, and C are the stations where staff readings were obtained with a level set up at E and D. The observed readings are given in Table 3.14.

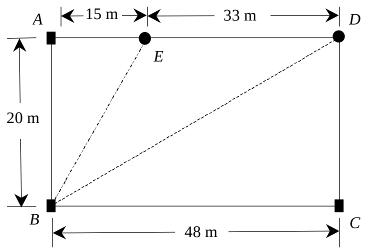

Fig. 3.11

Table 3.14

Level at	Staff reading at		
	A	*B*	*C*
E	1.856	0.809	–
D	2.428	1.369	1.667

If A is a bench mark having elevation of 150 m, calculate the correct elevations of B and C.

Solution: Since $\angle DAB = 90°$

$$EB = \sqrt{(15^2 + 20^2)} = 25 \text{ m}$$

$$DB = \sqrt{(48^2 + 20^2)} = 52 \text{ m}$$

Assuming the line of sight is inclined upwards by angle θ,

the correct staff reading on A when level at $E = 1.856 - 15\theta$

the correct staff reading on B when level at $E = 0.809 - 25\theta$

the correct staff reading on A when level at $D = 2.428 - 48\theta$

the correct staff reading on B when level at $D = 1.369 - 52\theta$

The correct differences in level of A and B from the two instrument positions must be equal. Therefore

$$(1.856 - 15\theta) - (0.809 - 25\theta) = (2.428 - 48\theta) - (1.369 - 52\theta)$$

$$= \frac{0.012}{6} = 0.002 \text{ radians}$$

The correct level difference of A and B = $(1.856 - 15\theta) - (0.809 - 25\theta)$

$$= 1.047 + 10\theta$$

$$= 1.047 + 10 \times 0.002 = 1.067 \text{ m}$$

Reduced level of B = 150 + 1.067

= **151.067 m**.

The correct staff reading at C = $1.667 - 20\theta$

The correct level difference of A and C = $(2.428 - 48\theta) - (1.667 - 20\theta)$

$$= 0.761 - 28 \times 0.002 = 0.705 \text{ m}$$

Reduced level of C = 150 + 0.705 = **150.705 m**.

Example 3.14. To determine the difference in level between two stations A and B, 4996.8 m apart, the reciprocal trigonometric levelling was performed and the readings in Table 3.15,were obtained. Assuming the mean earth's radius as 6366.20 km and the coefficient of refraction as 0.071 for both sets of observations, compute the observed value of the vertical angle of A from B and the difference in level between A and B.

Table 3.15

Instrument at	Height of Instrument (m)	Target at	Height of Target (m)	Mean vertical angle
A	1.6	B	5.5	+ 1°15′32″
B	1.5	A	2.5	–

Solution (Fig. 3.9):

Height of instrument at A, $h_i = 1.6$ m

Height of target at B, $h_S = 5.5$ m

Correction for eye and object to the angle α' observed from A to B

$$\varepsilon_A = \frac{h_S - h_i}{d}.206265 \text{ seconds}$$

$$= \frac{5.5 - 1.6}{4996.8} \times 206265 \text{ seconds}$$

$$= 2'41''$$

Similarly, the correction for eye and object to the angle β' observed from B to A

$$\varepsilon_B = \frac{2.5 - 1.5}{4996.8} \times 206265 \text{ seconds} = 41.3''$$

Length of arc at mean sea level subtending an angle of 1″ at the centre of earth

$$= \frac{R \times 1''}{206265} \times 1000$$

$$= \frac{6366.2 \times 1''}{206265} \times 1000 = 30.86 \text{ m}$$

Therefore angle θ subtended at the centre of earth by AB

$$= \frac{4996.8}{30.86}$$

$$\theta = 2'41.9''$$

Refraction $\upsilon = K\theta$

$$= 0.071 \times 2'41.9'' = 11.5''$$

Therefore correction for curvature and refraction

$$\theta/2 - \upsilon = \frac{2'41.5''}{2} - 11.5'' = 1'9.5''$$

Corrected angle of elevation for eye and object

$$\alpha = \alpha' - \varepsilon_A$$

$$= 1°15'32'' - 2'41'' = 1°12'51''$$

Corrected angle of elevation for curvature and refraction

$$\alpha + \theta/2 - \upsilon = 1°12'51'' + 1'9.5''$$

$$= 1°14'0.5''$$

If b is the angle of depression at B corrected for eye and object then

$$\alpha + \theta/2 - \upsilon = \beta - (\theta/2 - \upsilon)$$

or $\beta = 1°14'0.5'' + 1'9.5'' = 1°15'10''$

If the observed angle of depression is β' then

$$\beta' = \beta' - \varepsilon_B$$

or $\beta' = \beta + \varepsilon_B$

$$= 1°15'10'' + 41.3'' = \mathbf{1°15'51.3''}$$

Now the difference in level

$$\Delta h = AC \tan\left(\frac{\alpha' + \beta'}{2}\right)$$

$$= 4996.8 \times \tan\left(\frac{1°12'51'' + 1°15'51.3''}{2}\right)$$

$$= \mathbf{108.1 \text{ m}}$$

OBJECTIVE TYPE QUESTIONS

1. A datum surface in levelling is a

(*a*) horizontal surface.

(*b*) vertical surface.

(*c*) level surface.

(*d*) non of the above.

2. Reduced level of a point is its height or depth above or below

(*a*) the ground surface.

(*b*) the assumed datum.

(*c*) assumed horizontal surface.

(*d*) the line of collimation.

3. The correction for the atmospheric refraction is equal to

(*a*) + 1/7 of the correction for curvature of the earth.

(*b*) 1/7 of the correction for curvature of the earth.

(*c*) + 6/7 of the correction for curvature of the earth.

(*d*) 6/7 of the correction for curvature of the earth.

4. If the back sight reading at point *A* is greater than the fore sight reading at point *B* then

(a) *A* is higher than *B*.

(b) *B* is higher than *A*.

(*c*) height of the instrument is required to know which point is higher.

(*d*) instrument position is required to know which point is higher.

5. Change points in levelling are

(*a*) the instrument stations that are changed from one position to another.

(*b*) the staff stations that are changed from point to point to obtain the reduced levels of the points.

(*c*) the staff stations of known elevations.

(*d*) the staff stations where back sight and fore sight readings are taken.

6. Balancing of sights mean

(*a*) making fore sight reading equal to back sight reading.

(*b*) making the line of collimation horizontal.

(*c*) making the distance of fore sight station equal to that of the back sight station from the instrument station.

(*d*) taking fore sight and back sight readings at the same station.

7. The height of instrument method of reducing levels is preferred when

(*a*) there are large numbers of intermediate sights.

(*b*) there are no intermediate sights.

(*c*) there are large numbers of fore sights.

(*d*) there are no fore sights.

8. Sensitivity of a bubble tube depends on

(*a*) the radius of curvature.

(*b*) the length of the vapour bubble.

(*c*) the smoothness of the inner surface of the buble tube.

(*d*) all the above.

9. Reciprocal levelling is employed to determine the accurate difference in level of two points which

(*a*) are quite apart and where it is not possible to set up the instrument midway between the points.

(*b*) are quite close and where it is not possible to set up the instrument midway between the points.

(*c*) have very large difference in level and two instrument settings are required to determine the difference in level.

(*d*) are at almost same elevation.

10. When a level is in adjustment, the line of sight of the instrument is

(*a*) perpendicular to the vertical axis of the instrument and parallel to the bubble tube axis.

(*b*) perpendicular to the vertical axis of the instrument and bubble level axis.

(*c*) perpendicular to the bubble tube axis and parallel to the vertical axis.

(*d*) none of the above.

11. A Dumpy level is preferred to determine the elevations of points

(*a*) lying on hills.

(*b*) lying on a line.

(*c*) lying in moderately flat terrain.

(*d*) on a contour gradient.

ANSWERS

1. (*c*)	**2.** (*b*)	**3.** (*a*)	**4.** (*b*)	**5.** (*d*)	**6.** (*c*)
7. (*a*)	**8.** (*d*)	**9.** (a)	**10.** (*a*)	**11.** (*c*)	